1. FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of resultant force :

- may change only speed
- may change only direction of motion.
- may change both the speed and direction of motion.
- may change size and shape of a body

unit of force : newton and $\frac{kg.m}{s^2}$ (MKS System)

dyne and $\frac{g.cm}{s^2}$ (CGS System)

1 newton = 10^5 dyne

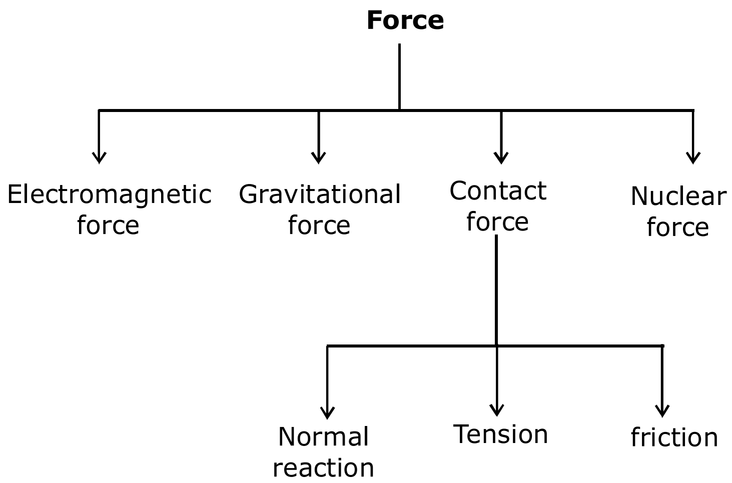
Kilogram force (kgf)

The force with which earth attracts a 1 kg body towards its centre is called kilogram force, thus

$$kgf = \frac{\text{Force in newton}}{g}$$

Dimensional Formula of force : [MLT⁻²]

- For full information of force we require
  - → Magnitude of force
  - → direction of force
  - → point of application of the force



1.1 Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force

- These can be attractive or repulsive
- These are long range forces
- These depend on the nature of medium between the charged particles.
- All macroscopic force (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions are repulsions between atoms/molecules.

1.2 Gravitational force :

It acts between any two masses kept anywhere in the universe. It follows inverse square rule ($F \propto \frac{1}{\text{distance}^2}$) and is attractive in nature.

$$F = \frac{GM_1M_2}{R^2}$$

The force mg, which Earth applies on the bodies, is gravitational force.

1.3 Nuclear force :

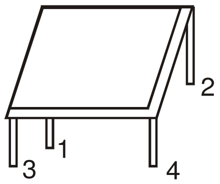
It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

1.4 Contact force :

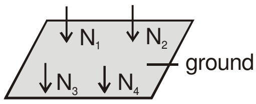
Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

1.4.1 Normal force (N) :

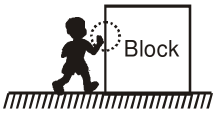
It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force. A table is placed on Earth as shown in figure.



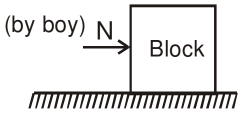
- Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



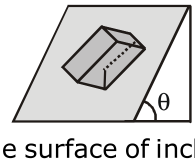
- Now a boy pushes a block kept on a frictionless surface.



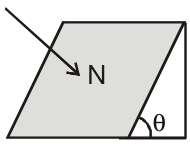
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



- A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



Normal force exerted by block on the surface of inclined plane is shown in figure.

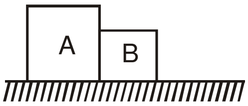


Force acts perpendicular to the surface

:

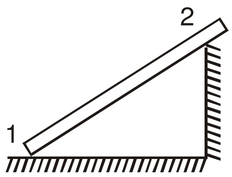
- Normal force acts in such a fashion that it tries to compress the body
- Normal is a dependent force, it comes in role when one surface presses the other.

Ex.1 Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by A on B.

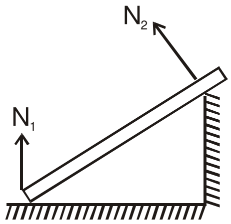


Sol. In above problem, block A does not push block B, so there is no molecular interaction between A and B. Hence normal force exerted by A on B is zero.

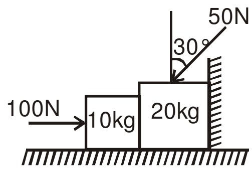
Ex.2 Draw normal forces on the massive rod at point 1 and 2 as shown in figure.



Sol. Normal force acts perpendicular to extended surface at point of contact.



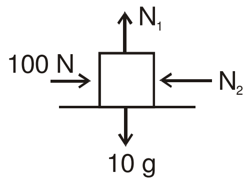
Ex.3 Two blocks are kept in contact as shown in figure. Find
(a) forces exerted by surfaces (floor and wall) on blocks
(b) contact force between two blcoks.



Sol. **F.B.D. of 10 kg block**

$N_1 = 10\text{ g} = 100\text{ N}$ ...(1)

$N_2 = 100\text{ N}$ ...(2)



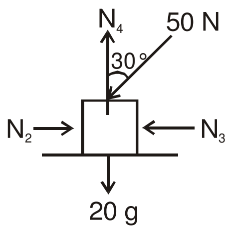
F.B.D. of 20 kg block

$N_2 = 50 \sin 30° + N_3$

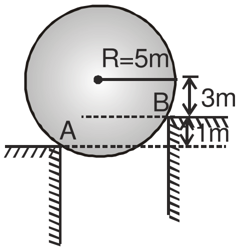
$\therefore\ N_3 = 100 - 25 = 75\text{ N}$ ...(3)

and $N_4 = 50 \cos 30° + 20\text{ g}$

$N_4 = 243.30\text{ N}$

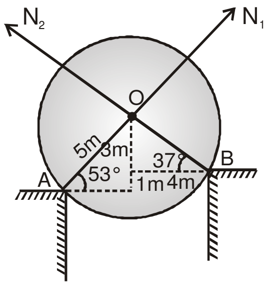


Ex.4

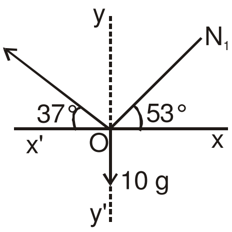


Find out the normal reaction at point A and B if the mass of sphere is 10 kg.

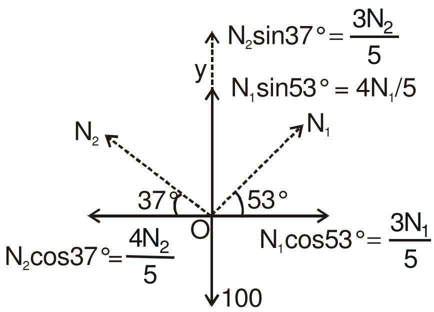
Sol.



Now F.B.D.



Now resolve the forces along x & y direction



$\because$ The body is in equilibrium so equate the force in x & y direction

In x-direction $\frac{3N_1}{5} = \frac{4N_2}{5}$ ...(1)

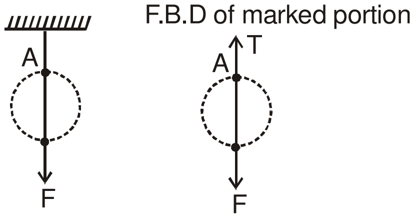
In y-direction $\frac{3N_2}{5} + \frac{4N_1}{5} = 100$ ...(2)

after solving above equation

$N_1 = 80$ N, $N_2 = 60$ N

1.4.2 Tension :

Tension in a string is an electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force 'F' as shown in figure, for calculating the tension at point 'A' we draw F.B.D. of marked portion of the string; Here string is massless.



