Volume II

New Simplified

PHYSICS

A Reference Book for Class XI

S.L. Arora

DHANPAT RAI & Co.

Scanned by CamScanner

Including Value Based Questions

new

SIMPLIFIED PHYSICS

[A REFERENCE BOOK FOR CLASS XI]

VOLUME II

- Properties of Bulk Matter
- Thermodynamics
- Behaviour of Perfect Gas & Kinetic Theory of Gases
- Oscillations & Waves

S.L. ARORA
M.Sc., B.Ed.
Head, Department of Physics
The Delhi United Christian Sr. Sec. School
17, Raj Niwas Marg, Delhi.

DHANPAT RAI & CO. (Pvt.) Ltd.
EDUCATIONAL & TECHNICAL PUBLISHERS

Acc. No 9385
LRC
Scanned by CamScanner

Published by : GAGAN KAPUR
Dhanpat Rai & Co. (P) Ltd., Delhi

Sales Office : 1682, Nai Sarak, Delhi-110006
Phone : 2325 0251
Regd. Office : 4576/15, Agarwal Road
Darya Ganj, New Delhi-110002
Phone : 2324 7736, 37, 38, dhanpatrai@gmail.com

THE SCHRAM ACADEMY LIBRARY
Ass. No. 9385 LRC
One Schram Avenue, Maduravoyal Che

Edited by : Bhupesh Arora

REFERENCE

© S.L. ARORA

No part and style of the book may be copied in any form without the written permission of the author.

EDITIONS : 2006, 2007, 2009, 2012
EDITION : 2014
REPRINT : 2015
REPRINT : 2016

PRICE : ₹ 820 (*For Vol. I & II*)

Typesetting by : North Delhi Computer Convention, Delhi–110009, ndcc.in@gmail.com
Printed at : Natraj Offset, Delhi

Scanned by CamScanner

Contents

Chapters 1 to 8 are in Vol. I

Scanned by CamScanner

Chapter 11 THERMAL PROPERTIES OF MATTER 11.1 – 11.98

Chapter 12

Chapte…

Scanned by CamScanner

Chapter 12 THERMODYNAMICS 12.1 – 12.78

Chapter 13 KINETIC THEORY OF GASES 13.1 – 13.61

Scanned by CamScanner

Chapter 14 OSCILLATIONS 14.1 – 14.98

Chapter 15 WAVES 15.1 – 15.130

SAMPLE VALUE BASED QUESTIONS

Scanned by CamScanner

CHAPTER

9

MECHANICAL PROPERTIES OF SOLIDS

9.1 ELASTIC AND PLASTIC BEHAVIOUR OF SOLIDS

1. *Define the terms deforming force, elasticity and plasticity. What are perfectly elastic and perfectly plastic bodies ? Give examples.*

Elastic and plastic behaviour of solids. By a rigid body, we generally mean a hard solid object having a definite shape and size. In reality, solid bodies are not perfectly rigid. They can be stretched, compressed and bent. When an external force is applied, a body may get deformed. When the deforming force is removed, some bodies tend to regain their original size and shape while others do not show any such tendency. Let us define few terms to explain this behaviour of bodies.

Deforming force. If a force is applied on a body which is neither free to move nor free to rotate, the molecules of the body are forced to undergo a change in their relative positions. As a result, the body may undergo a change in length, volume or shape. *A force which changes the size or shape of a body is called a deforming force.*

Elasticity. *If a body regains its original size and shape after the removal of deforming force, it is said to be elastic body and this property is called elasticity.* For example, if we stretch a rubber band and release it, it snaps back to its original length.

Perfectly elastic body. *If a body regains its original size and shape completely and immediately after the removal of deforming force, it is said to be a perfectly elastic body. The nearest approach to a perfectly elastic body is quartz fibre.*

Plasticity. *If a body does not regain its original size and shape even after the removal of deforming force, it is said to be a plastic body and this property is called plasticity.* For example, if we stretch a piece of chewing-gum and release it, it will not regain its original size and shape.

Perfectly plastic body. If *a body does not show any tendency to regain its original size and shape even after the removal of deforming force, it is said to be a perfectly plastic body.* Putty and paraffin wax are nearly perfectly plastic bodies.

For Your Knowledge

▲ No body is perfectly elastic or perfectly plastic. All the bodies found in nature lie between these two limits. When the elastic behaviour of a body decreases, its plastic behaviour increases.

Scanned by CamScanner

9.2 ▼ ELASTIC BEHAVIOUR IN TERMS OF INTERATOMIC FORCES

2. *Give an explanation of the elastic properties of materials in terms of interatomic/intermolecular forces.*

Elastic behaviour in terms of interatomic forces. The atoms in a solid are held together by interatomic forces. The variations of potential energy U and interatomic force F with interatomic separation r are shown in Figs. 9.1(*a*) and (*b*) respectively.

When the interatomic separation r is large, the potential energy of the atoms is negative and the interatomic force is *attractive*. At some particular separation r_0, the potential energy becomes minimum and the interatomic force becomes zero. This separation r_0 is called *normal or equilibrium separation*.

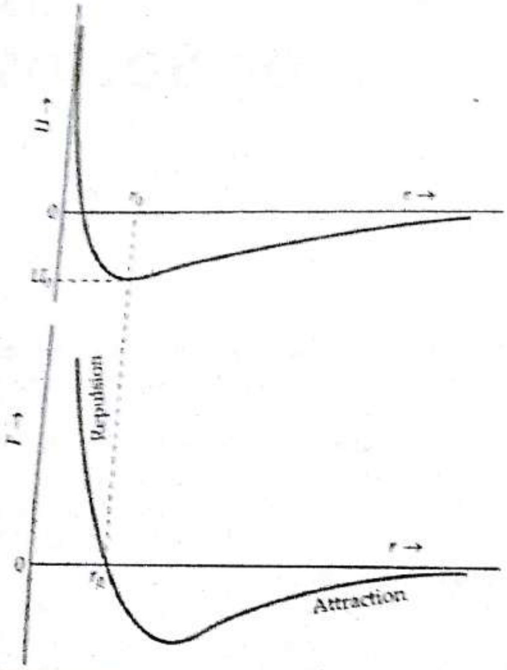


Fig. 9.1. (*a*) Interatomic potential energy *U*, (*b*) Force *F*, between two identical atoms as a function of interatomic separation *r*.

When separation reduces below r_0, the potential energy increases steeply and the interatomic force becomes *repulsive*.

Normally, the atoms occupy the positions ($r = r_0$) of minimum potential energy called the positions of stable equilibrium. When a tensile or compressive force is applied on a body, its atoms are pulled apart or pushed closer together to a distance r, greater than or smaller than r_0. When the deforming force is removed, the interatomic forces of attraction / repulsion restore the atoms to their equilibrium positions. The body regains its original size and shape. The stronger the interatomic forces, the smaller will be the displacements of atoms from the equilibrium positions and hence greater is the elasticity (or modulus of elasticity) of the material.

3. *Explain elastic behaviour of solids on the basis of mechanical spring-ball model of a solid.*

Elastic behaviour on the basis of spring-ball model of a solid. The atoms in a solid may be regarded as mass points or small balls connected in three-dimensional space through springs. The springs represent the interatomic forces. This is called spring-ball model of a solid, as shown in Fig. 9.2.

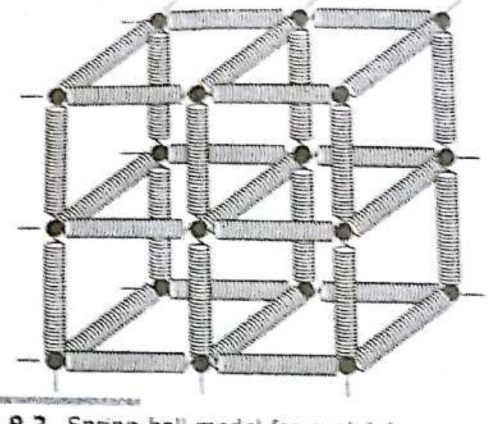


Fig. 9.2 Spring-ball model for explaining elastic behaviour of solids.

Normally, the balls occupy the positions of minimum potential energy or zero interatomic force. When any ball is displaced from its equilibrium position, the various springs connected to it exert a resultant force on this ball. This force tends to bring the ball to its equilibrium position. This explains the elastic behaviour of solid in terms of microscopic nature of the solid.

9.3 ▼ STRESS

4. *Define the term stress. Give its units and dimensions. Describe the different types of stress.*

Stress. If a body gets deformed under the action of an external force, then at each section of the body an internal force of reaction is set up which tends to restore the body into its original state. *The internal restoring force set up per unit area of cross-section of the deformed body is called stress.* As the restoring force is equal and opposite to the external deforming force, therefore

$$\text{Stress} = \frac{\text{Applied force}}{\text{Area}} = \frac{F}{A}$$

Scanned by CamScanner

The SI unit of stress is Nm^{-2} and the CGS unit is dyne cm^{-2}. The dimensional formula of stress is $[ML^{-1}T^{-2}]$.

Types of stress :

(*i*) **Tensile stress.** It is the restoring force set up per unit cross-sectional area of a body when the length of the body increases in the direction of the deforming force. It is also known as *longitudinal stress.*

(*ii*) **Compressional stress.** It is the restoring force set up per unit cross-sectional area of a body when its length decreases under a deforming force.

(*iii*) **Hydrostatic stress.** If a body is subjected to a uniform force from all sides, then the corresponding stress is called *hydrostatic stress.*

(*iv*) **Tangential or Shearing stress.** When a deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body. The tangential force applied per unit area is equal to the tangential stress.

9.4 STRAIN

5. *Define the term strain. Why it has no units and dimensions ? What are different types of strain ?*

Strain. When a deforming force acts on a body, the body undergoes a change in its shape and size. *The ratio of the change in any dimension produced in the body to the original dimension is called strain.*

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

As strain is the ratio of two like quantities, it has no units and dimensions.

Types of strain :

(*i*) **Longitudinal strain.** *It is defined as the increase in length per unit original length,* when the body is deformed by external forces.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

(*ii*) **Volumetric strain.** *It is defined as the change in volume per unit original volume,* when the body is deformed by external forces.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original Volume}} = \frac{\Delta V}{V}$$

(*iii*) **Shear strain.** *It is defined as the angle θ (in radian), through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.*

$$\text{Shear strain} = \theta = \tan\theta$$

$$= \frac{\text{Relative displacement between 2 parallel planes}}{\text{Distance between parallel planes}}$$

9.5 ELASTIC LIMIT

6. *What is meant by the term elastic limit ?*

Elastic limit. If a small load is suspended from a wire, its length increases. When the load is removed, the wire regains its original length. But if a sufficiently large force is suspended from the wire, it is found that the wire does not regain its original length after the load is removed. *The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit.* If the deforming force exceeds the elastic limit, the body acquires a *permanent set* or *deformation* and is said to be *overstrained.*

9.6 HOOKE'S LAW AND MODULUS OF ELASTICITY

7. *State Hooke's law. How can it be verified experimentally ?*

Hooke's law. From experimental investigations, *Robert Hooke,* an English physicist (1635-1703 A.D.), formulated in 1679 a law known after him as Hooke's law which states that *the extension produced in a wire is directly proportional to the load applied.*

In 1807, *Thomas Young* pointed out that the strain is proportional to the extension of the wire and the stress is proportional to the load applied. He, therefore, modified Hooke's law to the more general form as follows :

Within the elastic limit, the stress is directly proportional to strain. Thus within the elastic limit,

$$\text{Stress} \propto \text{Strain}$$

or $$\text{Stress} = \text{Constant} \times \text{Strain}$$

or $$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

The constant of proportionality is called **modulus of elasticity** or **coefficient of elasticity** of the material. Its value depends on the nature of the material of the body and the manner in which it is deformed.

Experimental verification of Hooke's law. As shown in Fig. 9.3, suspend a metallic spring from a rigid support and attach to its lower end a pan and a pointer. Arrange a scale in the vertical position so that a pointer is able to move along it. Read the position of the pointer on the scale when the pan is empty. Place a weight of 50 gram in the pan. Note the position of the pointer on the scale. The difference between the two readings gives the extension produced in the spring by the weight added in the pan.

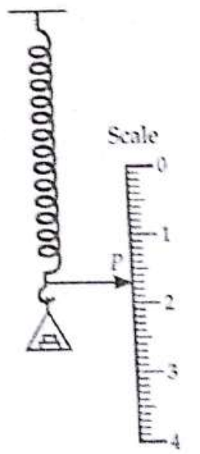


Fig. 9.3 Arrangement for studying Hooke's law.

Scanned by CamScanner

Increase the weight in the pan in steps of 50 gram and note the corresponding extensions. Plot a graph between the extension of spring and the total load producing it. The graph is a straight line, as shown in Fig. 9.4. This indicates that extension ∝ load applied. This verifies Hooke's law.

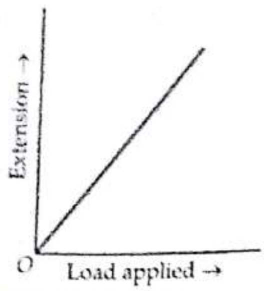


Fig. 9.4 Load-extension graph.

For Your Knowledge

- Like Boyle's law, Hooke's law is one of the earliest quantitative relationships in science.
- Hooke's law is valid only in the linear position of the stress-strain curve. The law is not valid for large values of strains.
- Stress is not a vector quantity since, unlike a force, the stress cannot be assigned a specific direction.
- When a wire, suspended from a ceiling, is stretched by a weight (F) suspended from its lower end, the ceiling exerts a force on the wire equal and opposite to the weight F. But the tension at any cross-section A of the wire is just F and not $2F$. Hence the tensile stress which is equal to the tension per unit area is equal to F/A.

8. *Define modulus of elasticity. Give its units and dimensions. What are different types of moduli of elasticity ?*

Modulus of elasticity. *The modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of stress to the corresponding strain, within the elastic limit.*

$$\text{Modulus of elasticity, } E = \frac{\text{Stress}}{\text{Strain}}$$

The SI unit of modulus of elasticity is Nm^{-2} and its dimensions are $[ML^{-1}T^{-2}]$.

Different types of moduli of elasticity. Corresponding to the three types of strain, we have three important moduli of elasticity :

(*i*) *Young's modulus* (Y), *i.e.*, the modulus of elasticity of length.

(*ii*) *Bulk modulus* (κ), *i.e.*, the modulus of elasticity of volume.

(*iii*) *Modulus of rigidity* or *shear modulus* (η), *i.e.*, modulus of elasticity of shape.

9.7 YOUNG'S MODULUS OF ELASTICITY

9. *Define Young's modulus of elasticity. Give its units and dimensions.*

Young's modulus of elasticity. *Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called Young's modulus of the material of the wire.*

As shown in Fig. 9.5, suppose a wire of length l and cross-sectional area A suffers an increase in length Δl under a force F acting along its length l. Then Young's modulus is given by

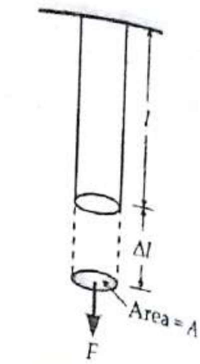


Fig. 9.5 Young's modulus of elasticity.

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta l/l}$$

or $$Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

If the wire has a circular cross-section of radius r, then

$$Y = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}.$$

If $l = 1$ m, $A = 1\ m^2$ and $\Delta l = 1$ m, then $Y = F$.

Thus, *Young's modulus of elasticity is equal to the force required to extend a wire of unit length and unit area of cross-section by unit amount, i.e., the force required to double the length of the wire.*

Units and dimensions of Y. The SI unit of Young's modulus is Nm^{-2} or pascal (Pa) and its CGS unit is dyne cm^{-2}. The dimensional formula of Y is $[ML^{-1}T^{-2}]$.

9.8 STRESS-STRAIN CURVE FOR A METALLIC WIRE

10. *Explain what happens when the load on a metal wire suspended from a rigid support is gradually increased. Illustrate your answer with a suitable stress-strain graph.*

Stress-strain curve for a metallic wire. Fig. 9.6, shows a stress-strain curve for a metal wire which is gradually being loaded.

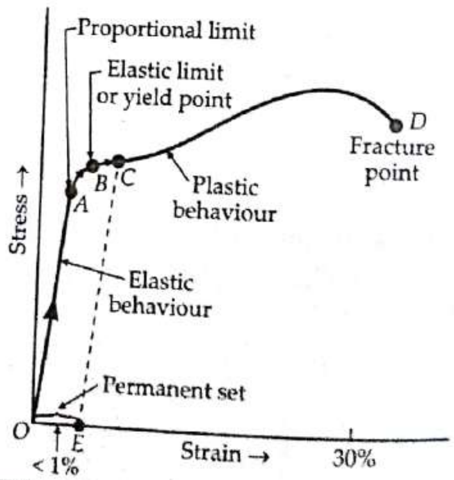


Fig. 9.6 A typical stress-strain curve for a ductile metal.

(*i*) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A, Hooke's law is obeyed. The point A is called the ***proportional limit***. In this region, the wire is perfectly elastic.

(*ii*) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called ***elastic region*** and the point B is called ***elastic limit*** or ***yield point***. The stress corresponding to the yield point is called ***yield strength*** (S_y). Upto point B, the elastic forces of the material are ***conservative*** *i.e.*, when the material returns to its original size, the work done in producing the deformation is completely recovered.

(*iii*) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line CE. Even on reducing the stress to zero, a residual strain equal to OE is left in the wire. The material is said to have acquired a ***permanent set***. The fact that the stress-strain curve is not retaced on reversing the strain is called ***elastic hysteresis***.

(*iv*) If the load is increased beyond the point C, there is large increase in the strain or the length of the wire. In this region, the constrictions (called necks and waists) develop at few points along the length of the wire and the wire ultimately breaks at the point D, called the ***fracture point***. In the region between B and D, the length of wire goes on increasing even without any addition of load. This region is called *plastic region* and the material is said to undergo ***plastic flow*** or ***plastic deformation***. The stress corresponding to the breaking point is called ***ultimate strength*** or ***tensile strength*** of the material.

9.9 DETERMINATION OF YOUNG'S MODULUS OF THE MATERIAL OF A WIRE

11. *Explain an experiment for the determination of Young's modulus of the material of a wire.*

Experiment to determine the Young's modulus of the material of a wire. A simple experimental arrangement used for the determination of Young's modulus of the material of a wire is shown in Fig. 9.7. It consists of two long straight wires of same length and equal radius suspended side by side from a fixed rigid support. The wire A, called the *reference wire*, carries a main milliammeter scale M and below it a heavy fixed load. This load keeps the wire taught and free from kinks. The wire B, called the *experimental wire*, carries a vernier scale at its bottom. The vernier scale can slide against the main scale attached to the reference wire. A hanger is attached at the lower end of the vernier scale. Slotted half kg weights can be slipped into this hanger.

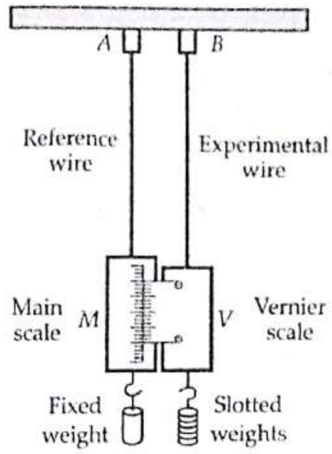


Fig. 9.7 Experimental arrangement for the determination of Young's modulus.

With the help of a screw gauge, the radius of the experimental wire is measured at several places. Let r be the initial average radius and L the initial length of the experimental wire. A small initial load, say 1 kg, is put on the hanger. This keeps the experimental wire straight and kink free. The vernier scale reading is noted. A half kg weight is added to the hanger. The wire is allowed to elongate for a minute. The vernier scale reading is again noted. The difference between the two vernier readings gives the extension produced due to the extra weight added. The weight is gradually increased in few steps and every time we note the extension produced.

A graph is plotted between the load applied and extension produced. It will be a straight line passing through the origin, as shown in Fig. 9.8.

Slope of the load-extension line

$$= \tan\theta = \frac{\Delta L}{Mg}$$

$$\text{Stress} = \frac{Mg}{\pi r^2}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

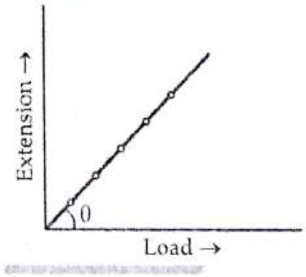


Fig. 9.8 Load-extension graph.

The Young's modulus of the material of the experimental wire will be

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{Mg}{\pi r^2} \cdot \frac{L}{\Delta L} = \frac{L}{\pi r^2 \tan\theta}$$

Scanned by CamScanner

9.10 CLASSIFICATION OF MATERIALS ON THE BASIS OF STRESS-STRAIN CURVE

12. *Distinguish between ductile and brittle materials on the basis of stress-strain curve.*

(*i*) **Ductile materials.** *The materials which have large plastic range of extension are called ductile materials.* As shown in the stress-strain curve of Fig. 9.9, their fracture point is widely separated from the elastic limit. Such materials undergo an irreversible increase in length before snapping. So they can be drawn into thin wires. For example, copper, silver, iron, aluminium, etc.

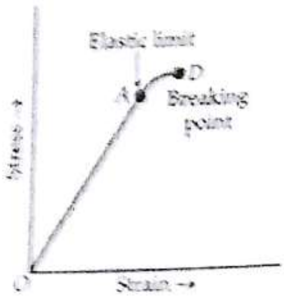


Fig. 9.9 Stress-strain curve for a brittle material.

(*ii*) **Brittle materials.** *The materials which have very small range of plastic extension are called brittle materials.* Such materials break as soon as the stress is increased beyond the elastic limit. Their breaking point lies just close to their elastic limit, as shown in Fig. 9.9. For example, cast iron, glass, ceramics, etc.

13. *Explain malleability on the basis of load-compression curve.*

Malleability. When a solid is compressed, a stage is reached beyond which it cannot recover its original shape after the deforming force is removed. This is the elastic limit (point A') for compression. The solid then behaves like a plastic body. The yield point (B') obtained under compression is called ***crushing point.*** After this stage, metals are said to be malleable *i.e.*, they can be hammered or rolled into thin sheets. For example, gold, silver, lead, etc.

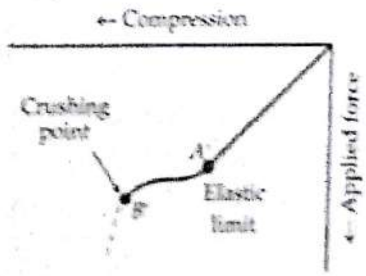


Fig. 9.10 Load-compression curve for a metal.

9.11 ELASTOMERS

14. *What are elastomers ? Give examples. Draw a stress-strain curve for an elastomer.*

Elastomers. *The materials which can be elastically stretched to large values of strain are called elastomers.* For example, rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed. There is no well defined plastic region, rubber just breaks when pulled beyond a certain limit. Its Young's modulus is very small, about 3×10^5 Nm^{-2} at slow strains. Elastic

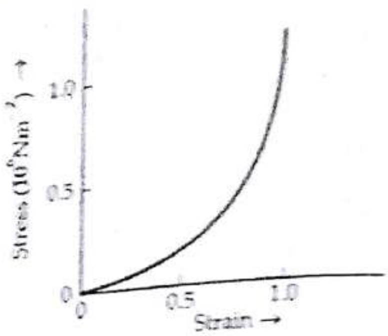


Fig. 9.11 Stress vs. strain curve for the elastic tissue of aorta.

region in such cases is very large, but the material does not obey Hooke's law. In our body, the elastic tissue of aorta (the large blood vessel carrying blood from the heart) is an elastomer, for which the stress-strain curve is shown in Fig. 9.11.

Table 9.1 Young's moduli, ultimate strengths and yield strengths of some materials

Substance	Density ρ (kgm^{-3})	Young's modulus Y (10^9 Nm^{-2})	Ultimate strength S_u (10^6 Nm^{-2})	Yield strength S_y (10^6 Nm^{-2})
Aluminium	2710	70	110	95
Copper	8890	110	400	200
Iron (wrought)	7800-7900	190	330	170
Steel	7860	200	400	250
Glass	2190	65	50	-
Concrete	2320	30	40	-
Wood	525	13	50	-
Bone	1900	9	170	-
Polystyrene	1050	3	48	-

The above table shows that metals have large Young's moduli. Such materials require large forces to produce small changes in length *i.e.*, they are highly elastic. Thus steel is more elastic than copper, brass and aluminium. That is why steel is preferred for making heavy-duty machines and structural designs. On the other hand, the materials like wood, bone, concrete and glass have small Young's moduli.

Examples based on Young's Modulus

FORMULAE USED

1. $\text{Stress} = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A}$
2. $\text{Longitudinal strain} = \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{\Delta l}{l}$
3. Young's modulus
$$= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} \quad \text{or} \quad Y = \frac{F/A}{\Delta l / l} = \frac{F}{A} \cdot \frac{l}{\Delta l}$$
4. Percentage increase in length,
$$\frac{\Delta l}{l} \times 100 = \frac{F}{AY} \times 100$$

UNITS USED

Force F is in newton, area A in m^2, stress in Nm^{-2}, Young's modulus Y in Nm^{-2} or Pa, strain $\Delta l / l$ has no units

EXAMPLE 1. *The length of a suspended wire increases by 10^{-4} of its original length when a stress of $10^7\ Nm^{-2}$ is applied on it. Calculate the Young's modulus of the material of the wire.* **[Delhi 03C, 05C]**

Solution. $\text{Strain} = \dfrac{\Delta l}{l} = 10^{-4}$, $\text{stress} = 10^7\ Nm^{-2}$

Young's modulus,
$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{10^7\ Nm^{-2}}{10^{-4}} = \mathbf{10^{11}\ Nm^{-2}}.$$

EXAMPLE 2. *A uniform wire of steel of length 2.5 m and density 8.0 gcm^{-3} weighs 50 g. When stretched by a force of 10 kgf, the length increases by 2 mm Calculate Young's modulus of steel.*

Solution. Here $l = 2.5\ m = 250\ cm$,
$$\Delta l = 2\ mm = 0.2\ cm,$$
$$F = 10\ kg\ f = 10 \times 9.8\ N = 10 \times 9.8 \times 10^5\ dyne$$
$$\text{Mass} = \text{Volume} \times \text{density}$$
$$\therefore \quad A = \frac{\text{Mass}}{l \times \rho} = \frac{50}{250 \times 8} = 0.025\ cm^2$$

Young's modulus,
$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{10 \times 9.8 \times 10^5 \times 250}{0.025 \times 0.2}$$
$$= \mathbf{4.9 \times 10^{11}\ dyne\ cm^{-2}}.$$

EXAMPLE 3. *A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force F stretches it along its length. Calculate (a) the stress, (b) elongation, and (c) strain on the rod. Given that the Young's modulus, Y, of the structural steel is $2.0 \times 10^{11}\ Nm^{-2}$.* **[NCERT]**

Solution. Here $r = 10\ mm = 0.01\ m$, $l = 1\ m$,
$F = 100\ kN = 10^5\ N$, $Y = 2.0 \times 10^{11}\ Nm^{-2}$

(a) Stress
$$= \frac{F}{A} = \frac{F}{\pi r^2} = \frac{10^5\ N}{(22/7) \times (0.01\ m)^2}$$
$$= \mathbf{3.18 \times 10^8\ Nm^{-2}}.$$

(b) As $\quad Y = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$\therefore$ Elongation,
$$\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{3.18 \times 10^8 \times 1}{2.0 \times 10^{11}}$$
$$= 1.59 \times 10^{-3}\ m = \mathbf{1.59\ mm}.$$

(c) Strain
$$= \frac{\Delta l}{l} = \frac{1.59 \times 10^{-3}\ m}{1\ m}$$
$$= 1.59 \times 10^{-3} = \mathbf{0.16\%}.$$

EXAMPLE 4. *What is the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg wt ? Young's modulus of elasticity of the wire is 12.5×10^{11} dyne cm^{-2}.*

Solution. Given $r = 1.25\ mm = 0.125\ cm$,
$$F = 100 \times 9.8 = 980\ N = 98 \times 10^6\ dyne$$
$$Y = 12.5 \times 10^{11}\ dyne\ cm^{-2}$$

As $\quad Y = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l} \quad \text{or} \quad \dfrac{\Delta l}{l} = \dfrac{F}{AY} = \dfrac{F}{\pi r^2 Y}$

$\therefore$ The percentage increase in length is
$$\frac{\Delta l}{l} \times 100 = \frac{F \times 100}{\pi r^2 Y} = \frac{98 \times 10^6 \times 7 \times 100}{22 \times (0.125)^2 \times 12.5 \times 10^{11}}$$
$$= 15.965 \times 10^{-2} = \mathbf{0.16\%}.$$

EXAMPLE 5. *The breaking stress for a metal is $7.8 \times 10^9\ Nm^{-2}$. Calculate the maximum length of the wire made of this metal which may be suspended without breaking. The density of the metal $= 7.8 \times 10^3\ kg\ m^{-3}$. Take $g = 10\ N\ kg^{-1}$.* **[Delhi 03]**

Solution. Breaking stress
$$= \text{Maximum stress that the wire can withstand}$$
$$= 7.8 \times 10^9\ Nm^{-2}.$$

When the wire is suspended vertically, it tends to break under its own weight. Let its length be l and cross-sectional area A.

Scanned by CamScanner

Weight of wire $= mg =$ volume × density × $g = Al\rho g$

$$\text{Stress} = \frac{\text{Weight}}{A} = \frac{Al\rho g}{A} = l\rho g$$

For the wire not to break,

$$l\rho g = \text{Breaking stress} = 7.8 \times 10^9 \text{ Nm}^{-2}$$

$$\therefore \quad l = \frac{7.8 \times 10^9}{\rho g} = \frac{7.8 \times 10^9}{7.8 \times 10^3 \times 10} = \mathbf{10^5 \text{ m}}.$$

EXAMPLE 6. *A rubber string 10 m long is suspended from a rigid support at its one end. Calculate the extension in the string due to its own weight. The density of rubber is 1.5×10^3 kg m^{-3} and Young's modulus for the rubber is 5×10^6 Nm^{-2}. Take $g = 10$ N kg^{-1}.* **[Delhi 03]**

Solution. Let the area of cross-section of the string be A m^2. Then the weight of the string is

$$W = mg = \text{volume} \times \text{density} \times g$$
$$= 10\ A \times 1.5 \times 10^3 \times 10 = 1.5 \times 10^5 A \text{ N}$$

Longitudinal stress

$$= \frac{W}{A} = 1.5 \times 10^5 \text{ Nm}^{-2}.$$

As the weight of the string acts on its centre of gravity, so it produces extension only in 5 m length of the string. If Δl be the extension in the string, then

$$\text{Longitudinal strain} = \frac{\Delta l}{l} = \frac{\Delta l}{5}$$

Young's modulus,

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

or

$$5 \times 10^6 = \frac{1.5 \times 10^5}{\Delta l / 5}$$

$$\therefore \quad \Delta l = \frac{1.5 \times 10^5 \times 5}{5 \times 10^6} = \mathbf{0.15 \text{ m}}.$$

EXAMPLE 7. *A silica glass rod has a diameter of 1 cm and is 10 cm long. The ultimate strength of glass is 50×10^6 Nm^{-2}. Estimate the largest mass that can be hung from it without breaking it. Take $g = 10$ N kg^{-1}.*

Solution. Radius, $r = \frac{1}{2}$ cm $= 0.5 \times 10^{-2}$ m, ultimate strength $= 50 \times 10^6$ Nm^{-2}.

Let M be the largest mass that can be hung. Then

$$\text{Ultimate strength} = \frac{Mg}{\pi r^2}$$

or

$$50 \times 10^6 = \frac{M \times 10}{3.14 \times (0.5 \times 10^{-2})^2}$$

or

$$M = \frac{50 \times 10^6 \times 3.14 \times 0.25 \times 10^{-4}}{10}$$

$$= \mathbf{392.5 \text{ kg}}.$$

EXAMPLE 8. *A composite wire of uniform diameter 3.0 mm consisting of a copper wire of length 2.2 m and a steel wire of length 1.6 m stretches under a load by 0.7 mm. Calculate the load, given that the Young's modulus for copper is 1.1×10^{11} Pa and for steel is 2.0×10^{11} Pa.* **[NCERT ; Delhi 09]**

Solution. Here $r = \frac{3}{2}$ mm $= 1.5 \times 10^{-3}$ m,

$$l_c = 2.2 \text{ m}, \ l_s = 1.6 \text{ m}$$
$$\Delta l_c + \Delta l_s = 0.7 \text{ mm} = 0.7 \times 10^{-3} \text{ m}$$
$$Y_c = 1.1 \times 10^{11} \text{ Pa}, \ Y_s = 2.0 \times 10^{11} \text{ Pa}$$

As same load (say F) is being applied on both the wires, which have same area of cross-section A, so stress is same for both wires.

$$\text{But} \quad \text{Stress} = \frac{F}{A}$$

$$= \text{Young's modulus} \times \text{strain} = Y \times \frac{\Delta l}{l}$$

Now, Stress on copper wire = Stress on steel wire

$$\therefore \quad Y_c \times \frac{\Delta l_c}{l_c} = Y_s \times \frac{\Delta l_s}{l_s}$$

or

$$\frac{\Delta l_c}{\Delta l_s} = \frac{Y_s \times l_c}{Y_c \times l_s} = \frac{2.0 \times 10^{11} \times 2.2}{1.1 \times 10^{11} \times 1.6} = 2.5$$

or

$$\Delta l_c = 2.5 \Delta l_s$$

But

$$\Delta l_c + \Delta l_s = 0.7 \times 10^{-3} \text{ m}$$

or

$$3.5\ \Delta l_s = 0.7 \times 10^{-3} \text{ m}$$

or

$$\Delta l_s = \frac{0.7 \times 10^{-3}}{3.5} = 2.0 \times 10^{-4} \text{ m}$$

and

$$\Delta l_c = 2.5 \times 2.0 \times 10^{-4} = 5.0 \times 10^{-4} \text{ m}$$

$$\text{Load,} \quad F = A \times Y_c \times \frac{\Delta l_c}{l_c} = \pi r^2\ Y_c \times \frac{\Delta l_c}{l_c}$$

$$= \frac{22}{7} \times (1.5 \times 10^{-3})^2 \times 1.1 \times 10^{11} \times \frac{5.0 \times 10^{-4}}{2.2}$$

$$= \mathbf{176.8 \text{ N}}.$$

EXAMPLE 9. *The maximum stress that can be applied to the material of a wire used to suspend an elevator is 1.3×10^8 Nm^{-2}. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 ms^{-2}, what is the minimum diameter of the wire ?*

Solution. As the elevator moves up, the tension in the wire is

$$F = mg + ma = m(g + a)$$
$$= 900 \times (9.8 + 2.2) = 10{,}800 \text{ N}$$

$$\text{Stress in the wire} = \frac{F}{A} = \frac{F}{\pi r^2}$$

Clearly, when the stress is maximum, r is minimum.

Scanned by CamScanner

$\therefore$ Maximum stress $= \dfrac{F}{\pi r_{min}^2}$

or $r_{min}^2 = \dfrac{F}{\pi \times \text{Maximum stress}}$

$= \dfrac{10800}{3.14 \times 1.3 \times 10^8} = 0.2645 \times 10^{-4}$ m

or $r_{min} = 0.5142 \times 10^{-2}$ m

Minimum diameter

$= 2\, r_{min} = 2 \times 0.5142 \times 10^{-2}$

$= \mathbf{1.0284 \times 10^{-2}}$ **m.**

Example 10. *A mass of 100 gram is attached to the end of a rubber string 49 cm long and having an area of cross-section 20 mm². The string is whirled round, horizontally at a constant speed of 40 rps in a circle of radius 51 cm. Find Young's modulus of rubber.*

Solution. When the mass is rotated at the end of the rubber string, the restoring force in the string is equal to the centripetal force.

$\therefore \quad F = mr\omega^2 = mr(2\pi\nu)^2$

$= 100 \times 51 \times (2 \times \pi \times 40)^2$ dyne

Also $\quad l = 49$ cm, $\Delta l = 51 - 49 = 2$ cm,

$A = 20 \text{ mm}^2 = 20 \times 10^{-2} \text{ cm}^2$

Hence $\quad Y = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$= \dfrac{100 \times 51 \times 4 \times 9.87 \times 1600 \times 49}{20 \times 10^{-2} \times 2} \quad [\pi^2 = 9.87]$

$= 3.95 \times 10^{10}$ dyne cm^{-2}

$= \mathbf{3.95 \times 10^9\ Nm^{-2}}$**.**

Example 11. *A uniform heavy rod of weight W, cross-sectional area A and length l is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglecting the lateral contraction, find the elongation produced in the rod.*

Solution. As shown in Fig. 9.12, consider a small element of thickness dx at distance x from the fixed support. Force acting on the element dx is

F = Weight of length $(l - x)$ of the rod

$= \dfrac{W}{l}(l - x)$

Elongation of the element

$=$ Original length $\times \dfrac{\text{stress}}{Y}$

$= dx \times \dfrac{F/A}{Y} = \dfrac{W}{l\,Ay}(l - x)\,dx$

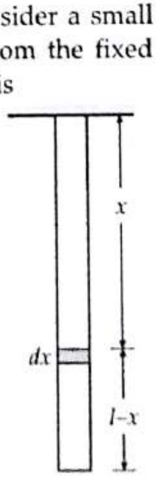


Fig. 9.12

Total elongation produced in the rod

$= \dfrac{W}{l\,AY}\int_0^l (l - x)\,dx = \dfrac{W}{l\,Ay}\left[lx - \dfrac{x^2}{2}\right]_0^l$

$= \dfrac{W}{l\,Ay}\left[l^2 - \dfrac{l^2}{2}\right] = \dfrac{Wl}{2Ay}.$

Example 12. *A steel wire of uniform cross-section of 1 mm² is heated to 70°C and stretched by tying its two ends rigidly. Calculate the change in the tension of the wire when the temperature falls from 70°C to 35°C. Coefficient of linear expansion of steel is $1.1 \times 10^{-5}\,{}^\circ C^{-1}$ and the Young's modulus is 2.0×10^{11} Nm^{-2}.*

Solution. Here $A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$,

$\Delta T = 70 - 35 = 35°$,

$\alpha = 1.1 \times 10^{-5}\,{}^\circ C^{-1}, Y = 2.0 \times 10^{11} \text{ Nm}^{-2}$

Increase in length $\Delta l = l\,\alpha\,\Delta T$

$\therefore$ Strain $= \dfrac{\Delta l}{l} = \alpha\,\Delta T = 1.1 \times 10^{-5} \times 35 = 38.5 \times 10^{-5}$.

If T is the tension in the wire due to the decrease in temperature, then

Stress $= \dfrac{T}{A} = \dfrac{T}{10^{-6}}$ Nm^{-2}

But Stress = Y × Strain

$\therefore \quad \dfrac{T}{10^{-6}} = 2.0 \times 10^{11} \times 38.5 \times 10^{-5}$

or $\quad T = 2.0 \times 38.5 = \mathbf{77.0\ N.}$

Example 13. *In a human pyramid in a circus, the entire weight of the balanced group is supported by the legs of a performer who is lying on his back (as shown in Fig. 9.13). The combined mass of all the persons performing the act, and the tables, plaques etc. involved is 280 kg. The mass of the performer lying on his back at the bottom of the pyramid is 60 kg. Each thighbone (femur) of this performer has a length of 50 cm and an effective radius of 2.0 cm. Determine the amount by which each thighbone gets compressed under the extra load.* [NCERT]

Fig. 9.13

Scanned by CamScanner

Solution. Total mass of all the performers, tables, plaques, etc.

$$= 280 \text{ kg}$$

Mass of the performer $= 60$ kg

Mass supported by the legs of the performer at the bottom of the pyramid,

$$M = 280 - 60 = 220 \text{ kg}$$

Weight of the supported mass,

$$W = Mg = 220 \times 9.8 = 2156 \text{ N}$$

Weight supported by each thighbone of the performer,

$$F = \frac{W}{2} = \frac{1}{2} \times 2156 = 1078 \text{ N}$$

Young's modulus of bone, $Y = 9.4 \times 10^9 \text{ Nm}^{-2}$

Length of each thighbone, $l = 0.5$ m

Radius of a thighbone, $r = 2.0 \text{ cm} = 2 \times 10^{-2}$ m

Cross-sectional area of the thighbone,

$$A = \pi r^2 = 3.14 \times (2 \times 10^{-2})^2$$
$$= 1.26 \times 10^{-3} \text{ m}^2$$

Compression produced in each thighbone of the performer,

$$\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{1078 \times 0.5}{1.26 \times 10^{-3} \times 9.4 \times 10^9} \text{ m}$$
$$= 4.55 \times 10^{-5} \text{ m} = \mathbf{4.55 \times 10^{-3}} \textbf{ cm.}$$

PROBLEMS FOR PRACTICE

1. A wire increases by 10^{-3} of its length when a stress of $1 \times 10^8 \text{ Nm}^{-2}$ is applied to it. What is the Young's modulus of the material of the wire ? **[Delhi 98]** (**Ans.** 10^{11} Nm^{-2})

2. What force is required to stretch a steel wire 1 cm^2 in cross-section to double its length ? Given $Y = 2 \times 10^{11} \text{ Nm}^{-2}$. (**Ans.** 2×10^7 N)

3. Find the stress to be applied to a steel wire to stretch it by 0.025% of its original length. Y for steel is $9 \times 10^{10} \text{ Nm}^{-2}$. (**Ans.** $2.25 \times 10^7 \text{ Nm}^{-2}$)

4. A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg-wt. Find the increase in its length, if the Young's modulus of steel wire is $2.4 \times 10^{-12} \text{ dyne cm}^{-2}$. **[Delhi 05]** (**Ans.** 0.0041 cm)

5. Two wires made of the same material are subjected to forces in the ratio of 1 : 4. Their lengths are in the ratio 8 : 1 and diameter in the ratio 2 : 1. Find the ratio of their extensions. (**Ans.** 1 : 2)

6. A wire elongates by 9 mm when a load of 10 kg is suspended from it. What is the elongation when its radius is doubled, if all other quantities are same as before ? (**Ans.** 2.25 mm)

7. The breaking stress of aluminium is $7.5 \times 10^7 \text{ Nm}^{-2}$. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is $2.7 \times 10^3 \text{ kg m}^{-3}$. (**Ans.** 2.83×10^3 m)

8. A steel wire of length 5.0 m and cross-section $3.0 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.0 m and cross-section $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of Young's modulus of steel to that of copper ? **[Punjab 91]** (**Ans.** 2.22)

9. A stress of 1 kg mm^{-2} is applied to a wire of which Young's modulus is 10^{11} Nm^{-2}. Find the percentage increase in length. (**Ans.** 0.0098%)

10. Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is 1 cm, find by how much is each wire elongated ? Given Y for steel $= 20 \times 10^{11} \text{ dyne cm}^{-2}$ and Y for copper $= 12 \times 10^{11} \text{ dyne cm}^{-2}$. (**Ans.** 0.375 cm and 0.625 cm)

11. Two parallel steel wires A and B are fixed to rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wires are in the ratio 4 : 5 and their radii are in the ratio 4 : 3. The increase in the length of the wire A is 1 mm. Calculate the increase in the length of the wire B. (**Ans.** 2.22 mm)

12. Two wires of equal cross-section but one made of steel and the other copper are joined end to end. When the combination is kept under tension, the elongation in the two wires is found to be equal. Given Young's moduli of steel and copper are $2.0 \times 10^{11} \text{ Nm}^{-2}$ and $1.1 \times 10^{11} \text{ Nm}^{-2}$. Find the ratio between the lengths of steel and copper wires. (**Ans.** 20 : 11)

13. A lift is tied with thick iron wires and its mass is 1000 kg. If the maximum acceleration of lift is 1.2 ms^{-2} and the maximum safe stress is $1.4 \times 10^8 \text{ Nm}^{-2}$, find the minimum diameter of the wire. Take $g = 9.8 \text{ ms}^{-2}$. (**Ans.** 0.01 m)

14. The length of a metal wire is l_1 when the tension in it is T_1 and l_2 when the tension in it is T_2. Find the original length of the wire. $\left(\textbf{Ans. } \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}\right)$

15. A metal bar of length l and area of cross-section A is rigidly clamped between two walls. The Young's modulus of the material is Y and the coefficient of linear expansion is α. The bar is heated so that its temperature is increased by ΔT. Find the force exerted at the ends of the bar. (**Ans.** $YA\,\alpha\,\Delta T$)

Scanned by CamScanner

✗ HINTS

1. Stress $= 10^8$ Nm^{-2}, Strain $= \frac{\Delta l}{l} = 10^{-3}$

Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{10^8}{10^{-3}} = \mathbf{10^{11}\ Nm^{-2}}$.

2. Here $l = \Delta l = x$ (say),

$$A = 1\,\text{cm}^2 = 10^{-4}\ \text{m}^2,\ Y = 2 \times 10^{11}\ \text{Nm}^{-2}.$$

As $Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$

$$F = \frac{YA \cdot \Delta l}{l} = \frac{2 \times 10^{11} \times 10^{-4} \times x}{x} = \mathbf{2 \times 10^7\ N.}$$

3. Here $\Delta l = \frac{0.025}{100} l$ or $\frac{\Delta l}{l} = \frac{0.025}{100}$

$$\text{Stress} = Y \times \text{strain} = 9 \times 10^{10} \times \frac{0.025}{100}$$

$$= \mathbf{2.25 \times 10^7\ Nm^{-2}}.$$

4. $\Delta l = \frac{Fl}{\pi r^2 Y} = \frac{(5000 \times 980) \times 400}{3.14 \times (0.025)^2 \times 2.4 \times 10^{12}} = \mathbf{0.0041\ cm.}$

5. $Y = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}$ or $\Delta l = \frac{Fl}{\pi r^2 Y}$

Both wires are of same material, so their Y is same.

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{F_2} \cdot \frac{l_1}{l_2} \cdot \frac{r_2^2}{r_1^2} = \frac{1}{4} \times \frac{8}{1} \times \frac{1}{4} = \mathbf{1 : 2.}$$

6. $Y = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}$ or $r^2 \Delta l = \frac{Fl}{\pi Y} =$ a constant

$$\therefore \quad r_1^2 \Delta l_1 = r_2^2\, \Delta l_2$$

Given $\Delta l_1 = 9$ mm, $r_2 = 2r_1$

$$\therefore\ r_1^2 \times 9\ \text{mm} = 4r_1^2 \times \Delta l_2 \quad \text{or} \quad \Delta l_2 = \frac{9}{4} = \mathbf{2.25\ mm.}$$

12. $Y_s = \frac{F}{A} \cdot \frac{l_s}{\Delta l_s}$ and $Y_c = \frac{F}{A} \cdot \frac{l_c}{\Delta l_c}$

$$\therefore \quad \frac{Y_s}{Y_c} = \frac{l_s}{l_c} \cdot \frac{\Delta l_c}{\Delta l_s} = \frac{l_s}{l_c} \qquad [\because \Delta l_c = \Delta l_s]$$

or $\frac{l_s}{l_c} = \frac{Y_s}{Y_c} = \frac{2.0 \times 10^{11}}{1.1 \times 10^{11}} = \mathbf{20 : 11.}$

13. Tension in the wire,

$$F = m(g + a) = 1000\,(9.8 + 1.2) = 11{,}000\ \text{N}$$

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi (d/2)^2} = \frac{4F}{\pi d^2}$$

or $d^2 = \frac{4F}{\pi \times \text{stress}} = \frac{4 \times 11000 \times 7}{22 \times 1.4 \times 10^8} = 10^{-4}$

or $d = 10^{-2}$ m $=$ **0.01 m.**

14. Let l be the original length and A the area of cross-section of the wire.

Change in length in first case $= l_1 - l$

Change in length in second case $= l_2 - l$

$$Y = \frac{T_1}{A} \cdot \frac{l}{l_1 - l} = \frac{T_2}{A} \cdot \frac{l}{l_2 - l}$$

or $T_1 l_2 - T_1 l = T_2 l_1 - T_2 l$

or $l(T_2 - T_1) = T_2 l_1 - T_1 l_2$ or $l = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

15. Change in length $\Delta l = l\,\alpha\,\Delta T$

$$\therefore\ Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{A} \cdot \frac{l}{l\,\alpha\,\Delta T} \quad \text{or} \quad F = YA\,\alpha\,\Delta T.$$

9.12 BULK MODULUS OF ELASTICITY

15. *Define bulk modulus of elasticity. Give its units and dimensions.*

Bulk modulus of elasticity. *Within the elastic limit, the ratio of normal stress to the volumetric strain is called bulk modulus of elasticity.*

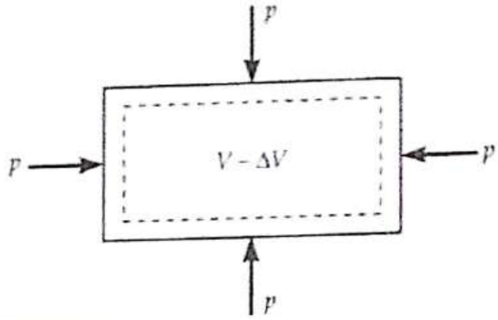


Fig. 9.14 Bulk modulus of elasticity.

Consider a body of volume V and surface area A. Suppose a force F acts uniformly over the whole surface of the body and it decreases the volume by ΔV as shown in Fig. 9.14. Then bulk modulus of elasticity is given by

$$\kappa = \frac{\text{Normal stress}}{\text{Volumetric strain}} = -\frac{F/A}{\Delta V/V}$$

or $$\kappa = -\frac{F}{A} \cdot \frac{V}{\Delta V} = -\frac{pV}{\Delta V}$$

where $p (= F/A)$ is the normal pressure. Negative sign shows that the volume decreases with the increase in stress.

Units and dimensions of κ. The SI unit of bulk modulus is Nm^{-2} or Pascal (Pa) and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$.

16. *Define the term compressibility. Give its units and dimensions.*

Compressibility. *The reciprocal of the bulk modulus of a material is called its compressibility.*

$$\text{Compressibility} = \frac{1}{\kappa}$$

SI unit of compressibility $= N^{-1}m^2$.

CGS unit of compressibility $=$ dyne^{-1}cm^2.

The dimensional formula of compressibility is $[M^{-1}LT^2]$.

Scanned by CamScanner

Table 9.2 Bulk moduli (κ) of some common materials

Material	κ (10^9 Nm^{-2})
Solids	
Aluminium	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260
Steel	160
Liquids	
Water	2.2
Ethanol	0.9
Carbon disulphide	1.56
Glycerine	4.76
Mercury	25
Gases	
Air (at STP)	1.0×10^{-4}

The above table shows that bulk moduli of the solids are in the range of 10^{11} Nm^{-2}, and are about 50 times larger than that of water. Thus solids are least compressible while gases are most compressible. Gases are about a million times more compressible than solids. The solids are incompressible because of tight coupling between the neighbouring atoms. The molecules in liquids are less tightly bound than in liquids. The molecules in gases are very poorly coupled to the neighbouring molecules.

Examples based on Bulk Modulus

FORMULAE USED

1. Volumetric stress $= \frac{F}{A} = p$, the applied pressure
2. Volumetric strain $= \frac{\Delta V}{V}$
3. Bulk modulus $= \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$

or $\kappa = -\frac{F/A}{\Delta V/V} = -\frac{p}{\Delta V/V} = -V\frac{p}{\Delta V}$

Negative sign indicates the decrease in volume with the increase in stress

4. Compressibility $= \frac{1}{\kappa} = -\frac{\Delta V}{pV}$

UNITS USED

Bulk modulus κ is in Nm^{-2} and compressibility in $N^{-1}m^2$ or Pa^{-1}.

EXAMPLE 14. *The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change is volume is 10% keeping temperature constant. Find the bulk modulus of the medium.* [IIT 05]

Solution. Here : $p = 1.165 \times 10^5 - 1.01 \times 10^5$

$= 0.155 \times 10^5$ Pa

$\frac{\Delta V}{V} = 10\% = 0.1$

Bulk modulus of the medium,

$$\kappa = \frac{p}{\Delta V/V} = \frac{0.155 \times 10^5}{0.1} = \mathbf{1.55 \times 10^5 Pa.}$$

EXAMPLE 15. *The average depth of Indian ocean is about 3000 m. Calculate the fractional compression $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^9 Nm^{-2}.* [NCERT]

Solution. Stress = Pressure exerted by a water column of height 3000 m

$= h\rho g = 3000 \text{ m} \times 1000 \text{ kg m}^{-3} \times 10 \text{ ms}^{-2}$

$= \mathbf{3 \times 10^7 \ Nm^{-2}}$

As bulk modulus, $\kappa = \frac{\text{Stress}}{\Delta V/V}$

∴ Fractional compression,

$$\frac{\Delta V}{V} = \frac{\text{Stress}}{\kappa} = \frac{3 \times 10^7 \text{ Nm}^{-2}}{2.2 \times 10^9 \text{ Nm}^{-2}}$$

$= 1.36 \times 10^{-2} = \mathbf{1.36\%}.$

EXAMPLE 16. *A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the bulk modulus of the material of the sphere. Density of sea water may be taken as 1.0×10^3 kgm^{-3}.*

Solution. Here $\frac{\Delta V}{V} = \frac{0.01}{100}$, $h = 1 \text{ km} = 10^3$ m,

$\rho = 1.0 \times 10^3 \text{ kg m}^{-3}$

$p = h\rho g = 10^3 \times 1.0 \times 10^3 \times 9.8 = 9.8 \times 10^6 \text{ Nm}^{-2}$

$$\kappa = \frac{p}{\Delta V/V} = \frac{9.8 \times 10^6 \times 100}{0.01} = \mathbf{9.8 \times 10^{10} \ Nm^{-2}}.$$

EXAMPLE 17. *If the normal density of sea water is 1.00 g cm^{-3}, what will be its density at a depth of 3 km ? Given compressibility of water = 0.0005 per atmosphere, 1 atmospheric pressure = 10^6 dyne cm^{-2}, $g = 980$ cms^{-2}.*

Solution. $\kappa = \frac{1}{\text{Compressibility}} = \frac{1}{0.00005}$

$= 2 \times 10^4 \text{ atm} = 2 \times 10^4 \times 10^6$

$= 2 \times 10^{10} \text{ dyne cm}^{-2}$

$p = h\rho g = 3 \times 10^5 \times 1 \times 980$

$= 294 \times 10^6 \text{ dyne cm}^{-2}$.

