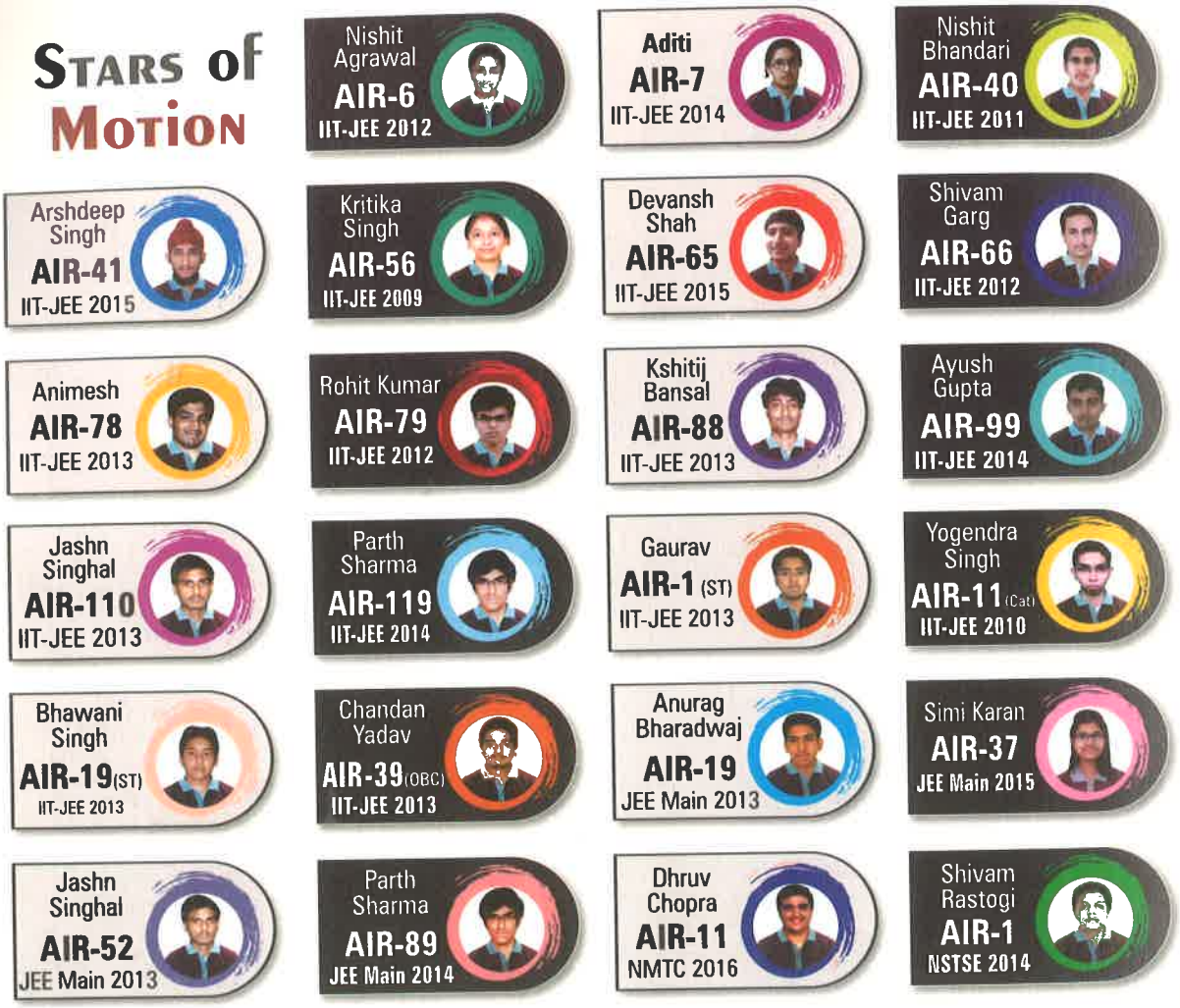
हमारा विश्वास... हर एक विद्यार्थी है ख़ास

IIT/NIT | AIPMT/AIIMS | BOARDS/KVPY | NTSE/OLYMPIADS

Classroom Coaching / School Integration / Distance Learning

11000+ selections in various competitive exams since 2007

STARS of MOTION



MOTION

Nurturing potential through education

JEE Class Companion Physics

for JEE Main and Advanced

Module-4

MOTION
Nurturing potential through education

Content Developers – Motion Education Pvt. Ltd

The content of this book is developed by Motion Education Pvt. Ltd with extensive research on various competitive exam papers and their pattern. The whole content is focused towards conceptual learning of the student for preparation of every engineering entrance exam. The unsolved questions are arranged in a way where confidence of a student is increased with easy questions in the start, brain teasers in the middle and previous year's questions at the end. This helps a student to gain a positive momentum while solving them. The content is the intellectual property of Motion and therefore cannot be copied and used for development under other brand name without prior permission from the content developers.

Motion Education Pvt. Ltd.
HO: 394, Rajeev Gandhi Nagar, Kota 324 002, Rajasthan, India
Ph: +91-744-2209 671, Toll free: 1800-3070-2299
Website: www.motion.ac.in, Email: info@motioniitjee.com

First Impression

Published by Pearson India Education Services Pvt. Ltd, CIN: U72200TN2005PTC057128, formerly known as TutorVista Global Pvt. Ltd, licensee of Pearson Education in South Asia.

Head Office: A-8(A), 7th Floor, Knowledge Boulevard, Sector 62, Noida 201 309, Uttar Pradesh, India.
Registered Office: 4th Floor, Software Block, Elnet Software City, TS-140, Block 2 & 9,
Rajiv Gandhi Salai, Taramani, Chennai 600 113, Tamil Nadu, India.
Fax: 080-30461003, Phone: 080-30461060
www.pearson.co.in, Email: companysecretary.india@pearson.com

Compositor: Macrotex Solutions.
Printed in India by Rahul Print O Pack

Contents

Chapter 1 Gravitation 1.1

Introduction 1.1
Newton's Law of Universal Gravitation 1.1
Gravitational Field 1.2
Strength of Gravitational Field 1.2
Gravitational Field Strength of Point Mass 1.2
Interaction Energy 1.3
Interaction Energy of a System of Particles 1.4
Gravitational Potential 1.5
Gravitational Potential due to a Point Mass 1.5
Gravitational Potential 1.6
Gravitational Lines of Forces 1.6
Gravitational Field Strength of Earth 1.7
Value of g on Earth's Surface 1.7
Value of g at a Height h Above the Earth's Surface 1.7
Value of g at a Depth h Below the Earth's Surface 1.7
Effect of Earth's Rotation on Value of g 1.8
Effect of Shape of Earth on Value of g 1.8
Satellite and Planetary Motion 1.10
Motion of a Satellite in a Circular Orbit 1.10
Energies of a Satellite in a Circular Orbit 1.10
Motion of a Satellite in Elliptical Path 1.13
Satellite Motion and Angular Momentum Conservation 1.13
Projection of Satellites and Spaceships from Earth 1.14
Communication Satellites 1.17
Geostationary Satellite and Parking Orbit 1.17
Broadcasting Region of a Satellite 1.17
Kepler's Laws of Planetary Motion 1.19
Kepler's First Law (The Law of Orbits) 1.19
Kepler's Second Law (The Law of Areas) 1.19
Kepler's Third Law (The Law of Periods) 1.20
Exercises 1.23
Answer Keys 1.36

Chapter 2 Current Electricity 2.1

Introduction 2.1
Conductor 2.1
Insulator 2.1
Semiconductor 2.1
Electric Current and Current Density 2.1
Drift Speed 2.2
Ohm's Law 2.3
Effect of Temperature on Resistance 2.5
Battery and Emf 2.6
Kirchhoff's Laws for Circuit Analysis 2.8
The Kirchhoff's Current Law 2.8
The Kirchhoff's Voltage Law 2.10
Combination of Resistance 2.11
Special Problems 2.14
Wheatstone's Bridge 2.15
Unbalanced Wheatstone's Bridge 2.16
Combinations of Cells 2.19
Electrical Power 2.22
Instruments 2.24
Exercises 2.31
Answer Keys 2.58

Chapter 3 Capacitance 3.1

Introduction 3.1
Capacitance of an Isolated Conductor 3.1
Definition of Capacitance 3.1
Important Point about the Capacitance of an Isolated Conductor 3.1
Capacitance of an Isolated Spherical Conductor 3.1
Capacitor 3.2
Parallel Plate Capacitor 3.2
Cylindrical Capacitor 3.3
Spherical Capacitor 3.3
Energy Stored in a Charged Capacitor 3.3
Energy Density of a Charged Capacitor 3.4

CHAPTER 1

Gravitation

INTRODUCTION

Newton observed that an object, an apple, when released near the earth surface is accelerated towards the earth. As acceleration is caused by an unbalanced force, there must be a force pulling objects towards the earth. If someone throws a projectile with some initial velocity, then instead of that object moving off into space in a straight line, it is continously acted on by a force pulling it back to earth. If we throw the projectile with greater velocity, then the path of projectile would be different as well and its range is also increased with initial velocity. If the projection velocity is further increased until at some initial velocity, the body would not hit the earth at all but would go right around it in an orbit. But at any point along its path the projectile would still have a force acting on it pulling it towards the surface of earth.

Newton was led to the conclusion that the same force that causes the apple to fall to the earth also causes the moon to be pulled to the earth. Thus, the moon moves in its orbit about the earth because it is pulled towards the earth. But if there is a force between the moon and the earth, why not a force between the sun and the earth or why not a force between the sun and the other planets? Newton proposed that the same force, named gravitational force which acts on objects near the earth surface also acts on all the heavenly bodies. He proposed that there was a force of gravitation between each and every mass in the universe.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

All physical bodies are subjected to the action of the forces of mutual gravitational attraction. The basic law describing the gravitational forces was stated by Sir Issac Newton and it is called Newton's Law of Universal gravitation.

The law is stated as: "Between any two particles of masses m_1 and m_2 at separation r from each other there exist attractive forces $\vec{F}_{AB}$ and $\vec{F}_{BA}$ directed from one body to the other and equal in magnitude which is directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between the two". Thus, we can write as

$$F_{AB} = F_{BA} = G\frac{m_1 m_2}{r^2} \qquad (1)$$

where G is called the universal gravitational constant. Its value is equal to 6.67×10^{-11} Nm^2/kg. The law of gravitation can be applied to the bodies whose dimensions are small as compared to the separation between the two or when bodies can be treated as point particles.

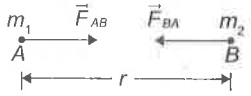


Figure 1.1

If the bodies are not very small sized, we cannot directly apply the expression in Eq. (1) to find their natural gravitational attraction. In this case, we use the following procedure to find the same. The bodies are initially split into small parts or a large number of point masses. Now using Eq. (1), the force of attraction exerted on a particle of one body by a particle of another body can be obtained. Now we add all forces vectorially which are exerted by all independent particles of the second body on the particle of the first body. Finally, the resultant of these forces is summed over all particles of the first body to obtain the net force experinced by the bodies. In general, we use integration or basic summation of these forces.

- Gravitational force is a conservative force.
- Gravitational force is a central force.

- Gravitational force is equal in magnitude and opposite in direction.
- Gravitational forces are action–reaction pair.
- Gravitational force acts along the line joining the two masses.
- Gravitational force does not depend upon the medium.
- Gravitational force is an attractive force.

$$\vec{F} = \frac{-Gm_1m_2\vec{r}}{|\vec{r}|^3}$$

[Head of $\vec{r}$ is placed at that position where we have to evaluate force]

GRAVITATIONAL FIELD

We can state by Newton's universal law of gravitation that every mass M produces, in the region around it, a physical situation in which, whenever any other mass is placed, force acts on it, is called gravitational field. This field is recognized by the force that the mass M exerts another mass, such as m, brought into the region.

Strength of Gravitational Field

We define gravitational field strength at any point in space to be the gravitational force per unit mass on a test mass (mass brought into the field for experimental observation). Thus for a point in space if a test mass m_0 experiences a force $\vec{F}$, then at that point in space, gravitational field strength which is denoted by $\vec{g}$, is given as $\vec{g} = \frac{\vec{F}}{m_0}$.

Gravitational field strength $\vec{g}$ is a vector quantity and has the same direction as that of the force on the test mass in the field.

Generally, magnitude of test mass is very small such that its gravitational field does not modify the field that is being measured. It should also be noted that gravitational field strength is just the acceleration that a unit mass would experience at that point in space.

Gravitational Field Strength of Point Mass

As per our previous discussion, we can state that every point mass also produces a gravitational field in its surrounding. To find the gravitational field strength due to a point mass, we put a test mass m_0 at a point P at distance x from a point mass m then force on m_0 is given as

$$F_g = \frac{Gmm_0}{x^2}$$

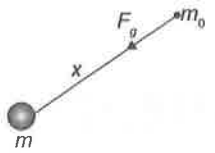


Figure 1.2

Now if at point P, gravitational field strength due to m is g_p then it is given as

$$g_p = \frac{F_g}{m_0} = \frac{Gm}{x^2}$$

The expression written in the above equation gives the gravitational field strength at a point due to a point mass.

It should be noted that the expression in the above equation is only applicable for gravitational field strength due to point masses. It should not be used for extended bodies.

However, the expression for the gravitational field strength produced by extended masses has already been derived in electrostatics section.

[Just replace k by $-G$ and Q by M in those expressions]

So we will just revise the expression of gravitational field strength at points due to various extended masses. Gravitational field strength:

1. At a point on the axis of ring $= \frac{-GMx}{\left(x^2+R^2\right)^{3/2}}$
2. At a point on the axis of disc $= \frac{2GM}{R^2}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$
3. At an axial point of a rod $= \frac{-GM}{L}\left[\frac{1}{x}-\frac{1}{x+L}\right]$
4. Due to a circular arc $= \frac{-2GM\sin(\phi/2)}{\phi R^2}$
5. Due to a long infinite thread $= \frac{-2G\lambda}{x}$
6. Due to long solid cylinder
   - **(a)** at an outer point $= \frac{-2G\rho\pi R^2}{x}$ (where ρ is the mass density per volume)
   - **(b)** at an inner point $= -2G\rho\pi x$

In the above example, as work is done by the gravitational forces of the system of two masses, the interaction energy of system must be negative and it can be given as

$$U_{12} = -\frac{Gm_1m_2}{r} \quad (1)$$

As gravitational forces are always attractive, the gravitational potential energy is always taken negative.

Interaction Energy of a System of Particles

If in a system there are more than two particles, then we can find the interaction energy of particle in pairs using Eq. (1) and finally sum up all the results to get the total energy of the system. For example in a system of N particles with masses $m_1, m_2, \ldots, m_n$ separated from each other by a distance $r_{12}\ldots$ where r_{12} is the separation between m_1 and m_2 and so on.

In the above case, the total interaction energy of system is given as

$$U = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{Gm_im_j}{r_{ij}}$$

In this expression, the factor $\frac{1}{2}$ is taken because the interaction energy for each possible pair of system is taken twice during summation as for masses m_1 and m_3

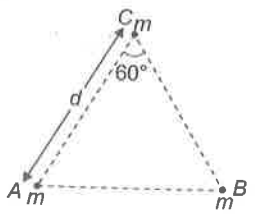
$$U = -\frac{Gm_1m_3}{r_{13}} = -\frac{Gm_3m_1}{r_{31}}$$

Now to understand the applications of interaction energy we take few examples.

SOLVED EXAMPLES

EXAMPLE 1

Three particles each of mass m are placed at the corners of an equilateral triangle of side d and shown in the figure. Calculate (a) the potential energy of the system, (b) work done on this system if the side of the triangle is changed from d to $2d$.



SOLUTION

(a) As in the case of two-particle system potential energy is given by $(-Gm_1m_2/r)$, so

$$U = U_{12} + U_{23} + U_{31}$$

or $$U_i = -3\left|\frac{Gmm}{d}\right| = -\frac{3Gm^2}{d}$$

(b) When d is changed to $2d$,

$$U_f = -\frac{3Gm^2}{2d}$$

Thus, work done in changing potential energy is given as

$$W = U_f - U_i = \frac{3Gm^2}{2d}$$

EXAMPLE 2

Two particles m_1 and m_2 are initially at rest at infinite distance. Find their relative velocity of approach due to gravitational atraction when their separation is d.

SOLUTION

Initiallly when the separation was large there was no interaction energy and when they get closer the system gravitational energy decreases and the kinetic energy increases.

When separation between the two particles is d, then according to energy conservation we have

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} = 0$$

As no other force is present, we have according to momentum conservation

$$m_1v_1 = m_2v_2$$

From the above equations,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}\frac{m_1^2}{m_2}v_1^2 = \frac{Gm_1m_2}{d}$$

or $$v_1 = \sqrt{\frac{2Gm_2^2}{d(m_1+m_2)}} = \sqrt{\frac{2G}{d(m_1+m_2)}}m_2$$

And on further solving we get

$$v_2 = \sqrt{\frac{2G}{d(m_1+m_2)}}m_1$$

$$U = -\frac{Gmm_0}{r}$$

Now the gravitational potential at P due to m can be written as

$$V = \frac{U}{m_0} = -\frac{Gm}{r}$$

The expression of gravitational potential in equation is a standard result due to a point mass which can be used as an elemental form to find other complex results, which we will see later.

The same thing can also be obtained by using the equation

$$V_p = \int_{\infty}^{p} \vec{g} \cdot \vec{dx}$$

or $$V_p = \int_{\infty}^{r} \frac{Gm}{x^2} dx$$

or $$V_p = -\frac{Gm}{r}$$

Gravitational Potential

1. Due to a rod at an axial point $= -G\lambda l \, n\left(\frac{a+l}{a}\right)$
2. Due to ring at an axial point $= \frac{-GM}{\sqrt{R^2 + x^2}}$
3. Due to ring at the centre $= \frac{-GM}{R}$
4. Due to disc $= -G\sigma 2\pi\left[\sqrt{R^2 + x^2} - x\right]$ (where σ is the mass density per unit area)
5. Due to hollow sphere:

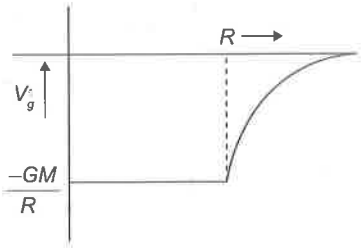


Figure 1.7

For outer points $= \frac{-GM}{r}$

For surface points $= \frac{-GM}{R}$

For inner points $= \frac{-GM}{R}$

6. Due to solid sphere:

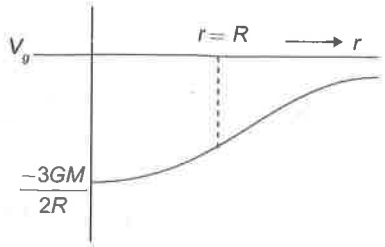


Figure 1.8

For outer points $= \frac{-GM}{r}$

For surface points $= \frac{-GM}{R}$

For inner points $= \frac{-GM}{2R^3}(3R^2 - r^2)$

Potential energy of hollow sphere $= \frac{-GM^2}{2R}$

Potential energy of solid sphere $= \frac{-3GM^2}{5R}$

GRAVITATIONAL LINES OF FORCES

Gravitational field can also be represented by lines of force. A line of force is drawn in such a way that at each point the direction of field is tangent to a line that passes through the point. Thus tangent to any point on a line of force gives the direction of gravitational field at that point. By convention lines of force are drawn in such a way that their density is proportional to the strength of field. Figure 1.9 shows the field of a point mass in its surrounding. We can see that the lines of force are radially inward giving direction of field and as we go closer to the mass the density of lines is more which shows that field strength is increasing.

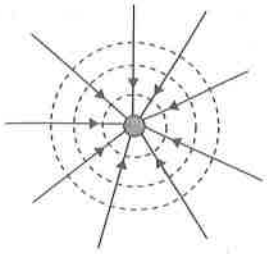


Figure 1.9

Figure 1.10 shows the configuration of field lines for a system of two equal masses separated by a given distance. Here we can see that there is no point where any two lines of force intersect or meet. The reason is obvious that at one point in space there can never be two directions of gravitational fields. It should be noted that a line of force gives the direction of net gravitational field in the region. Like electric field gravitational field never exists in closed loops.

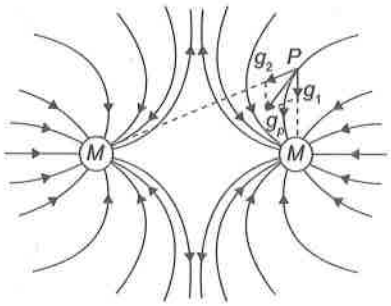


Figure 1.10

- **Gravitational flux:** $\phi = \int \vec{g} \cdot \vec{ds}$
- **Gravitational Gauss law:** $\oint \vec{g} \cdot \vec{ds} = -4\pi G M_{in}$

Here $\vec{g}$ is the gravitational field due to all the masses. M_{in} is the mass inside the assumed Gaussian surface.

Gravitational Field Strength of Earth

We can consider earth to be a very large sphere of mass M_e and radius R_e. Gravitational field strength due to earth is also regarded as acceleration due to gravity or gravitational acceleration. Now we find values of g at different points due to earth.

- Earth behaves as a non-conducting solid sphere.

Value of *g* on Earth's Surface

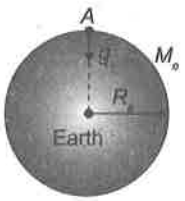


Figure 1.11

If g_s be the gravitational field strength at a point A on the surface of earth, then it can be easily obtained by using the result of a solid sphere. Thus for earth, value of g_s can be given as

$$g_s = \frac{GM_e}{R_e^2} \qquad (1)$$

Value of *g* at a Height *h* Above the Earth's Surface

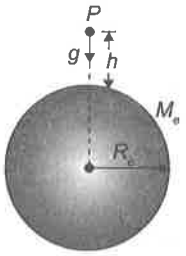


Figure 1.12

If we find the value of g at a point P as shown in Fig. 1.12 at a height h above the earth's surface. Then the value can be obtained as

$$g_s = \frac{GM_e}{(R_e + h)^2}$$

or

$$g_s = \frac{GM_e}{R_e^2\left(1+\frac{h}{R_e}\right)^2} = \frac{g_s}{\left(1+\frac{h}{R_e}\right)^2}$$

If point P is very close to earth's surface, then for $h << R_e$ we can rewrite the expression in given equation as

$$g_h = g_s\left(1+\frac{h}{R_e}\right)^{-2} \simeq g_s\left(1-\frac{2h}{R_e}\right) \qquad (2)$$

[Using binomial approximation]

Value of *g* at a Depth *h* Below the Earth's Surface

If we find the value of g inside the volume of earth at a depth h below the earth's surface at point P as shown in Fig. 1.13, then we can use the result of g inside a solid sphere as

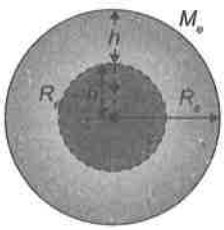


Figure 1.13

$$g_{in} = \frac{GM_e x}{R_e^3}$$

Here x, the distance of point from the centre of earth is given as

$$x = R_e - h$$

Thus, we have

$$g_h = \frac{GM_e(R_e - h)}{R_e^3} = g_s\left(1 - \frac{h}{R_e}\right) \quad (3)$$

From Eqs. (1), (2) and (3), we can say that the value of g at earth's surface is maximum and as we move above the earth's surface or we go below the surface of earth, the value of g decreases.

Effect of Earth's Rotation on Value of g

Let us consider a body of mass m placed on earth's surface at a latitude q as shown in Fig. 1.14. This mass experiences a force mg_s towards the centre of earth and a centrifugal force $m\omega_e^2 R_e \sin\theta$ relative to earth's surface.

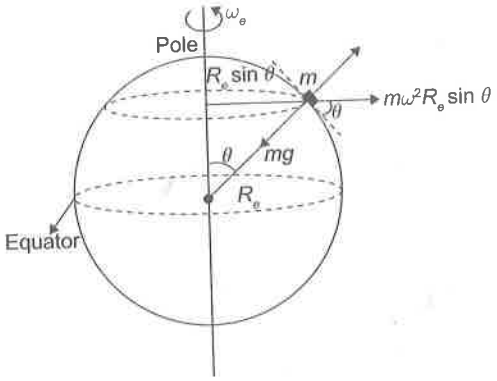


Figure 1.14

If we consider g_{eff} as the effective value of g on earth's surface at a latitude θ, then we can write as

$$g_{eff} = \frac{\vec{F}_{net}}{m} = g_{eff}$$

$$= \sqrt{(\omega_e^2 R_e \sin\theta)^2 + g^2 + 2\omega_e^2 R_e \sin\theta \cdot g\cos(90+\theta)}$$

ω_e^4 is very very small.

So we can write as

$$g_{eff} = \sqrt{g^2 - 2\omega_e^2 R_e \sin^2\theta g}$$

$$g_{eff} = g\left(1 - \frac{2\omega_e^2 R_e \sin^2\theta}{g}\right)^{1/2} \approx g - \omega_e^2 R\sin^2\theta \quad (1)$$

From Eq. (1), we can find the value of effective gravity at poles and equatorial points on the earth as

At poles $\theta = 0$

$\Rightarrow$ $g_{poles} = g_s = 9.83$ m/s^2

At equator $\theta = \frac{\pi}{2}$

$$g_{equator} = g_s - \omega^2 R_e = 9.78 \text{ m/s}^2$$

Thus, we can see that the body if placed at poles of the earth, it will only have a spin, not circular motion; so there is no reduction in value of g at poles due to rotation of earth. Thus at poles the value of g on earth's surface is maximum and at equator it is minimum. But on an average we take 9.8 m/s^2, the value of g everywhere on earth's surface.

Effect of Shape of Earth on Value of g

Till now we considered that earth is spherical in shape but this is not actually true. Due to some geological and astromonical reasons, the shape of earth is not exactly spherical. It is ellipsoidal.

As we have discussed, the value of g at a point on earth's surface depends on radius of earth. It is observed that the approximate difference in earth's radius at different points on equator and poles is $r_e - r_p \simeq 21$ to 34 km. Due to this, the difference in value of g at poles and equatorial points is approximately $g_p - g_e \simeq 0.02$ to 0.04 m/s^2, which is very small. So for numerical calculations, generally, we ignore this factor while taking the value of g and we assume that earth is spherical in shape.

SOLVED EXAMPLES

EXAMPLE 4

Calculate the mass and density of the earth. Given that gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2, the radius of the earth $= 6.37 \times 10^6$ m and $g = 9.8$ m/s^2.

SOLUTION

The acceleration due to gravity on earth's surface is given as

$$g_e = \frac{GM_e}{R_e^2}$$

or $$M_e = \frac{g_s R_e^2}{G} = \frac{9.8\times(6.37\times10^6)^2}{6.67\times10^{-11}}$$

$$= 6 \times 10^{24} \text{ kg}$$

If ρ be the density of earth, then

$$M = \frac{4}{3}\pi R \times \rho$$

or $$\rho = \frac{3M}{4\pi R^3} = \frac{3\times(6\times10^{24})}{4\times3.14\times(6.37\times10^6)^3}$$

$$= 5.5 \times 10^3 \text{ kg/m}^3$$

EXAMPLE 5

If the radius of the earth were to shrink by 1%, and its mass remaining the same, what would happen to the acceleration due to gravity on the earth's surface?

SOLUTION

Consider the case of body of mass m placed on the earth's surface (mass of the earth M and radius R). If g is acceleration due to gravity, then we know that

$$g_s = \frac{GM_e}{R_e^2} \qquad (1)$$

Now, when the radius is reduced by 1%, i.e. radius becomes 0.99 R, let acceleration due to gravity be g', then

$$g' = \frac{GM}{(0.99R)^2} \qquad (2)$$

From Eqs. (1) and (2), we get

$$\frac{g'}{g} = \frac{R^2}{(0.99R)} = \frac{1}{(0.99)^2}$$

or $$g' = g \times \left(\frac{1}{0.99}\right)^2$$

or $$g' = 1.02g$$

Thus, the value of g is increased by 2%.

EXAMPLE 6

At what rate should the earth rotate so that the apparent g at the equator becomes zero? What will be the length of the day in this situation?

SOLUTION

At earth's equator, the effective value of gravity is

$$g_{eq} = g_s - \omega^2 R_e$$

If g_{eff} at equator is zero, we have

$$g_s = \omega^2 R_e$$

or $$\omega = \sqrt{\frac{g_s}{R_e}}$$

Thus, the length of the day will be

$$T = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{R_e}{g_s}} = 2 \times 3.14\sqrt{\frac{6.14 \times 10^6}{9.8}}$$

$$= 5074.77s \simeq 84.57 \text{ min}.$$ ■

EXAMPLE 7

Calculate the acceleration due to gravity at the surface of Mars if its diameter is 6760 km and mass is one-tenth that of the earth. The diameter of earth is 12,742 km and the acceleration due to gravity on earth is 9.8 m/s².

SOLUTION

We know that

$$g = \left(\frac{GM}{R^2}\right)$$

So $$\frac{g_M}{g_E} = \left(\frac{M_M}{M_E}\right)\left(\frac{R_E}{R_M}\right)^2 = \left(\frac{1}{10}\right)\left(\frac{12,742}{6760}\right)^2$$

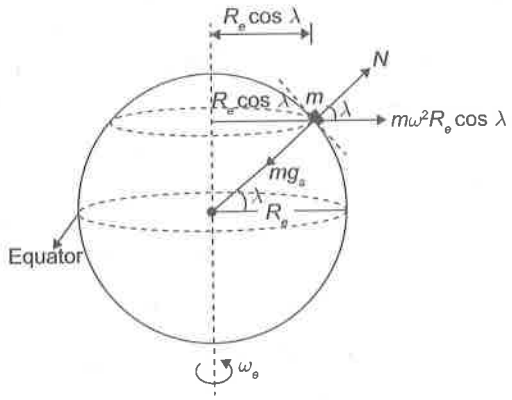
$$\frac{g_M}{g_E} = 0.35$$

or $$g_M = 9.8 \times 0.35 = 3.48 \text{ m/s}^2$$ ■

EXAMPLE 8

Calculate the apparent weight of a body of mass m at a latitude l when it is moving with speed v on the surface of the earth from west to east at the same latitude.

SOLUTION



If W be the apparent weight of the body at a latitude λ, then from the figure shown, we have

$$W = mg - m\omega^2 R \cos^2\lambda \qquad (1)$$

When body moves at speed v from west to east relative to earth, its net angular speed ω can be given as

$$\omega = \omega_e + \frac{v}{R\cos\lambda} \quad [\omega_e \rightarrow \text{earth's angular velocity}]$$

Now from Eq. (1), we have

$$W = mg - m\left[\omega_e + \frac{v}{R\cos\lambda}\right]^2 R\cos^2\lambda$$

or $$W = mg - m\left[\omega_e^2 + \frac{v^2}{R^2\cos^2\lambda} + \frac{2\omega_e v}{R\cos\lambda}\right] R\cos^2\lambda$$

$$= mg - m\omega_e^2 R\cos^2\lambda - \frac{mv^2}{R} - 2m\omega_e v\cos\lambda$$

$$= mg\left(1 - \frac{\omega_e^2 R\cos^2\lambda}{g} - \frac{2\omega_e v\cos\lambda}{g}\right)$$

[Neglecting $\frac{mv^2}{R}$ as being very small]

SATELLITE AND PLANETARY MOTION

Motion of a Satellite in a Circular Orbit

To understand how a satellite continously moves in its orbit, we consider the projection of a body horizontally from the top of a high mountain on earth as shown in Fig. 1.15. Here till our discussion ends we neglect air friction. The distance the projectile travels before hitting the ground depends on the launching speed. The greater the speed, the greater the distance. The distance the projectile travels before hitting the ground is also affected by the curvature of earth as shown in the figure. This figure was given by Newton in his explanantion of laws of gravitation. It shows different trajectories for diferent launching speeds. As the launching speed is made greater, a speed is reached whereby the projectile's path follows the curvature of the earth. This is the launching speed which places the projectile in a circular orbit. Thus, an object in circular orbit may be regarded as falling, but as it falls its path is concentric with the earth's spherical surface and the object maintains a fixed distance from the earth's centre. Since the motion may continue indefinitely, we may say that the orbit is stable.

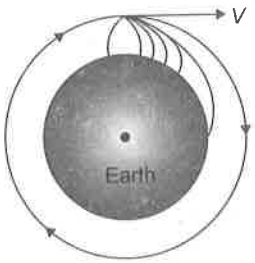


Figure 1.15

Let us find the speed of a satellite of mass m in a circular orbit around the earth. Consider a satellite revolving around the earth in a circular orbit of radius r as shown in Fig. 1.16. If its orbit is stable during its motion, the net gravitational force on it must be balanced by the centrifugal force on it relative to the rotating frame as

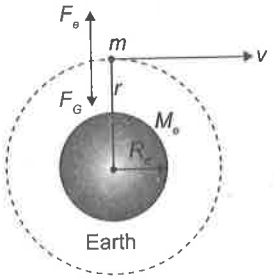


Figure 1.16

$$\frac{GM_e m}{r^2} = \frac{mv^2}{r}$$

or $$v = \sqrt{\frac{GM_e}{r}} \quad (1)$$

The expression in Eq. (1) gives the speed of a statellite in a stable circular orbit of radius r.

Energies of a Satellite in a Circular Orbit

When there is a satellite revolving in a stable circular orbit of radius r around the earth, its speed is given by Eq. (1). During its motion, the kinetic energy of the satellite can be given as

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GM_e m}{r} \quad (2)$$

As gravitational force on satellite due to earth is the only force it experiences during motion, it has gravitational interaction energy in the field of earth, which is given as

$$U = -\frac{GM_e m}{r} \quad (3)$$

Thus, the total energy of a satellite in an orbit of radius r can be given as

Total energy E = Kinetic energy K + Potential energy U

$$= \frac{1}{2}\frac{GM_e m}{r} - \frac{GM_e m}{r}$$

or $$E = -\frac{1}{2}\frac{GM_e m}{r} \quad (4)$$

From Eqs. (1), (2) and (3) we can see that

$$|k| = \left|\frac{U}{2}\right| = |E|$$

it reaches point A, it is imparted a velocity in tangential direction to start its revolution around the earth in its orbit.

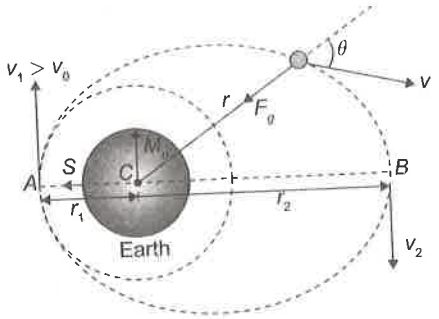


Figure 1.19

This velocity is termed as insertion velocity, if the velocity imparted to satellite is $v_0 = \sqrt{\frac{GM_e}{r_1}}$ then it starts following the circular path as shown in Fig. 1.19. If velocity imparted is $v_1 > v_0$, then it will trace the elliptical path shown. During this motion, the only force acting on satellite is the gravitational force due to earth which is acting along the line joining satellite and centre of earth.

As the force on satellite always passes through centre of earth during motion, we can say that on satellite there is no torque acting about centre of earth; thus, total angular momentum of satellite during orbital motion remains constant about earth's centre.

As no external force is involved for earth–satellite system, no external work is being done here; so we can also state that total mechanical energy of system also remains conserved.

In the elliptical path of satellite shown in Fig. 1.19 if r_1 and r_2 are the shortest distance (perigee) and farthest distance (appogee) of satellite from earth and at the points, velocities of satellite are v_1 and v_2 then we have according to conservation of angular momentum, the angular momentum of satellite at a general point is given as

$$L = mv_1r_1 = mv_2r_2$$
$$= mvr \sin\theta$$

During motion the total mechanical energy of satellite (kinetic + potential) also remains conserved. Thus, the total energy of satellite can be given as

$$E = \frac{1}{2}mv_1^2 - \frac{GM_em}{r_1}$$
$$= \frac{1}{2}mv_2^2 - \frac{GM_em}{r_2} = \frac{1}{2}mv^2 - \frac{GM_em}{r}$$

Using the above relations in equation written above we can find velocities v_1 and v_2 of satellite at nearest and farthest locations in terms of r_1 and r_2.

Projection of Satellites and Spaceships from Earth

To project a body into space, first it should be taken to a height where no atmopshere is present then it is projected with some initial speed. The path followed by the body also depends on the projection speed. Let us discuss the cases step by step.

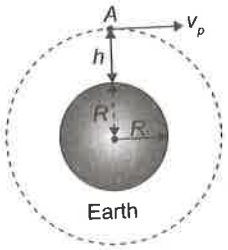


Figure 1.20

Consider the situation shown in Fig. 1.20. A body of mass m is taken to a height h above the surface of earth to a point A and then projected with an insertion velocity v_p as shown in the figure.

If we wish to launch the body as an earth's satellite in circular path, the velocity of projection must be

$$v_p = \sqrt{\frac{GM_e}{R_e + h}} \quad (1)$$

If h is small compared to radius of earth, we have

$$v_1 = v_p = \sqrt{\frac{GM_e}{R_e}}$$
$$= \sqrt{g_sR_e} = 7.93 \text{ km/s}$$

This velocity $v_1 = 7.93$ km/s with which, when a body is thrown from earth's surface tangentially so that after projection it becomes a satellite of earth in a circular orbit around it, is called orbital speed or first cosmic velocity.

We have already discussed that if projection speed is lesser than the orbital speed, the body will start following the inner ellipse and if velocity of projection is increased the body will follow the outer ellipse. If projection speed of the satellite is further increased, the outer ellipse will also become bigger and at a particular higher projection speed, it may also be possible that body will go to infinity and will never come back to earth again.

We have discussed that negative total energy of a body shows its boundness. If we write the total energy of a body projected from point A as shown in the figure is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_e m}{R_e + h}$$

If after projection body becomes a satellite of earth, then it implies that it is bounded to earth and its total energy is negative. If at point A, that much of kinetic energy is imparted to the body so that total energy of body becomes zero then it implies that the body will reach to infinity and escape from gravitational field of earth. If v_{II} is such a velocity, then we have

$$\frac{1}{2}mv_{II}^2 - \frac{GM_e m}{R_e + h} = 0$$

or
$$v_{II} = \sqrt{\frac{2GM_e}{R_e + h}} = \sqrt{2}v_I \qquad (2)$$

For $h << R_e$, we have

$$v_{II} = \sqrt{\frac{2GM_e}{R_e}}$$
$$= \sqrt{2g_s R_e} = 11.2 \text{ km/s} \qquad (3)$$

Thus from earth's surface a body is thrown at a speed of 11.2 km/s, it will escape from earth's gravitation. If the projection speed of body is less than this value total energy of body is negative and it wil orbit the earth in elliptical orbit. This velocity is referred as the second cosmic velocity or escape velocity. When a body is thrown with this speed, it follows a parabolic trajectory and will become free from earth's gravitational attraction.

When body is thrown with speed more then v_{II} then it moves along a hyperbolic trajectory and also leaves the region where the earth's gravitational attraction acts. Also when it reaches infinity some kinetic energy will be left in it and it becomes a satellite of sun, that is small artificial planet.

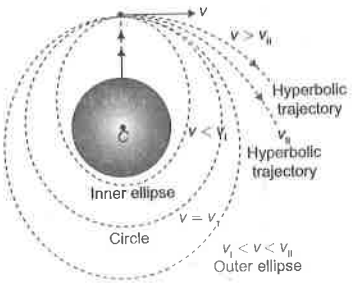


Figure 1.21

All the calculations we have performed till now do not take into account the influence of the sun and of the planets on the motion of the projected body. In other words, we have assumed that the reference frame connected with the earth is an inertial frame and the body moves relative to it. But in reality the whole system body and the earth are in a non-inertial frame which is permanently accelerated relative to sun.

Let us consider some examples to understand some basic concepts related to gravitational energy and projection.

SOLVED EXAMPLES

EXAMPLE 14

A spaceship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull. (Radius of the earth = 6400 km and $g = 9.8$ m/s)

SOLUTION

In an orbit close to earth's surface, the velocity of space ship is

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

We know that the escape velocity is $v_{II} = \sqrt{2gR}$.

Hence, additional velocity required to be imparted is

$$\Delta v = v_{II} - v = \left(\sqrt{2} - 1\right)\sqrt{gR}$$
$$= \left(\sqrt{2} - 1\right)\sqrt{9.8 \times 6400 \times 10^3} = 3.28 \times 10^3 \text{ m/s} \quad \blacksquare$$

EXAMPLE 15

A particle is fired vertically upwards with a speed of 9.8 km/s. Find the maximum height attained by the particle. Radius of the earth = 6400 km and g at the surface = 9.8 m/s^2. Consider only earth's gravitation.

SOLUTION

Initial energy of particle on earth's surface is

$$E_i = \frac{1}{2}mu^2 - \frac{GMm}{R}$$

If the particle reaches upto a height h above the surface of earth, then its final energy will only be the gravitational potential energy.

$$E_f = -\frac{GMm}{R + h}$$

According to energy conservation, we have

$$E_i = E_f$$

or $$\frac{1}{2}mu^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or $$\frac{1}{2}u^2 - gR = -\frac{gR^2}{R+h}$$

or $$h = \frac{2gR^2}{2gR-u^2} - R$$

$$= \frac{2\times 9.8\times(6400\times10^3)^2}{2\times9.8\times6400\times10^3 - (9.8)^2} - 6400\times10^3$$

$$= (27300-6400)\times10^3 = 20900\text{ km}$$ ■

EXAMPLE 16

A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy slowly at a constant rate C due to friction. If M_e and R_e denote the mass and radius of the earth, respectively, show that the satellite falls on the earth in a limit t given by

$$t = \frac{GmM_e}{2C}\left(\frac{1}{R_e} - \frac{1}{r}\right)$$

SOLUTION

Let velocity of satellite in its orbit of radius r be v. Then we have

$$v = \sqrt{\frac{GM_e}{r}}$$

When satellite approaches earth's surface, if its velocity becomes v', then it is given as

$$v' = \sqrt{\frac{GM_e}{R_e}}$$

The total initial energy of satellite at a distance r is

$$E_{T_f} = K_f + U_r$$

$$= \frac{1}{2}mv^2 - \frac{GM_em}{R_e} = -\frac{1}{2}\frac{GM_em}{r}$$

The total final energy of satellite at a distance R_e is

$$E_{T_f} = K_f + U_r$$

$$= \frac{1}{2}mv'^2 - \frac{GM_em}{R_e} = -\frac{1}{2}\frac{GM_em}{R_e}$$

As satellite is loosing energy at rate C, if it takes a time t in reaching earth, we have

$$Ct = E_{T_i} - E_{T_f}$$

$$= \frac{1}{2}GM_em\left[\frac{1}{R_e} - \frac{1}{r}\right]$$

$$\Rightarrow \quad t = \frac{GM_em}{2C}\left[\frac{1}{R_e} - \frac{1}{r}\right]$$ ■

EXAMPLE 17

An artifical satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth.

(a) Determine the height of the satellite above earth's surface.

(b) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth.

SOLUTION

(a) Let M and R be the mass and radius of the earth, respectively. If m be the mass of satellite, then escape velocity from earth $v_c = \sqrt{2gR_e}$

$$\text{velocity of satellite} = \sqrt{\frac{gR_e}{2}}$$

Further, we know orbital speed of satallite at a height h is

$$v_s = \sqrt{\left(\frac{GM_e}{r}\right)} = \sqrt{\left(\frac{R_e^2 g}{R_e+h}\right)}$$

or $$v_s^2 = \frac{R^2 g}{R+h}$$

From the equation written above, we get

$$h = R = 6400\text{ km}$$

(b) Now total energy at height h = total energy at earth's surface (principle of conservation of energy)

or $$0 - GM_e\frac{m}{R+h} = \frac{1}{2}mv^2 - GM_e\frac{m}{R_E}$$

or $$\frac{1}{2}mv^2 = \frac{GM_em}{R_e} - \frac{GM_em}{2R_e} \quad [\text{As } h = R]$$

Solving we get

$$v = \sqrt{gR}$$

or $\sqrt{9.8 \times 6400 \times 10^3} = 7.919\,\text{km/s}$ ■

COMMUNICATION SATELLITES

Communication satellites around the earth are used by Information Technology for spreading information throughout the globe.

Figure 1.22 shows as to how using satellites an information from an earth station, located at a point on earth's surface can be sent throughout the world.

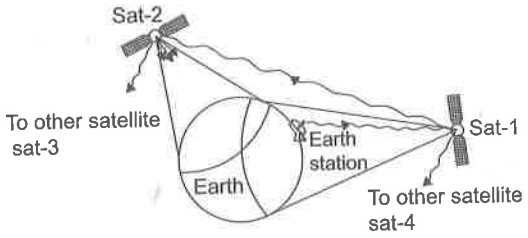


Figure 1.22

First, the information is sent to the nearest satellite in the range of earth station by means of electromagnetic waves then that satellite broadcasts the signal to the region of earth exposed to this satellite and also send the same signal to other satellite for broadcasting in other parts of the globe.

Geostationary Satellite and Parking Orbit

There are so many types of communication satellites revolving around the earth in different orbits at different heights depending on their utility. Some of which are geostationary satellites, which appear at rest relative to earth or which have same angular velocity as that of earth's rotation, i.e., with a time peiod of 24 hour. Such satellite must be orbiting in an orbit of specific radius. This orbit is called parking orbit. If a geostationary satellite is at a height h above the earth's surface, then its orbiting speed is given as

$$v_{gs} = \sqrt{\frac{GM_e}{(R_e + h)}}$$

The time period of its revolution can be given Kepler's third law as

$$T^2 = \frac{4\pi^2}{GM_e}(R_e + h)^3$$

or $$T^2 = \frac{4\pi^2}{g_s R_e^2}(R_e + h)^3$$

or $$h = \left(\frac{g_s R_e^2}{4\pi^2} T^2\right)^{1/3} - R_e$$

or $$h = \left[\frac{9.8 \times (6.4 \times 10^6) \times [86,400]}{4 \times (3.14)^2}\right]^{1/3} - 6.4 \times 10^6$$

$$= 35,954.6\,\text{km} \simeq 36,000\,\text{km}$$

Thus when a satellite is launched in an orbit at a height of about 36,000 km above the quator then it will appear to be at rest with respect to a point on Earth's surface. A geostationary satellite must have in orbit in equatorial plane due to the geographic limitation because of irregular geometry of earth (ellipsoidal shape).

In short

- Plane of the satellite should pass through centre of the planet.
- For geostationary satallites, plane should be equatorial plane.
- Time peirod should be 24 hours and direction should be west to east.
- For any point on the earth, geostationary satellite is stationary.

Broadcasting Region of a Satellite

Now as we know the height of a geostationary satellite we can easily find the area of earth exposed to the satellite or area of the region in which the communication can be mode using this satellite.

Figure 1.23 shows earth and its exposed area to a geostationary satellite. Here, the angle θ can be given as

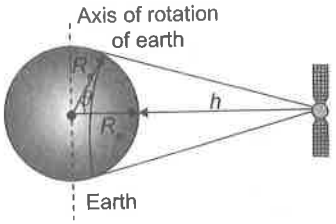


Figure 1.23

$$\theta = \cos^{-1}\left(\frac{R_e}{R_e + h}\right)$$

Now we can find the solid angle Ω which the exposed area subtends on earth's centre as

$$\Omega = 2\pi(1 - \cos\theta)$$

$$= 2\pi\left(1-\frac{R_e}{R_e+h}\right)=\frac{2\pi h}{R_e+h}$$

Thus, the area of earth's surface to geostationary satellite is

$$S=\Omega R_e^2=\frac{2\pi hR_e^2}{R_e+h}$$

Let us consider some examples to understand the concept in detail.

SOLVED EXAMPLES

EXAMPLE 18

A satellite is revolving around the earth in an orbit of radius double that of the parking orbit and revolving in same sense. Find the periodic time duration between two instants when this satellite is closest to a geostationary satellite.

SOLUTION

We know that the time period of revolution of a satellite is given as

$$T^2=\frac{4\pi^2}{GM_e}r^3 \qquad \text{[Kepler's III law]}$$

For the satellite given in the problem and for a geostationary satellite, we have

$$\frac{T_1}{T_2}=\left(\frac{r_1}{r_2}\right)^3$$

or $$T_1=\left(\frac{r_1}{r_2}\right)^3\times T_2$$

$$=(2)^3\times 24=192\,\text{hr}$$

If Δt be the time between two sucessive instants when the satellite is closed then we must have

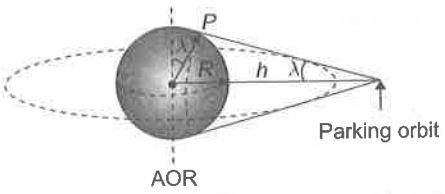
$$\Delta t=\frac{\theta}{\omega_1}=\frac{2\pi+\theta}{\omega_2}$$

$$=\frac{2\pi}{\omega_2-\omega_1}$$

where ω_1 and ω_2 are the angular speeds of the two planets. ■

EXAMPLE 19

Find the minimum colatitude which can directly receive a signal from a geostationary satellite.



SOLUTION

The farthest point on earth, which can receive signals from the parking orbit, is the point where a length is drawn on earth surface from satellite as shown in the figure.

The colatitude λ of point P can be obtained from the figure as

$$\sin\lambda=\frac{R_e}{R_e+h}\simeq\frac{1}{7}$$

We know for a parking orbit $h\simeq 6R_e$

Thus, we have $$\lambda=\sin^{-1}\left(\frac{1}{7}\right)$$

EXAMPLE 20

If a satellite is revolving around the earth in a circular orbit in a plane containing earth's axis of rotation, if the angular speed of satellite is equal to that of earth, find the time it takes to move from a point above north pole of a point above the equator.

SOLUTION

A satellite which rotates with angular speed equal to earth's rotation has an orbit radius $7R_e$ and the angular speed of revolution is

$$\omega=\frac{2\pi}{T}=\frac{2\pi}{86400}$$

$$=7.27\times10^{-5}\,\text{rad/s}$$

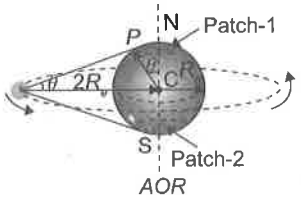
When satellite moves from a point above north pole to a point above equator, it traverses an angle $\frac{\pi}{2}$, this time taken is

$$t=\frac{\pi/2}{\omega}=21,600\,\text{s}$$

$$=6\,\text{hr}$$ ■

EXAMPLE 21

A satellite is orbiting around the earth in an orbit in equatorial plane of radius $2R_e$ where R_e is the radius of earth. Find the area on earth, this satellite covers for communication purpose in its complete revolution.



SOLUTION

As shown in the figure when statelite S revolves, it covers a complete circular belt on earth's surface for communication. If the colatitude of the farthest point on surface upto which singals can be received (point P) is q, then we have

$$\sin\theta = \frac{R_e}{2R_e} = \frac{1}{2}$$

or $$\theta = \frac{\pi}{6}$$

During revolution satellite leaves two spherical patches 1 and 2 on earth's surface at north and south poles where no signals can be transmitted due to curvature. The areas of these patches can be obtained by solid angles.

The solid angle subtended by a patch on earth's centre is

$$\Omega = 2\pi(1-\cos\theta) = \pi\left(2-\sqrt{3}\right)st.$$

Area of patches 1 and 2 is

$$A_p = \Omega R_e^2 = \pi\left(2-\sqrt{3}\right)R_e^2$$

Thus, the total area on earth's surface to which communication can be made is

$$A_c = 4\pi R_e^2 - 2A_p = 4\pi R_e^2 - 2\pi\left(2-\sqrt{3}\right)R_e^2$$

$$2\pi R_e^2\left(2-2+\sqrt{3}\right) = 2\sqrt{3}\pi R_e^2$$ ■

KEPLER'S LAWS OF PLANETARY MOTION

The motions of planet in universe have always been a puzzle. In 17th century, Johannes Kepler, after a life time of study worded out some empirical laws based on the analysis of astronomical measurements of Tycho Brahe. Kepler formulated his laws, which are kinematical description of planetary motion. Now we discuss these laws step by step.

Kepler's First Law (The Law of Orbits)

Kepler's first law is illustrated in the image shown in Fig. 1.24. It states that "All the planets move around the sun in ellipitcal orbits with sun at one of the focus not at centre of orbit."

It is observed that the orbits of planets around the sun are very less ecentric or approximately circular.

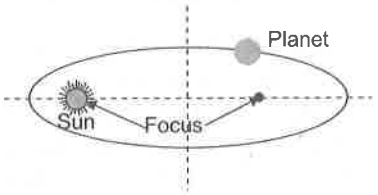
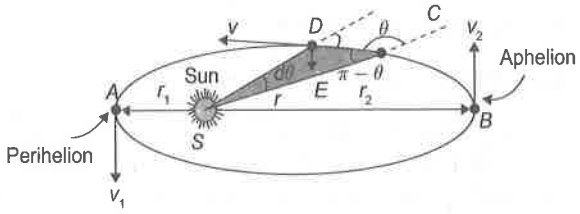


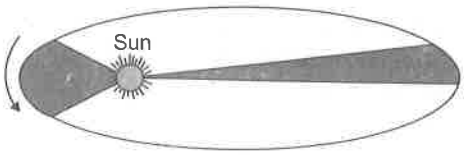
Figure 1.24

Kepler's Second Law (The Law of Areas)

Kepler's second law is basically an alternative statement of law of conservation of momentum. It is illustrated in the image shown in Fig. 1.25(a). We know from angular momentum conservation, in elliptical orbit plane will move faster when it is nearer to the sun. Thus when a planet executes elliptical orbit its angular speed changes continuously as it moves in the orbit. The point of nearest approach of the planet to the sun is termed as perihelion. The point of greatest seperation is termed as aphelion. Hence by angular momentum conservation we can state that the planet moves with maximum speed when it is near perihelion and moves with slowest speed when it is near aphelion.



(a)



(b)

Figure 1.25

Kepler's second law states that "The line joining the sun and planet sweeps out equal areas in equal time or the rate of sweeping area by the position vector of the planet with respect to sun remains constant." This is shown in Fig. 1.25(b).

The above statement of Kepler's second law can be verified by the law of conservation of angular momentum. To verify this, consider the moving planet around the sun at a general point C in the orbit at speed v. Let at this instant the distance of planet from the sun is r. If θ be the angle between position vector $\vec{r}$ of planet and its velocity vector then the angular momentum of planet at this instant is

$$L = mvr\sin\theta \quad (1)$$

In an elemental time, the planet will cover a small distance $CD = dl$ and will travel to another adjacent point D as shown in Fig. 1.25(a). Thus, the distance $CD = vdt$. In this duration dt, the position vector sweeps out an area equal to that of triangle SCD, which is calculated as follows:

Area of triangle SCD is

$$dA = \frac{1}{2}\times r\times vdt\sin(\pi-\theta)$$

$$= \frac{1}{2}rv\sin\theta\cdot dt$$

Thus, the rate of sweeping area by the position vector $\vec{r}$ is

$$\frac{dA}{dt} = \frac{1}{2}rv\sin\theta$$

Now from Eq. (1)

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant} \quad (2)$$

The expression in Eq. (2) verifies the statement of Kepler's second law of planetary motion.

Kepler's Third Law (The Law of Periods)

Kepler's third law is concerned with the time period of revolution of planets. It states that "The time period of revolution of a planet in its orbit around the sun is directly proportional to the cube of semi-major axis of the elliptical path around the sun."

If 'T' is the period of revolution and 'a' be the semi-major axis of the path of planet, then according to Kepler's third law, we have

$$T^2 \propto a^3$$

For circular orbits, it is a special case of ellipse when its major and minor axes are equal. If a planet is in a circular orbit of radius r around the sun, then its revolution speed must be given as

$$v = \sqrt{\frac{GM_s}{r}}$$

where M_s is the mass of the sun. Here you can recall that this speed is independent from the mass of planet. The time period of revolution can be given as

$$T = \frac{2\pi r}{v} \quad (1)$$

or

$$T = \frac{2\pi r}{\sqrt{\frac{GM_s}{r}}}$$

Squaring Eq. (1), we get

$$T^2 = \frac{4\pi^2}{GM_s}r^3 \quad (2)$$

Equation (2) verifies the statement of Kepler's third law for circular orbits. Similarly we can also verify it for elliptical orbits. For this we start from the relation we have derived earlier for rate of sweeping area by the position vector of planet with respect to sun which is given as

$$\frac{dA}{dt} = \frac{L}{2m}$$

SOLVED EXAMPLES

EXAMPLE 22

The moon revolves around the earth 13 times per year. If the ratio of the distance of the earth from the sun to the distance of the moon from the earth is 392, find the ratio of mass of the sun to the mass of the earth.

SOLUTION

The time period T_e of the earth around the sun of mass M_s is given by

$$T_e^2 = \frac{4\pi^2}{GM_s}\times r_e^3 \quad (1)$$

where r_e is the radius of the earth.

Similarly, time period T_m of moon around earth is given by

$$T_m^2 = \frac{4\pi^2}{GM_e}\times r_m^3 \quad (2)$$

Dividing Eq. (1) by Eq. (2), we get

$$\left(\frac{T_e}{T_m}\right)^2 = \left(\frac{M_e}{M_s}\right)\left(\frac{r_e}{r_m}\right)^3$$

or

$$\left(\frac{M_s}{M_e}\right) = \left(\frac{T_m}{T_e}\right)^2\times\left(\frac{r_e}{r_m}\right)^3 \quad (3)$$

$$T = 2\pi\sqrt{\left(\frac{d^3}{G(m_1+m_2)}\right)}$$

$$\frac{J_1}{J_2} = \frac{I_1\omega}{I_2\omega} = \frac{I_1}{I_2}$$

pplying the law of conservation of energy

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r} = \frac{1}{2}m(v_0+v)^2 - \frac{GMm}{a}$$

$$\frac{1}{2}m\left(\frac{5}{4}\frac{GMa}{r^2}\right) - \frac{GMm}{r} = \frac{1}{2}m\left(\frac{5}{4}\frac{GM}{a}\right) - \frac{GMm}{a}$$

$$\frac{5}{8}\times\frac{a}{r^2} - \frac{1}{r} = \frac{5}{8}\times\frac{1}{a} - \frac{1}{a} = -\frac{3}{8a}$$

r $\quad 3r^2 - 8ar + 5a^2 = 0$

r $\quad r = a \text{ or } \frac{5a}{3}$

Thus minimum distance of the particle $= a$

And maximum distance of the particle $= \frac{5a}{3}$ ■

EXAMPLE 26

A sky lab of mass 2×10^3 kg is first launched from the surface of earth in a circular orbit of radius $2R$ (from the centre of earth) and then it is shifted from this circular orbit to another circular orbit of radius $3R$. Calculate the minimum energy required (a) to place the lab in the first orbit and (b) to shift the lab from the first orbit to the second orbit. Given, $R = 6400$ km and $g = 10$ m/s^2.

SOLUTION

(a) The energy of the sky lab on the surface of earth

$$E_s = KE + PE = 0 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{R}$$

And the total energy of the sky lab in an orbit of radius $2R$ is

$$E_1 = -\frac{GMm}{4R}$$

So the energy required to place the lab from the surface of earth to the orbit of radius $2R$ is given as

$$E_1 - E_s = -\frac{GMm}{4R} - \left[-\frac{GMm}{R}\right] = \frac{3}{4}\frac{GMm}{R}$$

or $\quad \Delta E = \frac{3}{4}\frac{m}{R}\times gR^2 = \frac{3}{4}mgR \quad \text{as} \quad \left[g = \frac{GM}{R^2}\right]$

or $\quad \Delta E = \frac{3}{4}\left(2\times10^3\times10\times6.4\times10^6\right)$

$$= \frac{3}{4}\left(12.8\times10^{10}\right) = 9.6\times10^{10}\text{ J}$$

(b) As for II orbit of radius $3R$, the total energy of sky lab is

$$E_2 = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$$

or $\quad E_2 - E_1 = -\frac{GMm}{6R} - \left(-\frac{GMm}{4R}\right) = \frac{1}{12}\frac{GMm}{R}$

or $\quad \Delta E = \frac{1}{12}mgR = \frac{1}{12}\left(12.8\times10^{10}\right)$

$$= 1.1 \times 10^{10}\text{J}$$ ■

EXAMPLE 27

A satellite is revolving around a planet of mass M in an elliptic orbit of semi-major axis a. Show that the orbital speed of the satellite when it is at a distance r from the focus will be given by:

$$v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$

SOLUTION

As in case of elliptic orbit with semi-major axes a, of a satellite total mechanical energy remains constant, at any position of satellite in the orbit, given as

$$E = -\frac{GMm}{2a}$$

or $\quad KE + PE = -\frac{GMm}{2a} \quad (1)$

Now, if at position r, v is the orbital speed of satellite, we have

$$KE = \frac{1}{2}mv^2$$

and $\quad PE = -\frac{GMm}{r} \quad (2)$

So from Eqs. (1) and (2), we have

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a},$$

i.e., $\quad v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$ ■

JEE Main

1. If R is the radius to gravity on th the earth is
   (A) $4\pi G/3gR$
   (C) $3g/4\pi RG$

2. The height ab of gravitationa surface, will be
   (A) $R_e/4$
   (C) $3R_e/4$

3. The decrease i above the earth
   (A) $g/2$
   (C) $\frac{4g}{9}$

4. At what altitu 25% of that at is R)?
   (A) $R/4$
   (C) $3R/8$

5. If the rotatio weight of the
   (A) will rem
   (C) will dec

6. On doublin gravitational
   (A) remain
   (B) become
   (C) become
   (D) becom

7. If the accel be kept con d and the d
   (A) $d \propto r$
   (C) $d \propto 1/$

EXERCISES

JEE Main

1. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is
   (A) $4\pi G/3gR$ (B) $3\pi R/4gG$
   (C) $3g/4\pi RG$ (D) $\pi Rg/12G$

2. The height above surface of earth where the value of gravitational acceleration is one fourth of that at surface, will be
   (A) $R_e/4$ (B) $R_e/2$
   (C) $3R_e/4$ (D) R_e

3. The decrease in the value of g on going to a height $R/2$ above the earth's surface will be
   (A) $g/2$ (B) $\frac{5g}{9}$
   (C) $\frac{4g}{9}$ (D) $\frac{g}{3}$

4. At what altitude will the acceleration due to gravity be 25% of that at the earth's surface (given radius of earth is R)?
   (A) $R/4$ (B) R
   (C) $3R/8$ (D) $R/2$

5. If the rotational motion of earth increases, then the weight of the body
   (A) will remain same (B) will increase
   (C) will decrease (D) none of these

6. On doubling the distance between two masses the gravitational force between them will
   (A) remain unchanged
   (B) become one-fourth
   (C) become half
   (D) become double

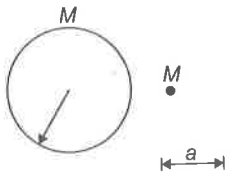
7. If the acceleration due to gravity inside the earth is to be kept constant, then the relation between the density d and the distance r from the centre of earth will be
   (A) $d \propto r$ (B) $d \propto r^{1/2}$
   (C) $d \propto 1/r$ (D) $d \propto \frac{1}{r^2}$

8. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole ($d << R$). The value of d is
   (A) $\frac{\omega^2 R^2}{g}$ (B) $\frac{\omega^2 R^2}{2g}$
   (C) $\frac{2\omega^2 R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$

9. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same ?
   (A) 1/25 (B) 1/5
   (C) $1/\sqrt{5}$ (D) 5

10. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth ?
    (A) $\sqrt{2}$ second (B) $2\sqrt{2}$ seconds
    (C) $\frac{1}{\sqrt{2}}$ second (D) $\frac{1}{2\sqrt{2}}$ second

11. A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a.



    (A) Gravitational field and potential both are zero at centre of the shell
    (B) Gravitational field is zero not only inside the shell but at a point outside the shell also
    (C) Inside the shell, gravitational field alone is zero
    (D) Neither gravitational field nor gravitational potential is zero inside the shell

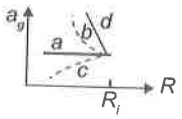
12. If the kinetic energy of a satellite orbiting around the earth is doubled then
    (A) the satellite will escape into the space.
    (B) the satellite will fall down on the earth
    (C) radius of its orbit will be doubled
    (D) radius of its orbit will become half.

13. The escape velocity from a planet is v_0. The escape velocity from a planet having twice the radius but same density will be
    (A) $0.5\,v_0$ (B) v_0
    (C) $2v_0$ (D) $4v_0$

14. Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity $\frac{v_A}{v_B}$ is
    (A) 2 (B) $\sqrt{2}$
    (C) $1/\sqrt{2}$ (D) 1/2

15. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre
    (A) increases
    (B) decreases
    (C) remains same
    (D) during the compression increases then returns at the previous value

16. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse?



    (A) a (B) b
    (C) c (D) d

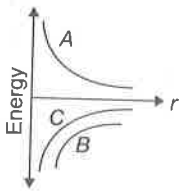
17. A satellite of the earth is revolving in circular orbit with a uniform velocity V. If the gravitational force suddenly disappears, the satellite will
    (A) continue to move with the same velocity in the same orbit
    (B) move tangentially to the original orbit with velocity V
    (C) fall down with increasing velocity
    (D) come to a stop somewhere in its original orbit

18. A satellite revolves in the geostationary orbit but in a direction east to west. The time interval between its successive passing about a point on the equator is
    (A) 48 hrs (B) 24 hrs
    (C) 12 hrs (D) never

19. Two point masses of mass 4 m and m respectively separated by d distance are revolving under mutual force of attraction. Ratio of their kinetic energies will be
    (A) 1:4 (B) 1:5
    (C) 1:1 (D) 1:2

20. Select the correct choice(s):
    (A) The gravitational field inside a spherical cavity, within a spherical planet must be non zero and uniform.
    (B) When a body is projected horizontally at an appreciable large height above the earth, with a velocity less than for a circular orbit, it will fall to the earth along a parabolic path
    (C) A body of zero total mechanical energy placed in a gravitational field will escape the field
    (D) Earth's satellite must be in equatorial plane.

21. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



    (A) A shows the kinetic energy, B the total energy and C the potential energy of the system
    (B) C shows the total energy, B the kinetic energy and A the potential energy of the system

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system

(D) A and B are kinetic and potential energies and C is the total energy of the system

22. Two different masses are dropped from same heights, then just before these strike the ground, the following is same :

(A) kinetic energy

(B) potential energy

(C) linear momentum

(D) Acceleration

23. When a satellite moves around the earth in a certain orbit, the quantity which remains constant is

(A) angular velocity (B) kinetic energy

(C) aerial velocity (D) potential energy

24. A body of mass m rises to height $h = R/5$ from the earth's surface, where R is earth's radius. If g is acceleration due to gravity at earth's surface, the increase in potential energy is

(A) mg/h (B) $\frac{5}{6}mgh$

(C) $\frac{3}{5}mgh$ (D) $\frac{6}{7}mgh$

25. A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of 90 m, then he will be able to throw the stone on that planet to a height

(A) 90 m (B) 40 m

(C) 100 m (D) 45 m

26. Work done in taking a body of mass m to a height nR above surface of earth will be: (R = radius of earth)

(A) $mgnR$ (B) $mgR\,(n/n+1)$

(C) $mgR\frac{(n+1)}{n}$ (D) $\frac{mgR}{n(n+1)}$

27. The potential energy of a body of mass 3 kg on the surface of a planet is 54 joule. The escape velocity will be

(A) 18 m/s (B) 162 m/s

(C) 36 m/s (D) 6 m/s

28. A body of mass m is situated at a distance $4R_e$ above the earth's surface, where R_e is the radius of earth. How much minimum energy be given to the body so that it may escape

(A) mgR_e (B) $2mgR_e$

(C) $\frac{mgR_e}{5}$ (D) $\frac{mgR_e}{16}$

29. A satellite of earth is moving in its orbit with a constant speed v. If the gravity of earth suddenly vanishes, then this satellite will

(A) continue to move in the orbit with velocity v.

(B) start moving with velocity v in a direction tangential to the orbit

(C) fall down with increased velocity

(D) be lost in outer space.

30. The ratio of distances of satellites A and B from the centre of the earth is 1.4:1, then the ratio of energies of satellites B and A will be-

(A) 1.4:1 (B) 2:1

(C) 1:3 (D) 4:1

31. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$

(C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

32. A body is dropped by a satellite in its geo-stationary orbit

(A) it will burn on entering into the atmosphere.

(B) it will remain in the same place with respect to the earth.

(C) it will reach the earth in 24 hours

(D) it will perform uncertain motion

33. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

(A) $\frac{3}{2}v$ (B) $\sqrt{\frac{3}{2}}v$

(C) $v\sqrt{\frac{2}{3}}$ (D) $v\frac{2}{3}$

34. Two artificial satellites A and B are at a distances r_A and r_B above the earth's surface. If the radius of earth is R, then the ratio of their speeds will be

(A) $\left(\frac{r_B+R}{r_A+R}\right)^{1/2}$ (B) $\left(\frac{r_B+R}{r_A+R}\right)^{2}$

(C) $\left(\frac{r_B}{r_A}\right)^{2}$ (D) $\left(\frac{r_B}{r_A}\right)^{1/2}$

35. A planet of mass m is in an elliptical orbit about the sun ($m << M_{sun}$) with an orbital period T. If A be the area of orbit, then its angular momentum would be

(A) $\frac{2mA}{T}$ (B) mAT

(C) $\frac{mA}{2T}$ (D) $2mAT$

36. If the distance between sun and earth is made 3 times of the present value then gravitational force between them will become:

(A) 9 times (B) $\frac{1}{9}$ times

(C) $\frac{1}{3}$ times (D) 3 times

37. If a body is carried from surface of earth to moon, then

(A) the weight of a body will continuously increase

(B) the mass of a body will continuously increase

(C) the weight of a body will decrease first, become zero and then increase,

(D) the mass of a body will decrease first, become zero and then increase.

38. The period of revolutions of two satellites are 3 hours and 24 hours. The ratio of their velocities will be

(A) 1:8 (B) 1:2

(C) 2:1 (D) 4:1

39. **Statement 1 :** Assuming zero potential at infinity, gravitational potential at a point cannot be positive.

Statement 2 : Magnitude of gravitational force between two particle has inverse square dependence on distance between two particles.

(A) Statement 1 is true, statement 2 is true and statement 2 is correct explanation for statement 1

(B) Statement 1 is true, statement 2 is true and statement 2 is NOT the correct explanation for statement 1

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

JEE Advanced

Single Correct

1. Two masses m_1 and m_2 are initially at rest and are separated by a very large distance. If the masses approach each other subsequently, due to gravitational attraction between them, their relative velocity of approach at a separation distance of d is

(A) $\frac{2Gd}{(m_1+m_2)}$ (B) $\frac{(m_1+m_2)G}{2d}$

(C) $\left[(m_1+m_2)\frac{2G}{d}\right]^{1/2}$ (D) $(m_1+m_2)^{1/2}\,2Gd$

2. A man of mass m starts falling towards a planet of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he sees that the planet is really made of two pieces a spherical shell of negligible thickness of mass $2M/3$ and a point mass $M/3$ at the centre. Change in the force of gravity experienced by the man is

(A) $\frac{2}{3}\frac{GMm}{R^2}$ (B) 0

(C) $\frac{1}{3}\frac{GMm}{R^2}$ (D) $\frac{4}{3}\frac{GMm}{R^2}$

3. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V.

(A) $V_e = 2V$ (B) $V_e = V$

(C) $V_e = V/2$ (D) $V_e = \sqrt{3}V$

4. The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(A) v_e (B) $\frac{v_e}{\sqrt{2}}$

(C) $\frac{v_e}{2}$ (D) zero

5. If a tunnel is cut at any orientation through earth, then a ball released from one end will reach the other end in time (neglect earth rotation)

(A) 84.6 minutes

(B) 42.3 minutes

(C) 8 minutes

(D) depends on orientation

Question Nos. 6 and 7

Two uniform spherical stars made of same material have radii R and $2R$. Mass of the smaller planet is m. They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution 1/2.

6. Kinetic energy of the system just after the collision is

(A) $\frac{8Gm^2}{3R}$

(B) $\frac{2Gm^2}{3R}$

(C) $\frac{4Gm^2}{3R}$

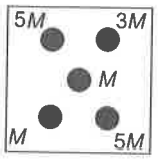
(D) cannot be determined

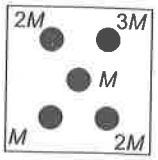
7. The maximum separation between their centres after their first collision

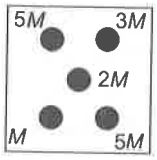
(A) $4R$ (B) $6R$

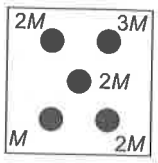
(C) $8R$ (D) $12R$

8. A mass is at the center of a square, with four masses at the corners as shown.

(A) 

(B) 

(C) 

(D) 

Rank the choices according to the magnitude of the gravitational force on the centre mass.

(A) $F_A = F_B < F_C = F_D$

(B) $F_A > F_B < F_D < F_C$

(C) $F_A = F_B > F_C = F_D$

(D) none

9. A satellite of mass m, initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is

(A) $\frac{\sqrt{3}}{4}mgR$ (B) $\frac{1}{2}mgR$

(C) $\frac{1}{4}mgR$ (D) $\frac{3}{4}mgR$

10. A satellite of mass $5M$ orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass $4M$. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass $4M$ is in

(A) bound orbit

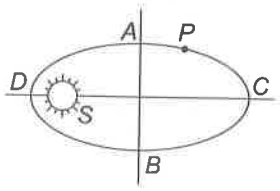
(B) unbound orbit

(C) partially bound orbit

(D) data is insufficient to determine the nature of the orbit

Question Nos. 11 and 12

Figure shows the orbit of a planet P round the sun S. AB and CD are the minor and major axes of the ellipse.

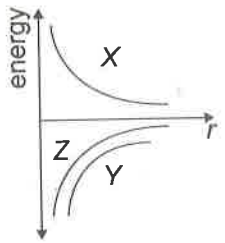


11. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then
    (A) $t_1 = t_2$
    (B) $t_1 > t_2$
    (C) $t_1 < t_2$
    (D) nothing can be concluded

12. If U is the potential energy and K kinetic energy then $|U| > |K|$ at
    (A) Only D
    (B) Only C
    (C) both D and C
    (D) neither D nor C

13. Satellites A and B are orbiting around the earth in orbits of ratio R and $4R$ respectively. The ratio of their areal velocities is
    (A) 1:2
    (B) 1:4
    (C) 1:8
    (D) 1:16

14. Figure shows the variation of energy with the orbit radius r of a satellite in a circular motion. Select the correct statement.



    (A) Z is total energy, Y is kinetic energy and X is potential energy
    (B) X is kinetic energy, Y is total energy and Z is potential energy
    (C) X is kinetic energy, Y is potential energy and Z is total energy
    (D) Z is kinetic energy, X is potential energy and Y is total energy

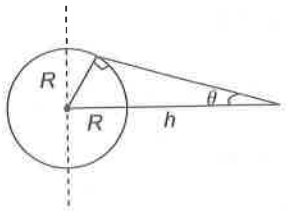
Multiple Correct

15. Assuming the earth to be a sphere of uniform density the acceleration due to gravity
    (A) at a point outside the earth is inversely proportional to the square of its distance from the centre
    (B) at a point outside the earth is inversely proportional to its distance from the centre
    (C) at a point inside is zero
    (D) at a point inside is proportional to its distance from the centre

16. Two masses m_1 and m_2 ($m_1 < m_2$) are released from rest from a finite distance. They start under their mutual gravitational attraction
    (A) acceleration of m_1 is more than that of m_2
    (B) acceleration of m_2 is more than that of m_1
    (C) centre of mass of system will remain at rest in all the references frame
    (D) total energy of system remains constant

17. Inside a hollow spherical shell
    (A) everywhere gravitational potential is zero
    (B) everywhere gravitational field is zero
    (C) everywhere gravitational potential is same
    (D) everywhere gravitational field is same

18. A geostationary satellite is at a height h above the surface of earth. If earth radius is R



    (A) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1}(R/R+h)$
    (B) The maximum colatitudes on earth upto which the satellite can be used for communication is $\sin^{-1}(R/R+h)$
    (C) The area on earth escaped from this satellite is given as $2\pi R^2(1+\sin\theta)$
    (D) The area on earth escaped from this satellite is given as $2\pi R^2(1+\cos\theta)$

19. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then
    (A) its kinetic energy increases
    (B) its kinetic energy decreases
    (C) its angular momentum about the earth decreases
    (D) its period of revolution around the earth increases

20. A communications Earth satellite
(A) goes round the earth from east to west
(B) can be in the equatorial plane only
(C) can be vertically above any place on the earth
(D) goes round the earth from west to east

21. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change
(A) gravitational potential energy
(B) angular velocity
(C) linear orbital velocity
(D) centripetal acceleration

22. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth
(A) the acceleration of S is always directed towards the centre of the earth
(B) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(C) the total mechanical energy of S varies periodically with time
(D) the linear momentum of S remains constant in magnitude

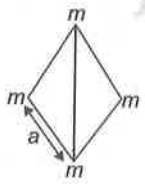
23. If a satellite orbits as close to the earth's surface as possible,
(A) its speed is maximum
(B) time period of its rotation is minimum
(C) the total energy of the 'earth plus satellite' system is minimum
(D) the total energy of the 'earth plus satellite' system is maximum

24. For a satellite to orbit around the earth, which of the following must be true ?
(A) It must be above the equator at some time
(B) It cannot pass over the poles at any time
(C) Its height above the surface cannot exceed 36,000 km
(D) Its period of rotation must be $> 2\pi\sqrt{R/g}$ where R is radius of earth

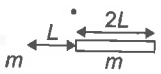
JEE Advanced

Level I

1. Four masses (each of m) are placed at the vertices of a regular pyramid (triangular base) of side 'a'. Find the work done by the system while taking them apart so that they form the pyramid of side '$2a$'.



2. A small mass and a thin uniform rod each of mass 'm' are positioned along the same straight line as shown. Find the force of gravitational attraction exerted by the rod on the small mass.



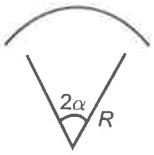
3. An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate :
(i) the initial speed of projection
(ii) the speed at half the maximum height.

4. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time _____.

5. A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5$ J. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2 \times 10^5$ J is equal to _____.

30

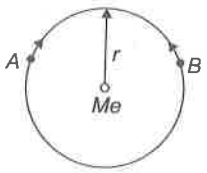
6. Find the gravitational field strength and potential at the centre of arc of linear mass density λ subtending an angle 2α at the centre.



7. A point P lies on the axis of a fixed ring of mass M and radius a, at a distance a from its centre C. A small particle starts from P and reaches C under gravitational attraction only. Its speed a C will be _____.

8. Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

9. Consider two satellites A and B of equal mass m, moving in the same circular orbit of radius r around the earth E but in opposite sense of rotation and therefore on a collision course (see figure).



(a) In terms of G, M_e, m and r find the total mechanical energy $E_A + E_B$ of the two satellite plus earth system before collision.

(b) If the collision is completely inelastic so that wreckage remains as one piece of tangled material (mass = 2 m), find the total mechanical energy immediately after collision.

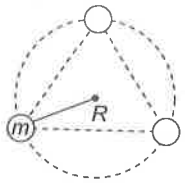
(c) Describe the subsequent motion of the wreckage.

10. A particle is fired vertically from the surface of the earth with a velocity kv_e, where v_e is the escape velocity and $k < 1$. Neglecting air resistance and assuming earth's radius as R_e. Calculate the height to which it will rise from the surface of the earth.

11. A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/s. Then the time taken for the satellite to reach the earth is _____.

12. Find the potential energy of a system of eight particles placed at the vertices of a cube of side L. Neglect the self-energy of the particles.

13. A hypothetical planet of mass M has three moons each of equal mass 'm' each revolving in the same circular orbit of radius R. The masses are equally spaced and thus form an equilateral triangle. Find

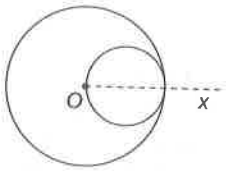


(i) the total P.E. of the system

(ii) the orbital speed of each moon such that they maintain this configuration.

14. Two small dense stars rotate about their common centre of mass as a binary system with the period 1 year for each. One star is of double the mass of the other and the mass of the lighter one is 1/3 of the mass of the sun. Find the distance between the stars if distance between the earth and the sun is R.

15. A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius $r = R/2$ whose centre is at $x = R/2$ as in fig. (a) Find gravitational field at points on the axis for $x > R$



(ii) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.

16. A body moving radially away from a planet of mass M, when at distance r from planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be

(a) then velocity in circular orbits.

(b) maximum distance between the two fragments before collision and

(c) magnitude of their relative velocity just before they collide.

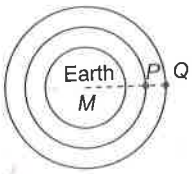
17. A thin spherical shell of total mass M and radius R is held fixed. There is a small hole in the shell. A mass m is released from rest a distance R from the hole along

a line that passes through the hole and also through the centre of the shell. This mass subsequently moves under the gravitational force of the shell. How long does the mass take to travel from the hole to the point diametrically opposite.

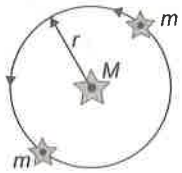
18. A remote sensing satellite is revolving in an orbit of radius x the equator of earth. Find the area on earth surface in which satellite cannot send message.

Level II

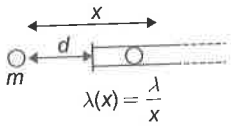
1. A satellite P is revolving around the earth at a height h = radius of earth (R) above equator. Another satellite Q is at a height $2h$ revolving in opposite direction. At an instant the two are at same vertical line passing through centre of sphere. Find the least time of after which again they are in this is situation.



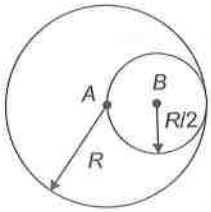
2. A certain triple-star system consists of two stars, each of mass m, revolving about a central star, mass M, in the same circular orbit. The two stars stay at opposite ends of a diameter of the circular orbit, see figure. Derive an expression for the period of revolution of the stars; the radius of the orbit is r.



3. Find the gravitational force of interaction between the mass m and an infinite rod of varying mass density λ such that $\lambda(x) = \lambda/x$, where x is the distance from mass m. Given that mass m is placed at a distance d from the end of the rod on its axis as shown in figure.



4. Inside an isolated fixed sphere of radius R and uniform density r, there is a spherical cavity of radius $R/2$ such that the surface of the cavity passes through the centre of the sphere as in figure. A particle of mass m is released from rest at centre B of the cavity. Calculate velocity with which particle strikes the centre A of the sphere.



5. A ring of radius R is made from a thin wire of radius r. If ρ is the density of the material of wire then what will be the gravitational force exerted by the ring on the material particle of mass m placed on the axis of ring at a distance x from its centre. Show that the force will be maximum when $x = R/\sqrt{2}$ and the maximum value of force will be given as $F_{max} = \dfrac{4\pi^2 Gr^2 \rho m}{(3)^{3/2} R}$

6. A man can jump over $b = 4$ m wide trench on earth. If mean density of an imaginary planet is twice that of the earth, calculate its maximum possible radius so that he may escape from it by jumping. Given radius of earth = 6400 km.

7. A launching pad with a spaceship is moving along a circular orbit of the moon, whose radius R is triple that of moon Rm. The ship leaves the launching pad with a relative velocity equal to the launching pad's initial orbital velocity $\vec{v}_0$ and the launching pad then falls to the moon. Determine the angle θ with the horizontal at which the launching pad crashes into the surface if its mass is twice that of the spaceship m.

8. A satellite of mass m is in an elliptical orbit around the earth of mass $M(M >> m)$. The speed of the satellite at its nearest point to the earth (perigee) is $\sqrt{\dfrac{6GM}{5R}}$ where R = its closest distance to the earth. It is desired to transfer this satellite into a circular orbit around the earth of radius equal its largest distance from the earth. Find the increase in its speed to be imparted at the apogee (farthest point on the elliptical orbit).

9. A body is launched from the earth's surface a an angle $\alpha = 30°$ to the horizontal at a speed $v_0 = \sqrt{\frac{1.5GM}{R}}$. Neglecting air resistance and earth's rotation, find (a) the height to which the body will rise. (ii) The radius of curvature of trajectory at its top point.

10. Assume that a tunnel is dug across the ear (radius = R) passing through its centre. Find the tim a particle takes to reach centre of earth if it is project into the tunnel from surface of earth with speed need for it to escape the gravitational field of earth.

Previous Year Questions

JEE Main

1. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will **[AIEEE 2002]**

(A) continue to move in its orbit with same velocity

(B) move tangentially to the original orbit with the same velocity

(C) become stationary in its orbit

(D) move towards the earth

2. The escape velocity of a body depends upon mass as **[AIEEE 2002]**

(A) m^0 (B) m^1

(C) m^2 (D) m^3

3. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is **[AIEEE 2002]**

(A) $\frac{mgR}{2}$ (B) $2mgR$

(C) mgR (D) $\frac{mgR}{4}$

4. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is **[AIEEE 2002]**

(A) $\frac{GMm}{12R^2}$ (B) $\frac{GMm}{3R^2}$

(C) $\frac{GMm}{8R}$ (D) $\frac{GMm}{6R}$

5. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be **[AIEEE 2003]**

(A) $11\sqrt{2}$km/s (B) 22 km/s

(C) 11 km/s (D) $\frac{11}{\sqrt{2}}$ m/s

6. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only then the distance covered by the smaller body just before collision is **[AIEEE 2003]**

(A) 2.5 R (B) 4.5 R

(C) 7.5 R (D) 1.5 R

7. The time period of a satellite of earth is $5h$. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become **[AIEEE 2003]**

(A) $10h$ (B) $80h$

(C) $40h$ (D) $20h$

8. Suppose the gravitational forces varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to **[AIEEE 2004]**

(A) $R\left(\frac{n+1}{2}\right)$ (B) $R\left(\frac{n-1}{2}\right)$

(C) R^n (D) $R\left(\frac{n-2}{2}\right)$

9. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is **[AIEEE 2004]**

(A) $2\,mgR$ (B) $\frac{1}{2}mgR$

(C) $\frac{1}{4}mgR$ (D) mgR

10. The time period of an earth satellite in circular orbit is independent of **[AIEEE 2004]**

(A) the mass of the satellite

(B) radius of its orbit

(C) both the mass and radius of the orbit

(D) neither the mass of the satellite nor the radius of its orbit

11. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[AIEEE 2004]**

(A) gx (B) $\frac{gR}{R-x}$

(C) $\frac{gR^2}{R+x}$ (D) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

12. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere, (you may take $G = 6.67 \times 10^{-11}$ Nm^2/kg^{-2}) **[AIEEE 2005]**

(A) 13.34×10^{-10} J (B) 3.33×10^{-10} J

(C) 6.67×10^{-9}J (D) 6.67×10^{-10} J

13. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct? **[AIEEE 2005]**

(A) $d = \frac{h}{2}$ (B) $d = \frac{3h}{2}$

(C) $d = 2h$ (D) $d = h$

14. Average density of the earth **[AIEEE 2005]**

(A) does not depend on g

(B) is a complex function of g

(C) is directly proportional to g

(D) is inversely proportional to g

15. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be **[AIEEE 2007]**

(A) 1 (B) zero

(C) $\frac{g_E}{g_M}$ (D) $\frac{g_M}{g_E}$

Directions Question number 6 is Assertion-Reason type question. This question contains two statements:

Statement I (Assertion) and Statement II (Reason). The question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(A) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I.

(B) Statement I is true; Statement II is false.

(C) Statement I is false; Statement II is true.

(D) Statement I is true; Statement II is true; Statement II is a correct explanation for Statement I.

16. **Statement I:** For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is 4π GM.

and

Statement II: If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface. **[AIEEE 2008]**

17. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1}, the escape velocity from the surface of the planet would be **[AIEEE 2008]**

(A) 1.1 kms^{-1} (B) 11 kms^{-1}

(C) 110 kms^{-1} (D) 0.11 kms^{-1}

18. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is **[AIEEE 2009]**

(A) $2R$ (B) $\frac{R}{\sqrt{3}}$

(C) $\frac{R}{2}$ (D) $\sqrt{2}R$

19. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero, is **[AIEEE 2011]**

(A) $-\frac{4Gm}{r}$ (B) $-\frac{6Gm}{r}$

(C) $-\frac{9Gm}{r}$ (D) zero

20. Two particles of equal mass m go around a circle of radius R under action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **[AIEEE 2011]**

(A) $\sqrt{\frac{Gm}{R}}$ (B) $\sqrt{\frac{Gm}{4R}}$

(C) $\sqrt{\frac{Gm}{3R}}$ (D) $\sqrt{\frac{Gm}{2R}}$

21. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 Km respectively. The required energy for this work will be **[AIEEE 2012]**

(A) 6.4×10^{11} J (B) 6.4×10^{8} J

(C) 6.4×10^{9}J (D) 6.4×10^{10}J

22. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? **[JEE Main 2013]**

(A) $\frac{GmM}{2R}$ (B) $\frac{GmM}{3R}$

(C) $\frac{5GmM}{6R}$ (D) $\frac{2GmM}{3R}$

23. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is: **[JEE Main 2014]**

(A) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

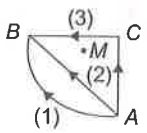
(B) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

(C) $\sqrt{\frac{GM}{R}}$

(D) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

JEE Advanced

1. In a region of only gravitational field of mass M a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then **[JEE (Scr.) 2003]**



(A) $W_1 = W_2 = W_3$ (B) $W_1 > W_2 > W_3$

(C) $W_1 = W_2 > W_3$ (D) $W_1 < W_2 < W_3$

2. A body is projected vertically upwards from the bottom of a crater of moon of depth $R/100$ where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. **[JEE 2003]**

3. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then **[JEE 2006]**

(A) $T_A > T_B$ (if $r_A > r_B$)

(B) $T_A > T_B$ (if $m_A > m_B$)

(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

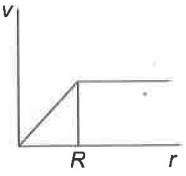
(D) $T_A = T_B$

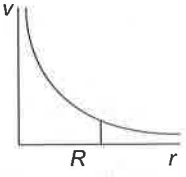
4. A spherically symmetric gravitational system of particles has a mass density **[JEE 2008]**

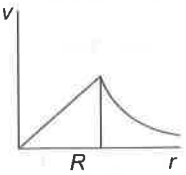
$$\rho = \begin{cases} \rho_0 \text{ for } r \le R \\ 0 \text{ for } r > R \end{cases}$$

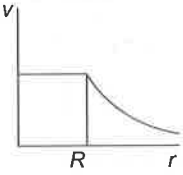
where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational

field of particles. Its speed V as a function of distance r $(0 < r < \infty)$ from the centre of the system is represented by

(A) 

(B) 

(C) 

(D) 

5. Statement 1
An astronaut in an orbiting space station above the Earth experiences weightlessness. **[JEE 2008]**
and
Statement 2
An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'.

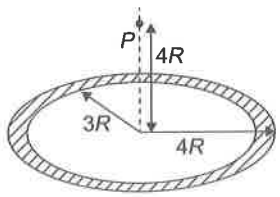
(A) Statement 1 is True, Statement 2 is True; Statement 2 is a correct explanation for Statement 1

(B) Statement 1 is True, Statement 2 is True; Statement 2 is NOT a correct explanation for Statement 1

(C) Statement 1 is True, Statement 2 is False

(D) Statement 1 is False, Statement 2 is True

6. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is **[JEE 2010]**



(A) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$

(B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$

(C) $\frac{GM}{4R}$

(D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

7. A binary star consists of two stars A (mass 2.2 M_s) and B (mass 11 M_s), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is. **[JEE 2010]**

8. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$ where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **[JEE 2010]**

9. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **[JEE 2011]**

(A) $\frac{1}{2}mV^2$ (B) mV^2

(C) $\frac{3}{2}mV^2$ (D) $2mV^2$

10. Two spherical planets P and Q have the same uniform density ρ, masses M_p and M_Q, and surface areas A and $4A$, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then **[JEE 2012]**

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$

(C) $V_R / V_P = 3$ (D) $V_P / V_Q = \frac{1}{2}$

11. Two bodies, each of mass M, are kept fixed with a separation $2L$. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) **[JEE 2013]**

36

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$.

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$.

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$.

(D) The energy of the mass m remains constant.

12. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius oif Earth $= 6 \times 10^6$ m and the acceleration due to gravity on Earth is 10 ms^{-2}) **[JEE Main 2014]**

(A) 96 N (B) 108 N

(C) 120 N (D) 150 N

ANSWER KEYS

Exercises

JEE Main

1. C	**2.** D	**3.** B	**4.** B	**5.** C	**6.** B	**7.** C	**8.** A	**9.** B	**10.** B
11. D	**12.** A	**13.** C	**14.** A	**15.** B	**16.** B	**17.** B	**18.** C	**19.** A	**20.** C
21. D	**22.** D	**23.** C	**24.** B	**25.** C	**26.** B	**27.** D	**28.** C	**29.** B	**30.** A
31. C	**32.** B	**33.** C	**34.** A	**35.** A	**36.** B	**37.** C	**38.** C	**39.** B	

JEE Advanced

1. C	**2.** A	**3.** A	**4.** B	**5.** B	**6.** B	**7.** A	**8.** A	**9.** D	**10.** B
11. B	**12.** C	**13.** A	**14.** C	**15.** A, D	**16.** A, D	**17.** B, C, D	**18.** A, C	**19.** A, C	**20.** B, D
21. A	**22.** A	**23.** A, B, C	**24.** A, D						

JEE Advanced

Level I

1. $-\frac{3Gm^2}{a}$ **2.** $\frac{Gm^2}{3L^2}$ **3.** (i) $\frac{4}{3}\sqrt{\frac{Gm}{R}}$, (ii) $\frac{2}{3}\sqrt{\frac{2Gm}{5R}}$

4. 1.6 hours if is rotating from west to east, 24/17 hours if it is rotating from west to east.

5. 1×10^5 J **6.** $\frac{2G\lambda}{R}(\sin\alpha), (-G\lambda\, 2\alpha)$ **7.** $\sqrt{\frac{2GM}{a}\left(1-\frac{1}{\sqrt{2}}\right)}$ **8.** $h = \frac{\sqrt{5}-1}{2}R$

9. (a) $-GmM_e/r$, (b) $-2GmM_e/r$ **10.** $\frac{R_e k^2}{1-k^2}$ **11.** $t = \frac{GMm}{2C}\left(\frac{1}{R_e}-\frac{1}{r}\right)$ **12.** $\frac{-4GM^2}{L}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]$

13. (i) $-\frac{3Gm}{R}\left(\frac{m}{\sqrt{3}}+M\right)$, (ii) $\sqrt{\frac{G}{R}\left(\frac{m}{\sqrt{3}}+M\right)}$ **14.** R **15.** $\vec{g}=+\frac{\pi G\rho_0 R^3}{6}\left[\frac{1}{\left(x-\frac{R}{2}\right)^2}-\frac{8}{x^2}\right]\hat{i}, \vec{g}=-\frac{2\pi G\rho_0 R}{3}\hat{i}$

16. (a) $\sqrt{\frac{Gm}{r}}$; (b) $r\sqrt{2}$; (c) $\sqrt{\frac{2GM}{r}}$ **17.** $2\times\sqrt{R^3/GM}$ **18.** $\left(1-\frac{\sqrt{x^2-R^2}}{x}\right)4\pi R^2$

Level II

1. $\frac{2\pi R^{3/2}\left(6\sqrt{6}\right)}{\sqrt{GM}\left(2\sqrt{2}+3\sqrt{3}\right)}$ **2.** $\frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$ **3.** $\frac{Gm\lambda}{2d^2}$ **4.** $\sqrt{\frac{2}{3}\pi G\rho R^2}$ **6.** $\sqrt{6.4}$ km

7. $\cos\theta=\frac{3}{\sqrt{10}}$ **8.** $\sqrt{\frac{GM}{R}}\left[\sqrt{\frac{2}{3}}-\sqrt{\frac{8}{15}}\right]$ **9. (a)** $h=\left(\frac{\sqrt{7}}{2}+1\right)R$, **(b)** $1.13\,R$ **10.** $T=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{g}}$

Previous Year Questions

JEE Main

1. B	**2.** A	**3.** C	**4.** D	**5.** C	**6.** C	**7.** C	**8.** A	**9.** B	**10.** A
11. D	**12.** D	**13.** C	**14.** C	**15.** A	**16.** A	**17.** C	**18.** A	**19.** C	**20.** B
21. D	**22.** C	**23.** B							

JEE Advanced

1. A	**2.** $h=99R$	**3.** D	**4.** C	**5.** A	**6.** A	**7.** 6	**8.** 3	**9.** B	**10.** B, D
11. B	**12.** B								

Chapter 2

Current Electricity

INTRODUCTION

Conductor

In some materials, the outer electrons of each atoms or molecules are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.

Insulator

Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

Semiconductor

In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller then that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

ELECTRIC CURRENT AND CURRENT DENSITY

When there is a transfer of charge from one side of an area to the other, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion, if they are negative, the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt, we define the average electric current through the area during this time as

$$i = \frac{\Delta Q}{\Delta t}$$

The current at time t is

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Thus, electric current through an area is the rate of transfer of charge from one side of the area to the other. The SI unit of current is ampere. If one coulomb of charge crosses an area in one second, the current is one ampere. It is one of the seven base units accepted in SI.

SOLVED EXAMPLES

EXAMPLE 1

If $q = 2t^2 + 3$, find current at $t = 2$ sec?

SOLUTION

$$i = \frac{dq}{dt}$$

$$i = 4t$$

$$\therefore \quad i \text{ at } 2 \text{ sec} = 4 \times 2 = 8 \text{ A}$$ ■

We shall now define a vector quantity known as electric current density at a point. To define the current density at a point P, we draw a small area ΔS through P perpendicular

to the flow of charges (shown in figure). If Δi be the current through the area ΔS, the average current density is

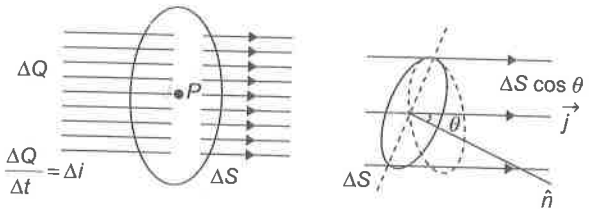
$$\vec{j} = \frac{\Delta i}{\Delta \vec{S}}$$

The current density at the point P is

$$j = \lim_{\Delta S \to 0} \frac{\Delta i}{\Delta S} = \frac{di}{dS}$$

The direction of the current density is the same as the direction of the current. Thus, it is along the motion of the moving charges, if the charges are positive and opposite to the motion of the charges, if the charges are negative. If a current i is uniformly distributed over an area S and is perpendicular to it,

$$j = \frac{i}{S}$$



Now let us consider an area ΔS which is not necessarily perpendicular to the current (figure shown). If the normal to the area makes an angle θ with the direction of the current, the current density is,

$$j = \frac{\Delta i}{\Delta S \cos\theta}$$

or, $$\Delta i = j\, \Delta S \cos\theta$$

where Δi is the current through ΔS. If $\Delta \vec{S}$ be the area vector corresponding to the area ΔS, we have

$$\Delta i = \vec{j} \cdot \Delta \vec{S}$$

For a finite area,

$$i = \int \vec{j} \cdot d\vec{S}$$

Note carefully that an electric current has direction as well as magnitude but it is not a vector quantity. It does not add like vectors. Therefore current is neither a vector quantity nor a scalar quantity but a tensor quantity. The current density is a vector quantity.

EXAMPLE 2

An electron beam has an aperture 1.0 mm^2. A total of 6.0×10^{10} electrons go through any perpendicular cross-section per second. Find (a) the current and (b) the current density in the beam.

SOLUTION

The total charge crossing a perpendicular cross-section in one second is

$$q = ne$$
$$= 6.0 \times 10^{16} \times 1.6 \times 10^{-19}\ \text{C}$$
$$= 9.6 \times 10^{-3}\ \text{C}$$

(a) The current is

$$i = \frac{q}{t} = \frac{9.6 \times 10^{-3}\,\text{C}}{1\,s}$$

As the charge is negative, the current is opposite to be direction of motion of the beam.

(b) The current density is

$$j = \frac{i}{S} = \frac{9.6 \times 10^{-3}\,\text{A}}{(1.0\ \text{mm})^2}$$
$$= \frac{9.6 \times 10^{-3}\,\text{A}}{1.0 \times 10^{-6}\ \text{m}^2} = 9.6 \times 10^3\ \text{A/m}^2$$

DRIFT SPEED

A conductor contains a large number of loosely bound electrons which we call free electrons or conduction electrons. The remaining material is a collection of relatively heavy positive ions which we call lattice. These ions keep on vibrating about their mean positions. The average amplitude depends on the temperature. Occasionally, a free electron collides or interacts in some other fashion with the lattice. The speed and direction of the electron changes randomly at each such event. As a result, the electron moves in a zig-zag path. As there is a large number of free electrons moving in random directions, the number of electrons crossing an area ΔS from one side very nearly equals the number crossing from the other side in any given time interval. The electric current through the area is, therefore, zero.

When there is an electric field inside the conductor, a force acts on each electron in the direction opposite to the field. The electrons get biased in their random motion in favour of the force. As a result, the electrons drift slowly in this direction. At each collision, the electron starts afresh

in a random direction with a random speed but gains an additional velocity v' due to the electric field. This velocity v' increases with time and suddenly becomes zero as the electron makes a collision with the lattice and starts afresh with a random velocity.

To derive an expression for drift velocity in terms of atomic constants. Consider a free electron in the lattice to which an electric field $\vec{E}$ is applied. This applied field will exert a force $\vec{F} = (-e)\vec{E}$ on the electron and so if m is the mass of electron, its acceleration

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m} \quad i.e., \quad \frac{d\vec{v}}{dt} = \frac{-e\vec{E}}{m} \left[\text{as } \vec{a} = \frac{d\vec{v}}{dt}\right]$$

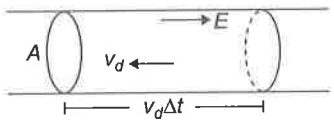
If $\vec{v}_{rms}$ is the thermal velocity of electron and t the time between two successive collisions,

$$\int_{\vec{v}_{rms}}^{\vec{v}} d\vec{v} = \frac{-e\vec{E}}{m} \int_0^t dt, \; i.e., \; \vec{v} = \vec{v}_{rms} - \frac{e\vec{E}}{m} t$$

Averaging $\vec{v}$ over large number of electrons as $(\vec{v}_{rms})_{av} = 0$ and $t_{av} = \tau =$ relaxation time.

The average drift velocity

$$\vec{v}_d = -\left[\frac{e\tau}{m}\right]\vec{E}$$



Let us now find the relation between the current density and the drift speed. Consider a cylindrical conductor of cross-sectional area A in which an electric field E exists. Consider a length $v_d \Delta t$ of the conductor (figure shown). The volume of this portion is $Av_d\Delta t$. If there are n free electrons per unit volume of the wire, the number of free electrons in this portion is $nAv_d\Delta t$. All these electrons cross the area A in time Δt. Thus, the charge crossing this area in time Δt is

$$\Delta Q = nAv_d \Delta te$$

or,

$$i = \frac{\Delta Q}{\Delta t} = nAv_d e \quad \text{and} \quad j = \frac{i}{A} = nev_d \qquad (2)$$

SOLVED EXAMPLES

EXAMPLE 3

Calculate the drift speed of the electrons when 1 A of current exists in a copper wire of cross-section 2 mm^2. The number of free electrons in 1 cm^3 of copper is 8.5×10^{22}.

SOLUTION

We have

$$j = nev_d$$

$$\text{or, } v_d = \frac{j}{ne} = \frac{i}{Ane}$$

$$= \frac{1\,\text{A}}{(2 \times 10^{-6}\,\text{m}^2)(8.5 \times 10^{22} \times 10^6\,\text{m}^{-3})(1.6 \times 10^{-19}\,\text{C})}$$

$$= 0.036\,\text{mm/s}$$

We see that the drift speed is indeed small. ■

OHM'S LAW

Using equations (1) and (2)

$$j = nev_d = \frac{ne^2\tau}{2m} E$$

$$\text{or,} \qquad j = \sigma E \qquad (3)$$

$$\text{where} \qquad \sigma = \frac{ne^2\tau}{2m}$$

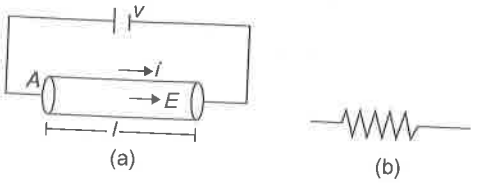
where σ depends only on material of the conductor and its temperature. This constant is called the electrical conductivity of the material. Equation (3) is known as Ohm's law.

The resistivity of a material is defined as

$$\rho = \frac{1}{\sigma} = \frac{2m}{ne^2\tau}. \qquad (4)$$

Ohm's law tells us that the conductivity (or resistivity) of a material is independent of the electric field existing in the material. This is valid for conductors over a wide range of field.

Suppose we have a conductor of length l and uniform cross-sectional area A (figure shown). Let us apply a potential difference V between the ends of the conductor. The electric field inside the conductor is $E = \frac{V}{l}$. If the current in the conductor is i, the current density is $j = \frac{i}{A}$. Ohm's law $j = \sigma E$ then becomes



$$\frac{i}{A} = \sigma \frac{V}{l}$$

or, $$V = \frac{1}{\sigma}\frac{l}{A} i = \rho \frac{l}{A} i \qquad (5)$$

or, $$V = Ri \qquad (6)$$

R is called the resistance of the given conductor. The quantity $\frac{1}{R}$ is called conductance.

Equation (5) is another form of Ohm's law which is widely used in circuit analysis. The unit of resistance is called ohm and is denoted by symbol Ω. An object of conducting material, having a resistance of desired value, is called a resistor.

From equation (5) and (6)

$$R = \frac{\rho \ell}{A}. \qquad (7)$$

From equation (7), the unit of resistivity ρ is ohm-metre, also written as Ω-m. The unit of conductivity (σ) is (ohm-m) written as mho/m.

From equation (4) and (7)

$$R = \frac{2m\ell}{ne^2\tau A} = \frac{\rho \ell}{A}$$

where ρ = resistivity $\left(\text{where } \rho = \frac{2m}{ne^2\tau} \right)$

ℓ = length along the direction of current

A = Area of the cross section perpendicular to direction of current

n = number of free charges per unit volume.

τ = relaxation time

m = mass of electron

SOLVED EXAMPLES

EXAMPLE 4

Calculate the resistance of an aluminium wire of length 50 cm and cross-sectional area 2.0 mm^2. The resistivity of aluminium is $\rho = 2.6 \times 10^{-8}$ Ω-m.

SOLUTION

The resistance is

$$R = \rho \frac{\ell}{A}$$

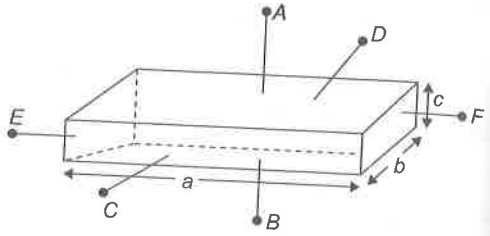
$$= \frac{(2.6 \times 10^{-8}\ \Omega\text{-m}) \times (0.50\ \text{m})}{2 \times 10^{-6}\ \text{m}^2}$$

$$= 0.0065\ \Omega.$$

We arrived at Ohm's law by making several assumptions about the existence and behaviour of the free electrons. These assumption are not valid for semiconductors, insulators, solutions etc. Ohm's law cannot be applied in such cases.

EXAMPLE 5

The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.



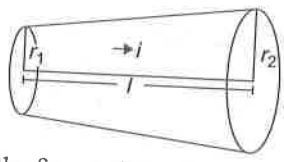
SOLUTION

$$R_{AB} = \frac{\rho c}{ab}, \quad R_{CD} = \frac{\rho b}{ac}, \quad R_{EF} = \frac{\rho a}{bc}$$

EXAMPLE 6

A portion of length L is cut out of a conical solid wire. The two ends of this portion have circular cross-sections of radii r_1 and $r_2 (r_2 > r_1)$. It is connected lengthwise to a circuit and a current i is flowing in it. The resistivity of the material of the wire is ρ. Calculate the resistance of the considered portion and the voltage developed across it.

SOLUTION



If follows from the figure, that

$$\tan\theta = \frac{r_2 - r_1}{L}$$

$$\therefore \quad r = r_1 + x \tan\theta = r_1 + x$$

$$= \left(\frac{r_2 - r_1}{L} \right) = \frac{r_1 L + x(r_2 - r_1)}{L}$$

$$\therefore \quad A = \pi r^2 = \frac{\pi}{L^2}[r_1 L + (r_2 - r_1)x]^2$$

$$dR = \frac{\rho dx}{\pi r^2} = \frac{\rho dx L^2}{\pi[r_1 L + (r_2 - r_1)x]^2}$$

temperature. As $\rho \propto R$ and $\sigma \propto \left(\frac{1}{\rho}\right)$, hence resistivity increases and conductivity decreases with rise in temperature of the metallic of the metallic wires.

(c) If R_0 and R_t represent the resistances of metallic wire at 0°C and t°C respectively then R_t is given by the following formula:

$$R_t = R_0(1 + \alpha t)$$

where α is called as the **Temperature coefficient of resistance** of the material of the wire.

α depends on material and temperature but generally it is taken as a constant for a particular material for small change.

$$R_t - R_0 = R_0 \alpha t$$

for very small change in temperature $dR = R_0\, \alpha\, dt$

Resistance of semiconductors

(a) There are certain substances whose conductivity lies in between that of insulators and conductors, higher than that of insulators but lower than that of conductors. These are called as semiconductors, e.g., silicon, germanium, carbon etc.

(b) The resistivity of semiconductors decreases with increase in temperature i.e., **α for semiconductors is –ve and high.**

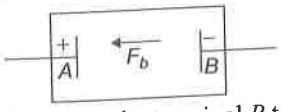
(c) Though at ordinary temperature the value of n (no. of free electrons per unit volume) for these materials is very small as compared to metals, but increases very rapidly with rise in temperature (this happens due to breaking of covalent bonds). Though τ decreases but factor of n dominates. Therefore, the resistance

$$R = \frac{ml}{ne^2\tau A}$$

goes on decreasing with increase in temperature.

BATTERY AND EMF

A battery is a device which maintains a potential difference between its two terminals A and B. Figure shows a schematic diagram of a battery. Some internal mechanism exerts forces on the charges of the battery material. This force drives the positive charges of the battery material towards A and the negative charges of the battery material towards B. We show the force on a positive charge q as $\vec{F}_b$. As positive charge accumulates on A and negative charge on B, a potential difference develops and grows between A and B. An electric field $\vec{E}$ is developed in the battery material from A to B and exerts a force $\vec{F}_e = q\vec{E}$ on a charge q. The direction of this force is opposite to that of $\vec{F}_b$. In steady state, the charge accumulation on A and B is such that $F_b = F_e$. No further accumulation takes place.



If a charge q is taken from the terminal B to the terminal A, the work done by the battery force F_b is $W = F_b\, d$ where d is the distance between A and B. The work done by the battery force per unit charge is

$$E = \frac{W}{q} = \frac{F_b d}{q}$$

This quantity is called the emf of the battery. The full form of emf is electromotive force. The name is misleading in the sense that emf is not a force, it is work done/charge. We shall continue to denote this quantity by the short name emf. If nothing is connected externally between A and B,

$$F_b = F_e = qE$$

or,
$$F_b d = qEd = qV$$

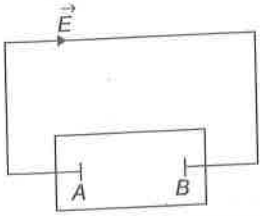
where $V = Ed$ is the potential difference between the terminals. Thus,

$$E = \frac{F_b d}{q} = V$$

Thus, the emf of a battery equals the potential difference between its terminals when the terminals are not connected externally.

Potential difference and emf are two different quantities whose magnitudes may be equal in certain conditions. The emf is the work done per unit charge by the battery force F_b which is non-electrostatic in nature. The potential difference originates from the electrostatic field created by the charges accumulated on the terminals of the battery.

A battery is often prepared by putting two rods or plates of different metals in a chemical solution. Such a battery, using chemical reactions to generate emf, is often called a cell.



Now suppose the terminals of a battery are connected by a conducting wire as shown in above figure. As the terminal A is at a higher potential than B, there is an electric field in the wire in the direction shown in the figure. The free electrons

in the wire move in the opposite direction and enter the battery at the terminal A. Some electrons are withdrawn from the terminal B which enter the wire through the right end. Thus, the potential difference between A and B tends to decrease. If this potential difference decreases, the electrostatic force F_e inside the battery also decreases. The force F_b due to the battery mechanism remains the same. Thus, there is a net force on the positive charges of the battery material from B to A. The positive charges rush towards A and neutralise the effect of the electrons coming at A from the wire. Similarly, the negative charges rush towards B. Thus, the potential difference between A and B is maintained.

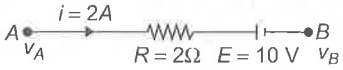
For calculation of current, motion of a positive charge in one direction is equivalent to the motion of a negative charge in opposite direction. Using this fact, we can describe the above situation by a simpler model. The positive terminal of the battery supplies positive charges to the wire. These charges are pushed through the wire by the electric field and they reach the negative terminal of the battery. The battery mechanism drives these charges back to the positive terminal against the electric field existing in the battery and the process continues. This maintains a steady current in the circuit.

Current can also be driven into a battery in the reverse direction. In such a case, positive charge enters the battery at the positive terminal, moves inside the battery to the negative terminal and leaves the battery from the negative terminal. Such a process is called charging of the battery. The more common process in which the positive charge comes out of the battery from the positive terminal is called discharging of the battery.

SOLVED EXAMPLES

EXAMPLE 8

Find $v_A - v_B$



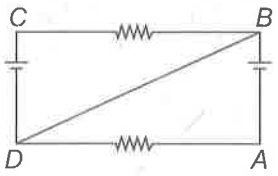
SOLUTION

$$v_A - iR - E = v_B$$

$$v_A - v_B = iR + E = 4 + 10 = 14 \text{ V}$$ ■

EXAMPLE 9

Shown in the figure. Find out the current in the wire BD

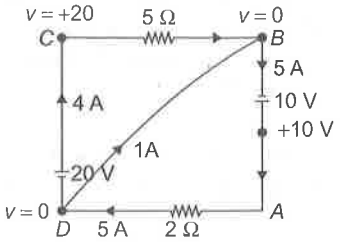


SOLUTION

Let at point D potential = 0 and write the potential of other points then current in wire $AD = \frac{10}{2} = 5$ A from A to D

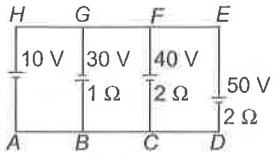
current in wire $CB = \frac{20}{5} = 4$ A from C to B

$\therefore$ current in wire $BD = 1$ A from D to B



EXAMPLE 10

Find the current in each wire



SOLUTION

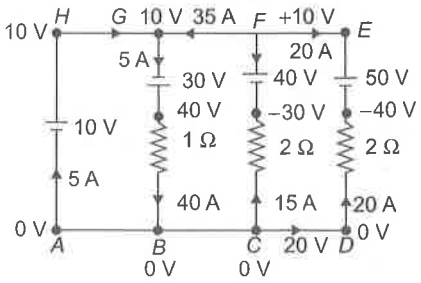
Let potential at point A is 0 volt then potential of other points is shown in figure.

Current in $BG = \frac{40 - 0}{1} = 40$ A from G to B

Current in $FC = \frac{0 - (-30)}{2} = 15$A from C to K

Current in $DE = \frac{0 - (-40)}{2} = 20$ A from D to E

Current in wire $AH = 40 - 35 = 5$ A from A to H

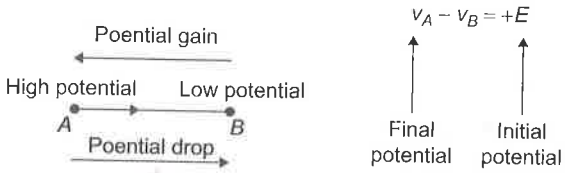


KIRCHHOFF'S LAWS FOR CIRCUIT ANALYSIS

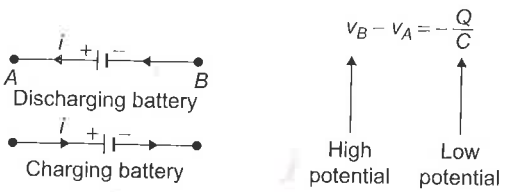
Before moving on to the statement of Kirchhoff's law, we state some conventions to be followed in circuit analysis:

1. Direction of conventional current is from high potential to low potential terminal.
2. Current flows from high potential node A to low potential node B. If we traverse from point A to B, there is drop of potential; similarly from B to A, there is gain of potential.

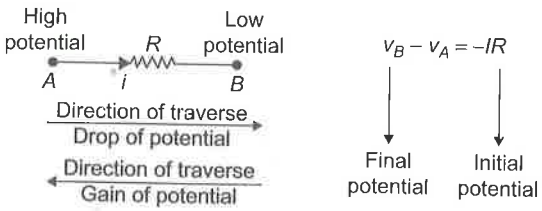
If a source of emf is traversed from negative to positive terminal, the change in potential is $+E$.



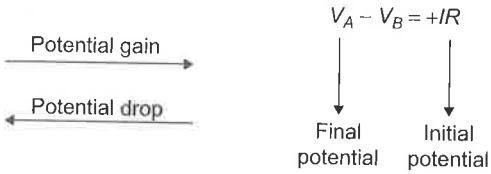
While discharging, current is drawn from the battery, the current comes out from positive terminal and enters negative terminal, while charging of battery current is forced from positive terminal of the battery to negative terminal. Irrespective of direction of current through a battery the sign convention mentioned above holds.



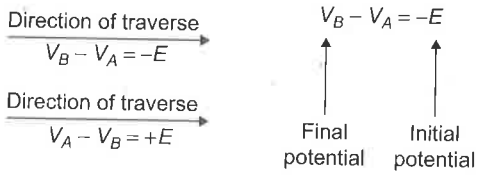
If we traverse a resistor in the direction of current, the change in potential is $-IR$.



If we traverse a resistor in the direction opposite to the direction of current, the change in potential is $+IR$.

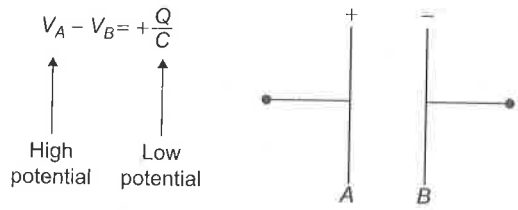


Positive terminal of source of emf is at high potential and negative terminal at low potential. If we traverse a source of emf from the positive terminal to negative terminal, the change in potential is $-E$.



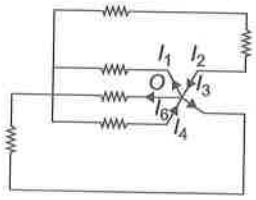
The positive plate of a capacitor is at high potential and negative plate at low potential. If we traverse a capacitor from positive plate to negative plate, the change in potential is $-\frac{Q}{C}$.

If a capacitor is traversed from negative plate to positive plate, the change in potential is $+\frac{Q}{C}$.

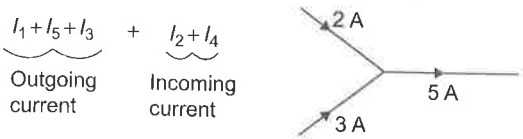


The Kirchhoff's Current Law

The Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering the junction must equal to sum of the currents leaving the junction. From the standard point of physics, KCL is a statement of charge conservation.



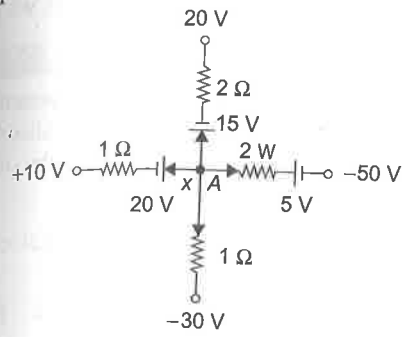
The KCL applied to junction O yields.



SOLVED EXAMPLES

EXAMPLE 11

Find the potential at point A



SOLUTION

Let potential at $A = x$, applying Kirchhoff current law at junction A

$$\frac{x-20-10}{1}+\frac{x-15-20}{2}+\frac{x-5+50}{2}+\frac{x+30}{1}=0$$

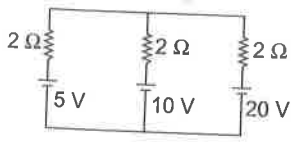
$$\Rightarrow \quad \frac{2x-60+x-35+x+45+2x+60}{2}=0$$

$$\Rightarrow \quad 6x+10=0 \quad \Rightarrow \quad x=-\frac{5}{3}$$

Potential at $A = \frac{-5}{3}$ V. ■

EXAMPLE 12

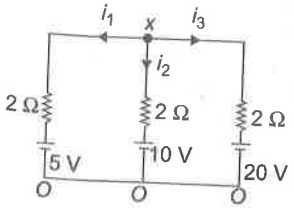
Find the current in every branch?