$\Rightarrow$ T = F

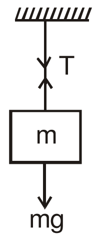
String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature. The attraction force between two segments is equal and opposite due to newton's third law.

Conclusion :

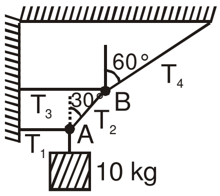
T = mg

(i) Tension always acts along the string and in such a direction that it tries to reduce the length of string

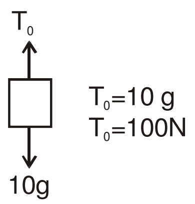
(ii) If the string is massless then the tension will be same along the string but if the string have some mass then the tension will continuously change along the string.



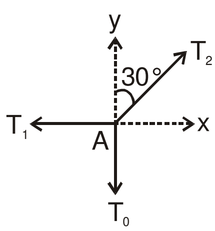
Ex.5 The system shown in figure is in equilibrium. Find the magnitude of tension in each string ; T_1, T_2, T_3 and T_4. (g = 10 m/s^{-2})



Sol. F.B.D. of block 10 kg



F.B.D. of point 'A'



$\sum F_y = 0$

$T_2 \cos 30° = T_0 = 100$ N

$T_2 = \frac{200}{\sqrt{3}}$N

$\sum F_x = 0$

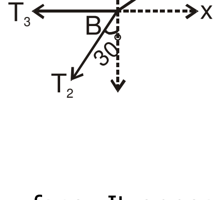
$T_1 = T_2 \sin 30° = \frac{200}{\sqrt{3}} \cdot \frac{1}{2} = \frac{100}{\sqrt{3}}$N

F.B.D of point of 'B'

$\sum F_y = 0 \Rightarrow T_4 \cos 60° = T_2 \cos 30°$

and $\sum F_x = 0 \Rightarrow T_3 + T_2 \sin 30° = T_4 \sin 60°$

$\therefore\ T_3 = \frac{200}{\sqrt{3}}$N, $T_4 = 200$ N



1.4.3 Frictional force :

It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact. (which is explained later)

2. NEWTON'S FIRST LAW OF MOTION :

According to this law "A system will remain in its state of rest or of uniform motion unless a net external force act on it.

1st law can also be stated as "If the net external force acting on a body is zero, only then the body remains at rest."

- The word external means external to the system (object under observation), interactions within the system has not to be considered.
- The word net means the resultant of all the forces acting on the system.
- Newton's first law is nothing but Galileo's law of inertia.
- Inertia means inability of a body to change its state of motion or rest by itself.
- The property of a body that determines its resistance to a change in its motion is its mass (inertia). Greater the mass, greater the inertia.
- An external force is needed to set the system into motion, but no external force is needed to keep a body moving with constant velocity in its uniform motion.
- Newton's laws of motion are valid only in a set of frame of references, these frames of reference are known as inertial frames of reference.
- Generally, we take earth as an inertial frame of reference, but strictly speaking it is not an inertial frame.
- All frames moving uniformly with respect to an inertial frame are themselves inertial.
- We take all frames at rest or moving uniformly with respect to earth, as inertial frames.

3. NEWTON'S SECOND LAW OF MOTION :

Newton's second law states, "The rate of change of a momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts"

i.e., $\vec{F} \propto \frac{d\vec{p}}{dt}$ or $\vec{F} = k\frac{d\vec{p}}{dt}$

where k is a constant of proportionality.

$\vec{p} = m\vec{v}$, So $\vec{F} = k\frac{(dm\vec{v})}{dt}$

For a body having constant mass,

$\Rightarrow \quad \vec{F} = km\frac{d\vec{v}}{dt} = km\vec{a}$

From experiments, the value of k is found to be 1.

So, $\vec{F}_{net} = m\vec{a}$

- Force can't change the momentum along a direction normal to it, i.e., the component of velocity normal to the force doesn't change.
- Newton's 2nd law is strictly applicable to a single point particle. In case of rigid bodies or system of particles or system of rigid bodies, $\vec{F}$ refers to total external force acting on system and $\vec{a}$ refers to acceleration of centre of mass of the system. The internal forces, if any, in the system are not to be included in $\vec{F}$.
- Acceleration of a particle at any instant and at a particular location is determined by the force (net) acting on the particle at the same instant and at same location and is not in any way depending on the history of the motion of the particle.

PROBLEM SOLVING STRATEGY :

Newton's laws refer to a particle and relate the forces acting on the particle to its mass and to its acceleration. But before writing any equation from Newton's law, you should be careful about which particle you are considering. The laws are applicable to an extended body too which is nothing but collection of a large number of particles.

Follow the steps given below in writing the equations :

Step 1 : Select the body

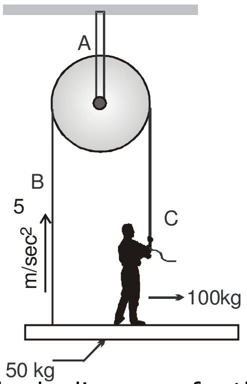
The first step is to decide the body on which the laws of motion are to be applied. The body may be a single particle, an extended body like a block, a combination of two blocks-one kept over another or connected by a string. The only condition is that **all the parts of the body or system must have the same acceleration.**

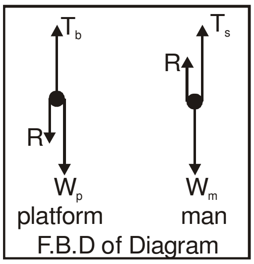
Step 2 : Identify the forces

Once the system is decided, list down all the force acting on the system due to all the objects in the environment such as inclined planes, strings, springs etc. However, any force applied by the system shouldn't be included in the list. You should also be clear about the nature and direction of these forces.

Step 3 : Make a Free-body diagram (FBD)

Make a separate diagram representing the body by a point and draw vectors representing the forces acting on the body with this point as the common origin.
This is called a free-body diagram of the body.





Look at the adjoining free-body diagrams for the platform and the man. Note that the force applied by the man on the rope hasn't been included in the FBD.
Once you get enough practice, you'd be able to identify and draw forces in the main diagram itself instead of making a separate one

Step 4 : Select axes and Write equations

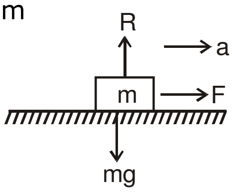
When the body is in equillibrium then choose the axis in such a fashion that maximum number of force lie along the axis.
If the body is moving with some acceleration then first find out the direction of real acceleration and choose the axis one is along the real acceleration direction and other perpendicular to it.
Write the equations according to the newton's second law (F_{net} = ma) in the corresponding axis.

