JEE Class Companion Physics

for JEE Main and Advanced

Module-5

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First Impression

Published by Pearson India Education Services Pvt. Ltd, CIN: U72200TN2005PTC057128, formerly known as TutorVista Global Pvt. Ltd, licensee of Pearson Education in South Asia.

Head Office: A-8(A), 7th Floor, Knowledge Boulevard, Sector 62, Noida 201 309, Uttar Pradesh, India.
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Compositor: Macrotex Solutions.
Printed in India by Rahul Print O Pack

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Chapter 1

Fluid

FLUID

Fluid mechanics deals with the behaviour of fluids at rest and in motion. A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be.

Thus, fluids comprise the liquid and gas (or vapor) phase of the physical forms in which matter exists.

Density (ρ): Mass of unit volume, Called density

Density at a point of liquid described by

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

density is a positive scalar quantity.

SI unit = Kg/m^3

CGS unit = gm/cm^3

Dimension = $[ML^{-3}]$

Relative Density: It is the ratio of density of given liquid to the density of pure water at 4°C

$$RD = \frac{\text{Density of given liquid}}{\text{Density of pure water at } 4°C}$$

Relative density or specific gravity is unit less, dimensionless. It is a positive scalar physical Quantity Value of RD is same in SI and CGS system due to dimensionless/unit less

Specific Gravity: It is the ratio of weight of given liquid to the weight of pure water at 4°C

Specific Gravity

$$= \frac{\text{Weight of given liquid}}{\text{Weight of pure water at } 4°C (9.81\ kN/m^3)}$$

$$= \frac{\rho_\ell \times g}{\rho_w \times g} = \frac{\rho_\ell}{\rho_w} = \text{Relative density of liquid}$$

i.e., than specific gravity of a liquid is approximately equal to the relative density. For calculation they can be interchange.

PRESSURE IN A FLUID

When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.

While the fluid as a whole is at rest, the molecules that makes up the fluid are in motion, the force exerted by the fluid is due to molecules colliding with their surroundings.

If we think of an imaginary surface within the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface, otherwise the surface would accelerate and the fluid would not remain at rest.

Consider a small surface of area dA centered on a point on the fluid, the normal force exerted by the fluid on each side is $dF_\perp$. The pressure P is defined at that point as the normal force per unit area, i.e.,

$$P = \frac{dF_\perp}{dA}$$

If the pressure is the same at all points of a finite plane surface with area A, then

$$P = \frac{F_\perp}{A}$$

where $F_\perp$ is the normal force on one side of the surface. The SI unit of pressure is pascal, where

$$1 \text{ pascal} = 1 \text{ Pa} = 1.0 \text{ N/m}^2$$

One unit used principally in meteorology is the Bar which is equal to 10^5 Pa.

$$1 \text{ Bar} = 10^5 \text{ Pa}$$

Atmospheric Pressure (P_0)

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value) is 1.013×10^5 Pa. Thus

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

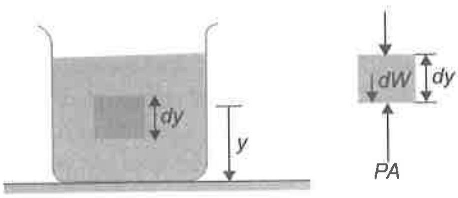
Notes

Fluid pressure acts perpendicular to any surface in the fluid no matter how that surface is oriented. Hence, pressure has no intrinsic direction of its own, its a **scalar**. By contrast, force is a vector with a definite direction.

Variation in Pressure with Depth

If the weight of the fluid can be neglected, the pressure in a fluid is the same throughout its volume. But often the fluid's weight is not negligible and under such condition pressure increases with increasing depth below the surface.

Let us now derive a general relation between the pressure P at any point in a fluid at rest and the elevation y of that point. We will assume that the density ρ and the acceleration due to gravity g are the same throughout the fluid. If the fluid is in equilibrium, every volume element is in equilibrium.



Consider a thin element of fluid with height dy. The bottom and top surfaces each have area A, and they are at elevations y and $y + dy$ above some reference level where $y = 0$. The weight of the fluid element is

$$dW = \text{(volume) (density) } (g) = (Ady)(\rho)(g)$$

or $$dW = \rho gAdy$$

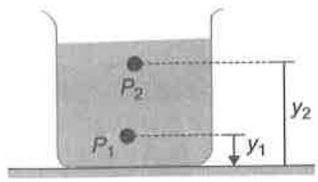
What are the other forces in y-direction of this fluid element? Call the pressure at the bottom surface P, the total y component of upward force is PA. The pressure at the top surface is $P + dP$ and the total y-component of downward force on the top surface is $(P + dP)\,A$. The fluid element is in equilibrium, so the total y-component of force including the weight and the forces at the bottom and top surfaces must be zero.

$$\Sigma F_y = 0$$

$\therefore$ $$PA - (P + dP)\,A - \rho gAdy = 0$$

or $$\frac{dP}{dy} = -\rho g \qquad (1)$$

This equation shows that when y increases, P decreases, i.e., as we move upward in the fluid, pressure decreases.



If P_1 and P_2 be the pressures at elevations y_1 and y_2 and if ρ and g are constant, then integrating Eq. (1), we get

$$\int_{P_1}^{P_2} dP = -\rho g \int_{y_1}^{y_2} dy$$

or $$P_2 - P_1 = -\rho g(y_2 - y_1) \qquad (2)$$

It's often convenient to express Eq. (2) in terms of the depth below the surface of a fluid. Take point 1 at depth h below the surface of fluid and let P represents pressure at this point. Take point 2 at the surface of the fluid, where the pressure is P_0 (subscript zero for zero depth). The depth of point 1 below the surface is,

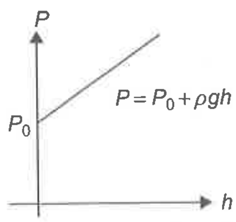
$$h = y_2 - y_1$$

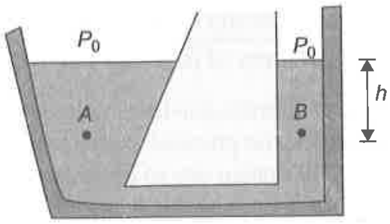
and Eq. (2) becomes

$$P_0 - P = -\rho g\,(y_2 - y_1) = -\rho gh$$

$\therefore$ $$P = P_0 + \rho gh \qquad (3)$$

Thus, pressure increases linearly with depth, if ρ and g are uniform, a graph between P and h is shown below.



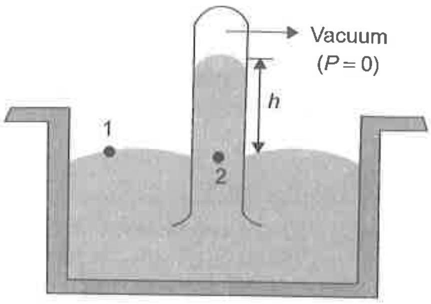


$P_A = P_B = P_0 + \rho gh$

Further, the pressure is the same at any two points at the same level in the fluid. The shape of the container does not matter.

Barometer

It is a device used to measure atmospheric pressure. In principle, any liquid can be used to fill the barometer, but mercury is the substance of choice because its great density makes possible an instrument of reasonable size.



$$P_1 = P_2$$

Here, P_1 = atmospheric pressure (P_0)
and $P_2 = 0 + \rho gh = \rho gh$
Here, ρ = density of mercury
$P_0 = \rho gh$

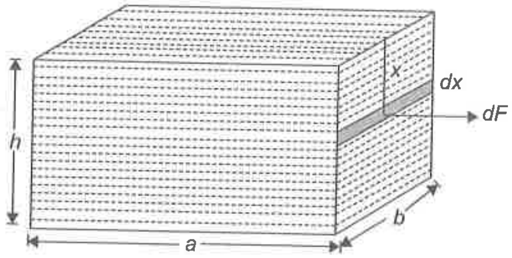
Thus, the mercury barometer reads the atmospheric pressure (P_0) directly from the height of the mercury column.

For example if the height of mercury in a barometer is 760 mm, then atmospheric pressure will be,

$$P_0 = \rho gh = (13.6 \times 10^3)(9.8)(0.760) = 1.01 \times 10^5 \text{ N/m}^2$$

Force on Side Wall of Vessel

Force on the side wall of the vessel can not be directly determined as at different depths pressures are different. To find this we consider a strip of width dx at a depth x from the surface of the liquid as shown in figure, and on this strip the force due to the liquid is given as:



$$dF = x\rho g \times bdx$$

This force is acting in the direction normal to the side wall.

Net force can be evaluated by integrating equation

$$F = \int dF = \int_0^h x\rho gbdx \qquad F = \frac{\rho gbh^2}{2} \tag{2.4}$$

Average Pressure on Side Wall

The absolute pressure on the side wall cannot be evaluated because at different depths on this wall pressure is different. The average pressure on the wall can be given as:

$$\langle p \rangle_{av} = \frac{F}{bh} = \frac{1}{2}\frac{\rho gbh^2}{bh} = \frac{1}{2}\rho gh \tag{2.5}$$

Equation (2.5) shows that the average pressure on side vertical wall is half of the net pressure at the bottom of the vessel.

Torque on the Side Wall due to Fluid Pressure

As shown in figure, due to the force dF, the side wall experiences a torque about the bottom edge of the side which is given as

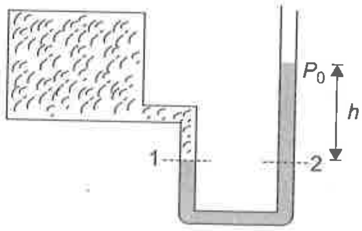
$$d\tau = dF \times (h - x) = x\rho gb\, dx\, (h - x)$$

This net torque is $\tau = \int d\tau = \int_0^h \rho gb(hx - x^2)dx$

$$= \rho gb\left[\frac{h^3}{2} - \frac{h^3}{3}\right] = \frac{1}{6}\rho gbh^3$$

Manometer

It is a device used to measure the pressure of a gas inside a container. The U-shaped tube often contains mercury.



$$P_1 = P_2$$

Here, P_1 = pressure of the gas in the container (P)
and P_2 = atmospheric pressure (P_0) + ρgh
$\therefore$ $P = P_0 + h\rho g$

This can also be written as

$$P - P_0 = \text{gauge pressure} = h\rho g$$

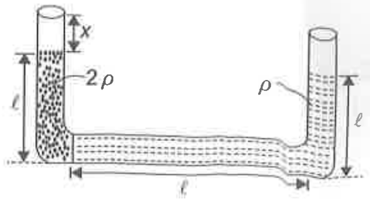
Here, ρ is the density of the liquid used in U-tube

Thus by measuring h we can find absolute (or gauge) pressure in the vessel.

SOLVED EXAMPLES

EXAMPLE 1

Two liquid which do not react chemically are placed in a bent tube as shown in figure. Find out the displacement of the liquid in equilibrium position.



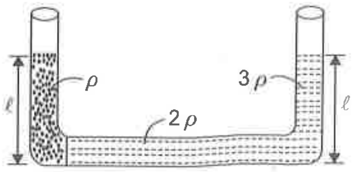
SOLUTION

The pressure at the interface must be same, calculated via either tube. Since both tube all open to the atmosphere, we must have.

$$2\rho g(\ell - x) = \rho g(\ell + x) \quad \Rightarrow \quad x = \ell/3$$

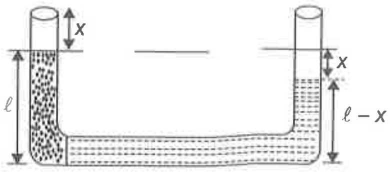
EXAMPLE 2

Three liquid which do not react chemically are placed in a bent tube as shown in figure (initially) then fluid out the displacement of the liquid in equilibrium position.



SOLUTION

Let us assume that level of liquid having density 3ρ displaced below by x as shown in figure below.



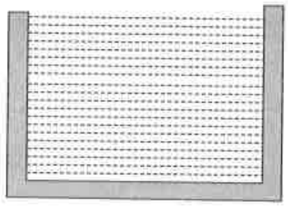
$$\Rightarrow \qquad \rho\ell g + 2\rho g x = 3\rho(\ell - x)g$$

$$x = 2\ell/5$$

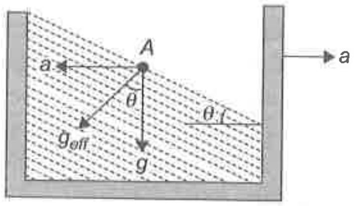
Pressure Distribution in an Accelerated Frame

We've already discussed that when a liquid is filled in a container, generally its free surface remains horizontal as shown in figure (a) as for its equilibrium its free surface must be normal to gravity i.e., horizontal. Due to the same reason we said that pressure at every point of a liquid layer parallel to its free surface remains constant. Similar situation exist when liquid is in an accelerated frame as shown in figure (b). Due to acceleration of container, liquid filled in it experiences a pseudo force relative to container and due to this the free surface of liquid which normal to the gravity now is filled as

$$\theta = \tan^{-1}\left(\frac{a}{g}\right) \tag{2.22}$$



(a)

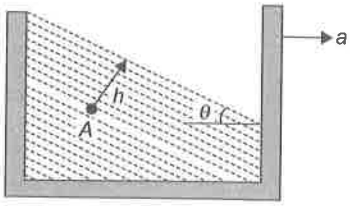


(b)

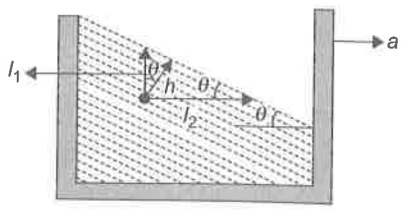
Now from equilibrium of liquid we can state that pressure at every point in a liquid layer parallel to the free surface (which is not horizontal), remains same for example if we find pressure at a point A in the accelerated container as shown in figure (a) is given as

$$P_A = P_0 + h_\rho\sqrt{a^2 + g^2} \tag{2.23}$$

Where h is the depth of the point A below the free surface of liquid along effective gravity and P_0 is the atmospheric pressure acting on free surface of the liquid.



(c)



(d)

The pressure at point A can also obtained in an another way as shown in figure (b). If l_1 and l_2 are the vertical and horizontal distances of point A from the surface of liquid then pressure at point A can also be given as

$$P_A = P_0 + l_1 \rho g = P_0 + l_2 \rho a \quad (2.24)$$

Here $l_1 \rho g$ is the pressure at A due to the vertical height of liquid above A and according to Pascal's Law pressure at A is given as

$$P_A = P_0 + l_1 \rho g \quad (2.25)$$

Here we can write l_1 as $l_1 = h \sec\theta = \dfrac{h\sqrt{a^2 + g^2}}{g}$

or from equation (2.25) $P_A = P_0 + h\rho\sqrt{a^2 + g^2}$

Similarly if we consider the horizontal distance of point A from free surface of liquid, which is l_2 then due to pseudo acceleration of container the pressure at point A is given as

$$P_A = P_0 + l_2 \rho a \quad (2.26)$$

Here l_2 is given as $l_2 = h \operatorname{cosec}\theta = \dfrac{h\sqrt{g^2 + a^2}}{a}$

From equation (2.24), we have

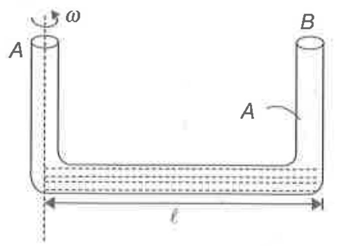
$$P_A = P_0 + h\rho\sqrt{g^2 + a^2}$$

Here students should note that while evaluating pressure at point A from vertical direction we haven't mentioned any thing about pseudo acceleration as along vertical length l_1, due to pseudo acceleration at every point pressure must be constant similarly in horizontal direction at every point due to gravity pressure remains constant.

SOLVED EXAMPLES

EXAMPLE 3

Figure shows a tube in which liquid is filled at the level. It is now rotated at an angular frequency w about an axis passing through arm A find out pressure difference at the liquid interfaces.



SOLUTION

To solve the problem we take a small mass dm from the axis at 'a' distance x in displaced condition.

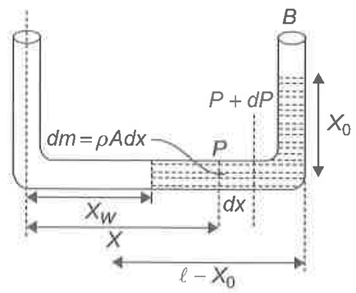
$$\text{Net inward force} = (P + dP)A - PA = AdP$$

This force is balanced by centripetal force in equilibrium

$$\Rightarrow \quad AdP = dm\,\omega^2 x = \rho A dx\,\omega^2 x$$

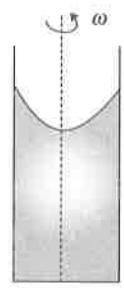
$$\Rightarrow \quad \int dP = \int_{x_0}^{\ell} \rho\omega^2 x\,dx$$

$$\Delta P = \rho\omega^2 \int_{x_0}^{\ell} x\,dx = x_0 \rho g$$



EXAMPLE 4

A liquid of density ρ is in a bucket that spins with angular velocity ω as shown in figure. Show that the pressure at a radial distance r from the axis is

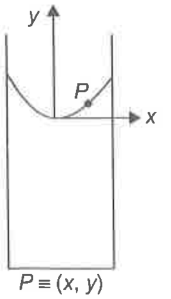


$$P = P_0 + \frac{\rho\omega^2 r^2}{2}$$

where P_0 is the atmospheric pressure.

SOLUTION

Consider a fluid particle P of mass m at coordinates (x, y). From a non-inertial rotating frame of reference two forces are acting on it.



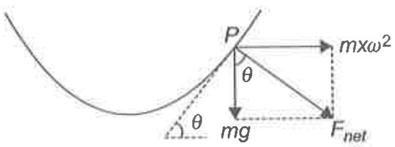
(i) pseudo force ($mx\omega^2$)
(ii) weight (mg)
in the directions shown in figure.
Net force on it should be perpendicular to the free surface (in equilibrium). Hence.

$$\tan\theta = \frac{mx\omega^2}{mg} = \frac{x\omega^2}{g} \quad \text{or} \quad \frac{dy}{dx} = \frac{x\omega^2}{g}$$

$$\therefore \quad \int_0^y dy = \int_0^x \frac{x\omega^2}{g}.dx$$

$$\therefore \quad y = \frac{x^2\omega^2}{2g}$$

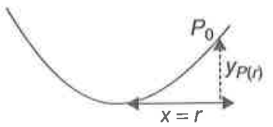
This is the equation of the free surface of the liquid, which is a parabola.



At $x = r$, $\quad y = \dfrac{r^2\omega^2}{2g}$

$$\therefore \quad P(r) = P_0 + \rho g y$$

$$\text{or} \quad P(r) = P_0 + \frac{\rho\omega^2 r^2}{2}$$

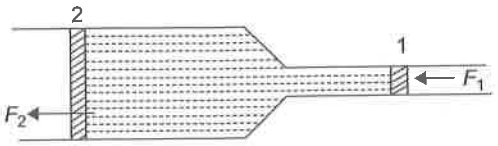


PASCAL'S PRINCIPLE

Some times while dealing with the problems of fluid it is desirable to know the pressure at one point is pressure at any other point in a fluid is known. For such types of calculations Pascal's Law is used extensively in dealing of static fluids. It is stated as

'The pressure applied at one point in an enclosed fluid is transmitted uniformly to every part of the fluid and to the walls of the container.'

One more example can be considered better to explain the concept of Pascal's Principle. Consider the situation shown in figure, a tube having two different cross section S_1 and S_2, with pistons of same cross sections fitted at the two ends.



If an external force F_1 is applied to the piston 1, it creates a pressure $p_1 = F_1/S_1$ on the liquid enclosed. As the whole liquid is at the same level, everywhere the pressure in the liquid is increased by p_1. The force applied by the liquid on the piston 2 can be given as $F_2 = p_2 \times S_2$, and as the two pistons are at same level $p_2 = p_1$. Thus

$$F_2 = p_2 \times S_2$$

$$F_2 = \frac{F_1}{S_1} \times S_2 \qquad (2.21)$$

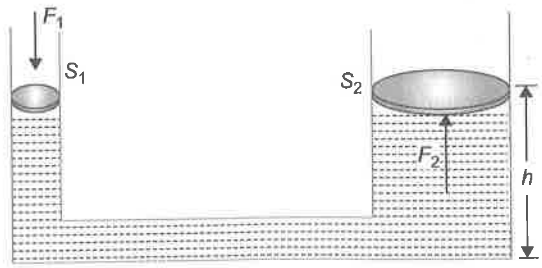
Equation (2.21) shows that by using such a system the force can be amplified by an amount equal to the ratio of the cross section of the two pistons. This is the principle of hydraulic press, we'll encounter in next few pages.

The Hydraulic Lift

Figure shows how Pascal's principle can be made the basis for a hydraulic lift. In operation, let an external force of magnitude F_1 be exerted downward on the left input piston, whose area is S_1. It result a force F_2 which will act on piston 2 by the incompressible liquid in the device.

Here $\quad F_2 = p_2 \times S_2$

And $\quad p_2 = p_B - \rho g h$



Where p_B is the pressure on the bottom of the device which can be given as:

$$p_B = p_1 + \rho g h$$

Thus $\quad p_2 = p_1$ and $F_2 = p_1 S_2$

or $$F_2 = F_1 \times \frac{S_2}{S_1}$$

If $S_2 >> S_1 \Rightarrow F_2 >> F_1$

ARCHIMEDES'S PRINCIPLE

If a heavy object is immersed in water, it seems to weight less than when it is in air. This is because the water exerts an upward force called **buoyant force**. It is equal to the weight of the fluid displaced by the body.

A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

This result is known as **Archimedes' principle**.
Thus, the magnitude of buoyant force (F) is given by,

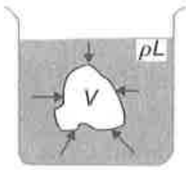
$$F = V_i \rho_L g$$

Here, V_i = immersed volume of solid ρ_L = density of liquid and g = acceleration due to gravity

Notes

Point of Application of buoyant force is centre of liquid displaced.

Proof Consider an arbitrarily shaped body of volume V placed in a container filled with a fluid of density ρ_L. The body is shown completely immersed, but complete immersion is not essential to the proof. To begin with, imagine the situation before the body was immersed. The region now occupied by the body was filled with fluid, whose weight was $V\rho_L g$. Because the fluid as a whole was in hydrostatic equilibrium, the net upwards force (due to difference in pressure at different depths) on the fluid in that region was equal to the weight of the fluid occupying that region.



Now, consider what happens when the body has displaced the fluid. The pressure at every point on the surface of the body is unchanged from the value at the same location when the body was not present. This is because the pressure at any point depends only on the depth of that point below the fluid surface. Hence, the net force exerted by the surrounding fluid on the body is exactly the same as the exerted on the region before the body was present. But we know the latter to be, the weight of the displaced fluid. Hence, this must also be the buoyant force exerted on the body. Archimedes' principle is thus, proved.

SOLVED EXAMPLES

EXAMPLE 5

Beaker circular cross-section of radius 4 cm is filled with mercury upto a height of 10 cm. Find the force exerted by the mercury on the bottom of the beaker. The atmospheric pressure = 10^5 N/m^2. Density of mercury = 13600 kg/m^3. Take g = 10 m/s^2

SOLUTION

The pressure at the surface = atmospheric pressure = 10^5 N/m^2.
The pressure at the bottom = 10^5 N/m^2 + $h\rho g$

$$= 10^5 \text{ N/m}^2 + (0.1 \text{ m})\left(13600 \frac{\text{kg}}{\text{m}^3}\right)\left(10 \frac{\text{m}}{\text{s}^2}\right)$$

$$= 10^5 \text{ N/m}^2 + 13600 \text{ N/m}^2 = 1.136 \times 10^5 \text{ N/m}^2$$

The force exerted by the mercury on the bottom

$$= (1.136 \times 10^5 \text{ N/m}^2) \times (3.14 \times 0.04 \text{ m} \times 004 \text{ m})$$

$$= 571 \text{ N}$$

EXAMPLE 6

A cubical block of iron 5 cm on each side is floating on mercury in a vessel.

(i) What is the height of the block above mercury level?
(ii) What is poured in the vessel until it just covers the iron block. What is the height of water column.

Density of mercury = 13.6 gm/cm^3
Density of iron 7.2 gm/cm^3

SOLUTION

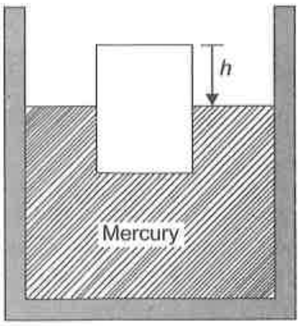
Case I: Suppose h be the height of cubical block of iron above mercury.
Volume of iron block = 5 × 5 × 5 = 125 cm^3
Mass of iron block = 125 × 7.2 = 900 gm
Volume of mercury displaced by the block = 5 × 5 × (5 – h) cm^3
Mass of mercury displaced = 5 × 5 (5 – h) × 13.6 gm
By the law floatation,

weight of mercury displaced = weight of iron block

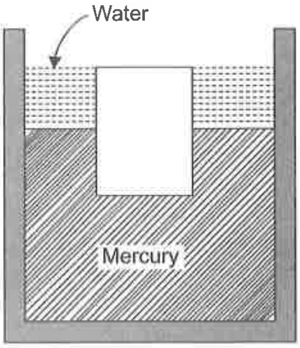
$$5 \times 5\,(5 - h) \times 13.6 = 900$$

or $$(5 - h) = \frac{900}{25 \times 13.6} = 2.65 \Rightarrow h = 5 - 2.65 = 2.35 \text{ cm}$$

Case II: Suppose in this case height of iron block in water be x. The height of iron block in mercury will be $(5 - x)$ cm.



(a)



(b)

Mass of the water displaced $= 5 \times 5 \times (x) \times 1$

Mass of mercury displaced $= 5 \times 5 \times (5 - x) \times 13.6$

So, weight of water displaced + weight of mercury displaced = weight of iron block

or $\quad 5 \times 5 \times x \times 1 + 5 \times 5 \times (5 - x) \times 13.6 = 900$

or $\quad x = (5 - x) \times 13.6 = 36$

$\quad x = 2.54$ cm

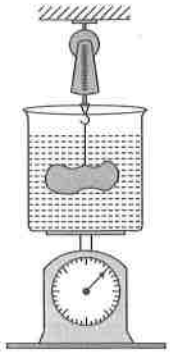
EXAMPLE 7

A tank containing water is placed on spring balanced. A stone of weight w is hung and lowered into the water without touching the sides and the bottom of the tank. Explain how the reading will change.

SOLUTION

The situation is shown in figure. Make free-body diagrams of the bodies separately and consider their equilibrium. Like all other forces, buoyancy is also exerted equally on the two bodies in contact. Hence it the water exerts a buoyant force, say, B on the stone upward, the stone exerts the same force on the water downward. The forces acting on the 'water + container' system are: W, weight of the system downward, B, buoyant force of the stone downward, and the force R of the spring in the upward direction. For equilibrium

$$R = W + B$$



Thus the reading of the spring scale will increase by an amount equal to the weight of the liquid displaced, that is, by an amount equal to the buoyant force.

EXAMPLE 8

A cylindrical vessel containing a liquid is closed by a smooth piston of mass m as shown in figure. The area of cross-section of the piston is A. If the atmospheric pressure is P_0, find the pressure of the liquid just below the prism.

SOLUTION

Let the pressure of the liquid just below the piston be P. The forces acting on the piston are

(a) its weight, mg (downward)

(b) force due to the air above it, P_0A (downward)

(c) force due to the liquid below it, PA (upward)

If the piston is in equilibrium

$$PA = P_0A + mg \quad \text{or} \quad P = P_0 + \frac{mg}{A}$$

EXAMPLE 9

A rubber ball of mass m and radius r is submerged in water to a depth h released. What height will the ball jump up to above the surface of the water? Neglect the resistance of water and air.

SOLUTION

Let the ball go up by x above the level of water.

Let us now consider energy conservation between the initial and final positions. In both the positions kinetic energy of the body is zero. The potential energy in the first position with reference to the water level is $-mgh$ plus the work done by an external agent against the buoyant force which is $\left(\frac{4}{3}\pi r^3 \rho g\right)h$, where ρ is the density of the water

or $$-mgh+\left(\frac{4}{3}\pi r^3 \rho g\right)h = mgx$$

$\Rightarrow$ $$x = \frac{(4/3)\pi r^3 \rho - m}{m} \times h$$

EXAMPLE 10

A cube of wood supporting a 200 g mass just floats in water. When the mass is removed, the cube rises by 2 cm. What is the size of the cube?

SOLUTION

If, l = side of cube, h = height of cube above water and ρ = density of wood.

Mass of the cube = $l^3\rho$

Volume of cube in water = $l^2(l - h)$

Volume of the displaced water = $l^2(l - h)$

As the tube is floating

weight of cube + weight of wood = weight of liquid displaced

or $$l^3\rho + 200 = l^2(l - h) \quad (2.10)$$

After the removal of 200 gm mass, the cube rises 2 cm.

$$= l^2 \times \{l - (h + 2)\}$$

Volume of cube in water

or $$l^2 \times \{l - (h + 2)\} = l^3\rho \quad (2.11)$$

Substituting the value of $l^3\rho$ from equation (2.11) in equation (2.10), we get

$$l^2 \times \{l - (h + 2)\} + 200 = l^2(l - h)$$

or $$l^3 - l^2h - 2l^2 + 200 = l^3 - l^2$$

$$2l^2 = 200 \quad \Rightarrow \quad l = 10 \text{ cm}$$

EXAMPLE 11

A boat floating in water tank is carrying a number of large stones. If the stones were unloaded into water, what will happen to water level? Given the reason in brief.

SOLUTION

Suppose W and w be the weights of the boat and stones respectively.

First, we consider that the boat is floating. It will displaced $(W + w) \times 1$ cm^3 of water.

Thus displaced water = $(W + w)$ cm^3 [As density of water = 1 gm/cm^3]

Secondly, we consider that the stones are unloaded into water.

Now the boat displaces only $W \times 1$ cm^3 of water. If ρ be the density of stones, the volume of water displaced by stones

$$= w/\rho \text{ cm}^3$$

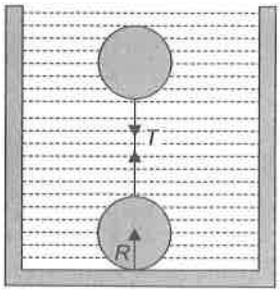
As $\rho > 1$, hence $w/\rho < w$, thus we have

Now $$(W + w/\rho) < (W + w)$$

This shows that the volume of water displaced in the second case is less than the volume of water displaced in the first case. Hence the level of water will come down.

EXAMPLE 12

Two solid uniform spheres each of radius 5 cm are connected by a light string and totally immersed in a tank of water. If the specific gravities of the sphere are 0.5 and 2, find the tension in the string and the contact force between the bottom of tank and the heavier sphere.



SOLUTION

The situation is shown in figure

Let the volume of each sphere be V m^3 and density of water be ρ kg/m^3.

Upward thrust on heavier sphere = $V\rho g$

Weight of the heavier sphere = $V \times 2 \times \rho g$

For heavier sphere,

$$T + R + V\rho g = V \times 2 \times \rho g \quad (2.12)$$

where R is the reaction at the bottom.

Similarly for lighter sphere

$$T + V \times 0.5 \times \rho g = V\rho g \quad (2.13)$$

Subtracting equation (2.13) from equation (2.12), we have

$$R + 0.5\, V\rho g = V\rho g \quad (2.14)$$

or $$R = 0.5\, V\rho g \quad (2.15)$$

From equation (2.13) $T = 0.5\, V\rho g$

$$= 0.5 \times \left(\frac{4}{3} \times 3.14 \times 5^3 \times 10^6\right) \times 1000 \times 9.8 = 2.565 \text{ N}$$

$$R = 2.565 \text{ N}$$

Similarly

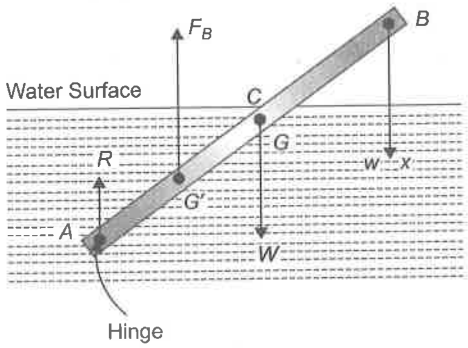
A rod of length 6 m has a mass of 12 kg. If it is hinged at one end at a distance of 3 m below a water surface,

(i) What weight must be attached to other end of the rod so that 5 m of the rod is submerged?

(ii) Find the magnitude and direction of the force exerted by the hinge on the rod. The specific gravity of the material of the rod is 0.5.

SOLUTION

Let AC be the submerged part of the rod AB hinged at A as shown in figure. G is the centre of gravity of the rod and G' is the centre of buoyancy through which force of buoyancy F_B acts vertically upwards.



Since the rod is uniform,

The weight of part AC will be $\frac{5}{6} \times 12 = 10$ kg

[Because $AB = 6$ m and $AC = 5$ m]

The buoyance force on rod at G' is

$$F_B = \frac{10}{0.5} = 20 \text{ kg weight}$$

(i) Let x be weight attached at the end B. Balancing torques about A, we get

$$W \times AG + x \times AB = F_B \times AG'$$

$$12 + 3 + x \times 6 = 20 \times (5/2) \quad [\text{As } AG' = 5/2]$$

Solving we get $x = 2.33$ kg

(ii) Suppose R be the upward reaction on the hinge, then in equilibrium position, we have

$$W + x = F_B + R$$

or

$$R = W + x - F_B$$

$$= 12 + 2.33 - 20 = -5.67 \text{ kg. wt.}$$

Negative sign shows that the reaction at the hinge is acting in the downward direction. The magnitude of the reaction is 5.67 kg. wt. ■

EXAMPLE 13

A cylinder of area 300 cm^2 and length 10 cm made of material of specific gravity 0.8 is floated in water with its axis vertical. It is then pushed downward, so as to be just immersed. Calculate the work done by the agent who pushes the cylinder into the water.

SOLUTION

Weight of the cylinder $= (300 \times 10^{-4}) \times (10 \times 10^{-2}) \times 800$ kgf $= 2.4$ kgf

Let x be the length of the cylinder inside the water. Then by the law of floatation

$$2.4 \text{ g} = (300 \times 10^{-4} x) \times 1000 \text{ g}$$

or

$$x = 0.08 \text{ m}$$

When completely immersed,

F_b(buoyant force) $= (300 \times 10^{-4} \times 0.1) \times 1000 \times g = 3$ gN

Thus to immerse the cylinder inside the water the external agent has to push it by 0.02 m against average upward thrust.

Increase in upward thrust $= 3\text{g} - 2.4 \text{ g} = 0.6$ gN

Since this increase in upthrust takes place gradually from 0 to 0.6 g, we may take the average upthrust against which work is done as 0.3 gN.

$$\text{work done} = 0.3 \text{ g} \times 0.02 = 0.0588 \text{ J}$$ ■

EXAMPLE 14

A piece of an alloy of mass 96 gm is composed of two metals whose specific gravities are 11.4 and 7.4. If the weight of the alloy is 86 gm in water, find the mass of each metal in the alloy.

SOLUTION

Suppose the mass of the metal of specific gravity 11.4 be m and the mass of the second metal of specific gravity 7.4 will be $(96 - m)$

$$\text{Volume of first metal} = \frac{m}{11.4} \text{ cm}^3$$

$$\text{Volume of second metal} = \frac{96 - m}{7.4} \text{ cm}^3$$

$$\text{Total volume} = \frac{m}{11.4} + \frac{96 - m}{7.4}$$

$$\text{Buoyancy force in water} = \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right) \text{gm weight}$$

$$\text{Apparent wt. in water} = 96 - \left[\left(\frac{m}{11.4}\right) + \frac{(96 - m)}{7.4}\right]$$

According to the given problem,

$$96 - \left[\left(\frac{m}{11.4}\right) + \frac{(96 - m)}{7.4}\right] = 86$$

or

$$\frac{m}{11.4} + \frac{(96 - m)}{7.4} = 10$$

Solving we get, $m = 62.7$ gm

Thus mass of second metal is $= 96 - 62.7 = 33.3$ gm ■

Notes

Assumptions of Ideal Fluid

1. Fluid is incompressible: density of fluid remain constant through out the fluid.
2. Fluid is non-viscous: fluid friction is absent
3. Doesn't show rotational effect: If we release any body in the flowing section there it will not rotate about its C.O.M.
4. Stream line flow: velocity of fluid at any particular point remains constant with time It may vary with position.

EQUATION OF CONTINUITY

This equation defines the steady flow of fluid in a tube. It states that if flow of a fluid is a steady then the mass of fluid entering per second at one end is equal to the mass of fluid leaving per second at the other end.

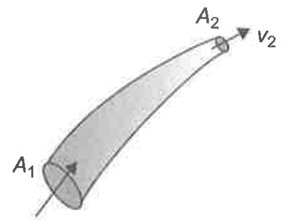


Figure shown a section of a tube in which at the ends, the cross sectional area are A_1 and A_2 and the velocity of the fluid are V_1 and v_2 respectively.

According to the equation of continuity, if flow is steady mass of fluid entering at end A_1 per second = mass of fluid leaving the end A_2 per second.

$$\frac{dV}{dt} = A_1 v_1$$

Hence mass entering per second at A_1 is $= A_1 v_1 \rho$

Similarly mass leaving per second at A_2 is $= A_2 v_2 \rho$

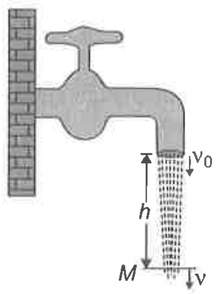
According to the definition of steady flow

$$A_1 v_1 \rho = A_2 v_2 \rho \quad \text{or} \quad A_1 v_1 = A_2 v_2$$

Equation above in known as equation of continuity, which gives that in steady flow the product of cross-section and the speed of fluid everywhere remains constant.

Freely Falling Liquid

When liquid falls freely under gravity, the area of cross section of the stream continuously decreases, as the velocity increases.



For example, we consider water coming out from a tap, as shown in figure. Let its speed near the mouth of tap is v_0 and at a depth h it is v, then we have

$$v^2 = v_0^2 + 2gh$$

If cross section of tap is A then according to the equation of continuity, the cross section at point M (say a) can be given as

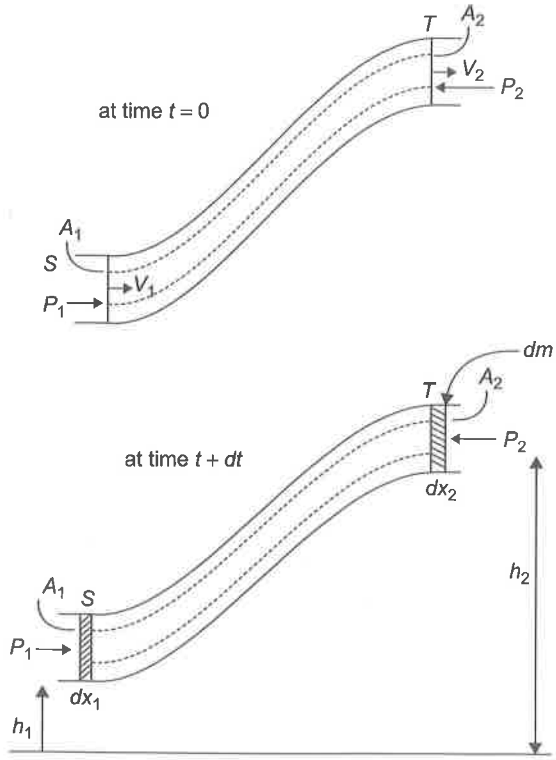
$$v_0 A = a\sqrt{v_0^2 + 2gh} \quad \text{or} \quad a = \frac{v_0 A}{\sqrt{v_0^2 + 2gh}}$$

BERNOULLI'S EQUATION

It relates the variables describing the steady laminer of liquid. It is based on energy conservation.

Assumptions

The fluid is incompressible, non-viscous, non rotational and streamline flow.



Mass of the fluid entering from side S

$$dm_1 = \rho A_1 dx_1 = \rho dv_1$$

The work done in this displacement dx_1 at point S is

$$W_{P_1} = F_1 dx_1 = P_1 A_1 dx_1$$

$$W_{P_1} = P_1 dV_1 \quad \{\because \quad A_1 dx_1 = dV_1\}$$

At the same time the amount of fluid moves out of the tube at point T is

$$dm_2 = \rho dV_2$$

According to equation of continuity

$$\frac{dm_1}{dt} = \frac{dm_2}{dt} \quad \Rightarrow \quad dV_1 = dV_2 = dv$$

The work done in the displacement of dm_2 mass at point T

$$W_{P_2} = P_2 dV_2$$

Now applying work energy theorem.

$$W_{P_1} + W_{P_2} = (K_f + U_f) - (K_i + U_i)$$

$$\Rightarrow \quad P_1 dV - P_2 dV = \left(\frac{1}{2}\rho dV v_2^2 + \rho dV g h_2\right) - \left(\frac{1}{2}\rho dV v_1^2 + \rho dV g h_1\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 + \rho g h_2 - \frac{1}{2}\rho V_1^2 + \rho g h_1$$

$$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V^2$$

$$\Rightarrow \quad P + \rho g h + \frac{1}{2}\rho V^2 = \text{constant}$$

$$\frac{P}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$

where $\frac{P}{\rho g}$ = pressure head

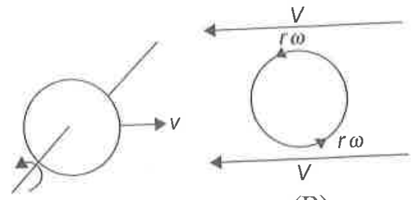
h = Gravitational head

$\frac{1}{2}\frac{v^2}{g}$ = volume head

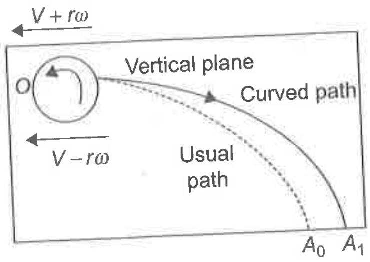
Application of Bernoulli's principle

Magnus effect: When a spinning ball is thrown, it deviates from its usual path in flight. This effect is called Magnus effect and plays an important role in tennis, cricket and soccer, etc., as by applying appropriate spin the moving ball can be made to curve in any desired direction.

If a ball is moving from left to right and also spinning about a horizontal axis perpendicular to the direction of motion as shown in figure, then relative to the ball air will be moving from right to left.



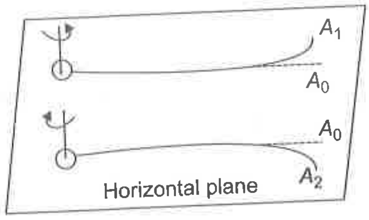
(A) (B)



(C)

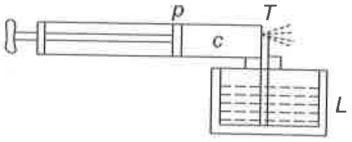
The resultant velocity of air above the ball will be $(V + r\omega)$ while below it $(V - r\omega)$ (shown figure). So in accordance with Bernoulli's principle pressure above the ball will be less than below it. Due to this difference of pressure an upward force will act on the ball and hence the ball will deviate from its usual path OA_0 and will hit the ground at A_1 following the path OA_1 (figure shown) i.e., if a ball is thrown with back spin, the pitch will curve less sharply prolonging the flight.

Similarly if the spin is clockwise, i.e., the ball i thrown with top-spin, the force due to pressure differenc will act in the direction of gravity and so the pitch wi curve more sharply shortening the flight.

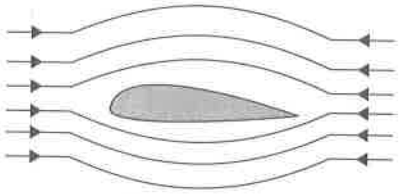


Furthermore, if the ball is spinning about a vertical ax the curving will be sideways as shown in figure. produci the so called out swing or in swing.

Action of Atomiser: The action of aspirator, carburett paint-gun, scent-spray or insect-sprayer is based on B noulli's principle. In all these by means of motion of a p ton P in a cylinder C high speed air is passed over a t T dipped in liquid L to be sprayed. High speed air cre low pressure over the tube due to which liquid (paint, sc insecticide or petrol) rises in it and is then blown off in v small droplets with expelled air.



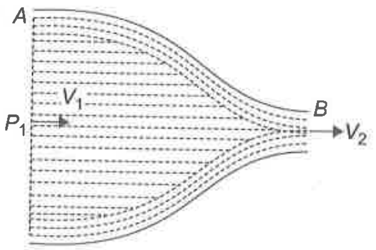
Working of Aeroplane: This is also based on Bernoulli's principle. The wings of the aeroplane are having tapering as shown in figure. Due to this specific shape of wings when the aeroplane runs, air passes at higher speed over it as compared to its lower surface. This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called 'dynamic lift' (= pressure difference × area of wing) acts on the plane. If this force becomes greater than the weight of the plane, the plane will rise up.



SOLVED EXAMPLES

EXAMPLE 15

If pressure and velocity at point A is P_1 and V_1 respectively and at point B is P_2, V_2 is the figure as shown. Comment on P_1 and P_2.



SOLUTION

From equation of continuity $A_1V_1 = A_2V_2$

here $\quad A_1 > A_2$

$\Rightarrow \quad V_1 < V_2 \quad (1)$

from Bernoulli's equation. We can write

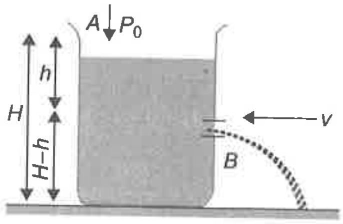
$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 =$$

after using equation (1) $P_1 > P_2$ ■

Torricelli's Law of Efflux (Velocity of efflux)

Cross-sectional Area of hole at A is greater than B. If water is come in tank with velocity v_A and going out side with velocity v_B then $A_1v_A = A_2v_B$

$\therefore \quad A_1 > A_2 \quad \because v_B >> v_A$



on applying Bernoulli theorem at A and B

$$P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2$$

$$P_A = P_B = P_0$$

and $\quad h_A - h_B = h$

$\Rightarrow \quad \rho g h = \frac{1}{2}\rho(v_B^2 - v_A^2) \quad [v_B >> v_A]$

$\Rightarrow \quad \rho g h = \frac{1}{2}\rho\, v^2 \quad [{v_B}^2 - {v_A}^2 = v^2]$

$\Rightarrow \quad v = \sqrt{2}\, gh$

Range (R)

Let us find the range R on the ground.

Considering the vertical motion of the liquid.

$$(H - h) = \frac{1}{2}gt^2 \quad \text{or} \quad t = \sqrt{\frac{2(H-h)}{g}}$$

Now, considering the horizontal motion,

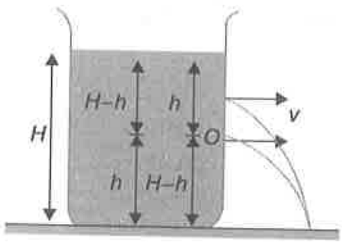
$$R = vt \quad \text{or} \quad R = (\sqrt{2gh})\left(\sqrt{\frac{2(H-h)}{g}}\right) \quad \text{or} \quad R = 2\sqrt{h(H-h)}$$

From the expression of R, following conclusions can be drawn,

(i) $R_h = R_{H-h}$

as $R_h = 2\sqrt{h(H-h)}$ and $R_{H-h} = 2\sqrt{(H-h)h}$

This can be shown as in Figure



(ii) R is maximum at $h = \frac{H}{2}$ and $R_{max} = H$.

Proof: $R^2 = 4\,(Hh - h^2)$

For R to be maximum. $\dfrac{dR^2}{dh} = 0$

or $\quad H - 2h = 0 \quad$ or $\quad h = \dfrac{H}{2}$

That is, R is maximum at $h = \dfrac{H}{2}$

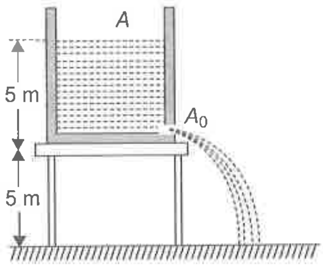
and $\quad R_{max} = 2\sqrt{\dfrac{H}{2}\left(H - \dfrac{H}{2}\right)} = H$ **Proved**

EXAMPLE 16

A cylindrical dark 1 m in radius rests on a platform 5 m high. Initially the tank is filled with water up to a height of 5 m. A plug whose area is 10^{-4} m^2 is removed from an orifice on the side of the tank at the bottom Calculate (a) initial speed with which the water flows from the orifice (b) initial speed with which the water strikes the ground and (c) time taken to empty the tank to half its original volume (d) Does the time to be emptied the tank depend upon the height of stand.

SOLUTION

The situation is shown in figure



(a) As speed of flow is given by

$$v_H = \sqrt{(2gh)}$$

or $$= \sqrt{2 \times 10 \times 5} \simeq 10 \text{ m/s}$$

(b) As initial vertical velocity of water is zero, so its vertical velocity when it hits the ground

$$v_V = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} \simeq 10 \text{ m/s}$$

So the initial speed with which water strikes the ground.

$$v = \sqrt{v_H^2 + v_V^2} = 10\sqrt{2} = 14.1 \text{ m/s}$$

(c) When the height of water level above the hole is y, velocity of flow will be $v = \sqrt{2gy}$ and so rate of flow

$$\frac{dV}{dt} = A_0 v = A_0\sqrt{2gy}$$

or $$-\,Ady = (\sqrt{2gy})A_0\,dt \qquad [\text{As } dV = -Ady]$$

Which on integration improper limits gives

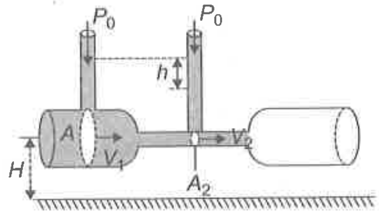
$$\int_H^0 \frac{Ady}{\sqrt{2gy}} = \int_0^t A_0 dt \quad \Rightarrow \quad t = \frac{A}{A_0}\sqrt{\frac{2}{g}}[\sqrt{H} - \sqrt{H'}]$$

$$\text{So } t = \frac{\pi \times 1^2}{10^{-4}}\sqrt{\frac{2}{10}}[\sqrt{5} - \sqrt{(5/2)}] = 9.2 \times 10^3 s \simeq 2.5 \text{ h}$$

(d) No, as expression of t is independent of height of stand. ■

Venturimeter

Figure shows a venturimeter used to measure flow speed in a pipe of non-uniform cross-section. We apply Bernoulli's equation to the wide (point 1) and narrow (point 2) parts of the pipe, with $h_1 = h_2$



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

From the continuity equation $v_2 = \dfrac{A_1 v_1}{A_2}$

Substituting and rearranging, we get

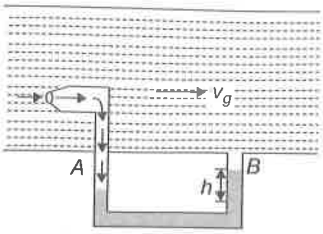
$$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

Because A_1 is greater than A_2, v_2 is greater than v_1 and hence the pressure P_2 is less than P_1. A net force to the right acceleration the fluid as it enters the narrow part of the tube (called throat) and a net force to the left slows as it leaves. The pressure difference is also equal to ρgh, where h is the difference in liquid level in the two tubes. Substituting in Eq. (1), we get

$$v_1 = \sqrt{\frac{2gh}{\left(\dfrac{A_1}{A_2}\right)^2 - 1}}$$

Pitot Tube

It is a device used to measure flow velocity of fluid. It is a U shaped tube which can be inserted in a tube or in the fluid flowing space as shown in figure shown. In the U tube a liquid which is immiscible with the fluid is filled upto a level C and the short opening M is placed in the fluid flowing space against the flow so that few of the fluid particles entered into the tube and exert a pressure on the liquid in limb A of U tube. Due to this the liquid level changes as shown in figure shown.



At end B fluid is freely flowing, which exert approximately negligible pressure on this liquid. The pressure difference at ends A and B can be given by measuring the liquid level difference h as

It is a gas, then $P_A - P_B = h\rho g$

It if the a liquid of density ρ, then

$$P_A - P_B = h(\rho - \rho_g)g$$

Now if we apply Bernoulli's equation at ends A and B we'l have

$$0 + 0 + P_A = \frac{1}{2}\rho v_g^2 + 0 + P_B \quad \text{or} \quad \frac{1}{2}\rho v_g^2 = P_A - P_B = h\rho g$$

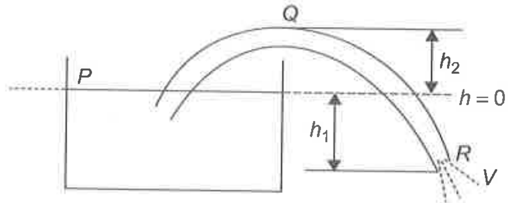
Now by using equations, we can evaluate the velocity v, with which the fluid is flowing.

Notes

Pitot tube is also used to measure velocity of aeroplanes with respect to wind. It can be mounted at the top surface of the plain and hence the velocity of wind can be measured with respect to plane.

Siphon

It is a pipe used to drain liquid at a lower height but the pipe initially rises and then comes down let velocity of outflow is v and the pipe is of uniform cross-section A. Applying bernoulli's equation between P (top of tank) and R (opening of pipe) we get



$$(P + \rho gh + \frac{1}{2}\rho v^2)_P = (P + \rho gh + \frac{1}{2}\rho v^2)_R$$

$$\Rightarrow \quad P + 0 + 0 = P_0 - \rho g h_1 + \frac{1}{2}\rho v^2$$

here velocity is considered zero at P since area of tank is very large compared to area of pipe

$$\Rightarrow \quad v = \sqrt{2gh}$$

Naturally for siphon to work $h_1 > 0$

Now as area of pipe is constant so by equation of continuity

as Av = constant so velocity of flow inside siphon is also constant between Q and R

$$(P + \rho gh + 1/2\, \rho v^2)_Q = (P + \rho gh + 1/2\, \rho v^2)_R$$

$\Rightarrow \quad P_Q + \rho g h_2 = P_0 - \rho g h_1$ (v is same)

$\Rightarrow \quad P_Q = P_0 - \rho g\,(h_1 + h_2) \quad$ as $P_Q = 0$

$\Rightarrow \quad P_0 \geq \rho g\,(h_1 + h_2)$

means $(h_1 + h_2)$ should not be more than $P/\rho g$ for siphon to work

EXERCISES

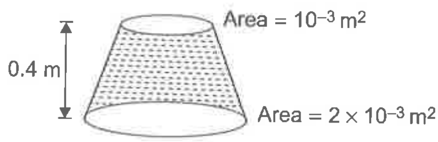
JEE Main

Static Fluid

1. A bucket contains water filled upto a height = 15 cm. The bucket is tied to a rope which is passed over a frictionless light pulley and the other end of the rope is tied to a weight of mass which is half of that of the (bucket + water). The water pressure above atmosphere pressure at the bottom is:

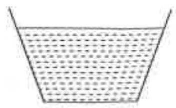
(A) 0.5 kPa (B) 1 kPa
(C) 5 kPa (D) None

2. A uniformly tapering vessel shown in Fig. is filled with liquid of density 900 kg/m^3. The force that acts on the base of the vessel due to liquid is (take g = 10 m/s^2)



(A) 3.6 N (B) 7.2 N
(C) 9.0 N (D) 12.6 N

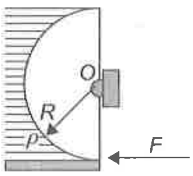
3. A liquid of mass 1 kg is filled in a flask as shown in figure. The force exerted by the flask on the liquid is ($g = 10$ m/s^2) [Neglect atmospheric pressure]



(A) 10 N (B) greater than 10 N
(C) less than 10 N (D) zero

4. A U-tube having horizontal arm of length 20 cm, has uniform cross-sectional area = 1 cm^2. It is filled with water of volume 60 cc. What volume of a liquid of density 4 g/cc should be poured from one side into the U-tube so that no water is left in the horizontal arm of the tube?
(A) 60 cc (B) 45 cc
(C) 50 cc (D) 35 cc

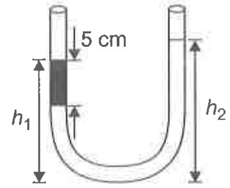
5. A light semi cylindrical gate of radius R is pivoted at its mid point O, of the diameter as shown in the figure holding liquid of density ρ. The force F required to prevent the rotation of the gate is equal to



(A) $2\pi R^3 \rho g$ (B) $2\rho g R^3 l$
(C) $\frac{2R^2 l \rho g}{3}$ (D) none of these

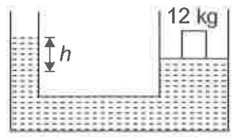
6. The pressure at the bottom of a tank of water is $3P$ where P is the atmospheric pressure. If the water is drawn out till the level of water is lowered by one fifth., the pressure at the bottom of the tank will now be
(A) $2P$ (B) $\left(\frac{13}{5}\right)P$
(C) $\left(\frac{8}{5}\right)P$ (D) $\left(\frac{4}{5}\right)P$

7. An open-ended U-tube of uniform cross-sectional area contains water (density 1.0 g/cm^3) standing initially 20 centimeters from the bottom in each arm. An immiscible liquid of density 4.0 g/cm^3 is added to one arm until a layer 5 centimeters high forms, as shown in the figure above. What is the ratio h_2/h_1 of the heights of the liquid in the two arms?