SOLUTION

Let we assume x potential at the top junction and zero potential at lower junction

As from KCL, net current on a junction is O



$$i_1+i_2+i_3=0$$

$$\frac{x-5}{2}+\frac{x-10}{2}+\frac{x-20}{2}=0$$

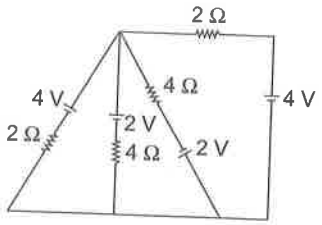
$$3x=35 \quad \Rightarrow \quad x=\frac{35}{3}$$

$$\therefore \quad i_1=\frac{\frac{35}{3}-5}{2}=\frac{10}{3}\text{ A}$$

Similarly, $i_2 = \frac{5}{6}$ A; $i_3 = -\frac{25}{6}$ A. ■

EXAMPLE 13

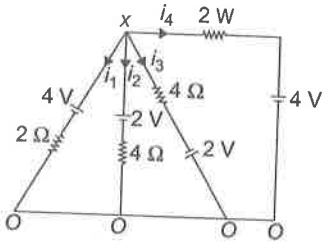
Find the current in every branch?



SOLUTION

Assume x potential at the upper junction and zero potential at the lower junction.

By KCl, we know that net current on a junction is zero.



$$\therefore \quad i_1+i_2+i_3+i_4=0$$

$$\frac{x-4}{2}+\frac{x-2}{4}+\frac{x+2}{4}+\frac{x-4}{2}=0$$

$$2x-8+x-2+x+2+2x-8=0$$

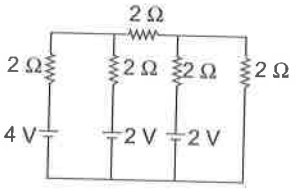
$$6x-16=0 \quad \Rightarrow \quad x=\frac{8}{3}\text{ V}$$

$$\therefore \quad i_1=-\frac{2}{3}\text{ A},\ i_2=+\frac{1}{6}\text{ A},\ i_3=\frac{7}{6}\text{ A},\ i_4=-\frac{2}{3}\text{ A}$$ ■

48

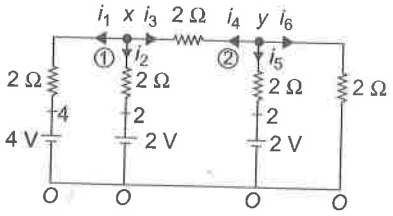
EXAMPLE 14

Find the current in every branch?



SOLUTION

The above question could be solved by assuming potential x and y at the top junctions and zero potential at lower junctions



At the junction 1 applying KCL,

$$i_1 + i_2 + i_3 = 0$$

$$\frac{x-4}{2} + \frac{x-2}{2} + \frac{x-y}{2} = 0$$

$$\Rightarrow \quad 3x - y = 6. \qquad (1)$$

At the junction 2 applying KCL,

$$i_4 + i_5 + i_6 = 0$$

$$\frac{y-x}{2} + \frac{y-2}{2} + \frac{y}{2} = 0$$

$$\Rightarrow \quad 3y - x = 2. \qquad (2)$$

Solving (1) and (2)

$$9x - 3y = 18$$

$$\Rightarrow \quad 3y - x = 2$$

$$\Rightarrow \quad 8x = 20$$

$$x = \frac{5}{2},\ y = \frac{3}{2}$$

Just put the values of x and y and then the evaluate the current in every branch ■

The Kirchhoff's Voltage Law

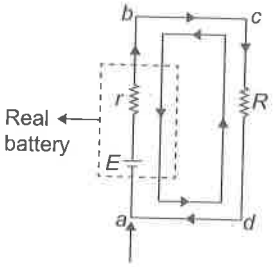
The Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential difference around any closed loop of an electric circuit is zero. **The KVL is a statement of conservation of energy.** The KVL reflects that electric force is conservative, the work done by a conservative force on a charge taken around a closed path is zero.

- We can move clockwise or anticlockwise, it will make no difference because the overall sum of the potential difference is zero.
- We can start from any point on the loop, we just have to finish at the same point.
- An ideal battery is modelled by an independent voltage source of emf E and an internal resistance r as shown in figure A real battery always absorbs power when there is a current through it, thereby offering resistance to flow of current.

Applying KVL around the single loop in anticlockwise direction, starting from point A, we have

$$+IR + Ir - E = 0$$

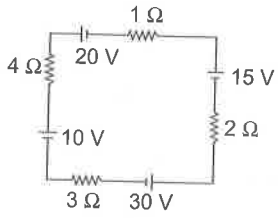
$+IR$	Ir	E	$= 0$
In the opposite direction to current	In the opposite direction to current	From positive to negative terminal	



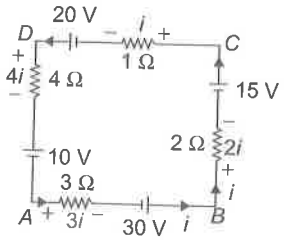
Hence, $$I = \frac{E}{R + r}.$$

EXAMPLE 15

Find current in the circuit



SOLUTION



$\therefore$ all the elements are connected in series

$\therefore$ current in all of them will be same let current $= i$

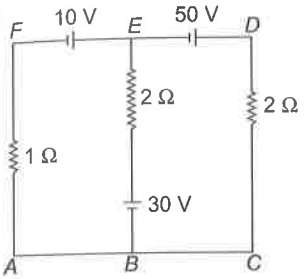
Applying Kirchhoff voltage law in $ABCDA$ loop

$$10 + 4i - 20 + i + 15 + 2i - 30 + 3i = 0$$

$$10i = 25 \quad \Rightarrow \quad i = 2.5 \text{ A}$$ ■

EXAMPLE 16

Find the current in each wire applying only Kirchhoff voltage law



SOLUTION

Applying Kirchhoff voltage law in loop $ABEFA$

$$i_1 + 30 + 2(i_1 + i_2) - 10 = 0$$

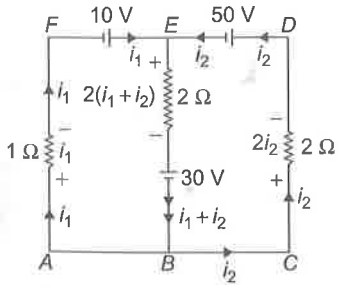
$$3i_1 + 2i_2 + 20 = 0 \qquad (1)$$

Applying Kirchhoff voltage law in $BCDEB$

$$+ 30 + 2(i_1 + i_2) + 50 + 2i_2 = 0$$

$$4i_2 + 2i_1 + 80 = 0$$

$$2i_2 + i_1 + 40 = 0 \qquad (2)$$



Solving (1) and (2)

$$3[-40 - 2i_2] + 2i_2 + 20 = 0$$

$$-120 - 4i_2 + 20 = 0$$

$$i_2 = -25 \text{ A} \quad \text{and} \quad i_1 = 10 \text{ A}$$

$\therefore$ $i_1 + i_2 = -15$ A

current in wire $AF = 10$ A from A to E

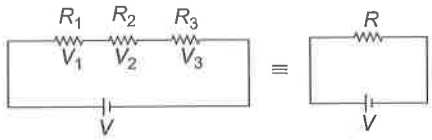
current in wire $EB = 15$ A from B to E

current in wire $DE = 25$ A from D to C ■

COMBINATION OF RESISTANCE

A number of resistance can be connected in a circuit and any complicated combination can be, in general, reduced essentially to two different types, namely series and parallel combinations.

(a) Resistance in Series



(i) In this combination the resistance are joined end to end. The second end of each resistance is joined to first end of the next resistance and so on. A cell is connected between the first end of first resistance and second end of last resistance. Figure shows three resistances R_1, R_2 and R_3 connected in this way. Let V_1, V_2 and V_3 are the potential differences across these resistances.

(ii) In this combination current flowing through each resistance will be same and will be equal to current supplied by the battery.

(iii) As resistances are different and current flowing through them is same, hence potential differences across them will be different. Applied potential difference will be distributed among three resistances directly in their ratio.

As i is constant, hence $V \propto$ R

i.e., $V_1 = iR_1$, $V_2 = iR_2$, $V_3 = iR_3$

(iv) If the potential difference between the points A and D is V, then

$$V = V_1 + V_2 + V_3 = i(R_1 + R_2 + R_3)$$

(v) If the combination of resistances between two points is replaced by a single resistance R such that there is no change in the current of the circuit in the potential difference between those two points, then the single resistance R will be equivalent to combination and $V = iR$ i.e.,

$$iR = i(R_1 + R_2 + R_3) \quad \text{or} \quad \boldsymbol{R = R_1 + R_2 + R_3}$$

(vi) Thus in series combination of resistances, important conclusion are

(a) Equivalent Resistance > highest individual resistance

(b) Current supplied by source = Current in each resistance

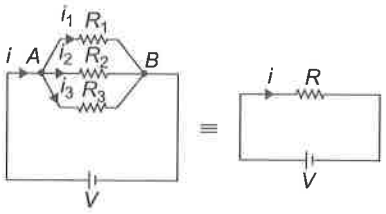
or, $$\frac{V}{R_1 + R_2 + R_3} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3}$$

50

(c) The total potential difference V between points A and B is shared among the three resistances directly in their ratio

$$V_1 : V_2 : V_3 = R_1 : R_2 : R_3$$

(b) Resistance in Parallel



(i) When two or more resistance are combined in such a way that their first ends are connected to one terminal of the battery while other ends are connected to other terminal, then they are said to be connected in parallel. Figure shows three resistances R_1, R_2 and R_3 joined in parallel between two points A and B. Suppose the current flowing from the battery is i. This current gets divided into three parts at the junction A. Let the currents in three resistance R_1, R_2 and R_3 are i_1, i_2, i_3 respectively.

(ii) Suppose potential difference between points A and B is V. Because each resistance is connected between same two points A and B, hence potential difference across each resistance will be same and will be equal to applied potential difference V.

(iii) Since potential difference across each resistance is same, hence current approaching the junction A is divided among three resistances reciprocally in their ratio.

As V is constant, hence $i \propto \left(\frac{1}{R}\right)$ i.e.,

$$i_1 = \frac{V}{R}, \quad i_2 = \frac{V}{R_2} \quad \text{and} \quad i_3 = \frac{V}{R_3}.$$

(iv) Because i the main current which is divided into three parts i_1, i_2 and i_3 at the junction A.

$$\text{hence, } i = i_1 + i_2 + i_3 = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

If the equivalent resistance between the points A and B is R, then $i = \frac{V}{R}$.

Thus, $$\frac{V}{R} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

or, $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

(v) Thus in parallel combination of resistance important conclusion are:

(a) Equivalent resistance < lowest individual resistance

(b) Applied potential difference = Potential difference across each resistance.

or, $$iR = i_1R_1 = i_2R_2 = i_3R_3$$

(c) Current approaching the junction A = Current leaving the junction B and current is shared among the three resistances in the inverse ratio of resistances

$$i_1 : i_2 : i_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

(i) If two or more resistance are joined in parallel then $i_1R = i_1R_2 = i_3R_3 \ldots$

i.e., iR = constant i.e., a low resistance joined in parallel always draws a higher current.

(ii) When two resistance R_1 and R_2 are joined in parallel, then

$$\frac{i_1R_1}{i_2R_2} = 1 \quad \text{or} \quad \frac{i_1^2R_1^2}{i_2^2R_2^2} = 1 \quad \text{or}$$

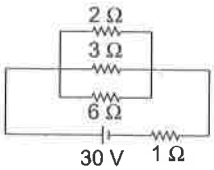
$$\frac{i_1^2R_1t}{i_2^2R_2t} = \frac{R_2}{R_1} \quad \text{or} \quad \frac{H_1}{H_2} = \frac{R_2}{R_1}$$

i.e., heat produced will be maximum in the lowest resistance.

SOLVED EXAMPLES

EXAMPLE 17

Find current which is passing through battery.



SOLUTION

Here potential difference across each resistor is not 30 V

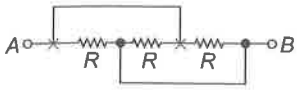
∵ battery has internal resistance here the concept of combination of resistors is useful.

$$R_{eq} = 1 + 1 = 2\ \Omega$$

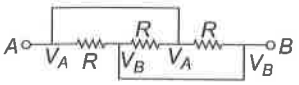
$$i = \frac{30}{2} = 15\ \text{A}$$

EXAMPLE 18

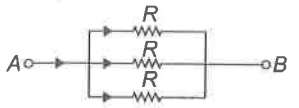
Find equivalent Resistance



SOLUTION



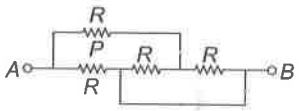
Here all the Resistance are connected between the terminals A and B. So, Modified circuit is



So, $$R_{eq} = \frac{R}{3}$$

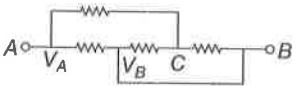
EXAMPLE 19

Find the current in Resistance P if voltage supply between A and B is V volts



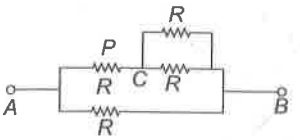
SOLUTION

$$R_{eq} = \frac{3R}{5}$$

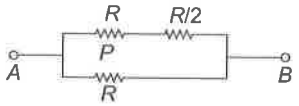


$$I = \frac{5V}{3R}$$

Modified circuit

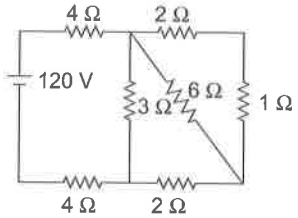


$$\text{Current in} = P = \frac{R \times \frac{5V}{3R}}{1.5R + R} = \frac{2V}{3R}$$

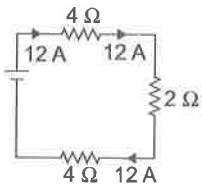


EXAMPLE 20

Find the current in 2 Ω resistance.



SOLUTION



2 Ω, 1 Ω in series = 3 Ω

3 Ω, 6 Ω in parallel = $\frac{18}{9} = 2\ \Omega$

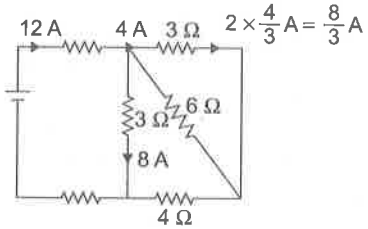
2 Ω, 4 Ω in series = 6 Ω

6 Ω, 3 Ω is parallel = 2 Ω

$R_{eq} = 4 + 4 + 2 = 10\ \Omega$

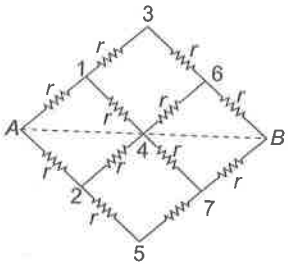
$$i = \frac{120}{10} = 12\ \text{A}$$

So current in 2 Ω Resistance = $\frac{8}{3}$ A



52

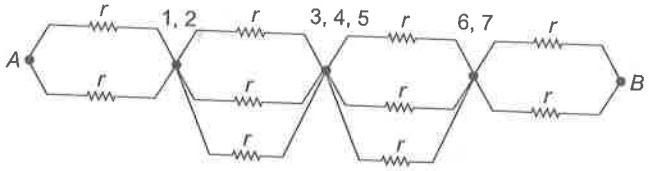
SPECIAL PROBLEMS



We wish to determine equivalent resistance between A and B. In figure shown points (1, 2) (3, 4, 5) and (6, 7) are at same potential equivalent circuit can be redrawn as in figure shown.

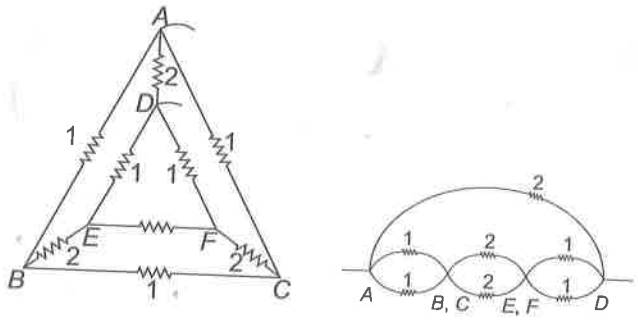
The equivalent resistance of this series combination is

$$R_{eq} = \frac{r}{2} + \frac{r}{4} + \frac{r}{4} + \frac{r}{2} = \frac{3r}{2}$$

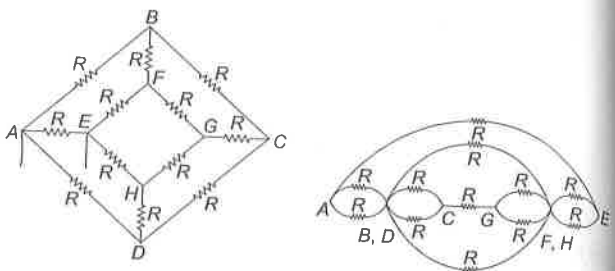


❖ In the figure shown, the resistances specified are in ohms. We wish to determine the equivalent resistance between point A and D. Point B and C, E and F are the the same potential so the circuit can be redrawn as in figure shown.

Thus the equivalent resistance is 1 Ω.



❖ In the network shown in figure shown all the resistances are equal, we wish to determine equivalent resistance between A and E. Point B and D have same potential, similarly F and H have same potential. The equivalent circuit is shown in figure shown. The equivalent resistance of network is $\frac{7R}{2}$.



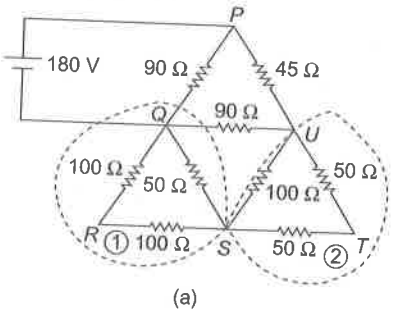
SOLVED EXAMPLES

EXAMPLE 21

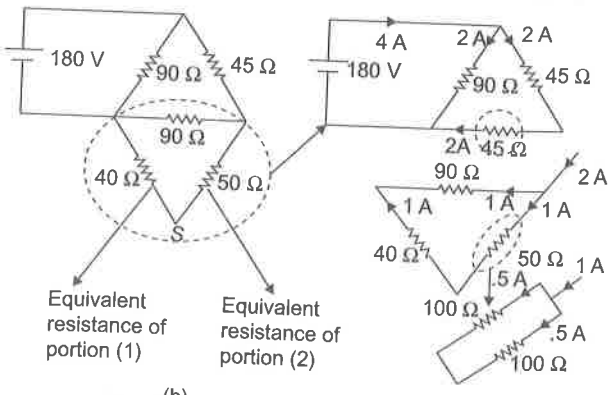
In the circuit shown in figure. (a) find the current flowing through the 100 Ω resistor connecting points U and S.

SOLUTION

Figure (b) shows simplified circuit. The battery is directly attached to resistor 90 Ω hence current in it is 2 A, see figure (c). The total resistance of second branch is also 90 Ω, hence current divides equally. Now current through 45 Ω resistor is 2 A and it is a combination of two equal 90 Ω resistors. Once again current divides equally. 90 Ω resistor is a series combination of 40 Ω and 50 Ω, hence current through them is equal, i.e.,



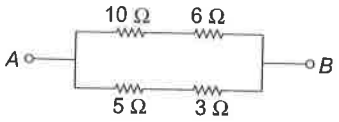
(a)



(b) (c)

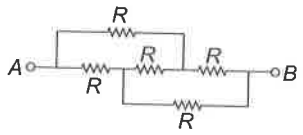
1 A. As 50 Ω resistor is a parallel combination of two equal 100 Ω resistors, they must have the same current i.e., 0.5 A ■

54



EXAMPLE 23

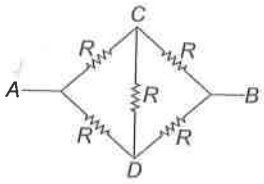
Find the equivalent resistance between A and B



SOLUTION

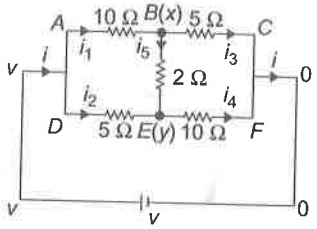
This arrangement can be modified as shown in figure since it is balanced wheat stone bridge.

$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$



Unbalanced Wheatstone's Bridge

EXAMPLE 24



Find equivalent resistance?

SOLUTION

Let potential at point B is x and E is Y

$$R_{eq} = \frac{v}{i}$$

Applying KCl at point B

$$\frac{x - v}{10} + \frac{x - y}{2} + \frac{x - 0}{5} = 0$$

$$8x - 5y = v \qquad (1)$$

Applying KCL at point E

$$\frac{y - v}{5} + \frac{y - x}{2} + \frac{y - 0}{10} = 0$$

$$\Rightarrow \quad 8y - 5x = 2v \qquad (2)$$

solving x and y

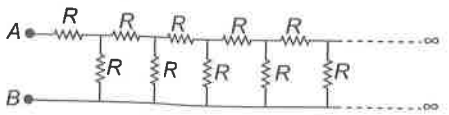
$$x = \frac{6v}{13}, \quad y = \frac{7v}{13}$$

current from branches BC and EF adds up to give total current (i) flowing in the circuit.

$$i = i_3 + i_4 = \frac{x - 0}{5} + \frac{y - 0}{10} = \frac{19v}{130}$$

$$\because \quad i = \frac{V}{R_{eq}} \quad \therefore \quad R_{eq} = \frac{130}{19}$$

Ladder Problem:

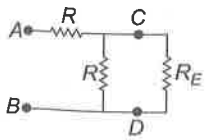


Find the effective resistance between A and B?

SOLUTION

Let the effective resistance between A and B be R_E since the network is infinite long, removal of one cell from the chain will not change the network. The effective resistance between points C and D would also be R_E.

The equivalent network will be as shown below



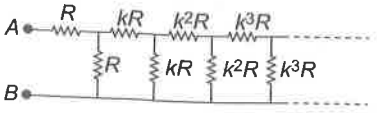
The original infinite chain is equivalent to R in series with R and R_E in parallel.

$$R_E = R + \frac{RR_E}{R + R_E}$$

$$R_E R + R_E^2 = R^2 + 2RR_E \quad \Rightarrow \quad R_E^2 - RR_E - R^2 = 0$$

$$R_E = \frac{R(1 + \sqrt{5})}{2}$$

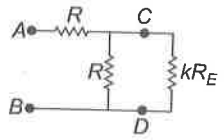
EXAMPLE 25



Find the equivalent resistance between A and B?

SOLUTION

As moving rom one section to next one, resistance is increasing by k times. Since the network is infinitely long, removal of one section from the chain will bring a little change in the network. The effective resistance between points C and D would be kR_E (where R_E is the effective resistance)



$\therefore$ Effective R between A and B.

$$R_E = R + \frac{R(kR_E)}{kR_E + R}$$

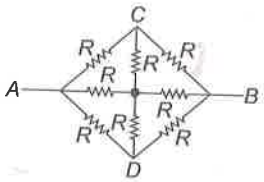
On solving we get

$$R_E = \frac{2kR - R + \sqrt{(R - 2kR)^2 + 4kR^2}}{2k}$$

Symmetrical Circuits: Some circuits can be modified to have simpler solution by using symmetry if they are solved by traditional method of KVL and KCL then it would take much time. ■

EXAMPLE 26

Find the equivalent Resistance between A and B



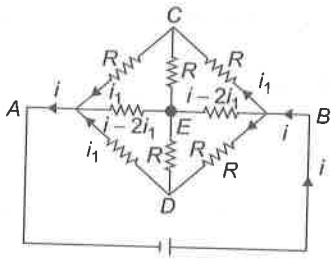
SOLUTION

Method 1: Mirror Symmetry

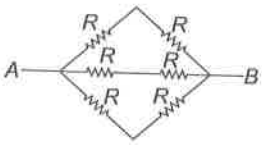
The branches AC and AD are symmetrical

$\therefore$ current through them will be same.

The circuit is also similar from left side and right side like mirror images with a mirror placed alone CD therefore current distribution while entering through B and an exiting from A will be same. Using all these facts the currents are as shown in the figure. It is clear that current in resistor between C and E is 0 and also in ED is 0. It's equivalent is shown in figure (b).



(a)

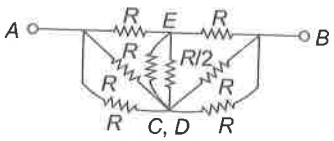


(b)

Method 2: Folding Symmetry

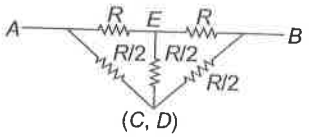
$\therefore$ The potential difference in R between (B, C) and between (B, D) is same $V_C = V_D$

Hence the point C and D are same hence circuit can be simplified as



This called folding.

Now, it is Balanced Wheatstone bridge

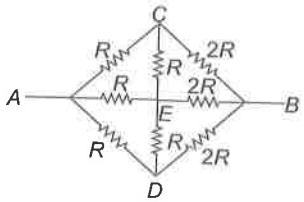


$$R_{eq} = \frac{2R \times R}{2R + R} = \frac{2R}{3}$$

In Method 2 it is not necessary to know the currents in CA and DA ■

EXAMPLE 27

Find the equivalent Resistance between A and B



SOLUTION

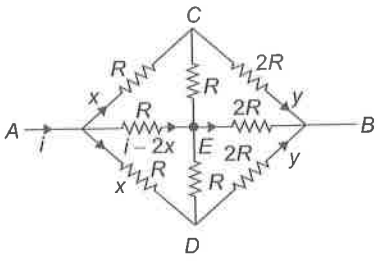
In this case the circuit has symmetry in the two branches AC and AD at the input

$\therefore$ current in them are same but from input and from exit the circuit is not similar ($\because$ on left R and on right $2R$)

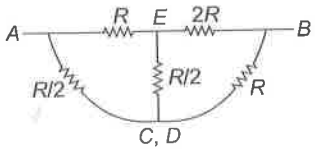
56

$\therefore$ on both sides the distribution of current will not be similar.

Here, $V_c = V_d$, Hence C and D are same point. So, the circuit can be simplified as



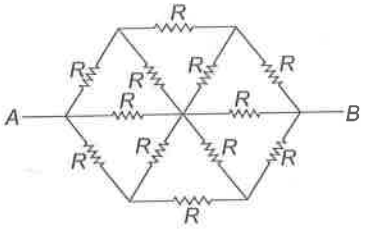
Now it is balanced wheat stone bridge.



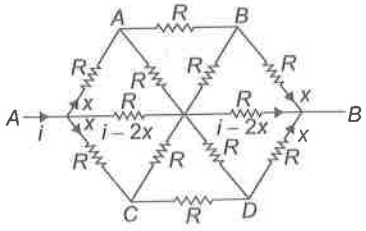
$$R_{eq} = \frac{3R \times \frac{3R}{2}}{3R + \frac{3R}{2}} = \frac{\frac{9}{2}R}{\frac{9}{2}} = R$$

EXAMPLE 28

Find the equivalent Resistance between A and B

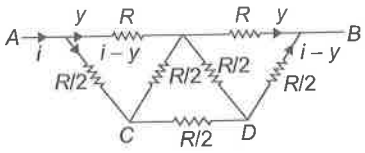


SOLUTION

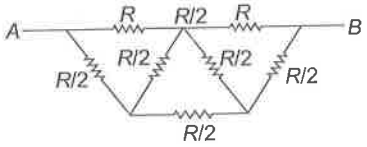


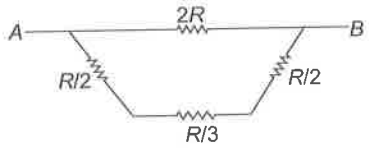
Here $V_A = V_C$ and $V_B = V_D$

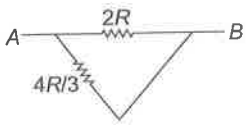
Here the circuit can be simplified as



$$R_{eq} \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}} = \frac{4R}{5}$$ **Ans.**

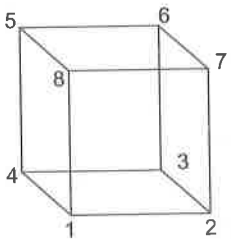






EXAMPLE 29

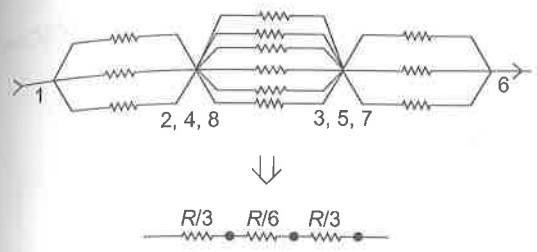
Twelve equal resistors each $R\ \Omega$ are connected to form the edges of a cube. Find the equivalent resistances of the network.



(a) When current enters at 1 and leaves at 6 (body diagonal)

SOLUTION

Here 2, 4, 8 are equipotential points (if we move from 1 $\rightarrow$ 2, 4, 8 it comes along the edge and 6 $\rightarrow$ 2, 4, 8 it comes along face diagonal). Similarly 3, 5, 7 are equipotential points.

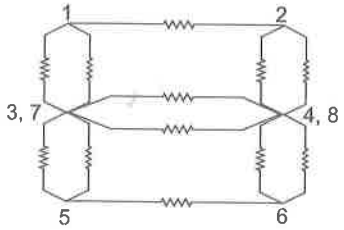


$$R_{eq} = \frac{5R}{6}$$

(b) When current enters at 1 and leaves at 2

SOLUTION

Here 3, 7 are equipotential surface (if we move from 1 → 3, 7 we have along face and 2, → 3, 7 we move along edge) similarly 4, 8 are equipotential surface.

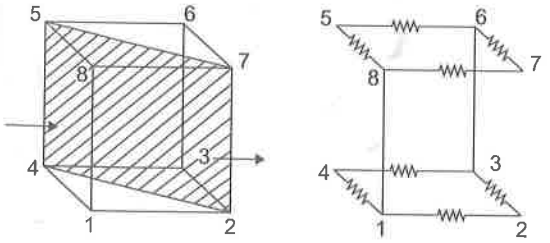


$$R_{eq} = \frac{7R}{12}$$

(c) When current enters at 1 and leaves at 3

SOLUTION

If we cut the cube along the plane passing through 2, 4, 5, 7 then by mirror symmetry, the final configuration will be

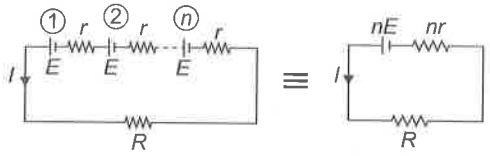


$$R_{eq} = \frac{3R}{4}$$

COMBINATIONS OF CELLS

A cell is used to maintain current in an electric circuit. We cannot obtain a strong current from a single cell. Hence need arises to combine two or more cells to obtain a strong current. Cells can be combined in three possible ways: (A) In series, (B) In parallel, and (C) In mixed grouping.

(A) Cells in Series



In this combination, cells are so connected that –ve terminal of each cell is connected with the +ve terminal of next and so on. Suppose n cells are connected in this way. Let emf and internal resistance of each cell are E and r respectively.

Net emf of the cells = nE. Total internal resistance = nr. Hence total resistance of the circuit = $nr + R$.

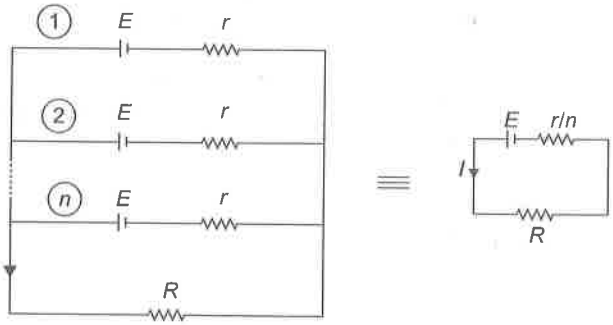
If total current in the circuit is I,

$$\text{then} \qquad I = \frac{\text{net emf}}{\text{Total Resistance}} = \frac{nE}{nr + R} \qquad (1)$$

Case (I): If $nr << R$, then $I \cong n$ E/R i.e., if total internal resistance of the cells is far less than external resistance, then current obtained from the cells is approximately equal to n times the current obtained from a single cell. Hence cells, whose total internal resistance is less than external resistance, just be joined in series to obtain strong current.

Case (II): If $nr >> R$, then $I \cong \frac{nE}{nr} = \frac{E}{r}$ i.e., if total internal resistance of the cells is much greater than the external resistance, then current obtained from the combination of n cells is nearly the same as obtained from a single cell. Hence there is no use of joining such cells in series.

(B) Cells in Parallel



(I) When emf's and internal resistance of all the cells are equal: In this combination, positive terminals of all the cells are connected at one point and negative terminals at other point. Figure shown such cells connected in parallel across some external

resistance R. Let emf and internal resistance of each cell are E and r respectively.

Because all the cells are connected in parallel between two points, hence emf of battery = E.

Total internal resistance of the combination of n cells = $\frac{r}{n}$.

Because external resistance R is connected in series with internal resistance, hence total resistance of the circuit = $\left(\frac{r}{n}\right) + R$.

If current in external resistance is I, then

$$I = \frac{\text{net emf}}{\text{Total resistance}} = \frac{E}{\left(\frac{r}{n}\right) + R} = \frac{nE}{r + nR}$$

Case (I): If $r << R$, the $I \cong \frac{nE}{nR} = \frac{E}{R}$ i.e., if internal resistance of the cells is much less than external resistance, then total current obtained from combination is nearly equal to current given by one cells only. Hence there is no use of joining cells of low internal resistance in parallel.

Case (II): If $r >> R$, then $I \cong \frac{nE}{r}$ i.e., if the internal resistance of the cells is much higher than the external resistance, then total current is nearly equal to n times the current given by one cell. Hence cells of high internal resistance must be joined in parallel to get a strong current.

(II) When emf's and internal resistance of all the cells connected in parallel are different.

In this case, total current in external resistance is obtained with the help of Kirchhoff's laws. Figure shows three cells of emf E_1, E_2 and E_3 and internal resistances r_1, r_2 and r_3 connected in parallel across some external resistance R. Suppose currents given by three cells are i_1, i_2 and i_3. Hence according to Kirchhoff's first law, total current I in external resistance R, is given by

$$I = i_1 + i_2 + i_3 \qquad (1)$$

Applying Kirchhoff's 2nd law to closed mesh $ABEF$ we get

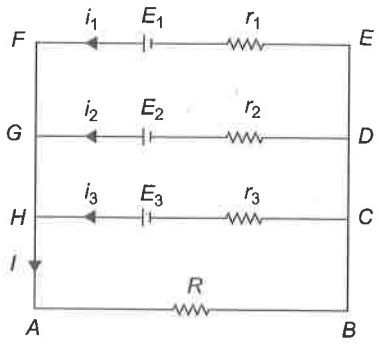
$$IR + i_1 r_1 = E_1 \quad \text{or} \quad i_2 = \left[\frac{(E_1 - IR)}{r_1}\right] \qquad (2)$$

Similarly, for closed meshes $ABDG$ and $ABCH$, we get

$$i_2 = \frac{E_2 - IR}{r_2} \qquad (3)$$

and

$$i_3 = \frac{E_2 - IR}{r_3} \qquad (4)$$



Substituting equation (2), (3) and (4) in equation (1), we have

$$I = \frac{E_1 - IR}{r_1} + \frac{E_2 - IR}{r_2} + \frac{E_3 - IR}{r_3}$$

$$= \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} - IR\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

or $$I\left[1 + R\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)\right] = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}$$

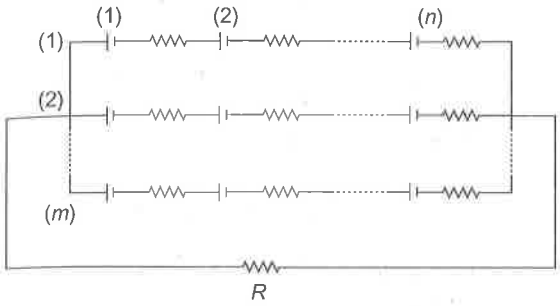
or $$I = \frac{\left(\frac{E_1}{r_1}\right) + \left(\frac{E_2}{r_2}\right) + \left(\frac{E_2}{r_3}\right)}{1 + R\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)}$$

If n cells are joined in parallel, then

$$I = \frac{\sum_i \frac{E_i}{r_i}}{1 + R\sum_i \frac{1}{r_i}}$$

and $$E_{eq} = \frac{\sum \frac{E_i}{r_i}}{\sum \frac{1}{r_i}}, \quad r_{eq} = \frac{1}{\sum \frac{1}{r_i}}$$

(C) Cells in Mixed Grouping



In this combination, a certain number of cells are joined in series in various rows, and all such rows are then connected in parallel with each other.

Suppose n cells, each of emf E and internal resistance r, are connected in series in every row and m such rows are connected in parallel across some external resistance R, as shown in figure.

Total number of cells in the combination = mn. As emf. of each row = nE and all the rows are connected in parallel, hence net emf of battery = nE.

Internal resistance of each row = nr. As m such rows are connected in parallel, hence total internal resistance of battery = $\left(\frac{nr}{m}\right)$

Hence total resistance of the circuit = $\left[\left(\frac{nr}{m}\right) + R\right]$

If the current in external resistance is I, then

$$I = \frac{\text{net emf}}{\text{Total resistance}}$$

$$= \frac{nE}{\left(\frac{nr}{m}\right) + R} = \frac{mnE}{nr + mR}$$

$$= \frac{mnE}{(\sqrt{nr} - \sqrt{mr})^2 + 2\sqrt{nmrR}}$$

It is clear from above equation that I will be maximum when $[(\sqrt{nr} - \sqrt{mR})^2 + 2\sqrt{nmrR}]$ is minimum.

This will be possible when the quantity $[\sqrt{nr} - \sqrt{mR}]^2$ is minimum. Because this quantity is in square, it can not be negative, hence its minimum value will be equal to zero, i.e.,

$$mR = nr \quad \text{or} \quad R = \frac{nr}{m}$$

i.e., in mixed grouping of cells, current in external resistance will be maximum when total internal resistance of battery is equal to external resistance.

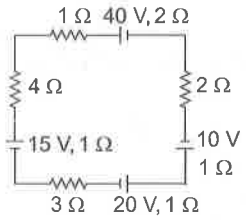
Because power consumed in the external resistance or load = I^2R, hence when current in load is maximum, consumed power in it is also maximum, Hence consumed power in the load will also be maximum when $R = \frac{nr}{m}$.

$$I_{max} = \frac{mnE}{2mR} \quad \text{or} \quad \frac{mnE}{2nr} = \frac{nE}{2R} \quad \text{or} \quad \frac{mE}{2r}$$

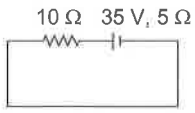
SOLVED EXAMPLES

EXAMPLE 30

Find the current in the loop.



SOLUTION



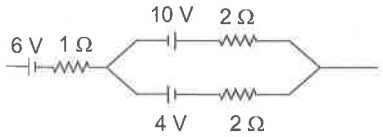
The given circuit can be simplified as

$$i = \frac{35}{10 + 5} = \frac{35}{15}$$

$$= \frac{7}{3}\text{ A} \quad \Rightarrow \quad I = \frac{7}{3}\text{ A}$$ ■

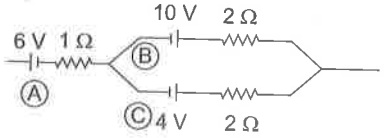
EXAMPLE 31

Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



60

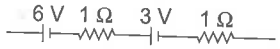
SOLUTION



Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent

$$\in_{BC} = \frac{\frac{10}{2} + \frac{-4}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{5-2}{1} = 3 \text{ V}$$

$$r_{BC} = 1\ \Omega$$



Now, $\in_{ABC} = 6 - 3 = 3$ V

$r_{ABC} = 2\ \Omega$ **Ans.**

ELECTRICAL POWER

The energy liberated per second in a device is called its power, the electrical power P delivered by an electrical device is given by

$$P = \frac{dq}{dt} \text{V} = VI$$

Power consumed by a resistor.

$$P = VI = I^2R = \frac{V^2}{R} \text{ W}$$

The power P is in watts when I is in amperes, R is in ohms and V is in volts.

The practical unit of power is 1 kW = 1000 W.

The formula for power $P = I^2R = VI = \frac{V^2}{R}$ is true only when all the electrical power is dissipated as heat and not converted into mechanical work, etc. simultaneously.

If the current enters the higher potential point of the device then electric power is consumed by it (i.e., acts as load). If the current enters the lower potential point then the device supplies power (i.e., acts as source.)

(A) Joule's law of electrical heating

When an electric current flows through a conductor electrical energy is used in overcoming the resistance of the wire. If the potential difference across a conductor of resistance R is V volt and if a current of I ampere flows the energy expanded in time t seconds is given by

$$W = VIt \text{ joule} = I^2Rt \text{ joule} = \frac{V^2}{R}t$$

The electrical energy so expanded is converted into heat energy and this conversion is called the heating effect of electric current.

The heat generated in joules when a current of I amperes flows through a resistance of R ohm for t seconds is given by

$$H = I^2Rt \text{ joule} = \frac{I^2Rt}{4.2} \text{ cal.}$$

This relation is known as Joule's law of electrical heating.

Bulb:

Rated power = P_r

Rated voltage = V_r

Resistance of bulb

$$R = \frac{V_r^2}{P_r}$$

SOLVED EXAMPLES

EXAMPLE 32

If bulb rating is 100 W and 220 V then determine

(a) Resistance of filament

(b) Current through filament

(c) If bulb operate at 110 V power supply then find power consume by bulb.

SOLUTION

Bulb rating in 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consume is 100 W

Here $V = 220$

$P = 100$

$$\frac{V^2}{R} = 100$$

So $R = 484\ \Omega$

Since Resistance depends only on material hence it is constant for bulb

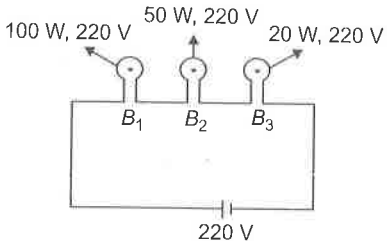
$$I = \frac{V}{R} = \frac{220}{22 \times 22} = \frac{5}{11} \text{ A}$$

power consumed at 110 V

$$\therefore \text{ power consumed} = \frac{110 \times 110}{484} = 25 \text{ W}$$

EXAMPLE 33

In the following figure, grade the bulb in order of their brightness:



SOLUTION

$$P_{\text{rated}} = \frac{V_{\text{rated}}^2}{R}$$

$$R = \frac{V_{\text{rated}}^2}{P_{\text{rated}}}$$

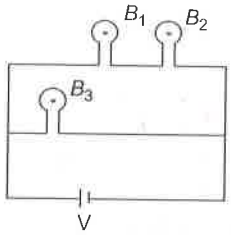
$\therefore \quad R_3 > R_2 > R_1$

$$\text{Power} = i^2R$$

As current passing through every bulb is same

$\therefore$ Brightness order is $B_3 > B_2 > B_1$ ■

EXAMPLE 34



The above configuration shows three identical bulbs, Grade them in order of their brightness.

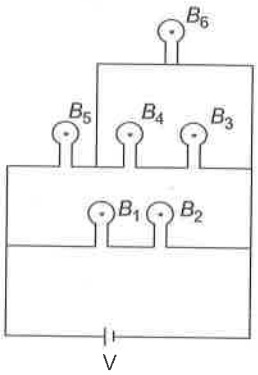
SOLUTION

B_1 and B_2 withdraw less current as compared to B_3 because in series they give $2R$ resistance where as R is the resistance dut to B_3.

$$\text{Power} = i^2R$$

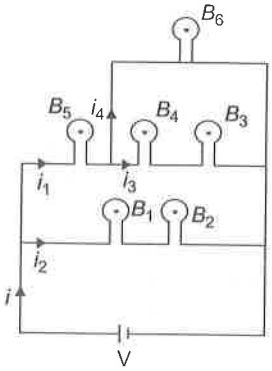
$\therefore$ Brightness order: $B_3 > B_2 = B_1$. ■

EXAMPLE 35



Grade the bulbs in order of their brightness (All bubls are identical)

SOLUTION



As $\quad i \propto \frac{1}{R}$

$$i_1 : i_2 = \frac{3}{5R} : \frac{1}{2R} = 6 : 5$$

$\therefore \quad i_1 = \frac{6i}{11}, \quad i_2 = \frac{5i}{11}$

$$i_3 : i_4 = \frac{1}{2R} : \frac{1}{R} = 1 : 2$$

As $\quad i_3 + i_4 = i_1$

$\therefore \quad i_3 = \frac{2i}{11}; \quad i_4 = \frac{4i}{11}$

$$\text{power} = i^2R$$

$\therefore$ Order of Brightness: $B_5 > B_1 = B_2 > B_6 > B_4 = B_3$

(B) Maximum power transfer theorem

Let E be emf and r internal resistance of the battery. It is supplying current to an external resistance R

62

current in circuit $I = \dfrac{E}{(R+r)}$

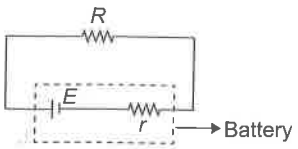
The power absorbed by load resistor R is

$$P = I^2R = \left(\frac{E}{R+r}\right)^2 R$$

For maximum power transfer we take the derivative of P wrt R, set it equal to zero and solve the equation for R

$$\frac{dP}{dR} = 0$$

$$\frac{dP}{dR} = E^2 \frac{(R+r)^2 - R[2(R+r)]}{(R+r)^2} = 0$$



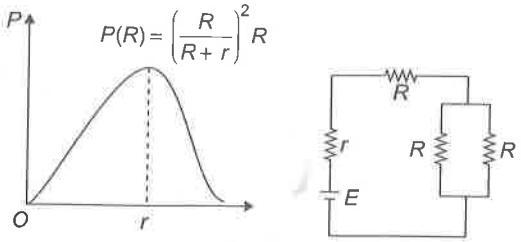
Solving for R, we have

$$(R+r)2 - R(2)(R+r) = 0$$

$$(R+r) - 2R = 0$$

$$R = r$$

For a given real battery the load resistance maximizes the power if it is equal to the internal resistance of the battery.



The maximum power transfer theorem in general, holds for any real voltage source. The resistance R may be a single resistor or R may be the equivalent resistance of a collection of resistors. ■

INSTRUMENTS

(A) Ammeter

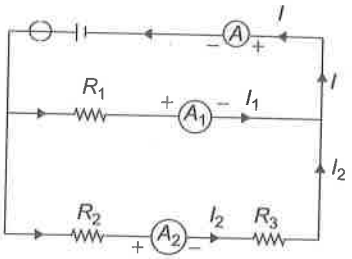
It is a device used to measure current and its always connected in series with the 'element' through which current is to be measured, e.g., in figure (A) ammeter A_1 will measure the current (I_1) through resistance R_1, A_2 measures current (I_2) through R_2 and R_3 while A, measures current $I(I_1 + I_2)$.

Regarding an ammeter it is worth noting that:

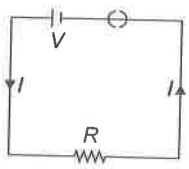
1. The reading of an ammeter is always lesser than actual current in the circuit, e.g., true current in the resistance R in the circuit shown in figure (B) is $I = \dfrac{V}{R}$

   However, when an ammeter of resistance r is used to measure current as shown in figure (C), the reading will be

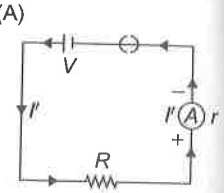
$$I' = \frac{V}{(R+r)} < I\left(=\frac{V}{R}\right)$$



(A)



(B)



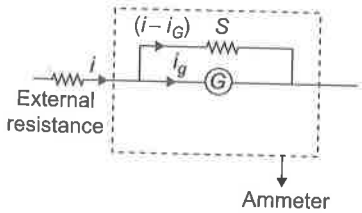
(C)

2. Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be **ideal** if its resistance (r) is zero. However, as practically $r \neq 0$, ideal ammeter cannot be realized in practice.

3. To convert a galvanometer into an ammeter of a certain range say I, a small resistance S (called shunt) is connected in parallel with the galvanometer so that the current passing through the galvanometer of resistance G becomes equal to its full scale deflection value I_g. This is possible only if

$$I_g G = (I - I_g)S$$

i.e., $$S = \frac{I_g}{(I - I_g)} G$$

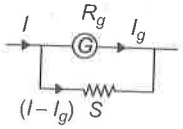


SOLVED EXAMPLES

EXAMPLE 36

What is the value of shunt which passes 10% of the main current through a galvanometer of 99 Ω?

SOLUTION

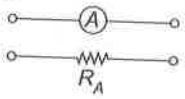


As in figure $R_g I_g = (I - I_g)S$

$$\Rightarrow \quad 99 \times \frac{I}{10} = \left(I - \frac{I}{10}\right) \times S$$

$$\Rightarrow \quad S = 11\ \Omega$$

For calculation it is simply a resistance



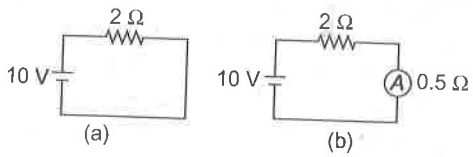
Resistance of ammeter

$$R_A = \frac{R_G \cdot S}{R_G + S}$$

for $\quad S \ll R_G \quad \Rightarrow \quad R_A = S$ ■

EXAMPLE 37

Find the current in the circuit also determine percentage error in measuring in current through an ammeter (a) and (b).



SOLUTION

In A $\quad I = \dfrac{10}{2} = 5\ \text{A}$

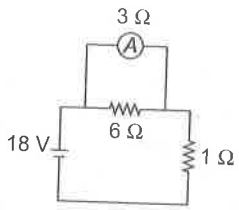
In B $\quad I = \dfrac{10}{2.5} = 4\ \text{A}$

Percentage error is $\dfrac{i - i'}{i} \times 100 = 20\%$ **Ans.**

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected. ■

EXAMPLE 38

Find the reading of ammeter. Is this the current through 6 Ω?



SOLUTION

$$R_{eq} = \frac{3 \times 6}{3 + 6} + 1 = 3\ \Omega$$

Current through battery

$$I = \frac{18}{3} = 6\ \text{A}$$

So, current through ammeter

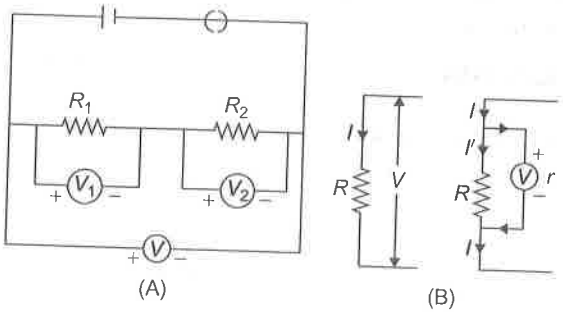
$$= 6 \times \frac{6}{9} = 4\ \text{A}$$

No, it is not the current through the 6 Ω resistor. ■

Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.

(B) Voltmeter

It is a device used to measure potential difference and is



always put in parallel with the 'circuit element' across which potential difference is to be measured e.g., in Figure (A) voltmeter V_1 will measure potential difference across resistance R_1, V_2 across resistance R_2 and V across $(R_1 + R_2)$ with $V = V_1 + V_2$

Regarding a voltmeter it is worth noting that:

1. The reading of a voltmeter is always lesser than true value, e.g., if a current I is passing through a resistance R [Fig. (B)], the true value $V = IR$. However, when a voltmeter having resistance

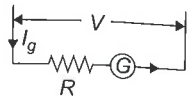
64

r is connected across R, the current through R will become

$$I' = \frac{r}{(R+r)} I \quad \text{and} \quad \text{so } V' = I'R = \frac{V}{\left[1 + \left(\frac{R}{r}\right)\right]}$$

and as voltmeter is connected across R its reading V' is lesser than V.

2. Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be **ideal** if its resistance r is infinite, i.e., it draws no current from the circuit element for its operation. Ideal voltmeter has been realized in practice in the form of potentiometer.
3. To convert a galvanometer into a voltmeter of certain range say V, a high resistance R is connected in series with the galvanometer so that current passing through the galvanometer of resistance G becomes equal to its full scale deflection value I_g. This is possible only if



$$V = I_g (G+R) \quad \text{i.e.,} \quad R = \frac{V}{I_g} - G$$

EXAMPLE 39

A voltmeter has a resistance of G ohm and range of V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

SOLUTION

Full scale current $i_g = \dfrac{V}{G}$

to change its range

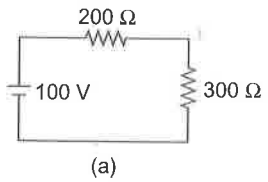
$$V_1 = (G + R_s)\, i_g$$

$$\Rightarrow \quad nV = (G + R_s)\frac{V}{G}$$

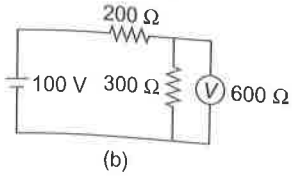
$$R_s = G(n-1) \qquad \textbf{Ans.}$$

EXAMPLE 40

Find potential difference across the resistance 300 Ω in A and B.



(a)



(b)

SOLUTION

In (a): Potential difference $= \dfrac{100}{200 + 300} = 300 = 60$ V

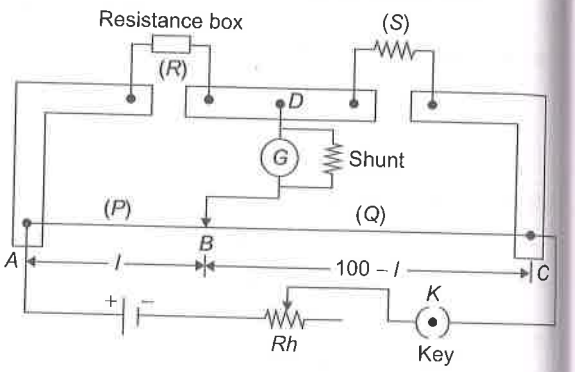
In (b): Potential difference

$$= \frac{100}{200 + \dfrac{300 \times 600}{300 + 600}} \times \left[\frac{300 \times 600}{300 + 600}\right] = 50 \text{ V}$$

We see that by connected voltmeter the voltage which was to be measured has charged. Such voltmeters are not good. If its resistance had been very large than 300 Ω then it would have not affected the voltage by much amount.

(C) Metre-Bridge

Metre-bridge is a sensitive device based on the principle of wheatstone-bridge, for the determination of the resistance of a conductor (wire). Its sensitivity is much more than that of the post-office box.



Metre-bridge is shown in figure AC is one metre long wire of manganin or constantan which is fixed along a scale on a wooden base. The area of cross-section of the wire is same at all places. The ends A and C of the wire are joined to two L-shaped copper strips carrying binding-screws as shown. In between these strips, leaving a gap on either side, there is a third copper strip having three binding screws. The middle screw D is connected to a sliding jockey B through a shunted-galvanometer G. The knob of the jockey can be made to touch at any point on the wire.

To measure the unknown resistance, the connection as shown in figure are made.

A resistance R is taken out from the resistance box and the key K is closed. Now the jockey is slided along the wire and a point is determined such that, on pressing the jockey on the wire at that point there is no deflection in the galvanometer G. In this position the points B and D are at the same potential. The point B is called 'null-point'. The lengths of both the parts AB and BC of the wire are read on

the scale. Suppose the resistance of the length AB of the wire is P and that of the length BC is Q. Then, by the principle of Wheatstone-bridge. We have,

$$\frac{P}{Q} = \frac{R}{S}$$

Let the length AB be l cm. Then the length BC will be = (100 – l) cm.

$\therefore$ resistance of AB, i.e., $P = \frac{\rho l}{A}$, and resistance of BC, $Q = \rho(100 - l)/A$

where ρ is the specific resistance of the material of the wire and 'A' is the area of cross-section of the wire. Thus

$$\frac{P}{Q} = \frac{l}{(100 - l)} \qquad \text{(i)}$$

Substituting this value of $\frac{P}{Q}$ in eq. (i), we get

$$\frac{l}{(100 - l)} = \frac{R}{S} \quad \text{or} \quad S = \frac{R(100 - l)}{l}$$

R is the resistance taken in the resistance box and l is the length measured. Hence, the value of resistance S can be determined from the above formula.

A number of observations are taken for different resistances in the resistance box and for each observation the value of S is calculated.

Finally, the experiment is repeated by interchanging the unknown resistances S and the resistance box. The mean of the values of S is then obtained.

EXAMPLE 41

In a meter bridge experiment, the value of unknown resistance is 2 Ω. To get the balancing point at 40 cm distance from the same end, then what will be the resistance in the resistance box?

SOLUTION

Apply condition for balance Wheatstone bridge,

$$\frac{P}{Q} = \frac{\ell}{100 - \ell} = \frac{P}{2} = \frac{100 - 40}{40}$$

$P = 3\ \Omega$ **Ans.** ■

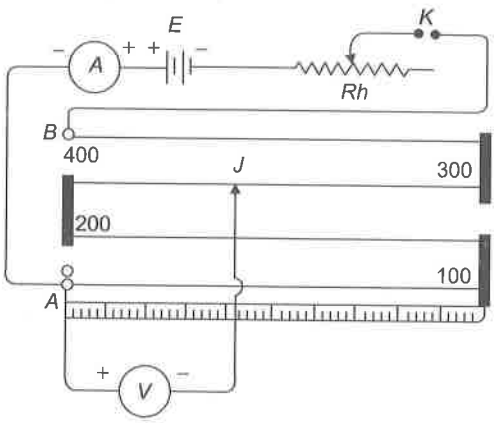
(D) Potentiometer

A potentiometer is used to compare emfs of two cells or to measure internal resistance of a cell.

Principle: The potentiometer is based upon the principle that when a constant current is passed through a wire of uniform area of cross-section, the potential drop across any portion of the wire is directly proportional to the length of that portion.

Construction: A potentiometer consists of a number of segments of wire of uniform area of cross section stretched on a wooden board between two thick copper strips. Each segment of wire is 100 cm long. The wire is usually of constantan or manganin. A metre rod is fixed parallel to its length. A battery connected across the two end terminals sends current through the wire, which is kept constant by using a rheostat.

Theory: Let V be potential difference across certain portion of wire, whose resistance is R. If I is the current through the wire, then $V = IR$



We know that $R = \rho \frac{l}{A}$,

where l, A and ρ are length, area of cross-section and resistivity of the material of wire respectively.

$\therefore$ $$V = i\rho \frac{l}{A}$$

If a constant current is passed through the wire of uniform area of cross-section, then i and A are constants. Since, for a given wire, ρ is also constant, we have

$$V = \text{constant} \times l$$

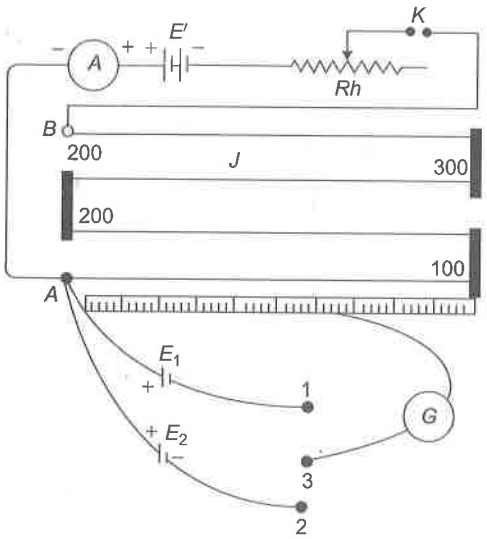
or $$V \propto l$$

Hence, if a constant current flows through a wire of uniform area of cross-section, then potential drop along the wire is directly proportional to the length of the wire.

Applications of a potentiometer. A potentiometer can be put to following uses:

1. **To compare emfs of two cells:** Two cells, whose emfs are E_1 and E_2, can be compared by making use of the ciruit as shown in figure. The positive

poles of both the cells are connected to the terminal A of the potentiometer. The negative poles of the two cells are connected to terminals 1 and 2 of a two way key. while its common terminal is connected to a jockey j through a galvanometer G. An auxiliary or driver battery of emf E', an ammeter A, rheostat Rh and a one way key K are connected between the end terminals A and B of the potentiometer. Thus, the positive poles of the two cells as well as the positive pole of auxiliary battery are connected at the common point A. It may be pointed that the emf of auxiliary battery should always be greater than the emf of either of the two cells.



To compare the emfs of the two cells, a constant current is passed through the potentiometer wire between points A and B. The current is kept constant by using the rheostat.

When the plug is put in the gap between the terminals 1 and 3 of the two way key, the cell of emf. E_1 will come in the circuit. Suppose the balancing length (between points A and J) is l_1. If x is the resistance per unit length of the potentiometer wire and I, the constant current flowing through it, then

$$E_1 = (xl_1)I$$

When the key is put in the gap between the terminals 2 and 3 and removed from the gap between 1 and 3, the cell of emf E_2 wil be included in the circuit. Let the balancing length be l_2 in this case. Then,

$$E_2 = (xl_2)I$$

Dividing above equation

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Notes

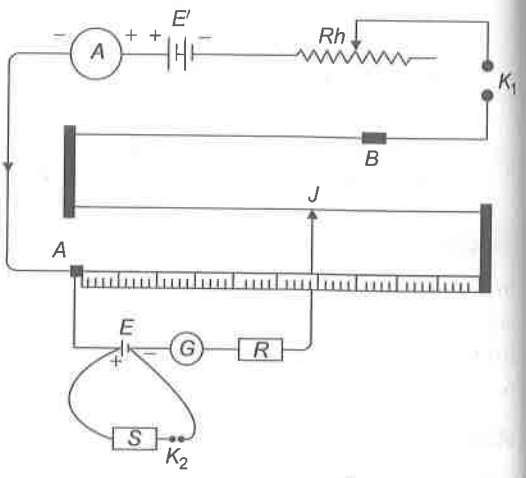
It may be pointed out that the emf of auxiliary battery should always be greater than the emf. of the either of the two cells.

2. **To measure internal resistance of a cell:** The internal resistance of a cell may be found by using a potentiometer by setting up the circuit as shown in figure.

A constant current I is maintained through the potentiometer wire with the help of the rheostat.

Plug in the key K_2 is kept out and the jockey is moved over the potentiometer wire so as to balance the emf E of the cell, whose internal resistance is to be found. Let l_1 be the balancing length of the potentiometer wire between point A and jockey J. If x is resistance per unit length of the wire, then

$$E = (xl_1)I$$



With the help of resistance box S, introduce resistance say S and then put the plug in key K_2. Now find the balance point for the terminal potential difference V between the two poles of the cell. If l_2 is the balancing length, then

$$V = (xl_2)I$$

Dividing above equation, we have

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

The internal resistance* of the cell is given by

$$r = \left(\frac{E}{V} - 1\right) S$$

Using above equation, we have

$$r = \left(\frac{l_1}{l_2} - 1\right) S \quad \text{or} \quad r = \frac{l_1 - l_2}{l_2} \times S$$

Knowing the values of l_1, l_2 and S, the internal resistance r of the cell can be found.

Notes

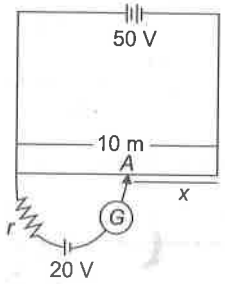
Apart from uses, a potentiometer can be used to compare unknown resistances and to calibrate a voltmeter or an ammeter.

We use potentiometer for two tasks:

(i) to find emf of a cell
(ii) to find internal resistance of a cell

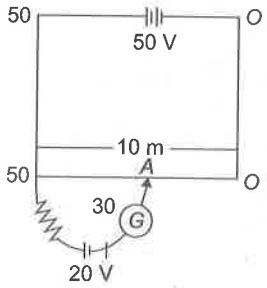
We will first analyze the first task → to find emf of a cell through some examples

EXAMPLE 42



Find the value of x if A is the null point?

SOLUTION

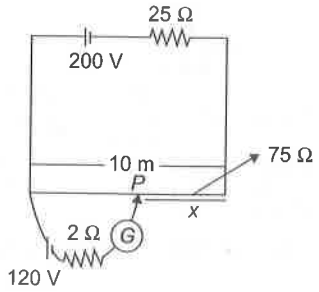


$$\text{Potential gradient} = \frac{50}{10} = 5 \text{ V/m}$$

$\therefore$ For 20 V potential difference

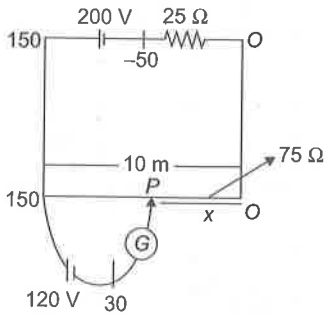
$$\Rightarrow \quad \frac{20}{5} = 4 \text{ m} \qquad \therefore \quad x = 10 - 4 = 6 \text{ m}$$

EXAMPLE 43



Find the value of x if P is a null point.

SOLUTION



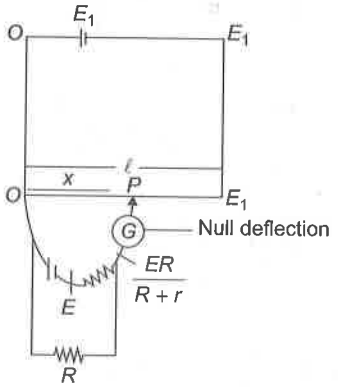
Current in the main circuit

$$200 - 25i - 75i = 0$$
$$i = 2 \text{ A}$$

$$\text{Potential gradient} = \frac{150 \text{ V}}{10 \text{ m}} = 15 \text{ V/m}$$

$$\text{For 120 V} \quad \Rightarrow \quad \frac{120 \text{ V/m}}{15 \text{ V}} = 8 \text{ m}$$

$$\therefore \quad x = 10 - 8 = 2 \text{ m}$$



Now we will analyze the other task to find internal resistance of the cell using potentiometer.

The main key point is that first analyze the main circuit then the auxiliary circuit (supplementary circuit)

Potential gradient $= \frac{E_1}{\ell}$

Now for the auxiliary circuit

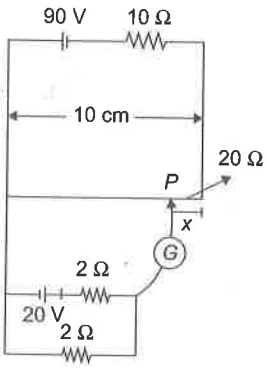
$$i = \frac{E}{R+r}$$

$$E - ir = E - \frac{Er}{R+r} = \frac{ER}{R+r}$$

$$\therefore \quad \frac{ER}{R+r} = \frac{E_1}{\ell}x$$

Let we take some examples to understand the topic in better way.

EXAMPLE 44



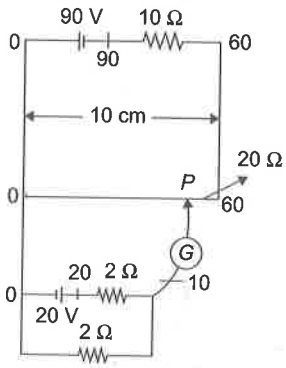
Find x if P is a null point?

SOLUTION

First analyzing the main circuit,

$$90 - 10i_1 - 20i_1 = 0$$

$$i_1 = 3 \text{ A}.$$



Potential gradient $= \frac{60 \text{ V}}{10 \text{ m}} = 6 \text{ V/m}.$

Now analyzing the auxiliary circuit

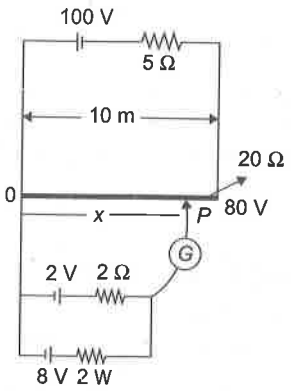
$$20 - 2i_2 - 2i_2 = 0$$

$$i_2 = 5 \text{ A}.$$

For $10V_e \quad \Rightarrow \quad \frac{10}{6} = \frac{5}{3}$ m

$$\therefore \quad x = 10 - \frac{5}{3} = \frac{25}{3} \text{ m}$$

EXAMPLE 45

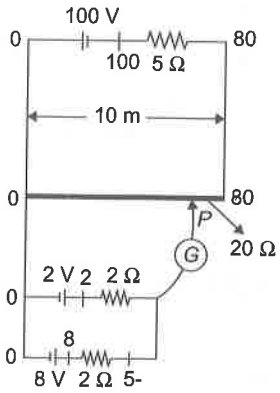


Find x if P is a null point?

SOLUTION

$$100 - 5i_1 - 20i_1 = 0$$

$$i_1 = 4 \text{ A}$$



Potential gradient $= \frac{80}{10} = 8 \text{ V/m}$

$$8 - 2i_2 - 2i_2 - 2 = 0$$

$$i_2 = \frac{3}{2} \text{ A}$$

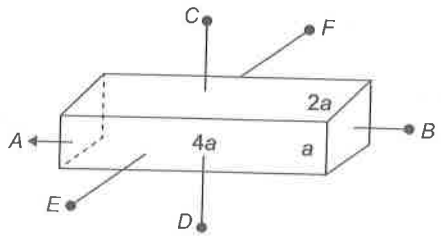
for 5 volts $\Rightarrow \quad \frac{5v}{8 \text{ V/m}} = \frac{5}{8}$ m

$$\therefore \quad x = \frac{5}{8} \text{ m}$$

EXERCISES

JEE Main

1. The electric current in a liquid is due to the flow of
   (A) electron only
   (B) positive ions only
   (C) negative and positive ions both
   (D) electrons and positive ions both
2. The electric current in a discharge tube containing a gas is due to
   (A) electron only
   (B) positive ions only
   (C) negative ion and positive ions both
   (D) electrons and positive ions both
3. A current (I) flows through a uniform wire of diameter (d) when the mean drift velocity is V. The same current will flow through a wire of diameter $\frac{d}{2}$ made of the same material if the mean drift velocity of the electron is
   (A) $\frac{V}{4}$ (B) $\frac{V}{2}$
   (C) 4 V (D) 2 V
4. When a potential difference (V) is applied across a conductor, the thermal speed of electrons is
   (A) zero (B) proportional to $\sqrt{T}$
   (C) proportional to (T) (D) proportional to V
5. A steady current is passing through a linear conductor of non-uniform cross-section. The current density in the conductor is
   (A) independent of area of cross-section
   (B) directly proportional to area of cross-section
   (C) inversely proportional to area of cross-section
   (D) inversely proportional to the square root of area of cross-section
6. A metallic block has no potential difference applied across it. Then the mean velocity of free electron is
   (A) proportional to T
   (B) proportional to $\sqrt{T}$
   (C) zero
   (D) finite but independent of temperature
7. The resistance of some substances become zero at very low temperature, then these substances are called –
   (A) good conductors (B) super conductors
   (C) bad conductors (D) semi conductors
8. The resistance of wire is 20 Ω. The wire is stretched to three times its length. Then the resistance will now be –
   (A) 6.67 Ω (B) 60 Ω
   (C) 120 Ω (D) 180 Ω
9. The resistance of a semi-conductors
   (A) increases with increase of temperature
   (B) decreases with increase of temperature
   (C) does not charge with charge of temperature
   (D) first decreases and then increases with increase of temperature
10. A hollow copper tube of 5 m length has got external diameter equal to 10 cm and the walls are 5 mm thick. If specific resistance of copper is 1.7×10^{-8} Ω × metre. Calculate the resistance of the tube.
    (A) 5.77×10^{5} Ω (B) 5.77×10^{-5} Ω
    (C) 5.77×10^{-7} Ω (D) 5.77×10^{7} Ω
11. In order to increase the resistance of a given wire of uniform cross section to four times its value, a fraction of its length is stretched uniformly till the full length of the wire becomes $\frac{3}{2}$ times the original length what is the value of this fraction?
    (A) $\frac{1}{4}$ (B) $\frac{1}{8}$
    (C) $\frac{1}{16}$ (D) $\frac{1}{6}$
12. A conductor with rectangular cross section has dimensions ($a \times 2a \times 4a$) as shown in figure. Resistance across AB is x, across CD is y and across EF is z. Then



    (A) $x = y = z$ (B) $x > y > z$
    (C) $y > z > x$ (D) $x > z > y$
13. A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m^2.

70

The measured resistance of the wire is 5.0 Ω. The resistivity of the material is

(A) 1.8×10^{-6} Ω-m (B) 0.2×10^{-6} Ω-m

(C) 0.6×10^{-6} Ω-m (D) 0.9×10^{-6} Ω-m

14. The resistance of a wire of length 20 cm is 5 Ω. It is stretched uniformly to a length of 40 cm. The resistance now becomes:

(A) 5 Ω (B) 10 Ω

(C) 20 Ω (D) 200 Ω

15. Two wires each of radius of cross section r but of different materials are connected together end to end (in series). If the densities of charge carriers in the two wires are in the ratio 1:4, the drift velocity of electrons in the two wires will be in the ratio.

(A) 1:2 (B) 2:1

(C) 4:1 (D) 1:4

16. In a wire of cross-section radius r, free electrons travel with drift velocity v when a current I flows through the wire. What is the current in another wire of half the radius and of the same material when the drift velocity is $2v$?

(A) $2I$ (B) I

(C) $\frac{I}{2}$ (D) $\frac{I}{4}$

17. Read the following statements carefully:

Y: The resistivity of a semiconductor decreases with increases of temperature.

Z: In a conducting solid, the rate of collision between free electrons and ions increases with increase of temperature.

Select the correct statement from the following

(A) *Y* is true but *Z* is false

(B) *Y* is false but *Z* is true

(C) Both *Y* and *Z* are true

(D) *Y* is true and *Z* is the correct reason for *Y*

18. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

(A) each of them increases

(B) each of them decreases

(C) copper increases and germanium decreases

(D) copper decreases and germanium increases

19. An insulating pipe of cross-section area 'A' contains an electrolyte which has two types of ions → their charges being $-e$ and $+2e$. A potential difference applied between the ends of the pipe result in the drifting of the two types of ions, having drift speed = v (−ve ion) and $\frac{v}{4}$ (+ve ion). Both ions have the same number per unit volume = n. The current flowing through the pipe is

(A) nev $\frac{A}{2}$ (B) nev $\frac{A}{4}$

(C) 5 nev $\frac{A}{2}$ (D) 3 nev $\frac{A}{2}$

20. Two wires of the same material having radii in the ratio 1:2, carry currents in the ratio 4:1. The ratio of drift velocities of electrons in them is:

(A) 1:16 (B) 16:1

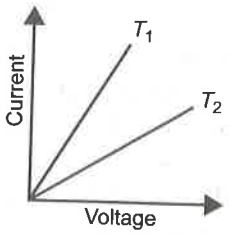
(C) 1:4 (D) 4:1

21. A current I flows through a uniform wire of diameter d when the mean electron drift velocity is V. The same current will flow through a wire of diameter $\frac{d}{2}$ made of the same material if the mean drift velocity of the electron is

(A) $\frac{v}{4}$ (B) $\frac{v}{2}$

(C) $2v$ (D) $4v$

22. The current in a metallic conductor is plotted against voltage at two different temperatures T_1 and T_2. Which is correct



(A) $T_1 > T_2$ (B) $T_1 < T_2$

(C) $T_1 = T_2$ (D) None

23. A uniform copper wire carries a current i amperes and has p carriers per metre^3. The length of the wire is ℓ metres and its cross-section area is s metre^2. If the charge on a carrier is q coulombs, the drift velocity in ms^{-1} is given by

(A) $\frac{i}{\ell sq}$ (B) $\frac{i}{psq}$

(C) $\frac{psq}{i}$ (D) $\frac{i}{ps\ell q}$

24. In the presence of an applied electric field ($\vec{E}$) in a metallic conductor.

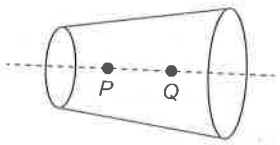
(A) The electrons move in the direction of $\vec{E}$

(B) The electrons move in a direction opposite to $\vec{E}$

(C) The electrons may move in any direction randomly, but slowly drift in the direction of $\vec{E}$

(D) The electrons move randomly but slowly drift in a direction opposite to $\vec{E}$

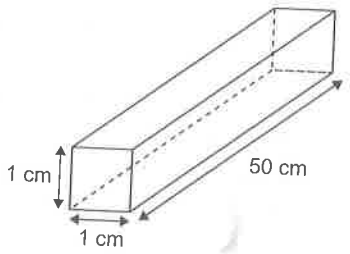
25. A wire has a non-uniform cross-section as shown in figure. A steady current flows through it. The drift speed of electrons at points P and q is v_p and v_Q.



(A) $v_p = v_Q$ (B) $v_p < v_Q$

(C) $v_p > v_Q$ (D) Data insufficient

26. A rectangular carbon block has dimensions 1.0 cm × 1.0 cm × 50 cm. Resistances are measured, first across two square ends and then across two rectangular ends, respectively. If resistivity of carbon is 3.5×10^{-5} Ω-m, then values of measured resistances respectively are:



(A) $\frac{35}{2} \times 10^{-2}\ \Omega, 7 \times 10^{-5}\ \Omega$

(B) $7 \times 10^{-5}\ \Omega, \frac{15}{2} \times 10^{-2}\ \Omega$

(C) $\frac{35}{2} \times 10^{-4}\ \Omega, 7 \times 10^{-7}\ \Omega$

(D) $\frac{15}{2}\ \Omega, 7 \times 10^{-2}\ \Omega$

27. A storage battery is connected to a charger for charging with a voltage of 12.5 V. The internal resistance of the storage battery is 1 Ω. When the charging current is 0.5 A, the emf of the storage battery is:

(A) 13 V (B) 12.5 V

(C) 12 V (D) 11.5 V

28. The terminal voltage across a battery of emf E can be

(A) 0 (B) $> E$

(C) $< E$ (D) All of above

29. In order to determine the emf of a storage battery it was connected in series with a standard cell in a certain circuit and a current I_1 was obtained. When the battery is connected to the same circuit opposite to the standard cell a current I_2 flow in the external circuit from the positive pole of the storage battery was obtained. What is the emf ε_1 of the storage battery? The emf of the standard cell is ε_2.

(A) $\varepsilon_1 = \frac{I_1 + I_2}{I_1 - I_2}\varepsilon_2$ (B) $\varepsilon_1 = \frac{I_1 + I_2}{I_2 - I_1}\varepsilon_2$

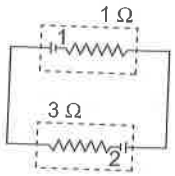
(C) $\varepsilon_1 = \frac{I_1 - I_2}{I_1 + I_2}\varepsilon_2$ (D) $\varepsilon_1 = \frac{I_2 - I_1}{I_1 + I_2}\varepsilon_2$

30. One end of a Nichrome wire of length $2L$ and cross-sectional area A is attached to an end of another Nichrome wire of length L and cross-sectional area $2A$. If the free end of the longer wire is at an electric potential of 8.0 V, and the free end of the shorter wire is at an electric potential of 1.0 V, the potential at the junction of the two wires is equal to

(A) 2.4 V (B) 3.2 V

(C) 4.5 V (D) 5.6 V

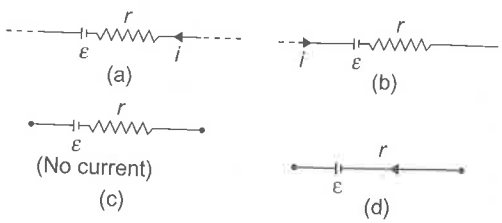
31. In the figure shown, battery 1 has emf = 6 V and internal resistance = 1 Ω. Battery 2 has emf = 2 V and internal resistance = 3 Ω. The wires have negligible resistance. What is the potential difference across the terminals of battery 2?



(A) 4 V (B) 1.5 V

(C) 5 V (D) 0.5 V

32.



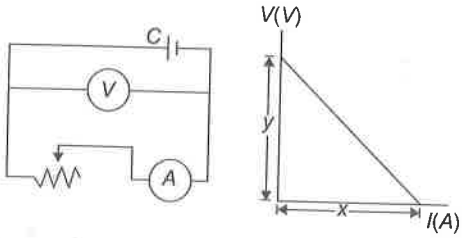
In which of the above cells, the potential difference between the terminals of a cell exceeds its emf.

(A) a (B) b

(C) c (D) d

72

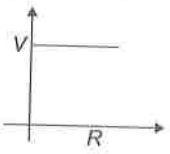
33. The diagram besides shows a circuit used in an experiment to determine the emf and internal resistance of the cell C. A graph was plotted of the potential difference V between the terminals of the cell against the current I, which was varied by adjusting the rheostat. The graph is shown on the right; x and y are the intercepts of the graph with the axes as shown. What is the internal resistance of the cell?

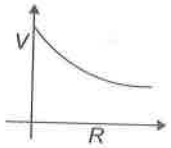


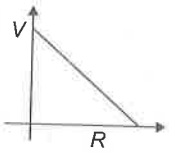
(A) x (B) y

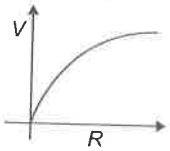
(C) $\frac{x}{y}$ (D) $\frac{y}{x}$

34. A cell of emf E has an internal resistance r and is connected to rheostat. When resistance R of rheostat is changed correct graph of potential difference across it is

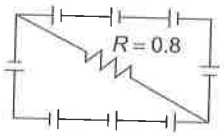
(A)


(B)


(C)


(D)


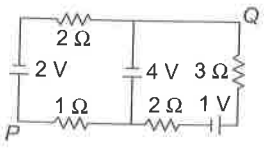
35. A circuit is comprised of eight identical batteries and a resistor $R = 0.8\ \Omega$. Each battery has an emf of 1.0 V and internal resistance of 0.2 Ω. The voltage difference across any of the battery is



(A) 0.5 V (B) 1.0 V

(C) 0 V (D) 2 V

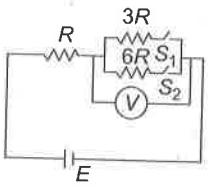
36. In the circuit shown, what is the potential difference V_{PQ}?



(A) +3 V (B) +2 V

(C) −2 V (D) None

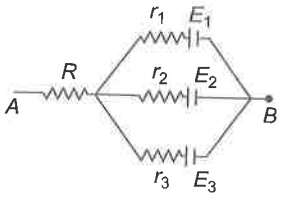
37. In the circuit shown in figure reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed. The reading of voltmeter is V_3 when both S_1 and S_2 are closed then



(A) $V_2 > V_1 > V_3$ (B) $V_3 > V_2 > V_1$

(C) $V_3 > V_1 > V_2$ (D) $V_1 > V_2 > V_3$

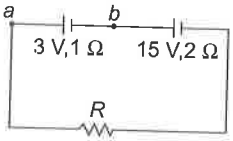
38. In the network shown the potential difference between A and B is ($R = r_1 = r_2 = r_3 = 1\ \Omega$, $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V)



(A) 1 V (B) 2 V

(C) 3 V (D) 4 V

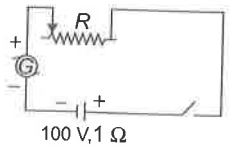
39. Two batteries one of the emf 3 V, internal resistance 1 ohm and the other of emf 15 V, internal resistance 2 ohm are connected in series with a resistance R as shown. If the potential difference between a and b is zero the resistance of R in ohm is



(A) 5 (B) 7

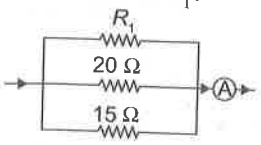
(C) 3 (D) 1

40. The battery in the diagram is to be charged by the generator G. The generator has a terminal voltage of 120 V when the charging current is 10 A. The battery has an emf of 100 V and an internal resistance of 1 Ω. In order to charge the battery at 10 A charging current, the resistance R should be set at



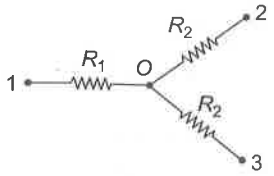
(A) 0.1 Ω (B) 0.5 Ω
(C) 1.0 Ω (D) 5.0 Ω

41. In the given circuit the current flowing through the resistance 20 Ω is 0.3 A while the ammeter reads 0.8 A. What is the value of R_1?



(A) 30 Ω (B) 40 Ω
(C) 50 Ω (D) 60 Ω

42. Find the current flowing through the resistance R_1 of the circuit shown in figure if the resistance are equal to $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $R_3 = 30\ \Omega$, and the potential of points 1, 2 and 3 are equal to $\varphi_1 = 10$ V, $\varphi_2 = 6$ V and $\varphi_3 = 5$ V.

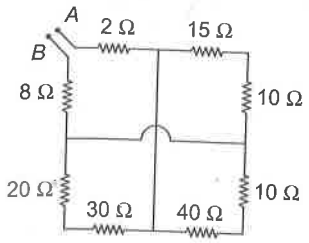


(A) 0.1 A (B) 0.2 A
(C) 0.3 A (D) 0.4 A

43. A piece of copper wire having a resistance R is cut into 10 pieces of equal length. These pieces are connected in parallel. The effective resistance of the combination will be:

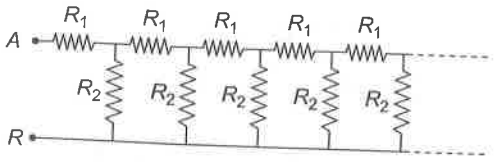
(A) $\frac{R}{100}$ (B) $\frac{R}{10}$
(C) $10R$ (D) $100R$

44. The equivalent resistance between points A and B is:



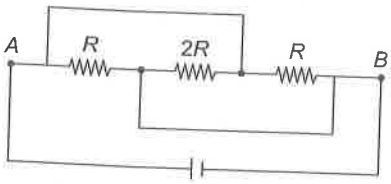
(A) $\frac{65}{2}\ \Omega$ (B) $\frac{45}{2}\ \Omega$
(C) $\frac{5}{2}\ \Omega$ (D) $\frac{91}{2}\ \Omega$

45. Consider an infinite ladder network shown in figure. A voltage V is applied between the points A and B. This applied value of voltage is halved after each section.



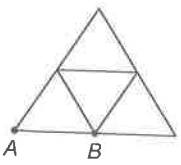
(A) $\frac{R_1}{R_2} = 1$ (B) $\frac{R_1}{R_2} = \frac{1}{2}$
(C) $\frac{R_1}{R_2} = 2$ (D) $\frac{R_1}{R_2} = 3$

46. In the figure shown the current flowing through $2R$ is:



(A) from left to right (B) from right to left
(C) no current (D) None of these

47. In the diagram resistance between any two junctions is R. Equivalent resistance across terminals A and B is

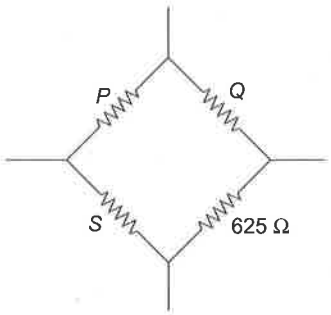


(A) $\frac{11R}{7}$ (B) $\frac{18R}{11}$
(C) $\frac{7R}{11}$ (D) $\frac{11R}{18}$

48. A line having a total resistance of 0.5 Ω deliver 15 kW at 240 V to a small factory. The efficiency of transmission is:

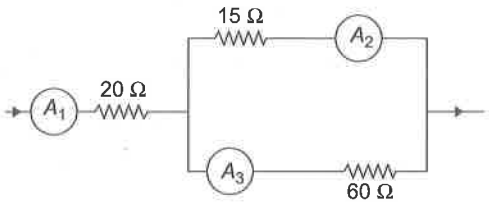
(A) 60% (B) 79%
(C) 89% (D) 97%

49. A Wheatstone's bridge is balanced with a resistance of 625 Ω in the third arm, where P, Q and S are in the 1st, 2nd and 4th arm respectively. If P and Q are interchanged, the resistance in the third arm has to be increased by 51 Ω to secure balance. The unknown resistance in the fourth arm is



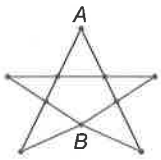
(A) 625 Ω (B) 650 Ω
(C) 676 Ω (D) 600 Ω

50. If the reading of ammeter A_3 in figure is 0.75 A. Neglecting the resistance of the ammeters, the reading of ammeter A_2 will be :



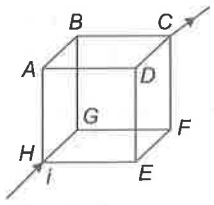
(A) 1.5 A (B) 3 A
(C) 4.5 A (D) 6 A

51. The resistance of all the wires between any two adjacent dots is R. Then equivalent resistance between A and B as shown in figure is:



(A) $\frac{7}{3}R$ (B) $\frac{7}{6}R$
(C) $\frac{14}{8}R$ (D) None of these

52. In the box shown current i enters at H and leaves at C. If $i_{AB} = \frac{i}{6}$, $i_{DC} = \frac{2i}{3}$, $i_{HA} = \frac{i}{2}$, $i_{GF} = \frac{i}{6}$, $i_{HE} = \frac{i}{6}$, choose the branch in which current is zero



(A) BG (B) FC
(C) ED (D) None

53. A resistor of resistance R is connected to a cell of internal resistance 5 Ω. The value of R is varied from 1 Ω to 5 Ω. The power consumed by R:
(A) increases continuously
(B) decreases continuously
(C) first decreases then increases
(D) first increases then decreases.

54. Power generated across a uniform wire connected across a supply is H. If the wire is cut into n equal parts and all the parts are connected in parallel across the same supply, the total power generated in the wire is

(A) $\frac{H}{n^2}$ (B) n^2H
(C) nH (D) $\frac{H}{n}$

55. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if
(A) both the length and the radius of the wire are halved
(B) both the length and the radius of the wire are doubled
(C) the radius of the wire is doubled
(D) the length of the wire is doubled

56. When electric bulbs of same power, but different marked voltage are connected in series across the power line, their brightness will be
(A) proportional to their marked voltage
(B) inversely proportional to their marked voltage
(C) proportional to the square of their marked voltage
(D) inversely proportional to the square of their marked voltage

57. Rate of dissipation of Joule's heat in resistance per unit volume is (symbols have usual meaning)
(A) σE (B) σJ
(C) JE (D) None

58. The charge flowing through a resistance R varies with time as $Q = 2t - 8t^2$. The total heat produced in the resistance is $\left(\text{for } 0 \le t \le \frac{1}{8}\right)$

(A) $\frac{R}{6}$ J (B) $\frac{R}{3}$ J
(C) $\frac{R}{2}$ J (D) RJ

59. If the length of the filament of a heater is reduced by 10%, the power of the heater will

(A) increase by about 9%
(B) increase by about 11%
(C) increase by about 19%
(D) decrease by about 10%

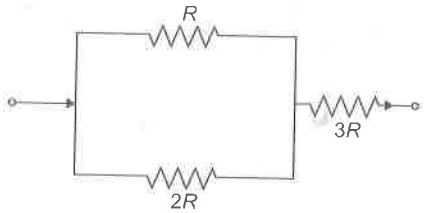
60. If the current in a electric bulb drops by 2% then the power decreases by
(A) 1% (B) 2%
(C) 4% (D) 16%

61. A heater A gives out 300 W of heat when connected to a 200 V DC supply. A second heater B gives out 600 W when connected to a 200 V DC supply. If a series combination of the two heaters is connected to a 200 V DC supply the heat output will be
(A) 100 W (B) 450 W
(C) 300 W (D) 200 W

62. Two bulbs one of 200 V, 60 W and the other of 200 V, 100 W are connected in series to a 200 V supply. The power consumed will be
(A) 37.5 W (B) 160 W
(C) 62.5 W (D) 110 W

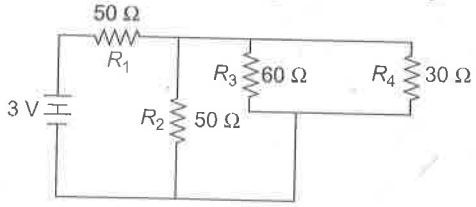
63. Three 60 W light bulbs are mistakenly wired in series and connected to a 120 V power supply. Assume the light bulbs are rated for single connection to 120 V. With the mistaken connection, the power dissipated by each bulb is
(A) 6.7 W (B) 13.3 W
(C) 20 W (D) 40 W

64. The ratio of powers dissipated respectively in R and $3R$, as shown is



(A) 9 (B) $\frac{27}{4}$
(C) $\frac{4}{9}$ (D) $\frac{4}{27}$

65. In the circuit shown, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3.0 V. The resistor that dissipates the most power is



(A) R_1 (B) R_2
(C) R_3 (D) R_4

66. The variation of current (I) and voltage (V) is as shown in figure A. The variation of power P with current I is best shown by which of the following graph

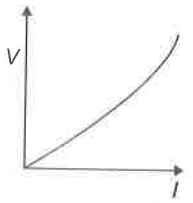
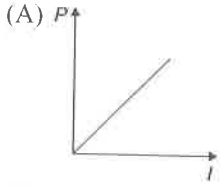
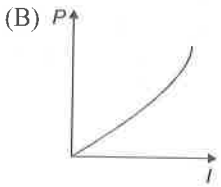
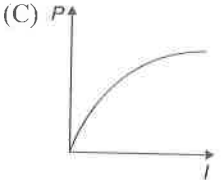
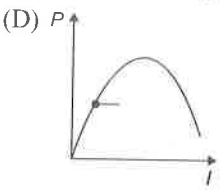


Figure (A)

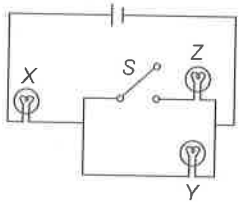
(A) 

(B) 

(C) 

(D) 

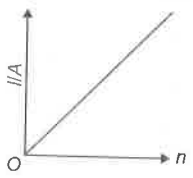
67. If X, Y and Z in figure are identical lamps, which of the following changes to the brightnesses of the lamps occur when switch S is closed?

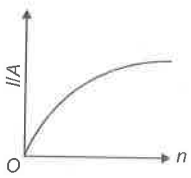


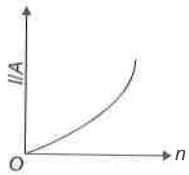
(A) X stays the same, Y decreases
(B) X increases, Y decreases
(C) X increases, Y stays the same
(D) X decreases, Y increases

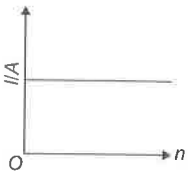
68. A battery consists of a variable number n of identical cells having internal resistance connected in series. The terminals of the battery are short circuited and the current I measured.

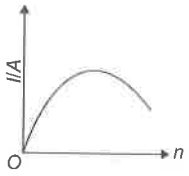
Which one of the graph below shows the relationship between I and n?

(A) 

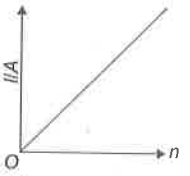
(B) 

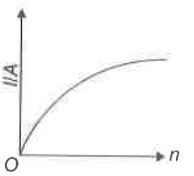
(C) 

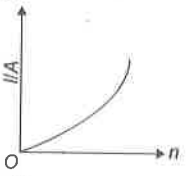
(D) 

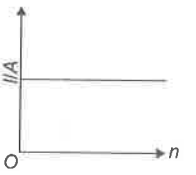
(E) 

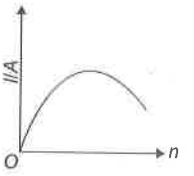
69. In previous problem, if the cell had been connected in parallel (instead of in series) which of the above graphs would have shown the relationship between total current I and n?

(A) 

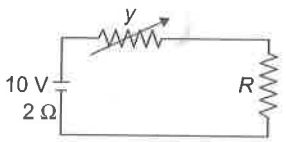
(B) 

(C) 

(D) 

(E) 

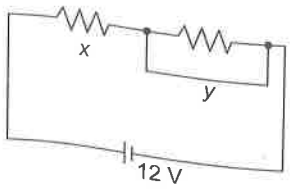
70. In the figure shown the power generated in y is maximum when $y = 5\ \Omega$. Then R is



(A) 2 Ω (B) 6 Ω
(C) 5 Ω (D) 3 Ω

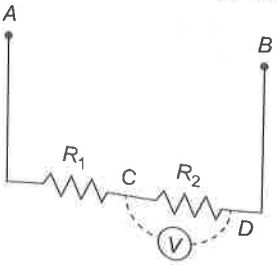
71. If an ammeter is to be used in place of a voltmeter then we must connect with the ammeter a
(A) Low resistance in parallel
(B) High resistance in parallel
(C) High resistance in series
(D) Low resistance in series

72. When an ammeter of negligible internal resistance is inserted in series with circuit it reads 1 A. When the voltmeter of very large resistance is connected across X it reads 1 V. When the point A and B are shorted by a conducting wire, the voltmeter measures 10 V across the battery. The internal resistance of the battery is equal to



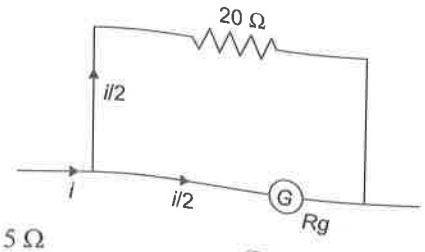
(A) zero
(B) 0.5 Ω
(C) 0.2 Ω
(D) 0.1 Ω

73. Resistances R_1 and R_2 each 60 Ω are connected in series as shown in figure. The Potential difference between A and B is kept 120 V. Then what will be the reading of voltmeter connected between the point C and D if resistance of voltmeter is 120 Ω.



(A) 48 V
(B) 24 V
(C) 40 V
(D) None

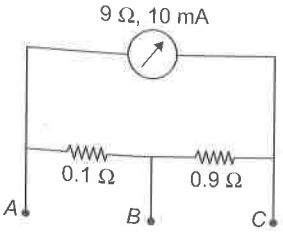
74. In a galvanometer, the deflection becomes one half when the galvanometer is shunted by a 20 Ω resistor. The galvanometer resistance is



(A) 5 Ω
(B) 10 Ω
(C) 40 Ω
(D) 20 Ω

75. A galvanometer has a resistance of 20 Ω and reads full-scale when 0.2 V is applied across it. To convert it into a 10 A ammeter, the galvanometer coil should have a
(A) 0.01 Ω resistor connected across it
(B) 0.02 Ω resistor connected across it
(C) 200 Ω resistor connected in series with it
(D) 2000 Ω resistor connected in series with it

76. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown. The metre gives full-scale deflection for current I when A and B are used as its terminals, i.e., current enters at A and leaves at B (C is left isolated). The value of I is

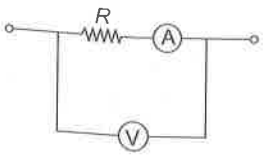


(A) 100 mA (B) 900 mA
(C) 1 A (D) 1.1 A

77. A galvanometer coil has a resistance 90 Ω and full scale deflection current 10 mA. A 910 Ω resistance is connected in series with the galvanometer to make a voltmeter. If the least count of the voltmeter is 0.1 V, the number of divisions on its scale is

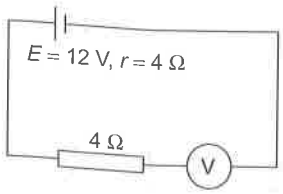
(A) 90 (B) 91
(C) 100 (D) None

78. In the circuit shown the resistance of voltmeter is 10,000 Ω and that of ammeter is 20 Ω. The ammeter reading is 0.10 A and voltmeter reading is 12 V. Then R is equal to



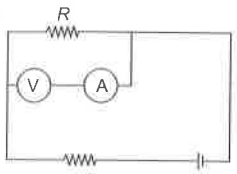
(A) 122 Ω (B) 140 Ω
(C) 116 Ω (D) 100 Ω

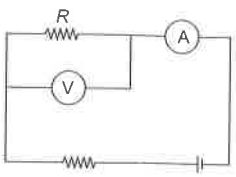
79. By error, a student places moving-coil voltmeter V (nearly ideal) in series with the resistance in a circuit in order to read the current, as shown. The voltmeter reading will be

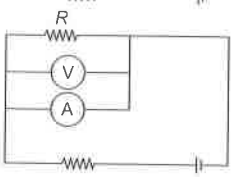


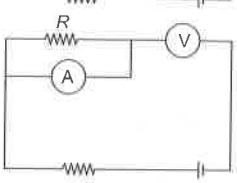
(A) 0 (B) 4 V
(C) 6 V (D) 12 V

80. Which of the following wiring diagrams could be used to experimentally determine R using ohm's law? Assume an ideal voltmeter and an ideal ammeter.

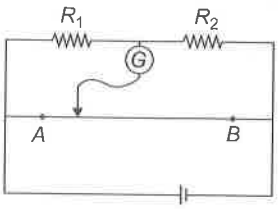
(A) 

(B) 

(C) 

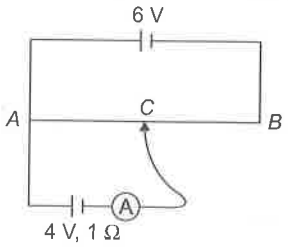
(D) 

81. In the figure shown for gives values of R_1 and R_2 the balance point for Jockey is at 40 cm from A. When R_2 is shunted by a resistance of 10 Ω, balance shifts to 50 cm. R_1 and R_2 are (AB = 1 m)



(A) $\frac{10}{3}$ Ω, 5 Ω (B) 20 Ω, 30 Ω
(C) 10 Ω, 15 Ω (D) 5 Ω, $\frac{15}{2}$ Ω

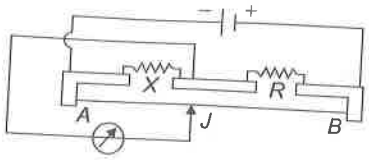
82. A 6 V battery of negligible internal resistance is connected across a uniform wire of length 1 m. The positive terminal of another battery of emf 4 V and internal resistance 1 Ω is joined to the point A as shown in figure. The ammeter shows zero deflection when the jockey touches the wire at the point C. The AC is equal to



(A) $\frac{2}{3}$ m (B) $\frac{1}{3}$ m
(C) $\frac{3}{5}$ m (D) $\frac{1}{2}$ m

83. The figure shows a metre-bridge circuit, with AB = 100 cm, X = 12 Ω and R = 18 Ω, and the jockey J in the position of balance. If R is now made 8 Ω, through what distance will J have to be moved to obtain balance?

78

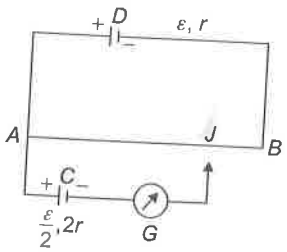


(A) 10 cm (B) 20 cm
(C) 30 cm (D) 40 cm

84. A potentiometer wire has length 10 m and resistance 10 Ω. It is connected to a battery of emf 11 V and internal resistance 1 Ω, then the potential gradient in the wire is
(A) 10 V/m (B) 1 V/m
(C) 0.1 V/m (D) None

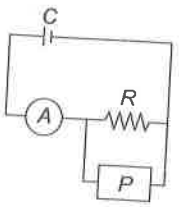
85. The length of a potentiometer wire is l. A cell of emf E is balanced at a length $\frac{l}{3}$ from the positive end of the wire. if the length of the wire is increased by $\frac{l}{2}$. At what distance will the same cell give a balance point.
(A) $\frac{2l}{3}$ (B) $\frac{l}{2}$
(C) $\frac{l}{6}$ (D) $\frac{4l}{3}$

86. In the figure, the potentiometer wire AB of length L and resistance $9r$ is joined to the cell D of emf ε and internal resistance r. The cell C's emf is $\frac{\varepsilon}{2}$ and its internal resistance is $2r$. The galvanometer G will show no deflection when the length AJ is



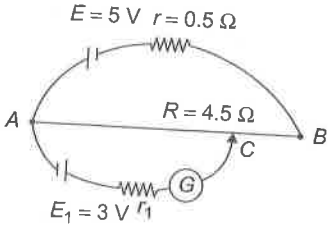
(A) $\frac{4L}{9}$ (B) $\frac{5L}{9}$
(C) $\frac{7L}{18}$ (D) $\frac{11L}{18}$

87. An ammeter A of finite resistance, and a resistor R are joined in series to an ideal cell C. A potentiometer P is joined in parallel to R. The ammeter reading is I_0 and the potentiometer reading is V_0. P is now replaced by a voltmeter of finite resistance. The ammeter reading now is I and the voltmeter reading is V.



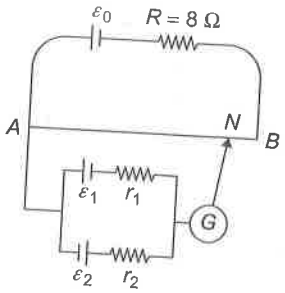
(A) $I > I_0, V < V_0$ (B) $I > I_0, V = V_0$
(C) $I = I_0, V < V_0$ (D) $I < I_0, V = V_0$

88. In the given potentiometer circuit length of the wire AB is 3 m and resistance is $R = 4.5\ \Omega$. The length AC for no deflection in galvanometer is



(A) 2 m (B) 1.8 m
(C) dependent on r_1 (D) None of these

89. A battery of emf $E_0 = 12$ V is connected across a 4 m long uniform wire having resistance 4 Ω/m. The cells of small emfs $\varepsilon_1 = 2$ V and $\varepsilon_2 = 4$ V having internal resistance 2 Ω and 6 Ω respectively, are connected as shown in the figure. If galvanometer shows no deflection at the point N, the distance of point N from the point A is equal to



(A) $\frac{1}{6}$ m (B) $\frac{1}{3}$ m
(C) 25 cm (D) 50 cm

90. Statement-1: When two conducting wires of different resistivity having same cross section area are joined in series, the electric field in them would be equal when they carry current.

Statement-2: When wires are in series they carry equal current.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

91. Statement-1: Potential difference across the terminals of a battery is always less than its emf.
Statement-2: A battery always has some internal resistance.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

92. Statement-1: Knowing that rating is done at steady state of the filament, an electric bulb connected to a source having rated voltage consumes more than rated power just after it is switched on.
Statement-2: When filament is at room temperature its resistance is less than its resistance when the bulb is fully illuminated

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

93. Statement-1: When a battery is supplying power to a circuit, work done by electrostatic forces on electrolyte ions inside the battery is positive
Statement-2: Electric field is directed from positive to negative electrode inside a battery

(A) Statement-1 **is** True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

94. Statement-1: Conductivity of a metallic conductor decreases with increase in temperature.
Statement-2: On increasing temperature the number of free electrons in the metallic conductar decreases.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

JEE Advanced

Single Correct Option Type Questions

1. A brass disc and a carbon disc of same radius are assembled alternatively to make a cylindrical conductor. The resistance of the cylinder is independent of the temperature. The ratio of thickness of the brass disc to that of the carbon disc is [α is temperature coefficient of resistance and Neglect linear expansion]

(A) $\left|\frac{\alpha_C \rho_C}{\alpha_B \rho_B}\right|$ (B) $\left|\frac{\alpha_C \rho_B}{\alpha_B \rho_C}\right|$

(C) $\left|\frac{\alpha_B \rho_C}{\alpha_C \rho_B}\right|$ (D) $\left|\frac{\alpha_B \rho_B}{\alpha_C \rho_C}\right|$

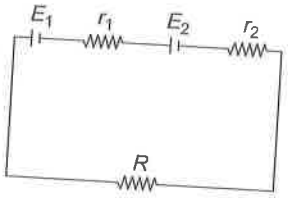
2. Current density in a cylindrical wire of radius R is given as $J = \begin{cases} J_0\left(\frac{x}{R} - 1\right) & \text{for } 0 \le x < \frac{R}{2} \\ J_0 \frac{x}{R} & \text{for } \frac{R}{2} \le x \le R \end{cases}$. The current flowing in the wire is

(A) $\frac{7}{24}\pi J_0 R^2$ (B) $\frac{1}{6}\pi J_0 R^2$

(C) $\frac{7}{12}\pi J_0 R^2$ (D) $\frac{5}{12}\pi J_0 R^2$

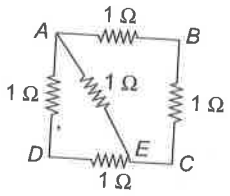
3. Under what condition current passing through the resistance R can be increased by short circuiting the battery of emf E_2. The internal resistances of the two batteries are r_1 and r_2 respectively.

80



(A) $E_2r_1 > E_1(R + r_2)$ (B) $E_1r_2 > E_2(R + r_1)$
(C) $E_2r_2 > E_1(R + r_2)$ (D) $E_1r_1 > E_2(R + r_1)$

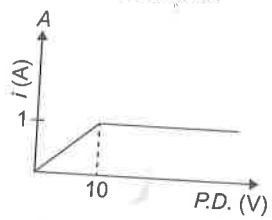
4. $ABCD$ is a square where each side is a uniform wire of resistance 1 Ω. A point E lies on CD such that if a uniform wire of resistance 1 Ω is connected across AE and constant potential difference is applied across A and C then B and E are equipotential.

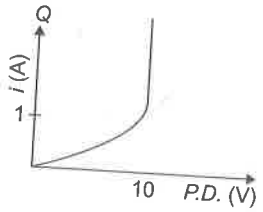


(A) $\frac{CE}{ED} = 1$ (B) $\frac{CE}{ED} = 2$

(C) $\frac{CE}{ED} = \frac{1}{\sqrt{2}}$ (D) $\frac{CE}{ED} = \sqrt{2}$

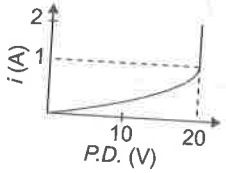
5. Two current elements P and Q have current voltage characteristics as shown below:



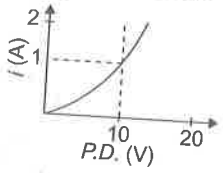


Which of the graphs given below represents current voltage characteristics when P and Q are in series.

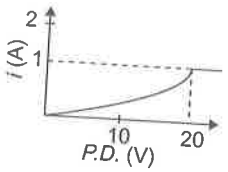
(A)



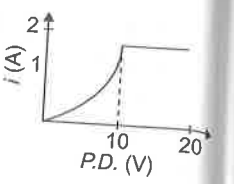
(B)



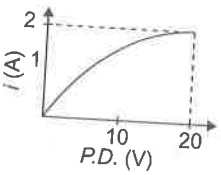
(C)



(D)



(E)



6. In a balanced wheat stone bridge, current in the galvanometer is zero. It remains zero when

(1) battery emf is increased
(2) all resistances are increased by 10 Ω
(3) all resistances are made five times
(4) the battery and the galvanometer are interchanged

(A) only (1) is correct
(B) (1), (2) and (3) are correct
(C) (1), (3) and (4) are correct
(D) (1) and (3) are correct

7. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases to zero uniformly during a time interval Δt

(A) $\frac{4}{3}\frac{q^2R}{\Delta t}$ (B) $\ln\frac{q^2R}{2\Delta t}$

(C) $\frac{2q^2R}{3\Delta t}$ (D) $\ln\frac{(2\Delta t)}{q^2R}$

8. n identical cells are joined in series with its two cells A and B in the loop with reversed polarities. emf of each shell is E and internal resistance r. Potential difference across cell A or B is (here $n > 4$)

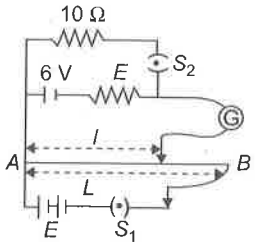
(A) $\frac{2E}{n}$ (B) $2E\left(1 - \frac{1}{n}\right)$

(C) $\frac{4E}{n}$ (D) $2E\left(1 - \frac{2}{n}\right)$

9. A wire of length L and 3 identical cells of negligible internal resistances are connected is series. Due to the current, the temperature of the wire is raised by ΔT in time t. N number of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is:

(A) 4 (B) 6
(C) 8 (D) 9

10. In the arrangement shown in figure when the switch S_2 is open, the galvanometer shows no deflection for $l = \frac{L}{2}$. When the switch S_2 is closed, the galvanometer shows no deflection for $l = \frac{5L}{12}$. The internal resistance (r) of 6 V cell, and the emf E of the other battery are respectively.

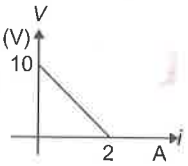


(A) 3 Ω, 8 V (B) 2 Ω, 12 V
(C) 2 Ω, 24 V (D) 3 Ω, 12 V

Multiple Correct Option Type Questions

11. A battery is of emf E is being charged from a charger such that positive terminal of the battery is connected to terminal A of charger and negative terminal of the battery is connected to terminal B of charger. The internal resistance of the battery is r.
(A) Potential difference across points A and B must be more than E
(B) A must be at higher potential than B
(C) In battery, current flows from positive terminal to the negative terminal
(D) No current flows through battery

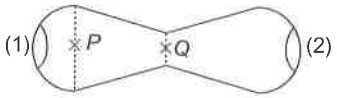
12. A battery of emf E and internal resistance r is connected across a resistance R. Resistance R can be adjusted to any value greater than or equal to zero. A graph is plotted between the current (i) passing through the resistance and potential difference (V) across it. Select the correct alternative (s).



(A) internal resistance of battery is 5 Ω
(B) emf of the battery is 20 V
(C) maximum current which can be taken from the battery is 4 A
(D) V-i graph can never be a straight line as shown in figure.

13. Which of the following quantities do not change when a resistor connected to a battery is heated due to the current?
(A) drift speed
(B) resistivity
(C) resistance
(D) number of free electrons.

14. A metallic conductor of irregular cross-section is as shown in the figure. A constant potential difference is applied across the ends (1) and (2). Then:

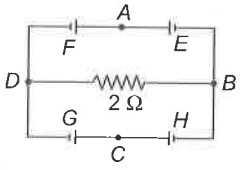


(A) the current at the cross-section P equals the current at the cross-section Q
(B) the electric field intensity at P is less than that at Q.
(C) the rate of heat generated per unit time at Q is greater than that at P
(D) the number of electrons crossing per unit area of cross-section at P is less than that at Q.

15. A current passes through a wire of nonuniform cross section. Which of the following quantities are independent of the cross-section?
(A) the charge crossing in a given time interval.
(B) drift speed
(C) current density
(D) free-electron density

16. A simple circuit contains an ideal battery and a resistance R. If a second resistor is placed in parallel with the first.
(A) the potential across R will decrease
(B) the current through R will decreased
(C) the current delivered by the battery will increase
(D) the power dissipated by R will increased.

17. In the circuit shown E, F, G and H are cells of emf 2 V, 1 V, 3 V and 1 V respectively and their internal resistances are 2 Ω, 1 Ω, 3 Ω and 1 Ω respectively.



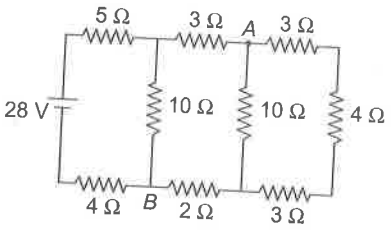
(A) $V_D - V_B = -\frac{2}{13}$ V

(B) $V_D - V_B = \frac{2}{13}$ V

(C) $V_G = \frac{21}{13}$ V = potential difference across G.

(D) $V_H = \frac{19}{13}$ V = potential difference across H.

18. Consider the circuit shown in the figure

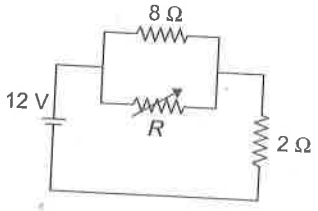


(A) the current in the 5 Ω resistor is 2 A
(B) the current in the 5 Ω resistor is 1 A
(C) the potential difference $V_A - V_B$ is 10 V
(D) the potential difference $V_A - V_B$ is 5 V

19. The equivalent resistance of a group of resistances is R. If another resistance is connected in parallel to the group, its new equivalent becomes R_1 and if it is connected in series to the group, its new equivalent becomes R_2 we have

(A) $R_1 > R$ (B) $R_1 < R$
(C) $R_2 > R$ (D) $R_2 < R$

20. The value of the resistance R in figure is adjusted such that power dissipated in the 2 Ω resistor is maximum. Under this condition



(A) $R = 0$
(B) $R = 8\ \Omega$
(C) power dissipated in the 2 Ω resistor is 72 W
(D) power dissipated in the 2 Ω resistor is 8 W

21. A bulb is connected to a battery of emf 10 V so that the resulting current is 10 mA. When the bulb is connected to 220 V mains, the current is 50 mA. Choose the correct alternative (s)

(A) In the first case, the resistance of the bulb is 1 kΩ and in second case, it is 4.4 kΩ
(B) It is not possible since ohm's law is not followed
(C) The increase in resistance is due to heating of the filament of the bulb when it is connected to 220 V mains
(D) None of these

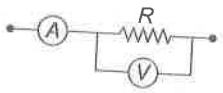
22. A galvanometer may be converted into ammeter or voltmeter. In which of the following cases the resistance of the device will be the largest? (Assume maximum range of galvanometer = 1 mA)

(A) An ammeter of range 10 A
(B) A voltmeter of range 5 V
(C) An ammeter of range 5 A
(D) A voltmeter of range 10 V

23. Mark out the correct options.

(A) An ammeter should have small resistance.
(B) An ammeter should have large resistance.
(C) A voltmeter should have small resistance.
(D) A voltmeter should have large resistance.

24. In the circuit shown the readings of ammeter and voltmeter are 4 A and 20 V respectively. The meters are non ideal, then R is:

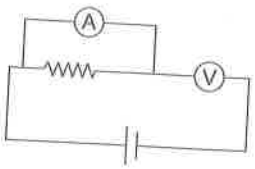


(A) 5 Ω (B) less than 5 Ω
(C) greater than 5 Ω (D) between 4 Ω and 5 Ω

25. A micrometer has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s).

(A) 50 V range with 10 kΩ resistance in series
(B) 10 V range with 200 kΩ resistance in series
(C) 5 mA range with 1 Ω resistance in parallel
(D) 10 mA range with 1 kΩ resistance in parallel.

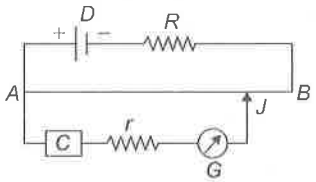
26. By mistake, a voltmeter is placed in series and an ammeter is parallel with a resistance in an electric circuit, with a cell in series.



(A) If the devices are ideal, ammeter will read zero current and voltmeter will read the emf of cell.

(B) If the devices are ideal, a large current will flow through the ammeter and it will be damaged.
(C) The main current in the circuit will be very low and practically all current will flow through the ammeter, if resistance of ammeter is much smaller than the resistance in parallel.
(D) The devices may get damaged if emf of the cell is very high and the meters and nonideal.

27. In the given potentiometer circuit, the resistance of the potentiometer wire AB is R_0. C is a cell of internal resistance r. The galvanometer G does not give zero deflection for any position of the Jockey J. Which of the following cannot be a reason for this?



(A) $r > R_0$
(B) $R >> R_0$
(C) emf of $C >$ emf of D
(D) The negative terminal of C is connected A.

28. In a potentiometer arrangement. E_1 is the cell establishing current in primary circuit. E_2 is the cell to be measured. AB is the potentiometer wire and G is a galvanometer. Which of the following are the essential condition for balance to be obtained.
(A) The emf of E_1 must be greater than the emf of E_2
(B) Either the positive terminals of both E_1 and E_2 or the negative terminals of both E_1 and E_2 must be joined to one end of potentiometer wire.
(C) The positive terminals of E_1 and E_2 must be joined to one end of potentiometer wire.
(D) The resistance of G must be less than the resistance of AB.

29. In a potentiometer wire experiment the emf of a battery in the primary circuit is 20 V and its internal resistance is 5 Ω. There is a resistance box in series with the battery and the potentiometer wire, whose resistance can be varied from 120 Ω to 170 Ω. Resistance of the potentiometer wire is 75 Ω. The following potential differences can be measured using this potentiometer.
(A) 5 V (B) 6 V
(C) 7 V (D) 8 V

30. Two indentical fuses are rated at 10 A. If they are joined
(A) in parallel, the combination acts as a fuse of rating 20 A
(B) in parallel, the combination acts as a fuse of rating 5 A
(C) in series, the combination acts as a fuse of rating 10 A
(D) in series, the combination acts as a fuse of rating 20 A

JEE Advanced

Single Correct Option Type Questions

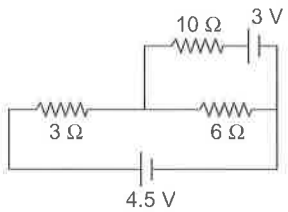
1. A current of 0.50 A is passing through a $CuSO_4$ solution. How many Cu^{++} ions will be deposited on cathode in 10 seconds?

2. A copper wire of radius 0.1 mm and resistance 1 kΩ is connected across a power supply of 20 V. (a) How many electrons are transferred per second between the supply and the wire at one end? (b) Write down the current density in the wire.

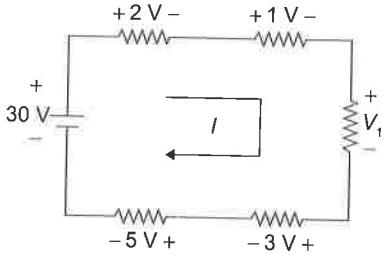
3. A wire has a length of 2.0 m and a resistance of 5.0 Ω. Find the electric field existing inside the wire if it carries a current of 10 A.

4. (a) A car has a fresh storage battery of emf 12 V and internal resistance 5.0×10^{-2} Ω. If the starter draws a current of 90 A, what is the terminal voltage of the battery when the starter is on?
(b) After long use, the internal resistance of the storage battery increases to 500 Ω. What maximum current can be drawn from the battery? Assume the emf of the battery to remains unchanged.
(c) If the discharged battery is charged by an external emf source, is the terminal voltage of the battery during charging greater of less than its emf 12 V?