4. APPLICATIONS :

4.1 Motion of a Block on a Horizontal Smooth Surface.

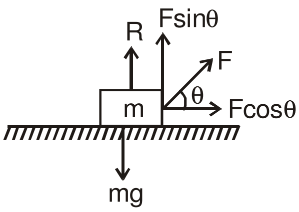
Case (i) : When subjected to a horizontal pull :

The distribution of forces on the body are shown. As there is no motion along vertical direction, hence, R = mg
For horizontal motion F = ma or a = F/m



Case (ii) : When subjected to a pull acting at an angle (θ) to the horizontal :

Now F has to be resolved into two components, F cosθ along the horizontal and F sin θ along the vertical direction.



For no motion along the vertical direction.

we have $R + F \sin\theta = mg$
or $R = mg - F \sin\theta$

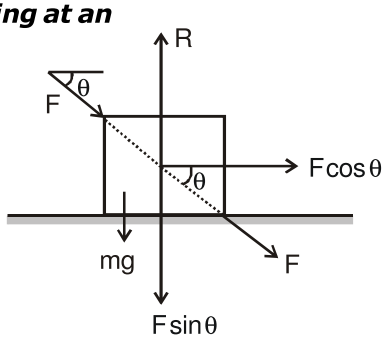
: Hence $R \neq mg$. $R < mg$

For horizontal motion

$F \cos\theta = ma$, $a = \frac{F\cos\theta}{m}$

Case (iii) : When the block is subjected to a push acting at an angle θ to the horizontal : (down ward)

The force equation in this case
$R = mg + F \sin\theta$



: $R \neq mg$, $R > mg$

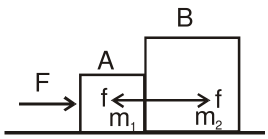
For horizontal motion

$F \cos\theta = ma$, $a = \frac{F\cos\theta}{m}$

4.2 Motion of bodies in contact.

Case (i) : Two body system :

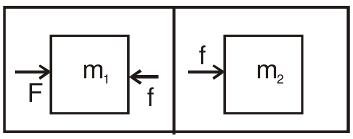
Let a force F be applied on mass m_1



Free body diagrams :

(vertical force do not cause motion, hence they have not been shown in diagram)

$\Rightarrow$ $a = \frac{F}{m_1 + m_2}$ and $f = \frac{m_2 F}{m_1 + m_2}$



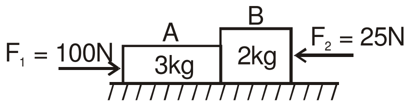
(i) Here f is known as force of contact.
(ii) Acceleration of system can be found simply by

$a = \frac{\text{force}}{\text{total mass}}$

: If force F be applied on m_2, the acceleration will remain the same, but the force of contact will be different

i.e., $f' = \frac{m_1 F}{m_1 + m_2}$

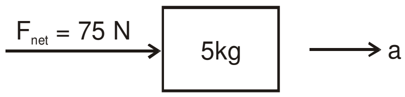
Ex.6 Find the contact force between the 3 kg and 2kg block as shown in figure.



Sol. Considering both blocks as a system to find the common acceleration

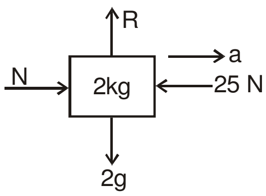
$F_{net} = F_1 - F_2 = 100 - 25 = 75$ N

common acceleration



$$a = \frac{F_{net}}{5kg} = \frac{75}{5} = 15\,m/s^2$$

To find the contact force between A & B we draw F.B.D of 2 kg block

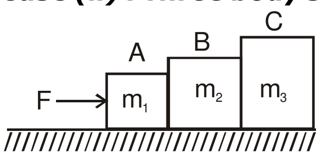


from $(\Sigma F_{net})_x = ma_x$

$\Rightarrow$ N − 25 = (2) (15)

$\Rightarrow$ N = 55 N

Case (ii) : Three body system :

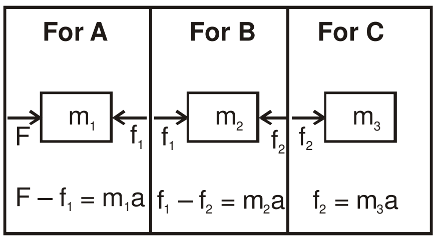


Free body diagrams :

$$\Rightarrow a = \frac{F}{m_1 + m_2 + m_3}$$

$$\text{and } f_1 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

$$f_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

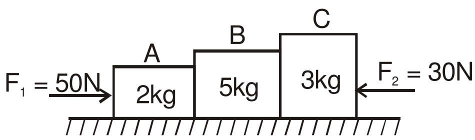


f_1 = contact force between masses m_1 and m_2

f_2 = contact force between masses m_2 and m_3

***Remember* :** Contact forces will be different if force F will be applied on mass C

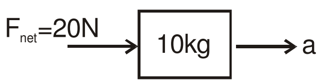
Ex.7 Find the contact force between the block and acceleration of the blocks as shown in figure.



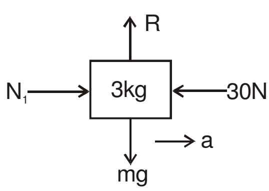
Sol. Considering all the three block as a system to find the common acceleration

$F_{net} = 50 - 30 = 20$ N

$$a = \frac{20}{10} = 2\,m/s^2$$



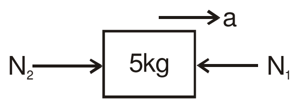
To find the contact force between B & C we draw F.B.D. of 3 kg block.



$$\left(\sum F_{net}\right)_x = ma$$

$\Rightarrow$ $N_1 - 30 = 3(2) \Rightarrow N_1 = 36$ N

To find contact force between A & B we draw F.B.D. of 5 kg block



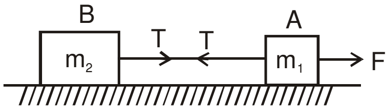
$\Rightarrow$ $N_2 - N_1 = 5a$

$N_2 = 5 \times 2 + 36 \Rightarrow N_2 = 46$ N

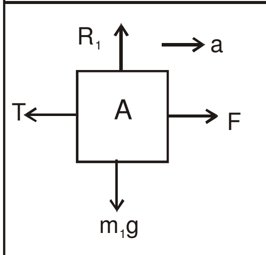
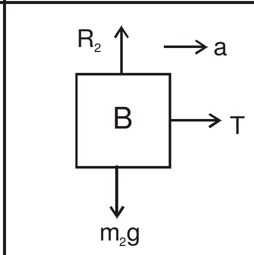
4.3 Motion of connected Bodies

Case (i) For Two Bodies :

F is the pull on body A of mass m_1. The pull of A on B is exercised as tension through the string connecting A and B. The value of tension throughout the string is T only.