$[\because h = 3 \text{ km} = 3 \times 10^5 \text{ cm}, \rho \text{ (water)} = 1 \text{ g cm}^{-3}]$

Scanned by CamScanner

As $\kappa = \dfrac{pV}{\Delta V}$

$\therefore \quad \Delta V = \dfrac{pV}{\kappa} = \dfrac{294 \times 10^6 \times 1}{2 \times 10^{10}}$

$= 1.47 \times 10^{-2}\ cm^3 \qquad [\because V = 1\ cm^3]$

Volume of 1 g of water at a depth of 3 km,

$V' = V - \Delta V = 1 - 1.47 \times 10^{-2}$

$= 0.9853\ cm^3$

$\text{Density} = \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{1\ g}{0.9853\ cm^3}$

$= \mathbf{1.0149\ g\ cm^{-3}}$.

EXAMPLE 18. *A solid cube is subjected to a pressure of $5 \times 10^5\ Nm^{-2}$. Each side of the cube is shortened by 1%. Find volumetric strain and bulk modulus of elasticity of the cube.*

Solution. Let l be the initial length of each side of cube.

Final length of the cube

$= l - 1\%\ \text{of}\ l = \left(1 - \dfrac{1}{100}\right) l$

Initial volume,

$V_i = l^3 = V$ (say)

Final volume,

$$V_f = \left(1 - \frac{1}{100}\right)^3 l^3 = \left(1 - \frac{1}{100}\right)^3 V$$

Change in volume,

$$\Delta V = V_f - V_i = V\left[\left(1 - \frac{1}{100}\right)^3 - 1\right]$$

Volumetric strain

$$= \frac{\Delta V}{V} = \left(1 - \frac{1}{100}\right)^3 - 1 \simeq \left[1 - 3 \times \frac{1}{100}\right] - 1$$

$[(1-x)^n \simeq 1 - nx \text{ for } x << 1]$

$= -\dfrac{3}{100} = \mathbf{0.03}.$

Normal stress = Applied pressure = $5 \times 10^5\ Nm^{-2}$

Bulk modulus,

$$\kappa = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

$$= \frac{5 \times 10^5}{0.03} = \mathbf{1.67 \times 10^7\ Nm^{-2}}$$

EXAMPLE 19. *Calculate the pressure required to stop the increase in volume of a copper block when it is heated from 50° to 70°C. Coefficient of linear expansion of copper $= 8.0 \times 10^{-6}\ {}^\circ C^{-1}$ and bulk modulus of elasticity $= 3.6 \times 10^{11}\ Nm^{-2}$.*

Solution. When a block of volume V is heated through a temperature of ΔT, the change in volume is

$$\Delta V = \gamma V \Delta T$$

where $\gamma\ (= 3\alpha)$ is the coefficient of cubical expansion.

$\therefore$ Volume strain $= \dfrac{\Delta V}{V} = \gamma \Delta T$

Bulk modulus, $\kappa = \dfrac{p}{\Delta V / V} = \dfrac{p}{\gamma \Delta T}$

Pressure, $p = \kappa \gamma \Delta T$

Here $\kappa = 3.6 \times 10^{11}\ Nm^{-2}$,

$\gamma = 3\alpha = 3 \times 8.0 \times 10^{-6} = 24 \times 10^{-6}\ {}^\circ C^{-1}$,

$\Delta T = 70 - 50 = 20°C$

$\therefore \quad p = 3.6 \times 10^{11} \times 24 \times 10^{-6} \times 20$

$= \mathbf{1.728 \times 10^8\ Nm^{-2}}$.

PROBLEMS FOR PRACTICE

1. A solid sphere of radius 10 cm is subjected to a uniform pressure $= 5 \times 10^8\ Nm^{-2}$. Determine the consequent change in volume. Bulk modulus of the material of the sphere is equal to $3.14 \times 10^{11}\ Nm^{-2}$. (**Ans.** $6.67 \times 10^{-6} m^3$)

2. Find the change in volume which $1\ m^3$ of water will undergo when taken from the surface to the bottom of a lake 100 m deep. Given volume elasticity of water is 22,000 atmosphere. (**Ans.** $4.4 \times 10^{-4}\ m^3$)

3. A solid ball 300 cm in diameter is submerged in a lake at such a depth that the pressure exerted by water is $1.00\ kgf\ cm^{-2}$. Find the change in volume of the ball at this depth. κ for material of the ball $= 1.00 \times 10^{13}$ dyne cm^{-2}. (**Ans.** $1.385\ cm^3$)

4. A spherical ball contracts in volume by 0.0098% when subjected to a pressure of 100 atm. Calculate its bulk modulus. Given 1 atm $= 1.01 \times 10^5\ Nm^{-2}$. (**Ans.** $1.033 \times 10^{11}\ Nm^{-2}$)

5. What increase in pressure will be needed to decrease the volume of $1.0\ m^3$ of water by 10 c.c. ? The bulk modulus of water is $0.21 \times 10^{10}\ Nm^{-2}$. (**Ans.** $2.1 \times 10^4\ Nm^{-2}$)

6. Determine the fractional change in volume as the pressure of the atmosphere (1.0×10^5 Pa) around a metal block is reduced to zero by placing the block in vacuum. The bulk modulus for the block is $1.25 \times 10^{11}\ Nm^{-2}$. (**Ans.** 8×10^{-7})

7. Find the density of the metal under a pressure of $20{,}000\ N\ cm^{-2}$. Given density of the metal $= 11\ g\ cm^{-3}$, bulk modulus of the metal $= 8 \times 10^9\ Nm^{-2}$. (**Ans.** $11.28\ g\ cm^{-3}$)

Scanned by CamScanner

8. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. What will be the decrease in volume of 100 cm^3 of water under pressure of 100 atmosphere ? **(Ans. 0.4 cm^3)**

9. On taking a solid ball of rubber from the surface to the bottom of a lake of 180 m depth, the reduction of the volume of the ball is 0.1%. The density of water of the lake is 1.0×10^3 kg m^{-3}. Determine the value of the bulk modulus of elasticity of rubber. Take $g = 10$ ms^{-2}. **(Ans. 1.8×10^9 Nm^{-2})**

10. A uniform pressure P is exerted on all sides of a solid cube at temperature t°C. By what amount should the temperature of the cube be raised in order to bring its volume back to the volume it had before the pressure was applied, if the bulk modulus and coefficient of volume expansion of the material are κ and γ respectively ? $\left(\textbf{Ans. } \frac{P}{\gamma\kappa}\right)$

11. A solid sphere of radius R made of a material of bulk modulus κ is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, find fractional change in the radius of the sphere. [IIT 88] $\left(\textbf{Ans. } \frac{\Delta R}{R} = \frac{Mg}{3A\kappa}\right)$

HINTS

1. $\Delta V = \frac{pV}{\kappa} = \frac{p \times \frac{4}{3}\pi r^3}{\kappa} = \frac{5 \times 10^8 \times 4 \times 3.14 \times (0.1)^3}{3 \times 3.14 \times 10^{11}}$

$= \mathbf{6.67 \times 10^{-6}\ m^3}$.

2. Here $V = 1\ m^3$, $h = 100$ m, $g = 9.8\ ms^{-2}$,

ρ (water) $= 1000$ kg m^{-3},

$p = h\rho g = 100 \times 1000 \times 9.8 = 9.8 \times 10^5\ Nm^{-2}$

$\kappa = 22{,}000$ atm $= 22{,}000 \times 1.013 \times 10^5\ Nm^{-2}$

$= 22.286 \times 10^8\ Nm^{-2}$

$\Delta V = \frac{pV}{\kappa} = \frac{9.8 \times 10^5 \times 1}{22.286 \times 10^8} = \mathbf{4.4 \times 10^{-4}\ m^3}$.

3. Here $\kappa = 1.00 \times 10^{13}$ dyne cm^{-2}, $r = 300/2 = 150$ cm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (150)^3 = 1.413 \times 10^7\ cm^3$

$p = 1.00$ kg f $cm^{-2} = 1000$ g f cm^{-2}

$= 1000 \times 980$ dyne cm^{-2}

$\Delta V = \frac{pV}{\kappa} = \frac{1000 \times 980 \times 1.413 \times 10^7}{1.00 \times 10^{13}} = \mathbf{1.385\ cm^3}$.

4. Here $\frac{\Delta V}{V} = \frac{0.0098}{100}$,

$p = 100$ atm $= 100 \times 1.01 \times 10^5\ Nm^{-2}$

$\therefore \quad \kappa = p \times \frac{V}{\Delta V} = \frac{100 \times 1.01 \times 10^5 \times 100}{0.0098}$

$= \mathbf{1.033 \times 10^{11}\ Nm^{-2}}$.

5. Here $V = 1.0\ m^3$,

$\Delta V = 10$ c.c. $= 10 \times 10^{-6}\ m^3 = 10^{-5}\ m^3$

$\kappa = 0.21 \times 10^{10}\ Nm^{-2}$

$\therefore \quad p = \kappa \times \frac{\Delta V}{V} = \frac{0.21 \times 10^{10} \times 10^{-5}}{1.0}$

$= \mathbf{2.1 \times 10^4\ Nm^{-2}}$.

6. $\frac{\Delta V}{V} = \frac{p}{\kappa} = \frac{1.0 \times 10^5}{1.25 \times 10^{11}} = \mathbf{8 \times 10^{-7}}$.

7. Here $p = 2 \times 10^4\ N\,cm^{-2} = 2 \times 10^8\ Nm^{-2}$,

$\kappa = 8 \times 10^9\ Nm^{-2}$

$\Delta V = \frac{pV}{\kappa} = \frac{2 \times 10^8 \times V}{8 \times 10^9} = \frac{V}{40}$

Final volume,

$V' = V - \Delta V = V - \frac{V}{40} = \frac{39V}{40}$

As the mass of the metal remains constant, so

$m = V\rho = V'\rho'$

or $V \times 11 = \frac{39\,V}{40} \times \rho'$

or $\rho' = \frac{40 \times 11}{39} = \mathbf{11.28\ gcm^{-3}}$.

8. $\kappa = \frac{1}{\text{Bulk modulus}} = \frac{1}{4 \times 10^{-5}} = 0.25 \times 10^5$ atm

$= 0.25 \times 10^5 \times 1.013 \times 10^5\ Nm^{-2}$

$= 2.533 \times 10^9\ Nm^{-2}$

$V = 100\ cm^3 = 10^{-4}\ m^3$,

$p = 100$ atm $= 100 \times 1.013 \times 10^5$

$= 1.013 \times 10^7\ Nm^{-2}$

$\Delta V = \frac{pV}{\kappa} = \frac{1.013 \times 10^7 \times 10^{-4}}{2.533 \times 10^9}$

$= 0.4 \times 10^{-6}\ m^3 = \mathbf{0.4\ cm^3}$.

9. Here $h = 180$ m, $\rho = 1.0 \times 10^3\ kgm^{-3}$, $g = 10\ ms^{-2}$

$p = h\rho g = 180 \times 1.0 \times 10^3 \times 10 = 1.8 \times 10^6\ Nm^{-2}$

Volume strain $= 0.1\% = \frac{0.1}{100} = 10^{-3}$

$\kappa = \frac{p}{\text{Volume strain}} = \frac{1.8 \times 10^6}{10^{-3}} = \mathbf{1.8 \times 10^9\ Nm^{-2}}$.

10. As $\Delta V = \gamma V \Delta T \quad \therefore \quad \frac{\Delta V}{V} = \gamma\,\Delta T$

$\kappa = \frac{p}{\Delta V / V} = \frac{p}{\gamma \Delta T}$ or $\Delta T = \frac{p}{\gamma K}$.

11. When mass M is placed on the piston, the excess pressure, $p = mg/A$. This pressure acts equally from all directions on the sphere. The volume of the sphere decreases due to the decrease in its radius.

Scanned by CamScanner

As $V = \frac{4}{3}\pi R^3$

$\therefore$ Fractional decrease in volume, $\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$

Also $\kappa = \frac{p}{\Delta V / V}$ or $\frac{\Delta V}{V} = \frac{p}{\kappa} = \frac{Mg}{A\kappa}$

Hence $3\frac{\Delta R}{R} = \frac{Mg}{A\kappa}$ or $\frac{\Delta R}{R} = \frac{Mg}{3A\kappa}$.

9.13 MODULUS OF RIGIDITY OR SHEAR MODULUS

17. *Define modulus of rigidity. Give its units and dimensions.*

Modulus of rigidity or shear modulus. *Within the elastic limit, the ratio of tangential stress to shear strain is called modulus of rigidity.*

As shown in Fig. 9.15, consider a rectangular block whose lower face is fixed and a tangential force F is applied over its upper face of area A. An equal and opposite force F comes into play on its lower fixed face. The two equal and opposite forces form a couple which exerts a torque. As the lower face of the block is fixed, the couple shears the block into a parallelopied by displacing its upper face through distance $AA' = \Delta l$. Let $AB = DC = l$ and $\angle ABA' = \theta$.

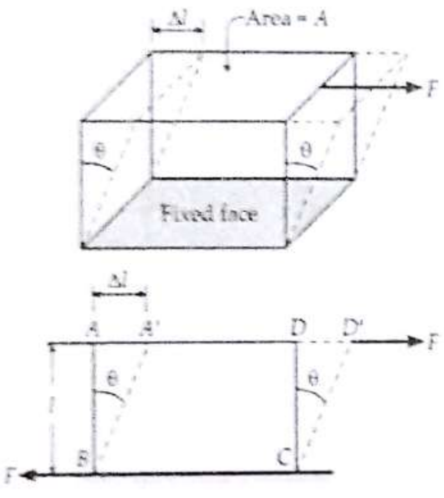


Fig. 9.15 Shear modulus of rigidity

Tangential stress $= \frac{F}{A}$

Shear strain $= \theta \simeq \tan\theta = \frac{AA'}{AB} = \frac{\Delta l}{l}$

The modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

Units and dimensions of η. The SI unit of modulus of rigidity is Nm^{-2} and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$.

18. *The shear modulus of a material is always considerably smaller than the Young modulus for it. What does it signify ?*

η of a material is smaller than its Y. This shows that it is easier to slide layers of atoms of solids over one another than to pull them apart or to squeeze them close together.

Table 9.3 Shear moduli (η) of some common materials

Material	η ($10^9 Nm^{-2}$ *or GPa*)
Aluminium	25
Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6
Nickel	77
Steel	84
Tungsten	150
Wood	10

If compare the values of η of Table 9.3 with the values of Y of Table 9.1, we see that, in general, the shear modulus is less than Young's modulus. For most of the materials, $\eta = Y/3$.

For Your Knowledge

- Elastic deformations in all bodies become plastic deformations with time.
- As only solids have length and shape, Young's modulus and shear modulus are relevant only for solids.
- As solids, liquids and gases all have volume elasticity, bulk modulus is relevant for all three states of matter.
- Metals have large values of Young's modulus than alloys and elastomers. A material with large Y requires a large force to produce small changes in length.
- Elastic has a different meaning in physics than that in daily life. In daily life, a material which stretches more is said to be more elastic, but it is a misnomer. In physics, a material which stretches to a lesser extent for a given load is considered to be more elastic.

Scanned by CamScanner

Examples based on Modulus of Rigidity

FORMULAE USED

1. Shearing stress $= \frac{\text{Tangential Force}}{\text{Area}} = \frac{F}{A}$
2. Shearing strain $= \theta = \frac{\Delta l}{l}$
3. Modulus of rigidity $= \frac{\text{Shearing stress}}{\text{Shearing strain}}$

or $\quad \eta = \frac{F/A}{\theta} = \frac{F/A}{\Delta l / l}$

UNITS USED

Modulus of rigidity η is in Nm^{-2} or Pa^{-1}.

EXAMPLE 20. *A cube of aluminium of each side 4 cm is subjected to a tangential (shearing) force. The top face of the cube is sheared through 0.012 cm with respect to the bottom face. Find (i) shearing strain (ii) shearing stress and shearing force. Given $\eta = 2.08 \times 10^{11}$ dyne cm^{-2}.*

Solution. Here $l = 4$ cm, $\Delta l = 0.012$ cm,

$$\eta = 2.08 \times 10^{11} \text{ dyne cm}^{-2}$$

(*i*) Shearing strain,

$$\theta = \frac{\Delta l}{l} = \frac{0.012}{4} = \mathbf{0.003\ rad.}$$

(*ii*) Area of top face

$$= l^2 = (4\text{ cm})^2 = 16\text{ cm}^2$$

Modulus of rigidity, $\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$

∴ Shearing stress

$$= \eta \times \text{Shearing strain}$$

$$= 2.08 \times 10^{11} \times 0.003 = \mathbf{6.24 \times 10^8\ dyne\ cm^{-2}}.$$

Shearing force,

$$F = \text{Shearing stress} \times \text{area}$$

$$= 6.24 \times 10^8 \times 16 = \mathbf{9.984 \times 10^9\ dyne.}$$

EXAMPLE 21. *A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4 N. The lower edge is riveted to the floor. How much is the upper edge displaced, if the shear modulus of lead is 5.6×10^9 Pa?* [NCERT]

Solution. Here $l = 50$ cm $= 0.50$ m, $F = 9.0 \times 10^4$ N

$$\eta = 5.6 \times 10^9 \text{ Pa}$$

Area of the face on which force is applied,

$$A = 50\text{ cm} \times 10\text{ cm}$$

$$= 500\text{ cm}^2 = 500 \times 10^{-4}\text{ m}^2 = 0.05\text{ m}^2$$

If Δl is the distance through which the upper edge is displaced relative to the lower fixed edge, then

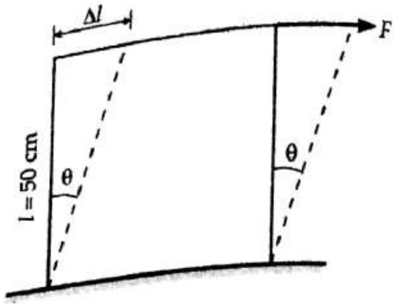


Fig. 9.16

$$\eta = \frac{F/A}{\Delta l / l} = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

or $\quad \Delta l = \frac{F}{A} \cdot \frac{l}{\eta} = \frac{9.0 \times 10^4 \times 0.50}{0.05 \times 5.6 \times 10^9}$

$$= \mathbf{1.6 \times 10^{-4}\ m = 0.16\ mm.}$$

EXAMPLE 22. *A rubber block 1 cm × 3 cm × 10 cm is clamped at one end with its 10 cm side vertical. A horizontal force of 30 N is applied to the free surface. What is the horizontal displacement of the top face? Modulus of rigidity of rubber $= 1.4 \times 10^5\ Nm^{-2}$.*

Solution. Area of the upper face,

$$A = 1\text{ cm} \times 3\text{ cm} = 3 \times 10^{-4}\text{ m}^2$$

$$F = 30\text{ N}, \ \eta = 1.4 \times 10^5\text{ Nm}^{-2},$$

$$l = 10\text{ cm} = 0.10\text{ m}$$

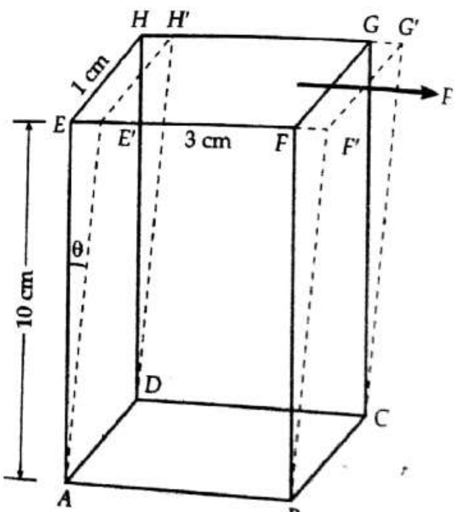


Fig. 9.17

As $\quad \eta = \frac{F}{A} \cdot \frac{l}{\Delta l}$

∴ $\quad \Delta l = \frac{F}{A} \cdot \frac{l}{\eta} = \frac{30 \times 0.10}{3 \times 10^{-4} \times 1.4 \times 10^5}$

$$= \frac{1}{14} = 0.0714\text{ m} = \mathbf{7.14\ cm.}$$

Scanned by CamScanner

EXAMPLE 23. *A 60 kg motor rests on four cylindrical rubber blocks. Each cylinder has a height of 3 cm and a cross-sectional area of 15 cm^2. The shear modulus for this rubber is 2×10^6 Nm^{-2}. If a sideways force of 300 N is applied to the motor, how far will it move sideways ?*

Solution. Tangential force on each block,

$$F = (1/4) \times 300 = 75 \text{ N}, \quad l = 3 \text{ cm} = 3 \times 10^{-2} \text{ m},$$

$$A = 15 \text{ cm}^2 = 15 \times 10^{-4} \text{ m}^2, \quad \eta = 2 \times 10^6 \text{ Nm}^{-2}$$

As $\eta = \dfrac{F/A}{\Delta l / l} = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$$\therefore \quad \Delta l = \frac{F}{A} \cdot \frac{l}{\eta} = \frac{75 \times 3 \times 10^{-2}}{15 \times 10^{-4} \times 2 \times 10^6}$$

$$= 7.5 \times 10^{-4} \text{ m} = \mathbf{0.075 \text{ cm}}.$$

✗ PROBLEMS FOR PRACTICE

1. A metallic cube whose each side is 10 cm is subjected to a shearing force of 100 kg f. The top face is displaced through 0.25 cm with respect to the bottom. Calculate the shearing stress, strain and shear modulus.

   (**Ans.** 9.8×10^4 Nm^{-2}, 0.025 rad, 3.92×10^6 Nm^{-2})

2. An Indian rubber cube of side 7 cm has one side fixed, while a tangential force equal to the weight of 200 kilogram is applied to the opposite face. Find the shearing strain produced and distance through which the strained side moves. Modulus of rigidity for rubber is 2×10^7 dyne cm^{-2}.

   (**Ans.** 0.2 radian, 1.4 cm)

3. A metal cube of side 10 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity if the top of the cube is displaced by 0.05 cm with respect to its bottom. (**Ans.** 2×10^6 Nm^{-2})

4. Two parallel and opposite forces, each 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block 25 cm on a side. Find the angle of shear and the displacement of the upper surface relative to the lower surface. The shear modulus for the metal is 8×10^{10} Nm^{-2}.

   (**Ans.** 8.0×10^{-7} rad, 2.0×10^{-7} m)

✗ HINTS

1. Here $l = 10 \text{ cm} = 0.10 \text{ m}$, $F = 100 \text{ kg f} = 100 \times 9.8$ N,

   $\Delta l = 0.25 \text{ cm} = 0.25 \times 10^{-2}$ m

   Shearing stress

   $$= \frac{F}{A} = \frac{F}{l^2} = \frac{100 \times 9.8}{0.10 \times 0.10} = \mathbf{9.8 \times 10^4 \text{ Nm}^{-2}}$$

   Shearing strain

   $$= \frac{\Delta l}{l} = \frac{0.25 \times 10^{-2}}{0.10} = \mathbf{0.025 \text{ rad}}$$

   Shear modulus,

   $$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{9.8 \times 10^4}{0.025} = \mathbf{3.92 \times 10^6 \text{ Nm}^{-2}}.$$

2. Here $l = 7$ cm, $F = 200 \text{ kg f} = 200 \times 1000 \times 981$ dyne,

   $\eta = 2 \times 10^7$ dyne cm^{-2}

   Area of the free face,

   $$A = l^2 = 7 \text{ cm} \times 7 \text{ cm} = 49 \text{ cm}^2$$

   As $\eta = \dfrac{F}{A\theta}$

   $$\therefore \quad \theta = \frac{F}{A\eta} = \frac{200 \times 1000 \times 981}{49 \times 2 \times 10^7} = \mathbf{0.2 \text{ rad}}.$$

   $$\Delta l = l\theta = 7 \times 0.2 = \mathbf{1.4 \text{ cm}}.$$

3. Here $l = 10$ cm, $\Delta l = 0.05$ cm

   Shearing stress = 10^4 Nm^{-2}

   Shearing strain $= \dfrac{\Delta l}{l} = \dfrac{0.05}{10} = 0.005$

   $$\therefore \quad \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{10^4}{0.005} = \mathbf{2 \times 10^6 \text{ Nm}^{-2}}.$$

4. Shearing stress $= \dfrac{F}{l^2} = \dfrac{4000}{0.25 \times 0.25} = 64000$ Nm^{-2}

   Shearing strain,

   $$\theta = \frac{\text{Shearing stress}}{\text{Shear modulus}} = \frac{64000}{8 \times 10^{10}} = \mathbf{8 \times 10^{-7} \text{ rad}}.$$

   $$\Delta l = l\theta = 0.25 \times 8 \times 10^{-7} = \mathbf{2.0 \times 10^{-7} \text{ m}}.$$

9.14 SOME OTHER ELASTIC EFFECTS

19. *What is elastic after effect ? What is its importance ?*

Elastic after effect. The bodies return to their original state on the removal of the deforming force. Some bodies return to their original state immediately after the removal of the deforming force while some bodies take longer time to do so. *The delay in regaining the original state by a body on the removal of the deforming force is called elastic after effect.*

In galvanometers, suspensions made from quartz or phosphor-bronze alloy are used because their elastic after effect is small. On the contrary, a glass fibre takes hours to regain its original state.

20. *What is elastic fatigue ? What is its importance ?*

Elastic fatigue. As shown in Fig. 9.18, in a torsion pendulum, a disc oscillates in a horizontal plane. The elastic twist of the suspension wire provides the restoring torque. During torsional vibrations, the wire is subjected to repeated alternating strains. If we set the wire into torsional vibrations, it will continue vibrating

Scanned by CamScanner

for a long time before its vibrations die out. If it is again made to vibrate, its vibrations will die out in a lesser time. Due to continuous alternating strains, the wire is said to have been *tired* or *fatigued*.

Elastic fatigue is defined as loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

A hard wire can be broken by bending it repeatedly in opposite directions, as it loses strength due to elastic fatigue. For the same reason, the railway bridges are declared unsafe after a reasonably good period to avoid the risk of a mishap.

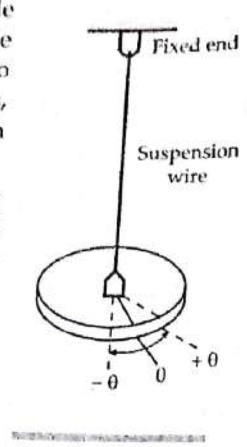


Fig. 9.18 Oscillations of a torsion pendulum.

21. *Describe elastic hysteresis. Mention its few applications.*

Elastic hysteresis. Fig. 9.19 shows the stress-strain curve for a rubber sample when loaded and then unloaded. For increasing load, the stress-strain curve is *OAB* and for decreasing load, the curve is *BCO*. *The fact that the stress-strain curve is not retraced on reversing the strain is known as elastic hysteresis.*

The area under the curve *OAB* represents the work done per unit volume in stretching the rubber. The area under *BCO* represents the energy given up by rubber on unloading. So the shaded area of the hysteresis loop represents the energy lost as heat during the loading-unloading cycle.

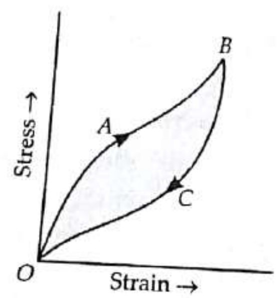


Fig. 9.19

Applications of elastic hysteresis :

(*i*) Car tyres are made with synthetic rubbers having small-area hysteresis loops because a car tyre of such a rubber will not get excessively heated during the journey.

(*ii*) A padding of vulcanized rubber having large-area hysteresis loop is used in shock absorbers between the vibrating system and the flat board. As the rubber is compressed and released during each vibration, it dissipates a large amount of vibrational energy.

9.15 APPLICATIONS OF ELASTICITY

22. *Why is any metallic part of a machinery never subjected to a stress beyond the elastic limit ?*

Any metallic part of a machinery is never subjected to a stress beyond the elastic limit. This is because a stress beyond elastic limit will permanently deform that metallic part.

23. *How is the knowledge of elasticity useful in selecting metal ropes used in cranes for lifting heavy loads ?*

The thickness of metallic ropes used in cranes to lift heavy loads is decided from the knowledge of the elastic limit of the material and the factor of safety. Suppose a crane having steel ropes is required to lift load of ten ton *i.e.*, 10^4 kg. The rope is usually designed for a safety factor of 10 *i.e.*, it should not break even when a load of $10^4 \times 10 = 10^5$ kg is applied to it. If r is the radius of the rope, then

$$\text{Ultimate stress} = \frac{F}{A} = \frac{Mg}{\pi r^2} = \frac{10^5 \times 9.8}{\pi r^2}$$

The ultimate stress should not exceed the elastic limit ($= 30 \times 10^7$ Nm^{-2}) for steel.

$$\therefore \quad \frac{10^5 \times 9.8}{\pi r^2} = 30 \times 10^7 \quad \text{or} \quad r = 0.032 \text{ m} = 3.2 \text{ cm}.$$

A single wire of this much radius would be a rigid rod. For the ease in manufacture and to impart flexibility and strength to the rope, it is always made of a large number of thin wires braided together.

24. *Explain why should the beams used in the construction of bridges have large depth and small breadth.*

Or

Explain why are girders given I shape.

The knowledge of elasticity is applied in designing a bridge such that it does not bend too much or break under the load of traffic, the force of wind and under its own weight. Consider a rectangular bar of length l, breadth b and thickness d supported at both ends, as shown in Fig. 9.20. When a load W is suspended at its middle, the bar gets depressed by an amount given by

$$\delta = \frac{Wl^3}{4Y\, bd^3}$$

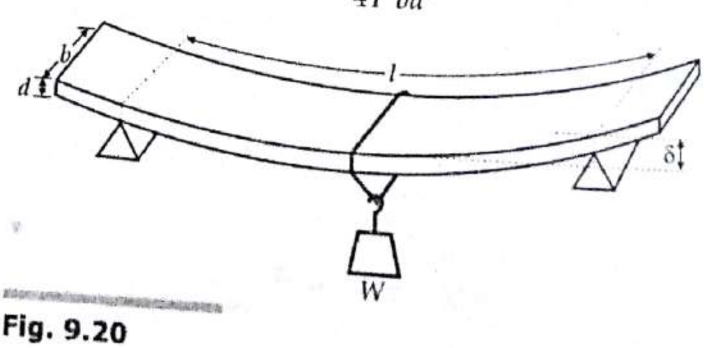


Fig. 9.20

Scanned by CamScanner

Bending can be reduced by using a material with a large Young's modulus Y. As δ is proportional to d^{-3} and only to b^{-1}, *so depression can be decreased more effectively by increasing the depth d rather than the breadth b*. But a deep bar has a tendency to bend under the weight of a moving traffic, as shown in Fig. 9.21(*b*). This bending is called ***buckling***. Hence a better choice is to have a bar of I-shaped cross-section, as shown in Fig. 9.21(*c*). This section provides a large load bearing surface and enough depth to prevent bending. Also, this shape reduces the weight of the beam without sacrificing its strength and hence reduces the cost.

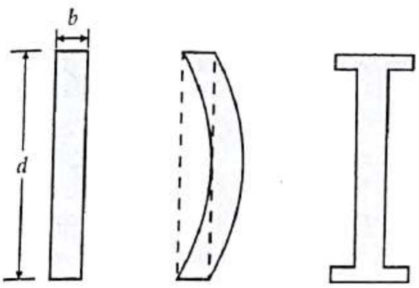


Fig. 9.21. (*a*) Rectangular cross-section of bar, (*b*) Buckling of a deep bar, (*c*) I-shaped cross-section of a bar.

25. *How can the knowledge of elasticity be used to estimate the maximum height of a mountain on earth ?*

The maximum height of mountain on earth depends upon shear modulus of rock. At the base of the mountain, the stress due to all the rock on the top should be less than the critical shear stress at which the rock begins to flow. Suppose the height of the mountain is h and the density of its rock is ρ. Then force per unit area (due to the weight of the mountain) at the base $= h\rho g$. The material at the base experiences this force per unit area in the vertical direction, but sides of the mountain are free. Hence there is a tangential shear of the order of $h\rho g$. The elastic limit for a typical rock is about 3×10^8 Nm^{-2} and its density is 3×10^3 kg m^{-3}. Hence

$$h_{max}\, \rho g = 3 \times 10^8$$

or
$$h_{max} = \frac{3 \times 10^8}{\rho g} = \frac{3 \times 10^8}{3 \times 10^3 \times 9.8}$$

$$= 10{,}000 \text{ m} = 10 \text{ km}$$

This is nearly the height of the Mount Everest. A height greater than this will not be able to withstand the shearing stress due to the weight of the mountain.

26. *Explain why hollow shafts are preferred to solid shafts for transmitting torque.*

A hollow shaft is stronger than a solid shaft made of equal quantity of same material. The torque required to produce unit twist in a solid shaft of radius r, length l and made of material of modulus of rigidity η is given by

$$\tau = \frac{\pi \eta r^4}{2l}$$

The torque required to produce a unit twist in a hollow shaft of internal and external radii r_1 and r_2 is given by

$$\tau' = \frac{\pi \eta (r_2^4 - r_1^4)}{2l}$$

$$\therefore \quad \frac{\tau'}{\tau} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4}$$

If the two shafts are made from equal amounts of materials, then

$$\pi r^2 l = \pi (r_2^2 - r_1^2)\, l \quad \text{or} \quad r_2^2 - r_1^2 = r^2$$

$$\therefore \quad \frac{\tau'}{\tau} = \frac{r_2^2 + r_1^2}{r^2}$$

As $\quad r^2 = r_2^2 - r_1^2$

so $\quad r_2^2 + r_1^2 > r^2 \quad$ and hence $\tau' > \tau$.

Thus torque required to twist hollow cylinder through a certain angle is greater than the torque necessary to twist a solid cylinder of same mass, length and material through the same angle. Hence a *hollow shaft is stronger than a solid shaft*. For this reason, electric poles are given hollow structures.

9.16 ELASTIC POTENTIAL ENERGY OF A STRETCHED WIRE

27. *What is meant by elastic potential energy ? Derive an expression for the elastic potential energy of stretched wire. Prove that its elastic energy density is equal to $\frac{1}{2}$ stress × strain.*

Elastic potential energy. When a wire is stretched, interatomic forces come into play which oppose the change. Work has to be done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

Expression for elastic potential energy. Suppose a force F applied on a wire of length l increases its length by Δl. Initially, the internal restoring force in the wire is zero. When the length is increased by Δl, the internal force increases from zero to F (= applied force).

∴ Average internal force for an increase in length Δl of wire

$$= \frac{0 + F}{2} = \frac{F}{2}$$

Work done on the wire is

$$W = \text{Average force} \times \text{increase in length} = \frac{F}{2} \times \Delta l$$

Scanned by CamScanner

This work done is stored as elastic potential energy U in the wire.

$$\therefore \quad U = \frac{1}{2} F \times \Delta l$$

$$= \frac{1}{2} \text{ Stretching force} \times \text{increase in length}$$

Let A be the area of cross-section of the wire. Then

$$U = \frac{1}{2} \frac{F}{A} \times \frac{\Delta l}{l} \times Al$$

$$= \frac{1}{2} \text{ Stress} \times \text{strain} \times \text{volume of wire}$$

Elastic potential energy per unit volume of the wire or elastic energy density is

$$u = \frac{U}{\text{Volume}}$$

or $$u = \frac{1}{2} \text{ stress} \times \text{strain}$$

But stress = Young's modulus × strain

$$\therefore \quad u = \frac{1}{2} \text{ Young's modulus} \times \text{strain}^2$$

Examples based on Elastic Potential Energy

Formulae Used

1. Total P.E. stored in a stretched wire,

$$U = \frac{1}{2} \text{ Stretching force} \times \text{extension} = \frac{1}{2} F \Delta l$$

or $$U = \frac{1}{2} \text{ Stress} \times \text{strain} \times \text{volume of wire}$$

2. P.E. stored per unit volume of a stretched wire,

$$u = \frac{1}{2} \text{ Stress} \times \text{strain}$$

or $$u = \frac{1}{2} \text{ Young's modulus} \times \text{strain}^2$$

Units Used

Elastic P.E. is in joule, elastic P.E. per unit volume is in Jm^{-3}.

Example 24. *A steel wire of 4.0 m is stretched through 2.0 mm. The cross-sectional area of the wire is 2.0 mm². If Young's modulus of steel is 2.0×10^{11} Nm^{-2}, find (i) the energy density of the wire and (ii) the elastic potential energy stored in the wire.*

Solution. Here $l = 4.0$ m,

$\Delta l = 2.0$ mm $= 2.0 \times 10^{-3}$ m,

$A = 2.0$ mm$^2 = 2.0 \times 10^{-6}$ m^2, $Y = 2.0 \times 10^{11}$ Nm^{-2}

(i) Energy density,

$$u = \frac{1}{2} Y \times (\text{strain})^2 = \frac{1}{2} Y \times \left(\frac{\Delta l}{l}\right)^2$$

$$= \frac{1}{2} \times 2 \times 10^{11} \times \left[\frac{2 \times 10^{-3}}{4.0}\right]^2 = \mathbf{2.5 \times 10^4 \text{ Jm}^{-3}}.$$

(ii) Elastic potential energy,

$$U = \text{Energy density} \times \text{volume}$$

$$= u \times A \times l = 2.5 \times 10^4 \times 2.0 \times 10^{-6} \times 4$$

$$= \mathbf{0.2 \text{ J}}.$$

Example 25. *Calculate the increase in energy of a brass bar of length 0.2 m and cross-sectional area 1 cm² when compressed with a load of 5 kg weight along its length. Young's modulus of brass $= 1.0 \times 10^{11}$ Nm^{-2} and $g = 9.8$ ms^{-2}.*

Solution. Increase in the energy of the bar,

$$U = \frac{1}{2} \times \text{Stretching force} \times \text{extension} = \frac{1}{2} F \times \Delta l$$

As $$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} \qquad \therefore \Delta l = \frac{F}{A} \cdot \frac{l}{Y}$$

Hence $$U = \frac{1}{2} F \times \frac{F \cdot l}{AY} = \frac{F^2 l}{2AY}$$

Here $F = 5$ kg wt $= 5 \times 9.8 = 49$ N,

$l = 0.2$ m, $A = 1$ cm$^2 = 10^{-4}$ m^2,

$Y = 1.0 \times 10^{11}$ Nm^{-2}

$$\therefore \quad U = \frac{(49)^2 \times 0.2}{2 \times 10^{-4} \times 1.0 \times 10^{11}} = \mathbf{2.4 \times 10^{-5} \text{ J}}.$$

Example 26. *When the load on a wire is increased from 3 kg wt to 5 kg wt, the elongation increases from 0.61 mm to 1.02 mm. How much work is done during the extension of the wire ?*

Solution. Work done in stretching the wire through 0.61 mm under the load of 3 kg wt,

$$W_1 = \frac{1}{2} \text{ Stretching force} \times \text{extension}$$

$$= \frac{1}{2} \times 3 \times 9.8 \times 0.61 \times 10^{-3} = 8.967 \times 10^{-3} \text{ J}$$

Work done in stretching the wire through 1.02 mm under the load of 5 kg wt,

$$W_2 = \frac{1}{2} \times 5 \times 9.8 \times 1.02 \times 10^{-3} = 24.99 \times 10^{-3} \text{ J}$$

Hence the work done in stretching the wire from 0.61 mm to 1.02 mm,

$$\Delta W = W_2 - W_1 = (24.99 - 8.967) \times 10^{-3}$$

$$= \mathbf{16.023 \times 10^{-3} \text{ J}}.$$

Example 27. *A 40 kg boy whose leg bones are 4 cm² in area and 50 cm long falls through a height of 50 cm without breaking his leg bones. If the bones can stand a stress of 0.9×10^8 Nm^{-2}, calculate the Young's modulus for the material of the bone. Take $g = 10$ ms^{-2}.*

Solution. Here $m = 40$ kg, $h = 2$ m, $l = 0.50$ m,

$$A = 4 \times 10^{-4} \text{ m}^2$$

Volume of leg $= Al = 4 \times 10^{-4} \times 0.50$

$$= 2 \times 10^{-4} \text{ m}^3$$

Scanned by CamScanner

Loss in gravitational P.E.

= Gain in elastic P.E. by both legs

$$mgh = 2 \times \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$40 \times 10 \times 2 = 2 \times \frac{1}{2} \times 0.9 \times 10^8 \times \text{strain} \times 2 \times 10^{-4}$$

or $$\text{strain} = \frac{40 \times 10 \times 2}{0.9 \times 2 \times 10^4} = \frac{2}{45}$$

Young's modulus,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{0.9 \times 10^8 \times 45}{2}$$

$$= \mathbf{2.025 \times 10^9\ Nm^{-2}}.$$

PROBLEMS FOR PRACTICE

1. A steel wire of length 2.0 m is stretched through 2.0 mm. The cross-sectional area of the wire is 4.0 mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel is 2.0×10^{11} Nm^{-2}. (**Ans.** 0.8 J)
2. If the Young's modulus of steel is 2×10^{11} Nm^{-2}, calculate the work done in stretching a steel wire 100 cm in length and of cross-sectional area 0.03 cm^2 when a load of 20 kg is slowly applied without the elastic limit being reached. (**Ans.** 0.032 J)
3. The limiting stress for a typical human bone is 0.9×10^8 Nm^{-2} while Young's modulus is 1.4×10^{10} Nm^{-2}. How much energy can be absorbed by two legs (without breaking) if each has a typical length of 50 cm and an average cross-sectional area of 5 cm^2 ? (**Ans.** 144.7 J)

HINTS

1. $$\text{Strain} = \frac{\Delta l}{l} = \frac{2.0 \times 10^{-3}}{2.0} = 10^{-3}$$

$$\text{Stress} = Y \times \frac{\Delta l}{l} = 2.0 \times 10^{11} \times 10^{-3} = 2.0 \times 10^8\ Nm^{-2}$$

Volume of the wire

$$= Al = 4.0 \times 10^{-6} \times 2.0 = 8.0 \times 10^{-6}\ m^3$$

$$U = \frac{1}{2} \text{Stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times 2.0 \times 10^8 \times 10^{-3} \times 8.0 \times 10^{-6} = \mathbf{0.8\ J.}$$

2. $$\text{Work done} = \frac{1}{2} \text{Stretching force} \times \text{extension}$$

$$= \frac{1}{2} F\Delta l = \frac{1}{2} F . \frac{Fl}{Ay}$$

$$= \frac{F^2 l}{2\,AY} = \frac{(20 \times 9.8)^2 \times 1.00}{2 \times 0.03 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= \mathbf{0.032\ J.}$$

3. Limiting stress $= 0.9 \times 10^8\ Nm^{-2}$,

$$Y = 1.4 \times 10^{10}\ Nm^{-2}$$

Length of both the legs, $l = 2 \times 50 = 100$ cm $= 1.0$ m,

$A = 5\ cm^2 = 5 \times 10^{-4}\ m^2$

Stretching force,

$$F = \text{Stress} \times \text{area} = 0.9 \times 10^8 \times 5 \times 10^{-4}$$

$$= 4.5 \times 10^4\ N$$

As $$Y = \frac{\text{Stress}}{\Delta l / l}$$

$$\therefore \quad \Delta l = \frac{\text{Stress} \times l}{Y} = \frac{0.9 \times 10^8 \times 1.0}{1.4 \times 10^{10}}$$

$$= 6.43 \times 10^{-3}\ m$$

Elastic P.E.,

$$U = \frac{1}{2} F \times \Delta l = \frac{1}{2} \times 4.5 \times 10^4 \times 6.43 \times 10^{-3}$$

$$= \mathbf{144.7\ J.}$$

9.17 POISSON'S RATIO

28. *Define Poisson's ratio. Write an expression for it. What is the significance of negative sign in this expression ?*

Poisson's ratio. When a wire is loaded, its length increases but its diameter decreases. The strain produced in the direction of applied force is called longitudinal strain and that produced in the perpendicular direction is called lateral strain.

Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

Suppose the length of the loaded wire increases from l to $l + \Delta l$ and its diameter decreases from D to $D - \Delta D$.

$$\text{Longitudinal strain} = \frac{\Delta l}{l}$$

$$\text{Lateral strain} = -\frac{\Delta D}{D}$$

Poisson's ratio is

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{-\Delta D / D}{\Delta l / l}$$

or $$\sigma = -\frac{l}{D} \cdot \frac{\Delta D}{\Delta l}$$

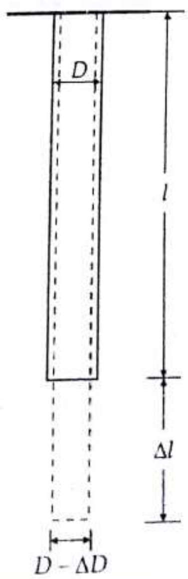


Fig. 9.22 Poisson's ratio.

The negative sign indicates that longitudinal and lateral strains are in opposite sense.

As the Poisson's ratio is the ratio of two strains, it has no units and dimensions.

Scanned by CamScanner

For Your Knowledge

- For all substances, the theoretical value of σ lies between -1 and $+0.5$. In actual practice, the value of σ lies between 0 and 0.5 for most of the substances.
- **Relations between Y, K, η and σ**

(i) $Y = 3\kappa(1-2\sigma)$ (ii) $Y = 2\eta(1+\sigma)$

(iii) $\sigma = \dfrac{3\kappa - 2\eta}{6\kappa + 2\eta}$ (iv) $\dfrac{9}{Y} = \dfrac{3}{\eta} + \dfrac{1}{\kappa}$

Examples based on Poisson's Ratio

FORMULA USED

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

or $$\sigma = \frac{\Delta D / D}{\Delta l / l}$$

UNITS USED

Length l and diameter D are in metre, Poisson's ratio σ has no units.

EXAMPLE 28. Determine the Poisson's ratio of the material of a wire whose volume remains constant under an external normal stress.

Solution. Volume of a wire, $V = \pi \dfrac{D^2}{4} l$

As volume remains constant, the differentiation of the above equation gives

$$0 = \frac{\pi l}{4} 2D\,dD + \frac{\pi D^2}{4} . dl$$

or $$-2l\,dD = D\,dl \quad \text{or} \quad \frac{dD}{D} = -\frac{1}{2}\frac{dl}{l}$$

By definition, Poisson's ratio is

$$\sigma = \frac{-dD/D}{dl/l} = \frac{+\frac{1}{2}\frac{dl}{l}}{dl/l} = \frac{1}{2} = 0.5.$$

EXAMPLE 29. One end of a nylon rope of length 4.5 m and diameter 6 mm is fixed to a free limb. A monkey weighing 100 N jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in diameter. Given Young's modulus of nylon $= 4.8 \times 10^{11}\ Nm^{-2}$ and Poisson's ratio of nylon = 0.2.

Solution. Here $l = 4.5$ m,

$$D = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}, \; F = 100 \text{ N},$$
$$Y = 4.8 \times 10^{11}\ \text{Nm}^{-2}, \quad \sigma = 0.2$$

As $$Y = \frac{F}{A} . \frac{l}{\Delta l}$$

$$\therefore \Delta l = \frac{F}{A} . \frac{l}{Y} = \frac{100 \times 4.5}{3.14 \times (3 \times 10^{-3})^2 \times 4.8 \times 10^{11}}$$

$$= \mathbf{3.32 \times 10^{-5}\ m.}$$

Poisson's ratio,

$$\sigma = \frac{\Delta D / D}{\Delta l / l} = \frac{\Delta D}{D} . \frac{l}{\Delta l}$$

$$\therefore \Delta D = \frac{\sigma D \Delta l}{l} = \frac{0.2 \times 6 \times 10^{-3} \times 3.32 \times 10^{-5}}{4.5}$$

$$= \mathbf{8.8 \times 10^{-9}\ m.}$$

EXAMPLE 30. A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, what is the percentage increase in volume ?

Solution. Longitudinal strain,

$$\frac{\Delta l}{l} = 2 \times 10^{-3}$$

Poisson's ratio, $\sigma = 0.5$

As $$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta R / R}{\Delta l / l}$$

$$\therefore \frac{\Delta R}{R} = -\sigma \frac{\Delta l}{l} = -0.5 \times 2 \times 10^{-3} = -1 \times 10^{-3}$$

Volume of rod,

$$V = \pi R^2 l$$

Percentage increase in volume is

$$\frac{\Delta V}{V} \times 100 = \left(2\frac{\Delta R}{R} + \frac{\Delta l}{l}\right) \times 100$$

$$= [2 \times (-1) \times 10^{-3} + 2 \times 10^{-3}] \times 100 = \mathbf{0}.$$

PROBLEMS FOR PRACTICE

1. Calculate the Poisson's ratio for silver. Given its Young's modulus $= 7.25 \times 10^{10}\ \text{Nm}^{-2}$ and bulk modulus $= 11 \times 10^{10}\ \text{Nm}^{-2}$. **(Ans. 0.39)**
2. A material has Poisson's ratio 0.2. If a uniform rod of it suffers longitudinal strain 4.0×10^{-3}, calculate the percentage change in its volume. **(Ans. 0.24%)**

HINTS

1. As $Y = 3\kappa(1 - 2\sigma)$

$$\therefore \sigma = \frac{1}{2}\left(1 - \frac{Y}{3\kappa}\right) = \frac{1}{2}\left(1 - \frac{7.25 \times 10^{10}}{3 \times 11 \times 10^{10}}\right) = \mathbf{0.39.}$$

2. As $$\sigma = -\frac{\Delta R / R}{\Delta l / l}$$

$$\therefore \frac{\Delta R}{R} = -\sigma \frac{\Delta l}{l} = -0.2 \times 4.0 \times 10^{-3}$$
$$= -0.8 \times 10^{-3}$$
$$V = \pi R^2 l$$
$$\therefore \frac{\Delta V}{V} \times 100 = \left(2\frac{\Delta R}{R} + \frac{\Delta l}{l}\right) \times 100$$
$$= [2 \times (-0.8 \times 10^{-3}) + 4.0 \times 10^{-3}] \times 100$$
$$= 2.4 \times 10^{-3} \times 100 = \mathbf{0.24\%}.$$

Scanned by CamScanner

Very Short Answer Conceptual Problems

Problem 1. What is the nature of intermolecular forces ?

Solution. Generally the intermolecular forces are attractive, but they are repulsive for intermolecular separations less than 10^{-10} m.

Problem 2. What is the origin of interatomic force ?

Solution. Interatomic force arises due to the electrostatic interaction between the nuclei of two atoms, their electron clouds and between the nucleus of one atom and the electron cloud of the other atom.

Problem 3. What is the origin of intermolecular force ?

Solution. Intermolecular force arises due to the electrostatic interaction between the opposite charged ends of molecular dipoles.

Problem 4. Are the intermolecular forces involved in the formation of liquids and solids different in nature ? If yes, how ?

Solution. Yes. The intermolecular forces involved in the formation of liquids are attractive in nature while in the formation of solids, the repulsive intermolecular forces are more important.

Problem 5. State the factors due to which three states of matter differ from each other.

Solution. The three states of matter differ from each other due to the following *two* factors :

(*i*) The different magnitudes of interatomic and intermolecular forces.

(*ii*) The degree of random thermal motion of the atoms and the molecules of a substance depending on temperature.

Problem 6. What do you mean by long range order in a crystalline structure ?

Solution. Long range order means that similar patterns of atoms or molecules repeat over a large distance in a crystal.

Problem 7. What is the important structural difference between crystalline and glassy solids ?

Solution. In crystalline solids the atoms or molecules are arranged in a definite and long range order, but in glassy solids there exists no such long range order in the arrangement of atoms or molecules.

Problem 8. Amorphous solids do not melt at a sharp temperature, rather these have softening range. Explain this observation.

Solution. All bonds in an amorphous solid are not equally strong. When the solid is heated, weaker bonds get ruptured at lower temperatures and the stronger ones at higher temperatures. So the solid first softens and then finally melts.

Problem 9. In what respect, the behaviour of glassy solids is similar to that of the liquids ?

Solution. In case of glassy solids, the orderly arrangement of atoms is limited to a very short range and in this respect they are similar to liquids.

Problem 10. Why do crystalline solids have well defined geometrical external shapes ?

Solution. This is because the atoms and molecules are arranged in a definite geometrical repeating manner throughout the body of the crystal.

Problem 11. Amorphous solids are not true solids. Why and what are they called then ?

Solution. Like liquids, amorphous solids have disordered arrangement of atoms or molecules. The molecules of a liquid are free to move but the molecules of an amorphous solid are almost fixed at their positions *i.e.*, amorphous solids are rigid due to their high viscosity. That is why, we say amorphous solids are super-cooled liquids of high viscosity.

Problem 12. Our knowledge about crystalline solids is better than amorphous solids. Why ?

Solution. As crystalline solids possess long range and regular arrangement of atoms, hence their behaviour can be easily understood.

Problem 13. Crystalline solids are called true solids. Why ?

Solution. This is because crystalline solids have well defined, regularly repeated three-dimensional arrangement of atoms or molecules.

Problem 14. What is a perfectly elastic body ? Give an example.

Solution. If, on removal of deforming force, a body completely regains its original configuration, then it is said to be perfectly elastic. For example, quartz.

Problem 15. What is a perfectly plastic body ? Give an example.

Solution. If, on removal of deforming force, a body does not regain its original configuration even a little, then it is said to be perfectly plastic. For example, putty.

Problem 16. No material is perfectly elastic. Why ?

Solution. All materials undergo a change in their original state, howsoever small it may be, after the removal of deforming force. Hence, there is no such material which is perfectly elastic.

Problem 17. When does a body acquire a permanent set ?

Solution. When the deforming force exceeds the elastic limit, the body acquires the permanent set.

Scanned by CamScanner

Problem 18. A thick wire is suspended from a rigid support, but no load is attached to its free end. Is this wire under stress ?

Solution. Yes, the wire is under stress due to its own weight.

Problem 19. State the two factors on which the modulus of elasticity depends.