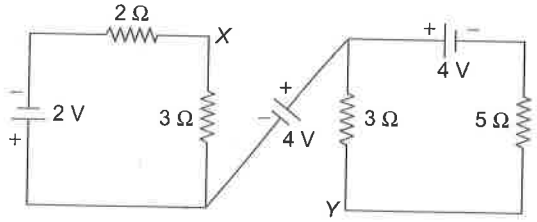
5. Find the current through the 10 Ω resistor shown in figure.



6. For the circuit shown in figure, determine the unknown voltage drop V_1.

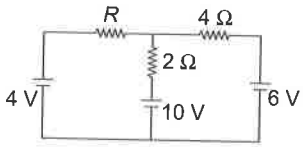


7. (a) Determine the potential difference between X and Y in the circuit shown in figure.

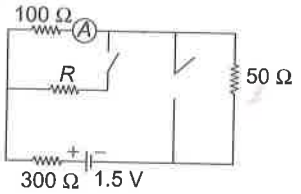


(b) If intermediate cell has internal resistance $r = 1\ \Omega$ then determine the potential difference between X and Y.

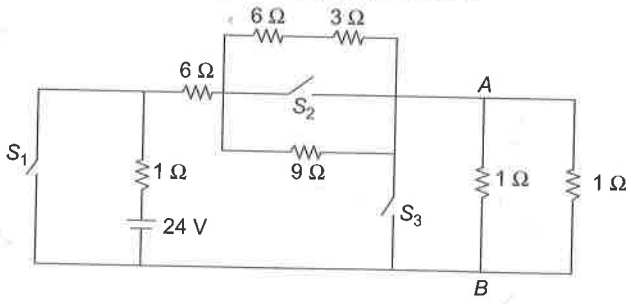
8. For what value of R in circuit, current through 4 Ω resistance is zero.



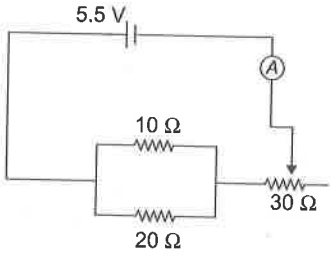
9. In the circuit shown in figure the reading of ammeter is the same with both switches open as with both closed. Then find the resistance R. (ammeter is ideal)



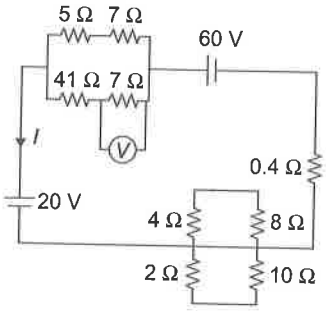
10. If the switches S_1, S_2 and S_3 in the figure are arranged such that current through the battery is minimum, find the voltage across points A and B.



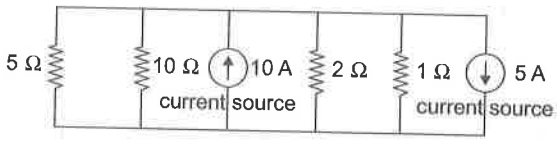
11. The resistance of the rheostat shown in figure is 30 Ω. Neglecting the meter resistance, find the minimum and maximum currents through the ammeter as the rheostat is varied.



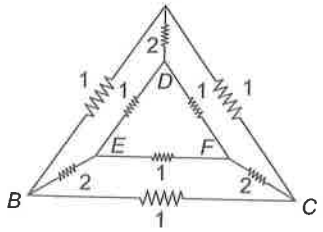
12. Find the current I and voltage V in the circuit shown.



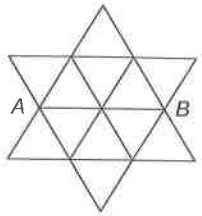
13. For the circuit shown in the figure, find the voltage across 10 Ω resistor and the current passing through it.



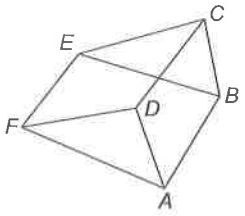
14. A network of nine conductors connects six points A, B, C, D, E and F as shown in figure. The figure denotes resistances in ohms. Find the equivalent resistance between A and D.



15. Find the equivalent resistance of the circuit between points A and B shown in figure is: (each branch is of resistance = 1 Ω)



16. In the circuit shown in figure, all wires have equal resistance r. Find the equivalent resistance between A and B.



17. In given circuit determine.

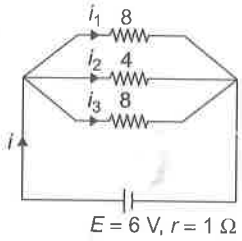
(a) The rate at which the chemical energy of the cell is consumed

(b) The rate at which heat is generated inside the battery

(c) Electric power output

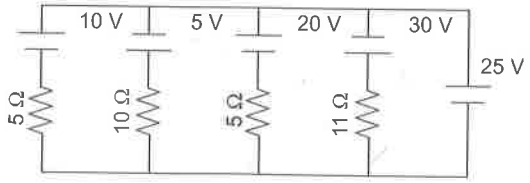
(d) Which resistance consumes maximum power

(e) Power dissipated across 4 Ω resistance



18. Three equal resistors connected in series across a source of emf together dissipate 10 watts of power. What would be the power dissipated if the same resistors are connected in parallel across the same source of emf?

19. Find the current through 25 V cell and power supplied by 20 V cell in the figure shown.



20. If a cell of constant emf produces the same amount of the heat during the same time in two in dependent resistors R_1 and R_2, when they are separately connected across the terminals of the cell, one after the another, find the internal resistance of the cell.

21. One kilowatt electric heater is to be used with 220 V.D.C supply.

(a) What is the current in the heater

(b) What is its resistance.

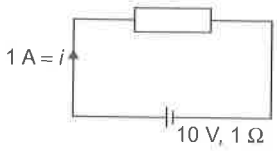
(c) What is the power dissipated in the heater.

(d) How much heat in calories is produced per second.

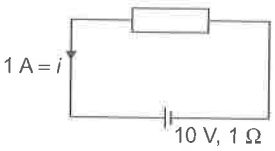
(e) How many grams of water at 100°C will be converted per minute into steam at 100°C with the heater. [(latent heat of vaporisation of water = 540 cal/g)]

22. The efficiency of a cell when connected to a resistance R is 60%. What will be its efficiency if the external resistance is increased to six times.

23. In following diagram boxes may contain resistor or battery or any other element



(A)



(B)

then determine in each case

(A) emf of battery

(B) Battery is acting as a source or load

(C) Potential difference across each battery.

(D) Power input to the battery or output by the battery.

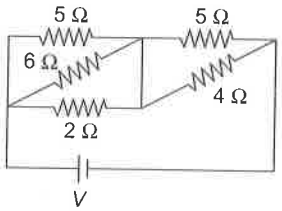
(E) The rate at which heat is generated inside the battery.

(F) The rate at which the chemical energy of the cell is consumed or increased.

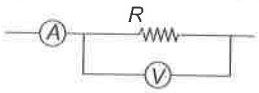
(G) Potential difference across box.

(H) Electric power output across box.

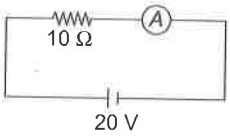
24. Find the resistor in which maximum heat will be produced.



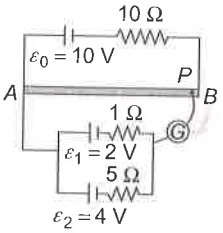
25. A part of a circuit is shown in figure. Here reading of ammeter is 5 A and voltmeter is 96 V and voltmeter resistance is 480 Ω. Then find the resistance R



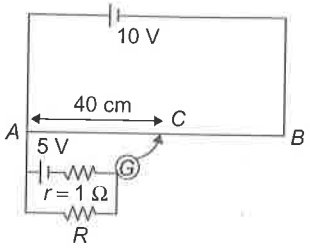
26. The ammeter shown in figure consists of a 480 Ω coil connected in parallel to a 20 Ω shunt. Find the reading of the ammeter.



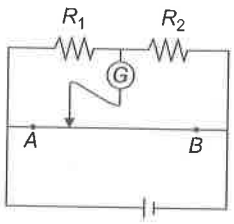
27. A battery of emf $\varepsilon_0 = 10$ V is connected across a 1 m long uniform wire having resistance 10 Ω/m. Two cells of emf $\varepsilon_1 = 2$ V and $\varepsilon_2 = 4$ V having internal resistances 1 Ω and 5 Ω respectively are connected as shown in the figure. If a galvanometer shows no deflection at the point P, find the distance of point P from the point a.



28. A potentiometer wire AB is 100 cm long and has a total resistance of 10 ohm. If the galvanometer shows zero deflection at the position C, then find the value of unknown resistance R:



29. In the figure shown for which values of R_1 and R_2 the balance point for Jockey is at 40 cm from A. When R_2 is shunted by a resistance of 10 Ω, balance shifts to 50 cm. Find R_1 and R_2. (AB = 1 m)



30. An accumulator of emf 2 V and negligible internal resistance is connected across a uniform wire of length 10 m and resistance 30 Ω. The appropriate terminals of a cell of emf 1.5 V and internal resistance 1 Ω is connected to one end of the wire, and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire. What length of the wire will be required to produce zero deflection of the galvanometer? How will the balancing change (a) when a coil of resistance 5 Ω is placed in series with the accumulator, (b) the cell of 1.5 V is shunted with 5 Ω resistor?

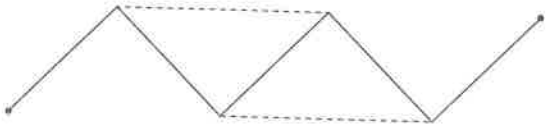
Subjective Questions

1. A triangle is constructed using the wires AB, BC and CA of same material and of resistance α, 2α and 3α respectively. Another wire of resistance $\frac{\alpha}{3}$ from A can make a sliding contact with wire BC. Find the maximum resistance of the network between points A and the point of sliding wire with BC.

2. (a) The current density across a cylindrical conductor of radius R varies according to the equation $J = J_0\left(1 - \frac{r}{R}\right)$, where r is the distance from the axis. Thus the current density is a maximum J_0 at the axis $r = 0$ and decreases linearly to zero at the surface $r = R$. Calculate the current in terms of J_0 and the conductor's cross sectional area is $A = \pi R^2$.

(b) Suppose that instead the current density is a maximum J_0 at the surface and decreases linearly to zero at the axis so that $J = J_0 \frac{r}{R}$. Calculate the current.

3. What will be the change in the resistance of a circuit consisting of five identical conductors if two similar conductors are added as shown by the dashed line in figure

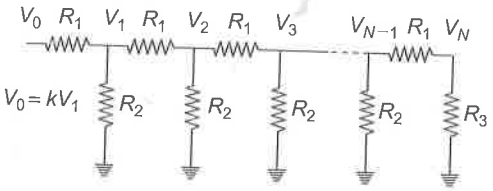


4. The current I through a rod of a certain metallic oxide is given by $I = 0.2\,V^{5/2}$, where V is the potential difference across it. The rod is connected in series with a resistance to a 6 V battery of negligible internal resistance. What value should the series resistance have so that
   (i) the current in the circuit is 0.44
   (ii) the power dissipated in the rod is twice that dissipated in the resistance.

5. A piece of resistive wire is made up into two squares with a common side of length 10 cm. A current enters the rectangular system at one of the corners and leaves at the diagonally opposite corners. Show that the current in the common side is 1/5th of the entering current. What length of wire connected between input and output terminals would have an equivalent effect.

6. A network of resistance is constructed with R_1 and R_2 as shown in the figure. The potential at the points 1, 2, 3,..., N and V_1, V_2, V_3,..., V_n respectively each having a potential k time smaller than previous one.

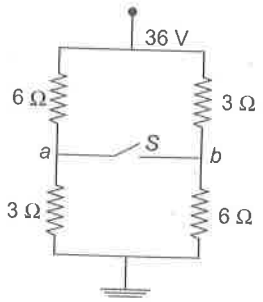
   Find (i) $\frac{R_1}{R_2}$ and $\frac{R_2}{R_3}$ in terms of k



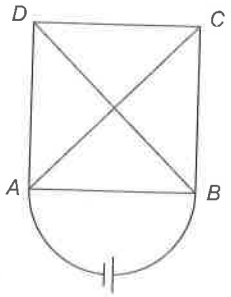
   (ii) current that passes through the resistance R_2 nearest to the V_0 in terms V_0, k and R_3.

7. A person decides to use his bath tub water to generate electric power to run a 40 W bulb. The bath tube is located at a height of 10 m from the ground and it holds 200 litres of water. If we install a water driven wheel generator on the ground, at what rate should the water drain from the bath tube to light bulb? How long can we keep the bulb on, if the bath tub was full initially. The efficiency of generator is 90%. ($g = 10$ m/s^{-2})

8. In the circuit shown in figure, calculate the following
   (i) Potential difference between points a and b when switch S is open.
   (ii) Current through S in the circuit when S is closed.



9. The circuit shown in figure is made of a homogeneous wire of uniform cross-section. $ABCD$ is a square. Find the ratio of the amounts of heat liberated per unit time in wire A-B and C-D.



10. A rod of length L and cross-section area A lies along the x-axis between $x = 0$ and $x = L$. The material obeys Ohm's law and its resistivity varies along the rod according to $\rho(x) = \rho_0 e^{-x/L}$. The end of the rod at $x = 0$ is at a potential V_0 and it is zero at $x = L$.
    (a) Find the total resistance of the rod and the current in the wire.
    (b) Find the electric potential in the rod as a function of x.

11. An ideal cell having a steady emf of 2 V is connected across the potentiometer wire of length 10 m. The potentiometer wire is of magnesium and having resistance of 11.5 Ω/m. An another cell gives a null point at 6.9 m. If a resistance of 5 Ω is put in series with potentiometer wire, find the new position of the null point.

12. An enquiring physics student connects a cell to a circuit and measures the current drawn from the cell to I_1. When he joins a second identical cell is series with the first, the current becomes I_2. When the cells are connected are in parallel, the current through the circuit is I_3. Show that relation between the current is $3I_3I_2 = 2I_1(I_2 + I_3)$.

(A) 40 Ω (B) 20 Ω
(C) 400 Ω (D) 200 Ω

16. An electric bulb is rated 220 V-100 W. The power consumed by it when operated on 110 V will be **[AIEEE-2006]**

(A) 75 W (B) 40 W
(C) 25 W (D) 50 W

17. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200 Ω at temperature of **[AIEEE-2006]**

(A) 300°C (B) 400°C
(C) 500°C (D) 200°C

18. In a Wheatstone's bridge, three resistances P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be **[AIEEE-2006]**

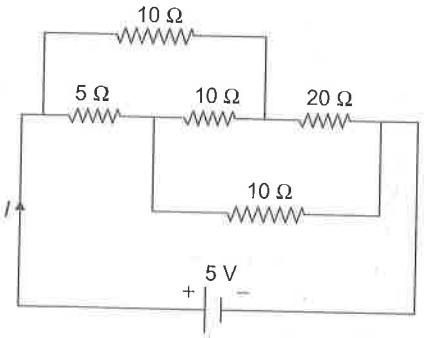
(A) $\frac{P}{Q} = \frac{2R}{S_1 + S_2}$ (B) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$

(C) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{2S_1 S_2}$ (D) $\frac{P}{Q} = \frac{R}{S_1 + S_2}$

19. The Kirchhoff's first $(\Sigma i = 0)$ law and second law $(\Sigma iR = \Sigma E)$
When the symbols have their usual meanings, are respectively based on **[AIEEE-2006]**

(A) Conservation of charge, conservation of momentum
(B) Conservation of energy, conservation of charge
(C) Conservation of momentum, conservation of charge
(D) Conservation of charge, conservation of energy

20. The current I drawn from the 5 V source will be **[AIEEE-2006]**



(A) 0.33 A (B) 0.5 A
(C) 0.67 A (D) 0.17 A

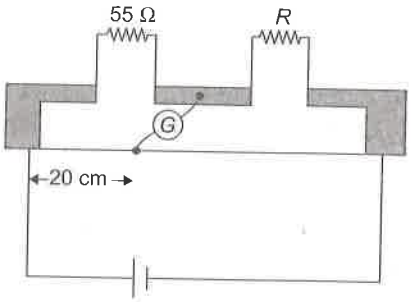
21. The resistance of a wire is 5 Ω at 50°C and 6 Ω at 100°C. The resistance of the wire at 0°C will be **[AIEEE-2007]**

(A) 2 Ω (B) 1 Ω
(C) 4 Ω (D) 3 Ω

22. A material B has twice the specific resistance of A. A circular wire made of B has twice the diameter of a wire made of A. Then for the two wires to have the same resistance, the ratio $\frac{l_B}{l_A}$ of their respective lengths must be **[AIEEE-2007]**

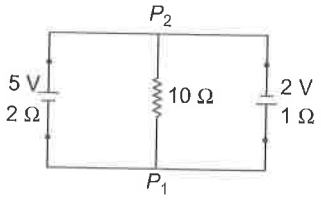
(A) 1 (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) 2

23. Shown in the figure adjacent is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is **[AIEEE-2008]**



(A) 13.75 Ω (B) 220 Ω
(C) 110 Ω (D) 55 Ω

24. A 5 V battery with internal resistance 2 Ω and a 2 V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. **[AIEEE-2008]**



The current in the resistor is

(A) 0.27 A, P_2 to P_1 (B) 0.03 A, P_1 to P_2
(C) 0.03 A, P_2 to P_1 (D) 0.27 A, P_1 to P_2

25. This question contains Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I: The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω

when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}/°C$.

Statement II: $R = R_i(1 + \alpha\Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) \ll R_0$. **[AIEEE-2009]**

(A) Statement I is true, Statement II is false.
(B) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I.
(C) Statement I is true. Statement II is true; Statement II is not the correct explanation of Statement I.
(D) Statement I is false, Statement II is true.

26. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly **[AIEEE-2010]**

(A) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$ (B) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(C) $\alpha_1 + \alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$ (D) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

27. If 400 Ω of resistance is made by adding four 100 Ω resistance of tolerance 5%, then the tolerance of the combination is **[AIEEE-2011]**

(A) 20% (B) 5%
(C) 10% (D) 15%

28. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} Ω-m and 8×10^{-7} m^2 respectively. The potential gradient will be equal to **[AIEEE-2011]**

(A) 0.2 V/m (B) 1 V/m
(C) 0.3 V/m (D) 0.1 V/m

29. If a wire is stretched to make it 0.1% longer, its resistance will **[AIEEE-2011]**

(A) increase by 0.2% (B) decrease by 0.2%
(C) decrease by 0.05% (D) increase by 0.05%

30. Two electric bulbs marked 25 W-220V and 100 W-220V are connected in series to a 440V supply. Which of the bulbs will fuse? **[AIEEE-2012]**

(A) Both (B) 100 W
(C) 25 W (D) Neither

31. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then errors in the value of resistance of the wire is **[AIEEE-2012]**

(A) 6% (B) zero
(C) 1% (D) 3%

32. In a large building, there are 15 bulbs of 40 W, 5 bulbs 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be:

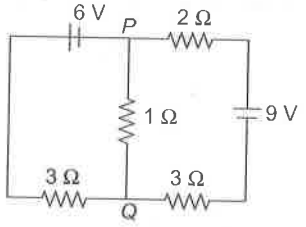
(A) 12 A (B) 14 A
(C) 8 A (D) 10 A

[JEE MAIN-2014]

33. When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to: **[JEE MAIN-2015]**

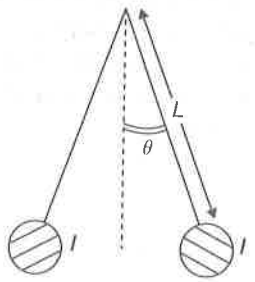
(A) 1.6×10^{-6} Ωm (B) 1.6×10^{-5} Ωm
(C) 1.6×10^{-8} Ωm (D) 1.6×10^{-7} Ωm

34. In the circuit shown, the current in the 1 Ω resistor is: **[JEE MAIN-2015]**



(A) 0.13 A, from Q to P (B) 0.13 A, from P to Q
(C) 1.3 A, from P to Q (D) 0A

35. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is: (g = gravitational acceleration) **[JEE MAIN-2015]**



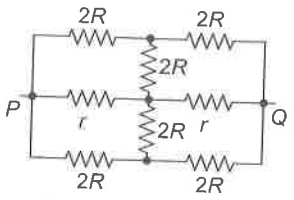
(A) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (B) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

(C) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (D) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

92

JEE Advanced

1. The effective resistance between the points P and Q of the electrical circuit shown in the figure is

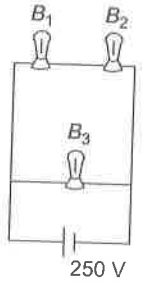


(A) $\frac{2Rr}{(R+r)}$ (B) $\frac{8R(R+r)}{(3R+r)}$

(C) $2r+4R$ (D) $\frac{5R}{2+2r}$

[JEE-2002]

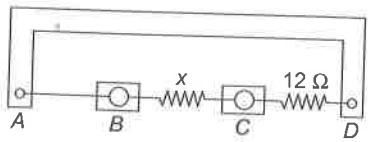
2. A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source, as shown in the figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3 respectively. Then: **[JEE-2002]**



(A) $W_1 > W_2 = W_3$ (B) $W_1 > W_2 > W_3$

(C) $W_1 < W_2 = W_3$ (D) $W_1 < W_2 < W_3$

3. A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following question.

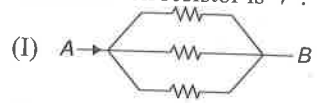


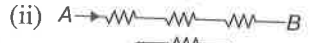
(A) Are there positive and negative terminals on the galvanometer?

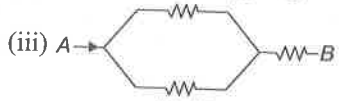
(B) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

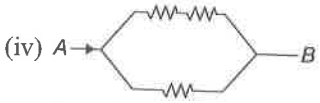
(C) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of resistance X. **[JEE-2002]**

4. Arrange the order of power dissipated in the given circuits, if the same current is passing through all circuit and each resistor is 'r': **[JEE-2003]**

(I) 

(ii) 

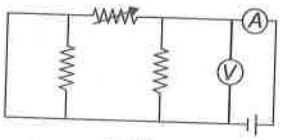
(iii) 

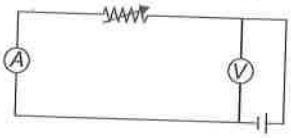
(iv) 

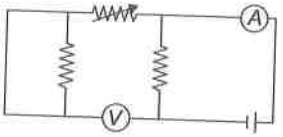
(A) $P_2 > P_3 > P_4 > P_1$ (B) $P_3 > P_2 > P_4 > P_1$

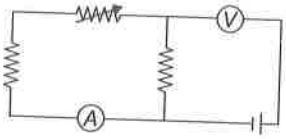
(C) $P_4 > P_3 > P_2 > P_1$ (D) $P_1 > P_2 > P_3 > P_4$

5. Which of the following set-up can be used to verify Ohm's law? **[JEE-2003]**

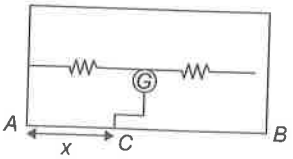
(A) 

(B) 

(C) 

(D) 

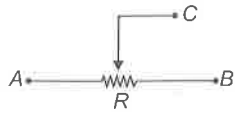
6. In the given circuit, no current is passing through the galvanometer. If the cross-sectional diameter of AB is doubled the for null point of galvanometer the value of AC would **[JEE-2003]**



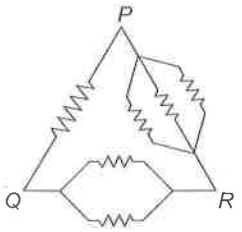
(A) x (B) $\frac{x}{2}$

(C) $2x$ (D) none

7. How a battery is to be connected so that shown rheostat will behave like a potential divider? Also indicate the points about which output can be taken. **[JEE-2003]**



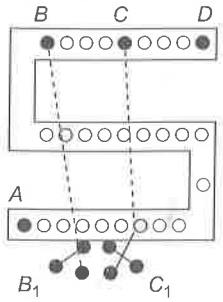
8. Six equal resistances are connected between points P, Q and R as shown in the figure. Then the net resistance will be maximum between **[JEE-2004]**



(A) P and Q (B) Q and R

(C) P and R (D) any two points

9. For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between **[JEE-2004]**

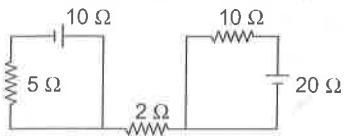


(A) B and C (B) C and D

(C) A and D (D) B_1 and C_1

10. Draw the circuit for experimental verification of Ohm's law using a source of variable D.C. voltage, a main resistance of 100 Ω, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter. **[JEE-2004]**

11. In the figure shown the current through 2 Ω resistor is



(A) 2 A (B) 0 A

(C) 4 A (D) 6 A

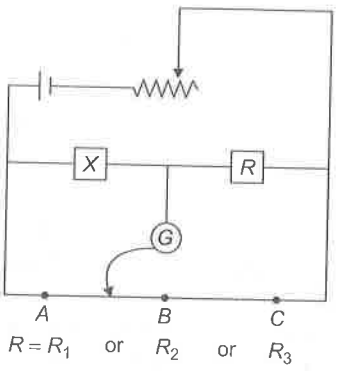
[JEE-2004]

12. A galvanometer has resistance 100 Ω and it requires current 100 μA for full scale deflection. A resistor 0.1 Ω is connected to make it an ammeter. The smallest current required in the circuit to produce the full scale deflection is **[JEE-2005]**

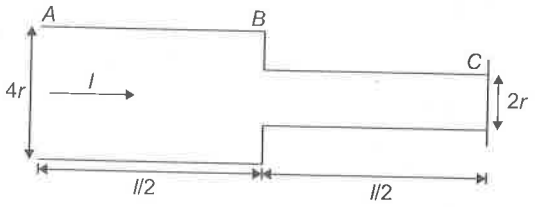
(A) 1000.1 mA (B) 1.1 mA

(C) 10.1 mA (D) 100.1 mA

13. An unknown resistance X is to be determined using resistances R_1, R_2 or R_3. Their corresponding null points are A, B and C. Find which of the above will give the most accurate reading and why? **[JEE-2005]**



14. Consider a cylindrical element as shown in the figure. Current flowing the through element is I and resistivity of material of the cylinder is ρ. Choose the correct option out the following **[JEE-2006]**



(A) Power loss in second half is four times the power loss in first half

(B) Voltage drop in first half is twice of voltage drop in second half

(C) Current density in both halves are equal

(D) Electric field in both halves is equal

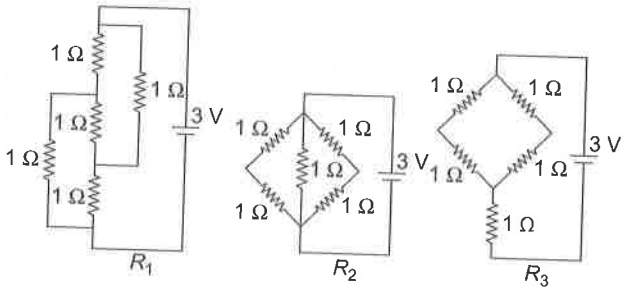
15. A resistance of 2 Ω is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances

94

are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is **[JEE-2007]**

(A) 3 Ω (B) 4 Ω
(C) 5 Ω (D) 6 Ω

16. Figure shows three resistor configurations R_1, R_2 and R_3 connected to 3 V battery. If the power dissipated by the configuration R_1, R_2 and R_3 is P_1, P_2 and P_3, respectively, then



[JEE-2008]

(A) $P_1 > P_2 > P_3$ (B) $P_1 > P_3 > P_2$
(C) $P_2 > P_1 > P_3$ (D) $P_3 > P_2 > P_1$

17. STATEMENT-1

In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

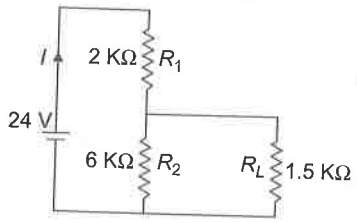
STATEMENT-2

Resistance of a metal increases with increase in temperature.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

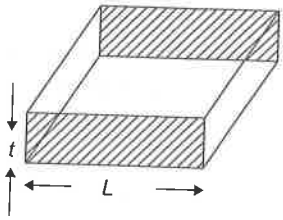
[JEE-2008]

18. For the circuit shown in the figure



(A) the current I through the battery is 7.5 mA
(B) the potential difference across R_L is 18 V
(C) ratio of powers dissipated in R_1 and R_2 is 3
(D) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9 **[JEE-2009]**

19. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces shown by the shaded area in the figure is: **[JEE-2010]**



(A) directly proportional to L
(B) directly proportional to t
(C) independent of L
(D) independent of t

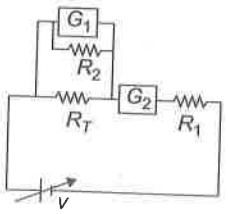
20. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40} respectively, the relation between these resistance is **[JEE-2010]**

(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (B) $R_{100} = R_{40} + R_{60}$

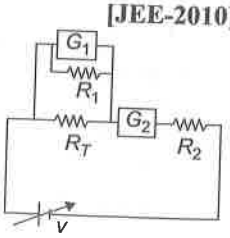
(C) $R_{100} > R_{60} > R_{40}$ (D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

21. To verify Ohm's law, a student is provided with a test resistor R_T. A high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2 and a variable voltage source V. The correct circuit to carry out the experiment is: **[JEE-2010]**

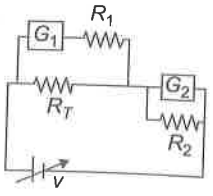
(A)



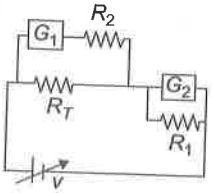
(B)



(C)



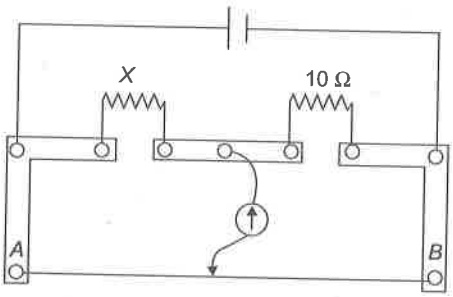
(D)



22. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\, J_2$ then the value of R in Ω is

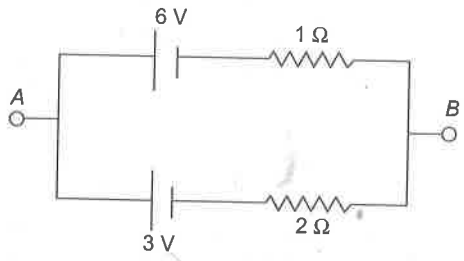
[JEE-2010]

23. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 Ω resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is **[JEE-2011]**

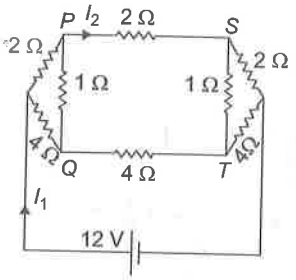


(A) 10.2 Ω (B) 10.6 Ω
(C) 10.8 Ω (D) 11.1 Ω

24. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is **[JEE-2011]**

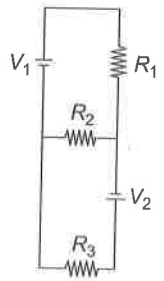


25. For the resistance network shown in the figure, choose the correct option(s). **[JEE 2012]**



(A) The current through PQ is zero
(B) $I_1 = 3$ A
(C) The potential at S is less than that at Q
(D) $I_2 = 2$ A

26. Two ideal batteries of emf V_1 and V_2 and these resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

[JEE-2014]

27. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. Howe much time in minutes will it take to raise the temperature of the same amount of water by 40 K?
(A) 4 if wires are in parallel
(B) 2 if wires are in series
(C) 1 if wires are in series
(D) 0.5 if wires are in parallel

[JEE-2014]

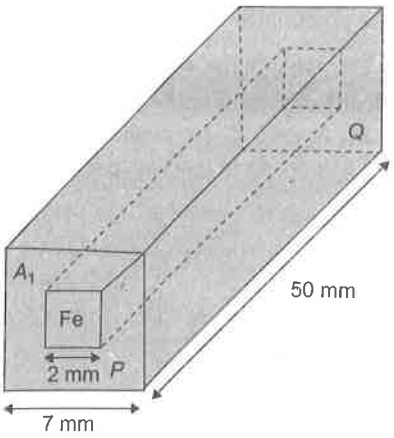
28. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0–30 V. If connected to a $\frac{2n}{246}\Omega$ resistance, it becomes an ammeter of range 0–1.5 A. The value of n is

[JEE-2014]

29. In an aluminum (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ωm and 1.0×10^{-7} WΩm, respectively. The electrical resistance between the two faces P and Q of the composite bar is

[JEE Advanced-2015]

96

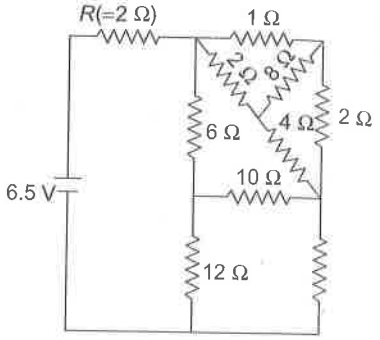


(A) $\frac{2475}{64}\mu\Omega$ (B) $\frac{1875}{64}\mu\Omega$

(C) $\frac{1875}{49}\mu\Omega$ (D) $\frac{2475}{132}\mu\Omega$

30. In the following circuit, the current through the resistor R (= 2 Ω) is I A. The value of I is

[JEE Advanced 201



ANSWERS KEYS

Exercises

JEE Main

1. C	**2.** D	**3.** C	**4.** B	**5.** C	**6.** C	**7.** B	**8.** D	**9.** B	**10.** B
11. B	**12.** D	**13.** B	**14.** C	**15.** C	**16.** C	**17.** C	**18.** D	**19.** D	**20.** B
21. D	**22.** B	**23.** B	**24.** D	**25.** C	**26.** A	**27.** C	**28.** D	**29.** A	**30.** A
31. A	**32.** B	**33.** D	**34.** D	**35.** C	**36.** B	**37.** A	**38.** B	**39.** C	**40.** C
41. D	**42.** B	**43.** A	**44.** B	**45.** B	**46.** B	**47.** D	**48.** C	**49.** B	**50.** B
51. B	**52.** B	**53.** A	**54.** B	**55.** B	**56.** C	**57.** C	**58.** A	**59.** B	**60.** C
61. D	**62.** A	**63.** A	**64.** D	**65.** A	**66.** B	**67.** B	**68.** D	**69.** A	**70.** D
71. C	**72.** C	**73.** A	**74.** D	**75.** B	**76.** C	**77.** C	**78.** D	**79.** D	**80.** B
81. A	**82.** A	**83.** B	**84.** B	**85.** B	**86.** B	**87.** A	**88.** D	**89.** C	**90.** D
91. D	**92.** A	**93.** D	**94.** C						

JEE Advanced

Single Correct Option-type Questions

1. A	**2.** D	**3.** B	**4.** D	**5.** C	**6.** C	**7.** A	**8.** D	**9.** B
10. B	**11.** A, B, C	**12.** A	**13.** D	**14.** A, B, C, D	**15.** A, D	**16.** C	**17.** A, C, D	**18.** A
19. C	**20.** A, C	**21.** A, C	**22.** D	**23.** A, D	**24.** C	**25.** B, C	**26.** A, C, D	**27.** A
28. A, B	**29.** A, B, C	**30.** A, C						

Multiple Correct Option Type Questions

1. 1.5625×10^{19} **2.** 1.25×10^{-17}, **(b)** $\frac{2}{\pi} \times 10^6$ A/m^2 **3.** 25 V/m

4. (a) 7.5 V, **(b)** 24 mA **(c)** greater than 12 V

5. zero **6.** 19 V **7. (a)** 3.7 V **(b)** 3.7 V **8.** 1 Ω **9.** 600 Ω

10. 1 V **11.** $\frac{3}{20} = 0.15$ A, $\frac{3 \times 5.5}{20} = 0.83$ A **12.** $I = 2.5$ A, $V = 3.5$ V

13. $\frac{25}{9}$ V $= 2.78$ V, $\frac{5}{18}$ A $= 0.278$ A **14.** 1 Ω **15.** $\frac{22}{35}$ Ω **16.** $\frac{3r}{5}$

17. (a) 12 W **(b)** 4W **(c)** 8 W **(d)** 4 Ω **(e)** 4 W

18. 90 W **19.** 12 A, –20 W **20.** $\sqrt{R_1 R_2}$

21. (a) $\frac{50}{11} = 4.55$ A **(b)** $\frac{22 \times 11}{5} = 48.4\,\Omega$ **(c)** 1000 W **(d)** 240 cal s^{-1} **(e)** 80/3 gm **22.** 90%

23. (a) $E = 10$ V each **(b)** (A) act as a source and (B) act as load

(c) $V_A = 9$ V, $V_B = 11$ V **(d)** $P_A = 9$ W, $P_B = 11$ W **(e)** Heat rate = 1 W each

(f) 10 W. **(g)** 9 V, 11 V **(b)** –9 W, 11 W

24. 4 Ω **25.** 20 Ω **26.** $\frac{50}{73}$ A **27.** 46.67 cm

28. 4 Ω **29.** $\frac{10}{3}$ Ω, 5 Ω **30.** 7.5 m, 8.75 m, 6.25 m

Subjective Questions

1. $(3/11)\alpha$ **2. (a)** $J_0 A/3$; (b) $2J_0 A/3$ **3.** $\frac{R_2}{R_1} = \frac{3}{5}$ **4. (i)** 10.52 Ω; **(ii)** 0.3125 Ω

5. $\frac{7}{5}$ times the length of any side of the square

6. (i) $\frac{(k-1)^2}{k}; \frac{k}{(k-1)}$ **(ii)** $\frac{((k-1)/k^2)V_0}{R^3}$ **7.** $\frac{4}{9}$ kg/sec, 450 sec **8. (i)** $V_{ab} = -12$ V, (ii) 3 A from b to a

9. $11 + 6\sqrt{2}$ **10.** $R = \frac{\rho_0 L}{A}\left(1 - \frac{1}{e}\right); I = \frac{V_0 A}{\rho_0 L}\left(\frac{e}{e-1}\right); V = \frac{V_0(e^{-x/L} - e^{-1})}{1 - e^{-1}}$ **11.** 7.2 m **13.** $-\frac{22}{9}V_n$

14. $\alpha_{\text{eff}} = \frac{5}{4}\alpha$ **15.** 233.3 Ω, 144 V **16. (a)** 6 m, **(b)** 1 Ω **17. (a)** 1.01 W, **(b)** 0-5 A, 0-10 V, **(c)** 0.05 A

Previous Years' Questions

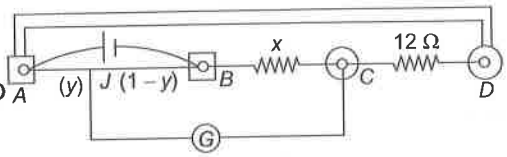
JEE Main

1. B	**2.** B	**3.** D	**4.** B	**5.** D	**6.** C	**7.** C	**8.** A	**9.** A	**10.** B
11. B	**12.** A	**13.** C	**14.** D	**15.** A	**16.** C	**17.** B	**18.** B	**19.** D	**20.** A
21. C	**22.** D	**23.** B	**24.** C	**25.** D	**26.** D	**27.** B	**28.** D	**29.** A	**30.** C
31. A	**32.** A	**33.** B	**34.** A	**35.** D					

JEE Advanced

1.

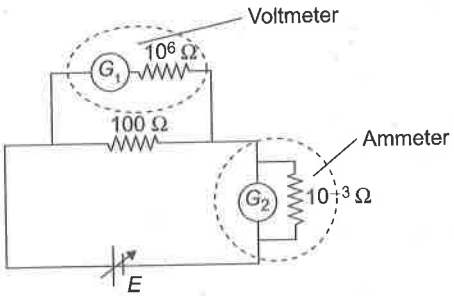
2. D



3. (a) No, (b) (c) 8 Ω **4.** A **5.** A **6.** A

7. Battery should be connected across A and B. Out put can be taken across the terminals A and C or B and C

8. A **9.** C **10.**



11. B **12.** D **13.** This is true for $r_1 = r_2$; So R_2 given most accurate value

14. A **15.** A **16.** C **17.** D **18.** A, D **19.** C **20.** D

21. C **22.** 4 **23.** B **24.** 5 **25.** A, B, C, D **26.** A, B, D

27. B, D **28.** 5 **29.** B **30.** 1

Chapter 3

Capacitance

INTRODUCTION

A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

Capacitance of an isolated conductor

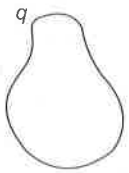
When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.

q = charge on conductor

V = potential of conductor

$q \propto V$

$\Rightarrow \quad q = CV$



Isolated conductor

Where C is proportionally constant called capacitance of the conductor.

Definition of Capacitance

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

Important Point about the Capacitance of an Isolated Conductor

- It is a scalar quantity.
- Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1}L^{-2}I^2T^4$
- **1 Farad:** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 V.

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$1\ \mu\text{F} = 10^{-6}$ F, $\quad 1$ nF $= 10^{-9}$ F $\quad$ or $\quad 1$ pF $= 10^{-12}$ F

- **Capacitance of an isolated conductor depends on following factors:**
  - **(a) Shape and size of the conductor:** On increasing the size, capacitance increase.
  - **(b) On surrounding medium:** With increase in dielectric constant K, capacitance increases.
  - **(c) Presence of other conductors:** When a neutral conductor is placed near a charged conductor capacitance of conductors increases.
- Capacitance of a conductor does not depend on
  - **(a)** Charge on the conductor
  - **(b)** Potential of the conductor
  - **(c)** Potential energy of the conductor.

Capacitance of an Isolated Spherical Conductor

SOLVED EXAMPLES

EXAMPLE 1

Find out the capacitance of an isolated spherical conductor of radius R.

SOLUTION

Let there is charge Q on sphere.

$$\therefore \qquad \text{Potential } V = \frac{KQ}{R}$$

Hence by formula: $Q = CV$

$$Q = \frac{CKQ}{R}$$

$$C = 4\pi\varepsilon_0 R$$

100

(i) If the medium around the conductor is vacuum or air:

$$C_{vacuum} = 4\pi\varepsilon_0 R$$

R = Radius of spherical conductor. (may be solid or hollow)

(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity then

$$C_{medium} = 4\pi\varepsilon_0 KR$$

(iii) $\dfrac{C_{medium}}{C_{air/vaccum}} = K$ = dielectric constant.

CAPACITOR

A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

(i) When uncharged conductor is brought near to a charged conductor, the charge on conductors remains same but its potential decreases resulting in the increase of capacitance.

(ii) In capacitor two conductors have equal but opposite charges.

(iii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iv) Formulae related with capacitors:

(a) $Q = CV$

$$\Rightarrow \qquad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

Q = Charge of positive plate of capacitor.

V = Potential difference between positive and negative plates of capacitor

C = Capacitance of capacitor.

(v) The capacitor is represented as following:

(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors:

(a) Parallel plate capacitor.

(b) Spherical capacitor.

(c) Cylindrical capacitor.

(v) Capacitance of a capacitor depends on

(a) Area of plates.

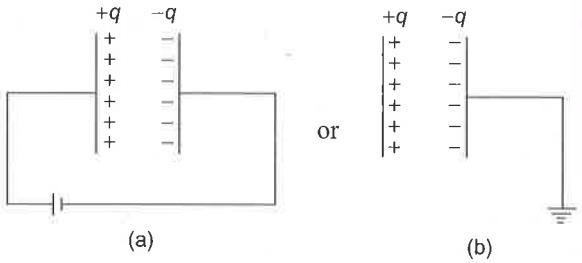
(b) Distance between the plates.

(c) Dielectric medium between the plates.

Parallel Plate Capacitor

Two metallic parallel plates of any shape but of same size and separated by small distance constitute parallel plate capacitor. Suppose the area of each plate is A and the separation between the two plates is d. Also assume that the space between the plates contains vacuum.

We put a charge q on one plate and a charge $-q$ on the other. This can be done either by connecting one plate with the positive terminal and the other with negative plate of a battery (as shown in figure a) or by connecting one plate to the earth and by giving a charge $+q$ to the other plate only. This charge will induce a charge $-q$ on the earthed plate. The charges will appear on the facing surfaces. The charges density on each of these surfaces has a magnitude $\sigma = q/A$.



If the plates are large as compared to the separation between them, then the electric field between the plates (at point B) is uniform and perpendicular to the plates except for a small region near the edge. The magnitude of this uniform field E may be calculated by using the fact that both positive and negative plates produce the electric field in the same direction (from positive plate towards negative plate) of magnitude $\sigma/2\varepsilon_0$ and therefore, the net electric field between the plates will be,

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Outside the plates (at point A and C) the field due to positive sheet of charge and negative sheet of charge are in opposite directions. Therefore, net field at these points is zero.

The potential difference between the plates is,

$$\therefore \qquad V = E \cdot d = \left(\frac{\sigma}{\varepsilon_0}\right) d = \frac{qd}{A\varepsilon_0}$$

$\therefore$ The capacitance of the parallel plate capacitor is,

$$C = \frac{q}{V} = \frac{A\varepsilon_0}{d} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

Cylindrical Capacitor

Cylindrical capacitor consists of two co-axial cylinders of radii a and b and length l. If a charge q is given to the inner cylinder, induced change $-q$ will reach the inner surface of the outer cylinder. By symmetry, the electric field in region between the cylinders is radially outwards.

By Gauss's theorem, the electric field at a distance r from the axis of the cylinder is given by

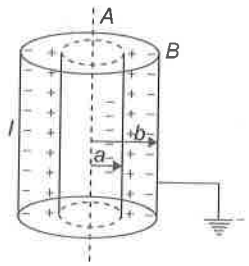
$$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}$$

The potential difference between the cylinders is given by

$$V = -\int_b^a \vec{E}\,\vec{dr} = -\frac{1}{2\pi\varepsilon_0 l}q\int_b^a \frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0 l}\left(\ln\frac{a}{b}\right)$$

or,

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\left(\ln\frac{a}{b}\right)}$$



Spherical Capacitor

A spherical capacitor consists of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential.

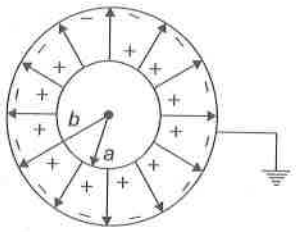
The inner surface of the outer sphere has an equal negative charge.

The potential difference between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

Hence, capacitance

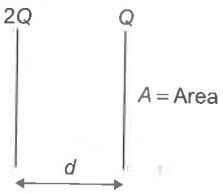
$$C = \frac{Q}{V}\frac{4\pi\varepsilon_0 ab}{(b-a)}$$



SOLVED EXAMPLES

EXAMPLE 2

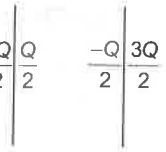
Find capacitance of the given system.



SOLUTION

Arranging charges

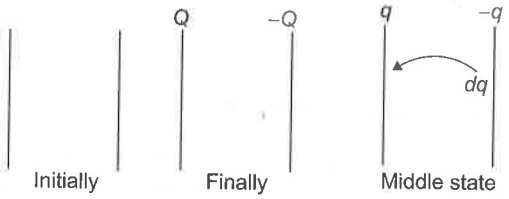
$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$



Now, $V = Ed = \dfrac{Qd}{2A\varepsilon_0}$

$$\Rightarrow \quad C = \frac{Q/2}{V} = \frac{A\varepsilon_0}{d}$$

ENERGY STORED IN A CHARGED CAPACITOR



Work has to be done in charging a conductor against the force of repulsion by the already existing charges on it. The work is stored as a potential energy in the electric field of the conductor. Suppose a conductor of capacity C is charged to a potential V_0 and let q_0 be the charge on the conductor at this instant. The potential of the conductor when (during charging) the charge on it was $q(< q_0)$ is,

$$V = \frac{q}{C}$$

102

Now, work done in bringing a small charge dq at this potential is,

$$dW = Vdq = \left(\frac{q}{C}\right)dq$$

$\therefore$ total work done in charging it from 0 to q_0 is,

$$W = \int_0^{q_0} dW = \int_0^{q_0} \frac{q}{C}dq = \frac{1}{2}\frac{q_0^2}{C}$$

This work is stored as the potential energy,

$$\therefore \quad U = \frac{1}{2}\frac{q_0^2}{C}$$

Further by using $q_0 = CV_0$ we can write this expression also as,

$$U = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

In general if a conductor of capacity C is charged to a potential V by giving it a charge q,

then $$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

Energy Density of a Charged Capacitor

This energy is localized on the charges or the plates but is distributed in the field. Since in case of a parallel plate capacitor, the electric field is only between the plates, i.e., in a volume $(A \times d)$, the energy density

$$U_E = \frac{U}{\text{volume}} = \frac{\frac{1}{2}CV^2}{A \times d} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad}$$

or $$U_E = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2 \left[\because \frac{v}{d} = E\right]$$

Calculation of Capacitance

The method for the calculation of capacitance involves integration of the electric field between two conductors or the plates which are just equipotential surfaces to obtain the potential difference V_{ab}. Thus,

$$V_{ab} = -\int_b^a \vec{E}\cdot\vec{dr}$$

$$\therefore \quad C = \frac{q}{V_{ab}} = \frac{q}{-\int_b^a \vec{E}\cdot\vec{dr}}$$

Heat Generated

1. Work done by battery
   $W = QV$
   Q = charge flow in the battery
   V = EMF of battery
2. W = +Ve (When Battery discharging)
   W = –Ve (When Battery charging)
3. $\because$ $Q = CV$ (C = equivalent capacitance)
   so $W = CV \times V = CV^2$
   Now energy on the capacitor $= \frac{1}{2}CV^2$
   $\therefore$ Energy dissipated in form of heat (due to resistance)
   H = Work done by battery—{final energy of capacitor – initial energy of capacitor}

FORCE BETWEEN THE PLATES OF A CAPACITOR

In a capacitor as plates carry equal and opposite charges, there is a force of attraction between the plates. To calculate this force, we use the fact that the electric field is conservative and in a conservative field $F = -\frac{dU}{dx}$. In case of parallel plate capacitor:

$$U = \frac{q^2}{2C} = \frac{1}{2}\frac{q^2x}{\varepsilon_0 A}\left[\text{as } C = \frac{\varepsilon_0 A}{x}\right]$$

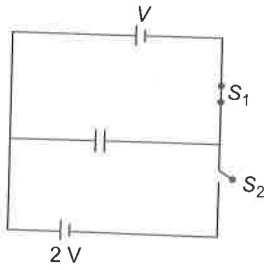
So, $$F = -\frac{d}{dx}\left[\frac{q^2}{2\varepsilon_0 A}x\right] = \frac{-1}{2}\frac{q^2}{\varepsilon_0 A}$$

The negative sign implies that the force is attractive.

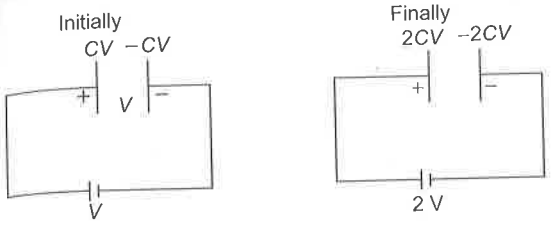
SOLVED EXAMPLES

EXAMPLE 3

At any time S_1 switch is opened and S_2 is closed then find out heat generated in circuit.



SOLUTION



Charge flow through battery $= Q_f - Q_i = 2CV - CV = CV$

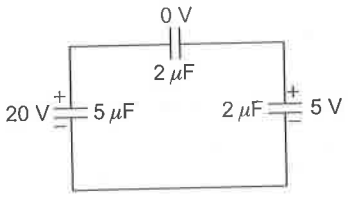
$$H = (CV \times 2V) - \left\{\frac{1}{2}C(2V)^2 - \frac{1}{2}CV^2\right\}$$

$$= 2CV^2 - \left\{2CV^2 - \frac{1}{2}CV^2\right\}$$

$$H = \frac{1}{2}CV^2$$ ■

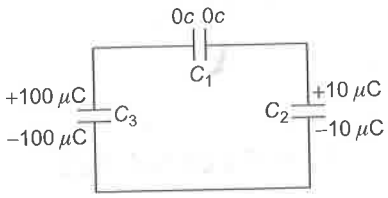
EXAMPLE 4

(a) Find the final charge on each capacitor if they are connected as shown in the figure.



SOLUTION

Initially



Finally let q charge flows clockwise then

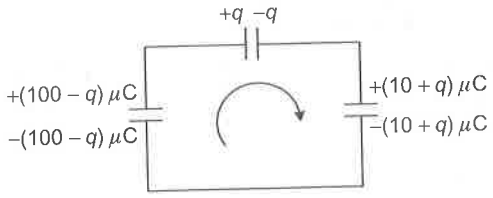
Now applying KVL

$$\frac{+q}{C_1} + \frac{(10+q)}{C_2} - \frac{(100-q)}{C_3} = 0$$

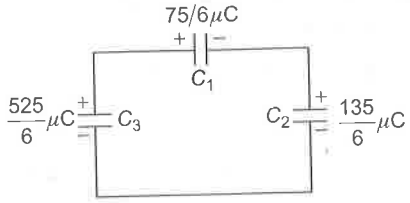
$$\Rightarrow \quad \frac{q}{2} + \frac{10+q}{2} - \frac{100-q}{5} = 0$$

$$5q + 50 + 5q - 200 + 2q = 0$$

$$12q - 150 = 0 \quad \Rightarrow \quad q = \frac{75}{6}\,\mu C$$



so finally



(b) Find heat loss in the above circuit.

ΔH = Energy [initially − finally] on capacitor

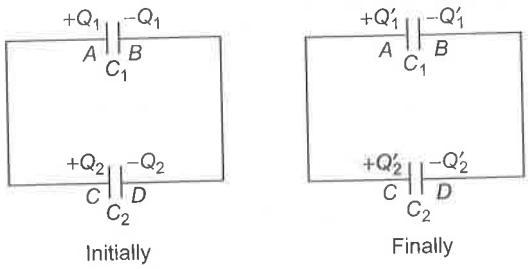
$$= \left[\left\{\frac{1}{2}\times 5\times(20)^2 + \frac{1}{2}\times 2\times(5)^2\right\}\right.$$

$$-\left\{\frac{1}{2}\times\left(\frac{525}{6}\right)^2\times\frac{1}{5} + \frac{1}{2}\times\left(\frac{75}{6}\right)^2\right.$$

$$\left.\left.\times\frac{1}{2} + \frac{1}{2}\left(\frac{135}{6}\right)^2\times\frac{1}{2}\right\}\right]\times 10^{-6}\ \text{J}$$

DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS

When two capacitors C_1 and C_2 are connected as shown in figure



Before connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

104

	After connecting the capacitors	
Parameter	Ist Capacitor	IInd Capacitor
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

(a) Common potential:

By charge conservation on plates A and C before and after connection.

$$Q_1 + Q_2 = C_1V + C_2V$$

$$\Rightarrow \quad V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $Q_1' = C_1V = \dfrac{C_1}{C_1 + C_2}(Q_1 + Q_2)$

$$Q_2' = C_2V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

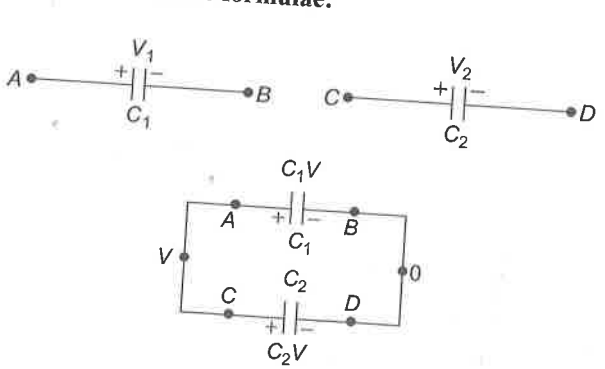
(c) Heat loss during redistribution:

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

- ❖ When plates of similar charges are connected with each other (+with + and – with –) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.
- ❖ When plates of opposite polarity are connected with each other (+with –) then take charge and potential of one of the plate to be negative.

Derivation of above formulae:



Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

$$C_1V + C_2V = C_1V_1 + C_2V_2$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

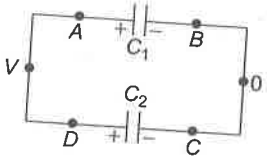
$$H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2}\left[\frac{\begin{array}{l}C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 \\ - C_1^2V_1^2 - C_2V_2^2 - 2C_1C_2V_1V_2\end{array}}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$



when oppositely charged terminals are connected then

$$\therefore \quad C_1V + C_2V = C_1V_1 - C_2V_2$$

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$$

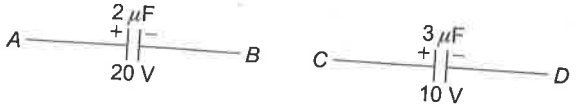
SOLVED EXAMPLES

EXAMPLE 5

Find out the following if A is connected with C and B is connected with D.

(i) How much charge flows in the circuit.

(ii) How much heat is produced in the circuit.



SOLUTION

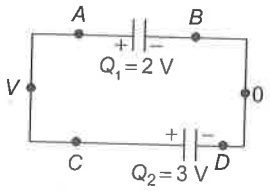
Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

By charge conservation,

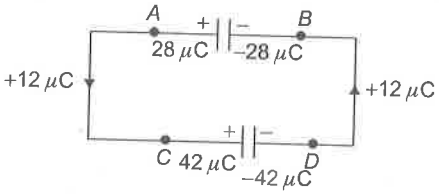
$$3V + 2V = 40 + 30$$

$$5V = 70 \Rightarrow V = 14 \text{ V}$$

Charge flow = 40 – 28 = 12 μC



Now final charges on each plate is shown in the figure.



(ii) Heat produced $= \frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2$

$$- \frac{1}{2} \times 5 \times (14)^2$$

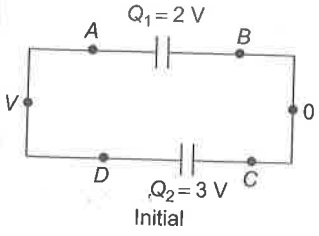
$$= 400 + 150 - 490$$

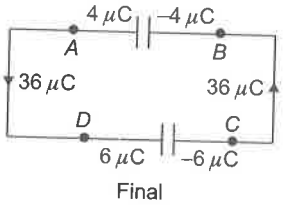
$$= 550 - 490 = 60 \ \mu\text{J}$$

- When capacitor plates are joined then the charge remains conserved.
- We can also use direct formula of redistribution as given above. ■

EXAMPLE 6

Repeat above question if A is connected with D and B is connected with C.





SOLUTION

Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

$$2V + 3V = 10 \Rightarrow V = 2 \text{ V}$$

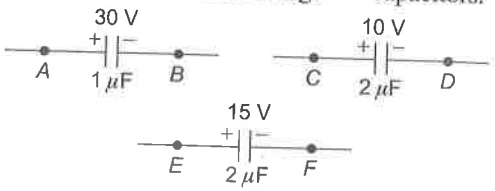
Now charge on each plate is shown in the figure.

Heat produced $= 400 + 150 - \frac{1}{2} \times 5 \times 4 = 550 - 10 = 540 \ \mu\text{J}$

- Here heat produced is more. Think why? ■

EXAMPLE 7

Three capacitors as shown of capacitance 1 μF, 2 μF and 2 μF are charged upto potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.



SOLUTION

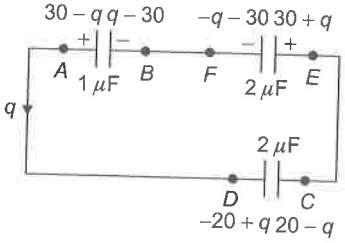
Let charge flow is q.

Now applying Kirchhoff's voltage low

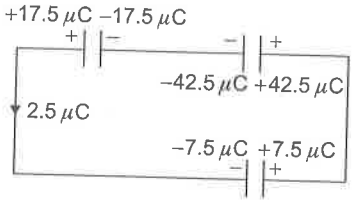
$$-\frac{(q-20)}{2} - \frac{(30+q)}{2} + \frac{30-q}{1} = 0$$

$$-2q = -25$$

$$q = 12.5 \ \mu\text{C}$$



Final charges on plates



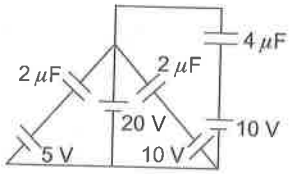
■

CAPACITOR CIRCUITS

SOLVED EXAMPLES

EXAMPLE 8

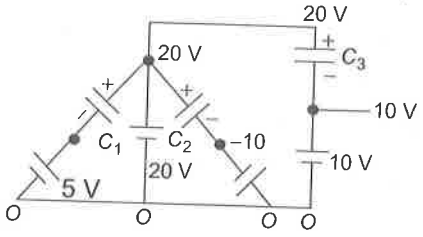
Find charge on each capacitor.



106

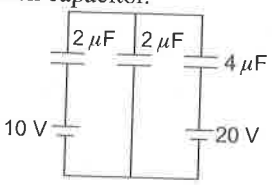
SOLUTION

Charge on $C_1 = C_1V_1 = 2 \times (20 - 5)\ \mu C = 30\ \mu C$
Charge on $C_2 = C_2V_2 = 2 \times (20 - (-10))\ \mu C = 60\ \mu C$
Charge on $C_3 = C_3V_3 = 4 \times (20 - 10)\ \mu C = 40\ \mu C$



EXAMPLE 9

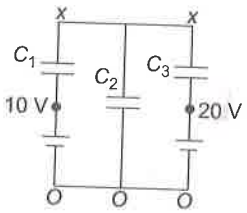
Find charge on each capacitor.



SOLUTION

Charge on $C_1 = (x - 10)C_1$
Charge on $C_2 = (x - 0)C_2$
Charge on $C_3 = (x - 20)C_3$
Now from charge conservation at node x

$$(x - 10)C_1 + (x - 0)C_2 + (x - 20)C_3 = 0$$
$$\Rightarrow \quad 2x - 20 + 2x + 4x - 80 = 0$$
$$\Rightarrow \quad x = \frac{25}{2}$$

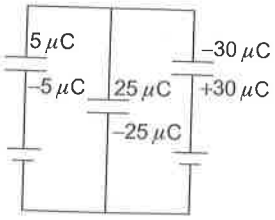


so

$$Q_{C_1} = \left(\frac{25}{2} - 10\right) 2\ \mu C = 5\ \mu C$$

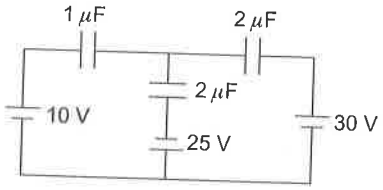
$$Q_{C_2} = \frac{25}{2} \times 2\ \mu C = 25\ \mu C$$

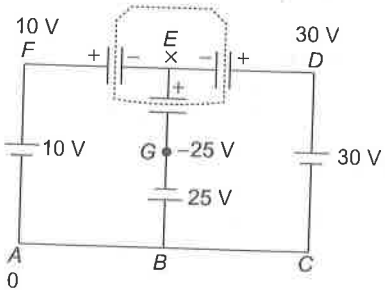
$$Q_{C_3} = \left(\frac{25}{2} - 40\right) \times 4\ \mu C = -30\ \mu C$$



EXAMPLE 10

In the given circuit find out the charge on each capacitor. (Initially they are uncharged)

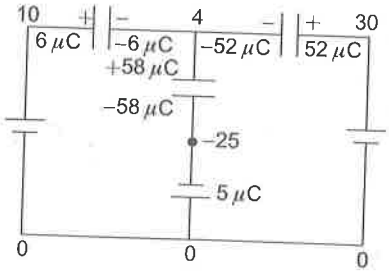




SOLUTION

Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is –25 V. Now apply Kirchhoff's 1st law at point E. (total charge of all the plates connected to 'E' must be same as before i.e., 0)

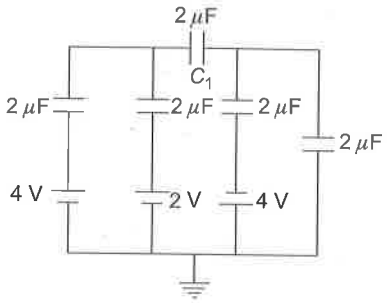
$$\therefore \quad (x - 10) + (x - 30)\,2 + (x + 25)\,2 = 0$$
$$5x = 20$$
$$x = 4$$



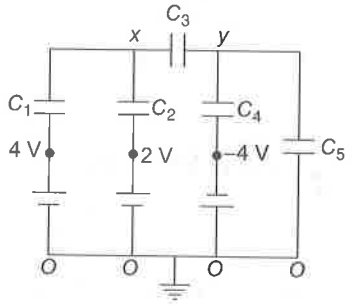
Final charges:

$$Q_{2\mu F} = (30 - 4)\,2$$
$$= 52\ \mu C$$
$$Q_{1\mu F} = (10 - 4)$$
$$= 6\ \mu C$$
$$Q_{2\mu F} = (4 - (-25))\,2$$
$$= 58\ \mu C$$

EXAMPLE 11



Find voltage across capacitor C_1.



SOLUTION

Now from charge conservation at node x and y

for x

$$(x-4)C_1 + (x-2)C_2 + (x-y)C_3 = 0$$

$$\Rightarrow \quad 2(x-4) + 2(x-2) + (x-y)2 = 0$$

$$6x - 2y - 12 = 0 \qquad (1)$$

For y

$$(y-x)C_3 + [y-(-4)]C_4 + (y-0)C_5 = 0$$

$$\Rightarrow \quad (y-x)2 + (y+4)2 + y2 = 0$$

$$= 6y - 2x + 8 = 0 \qquad (2)$$

Eq. (1) and (2)

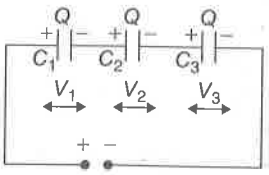
$$y = -\frac{3}{4}$$

$$x = \frac{7}{4}$$

So potential difference $= x - y = \frac{7}{4} - \left(-\frac{3}{4}\right) = \frac{5}{3}$ volt ■

COMBINATION OF CAPACITORS

Series Combination



(i) When initially uncharged capacitors are connected as shown, then the combination is called series combination.

(ii) All capacitors will have same charge but different potential difference across then.

(iii) We can say that

$$V_1 = \frac{Q}{C_1}$$

V_1 = potential across C_1

Q = charge on positive plate of C_1

C_1 = capacitance of capacitor similarly

$$V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3} \cdots$$

(iv) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination.

$$V \propto \frac{1}{C}$$

Notes

In series combination the smallest capacitor gets maximum potential.

(v) $$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots} \text{ V},$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots} \text{ V},$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots} \text{ V}$$

Where $V = V_1 + V_2 + V_3$

(vi) Equivalent Capacitance:
Equivalent capacitance of any combination is that capacitance which when connected in place of the

combination stores same charge and energy as that of the combination

In series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots$$

❖ In series combination equivalent is always less then smallest capacitor of combination.

(vii) Energy stored in the combination

$$U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$U_{combination} = \frac{Q^2}{2C_{eq}}$$

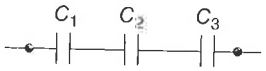
Energy supplied by the battery in charging the combination

$$U_{battery} = Q \times V = Q \cdot \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$$

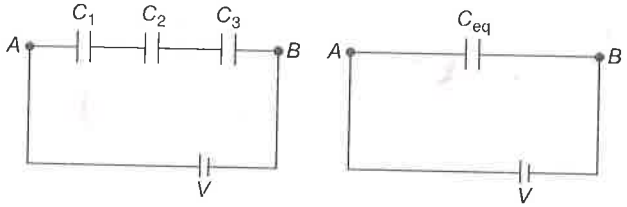
$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

❖ Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

Derivation of Formulae:



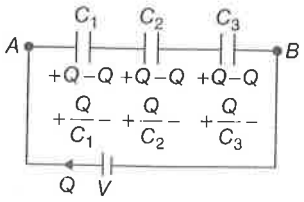
Meaning of equivalent capacitor



$$C_{eq} = \frac{Q}{V}$$

Now,

Initially, the capacitor has no charge. Applying Kirchhoff's voltage law



$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0$$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

in general

$$\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

SOLVED EXAMPLES

EXAMPLE 12

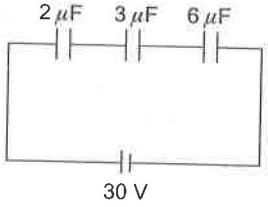
Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30 V. Find out following:

(i) charge flow through the battery.

(ii) potential energy in 3 μF capacitor.

(iii) U_{total} in capacitors

(iv) heat produced in the circuit



SOLUTION

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$$

$$C_{eq} = 1\ \mu F.$$

(i) $Q = C_{eq} \quad V = 30\ \mu C$

(ii) charge on 3 μF capacitor = 30 μC

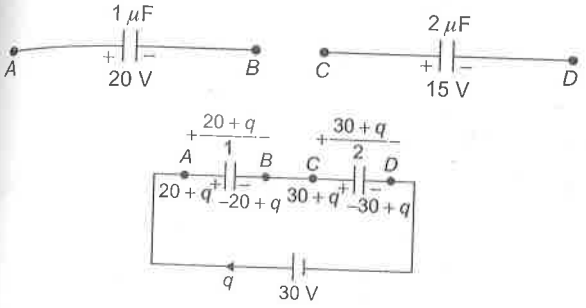
$$\text{energy} = \frac{Q^2}{2C} = \frac{30 \times 30}{2 \times 3} = 150\ \mu J$$

(iii) $U_{total} = \frac{30 \times 30}{2}\ \mu J = 450\ \mu J$

(iv) Heat produced = (30 μC)(30) − 450 μJ = 450 μJ ■

EXAMPLE 13

Two capacitors of capacitance 1 μF and 2 μF are charged to potential difference 20 V and 15 V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor.



SOLUTION

Now applying Kirchoff voltage law

$$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$$-40 - 2q - 30 - q = -60$$

$$3q = -10$$

Charge flow $= -\frac{10}{3}$ μC.

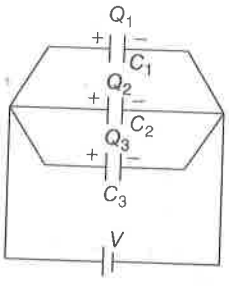
Charge on capacitor of capacitance 1 μF $= 20 + q = \frac{50}{3}$

Charge on capacitor of capacitance 2 μF $= 30 + q = \frac{80}{3}$

Parallel Combination

(i) When one plate of one capacitor is connected with one plate of the other capacitor, such combination is called parallel combination.

(ii) All capacitors have same potential difference but different charges.



(iii) We can say that:

$Q_1 = C_1 V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

(iv) $Q_1 : Q_2 : Q_3 : C_1 : C_2 : C_3$

The charge on the capacitor is proportional to its capacitance $Q \propto C$

(v) $$Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q$$

$$Q_2 = \frac{C_2}{C_1 + C_2 + C_3} Q$$

$$Q_3 = \frac{C_3}{C_1 + C_2 + C_3} Q$$

Where $Q = Q_1 + Q_2 + Q_3 \cdots$

❖ Maximum charge will flow through the capacitor of largest value.

(vi) Equivalent capacitance of parallel combination

$$C_{eq} = C_1 + C_2 + C_3$$

❖ Equivalent capacitance is always greater then the largest capacitor of combination.

(vii) Energy stored in the combination:

$$V_{combination} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 + \cdots$$

$$= \frac{1}{2}(C_1 + C_2 + C_3 \ldots) V^2 = \frac{1}{2} C_{eq} V^2$$

$$U_{battery} = QV = CV^2$$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

❖ Half of the energy supplied by the battery is stored in the form of electrostatic energy and half of the energy is converted into heat through resistance.

Formulae Derivation for Parallel Combination

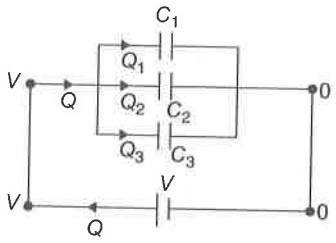
$$Q = Q_1 + Q_2 + Q_3$$

$$= C_1 V + C_2 V + C_3 V$$

$$= V(C_1 + C_2 + C_3)$$

$$\frac{Q}{V} = C_1 + C_2 + C_3$$

$$C_{eq} = C_1 + C_2 + C_3$$



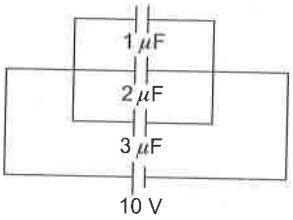
In general

$$C_{eq} = \sum_{n=1}^{n} C_n$$

EXAMPLE 14

Three initially uncharged capacitors are connected to a battery of 10 V in parallel combination. Find out the following?

(i) charge flow from the battery
(ii) total energy stored in the capacitors
(iii) heat produced in the circuit
(iv) potential energy in the 3 μF capacitor.



SOLUTION

(i) $Q = (30 + 20 + 10)\ \mu C = 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) Heat produced $= 60 \times 10 - 300 = 300\ \mu J$

(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

Mixed Combination

The combination which contains mixing of series combinations or other complex combinations fall in mixed category.

There are two types of mixed combinations

(i) Simple
(ii) Complex.

EXAMPLE 15

Two capacitors of capacitances $C_1 = 6\ \mu F$ and $C_2 = 3\ \mu F$ are connected in series across a cell of emf 18 V. Calculate

(i) the equivalent capacitance,
(ii) the potential difference across each capacitor,
(iii) the charge on each capacitor.

SOLUTION

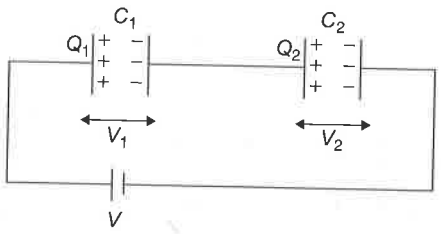
(i) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$\Rightarrow C = \frac{C_1 C_2}{C_1 + C_2} = \frac{6 \times 3}{6 + 3} = 2\ \mu F$

(ii) $V_1 = \left(\frac{C_2}{C_1 + C_2}\right) V = \left(\frac{6}{6 + 3}\right) \times 18 = 12\ V$

(iii) $v_2 = 18 - v_1 = 6v$

(iv) $Q_1 = Q_2 = C_1 V_1 = C_2 V_2 = CV$

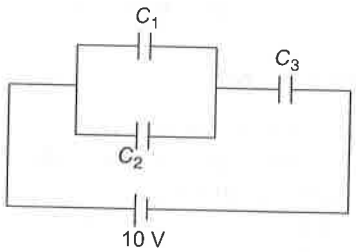


EXAMPLE 16

In the circuit shown the capacitors are

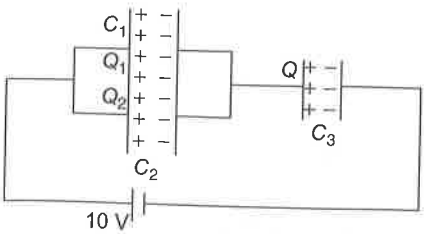
$C_1 = 15\ \mu F$; $C_2 = 10\ \mu F$ and $C_3 = 25\ \mu F$. Find

(i) the equivalent capacitance of the circuit,
(ii) the charge on each capacitor and
(iii) the potential difference across each capacitor.



SOLUTION

(i) $\frac{(C_1 + C_2) C_3}{(C_1 + C_2) + C_3} = \left(\frac{25 \times 25}{25 + 25}\right) \mu F = 12.5\ \mu F$



(ii) Q = Total charge supplied by the cell = CV = (12.5 × 10) μC = 125 μC

Charge on

$$C_1 = Q_1 = \left(\frac{C_1}{C_1 + C_2}\right)Q = \left(\frac{15}{15+10}\right) \times 125 = 75\,\mu C$$

Charge on

$$C_2 = Q_2 = \left(\frac{C_2}{C_1 + C_2}\right)Q = \left(\frac{10}{15+10}\right) \times 125 = 50\,\mu C$$

Charge on $C_3 = Q = 125\ \mu C$

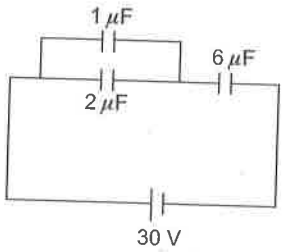
(iii) p.d. across $C_1 = V_1 = \frac{Q_1}{C_1} = \frac{75}{15} = 5\ V$

p.d. across $C_2 = V_2 = V_1 = 5\ V$

p.d. across $C_3 = V_3 = \frac{Q_3}{C_3} = \frac{125}{25} = 5\ V$

EXAMPLE 17

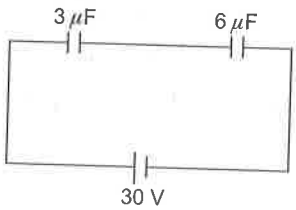
In the given circuit find out charge across 6 μF and 1 μF capacitor.



SOLUTION

It can be simplified as

$$C_{eq} = \frac{18}{9} = 2\ \mu F$$



Charge flow through the cell = 30 × 2 μC

$$Q = 60\ \mu C$$

Now charge on 3 μF = Charge on 6 μF = 60 μC

Potential difference across 3 μF

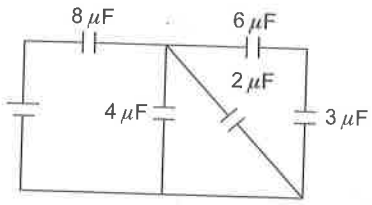
$$= \frac{60}{3} = 20\ V$$

∴ Charge on 1 μF = 20 μC

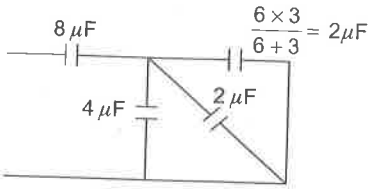
Series-Parallel

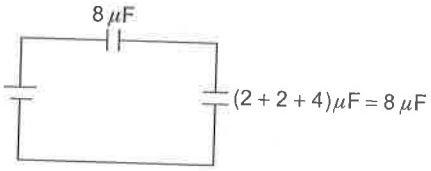
EXAMPLE 18

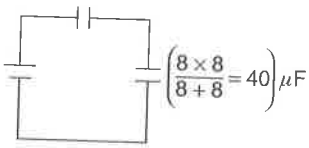
Find equivalent capacitance of the circuit



SOLUTION





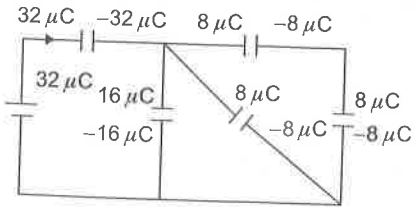


EXAMPLE 19

Find charge in each capacitor:

SOLUTION

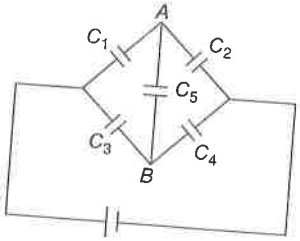
In series charge will be same on all capacitors and in parallel charge will be proportional to capacitance.



EXAMPLE 20

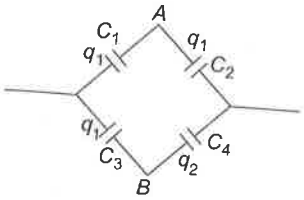
Find out the Relation between C_1, C_2, C_3 and C_4 such that point A and B are equipotential. [Balanced wheat stone bridge]

112



SOLUTION

When A and B are equipotential then there will be no charge on capacitor C_5. So remove it. Now C_1, C_2 are in series and C_3, C_4 are in series so they will have same charges respectively.



$$\Rightarrow \quad \frac{q_1}{C_1} = \frac{q_2}{C_3} \qquad (1)$$

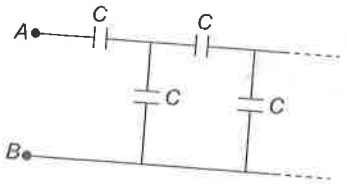
and

$$\frac{q_1}{C_2} = \frac{q_2}{C_4} \qquad (2)$$

$$\frac{C_2}{C_1} = \frac{C_4}{C_3} \quad \Rightarrow \quad C_2C_3 = C_1C_4$$

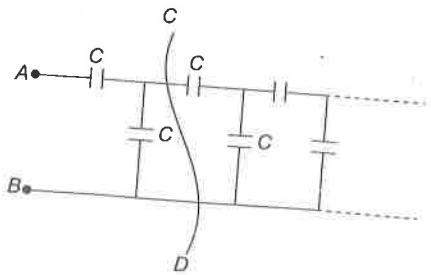
■

Ladder Problems



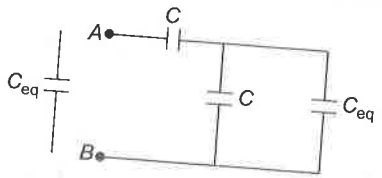
EXAMPLE 21

Find equivalent capacitance between point A and B.



SOLUTION

Let Capacitance is C_{eq} then after line CD capacitance agai will be equal to C_{eq}, because circuit is infinite. Then

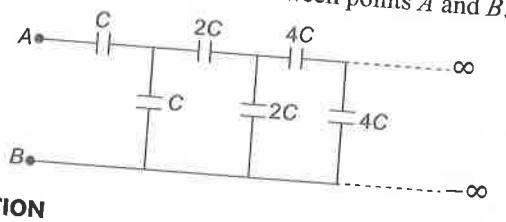


$$\Rightarrow \quad C_{eq} = \frac{(C_{eq} + C) \times C}{C + C + C_{eq}} \quad \Rightarrow \quad C_{eq} = \frac{\sqrt{5}-1}{2}C$$

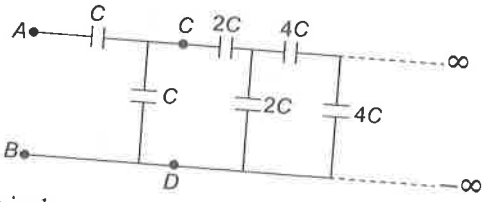
■

EXAMPLE 22

Find equivalent capacitance between points A and B.

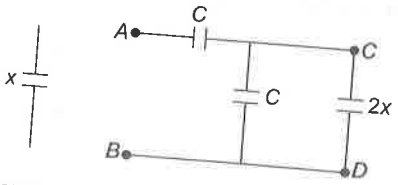


SOLUTION



Let equivalent capacitance x then capacitance after C, D point will be $2x$ because every capacitance becomes 2 times as compared to A, B.

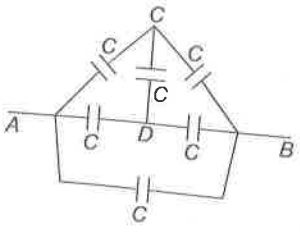
So



$$\Rightarrow \quad x = \frac{(2x + C)C}{(2x + C + C)} \quad \Rightarrow \quad X = \frac{C}{\sqrt{2}}$$

■

SYMMETRIC CIRCUITS



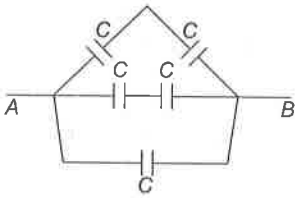
SOLVED EXAMPLES

EXAMPLE 23

Find equivalent Capacitance between A and B.

SOLUTION

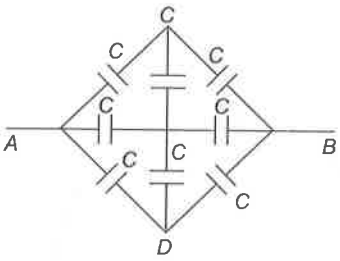
Because line CD is symmetric about A and B so we remove it then



$\Rightarrow \quad C_{eq} = 2C$ ■

EXAMPLE 24

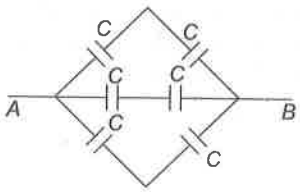
Find equivalent Capacitance between A and B.



SOLUTION

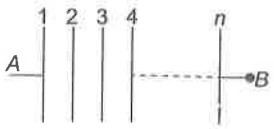
Because line CD is symmetric about points A and B so we remove it.

Now



So $$C_{eq} = \frac{3C}{2}$$ ■

EXAMPLE 25



Find equivalent Capacitance between point A and B if Capacitance between any two plates is C.

SOLUTION

There are total $(n-1)$ capacitors which are in series.

so $$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \cdots (n-1) \text{ times}$$

$$\frac{1}{C_{eq}} = \frac{(n-1)}{C} \quad \Rightarrow \quad C_{eq} = \frac{C}{(n-1)}$$ ■

EXAMPLE 26

Find out equivalent capacitance between A and B.

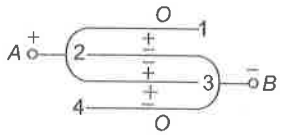
SOLUTION

Put numbers on the plates The charges will be as shown in the figure.

$$V_{12} = V_{32} = V_{34}$$

So all the capacitors are in parallel combination.

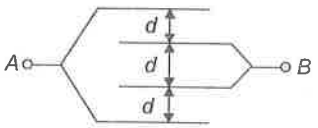
$$C_{eq} = C_1 + C_2 + C_3$$



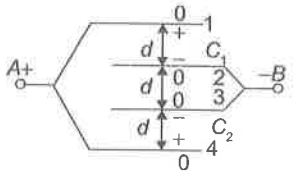
■

EXAMPLE 27

Find out equivalent capacitance between A and B.



SOLUTION

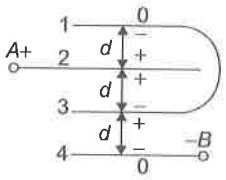


There are only two capacitors. $C_{eq} = C_1 + C_2$ ■

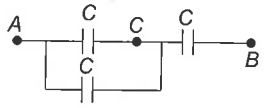
114

EXAMPLE 28

Find out equivalent capacitance between A and B.

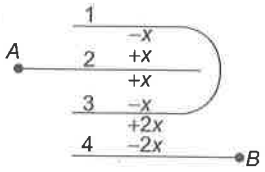


The modified circuit is



$$C_{eq} = \frac{2C}{3}$$

Other method:



$$C_{eq} = \frac{Q}{V} = \frac{2xA}{V}$$

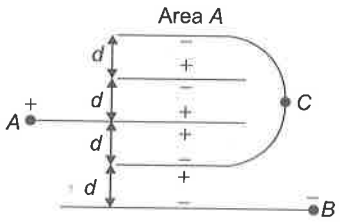
$$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_4)$$

$$= \frac{xd}{\varepsilon_0} + \frac{2xd}{\varepsilon_0} = \frac{3xd}{\varepsilon_0}$$

$$\therefore \quad C_{eq} = \frac{2Ax\varepsilon_0}{3xd} = \frac{2A\varepsilon_0}{3d} = \frac{2C}{3}$$

EXAMPLE 29

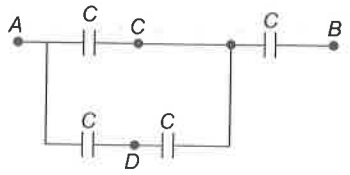
Find out equivalent capacitance between A and B.



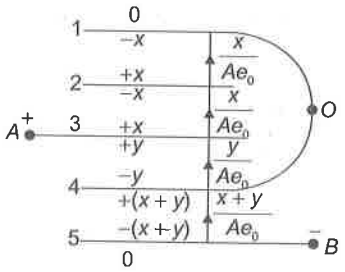
SOLUTION

$$C = \frac{A\varepsilon_0}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C} \quad \Rightarrow \quad C_{eq} = \frac{3C}{5} = \frac{3A\varepsilon_0}{5d}$$



Alternative method:



$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

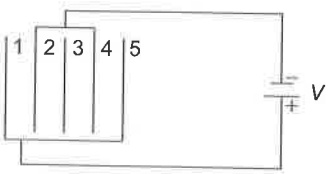
Potential of 1 and 4 is same

$$\frac{y}{A\varepsilon_0} = \frac{2x}{A\varepsilon_0} \qquad y = 2x$$

$$V = \left(\frac{2y+x}{A\varepsilon_0}\right)d \qquad C = \frac{(x+2x)\,A\varepsilon_0}{(5x)d} = \frac{3A\varepsilon_0}{5d}$$

EXAMPLE 30

Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of emf E as shown in the following diagram. The charge on the plates 1 and 4 will be

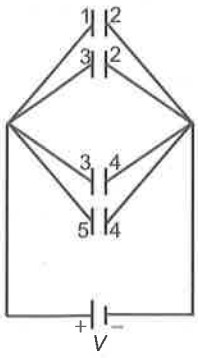


(A) $\frac{\varepsilon_0 A}{d}, \frac{-2\varepsilon_0 A}{d}$ (B) $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$

(C) $\frac{-\varepsilon_0 AV}{d}, \frac{-3\varepsilon_0 AV}{d}$ (D) $\frac{\varepsilon_0 AV}{d}, \frac{-4\varepsilon_0 AV}{d}$

SOLUTION

From equivalent circuit diagram Charge on first plate



$$Q = CV \quad \Rightarrow \quad Q = \frac{\varepsilon_0 AV}{d}$$

Charge on fourth plate

$$Q' = C(-V) \quad Q' = \frac{-\varepsilon_0 AV}{d}$$

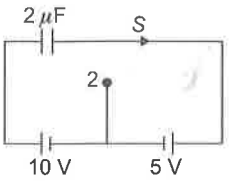
As plate 4 is repeated twice, hence charge on 4 will be $Q'' = 2Q'$

$$Q'' = -\frac{2\varepsilon_0 AV}{d}$$

Hence the correct answer will be (B). ■

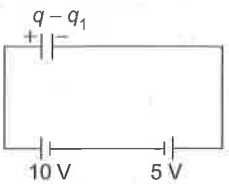
EXAMPLE 31

When switch is shifted from 1 to 2 then find charge flown in circuit and heat generated.



SOLUTION

Initially



$$\frac{q}{C} 1 - 5 + 10 = 0 \quad \Rightarrow \quad q_1 = 10\ \mu\text{F}$$

Finally

$$\Rightarrow \quad q_2 = 10 \times 2\ \mu\text{C} = 20\ \mu\text{C}$$

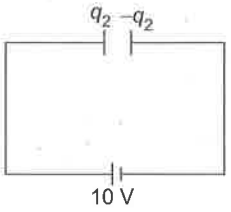
So charge Flown $(Q) = (q_2 - q_1) = 10\ \mu\text{C}$

Work done by battery $= QV = 10 \times 10 = 100\ \mu\text{J}$

So Heat = [W.D$_B$ – {final initial} energy of capacitor]

$$= \left[100 - \left\{\frac{1}{2} \times 2 \times (0)^2 - \frac{1}{2} \times 2 \times 5\right\}\right] \mu\text{J}$$

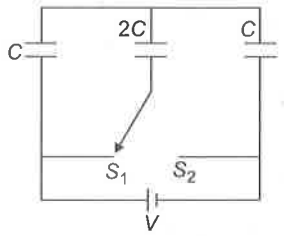
$$= [100 - \{100 - 25\}]\mu\text{J} = 25\ \mu\text{J}$$



■

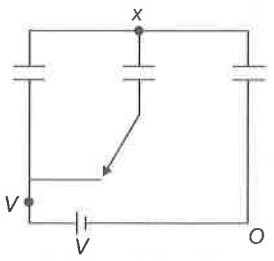
EXAMPLE 32

Find charge flown through battery and heat generated in the circuit after shifting switch from S_1 to S_2.



SOLUTION

Initially

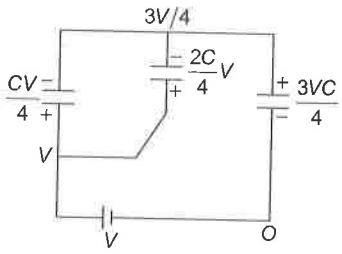


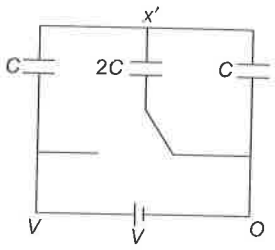
For node x

$$(x - v)\,2c + (x - v)\,c + (x + 0)\,c = 0$$

$$\Rightarrow \quad 4x = 3v \quad \Rightarrow \quad x = \frac{3v}{4}$$

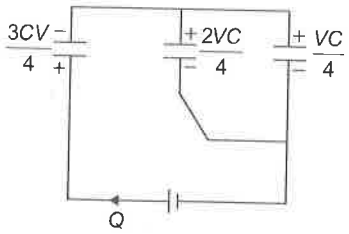
116





$$(x' - 0)C + (x' - 0)2C + (x' - V)C + = 0$$

$$\Rightarrow \quad x' = \frac{V}{4}$$



So now charge on capacitor

So charge flow from battery

$$Q = \left(\frac{3CV}{4} - \frac{CV}{4}\right) = \frac{CV}{2}$$

So heat generated

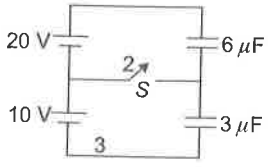
$$\Delta H = QV - \left[\left\{\frac{1}{2}C\left(\frac{3V}{4}\right)^2 + \frac{1}{2}(2C)\left(\frac{V}{4}\right)^2 + \frac{1}{2}\times C\times\left(\frac{V}{4}\right)^2\right\} - \left\{\frac{1}{2}C\left(\frac{V}{4}\right)^2 + \frac{1}{2}(2C)\left(\frac{V}{4}\right)^2 + \frac{1}{2}\times C\times\left(\frac{3V}{4}\right)^2\right\}\right]$$

$$\Delta H = QV = \frac{CV^2}{2}$$

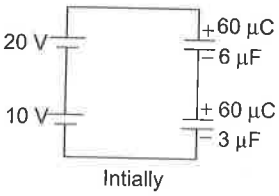
■

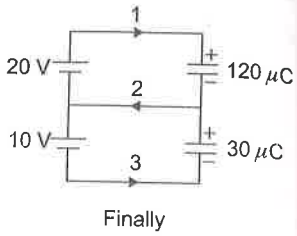
EXAMPLE 33

Find the charge flown through the path 1, 2, 3 as shown in figure after closing switch S and heat generated in the circuit.



SOLUTION





So charge flow in path 1 = 120 – 60 = 60 μC

So charge flow in path 2 = 60 + (60 – 30) = 90 μC

So charge flow in path 3 = (60 – 30) = 30 μC

So work done by battery of 20 V = 60 × 20 μJ = 1200 μJ

Work done by battery of 10 V = –30 × 10 μJ = –300 μJ

$$\text{Initial energy of capacitors} = \left[\frac{1}{2}\frac{(60)^2}{6} + \frac{1}{2}\frac{(60)^2}{3}\right]\mu\text{J}$$

$$= 900\ \mu\text{J}$$

$$\text{Final energy of capacitor} = \left[\frac{1}{2}6\times(20)^2 + \frac{1}{2}\times 3\times(10)^2\right]\mu\text{J}$$

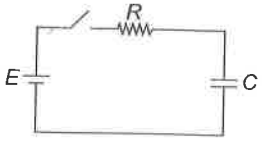
$$= 1350\ \mu\text{J}$$

So heat loss = (1200 – 300) – (1350 – 900) μJ = 450 μJ ■

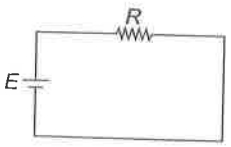
CIRCUIT SOLUTION FOR $R - C$ CIRCUIT AT $T = 0$ (INITIAL STATE) AND $T = \infty$ (FINAL STATE)

Notes

- Charge on the capacitor does not change instantaneously or suddenly if there is a resistance in the path (series) of the capacitor.
- When an uncharged capacitor is connected with battery then its charge is zero initially hence potential difference across it is zero initially. At this time the capacitor can be treated as a conducting wire

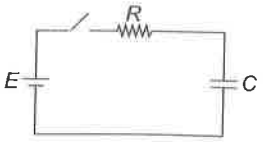


Before connection

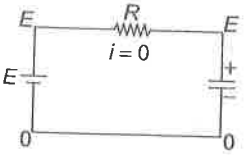


Just after connection at $t = 0$

- The current will become zero finally (that means in steady state) in the branch which contains capacitor.



Before connection

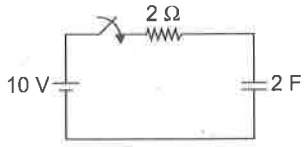


After connection at $t = \infty$

SOLVED EXAMPLES

EXAMPLE 34

Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.



(a) Just after the switch is closed

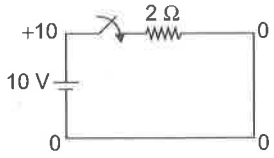
(b) After a long time when switch was closed.

SOLUTION

(a) For just after closing the switch:

Potential difference across capacitor = 0

$$\therefore \quad Q_C = 0 \qquad \therefore \quad i = \frac{10}{2} = 5 \text{ A}$$

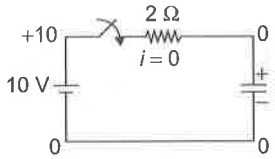


(b) After a long time

At steady state current $i = 0$

and potential difference across capacitor = 10 V

$$\therefore \quad Q_C = 3 \times 10 = 30 \text{ C}$$



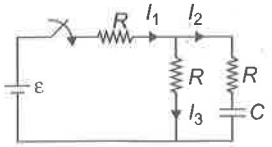
■

EXAMPLE 35

Find out current I_1, I_2, I_3, charge on capacitor and $\frac{dQ}{dt}$ of capacitor in the circuit when it is initially uncharged in the following situations

(a) Just after the switch is closed

(b) After a long time when switch is closed.



SOLUTION

(a) Initially the capacitor is uncharged so its behaviour is like a conductor

Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x. Apply Kirchoff's 1st law at point E:

$$\frac{x-\varepsilon}{R} + \frac{x-0}{R} + \frac{x-0}{R} = 0$$

$$\frac{3x}{R} = \frac{\varepsilon}{R}$$

$$x = \frac{\varepsilon}{3} \qquad Q_c = 0$$

$$\therefore \quad I_1 = \frac{-\varepsilon/3 + \varepsilon}{R} = \frac{2\varepsilon}{3R}$$

$$I_2 = \frac{dQ}{dt} = \frac{\varepsilon}{3R}, \qquad I_3 = \frac{\varepsilon}{3R}$$

■

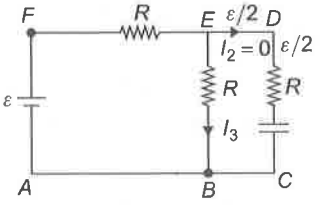
Alternatively

$$i_1 = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R + \frac{R}{2}} = \frac{2\varepsilon}{3R}$$

$$i_2 = i_3 = \frac{i_1}{2} = \frac{\varepsilon}{3R}$$

(b) at $t = \infty$ (finally)

Capacitor is completely charged. So their will be no current through it.



$$I_2 = 0, \quad I_1 = I_3 = \frac{\varepsilon}{2R}$$

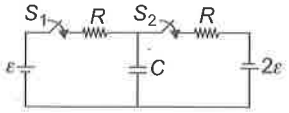
$$V_E - V_B = V_D - V_C = \left(\frac{\varepsilon}{2R}\right) R = \frac{\varepsilon}{2}$$

$$\Rightarrow \qquad Q_C = \frac{\varepsilon C}{2}, \quad \frac{dQ}{dt} = I_2 = 0$$

	I_1	I_2	I_3	Q	dQ/dt
Time $t = 0$	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally $t =$	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2R}$	0

EXAMPLE 36

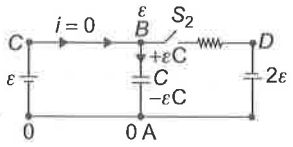
At $t = 0$ switch S_1 is closed and remains closed for a long time and S_2 remains open. Now S_1 is opened and S_2 is closed. Find out



(i) The current through the capacitor immediately after that moment.
(ii) Charge on the capacitor long after that moment.
(iii) Total charge flown through the cell of emf 2ε after S_2 is closed.

SOLUTION

(i) Let Potential at point A is zero. The potential at point B and C will be ε (because current through the circuit is zero.)



$$V_B - V_A = (\varepsilon - 0)$$

$\therefore$ Charge on capacitor $= C(\varepsilon - 0) = C_\varepsilon$

Now S_2 is closed and S_1 is open.

(p.d. across capacitor and charge on it will not change suddenly)

Potential at A is zeo so at D it is -2ε

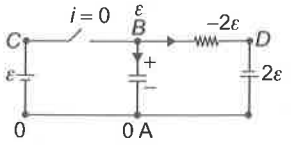
$\therefore$ current through the capacitor

$$= \frac{\varepsilon - (-2\varepsilon)}{R} = \frac{3\varepsilon}{R} (B \text{ to } D)$$

(ii) after a long time, $i = 0$

$$V_B - V_A = V_D - V_A = -2\varepsilon$$

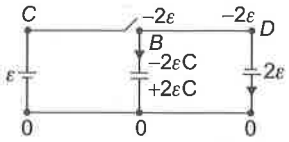
$\therefore$ $$Q = C(-2\varepsilon - 0) = -2\varepsilon C$$



(iii) The charge on the lower plate (which is connected to the battery) changes from $-\varepsilon C$ to $2\varepsilon C$.

$\therefore$ this charge will come from the battery,

$\therefore$ charge flown from that cell is $2\varepsilon C$ downward.

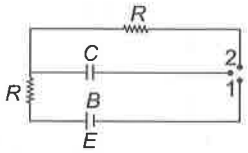


CHARGING AND DISCHARGING OF A CAPACITOR

Charging of a condenser

❖ In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by

$$q = q_0[1 - e^{-(t/RC)}]$$



Where q_0 = maximum final value of charge at $t = \infty$.

According to this equation the quantity of charge on the condenser increases exponentially with increase of time.

❖ If $t = RC = \tau$ then

$$q = q_0[1 - e^{-(RC/RC)}] = q_0\left[1 - \frac{1}{e}\right]$$

or $q = q_0\,(1 - 0.37) = 0.63q_0 = 63\%$ of q_0

❖ Time $t = RC$ is known as Time Constant.
i.e., the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

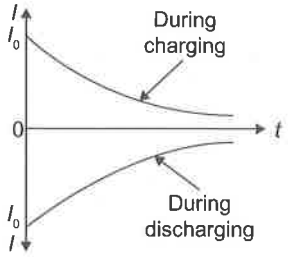
❖ The potential difference across the condenser plates at any instant of time is given by

$$V = V_0[1 - e^{-(t/RC)}] \text{ V}$$

❖ The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$$I = I_0[e^{-(t/RC)}] \text{ A}$$

According to this equation the current falls in the circuit exponentially (Fig.)



❖ If $t = RC = \tau$ = Time constant

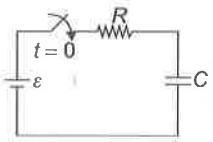
$$I = I_0[e^{-(RC/RC)} = \frac{I_0}{e} = 0.37I_0 = 37\% \text{ of } I_0$$

i.e., time constant is that time during which current in the circuit falls to 37% of its maximum value.

Derivation of formulae for charging of capacitor:

It is given that initially capacitor is uncharged. Let at any time t

Applying Kirchoff voltage law



$$\varepsilon - iR - \frac{q}{C} = 0 \quad \Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$$

$$i = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

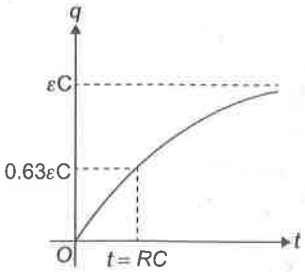
$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q} \cdot dq = dt$$

$$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}$$

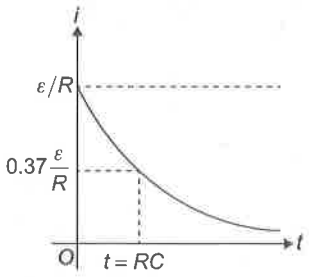
$$\varepsilon C - q = \varepsilon C \cdot e^{-t/RC} \quad \Rightarrow \quad q = \varepsilon C(1 - e^{-t/RC})$$

RC = time constant of the RC series circuit.



After one time constant

$$q = \varepsilon C\left(1 - \frac{1}{e}\right) = \varepsilon C\,(1 - 0.37) = 0.63\varepsilon C$$



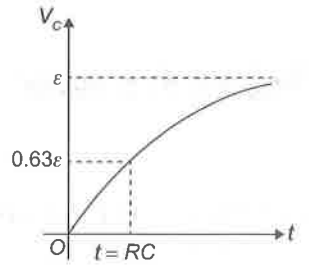
Current at any time t

$$i = \frac{dq}{dt} = \varepsilon C\left(\square e^{-t/RC}\left(-\frac{1}{RC}\right)\right) = \frac{\varepsilon}{R}e^{-t/RC}$$

Voltage across capacitor after one time constant $V = 0.63\varepsilon$

$$Q = CV$$

$$V_C = \varepsilon(1 - e^{-t/RC})$$



Voltage across the resistor

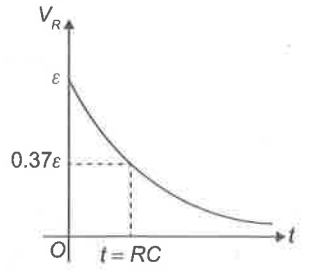
$$V_R = iR = \varepsilon e^{-t/RC}$$

By energy conservation,

Heat dissipated = work done by battery $-\Delta U$ capacitor

$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2}C\varepsilon^2 - 0\right) = \frac{1}{2}C\varepsilon^2$$

Alternatively:



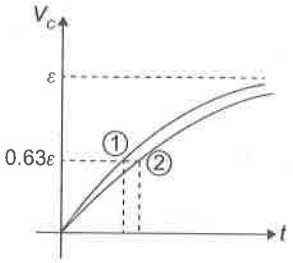
$$\text{Heat} \quad = H = \int_0^\infty i^2 R\,dt$$

$$= \int_0^\infty \frac{\varepsilon^2}{R^2} e^{\frac{2t}{RC}} R\,dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC}\,dt$$

$$= \frac{\varepsilon^2}{R}\left[\frac{e^{-\frac{2t}{RC}}}{-2/RC}\right]_0^\infty$$

$$= -\frac{\varepsilon^2 RC}{2R}\left[e^{-\frac{2t}{RC}}\right]_0^\infty = \frac{\varepsilon^2 C}{2}$$

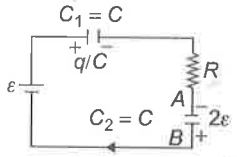
In the figure, time constant of (2) is more than (1)



SOLVED EXAMPLES

EXAMPLE 37

In the figure shown below, find out the current as function of time and charge on capacitor C_1 and also plot the graph of charge on plate A and B of capacitor C_2 as a function of time.



SOLUTION

Let at any time charge on $C_1 = q$

Now apply, KVL

$$-iR - \frac{q}{c} - \frac{(q - 2\varepsilon C)}{C} + \varepsilon = 0$$

$$3\varepsilon = \frac{2q}{c} + iR \quad \Rightarrow \quad 3\varepsilon C = 2q + iRC$$

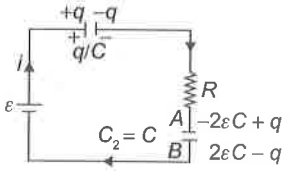
$$3\varepsilon C = 2q + iRC \quad \Rightarrow \quad \int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC}$$

$$\frac{-\ell n(3\varepsilon C - 2q)}{2}\Big|_0^q = \frac{t}{RC} \quad \Rightarrow \quad \ell n\left(\frac{3\varepsilon C - 2q}{3\varepsilon c}\right) = \frac{-2t}{RC}$$

$$3\varepsilon C - 2q = 3\varepsilon C e^{-2t/RC} \quad \Rightarrow \quad q = \frac{3\varepsilon C}{2}(1 - e^{-2t/RC})$$

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R} e^{-2t/RC}$$

At plate A,

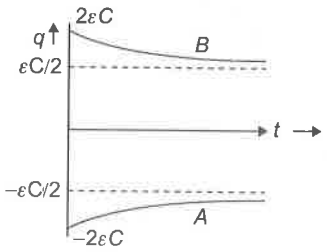


Charge $q_A = q - 2\varepsilon C = \frac{3}{2}\varepsilon C(1 - e^{-2t/RC}) - 2\varepsilon C$

$$= -\frac{\varepsilon C}{2} - \frac{3\varepsilon C}{2} e^{-2t/RC}$$

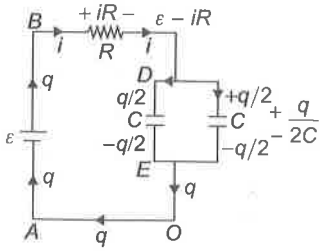
$$q_A = -\frac{\varepsilon C}{2}(1 + 3e^{-2t/RC})$$

$$q_B = \frac{\varepsilon C}{2}(1 + 3e^{-2t/RC})$$



EXAMPLE 38

Without using the formula of equivalent find out the charge on capacitor and current in all the branches as a function of time.



SOLUTION

Applying KVL in ABDEA

$$\varepsilon - iR = \frac{q}{2C} \quad \Rightarrow \quad i = \frac{\varepsilon}{R} - \frac{q}{2CR} = \frac{2C\varepsilon - q}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR} \Rightarrow \int_0^q \frac{dq}{(2\varepsilon C - q)} = \frac{t}{2CR}$$

$$\frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC} \Rightarrow q = 2\varepsilon C(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C(1 - e^{-t/RC}) \Rightarrow i_1 = \frac{\varepsilon}{2R} e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C(1 - e^{-t/2RC}) \Rightarrow i_2 = \frac{\varepsilon}{2R} e^{-t/RC}$$

Alternate solution

By equivalent

Time constant of circuit = $2C \times R = 2RC$

maximum charge on capacitor = $2C \times \varepsilon = 2C\varepsilon$

Hence equations of charge and current are as given below

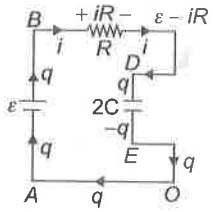
$$q = 2\varepsilon C(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C(1 - e^{-t/2RC})$$

$$\Rightarrow \quad i_1 = \frac{\varepsilon}{2R} e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C(1 - e^{-t/2RC})$$

$$\Rightarrow \quad i_2 = \frac{\varepsilon}{2R} e^{-t/RC}$$



EXAMPLE 39

A capacitor is connected to a 12 V battery through a resistance of 10 Ω. It is found that the potential difference across the capacitor rises to 4.0 V in 1 μs. Find the capacitance of the capacitor.

SOLUTION

The charge on the capacitor during charging is given by

$$Q = Q_0(1 - e^{-t/RC})$$

Hence, the potential difference across the capacitor is

$$V = \frac{Q}{C} = \frac{Q_0}{C}(1 - e^{-t/RC})$$

Here, at $t = 1$ μs, the potential differences is 4 V whereas the steady potential difference is $\frac{Q_0}{C} = 12$ V. So,

$$4V = 12V(1 - e^{-t/RC})$$

or, $$1 - e^{-t/RC} = \frac{1}{3}$$

or, $$e^{-t/RC} = \frac{2}{3}$$

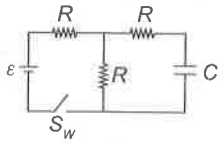
or, $$\frac{t}{RC} = \ln\left(\frac{3}{2}\right) = 0.405$$

or, $$RC = \frac{t}{0.405} = \frac{1\ \mu s}{0.405} = 2.469\ \mu s$$

or, $$C = \frac{2.469\ \mu s}{10\ \Omega} = 0.25\ \mu F.$$

EXAMPLE 40

Initially the capacitor is uncharged find the charge on capacitor as a function of time, if switch is closed at $t = 0$.

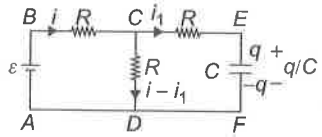


SOLUTION

Applying KVL in loop ABCDA

$$\varepsilon - iR - (i - i_1)R = 0$$

$$\varepsilon - 2iR + i_1R = 0$$



Applying KVL in loop ABCEFDA

$$\varepsilon - iR - i_1R - \frac{q}{C} = 0$$

$$\frac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \frac{q}{C}$$

$$\varepsilon C - 3i_1RC = 2q$$

$$\varepsilon C - 2q = 3\frac{dq}{dt} \cdot RC$$

$$\int_0^q \frac{dq}{\varepsilon C - 2q} = \int_0^t \frac{dt}{3RC}$$

$$\Rightarrow \quad -\frac{1}{2}\ln\frac{\varepsilon C - 2q}{\varepsilon C} = \frac{t}{3RC}$$

$$q = \frac{\varepsilon C}{2}(1 - e^{-2t/3RC})$$

Method for objective:

In a circuit when there is only one capacitor then

$q = Q_{st}(1 - e^{-t/\tau})$: Q_{st} = steady state change on capacitor

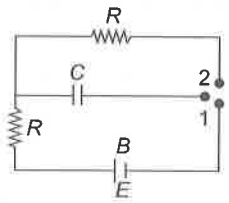
$$\tau = R_{eff}C$$

$R_{effective}$ is the resistance between the capacitor when battery is replaced by its internal resistance.

Discharging of a condenser

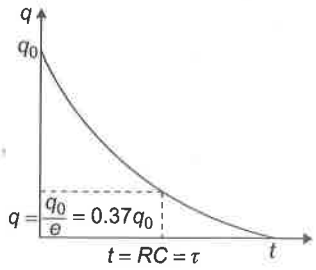
(i) In the given circuit if key 1 is opened and key 2 is closed then the condenser gets discharged.

(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0 e^{-(t/RC)}$ i.e., the charge falls exponentially.



(iii) If $t = RC = \tau$ = time constant, then $q = \frac{q_0}{e} = 0.37q_0$ = 37% of q_0

i.e., the time constant is that time during which the charge on condenser plates falls to 37%

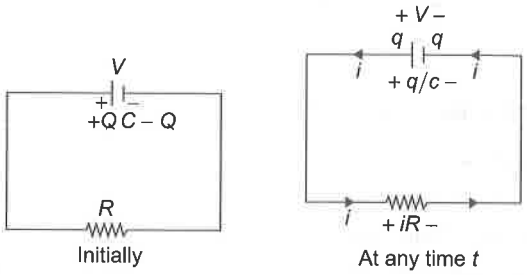


(iv) The dimensions of RC are those of time i.e., $M^{\circ}L^{\circ}T^{1}$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e., $M^{\circ}L^{\circ}T^{-1}$

(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0 e^{-(t/RC)}$ volt

(vi) The transient current at any instant of time is given by $I = -I_0 e^{-(t/RC)}$ ampere. i.e., the current in the circuit decreases exponentially but its direction is opposite to that of charging current.

Derivation of equation of discharging circuit

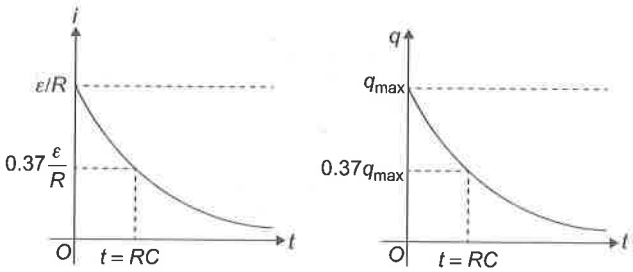


Applying KVL $+\frac{q}{C} - iR = 0 \quad \Rightarrow \quad i = \frac{q}{CR}$

$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR} \quad \Rightarrow \quad -\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q \cdot e^{-t/RC}$$

$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC}$$



EXAMPLE 41

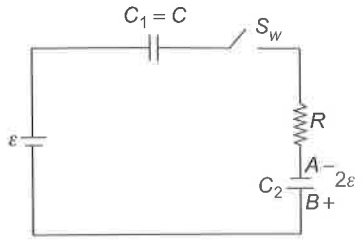
At $t = 0$ S_w is closed, if initially C_1 is uncharged and C_2 is charged to a potential difference 2ε then find out following

(Given $C_1 = C_2 = C$)

(a) Charge on C_1 and C_2 as a function of time.

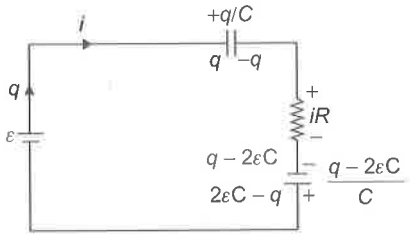
(b) Find out current in the circuit as a function of time.

(c) Also plot the graphs for the relations derived in part (a)



SOLUTION

Let q charge flow in time 't' from the battery as shown. The charge on various plates of the capacitor is as shown in the figure.



Now applying KVL

$$\varepsilon - \frac{q}{C} - iR - \frac{q - 2\varepsilon C}{C} = 0$$

$$\varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$$

$$3\varepsilon = \frac{2q}{C} + iR$$

$$3\varepsilon - iR = \frac{2q}{C}$$

$$3\varepsilon - iRC = 2q$$

$$\frac{dq}{dt} RC = 3\varepsilon C - 2q$$

$$\int_0^q \frac{dq}{2\varepsilon C - 2q} = \int_0^t \frac{dt}{RC}$$

$$-\frac{1}{2} \ell n\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{RC}$$

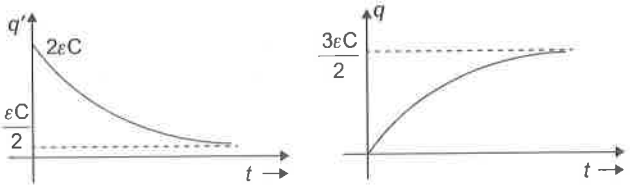
$$\ln\left(\frac{3\varepsilon C - 2q}{3\varepsilon C}\right) = -\frac{2t}{RC}$$

$$3\varepsilon C - 2q = 3\varepsilon C e^{-2t/RC}$$

$$3\varepsilon C(1 - e^{-2t/RC}) = 2q$$

$$q = \frac{3}{2}\varepsilon C(1 - e^{-2t/RC}) \quad \textbf{Ans.}$$

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R} e^{-2t/RC} \quad \textbf{Ans.}$$



On the plate B

$$q' = 2\varepsilon C - q$$

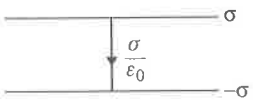
$$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\varepsilon C e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2} + \frac{3}{2}\varepsilon C e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2}[1 + 3e^{-2t/RC}]$$

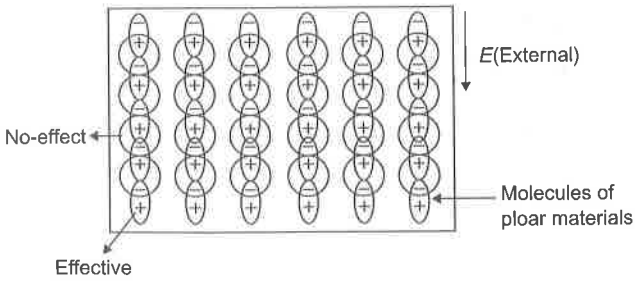
CAPACITORS WITH DIELECTRIC

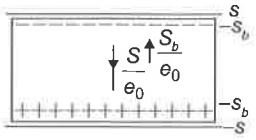
(i) In absence of dielectric



$$E = \frac{\sigma}{\varepsilon_0}$$

(ii) When a dielectric fills the sapce between the plates then molecules having dipole moment align themselves in the direction of electric field.





σ_b = induced charge density (called bound charge because it is not due to free electrons).

- For polar molecules dipole moment $\neq 0$
- For non-polar molecules dipole moment = 0

(iii) Capacitance in the presence of dielectric

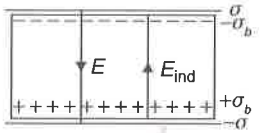
$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K\varepsilon_0} \cdot d} = \frac{AK\varepsilon_0}{d} = \frac{AK\varepsilon_0}{d}$$

Here capacitance is increased by a factor K.

$$C = \frac{AK\varepsilon_0}{d}$$

(iv) Polarization of material:

When a nonpolar substance is placed in an electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarization of material. The induced charge also produces electric field.



σ_b = induced (bound) charge density.

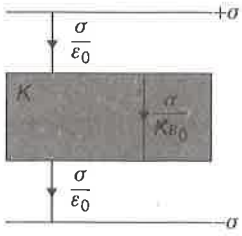
$$E_{in} = E - E_{ind} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0}$$

It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by ε_r or k.

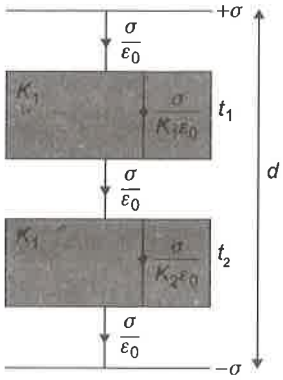
$$E_{in} = \frac{\sigma}{K\varepsilon_0} \Rightarrow \sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

(v) If the medium is not filled between the plates completely then electric filed will be as shown in figure.

Case I: The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.