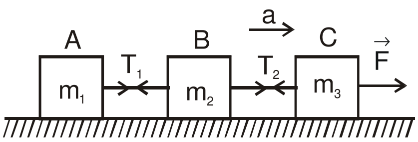


Free body diagrams :

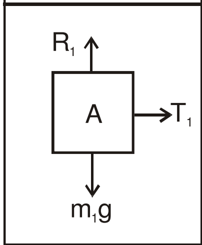
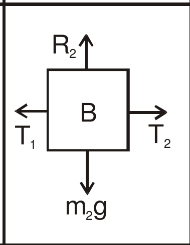
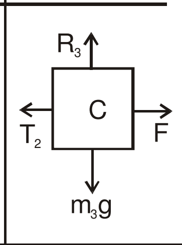
For body A	For body B
	
$R_1 = m_1g$ $F - T = m_1a$	$R_2 = m_2g$ $T = m_2a$

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2}$$

Case (ii) : For Three bodies :

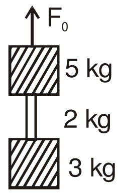


Free body diagrams :

For A	For B	For C
		
$R_1 = m_1g$ $T_1 = m_1a$	$R_2 = m_2g$ $T_2 - T_1 = m_2a$ $\Rightarrow T_2 = m_2a + T_1$ $T_2 = (m_2 + m_1)a$	$R_3 = m_3g$ $F - T_2 = m_3a$ $\Rightarrow F = m_3a + T_2$ $= m_3a + (m_1 + m_2)a$ $F = (m_1 + m_2 + m_3)a$

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2 + m_3}$$

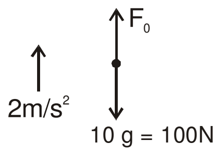
Ex.8 ***A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward is 2 m/s² by an external force F_0.***



(a) What is F_0?
(b) What is the force on rope?
(c) What is the tension at middle point of the rope?
(g = 10 m/s²)

Sol. For calculating the value of F_0, consider two blocks with the rope as a system.
F.B.D. of whole system

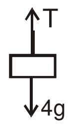
(a)



$F_0 - 100 = 10 \times 2$
$F = 120$ N ...(1)

(b) According to Newton's second law, net force on rope.
$F = ma = (2)\,(2) = 4$ N ...(2)

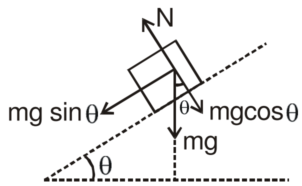
(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.



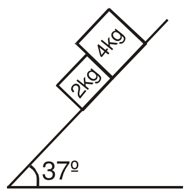
$T - 4g = 4.(2) = 48$ N

4.4 Motion of a body on a smooth inclined plane :

Natural acceleration down the plane = g sin θ
Driving force for acceleration a up the plane, F=m(a+ gsinθ)
and for an acceleration a down the plane, F=m(a – gsinθ)

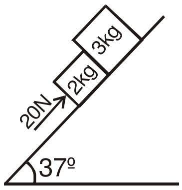


Ex.9 ***Find out the contact force between the 2kg & 4kg block as shown in figure.***

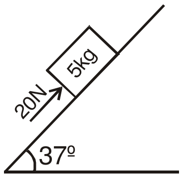


Sol. On an incline plane acceleration of the block is independent of mass. So both the blocks will move with the same acceleration (gsin 37º) so the contact force between them is zero.

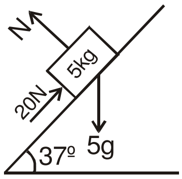
Ex.10 ***Find out the contact force between 2kg & 3kg block placed on the incline plane as shown in figure.***



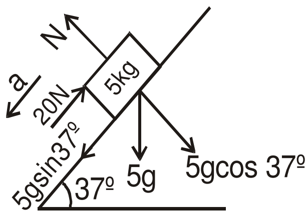
Sol. Considering both the block as a 5kg system because both will move the same acceleration.



Now show forces on the 5 kg block



$\because$ Acceleration of 5kg block is down the incline.
So choose one axis down the incline and other perpendicular to it



From Newton's second Law

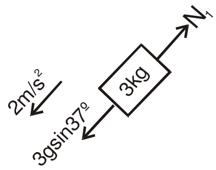
$N = 5g \cos 37º$ ...(i)

$5g\sin 37º - 20 = 5a$ ..(ii)

$30 - 20 = 5a$

$a = 2 m/s^2$ (down the incline)

For contact force (N_1) between 2kg & 3kg block we draw F.B.D. of 3kg block



From

$F_{net} = ma$

$\Rightarrow 3g\sin 37º - N_1 = 3 \times 2$

$18 - N_1 = 6$

$N_1 = 12$ N

4.5 Pulley block system :

Ex.11 ***One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass. ($g = 9.8 m/s^2$)***

Sol. Since string is pulled by 100N force.

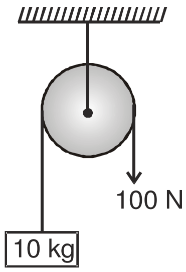
So tension in the string is 100 N.

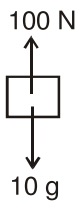
F.B.D. of 10 kg block

$100 - 10 g = 10 a$

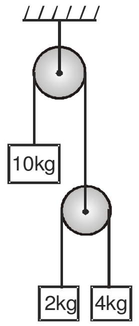
$100 - 10 \times 9.8 = 10 a$

$a = 0.2 m/s^2$

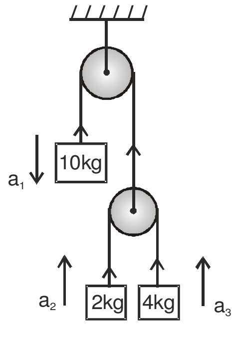




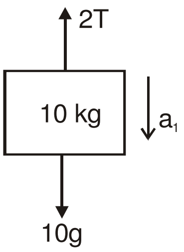
Ex.12 In the figure shown, find out acceleration of each block.



Sol. Now F.B.D. of each block and apply Newton's second law on each F.B.D

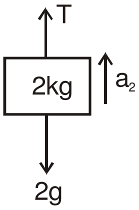


(1)



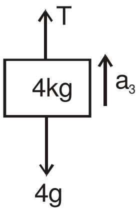
$\Rightarrow$ $10g - 2T = 10a_1$ ...(i)

(2)



$\Rightarrow$ $T - 2g = 2a_2$ ...(2)

(3)



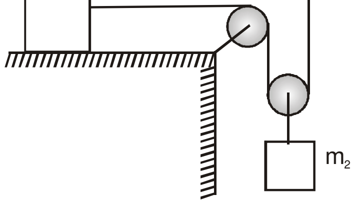
$\Rightarrow$ $T - 4g = 4a_3$ ...(3)

from constrain relation $2a_1 = a_2 + a_3$ ...(4)
Solving equations (1), (2), (3) and (4) we get

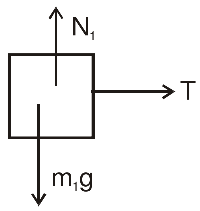
$$T = \frac{800}{23} N$$

$a_1 = 70/23$ m/s^2 (downward), $a_2 = 170/23$ m/s^2 (upward), $a_3 = 30/23$ m/s^2 (downward)

Ex.13 Find the acceleration of each block in the figure shown below; in terms of their masses m_1, m_2 and g. Neglect any friction.



Sol. Let T be the tension in the string that is assumed to be massless.



For mass m_1, the FBD shows that

$N_1 = m_1g$

Where N_1 is the force applied upward by plane on the mass m_1.