(A) $\frac{3}{1}$ (B) $\frac{5}{2}$
(C) $\frac{2}{1}$ (D) $\frac{3}{2}$

8. The area of cross-section of the wider tube shown in figure is 800 cm^2. If a mass of 12 kg is placed on the massless piston, the difference in heights h in the level of water in the two tubes is:



(A) 10 cm (B) 6 cm
(C) 15 cm (D) 2 cm

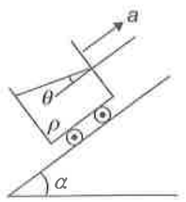
9. A body is just floating in a liquid (their densities are equal). If the body is slightly pressed down and released it will
(A) start oscillating
(B) sink to the bottom
(C) come back to the same position immediately
(D) come back to the same position slowly

10. Two stretched membranes of areas 2 and 3 m^2 are placed in a liquid at the same depth. The ratio of the pressure on them is
(A) 1:1 (B) 2:3
(C) $\sqrt{2}:\sqrt{3}$ (D) 22:32

11. An ice block floats in a liquid whose density is less than water. A part of block is outside the liquid. When whole of ice has melted, the liquid level will
(A) rise
(B) go down
(C) remain same
(D) first rise then go down

Accelerated Fluid

12. A fluid container is containing a liquid of density ρ is accelerating upward with acceleration a along the inclined place of inclination a as shown. Then the angle of inclination θ of free surface is:



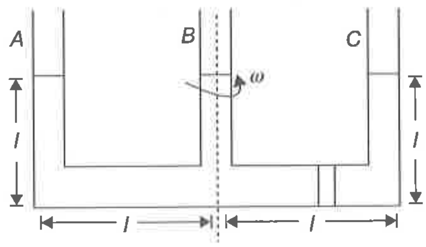
(A) $\tan^{-1}\left[\dfrac{g}{g\cos\alpha}\right]$

(B) $\tan^{-1}\left[\dfrac{a+g\sin\alpha}{g\cos\alpha}\right]$

(C) $\tan^{-1}\left[\dfrac{a-g\sin\alpha}{g(1+\cos\alpha)}\right]$

(D) $\tan^{-1}\left[\dfrac{a-g\sin\alpha}{g(1-\cos\alpha)}\right]$

13. Figure shows a three arm tube in which a liquid is filled upto levels of height l. It is now rotated at an angular frequency ω about an axis passing through arm B. The angular frequency ω at which level of liquid of arm B becomes zero.



(A) $\sqrt{\dfrac{2g}{3l}}$ (B) $\sqrt{\dfrac{g}{l}}$

(C) $\sqrt{\dfrac{3g}{l}}$ (D) $\sqrt{\dfrac{3g}{2l}}$

Pascal's Law and Archimedes's Principle

14. Two cubes of size 1.0 m sides, one of relative density 0.60 and another of relative density = 1.15 are connected by weightless wire and placed in a large tank of water. Under equilibrium the lighter cube will project above the water surface to a height of
(A) 50 cm (B) 25 cm
(C) 10 cm (D) zero

15. A cuboidal piece of wood has dimensions a, b and c. Its relative density is d. It is floating in a larger body of water such that side a is vertical. It is pushed down a bit and released. The time period of SHM executed by it is:

(A) $2\pi\sqrt{\dfrac{abc}{g}}$ (B) $2\pi\sqrt{\dfrac{g}{da}}$

(C) $2\pi\sqrt{\dfrac{bc}{dg}}$ (D) $2\pi\sqrt{\dfrac{da}{g}}$

16. Two bodies having volumes V and $2V$ are suspended from the two arms of a common balance and they are found to balance each other. If larger body is immersed in oil (density d_1 = 0.9 g/cm^3) and the smaller body is immersed in an unknown liquid, then the balance remain in equilibrium. The density of unknown liquid is given by:
(A) 2.4 g/cm^3 (B) 1.8 g/cm^3
(C) 0.45 g/cm^3 (D) 2.7 g/cm^3

17. A boy carries a fish in one hand and a bucket (not full) of water in the other hand. If the places the fish in the bucket, the weight now carried by him (assume that water does not spill):
(A) is less than before
(B) is more than before
(C) is the same as before
(D) depends upon his speed

18. A piece of steel has a weight W in air, W_1 when completely immersed in water and W_2 when completely immersed in an unknown liquid. The relative density (specific gravity) of liquid is:

(A) $\dfrac{W-W_1}{W-W_2}$ (B) $\dfrac{W-W_2}{W-W_1}$

(C) $\dfrac{W_1-W_2}{W-W_1}$ (D) $\dfrac{W_1-W_2}{W-W_2}$

19. A ball of relative density 0.8 falls into water from a height of 2 m. The depth to which the ball will sink is (neglect viscous forces):
(A) 8 m (B) 2 m
(C) 6 m (D) 4 m

20. A cube of iron whose sides are of length L, is put into mercury. The weight of iron cube is W. The density of iron is ρ_I, that of mercury is ρ_M. The depth to which the cube sinks is given by the expression
(A) $WL^2\rho I$ (B) $WL^2\rho M$

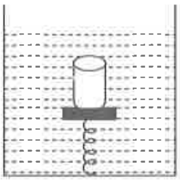
(C) $\dfrac{W}{L^2\rho_I}$ (D) $\dfrac{W}{L^2\rho_M g}$

21. A metal ball of density 7800 kg/m^3 is suspected to have a large number of cavities. It weighs 9.8 kg when weighed directly on a balance and 1.5 kg less when immersed in water. The fraction by volume of the cavities in the metal ball is approximately:
(A) 20% (B) 30%
(C) 16% (D) 11%

22. A sphere of radius R and made of material of relative density s has a concentric cavity of radius r. It just floats when placed in a tank full of water. The value of the ratio R/r will be
(A) $\left(\frac{\sigma}{\sigma-1}\right)^{1/3}$ (B) $\left(\frac{\sigma-1}{\sigma}\right)^{1/3}$
(C) $\left(\frac{\sigma+1}{\sigma}\right)^{1/3}$ (D) $\left(\frac{\sigma-1}{\sigma+1}\right)^{1/3}$

23. A beaker containing water is placed on the platform of a spring balance. The balance reads 1.5 kg. A stone of mass 0.5 kg and density 500 kg/m^3 is immersed in water without touching the walls of beaker. What will be the balance reading now?
(A) 2 kg (B) 2.5 kg
(C) 1 kg (D) 3 kg

24. A cylindrical block of area of cross-section A and of material of density ρ is placed in a liquid of density one-third of density of block. The block compresses a spring and compression in the spring is one-third of the length of the block. If acceleration due to gravity is g, the spring constant of the spring is



(A) ρAg (B) $2\rho Ag$
(C) $\frac{2\rho Ag}{3}$ (D) $\frac{\rho Ag}{3}$

25. A body of density ρ' is dropped from rest at a height h into a lake of density ρ, where $\rho > \rho'$. Neglecting all dissipative forces, calculate the maximum depth to which the body sinks before returning of float on the surface.
(A) $\frac{h}{\rho-\rho'}$ (B) $\frac{h\rho'}{\rho}$
(C) $\frac{h\rho'}{\rho-\rho'}$ (D) $\frac{h\rho}{\rho-\rho'}$

Fluid Flow and Bernoulli's Principle

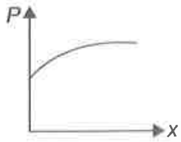
26. Water is flowing in a horizontal pipe of non-uniform cross-section. At the most contracted place of the pipe
(A) Velocity of water will be maximum and pressure minimum
(B) Pressure of water will be maximum and velocity minimum
(C) Both pressure and velocity of water will be maximum
(D) Both pressure and velocity of water will be minimum

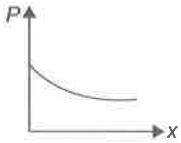
27. Water is flowing in a tube of non-uniform radius. The ratio of the radii at entrance and exit ends of tube is 3:2. The ratio of the velocities of water entering in and exiting from the tube will be
(A) 8:27 (B) 4:9
(C) 1:1 (D) 9:4

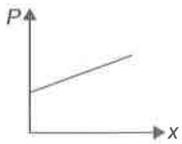
28. A rectangular tank is placed on a horizontal ground and is filled with water to a height H above the base. A small hole is made on one vertical side at a depth D below the level of the water in the tank. The distance x from the bottom of the tank at which the water jet from the tank will hit the ground is
(A) $2\sqrt{D(H-D)}$ (B) $2\sqrt{DH}$
(C) $2\sqrt{D(H+D)}$ (D) $\frac{1}{2}\sqrt{DH}$

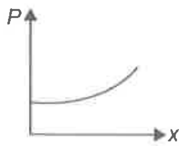
29. A jet of water with cross section of 6 cm^2 strikes a wall at an angle of 60° to the normal and rebounds elastically from the wall without losing energy. If the velocity of the water in the jet is 12 m/s, the force acting on the wall is
(A) 0.864 Nt (B) 86.4 Nt
(C) 72 Nt (D) 7.2 Nt

30. The cross sectional area of a horizontal tube increases along its length linearly, as we move in the direction of flow. The variation of pressure, as we move along its length in the direction of flow (x-direction), is best depicted by which of the following graphs

(A)


(B)


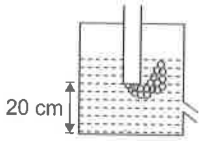
(C)


(D)


31. Water is flowing steadily through a horizontal tube of non uniform cross-section. If the pressure of water is 4×10^4 N/m^2 at a point where cross-section is 0.02 m^2 and velocity of flow is 2 m/s, what is pressure at a point where cross-section reduces to 0.01 m^2

(A) 1.4×10^4 N/m^2
(B) 3.4×10^4 N/m^2
(C) 2.4×10^{-4} N/m^2
(D) none of these

32. A tube is attached as shown in closed vessel containing water. The velocity of water coming out from a small hole is:



(A) $\sqrt{2}$ m/s
(B) 2 m/s
(C) depends on pressure of air inside vessel
(D) None of these

33. In the case of a fluid, Bernoulli's theorem expresses the application of the principle of conservation of

(A) linear momentum (B) energy
(C) mass (D) angular momentum

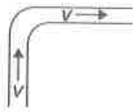
34. Fountains usually seen in gardens are generated by a wide pipe with an enclosure at one end having many small holes. Consider one such fountain which is produced by a pipe of internal diameter 2 cm in which water flows at a rate 3 ms^{-1}. The enclosure has 100 holes each of diameter 0.05 cm. The velocity of water coming out of the holes is (in ms^{-1}):

(A) 0.48 (B) 96
(C) 24 (D) 48

35. A cylindrical vessel open at the open at the top is 20 cm high and 10 cm in diameter. A circular hole whose cross-sectional area 1 cm^2 is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate 100 cm^{3}s^{-1}. The height of water in the vessel under state is (Take $g = 1000$ cms^{-2})

(A) 20 cm (B) 15 cm
(C) 10 cm (D) 5 cm

36. A fire hydrant delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed V. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is:



(A) ρVL (B) zero
(C) $2\rho VL$ (D) $\sqrt{2}\,\rho VL$

37. A vertical tank, open at the top, is filled with a liquid and rests on a smooth horizontal surface. A small hole is opened at the centre of one side of the tank. The area of cross-section of the tank is N times the area of the hole, where N is a large number. Neglect mass of the tank itself. The initial acceleration of the tank is

(A) $\frac{g}{2N}$ (B) $\frac{g}{\sqrt{2}N}$
(C) $\frac{g}{N}$ (D) $\frac{g}{2\sqrt{N}}$

38. Two water pipes P and Q having diameters 2×10^{-2} m and 4×10^{-2} m, respectively, are joined in series with the main supply line of water. The velocity of water flowing in pipe P is

(A) 4 times that of Q
(B) 2 times that of Q
(C) $\frac{1}{2}$ times of that of Q
(D) $\frac{1}{4}$ times that of Q

39. Water flows into a cylindrical vessel of large cross-sectional area at a rate of 10^{-4}m^3/s. It flows out from a hole of area 10^{-4} m^2, which has been punched through the base. How high does the water rise in the vessel?

(A) 0.075 m (B) 0.051 m
(C) 0.031 m (D) 0.025 m

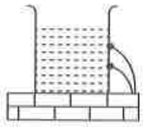
40. A tank has an orifice near its bottom. The volume of the liquid flowing per second out of the orifice does not depend upon

(A) Area of the orifice
(B) Height of the liquid level above the orifice
(C) Density of liquid
(D) Acceleration due to gravity

41. The rate of flowing of water from the orifice in a wall of a tank will be more if the orifice is

(A) Near the bottom
(B) Near the upper end
(C) Exactly in the middle
(D) Does not depend upon the position of orifice

42. In a cylindrical vessel containing liquid of density ρ, there are two holes in the side walls at heights of h_1 and h_2 respectively such that the range of efflux at the bottom of the vessel is same. The height of a hole, for which the range of efflux would be maximum will be.



(A) $h_2 - h_1$ (B) $h_2 + h_1$
(C) $\frac{h_2 - h_1}{2}$ (D) $\frac{h_2 + h_1}{2}$

43. A water barrel stands on a table of height h. If a small hole is punched in the side of the barrel at its base, it is found that the resultant stream of water strikes the ground at a horizontal distance R from the barrel. The depth of water in the barrel is
(A) $\frac{R}{2}$ (B) $\frac{R^2}{4h}$
(C) $\frac{R^2}{h}$ (D) $\frac{h}{2}$

44. A cylindrical vessel of cross-sectional area 1000 cm^2, is fitted with a frictionless piston of mass 10 kg, and filled with water completely. A small hole of cross-sectional area 10 mm^2 is opened at a point 50 cm deep from the lower surface of the piston. The velocity of efflux from the hole will be
(A) 10.5 m/s (B) 3.4 m/s
(C) 0.8 m/s (D) 0.2 m/s

45. A horizontal right angle pipe bend has cross-sectional area = 10 cm^2 and water flows through it at speed = 20 m/s. The force on the pipe bend due to the turning of water is:
(A) 565.7 N (B) 400 N
(C) 20 N (D) 282.8 N

46. A jet of water having velocity = 10 m/s and stream cross-section = 2 cm^2 hits a flat plate perpendicularly, with the water splashing out parallel to plate. The plate experiences a force of
(A) 40 N (B) 20 N
(C) 8 N (D) 10 N

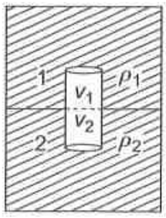
47. Water is pumped from a depth of 10 m and delivered through a pipe of cross section 10^{-2} m^2. If it is needed to deliver a volume of 10^{-1} m^3 per second the power required will be:
(A) 10 kW (B) 9.8 kW
(C) 15 kW (D) 4.9 kW

48. Which of the following is not an assumption for an ideal fluid flow for which Bernoulli's principle is valid
(A) Steady flow (B) Incompressible
(C) Viscous (D) Irrotational

Assertion-Reason

49. **Statement-1:** A helium filled balloon does not rise indefinitely in air but halts after a certain height.
Statement-2: Viscosity opposes the motion of balloon.
Choose any one of the following four responses:
(A) If both (A) and (R) are true and (R) is the correct explanation of (A)
(B) If both (A) and (R) are true but (R) is not correct explanation of (A)
(C) If both (A) if true but (R) is false
(D) If (A) is false and (R) is true

50. **Statement-1:** When a body floats such that it's parts are immersed into two immiscible liquids force exerted by liquid-1 is of magnitude $\rho_1 v_1 g$.
Statement-2: Total Buoyant force = $\rho_1 v_1 g + \rho_2 v_2 g$

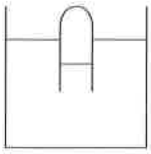


(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

51. **Statement-1:** When temperature rises the coefficient of viscosity of gases decreases.
Statement-2: Gases behave more like ideal gases at higher temperature
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

52. **Statement-1:** A partly filled test tube is floating in a liquid as shown. The tube will remain as atmospheric pressure changes.

Statement-2: The buoyant force on a submerged object is independent of atmospheric pressure



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

53. **Statement-1:** Submarine sailors are advised that they should not allow it to rest on floor of the occen.

Statement-2: The force exerted by a liquid on a submerged body may be downwards.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

54. **Statement-1:** The free surface of a liquid at rest with respect to stationary container is always normal to the $\vec{g}_{eff}$.

Statement-2: Liquids at rest cannot have shear stress.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

JEE Advanced

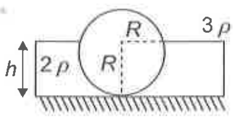
Single Correct Option Type Questions

STATIC FLUID

1. Some liquid is filled in a cylindrical vessel of radius R. Let F_1 be the force applied by the liquid on the bottom of the cylinder. Now the same liquid is poured into a vessel of uniform square cross-section of side R. Let F_2 be the force applied by the liquid on the bottom of this new vessel. Then:

(A) $F_1 = \pi F_2$ (B) $F_1 = \frac{F_2}{\pi}$
(C) $F_1 = \sqrt{\pi} F_2$ (D) $F_1 = F_2$

2. In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. The height 'h' for the equilibrium of cylinder must be



(A) $\frac{3R}{2}$ (B) $R\sqrt{\frac{3}{2}}$
(C) $R\sqrt{2}$ (D) None

3. The vertical limbs of a U shaped tube are filled with a liquid of density ρ upto a height h on each side. The horizontal portion of the U tube having length $2h$ contains a liquid of density $2\,\rho$. The U tube is moved horizontally with an accelerator $g/2$ parallel to the horizontal arm. The difference in heights in liquid levels in the two vertical limbs, at steady state will be

(A) $\frac{2h}{7}$ (B) $\frac{8h}{7}$
(C) $\frac{4h}{7}$ (D) None

ACCELERATED FLUID

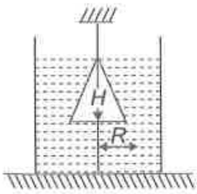
4. An open cubical tank was initially fully filled with water. When the tank was accelerated on a horizontal plane along one of its side it was found that one third of volume of water spilled out. The acceleration was

(A) $\frac{g}{3}$ (B) $\frac{2g}{3}$
(C) $\frac{3g}{2}$ (D) None

PASCAL'S LAW AND ARCHIMEDES'S PRINCIPLE

5. A cone of radius R and height H, is hanging inside a liquid of density ρ by means of a string as shown

in the figure. The force, due to the liquid acting on the slant surface of the cone is (neglect atmospheric pressure)

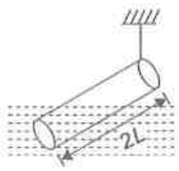


(A) $\rho\pi gHR^2$
(B) $\pi\rho HR^2$
(C) $\frac{4}{3}\pi\rho gHR^2$
(D) $\frac{2}{3}\pi\rho gHR^2$

6. A heavy hollow cone of radius R and height h is placed on a horizontal table surface, with its flat base on the table. The whole volume inside the cone is filled with water of density ρ. The circular rim of the cone's base has a watertight seal with the table's surface and the top apex of the cone has a small hole. Neglecting atmospheric pressure find the total upward force exerted by water on the cone is

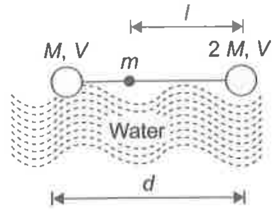
(A) $\left(\frac{2}{3}\right)\pi R^2 h\rho g$ (B) $\left(\frac{1}{3}\right)\pi R^2 h\rho g$
(C) $\pi R^2 h\rho g$ (D) None

7. A slender homogeneous rod of length 2 L floats partly immersed in water, being supported by a string fastened to one of its ends, as shown. The specific gravity of the rod is 0.75. The length of rod that extends out of water is



(A) L (B) $\frac{1}{2}L$
(C) $\frac{1}{4}L$ (D) $3L$

8. A dumbbell is placed in water of density ρ. It is observed that by attaching a mass m to the rod, the dumbbell floats with the rod horizontal on the surface of water and each sphere exactly half submerged as shown in the figure. The volume of the mass m is negligible. The value of length l is



(A) $\frac{d(V_\rho - 3M)}{2(V_\rho - 2M)}$ (B) $\frac{d(V_\rho - 2M)}{2(V_\rho - 3M)}$
(C) $\frac{d(V_\rho + 2M)}{2(V_\rho - 3M)}$ (D) $\frac{d(V_\rho - 2M)}{2(V_\rho + 3M)}$

9. A container of large surface area is filled with liquid of density ρ. A cubical block of side edge a and mass M is floating in it with four-fifth of its volume submerged. If a coin of mass m is placed gently on the top surface of the block is just submerged. M is

(A) $\frac{4m}{5}$ (B) $\frac{m}{5}$
(C) $4m$ (D) $5m$

10. A cork of density 0.5 gcm^{-3} floats on a calm swimming pool. The fraction of the cork's volume which is under water is

(A) 0% (B) 25%
(C) 10% (D) 50%

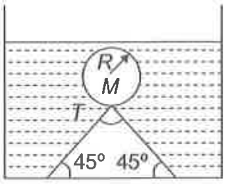
11. Two cylinders of same cross-section and length L but made of two material of densities d_1 and d_2 are cemented together to form a cylinder of length $2L$. The combination floats in a liquid of density d with a length $L/2$ above the surface of the liquid. If $d_1 > d_2$ then:

(A) $d_1 > \frac{3}{4}d$ (B) $\frac{d}{2} > d_1$
(C) $\frac{d}{4} > d_1$ (D) $d < d_1$

12. A small wooden ball of density ρ is immersed in water of density σ to depth h and then released. The height H above the surface of water up to which the ball will jump out of water is

(A) $\frac{\sigma h}{\rho}$ (B) $\left(\frac{\sigma}{\rho} - 1\right)h$
(C) h (D) zero

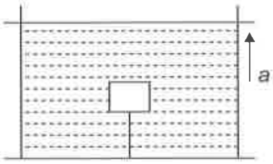
13. A hollow sphere of mass M and radius r is immersed in a tank of water (density ρ_w). The sphere would float if it were set free. The sphere is tied to the bottom of the tank by two wires which makes angle 45° with the horizontal as shown in the figure. The tension T_1 in the wire is:



(A) $\dfrac{\frac{4}{3}\pi R^3 \rho_w g - Mg}{\sqrt{2}}$ (B) $\frac{2}{3}\pi R^3 \rho_w g - Mg$

(C) $\dfrac{\frac{4}{3}\pi R^3 \rho_w g - Mg}{2}$ (D) $\frac{4}{3}\pi R^3 \rho_w g - Mg$

14. A body having volume V and density ρ is attached to the bottom of a container as shown. Density of the liquid is $d(>\rho)$. Container has a constant upward acceleration a. Tension in the string is

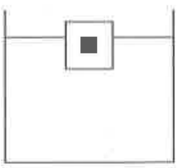


(A) $V[Dg - \rho(g + a)]$ (B) $V(g + a)(d - \rho)$
(C) $V(d - \rho)g$ (D) none

15. A hollow cone floats with its axis vertical upto one-third of its height in a liquid of relative density 0.8 and with its vertex submerged. When another liquid of relative density ρ is filled in it upto one-third of its height, the cone floats upto half its vertical height. The height of the cone is 0.10 m and radius of the circular base is 0.05 m. The specific gravity ρ is given by

(A) 1.0 (B) 1.5
(C) 2.1 (D) 1.9

16. There is a metal cube inside a block of ice which is floating on the surface of water. The ice melts completely and metal falls in the water. Water level in the container



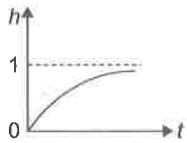
(A) Rises
(B) Falls
(C) Remains same
(D) Nothing can be concluded

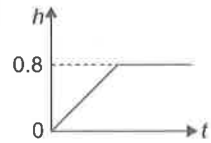
17. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquid A and B with its axis vertical. The densities of liquid A and B are 0.7 g/cm^3 and 1.2 g/cm^3. The height of liquid A is $h_A = 1.2$ cm and the length of the part of cylinder immersed in liquid B is $h_B = 0.8$ cm. Then the length of the cylinder in air is

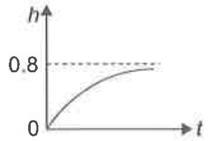
(A) 0.21 m (B) 0.25 cm
(C) 0.35 cm (D) 0.4 cm

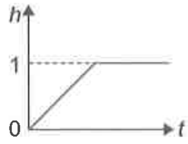
FLUID FLOW AND BERNOULLI'S PRINCIPLE

18. A cylindrical tank of height 1 m and cross section area $A = 4000$ cm^2 is initially empty when it is kept under a tap of cross sectional area 1 cm^2. Water starts flowing from the tap at $t = 0$, with a speed = 2 m/s. There is a small hole in the base of the tank of cross-sectional area 0.5 cm^2. The variation of height of water in tank (in meters) with time t is best depicted by

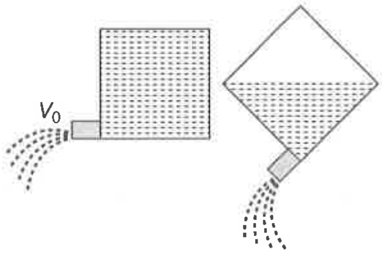
(A)


(B)


(C)


(D)


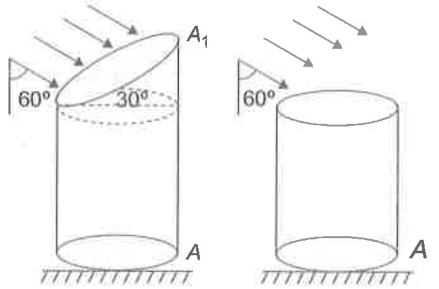
19. A cubical box of wine has a small spout located in one of the bottom corners. When the box is full and placed on a level surface, opening the spout results in a flow of wine with a initial speed of v_0 (see figure). When the box is half empty, someone tilts it at 45° so that the spout is at the lowest point (see figure). When the spout is opened the wine will flow out with a speed of



(A) v_0 (B) $\dfrac{v_0}{2}$

(C) $\dfrac{v_0}{\sqrt{2}}$ (D) $\dfrac{v_0}{\sqrt[4]{2}}$

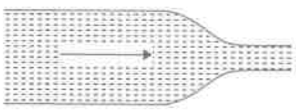
20. A vertical cylindrical container of base area A and upper cross-section area A_1 making an angle 30° with the horizontal is placed in an open rainy field as shown near another cylindrical container having same base area A. The ratio of rates of collection of water in the two containers will be.

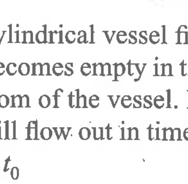
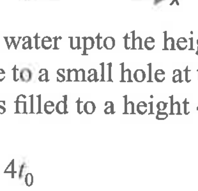
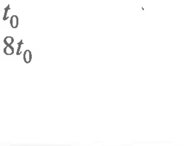
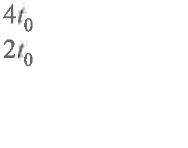


(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{4}{\sqrt{3}}$
(C) 2 (D) None

21. A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to H/η, ($\eta > 1$) and it takes T_2 time to take out the rest of water. If $T_1 = T_2$, then the value of η is:
(A) 2 (B) 3
(C) 4 (D) $2\sqrt{2}$

22. Water flows through a frictionless duct with a cross-section varying as shown in figure. Pressure p at points along the axis is represented by

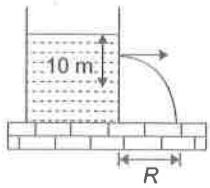


(A) 
(B) 
(C) 
(D) 

23. A cylindrical vessel filled with water upto the height H becomes empty in time t_0 due to a small hole at the bottom of the vessel. If water is filled to a height $4H$ it will flow out in time
(A) t_0 (B) $4t_0$
(C) $8t_0$ (D) $2t_0$

24. A tank is filled up to a height 2 H with a liquid and is placed on a platform of height H from the ground. The distance x from the ground where a small hole is punched to get the maximum range R is:
(A) H (B) 1.25 H
(C) 1.5 H (D) 2 H

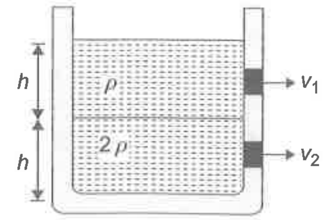
25. A large tank is filled with water (density = 10^3 kg/m^3). A small hole is made at a depth 10 m below water surface. The range of water issuing out of the hole is Ron ground. What extra pressure must be applied on the water surface so that the range becomes $2R$ (take 1 atm = 10^5 Pa and g = 10 m/s^2):



(A) 9 atm (B) 4 atm
(C) 5 atm (D) 3 atm

26. A laminar stream is flowing vertically down from a tap of cross-section area 1 cm^2. At a distance 10 cm below the tap, the cross-section area of the stream has reduced to 1/2 cm^2. The volumetric flow rate of water from the tap must be about
(A) 2.2 litre/min (B) 4.9 litre/min
(C) 0.5 litre/min (D) 7.6 litre/min

27. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depth $h/2$ and $3h/2$ from the surface of lighter liquid. If v_1 and v_2 are the velocities of a flux at these two holes, then v_1/v_2 is:



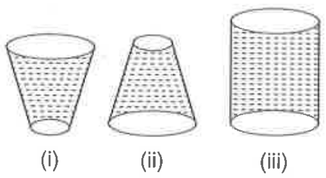
(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{1}{\sqrt{2}}$

28. A horizontal pipe line carries water in a streamline flow. At a point along the tube where the cross-sectional area is 10^{-2} m^2, the water velocity is 2 ms^{-1}

and the pressure is 8000 Pa. The pressure of water at another point where the cross-sectional area is 0.5×10^{-2} m^2 is:

(A) 4000 Pa (B) 1000 Pa
(C) 2000 Pa (D) 3000 Pa

29. The three water filled tanks shown have the same volume and height. If small identical holes are punched near this bottom, which one will be the first to get empty.



(A) (i) (B) (ii)
(C) (iii) (D) All will take same time

30. A cylindrical vessel filled with water upto height of H stands on a horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. The coefficient of friction between the bottom of vessel and plane is m and total mass of water plus vessel is M. What should be minimum diameter of hole so that the vessel begins to move on the floor if plug is removed (here density of water is ρ)

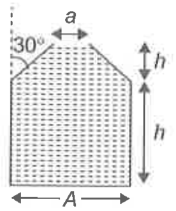
(A) $\sqrt{\dfrac{2\mu M}{\pi\rho H}}$ (B) $\sqrt{\dfrac{\mu M}{2\pi\rho H}}$

(C) $\sqrt{\dfrac{\mu M}{\rho H}}$ (D) none

MULTIPLE CORRECT

STATIC FLUID

31. The vessel shown in the figure has two sections. The lower part is a rectangular vessel with area of cross-section A and height h. The upper part is a conical vessel of height h with base area 'A' and top area 'a' and the walls of the vessel are inclined at an angle 30° with the vertical. A liquid of density ρ fills both the sections upto a height 2h. Neglecting atmospheric pressure.



(A) The force F exerted by the liquid on the base of the vessel is $2h\rho g\dfrac{(A+a)}{2}$

(B) the pressure P at the base of the vessel is $2h\rho g\dfrac{A}{2}$

(C) the weight of the liquid W is greater than the force exerted by the liquid on the base

(D) the walls of the vessel exert a downward force $(F - W)$ on the liquid.

ACCELERATED FLUID

32. A beaker is filled in with water is accelerated a m/s^2 in $+x$ direction. The surface of water shall make on angle

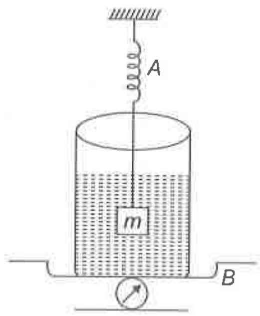
(A) $\tan^{-1}(a/g)$ backwards (B) $\tan^{-1}(a/g)$ forwards
(C) $\cot^{-1}(g/a)$ backwards (D) $\cot^{-1}(g/a)$ forwards

PASCAL'S LAW AND ARCHIMEDES'S PRINCIPLE

33. The weight of an empty balloon on a spring balance is w_1. The weight becomes w_2 when the balloon is filled with air. Let the weight of the air itself be w. Neglect the thickness of the balloon when it is filled with air. Also neglect the difference in the densities of air inside and outside the balloon. Then:

(A) $w_2 = w_1$ (B) $w_2 = w_1 + w$
(C) $w_2 < w_1 + w$ (D) $w_2 > w_1$

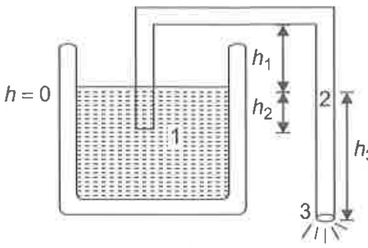
34. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure in this situation:



(A) the balance A will read more than 2 kg
(B) the balance B will read more than 5 kg
(C) the balance A will read less than 2 kg and B will read more than 5 kg
(D) the balances A and B will read 2 kg and 5 kg respectively

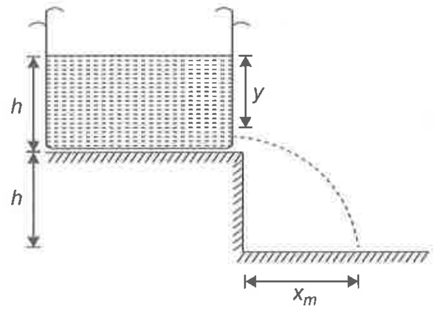
FLUID FLOW AND BERNOULLI'S PRINCIPLE

35. Figure shows a siphon. Choose the wrong statement:



(A) Siphon works when $h_3 > 0$
(B) Pressure at point 2 is $P_2 = P_0 - \rho g h_3$
(C) Pressure at point 3 is P_0
(D) None of the above
(P_0 = atmospheric pressure)

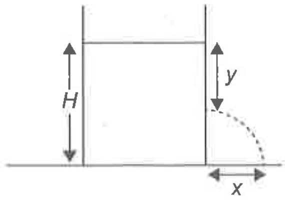
36. A tank is filled upto a height h with a liquid and is placed on a platform of height h from the ground. To get maximum range x_m a small hole is punched at a distance of y from the free surface of the liquid. Then



(A) $x_m = 2\,h$
(B) $x_m = 1.5\,h$
(C) $y = h$
(D) $y = 0.75\,h$

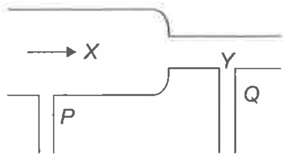
37. Water coming out of a horizontal tube at a speed v strikes normally a vertically wall close to the mouth of the tube and falls down vertically after impact. When the speed of water is increased to $2v$.
(A) the thrust exerted by the water on the wall will be doubled
(B) the thrust exerted by the water on the wall will be four times
(C) the energy lost per second by water strike up the wall will also be four times
(D) the energy lost per second by water striking the wall be increased eight times

38. A cylindrical vessel is filled with a liquid up to height H. A small hole is made in the vessel at a distance y below the liquid surface as shown in figure. The liquid emerging from the hole strike the ground at distance x



(A) if y is increased from zero to H, x will decrease and then increase
(B) x is maximum for $y = \dfrac{H}{2}$
(C) the maximum value of x is $\dfrac{H}{2}$
(D) the maximum value of x increases with the increases in density of the liquid

39. A steady flow of water passes along a horizontal tube from a wide section X to the narrower section Y, see figure. Manometers are placed at P and Q at the sections. Which of the statements A, B, C, D, E is most correct?



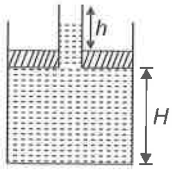
(A) water velocity at X is greater than at Y
(B) the manometer at P shows lower pressure than at Q
(C) kinetic energy per m^3 of water at X = kinetic energy per m^3 at Y
(D) the manometer at P shows greater pressure than at Y

JEE Advanced

Level I

Static Fluid

1. A piston of mass $M = 3$ kg and radius $R = 4$ cm has a hole into which a thin pipe of radius $r = 1$ cm is inserted. The piston can enter a cylinder tightly and without friction, and initially it is at the bottom of the cylinder. 750 gm of water is now poured into the pipe so that the piston and pipe are lifted up as shown. Find the height H of water in the cylinder and height h of water in the pipe. (Neglect width of piston)



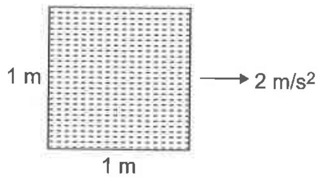
2. Compute the work which must be performed to slowly pump the water out of a hemispherical reservoir of radius $R = 0.6$ m.

3. A vertical uniform U tube open at both ends contains mercury. Water is poured in one limb until the level of mercury is depressed 2 cm in that limb. What is the length of water column when this happens.

ACCELERATED FLUID

4. A spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank.

5. An open cubical tank completely filled with water is kept on a horizontal surface. Its acceleration is then slowly increased to 2 m/s^2 as shown in the figure. The side of the tank is 1 m. Find the mass of water that would spill out of the tank.



6. Find the speed of rotation of 1 m diameter tank, initially full of water such that water surface makes an angle of 45° with the horizontal at a radius of 30 cm. What is the slope of the surface at the wall of the tank.

PASCAL'S LAW AND ARCHIMEDE'S PRINCIPLE

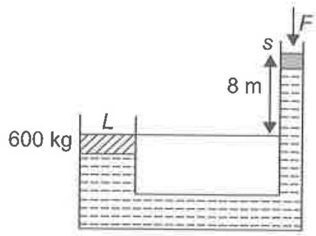
7. A solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enter water. Upto what depth will the ball go? How much time will it take to come again to the water surface? Neglect air resistance and velocity effects in water.

8. Place a glass beaker, partially filled with water, in a sink. The beaker has a mass 390 gm and an interior volume of 500 cm^3. You now start to fill the sink with water and you find, by experiment, that if the beaker is less than half full, it will float; but if it is more than half full, it remains on the bottom of the sink as the water rises to its rim. What is the density of the material of which the beaker is made?

9. Two spherical balls A and B made up of same material having masses 2 m and m are released from rest. Ball B lies at a distance h below the water surface while A is at a height of 2 h above water surface in the same vertical line at the instant they are released.
   (a) Obtain the position where they collide.
   (b) If the bodies stick together due to collision, to what maximum height above water surface does the combined mass rise?

   Specific gravity of the material of the balls is $\frac{2}{3}$. Neglect viscosity and loss due to splash.

10. For the system shown in the figure, the cylinder on the left at L has a mass of 600 kg and a cross sectional area of 800 cm^2. The piston on the right, at S, has cross sectional area 25 cm^2 and negligible weight. If the apparatus is filled with oil. (ρ = 0.75 gm/cm^3) Find the force F required to hold the system in equilibrium.

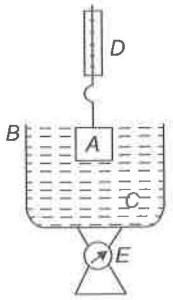


11. A test tube of thin walls floats vertically in water, sinking by a length l_0 = 10 cm. A liquid of density less than that of water, is poured into the tube till the levels inside and outside the tube are even. If the tube now sinks to a length l_0 = 40 cm, the specific gravity of the liquid is ______.

12. In air an object weighs 15 N, when immersed completely in water the same object weighs 12 N. When immersed in another liquid completely, it weighs 13 N. Find
   (a) the specific gravity of the object and
   (b) the specific gravity of the other liquid.

13. Block A in figure hangs by a cord from spring balance D and is submerged in a liquid C contained in a beaker B. The mass of the beaker is 1 kg and

the mass of the liquid is 1.5 kg. The balance D reads 2.5 kg and balance E reads 7.5 kg.



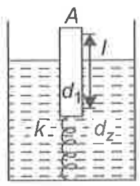
The volume of block A is $0.003\ m^3$.

(i) What is the density of block and the liquid.

(ii) What will each balance read if block is pulled out of the liquid.

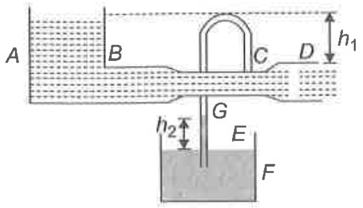
14. A solid cube, with faces either vertical or horizontal, is floating in a liquid of density 6 g/cc. It has two third of its volume submerged. If enough water is added from the top so as to completely cover the cube, what fraction of its volume will remain immersed in the liquid?

15. A uniform cylindrical block of length l density d_1 and area of cross section A floats in a liquid of density d_2 contained in a vessel ($d_2 > d_1$). The bottom of the cylinder just rests on a spring of constant k. The other end of the spring is fixed to the bottom of the vessel. The weight that may be placed on top of the cylinder such that the cylinder is just submerged in the liquid is ______

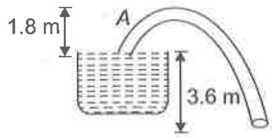


FLUID FLOW AND BERNOULLI'S PRINCIPLE

16. Two very large open tanks A and F both contain the same liquid. A horizontal pipe BCD, having a constriction at C leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section at C is one half that at D and if D is at a distance h_1 below the level of liquid in A, to what height h_2 (in terms of h_1) will liquid rise in pipe E? (above G and upto C there is air in the pipe)



17. A siphon has a uniform circular base of diameter $\frac{8}{\sqrt{\pi}}$ cm with its crest A 1.8 m above water level as in figure.



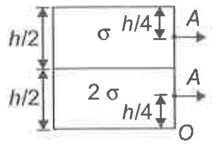
Find

(a) velocity of flow

(b) discharge rate of the flow in m^3/sec.

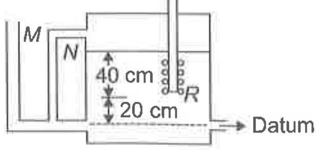
(c) absolute pressure at the crest level A.
[Use $P_0 = 10^5\ N/m^2$ and $g = 10\ m/s^2$]

18. A large tank is filled with two liquids of specific gravities 2σ and σ. Two holes are made on the wall of the tank as shown. Find the ratio of the distances from O of the points on the ground where the jets from holes A and B strike.



19. Calculate the rate of flow of glycerine of density $1.25 \times 10^3\ kg/m^3$ through the conical section of a pipe if the radii of its ends are 0.1 m and 0.04 m and the pressure drop across its length is $10\ N/m^2$

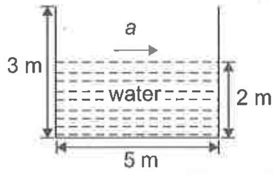
20. The tank in fig discharges water at constant rate for all water levels above the air inlet R. The height above datum to which water would rise in the manometer tubes M and N respectively are ______ and ______.



Level II

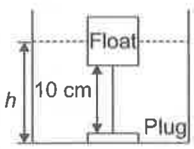
1. A solid block of volume $V = 10^{-3}\,m^3$ and density $d = 800\ kg/m^3$ is tied to one end of a string, the other end of which is tied to the bottom of the vessel. The vessel contains 2 immiscible liquids of density $\rho_1 = 1000\ kg/m^3$ and $\rho_2 = 1500\ kg/m^3$. The solid block is immersed with 2/5 th of its volume in the liquid higher density and 3/5 th in the liquid of lower density. The vessel is placed in an elevator which is moving up with an acceleration of $a = g/2$. Find the tension in the string. [$g = 10\ m/s^2$]

2. An open rectangular tank 5 m × 4 m × 3 m high containing water upto a height of 2 m is accelerated horizontally along the longer side.

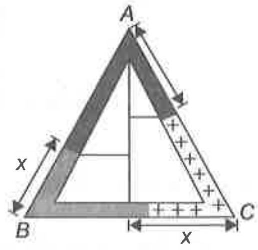


   (a) Determine the maximum acceleration that can be given without spilling the water.
   (b) Calculate the percentage of water split over, if this acceleration is increased by 20%
   (c) If initially, the tank is closed at the top and is accelerated horizontally by 9 m/s^2, find the gauge pressure at the bottom of the front and rear walls of the tank.

3. A level controller is shown in the figure. It consists of a thin circular plug of diameter 10 cm and a cylindrical float of diameter 20 cm tied together with a light rigid rod of length 10 cm. The plug fits in snugly in a drain hole at the bottom of the tank which opens into atmosphere. As water fills up and the level reaches height h, the plug opens. Find h. Determine the level of water in the tank when the plug closes again. The float has a mass 3 kg and the plug may be assumed as massless.

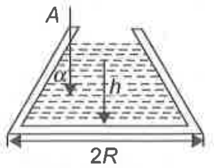


4. A closed tube in the form of an equilateral triangle of side l contains equal volumes of three liquids which do not mix and is placed vertically with its lowest side horizontal. Find x in the figure if the densities of the liquids are in A.P.

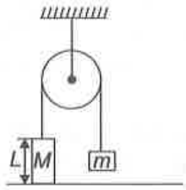


5. A ship sailing from sea into a river sinks X mm and on discharging the cargo rises Y mm. On proceeding again into sea the ship rises by Z mm. Assuming ship sides to be vertical at water line, find the specific gravity of sea water.

6. A conical vessel without a bottom stands on a table. A liquid is poured with the vessel and as soon as level reaches h, the pressure of the liquid raises the vessel. The radius of the base of vessel is R and half angle of the cone is α and the weight of the vessel is W. What is the density of the liquid?



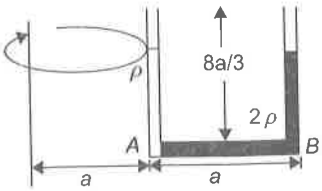
7. As the arrangement shown in the fig is released the rod of mass M moves down into the water. Friction is negligible and the string is in extensible



   (a) Find the acceleration of the system wrt the distance moved by each mass.
   (b) Find the time required to completely immerse the rod into water if $\dfrac{m}{M} = \dfrac{\rho - \rho_{water}}{\rho}$.

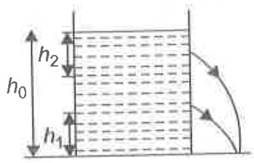
   ρ = density of rod; ρ_{water} = density of water

8. The interface of two liquids of densities ρ and 2ρ respectively lies at the point A in a U-tube at rest. The height of liquid column above A is $8a/3$ where $AB = a$. The cross sectional area of the tube is S. With what angular velocity the tube must be whirled about a vertical axis at a distance 'a' such that the interface of the liquids shifts towards B by $2a/3$.



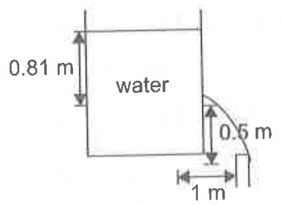
9. A closed cylindrical tank 2 m high and 1 m in diameter contains 1.5 m of water. When the angular velocity is constant at 20.0 rad/s, how much of the bottom of the tank is uncovered? (The cylinder is rotated about vertical axis of symmetry passing through its length.)

10. A cylinder of height H is filled with water to a height $h_0 (h_0 < H)$, and is placed on a horizontal floor. Two small holes are punched at time $t = 0$ on the vertical line along the length of the cylinder, one at a height h_1 from the bottom and the other a depth h_2 below the level of water in the cylinder. Find the relation between h_1 and h_2 such that the instantaneous water jets emerging



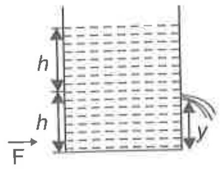
11. A cylindrical tank with a height of $h = 1$ m is filled with water upto its rim. What time is required to empty the tank through an orifice in its bottom? The cross sectional area of the orifice is (1/400)th of the tank. Find the time required for the same amount of water to flow out of the tank if the water level in the tank is maintained constant at a height of $h = 1$ m from the orifice.

12. For the arrangement shown in the figure. Find the time interval after which the water jet ceases to cross the wall.



Area of the tank = 0.5 m^2
Area of the orifice = 1 cm^2

13. A cylindrical tank having cross-sectional area A = 0.5 m^2 is filled with two liquids of densities $\rho_1 = 900$ kgm^{-3} and $\rho_2 = 600$ kgm^{-3}, to a height $h = 60$ cm as shown in the figure. A small hole having area $a = 5$ cm^2 is made in right vertical wall at a height $y = 20$ cm from the bottom. Calculate



(i) velocity of efflux.
(ii) horizontal force F to keep the cylinder in static equilibrium, it is placed on a smooth horizontal plane.
(iii) minimum and maximum value of F to keep the cylinder at rest. The coefficient of friction between cylinder and the plane is $\mu = 0.01$.
(iv) velocity of the top most layer of the liquid column and also the velocity of the boundary separating the two liquids.

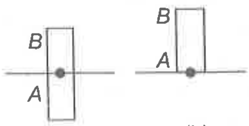
14. A cylindrical wooden float whose base area s = 4000 cm^2 and the altitude $H = 50$ cm drifts on the water surface. Specific weight of wood $d = 0.8$ gf/cm^3.
(a) What work must be performed to take the float out of the water?
(b) Compute the work to be performed to submerge completely the float into the water.

15. A 10 cm side cube weighing 5 N is immersed in a liquid of relative density 0.8 contained in a rectangular tank of cross sectional area 15 cm × 15 cm. If the tank contained liquid to a height of 8 cm before the immersion, determine the levels of the bottom of the cube and the liquid surface.

16. A jug contains 15 glasses of orange juice. When you open the tap at the bottom it takes 12 sec to fill a glass with juice. If you leave the tap open. how long will it take to fill the remaining 14 glasses and thus empty the jug?

17. An interstellar explorer discovers a remarkable planet made entirely of a uniform incompressible fluid on density ρ. The radius of the planet is R and the acceleration of gravity at its surface is g. What is the pressure at the center of the planet.

18. A cylindrical rod of length $l = 2$ m and density $\rho/2$ floats vertically in a liquid of density ρ as shown in fig.(a)

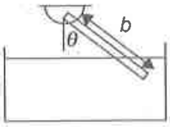


(a) (b)

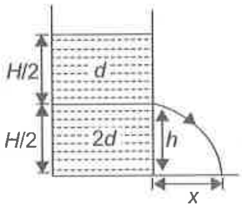
(a) Show that it performs SHM when pulled slightly up and released and find its time period. Neglect change in liquid level.

(b) Find the time taken by the rod to completely immerse when released from position shown in (b). Assume that it remains vertical throughout its motion. (take $g = \pi^2$ m/s^2)

19. A uniform rod of length b capable of tuning about its end which is out of water, rests inclined to the vertical. If its specific gravity is $\frac{5}{9}$, find the length immersed in water.



20. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in figure. The lower density liquid is open to the atmosphere having pressure P_0.



(a) A homogeneous solid cylinder of length $L\left(L < \frac{H}{2}\right)$ cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with the length $L/4$ in the denser liquid. Determine:

(i) The density D of the solid and

(ii) The total pressure at the bottom of the container.

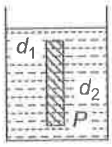
(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area $s(s << A)$ is punched on the vertical side of the container at a height $h\left(h < \frac{H}{2}\right)$. Determine :

(i) The initial speed of efflux of the liquid at the hole;

(ii) The horizontal distance x travelled by the liquid initially and

(iii) The height h_m at which the hold should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m. [Neglect the air resistance in these calculations]. **[JEE-1995, 2010]**

21. A thin rod of length L and area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous and non-viscous liquid (Figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the entity d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **[JEE-1995, 2005]**

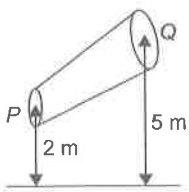


22. A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $A/100$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$, calculate **[JEE-1997, 2005]**

(i) the acceleration of the container and

(ii) its velocity when 75% of the liquid has drained out.

23. A nonviscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4×10^{-3} m^2 and 8×10^{-3} m^3. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. **[JEE-1997]**



24. A wooden stick of length l, and radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters that would make the stick float vertically in equilibrium in a liquid of density $\sigma(>\rho)$. **[JEE-1999, 2010]**

Previous Year Questions

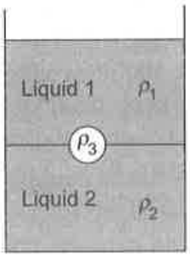
JEE Main

1. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is **[AIEEE-2002]**

(A) 10 (B) 20
(C) 25.5 (D) 5

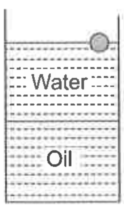
2. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure.

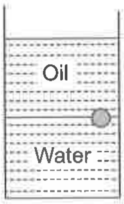
Which of the following is true for ρ_1, ρ_2 and ρ_3? **[AIEEE-2008]**

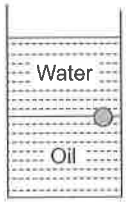


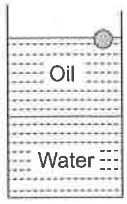
(A) $\rho_3 < \rho_1 < \rho_2$ (B) $\rho_1 > \rho_3 > \rho_2$
(C) $\rho_1 > \rho_2 > \rho_3$ (D) $\rho_1 < \rho_3 < \rho_2$

3. A ball is made of a material of density ρ where, $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium positions? **[AIEEE-2010]**

(A)


(B)


(C)


(D)


4. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to **[AIEEE-2011]**

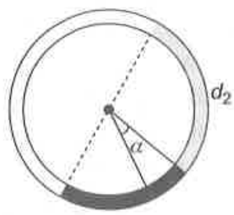
(A) 7.5×10^{-3} m (B) 9.6×10^{-3} m
(C) 3.6×10^{-3} m (D) 5.0×10^{-3} m

5. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: **[JEE Main-2013]**

(A) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (B) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

(C) $\frac{Mg}{k}$ (D) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

6. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is: **[JEE Main-2014]**



(A) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (B) $\frac{1+\sin\alpha}{1-\cos\alpha}$

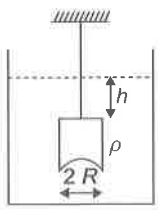
(C) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (D) $\frac{1+\cos\alpha}{1-\cos\alpha}$

JEE Advanced

1. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to: **[JEE 2000-(Scr.)]**

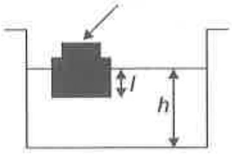
(A) $\frac{L}{\sqrt{2\pi}}$ (B) $2\pi L$
(C) L (D) $\frac{L}{2\pi}$

2. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is **[JEE-2001 (Scr.)]**



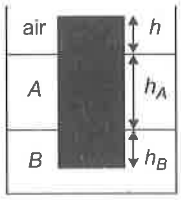
(A) Mg (B) $Mg - v\rho g$
(C) $Mg + \pi R^2 h\rho g$ (D) $\rho g(V + \pi R^2 h)$

3. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distances l and h are shown there. After some time the coin falls into the water. Then **[JEE-2002 (Scr.)]**



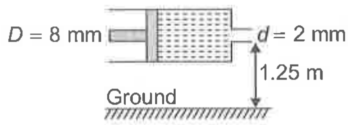
(A) l decreases and h increases
(B) l increases and h decreases
(C) both l and h increase
(D) both l and h decrease

4. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3, and 1.2 g/cm^3 respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm. **[JEE-2002]**



(a) Find the total force exerted by liquid A on the cylinder.
(b) Find h, the length of the part of the cylinder in air.
(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.

5. Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. Take $g = 10$ m/s^2. **[JEE-2004]**

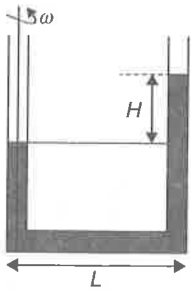


6. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[JEE-2004]**

7. Water is filled in a container upto height 3 m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $\frac{a}{A} = 0.1$ then v^2 is (where v is the velocity of water coming out of the hole) **[JEE-2005-(Scr.)]**

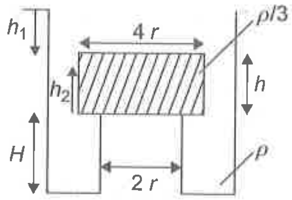
(A) 48 (B) 51
(C) 50 (D) 51.5

8. A U-tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube $d \ll L$. **[JEE-2005]**



Comprehension (9–11)

A wooden cylinder of diameter $4r$, height h and density $\rho/3$ is kept on a hole of diameter $2r$ of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H.



9. If level of liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder, the block just starts moving up. Then, value of h_1 is **[JEE-2006]**

(A) $\frac{2h}{3}$ (B) $\frac{5h}{4}$

(C) $\frac{5h}{3}$ (D) $\frac{5h}{2}$

10. Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h_2 in figure) for which the cylinder remains in original position without application of force is **[JEE-2006]**

(A) $\frac{h}{3}$ (B) $\frac{4h}{9}$

(C) $\frac{2h}{3}$ (D) h

11. If height h_2 of water level is further decreased, then **[JEE-2006]**

(A) cylinder will not move up and remains at its original position

(B) for $h_2 = h/3$, cylinder again starts moving up

(C) for $h_2 = h/4$, cylinder again starts moving up

(D) for $h_2 = h/5$, cylinder again starts moving up

12. Statement-1

The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

Statement-2

In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **[JEE-2008]**

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct expalantion for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is **NOT**, a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

13. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End I has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve. **[JEE-2008]**

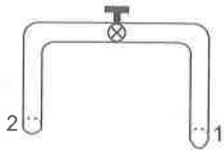


Figure.

(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles

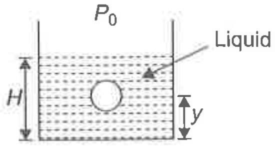
(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

(C) no changes occurs

(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

Common Data for Questions 14–16: A small spherical monoatomic ideal gas double $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ, (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure: **[JEE-2008]**



14. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

(A) Only the force of gravity

(B) The force due to gravity and the force due to the pressure of the liquid

(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(D) The force due to gravity and the force due to viscosity of the liquid

15. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0\left(\dfrac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{2/5}$

(B) $T_0\left(\dfrac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{2/5}$

(C) $T_0\left(\dfrac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{3/5}$

(D) $T_0\left(\dfrac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{3/5}$

16. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0\dfrac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$

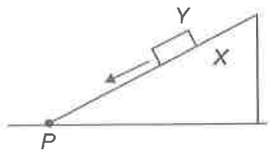
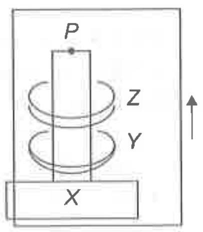
(B) $\dfrac{\rho\, nRgT_0}{(P_0+\rho\, gH)^{2/5}[P_0+\rho\, g(H-y)]^{3/5}}$

(C) $\rho_\ell nRgT_0\dfrac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$

(D) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}[P_0+\rho_\ell g(H-y)]^{2/5}}$

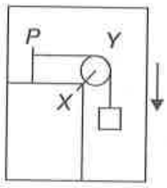
17. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. [Take atmospheric pressure = 1.0×10 5 Nm^{-2}, density of water = 1000 kg m^{-3} and $g = 10$ ms^{-2}. Neglect any effect of surface tension.] (Take temperature to be constant) **[JEE-2009]**

18. Column II shows five systems in which two objects are labeled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and/or Y Match these statements to the appropriate system(s) from **Column II**. **[JEE-2009]**

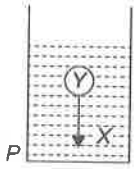
Column I	**Column II**
(A) The force exerted by X on Y has a magnitude Mg.	(P) Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity 
(B) The gravitational potential energy of X is continuously increasing.	(Q) Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity. 

(C) Mechanical energy of the system $X + Y$ is continuously decreasing

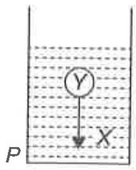
(R) A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.



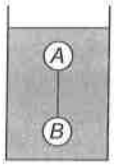
(S) A sphere Y of mass M is put is a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.



(T) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.



19. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if **[JEE-2011]**



(A) $d_A < d_F$ (B) $d_B > d_F$
(C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

Paragraph 20 and 21

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere. **[JEE-2014]**

20. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of
(A) 0.1 ms^{-1} (B) 1 ms^{-1}
(C) 2 ms^{-1} (D) 8 ms^{-1}

21. If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\dfrac{\rho_a}{\rho_l}}$ (B) $\sqrt{\rho_a \rho l}$

(C) $\sqrt{\dfrac{\rho l}{\rho_a}}$ (D) ρl

22. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in list I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those is List II and select the correct answer using the code given below the lists. **[JEE-2014]**

	List I		List II
P.	Lift is acceleration vertically up.	1.	d = 1.2 m
Q.	Lift is accelerating vertically with an acceleration less than the gravitational acceleration.	2.	$d > 1.2$ m
R.	Lift is moving vertically up with constant speed.	3.	$d < 1.2$ m
S.	Lift is falling freely.	4.	No water leaks out of the jar

Code:

(A) P-2, Q-3, R-2, S-4
(B) P-2, Q-3, R-1, S-4
(C) P-1, Q-1, R-1, S-4
(D) P-2, Q-3, R-1, S-1

ANSWER KEYS

Exercises

JEE Main

1. B	**2.** B	**3.** A	**4.** D	**5.** D	**6.** B	**7.** C	**8.** C	**9.** B	**10.** A
11. B	**12.** B	**13.** C	**14.** B	**15.** D	**16.** B	**17.** C	**18.** B	**19.** A	**20.** D
21. C	**22.** A	**23.** B	**24.** B	**25.** C	**26.** A	**27.** B	**28.** A	**29.** B	**30.** A
31. B	**32.** B	**33.** B	**34.** D	**35.** D	**36.** D	**37.** C	**38.** A	**39.** B	**40.** C
41. A	**42.** D	**43.** B	**44.** B	**45.** A	**46.** B	**47.** C	**48.** C	**49.** B	**50.** D
51. D	**52.** D	**53.** A	**54.** A						

JEE Advanced

1. D	**2.** B	**3.** B	**4.** B	**5.** D	**6.** A	**7.** A	**8.** B	**9.** C	**10.** D
11. A	**12.** B	**13.** A	**14.** B	**15.** D	**16.** B	**17.** B	**18.** C	**19.** D	**20.** C
21. C	**22.** A	**23.** D	**24.** C	**25.** D	**26.** B	**27.** D	**28.** C	**29.** A	**30.** A
31. D	**32.** A, C	**33.** A, C	**34.** B, C	**35.** D	**36.** A, C	**37.** B, D	**38.** B	**39.** D	

JEE Advanced

Level I

1. $h = \frac{2\text{ m}}{\pi}, H = \frac{11}{32\pi}$ m **2.** 101.8 Kgf – m **3.** 54.4 cm **4.** $45°, 9600\sqrt{2}$ (gauge) N/m^2

5. 100 kg **6.** $\omega = \frac{10}{\sqrt{3}}$ rad/s, $\tan\alpha = \frac{5}{3}$ **7.** 19.6 m, 4 sec **8.** 2.79 gm/cc

9. at the water surface, $h/2$ **10.** 37.5 N **11.** 0.75 **12.** **(a)** 5, **(b)** $\frac{2}{3}$

13. **(i)** 2500kg/m^3, $\frac{5000}{3}$ kg/m^3 **(ii)** $R_D = 7.5$ kg, $R_E = 2.5$ kg **14.** $\frac{3}{5}$ **15.** $\ell(d_2 - d_1)\left(\frac{k}{d_2} + Ag\right)$

16. $h_2 = 3\,h_1$ **17.** **(a)** $6\sqrt{2}$ m/s **(b)** $9.6\sqrt{2} \times 10^{-3}$ M^3/sec, **(c)** 4.6×10^4 N/m^2 **18.** $\sqrt{3}:\sqrt{2}$

19. 6.43×10^{-4} m^3/s **20.** 20 cm, 60 cm

Level II

1. 6 N **2.** 4 m/s^2, 10%, 0, 45 kPa **3.** $h_1 = \frac{2(3+\pi)}{15\pi} = 0.26$; $h_2 = \frac{3+\pi}{10\pi} = 0.195$ **4.** $x = \frac{1}{3}$

5. $\frac{Y}{y - x + z}$ **6.** $\rho = \frac{W}{\pi h^2 g \tan\alpha\left(R - \frac{1}{3}h\tan\alpha\right)}$ **7. (a)** $\left(\frac{M-m}{M+m}\right)g - \frac{(M-m)gx}{(M+m)L}$

(b) $t = \frac{\pi}{2}\sqrt{\frac{L}{g}\left(\frac{M+m}{M-m}\right)}$

8. $\sqrt{\frac{18g}{19a}}$ **9.** $\frac{\pi}{80}$ m^2 **10.** $h_1 = h_2$ **11.** $80\sqrt{5}$ sec, $40\sqrt{5}$ sec **12.** 431 sec

13. (i) 4 m/s, **(ii)** $F = 7.2$ N, **(iii)** $F_{min} = 0$, $F_{max} = 52.2$ N, **(iv)** both 4×10^{-3} m/s

14. (a) $\frac{d^2H^2S}{2\rho g} = 32\,\text{Kg}f - m$, **(b)** $\frac{1}{2}SH^2(1-d)^2 = 2\text{Kg}f - m$ **15.** $\frac{163}{36}$ cm, $\frac{388}{36}$ cm

16. $t = \frac{12\sqrt{14}}{\sqrt{15} - \sqrt{14}}$ **17.** $\frac{\rho g R}{2}$ **18.** 2 sec, 1 sec **19.** $\frac{b}{3}$

20. (a) (i) $D = \frac{5}{4}d$, **(ii)** $p = P_0 + \frac{1}{4}(6H + L)dg$; **(b) (i)** $v = \sqrt{\frac{g}{2}(3H - 4h)}$ **(ii)** $x = \sqrt{h(3H - 4h)}$

(iii) $x_{max}\ \frac{3}{4}H$

21. $w = \sqrt{\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)}$ **22. (i)** 0.2 m/s^2, **(ii)** $\sqrt{2g\frac{m_0}{A\rho}}$

23. +29625 J/m^3, – 30000 J/m^3

24. $m_{min} = \pi r^2 l(\sqrt{\rho\sigma} - \rho)$; if tilted then it's axis should become vertical C.M. should be lower than centre of buoyancy.

Previous Years' Questions

JEE Main

1. B **2.** D **3.** B **4.** C **5.** A **6.** A

JEE Advanced

1. A **2.** D **3.** D **4. (a)** 0, **(b)** $h = 0.25$ cm, **(c)** $a = \frac{g}{6}$ (upward)

5. $x = 2$ m **6.** $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$ **7.** C **8.** $H = \frac{L^2\omega^2}{2g}$

9. C **10.** B **11.** A **12.** A **13.** B **14.** D **15.** B **16.** B

17. 6 **18.** $A \to P, T$, $B \to Q, S, T$, $C \to P, R, T$, $D \to Q$ **19.** A, B, D

20. C **21.** A **22.** C

Chapter 2

Modern Physics–1

NATURE OF LIGHT

It was a matter of great interest for scientists of know that what exactly from the light is made up of or how the light behaves. This is briefly described over here

Newton's Corpuscular Theory

Newton was the first scientist who said that light is made 0 up tiny elastic particles called 'Corpuscles' which travels with the velocity of light. So according to Newtons, light is a particle.

Huygen's Wave Theory

Huygen was a scientist working parallel to Newton who come with a drastically different idea for nature of light and said that light is not a particle but a wave.

Maxwell's Electromagnetic Wave Theory

During the time of Huygen, his views regarding nature of light were not accepted as newton was a popular scientist of his time. but, when maxwell asserted that light is a electromagnetic wave, scientists started believing that light is a wave.

Max Planck's Quantum Theory of Light

Once again when scientists started believing that the light is a wave max Planck came with different idea and asserted that light is not a wave but a photon (i.e., a particle) which he proved through black body radiation spectrum. At this time there was a great confusion about the nature of light which was solved by de Broglie from where origin of theory of matter wave come into picture.

de Broglie Hypothesis

It supports dual nature of light (wave nature and particle nature). According to him the light consists of particles associated with definite amount of energy and momentum. These particles were later named as photons.

The photon posses momentum and is given by

$$P = \frac{h}{\lambda} \qquad (1)$$

P = momentum of one photon

λ = wavelength of wave.

h = Planck's constant = 6.62×10^{-34} Js.

A photon is a packet of energy. It posses energy given by

$$E = \frac{hc}{\lambda} \qquad (2)$$

where c = speed of light

de Broglie relates particle property (momentum) with wave property (wavelength) i.e., he favours dual nature of light.

Electron volt: It is the energy gained by an electron when it is accelerated through a potential difference of one volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}.$$

Now from eq. (2)

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} \text{ in Joule.}$$

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{\lambda \times 1.6 \times 10^{-19}} \text{ eV}$$

$$E = \frac{12400}{\lambda} \text{ eV}$$

where λ is in Å

Properties of Photon:

1. Photon travels with speed of light.
2. The rest mass of a photon is zero.

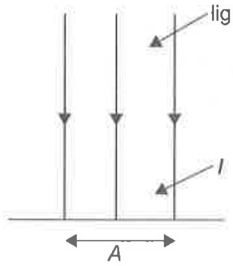
3. There is no concept of photon conservation.
4. All the photons of a particular frequency or wavelength posses the same energy irrespective of the intensity of the radiation.
5. The increase in the intensity of the radiation imply an increase in the number of photon's crossing a given area per second.

❖ When light travels from one medium to another medium then frequency = const (because it is the property of source) but v, λ changes

SOLVED EXAMPLES

EXAMPLE 1

A beam of light having wavelength λ and intensity 1 falls normally on an area A of a clean surface then find out the number of photon incident on the surface.



SOLUTION

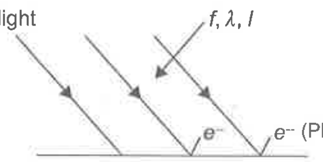
Total energy incident in time $t = IAt$

Energy of one photon $E = \dfrac{hc}{\lambda}$

Then number of photon incident in time

$$t = \frac{\text{Total energy incident}}{\text{energy of one photon}} = \frac{IAt\lambda}{hc}$$

Electron Emission Process:



When light is incident on a metal surface it was observed that electrons are ejected from a metal surface some times even when incredibly dim light such as that from starts and distance galaxies incident on it and some time electrons not comes out from the metal surface even high energetic or high intensity light falling on the metal surface.

This shows that the electron emission from a metal surface is not depends on the intensity of incident light but it is basically depends on the energy of the incident.

Photons no matters in number of photons are very less in a dim light, photo electric effect can be seen.

During the phenomenon of photoelectric effect one incident photon on metal surface can eject at most only one electron.

A photon is an energy packet which is fully absorbed not partially. Thus one photon can not be absorbed by more than one electron.

The minimum amount of energy of photon required to eject an electron out of a metal surface is called work function It is denoted by ϕ.

The work function depends on the nature of the metal.

1. The electron emission from a metal is only depends on the work function or energy of one photons.
2. But how many electrons comes out from the metal is depends on intensity of the falling light on energy of the light.
3. Energy of photon incident on metal will not necessarily cause emission of an electron even if its energy is more than work function. The electron after absorption may be involved in many other process like collision etc in which it can lose energy hence the ratio of no. of electrons emitted to the no. of photons incident on metal surface is less than unity.

Three Major Features of the Photoelectric Effect cannot be Explained in Terms of the Classical of the Wave Theory of Light.

(a) **The intensity problem:** Wave theory requires that the oscillating electric field vector E of the light wave increases in amplitude as the intensity of the light beam is increased. Since the force applied to the electron is eE, this suggests that the kinetic energy of the photoelectrons should also increased the light beam is made more intense. However observation shows that maximum kinetic energy is independent of the light intensity.

(b) **The frequency problem:** According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided only that the light is intense enough to supply the energy needed to eject the photoelectrons. However observations shows that there exists for each surface a characteristic cutoff frequency ν_{th}, for frequency less than ν_{th}, the photoelectric effect does not occur, no matter how intense is light beam.

(c) The time delay problem: If the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, the 'effective target area' for an electron in the metal is limited and probably not much more than that of a circle of diameter roughly equal to that of an atom. In the classical theory, the light energy is uniformly distributed over the wave front. Thus, if the light is feeble enough, there should be a measurable time lag, between the impinging of the light on the surface and the ejection of the photoelectron. During this interval the electron should be absorbing energy from the beam until it had accumulated enough to escape. However, no detectable time lag has ever been measured.

Now, quantum theory solves these problems in providing the correct interpretation of the photoelectric effect.

Threshold Frequency and Threshold Wavelength

We have discussed that to start photoelectric emission the energy of incident photon on metal surface must be more than the work function of the metal. If ϕ is the work function of the metal then there must be a minimum frequency of the incident light photon which is just able to eject the electron from the metal surface. This minimum frequency or threshold frequency ν_{th} can be given as

$$h\nu_{th} = \phi$$

Threshold frequency ν_{th} is a characteristic property of a metal as it is the minimum frequency of the light radiation required to eject a free electron from the metal surface.

As the threshold frequency is defined, we can also define threshold wavelength λ_{th} for a metal surface. Threshold wavelength is also called cut off wavelength. For a given metal surface threshold wavelength is the longest wavelength at which photo electric effect is possible. Thus we have

$$\frac{hc}{\lambda_{th}} = \phi$$

So for wavelength of incident light $\lambda > \lambda_{th}$, the energy of incident photons will become less then the work function of the metal and hence photoelectric effect will not start.

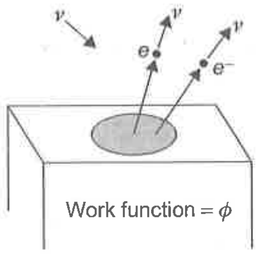
Thus for a given metal surface photoelectric emission will start at or $\nu > \nu_{th}$ or $\lambda < \lambda_{th}$.

Einstein Relation

Einstein suggested that the energy of photon ($h\nu > \phi$) which is more than work function of a metal when incident on the metal surface is used by the electron after absorption in two parts.

(i) A part of energy of absorbed photon is used by the free electron in work done in coming out from the metal surface as work function.

(ii) The remaining part of the photon energy will be gained by the electron in the form of kinetic energy after ejection from the metal surface.



If a light beam of frequency ν (each photon energy = $h\nu$) is incident on a metal surface having work function ϕ then for $h\nu > \phi$, we have

$$h\nu = \phi + \frac{1}{2}mv_{\max}^2 \qquad (1)$$

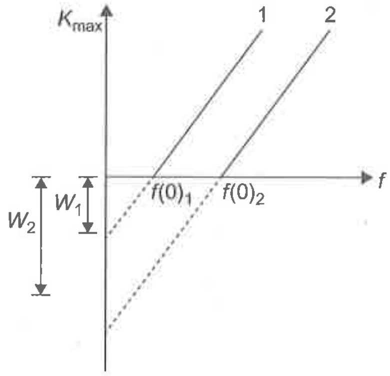
In equation (1) the second terms on right hand side of equation is $\frac{1}{2}mv_{\max}^2$, which is the maximum kinetic energy of the ejected electron.

In practical cases whenever an electron absorbs a photon from incident light, it comes out from the metal surface if $h\nu > \phi$ but in process of ejection it may collide with the neighbouring electrons and before ejection it may loose some energy during collisions with the neighbouring electrons. In this case after ejection the kinetic energy of ejected electrons will be certainly less then $(h\nu - \phi)$. If we assume there are some electrons which do not loose any energy in the process of ejection, will come out from the metal surface with the maximum kinetic energy given as

$$\frac{1}{2}mv_{\max}^2 = h\nu - \phi$$

Thus all the ejected electrons from the metal surface may have different kinetic energies, distributed from 0 to $\frac{1}{2}mv_{\max}^2$.

Graph between K_{max} and f



Let us plot a graph between maximum kinetic energy K_{max} of photoelectrons and frequency f of incident light. The equation between K_{max} and f is,

$$K_{max} = hf - W$$

comparing it with $y = mx + c$, the graph between K_{max} and f is a straight line with positive slope and negative intercept.

From the graph we can note the following points.

(i) $K_{max} = 0$ at $f = f_0$

(ii) Slope of the straight line is h, a universal constant. i.e., if graph is plotted for two different metals 1 and 2, slope of both the lines is same.

(iii) The negative intercept of the line is W, the work function, which is characteristic of a metal, i.e., intercepts for two different metals will be different. Further,

$$W_2 > W_1 \quad \therefore \quad (f_0)_2 > (f_0)_1$$

Here f_0 = threshold frequency as $W = hf_0$

SOLVED EXAMPLES

EXAMPLE 2

The photoelectric threshold of the photo electric effect of a certain metal is 2750 Å. Find

(i) The work function of emission of an electron from this metal,

(ii) Maximum kinetic energy of these electrons,

(iii) The maximum velocity of the electrons ejected from the metal by light with a wavelength 1800 Å.

SOLUTION

(i) Given that the threshold wavelength of a metal is λ_{th} = 2750 Å. Thus work function of metal can be given as

$$\phi = \frac{hc}{\lambda_{th}} = \frac{12431}{2750}\text{ eV} = 4.52\text{ eV}$$

(ii) The energy of incident photon of wavelength 1800 Å on metal in eV is

$$E = \frac{12431}{1800}\text{ eV} = 6.9\text{ eV}$$

Thus maximum kinetic energy of ejected electrons is

$$KE_{max} = E - \phi = 6.9 - 4.52\text{ eV} = 2.38\text{ eV}$$

(iii) If the maximum speed of ejected electrons is v_{max} then we have

$$\frac{1}{2}mv_{max}^2 = 2.38\text{ eV}$$

$$\text{or } v_{max} = \sqrt{\frac{2 \times 2.38 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 9.15 \times 10^5\text{ m/s}$$ ■

EXAMPLE 3

Light quanta with a energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electrons flies out.

SOLUTION

According to Einstein's photoelectric equation

$$E = \frac{1}{2}mv_{max}^2 = hv - \phi = 4.9 - 4.5 = 0.4\text{ eV}$$

If E be the energy of each ejected photo electron

momentum of electrons is $P = \sqrt{2mE}$.

We know that change of momentum is impulse. Here the whole momentum of electron is gained when it is ejected out thus impulse on surface is

$$\text{Impulse} = \sqrt{2mE}.$$

Substituting the values, we get

$$\text{Maximum impulse} = \sqrt{2 \times 9.1 \times 10^{-31} \times 0.4 \times 1.6 \times 10^{-19}}$$
$$= 3.45 \times 10^{-25}\text{ kg/sec}$$ ■

EXAMPLE 4

In a experiment tungsten cathode which has a threshold 2300 Å is irradiated by ultraviolet light of wavelength 1800 Å. Calculate

(i) Maximum energy of emitted photoelectron and

(ii) Work function for tungsten

(Mention both the results in electron-volts)

Given Planck's constant $h = 6.6 \times 10^{-34}$ J-s, 1 eV = 1.6 $\times 10^{-19}$ J and velocity of light $c = 3 \times 10^8$ m/sec

SOLUTION

The work function of tungsten cathode is

$$\phi = \frac{hc}{\lambda_{th}} = \frac{12431}{2300}\text{eV} = 5.4\,\text{eV}$$

The energy in eV of incident photons is

$$E = \frac{hc}{\lambda} = \frac{12431}{1800}\text{eV}$$

The maximum kinetic energy of ejected electrons can be given as

$$KE_{max} = E - \phi = 6.9 - 5.4\,\text{eV} = 1.5\,\text{eV}$$

EXAMPLE 5

Light of wavelength 1800 Å ejects photoelectrons from a plate of a metal whose work functions is 2 eV. If a uniform magnetic field of 5×10^{-5} T is applied parallel to plate, what would be the radius of the path followed by electrons ejected normally from the plate with maximum energy.

SOLUTION

Energy of incident photons in eV is given as

$$E = \frac{12431}{1800}\text{eV}$$

As work function of metal is 2 eV, the maximum kinetic energy of ejected electrons is

$$KE_{max} = E - \phi$$
$$= 6.9 - \text{eV} = 4.9 \text{ eV}$$

If v_{max} be the speed of fasted electrons then we have

$$\frac{1}{2}mv_{max}^2 = 4.9 \times 1.6 \times 10^{-19}\text{ J}$$

or $$v_{max} = \sqrt{\frac{2 \times 4.9 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.31 \times 10^6 \text{ m/s}$$

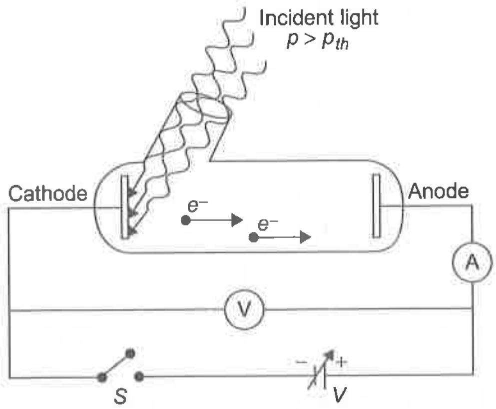
When an electron with this speed enters a uniform magnetic field normally it follows a circular path whose radius can be given by

$$r = \frac{mv}{qB} \quad \left[\text{As } qvB = \frac{mv^2}{r}\right]$$

or $$r = \frac{9.1 \times 10^{-31} \times 1.31 \times 10^6}{1.6 \times 10^{-19} \times 5 \times 10^{-5}} \quad \text{or} \quad r = 0.149 \text{ m}$$

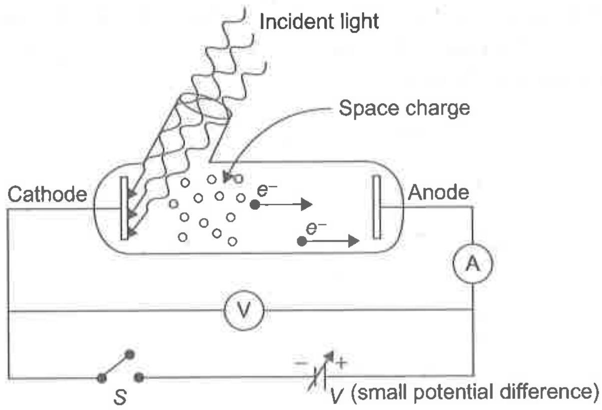
Experimental Study of Photo Electric Effect

Experiments with the photoelectric effect are performed in a discharge tube apparatus as illustrated in figure shown. The cathode of discharge tube is made up of a metal which shows photoelectric effect on which experiment is being carried out.

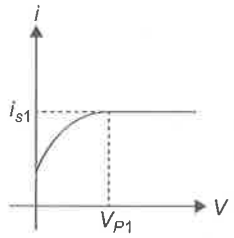


A high potential is applied to a discharge tube through a variable voltage source and a voltmeter and an ammeter are connected a measure the potential difference across the electrodes and to measure photoelectric current. Light with frequency more than threshold frequency of cathode metal is incident on it, due to which photoelectrons are emitted from the cathode. These electrons will reach the anode and constitute the photoelectric current which the ammeter will show.

Now we start the experiment by closing the switch S. Initially the variable battery source is set at zero potential. Even at zero potential variable source, ammeter will show some current because due to the initial kinetic energy some electrons will reach the anode and cause some small current will flow. But as we know majority of ejected electrons have low values of kinetic energies which are collected outside the cathode and create a could of negative charge, we call space charge, as shown in figure shown.

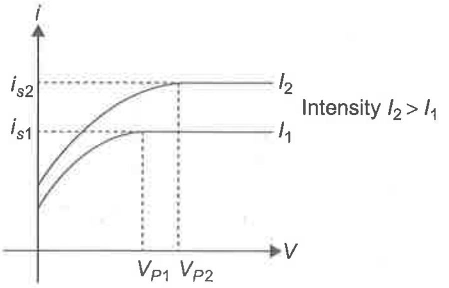


If the potential difference applied across the discharge tube is gradually increased from the variable source, positive potential of anode starts pulling electrons from the space charge. As potential difference increases, space charge decrease and simultaneously the photoelectric current in circuit also increases. This we can also see in the variation graph of current with potential difference as shown in figure shown.



A shown in graph, we can see as potential difference increases, current in circuit increases. But at a higher voltage V_{P1} space charge vanishes and at this voltage anode is able to pull the slowest electron (zero kinetic energy) ejected by the cathode. Now as all the ejected electrons from cathode start reading anode. If further potential difference is increased, it will not make any difference in the number of electrons reaching the anode hence, further increases in potential difference will not increases the current. This we can see in figure shown that beyond V_{P1} current in circuit becomes constant. This current i_{s1} is called saturation current. This potential difference V_{P1} at which current becomes saturated is called 'pinch off voltage'.

Now if the frequency of incident light is kept constant and its intensity is further increased, then the number of incident photons will increase which increases the number of ejected photo electrons so current in circuit increases and now in this case at higher intensity of incident light, current will not get saturated at potential difference V_{P1} as now due to more electron emission, space charge will be more and it will not vanish at V_{P1}. To pull all the electrons emitted from cathode more potential difference is required. This we can se from figure shown, that at higher intensity $I_2(I_2 > I_1)$ current becomes saturated at higher value of potential difference V_{P2}.



Beyond V_{P2}, we can see that all the electrons ejected from cathode are reaching the anode are current become saturated at i_{s2} because of more electrons. Another point we can see from figure shown that when $V = 0$ then also current is more at high intensity incident radiation as the number of electrons of high kinetic energy are also more in the beginning which will reach anode by penetrating the space charge.

Kinetic Energies of Electrons Reaching Anode

We know that when electrons are ejected from cathode then kinetic energies may vary from 0 to $\frac{1}{2}mv_{max}^2$. If V is the potential difference applied across the discharge tube then it will accelerates the electron while reaching the anode. the electron which is ejected from cathode with zero kinetic energy will be the slowest one reaching the anode if its speed is v_1 at anode then we have

$$0 + ve = \frac{1}{2}mv_1^2$$

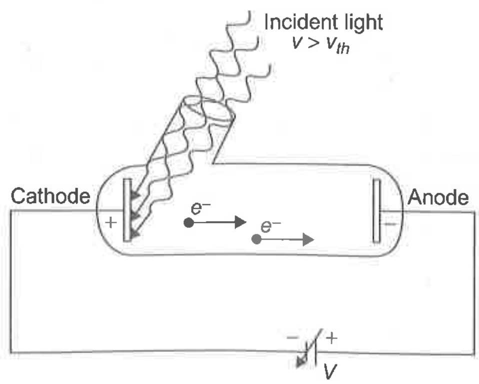
Similarly the electron ejected from cathode with maximum kinetic energy $\frac{1}{2}mv_{max}^2$ will be the fastest one when it will reach anode. If its speed is v_2 at anode then we have

$$\frac{1}{2}mv_{max}^2 + \mathrm{eV} = \frac{1}{2}mv_2^2$$

Thus we can say that all the electrons reaching anode will have their speeds distributed from v_1 to v_2.

Reversed Potential Across Discharge Tube

Now the experiment is repeated with charging the polarity of source across the discharge tube. Now positive terminal of source is connected to the cathode of discharge tube. When a light beam incident on the cathode with $(h\nu > \phi)$, photoelectrons are ejected and move towards anode with negative polarity.



Now the electrons which are ejected with very low kinetic energy are attracted back to the cathode because of its positive polarity. Those electrons which have high kinetic energies will rush toward, anode and may constitute the current in circuit.

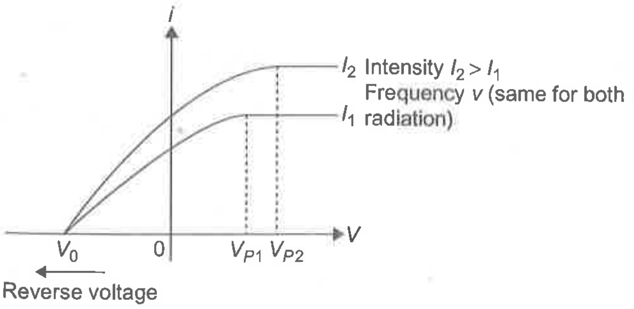
In this case the fastest electron ejected from cathode will be retarded during its journey to anode. As the maximum kinetic energy just after emission at cathode is $\frac{1}{2}mv_{max}^2$, if potential difference across the discharge tube is V then the seed v_f with which electrons will reach anode can be given as

$$\frac{1}{2}mv_{max}^2 - eV = \frac{1}{2}mv_f^2 \qquad (1)$$

Thus all the electrons which are reaching anode will have speed less then or equal to v_f. Remaining electrons which have relatively low kinetic energy will either be attracted to cathode just after ejection or will return during their journey from cathode to anode. Only those electrons will case current of flow in circuit which have high kinetic energies more then eV which can overcome the electric work against electric forces on electron due to opposite polarity of source.

Cut off Potential or Stopping Potential

We have seen with reverse polarity electrons are retarded in the discharge tube. If the potential difference is increased with reverse polarity, the number of electrons reaching anode will decrease hence photo electric current in circuit also decreases, this we can see from figure shown which shows variation of current with increase in voltage across discharge tube in opposite direction. Here we can see that at a particular reverse voltage V_0, current in circuit becomes zero. This is the voltage at which the faster electron from cathode will be retarded and stopped just before reaching the anode.



This voltage V_0, we can calculate from equation (1) by substituting $v_f = 0$ hence

$$\frac{1}{2}mv_{max}^2 - eV_0 = 0$$

or

$$eV_0 = \frac{1}{2}mv_{max}^2$$

or

$$V_0 = \frac{\frac{1}{2}mv_{max}^2}{e} \qquad (2)$$

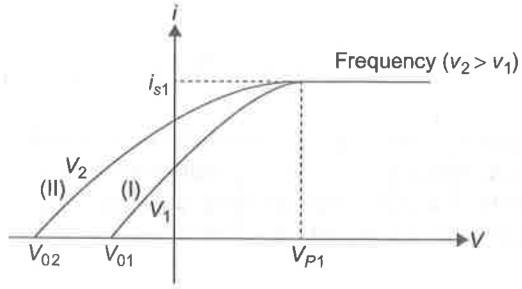
or

$$V_0 = \frac{hv - \phi}{e} \qquad (3)$$

We can see one more thing in figure shown that the graphs plotted for two different intensities I_1 and I_2, V_0 is same. Current in both the cases in cut off at same reverse potential V_0. The reason for this is equation (2) and (3). It is clear that the value of V_0 depends only on the maximum kinetic energy of the ejected electrons which depends only on frequency of light and not on intensity of light. Thus in above two graphs as frequency of incident light is same, the value of V_0 is also same. This reverse potential difference V_0 at which the fastest photoelectron is stopped and current in he circuit becomes zero is called cut off potential or stopping potential.

Effect of Change in Frequency of Light on Stopping Potential

If we repeat the experiment by increasing the frequency of incident light with number of incident photons constant, the variation graph of current with voltage will be plotted as shown in figure shown.



This graph is plotted for two incident light beams of different frequency v_1 and v_2 and having same photon flux. As the number of ejected photoelectrons are same in the two cases of incident light here we can see that the pinch off voltage V_{01} as well as saturation current i_{s1} are same. But as in the two cases the kinetic energy of fastest electron are different as frequencies are different, the stopping potential for the two cases will be different. In graph II as frequency of incident light is more, the maximum kinetic energy of photoelectrons will also be high and to stop it high value of stopping potential is needed. These here V_{01} and V_{02} can be given as

$$V_{01} = \frac{h\nu_1 - \phi}{e} \quad (4)$$

and

$$V_{02} = \frac{h\nu_2 - \phi}{e} \quad (5)$$

In general for a given metal with work function ϕ, if V_o is the stopping potential for an incident light of frequency ν then we have

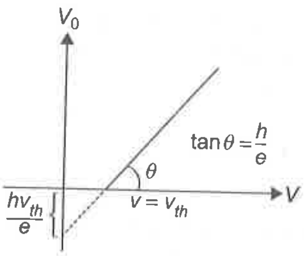
$$eV_0 = h\nu - \phi$$

or

$$eV_0 = h\nu - h\nu_{th} \quad (6)$$

or

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{h\nu_{th}}{e} \quad (7)$$



Equation (7) shows that stopping potential V_0 is linearly proportional to the frequency ν of incident light. The variation of stopping potential with frequency ν can be shown in figure shown.

Here equation (6) can be written as

$$\frac{1}{2}mv_{max}^2 = eV_0 = h(\nu - \nu_{th}) \quad (8)$$

This equation (8) is called Einstein's Photo Electric Effect equation which gives a direction relationship between the maximum kinetic energy stopping potential frequency of incident light and the threshold frequency.

SOLVED EXAMPLES

EXAMPLE 6

Find the frequency of light which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photo electric effect begins in this metal at frequency of 6×10^{14} sec^{-1}. Find the work function for this metal.

SOLUTION

The threshold frequency for the given metal surface is

$$\nu_{th} = 6 \times 10^{14} \text{ Hz}$$

Thus the work function for metal surface is

$$\phi = h\nu_{th} = 6.63 \times 10^{-34} \times 6 \times 10^{14} = 3.978 \times 10^{-19} \text{ J}$$

As stopping potential for the ejected electrons is 3 V, the maximum kinetic energy of ejected electrons will be

$$KE_{max} = 3 \text{ eV} = 3 \times 1.6 \times 10^{-19} \text{ J} = 4.8 \times 10^{-19} \text{ J}$$

According to photo electric effect equation, we have

$$h\nu = h\nu_{th} + KE_{max}$$

or frequency of incident light is

$$\nu = \frac{\phi + KE_{max}}{h} = \frac{3.978 \times 10^{-19} + 4.8 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 1.32 \times 10^{15} \text{ Hz}$$

EXAMPLE 7

Electrons with maximum kinetic energy 3 eV are ejected from a metal surface by ultraviolet radiation of wavelength 1500 Å. Determine the work function of the metal, the threshold wavelength of metal and the stopping potential difference required to stop the emission of electrons.

SOLUTION

Energy of incident photon in eV is

$$E = \frac{12431}{1500} \text{eV}$$

According to photo electric effect equation, we have

$$E = \phi + KE_{max} \quad \Rightarrow \quad \text{or} \quad \phi = E - KE_{max}$$

or $\quad = 8.29 - 3 \text{ eV} \quad$ or $\quad = 5.29 \text{ eV}$

Threshold wavelength for the metal surface corresponding to work function 5.29 eV is given as

$$\lambda_{th} = \frac{12431}{5.29} \text{Å} = 2349.9 \text{ Å}$$

Stopping potential for the ejected electrons can be given as

$$V_0 = \frac{KE_{max}}{e} = \frac{3 \text{ eV}}{e} = 3 \text{ V}$$

EXAMPLE 8

Calculate the velocity of a photo-electron, if the work function of the target material is 1.24 eV and the wavelength of incident light is 4360 Å. What retarding potential is necessary to stop the emission of the electrons?

SOLUTION

Energy of incident photons in eV on metal surface is

$$E = \frac{12431}{4360} \text{eV} = 2.85 \text{ eV}$$

According to photo electric effect equation we have

$$E = \phi + \frac{1}{2}mv_{max}^2 \quad \text{or} \quad \frac{1}{2}mv_{max}^2 = E - \phi$$

$$= 2.85 - 1.24 \text{ eV} = 1.61 \text{ eV}$$

The stopping potential for these ejected electrons can be given as

$$V_0 = \frac{1/2mv_{max}^2}{e} = \frac{1.61 \text{ eV}}{e} = 1.61 \text{ V}$$

■

EXAMPLE 9

Determine the Planck's constant h if photoelectrons emitted from a surface of a certain metal by light of frequency 2.2×10^{15} Hz are fully retarded by a reverse potential of 6.6 V and those ejected by light of frequency 4.6×10^{15} Hz by a reverse potential of 16.5 eV.

SOLUTION

From photo electric effect equation, we have

Here $\quad h\nu_1 = \phi + \text{eV}_{01} \quad (1)$

and $\quad h\nu_2 = \phi + 2\text{eV}_{02} \quad (2)$

Subtracting equation (1) from equation (2), we get

$$h(\nu_2 - \nu_1) = e(\nu_{02} - \nu_{01})$$

or $$h = \frac{(\nu_{02} - \nu_{01})(1.6 \times 10^{-19})}{(\nu_2 - \nu_1)}$$

or $$h = \frac{(16.5 - 6.6)(1.6 \times 10^{-19})}{(4.6 - 2.2) \times 10^{15}}$$

or $$= 6.6 \times 10^{-34} \text{ J-s}$$

■

EXAMPLE 10

When a surface is irradiated with light of wavelength 4950 Å, a photo current appears which vanishes if a retarding potential greater than 0.6 V is applied across the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1.1 V. Find the work function of the emitting surface and the wavelength of second source. If the photo electrons (after emission from the surface) are subjected to a magnetic field of 10 T, what changes will be observed in the above two retarding potentials.

SOLUTION

In first case the energy of incident photon in eV is

$$E_1 = \frac{12431}{4950} \text{eV} = 2.51 \text{ eV}$$

The maximum kinetic energy of ejected electrons is

$$KE_{max\,1} = \text{eV}_{01} = 0.6 \text{ eV}$$

Thus work function of metal surface is given as

$$\phi = E_1 - KE_{max\,1} = 2.51 - 0.6 \text{ eV} = 1.91 \text{ eV}$$

In second case the maximum kinetic energy of ejected electrons will become

$$KE_{max\,2} = \text{eV}_{02} = 1.1 \text{ eV}$$

Thus the incident energy of photons can be given as

$$E_2 = \phi + KE_{max\,2}$$

$$E_2 = 1.91 + 1.1 \text{ eV} = 3.01 \text{ eV}$$

Thus the wavelength of incident photons in second case will be

$$\lambda = \frac{12431}{3.01} \text{Å} = 4129.9 \text{ Å}$$

When magnetic field is present there will be no effect on the stopping potential as magnetic force can not change the kinetic energy of ejected electrons. ■

EXAMPLE 11

(a) If the wavelength of the light incident on a photoelectric cell be reduced from λ_1 to λ_2 Å, then what will be the change in the cut-off potential?

(b) Light is incident on the cathode of a photocell and the stopping voltages are measured from light of two difference wavelengths. From the data given below, determine the work function of the metal of the cathode in eV and the value of the universal constant hc/e.

Wavelength (Å)	Stopping voltage (volt)
4000	1.3
4500	0.9

SOLUTION

(a) Let the work function of the surface be ϕ. If ν be the frequency of the light falling on the surface, then according to Einstein's photoelectric equation, the maximum kinetic energy KE_{max} of emitted electron is given by

$$KE_{max} = h\nu - \phi = \frac{hc}{\lambda} - \phi$$

We know that, $KE_{max} = eV_0$

Where V_0 = cut-off potential.

or $\quad eV_0 = \frac{hc}{\lambda} - \phi \quad$ or $\quad V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$

Now, $\Delta V_0 = V_{02} - V_{01}$

$$= \left(\frac{hc}{e\lambda_2} - \frac{\phi}{e}\right) - \left(\frac{hc}{e\lambda_1} - \frac{\phi}{e}\right)$$

$$= \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = \frac{hc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right) \qquad (1)$$

(b) From equation (1), we have

$$\frac{hc}{e} = \frac{\Delta V_0(\lambda_1\lambda_2)}{\lambda_1 - \lambda_2}$$

$$= \frac{(1.3 - 0.9)[(4000 \times 10^{-10}) \times (4500 \times 10^{-10})]}{500 \times 10^{-10}}$$

$$= 1.44 \times 10^{-6}\ \text{V/m}$$

Now, $V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$

or $\quad \frac{\phi}{e} = \frac{hc}{e\lambda} - V_0 = \frac{1.44 \times 10^{-6}}{4000 \times 10^{-10}} - 1.3 = 2.3\ \text{V}$

or $\quad \phi = 2.3\ \text{eV}$ ■

EXAMPLE 12

A low intensity ultraviolet light of wavelength 2271 Å irradiates a photocell made of molybdenum metal. If the stopping potential is 1.3 V, find the work function of the metal. Will the photocell work if it is irradiated by a high intensity red light of wavelength 6328 Å?

SOLUTION

The energy in eV of incident photons is

$$E = \frac{12431}{2271}\ \text{eV} = 5.47\ \text{eV}$$

As stopping potential for ejected electrons is 1.3 V, the maximum kinetic energy of ejected electrons will be

$$KE_{max} = eV_0 = 1.3\ \text{eV}$$

Now from photoelectric effect equation, we have

$$E = \phi + KE_{max}$$

or $\quad \phi = E - KE_{max}$

or $\quad \phi = 5.47 - 1.3\ \text{eV} = 4.17\ \text{eV}$

Energy in eV for photons for red light of wavelength 6328 Å is

$$E' = \frac{12431}{6328}\ \text{eV} = 1.96\ \text{eV}$$

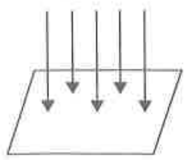
As $E' < \phi$, photocell will not work if irradiated by this red light no matter however intense the light will be. ■

FORCE DUE TO RADIATION (PHOTON)

Each photon has a definite energy and a definite linear momentum. All photons of light of a particular wavelength λ have the same energy $E = \frac{hc}{\lambda}$ and the same momentum $p = \frac{h}{\lambda}$.

When light of intensity I falls on a surface, it exerts force on that surface. Assume absorption and reflection coefficient of surface be 'a' and 'r' and assuming no transmission.

Assume light beam falls on surface of surface area 'A' perpendicularly as shown in figure.



For calculating the force exerted by beam on surface, we consider following cases.

Case I: $a = 1, r = 0$

initial momentum of the photon $= \frac{h}{\lambda}$

final momentum of photon = 0

change in momentum of photon $= \dfrac{h}{\lambda}$ (upward)

$$\Delta P = \frac{h}{\lambda}$$

energy incident per unit time = IA

number of photons incident per unit time $= \dfrac{IA}{h\nu} = \dfrac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time $= n\Delta P = \dfrac{IA\lambda}{hc} \times \dfrac{h}{\lambda} = \dfrac{IA}{c}$ (upward)

force on photons = total change in momentum per unit time

$= \dfrac{IA}{c}$ (upward)

$\therefore$ force on plate due to photon $(F) = \dfrac{IA}{c}$ (downward)

pressure $= \dfrac{F}{A} = \dfrac{IA}{cA} = \dfrac{I}{c}$

Case II: when $r = 1$, $a = 0$

initial momentum of the photon $= \dfrac{h}{\lambda}$ (downward)

final momentum of photon $= \dfrac{h}{\lambda}$ (upward)

change in momentum $= \dfrac{h}{\lambda} + \dfrac{h}{\lambda} = \dfrac{2h}{\lambda}$

$\therefore$ energy incident per unit time = IA

number of photons incident per unit time $= \dfrac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time $= n \cdot \Delta P = \dfrac{IA\lambda}{hc} \cdot \dfrac{2h}{\lambda} = \dfrac{2IA}{C}$

force = total change in momentum per unit time

$F = \dfrac{2IA}{c}$ (upward on photons and downward on the plate)

pressure $P = \dfrac{F}{A} = \dfrac{2IA}{cA} = \dfrac{2I}{c}$

Case III: When $0 < r < 1$ $a + r = 1$

change in momentum of photon when it is reflected $= \dfrac{2h}{\lambda}$ (upward)

change in momentum of photon when it is absorbed $= \dfrac{h}{\lambda}$ (upward)

number of photons incident per unit time $= \dfrac{IA\lambda}{hc}$

Number of photons reflected per unit time $= \dfrac{IA\lambda}{hc} \cdot r$

Number of photon absorbed per unit time $= \dfrac{IA\lambda}{hc}(1 - r)$

force due to absorbed photon $(F_a) = \dfrac{IA\lambda}{hc}(1 - r) . \dfrac{h}{\lambda}$

$= \dfrac{IA}{c}(1 - r)$ (downward)

Force due to reflected photon $(F_r) = \dfrac{IA\lambda}{hc} \cdot r \dfrac{2h}{\lambda} = \dfrac{2IA\lambda}{c}$ (downward)

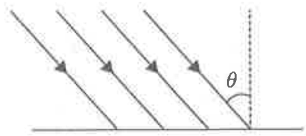
total force $= F_a + F_r = \dfrac{IA}{c}(1 - r) + \dfrac{2IAr}{c} = \dfrac{IA}{c}(1 + r)$

Now pressure $P = \dfrac{IA}{c}(1 + r) \times \dfrac{1}{A} = \dfrac{I}{c}(1 + r)$

SOLVED EXAMPLES

EXAMPLE 13

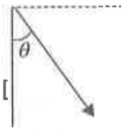
Calculate force exerted by light beam if light is incident on surface at an angle θ as shown in figure. Consider all cases.



SOLUTION

Case I: $a = 1$, $r = 0$

initial momentum of photon (in downward direction at an angle θ with vertical) is h/λ final momentum of photon = 0

change in momentum (in upward direction at an angle θ with vertical) $= \dfrac{h}{\lambda}$ []

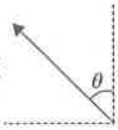
energy incident per unit time = $IA \cos \theta$

Intensity = power per unit normal area

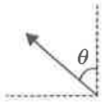
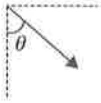
$$I = \frac{P}{A\cos\theta} \qquad P = IA\cos\theta$$

Number of photons incident per unit time $= \dfrac{IA\cos\theta}{hc} \cdot \lambda$

total change in momentum per unit time (in upward direction at an angle θ with vertical)

$$= \frac{IA\cos\theta\,\lambda}{hc} \cdot \frac{h}{\lambda} = \frac{IA\cos\theta}{c}$$ []

Force (F) = total change in momentum per unit time

$F = \dfrac{IA\cos\theta}{c}$ (direction  on photon and  on the plate)

Pressure = normal force per unit Area

$$\text{Pressure} = \frac{F\cos\theta}{A} \qquad P = \frac{IA\cos^2\theta}{cA} = \frac{I}{c}\cos^2\theta$$

Case II: When $r = 1$, $a = 0$

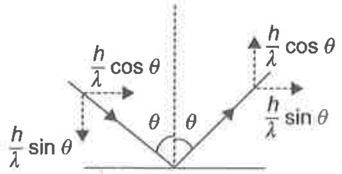
$\therefore$ change in momentum of one photon

$$= \frac{2h}{\lambda}\cos\theta \quad \text{(upward)}$$

Number of photons incident per unit time

$$= \frac{\text{energy incident per unit time}}{h\nu}$$

$$= \frac{IA\cos\theta \cdot \lambda}{hc}$$



$\therefore$ total change in momentum per unit time

$$= \frac{IA\cos\theta.\lambda}{hc} \times \frac{2h}{\lambda}\cos\theta = \frac{2IA\cos^2\theta}{c} \quad \text{(upward)}$$

$\therefore$ force on the plate $= \dfrac{2IA\cos^2\theta}{c}$ (downward)

$$\text{Pressure} = \frac{2IA\cos^2\theta}{cA} \qquad P = \frac{2I\cos^2\theta}{c}$$

Case III: $0 < r < 1$, $a + r = 1$

change in momentum of photon when it is reflected

$$= \frac{2h}{\lambda}\cos\theta \quad \text{(downward)}$$

change in momentum of photon when it is absorbed $= \dfrac{h}{\lambda}$

(in the opposite direction of incident beam)

energy incident per unit time = $IA\cos\theta$)

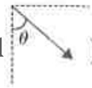
number of photons incident per unit time $= \dfrac{IA\cos\theta \cdot \lambda}{hc}$

number of reflected photon $(n_r) = \dfrac{IA\cos\theta \cdot \lambda r}{hc}$

number of absorbed photon $(n_Q) = \dfrac{IA\cos\theta \cdot \lambda}{hc}(1-r)$

force on plate due to absorbed photons $F_a = n_a \cdot \Delta P_a$

$$= \frac{IA\cos\theta.\lambda}{hc}(1-r)\frac{h}{\lambda}$$

$= \dfrac{IA\cos\theta}{hc}(1-r)$ (at an angle θ with vertical )

force on plate due to reflected photons $F_r = n_r\,\Delta P_r$

$$= \frac{IA\cos\theta \cdot \lambda}{hc} \times \frac{2h}{\lambda}\cos\theta \quad \text{(vertically downward)}$$

$$= \frac{IA\cos^2\theta}{c}2r$$

now resultant force is given by

$$F_R = \sqrt{F_r^2 + F_a^2 + 2F_aF_r\cos\theta}$$

$$= \frac{IA\cos\theta}{c}\sqrt{(1-r)^2 + (2r)^2\cos^2\theta + 4r(r-1)\cos^2\theta}$$

and, pressure $P = \dfrac{F_a\cos\theta + F_r}{A}$

$$= \frac{IA\cos\theta(1-r)\cos\theta}{cA} + \frac{IA\cos^2\theta \cdot 2r}{cA}$$

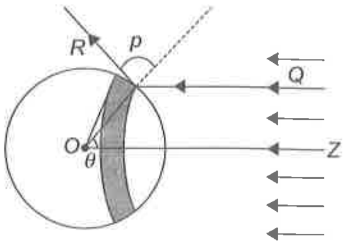
$$= \frac{I\cos^2\theta}{c}(1-r) + \frac{I\cos^2\theta}{c}2r = \frac{I\cos^2\theta}{c}(1+r)$$ ■

EXAMPLE 14

A perfectly reflecting solid sphere of radius r is kept in the path of a parallel beam of light of large aperture. If the beam carries an intensity I, find the force exerted by the beam on the sphere.

SOLUTION

Let O be the centre of the sphere and OZ be the line opposite to the incident beam (figure). Consider a radius about OZ to get a making an angle θ with OZ. Rotate this radius about OZ to get a circle on the sphere. Change θ to $\theta + d\theta$ and rotate the radius about OZ to get another circle on the sphere. The part of the sphere between these circles is a ring of area $2\pi r^2 \sin\theta\, d\theta$. Consider a small part ΔA of this ring at P. Energy of light falling on this part in time Δt is



$$\Delta U = I\Delta t(\Delta A \cos\theta)$$

The momentum of this light falling on ΔA is $\Delta U/c$ along QP. The light is reflected by the sphere along PR. The change in momentum is

$$\Delta p = 2\frac{\Delta U}{c}\cos\theta = \frac{2}{c}\Delta t\ (\Delta A\cos^2\theta)\ (\text{direction along } \vec{OP})$$

The force on ΔA due to the light falling on it, is

$$\frac{\Delta p}{\Delta t} = \frac{2}{c}\Delta A\ \cos^2\theta \quad (\text{direction along } \vec{OP})$$

The resultant force on the ring as well as on the sphere is along ZO by symmetry. The component of the force on ΔA along ZO

$$\frac{\Delta p}{\Delta t}\cos\theta = \frac{2}{c}I\Delta A\cos^2\theta \quad (\text{along } \vec{ZO})$$

The force acting on the ring is

$$dF = \frac{2}{c}I(2\pi r^2\sin\theta d\theta)\cos^3\theta$$

The force on the entire sphere is $F = \int_0^{\pi/2}\frac{4\pi r^2 I}{c}\cos^3\theta d\theta$

$$F = \int_0^{\pi/2}\frac{4\pi r^2 I}{c}\cos^3\theta d(\cos\theta)$$

$$= -\int_{0-0}^{\pi/2}\frac{4\pi r^2 I}{c}\left[\frac{\cos^4\theta}{4}\right]_0^{\pi/2} = \frac{\pi r^2 I}{c}$$

Note that integration is done only for the hemisphere that faces the incident beam.

DE BROGLIE WAVELENGTH OF MATTER WAVE

A photon of frequency ν and wavelength λ has energy.

$$E = h\nu = \frac{hc}{\lambda}$$

By Einstein's energy mass relation, $E = mc^2$ the equivalent mass m of the photon is given by.

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}. \quad (1)$$

or $$\lambda = \frac{h}{mc} \quad \text{or} \quad \lambda = \frac{h}{p} \quad (2)$$

Here p is the momentum of photon. By analogy de Broglie suggested that a particle of mass m moving with speed v behaves in some ways like waves of wavelength λ (called de Broglie wavelength and the wave is called matter wave) given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad (3)$$

where p is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2\text{ Km}}$$

and a charge q when accelerated by a potential difference V gains a kinetic energy $K = qV$. Combining all these relations Eq. (3), can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2\text{ Km}}} = \frac{h}{\sqrt{2qVm}}$$

(de = Broglie wavelength) (4)

de Broglie Wavelength for an Electron

If an electron (charge = e) is accelerated by a potential of V volts, it acquires a kinetic energy,

$$K = \text{eV}$$

Substituting the value of h, m and q in Eq. (4), we get a simple formula for calculating de Broglie wavelength of an electron.

$$\lambda(\text{in Å}) = \sqrt{\frac{150}{V(\text{in volts})}}$$

de Broglie Wavelength of a Gas Molecule

Let us consider a gas molecule at absolute temperature T. Kinetic energy of gas molecule is given by

$$\text{KE} = \frac{3}{2}kT;\ k = \text{Boltzmann constant}$$

$$\lambda_{\text{gas molecules}} = \frac{h}{\sqrt{2\text{mkT}}}$$

ATOMIC MODEL

A model is simply a set of hypothesis based on logical and scientific facts.

Theory: When any model satisfies majority of scientific queries by experimental verification then it is termed as theory otherwise, model is simply not accepted.

In Nutshell we can say that every theory is a model but every model is not a theory. So, after more and more clarity about the substances, various new models like Dalton, Thomson, Rutherford, Bohr etc came into the pictures.

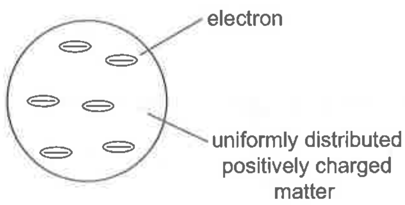
Dalton's Atomic Model

(i) Every element is made up of tiny indivisible particles called atoms.
(ii) Atoms of same element are identical both in physical and chemical properties while atoms of different elements are different in their properties.
(iii) All elements are made up of hydrogen atom. The mass of heaviest atom is about 250 times the mass of hydrogen atom while radius of heaviest atom is about 10 times the radius of hydrogen atom.
(iv) Atom is stable and electrically neutral.

***Reason of Failure of model*:** After the discovery of electron by *U.* Thomson (1897), it was established that atom can also be divide. Hence the model was not accepted.

Thomson's Atomic Model (or Plum-pudding Model)

(i) Atom is a positively charged solid sphere of radius of the order of 10^{-10} m in which electrons are embedded as seeds in a watermelon
(ii) Total charge in a atom is zero and so, atom is electrically neutral.



Achievements of model: Explained successfully the phenomenon of thermionic emission, photoelectric emission and ionization

Type of Line Spectrum

(a) Emission line spectrum: When an atomic gas or vapour at a pressure less than the atmospheric pressure is excited by passing electric discharge, the emitted radiation has spectrum which contains certain specific bright lines only. These emission lines constitute emission spectrum. These are obtained when electron jumps from excited states to lower states. The wavelength of emission lines of different elements are different. For one element the emission spectrum is unique. It is used for the determination of composition of an unknown substance.

(b) Absorption line spectrum: When white light is passed through a gas, the gas is found to absorb light of certain wavelength, the bright background on the photographic plate is then crossed by dark lines that corresponds to those wavelengths which are absorbed by the gas atoms.

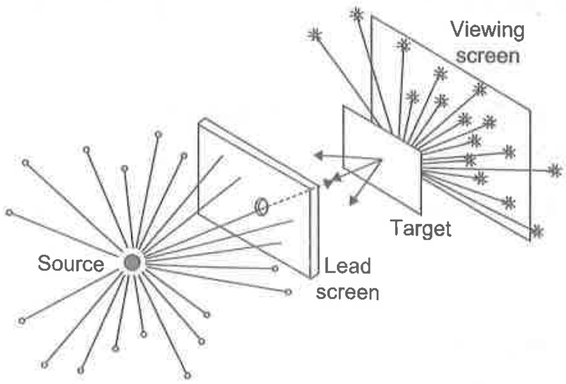
The absorption spectrum consists of dark lines on bright background. These are obtained due to absorption of certain wavelengths, resulting into transition of atom from lower energy states to higher energy states. (The emission spectrum consists of bright lines on dark background.)

Failure of the model:

(i) It could not explain the line spectrum of H-atom
(ii) It could not explain the Rutherford's α-particle scattering experiment

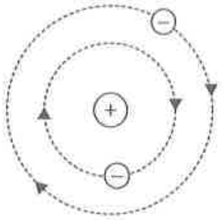
Rutherford's Atomic Model

In 1911, Earnest Rutherford performed a critical experiment that showed that Thomson's model could not be correct. In this experiment a beam of positively charged alpha particles (helium nuclei) was projected into a thin gold foil. It is observed that most of the alpha particles passed through the foil as if it were empty space. But some surprising results are also seen. Several alpha particles are deflected from their original direction by large angles. Few alpha particles are observed to be reflected back, reversing their direction of travel as shown in figure-1.2.



If Thomson model is assumed true that the positive charge is spreaded uniformly in the volume of an atom then the alpha particle can never experience such a large repulsion due to which it will be deflected by such large angles as observed in the experiment. On the basis of this experiment Rutherford presented a new atomic model.

In this new atomic model it was assumed that the positive charge in the atom was concentrated in a region that was small relative to the size of atom. He called this concentration of positive charge, the nucleus of the atom. Electrons belonging to the atom were assumed to be moving in the large volume of atom outside the nucleus. To explain why these electrons were not pulled into the nucleus, Rutherford said that electrons revolve around the 'nucleus in orbits around the positively charged nucleus in the same manner as the planets orbit the sun. The corresponding atomic model can be approximately shown in figure.



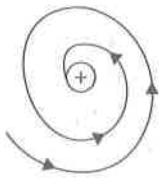
Reason of Failure of Model:

1. It could not explain the line spectrum of H-atom.

   Justification: According to Maxwell's electromagnetic theory every accelerated moving charged particle radiates energy in the form of electromagnetic waves and therefore during revolution of e^- in circular orbit its frequency will continuously vary (i.e., decrease) which will result in the continuous emission of lines and therefore spectrum of atom must be continuous but in reality, one obtains line spectrum for atoms.

2. It could not explain the stability of atoms.

   Justification: Since revolving electron will continuously radiates energy and therefore radii of circular path will continuously decrease and in a time of about 10^{-8} sec revolving electron must fall down in a nucleus by adopting a spiral path



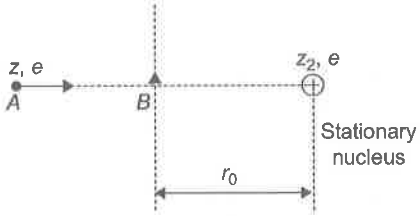
Path of electron spiral

***Determination of distance of closest approach*:** When a positively charged particle approaches towards stationary nucleus then due to repulsion between the two, the kinetic energy of positively charged particle gradually decreases and a stage comes when its kinetic energy becomes zero and from where it again starts retracting its original path.

***Definition*:** The distance of closest approach is the minimum distance of a stationary nucleus with a positively charged particle making head on collision from a point where its kinetic energy becomes zero.