Case II:



so potential difference between plates

$$V = \frac{\sigma}{\varepsilon_0}[d - t_1 - t_2] + \frac{\sigma}{k_1\varepsilon_0}t_1 + \frac{\sigma}{k_2\varepsilon_0}t_2$$

so equivalent capacitance

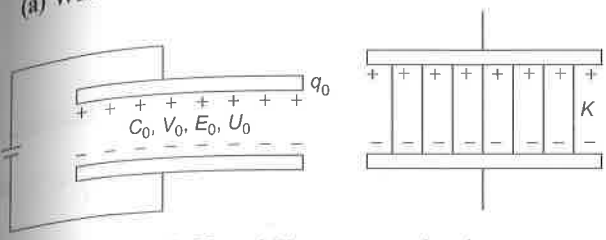
$$C = \frac{Q}{v}$$

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[d - t_1 - t_2 + \frac{t_1}{k_1} + \frac{t_2}{k_2}\right]}$$

$$C = \frac{A\varepsilon_0}{d - t_1\left(1 - \frac{1}{k_1}\right) - t_2\left(1 - \frac{1}{k_2}\right)}$$

Introduction of a Dielectric slab of dielectric constant K between the plates

(a) When the battery is disconnected



Let q_0, C_0, V_0, E_0 and U_0 represents the charge, capacity, potential difference, electric field and energy associated with charged air capacitor respectively. With the introduction of a dielectric slab of dielectric constant K between the plates and the battery disconnected.

(i) Charge remains constant, i.e., $q = q_0$, as in an isolated system charge is conserved.

(ii) Capacity increases, i.e., $C = KC_0$, as by the presence of a dielectric capacity becomes K times.

(iii) Potential difference between the plates decreases, i.e., $V = \left(\frac{V_0}{K}\right)$, as

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K} \quad [\because\ q = q_0 \text{ and } C = KC_0]$$

(iv) Field between the plates decreases, i.e., $E = \frac{E_0}{K}$, as

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \left[\text{as } V = \frac{V_0}{K}\right]$$

and $E_0 = \frac{V_0}{d}$

(v) Energy stored in the capacitor decreases i.e., $U = \left(\frac{U_0}{K}\right)$, as

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{V_0}{K} \text{ (as } q = q_0 \text{ and } C = KC_0]$$

(b) When the battery remains connected (potential is held constant)

(i) Potential difference remains constant, i.e., $V = V_0$, as battery is a source of constant potential difference.

(ii) Capacity increases, i.e., $C = KC_0$, as by presence of a dielectric capacity becomes K times.

(iii) Charge on capacitor increases, i.e., $q = Kq_0$, as

$$q = CV = (KC_0)V = Kq_0 \ [\because\ q_0 = C_0V]$$

(iv) Electric field remains unchanged, i.e., $E = E_0$, as

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0 \left[\text{as } V = V_0 \text{ and } \frac{V_0}{d} = E_0\right]$$

(v) Energy stored in the capacitor increases, i.e., $U = KU_0$, as

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2 = \frac{1}{2}KU_0$$

$$\left[\text{as } C = KC_0 \text{ and } U_0 = \frac{1}{2}C_0V_0^2\right]$$

SOLVED EXAMPLES

EXAMPLE 42

A parallel plate air capacitor is made using two square plates each of side 0.2 m, spaced 1 cm apart. It is connected to a 50 V battery.

(a) What is the capacitance?
(b) What is the charge on each plate?
(c) What is the energy stored in the capacitor?
(d) What is the electric field between the plates?
(e) If the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the answers to the above parts?

SOLUTION

(a) $C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}) \times 0.2 \times 0.2}{0.01}$

$= 3.54 \times 10^{-5}\ \mu\text{F}$

(b) $Q_0 = C_0V_0 = 3.54 \times 10^{-5} \times 50 = 1.77 \times 10^{-3}\ \mu\text{C}$

(c) $U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2} \times (3.54 \times 10^{-11})(50)^2$

$= 4.42 \times 10^{-8}\ \text{J}$

(d) $E_0 = \frac{V_0}{d} = \frac{50}{0.01} = 5000\ \text{V/m}$

(e) If the battery is disconnected the charge on the capacitor plates remains constant while the potential difference between the plates can change.

$$C = \frac{\varepsilon_0 A}{d} = \frac{C_0}{2} = 1.77 \times 10^{-5}\ \mu\text{F}$$

$$Q = Q_0 = 1.77 \times 10^{-3}\ \mu\text{C}$$

126

$$V = \frac{Q}{C} = \frac{Q_0}{C_0/2} = 2V_0$$

$$U = \frac{1}{2}CV^2 = C_0V_0^2 = 8.84 \times 10^{-8}\ \text{J}$$

$$E = \frac{V}{d} = \frac{2V_0}{2d_0} = E_0 = 5000\ \text{V/m}$$

EXAMPLE 43

In the last illustration, suppose that the battery is kept connected while the plates are pulled apart. What are the answers to the parts (a), (b), (c) and (d) in that case?

SOLUTION

If the battery is kept connected, the potential difference across the capacitor plates always remains equal to the emf of battery and hence is constant.

$$V = V_0 = 50\ \text{V}$$

$$C = \frac{\varepsilon_0 A}{d'} = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2} = 1.77 \times 10^{-5}\ \mu\text{F}$$

$$Q = CV = \frac{C_0V_0}{2} = \frac{Q_0}{2} = 8.85 \times 10^{-4}\ \mu\text{C}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{C_0}{2}\right)V_0^2 = \frac{U_0}{2} = 2.21 \times 10^{-8}\ \text{J}$$

$$E = \frac{V}{d} = \frac{V_0}{2d_0} = \frac{E_0}{2} = 2500\ \text{V/m}$$

EXAMPLE 44

A parallel plate capacitor has plates of area 4 m^2 separated by distance of 0.5 mm. The capacitor is connected across a cell of emf 100 V.

(a) Find the capacitance, charge and energy stored in the capacitor.

(b) A dielectric slab of thickness 0.5 mm is inserted inside this capacitor after it has been disconnected from the cell. Find the answers to part (a) if $K = 3$.

SOLUTION

(a) $$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 4}{0.5 \times 10^{-3}} = 7.08 \times 10^{-2}\ \mu\text{F}$$

$$Q_0 = C_0V_0 = (7.08 \times 10^{-2} \times 100)\mu\text{C} = 7.08\ \mu\text{C}$$

$$U_0 = \frac{1}{2}C_0V_0^2 = 3.54 \times 10^{-4}\ \text{J}$$

(b) As the cell has been disconnected

$$Q = Q_0$$

$$C = \frac{K\varepsilon_0 A}{d} = KC_0 = 0.2124\ \mu\text{F}$$

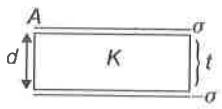
$$V = \frac{Q}{C} = \frac{Q_0}{KC_0} = \frac{V_0}{K} = \frac{100}{3}\ \text{V}$$

$$U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K} = 118 \times 10^{-6}\ \text{J}$$

EXAMPLE 45

If a dielectric slab of thickness t and area A is inserted in between the plates of a parallel plate capacitor of plate area A and distance between the plates $d(d > t)$ then find out capacitance of system. What do you predict about the dependence of capacitance on location of slab?

SOLUTION



$$C = \frac{Q}{V} = \frac{\sigma A}{V}$$

$$V = \frac{\sigma t_1}{\varepsilon_0} + \frac{\sigma t}{K\varepsilon_0} + \frac{\sigma t_2}{\varepsilon_0} \quad (\because\ t_1 + t_2 = d - t)$$

$$= \frac{\sigma}{\varepsilon_0}\left[t_1 + t_2 + \frac{t}{k}\right]$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{k}\right] = \frac{Q}{C} = \frac{\sigma A}{C}$$

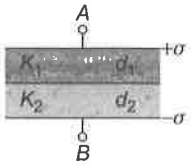
$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

- Capacitance does not depend upon the position of dielectric (it can be shifted up or down and still capacitance does not change).
- If the slab is of metal then

$$C = \frac{A\varepsilon_0}{d - t}$$

EXAMPLE 46

Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.

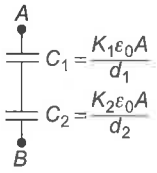


SOLUTION

$$C = \frac{\sigma A}{V};$$

$$V = E_1 d_1 + E_2 d_2 = \frac{\sigma d_1}{K_1 \varepsilon_0} + \frac{\sigma d_2}{K_2 \varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)$$

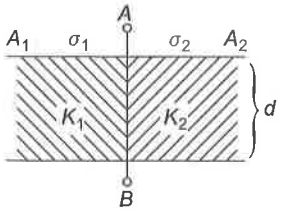
$$\therefore \quad C = \frac{A\varepsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \quad \Rightarrow \quad \frac{1}{C} = \frac{d_1}{AK_1\varepsilon_0} + \frac{d_2}{AK_2\varepsilon_0}$$

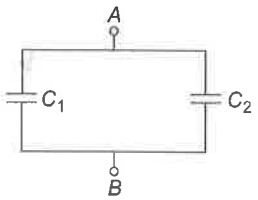


This formula suggests that the system between A and B can be considered as series combination of two capacitors.

EXAMPLE 47

Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of area A_1 and A_2 and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.





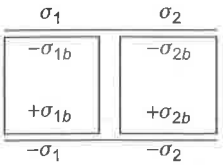
SOLUTION

$$C_1 = \frac{A_1 K_1 \varepsilon_0}{d}, \qquad C_2 = \frac{A_2 K_2 \varepsilon_0}{d}$$

$$E_1 = \frac{V}{d} = \frac{\sigma_1}{K_1\varepsilon_0}; \quad E_2 = \frac{V}{d} = \frac{\sigma_2}{K_2\varepsilon_0}$$

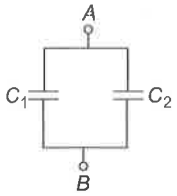
$$\sigma_1 = \frac{K_1\varepsilon_0 V}{d} \qquad \sigma_2 = \frac{K_2\varepsilon_0 V}{d}$$

$$C = \frac{Q_1 + Q_2}{V} = \frac{\sigma_1 A_1 + \sigma_2 A_2}{V} = \frac{K_1\varepsilon_0 A_1}{d} + \frac{K_2\varepsilon_0 A_2}{d}$$



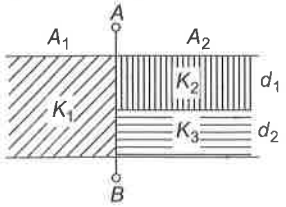
The combination is equivalent to:

$$\therefore \qquad C = C_1 + C_2$$



EXAMPLE 48

Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d_1 K_2 of area A_2 and thickness d_2 and K_3 of area A_2 and thickness d_3 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1 + d_2$)

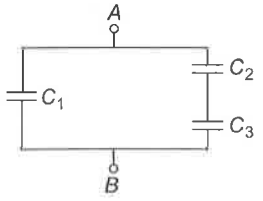


SOLUTION

It is equivalent to

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

$$C = \frac{A_1 K_1 \varepsilon_0}{d_1 + d_2} + \frac{\frac{A_2 K_2 \varepsilon_0}{d_1} \cdot \frac{A_2 K_3 \varepsilon_0}{d_2}}{\frac{A_2 K_2 \varepsilon_0}{d_1} + \frac{A_2 K_3 \varepsilon_0}{d_2}}$$



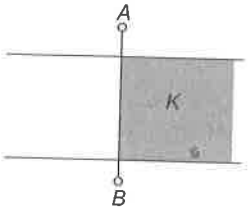
128

$$= \frac{A_1 K_1 \varepsilon_0}{d_1 + d_2} + \frac{A_2^2 K_2 K_3 \varepsilon_0^2}{A_2 K_2 \varepsilon_0 d_2 + A_2 K_3 \varepsilon_0 d_1}$$

$$= \frac{A_1 K_1 \varepsilon_0}{d_1 + d_2} + \frac{A_2 K_2 K_3 \varepsilon_0}{K_2 d_2 + K_3 d_1}$$

EXAMPLE 49

A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be



(A) $\frac{C}{2}$ (B) $\frac{C}{2K}$

(C) $\frac{C}{2}[1+K]$ (D) $\frac{2[1+K]}{C}$

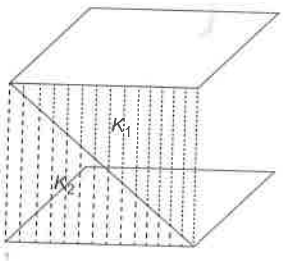
SOLUTION

This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

$$C' = C_1 + C_2$$

$$= \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 AK}{2d} = \frac{\varepsilon_0 A}{2d}[1+K] = \frac{C}{2}[1+K]$$

Hence the correct answer will be (C)



EXAMPLE 50

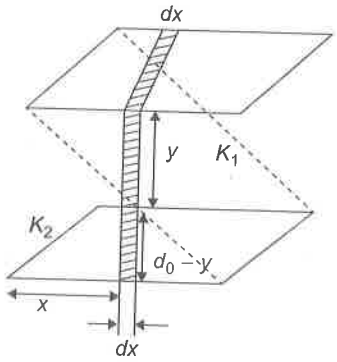
Find the equivalent capacitance of the given figure.

SOLUTION

$$\frac{1}{dc} = \frac{y}{\varepsilon_0 k_1 b dx} + \frac{d_0 - y}{\varepsilon_0 k_2 b dx}$$

$$\frac{1}{dc} = \frac{d_0 k_1 + y(k_2 - k_1)}{\varepsilon_0 k_1 k_2 b dx}$$

$$dc = \frac{\varepsilon_0 k_1 k_2 b dx}{d_0 k_1 + y(k_2 - k_1)}$$



All these capacitors (small) are parallel

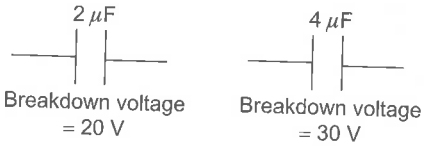
so $$C_{eq} = \int_0^{C_{eq}} dC = \int_0^a \frac{\varepsilon_0 k_1 k_2 b dx}{d_0 k_1 + y(k_2 - k_1)}$$

Now $$\frac{d_0}{a} = \frac{y}{x} \Rightarrow C_{eq} = \int_0^a \frac{\varepsilon_0 k_1 k_2 b dx}{d_0 k_1 + \frac{d_0}{a} x(k_2 - k_1)}$$

$$C_{eq} = \frac{a\varepsilon_0 k_1 k_2 b}{d_0 (k_2 - k_1)} \ln\left[\frac{(k_2 - k_1)}{k_1}\right]$$

❖ **Breakdown voltage**

The voltage across the capacitor at which the current starts flowing through capacitor is called Breakdown voltage.



EXAMPLE 51

Find the breakdown voltage for the capacitors if they are connected in series

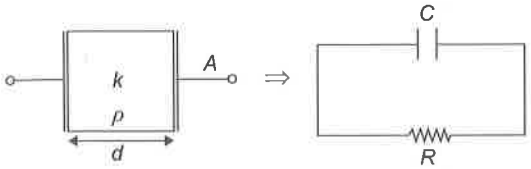
Charge will be same on both the capacitors so 2 μF capacitor will reach at breakdown voltage first, so maximum charge possible = 40 μC

So Breakdown voltage = $V_{C_1} + V_{C_2}$ $\frac{Q_1}{C_1} + \frac{Q_1}{C_2}$

$$V = \frac{40}{2} + \frac{40}{4} \text{ V}$$

$$V = \frac{120}{4} \text{ V} = 30 \text{ V}$$

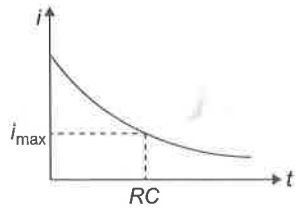
Leakage current:



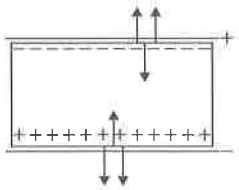
here $C = \frac{\varepsilon_0 kA}{d}$

Resistance $(R) = \frac{\rho d}{A}$

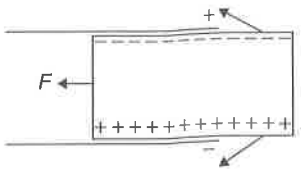
Theoretically after disconnection from battery charge should remain as it is but due to the material's resistance discharging takes place. This discharging current is called leakage current. So R should be high for a good capacitor so that leakage current is minimum.



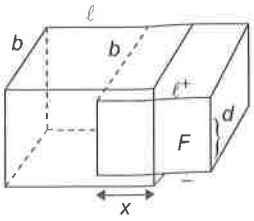
(viii) Force on a dielectric due to charged capacitor



If dielectric is completely inside the capacitor then force is equal to zero.



Case I: Voltage source remains connected



V = constant.

$$U = \frac{1}{2}CV^2$$

$$F = \left(\frac{dU}{dx}\right) = \frac{v^2}{2}\frac{dC}{dx}$$

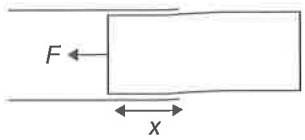
where $C = \frac{xb\varepsilon_0 K}{d} + \frac{\varepsilon_0(\ell - x)b}{d}$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}[Kx + \ell - x]$$

$$\frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K - 1)$$

$$\therefore \quad F = \frac{\varepsilon_0 b(K-1)V^2}{2d} = \text{constant (does not depends on } x)$$

Case II: When charge on capacitor is constant



$$C = \frac{xb\varepsilon_0 x}{d} + \frac{\varepsilon_0(\ell - x)b}{d}, \quad U = \frac{Q^2}{2C}$$

$$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \quad \left[\text{where, } \frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K - \ell)\right]$$

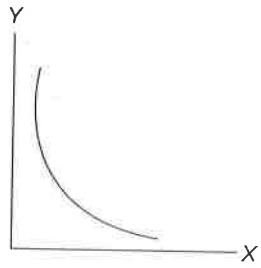
$$= \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \quad \text{(here force '}F\text{' depends on } x)$$

130

EXERCISES

JEE Main

1. What physical quantities may X and Y represent?



(A) pressure v/s temperature of a given gas (constant volume)
(B) kinetic energy v/s velocity of a particle
(C) capacitance v/s charge at a constant potential
(D) potential v/s capacitance at a constant charge

2. The capacitance of a metallic sphere will be 1 μF, if its radius is nearly
(A) 9 km (B) 10 m
(C) 1.11 m (D) 1.11 cm

3. No current flows between two charged bodies connected together when they have the same
(A) capacitance or Q/V ratio
(B) charge
(C) resistance
(D) potential or Q/C ratio

4. Two spherical conductors A and B of radii R and $2R$ respectively are each given a charge Q. When they are connected by a metallic wire. The charge will
(A) flow from A to B
(B) flow from B to A
(C) remain stationary on conductor
(D) none of these

5. The capacity of a parallel plate condenser is C. Its capacity when the separation between the plates is halved will be
(A) 4C (B) 2C
(C) $\frac{C}{2}$ (D) $\frac{C}{4}$

6. If the p.d. across the ends of a capacitor 4 μF is 1.0 kV. Then its electrical potential energy will be
(A) 4×10^{-3} ergs (B) 2 ergs
(C) 2 J (D) 4 J

7. The energy of a charged capacitor resides in
(A) the electric field only
(B) the magnetic field only
(C) both the electric and magnetic field
(D) neither in electric nor magnetic field

8. The capacitance of a parallel plate condenser does not depend upon
(A) the distance between the plates
(B) area of the plates
(C) medium between the plates
(D) metal of the plates

9. The energy density in a parallel plate capacitor is given as 2.2×10^{-10} J/m^3. The value of the electric field in the region between the plates is -
(A) 7 NC^{-1} (B) 3.6 NC^{-1}
(C) 72 NC^{-1} (D) 8.4 NC^{-1}

10. Two capacitances of capacity C_1 and C_2 are connected in series and potential difference V is applied across it. Then the potential difference acros C_1 will be
(A) $V\frac{C_2}{C_1}$ (B) $V\frac{C_1+C_2}{C_1}$
(C) $V\frac{C_2}{C_1+C_2}$ (D) $V\frac{C_1}{C_1+C_2}$

11. A conductor of capacitance 0.5 μF has been charged to 100 V. It is now connected to uncharged conductor of capacitance 0.2 μF. The loss in potential energy is nearly -
(A) 7×10^{-4} J (B) 3.5×10^{-4} J
(C) 14×10^{-4} J (D) 7×10^{-3} J

12. Two spherical conductors of capacitance 3.0 μF and 5.0 μF are charged to potentials of 300 V and 500 V. The two are connected resulting in redistribution of charges. Then the final potential is -
(A) 300 V (B) 500 V
(C) 425 V (D) 400 V

13. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference 2 V. The charging battery is now disconnected and the capacitors are connect

in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(A) zero (B) $\frac{3}{2}CV^2$

(C) $\frac{25}{6}CV^2$ (D) $\frac{9}{2}CV^2$

14. A 2 μF capacitor is charged to a potential = 10 V. Another 4 μF capacitor is charged to a potential = 20 V. The two capacitors are then connected in a single loop, with the positive plate of one connected with negative plate of the other. What heat is evolved in the circuit?

(A) 300 μJ (B) 600 μJ

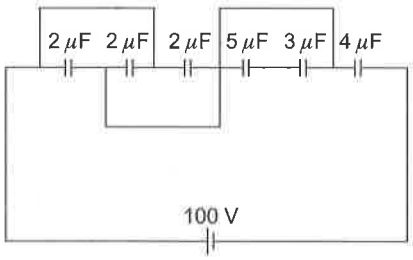
(C) 900 μJ (D) 450 μJ

15. N drops of mercury of equal radii and possessing equal charges combine to form a big spherical drop. Then the capacitance of the bigger drop compared to each individual drop is

(A) N times (B) $N^{2/3}$ times

(C) $N^{1/3}$ times (D) $N^{5/3}$ times

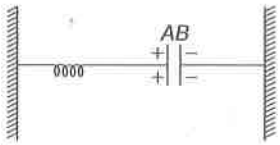
16. In the circuit shown in figure charge stored in the capacitor of capacity 5 μF is



(A) 60 μC (B) 20 μC

(C) 30 μC (D) zero

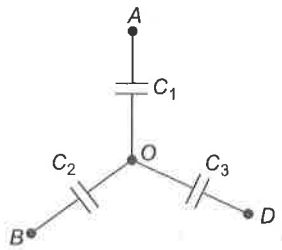
17. Plate A of a parallel air filled capacitor is connected to a spring having force constant k and plate B is fixed. If a charge $+q$ is placed on plate A and charge $-q$ on plate B then find out extension in spring in equilibrium. Assume area of plate is 'A'



(A) $\frac{Q^2}{k\epsilon_0 A}$ (B) $\frac{Q^2}{4k\epsilon_0 A}$

(C) $\frac{Q^2}{2k\epsilon_0 A}$ (D) none of these

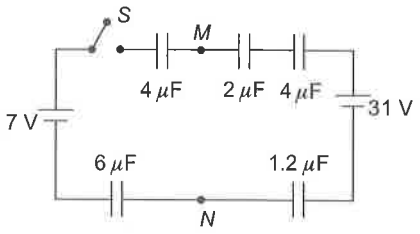
18. Three uncharged capacitors of capacitance $C_1 = 1$ μF, $C_2 = 2$ μF and $C_3 = 3$ μF are connected as shown in figure to one another and to points A, B and D potential $\phi_A = \phi 10$ V, $\phi_B = 25$ V and $\phi_D = 20$ V. Determine the potential (ϕ_0) at point O.



(A) 20 V (B) 30 V

(C) 40 V (D) 10 V

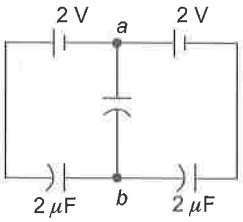
19. Five capacitors are connected as shown in the figure. Initially S is opened and all capacitors are uncharged. When S is closed, steady state is obtained. Then find out potential difference between the points M and N.



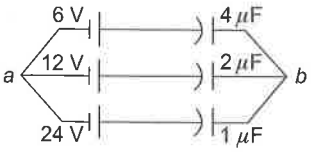
(A) 14 (B) 12

(C) 10 (D) 15

20. Find the potential difference $V_a - V_b$ between the points a and b shows in each parts of the figure.



(a)



(b)

(A) zero, $-\frac{7}{72}$ V = −10.3 V

(B) one, $-\frac{72}{7}$ V = −10.3 V

(C) one, $-\frac{7}{72}$ V = −10.3 V

(D) zero, $-\frac{72}{7}$ V = −10.3 V

132

21. Each plate of a parallel plate air capacitor has an area S. What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2. If:

(i) the charge of the capacitor, which is equal to q or

(A) $\frac{q^2(x_2 + x_1)}{2 \epsilon_0 S}$ (B) $\frac{q^2(x_2 - x_1)}{2 \epsilon_0 S}$

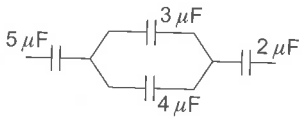
(C) $\frac{q^2(x_2 - x_1)}{\epsilon_0 S}$ (D) $\frac{q^2(x_1 - x_2)}{2 \epsilon_0 S}$

(ii) the voltage across the capacitor, which is equal to V, is kept constant in the process.

(A) $\frac{\epsilon_0 SV^2\left(\frac{1}{x_1} - \frac{1}{x_2}\right)}{2}$ (B) $\frac{\epsilon_0 SV^2\left(\frac{1}{x_2} - \frac{1}{x_1}\right)}{4}$

(C) $\frac{\epsilon_0 SV^2\left(\frac{1}{x_2} - \frac{1}{x_1}\right)}{2}$ (D) $\frac{\epsilon_0 SV^2\left(\frac{1}{x_2} + \frac{1}{x_1}\right)}{2}$

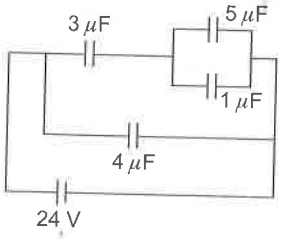
22. If charge on left plane of the 5 μF capacitor in the circuit segment shown in the figure is –20 μC, the charge on the right plate of 3 μF capacitor is



(A) +8.57 μC (B) –8.57 μC

(C) +11.42 μC (D) –11.42 μC

23. In the circuit shown, the energy stored in 1 μF capacitor is



(A) 40 μJ (B) 64 μJ

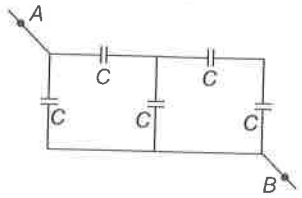
(C) 32 μJ (D) none

24. A capacitor $C_1 = 4$ μF is connected in series with another capacitor $C_2 = 1$ μF. The combination is connected across a DC source of voltage 200 V. The ratio of potential across C_1 and C_2 is -

(A) 1:4 (B) 4:1

(C) 1:2 (D) 2:1

25. What is the equivalent capacitance of the system of capacitors between A and B



(A) $\frac{7}{6}C$ (B) $1.6C$

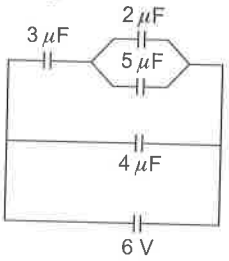
(C) C (D) none

26. From a supply of identical capacitors rated 8 μF, 250 V, the minimum number of capacitors required to form a composite 16 μF, 1000 V is:

(A) 2 (B) 4

(C) 16 (D) 32

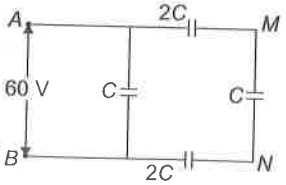
27. In the circuit shown in figure, the ratio of charges on 5 μF and 4 μF capacitor is:



(A) $\frac{4}{5}$ (B) $\frac{3}{5}$

(C) $\frac{3}{8}$ (D) $\frac{1}{2}$

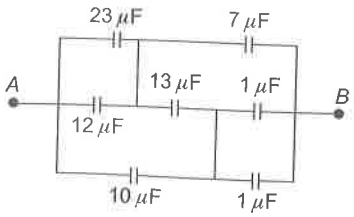
28. In the circuit shown, a potential difference of 60 V is applied across AB. The potential difference between the point M and N is



(A) 10 V (B) 15 V

(C) 20 V (D) 30 V

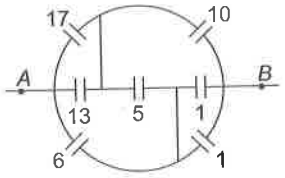
29. Find the equivalent capacitance across A and B



(A) $\frac{28}{3}\mu F$ (B) $\frac{15}{2}\mu F$

(C) 15 μF (D) none

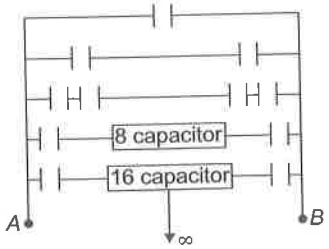
30. Find equivalent capacitance across AB (all capacitances in μF)



(A) $\frac{20}{3}\mu F$ (B) 9 μF

(C) 48 μF (D) none

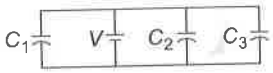
31. An infinite number of identical capacitors each of capacitance 1 μF are connected as in adjoining figure. Then the equivalent capacitance between A and B is



(A) 1 μF (B) 2μF

(C) $\frac{1}{2}\mu F$ (D) ∞

32.



(i) Find the charges on the three capacitors connected to a battery as shown in figure. Take $C_1 = 2.0\ \mu F$, $C_2 = 4.0\ \mu F$, $C_3 = 6.0\ \mu F$ and $V = 10$ V.

(A) 20 μC, 40 μC, 60 μC

(B) 40 μC, 20 μC, 60 μC

(C) 60 μC, 40 μC, 20 μC

(D) 40 μC, 60 μC, 20 μC

(ii) Find out work done by the battery during the process of charging

(A) 1500 μJ (B) 1200 μJ

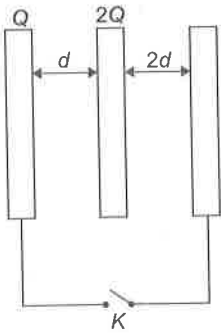
(C) 1600 μJ (D) 1300 μJ

(iii) Find out total energy stored in the capacitors.

(A) 500 μJ (B) 300 μJ

(C) 600 μJ (D) 800 μJ

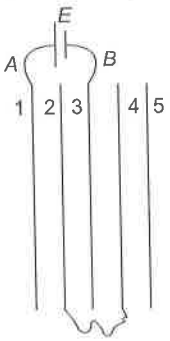
33. Three large plates are arranged as shown. How much charge will flow through the key k if it is closed?



(A) $\frac{5Q}{6}$ (B) $\frac{4Q}{3}$

(C) $\frac{3Q}{2}$ (D) none

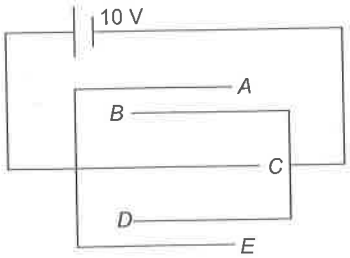
34. Five conducting parallel plates having area A and separation between them d, are placed as shown in the figure. Plate number 2 and 4 are connected wire and between point A and B, a cell of emf E is connected. The charge flown through the cell is



(A) $\frac{3}{4}\frac{\varepsilon_0 AE}{d}$ (B) $\frac{2}{3}\frac{\varepsilon_0 AE}{d}$

(C) $\frac{4\varepsilon_0 AE}{d}$ (D) $\frac{\varepsilon_0 AE}{2d}$

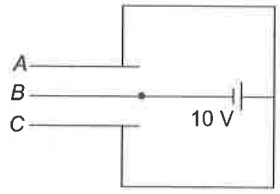
35. Five identical capacitor plates are arranged such that they make capacitors each of 2 μF. The plates are connected to a source of emf 10 V. The charge on plate C is



134

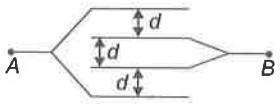
(A) +20 μC (B) +40 μC
(C) +60 μC (D) +80 μC

36. Three plates A, B and C each of area 0.1 m^2 are separated by 0.885 mm from each other as shown in the figure. A 10 V battery is used to charge the system. The energy stored in the system is



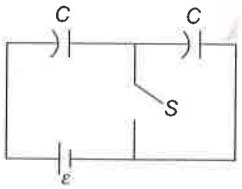
(A) 1 μJ (B) 10^{-1} μJ
(C) 10^{-2} μJ (D) 10^{-3} μJ

37. Four metallic plates are arranged as shown in the figure. If the distance between each plate then capacitance of the given system between points A and B is (Given $d << A$)



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{2\varepsilon_0 A}{d}$
(C) $\frac{3\varepsilon_0 A}{d}$ (D) $\frac{4\varepsilon_0 A}{d}$

38. Consider the situation shown in the figure. The switch S is open for a long time and then closed.



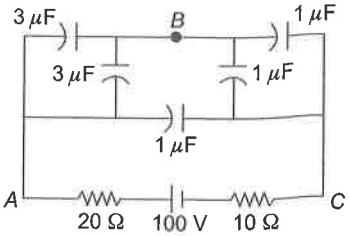
(a) Find the charge flown through the battery when the switch S is closed.
(A) $C\varepsilon/2$ (B) $C\varepsilon/4$
(C) $C\varepsilon$ (D) none of these

(b) Find the work done by the battery.
(A) $C\varepsilon^2$ (B) $\frac{C\varepsilon^2}{2}$
(C) $\frac{C\varepsilon^2}{4}$ (D) none of these

(c) Find the change in energy stored in the capacitors.
(A) $C\varepsilon^2$ (B) $\frac{C\varepsilon^2}{2}$
(C) $\frac{C\varepsilon^2}{4}$ (D) none of these

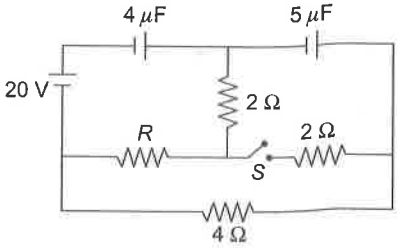
(d) Find the heat developed in the system.
(A) $C\varepsilon^2$ (B) $\frac{C\varepsilon^2}{2}$
(C) $\frac{C\varepsilon^2}{4}$ (D) none of these

39. Find the potential difference between the points A and B and between the points B and C figure in steady



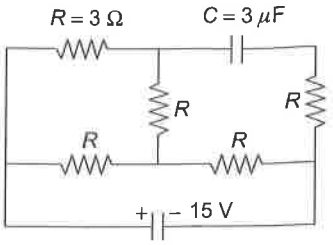
(A) 75 V and 25 V (B) 35 V and 65 V
(C) 25 V and 75 V (D) 65 V and 35 V

40. Find heat produced in the capacitors on closing the switch S



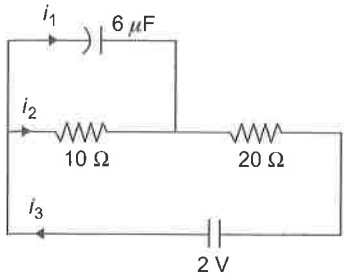
(A) 0.0002 J (B) .0005 J
(C) 0.00075 J (D) zero

41. In the circuit shown, the cell is ideal, with emf = 15 V. Each resistance is of 3 Ω. The potential difference across the capacitor is



(A) zero (B) 9 V
(C) 12 V (D) 15 V

42.



(i) In steady state, find the charge on the capacitor shown in figure.

(A) 4 μC (B) 5 μC
(C) 6 μC (D) 7 μC

(ii) Find out values of i_1, i_2 and i_3

(A) $0, \frac{1}{15}\text{A}, \frac{1}{15}\text{A}$ (B) $\frac{1}{15}\text{A}, 0, \frac{1}{15}\text{A}$

(C) $0, \frac{1}{15}\text{A}, 0$ (D) $\frac{1}{15}\text{A}, \frac{1}{15}\text{A}, 0$

43. A capacitor of capacitance 100 μF is connected across a battery of emf 6.0 V through a resistance of 20 kΩ for 4.0 s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 4.0 s after the battery is disconnected?

(A) 70 μC (B) 80 μC
(C) 60 μC (D) none of these

44. A capacitor of capacitance 0.1 μF is charged to certain potential and allow to discharge through a resistance of 10 MΩ. How long will it take for the potential to fall to one half of its original value-

(A) 0.1 s (B) 0.2346 s
(C) 1.386 s (D) 0.693 s

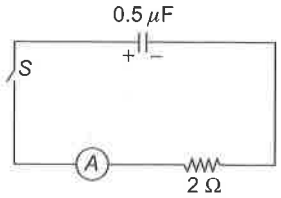
45. The electric field between the plates of a parallel-plate capacitance 2.0 μF drops to one third of its initial value in 4.4 μs when the plates are connected by a thin wire. Find the resistance of the wire.

(A) 3.0 Ω (B) 2.0 Ω
(C) 4.0 Ω (D) 1.0 Ω

46. A 5.0 μF capacitor having a charge of 20 μC is discharged through a wire of resistance of 5.0 Ω. Find the heat dissipated in the wire between 25 to 50 μs after the capactions are made.

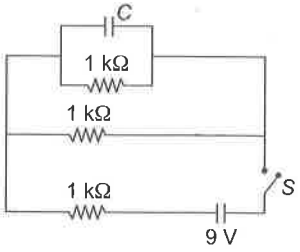
(A) 4.7 μJ (B) 3.7 μJ
(C) 5.7 μJ (D) 2.7 μJ

47. A charged capacitor is allowed to discharge through a resistance 2 Ω by closing the switch S at the instant $t = 0$. At time $t = \ln 2$ μs, the reading of the ammeter falls half of its initial value. The resistance of the ammeter equal to



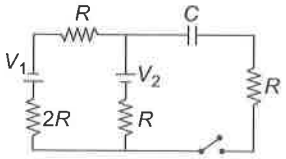
(A) 0 (B) 2 Ω
(C) ∞ (D) 2 MΩ

48. A capacitor $C = 100$ μF is connected to three resistor each of resistance 1 kΩ and a battery of emf 9 V. The switch S has been closed for long time so as to charge the capacitor. When switch S is opened, the capacitor. Discharges with time constant



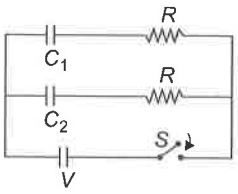
(A) 33 ms (B) 5 ms
(C) 3.3 ms (D) 50 ms

49. In the transient shown the time constant of the circuit is:



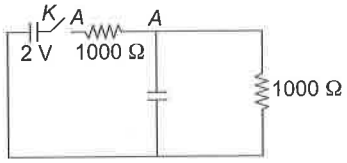
(A) $\frac{5}{3}RC$ (B) $\frac{5}{2}RC$
(C) $\frac{7}{4}RC$ (D) $\frac{7}{3}RC$

50. In the circuit shown in figure $C_1 = 2C_2$. Switch S is closed at time $t = 0$. Let i_1 and i_2 be the currents flowing through C_1 and C_2 at any time t, then the ratio $\frac{i_1}{i_2}$



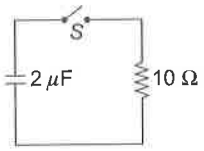
(A) is constant
(B) increases with increase in time t
(C) decreases with increase in time t
(D) first increases then decreases

51. In the circuit shown, when the key k is pressed at time $t = 0$, which of the following statements about current I in the resistor AB is true



(A) $I = 2$ mA at all t
(B) I oscillates between 1 mA and 2 mA
(C) $I = I$ mA at all t
(D) At $t = 0$, $I = 2$ mA and with time it goes to 1 mA

52. In the R-C circuit shown in the figure the total energy of 3.6×10^{-3} J is dissipated in the 10 Ω resistor when the switch S is closed. The initial charge on the capacitor is



(A) 60 μC (B) 120 μC
(C) $60\sqrt{2}$ μC (D) $\frac{60}{\sqrt{2}}$ μC

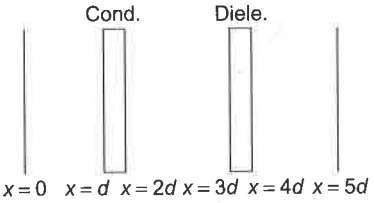
53. The capacity of a parallel plate condenser is 5 μF. When glass plate is placed between the plates of the conductor, its potential becomes $\frac{1}{8}$ th of the original value. The value of dielectric constant will be -
(A) 1.6 (B) 5
(C) 8 (D) 40

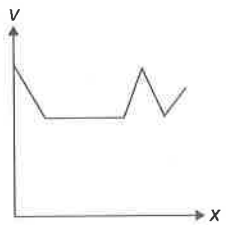
54. A metallic plate of thickness (t) and face area of one side (A) is inserted between the plates of a parallel plate air capacitor with a separation (d) and face are (A). Then the equivalent capacitance is:

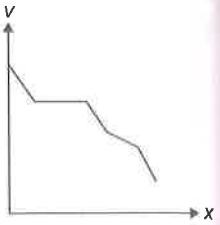
(A) $\frac{\epsilon_0 A}{d}$ (B) $\frac{\epsilon_0 A}{(dxt)}$
(C) $\frac{\epsilon_0 A}{(d-t)}$ (D) $\frac{\epsilon_0 A}{(d+t)}$

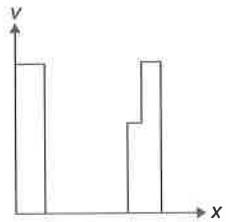
55. Two metal plates form a parallel plate condenser. The distance between the plates in d. Now a metal plate of thickness $\frac{d}{2}$ and of same area is inserted completely between the plates, the capacitance -
(A) remains unchanged (B) is doubled
(C) is halved (D) reduced to one fourth

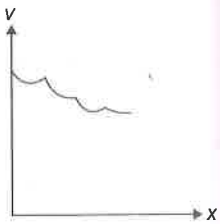
56. The distance between plates of a parallel plate capacitor is $5d$. Let the positively charged plate is at $x = 0$ and negatively charged plate is at $x = 5d$. Two slabs one of conductor and other of a dielectric of equal thickness d are inserted between the plates as shown in figure. potential versus distance graph will look like:



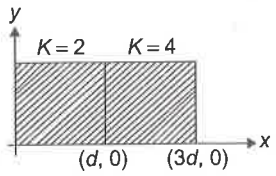
(A) 

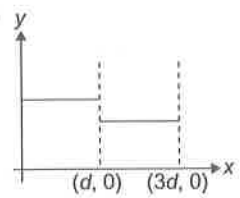
(B) 

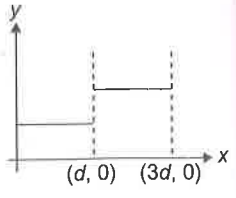
(C) 

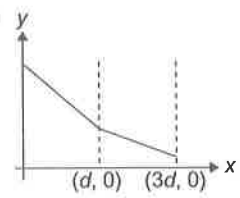
(D) 

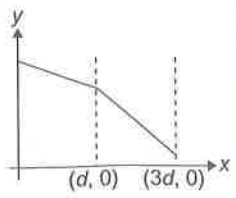
57. A parallel plate capacitor has two layers of dielectric as shown in figure. This capacitor is connected across a battery. The graph which shows the variation of electric field (E) and distance (x) from left plate.



(A) 

(B) 

(C) 

(D) 

58. The parallel plates of a capacitor have an area 0.2 m^2 and are 10^{-2} m apart. The original potential difference between them is 3000 V, and it decreases to 1000 V when a sheet of dielectric is inserted between the plates filling the full space.

Compute: ($\in_0 = 9 \times 10^{-12}$ S.I. units)

(i) Permittivity $\in$ of the dielectric.

(A) 25×10^{-12} C^2 N^{-1} m^{-2}

(B) 37×10^{-12} C^2 N^{-1} m^{-2}

(C) 27×10^{-12} C^2 N^{-1} m^{-2}

(D) 28×10^{-12} C^2 N^{-1} m^{-2}

59. A parallel plate isolated condenser consists of two metal plates of area A and separation 'd'. A slab of thickness 't' and dielectric constant K is inserted between the plates with its faces parallel to the plates and having the same surface area as that of the plates. Find the capacitance of the system. If $K = 2$, for what value of $\frac{t}{d}$ will the capacitance of the system be $\frac{3}{2}$ times that of the condenser with air filling the full space?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

60. Hard rubber has a dielectric constant of 2.8 and a dielectric strength of 18×10^8 V/m. If it is used as the dielectric material filling the full space in a parallel plate capacitor. What minimum area may the plates of the capacitor have in order that the capacitance be 7.0×10^{-2} μF and that the capacitor be able to withstand a potential difference of 4000 V.

(A) 0.62 m^{-2} (B) 0.32 m^{-2}

(C) 0.42 m^{-2} (D) 0.52 m^{-2}

61. Two parallel plate air capacitors each of capacitance C were connected in series to a battery with emf ε. Then one of the capacitors was filled up with uniform dielectric with relative permittivity k. What amount of charge flows through the battery?

(A) $\Delta q = \frac{1}{2} CE \frac{k+1}{k-1}$ (B) $\Delta q = \frac{1}{2} CE \frac{k-1}{k+1}$

(C) $\Delta q = \frac{1}{2} CE \frac{k-1}{k+1}$ (D) none of these

62. A parallel-plate capacitor of plate area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the whole space between the plates. Find the work done on the system in the process of inserting the slab.

(A) $\frac{\varepsilon_0 AV^2}{2d}\left(\frac{1}{K} - 1\right)$ (B) $\frac{\varepsilon_0 AV^2}{d}\left(\frac{1}{K} - 1\right)$

(C) $\frac{\varepsilon_0 AV^2}{2d}\left(\frac{1}{K} + 1\right)$ (D) $\frac{\varepsilon_0 AV^2}{d}\left(\frac{1}{K} + 1\right)$

63. Two capacitor having capacitances 8 μF and 16 μF have breaking voltages 20 V and 80 V. They are combined in series. The maximum charge they can store individually in the combination is

(A) 160 μC (B) 200 μC

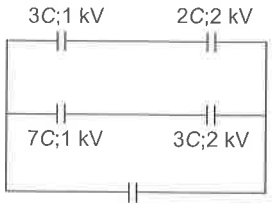
(C) 1280 μC (D) none of these

64. A capacitor of capacitance 1 μF withstands the maximum voltage 6 kV while a capacitor of 2 μF withstands the maximum voltage 4 kV. What maximum voltage will the system of these two capacitor withstands if they are connected in series?

(A) 10 kV (B) 12 kV

(C) 8 kV (D) 9 kV

65. The diagram shows four capacitors with capacitances and break down voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor breaks down? [Hint: First of all find out the break down voltages of each branch. After that compare them.]



(A) 2.5 KV (B) $\frac{10}{3}$ KV

(C) 3 KV (D) 1 KV

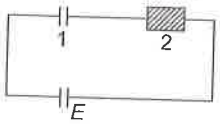
66. Three capacitors 2 μF, 3 μF and 5 μF can withstand voltages to 3 V, 2 V and 1 V respectively. Their series combination can withstand a maximum voltage equal to

(A) 5 V (B) $\frac{31}{6}$ V

(C) $\frac{26}{5}$ V (D) none

67. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of dielectric constant k as shown.

Q_1 and Q_2 are the charges stored in the capacitors. Now the dielectric slab is removed and the corresponding charges are Q_1' and Q_2'. Then



(A) $\frac{Q_1'}{Q_1} = \frac{K+1}{K}$ (B) $\frac{Q_2'}{Q_2} = \frac{K+1}{2}$

(C) $\frac{Q_2'}{Q_2} = \frac{K+1}{2K}$ (D) $\frac{Q_1'}{Q_1} = \frac{K}{2}$

68. The area of the plates of a parallel plate capacitor is A and the gap between them is d. The gap is filled with a non homogeneous dielectric whose dielectric constant varies with the distance 'y' from one plate as: $K = \lambda \sec(\pi y/2d)$, where λ is a dimensionless constant. The capacitance of this capacitor is

(A) $\pi\varepsilon_0\lambda\frac{A}{2d}$ (B) $\pi\varepsilon_0\lambda\frac{A}{d}$

(C) $2\pi\varepsilon_0\lambda\frac{A}{d}$ (D) none

69. A capacitor stores 60 μC Charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 120 μC flows through the battery. The dielectric constant of the material inserted is

(A) 1 (B) 2

(C) 3 (D) none

70. In the **above question,** if the initial capacitance of the capacitor was 2 μF, the amount of heat produced when the dielectric is inserted.

(A) 3600 μJ (B) 2700 μJ

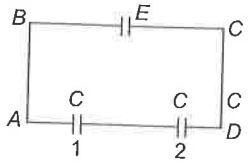
(C) 1800 μJ (D) none

71. Condenser A has a capacity of 15 μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity 1 μF with air between the plates. Both are charged separately by a battery of 100 V. After charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is:

(A) 400 V (B) 800 V

(C) 1200 V (D) 1600 V

72. In the adjoining figure, capacitor (1) and (2) have a capacitance 'C' each. When the dielectric of dielectric constant K is inserted between the plates of one of the capacitor, the total charge flowing through battery is:



(A) $\frac{KCE}{K+1}$ from B to C

(B) $\frac{KCE}{K+1}$ from C to B

(C) $\frac{(K-1)CE}{2(K+1)}$ from B to C

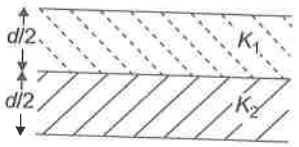
(D) $\frac{(K-1)CE}{2(K+1)}$ from C to B

73. The distance between the plates of a charged parallel plate capacitor is 5 cm and electric field inside the plates is 200 Vcm^{-1}. An uncharged metal bar of width 2 cm is fully immersed into the capacitor. The length of the metal bar is same as that of plate of capacitor. The voltage across capacitor after the immersion of the bar is:

(A) zero (B) 400 V

(C) 600 V (D) 100 V

74. Two dielectric slabs of constant K_1 and K_2 have been filled in between the plates of a capacitor as shown below. What will be the capacitance of the capacitor



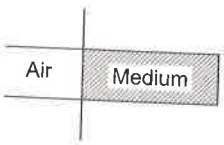
(A) $\frac{2\varepsilon_0 A}{d}(K_1 + K_2)$ (B) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{K_1 \times K_2}\right)$

(C) $\frac{d}{2\varepsilon_0 A}$ (D) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 \times K_2}{K_1 + K_2}\right)$

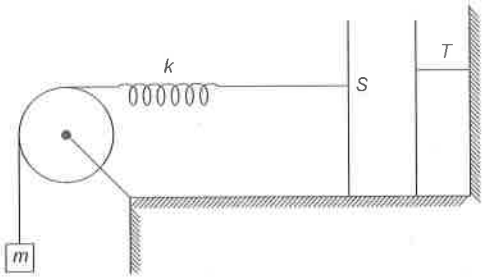
75. The capacitance of a parallel plate capacitor is 2.5 μF when it is half filled with a dielectric as shown in the figure, its capacitance becomes 5 μF, the dielectric constant of the dielectric is

(A) 7.5 (B) 3.0

(C) 0.33 (D) 4.0



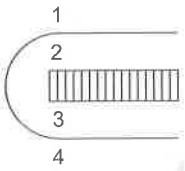
76. The plates S and T of an uncharged parallel plate capacitor are connected across a battery. The battery is then disconnected and the charged plates are now connected in a system as shown in the figure. The system shown is in equilibrium. All the strings are insulating and massless. The magnitude of charge on one of the capacitor plates is: [Area of plates = A]



(A) $\sqrt{2mgA\epsilon_0}$ (B) $\sqrt{\dfrac{4mgA\epsilon_0}{k}}$

(C) $\sqrt{mgA\epsilon_0}$ (D) $\dfrac{\sqrt{2mgA\epsilon_0}}{k}$

77. Four identical plates 1, 2, 3 and 4 are placed parallel to each other at equal distance as shown in the figure. Plates 1 and 4 are joined together and the space between 2 and 3 is filled with a dielectric of dielectric constant $k = 2$. The capacitance of the system between 1 and 3 and 2 and 4 are C_1 and C_2 respectively. The ratio $\dfrac{C_1}{C_2}$ is:



(A) $\dfrac{5}{3}$ (B) 1

(C) $\dfrac{3}{5}$ (D) $\dfrac{5}{7}$

78. A parallel plate capacitor has an electric field of 10^5 V/m between the plates. If the charge on the capacitor plate is 1 μC, then the force on each capacitor plate is

(A) 0.1 Nt (B) 0.05 Nt

(C) 0.02 Nt (D) 0.01 Nt

79. A capacitor is connected to a battery. The force of attraction between the plates when the separation between them is halved

(A) remains the same (B) becomes eight times

(C) becomes four times (D) becomes two times

80. A_1 is a spherical conductor of radius (r) placed concentrically inside a thin spherical hollow conductor A_2 of radius (R). A_1 is earthed and A_2 is given a charge $+Q$ then the charge on A_1 is -

(A) $-Q$ (B) $\dfrac{Qr}{R}$

(C) $-\dfrac{rQ}{R}$ (D) $\dfrac{-Q(R-r)}{R}$

81. Two spherical conductors A_1 and A_2 of radii (r_1) and (r_2) ($r_2 > r_1$) are placed concentrically in air. A_1 is given a charge $+Q$ while A_2 in earthed. Then the capacitance of the system is -

(A) $4\pi\epsilon_0 \dfrac{r_1 \cdot r_2}{r_2 - r_1}$ (B) $4\pi\epsilon_0 (r_1 + r_2)$

(C) $4\pi\epsilon_0 \cdot r_2$ (D) $4\pi\epsilon_0 \dfrac{r_2^2}{r_2 - r_1}$

JEE Advanced

Multiple Correct Option Type questions

1. The two plates X and Y of a parallel plate capacitor of capacitance C are given a charge of amount Q each. X is now joined to the positive terminal and Y to the negative terminal of a cell of emf $E = \dfrac{Q}{C}$.

(A) Charge of amount Q will flow from the negative terminal to the positive terminal of the cell inside it.

(B) The total charge on the plate X will be $2Q$

(C) The total charge on the plate Y will be zero

(D) The cell will supply CE^2 amount of energy

2. Two capacitors of capacitances 1 μF and 3 μF are charged to the same voltages 5 V. They are connected in parallel with oppositely charged plates connected together. Then

(A) Final common voltage will be 5 V

(B) Final common voltage will be 2.5 V

(C) Heat produced in the circuit will be zero

(D) Heat produced in the circuit will be 37.5 μJ

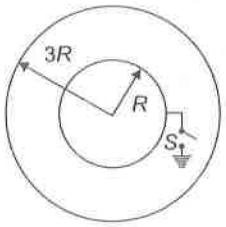
3. Each plate of a parallel plate capacitor has a charge q on it. The capacitor is now connected to a battery. Now,

(A) the facing surfaces of the capacitor have equal and opposite charges
(B) the two plates of the capacitor have equal and opposite charges
(C) the battery supplies equal and opposite charges to the two plates
(D) the outer surfaces of the plates have equal charges

4. When a parallel plates capacitor is connected to a source of constant potential difference,
(A) all the charge drawn from the source is stored in the capacitor.
(B) all the energy drawn from the source is stored in the capacitor.
(C) the potential difference across the capacitor grows very rapidly initially and this rate decreases to zero eventually.
(D) the capacity of the capacitor increases with the increase of the charge in the capacitor

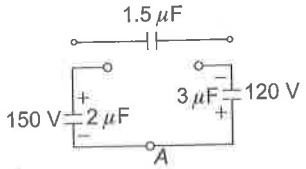
5. When two identical capacitors are charged individually to different potentials and connected parallel to each other, after disconnecting them from the source:
(A) net charge on connected plates is less than the sum of initial individual charges.
(B) net charge on connected plates equals the sum of initial charges.
(C) the net potential difference across them is different from the sum of the individual initial potential differences.
(D) the net energy stored in the two capacitors is less than the sum of the initial individual energies.

6. Two thin conducting shells of radii R and $3R$ are shown in the figure. The outer shell carries a charge $+Q$ and the inner shell is neutral. The inner shell is earthed with the help of a switch S.



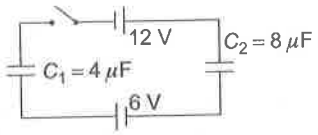
(A) With the switch S open, the potential of the inner sphere is equal to that of the outer
(B) When the switch S is closed, the potential of the inner sphere becomes zero
(C) With the switch S closed, the charge attained by the inner sphere is $-\frac{q}{3}$
(D) By closing the switch the capacitance of the system increases

7. Two capacitors of 2 μF and 3 μF are charged to 150 V and 120 V respectively. The plates of capacitor are connected as shown in the figure. A discharged capacitor of capacity 1.5 μF falls to the free ends of the wire. Then



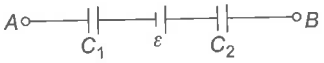
(A) charge on the 1.5 μF capacitor is 180 μC
(B) charge on the 2 μF capacitor is 120 μC
(C) charge flows through A from right to left
(D) charge flows through A from left to right

8. In the circuit shown initially C_1, C_2 are uncharged. After closing the switch



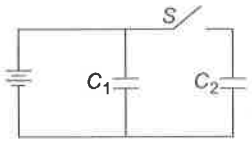
(A) The charge on C_2 is greater that on C_1
(B) The charge on C_1 and C_2 are the same
(C) The potential drops across C_1 and C_2 are the same
(D) The potential drops across C_2 is greater than that across C_1

9. A circuit shown in the figure consists of a battery of emf 10 V and two capacitance C_1 and C_2 of capacitances 1.0 μF and 2.0 μF respectively. The potential difference $V_A - V_B$ is 5 V



(A) charge on capacitor C_1 is equal to charge on capacitor C_2
(B) Voltage across capacitor C_1 is 5 V
(C) Voltage across capacitor C_2 is 10 V
(D) Energy stored in capacitor C_1 is two times the energy stored in capacitor C_2

10. Two capacitors of equal capacitance ($C_1 = C_2$) are shown in the figure. Initially, while the switch S is open, one of the capacitors is uncharged and the other carries charge Q_0. The energy stored in the charged capacitor is U_0. Sometimes after the switch is colsed, the capacitors C_1 and C_2 carry charges Q_1 and Q_2, respectively, the voltages across the capacitors are V_1 and V_2, and the energies stored in the capacitors are U_1 and U_2. Which of the following statements is INCORRECT?

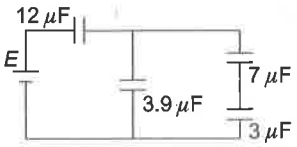


(A) $Q_0 = \frac{1}{2}(Q_1 + Q_2)$ (B) $Q_1 = Q_2$

(C) $V_1 = V_2$ (D) $U_1 = U_2$

(E) $U_0 = U_1 + U_2$

11. Four capacitors and a battery are connected as shown. The potential drop across the 7 μF capacitor is 6 V. Then the



(A) potential difference across the 3 μF capacitor is 10 V

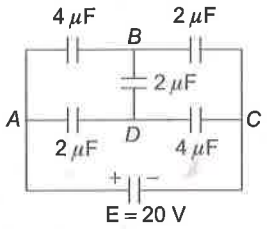
(B) charge on the 3 μF capacitor is 42 μC

(C) emf of the battery is 30 V

(D) potential difference across the 12 μF capacitor is 10 V

Common Data for Questions 12–15: The figure shows a diagonal symmetric arrangement of capacitors and a battery

12. Identify the correct statements.



(A) Both the 4 μF capacitors carry equal charges in opposite sense

(B) Both the 4 μF capacitors carry equal charges in same sense

(C) $V_B - V_D > 0$

(D) $V_D - V_B > 0$

13. If the potential of C is zero, then

(A) $V_A = +20$ V

(B) $4(V_A - V_B) + 2(V_D - V_B) = 2V_B$

(C) $2(V_A - V_D) + 2(V_B - V_D) = 4V_D$

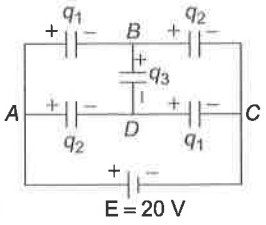
(D) $V_A = V_B + V_D$

14. The potential of the point B and D are

(A) $V_B = 8$ V (B) $V_B = 12$ V

(C) $V_D = 8$ V (D) $V_D = 12$ V

15. The value of charge q_1, q_2 and q_3 as shown in the figure are



(A) $q_1 = 32\ \mu C; q_2 = 24\ \mu C; q_3 = -8\ \mu C$

(B) $q_1 = 48\ \mu C; q_2 = 16\ \mu C; q_3 = +8\ \mu C$

(C) $q_1 = 32\ \mu C; q_2 = 24\ \mu C; q_3 = +8\ \mu C$

(D) $q_1 = 3\ \mu C; q_2 = 4\ \mu C; q_3 = +2\ \mu C$

16. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges $+Q$ and $-Q$ respectively. A third plate C, identical to A and B, with charge $+Q$, is now introduced midway between A and B, parallel to them. Which of the following are correct?

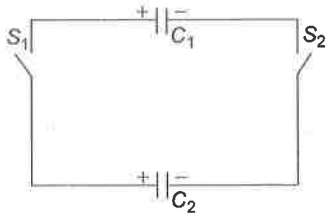
(A) The charge on the inner face of B is now $-\frac{3Q}{2}$

(B) There is no change in the potential difference between A and B

(C) The potential difference between A and C is one-third of the potential difference between B and C

(D) The charge on the inner face of A is now $\frac{Q}{2}$

17. Two capacitors $C_1 = 4\ \mu$F and $C_2 = 2\ \mu$F are charged to same potential $V = 500$ V, but with opposite polarity as shown in the figure. The switches S_1 and S_2 are closed.



(A) The potential difference across the two capacitors are same and is given by $\frac{500}{3}$ V

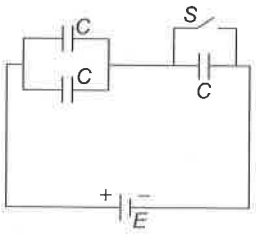
(B) The potential difference across the two capacitors are same and is given by $\frac{1000}{3}$ V

142

(C) The ratio of final energy to initial energy of the system is $\frac{1}{9}$

(D) The ratio of final energy to initial energy of the system is $\frac{4}{9}$

18. In the circuit shown, each capacitor has a capacitance C. The emf of the cell is E. If the switch S is closed



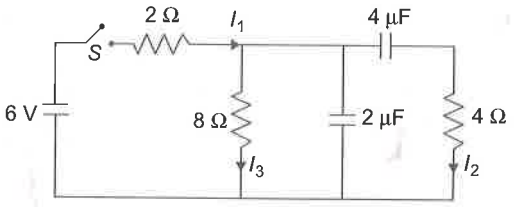
(A) positive charge will flow out of the positive terminal of the cell

(B) positive charge will enter the positive terminal of the cell

(C) the amount of charge flowing through the cell will be CE

(D) the amount of charge flowing through the cell will be $\frac{4}{3}CE$

19. In the circuit shown in the figure, the switch S is initially open and the capacitor is initially uncharged. I_1, I_2 and I_3 represent the current in the resistance 2 Ω, 4 Ω and 8 Ω respectively.



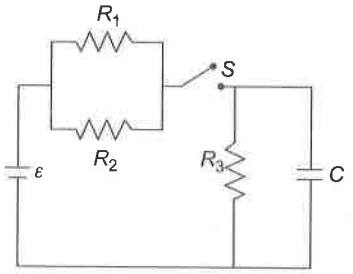
(A) Just after the switch S is closed, $I_1 = 3$A, $I_2 = 3$A and $I_3 = 0$

(B) Just after the switch S is closed, $I_1 = 3$ A, $I_2 = 0$ and $I_3 = 0$

(C) long time after the switch S is closed, $I_1 = 0.6$ A, $I_2 = 0$ and $I_3 = 0$

(D) long after the switch S is closed, $I_1 = I_2 = I_3 = 0.6$ A

20. The circuit shown in the figure consists of a battery of emf $\varepsilon = 10$ V; a capacitor of capacitance $C = 1.0$ μF and three resistor of values $R_1 = 2$ Ω, $R_2 = 2$ Ω and $R_3 = 1$ Ω. Initially the capacitor is completely uncharged and the switch S is open. The switch S is closed at $t = 0$.



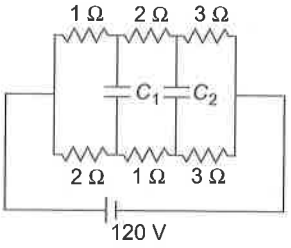
(A) The current through resistor R_3 at the moment the switch closed is zero

(B) The current through resistor R_3 a long time after the switch closed is 5 A

(C) The ratio of current through R_1 and R_2 is always constant

(D) The maximum charge on the capacitor during the operation is 5 μC

21. In the circuit shown in figure $C_1 = C_2 = 2$ μF. Then charge stored in



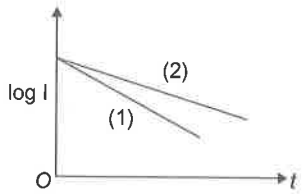
(A) capacitor C_1 is zero

(B) capacitor C_2 is zero

(C) both capacitor is zero

(D) capacitor C_1 is 40 μC

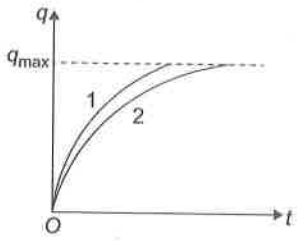
22. A capacitor of capacity C is charged to a steady potential difference V and connected in series with an open key and a pure resistor 'R'. At time $t = 0$, the key is closed. If I = current at time t, a plot of log I against 't' is as shown in (1) in the graph. Later one of the parameters i.e., V, R or C is changed keeping the other two constant, and the graph (2) is recorded. Then



(A) C is reduced (B) C is increased

(C) R is reduced (D) R is increased

Common Data for Questions 23–24: The charge across the capacitor in two different *RC* circuits 1 and 2 are plotted as shown in figure.



23. Choose the correct statement(s) related to the two circuits
(A) Both the capacitors are charged to the same charge
(B) The emf's of cells in both the circuit are equal
(C) The emf's of the cells may be different
(D) The emf E_1 is more than E_2

24. Identify the correct statement(s) related to the R_1, R_2, C_1 and C_2 of the two *RC* circuits.
(A) $R_1 > R_2$ if $E_1 = E_2$ (B) $C_1 < C_2$ if $E_1 = E_2$
(C) $R_1C_1 > R_2C_2$ (D) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$

25. A parallel plate capacitor is charged by connecting it to a battery. The battery is disconnected and the plates of the capacitor are pulled apart to make the separation between the plates twice. Again the capacitor is connected to the battery (with same polarity) then
(A) Charge from the battery flows into the capacitor after reconnection
(B) Charge from capacitor flows into the battery after reconnection
(C) The potential difference between the plates increases when the plates are pulled apart
(D) After reconnection of battery potential difference between the plate will immediately becomes half of the initial potential difference. (Just after disconnecting the battery)

26. A capacitor C is charged to a potential difference V and battery is disconnected. Now if the capacitor plates are brought close slowly by some distance
(A) some +ve work is done by external agent
(B) energy of capacitor will decrease
(C) energy of capacitor will increase
(D) none of the above

27. The separation between the plates of a isolated charged parallel plate capacitor is increased. Which of the following quantities will change?
(A) charge on the capacitor
(B) potential difference across the capacitor
(C) energy of the capacitor
(D) energy density between the plates

28. A parallel plate capacitor is charged and then disconnected from the source of potential difference. If the plates of the condenser are then moved farther apart by the use of insulated handle, which one of the following is true?
(A) the charge on the capacitor increases
(B) the charge on the capacitor decreases
(C) the capacitance of the capacitor increases
(D) the potential difference across the plate increases

29. A parallel plate capacitor A is filled with a dielectric whose dielectric constant varies with applied voltage as $K = V$. An identical capacitor B of capacitance C_0 with air as dielectric is connected to voltage source $V_0 = 30$ V and then connected to the first capacitor after disconnecting the voltage source. The charge and voltage on capacitor.
(A) A are 25 C_0 and 25 V
(B) A are 25 C_0 and 5 V
(C) B are $5C_0$ and 5 V
(D) B are $5C_0$ and 25 V

30. A parallel plate air-core capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then:
(A) some charge from the capacitor will flow back into the source
(B) some extra charge from the source will flow back into the capacitor
(C) the electric field intensity between the two plate does not change
(D) the electric field intensity between the two plates will decrease

31. A parallel plate capacitor has a parallel sheet of copper inserted between and parallel to the two plates, without touching the plates. The capacity of the capacitor after the introduction of the copper sheet is:
(A) minimum when the copper sheet touches one of the plates
(B) maximum when the copper sheet touches one of the plates
(C) invariant for all positions of the sheet between the plates
(D) greater than that before introducing the sheet

32. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor.

(A) the energy stored in the capacitor will become K-times

(B) the electric field inside the capacitor will decrease to K-times

(C) the force of attraction between the plates will increase to K^2-times

(D) the charge on the capacitor will increase to K-times

33. The capacitance of a parallel plate capacitor is C when the region between the plate has air. This region is now filled with a dielectric slab of dielectric constant k. The capacitor is connected to a cell of emf E, and the slab is taken out

(A) charge $CE(k-1)$ flows through the cell

(B) energy $E^2C(k-1)$ is absorbed by the cell

(C) the energy stored in the capacitor is reduced by $E^2C(k-1)$

(D) the external agent has to do $\frac{1}{2}E^2C(k-1)$ amount of work to take the slab out

34. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain the same?

(A) the electric field in the capacitor

(B) the charge on the capacitor

(C) the potential difference between the plates

(D) the stored energy in the capacitor

35. Following operations can be performed on a capacitor X – connect the capacitor to a battery of emf
EY – disconnect the battery
Z – reconnect the battery with polarity reversed
W – insert a dielectric slab in the capacitor

(A) In XYZ (perform X, then Y, then Z) the stored electric energy remains unchanged and no thermal energy is developed

(B) The charge appearing on the capacitor is greater after the action XWY than after the action XYW.

(C) The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.

(D) The electric field in the capacitor after the action XW is the same as that after WX

36. A parallel plate capacitor of plate area A and plate seperation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and the work done on the system, in question, in the process of inserting the slab, then

(A) $Q = \dfrac{\varepsilon_0 AV}{d}$

(B) $Q = \dfrac{\varepsilon_0 KAV}{d}$

(C) $E = \dfrac{V}{Kd}$

(D) $W = -\dfrac{\varepsilon_0 AV^2}{2d}\left(1 - \dfrac{1}{K}\right)$

37. A parallel plate capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with the capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is introduced between plates of capacitor but battery is still in connection. The corresponding quantities now given by Q, V, E and U related to previous ones are

(A) $Q > Q_0$ (B) $V > V_0$

(C) $E > E_0$ (D) $U < U_0$

38. If Q is the charge on the plates of a capacitor of capacitance C, V the potential difference between the plates, A the area of each plate and d the distance between the plates, the force of attraction between the plates is

(A) $\dfrac{1}{2}\left(\dfrac{Q^2}{\varepsilon_0 A}\right)$ (B) $\dfrac{1}{2}\left(\dfrac{CV^2}{d}\right)$

(C) $\dfrac{1}{2}\left(\dfrac{CV^2}{A\varepsilon_0}\right)$ (D) $\dfrac{1}{4}\left(\dfrac{Q^2}{\pi\varepsilon_0 d^2}\right)$

39. The capacitance (C) for an isolated conducting sphere of radius (a) is given by $4\pi\varepsilon_0 a$. If the sphere is enclosed with an earthed concentric sphere. The ratio of the radii of the spheres being $\dfrac{n}{(n-1)}$ then the capacitance of such a sphere will be increased by a factor

(A) n (B) $\dfrac{n}{(n-1)}$

(C) $\dfrac{(n-1)}{n}$ (D) $a \cdot n$

40. Three long concentric conducting cylindrical shells have radii R, $2R$ and $2\sqrt{2}R$. Inner and outer shells are connected to each other. The capacitance across middle and inner shells per unit length is:

(A) $\frac{\frac{1}{3}\epsilon_0}{\ln 2}$ (B) $\frac{6\pi\epsilon_0}{\ln 2}$

(C) $\frac{\pi\epsilon_0}{2\ln 2}$ (D) None

41. When 30 μC charge is given to an isolated conductor of capacitance 5 μF. Find out following

(i) Potential of the conductor

(A) 6 V (B) 7 V

(C) 8 V (D) 9 V

(ii) Energy stored in the electric field of conductor

(A) 90 μJ (B) 50 μJ

(C) 60 μJ (D) 70 μJ

(iii) It this conductor is now connected to another isolated conductor by a conducting wire (at large distance) of total charge 50 μC and capacity 10 μF then

(a) Find out common potential of both the conductors.

(A) $\frac{17}{3}$ (B) $\frac{16}{3}$

(C) $\frac{14}{3}$ (D) none of these

(b) Find out heat dissipated during the process of charge distribution.

(A) $\frac{5}{3}$ μJ (B) $\frac{16}{3}$ μJ

(C) $\frac{14}{3}$ μJ (D) none of these

(c) Find out ratio of final charges on conductors.

(A) 1:2 (B) 2:1

(C) 1:4 (D) 1:3

(b) Find out final charges on each conductor.

(A) $\frac{80}{3}, \frac{150}{3}$ μC

(B) $\frac{80}{3}, \frac{160}{3}$ μC

(C) $\frac{80}{3}, \frac{60}{3}$ μC

(D) $\frac{80}{3}, \frac{90}{3}$ μC

42. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. The potential difference across the capacitor is now

(A) V (B) $V + \frac{Q}{C}$

(C) $V + \frac{Q}{2C}$ (D) $V - \frac{Q}{C}$, if $V < CV$

43. A conducting body 1 has some initial charge Q, and its capacitance is C. There are two other conducting bodies, 2 and 3, having capacitances: $C_2 = 2C$ and $C_3 \to \infty$. Bodies 2 and 3 are initially uncharged. 'Body 2 is touched with body 1. Then, body 2 is removed from body 1 and touched with body 3, and then removed.' This process is repeated N times. Then, the charge on body 1 at the end must be

(A) $\frac{Q}{3^N}$ (B) $\frac{Q}{3^{N-1}}$

(C) $\frac{Q}{N^3}$ (D) None

44. A capacitor of capacitance C is initially charged to a potential difference of V V. Now it is connected to a battery of 2 V with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor will be

(A) 1.75 (B) 2.25

(C) 2.5 (D) $\frac{1}{2}$

JEE Advanced

Level I

Subjective Type Questions

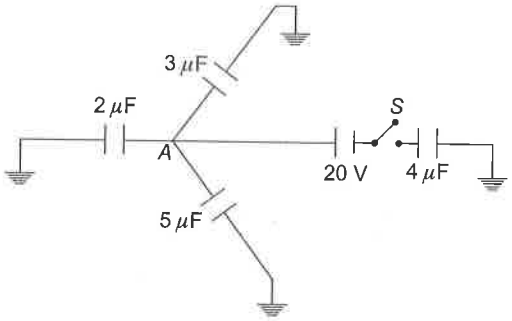
1. A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge $q = 20$ μC is given to the inner sphere. Find the heat generated in the process, the inner sphere is connected to the shell by a conducting wire -

2. The plates of a parallel plate capacitor are given charges $+4Q$ and $-2Q$. The capacitor is then connected across an uncharged capacitor of same capacitance as first one ($= C$). Find the final potential difference between the plates of the first capacitor.

3. Three capacitors of 2 μF, 3 μF and 5 μF are independently charged with batteries of emf's 5 V, 20 V and 10 V respectively. After disconnecting from the voltage sources. These capacitors are connected as shown in figure with their positive polarity plates are

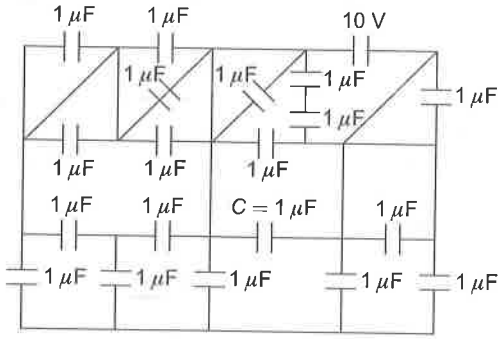
146

connected to A and negative polarity is earthed. Now a battery of 20 V and an uncharged capacitor of 4 μF capacitance are connected A as shown with a switch S. When switch is closed, find:

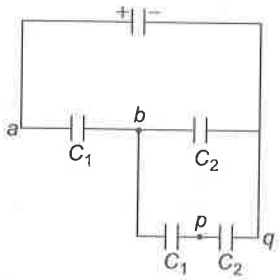


(A) the potential of the junction A.
(B) final charges on all four capacitors.

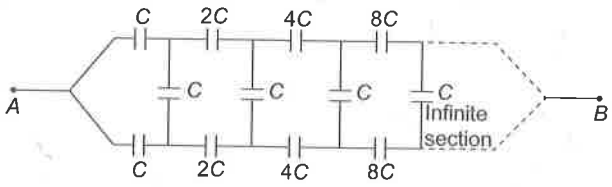
4. Find the charge on the capacitor $C = 1$ μF in the circuit shown in the figure.



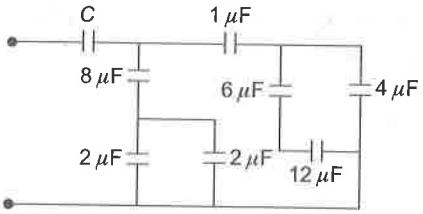
5. In the given network if potential difference between p and q is 2 V and $C_2 = 3C_1$. Then find the potential difference between a and b.



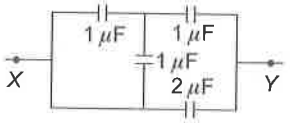
6. Find the equivalent capacitance of the circuit between point A and B.



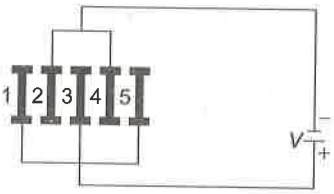
7. In the following circuit, the resultant capacitance between A and B is 1 μF. Find the value of C.



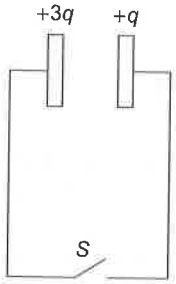
8. The figure shows a circuit consisting of four capacitors. Find the effective capacitance between X and Y.



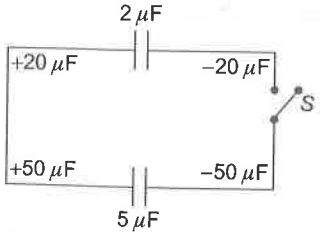
9. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at a distance 'd' apart, the plates are connected to a source of emf V as shown in figure. The charge on plate 1 is _____ and that on plate 4 is _____.



10. The two identical parallel plates are given charges as shown in figure. If the plate area of either face of each plate is A and separation between plates is d, then find the amount of heat liberate after closing the switch.



11. Find heat produced in the circuit shown in figure on closing the switch S.



12. The
con
figu

(A)
(B)
(C)
(D)

13. In t
oper
thro
to _

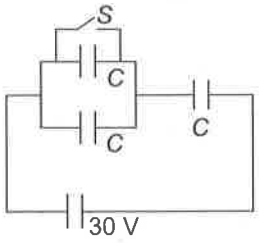
14. In th
gene

Level II

1. The
fille
cons
plate
whe
is a
the c

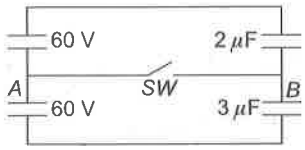
2. Five
fixe
figu
whil
junc
sour

12. The capacitor each having capacitance $C = 2\ \mu F$ are connected with a battery of emf 30 V as shown in figure. When the switch S is closed. Find

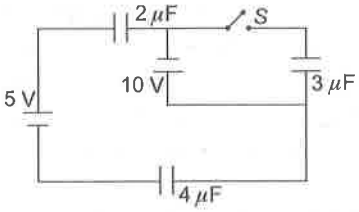


(A) the amount of charge flown through the battery
(B) the heat generated in the circuit
(C) the energy supplied by the battery
(D) the amount of charge flown through the switch S

13. In the circuit shown in the figure, initially SW is open. When the switch is closed, the charge passing through the switch _____ in the direction _____ to _____.

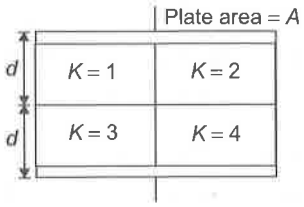


14. In the circuit shown in figure, find the amount of heat generated when switch s is closed.



15. A 10 μF and 20 μF capacitor are connected to a 10 V cell in parallel for some time after which the capacitors are disconnected from the cell and reconnected at $t = 0$ with each other, in series, through wires of finite resistance. The +ve plate of the first capacitor is connected to the –ve plate of the second capacitor. Draw the graph which best describes the charge on the +ve plate of the 20 μF capacitor with increasing time.

16. Find the capacitance of the system shown in figure.

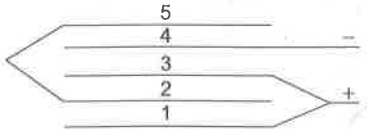


17. The two parallel plates of a capacitor have equal and opposite charges Q. The dielectric has a dielectric constant K and resistivity ρ. Show that the 'leakage' current carried by the dielectric is given by the relationship $i = \dfrac{Q}{K \in_0 \rho}$

Level II

1. The gap between the plates of a plane capacitor is filled with an isotropic insulator whose di-electric constant varies in the direction perpendicular to the plates according to the law $K = K_1\left[1 + \sin\dfrac{\pi}{d}X\right]$, where d is the separation, between the plates and K_1 is a constant. The area of the plates is S. Determine the capacitance of the capacitor.

2. Five identical conducting plates 1, 2, 3, 4 and 5 are fixed parallel to and equidistant from each other (see figure). Plates 2 and 5 are connected by a conductor while 1 and 3 are joined by another conductor. The junction of 1 and 3 and the plate 4 are connected to a source of constant emf V_0. Find:



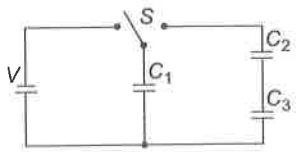
(i) the effective capacity of the system between the terminals of the source.
(ii) the charges on plates 3 and 5 Given d = distance between any 2 successive plates and A = area of either face of each plate.

3. A potential difference of 300 V is applied between the plates of a plane capacitor spaced 1 cm apart. A plane parallel glass plate with a thickness of 0.5 cm and a plane parallel paraffin plate with a thickness of 0.5 cm are placed in the space between the capacitor plates find:
(i) Intensity of electric field in each layer.
(ii) The drop of potential in each layer.
(iii) The surface charge density of the charge on capacitor the plates. Given that: $k_{glass} = 6$, $k_{paraffin} = 2$

4. A parallel plate capacitor has plates with area A and separation d. A battery charges the plates to a potential difference of V_0. The battery is then disconnected and

a di-electric slab of constant K and thickness d is introduced. Calculate the positive work done by the system (capacitor + slab) on the man who introduces the slab.

When the switch S in the figure is thrown to the left, the plates of capacitors C_1 acquire a potential difference V. Initially the capacitors C_2C_3 are uncharged. The switch is now thrown to the right. What are the final charges q_1, q_2 and q_3 on the corresponding capacitors.



An insolated conductor initially free from charge is charged by repeated contacts with a plate which after each contact has a charge Q due to some mechanism. If q is the charge on the conductor after the first operation, prove that the maximum charge which can be given to the conductor in this way is $\frac{Qq}{Q-q}$.

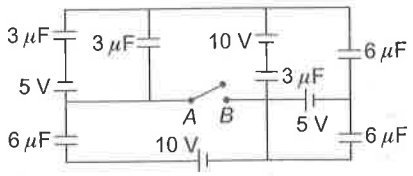
A parallel plate capacitor is filled by a di-electric whose relative permittivity varies with the applied voltage according to the law = α V, where α = 1 per V. The same (but containing no di-electric) capacitor charged to a voltage V = 156 V is connected in parallel to the first 'non-linear' uncharged capacitor. Determine the final voltage V_f across the capacitors.

A capacitor consists of two air spaced concentric cylinders. The outer of radius b is fixed, and the inner is of radius a. If breakdown of air occurs at field strengths greater than E_b, show that the inner cylinder should have

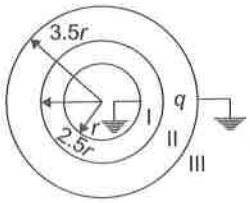
(i) radius $a = \frac{b}{e}$ if the potential of the capacitor is to be maximum

(ii) radius $a = \frac{b}{\sqrt{e}}$ if the energy per unit length of the system is to be maximum.

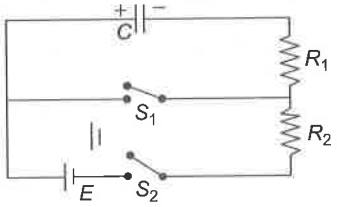
Find the charge flown through the switch from A to B when it is closed.



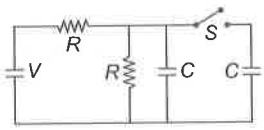
10. Figure shows three concentric conducting spherical shells with inner and outer shells earthed and the middle shell is given a charge q. Find the electrostatic energy of the system stored in the region and II.



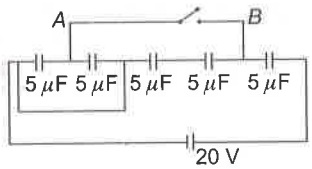
11. The capacitors shown in figure has been charged to a potential difference of V volts, so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at $t = 0$. At $t = R_1C$ switch S_1 is opened and S_2 is closed. Find the charge on the capacitor at $t = 2R_1C + R_2C$.



12. In the figure shown initially switch is open for a long time. Now the switch is closed at $t = 0$. Find the charge on the right most capacitor as a function of time given that it was initially unchanged.



13. Find the charge which flows from point A to B, when switch is closed.



14. Two parallel plate capacitors of capacitance C and $2C$ are connected in parallel then following steps are performed.

(i) A battery of voltage V is connected across the capacitors.

(ii) A dielectric slab of relative permittivity k is slowly inserted in capacitor C.

(iii) Battery is disconnected.

(iv) Dielectric slab is slowly removed from capacitor.

Find the heat produced in (i) and work done by external agent in step (ii) and (iv).

Previous Year Que

JEE Main

1. A resistor R and
through a switch
capacitor is a ne
culate the value
after the switch

(A) $1.7 \times 10^5\ \Omega$

(C) $3.3 \times 10^7\ \Omega$

2. Let C be that ca
through a resisto
the energy stored
initial value and
reduce to one-fo
$\frac{t_1}{t_2}$ will be

(A) 1

(C) $\frac{1}{4}$

3. A parallel plate c
has a capacitanc
its plates is d. T
filled with two di

dielectric constan

other one has diel
$\frac{2d}{3}$. Capacitanc

(A) 1.8 pF

(C) 40.5 pF

4. A parallel plate c
tric constant K b
and is charged to
is slowly remove
reinserted. The n
process is

(A) $\frac{1}{2}(K-1)CV$

(C) $(K-1)CV^2$

5. A battery is use
tor till the potenti
comes equal to th
The ratio of the e
work done by the

Previous Year Questions

JEE Main

1. A resistor R and 2 μF capacitor in series in connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed ($\log_{10} 2.5 = 0.4$) **[AIEEE-2011]**

(A) $1.7 \times 10^5\ \Omega$ (B) $2.7 \times 10^6\ \Omega$

(C) $3.3 \times 10^7\ \Omega$ (D) $1.3 \times 10^4\ \Omega$

2. Let C be that capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio $\frac{t_1}{t_2}$ will be **[AIEEE-2010]**

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) 2

3. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plate is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now **[AIEEE-2008]**

(A) 1.8 pF (B) 45 pF

(C) 40.5 pF (D) 20.25 pF

4. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V V. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[AIEEE-2007]**

(A) $\frac{1}{2}(K-1)CV^2$ (B) $CV^2(K-1)/K$

(C) $(K-1)CV^2$ (D) zero

5. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be **[AIEEE-2007]**

(A) 1 (B) 2

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

6. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is **[AIEEE-2005]**

(A) $(n-1)C$ (B) $(n+1)C$

(C) C (D) nC

7. A full charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is **[AIEEE-2005]**

(A) $\sqrt{\frac{2mC\Delta T}{s}}$ (B) $\frac{mC\Delta T}{s}$

(C) $\frac{ms\Delta T}{C}$ (D) $\sqrt{\frac{2ms\Delta T}{C}}$

8. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor **[AIEEE-2003]**

(A) decreases (B) remains unchanged

(C) becomes infinite (D) increases

9. The work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 μF is **[AIEEE-2003]**

(A) 16×10^{-32} J (B) 3.1×10^{-26} J

(C) 4×10^{-10} J (D) 32×10^{-32} J

10. If there are n capacitors in parallel connected to V V source, then the energy stored is equal to **[AIEEE-2002]**

(A) CV (B) $\frac{1}{2}nCV^2$

(C) CV^2 (D) $\frac{1}{2n}CV^2$

11. Capacitance (in F) of a spherical conductor having radius 1m, is **[AIEEE-2002]**

(A) 1.1×10^{-10} (B) 10^{-6}

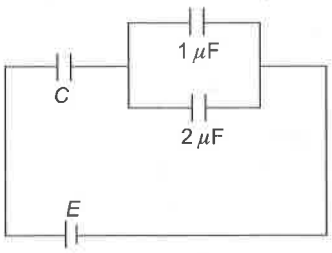
(C) 9×10^{-9} (D) 10^{-3}

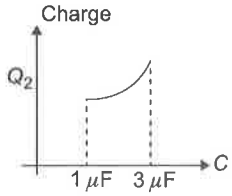
12. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the change density of the positive plate will be close to: **[JEE MAIN-2014]**

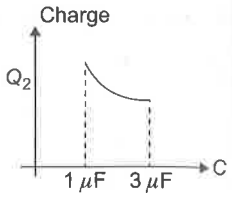
(A) 3×10^4 C/m^2 (B) 6×10^4 C/m^2
(C) 6×10^{-7} C/m^2 (D) 3×10^{-7} C/m^2

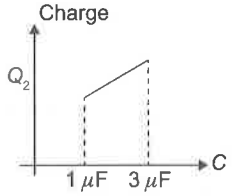
13. In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of 'C' is given properly by: (figure are drawn schematically and are not to scale)

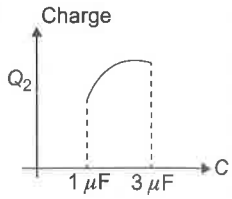
[JEE MAIN-2015]



(A) 

(B) 

(C) 

(D) 

JEE Advanced

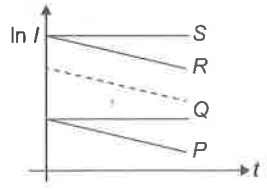
1. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. Likely charged plates are then connected. Then, the decrease in energy of the combined system is: **[JEE-2002]**

(A) $\frac{1}{4}C(V_1^2 - V_2^2)$ (B) $\frac{1}{4}C(V_1^2 - V_2^2)^2$

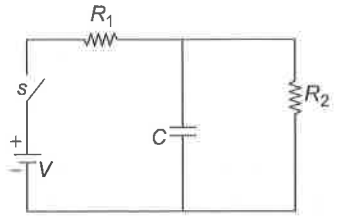
(C) $\frac{1}{4}C(V_1 + V_2)^2$ (D) $\frac{1}{4}C(V_1 + V_2)^2$

2. In an RC circuit while charging, the graph of ln I versus time is as shown by the dotted line in the adjoining diagram where I is the current. When the value of the resistance is doubled, which of the solid curves best represents the variation of ln I versus time? **[JEE 2004 (Scr.)]**

(A) P (B) Q
(C) R (D) S



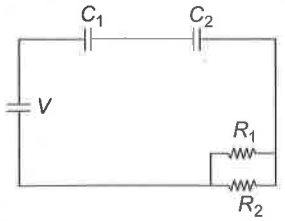
3. In the given circuit, the switch S is closed at time $t = 0$. The charge Q on the capacitor at any instant t is given by $Q(t) = Q_0(1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters shown in the circuit. **[JEE 2005 (Scr.)]**



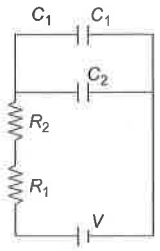
4. An uncharged capacitor of capacitance 4 μF, a battery of emf 12 V and a resistor of 2.5 MΩ are connected in series. The time after which $v_C = 3v_R$ is (take ln 2 = 0.693) **[JEE 2005 (Scr.)]**

(A) 6.93 sec. (B) 13.86 sec
(C) 20.52 sec. (D) none of these

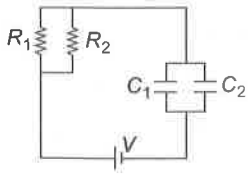
5. Given: $R_1 = -1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 2\ \mu$F, $C_2 = 4\ \mu$F. The time constants (in μS) for the circuits I, II, III are respectively **[JEE 2006]**



(I)



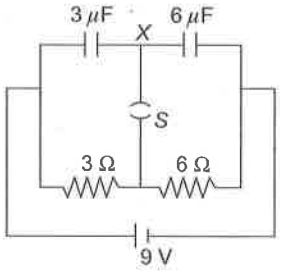
(II)



(III)

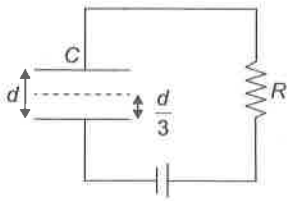
(A) 18, 8/9, 4 (B) 18, 4, 8/9
(C) 4, 8/9, 18 (D) 8/9, 18, 4

6. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is **[JEE 2007]**



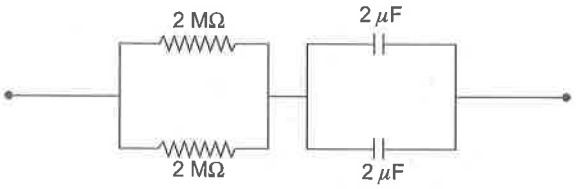
(A) 0 (B) 54 μC
(C) 27 μC (D) 81 μC

7. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is **[JEE 2008]**



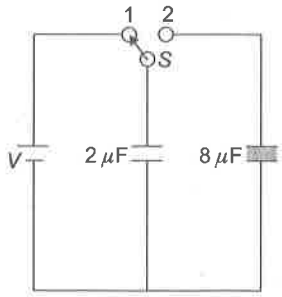
(A) $\frac{6\varepsilon_0 R}{5d + 3Vt}$ (B) $\frac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$
(C) $\frac{6\varepsilon_0 R}{5d - 3Vt}$ (D) $\frac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

8. At time $t = 0$, a battery of 10 V is connected across point A and B in the given circuit. It the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V? **[JEE 2010]**



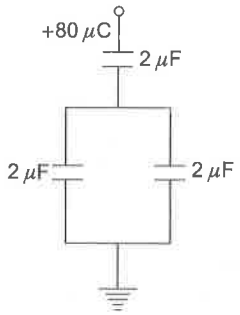
[Take: $\ell n\, 5 = 1.6$, $\ell n\, 3 = 1.1$]

9. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **[JEE 2011]**



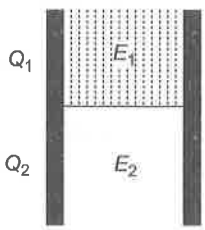
(A) 0% (B) 20%
(C) 75% (D) 80%

10. In the given circuit a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is. **[JEE 2012]**



(A) +32 μC (B) +40 μC
(C) +48 μC (D) +80 μC

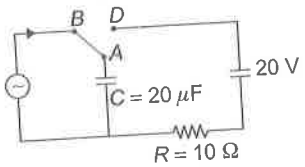
11. A parallel plate capacitor has a dielectric slab of dielectric constant K between it plates that covers $\frac{1}{3}$ of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between C_1. When the capacitor is charged, the plate area covered by the dielectric get charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options. ignoring edge effects. **[JEE 2014]**



(A) $\frac{E_1}{E_2}=1$ (B) $\frac{E_1}{E_2}=\frac{1}{K}$

(C) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (D) $\frac{C}{C_1}=\frac{2+K}{K}$

12. At time $t=0$, terminal A is the circuit shown in the figure is connected to B by a key and an alternating current $I(t)=I_0\cos(\omega t)$ with $I_0=1$ A and $\omega=500$ rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t=\frac{7\pi}{6\omega}$. the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C=20\ \mu$F, $R=10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s). **[JEE 2014]**



(A) Magnitude of the maximum charge on the capacitor before $t=\frac{7\pi}{6\omega}$ is $1\times10^{-3}C$.

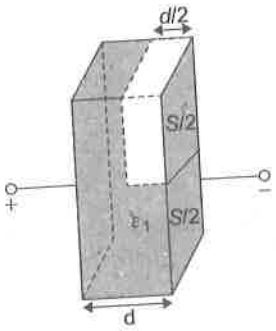
(B) The current in the left part of the circuit just before $t=\frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10 A.

(D) $Q=2\times10^{-3}C$.

13. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative primitivities ($\varepsilon_1=2$ and $\varepsilon_2=4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2.

The ratio $\frac{C_2}{C_1}$ is- **[JEE Advanced 2015]**



(A) $\frac{6}{5}$ (B) $\frac{6}{5}$

(C) $\frac{7}{5}$ (D) $\frac{7}{3}$

ANSWER KEYS

Exercises

JEE Main

1. D	2. A	3. D	4. A	5. B	6. C	7. A	8. D	9. A	10. C
11. A	12. C	13. B	14. B	15. C	16. D	17. C	18. A	19. B	20. D
21. (i) B	(ii) C	22. A	23. C	24. A	25. B	26. D	27. C	28. D	29. B
30. B	31. B	32. (i) A,	(ii) B	(iii) C	33. A	34. B	35. B	36. B	37. B
38. (a) A	(b) B	(c) C (d) C		39. C	40. D	41. C	42. (i) A	(ii) A	43. A
44. D	45. B	46. A	47. A	48. D	49. C	50. B	51. D	52. B	53. C
54. C	55. B	56. B	57. A	58. C	59. B	60. A	61. B	62. A	63. A
64. D	65. A	66. B	67. C	68. A	69. C	70. C	71. B	72. D	73. C
74. D	75. B	76. A	77. B	78. B	79. C	80. C	81. A		

JEE Advanced

Multiple Corre

1. A, B, C, D
8. B
15. C
22. B
29. B, C
36. A, C, D
41. (i) A

JEE Advance

Level I

1. 9 J
5. 30 V
11. 0
14. 150 μJ

Level II

1. $C=\frac{\epsilon S\pi}{2d}$
3. (i) 1.5 ×
5. $q_1=\frac{}{C_1C}$
9. 69 μC
12. $q=\frac{CV}{2}$

onnected to D, the

ing plates of area S
itance C_1 in air. W
ative primitivities (
between the two pl
pacitance becomes

[JEE Advanced 20

$\frac{6}{5}$

$\frac{7}{3}$

Advanced

tiple Correct Option Type Questions

A, B, C, D	**2.** B, D	**3.** A, C, D	**4.** A, C	**5.** B, C, D	**6.** A, B, C, D	**7.** A, B, C
B	**9.** A, D	**10.** E	**11.** B, C, D	**12.** B, C	**13.** A, B, C, D	**14.** B, C
C	**16.** A, B, C, D	**17.** A, C	**18.** A, D	**19.** B	**20.** A, B, C, D	**21.** B, D
B	**23.** A, C	**24.** D	**25.** B, C	**26.** B	**27.** B, C	**28.** D
B, C	**30.** B	**31.** C, D	**32.** A, C, D	**33.** A, B, D	**34.** B	**35.** B, C, D
A, C, D	**37.** A	**38.** A, B	**39.** A	**40.** B		

(i) A (ii) A (iii) (a) B (b) A (c) A (d) B **42.** C **43.** A **44.** B

Advanced

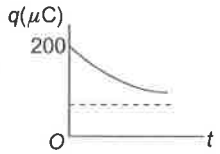
el I

9 J **2.** 3Q/2C **3.** (a) $\frac{100}{7}$ V; (b) 28.56 μC, 42.84 μC, 71.4 μC, 22.88 μC **4.** 10 μC

30 V **6.** C **7.** $\frac{32}{23}\mu$F **8.** $\frac{8}{3}\mu$F **9.** $\frac{A\epsilon_0 V}{d}, -\frac{2A\epsilon_0 V}{d}$ **10.** $\frac{1}{2}\frac{q^2 d}{\epsilon_0 A}$

0 **12.** (a) 20 μC, (b) 0.3 mJ, (c) 0.6 mJ (d) 60 μC **13.** 60 μc, A to B

150 μJ **15.**



16. $\frac{25}{24}\frac{\varepsilon_0 A}{d}$ **17.** Proof

el II

$C = \frac{\epsilon S\pi K_1}{2d}$ **2.** (i) $\frac{5}{3}\left(\frac{\epsilon_0 A}{d}\right)$; (ii) $Q_3 = \frac{4}{3}\left(\frac{\epsilon_0 A V_a}{d}\right), Q_5 = \frac{2}{3}\left(\frac{\epsilon_0 A V_a}{d}\right)$

(i) 1.5×10^4 V/m, 4.5×10^4 V/m, (ii) 75 V, 225 V, (iii) 8×10^{-7} C/m^2 **4.** $W = \frac{1}{2}C_0 V_0^2\left(1 - \frac{1}{K}\right)$

A **10.** C
B **20.** D
D **29.** B
B **37.** B
A **43.** A
B **53.** C
A **63.** A
D **73.** C

$q_1 = \frac{C_1^2 V(C_2 + C_3)}{C_1C_2 + C_2C_3 + C_1C_3}$ $q_2 = q_3\frac{C_1C_2C_3V}{C_1C_2 + C_2C_3 + C_3C_1}$ **6.** Proof **7.** 12 V **8.** Proof

69 μC **10.** $U_I = \frac{3kq_1^2}{10r}$ where $q_1 = -\frac{4q}{25}; U_{II} = 2K(q - q_1)^2/35r$ **11.** $q = CE\left(1 - \frac{1}{e}\right) + \frac{CV}{e^2}$

$q = \frac{CV}{2}\left(1 - \frac{1}{2}e^{-t/RC}\right)$ **13.** $-\frac{400}{7}\mu$C **14.** (i) $\frac{3}{2}CV^2$; (ii) $-\frac{1}{2}CV^2(K-1); \frac{1}{6}(K+2)(K-1)CV^2$;

Previous Year Questions

JEE Main

1. B 2. C 3. C 4. D 5. D 6. A 7. D 8. B 9. D 10. B
11. A 12. C 13. D

JEE Advanced

1. C 2. B 3. $Q_0 = \frac{CVR_2}{R_1 + R_2}$ and $a = \frac{R_1 + R_2}{CR_1R_2}$ 4. B 5. D 6. C 7. A 8. 2 sec
9. D 10. C 11. A, D 12. C, D 13. D

Chapter 4

Magnetism

THE MAGNETIC FIELD

In earlier lessons we found it convenient to describe the interaction between charged objects in terms of electric fields. Recall that an electric field surrounding an electric charge. The region of space surrounding a moving charge includes a magnetic field in addition to the electric field. A magnetic field also surrounds a magnetic substance.

In order to describe any type of field, we must define its magnitude, or strength, and its direction.

Magnetic field is the region surrounding a moving charge in which its magnetic effects are perceptible on a moving charge (electric current). Magnetic field intensity is a vector quantity and also known as magnetic induction vector. It is represented by $\vec{B}$.

Lines of magnetic induction may be drawn in the same way as lines of electric field. The number of lines per unit area crossing a small area perpendicular to the direction of the induction being numerically equal to $\vec{B}$. The number of lines of $\vec{B}$ crossing a given area is referred to as the magnetic flux linked with that area. For this reason $\vec{B}$ is also called magnetic flux density.

There are two methods of calculating magnetic field at some point. One is **Biot-Savart law** which gives the magnetic field due to an infinitesimally small current carrying wire at some point and the another is **Ampere' law,** which is useful in calculating the magnetic field of a symmetric configuration carrying a steady current.

The unit of magnetic field is weber/m^2 and is known as tesla (T) in the SI system.

BIOT-SAVART LAW

Biot-Savart law gives the magnetic induction due to an infinitesimal current element.

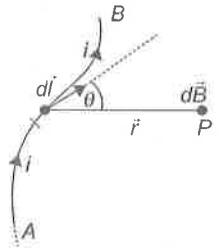
Let AB be a conductor of an arbitrary shape carrying a current i, and P be a point in vacuum at which the field is to be determined. Let us divide the conductor into infinitesimal current-elements. Let $\vec{r}$ be a displacement vector from the element to the point P.

According to 'Biot-Savart Law', the magnetic field induction $d\vec{B}$ at P due to the current element $d\vec{l}$ is given by

$$d\vec{B} \propto \frac{i(d\vec{l} \times \vec{r})}{r^3} \quad \text{or} \quad d\vec{B} = k\frac{i(d\vec{l} \times \vec{r})}{r^3}.$$

Where k is a proportionality constant.

Here $d\vec{l}$ vector points in the direction of current i.



In SI units, $k = \frac{\mu_0}{4\pi} = 10^{-7}$ Wb/A-m

$$\therefore \quad d\vec{B} = \frac{\mu_0}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3} \qquad (1)$$

Equation (1) is the vector form of the Biot-Savart Law. The magnitude of the field induction at P is given by

$$dB = \frac{\mu_0}{4\pi}\frac{idl \times \sin\theta}{r^2},$$

where θ is the angle between $d\vec{l}$ and $\vec{r}$.

If the medium is other than air or vacuum, the magnetic induction is

$$d\vec{B} = \frac{\mu_0\mu_r}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3} \qquad (2)$$

where μ_r is relative permeability of the medium and is a dimensionless quantity.

FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE

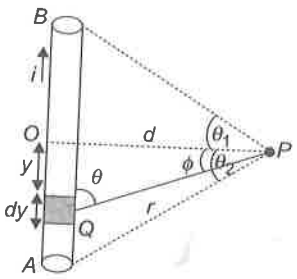
When the Wire Is of Finite Length

Consider a straight wire segment carrying a current i and there is a point P at which magnetic field to be calculated as shown in the figure. This wire segment makes angle θ_1 and θ_2 at that point with normal OP. Consider an element of length dy at a distance y from O and distance of this element from point P is r and line joining P to Q makes an angle θ with the direction of current as shown in figure. Using Biot-Savart Law magnetic field at point P due to small current element is given by

$$dB = \frac{\mu_0 i}{4\pi}\left(\frac{dy \sin\theta}{r^2}\right)$$

As every element of the wire contributes to $\vec{B}$ in the same direction, we have

$$B = \frac{\mu_0 i}{4\pi}\int_A^B \frac{dy \sin\theta}{r^2} \qquad (1)$$



From the triangle OPQ as shown in diagram, we have

$$y = d\tan\phi \quad \text{or} \quad dy = d\sec^2\phi\, d\phi$$

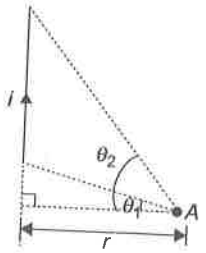
and is same triangle, $r = d\sec\phi$ and $\theta = (90° - \phi)$, where ϕ is angle between line OP and PQ.

Now equation (i) can be written in this form

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{i}{d}\int_{-\theta_2}^{\theta_1} \cos\phi\, d\phi$$

or

$$B = \frac{\mu_0}{4\pi}\frac{i}{d}[\sin\theta_1 + \sin\theta_2] \qquad (3)$$



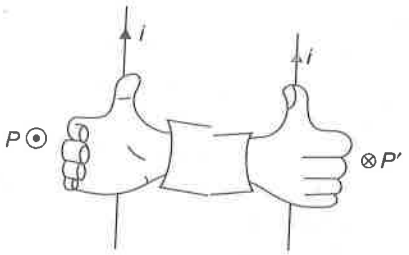
Notes

θ_1 and θ_2 must be taken with sign
For the case shown in figure

$$B \text{ at } A = \frac{\mu_0 i}{4\pi r}(\sin\theta_2 - \sin\theta_1) \otimes$$

Direction of $\vec{B}$: The direction of magnetic field is determined by the cross product of the vector $id\vec{l}$ with $\vec{r}$. Therefore, at point P, the direction of the magnetic field due to the whole conductor will be perpendicular to the plane of paper and going into the plane.

Right-hand Thumb Rule: The direction of B at a point P due to a long, straight wire can be found by the right-hand thumb rule. The direction of magnetic field is perpendicular to the plane containing wire and perpendicular from the point. The orientation of magnetic field is given by the direction of curl fingers if we stretch thumb along the wire in the direction of current. Refer figure.

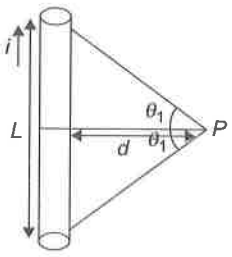


Conventionally, the direction of the field perpendicular to the plane of the paper is represented by ⊗ if into the page and by ⊙ if out of the page.

Now consider some special cases involving the application of equation (3).

Case I: When the point P is on the perpendicular bisector

In this case angle $\theta_1 = \theta_2$, using result of equation (3), the magnetic field is



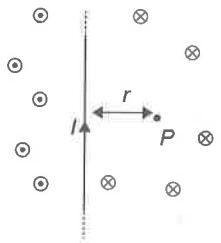
$$B = \frac{\mu_0}{4\pi}\frac{2i}{d}\sin\theta_1$$

here $$\sin\theta_1 = \frac{L}{\sqrt{L^2 + 4d^2}}$$

ase II:

(i) If the wire is infinitely long then the magnetic field at '*P*' (as shown in the figure) is given by (using $\theta_1 = \theta_2 = 90°$ and the formula '*B*' due to straight wire)

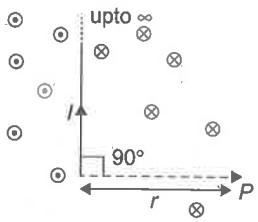
$$B = \frac{\mu_0 I}{2\pi r} \quad \Rightarrow \quad B \propto \frac{1}{r}$$



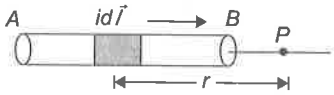
The direction of $\vec{B}$ at various is as shown in the figure. The magnetic lines of force will be concentric circles around the wire (as shown earlier).

(ii) If the wire is infinitely long but '*P*' is as shown in the figure. The direction of $\vec{B}$ at various points is as shown in the figure. At '*P*'

$$B = \frac{\mu_0 I}{4\pi r}$$



Case III: When the point lies along the length of wire (but not on it)



If the point *P* is along the length of the wire (but not one it), then as $d\vec{l}$ and $\vec{r}$ will either be parallel or antiparallel, i.e., $\theta = 0$ or π, so $id\vec{l} \times \vec{r} = 0$ and hence using equation (1)

$$\vec{B} = \int_A^B d\vec{B} = 0$$

SOLVED EXAMPLES

EXAMPLE 1

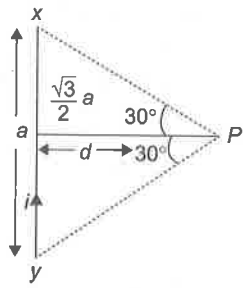
Calculate the magnetic field induction at a point distance, $\frac{a\sqrt{3}}{2}$ metre from a straight wire of length '*a*' metre carrying a current of *i* amp. The point is on the perpendicular bisector of the wire.

SOLUTION

$$B = \frac{\mu_0}{4\pi}\frac{i}{d}[\sin\theta_1 + \sin\theta_2]$$

$$= 10^{-7}\left[\frac{i}{\left(a\frac{\sqrt{3}}{2}\right)}\left(\frac{1}{2} + \frac{1}{2}\right)\right]$$

$$= \frac{2i}{a\sqrt{3}} \times 10^{-7} T$$



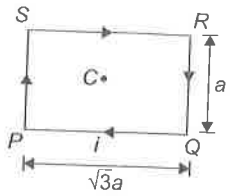
Perpendicular to the plane of figure (inward). ■

158

EXAMPLE 2

Find resultant magnetic field at 'C' in the figure shown.

SOLUTION



It is clear that 'B' at 'C' due all the wires is directed $\otimes$. Also B at C due PQ and SR is same. Also due to QR and PS is same

$$\therefore \quad B_{res} = 2(B_{PQ} + B_{SP})$$

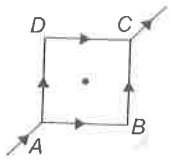
$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}}(\sin 60° + \sin 60°)$$

$$B_{SP} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}}(\sin 30° + \sin 30°)$$

$$\Rightarrow \quad B_{res} = 2\left(\frac{\sqrt{3}\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$ ■

EXAMPLE 3

Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points A and C.

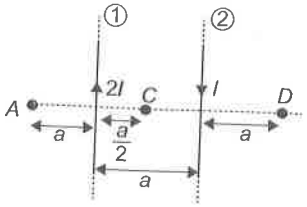


SOLUTION

The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero. ■

EXAMPLE 4

In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents $2I$ and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned.



Find

(i) $\vec{B}$ at A, C, D

(ii) position of point on line ACD where $\vec{B}$ is zero.

SOLUTION

(i) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then at A: $\vec{B}_1$ is $\odot$ and $\vec{B}_2$ is $\otimes$

$$B_1 = \frac{\mu_0 2I}{2\pi a} \quad \text{and} \quad B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$$\therefore \quad B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot \quad \textbf{Ans.}$$

at C: $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ also $\otimes$

$$\therefore \quad B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi\frac{a}{2}} + \frac{\mu_0 I}{2\pi\frac{a}{2}}$$

$$= \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes \qquad \textbf{Ans.}$$

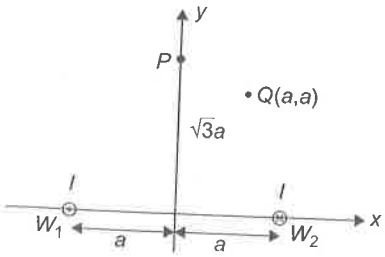
at D: $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ is $\odot$ and both are equal in magnitude

$$\therefore \quad B_{res} = 0 \quad \textbf{Ans.}$$

(ii) It is clear from the above solution that $B = 0$ at point 'D'. ■

EXAMPLE 5

In the figure shown two long wires W_1 and W_2 each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find the $\vec{B}$ at 'P' and 'Q'



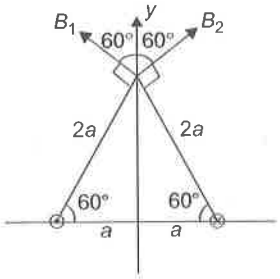
SOLUTION

Let $\vec{B}$ due to W_1 be $\vec{B}_1$ and due to W_2 be $\vec{B}_2$.

By symmetry $|\vec{B}_1| = |\vec{B}_2| = B$

$$B_p = 2B\cos 60° = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \quad \vec{B}_p = \frac{\mu_0 I}{4\pi a}\hat{j} \quad \textbf{Ans.}$$

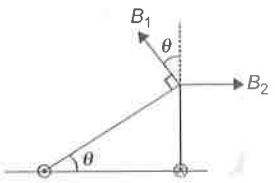


For Q $B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}$, $\Rightarrow$ $B_2 = \frac{\mu_0 I}{2\pi a}$

$$\tan\theta = \frac{a}{2a} = \frac{1}{2} \quad \Rightarrow \quad \vec{B} = (B_1\cos\hat{j}) + (B_2 - B_1\sin\theta)\hat{i}$$

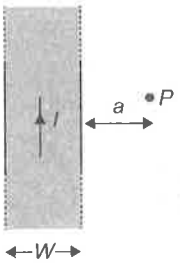
$$\sin\theta = \frac{1}{\sqrt{5}} \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a}\right)\hat{i}$$

$$\cos\theta = \frac{2}{\sqrt{5}} \quad \Rightarrow \quad \vec{B} = \frac{2\mu_0 I}{5\pi a}\hat{i} + \frac{\mu_0 I}{5\pi a}\hat{j}$$



EXAMPLE 6

In the figure shown a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'. Find the magnetic field at point 'P' which lies in the plane of the sheet

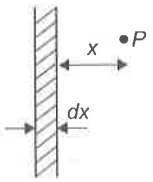


SOLUTION

To find 'B' at 'P' the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'

$$dB = \frac{\mu_0\left(\frac{I}{w}dx\right)}{2\pi x}\otimes$$

due to each such wire $\vec{B}$ will be directed inwards

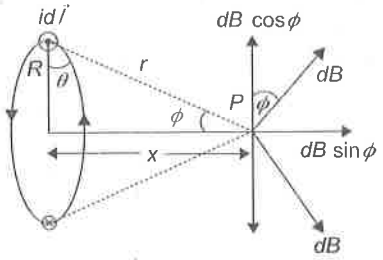


$$\therefore \quad B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}$$

$$\int_{x=a}^{x=a+w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}\cdot\ln\frac{a+w}{a} \quad \textbf{Ans.}$$

MAGNETIC FIELD AT AN AXIAL POINT OF A CIRCULAR COIL

Consider a circular loop of radius R and carrying a steady current i. We have to find out magnetic field at the axial point P, which is at distance x from the centre of the loop.



Consider an element $id\vec{l}$ of the loop as shown in figure, and the distance of point P from current element is r. The magnetic field at P due to this current element from the equation (1) can be given by,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{id\vec{l}\times\vec{r}}{r^3}$$

In case of point on the axis of a circular coil, as for every current element there is a symmetrically situated opposite element, the component of the field perpendicular to the axis cancel each other while along the axis add up.

160

$$\therefore \quad B = \int dB \sin\phi = \frac{\mu_0}{4\pi}\int \frac{idl\sin\theta}{r^2}\sin\phi$$

Here, θ is angle between the current element $id\vec{l}$ and $\vec{r}$, which is $\frac{\pi}{2}$ everywhere and

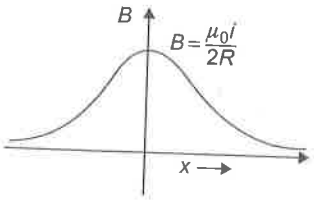
$$\sin\phi = \frac{R}{r} = \frac{R}{\sqrt{R^2 + x^2}}$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{iR}{(R^2+x^2)^{3/2}}\int_0^{2\pi R} dL$$

or,
$$B = \frac{\mu_0}{4\pi}\frac{iR}{(R^2+x^2)^{3/2}}(2\pi R)$$

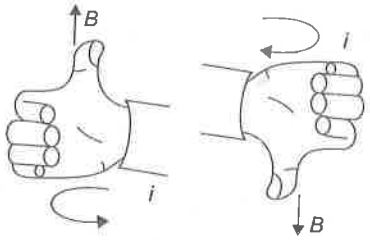
or,
$$B = \frac{\mu_0}{4\pi}\frac{2\pi iR^2}{(R^2+x^2)^{3/2}} \qquad (4)$$

If the coil has N turns, then



$$B = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{(R^2+x^2)^{3/2}}$$

Direction of $\vec{B}$: Direction of magnetic field at a point the axis of a circular coil is along the axis and its orientation can be obtained by using the right-hand thumb rule. If the figures are curled along the current, the stretched thumb will point towards the magnetic field.



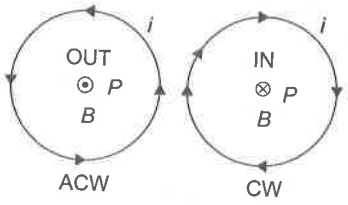
Magnetic field will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure given.

Now consider some special cases involving the application of equation (4).

Case I: Field at the centre of the coil.

In this case distance of the point P from the centre $(x) = 0$, the magnetic field

$$B = \frac{\mu_0}{4\pi}\frac{2\pi i}{R} = \frac{\mu_0}{2}\frac{i}{R}$$



Case II: Field at a point far away from the centre.

It means $x >> R$, $B = \frac{\mu_0}{4\pi}\frac{2\pi R^2 i}{x^3}$

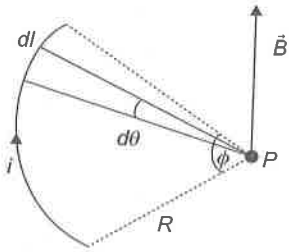
Field at the Centre of a Current Arc

Consider an arc of radius R carrying current i and subtending an angle ϕ at the centre.

According to Biot-Savart Law, the magnetic field induction at the point P is given by

$$B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{idl}{R^2}$$

Here, $dl = Rd\theta$



$$\therefore \quad B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{iRd\theta}{R^2}$$

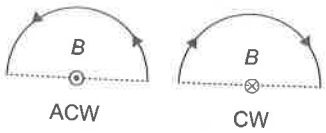
or,
$$B = \frac{\mu_0}{4\pi}\frac{i\phi}{R} \qquad (5)$$

It 'l' is the length of the circular arc, we have

$$B = \frac{\mu_0}{4\pi}\frac{il}{R^2} \qquad (6)$$

Consider some special cases involving the application of equation (5).

Case I: If the loop is semicircular



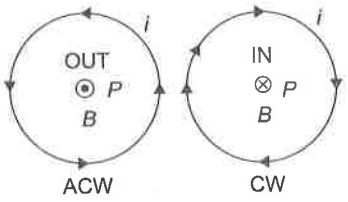
In this case $\phi = \pi$, so

$$B = \frac{\mu_0}{4\pi}\frac{\pi i}{R}$$

and will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure.

Case II: If the loop is a full circle with N turns.

In this case $\phi = 2\pi$, so



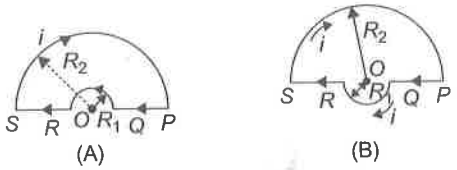
$$B = \frac{\mu_0}{4\pi}\frac{2\pi Ni}{R}$$

and will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure.

SOLVED EXAMPLES

EXAMPLE 7

Two wire loop $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current i as shown in the figure given below. What is the magnetic field induction at the centre O in cases (A) and (B)?