Solution. The modulus of elasticity depends upon (*i*) nature of the material and (*ii*) type of stress used in producing the strain.

Problem 20. Is it possible to double the length of a metallic wire by applying a force over it ?

Solution. No, it is not possible because within elastic limit strain is only of the order of 10^{-3}. Wires actually break much before it is stretched to double the length.

Problem 21. Is elastic limit a property of the material of the wire ?

Solution. No. It also depends on the radius of the wire.

Problem 22. Stress and pressure are both forces per unit area. Then in what respect does stress differ from pressure ?

Solution. Pressure is the external force per unit area, while stress is the internal restoring force which comes into play in a deformed body acting transversely per unit area of the body.

Problem 23. Among solids, liquids and gases, which can have all the three moduli of elasticity ?

Solution. Only solids. Liquids and gases have only bulk modulus.

Problem 24. Among solids, liquids and gases, which possess the greatest bulk modulus ?

Solution. Solids.

Problem 25. Which type of elasticity is involved in the following cases ?

(*i*) Compressing of gas (*ii*) Compressing a liquid (*iii*) Stretching a wire (*iv*) Tangential push on the upper face of a block.

Solution. (*i*) Bulk modulus (*ii*) Bulk modulus (*iii*) Young's modulus (*iv*) Modulus of rigidity.

Problem 26. What does the slope of stress versus strain graph give ?

Solution. The slope of stress-strain gives modulus of elasticity.

Problem 27. How does Young's modulus change with the rise of temperature ?

Solution. Young's modulus decreases with the rise of temperature.

Problem 28. Write copper, steel, glass and rubber in the order of increasing coefficient of elasticity.

Solution. Rubber < glass < copper < steel.

Problem 29. Which is more elastic – water or air ?

Solution. Water is more elastic than air. Air can be easily compressed while water is incompressible and bulk modulus is reciprocal of compressibility.

Problem 30. Why are springs made of steel and not of copper ? *[Himachal 04]*

Solution. Young's modulus of steel is greater than that of copper. So steel spring is stretched lesser than a copper spring under the same deforming force. Moreover, steel returns to its original state more quickly than copper on the removal of deforming force.

Problem 31. In stretching a wire, we have to perform work. Why ?

Solution. When a wire is stretched, interatomic forces of attraction come into play. In order to stretch the wire, work has to be done against these forces.

Problem 32. What happens to the work done in stretching a wire ?

Solution. The work done in stretching a wire is stored in it as elastic potential energy.

Problem 33. Two identical springs of steel and copper are equally stretched. On which more work will have to be done ?

Solution. Young's modulus of steel is greater than that of copper. In order to produce same extension, large force will have to be applied on the steel spring than that on the copper spring. Hence more work will be done on the steel spring.

Problem 34. If two identical springs of steel and copper are pulled by applying equal forces, then in which case more work will have to be done ?

Solution. Steel spring will be stretched to a lesser extent. Now more work will be done on the copper spring.

Problem 35. Why does a wire get heated when it is bent back and forth ?

Solution. When a wire is bent back and forth, its deformations are beyond elastic limit. The work done against interatomic forces is no longer stored totally in the form of potential energy. The crystalline structure of the wire gets affected and work done is converted into heat energy.

Problem 36. A hard wire is broken by bending it repeatedly in alternating directions. Why ?

Solution. When the wire is subjected to repeated alternating strains, the strength of its material decreases and the wire breaks.

Problem 37. Why is the longer side of cross-section of girder used as depth ?

Solution. Depression, $\delta = \dfrac{WL^3}{4\,bd^3}$

Clearly, the depression of the girder will be small when depth d is large, because $\delta \propto d^{-3}$.

Scanned by CamScanner

Problem 38. The ratio stress/strain remains constant for a small deformation. What happens to this ratio if deformation is made very large ?

Solution. When the deforming force exceeds the elastic limit, the strain increases more rapidly than stress. Hence the ratio of stress/strain decreases.

Problem 39. Why are electric poles given hollow structure ?

Solution. This is because a hollow shaft is stronger than a solid shaft made from the same and equal amount of material.

Problem 40. The Young's modulus of a wire of length L and radius r is Y. If the length is reduced to $L/2$ and radius $r/4$, what will be its Young's modulus ? *[Central Schools 04]*

Solution. Young's modulus is a material constant. It is not affected by the change in dimensions of the wire. It will remain equal to Y.

Problem 41. A wire fixed at the upper end stretches by length l by applying a force F. What is the work done in stretching the wire ? *[AIEEE 04]*

Solution. Work done in stretching the wire,

$$W = \frac{1}{2}\,\text{Stretching force} \times \text{increase in length} = \frac{1}{2}Fl.$$

Problem 42. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Find the elastic energy stored in the wire. *[AIEEE 03]*

Solution. Here $F = 200$ N, $l = 1\text{ mm} = 10^{-3}$ m

Elastic potential energy stored in the wire,

$$U = \frac{1}{2}Fl = \frac{1}{2} \times 200 \times 10^{-3} = \mathbf{0.1\ J.}$$

Problem 43. If S is the stress and Y is Young's modulus of the material of a wire, what is the energy stored in the wire per unit volume in terms of S and Y ? *[AIEEE 05]*

Solution. Elastic potential energy stored per unit volume

$$u = \frac{1}{2}\,\text{Stress} \times \text{Strain} = \frac{1}{2} \times \text{Stress} \times \frac{\text{Stress}}{Y} = \frac{S^2}{2Y}.$$

Problem 44. Following are the graphs of elastic materials. Which one corresponds to that of brittle material ? *[Central Schools 08]*

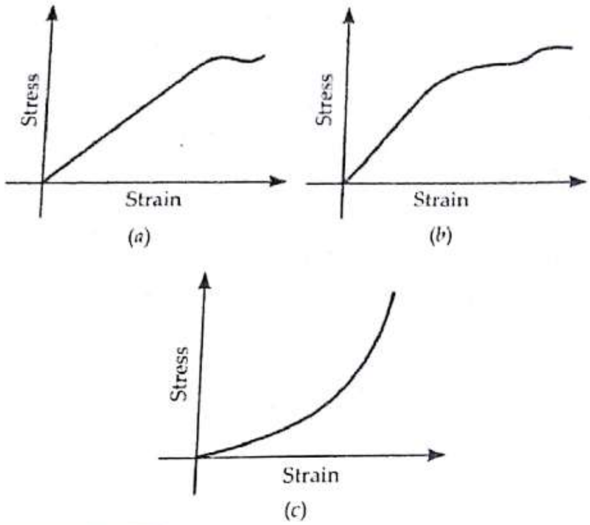


Fig. 9.23

Solution. Graph (*a*) represents a brittle material as it indicates a very small plastic range of extension.

Problem 45. A wire stretches by a certain amount under a load. If the load and radius both are increased to four times, find the stretch caused in the wire. *[Chandigarh 08]*

Solution. Young's modulus, $Y = \frac{W}{A} \times \frac{L}{l}$

$\therefore$ Elongation, $l = \frac{WL}{AY} = \frac{WL}{\pi r^2 Y}$

When both load and radius are increased to four times, the elongation becomes

$$l' = \frac{4W \times L}{\pi(4r)^2 Y} = \frac{WL}{4\pi r^2 Y} = \frac{l}{4}.$$

Short Answer Conceptual Problems

Problem 1. In the diagram a graph between the inter-molecular force F acting between the molecules of a solid and the distance r between them is shown. Explain the graph.

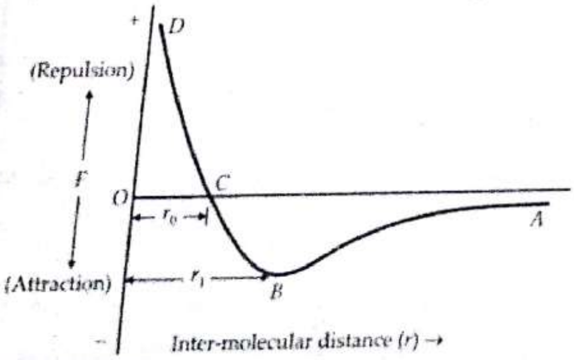


Fig. 9.24

Solution. (*i*) As the intermolecular distance r decreases, the force of attraction between the molecules increases.

(*ii*) When the distance decreases to r_1, the force of attraction is maximum.

(*iii*) As the distance further decreases, the attractive force goes on decreasing and when the distance decreases to r_0, the force becomes zero. When the distance decreases below r_0, the molecules begin to repel and the repulsive force increases rapidly.

Problem 2. Crystalline solids have sharp melting points. Amorphous solids do not melt at a sharp temperature ; rather these have a softening range of temperature. Explain.

Scanned by CamScanner

Solution. All bonds in a crystalline solid are equally strong. When the solid is heated, these bonds ruptured at the same temperature. So crystalline solids have sharp melting points.

On the other hand, all bonds in an amorphous solid are not equally strong. When the solid is heated, weaker bonds break at lower temperatures and the stronger ones at higher temperatures. So the solid first softens and then finally melts, *i.e.*, the amorphous solids do not have sharp melting points.

Problem 3. Which is more elastic—rubber or steel ? Explain. *[Delhi 96 ; Himachal 03, 05, 07C]*

Solution. Consider two rods of steel and rubber, each having length l and area of cross-section A. If they are subjected to the same deforming force F, then the extension Δl_s produced in the steel rod will be less than the extension Δl_r in the rubber rod, *i.e.*, $\Delta l_s < \Delta l_r$. Now

$$Y_s = \frac{F}{A} \cdot \frac{l}{\Delta l_s} \quad \text{and} \quad Y_r = \frac{F}{A} \cdot \frac{l}{\Delta l_r}$$

$$\therefore \quad \frac{Y_s}{Y_r} = \frac{\Delta l_r}{\Delta l_s}$$

As $\Delta l_s < \Delta l_r$, so $Y_s > Y_r$

i.e., Young's modulus for steel is greater than that of rubber. Hence *steel is more elastic than rubber.*

Problem 4. The stress-strain graph for a metal wire is shown in Fig. 9.25. Up to the point E, the wire returns to its original state O along the curve EPO when it is gradually unloaded. Point B corresponds to the fracture of the wire.

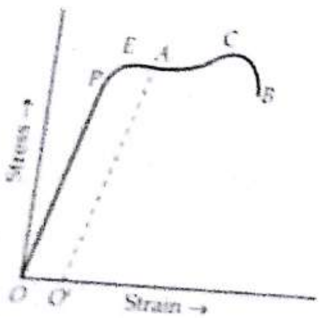


Fig. 9.25

(a) Up to which point on the curve is Hooke's law obeyed ? This point is sometimes called "Proportionality limit".

(b) Which point on the curve corresponds to elastic limit and yield point of the wire ?

(c) Indicate the elastic and plastic regions of the stress-strain graph.

(d) Describe what happens when the wire is loaded up to a stress corresponding to the point A on the graph, and then unloaded gradually. In particular, explain the dotted curve.

(e) What is peculiar about the portion of the stress-strain graph from C to B ? Up to what stress can the wire be subjected without causing fracture ? *[Delhi 03]*

Solution. (*a*) Hooke's law is obeyed upto the point P, because upto this point, stress ∝ strain.

(*b*) Point E corresponds to elastic limit because the wire returns to original state O along EPO if it is gradually unloaded.

(*c*) The elastic region is from O to E and the plastic region is from E to B.

(*d*) Upto point P, stress is proportional to strain. Between P and E, strain increases more rapidly than stress and Hooke's law is not obeyed. When the wire is unloaded at any point A beyond E, the wire does not retrace the curve $AEPO$ but follows the dashed curve AO'. When the stress becomes zero, a residual strain OO' is left in the wire.

(*e*) Between C and B the wire virtually flows out, *i.e.*, the strain increases even when the wire is being unloaded. Fracture takes place at point B. The stress can be applied to the value corresponding to the point C without causing fracture.

Problem 5. Two different types of rubber are found to have the stress-strain curves as shown in Fig. 9.26.

(a) In which significant ways do these curves differ from the stress-strain curve of a metal wire shown in Fig. 9.26 ?

(b) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers A and B would you prefer to use for this purpose ? Why ?

(c) Which of the two rubber materials would you choose for a car tyre ? *[Central Schools 07]*

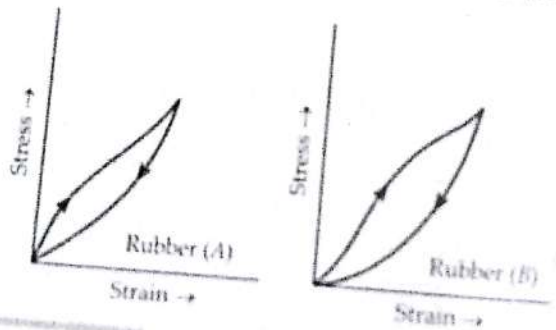


Fig. 9.26

Solution. (*a*) The stress-strain curves for rubber differ from the stress-strain curve for a metal in following respects :

(*i*) Hooke's law is not obeyed even for small stresses.

(*ii*) There is no permanent set (residual strain) even for large stresses.

(iii) There is large elastic region for both types of rubber.

(iv) Neither material retraces the curve during unloading. Thus both materials exhibit elastic hysteresis.

(b) The area of the hysteresis loop is proportional to the energy dissipated by the material as heat when the material undergoes loading and unloading. A material for which the hysteresis loop has larger area would absorb more energy when subjected to vibrations. Therefore to absorb vibrations, we would prefer rubber *B*.

(c) In car tyre, the energy dissipation must be minimised to avoid excessive heating of the car tyre.

As rubber *A* has smaller hysteresis loop area (and hence smaller energy loss), so it is preferred to *B* for a car tyre.

Problem 6. Read each of the statements below carefully and state, with reasons, if it is true or false.

(a) When a material is under tensile stress, the restoring forces are caused by interatomic attraction while under compressional stress, the restoring forces are due to inter-atomic repulsion.

(b) A piece of rubber under an ordinary stress can display 1000% strain : yet when unloaded returns to its original length. This shows that the elastic restoring forces in a rubber piece are strictly conservative.

(c) Elastic restoring forces are strictly conservative only when Hooke's law is obeyed.

Solution. (a) **True.** In tensile stress, the interatomic separation becomes greater than equilibrium separation and the interatomic forces are attractive. In compressional stress, the interatomic separation becomes less than r_0 and the interatomic forces are repulsive.

(b) **False.** As the piece of rubber returns to its original length when unloaded, it is a case of elastic hysteresis in which there is some loss of energy. This signifies non-conservative forces.

(c) **False.** Even if the stress-strain curves are non-linear, the elastic forces are conservative as long as loading and unloading curves are identical.

Problem 7. Two wires of different materials are suspended from a rigid support. They have the same length and diameter and carry the same load at their free ends. (a) Will the stress and strain in each wire be the same ? (b) Will the extension in both wires be the same ?

Solution. (a) Stress in both the wires is the same as both the wires have the same diameter and carry the same load at their free ends. Strain will be different in the two wires as the wires are of different materials, even though the stress is the same.

(b) Because the original lengths of the two wires are equal and strains produced in them are different, hence extensions in the two wires will not be same.

Problem 8. A cable is replaced by another of the same length and material but of twice the diameter. (a) How does this affect its elongation under a given load ? (b) How many times will be the maximum load it can now support without exceeding the elastic limit ?

Solution. (a) Young's modulus,

$$Y = \frac{Mgl}{\pi r^2 \cdot \Delta l} = \frac{Mgl}{\pi\left(\frac{D}{2}\right)^2 \cdot \Delta l} = \frac{4\,Mgl}{\pi D^2 \cdot \Delta l}$$

where *D* is the diameter of the wire.

Elongation, $\Delta l = \frac{4\,Mgl}{\pi\,D^2 Y}$ i.e., $\Delta l \propto \frac{1}{D^2}$

Clearly, if the diameter is doubled, the elongation will become one-fourth.

(b) Also load, $Mg = \frac{\pi D^2 \cdot \Delta l \cdot Y}{4l}$ i.e., $Mg \propto D^2$

Clearly, if the diameter is doubled, the wire can support 4 times the original load.

Problem 9. Two wires of same length and material but of different radii are suspended from a rigid support. Both carry the same load. Will the stress, strain and extension in them be same or different ?

Solution. Let r_1 and r_2 be the radii of the two wires.

(i) Stress $= \frac{F}{A} = \frac{F}{\pi r^2}$. For same load *F*, $\frac{(\text{stress})_1}{(\text{stress})_2} = \frac{r_2^2}{r_1^2}$

(ii) Strain, $\frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi r^2\,Y}$

For the two wires *F* and *Y* are same, so $\frac{(\text{strain})_1}{(\text{strain})_2} = \frac{r_2^2}{r_1^2}$

(iii) Extension, $\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{F}{\pi r^2} \cdot \frac{l}{Y}$

For the two wires *F*, *L* and *Y* are same, so $\frac{(\Delta l)_1}{(\Delta l)_2} = \frac{r_2^2}{r_1^2}$

Hence stress, strain and extension are all different for the two wires.

Problem 10. A uniform plank of Young's modulus *Y* is moved over a smooth horizontal surface by a constant horizontal force *F*. The area of transverse section of the plank is *A*. Find the compressive strain on the plank in the direction of the force.

Solution. As the force at the other end of the plank is zero, so the average stretching force $= \frac{F+0}{2} = \frac{F}{2}$

$\therefore$ Stress $= \frac{F}{2A}$

Strain $= \frac{\text{Stress}}{Y} = \frac{1}{2}\frac{F}{AY}$.

Scanned by CamScanner

Problem 11. What are the factors which affect the elasticity of a material ?

Solution. The following factors affect the elasticity of a material :

(*i*) ***Hammering and rolling.*** In both of these processes, the crystal grains are broken into small units and the elasticity of the material increases.

(*ii*) ***Annealing.*** This process results in the formation of larger crystal grains and elasticity of the material decreases.

(*iii*) ***Presence of impurities.*** Depending on the nature of the impurity, the elasticity of a material can be increased or decreased.

(*iv*) ***Temperature.*** Elasticity of most of the materials decreases with the increase in temperature. The elasticity of in var is not affected by temperature.

Problem 12. Elasticity has a different meaning in physics than that in daily life. Comment.

Solution. In daily life, a body is said to be elastic if a large deformation or strain is produced on applying a given stress on it. In physics, elasticity is the property of the material of a body by virtue of which it opposes any change in its size or shape when a stress is applied on it. Thus a body will be more elastic if a small strain is produced on applying a given stress on it.

Problem 13. Why a spring balance does not give correct measurement, when it has been used for a long time ?

[Himachal 05C, 07C]

Solution. When a spring balance has been used for a long time, it develops an elastic fatigue, the spring of such a balance takes longer time to recover its original configuration and therefore it does not give correct measurement.

Problem 14. Why the bridges are declared unsafe after long use ?

[Himachal 03, 07C]

Solution. During its long use, a bridge suffers alternating strains continuously. Consequently, the elastic strength of the bridge gets reduced. After a long time, the bridge develops elastic fatigue and there occurs a permanent change in its structure. This permanent change ultimately leads the bridge to collapse. In order to avoid this event, the bridges are declared unsafe after long use.

Problem 15. Two identical solid balls, one of ivory and the other of wet-clay, are dropped from the same height on the floor. Which will rise to a greater height after striking the floor and why ?

Solution. The ball which is more elastic rises to a greater height after striking the floor. Ivory is more elastic than wet-clay. Hence the ball of ivory will rise to a greater height. In fact, the ball of wet-clay will not rise at all, it will get flattened.

Problem 16. The breaking force for a wire is *F*. What will be the breaking force for (*a*) two parallel wires of the same size (*b*) for a single wire of double the thickness?

Solution. (*a*) When two wires of same size are suspended in parallel, a force *F* equal to the breaking force will act on each wire if a breaking force of 2*F* is applied on the parallel combination.

(*b*) $F = \dfrac{YA\,\Delta l}{l} = \dfrac{Y \cdot \pi r^2\,\Delta l}{l}$ *i.e.*, $F \propto r^2$

Thus for a single wire of double the thickness, the breaking force will be **4*F***.

Problem 17. Graphite consists of planes of carbon atoms. Between atoms in the planes there are only weak forces. What kind of elastic properties do you expect from graphite ?

Solution. Due to weak attractive forces between carbon atoms of different planes, it is easier to produce a large shearing strain by moving one plane of atoms over the other with the application of a small tangential stress. Now

$$\text{Modulus of rigidity, } \eta = \frac{\text{Tangential stress}}{\text{Shear strain}}$$

Hence graphite should possess a small modulus of rigidity.

Problem 18. Why does modulus of elasticity of most of the materials decrease with the increase of temperature ?

Solution. As the temperature increases, the inter-atomic forces of attraction become weaker. For given stress, a larger strain or deformation is produced at a higher temperature. Hence the modulus of elasticity (stress/strain) decreases with the increase of temperature.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, what will be the elongation of the wire in mm ?*

[AIEEE 06]

Solution. Young's modulus, $Y = \dfrac{W}{A} \times \dfrac{L}{l}$

$\therefore$ Elongation, $l = \dfrac{WL}{AY}$

On either side of the wire, tension is W but length is $l/2$.

$\therefore$ Elongation produced along either side $= l/2$ mm

Total elongation produced $= l/2 + l/2 = l$ **mm.**

Scanned by CamScanner

Problem 2. *A wire is cut to half its original length. (a) How would it affect the elongation under a given load ? (b) How does it affect the maximum load it can support without exceeding the elastic limit ?*

Solution. Young's modulus, $Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$

(*a*) $\Delta l = \frac{F}{A} \cdot \frac{l}{Y}$ *i.e.,* $\Delta l \propto l$

So when the wire is cut half to its original length, extension is halved.

(*b*) Maximum load, $F = \frac{YA\,\Delta l}{l}$

Here Y and A are constant. When the wire is cut to half its original length, there is no change on the value of $\Delta l / l$. Hence there is no effect on the maximum load.

Problem 3. *A bar of cross-section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.*

(*a*) *What is the tensile stress on this plane ?*

(*b*) *What is the shearing stress on this plane ?*

(*c*) *For what value of θ is the tensile stress maximum ?*

(*d*) *For what value of θ is the shearing stress maximum ?*

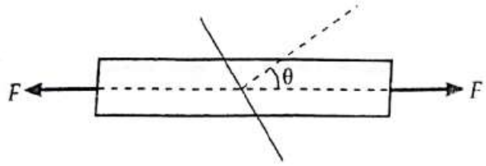


Fig. 9.27

Solution. (*a*) The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on the plane.

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\cos^2\theta$$

[∵ Area of plane section = $A \sec\theta$]

(*b*) Shearing stress

$$= \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\sin\theta\cos\theta = \frac{F}{2A}\sin 2\theta.$$

(*c*) Tensile stress will be maximum when $\cos^2\theta$ is maximum, *i.e.,* $\cos\theta = 1$ or $\theta = 0°$.

(*d*) Shearing stress will be maximum when $\sin 2\theta$ is maximum, *i.e.,* $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = 45°$.

Problem 4. *The graph (Fig. 9.28) shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end with a load W connected to the other end. If the cross-sectional area of the wire is $10^{-6} m^2$, calculate the Young's modulus of the material of the wire.* **[IIT Screening 03]**

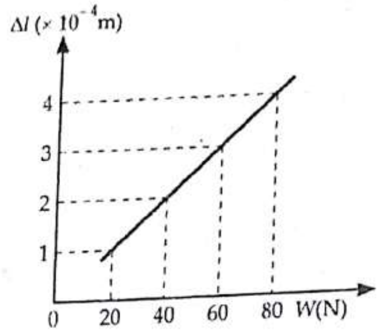


Fig. 9.28

Solution. When $W = 20$ N, $\Delta l = 1 \times 10^{-4}$ m

$$\therefore \quad Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{W}{A} \cdot \frac{l}{\Delta l} = \frac{20}{10^{-6}} \times \frac{1}{1 \times 10^{-4}}$$

$$= 2 \times 10^{11}\ \text{Nm}^{-2}.$$

Problem 5. *A metallic wire is stretched by suspending weight from it. If α is the longitudinal strain and Y is the Young's modulus, show that elastic potential energy per unit volume is given by $\frac{1}{2}Y\alpha^2$.* **[Roorkee 82]**

Solution. Elastic P.E. per unit volume,

$$u = \frac{1}{2}\ \text{stress} \times \text{strain}$$

But $\quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Stress}}{\alpha} \qquad$ [∵ Strain = α]

∴ $\quad$ Stress = $Y\alpha$

Hence $\quad u = \frac{1}{2}Y\alpha \times \alpha = \frac{1}{2}Y\alpha^2.$

Problem 6. *A copper wire of negligible mass, 1 m length and cross-sectional area $10^{-6}\ m^2$ is kept on a smooth horizontal table with one end fixed. A ball of mass 1 kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad s^{-1}. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus. If on increasing the angular velocity to 100 rad s^{-1}, the wire breaks down, obtain the breaking stress.* **[Roorkee 92]**

Solution. When the ball is rotated at the end of copper wire, restoring force in the wire is equal to the centripetal force on the ball.

Centripetal force,

$$F = mr\omega^2 = ml\omega^2 \qquad [\because r = l]$$

As $\quad Y = \frac{F}{A} \cdot \frac{l}{\Delta l} \quad$ or $\quad F = \frac{YA\,\Delta l}{l}$

$$\therefore \quad \frac{YA\,\Delta l}{l} = ml\omega^2 \quad \text{or} \quad Y = \frac{ml^2\omega^2}{A\,\Delta l}$$

But $m = 1$ kg, $l = 1$ m, $\omega = 20$ rad s^{-1}, $A = 10^{-6}$ m^2, $\Delta l = 10^{-3}$ m

$$\therefore \quad Y = \frac{1 \times (1)^2 \times (20)^2}{10^{-6} \times 10^{-3}} = 4 \times 10^{11}\ \text{Nm}^{-2}.$$

Scanned by CamScanner

Breaking force

$= m r\omega^2_{max} = m l \omega^2_{max} = 1 \times 1 \times (100)^2 = 10^4$ N

Breaking stress

$$= \frac{\text{Breaking force}}{\text{Area}} = \frac{10^4}{10^{-6}} = 10^{10}\ \text{Nm}^{-2}.$$

Problem 7. *A load of 31.4 kg is suspended from a wire of radius 10^{-3} m and density 9×10^3 kg m^{-3}. Calculate the change in temperature of the wire if 75% of the work done is converted into heat. The Young's modulus and the specific heat of the material of the wire are 9.8×10^{10} Nm^{-2} and 490 J kg^{-1} K^{-1} respectively.* **[Roorkee 93]**

Solution. As $Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$ $\quad \therefore \quad \Delta l = \frac{F}{A} \cdot \frac{l}{Y}$

Work done,

$$W = \frac{1}{2} \text{Stretching force} \times \text{extension} = \frac{1}{2} F \Delta l$$

$$= \frac{1}{2} F \times \frac{F}{A} \frac{l}{Y} = \frac{F^2 l}{2AY}$$

Let ΔT be the rise in temperature when 75% of the work done changes into heat. Then

$$0.75\, W = mc\, \Delta T$$

where $m (= A l \rho)$ is the mass of the wire and c its specific heat.

$$\therefore \quad 0.75 \times \frac{F^2 l}{2AY} = A l \rho c \Delta T \quad \text{or} \quad \Delta T = \frac{0.75\, F^2}{2 A^2 Y \rho c}$$

But $F = Mg = 31.4 \times 9.8$ N, $A = 3.14 \times 10^{-6}\ \text{m}^2$,

$Y = 9.8 \times 10^{10}\ \text{Nm}^{-2}$, $\rho = 9 \times 10^3\ \text{kg m}^{-3}$,

$c = 490\ \text{J kg}^{-1}\ \text{K}^{-1}$

$$\therefore \Delta T = \frac{0.75 \times (31.4 \times 9.8)^2}{2 \times (3.14 \times 10^{-6})^2 \times 9.8 \times 10^{10} \times 9 \times 10^3 \times 490}$$

$$= \frac{1}{120}\ \text{K}.$$

Problem 8. *A light rod of length 2 m is suspended horizontally by means of two vertical wires of equal lengths tied to its ends. One of the wires is made of steel and is of cross-section $A_1 = 0.1\ cm^2$ and the other is of brass and is of cross-section $A_2 = 0.2\ cm^2$. Find out the position along the rod at which a weight must be suspended to produce (i) equal stresses in both wires, (ii) equal strains in both wires. For steel, $Y = 20 \times 10^{10}\ Nm^{-2}$ and for brass $Y = 10 \times 10^{10}\ Nm^{-2}$.* **[IIT]**

Solution. The situation is shown in Fig. 9.29. Let AB be the rod of length 2 m. Suppose a weight W is hung at C at distance x from A. Let T_1 and T_2 be the tensions in the steel and brass rods respectively.

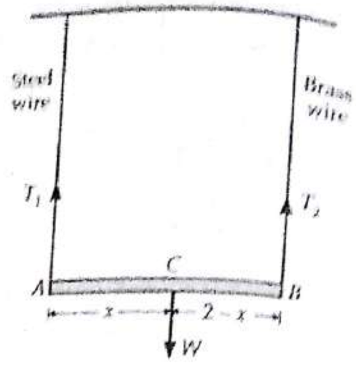


Fig. 9.29

(*i*) Stress in steel wire $= \frac{T_1}{A_1}$,

Stress in brass wire $= \frac{T_2}{A_2}$

As both the stresses are equal, so

$$\therefore \quad \frac{T_1}{A_1} = \frac{T_2}{A_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{0.1}{0.2} = \frac{1}{2}$$

Now moments about C are equal as the system is in equilibrium

$$\therefore \quad T_1 x = T_2 (2 - x) \quad \text{or} \quad \frac{T_1}{T_2} = \frac{2 - x}{x}$$

$$\text{or} \quad \frac{1}{2} = \frac{2 - x}{x} \qquad \left(\because \frac{T_1}{T_2} = \frac{1}{2} \right)$$

$$x = 4 - 2x$$

$$\therefore \quad 3x = 4 \quad \text{or} \quad x = \frac{4}{3} = \mathbf{1.33\ m.}$$

(*ii*) Now Y = Stress/Strain

$\therefore$ Strain = Stress/Y

$$\text{Strain in steel wire} = \frac{T_1 / A_1}{Y_1}$$

$$\text{Strain in brass wire} = \frac{T_2 / A_2}{Y_2}$$

$$\text{Now} \quad \frac{T_1}{A_1 Y_1} = \frac{T_2}{A_2 Y_2}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{0.1\ \text{cm}^2 \times 20 \times 10^{10}\ \text{Nm}^{-2}}{0.2\ \text{cm}^2 \times 10 \times 10^{10}\ \text{Nm}^{-2}}$$

$$= 1$$

$$\text{Again,} \quad T_1 x = T_2 (2 - x) \quad \text{or} \quad \frac{T_1}{T_2} = \frac{2 - x}{x}$$

$$\text{or} \quad 1 = \frac{2 - x}{x} \qquad \left[\because \frac{T_1}{T_2} = 1 \right]$$

$$\therefore \quad x = 2 - x$$

$$\text{or} \quad 2x = 2 \quad \text{or} \quad x = \mathbf{1\ m.}$$

Scanned by CamScanner

Problem 9. *A thin rod of negligible mass and area of cross-section 4×10^{-6} m^2, suspended vertically from one end has a length of 0.5 m at 100°C. The rod is cooled at 0°C, but prevented from contracting by attaching a mass at the lower end. Find (i) this mass and (ii) the energy stored in the rod. Given for this rod, $Y = 10^{11}$ Nm^{-2}, coefficient of linear expansion $= 10^{-5}$ K^{-1} and $g = 10$ ms^{-2}.* **[IIT 97]**

Solution. Here

$$A = 4 \times 10^{-6} \text{ m}^2, \quad l = 0.5 \text{ m},$$
$$\Delta T = 100 - 0 = 100°\text{C} = 100 \text{ K}$$
$$Y = 10^{11} \text{ Nm}^{-2}, \quad \alpha = 10^{-5} \text{ K}^{-1}$$

Change in length,

$$\Delta l = l \alpha \Delta T = 0.5 \times 10^{-5} \times 100 = 5 \times 10^{-4} \text{ m}$$

As $$Y = \frac{\text{Stress}}{\Delta l / l}$$

$$\text{Stress} = Y \frac{\Delta l}{l} = Y \times \alpha \Delta T = 10^{11} \times 10^{-5} \times 100 = 10^8 \text{ Nm}^{-2}$$

Stretching force,

$$F = \text{Stress} \times \text{area} = 10^8 \times 4 \times 10^{-6} = 4 \times 10^2 \text{ N}$$

But $F = Mg \quad \therefore \quad M = 4 \times 10^2$

or $$M = \frac{4 \times 10^2}{g} = \frac{4 \times 10^2}{10} = \mathbf{40\ kg.}$$

(*ii*) Energy stored in the rod

$$= \frac{1}{2} F \times \Delta l = \frac{1}{2} \times 4 \times 10^2 \times 5 \times 10^{-4} = \mathbf{0.1\ J.}$$

Problem 10. *A wire of cross-sectional area A is stretched horizontally between two clamps located at a distance 2l metres from each other. A weight W kg is suspended from the midpoint of the wire. If the vertical distance through which the mid-point of the wire moves down be $x < l$, then find (i) the strain produced in the wire. (ii) the stress in the area. (iii) If Y is the Young's modulus of wire, then find the value of x.*

Solution. The situation is shown in Fig. 9.30. The increase in length of the wire when it is pulled down into shape *BOC* is

$$\Delta l = BO + OC - 2l = 2BO - 2l = 2(l^2 + x^2)^{1/2} - 2l$$
$$= 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l = 2l\left[1 + \frac{x^2}{2l^2}\right] - 2l = \frac{x^2}{l}$$

$$\therefore \text{Strain} = \frac{\Delta l}{2l} = \frac{x^2}{2l^2}.$$

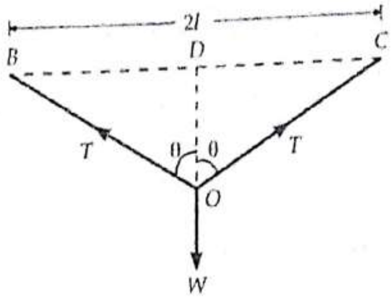


Fig. 9.30

(*ii*) If T is the tension in the string, then

$$2T \cos\theta = W \quad \text{or} \quad T = \frac{W}{2\cos\theta}$$

Now $$\cos\theta = \frac{x}{OB} = \frac{x}{\sqrt{l^2 + x^2}} = \frac{x}{l\left[1 + \frac{x^2}{l^2}\right]^{1/2}} = \frac{x}{l\left[1 + \frac{x^2}{2l^2}\right]}$$

As $\frac{x^2}{2l^2} << 1$, so $1 + \frac{x^2}{2l^2} = 1$

$$\therefore \quad \cos\theta = \frac{x}{l} \quad \text{and} \quad T = \frac{W}{2(x/l)} = \frac{Wl}{2x}$$

$$\text{Stress} = \frac{T}{A} = \frac{Wl}{2Ax}.$$

$$(iii)\ Y = \frac{\text{Stress}}{\text{Strain}} = \frac{Wl}{2Ax} \times \frac{2l^2}{x^2} = \frac{Wl^3}{Ax^3}$$

or $$x = l\left[\frac{W}{YA}\right]^{1/3}.$$

Problem 11. *A stone of 0.5 kg mass is attached to one end of a 0.8 m long aluminium wire of 0.7 mm diameter and suspended vertically. The stone is now rotated in a horizontal plane at a rate such that the wire makes an angle of 85° with the vertical. Find the increase in the length of the wire. The Young's modulus of aluminium $= 7 \times 10^{10}$ Nm^{-2}, $\sin 85° = 0.9962$, $\cos 85° = 0.0872$)* **[Roorkee 90]**

Solution. As shown in Fig. 9.31, let the stone be rotated in a circle of radius r with speed v. Then the forces acting on the stone are

(*i*) its weight mg acting vertically downwards,

(*ii*) centrifugal force mv^2/r in horizontal direction, and

(*iii*) tension T in the wire.

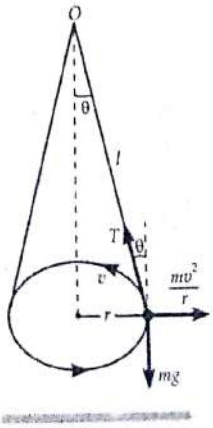


Fig. 9.31

Scanned by CamScanner

Resolving T in horizontal and vertical directions, we get

$$T \cos \theta = mg \qquad ...(i)$$

and

$$T \sin \theta = \frac{mv^2}{r} \qquad ...(ii)$$

$$\therefore \quad T = \frac{mg}{\cos \theta} = \frac{mg}{\cos 85^\circ} = \frac{0.5 \times 9.8}{0.0872} = 56.19 \text{ N}.$$

As
$$Y = \frac{T/A}{\Delta l / l} = \frac{T}{A} \cdot \frac{l}{\Delta l}$$

$$\therefore \quad \Delta l = \frac{Tl}{YA} = \frac{56.19 \times 0.8}{7 \times 10^{10} \times \pi \times (0.35 \times 10^{-3})^2}$$

$$= 1.67 \times 10^{-3} \text{ m} = \mathbf{1.67\ mm}.$$

Problem 12. *Two rods of different materials but of equal cross-sections and lengths (1.0 m each) are joined to make a rod of length 2.0 m. The metal of one rod has coefficient of linear thermal expansion $10^{-5}\,{}^\circ C^{-1}$ and Young's modulus 3×10^{10} Nm^{-2}. The other metal has the values $2 \times 10^{-5}\,{}^\circ C^{-1}$ and 10^{10} Nm^{-2} respectively. How much pressure must be applied to the ends of the composite rod to prevent its expansion when the temperature is raised by 100°C ?*

[Roorkee 91]

Solution. When the temperature is raised by 100°C, the extensions in the two rods are

$$\Delta l_1 = \alpha_1 \, l \, \Delta T = 10^{-5} \times 1.0 \times 100 = 10^{-3} \text{ m}$$

$$\Delta l_2 = \alpha_2 \, l \, \Delta T = 2 \times 10^{-5} \times 1.0 \times 100 = 2 \times 10^{-3} \text{ m}$$

Tensions produced in the rods are

$$T_1 = Y_1 \, A . \frac{\Delta l_1}{l} = \frac{3 \times 10^{10} \times A \times 10^{-3}}{1.0}$$

$$= 3 \times 10^7 \, A \text{ newton}$$

$$T_2 = Y_2 \, A . \frac{\Delta l_2}{l} = \frac{10^{10} \times A \times 2 \times 10^{-3}}{1.0}$$

$$= 2 \times 10^7 \, A \text{ newton}$$

where A = area of cross-section of each rod.

Total force needed to be applied at the ends to prevent expansion

$$= T_1 + T_2 = 5 \times 10^7 \, A \text{ newton}$$

$$\therefore \quad \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{5 \times 10^7 \, A}{A} = \mathbf{5 \times 10^7\ Nm^{-2}}.$$

Guidelines to NCERT Exercises

9.1. *A steel wire of length 4.7 m and cross-section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-section 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper ?*

Ans. For steel : $l = 4.7$ m, $A = 3.0 \times 10^{-5}$ m^2

For copper : $l = 3.5$ m, $A = 4.0 \times 10^{-5}$ m^2

Applied force F and extension Δl are same for both wires.

$\therefore$ Young's modulus of steel,

$$Y_S = \frac{Fl}{A . \Delta l} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l}$$

Young's modulus of copper,

$$Y_C = \frac{Fl}{A . \Delta l} = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l}$$

$$\frac{Y_S}{Y_C} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l} \times \frac{4.0 \times 10^{-5} \times \Delta l}{F \times 3.5}$$

$$= \mathbf{1.79}.$$

9.2. *Fig. 9.32 shows the stress-strain curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material ?*

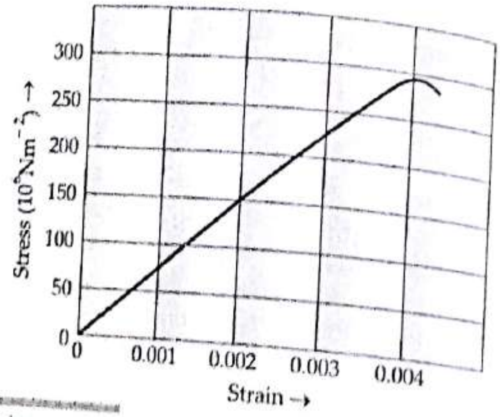


Fig. 9.32

Ans. (*a*) From the given graph for a stress of 150×10^6 Nm^{-2}, the strain is 0.002.

$\therefore$ Young's modulus,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150 \times 10^6 \text{ Nm}^{-2}}{0.002}$$

$$= \mathbf{7.5 \times 10^{10}\ Nm^{-2}}.$$

(*b*) Near the bend of the curve, the stress is nearly 300×10^6 Nm^{-2}.

$\therefore$ Approximate yield strength of the material

$$= \mathbf{3 \times 10^8\ Nm^{-2}}.$$

Scanned by CamScanner

9.3. *The stress-strain graphs for materials A and B are shown in Fig. 9.33.*

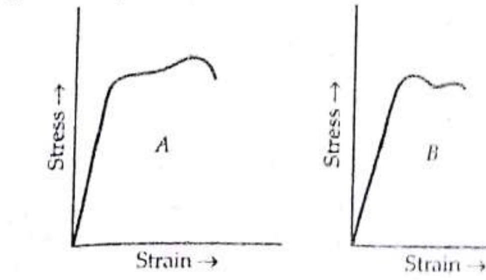


Fig. 9.33

The graphs are drawn to the same scale.

(a) *Which of the material has greater Young's modulus ?*

(b) *Which material is more ductile ?*

(c) *Which is more brittle ?*

(d) *Which of the two is stronger material ?*

[Delhi 09 ; Central Schools 12, 14]

Ans. (a) Young's modulus $= \frac{\text{Stress}}{\text{Strain}} =$ slope of stress-strain graph.

As the slope of stress-strain graph for material A is higher than that for material B, so **material A has greater Young's modulus than B.**

(b) **Material A is more ductile than B,** because it has larger range of plastic extension between its elastic limit and fracture point.

(c) **Material B is more brittle than A,** because its plastic range of extension is very small.

(d) **Material A is stronger than B,** because it can withstand greater stress before breaking.

9.4. *Read each of the statements below carefully and state, with reasons, if it is true or false.*

(a) *The modulus of elasticity of rubber is greater than that of steel.*

(b) *The stretching of a coil is determined by its shear modulus.*

Ans. (a) **False.** When steel and rubber are subjected to the same deforming force, less extension and hence less strain is produced in the steel than the rubber and as

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

so Y is more in case of steel than in the case of rubber.

(b) **True.** When the coil is stretched, there is no change in the length or the volume of the wire used in the coil. There is only a change in the shape of the spring, so shear modulus is involved.

9.5. *Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in Fig. 9.34. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Compute the elongations of steel and brass wires. (1 Pa = 1 Nm^{-2}).*

Ans. *For steel wire :*

$$l_1 = 1.5 \text{ m}, \quad r_1 = \frac{0.25}{2} \text{ cm} = 0.125 \times 10^{-2} \text{ m}$$

$$F_1 = 6 + 4 = 10 \text{ kg f} = 10 \times 9.8 \text{ N}$$

$$Y_1 = 2.0 \times 10^{11} \text{ Pa}$$

As $Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}$

$$\Delta l = \frac{F}{\pi r^2} \cdot \frac{l}{Y}$$

$$\therefore \Delta l_1 = \frac{F_1}{\pi r_1^2} \cdot \frac{l_1}{Y_1}$$

$$= \frac{10 \times 9.8 \times 1.5}{3.14 \times (0.125 \times 10^{-2})^2 \times 2.0 \times 10^{11}}$$

$$= \mathbf{1.5 \times 10^{-4}\ m.}$$

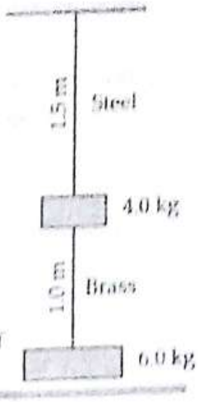


Fig. 9.34

For brass wire :

$$l_2 = 1.0 \text{ m}, \quad r_2 = 0.125 \times 10^{-2} \text{ m}$$

$$F_1 = 6 \text{ kg f} = 6 \times 9.8 \text{ N}, \quad Y_2 = 0.91 \times 10^{11} \text{ Pa}$$

$$\therefore \quad \Delta l_2 = \frac{F_2}{\pi r_2^2} \cdot \frac{l_2}{Y_2}$$

$$= \frac{6 \times 9.8 \times 1.0}{3.14 \times (0.125 \times 10^{-2})^2 \times 0.91 \times 10^{11}}$$

$$= \mathbf{1.3 \times 10^{-4}\ m.}$$

9.6. *The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 G Pa. What is the vertical deflection of this face ? (1 Pa = 1 Nm^{-2}).*

Ans. Area of the face on which force is applied,

$$A = 10 \text{ cm} \times 10 \text{ cm} = 100 \text{ cm}^2$$

$$= 100 \times 10^{-4} \text{ m}^2 = 10^{-2} \text{ m}^2$$

$$F = \text{L.I.} = 100 \times 10 = 1000 \text{ N},$$

$$\eta = 25 \text{ G Pa} = 25 \times 10^9 \text{ Pa}$$

$$l = 10 \text{ cm} = 0.10 \text{ m}$$

As $\eta = \frac{F/A}{\Delta l / l} = \frac{F}{A} \cdot \frac{l}{\Delta l}$

$$\therefore \quad \Delta l = \frac{F}{A} \cdot \frac{l}{\eta} = \frac{1000 \times 0.10}{10^{-2} \times 25 \times 10^9} = \mathbf{4 \times 10^{-7}\ m.}$$

9.7. *Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.*

Ans. Here $r_1 = 30 \text{ cm} = 0.3 \text{ m}$,

$$r_2 = 40 \text{ cm} = 0.4 \text{ m}, \quad Y = 2.0 \times 10^{11} \text{ Pa}$$

Scanned by CamScanner

As the load is uniformly distributed among the four columns, hence the load on each column

$$= \frac{50,000}{4}\ \text{kg} = 12500\ \text{kg}$$

$$\therefore \quad F = 12500 \times 9.8\ \text{N}$$

Also A = Area of cross-section of each column

$$= \pi r_2^2 - \pi r_1^2 = \pi (r_2^2 - r_1^2)$$

$$= \frac{22}{7}[(0.4)^2 - (0.3)^2] = \frac{22}{7} \times 0.07 = 0.22\ \text{m}^2$$

∴ Compressional strain

$$= \frac{\text{Stress}}{\text{Young's modulus}} = \frac{F/A}{Y} = \frac{F}{AY}$$

$$= \frac{12500 \times 9.8}{0.22 \times 2.0 \times 10^{11}} = \mathbf{2.8 \times 10^{-6}}.$$

9.8. *A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.*

Ans. Here $F = 44,\ 500\ \text{N}$,

$$A = 15.2\ \text{mm} \times 19.1\ \text{mm}$$
$$= 15.2 \times 19.1 \times 10^{-6}\ \text{m}^2$$

For copper,

$$Y = 1.2 \times 10^{11}\ \text{Nm}^{-2}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A \times \text{strain}}$$

$$\therefore \quad \text{Strain} = \frac{F}{AY} = \frac{44500}{15.2 \times 19.1 \times 10^{-6} \times 1.2 \times 10^{11}}$$

$$= \mathbf{0.001277}.$$

9.9. *A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed $10^8\ Nm^{-2}$, what is the maximum load the cable can support ?*

Ans. Maximum stress

$$= \frac{\text{Maximum load}}{\text{Area of cross-section}} = \frac{\text{Maximum load}}{\pi r^2}$$

$$\therefore \text{Maximum load} = \pi r^2 \times \text{Maximum stress}$$

$$= 3.142 \times (1.5 \times 10^{-2})^2 \times 10^8\ \text{N}$$

$$= \mathbf{7.07 \times 10^4\ N}.$$

9.10. *A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.*

Ans. Let T be the tension in each wire. As the bar is supported symmetrically by the three wires, the increase in length Δl of each wire should be same.

$$\text{Now} \quad Y = \frac{T}{A}\frac{l}{\Delta l}$$

For all wires, we have same l, Δl and T.

$$\text{Hence} \quad Y \propto \frac{1}{A} \quad \text{or} \quad A \propto \frac{1}{Y}$$

$$\text{or} \quad \frac{\pi D^2}{4} \propto \frac{1}{Y} \quad \text{or} \quad D \propto \frac{1}{\sqrt{Y}}$$

$$\therefore \quad \frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{1.9 \times 10^{11}\ \text{Pa}}{1.1 \times 10^{11}\ \text{Pa}}} = 1.3.$$

9.11. *A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is $0.005\ cm^2$. Calculate the elongation of the wire when the mass is at the lowest point of its path.*

Ans. The centripetal force at the lowest point is given by

$$mr\omega^2 = T - mg$$

where T is the tension in the wire when the mass is at the lowest point.

∴ Tension,

$$T = mg + mr\omega^2 = m[g + r(2\pi\nu)^2]$$
$$= 14.5[9.8 + 1.0 \times 4 \times \pi^2 \times (2)^2]$$
$$= 14.5[9.8 + 16 \times 9.87] = 14.5 \times 167.72$$
$$= 2431.94\ \text{N}.$$

$$\text{Now} \quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta l/l} = \frac{T/A}{\Delta l/l}$$

$$\therefore \quad \Delta l = \frac{Tl}{AY} = \frac{2431.94 \times 1.0}{0.065 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= \mathbf{1.87 \times 10^{-3}\ m}.$$

9.12. *Compute the bulk modulus of water from the following data : Initial volume = 100.0 litre, pressure increase = 100.0 atm, final volume = 100.5 litre (1 atm = 1.013×10^5 Pa).*

Ans. Here $p = 100\ \text{atm} = 100 \times 1.013 \times 10^5\ \text{Pa}$

Initial volume,

$$V = 100.0\ \text{litre} = 100.0 \times 10^{-3}\ \text{m}^3$$

Final volume,

$$V' = 100.5\ \text{litre} = 100.5 \times 10^{-3}\ \text{m}^3$$

Increase in volume,

$$\Delta V = (100.5 - 100.0) \times 10^{-3} = 0.5 \times 10^{-3}\ \text{m}^3$$

Bulk modulus of water,

$$\kappa = \frac{pV}{\Delta V} = \frac{100 \times 1.013 \times 10^5 \times 100.0 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$= \mathbf{2.026 \times 10^9\ Pa}.$$

9.13. *What is the density of ocean water at a depth, where the pressure is 80.0 atm, given that its density at the surface is $1.03 \times 10^3\ kgm^{-3}$? Compressibility of water $= 45.8 \times 10^{-11}\ Pa^{-1}$. Given 1 atm = 1.013×10^5 Pa.*

Ans. Compressibility

$$= \frac{1}{\kappa} = 45.8 \times 10^{-11}\ \text{Pa}^{-1}$$

Scanned by CamScanner

Change in pressure,

$$p = 80 - 1 = 79 \text{ atm} = 79 \times 1.013 \times 10^5 \text{ Pa}$$

Density at the surface, $\rho = 1.03 \times 10^3 \text{ kgm}^{-3}$

As $\kappa = \frac{p}{\Delta V / V}$

$\therefore \quad \frac{\Delta V}{V} = p \times \frac{1}{\kappa}$

$$= 79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11}$$

$$= 3.665 \times 10^{-3}$$

Also, $\frac{\Delta V}{V} = \frac{V - V'}{V} = \frac{M/\rho - M/\rho'}{M/\rho} = 1 - \frac{\rho}{\rho'}$

or $\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}$.

or $\rho' = \frac{\rho}{1 - \frac{\Delta V}{V}} = \frac{1.03 \times 10^3}{1 - 3.665 \times 10^{-3}}$

$$= \frac{1.03 \times 10^3}{0.996} = \mathbf{1.034 \times 10^3 \text{ kgm}^{-3}}.$$

9.14. *Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.*

Ans. Here $p = 10 \text{ atm} = 10 \times 1.013 \times 10^5 \text{ Nm}^{-2}$, $\kappa = 37 \times 10^9 \text{ Nm}^{-2}$

Bulk modulus, $\kappa = \frac{pV}{\Delta V}$

Fractional change in volume of glass slab,

$$\frac{\Delta V}{V} = \frac{p}{\kappa} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9} = \mathbf{2.74 \times 10^{-5}}$$

9.15. *Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.*

Ans. Original volume

$$V = (10 \text{ cm})^3 = 1000 \text{ cm}^3$$

$$= 1000 \times 10^{-6} \text{ m}^3 = 10^{-3} \text{ m}^3$$

Pressure, $p = 7.0 \times 10^6$ Pa

For copper, $\kappa = 140 \times 10^9$ Pa

Bulk modulus, $\kappa = \frac{pV}{\Delta V}$

Volume contraction of copper cube,

$$\Delta V = \frac{pV}{\kappa} = \frac{7.0 \times 10^6 \times 10^{-3}}{140 \times 10^9}$$

$$= 0.05 \times 10^{-6} \text{ m}^3 = \mathbf{0.05 \text{ cm}^3}.$$

9.16. *How much should the pressure on a litre of water be changed to compress it by 0.10% ?*

Ans. Here $V = 1 \text{ litre} = 10^{-3} \text{m}^3$,

$$\frac{\Delta V}{V} = 0.10\% = \frac{0.10}{100} = 0.001$$

For water, $\kappa = 2.2 \times 10^9 \text{ Nm}^{-2}$

As $\kappa = \frac{pV}{\Delta V}$

$\therefore \quad p = \frac{\kappa \Delta V}{V} = 2.2 \times 10^9 \times 0.001$

$$= \mathbf{2.2 \times 10^6 \text{ Nm}^{-2}}.$$

9.17. *Anvils made of single crystals of diamond, with the shape as shown in Fig. 9.35, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil ?*

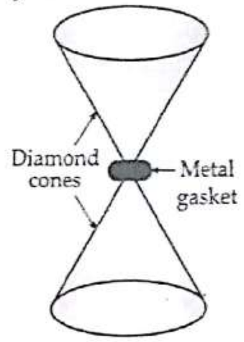


Fig. 9.35

Ans. Here $r = 0.25 \text{ mm} = 0.25 \times 10^{-3}$ m

$$F = 50{,}000 \text{ N}$$

Pressure at the tip of the anvil

$$= \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2} = \frac{50{,}000}{3.14 \times (0.25 \times 10^{-3})^2}$$

$$= \mathbf{2.55 \times 10^{11} \text{ Nm}^{-2}}.$$

9.18. *A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. 9.36. The cross-sectional areas of wires A and B are 1.0 mm² and 2.0 mm² respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires ?*

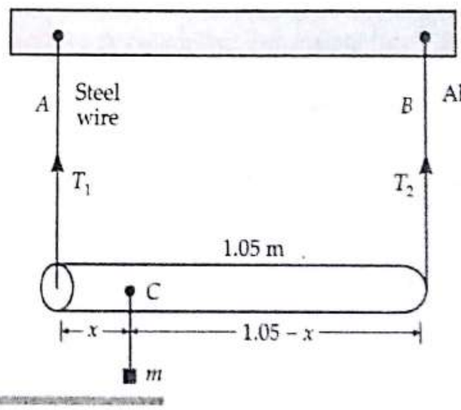


Fig. 9.36

Ans. Suppose the mass m is suspended at distance x from the wire A. Let T_1 and T_2 be the tensions in the steel and aluminium wires respectively.