Suppose a positively charged particle A of charge $q_1(= z_1e)$ approaches from infinity towards a stationary nucleus of charge z_2e then,

Suppose a positively charged particle A of charge $q_1(= z_1e)$ approaches from infinity towards a stationary nucleus of charge z_2e then,



Let at point B, kinetic energy of particle A becomes zero then by the law of conservation of energy at point A and B,

$$TE_A = TE_B$$

$$KE_A + PE_A = KE_B + PE_B$$

$$E + 0 = 0 + \frac{k(z_1e)(z_2e)}{r_0} \text{(in joule)} \quad \therefore \quad r_0 = \frac{k(z_1e)(z_2e)}{E} \text{ m}$$

SOLVED EXAMPLES

EXAMPLE 15

An α-particle with kinetic energy 10 MeV is heading towards a stationary point-nucleus of atomic number 50. Calculate the distance of closest approach.

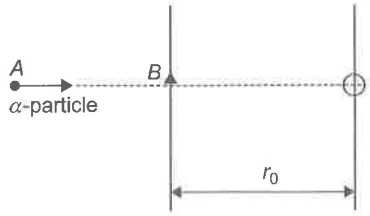
SOLUTION

$$\therefore \quad TE_A = TE_B$$

$$\therefore \quad 10 \times 10^6 e = \frac{K \times (2e)(50e)}{r_0}$$

$$r_0 = 1.44 \times 10^{-14} \text{ m}$$

$$r_0 = 1.44 \times 10^{-4} \text{ A}$$



EXAMPLE 16

A beam of α-particles of velocity 2.1×10^7 m/s is scattered by a gold ($z = 79$) foil. Find out the distance of closest approach of the α-particle to the gold nucleus. The value of charge/mass for α-particle is 4.8×10^7 c/kg.

SOLUTION

$$\frac{1}{2} m_\alpha v_\alpha^2 = \frac{K(2e)(ze)}{r_0}$$

$$r_0 = \frac{2K\left(\dfrac{2e}{m_\alpha}\right)(79\,e)}{v_\alpha^2}$$

$$= \frac{2 \times (9 \times 10^8)(4.8 \times 10^7)(79 \times 1.6 \times 10^{-19})}{(2.1 \times 10^7)^2};$$

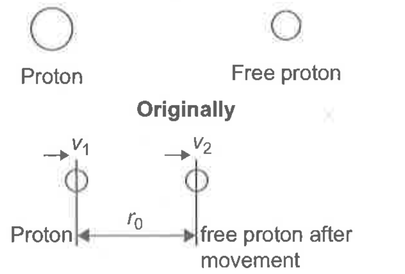
$$r_0 = 2.5 \times 10^{-14}\text{ m}$$

EXAMPLE 17

A proton moves with a speed of 7.45×10^5 m/s directing towards a free proton originally at rest. Find the distance of closest approach for the two protons.

SOLUTION

$\rightarrow v = 7.45 \times 10^5$ m/s $u = 0$



At the time of distance of closest approach

By the law of cons. of energy

$$\frac{1}{2}mv^2 + 0 = \frac{ke^2}{r_0} + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2 \quad (1)$$

By the cons. of momentum $mv + 0 = mv_1 + mv_1$

$$\therefore \quad v_1 = \frac{v}{2}$$

From equation (1)

$$\frac{1}{2}mv^2 = \frac{ke^2}{r_0} + m\left(\frac{v}{2}\right)^2$$

$$r_0 = \frac{4}{mv^2} \times ke^2 = \frac{4 \times (9 \times 10^9)(1.6 \times 10^{-19})^2}{(1.66 \times 10^{-27})(7.45 \times 10^5)^2};$$

$$r_0 = 1.0 \times 10^{-12}\text{ m}$$

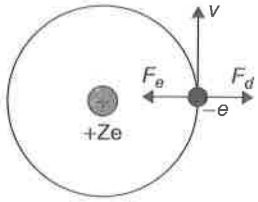
BOHR'S MODEL OF AN ATOM

Between 1913 and 1915 Niels Bohr developed a quantitative atomic model to the Hydrogen atom that could account for its spectrum. The model incorporated the nuclear model of the atom proposed by Rutherford on the basis of his experiments. We shall see that this model was successful in its ability to predict the gross features of the spectrum emitted by Hydrogen atom. This model was developed specifically for Hydrogenic atoms. Hydrogenic atoms are those which consist of a nucleus with positive charge +Ze (Z = atomic number, e = charge of electron) and a single electron. More complex electron-electron interactions in an atom are not accounted in the Bohr's Model that's why it was valid only for one electron system or hydrogenic atoms.

The Bohr model is appropriate for one electron systems like H, He^+, Li^{+2} etc. and it was successful upto some extent in explaining the features of the spectrum emitted by such hydrogenic atoms. However this model is not giving a true picture of even these simple atoms. The true picture is fully a quantum mechanical affair which is different from Bohr model in several fundamental ways. Since Bohr model incorporates aspects of some classical and some modern physics it is now called semiclassical model Bohr has explained his atomic model in three steps called postulates of Bohr's atomic model. Lets discuss one by one.

First Postulate

In this postulate Bohr incorporate and analyses features of the Rutherford nuclear model of atom. In this postulate it was taken that as the mass of nucleus is so much greater then the mass of electron, nucleus was assumed to be at rest and electron revolves around the nucleus in an orbit. The orbit of electron is assumed to be circular for simplicity. Now the statement of first postulate is 'During revolution of electron around the nucleus in circular orbit, the electric coulombian force on electron is balanced by the centrifugal force acting on it in the rotating frame of reference.'



If electron revolves with speed v in the orbit of radius r. Then relative to rotating frame attached with electron, the centrifugal force acting on it is

$$F_{cf} = \frac{mv^2}{r} \quad (1)$$

The coulombian force acting on electron due to charge of nucleus (+Ze) is

$$F_{electric} = \frac{K(e)(Ze)}{r^2} \quad (2)$$

Now according to first postulate from equation (1) and (2) we have

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

or

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2} \quad (3)$$

Equation (3) is called equation of Bohr's first postulate.

Second Postulate

In the study of atom, Bohr found that while revolving around the nucleus the orbital angular momentum of the electron was restricted to only certain values, we say that the orbital angular momentum of the electron is quantized. He therefore took this as a second postulate of the model. The statement of second Postulate is, Bohr proposed that 'During revolution around the nucleus, the orbital angular momentum of electron L could not have just any value, it can take up only those values which are integral multiples of Planck's Constant divided by 2π i.e., $h/2\pi$'.

Thus the angular momentum of electron can be written as

$$L = \frac{nh}{2\pi}. \quad (1)$$

Where n is a positive integer, known as quantum number. In an orbit of radius r if an electron (mass m) revolves at speed v, then its angular momentum can be given as

$$L = mvr \quad (2)$$

Now from equation (1) and (2), we have for a revolving electron

$$mvr = \frac{nh}{2\pi}. \quad (3)$$

Equation (3) is known as equation of second postulate of Bohr model. Here the quantity $\frac{h}{2\pi}$ occurs so frequently in modern physics that, for convenience, it is given its own designation h, pronounced as 'h-bar.'

$$\hbar = \frac{h}{2\pi} \simeq 1.055 \times 10^{-34} \text{ J-s} \quad (4)$$

Third Postulate

While revolution of an electron in an orbit its total energy is taken as sum of its kinetic and electric potential energy due to the interaction with nucleus. Potential energy of electron revolving in an orbit of radius r can be simply given as

$$U = -\frac{K(e)(Ze)}{r} = -\frac{Kze^2}{r} \quad (1)$$

For kinetic energy of electron, we assume that relativistic speeds are not involved so we can use the classical expression for kinetic energy. Thus kinetic energy of electron in an orbit revolving at speed v can be given as

$$K = \frac{1}{2}mv^2 \quad (2)$$

Thus total energy of electron can be given as

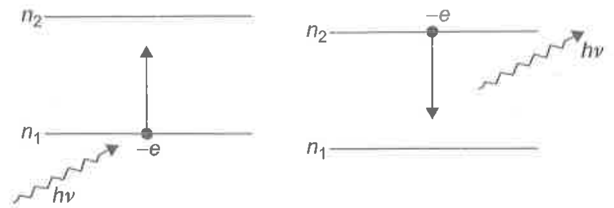
$$E = K + U = \frac{1}{2}mv^2 - \frac{KZe^2}{r}$$

Here we can see that while revolving in a stable orbit, the energy of electron remains constant. From the purely classical viewpoint, during circular motion, as electron is accelerated, it should steadily loose energy by emitting electromagnetic radiations and it spiraled down into the nucleus and collapse the atom.

Bohr in his third postulate stated that 'While revolving around the nucleus in an orbit, it is in stable state, it does not emit any energy radiation during revolution. It emits energy radiation only when it makes a transition from higher energy level (upper orbit) to a lower energy level (lower orbit) and the energy of emitted radiation is equal to the difference in energies of electron in the two corresponding orbits in transition.'

If an electron makes a transition form a higher orbit n_2 to a lower orbit n_1 as shown in figure. Then the electron radiates a single photon of energy

$$\Delta E = E_{n_2} - E_{n_1} = h\nu$$



Here E_{n_2} and E_{n_1} are the total energies of electron in the two orbits n_2 and n_1. The emitted photon energy can be expressed as $h\nu$ where ν is the frequency of radiated energy photon. If A be the wavelength of photon emitted then the energy of emitted photon can also be given as

$$\Delta E = h\nu = \frac{hc}{\lambda} \quad (1)$$

Similarly when energy is supplied to the atom by an external source then the electron will make a transition from lower energy level to a higher energy level. This process is called excitation of electron from lower to higher energy level. In this process the way in which energy is supplied to the electron is very important because the behaviour of the electron in the excitation depends only on the process by which energy as supplied from an external source. This we'll discuss in detail in later part of this chapter.

First we'll study the basic properties of an electron revolving around the nucleus of hydrogenic atoms.

Properties of Electron in Bohr's Atomic Model

Now we'll discuss the basic properties of an electron revolving in stable orbits, we call Bohr energy level. We have discussed that there are some particular orbits in which electron can revolve around the nucleus for which first and second postulates of Bohr model was satisfied. Thus only those orbits are stable for which the quantum number $n = 1, 2, 3, \ldots$. Now for nth orbit if we assume its radius is denoted by r_n and electron is revolving in this orbit with speed v_n. We can represent all the physical parameters associated with the electron in nth orbit by using a subscript n with the symbol of the physical parameters like r_n, v_n etc.

(a) Radius of nth Orbit in Bohr Model

Radius of electron in nth Bohr's orbit can be calculated using the first two postulates of the Bohr's model, using previous equations, we get

$$v_n = \frac{nh}{2\pi m r_n}$$

Substituting this value of v_n in equation $mvr = \frac{nh}{2\pi}$, we get

$$r_n = \frac{n^2h^2}{4\pi^2 KZe^2 m}$$

or $$r_n = \frac{h^2}{4\pi^2 Ke^2 m} \times \frac{n^2}{Z}$$

or $$r_n = 0.529 \times \frac{n^2}{Z} A$$

Velocity of Electron in nth Bohr's Orbit

By substituting the value of r_n we can calculate the value of v_n as

$$v_n = \frac{2\pi KZe^2}{nh}$$

or $$v_n = \frac{2\pi Ke^2}{h} \times \frac{Z}{n}$$

or $$v_n = 2.18 \times 10^6 \times \frac{Z}{n} \text{ m/s}$$

(b) Time period of Electron in nth Bohr's Orbit

Time period of electron of nth orbit is given by

$$T_n = \frac{1}{f_n} \quad \text{or} \quad T_n = \frac{n^3h^3}{4\pi^2 K^2 Z^2 e^4 m}$$

(c) Current in nth Bohr's Orbit

Electrons revolve around the nucleus in the nth Bohr's Orbit then due to revolution there is current in the orbit and according to the definition of current, the current in the nth orbit will be total coulombs passing through a point in one seconds, and in an orbit an electron passes through a point f_n times in one second so the current in the nth orbit will be

$$I_n = f_n \times e$$

or $$I_n = \frac{4\pi^2 K^2 Z^2 e^5 m}{n^3 h^3}$$

(d) Energy of Electron in nth Orbit

We've discussed that in nth orbit during revolution the total energy of electron can be given as sum of kinetic and potential energy of the electron as

$$E_n = K_n + U_n$$

Kinetic energy of electron in nth orbit can be given as

$$K_n = \frac{1}{2} m v_n^2$$

From equation of first postulate of Bohr Model we have for nth orbit

$$mv_n^2 = \frac{KZe^2}{r_n}$$

From equation

$$K_n = \frac{1}{2}mv_n^2 = \frac{1}{2}\frac{KZe^2}{r_n}$$

the potential energy of electron in nth orbit is given as

$$U_n = -\frac{KZe^2}{r_n}$$

Thus total energy of e^- in nth orbit can given as

$$E_n = K_n + U_R = \frac{1}{2}\frac{KZe^2}{r_n} - \frac{KZe^2}{r_n} = -\frac{1}{2}\frac{KZe^2}{r_n}$$

- Here we can see that $|E_n| = |K_n| = \frac{1}{2}|U_n|$ which is a very useful relation, always followed by a particle revolving under the action of a force obeying inverse square law.

Now substituting the value of r_n we get

$$E_n = -\frac{1}{2}KZe^2 \times \frac{4\pi^2 KZe^2 m}{n^2h^2}$$

or $$= -\frac{2\pi^2K^2Z^2e^4m}{n^2h^2}$$

or $$E_n = \frac{2\pi^2K^2Z^2e^4m}{h^2} \times \frac{Z^2}{h^2}$$

Substituting the value of constants in above equation we get

or $$E_n = -13.6 \times \frac{Z^2}{n^2}\text{ eV}$$

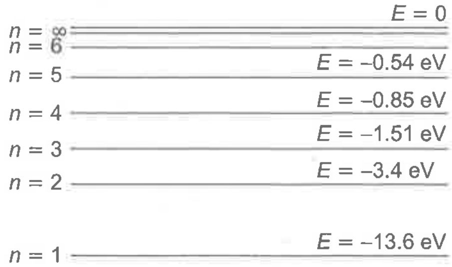
The above equation can be used to find out energies of electron in different energy level of different hydrogen atoms.

(e) Energies of Different Energy Level in Hydrogenic Atoms

By the use of above equation we can find out the energies of different energy levels. Students should remember these energies for first six level as

$$E_1 = -13.6\ Z^2\text{ eV}$$
$$E_2 = -3.40\ Z^2\text{ eV}$$
$$E_3 = -1.51\ Z^2\text{ eV}$$
$$E_4 = -0.85\ Z^2\text{ eV}$$
$$E_5 = -0.54\ Z^2\text{ eV}$$
$$E_6 = -0.36\ Z^2\text{ eV}$$

The above equations clearly shows that as the value of n increase, the difference between two consecutive energy levels decreases. It can be shown with the the help of figure, which shows the energy level diagram for a hydrogen atom.



Now if we multiply the numerator and denominator of above equation by ch we get

$$E_n = -\frac{2\pi^2K^2e^4m}{ch^3} \times ch \times \frac{Z^2}{n^2}$$

or $$E_n = -Rch \times \frac{Z^2}{n^2}\text{ eV}$$

Where $R = \frac{2\pi^2K^2e^4m}{ch^3}$ is defined as Rydberg Constant and the value of it is given as $R = 10967800\ \text{m}^{-1}$, which can be taken approximately as $10^7\ \text{m}^{-1}$. For $n = 1$ and $Z = 1$ the energy is given as

$E = -Rch$ joules and is called as One Rydberg Energy

1 Rydberg $= 13.6\text{ eV} = 2.17 \times 10^{-18}$ J

Lets discuss some examples on Bohr's atomic model to understand it better.

SOLVED EXAMPLES

EXAMPLE 18

What is the angular momentum of an electron in Bohr's Hydrogen atom whose energy is –3.4 eV?

SOLUTION

Energy of electron in nth Bohr orbit of hydrogen atom is given by,

$$E = -\frac{13.6}{n^2}\text{ eV}$$

Hence, $-3.4 = -\frac{13.6}{n^2}$

or $n^2 = 4$

or $n = 2$

The angular momentum of an electron in nth orbit is given as $L = \frac{nh}{2\pi}$. Putting $n = 2$, we obtain

$$L = \frac{2h}{2\pi} = \frac{h}{\pi} \quad \Rightarrow \quad T = \frac{n^3h^3}{4\pi^2k^2Z^2e^4m}$$

Thus, $T \propto n^3$ or $\frac{T_1}{T_2} = \frac{n_1^3}{n_2^3}$

As $T_1 = 8T_2$, the above relation gives

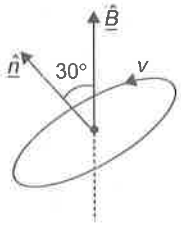
$$\left(\frac{n_1}{n_2}\right)^3 = 8 \quad \text{or} \quad n_1 = 2n_2$$

Thus the possible values of n_1 and n_2 are

$n_1 = 2, n_2 = 1; n_1 = 4, n_2 = 2; n_1 = 6, n_2 = 3$ and so on … ■

EXAMPLE 19

An electron in the ground state of hydrogen atom is revolving in anti-clockwise direction in the circular orbit of radius R as shown in figure



(i) Obtain an expression for the orbital magnetic dipole moment of the electron.

(ii) The atom is placed in a uniform magnetic induction B such that the plane normal of the electron orbit makes an angle 30° with the magnetic induction. Find the torque experienced by the orbiting electron.

SOLUTION

(i) According to Bohr's second postulate

$$mvr = n\frac{h}{2\pi} = \frac{h}{2\pi} \quad \text{(As for } n = 1 \text{ first only)}$$

or $$v_1 = \frac{h}{2\pi m r_1}$$

We know that the rate of flow of charge is current. Hence current in first orbit is

Now from each equation

$$i = ev_1 = e\left(\frac{v_1}{2\pi r_1}\right) = \frac{e}{2\pi r_1} \times v_1$$

$$i = \frac{e}{2\pi r_1} \times \frac{h}{2\pi n r_1} = \frac{eh}{4\pi^2 m r^2}$$

Magnetic dipole moment, $M_1 = i \times A_1$

or $$M_1 = \frac{eh}{4\pi^2 m r^2} \times \pi r_1^2 = \frac{eh}{4\pi m}$$

(ii) Torque on the orbiting electron in uniform magnetic field is

$$\vec{\tau} = \vec{M} \times \vec{B}$$

or $$\tau = MB \sin 30°$$

or $$\tau = \frac{eh}{4\pi m} \times \frac{B}{2} = \frac{ehB}{8\pi m}$$ ■

EXCITATION AND IONIZATION OF AN ATOM

According to third postulate of Bohr model we've discussed when some energy is given to an electron of atom from an external source it may make a transition to the upper energy level. This phenomenon we call excitation of electron or atom and the upper energy level to which the electron is excited is called excited state. To excite an electron to a higher state energy can be supplied to it ijn two ways. Here we'll discuss only the energy supply by an electromagnetic photon. Other method of energy supply we'll discuss later in this chapter.

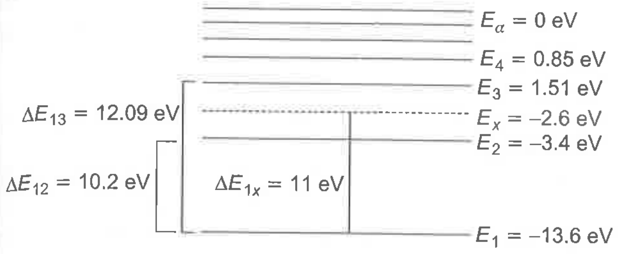
According to Planck's quantum theory photon is defined as a packet of electromagnetic energy, which when absorbed by a physical particle, its complete electromagnetic energy is converted into the mechanical energy of particle or the particle utilizes the energy of photon in the form of increment in its mechanical energy. When a photon is supplied to an atom and an electron absorbs this photon, then the electron gets excited to a higher energy level only if the photon energy is equal to the difference in energies of the two energy levels involved in the transition.

For example say in hydrogen atom an electron is in ground state (energy $E_1 = -13.6$ eV). Now it absorbs a photon and makes a transition to $n = 3$ state (energy $E_3 = -1.51$ eV) then the energy of incident photon must be equal to

$$\Delta E_{13} = E_3 - E_1 = (-1.51) - (13.6) \text{ eV} = 12.09 \text{ eV}$$

Now we'll see what will happen when a photon of energy equal to 11 eV incident on this atom. From the above calculation of energy differences of different energy levels we can say that if the electron in ground state absorbs this photon it will jump to a state some where between energy level $n = 2$ and $n = 3$ as shown in figure. When electron in ground state absorbs a photon of 11 eV energy, its total energy becomes

$$E = E_1 + 11 = -13.6 + 11 \text{ eV} = -2.6 \text{ eV}$$



As discussed in previous sections, in an atom electron can not take up all energies. It can exist only in some particular energy levels which have energy given as $-13.6/n^2$eV.

When a photon of energy 11 eV is absorbed by an electron in ground state. The energy of electron becomes –2.6 eV of it will excite to a hypothetical energy level X some where between $n = 2$ and $n = 3$ as shown in figure, which is not permissible for an electron. Thus when in ground state electron can absorb only those photons which have energies equal to the difference in energies of the stable energy level with ground state. If a photon beam incident on H-atoms having photon energy not equal to the difference of energy levels of H-atoms such as 11 eV, the beam will just be transmitted without any absorption by the H-atoms.

Thus to excite an electron from lower energy level to higher levels by photons, it is necessary that the photon must be of energy equal to the difference in energies of the two energy levels involved in the transition.

As we know that for higher energy levels, energy of electrons is less. When an electron is moved away from the nucleus to ∞th energy level or at $n = \infty$, the energy becomes (zero) or the electron becomes free from the attraction of nucleus or it is removed from the atom. In fact when an electron is in an atom, its total energy is negative $\left(E_n = -\frac{13.6}{n^2}Z^2\right)$. This negative sign shows that electron in under the influence of attractive forces of nucleus. When energy equal in magnitude to the total energy of an electron in a particular energy level is given externally, its total energy becomes zero or we can say that electron gets excited to ∞th energy level or the electron is removed from the atom and atom is said to be ionized.

We know that removal of electron from an atom is called ionization. In other words, ionization is the excitation of an electron to $n = \infty$ level. The energy required to ionize an atom is called ionization energy of atom for the particular energy level from which the electron is removed. In hydrogen atoms, the ionization energy for nth state can be given as

$$\Delta E_{n\to\infty} = E_\infty - E_n$$

or $$\Delta E_{n\to\infty} = 0 - \left(-\frac{13.6Z^2}{n^2}\right)\text{eV}$$

or $$\Delta E_{n\to\infty} = \frac{13.6Z^2}{n^2}\text{eV}.$$

When an electron absorbs a monochromatic radiation from an external energy source then it makes a transition from a lower energy level to a higher level. But this state of the electron is not a stable one. Electron can remain in this excited state for a very small internal at most of the order of 10^{-1} second. The time period for which this excited state of the electron exists is called the life time of the excited state. After the life time of the excited state the electron must radiate energy and it will jump to the ground state.

Let us assume that the electron is initially in n_2 state and it will jump to a lower state n_1 then it will emit a photon of energy equal to the energy difference of the two states n_1 and n_2 as

$$\Delta E = E_{n_2} - E_{n_1}.$$

When ΔE is the energy of the emitted photon. Now substituting the values of E_{n_2} and E_{n_1} in above equation, we get

$$\Delta E = -\frac{2\pi^2K^2Z^2e^4m}{n_2^2h^2} + \frac{2\pi^2K^2Z^2e^4m}{n_1^2h^2}$$

or $$\Delta E = \frac{2\pi^2K^2Z^2e^4m}{h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or $$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$$

Here $13.6Z^2$ can be used as ionization energy for $n = 1$ state for a hydrogenic atom thus the energy of emitted photon can also be written as

$$\Delta E = IP\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Equation can also be used to find the energy of emitted radiation when an electron jumps from a higher orbit n_2 to a lower orbit n_1. If λ be the wavelength of the emitted radiation then

$$\Delta E = \frac{hc}{\lambda}$$

This energy can be converted to eV by dividing this energy by the electronic charge e, as if wavelength is given in A, the energy in eV can be given as

$$\Delta E = \frac{hc}{\lambda e}(\text{in eV})$$

Substituting the values of h, c and e we get

$$\Delta E = \frac{(6.63\times10^{-34})\times(3\times10^{8})}{\lambda\times(1.6\times10^{-19})\times10^{-10}}\text{ eV}$$

$$\Delta E = \frac{12431}{\lambda}\text{ eV} \qquad (1)$$

Here in above equation, lambda is in Å units

This equation is the most important in numerical calculations, as it will be very frequency used. From equation we have

$$\frac{hc}{\lambda} = -13.6Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\text{eV}$$

or
$$\frac{1}{\lambda} = \frac{-13.6Z^2}{hc}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\text{eV}$$

or
$$\bar{v} = \frac{1}{\lambda} = -RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

(As Rydberg Constant $R = \frac{13.6}{hc}$ eV)

Here v is called wave number of the emitted radiation and is defined as number of waves per unit length and the above relation is used to find the wavelength of emitted radiation when an electron makes a transition from higher level n_2 to lower level n_1 is called Rydberg formula. But students are advised to use equation (1) is numerical calculations to find the wavelength of emitted radiation using the energy difference in electron volt. If can be rearranged as

$$\lambda = \frac{12431}{\Delta E\,(\text{in eV})}\text{ Å}$$

Number of emission spectral lines: If the electron is excited to state with principal quantum number n then from the nth state, the electron may go to $(n-1)$th state, ..., 2nd state or 1st state. So there are $(n-1)$ possible transitions starting from the nth state. The electron reaching $(n-1)$th state may make $(n-2)$ different transitions. Similarly for other lower states. The total no. of possible transitions is

$$(n-1)+(n-2)+(n-3)+\cdots 2+1 = \frac{n(n-1)}{2}$$

Number of absorption spectral lines: Since at ordinary temperatures, almost all the atoms remain in their lowest energy level ($n = 1$) and so absorption transition can start only from $n = 1$ level (not from $n = 2, 3, 4, \ldots$ levels). Hence, only Lyman series is found in the absorption spectrum of hydrogen atom (which as in the emission spectrum, all the series are found No. of absorption spectral lines = $(n-1)$)

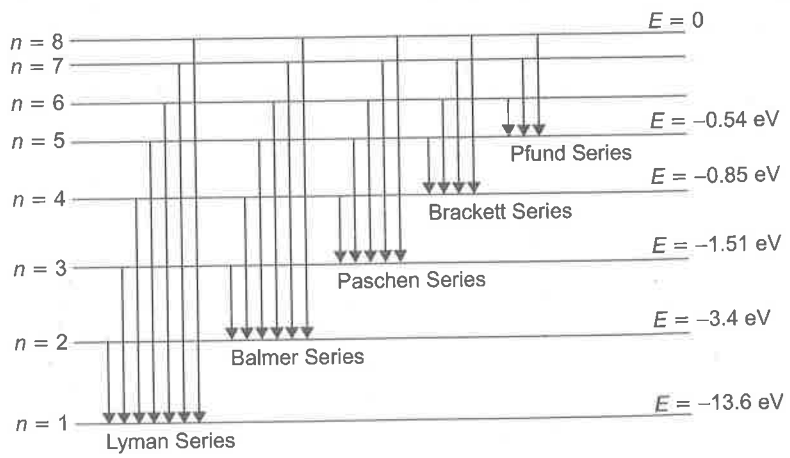
SPECTRAL SERIES OF HYDROGEN ATOM

The wavelength of the lines of every spectral series can be calculated using the formula given by equation (1).

Five special series are observed in the Hydrogen Spectrum corresponding to the five energy levels of the Hydrogen atom and these five series are named as on the names of their inventors. These series are

1. Lyman Series
2. Balmer Series
3. Paschen Series
4. Brackett Series
5. Pfund Series

These spectral series are shown in figure-1.11.



1. **Lyman Series:** The series consists of wavelength of the radiations which are emitted when electron jumps from a higher energy level to $n = 1$ orbit. The wavelength constituting this series lie in the Ultra Violet region of the electromagnetic spectrum.

   For Lyman Series $n_1 = 1$

   and $n_2 = 2, 3, 4, \ldots$

   First line of Lyman series is the line corresponding to the trnasition $n_2 = 2$ to $n_1 = 1$, similarly second line of the Lyman series is the line corresponding to the transition $n_2 = 3$ to $n_1 = 1$.

2. **Balmer Series:** The series consists of wavelengths of the radiations which are emitted when electron jumps from a higher energy level to $n = 2$ orbit. The wavelengths consisting this series lie in the visible region of the electromagnetic spectrum.

3. **Paschen Series:** The series consists of wavelengths of the radiations which are emitted when electron jumps from a higher energy level to $n = 3$ orbit. The wavelengths constituting this series lie in the Near Infra Red region of the electromagnetic spectrum.

4. **Brackett Series:** The series consists of wavelengths of the radiations which are emitted when electron jumps from a higher energy level to $n = 4$ orbit. The wavelengths constituting this series lie in the Infra Red region of the electromagnetic spectrum.

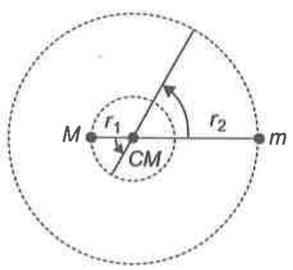
5. **Pfund Series:** The series consists of wavelengths of the radiations which are emitted when electron jumps from a higher energy level to $n = 5$ orbit. The wavelengths constituting this series lie in the Deep Infra Red region of the electromagnetic spectrum.

   We can find out the wavelengths corresponding to the first line and the last line for remaining four spectral series as mentioned in the case of Lyman Series.

APPLICATION OF NUCLEUS MOTION ON ENERGY OF ATOM

Let both the nucleus of mass M, charge Ze and electron of mass m, and charge e revolve about their centre of mass (CM) with same angular velocity (ω) but different linear speeds. Let r_1 and r_2 be the distance of CM from nucleus and electron. Their angular velocity should be same then only their separation will remain unchanged in an energy level.

Let r be the distance between the nucleus and the electron. Then



$$Mr_1 = mr_2$$

$$r_1 + r_2 = r$$

$$\therefore \quad r_1 = \frac{mr}{M+m} \quad \text{and} \quad r_2 = \frac{Mr}{M+m}$$

Centripetal force to the electron is provided by the electrostatic force. So,

$$mr_2\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

or
$$m\left(\frac{Mr}{M+m}\right)\omega^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r^2}$$

or
$$\left(\frac{Mr}{M+m}\right)r^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$$

or
$$\mu r^3\omega^2 = \frac{e^2}{4\pi\varepsilon_0}$$

where
$$\frac{Mm}{M+m} = \mu$$

Moment of inertia of atom about CM,

$$I + Mr_1^2 + mr_2^2 = \left(\frac{Mm}{M+m}\right)r^2 = \mu r^2$$

According to Bohr's theory, $\frac{nh}{2\pi} = I\omega$

or
$$\mu r^2\omega = \frac{nh}{2\pi}$$

Solving above euqations for r, we get

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi\mu e^2 Z} \quad \text{and} \quad \mathbf{r = (0.529\ A)}\frac{n^2}{Z}\cdot\frac{m}{\mu}$$

Further electrical potential energy of the system.

$$U = \frac{-Ze^2}{4\pi\varepsilon r} \quad U = \frac{-Z^2e^4\mu}{4\varepsilon_0^2 n^2 h^2}$$

and kinetic energy.

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\mu r^2\omega^2 \quad \text{and} \quad K = \frac{1}{2}\mu v^2$$

v-speed of electron with respect to nucleus. ($v = r\,\omega$)

here $$\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0 \mu r^3}$$

$\therefore$ $$K = \frac{Ze^2}{8\pi\varepsilon_0 r} = \frac{Z^2 e^4 \mu}{8\pi\varepsilon_0^2 n^2 h^2}$$

$\therefore$ Total energy of the system $E_n = K + U$

$$E_n = -\frac{\mu e^4}{8\varepsilon_0^2 n^2 h^2}$$

this expression can also be written as

$$E_n = -(13.6\,\text{eV})\frac{Z^2}{n^2}\cdot\left(\frac{\mu}{m}\right)$$

The expression for E_n without considering the motion of proton is $E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$, i.e., m is replaced by μ while considering the motion of nucleus.

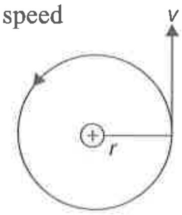
IMPORTANT FORMULA

1. Time period (*T*): distance = time *x* speed

$$2\pi r = T \times v$$

$$T = \frac{2\pi r}{v}$$

$$T_n = \frac{n^2 h^2}{4\pi k^2 z^2 m e^4}$$



$T_n \propto \frac{n^3}{Z^2}$

2. Frequency of revolution

$$f_n = \frac{v_n}{2\pi r_0} = \frac{4\pi k^2 z^2 m e^4}{n^3 h^3}; \quad f_n \propto \frac{z^2}{n^3}$$

3. Momentum of electron

$$P_n = \frac{2\pi m k z^2}{nh}\; P_n \propto \frac{1}{n}$$

4. Angular velocity of electron

$$\omega_n = \frac{8\pi^3 k^2 z^2 m e^4}{n^3 h^3}, \; \omega_n \propto \frac{z^2}{n^3}$$

5. Current (1)

$$I = \frac{e}{T} = ev = \frac{ev}{2\pi r}; \; I \propto \frac{z^2}{n^3}$$

6. Magnetic moment of electron (*M*)

$$M = iA$$

$$M = \frac{ev}{2\pi r} \times \pi r^2, \quad M = \frac{evr}{2}$$

$$M = \frac{e(mvr)}{2m} = \frac{eJ}{2m}; \quad \frac{M}{J} = \frac{e}{2m},$$

$$M = \frac{e}{2m}\left(\frac{nh}{2\pi}\right) = n\left(\frac{eh}{4\pi m}\right)$$

$M = n\mu_B$ $\quad \mu_B$ = Bohr magneton = 9.3 $\times 10^{-24}$ A m^2.

$$M \propto n$$

7. Magnetic field of Magnetic induction at the centre

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 ev}{4\pi r^2}$$

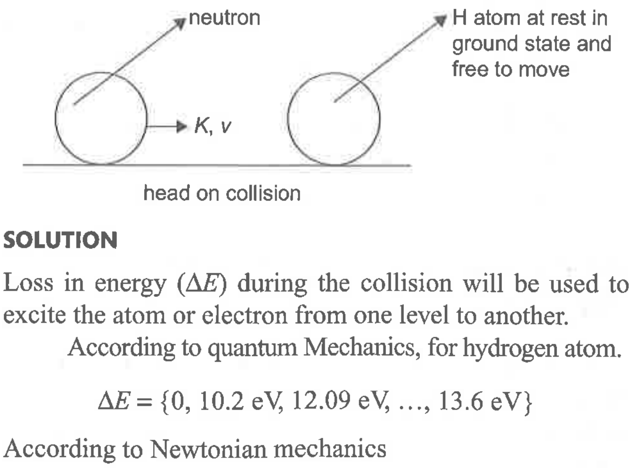
ATOMIC COLLISION

In such collisions assume that the loss in the kinetic energy of system is possible only if it can excite or ionise.

SOLVED EXAMPLES

EXAMPLE 20

What will be the type of collision, if K = 14 eV, 20.4 eV, 22 eV, 24.18 eV (elastic/inelastic/perfectly inelastic)



head on collision

SOLUTION

Loss in energy (ΔE) during the collision will be used to excite the atom or electron from one level to another.

According to quantum Mechanics, for hydrogen atom.

$$\Delta E = \{0, 10.2\text{ eV}, 12.09\text{ eV}, \ldots, 13.6\text{ eV}\}$$

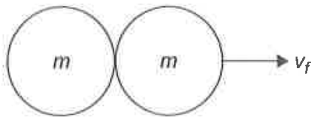
According to Newtonian mechanics

minimum loss = 0, (elastic collision)

for maximum loss collision will be perfectly inelastic

if neutron collides perfectly inelastically

then, Applying momentum conservation



$$mv_0 = 2mv_f \quad \Rightarrow \quad v_f = \frac{v_0}{2}$$

final $$KE = \frac{1}{2} \times 2m \times \frac{v_0^2}{4} = \frac{\frac{1}{2}mv_0^2}{2} = \frac{k}{2}$$

maximum loss $= \dfrac{K}{2}$

According to classical mechanics $(\Delta E) = \left[0, \dfrac{K}{2}\right]$

(a) If $K = 14$ eV, According to quantum mechanics $(\Delta E) = \{0, 10, 2\text{eV}, 12.09 \text{ eV}\}$

According to classical mechanics
$\Delta E = [0, 7 \text{ eV}]$

loss = 0, hence it is elastic collision speed of particle changes

(b) If $K = 20.4$ eV

According to classical mechanics

loss = [0, 10.2 eV]

According to quantum mechanics

loss = {0, 10.2 eV, 12.09 eV, ...}

loss = 0 elastic collision.

loss = 10.2 eV perfectly inelastic collision

(c) If $K = 22$ eV

Classical mechanics $\Delta E = [0, 1]$

Quantum mechanics $\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots\}$

loss = 0 elastic collision

loss = 10.2 eV inelastic collision

(d) If $K = 24.18$ eV

According to classical mechanics $\Delta E = [0, 12.09 \text{ eV}]$

According to quantum mechanics $\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots 13.6 \text{ eV}\}$

loss = 0 elastic collision

loss = 10.2 eV inelastic collision

loss = 12.09 eV perfectly inelastic collision ■

EXAMPLE 21

A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particle.

SOLUTION

Here the loss during the collision can only be used to excite the atoms or electrons.

So according to quantum mechanics

$$\text{loss} = \{0, 40.8 \text{ eV}, 48.3 \text{ eV}, \ldots, 54.4 \text{ eV}\} \qquad (1)$$

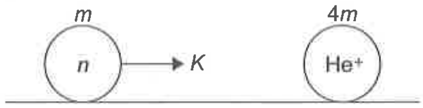
$$E_n = -13.6\frac{Z^2}{n^2} \text{ eV}$$

Now according to Newtonian mechanics.

Minimum loss = 0.

Maximum loss will be for perfectly inelastic collision.

Let v_0 be the initial speed of neutron and v_f be the final common speed.



so by momentum conservation $mv_0 = mv_f + 4mv_f$ $v_f = \dfrac{v_0}{5}$

where m = mass of Neutron

$\therefore$ mass of He^+ ion $= 4m$

so final kinetic energy of system

$$KE = \frac{1}{2}mv_f^2 + \frac{1}{2}4mv_f^2 = \frac{1}{2}(5m)\frac{v_0^2}{25} = \frac{1}{5}\cdot\left(\frac{1}{2}mv_0^2\right) = \frac{K}{5}$$

$$\text{maximum loss} = K - \frac{K}{5} = \frac{4K}{5}$$

so loss will be $\left[0, \dfrac{4K}{5}\right]$

For inelastic collision there should be at least one common value other than zero is set (1) and (2)

$$\therefore \qquad \frac{4K}{5} > 40.8 \text{ eV}$$

$$K > 51 \text{ eV}$$

minimum value of $K = 51$ eV ■

EXAMPLE 22

How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n?

SOLUTION

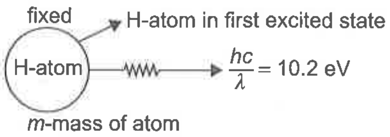
From the nth state, the atom may go to $(n - 1)$th state, ... 2nd state or 1st state. So there are $(n - 1)$ possible transitions starting from the nth state. The atoms reaching $(n - 1)$th state may make $(n - 2)$ different transitions. Similarly for other lower states. The total number of possible transitions is

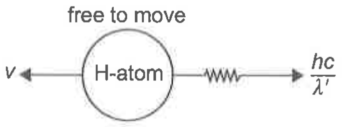
$$(n-1)+(n-2)+(n-3)+\cdots 2+1$$

$$=\frac{n(n-1)}{2} \quad \textbf{(Remember)}$$

❖ **Calculation of recoil speed of atom on emission of a photon**

momentum of photon $= mc = \dfrac{h}{\lambda}$

(a)


(b)


According to momentum conservation

$$mv = \frac{h}{\lambda'} \qquad (1)$$

According to energy conservation

$$\frac{1}{2}mv^2 + \frac{hc}{\lambda'} = 10.2\,\text{eV}$$

Since mass of atom is very large than photon

hence $\frac{1}{2}mv^2$ can be neglected

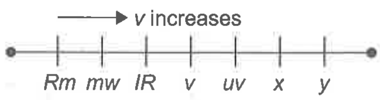
$$\frac{hc}{\lambda'} = 10.2\,\text{eV} \qquad \frac{h}{\lambda} = \frac{10.2}{c}\,\text{eV}$$

$$mv = \frac{10.2}{c}\,\text{eV} \qquad v = \frac{10.2}{cm}$$

recoil speed of atom $= \dfrac{10.2}{cm}$ ■

X-RAYS

It was discovered by **ROENTGEN.** The wavelenth of x-rays of found between 0.1 Å to 10 Å. These rays are invisible to eye. They are electromagnetic waves and have speed $c = 3 \times 10^8$ m/s in vacuum. Its photons have energy around 1000 times move than the visible light.

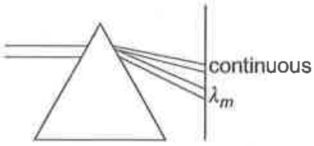


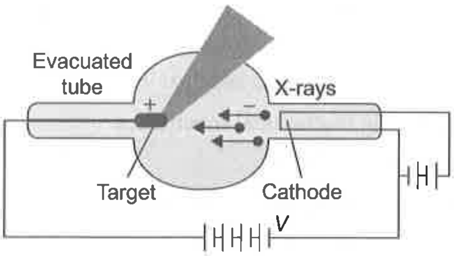
When fast moving electrons having energy of order of several KeV strike the metallic target then x-rays are produced.

Production of X-rays by Coolidge Tube

The melting point, specific heat capacity and atomic num ber of target should be high. When voltage is applied acros the filament then filament on being heated emits electron from it. Now for giving the beam shape of electrons, col limator is used. Now when electron strikes the target the x-rays are produced. When electrons strike with the targe some part of energy is lost and converted into heat. Sinc target should not melt or it can absorb heat so that the melt ing point, specific heat of target should be high.

Here copper rod is attached so that heat produced ca go behind and it can absorb heat and target does not ge heated very high.



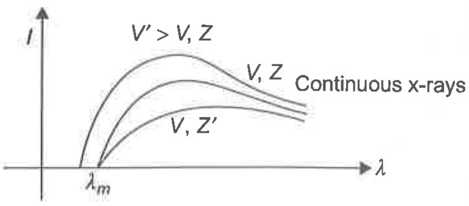


Form one energetic electron, accelerating voltage is in creased.

Form one no. of photons voltage across filament i increased.

The x-ray were analyzed by mostly taking their spectrum

Variation of Intensity of x-rays with *I* Is Plotted as Shown in Figure



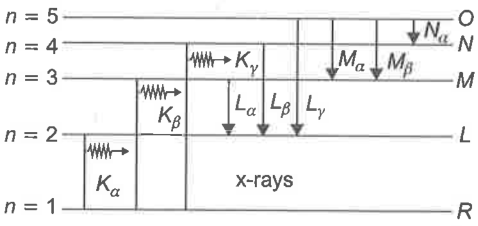
1. The minimum wavelength corresponds to the maxi mum energy of the x-rays which in turn is equal t the maximum kinetic energy eV of the striking elec trons thus

$$\text{eV} = h\nu_{max} = \frac{hc}{\lambda_{min}} \Rightarrow \lambda_{min} = \frac{hc}{\text{eV}} = \frac{12400}{V(\text{in volts})}\ \text{Å}$$

We see that cutoff wavelength λ_{min} depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z′).

2. **Characteristic X-rays**

   The sharp peaks obtained in graph are known as characteristic x-rays because they are characteristic of target material $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \ldots$ = characteristic wavelength of material having atomic number Z are called **characteristic x-rays** and the spectrum obtained is called **characteristic spectrum.** If target of atomic number Z′ is used then peaks are shifted.

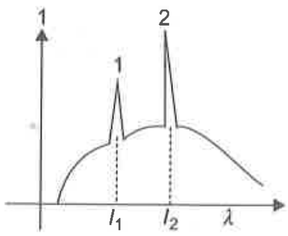


Characteristic x-rays emission occurs when an energetic electron collides with target and remove an inner shell electron from atom, the vacancy created in the shell is filled when an electron from higher level drops into it. Suppose vacancy created in innermost *K*-shell is filled by an electron dropping from next higher level *L*-shell then K_α characteristic x-ray is obtained. If vacancy in *K*-shell is filled by an electron from *M*-shell, K_β line is produced and so on similarly L_α, L_β, ... M_α, M_β lines are produced.

SOLVED EXAMPLES

EXAMPLE 23

Find which is K_α and K_β



SOLUTION

$$\Delta E = \frac{hc}{\lambda}, \quad \lambda = \frac{hc}{\Delta E}$$

since energy difference of K_α is less than K_β

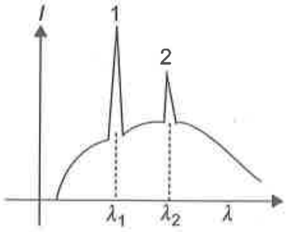
$$\Delta E_{k\alpha} < \Delta K_{k\beta}$$

1 is K_β and 2 is K_α ■

EXAMPLE 24

Find which is K_α and L_α

SOLUTION

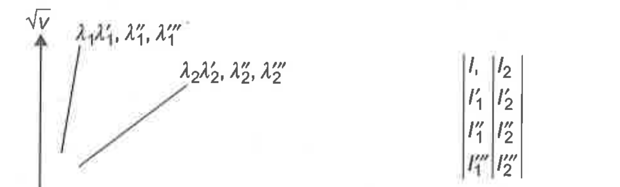


$$\therefore \quad \Delta E_{K\alpha} > \Delta E_{L\alpha}$$

1 is K_α and 2 is L_α ■

MOSELEY'S LAW

Moseley measured the frequencies of characteristic x-rays for a large number of elements and plotted the square root of frequency against position number in periodic table. He discovered that plot is very closed to a straight line not passing through origin.



Wavelength of charactristic wavelengths.

Moseley's observations can be mathematically expressed as

$$\sqrt{v} = a(Z - b)$$

a and *b* are positive constants for one type of x-rays and for all elements (independent of *Z*).

Moseley's Law can be derived on the basis of Bohr's theory of atom, frequency of x-rays is given by

$$\sqrt{v} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \cdot (Z - b)$$

by using the formula $\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ with modification for multi electron system.

$b \rightarrow$ known as screening constant or shielding effect, and $(Z - b)$ is effective nuclear charge.

For K_α line

$$n_1 = 1, \quad n_2 = 2$$

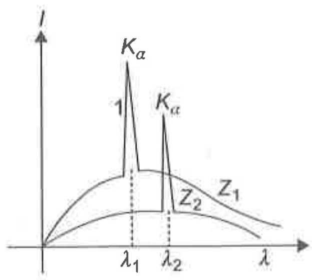
$$\therefore \quad \sqrt{v} = \sqrt{\frac{3RC}{4}}(Z - b) \qquad \sqrt{v} = a(Z - b)$$

Here $\quad a = \sqrt{\frac{3RC}{4}}, \quad [b = 1 \text{ for } K_\alpha \text{ lines}]$

SOLVED EXAMPLES

EXAMPLE 25

Find in Z_1 and Z_2 which one is greater.



SOLUTION

$$\therefore \quad \sqrt{v} = \sqrt{cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \cdot (Z - b)$$

If Z is greater then v will be greater, λ will be less

$$\therefore \quad \lambda_1 < \lambda_2$$

$$\therefore \quad Z_1 > Z_2$$

EXERCISES

JEE Main

Photo Electric Effect

1. Let n_r and n_b be respectively the number of photons emitted by a red bulb and a blue bulb of equal power in a given time.

(A) $n_r = n_b$ (B) $n_r < n_b$
(C) $n_r > n_b$ (D) data insufficient

2. 10^{-3} W of 5000 Å light is directed on a photoelectric cell. If the current in the cell is 0.16 mA, the percentage of incident photons which produce photoelectrons, is

(A) 0.4% (B) .04%
(C) 20% (D) 10%

3. If the frequency of light in a photoelectric experiment is doubled, the stopping potential will

(A) be doubled
(B) halved
(C) become more than doubled
(D) become less than double

4. The stopping potential for the photo electrons emitted from a metal surface of work function 1.7 eV is 10.4 V. Identify the energy levels corresponding to the transitions in hydrogen atom which will result in emission of wavelength equal to that of incident radiation for the above photoelectric effect

(A) $n = 3$ to 1 (B) $n = 3$ to 2
(C) $n = 2$ to 1 (D) $n = 4$ to 1

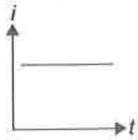
5. When a photon of light collides with a metal surfac number of electrons, (if any) coming out is

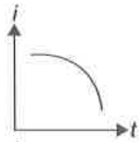
(A) only one (B) only two
(C) infinite (D) depends upon facto

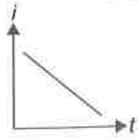
6. A point source of light is used in photoelectric effec If the source is removed farther from the emittir metal, the stopping potential:

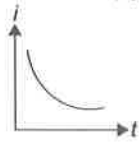
(A) will increase
(B) will decrease
(C) will remain constant
(D) will either increase or decrease

7. A point source causes photoelectric effect from small metal plate. Which of the following curves m represent the saturation photocurrent as a function the distance between the source and the metal?

(A)


(B)


(C)


(D)


8. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential is Volts is:
(A) 2 (B) 4
(C) 6 (D) 10

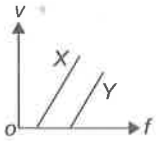
9. Radiation of two photon energies twice and five times the work function of metal are incident successively on the metal surface. The ratio of the maximum velocity of photoelectrons emitted is the two cases will be
(A) 1:2 (B) 2:1
(C) 1:4 (D) 4:1

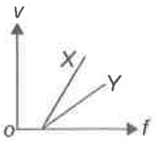
10. Cut off potentials for a metal in photoelectric effect for light of wavelength λ_1, λ_2 and λ_3 is found to be V_1, V_2 and V_3 volts if V_1, V_2 and V_3 are in Arithmetic Progression and λ_1, λ_2 and λ_3 will be:
(A) Arithmetic Progression
(B) Geometric Progression
(C) Harmonic Progression
(D) None

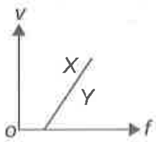
11. Photons with energy 5 eV are incident on a cathode C, on a photoelectric cell. The maximum energy of the emitted photoelectrons is 2 eV. When photons of energy 6 eV are incident on *C*, no photoelectrons will reach the anode *A* if the stopping potential of *A* relative to *C* is
(A) 3 V (B) –3 V
(C) –1 V (D) 4 V

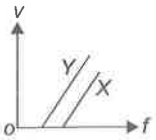
12. By increasing the intensity of incident light keeping frequency ($\nu > \nu_0$) fixed on the surface of metal
(A) kinetic energy of the photoelectrons increases
(B) number of emitted electrons increases
(C) kinetic energy and number of electrons increases
(D) no effect

13. In a photoelectric experiment, electrons are ejected from metals *X* and *Y* by light of intensity *I* and frequency *f*. The potential difference *V* required to stop the electrons is measured for various frequencies. If *Y* has a greater work function than *X*; which one of the following graphs best illustrates the expected results?

(A)


(B)


(C)


(D)


14. A image of the sun of formed by a lens of focal-length of 30 cm on the metal surface of a photo-electric cell and a photo-electric current is produced. The lens forming the image is then replaced by another of the same diameter but of focal length 15 cm. The photo-electric current in this case is
(A) $\frac{I}{2}$ (B) I
(C) $2I$ (D) $4I$

Matter Waves

15. A proton and an electron are accelerated by same potential difference have de Broglie wavelength λ_p and λ_e.
(A) $\lambda_e = \lambda_p$ (B) $\lambda_e < \lambda_p$
(C) $\lambda_e > \lambda_p$ (D) none of these

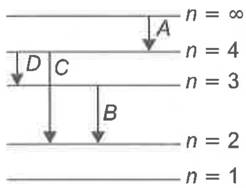
16. An electron with initial kinetic energy of 100 eV is acceleration through a potential difference of 50 V. Now the de Broglie wavelength of electron becomes.
(A) 1 Å (B) $\sqrt{1.5}$ Å
(C) $\sqrt{3}$ Å (D) 12.27 Å

17. If *h* is Planck's constant is SI system, the momentum of a photon of wavelength 0.01 Å is:
(A) $10^{-2}\,h$ (B) h
(C) $10^2\,h$ (D) $10^{12}\,h$

Atomic Structure

18. The angular momentum of an electron in the hydrogen atom is $\frac{3h}{2\pi}$. Here *h* is Planck's constant. The kinetic energy of this electron is:
(A) 4.53 eV (B) 1.51 eV
(C) 3.4 eV (D) 6.8 eV

19. Consider the following electronic energy level diagram of H-atom: Photons associated with shortest and longest wavelengths would be emitted from the atom by the transitions labelled.



(A) *D* and *C* respectively
(B) *C* and *A* respectively
(C) *C* and *D* respectively
(D) *A* and *C* respectively

20. If the electron in a hydrogen atom were in the energy level with $n = 3$, how much energy in joule would be required to ionize the atom? (Ionization energy of H-atom is 2.18×10^{-18} J):
(A) 6.54×10^{-19} (B) 1.43×10^{-19}
(C) 2.42×10^{-19} (D) 3.14×10^{-20}

21. In a hydrogen atom, the electron is in nth excited state. It may come down to second excited state by emitting ten different wavelengths. What is the value of n:
(A) 6 (B) 7
(C) 8 (D) 5

22. Difference between nth and $(n + 1)$th Bohr's radius of 'H' atom is equal to it's $(n - 1)$th Bohr's radius. The value of n is:
(A) 1 (B) 2
(C) 3 (D) 4

23. The electron in a hydrogen atom makes transition from M shell to L. The ratio of magnitudes of initial to final centripetal acceleration of the electron is
(A) 9:4 (B) 81:16
(C) 4:9 (D) 16:81

24. The electron in a hydrogen atom makes a transition $n_1 \to n_2$ whose n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The frequency of orbital motion of the electron in the initial state is $\frac{1}{27}$ of that in the final state. The possible values of n_1 and n_2 are
(A) $n_1 = 4, n_2 = 2$ (B) $n_1 = 3, n_2 = 1$
(C) $n_1 = 8, n_2 = 1$ (D) $n_1 = 6, n_2 = 3$

25. The radius of Bohr's first orbit is a_0. The electron in nth orbit has a radius:
(A) na_0 (B) $\frac{a_0}{n}$
(C) n^2a_0 (D) $\frac{a_0}{n^2}$

26. The ionization potential of hydrogen atom is 13.6 V. The energy required to remove an electron from the second orbit of hydrogen is:
(A) 3.4 eV (B) 6.8 eV
(C) 13.6 eV (D) 27.2 eV

27. In a sample of hydrogen like atoms all of which are in ground state, a photon beam containing photons of various energies is passed. In absorption spectrum, five dark lines are observed. The number of bright lines in the emission spectrum will be (Assume that all transitions take place)
(A) 5 (B) 10
(C) 15 (D) none of these

28. When a hydrogen atom, initially at rest emits, a photon resulting in transition $n = 5 \to n = 1$, its recoil speed is about
(A) 10^{-4} m/s (B) 2×10^{-2} m/s
(C) 4.2 m/s (D) 3.8×10^{-2} m/s

29. Consider the spectral line resulting from the transition $n = 2 \to n = 1$ in the atoms and ions given below. The shortest wavelength is produced by:
(A) hydrogen atom
(B) deuterium atom
(C) singly ionized helium
(D) doubly ionized lithium

30. In an atom, two electrons move around the nucleus in circular orbits of radii R and $4R$. The ratio of the time taken by them to complete one revolution is:(neglect electric interaction)
(A) 1:4 (B) 4:1
(C) 1:8 (D) 8:1

31. The electron in hydrogen atom in a sample is in nth excited state, then the number of different spectrum lines obtained in its emission spectrum will be:
(A) $1 + 2 + 3 + \cdots + (n - 1)$
(B) $1 + 2 + 3 + \cdots + (n)$
(C) $1 + 2 + 3 + \cdots + (n + 1)$
(D) $1 \times 2 \times 3 \times \cdots \times (n - 1)$

32. A neutron collides head on with a stationary hydrogen atom in ground state
(A) If kinetic energy of the neutron is less than 13.6 eV, collision must be elastic
(B) if kinetic energy of the neutron is less than 13.6 eV, collision may be inelastic.
(C) inelastic collision takes place when initial kinetic energy of neutron is greater than 13.6 eV.
(D) perfectly inelastic collision cannot take place.

33. The electron in a hydrogen atom make a transition from an excited state to the ground state. Which of the following statement is true?
(A) Its kinetic energy increases and its potential and total energies decrease
(B) Its kinetic energy decreases, potential energy increases and its total energy remains the same.
(C) Its kinetic and total energies decrease and its potential energy increases.
(D) Its kinetic potential and total energies decreases.

34. Radius of the second Bohr orbit of singly ionized helium atom is
(A) 0.53 Å (B) 1.06 Å
(C) 0.265 Å (D) 0.132 Å

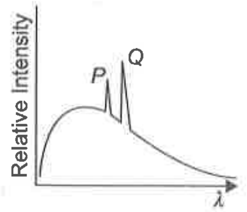
35. An electron in Bohr's hydrogen atom has an energy of −3.4 eV. The angular momentum of the electron is

(A) $\frac{h}{\pi}$ (B) $\frac{h}{2\pi}$

(C) $\frac{nh}{2\pi}$ (n is an integer) (D) $\frac{2h}{\pi}$

-Rays

36. In a characteristic X-ray spectra of some atom superimposed on continuous X-ray spectra



(A) P represents K_α line
(B) Q represents K_β line
(C) Q and P represents K_α and K_β lines respectively
(D) Relative positions of K_α and K_β depend on the particular atom

37. Which of the following wavelength falls in a X-rays region?
(A) 10,000 Å (B) 1000 Å
(C) 1 Å (D) 10^{-2} Å

38. The penetrating power of X-ray increases with the
(A) Increases of its velocity
(B) Increase in its intensity
(C) Decrease in its velocity
(D) Increases in its frequency.

