BRAIN MAP

MECHANICAL PROPERTIES OF SOLIDS AND FLUIDS

A solid vertical column can rest on table without external support but a liquid column cannot because it cannot withstand shear stresses

Properties of Solids

Elasticity and Plasticity

- Elasticity is the property of the body by virtue of which it tends to regain its original size and shape, when applied force is removed.
- Plasticity is the inability of a body to regain its original size and shape on the removal of the deforming forces.

Stress and Strain

- Stress is the internal restoring force acting per unit area of a deformed body.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

- Strain is the ratio of change in configuration to the original configuration.

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

- According to Hooke's law, within elastic limit stress is directly proportional to strain, that is the extension produced in a wire is directly proportional to the load applied to the wire.
i.e., Stress ∝ Strain or Stress = E × Strain, where E is modulus of elasticity.

Types of Modulus of Elasticity

- Young's modulus, $Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

or $Y = \frac{F/A}{\Delta l/l}$; $\frac{Fl}{A\Delta l}$

- Bulk modulus, $B = \frac{\text{Normal stress}}{\text{Volumetric strain}}$

or $B = \frac{+F/A}{\Delta V/V} = \frac{-FV}{A\Delta V} = \frac{-PV}{\Delta V}$

- Compressibility, $k = \frac{1}{B} = \frac{+\text{B}V}{PV}$.
- Modulus of rigidity, G ; $\frac{\text{Shearing stress}}{\text{Shearing strain}}$

or $G = \frac{\theta_s}{\theta} = \frac{F/A}{\Delta x/L} = \frac{FL}{A\Delta x}$

Elastic Potential Energy

- Work done against the internal restoring forces acting between the various particles of the wire when it is stretched, is stored in the form of potential energy in the wire and is known as elastic potential energy,

$$U = \frac{1}{2}F..\Delta L = \frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume of wire}$$

- Potential energy stored per unit volume of stretched wire,

$$u = \frac{1}{2}..\text{stress}\times\text{strain} = \frac{1}{2}\times\text{Young's modulus}\times(\text{strain})^2$$

Poisson's Ratio

- Poisson's ratio (σ) ; $\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$$= \frac{+\Delta R/R}{\Delta L/L} = \frac{-L\Delta R}{R\Delta L}$$

Relations between Y, B, G and σ

- $Y = 3B(1-2\sigma)$
- $Y = 2G(1+\sigma)$
- $\sigma = \frac{3B+2G}{2G+6B}$
- $\frac{9}{Y}$; $\frac{1}{B} + \frac{3}{G}$

Properties of Fluids

Pressure

- It is the normal force or thrust exerted by a liquid at rest per unit area of the surface in contact with it. $P = F/A$
- Gauge pressure = Total pressure – atmospheric pressure

$$= P - P_0 = h\rho g.$$

- The pressure is same at all points inside the liquid lying at the same depth in a horizontal plane.

Archimede's Principle

- It states that when a body is immersed wholly or partly in a liquid at rest, it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.
Apparent weight = Actual weight –Buoyant force $= W - W'$
(ρ′ is density of the liquid) ; $V\rho g - V\rho' g$

$$= V\rho g\left(1-\frac{\pi'}{\rho}\right) = mg\left(1-\frac{\rho'}{\rho}\right)$$

- When $W > W'$, the body sinks down.
- When $W = W'$, the body floats completely immersed in the liquid.
- When $W < W'$, the body floats partly immersed.

Fluid in Motion

- **Bernoulli's Theorem :** It states that for the streamline flow of an ideal liquid, the total energy per unit mass remains constant at every cross-section throughout the liquid flow.

$$P\ )\ \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

- **Equation of continuity :**
$Av = \text{constant}$
It is a statement of conservation of mass in flow of incompressible fluids.
- **Torricelli's law :**
Velocity of efflux is the velocity with which the liquid flows out of a narrow hole at depth h below the free surface of the liquid.

$$v = \sqrt{2gh + \frac{2(P+P_0)}{\rho}}.$$

- If the hole is open to the atmosphere, then $P = P_0$, so v ; $\sqrt{2gh}$

Capillarity

- The phenomenon of rise or fall of liquid in a capillary tube is called capillarity.
- Ascent formula : Height of the liquid within the capillary tube,

$$h = \frac{2S\cos o}{a\rho g}$$

where a is the radius of the capillary tube.

Viscosity

- Coefficient of viscosity of a liquid is the tangential force required to maintain a unit velocity gradient between two parallel layers of liquid, each of unit area.

$$\eta = \frac{+F}{A\left(\frac{dv}{dx}\right)}$$

where $\frac{dv}{dx}$ is the velocity gradient between two layers of liquid.

- **Poiseuille's formula :** Volume of the liquid flowing per second through a narrow tube, $Q = \frac{v}{8}\frac{Pr^4}{\eta l}$.
- **Stoke's law :** Backward dragging force F on a small spherical body of radius r, moving through a fluid of coefficient of viscosity η, with velocity v is given by $F = 6\pi\eta rv$.
- Terminal velocity, $v_T = \frac{2r^2(\pi+\theta)}{9\eta}g$
- Reynold's number is a number which determines the nature of flow of liquid through a pipe.

$$R = \frac{\pi vd}{\eta}$$

Surface Tension, Surface Energy and Angle of Contact

- **Surface tension(S)** is the property of the liquid by virtue of which the free surface of liquid at rest tends to have minimum surface area.
- **Surface energy** of given liquid surface is defined as the amount of work done against the force of surface tension in forming the liquid surface of given area at a constant temperature.
- **Angle of contact** (θ) is the angle enclosed between the tangents to the liquid surface at the point of contact and the solid surface inside the liquid. It determines whether a liquid will spread on the surface of a solid or it will form droplets on it.

Excess Pressure in Drops

- Excess pressure inside a liquid drop

P ; $\frac{2S}{R}$ (with one free surface)

- Excess pressure inside a soap bubble

P ; $\frac{4S}{R}$ (with two free surface)

- Excess pressure in an air bubble

P ; $\frac{2S}{R}$ (with one free surface)