Scanned by CamScanner

(a) Stress in steel wire $= \frac{T_1}{A_1}$

Stress in aluminium wire $= \frac{T_2}{A_2}$

As both the stresses are equal, so

$$\frac{T_1}{A_1} = \frac{T_2}{A_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{1.0\ \text{mm}^2}{2.0\ \text{mm}^2} = \frac{1}{2}$$

Now the moments about point C are equal because the system is in equilibrium.

$$\therefore \quad T_1 x = T_2 (1.05 - x)$$

$$\text{or} \quad \frac{T_1}{T_2} = \frac{1.05 - x}{x} \quad \text{or} \quad \frac{1}{2} = \frac{1.05 - x}{x}$$

$$\text{or} \quad x = 2.10 - 2x$$

$$\text{or} \quad x = \mathbf{0.7\ m} \quad \text{(from steel wire)}$$

(b) Strain $= \frac{\text{Stress}}{\text{Young's modulus}}$

$\therefore$ Strain in steel wire $= \frac{T_1 / A_1}{Y_1} = \frac{T_1}{A_1 Y_1}$

Strain in aluminium wire $= \frac{T_2}{A_2 Y_2}$

For the two strains to be equal,

$$\frac{T_1}{A_1 Y_1} = \frac{T_2}{A_2 Y_2}$$

$$\text{or} \quad \frac{T_1}{T_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{1.0\ \text{mm}^2 \times 200 \times 10^9\ \text{Pa}}{2.0\ \text{mm}^2 \times 70 \times 10^9\ \text{Pa}} = \frac{10}{7}$$

Again, $T_1 x = T_2 (1.05 - x)$

$$\text{or} \quad \frac{T_1}{T_2} = \frac{1.05 - x}{x} \quad \text{or} \quad \frac{10}{7} = \frac{1.05 - x}{x}$$

$$\text{or} \quad 10x = 7.35 - 7x$$

$$\text{or} \quad x = \frac{7.35}{17} = \mathbf{0.43\ m} \quad \text{(from steel wire)}$$

9.19. *A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of wire. Calculate the depression at the mid-point.*

Ans. The situation is shown in Fig. 9.37. The increase in length of the wire when it is pulled down into shape *BOC* is

$$\Delta l = BO + OC - 2l = 2BO - 2l$$

$$= 2(l^2 + x^2)^{1/2} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l$$

$$= 2l\left(1 + \frac{x^2}{2l^2}\right) - 2l = \frac{x^2}{l}$$

$$\therefore \quad \text{Strain} = \frac{\Delta l}{2l} = \frac{x^2}{2l^2}$$

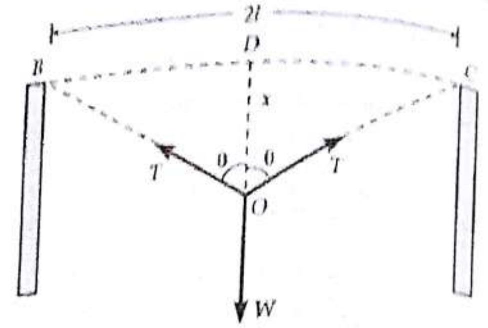


Fig. 9.37

Let T be the tension in the wire. Equating the vertical components of the forces, we get

$$2T\cos\theta = W \quad \text{or} \quad T = \frac{W}{2\cos\theta}$$

Now,

$$\cos\theta = \frac{x}{OB} = \frac{x}{\sqrt{l^2 + x^2}} = \frac{x}{l\left[1 + \frac{x^2}{l^2}\right]^{1/2}} = \frac{x}{l\left[1 + \frac{x^2}{2l^2}\right]}$$

As $\frac{x^2}{2l^2} << 1$ so $1 + \frac{x^2}{2l^2} \approx 1$

$$\therefore \quad \cos\theta = \frac{x}{l} \quad \text{and} \quad T = \frac{W}{2(x/l)} = \frac{Wl}{2x}$$

$$\text{Stress} = \frac{T}{A} = \frac{Wl}{2Ax}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{Wl}{2Ax} \times \frac{2l^2}{x^2} = \frac{Wl^3}{Ax^3}$$

$$\text{or} \quad x = l\left[\frac{W}{YA}\right]^{1/3} = \frac{1.0}{2}\left[\frac{0.100 \times 9.8}{2 \times 10^{11} \times 0.5 \times 10^{-6}}\right]^{1/3}$$

$$= 0.5(9.8 \times 10^{-6})^{1/3} = 0.5 \times 2.14 \times 10^{-2}$$

$$= \mathbf{1.07 \times 10^{-2} = 1.07\ cm.}$$

9.20. *Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 2.3×10^9 Pa ? Assume that each rivet is to carry one quarter of the load.*

Ans. Let the tension exerted by riveted strip = F

This tension would provide shearing force on the four rivets, which share it equally.

$\therefore$ Shearing force on each rivet $= \frac{F}{4}$

and shearing stress on each rivet $= \frac{F/4}{A} = \frac{F}{4A}$

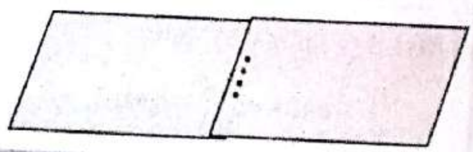


Fig. 9.38

Scanned by CamScanner

As the maximum shearing stress on each rivet is given to be 2.3×10^9 Pa, so we have

$$\frac{F_{max}}{4A} = 2.3 \times 10^9$$

or $$F_{max} = 4A \times 2.3 \times 10^9 = 4 \times \pi r^2 \times 2.3 \times 10^9$$

$$= 4 \times \frac{22}{7} \times (3.0 \times 10^{-3})^2 \times 2.3 \times 10^9$$

$$= 260.2 \times 10^3 \text{ N} = \mathbf{260 \text{ kN}}.$$

9.21. *The Marina trench is located in the Pacific Ocean and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom ?*

Ans. Here $p = 1.01 \times 10^8$ Pa, $V = 0.32$ m^3

For steel, $\kappa = 160 \times 10^9$ Pa

As $$\kappa = \frac{p}{\Delta V / V}$$

$\therefore$ $$\Delta V = \frac{pV}{\kappa} = \frac{1.01 \times 10^8 \times 0.32}{160 \times 10^9}$$

$$= \mathbf{2.02 \times 10^{-4} \text{ m}^3}.$$

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. Among the interatomic and intermolecular forces, which are the stronger ones ? How much ?
2. Write a relation between interatomic force and potential energy.
3. Write an expression showing the dependence of potential energy on interatomic separation *r*.
4. Among the three states of matter : solid, liquid and gas ; which has got its own shape ?
5. Give another name for amorphous solids.
6. What is an isotropic medium ?
7. What is an anisotropic solid ?
8. What type of solids : crystalline or amorphous, are anisotropic ?
9. Give one example each of isotropic and anisotropic substance ?
10. What is the meaning of word 'amorphous' ?
11. Which state of the solid is more stable : crystalline or amorphous ?
12. Give an example of semi-crystalline solid.
13. What is a deforming force ? **[Himachal 07]**
14. What is restoring force ? **[Himachal 07]**
15. Are the elastic restoring forces conservative in nature ?
16. Define stress and strain. **[Delhi 95, 05C]**
17. Define elastic limit. **[Meghalaya 99]**
18. Define yield point.
19. Define Young's modulus of elasticity. **[Delhi 03]**
20. Write dimensional formula of Young's modulus.
21. What is the value of Young's modulus for a perfectly rigid body ?
22. What is the limitation of Hooke's law ?
23. Why any metallic part of a mechinery is never subjected to a stress beyond the elastic limit of the material ?
24. Define bulk modulus of elasticity. Give its units and dimensions. **[Central Schools 13]**
25. What is the reciprocal of bulk modulus known as ?
26. Give the SI unit and dimensions of compressibility ?
27. What is the value of bulk modulus for an incompressible liquid ?
28. What is the value of modulus of rigidity for an incompressible liquid ? **[Central Schools 09]**
29. What a Poisson's ratio ? Does it have any unit ?
30. Young's modulus of the material of a wire is *Y*. On pulling the wire by a force *F*, the increase in its length is *x*. What will be the potential energy of the stretched wire ?
31. Define modulus of rigidity. What is its SI unit ?
32. What is breaking stress for a wire of unit cross-section called ?
33. What is elastic after effect ?
34. What is elastic fatigue ?
35. What is elastic hysteresis ?
36. If the length of a wire increases by 1 mm under 1 kg wt, what will be the increase under

   (*i*) 2 kg wt (*ii*) 100 kg wt ?

Scanned by CamScanner

37. What will happen to the potential energy of the atoms of a solid when it is compressed ? What happens when a wire is stretched ?

38. A spiral spring is stretched by a force. What type of strain is produced in it ?

39. State Hooke's law. [Himachal 03, 05 ; Delhi 05]

40. Which of the two forces-deforming or restoring force is responsible for elastic behaviour of substance. [Himachal 01]

Answers

1. Interatomic forces are 50 to 100 times stronger than the intermolecular forces.
2. $F = -\frac{dU}{dr}$
3. $U(r) = \frac{A}{r^n} - \frac{B}{r^m}$

   For most of the substances, exponents n and m are 12 and 6 respectively.
4. Solid has its own shape.
5. Amorphous solids are also called glassy solids.
6. Any medium which has the same physical properties in all directions is called an isotropic medium.
7. If a solid has different physical properties (thermal, electrical, mechanical and optical) in different directions, then it is said to be an anisotropic solid.
8. Crystalline solids are anisotropic.
9. Glass is an isotropic substance and quartz is anisotropic substance.
10. The word amorphous means without any form.
11. Crystalline state is more stable.
12. Polyethylene.
13. A force which produces a change in the size or shape of a body is called deforming force.
14. The intermolecular force developed within a body due to relative molecular displacements is called restoring force.
15. The elastic restoring forces are conservative only when the loading and unloading curves coincide.
16. *Stress* is the internal restoring force set up per unit area of a deformed body.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

Strain. When external deforming forces act upon a body, the fractional change produced in the body is called strain.

$$\text{Strain} = \frac{\text{Change in any dimension}}{\text{Original dimension}}$$

17. That maximum stress within which a body completely regains its original size and shape after the removal of the deforming force is called elastic limit.
18. The stage of a material when it yields to the deforming force and goes on increasing in length even when the load is kept constant is called yield point.
19. Young's modulus of elasticity is defined as the ratio of the stress to the longitudinal strain, within the elastic limit.
20. $[ML^{-1}T^{-2}]$.
21. Infinite.
22. Hooke's law is obeyed upto the proportionality limit of the material.
23. When a metallic part is subjected to a stress beyond the elastic limit, a permanent deformation is set up in it.
24. Bulk modulus of elasticity is defined as the ratio of tangential stress to shear strain within the elastic limit. Its SI unit is Nm^{-2} and its dimensional formula is $[ML^{-1}T^{-2}]$.
25. Compressibility.
26. The SI unit of compressibility is $N^{-1}m^2$ and its dimensional formula is $[M^{-1}LT^2]$.
27. Infinite.
28. Zero.
29. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio. It has no units.
30. P.E. of stretched wire $= \frac{1}{2} Fx$.
31. Modulus of rigidity is the ratio of tangential stress to the shearing strain within the elastic limit. Its SI unit is Nm^{-2}.
32. Tensile strength.
33. The delay on the part of the body in regaining its original configuration on the removal of the deforming force is called the elastic after effect.
34. Elastic fatigue is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.
35. The fact that the stress-strain curve is not retraced on reversing the strain (for a material like rubber) is called elastic hysteresis.

Scanned by CamScanner

36. (i) 2 mm

(ii) The wire will break, because the stretching force is increased beyond breaking force.

37. Potential energy increases in both cases.

38. Shear strain.

39. Refer to point 15 of Glimpses.

40. Restoring force.

Type B : Short Answer Questions

2 or 3 Marks Each

1. Distinguish between elasticity and plasticity.
2. State Hooke's law and hence define modulus of elasticity. [Meghalaya 96]
3. Define stress and strain and derive their units. What is Hooke's law ? Write its one limitation. [Delhi 04]
4. Which is more elastic – iron or rubber ? Why ? [Delhi 96 ; Himachal 05, 07]
5. Define the terms stress and strain and also state their SI units. Draw the stress versus strain graph for a metallic wire, when stretched upto the breaking point. [Himachal 06]
6. Draw stress-strain curve for a loaded wire. On the graph mark :
   (a) Hooke's limit (b) Elastic limit.
   (c) Yield point. (d) Breaking point. [Central Schools 16]
7. Define the terms Young's modulus, bulk modulus and modulus of rigidity. Also give their units. [Himachal 06, 07]
8. What are elastomers ? Draw a stress-strain graph for an elastomer.
9. Define elastic limit and elastic fatigue. What are ductile and brittle substances ? [Himachal 06]
10. What is elastic after effect ? What is its importance ?
11. Describe elastic hysteresis. Mention its two applications.
12. Explain how is the knowledge of elasticity useful in selecting metal ropes used in cranes for lifting heavy loads.
13. Explain why should the beams used in the construction of bridges have large depth and small breadth.

    Or

    Why are girders given I shape ?
14. Show that the maximum height of any mountain on the earth cannot exceed 10 km.
15. Explain why hollow shafts are preferred to solid shafts for transmitting torque.
16. What is elastic potential energy ? Prove that the work done by a stretching force to produce certain tension in a wire is

$$W = \frac{1}{2} \text{Stretching force} \times \text{extension}.$$

17. Derive an expression for energy stored in a wire due to extension. [Chandigarh 07]
18. Define Poisson's ratio. Write an expression for it. What is the significance of negative sign in this expression ?

Answers

1. Refer answer to Q. 1 on page 9.1.
2. Refer answer to Q.7 on page 9.3 and Q. 8 on page 9.4.
3. Refer to points 12, 13 and 15 of Glimpses.
4. Refer to the solution of Problem 3 on page 9.26.
5. Refer to points 12 and 13 of Glimpses and see Fig. 9.6 on page 9.4.
6. See Fig. 9.6 on page 9.4.
7. Refer to points 17, 18, 19 and 20 of Glimpses.
8. Refer answer to Q. 16 on page 9.6.
9. Refer to points 14, 25, 26 and 30 of Glimpses.
10. Refer answer to Q. 19 on page 9.17.
11. Refer answer to Q. 21 on page 9.18.
12. Refer answer to Q. 23 on page 9.18.
13. Refer answer to Q. 24 on page 9.18.
14. Refer answer to Q. 25 on page 9.19.
15. Refer answer to Q. 26 on page 9.19.
16. Refer answer to Q. 27 on page 9.19.
17. Refer answer to Q.27 on page 9.19.
18. Refer answer to Q. 28 on page 9.21.

Scanned by CamScanner

Type C : Long Answer Questions

5 Marks Each

1. What is interatomic force ? Discuss the variation of interatomic force with the interatomic separation. [Himachal 02]
2. Define the term elasticity. Give an explanation of the elastic properties of materials in terms of interatomic forces ?
3. State Hooke's law. How can it be verified experimentally ?
4. Define Young's modulus of elasticity. Describe an experiment for the determination of Young's modulus of the material of a wire.
5. Define Young's modulus, bulk modulus and modulus of rigidity. Write mathematical expressions for these moduli. What is compressibility.
6. Discuss stress vs. strain graph, explaining clearly the terms elastic limit, permanent set, elastic hysteresis and tensile strength. [Delhi 06]
7. Describe stress-strain relationship for a loaded steel wire and hence explain the terms elastic limit, yield point, tensile strength. [Chandigarh 03]
8. On the basis of stress-strain curves, distinguish between ductile, brittle and malleable materials.
9. Derive an expression for the elastic potential energy stored in a stretched wire under stress. Define the terms elastic after effect and elastic fatigue. [Himachal 07C]

Answers

1. Refer to point 1 of Glimpses and answer to Q. 2 on page 9.2.
2. Refer answer to Q. 2 on page 9.2.
3. Refer answer to Q. 7 on page 9.3.
4. Refer answer to Q. 11 on page 9.5.
5. Refer to points 17, 18, 19 and 21 of Glimpses.
6. Refer answer to Q. 10 on page 9.4.
7. Refer answer to Q. 10 on page 9.4.
8. Refer answer to Q. 12 and Q. 13 on page 9.6.
9. Refer answer to Q.27 on page 9.19 and refer to points 29 and 30 of Glimpses.

Scanned by CamScanner

Competition Section

Mechanical Properties of Solids

GLIMPSES

1. **Interatomic force.** It is the force between the atoms of a molecule. It arises due to the electrostatic interaction between the nuclei of two atoms, their electron clouds and between the nucleus of one atom and the electron cloud of the other atom.
2. **Intermolecular forces.** It is the force acting between the two molecules of a substance due to electrostatic interaction between their oppositely charged ends. Such forces operate over distances of 10^{-9} m and are weaker than interatomic forces.
3. **Solids.** A solid is a large accumulation of ($\sim 10^{23}$) atoms or molecules. It has definite shape and size. The solids we come across in daily life can be classified into three groups :

   (*i*) Crystalline solids.

   (*ii*) Amorphous solids.

   (*iii*) Semi-crystalline solids.
4. **Crystalline solids.** Those solids in which the atoms or molecules are arranged in a regular and repeated geometrical pattern are called crystalline solids. Such solids are bounded by flat surfaces, are anisotropic, have sharp melting points and have long range order in their structure.

   Examples. Rock salt, quartz, mica, calcite, diamond, etc.
5. **Amorphous solids.** These are the solids in which the atoms or molecules are not arranged according to certain definite geometrical order, *i.e.*, the atoms or molecules are arranged in a random order. Such solids are isotropic, do not have flat surfaces and their melting points are not sharp. They are super-cooled liquids.

   Examples. Glass, rubber, cellulose, bitumen, bone and many plastics.
6. **Semi-crystalline solids.** These are the solids in which the crystalline phase is inter-dispersed in the amorphous phase, *i.e.*, in which crystalline and amorphous phases co-exist. Polyethylene and protein are such solids.
7. **Deforming force.** A force which changes the size and shape of body is called deforming force.
8. **Elasticity.** The property by virtue of which a body regains its original size and shape after the removal of deforming force is called elasticity.
9. **Perfectly elastic body.** If a body regains its original size and shape completely and immediately after the removal of deforming force, it is said to be perfectly elastic body. The nearest approach to a perfectly elastic body is quartz fibre.
10. **Plasticity.** The property by virtue of which a body does not regain its original size and shape even after the removal of the deforming force is called plasticity.
11. **Perfectly plastic body.** If a body does not show any tendency to regain its original size and shape even after the removal of deforming force, it is said to be perfectly plastic body. Putty and paraffin wax are nearly perfectly elastic bodies.
12. **Stress.** The restoring force set up per unit area of a deformed body is called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{\text{Applied force}}{\text{Area}} = \frac{F}{A}$$

The SI unit of stress is Nm^{-2} and the CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$.

Stress is of *two* types :

(*i*) Normal stress

(*ii*) Tangential stress.

Scanned by CamScanner

13. **Strain.** The ratio of the change in any dimension produced in the body to the original dimension is called strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

As strain is the ratio of two like quantities, it has no units and dimensions. Strain is of *three* types :

(*i*) **Longitudinal strain.** It is defined as the ratio of change in length to the original length.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

(*ii*) **Volumetric strain.** It is defined as the ratio of the change in volume to the original volume.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}.$$

(*iii*) **Shear strain.** It is defined as the angle θ (in radian) through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.

$$\text{Shear strain} = \theta \simeq \tan\theta$$

$$= \frac{\text{Relative displacement between two} \parallel \text{planes}}{\text{Distance between} \parallel \text{planes}}$$

$$= \frac{\Delta l}{l}$$

14. **Elastic limit.** The maximum stress upto which stress is proportional to strain is called elastic limit.

15. **Hooke's law.** It states that within the elastic limit, stress is proportional to strain.

$$\text{Stress} \propto \text{Strain}$$

or $$\text{Stress} = E \times \text{strain}$$

The constant E is called modulus of elasticity of the material of the body.

16. **Modulus of elasticity or coefficient of elasticity.** It is defined as the ratio of stress to the corresponding strain, within the elastic limit.

It is of *three* types :

(*i*) Young's modulus

(*ii*) Bulk modulus

(*iii*) Modulus of rigidity.

17. **Young's modulus of elasticity.** It is defined as the ratio of longitudinal stress to the longitudinal strain within the elastic limit.

It is given by

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta l/l} = \frac{F}{A}\cdot\frac{l}{\Delta l}$$

18. **Bulk modulus of elasticity.** It is defined as the ratio of normal stress to volumetric strain within the elastic limit.

It is given by

$$\kappa = \frac{\text{Normal stress}}{\text{Volumetric strain}} = -\frac{F/A}{\Delta V/V}$$

$$= -\frac{p}{\Delta V/V} = -\frac{pV}{\Delta V}.$$

The negative sign shows that the volume decreases with the increase in stress.

19. **Modulus of rigidity.** It is defined as the ratio of tangential stress to shear strain within the elastic limit.

It is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F/A}{\Delta l/l} = \frac{F}{A}\cdot\frac{l}{\Delta l}$$

20. **Units of moduli of elasticity.** As strain is a pure ratio, the unit of elasticity is same as that of stress. So SI unit of Y, κ or η is Nm^{-2} and the CGS unit is dyne cm^{-2}.

21. **Compressibility.** The reciprocal of the bulk modulus of a material is called its compressibility.

Compressibility = $1/\kappa$

The SI unit of compressibility is $N^{-1}\,m^2$ or Pa^{-1}.

22. **Yield point.** The stress beyond which a solid flows is called yield point. For example, a paste of flour and water flows under its own weight.

23. **Breaking stress.** The stress corresponding to which a wire breaks is called breaking stress.

Breaking force = Breaking stress × area of cross-section of the wire.

24. **Plastic region.** The region of stress-strain curve between the elastic limit and the breaking point is called plastic region.

25. **Ductile materials.** The materials which have large plastic range of extension are called ductile materials. Such materials can be drawn into thin wires.

Examples. Copper, silver, iron, aluminium, etc.

26. **Brittle materials.** The materials which have very small range of plastic extension are called brittle materials. Such materials break as soon as the stress is increased beyond the elastic limit.

Examples. Cast iron, glass, ceramics, etc.

27. **Malleable metals.** The metals which can be hammered or rolled into thin sheets are called malleable metals.

Examples. Gold, silver, lead, etc.

28. **Elastomers.** The materials which can be elastically stretched to large values of strain are called elastomers. They have large elastic region but do not obey Hooke's law.

Examples. Rubber and elastic tissue of aorta.

Scanned by CamScanner

29. **Elastic after effect.** The delay in regaining the original state by a body on the removal of deforming force is called elastic after effect. This effect is minimum for quartz and phosphor bronze and maximum for glass fibre.

30. **Elastic fatigue.** It is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

31. **Elastic hysteresis.** The fact that the stress-strain curve is not retraced on reversing the strain (for a material like rubber) is called elastic hysteresis.

32. **Poisson's ratio.** When the length of a loaded wire increases, its diameter decreases. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio. It is given by

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= -\frac{\Delta D/D}{\Delta l/l}$$

Poisson's ratio σ is a pure number. It has no units or dimensions.

33. **Elastic potential energy in a stretched wire.** The work done against the internal restoring forces in stretching a wire is stored as its elastic potential energy.

It is given by

$$U = \frac{1}{2}\text{Stretching force} \times \text{extension}$$

$$= \frac{1}{2}F\,\Delta l$$

$$= \frac{1}{2}\cdot\frac{F}{A}\cdot\frac{\Delta l}{l}\cdot Al$$

$$= \frac{1}{2}\text{Stress} \times \text{strain} \times \text{volume of the wire}$$

P.E. stored per unit volume of the wire or elastic energy density is

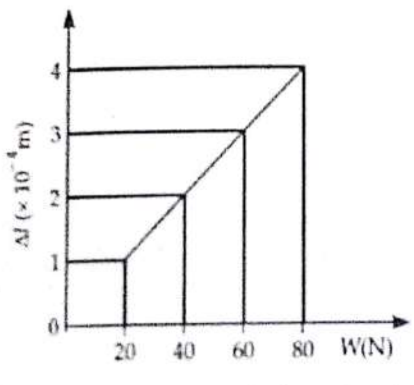
$$u = \frac{U}{V} = \frac{1}{2}\text{Stress} \times \text{strain}$$

$$= \frac{1}{2}\text{Young's modulus} \times \text{strain}^2.$$

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. The adjacent graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end.



If the cross-sectional area of the wire is $10^{-6}\,m^2$, calculate the Young's modulus of the material of the wire

(a) $2\times10^{11}\,N/m^2$

(b) $2\times10^{-11}\,N/m^2$

(c) $3\times10^{-12}\,N/m^2$

(d) $2\times10^{-13}\,N/m^2$ [IIT 04]

2. The following four wires are made of the same material. Which of these will have the largest extension, when the same tension is applied ?

(a) length = 50 cm, diameter = 0.5 mm

(b) length = 100 cm, diameter = 1 mm

(c) length = 200 cm, diameter = 2 mm

(d) length = 300 cm, diameter = 3 mm [IIT 81]

3. A wire of length L, and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is

(a) $YAx^2/2L$ (b) YAx^2/L

(c) $YAx/2L$ (d) YAx^2L [IIT 87]

4. The pressure of a medium is changed from 1.01×10^5 Pa to 1.65×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is

(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa

(c) 51.2×10^5 Pa (d) 1.55×10^5 Pa [IIT 05]

5. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is

(a) $2P/3$ (b) P

(c) $3P/2$ (d) $2P$ [IIT 98]

Scanned by CamScanner

Answers and Explanations

1. (*a*) When $W = 20$ N, $\Delta l = 1 \times 10^{-4}$ m

$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{W}{A} \cdot \frac{l}{\Delta l}$$
$$= \frac{20}{10^{-6}} \times \frac{1}{1 \times 10^{-4}}$$
$$= 2 \times 10^{11} \text{ Nm}^{-2}.$$

2. (*a*)
$$\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{4F}{\pi D^2} \cdot \frac{l}{Y}$$
$$\therefore \quad \Delta l \propto \frac{l}{D^2}$$

In (*a*), $\frac{l}{D^2} = \frac{50}{(0.05)^2} = 2 \times 10^4 \text{ cm}^{-1}$

In (*b*), $\frac{l}{D^2} = \frac{100}{(0.1)^2} = 10^4 \text{ cm}^{-1}$

In (*c*), $\frac{l}{D^2} = \frac{200}{(0.2)^2} = 5 \times 10^3 \text{ cm}^{-1}$

In (*d*), $\frac{l}{D^2} = \frac{300}{(0.3)^2} = 3.3 \times 10^3 \text{ cm}^{-1}$

Hence Δl is maximum in (*a*).

3. (*a*) W = Average force × increase in length,
$$= \frac{1}{2} F \times x = \frac{1}{2} \frac{F}{A} \cdot \frac{L}{x} \cdot \frac{Ax^2}{L}$$
$$= \frac{1}{2} Y \cdot \frac{Ax^2}{L} = \frac{YAx^2}{2L}.$$

4. (*d*) Bulk modulus $= \frac{\Delta P}{\Delta V / V}$
$$= \frac{(1.165 - 1.01) \times 10^5}{10/100}$$
$$= 1.55 \times 10^5 \text{ Pa}.$$

5. (*b*) Isothermal bulk modulus of a gas
= Pressure of the gas = P.

AIEEE

1. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)

(*a*) $l/2$ (*b*) l
(*c*) $2l$ (*d*) zero [AIEEE 06]

2. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is

(*a*) $F/2l$ (*b*) Fl
(*c*) $2Fl$ (*d*) $Fl/2$

[AIEEE 04]

3. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is

(*a*) 0.2 J (*b*) 10 J
(*c*) 20 J (*d*) 0.1 J [AIEEE 03]

4. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(*a*) $2Y/S$ (*b*) $S/2Y$
(*c*) $2S^2Y$ (*d*) $\frac{S^2}{2Y}$ [AIEEE 05]

5. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount ?

(*a*) F (*b*) $4F$
(*c*) $6F$ (*d*) $9F$ [AIEEE 09]

6. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6},$$

where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(*a*) $\frac{b^2}{6a}$ (*b*) $\frac{b^2}{2a}$
(*c*) $\frac{b^2}{12a}$ (*d*) $\frac{b^2}{4a}$

[AIEEE 2010]

Scanned by CamScanner

Answers and Explanations

1. (*b*) Refer to the solution of Problem 1 on page 9.28.

2. (*d*) $W = \frac{1}{2}$ Stretching force × increase in length $= \frac{1}{2}Fl$.

3. (*d*) Here $F = 200$ N, $l = 1$ mm $= 10^{-3}$ m

$$U = \frac{1}{2}Fl = \frac{1}{2} \times 200 \times 10^{-3} = 0.1 \text{ J}.$$

4. (*d*) Elastic P.E. stored per unit volume,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times \text{Stress} \times \frac{\text{Stress}}{Y} = \frac{S^2}{2Y}.$$

5. (*d*) $$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{A^2}\frac{Al}{\Delta x} = \frac{FV}{A^2 \Delta x}$$

$$\therefore \quad F = \frac{Y \Delta x}{V} A^2 \quad \text{or} \quad F \propto A^2$$

$$F' = (3A)^2$$

$$\frac{F'}{F} = 9$$

$$F' = 9F.$$

6. (*d*) $$U = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$F = -\frac{dU}{dx} = \frac{12a}{x^{13}} - \frac{6b}{x^7}$$

At equilibrium,

$$F = \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$\therefore \quad x^6 = \frac{2a}{b}$$

$$\Rightarrow \quad x = \left(\frac{2a}{b}\right)^{1/6}$$

$$U(x = \infty) = 0$$

$$U_{\text{equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$$

$$\therefore \quad D = U(x = \infty) - U_{\text{equilibrium}}$$

$$= 0 - \left(-\frac{b^2}{2a}\right) = \frac{b^2}{4a}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. In solids interatomic forces are

(*a*) totally repulsive (*b*) totally attractive

(*c*) both (*a*) and (*b*) (*d*) none of these [DCE]

2. The term liquid crystal refers to a state that is intermediate between

(*a*) crystalline solid and amorphous liquid

(*b*) crystalline solid and vapour

(*c*) amorphous liquid and its vapour

(*d*) a crystal immersed in a liquid [IPUEE 07]

3. Which one of the following is not a unit of Young's modulus ?

(*a*) Nm^{-1} (*b*) Nm^{-2}

(*c*) dyne cm^{-2} (*d*) mega pascal [IPUEE 06]

4. Two wires A and B are of same material. Their lengths are in the ratio 1 : 2 and diameters are in the ratio 2 : 1. When stretched by forces F_A and F_B respectively, they get equal increase in their lengths. Then the ratio F_A / F_B should be

(*a*) 1 : 2 (*b*) 1 : 1

(*c*) 2 : 1 (*d*) 8 : 1 [IPUEE 04]

5. There are two wires of same material and same length while the diameter of second wire is two times the diameter of first wire, then the ratio of extension produced in the wire by applying same load will be

(*a*) 1 : 1 (*b*) 2 : 1

(*c*) 1 : 2 (*d*) 4 : 1 [DCE 2K, 03]

6. A wire of diameter 1 mm breaks under a tension of 1000 N. Another wire, of same material as that of the first one, but of diameter 2 mm breaks under a tension of

(*a*) 500 N (*b*) 100 N

(*c*) 1000 N (*d*) 4000 N [IPUEE 04]

7. A steel rod has a radius of 10 mm and a length of 1.0 m. A force stretches it along its length and produces a strain of 0.16%. Young's modulus of the

Scanned by CamScanner

steel is $2.0 \times 10^{11} Nm^{-2}$. What is the magnitude of the force stretching the rod ?

(a) 100 kN (b) 314 kN

(c) 31.4 kN (d) 200 kN [DCE 08]

8. A cube is subjected to a uniform volume compression. If the side of the cube decreases by 2%, the bulk strain is

(a) 0.02 (b) 0.03

(c) 0.04 (d) 0.06 [DCE 07]

9. The work done per unit volume in deforming a body is given by

(a) stress × strain (b) 1/2 (stress × strain)

(c) stress/strain (d) strain/stress [DCE 03]

10. Energy stored in stretching a string per unit volume is

(a) $\frac{1}{2} \times$ stress × strain (b) stress × strain

(c) $Y(\text{Strain})^2$ (d) $\frac{1}{2}Y(\text{Stress})^2$ [DCE 07]

11. A body of weight mg is hanging on a string which extends in length by l. The work done in extending the string is

(a) mgl (b) $mgl/2$

(c) $2mgl$ (d) none of these [DCE 09]

12. Minimum and maximum values of Possion's ratio for a metal lies between

(a) $-\infty$ to $+\infty$ (b) 0 to 1

(c) $-\infty$ to 1 (d) 0 to 0.5 [IPUEE 05]

13. A long piece of rubber is wider than it is thick. When it is stretched in length by some amount

(a) its thickness decreases but its width increases

(b) its thickness decreases but its width remains constant

(c) its thickness increases but its width decreases

(d) both its thickness and width decrease. [DCE 08]

14. A metallic rod of length l and cross-sectional area A is made of a material of Young's modulus Y. If the rod is elongated by an amount y, then the work done is proportional to

(a) y (b) $1/y$

(c) y^2 (d) $1/y^2$ [DCE 09]

Answers and Explanations

1. (c) The interatomic forces in solids are both attractive and repulsive.

2. (a) Liquid crystal is a state intermediate between crystalline solid and amorphous liquid.

3. (a) Nm^{-1} is not a unit of Young's modulus.

4. (d) As both wires of same material,

$$Y_A = Y_B$$

$$\frac{F_A}{\pi r_A^2} \cdot \frac{l_A}{\Delta l_A} = \frac{F_B}{\pi r_B^2} \cdot \frac{l_B}{\Delta l_B}$$

But $\Delta l_A = \Delta l_B$

$$\therefore \quad \frac{F_A}{F_B} = \frac{l_B}{l_A} \cdot \left(\frac{r_A}{r_B}\right)^2$$

$$= \frac{2}{1}\left(\frac{2}{1}\right)^2 = \mathbf{8:1}.$$

5. (d) Extension,

$$\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{F}{\pi r^2} \cdot \frac{l}{Y}$$

For the two wires F, l and Y are same, so

$$\frac{\Delta l_1}{\Delta l_2} = \frac{r_2^2}{r_1^2} = \left(\frac{2}{1}\right)^2 = \mathbf{4:1}.$$

6. (d) Breaking tension $\propto r^2$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$T_2 = 4T_1 = 4 \times 1000 = \mathbf{4000\ N}.$$

7. (a) Here $r = 10$ mm $= 10^{-2}$ m, $l = 1$ m

$$\Delta l = \frac{0.16}{100} l, \; Y = 2 \times 10^{11}\ Nm^{-2}$$

$$F = \frac{YA\Delta l}{l}$$

$$= 2 \times 10^{11} \times 3.14 \times 10^{-4} \times 0.16 \times 10^{-2}$$

$$= 10^5\ N = \mathbf{100\ kN}.$$

8. (d) $V = l^3$

$$\frac{\Delta V}{V} = 3\frac{\Delta l}{l} = 3\left(\frac{2}{100}\right) = \mathbf{0.06}.$$

9. (b) W = Average force × extension

$$= \frac{1}{2} F \times \Delta l = \frac{1}{2} \frac{F}{A} \cdot \frac{\Delta l}{l} \cdot Al$$

$$= \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{volume of wire}$$

$\therefore$ Work done per unit volume

$$= \frac{1}{2} \times \text{Stress} \times \text{strain}$$

Scanned by CamScanner

10. (a) From the above problem,

Energy stored per unit volume

= Work done per unit volume

$= \frac{1}{2} \times$ Stress × strain.

11. (b) W = Average force × extension

$= \frac{1}{2} F \times l = \frac{1}{2} mg \times l = \frac{mgl}{2}$.

12. (d) Poission's ratio for a metal lies between 0 and 0.5.

13. (d) When the length increases, both thickness and width decrease.

14. (c) W = Average force × extension

$= \frac{1}{2} F \times y = \frac{1}{2} \frac{F}{A} \cdot \frac{l}{y} \cdot \frac{Ay^2}{l}$

$= \frac{1}{2} Y \cdot \frac{Ay^2}{l} = \left(\frac{YA}{2l}\right) y^2$

$\therefore$ $W \propto y^2$.

AIIMS Entrance Exam

1. According to Hooke's law of elasticity, if stress is increased, the ratio of stress to strain

(a) increases (b) decreases
(c) becomes zero (d) remains constant [AIIMS 01]

2. A thick copper rope of density $1.5 \times 10^3\ kgm^{-3}$ and Young's modulus $5 \times 10^6\ Nm^{-2}$, 8 m in length, when hung from the ceiling of a room, the increase in its length due to its own weight is

(a) 9.6×10^{-5} m (b) 19.2×10^{-7} m
(c) 9.6×10^{-2} m (d) 9.6 m [AIIMS 82]

3. If in a wire of Young's modulus Y, longitudinal strain X is produced, then the value of potential energy stored in its unit volume will be

(a) YX^2 (b) $2YX^2$
(c) $0.5Y^2X$ (d) $0.5YX^2$ [AIIMS 01]

4. A metal ring of initial radius r and cross-sectional area A is fitted onto a wooden disc of radius $R > r$. If Young's modulus of the metal is Y, then the tension in the ring is

(a) $\frac{AYR}{r}$ (b) $\frac{Yr}{AR}$
(c) $\frac{AY(R-r)}{r}$ (d) $\frac{Y(R-r)}{Ar}$ [AIIMS 2K]

5. For a constant hydraulic stress on an object, the fractional change in the object's volume $(\Delta V / V)$ and its bulk modulus (B) are related as

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$
(c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto \frac{1}{B^2}$ [AIIMS 05]

6. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of $100\ cm^3$ of water under a pressure of 100 atmosphere will be

(a) $0.4\ cm^3$ (b) $4 \times 10^{-5}\ cm^3$
(c) $0.025\ cm^3$ (d) $0.004\ cm^3$ [AIIMS 2K]

7. A stretched rubber has

(a) increased kinetic energy
(b) increased potential energy
(c) decreased kinetic energy
(d) decreased potential energy [AIIMS 99]

8. The breaking stress of a wire depends upon

(a) length of the wire (b) radius of the wire
(c) material of the wire
(d) shape of the cross-section [AIIMS 02]

9. Which of the following affects the elasticity of a substance ?

(a) hammering and annealing
(b) change in temperature
(c) impurity in substance
(d) all of these. [AIIMS.99]

Assertions and Reasons

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion.
(c) If assertion is true, but reason is false.
(d) If both assertion and reason are false.

10. **Assertion.** Lead is more elastic than rubber.

Reason. If same load is loaded on the lead and rubber wire of same cross-sectional area, the strain of lead is very much less than that of rubber. [AIIMS 97]

Scanned by CamScanner

11. Assertion. Stress is the internal force per unit area of a body.

Reason. Rubber is more elastic than steel. [AIIMS 02]

12. Assertion. A hollow shaft is found to be stronger than a solid shaft made of same material.

Reason. The torque required to produce a given twist in hollow cylinder is greater than that required to twist a solid cylinder of same size and material. [AIIMS 2010]

Answers and Explanations

1. (*d*) According to Hooke's law,

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant (within the elastic limit)}$$

2. (*c*) $W = mg = Al\rho g$

As weight of the wire acts on its CG, so it produces extension only in length $l/2$.

$$Y = \frac{W}{A} \cdot \frac{l/2}{\Delta l} = \frac{Al\rho g}{A} \cdot \frac{l}{2\Delta l}$$

$$\Delta l = \frac{l^2 \rho g}{2Y}$$

$$= \frac{64 \times 1.5 \times 10^3 \times 10}{2 \times 5 \times 10^6} \text{ m}$$

$$= 9.6 \times 10^{-2} \text{ m.}$$

3. (*d*) P.E. stored per unit volume,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2}(Y \times \text{Strain}) \times \text{Strain}$$

$$= 0.5\, YX^2. \qquad [\text{strain} = X]$$

4. (*c*) Strain $= \frac{2\pi R - 2\pi r}{2\pi r} = \frac{R - r}{r}$

$$\text{Stress} = Y \times \text{strain} = \frac{F}{A} = \frac{T}{A}$$

$$T = AY \times \text{strain} = \frac{AY(R-r)}{r}.$$

5. (*b*) Bulk modulus $= \frac{\text{Hydraulic stress}}{\text{Volumetric strain}}$

$$B = \frac{\text{Hydraulic stress}}{\Delta V / V}$$

or $$\frac{\Delta V}{V} = \text{Hydraulic stress} \times \frac{1}{B}$$

∴ For constant hydraulic stress

$$\frac{\Delta V}{V} \propto \frac{1}{B}.$$

6. (*a*) Compressibility $= \frac{\Delta V}{pV}$

$$4 \times 10^{-5} = \frac{\Delta V}{100 \times 100}$$

$$\Delta V = 0.4 \text{ cm}^3.$$

7. (*b*) The work done in stretching the rubber is stored as its potential energy.

8. (*c*) The stress at which rupture of the wire occurs is called its breaking stress. Its value depends on the material of the wire.

9. (*d*) All the factors mentioned in options (*a*), (*b*) and (*c*) affect the elasticity of a substance.

10. (*a*) Both the assertion and reason are true and the reason is a correct explanation of the assertion.

11. (*c*) The assertion is true but the reason is false. Steel is more elastic than rubber.

12. (*a*) Assertion is true and the reason is its correct explanation.

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. Which of the following has no dimensions ?

(*a*) strain (*b*) angular velocity
(*c*) momentum (*d*) angular momentum [DPMT 94]

2. The diameter of brass rod is 4 mm. Young's modulus of brass is $9 \times 10^9 \text{ N/m}^2$. The force required to stretch 0.1% of its length is

(*a*) 360πN (*b*) 36 N
(*c*) $36\pi \times 10^5$N (*d*) $144\pi \times 10^3$N [VMMC 05]

3. When a body of mass M is hung from a spring, the spring extends by 1 cm. If the body of mass 2 M be hung from the same spring, the extension of spring will be

(*a*) 1 cm (*b*) 2 cm
(*c*) 0.5 cm (*d*) 4 cm [VMMC 07]

Scanned by CamScanner

4. A wire whose cross-sectional area is 2 mm^2 is stretched by 0.1 mm by a certain load, and if a similar wire of triple the area of cross-section is stretched by the same load, then the elongation of the second wire would be

(a) 3.3 mm (b) 0.033 mm
(c) 0.33 mm (d) 0.0033 mm [DPMT 92]

5. A substance breaks down by a stress of $10^6\ N/m^2$. If the density of the material of the wire is $3\times10^3\ kg/m^3$, then the length of the wire of the substance which will break under its own weight when suspended vertically will be

(a) 66.6 m (b) 60.0 m
(c) 33.3 mm (d) 30.3 mm [DPMT 04]

6. With what minimum acceleration can a fireman slide down a rope whose breaking strength is two third of his weight :

(a) $\frac{g}{3}$ (b) $\frac{2}{3}g$
(c) $\frac{3}{2}g$ (d) $\frac{g}{2}$ [VMMC 05]

7. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by the amount x, the work done is

(a) $\frac{YAx^2}{2L}$ (b) $\frac{YAx^2}{L}$
(c) YAx^2L (d) $\frac{YAx}{2L}$ [VMMC 05]

8. When a sphere is taken to bottom of sea 1 km deep, it contracts by 0.01%. The bulk modulus of elasticity of the material of sphere is (Given : Density of water = 1 g / cm^3)

(a) $9.8\times10^{10}\ N/m^2$ (b) $10.2\times10^{10}\ N/m^2$
(c) $0.98\times10^{10}\ N/m^2$ (d) $8.4\times10^{10}\ N/m^2$ [NMOC 06]

9. According to C.E. van der Wall, the interatomic potential varies with the average interatomic distance (R) as

(a) R^{-1} (b) R^{-2}
(c) R^{-4} (d) R^{-6} [DPMT 2010]

10. A sphere of radius 3 cm is subjected to a pressure of 100 atm. Its volume decreases by 0.3 c.c. . What will be its bulk modulus ?

(a) $4\pi\times10^5$ atm (b) $4\pi\times3\times10^3$ atm
(c) $4\pi\times10^6$ atm (d) $4\pi\times10^8$ atm [DPMT 2011]

Answers and Explanations

1. (a) As strain is ratio of two like quantities, it has no dimensions.

2. (a) $\frac{\Delta l}{l}=\frac{0.1}{100}$

$$F=\frac{YA\Delta l}{l}=\frac{Y\times\pi r^2\times\Delta l}{l}$$

$$=\frac{9\times10^9\times\pi\times(2\times10^{-3})^2\times0.1}{100}\ N=\mathbf{360\ \pi\ N.}$$

3. (b) $F=Mg=kx$

$$2Mg=kx'$$
$$2kx=kx'$$
$$x'=2x=2\times1\ cm=\mathbf{2\ cm.}$$

4. (b) $\Delta l=\frac{F}{A}\cdot\frac{l}{Y}$

For same F, l and Y,

$$\Delta l\propto\frac{1}{A}$$

$$\frac{\Delta l_2}{\Delta l_1}=\frac{A_1}{A_2}=\frac{2\ mm^2}{6\ mm^2}$$

$$\Delta l_2=\frac{1}{3}\Delta l_1=\frac{1}{3}\times0.1\ mm=\mathbf{0.033\ mm.}$$

5. (c) Breaking stress $=\frac{mg}{area}=\frac{Al\rho g}{A}=l\rho g$

$$l=\frac{Stress}{\rho g}=\frac{10^6}{3\times10^3\times10}=\mathbf{33.3\ m.}$$

6. (a) Net downward force in the string,

$$ma=mg-T$$

But $T_{min}=\frac{2}{3}mg \quad \therefore \quad ma_{min}=mg-\frac{2}{3}mg$

$$a_{min}=g-\frac{2}{3}g=\frac{g}{3}.$$

7. (c) Refer to the solution of Problem 3 on page 9.44.

8. (a) Refer to the solution of Example 16 on page 9.12.

9. (d) $U(r)=\frac{A}{r^n}-\frac{B}{r^m}$. For most of the substances, exponents n and m are 12 and 6 respectively.

10. (b) $K=\frac{p}{\Delta V/V}=\frac{pV}{\Delta V}$

$$=\frac{100\times\frac{4}{3}\pi(3)^3}{0.3}\ atm=\mathbf{4\pi\times3\times10^3\ atm}$$

Scanned by CamScanner

CHAPTER

10

MECHANICAL PROPERTIES OF FLUIDS

10.1 WHAT IS A FLUID ?

1. *What are fluids ? Give their important characteristics.*

Fluid. *A fluid is a substance that can flow.* It ultimately assumes the shape of the containing vessel because it cannot withstand shearing stress. Thus, both liquids and gases are fluids.

Important characteristics of fluids :

(*i*) The atoms or molecules in a fluid are arranged in a random manner.

(*ii*) A fluid cannot withstand tangential or shearing stress for an indefinite period. It begins to flow when a shearing stress is applied.

(*iii*) A fluid has no definite shape of its own. It ultimately assumes the shape of the containing vessel. So a fluid has no modulus of rigidity.

(*iv*) A fluid can exert/withstand a force in a direction perpendicular to its surface. So a fluid does have a bulk modulus of rigidity.

2. *Both liquids and gases are fluids. What is the main difference between them ?*

Difference between liquid and gas. A liquid is incompressible and has a definite volume and a free surface of its own. A gas is compressible and it expands to occupy all the space available to it.

3. *Distinguish between the terms fluid statics and fluid dynamics.*

Fluid statics. *The branch of physics that deals with the study of fluids at rest is called fluid statics or hydrostatics.* Its study includes hydrostatic pressure, Pascal's law, Archimedes' principle, floatation of bodies and surface tension.

Fluid dynamics. *The branch of physics that deals with the study of fluids in motion is called fluid dynamics or hydrodynamics.* Its study includes equation of continuity, Bernoulli's theorem, Toricelli's theorem, viscosity, etc.

10.2 THRUST OF A LIQUID

4. *Define the term thrust. Give its SI unit.*

Thrust. A liquid in equilibrium has a fundamental property that it exerts a force on any surface in contact with it and this force acts perpendicular to the surface. *The total force exerted by a liquid on any surface in contact with it is called thrust.* It is because of this thrust that a liquid flows out through the holes of the containing vessel.

As thrust is a force, so its SI unit is newton (N).

Scanned by CamScanner

5. *Show that a liquid at rest exerts force perpendicular to the surface of the container at every point.*

Liquid in equilibrium. Consider a liquid contained in a vessel in the equilibrium state of rest. As shown in Fig. 10.1, suppose the liquid exerts a force F on the bottom surface in an inclined direction OA. The surface exerts an equal reaction R to water along OB.

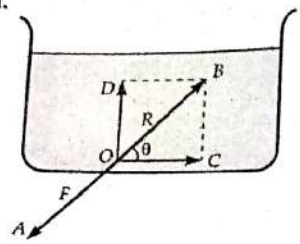


Fig. 10.1 Liquid in equilibrium.

The reaction R along OB has two rectangular components :

(*i*) Tangential component, $OC = R \cos \theta$

(*ii*) Normal component, $OD = R \sin \theta$

Since a liquid cannot resist any tangential force, so the liquid near O should begin to flow along OC. But the liquid is at rest, the force along OC must be zero.

$\therefore \quad R \cos \theta = 0$

As $R \neq 0$, so $\cos \theta = 0$ or $\theta = 90°$

Hence a *liquid always exerts force perpendicular to the surface of the container at every point.*

10.3 PRESSURE

6. *Define the term pressure. Is it a scalar or a vector ? Give its units and dimensions.*

Pressure. *The pressure at a point on a surface is the thrust acting normally per unit area around that point.* If a total force F acts normally over a flat area A, then the pressure is

$$P = \frac{F}{A}$$

If the force is not distributed uniformly over the given surface, then pressure will be different at different points. If a force ΔF acts normally on a small area ΔA surrounding a given point, then pressure at that point will be

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

Pressure is a scalar quantity, because fluid pressure at a particular point in fluid has same magnitude in all directions. This shows that a definite direction is not associated with fluid pressure. Though force is a vector quantity, only the magnitude of the normal component of the force appears in the above equations for pressure.

Units and dimensions of pressure :

SI unit of pressure = Nm^{-2} or Pascal (Pa)

CGS unit of pressure = dyne cm^{-2}

Dimensional formula of pressure is $[ML^{-1}T^{-2}]$.

7. *Briefly explain a method for measuring fluid pressure at any point inside the fluid.*

Measurement of fluid pressure. Fig. 10.2 shows a small pressure measuring device placed in a fluid-filled vessel. This pressure sensor consists of a small piston of area ΔA moving in a close fitting cylinder and resting against a spring. The cylinder is evacuated and the spring is calibrated to measure the force acting on the cylinder. The inward force exerted by the fluid on the piston is balanced by the outward spring force and is thereby measured. If ΔF is the magnitude of this normal force acting on the piston of area ΔA, the fluid pressure is

$$p = \frac{\Delta F}{\Delta A}$$

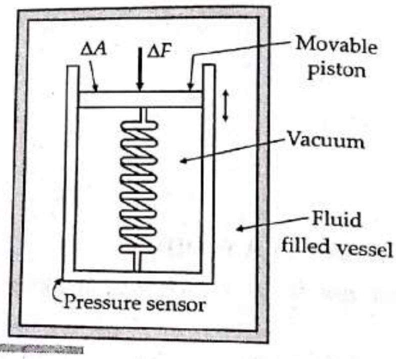


Fig. 10.2 Measurement of fluid pressure.

10.4 PRACTICAL APPLICATIONS OF PRESSURE

8. *Describe some practical applications from daily life which make use of the concept of pressure.*

Practical applications based on the concept of pressure :

(*i*) **A sharp knife cuts better than a blunt one.** The area of a sharp edge is much less than the area of a blunt edge. For the same total force, the effective force per unit area (or pressure) is more for the sharp edge than the blunt edge. Hence a sharp knife cuts better.

(*ii*) **Railway tracks are laid on wooden sleepers.** This spreads force due to the weight of the train on a larger area and hence reduces the pressure considerably. This, in turn, prevents the yielding of the ground under the weight of the train.

(*iii*) **It is difficult for a man to walk on sand while a camel walks easily on sand inspite of the fact that a**

Scanned by CamScanner

camel is much heavier than a man. This is because camel's feet have a larger area than the feet of man. Due to larger area, pressure is less.

• (*iv*) **Pins and nails are made to have pointed ends.** Their pointed ends have very small area. When a force is applied over head of a pin or a nail, it transmits a large pressure (= force/area) on the surface and hence easily penetrate the surface.