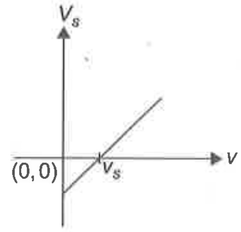
39. If the frequency of K_α, K_β and L_α X-rays for a material $\nu_{K_\alpha}, \nu_{K_\beta}, \nu_{L_\alpha}$ respectively, then

(A) $\nu_{K_\alpha} = \nu_{K_\beta} + \nu_{L_\alpha}$ (B) $\nu_{L_\alpha} = \nu_{K_\alpha} + \nu_{K_\beta}$
(C) $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$ (D) none of these

40. In X-ray tube, when the accelerating voltage V is doubled, the different between the wavelength of K_α line and the minimum cut off of continuous X-ray spectrum:
(A) remains constant
(B) becomes more than two times
(C) becomes half
(D) becomes less than 2 times.

asoning Type

41. **Statement-1:** Figure shows graph of stopping potential and frequency of incident light in photoelectric effect. For values of frequency less than threshold frequency (ν_0) stopping potential is negative.



Statement-2: Lower the value of frequency of incident light (for $n > n_0$) the lower is the maxima of kinetic energy of emitted photoelectrons.
(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

42. **Statement-1:** In the process of photo electric emission, all the emitted photoelectrons have same *KE*.

Statement-2: According to einstein's photo electric equation $KE_{max} = h\nu - \phi$.
(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

43. **Statement-1:** Work function of aluminum is 4.2 eV. If two photons each of energy 2.5 eV strikes on a piece of aluminum, the photo electric emission does not occur.

Statement-2: In photo electric effect a single photon interacts with a single electron and electron is emitted only if energy of each incident photon is greater then the work function.
(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

44. **Statement-1:** An electron and a proton are accelerated through the same potential difference. The de Broglie wavelength associated with the electron is longer.

Statement-2: de Broglie wavelength associated with a moving particle is $\lambda = \frac{h}{p}$ where, p is the linear momentum and both have same KE.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

45. Statement-1: Two photons having equal wavelengths have equal linear momenta.

Statement-2: When light shows its photon character, each photon has a linear momentum $p = \frac{h}{\lambda}$.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

46. Statement-1: If the accelerating potential of a X-ray tube is increased then the characteristic wavelength decreases.

Statement-2: The cut-off wavelength for X-ray tube is given by $\lambda_{min} = \frac{hc}{eV}$, where V is accelerating potential.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

JEE Advanced

Single Correct Option Type Questions

Photo Electric Effect

1. In a photo-emissive cell, with exciting wavelength λ, the maximum kinetic energy of electron is K. If the exciting wavelength is changed to $\frac{3\lambda}{4}$ the kinetic energy of the fastest emitted electron will be:

(A) $\frac{3K}{4}$ (B) $\frac{4K}{3}$

(C) less than $\frac{4K}{3}$ (D) greater than $\frac{4K}{3}$

2. The frequency and the intensity of a beam of light falling on the surface of photoelectric material are increased by a factor of two (Treating efficiency of photoelectron generation as constant). This will:

(A) increase the maximum energy of the photoelectrons, as will as photoelectric current by a factor of two
(B) increase the maximum kinetic energy of the photo electrons and would increase the photoelectric current by a factor of two
(C) increase the maximum kinetic energy of the photoelectrons by a factor of greater than two and will have no effect on the magnitude of photoelectric current produced.
(D) not produce any effect on the kinetic energy of the emitted electrons but will increase the photoelectric current by a factor of two

3. Light coming from a discharge tube filled with hydrogen falls on the cathode of the photoelectric cell. The work function of the surface of cathode is 4 eV. Which one of the following values of the anode voltage (in Volts) with respect to the cathode will likely to make the photo current zero.

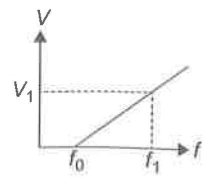
(A) –4 (B) –6
(C) –8 (D) –10

4. Let K_1 be the maximum kinetic energy of photoelectrons emitted by a light of wavelength λ_1 and K_2 corresponding to λ_2. If $\lambda_1 = 2\lambda_2$, then:

(A) $2K_1 = K_2$ (B) $K_1 = 2K_2$

(C) $K_1 < \frac{K_2}{2}$ (D) $K_1 > 2K_2$

5. In a photoelectric experiment, the potential difference V that must be maintained between the illuminated surface and the collector so as just to prevent any electron from reaching the collector is determined for different frequencies f of the incident illumination. The graph obtained is shown. The maximum kinetic energy of the electrons emitted at frequency f_1 is



(A) hf_1 (B) $\frac{V_1}{(f_1 - f_0)}$

(C) $h(f_1 - f_0)$ (D) $eV_1(f_1 - f_0)$

6. In a photoelectric experiment, the collector plate is at 2.0 V with respect to the emitter plate made of copper ($\varphi = 4.5$ eV). The emitter is illuminated by a source of monochromatic light of wavelength 200 nm.

(A) the minimum kinetic energy of the photoelectrons reaching the collector is 0.

(B) the maximum kinetic energy of the photoelectrons reaching the collector is 3.7 eV.

(C) if the polarity of the battery is reversed then answer to part *A* will be 0.

(D) if the polarity of the battery is reversed then answer to part *B* will be 1.7 eV.

7. Monochromatic light with a frequency well above the cutoff frequency is incident on the emitter in a photoelectric effect apparatus. The frequency of the light is then doubled while the intensity is kept constant. How does this effect the photoelectric current?

(A) The photoelectric current will increase

(B) The photoelectric current will decrease

(C) The photoelectric current will remain the same

(D) None of these

8. If a parallel beam of light having intensity *I* is incident normally on a perfectly reflecting surface, the force exerted on the surface, equals *F* (Assume that the cross section of beam remains constant).

(A) $2F \tan\theta$ (B) $F \cos\theta$

(C) $F \cos^2\theta$ (D) $2F$

Matter Waves

9. Imagine a Young's double slit interference experiment performed with waves associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease maximum if

(A) the accelerating voltage in the electron gun is decreased

(B) the accelerating voltage is increased and the distance of the screen from the slits is decreased

(C) the distance of the screen from the slits is increased.

(D) the distance between the slits is decreased.

Atomic Structure

10. In a hypothetical system of particle a mass *m* and charge $-3q$ is moving around a very heavy particle having charge *q*. Assuming Bohr's model to be true to this system, the orbital velocity of mass *m* when it is nearest to heavy particle is

(A) $\frac{3q^2}{2\varepsilon_0 h}$ (B) $\frac{3q^2}{4\varepsilon_0 h}$

(C) $\frac{3q}{2\varepsilon_0 h}$ (D) $\frac{3q}{4\varepsilon_0 h}$

11. de Broglie wavelength of an electron in the *n*th Bohr orbit is λ_n and the angular momentum is J_n, then:

(A) $J_n \,\mu\, \lambda_n$ (B) $\lambda_n \propto \frac{1}{J_n}$

(C) $\lambda_n \propto J_n^2$ (D) none of these

12. In a hydrogen atom, the binding energy of the electron in the *n*th state is E_n, then the frequency of revolution of the electron in the *n*th orbits is:

(A) $\frac{2E_n}{nh}$ (B) $\frac{2E_n n}{h}$

(C) $\frac{E_n}{nh}$ (D) $\frac{E_n n}{h}$

13. In hydrogen and hydrogen like atoms, the ratio of difference of energies $E_{4n} - E_{2n}$ and $E_{2n} - E_n$ varies with its atomic number *z* and *n* as:

(A) $\frac{z^2}{n^2}$ (B) $\frac{z^4}{n^4}$

(C) $\frac{z}{n}$ (D) $\frac{z^0}{n^0}$

14. An electron in hydrogen atom after absorbing energy photons can jump between energy states n_1 and n_2($n_2 > n_1$). Then it may return to ground state after emitting six different wavelengths in emission spectrum. The energy of emitted photons is either equal to, less than or greater than the absorbed photons. Then n_1 and n_2 are:

(A) $n_2 = 4, n_1 = 3$ (B) $n_2 = 5, n_1 = 3$

(C) $n_2 = 4, n_1 = 2$ (D) $n_2 = 4, n_1 = 1$

15. Electron in a hydrogen atom is replaced by an identically charged particle muon with mass 207 times that of electron. Now the radius of *K* shell will be

(A) 2.56×10^{-3} Å (B) 109.7 Å

(C) 1.21×10^{-3} Å (D) 22174.4 Å

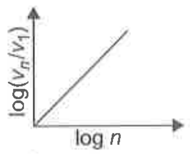
16. Monochromatic radiation of wavelength λ is incident on a hydrogen sample containing in ground state. Hydrogen atoms absorb the light and subsequently emit radiations of ten different wavelengths. The value of λ is

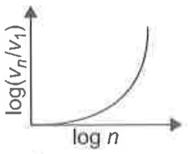
(A) 95 nm (B) 103 nm

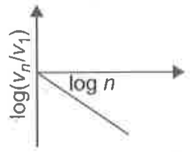
(C) 73 nm (D) 88 nm

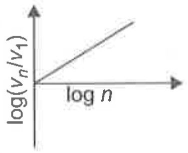
17. An electron collides with a fixed hydrogen atom in its ground state. Hydrogen atom gets excited and the colliding electron loses all its kinetic energy. Consequently the hydrogen atom may emit a photon corresponding to the largest wavelength of the Balmer series. The min. *KE* of colliding electron will be
(A) 10.2 eV (B) 1.9 eV
(C) 12.1 eV (D) 13.6 eV

18. The frequency of revolution of electron in *n*th Bohr orbit is v_n. The graph between log *n* and log $\left(\frac{v_n}{v_1}\right)$ may be

(A)


(B)


C)


(D)


19. The total energy of a hydrogen atom in its ground state is –13.6 eV. If the potential energy in the first excited state is taken as zero then the total energy in the ground state will be:
(A) –3.4 eV (B) 3.4 eV
(C) –6.8 eV (D) 6.8 eV

20. The magnitude of angular momentum, orbit radius and frequency of revolution of electron in hydrogen atom corresponding to quantum number *n* are *L*, *r* and *f* respectively. Then according to Bohr's theory of hydrogen atom,
(A) fr^2L is constant for all orbits
(B) frL is constant for all orbits
(C) f^2rL is constant for all orbits
(D) frL^2 is constant for all orbits

X-Rays

21. The 'K_α' X-rays emission line of tungsten occurs at $\lambda = 0.021$ nm. The energy difference between *K* and *L* levels in this atom is about
(A) 0.51 MeV (B) 1.2 MeV
(C) 59 keV (D) 13.6 eV

22. The wavelength of the K_α line for an element of atomic number 57 is α. What is the wavelength of the K_α line for the element of atomic number 29?
(A) α (B) 2α
(C) 4α (D) 8α

Multiple Correct Option Type Questions

Photo Electric Effect

23. In photoelectric effect, stopping potential depends on
(A) frequency of the incident light
(B) intensity of the incident light by varies source distance
(C) emitter's properties
(D) frequency and intensity of the incident light

24. In the experiment on photoelectric effect using light having frequency greater than the threshold frequency, the photocurrent will certainly increase when
(A) Anode voltage is increased
(B) Area of cathode surface is increased
(C) Intensity of incident light is increased
(D) Distance between anode and cathode is increased.

Matter Waves

25. Two electrons are moving with the same speed *v*. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after sometime if the de Broglie wavelengths of the two are λ_1 and λ_2 then:
(A) $\lambda_1 = \lambda_2$ (B) $\lambda_1 > \lambda_2$
(C) $\lambda_1 < \lambda_2$ (D) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

Atomic Structure

26. An electron in hydrogen atom first jumps from second excited state to first excited state and then, from first excited state to ground state. Let the ratio of wavelength, momentum and energy of photons in the two cases by *x*, *y* and *z*, then select the wrong answers:
(A) $z = \frac{1}{x}$ (B) $x = \frac{9}{4}$
(C) $y = \frac{5}{27}$ (D) $z = \frac{5}{27}$

27. An electron is in an excited state in hydrogen-like atom. It has a total energy of –3.4 eV. If the kinetic energy of the electron is *E* and its de Broglie wavelength is λ, then
(A) $E = 6.8$ eV, $\lambda = 6.6 \times 10^{-10}$ m
(B) $E = 3.4$ eV, $\lambda = 6.6 \times 10^{-10}$ m
(C) $E = 3.4$ eV, $\lambda = 6.6 \times 10^{-11}$ m
(D) $E = 6.8$ eV, $\lambda = 6.6 \times 10^{-11}$ m

28. A particular hydrogen like atom has its ground state binding 'energy 122.4 eV. Its is in ground state. Then:

(A) Its atomic number is 3
(B) An electron of 90 eV can excite it.
(C) An electron of kinetic energy nearly 91.8 eV can be brought to almost rest by this atom.
(D) An electron of kinetic energy 2.6 eV may emerge from the atom when electron of kinetic energy 125 eV collides with this atom.

29. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are:
(A) $n_1 = 4, n_2 = 2$
(B) $n_1 = 8, n_2 = 2$
(C) $n_1 = 8, n_2 = 1$
(D) $n_1 = 6, n_2 = 3$

30. A beam of ultraviolet light of all wavelengths passes through hydrogen gas at room temperature, in the x-direction. Assume that all photons emitted due to electron transition inside the gas emerge in the y-direction. Let A and B denote the lights emerging from the gas in the x and y directions respectively.
(A) Some of the incident wavelengths will be absent in A
(B) Only those wavelengths will be present in B which are absent in A
(C) B will contain some visible light.
(D) B will contain some infrared light.

31. If radiation of allow wavelengths from ultraviolet to infrared is passed through hydrogen agas at room temperature, absorption lines will be observed in the:
(A) Lyman series
(B) Balmer series
(C) Both (A) and (B)
(D) neither (A) nor (B)

32. In the hydrogen atom, if the reference level of potential energy is assumed to be zero at the ground state level. Choose the incorrect statement.
(A) The total energy of the shell increases with increase in the value of n
(B) The total energy of the shell decrease with increase in the value of n.
(C) The difference in total energy of any two shells remains the same.
(D) The total energy at the ground state becomes 13.6 eV.

33. Choose the correct statement(s) for hydrogen and deuterium atoms (considering motion of nucleus)
(A) The radius of first Bohr orbit of deuterium is less than that of hydrogen
(B) The speed of electron in the first Bohr orbit of deuterium is more than that of hydrogen.
(C) The wavelength of first Balmer line of deuterium is more than that of hydrogen
(D) The angular momentum of electron in the first Bohr orbit of deuterium is more than that of hydrogen.

34. Let A_n be the area enclosed be the nth orbit in a hydrogen atom. The graph of $\ln\left(\frac{A_n}{A_1}\right)$ against $\ln(n)$.
(A) will pass through origin
(B) will be a straight line will slope 4
(C) will be a monotonically increasing nonlinear curve
(D) will be a circle.

35. A free hydrogen atom in ground state is at rest. A neutron of kinetic energy 'K' collides with the hydrogen atom. After collision hydrogen atom emits two photons in succession one of which has energy 2.55 eV. (Assume that the hydrogen atom and neutron has same mass)
(A) minimum value of 'K' is 25.5 eV.
(B) minimum value of 'K' is 12.75 eV.
(C) the other photon has energy 10.2 eV
(D) the upper energy level is of excitation energy 12.75 eV.

36. A neutron collides head-on with a stationary hydrogen atom in ground state. Which of the following statements are correct (Assume that the hydrogen atom and neutron has same mass)
(A) If kinetic energy of the neutron is less than 20.4 eV collision must be elastic
(B) If kinetic energy of the neutron is less than 20.4 eV collision may be inelastic
(C) Inelastic collision may be take place only when initial kinetic energy of neutron is greater than 20.4 eV.
(D) Perfectly inelastic collision can not take place.

X-Rays

37. In a Coolidge tube experiment, the minimum wavelength of the continuous X-ray spectrum is equal to 66.3 pm, then
(A) electrons accelerate through a potential difference of 12.75 kV in the Coolidge tube
(B) electrons accelerate through a potential difference of 18.75 kV in the Coolidge tube
(C) de Broglie wavelength of the electrons reaching the anticathode is of the order of 10 μm
(D) de Broglie wavelength of the electrons reaching the anticathode is 0.01 Å.

38. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation:
(A) the intensity increases
(B) the minimum wave length increases
(C) the intensity decreases
(D) the minimum wave length decreases

39. A X-ray tube operates at an accelerating potential of 20 kV. Which of the following wavelengths will be absent in the continuous spectrum of X-ray.
(A) 12 pm (B) 45 pm
(C) 65 pm (D) 95 pm

40. X-ray are produced by accelerating electrons across a given potential difference to strike a meta target of high atomic number. If the electrons have same speed when they strike the target, the X-ray spectrum will exhibit.
(A) a minimum wavelength
(B) a continuous spectrum
(C) some discrete comparatively prominent wavelength
(D) uniform density over the whole spectrum

JEE Advanced

Level I

Photo Electric Effect

1. A parallel beam of uniform, monochromatic light of wavelength 2640 A has an intensity of 200 W/m^2. The number of photons in 1 mm^3 of this radiation are ______

2. When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_a eV and de Broglie wavelength λ_a. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.7 eV is $T_b = (T_a - 1.5)$ eV. If the de Broglie wavelength of these photoelectrons is $\lambda_b = 2\,\lambda_a$, then find
(a) The work function of a
(b) The work function of b is
(c) T_a and T_b

3. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut off voltage and the saturation current are respectively 0.6 volt and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then find
(a) the stopping potential
(b) the saturation current

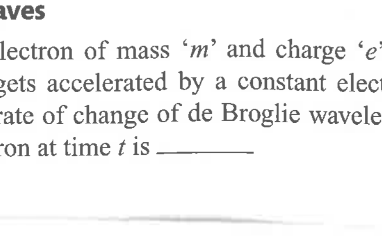
4. An isolated metal body is illuminated with monochromatic light and is observed to become charged to a steady positive potential 1.0 V with respect to the surrounding. The work function of the metal is 3.0 eV. The frequency of the incident light is ______.

5. 663 mW of light from a 540 nm source is incident on the surface of a metal. If only 1 of each 5 × 10^9 incident photons in absorbed and causes an electron to be ejected from the surface, the total photocurrent in the circuit is ______.

6. Light of wavelength 330 nm falling on a piece of metal ejects electrons with sufficient energy which requires voltage V_0 to prevent a collector. In the same setup, light of wavelength 220 nm, ejects electrons which require twice the voltage V_0 to stop them in reaching a collector. Find the numerical value of voltage V_0. (Take Planck's constant, $h = 6.6 \times 10^{-34}$ Js and 1 eV = 1.6×10^{-19} J)

7. A small 10W source of ultraviolet light of wavelength 99 nm is held at a distance 0.1 m from a metal surface. The radius of an atom of the metal is approximately 0.05 nm. Find
(i) the average number of photons striking an atom per second.
(ii) the number of photoelectrons emitted per unit area per second it the efficiency of liberation of photoelectrons is 1%

8. The surface of cesium is illuminated with monochromatic light of various wavelengths and the stopping potentials for the wavelengths are measured. The results of this experiment is plotted as shown in the figure. Estimate the value of work function of the cesium and Planck's constant.



Matter Waves

9. An electron of mass 'm' and charge 'e' initially at rest gets accelerated by a constant electric field E. The rate of change of de Broglie wavelength of this electron at time t is ______

10. Assume that a particle cannot be confined to a spherical volume of diameter less then de Broglie wavelength of the particle. Estimate the minimum kinetic energy a proton confined to a nucleus *f* diameter 10^{-14} m may have.

Atomic Structure

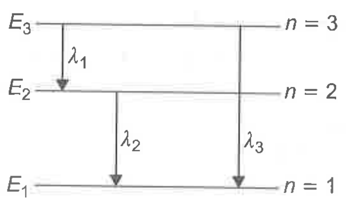
11. A hydrogen atom in a state having a binding energy 0.85 eV makes a transition to a state of excitation energy 10.2 eV. The wave length of emitted photon is ______ nm.

12. A hydrogen atom is in 5in excited state. When the electron jumps to ground state the velocity of recoiling hydrogen atom is ______ m/s and the energy of the photon is ______ eV.

13. The ratio of series limit wavelength of Balmer series to wavelength of first line of Paschen series is ______

14. An electron joins a helium nucleus to form a He^+ ion. The wavelength of the photon emitted in this process if the electron is assumed to had has no kinetic energy when it combines with nucleus is ______ nm.

15. Three energy levels of an atom are shown in the figure. The wavelength corresponding to three possible transition are λ_1, λ_2 and λ_3. The value of λ_3 in terms of λ_1 λ_2 is given by



16. Imagine an atom made up of a proton and a hypothetical particle of double the mass of an electron but having the same charge as the electron. Apply the Bohr atom model and consider a possible transitions of this hypothetical particle to the first excited level. Find the longest wavelength photon that will be emitted λ (in terms of the Rydberg constant *R*.)

17. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10^{16} revolution per second. The magnetic moment associated with the orbital motion of the electron is ______.

18. The positron is a fundamental particle with the same mass as that of the electron and with a charge equal to that of an electron but of opposite sign. When a positron and an electron collide, they may annihilate each other. The energy corresponding to their mass appears in two photons of equal energy. Find the wavelength of the radiation emitted.

[Take: mass of electron = $(0.5/C^2)$ MeV and $hC = 1.2 \times 10^{-12}$ MeVm where *h* is the Planck's constant and *C* is the velocity of light in air]

19. A hydrogen like atom has its single electron orbiting around its stationary nucleus. The energy to excite the electron from the second Bohr orbit to the third Bohr orbit is 47.2 eV. The atomic number of this nucleus is ______.

20. A single electron orbits a stationary nucleus of charge *Ze* where *Z* is a constant and e is the electronic charge. It requires 47.2 eV to excite the electron from the 2nd Bohr orbit to 3rd Bohr orbit. Find
    (i) the value of *Z*,
    (ii) energy required to excite the electron from the third to the fourth orbit.
    (iii) the wavelength of radiation required to remove the electron from the first orbit to infinity
    (iv) the kinetic energy, potential energy and angular momentum in the first Bohr orbit
    (v) the radius of the first Bohr orbit.

21. A hydrogen like atom (atomic number *Z*) is in higher excited state of quantum number *n*. This excited atom can make a transition of the first excited state by successively emitting two photons of energy 22.95 eV and 5.15 eV respectively. Alternatively, the atom from the same excited state can make transition to the second excited state by successively emitting two photons of energies 2.4 eV and 8.7 eV respectively. Find the values of n and *Z*.

22. Find the binding energy of an electron in the ground state of a hydrogen like atom in whose spectrum the third of the corresponding Balmer series is equal to 108.5 nm.

23. Which level of the doubly ionized lithium has the same energy as the ground state energy of the hydrogen atom. Find the ratio of the two radii of corresponding orbits.

24. Determine the number of lines in Paschen series which have a wavelength greater than 1000 nm.

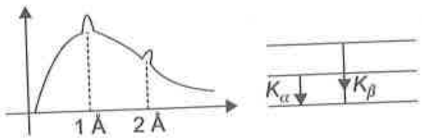
25. In a block body radiation at certain temperature T_1, the wavelength having maximum intensity of radiation equals 9000 Å. When the temperature is increased from T_1 to T_2 the total radiation increases 16 times. The peak radiation at T_2 is found to be capable of ejecting photoelectrons. The maximum kinetic energy of the photoelectrons is the same as the energy of photon that one gets when one of electrons in the M-shell of hydrogen atom jumps to L-shell. What is the work function of the metal?

Section-D X-Ray

26. Obtain a relation between the frequencies of K_α, K_β and L_α lines for a target material.

27. A 20 KeV energy electron is brought to rest in an X-ray tube, by undergoing two successive bremsstrahlung events, thus emitting two photons. The wavelength of the second photon is 130×10^{-12} m greater than the wavelength of the first emitted photon. Calculate the wavelengths of the two photons.

28. Figures shows K_α and K_β X-rays along with continuous X-ray. Find the energy of L_α X-ray. (Use hc = 12420 eVÅ).



29. Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α-particles. A maximum energy electron combine with an α-particle to form a He^+ ion, emitting a singl photon in this process. He^+ ions thus formed are i their fourth excited state. Find the energies in eV o the photons, lying in the 2 to 4 eV range, that are like to be emitted during and after the combination. [Tak $h = 4.14 \times 10^{-15}$ eV-s]

30. The wavelength of K_α x-ray of tungsten is 21.3 pı It takes 11.3 KeV to knock out an electron from t L shell of a tungsten atom. What should be the mi mum accelerating voltage across *ab* x-ray tube ha ing tungsten target which allows production of x-ray.

31. An electron, in a hydrogen like atom, is in an exci state. It has a total energy of –3.4 eV. Calculate:
    (i) The kinetic energy and
    (ii) The de Broglie wave length of the electron.

32. A pontial difference of 20 KV is applied across x-ray tube. The minimum wave length of X-rays g erated is ______.

Level II

1. In a photo electric effect set-up, a point source of light of power 3.2×10^{-3} W emits mono energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius 8.0×10^{-3} m. The efficiency of photo electrons emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photo electrons are instantly swept away after emission.
    (A) Calculate the number of photo electrons emitted per second.
    (B) Find the ratio of the wavelength of incident light to the de Broglie wave length of the fastest photo electrons emitted.
    (C) It is observed that the photo electron emission stops at a certain time t after the light source is switched on. Why?
    (D) Evaluate the time t.

2. A stationary He^+ ion emitted a photon corresponding to the first line its Lyman series. That photon liberated a photoelectron from a stationary hydrogen atom in the ground state. Find the velocity of the photoelectron.

3. A gas of identical hydrogen like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. atoms of the gas make transition to a higher en level by the absorbing monochromatic light of p ton energy 2.7 eV. Subsequently, the atoms emi diation of only six different photon energies. Son the emitted photons have energy 2.7 eV. Some energy more and some have less than 2.7 eV.
    (i) Find the principal quantum number of the tially excited level B.
    (ii) Find the ionization energy for the gas atom
    (iii) Find the maximum and the minimum ene of the emitted photons.

4. An energy of 68.0 eV is required to excite a h gen like atom from its second Bohr orbit to the The nuclear charge Ze. Find the value of Z, the k energy of the electron in the first Bohr orbit ar wavelength of the electromagnetic radiation re to eject the electron from the first Bohr orbit to in

5. Electrons in hydrogen like atoms ($Z = 3$) make t tions from the fifth to the fourth orbit and fro fourth to the third orbit. The resulting radiatio incident normally on a metal plate and eject electrons. The stopping potential for the phot trons ejected by the shorter wavelength is 3 Calculate the work function of the metal, a stopping potential for the photoelectrons ejec

the longer wavelength.(Rydberg constant $=1.094 \times 10^7 m^{-1}$)

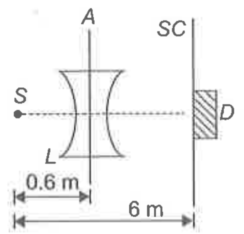
6. A beam of light has three wavelengths 4144 Å, 4972 Å and 6216Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds.

7. Monochromatic radiation of wavelength $\lambda_1 = 3000$ Å falls on a photocell operating in saturating mode. The corresponding spectral sensitivity of photocell is $J = 4.8 \times 10^{-3}$ A/w. When another monochromatic radiation of wavelength $\lambda_2 = 1650$ Å and power $P = 5 \times 10^{-3}$ W is incident, it is found that maximum velocity of photoelectrons increases $n = 2$ times. Assuming efficiency of photoelectron generation per incident photon to be same for both the cases, calculate

(i) threshold wavelength for the cell.

(ii) saturation current in second case.

8. A monochromatic point source S radiating wavelength 6000 Å with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in figure. A photoemissive detector D of surface area 0.5 cm^2 is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9.



(i) Calculate the photon flux density at the centre of the screen and the photocurrent in the detector.

(ii) If a concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux density and photocurrent Assume a uniform average transmission of 80% for the lens.

(iii) If the work-function of the photoemissive surface is 1 eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture.)

9. A small 10 W source of ultraviolet light of wavelength 99 nm is held at a distance 0.1 m from a metal surface. The radius of an atom of the metal is approximately 0.05 nm. Find:

(i) the number of photons striking an atom per second.

(ii) the number of photoelectrons emitted per second it the efficiency of liberation of photoelectrons is 1%

10. A monochromatic light source of frequency ν illuminates a metallic surface and ejects photoelectrons. The photoelectrons having maximum energy are just able to ionize the hydrogen atoms in ground state. When the whole experiment is repeated with an incident radiation of frequency $\left(\frac{5}{6}\right)\nu$, the photoelectrons so emitted are able to excite the hydrogen atom beam which then emits a radiation of wavelength of 1215 Å. Find the work function of the metal and the frequency ν.

11. A neutron of kinetic energy 65 eV collides inelastically with a single ionized helium atom at rest. It is scattered at an angle of 90° with respect of its original direction.

(i) Find the allowed values of the energy of the neutron and that of the atom after collision.

(ii) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation.

(Given: Mass of he atom = 4 × (mass of neutron), ionization energy of H atom = 13.6 eV)

12. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV respectively. Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the values of n and Z. (Ionization energy of hydrogen atom = 13.6 eV)

13. Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance 'd' between the atoms of the array is 2 Å. A similar standing wave is again formed if 'd' is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form.

14. A beam of ultraviolet light of wavelength 100 nm – 200 nm is passed through a box filled with hydrogen gas in ground state. The light coming out of the box is split into two beams '*A*' and '*B*'. A contains unabsorbed light from the incident light and 'B' contains emitted light by hydrogen atoms. The beam '*A*' is incident on the emitter in a photoelectric tube. The stopping potential the case is 5 volts. Find the work function of t emitter. In the second case the beam '*B*' is incid on the same emitter. Find the stopping potent in this case. You can assume that the transition higher energy states are not permitted from the e cited states. Use $hc = 12400$ eVÅ.

Previous Years' Questions

JEE Main

1. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then the ratio of the wavelengths is nearest to **[AIEEE-2002]**
   (A) 1:2 (B) 4:1
   (C) 2:1 (D) 1:4

2. Formation of covalent bonds in compounds exhibits **[AIEEE-2002]**
   (A) wave nature of electron
   (B) particle nature of electron
   (C) both wave and particle nature of electron
   (D) None of the above

3. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is **[AIEEE-2002]**
   (A) 10.2 eV (B) zero
   (C) 3.4 eV (D) 6.8 eV

4. Two identical, photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2 then **[AIEEE-2003]**
   (A) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$
   (B) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$
   (C) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$
   (D) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

5. Which of the following atoms has the lowest ionization potential? **[AIEEE-2003]**
   (A) $^{14}_{7}N$ (B) $^{133}_{55}Cs$
   (C) $^{40}_{18}Ar$ (D) $^{16}_{8}O$

6. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **[AIEEE-2004]**
   (A) 540 nm (B) 400 nm
   (C) 310 nm (D) 220 nm

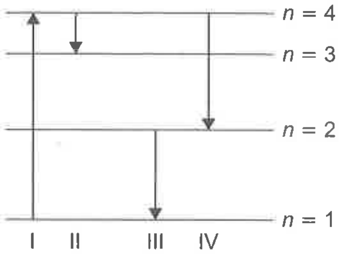
7. According to Einstein's photoelectric equation, plot of the kinetic energy of the emitted photoelectr from a metal Vs the frequency of the incident radiat gives a straight line whose slope **[AIEEE-20**
   (A) depends on the nature of the metal used
   (B) depends on the intensity of the radiation
   (C) depends both on the intensity of the radiat and the metal used
   (D) is the same for all metal and independent of intensity of the radiation

8. When U^{238} nucleus originally at rest, decays by em ting an alpha particle having a speed u, the re speed of the residual nucleus is **[AIEEE-20**
   (A) $\frac{4u}{238}$ (B) $-\frac{4u}{234}$
   (C) $\frac{4u}{234}$ (D) $-\frac{4u}{238}$

9. If the kinetic energy of a free electron doubles, it Broglie wavelength changes by the factor **[AIEEE-20**
   (A) $\frac{1}{2}$ (B) 2
   (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

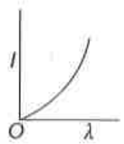
10. A photocell is illuminated by a small bright so placed 1 m away. When the same source of lig placed away, $\frac{1}{2}$ m the number of electrons emitte photocathode would **[AIEEE-2**
    (A) decrease by a factor of 4
    (B) increase by a factor of 4
    (C) decrease by a factor of 2
    (D) increase by a factor of 2

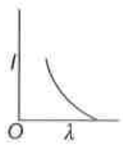
11. The diagram shows the energy levels for an elec in a certain atom. Which transition shown repres the emission of a photon with the most energy? **[AIEEE-2**

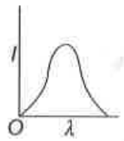


(A) III (B) IV
(C) I (D) II

12. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows **[AIEEE-2006]**

(A)


(B)


(C)


(D) None

13. The time taken by a photoelectron to come out after the photon strikes is approximately **[AIEEE-2006]**

(A) 10^{-4} s (B) 10^{-10} s
(C) 10^{-16} s (D) 10^{-1} s

14. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5 V. The incident radiation lies in **[AIEEE-2006]**

(A) ultra-violet region (B) infra-red region
(C) visible region (D) X-ray region

15. Photon of frequency v has a momentum associated with it. If c is the velocity of light, the momentum is **[AIEEE-2007]**

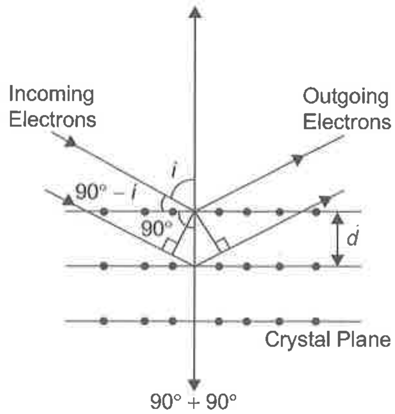
(A) $\frac{v}{c}$ (B) hvc
(C) $\frac{hv}{c^2}$ (D) $\frac{hv}{c}$

16. Which of the following transitions in hydrogen atoms emit photons of highest frequency? **[AIEEE-2007]**

(A) $n = 2$ to $n = 6$ (C) $n = 6$ to $n = 2$
(C) $n = 2$ to $n = 1$ (D) $n = 1$ to $n = 2$

Directions: Questins No. 17, 18, and 19 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure). **[AIEEE-2008]**



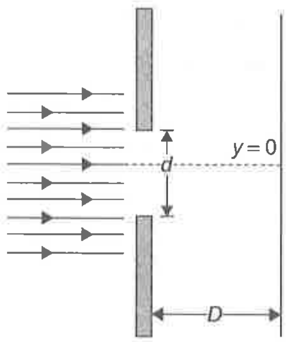
17. Electrons accelerated by potential V are diffracted from a crystal. If $d = 1$ Å and $i = 30°$ V should be about ($h = 6.6 \times 10^{-34}$ J-s, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C) **[AIEEE-2008]**

(A) 2000 V (B) 50 V
(C) 500 V (D) 1000 V

18. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer) **[AIEEE-2008]**

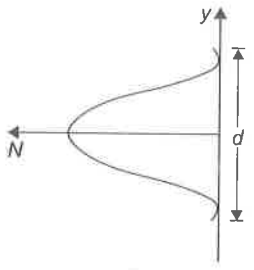
(A) $d \sin i = n\lambda_{dB}$ (B) $2d \cos i = n\lambda_{dB}$
(C) $2d \sin i = n\lambda_{dB}$ (D) $d \cos i = n\lambda_{dB}$

19. In an experiment electrons are made to pass through a narrow slit of width d comparable to their de Broglie wavelength. They are detected on a screen at a distance D from the slit (see figure).

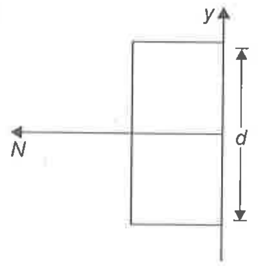


Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y = 0$ corresponds to the middle of the slit)? **[AIEEE-2008]**

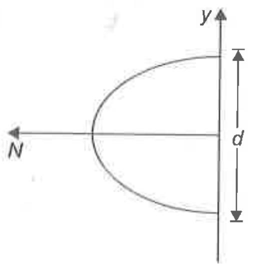
(A)



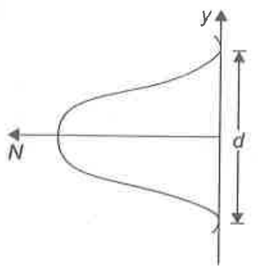
(B)



(C)



(D)



20. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$, where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this systems, the radius of the nth orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then which of the following is true? **[AIEEE-2008]**

(A) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$

(B) T_n is independent of n, $r_n \propto n$

(C) $T_n \propto \frac{1}{n}, r_n \propto n$

(D) $T_n \propto \frac{1}{n}, r_n \propto n^2$

21. The surface a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is ($hc = 1240$ eV-nm) **[AIEEE-2009]**

(A) 3.09 eV (B) 1.42 eV
(D) 151 eV (D) 1.68 eV

22. The transition from the state $n = 4$ to a $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from **[AIEEE-2009]**

(A) $2 \to 1$ (B) $3 \to 2$
(C) $4 \to 2$ (D) $5 \to 4$

23. If a source of power 4 kW produces 10^{20} photons/second, the radiation belong to a part of the spectrum called **[AIEEE-2010]**

(A) X-rays (B) ultraviolet rays
(C) microwaves (D) γ-rays

24. This question has Statement I and Statement II of the four choices given after the statements, choose the one that best describes the two statements.

Statement I: A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement II: The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light. **[AIEEE-2011]**

(A) Statement I is the true, Statement II is ture, Statement II is the correct explanation of Statement I
(B) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I
(C) Statement I is false, Statement II is true
(D) Statement I is true, Statement II is false

25. Energy required for the electron excitation in Li^{2+} from the first to the third Bohr orbit is **[AIEEE-2011]**

(A) 36.3 eV (B) 108.8 eV
(C) 122.4 eV (D) 12.1 eV

26. After absorbing a slowly moving neutron of mass m_N (momentum ~ 0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1$ ($6m_1 = M + m_N$), respectively. If the de Broglie wavelength of the nucleus with mass m_1 is , then de Broglie wavelength of the other nucleus will be **[AIEEE-2011]**

(A) 25λ (B) 5λ

(C) $\frac{\lambda}{5}$ (D) λ

27. This question has statement 1 and statement 2. Of the four choices given the statements, choose the one that describes the two statements. **[AIEEE-2012]**

Statement 1: Davission-Germer experiment established the wave nature of electrons.

Statement 2: If electrons have wave nature, they can interfere and show diffraction.

(A) Statement 1 is false, Statement 2 is true.
(B) Statement 1 is true, Statement 2 is false.
(C) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(D) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

28. Hydrogen atom is excited from ground state to another state with principle quantum number equal to 4. Then, the number of spectral lines in the emission spectra will be **[AIEEE-2012]**

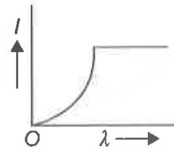
(A) 2 (B) 3
(C) 5 (D) 6

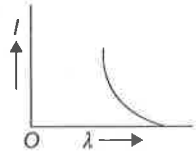
29. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer) **[AIEEE-2012]**

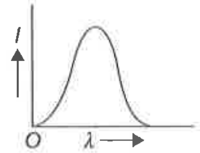
(A) $\frac{(m_1+m_2)^2 n^2 h^2}{2m_1^2 m_2^2 r^2}$ (B) $\frac{n^2 h^2}{2(m_1+m_2)r^2}$

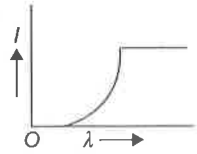
(C) $\frac{2n^2 h^2}{(m_1+m_2)r^2}$ (D) $\frac{(m_1+m_2)n^2 h^2}{2m_1 m_2 r^2}$

30. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows: **[JEE Main-2013]**

(A)


(B)


(C)


(D)


31. In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted it proportional to **[JEE Main-2013]**

(A) $\frac{1}{n^{3/2}}$ (B) $\frac{1}{n^3}$

(C) $\frac{1}{n}$ (D) $\frac{1}{n^2}$

32. Hydrogen $({}_1H^1)$, Deuterium $({}_1H^2)$, singly ionised Helium $({}_2He^4)^+$ and doubly ionized lithium $({}_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct? **[JEE Main-2014]**

(A) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(B) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$
(C) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(D) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

33. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to: **[JEE Main-2014]**

(A) 0.8 eV (B) 1.6 eV
(C) 1.8 eV (D) 1.1 eV

JEE Advanced

1. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Then the number of electrons striking the target per second is **[JEE-2002 (Scr.)]**
(A) 2×10^{16} (B) 5×10^{16}
(C) 1×10^{17} (D) 4×10^{15}

2. A Hydrogen atom and Li^{++} ion are both in the second excited state. If l_H and l_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then **[JEE-2002 (Scr.)]**
(A) $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$
(B) $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$
(C) $l_H = l_{Li}$ and $|E_H| > |E_{Li}|$
(D) $l_H < l_{Li}$ and $|E_H| < |E_{Li}|$

3. A hydrogen like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transition between a group of levels. These levels have energies between –0.85 eV and –0.544 eV (including both these values) **[JEE-2002]**
(A) Find the atomic number of the atom.
(B) Calculate the smallest wavelength emitted in these transitions.

4. Two metallic plates A and B each of area 5×10^{-4} m^2, are placed at a separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at $t = 0$ so that 10^{16} photon fall on it per square meter per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV. Determine **[JEE-2002]**
(A) the number of photoelectrons emitted up to t = 10 sec.
(B) the magnitude of the electric field between the plates A and B at t = 10 s and
(C) the kinetic energy of the most energetic photoelectron emitted at t = 10 s when it reaches plate B.
(Neglect the time taken by photoelectron to reach plate B)

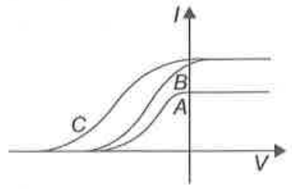
5. If the atom $_{100}Fm^{257}$ follows Bohr model and the radius of last orbit of $_{100}Fm^{257}$ is n times the Bohr radius, then find n **[JEE-2003]**
(A) 100 (B) 200
(C) 4 (D) $\frac{1}{4}$

6. The attractive potential for an atom is given by $v = v_0 / n\,(r/r_0)$, v_0 and r_0 are constant and r is the radius of the orbit. The radius r of the nth Bohr's orbit depends upon principal quantum number n as: **[JEE-2003 (Scr.)]**
(A) $r \propto n$ (B) $r \propto 1/n^2$
(C) $r \propto n^2$ (D) $r \propto 1/n$

7. Frequency of a photon emitted due to transition of electron of certain element from L to K shell is found to be 4.2×10^{18} Hz. Using Moseley's law, find the atomic number of the element, given that the Rydberg's constant $R = 1.1\times10^7$ m^{-1}. **[JEE-2003]**

8. In a photoelectric experiment set up, photons of energy 5 eV falls on the cathode having work function 3 eV.
(A) If the saturation current is $i_A = 4$ μA for intensity 10^{-5} W/m^2, then plot a graph between anode potential and current.
(B) Also draw a graph for intensity of incident radiation of 2×10^{-5} W/m^2? **[JEE-2003]**

9. In a photoelectric experiment anode potential is plotted against plate current **[JEE-2004 (Scr.)]**



(A) A and B will have different intensities while B and C will have different frequencies.
(B) B and C will have different intensities while A and C will have different frequencies.
(C) A and B will have different intensities while A and C will have equal frequencies.
(D) A and B will have equal intensities while B and C will have different frequencies.

10. A proton has kinetic energy E = 100 keV which is eqal to that of a photon. The wavelength of photon is λ_2 and that of proton is λ_1. The ratio of $\frac{\lambda_2}{\lambda_1}$ is proportional to **[JEE-2004 (Scr.)]**
(A) E^2 (B) $E^{-\frac{1}{2}}$
(C) E^{-1} (D) $E^{\frac{1}{2}}$

11. In a photoelectric setup, the radiations from the Balmer series of hydrogen atom are incident on a metal surface of work function 2 eV. The wavelength of incident radiations lies between 450 nm to 700 nm.

Find the maximum kinetic energy of photoelectron emitted. (Given $hc/e = 1242$ eV-nm). **[JEE-2004]**

12. The wavelength of K_α X-ray of an element having atomic number $z = 11$ is λ. The wavelength of K_α X-ray of another element of atomic number z' is $4l$. Then z′ is **[JEE-2005 (Scr.)]**

(A) 11 (B) 44
(C) 6 (D) 4

13. A photon of 10.2 eV energy collides with a hydrogen atom in ground state inelastically. After few micro-seconds one more photon of energy 15 eV collides with the same hydrogen atom. Then what can be detected by a suitable detector. **[JEE-2005 (Scr.)]**

(A) one photon of 10.2 eV and an electron of energy 1.4 eV
(B) 2 photons of energy 10.2 eV
(C) 2 photons of energy 3.4 eV
(D) 1 photon of 3.4 eV and one electron of 1.4 eV

14. In Young's double slit experiment an electron beam is used to form a fringe pattern instead of light. If speed of the electrons is increased then the fringe width will:

(A) increase
(B) decrease
(C) remains same
(D) no fringe pattern will be formed

15. The potential energy of a particle of mass m is given by

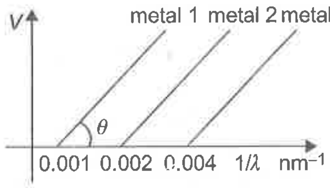
$$V(x) = \begin{Bmatrix} E_0 & 0 \le x \le 1 \\ 0 & x > 1 \end{Bmatrix}$$

λ_1 and λ_2 are the de Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, find $\dfrac{\lambda_1}{\lambda_2}$ **[JEE-2005]**

16. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of helium nucleus is $(14)^{1/3}$. Find **[JEE-2005]**

(A) atomic number of the nucleus
(B) the frequency of K_α line of the X-ray produced. ($R = 1.1 \times 10^7$ m^{-1} and $c = 3 \times 10^8$ m/s)

17. The graph between $\dfrac{1}{\lambda}$ and stopping potential (V) of three metals having work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statement(s) is/are correct? [Here λ is the wavelength of incident ray].



(A) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 1:2:4$
(B) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 4:2:1$
(C) $\tan\theta$ is directly proportional to hc/e, where h is Planck's constant and c is the speed of light
(D) The violet colour light can eject photoelectrons from metals 2 and 3. **[JEE-2006]**

18. In hydrogen-like atom ($z = 11$), nth line of Lyman series has wavelength λ equal to the de Broglie's wavelength of electron in the level from which it originated. What is the value of n?

[Take: Bohr radius $(r_0) = 0.53$ Å and Rydberg constant $(R) = 1.1 \times 10^7$ m^{-1}] **[JEE-2006]**

19. STATEMENT-1

If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change. **[JEE-2007]**

because

STATEMENT-2

When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

20. Electrons with de Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is **[JEE-2007]**

(A) $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ (B) $\lambda_0 = \dfrac{2h}{mc}$

(C) $\lambda_0 = \dfrac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0 = \lambda$

21. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is: **[JEE-2007]**

(A) 802 nm (B) 823 nm
(C) 1882 nm (D) 1648 nm

22. Which one of the following statements is wrong in the context of X-rays generated from a X-ray tube? **[JEE-2008]**

(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases.

(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.

(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube

(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-rays tube

Common Data for Questions 23–25: In mixture of H^- He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^- ions (by collision). Assume that the Bohr model of atom is exactly valid. **[JEE-2008]**

23. The quantum number n of the state finally poulated in He^+ ions is

(A) 2 (B) 3

(C) 4 (D) 5

24. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is

(A) 6.5×10^{-7} m (B) 5.6×10^{-7}m

(C) 4.8×10^{-7}m (D) 4.0×10^{-7}m

25. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He^+ ion is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 1 (D) 2

Common Data for Questions 26–28: When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3 … ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave. Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C. **[JEE-2009]**

26. The allowed energy for the particle for a particula value of n is proportional to

(A) a^{-2} (B) $a^{-3/2}$

(C) a^{-1} (D) a^2

27. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg an $a = 6.6$ nm, the energy of the particle in its groun state is closest to

(A) 0.8 meV (B) 8 meV

(C) 80 meV (D) 800 meV

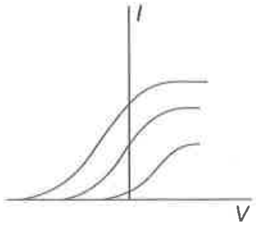
28. The speed of the particle, that can take discrete val ues, is proportional to

(A) $n^{-3/2}$ (B) n^{-1}

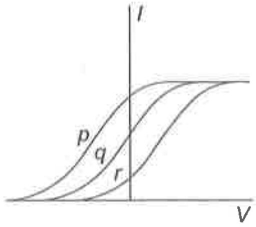
(C) $n^{1/2}$ (D) n

29. Photoelectric effect experiments are performed usin three different metal plates p, q and r having wor functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ e respectively. A light beam containing wavelengths o 550 nm, 450 nm and 350 nm with equal intensitie illuminates each of the plates. The correct I–V grap for the experiment is **[JEE-2009]**

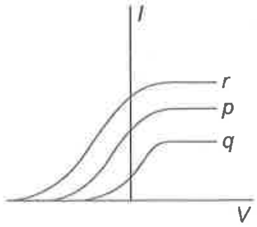
(A)



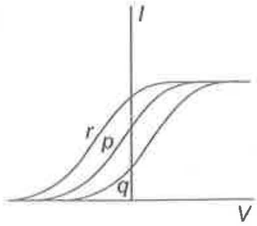
(B)



(C)



(D)



30. An $\propto$-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_a and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_a}$, to the nearest integer, is **[JEE-2010]**

Common Data for Questions 31–33: The key feature of Bohr's theory of spectrum of hydrogen atoms is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition. **[JEE-2010]**

31. diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the nth level ($n = 0$ is not allowed) is

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

32. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take $h = 2\pi \times 10^{-34}$ Js)

(A) 2.76×10^{-46} kg m^2 (B) 1.87×10^{-46} kg m^2
(C) 4.67×10^{-47} kg m^2 (D) 1.17×10^{-47} kg m^2

33. In a CO molecule, the distance between C (mass = 12 amu) and O (mass = 16 amu) where 1 amu = $\frac{5}{3}\times 10^{-27}$ kg, is close to

(A) 2.4×10^{-10} m (B) 1.9×10^{-10} m
(C) 1.3×10^{-10} m (D) 4.4×10^{-11} m

34. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is **[JEE-2011]**

(A) 1215 Å (B) 1640 Å
(C) 2430 Å (D) 4687 Å

35. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^z$ (where $1 < A < 10$). The value of 'Z' is **[JEE-2011]**

36. A proton is fired from very far away towards a nucleus with charge $Q = 120$ e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = \left(\frac{5}{3}\right)\times 10^{-27}$ kg; $\frac{h}{e} = 4.2 \times 10^{-15}$ Js/C; $\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m) **[JEE-2012]**

37. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is **[JEE-2013]**

(A) 0.3×10^{-17} kg ms^{-1} (B) 1.0×10^{-17} kg ms^{-1}
(C) 3.0×10^{-17} kg ms^{-1} (D) 9.0×10^{-17} kg ms^{-1}

38. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is **[JEE-2013]**

39. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are) **[JEE-2013]**

(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$

(C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

40. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2:1$ and and $hc = 1240$ eV nm, the work function of the metal is nearly

(A) 3.7 eV (B) 3.2 eV
(C) 2.8 eV (D) 2.5 eV

41. If λ_{Cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{M0} is the wavelength i of the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to

(A) 1.99 (B) 2.14
(C) 0.50 (D) 0.48

ANSWER KEYS

Exercises

JEE Main

1. C	**2.** B	**3.** C	**4.** A	**5.** A	**6.** C	**7.** D	**8.** B	**9.** A	**10.** C
11. B	**12.** B	**13.** A	**14.** B	**15.** C	**16.** A	**17.** D	**18.** B	**19.** C	**20.** C
21. A	**22.** D	**23.** D	**24.** B	**25.** C	**26.** A	**27.** C	**28.** C	**26.** D	**30.** C
31. B	**32.** A	**33.** A	**34.** B	**35.** A	**36.** C	**37.** C	**38.** D	**39.** C	**40.** B
41. D	**42.** B	**43.** A	**44.** A	**45.** D	**46.** D				

JEE Advanced

Single Correct Option Type Questions

1. D	**2.** C	**3.** D	**4.** C	**5.** C	**6.** B	**7.** B	**8.** C	**9.** B	**10.** A
11. A	**12.** A	**13.** D	**14.** C	**15.** A	**16.** A	**17.** C	**18.** C	**19.** C	**20.** B
21. C	**22.** C								

Multiple Correct Option Type Questions

23. a, c **24.** B, C **25.** D **26.** B **27.** B **28.** A, C, D **29.** A, D **30.** A, C, D
31. A **32.** B **33.** A **34.** A, B **35.** A, C, D **36.** A,C **37.** B
38. A, D **39.** A, B **40.** A, B, C

JEE Advanced

Level I

1. 885 **2.** **(A)** 2.25 eV, **(B)** 4.2 eV, **(C)** 2.0 eV, 0.5 eV **3.** **(A)** 0.6 V, **(B)** 2.0 mA

4. when the potential is steady, photo electric emission just stop when $hv = (3 + 1)$ eV = 4.0 eV

5. 5.76×10^{-11} A **6.** $\frac{15}{8}$ V **7.** $\frac{5}{16}, \frac{10^{20}}{80\pi}$ **8.** 2 eV, 6.53×10^{-34} J-s **9.** $\frac{-h}{e} ET^2$

10. 8.6 MeV **11.** 487.06 nm **12.** 4.26 m/s, 13.2 eV **13.** 7:36

14. 22.8 nm **15.** $\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ **16.** $\frac{18}{(5R)}$ **17.** 1.257×10^{-23} Am^2 **18.** 2.48×10^{-12} m

19. 5 **20.** **(i)** 5, 16.5 eV, 36.4 A, 340 eV, –680 eV, $\frac{h}{2\pi} 1.06 \times 10^{-11}$ m **21.** $z = 3, n = 7$

22. 54.4 eV **23.** $n = 3$, 3:1 **24.** 0004 **25.** 0.88 eV **26.** $f_b = f_a + f_\alpha'$

27. 62.5×10^{-12}, 192.5×10^{-12} **28.** 6210 eV

29. during combination = 3.365 eV; after combination = 3.88 eV ($5 \rightarrow 3$) and 2.63 eV ($4 \rightarrow 3$)

30. 69.5 keV **31.** **(i)** $KE = 3.4$ eV, **(ii)** $\lambda = 6.66$ Å **32.** 0.61 Å

Level II

1. **(A)** 10^5 s^{-1}; **(B)** 286.18; **(D)** 111 s **2.** 3.1×10^6 m/s **3.** **(i)** 2; **(ii)** 23.04×10^{-19} J; **(iii)** $4 \rightarrow 1, 4 \rightarrow 3$

4. 489.6 eV, 25.28 Å **5.** 1.99 eV, 0.760 V **6.** 1.1×10^{12} **7.** **(i)** 4125 Å, **(ii)** 13.2 μA

8. **(i)** 1.33×10^{16} photons/m^2-s ; 0.096 μÅ **(ii)** 2.956×10^{15} photons/m^{2}s ; 0.0213 μA **(iii)** 1.06 V

9. **(i)** $\frac{5}{16}$ photon/sec, **(ii)** $\frac{5}{1600}$ electrons/sec **10.** 6.8 eV, 5×10^{15} Hz

11. **(i)** Allowed values of energy of neutron = 6.36 eV and 0.312 eV; Allowed values of energy of He atom = 17.84 and 16.328 eV, **(ii)** 18.23×10^{14} Hz, 9.846×10^{15} Hz, 11.6×10^{15} Hz **12.** $n = 6, Z = 3$

13. $KE \cong 151$ eV, $d_{least} = 0.5$ Å **14.** 7.4 eV, 4.7 V

Previous Years' Questions

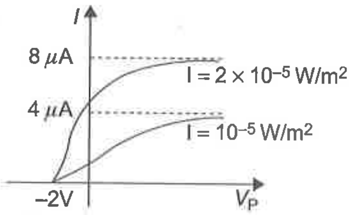
JEE Main

1. C	**2.** A	**3.** C	**4.** A	**5.** B	**6.** C	**7.** D	**8.** C	**9.** C	**10.** B
11. A	**12.** D	**13.** B	**14.** A	**15.** D	**16.** C	**17.** B	**18.** B	**19.** D	**20.** B
21. B	**22.** D	**23.** A	**24.** C	**25.** B	**26.** D	**27.** C	**28.** D	**29.** D	**30.** B
31. B	**32.** A	**33.** D							

JEE Advanced

1. A **2.** B **3.** 3, 4052.3 nm **4.** 5×10^7, 2000 N/C, 23 eV **5.** D **6.** A **7.** $z = 42$

8.



9. A **10.** B **11.** 0.55 eV **12.** C **13.** A **14.** B

15. $\sqrt{2}$ **16.** $v = 1.546 \times 10^{18}$ Hz **17.** A, C **18.** $n = 24$ **19.** B **20.** A **21.** B **22.** B

23. C **24.** C **25.** A **26.** A **27.** B **28.** D **29.** A **30.** 3 **31.** D **32.** B

33. C **34.** A **35.** 7 **36.** 7 **37.** B **38.** 1 **39.** AC **40.** A **41.** B

Chapter 3

Modern Physics-2

NUCLEAR PHYSICS

It is the branch of physics which deals with the study of nucleus.

NUCLEUS

(a) Discoverer: Rutherford

(b) Constituents: neutrons (n) and protons (p) [collectively known as nucleons]

1. **Neutron:** It is a neutral particle. It was discovered by J. Chadwick. Mass of neutron,

$$m_n = 1.6749286 \times 10^{-27} \text{ kg}$$

2. **Proton:** It has a charge equal to +e. It was discovered by Goldstein.
Mass of proton, $m_p = 1.6726231 \times 10^{-27}$ kg
$(m_p \lessapprox m_n)$

(c) Representation:

$_Z X^A$ or $^A_Z X$

where X ⇒ symbol of the atom

Z ⇒ Atomic number = number of protons

A ⇒ Atomic mass number = total number of nucleons.

= no. of protons + no. of neutrons.

Atomic mass number:

It is the nearest integer value of mass represented in amu (atomic mass unit)

1 amu $= \frac{1}{12}$ [mass of one atom of $_6C^{12}$ atom at rest and in ground state]

1.6603×10^{-27} kg; 931.478 MeV/c^2

mass of proton (m_o) = mass of neutron (m_n) = 1 amu.

Some definitions

1. **Isotopes:**
The nuclei having the same number of protons but different number of neutrons are called isotopes.
2. **Isotones:**
Nuclei with the same neutron number N but different atomic number Z are called isotones.
3. **Isobars:**
The nuclei with the same mass number but different atomic number are called isobars

(d) Size of nucleus: Order of 10^{-15} m (fermi)
Radius of nucleus ; $R = R_0 A^{1/3}$
where $R_0 = 1.1 \times 10^{-15}$ m (which is an empirical constant)
A = Atomic mass number of atom.