SOLUTION

(a) As the point O is along the length of the straight wires, so the field at O due to them will be zero and hence magnetic field is only due to semicircular portions

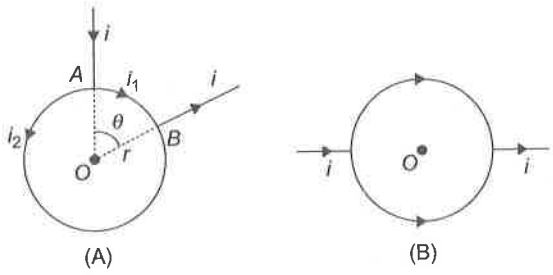
$$\therefore \quad |\vec{B}| = \frac{\mu_0}{4\pi}\left[\frac{\pi i}{R_2}\otimes + \frac{\pi i}{R_1}\odot\right]$$

$$\text{or,} \quad |\vec{B}| = \frac{\mu_0}{4\pi}\pi i\left[\frac{1}{R_2} - \frac{1}{R_1}\right] \text{ out of the page}$$

(b) $|\vec{B}| = \frac{\mu_0}{4\pi} i\pi\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$ into the page ■

EXAMPLE 8

A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R as shown in the figure given below. One of the arcs AB of the ring subtends an angle θ at the centre. What is the value of the magnetic field at the centre due to the current in the ring?



SOLUTION

(a) As the field due to arc at the centre is given by

$$B = \frac{\mu_0}{4\pi}\frac{i\phi}{r}$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{i_1\theta}{r}\otimes + \frac{\mu_0}{4\pi}\frac{i_2(2\pi-\theta)}{r}\odot$$

But $(V_A - V_B) = i_1R_1 = i_2R_2$

$$\text{or,} \quad i_2 = i_1\frac{R_1}{R_2} = i_1\frac{L_1}{L_2} \quad [\because \ R \propto L]$$

$$i_2 = i_1\frac{\theta}{(2\pi-\theta)} \quad [\because \ L = r\theta]$$

$$\therefore \quad B_R = \frac{\mu_0}{4\pi}\frac{i_1\theta}{r}\otimes + \frac{\mu_0}{4\pi}\frac{i_1\theta}{r}\odot = 0$$

i.e., the field at the centre of the coil is zero and is independent of θ. ■

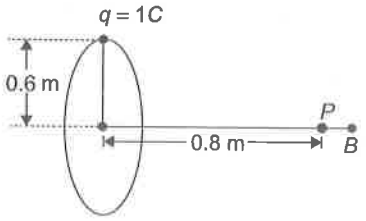
EXAMPLE 9

A charge of one coulomb is placed at one end of a nonconducting rod of length 0.6 m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with angular frequency $10^4\pi$ rad/s. Find the magnetic field at a point on the axis of rotation at a distance of 0.8 m from the centre of the path.

Now half of the charge is removed from one end and placed on the other end. The rod is rotated in a vertical plane about horizontal axis passing through the mid-point of the rod with the same angular frequency. Calculate the magnetic field at a point on the axis at a distance of 0.4 m from the centre of the rod.

SOLUTION

As the revolving charge q is equivalent to a current

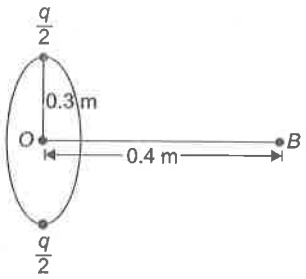


$$i = qf = q \times \frac{\omega}{2\pi} = 1 \times \frac{10^4 \pi}{2\pi} = 5 \times 10^3 \text{ A}$$

Now $$B = \frac{\mu_0}{4\pi} \frac{2\pi i R^2}{(R^2 + x^2)^{3/2}}$$

$$\therefore \quad B = 10^{-7} \times \frac{2\pi \times 5 \times 10^3 (0.6)^2}{[(0.6)^2 + (0.8)^2]^{3/2}} = 1.13 \times 10^{-3} \text{ T}$$

If half of the charge is placed at the other end and the rod is rotated at the same frequency, the equivalent current.



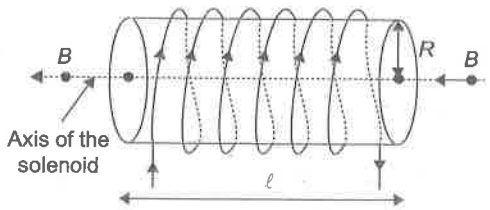
$$i' = \left(\frac{q}{2}\right) f + \left(\frac{q}{2}\right) f = qf = i = 5 \times 10^3 \text{ A}$$

In this case, $R' = 0.3$ m and $x' = 0.4$ m

$$\therefore \quad B' = 10^{-7} \times \frac{2\pi \times 5 \times 10^3 \times (0.3)^2}{[(0.3)^2 + (0.4)^2]^{3/2}} = 2.3 \times 10^{-3} \text{T}$$

SOLENOID

(i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)

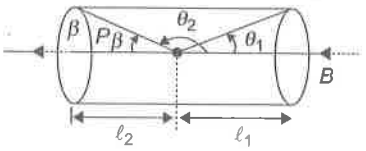


(ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) $\vec{B}$ on axis (turns should be very close to each others).

$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2) \qquad (7)$$

where n: number of turns per unit length.



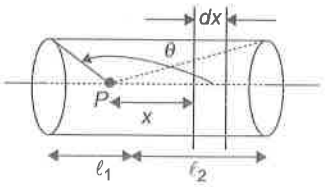
$$\cos\theta_1 = \frac{\ell_1}{\sqrt{\ell_1^2 + R^2}}; \quad \cos\beta = \frac{\ell_1}{\sqrt{\ell_2^2 + R^2}} = -\cos\theta_2$$

$$B = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right]$$

$$= \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$$

Derivation:

Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element $dn = ndx$ where n: number of turns per unit length]



$$dB = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}(ndx)$$

$$B = \int dB = \int_{-\ell_1}^{\ell_2} \frac{\mu_0 i R^2 n dx}{2(R^2 + x^2)^{3/2}}$$

$$= \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right]$$

$$= \frac{\mu_0 ni}{2}[\cos\theta_1 - \cos\theta_2]$$

(iv) For 'Ideal Solenoid':

*** Inside** (at the mid point)

$\ell >> R$ or length is infinite

$\theta_1 \to 0$

$\theta_2 \to \pi$

$B = \dfrac{\mu_0 ni}{2}[1-(-1)]$

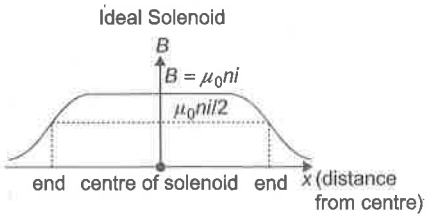
$B = \mu_0 ni$

If material of the solid cylinder has relative permeability 'μ_r' then $B = \mu_0 \mu_r ni$

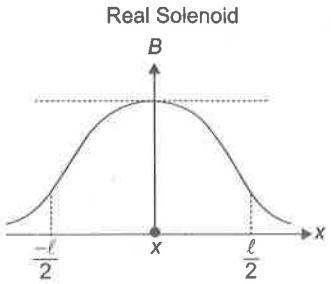
At the ends $B = \dfrac{\mu_0 ni}{2}$

(v) Comparison between ideal and real solenoid:

(a)



(b)



SOLVED EXAMPLES

EXAMPLE 10

A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0×10^{-3} ampere. Find the magnetic field on the axis at the middle and at the ends of the solenoid.

$$\left(\text{Gives } \mu_0 = 4\pi \times 10^{-7}\,\frac{V-s}{A-m}\right)$$

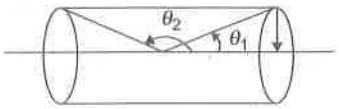
SOLUTION

$$, B = \frac{1}{2}\mu_0 ni[\cos\theta_1 - \cos\theta_2]$$

$$\Rightarrow \quad n = \frac{1000}{0.4} = 2500 \text{ per meter}$$

$$i = 5 \times 10^{-3} \text{ A.}$$

(i)

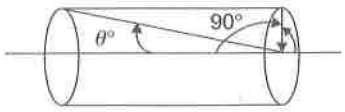


$$\cos\theta_1 = \frac{0.2}{\sqrt{(0.3)^2+(0.2)^2}} = \frac{0.2}{\sqrt{0.13}}$$

$$\cos\theta_2 = \frac{-0.2}{\sqrt{0.13}}$$

$$\Rightarrow \quad B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3}\,\frac{2 \times 0.2}{\sqrt{0.13}} = \frac{\pi \times 10^{-5}}{\sqrt{13}}\text{ T}$$

(ii)



At the end

$$\cos\theta_1 = \frac{0.4}{\sqrt{(0.3)^2+(0.4)^2}} = 0.8$$

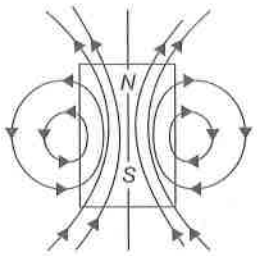
$$\cos\theta_2 = \cos 90° = 0$$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$$

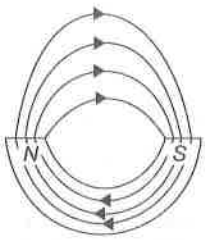
$$\Rightarrow \quad B = 2\pi \times 10^{-6} \text{ Wb/m}^2$$

MAGNETIC LINES AND THEIR CHARACTERISTICS

The space surrounding a magnet or magnetic configuration in which its effects are perceptible is called the **magnetic field** of the given magnet or magnetic configuration.



Bar-magnet
(A)



U-Shape magnet
(B)

In order to visualize a magnetic field graphically, **Michael Faraday** introduced the concept of lines. According to him a line is an imaginary curve the tangent to which at a point gives the direction of the field at that point.

Regarding magnetic field it is worth noting that:

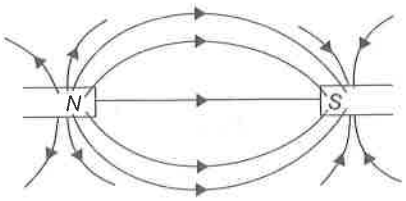
(i) Outside a magnet, field are from north to to south pole while inside from south to north, i.e., magnetic lines are closed curves i.e., they appear to converge or diverge at poles.

164

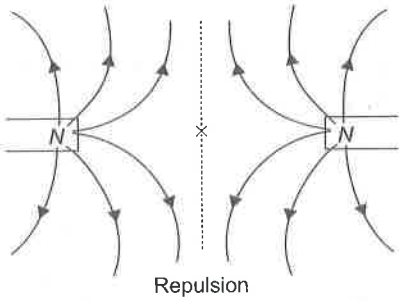
(ii) The number of magnetic lines of field originating or terminating on a pole is proportional to its strength. μ_0 lines are assumed to be associated with a unit pole. So if a body encloses a pole of strength m, total lines linked with the body (called **magnetic flux**) will be $\mu_0(m)$.

(iii) Magnetic lines of field can never intersect each other because if they intersect at a point, intensity at that point will have two directions which is absurd.

(iv) Magnetic lines of field have a tendency to contract longitudinally like a stretched elastic string (producing attraction between opposite poles) and repel each other laterally (resulting in repulsion between similar poles).

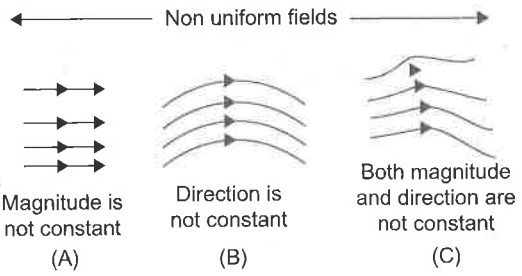


Attraction
(A)

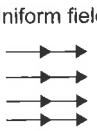


Repulsion
(B)

(v) Number of lines of field per unit area, normal to the area at a point, represents the magnitude of field at that point. So crowded lines represent a strong field while distant lines represent weak field. Further, if the lines of force are equidistant and straight the field is uniform otherwise not



(A) (B) (C)



(D)

(vi) In a region of space where there is no magnetic field, there will be no lines of field. This is why, at a **neutral point** (where resultant field is zero) there cannot be any line of field.

(vii) Magnetic lines of field originate from or enter in the surface of a magnetic material at any angle.

(viii) Magnetic lines of field exist inside every magnetized material.

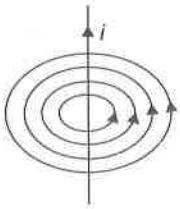
(ix) As monopoles do not exist, the total magnetic flux linked with a closed surface is always zero, i.e.,

$$\oint \vec{B} \cdot \vec{ds} = \mu_0(0) = 0$$

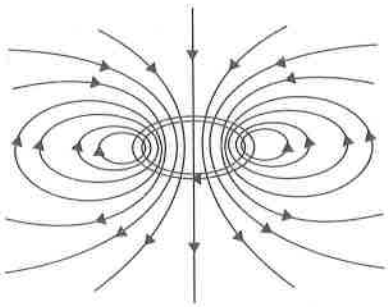
This law is called Gauss's law for magnetism.

Magnetic field line due to some important structure

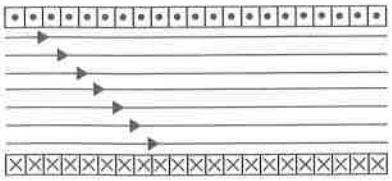
1. Straight current carrying wire



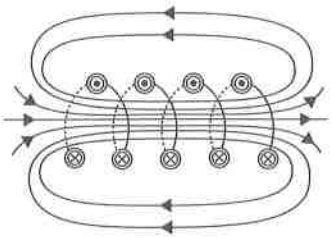
2. Circular coil



3. Solenoid



(ideal)



(Real)

AMPERE'S LAW

This law is useful in finding the magnetic field due to currents under certain conditions of symmetry. Consider a closed plane curve enclosing some current-carrying conductors.

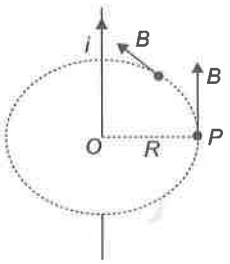
The line integral $\oint \vec{B} \cdot \vec{dl}$ taken along this closed curve is equal to μ_0 times the total current crossing the area bounded by the curve.

i.e., $$\oint \vec{B} \cdot \vec{dl} = \mu_0 i \qquad (8)$$

where i = total current (algebraic sum) crossing the area.

As a simple application of this law, we can derive the magnetic induction due to a long straight wire carrying current i.

Suppose the magnetic induction at point P, distance R from the wire is required.



Draw the circle through P with centre O and radius R as shown in figure.

The magnetic induction $|\vec{B}|$ at all points along this circle will be the same and will be tangential to the circle, which is also the direction of the length element $\vec{dl}$.

Thus, $\oint \vec{B} \cdot \vec{dl} = \oint B dl = B \oint dl = B \times 2\pi R$

The current crossing the circular area is i.

Thus, by Ampere's law, $B \times 2\pi R = \mu_0 i$

$$\Rightarrow \qquad B = \frac{\mu_0 i}{2\pi R}$$

Notes

- Line integral is independent of the shape of path and position of wire with in it.
- The statement $\oint \vec{B} \cdot \vec{d\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is passing through the path.
- **Sign of current:** The current due to which $\vec{B}$ is produced in the same sense as $\vec{d\ell}$ (i.e., $\vec{B} \cdot \vec{d\ell}$) positive will be taken positive and the current which produces $\vec{B}$ in the sense opposite to $\vec{d\ell}$ will be negative.

SOLVED EXAMPLES

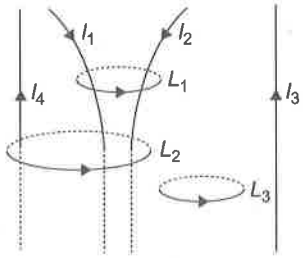
EXAMPLE 11

Find the value of $\oint \vec{B} \cdot \vec{d\ell}$ for the loops L_1, L_2, L_3 in the figure shown. The sense of $\vec{d\ell}$ is mentioned in the figure.

SOLUTION

for $$L_1 \quad \oint \vec{B} \cdot \vec{d\ell} = \mu_0 (I_1 - I_2)$$

here I_1 is taken positive because magnetic lines of force produced by I_1 is anticlockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{d\ell}$ is anticlockwise as seen from top.



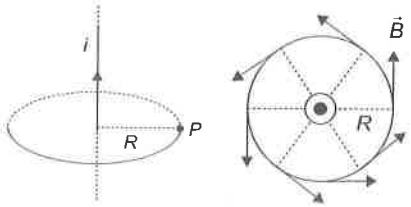
for $$L_2: \oint \vec{B} \cdot \vec{dl} = \mu_0 (I_1 - I_2 + I_4)$$

for $$L_3: \oint \vec{B} \cdot \vec{d\ell} = 0$$

Uses of Ampere's Current Law

To find out magnetic field due to infinite current carrying wire

166



By B.S.L. $\vec{B}$ will have circular lines. $\vec{d\ell}$ is also taken tangent to the circle

$$\oint \vec{B}\cdot\vec{d\ell} = \oint B\cdot d\ell \quad \therefore \quad \theta = 0°$$

so $$\oint B\cdot d\ell = B2\pi R \quad (\because \quad B = \text{const.})$$

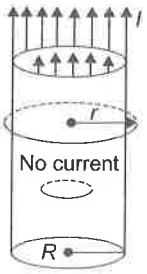
Now by amperes law: $B2\pi R = \mu_0 I$

$$\therefore \qquad B = \frac{\mu_0 i}{2\pi R}$$

Hollow current carrying infinitely long cylinder: (I is uniformly distributed on the) whole circumference

(i) for $r \geq R$

By symmetry the amperian loop is a circle.



$$= \oint \vec{B}\cdot\vec{d\ell} = \oint B d\ell \quad \because \quad \theta = 0$$

$$= B\int_0^{2\pi r} d\ell \quad \because \quad B = \text{const.}$$

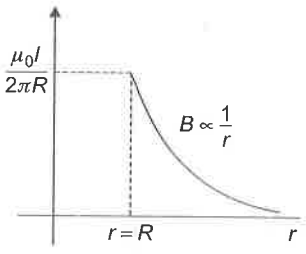
$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

(ii) $r < R$

$$\oint \vec{B}\cdot\vec{d\ell} = \oint B\cdot d\ell = \mu_0(0) = B(2\pi r) = 0$$

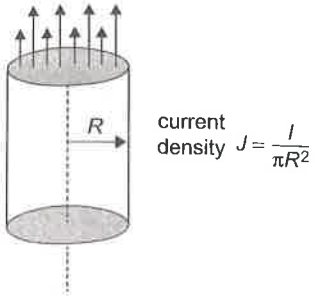
$$\Rightarrow \quad B_{in} = 0$$

Graph



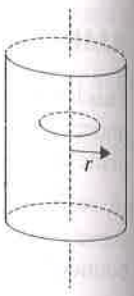
Solid Infinite Current Carrying Cylinder

Assume current is uniformly distributed on the whole cross section area



Case I: $r \leq R$

Taken an amperian loop inside the cylinder. By symmetry it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.

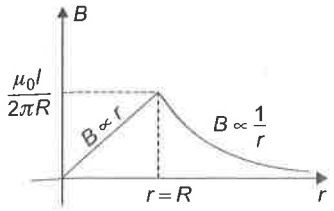


$$\oint \vec{B}\cdot\vec{d\ell} = \oint B\cdot d\ell = B\oint d\ell$$

$$= B\cdot 2\pi r = \mu_0 \frac{I}{\pi R^2}\pi r^2$$

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2} \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 \vec{J}\times\vec{r}}{2}$$

Case II:



$$r \geq R \oint \vec{B}\cdot d\vec{\ell} = B\oint d\ell = B\cdot(2\pi r) = \mu_0 \cdot I$$

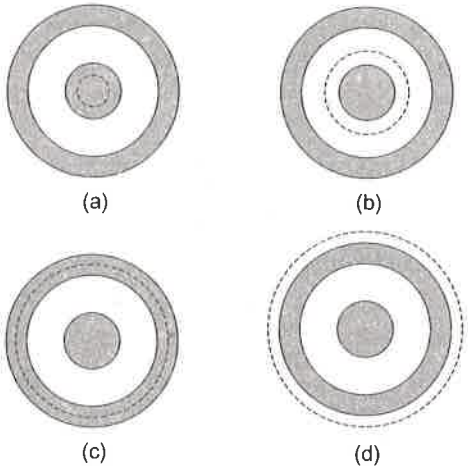
$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r} \text{ also } \vec{B}\frac{\mu_0 I}{2\pi r}(\hat{j}\times\hat{r}) = \frac{\mu_0 J\pi R^2}{2\pi r}$$

$$\vec{B} = \frac{\mu_0 R^2}{2r^2}(\vec{J}\times\vec{r})$$

EXAMPLE 12

Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where (a) $x < a$, (b) $a < x < b$ (c) $b < x < c$ and (d) $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

SOLUTION



A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B} \cdot d\vec{\ell} = B2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle in part b is i_0 so that

$$\frac{i_0}{\pi a^2} \cdot \pi x^2 = \frac{i_0}{a^2} x^2$$

Ampere's law

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i$ gives

$$B \cdot 2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or,} \quad B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is i_0 so that

$$B2\pi x = \mu_0 i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{2\pi x}$$

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell with in the circle in part c of the figure is $\pi x^2 - \pi b^2$.

Thus, the current through this part is $\frac{i_0(x^2 - b^2)}{(c^2 - b^2)}$.

This is in the opposite direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

$$i_{\text{net}} = i_0 - \frac{i_0(x^2 - b^2)}{c^2 - b^2} = \frac{i_0(c^2 - x^2)}{c^2 - b^2}$$

From Ampere's law,

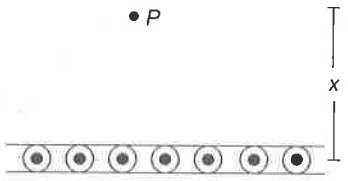
$$B2\pi x = \frac{\mu_0 i_0 (c^2 - x^2)}{c^2 - b^2} \quad \text{or} \quad B = \frac{\mu_0 i_0 (c^2 - x^2)}{2\pi x (c^2 - b^2)}$$

(d) The net current enclosed by the circle in part d of the figure is zero and hence

$$B2\pi x = 0 \quad \text{or,} \quad B = 0.$$

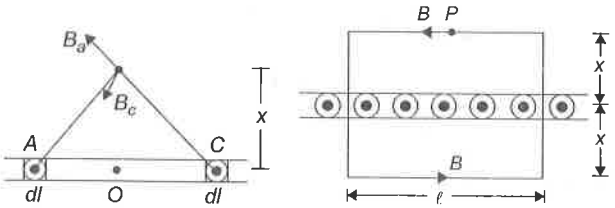
EXAMPLE 13

(a) Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.



SOLUTION

Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is B_0 perpendicular to AP and that due to the strip C is B_c perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.



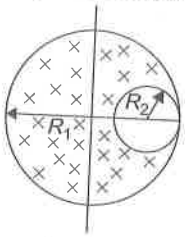
The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

$$2B\ell = \mu_0 K \ell \quad \text{or,} \quad B = \frac{1}{2}\mu_0 K$$

Note that it is independent of x.

168

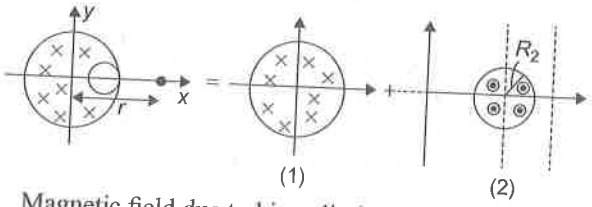
(b) A cylinder of radius R_1 have cylindrical cavity of radius R_2 as shown in the figure and have current density J (down ward). Find Magnetic field when



(i) $r > R_1$ (on x-axis towards right)
(ii) $r < R_1$ (on x-axis towards left)
(iii) when point is in cavity

SOLUTION

(i) $r > R_1$ (on x-axis towards right)



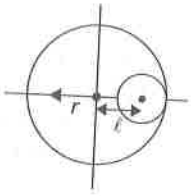
Magnetic field due to big cylinder $= B_1$
Magnetic field due to smaller cylinder $= B_2$

$$B_r = B_1 + B_2 = \left[\frac{\mu_0 j\pi R_1^2}{2\pi r} + \left\{-\frac{\mu_0 J\pi R_2^2}{2\pi(r-\ell)}\right\}\right]$$

(ii) $r < R_1$ (on x-axis) towards left of centre Magnetic field due to big cylinder

$$B_1 = \frac{\mu_0 J\pi r^2}{2\pi r}$$

Magnetic field due to smaller cylinder

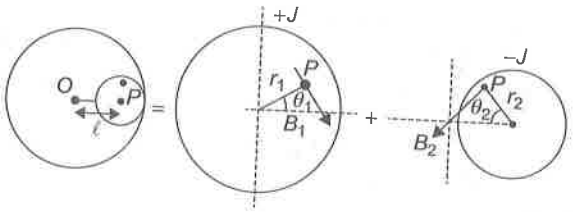


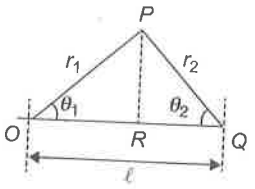
$$B_2 = \frac{\mu_0(-J)\pi R_2^2}{2\pi(r+\ell)}$$

So net Magnetic field

$$B = B_1 + B_2 = \frac{\mu_0 J}{2}\left[r - \frac{R_2^2}{r+\ell}\right]$$

(iii) When the point (p) is in cavity





$$PR = r_1 \sin\theta_1 = r_2 \sin\theta_2 \quad (1)$$

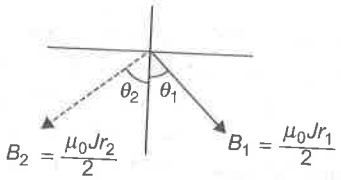
and $$\ell = r_1 \cos\theta_1 + r_2 \cos\theta_2 \quad (2)$$

Horizontal component

$$B_H = B_1 \sin\theta_1 - B_2 \sin\theta_2$$

$$= \frac{\mu_0 J}{2}[r_1 \sin\theta_1 - r_2 \sin\theta_2]$$

$B_H = 0$ from eq. (1)



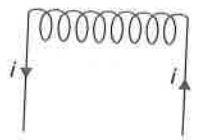
Vertical component $B_v = B_1 \cos\theta_1 + B_2 \cos\theta_2$

$$= \frac{\mu_0 J}{2}[r_1 \cos\theta_1 + r_2 \cos\theta_2]$$

$$B_v = \frac{\mu_0 i}{2} \cdot \ell$$ from eq. (2) ■

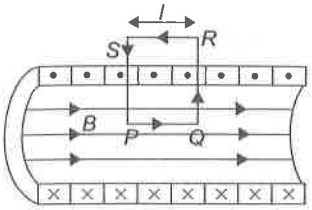
MAGNETIC FIELD INSIDE A LONG SOLENOID

The magnetic field inside a very tightly wound long solenoid is uniform everywhere along the axis of the solenoid and is zero outside it.



To calculate the magnetic field at a point P inside the solenoid, let us draw a rectangle $PQRS$ as shown in figure. The line PQ is parallel to the solenoid axis and hence parallel to the magnetic field $\vec{B}$ inside the solenoid.

$$\Rightarrow \int_P^Q \vec{B} \cdot \vec{d}\ell = Bl$$



On the remaining three sides, $\vec{B} \cdot d\vec{l}$ is zero everywhere as $\vec{B}$ is either zero (outside the solenoid) or perpendicular to $\vec{dl}$ (inside the solenoid).

$\Rightarrow$ the circulation of $\vec{B}$ along $PQRS$ is

$$\oint \vec{B} \cdot d\vec{l} = Bl$$

If n is the number of turns per unit length along the length of solenoid, total nl turns cross the rectangle $PQRS$. Each turn carries a current i.

$\Rightarrow$ Net current crossing $PQRS = nli$

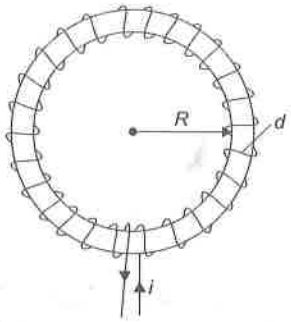
Using Ampere's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 nil$$

$$\Rightarrow \quad Bl = \mu_0 nil \quad \Rightarrow \quad B = \mu_0 ni \qquad (9)$$

TOROID

It is on hollow circular tube have windings of conducting wire closely attached to each other circularly on it (as shown below)



for ideal Toroid $d << R$

Magnetic field in Toroid

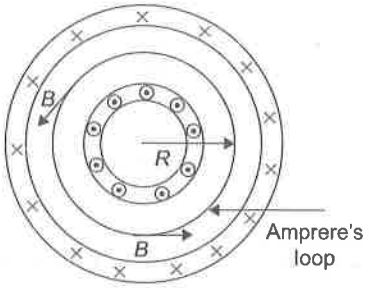
Let N = Total number of turns

Now from Ampere's circuital law

$$\oint B \cdot d\ell = \mu_0 i_{in}$$

$$B \cdot 2\pi R = \mu_0 i_{in} = \mu_0 Ni$$

$$\Rightarrow \quad B = \frac{\mu_0 Ni}{2\pi R}$$

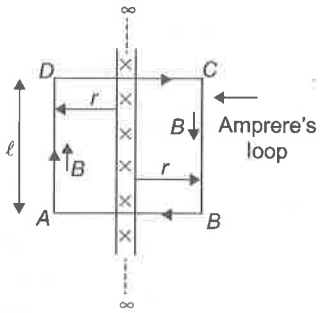


$$n = \frac{N}{2\pi R} = \text{Number of turns per unit length}$$

so $B = \mu_0 ni$

INFINITE CURRENT CARRYING SHEET

Now from Ampere's loop



$$\oint B \cdot d\ell = \mu_0 \lambda \ell$$

$$\int_{AB} B \cdot d\ell + \int_{BC} B \cdot d\ell + \int_{CD} B \cdot d\ell + \int_{DA} B \cdot d\ell = \mu_0 \lambda \ell$$

$$B\int d\ell + \int B' d\ell \cos 90° + B\int d\ell + \int B' d\ell \cos 90° = \mu_0 \lambda \ell$$

$$B = \frac{\mu_0 \lambda}{2}$$

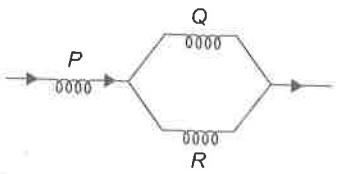
SOLVED EXAMPLES

EXAMPLE 14

Three identical long solenoids P, Q and R are connected to each other as shown in figure. If the magnetic field at the centre of P is 2.0 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.

170

SOLUTION



As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be 1.0 T. ■

MAGNETIC FORCE ON MOVING CHARGE

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula:

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{Put } q \text{ with sign.} \qquad (9)$$

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point.

Difference Between Magnetic Force and Electric Force

1. Magnetic force is always perpendicular to the field while electric force is collinear with the field.
2. Magnetic force is velocity dependent, i.e., acts only when the charged particle is in motion while electric force (qE) is independent of the state of rest or motion of the charged particle.
3. Magnetic force does no work when the charged particle is displaced while the electric force does work in displacing the charged particle.

Notes

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\therefore \quad \vec{F} \perp \vec{v} \quad \therefore$ power due to magnetic force on a charged particle is zero. (use the formula of power $P = \vec{F} \cdot \vec{v}$ for its proof)
- Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.
- On a stationary charged particle, magnetic force is zero.
- If $\vec{V} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

SOLVED EXAMPLES

EXAMPLE 15

A charged particle of mass 5 mg and charge $q = +2\mu C$ has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k} \cdot \vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

SOLUTION

$$\vec{F} = q\vec{v} \times \vec{B} = 2 \times 10^{-6}(2\hat{i} - 3\hat{j} + 4\hat{k}) \times (3\hat{j} - 2\hat{k})$$
$$= 2 \times 10^{-6}[-6\hat{i} + 4\hat{j} + 6\hat{k}]N$$

By Newton's Law $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{2 \times 10^{-6}}{5 \times 10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k})$

$$= 0.8(-3\hat{i} + 2\hat{j} + 3\hat{k}) \text{ m/s}^2$$ ■

EXAMPLE 16

A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

SOLUTION

$\because \quad \vec{F} \perp \vec{B}$

$\therefore \quad \vec{a} \perp \vec{B}$

$\therefore \quad \vec{a} \cdot \vec{B} = 0$

$\therefore \quad (2\hat{i} + x\hat{j}) \cdot (-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0$

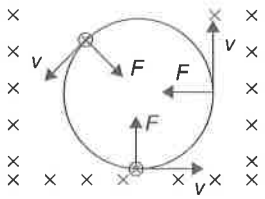
$\Rightarrow \quad -6 + 2x = 0 \quad \Rightarrow \quad x = 3.$ ■

MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

When the Charged Particle Is Given Velocity Perpendicular to the Field

Let a particle of charged q and mass m is moving with a velocity v and enters at right angles to a uniform magnetic field $\vec{B}$ as shown in figure.

The force on the particle is qvB and this force will always act in a direction perpendicular to v. Hence, the particle will move on a circular path. If the radius of the path is r then



$$\frac{mv^2}{r} = Bqv \quad \text{or,} \quad r = \frac{mv}{qB} \tag{10}$$

Thus, radius of the path is proportional to the momentum mv of the particle and inversely proportional to the magnitude of magnetic field.

Time period: The time period is the time taken by the charged particle to complete one rotation of the circular path which is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \tag{11}$$

The time period is independent of the speed v.

Frequency: The frequency is number of revolution of charged particle in one second, which is given by,

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m} \tag{12}$$

and angular frequency $= \omega = 2\pi\nu$

SOLVED EXAMPLES

EXAMPLE 17

A proton (p), α-particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

SOLUTION

$$R = \frac{\sqrt{2mK}}{qB}$$

$$\therefore \quad R_p : R_\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{qB} : \frac{\sqrt{2.2mK}}{qB}$$

$$= 1 : 1 : \sqrt{2}$$

$$= T = \frac{2\pi m}{qB}$$

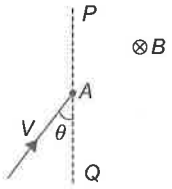
$$\therefore T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB} = 1 : 2 : 2 \quad \textbf{Ans.}$$

EXAMPLE 18

A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ.

Find

(i) time spent,
(ii) distance travelled in the magnetic field
(iii) impulse of magnetic force.



SOLUTION

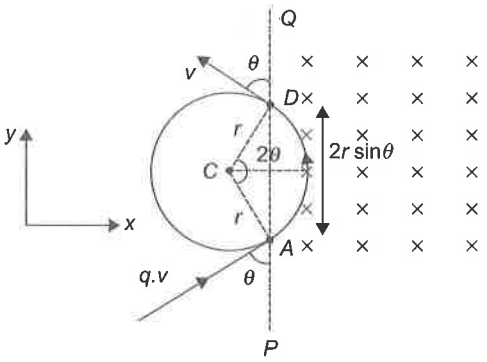
The particle will move in the field as shown. Angle subtended by the arc at the centre $= 2\theta$

(i) Time spent by the charge in magnetic field

$$\omega t = \theta \quad \Rightarrow \quad \frac{qB}{m}t = \theta \quad \Rightarrow \quad t = \frac{m\theta}{qB}$$

(ii) Distance travelled by the charge in magnetic field:

$$= r(2\theta) = \frac{mv}{qB} \cdot 2\theta$$



(iii) Impulse = change in momentum of the charge

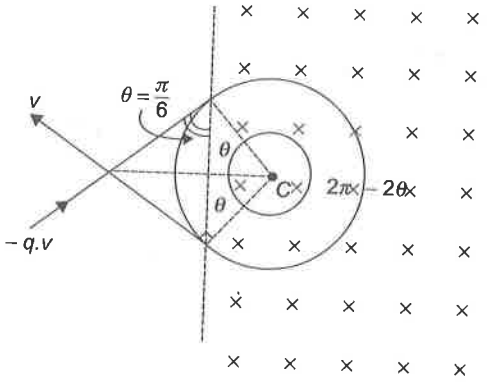
$$= (-mv\sin\theta\hat{i} + mv\cos\theta\hat{j})$$

$$-(mv\sin\theta\hat{i} + mv\cos\theta\hat{j}) = -2mv\sin\theta\hat{i}$$

EXAMPLE 19

Repeat above question if the charge is –ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

172



SOLUTION

(i) $2\pi - 2\theta = 2\pi - 2 \cdot \frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$= \omega t = \frac{qBt}{m} \quad \Rightarrow \quad t = \frac{5\pi m}{3qB}$$

(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

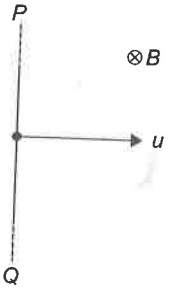
(iii) Impulse = charge in linear momentum

$= m(-v\sin\theta\hat{i} + v\cos\theta\hat{j}) - m(v\sin\theta\hat{i} + v\cos\theta\hat{j})$

$= -2mv\sin\theta\hat{i} = -2mv\sin\frac{\pi}{6}\hat{i} = -mv\hat{i}$

EXAMPLE 20

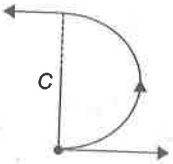
In the figure shown the magnetic field on the left on 'PQ' is zero and on the right of 'PQ' it is uniform.



Find the time spent in the magnetic field.

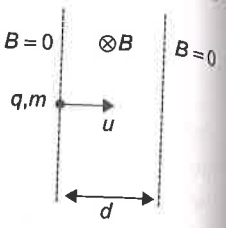
SOLUTION

The path will be semicircular time spent $= \frac{T}{2} = \frac{\pi m}{qB}$



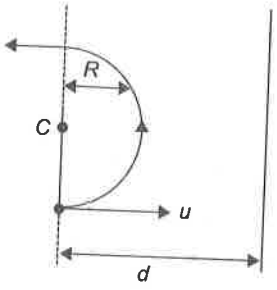
EXAMPLE 21

A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perciculariy (as shown in the figure) into the magnetic field. Find the time spend by the particle in the magnetic field if



(i) $d > \frac{mu}{qB}$ (ii) $d < \frac{mu}{qB}$

SOLUTION



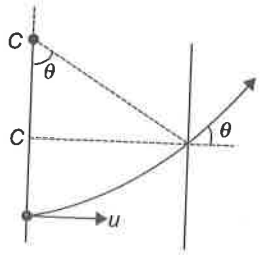
(i) $d > \frac{mu}{qB}$ means $d > R \quad \therefore \quad t = \frac{T}{2} = \frac{\pi m}{qB}$

(ii) $\sin\theta = \frac{d}{R}$

$$\theta = \sin^{-1}\left(\frac{d}{R}\right)$$

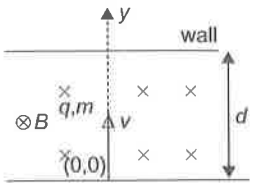
$$\omega t = \theta$$

$$\Rightarrow \quad t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$$



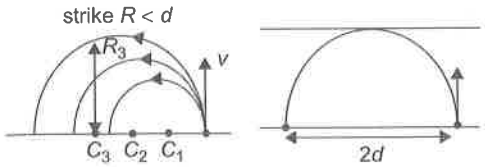
EXAMPLE 22

What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.



SOLUTION

(i) The path of the particle will be circular larger the velocity, larger will be the radius. For particle not to s



$$\therefore \quad \frac{mv}{qB} < d \quad \Rightarrow \quad v < \frac{qBd}{m}$$

(ii) for limiting case $v = \frac{qBd}{m}$

$$R = d$$

$\therefore$ coordinate = $(-2d, 0, 0)$ ■

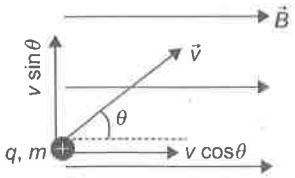
When the Charged Particle Is Moving at an Angle to the Field

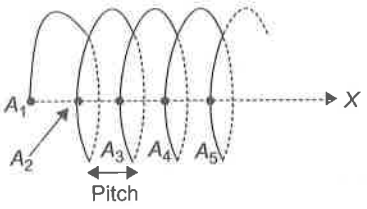
in this case the charged particle having charge q and mass m is moving with velocity v and it enter the magnetic field B at angle θ as shown in figure. Velocity can be resolved in two components, one along magnetic field and the other perpendicular to it. Let these components are $v_{\|}$ and $v_{\perp}$

$$v_{\|} = v\cos\theta$$

and $v_{\perp} = v\sin\theta$

The parallel component $v_{\|}$ of velocity remains unchanged as it is parallel to $\vec{B}$. Due to the $v_{\perp}$ the particle will move on a circular path. So the resultant path will be combination of straight-line motion and circular motion, which will be helical as shown in figure.





The radius of path is $(r) = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$ (13)

Time period $(T) = \frac{2\pi r}{v_{\perp}} = \frac{2\pi mv\sin\theta}{v\sin\theta qB} = \frac{2\pi m}{qB}$ (14)

Frequency $(f) = \frac{Bq}{2\pi m}$ (15)

Pitch: Pitch of helix described by charged particle is defined as the distance moved by the centre of circular path in the time in which particle completes one revolution.

Pitch = distance $A_1A_2 = A_3A_4 = \cdots = v\cos\theta \cdot T$

$$V_{\|} \cdot T = v\cos\theta \frac{2\pi m}{Bq} = \frac{2\pi mv\cos\theta}{qB} \quad (16)$$

If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components $-v_{\|}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The components $v_{\|}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

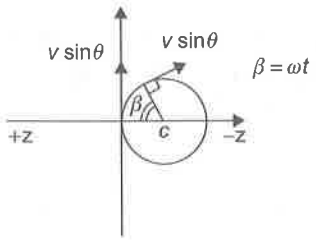
Complete analysis: Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper. The particle starts from point A_1.

It completes its one revolution at A_2 and 2nd revolution at A_3 and so on. X-axis is the tangent to the helix points A_1, A_2, A_3, ... all are on the x-axis.

distance $A_1A_2 = A_3A_4 = \cdots = v\cos\theta \cdot T$ = pitch

where T = Time period

Let the initial position of the particle be (0, 0, 0) and $v\sin\theta$ *in* $+y$ direction. Then



174

in x: $F_x = 0$, $a_x = 0$, v_x = constant = $v\cos\theta$, $x = (v\cos\theta)t$

In y-z plane: From figure it is clear that

$$y = R\sin\beta, \text{ here } \beta = \omega t$$
$$v_y = v\sin\theta\cos\beta$$
$$z = -(R - R\cos\beta)$$
$$v_z = v\sin\theta\sin\beta$$

acceleration towards centre

$$= \frac{(v\sin\theta)^2}{R} = \omega^2 R$$

$$\therefore \quad a_y = -\omega^2 R\sin\beta,\ a_z = -\omega^2 R\cos\beta$$

At any time: the position vector of the particle (or its displacement wrt initial position)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},\ x, y, z \text{ already found}$$

velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, v_x, v_y, v_z already found

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k},\ a_x, a_y, a_z \text{ already found}$$

EXAMPLE 23

A beam of protons with a velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 T at an angle 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix $m_p = 1.67 \times 10^{-27}$ kg

SOLUTION

Radius of the helical path taken by the proton beam

$$r = \frac{m(v\sin\theta)}{qB} = 1.2 \text{ cm}$$

Time period $T = \dfrac{2\pi r}{v\sin\theta} = 2.175 \times 10^{-7}$ s

$\therefore$ pitch of the helix $p = v\cos\theta \cdot T$

$$\Rightarrow \qquad p = 4 \times 10^5 \times \frac{1}{2} \times 2.175 \times 10^{-7} = 4.35 \text{ cm} \quad \blacksquare$$

MOTION OF A CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELD

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$

$$\vec{F} = q[\vec{E} + \vec{V} \times \vec{B}] \qquad (17)$$

which is **'Lorentz force equation'**.

Now let us consider two special cases involving the application of above equation

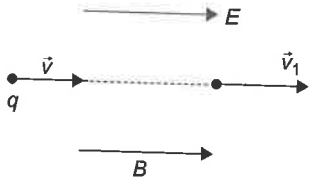
Notes

- Magnetic force is frame dependent, Electric force is frame dependent but Lorentz force is frame independent.

Case I: When $\vec{v}$, $\vec{E}$ and $\vec{B}$ all the three are collinear.

In this situation as the particle is moving parallel or anti-parallel to the field, the magnetic force on it will be zero and only electric force will act, so

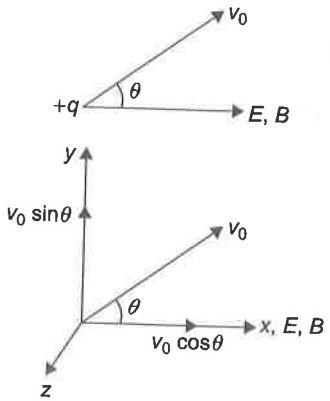
$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

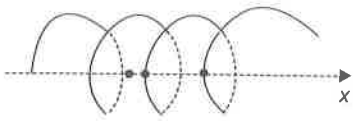


Hence the particle will pass through the field following a straight-line path (parallel to the field) with change in its speed. So in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown in the figure.

$\vec{v}$, $\vec{E}$ and $\vec{B}$ are collinear.

Case II: $\vec{E} \| \vec{B}$ and uniform $\theta \neq 0, 180°$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)





along x-axis: $F_x = qE$, $a_x = \frac{qE}{m}$, $v_x = v_0 \cos\theta + a_x t$,

$$x = v_0 \cos\theta t + \frac{1}{2} a_x t^2$$

in y-z plane:

$$qv_0 \sin\theta B = \frac{m(v_0 \sin\theta)^2}{R}$$

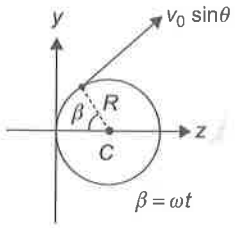
$$\Rightarrow \quad R = \frac{mv_0 \sin\theta}{qB}$$

$$\omega = \frac{v_0 \sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$$

$$\vec{r} = \left\{(V_0 \cos\theta)t + \frac{1}{2}\frac{qE}{m}t^2\right\}\hat{i} + R\sin\omega t\hat{j} + (R - R\cos\omega t)(-\hat{k})$$

$$\vec{V} = \left(v_0 \cos\theta = \frac{qE}{m}t\right)\hat{i} + (V_0 \sin\theta)\cos\omega t\hat{j} + V\sin\theta\sin\omega t - \hat{k}$$

$$\vec{a} = \frac{qE}{m}\hat{i} + \omega^2 R[-\sin\beta\hat{j} - \cos\beta\hat{k}]$$



Case III: $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular

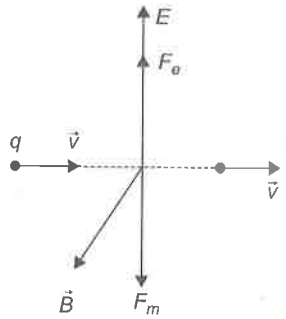
$\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular. In case situation of $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0$$

or, $\vec{a} = \left(\frac{\vec{F}}{m}\right) = 0$, then the particle will pass through the field with the same velocity.

In this situation,

$$F_e = F_m \quad \text{or,} \quad qE = qvB \quad \text{or,} \quad v = \frac{E}{B}$$



This principle is used in velocity-selector to get a charged beam having a specific velocity.

If a charge particle (Q) inters into the magnetic field at origin with velocity v_p, then find the maximum possible positive x co-ordinate of particle if magnetic field is given as

$$B = B_0 x \qquad (-\hat{k})\, x > 0$$
$$= 0 \qquad x < 0$$

SOLUTION

Because $F = qv_0 B_0 x$ at any time t at position x

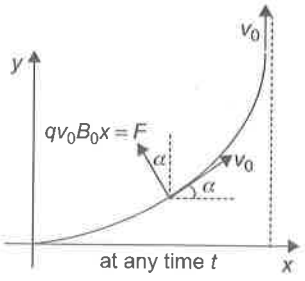
$$F_{\text{vertical}} = qB_0 v_0 x \cos\alpha$$

so $$a_y = \frac{dy}{dt} = \frac{qB_0 v_0 x \cos\alpha}{m} \qquad (1)$$

$$\frac{dv_y}{dx}\left(\frac{dx}{dt}\right) = \frac{qB_0 x}{m}(v_0 \cos\alpha)$$

$$\frac{dx}{dt} = v_0 \cos\alpha$$

$$\int_0^{v_0} dv_y = \int_0^{x_{\max}} \frac{q\{B_0 \cdot x\}}{m} dx \quad \Rightarrow \quad v_0 = \frac{qB_0}{m} \cdot \frac{x_{\max}^2}{2}$$



176

EXAMPLE 24

A long, straight wire carries a current i. A particle having a positive charge q and mass m kept at a distance x_0 from the wire is projected towards it with a speed v. Find the minimum separation between the wire and the particle.

SOLUTION

Let the particle be initially at P(figure). Take the wire as the Y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the X-axis. We have, $OP = x_0$. The magnetic field B at any point to the right of the particle is, therefore, in the X-Y plane. As there is no initial velocity along the Z-axis, the motion will be in the X-Y plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle.

The force at time t is $\vec{F} = q\vec{v} \times \vec{B}$

$$= q(v_x\vec{i} + v_y\vec{j}) \times \left(-\frac{\mu_0 i}{2\pi x}\vec{k}\right)$$

$$= \vec{j}qv_x\frac{\mu_0 i}{2\pi x} - \vec{i}qv_y\frac{\mu_0 i}{2\pi x}$$

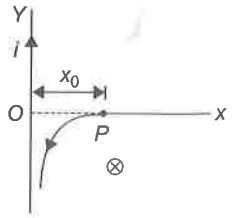
Thus $$a_x = \frac{F_x}{m} = -\frac{\mu_0 qi}{2\pi m}\frac{v_y}{x} = -\lambda\frac{v_y}{x} \quad (1)$$

where $$\lambda = \frac{\mu_0 qi}{2\pi m}$$

Also $$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{v_x dv_x}{dx} \quad (2)$$

As, $v_x^2 + v_y^2 = v^2$,

giving $$v_x\, dv_x = -v_y\, dv_y \quad (3)$$



From (1), (2) and (3),

$$\frac{v_y dv_y}{dx} = \frac{\lambda v_y}{x}$$

or, $$\frac{dx}{x} = \frac{dv_y}{\lambda}$$

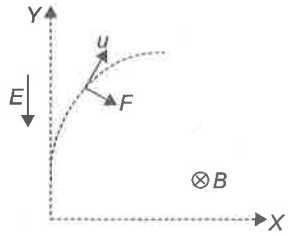
Initially $x = x_0$ and $v_y = 0$. At minimum separation from the wire, $v_x = 0$ so that $v_y = -v$.

Thus $$\int_{x_0}^{x}\frac{dx}{x} = \int_{0}^{-v}\frac{dv_y}{\lambda} \quad \text{or,} \quad \ln\frac{x}{x_0} = -\frac{v}{\lambda}$$

or; $$x = x_0 e^{-v/\lambda} = x_0 e^{-\frac{2\pi mv}{\mu_0 qi}}$$

EXAMPLE 25

An electron is released from the origin at a place where a uniform electric field E and a uniform magnetic field B exist along the negative Y-axis and the negative Z-axis respectively. Find the displacement of the electron along the Y-axis when its velocity becomes perpendicular to the electric field for the first time.



SOLUTION

Let us take axes as shown in figure. According to the right-handed system, the Z-axis is upward in the figure and hence the magnetic field is shown downwards. At any time, the velocity of the electron may be written as

$$\vec{u} = u_x\vec{i} + u_y\vec{j}$$

The electric and magnetic fields may be written as

$$\vec{E} = -E\vec{j} \text{ and } \vec{B} = -B\vec{k}$$

respectively. The force on the electron is

$$\vec{F} = -e(\vec{E} + \vec{u} \times \vec{B}) = e\vec{E}\,\vec{j} + eB(u_y\vec{i} - u_x\vec{j})$$

Thus, $F_x = eu_y B$ and $F_y = e(E - u_x B)$.

The components of the acceleration are

$$a_x = \frac{du_x}{dt} = \frac{eB}{m}u_y \quad (1)$$

and $$a_y = \frac{du_y}{dt} = \frac{e}{m}(E - u_x B) \quad (2)$$

We have,

$$\frac{d^2u_y}{dt^2} = -\frac{eB}{m}\frac{du_x}{dt} = -\frac{eB}{m}\cdot\frac{eB}{m}u_y = -\omega^2 u_y$$

where $$\omega = \frac{eB}{m} \quad (3)$$

This equation is similar to that for a simple harmonic motion. Thus,

$$u_y = A\sin(\omega t + \delta) \tag{4}$$

and hence,

$$\frac{du_y}{dt} = A\omega\cos(\omega t + \delta) \tag{5}$$

At $t = 0,\ u_y = 0$ and $\dfrac{du_y}{dt} = \dfrac{F_y}{dt} = \dfrac{eE}{m}$

Putting in (4) and (5)

$$\delta = 0 \quad \text{and} \quad A = \frac{eE}{m\omega}\frac{E}{B}$$

Thus, $$u_y = \frac{E}{B}\sin\omega t$$

The path of the electron will be perpendicular to the Y-axis when $u_y = 0$. This will be the case for the first time at t where

$$\sin\omega t = 0$$

or, $$\omega t = \pi \quad \text{or,} \quad t = \frac{\pi}{\omega} = \frac{\pi m}{eB}$$

Also, $$u_y = \frac{dy}{dt} = \frac{E}{B}\sin\omega t$$

or, $$\int_0^y dy = \frac{E}{B}\sin\omega t\, dt \quad \text{or} \quad y = \frac{E}{B\omega}(1 - \cos\omega t)$$

At $$t = \frac{\pi}{\omega},$$

$$y = \frac{E}{B\omega}(1 - \cos\pi) = \frac{2E}{B\omega}$$

Thus, the displacement along the Y-axis is

$$\frac{2E}{B\omega} = \frac{2Em}{BeB} = \frac{2Em}{eB^2}$$

(1)

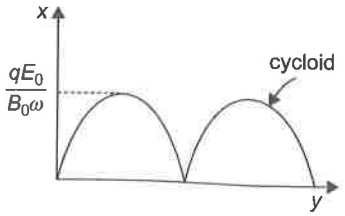
(2)

$$a_x = \frac{q}{m}E_0\sin\omega t$$

$$\int_0^{v_x} dv_x = \frac{qE_0}{m}\int_0^t \sin\omega t\, dt \quad \Rightarrow \quad v_x = \frac{qE_0}{m\omega}(1 - \cos\omega t)$$

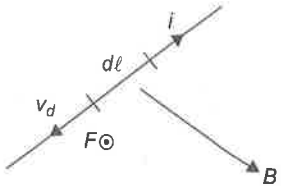
(3)

$$\int_0^x dx = \frac{qE_0}{m\omega}\int_0^t (1 - \cos\omega t)\cdot dt \quad \Rightarrow \quad x = \frac{qE_0}{m\omega}\left[t - \frac{\sin\omega t}{\omega}\right]$$



MAGNETIC FORCE ON A CURRENT CARRYING WIRE

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element $d\ell$ of the wire (figure). The free electrons drift



with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is

$$i = jA = nev_dA$$

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$$\vec{f} = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered in $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$$

If we denote the length $d\ell$ along the direction of the current by $\vec{d}\ell$, the above equation becomes

$$d\vec{F} = nAev_d d\vec{\ell} \times \vec{B}$$

Using (1), $$d\vec{F} = id\vec{\ell} \times \vec{B}$$

The quantity $id\vec{\ell}$ is called a current element

$$\vec{F}_{res} = \int d\vec{F} = \int id\vec{\ell} \times \vec{B} = i\int \vec{d}\ell \times B$$

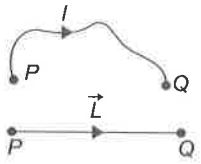
($\because$ i is same at all points of the wire)

If $\vec{B}$ is uniform then $\vec{F}_{res} = i(\int \vec{d}\ell) \times \vec{B}$

$$\vec{F}_{res} = i\vec{L} \times \vec{B} \tag{18}$$

178

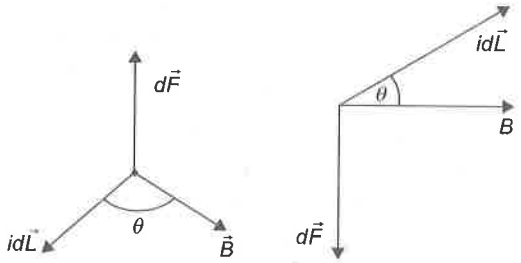
Here $\vec{L} = \int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.



Notes

If a current loop of any shape is placed in a uniform $\vec{B}$ then $(\vec{F}_{res})_{magnetic}$ on it $= 0$ ($\because \quad \vec{L} = 0$)

Direction of force: The direction of force is always perpendicular to the plane containing $id\vec{L}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{a} \times \vec{b})$ with $\vec{a} = id\vec{L}$ and $\vec{b} = \vec{B}$



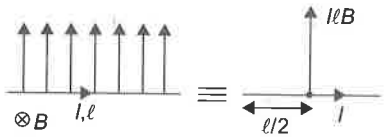
The direction of force when current element $id\vec{L}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules.

(a) ***Fleming's Left-hand Rule:*** Stretch the fore-finger, central finger and thumb of the left hand mutually perpendicular. Then if the fore-finger points in the direction of the field $(\vec{B})$ and the central in the direction of current i, the thumb will point in the direction of force (or motion).

(b) ***Right-hand Palm rule:*** Stretch the fingers and thumb of the right-hand at right angles to each other. If the fingers point in the direction of current i, and the palm in the direction of the field $\vec{B}$ then thumb will point in the direction of force.

Point of Application of Magnetic Force

On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.

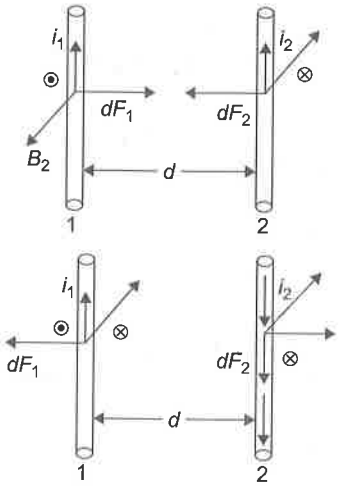


This can be used for calculation of torque.

Force Between Two Long Straight Parallel Current Carrying Conductors

Let us consider two very long parallel straight wires carrying currents i_1 and i_2.

Each wire is placed in the region of magnetic induction of other and hence will experience a force. The net force on a current-carrying conductor due to its own field is zero. So if there are two long parallel current-carrying wires 1 and 2 (as shown below), the wire-1 will be in the field of wire-2 and vice versa.



The force on dl_2 length of wire-2 due to field of wire-1, $dF_2 = i_2\, dL_2\, B_1$

$$dF_2 = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d}\, dL_2 \qquad \left[\because \quad B_1 = \frac{\mu_0}{4\pi}\frac{2i_1}{d}\right]$$

or $$\frac{dF_2}{dL_2} = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d}. \qquad (19)$$

It will be true for wire-1 in the field of wire-2. The direction of force in accordance with the right-hand screw rule will be as shown above.

So the force per unit length in case of two parallel current-carrying wires separated by a distance 'd' is

$$\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d}.$$

If i_1 and i_2 are along the same direction, the forces between the wires is attractive in nature and if i_1 and i_2 are oppositely directed the force is repulsive. The direction of forces is given by Fleming's left-hand rule.

Definition of 'ampere'

We have $\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d}$.

If $i_1 = i_2 = 1$ A; $d = 1$ m; $dL = 1$ m; then

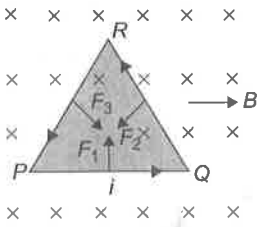
$dF = 2 \times 10^{-7}$ N

Hence, 'ampere' is defined as the current which when passing through each of two parallel infinitely long straight conductors placed in free space at a distance of 1 m from each other produces between them force of 2×10^{-7} N for one metre of their length.

SOLVED EXAMPLES

EXAMPLE 26

A wire is bent in the form of an equilateral triangle PQR of side 10 cm and carries a current of 5.0 A. It is placed in magnetic field B of magnitude $2.0T$ direction perpendicular to the plane of the loop. Find the forces on the three sides of the triangle.



SOLUTION

Suppose the field and the current have directions as shown in figure. The force on PQ is

$$\vec{F}_1 = i\vec{\ell} \times \vec{B}$$

or, $F_1 = 5.0\text{ A} \times 10\text{ cm} \times 2.0T = 1.0$ N

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

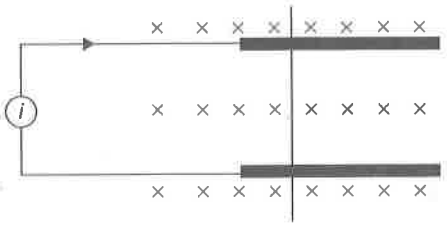
The three forces $\vec{F}_1$, $\vec{F}_2$ and F_3 will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalized. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force. ■

EXAMPLE 27

Figure shown two long metal rails placed horizontally and parallel to each other at a separation ℓ. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.

(a) What should the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the pervious part



SOLUTION

(a) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

or, $F = i\ell B$

It acts towards right in the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to μ_0 mg. Thus,

$$\mu_0 mg = i\ell B$$

or, $$\mu_0 = \frac{i\ell B}{mg}$$

(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{i\ell B}{2mg}$, the wire will slide towards right. The frictional force by the rails is

$$f = \mu mg = \frac{i\ell B}{2} = \text{towards left.}$$

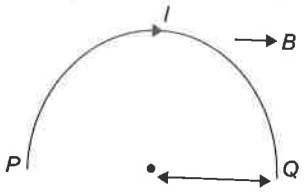
180

The resultant force is $i\ell B - \frac{i\ell B}{2} = \frac{i\ell B}{2}$ towards right.

The acceleration will be $a = \frac{i\ell B}{2m}$. The wire will slide towards right with this acceleration. ■

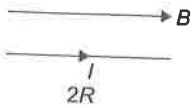
EXAMPLE 28

In the figure shown a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.



SOLUTION

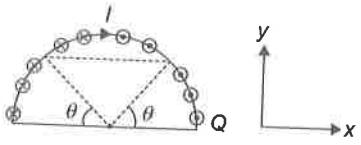
The wire is equivalent to



$\because \quad \theta = 0$

$\because \quad F_{res} = 0$ **Ans.**

forces on individual parts are marked in the figure by ⊗ and ⊙. By symmetry their will be pair of forces forming couples.



$$\tau = \int_0^{\pi/2} i(Rd\theta)\, B \sin(90 - \theta) \cdot 2R\cos\theta$$

$$\tau = \frac{i\pi R^2}{2} B \quad \Rightarrow \quad \vec{\tau} = \frac{i\pi R^2}{2} B(-\hat{j}) \quad \textbf{Ans.}$$ ■

EXAMPLE 29

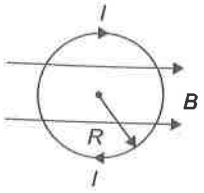
Find the resultant magnetic force and torque on the loop.

SOLUTION

$$\vec{F}_{res} = 0, \qquad (\because \text{ loop})$$

and $$\vec{\tau} = i\pi R^2 B(-\hat{j})$$

using the above method



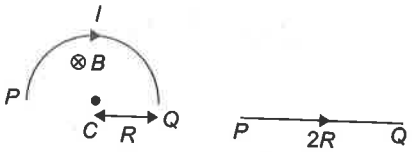
EXAMPLE 30

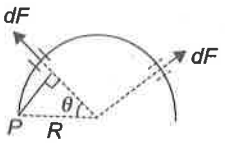
In the figure shown find the resultant magnetic force and torque about 'C', and 'P'.

SOLUTION

$$\vec{F}_{nett} = I \cdot 2R \cdot B$$

$\because$ wire is equivalent to





Force on each element is radially outward: $\tau_c = 0$ about point P

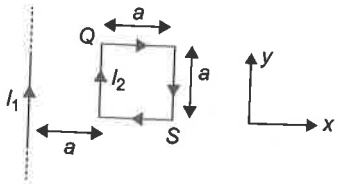
$$P = \int_0^{\pi} [i/(Rd\theta)B\sin 90°]R\sin\theta = 2IBR^2 \quad \textbf{Ans.}$$ ■

EXAMPLE 31

Find the magnetic force on the loop '$PQRS$' due to the straight wire.

SOLUTION

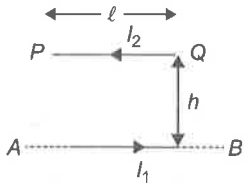
$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a}a(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)}a(\hat{i}) = \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$$



■

EXAMPLE 32

In the figure shown the wires AB and PQ carry constant currents I_1 and I_2 respectively. PQ is of uniformly distributed mass 'm' and length 'ℓ' AB and PQ are both horizontal and kept in the same vertical plane. The PQ is in equilibrium at height 'h'. Find



(i) 'h' is terms of I_1 I_2, ℓ, m, g and other standard constants.

(ii) If the wire PQ is displaced vertically by small distance prove that it performs SHM. Find its time period in terms of h and g.

SOLUTION

(i) Magnetic repulsive force balances the weight.

$$\frac{\mu_0 I_1 I_2}{2\pi h}\ell = mg \quad \Rightarrow \quad h = \frac{\mu_0 I_1 I_2 \ell}{2\pi mg}$$

(ii) Let the wire be displaced downward by distance $x(<< h)$.

Magnetic force on it will increase, so it goes back towards its equilibrium position. Hence it performs oscillations.

$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi(h-x)}\ell - mg$$

$$= \frac{mgh}{h-x} - mg = \frac{mg(h-h+x)}{h-x}$$

$$= \frac{mg}{h-x}x \cong \frac{mg}{h}x \quad \text{for} \quad x << h$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}} \quad \textbf{Ans.}$$

CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

Magnetic Moment

According to magnetic effects of current, in case of current-carrying coil for axial point,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{(R^2+x^2)^{3/2}}$$

when $x >> R$, $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{x^3}$

If we compare this result with the field due to a small bar magnet for a distant axial point, i.e.,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3}$$

where M is magnetic moment of the bar magnet.

We find that a current-carrying coil for a distant point behaves as a magnetic dipole of moment

$$\vec{M} = Ni\pi R^2 = NiA \qquad (20)$$

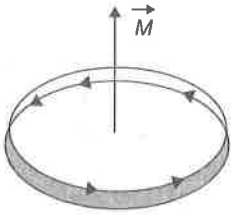
where A is area of the loop. So the magnetic moment of a current carrying coil is defined as the product of current in the coil with the area of coil in the vector form.

Magnetic moment of a current loop is a vector quantity and direction is perpendicular to the plane of the loop. Its dimensions are $[L^2A]$ and units are A-m^2.

Magnetic moment is case of a charged particle having charge q and moving in a circle of radius R with speed v is given by $\frac{1}{2}qvR$.

As we know, the equivalent current $i = qf = q\frac{v}{2\pi R}$ and $|A| = \pi R^2$

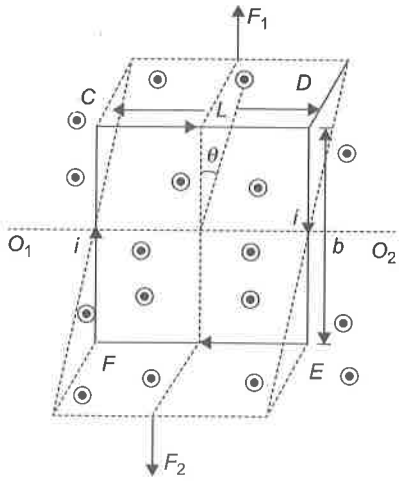
$$\therefore \quad M = i|\vec{A}| = \frac{1}{2}qvR$$



Torque on a Current Loop

Consider a rectangular coil $CDEF$ of length L and width b is placed vertically, while a uniform magnetic induction B passes normally through it as shown. The coil is capable of rotation about an axis O_1O_2.

If the loop is oriented in the magnetic field such that the normal to the plane of the coil makes an angle θ with the direction of $\vec{B}$, then the torque experienced by the loop



$$\tau = \frac{b}{2}(iLB)\sin\theta + \frac{b}{2}(iLB)\sin\theta$$

i.e., $\tau = iLbB\sin\theta = iAB\sin\theta$

where $A = Lb$ is the area of the loop.

The maximum torque experienced is $\tau = iAB$, when $\theta = 90°$ and for a coil of N turns

$$\tau = NiAB$$

Here $NiA = M$ = Magnetic moment of the loop.

In vector notation $\vec{\tau} = \vec{M} \times \vec{B}$ (21)

This result holds good for plane loops of all shapes rectangular, circular or otherwise.

Work Done in Rotating a Current Loop

When a current loop is rotated in a uniform magnetic field through an angle θ about an axis then work done will be

$$\int_0^W dW = \int \tau d\theta = \int_0^\theta MB\sin\theta d\theta$$

$$W = -[MB\cos\theta]_0^\theta = MB(1 - \cos\theta) \quad (22)$$

SOLVED EXAMPLES

EXAMPLE 33

For a given length L of a wire carrying a current i, how many circular turns would produce the maximum magnetic moment and of what value?

SOLUTION

For a cicular coil having N turns, magnetic moment

$$M = \pi R^2 iN \quad (1)$$

Now, length of wire $L = (2\pi R)N$

$$\therefore \quad R = \frac{L}{2\pi N}$$

Substituting the above value of R in equation (i), we get

$$M = \pi Ni \times \frac{L^2}{4\pi^2 N^2} \quad \text{or,} \quad M = \frac{iL^2}{4\pi N}$$

From equation (ii), it is clear that M will be maximum when N = minimum = 1, i.e., the coil has only one turn and

$$(M)_{max} = \frac{1}{4\pi} iL^2$$

EXAMPLE 34

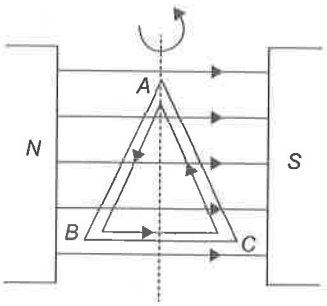
A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical in plane magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 A is passed through it and the magnetic field is parallel to its plane.

SOLUTION

As the coil is in the form of an equilateral triangle, its area

$$A = \frac{1}{2} \times L \times L\sin 60°$$

$$= \frac{1}{2} \times (0.02)^2 \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-4} \text{ m}^2$$



So its magnetic moment

$$M = iA = 0.1 \times \sqrt{3} \times 10^{-4}$$
$$= \sqrt{3} \times 10^{-5} \text{ A-m}^2$$

Now, the couple on a current-carrying coil in a magnetic field is given by $\tau = MB\sin\theta$

Since the plane of the coil is parallel to the magnetic field, the angle between M and $\vec{B}$ will be 90° and hence $\tau = MB\sin 90° = MB$

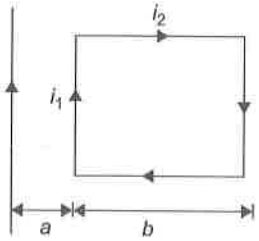
$$\therefore \quad \tau = (\sqrt{3} \times 10^{-5}) \times 5 \times 10^{-2} = 5\sqrt{3} \times 10^{-7} \text{ N-m}$$

EXAMPLE 35

The arrangement is as shown below

(a) Find the potential energy of the loop.

(b) Find the work done to increase the spacing between the wire and the loop from a to $2a$.



SOLUTION

(a) Magnetic moment of a small element of the loop.

$$dM = i_2\, Ldx$$

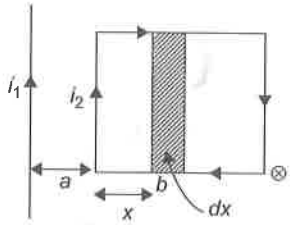
The direction of the magnetic moment is perpendicular to the plane of paper pointing inwards.

$dU = -d\vec{M}\vec{B} = -dMB$, where B is the magnetic field at the position of this element.

i.e., $B = \dfrac{\mu_0}{4\pi}\dfrac{2i_1}{a+x}$

$$\therefore \quad dU = -\frac{\mu_0}{4\pi} 2i_1 i_2 l\left(\frac{dx}{a+x}\right)$$

$$\therefore \quad U = -\frac{\mu_0}{4\pi} 2i_1 i_2 l \int_0^b \frac{dx}{a+x} = \frac{\mu_0}{4\pi} 2i_1 i_2 l \log_e\left(\frac{a+b}{a}\right)$$



(b) $U_i = -\dfrac{\mu_0}{4\pi} 2i_1 i_2 l \log_e\left(\dfrac{a+b}{a}\right)$

$$U_f = -\frac{\mu_0}{4\pi} 2i_1 i_2 l\left(\log_e \frac{2a+b}{2a}\right)$$

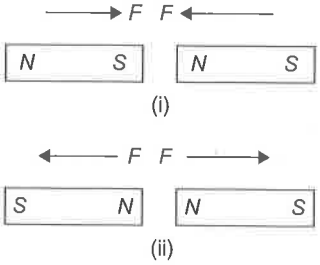
$$\therefore \quad W = \Delta U = U_f - U_i$$

$$= \frac{\mu_0}{4\pi} 2i_1 i_2 l \log_e\left(\frac{2(a+b)}{2a+b}\right)$$

MAGNET

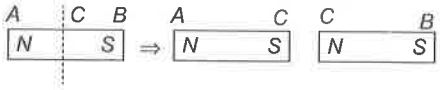
Pole Strength, Magnetic Dipole and Magnetic Dipole Moment

A magnet always has two poles 'N' and 'S' and like poles of two magnets repel other and the unlike poles of two magnets attract each other they form action reaction pair.



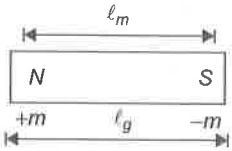
The poles of the same magnet do not comes to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. SO, 'N' and 'S' always exist together.

$\therefore$ they are



Know as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" $+m$ and $-m$ respectively (just like we have charge $+q$ and $-q$ in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges $-q$ and $+q$). It is called MAGNETIC DIPOLE and it has a direction is from $-m$ to $+m$ that means from 'S' to 'N')



$M = m \cdot l_m$ here l_m = magnetic length of the magnet. l_m is slightly less than l_g (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].

The units of m and M will be mentioned afterwards where you can remember and understand.

Magnetic Field and Strength of Magnetic Field

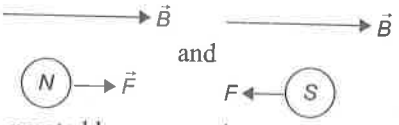
The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is quantitatively represented by 'STRENGTH OF MAGNETIC FIELD' or 'MAGNETIC INDUCTION' or 'MAGNETIC FLUX DENSITY' It is represented by $\vec{B}$. It is a vector quantity.

Definition of $\vec{B}$: The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \dfrac{\vec{F}}{m}$

Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value. SI unit of $\vec{B}$ is **Tesla** or **Weber/m^2** (abbreviated as T and Wb/m^2).

We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$.

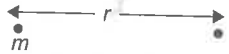


The field generated by sources does not depend on the test pole (for its any value and any sign).

(A) $\vec{B}$ due to various source

(i) Due to a single pole:

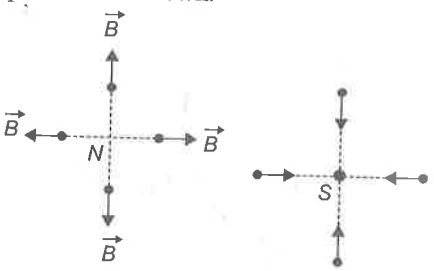
(Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2} \qquad (23)$$

This is magnitude.

Direction of B due to north pole and due to south poles are as shown.



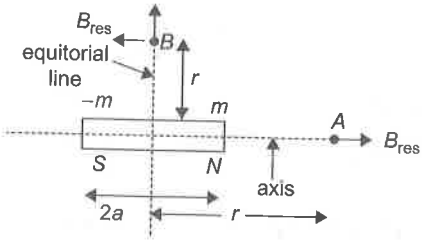
in vector from $\vec{B} = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{r^3}\vec{r}$ (24)

here m is with sign and $\vec{r}$ = position vector of the test point with respect to the pole.

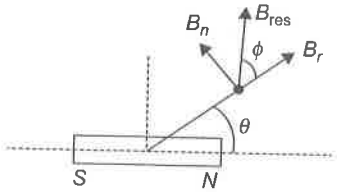
(ii) Due to a bar magnet:

(Same as the case of electric dipole in electrostatics) independent case never found. Always 'N' and 'S' exist together as magnet.

at A (on the axis) $= 2\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{\vec{M}}{r^3}$ for $a << r$ (25)



at B (on the equatorial) $= -\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{\vec{M}}{r^3}$ for $a << r$ (26)



At General point:

$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$$

$$B_n = \left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

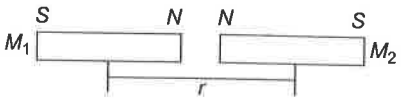
$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta} \qquad (27\ (a))$$

$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2} \qquad (28\ (b))$$

SOLVED EXAMPLES

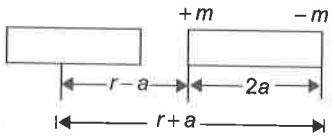
EXAMPLE 36

Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



SOLUTION

To find the magnetic force we will use the formula of 'B' due to a magnet. We will also assume m and $-m$ as pole strengths of 'N' and 'S' of M_2. Also length of M_2 as $2a$. B_1 and B_2 are the strengths of the magnetic field due to M_1 at $+m$ and $-m$ respectively. They experience magnetic forces F_1 and F_2 as shown.



$$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m \quad \text{and} \quad F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$$

$$\therefore \quad F_{res} = F_1 - F_2 = 2\left(\frac{\mu_0}{4\pi}\right) M_1 m\left[\left(\frac{1}{(r-a)^3}\right) - \left(\frac{1}{(r+a)^3}\right)\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3} - \left(1+\frac{a}{r}\right)^{-3}\right]$$

By using, Binomial expansion, and neglecting terms of high power we get

$$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4}$$

$$= 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$$

Direction of F_{res} is towards right.

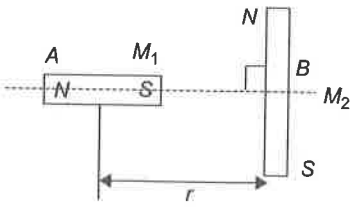
Alternative Method:

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2M_1}{r^3} \quad \Rightarrow \quad \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$$

$$F = -M_2 \times \frac{dB}{dr} \quad \Rightarrow \quad F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1 M_2}{r^4}$$ ■

EXAMPLE 37

Two short magnets A and B of magnetic dipole moments M_1 and M_2 respectively are placed as shown. The axis of 'A' and the equatorial line of 'B' are the same. Find the magnetic force on one magnet due to the other.



SOLUTION

$$F = 3\left(\frac{\mu_0}{4\pi}\right)\frac{M_2 M_1}{r^4}$$ upwards on M_1

down wards on M_2 ■

EXAMPLE 38

A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

SOLUTION

The pole strength is m = 120 CGS units = 12 A-m

Magnetic length is $2l$ = 10 cm or l = 0.05 m

Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\ell^2)^2} = \frac{\mu_0}{4\pi}\frac{4m\ell d}{(d^2-\ell^2)^2}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12A-m)\times(0.05m)\times(0.2m)}{[(0.2m)]^2-(0.05m)^2]^2}$$

$$= 3.4\times10^{-5}T.$$ ■

EXAMPLE 39

Find the magnetic field due to a dipole of magnetic moment 1.2 A-m^2 at a point 1 m away from it in a direction making an angle of 60° with the dipole-axis.

SOLUTION

The magnitude of the field is

$$B = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{1.2A-m^2}{1m^2}\sqrt{1+3\cos^2 60°}$$

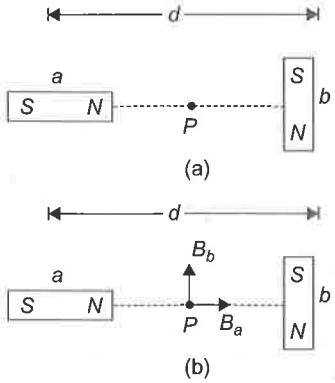
186

The direction of the field makes an angle α with the radial line where

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$ ■

EXAMPLE 40

Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



SOLUTION

The point p is in end-on position for the dipole a and in broadside-on position for the dipole b'. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore

$$B = \sqrt{B_a^2 + B_b^2} = \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2 + 2^2} = \frac{2\sqrt{5}\mu_0 M}{\pi d^2}$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = \frac{B_b}{B_a} = \frac{1}{2}$. ■

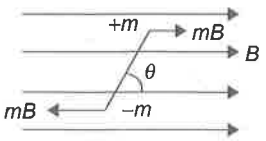
Magnet in an External Uniform Magnetic Field

(same as case of electric dipole)

$F_{res} = 0$ (for any angle)

$\tau = MB\sin\theta$

* here θ is angle between $\vec{B}$ and $\vec{M}$



Notes

- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$
- $\vec{\tau}$ is same about every point of the dipole it's potential energy is

$$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$$

$\theta = 0°$ is stable equilibrium

$\theta = \pi$ is unstable equilibrium

for small 'θ' the dipole performs SHM about $\theta = 0°$ position

$$\tau = -MB\sin\theta;$$

$$I\alpha = -MB\sin\theta$$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

for small θ, $\sin\theta \simeq \theta$

Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB}}$$

here $I = I_{cm}$ if the dipole is free to rotate $= I_{hinge}$ if the dipole is hinged.

EXAMPLE 41

A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.

SOLUTION

The work done by the external agent = change in potential energy

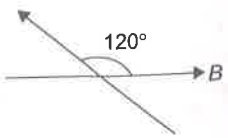
$$= (-MB\cos\theta_2) - (-MB\cos\theta_1)$$

$$= -MB(\cos 60° - \cos 0°)$$

$$= \frac{1}{2}MB = \frac{1}{2} \times (1.0 \times 10^4\,\text{J/T})(4 \times 10^{-5}\,\text{T}) = 0.2\,\text{J}$$ ■

EXAMPLE 42

A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure.



Find:

(i) Its kinetic energy at $\theta = 90°$

(ii) its maximum kinetic energy during the motion.

(iii) will it perform SHM? oscillation? Periodic motion? What is its amplitude?

SOLUTION

(i) Apply energy conservation at $\theta = 120°$ and $\theta = 90°$

$$-MB\cos 120° + 0$$
$$= -MB\cos 90° + (\text{KE})$$
$$\text{KE} = \frac{MB}{2} \qquad \textbf{Ans.}$$

(ii) KE will be maximum where PE is minimum. PE is minimum at $\theta = 0°$. Now apply energy conservation between $\theta = 120°$ and $\theta = 0°$.

$$-mB\cos 120° + 0$$
$$= -mB\cos 0° + (\text{KE})_{\max}$$
$$(\text{KE})_{\max} = \frac{3}{2}MB \qquad \textbf{Ans.}$$

The KE is max at $\theta = 0°$ can also be proved by torque method. From $\theta = 120°$ to $\theta = 0°$ the torque always acts on the dipole in the same direction (here it is clockwise) so its KE keeps on increases till $\theta = 0°$. Beyond that τ reverses its direction and then KE starts decreasing.

$\therefore$ $\theta = 0°$ is the orientation of M to here the maximum KE.

(iii) Since 'θ' is not small.

$\therefore$ the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120°. ■

EXAMPLE 43

A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25 μT.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

SOLUTION

(a) The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2)$$
$$= \frac{100 \times 10^{-3}}{12}[(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]\ \text{kg} - \text{m}^2$$
$$= \frac{25}{6} \times 10^{-5}\ \text{kg} - \text{m}^2$$

We have, $T = 2\pi\sqrt{\frac{I}{MB}}$

or, $$M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5}\ \text{kg/m}^2}{6 \times (25 \times 10^{-6}T) \times \frac{\pi^2}{4}s^2}$$
$$= 27\ \text{A-m}^2$$

(b) In this case the moment of inertia becomes

$$I' = \frac{m'}{12}(L^2 + b^2) \quad \text{where } b' = 0.5\ \text{cm}.$$

The time period would be

$$T' = \sqrt{\frac{I'}{MB}} \qquad (2)$$

Dividing by equation (1),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2 + b'^2)}}{\sqrt{\frac{m'}{12}(L^2 + b^2)}}$$
$$= \frac{\sqrt{(7\ \text{cm})^2 + (0.5\ \text{cm})^2}}{\sqrt{(7\ \text{cm})^2 + (1.0\ \text{cm})^2}} = 0.992$$

or, $$T' = \frac{0.992 \times \pi}{2}s = 0.496\,\pi s.$$ ■

Magnet in an External Nonuniform Magnetic Field

No special formula are applied is such problems. Instead see the force on individual poles and calculate the resultant force torque on the dipole.

Notes

❖ Force due to Non-uniform Magnetic field

$$F = -M\frac{dB}{dr}$$

❖ If a source of Magnetic Moment have $\vec{M}$ dimension very less than the distance of point of application then we can replace it with magnet of magnetic moment equal to $\vec{M}$.

TERRESTRIAL MAGNETISM

Earth is a natural source of magnetic field.

Elements of the Earth's Magnetic Field

The earth's magnetic field at a point on its surface is usually characterized by three quantities: (a) declination (b) inclination or dip and (c) horizontal component of the field. These are known as the elements of the earth's magnetic field.

(a) Declination

A plane passing through the geographical poles (that is, through the axis of rotation of the earth) and a given point P on the earth's surface is called the geographical meridian at the point P. Similarly, the plane passing through the geomagnetic poles (that is, through the dipole-axis of the earth) and the point P is called the magnetic meridian at the point P.

The angle made by the magnetic meridian at a point with the geographical meridian is called the declination at that point.

(b) Inclination or dip

The angle made by the earth's magnetic field with the horizontal direction in the magnetic meridian, is called the inclination or dip at that point.

(c) Horizontal component of the earth's magnetic field

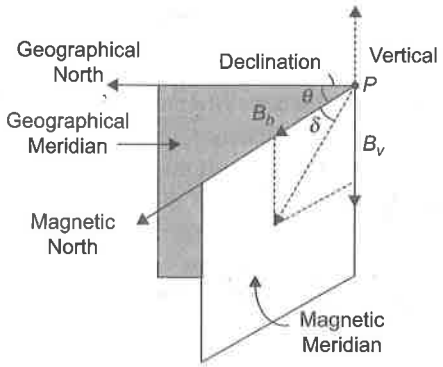
As the name indicates, the horizontal component is component of the earth's magnetic field in the horizontal direction in the magnetic meridian. This direction is towards the magnetic north.

Figure shows the three elements. Starting from the geographical meridian we draw the magnetic meridian at an angle θ(declination). In the magnetic meridian we draw the horizontal direction specifying magnetic north. The magnetic field is at an angle δ (dip) from this direction. The horizontal component B_H and the total field B are related as

$$B_H = B\cos\delta$$

or,
$$B = B_H/\cos\delta.$$

Thus, from the knowledge of the three elements, both the magnitude and direction of the earth's magnetic field can be obtained.



SOLVED EXAMPLES

EXAMPLE 45

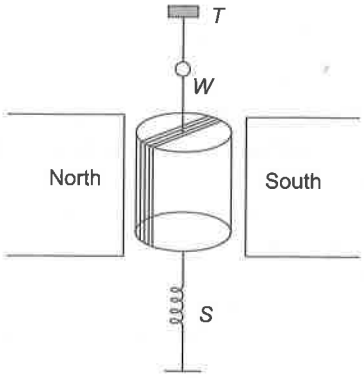
The horizontal component of the earth's magnetic field is 3.6×10^{-5}T where the dip is 60°. Find the magnitude of the earth's magnetic field.

SOLUTION

We have $B_H = B\cos\delta$

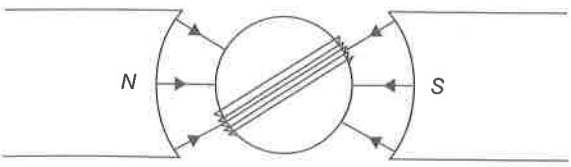
or,
$$B = \frac{B_H}{\cos\delta} = \frac{3.6\times10^{-5}\text{T}}{\cos 60^\circ} = 7.2\times10^{-5}\text{T}$$

MOVING-COIL GALVANOMETER



The main parts of a moving-coil galvanometer are shown in figure. A rectangular coil of several turns is wound over a soft-iron core. The wire of the coil is coated with an insulating material so that each turn is insulated from the other and from the iron core. The coil is suspended between the two pole pieces of a strong permanent magnet.

The current to be measured is passed through the galvanometer. As the coil is in the magnetic field $\vec{B}$ of the permanent magnet, a torque $\vec{\Gamma} = ni\vec{A}\times\vec{B}$ acts on the coil. Here n = number of turns, i = current in the coil, $\vec{A}$ = area-vector of the coil and $\vec{B}$ = magnetic field at the site of the coil. This torque deflects the coil from its equilibrium position.



The pole pieces are made cylindrical. As a result, the magnetic field at the arms of the coil remains parallel to the plane of the coil everywhere even as the coil rotates. The deflecting torque is then $\Gamma = niAB$. As the upper end of the suspension strip W is fixed, the strip gets twisted when the coil rotates. This produces a restoring torque acting on the coil. If the deflection of the coil is θ and the torsional constant of the suspension strip is k, the restoring torque is $k\theta$. The coil will stay at a deflection θ where

$$niAB = k\theta \quad \text{or,} \quad i = \frac{k}{nAB}\theta$$

Hence, the current is proportional to the deflection. The constant $\frac{k}{nAB}$ is called the galvanometer constant, and may be found by passing a known current, measuring the deflection θ and putting these values in equation.

Sensitivity

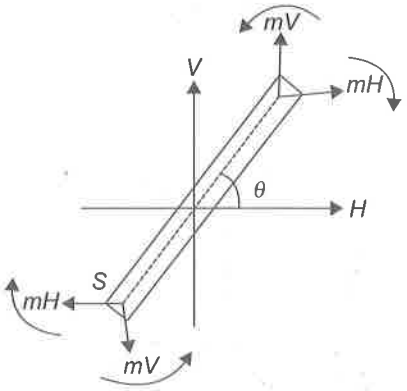
The sensitivity of a moving-coil galvanometer is defined as θ/i. From equation, the sensitivity is $\frac{nAB}{k}$. For large sensitivity, the field B should be large. The presence of soft-core increases the magnetic field.

EXTRA PORTION OF JEE MAIN

TANGENT LAW

When a magnet (e.g. a compass needle) is placed under the combination of two perpendicular magnetic fields of intensities V and H. Let the magnet make an angle θ with H at equilibrium position then

$$V = H\tan\theta.$$



SOLVED EXAMPLES

Tangent Law

EXAMPLE 46

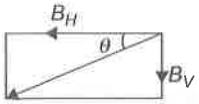
At any place on earth, the horizontal component of earth's magnetic field is $\sqrt{3}$ times the vertical component. The angle of dip at that place will be

(A) 60° (B) 45°
(C) 90° (D) 30°

SOLUTION

$$\tan\theta = \frac{B_v}{B_H} = \frac{B_v}{\sqrt{3}B_v}$$

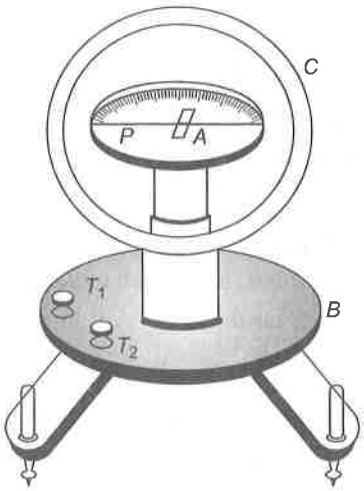
$$= \frac{1}{\sqrt{3}} = \tan 30° \quad \therefore \quad \theta = 30° \qquad (4)$$



TANGENT GALVANOMETER (MOVING MAGNET TYPE GALVANOMETER)

This is based on tangent law. It is used for detention and measurement of low electric currents.

If galvanometer has coil in which, n is the number of turns, r is radius of solid and μ_0 magnetic permittivity of free space.

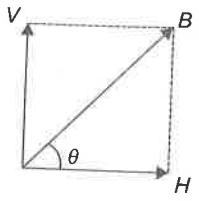


190

Then magnetic field intensity due to coil

$$B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$$

This field will be perpendicular to the horizontal component of earth's magnetic field so $B = H\tan\theta$



$$\frac{\mu_0}{4\pi}\frac{2n\pi I}{r} = H\tan\theta$$

$$I = K\tan\theta$$

$K = \frac{2rH}{\mu_0 n}$ is a constant for a galvanometer. It is called reduction factor of galvanometer.

SOLVED EXAMPLES

Tangent Galvanometer

EXAMPLE 47

The radius of the coil of a tangent galvanometer is 16 cm. How many turns of the wire should be used if a current of 40 mA is to produced a deflection of 45°. given, horizontal component of earth's field is 0.36×10^{-4}T.

(A) 458 (B) 229
(C) 200 (D) 115

SOLUTION

Here, $r = 16$ cm $= 0.16$ m, n?
$I = 40$ mA $= 40 \times 10^{-3}$ A,
$\theta = 45°$ $H = 0.36 \times 10^{-4}$ T

As $\frac{\mu_0 nI}{2r} = H\tan\theta$

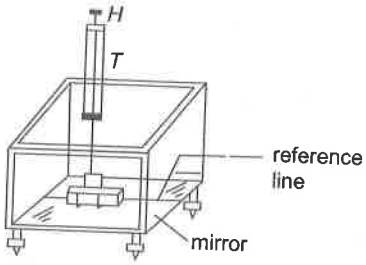
$\therefore$ $n = \frac{2rH\tan\theta}{\mu_0 I}$

$$= \frac{2 \times 0.16 \times 0.36 \times 10^{-4}\tan 45°}{4\pi \times 10^{-7} \times (40 \times 10^{-3})},$$

$n = 229$

OSCILLATION MAGNETOMETER

When a magnet is suspended freely in a uniform magnetic field, using an unspun thread, it comes to rest in a direction NS parallel to the direction of the field. If this magnet is slightly deflected from its equilibrium position and left, magnet begins to vibrate simple harmonically about the direction of field and its mean position.



The time period of T of vibration is given by

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

where, I = moment of inertia of magnetic about the suspension fibre as the axis of rotation,
M = magnetic moment of the magnet,
H = intensity of uniform magnetic field

Uses:

(A) To compare of horizontal component of earth's magnetic field at two places, horizontal component of earth's magnetic field be H_1 and H_2 and time period of oscillation T_1 and T_2 respectively.

Then $\frac{H_2}{H_1} = \frac{T_1^2}{T_2^2}$

(B) To compare magnetic moments of two magnets of same size and same mass

$$\frac{M_2}{M_1} = \frac{T_1^2}{T_2^2}$$

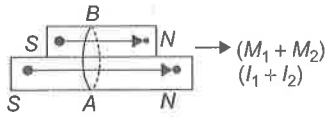
(C) To determine the magnetic moment of a magnet

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

If mass, length and breadth of magnet are m, ℓ and b respectively then $I = \frac{m(\ell^2 + b^2)}{12}$. If H is known, M can be calculated.

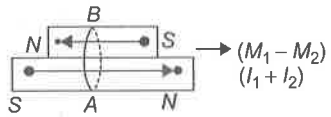
(D) To compare the magnetic moments of two magnets of unequal size and masses. When north pole of two magnets are in same direction, Then

Time period $T_1 = 2\pi\sqrt{\dfrac{I_1 + I_2}{(M_1 + M_2)H}}$ (1)



and when they are in opposite direction.

Time period $T_1 = 2\pi\sqrt{\dfrac{I_1 + I_2}{(M_1 + M_2)H}}$ (2)



By equation (1) and (2) $\dfrac{M_1}{M_2} = \dfrac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$

SOLVED EXAMPLES

Oscillation Magnetometer

EXAMPLE 48

The period of oscillation of a freely suspended bar magnet is 4 second. If it is cut into two equal parts length wise then the time period of each part will be

(A) 4 sec (B) 2 sec
(C) 0.5 sec (D) 0.25 sec

SOLUTION

$$T = 2\pi\sqrt{\frac{I}{MB}} = 2\pi\sqrt{\frac{m\ell^2}{12 \times m_p \ell B}} = 4 \text{ sec}$$

$$T' = 2\pi\sqrt{\frac{\frac{m}{2}\ell^2}{12 \times \frac{m_p}{2}\ell B}} = 4 \text{ sec}$$

■

EXAMPLE 49

The length, breadth and mass of two bar magnets are same but their magnetic moments are 3 M and 2 M respectively. These are joined pole to pole and are suspended by a string. When oscillated in a magnetic field of strength B, the time period obtained is 5 s. If the poles of either of the magnets are reverse then the time period of the combination in the same magnetic field will be

(A) $3\sqrt{3}$ s (B) $2\sqrt{2}$ s
(C) $5\sqrt{5}$ s (D) 1s

SOLUTION

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad \text{or} \quad T \propto \frac{1}{\sqrt{M}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{3M - 2M}{3M + 2M}} \quad \text{or} \quad T_2 = 5\sqrt{5} \text{ s}$$

■

EXAMPLE 50

A thin magnetic needle oscillates in a horizontal plane with a period T. It is broken into n equals parts. The time period of each part will be:

(A) T (B) $\dfrac{T}{n^2}$
(C) Tn^2 (D) $\dfrac{T}{n}$

SOLUTION

$$T = 2\pi\sqrt{\frac{I}{MB}} = 2\pi\sqrt{\frac{m\ell^2/12}{m_p \ell B}} \quad \text{or} \quad T \propto m\ell$$

$$\frac{T'}{T} = \left(\frac{\frac{m}{n}\frac{\ell}{n}}{m\ell}\right)^{1/2} \quad \text{or} \quad T' = \frac{T}{n}$$

■

SOME TERMS RELATED TO MAGNETISM

(A) Magnetic permeability: It is the ability of material to permit the passage of magnetic lines through it. If it denoted by μ. Its unit is weber/ammeter.

Relative magnetic permeability (μ_r), is a factor by which the magnetic field B is increased when a material is brought in the field. μ_r is unit less.

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$$

where μ_0 is permeability of vacuum and χ is susceptibly of material.

(B) Intensity of magnetization (I): It is defined as magnetic moment per unit volume.

192

$$\vec{I} = \frac{\text{magnetic moment}}{\text{volume}} = \frac{\vec{m}}{V}$$

$$I = \frac{m}{V} = \frac{2m\ell}{A(2\ell)} = \frac{m}{A}$$

For bar magnet of pole strength m, length x and area of cross-section A.

(C) Magnetic susceptibility: For paramagnetic and diamagnetic substances, the intensity of magnetization $(\vec{I})$ of a material is directly proportional to magnetic in intensity $(\vec{H})$

Thus $\vec{I} = \chi \vec{H}$

The proportionality content χ is called susceptibility. It is a dimensionless quantity.
For vacuum $\chi = 0$.

Notes

$\mu_r = 1 + \chi$ (in M.K.S. system)
$\mu_r = 1 + 4\pi\chi$ (in C.G.S. system)

(D) Magnetic Intensity: The actual magnetic field inside the material is the sum of the applied magnetic field and magnetic field due to magnetization.

Hence $$\vec{H} = \frac{\vec{B}}{\mu_0} - I = \frac{\vec{B}}{\mu}$$

where $\vec{B}$ is the resultant magnetic field, and μ permeability of the material and $\vec{I}$ intensity of magnetization. The quantity $\vec{H}$ is called magnetic field intensity or magnetizing field intensity. Unit of it is ampere/meter.

SOLVED EXAMPLES

Magnetic Susceptibility

EXAMPLE 51

The magnetic susceptibility of a paramagnetic substance is 3×10^{-4}. It is placed in a magnetizing field of 4×10^4 amp/m. The intensity of magnetization will be

(A) 3×10^8 A/m (B) 12×10^8 A/m
(C) 12 A/m (D) 24 A/m

SOLUTION

$$I = XH = 3 \times 10^{-4} \times 4 \times 10^3 = 12 \text{ A/m}$$

Magnetic Intensity

EXAMPLE 52

The horizontal component of flux density of earth's magnetic field is 1.7×10^{-5} tesla. The value of horizontal component of intensity of earth's magnetic field will be?

(A) 24.5 A/m (B) 13.5 A/m
(C) 0.135 A/m (D) 1.35 A/m

SOLUTION

$$H = \frac{B}{\mu_0} = \frac{1.7 \times 10^{-5} \text{ Wb/m}^2}{4\pi \times 10^{-7} \text{ Wb/A-m}} = 13.5 \text{ A/m}$$

Hence the correct answer will be (B).

Magnetic Permeability

EXAMPLE 53

A magnetizing field of 2×10^3 A/m produces a magnetic flux density of 8π Tesla in an iron rod. The relative permeability of the rod will be

(A) 10^2 (B) 10^0
(C) 10^4 (D) 10^1

SOLUTION

$$\because \quad \mu_r = \frac{\mu}{\mu_0} = \frac{B}{H\mu_0}$$

or $$\mu_r = \frac{8\pi}{2 \times 10^3 \times 4\pi \times 10^{-7}} = 10^4$$

Magnetization

EXAMPLE 54

A bar magnet made of steel has a magnetic moment of 2.5 A-m^2 and a mass of 6.6×10^3 kg. If the density of steel is 7.9×10^9 kg/m^3, find the intensity of magnetization of the magnet.

SOLUTION

The volume of the bar magnet is

$$V = \frac{\text{mass}}{\text{density}} = \frac{6.6 \times 10^{-3}\ \text{kg}}{7.9 \times 10^{3}\ \text{kg/m}^3}$$

$$= 8.3 \times 10^{-7}\ \text{m}^3.$$

The intensity of magnetization is

$$I = \frac{M}{V} = \frac{2.5\text{A/m}^2}{8.3 \times 10^{-7}\ \text{m}^2} = 3.0 \times 10^{6}\ \text{A/m}$$ ■

MAGNETIC BEHAVIOUR OF ATOM

If an electron having charge e is revolving in an orbit of radius r having uniform angular velocity ω. Then

Magnetic moment of atom contributed by the electron $M = \frac{1}{2}e\omega r^2$ and magnetic field

$$B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{A/m}^2$$

If source as the unit of atomic magnetic dipole moment. Where h is plank's constant.

MOLECULAR/ATOMIC THEORY OF MAGNETIZATION

According to This Theory

(i) Every molecule/atom of a magnetic substance has north pole as well as south pole of equal strength, so it is a magnetic dipole.

(ii) In unmagnetized condition, atomic dipoles are randomly oriented and they cancel dipole moment of one another.

(iii) After applying magnetic field, atomic dipoles align in its direction.

(iv) Extent of magnetization depends on extent of realignment of the molecular dipoles.

(v) On heating magnetization reduces or vanish completely. On the basis of this theory magnetic materials are classified into three categories

(a) **Paramagnetic substances:** Every molecules/atoms of these substances has its own dipole moment. When we place such substances in magnetic field tries to align every individual molecule/atom in its direction e.g. chromium, CuSO4, crown glass, aluminium, platinum, manganese, solution of salts of iron and nickel are paramagnetic substances.

Permeability of paramagnetic substance is greater than unity. Susceptibility of paramagnetic substances varies inversely to the temperature of substance.

(b) **Diamagnetic substances:** Molecules/atoms of this type of substances do not have dipole moment individually. When they are kept in any magnetic field, magnetic moment induces but it is in opposite direction to the applied magnetic field. e.g. copper, gold, mercury, quart, antimony and Bismath etc.

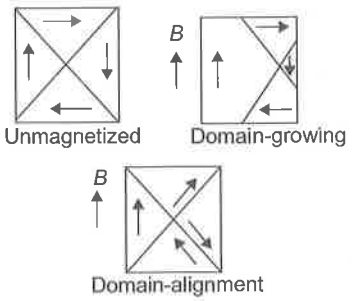
Permeability of diamagnetic substances is always less than unity.

Susceptibility of diamagnetic substances does not change with temperature.

(c) **Ferromagnetic substances:** In these substance each individual atom/molecule has magnetic moment. They have strong tendency to align themselves so they make domain. In a domain, a large no. of atoms/molecules align in same direction.

Different domains have different directions of magnetic moment hence, the material remains un magnetized.

When a magnetic field is applied, the domains which are aligned along the direction of the field grown in size and those opposite to it get reduced. iron nickel and cobalt etc. are examples of such materials.

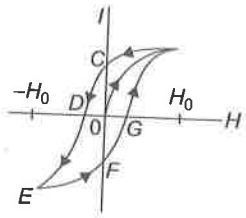


HYSTERESIS

As shown in adjacent curve, as we change field intensity, magnetization changes.

In the beginning, the field is zero and sample has no magnetization, this is corresponding to point O.

194



As we increase field intensity, magnetization point A corresponds to saturation magnetization. As value of and direction of field intensity varies, magnetization along path ACDEFGA is followed.

Hence as field intensity increased or decreased, magnetization does not return to its initial value. This fact is called hysteresis. The curve ACDEFGA is called hysteresis loop. This are of the hysteresis loop is proportional to the thermal energy developed per unit volume of the material as it goes through the hysteresis cycle.

Hysteresis loop for soft iron is narrow and large, whereas hysteresis loop for steel is wide and short.

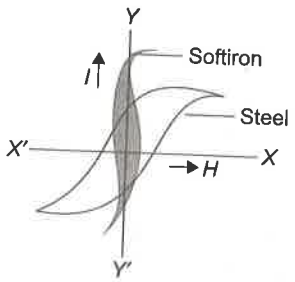
SOLVED EXAMPLES

Hysteresis

EXAMPLE 55

The mass of a specimen of a ferromagnetic material is 0.6 kg. and its density is 7.8×10^3 kg/m^3. If the area of hysteresis loop of alternating magnetizing field of frequency 50 Hz is 0.722 MKS units then the hysteresis loss per second will be

(A) 277.7×10^{-5} J
(B) 277.7×10^{-6} J
(C) 277.7×10^{-4} J
(D) 277.7×10^{-4} J



SOLUTION

$$W_H = \text{VAft}$$

$$= \frac{m}{d}\text{aft} \quad \text{or} \quad W_H = \frac{0.6}{7.8 \times 10^3} \times 0.722 \times 50$$

$$= 277.7 \times 10^{-5}\ \text{J}$$

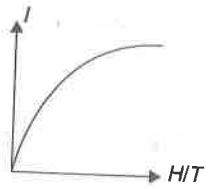
CURIE LAW

According to this law, intensity of magnetization (I) of a magnetic material is directly proportional to magnetic induction (B) and inversely proportional to the temperature of magnetic material.

$$I\alpha \frac{B}{T}$$

we know $B\,\alpha H$

So $$\frac{I}{H}\alpha \frac{1}{T} \quad \frac{I}{H} = \chi = \frac{C}{T}$$



where χ is susceptibility of material and C is a constant called curie constant.

The variation of I with graph it is clear that saturation is reached when H/T exceeds a certain limit. Magnetic thermometers are based on curie law and filled with paramagnetism substance (e).

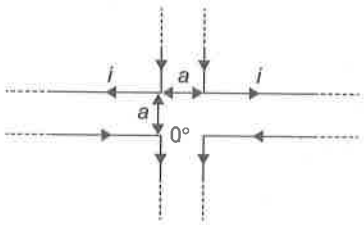
EXERCISES

JEE Main

Section A: Magnetic Field Due to a Straight Wire

1. A point charge is moving in a circle with constant speed. Consider the magnetic field produced by the charge at a fixed point P (not centre of the circle) on the axis of the circle.
   (A) it is constant in magnitude only
   (B) it is constant in direction only
   (C) it is constant in direction and magnitude both
   (D) it is not constant in magnitude and direction both.

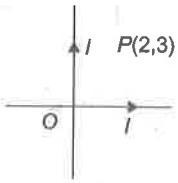
2. Four infinitely long 'L' shaped wires, each carrying a current i have been arranged as shown in the figure. Obtain the magnetic field intensity at the point 'O' equidistant from all the four corners.



   (A) 1 Wb/m²
   (B) 0 Wb/m²
   (C) 2 Wb/m²
   (D) none of these

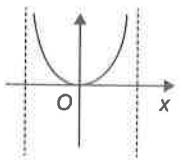
3. Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the current are in the same direction, the magnetic field at a point midway between the wire is 10 mT. If the direction of i_2 is reversed, the field becomes 30 mT. The ratio i_1/i_2 is
   (A) 4 (B) 3
   (C) 2 (D) 1

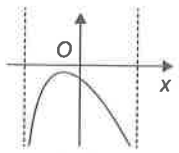
4. Two mutually perpendicular insulated long conducting wires carrying equal currents I, intersect at origin. Then the resultant magnetic induction at point P (2 m, 3 m) will be

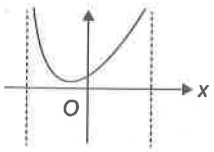


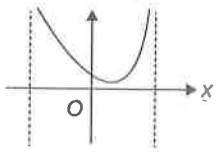
   (A) $\frac{\mu_0 I}{5a}$ (B) $\frac{5\mu_0 I}{2\pi}$
   (C) $\frac{\mu_0 I}{I2\pi}$ (D) 0

5. Two parallel straight long conducting wires, which are placed at a distance r from each other, are carrying equal currents i in opposite directions. The value of magnetic induction at a point situated at a point situated x from one wire in between the wires will be:
   (A) $\frac{\mu_0 i}{2\pi}\left\{\frac{1}{r-x}-\frac{1}{x}\right\}$ (B) $\frac{\mu_0 i}{2\pi}\left\{\frac{1}{r-x}+\frac{1}{x}\right\}$
   (C) $\frac{\mu_0 i}{2\pi(r-x)}$ (D) $\frac{\mu_0 i}{2\pi x}$

6. Two very long straight parallel wires, parallel to y-axis, carry currents $4I$ and I, along $+y$ direction and $-y$ direction respectively. The wires are passes through the x-axis at the points $(d, 0, 0)$ and $(-d, 0, 0)$ respectively. The graph of magnetic field z-component as one moves along the x-axis from $x = -d$ to $x = +d$, is best given by
   (A)


   (B)


   (C)


   (D)


7. If the ratio of magnetic fields at two point in a definite direction due to a long current carrying wire is $\frac{3}{4}$, then the ratio of the distances of these points from the wire will be:
   (A) $\frac{2}{\sqrt{3}}$ (B) $\frac{4}{3}$
   (C) $\sqrt{\frac{3}{4}}$ (D) $\sqrt{\frac{3}{2}}$

196

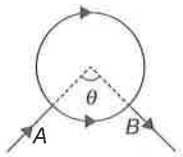
8. The points A and B are situated perpendicular to the axis of 2 cm long bar magnet at large distances x and $3x$ from the centre on opposite sides. The ratio of magnetic fields at A and B will be approximately equal to
(A) 27:1 (B) 1:27
(C) 9:1 (D) 1:9

9. Magnetic lines of force are
(A) continuous
(B) discontinuous
(C) sometimes continuous and sometimes discontinuous
(D) nothing can be said

10. Magnetic monopoles cannot exist
(A) true
(B) false
(C) may be true or false
(D) nothing can be said

Section B: Magnetic Field Due to a Circular Loop

11. A battery is connected between two points A and B the circumference of a uniform conducting ring of radius r and resistance R. One o the arcs AB of the ring subtends an angle q at the centre. The value of the magnetic induction at the centre due to the current in the ring is:



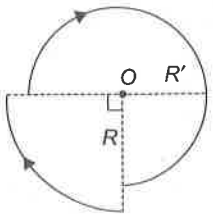
(A) zero, only if $q = 180°$
(B) zero for all values of q
(C) proportional to $2(180° - q)$
(D) inversely proportional to r

12. Two circular coils of wire each having a radius of 4 cm and 10 turns have a common axis and are 6 cm apart. If a current of 1 A passes through each coil in the opposite direction find the magnetic induction.
At a point on the axis, midway between them.
(A) 13×10^{-5} T (B) zero
(C) 15×10^{-5} T (D) none of these

13. A direct current is passing through a wire. It is bent to form a coil of one turn. Now it is further bent to form a coil of two turns but at smaller radius. The ratio of the magnetic induction at the centre of this coil and at the centre of the coil of two turn is
(A) 1:4 (B) 4:1
(C) 2:1 (D) 1:1

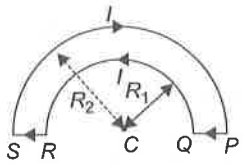
Section C: Magnetic Field Due to a Straight Wire and Circular Arc

14. A current of i ampere is flowing through each of the bent wires as shown the magnitude and direction of magnetic field at O is



(A) $\frac{\mu_0 i}{4}\left(\frac{1}{R} + \frac{2}{R'}\right)$
(B) $\frac{\mu_0 i}{4}\left(\frac{1}{R} + \frac{3}{R'}\right)$
(C) $\frac{\mu_0 i}{8}\left(\frac{1}{R} + \frac{3}{2R'}\right)$
(D) $\frac{\mu_0 i}{8}\left(\frac{1}{R} + \frac{3}{R'}\right)$

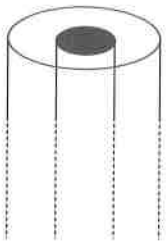
15. Two wire loops $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown in (fig.) The magnitude of the magnetic induction at the centre C is …



(A) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (B) $\frac{\mu_0 I}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
(C) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ (D) none of these

Section D: Magnetic Field Due to a Cylinder, Large Sheet, Solenoid, Toroid and Ampere's Law

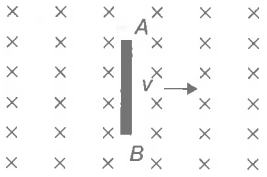
16. In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero.



(A) outside the cable
(B) inside the inner conductor
(C) inside the outer conductor
(D) in between the two conductors.

17. The magnetic field B inside a long solenoid, carrying a current of 10 A, is 3.14×10^{-2} T. Find the number of turns per unit length of the solenoid.
(A) 1500 turns/m. (B) 3500 turns/m.
(C) 2000 turns/m. (D) 2500 turns/m.

18. A rod AB moves with a uniform velocity v in a uniform magnetic field as shown in figure.

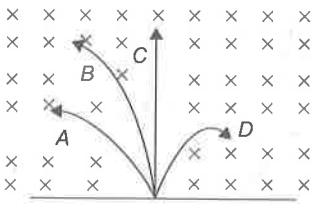


(A) The rod becomes electrically charged.
(B) The end A becomes positively charged.
(C) The end B becomes positively charged.
(D) The rod becomes hot because of Joule heating.

Section E: Magnetic Force on a Charge

19. A charged particle is accelerated through a potential difference of 12 kV and acquires a speed of 1.0×10^6 m/s. It is then injected perpendicularly into a magnetic field of strength 0.2 T. Find the radius of the circle described by it.
(A) 12 cm (B) 14 cm
(C) 10 cm (D) none of these

20. A neutron, a proton, an electron and an a-particle enters a uniform magnetic field with equal velocities. The field is directed along the inward normal to the plane of the paper. Which of these tracks followed are by a-particle.



(A) A (B) B
(C) C (D) D

21. A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C is projected with a speed of 2×10^6 m/s at an angle of 60° to the x-axis. If a uniform magnetic field of 0.104 T is applied along the y-axis, the path of the proton is:
(A) A circle of radius 0.2 m and time period $\pi \times 10^{-7}$s
(B) A circle of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(C) A helix of radius 0.1 m and time period of $2\pi \times 10^{-7}$ s
(D) A helix of radius of 0.2 m and time period $4\pi \times 10^{-7}$ s

22. Three ions H^+, He^+ and O^{+2} having same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity, then:
(A) H^+ will be least deflected
(B) He^+ and O^{+2} will be deflected equally.
(C) O^{+2} will be deflected most
(D) all will be deflected equally.

23. A charged particle moves in a magnetic field $\vec{B} = 10\hat{i}$ with initial velocity $\vec{u} = 5\hat{i} + 4\hat{j}$. The path of the particle will be
(A) straight line (B) circle
(C) helical (D) none

24. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths.
(A) of same radius
(B) with larger radii for the faster electrons.
(C) with smaller radii for the faster electrons.
(D) either (B) or (C) depending on the magnitude of the magnetic field

25. In the previous question, time periods of rotation will be:
(A) same for all electrons.
(B) greater for the faster electrons
(C) smaller for the faster electrons.
(D) either (B) or (C) depending on the magnitude of the magnetic field

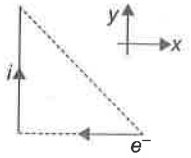
26. A particle having charge of 1 C, mass 1 kg and speed 1 m/s enters a uniform magnetic field, having magnetic induction of 1 T, at an angle $q = 30°$ between velocity vector and magnetic induction. The pitch of its helical path is (in meters)

198

(A) $\frac{\sqrt{3}\pi}{2}$ (B) $\sqrt{3}\pi$

(C) $\frac{\pi}{2}$ (D) p

27. The direction of magnetic force on the electron as shown in the diagram is along



(A) y-axis (B) $-y$-axis

(C) z-axis (D) $-z$-axis

Section F: Electric and Magnetic Force on a Charge

28. A particle moves in a region having a uniform magnetic field and a parallel, uniform electric field. At some instant, the velocity of the particle is perpendicular to the field direction. The path of the particle will be

(A) a straight line

(B) a circle

(C) a helix with uniform pitch

(D) a helix with nonuniform pitch

29. An electron is moving along positive x-axis. A uniform electric field exists towards negative y-axis. What should be the direction of magnetic field of suitable magnitude so that net force of electron is zero.

(A) positive z-axis (B) negative z-axis

(C) positive y-axis (D) negative y-axis

30. A charged particle is released from rest in a region of uniform electric and magnetic fields, which are parallel to each other. The locus of the particle will be

(A) helix of constant pitch

(B) straight line

(C) helix of varying pitch

(D) cycloid

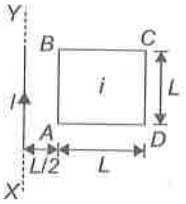
Section G: Magnetic Force on a Current Carrying Wire

31. A straight current carrying conductor is placed in such a way that the current in the conductor flows in the direction out of the plane of the paper. The conductor is placed between two poles of two magnets, as shown. The conductor will experience a force in the direction towards

(A) P (B) Q

(C) R (D) S

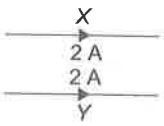
32. A square loop $ABCD$, carrying a current i, is placed near and coplanar with a long straight conductor XY carrying a current I, the net force on the loop will be



(A) $\frac{2\mu_0 Ii}{3\pi}$ (B) $\frac{\mu_0 Ii}{2\pi}$

(C) $\frac{2\mu_0 IiL}{3\pi}$ (D) $\frac{\mu_0 IiL}{2\pi}$

33. In given figure, X and Y are two long straight parallel conductors each carrying a current of 2 A. The force on each conductor is F newtons. When the current in each is changed to 1 A and reversed in direction, the force on each is now



(A) $\frac{F}{4}$ and unchanged in direction

(B) $\frac{F}{2}$ and reversed in direction

(C) $\frac{F}{2}$ and unchanged in direction

(D) $\frac{F}{4}$ and reversed in direction

Section H: Magnetic Force and Torque on a Current Carrying Loop and Magnetic Dipole Moment

34. The magnetic moment of a circular orbit of radius 'r' carrying a charge 'q' and rotating with velocity v is given by

(A) $\frac{qvr}{2\pi}$ (B) $\frac{qvr}{2}$

(C) $qv\pi r$ (D) $qv\pi r^2$

35. A current carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend upon

(A) area of loop (B) shape of loop
(C) value of current (D) magnetic field

Section I: Magnetic Field Due to a Magnet and Earth

36. The time period of vibration of two magnets in same position is 3 sec. When polarity of weaker magnet is reversed, the combination makes 12 oscillations per minute. Compare the magnetic moments of two magnets.

(A) 4 (B) $\frac{17}{8}$

(C) $\frac{13}{8}$ (D) $\frac{1}{4}$

37. On placing a piece of ferromagnetic material of cross-sectional area 1cm^2 in magnetic field of 200 oersted, flux density of 3000 Gauss is produced in it. The values of relative permeability and magnetic susceptibility of the material will respectively be
(A) 11.9 and 150 (B) 150 and 11.9
(C) 50 and 11.9 (D) 15 and 14

38. Every atom of every substance is a complete magnet in itself.
(A) true
(B) false
(C) may be true or false
(D) nothing can be said

39. The most suitable metal for permanent magnets is
(A) steel (B) iron
(C) copper (D) aluminium

40. The permanent magnetic moment of atoms or molecules of a diamagnetic material is
(A) small (B) large
(C) negative (D) zero.

41. The relation between magnetic susceptibility x_m and relative permeability μ_r, is
(A) $x_m = \mu_r$ (B) $x_m - 1 = \mu_r$
(C) $\mu_r = 1 + x_m$ (D) $\mu_r = 1 - x_m$

42. When a ferromagnetic material is heated above its Curie temperature,
(A) it gets demagnetized
(B) it becomes diamagnetic
(C) behaves like a paramagnetic substance
(D) remains unaffected

43. As magnetizing field on a ferromagnetic material is increased, its permeability
(A) increases (B) decreases
(C) remains constant (D) cannot say.