Examples based on Thrust and Pressure

FORMULAE USED

1. Thrust = Total force exerted by a liquid on the surface in contact
2. $\text{Pressure} = \dfrac{\text{Thrust}}{\text{Area}}$ or $P = \dfrac{F}{A}$

UNITS USED

Thrust is in newton and pressure in Nm^{-2} or pascal (Pa).

EXAMPLE 1. *The two thigh bones (femurs), each of cross-sectional area 10 cm² support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs. Take $g = 10\ ms^{-2}$.* **[NCERT]**

Solution. Total cross-sectional area of the femurs,

$$A = 2 \times 10\ \text{cm}^2 = 20 \times 10^{-4}\ \text{m}^2$$

Force acting vertically downwards and hence normally on femurs,

$$F = mg = 40 \times 10 = 400\ \text{N}$$

$$\therefore \quad P_{av} = \frac{F}{A} = \frac{400}{20 \times 10^{-4}} = \mathbf{2 \times 10^5\ Nm^{-2}}.$$

EXAMPLE 2. *How much pressure will a man of weight 80 kgf exert on the ground when (i) he is lying and (ii) he is standing on his feet ? Given that the area of the body of the man is 0.6 m² and that of a foot is 80 cm².*

Solution. Force, $F = 80\ \text{kgf} = 80 \times 9.8\ \text{N}$

(*i*) When the man is lying on the ground,

$$A = 0.6\ \text{m}^2$$

$$\therefore \quad P = \frac{F}{A} = \frac{80 \times 9.8}{0.6} = \mathbf{1.307 \times 10^3\ Nm^{-2}}.$$

(*ii*) When the man is standing on his feet,

$$A = 2 \times 80\ \text{cm}^2 = 160 \times 10^{-4}\ \text{m}^2$$

$$P = \frac{80 \times 9.8}{160 \times 10^{-4}} = \mathbf{4.9 \times 10^4\ Nm^{-2}}.$$

PROBLEMS FOR PRACTICE

1. Find the pressure exerted at the tip of a drawing pin if it is pushed against a board with a force of 20 N. Assume the area of the tip to be 0.1 mm². **(Ans. 2×10^8 Pa)**
2. Atmospheric pressure is nearly 100 kPa. How large the force does the air in a room exert on the inside of a window pan that is 40 cm × 80 cm ? **(Ans. 32 kN)**
3. The force on a phonograph needle is 1.2 N. The point has a circular cross-section whose radius is 0.1 mm. Find the pressure (in atm) it exerts on the records. Given 1 atm = 1.013×10^5 Pa. **(Ans. 377 atm)**
4. A cylindrical vessel containing liquid is closed by a smooth piston of mass m. The area of cross-section of the piston is A. If the atmospheric pressure is P_0, find the pressure of the liquid just below the piston. **(Ans. $P_0 + mg/A$)**

HINTS

1. $P = \dfrac{F}{A} = \dfrac{20\ \text{N}}{0.1\ \text{mm}^2} = \dfrac{20\ \text{N}}{0.1 \times 10^{-6}\ \text{m}^2} = \mathbf{2 \times 10^8\ Pa}.$
2. $P = 100\ \text{kPa} = 10^5\ \text{Pa}$,

   $A = 40\ \text{cm} \times 80\ \text{cm} = 40 \times 80 \times 10^{-4}\ \text{m}^2$

   $F = P \times A = 10^5 \times 40 \times 80 \times 10^{-4}$

   $= 32 \times 10^3\ \text{N} = \mathbf{32\ kN}.$
3. $P = \dfrac{F}{A} = \dfrac{F}{\pi r^2} = \dfrac{1.2}{3.14 \times (10^{-4})^2}\ \text{Pa}$

   $= \dfrac{1.2}{3.14 \times 10^{-8} \times 1.013 \times 10^5}\ \text{atm.} = \mathbf{377\ atm.}$

10.5 DENSITY

9. *Define the term density. Give its units and dimensions.*

Density. *The density of any material is defined as its mass per unit volume.* If a body of mass M occupies volume V, then its density is

$$\rho = \frac{M}{V} \quad i.e., \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}}.$$

Density is a positive scalar quantity. As liquids are incompressible, their density remains constant at all pressures. Density of a gas varies largely with pressure.

Units and dimensions of density :

SI unit of density = kg m^{-3}

CGS unit of density = g cm^{-3}

Dimensional formula of density is $[\text{ML}^{-3}]$.

10. *What do you mean by specific gravity or relative density of a substance ?*

Specific gravity or relative density. *The specific gravity or relative density of a substance is defined as the ratio of the density of the substance to the density of water at* 4°C. The density of water at 4°C is $1.0 \times 10^3\ \text{kg m}^{-3}$.

Scanned by CamScanner

$$\text{Specific gravity} = \frac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}}$$

Specific gravity is a dimensionless positive scalar quantity. Clearly,

Density of a substance

= Specific gravity × Density of water at 4°C

Table 10.1 *Densities of some common fluids at STP*

Fluid	Density (kg m^{-3})
Water	1.00×10^3
Sea water	1.03×10^3
Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3
Whole blood	1.06×10^3
Air	1.29
Oxygen	1.43
Hydrogen	9.0×10^{-2}
Intersteller space	$\approx 10^{-20}$

10.6 PASCAL'S LAW

11. *State and prove Pascal's law of transmission of fluid pressure.*

Pascal's law. This law tells as how pressure can be transmitted in a fluid. It can be stated in a number of equivalent ways as follows :

(*i*) *The pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.*

(*ii*) *A change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.*

(*iii*) *The pressure in a fluid at rest is same at all points if we ignore gravity.*

Proof of Pascal's law. Pascal law can be proved by using two principles :

(*i*) *The force on any layer of a fluid at rest is normal to the layer* and

(*ii*) *Newton's first law of motion.*

As shown in Fig. 10.3, consider a small element ABC – DEF in the form of a right angled prism in the interior of a fluid at rest. The element is so small that its parts can be assumed to be at same depth from liquid surface and, therefore, the effect of gravity is same for all of its points. Suppose the fluid exerts pressure P_a, P_b and P_c on the faces BEFC, ADFC and ADEB respectively of the this element and the corresponding normal forces on these faces are F_a, F_b and F_c. Let A_a, A_b and A_c be the respective areas of the three faces. In right ΔABC, let $\angle ACB = \theta$.

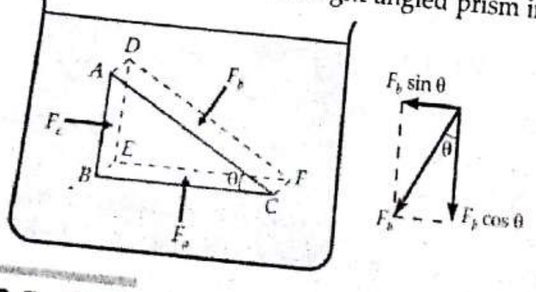


Fig. 10.3 Proof of Pascal's law.

As the prismatic element is in equilibrium with the remaining fluid, by Newton's law, the fluid forces should balance in various directions.

Along horizontal direction, $F_b \sin\theta = F_c$

Along vertical direction, $F_b \cos\theta = F_a$

From the geometry of the figure, we get

$$A_b \sin\theta = A_c$$

and $$A_b \cos\theta = A_a$$

From the above equations, we get

$$\frac{F_b \sin\theta}{A_b \sin\theta} = \frac{F_c}{A_c}$$

and $$\frac{F_b \cos\theta}{A_b \cos\theta} = \frac{F_a}{A_a}$$

$$\therefore \quad \frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c}$$

or $$P_a = P_b = P_c$$

Hence, pressure exerted is same in all directions in a fluid at rest. This proves Pascal's law of transmission of fluid pressure.

The above discussion again shows that pressure is not a vector quantity. No direction can be assigned to it.

12. *How will you experimentally verify the Pascal's law of transmission of fluid pressure ?*

Experimental verification of Pascal's law. As shown in Fig. 10.4, take a vessel having three openings

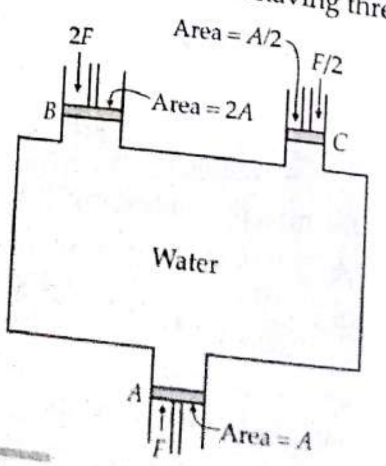


Fig. 10.4 Experimental verification of Pascal's law.

Scanned by CamScanner

A, B and C and provided with frictionless and water tight pistons. Let their cross-sectional areas be $A, 2A$ and $\frac{A}{2}$ respectively. Fill the vessel with water and apply an additional force F on piston A. To keep the pistons B and C in their positions, forces equal to $2F$ and $\frac{F}{2}$ respectively have to be applied on them. This shows that the pressure P is transmitted equally in all directions because

$$P = \frac{F}{A} = \frac{2F}{2A} = \frac{F/2}{A/2}.$$

10.7 APPLICATIONS OF PASCAL'S LAW

13. *Explain how is Pascal's law applied in a hydraulic lift.*

Hydraulic lift. Hydraulic lift is an application of Pascal's law. It is used to lift heavy objects. It is a force multiplier.

It consists of two cylinders C_1 and C_2 connected to each other by a pipe. The cylinders are fitted with water-tight frictionless pistons of different cross-sectional areas. The cylinders and the pipe contain a liquid. Suppose a force f is applied on the smaller piston of cross-sectional area a. Then

Pressure exerted on the liquid,

$$P = \frac{f}{a}$$

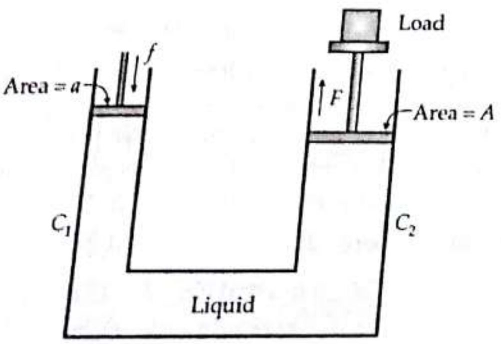


Fig. 10.5 Hydraulic lift.

According to Pascal's law, the same pressure P is also transmitted to the larger piston of cross-sectional area A.

$\therefore$ Force on larger piston is

$$F = P \times A = \frac{f}{a} \times A = \frac{A}{a} \times f$$

As $A > a$, therefore, $F > f$.

Hence by making the ratio A/a large, very heavy loads (like cars and trucks) can be lifted by the application of a small force. However, there is no gain of work. The work done by force f is equal to the work done by F. The piston P_1 has to be moved down by a larger distance compared to the distance moved up by piston P_2.

14. *With the help of a labelled diagram, explain the working of hydraulic brakes.*

Hydraulic brakes. The hydraulic brakes used in automobiles are based on Pascal's law of transmission of pressure in a liquid.

Construction. As shown in Fig. 10.6, a hydraulic brake consists of a tube T containing brake oil. One end of this tube is connected to a master cylinder fitted with piston P. The piston P is attached to the brake pedal through a lever system. The other end of the tube is connected to the wheel cylinder having two pistons P_1 and P_2. The pistons P_1 and P_2 are connected to the brake shoes S_1 and S_2 respectively. The area of cross-section of the wheel cylinder is larger than that of master cylinder.

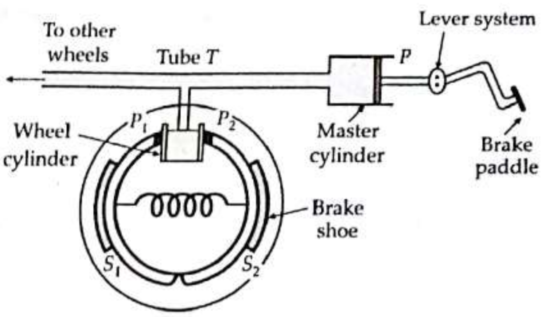


Fig. 10.6 Hydraulic brakes.

Working. When the pedal is pressed, its lever system pushes the piston P into the master cylinder. The pressure is transmitted through the oil to the pistons P_1 and P_2 in the wheel cylinder, in accordance with Pascal's law. The pistons P_1 and P_2 are pushed outwards. The brake shoes get pressed against the inner rim of the wheel, retarding the motion of the wheel. As the cross-sectional area of wheel cylinder is larger than that of master cylinder, a small force applied to the pedal produces a large retarding force.

When the paddle is released, a spring pulls the brake shoes away from the rim. The pistons in both cylinders move towards their normal positions and the oil is forced back into the master cylinder.

Advantages of hydraulic brakes :

(*i*) The master cylinder transmits equal retarding force on each wheel. So a hydraulic brake operates uniformly and hence prevents skidding.

(*ii*) A small force applied to the pedal exerts a much larger force on the wheel drums. It enables the driver to keep the vehicle under control.

Scanned by CamScanner

Examples based on Pascal's Law and Hydraulic Lift

FORMULAE USED

1. According to Pascal's law, pressure applied at any point of an enclosed mass of fluid is transmitted equally in all directions.
2. For a hydraulic lift, $P = \frac{f}{a} = \frac{F}{A}$.

UNITS USED

Forces f and F are in newton, area of cross-sections a and A in m^2.

EXAMPLE 3. *In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, what is F_1 ? What is the pressure necessary to accomplish this task ? Take $g = 9.81\ ms^{-2}$.* [NCERT ; Central Schools 08]

Solution. Here $r_1 = 5$ cm, $r_2 = 15$ cm,

$$F_2 = mg = 1350 \times 9.81 \text{ N}$$

As the pressure through air is transmitted equally in all directions, so

$$\frac{F_1}{A_1} = \frac{F}{A_2} \quad \text{or} \quad \frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

or
$$F_1 = F_2 \times \frac{r_1^2}{r_2^2} = 1350 \times 9.81 \times \frac{5 \times 5}{15 \times 15}$$

$$= \mathbf{1.5 \times 10^3\ N.}$$

Required air pressure,

$$P = \frac{F_1}{A_1} = \frac{F_1}{\pi r_1^2} = \frac{1.5 \times 10^3 \text{ N}}{3.14 \times (5 \times 10^{-2} \text{ m})^2} = \mathbf{1.9 \times 10^5\ Pa.}$$

EXAMPLE 4. *The neck and bottom of a bottle are 2 cm and 10 cm in diameter respectively. If the cork is pressed with a force of 1.2 kg f in the neck of the bottle, calculate the force exerted on the bottom of the bottle.*

Solution. Here $2r = 2$ cm or $r = 1$ cm

and
$$2R = 10 \text{ cm} \quad \text{or} \quad R = 5 \text{ cm}$$

$$a = \pi r^2 \text{ and } A = \pi R^2$$

$$f = 1.2 \text{ kg f} = 1.2 \times 9.8 \text{ N}$$

$$F = \frac{f}{a} \times A = \frac{1.2 \times 9.8}{\pi r^2} \times \pi R^2$$

$$= 1.2 \times 9.8 \times \frac{R^2}{r^2}$$

$$= 1.2 \times 9.8 \times \frac{25}{1} \text{ N} = 1.2 \times 25 \text{ kgf} = \mathbf{30\ kg\ f.}$$

EXAMPLE 5. *Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively. (a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston. (b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out ?* [NCERT ; Chandigarh 07]

Solution. (*a*) As the pressure is transmitted undiminished through water, so

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad \frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

$$\therefore \quad F_2 = \frac{r_2^2}{r_1^2} F_1 = \frac{\left(\frac{3}{2} \times 10^{-2}\right)^2}{\left(\frac{1}{2} \times 10^{-2}\right)^2} \times 10 = \mathbf{90\ N.}$$

(*b*) As water is incompressible, so

Volume covered by inward movement of smaller piston
= Volume covered by outward movement of larger piston

or
$$L_1 A_1 = L_2 A_2$$

or
$$L_1 \times \pi r_1^2 = L_2 \times \pi r_2^2$$

or
$$L_2 = \frac{r_1^2}{r_2^2} L_1 = \frac{\left(\frac{1}{2} \times 10^{-2}\right)^2}{\left(\frac{3}{2} \times 10^{-2}\right)^2} \times 6.0 \times 10^{-2}$$

$$= 0.67 \times 10^{-2} \text{ m} = \mathbf{0.67\ cm.}$$

EXAMPLE 6. *Two pistons of hydraulic press have diameters of 30.0 cm and 2.5 cm. What is force exerted by larger piston, when 50.0 kg wt. is placed on the smaller piston ? If the stroke of the smaller piston is 4.0 cm, through what distance will the larger piston move after 10 strokes ?*

Solution. Here $2r = 2.5$ cm, $r = 1.25$ cm

and
$$2R = 30.0 \text{ cm}, \quad R = 15.0 \text{ cm}$$

$$f = 50.0 \text{ kg wt}, \quad F = ?$$

As
$$\frac{f}{a} = \frac{F}{A} \quad \text{or} \quad \frac{f}{\pi r^2} = \frac{F}{\pi R^2}$$

$$\therefore \quad F = f \times \frac{R^2}{r^2} = 50 \times \left(\frac{15.0}{1.25}\right)^2$$

$$= 50 \times 144 = 7200 \text{ kg wt}$$

Also $f \times l = F \times L$

$$\therefore \quad L = \frac{f \times l}{F} = \frac{50 \times 4.0}{7200} = 0.028 \text{ cm}$$

Distance through which larger piston moves in 10 strokes

$$= 10 \times 0.028 = \mathbf{0.28\ cm.}$$

Scanned by CamScanner

PROBLEMS FOR PRACTICE

1. The area of the smaller piston of a hydraulic press is 1 cm and that of larger piston is 22 cm^2. How much weight can be raised on the larger piston by a 200 kg f exerted on the smaller piston ? **(Ans. 4400 kg)**
2. The average mass that must be lifted by a hydraulic press is 80 kg. If the radius of the larger piston is five times that of the smaller piston, what is the minimum force that must be applied ? **(Ans. 31.4 N)**
3. In a hydraulic press used for compressing cotton, the area of the piston is 0.1 m^2 and the force exerted along the piston rod is 200 N. If the area of the larger cylinder is 0.8 m^2, find the pressure produced in the cylinder and the total crushing force exerted on the bale of cotton. **(Ans. 2000 Nm^{-2}, 1600 N)**
4. An automobile back is lifted by a hydraulic jack that consists of two pistons. The large piston is 1 m in diameter and the small piston is 10 cm in diameter. If W be the weight of the car, how much smaller a force is needed on the small piston to lift the car ? **(Ans. 1% of the weight of the car)**

HINTS

1. Here $a = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$,

$$A = 22 \text{ cm}^2 = 22 \times 10^{-4} \text{ m}^2,$$

$$f = 200 \text{ kg f} = 200 \times 9.8 \text{ N}$$

Suppose m mass can be raised on the larger piston. Then

$$F = m \times 9.8 \text{ newton}$$

As $\frac{f}{a} = \frac{F}{A} \quad \therefore \quad \frac{200 \times 9.8}{10^{-4}} = \frac{m \times 9.8}{22 \times 10^{-4}}$

or $m = 200 \times 22 = \mathbf{4400 \text{ kg}}$.

4. As $\frac{f}{\pi r^2} = \frac{F}{\pi R^2}$

$$\therefore \quad f = F \times \left(\frac{r}{R}\right)^2 = W \times \left(\frac{0.05}{0.5}\right)^2$$

$$= 0.01 \text{ W} = 1\% \text{ of the weight of the car.}$$

10.8 PRESSURE EXERTED BY A LIQUID COLUMN

15. *Derive an expression for the pressure exerted by a liquid column of height h.*

Pressure exerted by a liquid column. Consider a vessel of height h and cross-sectional area A filled with a liquid of density ρ. The weight of the liquid column exerts a downward thrust on the bottom of the vessel and the liquid exerts pressure.

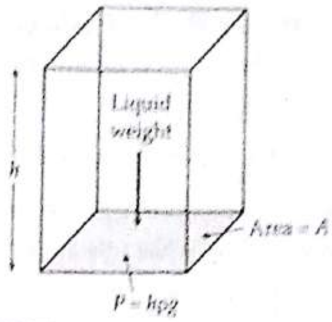


Fig. 10.7 Pressure exerted by a liquid column.

Weight of liquid column,

$$W = \text{Mass of liquid} \times g$$
$$= \text{Volume} \times \text{density} \times g$$
$$= Ah \times \rho \times g = Ah\rho g$$

Pressure exerted by the liquid column on the bottom of the vessel is

$$P = \frac{\text{Thrust}}{\text{Area}} = \frac{W}{A} = \frac{Ah\rho g}{A}$$

or $P = h\rho g$

Thus the pressure exerted by a liquid column at rest is proportional to (*i*) height of the liquid column and (*ii*) density of the liquid.

10.9 EFFECT OF GRAVITY ON FLUID PRESSURE

16. *Discuss the variation of fluid pressure with depth. Hence explain how is Pascal's law affected in the presence of gravity.*

Variation of liquid pressure with depth. As shown in Fig. 10.8, consider a liquid at rest in a container. The liquid pressure must be same at all points which are at the same depth, as otherwise liquid will not be in equilibrium. Imagine a cylindrical element of the liquid of cross-sectional area A and height h. Let P_1 and P_2 be the liquid pressures at its top point 1 and bottom point 2 respectively.

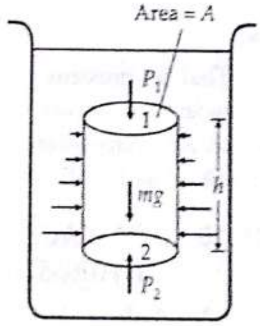


Fig. 10.8 Variation of liquid pressure with depth.

As the liquid cylinder is at rest, the resultant horizontal force should be zero. Various force acting on it in the vertical direction are :

1. Force due to the liquid pressure at the top,

$$F_1 = P_1 A, \text{ acting downwards}$$

2. Force due to the liquid pressure at the bottom,

$$F_2 = P_2 A, \text{ acting upwards}$$

Scanned by CamScanner

3. Weight of the liquid cylinder acting downwards

$$W = \text{Mass} \times g = \text{Volume} \times \text{density} \times g$$
$$= Ah\rho g$$

where ρ is the density of the liquid.

As the liquid cylinder is in equilibrium,

Net downward force = Net upward force

or $\quad F_1 + W = F_2$

or $\quad F_2 - F_1 = W$

or $\quad P_2A - P_1A = Ah\rho g$

or $\quad P_2 - P_1 = h\rho g$

If we shift point 1 to the liquid surface, which is open to the atmosphere, then we can replace P_1 by atmospheric pressure P_a and P_2 by P in the above equation and we get

$$P - P_a = h\rho g$$

or $\quad P = P_a + h\rho g$

We can note the following points :

(*i*) The liquid pressure is the same at all points at the same horizontal level or at same depth.

(*ii*) Pressure at any point inside the fluid depends on the depth h.

(*iii*) The absolute (actual) pressure P, at a depth h below the liquid surface open to the atmosphere is greater than the atmospheric pressure by an amount $h\rho g$. The excess pressure $P - P_a$, at depth h is called a ***gauge pressure*** at that point.

(*iv*) Pressure does not depend on the cross-section or base-area or the shape of the vessel.

Effect of gravity on Pascal's law. If we neglect the effect of gravity, then

$$P_2 - P_1 = h\rho g = 0$$

or $\quad P_2 = P_1$

That is, pressure at all points inside the liquid is same in the absence of gravity. This is Pascal's law. However, in the presence of gravity, Pascal's law gets modified as $P_2 - P_1 = h\rho g$.

10.10 PASCAL'S VASES : HYDROSTATIC PARADOX

17. *Explain hydrostatic paradox with suitable example.*

Hydrostatic paradox. Pascal demonstrated experimentally that *the pressure exerted by a liquid column depends only on the height of the liquid column and not on the shape of the containing vessel.* As shown in Fig. 10.9, the experimental arrangement consists of three glass vessels *A*, *B* and *C* of different shapes. The area of the lower open end of all the vessels is same. The lower end of each vessel is closed by supporting a disc against it. Each disc is connected to a pressure-meter. When the three vessels are filled with the same liquid upto the same height, all the three meters record the same pressure. This appears anomalous because the three vessels have different shapes and contain different amounts of liquid. This apparently unexpected result is known as **hydrostatic paradox.**

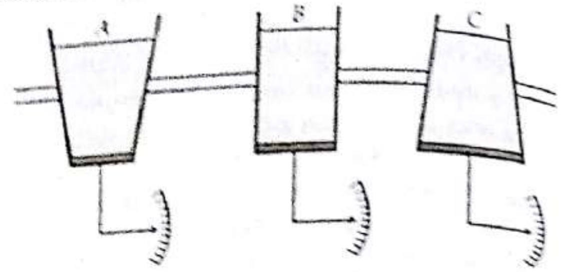


Fig. 10.9 Pascal's Vases.

Explanation. First consider the vessel *A*. Its one wall is shown in Fig. 10.10.(*a*). The liquid presses normally on the surface of the container. In the case of vessel *A*, the reaction *R* of the wall is inclined upwards. The vertical component *V* reduces the downward thrust of liquid. In the case of vessel *B*, the pressure is normal to the walls. The pressure acts horizontally on the walls. The reaction *R* of the walls is also horizontal [Fig. 10.10.(*b*)]. In the case of vessel *C*, the reaction *R* is inclined in the downward direction as shown in Fig. 10.10(*c*). The vertical component *V* increases the downward thrust of the liquid.

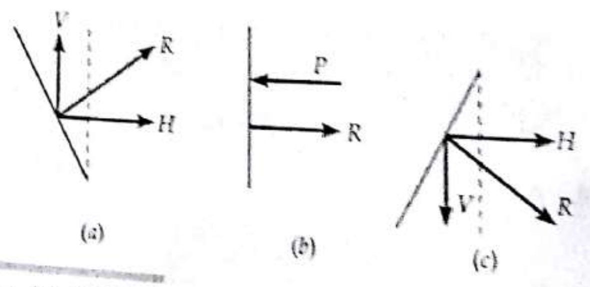


Fig. 10.10 Explanation of hydrostatic paradox.

10.11 ATMOSPHERIC PRESSURE AND GAUGE PRESSURE

18. *What is atmospheric pressure ?*

Atmospheric pressure. The gaseous envelope surrounding the earth is called the atmosphere. *The pressure exerted by the atmosphere is called atmospheric pressure.* The force exerted by air column of air on a unit area of the earth's surface is equal to the atmospheric pressure. The atmospheric pressure at sea level is 1.013×10^5 Nm^{-2} or Pa.

Scanned by CamScanner

19. *Describe mercury barometer for measuring atmospheric pressure.*

Or

Describe Torricelli's experiment of measuring atmospheric pressure.

Mercury barometer. An Italian scientist *E. Torricelli* was first to device a method for measuring atmospheric pressure accurately. It is called a simple barometer. A 1 m long glass tube closed at one end is filled with clean and dry mercury. After closing the end of the tube with the thumb, the tube is inverted into a dish of mercury. As the thumb is removed, the mercury level in the tube falls down a little and comes to rest at a vertical height of 76 cm above the mercury level in the dish.

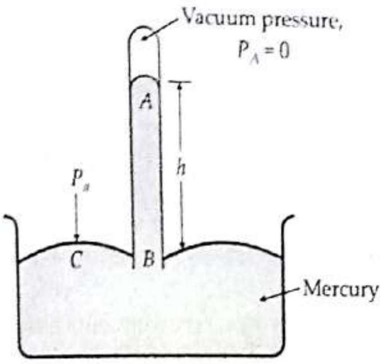


Fig. 10.11 Mercury barometer.

The space above mercury in the tube is almost a perfect vacuum and is called Torricellian vacuum. Therefore, pressure $P_A = 0$. Consider a point C on the mercury surface in the dish and point B in the tube at the same horizontal level. Then

$$P_B = P_C = \text{Atmospheric pressure, } P_a$$

If h is the height of mercury column and ρ is the density of mercury, then

$$P_B = P_A + h\rho g$$

or $$P_a = 0 + h\rho g$$

or $$P_a = h\rho g$$

For a mercury barometer, $h = 76 \text{ cm} = 0.76 \text{ m}$, $\rho = 13.6 \times 10^3 \text{ kg m}^{-3}$, $g = 9.8 \text{ m s}^{-2}$, therefore

Atmospheric pressure,

$$P_a = 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5 \text{ Pa}.$$

20. *Describe how an open tube manometer can be used to measure the pressure of a gas. Distinguish between absolute pressure and gauge pressure.*

Open-tube manometer. It is a simple device used to measure the pressure of a gas enclosed in a vessel. It consists of a *U*-tube containing some liquid. One end of the tube is open to the atmosphere and the other end is connected to the vessel.

The total pressure P of the gas is equal to the pressure at A. Thus

$$P = P_A = P_C + h\rho g \quad \text{or} \quad P = P_a + h\rho g$$

where P_a is the atmospheric pressure, $h = BC$ = difference in the levels of the liquid in the two arms and ρ is the density of the liquid.

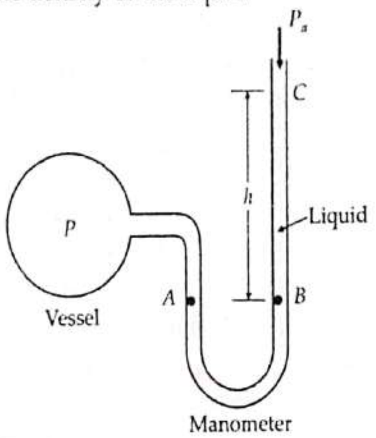


Fig. 10.12 Open tube manometer.

Absolute pressure and gauge pressure. *The total or actual pressure P at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (or absolute pressure) at a point and the atmospheric pressure, i.e.,* $P_g = P - P_a = h\rho g$

The gauge pressure is proportional to h. Many pressure measuring devices directly measure the gauge pressure. These include the tyre pressure gauge and the blood pressure gauge (sphygmomanometer).

10.12 HEIGHT OF ATMOSPHERE

21. *Calculate the height of the atmosphere above the earth's surface. Also state the assumptions used.*

Height of atmosphere. For calculating the height of atmosphere, we make use of the following assumptions :

(*i*) The value of g does not change appreciably upto a certain height.

(*ii*) Temperature remains uniform throughout.

(*iii*) Although density of air decreases with height, we assume it to be uniform and take $\rho = 1.3 \text{ kg m}^{-3}$.

Pressure exerted by h height of air column

= Pressure exerted by 0.76 m of Hg

or $$h\rho g = 1.013 \times 10^5 \text{ Pa}$$

or $$h = \frac{1.013 \times 10^5}{\rho g} = \frac{1.013 \times 10^5}{1.3 \times 9.8}$$

$$= 7951 \text{ m} \simeq 8 \text{ km}.$$

In actual practice, both the density of air and the value of g decrease with height, so the atmospheric cover extends with decreasing pressure even beyond 100 km.

Scanned by CamScanner

10.13 DIFFERENT UNITS OF PRESSURE

22. *Name the various units used for measuring pressure.*

Various units for pressure :

(*i*) SI unit of pressure = Nm^{-2} or **Pascal (Pa)**.

(*ii*) CGS unit of pressure = **dyne** cm^{-2}.

(*iii*) **Atmosphere** (atm). It is the pressure exerted by 76 cm of Hg column (at 0°C, 95° latitude and mean sea level).

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1.013 \times 10^6 \text{ dyne cm}^{-2}.$$

(*iv*) In meteorology, the atmospheric pressure is measured in bar and millibar.

$$1 \text{ bar} = 10^5 \text{ Pa} = 10^6 \text{ dyne cm}^{-2}$$

$$1 \text{ millibar} = 10^{-3} \text{ bar} = 100 \text{ Pa}$$

(*v*) Atmospheric pressure is also measured in **torr**, a unit named after Torricelli.

$$1 \text{ torr} = 1 \text{ mm of Hg}$$

$$1 \text{ atm} = 1.013 \text{ bar} = 760 \text{ torr}$$

23. *In what units is the blood pressure measured ?*

Units for blood pressure. The blood pressure is measured in mm of Hg. When the heart is contracted to its smallest size, the pumping is hardest and the pressure of blood flowing in major arteries is nearly 120 mm of Hg. This is known as ***systolic pressure***. When the heart is expanded to its largest size, the blood pressure is nearly 80 mm of Hg. This is known as ***diastolic pressure***.

For Your Knowledge

- While describing a fluid, we are concerned with properties that vary from point to point and not with properties associated with a specific piece of matter. So the role of force in a solid is replaced in a fluid by pressure and that of mass by density.
- A fluid exerts pressure not only on a solid piece immersed in fluid or on the walls of container, fluid pressure exists at all points in a fluid. A volume element (of fluid) inside a fluid is in a equilibrium because the pressures exerted on its various faces get balanced.
- Pressure at a point in a liquid acts equally in all directions.
- Pressure in a liquid is the same for all points at the same horizontal level.
- Pressure in a liquid increases with depth h according to the relation $P = P_a + h\rho g$
  This expression is valid only for incompressible fluids *i.e.*, liquids.
- Liquid pressure is independent of the area and the shape of the containing vessel.
- The mean pressure on the walls of a vessel containing liquid upto height h is $h\rho g/2$.
- Most of the pressure-measuring devices measure the pressure difference between the true pressure and the atmospheric pressure. This difference is called *gauge pressure* and the pressure is called *absolute pressure*.
  Absolute pressure
  = Gauge pressure + Atmospheric pressure
  i.e., $P = P_g + P_a$
- The gauge pressure may be positive or negative depending on $P > P_a$ or $P < P_a$. In inflated tyres or the human circulatory system, the absolute pressure is greater than atmospheric pressure, so gauge pressure is positive, called the *overpressure*. However, when we suck a fluid through a straw, the absolute pressure in our lungs is less than atmospheric pressure and so the gauge pressure is negative.
- A diver in water at a depth of 10 m is under twice the atmospheric pressure.
- At a depth of 100 km in a sea, the increase in pressure is 100 atm. Submarines are designed to withstand such high pressures.
- The pressure at the centre of the earth is estimated to be 3 million atmospheres.
- The atmospheric pressure is nearly 100 kPa. The tyres of a car are usually inflated to a pressure of about 200 kPa.
- It is because of the blood pressure from inside that we do not feel such a high atmospheric pressure.
- A drop in the atmospheric pressure by 10 mm of Hg or more is a sign of an approaching storm.

Examples based on Pressure Exerted by a Liquid Column and Gauge Pressure

FORMULAE USED

1. Pressure exerted by a liquid column of height h and density ρ is $P = h\rho g$
2. Absolute pressure
   = Atmospheric pressure + Gauge pressure
   $P = P_a + P_g$

UNITS USED

Height h is in metre, density ρ in kg m^{-3} and pressure P in Nm^{-2} or Pa.

CONVERSIONS USED

$$1 \text{ atm} = 1.013 \times 10^6 \text{ dyne cm}^{-2} = 1.013 \times 10^5 \text{ Nm}^{-2} \text{ (or Pa)}$$

$$1 \text{ bar} = 10^6 \text{ dyne cm}^{-2} = 10^5 \text{ Nm}^{-2}$$

$$1 \text{ millibar (m bar)} = 10^{-3} \text{ bar} = 10^3 \text{ dyne cm}^{-2} = 10^2 \text{ Nm}^{-2}$$

$$1 \text{ torr} = 1 \text{ mm Hg}$$

$$1 \text{ atm} = 101.3 \text{ kPa} = 1.013 \text{ bar} = 760 \text{ torr.}$$

Scanned by CamScanner

EXAMPLE 7. *Express standard atmospheric pressure in (i) Nm^{-2} (ii) bars and (iii) torr.*

Solution. (*i*) Standard atmospheric pressure

$= 76$ cm of Hg

We know that,

$$P = h\rho g$$

Here $h = 76$ cm $= 0.76$ m,

ρ (Hg) $= 13.6 \times 10^3$ kg m^{-3}, $g = 9.8$ ms^{-2}

$\therefore$ 1 atm $= 0.76 \times 13.6 \times 10^3 \times 9.8$

$= \mathbf{1.013 \times 10^5}$ **Nm^{-2}**.

(*ii*) As 1 Nm^{-2} $= 10^{-5}$ bar

$\therefore$ 1 atm = **1.013 bar = 1013 millibar.**

(*iii*) As 1 torr = 1 mm of Hg

$\therefore$ 1 atm = 760 mm of Hg = **760 torr.**

EXAMPLE 8. *What will be the length of mercury column in a barometer tube, when the atmospheric pressure is 75 cm of mercury and the tube is inclined at an angle of 60° with the horizontal direction ?*

Solution. Here $h = 75$ cm, $\theta = 60$

If l is the length of mercury coloumn in the barometer tube, then

$$\frac{h}{l} = \sin 60 \quad \text{or} \quad \frac{75}{l} = \frac{\sqrt{3}}{2}$$

$$\therefore \quad l = \frac{75 \times 2}{\sqrt{3}} = \mathbf{86.6\ cm}.$$

EXAMPLE 9. *What is the pressure on a swimmer 10 m below the surface of a lake ?* **[NCERT]**

Solution. Here $h = 10$ m,

ρ (water) $= 1000$ kg m^{-3}, $g = 9.8$ ms^{-2}

Pressure on a swimmer 10 m below the surface of the lake,

$$P = P_a + h\rho g$$

$= 1.0 \times 10^5 + 10 \times 1000 \times 9.8$

$= 1.98 \times 10^5$ Pa $\simeq$ **2 atm.**

EXAMPLE 10. *The density of the atmosphere at sea level is 1.29 kg m^{-3}. Assume that it does not change with altitude. Then how high would the atmosphere extend ? Take $g = 9.81$ ms^{-2}.* **[NCERT]**

Solution. Here $\rho = 1.29$ kg m^{-3}, $g = 9.81$ ms^{-2}, $P_a = 1.01 \times 10^5$ Pa

As $P_a = h\rho g$

$$\therefore \quad h = \frac{P_a}{\rho g} = \frac{1.01 \times 10^5}{1.29 \times 9.81} = 7981 \text{ m} \simeq \mathbf{8\ km}.$$

EXAMPLE 11. *A rectangular tank is 10 m long, 10 m broad and 3 m high. It is filled to the rim with water of density 10^3 kg m^{-3}. Calculate the thrust at the bottom and walls of the tank due to hydrostatic pressure. Take $g = 9.8$ ms^{-2}.*

Solution. Pressure on the bottom of the tank

$= h\rho g = 3 \times 10^3 \times 9.8 = 2.94 \times 10^3$ Nm^{-2}

Area of bottom = Length × Breadth

$= 10 \times 5 = 50$ m^2

$\therefore$ Thrust on the bottom

= Pressure × Area

$= 2.94 \times 10^3 \times 50 = 1.47 \times 10^6$ N

The hydrostatic pressure on the walls of the tank increases uniformly from zero at the free surface of water to $h\rho g$ at the bottom of the tank.

$\therefore$ Average hydrostatic pressure on the walls

$$= \frac{0 + h\rho g}{2} = \frac{1}{2} h\rho g = \frac{1}{2} \times 3 \times 10^3 \times 9.8$$

$= 1.47 \times 10^4$ Nm^{-2}

Now, area of broad walls

$= 2 \times$ Length × Height

$= 2 \times 10 \times 3 = 60$ m^2

Area of narrow walls

$= 2 \times$ Breadth × Height

$= 2 \times 5 \times 3 = 30$ m^2

Total area of walls $= 90$ m^2

$\therefore$ Thrust on the walls

= Average pressure × Area

$= 1.47 \times 10^4 \times 90 = 1.323 \times 10^6$ N

Total thrust on walls and bottom

$= 1.47 \times 10^6 + 1.323 \times 10^6$

$= \mathbf{2.793 \times 10^6}$ **N.**

EXAMPLE 12. *The manual of a car instructs the owner to inflate the tyres to a pressure of 200 kPa. (a) What is the recommended gauge pressure ? (b) What is the recommended absolute pressure ? (c) If, after the required inflation of the tyres, the car is driven to a mountain peak where the atmospheric pressure is 10% below that at sea level, what will the tyre gauge read ?*

Solution. (*a*) The pressure instructed by a manual is the gauge pressure.

$\therefore$ P_g = **200 kPa.**

(*b*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $P = P_a + P_g = 101$ kPa + 200 kPa = **301 kPa.**

(*c*) At the mountain peak, the atmospheric pressure P_a' is 10% less.

$\therefore$ $P_a' = 90$ kPa.

Scanned by CamScanner

If we assume that the absolute pressure in the tyres does not change during the driving, then

$$P_g = P - P'_a = 301 - 90 = \mathbf{211\ kPa}.$$

As the tyre gauge reads the gauge pressure, so it will read 211 kPa.

Example 13. *At a depth of 1000 m in an ocean (a) What is the absolute pressure ? (b) What is the gauge pressure ? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure. (The density of sea water is 1.03×10^3 kg m^{-3}, g = 10 ms^{-2})* [NCERT]

Solution. Here $h = 1000$ m, $\rho = 1.03 \times 10^3$ kg m^{-3}

Atmospheric pressure, $P_a = 1$ atm $= 1.01 \times 10^5$ Pa

The absolute pressure,

$$P = P_a + h\rho g = 1.01 \times 10^5 + 1000 \times 1.03 \times 10^3 \times 10$$

$$= 104.01 \times 10^5 \text{ Pa} = \frac{104.01 \times 10^5}{1.01 \times 10^5} \text{ atm} = \mathbf{104\ atm.}$$

(b) Gauge pressure,

$$P_g = P - P_a = h\rho g = 100 \times 1.03 \times 10^3 \times 10 \text{ Pa}$$

$$= 103 \times 10^5 \text{ Pa} \approx \mathbf{103\ atm.}$$

(c) Pressure outside the submarine,

$$P = P_a + h\rho g$$

Pressure inside the submarine $= P_a$

Net pressure on the window

= Gauge pressure

$$P_g = h\rho q$$

Area of window,

$$A = 20 \text{ cm} \times 20 \text{ cm} = 0.04 \text{ m}^2$$

Force acting on the window,

$$F = P_g A = 103 \times 10^5 \text{ Pa} \times 0.04 \text{ m}^2 = \mathbf{4.12 \times 10^5\ N.}$$

Example 14. *What is the absolute and gauge pressure of the gas above the liquid surface in the tank shown in Fig. 10.13 ? Density of oil = 820 kg m^{-3}, density of mercury = 13.6×10^3 kg m^{-3}. Given 1 atmospheric pressure = 1.01×10^5 Pa.* [NCERT]

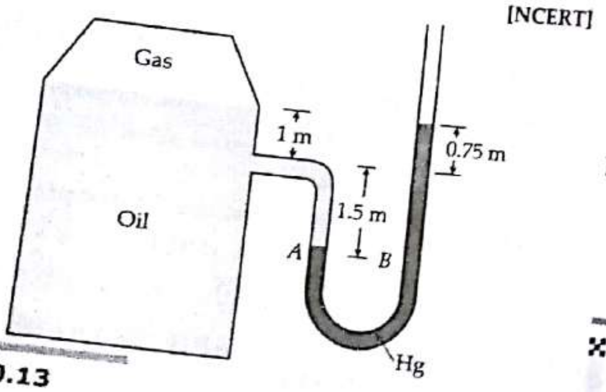


Fig. 10.13

Solution. As the points A and B are at the same level in the mercury coloumn, so

$$P_A = P_B$$

Now $P_A = P + (1.50 + 1.00) \times 820 \times 9.8$

where P is the pressure of the gas in the tank.

And $P_B = P' + (1.50 + 0.75) \times 13.6 \times 10^3 \times 9.8$

where P' is the atmospheric pressure.

As $P_A = P_B$

$$P - P' = 2.25 \times 13.6 \times 10^3 \times 9.8 - 2.50 \times 820 \times 9.8$$

$$= 3 \times 10^5 - 0.2 \times 10^5 = 2.8 \times 10^5 \text{ Pa}$$

∴ Gauge pressure

= Absolute pressure − Atmospheric pressure

$$P_g = P - P' = \mathbf{2.8 \times 10^5\ Pa.}$$

Absolute pressure

= Gauge pressure + Atmospheric pressure

$$P = 2.8 \times 10^5 + 1.01 \times 10^5 = \mathbf{3.81 \times 10^5\ Pa.}$$

Example 15. *A liquid stands at the same level in the U-tube when at rest. If A is the area of cross-section and g the acceleration due to gravity, what will be the difference in height h of the liquid in the two limbs of U-tube, when the system is given an acceleration 'a' towards right, as shown in Fig. 10.14 ?*

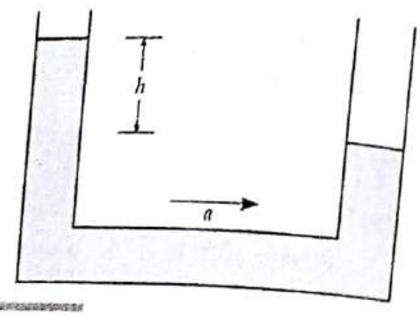


Fig. 10.14

Solution. If L is the length of the horizontal portion of the tube, then mass of liquid in this portion

$$= AL\rho.$$

Force exerted on the above mass towards left

$$= AL\rho \times a$$

Due to the difference h in the height of the liquid in the two limbs, the downward force exerted on the liquid in the horizontal portion

$$= h\rho g \times A$$

$$\therefore \quad h\rho g A = AL\rho a \quad \text{or} \quad h = \frac{La}{g}.$$

Problems For Practice

1. What is the minimum pressure required to force blood from the heart to the top of the head (a

Scanned by CamScanner

vertical distance of 50 cm) ? Assume that the density of blood is 1.04 g cm^{-3} and neglect friction. (**Ans.** 50960 dyne cm^{-2} or 38 mm of Hg)

2. A column of water 40 cm high supports a 30 cm column of an unknown liquid. What is the density of the liquid ? (**Ans.** 1.33×10^3 kgm^{-3})

3. If the water pressure gauge shows the pressure at ground floor to be 270 k Pa, how high would water rise in the pipes of a building ? (**Ans.** 27.6 m)

4. A cylindrical jar of cross-sectional area of 50 cm^2 is filled with water to a height of 20 cm. It carries a tight fitting piston of negligible mass. Calculate the pressure at the bottom of the jar when a mass of 1 kg is placed on the piston. Ignore atmospheric pressure. (**Ans.** 3920 Nm^{-2})

5. Water is filled in a flask upto a height of 20 cm. The bottom of the flask is circular with radius 10 cm. If the atmospheric pressure is 1.013×10^5 Pa, find the force exerted by the water on the bottom. Take $g = 10\ ms^{-2}$ and density of water = 1000 kgm^{-3}. (**Ans.** 3246 N)

6. A vertical U-tube of uniform inner cross-section contains mercury in both of its arms. A glycerine (density 1.3 g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8 g cm^{-3} is poured in the other arm until the upper surfaces of the oil and glycerine are in the same horizontal level. Find the length of the oil column. [IIT] (**Ans.** 9.6 cm)

7. The area of cross-section of the wider tube shown in Fig. 10.15 is 800 cm^2. If a mass of 12 kg is placed on the massless piston, what is the difference h in the level of water in the two tubes ? (**Ans.** 15.0 cm)

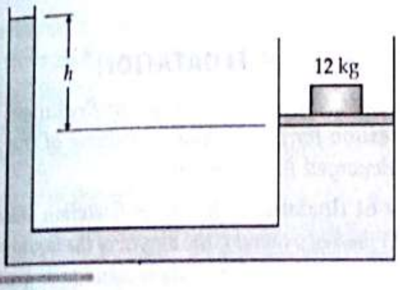


Fig. 10.15

8. A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. If the elevator is accelerating upwards at 4.9 ms^{-2}, what will be the air pressure in the elevator ? (**Ans.** 114 cm of Hg)

HINTS

1. $P = h\rho g = 50 \times 1.04 \times 980 =$ **50960 dyne** cm^{-2}

= **38 mm of Hg.**

[$\because$ 1 mm of Hg = 1333 dyne cm^{-2}]

2. As $h_1 \rho_1 g = h_2 \rho_2 g$

$\therefore \quad \rho_2 = \frac{h_1}{h_2} \times \rho_1 = \frac{0.40}{0.30} \times 10^3 =$ **1.33×10^3 kg m^{-3}.**

3. $P = h\rho g = 270$ kPa $= 270 \times 10^3$ Pa

or $\quad h = \frac{270 \times 10^3}{\rho g} = \frac{270 \times 10^3}{10^3 \times 9.8} =$ **27.6 m.**

4. Here $A = 50\ cm^2 = 50 \times 10^{-4}\ m^2$,

$h = 20$ cm $= 0.20$ m,

$m = 1$ kg, ρ (water) $= 10^3$ kg m^{-3}

Total force on the bottom $= mg + h\rho g \times A$

$= 1 \times 9.8 + 0.20 \times 10^3 \times 9.8 \times 50 \times 10^{-4}$

$= 9.8 + 9.8 = 19.6$ N

Pressure at the bottom

$= \frac{\text{Force}}{\text{Area}} = \frac{19.6}{50 \times 10^{-4}} =$ **3920 Nm^{-2}.**

5. Pressure at the bottom,

$P = P_0 + h\rho g = 1.013 \times 10^5 + 0.20 \times 1000 \times 10$

$= 1.033 \times 10^5$ Pa

Area of the bottom,

$A = \pi r^2 = 3.142 \times (0.1)^2 = 0.03142\ m^2$

$F = PA = 1.033 \times 10^5 \times 0.03142 =$ **3246 N.**

6. The situation is shown in Fig. 10.16. Let h be the length of oil column. As B and E are the two points in mercury at the same level, so

$P_E = P_B$

$h \times 0.8 \times g + (10 - h) \times 13.6 \times g = 10 \times 1.3 \times g$

or $\quad h =$ **9.6 cm.**

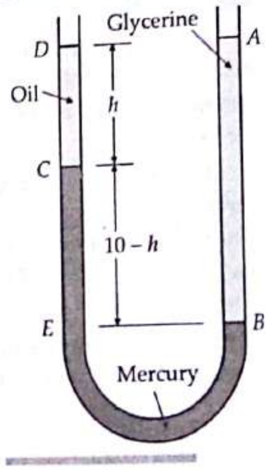


Fig. 10.16

7. Area of cross-section of the piston, $A = 800\ cm^2$

$F = mg$

$= 12 \times 1000 \times 980$ dyne

Pressure on the liquid,

$P = \frac{F}{A} = \frac{12 \times 1000 \times 980}{800}$ dyne cm^{-2}

But $\quad P = h\rho g = h \times 1 \times 980$

$\therefore \quad h \times 1 \times 980 = \frac{12 \times 1000 \times 980}{800}$

or $\quad h =$ **15.0 cm.**

8. When the elevator moves upward with acceleration a, net acceleration $= g + a$

$\therefore$ Pressure $= h\rho(g + a)$ dyne cm^{-2}

$= \frac{76 \times 13.6 \times (9.8 + 4.9)}{13.6 \times 9.8} =$ **114 cm of Hg.**

Scanned by CamScanner

10.14 BUOYANCY

24. *What do you understand by buoyancy and centre of buoyancy ?*

Buoyancy. When body is immersed in a fluid, the fluid exerts pressure on all faces of the body. But the fluid pressure increases with depth. The upward thrust at the bottom is more than the downward thrust on the top because the bottom is at the greater depth than the top. Hence a resultant upward force acts on the body. *The upward force acting on a body immersed in a fluid is called upthrust or buoyant force and the phenomenon is called buoyancy.* For example, a cork taken inside water experiences an upward thrust and comes to the surface. Similarly, while drawing water from a well, a bucket is found too much lighter when it is inside water than when it comes out of it.

The force of buoyancy acts through the centre of gravity of the displaced fluid which is called centre of buoyancy.

10.15 ARCHEMEDES' PRINCIPLE*

25. *State Archemedes' principle and prove it mathematically. Deduce an expression for the apparent weight of the immersed body.*

Archemedes' principle. This principle was discovered by the great Greek scientist, *Archimedes* around 225 B.C. and it gives the magnitude of buoyant force on a body.

Archimedes' principle *states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the displaced fluid.*

Proof. As shown in Fig. 10.17, consider a body of height h lying inside a liquid of density ρ, at a depth x below the free surface of the liquid. Area of cross-section of the body is a. The forces on the sides of the body cancel out.

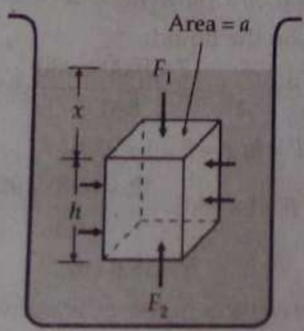


Fig. 10.17 Buoyant force on a body.

Pressure at the upper face of the body,

$$P_1 = x\rho g$$

Pressure at the lower face of the body,

$$P_2 = (x+h)\rho g$$

Thrust acting on the upper face of the body is

$$F_1 = P_1\, a = x\rho g a,$$

acting vertically downwards.

Thrust acting on the lower face of the body is

$$F_2 = P_2\, a = (x+h)\rho g a,$$

acting vertically upwards.

The resultant force $(F_2 - F_1)$ is acting on the body in the upward direction and is called upthrust (U).

$$\therefore \quad U = F_2 - F_1 = (x+h)\rho g a - x\rho g a = ah\rho g$$

But $ah = V$, the volume of the body = Volume of liquid displaced

$$\therefore \quad U = V\rho g = Mg$$

$[\because M = V\rho = \text{mass of liquid displaced}]$

i.e., Upthrust or buoyant force

= Weight of liquid displaced

This proves the Archimedes' principle.

Apparent weight of immersed body. The actual weight W of the immersed body acts downwards and the upthrust U acts upwards.

$\therefore$ Apparent weight

= Actual weight – Buoyant force

$$W_{app} = W - U = V\sigma g - V\rho g = V\sigma g\left(1 - \frac{\rho}{\sigma}\right)$$

or $$W_{app} = W\left(1 - \frac{\rho}{\sigma}\right)$$

Here $W = V\sigma g$ is the true weight of the body and σ is its density.