(e) Density:

$$\text{density} = \frac{\text{mass}}{\text{volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

$$= \frac{3m_p}{4\pi R_0^3} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.1 \times 10^{-15})^3}$$

$$= 3 \times 10^{17} \text{ kg/m}^3$$

Nuclei of almost all atoms have almost same density as nuclear density is independent of the mass number (A) and atomic number (Z).

MASS DEFECT

It has observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{\text{expected}} = zm_p + (A - Z)m_n$$

$$M_{\text{observed}} = M_{\text{atom}} - Zm_e$$

It is found that

$$M_{\text{observed}} < M_{\text{expected}}$$

Hence, mass defect is defined as

$$\text{Mass defect} = M_{\text{expected}} - M_{\text{observed}}$$

$$\Delta m = [Zm_p + (A - Z)m_n - [M_{\text{atom}} - Zm_e]$$

BINDING ENERGY

It is the minimum energy required to break the nucleus into its constituent particles.

or

Amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

$$\text{Binding Energy } (BE) = \Delta mc^2$$

$$BE = \Delta m \text{ (in amu)} \times 931.5 \text{ MeV/amu}$$
$$= \Delta m \times 931.5 \text{ MeV}$$
$$= \Delta m \times 931 \text{ MeV}$$

Notes

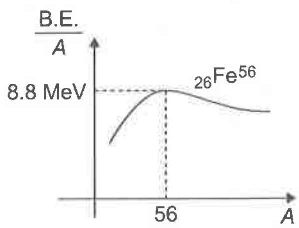
If binding energy per nucleon is more for a nucleus then it is more stable.

For Example

If $\left(\frac{BE_1}{A_1}\right) > \left(\frac{BE_2}{A_2}\right)$ then nucleus 1 would be more stable.

Variation of Binding Energy per Nucleon with Mass Number

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.7 MeV for A. 50 → 80. For still heavier nuclei, the binding energy per nucleon slowly decreases as A increases.



Binding energy per nucleon is maximum for ${}_{26}Fe^{56}$, which is equal to 8.8 MeV. Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.

- The heavier nuclei being unstable have tendency to split into medium nuclei. This process is called **Fission.**
- The Lighter nuclei being unstable have tendency to fuse into a medium nucleus. This process is called **Fusion.**

RADIOACTIVITY

It was discovered by Henry Becquerel.

Spontaneous emission of radiations (α, β, γ) from unstable nucleus is called **radioactivity.** Substances which shows radioactivity are known as **radioactive substance.**

Radioactivity was studied in detail by Rutherford.

In radioactive decay, an unstable nucleus emits α-particle or β-particle. After emission of α or β the remaining nucleus may emit γ-particle, and converts into more stable nucleus.

α-particle:

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

$$\text{Mass of } \alpha\text{-particle} = \text{Mass of } {}_2He^4 \text{ atom} - 2m_e = 4m_p$$

$$\text{Charge of } \alpha\text{-particle} = +2e$$

β-particle:

(a) β^- (electron):

Mass = m_e; Charge = $-e$

(b) β^+ (positron):

Mass = m_e; Charge = $+e$

positron is an antiparticle of electron.

Antiparticle

A particle is called antiparticle of other if on collision both can annihilate (destroy completely) and converts into energy. For example: (i) electron ($-e$, m_e) and positron ($+e$, m_e) are anti particles, (ii) neutrino (ν) and antineutrino ($\bar{\nu}$) are anti particles.

γ-particle: They are energetic photons of energy of the order of MeV and having rest mass zero.

RADIOACTIVE DECAY (DISPLACEMENT LAW)

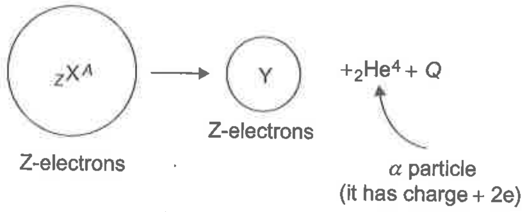
α-decay

$${}_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$$

Q value: It is defined as energy released during the decay process.

Q value = rest mass energy of reactants − rest mass energy of products.

This energy is available in the form of increase in KE of the products



Let, M_x = mass of atom $_zX^A$

M_y = mass of atom $_{z-2}Y^{A-4}$

M_{He} = mass of atom $_2He^4$

$$Q \text{ value} = [(M_x - Zm_e) - \{(M_y - (Z-2)m_e) + (M_{He} - 2m_e)\}]c^2$$

$$= [M_x - M_y - M_{He}]\, c^2$$

Considering actual number of electrons in α-decay

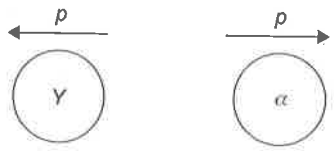
$$Q \text{ value} = [M_x - (M_y + 2m_e) - (M_{He} - 2m_e)]\, c^2$$

$$= [M_x - M_y - M_{He}]c^2$$

Calculation of Kinetic energy of final products:

As atom X was initially at rest and no external forces are acting, so final momentum also has to be zero.

Hence both Y and α-particle will have same momentum in magnitude but in opposite direction.



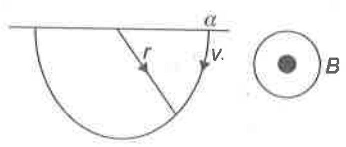
$p_\alpha^2 = p_Y{}^2$ $\quad 2m_\alpha T_\alpha = 2\, m_Y T_Y$ (Here we are representing T for kinetic energy)

$Q = T_y + T_\alpha$ $\qquad m_\alpha T_\alpha = m_Y T_y$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y} Q; \qquad T_Y = \frac{m_\alpha}{m_\alpha + m_Y} Q$$

$$T_\alpha = \frac{A-4}{A} Q; \qquad T_Y = \frac{4}{A} Q$$

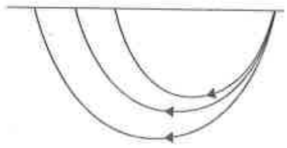
From the above calculation, one can see that all the α-particles emitted should have same kinetic energy. Hence, if they are passed through a region of uniform magnetic field having direction perpendicular to velocity, they should move in a circle of same radius.



$$r = \frac{mv}{qB} = \frac{mv}{2eB} = \frac{\sqrt{2\text{ Km}}}{2eB}$$

Experimental Observation:

Experimentally it has been observed that all the α-particles do not move in the circle of same radius, but they move in 'circles having different radii.



This shows that they have different kinetic energies. But it is also observed that they follow circular paths of some fixed values of radius i.e., yet the energy of emitted α-particles is not same but it is quantized. The reason behind this is that all the daughter nuclei produced are not in their ground state but some of the daughter nuclei may be produced in their excited states and they emits photon to acquire their ground state.

$X \longrightarrow Y^*$ (excited state) $+ \alpha + Q$

$\downarrow$

Y + photon (γ particle)

The only difference between Y and Y^* is that Y^* is in excited state and Y is in ground state.

Let, the energy of emitted γ-particles be E

$$\therefore \qquad Q = T_\alpha + T_Y + E$$

where $\qquad Q = [M_x - M_Y - M_{He}]\, c^2$

$$T_\alpha + T_Y = Q - E$$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y}(Q - E); \quad T_Y = \frac{m_\alpha}{m_\alpha + m_Y}(Q - E)$$

β^--decay

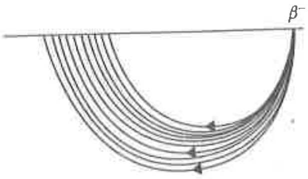
$$_zX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q$$

$_{-1}e^0$ can also be written as $_{-1}\beta^0$

Here also one can see that by momentum and energy conversion, we will get

$$T_e = \frac{m_Y}{m_e + m_Y} Q, \quad T_Y = \frac{m_e}{m_e + m_Y} Q$$

as $m_e << m_Y$, we can consider that all the energy is taken away by the electron. From the above results, we will find that all the β-particles emitted will have same energy and hence they have same radius if passed through a region of perpendicular magnetic field. But, experimental observations were completely different. On passing through a region of uniform magnetic field perpendicular to the velocity, it was observed that β-particles take circular paths of different radius having a continuous spectrum.



To explain this, Paulling has introduced the extra particles called neutrino and antineutrino (antiparticle of neutrino).

$$\bar{\nu} \rightarrow \text{antineutrino}, \ \nu \rightarrow \text{neutrino}$$

Properties of antineutrino ($\bar{\nu}$) and neutrino (n)

1. They are like photons having rest mass = 0
   speed = c
   Energy, $E = mc^2$
2. They are charge less (neutral)
3. They have spin quantum number, $s = \pm\frac{1}{2}$

   Considering the emission of antineutrino, the equation of β^--decay can be written as

$$_Z X^A \rightarrow {}_{z+1}Y^A + {}_{-1}e^0 + Q + \bar{\nu}$$

Production of antineutrino along with the electron helps to explain the continuous spectrum because the energy is distributed randomly between electron and $\bar{\nu}$ and it also helps to explain the spin quantum number balance (p, n and $\pm e$ each has spin quantum number ±12)

During β^--decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$$n \rightarrow p + {}_{-1}e^0 + \bar{\nu}$$

Let, M_x = mass of atom $_zX^A$
M_Y = mass of atom $_{z+1}Y^A$
m_e = mass of electron

$$Q \text{ value} = [(M_x - Zm_e) - \{(M_Y - (z+1)m_e) + m_e\}]\, c^2$$
$$= [M_x - M_Y]\, c^2$$

Considering actual number of electrons.

$$Q \text{ value} = [M_x - \{(M_Y - m_e) + m_e\}]\, c^2 = [M_x - M_y]\, c^2$$

β^+-decay

$$_Z X^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$$

In β^+ decay, inside a nucleus a proton is converted into a neutron, positron and neutrino.

$$p \rightarrow n + {}_{+1}e^0 + \nu$$

As mass increases during conversion of proton to a neutron, hence it requires energy for β^+-decay to take place,

$\therefore$ β^+-decay is rare process. It can take place in the nucleus where a proton can take energy from the nucleus itself.

$$Q \text{ value} = [(M_X - Zm_e) - \{(M_Y - (Z-1)\, m_e) + m_e\}]\, c^2$$
$$= [M_X - M_Y - 2m_e]\, c^2$$

Considering actual number of electrons.

$$Q \text{ value} = [M_X - \{(M_Y + m_e) + m_e\}]\, c^2$$
$$= [M_X - M_Y - 2m_e]\, c^2$$

K Capture

It is rare process which is found only in few nucleus. In thi[s] process the nucleus captures one of the atomic electron[s] from the K shell. A proton in the nucleus combines wit[h] this electron and converts itself into a neutron. A neutrino i[s] also emitted in the process and is emitted from the nucleu[s]

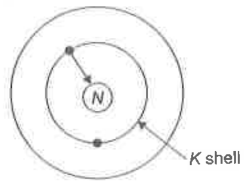
$$p + {}_{-1}e^0 \rightarrow n + \nu$$

If X and Y are atoms then reactions is written as:

$$_z X^A \rightarrow {}_{z-1}Y^A + \nu + Q + \text{characteristic x-rays of } Y.$$

If X and Y are taken as nucleus, then reactions is written a[s]

$$_z X^A + {}_{-1}e^0 \rightarrow {}_{z-1}Y^A + \nu$$

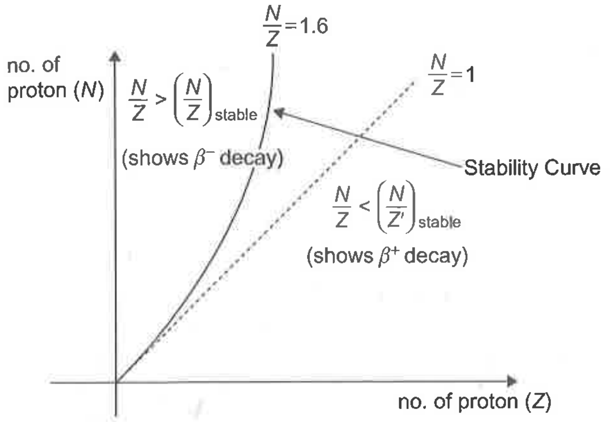


Notes

1. Nuclei having atomic numbers from Z = 84 to 11[…] shows radioactivity.
2. Nuclei having Z = 1 to 83 are stable (only few excep[…]tions are there)
3. Whenever a neutron is produced, a neutrino is als[o] produced.
4. Whenever a neutron is converted into a proton, a an[…]tineutrino is produced.

NUCLEAR STABILITY

Figure shows a plot of neutron number N versus proton number Z for the nuclides found in nature. The solid line in the figure represents the stable nuclides. For light stable nuclides, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1.6 for the heaviest stable nuclides.



The points (Z, N) for stable nuclides fall in a rather well-defined narrow region. There are nuclides to the left of the stability belt as well as to the right of it. The nuclides to the left of the stability region have excess neutrons, whereas, those to the right of the stability belt have excess protons. These nuclides are unstable and decay with time according to the laws of radioactive disintegration. Nuclides with excess neutrons (lying above stability belt) show β^- decay while nuclides with excess protons (lying below stability belt) show β^+ decay and K-capture.

NUCLEAR FORCE

(i) Nuclear forces are basically attractive and are responsible for keeping the nucleons bound in a nucleus in spite of repulsion between the positively charge protons.

(ii) It is strongest force with in nuclear dimensions ($F_n - 100F_e$)

(iii) It is short range force (acts only inside the nucleus)

(iv) It acts only between neutron-neutron, neutron-proton and proton-proton i.e., between nucleons.

(v) It does not depend on the nature of nucleons

(vi) An important property of nuclear force is that it is not a central force. The force between a pair of nucleons is not solely determined by the distance between the nucleons. For example, the nuclear force depends on the directions of the spins of the nucleons. The force is stronger if the spins of the nucleons are parallel (i.e., both nucleons have $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$) and is weaker if the spins are antiparallel (i.e., one nucleon has $m_s = +\frac{1}{2}$ and the other has $m_s = -\frac{1}{2}$). Here m_s is spin quantum number.

RADIOACTIVE DECAY: STATISTICAL LAW

(Given by Rutherford and Soddy)
Rate of radioactive decay $\propto N$
where N = number of active nuclei = λN
where λ = decay constant of the radioactive substance.
Decay constant is different for different radioactive substances, but it does not depend on amount of substances and time.
SI unit of λ is s^{-1}

If $\lambda_1 > \lambda_2$ then first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ.

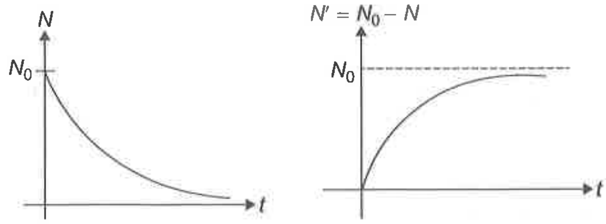
$$A \xrightarrow{\lambda} B$$

$t = 0$ N_0 0   where N_0 = number of active nuclei of A at $t = 0$

$t = t$ N N' where N = number of active nuclei of A at $t = t$

Rate of radioactive decay of $A = -\frac{dN}{dt} = \lambda N$

$$-\int_{N_0}^{N} \frac{dN}{N} = \int_0^t \lambda dt \quad \Rightarrow \quad N = N_0 e^{-\lambda t} \text{ (it is exponential decay)}$$



Number of nuclei decayed (i.e., the number of nuclei of B formed)

$$N' = N_0 - N = N_0 - N_0 e^{-\lambda t}$$

$$N' = N_0(1 - e^{-\lambda t})$$

Half Life ($T_{1/2}$)

It is the time in which number of active nuclei becomes half.

$$N = N_0 e^{-\lambda t}$$

After one half life, $N = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda t} \Rightarrow t = \frac{\ln 2}{\lambda} \Rightarrow \frac{0.693}{\lambda} = t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \text{ (to be remembered)}$$

Number of nuclei present after n half lives i.e., after a time $t = nt_{1/2}$

$$N = N_0 e^{-\lambda t} = N_0 e^{-\lambda n t_{1/2}} = N_0 e^{-\lambda n \frac{\ln 2}{\lambda}}$$

$$= N_0 e^{\ln 2^{(-n)}} = N_0 (2)^{-n} = N_0 \left(\frac{1}{2}\right)^n = \frac{N_0}{2^n}$$

$\{ n = \frac{t}{t_{1/2}}$. It may be a fraction, need not to be an integer$\}$

or $$N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3 \cdots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$$

Activity

Activity is defined as rate of radioactive decay of nuclei.

It is denoted by A or R $\quad A = \lambda N$.

If a radioactive substance changes only due to decay then

$$A = -\frac{dN}{dt}$$

As in that case, $N = N_0 e^{-\lambda t}$

$$A = \lambda N = \lambda N_0 e^{-\lambda t} \quad \Rightarrow \quad A = A_0 e^{-\lambda t}$$

SI unit of activity: Becquerel (Bq) which is same as 1 dps (disintegration per second)

The popular unit of activity is Curis which is defined as 1 curie = 3.7×10^{10} dps (which is activity of 1 gm Radium)

specific activity: The activity per unit mass is called specific activity.

Average Life

$$T_{avg} = \frac{\text{sum of ages of all the nuclei}}{N_0} = \frac{\int_0^\infty \lambda N_0 e^{-\lambda t} dt \cdot t}{N_0} = \frac{1}{\lambda}$$

SOLVED EXAMPLES

EXAMPLE 17

A radioactive nucleus can decay be two different processes. The half-life for the first process is t_1 and that for the second process is t_2. Show that the effective half-life t of the nucleus is given by

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

SOLUTION

The decay constant for the first process is $\lambda_1 = \frac{\ln 2}{t_1}$ and for the second process it is $\lambda_2 = \frac{\ln 2}{t_1}$. The probability that an active nucleus decays by the first process in a time interval it is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ, this probability is also equal to λdt. Thus

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or, $$\lambda = \lambda_1 + \lambda_2$$

or, $$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \text{ (To be remembered)}$$

EXAMPLE 18

A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find (i) the number of nuclei of A and (ii) Number of nuclei of B, at any time t assuming the production of A starts $t = 0$. (iii) Also find out the maximum number of nuclei of 'A' present at any time during its formation.

SOLUTION

$$\text{Factory} \xrightarrow[\text{constant}]{R} A \xrightarrow[\text{decay}]{\lambda} B$$

Let N be the number of nuclei of A at any time t

$$\therefore \quad \frac{dN}{dt} = R - \lambda N \int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt$$

On solving we will get

$$N = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

(ii) Number of nuclei of B at any time t, $N_B = Rt - N_A$

$$= Rt - \frac{R}{\lambda}(1 - e^{-\lambda t}) = \frac{R}{\lambda}(\lambda t - 1 + e^{-\lambda t})$$

(iii) Maximum number of nuclei of 'A' present at any time during its formation $= \frac{R}{\lambda}$ ■

EXAMPLE 19

A radioactive substance 'A' having N_0 active nuclei at $t = 0$, decays to another radioactive substance 'B' with decay constant λ_1. B further decays to a stable substance 'C' with decay constant λ_2. (a) Find the number of nuclei of A, B and C after time t, (b) What would be answer of part (a) if $\lambda_1 >> \lambda_2$ and $\lambda_1 << \lambda_2$

SOLUTION

The decay scheme is as shown

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C \text{ (stable)}$$

	A	B	C
$t = 0$	N_0	0	0
t	N_1	N_2	N_3

Here N_1, N_2 and N_3 represent the nuclei of A, B and C at any time t.

For A, we can write

$$N_1 = N_0 e^{-\lambda_1 t} \quad (1)$$

For B, we can write

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad (2)$$

or,

$$\frac{dN_2}{dt} + \lambda_2 N_2 - \lambda_1 N_1$$

This is a linear differential equation with integrating factor

$$IF = e^{\lambda} 2^t$$

$$e^{\lambda_2 t}\frac{dN_2}{dt} + e^{\lambda_2 t}\lambda_2 N_2 = \lambda_1 N_1 e^{\lambda_2 t}$$

$$\int d(N_2 e^{\lambda_2 t}) = \int \lambda_1 N_1 e^{\lambda_2 t} dt$$

$$N_2 e^{\lambda_2 t} = \lambda_1 N_0 \int e^{-\lambda_1 t} e^{\lambda_2 t} dt \quad \text{using (1)}$$

$$N_2 e^{\lambda_2 t} = \lambda_1 N_0 \frac{e^{(\lambda_2 - \lambda_1)t}}{\lambda_2 - \lambda_1} + C \quad (3)$$

At $t = 0$, $N_2 = 0$ $\quad 0 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} + C$

Hence $\quad C = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2}$

Using C in eqn. (3), we get

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

and $\quad N_1 + N_2 + N_3 = N_0$

$\therefore \quad N_3 = N_0 - (N_1 + N_2)$

(b) For $\lambda_1 >> \lambda_2$ $\quad N_2 = \frac{\lambda_1 N_0}{-\lambda_1}(-e^{-\lambda_2 t}) = N_0 e^{-\lambda_2 t}$

For $\lambda_1 << \lambda_2$ $\quad N_2 = \frac{\lambda_1 N_0}{\lambda_2}(e^{-\lambda_1 t}) = 0$ ■

NUCLEAR FISSION

In nuclear fission heavy nuclei of A, above 200, break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use $_{92}U^{236}$ as the fission material. The technique is to hit a uranium sample by sample by slow moving neutrons (kinetic energy ≈ 0.04 eV, also called thermal neutrons.) $A_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming $_{92}U^{236}$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$$_{92}U^{235} + {_0n^1} \rightarrow {_{92}U^{236}} \rightarrow X + Y + 2\,{_0n^1}$$

or

$$_{92}U^{235} + {_0n^1} \rightarrow {_{92}U^{236}} \rightarrow X' + Y' + 2\,{_0n^1}$$

and a number of other combinations.

- On an average 2.5 neutrons are emitted in each fission event.
- Mass lost per reaction 0.2 amu.
- In nuclear fission the total BE increases and excess energy is released.
- In each fission event, about 200 MeV of energy is released a large part of which appears in the form of kinetic energies of the two arrangements. Neutrons take away about 5 MeV.

EXAMPLE

$${}^{235}_{92}U + {_0n^1} \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{_0n^1} + \text{energy}$$

$$Q \text{ value} = [(M_U - 92m_e + m_n) - \{(M_{Ba} - 56m_e) + (M_{Kr} - 36m_e) + 3m_n\}]\,c^2$$
$$= [(M_U + m_n) - (M_{Ba} + M_{Kr} + 3m_n)]\,c^2$$

- A very important and interesting feature of neutron-induced fission is the chain reaction. For working of nuclear reactor refer your text book.

NUCLEAR FUSION (THERMO-NUCLEAR REACTION)

(a) Some unstable light nuclei of A below 20, fuse together, the BE per nucleon increases and hence the excess energy is released. The easier thermonuclear reaction that can be handled on earth is the fusion of two deuterons ($D - D$ reaction) or fusion of a deuteron with a triton ($D - T$ reaction).

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1 + 3.3\,\text{MeV}(D - D)$$

$$Q\text{value} = [2((M_D - m_e) - \{(M_{He^3} - 2m_e) + m_n\}]c^2$$
$$= [2M_D - (M_{He^3} + m_n)]c^2$$

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4.0\,\text{MeV}\,(D - D)$$

$$Q \text{ value} = [2(M_D - m_e) - \{(M_T - m_e) + (M_H - m_e)\}]c^2$$
$$= [2M_D - (M_T + M_H)]c^2$$

$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n + 17.6\,\text{MeV}\,(D - T)$$

$$Q \text{ value} = [\{(M_D - m_e) + (M_T - m_e)\} - \{(M_{He^4} - 2m_e) + m_n\}]c^2$$
$$= [(M_D + M_T) - (M_{He^4} + m_n)]c^2$$

Notes

- In case of fission and fusion, $\Delta m = \Delta m_{\text{atom}} = \Delta m_{\text{nucleus}}$
- These reactions take place at ultra high temperature ($\cong 10^7$ to 10^8). At high pressure it can take place at low temperature also. For these reactions to take place nuclei should be brought upto 1 fermi distance which requires very high kinetic energy.
- Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.

EXERCISES

JEE Main

1. Let u be denote one atomic mass unit. One atom of an element of mass number A has mass exactly equal to Au.
(A) for any value of A
(B) only for $A = 1$
(C) only for $A = 12$
(D) for any value of A provided the atom is stable

2. Consider the nuclear reaction

$$X^{200} \rightarrow A^{110} + B^{90}$$

If the binding energy per nucleon for X, A and B is 7.4 MeV, 8.2 MeV and 8.2 MeV respectively, what is the energy released?
(A) 200 MeV (B) 160 MeV
(C) 110 MeV (D) 90 MeV

3. The binding energies of nuclei X and Y are E_1 and E_2 respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Then
(A) $Q = 2E_1 - E_2$
(B) $Q = E_2 - 2E_1$
(C) $Q = 2E_1 + E_2$
(D) $Q = 2E_2 + E_1$

4. The binding energies of the atom of elements A and B are E_a and E_b respectively. Three atom of the element B fuse to give one atom of element A. This fusion process is accompanied by release of energy e. Then E_a, E_b are related to each other as
(A) $E_a + e = 3E_b$
(B) $E_a = 3E_b$
(C) $E_a - e = 3E_b$
(D) $E_a + 3E_b + e = 0$

5. The binding energies of the nuclei of ${}^{4}_{2}He$, ${}^{7}_{3}Li$, ${}^{12}_{6}C$ and ${}^{14}_{7}N$ are 28, 52, 90, 98 MeV respectively. Which of these is most stable.

(A) ${}^{4}_{2}He$ (B) ${}^{7}_{3}Li$

(C) ${}^{12}_{6}C$ (D) ${}^{14}_{7}N$

6. The following nuclear reaction is an example of ${}^{12}_{6}C + {}^{4}_{2}H \rightarrow {}^{16}_{8}O$ + energy

(A) fission (B) fusion

(C) alpha decay (D) beta decay

7. The rest mass of the deuteron, ${}^{2}_{1}H$, is equivalent to an energy of 1876 MeV, the rest mass of a proton is equivalent to 939 MeV and that of a neutron to 940 MeV. A deuteron may disintegrate to a proton and a neutron if it:

(A) emits a γ-ray photon of energy 2 MeV

(B) captures a γ-ray photon of energy 2 MeV

(C) emits a γ-ray photon of energy 3 MeV

(D) captures a γ-ray photon of energy 3 MeV

8. In an α-decay the Kinetic energy of α-particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is: (Assume that daughter nucleus is in ground state)

(A) 96 (B) 100

(C) 104 (D) none of these

9. A certain radioactive nuclide of mass number m_x disintegrates, with the emission of an electron and γ radiation only, to give second nuclide of mass number m_y. Which one of the following equation correctly relates m_x and m_y?

(A) $m_y = m_x + 1$ (B) $m_y = m_x - 2$

(C) $m_y = m_x - 1$ (D) $m_y = m_x$

10. The number of α and β^- emitted during the radioactive decay chain starting from ${}^{226}_{88}Ra$ and ending at ${}^{206}_{82}Pb$ us

(A) 3α and $6\beta^-$ (B) 4α and $5\beta^-$

(C) 5α and $4\beta^-$ (D) 6α and $6\beta^-$

11. Two radioactive material A_1 and A_2 have decay constants of $10\,\lambda_0$ and λ_0. If initially they have same number of nuclei, the ratio of number of their undecayed nuclei will be $(1/e)$ after a time

(A) $\frac{1}{\lambda_0}$ (B) $\frac{1}{9\lambda_0}$

(C) $\frac{1}{10\lambda_0}$ (D) 1

12. The radioactive sources A and B of half lives of 2 hr and 4 hr respectively, initially contain the same number of radioactive atoms. At the end of 2 hours, their rates of disintegration are in the ratio:

(A) 4:1 (B) 2:1

(C) $\sqrt{2}:1$ (D) 1:1

13. In a RA element the fraction of initiated amount remaining after its mean life time is:

(A) $1 - \frac{1}{e}$ (B) $\frac{1}{e^2}$

(C) $\frac{1}{e}$ (D) $1 - \frac{1}{e^2}$

14. 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time $2t$:

(A) 20% (B) 19%

(C) 40% (D) 38%

15. Activity of a radioactive substance is R_1 at time t_1 and R_2 at time $t_2 (t_2 > t_1)$. Then the ratio $\frac{R_2}{R_1}$ is:

(A) $\frac{t_2}{t_1}$ (B) $e^{-\lambda(t_1+t_2)}$

(C) $e\left(\frac{t_1 - t_2}{\lambda}\right)$ (D) $e^{\lambda(t_1-t_2)}$

16. There are two radio-nuclei A and B. A is an alpha emitter and B is a beta emitter. Their disintegration constants are in the ratio of 1:2. What should be the ratio of number of atoms of two at time $t = 0$ so that probabilities of getting α and β-particles are same at time $t = 0$

(A) 2:1 (B) 1:2

(C) e (D) e^{-1}

17. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time $t_2 (t_2 > t_1)$. Its mean life is T.

(A) $A_1 t_1 = A_2 t_2$ (B) $\frac{A_1 - A_2}{t_2 - t_1}$ = constant

(C) $A_2 = A_1 e^{(t_1 - t_2)/T}$ (D) $A_2 = A_1 e^{(t_1/Tt_2)}$

18. A fraction f_1 of a radioactive sample decays in one mean life, and a fraction f_2 decays in one half-life.

(A) $f_1 > f_2$

(B) $f_1 < f_2$

(C) $f_1 = f_2$

(D) May be (A), (B) or (C) depending on the values of the mean life and half life.

19. The radioactivity of a sample is R_1 at time T_1 and R_2 at time T_2. If the half life of the specimen is T. Number of atoms that have disintegrated in time $(T_2 - T_1)$ is proportional to

(A) $(R_1T_1 - R_2T_2)$ (B) $(R_1 - R_2)\,T$

(C) $\dfrac{(R_1 - R_2)}{T}$ (D) $(R_1 - R_2)(T_1 - T_2)$

20. The decay constant of the end product of a radioactive series is

(A) zero

(B) infinite

(C) finite (non zero)

(D) depends on the end product.

21. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that:

(A) no nucleus will decays before $t = 4$ days

(B) no nucleus will decays before $t = 8$ days

(C) all nuclei will decays before $t = 16$ days

(D) a given nucleus may decay at any time after $t = 0$.

22. There are two radio-nuclei A and B. A is an alpha emitter and B is a beta emitter. Their disintegration constants are in the ratio of 1:2. What should be the ratio of number of atoms of two at time $t = 0$ so that probabilities of getting α and β-particles are same at time $t = 0$.

(A) 2:1 (B) 1:2

(C) e (D) e^{-1}

23. A certain radio active substance has a half life of 5 years. Thus for a particular nucleus in a sample of the element, the probability of decay in ten years is

(A) 50% (B) 75%

(C) 100% (D) 60%

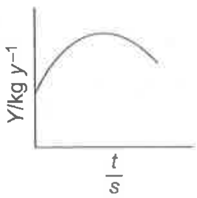
24. A radioactive nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2. The effective decay constant of the nuclide is λ, then:

(A) $\lambda = \lambda_1 + \lambda_2$ (B) $\lambda = \dfrac{1}{2}(\lambda_1 + \lambda_2)$

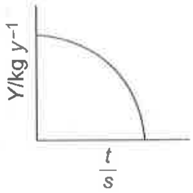
(C) $\dfrac{1}{\lambda} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$ (D) $\lambda = \sqrt{\lambda_1\lambda_2}$

25. The radioactive nucleus of an element X decays to a stable nucleus of element Y. a graph of the rate of formation of Y against time would look like

(A)



(B)



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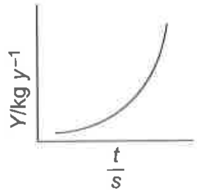

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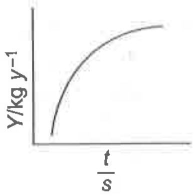
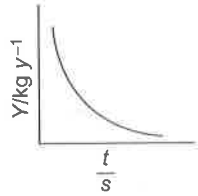


(E)



26. A radioactive substance is dissolved in a liquid and the solution is heated. The activity of the solution

(A) is smaller than that of element

(B) is greater than that of element

(C) is equal to that of element

(D) will be smaller or greater depending upon whether the solution is weak or concentrated

27. The half life of a neutron is 800 sec. 10^8 neutrons at a certain instant are projected from one space station towards another space station, situated 3200 km away, with a velocity 2000 m/s. Their velocity remains constant during the journey. How many neutrons reach the other station?

(A) 50×10^6 (B) 25×10^6

(C) 80×10^5 (D) 25×10^5

28. An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (In eV) required to remove both the electrons from a neutral helium atom is:

(A) 38.2 (B) 49.2

(C) 51.8 (D) 79.0

REASONING TYPE

29. Statement-1: It is easy to remove a proton from $^{40}_{20}$Ca nucleus as compared to a neutron.

Statement-2: Inside nucleus neutrons are acted on only attractive forces but protons are also acted on by repulsive forces.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

30. Statement-1: It is possible for a thermal neutron to be absorbed by a nucleus whereas a proton or an α-particle would need a much larger amount of energy for being absorbed by the same nucleus.

Statement-2: Neutron is electrically neutral but proton and α-particle are positively charged.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true and Statement-2 is NOT the correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

31. Statement-1: Consider the following nuclear reaction of an unstable $^{14}_{6}C$ nucleus initially at rest. The decay $^{14}_{6}C \longrightarrow ^{14}_{7}N + ^{0}_{-1}e + \bar{\nu}$. In a nuclear reaction total energy and momentum is conserved experiments show that the electrons are emitted with a continuous range of kinetic energies upto some maximum value.

Statement-2: Remaining energy is released as thermal energy.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

32. Half life for certain radioactive element is 5 min. Four nuclei of that element are observed at a certain instant of time. After five minutes

Assertion (A): It can be definitely said that two nuclei will be left undecayed.

Reasoning (R): After half life i.e., 5 minutes, half of total nuclei will disintegrate. So only two nuclei will be left undecayed. Then

(A) A is correct and R is correct explanation of A.

(B) Both are correct. But R is not correct explanation of A.

(C) A is incorrect and R is correct.

(D) Both are incorrect.

JEE Advanced

Single Correct Option Type Questions

1. The surface area of a nucleus varies with mass number A as

(A) $A^{2/3}$ (B) $A^{1/3}$

(C) A (D) None

2. The binding energy per nucleon for C^{12} is 7.68 MeV and that for C^{13} is 7.5 MeV. The energy required to remove a neutron from C^{13} is

(A) 5.34 MeV (B) 5.5 MeV

(C) 9.5 MeV (D) 9.34 MeV

3. If each fission in a U^{235} nucleus releases 200 MeV, how many fissions must occurs per second to produce a power of 1 KW

(A) 1.325×10^{13}

(B) 3.125×10^{13}

(C) 1.235×10^{13}

(D) 2.135×10^{13}

4. A star initially has 10^{40} deutrons. It produces energy via, the processes $_1H^2 + {_1H^2} \rightarrow {_1H^3} + p$ and $_1H^2 + {_1H^3} \rightarrow {_2He^4} + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of:

(A) 10^6 sec (B) 10^8 sec

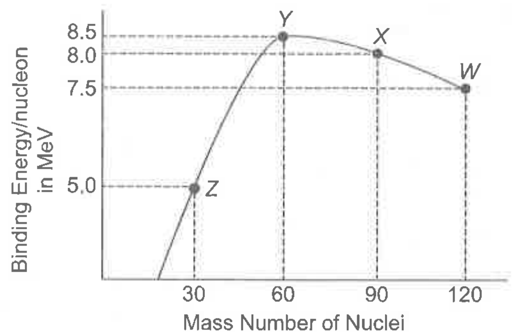
(C) 10^{12} sec (D) 10^{16} sec

5. In the uranium radioactive series the initial nucleus is $_{92}U^{238}$, and the final nucleus is $_{82}Pb^{206}$. When the uranium nucleus decays to lead, the number of α-particles emitted is. and the number of β-particles emitted.

(A) 6, 8 (B) 8, 6

(C) 16, 6 (D) 32, 12

6. Binding energy per nucleon *vs.* mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy *s*



(A) $Y \to 2Z$ (B) $W \to X + Z$
(C) $W \to 2Y$ (D) $X \to Y + Z$

7. A radioactive material of half-life T was produced in a nuclear reactor at different instants, the quantity produced second time was twice of that produced first time. If now their present activities are A_1 and A_2 respectively then their age difference equals:

(A) $\frac{T}{\ln 2}\left|\ln\frac{A_1}{A_2}\right|$ (B) $T\left|\ln\frac{A_1}{A_2}\right|$

(C) $\frac{T}{\ln 2}\left|\ln\frac{A_2}{2A_1}\right|$ (D) $T\left|\ln\frac{A_2}{2A_1}\right|$

8. The activity of a sample reduces from A_0 to $\frac{A_0}{\sqrt{3}}$ in one hour. The activity after 3 hours more will be

(A) $\frac{A_0}{3\sqrt{3}}$ (B) $\frac{A_0}{9}$

(C) $\frac{A_0}{9\sqrt{3}}$ (D) $\frac{A_0}{27}$

9. Half life of radium is 1620 years. How many radium nuclei decay in 5 hours in 5 gm radium? (Atomic weight of radium = 223)
(A) 9.1×10^{12} (B) 3.23×10^{15}
(C) 1.72×10^{20} (D) 3.3×10^{17}

10. A radioactive substance is being produced at a constant rate of 10 nuclei is. The decay constant of the substance is 1/2 $\sec^{-1}$. After what time the number of radioactive nuclei will become 10? Initially there are no nuclei present. Assume decay law holds for the sample.
(A) 2.45 sec (B) log (2) sec
(C) 1.386 sec (D) $\frac{l}{\ln(2)}$ sec

11. At time $t = 0$, N_1 nuclei of decay constant λ_1 and N_2 nuclei of decay constant λ_2 are mixed. The decay rate of the mixture is:

(A) $N_1N_2e^{-(\lambda_1+\lambda_2)t}$

(B) $+\left(\frac{N_1}{N_2}\right)e^{-(\lambda_1-\lambda_2)t}$

(C) $+(N_1\lambda_1e^{-\lambda_1 t} + N_2\lambda_2e^{-\lambda_2 t})$

(D) $+N_1\lambda_1\ N_2\lambda_2e^{-(\lambda_1+\lambda_2)_t}$

12. A particular nucleus in a large population of identical radioactive nuclei did survive 5 half lives of that isotope. Then the probability that this surviving nucleus will survive the next half life:

(A) $\frac{1}{32}$ (B) $\frac{1}{5}$

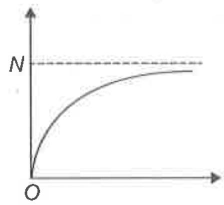
(C) $\frac{1}{2}$ (D) $\frac{1}{10}$

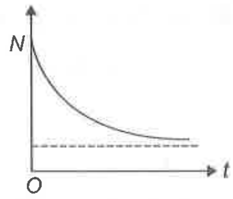
(E) $\frac{5}{2}$

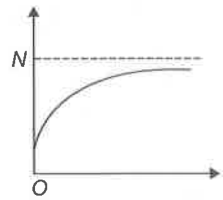
13. The half-life of substance X is 45 years, and it decomposes to substance Y. A sample from a meteorite was taken which contained 2% of X and 14% Y by quantity of substance. If substance Y is not normally found on a meteorite, what is the approximate age of the meteorite?
(A) 270 years (B) 135 years
(C) 90 years (D) 45 years

14. In a certain nuclear reactor, a radioactive nucleus is being produced at a constant rate = 1000/s. The mean life of the radionuclide is 40 minutes. A steady state, the number of radionuclide will be
(A) 4×10^4 (B) 24×10^4
(C) 24×10^5 (D) 24×10^6

15. In the above question, if there were 20×10^5 radionuclide at $t = 0$, then the graph of N v/s t is

(A)


(B)


(C)


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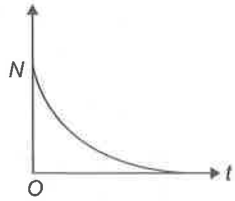


16. A radioactive source in the form of a metal sphere of diameter 3.2×10^{-3} m emits β-particle at a constant rate of 6.25×10^{10} particle/sec. The source is electrically insulated and all the β-particle are emitted from the surface. The potential of the sphere will raise to 1 V in time
(A) 180 μ sec (B) 90 μ sec
(C) 18 μ sec (D) 9 μ sec

17. When a nucleus with atomic number Z and mass number A undergoes a radioactive decay process:

(A) both Z and A will decrease, if the process is α decay
(B) Z will decrease but A will not change, if the process is β^+ decay
(C) Z will decrease but A will not change, if the process is β^- decay
(D) Z and A will remain unchanged, if the process is γ decay.

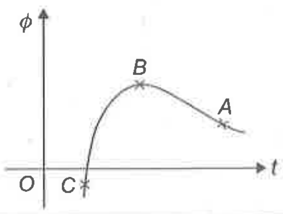
18. When the atomic number A of the nucleus increases
(A) initially the neutron-proton ratio is constant = 1
(B) initially neutron-proton ratio increases and later decreases
(C) initially binding energy per nucleon increases and later decreases
(D) the binding energy per nucleon increases when the neutron-proton ratio increases.

19. Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a $^{20}_{10}$Ne nucleus and M_2 the mass of a $^{40}_{20}$Ca nucleus. Then
(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$
(C) $M_2 < 2M_1$ (D) $M_1 < 10\,(m_n + m_p)$

20. Which of the following statement(s) is (are) correct?
(A) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons.
(B) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.
(C) In nuclear fusion, energy is released by fusion two nuclei of medium mass (approximately 100 amu).
(D) In nuclear fusion, energy is released by fragmentation of a very heavy nucleus.

21. The graph shown by the side shows the variation of potential energy ϕ of proton with its distance 'r' from a fixed sodium nucleus, as it approaches the nucleus, placed at origin O. Then the portion.
(A) AB indicates nuclear repulsion
(B) AB indicates electrostatic repulsion
(C) BC indicates nuclear attraction
(D) BC represents electrostatic interaction

22. A nitrogen nucleus $_7N^{14}$ absorbs a neutron and can transform into lithium nucleus $_3Li^7$ under suitable conditions, after emitting:



(A) 4 protons and 3 neutrons
(B) 5 protons and 1 negative beta particle
(C) 1 alpha particles and 2 gamma particles
(D) 1 alpha particle, 4 protons and 2 negative beta particles
(E) 4 protons and 4 neutrons

23. The instability of the nucleus can be due to various causes. An unstable nucleus emits radiations if possible of transform into less unstable state. Then the cause and the result can be
(A) a nucleus of excess nucleons is α active
(B) an excited nucleus of excess protons is β^- active
(C) an excited nucleus of excess protons is β^- active
(D) an nucleus of excess neutrons is β^- active

24. In β-decay, the Q-value of the process is E. Then
(A) KE of a β-particle cannot exceed E.
(B) KE of anti neutrino emitted lies between Zero and E.
(C) N/Z ratio of the nucleus is altered.
(D) Mass number (A) of the nucleus is altered.

25. Consider the following nuclear reactions and select the correct statements from the options that follow.

Reaction I: $n \rightarrow p + e^- + \bar{v}$

Reaction II: $p \rightarrow n + e^+ + v$

(A) Free neutron is unstable, therefore reaction I is possible
(B) Free proton is stable, therefore reaction II is not possible
(C) Inside a nucleus, both decays (reaction I and II) are possible
(D) Inside a nucleus, reaction I is not possible but reaction II is possible.

26. When the nucleus of an electrically neutral atom undergoes a radioactive decay process, it will remain neutral after the decay if the process is:
(A) α decay (B) β^- decay
(C) γ decay (D) K-capture

27. The decay constant of radio active substance is 0.173 (years)$^{-1}$. Therefore:
(A) Nearly 63% of the radioactive substance will decay in (1/0.173) year.
(B) half life of the radio active substance is (1/0.173) year.
(C) one-forth of the radioactive substance will be left after nearly 8 years.
(D) all the above statements are true.

JEE Advanced

Level I

Nuclear Physics

1. Calculate the mass of an α-particle. Its binding energy is 28.2 MeV.

2. Find the binding of $^{56}_{20}$Fe. Atomic mass of ^{56}Fe is 55.9349u and that of ^{1}H is 1.00783u. Mass of neutron = 1.00867u.

3. Find the kinetic energy of the α-particle emitted in the decay ^{238}Pu $\rightarrow$ ^{234}U + α. The atomic masses needed are as follows:

^{238}Pu	^{234}U	^{4}H
238.04955u	234.04095u	4.002603u

Neglect any recoil of the residual nucleus.

4. How much energy is released in the following reaction?

$$^{7}\text{Li} + p \rightarrow \alpha + \alpha$$

Atomic mass of ^{7}Li = 7.0160u and that of ^{4}He = 4.0026u.

5. ^{32}P beta-decays to ^{32}S. Find the sum of the energy of the antineutrino and the kinetic energy of the β-particle. Neglect the recoil of the daughter nucleus. Atomic mass of ^{32}P = 31.974u and that of ^{32}S = 31.972u.

6. Potassium-40 can decay in three modes. It can decay by β^--emission, β^+-emission or electron capture.
   (A) Write the equations showing the end products
   (B) Find the Q-values in each of the three cases. Atomic masses of $^{40}_{18}$Ar, $^{40}_{19}$K and $^{40}_{20}$C are 39.9624 u, 39.9640u and 39.9626u respectively.

7. ^{228}Th emits an alpha particle to reduce to ^{224}Ra. Calculate the kinetic energy of the alpha particle emitted in the following decay.

$$^{228}\text{Th} \rightarrow {}^{224}\text{Ra}^* + \alpha$$
$$^{224}\text{Ra}^* \rightarrow {}^{224}\text{Ra} + \gamma(217\text{ keV}).$$

Atomic mass of ^{228}Th is 228.028726u, that of ^{224}Ra is 224.020196u and that of $^{4}_{2}$He is 4.00260u.

8. Calculate the maximum kinetic energy of the beta particle emitted in the following decay scheme:

$$^{12}\text{N} \rightarrow {}^{12}\text{C}^* + e^+ + v$$
$$^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \gamma(4.43\text{ MeV})$$

The atomic mass of ^{12}N is 12.018613u.

9. Calculate the Q-value in the following decays:
   (A) $^{19}\text{O} \rightarrow {}^{19}\text{F} + e + \bar{v}$
   (B) $^{25}\text{Al} \rightarrow {}^{25}\text{Mg} + e^+ + v$

The atomic masses needed are as follows:

^{19}O	^{19}F
19.003576u	18.998403u
^{25}Al	^{25}Mg
24.990432u	24.985839u

10. Find the maximum energy that a beta particle can have in the following decay

$$^{176}\text{Lu} \rightarrow {}^{176}\text{Hf} + e + \bar{v}$$

Atomic mass of ^{176}Lu is 175.942694u and that of ^{176}Hf is 175.941420u.

11. Consider the beta decay

$$^{198}\text{Au} \rightarrow {}^{196}\text{Hg}^* + \beta^- + \bar{v}$$

where ^{198}Hg* represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass of ^{198}Au is 197.968233u and that of ^{198}Hg is 197.966760u.

12. A uranium reactor develops thermal energy at a rate of 300 MW. Calculate the amount of ^{235}U being consumed every second. Average energy released per fission is 200 MeV.

13. Calculate the Q-value of the fusion reaction

$$^{4}\text{He} + {}^{4}\text{He} = {}^{8}\text{Be}.$$

Is such a fusion energetically favourable? Atomic mass of ^{8}Be is 8.0053u and that of ^{4}He is 4.0026u.

14. The binding energies per nucleon for deuteron ($_1$H^2) and helium ($_2$He^4) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus ($_2$He^4) is ______.

15. Suppose that the Sun consists entirely of hydrogen atom and releases the energy by the nuclear reaction, $4{}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He}$ with 26 MeV of energy released. If the total output power of the Sun is assumed to remain constant at 3.9×10^{26} W, find the time it will take to burn all the hydrogen. Take the mass of the Sun as 1.7×10^{30} kg.

16. To positron is a fundamental particle with the same mass as that of the electron and with a charge equal to that of an electron but of opposite sign. When a positron and an electron collide, they may annihilate each other. The energy corresponding to their mass appears in two photons of equal energy. Find the wavelength of the radiation emitted.

[Take: mass of electron = $(0.5/C^2)$ MeV and $hC = 1.2 \times 10 - 12$ MeV·m where h is the Plank's constant and C is the velocity of light in air]

17. When two deutrons $({}_1H^2)$ fuse to from a helium nucleus ${}_2He^4$, 23.6 MeV energy is released. Find the binding energy of helium if it is 1.1 MeV for each nucleon of deutrim.

18. A π^+ meson of negligible initial velocity decays to a μ^+ (muon) and a neutrino. With what kinetic energy (in eV) does the muon move? (The rest mass of neutrino can be considered zero. The rest mass of the π^+ meson is 150 MeV and the rest mass of the muon is 100 MeV.) Take neutrino to behave like a photon.

Take $= \sqrt{2} = 1.41$.

19. Consider the following reaction;

$$^2H_1 + {}^2H_1 = {}^4He_2 + Q.$$

Mass of the deuterium atom = $2.0141u$; Mass of the helium atom = $4.0024u$

This is a nuclear ________ reaction in which the energy Q is released is ________ MeV.

20. The activity of a radioactive sample falls from 600 s^{-1} to 500 s^{-1} in 40 minutes. Calculate its half-life.

21. The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is $4.5 \times 10^9 y$. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U.

22. A radioactive decay counter is switched on at $t = 0$. A b-active sample is present near the counter. The counter registers the number of b-particles emitted by the sample. The counter registers 1×10^5 b-particles at $t = 36$ s and $1.11 \times 10^5 b$-particles at $t = 108$s. Find $T_{1/2}$ of this sample

23. An isotopes of Potassium $^{40}_{19}K$ has a half life of 1.4×10^9 year and decays to Argon $^{40}_{18}Ar$ which is stable.

(i) Write down the nuclear reaction representing this decay.

(ii) A sample of rock taken from the moon contains both potassium and argon in the ratio $\frac{1}{7}$. Find age of rock

24. At $t = 0$, a sample is placed in a reactor. An unstable nuclide is produced at a constant rate R in the sample by neutron absorption. This nuclide β-decays with half life τ. Find the time required to produce 80% of the equilibrium quantity of this unstable nuclide.

25. Radioactive ^{131}I has a half-life of 8.0 days. A sample containing ^{131}I has activity 20 μCi at $t = 0$.

(A) What is its activity at $t = 4.0$ days?

(B) What is its decay constant at $t = 4.0$ days?

26. The decay constant of ^{238}U is 4.9×10^{-18} s^{-1}.

(A) What is the average-life of ^{238}U?

(B) What is the half-life of ^{238}U?

(C) By what factor does the activity of a ^{238}U sample decrease in 9×10^9 years?

27. Carbon ($Z = 6$) with mass number 11 decays to boron ($Z = 5$).

(A) Is it a β^+-decay or a β^- decay?

(B) The half-life of the decay scheme is 20.3 minutes. How much time will elapse before a mixture of 90% carbon-11 and 10% born 11 converts itself into a mixture of 10 % Carbon-11 and 90% Boron-11.

28. ^{238}U decays to ^{206}Pb with a half-life of $4.47 \times 10^9 y$. This happens in a number of steps. Can you justify a single half-life for this chain of processes? A sample of rock is found to contain 2.00 mg of ^{238}U and 0.600 mg of ^{206}Pb. Assuming that all the lead has come from uranium. find the life of the rock.

29. Nuclei of radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element.

(A) Calculate the number N of nuclei of A at time t.

(B) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N as $t \to \infty$.

Level II

1. The kinetic energy of an α-particle which flies out of the nucleus of a Ra^{226} atom in radioactive disintegration is 4.78 MeV. Find the total energy evolved during the escape of the α-particle.

2. A small bottle contains powdered beryllium Be and gaseous radon which is used as a source of α-particles. Neutrons are produced when α-particles of the radon react with beryllium. The yield of this reaction is $\left(\frac{1}{4000}\right)$ i.e,. only one α-particle out of 4000 induces the reaction. Find the amount of radon (Rn^{222}) originally introduced into the source, if it produces 1.2×10^6 neutrons per second after 7.6 days. [$T_{1/2}$ of R_n = 3.8 days]

3. When thermal neutrons (negligible kinetic energy) are used to induce the reaction;
$^{10}_{5}B + ^{1}_{0}H \rightarrow ^{7}_{3}Li + ^{4}_{2}He$ α-particles are emitted with an energy of 1.83 MeV.

Given the masses of boron neutron and He^4 as 10.01167, 1.00894 and 4.00386u respectively. What is the mass of $^{7}_{3}Li$? Assume that particles are free to move after the collision.

4. Show that in a nuclear reaction where the outgoing particle is scattered at an angle of 90° with the direction of the bombarding particle, the Q-value is expressed as

$$Q = K_P\left(1 + \frac{m_P}{M_O}\right) - K_1\left(1 + \frac{m_1}{M_O}\right)$$

Where, I = incoming particle, P = product nucleus, T = target nucleus, O = outgoing particle.

5. A body of mass m_0 is placed on a smooth horizontal surface. The mass of the body is decreasing exponentially with disintegration constant λ. Assuming that the mass is ejected backward with a relative velocity u. Initially the body was at rest. Find the velocity of body after time t.

6. A radionuclide with disintegration constant λ is produced in a reactor at a constant rate α nuclei per sec. During each decay energy E_0 is released. 20% of this energy is utilized in increasing the temperature of water. Find the increase in temperature of m mass of water in time t. Specific heat of water is S. Assume that there is no loss of energy through water surface.

7. U^{238} and U^{235} occur in nature in an atomic ratio 140 : 1. Assuming that at the time of earth's formation the two isotopes were present in equal amounts. Calculate the age of the earth.
(Half life of $u^{238} = 4.5 \times 10^9$ yrs and that of U^{235} = 7.13×10^8 yrs).

8. An experiment is done to determine the half-life of radioactive substance that emits one β-particle for each decay process. Measurement show that an average of 8.4 β are emitted each second by 2.5 mg of the substance. The atomic weight of the substance is 230. Find the half life of the substance.

9. A wooden piece of great antiquity weight 50 gm and shows C^{14} activity of 320 disintegrations per minute. Estimate the length of the time which has elapsed since this wood was part of living tree, assuming that living plants show a C^{14} activity of 12 disintegrations per minute per gm. The half life of C^{14} is 5730 yrs.

10. The element Curium $^{248}_{96}Cm$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in α decay are as follows:

$^{248}_{96}Cm$ = 248.0.072220u, $^{248}_{96}Pu = 244.064100u$

and $^{4}_{2}He = 4.002603u$.

Calculate the power output from a sample of 10^{20} Cm atoms. ($1u = 931$ MeV/c^2)

11. A small quantity of solution containing ^{24}Na radionuclide (half life 15 hours) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hours shows an activity of 296 disintegrations per minute. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.
(1 Curie = 3.7×10^{10} disintegrations per second)

12. At a given instant there are 25% undecayed radio–active nuclei in a sample. After 10 sec the number of undecayed nuclei remains to 12.5%. Calculate:
(i) mean–life of the nuclei and
(ii) The time in which the number undecayed nuclear will further reduce to 6.25% of the reduced number.