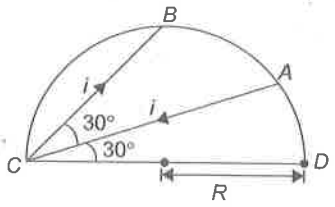
44. The vertical component of earth's magnetic field is zero at
(A) magnetic equator (B) magnetic pole
(C) geographic poles (D) at 90° latitude

JEE Main

Single Correct Option Type Questions

Section A: Magnetic Field Due to a Straight Wire

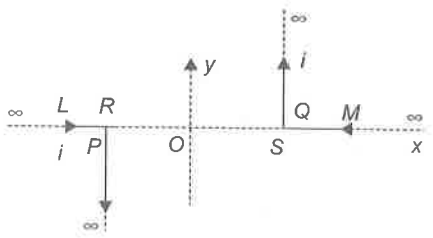
1. A current carrying wire is placed in the grooves of an insulating semi circular disc of radius 'R', as shown. The current enters at point A and leaves from point B. Determine the magnetic field at point D.



(A) $\frac{\mu_0 I}{8\pi R\sqrt{3}}$ (B) $\frac{\mu_0 I}{4\pi R\sqrt{3}}$

(C) $\frac{\sqrt{3}\mu_0 I}{4\pi R}$ (D) none of these

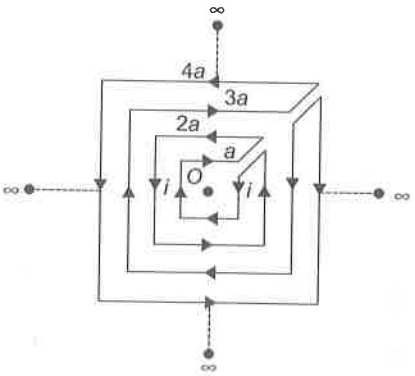
2. A pair of stationary and infinitely long bent wires is placed in the X-Y plane as shown in figure. The wires carry currents of 10 A each as shown. The segments L and M are along the x-axis. The segments P and Q are parallel to the y-axis such that $OS = OR = 0.02$ m. Find the magnetic induction at the origin O.



(A) 1×10^{-4} Wb/m^2, towards the reader.
(B) 1×10^{-4} Wb/m^2, opposite to the reader.
(C) 2×10^{-4} Wb/m^2, towards the reader.
(D) zero

200

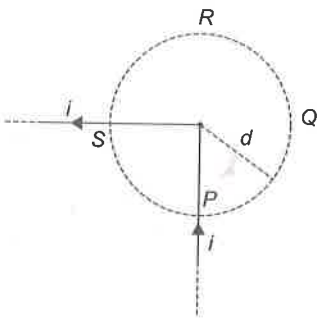
3. Determine the magnitude of magnetic field at the centre of the current carrying wire arrangement shown in the figure. The arrangement extends to infinity. (The wires joining the successive square are along the line passing through the centre)



(A) $\frac{\mu_0 i}{\sqrt{2}\pi a}$ (B) 0

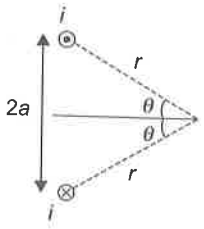
(C) $\frac{2\sqrt{2}\mu_0 i}{\pi a}\ln 2$ (D) none of these

4. Figures shows a long wire bent at the middle to form a right angle. The magnitudes of the magnetic fields at the points P, Q, R and S are B_1, B_2, B_3, B_4 respectively. The wire and the circumference of circle are coplanar. T



(A) $B_1 = B_2 = B_3 = B_4$
(B) $B_1 = 2B_2$
(C) $B_3 = 2B_4$
(D) none of these

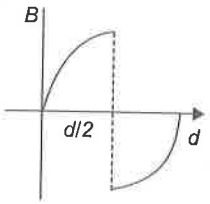
5. Two long parallel wires situated at a distance $2a$ are carrying equal current 'i' in opposite direction as shown in figure. The value of magnetic field at a point P situated at equal distances from both the wires will be:

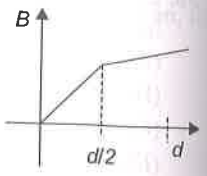


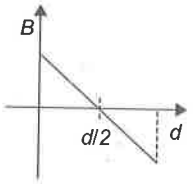
(A) $\frac{\mu_0 ia}{\pi r}$ (B) $\frac{\mu_0 ia^2}{\pi r}$

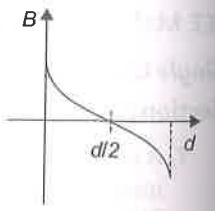
(C) $\frac{\mu_0 ia^2}{\pi r^2}$ (D) $\frac{\mu_0 ia}{\pi r^2}$

6. A uniform beam of positively charged particles is moving with a constant velocity parallel to another beam of negatively charged particles moving with the same velocity in opposite direction separated by a distance d. The variation of magnetic field B along a perpendicular line draw between the two beams is best represented by

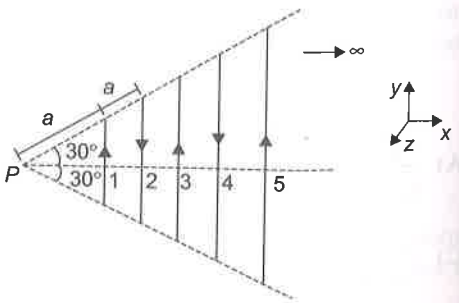
(A)


(B)


(C)


(D)


7. Infinite number of straight wires each carrying current I are equally placed as shown in the figure. Adjacent wires have current in opposite direction. Net magnetic field at point P is



(A) $\frac{\mu_0 I}{4\pi}\frac{\ln 2}{\sqrt{3}a}\hat{k}$ (B) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}\hat{k}$

(C) $\frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}(-\hat{k})$ (D) Zero

8. Two mutually perpendicular conductors carrying currents I_1 and I_2 lie in one plane. Locus of the point at which the magnetic induction is zero, is a

(A) circle with centre as the point of intersection of the conductor.

(B) parabola with vertex as the point of intersection of the conductors

(C) straight line passing through the point of intersection of the conductors

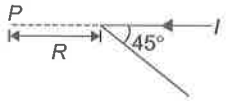
(D) rectangular hyperbola

9. Equal current i is flowing in three infinitely long wires along positive x, y and z directions. The magnitude field at a point $(0, 0, -a)$ would be

(A) $\frac{\mu_0 i}{2\pi a}(\hat{j}-\hat{i})$ (B) $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j})$

(C) $\frac{\mu_0 i}{2\pi a}(\hat{i}-\hat{j})$ (D) $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j}+\hat{k})$

10. A long straight wire, carrying current I, is bent at its midpoint to form an angle of 45°. Induction of magnetic field at point P, distant R from point of bending is equal to

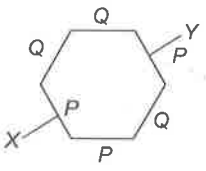


(A) $\frac{(\sqrt{2}-1)\mu_0 I}{4\pi R}$ (B) $\frac{(\sqrt{2}+1)\mu_0 I}{4\pi R}$

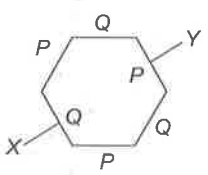
(C) $\frac{(\sqrt{2}-1)\mu_0 I}{4\sqrt{2}\pi R}$ (D) $\frac{(\sqrt{2}+1)\mu_0 I}{4\sqrt{2}\pi R}$

11. In the following hexagons, made up of two different material P and Q, current enters and leaves from points X and Y respectively. In which case the magnetic field at its centre is not zero.

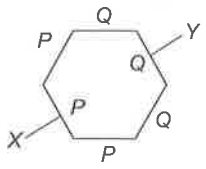
(A)



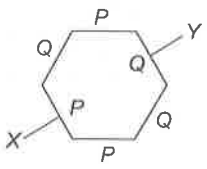
(B)



(C)



(D)



12. Consider the magnetic field produced by a finitely long current carrying wire.

(A) the lines of field will be concentric circles with centres on the wire.

(B) There can be two points in the same plane where magnetic fields are same.

(C) There can be large number of points where the magnetic field is same.

(D) The magnetic field at a point is inversely proportional to the distance of the point from the wire.

13. Consider three quantities $x = \frac{E}{B}$, $y = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$ and $z = \frac{l}{CR}$. Here, l is the length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings.

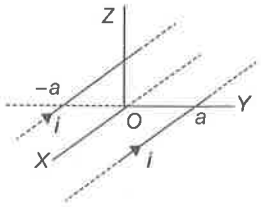
(A) x, y have the same dimensions

(B) y, z have the same dimensions

(C) z, x have the same dimensions

(D) none of the three pairs have the same dimensions.

14. Two long thin, parallel conductors carrying equal currents in the same direction are fixed parallel to the x-axis, one passing through $y = a$ and the other through $y = -a$. The resultant magnetic field due to the two conductors at any point is B. Which of the following are correct?



(A) $B = 0$ for all points on the x-axis

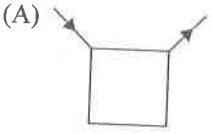
(B) At all points on the y-axis, excluding the origin, B has only a z-component.

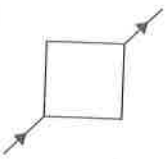
(C) At all point of the z-axis, excluding the origin, B has only a y-component.

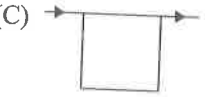
(D) B cannot have an x-component.

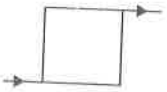
202

15. Current flows through uniform, square frames as shown. In which case is the magnetic field at the centre of the frame not zero?

(A) 

(B) 

(C) 

(D) 

16. A long straight wire carries a current along the x-axis. Consider the points $A(0, 1, 0)$, $B(0, 1, 1)$, $C(1, 0, 1)$ and $D(1, 1, 1)$. Which of the following pairs of points will have magnetic fields of the same magnitude?

(A) A and B (B) A and C
(C) B and C (D) B and D

17. In the previous question, if the current is i and the magnetic field at D has magnitude B,

(A) $B = \dfrac{\mu_0 i}{2\sqrt{2}\pi}$

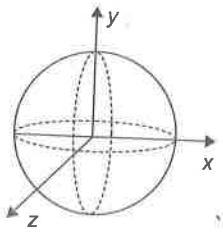
(B) $B = \dfrac{\mu_0 i}{2\sqrt{3}\pi}$

(C) B is parallel to the x-axis

(D) B makes an angle of 45° with the xy plane

Section B: Magnetic Field Due to a Circular Loop

18. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its centre at the origin of co-ordinate system. If current I is flowing through each ring then the magnitude of the magnetic field at the common centre is



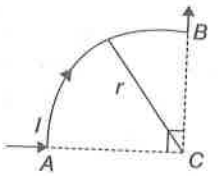
(A) $\sqrt{3}\dfrac{\mu_0 I}{2R}$ (B) zero

(C) $(\sqrt{2}-1)\dfrac{\mu_0 I}{2R}$ (D) $(\sqrt{3}-\sqrt{2})\dfrac{\mu_0 I}{2R}$

19. Two concentric coils X and Y of radii 16 cm and 10 cm lie in the same vertical plane containing N-S direction. X has 20 turns and carries 16 A. Y has 25 turns and carries 18 A. X has current in anticlockwise direction and Y has current in clockwise direction for an observer, looking at the coils facing the west. The magnitude of net magnetic field at their common centre is

(A) $5\pi\times 10^{-4}$ T towards west
(B) $13\pi\times 10^{-4}$ T towards east
(C) $13\pi\times 10^{-4}$ T towards west
(D) $5\pi\times 10^{-4}$ T towards east

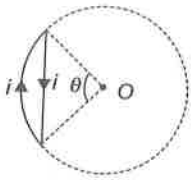
20. A wire carrying I is shaped as shown. Section AB is a quarter circle of radius r. The magnetic field at C is directed



(A) along the bisector of the angle ACB, away from AB
(B) along the bisector of the angle ACB, towards AB
(C) perpendicular to the plane of the paper, directed into the paper
(D) at an angle $\dfrac{\pi}{4}$ to the plane of the paper

Section C: Magnetic Field Due to a Straight Wire and Circular Arc

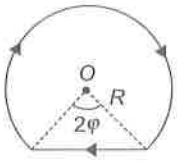
21. Net magnetic field at the centre of the circle O due to a current carrying loop as shown in figure is ($\theta < 180°$)



(A) zero
(B) perpendicular to paper inwards
(C) perpendicular to paper outwards
(D) is perpendicular to paper inwards if $\theta \le 90°$ and perpendicular to paper outwards if $90° \le \theta < 180°$

22. Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated A current I flows along a thin wire shaped as shown in figure. The radius of a curved part of the wire is equal

to R the angle is 2ϕ. Find the magnetic induction of the field at the point O.



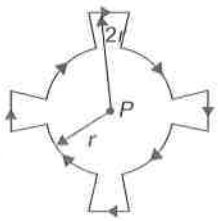
(A) $B = (\pi - \phi + \sin\phi)\mu_0 \dfrac{I}{2\pi R}$

(B) $B = (\pi - \phi + \tan\phi)\mu_0 \dfrac{I}{2R}$

(C) $B = (\pi - \phi + \tan\phi)\mu_0 \dfrac{I}{2\pi R}$

(D) none of these

23. A current I flows around a closed path in the horizontal plane of the circle as shown in the figure. The path consists of eight arcs with alternating radii r and $2r$. Each segment of arc subtends equal angle at the common centre P. The magnetic field produced by current path at point P is



(A) $\dfrac{3}{8}\dfrac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed inward

(B) $\dfrac{3}{8}\dfrac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed outward

(C) $\dfrac{1}{8}\dfrac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed inward.

(D) $\dfrac{1}{8}\dfrac{\mu_0 I}{r}$; perpendicular to the plane of the paper and directed outward.

24. Which of the following statement is correct:

(A) A charged particle enters a region of uniform magnetic field at an angle 85° to magnetic lines of force. The path of the particle is a circle.

(B) An electron and proton are moving with the same kinetic energy along the same direction. When they pass through uniform magnetic field perpendicular to their direction of motion, they describe circular path.

(C) There is no change in the energy of a charged particle moving in a magnetic field although magnetic force acts on it.

(D) Two electrons enter with the same speed but in opposite direction in a uniform transverse magnetic field. Then the two describe circle of the same radius and these move in the same direction.

25. Two identical charged particles enter a uniform magnetic field with same speed but at angles 30° and 60° with field. Let a, b and c be the ratio of their time periods, radii and pitches of the helical paths than

(A) $abc = 1$ (B) $abc > 1$

(C) $abc < 1$ (D) $a = bc$

26. Consider the following statements regarding a charged particle in a magnetic field. Which of the statements are true:

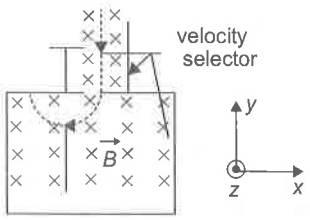
(A) Starting with zero velocity, it accelerates in a direction perpendicular to the magnetic field.

(B) While deflecting in magnetic field its energy gradually increases.

(C) Only the component of magnetic field perpendicular to the direction of motion of the charged particle is effective in deflecting it.

(D) Direction of deflecting force on the moving charged particle is perpendicular to its velocity.

27. Two charged particle A and B each of charge $+e$ and masses 12 amu and 13 amu respectively follow a circular trajectory in chamber X after the velocity selector as shown in the figure. Both particles enter the velocity selector with speed 1.5×10^6 ms^{-1}. A uniform magnetic field of strength 1.0 T maintained within the chamber X and in the velocity selector.



(A) Electric field across the conducting plate of the velocity selector is -10^6 NC^{-1} $\hat{i}$.

(B) Electric field across the conducting plate of the velocity selector is 10^6 NC^{-1} $\hat{i}$.

204

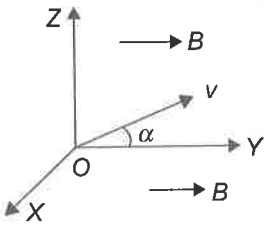
(C) The ratio $\frac{r_A}{r_B}$ of the radii of the circular paths for the two particles is $\frac{12}{13}$.

(D) The ratio $\frac{r_A}{r_B}$ of the radii of the circular paths for the two particles is $\frac{13}{12}$.

28. An electron is moving along the positive X-axis. you want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative X-axis. This can be done by applying the magnetic field along.

(A) Y-axis
(B) Z-axis
(C) Y-axis only
(D) Z-axis only

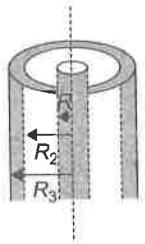
29. In a region of space, a uniform magnetic field B exists in the y-direction. A proton is fired from the origin, with its initial velocity v making a small angle α with the y-direction in the yz plane. In the subsequent motion of the proton,

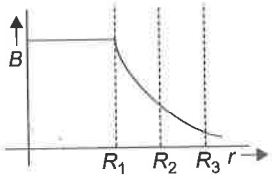


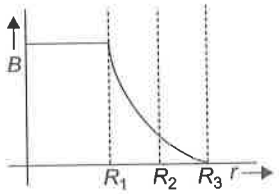
(A) its x-coordinate can never be positive
(B) its x-and z-coordinates cannot both be zero at the same time
(C) its z-coordinate can never be negative
(D) its y-coordinate will be proportional to the square of its time of flight

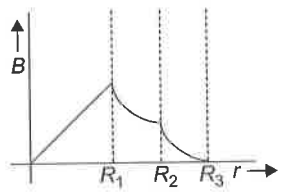
Section D: Magnetic Field Due to a Cylinder, Large Sheet, Solenoid, Toroid and Ampere's Law

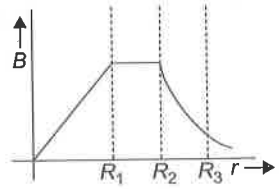
30. A coaxial cable is made up of two conductors. The inner conductor is solid and is of radius R_1 and the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then the variation of magnetic field with distance from the axis is best plotted as:



(A)


(B)


(C)


(D)


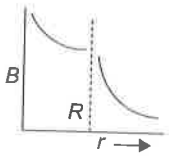
31. Axis of a solid cylinder of infinite length and radius R lies along y-axis it carries a uniformly distributed current 'i' along $+y$ direction. Magnetic field at a point $\left(\frac{R}{2}, y, \frac{R}{2}\right)$ is:

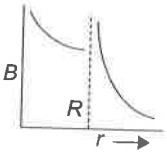
(A) $\frac{\mu_0 I}{4\pi R}(\hat{i} - \hat{k})$ (B) $\frac{\mu_0 i}{2\pi R}(\hat{j} - \hat{k})$

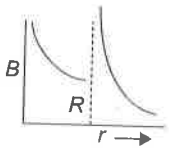
(C) $\frac{\mu_0 i}{4\pi R}\hat{j}$ (D) $\frac{\mu_0 i}{4\pi R}(\hat{i} + \hat{k})$

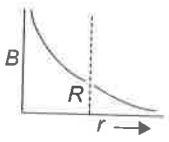
32. A thin, straight conductor lies along the axis of a hollow conductor of radius R. The two carry equal currents in the same direction. The magnetic field B

is plotted against the distance r from the axis. Which of the following best represents the resulting curve?

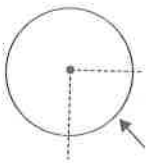
(A) 

(B) 

(C) 

(D) 

33. A long thin walled pipe of radius R carries a current I along its length. The current density is uniform over the circumference of the pipe. The magnetic field at the center of the pipe due to quarter portion of the pipe shown, is



(A) $\frac{\mu_0 I\sqrt{2}}{4\pi^2 R}$ (B) $\frac{\mu_0 I}{\pi^2 R}$

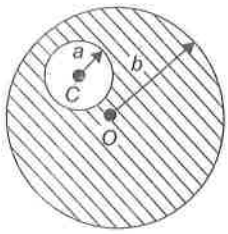
(C) $\frac{2\mu_0 I\sqrt{2}}{\pi^2 R}$ (D) None

34. A hollow cylinder having infinite length and carrying uniform current per unit length λ along the circumference as shown. Magnetic field inside the cylinder is

(A) $\frac{\mu_0\lambda}{2}$ (B) $\mu_0\lambda$

(C) $2\mu_0\lambda$ (D) none

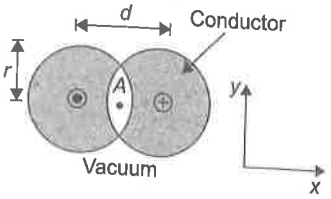
35. A long straight metal rod has a very long hole of radius 'a' drilled parallel to the rod axis as shown in the figure. If the rod carries a current 'i' find the value of magnetic induction on the axis of the hole, where $OC = c$



(A) $\frac{\mu_0 ic}{\pi(b^2 - a^2)}$ (B) $\frac{\mu_0 ic}{2\pi(b^2 - a^2)}$

(C) $\frac{\mu_0 i(b^2 - a^2)}{2\pi c}$ (D) $\frac{\mu_0 ic}{2\pi a^2 b^2}$

36. Two long conductors are arranged as shown below to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J flows into the plane of the page along the shaded part of the one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown. What is the magnitude and direction of the magnetic field at point A?



(A) $\left(\frac{\mu_0}{2\mu}\right)\pi dJ$, in the $+y$ direction

(B) $\left(\frac{\mu_0}{2\mu}\right)\frac{d^2}{r}$, in the $+y$ direction

(C) $\left(\frac{\mu_0}{2\mu}\right)\frac{4d^2 J}{r}$, in the $-y$ direction

(D) $\left(\frac{\mu_0}{2\mu}\right)\frac{Jr^2}{d}$, in the $-y$ direction

(E) There is no magnetic field at A.

206

37. A particle of the charge q and velocity v passes undeflected through a space with non-zero electric field E and magnetic field B. The undeflecting conditions will hold if.

(A) signs of both q and E are reversed.

(B) signs of both q and B are reversed

(C) both B and E are changed in magnitude, but keeping the product of $|B|$ and $|E|$ fixed,

(D) both B and E are doubled in magnitude.

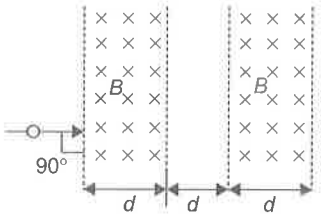
Section E: Magnetic Force on a Charge

38. A uniform magnetic field $\vec{B} = B_0\hat{j}$ exists in a space. A particle of mass m and charge q is projected towards negative x-axis with speed v from the a point $(d, 0, 0)$. The maximum value v for which the particle does not hit y-z plane is

(A) $\frac{2Bq}{dm}$ (B) $\frac{Bqd}{m}$

(C) $\frac{Bq}{2dm}$ (D) $\frac{Bqd}{2m}$

39. A positive charge particle of mass m and charge q is projected with velocity v as shown in the figure. If radius of curvature of charge particle in magnetic field is R $(2d < R < 3d)$, then time lapse by charge particle in magnetic field regions is



(A) $\frac{m}{2qB}$ (B) $\frac{m}{qB}\sin^{-1}\left(\frac{2d}{R}\right)$

(C) $\frac{m}{qB}$ (D) $\frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$

40. An electron having kinetic energy T is moving in a circular orbit of radius R perpendicular to a uniform magnetic induction $\vec{B}$. If kinetic energy is doubled and magnetic induction tripled, the radius will become

(A) $\frac{3R}{2}$ (B) $\sqrt{\frac{3}{2}}R$

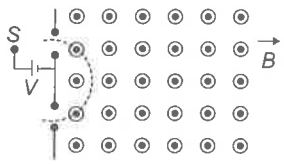
(C) $\sqrt{\frac{2}{9}}R$ (D) $\sqrt{\frac{4}{3}}R$

41. A electron experiences a force $(4.0\hat{i} + 3.0\hat{j}) \times 10^{-13}$ N in a uniform magnetic field when its velocity is $2.5\hat{k} \times 10^7\ \text{ms}^{-1}$. When the velocity is redirected and becomes $(1.5\hat{i} - 2.0\hat{j}) \times 10^7\ \text{ms}^{-1}$, the magnetic force of the electron is zero. The magnetic field vector $\vec{B}$ is:

(A) $-0.075\hat{i} + 0.1\hat{j}$ (B) $0.1\hat{i} + 0.075\hat{j}$

(C) $0.075\hat{i} - 0.1\hat{j} + \hat{k}$ (D) $0.075\hat{i} - 0.1\hat{j}$

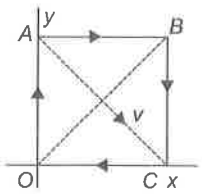
42. A mass spectrometer is a device which select particle of equal mass. An ion with electric charge $q > 0$ starts at rest from a source S and is accelerated through a potential difference V. It passes through a hole into a region of constant magnetic field $\vec{B}$ perpendicular to the plane of the paper as shown in the figure. The particle is deflected by the magnetic field and emerges through the bottom hole at a distance d from the top hole. The mass of the particle is



(A) $\frac{qBd}{V}$ (B) $\frac{qB^2d^2}{4\text{ V}}$

(C) $\frac{qB^2d^2}{8\text{ V}}$ (D) $\frac{qBd}{2\text{ V}}$

43. $OABC$ is a current carrying square loop an electron is projected from the centre of loop along its diagonal AC as shown. Unit vector in the direction of initial acceleration will be



(A) $\hat{k}$ (B) $-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(C) $-\hat{k}$ (D) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

44. An electron moving with a velocity $\vec{V}_1 = 2\hat{i}$ m/s at a point in a magnetic field experiences a force $\vec{F}_1 = -2\hat{j}N$. If the electron is moving with a velocity

$\vec{V}_2 = 2\hat{j}$ m/s at the same point, it experiences a force $\vec{F}_2 = +2\hat{i}N$. The force the electron would experience if it were moving with a velocity $\vec{V}_3 = 2\hat{k}$ m/s at the same point is

(A) zero

(B) $2\hat{k}N$

(C) $-2\hat{k}N$

(D) information is insufficient

45. Two particles of charges $+Q$ and $-Q$ are projected from the same point with a velocity v in a region of uniform magnetic field B such that the velocity vector makes an angle θ with the magnetic field. Their masses are M and $2M$, respectively. Then, they will meet again for the first time at a point whose distance from the point of projection is

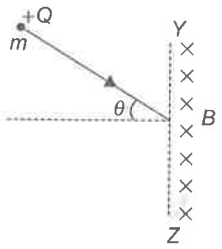
(A) $2\pi M v \cos\theta/QB$

(B) $8\pi M v \cos\theta/QB$

(C) $\pi M v \cos\theta/QB$

(D) $4\pi M v \cos\theta/QB$

46. A particle with charge $+Q$ and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of B. Let $T = 2\pi\dfrac{m}{QB}$. The time spent by the particle in the field will be



(A) $T\theta$ (B) $2T\theta$

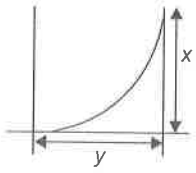
(C) $T\left(\dfrac{\pi + 2\theta}{2\pi}\right)$ (D) $T\left(\dfrac{\pi - 2\theta}{2\pi}\right)$

47. In the previous question, if the particle has $-Q$ charge, the time spend by the particle in the field will be

(A) $T\theta$ (B) $2T\theta$

(C) $T\left(\dfrac{\pi + 2\theta}{2\pi}\right)$ (D) $T\left(\dfrac{\pi - 2\theta}{2\pi}\right)$

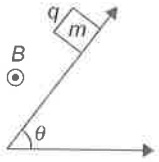
48. A particle having charge q enters a region of uniform magnetic field $\vec{B}$ (directed inwards) and is deflected a distance x after travelling a distance y. The magnitude of the momentum of the particle is



(A) $\dfrac{qBy}{2}$ (B) $\dfrac{qBy}{x}$

(C) $\dfrac{qB}{2}\left(\dfrac{y^2}{x} + x\right)$ (D) $\dfrac{qBy^2}{2x}$

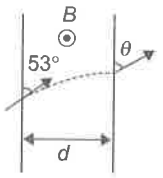
49. A block of mass m and charge q is released on a long smooth inclined plane magnetic field B is constant, uniform, horizontal and parallel to surface as shown. Find the time from start when block loses contact with the surface.



(A) $\dfrac{m\cos\theta}{qB}$ (B) $\dfrac{m\,\text{cosec}\,\theta}{qB}$

(C) $\dfrac{m\cot\theta}{qB}$ (D) none

50. A particle moving with velocity v having specific charge (q/m) enters a region of magnetic field B having width $d = \dfrac{3mv}{5qB}$ at angle 53° to the boundary of magnetic field. Find the angle θ in the diagram.



(A) 37° (B) 60°

(C) 90° (D) none

Section F: Electric and Magnetic Force on a Charge

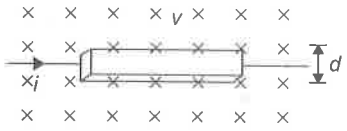
51. A particle of specific charge (charge/mass) α starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{k}$. Its velocity at $(x_0, y_0, 0)$ is $(4\hat{i} - 3\hat{j})$. The value of x_0 is

208

(A) $\frac{13}{2}\frac{\alpha E_0}{B_0}$ (B) $\frac{16\alpha B_0}{E_0}$

(C) $\frac{25}{2\alpha E_0}$ (D) $\frac{5\alpha}{2B_0}$

Common Data for Questions 52–55: A current i is passed through a silver strip of width d and area of cross-section A. The number of free electrons per unit volume is n.



52. Find the drift velocity v of the electrons.

(A) $\frac{i}{Ane}$ (B) $\frac{iB}{An}$

(C) $\frac{iB}{Ane}$ (D) $\frac{iBd}{Ane}$

53. If a magnetic field B exists in the region as shown in figure, what is the average magnetic force on the free electrons?

(A) $\frac{i}{Ane}$ (B) $\frac{iB}{An}$

(C) $\frac{iB}{Ane}$ (D) $\frac{iBd}{Ane}$

54. Due to the magnetic force, the free electrons get accumulated on one side of the conductor along its length. This produces a transverse electric field in the conductor which opposes the magnetic force on the electrons. Find the magnitude of the electric field which will stop further accumulation of electrons.

(A) $\frac{i}{Ane}$ (B) $\frac{iB}{An}$

(C) $\frac{iB}{Ane}$ (D) $\frac{iBd}{Ane}$

55. What will be the potential difference developed across the width of the conductor due to the electron accumulation? The appearance of a transverse emf, when a current-carrying wire is placed in a magnetic field, is called Hall effect.

(A) $\frac{i}{Ane}$ (B) $\frac{iB}{An}$

(C) $\frac{iB}{Ane}$ (D) $\frac{iBd}{Ane}$

56. A particle of specific charge (q/m) is projected from the origin of coordinates with initial velocity $[ui - v\hat{j}]$. Uniform electric magnetic fields exist in the region along the $+y$ direction, of magnitude E and B. The particle will definitely return to the origin once if

(A) $[vB/2\pi E]$ is an integer

(B) $(u^2 + v^2)^{1/2}\,[B/\pi E]$ is an integer

(C) $[vB/\pi E]$ in an integer

(D) $[uB/\pi E]$ is an integer

57. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{i}$ with velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time

(A) $t = \frac{2mv_0}{qE}$ (B) $t = \frac{2Bq}{mv_0}$

(C) $t = \frac{\sqrt{3}Bq}{mv_0}$ (D) $t = \frac{\sqrt{3}mv_0}{qE}$

58. An electron is projected with velocity v_0 in a uniform electric field E perpendicular to the field. Again it is projected with velocity v_0 perpendicular to a uniform magnetic field B. If r_1 is initial radius of curvature just after entering in the electric field and r_2 is initial radius of curvature just after entering in magnetic field then the ratio r_1/r_2 is equal to

(A) $\frac{Bv_0^2}{E}$ (B) $\frac{B}{E}$

(C) $\frac{Ev_0}{B}$ (D) $\frac{Bv_0}{E}$

59. An electron (mass = 9.1×10^{-31}; charge = -1.6×10^{-19} C) experiences no deflection if subjected to an electric field of 3.2×10^5 V/m and magnetic field of 2.0×10^{-3} Wb/m^2. Both the fields are normal to the path of electron and to each other. If the electric field is removed, then the electron will revolve in an orbit of radius:

(A) 45 m (B) 4.5 m

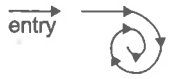
(C) 0.45 m (D) 0.045 m

60. A charged particle enters a uniform magnetic field perpendicular to its initial direction travelling in air. The path of the particle is seen to follow the path in figure. Which of statements 1–3 is/are correct?

(A) The magnetic field strength may have been increased while the particle was travelling in air

(B) The particle lost energy by ionizing the air

(C) The particle lost charge by ionizing the air



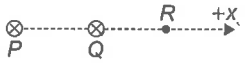
(A) 1, 2, 3 are correct
(B) 1, 2 only are correct
(C) 2, 3 only are correct
(D) 1 only

61. Two protons move parallel to each other, keeping distance r between them, both moving with same velocity $\vec{V}$. Then the ratio of the electric and magnetic force of interaction between them is

(A) c^2/V^2 (B) $2c^2/V^2$
(C) $c^2/2V^2$ (D) None

Section G: Magnetic Force on a Current Carrying Wire

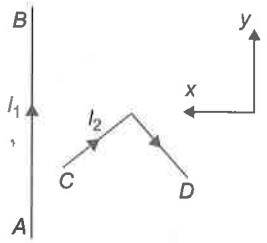
62. Two long parallel wires carrying currents 2.5 A and I A in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 m and 2 m respectively from a collinear point R.



An electron moving with a velocity of 4×10^5 m/s along the positive X-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(A) 4 Å (B) 8 Å
(C) 16 Å (D) none

63. In the figure shown a current I_1 is established in the long straight wire AB. Another wire CD carrying current I_2 is placed in the plane of the paper. The line joining the ends of the this wire is perpendicular to the wire AB. The resultant force on the wire CD is:

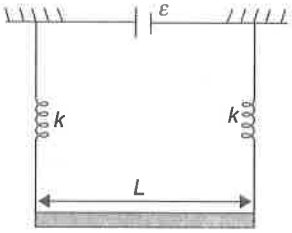


(A) zero
(B) towards negative x-axis
(C) towards positive y-axis
(D) none of these

64. A straight current carrying conductor is placed in such a way that the current in the conductor flows in the direction out of the plane of the paper. The conductor is placed between two poles of two magnets, as shown. The conductor will experience a force in the direction towards

(A) P (B) Q
(C) R (D) S

65. A straight rod of mass m and length L is suspended from the identical spring as shown in the figure. The spring stretched by a distance of x_0 due to the weight of the wire. The circuit has total resistance $R\Omega$. When the magnetic field perpendicular to the plane of the paper is switched on, springs are observed to extend further by the same distance. The magnetic field strength is



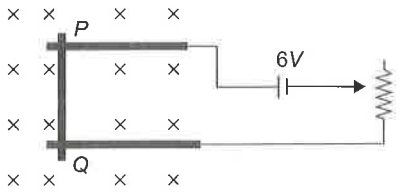
(A) $\frac{mgR}{\varepsilon L}$; directed outward from the plane of the paper

(B) $\frac{mgR}{2\varepsilon x_0}$; directed outward from the plane of the paper

(C) $\frac{mgR}{\varepsilon L}$; directed into the plane of the paper

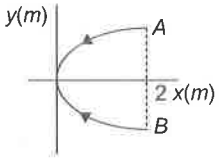
(D) $\frac{mgR}{\varepsilon x_0}$; directed into the plane of the paper

66. A metal wire PQ of mass 10 g lies at rest on two horizontal metal rails separated by 4.90 cm (figure). A vertically downward magnetic field of magnitude 0.800 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20.0 Ω, the wire PQ starts sliding on the rails. Find the coefficient of friction. Neglect magnetic force acting on wire PQ due to metal rails ($g = 9.8$ m/s^2)



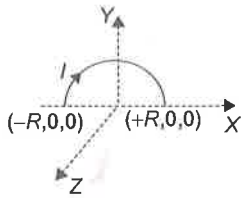
(A) 0.12 (B) 0.10
(C) 0.24 (D) none of these

67. A conducting wire bent in the form of a parabola $y^2 = 2x$ carries a current $i = 2$ A as shown in figure. This wire is placed in a uniform magnetic field $\vec{B} = -4\hat{k}$ Tesla. The magnetic force on the wire is (in newton)



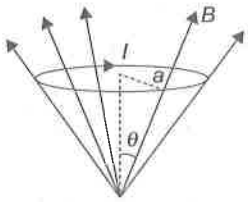
(A) $-16\hat{i}$ (B) $32\hat{i}$
(C) $-32\hat{i}$ (D) $16\hat{i}$

68. A semi-circular current carrying wire having radius R is placed in x-y plane with its centre at origin 'O'. There is non-uniform magnetic field $\vec{B} = \frac{B_0 x}{2R}\hat{k}$ (here B_0 is +ve constant) is existing in the region. The magnetic force acting on semi-circular wire will be along



(A) $-x$-axis (B) $+y$-axis
(C) $-y$-axis (D) $+x$-axis

69. A circular current loop of radius a is placed in a radial field B as shown. The net force acting on the loop is



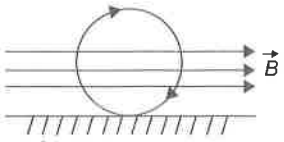
(A) zero (B) $2\pi BaI\cos\theta$
(C) $2\pi aIB\sin\theta$ (D) none

70. A metal ring of radius $r = 0.5$ m with its plane normal to a uniform magnetic field B of induction $0.2T$ carries a current $I = 100$ A. The tension in newtons developed in the ring is
(A) 100 (B) 50
(C) 25 (D) 10

Section H: Magnetic Force and Torque on a Current Carrying Loop and Magnetic Dipole Moment

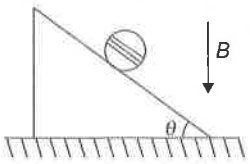
71. A toroid of mean radius 'a', cross section radius 'r' and total number of turns N. It carries a current 'i'. The torque experienced by the toroid if a uniform magnetic field of strength B is applied:
(A) is zero
(B) is $BiN\pi r^2$
(C) is $BiN\pi a^2$
(D) depends on the direction of magnetic field.

72. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current $i = 4$ A. A horizontal magnetic field $B = 10T$ is switched on at time $t = 0$ as shown in figure. The initial angular acceleration of the ring will be



(A) 40π rad/s^2 (B) 20π rad/s^2
(C) 5π rad/s^2 (D) 15π rad/s^2

73. In the figure shown a coil of single turn is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the plane and lies in the equatorial plane of the sphere. Current in the coil is i. The value of B if the sphere is in equilibrium is

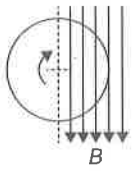


(A) $\frac{mg\cos\theta}{\pi iR}$ (B) $\frac{mg}{\pi iR}$
(C) $\frac{mg\tan\theta}{\pi iR}$ (D) $\frac{mg\sin\theta}{\pi iR}$

74. The dimensional formula for the physical quantity $\frac{E^2\mu_0\varepsilon_0}{B^2}$ is (E = electric field and B = magnetic field)

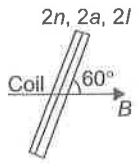
(A) $L^0M^0T^0$ (B) $L^1M^0T^{-1}$
(C) $L^{-1}M^0T^1$ (D) $L^{1/2}M^0T^{-1/2}$

75. A thin non conducting disc of radius R is rotating clockwise (see figure) with an angular velocity ω about its central axis, which is perpendicular to its plane. Both its surfaces carry +ve charges of uniform surface density. Half the disc is in a region of a uniform, unidirectional magnetic field B parallel to the plane of the disc, as shown. Then,



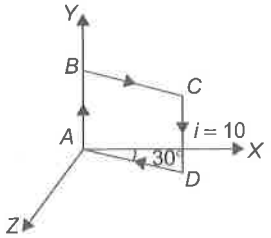
(A) The net torque on the disc is zero
(B) The net torque vector on the disc is directed left wards
(C) The net torque vector on the disc is directed
(D) The net torque vector on the disc is parallel to B

76. A rectangular coil PQ has $2n$ turns, an area $2a$ and carries a current $2I$, (refer figure). The plane of the coil is at 60° to a horizontal uniform magnetic field of flux density B. The torque on the coil due to magnetic force is



(A) $BanI\sin 60°$ (B) $8BanI\cos 60°$
(C) $4anIB\sin 60°$ (D) none

77. Figure shows a square current carrying loop $ABCD$ of side 10 cm and current $i = 10$ A. The magnetic moment $\vec{M}$ of the loop is



(A) $(0.05)(\hat{i} - \sqrt{3}\hat{k})$A-m^2
(B) $(0.05)(\hat{j} + \hat{k})$A-m^2
(C) $(0.05)(\hat{j} + \hat{k})$A-m^2
(D) $(\hat{i} + \hat{k})$A-m^2

Section I: Magnetic Field Due to a Magnet and Earth

78. A conductor of length l and mass m is placed along the east-west line on a table. Suddenly a certain amount of charge is passed through it and it is found to jump to a height h. The earth's magnetic induction is B. The charge passed through the conductor is

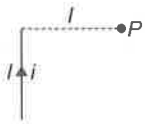
(A) $\frac{1}{Bmgh}$ (B) $\frac{\sqrt{2gh}}{Blm}$
(C) $\frac{gh}{Blm}$ (D) $\frac{m\sqrt{2gh}}{Bl}$

JEE Advanced

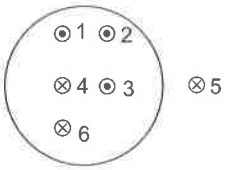
Level I

Section A: Magnetic Field Due to a Straight Wire

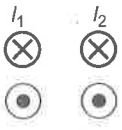
1. Figure shows a straight wire of length l carrying a current i. Find the magnitude of magnetic field produced by the current at point P.



2. Six wires of current $I_1 = 1$ A, $I_2 = 2$ A, $I_3 = 3$ A, $I_4 = 1$ A, $I_5 = 5$ A and $I_6 = 4$ A cut the page perpendicularly at the points 1, 2, 3, 4, 5 and 6 respectively as shown in the figure. Find the value of the integral $\oint \vec{B} \cdot d\vec{l}$ around the closed path.



3. A system of long four parallel conductors whose sections with the plane of the drawing lie at the vertices of a square there flow four equal currents.



The directions of these currents are as follows: those marked ⊗ point away from the reader, while those

912

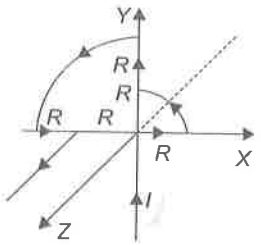
marked with a dot point towards the reader. How is the vector of magnetic induction directed at the centre of the square?

Section B: Magnetic Field Due to a Circular Loop

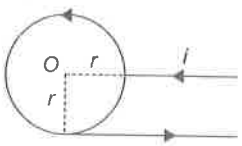
4. Two circular coils A and B of radius $\frac{5}{\sqrt{2}}$ cm and 5 cm respectively carry current 5 A and $\frac{5}{\sqrt{2}}$ A respectively. The plane of B is perpendicular to plane of A and their centres coincide. Find the magnetic field at the centre.
5. Two circular coils of wire each having a radius of 4 cm and 10 turns have a common axis and are 6 cm apart. If a current of 1 A passes through each coil in the opposite direction find the magnetic induction.
   (i) At the centre of either coil;
   (ii) At a point on the axis, midway between them.
6. Find the ratio of magnetic field magnitudes at a distance 10 m along the axis and at 60° from the axis, from the centre of a coil of radius 1 cm, carrying a current 1 A.

Section C: Magnetic Field Due to a Straight Wire and Circular Arc

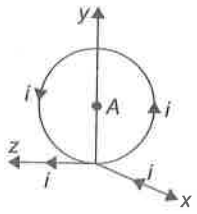
7. Find the magnetic induction at the origin in the figure shown.



8. Find the magnetic induction at point O, if the current carrying wire is in the shape shown in the figure.

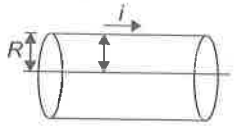


9. Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors along which a current i is flowing at a point $A(O, R, O)$, that is the centre of a circular conductor of radius R. The ring is in yz plane.



Section D: Magnetic Field Due to a Cylinder, Large Sheet, Solenoid, Toroid and Ampere's Law

10. A cylindrical conductor of radius R carries a current its length. The current density J, however, it is not uniform over the cross section of the conductor but is a function of the radius according to $J = br$, where b is a constant. Find an expression for the magnetic field B.



   (a) at $r_1 < R$ and
   (b) at distance $r_2 > R$, measured from the axis

Section E: Magnetic Force on a Charge

11. Electric charge q is uniformly distributed over a rod of length l. The rod is placed parallel to a long wire carrying a current i. The separation between the rod and the wire is a. Find the force needed to move the rod along its length with a uniform velocity v.
12. A charged particle (charge q, mass m) has velocity v_0 at origin in $+x$ direction. In space there is a uniform magnetic field B in $-z$ direction. Find the y coordinate of particle when is crosses y-axis.
13. A charged particle having mass m and charge q is accelerated by a potential difference V, it flies through a uniform transverse magnetic field B. The field occupies a region of space d. Find the time interval for which it remains inside the magnetic field.

Section F: Electric and Magnetic Force on a Charge

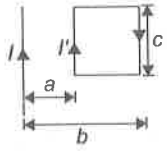
14. An electron moving with a velocity $5 \times 10^6\,\text{ms}^{-1}\hat{i}$. In the uniform electric field of $5 \times 10^7\,vm^{-1}\hat{j}$. Find the magnitude and direction of a minimum uniform magnetic field in tesla that will cause the electron to move undeviated along its original path.
15. A particle of charge $+q$ and mass m moving under the influence of a uniform electric field $E\hat{i}$ and a magnetic field $B\hat{k}$ enters in I quadrant of a coordinate system at a point $(0, a)$ with initial velocity $v\hat{i}$ and leaves the quadrant at a point $(2a, 0)$ with velocity $-2v\hat{j}$. Find

(a) Magnitude of electric field
(b) Rate of work done by the electric field at point $(0, a)$
(c) Rate of work done by both the fields at $(2a, 0)$

16. A proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic field with E and B. Then the beam strikes a grounded target. Find the force imparted by the beam on the target if the beam current is equal to I.

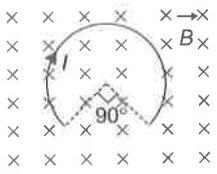
Section G: Magnetic Force on a Current Carrying Wire

17. An infinitely long straight wire carries a conventional current I as shown in the figure.



The rectangular loop carries a conventional current I' in the clockwise direction. Find the net force on the rectangular loop.

18. An arc of a circular loop of radius R is kept in the horizontal plane and a constant magnetic field B is applied in the vertical direction as shown in the figure. If the arc carries current I then find the force on the arc.

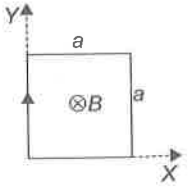


19. Two long straight parallel conductors are separated by a distance of $r_1 = 5$ cm and carry currents $i_1 = 10$ A and $i_2 = 20$ A. What work per unit length of a conductor must be done to increase the separation between the conductors to $r_2 = 10$ cm if, currents flow in the same direction?

Section H: Magnetic Force and Torque on a Current Carrying Loop and Magnetic Dipole Moment

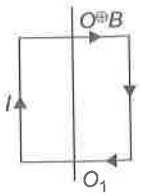
20. A conducting circular loop of radius r carries a constant current I. It is placed in a uniform magnetic field $\vec{B}_0$ such that $\vec{B}_0$ is perpendicular to the plane of the loop. Find the magnetic force acting on the loop is

21. A rectangular loop of wire is oriented with the left corner at the origin, one edge along X-axis and the other edge along Y-axis as shown in the figure. A magnetic field is into the the page and has a magnitude that is given by $\beta = \alpha y$



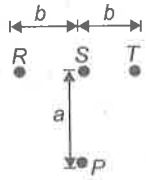
where α is constant. Find the total magnetic force on the loop if it carries current i.

22. A square current carrying loop made of thin wire and having a mass $m = 10$ g can rotate without friction with respect to the vertical axis OO_1, passing through the centre of the loop at right angles to two opposite sides of the loop. The loop is placed in a homogeneous magnetic field with an induction $B = 10^{-1}$T directed at right angles to the plane of the drawing. A current $I = 2$ A is flowing in the loop. Find the period of small oscillations that the loop performs about its position of stable equilibrium.



Level II

1. Three infinitely long conductors R, S and T are lying in a horizontal plane as shown in the figure. The currents in the respective conductors are



$$I_R = I_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$

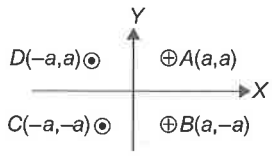
$$I_S = I_0 \sin(\omega t)$$

$$I_T = I_0 \sin\left(\omega t - \frac{2\pi}{3}\right)$$

214

Find the amplitude of the vertical component of the magnetic field at a point P, distance 'a' away from the central conductor S.

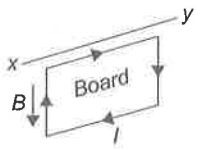
2. Four long wires each carrying current I as shown in the figure are placed at the points A, B, C and D. Find the magnitude and direction of



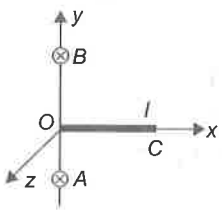
(i) magnetic field at the centre of the square.
(ii) force per metre acting on wire at point D.

3. An infinite wire, placed along z-axis, has current I_1 in positive z-direction. A conducting rod placed in xy plane parallel to y-axis has current I_2 in positive y-direction. The ends of the rod subtend +30° and –60° at the origin with positives x-direction. The rod is at a distance a from the origin. Find net force on the rod.

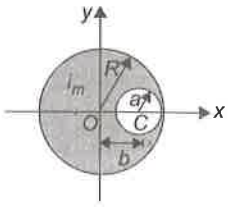
4. A square cardboard of side l and mass m is suspended from a horizontal axis XY as shown in figure. A single wire is wound along the periphery of board and carrying a clockwise current I. At $t = 0$, a vertical downward magnetic field of induction B is switched on. Find the minimum value of B so that the board will be able to rotate up to horizontal level.



5. A straight segment OC (of length L meter) of a circuit carrying a current I A is placed along the x-axis. Two infinitely line straight wires A and B, each extending form $z = -\infty$ to $+\infty$, are fixed at $y = -a$ metre and $y = +a$ metre respectively, as shown in the figure. If the wires A and B each carry a current I A into plane of the paper. Obtain the expression for the force acting on the segment OC. What will be the force OC if current in the wire B is reversed?

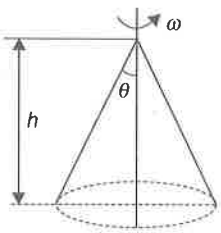


6. A very long straight conductor has a circular cross-section of radius R and carries a current density J. Inside the conductor there is a cylindrical hole of radius a whose axis is parallel to the axis of the conductor and a distance b from it. Let the z-axis be the axis of the conductor, and let the axis of the hole be at $x = b$. Find the magnetic field

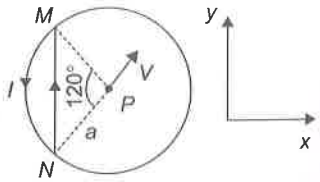


(a) on the x = axis at $x = 2R$
(b) on the y = axis at $y = 2R$.

7. Q charge is uniformly distributed over the same surface of a right circular cone of semi-vertical angle θ and height h. The cone is uniformly rotated about its axis at angular velocity ω. Calculated associated magnetic dipole moment.

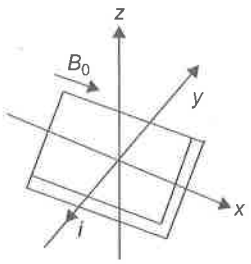


8. A wire loop carrying current I is placed in the X-Y plane as shown in the figure.



(a) If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity along NP (fig). Find its instantaneous acceleration
(b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the torque acting on the loop due to the field.

9. A long straight wire carries a current of 10 A directed along the negative y-axis as shown in figure. A uniform magnetic field B_0 of magnitude 10^{-6}T is directed parallel to the x-axis. What is the resultant magnetic field at the following points?

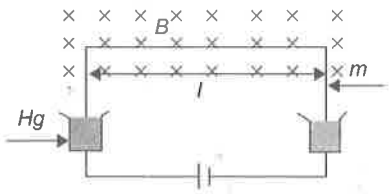


(a) $x = 0, z = 2$ m;
(b) $x = 2$ m, $z = 0$;
(c) $x = 0, z = -0.5$ m

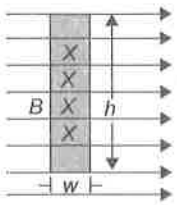
10. A stationary, circular wall clock has a face with a radius of 15 cm. Six turns of wire are wound around its perimeter, the wire carries a current 2.0 A in the clockwise direction. The clock is located, where there is a constant, uniform external magnetic field of 70 mT (but the clock still keeps perfect time) at exactly 1:00 pm, the hour hand of the clock points in the direction of the external magnetic field
(a) After how many minutes will the minute hand point in the direction of the torque on the winding due to the magnetic field?
(b) What is the magnitude of this torque.

11. A U-shaped wire of mass m and length l is immersed with its two ends in mercury (see figure). The wire is in a homogeneous field of magnetic induction B. If a charge, that is, a current pulse $q = \int i dt$, is sent through the wire, the wire will jump up.

Calculate, from the height h that the wire reaches, the size of the charge or current pulse, assuming that the time of the current pulse is very small in comparison with the time of flight. Make use of the fact that impulse of force equals $\int F dt$, which equals mv. Evaluate q for $B = 0.1$ Wb/m^2, $m = 10$ gm $\ell = 20$ cm and $h = 3$ meters [$g = 10$ m/s^2]



12. A current i, indicated by the crosses in fig. is established in a strip of copper of height h and width w. A uniform field of magnetic induction $\boldsymbol{B}$ is applied at right angles to the strip.



(a) Calculate the drift speed v_d for the electrons.
(b) What are the magnitude and direction of the magnetic force $\boldsymbol{F}$ acting on the electrons?
(c) What would the magnitude and direction of homogeneous electric field $\boldsymbol{E}$ have to be in order to counter balance the effect of the magnetic field?
(d) What is the voltage V necessary between two sides of the conductor in order to create this field E?
(e) If no electric field is applied form the outside the electrons will be pushed somewhat to one side and therefore will give rise to a uniform electric field E_H across the conductor until the force of this electrostatic field $\boldsymbol{E}_H$ balance the magnetic forces encountered in part (b). What will be the magnitude and direction of the field $\boldsymbol{E}_H$? Assume that n, the number of conduction electrons per unit volume, is 1.1×10^{29}/m^3 and that $h = 0.02$ m, $w = 0.1$ cm, $i = 50$ A, and $B = 2$ W/m^2.

13. (a) A rigid circular loop of radius r and mass m lies in the xy plane on a flat table and has a current I flowing in it. At this particular place, the earth's magnetic field is $\vec{B} = \vec{B}_x\hat{i} + \vec{B}_y\hat{j}$. How large must I be before one edge of the loop will lift from table?
(b) Repeat if, $\vec{B} = \vec{B}_x\hat{i} + \vec{B}_z\hat{k}$.

14. Zeeman effect. In Bohr's theory of the hydrogen atom the electron can be thought of as moving in a circular orbit of radius r about the proton. Suppose that such an atom is placed in a magnetic field, with the plane of the orbit at right angle to B.
(a) If the electron is circulating clockwise, as viewed by an observer sighting along B, will the angular frequency increase or decrease?
(b) What if the electron is circulating counterclockwise? Assume that the orbit radius does not change.

15. In above problem show that the change in frequency of rotation caused by the magnetic field is given approximately by $\Delta\nu = \pm\dfrac{Be}{4\pi m}$. Such frequency shifts were actually observed by Zeeman in 1896.

216

16. A square loop of wire of edge ***a*** carries a current i.

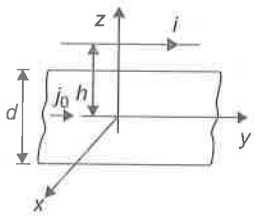
(a) Show that B for a point on the axis of the loop and a distance x from its centre is given by,

$$B = \frac{4\mu_0 i a^2}{\pi(4x^2 + a^2)(4x^2 + 2a^2)^{1/2}}.$$

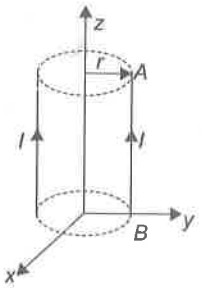
(b) Can the result of the above problem be reduced to give field at $x = 0$?

(c) Does the square loop behave like a dipole for points such that $x >> a$? If so, what is its dipole moment?

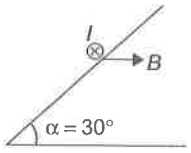
17. A conductor carrying a current i is placed parallel to a current per unit width j_0 and width d, as shown in the figure. Find the force per unit length on the conductor.



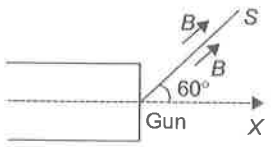
18. Find the work and power required to move the conductor of length l shown in the figure. One full turn in the anticlockwise direction at a rotationl frequency of n revolutions per second if the magnetic field is of magnitude B_0 everywhere and points radially outwards from Z-axis. The figure shows the surface traced by the wire AB.



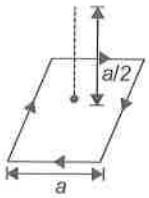
19. The figure shows a conductor of weight 1.0 N and length L = 0.5 m placed on a rough inclined plane making an angle 30° with the horizontal so that conductor is perpendicular to a uniform horizontal magnetic field of induction $B = 0.10T$. The coefficient of static friction between the conductor and the plane is 0.1. A current of $I = 10$ A flows through the conductor inside the plane of this paper as shown. What is the force needed to be the applied parallel to the inclined plane to sustaining the conductor at rest?



20. An electron gun G emits electron of energy 2 kev traveling in the (+)ve x-direction. The electron are required to hit the spot S where $GS = 0.1$ m and the line GS makes an angle of 60° with the x-axis, as shown in the figure. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region out sides to electron gun. Find the minimum value of B needed to make the electron hit S.



21. The magnetic field due to a current carrying square loop of side a at a point located symmetrically at a distance of $a/2$ from its centre (as shown is)

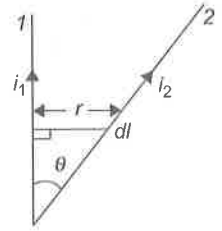


22. A charged particle of specific charge α is released from origin at time $t = 0$ with velocity $\vec{V} = V_0\hat{i} + V_0\hat{j}$ in magnetic field $\vec{B} = B_0\hat{i}$. The coordinates of the particle at time $t = \frac{\pi}{B_0\alpha}$ are (specific charge $\alpha = q/m$)

Previous Years' Questions

JEE Main

1. Wires 1 and 2 carrying currents I_1 and I_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance r from wire 1 (as shown in figure) due to the magnetic field of wire 1? **[AIEEE-2002]**



(A) $\frac{\mu_0}{2\pi r} I_1 I_2 dl \tan\theta$ (B) $\frac{\mu_0}{2\pi r} I_1 I_2 dl \sin\theta$

(C) $\frac{\mu_0}{2\pi r} I_1 I_2 dl \cos\theta$ (D) $\frac{\mu_0}{4\pi r} I_1 I_2 dl \sin\theta$

2. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit **[AIEEE-2002]**

(i) electrons (ii) protons

(iii) He^{2+} (iv) neutrons

The emission at the instant can be

(A) i, ii, iii (B) i, ii, iii, iv

(C) iv (D) ii, iii

3. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its **[AIEEE-2002]**

(A) speed (B) mass

(C) charge (D) magnetic induction

4. If an electron and a proton having same momenta enter perpendicularly to a magnetic field, then **[AIEEE-2002]**

(A) curved path of electron and proton will be same (ignoring the sense of revolution)

(B) they will move undeflected

(C) curved path of electron is more curved than that of proton

(D) path of proton is more curved

5. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields, B_A and B_B produced by them will be **[AIEEE-2002]**

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) 4

6. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a **[AIEEE-2002]**

(A) low resistance in parallel

(B) high resistance in parallel

(C) high resistance in series

(D) low resistance in series

7. An ammeter reads upto 1 A. Its internal resistance is 0.81 Ω. To increase the range to 10 A, the value of the required shunt is **[AIEEE-2003]**

(A) 0.03 Ω. (B) 0.3 Ω

(C) 0.9 Ω (D) 0.09 Ω

8. A particle of charge -16×10^{-18} C moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is **[AIEEE-2003]**

(A) 10^3 Wb/m^2 (B) 10^5 Wb/m^2

(C) 10^{16} Wb/m^2 (D) 10^{-3} Wb/m^2

9. A particle fo mass M and charge Q moving with velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is **[AIEEE-2003]**

(A) $\left(\frac{Mv^2}{R}\right) 2\pi R$ (B) zero

(C) $BQ2\pi R$ (D) $BQv2\pi R$

10. Curie temperature is the temperature above which **[AIEEE-2003]**

(A) a ferromagnetic material becomes para magnetic

(B) a paramagnetic material becomes diamagnetic

(C) a ferromagnetic material becomes diamagnetic

(D) a paramagnetic material becomes ferromagnetic

11. The magnetic lines of force inside a bar magnet **[AIEEE-2003]**

(A) are from north-pole to south-pole of the magnet

(B) do not exist

(C) depend upon the area fo cross-section of the bar magnet

(D) are from south-pole to north-pole of the magnet

218

12. A magnetic needle lying parallel to a magnetic field requires W unit of work to turn it through 60°. The torque needed to maintain the needle in this position will be **[AIEEE-2003]**

(A) $\sqrt{3}$ W (B) W

(C) $\left(\frac{\sqrt{3}}{2}\right)$W (D) 2 W

13. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now, it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio T'/T is **[AIEEE-2003]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2}$

(C) 2 (D) $\frac{1}{4}$

14. The materials suitable for making electromagnets should have **[AIEEE-2004]**

(A) high retentivity and high coercivity

(B) low retentivity and low coercivity

(C) high retentivity and low coercivity

(D) low retentivity and high coercivity

15. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be **[AIEEE-2004]**

(A) 2 s (B) $\frac{2}{3}$s

(C) $2\sqrt{3}$ s (D) $\frac{2}{\sqrt{3}}$s

16. Two long conductors, separated by a distance ***d*** carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its directions is reversed. The distances is also increased to $3d$. The new value of the force between them is **[AIEEE-2004]**

(A) $-2F$ (B) $\frac{F}{3}$

(C) $-\frac{2F}{3}$ (D) $-\frac{F}{3}$

17. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is $54\mu T$. What will be its value at the centre of the loop? **[AIEEE-2004]**

(A) $250\mu T$ (B) $150\mu T$

(C) $125\mu T$ (D) $75\mu T$

18. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be **[AIEEE-2004]**

(A) nB (B) n^2B

(C) $2nB$ (D) $2n^2B$

19. A current i A flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is **[AIEEE-2004]**

(A) infinite (B) zero

(C) $\frac{\mu_0}{4\pi}\cdot\frac{2i}{r}$ T (D) $\frac{2i}{r}$ T

20. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is **[AIEEE-2005]**

(A) $\frac{2\pi mq}{B}$ (B) $\frac{2\pi q^2B}{m}$

(C) $\frac{2\pi qB}{m}$ (D) $\frac{2\pi m}{qB}$

21. A magnetic needle is kept in a non-uniform magnetic field. It experiences **[AIEEE-2005]**

(A) a torque but not a force

(B) neither a force nor a torque

(C) a force and a torque

(D) a force but not a torque

22. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 A and 4 A are the currents flowing in each coil respectively. The magnetic induction in Wb/m^2 at the centre of the coils will be **[AIEEE-2005]**

($\mu_0 = 4\pi \times 10^{-7}$ Wb/Am)

(A) 12×10^{-5} (B) 10^{-5}

(C) 5×10^{-5} (D) 7×10^{-5}

23. Two thin, long parallel wires, separated by a distance d carry a current of I ampere in the same direction. They will **[AIEEE-2005]**

(A) attract each other with a force of $\frac{\mu_0 I^2}{(2\pi d)}$

(B) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d)}$

(C) attract each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

(D) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

24. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V, the resistance in ohms needed to be connected in series with the coil will be **[AIEEE-2005]**

(A) 10^3 (B) 10^5

(C) 99995 (D) 9995

25. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a **[AIEEE-2006]**

(A) helix (B) straight line

(C) ellipse (D) circle

26. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will **[AIEEE-2006]**

(A) attract N_1 and N_2 strongly but repel N_3

(B) attract N_1 strongly, N_2 weakly and repe N_3 weakly

(C) attract N_1 strongly, but repel N_2 and N_3 weakly

(D) attract all three of them

27. A charged particle moves through a magnetic field perpendicular to its direction. Then **[AIEEE-2007]**

(A) the momentum changes but the kinetic energy is constant

(B) both momentum and kinetic energy of the particle are not constant

(C) both momentum and kinetic energy of the particle are constant

(D) kinetic energy changes but the momentum is constant

28. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then **[AIEEE-2007]**

(A) the magnetic field is zero only on the axis of the pipe

(B) the magnetic field is different at different points inside the pipe

(C) the magnetic field at any point inside the pipe is zero

(D) the magnetic field at all points inside the pipe is the same, but not zero

29. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then **[AIEEE-2007]**

(A) $\vec{v} = \vec{E} \times \frac{\vec{B}}{B^2}$ (B) $\vec{v} = \vec{B} \times \frac{\vec{E}}{B^2}$

(C) $\vec{v} = \vec{E} \times \frac{\vec{B}}{E^2}$ (D) $\vec{v} = \vec{B} \times \frac{\vec{E}}{E^2}$

30. A long straight wire of radius **a** carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is **[AIEEE-2007]**

(A) $\frac{1}{4}$ (B) 4

(C) 1 (D) $\frac{1}{2}$

31. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by **[AIEEE-2007]**

(A) $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{1/2}$ (B) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$

(C) $\frac{\mu_0}{2\pi d}(I_1 + I_2)$ (D) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)$

32. A horizontal overhead power line is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is **[AIEEE-2008]**

$(\mu = 4\pi \times 10^{-7}\,\text{TmA}^{-1})$

(A) 2.5×10^{-7}T, southward

(B) 5×10^{-6}T, northward

220

(C) 5×10^{-6}T, southward

(D) 2.5×10^{-7}T, northward

33. Relative permittivity and permeability of a material are ε_r and μ_r respectively. Which of the following values of these quantities are allowed for a diamagnetic material? **[AIEEE-2008]**

(A) $\varepsilon_r = 0.5, \mu_r = 1.5$

(B) $\varepsilon_r = 1.5, \mu_r = 0.5$

(C) $\varepsilon_r = 0.5, \mu_r = 0.5$

(D) $\varepsilon_r = 1.5, \mu_r = 1.5$

34. Due to the presence of the current I_1 at the origin **[AIEEE-2009]**

(A) The forces on AB and DC are zero

(B) The forces on AD and BC are zero

(C) The magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

(D) The magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$

35. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is **[AIEEE-2009]**

(A) zero

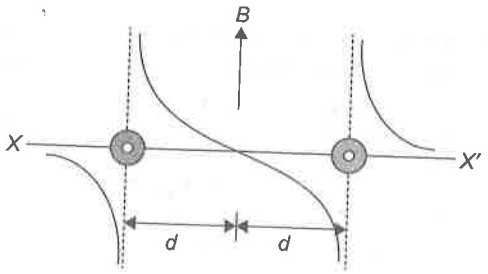
(B) $\frac{\mu_0(b-a)}{24ab}$

(C) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$

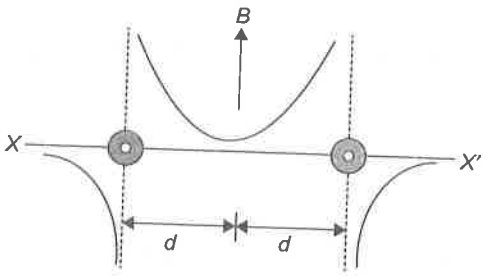
(D) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

36. Two long parallel wires are at a distance $2d$ apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by **[AIEEE-2010]**

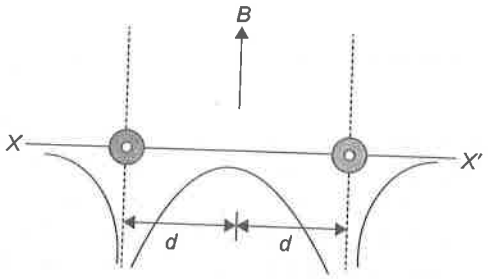
(A)



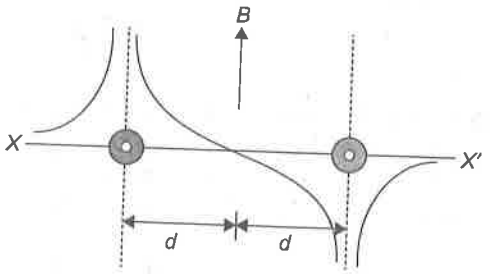
(B)



(C)



(D)



37. A current I flows in a infinitely long wire with cross section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is **[AIEEE-2011]**

(A) $\frac{\mu_0 I}{2\pi^2 R}$ (B) $\frac{\mu_0 I}{2\pi R}$

(C) $\frac{\mu_0 I}{4\pi R}$ (D) $\frac{\mu_0 I}{\pi^2 R}$

38. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is **[AIEEE-2011]**

(A) 0.75 mV (B) 0.50 mV

(C) 0.15 mV (D) 1 mV

39. A thin circular disk of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disk rotates about its axis with a uniform angular speed. The magnetic moment of the disk is **[AIEEE-2011]**

(A) $2\pi R^4\sigma\omega$ (B) $\pi R^4\sigma\omega$

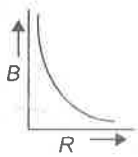
(C) $\frac{\pi R^4}{2}\sigma\omega$ (D) $\frac{\pi R^4}{4}\sigma\omega$

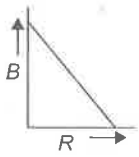
40. A horizontal straight wire 20 m long extending from east to west falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wb/m^2. The instantaneous value of the emf induced in the wire will be **[AIEEE-2011]**

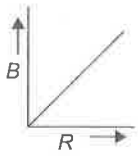
(A) 6.0 mV (B) 3 mV

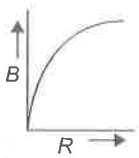
(C) 4.5 mV (D) 1.5 mV

41. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation, a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure. **[AIEEE-2012]**

(A) 

(B) 

(C) 

(D) 

42. Two short bar magnetic of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centre is close to: **[JEE Main-2013]**

(Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

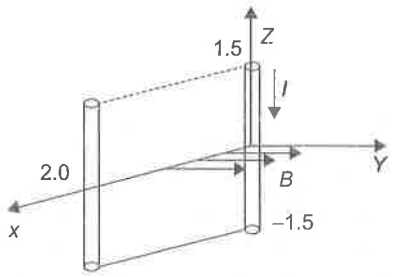
(A) 3.50×10^{-4} Wb/m^2

(B) 5.80×10^{-4} Wb/m^2

(C) 3.6×10^{-5} Wb/m^2

(D) 2.56×10^{-4} Wb/m^2

43. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{0.2x} \hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis. **[JEE Main-2014]**



(A) 14.85 W (B) 29.7 W

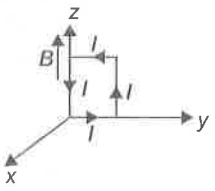
(C) 1.57 W (D) 2.97 W

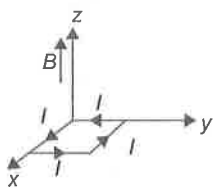
44. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 A m^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is: **[JEE Main-2014]**

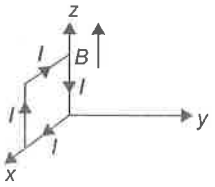
(A) 3 A (B) 6 A

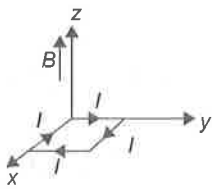
(C) 30 mA (D) 60 mA

45. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below;

(a) 

(b) 

(c) 

(d) 

If there is a uniform magnetic field of $0.3T$ in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium? **[JEE Main-2015]**

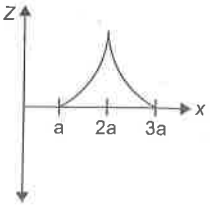
(A) (b) and (d), respectively

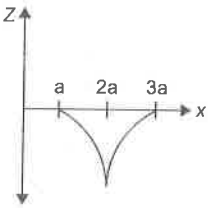
(B) (b) and (c), respectively

(C) (a) and (b), respectively

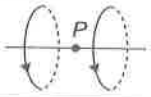
(D) (a) and (c), respectively

224

(C) 

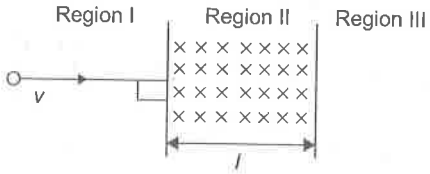
(D) 

13. Two wires each carrying a steady current I are shown in four configurations in **Column I.** Some of the resulting effects are described in **Column II.** Match the statements in **Column I** with the statement in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[JEE-2007]**

	Column I		Column II
(A)	Point P is situated midway between wires	(P)	The magnetic the fields (B) at P due to the currents in the wires are in the same direction.
(B)	Point P is situated at the mid-point of the of line joining the centers the circular wires, which have same radii. 	(Q)	The magnetic fields (B) at P due to the currents in the wires are in opposite directions
(C)	Point P is situated at the mid-point of the line joining the centers of the circular	(R)	There is no magnetic field at P
(D)	Point P is situated at the common center of each other	(S)	The wires repel the wires

14. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is l. Choose the correct choice(s). **[JEE-2008]**

Figure:



(A) The particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

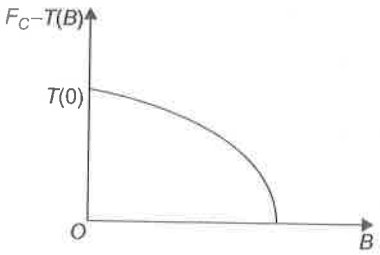
(B) The particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) Path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

(D) Time spent in Region II is same for any velocity V as long as the particle returns to Region I

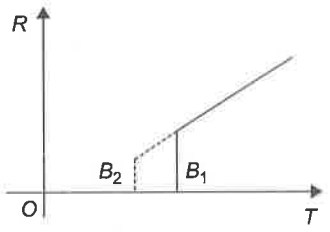
15. A steady current I goes through a wire loop PQR having shape of a right angle triangle with $PQ = 3x$, $PR = 4x$ and $QR = 5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k. **[JEE-2009]**

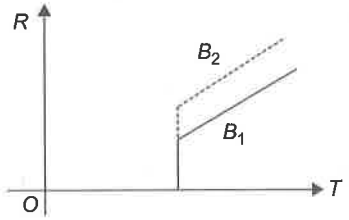
Common Data for Questions 16–17: Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T(0)$ if they are placed in a magnetic field, i.e, the the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_c(B)$ on B is shown in the figure.

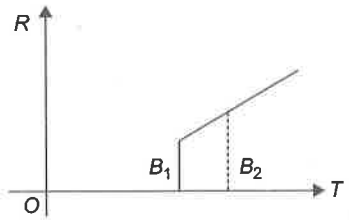


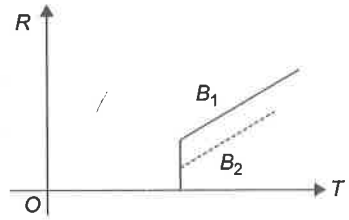
16. In the graph below, the resistance R of a super conductor is shown as function of its temperature T for two different magnetic field B_1 (solid line) and B_2

(dashed line). If B_2 is larger than B_1. Which of the follwing graphs shows the correct variation of R with T in these fields? **[JEE-2010]**

(A) 

(B) 

(C) 

(D) 

17. A superconductor this $T_0(0) - 100$ K. When a magnetic field of 7.5 T is applied. Its T_c decreases to 75 K. For this material one can definitely say that when **[JEE-2010]**

(A) $B = 5$ T, $T_C(B) = 80$ K

(B) $B = 5$ T, 75 K $< T_c(B) <$ 100 K

(C) $B = 10$ T, 75 K $< T_C(B) <$ 100 K

(D) $B = 10$ T, $T_C(B) = 70$ K

18. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? **[JEE-2011]**

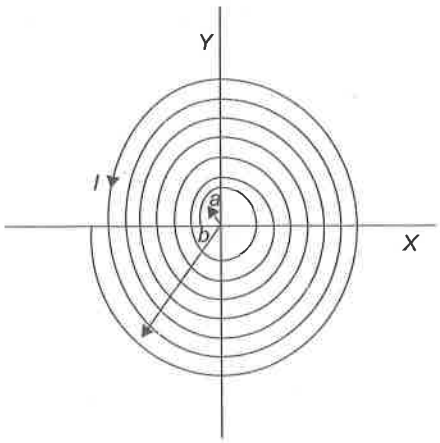
(A) They will never come out of the magnetic field region.

(B) They will come out travelling along the parallel paths.

(C) They will come out at the same time.

(D) They will come out at different times.

19. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X-Y plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the center of the spiral is: **[JEE-2011]**



(A) $\dfrac{\mu_0 NI}{2(b-a)} \ln\left(\dfrac{b}{a}\right)$

(B) $\dfrac{\mu_0 NI}{2(b-a)} \ln\left(\dfrac{b+a}{b-a}\right)$

(C) $\dfrac{\mu_0 NI}{2b} \ln\left(\dfrac{b}{a}\right)$

(D) $\dfrac{\mu_0 NI}{2b} \ln\left(\dfrac{b+a}{b-a}\right)$

20. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0 \hat{j}$ and $\vec{B} = B_0 \hat{j}$. At time $t = 0$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making and angle with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$? **[JEE-2012]**

(A) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.

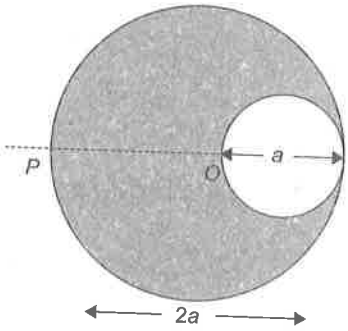
(B) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.

226

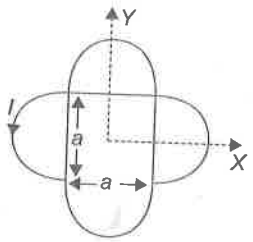
(C) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(D) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

21. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aj$, then the value of N is: **[JEE-2012]**



22. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{K}$ is coming out of the plane of the paper. The magnetic moment of the current loop is: **[JEE-2012]**



(A) $a^2 I\hat{K}$

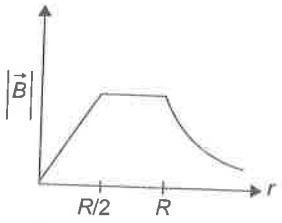
(B) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{K}$

(C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{K}$

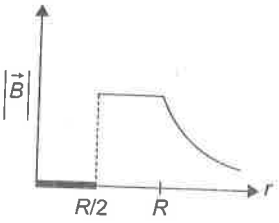
(D) $(2\pi+1)a^2 I\hat{K}$

23. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by: **[JEE-2012]**

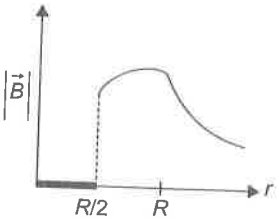
(A)



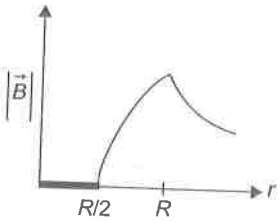
(B)



(C)



(D)



24. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}$ ms^{-1}, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i} + \hat{j})$ ms^{-1}. The correct statement (s) is (are) **[JEE-2013]**

(A) The direction of the magnetic field is -z direction.

(B) The direction of the magnetic field is $+z$ direction.

(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.

(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

25. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are) **[JEE-2013]**

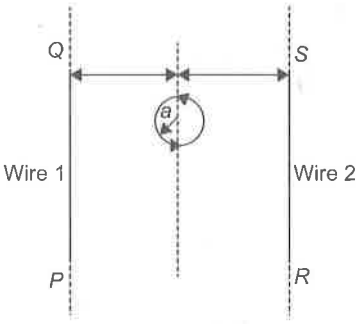
(A) In the region $0 < r < R$, the magnetic field is non-zero.

(B) In the region $R < r < 2R$, the magnetic field is along the common axis.

(C) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis.

(D) In the region $r > 2R$, the magnetic field is non-zero.

Common Data for Questions 26–27: The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above. **[JEE Advanced-2014]**



26. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$

(B) current in wire 1 and wire 2 is the direction PQ and SR, respective and $h \approx a$

(C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$

(D) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

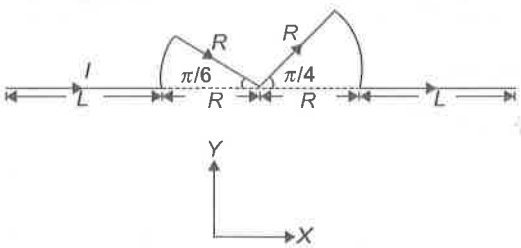
27. consider $d >> a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$

(C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

28. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, it the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{x_1} = 3$, the value of $\frac{R_1}{R_2}$ is **[JEE Advanced-2014]**

29. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are) **[JEE Advanced-2015]**



(A) If $\vec{B}$ is along, $\hat{z}, F\alpha(L + R)$

(B) If $\vec{B}$ is along, $\hat{x}, F = 0$

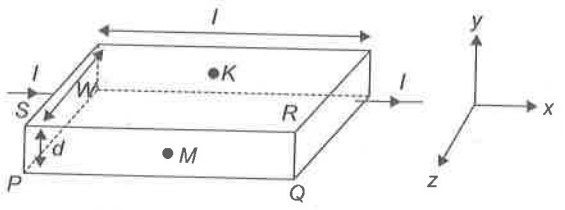
(C) If $\vec{B}$ is along, $\hat{y}, F\alpha(L + R)$

(D) If $\vec{B}$ is along, $\hat{z}, F = 0$

Common Data for Questions 30–31: In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively. A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carries experience a net deflection along the z-direction. This results in accumulation of charge caries on the surface $PQRS$ and appearance of equal and opposite charges on the

face opposite to *PQRS*. A potential difference along the *z*-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.

[JEE Advanced-2015]



30. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is (are).

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

31. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option (s) is (are).

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

ANSWER KEYS

Exercises

JEE Main

1. A	**2.** B	**3.** C	**4.** C	**5.** B	**6.** C	**7.** B	**8.** A	**9.** A	**10.** A
11. B	**12.** B	**13.** B	**14.** D	**15.** A	**16.** A	**17.** D	**18.** B	**19.** A	**20.** B
21. C	**22.** B	**23.** C	**24.** B	**25.** A	**26.** B	**27.** A	**28.** D	**29.** B	**30.** B
31. B	**32.** A	**33.** A	**34.** B	**35.** B	**36.** B	**37.** D	**38.** B	**39.** A	**40.** D
41. C	**42.** C	**43.** A	**44.** A						

JEE Advanced

1. B	**2.** A	**3.** C	**4.** A	**5.** D	**6.** D	**7.** B	**8.** C	**9.** A	**10.** A
11. A	**12.** A, B, C		**13.** A, B, C		**14.** A, B, C, D		**15.** C	**16.** B, D	**17.** A, D
18. A	**19.** A	**20.** C	**21.** C	**22.** C	**23.** A	**24.** B, C	**25.** A, D	**26.** C, D	**27.** C
28. A, B	**29.** A	**30.** C	**31.** A	**32.** B	**33.** A	**34.** B	**35.** B	**36.** A	**37.** D
38. B	**39.** B	**40.** C	**41.** A	**42.** C	**43.** B	**44.** A	**45.** D	**46.** C	**47.** D
48. C	**49.** C	**50.** C	**51.** C	**52.** A	**53.** B	**54.** C	**55.** D	**56.** C	**57.** D
58. D	**59.** C	**60.** B	**61.** A	**62.** A	**63.** D	**64.** B	**65.** A	**66.** A	**67.** B
68. A	**69.** C	**70.** D	**71.** A	**72.** A	**73.** B	**74.** A	**75.** B	**76.** B	**77.** A
78. D									

JEE Advanced

Level I

1. $\frac{\sqrt{2}\mu_0 i}{8\pi l}$ 2. μ_0 weber $\cdot$ m^{-1} 3. In the plane of the drawing from right to left

4. $\frac{\sqrt{5}}{2\sqrt{2}} 4\pi \times 10^{-5} T$ 5. (i) $1.3 \times 10^{-4} T$, (ii) zero 6. $\frac{4}{\sqrt{7}}$ 7. $\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k} + \frac{1}{\pi}\hat{j}\right)$

8. $\frac{\mu_0 I}{4\pi r}\left[\frac{3}{2}\pi + 1\right]$ 9. $B = \frac{\mu_0 i}{4\pi R}\sqrt{2(2\pi^2 - 2\pi + 1)}$ 10. $B_1 = \frac{\mu_0 b r_1^2}{3}$, $B_2 = \frac{\mu_0 b R^3}{3r_2}$

11. $\frac{\mu_0 iqv}{2\pi a}$ 12. $\frac{2mv_0}{qB}$ 13. $t = m\frac{\alpha}{qB}$, where $\alpha = \sin^{-1}\left(\frac{dB\sqrt{q}}{\sqrt{2mV}}\right)$ if $d < R$; $T/2$ if $d > R$

14. $10\hat{k}$ 15. (a) $\frac{3mv^2}{4qa}$, (b) $\frac{3mv^3}{4a}$, (c) zero 16. $\frac{mEl}{Be}$

17. $\frac{\mu_0 II'C}{2\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$ to the left 18. $\sqrt{2}IRB$ 19. $\frac{W}{l} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\frac{r_2}{r_1} = 27.6\mu$ J/m

20. zero 21. $F = \alpha a^2 \hat{ij}$ 22. $T_0 = 2\pi\sqrt{\frac{m}{61B}} = 0.57s$

Level II

1. $\frac{\mu_0 I_0}{2\pi}\frac{\sqrt{3}b}{(a^2 + b^2)}$ 2. (i) $\frac{\mu_0}{4\pi}\left(\frac{4I}{a}\right)$ along Y-axis, (ii) $\frac{\mu_0}{4\pi}\left(\frac{I^2}{2a}\right)\sqrt{10}, \tan^4\left(\frac{1}{3}\right) + \pi$ with positive axis

3. $\frac{\mu_0 I_1 I_2}{4\pi}\ln(3)$ along – ve z direction 4. $\frac{mg}{2Il}$ 5. $F = \left(\frac{\mu_0 I^2}{2\pi}\right)\ln\left(\frac{L^2 + a^2}{a^2}\right)(-\hat{k})$, zero

6. (a) $B = \frac{\mu_0 J}{2}\left(\frac{a^2}{2R - b} - \frac{R}{2}\right)$, (b) $B_x = \mu_0 JR\left(\frac{1}{4} - \frac{a^2}{4R^2 + b^2}\right)$, $B_y = -\frac{\mu_0 J}{2}\left(\frac{a^2 b}{4R^2 + b^2}\right)$

7. $\frac{Q\omega}{4}h^2\tan^2\theta$ 8. (a) $\frac{QV}{m}\frac{\mu_0 I}{6a}\left(\frac{3\sqrt{3}}{\pi} - 1\right)$, (b) $\vec{\tau} = BI\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)a^2\hat{j}$

9. (a) 0 (b) $1.41 \times 10^{-6} T$, 45° in xz-plane, (c) $5 \times 10^{-6} T$, $+x$ direction]

10. (a) 20 min. (b) 5.94×10^{-2} Nm 11. $\sqrt{15}$ C

12. (a) 1.4×10^{-4} m/s (b) 4.5×10^{-23} N (down) (c) 2.8×10^{-4} V/m (down)
(d) 5.7×10^{-6} V (top +, bottom–) (e) same as (c)

13. (a) $I = \frac{mg}{\pi r\left(B_x^2 + B_y^2\right)^{1/2}}$ (b) $I = \frac{mg}{\pi r B_x}$ 14. (a) increase, (b) decrease

17. $\frac{\mu_0 i J_0}{\pi}\tan^{-1}\left(\frac{d}{2h}\right)(-\hat{k})$ 18. $-2\pi r B_0 il, -2\pi r B_0 iln$ 19. 0.62 N $< F <$ 0.88 N 20. $B_{min} = 4.7 \times 10^{-3} T$

21. $\frac{2\mu_0 i}{\sqrt{3}\pi a}$ 22. $\left(\frac{V_0\pi}{B_0\alpha}, 0, -\frac{2V_0}{B_0\alpha}\right)$

230

Previous Years' Questions

JEE Main

1. C	2. A	3. A	4. A	5. A	6. A	7. D	8. A	9. B	10. A
11. D	12. A	13. B	14. C	15. B	16. C	17. A	18. B	19. B	20. D
21. C	22. C	23. A	24. D	25. B	26. B	27. A	28. C	29. A	30. C
31. B	32. C	33. B	34. B	35. B	36. A	37. D	38. C	39. D	40. B
41. A	42. D	43. D	44. A	45. A	46. C				

JEE Advanced

1. B 2. A 3. D 4. (a) current in loop $PQRS$ is clockwise from P to QRS.,

(b) $\vec{F} = BI_0b(3\hat{k} - 4\hat{i})$, (c) $I = \frac{mg}{6bB_0}$ 5. A 6. A 7. $\omega = \frac{dT_0}{QR^2B}$

8. $\frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha} \cdot \frac{q_\alpha}{q_p}} = \frac{1}{\sqrt{2}}$ 9. D 10. (a) $k = NAB$, (b) $C = \frac{2i_0NAB}{\pi}$, (c) $Q \times \sqrt{\frac{NAB\pi}{2li_0}}$

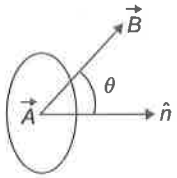
11. A, C 12. A 13. (A) Q, R; (B) P; (C) Q, R; (D) Q or (A) Q, R; (B) P; (C) Q, R; (D) Q, S 14. (A), (C) and (D) 15. 7 16. A 17. B 18. B, D 19. A 20. C, D 21. 5 22. B 23. D 24. AC 25. AD 26. C 27. B 28. 3 29. A, B, C 30. A, D 31. A, C

Chapter 5

Electromagnetic Induction

MAGNETIC FLUX

Consider a closed curve enclosing an area A (as shown in the figure). Let there be a uniform magnetic field $\vec{B}$ in that region. The magnetic flux through the area $\vec{A}$ is given by



$$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta \qquad (1)$$

where θ is the angle which the vector B makes with the normal to the surface. If $\vec{B}$ is perpendicular to $\vec{A}$, then the flux through the closed area $\vec{A}$ is zero. SI unit of magnetic flux is weber (Wb).

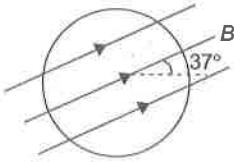
Notes

- Area vector is $\perp$ to the surface
- For open surface choose one direction as the area vector direction and stick to it for the whole problem.
- For closed surfaces outward normal is taken as area vector direction
- Flux is basically count of number of lines crossing a surface
- $\oint \vec{B} \cdot \vec{ds} = 0$ because magnetic field lines exists in closed loop.

SOLVED EXAMPLES

EXAMPLE 1

Find flux passing through Area?



SOLUTION

Since $\vec{A}$ is $\perp$ to $\vec{B}$

$$\varphi = \vec{B} \cdot \vec{A} = 0$$ ■

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$\varepsilon = -\frac{d\varphi}{dt} \qquad (2)$$

where $\phi = \int \vec{B} \cdot d\vec{A}$ the flux of magnetic field through the area.

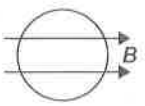
The emf so produced drives an electric current through the loop. If the resistance of the loop is R, then the current

$$i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt} \qquad (3)$$

SOLVED EXAMPLES

EXAMPLE 2

A coil is placed in a constant magnetic field. The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.



SOLUTION

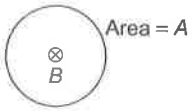
$\phi = 0$ (always) since area is perpendicular to magnetic field.

$\therefore$ emf $= 0$ ■

232

EXAMPLE 3

Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.



SOLUTION

$\phi = BA$ (always) = const.

$\therefore$ emf = 0 ■

EXAMPLE 4

Show that if the flux of magnetic induction through a coil changes from ϕ_1 to ϕ_2, then the charge q that flows through the circuit of total resistance R is given by $q = \frac{\phi_2 - \phi_1}{R}$, where R is the resistance of the coil.

SOLUTION

Let ϕ be the instantaneous flux. Then $\frac{d\phi}{dt}$ is the instantaneous rate of change of flux which is equal to the magnitude of the instantaneous emf so the current in the circuit $|i| = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$, since the current is the rate of flow of charge, that is, $i = \frac{dq}{dt}$

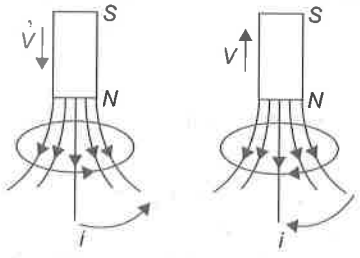
$$q = \int i\,dt \quad \text{or} \quad q = \int_{t=0}^{t=t} \left(\frac{1}{R} \cdot \frac{d\phi}{dt}\right) dt$$

where τ is the time during which change takes place, but at $t = 0$, $\phi = \phi_1$, and at $t = t$, $\phi = \phi_2$

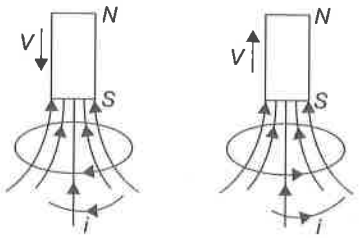
$$\therefore \quad q = \frac{1}{R}\int_{\phi=\phi_1}^{\phi=\phi_2} d\phi = \frac{\phi_2 - \phi_1}{R}$$ ■

LENZ'S LAW

The effect of the induced emf is such as to oppose the change in flux that produces it.



(a) (c)



(b) (d)

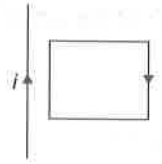
In figure (a and b) as the magnet approaches the loop, the flux through the loop increases. The induced current sets up an induced magnetic field B_{ind} whose flux opposes this change. The direction of B_{ind} is opposite to that of external field B_{ext} due to the magnet.

In figure (c and d) the flux through the loop decreases as the magnet moves away from the loop, the flux due to the induced magnetic field tries to maintain the flux through the loop. The direction of B_{ind} is same as that of B_{ext} due to magnet.

Lenz's law is closely related to the law of conservation of energy and is actually a consequence of this general law of nature. As the north pole of the magnet moves towards the loop an induced current is produced. This opposes the motion of N-pole of the bar magnet. Thus, in order to move the magnet toward the loop with a constant velocity an external force is to be applied. The work done by this external force gets transformed into electric energy, which induces current in the loop.

There is another alternative way to find the direction of current inside the loop which is described below.

Figure shows a conducting loop placed near a long, straight wire carrying a current i as shown. If the current increases continuously, then there will be an emf induced inside the loop. Due to this induced emf, an electric current is induced. To determine the direction of current inside the loop we put an arrow as shown. The right hand thumb rule shows that the normal to the loop is going into the plane. Again the same rule shows that the magnetic field at the site of the loop is also going into the plane of the diagram.



Thus $\vec{B}$ and $d\vec{A}$ are in the same direction. Therefore $\int \vec{B} \cdot d\vec{A}$ is positive if i increases, the magnitude of flux ϕ increases. Since magnetic flux ϕ is positive and its magnitude

increases, $\frac{d\phi}{dt}$ is positive. Thus ε is negative and hence the current is negative. Thus the current induced is opposite, to that of arrow.

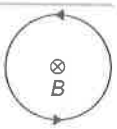
Brain Teaser

Two identical coaxial circular loops carry equal currents circulating in the same direction. What will happen to the current in each loop if the loops approach each other?

SOLVED EXAMPLES

EXAMPLE 5

Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.

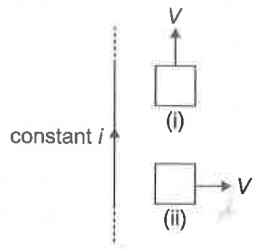


SOLUTION

Inward flux is increasing with time. To oppose it outward magnetic field should be induced. Hence current will flow in anticlockwise. ■

EXAMPLE 6

Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



SOLUTION

In loop (i) no emf will be induced because there is no flux change.

In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise. ■

CALCULATION OF INDUCED EMF

As we know that magnetic flux (ϕ) linked with a closed conducting loop = $BA\cos\theta$ where B is the strength of the magnetic field, A is the magnitude of the area vector and θ is the angle between magnetic field vector and area vector.

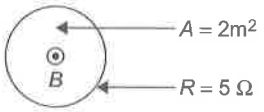
Hence flux will be affected by change in any of them.

By Changing the Magnetic Field

SOLVED EXAMPLES

EXAMPLE 7

Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil. The magnetic field is decreasing at a rate of 10 T/s. Find out current in magnitude and direction



SOLUTION

$$\phi = B \cdot A$$

$$\text{emf} = A \cdot \frac{dB}{dt} = 2 \times 10 = 20 \text{ V}$$

$\therefore$ $i = 20/5 = 4$ A From Lenz's law direction of current will be anticlockwise. ■

EXAMPLE 8

The magnetic flux (ϕ_2) in a closed circuit of resistance 20 Ω varies with time (t) according to the equation $\phi = 7t^2 - 4t$ where ϕ is in weber and t is in seconds. The magnitude of the induced current at $t = 0.25$ s is

(A) 25 mA (B) 0.025 mA
(C) 47 mA (D) 175 mA

SOLUTION

$$\phi = 7t^2 - 4t$$

$\Rightarrow$ Induced emf: $|e| = \frac{d\phi}{dt} = 14t - 4$

$\Rightarrow$ Induced current:

$$i = \frac{|e|}{R} = \frac{|14t - 4|}{20} = \frac{|14 \times 0.25 - 4|}{20} \quad (\text{at } t = 0.25 \text{ s})$$

$$= \frac{0.5}{20} = 2.5 \times 10^{-2} \text{ A}$$

$\therefore$ (A) ■

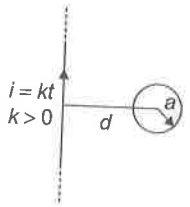
EXAMPLE 9

Consider a long infinite wire carrying a time varying current $i = kt(k > 0)$. A circular loop of radius a and resistance R is placed with its centre at a distance d from the wire ($a << d$). Find out the induced current in the loop?

SOLUTION

Since current in the wire is continuously increasing therefore we conclude that magnetic field due to this wire in the region is also increasing.

Magnetic field B due to wire $= \frac{\mu_0 i}{2\pi d}$ going into and perpendicular to the plane of the paper



Flux through the circular loop,

$$\phi = \frac{\mu_0 i}{2\pi d} \times \pi a^2$$

$$\phi = \frac{\mu_0 a^2 kt}{2d}$$

Induced emf in the loop

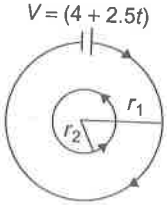
$$\varepsilon = -\frac{d\phi}{dt} = \frac{-\mu_0 a^2 k}{2d}$$

Induced current in the loop $i = \frac{|\varepsilon|}{R} = \frac{\mu_0 a^2 k}{2dR}$

Direction of induced current in the loop is anticlockwise. ■

EXAMPLE 10

Two concentric coplanar circular loops have diameters 20 cm and 2 m and resistance of unit length of the wire $= 10^{-4}$ Ω/m. A time-dependent voltage $V = (4 + 2.5t)$ V is applied to the larger as shown. The current in the smaller loop is



SOLUTION

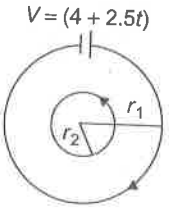
$$r_1 = 1.0 \text{ m}, r_2 = 10^{-1} \text{ m}$$

Resistance of outer loop $= 2\pi \times 10^{-4}$ Ω

Resistance of inner loop $= 0.2\pi \times 10^{-4}$ Ω

Current in outer loop $= \frac{V}{R} = \frac{(4 + 2.5t)}{2\pi \times 10^{-4}}$ A

or $$i_0 = \left[\left(\frac{2}{\pi}\right) \times 10^4 + \left(\frac{1.25}{\pi}\right) \times 10^4 \times t\right] \text{A}$$



Magnetic field produced at the common centre (see figure)

$$B = \frac{\mu_0 i}{2r_1}$$

or

$$B = \frac{4\pi}{2} \times 10^{-7} \times \frac{[(2 + 1.25t) \times 10^4]}{\pi} = 2 \times 10^{-3}(2 + 1.25t) \text{ T}$$

Hence, flux linked with the inner loop,

$$\phi = BA = 2 \times 10^{-3} (2 + 1.25t) \times \pi(0.1)^2$$

$$= 2\pi \times 10^{-5} (2 + 1.25t) \text{ Wb}$$

Hence, the emf induced in smaller loop =

$$\varepsilon = -\frac{d\phi}{dt} = -2\pi \times 10^{-5} \times 1.25 = -2.5\pi \times 10^{-5} \text{ V}$$

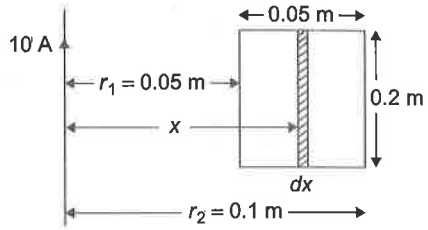
The negative sign indicates that the induced emf (or current) is opposite to applied emf (or current)

Hence, the current induced in the inner (smaller) loop is

$$i = \frac{|\varepsilon|}{R} = \frac{2.5\pi \times 10^{-5} \text{ V}}{(0.2\pi \times 10^{-4})\Omega} = 1.25 \text{ A}$$ ■

EXAMPLE 11

A rectangular wire frame of length 0.2 m, is located at a distance of 5×10^{-2} m from a long straight wire carrying a current of 10 A as shown in the figure. The width of the frame = 0.05 m. The wire is in the plane of the rectangle. Find the magnetic flux through the rectangular circuit. If the current decays uniformly to 0 in 0.2 s, find the emf induced in the circuit.



SOLUTION

A current, $i = 10$ A is flowing in the long straight wire. Consider a small rectangular strip (in the rectangular wire frame) of width dx at a distance x from the straight wire.

The magnetic flux at the location of the strip,

$$B_x = \frac{\mu_0 i}{2\pi x}$$

The flux linked with the infinitesimally small rectangular strip

$$= B_x \times \text{Area of the strip} = d\phi_x = \frac{\mu_0 i}{2\pi x} l dx$$

where l is the length of the rectangular wire circuit

$$= 2 \times 10^{-1} \text{ m}$$

or
$$d\phi_x = \left(\frac{\mu_0 il}{2\pi}\right)\left(\frac{dx}{x}\right)$$

Hence, the total magnetic flux linked with the rectangular frame

$$= \int d\phi_x = \phi = \frac{\mu_0 il}{2\pi}[\log_e x]_{r_1}^{r_2}$$

or
$$\phi = \frac{\mu_0 il}{2\pi}[\log_e r_2 - \log_e r_1] = \frac{\mu_0 il}{2\pi}\log_e\left(\frac{r_2}{r_1}\right)$$

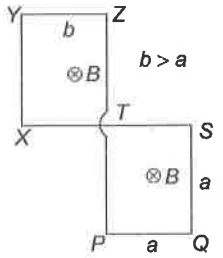
Substituting values, we get

$$\phi = 2 \times 10^{-7} \times 10 \times 2 \times 10^{-1} \times \log_e 2$$
$$= 2.772 \times 10^{-7} \text{ Wb}$$

$$\text{Induced emf } |\varepsilon| = \frac{d\phi}{dt} = \frac{\mu_o l \log_e\left(\frac{r_2}{r_1}\right)}{2\pi}\frac{di}{dt}$$
$$= (2 \times 10^{-7} \times 2 \times 10^{-1} \log_e 2)\frac{10}{0.2}$$
$$= 1.386 \times 10^{-6} \text{ V} = 1.386\ \mu\text{V}$$

EXAMPLE 12

Figure shows a wire frame $PQSTXYZ$ placed in a time varying magnetic field given as $B = \beta t$, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.



SOLUTION

Induced emf in part $PQST = \beta a^2$ (in anticlockwise direction, from Lenz's Law) Similarly induced emf in part $TXYZ = \beta b^2$ (in anticlockwise direction, from Lenz's Law)

Total resistance of the part $PQST = \lambda 4a$

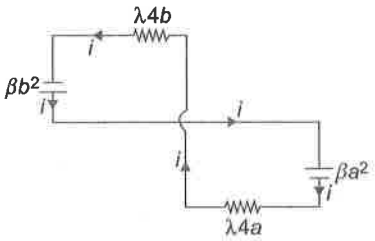
Total resistance of the part $PQST = 14b$.

The equivalent circuit it is shown in the diagram.

Writing KVL along the current flow

$$\beta b^2 - \beta a^2 - \lambda 4ai - \lambda 4bi = 0$$

$$i = \frac{\beta}{4\lambda}(b - a)$$



Brain Teaser:

A copper ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. Will the acceleration of the falling magnet be equal to, greater than or lesser than the acceleration due to gravity?

By Changing the Area

EXAMPLE 13

A space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop

936

being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O, and perpendicular to the plane of the paper in the clockwise direction. The effective resistance of the loop is R.

(a) Obtain an expression for the magnitude of the induced current in the loop.
(b) Show the direction of the current when the loop is entering into the region II.
(c) Plot a graph between the induced emf and the time of rotation for two periods of rotation.

SOLUTION

(a) As in time t, the arc swept by the loop in the field, i.e., region II.

$$A = \frac{1}{2} r(r\theta) = \frac{1}{2} r^2 \omega t$$

So the flux linked with the rotating loop at time t,

$$\phi = BA = \frac{1}{2} B\omega r^2 t \quad [\theta = \omega t]$$

and hence the induced emf in the loop,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{1}{2} B\omega r^2 = \text{constant.}$$

And as the resistance of the loop is R, the induced current in it,

$$i = \frac{\varepsilon}{R} = -\frac{B\omega r^2}{2R}$$

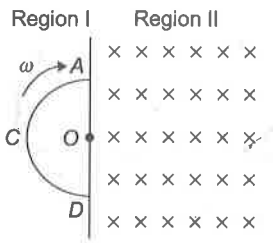


Figure (a)

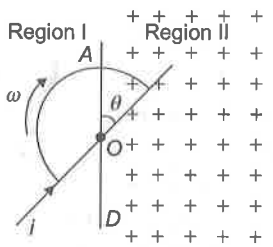


Figure (b)

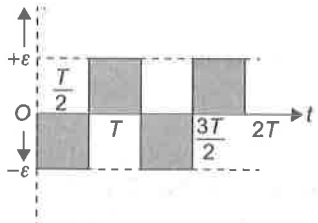


Figure (c)

(b) When the loop is entering the region II, i.e., the field figure (b), the inward flux linked with it will increase, so in accordance with Lenz's law an anticlockwise current will be induced in it.
(c) Taking induced emf to the negative when flux linked with the loop is increasing and positive when decreasing, the emf versus time graph will be, as shown in figure (c) ■

EXAMPLE 14

Two parallel, long, straight conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. Now they start moving out with a constant velocity (v).

(a) Will the induced emf be time dependent?
(b) Will the current be time dependent?

SOLUTION

(a) Yes, φ (instantaneous flux) $= B(a + 2vt)^2$

$$\therefore \quad \varepsilon = \frac{d\varphi}{dt} = 4Bv(a + 2vt)$$

(b) No, (instantaneous current) $i = \frac{\varepsilon}{R}$

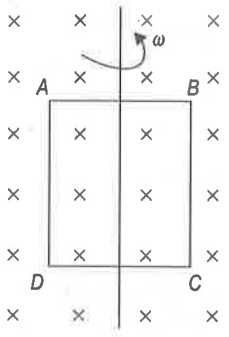
Now $R = 4(a + 2vt)r$ where r = resistance per unit length

$$\therefore \quad i = \frac{4Bv(a + 2vt)}{4r(a + 2vt)} = \frac{Bv}{r} \quad \text{(a constant)}$$

The current will be time independent ■

EXAMPLE 15

Find the direction of induced of current in the wire AB. When rotated anticlockwise through angle θ, if it is placed initially as shown in the figure

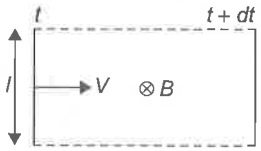


SOLUTION

Range of Angle Rotated	Direction of Induced Current
0–90	A to B
90–180	A to B
180–270	B to A
270–360	B to A

Motional emf

We can find emf induced in a moving conducting rod by considering the number of lines cut by it per sec assuming there are 'B' lines per unit area. Thus when a rod of length l moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to lv and hence it will cut Blv lines per unit time.

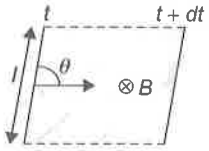


Hence emf induced between the ends of the rod = Bvl

Also emf = $\frac{d\phi}{dt}$. Here ϕ denotes flux passing through the area, swept by the rod. The rod sweeps an area equal to $lvdt$ in time interval dt. Flux through this area = $Blv\,dt$. Thus

$$\frac{d\phi}{dt} = \frac{Blv\,dt}{dt} = Bvl$$

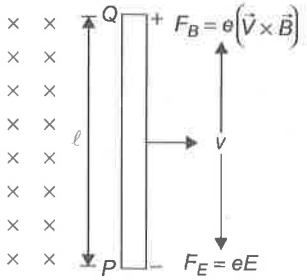
If the rod is moving as shown in the following figure, it will sweep area per unit time = $vl \sin\theta$



and hence it will cut $Bvl \sin\theta$ lines per unit time. Thus emf = $Bvl \sin\theta$. ■

Mechanism of the induced emf a cross the ends of a moving rod

Figure shows a conducting rod of length l moving with a constant velocity v in a uniform magnetic field. The length of the rod is perpendicular to magnetic field, and velocity is perpendicular to both the magnetic field and the length of the rod. An electron inside the conductor experiences a magnetic force $\vec{F_B} = -e(\vec{V} \times \vec{B})$ directed downward along the rod. As a result electrons migrate towards the lower end and leave unbalanced positive charges at the top. This redistribution of charges sets up an electric field E directed downward. This electric field exerts a force on free electrons in the upward direction. As redistribution continues electric field grows in magnitude until a situation, when

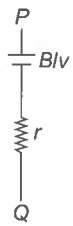


$$|q\vec{v} \times \vec{B}| = |q\vec{E}|$$

After this, there is no resultant force on the free electrons and the potential difference across the conductor is

$$\int d\varepsilon = -\vec{E} \cdot d\vec{l} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l} \qquad (4)$$

Thus it is the magnetic force on the moving free electrons that maintains the potential difference. So emf developed across the ends of the rod moving perpendicular to magnetic field with velocity perpendicular to the rod,



$$\varepsilon = vBl \qquad (5)$$

As this emf is produced due to the motion of the conductor, it is called motional emf.

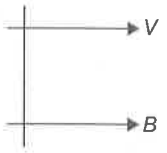
In the problems related to motional emf we can replace the rod by a battery of emf vBl.

The moving rod can be represented (or equivalent) as electrical circuit as shown in figure.

EXAMPLE 16

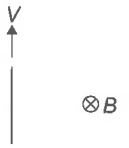
Find the value of emf induced in the rod for the following cases. The figures are self explanatory.

(a)

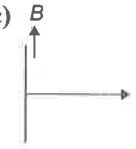


Ans. emf = 0

(b)

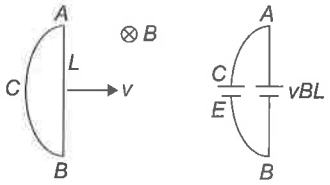


Ans. emf = 0

(c) 

Ans. emf = 0

Figure shows a closed coil *ABCA* moving in a uniform magnetic field *B* with a velocity *v*. The flux passing through the coil is a constant and therefore the induced emf is zero.



Now consider rod *AB*, which is a part of the coil. Emf induced in the rod = *BLv*. Now suppose the emf induced in part *ACB* is *E*, as shown in figure.

Since the emf in the coil is zero, emf (in *ACB*) + emf (in *BA*) = 0

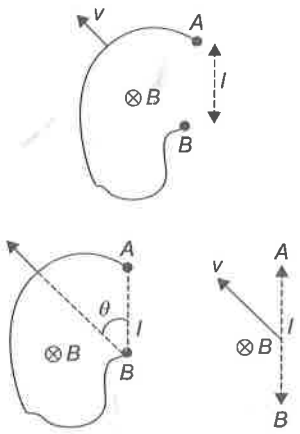
or $$-E + vBL = 0$$

or $$E = vBL$$

Thus emf induced in any path joining *A* and *B* is same, provided the magnetic field is uniform. Also the equivalent emf between *A* and *B* is *BLv* (here the two emf's are in parallel) ■

EXAMPLE 17

Figure shows an irregular shaped with *AB* moving with velocity *v*, as shown. Find the emf induced in the wire.



SOLUTION

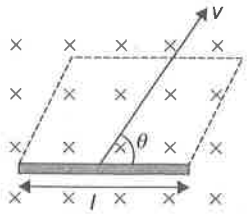
The same emf will be induced in the straight imaginary wire joining *A* and *B*, which is *Bvl* sin *θ*. ■

EXAMPLE 18

A 0.4 m long straight conductor moves in a magnetic field of magnetic induction 0.9 Wb/m^2 with a velocity of 7 m/sec. Calculate the emf induced in the conductor under the condition when it is maximum.

SOLUTION

If a rod of length *l* is moved with velocity $\vec{v}$ and at angle *θ* to the length of the rod in a field $\vec{B}$ which is perpendicular to the plane of the motion, the flux linked with the area generated by the motion of rod in time *t*,



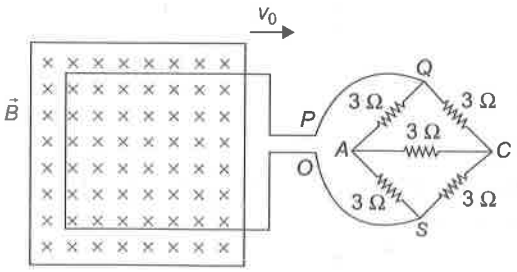
$$\phi = Bl(v \sin\theta)t \quad \text{so, } |\varepsilon| = \frac{d\phi}{dt} = Bvl \sin\theta$$

This will be maximum when sin *θ* = max = 1, i.e., the rod is moving perpendicular to its length and then $(\varepsilon)_{max} = Bvl$

$$E_{max} = 0.9 \times 7 \times 0.4 = 2.52 \text{ V}$$ ■

EXAMPLE 19

A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction *B* = 2 Wb/m^2 as shown in figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value 3 Ω. The resistances of the lead wires *OS* and *PQ* are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop? Find the direction of current in the loop?



SOLUTION

As the network *AQCS* is a balanced Wheatstone bridge, no current will flow through *AC* and hence the effective resistance of the network between *QS*,

$$R_{QS} = \frac{6 \times 6}{6 + 6} = 3\ \Omega$$

and as the resistance of the square metal wire loop is 1 Ω, the total resistance of the circuit,

$$R = 3 + 1 = 4\ \Omega$$

Now if the loop moves with speed v_0, the emf induced in the loop,

$$\varepsilon = Bv_0 l$$

So the current in the circuit, $i = \frac{\varepsilon}{R} = \frac{Bv_0 l}{R}$

Substituting the given data,

In accordance with Lenz's law, the induced current in the loop will be in clockwise direction. ■

EXAMPLE 20

A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

SOLUTION

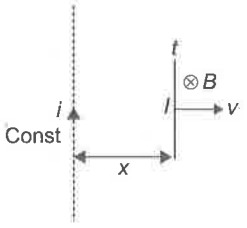
$$E = Bvl$$
$$= \frac{\mu_0 i}{2\pi x} \cdot vl$$

Emf is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is $lv\,dt$. The magnetic field lines cut in dt time

$$Blv\,dt = \frac{\mu_0 ilv\,dt}{2\pi x}$$

∴ The rate with which magnetic field lines are cut

$$= \frac{\mu_0 ilv}{2\pi x}$$

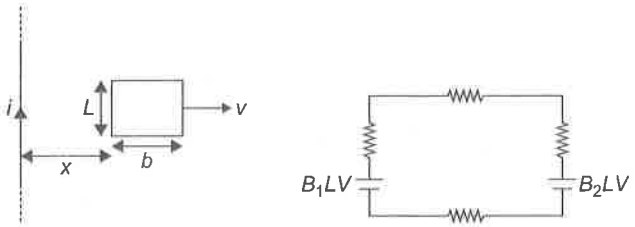


■

EXAMPLE 21

A rectangular loop, as shown in the figure, moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.

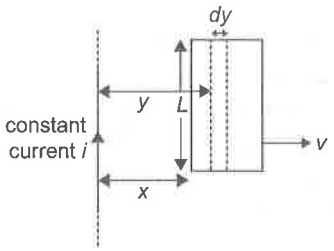
$$E = B_1 LV - B_2 Lv$$



$$\because \quad B_1 > B_2$$

$$= \frac{\mu_0 i}{2\pi x} Lv - \frac{\mu_0 i}{2\pi(x + b)} Lv = \frac{\mu_0 iLbv}{2\pi x(x + b)}$$

Aliter: Consider a small segment of width dy at a distance y from the wire. Let flux through the segment be $d\phi$.



$$\therefore \quad d\phi = \frac{\mu_0 i}{2\pi y} Ldy$$

$$\therefore \quad \phi = \frac{\mu_0 iL}{2\pi} \int_x^{x+b} \frac{dy}{y} = \frac{\mu_0 iL}{2\pi} (\ln(x + b) - \ln x)$$

$$\text{Now} \quad \frac{d\phi}{dt} = \frac{\mu_0 iL}{2\pi} \left[\frac{1}{x + b} \frac{dx}{dt} - \frac{1}{x} \frac{dx}{dt} \right] = \frac{\mu_0 iL}{2\pi} \left[\frac{(-b)}{x(x + b)} \right] v$$

$$= \frac{-\mu_0 ibLv}{2\pi x(x + b)}$$

$$\therefore \quad \text{induced emf} = \frac{\mu_0 ibLv}{2\pi x(x + b)}$$ ■

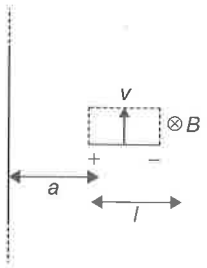
EXAMPLE 22

A rod of length l is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod, if its nearest end is at a distance 'a' from the wire.

SOLUTION

Consider a segment of rod of length dx, at a distance x from the wire. Emf induced in the segment

240



$$de = \frac{\mu_0 i}{2\pi x} dx \cdot v$$

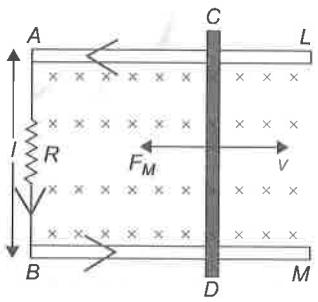
$$\therefore \quad E = \int_a^{a+l} \frac{\mu_0 iv\,dx}{2\pi x} = \frac{\mu_0 iv}{2\pi} \ln\left(\frac{l+a}{a}\right)$$

EXAMPLE 23

Two parallel wires AL and BM placed at a distance l are connected by a resistor R and placed in a magnetic field B which is perpendicular to the plane containing the wires. Another wire CD now connects the two wires perpendicularly and made to slide with velocity v. Calculate the work done per second needed to slide the wire CD. Neglect the resistance of all the wires.

SOLUTION

When a rod of length l moves in a magnetic field with velocity v as shown in figure, an emf $\varepsilon = Bvl$ will be induced in it. Due to this induced emf, a current $i = \frac{\varepsilon}{R} = \frac{Bvl}{R}$ will flow in the circuit as shown in figure. Due to this induced current, the wire will experience a magnetic force



$$F_M = Bil = \frac{B^2 l^2 v}{R}$$

which will oppose its motion. So to maintain the motion of the wire CD, a force $F = F_M$ must be applied in the direction of motion.

The work done per second, i.e., power needed to slide the wire is given by

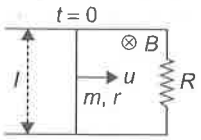
$$P = \frac{dW}{dt} = Fv = F_M v = \frac{B^2 v^2 l^2}{R}$$

Notes

❖ *The power delivered by the external agent is converted into joule heating in a by using the circuit(as shown above). It means magnetic field helps in converting the mechanical energy into joule heating.*

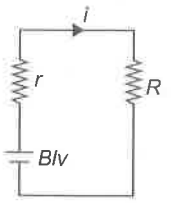
EXAMPLE 24

A rod of mass m and resistance r is placed on fixed, resistanceless, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u. Find its velocity as a function of time.



SOLUTION

Let at an instant the velocity of the rod be v. The emf induced in the rod will be vBl. The electrically equivalent circuit is shown in the following diagram.