10.16 LAW OF FLOATATION*

26. *State and explain the law of floatation. Deduce an expression for the fraction of volume of the floating body submerged in the liquid.*

Law of floatation. *The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.*

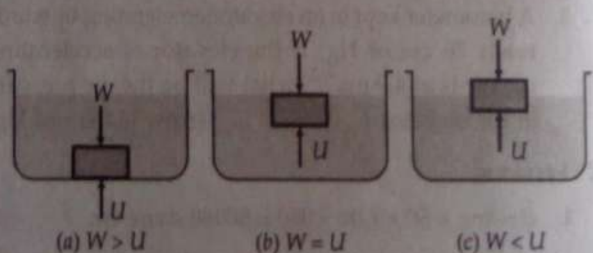


Fig. 10.18 Law of floatation.

Explanation. When a body is immersed fully or partly in a liquid, the following two vertical forces act on it :

(*i*) Its true weight W which acts vertically downward through its centre of gravity.

(*ii*) Force of buoyancy or upthrust U which acts vertically upwards through the centre of buoyancy.

As shown in Fig. 10.18, three cases are possible :

(*a*) **When $W > U$.** The downward pull of the weight of the body is higher than the upthrust. The net force $(W - U)$ acts in the downward direction and hence the body sinks.

$$W > U \quad \Rightarrow \quad V\sigma g > V\rho g \quad \text{or} \quad \sigma > \rho$$

Thus *a body sinks in a liquid if its density greater than the density of the liquid.* That is why an iron piece or a stone sinks in water.

(*b*) **When $W = U$.** The weight of the body is just balanced by the upthrust. No net force acts on the body. The body floats fully immersed.

$$W = U \quad \Rightarrow \quad V\sigma g = V\rho g \quad \text{or} \quad \sigma = g$$

Thus a drop of olive oil stands at rest anywhere in a mixture of equal quantities of water and alcohol because the density of olive oil is equal to that of the mixture.

(*c*) **When $W < U$.** The gravitational force W is less than the upward force U. The body floats partly immersed. This is because the body sinks only to the extent that $W = U$.

Here $\sigma < \rho$. The density of the floating body is less than that of liquid. That is why a piece of cork floats on water.

Fractional submerged volume of floating body. When the weight of the body is less than the weight of the liquid displaced, the body floats partially submerged. If V is the total volume of the body and V' is the submerged volume, then at equilibrium,

Weight of the body

= Weight of liquid displaced

or $$V\sigma g = V'\rho g$$

or $$\frac{V'}{V} = \frac{\sigma}{\rho}$$

or $$\frac{\text{Volume of submerged part}}{\text{Total volume of the body}} = \frac{\text{Density of body}}{\text{Density of liquid}}$$

27. *Give some examples of floating bodies.*

Some examples of floating bodies :

(*i*) The ship is made of steel (8 times denser than water) but its interior is made hollow by giving it a concave shape. It can displace much more water than its own weight. So the ship floats and can carry a lot of cargo.

(*ii*) Ice floats on water because the density of ice is less than that of water.

(*iii*) Human body is slightly more denser than water. An inflated rubber tube has low weight and large volume and increases the upthrust. It helps a person to float.

(*iv*) A person can swim in sea water more easily than in river water. The density of sea water is more than that of river water and so it exerts a greater upthrust.

(*v*) The average density of a fish is slightly greater than water. By means of an anatomical attachment called swim bladder whose size it can adjust, the fish is able to swim with ease.

10.17 EQUILIBRIUM OF FLOATING BODIES*

28. *State the conditions for the equilibrium of floating bodies. Also discuss the stability of a floating body.*

Conditions for the equilibrium of a floating body :

(*i*) Weight of the liquid displaced must be equal to the weight of the body.

(*ii*) The centre of gravity of the body and the centre of buoyancy must lie on the same vertical line.

Stability of a floating body. When the centre of gravity of the body and the centre of buoyancy do not lie on the same vertical line, the two forces ; the weight (W) of the body and the upthrust (U) form a couple which produces rotation. As the floating body is slightly displaced from the equilibrium position, the

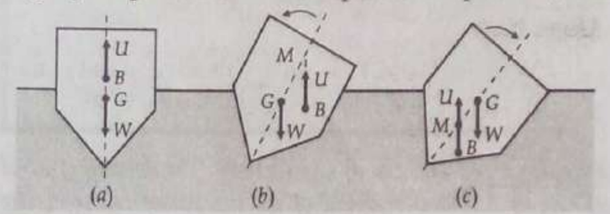


Fig. 10.19 Stability of a floating body.

centre of buoyancy shifts to a new position. *The point at which the vertical line passing through the new centre of buoyancy meets the initial vertical line is called* **metacentre** (M), as shown in Fig. 10.19.

(*i*) If the metacentre M lies above the centre of gravity G, the couple tends to bring the body back to its original position, as shown in Fig. 10.19(*b*). The floating body is in **stable equilibrium**.

(*ii*) If the metacentre M lies below the centre of gravity G, the couple tends to rotate the body away from the original position, as shown in Fig. 10.19(*c*). The floating body is in **unstable equilibrium**. The couple topples the floating body.

Scanned by CamScanner

Examples based on Archemedes' Principle and Law of Floatation

Formulae Used

1. According to Archemedes' principle, Loss in weight of a body in a liquid = Weight of liquid displaced = Volume × Density of liquid × g.
2. Apparent weight of solid in a liquid
$$= \text{True weight} - \text{Weight of liquid displaced}$$
$$= mg - V\rho' g = mg - \frac{m}{\rho}\rho' g = mg\left(1 - \frac{\rho'}{\rho}\right),$$
where ρ' is the density of the liquid and ρ that of solid.
3. What a body just floats, Weight of the body
$$= \text{Weight of liquid displaced}$$
or $V\rho g = V'\rho' g$ or $\frac{V'}{V} = \frac{\rho}{\rho'}$
$$\frac{\text{Volume of immersed part}}{\text{Total volume of the solid}} = \frac{\text{Density of solid}}{\text{Density of liquid}}$$
4. Relative density $= \dfrac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}}$
5. Relative density of a solid
$$= \frac{\text{Weight of solid in air}}{\text{Loss in weight in water}}$$
6. Relative density of a liquid
$$= \frac{\text{Loss in weight in liquid}}{\text{Loss in weight in water}}$$

Units Used

Volumes V and V' are in m^3, densities ρ and ρ' are in kgm^{-3} and relative density has no units.

Example 16. **The tip of the iceberg.** *The density of ice is 917 kg m^{-3}. What fraction of ice lies below water ? The density of sea water is 1024 kg m^{-3}. What fraction of the ice berg do we see assuming that it has the same density as ordinary ice (917 kg m^{-3}) ?* **[NCERT]**

Solution. Density of ice, $\rho = 917$ kg m^{-3}

Density of water = 1000 kg m^{-3}

According to the law of floatation,

Weight of the piece of ice

= Weight of liquid displaced

$$V\rho g = V'\rho' g$$

$$\frac{V'}{V} = \frac{\rho}{\rho'} = \frac{917}{1000} = 0.917$$

So 91.7% of the ice lies below water.

In the case of the iceberg at sea, the fraction visible to us is given by

$$f = 1 - \frac{V'}{V} = 1 - \frac{\rho}{\rho'} = 1 - \frac{917}{1024} = 0.105$$

$[\rho'$ (sea water) $= 1024$ kg $m^{-3}]$

So 10.5% of ice berg is visible to us. As most of the ice lies below the surface of the sea, hence the phrase "The tip of the ice berg".

Example 17. The density of ice is 0.918 g cm^{-3} and that of water is 1.03 g cm^{-3}. An iceberg floats with a portion of 224 m^3 outside the surface of water. Find the total volume of the iceberg.

Solution. Density of ice, $\rho = 0.918 \times 10^3$ kgm^{-3}

Density of water, $\rho' = 1.03 \times 10^3$ kgm^{-3}

Let volume of the iceberg = V m^3

Then volume of water displaced,

$$V' = (V - 224) \text{ m}^3$$

$\because$ Weight of iceberg = Weight of water displaced

$$V\rho g = V'\rho' g$$

or $$V \times 0.918 \times 10^3 \times g = (V - 224) \times 1.03 \times 10^3 \times g$$

or $$V(1.03 - 0.918) = 224 \times 1.03$$

or $$V = \frac{224 \times 1.03}{0.112} = 2060 \text{ m}^3.$$

Example 18. A body of mass 6 kg is floating in a liquid with 2/3 of its volume inside the liquid. Find (i) buoyant force acting on the body, and (ii) ratio between the density of body and density of liquid. Take g = 10 ms^{-2}. **[Delhi 04]**

Solution. When a body floats, its apparent weight is zero.

$\therefore$ Buoyant force = Weight of body

$$= 6 \text{ kg} \times 10 \text{ ms}^{-2} = 60 \text{ N}.$$

Also, Buoyant force = Weight of liquid displaced

or $$V\rho_b g = \frac{2}{3} V\rho_l g$$

or $$\frac{\rho_b}{\rho_l} = \frac{2}{3}.$$

Example 19. A piece of pure gold (ρ = 19.3 g cm^{-3}) is suspected to be hollow from inside. It weighs 38.250 g in air and 33.865 g in water. Calculate the volume of the hollow portion in gold, if any. **[NCERT]**

Solution. Density of pure gold, ρ = 19.3 g cm^{-3}

Weight of gold piece, M = 38.250 g

$\therefore$ Volume of gold piece,

$$V = \frac{M}{\rho} = \frac{38.250}{19.3} = 1.982 \text{ cm}^3$$

Scanned by CamScanner

Mass of gold piece in water,

$$M' = 33.865 \text{ g}$$

$\therefore$ Apparent loss in weight of the gold piece in water

$$= 38.250 - 33.865 = 4.385 \text{ g}.$$

Density of water $= 1 \text{ g cm}^{-3}$

$\therefore$ Volume of water displaced

$$= \frac{4.385}{1} = 4.385 \text{ cm}^3$$

$\therefore$ Volume of hollow portion of the gold piece

$$= 4.385 - 1.982 = 2.403 \text{ cm}^3.$$

EXAMPLE 20. *A solid body floating in water has 1/6th of the volume above surface. What fraction of its volume will project upward if it floats in a liquid of specific gravity 1.2 ?*

Solution. Let volume of the body $= V \text{ m}^3$

Then volume of body lying above surface

$$\frac{V}{6} \text{ m}^3$$

Volume of water displaced $= V - \frac{V}{6} = \frac{5}{6} V \text{ m}^3$

$\therefore$ Weight of body = Weight of water displaced

or $$V\rho g = \frac{5}{6} V \times 10^3 \times g \quad ...(i)$$

Let V' be the volume of the body that lies outside the liquid of specific gravity 1.2. Then

Volume of liquid displaced $= V - V'$

Again, weight of body

= Weight of liquid displaced

$$\therefore \quad V\rho g = (V - V') \times 1.2 \times 10^3 \times g \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$(V - V') \times 1.2 \times 10^3 \times g = \frac{5}{6} V \times 10^3 \times g$$

or $$\frac{V - V'}{V} = \frac{5}{6} \times \frac{10}{12} = \frac{25}{36}$$

or $$1 - \frac{V'}{V} = \frac{25}{36}$$

or $$\frac{V'}{V} = 1 - \frac{25}{36} = \frac{11}{36}.$$

EXAMPLE 21. *A spring balance reads 10 kg when a bucket of water is suspended from it. What is the reading on the spring balance when*

(*i*) *an ice cube of mass 1.5 kg is put into the bucket*

(*ii*) *an iron piece of mass 7.8 kg suspended by another spring is immersed with half its volume inside the water in the bucket ?*

Relative density of iron = 7.8. **[NCERT ; Delhi 06]**

Solution. (*i*) When the ice is put into the bucket, its total weight $= 10 + 1.5 = 11.5$ kg f

The spring balance shows the reaction of the above force, $= 11.5$ kg f

(*ii*) Density of iron

$$= \text{R.D.} \times \text{Density of water} = 7.8 \times 10^3 \text{ kgm}^{-3}.$$

Volume of iron piece

$$= \frac{\text{Mass}}{\text{Density}} = \frac{7.8}{7.8 \times 1000} = 0.001 \text{ m}^3$$

As only half iron piece is immersed,

Volume of water displaced $= \frac{0.001}{2} \text{ m}^3$

Upthrust = Weight of water displaced

$$= \text{Volume} \times \text{Density} \times g$$

$$= \frac{0.001}{2} \times 1000 \times g \text{ newton}$$

$$= 0.5 g \text{ newton} = 0.5 \text{ kg f}$$

$\therefore$ Total upward reaction $= 10 + 0.5 = 10.5$ kg f

$\therefore$ Reading on the spring balance $=$ **10.5 kg f.**

EXAMPLE 22. *A cube of wood floating in water supports a 200 g mass at the centre of its top face. When the mass is removed, the mass rises by 2 cm. Determine the volume of cube.* **[Chandigarh 03]**

Solution. Let the side of the cube be l cm. As the mass of 200 g is removed, the cube rises by 2 cm. So by law of floatation,

Upthrust on cube due to displaced volume

$$(V = l \times l \times 2 \text{ cm}^3) \text{ of water} = 200 \text{ gf}$$

or $$l \times l \times 2 \times 1 \times g = 200 \times g$$

or $$l^2 = 100, \text{ or } l = 10 \text{ cm}$$

$\therefore$ Volume of the cube $= (10)^3 =$ **1000 cm^3.**

EXAMPLE 23. *A block weighs 15 N in air. It weighs 12 N when immersed in water. When immersed in another liquid, it weighs 13 N. Calculate the relative density (specific gravity) of (i) the block and (ii) the other liquid.* **[REC 89]**

Solution. (*i*) Relative density of the block

$$= \frac{\text{Weight of the block in air}}{\text{Loss in weight when immersed in water}}$$

$$= \frac{15}{15 - 12} = \mathbf{5}.$$

(*ii*) Relative density of the liquid

$$= \frac{\text{Loss in weight when immersed in liquid}}{\text{Loss in weight when immersed in water}}$$

$$= \frac{15 - 13}{15 - 12} = \frac{\mathbf{2}}{\mathbf{3}}.$$

EXAMPLE 24. *A jeweller claims that he sells ornaments made of pure gold that has the relative density of 19.3. He sells a necklace weighing 25.250 g f to a person. The clever*

Scanned by CamScanner

customer weighs the necklace when immersed in pure water and finds that it weighs 23.075 g f in water. Is the ornament made of pure gold ?

Solution. Weight of ornament in air = 25.250 g f

Weight of ornament in water = 23.075 g f

Loss of weight in water

$= 25.250 - 23.075 = 2.175$ g f

Relative density of the ornament

$$= \frac{\text{Weight in air}}{\text{Loss of weight in water}} = \frac{25.250}{2.175} = 11.61$$

As the relative density of the ornament is much less than that of pure gold which is 19.3, therefore, the ornament is not made of pure gold.

EXAMPLE 25. *A tank contains water and mercury as shown in Fig. 10.20. An iron cube of edge 6 cm is in equilibrium as shown. What is the fraction of cube inside the mercury ? Given density of iron $= 7.7 \times 10^3$ kgm^{-3} and density of mercury $= 13.6 \times 10^3$ kg m^{-3}.*

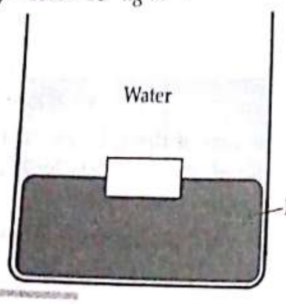


Fig. 10.20

Solution. Let x be the depth of cube in mercury. The depth of cube in water will be $(0.06 - x)$ m. From Archemedes' principle, the buoyant force on cube due to mercury is

$$B_1 = (0.06)^2 \times x \times (13.6 \times 10^3) \times 9.8 \text{ N}$$

Similarly, the buoyant force on cube due to water is

$$B_2 = (0.06)^2 \times (0.06 - x) \times 10^3 \times 9.8 \text{ N}$$

When cube is in equilibrium

$$B_1 + B_2 = \text{Weight of iron cube}$$

or $$(0.06)^2 \times 10^3 \times 9.8\,[13.6\,x + (0.06 - x)] = (0.06)^3 \times 7.7 \times 10^3 \times 9.8$$

On Solving, $x = \dfrac{0.40}{12.6} = 0.032$ m

Fraction of cube inside mercury $= \dfrac{0.032}{0.06} = \mathbf{0.533.}$

EXAMPLE 26. *A body of density ρ floats with a volume V_1 of its total volume V immersed in one liquid of density ρ_1 and with the remainder of volume V_2 immersed in another liquid of density ρ_2, where $\rho_1 > \rho_2$. Find the relative volumes immersed in two liquids.*

Solution. Weight of the body $= V\rho g$

Weight of first liquid displaced $= V_1 \rho_1 g$

Weight of second liquid displaced $= V_2 \rho_2 g$

When the body floats,

Weight of the body = Weight of two liquids displaced

$$V\rho g = V_1\rho_1 g + V_2\rho_2 g$$

or $$V\rho = V_1\rho_1 + V_2\rho_2 \quad ...(i)$$

Also $$V = V_1 + V_2 \quad ...(ii)$$

or $$V\rho_1 = V_1\rho_1 + V_2\rho_1 \quad ...(iii)$$

Subtracting (*iii*) from (*ii*), we get

$$V(\rho - \rho_1) = V_2(\rho_2 - \rho_1)$$

or $$V(\rho_1 - \rho) = V_2(\rho_1 - \rho_2)$$

or $$V_2 = \left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right) V \quad ...(iv)$$

Substituting the value of V_2 in (*ii*), we get

$$V = V_1 + \left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right) V$$

or $$V_1 = V\left(1 - \frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right) = \left(\frac{\rho - \rho_2}{\rho_1 - \rho_2}\right) V. \quad ...(v)$$

EXAMPLE 27. *A sample of milk diluted with water has a density of 1032 kgm^{-3}. If pure milk has a density of 1080 kgm^{-3}, find the percentage of water by volume in milk.*

Solution. Let volume of diluted sample of milk $= V$

Volume of water in the sample $= v$

Volume of pure milk in the sample $= V - v$

Density of diluted milk = 1032 kgm^{-3}

Density of pure milk = 1080 kgm^{-3}

Density of water = 1000 kgm^{-3}

Density of diluted milk

$$= \frac{\text{Mass of pure milk + Mass of water}}{\text{Volume of diluted milk}}$$

$$1032 = \frac{(V - v) \times 1080 + v \times 1000}{V}$$

or $$1032\,V = 1080\,V - 1080\,v + 1000\,v$$

or $$\frac{v}{V} = \frac{48}{80} = 0.6$$

Percentage of water by volume in milk

$= 0.6 \times 100 = 60\%$.

PROBLEMS FOR PRACTICE

1. How much will a body of 70 N weigh in water if it displaces 200 ml of water ? **(Ans. 68.04 N)**

Scanned by CamScanner

2. A solid weighs 6 kg in air. If its density is 2000 kg m^{-3}, what will be its apparent weight in water ? (**Ans.** 3 kg)

3. A solid weighs 10 N in air. Its weight decreases by 2 N when weighed in water. What is the density of solid ? (**Ans.** 5000 kgm^{-3})

4. A copper cube of mass 0.50 kg is weighed in water. The mass comes out to be 0.40 kg. Is the cube hollow or solid ? Given density of copper $= 8.96 \times 10^3 \text{ kgm}^{-3}$ and density of water $= 10^3 \text{ kgm}^{-3}$.
(**Ans.** Density of cube $= 5 \times 10^3 \text{ kgm}^{-3}$, so it is hollow)

5. A solid floats in water with 3/4 of its volume below the surface of water. Calculate the density of the solid. (**Ans.** 750 kg m^{-3})

6. A piece of wood of relative density 0.25 floats in a pail containing oil of relative density 0.81. What is the fraction of volume of the wood above the surface of the oil ? (**Ans.** 0.69)

7. A boat having a length of 3 m and breadth 2 m is floating on a lake. The boat sinks by one cm, when a man gets on it. What is the mass of the man ? (**Ans.** 60 kg)

8. When a boulder of mass 240 kg is placed on an iceberg floating in sea water it is found that the iceberg just sinks. What is the mass of the iceberg ? Take the relative density of ice as 0.9 and that of sea water as 1.02. (**Ans.** 1800 kg)

9. A cubical block of wood 10.0 cm on a side floats at the interface between oil and water, with its lower surface horizontal and 4.0 cm below the interface. What is the mass of the block ? The density of the oil is 0.6 g cm^{-3}. (**Ans.** 760 g)

10. A piece of brass (alloy of zinc and copper) weighs 12.9 g in air. When completely immersed in water it weighs 11.3 g. What is the mass of copper contained in the alloy ? Specific gravity of zinc and copper are 7.1 and 8.9 respectively. (**Ans.** 7.61 g)

11. A piece of iron floats in mercury. Given that the density of iron is $7.8 \times 10^3 \text{ kg m}^{-3}$ and that of mercury is $13.6 \times 10^3 \text{ kgm}^{-3}$, calculate the fraction of the volume of iron piece that remains outside the mercury. (**Ans.** 0.43)

12. A metal cube of 5 cm side and relative density 9 is suspended by a thread so as to be completely immersed in a liquid of density $1.2 \times 10^3 \text{ kg m}^{-3}$. Find the tension in the thread. (**Ans.** 9.56 N)

13. A cube of side 4 cm is just completely immersed in liquid A. When it is put in liquid B, it floats with 2 cm outside the liquid. Calculate the ratio of densities of two liquids. (**Ans.** 1 : 2)

14. An iron ball has an air space in it. It weighs 1 kg in air and 0.6 kg in water. Find the volume of air space. Density of iron $= 7200 \text{ kgm}^{-3}$. (**Ans.** $0.262 \times 10^{-3} \text{ m}^3$)

HINTS

1. Volume of water displaced = 200 ml = 0.2 litre

$\therefore$ Mass of water displaced = 0.2 kg

Loss in weight of body in water

= Weight of water displaced

$= 0.2 \times 9.8 \text{ N} = 1.96 \text{ N}$

Apparent weight of body = 70 − 1.96 = **68.04 N.**

2. $$\text{Volume of solid} = \frac{\text{Mass}}{\text{Density}} = \frac{6}{2000} \text{ m}^3$$

= Volume of water displaced

Mass of water displaced

$$= \text{Volume} \times \text{Density} = \frac{6}{2000} \times 1000 = 3 \text{ kg}$$

Apparent weight in water = 6 − 3 = **3 kg.**

3. Relative density of solid

$$= \frac{\text{Weight of solid in air}}{\text{Loss in weight in water}} = \frac{10}{2} = 5$$

Density of solid = R.D. × Density of water

$= \mathbf{5 \times 10^3 \text{ kg m}^{-3}}$.

4. Loss of weight in water

= Weight of water displaced

$$(0.50 - 0.40) \times g = V \times 10^3 \times g$$

Volume of cube, $V = 10^{-4} \text{ m}^3$

$$\text{Density of cube} = \frac{0.50}{10^{-4}} = 5 \times 10^3 \text{ kg m}^{-3}$$

which is less than the density of copper.

5. Let the density and volume of the solid be ρ and V. Then

Weight of solid $= V\rho g$

Volume of block in water $= \frac{3}{4} V$

= Volume of water displaced

Weight of water displaced

$$= V'\rho' g = \frac{3}{4} V \times 10^3 \times g$$

$\because$ Weight of body = Weight of water displaced

or $V\rho g = \frac{3}{4} V \times 10^3 g \quad \therefore \quad \rho = \mathbf{750 \text{ kg m}^{-3}}$.

6. Density of wood, $\rho = 0.25 \times 10^3 \text{ kgm}^{-3}$

Density of oil, $\rho' = 0.81 \times 10^3 \text{ kg m}^{-3}$

According to the law of floatation,

Weight of the piece of wood = Weight of liquid displaced

Scanned by CamScanner

or $\quad V\rho g = V'\rho' g$

or $\quad \frac{V'}{V} = \frac{\rho}{\rho'} = \frac{0.25 \times 10^3}{0.18 \times 10^3} = 0.31.$

i.e. fraction of volume of the wood submerged under the oil = 0.31

∴ Fraction of volume of the wood above the surface of the oil = 1 – 0.31 = **0.69.**

7. Fraction of volume above water surface

$= 1 - 0.90 = 0.10$

Weight of man = Weight of water displaced by boat when the man gets in

or $\quad m \times 9.8 = (3 \times 2 \times 0.01) \times 10^3 \times 9.8$

Hence $\quad$ **m = 60 kg.**

8. R.D. of ice = 0.9

∴ $\quad$ Density of ice $= 0.9 \times 10^3$ kgm^{-3}

Let mass of iceberg = m kg

Volume of iceberg, $V = \frac{\text{Mass}}{\text{Density}} = \frac{m}{0.9 \times 10^3}$ m^3

When this volume just sinks, mass of sea-water displaced

$= V \times$ Density of sea-water

$= \frac{m}{0.9 \times 10^3} \times 1.02 \times 10^3 = \frac{102}{90}$ m

According to Archimedes' principle,

$m + 240 = \frac{102}{90} m$ or $m\left(\frac{102}{90} - 1\right) = 240$

or $\quad m = \frac{240 \times 90}{12} = \mathbf{1800\ kg.}$

9. Volume of block = $(10.0\text{ cm})^3 = 1000\text{ cm}^3$

Volume of block in water

$= 10.0 \times 10.0 \times 4.0 = 400\text{ cm}^3$

Volume of block in oil = $1000 - 400 = 600\text{ cm}^3$

According to Archimedes' principle,

Weight of the block = Weight of water displaced + Weight of oil displaced

$mg = 400 \times 1 \times g + 600 \times 0.6 \times g$

or $\quad m = 400 + 360 = \mathbf{760\ g.}$

10. Let m be the mass of copper. Then the mass of zinc in the alloy = $(12.9 - m)$ g

Volume of copper in the alloy $= \frac{m}{8.9}$

Volume of zinc in the alloy $= \frac{(12.9 - m)}{7.1}$

Total volume of the alloy $= \frac{m}{8.9} + \frac{(12.9 - m)}{7.1}$

Apparent loss of weight of alloy

$= 12.9 - 11.3 = 1.6$ g

∴ Volume of water displaced by the alloy $= 1.6\text{ cm}^3$

Total volume of the alloy = Total volume of water displaced

$\frac{m}{8.9} + \frac{(12.9 - m)}{7.1} = 1.6$

On solving, $\quad$ **m = 7.61 g.**

11. Fraction of iron piece that remains inside mercury,

$\frac{V'}{V} = \frac{\rho}{\rho'} = \frac{7.8 \times 10^3}{13.6 \times 10^3} = 0.57$

Fraction of iron piece that remains outside mercury

$= 1 - 0.57 = \mathbf{0.43.}$

12. Tension in the thread = Weight in air – Upthrust of liquid

$= (0.05)^3 \times 9 \times 10^3 \times 9.8 - (0.05)^3 \times 1.2 \times 10^3 \times 9.8$

$= 7.8 \times (0.5)^3 \times 10^3 \times 9.8 = \mathbf{9.56\ N.}$

13. In each case, mass of cube

= mass of liquid displaced

$m = V\rho_A = \frac{2}{4} V\rho_B$ or $\frac{\rho_A}{\rho_B} = \frac{2}{4} = \mathbf{1:2.}$

14. Loss of weight of ball in water = 1 – 0.6 = 0.4 kg f

Loss of weight of ball

= Weight of water displaced = 0.4 kg f

$= V \times 1000$ kg f

Volume of iron ball with air space,

$V = \frac{0.4}{1000} = 0.4 \times 10^{-3}\text{ m}^3$

Volume of iron alone

$= \frac{\text{Mass}}{\text{Density}} = \frac{1}{7200} = 0.138 \times 10^{-3}\text{ m}^3$

∴ Volume of air space

= Volume of iron ball with air space – Volume of iron alone

$= 0.4 \times 10^{-3} - 0.138 \times 10^{-3}$

$= \mathbf{0.262 \times 10^{-3}\ m^3.}$

10.18 VISCOSITY

29. *What is viscosity ? Explain the cause of viscosity.*

Viscosity. *Viscosity is the property of fluid by virtue of which an internal force of friction comes into play when a fluid is in motion and which opposes the relative motion between its different layers.* The backward dragging force, called ***viscous drag*** or ***viscous force,*** acts tangentially on the layers of the fluid in motion and tends to destroy its motion.

Cause of viscosity. Consider a liquid moving slowly and steadily over a fixed horizontal surface. Each layer moves parallel to the fixed surface. The layer in contact with the fixed surface is at rest and the velocity of the every other layer increases uniformly upwards, as shown by arrows of increasing lengths in Fig. 10.21.

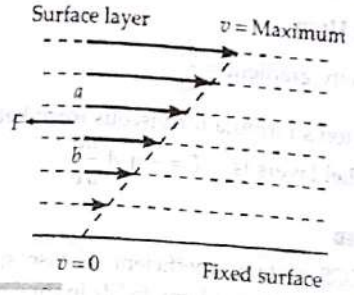


Fig. 10.21 Adjacent layers of a liquid in motion.

Consider any two adjacent layers *a* and *b*. The upper fast moving layer *a* tends to accelerate the lower slow moving layer *b* while the slow moving layer *b* tends to retard the fast moving layer *a*. As a result, a backward dragging tangential force *F*, called viscous drag comes into play which tends to destroy the relative motion. To maintain the motion, an external force has to be applied to overcome the backward viscous force.

30. *Give some examples in which the effect of viscosity can be easily seen.*

Examples of viscosity :

(*i*) When we stir a liquid contained in a beaker with a glass rod, it starts rotating in coaxial cylindrical layers as shown in Fig. 10.22. When we stop stirring, the speed of different layers gradually decreases and finally the water comes to rest, showing that an internal friction comes into play which destroys the relative motion between different layers.

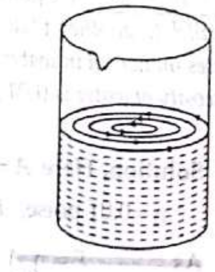


Fig. 10.22 Cylindrical layers in a stirred liquid.

(*ii*) When we swim in a pool of water, we experience some resistance to our motion. This is on account of viscous forces of water.

(*iii*) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey trickles down drop by drop very slowly. This is because honey is much more viscous than water. The relative motion between the layers of honey is strongly opposed.

(*iv*) The cloud particles fall down very slowly on account of the viscosity of air and hence seen floating in the sky.

(*v*) We can walk fast in air, but not in water. This is because viscosity of air is much smaller than that of water.

10.19 COEFFICIENT OF VISCOSITY

31. *What is meant by coefficient of viscosity ? Give its dimensions and units.*

Coefficient of viscosity. As shown in Fig. 10.23, suppose a liquid is flowing steadily in the form of parallel layers on a fixed horizontal surface. Consider two layers P and Q at distances x and $x + dx$ from the solid surface and moving with velocities v and $v + dv$ respectively. Then $\frac{dv}{dx}$ is the *rate of change of velocity with distance in the direction of increasing distance* and is called **velocity gradient**.

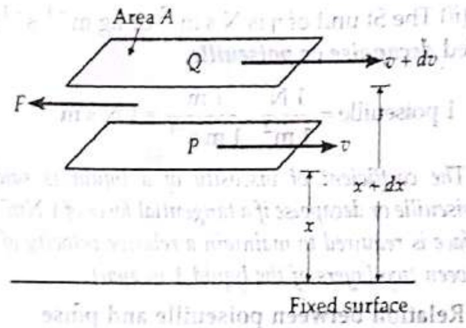


Fig. 10.23 Coefficient of viscosity.

According to Newton, a force of viscosity F acting tangentially between two layers is

(*i*) proportional to the area A of the layers in contact.

$$F \propto A$$

(*ii*) proportional to the velocity gradient $\frac{dv}{dx}$ between the two layers

$$F \propto \frac{dv}{dx}$$

$$\therefore \quad F \propto A \frac{dv}{dx}$$

$$\text{or} \quad F = -\eta A \frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid. It depends on the nature of the liquid and gives a measure of viscosity. Negative sign shows that the viscous force acts in a direction opposite to the direction of motion of the liquid.

$$\text{If} \quad A = 1 \quad \text{and} \quad \frac{dv}{dx} = 1$$

then $\quad F = \eta \quad$ (numerically)

Hence *coefficient of viscosity of a liquid may be defined as the tangential viscous force required to maintain a unit velocity gradient between its two parallel layers each of unit area.*

Scanned by CamScanner

Dimensions of η. Clearly,

$$\eta = \frac{F}{A} \cdot \frac{dx}{dv} \quad \therefore \quad [\eta] = \frac{MLT^{-2} \cdot L}{L^2 \cdot LT^{-1}} = [ML^{-1}T^{-1}].$$

Units of coefficient of viscosity. (*i*) The CGS unit of η is dyne s cm^{-2} or g $cm^{-1}s^{-1}$ and is called *poise*.

$$1 \text{ poise} = \frac{1 \text{ dyne}}{1 \text{ cm}^2} \cdot \frac{1 \text{ cm}}{1 \text{ cm s}^{-1}} = 1 \text{ dyne s cm}^{-2}$$

The coefficient of viscosity a liquid is said to be 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cm s^{-1} between two layers of the liquid 1 cm apart.

(*ii*) The SI unit of η is N s m^{-2} or kg m^{-1} s^{-1} and is called ***decapoise*** or ***poiseuille***.

$$1 \text{ poiseuille} = \frac{1 \text{ N}}{1 \text{ m}^2} \cdot \frac{1 \text{ m}}{1 \text{ m s}^{-1}} = 1 \text{ N s m}^{-2}$$

The coefficient of viscosity of a liquid is said to be 1 poiseuille or decapoise if a tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

Relation between poiseuille and poise.

1 poiseuille or 1 decapoise $= 1 \text{ Ns m}^{-2}$

$= (10^5 \text{ dyne}) \times s \times (10^2 \text{ cm})^{-2}$

$= 10 \text{ dyne s cm}^{-2} = 10$ poise.

For Your Knowledge

- Viscosity is like friction and converts kinetic energy into heat energy.
- No fluid has zero viscosity.
- Thin liquids like water, alcohol etc. ; are less viscous than thick liquids coal tar, blood, honey, glycerine etc.
- Unlike solids, the strain in a flowing liquid increases with time continuously. So for solids with elastic modulus of rigidity, the shearing stress is proportional to shear strain, while for fluids it is proportional to the *time rate of change of strain* or *strain rate*. The coefficient of viscosity of a fluid can be defined as the ratio of shearing stress to the strain rate. More over,

Coefficient of viscosity,

$$\eta = \frac{F/A}{v/x} = \frac{F/A}{\dfrac{\Delta x/x}{t}}$$

$$= \frac{\text{Shearing stress}}{\text{Shear strain}/t} = \frac{\text{Shearing stress}}{\text{Strain rate}}$$

Modulus of rigidity, $\eta = \dfrac{F/A}{\Delta x/x} = \dfrac{\text{Shearing stress}}{\text{Shear strain}}$

Thus *the coefficient of viscosity of liquids is analogous to the modulus of rigidity of solids*.

Examples based on Coefficient of Viscosity

FORMULAE USED

1. Velocity gradient $= \dfrac{dv}{dx}$
2. Newton's formula for viscous force between two parallel layers is $F = -\eta A \dfrac{dv}{dx}$

UNITS USED

In CGS system, coefficient of viscosity η is in poise, velocity gradient dv/dx in cm s^{-1}/cm, area *A* in cm^2 and force *F* in dyne. In SI, η is in decapoise or Pascal second (Pa s), *A* in m^2 and *F* in newton.

1 decapoise = 1 Pa s = 10 poise.

EXAMPLE 28. *A metal plate 5 cm × 5 cm rests on a layer of castor oil 1 mm thick whose coefficient of viscosity is 1.55 Nsm^{-2}. Find the horizontal force required to move the plate with a speed of 2 cms^{-1}.*

Solution. Here $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$,

$dx = 1 \text{ mm} = 10^{-3} \text{ m}, \eta = 1.55 \text{ Nsm}^{-2}, dv = 2 \times 10^{-2} \text{ ms}^{-1}$

$$F = \eta A \frac{dv}{dx} = 1.55 \times 25 \times 10^{-4} \times \frac{2 \times 10^{-2}}{1 \times 10^{-3}} = \mathbf{0.0775 \ N.}$$

EXAMPLE 29. *A square metal plate of 10 cm side moves parallel to another plate with a velocity of 10 cms^{-1}, both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, what is their distance apart?* **[NCERT]**

Solution. Here $A = 10 \times 10 = 100 \text{ cm}^2$,

$\eta = 0.01$ poise, $F = 200$ dyne, $dv = 10 \text{ cms}^{-1}$

As $\quad F = \eta A \dfrac{dv}{dx}$

$$\therefore \quad dx = \frac{\eta A \, dv}{F} = \frac{0.01 \times 100 \times 10}{200} = \mathbf{0.05 \ cm.}$$

EXAMPLE 30. *A flat square plate of side 20 cm moves over another similar plate with a thin layer of 0.4 cm of a liquid between them. If a force of one kg wt moves one of the plates uniformly with a velocity of 1 ms^{-1}, calculate the coefficient of viscosity of the liquid.*

Solution. Here

$A = 20 \times 20 = 400 \text{ cm}^2 = 400 \times 10^{-4} \text{ m}^2$,

$dx = 0.4 \text{ cm} = 0.4 \times 10^{-2} \text{ m}$

$F = 1 \text{ kg wt} = 9.8 \text{ N}, \ dv = 1 \text{ ms}^{-1}$

As $F = \eta A \dfrac{dv}{dx}$

$$\therefore \quad \eta = \frac{F}{A} \cdot \frac{dx}{dv} = \frac{9.8 \times 0.4 \times 10^{-2}}{400 \times 10^{-4} \times 1} = \mathbf{0.98 \ Pa \ s.}$$

EXAMPLE 31. The velocity of water in a river is 180 ... near the surface. If the river is 5 m deep, find the ... stress between horizontal layers of water. Coeff... viscosity of water $= 10^{-2}$ poise.

Solution. As the velocity of water at the ... the river is zero,

$dv = 18 \text{ kmh}^{-1} = 18 \times \frac{5}{18} = 5$ m...

Also $\quad dx = 5$ m, $\eta = 10^{-2}$ poise = 10...

Force of viscosity,

$F = \eta A \frac{dv}{dx}$

∴ Shearing stress

$= \frac{F}{A} = \eta \frac{dv}{dx} = \frac{10^{-3} \times 5}{5}$...

EXAMPLE 32. A metal plate of area 0.1... 0.01 kg mass via a string that passes... (considered massless and frictionless),... A liquid with a film thickness of 0.3... the plate and the table. When release... right with a constant speed of 0.08... cient of viscosity of the liquid. [N...

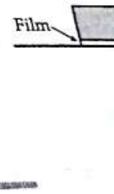


Fig. 10.24

Solution. Here A = ...

dx = 0.3 mm = 0.3 × ...

The metal plate m... tension T in the strin... the suspended mas... moves with uniform... the force of viscosit...

F = ...

Taking velocit...

η ...

PROBLE...

1. The rel... water... betw... grad...

Scanned by CamScanner

EXAMPLE 31. *The velocity of water in a river is 180 kmh^{-1} near the surface. If the river is 5 m deep, find the shearing stress between horizontal layers of water. Coefficient of viscosity of water = 10^{-2} poise.*

Solution. As the velocity of water at the bottom of the river is zero,

$$dv = 18 \text{ kmh}^{-1} = 18 \times \frac{5}{18} = 5 \text{ ms}^{-1}$$

Also $\quad dx = 5 \text{ m}, \eta = 10^{-2}$ poise $= 10^{-3}$ Pa s

Force of viscosity,

$$F = \eta A \frac{dv}{dx}$$

∴ Shearing stress

$$= \frac{F}{A} = \eta \frac{dv}{dx} = \frac{10^{-3} \times 5}{5} = \mathbf{10^{-3} \ Nm^{-2}}.$$

EXAMPLE 32. *A metal plate of area 0.10 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as shown in Fig. 10.24. A liquid with a film thickness of 0.3 mm is placed between the plate and the table. When released the plate moves to the right with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid.* **[NCERT ; Central Schools 14]**

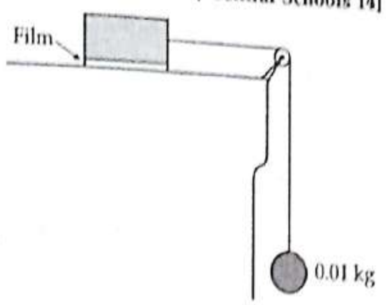


Fig. 10.24

Solution. Here $A = 0.10 \text{ m}^2$, $m = 0.01$ kg,

$dx = 0.3 \text{ mm} = 0.3 \times 10^{-3}$ m, $\quad dv = 0.085 \text{ ms}^{-1}$.

The metal plate moves towards right due to the tension T in the string which is equal to the weight of the suspended mass m. Assuming that the mass m moves with uniform velocity or zero acceleration, then the force of viscosity will be

$$F = T = mg = 0.01 \times 9.8 = 9.8 \times 10^{-2} \text{ N}.$$

Taking velocity gradient to be uniform, then

$$\eta = \frac{F}{A} \cdot \frac{dx}{dv} = \frac{9.8 \times 10^{-2} \times 0.3 \times 10^{-3}}{0.10 \times 0.085}$$

$$= \mathbf{3.45 \times 10^{-3} \ Pa \ s}.$$

✕ PROBLEMS FOR PRACTICE

1. The relative velocity between two parallel layers of water is 8 cm s^{-1} and the perpendicular distance between them is 0.1 cm. Calculate the velocity gradient. **(Ans.** 80 cm s^{-1}/cm)

2. A circular metal plate of radius 5 cm, rests on a layer of caster oil 2 mm thick, whose coefficient of viscosity is 15.5 poise. Calculate the horizontal force required to move the plate with a speed of 5 cm s^{-1}. **(Ans.** 3.04×10^4 dyne)

3. A metal plate of area 5 cm^2 is placed on a 0.5 mm thick castor oil layer. If a force of 22,500 dyne is needed to move the plate with a velocity of 3 cms^{-1}, calculate the coefficient of viscosity of castor oil. **(Ans.** 750 poise)

4. A metal plate of area 0.02 m^2 is lying on a liquid layer of thickness 10^{-3} m and coefficient of viscosity 120 poise. Calculate the horizontal force required to move the plate with a speed of 0.025 ms^{-1}. **(Ans.** 6 N)

✕ HINTS

2. Here $A = \pi r^2 = \frac{22}{7} \times 25 = \frac{550}{7} \text{ cm}^2$,

$dx = 2 \text{ mm} = 0.2 \text{ cm}, dv = 5 \text{ cm s}^{-1}, \ \eta = 15.5$ poise

$$F = \eta A \frac{dv}{dx} = 15.5 \times \frac{550}{7} \times \frac{5}{0.2} = \mathbf{3.04 \times 10^4 \ dyne}.$$

4. Here $A = 0.02 \text{ m}^2, dx = 10^{-3} \text{ m}, dv = 0.025 \text{ ms}^{-1}$,

$\eta = 120$ poise $= 12$ Pa s

$$\therefore \quad F = \eta A \frac{dv}{dx} = \frac{12 \times 0.02 \times 0.025}{10^{-3}} = \mathbf{6 \ N}.$$

10.20 COMPARISON BETWEEN VISCOUS FORCE AND SOLID FRICTION

32. *Give some points of similarity and differences between viscous force and solid friction.*

Points of similarity :

(*i*) Both viscous force and solid friction come into play whenever there is relative motion.

(*ii*) Both oppose the motion.

(*iii*) Both are due to molecular attractions.

Points of differences :

Viscous force	*Solid friction*
1. Viscous force is directly proportional to the area of layers in contact.	Solid friction is independent of the area of the surfaces in contact.
2. It is directly proportional to the relative velocity between the two liquid layers.	It is independent of the relative velocity between two solid surfaces.
3. It is independent of the normal reaction between the two liquid layers.	It is directly proportional to the normal reaction between the surfaces in contact.

Scanned by CamScanner

10.21 VARIATION OF VISCOSITY WITH TEMPERATURE AND PRESSURE

33. *Discuss the variation of fluid viscosity with temperature and pressure.*

Effect of temperature on viscosity. (*i*) When a liquid is heated, the kinetic energy of its molecules increases and the intermolecular attractions become weaker. Hence *the viscosity of a liquid decreases with the increase in its temperature.*

Slotle's empirical formula for the variation of viscosity of a liquid with temperature is

$$\eta_t = \frac{\eta_0}{1+\alpha t+\beta t^2}$$

where η_t and η_0 are the coefficients of viscosity at t°C and 0°C respectively, and α and β are constants.

(*ii*) Viscosity of gases is due to the diffusion of molecules from one moving layer to another. But the rate of diffusion of a gas is directly proportional to the square root of its absolute temperature, so viscosity of a gas increases with temperature as

$$\eta \propto \sqrt{T}$$

Effect of pressure. (*i*) Except water the viscosity of liquids increases with the increase in pressure. In case of water, viscosity decreases with the increase in pressure for first few hundred atmospheres of pressure.

(*ii*) The viscosity of gases is independent of pressure.

10.22 PRACTICAL APPLICATIONS OF THE KNOWLEDGE OF VISCOSITY

34. *Mention some important practical applications of the knowledge of viscosity in daily life.*

Practical applications of the knowledge of viscosity :

(*i*) The knowledge of viscosity and its variation with temperature helps us to select a suitable lubricant for a given machine in different seasons.

(*ii*) Liquids of high viscosity are used as buffers for absorbing shocks during the shunting of trains.

(*iii*) The knowledge of viscosity is used in determining the shape and molecular weight of some organic liquids like proteins, cellulose, etc.

(*iv*) The phenomenon of viscosity plays an important role in the circulation of blood through arteries and veins of human body.

(*v*) Millikan used the knowledge of viscosity in determining the charge on an electron.

10.23 POISEUILLE'S FORMULA

35. *State Poiseuille's formula. What are the assumptions used in the derivation of this formula ? Derive this formula on the basis of dimensional considerations.*

Poiseuille's formula. The volume of a liquid flowing out per second through a horizontal capillary tube of length l, radius r, under a pressure difference p applied across its ends is given by :

$$Q=\frac{V}{t}=\frac{\pi p r^4}{8\eta l}$$

This formula is called *Poiseulle's formula.*

Assumptions used in the derivation of Poiseuille's formula :

(*i*) The flow of the liquid is steady and parallel to the axis of the tube.

(*ii*) The pressure is constant over any cross-section of the tube.

(*iii*) The liquid velocity is zero at the walls of the tube and increases towards the axis of the tube.

(*iv*) The tube is held horizontal so that gravity does not influence the flow of liquid.

Derivation of Poiseuille's formula on the basis of dimensional analysis. The volume Q of liquid flowing out per second through a capillary tube depends on

(*i*) coefficient of viscosity η of the liquid,

(*ii*) radius r of the tube,

(*iii*) pressure gradient (p/l) set up along the capillary tube.

Let $Q \propto \eta^a r^b \left(\frac{p}{l}\right)^c$ or $Q = k\,\eta^a r^b \left(\frac{p}{l}\right)^c$ ...(1)

where k is a dimensionless constant. The dimensions of various quantities are

$$[Q]=\frac{\text{Volume}}{\text{Time}}=\frac{[L^3]}{[T]}=[L^3T^{-1}],$$

$$\left[\frac{p}{l}\right]=\frac{[ML^{-1}T^{-2}]}{[L]}=[ML^{-2}T^{-2}]$$

$$[\eta]=[ML^{-1}T^{-1}],\ [r]=L$$

Substituting these dimensions in equation (1), we get

$$[L^3T^{-1}]=[ML^{-1}T^{-1}]^a[L]^b[ML^{-2}T^{-2}]^c$$

or $[M^0L^3T^{-1}]=[M^{a+c}L^{-a+b-2c}T^{-a-2c}]$

Equating the powers of M, L and T on both sides, we get

$$a+c=0$$
$$-a+b-2c=3$$
$$-a-2c=-1$$

On solving, we get $a=-1$, $b=4$, and $c=1$

$$\therefore \quad Q=k\,\eta^{-1}r^4\left[\frac{p}{l}\right]^1=\frac{kpr^4}{\eta l}$$

Experimentally k is found to be $\pi/8$.

Scanned by CamScanner

$$\therefore \quad Q = \frac{\pi p r^4}{8 \eta l}$$

This is Poiseuille's formula for the flow of a liquid through a capillary tube.

Examples based on Poiseuille's Formula

FORMULA USED

Poiseuille's formula for the volume of a liquid flowing out per second through a narrow pipe is

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8 \eta l}$$

UNITS USED

In CGS system, coefficient of viscosity η is in poise, length in cm, radius r in cm and pressure p in dyne cm^{-2}. In SI, η is in decapoise or pascal second (Pa s), length l in m, r in metre and p in Nm^{-2}.

1 decapoise = 1 Pa s = 10 poise

EXAMPLE 33. *Check the dimensional consistency of the Poiseuille's formula for the laminar flow in a tube :*

$$Q = \frac{\pi R^4 (p_1 - p_2)}{8 \eta l}$$

Solution. LHS = Q = Volume of liquid flowing out per second $= \frac{\text{Volume}}{\text{Time}}$

$\therefore$ Dimensions of LHS = L^3T^{-1}

Dimensions of RHS

$$= \left[\frac{\pi R^4 (p_1 - p_2)}{8 \eta l}\right] = \frac{L^4 \times ML^{-1}T^{-2}}{ML^{-1}T^{-1} \times L} = L^3T^{-1}$$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence the Poiseuille's formula is dimensionally consistent.

EXAMPLE 34. *A capillary tube 1 mm in diameter and 20 cm in length is fitted horizontally to a vessel kept full of alcohol. The depth of the centre of capillary tube below the surface of alcohol is 20 cm. If the viscosity and density of alcohol are 0.012 cgs unit and 0.8 g cm^{-3} respectively, find the amount of the alcohol that will flow out in 5 minutes. Given that g = 980 cms^{-2}.*

Solution. Here $r = \frac{1}{2}$ mm = 0.05 cm, l = 20 cm,

ρ = 0.8 gcm^{-3}, t = 5 min = 300 s,

η = 0.012 cgs unit, pressure head = 20 cm of alcohol

$\therefore \quad p = h \rho g = 20 \times 0.8 \times 980$ dyne cm^{-2}

Volume of alcohol flowing out per second,

$$Q = \frac{\pi p r^4}{8 \eta l} = \frac{3.142 \times 20 \times 0.8 \times 980 \times (0.05)^4}{8 \times 0.012 \times 20} = 0.16 \text{ cm}^3$$

Mass of alcohol that flows out in 5 minutes

$= V \times \rho \times t = 0.16 \times 0.8 \times 300 =$ **38.4 g.**

EXAMPLE 35. *In giving a patient a blood transfusion, the bottle is set up so that the level of blood is 1.3 m above needle, which has an internal diameter of 0.36 mm and is 3 cm in length. If 4.5 cm^3 of blood passes through the needle in one minute, calculate the viscosity of blood. The density of blood is 1020 kgm^{-3}.*

Solution. Length of needle, l = 3 cm

Radius of needle, $r = \frac{0.36}{2}$ mm = 0.018 cm

Volume of blood flowing out per second,

$$Q = \frac{\text{Total Volume}}{\text{Time}} = \frac{4.5}{60} = 0.075 \text{ cm}^3 \text{ s}^{-1}$$

Density of blood,

$\rho = 1020 \text{ kg m}^{-3} = 1020 \times 10^{-3} \text{ g cm}^{-3} = 1.02 \text{ g cm}^{-3}$

Pressure difference,

p = 1.3 m column of blood

$= 1.3 \times 100 \times 1.02 \times 980$ dyne cm^{-2}

$$\eta = \frac{\pi p r^4}{8 Q l} = \frac{3.142 \times 1.3 \times 100 \times 1.02 \times 980 \times (0.018)^4}{8 \times 0.075 \times 3}$$

= **0.238 poise**

EXAMPLE 36. *A liquid flows through a pipe of 1.0 mm radius and 10 cm length under a pressure 10^4 dyne cm^{-2}. Calculate the rate of flow and the speed of the liquid coming out of the tube. The coefficient of viscosity of the liquid is 1.25 centipoise.*

Solution. Here r = 1.0 mm = 0.1 cm, l = 10 cm,

$p = 10^4$ dyne cm^{-2}, η = 1.25 centipoise = 0.0125 poise

Rate of flow,

$$Q = \frac{\pi p r^4}{8 \eta l} = \frac{3.142 \times 10^4 \times (0.1)^4}{8 \times 0.0125 \times 10}$$

= **3.142 cm^3s^{-1}.**

Speed of liquid,

$$v = \frac{Q}{\text{Cross - sectional area}} = \frac{3.142}{\pi r^2} = \frac{1}{r^2} = \frac{1}{(0.1)^2}$$

= 100 cms^{-1} = **1 ms^{-1}.**

EXAMPLE 37. *Two tubes A and B of lengths 100 cm and 50 cm have radii 0.1 mm and 0.2 mm respectively. If a liquid passing through the two tubes is entering A at a pressure of 80 cm of mercury and leaving B at a pressure of 76 cm of mercury, determine the pressure at the junction of A and B.*

Solution. Length of tube A, l_A = 100 cm

Radius of tube A, r_A = 0.1 mm = 0.01 cm

Pressure at which liquid enters tube A,

p_A = 80 cm of Hg

Let pressure at the junction of A and B = p_1

Scanned by CamScanner

Rate of flow through tube A,

$$Q_A = \frac{\pi(p_A - p_j)\, r_A^4}{8\eta\, l_A} = \frac{\pi(80 - p_j)(0.01)^4}{8\eta \times 100}\ \text{cm}^3\ \text{s}^{-1}$$

Length of tube B, $l_B = 50$ cm

Radius of tube B, $r_B = 0.2$ mm $= 0.02$ cm

Pressure at which liquid leaves tube B,

$$p_B = 76 \text{ cm of Hg}$$

Rate of flow through tube B,

$$Q_B = \frac{\pi(p_j - p_B)\, r_B^4}{8\,\eta\, l_B} = \frac{\pi(p_j - 76)(0.02)^4}{8\eta \times 50}\ \text{cm}^3\ \text{s}^{-1}$$

But $Q_A = Q_B$

$$\therefore \quad \frac{\pi(80 - p_j)(0.01)^4}{8\eta \times 100} = \frac{\pi(p_j - 76)(0.02)^4}{8\eta \times 50}$$

or $$(80 - p_j)(0.01)^4 = 2\,(p_j - 76)(0.02)^4$$

or $$80 - p_j = 32\,(p_j - 76)$$

or $$p_j = \mathbf{76.12 \text{ cm of Hg.}}$$

EXAMPLE 38. *Two capillary tubes AB and BC are joined end to end at B. AB is 16 cm long and of diameter 4 mm whereas BC is 4 cm long and of diameter 2 mm. The composite tube is held horizontally with A connected to a vessel of water giving a constant head of 3 cm and C is open to the air. Calculate the pressure difference between B and C.*

Solution. For tube AB :

$$l_1 = 16 \text{ cm},\ r_1 = 2 \text{ mm} = 0.2 \text{ cm}$$

For tube BC :

$$l_2 = 4 \text{ cm},\ r_2 = 1 \text{ mm} = 0.1 \text{ cm}$$

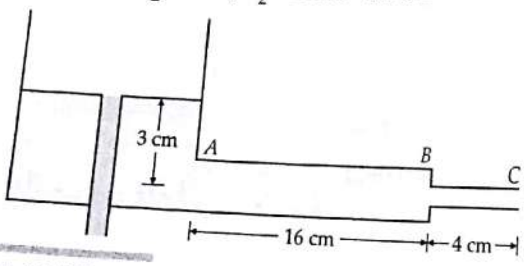


Fig. 10.25

Let h be the head of pressure at B over C, then it will be $(3 - h)$ at A over B.