Previous Year Questions

JEE Main

1. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ yr, then the amount of substance left after 15 yr, is **[AIEEE-2002]**

(A) $\frac{N_0}{8}$ (B) $\frac{N_0}{16}$

(C) $\frac{N_0}{2}$ (D) $\frac{N_0}{4}$

2. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{2+} is **[AIEEE-2003]**

(A) 30.6 eV (B) 13.6 eV

(C) 3.4 eV (D) 122.4 eV

3. The wavelengths involved in the spectrum of deuterium $\left({}_1^2D\right)$ are slightly different from that of hydrogen spectrum, because **[AIEEE-2003]**

(A) sizes of the two nuclei are different

(B) nuclear forces are different in the two cases

(C) masses of the two nuclei are different

(D) attraction between the electron and the nucleus is different in the two cases

4. In the nuclear fusion reaction, **[AIEEE-2003]**

$$ {}_1^2H + {}_1^3H \rightarrow {}_2^4He + n $$

given that the repulsive potential energy between the two nuclei is 7.7×10^{-14} J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K]

(A) 10^7 K (B) 10^5 K

(C) 10^3 K (D) 10^9 K

5. Which of the following cannot be emitted by radioactive substances during their decay? **[AIEEE-2003]**

(A) Protons (B) Neutrinos

(C) Helium nuclei (D) Electrons

6. A nucleus with $Z = 92$ emits the following in a sequence:

$\alpha, \alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha; \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha.$

The Z of the resulting nucleus is **[AIEEE-2003]**

(A) 76 (B) 78

(C) 82 (D) 74

7. A radioactive sample at any instant has its disintegration rate 5000 disintegrations/min. After 5 min, the rate is 1250 disintegrations/min. Then, the decay constant (per minute) is **[AIEEE-2003]**

(A) 0.4 ln 2 (B) 0.2 ln 2

(C) 0.1 ln 2 (D) 0.8 ln 2

8. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of **[AIEEE-2004]**

(A) 1 Å (B) 10^{-10} cm

(C) 10^{-12} cm (D) 10^{-15} cm

9. The binding energy per nucleon of deuteron (${}_1^2H$) and helium nucleus (${}_2^4He$) is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is **[AIEEE-2004]**

(A) 13.9 MeV (B) 26.9 MeV

(C) 23.6 MeV (D) 19.2 MeV

10. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2:1. The ratio of their nuclear sizes will be **[AIEEE-2004]**

(A) $2^{1/3}:1$ (B) $1:3^{1/3}$

(C) $3^{1/2}:1$ (D) $1:2^{1/3}$

11. A nuclear transformation is denoted by $X(n, \alpha) \rightarrow {}_3^7Li$. Which of the following is the nucleus of element X? **[AIEEE-2005]**

(A) ${}_6^{12}C$ (B) ${}_5^{10}B$

(C) ${}_5^9B$ (D) ${}_4^{11}Be$

12. If radius of the ${}_{13}^{27}Al$ nucleus is estimated to be 3.6 fermi, then the radius ${}_{52}^{125}Te$ of nucleus be nearly **[AIEEE-2005]**

(A) 6 fermi (B) 8 fermi

(C) 4 fermi (D) 5 fermi

13. Starting with a sample of pure ${}^{66}Cu$, 7/8 of it decays into Zn in 15 min. The corresponding half-life is **[AIEEE-2005]**

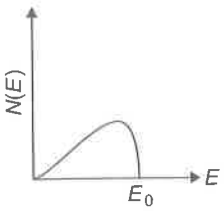
(A) 10 min (B) 15 min

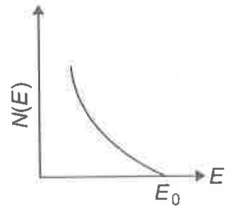
(C) 5 min (D) $7\frac{1}{2}$ min

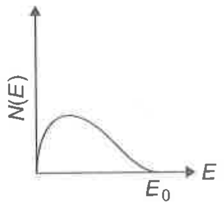
14. When ${}_3^7Li$ nuclei are bombarded by protons, and the resultant nuclei are ${}_4^8Be$, the emitted particles will be **[AIEEE-2006]**

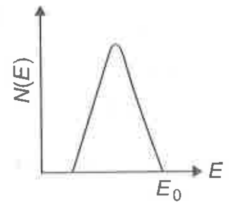
(A) alpha particles (B) beta particles
(C) gamma photons (D) neutrons

15. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is **[AIEEE-2006]**

(A)


(B)


(C)


(D)


16. If the binding energy per nucleon in ${}_3^7\text{Li}$ and ${}_2^4\text{He}$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction $p + {}_3^7\text{Li} \rightarrow 2\,{}_2^4\text{He}$, energy of proton must be **[AIEEE-2006]**
(A) 28.24 Mev (B) 17.28 MeV
(C) 1.46 MeV (D) 39.2 MeV

17. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closed approach for the alpha nucleus will be proportional to **[AIEEE-2006]**

(A) v^2 (B) $\frac{1}{m}$

(C) $\frac{1}{v^4}$ (D) $\frac{1}{Ze}$

18. In gamma ray emission from a nucleus **[AIEEE-2007]**
(A) both the neutron number and the proton number change
(B) there is no change in the proton number and the neutron number
(C) only the neutron number changes
(D) only the proton number changes

19. If M_0 is the mass of an oxygen isotope ${}_8\text{O}^{17}$, M_p and M_n are the masses of a proton and a neutron, respectively, the nuclear binding energy of the isotope is **[AIEEE-2007]**

(A) $(M_0 - 8M_p)c^2$

(B) $(M_0 - 8M_p - 9M_n)c^2$

(C) M_0c^2

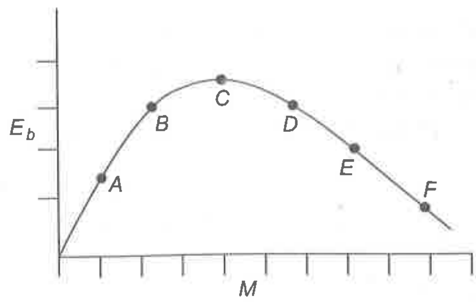
(D) $(M_0 - 17M_n)c^2$

20. The half-life period of a radioactive element X i same as the mean life time of another radioactiv element Y. Initially they have the same number o atoms. Then **[AIEEE-2007**
(A) X will decay faster than Y
(B) Y will decay faster than X
(C) Y and X have same decay rate initially
(D) X and Y decay at same rate always

21. Statement I Energy is released when heavy nucl undergo fission of light nuclei undergo fusion.
Statement II For heavy nuclei, binding energy p nucleon increases with increasing Z while for lig nuclei it decrease with increasing Z. **[AIEEE-200**
(A) Statement I is true, Statement II is true; Statement II is not a correct explanation f Statement I.
(B) Statement I is true, Statement II is false.
(C) Statement I is false, Statement II is true.
(D) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

22. The above is plot of binding energy per nucleon E against the nuclear mass M; A, B, C, D, E, F co respond to different nuclei. Consider four reactio **[AIEEE-200**

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$



where ε is the energy released? In which reactions ε positive?
(A) (i) and (iv) (B) (i) and (iii)
(C) (ii) and (iv) (D) (ii) and (iii)

23. A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **[AIEEE-2010]**

(A) $\frac{A-Z-8}{Z-4}$ (B) $\frac{A-Z-4}{Z-8}$

(C) $\frac{A-Z-12}{Z-4}$ (D) $\frac{A-Z-4}{Z-2}$

Directions Question No. 24-25 are based on the following paragraph.
A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $M/2$ each. Speed of light is C.

24. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then **[AIEEE-2010]**

(A) $E_2 = 2E_1$ (B) $E_1 > E_2$

(C) $E_2 > E_1$ (D) $E_1 = 2E_2$

25. The speed of daughter nuclei is **[AIEEE-2010]**

(A) $c\frac{\Delta m}{M + \Delta m}$ (B) $c\sqrt{\frac{2\Delta m}{M}}$

(C) $c\sqrt{\frac{\Delta m}{M}}$ (D) $c\sqrt{\frac{\Delta m}{M + \Delta m}}$

26. The half-life of a radioactive substance is 20 min. The approximate time interval $(t_2 - t_1)$ between the time t_2 when 2/3 of it had decayed and time t_1 when 1/3 of it had decayed is **[AIEEE-2011]**

(A) 14 min (B) 20 min
(C) 28 min (D) 7 min

27. **Statement I** A nucleus having energy E_1 decays be β^- emission to daughter nucleus having energy E_2, but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement II To conserve energy and momentum in β-decay at least three particles must take part in the transformation. **[AIEEE-2011]**

(A) Statement I is false, Statement II is true
(B) Statement I is true, Statement II is false
(C) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(D) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

28. If a simple pendulum has significant amplitude (upto a factor of $\frac{1}{e}$ of original) only in the period between $t = 0$ s to $t = \tau s$, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds **[AIEEE-2012]**

(A) $\frac{0.693}{b}$ (B) b

(C) $\frac{1}{b}$ (D) $\frac{2}{b}$

JEE Advanced

1. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is **[JEE-2002 (Scr.)]**

(A) 400 μs (B) 6.3 μs
(C) 40 μs (D) 300 μs

2. Which of the following processes represents a gamma-decay? **[JEE-2002 (Scr.)]**

(A) ${}^{A}X_{Z} + \gamma \rightarrow {}^{A}X_{Z-1} + a + b$

(B) ${}^{A}X_{Z} + {}^{1}n_{0} \rightarrow {}^{A-3}X_{Z-2} + c$

(C) ${}^{A}X_{Z} \rightarrow {}^{A}X_{Z} + f$

(D) ${}^{A}X_{Z} + e_{-1} \rightarrow {}^{A}X_{Z-1} + g$

3. The volume and mass of a nucleus are related as **[JEE-2003 (Scr.)]**

(A) $v \propto m$ (B) $v \propto 1/m$
(C) $v \propto m^2$ (D) $v \propto 1/m^2$

4. The nucleus of element X (A = 220) undergoes α-decay. If Q-value of the reaction is 5.5 MeV, then the kinetic energy of α-particle is: **[JEE-2003 (Scr.)]**

(A) 5.4 MeV (B) 10.8 MeV
(C) 2.7 MeV (D) None

5. A radioactive sample emits $n\beta$-particles in 2 sec. In next 2 sec it emits $0.75n$ β-particles, what is the mean life of the sample? **[JEE-2003]**

6. A 280 days old radioactive substance shows an activity of 6000 dps, 140 days later it's activity becomes 3000 dps. What was its initial activity. **[JEE-2004 (Scr)]**

(A) 20000 dps (B) 24000 dps
(C) 12000 dps (D) 6000 dps

7. The age of rock containing lead and uranium is equal to 1.5×10^9 years. The uranium is decaying into lead with half life equal to 4.5×10^9 years. Find the ratio of lead to uranium present in the rock, assuming initially no lead was present in the rock. $\left(\text{Given } 2\frac{1}{3} = 1.259\right)$. **[JEE-2004]**

8. Helium nuclei combines to form an oxygen nucleus. The binding energy per nucleon of oxygen nucleus is if $m_0 = 15.834$ amu and $m_{He} = 4.0026$ amu **[JEE-2005]**

(A) 10.24 MeV (B) 0 Me V
(C) 5.24 MeV (D) 4 Me V

9. In Young's double slit experiment an electron beam is used to form a fringe pattern instead of light. If speed of the electrons is increased then the fringe width will: **[JEE-2005 (Scr.)]**

(A) increase
(B) decrease
(C) remains same
(D) no fringe pattern will be formed

10. Given a sample of Radium –226 having half-life of 4 days. Find the probability, a nucleus disintegrates within 2 half lives. **[JEE-2006]**

(A) 1 (B) $\frac{1}{2}$
(C) $\frac{3}{4}$ (D) $\frac{1}{4}$

11. Match the following Columns **[JEE-2006]**

Column I	Column II
(A) Nuclear fusion	(P) Converts some matter into energy
(B) Nuclear fission	(Q) Generally occurs for nuclei with low atomic number
(C) β-decay	(R) Generally occurs for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(S) Essentially proceeds by weak nuclear forces

12. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is **[JEE-2007]**

(A) $E(^{236}_{92}U) > E(^{137}_{53}I) + E(^{97}_{39}Y) + 2E(n)$
(B) $E(^{236}_{92}U) < E(^{137}_{53}I) + E(^{97}_{39}Y) + 2E(n)$
(C) $E(^{236}_{92}U) < E(^{140}_{56}Ba) + E(^{94}_{36}Kr) + 2E(n)$
(D) $E(^{236}_{92}U) = E(^{140}_{56}Ba) + E(^{94}_{36}Kr) + 2E(n)$

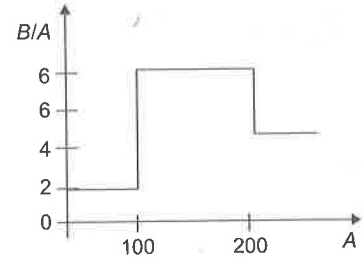
13. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **[JEE-2007]**

(A) 802 nm (B) 823 nm
(C) 1882 nm (D) 1648 nm

14. Some laws/processes are given in **Column I.** Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) Transition between two atomic energy levels	(P) Characteristic X-rays
(B) Electron emission from a material	(Q) Photoelectric effect
(C) Mosley's law	(R) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(S) β-decay

15. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use



this plot to choose the correct choice(s) given below Figure **[JEE-2008]**

(A) Fusion of two nuclei with mass numbers lying in the range $1 < A < 50$ will release energy
(B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy
(C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments
(D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

16. A radioactive sample $S1$ having an activity $5\mu Ci$ twice the number of nuclei as another sample $S2$ which as an activity of 10 μC. The half lives of $S1$ and $S2$ can be **[JEE-2008]**

(A) 20 years and 5 years, respectively

(B) 20 years and 10 years, respectively.

(C) 10 years each

(D) 5 years each

Common Data for Questions 17–20: Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ^2_1H, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The $D-D$ reaction is $^2_1H + ^2_1H \rightarrow ^3_2He + n$ energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ^2_1H nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm $^{-3}$.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K;

$$\frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9} \text{eVm}$$ **[JEE-2009]**

17. In the core of nuclear fusion reactor, the gas becomes plasma because of

(A) strong nuclear force acting between the deuterons

(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron electron pairs

(D) the high temperature maintained inside the reactor core

18. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(A) 1.0×10^9 K $< T < 2.0 \times 10^9$ K

(B) 2.0×10^9 K $< T < 3.0 \times 10^9$ K

(C) 3.0×10^9 K $< T < 4.0 \times 10^9$ K

(D) 4.0×10^9 K $< T < 5.0 \times 10^9$ K

19. Results of calculations for four different designs of a fusion reactor using $D-D$ reaction are given below. Which of these is most promising based on Lawson criterion?

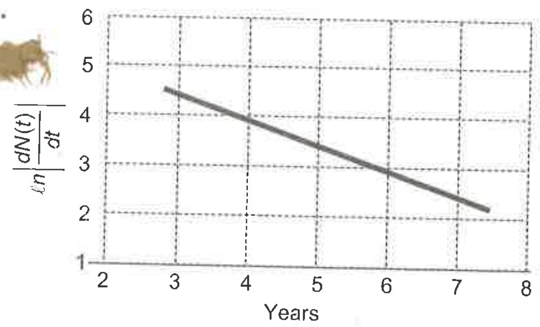
(A) deuteron density $= 2.0 \times 10^{12}$ cm^{-3}, confinement time $= 5.0 \times 10^{-3}$s

(B) deuteron density $= 8.0 \times 10^{14}$ cm^{-3}, confinement time $= 9.0 \times 10^{-1}$ s

(C) deuteron density $= 4.0 \times 10^{23}$ cm^{-3}, confinement time $= 1.0 \times 10^{-11}$ s

(D) deuteron density $= 1.0 \times 10^{24}$ cm^{-3}, confinement time $= 4.0 \times 10^{-12}$ s

20.



To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is: **[JEE-2010]**

21. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is **[JEE-2011]**

Common Data for Questions 22–24: The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e., $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

22. What is the maximum energy of the anti-neutrino?
(A) Zero
(B) Much less than 0.8×10^6 eV.
(C) Nearly 0.8×10^6 eV.
(D) Much larger than 0.8×10^6 eV.

23. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?
(A) $0 \le K\, 0.8 \times 10^6$ eV
(B) $3.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV
(C) $3.0 \text{ eV} \le K < 0.8 \times 10^6$ eV
(D) $0 \le K < 0.8 \times 10^6$ eV

24. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that ln 2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is **[JEE-2013]**

Common Data for Questions 25–26: The mass of nucleus ^A_ZX is less than the sum of the masses of $(A - Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

^1_1H	1.007825u	^2_1H	2.014102u	^3_1H	3.016050u	^4_2H	4.002603u
^6_3Li	6.015123u	^7_3Li	7.016004u	$^{70}_{30}Zn$	69.925325u	$^{82}_{34}Se$	81.916709u
$^{152}_{64}Gd$	151.919803u	$^{206}_{82}Pb$	205.974455u	$^{209}_{83}Bi$	208.980388u	$^{210}_{84}Po$	209.982876u

25. The correct statement is: **[JEE-2013]**
(A) The nucleus ^6_3Li can emit an alpha particle.
(B) The nucleus $^{210}_{84}Po$ can emit a proton.
(C) Deuteron and alpha particle can undergo complete fusion.
(D) The nuclei $^{70}_{30}Zn$ and $^{82}_{34}Se$ can undergo complete fusion.

26. The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}Po$ at rest undergoes alpha decay, is: **[JEE-2013]**
(A) 5319 (B) 5422
(C) 5707 (D) 5818

27. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists: **[JEE-2013]**

List I	List II
(P) Alpha decay	1. $^{15}_8O \rightarrow {}^{15}_7N + \cdots$
(Q) β^+ decay	2. $^{238}_{92}U \rightarrow {}^{234}_{90}Th + \cdots$
(R) Fission	3. $^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + \cdots$
(S) Proton emission	4. $^{239}_{94}Pu \rightarrow {}^{140}_{57}La + \cdots$

Codes:

	P	Q	R	S
(A)	4	2	1	3
(B)	1	3	2	4
(C)	2	1	4	3
(D)	4	3	2	1

ANSWER KEYS

Exercises

JEE Main

1. C	**2.** B	**3.** B	**4.** C	**5.** C	**6.** B	**7.** D	**8.** B	**9.** D	**10.** C
11. B	**12.** C	**13.** C	**14.** B	**15.** D	**16.** A	**17.** C	**18.** A	**19.** B	**20.** A
21. D	**22.** A	**23.** B	**24.** A	**25.** E	**26.** C	**27.** B	**28.** D	**29.** A	**30.** A
31. C	**32.** D								

JEE Advanced

Single Correct Option Type Questions

1. A	**2.** A	**3.** B	**4.** C	**5.** B	**6.** C	**7.** C	**8.** B	**9.** B	**10.** C
11. C	**12.** C	**13.** B	**14.** C	**15.** C	**16.** C	**17.** A, B, D		**18.** A, C	**19.** C
20. A, D	**21.** B, C	**22.** C, E	**23.** A, C, D		**24.** A, B, C	**25.** A, B, C		**26.** C, D	**27.** A, C

JEE Advanced

Level I

1. 4.0016u **2.** 492 MeV **3.** 5.58 MeV **4.** 17.34 MeV **5.** 1.86 MeV

6. **(A)** ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{20}\text{Ca} + e^- + \bar{\nu}, {}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + e^+ + \nu, {}^{40}_{19}\text{K} + e^- \rightarrow {}^{40}_{18}\text{Ar} + \nu$

(B) 1.3034 MeV, 0.4676 MeV, 1.490 MeV **7.** 5.304 MeV **8.** 11.88 MeV

9. **(A)** 4.816 MeV (B) 3.254 MeV **10.** 1.182 MeV **11.** 0.2806 MeV **12.** 3.7 mg

13. –93.1 KeV, No **14.** 23.6 MeV **15.** $\frac{8}{3} \times 10^{18}$ sec **16.** 2.48×10^{-12} m **17.** 28 MeV

18. 9×10^6 **19.** Fusion, 24 **20.** 152 min. **21.** $4.5 \times 10^{10} y$ old **22.** ($T_{1/2} = 10.8$ sec)

23. **(i)** ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + {}_{+1}e^0 + \nu$, **(ii)** 4.2×10^9 years **24.** $t = \left(\frac{\ln 5}{\ln 2}\right)\tau$ **25.** **(A)** 14 μCi,

(B) 1.4×10^{-6} sec^{-1} **26.** **(A)** $6.49 \times 10^9 y$, **(B)** $4.5 \times 10^9 y$, **(C)** 4

27. **(A)** β^+, (B)24 min **28.** $1.92 \times 10^9 y$ **29.** **(A)** $N = \frac{1}{\lambda}[\propto (1 - e^{-\lambda t}) + \lambda N_0\, e^{-\lambda t}]$ **(B)** $\frac{3N_0}{2}, 2N_0$

Level II

1. 4.87 MeV **2.** $3.3 \times 10^{-6} g$ **3.** 7.01366 amu **5.** $v = u\lambda t$

6. $\Delta T = \dfrac{0.2E_0\left[\alpha t - \frac{\alpha}{\lambda}(1 - e^{-\lambda t})\right]}{mS}$ **7.** 6.04×10^9 yrs **8.** 1.7×10^{10} yrs **9.** 5196 yrs

10. $\cong 33.298\ \mu$W **11.** 6 litre **12.** **(i)** $t_{1/2} = 10$ sec., $t_{means} = 14.43$ s **(ii)** 40 sec.

Previous Years' Questions

JEE Main

1. A	**2.** A	**3.** C	**4.** D	**5.** A	**6.** B	**7.** A	**8.** C	**9.** C	**10.** D
11. B	**12.** A	**13.** C	**14.** C	**15.** C	**16.** B	**17.** B	**18.** B	**19.** B	**20.** B
21. B	**22.** A	**23.** B	**24.** C	**25.** B	**26.** B	**27.** C	**28.** D		

JEE Advanced

1. A **2.** C **3.** A **4.** A **5.** $1.75n = N_0(1 - e^{-4\lambda})$, 6.95 sec, $\frac{2}{\ln\left(\frac{4}{3}\right)}$ **6.** B **7.** 0.25

8. A **9.** B **10.** C **11.** **(A)** P, Q; **(B)** P, R; **(C)** S, P; **(D)** P, Q, R **12.** A **13.** B

14. **(A)** R, P; **(B)** Q, S; **(C)** P; **(D)** Q **15.** B, D **16.** A **17.** D **18.** A **19.** B **20.** 8

21. 1 **22.** C **23.** D **24.** 4 **25.** C **26.** A **27.** C

Surface Tension and Viscosity

COHESIVE FORCE

The force of attraction between the molecules of the same substance is called cohesive force.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

EXAMPLE

(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

dhesive Force

he force at attraction between molecules of different subances is called adhesive force.

XAMPLE

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Adhesive force helps us to write on the paper with ink.

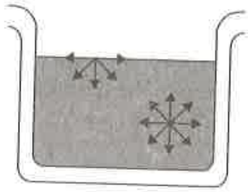
iii) Large force of adhesion between cement and bricks helps us in construction work.

iv) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

SURFACE TENSION

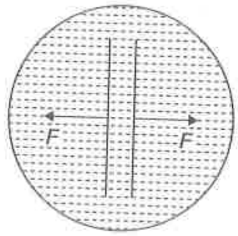
The property of a liquid at rest due to which its free surface tries to have minimum surface area and behaves as if it were under tension somewhat like a stretched elastic membrane is called surface tension.

The molecules of the liquid exert attractive forces on each other. There is zero net force on a molecule inside the volume of the liquid.



But a surface molecules is drawn into the volume. Thus, the liquid tends to minimize its surface area, just as a stretched membrane does.

Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being perpendicular to the line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line of length L, then



$$T = (F/L)$$

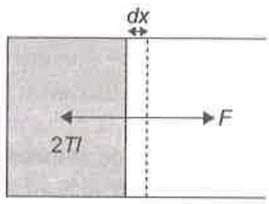
Regarding surface tension it is worth noting that:

1. It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

2. It is a scalar as it has a unique direction which is not to be specified.
3. It has dimension $[ML^{-2}]$ and SI units N/m while CGS unit dyne/cm, so that one MKS unit of surface tension = 10^3 dyne/cm
4. Surface tension of a liquid decreases with rise in temperature
5. The surface tension of a liquid is very sensitive to impurities on the surface (called *contamination*) and decreases with contamination of surface.
6. In case of soluble impurities surface tension may increase or decrease depending on the nature of impurity. Usually highly soluble salt such as sodium chloride increases surface tension while sparingly soluble salt such as soap decreases surface tension.

Surface Energy

When the surface area of a liquid is increased, the molecules from the interior rise to the surface. This requires work against force of attraction of the molecules just below the surface. This work is stored in the form of potential energy. Thus, the molecules in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called 'surface energy'. The surface energy is related to the surface tension as discussed below:



Let a liquid film be formed on a wire frame and a straight wire of length l can slide on this wire frame as shown in figure. The film has two surface and both the surface are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force Tl. Thus, net force on the wire due to both the surface is $2Tl$. One has to apply an external force F equal and opposite to it to keep the wire in equilibrium. Thus,

$$F = 2Tl$$

Now, suppose the wire is moved through a small distance dx, the work done by the force is,

$$dW = Fdx = (2Tl)dx$$

But $(2l)(dx)$ is the total increase in area of both the surface of the film. Let is be dA. Then,

$$dW = TdA \quad \text{or} \quad T = \frac{dW}{dA}$$

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity.

SOLVED EXAMPLES

EXAMPLE 1

Calculate the energy released when 1000 small water drops each of same radius 10^{-7} m coalesce to form one large drop. The surface tension of water is 7.0×10^{-2} N/m.

SOLUTION

Let r be the radius of smaller drops and R of bigger one. Equating the initial and final volumes, we have

$$\frac{4}{3}\pi R^3 = (1000)\left(\frac{4}{3}\pi r^3\right)$$

or $\quad R = 10r = (10)(10^{-7})$ m $\quad$ or $\quad R = 10^{-6}$ m

Further, the water drops have only one free surface. Therefore,

$$\Delta A = 4\pi R^2 - (1000)(4\pi r^2)$$
$$= 4\pi[(10^{-6})^2 - (10^3)(10^{-7})^2] = -36\pi(10^{-12})\ \text{m}^2$$

Here, negative sign implies that surface area is decreasing. Hence, energy released in the process.

$$U = T|\Delta A| = (7 \times 10^{-2})(36\pi \times 10^{-12})$$
$$\text{J} = 7.9 \times 10^{-12}\ \text{J} \qquad \textbf{Ans.}$$

EXAMPLE 2

A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculates the energy expanded if surface tension of mercury is 35×10^{-3} N/m.

SOLUTION

If drop of radius R is sprayed into n droplets of equal radius r, then as a drop has only surface, the initial surface area will be $4\pi R^2$ while final area is $n\,(4\pi r^2)$. So the increase in area

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

So energy expended in the process,

$$W = T\Delta S = 4\pi T\,[nr^2 - R^2] \qquad (1)$$

Now since the total volume of n droplets is the same as that of initial drop, i.e.,

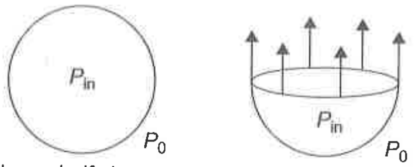
$$\frac{4}{3}\pi R^3 = n\left[\left(\frac{4}{3}\right)\pi r^3\right] \quad \text{or} \quad r = \frac{R}{n^{1/3}} \qquad (2)$$

Putting the value of r from equation (2) in (1)

$$W = 4\pi R^2 T((n)^{1/3} - 1)$$

Excess Pressure Inside a Liquid Drop

Consider a liquid drop of radius 'R' and surface tension 'T'. A liquid drop has only one surface film, hence the surface tension force is $T(2\pi R)$.



lower half shows in figure

Force due to inside pressure (P_{in}) is P_{in} × area i.e., $P_{in}\pi R^2$ similarly force due to outside pressure (P_0) is $P_0\pi R^2$ since each half of the liquid drop is in equilibrium

$$P_0\pi R^2 + T(2\pi R) = P_{in}(\pi R^2)$$

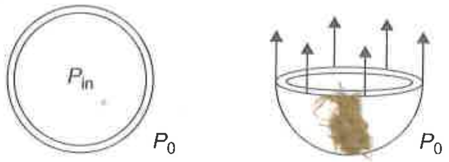
$$P_{in} - P_0 = \frac{2T}{R} = \text{Excess pressure}$$

Excess Pressure Inside a Bubble

Consider a bubble of radius 'R' and surface tension 'T'. A bubble consists of two spherical surface films with a thin layer of liquid between them.

The total surface tension force for each surface inner and outer $T(2\pi R)$ for a total of $(2T)(2\pi R)$.

Force due to inside pressure (P_{in}) is $P_{in}\pi R^2$ and due to outside pressure (P_0) is $P_0\pi R^2$



Since each half of bubble is in equilibrium (lower half shown in figure)

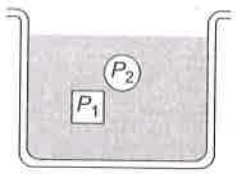
$$P_0\pi R^2 + 2T(2\pi R) = P_{in}\pi R^2$$

$$P_{in} - P_0 = \frac{4T}{R} = \text{Excess pressure}$$

Notes

1. If we have an air bubble inside a liquid, a single surface is formed. There is air on the concave side and liquid on the convex side. The pressure in the concave side (that is in the air) is greater than the pressure in the convex side (that is in the liquid) by an amount $\frac{2T}{R}$.

$$\therefore \qquad P_2 - P_1 = \frac{2T}{R}$$



The above expression has been written by assuming P_1 to be constat from all sides of the bubble. For small size bubbles this can be assumed.

2. From the above discussion, we can make a general statement. The pressure on the concave side of a spherical liquid surface is greater than the convex side by $\frac{2T}{R}$.

3. For any curved surface excess pressure on the concave side $= T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ where R_1 and R_2 are radius of curvature of the surface in two perpendicular direction of instead of liquid surface, liquid film is given then above expression will be

$$P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

For spherical curved surface R_1, R_2.

EXAMPLE 3

What should be the pressure inside a small air bubble of 0.1 mm radius situated just below the water surface. Surface tension of water = 7.2×10^{-2} N/m and atmospheric pressure = 1.013×10^5 N/m^2.

SOLUTION

Surface tension of water $T = 7.2 \times 10^{-2}$ N/m.

Radius of air bubble $R = 0.1$ mm $= 10^{-4}$ m.

The excess pressure inside the air bubble is given by,

$$P_2 - P_1 = \frac{2T}{R}$$

$\therefore$ Pressure inside the air bubble, $P_2 = P_1 + \frac{2T}{R}$

Substituting the values, we have

$$P_r = (1.013\times10^5) + \frac{2\times7.2\times10^{-2}}{10^{-4}} = 1.027\times10^3\ \text{N/m}^2$$

EXAMPLE 4

A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

SOLUTION

The total pressure inside the bubble at depth h_1 is (P is atmospheric pressure)

$$= (P + h_1\rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is

$$= (P + h_2\rho g) + \frac{2T}{r_2} = P_2$$

Now, according to Boyle's Law

$$P_1V_1 = P_2V_2 \quad \text{where, } V_1 = \frac{4}{3}\pi r_1^3, \quad \text{and} \quad V_2 = \frac{4}{3}\pi r_2^3$$

Hence we get

$$\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^3$$

or, $$\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]r_2^3$$

Given that: $h_1 = 100$ cm, $r_1 = 0.1$ mm $= 0.01$ cm, $r_2 = 0.126$ mm $= 0.0126$ cm, $T = 567$ dyne/cm, $P = 76$ cm of mercury. Substituting all the values, we get

$$h_2 = 9.48\ \text{cm}.$$

Pressure Inside a Charged Bubble

Consider a charged bubble of radius 'R', surface tension 'T' and surface charge density σ.

The total surface tension force for each surface (inner and outer) is $T(2\pi R)$ for a total of $2T(2\pi R)$.

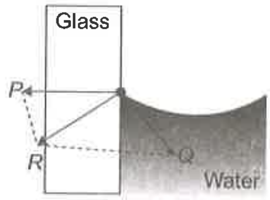
Force due to inside pressure (P_{in}) is $P_{in}\pi R^2$ and due to outside pressure (P_0) is $P_0\pi R^2$

$$2T(2\pi R) + P_0\pi R^2 = P_{in}\pi R^2 + \frac{\sigma^2}{2\varepsilon_0}\pi R^2$$

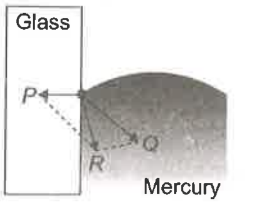
$$P_{in} = P_0 + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}.$$

CONTACT ANGLE AND SHAPE OF LIQUID SURFACE

The surface of a liquid when meets a solid, such as the wall of a container, it usually curves up or down near the solid surface. The angle which the tangent to the is called the contact angle. The curved liquid surface at the pt. of surface of the liquid is called meniscus. The shape of the meniscus contact of liquid surface with (convex or concave) is determined by the relative strengths of solid cohesive and adhesive forces surface with the solid surface inside the liquid.



When the adhesive force (P) between solid and liquid molecules is more than the cohesive force (Q) between liquid-liquid molecules (as with water and glass), shape of the meniscus is concave and the angle of contact θ is less than 90°. In this case the liquid wets or adheres to the solid surface. The resultant (R) of P and Q passes through the solid.



On the other hand when $P < Q$ (as with glass and mercury), shape of the meniscus is convex and the angle of contact $\theta > 90°$. The resultant (R) of P and Q in this case passes through the liquid.

Let us now see why the liquid surface bends near the contact with a solid. A liquid in equilibrium can not sustain tangential stress. The resultant force on any small part of the surface layer must be perpendicular to the surface at that point. Basically three forces are acting on a small part of the liquid surface near its contact with solid. These forces are,

(i) P, attraction due to the molecule of the solid surface near it i.e., adhesive force which acts outwards at right angle to the wall of tube.

(ii) Q, attraction due to liquid molecules near this part and i.e., cohesive force which acts at an angle of 45° to the vertical.

We have considered very small part, so weight of that part can be ignored for better understanding. As we have seen in the last figures, to make the resultant (R) of P and Q perpendicular to the liquid surface the surface becomes curved (convex or concave).

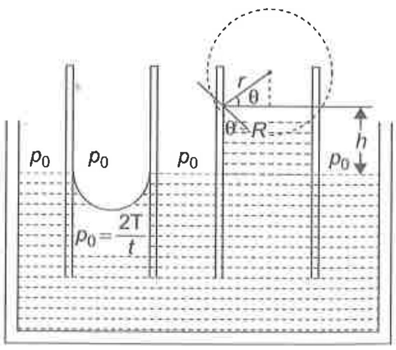
Note

The angle of contact between water and clean glass is zero.

CAPILLARY RISE

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called **capillarity**.

In order to calculate the height to which a liquid will rise in a capillary, consider a glass capillary of radius R dipped in water as shown in Figure. As the meniscus is concave and nearly spherical, the pressure below the meniscus will be $[p_0 - (2T/r)]$ with p_0 as atmospheric pressure and r as radius of meniscus. Now as liquid flows from higher to lower pressure and at same level in a liquid pressure must be same (this is because a liquid cannot sustain tangential stress), so the liquid will ascends in the capillary till hydrostatic pressure of the liquid compensates for the decrease in pressure. i.e.,



$$p_0 = \left[p_0 - \frac{2T}{r}\right] + h\rho g \quad \text{or} \quad h = \frac{2T}{r\rho g}. \qquad (1)$$

But from figure shown it is clear that radius of meniscus r is related to the radius of capillary through the relation

$$\left(\frac{R}{r}\right) = \cos\theta, \text{ i.e., } r = \frac{R}{\cos\theta} \qquad (2)$$

where θ is the angle of contact. *So substituting the value of from Equation (2) in (1), we get

$$h = \frac{2T}{r\rho g} = \frac{2T\cos\theta}{R\rho g} \qquad (3)$$

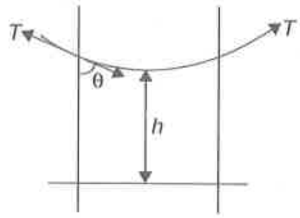
Alternate Method

As it can be seen from figure that $T\sin\theta$ cancels out:

The force due to $T\cos\theta$ balances the weight of liquid ($mg = \rho vg$) vol. of the curve is negligible

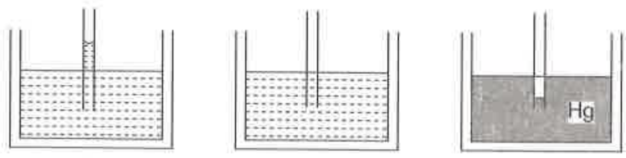
$\therefore$ vol. of liquid in $\pi r^2 h$

$$T\cos\theta = 2\pi r = \pi r^2 hg \quad \Rightarrow \quad h = \frac{2T\cos\theta}{r\rho g}$$



This is the desired result and from this it is clear that:

1. The capillarity depends on the nature of liquid and solid both, i.e., on T, ρ, θ and R. If $\theta > 90°$, i.e., meniscus is convex, h will be negative, i.e., the liquid will descends in the capillary as actually happens in case of mercury in a



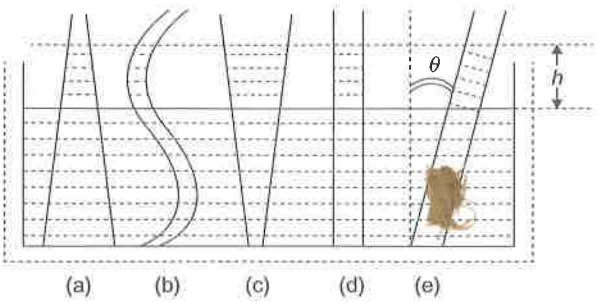
(A) $\theta < 90°$; $h = +\text{ve}$ (B) $\theta = 90°$; $h = 0$ (C) $\theta > 90°$; $h = -\text{ve}$

glass tube. However, if $\theta = 90°$, i.e., meniscus is plane, $h = 0$ and so no capillarity.

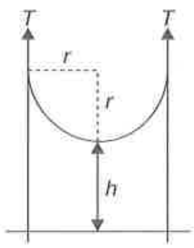
2. For a given liquid and solid at a given place as ρ, T, θ and g are constant, (figure shown) hr = constant

$\therefore$ lesser the radius of capillary greater will be the rise and vice-versa. (figure shown)

3. Here it is important to note that in equilibrium the height h is independent of the shape of capillary if the radius of meniscus remains the same. This is why the vertical height h of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same and also the vertical height of the liquid in a capillary does not change, when it is inclined to the vertical. (figure shown)



4. Capillarity has large number of applications in our daily life, e.g.,
   (a) The oil in the wick of a lamp rise due to capillary action of threads in the wick.
   (b) Action of towel in soaking up moisture from the body is due to capillary action of cotton in the towel.
   (c) Water is retained in a piece of sponge on account of capillarity.
   (d) A blotting paper soaks ink by capillary action of the pores in the blotting paper.
   (e) The root-hairs of plants drawn water from the soil through capillary action.
5. In Case of glass and water $\theta = 0'$ here force due to surface tension balances the weight of the liquid $(r \times v \times g)$



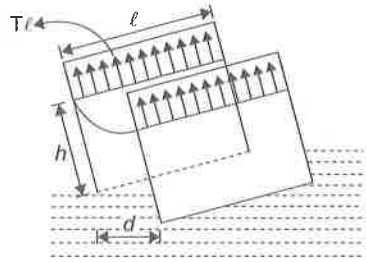
volume of the liquid $= \pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3$

where $\pi r^3 - \frac{2}{3}\pi r^3$ is the volume of the curve which is not negligible in this case

$$\therefore \quad T \cdot 2\pi r = \rho(\pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3)g$$

$$2T = rh\rho g + \frac{1}{3}\pi r^2 \rho g$$

6. If two parallel plates with the spacing 'd' are placed in water reservoir, then height or rise.

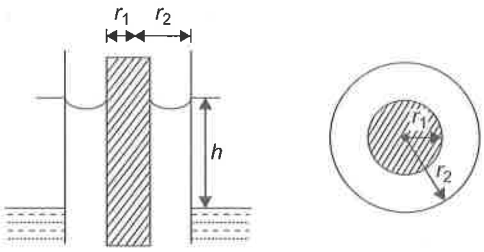


$$2T\ell = \rho \ell h d g \quad \Rightarrow \quad h = \frac{2T}{\rho d g}$$

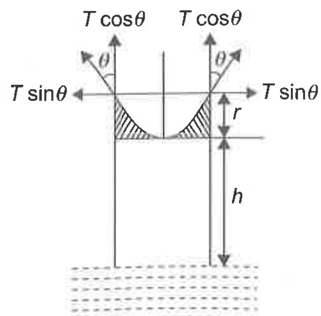
7. If two concentric tubes of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise?

$$\Rightarrow \quad T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g$$

$$h = \frac{2T}{(r_2 - r_1)\rho g}$$



8. If weight of the liquid in the meniscus is to be consider:

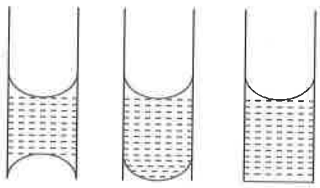


$$T\cos\theta \times 2\pi r = [\pi r^2 h + \frac{1}{3}\pi r^2 \times r]\rho g$$

$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$

9. When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = \frac{2T}{R_1}$, where R_1 = radius of curvature of upper meniscus.

The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards. If $p_1 > p_2$ i.e., resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (figure a). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$



(a) (b) (c)

If $p_1 < p_2$ i.e., resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (figure b).

The radius of lower meniscus can be given by

$$\frac{2T}{R_2} = p_2 - p_1.$$

If $p_1 = p_2$, then is no resulting pressure. then,

$$p_1 - p_2 = \frac{2T}{R_2} = 0 \quad \text{or,} \quad R_2 = \infty$$

i.e., lower surface will be FLAT (figure c).

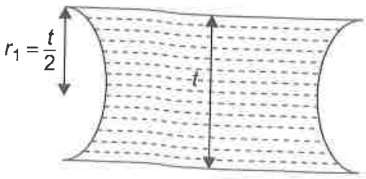
SOLVED EXAMPLES

EXAMPLE 5

A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to separate out the two glass plates in newton.

SOLUTION

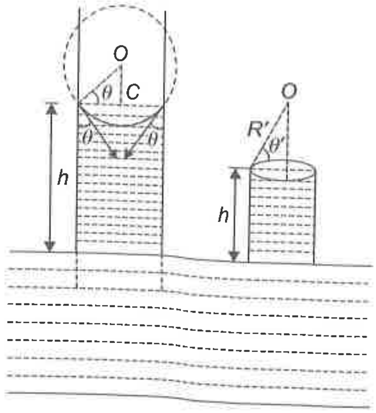
Pressure inside the film is less than outside by an amount, $P = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$, where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = \frac{t}{2}$ and $r_2 = \infty$, then the force required to separate the two glass plates, between which a liquid film is enclosed (figure) is, $F = P \times A = \frac{2AT}{t}$, where t is the thickness of the film, A = area of film.



$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V}$$

$$= \frac{2 \times (40 \times 10^{-4})^2 \times (70 \times 10^{-3})}{0.05 \times 10^{-6}} = 45 \text{ N}$$

Capillary Rise in a Tube of Insufficient Height



We know, the height through which a liquid rises in the capillary tube of radius r is given by

$$\therefore \quad h = \frac{2T}{R\rho g} \quad \text{or} \quad hR = \frac{2T}{\rho g} = \text{constant.}$$

When the capillary tube is cut an its length is less then h (i.e., h'), then the liquid rises upto the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus increases and it becomes more flat so that $hR = h'R'$ = Constant. Hence the liquid does not overflow.

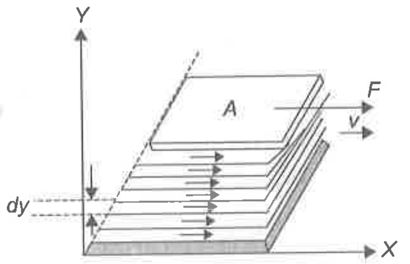
$$\text{If } h' < h \text{ then } R' > R \quad \text{or} \quad \frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$$

$$\Rightarrow \quad \cos\theta' < \cos\theta \quad \Rightarrow \quad \theta' > \theta.$$

VISCOSITY AND NEWTON'S LAW OF VISCOUS FORCE

In case of steady flow of a fluid when a layer of fluid slips or tends to slip on adjacent layer in contact, the two layers exert tangential force on each other which tries to destroy the relative motion between them. The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion *viscous force.* A briskly stirred fluid comes to rest after a short while because of viscosity.

As a result of large number of experiments Newton found that viscous force F acting on any layer of a fluid is directly proportional to its area A and to the velocity gradient $\frac{dv}{dy}$ * at the layer i.e.,



$$F \propto A\frac{dv}{dy} \quad \text{or} \quad F = -\eta A\frac{dv}{dy} \qquad (1)$$

when η is a constant called coefficient of viscosity or simply viscosity of the fluid. The negative sign shows that viscous force on a liquid layer acts in a direction opposite to the relative velocity of flow of fluid. The Eq. (1) is known as Newton's law of viscous force. Here y is taken from the layer of which velocity is zero.

Regarding viscosity of fluid it is worth noting that:

1. It depends only on the nature of fluid and is independent of area considered or velocity gradient.
2. Its dimensions are $[ML^{-1}T^{-1}]$ and SI unit poiseuille (PI) while CGS unit dyne-s/cm^2 called poise (P) with 1 Pl = 10 poise.
3. Viscosity of liquids is much greater (say about 100 times more) than that of gases i.e., $\eta_L > \eta_G$.

SOLVED EXAMPLES

EXAMPLE 6

A boat of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential water in contact with the bed is stationary, find the tangential force needed to keep the boat moving with same velocity. Viscosity of water is 0.01 poise.

SOLUTION

As velocity changes from 2 m/s at the surface to zero at the bed which is at a depth of 1 m. Velocity gradient

$$= \frac{dv}{dy} = \frac{2-0}{1} = 2s^{-1}$$

Now from Newton's law of viscous force,

$$|F| = \eta A\frac{dv}{dy} = (10^{-2} \times 10^{-1}) \times 10 \times 2 = 0.02\text{ N}$$

EXAMPLE 7

The velocity of water in a river is 18 km/hr at the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The viscosity of water is 10^{-3} poiseuuille.

SOLUTION

As velocity at the bottom of the river will be zero, velocity gradient

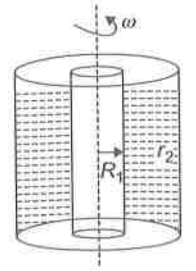
$$\frac{dv}{dy} = \frac{18 \times 10^3}{60 \times 60 \times 5} = 1s^{-1}$$

Now as the viscous force $F = \eta A\left(\frac{dv}{dy}\right)$ is tangential to the area,

$$\text{Shear stress} = \frac{F_{11}}{A} = \eta\frac{dv}{dy} = 10^{-3} \times 1 = 1 \times 10^{-3}\text{ N/m}^2$$

EXAMPLE 8

A cylinder of mass radius r_1 and length ℓ is kept inside another cylinder of radius r_2 and length ℓ. The space between them is filled with a liquid of viscosity η. The inner cylinder starts rotating with angular velocity ω while the other cylinder is at rest. Find time when inner cylinder stops.



SOLUTION

Viscous force $F = -\eta A \dfrac{dv}{dy}$

$$= -\eta 2\pi r_1 \ell \frac{\omega r_1^2}{r_2 - r_1} = -\eta 2\pi \ell \frac{\omega r_1^2}{r_2 - r_1}$$

$$\tau = |\vec{F} \times \vec{r}_1| = \vec{F} r_1 \sin 90° = F r_1 = -\eta 2\pi \ell \frac{\omega r_1^3}{r_2 - r_1} \quad (1)$$

$$\tau = I\alpha = \frac{M r_1^2}{2}\frac{d\omega}{dt} = -\eta 2\pi \ell \frac{\omega r_1^3}{r_2 - r_1}$$

from eq. (1)

$$\frac{a 4\pi \eta r_1 \ell}{M(r_2 - r_1)} \int_0^t dt = -\int_\omega^0 \frac{d\omega}{\omega}$$

$$\frac{a 4\pi \eta r_1 \ell}{M(r_2 - r_1)} t = \ell n \omega$$

STOKES LAW

When a body moves through a fluid, the fluid in contact with the body is dragged with it. This establishes relative motion in fluid layers near the body, due to which viscous force starts operating. The fluid exerts viscous force on the body to oppose its motion. The magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the fluid. Stokes established that if a sphere of radius r moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

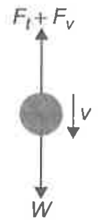
$$F = 6\pi\eta r v$$

TERMINAL VELOCITY (V_T)

Consider a small sphere falling from rest through a large column of viscous fluid. The forces acting on the sphere are,

(i) Weight W of the sphere acting vertically downwards
(ii) Upthrust F_t acting vertically upwards
(iii) Viscous force F_v acting vertically upwards, i.e., in a direction opposite to velocity of the sphere.

Initially, $F_v = 0$
and $W > F_t$



and the sphere accelerates downwards. As the velocity of the sphere increases, F_v increases, eventually a stage is reached when

$$W = F_t + F_v$$

After this net force on the sphere is zero and it moves downwards with a constant velocity called terminal velocity (v_T).

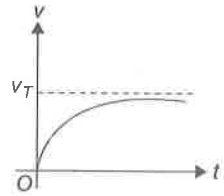
Substituting proper values in Equation (1) we have,

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_T$$

Here, ρ = density of sphere, σ = density of fluid and η = coefficient of viscosity of fluid

From Equation (2), we get $v_T = \dfrac{2}{9}\dfrac{r^2(\rho - \sigma)g}{\eta}$

Figure shows the variation of the velocity v of the sphere with time.



Notes

From the above expression we can see that terminal velocity of a spherical body is directly proportional to the difference in the densities of the body and the fluid ($\rho - \sigma$). If the density of fluid is greater than that of body (i.e., $\sigma > \rho$), the terminal velocity is negative. This means that the body instead of falling, moves upward. This is why air bubbles rise up in water.

SOLVED EXAMPLES

EXAMPLE 9

Two spherical raindrops of equal size are falling vertically through air with a terminal velocity of 1 m/s. What would be the terminal speed if these two drops were to coalesce to form a large spherical drop?

SOLUTION

$$v_T \propto r^2$$

Let r be the radius of small rain drops and R the radius of large drop. Equating the volumes, we have

$$\frac{4}{3}\pi R^2 = 2\left(\frac{4}{3}\pi r^3\right)$$

$$\therefore \quad R = (2)^{1/3} \cdot r \quad \text{or} \quad \frac{R}{r} = (2)^{1/3}$$

$$\therefore \quad \frac{v_{T'}}{v_{T'}} = \left(\frac{R}{r}\right)^2 = (2)^{2/3}$$

$$\therefore \quad v_{T'} = (2)^{2/3} v_T = (2)^{2/3}(1.0)\text{ m/s} = 1.587\text{ m/s} \quad \textbf{Ans.}$$

EXERCISES

JEE Main

1. There is a horizontal film of soap solution. On it a thread is placed in the form of a loop. The film is pierced inside the loop and the thread becomes a circular loop of radius R. If the surface tension of the loop be T, then what will be the tension in the thread?

(A) $\dfrac{\pi R^2}{T}$ (B) $\pi R^2 T$

(C) $2\pi RT$ (D) $2RT$

2. A container, whose bottom has round holes with diameter 0.1 mm is filled with water. The maximum height in cm upto which water can be filled without leakage will be what?

Surface tension = 75×10^{-3} N/m and $g = 10$ m/s^2:

(A) 20 cm (B) 40 cm

(C) 30 cm (D) 60 cm

3. If two soap bubbles of different radii are connected by a tube:

(A) air flows from the bigger bubble to the smaller bubble till the sizes become equal

(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

(C) air flows from the smaller bubble to the bigger

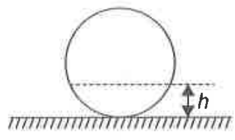
(D) there is no flow of air.

4. Two soap bubbles with radii r and $(r_1 > r_2)$ come in contact. Their common surface has radius of curvature r.

(A) $r = \dfrac{r_1 + r_2}{2}$ (B) $r = \dfrac{r_1 r_2}{r_1 - r_2}$

(C) $r = \dfrac{r_1 r_2}{r_1 + r_2}$ (D) $r = \sqrt{r_1 r_2}$

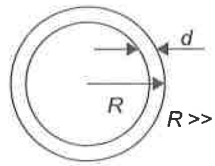
5. A liquid is filled in a spherical container of radius R till a height h. At this positions the liquid surface at the edges is also horizontal. The contact angle is



(A) 0 (B) $\cos^{-1}\left(\dfrac{R-h}{R}\right)$

(C) $\cos^{-1}\left(\dfrac{h-R}{R}\right)$ (D) $\sin^{-1}\left(\dfrac{R-h}{R}\right)$

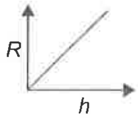
6. A soap bubble has radius R and thickness $d(<< R)$ as shown. It collapses into a spherical drop. The ratio of excess pressure in the drop to the excess pressure inside the bubble is.

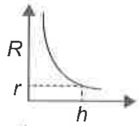


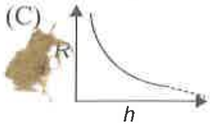
(A) $\left(\dfrac{R}{3d}\right)^{\frac{1}{3}}$ (B) $\left(\dfrac{R}{6d}\right)^{\frac{1}{3}}$

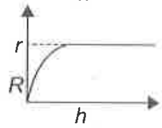
(C) $\left(\dfrac{R}{24d}\right)^{\frac{1}{3}}$ (D) None

7. A long capillary tube of radius 'r' is initially just vertically completely immerged inside a liquid of angle of contact 0°. If the tube is slowly raised then relation between radius of curvature of meniscus inside the capillary tube and displacement (h) of tube can be represented by

(A)


(B)


(C)


(D)


8. A Newtonian fluid fills the clearance between a shaft and a sleeve. When a force of 800 N is applied to the shaft, parallel to the sleeve, the shaft attains a speed of 1.5 cm/sec. If a force of 2.4 kN is applied instead, the shaft would move with a speed of

(A) 1.5 cm/sec (B) 13.5 cm/sec
(C) 4.5 cm/sec (D) None

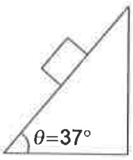
9. A solid metallic sphere of radius r is allowed to fall freely through air. If the frictional resistance due to air is proportional to the cross-sectional area and to the square of the velocity, then the terminal velocity of the sphere is proportional to which of the following?

(A) r^2 (B) r
(C) $r^{3/2}$ (D) $r^{1/2}$

10. Two drops of same radius are falling through air with steady velocity of v cm/s. If the two drops coalesce, what would be the terminal velocity?

(A) $4v$ (B) $(4)^{1/3}v$
(C) $2v$ (D) $64v$

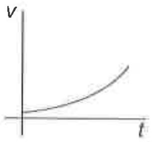
11. A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of viscous fluid of thickness 't' between the plane and the block. Then the coefficient of viscosity of the thin film will be:

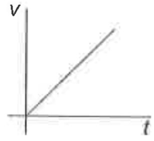


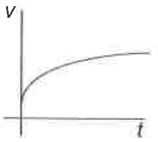
(A) $\dfrac{3\rho agt}{5v}$ (B) $\dfrac{4\rho agt}{5v}$

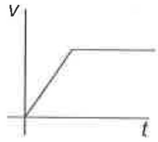
(C) $\dfrac{\rho agt}{v}$ (D) none of these

12. Which of the following graphs best represents the motion of a raindrop?

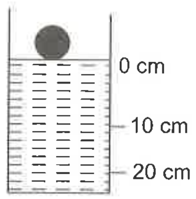
(A)


(B)


(C)


(D)


13. A spherical ball of density ρ and radius 0.003 m is dropped into a tube containing a viscous fluid filled up to the 0 cm mark as shown in the figure. Viscosity of the fluid = 1.260 $N \cdot m^{-2}$ and its density $\rho_L = \rho/2 = 1260$ $kg \cdot m^{-3}$. Assume the ball reaches a terminal speed by the 10 cm mark. The time taken by the ball to traverse the distance between the 10 cm and 20 cm mark is.



(A) 500 μs (B) 50 ms
(C) 0.5 s (D) 5 s

(g = acceleration due to gravity = 10 ms^{-2})

14. A sphere is dropped under gravity through a fluid of viscosity η. If the average acceleration is half of the initial acceleration, the time to attain the terminal velocity is (ρ = density of sphere, r = radius)

(A) $\dfrac{4\rho r^2}{9\eta}$ (B) $\dfrac{9\rho r^2}{4\eta}$

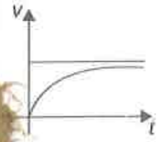
(C) $\dfrac{4\rho r}{9\eta}$ (D) $\dfrac{9\rho r}{4\eta}$

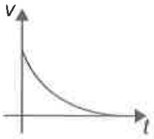
15. A ball of mass m and radius r is gently released in a viscous liquid. The mass of the liquid displaced by it is m' such that $m > m'$. The terminal velocity is proportional to

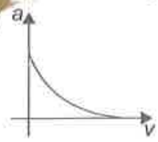
(A) $\dfrac{m - m'}{r}$ (B) $\dfrac{m + m'}{r}$

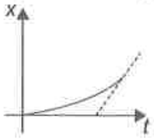
(C) $\dfrac{(m + m')}{r^2}$ (D) $(m - m')r^2$

16. Which of the following is the incorrect graph for a sphere falling in a viscous liquid?
(Given at $t = 0$, velocity $v = 0$ and displacement $x = 0$.)

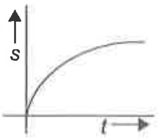
(A) 

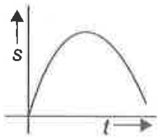
(B) 

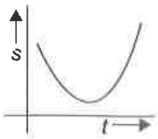
(C) 

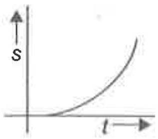
(D) 

17. The displacement of a ball falling from rest in a viscous medium is plotted against time. Choose a possible option.

(A) 

(B) 

(C) 

(D) 

18. There is a 1 mm thick layer of glycerine between a flat plate of area 100 cm^2 and a big fixed plate. If the coefficient of viscosity of glycerine is 1.0 kg/m-s then how much force is required to move the plate with a velocity of 7 cm/s?

(A) 3.5 N (B) 0.7 N

(C) 1.4 N (D) None

JEE Advanced

1. A film of water is formed between two straight parallel wires each 10 cm long and at separation 0.5 cm. Calculate the work required to increase 1 mm distance between wires. Surface tension of water = 72×10^{-3} Nm^{-1}.

2. Water rises in a capillary upto a certain height such that the upward force of surface tension balances the force of 75×10^{-4} N due to weight of the liquid. If the surface tension of water is 6×10^{-2} Nm^{-1}, what must be the internal circumference of the capillary?

3. A ring cut from a platinum tube, 8.5 cm internal diameter and 8.7 cm external diameter, is supported horizontally from the pair of a balance so that it comes in contact with the water in a vessel. If an extra weight of 3.97 g is required to pull it away from water, calculate the surface tension of water.

4. There is a soap bubble of radius 2.4×10^{-4} m in air cylinder which is originally at the pressure of 10^5 Nm^{-2}. The air in the cylinder is now compressed isothermally until the radius of the bubble is halved. Calculate now the pressure of air in the cylinder. The surface tension of the soap is 0.08 Nm^{-1}.

5. Two separate air bubbles (radii 0.002 m and 0.004 m) formed of the same liquid (surface tension 0.07 N/m) come together to form a double bubble. Find the radius and the sense of curvature of the internal film surface common to both the bubbles.

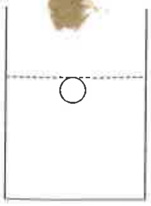
6. Two soap bubbles of radii a and b combine to form a single bubble of radius c. If the external pressure is P, then find the surface tension of soap solution.

7. A long capillary tube of radius 2 mm open at both ends is filled with water and placed vertically. What will be the height of the column of water left in the capillary? The thickness of the capillary walls is negligible. Surface tension of water 73.5 $\times 10^{-3}$ Nm^{-1}.

8. The limbs of a manometer consist of uniform capillary tubes of radii 1.44×10^{-3} m and 7.2×10^{-4} m. Find out the correct pressure difference if the level of the liquid (density 10^3 kgm^{-3}, surface tension 72 $\times$ 10^{-3} Nm^{-1}) in the narrower tube stands 0.2 m above that in the broader tube.

9. A glass capillary sealed at the upper end is of length 0.11 m and internal diameter 2×10^{-5} m. The tube is immersed vertically into a liquid of surface tension 5.06×10^{-2} Nm^{-1}. To what length the capillary has to be immersed so that the liquid level inside and outside and capillary becomes the same? What will happen to liquid level inside the capillary if the seal is now broken? Atmospspheric pressure is 1.012 $\times 10^5$ Nm^{-2}.

10. A ball is given velocity v_0 (greater than the terminal velocity v_T) in downward direction inside a highly viscous liquid placed inside a large container. The height of liquid in the container is H. The ball attains the terminal velocity just before striking at the bottom of the container. Draw graph between velocity of the ball and distance moved by the ball before getting terminal velocity.



11. Two arms of a U-tube have unequal diameters d_1 = 1.0 mm and d_2 = 1.0 cm. If water (surface tension 7×10^{-2} N/m) is poured into the tube held in the vertical position, find the difference of level of water in the U-tube. Assume the angle of contact to be zero.

12. A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. The viscosity of water is 9.8×10^{-6} N-s/m^2.

13. An expansible balloon filled with air floats on the surface of a lake with 2/3 of its volume submerged. How deep must it be sunk in the water so that it is just in equilibrium neither sinking further nor rising ? It is assumed that the temperature of the water is constant and that the height of the water barometer is 9 meters.