$\therefore$ Current in the circuit $i = \frac{Blv}{R+r}$

At time t

Magnetic force acting on the rod is $F = ilB$, opposite to the motion of the rod.

$$ilB = -m\frac{dv}{dt} \qquad (1)$$

$$i = \frac{Blv}{R+r} \qquad (2)$$

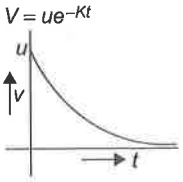
Now solving these two equation

$$\frac{B^2 l^2 v}{R+r} = -m \cdot \frac{dv}{dt} \quad \Rightarrow \quad -\frac{B^2 l^2}{(R+r)m} \cdot dt = \frac{dv}{v}$$

let $\Rightarrow \quad -K \cdot dt = \frac{dV}{V}$

$$\int_u^v \frac{dv}{v} = \int_0^t -K \cdot dt$$

$$\ln\left(\frac{v}{u}\right) = -Kt \qquad \Rightarrow \qquad V = ue^{-Kt}$$



EXAMPLE 25

In the above question if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

SOLUTION

$$m\frac{dv}{dt} = F - ilB \qquad (1)$$

$$i = \frac{Blv}{R+r} \qquad (2)$$

$$m\frac{dv}{dt} = F - \frac{B^2l^2v}{R+r}$$

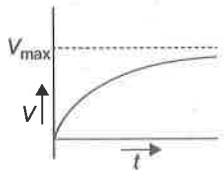
let $K = \frac{B^2l^2}{R+r} \quad \Rightarrow \quad \int_0^v \frac{dV}{F-Kv} = \int_0^t \frac{dt}{m}$

$$\Rightarrow \quad -\frac{1}{K}\ln(F-KV)\int_0^v \frac{t}{m}$$

$$\ln\left(\frac{F-kV}{F}\right) = -\frac{Kt}{m}$$

$$F - KV = Fe^{-kt/m}$$

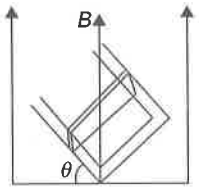
$$V = \frac{F}{K}(1 - {}^{-kt/m})$$



EXAMPLE 26

A square wire of length l, mass m and resistance R slides without friction down the parallel conducting wires of negligible resistance as shown in figure.

The rails are connected to each other at the bottom by a resistanceless rail parallel to the wire so that the wire and rails form a closed rectangular loop. The plane of the rails makes an angle θ with horizontal and a uniform vertical field of magnetic induction B exists throughout the region. Show that the wire acquires a steady state velocity of magnitude $v = \frac{mgR\sin\theta}{B^2l^2\cos^2\theta}$



SOLUTION

Force down the plane = $mg \sin\theta$

At any instant if the velocity is v the induced emf = $lB\cos\theta \times v$

$$\text{Current in the loop} \times \frac{Blv\cos\theta}{R}$$

Force on the conductor in the horizontal direction

$$= Bl \times \frac{Blv\cos\theta}{R}$$

The component of force parallel to the incline

$$= \frac{B^2l^2v\cos\theta}{R} \times \cos\theta$$

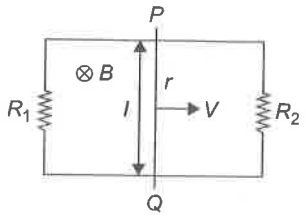
If v is constant, $\frac{B^2l^2\cos^2\theta}{R} \times v = mg\sin\theta$

$$\therefore \qquad v = \frac{mRg\sin\theta}{B^2l^2\cos^2\theta}$$

EXAMPLE 27

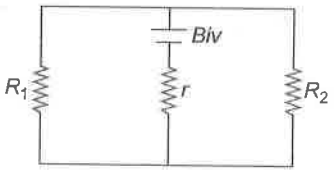
A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2). Find the current in the rod at the instant its velocity is v.

242



SOLUTION

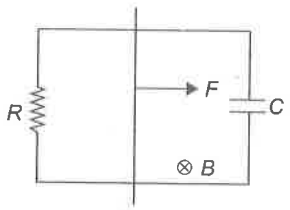
The equivalent circuit of above figure



$$i = \frac{Blv}{r + \frac{R_1 R_2}{R_1 + R_2}}$$

EXAMPLE 28

In the above question if one resistance is replaced by a capacitor of capacitance C as shown. Find the velocity of the moving rod at time t if the initial velocity of the rod is v and a constant force F is applied on the rod. Neglect the resistance of the rod.



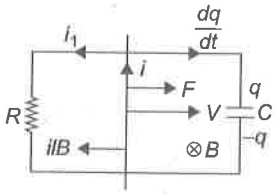
SOLUTION

At any time t, let the velocity of the rod be v.

Applying Newtons law:

$$F - ilB = ma \qquad (1)$$

Also $$Blv = i_1 R = \frac{q}{c}$$



Applying KCL,

$$i = i_1 + \frac{dq}{dt} = \frac{Blv}{R} + \frac{d}{dt}(BlvC)$$

or $$i = \frac{Blv}{R} + BlCa$$

Putting the value of i in eq. (1),

$$F - \frac{B^2 l^2 v}{R} = (m + B^2 l^2 C)a = (m + B^2 l^2 C)\frac{dv}{dt}$$

$$(m + B^2 l^2 C)\frac{dv}{F - \frac{B^2 l^2 v}{R}} = dt$$

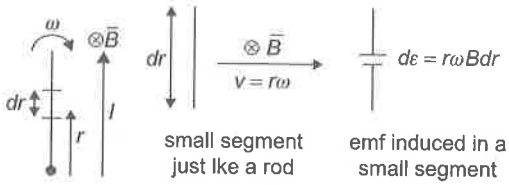
Integrating both sides, and solving we get

$$v = \frac{FR}{B^2 l^2}\left(1 - e^{\frac{tB^2 l^2}{R(m+CB^2 l^2)}}\right)$$

Induced emf due to Rotation

Rotation of the rod

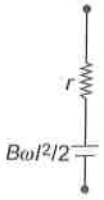
Consider a conducting rod of length l rotating in a uniform magnetic field.



Emf induced in a small segment of length dr, of the rod $= vB\,dr = r\omega B\,dr$

$\therefore$ emf induced in the rod

$$= \omega B \int_0^{\ell} r\,dr = \frac{1}{2} B\omega l^2$$



equivalent of this rod is as following

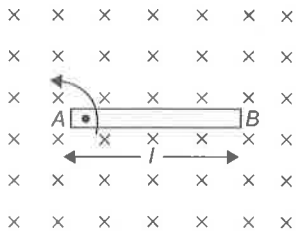
or

$$\varepsilon = \frac{d\phi}{dt} = \frac{\text{flux through the area swept by the rod in time } dt}{dt}$$

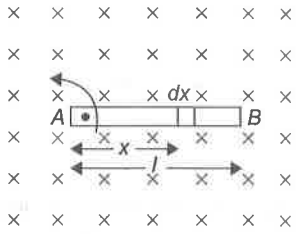
$$= \frac{B\frac{1}{2}l^2\omega\,dt}{dt} = \frac{1}{2}B\omega l^2$$

EXAMPLE 29

Find out the potential difference between A and B:



SOLUTION

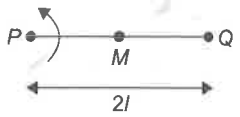


$$\int dE = \int_0^{\ell} B\omega x dx, \quad V_A - V_B = \frac{B\omega l^2}{2}$$ ■

EXAMPLE 30

A rod PQ of length $2l$ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M and Q if that between P and $Q = 100$ V.

$\otimes B$ = Uniform



SOLUTION

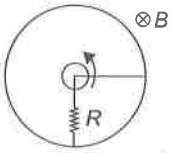
$$E_{MQ} + E_{PM} = E_{PQ} \qquad \text{corner} \rightarrow \frac{Bwl^2}{2} = 100$$

$$E_{MQ} + \frac{B\omega\left(\frac{l}{2}\right)^2}{2} = \frac{B\omega l^2}{2}$$

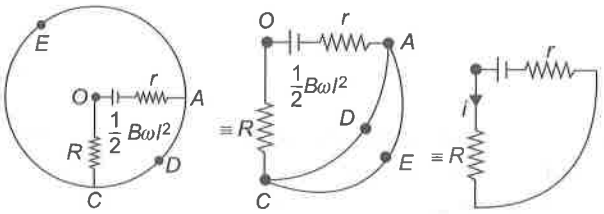
$$E_{MQ} = \frac{3}{8} B\omega l^2 = \frac{3}{4} \times 100 \text{ V} = 75 \text{ V}$$ ■

EXAMPLE 31

A rod of length L and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring a of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.



SOLUTION

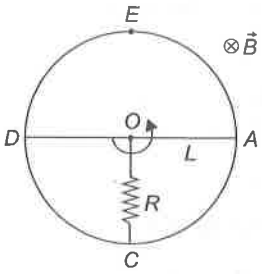


$$\text{current } i = \frac{\frac{1}{2} B\omega l^2}{R + r}$$ ■

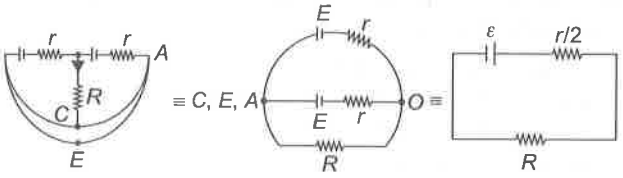
EXAMPLE 32

Solve the above question if the length of rod is $2L$ and resistance $2r$ and it is rotating about its centre.

Both ends of the rod now touch the conducting ring.



SOLUTION

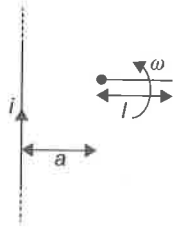


244

$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2} B\omega L^2}{R + \frac{r}{2}}$$

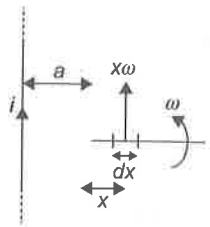
EXAMPLE 33

A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant as shown in the figure.



Consider a small segment of rod of length dx, at a distance x from one end of the rod.

Emf induced in the segment



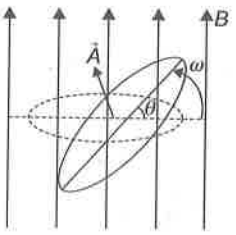
$$dE = \frac{\mu_0 i}{2\pi(x + a)} (x\omega)\, dx$$

$$\therefore \quad E = \int_0^l \frac{\mu_0 i}{2\pi(x + a)} (x\omega)\, dx = \frac{\mu_0 i\, \omega}{2\pi} \left[l - a \cdot \ln\left(\frac{l + a}{a} \right) \right]$$

By Changing the Angle

Let us consider the case when the magnitude of the magnetic field strength and the area of the coil remains constant. When the coil is rotated relative to the direction of the field, an induced current is produced which lasts as long as the coil is rotating.

We have, $\phi = BA \cos\theta$ [where B is the magnetic field strength, A is the magnitude of the area vector and θ is the angle between them] If the angular velocity with which the coil is rotating is ω, then $\theta = \omega t$



Induced emf in the coil

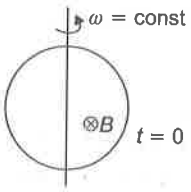
$$\varepsilon = -\frac{d\phi}{dt} = BA\omega \sin\omega t$$

Induced current in the coil

$$= i = \frac{|\varepsilon|}{R} = \frac{B\omega A}{R} \sin\omega t$$

EXAMPLE 34

A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.



SOLUTION

At any time t, $\phi = BA \cos\theta = BA \cos\omega t$

Now induced emf in the loop

$$e = \frac{-d\phi}{dt} = BA\omega \sin\omega t$$

If there are N turns

$$\text{emf} = BA\omega N \sin\omega t$$

$BA\omega N$ is the amplitude of the emf $e = e_m \sin\omega t$

$$i = \frac{e}{R} = \frac{e_m}{R} \sin\omega t = i_m \sin\omega t$$

$$i_m = \frac{e_m}{R}$$

The rotating coil thus produces a sinusoidally varying current or alternating current. This is the principle which is always used in generator. ■

INDUCED ELECTRIC FIELD DUE TO A TIME VARYING MAGNETIC FIELD

Consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Suppose, the field is constant till $t = 0$ and then changes with time. An induced current starts in the loop at $t = 0$.

The free electrons were at rest till $t = 0$ (we are not interested in the random motion of the electrons.) The magnetic field cannot exert force on electrons at rest. Thus, the magnetic force cannot start the induced current. The electrons may be forced to move only by an electric field. So we conclude that an electric field appears at time $t = 0$.

This electric field is produced by the changing magnetic field and not by charged particles. The electric field produced by the changing magnetic field is nonelectrostatic and nonconservative in nature. We cannot define a potential corresponding to this field. We call it induced electric field. The lines of induced electric field are closed curves. There are no starting and terminating points of the field lines.

If $\vec{E}$ be the induced electric field, the force on the charge q placed in the field of $q\vec{E}$. The work done per unit charge as the charge moves through $d\vec{l}$ is $E \cdot d\vec{l}$. The emf developed in the loop is, therefore,

$$\varepsilon = \oint \vec{E} \cdot d\vec{l}$$

Using Faraday's law of induction,

$$\varepsilon = -\frac{d\phi}{dt}$$

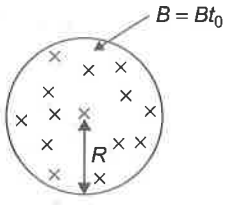
or,
$$\oint \vec{E} \cdot d\vec{l} = \frac{d\phi}{dt} \qquad (6)$$

The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

SOLVED EXAMPLES

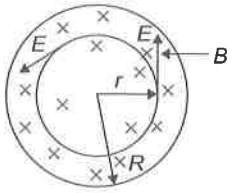
EXAMPLE 35

What will be the electric field at a distance r from axis of changing cylindrical magnetic field B, which is parallel to the axis of cylinder?



SOLUTION

(i) When $r < R$



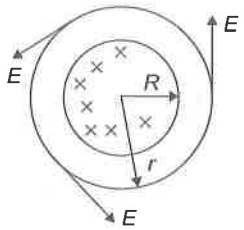
let at a distance r electric field is E

$$\varepsilon = \left|\oint \vec{E} \cdot d\vec{l}\right| = \left|-\frac{d\phi}{dt}\right|$$

$$E \cdot 2\pi r = \left|-\frac{d[B \cdot (\pi r^2)]}{dt}\right|$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{B_0 r}{2}$$

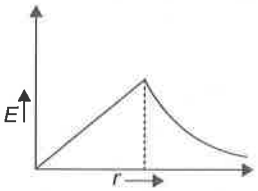
(ii) When $r > R$



$$\left|\oint \vec{E} \cdot d\vec{\ell}\right| = \left|\frac{-d\phi}{dt}\right|$$

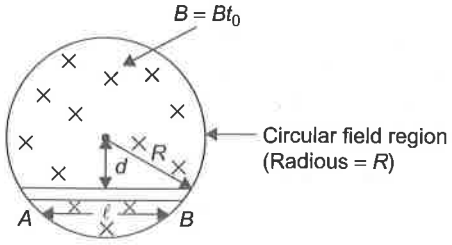
$$E \cdot 2\pi r = \frac{d(B\pi R^2)}{dt}$$

$$\Rightarrow \quad E = \frac{R^2}{2r} \cdot \frac{dB}{dt} = \frac{B_0 R^2}{2r}$$



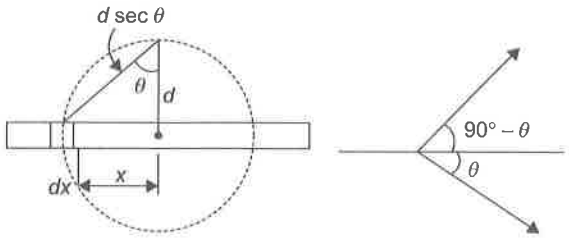
EXAMPLE 36

Find the emf induced in the rod as shown in the figure.



246

SOLUTION



$$E = \frac{r}{2}\frac{dB}{dt} = \frac{d\sec\theta}{2}B_0$$

$$dE = E \cdot dx\cos\theta = \frac{d\sec\theta B_0 dx\cos\theta}{2}$$

$$\Rightarrow \quad \int_0^E dE = \frac{B_0 d}{2}\int_{-\ell/2}^{\ell/2} dx \quad \Rightarrow \quad E = \frac{B_0 \ell d}{2}$$

Alternate

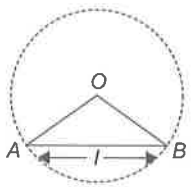
Induced emf in OA and OB is zero (because $\vec{B}$ and $d\vec{l}$ are perpendicular)

Total induced emf in OAB is in AB

$$\text{Area} = \frac{1}{2}ld$$

$$\phi = \frac{B_0 ldt}{2}$$

$$\frac{d\phi}{dt} = \frac{B_0 ld}{2}$$



EXAMPLE 37

A thin, nonconducting ring of mass m, carrying a charge q, can rotate freely about its axis. At the instant $t = 0$ the ring was at rest and no magnetic field was present. Then suddenly a magnetic field B was set perpendicular to the plane. Find the angular velocity acquired by the ring.

SOLUTION

Due to the sudden change of flux, an electric field is set up and the ring experiences an impulsive torque and suddenly acquires an angular velocity.

$$\varepsilon(\text{induced emf}) = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

Also $\varepsilon = \oint \vec{E}\cdot d\vec{l}$ where E is the induced electric field.

$$\therefore \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} \Rightarrow E\cdot 2\pi r = -\frac{d}{dt}(B\pi r^2)$$

$$\Rightarrow \qquad E = -\frac{r}{2}\frac{dB}{dt}$$

Force experienced by the ring $= q|\vec{E}|$

Torque experienced by the ring $\tau = (qE)r = \frac{qr^2}{2}\frac{dB}{dt}$

$\therefore$ Angular impulse experienced by the ring

$$= \int \tau\, dt = \frac{qr^2}{2}\int \frac{dB}{dt}dt = qr^2\frac{B}{2}$$

Also angular impulse acquired = $l\omega$ where l is moment of inertia of the ring about its axis = mr^2

$$\therefore \qquad mr^2\omega = qr^2\frac{B}{2}$$

$$\Rightarrow \text{ Angular velocity acquired by the ring } \omega = \frac{qB}{2m}$$

SELF INDUCTION

Self induction is induction of emf in a coil due to its own current change. Total flux $N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = Li$ where L is called coefficient of self induction or inductance. The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}$$

The instantaneous emf is given as

$$\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{Ldl}{dt}$$

SI unit of inductance is Wb/amp or Henry (H)

L - self inductance is +ve quantity.

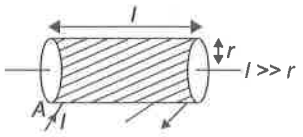
L depends on:

1. Geometry of loop
2. Medium in which it is kept. L does not depend upon current. L is a scalar quantity.

Brain Teaser

If a circuit has large self-inductance, what inference can you draw about the circuit.

Self Inductance of Solenoid



Let the volume of the solenoid be V, the number of turns per unit length be n. Let a current I be flowing in the solenoid. Magnetic field in the solenoid is given as $B = \mu_0 nl$. The magnetic flux through one turn of solenoid $\phi = \mu_0 nlA$.

The total magnetic flux through the solenoid $= N\phi$

$$= N\mu_0 nlA$$

$$= \mu_0 n^2 lAl$$

$$\therefore \quad L = \mu_0 n^2 lA = \mu_0 n^2 V$$

$$\phi = \mu_0 ni\pi r^2 (nl)$$

$$L = \frac{\phi}{i} = \mu_0 n^2 \pi r^2 l$$

Inductance per unit volume $= \mu_0 n^2$

SOLVED EXAMPLES

EXAMPLE 38

The current in a coil of self-inductance $L = 2$ H is increasing according to the law $i = 2 \sin t^2$. Find the amount of energy spent during the period when the current changes from 0 to 2 A.

SOLUTION

Let the current be 2 A at $t = \tau$

Then $\quad 2 = 2 \sin \tau^2 \quad \Rightarrow \quad \tau = \sqrt{\frac{\pi}{2}}$

When the instantaneous current is i, the self induced emf is $L\frac{di}{dt}$. If the amount of charge that is displaced in time dt is dq, then the elementary work done

$$= L \cdot \left(\frac{di}{dt}\right) dq = L \cdot \frac{di}{dt} idt = Lidi$$

$$W = \int_0^\tau Lidi = \int_0^\tau L(2\sin t^2)d(2\sin t^2)$$

$$W = \int_0^\tau 8L \sin t^2 \cos t^2 (tdt) = 4L \int_0^\tau \sin 2t^2 (tdt)$$

Let $\quad \theta = 2t^2$

Differentiating $d\theta = 4tdt$

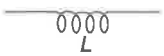
$$\therefore \quad W = 4L \int \frac{\sin\theta d\theta}{4}$$

$$= L(-\cos\theta) = -L\cos 2t^2$$

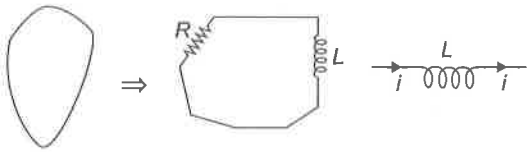
$$W = -L[\cos 2t^2]_0^{\sqrt{\pi/2}}$$

$$= 2L = 2 \times 2 = 4 \text{ J}$$

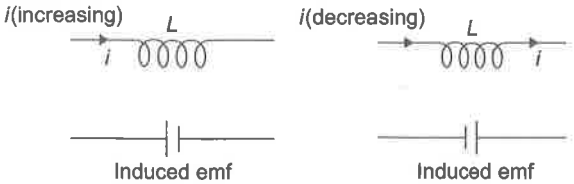
INDUCTOR

It is represent by 

electrical equivalence of loop

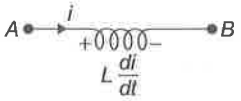


If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current. If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.



248

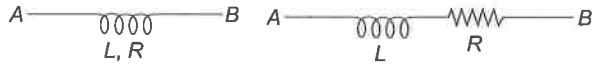
Over all result



$$V_A - L\frac{di}{dt} = V_B$$

Notes

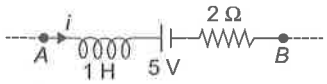
- If there is a resistance in the inductor (resistance of the coil of inductor) then:



SOLVED EXAMPLES

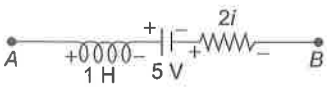
EXAMPLE 39

AB is a part of circuit. Find the potential difference $v_A - v_B$ if



(i) current $i = 2$ A and is constant
(ii) current $i = 2$ A and is increasing at the rate of 1 A/sec.
(iii) current $i = 2$ A and is decreasing at the rate 1 A/sec.

SOLUTION



$$L\frac{di}{dt} = 1\frac{di}{dt}$$

writing KVL from A to B

$$V_A - 1\frac{di}{dt} - 5 - 2i = V_B$$

(i) Put $i = 2, \frac{di}{dt} = 0$

$V_A - 5 - 4 = V_B \quad \therefore \quad V_A - V_B = 9$ V

(ii) Put $i = 2, \frac{di}{dt} = 1;$

$V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10V_0$

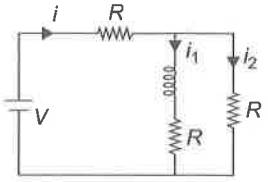
(iii) Put $i = 2, \frac{di}{dt} = -1$

$V_A + 1 - 5 - 2 \times 2 = V_B$

$V_A = 8$ V

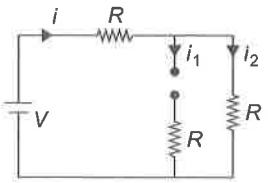
EXAMPLE 40

Find current i, i_1 and i_2 in the following circuit.



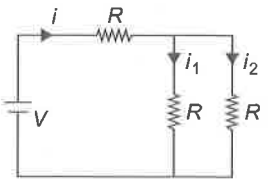
SOLUTION

At $t = 0$



$$i = i_2 = \frac{V}{2R} \quad \text{and} \quad i_1 = 0.$$

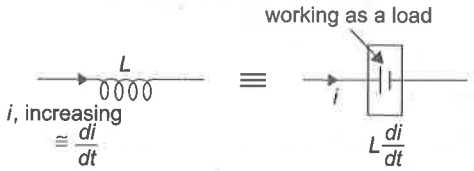
At $t = \infty$



$$\Rightarrow \quad i_1 = i_2 = \frac{i}{2} = \frac{V}{2R}.$$

Energy Stored in an Inductor

If current in an inductor at an instant is i and is increasing at the rate di/dt, the induced emf will oppose the current. Its behaviour is shown in the figure.



Power consumed by the inductor $= iL\frac{di}{dt}$

Energy consumed in dt time $= iL\frac{di}{dt}dt$

$\therefore$ total energy consumed as the current increases from 0 to $i = \int_0^i iL\,di = \frac{1}{2}Li^2 = \frac{1}{2}Li^2 \quad \Rightarrow \quad U = \frac{1}{2}Li^2$

Notes

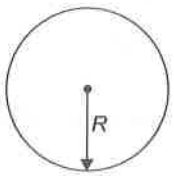
❖ This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy $U = \int \frac{B^2}{2\mu_0\mu_r} dV$

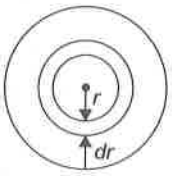
EXAMPLE 41

Find out the energy per unit length ratio inside the solid long wire having current density J.



SOLUTION

Take a ring of radius r and thickness dr as an element inside the wire



$$\frac{dE}{dv} = \frac{B^2}{2\mu_0}$$

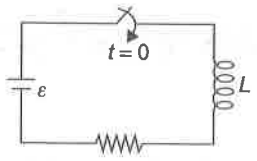
using $B = \frac{\mu_0 jr}{2}$

$$\frac{dE}{dv} = \frac{\mu_0^2 J^2 r^2}{4(2\mu_0)} \Rightarrow \int dE = \int \frac{\mu_0 j^2 r^2}{8} 2\pi r dr \ell$$

$$\Rightarrow \frac{E}{\ell} = \frac{\pi\mu_0 j^2 R^4}{16}$$ ■

EXAMPLE 42

A circuit contains an ideal cell and an inductor with a switch. Initially the switch is open. It is closed at $t = 0$. Find the current as a function of time.



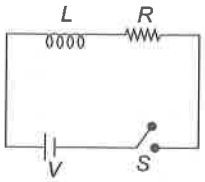
SOLUTION

$$\varepsilon = L\frac{di}{dt} \Rightarrow \int_0^i \varepsilon dt = \int_0^i L di$$

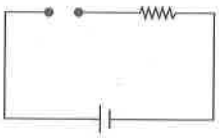
$$\varepsilon t = Li \Rightarrow i = \frac{\varepsilon t}{L}$$ ■

L.R. CIRCUIT

As the switch S is closed in given figure, current in circuit wants to rise upto $\frac{v}{R}$ in no time but inductor opposes it $\left(\frac{di}{dt} \to \frac{V}{L}\right)$



hence at time $t = 0$ inductor will behave as an open circuit.
At $t = 0$

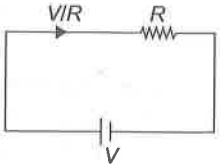


As the time passes, i in the circuit rises and $\frac{di}{dt}$ decreases.
At any instant t.

$$\frac{Ldi}{dt} + iR = V$$

current reaches the value $\frac{V}{R}$ at time $t = \infty$ or we can say, inductor will behave as a simple wire.

At $t = \infty$



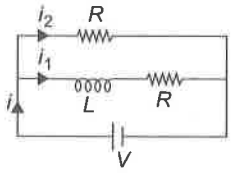
SOLVED EXAMPLES

EXAMPLE 43

Find value of current i, i_1 and i_2 in given figure at

(a) time $t = 0$

(b) time $t = \infty$



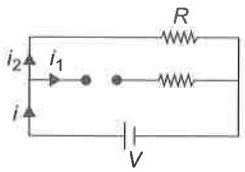
SOLUTION

(a) At time $t = 0$ inductor behaves as open circuit

$$i = \frac{V}{R}$$

$$i_1 = 0$$

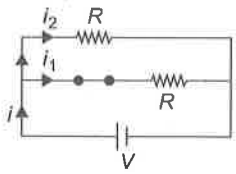
$$i_2 = i = \frac{V}{R}$$



(b) At time $t = \infty$. Inductor will behaves as simple wire

$$i = \frac{v}{\left(\frac{R}{2}\right)} = \frac{2V}{R}$$

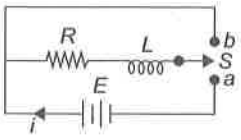
$$i_1 = i_2 = \frac{V}{R}$$



GROWTH AND DECAY OF CURRENT IN L-R CIRCUIT

Growth of Current

Consider a circuit containing a resistance R, an inductance L, a two way key and a battery of emf E connected in series as shown in figure. When the switch S is connected to a, the current in the circuit grows from zero value. The inductor opposes the growth of the current. This is due to the fact that when the current grows through inductor, a back emf is developed which opposes the growth of current in the circuit. So the rate of growth of current is reduced. During the growth of current in the circuit, let i be the current in the circuit at any instant t. Using Kirchhoff's voltage law in the circuit, we obtain



$$E - L\frac{di}{dt} = Ri \quad \text{or} \quad E - Ri = L\frac{di}{dt} \quad \text{or} \quad \frac{di}{E - Ri} = \frac{dt}{L}$$

Multiplying by $-R$ on both the sides, we get

$$\frac{-Rdi}{E - Ri} = \frac{-Rdt}{L}$$

Integrating the above equation, we have

$$\log_e(E - Ri) - \frac{R}{L}t + A \qquad (1)$$

where A is integration constant. The value of this constant can be obtained by applying the condition that current i is zero just at start i.e., at $t = 0$. Hence

$$\log_e E = 0 + A$$

or
$$A = \log_e E \qquad (2)$$

Substituting the value of A from equation (2) in equation (1), we get

$$\log_e(E - Ri) = -\frac{R}{L}t + \log_e E$$

or
$$\log_e\left(\frac{E - Ri}{E}\right) = -\frac{R}{L}t$$

or
$$\left(\frac{E - Ri}{E}\right) = \exp\left(-\frac{R}{L}t\right)$$

or
$$1 - \frac{Ri}{E} = \exp\left(-\frac{R}{L}t\right)$$

or
$$\frac{Ri}{E} = \left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$$

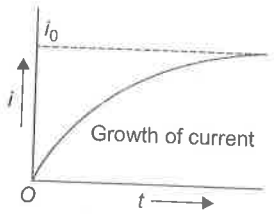
$\therefore$
$$i = \frac{E}{R}\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$$

The maximum current in the circuit $i_0 = \frac{E}{R}$. So

$$i = i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\} \qquad (3)$$

Equation (3) gives the current in the circuit at any instant t. It is obvious from equation (3) that $i = i_0$, when

$$\exp\left(-\frac{R}{L}t\right) = 0 \text{ i.e., at } t = \infty$$



Hence the current never attains the value i_0 but it approaches it asymptotically. A graph between current and time is shown in figure.

We observe the following points

(i) When $t = (L/R)$ then

$$i = i_0\left\{1 - \exp\left(-\frac{R}{L}\times\frac{L}{R}\right)\right\} = i_0\{1 - \exp(-1)\}$$

$$= i_0\left(1 - \frac{1}{e}\right) = 0.63i_0$$

Thus after an interval of (L/R) second, the current reaches to a value which is 63% of the maximum current. The value of (L/R) is known as time constant of the circuit and is represented by τ. Thus the time constant of a circuit may be defined as the time in which the current rises from zero to 63% of its final value. In terms of τ,

$$i = i_0\left(1 - e^{\frac{-t}{\tau}}\right)$$

(ii) The rate of growth of current (di/dt) is given by

$$\frac{di}{dt} = \frac{d}{dt}\left[i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}\right]$$

$$\Rightarrow \quad \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\exp\left(-\frac{R}{L}t\right) \qquad (4)$$

From equation (3), $\exp\left(-\frac{R}{L}t\right) = \frac{i_0 - i}{i_0}$

$$\therefore \quad \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\left(\frac{i_0 - i}{i_0}\right) = \frac{R}{L}(i_0 - i) \qquad (5)$$

This shows that the rate of growth of the current decreases as i tends to i_0. For any other value of current, it depends upon the value of R/L. Thus greater is the value of time constant, smaller will be the rate of growth of current.

(3)

Notes

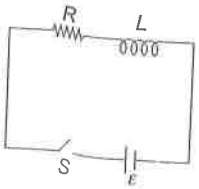
- Final current in the circuit $= \frac{E}{R}$, which is independent of L.
- After one time constant, current in the circuit = 63% of the final current (verify yourself)
- More time constant in the circuit implies slower rate of change of current.
- If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.

$$L_1 i_1 = L_2 i_2$$

SOLVED EXAMPLES

EXAMPLE 44

At $t = 0$ switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser $\left(\frac{L}{\eta}\right)$ then its initial value, find out instant current just after the operation.



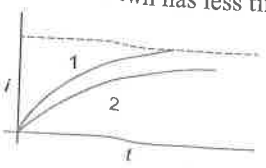
SOLUTION

Using above result (note 4)

$$L_1 i_1 = L_2 i_2 \quad \Rightarrow \quad i_2 = \frac{\eta\varepsilon}{R}$$

EXAMPLE 45

Which of the two curves shown has less time constant.



SOLUTION

Curve 1, growth in curve 1 is faster therefore less time constant.

Decay of Current

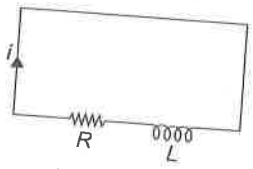
Let the circuit be disconnected from battery and switch S is thrown to point b in the figure. The current now begins to fall.

252

In the absence of inductance, the current would have fallen from maximum i_0 to zero almost instantaneously. But due to the presence of inductance, which opposes the decay of current, the rate of decay of current is reduced.

Suppose during the decay of current, i be the value of current at any instant t. Using Kirchhoff's voltage law in the circuit, we get

$$-L\frac{di}{dt} = Ri \quad \text{or} \quad \frac{di}{dt} = -\frac{R}{L}i$$



Integrating this expression, we get

$$\log_e i = -\frac{R}{L}t + B$$

where B is constant of integration. The value of B can be obtained by applying the condition that when $t = 0$, $i = i_0$

$\therefore$
$$\log_e i_0 = B$$

Substituting the value of B, we get

$$\log_e i = -\frac{R}{L}t + \log_e i_0$$

or
$$\log_e \frac{i}{i_0} = -\frac{R}{L}t$$

or
$$\left(\frac{i}{i_0}\right) = \exp\left(-\frac{R}{L}t\right) \qquad (6)$$

or
$$i = i_0 \exp\left(-\frac{R}{L}t\right) = i_0 \exp\left(-\frac{t}{\tau}\right)$$

where $\tau = L/R$ = inductive time constant of the circuit.

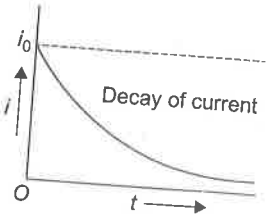
It is obvious from equation that the current in the circuit decays exponentially as shown in figure.

We observe the following points

(i) After $t = L/R$, the current in the circuit is given by

$$i = i_0 \exp\left(-\frac{R}{L} \times \frac{L}{R}\right) = i_0 \exp(-1)$$

$$= \left(\frac{i_0}{e}\right) = \frac{i_0}{2.718} = 0.37 i_0$$



So after a time (L/R) second, the current reduces to 37% of the maximum current i_0. (L/R) is known as time constant τ. This is defined as the time during which the current decays to 37% of the maximum current during decay.

(ii) The rate of decay of current in given by

$$\frac{di}{dt} = \frac{d}{dt}\left\{i_0 \exp\left(-\frac{R}{L}t\right)\right\}$$

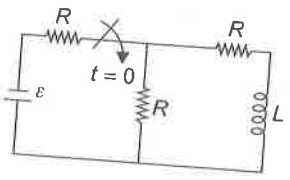
$$\Rightarrow \quad \frac{di}{dt} = \frac{R}{L} i_0 \exp\left(-\frac{R}{L}t\right) = -\frac{R}{L}i \qquad (7)$$

or
$$-\frac{di}{dt} = \frac{R}{L}i$$

This equation shows that when L is small, the rate of decay of current will be large i.e., the current will decay out more rapidly.

EXAMPLE 46

In the following circuit the switch is closed at $t = 0$. Initially there is no current in inductor. Find out current the inductor coil as a function of time.



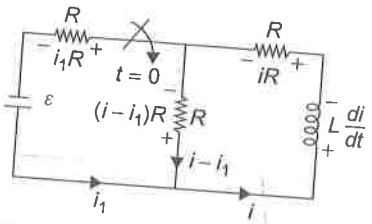
SOLUTION

At any time t

$$-\varepsilon + i_1 R - (i - i_1) R = 0$$

$$-\varepsilon + 2i_1 R - iR = 0$$

$$i_1 = \frac{iR + \varepsilon}{2R}$$



Now, $-\varepsilon + i_1 R + iR + L \cdot \frac{di}{dt} = 0$

$$-\varepsilon + \left(\frac{iR + \varepsilon}{2}\right) + iR + i \cdot \frac{di}{dt} = 0 \Rightarrow -\frac{\varepsilon}{2} + \frac{3IR}{2} = -L\frac{di}{dt}$$

$$\left(\frac{-\varepsilon + 3iR}{2}\right) dt = -L \cdot di \Rightarrow -\frac{-di}{dt} = \frac{di}{-\varepsilon + 3iR}$$

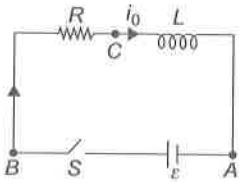
$$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon + 3iR} \Rightarrow -\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right)$$

$$-\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right) = \frac{3Rt}{2L}$$

$$i = +\frac{\varepsilon}{3R}\left(1 - e^{-\frac{3Rt}{2L}}\right)$$

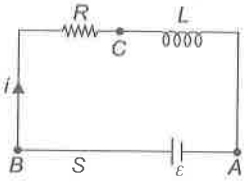
EXAMPLE 47

Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at $t = 0$. Suppose at $t = 0$ current in the inductor is i_0 then find out equation of current as a function of time



SOLUTION

Let an instant t current in the circuit is i which is increasing at the rate $\frac{di}{dt}$. Writing KVL along the circuit, we have



$$\varepsilon - L\frac{di}{dt} - iR = 0 \Rightarrow L\frac{di}{dt} = \varepsilon - iR$$

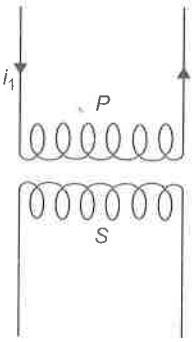
$$\Rightarrow \int_{i_0}^{i} \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L} \Rightarrow \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0 R}\right) = -\frac{Rt}{L}$$

$$\Rightarrow \varepsilon - iR = (\varepsilon - i_0 R)e^{-Rt/L} \Rightarrow i = \frac{\varepsilon - (\varepsilon - i_0 R)e^{-Rt/L}}{R}$$

MUTUAL INDUCTANCE

Consider two coils P and S placed close to each other as shown in the figure. When the current passing through a coil increases or decreases, the magnetic flux linked with the other coil also changes and an induced emf is developed in it. This phenomenon is known as mutual induction. This coil in which current is passed is known as primary and the other in which emf is developed is called as secondary.

Let the current through the primary coil at any instant be i_1. Then the magnetic flux ϕ_2 in the secondary at any time will be proportional to i_1 i.e., $\phi_2 \propto i_1$



Therefore the induced emf in secondary when i_1 changes is given by

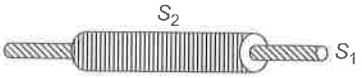
$$\varepsilon = -\frac{d\phi_2}{dt} \quad \text{i.e.,} \quad \varepsilon \propto -\frac{di_1}{dt}$$

$$\varepsilon = -M\frac{di_1}{dt} = -\frac{dMi_1}{dt} \Rightarrow \phi_2 = Mi_1$$

where M is the constant of proportionality and is known as mutual inductance of two coils. It is defined as the emf induced in the secondary coil by unit rate of change of current in the primary coil. The unit of mutual inductance is henry (H).

Mutual Inductance of a Pair of Solenoids One Surrounding the Other Coil

Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1 and number of turns N_1.



To calculate mutual inductance M between them, let us assume a current i_1 through the inner solenoid S_1.

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0\left(\frac{N_1}{l_1}\right)i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B_2} through the surrounding coil is, therefore,

$$\phi_{B_2} = B(\pi R_1^2) = \frac{\mu_0 N_1 i_1}{l_1}\pi R_1^2$$

Now, $M = \frac{N_2\phi_{B_2}}{i_1} = \left(\frac{N_2}{i_1}\right)\left(\frac{\mu_0 N_1 i_1}{l_1}\right)\pi R_1^2 \Rightarrow \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$

Notice that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior.

Brain Teaser

What is the meaning of the statement 'The coefficient of mutual inductance for a pair of coils is large'?

Notes

- $M \le \sqrt{L_1 L_2}$
- For two coils in series if mutual inductance is considered then

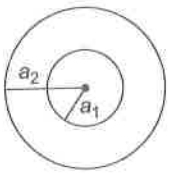
$$L_{eq} = L_1 + L_2 \pm 2M$$

SOLVED EXAMPLES

EXAMPLE 48

Find the mutual inductance of two concentric coils of radii a_1 and a_2.

($a_1 << a_2$) if the planes of coils are same.



SOLUTION

Let a current i flow in coil of radius a_2.

$$\text{Magnetic field at the centre of coil} = \frac{\mu_0 i}{2a_2}\pi a_1^2$$

$$\text{or} \quad Mi = \frac{\mu_0 i}{2a_2}\pi a_1^2 \quad \text{or} \quad M = \frac{\mu_0 \pi a_1^2}{2a_2}$$

EXAMPLE 49

Solve the above question, if the planes of coil are perpendicular.

SOLUTION

Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence $M = 0$.

EXAMPLE 50

Solve the above problem if the planes of coils make θ angle with each other.

SOLUTION

If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

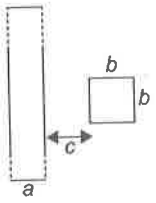
Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

$$\text{Thus flux} = \vec{B} \cdot \vec{A} = \frac{\mu_0 i}{2a_2} \cdot \pi a_1^2 \cos\theta$$

$$\text{or} \quad M = \frac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$$

EXAMPLE 51

Find the mutual inductance between two rectangular loops, shown in figure.



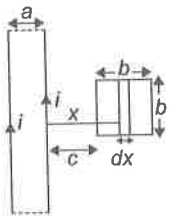
SOLUTION

Let current i flow in the loop having ∞– by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

$$d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx$$

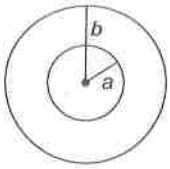
$$\Rightarrow \quad \phi = \int_c^{c+b} \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx$$

$$= \frac{\mu_0 i b}{2\pi}\left[\ln\frac{c+b}{c} - \ln\frac{a+b+c}{a+c}\right]$$



EXAMPLE 52

Figure shows two concentric coplanar coils with radii a and b ($a << b$). A current $i = 2t$ flows in the smaller loop. Neglecting self inductance of larger loop



(a) Find the mutual inductance of the two coils
(b) Find the emf induced in the larger coil
(c) If the resistance of the larger loop is R find the current in it as a function of time

SOLUTION

(a) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem). Let current i be flowing in the larger coil. Magnetic field at the centre $= \frac{\mu_0 i}{2b}$.

flux through the smaller coil $= \frac{\mu_0 i}{2b}\pi a^2$

$\therefore \quad M = \frac{\mu_0}{2b}\pi a^2$

(ii) |emf induced in larger coil|

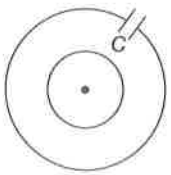
$$= M\left[\left(\frac{di}{dt}\right)\text{in smaller coil}\right]$$

$$= \frac{\mu_0}{2b}\pi a^2 \quad (2) \quad = \frac{\mu_0 \pi a^2}{b}$$

(iii) current in the larger coil $= \frac{\mu_0 \pi a^2}{bR}$ ■

EXAMPLE 53

If the current in the inner loop changes according to $i = 2t^2$ then, find the current in the capacitor as a function of time.

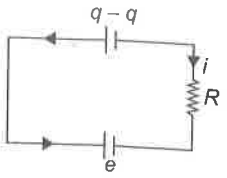


SOLUTION

$$M = \frac{\mu_0}{2b}\pi a^2$$

|emf induced in larger coil| $= M\left[\left(\frac{di}{dt}\right)\text{in smaller coil}\right]$

$$e = \frac{\mu_0}{2b}\pi a^2(4t) = \frac{2\mu_0 \pi a^2 t}{b}$$

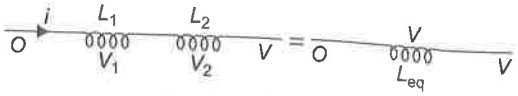


Applying KVL:

$$+e - \frac{q}{c} - iR = 0 \quad \Rightarrow \quad \frac{2\mu_0 \pi a^2 t}{b} - \frac{q}{c} - iR = 0$$

differentiate wrt time: $\frac{2\mu_0 \pi a^2}{b} - \frac{i}{c} - \frac{di}{dt}R = 0$ on solving it $i = \frac{2\mu_0 \pi a^2 C}{b}[1 - e^{-t/RC}]$ ■

SERIES COMBINATION OF INDUCTORS



$\therefore \quad V = V_1 + V_2$

$$L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

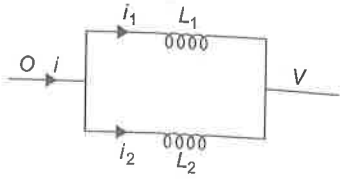
$$L_{eq} = L_1 + L_2 + \ldots$$

Parallel Combination of inductor

$$i = i_1 + i_2 \quad \Rightarrow \quad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{v}{L_{eq}} = \frac{v}{L_1} + \frac{v}{L_2}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots$$



EXERCISES

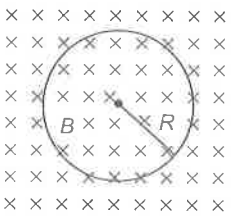
JEE Main

Section (a): Flux and Faraday's Laws of Electromagnetic Induction and Lenz's Law

1. A flux of 1 m Wb passes through a strip having an area $A = 0.02\ m^2$. The plane of the strip is at an angle of 60° to the direction of a uniform field B. The value of B is
   (A) 0.1 T (B) 0.058 T
   (C) 4.0 mT (D) none of the above.

2. The number of turns in a long solenoid is 500. The area of cross-section of solenoid is $2 \times 10^{-3}\ m^2$. If the value of magnetic induction, on passing a current of 2 A, through it is 5×10^{-3} T, the magnitude of magnetic flux connected with it in webers will be
   (A) 5×10^{-3} (B) 10^{-2}
   (C) 10^{-5} (D) 2.5

3. A conducting loop of radius R is present in a uniform magnetic field B perpendicular the plane of the ring. If radius R varies as a function of time 't', as $R = R_0 + t$. The emf induced in the loop is



   (A) $2\pi (R_0 + t) B$ clockwise
   (B) $\pi(R_0 + t) B$ clockwise
   (C) $2\pi(R_0 + t) B$ anticlockwise
   (D) zero

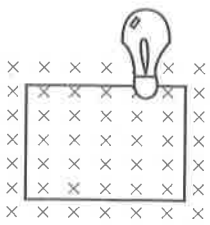
4. The instantaneous flux associated with a closed circuit of 10 Ω resistance is indicated by the following reaction $\phi = 6t^2 - 5t + 1$, then the value in amperes of the induced current at $t = 0.25$ sec will be:
   (A) 1.2 (B) 0.8
   (C) 6 (D) 0.2

5. When a magnet is being moved towards a coil, the induced emf does not depend upon
   (A) the number of turns of the coil
   (B) the motion of the magnet
   (C) the magnetic moment of the magnet
   (D) the resistance of the coil

6. An electron is moving in a circular orbit of radius R with an angular acceleration α. At the centre of the orbit is kept a conducting loop of radius r, ($r << R$). The emf induced in the smaller loop due to the motion of the electron is
   (A) zero, since charge on electron in constant
   (B) $\frac{\mu_0 e r^2}{4R}\alpha$
   (C) $\frac{\mu_0 e r^2}{4\pi R}\alpha$
   (D) none of these

7. A field of $5 \times 10^4/\pi$ ampere-turns/metre acts at right angles to a coil of 50 turns of area $10^{-2}\ m^2$. The coil is removed from the field in 0.1 second. Then the induced emf in the coil is
   (A) 0.1 V (B) 80 KV
   (C) 7.96 V (D) none of the above

8. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 20 T. A 10 V light bulb is in a series with the loop as shown in the fig. The magnetic field is decreasing steadily to zero over a time interval Δt. The bulb will shine with full brightness if Δt is equal to



   (A) 20 ms (B) 0.02 ms
   (C) 2 ms (D) 0.2 ms

9. The rate of change of magnetic flux density through a circular coil of area $10^{-2}\ m^2$ and number of turns 100 is 10^3 Wb/m^2/s. The value of induced emf will be
   (A) 10^{-2} V (B) 10^{-3} V
   (C) 10 V (D) 10^3 V

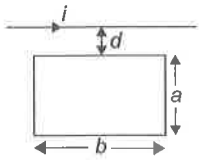
10. A long solenoid contains 1000 turns/cm and an alternating current of peak value 1A is flowing in it. A search coil of area of cross-section $1 \times 10^{-4}\ m^2$ and having 50 turns is placed inside the solenoid with

its plane perpendicular to the axis of the solenoid. A peak voltage of $2\pi^2 \times 10^{-2}$ V is produced in the search coil. The frequency of current in the solenoid will be

(A) 50 Hz (B) 100 Hz

(C) 500 Hz (D) 1000 Hz

11. A long straight wire is parallel to one edge as in fig. If the current in the long wire is varies in time as $I = I_0 e^{-t/\tau}$, what will be the induced emf in the loop?



(A) $\frac{\mu_0 bI}{\pi\tau}\ln\left(\frac{d+a}{d}\right)$ (B) $\frac{\mu_0 bI}{2\pi\tau}\ln\left(\frac{d+a}{d}\right)$

(C) $\frac{2\mu_0 bI}{\pi\tau}\ln\left(\frac{d+a}{d}\right)$ (D) $\frac{\mu_0 bI}{\pi\tau}\ln\left(\frac{d}{d+a}\right)$

12. The magnetic flux through a stationary loop with resistance R varies during interval of time T as $\phi = at(T - t)$. The heat generated during this time neglecting the inductance of loop will be

(A) $\frac{a^2T^3}{3R}$ (B) $\frac{a^2T^2}{3R}$

(C) $\frac{a^2T}{3R}$ (D) $\frac{a^2T^3}{R}$

13. The dimensions of permeability of free space can be given by

(A) $[MLT^{-2}A^{-2}]$ (B) $[MLA^{-2}]$

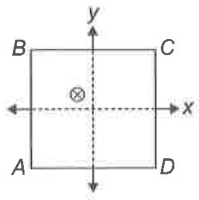
(C) $[ML^{-3}T^2A^2]$ (D) $[MLA^{-1}]$

14. A closed planar wire loop of area A and arbitrary shape is placed in a uniform magnetic field of magnitude B, with its plane perpendicular to magnetic field. The resistance of the wire loop is R. The loop is now turned upside down by 180° so that its plane again becomes perpendicular to the magnetic field. The total charge that must have flown through the wire ring in the process is

(A) $< \frac{AB}{R}$ (B) $= \frac{AB}{R}$

(C) $= \frac{2AB}{R}$ (D) None

15. A square coil $ABCD$ is placed in x-y plane with its centre at origin. A long straight wire, passing through origin, carries a current in negative z-direction. Current in this wire increases with time. The induced current in the coil is



(A) clockwise (B) anticlockwise

(C) zero (D) alternating

16. Two identical coaxial circular loops carry a current i each circulating in the same direction. If the loops approach each other

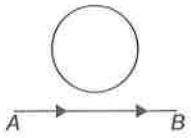
(A) the current in each will decrease

(B) the current in each will increase

(C) the current in each will remain the same

(D) the current in one will increase and in other will decrease

17. In the arrangement shown in given figure current from A to B is increasing in magnitude. Induced current in the loop will



(A) have clockwise direction

(B) have anticlockwise direction

(C) be zero

(D) oscillate between clockwise and anticlockwise

18. The north pole of a magnet is brought near a coil. The induced current in the coil as seen by an observer on the side of magnet will be

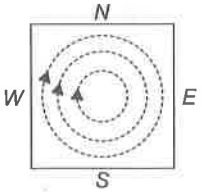
(A) in the clockwise direction

(B) in the anticlockwise direction

(C) initially in the clockwise and then anticlockwise direction

(D) initially in the anticlockwise and then clockwise direction.

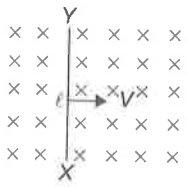
19. A metal sheet is placed in a variable magnetic field which is increasing from zero to maximum. Induced current flows in the directions as shown in figure. The direction of magnetic field will be



258

(A) normal to the paper, inwards
(B) normal to the paper, outwards.
(C) from east to west
(D) from north to south

20. A small conducting rod of length l, moves with a uniform velocity v in a uniform magnetic field B as shown in fig



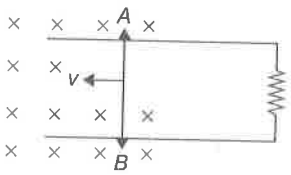
(A) Then the end X of the rod becomes positively charged
(B) the end Y of the rod becomes positively charged
(C) the entire rod is unevenly charged
(D) the rod becomes hot due to joule heating.

21. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge-length d is placed with its edge along x and y axis. The loop is moved with constant velocity $\vec{V} = V_0\hat{i}$. The emf induced in the loop is

(A) $\frac{V_0B_0d^2}{a}$ (B) $\frac{V_0B_0d^2}{2a}$

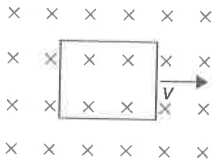
(C) $\frac{V_0B_0a^2}{d}$ (D) None

22. Consider the situation shown in fig. The resistanceless wire AB is slid on the fixed rails with a constant velocity. If the wire AB is replaced by a resistanceless semicircular wire, the magnitude of the induced current will



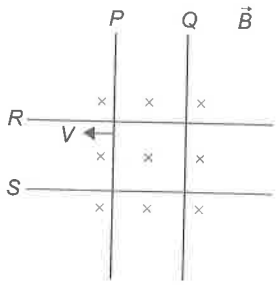
(A) increase
(B) remain the same
(C) decrease
(D) increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

23. A conducting square loop of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop in fig. The current induced in the loop is



(A) Blv/R clockwise (B) Blv/R anticlockwise
(C) $2Blv/R$ anticlockwise (D) zero

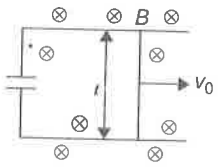
24. Two identical conductors P and Q are placed on two frictionless fixed conducting rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in figure with a constant speed, then rod Q



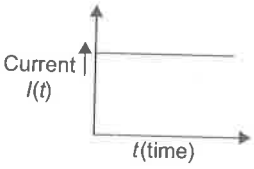
(A) will be attracted towards P
(B) will be repelled away from P
(C) will remain stationary
(D) may be repelled or attracted towards P

Section (B): Emf Induced in a Moving Rod

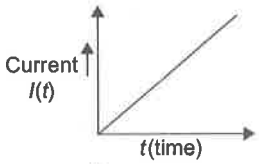
25. Two infinitely long conducting parallel rails are connected through a capacitor C as shown in the figure. A conductor of length l is moved with constant speed v_0. Which of the following graph truly depicts the variation of current through the conductor with time?



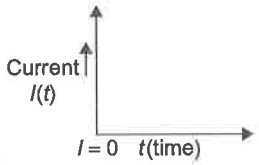
(A)



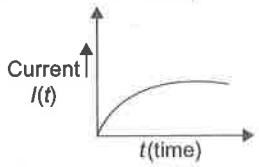
(B)



(C)



(D)



26. A thin wire of length 2 m is perpendicular to the xy plane. It is moved with velocity $v = (2\hat{i} + 3\hat{j} + \hat{k})$ m/s through a region of magnetic induction $\vec{B} = (\hat{i} + 2\hat{j})$ Wb/m^2. Then potential difference induced between the ends of the wire

(A) 2 V (B) 4 V

(C) 0 V (D) none of these

27. A long metal bar of 30 cm length is aligned along a north south line and moves eastward at a speed of 10 ms^{-1}. A uniform magnetic field of 4.0 T points vertically downwards. If the south end of the bar has a potential of 0 V, the induced potential at the north end of the bar is

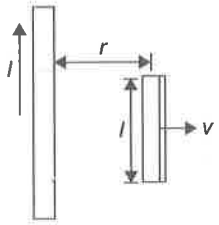
(A) +12 V

(B) −12 V

(C) 0 V

(D) cannot be determined since there is not closed circuit

28. A conducting rod moves with constant velocity v perpendicular to the long, straight wire carrying a current I as shown compute that the emf generated between the ends of the rod.



(A) $\frac{\mu_0 vIl}{\pi r}$ (B) $\frac{\mu_0 vIl}{2\pi r}$

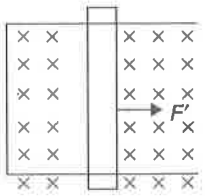
(C) $\frac{2\mu_0 v\, Il}{\pi r}$ (D) $\frac{\mu_0 v\, Il}{4\pi r}$

29. A square loop of side a and resistance R is moved in the region of uniform magnetic field B (loop remaining completely inside field), with a velocity v through a distance x. The work done is

(A) $\frac{Bl^2vx}{R}$ (B) $\frac{2B^2l^2vx}{R}$

(C) $\frac{4B^2l^2vx}{R}$ (D) none

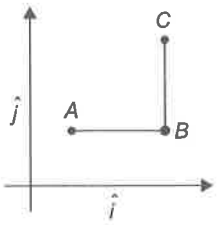
30. A rod closing the circuit shown in figure moves along a U shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicular to the plane. Calculate F if the rate of heat generation in the circuit is Q.



(A) $F = Qv$ (B) $F = \frac{Q}{v}$

(C) $F = \frac{v}{Q}$ (D) $F = \sqrt{Qv}$

31. There is a uniform magnetic field B normal to the xy plane. A conductor ABC has length $AB = l_1$, parallel to the x-axis, and length $BC = l_2$, parallel to the y-axis. ABC moves in the xy plane with velocity $v_x\hat{i} + v_y\hat{j}$. The potential difference between A and C is proportional to

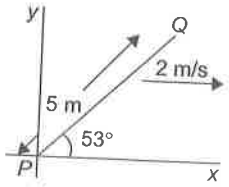


(A) $v_x l_1 + v_y l_2$ (B) $v_x l_2 + v_y l_1$

(C) $v_x l_2 - v_y l_1$ (D) $V_x l_1 - v_y l_2$

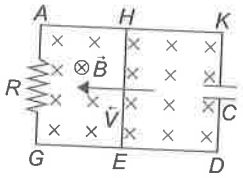
32. A conducting rod PQ of length 5 m oriented as shown in figure is moving with velocity $(2\text{m/s})\hat{i}$ without rotation in a uniform magnetic field $(3\hat{j} + 4\hat{k})$ T. Emf induced in the rod is

260



(A) 32 V (B) 40 V
(C) 50 V (D) none

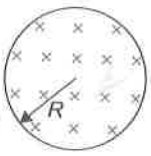
33. In the circuit shown in figure, a conducting wire HE is moved with a constant speed V towards left. The complete circuit is placed in a uniform magnetic field perpendicular to the plane of the circuit directed in inward direction. The current in HKDE is

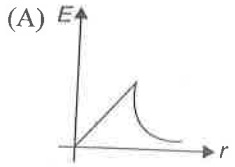


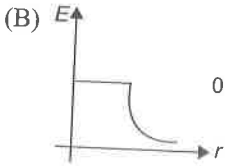
(A) clockwise (B) anticlockwise
(C) alternating (D) Zero

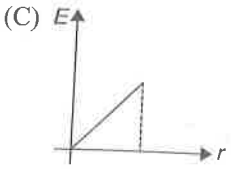
34. A conducting rod is moved with a constant velocity $\vec{v}$ in a magnetic field. A potential difference appears across the two ends
(A) if $\vec{v} \| \vec{l}$ (B) if $\vec{v} \| \vec{B}$
(C) if $\vec{l} \| \vec{B}$ (D) none of these

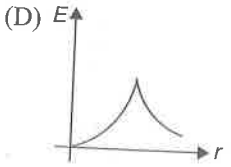
35. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is



(A) 

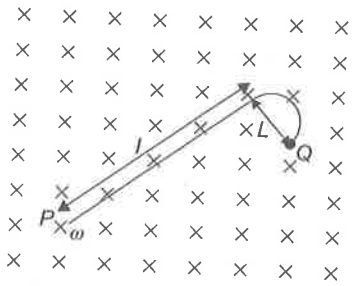
(B) 

(C) 

(D) 

Section (C): Induced emf in Rod, Ring Disc Rotating in a Uniform Magnetic Field

36. When a 'J' shaped conducting rod is rotating in its own plane with constant angular velocity ω, about one of its end P, in a uniform magnetic field $\vec{B}$ directed normally into the plane of paper) then magnitude of emf induced across it will be



(A) $B\omega\sqrt{L^2 + l^2}$ (B) $\frac{1}{2}B\omega L^2$
(C) $\frac{1}{2}B\omega(L^2 + l^2)$ (D) $\frac{1}{2}B\omega l^2$

37. A rectangular coil of single turn, having area A, rotates in a uniform magnetic field B an angular velocity ω about an axis perpendicular to the field. If initially the plane of coil is perpendicular to the field, then the average induced emf when it has rotated through 90° is

(A) $\frac{\omega BA}{\pi}$ (B) $\frac{\omega BA}{2\pi}$
(C) $\frac{\omega BA}{4\pi}$ (D) $\frac{2\omega BA}{\pi}$

38. A ring of resistance 10 Ω, radius 10 cm and 100 turns is rotated at a rate 100 revolutions per second about its diameter is perpendicular to a uniform magnetic field of induction 10 mT. The amplitude of the current in the loop will be nearly (Take: $\pi^2 = 10$)
(A) 200 A (B) 2 A
(C) 0.002 A (D) none of these

39. A circular copper disc of radius 25 cm is rotating about its own axis with a constant angular velocity of 130 rad/s. If a magnetic field of 5×10^{-3} T is applied at right angles to the disc, then the induced potential difference between the centre and the rim of the disc will approximately be
(A) 20×10^{-3} V (B) 20×10^{-6} V
(C) 20×10^{-9} V (D) Zero

40. The spokes of a wheel are made of metal and their lengths are of one metre. On rotating the wheel about its own axis in a uniform magnetic field of 5×10^{-5} T normal to the plane of wheel, a potential difference of 3.14 mV is generated between the rim and the axis. The rotational velocity of the wheel is

(A) 63 rev./s (B) 50 rev./s

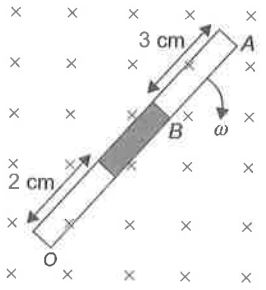
(C) 31.4 rev./s (D) 20 rev./s

41. A rod of length l rotates with a uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

(A) zero (B) $\frac{1}{2}\omega B\ell^2$

(C) $B\omega\ell^2$ (D) $2B\omega\ell^2$

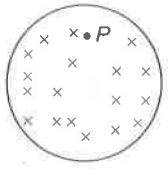
42. A rod of length 10 cm made up of conducting and non-conducting material (shaded part is non-conducting). The rod is rotated with constant angular velocity 10 rad/sec about point O, in constant magnetic field of 2 T as shown in the figure. The induced emf between the point A and B of rod will be



(A) 0.029 V (B) 0.1 V

(C) 0.051 V (D) 0.064 V

43. Figure shows a uniform magnetic field B confined to a cylindrical volume and is increasing at a constant rate. The instantaneous acceleration experienced by an electron placed at P is



(A) zero (B) towards right

(C) towards left (D) upwards

Section (D): Self Inductance, Mutual Inductance, Circuit Containing Resistance Inductance, and L.C. Oscillations

44. For L-R circuit, the time constant is equal to

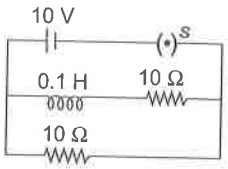
(A) twice the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(B) ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(C) half the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(D) square of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

45. In the adjoining circuit, initially the switch S is open. The switch 'S' is closed at $t = 0$. The difference between the maximum and minimum current that can flow in the circuit is



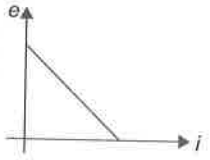
(A) 2 A

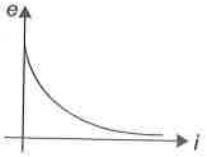
(B) 3 A

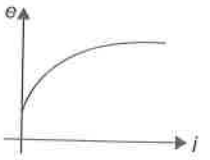
(C) 1 A

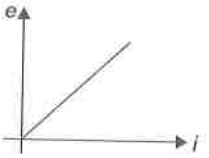
(D) nothing can be concluded

46. In an L-R circuit connected to a battery of constant emf E switch S is closed at time $t = 0$. If e denotes the magnitude of induced emf across inductor and i the current in the circuit at any time t. Then which of the following graphs shows the variation of e with i?

(A) 

(B) 

(C) 

(D) 

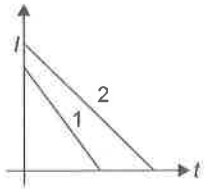
47. A current of 2 A is increasing at a rate of 4 A/s through a coil of inductance 2 H. The energy stored in the inductor per unit time is

(A) 2 J/s (B) 1 J/s

(C) 16 J/s (D) 4 J/s

262

48. Two identical inductance carry currents that vary with time according to linear laws (as shown in figure). In which of two inductance is the self induction emf greater?



(A) 1
(B) 2
(C) same
(D) data are insufficient to decide

49. L, C and R represent physical quantities inductance, capacitance and resistance. The combination which has the dimensions of frequency is

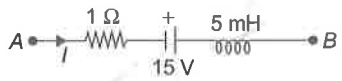
(A) $\frac{1}{RC}$ and $\frac{R}{L}$ (B) $\frac{1}{\sqrt{RC}}$ and $\sqrt{\frac{R}{L}}$

(C) $\sqrt{LC}$ (D) $\frac{C}{L}$

50. A long solenoid of N turns has a self inductance L and area of cross section A. When a current i flows through the solenoid, the magnetic field inside it has magnitude B. The current i is equal to:

(A) BAN/L (B) BANL
(C) BN/AL (D) B/ANL

51. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5 A and it is decreasing at a rate of 10^3 As^{-1} then $V_B - V_A$ equals



(A) 20 V (B) 15 V
(C) 10 V (D) 5 V

52. In **Previous Problem** if I is reversed in direction, then $V_B - V_A$ equals

(A) 5 V (B) 10 V
(C) 15 V (D) 20 V

53. On making a coil of copper wire of length ℓ and coil radius r, the value of self inductance is obtained as L. If the coil of same wire, but of coil radius $r/2$, is made, the value of self inductance will be

(A) $2L$ (B) L
(C) $4L$ (D) $L/2$

54. The coefficient of mutual induction between two coils is 4H. If the current in the primary reduces from 5 A to zero in 10^{-3} second then the induced emf in the secondary coil will be

(A) 10^4 V (B) 25×10^3 V
(C) 2×10^4 V (D) 15×10^3 V

55. The number of turns in a coil of wire of fixed radius is 600 and its self inductance is 108 mH. The self inductance of a coil of 500 turns will be

(A) 74 mH (B) 75 mH
(C) 76 mH (D) 77 mH

56. A thin copper wire of length 100 metres is wound as a solenoid of length ℓ and radius r. Its self inductance is found to be L. Now if the same length of wire is wound as a solenoid of length ℓ but of radius $r/2$, then its self inductance will be

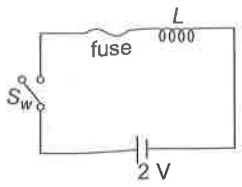
(A) $4L$ (B) $2L$
(C) L (D) $L/2$

57. A small coil of radius r is placed at the centre of a large coil of radius R, where $R >> r$. The coils are coplanar. The coefficient of mutual inductance between the coils is

(A) $\frac{\mu_0 \pi r}{2R}$ (B) $\frac{\mu_0 \pi r^2}{2R^2}$

(C) $\frac{\mu_0 \pi r^2}{2R}$ (D) $\frac{\mu_0 \pi r}{2R}$

58. In the circuit shown, the cell is ideal. The coil has an inductance of 4 H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5 A. The switch is closed at $t = 0$. The fuse will blow:



(A) just after $t = 0$ (B) after 2 s
(C) after 5 s (D) after 10 s

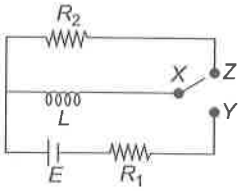
59. The self inductance and resistance of a coil are 5 H and 20 Ω respectively. On applying an emf of 100 V on it, the magnetic potential energy stored in the coil will be

(A) 62.5 J (B) 62.5 erg
(C) 62.5×10^{-3} J (D) zero

60. The current in an L-R circuit in a time $t = 2L/R$ reduces to

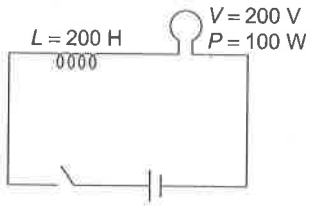
(A) 36.5% of maximum (B) 13.5% of maximum
(C) 0.50% of maximum (D) 63.2% of maximum

61. In the circuit shown, X is joined to Y for a long time, and then x is joined to Z. The total heat produced in R_2 is:



(A) $\frac{LE^2}{2R_1^2}$ (B) $\frac{LE^2}{2R_2^2}$
(C) $\frac{LE^2}{2R_1R_2}$ (D) $\frac{LE^2R_2}{2R_1^2}$

62. Calculate the ratio of power dissipated by the bulb at $t = 1$ sec and $t = 2$ sec after closing the switch



(A) $\frac{e^2}{e^4 - 1}$ (B) $\frac{e^2}{(e^2 + 1)}$
(C) $\frac{e^2 - 1}{e^4 + 1}$ (D) None of these

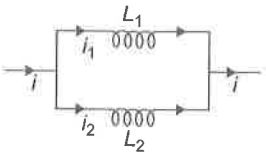
63. An inductor coil stores U energy when i current is passed through it and dissipates energy at the rate of P. The time constant of the circuit, when this coil is connected across a battery of zero internal resistance is

(A) $\frac{4U}{P}$ (B) $\frac{U}{P}$
(C) $\frac{2U}{P}$ (D) $\frac{2P}{U}$

64. A long straight wire is placed along the axis of a circular ring of radius R. The mutual inductance of this system is

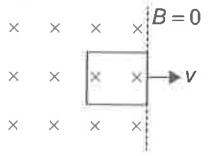
(A) $\frac{\mu_0 R}{2}$ (B) $\frac{\mu_0 \pi R}{2}$
(C) $\frac{\mu_0}{2}$ (D) 0

65. Two inductors L_1 and L_2 are connected in parallel and a time varying current i flows as shown. The ratio of currents $\frac{i_1}{i_2}$ at any time t is



(A) $\frac{L_1}{L_2}$ (B) $\frac{L_2}{L_1}$
(C) $\frac{L_1^2}{(L_1 + L_2)^2}$ (D) $\frac{L_2^2}{(L_1 + L_2)^2}$

66. Figure shows a square loop of side 0.5 m and resistance 10 Ω. The magnetic field has a magnitude B = 1.0 T. The work done in pulling the loop out of the field uniformly in 2.0 s is



(A) 3.125×10^{-3} J (B) 6.25×10^{-4} J
(C) 1.25×10^{-2} J (D) 5.0×10^{-4} J

67. In a series L-R growth circuit, if maximum current and maximum induced emf in an inductor of inductance 3 mH are 2 A and 6 V respectively, then the time constant of the circuit is
(A) 1 ms (B) 1/3 ms
(C) 1/6 ms (D) 1/2 ms

68. Two coils of self inductance 100 mH and 400 mH are placed very close to each other. Find the maximum mutual inductance between the two when 4 A current passes through them
(A) 200 mH (B) 300 mH
(C) $100\sqrt{2}$ mH (D) none of these

69. The value of coefficient of mutual induction in two coils can be increased by
(A) placing the coils mutually perpendicular.
(B) keeping the coils near to each other.
(C) keeping the coils considerably apart.
(D) winding the core on the common iron magnetic material and insulating them.

70. A solenoid having a core of cross-section 4 cm^2, half air and half iron (relative permeability = 500), is 22 cm long. If the number of turns on it is 10^3 its self inductance is

264

(A) 570 H
(B) 57 H
(C) 5.7 H
(D) 0.57 H

71. If two coils of negligible mutual induction and having coefficient of inductances L_1 and L_2 $(L_1 > L_2)$ are arranged in parallel, the value of the equivalent self-induction will be

(A) $\frac{L_1L_2}{(L_1 - L_2)}$
(B) $\frac{L_1L_2}{(L_1 + L_2)}$
(C) $\frac{(L_1 + L_2)}{L_1L_2}$
(D) $\frac{(L_1 - L_2)}{L_1L_2}$

Section (E) Transformer and AC Generator

72. To transmit electrical energy from a generator to distant consumers
(A) high voltage and low current are transmitted
(B) high voltage and high current are transmitted
(C) low voltage and low current are transmitted
(D) low voltage and high current are transmitted

73. A transformer transforms 220 V to 11 V. If the current strength in the primary coil is 5 A. and that in the secondary, 90 A, what is the efficiency of the transformer
(A) 90%
(B) 100%
(C) 211%
(D) 150%

74. A step up transformer operates on a 230 V line and supplies a load of 2 A. The ratio of primary and secondary windings is 1:25. Determine the primary current
(A) 12.5 A
(B) 50 A
(C) 8.8 A
(D) 25 A

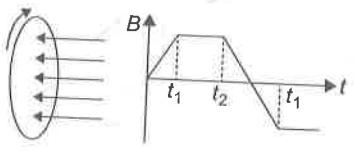
75. A transformer is used to light 140 W 24 V lamp from 240 V AC mains, the current in the main cable is 0.7 A. The efficiency of the transformer is
(A) 63.8%
(B) 84%
(C) 83.3%
(D) 48%

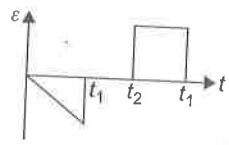
JEE Advanced

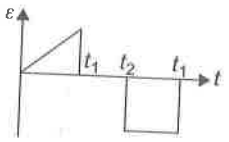
Single Correct Option Type Questions

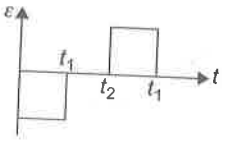
Section (A): Flux and Faraday's Laws of Electromagnetic Induction and Lenz's law

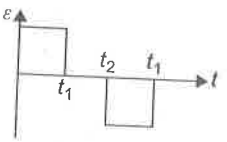
1. A wire loop is placed in a region of time varying magnetic field which is oriented orthogonally to the plane of the loop as shown in the figure. The graph shows the magnetic field variation as the function of time. Assume the positive emf is the one which drives a current in the clockwise direction and seen by the observer in the direction of B. Which of the following graphs best represents the induced emf as a function of time.



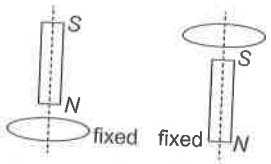
(A) 

(B) 

(C) 

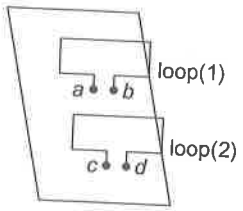
(D) 

2. A vertical bar magnet is dropped from position on the axis of a fixed metallic coil as shown in figure-I. In figure-II the magnet is fixed and horizontal coil is dropped. The acceleration of the magnet and coil are a_1 and a_2 respectively then



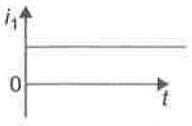
(A) $a_1 > g, a_2 > g$
(B) $a_1 > g, a_2 < g$
(C) $a_1 < g, a_2 < g$
(D) $a_1 < g, a_2 > g$

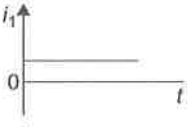
3. An electric current i_1 can flow either direction through loop (1) and induced current i_2 in loop (2). Positive i_1 is when current is from 'a' to 'b' in loop (1) and positive i_2 is when the current is from 'c' to 'd' in loop (2)

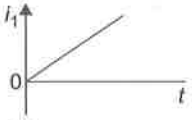


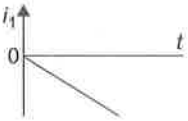
In an experiment, the graph of i_2 against time 't' is as shown below

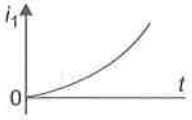
Which one(s) of the following graphs could have caused i_2 to behave as give above.

(A) 

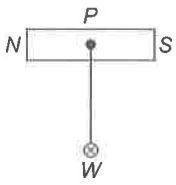
(B) 

(C) 

(D) 

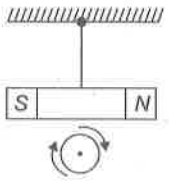
(E) 

4. Figure shows a bar magnet and a long straight wire W, carrying current into the plane of paper. Point P is the point of intersection of axis of magnet and the line of shortest distance between magnet and the wire. If p is the midpoint of the magnet, then which of the following statements is correct?



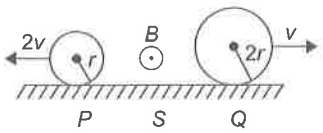
(A) magnet experiences a torque in clockwise direction
(B) magnet experiences a torque in anticlockwise direction
(C) magnet experiences a force, normal to the line of shortest distance
(D) magnet experiences a force along the line of shortest distance

5. A negative charge is given to a nonconducting loop and the loop is rotated in the plane of paper about its centre as shown in figure. The magnetic field produced by the ring affects a small magnet placed above the ring in the same plane:



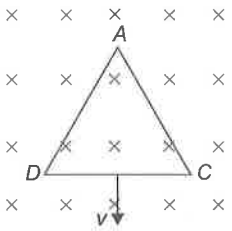
(A) the magnet does not rotate.
(B) the magnet rotates clockwise as seen from below.
(C) the magnet rotates anticlockwise as seen from below.
(D) no effect on magnet is there.

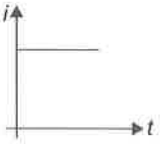
6. Two conducting rings P and Q of radii r and $2r$ rotate uniformly in opposite directions with centre of mass velocities $2v$ and v respectively on a conducting surface S. There is a uniform magnetic field of magnitude B perpendicular to the plane of the rings. The potential difference between the highest points of the two rings is

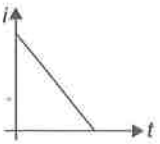


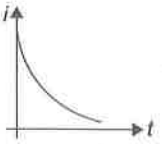
(A) zero
(B) $4\,Bvr$
(C) $8\,Bvr$
(D) $16\,Bvr$

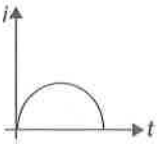
7. An equilateral triangular loop ADC having some resistance is pulled with a constant velocity v out of a uniform magnetic field directed into the paper. At time $t = 0$, side DC of the loop is at edge of the magnetic field. The induced current (i) versus time (t) graph will be as



(A) 

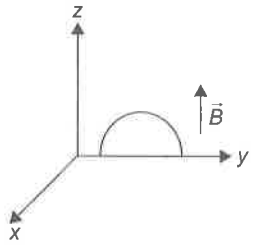
(B) 

(C) 

(D) 

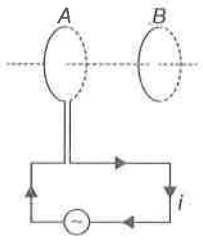
8. A semicircular conducting wire is placed in yz plane in a uniform magnetic field directed along positive z-direction. An induced emf will be developed between the ends of the wire if it is moved along.

266



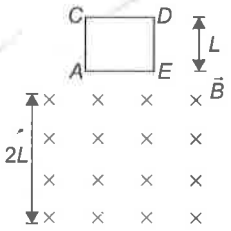
(A) positive x direction (B) positive y direction
(C) positive z direction (D) none of these

9. Two circular coils A and B are facing each other as shown in figure.



The current i through A can be altered
(A) there will be repulsion between A and B if i is increased
(B) there will be attraction between A and B if i is increased
(C) there will be neither attraction nor repulsion when i is changed
(D) attraction or repulsion between A and B depends on the direction of current.
It does not depend whether the current increased or decreased

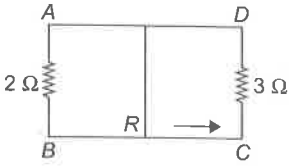
10. A square coil $ACDE$ with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length $2L$. The acceleration of the coil is



(A) less than g for all the time till loop crosses the magnetic field completely
(B) less than g when it enters the field and greater than g when it comes out of the field
(C) g all the time
(D) less than g when it enters and comes out of the field but equal to g when it is within the field

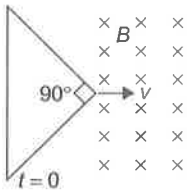
Section (B): Emf Induced in a Moving Rod

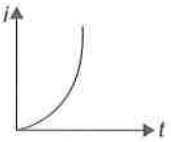
11. A rectangular loop with a sliding connector of length 10 cm is situated in uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 T and resistance of connector (R) is 1 Ω. The sides AB and CD have resistances 2 Ω and 3 Ω respectively. Find the current in the connector during its motion with constant velocity 1 m/sec.

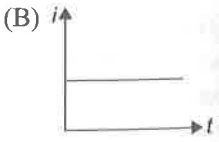


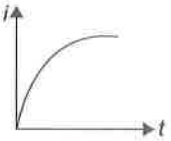
(A) $\frac{1}{110}$ A (B) $\frac{1}{220}$ A
(C) $\frac{1}{55}$ A (D) $\frac{1}{440}$ A

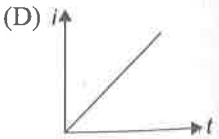
12. The figure shows an isosceles triangle wire frame with apex angle equal to $\frac{\pi}{2}$. The frame starts entering into the region of uniform magnetic field B with constant velocity v at $t = 0$. The longest side of the frame is perpendicular to the direction of velocity. If i is the instantaneous current through the frame then choose the alternative showing the correct variation of i with time.



(A) 

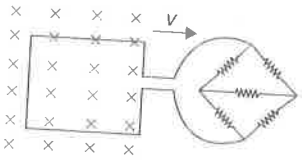
(B) 

(C) 

(D) 

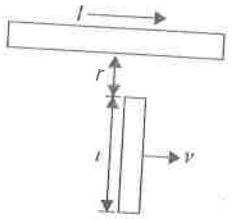
13. A square metal loop of side 10 cm and resistance 1 Ω is moved with a constant velocity partly inside a magnetic field of 2 Wbm^{-2}, directed into the paper, as shown in the figure. This loop is connected to

a network of five resistors each of value 3 Ω. If a steady current of 1 mA flows in the loop, then the speed of the loop is



(A) 0.5 cm s^{-1} (B) 1 cm s^{-1}
(C) 2 cm s^{-1} (D) 4 cm s^{-1}

14. A conducting rod of length l moves with velocity v a direction parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with near end a distance r away as shown in the figure. Find the emf induced in the rod.



(A) $\frac{\mu_0 Iv}{\pi}\ln\left(\frac{r+l}{r}\right)$ (B) $\frac{2\mu_0 Iv}{\pi}\ln\left(\frac{r+l}{r}\right)$
(C) $\frac{\mu_0 Iv}{\pi}\ln\left(\frac{l}{r+l}\right)$ (D) $\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{r+l}{r}\right)$

15. A metallic rod of length L and mass M is moving under the action of two unequal forces F_1 and F_2 (directed opposite to each other) acting at its ends along its length. Ignore gravity and any external magnetic field. If specific charge of electrons is (e/m), then the potential difference between the ends of the rod is steady state must be

(A) $|F_1 - F_2|$ mL/eM (B) $(F_1 - F_2)$ mL/eM
(C) [mL/eM] $\ln[F_1/F_2]$ (D) None

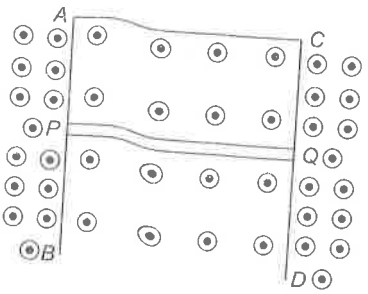
16. Two parallel long straight conductors lie on a smooth surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. They all start moving out with a constant velocity v. If r is the resistance per unit length of the wire the current in the circuit will be

(A) $\frac{Bv}{r}$ (B) $\frac{Br}{v}$
(C) Bvr (D) Bv

17. A uniform magnetic field exists in region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5 m is placed along y-axis is moved along x-axis with constant speed 1 m/sec. Then induced emf in the rod will be

(A) zero (B) 25 V
(C) 20 V (D) 15 V

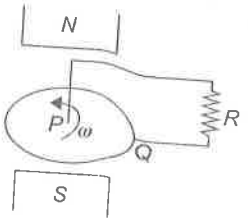
18. $BACD$ is a fixed conducting smooth rail placed in a vertical plane. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released from rest then,



(A) The rod PQ will move downward with constant acceleration
(B) The rod PQ will move upward with constant acceleration
(C) The rod will move downward with decreasing acceleration and finally acquire a constant velocity
(D) either A or B

Section (C): Induced emf in Rod, Ring Disc Rotating in a Uniform Magnetic Field

19. A metal disc rotates freely, between the poles of a magnet in the direction indicated. Brushes P and Q make contact with the edge of the disc and the metal axle. What current, if any, flows through R?



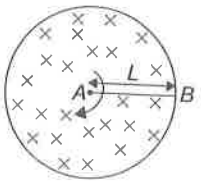
(A) a current from P to Q
(B) a current from Q to P
(C) no current, because the emf in the disc is opposed by the back emf

268

(D) no current, because the emf induced in one side of the disc is opposed by the emf induced in the other side.

(E) no current, because no radial emf is induced in the disc.

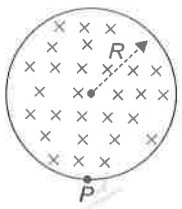
20. A copper rod AB of length L, pivoted at one end A, rotates at constant angular velocity ω, at right angles to a uniform magnetic field of induction B. The emf developed between the mid point C of the rod and end B is



(A) $\frac{B\omega l^2}{4}$ (B) $\frac{B\omega l^2}{2}$

(C) $\frac{3B\omega l^2}{4}$ (D) $\frac{3B\omega l^2}{8}$

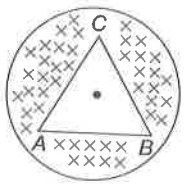
21. A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (T/s). An electron of charge q, placed at the point P on the periphery of the field experiences an acceleration



(A) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward left (B) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward right

(C) $\frac{eR}{m}\frac{dB}{dt}$ toward left (D) zero

22. A triangular wire frame (each side = 2 m) is placed in a region of time variant magnetic field having $\frac{dB}{dt} = \sqrt{3}$ T/s. The magnetic field is perpendicular to the plane of the triangle. The base of the triangle AB has a resistance 1 Ω while the other two sides have resistance 2 Ω each. The magnitude of potential difference between the points A and B will be

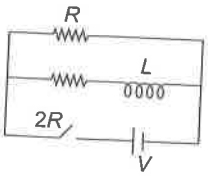


(A) 0.4 V (B) 0.6 V

(C) 1.2 V (D) None

Section (D): Self Inductance, Mutual Inductance, Circuit Containing Resistance Inductance, and L.C. Oscillations

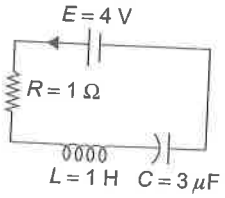
23. The ratio of time constant in charging and discharging in the circuit shown in figure is



(A) 1:1 (B) 3:2

(C) 2:3 (D) 1:3

24. The current in the given circuit is increasing with a rate $a = 4$ A/s. The charge on the capacitor at an instant when the current in the circuit is 2 A will be:



(A) 4 μC (B) 5 μC

(C) 6 μC (D) none of these

25. A coil of inductance 5 H is joined to a cell of emf 6 V through a resistance 10 Ω at time $t = 0$. The emf across the coil at time $t = \ln\sqrt{2}$ s is:

(A) 3 V (B) 1.5 V

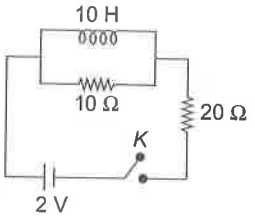
(C) 0.75 V (D) 4.5 V

26. A long straight wire of circular cross-section is made of a non-magnetic material. The wire is of radius a. The wire carries a current I which is uniformly distributed over its cross-section. The energy stored per unit length in the magnetic field contained within the wire is:

(A) $U = \frac{\mu_0 I^2}{8\pi}$ (B) $U = \frac{\mu_0 I^2}{16\pi}$

(C) $U = \frac{\mu_0 I^2}{4\pi}$ (D) $U = \frac{\mu_0 I^2}{2\pi}$

27. Two resistors of 10 Ω, and 20 Ω and an ideal inductor of 10 H are connected to a 2 V battery as shown. The key K is inserted at time $t = 0$. The initial ($t = 0$) and final ($t \to \infty$) currents through battery are



(A) $\frac{1}{15}$ A, $\frac{1}{10}$ A (B) $\frac{1}{10}$ A, $\frac{1}{15}$ A

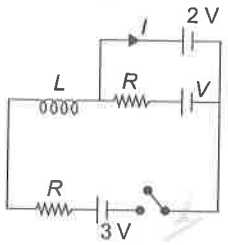
(C) $\frac{2}{15}$ A, $\frac{1}{10}$ A (D) $\frac{1}{15}$ A, $\frac{2}{25}$ A

28. Two long parallel wires whose centres are a distance d apart carry equal currents in opposite directions. If the flux within wires is neglected, the inductance of such arrangement of wire of length l and radius a will be

(A) $L = \frac{\mu_0 l}{\pi} \log_e \frac{d-a}{a}$ (B) $L = \frac{\mu_0 l}{\pi} \log_e \frac{d}{a}$

(C) $L = \frac{\mu_0 l}{\pi} \log_e \frac{a}{d}$ (D) none

29. In the LR circuit shown, what is the variation of the current I as a function of time? The switch is closed at time $t = 0$ sec.



(A) $\frac{V}{R}\left(1 - e^{\frac{Rt}{L}}\right)$ (B) $\frac{V}{R} e^{-\frac{Rt}{L}}$

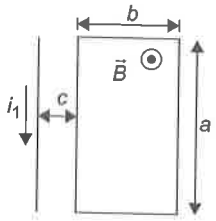
(C) $-\frac{V}{R} e^{-\frac{Rt}{L}}$ (D) None

30. In a L-R decay circuit, the initial current at $t = 0$ is I. The total charge that has flown through the resistor till the energy in the inductor has reduced to one-fourth its initial value, is

(A) $\frac{LI}{R}$ (B) $\frac{LI}{2R}$

(C) $LI\frac{\sqrt{2}}{R}$ (D) None

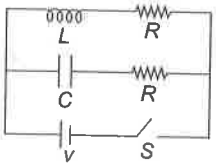
31. The mutual inductance between the rectangular loop and the long straight wire as shown in figure is M.



(A) M = Zero (B) $M = \frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{c}{b}\right)$

(C) $M = \frac{\mu_0 b}{2\pi} \ln\left(\frac{a+c}{b}\right)$ (D) $M = \frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{b}{c}\right)$

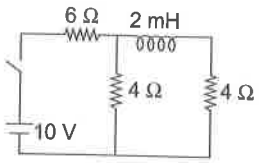
32. In the circuit shown in the figure, $R = \sqrt{\frac{L}{C}}$. Switch S is closed at time $t = 0$. The current through C and L would be equal after a time t equal to:



(A) CR (B) $CR \ln(2)$

(C) $\frac{L}{R \ln(2)}$ (D) LR

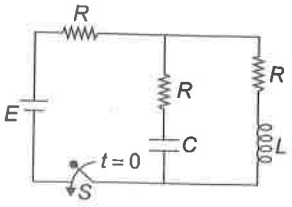
33. In the given circuit find the ratio of i_1 to i_2. Where i_1 is the initial (at $t = 0$) current, and i_2 is steady state (at $t = \infty$) current through the battery:



(A) 1.0 (B) 0.8

(C) 1.2 (D) 1.5

34. In the circuit shown in figure, switch S is closed at $t = 0$. Then:



270

(A) after a long time interval potential difference across capacitor and inductor will be equal
(B) after a long time interval charge on capacitor will be EC.
(C) after a long time interval current in the inductor will be E/R.
(D) after a long time interval current through battery will be same as the current through it initially.

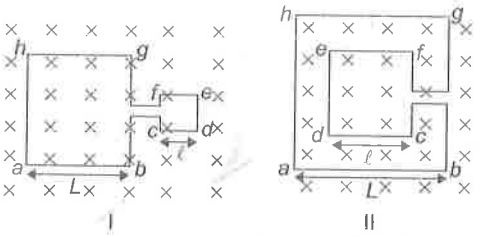
35. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s the emf in coil 1 in 25.0 mV, when coil 2 has no current and coil 1 has a current of 3.6 A, flux linkage in coil 2 is
(A) 16 mWb (B) 10 mWb
(C) 4.00 mWb (D) 6.00 mWb

Multiple correct

Section (A): Flux and Faraday's Laws of Electromagnetic Induction and Lenz's Law

36. The dimension of the ratio of magnetic flux and the resistance is equal to that of:
(A) induced emf (B) charge
(C) inductance (D) current

Common Data for Questions 37–40: The adjoining figure shows two different arrangements in which two square wire frames of same resistance are placed in a uniform constantly decreasing magnetic field B.



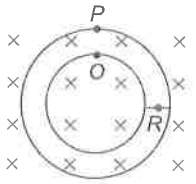
37. The value of magnetic flux in each case is given by
(A) Case I: $\phi = \pi(L^2 + \ell^2)\,B$; Case II: $\phi = \pi(L^2 - \ell^2)\,B$
(B) Case I: $\phi = \pi(L^2 + \ell^2)\,B$; Case II: $\phi = \pi(L^2 + \ell^2)\,B$
(C) Case I: $\phi = (L^2 + \ell^2)\,B$; Case II: $\phi = (L^2 - \ell^2)\,B$
(D) Case I: $\phi = (L + \ell)^2\,B$; Case II: $\phi = (L - \ell)^2\,B$

38. The direction of induced current in the case I is:
(A) from a to b and from c to d
(B) from a to b and from f to e
(C) from b to a and from d to c
(D) from b to a and from e to f

39. The direction of induced current in the case II is
(A) from a to b and from c to d
(B) from b to a and from f to e
(C) from b to a and from c to d
(D) from a to b and from d to c

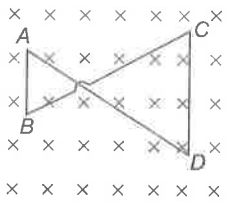
40. If I_1 and I_2 are the magnitudes of induced current in the cases I and II, respectively, then
(A) $I_1 = I_2$ (B) $I_1 > I_2$
(C) $I_1 < I_2$ (D) nothing can be said

41. Figure shown plane figure made of a conductor located in a magnetic field along the inward normal to the plane of the figure. The magnetic field starts diminishing. Then the induced current



(A) at point P is clockwise
(B) at point Q is anticlockwise
(C) at point Q is clockwise
(D) at point R is zero

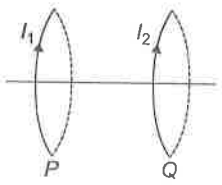
42. A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



(A) B to A and D to C (B) A to B and C to D
(C) A to B and D to C (D) B to A and C to D

43. A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?
(A) The south pole faces the ring and the magnet moves towards it.
(B) The north pole faces the ring and the magnet moves towards it.
(C) The south pole faces the ring and the magnet moves away from it.
(D) The north pole faces the ring and the magnet moves away from it.

44. Two circular coils P and Q are fixed coaxially and carry currents I_1 and I_2 respectively



(A) if $I_2 = 0$ and P moves towards Q, a current in the same direction as I_1 is induced in Q

(B) if $I_1 = 0$ and Q moves towards P, a current in the opposite direction to that of I_2 is induced in P.

(C) When $I_1 \neq 0$ and $I_2 \neq 0$ are in the same direction then the two coils tend to move apart.

(D) when $I_1 \neq 0$ and $I_2 \neq 0$ are in opposite directions then the coils tends to move apart.

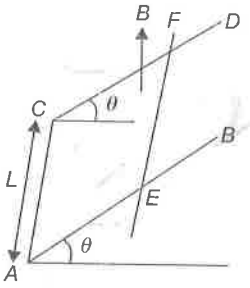
45. Which of the following quantities can be written in *SI* units in K $gm^2 A^{-2} S^{-3}$?

(A) Resistance (B) Inductance

(C) Capacitance (D) Magnetic flux

Section (B): Emf Induced in a Moving Rod

46. AB and CD are smooth parallel rails, separated by a distance l, and inclined to the horizontal at an angle θ. A uniform magnetic field of magnitude B, directed vertically upwards, exists in the region. EF is a conductor of mass m, carrying a current i. For EF to be in equilibrium,



(A) i must flow from E to F (B) Bil = mg tan θ

(C) Bil = mg sin θ (D) Bil = mg

47. In the **previous question,** if B is normal to the plane of the rails

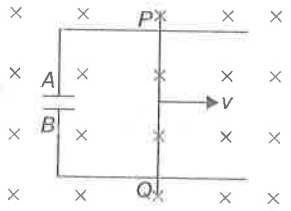
(A) Bil = mg tan θ

(B) Bil = mg sin θ

(C) Bil = mg cos θ

(D) equilibrium cannot be reached

48. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 20$ m/s in a uniform magnetic field $B = 4.0$ T directed into the paper. A capacitor of capacity $C = 10\ \mu$F is connected as shown in figure. Then



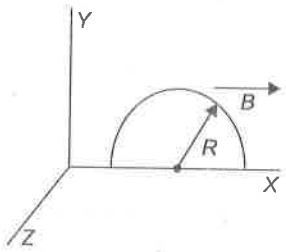
(A) $q_A = +800\ \mu$C and $q_B = -800\ \mu$C

(B) $q_A = -800\ \mu$C and $q_B = +800\ \mu$C

(C) $q_A = 0 = q_B$

(D) charged stored in the capacitor increases exponentially with time

49. A semicircle conducting ring of radius R is placed in the xy plane, as shown in the figure. A uniform magnetic field is set up along the x-axis. No net emf, will be induced in the ring. if



(A) it moves along the x-axis

(B) it moves along the y-axis

(C) it moves along the z-axis

(D) it remains stationary

Section (C): Induced emf in Rod, Ring Disc Rotating in a Uniform Magnetic Field

50. The emf induced in a coil of wire, which is rotating in a magnetic field, does not depend on

(A) the angular speed of rotation

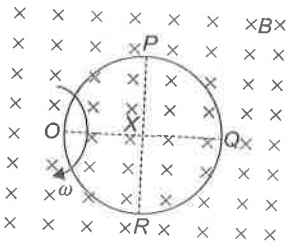
(B) the area of the coil

(C) the number of turns on the coil

(D) the resistance of the coil

Common Data for Questions 51–53: A conducting ring of radius a is rotated about a point O on its periphery as shown in the figure in a plane perpendicular to uniform magnetic field B which exists everywhere. The rotational velocity is ω.

272



51. Choose the correct statement(s) related to the potential of the points P, Q and R

(A) $V_p - V_O > 0$ and $V_R - V_0 < 0$

(B) $V_p = V_R > V_O$

(C) $V_O > V_P = V_Q$

(D) $V_Q - V_P = V_P - V_O$

52. choose the correct statement(s) related to the magnitude of potential differences

(A) $V_P - V_O = \frac{1}{2} B\omega a^2$ (B) $V_P - V_Q = \frac{1}{2} B\omega a^2$

(C) $V_Q - V_O = 2\, B\omega a^2$ (D) $V_P - V_R = 2B\omega a^2$

53. Choose the correct statement(s) related to the induced current in ring

(A) Current flows from $Q \to P \to O \to R \to Q$

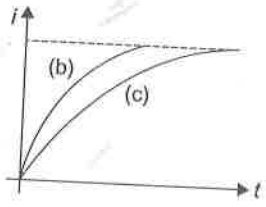
(B) Current flows from $Q \to R \to O \to P \to Q$

(C) Current flows from $Q \to P \to O$ and from $Q \to R \to O$

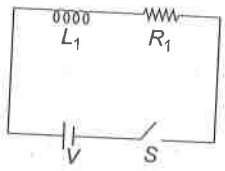
(D) No current flows

Section (D): Self Inductance, Mutual Inductance, Circuit Containing Resistance Inductance, and L.C. Oscillations

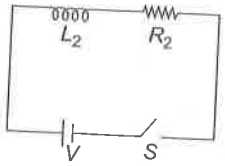
54. Current growth in two L–R circuits (b) and (c) as shown in figure (A). Let L_1, L_2, R_1 and R_2 be the corresponding values in two circuits. Then



(a)



(b)

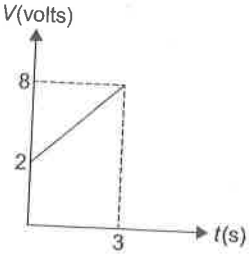


(c)

(A) $R_1 > R_2$ (B) $R_1 = R_2$

(C) $L_1 > L_2$ (D) $L_1 < L_2$

55. A circuit element is placed in a closed box. At time $t = 0$, constant current generator supplying a current of 1 A, is connected across the box. Potential difference across the box varies according to graph shown in figure. The element in the box is:



(A) resistance of 2 Ω (B) battery of emf 6 V

(C) inductance of 2 H (D) capacitance of 0.5 F

56. The symbols L, C, R represent inductance, capacitance and resistance respectively. Dimension of frequency are given by the combination

(A) $\frac{1}{RC}$ (B) $\frac{R}{L}$

(C) $\frac{1}{\sqrt{LC}}$ (D) $\frac{C}{L}$

57. An LR circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the circuit

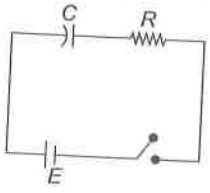
(A) current in the circuit

(B) magnetic field energy in the inductor

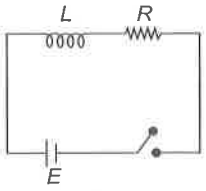
(C) power delivered by the battery

(D) emf induced in the inductor

58. The switches in figures (a) and (b) are closed at $t = 0$



(a)



(b)

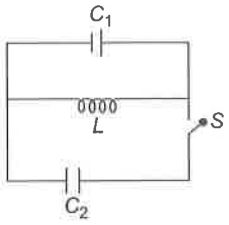
(A) The charge on C just after $t = 0$ is EC.

(B) The charge on C long after $t = 0$ is EC.

(C) The current in L just after $t = 0$ is $\frac{E}{R}$.

(D) The current in L long after $t = 0$ is $\frac{E}{R}$.

59. At a moment ($t = 0$) when charge on capacitor C_1 is zero, the switch is closed. If I_0 be the current through inductor at that instant, for $t > 0$,



(A) maximum current through inductor equals $\frac{I_0}{2}$.

(B) maximum current through inductor equals $\frac{C_1 I_0}{C_1 + C_2}$

(C) maximum charge on $C_1 = \frac{C_1 I_0 \sqrt{LC_1}}{C_1 + C_2}$

(D) maximum charge on $C_1 = I_0 C_1 \sqrt{\frac{L}{C_1 + C_2}}$

60. For L–R circuit, the time constant is equal to

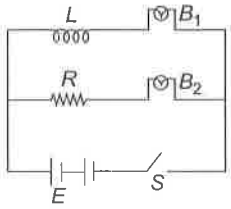
(A) twice the ratio of the energy stored in the magnetic field to the rate of the dissipation of energy in the resistance.

(B) the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(C) half of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

(D) square of the ratio of the energy stored in the magnetic field to the rate of dissipation energy in the resistance.

61. An inductor L, a resistance R and two identical bulbs B_1 and B_2 are connected to a battery through a switch S as shown in the figure. The resistance of coil having inductance L is also R. Which of the following statement gives the correct description of the happenings when the switch S is closed?



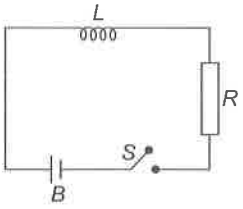
(A) The bulb B_2 lights up earlier than B_1 and finally both the bulbs shine equally bright.

(B) B_1 light up earlier and finally both the bulbs acquire equal brightness

(C) B_2 lights up earlier and finally B_1 shines brighter than B_2.

(D) B_1 and B_2 light up together with equal brightness all the time.

62. In figure, the switch S is closed so that a current flows in the iron-core inductor which has inductance L and the resistance R. When the switch is opened, a spark is obtained in it at the contacts. The spark is due to



(A) a slow flux change in L

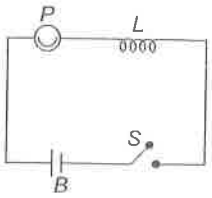
(B) a sudden increase in the emf of the battery B

(C) a rapid flux change in L

(D) a rapid flux change in R

63. In figure, a lamp P is in series with an iron-core inductor L. When the switch S is closed, the brightness of the lamp rises relatively slowly to its full brightness than it would do without the inductor. This is due to

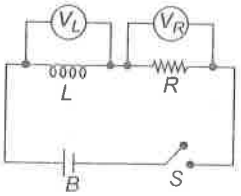
274



(A) the low resistance of P
(B) the induced-emf in L
(C) the low resistance of L
(D) the high voltage of the battery B

64. Two coil A and B have coefficient of mutual inductance $M = 2$ H. The magnetic flux passing through coil A changes by 4 Weber is 10 seconds due to the change in current in B. Then
(A) change in current in B in this time interval is 0.5 A
(B) the change in current in B in this time interval is 2 A
(C) the change in current in B in this time interval is 8 A
(D) a change in current of 1 A in coil A will produce a change in flux passing through B by 4 Weber.

65. An inductance L, resistance R, battery B and switch S are connected in series. Voltmeters V_L and V_R are connected across L and R respectively. When switch is closed:



(A) The initial reading in V_L will be greater than in V_R
(B) The initial reading in V_L will be lesser than V_R
(C) The initial readings in V_L and V_R will be the same.
(D) The reading in V_L will be decreasing as time increases.

66. Two different coils have self inductance 8 mH and 2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are I_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are I_2, v_2 and W_2 respectively. Then:

(A) $\frac{I_1}{I_2} = \frac{1}{4}$ (B) $\frac{I_1}{I_2} = 4$

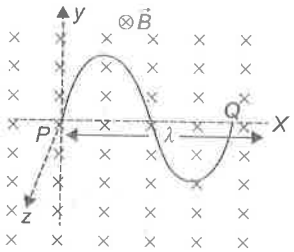
(C) $\frac{W_2}{W_1} = 4$ (D) $\frac{V_2}{V_1} = \frac{1}{4}$

JEE Advanced

Level I

1. The horizontal component of the earth's magnetic field at a place is 3×10^{-4} T and the dip is $\tan^{-1}\left(\frac{4}{3}\right)$. A metal rod of length 0.25 m placed in the north-south position is moved at a constant speed of 10 cm/s towards the east. Find the emf induced in the rod.

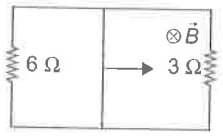
2. A wire forming one cycle of sine curve is moved in x-y plane with velocity $\vec{V} = V_x\hat{i} + V_y\hat{j}$. There exist a magnetic field $\vec{B} = -B_0\hat{k}$. Find the motional emf develop across the ends PQ of wire.



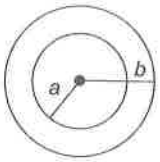
3. A conducting circular loop is placed in a uniform magnetic field of 0.02 T, with its plane perpendicular to the field. If the radius of the loop starts shrinking at a constant rate of 1.0 mm/s, then find the emf induced in the loop, at the instant when the radius is 4 cm.

4. Find the dimension of the quantity $\frac{L}{RCV}$, where symbols have usual meaning.

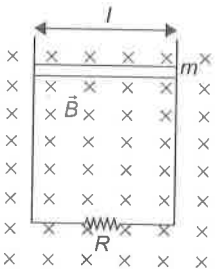
5. A rectangular loop with a sliding connector of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2$ T perpendicular to the plane of loop. Resistance of connector is $r = 2\ \Omega$. Two resistances of 6 Ω and 3 Ω are connected as shown in figure. Find the external force required to keep the connector moving with a constant velocity $v = 2$ m/s.



6. Two concentric and coplanar circular coils have radii a and b ($>> a$) as shown in figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then find the total charge circulating the inner coil.



7. A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in figure. The wire has a mass m and length l and the resistance of the circuit is R. If a uniform magnetic field B is directed perpendicular to the frame, then find the terminal speed of the wire as it falls under the force of gravity.

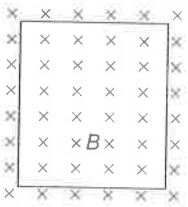


8. A metal rod of resistance 20 Ω is fixed along a diameter of a conducting ring of radius 0.1 m and lies on x-y plane. There is a magnetic field $\vec{B} = (50\text{T})\hat{k}$. The ring rotates with an angular velocity $\omega = 20$ rad/sec about its axis. An external resistance of 10 Ω is connected across the centre of the ring and rim. Find the current through external resistance.

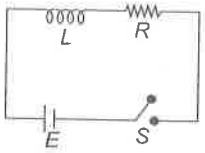
9. There exists a uniform cylindrically symmetric magnetic field directed along the axis of a cylinder but varying with time as $B = kt$. If an electron is released from rest in this field at a distance of 'r' from the axis of cylinder, its acceleration, just after it is released would be (e and m are the electronic charge and mass respectively)

10. An emf of 15 V is applied in a circuit containing 5 H inductance and 10 Ω resistance. Find the ratio of the currents at time $t = \infty$ and $t = 1$ second.

11. A uniform magnetic field of 0.08 T is directed into the plane of the page and perpendicular to it as shown in the figure. A wire loop in the plane of the page has constant area 0.010 m^2. The magnitude of magnetic field decrease at a constant rate of 3.0×10^{-4} Ts^{-1}. Find the magnitude and direction of the induced emf in the loop.



12. In the circuit shown in figure switch S is closed at time $t = 0$. Find the charge which passes through the battery in one time constant.



13. Two coils, 1 and 2, have a mutual inductance = M and resistance R each. A current flows in coil 1, which varies with time as: $I_1 = kt^2$, where K is a constant and 't' is time. Find the total charge that has flown through coil 2, between $t = 0$ and $t = T$.

14. A charged ring of mass $m = 50$ gm, charge 2 coulomb and radius $R = 2$ m is placed on a smooth horizontal surface. A magnetic field varying with time at a rate of (0.2 t) Tesla/sec is applied on to the ring in a direction normal to the surface of ring. Find the angular speed attained in a time $t_1 = 10$ sec.

15. A uniform but time varying magnetic field $B = Kt - C$; $\left(0 \le t \le \frac{C}{K}\right)$, where K and C are constant and t is time, is applied perpendicular to the plane of the circular loop of radius 'a' and resistance R. Find the total charge that will pass around the loop.

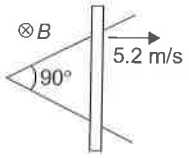
Level II

1. Two straight conducting rails from a right angle where their ends are joined. A conducting bar contact with the rails starts at vertex at the time $t = 0$ and moves symmetrically with a constant velocity of 5.2 m/s to the right as shown in figure. A 0.35 T magnetic field points out of the page. Calculate:

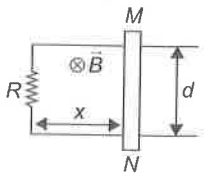
   (i) The flux through the triangle by the rails and bar at $t = 3.0$ s.

   (ii) The emf around the triangle at that time.

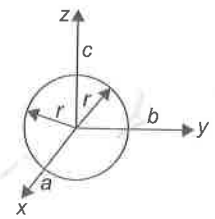
   (iii) In what manner does the emf around the triangle vary with time.



2. Two long parallel rails, a distance l apart and each having a resistance λ. per unit length are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction. There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current i flows through R. Find the velocity of the rod and the applied force F as function of the distance x of the rod from R



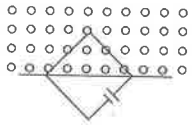
3. A wire is bent into 3 circular segments of radius r = 10 cm as shown in figure. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane and ca lying in the zx plane.
   (i) if a magnetic field B points in the positive x direction, what is the magnitude of the emf developed in the wire, when B increases at the rate of 3 mT/s?



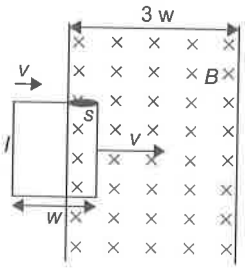
   (ii) what is the direction of the current in the segment bc.

4. Consider the possibility of a new design for an electric train. The engine is driven by the force due to the vertical component of the earths wheel and then back to the source via the other rail.
   (i) What current is needed to provide a modest 10 – KN force? Take the vertical component of the earth's field be 10 μT and the length of axle to be 3.0 m.
   (ii) How much power would be lost for each Ω of resistivity in the rails?
   (iii) is such a train realistic?

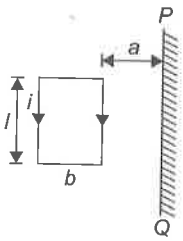
5. A square wire loop with 2 m sides in perpendicular to a uniform magnetic field, with half the area of the loop in the field. The loop contains a 20 V battery with negligible internal resistance. If the magnitude of the field varies with time according to $B = 0.042 - 0.87t$, with B in tesla and t in sec.
   (i) What is the total emf in the circuit?
   (ii) What is the direction of the current through the battery?



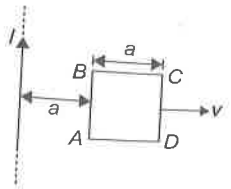
6. A rectangular loop of dimensions l and w and resistance R moves with constant velocity V to the right as shown in the figure. It continues to move with same speed through a region containing a uniform magnetic field B directed into the plane of the paper and extending a distance 3 W. Sketch the flux, induced emf and external force acting on the loop as a function of the distance.



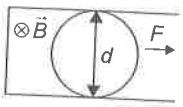
7. A rectangular loop with current I has dimension as shown in figure. Find the magnetic flux ϕ through the infinite region to the right of line PQ.



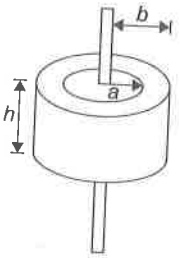
8. A square loop of side 'a' and resistance R moves with a uniform velocity v away from a long wire that carries current I as shown in the figure. The loop is moved away from the wire with side AB always parallel to the wire. Initially, distance between the side AB of the loop and wire is 'a'. Find the work done when the loop is moved through distance 'a' from the initial position



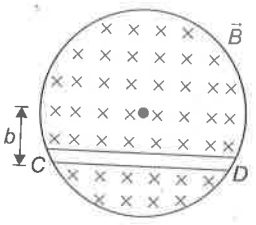
9. Two long parallel conducting horizontal rails are connected by a conducting wire at one end. A uniform magnetic field B exists in the region of space. A light uniform ring of diameter d which is practically equal to separation between the rails, is placed over the rails as shown in the figure. If resistance of ring is λ per unit length, calculate the force required to pull the ring with uniform velocity v.



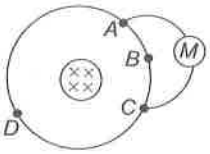
10. A long straight wire is arranged along the symmetry axis of a toroidal coil of rectangular cross-section, whose dimensions are given in the figure. The number of turns on the coil is N, and relative permeability of the surrounding medium is unity. Find the amplitude of the emf induced in this coil, if the current $i = i_m \cos \omega t$ flows along the straight wire.



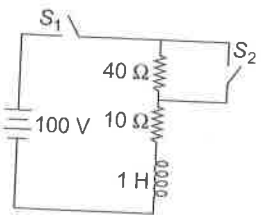
11. A uniform magnetic field $\vec{B}$ fills a cylindrical volumes of radius R. A metal rod CD of length l is placed inside the cylinder along a chord of the circular cross-section as shown in the figure. If the magnitude of magnetic field increases in the direction of field at a constant rate dB/dt, find the magnitude and direction of the EMF induced in the rod.



12. A variable magnetic field creates a constant emf E in a conductor $ABCDA$. The resistances of portion ABC, CDA and AMC are R_1, R_2 and R_3 respectively. What current will be shown by meter M? The magnetic field is concentrated near the axis of the circular conductor.

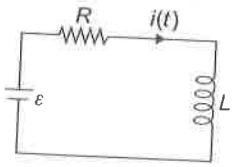


13. In the circuit shown in the figure the switched S_1 and S_2 are closed at time $t = 0$. After time $t = (0.1) \ln 2$ sec, switch S_2 is opened. Find the current in the circuit at time $t = (0.2) \ln 2$ sec.



14. Find the values of i_1 and i_2
- (i) immediately after the switch S is closed.
- (ii) long time later, with S closed.
- (iii) immediately after S is open.
- (iv) long time after S is opened

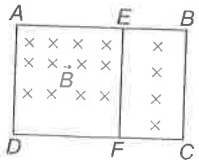
15. Suppose the emf of the battery, the circuit shown varies with time t so the current is given by $i(t) = 3 + 5t$, where i is in amperes and t is in seconds. Take $R = 4\,\Omega$, $L = 6$ H and find an expression for the battery emf as function of time.



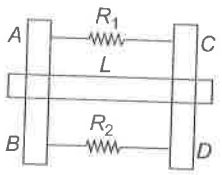
16. A rectangular frame $ABCD$ made of a uniform metal wire has a straight connection between E and F made of the same wire as shown in the figure. $AEFD$ is a square of side 1 m and $EB = FC = 0.5$ m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the place of the paper and normal to it. The rate of change of the magnetic field is 1 T/s, the resistance per unit length of the wire

278

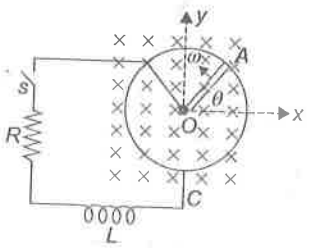
is 1 Ω/m. Find the current in segments AE, BE and EF.



17. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistance R_1 and R_2 as shown in the figure. A horizontally metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails, it is observed that when the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of bar L and value R_1 and R_2.



18. A metal rod OA of mass m and length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant, magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



(A) What is the induced emf across the terminals of the switch?

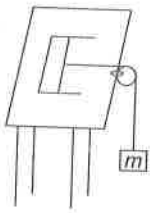
(B) (i) Obtain an expression for the current as a function of time after switch S is closed.

(ii) Obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at $t = 0$.

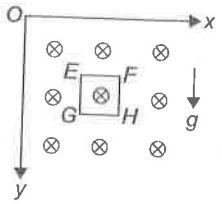
19. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table, The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate.

(i) the terminal velocity achieved by the rod.

(ii) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity. **[JEE-1997, 2005]**



20. A magnetic field $B = (B_0 y/a)\hat{k}$ is into the plane of paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m and resistance R, in x-y plane, starts falling under the influence of gravity. Note the directions of x and y in the figure. Find **[JEE-1999]**



(A) the induced current in the loop and indicate its direction,

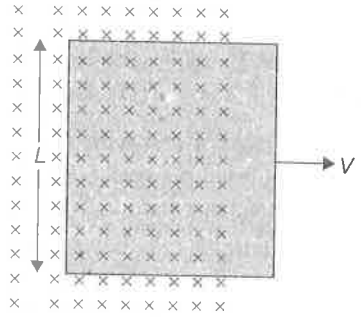
(B) the total Lorentz force acting on the loop and indicate its direction.

(C) an expression for the speed of the loop, $v(t)$ and its terminal value.