Rate of flow through AB,

$$Q_1 = \frac{\pi p_1\, r_1^4}{8\eta\, l_1} = \frac{\pi(3 - h)(0.2)^2}{8\eta \times 16}\ \text{cm}^3\ \text{s}^{-1}$$

Rate of flow through BC,

$$Q_2 = \frac{\pi p_2\, r_2^4}{8\eta l_2} = \frac{\pi h\,(0.1)^2}{8\eta \times 4}\ \text{cm}^3\ \text{s}^{-1}$$

But $Q_1 = Q_2$

or $$\frac{\pi(3 - h)(0.2)^4}{8\eta \times 16} = \frac{\pi\, h\,(0.1)^4}{8\eta \times 4}$$

or $$12 - 4h = h \quad \text{or} \quad 5h = 12$$

$$h = \mathbf{2.4 \text{ cm.}}$$

EXAMPLE 39. *A large bottle is fitted with a siphon made of capillary glass tubing. Compare the times taken to empty the bottle when it is filled (i) with water (ii) with petrol of density 0.8 cgs units. The viscosity of water and petrol are 0.01 and 0.02 cgs units respectively.*

Solution. The volume of liquid flowing in time t through a capillary tube is given by

$$V = Qt = \frac{\pi P r^4\, t}{8\eta l} = \frac{\pi h\rho\, g\, r^4\, t}{8\eta l}$$

$$\therefore \text{For water,} \quad V_1 = \frac{\pi h\rho_1\, g\, r^4\, t_1}{8l\eta_1}$$

$$\text{For petrol,} \quad V_2 = \frac{\pi h\rho_2\, g\, r^4\, t_2}{8l\eta_2}$$

But $V_1 = V_2$

$$\therefore \quad \frac{\pi h\rho_1 g\, r^4 t_1}{8l\,\eta_1} = \frac{\pi h\,\rho_2\, g\, r^4\, t_2}{8l\,\eta_2}$$

or $$\frac{t_1}{t_2} = \frac{\eta_1}{\eta_2} \times \frac{\rho_2}{\rho_1} = \frac{0.01}{0.02} \times \frac{0.8}{1.0} = \mathbf{0.4}$$

EXAMPLE 40. *The level of liquid in a cylindrical vessel is kept constant at 30 cm. It has three identical horizontal tubes of length 39 cm, each coming out at heights 0, 4 and 8 cm respectively. Calculate the length of a single overflow tube of the same radius as that of identical tubes which can replace the three when placed horizontally at bottom of the cylinder.*

Solution. Pressure head for tube A,

$$h_1 = 30 \text{ cm of water column}$$

Pressure head for tube B,

$$h_2 = 30 - 4 = 26 \text{ cm of water column}$$

Pressure head for tube C,

$$h_3 = 30 - 8 = 22 \text{ cm of water column}$$

Let radius of each tube $= r$ cm

Length of each tube, $l = 39$ cm

Rate of flow of liquid through different tubes will be

$$Q_1 = \frac{\pi p_1\, r_1^4}{8\eta l} = \frac{\pi \times h_1 \rho g \times r^4}{8\eta l} = \frac{\pi \times 30 \times \rho g \times r^4}{8\eta \times 39}$$

Similarly,

$$Q_2 = \frac{\pi \times 26 \times \rho g \times r^4}{8\eta \times 39}$$

and $$Q_3 = \frac{\pi \times 22 \times \rho g \times r^4}{8\eta \times 39}$$

Scanned by CamScanner

Total volume of liquid flowing through the three tubes per second is

$$Q = Q_1 + Q_2 + Q_3$$

$$= \frac{\pi \rho g r^4}{8\eta \times 39}(30 + 26 + 22) = \frac{\pi \rho g r^4}{4\eta} \quad ...(i)$$

Let l' be the length of the single tube that flows out the same volume per second. Then

$$Q = \frac{\pi h \rho g r^4}{8\eta l'} = \frac{\pi \times 30 \times \rho g r^4}{8\eta l'} \quad ...(ii)$$

From (*i*) and (*ii*), we have

$$\frac{\pi \rho g r^4}{4\eta} = \frac{\pi \times 30 \times \rho g r^4}{8\eta l'} \quad \text{or} \quad 2l' = 30 \text{ cm}$$

Hence $l' = \mathbf{15\ cm.}$

EXAMPLE 41. *Three capillary tubes of the same radius r but of lengths l_1, l_2 and l_3 are fitted horizontally to the bottom of a tall vessel containing a liquid at constant head and flowing through these tubes. Calculate the length of a single outflow tube of the same radius r which can replace the three capillaries.*

Solution. Let p be the liquid pressure at the bottom of the vessel. Then the rates of flow through the three tubes will be

$$Q_1 = \frac{\pi p r^4}{8\eta l_1}, \quad Q_2 = \frac{\pi p r^4}{8\eta l_2}, \quad Q_3 = \frac{\pi p r^4}{8\eta l_3}.$$

Let l be the length of the single tube of radius r which can replace the three tubes. The rate of flow through it will be

$$Q = \frac{\pi p r^4}{8\eta l}.$$

But $$Q = Q_1 + Q_2 + Q_3$$

$$\therefore \quad \frac{\pi p r^4}{8\eta l} = \frac{\pi p r^4}{8\eta l_1} + \frac{\pi p r^4}{8\eta l_2} + \frac{\pi p r^4}{8\eta l_3}$$

or $$\frac{1}{l} = \frac{1}{l_1} + \frac{1}{l_2} + \frac{1}{l_3} = \frac{l_2 l_3 + l_1 l_3 + l_1 l_2}{l_1 l_2 l_3}$$

or $$l = \frac{l_1 l_2 l_3}{l_2 l_3 + l_1 l_3 + l_1 l_2}.$$

PROBLEMS FOR PRACTICE

1. In an experiment with Poiseuille's apparatus, the following observations were noted :

   Volume of liquid collected per minute = 15 cm^3

   Head of liquid = 30 cm ; Length of tube = 25 cm

   Diameter of tube = 0.2 cm ;

   Density of liquid = 2.3 gcm^{-3}

   Find the coefficient of viscosity of the liquid. **(Ans. 0.425 poise)**

2. Water at 20° is escaping from a cistern by way of a horizontal capillary tube 10 cm long and 0.4 mm in diameter, at a distance of 50 cm below the free surface of water in the cistern. Calculate the rate at which the water is escaping. Coefficient of viscosity of water is 0.001 decapoise. **(Ans. $3.08 \times 10^{-8} m^3 s^{-1}$)**

3. Water is conveyed through a horizontal tube 8 cm in diameter and 4 kilometer in length at the rate of 20 litres/s. Assuming only viscous resistance, calculate the pressure required to maintain the flow. Coefficient of viscosity of water is 0.001 Pa s. **(Ans. 7.96×10^4 Nm^{-2})**

4. Alcohol flows through two capillary tubes under a constant pressure head. The diameters of the two tubes are in the ratio of 4 : 1 and the lengths are in the ratio 4 : 1. Compare the rates of flow of alcohols through the two tubes. **(Ans. 1024 : 1)**

5. Show that if two capillaries of radii r_1 and r_2 having lengths l_1 and l_2 respectively are set in series, the rate of flow Q is given by

$$Q = \frac{\pi p}{8\eta}\left[\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right]^{-1}$$

   where p is the pressure difference across the arrangement and η is the coefficient of viscosity of the liquid.

6. Three capillaries of lengths l, $2l$ and $l/2$ are connected in series. Their radii are r, $r/2$ and $r/3$ respectively. If stream-line flow is maintained and the pressure difference across the first capillary tube is p_1, find the pressure difference across (*i*) the second and (*ii*) the third capillary tube. **(Ans. 32 p_1, 40.5 p_1)**

HINTS

2. $$Q = \frac{\pi p r^4}{8\eta \rho} = \frac{\pi h \rho g r^4}{8\eta l}$$

$$= \frac{3.14 \times 0.50 \times 10^3 \times 9.8 \times (0.2 \times 10^{-3})^4}{8 \times 0.001 \times 0.1}$$

$$= \mathbf{3.08 \times 10^{-8}\ m^3\ s^{-1}}.$$

3. $$p = \frac{8\eta Q l}{\pi r^4} = \frac{8 \times 0.001 \times 20 \times 10^{-3} \times 4 \times 10^3}{3.142 \times (4 \times 10^{-2})^4}$$

$$= \mathbf{7.96 \times 10^4\ Nm^{-2}}.$$

4. $$\frac{r_1}{r_2} = \frac{d_1}{d_2} = \frac{4}{1} \quad \text{and} \quad \frac{l_1}{l_2} = \frac{1}{4}$$

   Now $Q = \frac{\pi p r^4}{8\eta l}$. For constant η and p, $Q \propto \frac{r^4}{l}$

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{r_1^4}{l_1} \times \frac{l_2}{r_2^4} = \left[\frac{r_1}{r_2}\right]^4 \times \frac{l_2}{l_1}$$

$$= \left[\frac{4}{1}\right]^4 \times \frac{4}{1} = \mathbf{1024 : 1.}$$

Scanned by CamScanner

5. As $Q = \frac{\pi p r^4}{8\eta l}$ or $p = \frac{8\eta Q l}{\pi r^4}$ or $p = k\frac{l}{r^4}$

$p = p_1 + p_2$ or $\frac{kl}{r^4} = \frac{kl_1}{r_1^4} + \frac{kl_2}{r_2^4}$

or $Q = \frac{\pi p}{8\eta}\left[\frac{l_1}{r_1^4} + \frac{l_2}{r_1^4}\right]^{-1}$

6. As the three tubes are connected in series, so the rate of flow of liquid through all the three tubes is same, say V. Let p_1, p_2 and p_3 be the pressure differences across the three tubes. Then

$$V = \frac{\pi p_1 r_1^4}{8\eta l_1} = \frac{\pi p_2 r_2^4}{8\eta l_2} = \frac{\pi p_3 r_3^4}{8\eta l_3}$$

But $l_1 = l,\ l_2 = 2l,\ l_3 = l/2,\ r_1 = r,\ r_2 = r/2,\ r_3 = r/3$

$$\therefore \quad \frac{p_1 r^4}{l} = \frac{p_2\left(\frac{r}{2}\right)^4}{2l} = \frac{p_3\left(\frac{r}{3}\right)^4}{l/2}$$

or $p_1 = \frac{p_2}{32} = \frac{2p_3}{81}$

Hence $p_2 = 32\,p_1$ and $p_3 = \frac{81}{2}p_1 = 40.5\,p_1$.

10.24 STOKES' LAW

36. *Explain the origin of viscous drag on a body falling through a fluid.*

Viscous drag on a body falling through a fluid. When a body falls through a viscous fluid, the layer of the fluid in contact with the body moves with its velocity. However, the fluid at large distance from it remains at rest. This produces relative motion between different layers of the fluid. As a result, the body experiences a viscous force which tends to retard its motion. This retarding force increases with the increase in velocity of the body.

37. *State Stokes' law. Deduce Stokes' law on the basis of dimensional considerations. State the conditions under which Stokes' law is valid.*

Stokes' law. *According to Stokes' law, the backward viscous force acting on a small spherical body of radius r moving with uniform velocity v through fluid of viscosity η is given by*

$$F = 6\,\pi\,\eta\,rv$$

Derivation of Stokes' law. The viscous force F acting on a sphere moving through a fluid may depend on

(*i*) coefficient of viscosity η of the fluid

(*ii*) radius r of the spherical body

(*iii*) velocity v of the body

Let $F = k\eta^a r^b v^c$

where k is dimensionless constant. The dimensions of various quantities are

$$[F] = [\text{MLT}^{-2}],\quad \eta = [\text{ML}^{-1}\text{T}^{-1}]$$
$$[r] = [\text{L}],\quad [v] = [\text{LT}^{-1}]$$

Substituting these dimensions in equation (1), we get

$$[\text{MLT}^{-2}] = [\text{ML}^{-1}\text{T}^{-1}]^a\,[\text{L}]^b\,[\text{LT}^{-1}]^c$$
$$= [\text{M}^a\text{L}^{-a+b+c}\text{T}^{-a-c}]$$

Equating the powers M, L and T on both sides, we get

$$a = 1$$
$$-a + b + c = 1$$
$$-a - c = -2$$

On solving, $a = b = c = 1$

$\therefore \quad F = k\eta\,r\,v$

For a small sphere, k is found to be equal to 6π.

Hence $F = 6\,\pi\,\eta\,rv$

This proves Stokes' law.

Conditions under which Stokes' law is valid :

(*i*) The fluid through which the body moves has infinite extension.

(*ii*) The body is perfectly rigid and smooth.

(*iii*) There is no slip between the body and fluid.

(*iv*) The motion of the body does not give rise to turbulent motion and eddies. Hence motion is streamlined.

(*v*) The size of the body is small but it is larger than the distance between the molecules of the liquid. Thus the medium is homogeneous and continuous for such a body.

10.25 TERMINAL VELOCITY

38. *Explain how does a body attain a terminal velocity when it is dropped from rest in a viscous medium. Derive an expression for the terminal velocity of a small spherical body falling through a viscous medium. Also discuss the result.*

Terminal velocity. When a body falls through a viscous fluid, it produces relative motion between its different layers. As a result, the body experiences a viscous force which tends to retard its motion. As the velocity of the body increases, the viscous force ($F = 6\,\pi\,\eta\,rv$) also increases. A stage is reached, when the weight of the body becomes just equal to the sum of the upthrust and viscous force. Then no net force acts on the body and it begins to move with a constant

velocity. *The maximum constant velocity ... body while falling through a viscous me... terminal velocity.*

Expression for terminal velo... spherical body of radius r falling t... liquid of density σ and coefficient o... be the density of the body.

As the body falls, the various f... body are as shown in Fig. 10.26. T...

(*i*) Weight of the body acting ve...

$W = mg = \frac{4}{3}\pi r^3 \rho g$

(*ii*) Upward thrust equal to th... displaced.

$U = \frac{4}{3}\pi r^3 \sigma g$

(*iii*) Force of viscosity F ... direction. According to...

$F = 6\pi\eta r v$

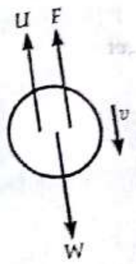


Fig. 10.26 Forces on a sphere falling in a viscous medium.

Clearly, the force... velocity of the body... when the weight of t... sum of the upthrus... body begins to fall w... called *terminal veloc...*

When the body...

$\frac{4}{3}\pi r^3 \sigma g + ...$

or

or

This is the ...

Discussio...

(i) Fig. 1... spher...

velocity. *The maximum constant velocity acquired by a body while falling through a viscous medium is called its terminal velocity.*

Expression for terminal velocity. Consider a spherical body of radius r falling through a viscous liquid of density σ and coefficient of viscosity η. Let ρ be the density of the body.

As the body falls, the various forces acting on the body are as shown in Fig. 10.26. These are

(*i*) Weight of the body acting vertically downwards.

$$W = mg = \frac{4}{3}\pi r^3 \rho g$$

(*ii*) Upward thrust equal to the weight of the liquid displaced.

$$U = \frac{4}{3}\pi r^3 \sigma g$$

(*iii*) Force of viscosity F acting in the upward direction. According to Stokes' law,

$$F = 6\pi\eta r v$$

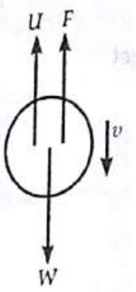


Fig. 10.26 Forces on a sphere falling in a viscous medium.

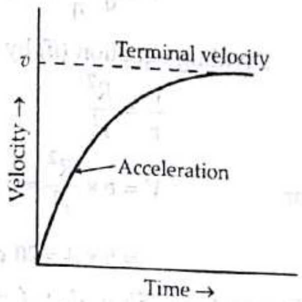


Fig. 10.27 Variation of v with t.

Clearly, the force of viscosity increases as the velocity of the body increases. A stage is reached, when the weight of the body becomes just equal to the sum of the upthrust and the viscous force. Then the body begins to fall with a constant maximum velocity, called *terminal velocity*.

When the body attains terminal velocity v,

$$U + F = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

or
$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

or
$$v = \frac{2}{9}\cdot\frac{r^2(\rho - \sigma) g}{\eta}$$

This is the expression for terminal velocity.

Discussion of the result :

(*i*) Fig. 10.27 shows how the velocity of a small sphere dropped from rest into a viscous medium varies with time. Initially the body is accelerated and after some time, it acquires terminal velocity v.

(*ii*) *The terminal velocity is directly proportional to the square of the radius of the body.* That is why bigger rain drops fall with a larger velocity compared to the smaller rain drops.

(*iii*) The terminal velocity is directly proportional the difference of the densities of the body and the fluid, *i.e.*, $(\rho - \sigma)$.

(*a*) If $\rho > \sigma$, the body will attain terminal velocity in the downward direction.

(*b*) If $\rho < \sigma$, the terminal velocity will be negative *i.e.*, the body will rise through the fluid. That is why, air bubble in a liquid and clouds in a sky are seen to move in the upward direction.

(*c*) If $\rho = \sigma$, the body remains suspended in the fluid.

(*iv*) The *terminal velocity is inversely proportional to the coefficient of viscosity of the fluid.* The more viscous the fluid, the smaller the terminal velocity attained by a body.

(*v*) The *terminal velocity is independent of the height through which a body is dropped.*

(*vi*) Knowing the values of ρ, σ, r and v, we can determine the coefficient of viscosity η as follows :

$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma) g}{v}.$$

Examples based on Stokes' Law and Terminal Velocity

FORMULAE USED

1. According to Stokes' law, force of viscosity acting on a spherical body of radius r moving with velocity v through a fluid of viscosity η is

$$F = 6\pi\eta r v$$

2. Terminal velocity of a spherical body of density ρ and radius r moving through a liquid of density ρ' is

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$$

UNITS USED

In SI, Force F is in newton, radius r in metre, velocity v in ms^{-1}, viscosity η in decapoise or Pa s, and density ρ in kgm^{-3}. In CGS system, F is in dyne, r in cm, v in cms^{-1}, η in poise and density ρ in gcm^{-3}.

EXAMPLE 42. *A rain drop of radius 0.3 mm falls through air with a terminal velocity of* $1\ ms^{-1}$. *The viscosity of air is* 18×10^{-5} *poise. Find the viscous force on the rain drop.*

Solution. Here $r = 0.3\ \text{mm} = 0.03\ \text{cm}$,

$$v = 1\ \text{ms}^{-1} = 100\ \text{cms}^{-1},\ \eta = 18 \times 10^{-5}\ \text{poise}.$$

According to Stokes' law, force of viscosity on rain drop is

$$F = 6\pi\eta r v$$
$$= 6 \times 3.142 \times 18 \times 10^{-5} \times 0.03 \times 100\ \text{dyne}$$
$$= \mathbf{1.018 \times 10^{-2}\ dyne.}$$

EXAMPLE 43. An iron ball of radius 0.3 cm falls through a column of oil of density 0.94 g cm^{-3}. It is found to attain a terminal velocity of 0.5 cms^{-1}. Determine the viscosity of the oil. Given that density of iron is 7.8 g cm^{-3}.

Solution. Here $r = 0.3$ cm, $v = 0.5\ \text{cms}^{-1}$,

$$\rho = 7.8\ \text{gcm}^{-3},\ \rho' = 0.94\ \text{gcm}^{-3}$$

As $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$

$$\therefore\ \eta = \frac{2}{9}\frac{r^2}{v}(\rho - \rho')g$$
$$= \frac{2 \times (0.3)^2 \times (7.8 - 0.94) \times 980}{9 \times 0.5}$$
$$= \frac{2 \times 0.09 \times 6.86 \times 980}{9 \times 0.5} = \mathbf{268.9\ poise.}$$

EXAMPLE 44. With what terminal velocity will an air bubble of density 1 kgm^{-3} and 0.8 mm in diameter rise in a liquid of viscosity 0.15 Nsm^{-2} and specific gravity 0.9 ? What is the terminal velocity of same bubble in water of $\eta = 1 \times 10^{-3}$ Nsm^{-2} ?

Solution. Here $r = \frac{0.8}{2} = 0.4\ \text{mm} = 0.4 \times 10^{-3}\ \text{m}$,

$\eta = 0.15\ \text{Nsm}^{-2}$, $g = 9.8\ \text{ms}^{-2}$

Specific gravity of liquid = 0.9

Density of liquid (medium),

$$\rho' = 0.9 \times 10^3\ \text{kg m}^{-3} = 900\ \text{kg m}^{-3}$$

Density of air bubble (spherical object),

$$\rho = 1\ \text{kg m}^{-3}$$

Terminal velocity of air bubble,

$$v = \frac{2r^2 g(\rho - \rho')}{9\eta} = \frac{2 \times (0.4 \times 10^{-3})^2 \times 9.8 \times (1 - 900)}{9 \times 0.15}$$
$$= \mathbf{-0.0021\ ms^{-1}}.$$

The negative sign shows that the air bubble will rise up.

Terminal velocity of air bubble in water :

Here $\rho' = 1000\ \text{kg m}^{-3}$, $\eta = 10^{-3}\ \text{Nsm}^{-2}$

$$\therefore\ v = \frac{2 \times (0.4 \times 10^{-3})^2 \times 9.8 \times (1 - 1000)}{9 \times 10^{-3}}$$
$$= \mathbf{-0.348\ ms^{-1}}.$$

EXAMPLE 45. Eight rain drops of radius 1 mm each falling down with terminal velocity of 5 cms^{-1} coalesce to form a bigger drop. Find the terminal velocity of the bigger drop.

[Delhi 95 ; Central Schools

Solution. Radius of each small drop,

$$r = 1\ \text{mm} = 0.1\ \text{cm}$$

Terminal velocity of each small drop,

$$v = 5\ \text{cms}^{-1}$$

Volume of bigger drop = Volume of 8 small drops

$$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

or $$R = 2r = 2 \times 0.1\ \text{cm} = 0.2\ \text{cm}$$

Terminal velocity of each small drop is given by

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g \quad ...(i)$$

Terminal velocity of bigger drop is given by

$$V = \frac{2}{9}\frac{R^2}{\eta}(\rho - \rho')g \quad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{V}{v} = \frac{R^2}{r^2}$$

or $$V = v \times \frac{R^2}{r^2} = 5 \times \frac{(0.2)^2}{(0.1)^2}$$
$$= 5 \times 4 = \mathbf{20\ cms^{-1}}.$$

EXAMPLE 46. Show that if n equal rain droplets falling through air with equal steady velocity of 10 cms^{-1} coalesce, the resultant drop attains a new terminal velocity of 10 $n^{2/3}$ cms^{-1}.

Solution. Volume of a bigger drop

= $n \times$ Volume of a smaller droplet

or $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$ or $R^3 = nr^3$

or $R = n^{1/3} r$

Terminal velocity of a small droplet is given by

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g \quad ...(i)$$

Terminal velocity of a bigger drop is given by

$$V = \frac{2}{9}\frac{R^2}{\eta}(\rho - \rho')g \quad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{V}{v} = \frac{R^2}{r^2}$$

But $R = n^{1/3} r$ and $v = 10\ \text{cms}^{-1}$

$$\therefore\ V = v \times \frac{R^2}{r^2} = 10 \times \frac{n^{2/3} r^2}{r^2} = \mathbf{10\, n^{2/3}\ cms^{-1}}.$$

Scanned by CamScanner

EXAMPLE 47. Fine particles of sand are shaken up in water contained in a tall cylinder. If the depth of water in the cylinder is 24 cm, calculate the size of the largest particle of sand that can remain suspended after the expiry of 40 minutes. Given density of sand = 2.6 g cm^{-3} and viscosity of water = 0.01 poise. Assume that all the particles are spherical and are of different sizes.

Solution. Largest particle which remains suspended is that which just covers 24 cm in 40 minutes. So terminal velocity of largest particle is

$$v = \frac{\text{Distance}}{\text{Time}} = \frac{24 \text{ cm}}{40 \times 60 \text{ s}} = 0.01 \text{ cms}^{-1}.$$

Also $\eta = 0.01$ poise, $\rho = 2.6 \text{ g cm}^{-3}$, $\rho' = 1 \text{ g cm}^{-3}$, $g = 980 \text{ cm s}^{-2}$

As $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$ or $r^2 = \frac{9}{2}\frac{\eta v}{(\rho - \rho')g}$

$$\therefore \quad r = \sqrt{\frac{9\eta v}{2(\rho - \rho')g}} = \sqrt{\frac{9 \times 0.01 \times 0.01}{2 \times (2.6 - 1) \times 980}}$$

$$= \frac{3 \times 0.01}{2 \times 4 \times 7} = \mathbf{5.357 \times 10^{-4}} \textbf{ cm.}$$

EXAMPLE 48. A sphere is dropped under gravity through a fluid of viscosity η. Taking the average acceleration as half of the initial acceleration, show that the time taken to attain the terminal velocity is independent of the fluid density. **[NCERT]**

Solution. Suppose a sphere of radius r and density ρ falls in a fluid of density ρ' and viscosity η. When the sphere just enters the fluid, the net downward force on it is

$$F = \text{Weight of the sphere} - \text{Weight of the fluid displaced}$$

$$= \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g = \frac{4}{3}\pi r^3 (\rho - \rho')g$$

$\therefore$ Initial acceleration,

$$a = \frac{F}{m} = \frac{\frac{4}{3}\pi r^3(\rho - \rho')g}{\frac{4}{3}\pi r^3 \rho} = \left(\frac{\rho - \rho'}{\rho}\right)g$$

When the sphere attains terminal velocity, its acceleration becomes zero.

$$\therefore \text{ Average acceleration} = \frac{a + 0}{2} = \left(\frac{\rho - \rho'}{2\rho}\right)g$$

Let the sphere take time t to attain the terminal velocity,

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$$

Initial velocity, $u = 0$

Hence by using first equation of motion,

$$v = u + at$$

$$\frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g = 0 + \left(\frac{\rho - \rho'}{2\rho}\right)gt$$

or

$$t = \frac{4}{9} \cdot \frac{r^2 \rho}{\eta}.$$

PROBLEMS FOR PRACTICE

1. Find the terminal velocity of a steel ball 2 mm in diameter falling through glycerine. Relative density of steel = 8, relative density of glycerine = 1.3 and viscosity of glycerine = 8.3 poise. **[Delhi 98]**
   (Ans. 1.758 cm s^{-1})

2. A gas bubble of diameter 2 cm rises steadily at the rate of 2.5 mm s^{-1} through a solution of density 2.25 g cm^{-3}. Calculate the coefficient of viscosity of the liquid. Neglect the density of the gas.
   (Ans. 1960 poise)

3. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} decapoise, what will be the terminal velocity of the drop ? Given density of water = 10^3 kgm^{-3} and $g = 9.8$ ms^{-2}. Density of air can be neglected.
   (Ans. 2.72×10^{-4} ms^{-1})

4. The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cms^{-1}. Compute the viscosity of the oil at 20°C. Density of oil = 1.5×10^3 kg m^{-3}, density of copper = 8.9×10^3 kg m^{-3}. **[NCERT]**
   (Ans. 0.992 decapoise)

5. A spherical glass ball of mass 1.34×10^{-4} kg and diameter 4.4×10^{-3} m takes 6.4 s to fall steadily through a height of 0.381 m inside a large volume of oil of specific gravity 0.943. Calculate the viscosity of oil. **(Ans. 0.8025 Nsm^{-2})**

6. Two exactly similar rain drops falling with terminal velocity of $(2)^{1/3}$ ms^{-1} coalesce to form a bigger drop. Find the new terminal velocity of the bigger drop. **(Ans. 2 ms^{-1})**

HINTS

1. $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g = \frac{2}{9}\frac{(0.1)^2}{8.3}(8 - 1.3) \times 980$

   $= \mathbf{1.758 \text{ cm s}^{-1}}.$

2. $\eta = \frac{2}{9}\frac{r^2}{v}(\rho - \rho')g = \frac{2}{9}\frac{(1)^2 \times (0 - 2.25) \times 980}{-0.25}$

   $= \mathbf{1960}$ **poise.**

Scanned by CamScanner

3. $v = \frac{2}{9} \times \frac{(1.5 \times 10^{-6})^2 (10^3 - 0) \times 9.8}{18 \times 10^{-5}}$

$= 2.72 \times 10^{-4}\ \text{ms}^{-1}$.

4. $\eta = \frac{2}{9}\frac{r^2}{v}(\rho - \rho')g$

$= \frac{2 \times (2.0 \times 10^{-3})^2 \times (8.9 \times 10^3 - 1.5 \times 10^3) \times 9.8}{9 \times 6.5 \times 10^{-2}}$

$= 0.992$ decapoise

5. Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{\frac{4}{3}\pi r^3}$

$= \frac{3 \times 134 \times 10^{-4}}{4 \times \pi \times (2.2 \times 10^{-3})^3} = 3003.1245\ \text{kgm}^{-3}$

Oil density, $\rho' = 0.943 \times 10^3 = 943\ \text{kgm}^{-3}$

Terminal velocity, $v = \frac{0.381}{6.4} = 0.05753\ \text{ms}^{-1}$

$\eta = \frac{2}{9}\frac{r^2}{v}(\rho - \rho')g$

$= \frac{2}{9} \times \frac{(2.2 \times 10^{-3})^2 (3003.1245 - 943) \times 9.8}{0.05953}$

$= 0.8025\ \text{Nsm}^{-2}$.

6. Volume of bigger drop = Volume of two smaller drops

$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3$ or $R^3 = 2r^3$ or $\frac{R}{r} = 2^{1/3}$

Terminal velocity of a smaller drop,

$v_1 = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$

Terminal velocity of the bigger drop,

$v_2 = \frac{2}{9}\frac{R^2}{\eta}(\rho - \rho')g$

$\therefore \quad \frac{v_2}{v_1} = \frac{R^2}{r^2} = (2^{1/3})^2 = 2^{2/3}$

or $\quad v_2 = 2^{2/3}\, v_1 = 2^{2/3} \times 2^{1/3} = 2\ \text{ms}^{-1}$.

10.26 STREAMLINE AND TURBULENT FLOWS

39. *Distinguish between streamline and turbulent flows. What do you understand by a streamline and tube of flow ? Give important properties of streamlines.*

Streamline flow. *When a liquid flows such that each particle of the liquid passing a given point moves along the same path and has the same velocity as its predecessor, the flow is called streamline flow or steady flow.*

Consider the flow of the liquid along the path ABC; where A, B and C are the points inside the liquid. If every successive particle passes through point A with constant velocity $\vec{v}_A$ directed along tangent at A, then through point B with constant velocity $\vec{v}_B$ directed along tangent at B and then through C with constant velocity $\vec{v}_C$, the flow is said to be *steady, orderly* or *streamlined*. The path ABC along which the particles move one after another is called a *streamline*. The particle velocity at a particular point remains constant with time but velocities at different points may or may not be the same. Streamline flow is possible only if the liquid velocity does not exceed a limiting value, called *critical velocity*. The fixed path followed by an orderly procession of particles in the steady flow is called a streamline. In Fig. 10.28(*a*), the curve ABC represents a streamline. *A streamline may be defined as the path, the tangent to which at any point gives the direction of the flow of liquid at that point.*

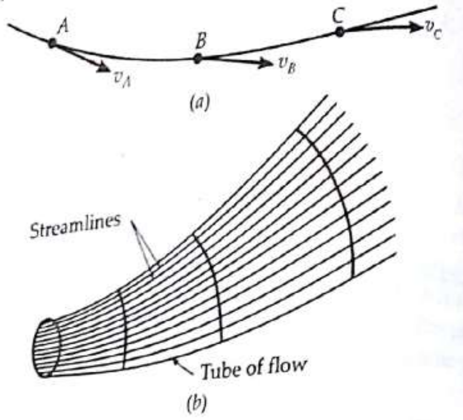


Fig. 10.28 (*a*) A streamline. (*b*) A tube of flow.

Tube of flow. *A bundle of streamlines forming a tubular region is called a tube of flow.* The boundary of such a tube is always parallel to the velocity of fluid particles. No fluid can cross the boundaries of a tube of flow, and the tube behaves somewhat like a tube. In a steady flow, the shape of the flow tube does not change with time.

Turbulent flow. When the liquid velocity exceeds a certain limiting value, called *critical velocity*, the liquid flow becomes zig-zag. The path and the velocity of a liquid particle changes continuously, haphazardly.

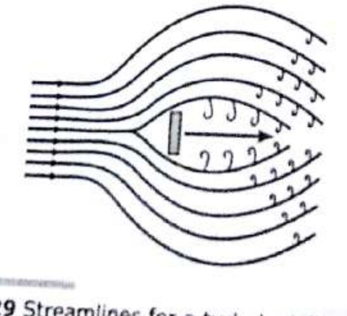


Fig. 10.29 Streamlines for a turbulent flow.

This flow is called *turbulent flow*. It is accompanied by random, irregular, local circular currents called vortices. As shown in Fig. 10.29, a jet of air striking a flat plate placed perpendicular to it causes a turbulent flow.

Properties of streamlines :

(*i*) In a steady flow, *no two streamlines can cross each other*. If they do so, the fluid particle at the point of intersection will have two different directions of flow. This will destroy the steady nature of the fluid flow.

(*ii*) The tangent at any point on the streamline gives the direction of velocity of fluid particle at that point.

(*iii*) Greater the number of streamlines passing normally through a section of the fluid, larger is the fluid velocity at that section.

(*iv*) Fluid velocity remains constant at any point of a streamline, but it may be different at different points of the same streamline.

10.27 LAMINAR FLOW

40. *What is laminar flow of a liquid ? Distinguish between the velocity profiles of non-viscous and viscous liquids.*

Laminar flow. When the velocity of the flow of a liquid is less than its critical velocity, the liquid flows steadily. Each layer of the liquid slides over the other layer. It behaves as if different lamina are sliding over one another. Such a flow is called laminar flow.

The surface obtained by joining the heads of the velocity vectors for the particles in a section of a flowing liquid is called a *velocity profile*.

(*i*) **Velocity profile for a non-viscous liquid.** In case of a non-viscous liquid, the velocity of all the particles at any section of a pipe is same, so the velocity profile is plane as shown in Fig. 10.30(*a*).

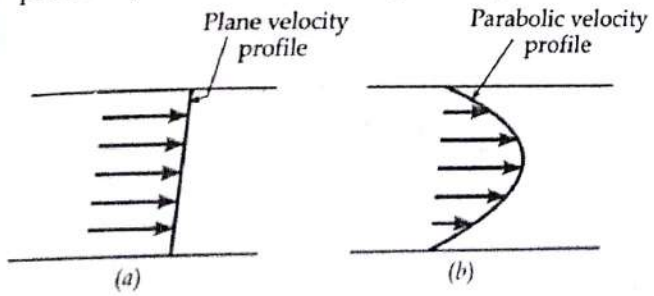


Fig. 10.30 Flow of (*a*) Non-viscous (*b*) Viscous liquid through a pipe.

(*ii*) **Velocity profile of a viscous liquid.** When a viscous liquid flows through a pipe, the velocity of layer at the axis is maximum, the velocity decreases as we go towards the wall of the pipe and becomes zero for the layer in contact with the pipe. Hence the velocity profile for a viscous liquid is parabolic, as shown in Fig. 10.30(*b*).

10.28 CRITICAL VELOCITY

41. *What do you mean by critical velocity of a liquid ? Derive an expression for it on the basis of dimensional considerations.*

Critical velocity. *The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent.*

The critical velocity v_c of a liquid flowing through a tube depends on

(*i*) coefficient of viscosity of the liquid (η)

(*ii*) density of the liquid (ρ)

(*iii*) diameter of the tube (D)

Let $v_c = k\,\eta^a\,\rho^b\,D^c$

where k is a dimensionless constant. Writing the above equation in dimensional form, we get

$$[M^0LT^{-1}] = [ML^{-1}T^{-1}]^a\,[ML^{-3}]^b\,[L]^c$$

$$[M^0LT^{-1}] = [M^{a+b}L^{-a-3b+c}T^{-a}]$$

Equating powers of M, L and T, we get

$$a + b = 0$$

$$-a - 3b + c = 1$$

$$-a = -1$$

On solving, we get $a = 1,\ b = -1,\ c = -1$

$$\therefore\ v_c = k\,\eta\,\rho^{-1}\,D^{-1} = \frac{k\eta}{\rho D}.$$

Clearly, the critical velocity v_c will be large if η is large, and ρ and D are small. So we can conclude that

(*i*) The flow of liquids of higher viscosity and lower density through narrow pipes tends to be streamlined.

(*ii*) The flow of liquids of lower viscosity and higher density through broad pipes tends to become turbulent, because in that case the critical velocity will be very small.

10.29 REYNOLD'S NUMBER

42. *What is Reynold's number ? What is its importance ?*

Reynold's number. *It is dimensionless parameter whose value decides the nature of flow of a liquid through a pipe. It is given by*

$$R_e = \frac{\rho\,vD}{\eta}$$

where ρ = density of the liquid

v = velocity of the liquid

η = coefficient of viscosity of the liquid

D = diameter of the pipe.

Importance of Reynold's number. If R_e lies between 0 and 2000, the liquid flow is streamlined or laminar. If $R_e > 3000$, the liquid flow is turbulent. If R_e

Scanned by CamScanner

lies between 2000 and 3000, the flow of liquid is unstable, it may change from laminar to turbulent and *vice-versa*. The exact value at which turbulence sets in a fluid is called *critical Reynold's number*.

43. *Show that the Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.*

Physical significance of Reynold's number. Consider a narrow tube having a cross-sectional area A. Suppose a fluid flows through it with a velocity v for a time interval Δt.

Length of the fluid = Velocity × time = $v\,\Delta t$

Volume of the fluid flowing through the tube in time $\Delta t = Av\,\Delta t$

Mass of the fluid,

$$\Delta m = \text{Volume} \times \text{density} = Av\,\Delta t \times \rho$$

Inertial force acting per unit area of the fluid

$$= \frac{F}{A} = \frac{\text{Rate of change of momentum}}{A}$$

$$= \frac{\Delta m \times v}{\Delta t \times A} = \frac{Av\,\Delta t\,\rho \times v}{\Delta t \times A} = \rho v^2$$

Viscous force per unit area of the fluid

$$= \eta \times \text{velocity gradient} = \eta\,\frac{v}{D}$$

$$\frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{\rho v^2}{\eta\, v/D} = \frac{\rho\, vD}{\eta} = R_e$$

Thus Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.

Examples based on Reynold's Number

FORMULAE USED

1. For a liquid of viscosity η, density ρ and flowing through a pipe of diameter D, Reynold's number is given by
$$R_e = \frac{\rho v D}{\eta}$$
2. Flow is laminar for R_e between 0 and 2000. The fluid velocity corresponding to $R_e = 2000$ is called critical velocity.
$$v_c = \frac{2000 \times \eta}{\rho D}$$
3. Flow is turbulent for R_e above 3000.
4. Flow is unstable for R_e between 2000 and 3000.

UNITS USED

Density ρ is in kg m^{-3}, diameter D in metre, viscosity η in Pa s and Reynold's number is dimensionless.

EXAMPLE 49. *Verify that the quantity* $\frac{\rho v D}{\eta}$ *(Reynold's number) is dimensionless.*

Solution. Dimensions of various quantities are

$$[\rho] = ML^{-3}, [v] = LT^{-1}, [D] = L, [\eta] = ML^{-1}T^{-1}$$

$$\therefore \quad \left[\frac{\rho v D}{\eta}\right] = \frac{ML^{-3}.\,LT^{-1}.\,L}{ML^{-1}T^{-1}} = M^{\circ}L^{\circ}T^{\circ}$$

Hence Reynold's number $\rho v D/\eta$ is dimensionless.

EXAMPLE 50. *What should be the average velocity of water in a tube of radius 0.005 m so that the flow is just turbulent? The viscosity of water is 0.001 Pa s.*

Solution. Here $D = 0.010$ m,

$$\eta = 0.001 \text{ Pa s}, \quad \rho = 1000 \text{ kgm}^{-3}$$

For flow to be just turbulent, $R_e = 3000$

$$\therefore \quad v = \frac{R_e\,\eta}{\rho\, D} = \frac{3000 \times 0.001}{1000 \times 0.010} = \mathbf{0.3\ ms^{-1}}.$$

EXAMPLE 51. *The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is 10^{-3} Pa s. After some time the flow rate is increased to 3 L/min. Characterise the flow for both the flow rates.* **[NCERT]**

Solution. $D = 1.25 \text{ cm} = 1.25 \times 10^{-2}$ m,

$$\eta = 10^{-3} \text{ Pa s}, \rho = 10^3 \text{ kg m}^{-3}$$

The volume of water flowing out per second is

$$Q = va = v \times \frac{\pi D^2}{4}$$

$$\therefore \text{ Speed of flow,} \quad v = \frac{4Q}{\pi D^2}$$

Reynold's number,

$$R_e = \frac{\rho\, vD}{\eta} = \frac{\rho\, D}{\eta} \cdot \frac{4Q}{\pi D^2} = \frac{4\rho Q}{\pi D \eta}$$

When $\quad Q = 0.48$ L/min

$$= \frac{0.48 \times 10^{-3}\text{ m}^3}{60 \text{ s}} = 8 \times 10^{-6} \text{ m}^3\text{ s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 8 \times 10^{-6}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} = 815$$

As $R_e < 1000$, the flow is steady.

When $\quad Q = 3\text{L/min} = \frac{3 \times 10^{-3}\text{ m}^3}{60 \text{ s}}$

$$= 5 \times 10^{-5} \text{ m}^3\text{ s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 5 \times 10^{-5}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} \simeq 5096$$

As $R_e > 3000$, so the flow will be turbulent.

Scanned by CamScanner

PROBLEMS FOR PRACTICE

1. What should be the maximum average velocity of water in a tube of diameter 0.5 cm so that the flow is laminar ? The viscosity of water is 0.00125 Ns m^{-2}. (**Ans.** 0.5 ms^{-1})

2. Water is flowing in a pipe of radius 1.5 cm with an average velocity of 15 cms^{-1}. What is the nature of flow ? Given coefficient of viscosity of water is 10^{-3} kg m^{-1} s^{-1} and its density is 10^3 kgm^{-3}. (**Ans.** $R_e = 4500 > 3000$, flow is turbulent)

3. Water flows at a speed of 6 cms^{-1} through a pipe of tube of radius 1 cm. Coefficient of viscosity of water at room temperature is 0.01 poise. What is the nature of flow ? (**Ans.** $R_e = 1200 < 2000$, so flow is laminar)

4. Find the critical velocity for air flowing through a tube of 2 cm diameter. For air, $\rho = 1.3 \times 10^{-3}\ g\ cm^{-3}$ and $\eta = 181 \times 10^{-6}$ poise. (**Ans.** 140 cms^{-1})

HINTS

1. $v_c = \frac{R_e\,\eta}{\rho\,D} = \frac{2000 \times 0.00125}{1000 \times 0.5 \times 10^{-2}} = 0.5\ ms^{-1}$.

2. Here $D = 2 \times 1.5\ cm = 3.0 \times 10^{-2}$ m

$v = 15\ cm\ s^{-1} = 0.15\ ms^{-1}$, $\eta = 10^{-3}\ kg\ m^{-1}\ s^{-1}$

$$R_e = \frac{\rho\,vD}{\eta} = \frac{10^3 \times 0.15 \times 3.0 \times 10^{-2}}{10^{-3}} = 4500.$$

10.30 IDEAL FLUID

44. *What is meant by an ideal fluid ?*

Ideal fluid. The motion of real fluids is very complicated. To understand fluid dynamics in a simpler manner, we assume that the fluid is ideal. *An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.* Thus an ideal fluid has the following features connected with its flow :

(*i*) **Steady flow.** In a steady flow, the fluid velocity at each point does not change with time, either in magnitude or direction.

(*ii*) **Incompressible flow.** The density of the fluid remains constant during its flow.

(*iii*) **Non-viscous flow.** The fluid offers no internal friction. An object moving through this fluid does not experience a retarding force.

(*iv*) **Irrotational flow.** This means that there is no angular momentum of the fluid about any point. A very small wheel placed at any point inside such a fluid does not rotate about its centre of mass.

10.31 EQUATION OF CONTINUITY

45. *Obtain the equation of continuity for the incompressible non-viscous fluid having a steady flow through a pipe.*

Equation of continuity. Consider a non-viscous and incompressible liquid flowing steadily between the sections A and B of a pipe of varying cross-section. Let a_1 be the area of cross-section, v_1 fluid velocity, ρ_1 fluid density at section A ; and the values of corresponding quantities at section B be a_2, v_2 and ρ_2.

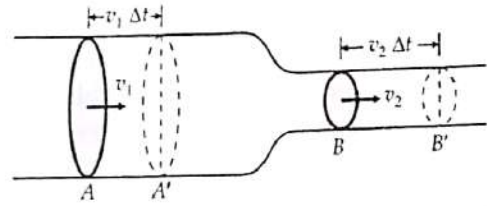


Fig. 10.31 Equation of continuity.

As m = Volume × density

= Area of cross-section × length × density

∴ Mass of fluid that flows through section A in time Δt,

$$m_1 = a_1\,v_1\,\Delta t\,\rho_1$$

Mass of fluid that flows through section B in time Δt,

$$m_2 = a_2\,v_2\,\Delta t\,\rho_2$$

By conservation of mass,

$$m_1 = m_2$$

or $$a_1\,v_1\,\Delta t\,\rho_1 = a_2\,v_2\,\Delta t\,\rho_2$$

As the fluid is incompressible, so $\rho_1 = \rho_2$, and hence

$$a_1\,v_1 = a_2\,v_2 \quad \text{or} \quad av = \text{constant}.$$

This is *the* ***equation of continuity***. *It states that during the streamlined flow of the non-viscous and incompressible fluid through a pipe of varying cross-section, the product of area of cross-section and the normal fluid velocity (av) remains constant throughout the flow.*

For Your Knowledge

- The equation of continuity is a special case of the law of conservation of mass.
- The equation of continuity shows that $v \propto 1/a$, *i.e.*, *the liquid velocity at any section of the pipe is inversely proportional to the area of cross-section of the pipe at that section.* This explains why the speed of water emerging from a PVC pipe increases when we press its outlet with our fingers and hence decrease its area of cross-section.

Scanned by CamScanner

46. *Why does deep water run slow ?*

Deep water runs slowly. As the depth of water in a river or a stream increases, the area of cross-section available to the flowing water increases. Consequently, velocity decreases in accordance with the equation of continuity. Thus deep water runs slowly.

10.32 ENERGY OF A FLUID IN A STEADY FLOW

47. *What are different forms of energy possessed by a flowing liquid ? Write expressions for them.*

Energies possessed by a flowing liquid. A liquid in a steady flow can have three kinds of energy (*i*) kinetic energy (*ii*) potential energy and (*iii*) pressure energy.

(*i*) ***Kinetic energy.*** *The energy possessed by a liquid by virtue of its motion is called its kinetic energy.*

$$\text{K.E.} = \frac{1}{2} mv^2$$

where m is the mass of the liquid and v is the velocity of the liquid.

$$\text{K.E. per unit mass of the liquid} = \frac{1}{2} v^2$$

The kinetic energy per unit weight of the liquid is known as *the velocity head.*

$$\therefore \quad \text{Velocity head} = \frac{v^2}{2g}$$

$$\text{K.E. per unit volume} = \frac{1}{2}\frac{mv^2}{V} = \frac{1}{2}\rho v^2$$

(*ii*) ***Potential energy.*** *The energy possessed by a liquid by virtue of its position above the earth's surface is called its potential energy.*

$$\text{P.E.} = mgh$$

where h is the average height of the liquid from the ground level.

$$\text{P.E. per unit mass of the liquid} = gh$$

The potential energy per unit weight of the liquid is known as the *potential head.*

$$\therefore \quad \text{Potential head} = \frac{mgh}{mg} = h$$

$$\text{P.E. per unit volume} = \frac{mgh}{V} = \rho gh$$

(*iii*) ***Pressure energy.*** *The energy possessed by a liquid by virtue of its pressure is called its pressure energy.* A liquid under pressure can do work and so possesses energy.

For example, a liquid in a cylinder can drive a piston as shown in Fig. 10.32. Let P be the pressure exerted by the liquid on a frictionless piston of area a. Suppose the piston moves through distance x under the pressure P.

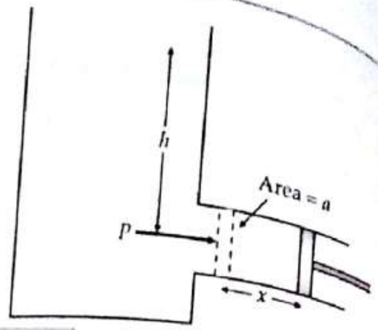


Fig. 10.32 Pressure energy of a liquid.

The work done is

$$W = \text{Force} \times \text{distance} = \text{Pressure} \times \text{area} \times \text{distance}$$
$$= Pax = PV$$

where $V = ax$ = volume swept by the piston.

This work done is stored as the pressure energy of liquid of volume V.

$$\therefore \quad \text{Pressure energy of volume } V = PV$$

Pressure energy per unit volume

$$= \frac{PV}{V} = P = \text{Excess pressure}$$

$$\text{Pressure energy per unit mass} = \frac{PV}{m} = \frac{P}{\rho}$$

Pressure energy per unit weight of the liquid is called *pressure head.*

$$\text{Pressure head} = \frac{P}{\rho g}$$

10.33 BERNOULLI'S PRINCIPLE

48. *State and prove Bernoulli's principle for the flow of non-viscous fluids. Give its limitations.*

Bernoulli's principle. The Swiss physicist *Daniel Bernoulli* first derived an expression relating the pressure to fluid speed and height in 1738. His result, called *Bernoulli's principle* is based on the law of conservation of energy and applies to ideal fluids.

Bernoulli's principle *states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant along a streamline.*

Mathematically, it can be expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Proof. Consider a non-viscous and incompressible fluid flowing steadily between the sections A and B of a pipe of varying cross-section. Let a_1 be the area of cross-section at A, v_1 the fluid velocity, P_1 the fluid pressure, and h_1 the mean height above the ground level. Let a_2, v_2, P_2 and h_2 be the values of the corresponding quantities at B.

Scanned by CamScanner

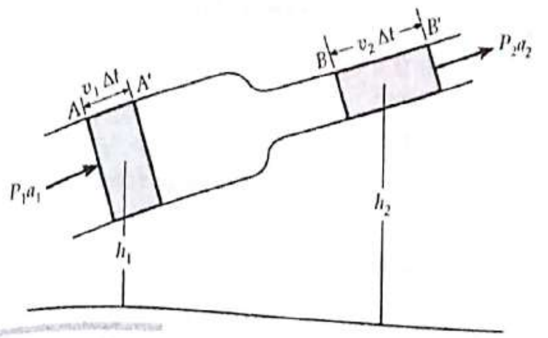


Fig. 10.33 Derivation of Bernoulli's principle.

Let ρ be the density of the fluid. As the fluid is incompressible, so whatever mass of fluid enters the pipe at section A in time Δt, an equal mass of fluid flows out at section B in time Δt. This mass is given by

m = Volume × density

= Area of cross-section × length × density

or $m = a_1 v_1 \Delta t\, \rho = a_2 v_2 \Delta t\, \rho$ ...(1)

or $a_1 v_1 = a_2 v_2$ ...(2)

∴ Change in K.E. of the fluid

= K.E. at B − K.E. at A

$= \frac{1}{2} m(v_2^2 - v_1^2) = \frac{1}{2} a_1 v_1 \Delta t\, \rho\, (v_2^2 - v_1^2)$ [Using (1)]

Change in P.E. of the fluid

= P.E. at B − P.E. at A

$= mg\,(h_2 - h_1) = a_1 v_1 \Delta t\, \rho\, g\,(h_2 - h_1)$ [Using (1)]

Net work done on the fluid

= Work done on the fluid at A
− Work done by the fluid at B

$= P_1 a_1 \times v_1 \Delta t - P_2 a_2 \times v_2 \Delta t$

$= P_1 a_1 v_1 \Delta t - P_2 a_1 v_1 \Delta t$ [Using (2)]

$= a_1 v_1 \Delta t\, (P_1 - P_2)$

By conservation of energy,

Net work done on the fluid

= Change in K.E. of the fluid
+ Change in P.E. of the fluid

$\therefore\ a_1 v_1 \Delta t\, (P_1 - P_2)$

$= \frac{1}{2} a_1 v_1 \Delta t\, \rho\, (v_2^2 - v_1^2) + a_1 v_1 \Delta t \rho g\, (h_2 - h_1)$

Dividing both sides by $a_1 v_1\, \Delta t$, we get

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$$

or $$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

or $$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant} \quad ...(3)$$

This proves Bernoulli's principle according to which the total energy per unit volume remains constant. Equation (3) can also be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant.}$$

This is another form of *Bernoulli's principle according to which the sum of pressure head, velocity head and gravitational head remains constant in the streamline flow of an ideal fluid.*

Limitations of Bernoulli's equation :

1. Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids. In case of viscous fluids, we need to take into account the work done against viscous drag.
2. Bernoulli's equation has been derived on the assumption that there is no loss of energy due to friction. But in practice, when fluids flow, some of their kinetic energy gets converted into heat due to the work done against the internal forces of friction or viscous forces.
3. Bernoulli's equation is applicable only to incompressible fluids because it does not take into account the elastic energy of the fluids.
4. Bernoulli's equation is applicable only to streamline flow of a fluid and not when the flow is turbulent.
5. Bernoulli's equation does not take into consideration the angular momentum of the fluid. So it cannot be applied when the fluid flows along a curved path.