If acceleration of m_1 along horizontal is a_1. then

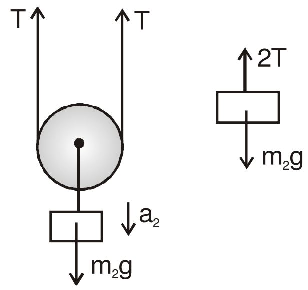
$T = m_1a_1$ ...(i)

For mass m_2, the FBD shows that

$m_2g - 2T = m_2a_2$ ...(ii)

Where a_2 is vertical acceleration of mass m_2.

Note that upward tension on m_2 is 2T applied by both sides of the string.



from constrain relation

$a_2 = \frac{a_1}{2}$

Thus, the acceleration of m_1 its twice that of m_2.

with this input, solving (i) and (ii) we find

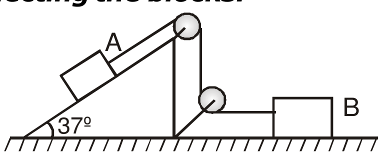
$a_1 = \frac{2m_2g}{4m_1 + m_2}$

$a_2 = \frac{m_2g}{4m_1 + m_2}$

Ex.14 Two blocks A and B each having a mass of 20 kg, rest on frictionless surfaces as shown in the figure below. Assuming the pulleys to be light and frictionless, compute :

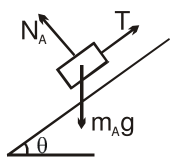
(a) the time required for block A, to move down by 2m on the plane, starting from rest,

(b) tension in the string, connecting the blocks.

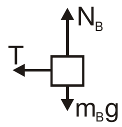


Sol.

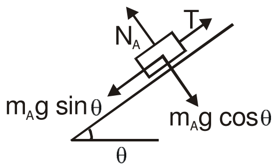
Step 1. Draw the FBDs for both the blocks. If tension in the string is T, then we have



and



Note that m_Ag, should better be resolved along and perpendicular to the plane, as the block A is moving along the plane.



Step 2. From FBDs, we write the force equations '

for block A where

$N_A = m_A\, g \cos\theta = 20 \times 10 \times 0.8 = 160$ N

and $m_Ag \sin\theta - T = m_A\, a$ ... (i)

Where 'a' is acceleration of masses of blocks A and B.

Similarly, force equations for block B are

$N_B = m_Bg = 20 \times 10 = 200$ N

and $T = m_Ba$ ...(ii)

From (i) and (ii), we obtain

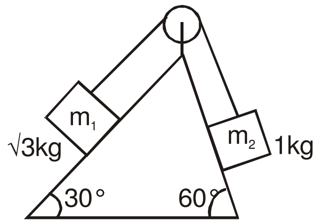
$$a = \frac{m_A g \sin\theta}{m_A + m_B} = \frac{20 \times 10 \times 0.6}{40} = 3\ ms^{-2}$$

$T = m_B a = 20 \times 3 = 60$ N

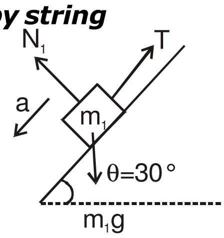
Step 3. With constant acceleration $a = 3\ ms^{-2}$, the block A moves down the inclined plane a distance S = 2 m in time t given by

$$S = \frac{1}{2}at^2 \text{ or } t = \sqrt{\frac{2S}{a}} = \frac{2}{\sqrt{3}} \text{ seconds.}$$

Ex.15 ***Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find :***
(i) acceleration of mass m_1 and m_2
(ii) tension in the string
(iii) net force on pulley exerted by string



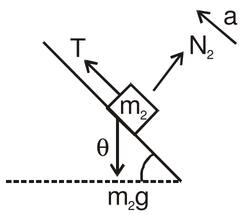
Sol. **F.B.D of m_1 :**



$m_1 g \sin\theta - T = m_1 a$

$$\frac{\sqrt{3}}{2}g - T = \sqrt{3}a \qquad ...(i)$$

F.B.D. of m_2 :



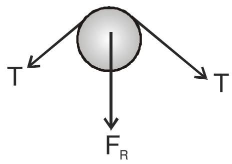
$T - m_2 g \sin\theta = m_2 a$

$$T - 1.\frac{\sqrt{3}}{2}g = 1.a \qquad ...(ii)$$

Adding eq. (i) and (ii) we get a = 0
Putting this value in eq. (i) we get

$$T = \frac{\sqrt{3}g}{2},$$

F.B.D. of pulley



$F_R = \sqrt{2}T$

$$F_R = \sqrt{\frac{3}{2}}g$$

5. NEWTONS' 3RD LAW OF MOTION :

Statement : "To every action there is equal and opposite reaction".

But what is the meaning of action and reaction and which force is action and which force is reaction? Every force that acts on body is due to the other bodies in environment. Suppose that a body A experiences a force $\vec{F}_{AB}$ due to other body B. Also body B will experience a force $\vec{F}_{BA}$ due to A. According to Newton third law two forces are equal in magnitude and opposite in direction Mathematically we write it as

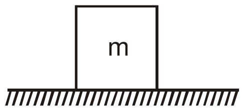
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Here we can take either $\vec{F}_{AB}$ or $\vec{F}_{BA}$ as action force and other will be the reaction force.

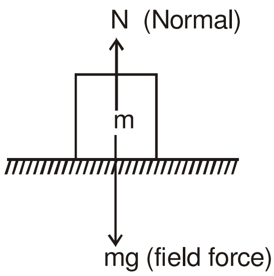
: (i) Action-Reaction pair acts on two different bodies.

(ii) Magnitude of force is same.
(iii) Direction of forces are in opposite direction.
(iv) For action-reaction pair there is no need of contact

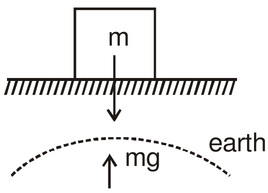
Ex.16 A block of mass 'm' is kept on the ground as shown in figure.
(i) Draw F.B.D. of block
(ii) Are forces acting on block action - reaction pair
(iii) If answer is no, draw action reaction pair.



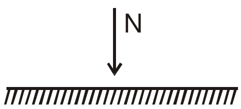
Sol. (i) F.B.D. of block



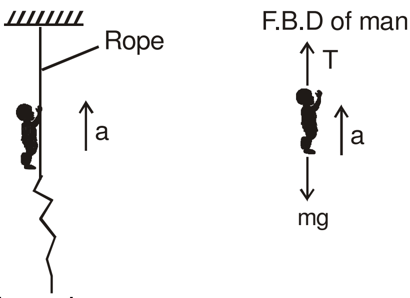
(ii) 'N' and Mg are not action - reaction pair. Since pair act on different bodies, and they are of same nature.
(iii) Pair of 'mg' of block acts on earth in opposite direction.



and pair of 'N' acts on surface as shown in figure.