Previous year questions

JEE Main

1. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is **[AIEEE-2002]**



(A) zero (B) RvB

(C) $\frac{vBL}{R}$ (D) vBL

2. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is **[AIEEE-2002]**

(A) 4 A (B) 2 A

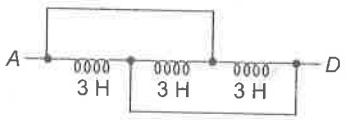
(C) 6 A (D) 10 A

3. If a current is passed through a spring then the spring will **[AIEEE-2002]**

(A) expand (B) compress

(C) remain same (D) None of the above

4. The inductance between A and D is **[AIEEE-2002]**



(A) 3.66 H (B) 9 H

(C) 0.66 H (D) 1 H

5. The core of any transformer is laminated so as to **[AIEEE-2003]**

(A) reduce the energy loss due to eddy currents

(B) make it light weight

(C) make it robust and strong

(D) increase the secondary voltage

6. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is **[AIEEE-2003]**

(A) $\frac{Q}{2}$ (B) $\frac{Q}{\sqrt{3}}$

(C) $\frac{Q}{\sqrt{2}}$ (D) Q

7. When the current changes from +2 A to –2 A in 0.05 s, an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is **[AIEEE-2003]**

(A) 0.2 H (B) 0.4 H

(C) 0.8 H (D) 0.1 H

8. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon **[AIEEE-2003]**

(A) the rates at which currents are changing in the two coils

(B) relative position and orientation of the two coils

(C) the materials of the wires of the coils

(D) the currents in the two coils

9. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 rad/s. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the emf developed between the two ends of the conductor is **[AIEEE-2004]**

(A) 5 μV (B) 50 μV

(C) 5 mV (D) 50 mV

10. In a uniform magnetic field of induction B, a wire in the form of semicircle of radius r rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per period of rotation is **[AIEEE-2004]**

(A) $\frac{B\pi r^2\omega}{2R}$ (B) $\frac{(B\pi r^2\omega)^2}{8R}$

(C) $\frac{(B\pi r\omega)^2}{2R}$ (D) $\frac{(B\pi r\omega^2)^2}{8R}$

11. A coil having n turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance 4 $R\ \Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is **[AIEEE-2004]**

(A) $\frac{W_2 - W_1}{5Rnt}$ (B) $-\frac{n(W_2 - W_1)}{5Rt}$

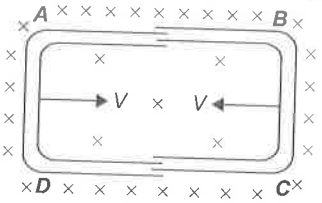
(C) $-\frac{(W_2 - W_1)}{Rnt}$ (D) $-\frac{n(W_2 - W_1)}{Rt}$

200

12. In an *LCR* series AC circuit, the voltage across each of the components, *L*, *C* and *R* is 50 V. The voltage across the *LC* combination will be **[AIEEE-2004]**

(A) 50 V (B) $50\sqrt{2}$ V
(C) 100 V (D) zero

13. One conducting *U*-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field *B* is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed *v*, then the emf induced in the circuit in terms of *B*, *l* and *v*, where *l* is the width of each tube, will be **[AIEEE-2005]**



(A) Blv (B) –Blv
(C) zero (D) 2 Blv

14. The phase difference between the alternating current and emf is $\frac{\pi}{2}$. Which of the following cannot be the constituent of the circuit? **[AIEEE-2005]**

(A) *C* alone (B) *R*, *L*
(C) *L*, *C* (D) *L* alone

15. A circuit has a resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be **[AIEEE-2005]**

(A) 0.8 (B) 0.4
(C) 1.25 (D) 0.125

16. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[AIEEE-2005]**

(A) 4 μF (B) 8 μF
(C) 1 μF (D) 2 μF

17. A coil of inductance 300 mH and resistance is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[AIEEE-2005]**

(A) 0.05 s (B) 0.1 s
(C) 0.15 s (D) 0.3 s

18. A long solenoid has 200 turns/cm and carries a current *I*. The magnetic field at its centre is 6.28×10^{-2} Wb/m^2. Another long solenoid has 100 turns/cm and it carries a current $\frac{I}{3}$. The value of the magnetic field at its centre is **[AIEEE-2006]**

(A) 1.05×10^{-2} Wb/m^2 (B) 1.05×10^{-5} Wb/m^2
(C) 1.05×10^{-3} Wb/m^2 (D) 1.05×10^{-4} Wb/m^2

19. The flux linked with a coil at any instant $\phi = 10t^2 - 50t + 250$ is given by The induced emf at $t = 3$s is **[AIEEE-2006]**

(A) –190 V (B) –10 V
(C) 10 V (D) 190 V

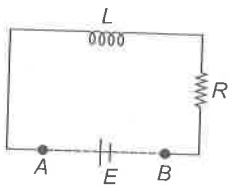
20. In a series resonant *LCR* circuit, the voltage across *R* is 100 V and $R = 1$ kΩ with $C = 2$ μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across *L* is **[AIEEE-2006]**

(A) 2.5×10^{-2} V (B) 40 V
(C) 250 V (D) 4×10^{-3} V

21. In an AC generator, a coil with *N* turns, all of the same area *A* and total resistance *R*, rotates with frequency ω in a magnetic field *B*. The maximum value of emf generated in the coil is **[AIEEE-2006]**

(A) *NABR*ω (B) *NAB*
(C) *NABR* (D) *NAB*ω

22. An inductor ($L = 100$ mH), a resistor ($R = 100$ Ω) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points *A* and *B*. The current in the circuit 1 ms after the short circuit is **[AIEEE-2006]**



(A) 1/*e* A (B) *e* A
(C) 0.1 A (D) 1 A

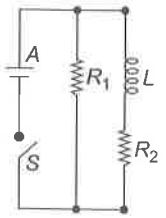
23. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. 2s after the connection is made, the current flowing (in ampere) in the circuit is **[AIEEE-2007]**

(A) $(1 - e)$ (B) e
(C) e^{-1} (D) $(1 - e^{-1})$

24. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10$ cm^2 and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}) **[AIEEE-2008]**

(A) $2.4\pi \times 10^{-5}$ H (B) $4.8\pi \times 10^{-4}$ H
(C) $4.8\pi \times 10^{-5}$ H (D) $2.4\pi \times 10^{-4}$ H

25. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 4\ \Omega$ and $R_2 = 2\ \Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[AIEEE-2009]**



(A) $6e^{-5t}$ V (B) $\frac{12}{t}e^{-3t}$ V

(C) $6(1 - e^{-t/0.2})$ V (D) $12e^{-5t}$ V

26. A coil is suspended in a uniform magnetic field with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil, it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops, This is due to **[AIEEE-2012]**

(A) development of air current when the plate is placed

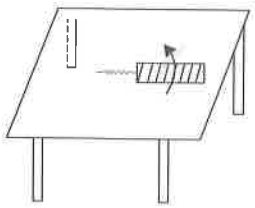
(B) induction of electrical charge on the plate

(C) shielding of magnetic lines of force as aluminium is a paramagnetic material

(D) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

27. A metallic rod of length 'l' is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the emf induced across the ends of the rod is:

[JEE Main-2013]

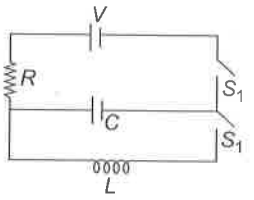


(A) $\frac{4B\omega l^2}{2}$ (B) $\frac{5B\omega l^2}{2}$

(C) $\frac{2B\omega l^2}{2}$ (D) $\frac{3B\omega l^2}{2}$

28. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which of the following statement is correct?

[JEE-2013]



(A) At $t = 2\tau$, $q = CV(1 - e^{-2})$

(B) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

(C) Work done by the battery is half of the energy dissipated in the resistor

(D) At $t = \tau$, $q = \frac{CV}{2}$

29. Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given below the lists:

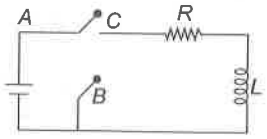
[JEE Main-2014]

List-I	List-II
(a) Infrared waves	(i) To treat muscular strain
(b) Radio waves	(ii) For broadcasting
(c) X-rays	(iii) To detect fracture of bones
(d) Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

	(a)	(b)	(c)	(d)
(A)	(iii)	(ii)	(i)	(iv)
(B)	(i)	(ii)	(iii)	(iv)
(C)	(iv)	(iii)	(ii)	(i)
(D)	(i)	(ii)	(iv)	(iii)

30. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to:

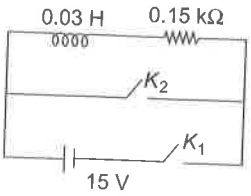
[JEE Main-2014]



(A) -1 (B) $\frac{1-e}{e}$

(C) $\frac{e}{1-e}$ (D) 1

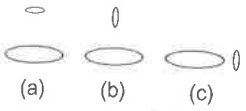
31. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ kΩ) are connected in series to a battery of 15 V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ ms, the current in the circuit will be ($e^5 \cong 150$) **[JEE Main-2015]**



(A) 6.7 mA
(B) 0.67 mA
(C) 100 mA
(D) 67 mA

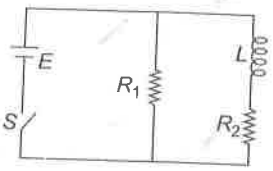
JEE advanced

1. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be **[JEE-2001 (Scr.)]**



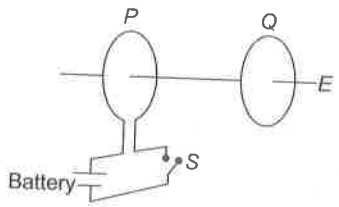
(A) maximum in situation (a)
(B) maximum in situation (b)
(C) maximum in situation (c)
(D) the same in all situations

2. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf $E = 12$ V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$. What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time? **[JEE-2001]**



3. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_p flows in P (as seen by E) and an induced current I_{Q1} flows in Q The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions of I_{Q1} and I_{Q2} (as seen by E) are:

[JEE-2002 (Scr.) 3]



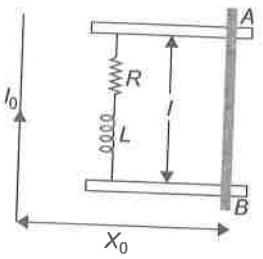
(A) respectively clockwise and anti-clockwise
(B) both clockwise
(C) both anti-clockwise
(D) respectively anti-clockwise and clockwise

4. A short-circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be **[JEE-2002 (Scr.) 3]**

(A) halved (B) the same
(C) doubled (D) quadrupled

5. A metal bar AB can slide on two parallel thick metallic tails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire, carrying a constant current I_0 is placed in the plane of the rails as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it made to slide on the rails away from the wire. Answer the following questions.

[JEE-2002]

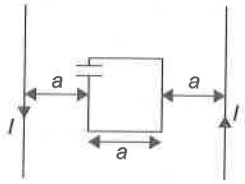


(A) Find a relation among i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$, where i is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(B) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.

(C) The bar is suddenly stopped at time T. The current through resistance R is found to be $i_1/4$ at time 2 T. Find the value of L/R in terms of the other give quantities.

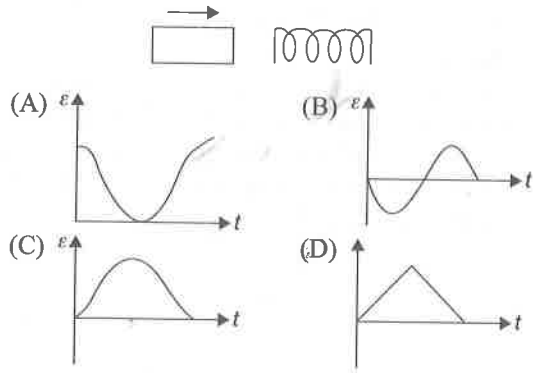
6. A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the is given as $I = I_0 \sin \omega t$. **[JEE-2003]**



(A) calculate maximum current in the square loop.

(B) Draw a graph between charge on the lower plate of the capacitor v/s time.

7. The variation of induced emf (ε) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as **[JEE-2004 (Scr.)]**



8. An infinitely long cylindrical conducting rod is kept along $+Z$ direction. A constant magnetic field is also present in $+Z$ direction. Then current induced will be **[JEE-2005 (Scr.)]**

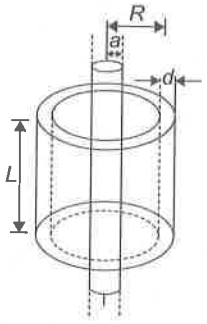
(A) 0

(B) along $+Z$ direction

(C) along clockwise as seen from $+Z$

(D) along anticlockwise as seen from $+Z$

9. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R, thickness d ($d << R$) and length L. A variable current $i = i_0 \sin \omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell. **[JEE-2005]**



10. In the given diagram, a line of force of a particular force field is shown. Out of the following options, it can never represent **[JEE-2006]**

(A) an electrostatic field

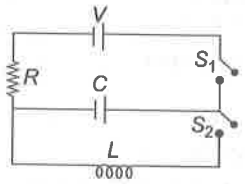
(B) a magnetostatic field

(C) a gravitational field of a mass at rest

(D) an induced electric field

Comprehension-I

The capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing switch S_1 while keeping switch S_2 open. The capacitor can be connected in series with an inductor 'L' by closing switch S_2 and opening S_1



11. Initially, the capacitor was uncharged. Now, switch S_1 is closed and S_2 is kept open. If time constant of this circuit is τ, then **[JEE-2006]**

(A) after time interval τ, charge on the capacitor is $CV/2$

(B) after time interval 2τ, charge on the capacitor is $CV(1-e^2)$

(C) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged.

(D) after time interval 2τ, charge on the capacitor is $CV(1-e^{-1})$

12. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then, **[JEE-2006]**

(A) at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy

(B) at any time $t > 0$, current in the circuit is in the same direction.

(C) at $t > 0$, there is no exchange of energy between the inductor and capacitor

(D) at any time $t > 0$, instantaneous current in the circuit may $V\sqrt{\frac{C}{L}}$

13. If the total charge stored in the LC circuit is Q_0, then for $t \geq 0$ **[JEE-2006]**

(A) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$

(B) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{LC}}\right)$

(C) the charge on the capacitor is $Q = -LC\frac{d^2Q}{dt^2}$

(D) the charge on the capacitor is $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

14. Match the following Columns **[JEE-2006]**

Column I	Column II
(A) Dielectric ring uniformly charged	(P) Time independent electrostatic field out of system
(B) Dielectric ring uniformly charged	(Q) Magnetic field rotating with angular velocity
(C) Constant current i_0 in ring	(R) Induced electric field
(D) Current $i = i_0 \cos \omega t$ in ring	(S) Magnetic moment

Common Data for Questions 15–17: Modern trains are based on Maglev technology in which trains are magnetically deviated, which runs its EDS Maglev system.

There are coils on both sides of wheels. Due to motion of train, current induces in the coil of track which levitate in. This is in accordance with Lenz's law. If trains lower down then due to Lenz's law a repulsive force increases due to which train gets uplifted and if it goes much high then there is a net downward force due to gravity. The advantage of Maglev train is that there is no friction between the train and the track, thereby reducing power consumption and enabling the train to attain very high speeds.

Disadvantage of Maglev train is that as it slows down the electromagnetic forces decreases and it becomes difficult to keep it deviated and as it moves forward according to Lenz's law ther is an electromagnetic drag force. **[JEE-2006]**

15. What is the advantage of this system?

(A) No friction hence no power consumption

(B) No electric power is used

(C) Gravitation force is zero

(D) Electrostatic force draws the train

16. What is the disadvantage of this system?

(A) Train experiences upward force according to Lenz's law

(B) Friction force create a drag on the train

(C) Retardation

(D) By Lenz's law train experience a drag

17. Which force causes the train to elevate up?

(A) Electrostatic force

(B) Time varying electric field

(C) Magnetic force

(D) Induced electric field

18. Statement-I

A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil

because

Statement-II

In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction. **[JEE-2007]**

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

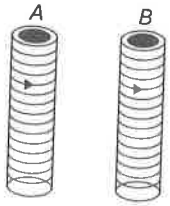
19. This section contains 2 questions. Each questions contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with one or more statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are $A - p$, s and t; $B - q$ and r; $C - p$ and q; and $D - s$ and t; then the correct darkening of bubbles will look like the following.

Column II gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process(es) from **Column II**. **[JEE-2009]**

Column I	Column II
(A) The energy of the system is Increased	(P) System: A capacitor initially uncharged Increased Process: It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(Q) System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston
(C) Internal energy of the system is converted into its mechanical energy.	(R) System: a gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it
(D) Mass of the system is decreased	(S) System: A heavy nucleus initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(T) System: A resistive wire loop Process: The loop is placed in a time varying magnetic field perpendicular to its plane.

20. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is (are) **[JEE-2009]**



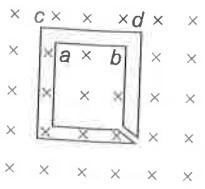
(A) $\rho_A > \rho_B$ and $m_A = m_B$

(B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$

(D) $\rho_A < \rho_B$ and $m_A < m_B$

21. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then, **[JEE-2009]**



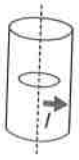
(A) $I_1 > I_2$

(B) $I_1 < I_2$

(C) I_1 is in the direction ba and I_2 is in the direction cd

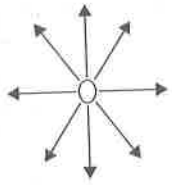
(D) I_1 is in the direction ab and I_2 is in the direction dc

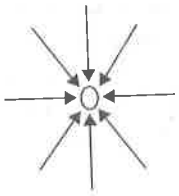
22. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N \mu_0 I_0 \sin(300t)$, then 'N' is **[JEE-2011]**

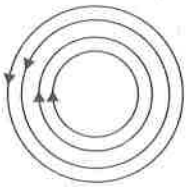


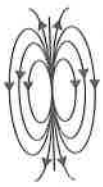
286

23. Which of the field patterns given below is valid for electric field as well as for magnetic field? **[JEE 2011]**

(A) 

(B) 

(C) 

(D) 

24. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are) **[JEE 2012]**

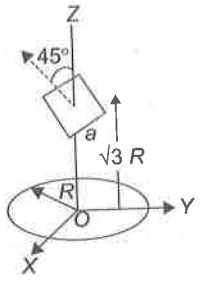
(A) The emf induced in the loop is zero if the current is constant.

(B) The emf induced in the loop is finite if the current is constant.

(C) The emf induced in the loop is zero if the current decreases at a steady rate.

(D) The emf induced in the loop is finite if the current decreases at a steady rate.

25. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side $a(a << R)$ having two turns is placed with its center at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then value of p is **[JEE 2012]**



Common Data for Questions 26–27: A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

26. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$

(C) BR (D) $2BR$

27. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

(A) $-\gamma BQR^2$ (B) $-\gamma \frac{BQR^2}{2}$

(C) $\gamma \frac{BQR^2}{2}$ (D) γBQR^2

ANSWER KEYS

Exercises

JEE Main

1. B	**2.** A	**3.** C	**4.** D	**5.** D	**6.** B	**7.** A	**8.** A	**9.** D	**10.** A
11. B	**12.** A	**13.** A	**14.** C	**15.** C	**16.** A	**17.** A	**18.** B	**19.** B	**20.** B
21. A	**22.** B	**23.** B	**24.** A	**25.** C	**26.** A	**27.** A	**28.** B	**29.** D	**30.** B
31. C	**32.** A	**33.** D	**34.** D	**35.** A	**36.** C	**37.** D	**38.** B	**39.** A	**40.** D
41. A	**42.** C	**43.** B	**44.** A	**45.** C	**46.** A	**47.** C	**48.** A	**49.** A	**50.** A
51. B	**52.** C	**53.** A	**54.** C	**55.** B	**56.** C	**57.** B	**58.** D	**59.** A	**60.** B
61. A	**62.** B	**63.** C	**64.** D	**65.** B	**66.** A	**67.** A	**68.** A	**69.** D	**70.** D
71. B	**72.** A	**73.** A	**74.** B	**75.** C					

JEE Advanced

1. C	**2.** C	**3.** D	**4.** D	**5.** B	**6.** C	**7.** B	**8.** A	**9.** A	**10.** D
11. B	**12.** D	**13.** C	**14.** D	**15.** A	**16.** A	**17.** B	**18.** C	**19.** A	**20.** D
21. C	**22.** A	**23.** B	**24.** C	**25.** A	**26.** B	**27.** A	**28.** A	**29.** C	**30.** B
31. D	**32.** B	**33.** B	**34.** D	**35.** D	**36.** B	**37.** C	**38.** C	**39.** B	**40.** B
41. A, C, D		**42.** A	**43.** B, C	**44.** B, D	**45.** A	**46.** A, B	**47.** B	**48.** A	
49. A, B, C, D		**50.** D	**51.** B, D	**52.** C	**53.** D	**54.** B, D	**55.** D	**56.** A, B, C	
57. D	**58.** B, D	**59.** D	**60.** A	**61.** A	**62.** C	**63.** B	**64.** B	**65.** A, D	
66. A, C, D									

JEE Advanced

Level I

1. 10 μV **2.** $\lambda V_y B_0$ **3.** 5.0 μV **4.** I^{-1} **5.** $2N$ **6.** $\frac{\mu_0 i a^2 \pi}{2Rb}$ **7.** $\frac{mgR}{B^2 l^2}$ **8.** $\frac{1}{3}$ A

9. $\frac{erk}{2m}$ directed along tangent to the circle of radius r, whose centre lies on the axis of cylinder.

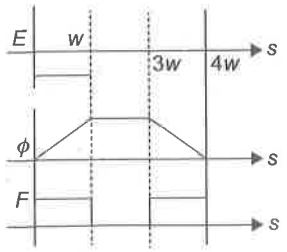
10. $\frac{e^2}{e^2 - 1}$ **11.** 3 μV, clockwise **12.** $\frac{EL}{eR^2}$ **13.** kMT^2/R **14.** 200 rad/sec **15.** $C\pi a^2/R$

Level II

1. **(i)** 85.22 Tm^2, **(ii)** 56.8 V; **(iii)** linearly **2.** $\frac{I(R + 2\lambda x)}{Bd}, \frac{2I^2 m\lambda(R + 2\lambda x)}{B^2 d^2} + BId$

3. **(i)** 2.4×10^{-5} V **(ii)** from c to b **4.** **(i)** 3.3×10^8 A, **(ii)** 1.1×10^{17} W **(iii)** totally unrealistic

5. 21.74 V, anticlockwise **6.**



7. $\phi = \frac{\mu_0}{2\pi} IL \ln \frac{a + b}{a}$

288

8. $\frac{\mu_0^2 I^2 a^2 V}{4\pi^2 R}\left[\frac{2}{3a}+\frac{2}{a}\ell n\frac{3}{4}\right]=\frac{\mu_0^2 I^2 aV}{2\pi^2 R}\left[\frac{1}{3}+\ell n\frac{3}{4}\right]$ **9.** $\frac{4B^2 vd}{\pi\lambda}$ **10.** $\frac{\mu_0 h\omega i_m N}{2\pi}\ln\frac{b}{a}$ **11.** $\frac{l}{2}\frac{dB}{dt}\sqrt{R^2-\frac{l^2}{4}}$

12. $\frac{E R_1}{R_1R_2+R_2R_3+R_3R_1}$ **13.** $\frac{67}{32}$ A **14.** **(i)** $i_1=i_2=\frac{10}{3}$ A, **(ii)** $i_1=\frac{50}{11}$ A; $i_2=\frac{30}{11}$ A,

(iii) $i_1=0, i_2=\frac{20}{11}$ A, **(iv)** $i_1=i_2=0$ **15.** $42+20t$ V **16.** $I_{EA}=\frac{7}{22}$ A; $I_{BE}=\frac{3}{11}$ A; $I_{FE}=\frac{1}{22}$ A

17. $V=1$ ms^{-1}, $R_1=0.47\ \Omega$, $R_2=0.30\ \Omega$ **18.** **(a)** $E=\frac{1}{2}B\omega r^2$ **(b)** **(i)** $I=\frac{B\omega r^2\,|1-e^{-Rt/L}|}{2R}$,

(ii) $\tau=\frac{mgr}{2}\cos\omega t+\frac{\omega B^2 r^4}{4R}(1-e^{-Rt/L})$ **19.** $V_{\text{terminal}}=\frac{mgR}{B^2l^2}$; **(ii)** $\frac{g}{2}$

20. **(a)** $i=\frac{B_0 av}{R}$ in anticlockwise direction, v = velocity at time t, **(b)** $F_{\text{nett}}=B_0{}^2a^2V/R$, **(c)** $V=\frac{mgR}{B_0^2a^2}\left(1-e^{\frac{B_0^2a^2t}{mR}}\right)$

Previous Years' Questions

JEE Main

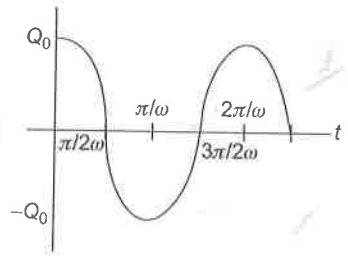
1. D	**2.** B	**3.** B	**4.** D	**5.** A	**6.** C	**7.** D	**8.** B	**9.** B	**10.** B
11. B	**12.** D	**13.** D	**14.** C	**15.** A	**16.** C	**17.** B	**18.** A	**19.** B	**20.** C
21. D	**22.** A	**23.** D	**24.** D	**25.** D	**26.** D	**29.** B	**30.** A	**31.** B	

JEE Advanced

1. A **2.** $12e^{-5}t, 6e^{-10}t$ **3.** D **4.** B **5.** **(A)** $\frac{d\phi}{dt}=iR+L\frac{di}{dt}$;

(B) $\frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}\ln(2)-Li_1\right]$; **(C)** $\frac{T}{\ln(4)}$ **6.** **(a)** $I_{\max}=\frac{\mu_0 a}{\pi}Cl_0\omega^2\ln 2$,

(b)



7. B **8.** A **9.** $I=\frac{(\mu_0 ni_0\omega\cos\omega t)\pi a^2(Ld)}{\rho 2\pi R}$ **10.** A, C

11. B **12.** D **13.** C **14.** **(A)** P; **(B)** P, Q, S; **(C)** Q, S; **(D)** Q, R, S **15.** A

16. D **17.** C **18.** A **19.** **(A)** $\to P, Q, S, T$; **(B)** $\to Q$; **(C)** $\to S$; **(D)** $\to S$ **20.** B, D

21. D **22.** 0006 **23.** C **24.** A, C **25.** 7 **26.** B **27.** BC

Chapter 6

Alternating Current

ALTERNATING CURRENT

Until now, we have studied only circuits with direct current (DC) which flows only in one direction (as shown in figure I and II). The primary source of emf in such circuit is a battery. When a resistance is connected across the terminals of the battery, a current is established in the circuits, which flows in a unique direction from the positive terminal to the negative terminal via the external resistance.

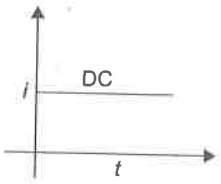


Figure I

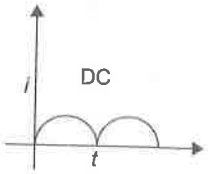


Figure II

But most of the electric power generated and used in the world is in the form of **alternating current (AC)**, the magnitude of which changes continuously with time and direction is reversed periodically (as shown in figure III and IV) and it is given by

$$i = i_0 \sin(\omega t + \phi)$$

Here i is instantaneous value of current i.e., the magnitude of current at any instant of time and i_0 is the maximum value of current which is called peak current or the current amplitude and the current repeats its value after each time interval $T = \frac{2\pi}{\omega}$ as shown in figure. This time interval is called the time period and ω is angular frequency which is equal to 2π times of frequency f.

$$\omega = 2\pi f$$

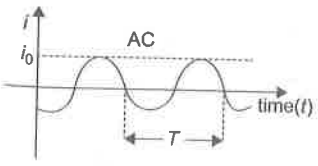


Figure III

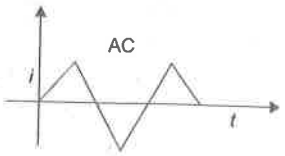


Figure IV

The current is positive for half the time period and negative for remaining half period. It means that the direction of current is reversed after each half time period. The frequency of AC in India is 50 Hz.

An alternating voltage is given by

$$V = V_0 \sin(\omega t + \phi)$$

It also varies alternatively as shown in the figure (b), where V is instantaneous voltage and V_0 is peak voltage. It is produced by AC generator also called as AC dynamo.

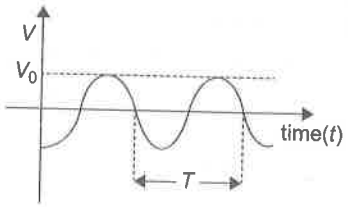
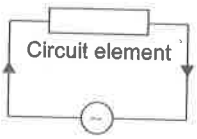


Figure (b)

AC Circuit: An AC circuit consists of circuit element i.e., resistor, capacitor, inductor or any combination of these and a generator that provides the alternating current as shown in figure. The AC source is represented by symbol —(~)— in the circuit.



AVERAGE AND RMS VALUE OF ALTERNATING CURRENT

Average Current (Mean Current)

As we know an alternating current is given by

$$i = i_0 \sin(\omega t + \phi) \quad (1)$$

990

The mean or the average value of AC over any time T is given by

$$i_{avg} = \frac{\int_0^T i\,dt}{\int_0^T dt}$$

Using equation (1)

$$i_{avg} = \frac{\int_0^T i_0 \sin(\omega t + \phi)\,dt}{\int_0^T dt}$$

In one complete cycle, the average current

$$i_{avg} = -\frac{i_0}{T}\left[\frac{\cos(\omega t + \phi)}{\omega}\right]_0^T = \frac{i_0}{T}\left[\frac{\cos(\omega t + \phi) - \cos\phi}{\omega}\right]$$

$$= -\frac{i_0}{T}\left[\frac{\cos(2\pi + \phi) - \cos\phi}{\omega}\right] = 0 \quad (\text{as } \omega T = 2\pi)$$

Since AC is positive during the first half cycle and negative during the other half cycle so i_{avg} will be zero for long time also. Hence the DC instrument will indicate zero deflection when connected to a branch carrying AC current. So it is defined for either positive half cycle or negative half cycle.

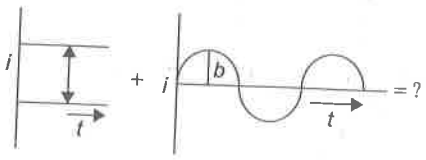
$$i_{avg} = \frac{\int_0^{T/2} i_0 \sin(\omega t + \phi)}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} \approx 0.637 i_0.$$

Similarly $V_{avg} = \frac{2V_0}{\pi} \approx 0.637 V_0$

SOLVED EXAMPLES

EXAMPLE 1

If a direct current of value a ampere is superimposed on an alternating current $i = b \sin \omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?



SOLUTION

As current at any instant in the circuit will be,

$$i = i_{DC} + i_{AC} = a + b \sin \omega t$$

$$\text{So, } i_{eff} = \left[\frac{\int_0^T i^2\,dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (a + b\sin\omega t)^2\,dt\right]^{1/2}$$

$$\text{i.e., } = i_{eff}\left[\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2\sin^2\omega t)\,dt\right]^{1/2}$$

but as

$$\frac{1}{T}\int_0^T \sin\omega t\,dt = 0 \quad \text{and} \quad \frac{1}{T}\int_0^T \sin^2\omega t\,dt = \frac{1}{2}$$

$$\text{So, } i_{eff} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$$

R.M.S Value of Alternating Current

The notation rms refers to root mean square, which is given by square root of mean of square current.

$$\text{i.e., } i_{rms} = \sqrt{i_{avg}^2}$$

$$i^2_{avg} = \frac{\int_0^T i^2\,dt}{\int_0^T dt} = \frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \phi)dt$$

$$= \frac{i_0^2}{2T}\int_0^T [1 - \cos 2(\omega t + \phi)]dt$$

$$= \frac{i_0^2}{2T}\left[T - \frac{\sin 2(\omega t + \phi)}{2\omega}\right]_0^T$$

$$= \frac{i_0^2}{2T}\left[T - \frac{\sin(4\pi + 2\phi) - \sin 2\phi}{2\omega}\right] = \frac{i_0^2}{2}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}} \approx 0.707 i_0$$

Similarly the rms voltage is given by

$$V_{rms} = \frac{V_0}{\sqrt{2}} \approx 0.707 V_0$$

The significance of rms current and rms voltage may be shown by considering a resistance R carrying a current $i = i_0 \sin(\omega t + \phi)$

The voltage across the resistor will be

$$V_R = Ri = (i_0 R)\sin(\omega t + \phi)$$

The thermal energy developed in the resistor during the time t to $t + dt$ is

$$i^2R\,dt = i_0^2R\sin^2(\omega t + \phi)\,dt$$

The thermal energy developed in one time period is

$$U = \int_0^T i^2R\,dt = R\int_0^T i_0^2\sin^2(\omega t + \phi)dt$$

$$= RT\left[\frac{1}{T}\int_0^T i_0^2\sin^2(\omega t + \phi)dt\right] = i_{rms}^2RT$$

- It means the root mean square value of AC is that value of steady current, which would generated the same amount of heat in a given resistance in a given time.
- So in AC circuits, current and AC voltage are measured in terms of their rms values. Likes when we say that the house hold supply is 220 V AC it means the rms value is 220 V and peak value is $220\sqrt{2} = 311$ V.

EXAMPLE 2

If the voltage in an AC circuit is represented by the equation,

$$V = 220\sqrt{2}\sin(314t - \phi),$$

calculate (a) peak and rms value of the voltage, (b) average voltage, (c) frequency of AC.

SOLUTION

(a) For AC voltage,

$$V = V_0\sin(\omega t - \phi)$$

The peak value of voltage

$$V_0 = 220\sqrt{2} = 311\text{ V}$$

The rms value of voltage

$$V_{rms} = \frac{V_0}{\sqrt{2}};\quad V_{rms} = 220\text{ V}$$

(b) Average voltage in full cycle is zero. Average voltage in half cycle is

$$V_{avg} = \frac{2}{\pi}V_0 = \frac{2}{\pi}\times 311 = 198.17\text{ V}$$

(c) As $\omega = 2\pi f$, $2\pi f = 314$

i.e., $f = \dfrac{314}{2\times\pi} = 50$ Hz ■

EXAMPLE 3

The electric current in a circuit is given by $i = i_0\,(t/T)$ for some time. Calculate the rms current for the period $t = 0$ to $t = T$.

SOLUTION

The mean square current is

$$(i^2)_{avg} = \frac{1}{T}\int_0^T i_0^2\left(\frac{t}{T}\right)^2 dt = \frac{i_0^2}{T^3}\int_0^T t^2dt = \frac{i_0^2}{3}$$

Thus, the rms current is

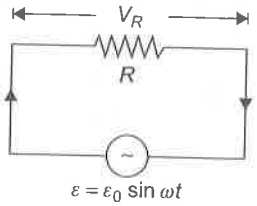
$$i_{rms} = \sqrt{i_{avg}^2} = \frac{i_0}{\sqrt{3}}$$ ■

SERIES AC CIRCUIT

When Only Resistance Is in an AC Circuit

Consider a simple AC circuit consisting of a resistor of resistance R and an AC generator, as shown in the figure.

According to Kirchhoff's loop law at any instant, the algebraic sum of the potential difference around a closed loop in a circuit must be zero.



$$\varepsilon - V_R = 0$$

$$\varepsilon - i_RR = 0$$

$$\varepsilon_0\sin\omega t - i_RR = 0$$

$$i_R = \frac{\varepsilon_0}{R}\sin\omega t = i_0\sin\omega t \qquad (1)$$

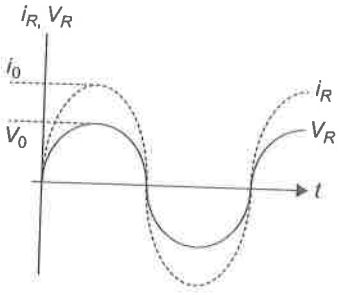
where i_0 is the maximum current. $i_0 = \dfrac{\varepsilon_0}{R}$

From above equations, we see that the instantaneous voltage drop across the resistor

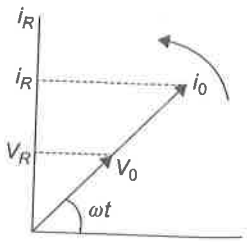
$$V_R = i_0R\sin\omega t \qquad (2)$$

We see in equation (1) and (2), i_R and V_R both vary as sin ωt and reach their maximum values at the same time as shown in figure (a), they are said to be in phase. A phasor diagram is used to represent phase relationships. The lengths of the arrows correspond to V_0 and i_0. The projections of the arrows onto the vertical axis give V_R and i_R. In case of the single-loop resistive circuit, the current and voltage phasors lie along the same line, as shown in figure (b), because **i_R and V_R are in phase.**

292



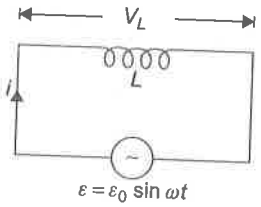
Wave diagram
Figure (a)



Phasor diagram
Figure (b)

When Only Inductor Is in an AC Circuit

Now consider an AC circuit consisting only of an Inductor of inductance L connected to the terminals of an AC generator, as shown in the figure. The induced emf across the inductor is given by Ldi/dt. On applying Kirchhoff's loop rule to the circuit



$$\varepsilon - V_L = 0 \quad \Rightarrow \quad \varepsilon - L\frac{di}{dt} = 0$$

When we rearrange this equation and substitute

$$\varepsilon = \varepsilon_0 \sin \omega t,$$

we get

$$L\frac{di}{dt} = \varepsilon_0 \sin \omega t. \qquad (3)$$

Integration of this expression gives the current as a function of time

$$i_L = \frac{\varepsilon_0}{L}\int \sin \omega t\, dt = -\frac{\varepsilon_0}{\omega L}\cos \omega t + C$$

For average value of current over one time period to be zero, $C = 0$

$$\therefore \qquad i_L = -\frac{\varepsilon_0}{\omega L}\cos \omega t$$

When we use the trigonometric identity

$\cos \omega t = -\sin\left(\omega t - \frac{\pi}{2}\right)$, we can express equation as

$$i_L = \frac{\varepsilon_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad (4)$$

From equation (4), we see that the current reaches its maximum values when $\cos \omega t = 1$.

$$i_0 = \frac{\varepsilon_0}{\omega L} = \frac{\varepsilon_0}{X_L} \qquad (5)$$

where the quantity X_L, called the inductive reactance, is

$$X_L = \omega L$$

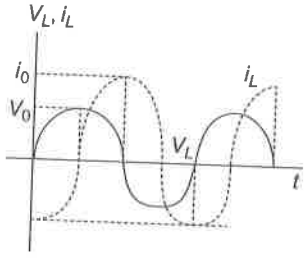
The expression for the rms current is similar to equation (5), with ε_0 replaced by ε_{rms}.

Inductive reactance, like resistance, has unit of ohm.

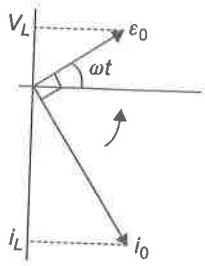
$$V_L = L\frac{di}{dt} = \varepsilon_0 \sin \omega t = i_0 X_L \sin \omega t$$

We can think of equation (5) as Ohm's law for an inductive circuit.

On comparing result of equation (4) with equation (3), we can see that the current and voltage are out of phase with each other by $\pi/2$ rad, or 90°. A plot of voltage and current versus time is given in figure (a). The voltage reaches its maximum value one quarter of an oscillation period before the current reaches its maximum value. The corresponding phasor diagram for this circuit is shown in figure (b). Thus, we see that for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90°.



Wave diagram
Figure (a)

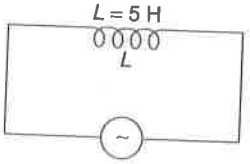


Phasor diagram
Figure (b)

SOLVED EXAMPLES

EXAMPLE 4

An inductor of inductance $L = 5\,\text{H}$ is connected to an AC source having voltage $V = 10 \sin\left(10t + \frac{\pi}{6}\right)$



Find

(i) Inductive Reactance (x_L)
(ii) Peak and Rms voltage (V_0 and V_{rms})
(iii) Peak and Rms current (I_0 and I_{rms})
(iv) Intstantaneous current ($I_{(t)}$)

SOLUTION

(i) $x_L = \omega_L = 10 \times 5 = 50$

(ii) $v_0 = 10$

$$V_{rms} = \frac{10}{\sqrt{2}}$$

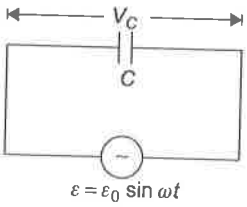
(iii) $I_0 = \frac{v_0}{x_L} = \frac{1}{5}$

$$I_{rms} = \frac{1}{5\sqrt{2}}$$

(iv) $I(t) = \frac{1}{5}\sin\left(10t + \frac{\pi}{6} - \frac{\pi}{2}\right)$ ■

When Only Capacitor Is in an AC Circuit

Figure shows an AC circuit consisting of a capacitor of capacitance C connected across the terminals of an AC generator. On applying Kirchhoff's loop rule to this circuit, we get



$$\varepsilon - V_C = 0$$

$$V_C = \varepsilon = \varepsilon_0 \sin \omega t \qquad (6)$$

where V_C is the instantaneous voltage drop across the capacitor. From the definition of capacitance, $V_C = Q/C$, and this value for V_C substituted into equation gives

$$Q = C\varepsilon_0 \sin \omega t$$

Since $i = dQ/dt$, on differentiating above equation gives the instantaneous current in the circuit.

$$i_C = \frac{dQ}{dt} = C\varepsilon_0 \omega \cos \omega t.$$

Here again we see that the current is not in phase with the voltage drop across the capacitor, given by equation (6). Using the trigonometric identity $\cos \omega t = \sin(\omega t + \pi/2)$, we can express this equation in the alternative from

$$i_C = \omega C\varepsilon_0 \sin\left(\omega t + \frac{\pi}{2}\right). \qquad (7)$$

From equation (7), we see that the current in the circuit reaches its maximum value when $\cos \omega t = 1$.

$$i_0 = \omega C\varepsilon_0 = \frac{\varepsilon_0}{X_C}.$$

Where X_C is called the capacitive reactance.

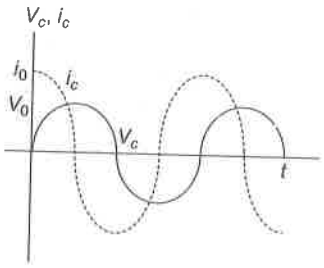
$$X_C = \frac{1}{\omega C}.$$

The SI unit of X_C is also Ω. The rms current is given by an expression similar to equation with V_0 replaced by V_{rms}.

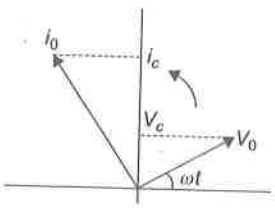
Combining equation (6) and (7), we can express the instantaneous voltage drop across the capacitor as

$$V_C = V_0 \sin \omega t = i_0 X_C \sin \omega t$$

Comparing the result of equation (5) with equation (6), we see that the current is $\pi/2$ rad = 90° out of phase with the voltage across the capacitor. A plot of current and voltage versus time, shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value. The corresponding phasor diagram is shown in the figure (b). Thus we see that for a sinusoidally applied emf, the current always leads the voltage across a capacitor by 90°.



Wave diagram
Figure (a)



Phasor diagram
Figure (b)

Brain Teaser

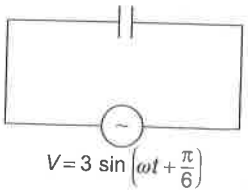
What is the reactance of a capacitor connected to a constant DC source?

SOLVED EXAMPLES

EXAMPLE 5

A capacitor of capacitive reactance 5 Ω is connected with AC source having voltage $V = 3 \sin(\omega t + \pi/6)$. Find rms and Peak voltage rms and peak current and instantaneous current

294



SOLUTION

On comparing with

$$v = v_0 \sin(\omega t + \phi) \Rightarrow V_0 = 3$$

$$V_{rms} = \frac{v_0}{\sqrt{2}} = \frac{3}{\sqrt{2}} \Rightarrow I_{rms} = \frac{v_{rms}}{x_c} = \frac{3}{5\sqrt{2}}$$

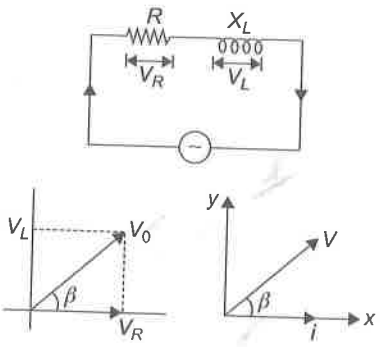
$$I_0 = \frac{v_0}{x_c} = \frac{3}{5} \Rightarrow I(t) = I_0 \sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right)$$ ■

SERIES L-R CIRCUIT

Now consider an AC circuit consisting of a resistor of resistance R and an inductor of inductance L in series with an AC source generator.

Suppose in phasor diagram, current is taken along positive x-direction. The V_R is also along positive x-direction and V_L along positive y-direction as we know that potential difference across a resistance in AC is in phase with current and it leads in phase by 90° with current across the inductor, and as we know $V_R = i_0 R$ and $V_0 = i_0 X_L$

$$i = i_0 \sin \omega t$$



$$V_R(t) = i_0 R \sin \omega t$$

$$V_L(t) = i_0 X_L \sin\left(\omega t + \frac{\pi}{2}\right)$$

hence we can write

$$V(t) = i_0 R \sin \omega t + i_0 X_L \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$V_0 = i_0 \sqrt{x_L^2 + R^2}$$

where $\sqrt{x_L^2 + R^2}$ is known as impedance (z) of the circuit.

now we can write

$$V_t = i_0 \sqrt{x_L^2 + R^2} \sin(\omega t + \beta)$$

where $\tan \beta = \frac{x_L}{R}$

hence $\beta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

SOLVED EXAMPLES

EXAMPLE 6

When 100 V DC is applied across a coil, a current of 1 A flows through it; when 100 V AC of 50 Hz is applied to the same coil, only 0.5 A flows. Calculate the resistance of inductance of the coil.

SOLUTION

In case of a coil, i.e., $L - R$ circuit.

$$i = \frac{V}{Z} \quad \text{with} \quad Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$$

So when DC is applied, $\omega = 0$, so $z = R$

and hence $i = \frac{V}{R}$ i.e., $R = \frac{V}{i} = \frac{100}{0.5} = 200\ \Omega$

but $Z = \sqrt{R^2 + \omega^2 L^2}$ i.e., $\omega^2 L^2 = Z^2 - R^2$

i.e., $(2\pi f L)^2 = 200^2 - 100^2 = 3 \times 10^4$ (as $\omega = 2\pi f$)

So, $$L = \frac{\sqrt{3} \times 10^2}{2\pi \times 50} = \frac{\sqrt{3}}{\pi} \text{H} = \mathbf{0.55\ H}$$ ■

EXAMPLE 7

A 12 Ω resistance and an inductance of $0.05/\pi$ henry with negligible resistance are connected in series. Across the end of this circuit is connected a 130 V alternating voltage of frequency 50 cycles/second. Calculate the alternating current in the circuit and potential difference across the resistance and that across the inductance.

SOLUTION

The impedance of the circuit is given by

$$Z = \sqrt{(R^2 + \omega^2 L^2)} = \sqrt{[R^2 + (2\pi fL)^2]}$$

$$= \sqrt{[(12)^2 + \{2 \times 3.14 \times 50 \times 0.05/3.14\}^2]}$$

$$= \sqrt{(144 + 25)} = 13\ \Omega$$

Current in the circuit $i = \frac{E}{Z} = \frac{130}{13} = 10$ A

Potential difference across resistance

$$V_R = i_R = 10 \times 12 = 120 \text{ V}$$

Inductive reactance of coil $X_L = \omega L = 2\pi fL$

$$\therefore \qquad X_L = 2\pi \times 50 \times = \left(\frac{0.05}{\pi}\right) = 5\ \Omega$$

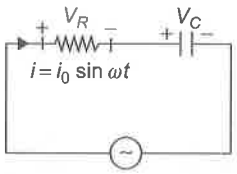
Potential difference across inductance

$$V_L = i \times X_L = 10 \times 5 = \mathbf{50\ V}$$

SERIES C-R CIRCUIT

Now consider an AC circuit consisting of a resistor of resistance R and an capacitor of capacitance C in series with an AC source generator.

Suppose in phasor diagram current is taken along positive x-direction. Then V_R is also along positive x-direction but V_C is along negative y-direction as potential difference across a capacitor in AC lags in phase by 90° with the current in the circuit. So we can write,



$$V_R = I_0 R \sin \omega t = V_{R_0} \sin \omega t$$

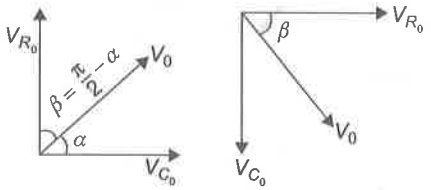
Potential difference across capacitor

$$V_C = I_0 X_C \sin\left(\omega t - \frac{\pi}{2}\right) = V_{C_0} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Potential at any instant t

$$V(t) = V_{c(t)} + V_{R(t)} = V_{C_0} \sin\left(\omega t - \frac{\pi}{2}\right) + V_{R_0} \sin \omega t$$

$$= V_0 \sin\left(\omega t - \frac{\pi}{2} + \alpha\right)$$



$$V(t) = V_0 \sin(\omega t + \beta)$$

$$\tan \alpha = \frac{V_{R_0}}{V_{C_0}} = \frac{R}{X_C}$$

SOLVED EXAMPLES

EXAMPLE 8

An AC source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is i. If now the frequency of the source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency.

SOLUTION

At angular frequency ω, the current in R-C circuit is given by

$$i_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{\sqrt{\left\{R^2 + \left(\frac{1}{\omega^2 C^2}\right)\right\}}} \qquad (1)$$

When frequency is changed to $\omega/3$, the current is halved. Thus

$$\frac{i_{\text{rms}}}{2} = \frac{\varepsilon_{\text{rms}}}{\sqrt{\left\{R^2 + \frac{1}{(\omega/3)^2 C^2}\right\}}} = \frac{\varepsilon_{\text{rms}}}{\sqrt{\left\{R^2 + \left(\frac{9}{\omega^2 C^2}\right)\right\}}} \qquad (2)$$

From equations (1) and (2), we have

$$\frac{1}{\sqrt{\left\{R^2 + \left(\frac{1}{\omega^2 C^2}\right)\right\}}} = \frac{2}{\sqrt{\left\{R^2 + \left(\frac{9}{\omega^2 C^2}\right)\right\}}}$$

Solving this equation, we get $3R^2 = \frac{5}{\omega^2 C^2}$

Hence, the ratio of reactance to resistance is

$$\frac{\left(\frac{1}{\omega C}\right)}{R} = \sqrt{\left(\frac{3}{5}\right)}$$

EXAMPLE 9

A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp?

SOLUTION

As resistance of the lamp $R = \frac{V_s^2}{W} = \frac{100^2}{50} = 200\ \Omega$ and the maximum current $i = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$ A; so when the

lamp is put in series with a capacitance and run at 200 V AC, from $V = iZ$ we have,

$$Z = \frac{V}{i} = \frac{200}{(1/2)} = 400\ \Omega$$

Now as in case of C-R circuit, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$,

i.e., $R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$

or, $\left(\frac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$

So, $\frac{1}{\omega C} = \sqrt{12} \times 10^2$

or $C = \frac{1}{100\pi \times \sqrt{12} \times 10^2}$ F

i.e., $C = \frac{100}{\pi\sqrt{12}}\ \mu\text{F} = 9.2\ \mu\text{F}$

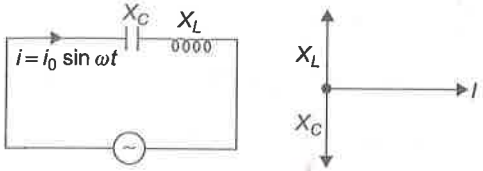
L.C. CIRCUIT

As shown in figure a capacitor and inductance are connected in series method and alternating voltage is applied across the circuit.

Let X_c is capacitance reactance,

X_L is Inductance reactance,

$i = i_0 \sin \omega t$ current flowing through the circuit



$$V_C(t) = i_0 X_C \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_L(t) = i_0 X_L \sin\left(\omega t + \frac{\pi}{2}\right)$$

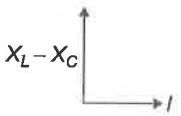
$$\vec{v}_t = \vec{v}_c + \vec{v}_L$$

$$= i_0 X_C \sin\omega t \cos\frac{\pi}{2} - i_0 X_C \cos\omega t \sin\frac{\pi}{2} + i_0 X_L \sin\omega t \cos\frac{\pi}{2} + i_0 X_L \cos\omega t \sin\frac{\pi}{2}$$

$$= i_0 \cos\omega t (X_L - X_C)$$

$$V(t) = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$V_0 = i_0 Z$ (Z is impedance)



$$Z = (X_L - X_C)$$

$$\cos\phi = 0$$

$$V_{CO} = i_0 X_C;\ X_{L_0} = i_0 X_L$$

SERIES L-C-R CIRCUIT

Now consider an AC circuit consisting of a resistor of resistance R, a capacitor of capacitance C and an inductor of inductance L are in series with an AC source generator.

Suppose in a phasor diagram current is taken along positive x-direction. Then V_R is along positive x-direction, V_L along positive y-direction and V_C along negative y-direction, as potential difference across an inductor leads the current by 90° in phase while that across a capacitor, lags by 90°.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

L-R-C circuit

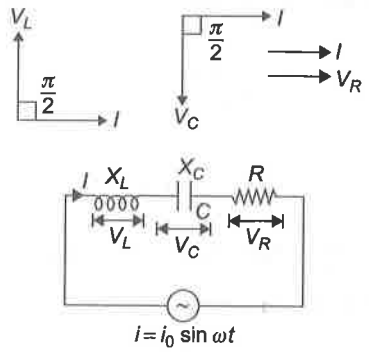
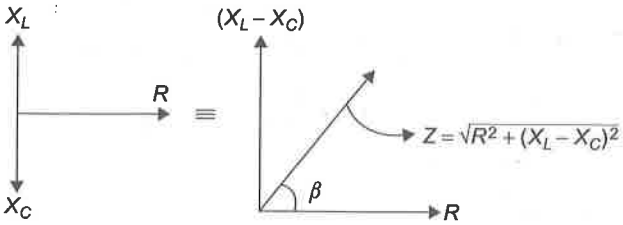


Figure A series L-C-R circuit

Impedance phasor of above circuit

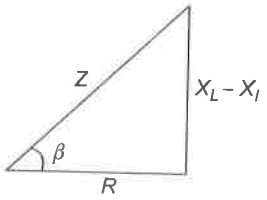


and Impedance triangle

$$z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

here B is phase angle. By triangle $\tan\beta = \frac{X_L - X_C}{R}$

Power factor $\cos\phi = \frac{R}{Z}$.



Let I be the current in the series circuit of any instant then

1. Voltage $V(t) = V_0 \sin(\omega t + \beta) = i_0 z \sin(\omega t + \beta)$, here $v_0 = i_0 z$ and $v_{rms} = i_{rms} z$
2. $v_L(t) = v_{O_L} \sin\left(\omega t + \frac{\pi}{2}\right)$ here voltage V_L across the inductance is ahead of current I in phase by $\frac{\pi}{2}$ rad
$$V_{O_L} = I_0 X_L$$
3. $V_C(t) = V_{OC} \sin\left(\omega t - \frac{\pi}{2}\right)$, here voltage V_C across the capacitance lags behind the current I in pase by $\frac{\pi}{2}$ rad
$$V_{OC} = I_0 X_C$$
4. $V_R(t) = i_0 R \sin \omega t$, here voltage V_R across the resistor R has same phase as I
$$V_{OR} = I_O R$$

Special Case:

1. When $X_L > X_C$ or $V_L > V_C$ then emf is ahead of current by phase β which is given by
$$\tan\beta = \frac{X_L - X_C}{R} \quad \text{or} \quad \cos\phi = \frac{R}{Z}$$
The series LCR circuit is said to be inductive
2. When $X_L < X_C$ or $V_L < V_C$ then current is ahead of emf by phase angle β which is given by
$$\tan\beta = \frac{X_C - X_L}{R} \quad \text{or} \quad \cos\phi = \frac{R}{Z}$$
The series LCR circuit is said to be capacitive
3. When $X_L = X_C$ or $V_L = V_C$, $\beta = 0$, the emf and current will be in the same phase. The series LCR circuit is said to be purely resistive. It may also be noted that
$$I_0 = \frac{\varepsilon_0}{Z} \quad \text{or} \quad \frac{I_0}{\sqrt{2}} = \frac{\varepsilon_0}{\sqrt{2}\,Z} \quad \text{or} \quad I_{rms} = \frac{E_{rms}}{Z}$$
Susceptance: The reciprocal of the reactance of an AC circuit is called its susceptance.
Admittance: The reciprocal of the impedance of an AC circuit is called its admittance.

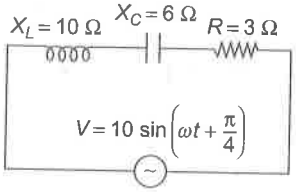
SOLVED EXAMPLES

EXAMPLE 10

Figure shows a series LCR circuit connected to a variable voltage source $V = 10 \sin\left(\omega t + \frac{\pi}{4}\right)$;

$$x_L = 10\ \Omega, X_C = 6\ \Omega, R = 3\ \Omega$$

Calculate Z, i_0, i_{rms}, v_{rms}, V_{LO}, V_{CO}, V_{RO}, β, V_{Lrms}, V_{Crms}, V_{rms}, $i(t)$, $V_L(t)$, $V_{c(t)}$, and $V_{R(t)}$



$X_L > X_C$

SOLUTION

$$V = 10 \sin\left(\omega t + \frac{\pi}{4}\right) \quad \text{so, } V_0 = 10\ \text{V}$$

$$V_{rms} = \frac{10}{\sqrt{2}}\ \text{V}$$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{9 + 16} = 5$$

$$i_0 = \frac{v_0}{z} = 2$$

$$i_{rms} = \sqrt{2}\ \text{A}$$

$$V_{LO} = i_0 X_L = 20\ \text{V};\ V_{CO} = i_0 X_L = 12\ \text{V}$$

$$V_{RO} = i_0 R = 6\ \text{V};\ V_{Lrms} = \frac{20}{\sqrt{2}}\ \text{V}$$

$$V_{Crms} = \frac{20}{\sqrt{2}}\ \text{V};\ V_{Rrms} = \frac{6}{\sqrt{2}}\ \text{V}$$

$$\cos\beta = \frac{R}{Z} = \frac{3}{5} \quad \Rightarrow \quad \beta = 53°$$

$$i(t) = 20 \sin\left(\omega t + \frac{\pi}{4} - 53°\right)$$

$$V_L(t) = 20 \sin\left(\omega t + \frac{\pi}{4} - 53° + \frac{\pi}{2}\right)$$

$$= 20 \sin\left(\omega t + \frac{3\pi}{4} - 53° - \frac{\pi}{2}\right)$$

$$= 12 \sin\left(\omega t - \frac{\pi}{4} - 53°\right)$$

298

$$V_{R(t)} = V_{RO}\sin\left(\omega t + \frac{\pi}{4} - 53°\right)$$
$$= 6\sin\left(\omega t + \frac{\pi}{4} - 53°\right)$$

EXAMPLE 11

A resistor of resistance R, an inductor of inductance L and a capacitor of capacitance C all are connected in series with an AC supply. The resistance of R is 16 Ω and for a given frequency the inductive reactance of L is 24 Ω and capacitive reactance of C is 12 Ω. If the current in the circuit is 5 A. Find

(a) the potential difference across R, L and C
(b) the impedance of the circuit
(c) the voltage of AC supply
(d) phase angle

SOLUTION

(a) Potential difference across resistance

$$V_R = iR = 5 \times 16 = \mathbf{80\ V}$$

Potential difference across inductance

$$V_L = i \times (\omega L) = 5 \times 24 = \mathbf{120\ V}$$

Potential difference across condenser

$$V_C = i \times \left(\frac{1}{\omega C}\right) = 5 \times 12 = \mathbf{60\ V}$$

(b) $$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$
$$= \sqrt{[(16)^2 + (24-12)^2]} = \mathbf{20\ \Omega}$$

(c) The voltage of AC supply is given by

$$V = iZ = 5 \times 20 = \mathbf{100\ V}$$

(d) Phase angle

$$\phi = \tan^{-1}\left[\frac{\omega L - (1/\omega C)}{R}\right] = \tan^{-1}\left[\frac{24-12}{16}\right]$$
$$= \tan^{-1}(0.75) = \mathbf{36°87'}$$

EXAMPLE 12

A series circuit consists of a resistance of 15 Ω, an inductance of 0.08 henry and a condenser of capacity 30 micro farad. The applied voltage has a frequency of 500 radian/s. Does the current lead or lag the applied voltage and by what angle?

SOLUTION

Here $\omega L = 500 \times 0.08 = 40\ \Omega$

and $$\frac{1}{\omega C} = \frac{1}{500 \times (30 \times 10^{-6})} = 66.7\ \Omega$$

$$\tan\phi = \frac{\left[\omega L - \left(\frac{1}{\omega C}\right)\right]}{R} = \frac{40 - 66.7}{15} = -1.78$$

$$\phi = -60.67°$$

Thus the current leads the applied voltage by 60.67°

EXAMPLE 13

A current of 4 A flows in a coil when connected to a 12 V DC source. If the same coil is connected to a 12 V, 50 rad/s, AC source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF condenser is connected in series with the coil.

SOLUTION

When the coil is connected to a DC source, its resistance R is given by

$$R = \frac{V}{i} = \frac{12}{4} = 3\ \Omega$$

When it is connected to AC source, the impedance Z of the coil is given by

$$Z = \frac{V_{rms}}{i_{rms}} = \frac{12}{2.4} = 5\ \Omega$$

For a coil, $Z = \sqrt{[R^2 + (\omega L)^2]}$

$\therefore$ $5 = \sqrt{[(3)^2 + (50L)^2]}$

or $25 = [(3)^2 + (50L)^2]$

Solving we get $L = \mathbf{0.08\ henry}$

When the coil is connected with a condenser in series, the impedance Z' is given by

$$Z' = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$
$$= \left[(3)^2 + \left(50 \times 0.08 - \frac{1}{50 \times 2500 \times 10^{-6}}\right)^2\right]^{1/2}$$
$$= 5\ \Omega$$

Power developed $P = V_{rms} \times i_{rms} \times \cos\phi$

where $\cos\phi = \frac{R}{Z'} = \frac{3}{5} = 0.6$

$\therefore$ $P = 12 \times 2.4 \times 0.6 = \mathbf{17.28\ W}$

EXAMPLE 14

A 20 V 5 W lamp is used in AC main of 220 V 50 c.p.s. Calculate the (i) Capacitance of capacitor. (ii) Inductance of inductor, to be put in series to run the lamp, (iii) What pure

resistance should be included in place of the above device so that the lamp can run on its voltage. (iv) Which of the above arrangements will be more economical and why?

SOLUTION

The current required by the lamp

$$i = \frac{\text{wattage}}{\text{voltage}} = \frac{5}{20} = 0.25\ \text{A}$$

The resistance of the lamp

$$R = \frac{\text{voltage}}{\text{current}} = \frac{20}{0.25} = 80\ \Omega$$

So for proper running of the lamp, the current through the lamp should be 0.25 A

(i) When the condenser C is placed in series with lamp, then

$$Z = \sqrt{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]}$$

The current through the circuit

$$i = \frac{200}{\sqrt{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]}} = 0.25$$

or $$\frac{200}{\sqrt{(80)^2 + \left(\frac{1}{4\pi^2 \times 50^2 \times C^2}\right)}} = 0.25$$

Solving it for C, we get

$$C = 4.0 \times 10^{-6}\ \text{F} = 4.0\ \mu\text{F}$$

(ii) When inductor L henry is placed in series with the lamp, then

$$Z = \sqrt{[R^2 + (\omega L)^2]} \quad \text{or} \quad \frac{200}{\sqrt{[R^2 + (\omega L)^2]}} = 0.25$$

or $$\frac{200}{\sqrt{[(80)^2 + (4\pi^2 \times 50^2 \times L^2)]}} = 0.25$$

Solving it for L, we get $L = 2.53$ henry

(iii) When resistance $r\ \Omega$ is placed in series with lamp of resistance R, then

$$\frac{200}{R + r} = 0.25$$

or $$\frac{200}{80 + r} = 0.25 \quad \Rightarrow \quad r = 720\ \Omega$$

(iv) It will be more economical to use inductance or capacitance is series with the lamp to run it as it consumes no power while there would be dissipation of power when resistance is inserted in series with the lamp. ■

Brain Teaser

Can the peak voltage across the inductor be greater than the peak voltage of the source in an LCR circuit?

POWER IN AN AC CIRCUIT

In case of a steady current the rate of doing work is given by,

$$P = Vi$$

In an alternating circuit, current and voltage both vary with time, so the work done by the source in time interval dt is given by

$$dW = Vidt$$

Suppose in an ac, the current is leading the voltage by an angle ϕ. Then we can write,

$$V = V_0 \sin \omega t$$

and $$i = i_0 \sin(\omega t + \phi)$$

$$dW = V_0 i_0 \sin \omega t \sin(\omega t + \phi)\, dt$$

$$= V_0 i_0 (\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi)\, dt$$

The total work done in a complete cycle is

$$W = V_0 i_0 \cos\phi \int_0^T \sin^2 \omega t\, dt + V_0 i_0 \sin\phi \int_0^T \sin \omega t \cos \omega t\, dt$$

$$= \frac{1}{2} V_0 i_0 \cos\phi \int_0^T (1 - \cos 2\omega t) dt + \frac{1}{2} V_0 i_0 \sin\phi$$

$$\int_0^T \sin 2\omega t\, dt + \frac{1}{2} V_0 i_0\, T \cos\phi$$

The average power delivered by the source is, therefore,

$$P = \frac{W}{T} = \frac{1}{2} V_0 i_0 \cos\phi = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)(\cos\phi)$$

$$= V_{\text{rms}}\, i_{\text{rms}} \cos\phi$$

or $$\langle P \rangle_{\text{one cycle}} = V_{\text{rms}}\, i_{\text{rms}} \cos\phi$$

Here, the term $\cos\phi$ is known as **power factor**.

It is said to be leading if current leads voltage, lagging if current lags voltage. Thus, a power factor of 0.5 lagging means current lags the voltage by 60° (as $\cos^{-1}0.5 = 60°$). The product of V_{rms} and i_{rms} gives the apparent power. While the true power is obtained by multiplying the apparent power by the power factor $\cos\phi$. Thus,

and apparent power = $V_{\text{rms}} \times i_{\text{rms}}$

True power = apparent power × power factor

For $\phi = 0°$, the current and voltage are in phase. The power is thus, maximum ($V_{\text{rms}} \times i_{\text{rms}}$). For $\phi = 90°$, the power is

300

zero. The current is then stated as wattless. Such a case will arise when resistance in the circuit is zero. The circuit is purely inductive or capacitive. Concept of wattless current, watlless component of current and impedance triangle can also be introduced.

RESONANT FREQUENCY

A series LCR circuit is said to be in the resonance condition when the current through it has its maximum value.

The current amplitude I_0 for a series LCR circuit is given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Clearly I_0 becomes zero both for $\omega \to 0$ and $\omega \to \infty$. The value of I_0 is maximum when

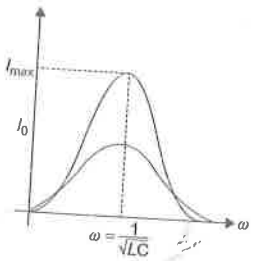
$$\omega L - \frac{1}{\omega C} = 0 \text{ or } \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}}$$

Then impedance will be minimum

$$Z_{min} = R$$

The circuit is purely resistive. The current and voltage are in the same phase and the current in the circuit is maximum. This condition of the *LCR* circuit is called resonance condition.



The variance of I_0 v/s ω shown in following figure

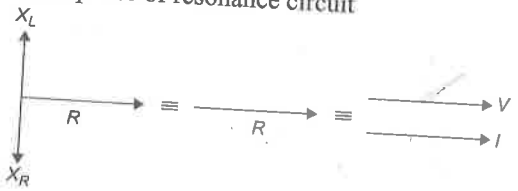
$$i_{0\,max} = \frac{v_0}{z_{min}} = \frac{v_0}{R}$$

Sq

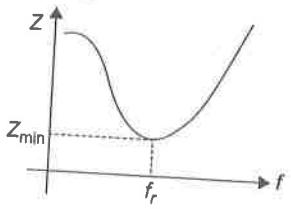
$$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

$$V = V_0 \sin(\omega t)$$

Impedance phase of resonance circuit



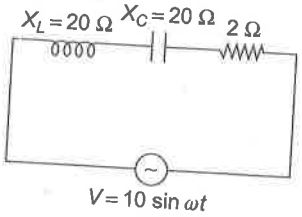
Impedance of the circuit is minimum and heat generated in the circuit is maximum.



SOLVED EXAMPLES

EXAMPLE 15

In following LCR circuit find Z, $i(t)$, V_{OC}, V_{OL} at resonance frequency



SOLUTION

$$Z = Z_{min} = R = 2\ \Omega$$

$$i_0 = \frac{E_0}{Z} = \frac{10}{2} = 5\text{ A}$$

$$i(t) = \frac{V(t)}{Z} = 5 \sin \omega t$$

$$V_{OL} = i_0 X_C = 100\text{ V}$$

$$V_{OL} = i_0 X_L = 100\text{ V}$$

Above circuit is used as voltage amplifier (magnification) as peak value of voltage by source is only 10 while we can have maximum voltage up to 100 (V_{OC} and V_{OL}) ■

EXAMPLE 16

A series LCR with $R = 20\ \Omega$, $L = 1.5$ H and $C = 35\ \mu$F is connected to a variable frequency 200 V AC supply. When the frequency of the supply equals the natural frequency of the circuit. What is the average power transferred to the circuit in one complete cycle?

SOLUTION

When the frequency of the supply equals the natural frequency of the circuit, resonance occurs.

$\therefore$

$$Z = R = 20\ \Omega$$

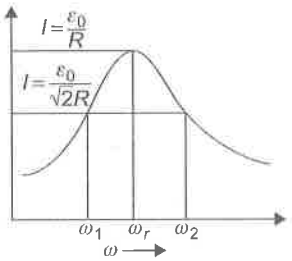
$$i_{rms} = \frac{E_{rms}}{Z} = \frac{200}{20} = 10\text{ A}$$

Average power transferred/cycle

$$P = E_{rms} i_{rms} \cos 0° = 200 \times 10 \times 1 = \mathbf{2000\ W}$$ ■

Sharpness of Resonance (Q-Factor)

The Q-factor of a series resonant circuit is defined as the ratio of the resonant frequency to the difference in two frequencies taken on the both sides of the resonant frequency such that at each frequency, the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value of current at resonant frequency.



Mathematically Q-factor.

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega} = \frac{\text{Resonant frequency}}{\text{Band width}}$$

or $$Q = \frac{x_L}{R} = \frac{\omega_r L}{R}$$

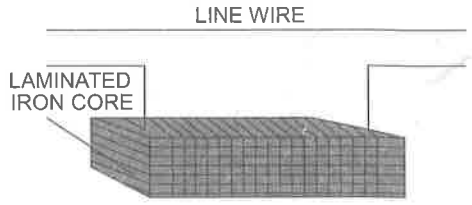
CHOKE COIL

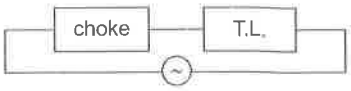
A choke coil is simply an inductor with large inductance which is used to reduce current in AC circuit without much loss of energy.

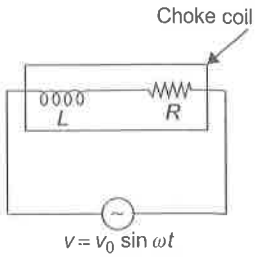
Principle: A choke coil is based upon the principle that when AC flows through an inductor, the current lags behind the emf by a phase angle $\pi/2$.

Construction: A choke coil is basically an inductance. It consists of a large number of turns of insulated copper wire wound over a soft iron core. In order to minimise loss of electrical energy due to production of eddy currents, a laminated iron core is used.

In practice, a low frequency choke coil is made of insulated copper wire wound on a soft iron core, while a high frequency choke coil has air as core materials





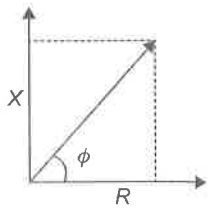


***Working*:** As shown in fig a choke is put in series across an electrical appliances of resistance R and is connected to an AC source.

Average power dissipated per cycle in the circuit is

$$P_{av} = V_{eff} I_{eff} \cos\phi = V_{eff} I_{eff} \frac{R}{\sqrt{R^2 + \omega^2 L^2}}.$$

Inductance L of the choke coil is very large so that $R << \omega L$. Then



Power factor $\cos\phi \simeq \frac{R}{\omega L} \simeq 0$

$$\tan\phi = \frac{x_L}{R}$$

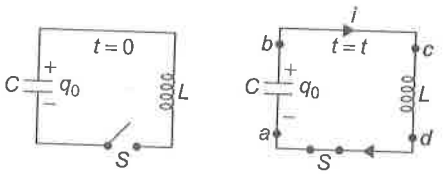
Uses: In AC circuit, a choke coil is used to control the current in place of a resistance. If a resistance is used to control the current, the electrical energy will be wasted in the form of heat. A choke coil decreases the current without wasting electrical energy in the form of heat.

OSCILLATIONS IN L-C CIRCUIT

If a charged capacitor C is short-circuited through an inductor L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. Assume an ideal situation in which energy is not radiated away from the circuit. With these idealizations-zero resistance and no

302

radiation, the oscillations in the circuit persist indefinitely and the energy is transferred from capacitor's electric field to the inductor's magnetic field back and forth. The total energy associated with the circuit is constant. This is analogous to the transfer of energy in an oscillating mechanical system from potential energy to kinetic energy and back, with constant total energy. Such an analogous mechanical system is an example of spring mass system.



Let us now derive an equation for the oscillations of charge and current in an L-C circuit. Refer figure (a): The capacitor is charged to a potential difference V such that charge on capacitor $q_0 = CV$.

Here q_0 is the maximum charge on the capacitor. At time $t = 0$, it is connected to an inductor through a switch S. At time $t = 0$, the switch S is closed.

Refer figure (b): When the switch is closed, the capacitor starts discharging. Let at time t charge on the capacitor is $q(<q_0)$ and since, it is further decreasing, there is a current i in the circuit in the direction shown in figure.

The potential difference across capacitor = potential difference across inductor, or

$$V_b - V_a = V_c - V_d$$

$$\therefore \quad \frac{q}{C} = L\left(\frac{di}{dt}\right) \tag{1}$$

Now, as the charge is decreasing, $i = \left(\frac{-dq}{dt}\right)$

or $$\frac{di}{dt} = -\frac{d^2q}{dt^2}$$

Substituting in equation (1), we get

$$\frac{q}{C} = -L\left(\frac{d^2q}{dt^2}\right)$$

or $$\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q \tag{2}$$

This is the standard equation of simple harmonic motion

$$\left(\frac{d^2x}{dt^2} = -\omega^2 x\right)$$

Here $$\omega = \frac{1}{\sqrt{LC}} \tag{3}$$

The general solution of equation (2), is

$$q = q_0 \cos(\omega t \pm \phi) \tag{4}$$

In our case $\phi = 0$ as $q = q_0$ at $t = 0$.

Thus, we can say that the charge in the circuit oscillates with angular frequency ω given by equation (3). Thus,

In L-C oscillations, q, i and $\frac{di}{dt}$ all oscillate simple harmonically with same angular frequency ω, but the phase difference between q and i or between i and $\frac{di}{dt}$ is $\frac{\pi}{2}$. Their amplitudes are $q_0 q_0 \omega$ are $\omega^2 q_0$ respectively. So

$$q = q_0 \cos \omega t, \tag{5}$$

then $$i = -\frac{dq}{dt} = q_0 \omega \sin \omega t \tag{6}$$

and $$\frac{di}{dt} = q_0 \omega^2 \cos \omega t \tag{7}$$

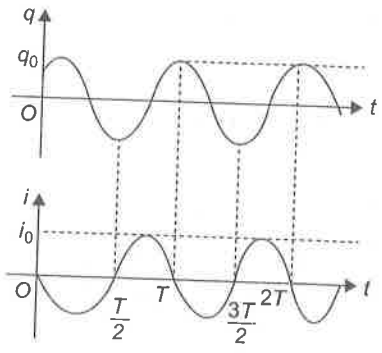
Potential energy in the capacitor

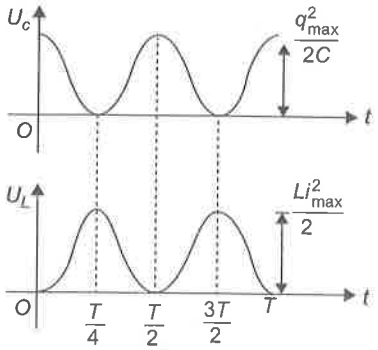
$$U_c = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{q_0^2}{C}\cos^2 \omega t \tag{8}$$

Potential energy in the inductor

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}\frac{q_0^2}{C}\sin^2 \omega t \tag{9}$$

Thus potential energy stored in the capacitor and that in the inductor also oscillates between maximum value and zero with double the frequency. All these quantities are shown in the figures that follows





SOLVED EXAMPLES

EXAMPLE 17

A capacitor of capacitance 25 μF is charged to 300 v. It is then connected across a 10 mH inductor. The resistance of the circuit is negligible.

(a) Find the frequency of oscillation of the circuit.

(b) Find the potential difference across capacitor and magnitude of circuit current 1.2 ms after the inductor and capacitor are connected.

(c) Find the magnetic energy and electric energy at $t = 0$ and $t = 1.2$ ms.

SOLUTION

(a) The frequency of oscillation of the circuit is,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Substituting the given values we have,

$$f = \frac{1}{2\pi\sqrt{(10\times10^{-3})(25\times10^{-6})}} = \frac{10^3}{\pi}\text{ Hz}$$

(b) Charge across the capacitor at time t will be,

$$q = q_0\cos\omega t$$

and $i = -q_0\,\omega\sin\omega t$

Here $q_0 = CV_0 = (25\times10^{-6})\,(300) = 7.5\times10^{-3}\,C$

Now, charge is the capacitor after $t = 1.2\times10^{-3}$ s is,

$q = (7.5\times10^{-3})\cos(2\pi\times318.3)\,(1.2\times10^{-3})C$

$= 5.53\times10^{-3}\,C$

$\therefore$ P.D. across capacitor,

$$V = \frac{|q|}{C} = \frac{5.53\times10^{-3}}{25\times10^{-6}} = 221.2\text{ V}$$

The magnitude of current in the circuit at $t = 1.2\times10^{-3}$ s is,

$$|i| = q_0\,\omega\sin\omega t$$

$= (7.5\times10^{-3})(2\pi)(318.3)\sin(2\pi\times318.3)$

$(1.2\times10^{-3})\text{ A} = 10.13\text{ A}$

(c) At $t = 0$: Current in the circuit is zero. Hence, $U_L = 0$

Charge on the capacitor is maximum

Hence, $U_c = \frac{1}{2}\frac{q_0^2}{C}$

or $U_c = \frac{1}{2}\times\frac{(7.5\times10^{-3})^2}{(25\times10^{-6})} = 1.125\text{ J}$

$\therefore$ Total energy $E = U_L + U_C = 1.125$ J

At $t = 1.2$ ms

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(10\times10^{-3})(10.13)^2 = 0.513\text{ J}$$

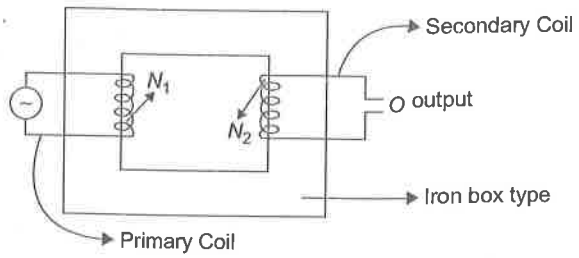
$U_C = E - U_L = 1.125 - 0.513 = 0.612$ J

Otherwise U_C can be calculated as,

$$U_c = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\times\frac{(5.53\times10^{-3})^2}{(25\times10^{-6})} = 0.612\text{ J}$$ ■

TRANSFORMER

A transformer is an electrical device for converting an alternating current at low voltage into that at high voltage or vice versa. If it increase the input voltage, it is called step up transformer and if it decreases the input voltage, it is called step down transformer.



Principle: It works on the principle of mutual induction, i.e., when a changing current is passed through one of the two inductively coupled coils, an induced emf is setup in the other coil.

Construction: A transformer essentially consists of two coils of insulated copper wire having different number of turns and wound on the same soft iron core. The coil to which electric energy is supplied is called the primary and the coil from which energy is drawn or output is obtained is called the secondary.

304

To prevent energy losses due to eddy currents, a laminated sheet is used. Because of high permeability of soft iron, the entire magnetic flux due to the current in the primary coil practically remains in the iron core and hence passes fully through the secondary.

Two types of arrangements are generally used for winding of primary and secondary coils in a transformer.

1. **Core type:** In this type, the primary and secondary coils are wound on separate limbs of the core.
2. **Shell type:** In this type the primary and secondary coils are wound one over another on the same limb of the iron core.

Theory: Consider the situation when no load is connected to the secondary, i.e., its terminals are open. Let N_1 and N_2 be the number of terminal in the primary and secondary respectively. Then Induced emf in the primary coil

$$v_p = -N_1 \frac{d\phi}{dt}$$

Induced emf in the secondary coil

$$v_s = -N_2 \frac{d\phi}{dt}$$

where ϕ is the magnetic flux linked with each turn of the primary or secondary at any instant. Thus

$$\frac{V_s}{V_p} = \frac{N_2}{N_1}$$

The ratio $\frac{N_2}{N_1}$ of the number of turns in the secondary to that in the primary called the turns ratio of the transformer. It is also called transformation ratio

for step up transformer: $N_2 > N_1$

for step down transformer: $N_1 > N_2$

Currents in primary and secondary: Assuming the transformer to be ideal one so that there are no energy loss, then

Input power = output power

or $$V_p I_p = V_s I_s$$

where I_p and I_S are the currents in the primary and secondary respectively.

Hence $$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_2}{N_1}$$

Efficiency: The efficiency of a transformer is defined as

$$\eta = \frac{\text{Power output}}{\text{Power input}} \times 100\%$$

The efficiency of real transformer is fairly high (90 – 98%) though not 100%.

EXTRA PORTION FOR JEE-MAIN

Explain with the help of a labelled diagram, the principle and working of an AC generator? Write the expression for the emf generated in the coil in terms of speed of rotation. Can the current produced by an AC generator be measured with a moving coil galvanometer.

SOLUTION

AC generator: A dynamo or generator is a device which converts mechanical energy into electrical energy. It is based on the principle of electromagnetic induction.

Construction: It consists of the four main parts:

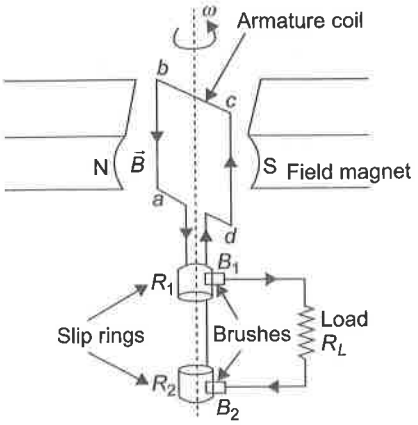
(i) **Field Magnet:** It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo, the magnetic field is produced by an electromagnet.

(ii) **Armature:** It consists of a large number of turns of insulated wire in the soft iron drum or ring. It can revolve round an axle between the two poles of the field magnet. The drum or ring serves the two purposes: (i) It serves as a support to coils and (ii) It increases the magnetic field due to air core being replaced by an iron core.

(iii) **Slip Rings:** The slip rings R_1 and R_2 are the two metal rings to which the ends of armature coil are connected. These rings are fixed to the shaft which rotates the armature coil so that the rings also rotate along with the armature.

(iv) **Brushes:** These are two flexible metal plates or carbon rods (B_1 and B_2) which are are fixed and constantly touch the revolving rings. The output current in external load R_L is taken through these brushes.

Working: When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming's right hand rule. Considering the armature to be in vertical position and as it rotates in anticlockwise direction, the wire ab moves upward and cd downward, so that the direction of induced current is shown in figure. In the external circuit, the current flows along $B_1 B_L B_2$. The direction of current remains unchanged during the first half turn of armature. During the second half revolution, the wire ab moves downward and cd upward, so the direction of current is reversed and in external circuit if flows along $B_2 R_L B_1$. Thus the direction of induced emf and current changes in the external circuit after each half revolution. If N is the number of turns in coil, f the frequency of rotation. A area of coil and B the magnetic induction, then induced emf



$$e = -\frac{d\phi}{dt} = \frac{d}{dt}(NBA(\cos 2\pi ft))$$

Obviously, the emf produced is alternating and hence the current is also alternating.

Current produced by an AC generator cannot be measured by moving coil ammeter, because the average value of AC over fully cycle is zero.

EXERCISES

JEE Main

1. An alternating current changes fiom a complete cycle in 1 μs, then the frequency in Hz will be
(A) 10^{-6} (B) 50
(C) 100 (D) 10^6

2. An AC circuit, the current is given by $i = 4 \sin (100 \pi t + 30°)$ A. The current becomes maximum first time (after $t = 0$) at t equal to
(A) $\frac{1}{200}$ sec (B) $\frac{1}{300}$ sec
(C) $\frac{1}{50}$ sec (D) None of the above

3. The instantaneous value of current in an AC circuit is $I = 2 \sin\left(100 \pi t + \frac{\pi}{3}\right)$ A. The current at the beginning ($t = 0$) will be
(A) $2\sqrt{3}$ A (B) $\sqrt{3}$ A
(C) $\frac{\sqrt{3}}{2}$ A (D) Zero

4. If instantaneous value of current is $I = 10 \sin (314 t)$ A, then the average current for the half cycle will be
(A) 10 A (B) 7.07 A
(C) 6.37 A (D) 3.53 A

5. In a circuit an AC current and a DC. current are supplied together. The expression of the instantaneous current is given as $i = 3 + 6 \sin \omega t$. Then the rms value of the current is
(A) 3 (B) 6
(C) $3\sqrt{2}$ (D) $3\sqrt{3}$

6. The emf and the current in a circuit are:
$E = 12 \sin(100 \pi t)$; $I = 4 \sin\left(100 \pi t + \frac{\pi}{3}\right)$ then
(A) The current leads the emf by 60°
(B) The current lags the emf by 60°
(C) The emf leads the current by 60°
(D) The phase difference between the current and the emf is zero

7. If the frequency of alternating potential is 50 Hz then the direction of potential, changes in one second by
(A) 50 times (B) 100 times
(C) 200 times (D) 500 times

8. The value of alternating emf is $e = 500 \sin 100\pi t$, then the frequency of this potential in Hz is
(A) 25 (B) 50
(C) 75 (D) 100

306

9. The domestic power supply is at 220 V. The amplitude of emf will be
(A) 220 V (B) 110 V
(C) 311 V (D) None of this

10. The average value or alternating current for half cycle in terms of I_0 is
(A) $\frac{2i_0}{\pi}$ (B) $\frac{i_0}{\pi}$
(C) $\frac{i_0}{\sqrt{2}}$ (D) 0

11. Sinusoidal peak potential is 200 V with frequency 50 Hz. It is represented by the equation
(A) $E = 200 \sin 50t$ (B) $E = 200 \sin 314t$
(C) $E = 200\sqrt{2} \sin 50t$ (D) $E = 200\sqrt{2} \sin 314t$

12. Rms value of AC $i = i_1 \cos \omega t + i_2 \sin \omega t$ will be
(A) $\frac{1}{\sqrt{2}}(i_1 + i_2)$ (B) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$
(C) $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$ (D) $\frac{1}{2}(i_1^2 + i_2^2)^{1/2}$

13. The phase difference between the alternating current and voltage represented by the following equation $I = I_0 \sin \omega t$, $E = E_0 \cos\left(\omega t + \frac{\pi}{3}\right)$, will be
(A) $\frac{\pi}{3}$ (B) $\frac{4\pi}{3}$
(C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{6}$

14. The inductive reactance of a coil is 1000 Ω. If its self inductance and frequency both are increased two times then inductive reactance will be
(A) 1000 Ω (B) 2000 Ω
(C) 4000 Ω (D) 16000 Ω

15. In an L-C-R series circuit $R = 1\ \Omega$, $X_L = 1000\ \Omega$ and $X_C = 1000\ \Omega$. A source of 100 mV is connected in the circuit the current in the circuit is
(A) 100 μA (B) 1 μA
(C) 0.1 μA (D) 10 μA

16. A coil of inductance 0.1 H is connected to an alternating voltage generator of voltage $E = 100 \sin (100\ t)$ volt. The current flowing through the coil will be
(A) $I = 10\sqrt{2} \sin(100t)$ A
(B) $I = 10\sqrt{2} \cos(100t)$ A
(C) $I = -10 \sin(100t)$ A
(D) $I = -10 \cos(100t)$ A

17. Alternating current lead the applied emf by $\pi/2$ when the circuit consists of
(A) only resistance
(B) only capacitor
(C) only an inductance coil
(D) capacitor and resistance both

18. A coil has reactance of 100 Ω when frequency is 50 Hz. If the frequency becomes 150 Hz, then the reactance will be
(A) 100 Ω (B) 300 Ω
(C) 450 Ω (D) 600 Ω

19. A resistance of 50 Ω, an inductance of $20/\pi$ henry and a capacitor of $5/\pi$ μF are connected in series with an AC source of 230 V and 50 Hz. The impedance of circuit is
(A) 5 Ω (B) 50 Ω
(C) 5 KΩ (D) 500 Ω

20. The potential difference between the ends of a resistance R is V_R between the ends of capacitor is $V_C = 2\ V_R$ and between the ends of inductance is $V_L = 3\ V_R$, then the alternating potential of the source in terms of V_R will be
(A) $\sqrt{2}V_R$ (B) V_R
(C) $V_R/\sqrt{2}$ (D) $5V_R$

21. The percentage increase in the impedance of an AC circuit, when its power factor changes form 0.866 to 0.5 is (Resistance constant)
(A) 73.2% (B) 86.6%
(C) 90.8% (D) 66.6%

22. In a series resonant L-C-R circuit, if L is increased by 25% and C is decreased by 20%, then the resonant frequency will
(A) Increase by 10% (B) Decrease by 10%
(C) Remain unchanged (D) Increase by 2.5%

23. Which of the following statements is correct for L-C-R series combination in the condition of resonance
(A) Resistance is zero
(B) Impedance is zero
(C) Reactance is zero
(D) Resistance, impedance and reactance all are zero

24. An AC circuit resonates at a frequency of 10 kHz. If its frequency is increased to 11 kHz, then:
(A) Impedance will increase by 1.1 times
(B) Impedance will remain remain unchanged
(C) Impedance will increase and become inductive
(D) Impedance will increase and become capacitive

25. In an AC circuit emf and current are $E = 5 \cos \omega t$ volt and $I = 2 \sin \omega t$ ampere respectively. The average power dissipated in this circuit will be
(A) 10 W (B) 2.5 W
(C) 5 W (D) Zero

26. A choke coil of negligible resistance carries 5 mA current when it is operated at 220 V. The loss of power in the choke coil is
(A) Zero (B) 11 W
(C) 44×10^3 W (D) 1.1 W

27. In an AC circuit, a resistance of 3 Ω, an inductance coil of 4 Ω and a condenser of 8 Ω are connected in series with an AC source of 50 V (rms). The average power loss in the circuit will be
(A) 600 W (B) 500 W
(C) 400 W (D) 300 W

28. Two bulbs of 500 W and 300 W work on 200 V rms the ratio of their resistances will be
(A) 25:9 (B) 3:5
(C) 9:25 (D) 5:9

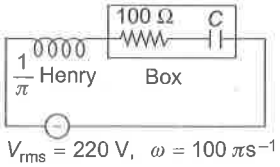
29. Three bulbs of 40, 60 and 100 W are connected in series with the source of 200 V. Then which of the bulb will be glowing the most
(A) 100 W
(B) 60 W
(C) 40 W
(D) All are glowing equally

30. Alternating current can not be measured by direct current meters, because
(A) alternating current can not pass through an ammeter
(B) the average value of current for complete cycle is zero
(C) some amount of alternating current is destroyed in the ammeter.
(D) None of these

31. In *LR* circuit the AC source has voltage 220 V. If the potential difference across the inductance is 176 V, the p.d. across the resistance will be:
(A) 44 V (B) 396 V
(C) 132 V (D) $\sqrt{(250 \times 176)}$ V

32. If the power factor changes from $\frac{1}{2}$ to $\frac{1}{4}$ then what is the increase in impedance in AC?
(A) 20% (B) 50%
(C) 25% (D) 100%

33. In the circuit, as shown in the figure, if the value of rms current is 2.2 A, the power factor of the box is



(A) $\frac{1}{\sqrt{2}}$ (B) 1
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

34. The power in AC circuit is given by $P = E_{rms} I_{rms} \cos \phi$. The value of $\cos \phi$ in series LCR circuit at resonance is
(A) zero (B) 1
(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

35. In AC circuit when AC ammeter is connected it reads i current if a student uses DC ammeter in place of AC ammeter the reading in the DC ammeter will be:
(A) $\frac{i}{\sqrt{2}}$ (B) $\sqrt{2}i$
(C) $0.637i$ (D) Zero

36. An AC current is given by $I = I_0 + I_1 \sin wt$ then its rms value will be
(A) $\sqrt{I_0^2 + 0.5I_1^2}$ (B) $\sqrt{I_0^2 + 0.5I_0^2}$
(C) 0 (D) $\frac{I_0}{\sqrt{2}}$

JEE Advanced

Single Correct Option Type Questions

1. In an AC circuit voltage V and current i are given by

$$V = 100 \sin 100t \text{ V}$$

$$i = 100 \sin\left(100\ t + \frac{\pi}{3}\right) \text{mA}.$$

The power dissipated in the circuit is:
(A) 10^4 W (B) 10 W
(C) 2.5 W (D) 5 W

2. The potential difference V and current i flowing through an AC circuit are given by $V = 5 \cos \omega t$ volt, $i = 2 \sin \omega t$ amp. the power dissipated in the circuit.

308

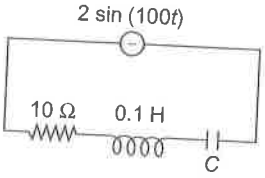
(A) 0 W (B) 10 W
(C) 5 W (D) 2.5 W

3. An AC circuit consists of an inductor of inductances 0.5 H and a capacitor of capacitance 8 μF in series. The current in the circuit is maximum when the angular frequency of an AC source is
(A) 500 Hz (B) 2×10^5 Hz
(C) 4000 Hz (D) 5000 Hz

4. The rms value of an AC of 50 Hz is 10 A. The time taken by an alternating current in reaching from zero to maximum value and the peak value will be;
(A) 2×10^{-2} sec and 14.14 A
(B) 1×10^{-2} sec and 7.07 A
(C) 5×10^{-3} sec and 7.07 A
(D) 5×10^{-3} sec and 14.14 A

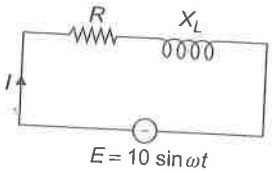
5. A voltage of peak value 283 V varying frequency is applied to a series L-C-R combination in which $R = 3\,\Omega$; $L = 25$ mH and $C = 400\ \mu$F. Then, the frequency (in Hz) of the source at which maximum power is dissipated in the above, is
(A) 51.5 (B) 50.7
(C) 51.1 (D) 50.3

6. The power factor of the circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(A) 400 μF (B) 300 μF
(C) 500 μF (D) 200 μF

7. An AC-circuit having supply voltage E consists of a resistor of resistance 3 Ω and an inductor of reactance 4 Ω as shown in the figure. The voltage across the inductor at $t = \pi/\omega$ is

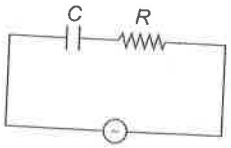


(A) 2 V (B) 10 V
(C) zero (D) 4.8 V

8. When 100 V DC is applied across a solenoid a current of 1 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are:
(A) 100 Ω, 0.93 H (B) 200 Ω, 1.0 H
(C) 10 Ω, 0.86 H (D) 200 Ω, 0.55 H

9. An inductive circuit contains resistance of 10 Ω and an inductance of 2.0 H. If an AC voltage of 120 V and frequency 60 Hz is applied to this circuit, the current would be nearly:
(A) 0.8 A (B) 0.48 A
(C) 0.16 A (D) 0.32 A

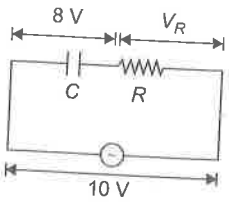
10. In the circuit shown if the emf of source at an instant is 5 V, the potential difference across capacitor at the same instant is 4 V. The potential difference across R at that instant may be



(A) 3 V (B) 9 V
(C) $\frac{3}{\sqrt{2}}v$ (D) none

11. Let $f = 50$ Hz, and $C = 100\ \mu$F in an AC circuit containing a capacitor only. If the peak value of the current in the circuit is 1.57 A at $t = 0$. The expression for the instantaneous voltage across the capacitor will be
(A) $E = 50 \sin\left(100\pi t - \frac{\pi}{2}\right)$
(B) $E = 100 \sin(50\pi t)$
(C) $E = 50 \sin 100\pi t$
(D) $E = 50 \sin\left(100\pi t + \frac{\pi}{2}\right)$

12. In a series CR circuit shown in figure, the applied voltage is 10 V and the voltage across capacitor is found to be 8 V. Then the voltage across R, and the phase difference between current and the applied voltage will respectively be



(A) 6 V, $\tan^{-1}\left(\frac{4}{3}\right)$ (B) 3 V, $\tan^{-1}\left(\frac{3}{4}\right)$
(C) 6 V, $\tan^{-1}\left(\frac{5}{3}\right)$ (D) none

13. The phase difference between current and voltage in an AC circuit is $\frac{\pi}{4}$ radian. If the frequency of AC is 50 Hz, then the phase difference is equivalent to the time difference:

(A) 0.78 s (B) 15.7 ms

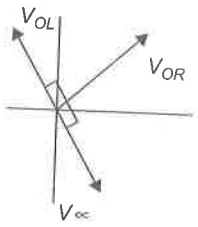
(C) 0.25 s (D) 2.5 ms

14. The given figure represents the phasor diagram of a series LCR circuit connected to an ac source. At the instant t' when the source voltage is given by $V = V_0 \cos \omega t$, the current in the circuit will be

$V_{OL} = 3$ V

$V_{OR} = \sqrt{3}$ V

$V_{OL} = 2$ V



(A) $I = I_0 \cos\left(\omega t' + \frac{\pi}{6}\right)$ (B) $I = I_0 \cos\left(\omega t' - \frac{\pi}{6}\right)$

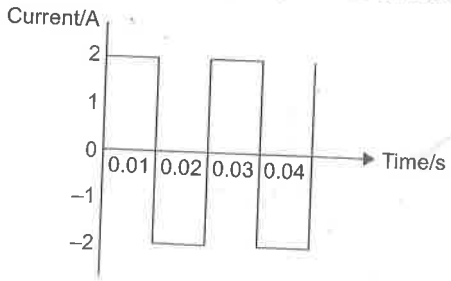
(C) $I = I_0 \cos\left(\omega t' + \frac{\pi}{3}\right)$ (D) $I = I_0 \cos\left(\omega t' - \frac{\pi}{3}\right)$

15. Power factor of an L-R series circuit is 0.6 and that of a C-R series circuit is 0.5. If the element (L, C, and R) of the two circuits are joined in series the power factor of this circuit is found to be 1. The ratio of the resistance in the L-R circuit to the resistance in the C-R circuit is

(A) $\frac{6}{5}$ (B) $\frac{5}{6}$

(C) $\frac{4}{3\sqrt{3}}$ (D) $\frac{3\sqrt{3}}{4}$

16. The direct current which Would give the same heating effect in an equal constant resistance as the current shown in figure, i.e., the rms current, is



(A) zero (B) $\sqrt{2}$ A

(C) 2 A (D) $2\sqrt{2}$ A

17. The effective value of current $i = 2 \sin 100\pi t + 2 \sin (100\pi t + 30°)$ is

(A) $\sqrt{2}$ A (B) $2\sqrt{2+\sqrt{3}}$

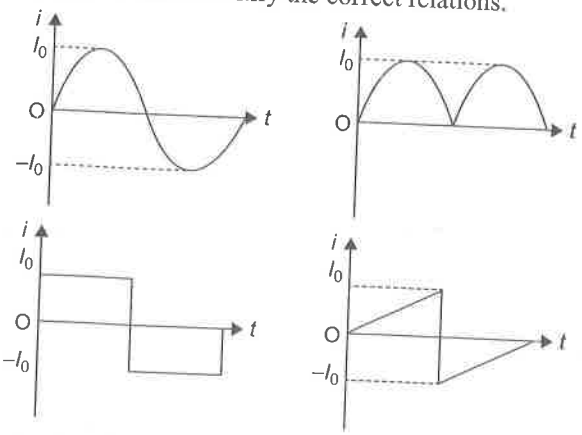
(C) 4 (D) None

18. In an L-R circuit, the value of L is $\left[\frac{0.4}{\pi}\right]$ henry and the value R is 30 Ω. If in the circuit, an alternating emf of 200 V rms value at 50 cycles per second is connected, the impedance of the circuit and current will be

(A) 11.4 Ω, 17.5 A (B) 30.7 Ω, 6.5 A

(C) 40.4 Ω, 5 A (D) 50 Ω, 4 A.

19. If I_1, I_2, I_3 and I_4 are the respective rms values of the time varying currents as shown in the four cases I, II, III and IV. Then identify the correct relations.



(A) $I_1 = I_2 = I_3 = I_4$ (B) $I_3 > I_1 = I_2 > I_4$

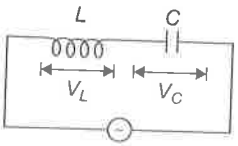
(C) $I_3 > I_4 > I_2 = I_1$ (D) $I_3 > I_2 > I_1 > I_4$

20. In series LR circuit $X_L = 3R$. Now a capacitor with $X_C = R$ is added in series. Ratio of new to old power factor is

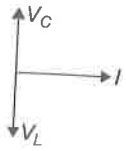
(A) 1 (B) 2

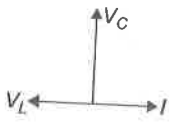
(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

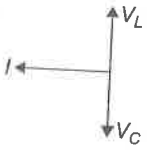
21. The current I, potential difference V_L across the inductor and potential difference V_C across the capacitor in circuit as shown in the figure are best represented vectorically as

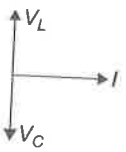


310

(A) 

(B) 

(C) 

(D) 

22. A series LCR circuit is tuned to resonance. The impedance of the circuit now is

(A) $\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]^{1/2}$

(B) $\left[R^2+(\omega L)^2+\left(\frac{1}{\omega C}\right)^2\right]^{1/2}$

(C) $\left[R^2+\left(\frac{1}{\omega C}-\omega L\right)^2\right]^{1/2}$

(D) R

23. A capacitor $C=2\ \mu F$ and an inductor with $L=10$ H and coil resistance 5 Ω are in series in a circuit. When an alternating current of r.m.s. value 2 A flows in the circuit, the average power in watts in the circuit is

(A) 100 (B) 50
(C) 20 (D) 10

Common Data for Questions 24–26: A student in a lab took a coil and connected it to a 12 V DC source. He measures the steady state current in the circuit to be 4 A. He then replaced the 12 V DC source by a 12 V, ($\omega=50$ rad/s) AC source and observes that the reading in the AC ammeter is 2.4 A. He then decides to connect a 2500 μF capacitor in series with the coil and calculate the average power developed in the circuit. Further he also decides to study the variation in current in the circuit (with the capacitor and the battery in series).

Based on the readings taken by the student answer the following questions.

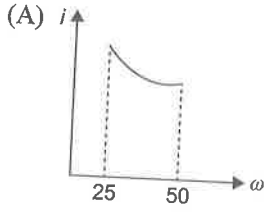
24. The value of resistance of the coil calculated by the student is

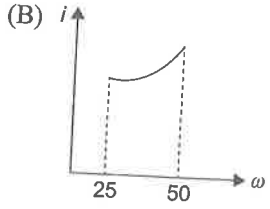
(A) 3 Ω (B) 4 Ω
(C) 5 Ω (D) 8 Ω

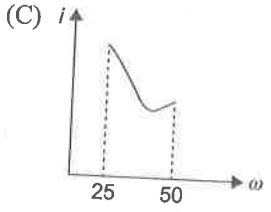
25. The power developed in the circuit when the capacitor of 2500 μF is connected in series with the coil is

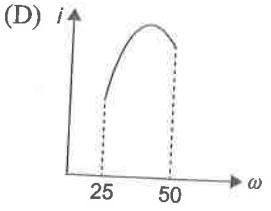
(A) 28.8 W (B) 23.04 W
(C) 17.28 W (D) 9.6 W

26. Which of the following graph roughly matches the variations of current in the circuit (with the coil and capacitor connected in the series) when the angular frequency is decreased from 50 rad/s to 25 rad/s?

(A) 

(B) 

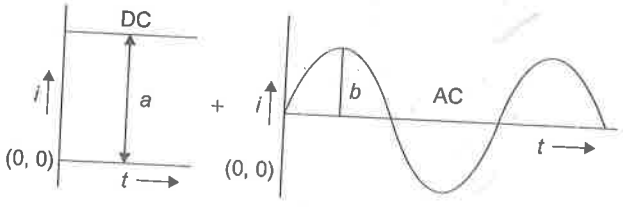
(C) 

(D) 

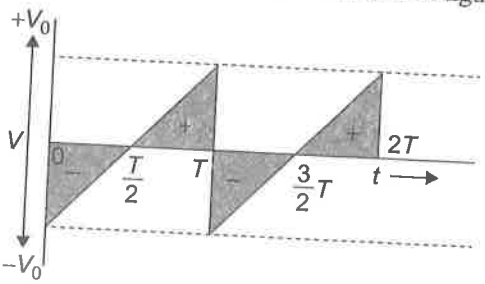
JEE Advanced

Level I

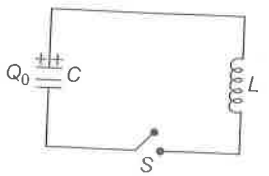
1. If a direct current of value 'a' ampere is superimposed on an alternating current $I=b\sin\omega t$ flowing through a wire, what is the effective (rms) value of the resulting current in the circuit?



2. Find the average for the saw-tooth voltage of peak value V_0 from $t=0$ to $t=2T$ as shown in figure.



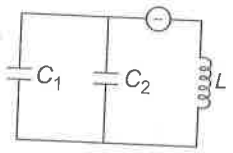
3. A circuit has a coil of resistance 50 Ω and inductance $\frac{3}{\pi}$ henry. It is connected in series with a condenser of $\frac{40}{\pi}\mu F$ and AC supply voltage of 200 V and 50 cycles/sec. Calculate
   (i) the impedance of the circuit.
   (ii) the p.d. across inductance coil and condenser.
4. A series circuit consists of a resistance, inductance and capacitance. The applied voltage and the current at any instant are given by $E = 141.4\cos(5000t - 10°)$ and $I = 5\cos(5000t - 370°)$ The inductance is 0.01 henry. Calculate the value of ***capacitance and resistance***.
5. A circuit takes A current of 3 a at a power factor of 0.6 lagging when connected to a 115 V – 50 Hz supply. Another circuit takes a current of 5 A at a power factor of 0.07 leading when connected to the same supply. If the two circuits are connected in series across a 230 V, 50 Hz supply. Calculate
   (a) the current
   (b) the power consumed and
   (c) the power factor.
6. In a L-R decay circuit, the initial current at $t = 0$ is I. Find the total charge that has flown through the resistor till the energy in the inductor has reduced to one-fourth its initial value.
7. A charged ring of mass $m = 50$ gm, charge 2 coulomb and radius $R = 2$ m is placed on a smooth horizontal surface. A magnetic field varying with time at a rate of $(0.2\,t)$ Tesla/sec is applied on to the ring in a direction normal to the surface of ring. Find the angular speed attained in a time $t_1 = 10$ sec.
8. A capacitor C with a charge Q_0 is connected across an inductor through a switch S. If at $t = 0$, the switch is closed, then find the instantaneous charge q on the upper plate of capacitor.



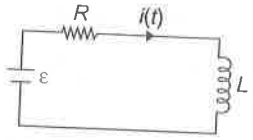
9. A coil of resistance 300 Ω and inductance 1.0 henry is connected across an alternating voltage of frequency $\frac{300}{2\pi}$ Hz. Calculate the phase difference between the voltage and current in the circuit.
10. Find the value of an inductance which should be connected in series with a capacitor of 5 μF, a resistance of 10 Ω and an AC source of 50 Hz so that the power factor of the circuit is unity.
11. In an L-R series A. C circuit the potential difference across an inductance and resistance joined in series are respectively 12 V and 16 V. Find the total potential difference across the circuit.
12. When 100 V DC is applied across a coil, a current of one ampere flows through it, when 100 V AC of 50 Hz is applied to the same coil, only 0.5 A flows. Calculate the resistance and inductance of the coil.
13. A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp.

Level II

1. Consider the circuit shown in figure. The oscillating source of emf deliver a sinusoidal emf of amplitude e_{max} and frequency ω to the inductor L and two capacitors C_1 to C_2. Find the maximum instantaneous current in each capacitor.



2. Suppose the emf of the battery, the circuit shown varies with time t so the current is given by $i(t) = 3 + 5t$, where i is in amperes and t is in seconds. Take $R = 4\ \Omega$, $L = 6$ H and find an expression for the battery emf as function of time.



3. A current of 4 A flows in a coil when connected to a 12 V DC source. If the same coil is connected to a 12 V, 50 rad/s AC source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.
4. An LCR series circuit with 100 Ω resistance is connected to an AC source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current

312

leads the voltage by 60°. Calculate the current and the power dissipated in the LCR circuit.

5. A box P and a coil Q are connected in series with an AC source of variable frequency. The emf of source at 10 V. Box P contains a capacitance of 1 μF in series with a resistance of 32 Ω coil Q has a self-inductance 4.9 mH and a resistance of 68 Ω series. The frequency is adjusted so that the maximum current flows in P and Q. Find the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively.

6. A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 red s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45°?

Previous Years' Questions

JEE Main

1. The power factor of an AC circuit having resistance R and inductance L (connected in series) and an angular velocity ω is **[AIEEE-2002]**

(A) $\frac{R}{\omega L}$ (B) $\frac{R}{(R^2+\omega^2L^2)^{1/2}}$

(C) $\frac{\omega L}{R}$ (D) $\frac{R}{(R^2-\omega^2L^2)^{1/2}}$

2. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is **[AIEEE-2002]**

(A) 4 A (B) 2 A
(C) 6 A (D) 10 A

3. Alternating current can not be measured by DC ammeter because **[AIEEE-2004]**

(A) AC cannot pass though DC ammeter
(B) AC changes direction
(C) average value of current for complete cycle is zero
(D) DC ammeter will get damaged

4. In an L-C-R series AC circuit, the voltage across each of the components. L, C and R is 50 V. The voltage across the LC combination will be **[AIEEE-2004]**

(A) 50 V (B) $50\sqrt{2}$ V
(C) 100 V (D) zero

5. In an L-C-R circuit, capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to **[AIEEE-2004]**

(A) $4L$ (B) $2L$
(C) $\frac{L}{2}$ (D) $\frac{L}{4}$

6. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[AIEEE-2005]**

(A) 0.05 s (B) 0.1 s
(C) 0.15 s (D) 0.3 s

7. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[AIEEE-2005]**

(A) 4 μF (B) 8 μF
(C) 1 μF (D) 2 μF

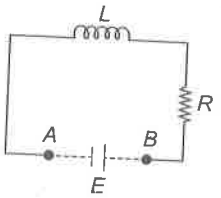
8. A circuit has a resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be **[AIEEE-2005]**

(A) 0.8 (B) 0.4
(C) 1.25 (D) 0.125

9. The phase difference between the alternating current and emf is $\frac{\pi}{2}$ Which of the following cannot be the constituent of the circuit? **[AIEEE-2005]**

(A) C alone (B) R, L
(C) L, C (D) L alone

10. An inductor ($L = 100$ mH), a resistor ($R = 100$ Ω) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is **[AIEEE-2006]**



(A) 1/eA (B) eA
(C) 0.1 A (D) 1 A

11. In a series resonant LCR circuit, the voltage across R is 100 V and $R = 1$ $k\Omega$ with $C = 1$ μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is **[AIEEE-2006]**

(A) 2.5×10^{-2} V (B) 40 V
(C) 250 V (D) 4×10^{-3} V

12. In an AC circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by. **[AIEEE-2007]**

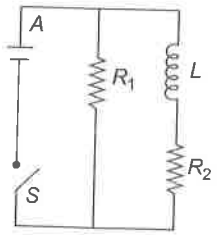
(A) $p = \frac{E_0 I_0}{\sqrt{2}}$ (B) $P =$ zero

(C) $p = \frac{E_0 I_0}{2}$ (D) $P = \sqrt{2} E_0 I_0$

13. An ideal coil of 10 H is connected in series with a resistance of and a battery of 5 V. 2 s after the connection is made, the current flowing (in ampere) in the circuit is **[AIEEE-2007]**

(A) $(1 - e)$ (B) e
(C) e^{-1} (D) $(1 - e^{-1})$

14. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 4\ \Omega$ and $R_2 = 2\ \Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[AIEEE-2009]**



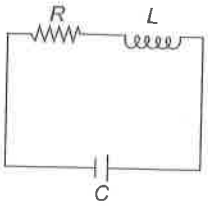
(A) $6e^{-5t}$ V (B) $\frac{12}{t} e^{-3t}$ V

(C) $6(1 - e^{-t/0.2})$ V (D) $12e^{-5t}$ V

15. In a series L-C-R circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the L-C-R circuit is **[AIEEE-2010]**

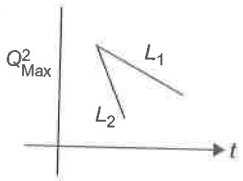
(A) 305 W (B) 210 W
(C) zero (D) 242 W

16. An L-C-R circuit is equivalent to a damped pendulum. In an L-C-R circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below: **[JEE Main-2015]**

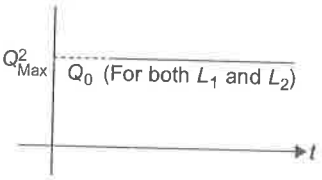


If a student plots graphs of the square of maximum charge (Q^2_{max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (Plots are schematic and not drawn to scale)

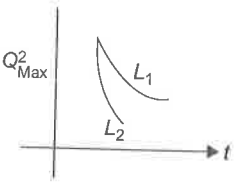
(A)



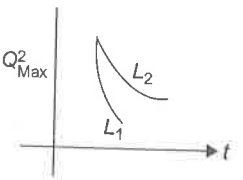
(B)



(C)



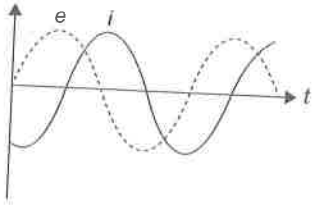
(D)



314

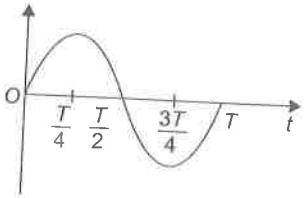
JEE Advanced

1. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\frac{\pi}{4}$ ahead, as shown in the diagram. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements: **[JEE-2003]**



(A) $R = 1\ k\Omega$, $C = 10\ \mu F$ (B) $R = 1\ k\Omega$, $C = 1\ \mu F$
(C) $R = 1\ k\Omega$, $L = 10$ H (D) $R = 1\ k\Omega$, $L = 1$ H

2. In an LR series circuit, a sinusoidal voltage $V = V_0 \sin \omega t$ is applied. It is given that $L = 35$ mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\frac{\omega}{2\pi} = 50$ Hz and $\pi = \frac{22}{7}$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph. **[JEE-2004]**



3. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero) when ω is increased **[JEE-2009]**
(A) the bulb glows dimmer
(B) the bulb glows brighter
(C) total impedance of the circuit is unchanged
(D) total impedance of the circuit increases

4. Your are given many resistance, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II.** When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2. (indicated in circuits) are related as shown in **Column I.** Match the two **[JEE-2009]**

Column I

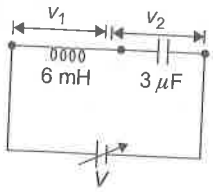
(A) $I \neq 0$, V_1 is proportional to I

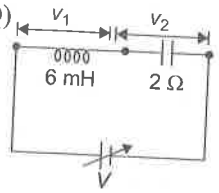
(B) $I \neq 0$, $V_2 > V_1$

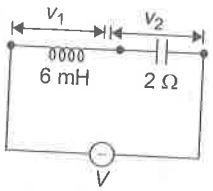
(C) $V_1 = 0$, $V_2 = V$

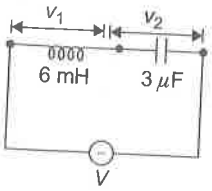
(D) $I \neq 0$, V_2 is proportional to I

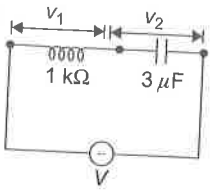
Column II

(P)


(Q)


(R)


(S)


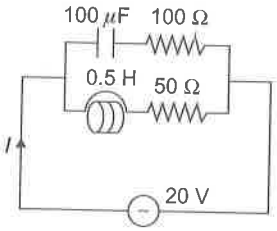
(T)


5. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true? **[JEE-2011]**

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$
(C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

6. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **[JEE-2011]**

7. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are) **[JEE-2012]**



(A) The current through the circuit, I is 0.3 A.
(B) The current through the circuit, I is $0.3\sqrt{2}$ A.
(C) The voltage across 100 Ω resistor = $10\sqrt{2}$ V.
(D) The voltage across 50 Ω resistor = 10 V.

ANSWER KEYS

Exercises

JEE Main

1. D	**2.** B	**3.** B	**4.** C	**5.** D	**6.** A	**7.** B	**8.** B	**9.** C	**10.** A
11. B	**12.** C	**13.** D	**14.** C	**15.** A	**16.** D	**17.** B	**18.** B	**19.** B	**20.** A
21. A	**22.** C	**23.** C	**24.** C	**25.** D	**26.** A	**27.** D	**28.** B	**29.** C	**30.** B
31. C	**32.** D	**33.** A	**34.** B	**35.** D	**36.** A				

JEE Advanced

1. C	**2.** A	**3.** A	**4.** D	**5.** D	**6.** C	**7.** D	**8.** D	**9.** C	**10.** B
11. C	**12.** A	**13.** D	**14.** B	**15.** D	**16.** C	**17.** D	**18.** D	**19.** B	**20.** D
21. D	**22.** D	**23.** C	**24.** A	**25.** C	**26.** B				

JEE Advanced

Level I

1. $I_{\text{eff}} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$ **2.** Zero **3.** $z = 50\sqrt{2}\,\Omega$, $V_C = 500\sqrt{2}$ V and $V_L = 600\sqrt{2}$ V **4.** $4\,\mu$F

5. **(a)** 5.5 A **(b)** 1.188 kw **(c)** 0.939 lag **6.** $LI/2R$ **7.** 200 rad/sec **8.** $q = Q_0 \sin\left(\sqrt{\frac{1}{LC}}t + \frac{\pi}{2}\right)$

9. $\frac{\pi}{4}$ **10.** $\frac{20}{\pi^2} \cong 2$ H **11.** 20 V **12.** $100\,\Omega, \frac{\sqrt{3}}{\pi}$ H **13.** C = 9.2 μF

Level II

1. $C_2 = \dfrac{\varepsilon_{\max}}{\left(1+\frac{C_1}{C_2}\right)\left(\omega L - \frac{1}{\omega(C_1+C_2)}\right)}$; $C_1 = \dfrac{\varepsilon_{\max}}{\frac{C_2}{C_1}\left(1+\frac{C_1}{C_2}\right)\left(\omega L - \frac{1}{\omega(C_1+C_2)}\right)}$ **2.** $42 + 20t$ V

3. 0.08 H, 17.28 W **4.** 2 A, 400 W **5.** 77 Ω, 97.6 Ω, 7.7 V, 9.76 V **6.** 0.2 mH, $\frac{1}{32}\,\mu$F, 8×10^5 rad/s

316

Previous Years' Questions

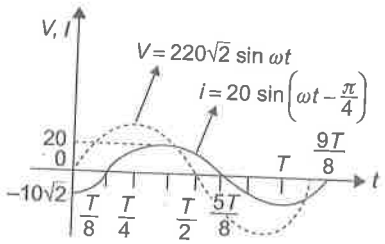
JEE Main

1. B	**2.** B	**3.** C	**4.** D	**5.** C	**6.** B	**7.** C	**8.** A	**9.** C	**10.** A
11. C	**12.** B	**13.** D	**14.** D	**15.** D	**16.** C				

JEE Advanced

1. A

2. 20 A, $\frac{\pi}{4}$, $\therefore$ Steady state current $i = 20\sin\pi\left(100t - \frac{1}{4}\right)$



3. B

4. **(A)** → R, S, T; **(B)** → Q, R, S, T; **(C)** → P, Q; **(D)** → Q, R, S, T

5. B, C **6.** 4 **7.** A, C

MOTION EDU-TAB

Facility for Every Classroom student

Learn: Start your learning with video lectures of total 300 + hours of Physics, Chemistry & Maths providing you in depth knowledge about every topic focusing towards concepts for very strong base.

Revise: After learning revise each topic with exercises and see how they are solved by experts in videos. This will improve your focus & chances of selection in competitive exams.

Examine: Examine yourself on competitive exam pattern based testing platform with more than 300 tests with AIR. After every test a detailed report is provided with analysis based on time taken, comparison with toppers & toughness level of questions.

MOTION ANDROID APPS

One click solution for students who wish to know information about various competitive exams, Boards and colleges

1. Notifies imminent exams
2. List of Engineering and Medical entrance exams and Top Colleges across India with required info.
3. List of national and international Olympiads with previous year papers.
4. Boards previous year papers available at one click.
5. Quick Info of other graduation courses and colleges of India
6. Rank Predictor, Branch predictor for engineering students.

For download search All About Exam Motion on Google Play Store

Student Performance Report Structure