Previous Years' Questions

JEE Main

1. If two soap bubbles of different radii are connected by a tube **[AIEEE-2004]**
(A) air flows from the smaller bubble to the bigger
(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(C) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(D) there is no flow of air.

2. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is **[AIEEE-2004]**
(A) directly proportional to R but inversely proportional to v.
(B) directly proportional to both radius R and velocity v.
(C) inversely proportional to both radius R and velocity v.
(D) inversely proportional to R but directly proportional to velocity v.

3. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be **[AIEEE-2005]**
(A) 8 cm (B) 10 cm
(C) 4 cm (D) 20 cm

4. If the terminal speed of a sphere of gold (density = 19.5 kgm^{-3}) is 0.2 ms^{-1} in a viscous liquid (density = 1.5kgm^{-3}), find the terminal speed of a sphere of silver (density = 10.5 kg/m^{-3}) of the same size in the same liquid. **[AIEEE-2006]**
(A) 0.4 ms^{-1} (B) 0.133 ms^{-1}
(C) 0.1 ms^{-1} (D) 0.2 ms^{-1}

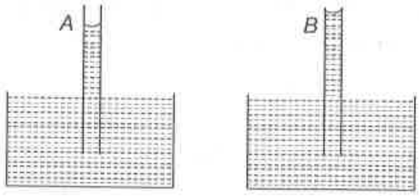
5. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2(\rho_2 < \rho_1)$. [Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{\text{viscous}} = -kv^2 (k > 0)$]. The terminal speed of the ball is **[AIEEE-2008]**

(A) $\sqrt{\dfrac{Vg(\rho_1 - \rho_2)}{k}}$ (B) $\dfrac{Vg\rho_1}{k}$

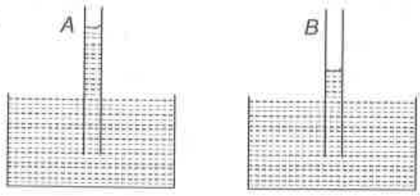
(C) $\sqrt{\dfrac{Vg\rho_1}{k}}$ (D) $\dfrac{Vg(\rho_1 - \rho_2)}{k}$

6. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[AIEEE-2008]**

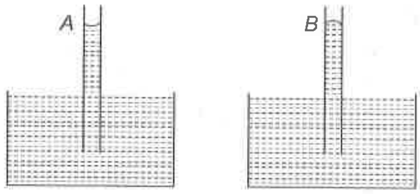
(A)



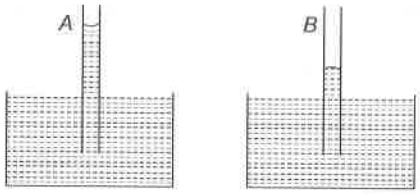
(B)



(C)



(D)



7. Work done in increasing the size of a soap bubble from radius of 3 cm to 5 cm is nearly (surface tension of soap = 0.03 Nm^{-1}) **[AIEEE-2011]**

(A) 0.2 πmJ (B) 2 πmJ
(C) 0.4 πmJ (D) 4 πmJ

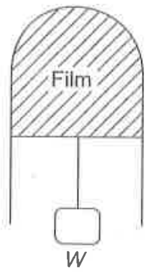
8. If a ball of steel (density $\rho = 7.8\ gcm^{-3}$) attains a terminal velocity of 10 cms^{-1} when falling in a tank of water (coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa-s) then its terminal velocity in glycerine (ρ = 12 gcm-3, n = 13.2 Pa-s) would be nearly **[AIEEE-2011]**

(A) $1.6 \times 10^{-5}\ cms^{-1}$ (B) $6.25 \times 10^{-4}\ cms^{-1}$
(C) $6.45 \times 10^{-4}\ cms^{-1}$ (D) $1.5 \times 10^{-5}\ cms^{-1}$

9. Two mercury drops (each of radius r) merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is **[AIEEE-2011]**

(A) $2^{5/3}\pi r^2 T$ (B) $4\pi r^2 T$
(C) $2\pi r^2 T$ (D) $2^{8/3}\pi r^2 T$

10. A thin liquid film formed between a U-shaped wire and a light supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is **[AIEEE-2012]**



(A) 0.0125 Nm^{-1} (B) 0.1 Nm^{-1}
(C) 0.05 Nm^{-1} (D) 0.025 Nm^{-1}

11. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for that to be possible? The surface tension is T, density of liquid is r and L is its latent heat of vaporization. **[JEE Main-2013]**

(A) $\dfrac{T}{\rho L}$ (B) $\dfrac{2T}{\rho L}$
(C) $\dfrac{\rho T}{T}$ (D) $\sqrt{\dfrac{T}{\rho T}}$

JEE Advanced

1. When an air bubble rises from the bottom of a deep lake to a point just below the water surface, the pressure of air inside the bubble

(A) is greater than the pressure outside it
(B) is less than the pressure outside it
(C) increases as the bubble moves up
(D) decreases as the bubble moves up

2. **Assertion:** A helium filled balloon does not rise indefinitely in air but halts after a certain height.
Reason: Viscosity opposes the motion of balloon.
Choose any one of the following four responses:

(A) if both (A) and (R) are true and (R) is the correct explanation of (A)
(B) if both (A) and (R) are true but (R) is not correct explanation of (A)
(C) if (A) is true but (R) is false
(D) if (A) is false and (R) is true

3. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\dfrac{81\pi}{7} \times 10^5\ Vm^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}\ ms^{-1}$. Given $g = 9.8\ ms^{-2}$, viscosity of the air = $1.8 \times 10^{-5}\ Ns\ m^{-2}$ and the density of oil = 900 kg m^{-5}, the magnitude of q is: **[JEE-2010]**

(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C
(C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

Common Data for Questions 4–6: When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper. **[JEE-2010]**

4. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is

(A) $2\pi rT$ (B) $2\pi RT$

(C) $\dfrac{2\pi r^2 T}{R}$ (D) $\dfrac{2\pi R^2 T}{r}$

5. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately.

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m

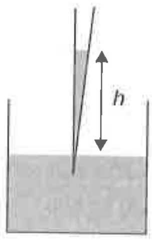
(C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

6. After the drop detaches, its surface energy is:

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J

(C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

7. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity) **[JEE-2014]**



(A) $\dfrac{2s}{b\rho g}\cos(\theta - \alpha)$ (B) $\dfrac{2s}{b\rho g}\cos(\theta + \alpha)$

(C) $\dfrac{2s}{b\rho g}\cos\left(\theta + \dfrac{\alpha}{2}\right)$ (D) $\dfrac{2s}{b\rho g}\cos\left(\theta + \dfrac{\alpha}{2}\right)$

ANSWER KEYS

Exercises

JEE Main

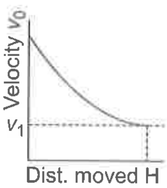
1. D	**2.** C	**3.** C	**4.** B	**5.** B	**6.** C	**7.** B	**8.** C	**9.** D	**10.** B
11. A	**12.** C	**13.** D	**14.** A	**15.** A	**16.** C	**17.** D	**18.** B		

JEE Advanced

1. 1.44×10^{-5} J **2.** 1.25×10^{-2} m **3.** 7.2×10^{-2} Nm^{-1} **4.** 8.08×10^5 Nm^{-2}

5. 0.004 m (common film will be concave towards the centre of the smaller bubble)

6. $\sigma = \dfrac{P(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$ **7.** $h = 1.5$ cm **8.** 1860 Nm^{-2} **9.** $\ell = 1$ cm

10. 

11. 2.5 cm **12.** 20.4 m **13.** 4.5 m

Previous Years' Questions

JEE Main

1. A **2.** B **3.** D **4.** C **5.** A **6.** B **7.** C **8.** B **9.** D **10.** D
11. B

JEE Advanced

1. A, D **2.** B **3.** D **4.** C **5.** A **6.** B **7.** D

Chapter 5

Errors

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity. (1) random and (2) systematic errors.

1. Random errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of random error can be some what reduced by taking the average of measured values. Random errors have no fixed sign or size.
2. Systematic error occur due to error in the procedure, or miscalibration of the instrument etc. Such errors have same size and sign for all measurements. Such errors can be determined.

A measurement with relatively small random error is said to have high precision. A measurement with small random error and small systematic error is said to have high accuracy.

The experimental error [uncertainty] can be expressed in several standard ways.

Error limits $Q \pm \Delta Q$ is the measured quantity and ΔQ is the magnitude of its limit of error. This expresses the experimenter's judgement that the 'true' value of Q lies between $Q - \Delta Q$ and $Q + \Delta Q$. This entire interval within which the measurement lies is called the range of error. Random errors are expressed in this form.

Absolute Error

Error may be expressed as absolute measures, giving the size of the error in a quantity in the same units as the quantity itself.

Least Count Error: If the instrument has known least count, the absolute error is taken to be half of the least count unless otherwise stated.

Relative (or Fractional) Error

Error may be expressed as relative measures, giving the ratio of the quantity's error to the quantity itself. In general,

$$\text{relative error} = \frac{\text{absolute error in a measurement}}{\text{size of the measurement}}$$

We should know the error in the measurement because these errors propagate through the calculations to produce errors in results.

A. **Systematic errors:** They have a known sign. The systematic error is removed before beginning calculations bench error and zero error are examples of systematic error.
B. **Random error:** They have unknown sign. Thus they are represented in the form $A \pm a$.

   Here we are only concerned with limits of error. We must assume a 'worst-case' combination. In the case of substraction, $A - B$, the worst-case deviation of the answer occurs when the errors are either $+a$ and $-b$ or $-a$ and $+b$. In either case, the maximum error will be $(a + b)$.

For example in the experiment on finding the focal length of a convex lens, the object distance (u) is found by subtracting the positions of the object needle and the lens. If the optical bench has a least count of 1 mrn, the error in each position will be 0.5 mm. So, the error in the value of u will be 1 mm.

1. ***Addition and subtraction rule:*** The absolute random errors add.

   Thus if $R = A + B, \quad r = a + b$
   and if $R = A - B, \quad r = a + b$

2. ***Product and quotient rule:*** The relative random errors add.

   Thus if $R = AB, \qquad \frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

   and if $R = \frac{A}{B}$, then also $\frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

3. **Power rule:** When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q. This also holds for negative powers.

If $R = Q^P$, $\quad \frac{r}{R} = P \times \frac{q}{Q}$

4. The quotient rule is not applicable if the numerator and denominator are dependent on each other. e.g., if $R = \frac{XY}{X+Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows $\frac{1}{R} = \frac{1}{X} + \frac{1}{Y}$. Differentiating both the sides, we get

$$-\frac{dR}{R^2} = -\frac{dX}{X^2} - \frac{dY}{Y^2}. \quad \text{Thus } \frac{r}{R^2} = \frac{x}{X^2} + \frac{y}{Y^2}$$

EXAMPLE 1

A student finds the constant acceleration of a slowly moving object with a stopwatch. The equation used is $S = \frac{1}{2}AT^2$. The time is measured with a stopwatch, the distance, S with a meter stick. What is the acceleration and its estimated error?

$$S = 2 \pm 0.005 \text{ m}$$

$$T = 4.2 \pm 0.2 \text{ s}$$

SOLUTION

We use capital letters for quantities, lower case for errors. Solve the equation for the result, a.

$A = \frac{2S}{T^2}$. Its random-error equation is $\frac{a}{A} = 2\frac{t}{T} + \frac{s}{S}$

Thus $A = 0.23 \pm 0.02 \text{ m/s}^2$.

SIGNIFICANT DIGITS

Significant figures are digits that are statistically significant. There are two kinds of values in science:

1. Measured Values
2. Computed Values

The way that we identify the proper number of significant figures in science are different for these two types.

Measured Values

Identifying a measured value with the correct number of significant digits requires that the instrument's calibration be taken into consideration. The last significant digit in a measured value will be the first estimated position. For example, a metric ruler is calibrated with numbered calibrations equal to 1 cm. In addition, there will be ten unnumbered calibration marks between each numbered position. (each equal to 0.1 cm). Then one could with a little practice estimate between each of those marking. (each equal to 0.05 cm). That first estimated position would be the last significant digit reported in the measured value. Let's say that we were measuring the length of a tube, and it extended past the fourteenth numbered calibration half way between the third and fourth unnumbered mark. The metric ruler was a meter stick with 100 numbered calibrations. The reported measured length would be 14.35 cm. Here the total number of significant digits will be 4.

Computed Value

The other type of value is a computed value. The proper number of significant figures that a computed value should have is decided by a set of conventional rules. However before we get to those rules for computed values we have to consider how to determine how many significant digits are indicated in the numbers being used in the math computation.

A. Rules for determining the number of significant digits in number with indicated decimals.

1. All non-zero digits (1–9) are to be counted as significant.
2. Zeros that have any non-zero digits anywhere to the LEFT of them are considered significant zeros.
3. All other zeros not covered in rule (2) above are NOT be considered significant digits.

For example: 0.0040000

The 4 is obviously to be counted significant (Rule-1), but what about the zeros? The first three zeros would not be considered significant since they have no non-zero digits anywhere to their left (Rule-3). The last four zeros would all be considered significant since each of them has the non-zero digit 4 to their left (Rule-2). Therefore the number has a total of five significant digits.

Here is another example: 120.00420

The digit 1, 2, 4 and 2 are all considered significant (Rule-1). All zeros are considered significant since they have non-zero digits somewhere to their left (Rule-2). So there are a total of eight significant digits. If in the question, we are given a number like 100, we will treat that the number has only one significant digit by convention.

B. Determining the number of significant digits if number is not having an indicated decimal.

The decimal indicated in a number tells us to what position of estimation the number has been indicated.

But what about 1,000,000?

Notice that there is no decimal indicated in the number. In other words, there is an ambiguity concerning the estimated position. This ambiguity can only be clarified by placing the number in exponential notation.

For example: If! write the number above in this manner.

$$1.00 \times 10^6$$

I have indicated that the number has been recorded with three significant digits. On the other hand, if write the same number as: 1.0000×10^6

I have identified the number to have 5 significant digits. Once the number has been expressed in exponential notation form then the digits that appear before the power of ten will all be considered significant.

So for example: 2.0040×10^4 will have five significant digits. Thus means that unit conversion will not change the number of significant digits. Thus 0.000010 km = 1.0 cm = 0.010 m = 1.0×10^{-2} m = 1.0×10^{-5} km.

Rule for expressing proper number of significant digits in an answer from multiplication or division

For multiplication AND division there is the following rule for expressing a computed product or quotient with the proper number of significant digits.

The product or quotient will be reported as having as many significant digits as the number involved in the operation with the least number of significant digits.

For example: $0.000170 \times 100.40 = 0.017068$

The product could be expressed with no more that three significant digits since 0.000170 has only three significant digits, and 100.40 has five. So according to the rule the product answer could only be expressed with three significant digits. Thus the answer should be 0.0171 (after rounding off)

Another example: $2.000 \times 10^4/6.0 \times 10^{-3} = 0.33 \times 10^7$

The answer could be expressed with no more that two significant digits since the least digited number involved in the operation has two significant digits.

Sometimes this would required expressing the answer in exponential notation.

For example: $3.0 \times 800.0 = 2.4 \times 10^3$

The number 3.0 has two significant digits and then number 800.0 has four. The rule states that the answer can have no more than two digits expressed. However the answer as we can all see would be 2400. How do we express the answer 2400 while obeying the rules? The only way is to express the answer in exponential notation so 2400 could be expressed as: 2.4×10^3.

Rule for expressing the correct number of significant digits in an addition or substraction:

The rule for expressing a sum or difference is considerably different than the one for multiplication of division. The sum or difference can be no more precise than the least precise number involved in the mathematical operation. Precision has to do with the number of positions to the RIGHT of the decimal. The more position to the right of the decimal, the more precise the number. So a sum or difference can have ho more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

For example: 160.45 + 6.732 = 167.18 (after rounding off)

The answer could be expressed only to two positions to the right of the decimal, since 160.45 is the least precise.

Another example: 45.621 + 4.3 − 6.41 = 43.5 (after rounding off)

The answer could be expressed only to one position to the right of the decimal, since the number 4.3 is the least precise number (i.e., having only one position to the right of its decimal). Notice we aren't really determining the total number of significant digits in the answer with this rule.

Rules for rounding off digits:

There are a set of conventional rules for rounding off.

1. Determine according to the rule what the last reported digit should be.
2. Consider the digit to the right of the last reported digit.
3. If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.
4. If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.
5. If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

For example if we wish to round off the following number to 3 significant digits: 18.3682
The last reported digits would be the 3. The digit to its right is a 6 which is greater than 5. According to the Rule-4 above, the digit 3 is increased by one and the answer is: 18.4

Another example: Round off 4.565 to three significant digits.

The last reported digit would be the 6. The digit to the right is a 5 followed by nothing. Therefore according to Rule-5 above since the 6 is even it remains so and the answer would be 4.56.

EXPERIMENT

MEASUREMENT OF LENGTH

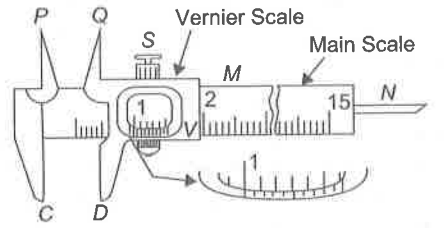
The simplest method measuring the length of a straight line by means of a meter scale. But there exists some limitation in the accuracy of the result:

(i) the dividing line... finite thickness
(ii) naked eye cannot correctly estimate less than 0.5 mm

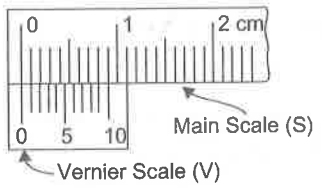
For greater accuracy devices like

(a) Vernier callipers
(b) micrometer scales (screw gauge) are used

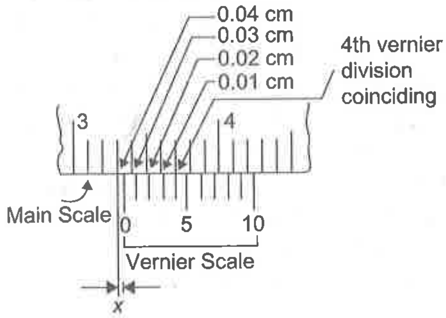
Vernier callipers:



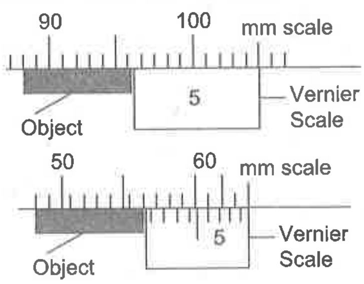
Vernier Callipers



Principle of Vernier



Reading a vernier with 4th division coinciding



It consists of a main scale graduated in cm/mm over whi an auxiliary scale (or Vernier scale) can slide along t length. The division of the Vernier scale being shorter th the divisions of the main scale.

Least count of Vernier Callipers

The least count of Vernier Constant (v.c) is the mi mum value of correct estimation of length without e estimation. If N division of vernier coincides with (N– division of main scale, then

$$N(VS) = (N-1)\,\text{ms} \;\Rightarrow\; 1VS = \frac{N-1}{N}\,\text{ms}$$

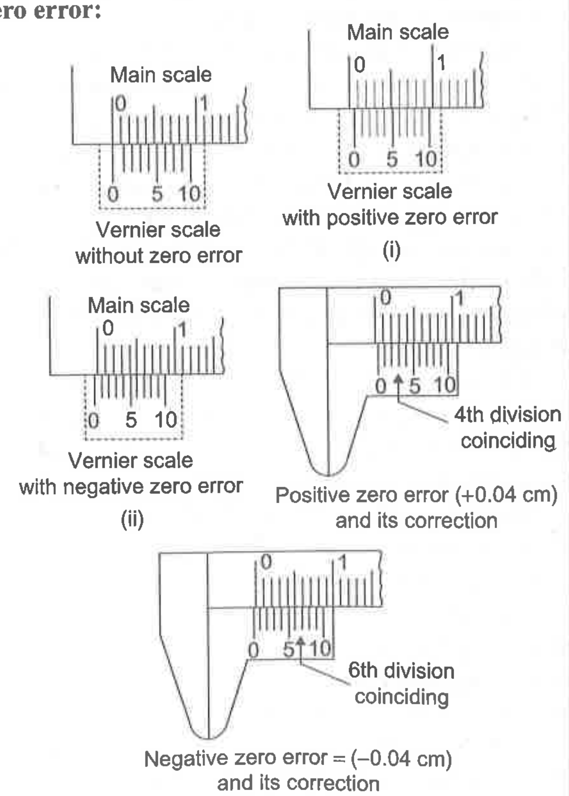
$$\text{Vernier constant} = 1\,\text{ms} - 1\,vs = \left(1 - \frac{N-1}{N}\right)\text{ms} = \frac{1\,\text{ms}}{N}$$

which is equal to the value of the smallest division on t main scale divided by total number of divisions on the v nier scale.

Length as measured by Vernier Callipers

The formula for measuring the length is L = main sc reading + least count of vernier scale × Vernier scale di sion coinciding with a main scale division Main scale rea ing is given by the zeroth division of the vernier scale shown in the figure.

Zero error:



If the zero marking of main scale and vernier callipers do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument.

If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

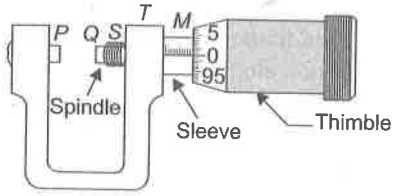
The zero error is always subtracted from the reading to get the corrected value.

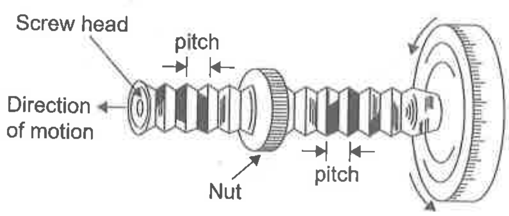
If the zero error is positive, its value is calculated as we take any normal reading. If the zero error negative (the zero of vernier scale lies to the left of the zero of main scale), negative zero error = –[Total no. of vsd – vsd coinciding] × L.C.

Do not try to read the main scale at the point where the lines match best. This has no meaning. Read from the vernier scale instead. Sometimes it is difficult to tell whether the best match of liners is for vernier marks 9, 0 or 1. Make your best estimate, but realize that the final result including the vernier must round off to the result you best estimate, but realize that the final result including the vernier must round off to the result you would choose if there was no vernier. If the mark is close to 3.20 on the main scale and the vernier is 9, the length is 3.19 cm. If the mark is close to 3.2 on the main scale and vernier 1, the length is 3.21 cm.

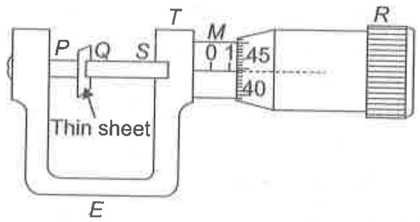
Screw Gauge (or Micrometer Screw)

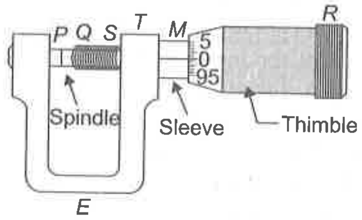
In general vernier callipers can measure accurately upto 0.01 cm and for greater accuracy micrometer screw devices e.g., screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by running it axially.



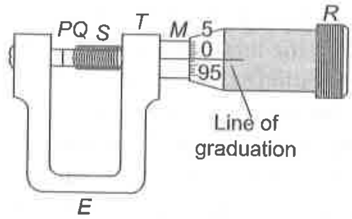


Principle of a micrometer

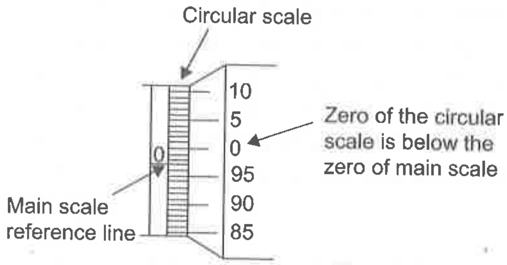


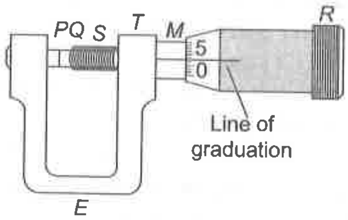


Screw gauge with no zero error

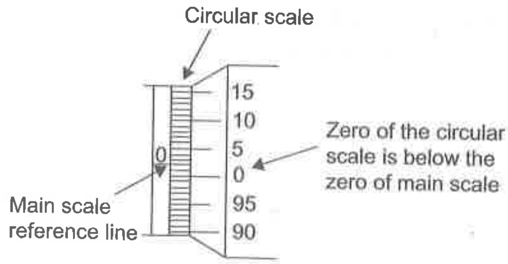


Negative zero error
(3 division error) i.e.,–0.003 cm





Positive zero error
(2 division error) i.e.,+0.002 cm



The instrument is provided with two scales:

(i) The main scale or pitch scale M graduated along the axis of screw.

(ii) The cap-scale or head scale H round the edge of the screw head.

Constants of the Screw Gauge

(a) Pitch: The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for 10 rotation of cap = 5 mm, then pitch = 0.5 mm.

(b) Least count: In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions. Thus in the Afore said Illustration.; if the total cap division is 100, then least count = 0.5 mm/100 = 0.005 mm

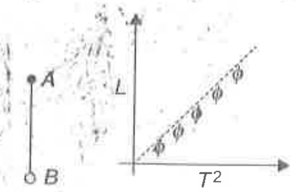
(c) Measurement of length by screw gauge:
$L = n \times$ pitch $+ f \times$ least count, where n = main scale reading and f = caps scale reading

Zero Error: In a perfect instrument the zero of the heat scale coincides with the line of gradiation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line of reference line of the cap lies below the line of graduation and vice versa. The corresponding corrections will be just opposite.

MEASUREMENT OF '*g*' USING A SIMPLE PENDULUM

A small spherical bob is attached to a cotton thread and the combination is suspended from a point A. The length of the thread (L) is read off on a meter scale. A correction is added to L to include the finite size of the bob and the hook. The corrected value of L is used for further calculation.

The bob is displaced slightly to one side and is allowed to oscillate, and the total time taken for 50 complete oscillations is noted on a stop-watch. The time period (T) of a single oscillation is now calculated by division.



Observations are now taken by using different lengths for the cotton thread (L) and pairs of values of L and T are taken. A plot of L v/s T^2, on a graph, is linear.

g is given by $g = 4\pi^2 \dfrac{L}{T^2}$

The major errors in this experiment are

(a) Systematic: Error due to finite amplitude of the pendulum (as the motion is not exactly SHM). This may be corrected for by using the correct numerical estimate for the time period. However the practice is to ensure that the amplitude is small.

(b) Statistical: Errors arising from measurement of length and time.

$$\frac{\delta g}{g} = \frac{\delta L}{L} + 2\left(\frac{\delta T}{T}\right)$$

The contributions to δL, δT are both statistical and systematic. These are reduced by the process of averaging.

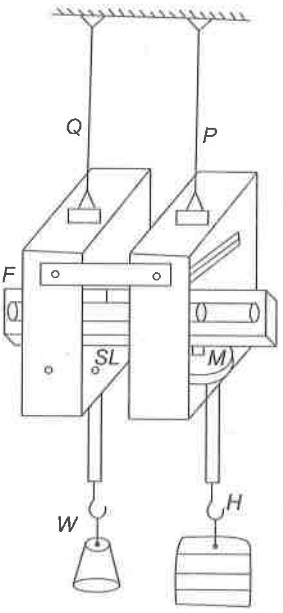
The systematic error in L can be reduced by plotting several values of L vs T^2 and fitting to a straight line.

The slope of this fit gives the correct value of L/T^2

DETERMINATION OF YOUNG'S MODULUS BY SEARLE'S METHOD

The experimental set up consists of two identical wires P and Q of uniform cross section suspended from a fixed rigid support. The free ends of these parallel wires are connected to a frame F as shown in the figure. The length of the wire Q remains fixed while the load L attached to the wire P through the frame F is varied in equal steps so as to produce extension along the length. The extension thus produced is measured with the help of spirit level SL and micrometer screw M attached to the F frame on the the side of experimental wire. On placing the slotted weights on the hanger H upto a permissible value (half of the breaking force) the wire gets extended by small amount and the spirit level gets disturbed from horizontal setting. This increase in length is measured by turning the micrometer screw M upwards so as to restore the balance of the spirit level. If n be the number of turns of the micrometer screw and f be the difference in the cap reading, the increase in length M is obtained by

$$\Delta l = n \times \text{pitch} + f \times \text{least count}$$



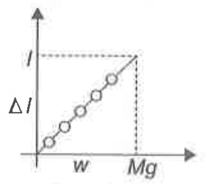
In some situations, the change in length is obtained by vernier arrangement instead of the screw gauge. The load on the hanger is reduced in the same steps and spirit level is restored to horizontal position. The mean of these two observations gives the true increase in length of the wire corresponding to the given value of load. This is to eliminate the effect of hysteresis.

From the data obtained, a graph showing extension (Δl) against the load (W) is plotted which is obtained as a straight line passing through the origin. The slope of the line gives

$$\tan\theta = \frac{l}{W} = \frac{l}{Mg}$$

Now, stress $= \dfrac{Mg}{\pi r^2}$ and strain $= \dfrac{l}{L}$

$$Y = \text{Stress/strain} = \frac{MgL}{\pi r^2 l} = \frac{L}{\pi^2 r \tan\theta}$$



With known values of initial length L, radius r of the experimental wire and tan θ, Young's modulus Y can be calculated.

SPECIFIC HEAT OF A LIQUID USING A CALORIMETER

The principle is to take a known quantity of liquid in an insulated calorimeter and heat it by passing a known current (i) though a heating coil immersed within the liquid for a known length of time (t). The mass of the calorimeter (m_1) and, the combined mass of the calorimeter and the liquid (m_2) are measured. The potential drop across the heating coil is V and the maximum temperature of the liquid is measured to θ_2.

The specific heat of the liquid (S_l) is found by using the relation

$$(m_2 - m_1)\ S_l(\theta_2 - \theta_0) + m_1 S_c(\theta_2 - \theta_0) = i.V.t$$

or $$(m_2 - m_1)S_l + m_1 S_c = i.V.t/(\theta_2 - \theta_0) \qquad (1)$$

Here, θ_0 is the room temperature, while S_c is the specific heat of the material of the calorimeter and the stirrer. If S_c is known, then S_l can be determined.

On the other hand, if S_c is unknown: one can either repeat the experiment with water or a different mass of the liquid and use the two equations to eliminate $m_1 S_c$.

The sources of error in this experiment are errors due to improper connection of the heating coil, radiation, apart from statistical errors in measurement.

Error analysis:

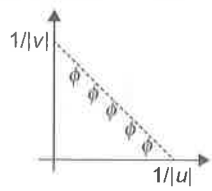
After correcting for systematic errors, equation (1) is used to estimate the remaining errors.

FOCAL LENGTH OF CONCAVE MIRROR AND A CONVEX LENS USING THE U-V METHOD.

In this method one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder. The position of the lens is noted by reading the scale at the bottom of the holder. A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).

The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).

For the concave mirror, the position of the image is determined by placing a sharp object (a pin) on the optical bench such that the parallax between the object pin and the image is nil.



A plot of $|u|$ versus $|v|$ gives a rectangular hyperbola.

A plot or $\frac{1}{|v|}$ vs $\frac{1}{|u|}$ gives a straight line.

The intercepts are equal to $\frac{1}{|f|}$, where f is the focal length.

Error: The systematic error in this experiment is mostly due to improper position of the object on the holder. This error may be eliminated by reversing the holder (rotating the holder by 180° about the vertical) and the taking the reading again. Averages are then taken.

The equation for random errors gives:

$$\frac{\delta f}{f^2} = \frac{\delta u}{u^2} + \frac{\delta v}{v^2}$$

The error δu, δv correspond to the error in the measurement of u and v. Actually, we know the errors in the object position, lens position and image position. So, the errors in u and v too be estimated as described before.

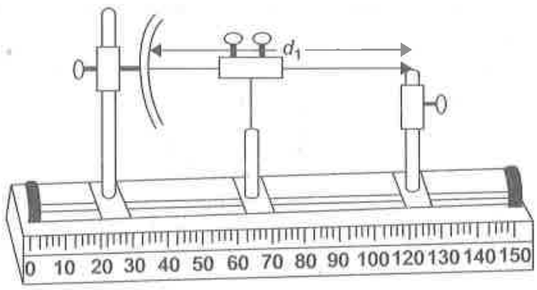
Index Error or Bench Error and its correction: In an experiment using an optical bench we are required to measure the object and image distances from the pole or vertex on the mirror. The distance between the tip of the needles and the pole of the mirror is the actual distance. But we practically measure distance between the indices with the help of the scale engraved on the bench. These distance are called the observed distances. The actual distances may not be equal to the observed distances and due to this reason an error creeps in the measurement of the distances. This error is called the index or the bench error. This error is estimated with the help of a needle of known length placed horizontally between the tip of the needle and the pole.

Index Error = Observed distance - actual distance and

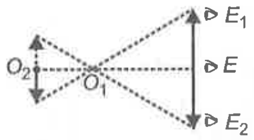
Index Correction = Actual - observed distance

Notes

Index correction whether position or negative, is always added algebraically to the observed distance to get the corrected distance.



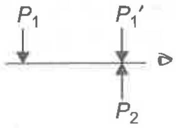
Parallax: When two object O_1 and O_2 are placed in such a way that both of them lie in the same line of sight as shown in figure, then the object nearer to the eye covers the object farther from it. Their image on the retina are superimposed and therefore, it is impossible to decide which is the nearer object. To identify this fact, the observer displaces his eye to a position E_1 or E_2 until he is able to see two distinct objects.



The more distant object O_2 apparently moves in the direction opposite to the displacement of the observer's eye with respect the nearer object O_1. This relative shift in the position of two object due to the shift in the position of the observer's eye is called parallax.

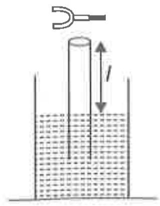
Parallax between the two objects disappear if they are at the same position.

The figure shows the tips of two pins P_1 and P_2 kept in the upright positions. The parallax between P_1 and P_2 is removed by shifting the position of observer's eye sideways. As the farther pin P_1 is displaced towards the pin P_2 the relative shift (parallax) between their position decreases as the position of eye is displaced sideways. The relative shift vanishes when the pin P_1 occupies the position P_1, that is, when the tips of the two are just coincident. At this position there is no parallax between the tips of the two pins.



SPEED OF SOUND USING RESONANCE COLUMN

A tuning fork of known frequency (f) is held at the mouth of a long tube, which is dipped into water as shown in the figure. The length (l_1) of the air column in the tube is adjusted until it resonance with the tuning fork. The air temperature and humidity are noted. The length of the tube is adjusted again until a second resonace length (l_2) is found (provided the tube is long)



Then $l_2 - l_1 = \frac{\lambda}{2}$, provided l_1, l_2 are resonance lengths for adjacent resonances.

$\therefore \quad \lambda = 2(l_2 - l_1)$, is the wavelength of sound.

Since the frequency f, is known; the velocity of sound in air at the temperature (θ) and humidity (h) is given by $C = f\lambda = 2(l_2 - l_1)f$

It is also possible to use a single measurement of the resonant length directly, but, then it has to be corrected for the 'end effect'.

λ (fundamental) $= 4(l_1 + 0.3d)$, where d = diameter

Errors: The major systematic error introduced are due to end effects in (end correction) and also due to excessive humidity.

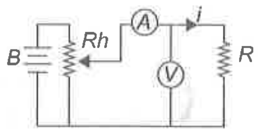
Random errors are given by

$$\frac{\delta C}{C} = \frac{\delta(l_2 - l_1)}{l_2 - l_1} = \frac{\delta l_2 + \delta l_1}{l_2 - l_1}$$

Verification of Ohm's law using voltmeter and ammeter

A voltmeter (V) and an ammeter (A) are connected in a circuit along with a resistance R as shown in the figure, along with a battery B and a rheostat, Rh

Simultaneous readings of the current i and the potential drop V are taken by changing the resistance in the rheostat (Rh). A graph of V vs i is plotted and it is found to be linear (within errors). The magnitude of R is determined by either



(a) taking the ratio $\frac{V}{i}$ and then

(b) fitting to a straight line: $V = iR$, and determining the slope R.

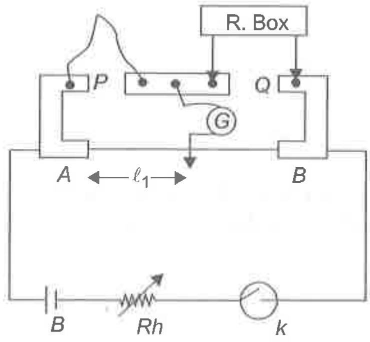
Errors: Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/connections of the resistance. The effect of Joule heating may be minimized by switching on the circuit for a short while only, while the effect of finite resistance of the voltameter can be overcome by using a high resistance instrument or a potentiometer. The lengths of connecting wires should be minimized as much as possible.

Error analysis: The error in computing the ratio $R = \frac{V}{i}$ is given by $\left|\frac{\delta R}{R}\right| = \left|\frac{\delta V}{V}\right| + \left|\frac{\delta i}{i}\right|$

where δV and δi are of the order of the least count of the instruments used.

SPECIFIC RESISTANCE OF THE MATERIAL OF A WIRE USING A METER BRIDGE

A known length (l) of a wire is connected in one of the gaps (P) of a meter bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completer by using a battery (B), a Rheostat (Rh), a Key (K) and a galvanometer (G). The balance length (l) is found by closing key k and momentarily connecting the galvanometer until it gives zero deflection (null point).



Then, $\frac{P}{Q} = \frac{l}{100 - l}$ (1)

using the expression for the meter bridge at balance. Here, represents the resistance of the wire while Q represents the resistance in the resistance box. The key K is open when the circuit is not in use.

The resistance of the wire, $P = \rho \frac{L}{\pi r^2} \quad \Rightarrow \quad \rho = \frac{\pi r^2}{L} P$ (2)

Where r is the radius or wire and L is the length of the wire, r is measured using a screw gauge while L is measured with a scale.

Errors: The major systematic errors in this experiment are due to the heating effect, end correction introduced due to shift of the zero of the scale at A and B, and stray resistances in P and Q, are errors due to non-uniformity of the meter bridge wire.

Error analysis: End corrections can be estimated by including know resistance P_1 and Q_1 in the two ends and finding the null point:

$$\frac{P_1}{Q_1} = \frac{l_1 + \alpha}{100 - l_1 + \beta} \qquad (2),$$

where α and β are the end corrections.

When the resistance Q_1 is placed in the left gap and P_1 in the right gap,

$$\frac{Q_1}{P_1} = \frac{l_2 + \alpha}{100 - l_2 + \beta} \qquad (3)$$

which give two linear equation for finding α and β.

In order that α and β be measured accurately, P_1 and Q_1 should be as different from each other as possible. For the actual balance point,

$$\frac{P}{Q} = \frac{\ell + a}{100 - \ell + \beta} = \frac{l_1'}{l_2'}$$

Error due to non-uniformity of the meter bridge wire can be minimized by interchanging the resistances in the gaps P and Q.

$$\therefore \quad \frac{\delta P}{P} = \left|\frac{\delta l'_1}{l'_1}\right| + \left|\frac{\delta l'_2}{l'_2}\right|$$

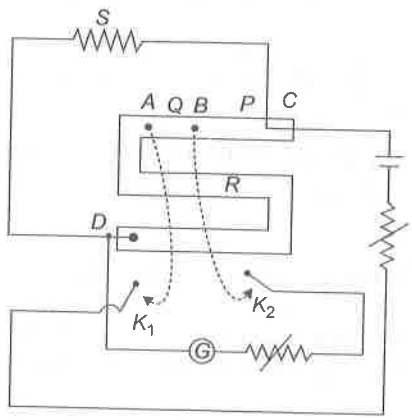
where $\delta l_1'$ and $\delta l_2'$ are of the order of the least count of the scale.

The error is, therefore, minimum if $l'_1 = l'_2$ i.e., when the balance point is in the middle of the bridge. The error is ρ is

$$\frac{\delta P}{P} = \frac{2\delta r}{r} + \frac{\delta L}{L} + \frac{\delta P}{P}$$

MEASUREMENT OF UNKNOWN RESISTANCE USING A P.O. BOX

A P.O. Box can also be used to measure an unknown resistance. It is a Wheatstone Bridge with three arms P, Q and R; while the fourth arm(s) is the unknown resistance. P and Q are known as the ratio arms while R is known at the rheostat arm.



At balance, the unknown resistance

$$S = \left(\frac{P}{Q}\right) R \qquad (1)$$

The ratio arms are first adjusted so that they carry 100 Ω each. The resistance in the rheostat arm is now adjusted so that the galvanometer deflection is in one direction, if $R = R_0(\Omega)$ and in the opposite direction when $R = R_0 + 1\ (\Omega)$.

This implies that the unknown resistance, S lies between R_0 and $R_0 + 1\ \Omega$. Now, the resistance in P and Q are made 100 Ω and 1000 Ω respectively, and the process is repeated.

Equation (1) is used to compute S.

The ratio P/Q is progressively made 1:10, and then 1:100. The resistance S can be accurately measured.

Errors: The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating, and the insensitivity of the Wheatstone bridge.

These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating) and calculating the sensitivity.

In order that the sensitivity is maximum, the resistance in the arm P is close to the value of the resistance S.

EXERCISES

JEE Main

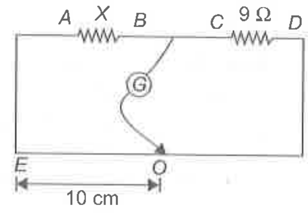
1. A vernier callipers having 1 main scale division = 0.1 cm is designed to have least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then
(A) $n = 10$, $m = 0.5$ cm
(B) $n = 9$, $m = 0.4$ cm
(C) $n = 10$, $m = 0.8$ cm
(D) $n = 10$, $m = 0.2$ cm

2. In a Vernier Calipers (VC), N divisions of the main scale coincide with $N + m$ division of the vernier scale. What is the value of m for which the instrument has minimum least count? ($N > 2$)
(A) 1
(B) N
(C) Infinity
(D) $\frac{N}{2}$

3. In the Searle's experiment, after every step of loading, why should we wait for two minutes before taking the readings? (More than one correct.)
(A) So that the wire can have its desired change in length
(B) So that the wire can attain room temperature.
(C) So that vertical oscillations can get subsided.
(D) So that the wire has no change in its radius.

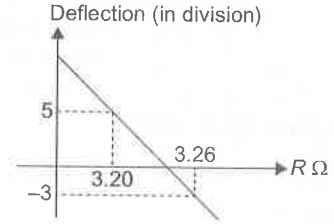
4. In a meter bridge set up, which of the following should be the properties of the one meter long wire?
(A) High resistivity and low temperature coefficient
(B) Low resistivity and low temperature coefficient
(C) Low resistivity and high temperature coefficient
(D) High resistivity and high temperature coefficient

5. Consider the MB shown in the diagram, let the resistance X have temperature coefficient α_1 and the resistance from the RB have the temperature coefficient α_2. Let the reading of the meter scale be 10 cm from the LHS. If the temperature of the two resistance increase by small temperature ΔT then what is the shift in the position of the null point?
Neglect all the other changes in the bridge due to temperature rise.

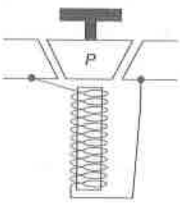


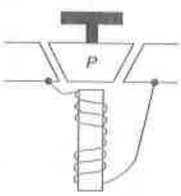
(A) $9(\alpha_1 - \alpha_2)\Delta T$
(B) $9(\alpha_1 + \alpha_2)\Delta T$
(C) $\frac{1}{9}(\alpha_1 + \alpha_2)\Delta T$
(D) $\frac{1}{9}(\alpha_1 - \alpha_2)\Delta T$

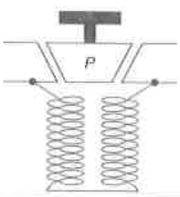
6. For a post office Box, the graph of galvanometer deflection versus R (resistance pulled out of RB) is given as shown. A careless student pulls out two non consecutive values R marked in the graph. Find the value of unknown resistance.

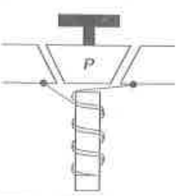


(A) 3.2 Ω
(B) 3.24 Ω
(C) 3.206
(D) 3.26

7. Identify which of the following diagrams represent the internal construction of the coils wound in a resistance box or PO box?
(A)


(B)


(C)


(D)


JEE Advanced

1. How many signficant figures are given in the following quantities?

(A) 343 g (B) 2.20
(C) 1.103 N (D) 0.4142 s
(E) 0.0145 m (F) 1.0080 V
(G) 9.1×10^4 km (H) 1.124×10^{-3} V

2. Perform the following operations:

(A) $703 + 7 + 0.66$ (B) 2.21×0.3
(C) 12.4×84 (D) $14.28/0.714$

3. Solve with due regard to significant digits

(i) $\sqrt{6.5 - 6.32}$ (ii) $\dfrac{2.91 \times 0.3842}{0.080}$

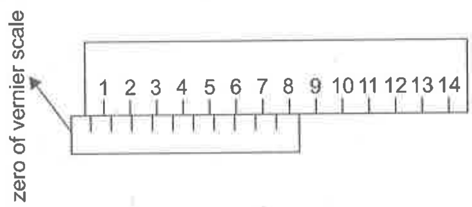
4. The main scale of a vernier calipers reads in millimeter and its vernier is divided into 10 divisions which coincide with 9 divisions of the main scale. When the two jaws of the instrument touch each other the seventh division of the vernier scale coincide with a scale division and the zero of the vernier lies to the right of the zero of main scale. Furthermore, when a cylinder is tightly placed along its length between the two jaws, the zero of the vernier scale lies slightly to the left of 3.2 cm; and the fourth vernier division coincides with a scale division. Calculate the measured length of the cylinder.

5. The VC shown in the diagram has zero error in it (a you can see).

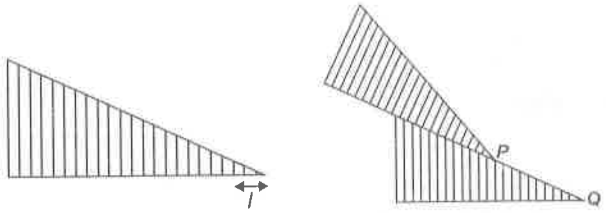
It is given that 9 msd = 10 vsd.

(i) What is the magnitude of the zero error? (1 msd = 1 mm)

(ii) The observed reading of the length of a rod measured by this VC comes out to be 5.4 mm. If the vernier had been error free then reading of main scale would be _____ and the division of vernier scale coinciding would be _____.



6. Consider a home made vernier scale as shown in the figure. In this diagram, we are interested in measuring the length of the line PQ. If both the inclines are identical and their angles are equal to θ then what is the least count of the instrument.



7. The pitch of a screw gauge is 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the pitch scale (or main scale) and thirty fourth division coincides with the reference line. Calculate the thickness of the metal plate.

8. The pitch of a screw gauge is 1 mm and there are 50 divisions on its cap. When nothing is put in between the studs, 44th division of the circular scale coincides with the reference line zero of the main scale is not visible. When a glass plate is placed between the studs, the main scale reads three divisions and the circular scale reads 26 divisions. Calculate the thickness of the plate.

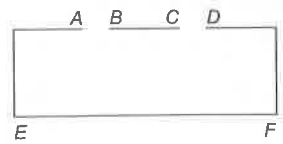
9. In a given optical bench, a needle of length 10 cm is used in between (object and lens) and lens and image to estimate bench error. The object needle, image needle and lens holder have their reading as shown.

$x_0 = 1.1$ cm, $x_1 = 21.0$ cm, $x_L = 10.9$ cm

Estimate the bench errors which are present in image needle holder and object needle holder. Also find the focal length of the convex lens when.

$x_0 = 0.6$ cm, $x_1 = 22.5$ cm, $x_L = 11.4$ cm

10. Make the appropriate connections in the meter bridge set up shown. Resistance box is connected between _____. Unknown resistance is connected between _____. Battery is connected between _____.



11. A body travels uniformly a distance of (13.8 ± 0.2) m in time (4.0 ± 0.3) sec. Calculate its velocity.

12. Consider $S = x \cos(\theta)$ for $x = (2.0 \pm 0.2)$ cm, $\theta = 53$ $\pm 2°$. Find S.

13. Two resistance R_1 and R_2 are connected in (i) series and (ii) parallel. What is the equivalent resistance with limit of possible percentage error in each case of $R_1 = 5.0 \pm 0.2\ \Omega$ and $R_2 = 10.0 \pm 0.1\ \Omega$.

14. 5.74 gm of a substance occupies a volume of 1.2 cm^3. Calculate its density with due regard for significant figures.

15. The time period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{l/g}$ The length of the pendulum is measured as $l = 10 \pm 0.1$ cm and the time periods as $T = 0.5 \pm 0.02$ s.
Determine percentage error in the value of g.

16. A physical quantity P is related to four observable A, B, C and D as $P = 4\pi^2 A^3 B^2 (\sqrt{C}D)$ The percentage error of the measurement in A, B, C and D are 1%, 3% and 2%, 4% respectively. Determine the percentage error and absolute error in the quantity P. Value of P is calculated 3.763.

17. A glass prism of angle $A = 60°$ (exact) gives minimum angle of deviation $\theta = 30°$ with the max. error of 1° when a beam of parallel light is passed through the prism during an experiment. Find the maximum permissible percentage error in the measurement of refractive index μ of the material of the prism.

18. In a vernier calipers the main scale and the vernier scale are made up different materials. When the room temperature increases by $\Delta T°C$, it is found the reading of the instrument remains the same. Earlier it was observed that the front edge of the wooden rod placed for measurement crossed the Nth main scale division and $N + 2$ msd coincided with the 2nd vsd. Initially, 10 vsd coincided with 9 msd. If coefficient of linear expansion of the main scale is α_1 and that of the vernier scale is α_2 then what is the value of α_1/α_2? (Ignore the expansion of the rod on heating)
(A) $\frac{1.8}{(N)}$ (B) $\frac{1.8}{(N+2)}$
(C) $\frac{1.8}{(N-2)}$ (D) None

Previous Years' Questions

1. The edge of a cube is $a = 1.2 \times 10^{-2}$ m. Then its volume will be recorded as: **[JEE 2003]**
(A) 1.7×10^{-6} m^3 (B) 1.70×10^{-6} m^3
(C) 1.70×10^{-7} m^3 (D) 1.78×10^{-6} m^3

2. In a vernier callipers, n divisions of its main scale match with $(n + 1)$ divisions on its vernier scale. Each division of the main scale is a units Using the vernier principle, calculate its least count. **[JEE 2003]**

3. A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is **[JEE 2004]**
(A) 1 (B) 2
(C) 3 (D) 4

4. In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of exactly 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data. **[JEE 2004]**

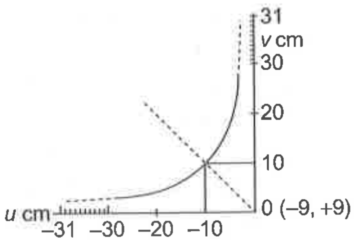
5. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in appropriate number of significant figures. **[JEE 2004]**

6. Draw the circuit for experimental verification of Ohm's law using a source of variable –DC voltage, a main resistance of 100 Ω, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter. **[JEE 2004]**

7. In a resonance column method, resonance occurs at two successive level of $l_1 = 30.7$ cm and $l_2 = 63.2$ cm using a tuning fork of $f = 512$ Hz(exact). What is the maximum error in measuring speed of sound using relations $v = f\lambda$ and $\lambda = 2(l_2 - l_1)$ **[JEE 2005]**
(A) 256 cm/sec (B) 92 cm/sec
(C) 128 cm/sec (D) 204.8 cm/sec

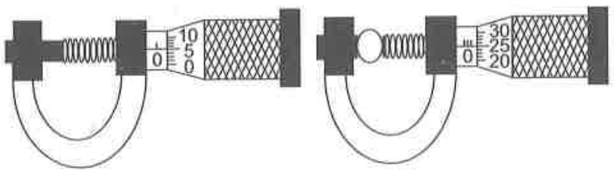
8. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures. **[JEE 2005]**

9. Graph of position of image vs position of point object from a convex lens is shown. Then, focal length of the lens is



(A) 0.50 ± 0.05 cm (B) 0.50 ± 0.10 cm
(C) 5.00 ± 0.05 cm (D) 5.00 ± 0.10 cm

10. The circular divisions of shown screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball is **[JEE 2006]**



(A) 2.25 mm (B) 2.20 mm
(C) 1.20 mm (D) 1.25 mm

11. A student performs an experiment for determination of $g\left(=\dfrac{4\pi^2 l}{T^2}\right)$ 1 = 1 m and he commits an error of Δl. For the experiment takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 sec. For which of the following data, the measurement of g will be most accurate?
Δl $\Delta T n$
Amplitude of oscillation **[JEE 2006]**
(A) 5 mm 0.2 sec 10 5 mm
(B) 5 mm 0.2 sec 20 5 mm
(C) 5 mm 0.1 sec 20 1 mm
(D) 1 mm 0.1 sec 50 1 mm

12. In an experiment of determine the focal length (f) a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then, **[JEE 2007]**
(A) $x < f$ (B) $f < x < 2f$
(C) $x = 2f$ (D) $x > 2f$

13. Some physical quantities are given in Column I and some possible SI units in which these quantities may be expressed are given in Column II. Match the physical quantities in Column I with the units in Column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS. **[JEE 2007]**

14. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained from the reading is **[JEE 2007]**
(A) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2
(B) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(C) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2
(D) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

15. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.
Least count for length = 0.1 cm
Least count for time = 0.1 s

Student	Length of the Pendulum (cm)	Number of oscilltions (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage error in g, i.e. $\left(\dfrac{\Delta g}{g} \times 100\right)$ for student I, II and III, respectively,
(A) $E_I = 0$ (B) E_I is minimum
(C) $E_I = E_{II}$ (D) E_{II} is maximum

16. A student performed the experiment of determination of focal length of a concave mirror by $u-v$ method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are (42, 56) (48, 48), (60, 40), (66, 33) (78, 39)

The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are)

(A) (42, 56) (B) (48, 48)
(C) (66, 38) (D) (78, 39)

17. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation which of the following statement(s) is (are) true?

(A) Error ΔT in measuring T, the time period, is 0.05 seconds
(B) Error ΔT in measuring T, the time period, is 1 second
(C) Percentage error in the the determination of g is 5%
(D) Percentage error in the determination of g is 2.5 %

18. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale division. For this Vernier calipers the least count is

(A) 0.02 mm (B) 0.05 mm
(C) 0.1 mm (D) 0.2 mm

19. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge. whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2% the relative percentage error in the density is:

(A) 0.9% (B) 2.4%
(C) 3.1% (D) 4.2%

20. In the determination of young's modulus $\left(Y = \dfrac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. The have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and l are the same
(B) due to the error in the measurement of d is twice that due to the error in the measurement of l.
(C) due to the error in the measurement of l is twice that due to the error in the measurement of d.
(D) due to the error in the measurement of d is four times that due to the error in the measurement of l.

21. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm
(C) 5.136 cm (D) 5.148 cm

Common Data for Questions 22–23: A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

22. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_p. **[JEE-2011]**

(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$

(C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

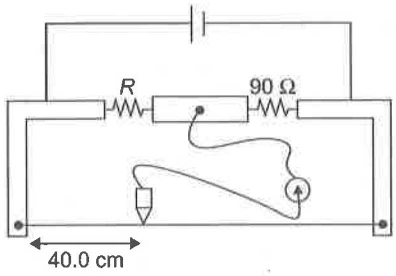
23. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ \text{m}^{-3}$. Take $\varepsilon_0 = 10^{-11}$ and $m = 10^{-30}$, where these quantities are in proper SI units **[JEE-2011]**

(A) 800 nm (B) 600 nm
(C) 300 nm (D) 200 nm

24. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? **[JEE Main 2014]**

(A) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(B) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.
(C) A meter scale
(D) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

25. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is



(A) $60 \pm 0.15\,\Omega$ (B) $135 \pm 0.56\,\Omega$
(C) $60 \pm 0.25\,\Omega$ (D) $135 \pm 0.23\,\Omega$

[JEE ADVANCED 2014]

26. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

[JEE ADVANCED 2014]

ANSWER KEYS

Exercises

JEE Main

1. C **2.** A **3.** A, B, C **4.** A **5.** A **6.** B **7.** D

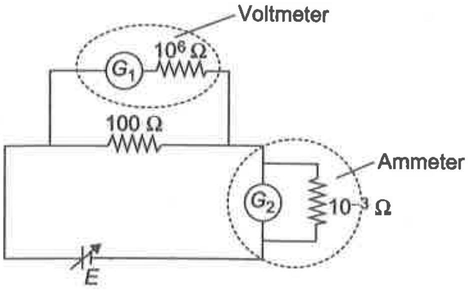
JEE Advanced

1. (A) 3, (B) 3, (C) 4, (D) 4, (E) 3, (F) 5, (G) 2, (H) 4
2. (A) 711, (B) 0.7, (C) 1.0×10^3, (D) 20.0
3. (i) 0.4, (ii) 14
4. 3.07 cm
5. (i) $x = -0.7$ msd, (ii) 6, 1
6. $\text{L.C.} = l\left[\dfrac{1-\cos\theta}{\cos\theta}\right]$
7. 2.84 mm
8. $R_t = 3.64$ mm
9. 5.5 ± 0.05 cm
10. CD, AB, EF
11. $v = (3.4 \pm 0.31)$ m/s
12. $S = (1.20 \pm 0.18)$ cm
13. $R_8 = 15\,\Omega \pm 2\%$, $R_p = 3.3\,\Omega \pm 3\%$
14. 4.8 g/cm^3
15. 9%
16. 14%, 0.53
17. $5\pi/18\%$
18. B

Previous Years' Questions

1. A **2.** $\frac{a}{n+1}$ **3.** D **4.** $\Delta Y = 6.47 \times 10^9$ N/m^2 $Y = 2.24 \times 10^{11}$ N/m^2

5. 2.6 cm^2 (in two significant figures)

6. 

7. D **8.** 2.66 g/cm^3

9. C **10.** C **11.** D **12.** B **13.** (A) P, Q; (B) R, S; (C) R, S; (D) R, S
14. B **15.** B **16.** CD **17.** A, C **18.** D **19.** C **20.** A **21.** B
22. C **23.** B **24.** D **25.** C **26.** 4

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