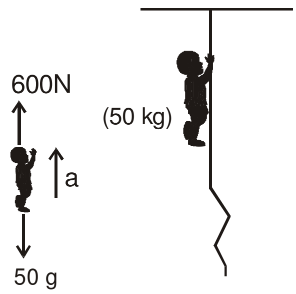
5.1 Climbing on the Rope :



Now three condition arises.
if T > mg ⇒ man accelerates in upward direction
T < mg ⇒ man accelerates in downward direction
T = mg ⇒ man's acceleration is zero

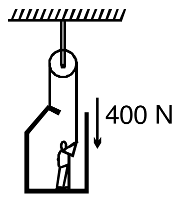
* Either climbing or decending on the rope man exerts force downward

Ex.17 If the breaking strength of string is 600N then find out the maximum acceleration of the man with which he can climb up the road



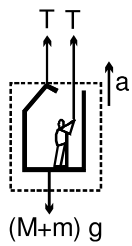
Sol. Maximum force that can be exerted on the man by the rope is 600 N.
F.B.D of man
$\Rightarrow$ 600 – 50 g = 50 a
a_{max} = 2 m/s^2

Ex.18 A 60 kg painter on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.



(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.
(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1 s ?
(iii) What force should apply now to maintain the constant speed of 1 m/s?

Sol. The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulles the rope.



(i) Applying Newton's Second Law
2T – (M + m)g = (M + m)a

or $a = \frac{2T-(M+m)g}{M+m}$

Here M = 60 kg; m = 15 kg ; T = 400 N
g = 10 m/s^2

$a = \frac{2(400)-(60+15)(10)}{60+15}$ = 0.67 m/s^2

(ii) To attain a speed of 1 m/s in one second the acceleration a must be 1 m/s^2
Thus, the applied force is

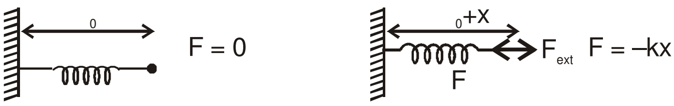
$F = \frac{1}{2}(M + m)(g + a) = (60 + 15)(10 + 1) = 412.5$ N

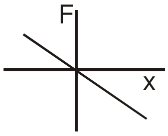
(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium
Thus, 2F – (M + m) g = 0

or $F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2}$ = 375 N

6. SPRING FORCE :

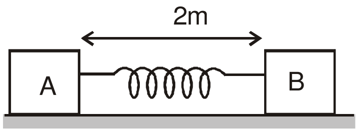
Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by F = –kx, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1})
When spring is in its natural length, spring force is zero.





Graph between spring force v/s x

Ex.19 Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m. Find spring force in following situations.



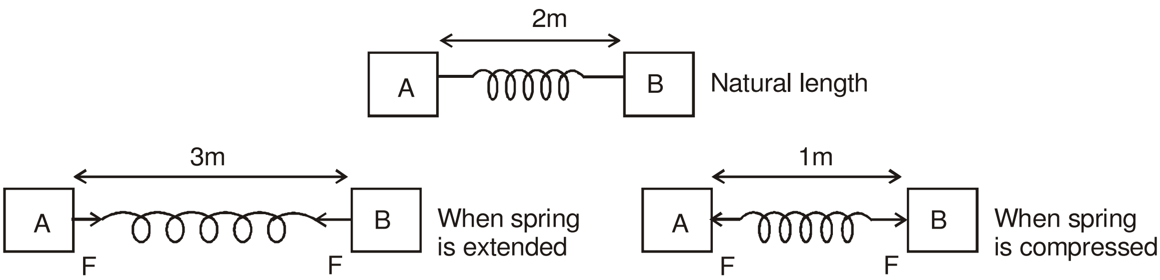
(a) If block 'A' and 'B' both are displaced by 0.5 m in same direction.
(b) If block 'A' and 'B' both are displaced by 0.5 m in opposite direction.

Sol. (a) Since both blocks are displaced by 0.5 m in same direcetion, so change in length of spring is zero. Hence, spring force is zero.

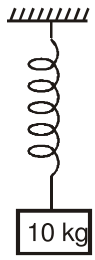
(b) In this case, change in length of spring is 1 m. So spring force is F = –Kx

= – (200). (1)

F = –200 N

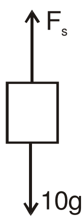


Ex.20 Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring (g = 10 m/s²)



Sol. In this situation, spring is in extended state so spring force acts in upward direction. Let x be the extension in the spring.

F.B.D. of 10 kg block :



$F_s = 10\,g$

$\Rightarrow$ Kx = 100

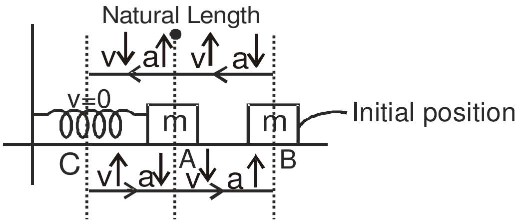
$\Rightarrow$ (100)x = (100)

$\Rightarrow$ x = 1 m

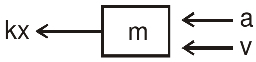
6.1 SPRING FORCE SYSTEM :

Initially the spring is in natural length at A with block m. But when the block displaced towards right then the spring is elongated and now block is released at B then the block move towards left due to spring force (kx).

Analysis of motion of block :

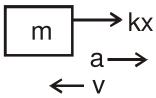


(i) From B to A speed of block increase and acceleration decreases. (due to decrease in spring force kx)

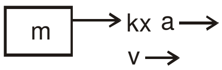


(ii) Due to inertia block crosses natural length at A.

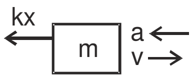
From A to C speed of the block decreases and acceleration increases.(due to increase in spring force kx)



(iii) At C the block stops momentarily at this instant and since the spring is compressed spring force is towards right and the block starts to move towards right. From C to A speed of block increases and acceleration decreases.(due to decrease in spring force kx)



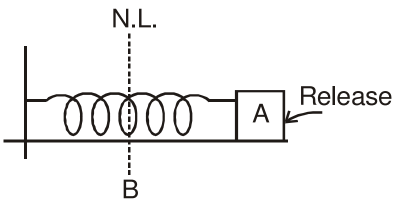
(iv) Again block crosses point A due to inertia then from A to B speed decreases and acceleration increases.



In this way block does SHM (to be expalined later) if no resistive force is acting on the block.

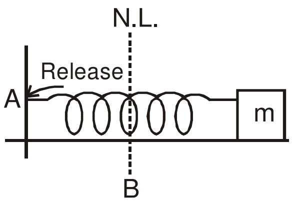
Note :

(i)



when the block A is released then it take some finite time to reach at B. i.e., spring force doesn't change instantaneously.

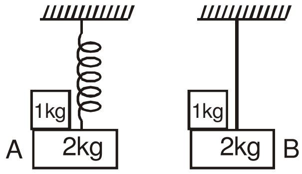
(2)



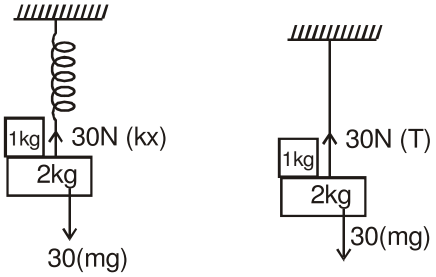
When point A of the spring is released in the above situation then the spring forces changes instantaneously and becomes zero because one end of the spring is free.

(3) In string tension may change instantaneously.

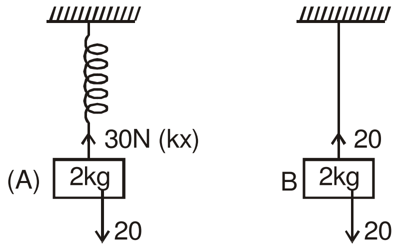
Ex.21 Find out the acceleration of 2 kg block in the figures shown at the instant 1 kg block falls from 2 kg block. (at t = 0)



Sol. F.B.D.s before fall of 1kg block

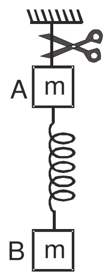


after the fall of the 1 kg block tension will change instantaneously but spring force (kx) doesn't change instantaneously. F.B.D.s just after the fall of 1 kg block



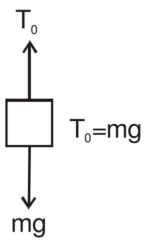
$a_A = \frac{30-20}{2} = 5\ m/s^2$ (upward) $\qquad a_B = 0\ m/s^2$

Ex.22 Two blocks 'A' and 'B' of same mass 'm' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block 'A' and 'B' just after the string is cut.



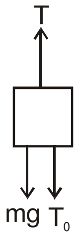
Sol. When block A and B are in equilibrium position

F.B.D of 'B'



...(i)

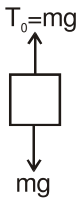
F.B.D of 'A'



$T = mg + T_0$ .....(ii)
$T = 2\ mg$

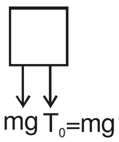
when string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of block A and B as shown in figure

F.B.D of 'B'



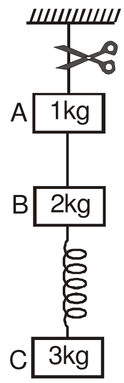
$T_0 - mg = m.a_B$
$a_B = 0$

F.B.D. of 'A'

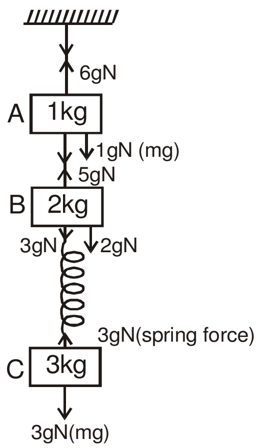


$mg + T_0 = m.\ a_A$
$2\ mg = m.\ a_A$
$a_A = 2g$ (downwards)

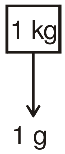
Ex.23 Find out the acceleration of 1kg, 2kg and 3kg block and tension in the string between 1 kg & 2 kg block just after cutting the string as shown in figure.



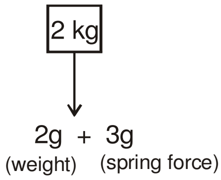
Sol. F.B.D before cutting of string



Let us assume the Tension in the string connecting blocks A & B becomes zero just after cutting the string then.



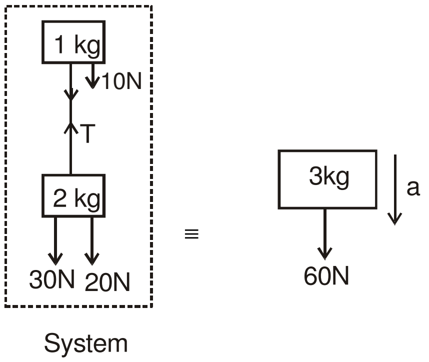
$a_1 = \frac{1g}{1} = g\ ms^{-2}$



$a_2 = \frac{5g}{2} = 2.5\ g\ ms^{-2}$

$\because$ $a_2 > a_1$ i.e., $\therefore$ $T \neq 0$

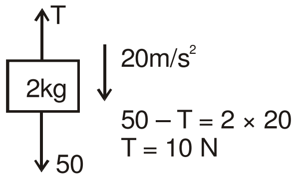
If $T \neq 0$ that means string is tight and Both block A & B will have same acceleration. So it will take as a system of 3 kg mass.



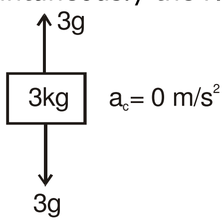
Total force down ward = 10 + 30 + 20 = 60 N

Total mass = 3 kg $\Rightarrow$ a = $\frac{60}{3}$ = 20 m/s^2

Now apply F_{net} = ma at block B.



$\because$ the spring force does not change instantaneously the F.B.D of 'C'



Reference Frame :

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) **Inertial reference frame :** Frame of reference moving with constant velocity or stationary

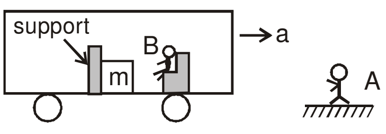
(b) **Non-inertial reference frame :** A frame of reference moving with non-zero acceleration

: (i) Although earth is a non inertial frame (due to rotation) but we always consider it as an inertial frame.

(ii) A body moving in circular path with constant speed is a non intertial frame (direction change cause acceleration)

7. PSEUDO FORCE :

Consider the following
example to understand the pseudo force concept



The block m in the bus is moving with constant acceleration a with respect to man A at ground. Force required for this acceleration is the normal reaction exerted by the support

So, $N = ma$ ..(i)

This block m is at rest with respect to man B who is in the bus (a non intertial frame). So the acceleration of the block with respect to man B is zero.

$N = m(0) = 0$ ..(ii)

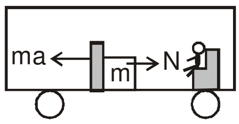
But the normal force is exerted in a non-inertial frame also. So the equation (ii) is wrong therefore we conclude that Newton's law is not valid in non-inertial frame.

If we want to apply Newton's law in non-inertial frame, then we can do so by using of the cencept pseudo force.

Pseudo force is an imaginary force, which in actual is not acting on the body. But after applying it on the body we can use Newton's laws in non-inertial frames.

This imaginary force is acting on the body only when we are solving the problem in a non-inertial frame of reference.

In the above example. The net force on the block m is zero with respect to man B after applying the pesudo force.



$N = ma$

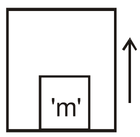
:

1. Direction of pseudo force is opposite to the acceleration of frame

2. Magnitude of pseudo force is equal to mass of the body which we are analyzing multiplied by acceleration of frame

3. Point of application of pseudo force is the centre of mass of the body which we are analysing

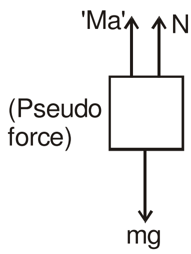
Ex.24 A box is moving upward with retardation 'a' < g, find the direction and magnitude of "pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



Sol. **Pseudo force** acts opposite to the direction of acceleration of reference frame.

pseudo force = ma in upward direction

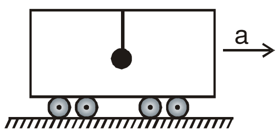
F.B.D of 'm' w.r.t box (non-inertial)



$N + ma = mg$

$N = mg - ma$

Ex.25 Figure shows a pendulum suspended from the roof of a car that has a constant acceleration a relative to the ground. Find the deflection of the pendulum from the vertical as observed from the ground frame and from the frame attached with the car.



Sol.

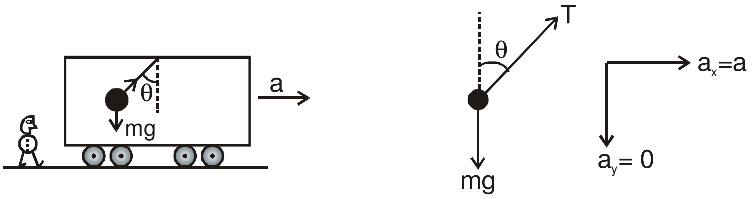


Figure represents free Body diagram of the bob w.r.t ground.

In an inertial frame the suspended bob has an acceleration a caused by the horizontal component of tension T.

$T \sin \theta = ma$ ...(i)

$T \cos\theta = mg$ ...(ii)

From equation (i) and (ii)

$$\tan \theta = \frac{a}{g} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

In a non-inertial frame

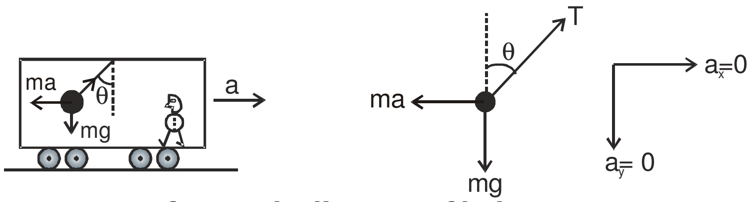


Figure represents free Body diagram of bob w.r.t car.

In the non-intertial frame of the car, the bob is in static equilibrium under the action of three froces, T, mg and ma (pseudo force)

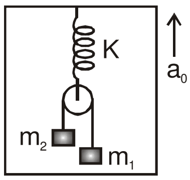
$T \sin \theta = ma$ ...(iii)

$T \cos \theta = mg$ ....(iv)

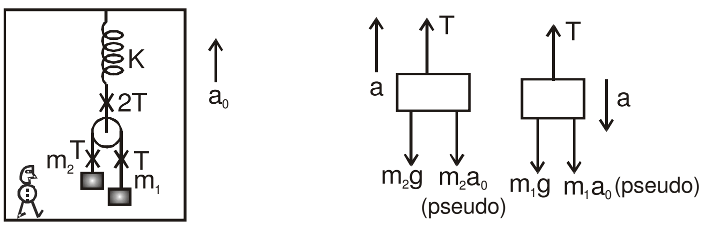
From equation (iii) and (iv)

$$\tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Ex.26 A pulley with two blocks system is attached to the ceiling of a lift moving upward with an acceleration a_0. Find the deformation in the spring.



Sol. Non-Inertial Frame



Let relative to the centre of pulley, m_1 accelerates downward with a and m_2 accelerates upwards with a. Applying Newton's 2nd law.

$m_1a + m_1a_0 - T = m_1a$ ...(i)

$T - m_2g - m_2a_0 = m_2a$ ...(ii)

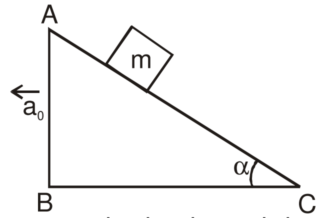
On adding (iv) and (v) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)(g + a_0) \qquad ...(iii)$$

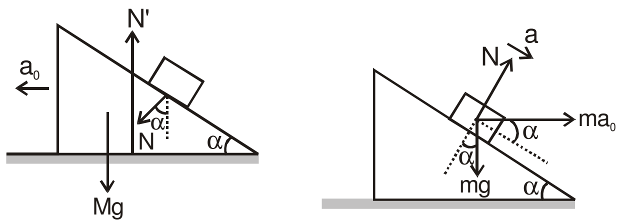
Substituting a in equation (i)

We get $T = \frac{2m_1m_2(g + a_0)}{m_1 + m_2}$ $\therefore\ x = \frac{F}{k} = \frac{2T}{k} = \frac{4m_1m_2(g + a_0)}{(m_1 + m_2)k}$

Ex.27 All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.



Sol. Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block from the frame of the prism The forces on the block are (figure)



(i) N normal force
(ii) mg downward (gravity),
(iii) ma_0 forward (Psuedo Force)
The block slides down the plane. Components of the forces parallel to the incline give

$ma_0 \cos\alpha + mg \sin \alpha = ma$

or, $a = a_0 \cos \alpha + g \sin \alpha$ ...(i)

Components of the forces perpendicular to the incline give

$N + ma_0 \sin \alpha = mg \cos \alpha$ ...(ii)

Now consider the motion of the prism from the ground frame. No pseudo force is needed as the frame used is inertial. The forces are (figure)
(i) Mg downward
(ii) N normal to the incline (by the block)
(iii) N' upward (by the horizontal surface)
Horizontal components give,
$N \sin \alpha = Ma_0$ or $N = Ma_0 / \sin \alpha$, ...(iii)
Putting in (ii)

$$\frac{Ma_0}{\sin\alpha} + ma_0 \sin \alpha = mg \cos \alpha$$

or, $a_0 = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$

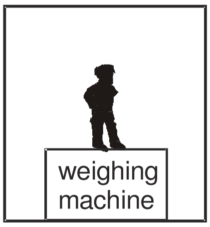
From (i) $a = \frac{mg\sin\alpha\cos^2\alpha}{M + m\sin^2\alpha} + g \sin \alpha = \frac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$

8. WEIGHING MACHING :

A weighing machine does not measure the weight but measures the force. exerted by object on its upper surface or we can say weighing machine measure normal force on the man.

8.1 Motion in a lift :

(A) If the lift is unaccelerated (v = 0 or constant)
In this case no pseudo force act on the man

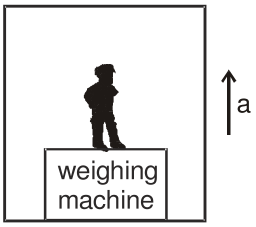


In this case the F.B.D. of the man

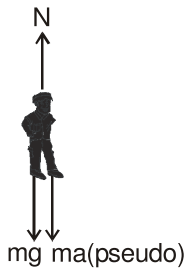
$N = mg$

In this case machine read the actual weight

(B) If the lift is accelerated upward.
(where a = constant)



F.B.D of man with respect to lift



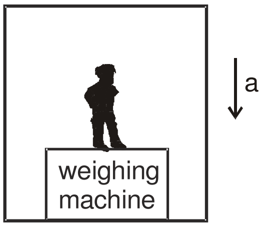
So weighing machine read

$N = m(g + a)$

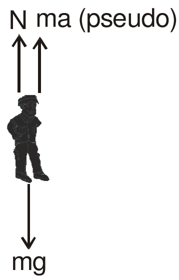
Apparent weight

N > Actual weight (mg)

(c) If the lift is accelerated down ward.



F.B.D of man with respect to lift



So weighing machine read

$N = m(g - a)$

Apparent weight

N < Actual weight (mg)

Note :

(i) If $a = g \Rightarrow N = 0$

Thus in a freely falling lift, the man will experience a state of weightlessness

(ii) If the lift is accelerated downwards such that a > g : So the man will be accelerated upward and will stay at the ceiling of the lift.

(iii) Apparent weight is greater than or less than actual weight only depends on the direction and magnitude of acceleration. Magnitude and direction of velocity doesn't play any roll in apparent weight.