For Your Knowledge

- Bernoulli's principle is a fundamental principle of fluid dynamics based on the law of conservation of energy.
- In Bernoulli's equation : $P + \rho g h + \frac{1}{2} \rho v^2$ = constant, the term $(P + \rho g h)$ is called *static pressure*, because it is the pressure of the fluid even if it is at rest, and the term $\frac{1}{2} \rho v^2$ is the *dynamic pressure* of the fluid which is the pressure by virtue of its velocity v. So Bernoulli's equation can be written as

  Static pressure + Dynamic pressure = Constant
- If a liquid is flowing through a horizontal tube, h remains constant and we can write

  $$P + \frac{1}{2} \rho v^2 = \text{constant}$$

  This shows that if v increases, P decreases and vice versa. Thus for the *streamline flow of an ideal liquid flowing horizontally, the pressure decreases where velocity increases and vice versa.* This is an important aspect of Bernoulli's principle which finds many applications.

We now consider some useful applications and phenomena which are based on Bernoulli's principle.

Scanned by CamScanner

10.34 TORRICELLI'S LAW OF EFFLUX

49. *Apply Bernoulli's principle to determine the speed of efflux from the side of a container both when its top is closed and open. Hence derive Torricelli's law.*

Speed of efflux. The word efflux means the outflow of a fluid. As shown in Fig. 10.34, consider a tank containing a liquid of density ρ with a small hole on its side at a height y_1 from the bottom. Let y_2 be the height of the liquid surface from the bottom and P be the air pressure above the liquid surface.

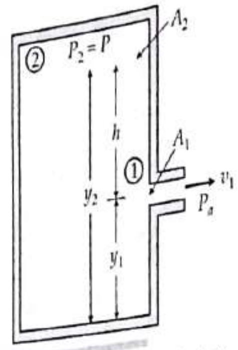


Fig. 10.34 Torricelli's law.

If A_1 and A_2 are the cross-sectional areas of the side hole and the tank respectively, and v_1 and v_2 are the liquid velocities at points 1 and 2, then from the equation of continuity, we get

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \frac{A_1}{A_2} v_1$$

As $A_2 >> A_1$, so the liquid may be taken at rest at the top, *i.e.*, $v_2 \approx 0$. Applying Bernoulli's equation at points 1 and 2, we get

$$P_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

or
$$\frac{1}{2}\rho v_1^2 = \rho g (y_2 - y_1) + (P - P_a)$$

If we take $y_2 - y_1 = h$, then

$$\frac{1}{2}\rho v_1^2 = \rho g h + (P - P_a)$$

or
$$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$$

Special cases (*i*) When $P >> P_a$, the term $2gh$ may be ignored.

$$v_1 = \sqrt{\frac{2(P - P_a)}{\rho}}.$$

Thus the speed of efflux is determined by container pressure P. Such a situation exists in rocket propulsion.

(*ii*) *When the tank is open to the atmosphere,*

$$P = P_a \quad \text{and} \quad v_1 = \sqrt{2gh}$$

Thus, *the velocity of efflux of a liquid is equal to the velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called* **Torricelli's law.**

10.35 THE VENTURIMETER

50. *What is a venturimeter ? Describe its construction and working.*

Venturimeter. *It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle.* It is also called **flow meter** or **venturi tube.**

Construction. It consists of a horizontal tube having wider opening of cross-section a_1 and a narrow neck of cross-section a_2. These two regions of the horizontal tube are connected to a manometer, containing a liquid of density ρ_m.

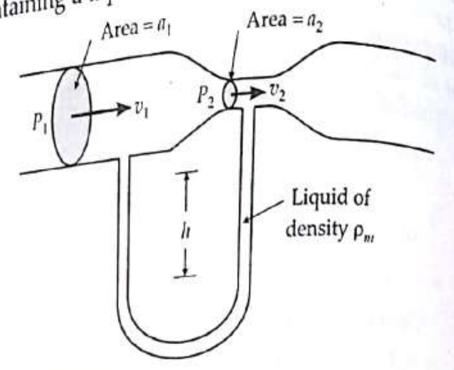


Fig. 10.35 Venturi tube.

Working. Let the liquid velocities be v_1 and v_2 at the wider and the narrow portions. Let P_1 and P_2 be the liquid pressures at these regions. By the equation of continuity,

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad \frac{a_1}{a_2} = \frac{v_2}{v_1}$$

If the liquid has density ρ and is flowing horizontally, then from Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or
$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right) \qquad \left[\because \frac{v_2}{v_1} = \frac{a_1}{a_2}\right]$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

If h is the height difference in the two arms of the manometer tube, then

$$P_1 - P_2 = h\rho_m g$$

$$\therefore \quad h\rho_m g = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

Scanned by CamScanner

$$\therefore \quad v_1 = \sqrt{\frac{2h\rho_m g}{\rho} \times \frac{a_2^2}{a_1^2 - a_2^2}}$$

Volume of the liquid flowing out per second,

$$Q = a_1 v_1 = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$$

10.36 ATOMIZER OR SPRAYER

51. *Briefly describe the working of an atomizer on the basis of Bernoulli's principle.*

Atomizer. The working of an atomizer which is used to spray liquids is based on Bernoulli's principle. Fig. 10.36 shows the essential parts of an atomizer. When the rubber balloon is pressed, the air rushes out

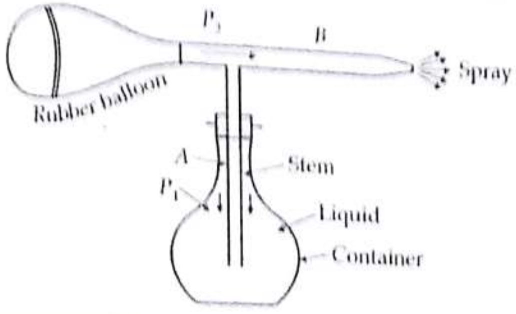


Fig. 10.36 Atomizer.

of the horizontal tube B decreasing the pressure to P_2 which is less than the atmospheric pressure P_1 in the container. As a result, the liquid rises up in the vertical tube A. When it collides with the high speed air in tube B it breaks up into a fine spray.

10.37 DYNAMIC LIFT

52. *What is dynamic lift ? If a ball is thrown and given a spin, then the path of the ball is curved more than in a usual spin free ball. Why ?*

Dynamic lift. *Dynamic lift is the force that acts on a body,* such as aeroplane wing, a hydrofoil or a spinning ball, *by virtue of its motion through a fluid.* It is responsible for the curved path of a spinning ball and the lift of an aircraft wing.

Curved path of a spinning ball : Magnus effect. When a ball is thrown horizontally with a large velocity and at the same time given a twisting motion to cause a spin, it deviates from its usual parabolic trajectory of spin free motion. This deviation can be explained on the basis of Bernoulli's principle.

When the ball spins about an axis perpendicular to its horizontal motion, it carries with itself an air of layer due to viscous drag. The streamlines around it are in the form of concentric circles, as shown in Fig. 10.37(*a*). When the ball moves forward with velocity v, the air ahead of the ball rushes backward with velocity v to fill the space left empty by the ball. Thus the streamlines in air due to translatory motion of the ball are of the form shown in Fig. 10.37(*b*). The layer above the ball moves in a direction opposite to that of the spinning ball, so the resultant velocity decreases and hence pressure increases in accordance with Bernoulli's principle. The layer below the ball moves in the direction of spin, the resultant velocity increases and hence pressure decreases. Due to the difference of pressure on the two sides of the ball, the ball curves downwards in the direction of spin, as shown in Fig. 10.37(*c*).

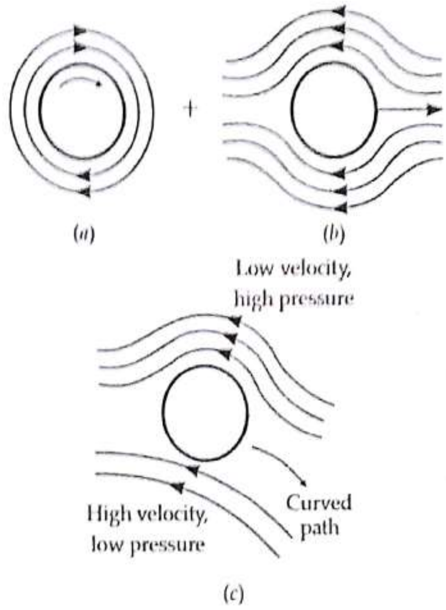


Fig. 10.37

The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards the greater pressure side, is called magnus effect. This effect was first noticed by German scientist H.G. Magnus in the mid-nineteenth century. The rougher the surface, the thicker is the layer of air dragged along by the spinning ball, and more curved the path.

53. *On the basis of Bernoulli's principle, explain the lift of an aircraft wing.*

Aerofoil : Lift of an aircraft wing. *Aerofoil is the name given to a solid object shaped to provide an upward vertical force as it moves horizontally through air.* This upward force (dynamic lift) makes aeroplanes fly.

As shown in Fig. 10.38, the cross-section of the wing of an aeroplane looks like an aerofoil. The wing is so designed that its upper surface is more curved (and hence longer) than the lower surface and the front edge

Scanned by CamScanner

is broader than the rear edge. As the aircraft moves, the air moves faster over the upper surface of the wing than on the bottom. According to Bernoulli's principle, the air pressure above the upper surface decreases below the atmospheric pressure and that on the lower surface increases above the atmospheric pressure. The difference in pressure provides an upward lift, called *dynamic lift*, to the aircraft.

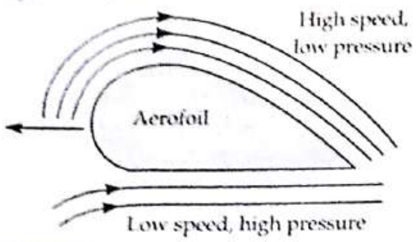


Fig. 10.38 Aerofoil.

10.38 BLOOD FLOW AND HEART ATTACK

54. *How does Bernoulli's principle help in explaining vascular flutter and heart attack ?*

Blood flow and heart attack. In persons suffering with advanced heart condition, the artery gets constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction, a greater demand is placed on the activity of the heart. The speed of blood flow increases in this region. From Bernoulli's principle, the inside pressure drops and the artery may collapse due to external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops leading to a repeat collapse. This phenomenon is called vascular flutter which can be heard on a stethoscope. This may result in a heart attack.

10.39 BLOWING OFF THE ROOFS DURING WIND STORM

55. *Why are the roofs of some houses blown off during a wind storm ?*

Blowing off the roof during wind storm. During certain wind storm or cyclone, the roofs of some houses are blown off without damaging the other parts of the house. The high wind blowing over the roof creates a low pressure P_2 in accordance with Bernoulli's principle. The pressure P_1 below the roof is equal to the atmospheric pressure which is larger than P_2. The difference of pressure $(P_1 - P_2)$ causes an upward thrust and the roof is lifted up. Once the roof is lifted up, it is blown off with the wind.

Fig. 10.39 Blowing off the roof during a wind storm.

Examples based on Equation of Continuity and Bernoulli's Theorem

FORMULAE USED

1. Volume of a liquid flowing per second through a pipe of cross-section a with velocity v, $Q = av$
2. Equation of continuity, av = constant
   or $a_1 v_1 = a_2 v_2$
3. First form of Bernoulli's theorem,
   $$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$
   or Pressure energy per unit mass + P.E. per unit mass + K.E. per unit mass = constant.
4. Second form of Bernoulli's theorem
   $$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$
   or Pressure head + Gravitational head + Velocity head = constant.
5. Volume of a liquid flowing out per second through a venturimeter,
   $$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$
   where a_1 and a_2 are the areas of cross-sections of bigger and smaller tubes respectively.
6. Torricell's theorem, velocity of efflux of a liquid through an orifice at depth h from the liquid surface, $v = \sqrt{2gh}$

UNITS USED

Rate of flow Q is in m^3s^{-1}, velocity v in ms^{-1}, area of cross-section a in m^2, pressure P in Nm^{-2}, density ρ in kgm^{-3} and height h in metre.

EXAMPLE 52. *Water flows through a horizontal pipe of varying area of cross-section at the rate of 10 cubic metre per minute. Determine the velocity of water at a point where radius of pipe is 10 cm.*

Solution. Rate of flow,

$$Q = \frac{\text{Volume}}{\text{Time}} = \frac{10\text{ m}^3}{60\text{ s}} = \frac{1}{6}\text{ m}^3\text{ s}^{-1}$$

Radius, $r = 10\text{ cm} = 0.1\text{ m}$

As $Q = av = \pi r^2 v$

$$\therefore \quad v = \frac{Q}{\pi r^2} = \frac{1 \times 7}{6 \times 22 \times (0.1)^2} = 5.303\text{ ms}^{-1}.$$

Scanned by CamScanner

EXAMPLE 53. Water flows through a horizontal pipe whose internal diameter is 2.0 cm, at a speed of 1.0 ms^{-1}. What should be the diameter of the nozzle, if the water is to emerge at a speed of 4.0 ms^{-1} ? [Central Schools 04]

Solution. Here $d_1 = 2$ cm $= 0.02$ m, $v_1 = 1$ ms^{-1}, $v_2 = 4$ ms^{-1}, $d_2 = ?$

Using equation of continuity,

$$a_1 v_1 = a_2 v_2 .$$

or
$$\frac{\pi d_1^2}{4} \times v_1 = \frac{\pi d_2^2}{4} \times v_2$$

or
$$d_2^2 = \frac{v_1}{v_2} \times d_1^2 = \frac{1}{4} \times (0.02)^2 = (0.01)^2$$

or
$$d_2 = 0.01 \text{ m} = \mathbf{1.0 \text{ cm}}.$$

EXAMPLE 54. At what speed will the velocity head of stream of water be 40 cm ?

Solution. Here $h = 40$ cm, $g = 980$ cms^{-2}

Velocity head, $h = \dfrac{v^2}{2g}$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 980 \times 40} = \mathbf{280 \text{ cms}^{-1}}.$$

EXAMPLE 55. At what speed will the velocity of a stream of water be equal to 20 cm of mercury column ? Take $g = 10$ ms^{-2}.

Solution. Velocity head = 20 cm of Hg

= 20 × 13.6 cm of water

But velocity head $= \dfrac{v^2}{2g}$

$$\therefore \quad 20 \times 13.6 = \frac{v^2}{2 \times 1000}$$

or
$$v = \sqrt{20 \times 13.6 \times 2 \times 1000} = 737.56 \text{ cms}^{-1}$$

$$= \mathbf{7.3756 \text{ ms}^{-1}}.$$

EXAMPLE 56. Calculate the total energy possessed by one kg of water at a point where the pressure is 20 gf / mm^2, velocity is 0.1 ms^{-1} and the height is 50 cm above the ground level.

Solution. Here

$$P = 20 \text{ gf mm}^{-2} = \frac{20}{1000} \times (10^{-3})^{-2} \text{ kg f m}^{-2}$$

$$= 20 \times 10^3 \times 9.8 \text{ Nm}^{-2} = 19.6 \times 10^4 \text{ Nm}^{-2}$$

$v = 0.1$ ms^{-1}, $h = 50$ cm $= 0.50$ m,

$\rho = 10^3$ kg m^{-3}

Pressure energy per kg $= \dfrac{P}{\rho} = \dfrac{19.6 \times 10^4}{10^3} = 196$ J

Gravitational P.E. per kg $= gh = 9.8 \times 0.50 = 4.90$ J

K.E. per kg $= \dfrac{1}{2} v^2 = \dfrac{1}{2} \times (0.1)^2 = 0.005$ J

Total energy possessed by per kg of water

$$= \frac{P}{\rho} + gh + \frac{1}{2} v^2 = 196 + 4.90 + 0.005 = 200.905 \text{ J}.$$

EXAMPLE 57. The reading of pressure meter attached with a closed pipe is 3.5×10^5 Nm^{-2}. On opening the valve of the pipe, the reading of the pressure meter is reduced to 3.0×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

Solution. *Before opening the valve :*

$$p_1 = 3.5 \times 10^5 \text{ Nm}^{-2}, \quad v_1 = 0$$

After opening the valve :

$$p_2 = 3.0 \times 10^5 \text{ Nm}^{-2}, \quad v_2 = ?$$

In horizontal flow, P.E. remains unchanged. So Bernoulli's theorem can be written as

$$p_2 + \frac{1}{2} \rho v_2^2 = p_1 + \frac{1}{2} \rho v_1^2$$

$$3.0 \times 10^5 + \frac{1}{2} \times 10^3 \times v_2^2 = 3.5 \times 10^5 + \frac{1}{2} \times 10^3 \times (0)^2$$

$$\frac{1}{2} \times 10^3 \times v_2^2 = (3.5 - 3.0) \times 10^5 = 0.5 \times 10^5$$

or
$$v_2^2 = 2 \times 0.5 \times 10^2 = 100$$

or
$$v_2 = \mathbf{10 \text{ ms}^{-1}}.$$

EXAMPLE 58. A fully loaded Boeing aircraft 747 has a mass of 3.3×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 960 km/h. (a) Estimate the pressure difference between the lower and upper surfaces of the wings. (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. The density of air is $\rho = 1.2$ kg m^{-3} and $g = 9.81$ ms^{-2}. [NCERT]

Solution. (*a*) For the Boeing aircraft in level flight, upward force due to the pressure difference = weight of the aircraft

or
$$\Delta p \times A = mg$$

or
$$\Delta p = \frac{mg}{A} = \frac{3.3 \times 10^5 \times 9.81}{500}$$

$$= 6.47 \times 10^3 \text{ Nm}^{-2} \simeq \mathbf{6.5 \times 10^3 \text{ Nm}^{-2}}.$$

(*b*) If v_1 and v_2 are the speeds of air on the lower and the upper surfaces of the wings of the aircraft and p_1 and p_2 are the corresponding pressures, then from Bernoulli's principle, we have

$$p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$$

or
$$p_1 - p_2 = \frac{\rho}{2} (v_2^2 - v_1^2)$$

or
$$\Delta p = \rho \left(\frac{v_2 + v_1}{2} \right) (v_2 - v_1) = \rho \, v_{av} (v_2 - v_1)$$

Scanned by CamScanner

Here $v_{av} = \dfrac{v_2 + v_1}{2} = 960\ \text{km h}^{-1} = 267\ \text{ms}^{-1}$

$$\therefore \quad \frac{v_2 - v_1}{v_{av}} = \frac{\Delta p}{\rho v_{av}^2} = \frac{6.5 \times 10^3}{1.2 \times (267)^2} = 0.08 = 8\%.$$

Thus the speed of air on the upper surface of the wing is about 8% higher than that below the lower surface.

EXAMPLE 59. *Calculate the minimum pressure required to force the blood from the heart to the top of the head (vertical distance = 50 cm). Assume the density of blood to be 1.04 g cm^{-3}. Friction is to be neglected.*

Solution. Here $h_2 - h_1 = 50$ cm, $\rho = 1.04$ g cm^{-3}

According to Bernouilli's theorem,

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

or
$$P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2}\rho (v_2^2 - v_1^2)$$

If $v_2 = v_1$,

then
$$P_1 - P_2 = \rho g (h_2 - h_1)$$
$$= 1.04 \times 981 \times 50 \text{ dyne cm}^{-2}$$
$$= \mathbf{5.1 \times 10^4\ dyne\ cm^{-2}}.$$

EXAMPLE 60. *Water is flowing through two horizontal pipes of different diameters which are connected together. In the first pipe the speed of water is 4 ms^{-1} and the pressure is 2.0×10^4 Nm^{-2}. Calculate the speed and pressure of water in the second pipe. The diameters of the pipes are 3 cm and 6 cm respectively.*

Solution. According to equation of continuity,

$$a_1 v_1 = a_2 v_2$$

or
$$\pi r_1^2 v_1 = \pi r_2^2 v_2 \quad \text{or} \quad v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Here
$r_1 = 1.5$ cm $= 1.5 \times 10^{-2}$ m,

$r_2 = 3$ cm $= 3 \times 10^{-2}$ m and $v_1 = 4$ ms^{-1}

$$\therefore \quad v_2 = \left(\frac{1.5 \times 10^{-2}}{3 \times 10^{-2}}\right)^2 \times 4 = \mathbf{1\ ms^{-1}}$$

Applying Bernouilli's theorem for horizontal flow,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or
$$P_2 = P_1 + \frac{1}{2}\rho (v_1^2 - v_2^2)$$

But ρ (water) $= 10^3$ kgm^{-3}, $v_1 = 4$ ms^{-1}, $v_2 = 1$ ms^{-1},
$P_1 = 2.0 \times 10^4$ Nm^{-2}

$$\therefore \quad P_2 = 2.0 \times 10^4 + \frac{1}{2} \times 10^3 \times (4^2 - 1^2)$$
$$= \mathbf{2.75 \times 10^4\ Nm^{-2}}.$$

EXAMPLE 61. *The cross-sectional area of water pipe entering the basement is 4×10^{-4} m^2. The pressure at this point is 3×10^5 Nm^{-2} and the speed of water is 2 ms^{-1}. This pipe tapers to a cross-sectional area of 2×10^{-4} m^2 when it reaches the second floor 8 m above. Calculate the speed and pressure at the second floor.*

Solution. Here $v_1 = 2$ ms^{-1}, $a_1 = 4 \times 10^{-4}$ m^2, $P_1 = 3 \times 10^5$ Nm^{-2} and $a_2 = 2 \times 10^{-4}$ m^2, $v_2 = ?$, $P_2 = ?$
$\rho = 1000$ kgm^{-3}, $(h_2 - h_1) = 8$ m

Using continuity equation,

$$a_1 v_1 = a_2 v_2$$

or
$$v_2 = \frac{a_1 v_1}{a_2} = \frac{4 \times 10^{-4} \times 2}{2 \times 10^{-4}} = \mathbf{4\ ms^{-1}}.$$

According to Bernouilli's theorem

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + g h_1 = \frac{P_2}{\rho} + g h_2 + \frac{1}{2}v_2^2$$

$$\frac{P_2}{\rho} = \frac{P_1}{\rho} - g(h_2 - h_1) - \frac{1}{2}(v_2^2 - v_1^2)$$

or
$$P_2 = P_1 - g\rho(h_2 - h_1) - \frac{1}{2}\rho(v_2^2 - v_1^2)$$
$$= 3 \times 10^5 - 9.8 \times 1000 \times 8 - \frac{1}{2} \times 1000(16 - 4)$$
$$= 3 \times 10^5 - 78.4 \times 1000 - \frac{1}{2} \times 12 \times 1000$$
$$= (3 - 0.784 - 0.06) \times 10^5 = \mathbf{2.156 \times 10^5\ Nm^{-2}}.$$

EXAMPLE 62. *The pressure difference between two points along a horizontal pipe, through which water is flowing, is 1.4 cm of mercury. If, due to non-uniform cross-section, the speed of flow of water at the point of greater cross-section is 60 cm s^{-1}, calculate the speed at the other point.*

Solution. Using Bernoulli's theorem for horizontal flow,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or
$$v_2^2 - v_1^2 = \frac{2}{\rho}(P_1 - P_2)$$

The speed of water will be greater at the place where the cross-section is smaller.

$$\therefore \quad v_2^2 = \frac{2}{\rho}(P_1 - P_2) + v_1^2$$

But $P_1 - P_2 = 1.4$ cm of mercury $= h\rho' g$
$$= 1.4 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8$$
$$= 1.866 \times 10^3 \text{ Nm}^{-2}$$

Also

ρ (water) $= 10^3$ kgm^{-3}, $v_1 = 60$ cms^{-1} $= 0.6$ ms^{-1}

$$\therefore \quad v_2^2 = \frac{2}{10^3} \times 1.866 \times 10^3 + (0.6)^2 = 4.092$$

or
$$v_2 = \mathbf{2\ ms^{-1}}.$$

Scanned by CamScanner

EXAMPLE 63. *A pitot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of 40 cm. What is the speed of the plane relative to air ? (sp. gr. of alcohol = 0.8 and density of air = 1 kg m^{-3}).*

Solution. The plane of aperture at A is parallel to the direction of air flow, so velocity of air at A is same as that of air in main pipe (which is equal to the velocity v of the plane w.r.t. air. The plane of aperture at B is perpendicular to the direction of air flow, so velocity of air entering the tube is reduced to zero. Applying Bernoulli's theorem at points A and B,

$$p_A + \frac{1}{2}\rho v_A^2 = p_B + \frac{1}{2}\rho v_B^2$$

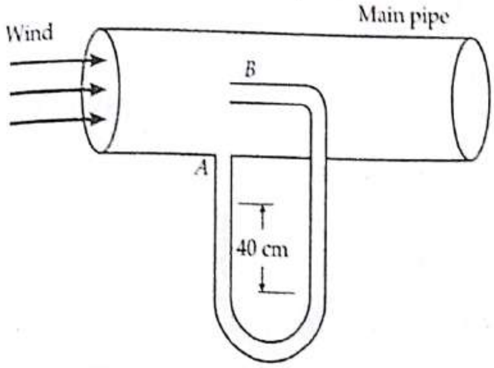


Fig. 10.40

But $v_1 = v$ and $v_B = 0$, so

$$\frac{1}{2}\rho v^2 = p_B - p_A = h\rho' g$$

where ρ' is the density of the liquid in the U-tube.

Hence $v = \sqrt{\dfrac{2h\rho' g}{\rho}}$

Now $h = 40\text{ cm} = 0.4\text{ m}$, $\rho' = 0.8 \times 10^3\text{ kgm}^{-3}$, $\rho = 1\text{ kgm}^{-3}$

$$\therefore \quad v = \sqrt{\frac{2 \times 0.4 \times 0.8 \times 10^3 \times 9.8}{1}}$$

$$= 56\sqrt{2} = 56 \times 1.414 = \mathbf{79.18\ ms^{-1}}.$$

EXAMPLE 64. *A pitot tube is fixed in a main pipe of diameter 20 cm and difference of pressure indicated by the gauge is 5 cm of water column. Find the volume of water passing through the main pipe in one minute.*

Solution. As both main pipe and U-tube contain water, so from the above example, we have

$$\frac{1}{2}\rho v^2 = h\rho g$$

or $\quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.05} = 0.7\sqrt{2}\text{ ms}^{-1}$

Volume of water flowing per second,

$$Q = av = \pi r^2 v$$

$$= \frac{22}{7} \times (0.10)^2 \times 0.7\sqrt{2} = 0.0311\text{ m}^3\text{ s}^{-1}$$

Volume of water passing in one minute,

$$V = Qt = 0.0311 \times 60 = \mathbf{1.866\ m^3}.$$

EXAMPLE 65. *A cylinder of height 20 m is completely filled with water. Find the velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom. Given $g = 10$ ms^{-2}.* **[AIEEE 02]**

Solution. Here $h = 20$ m, $g = 10\text{ ms}^{-2}$

Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = \mathbf{20\ ms^{-1}}.$$

EXAMPLE 66. *At what velocity does water emerge from an orifice in a tank in which gauge pressure is 3×10^5 Nm^{-2} before the flow starts ? Density of water = 1000 kgm^{-3}.*

Solution. Here $p = 3 \times 10^5\text{ Nm}^{-2}, \rho = 1000\text{ kgm}^{-3}$, $g = 9.8\text{ ms}^{-2}$

As $\quad p = h\rho g$

$$\therefore \quad h = \frac{p}{\rho g} = \frac{3 \times 10^5}{1000 \times 9.8}\text{ m}$$

Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{\frac{2 \times 9.8 \times 3 \times 10^5}{1000 \times 9.8}}$$

$$= \sqrt{600} = \mathbf{24.495\ ms^{-1}}.$$

EXAMPLE 67. *A boat strikes an under water rock which punctures a hole 5 cm in diameter in its hull which is 1.5 m below the water line. At what rate in litre per second does water enter ?*

Solution. Here $r = \frac{5}{2}\text{ cm} = 2.5\text{ cm}$, $h = 1.5\text{ m} = 150\text{ cm}$

Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{2 \times 980 \times 150} = 140\sqrt{15}\text{ cms}^{-1}$$

Rate at which water enters,

$$Q = av = \pi r^2 v = \frac{22}{7} \times (2.5)^2 \times 140\sqrt{15}\text{ cm}^3\text{ s}^{-1}$$

$$= 2750 \times 3.873 \times 10^{-3}\text{ litres s}^{-1}$$

$$= \mathbf{10.65\ litres\ s^{-1}}.$$

EXAMPLE 68. *A drum of 30 cm radius has a capacity of 220 dm^3 of water. It contains 198 dm^3 of water and is placed on a solid block of exactly the same size as of drum. If a small hole is made at lower end of drum perpendicular to its length, find the horizontal range of water on the ground in the beginning. Given $g = 980$ cm s^{-2}.*

Solution. Radius of the drum, $r = 30$ cm

Volume of the drum $= 220\text{ dm}^3 = 2.2 \times 10^5\text{ cm}^3$

Let l be the height of the drum. Then

$$\pi r^2 l = 2.2 \times 10^5$$

or $\quad l = \dfrac{2.2 \times 10^5 \times 7}{22 \times 30 \times 30} = \dfrac{700}{9}\text{ cm}$

Scanned by CamScanner

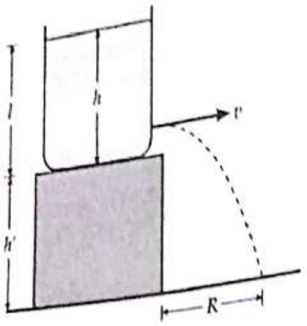


Fig. 10.41

Height of block $= h' = l = \frac{700}{9}$ cm

Volume of water $= 198\ dm^3 = 1.98 \times 10^5\ cm^3$

Let h be height of the water column in the drum. Then

$$\pi r^2 h = 1.98 \times 10^5$$

or
$$h = \frac{1.98 \times 10^5 \times 7}{22 \times 30 \times 30}\ \text{cm} = 70\ \text{cm}$$

Time taken by water to reach the ground,

$$t = \sqrt{\frac{2h'}{g}}$$

Efflux velocity of water $= \sqrt{2gh}$

If R be the horizontal range, then

R = Time taken by water to reach the ground × Efflux velocity of water

$$= \sqrt{\frac{2h'}{g}} \times \sqrt{2gh} = 2\sqrt{hh'}$$

$$= 2\sqrt{70 \times \frac{700}{9}} = \mathbf{147.6\ cm.}$$

Example 69. ***Blood velocity :*** *The flow of blood in a large artery of an anesthetized dog is diverted through a Venturi meter. The wider part of the meter has a cross-sectional area equal to that of the artery, $A = 8\ mm^2$. The narrower part has an area $a = 4\ mm^2$. The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery ?* **[NCERT]**

Solution. The Bernoulli's equation for the horizontal flow is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

By equation of continuity,

$$Av_1 = av_2 \quad \text{or} \quad v_2 = Av_1/a$$

$$\therefore \quad p_1 - p_2 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\frac{\rho A^2 v_1^2}{a^2}$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{A^2}{a^2} - 1\right)$$

Here $p_1 - p_2 = 24$ Pa,

ρ (blood) $= 1.06 \times 10^3\ kg\ m^{-3}$, $A/a = 8/4 = 2$

$$\therefore \quad v_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(\frac{A^2}{a^2} - 1\right)}}$$

$$= \sqrt{\frac{2 \times 24}{1.06 \times 10^3 \times (2^2 - 1)}} = \mathbf{0.125\ ms^{-1}}.$$

Example 70. *A horizontal tube has different cross-sectional areas at points A and B. The diameter of A is 4 cm and that of B is 2 cm. Two manometer limbs are attached at A and B. When a liquid of density 8.0 g cm^{-3} flows through the tube, the pressure difference between the limbs of the manometer is 8 cm. Calculate the rate of flow of the liquid in the tube.* **[ISM Dhanbad 82]**

Solution. Here $a_1 = \pi r_1^2 = \pi \times \left(\frac{4}{2}\right)^2 = 4\pi\ cm^2$,

$$a_2 = \pi r_2^2 = \pi\left(\frac{2}{2}\right)^2 = \pi\ cm^2,\ h = 8\ cm,\ g = 980\ cms^{-2}$$

Rate of flow of the liquid,

$$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}} = 4\pi \times \pi\sqrt{\frac{2 \times 980 \times 8}{(4\pi)^2 - (\pi)^2}}$$

$$= 4\pi\sqrt{\frac{2 \times 980 \times 8}{15}} = 4 \times 3.14 \times 32.3$$

$$= \mathbf{406\ cm^3\ ms^{-1}}.$$

Example 71. *Water is filled in a cylindrical container to a height of 3 m, as shown in Fig. 10.42. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. Find the speed of the liquid coming out from the orifice. Given $g = 10\ ms^{-2}$.* **[IIT 05]**

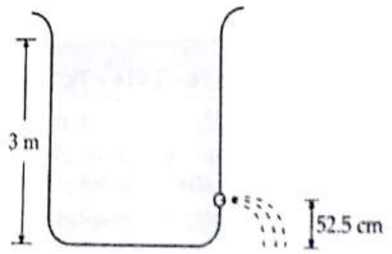


Fig. 10.42

Solution. Let a_1 and a_2 be the area of cross-sections, and v_1 and v_2 be the liquid velocities for the beaker and the orifice respectively.

According to the equation of continuity,

$$a_1 v_1 = a_2 v_2$$

or
$$v_1 = \frac{a_2}{a_1} v_2$$

According to Bernoulli's equation,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

But $p_1 = p_2$ = atmospheric pressure. Therefore

$$g h_1 + \frac{1}{2} v_1^2 = g h_2 + \frac{1}{2} v_2^2$$

or $$g h_1 + \frac{1}{2}\left(\frac{a_2}{a_1} v_2\right)^2 = g h_2 + \frac{1}{2} v_2^2$$

or $$v_2^2\left[1 - \left(\frac{a_2}{a_1}\right)^2\right] = 2g(h_1 - h_2)$$

or $$v_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

Here $\frac{a_2}{a_1} = 0.1$, $g = 10\ \text{ms}^{-2}$, $h_1 = 3$ m

$$h_2 = 52.5\ \text{cm} = 0.525\ \text{m}$$

$$\therefore \quad h_1 - h_2 = 3 - 0.525 = 2.475\ \text{m}$$

$$v_2 = \sqrt{\frac{2 \times 10 \times 2.475}{1 - (0.1)^2}} = \sqrt{50} = \mathbf{7.07\ ms^{-1}}.$$

PROBLEMS FOR PRACTICE

1. Water flows through a horizontal pipe of varying cross-section at the rate of 20 litres per minute. Determine the velocity of water at a point where diameter is 4 cm. (**Ans.** 2639 ms^{-1})
2. A garden hose having an internal diameter 2.0 cm is connected to a lawn sprinkler that consists of an enclosure with 24 holes, each 0.125 cm in diameter. If water in the hose has a speed of 90.0 cms^{-1}, find the speed of the water leaving the sprinkler holes. (**Ans.** 9.6 cms^{-1})
3. In a normal adult, the average speed of the blood through the aorta (which has a radius of 0.9 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.5 cm. Calculate the speed of blood through the arteries. (**Ans.** 0.036 ms^{-1})
4. Velocity of flow of water in a horizontal pipe is 4.9 ms^{-1}. Find the velocity head of water. (**Ans.** 1.225 m)
5. Water flows into a horizontal pipe whose one end is closed with a valve and the reading of a pressure gauge attached to the pipe is $3 \times 10^5\ \text{Nm}^{-2}$. This reading of the pressure gauge falls to $1 \times 10^5\ \text{Nm}^{-2}$ when the valve is opened. Calculate the speed of water flowing into the pipe. **[MNREC 91]** (**Ans.** 20 ms^{-1})
6. Water enters at one end of a horizontal pipe of non-uniform cross-section with a velocity of 0.4 ms^{-1} and leaves the other end with a velocity of 0.6 ms^{-1}. The pressure of water at the first end is 1500 Nm^{-2}. Calculate the pressure at the other end. Density of water = 1000 kgm^{-3}. (**Ans.** 1400 Nm^{-2})
7. Water is flowing with a speed of 2 m/s in a horizontal pipe with cross-sectional area decreasing from $2 \times 10^{-2}\ \text{m}^2$ to $0.01\ \text{m}^2$ at pressure 4×10^4 Pa. What will be the pressure at small cross-section ? **[Central Schools 05]** (**Ans.** 3.4×10^4 Pa)
8. A tank containing water has an orifice 10 m below the surface of water in the tank. If there is no wastage of energy, find the speed of discharge. (**Ans.** 14 ms^{-1})
9. Calculate speed of efflux of kerosene from an orifice of a tank in which pressure is 4 atmosphere. Density of kerosene is 0.72 kg per litre. One atmosphere = 1 kg f cm^{-2}. (**Ans.** 33.5 ms^{-1})
10. Water flows at the rate of 4 litres per second through an orifice at the bottom of tank which contains water 720 cm deep. Find the rate of escape of water if additional pressure of 16 kg f / cm^2 is applied at the surface of water. (**Ans.** 19.28 litre s^{-1})
11. The diameter of a pipe at two points, where a venturi meter is connected is 8 cm and 5 cm and the difference of levels in it is 4 cm. Calculate the volume of water flowing through the pipe per second. (**Ans.** 1889 cm^3s^{-1})
12. A venturi meter is 37.5 cm in diameter in mains and 15 cm diameter in throat. The difference between the pressure of water in the mains and the throat is 23 cm of Hg. Find the discharge in litres per minute. Sp. gravity of Hg = 13.56. (**Ans.** 8400 litre min^{-1})

HINTS

1. Rate of flow,

$$Q = \frac{20\ \text{litre}}{1\ \text{minute}} = \frac{20 \times 10^{-3}\ \text{m}^3}{60\ \text{s}} = \frac{1}{3} \times 10^{-3}\ \text{m}^3\ \text{s}^{-1}$$

$$v = \frac{Q}{a} = \frac{Q}{\pi r^2} = \frac{7 \times 10^{-3}}{3 \times 22 \times (0.02)^2} = \mathbf{0.2639\ ms^{-1}}.$$

2. Volume of water crossing hose per second
= Volume of water leaving the holes per second

$$\frac{\pi D^2}{4} \times V = n \frac{\pi d^2}{4} \times v$$

or $$v = \frac{D^2 V}{n d^2} = \frac{2^2 \times 90.0}{24 \times (0.125)^2} = \mathbf{9.6\ cms^{-1}}.$$

Scanned by CamScanner

3. $a_1 v_1 = 30\, a_2 v_2$ or $\pi r_1^2 v_1 = 30\, \pi r_2^2 v_2$

$$\therefore \quad v_2 = \left(\frac{r_1}{r_2}\right)^2 \frac{v_1}{30} = \left(\frac{0.9}{0.5}\right)^2 \times \frac{0.33}{30} = 0.036 \text{ ms}^{-1}.$$

4. Velocity head $= \dfrac{v^2}{2g} = \dfrac{(4.9)^2}{2 \times 9.8} = 1.225$ m.

5. Applying Bernoulli's theorem for horizontal flow, we get

$$p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

$$\therefore \quad 1 \times 10^5 + \frac{1}{2} \times 1000 \times v_2^2 = 3 \times 10^5 + \frac{1}{2} \times 1000 \times 0$$

or $$v_2^2 = \frac{(3 \times 10^5 - 1 \times 10^5) \times 2}{1000}$$

$$= \frac{2 \times 10^5 \times 2}{1000} = 400$$

or $v_2 = 20 \text{ ms}^{-1}$.

7. Here $v_1 = 2 \text{ ms}^{-1}$, $a_1 = 2 \times 10^{-2} \text{ m}^2$, $a_2 = 0.01 \text{ m}^2$

$p_1 = 3 \times 10^4$ Pa, $p_2 = ?$, $\rho = 10^3 \text{ kg m}^{-3}$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{2 \times 10^{-2} \times 2}{0.01} = 4 \text{ ms}^{-1}$$

As $p_1 + \dfrac{1}{2}\rho v_1^2 = p_2 + \dfrac{1}{2}\rho v_2^2$

$$\therefore \quad p_2 = p_1 - \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= 4 \times 10^4 - \frac{1}{2} \times 10^3 (2^2 - 4^2)$$

$$= 3.4 \times 10^4 \text{ Pa}.$$

9. Here $p = 4$ atmosphere $= 4 \times 1.03$ kg f cm^{-2}

$= 4 \times 1.03 \times 9.8 \times 10^4 \text{ Nm}^{-2}$.

Density, $\rho = 0.72$ kg litre^{-1} $= 0.72 \times 1000 \text{ kg m}^{-3}$

If the orifice be at depth h below the surface of oil, then

$$p = h\rho g \quad \text{or} \quad gh = \frac{p}{\rho}$$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{\frac{2p}{\rho}}$$

$$= \sqrt{\frac{2 \times 4 \times 1.03 \times 9.8 \times 10^4}{0.72 \times 1000}} = 33.5 \text{ ms}^{-1}.$$

10. Here $h = 720$ cm

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 980 \times 720} \text{ cms}^{-1}$$

Additional pressure $= 16$ kg f cm^{-2} $= 1600$ gf cm^{-2}

$$= \frac{16000 \times 980}{980} \text{ cm of water}$$

$= 16000$ cm of water

New pressure head, $h_1 = 16000 + 720 = 16720$ cm

New velocity,

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 980 \times 16720} \text{ cms}^{-1}$$

Rate of flow, $Q = av$ and $Q_1 = av_1$

$$Q_1 = \frac{v_1}{v} Q = \sqrt{\frac{16720}{720}} \times 4 \text{ litre s}^{-1}$$

$= 19.28$ litre s^{-1}.

11. Here $a_1 = \pi r_1^2 = \pi (4)^2 = 16\pi \text{ cm}^2$,

$a_2 = \pi r_2^2 = \pi (2.5)^2 = 6.25\, \pi \text{ cm}^2$

$$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

$$= 6.25\, \pi \times 16\, \pi \sqrt{\frac{2 \times 980 \times 4}{(16\, \pi)^2 - (6.25\, \pi)^2}}$$

$= 1889 \text{ cm}^3 \text{ s}^{-1}$.

12. Here $h = 23$ cm of Hg $= 23 \times 13.56$ cm of water

$$r_1 = \frac{37.5}{2} = 18.75 \text{ cm}, \quad r_2 = \frac{15}{2} = 7.5 \text{ cm}$$

$a_1 = \pi r_1^2 = \pi (18.75)^2 \text{ cm}^2$, $r_2 = \pi r_2^2 = \pi (7.5)^2 \text{ cm}^2$

Rate of flow,

$$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

$$= \pi (18.75)^2 \times \pi (7.5)^2 \sqrt{\frac{2 \times 980 \times 23 \times 13.56}{[\pi (18.75)^2]^2 - [\pi (7.5)^2]^2}}$$

$$= \pi \times (18.75)^2 \times (7.5)^2 \sqrt{\frac{2 \times 980 \times 23 \times 13.56}{(18.75)^4 - (7.5)^4}} \text{ cm}^3 \text{s}^{-1}$$

$$= \pi \times (18.75)^2 \times (7.5)^2 \sqrt{\frac{2 \times 980 \times 23 \times 13.56}{120432.13}}$$

$= 1.4 \times 10^5 \text{ cm}^3 \text{ s}^{-1}$

$$= \frac{1.4 \times 10^5 \times 60}{1000} = 8400 \text{ litre min}^{-1}.$$

10.40 COHESIVE AND ADHESIVE FORCES

56. *What are cohesive and adhesive forces ? Give examples.*

(*i*) **Cohesive force.** *It is the force of attraction between the molecules of the same substance.*

Example. Solids have definite shape and size due to strong forces of cohesion amongst their molecules.

(*ii*) **Adhesive force.** *It is the force of attraction between the molecules of two different substances.*

Example. It is due to force of adhesion that ink sticks to paper while writing.

Water wets the walls of its glass container because the force of adhesion between water and glass is greater than the force of cohesion between the water molecules.

Scanned by CamScanner

On the contrary, mercury does not wet glass because the force of cohesion between the mercury molecules is much greater than the force of adhesion between mercury and glass.

10.41 MOLECULAR RANGE

57. *Define the terms molecular range, sphere of influence and surface film.*

Molecular range. *It is the maximum distance upto which a molecule can exert some appreciable force of attraction on other molecules.* It is of the order of 10^{-9} m in solids and liquids.

Sphere of influence. *A sphere drawn around a molecule as centre and with a radius equal to the molecular range is called the sphere of influence of the molecule.* The molecule at the centre attracts all the molecules lying in its sphere of influence.

While studying the behaviour of a molecule under the influence of cohesive forces, we need to consider only the molecules lying in its sphere of influence.

Surface film. *A thin film of liquid near its surface having thickness equal to the molecular range for that liquid is called surface film.*

10.42 SURFACE TENSION

58. *Define surface tension. Give its units and dimensions.*

Surface tension. A steel needle may be made to float on water though the steel is more dense than water. This is because the water surface acts as a stretched elastic membrane and supports the needle. This property of a liquid is called surface tension.

***Surface tension** is the property by virtue of which the free surface of a liquid at rest behaves like an elastic stretched membrane tending to contract so as to occupy minimum surface area.*

As shown in Fig. 10.43, imagine a line *AB* on the free surface of a liquid. The small elements of the surface on this line are in equilibrium because they are acted upon by equal and opposite forces, acting perpendicular to the line from either side. The force acting on this line is proportional to the length of this line. If l is the length of the imaginary line and F the total force on either side of the line, then

$$F \propto l \quad \text{or} \quad F = \sigma l$$

or $$\sigma = \frac{F}{l}$$

or $$\text{Surface tension} = \frac{\text{Force}}{\text{Length}}$$

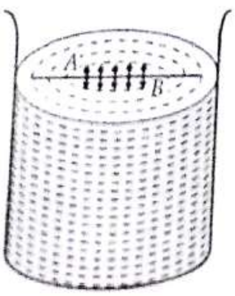


Fig. 10.43 Definition of surface tension.

Surface tension is measured as the force acting per unit length of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line and tangential to the liquid surface.

Units and dimensions of surface tension :

SI unit of surface tension $= \text{Nm}^{-1}$

CGS unit of surface tension = dyne cm^{-1}

Dimensions of surface tension

$$= \frac{[\text{Force}]}{[\text{Length}]} = \frac{\text{MLT}^{-2}}{\text{L}} = [\text{MT}^{-2}].$$

10.43 MOLECULAR THEORY OF SURFACE TENSION

59. *Explain surface tension on the basics of molecular theory.*

Molecular theory of surface tension. In Fig. 10.44, *PQRS* is the surface film of a liquid. Consider the molecule *A* well inside the liquid. It is attracted equally in all directions by the molecules lying in its sphere of influence. Net force on such a molecule is zero.

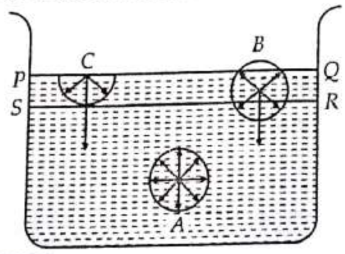


Fig. 10.44 Cohesive force acting on a molecule of water.

Now consider molecule *B* lying inside the surface film. Its sphere of influence lies partly outside. This molecule experiences less force upward and more force downward by the molecules in its sphere of influence. For molecule *C*, half its sphere of influence lies above the surface. The resultant downward force on such a molecule is maximum. Due to this downward force, the potential energy of the molecules of the surface film is higher than those lying well inside the liquid. For a system to be stable, potential energy must be minimum. For the surface film to have minimum energy, the number of molecules in it must be minimum. Thus the surface film tends to have minimum surface area. As a result, the free surface of a liquid at rest behaves like an elastic stretched membrane.

10.44 SOME PHENOMENA BASED ON SURFACE TENSION

60. *Explain some examples which illustrate the existence of surface tension.*

Examples to illustrate surface tension :

(*i*) **Needle supported on water surface.** Take a greased needle of steel on a piece of blotting paper and

Scanned by CamScanner

place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating. The floating needle causes a little depression. The forces F, F due to surface tension of the curved surface are inclined as shown in Fig. 10.45. The vertical components of these two forces support the weight of the needle.

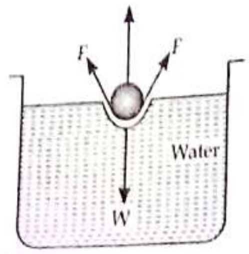


Fig. 10.45 A needle floating on water.

(*ii*) **Endless wet thread on a soap film.** If we take a circular frame of a stiff wire and dip it into a soap solution, a thin soap film is formed on the frame. If a wet endless thread loop is gently placed over the film, it takes any irregular shape. But when the film is pricked at the centre, the loop is stretched outwards and takes a symmetrical circular shape. This is because for a given length a circle has the maximum surface area and so the outer liquid film tries to occupy minimum possible area like a stretched elastic membrane.

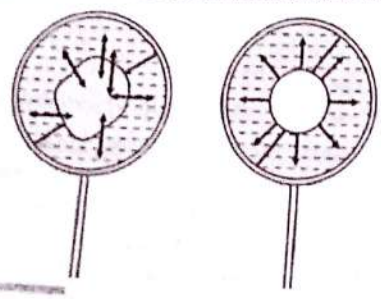


Fig. 10.46 A soap film in a circular frame.

(*iii*) **Rain drops are generally spherical in shape.** Due to surface tension, the rain drops tend to minimise their surface area and the surface area of a sphere is minimum for a given volume.

(*iv*) **Small mercury droplets are spherical and larger ones tend to flattened.** Small mercury droplets are spherical because the forces of surface tension tend to reduce their area to a minimum value and a sphere has minimum surface area for a given volume.

Fig. 10.47 Flattering of large mercury drops.

Larger drops of mercury are flattened due to the large gravitational force acting on them. Here the shape is such that the sum of the gravitational potential energy and the surface potential energy must be minimum. Hence the centre of gravity moves down as low as possible. This explains flattening of the larger drops.

(*v*) **The hair of a painting brush cling together when taken out of water.** This is because the water films formed on them tend to contract to minimum area.

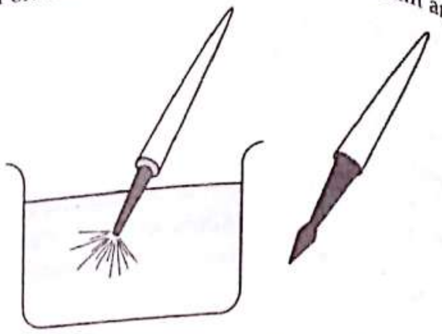


Fig. 10.48 Hair cling due to surface tension.

(*vi*) **A bug floats on water due to surface tension.** As shown in Fig. 10.49, a bug bends its legs on the surface of water such that the deformed surface gives rise to forces of surface tension which act tangential to the deformed surfaces. The weight of the bug is balanced by the upward components of these forces of surface tension.

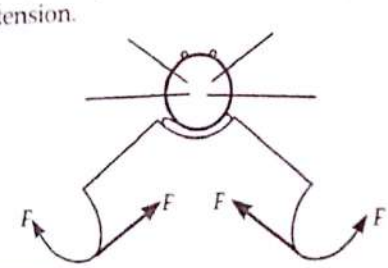


Fig. 10.49 A bug supported on water surface by surface tension.

(*vii*) **Plateau's experiment.** This experiment demonstrates beautifully that a liquid drop assumes spherical shape in the absence of gravitational forces. Prepare a mixture of alcohol and water such that its density is equal to that of olive oil and put it in a glass beaker. Introduce a large drop of olive oil in the

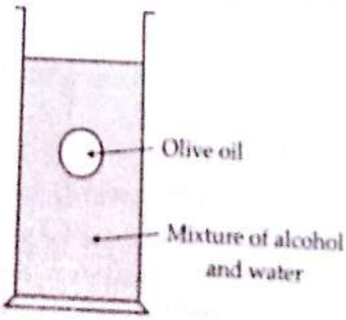


Fig. 10.50 Spherical shape in the absence of gravity.

mixture. The drop floa
ball. The upthrust on
weight. So gravitation
sufficiently large drop
(*viii*) **Oil spreads**
drop on hot water. T
of oil is less tha
than that of the hot

10.45 SURFAC

61. *Define surfa*
equal to the surfac

Surface energy
minimum area d
surface area, m
interior to the su
forces of attracti
energy of the m
cules at the surf
molecules in th

The extra e
film of unit area
called surface
increasing the

Surface e

The SI u

The rela
tension. C
the wire
solution.
inward d

Here
two free

Fig.

S
posi

Scanned by CamScanner

mixture. The drop floats in the mixture as a spherical ball. The upthrust on the drop becomes equal to its weight. So gravitational forces get eliminated. Even a sufficiently large drop is also spherical.

(*viii*) **Oil spreads on cold water but remains as a drop on hot water.** This is because the surface tension of oil is less than that of the cold water but it is greater than that of the hot water.

10.45 SURFACE ENERGY

61. *Define surface energy. Prove that it is numerically equal to the surface tension.*

Surface energy. The free surface of a liquid possesses minimum area due to surface tension. To increase the surface area, molecules have to be brought from interior to the surface. Work has to be done against the forces of attraction. This work is stored as the potential energy of the molecules on the surface. So the molecules at the surface have extra energy compared to the molecules in the interior.

The extra energy possessed by the molecules of surface film of unit area compared to the molecules in the interior is called surface energy. It is equal to the work done in increasing the area of the surface film by unit amount.

$$\text{Surface energy} = \frac{\text{Work done}}{\text{Increase in surface area}}$$

The SI unit of surface energy is Jm^{-2}.

The relation between surface energy and surface tension. Consider a rectangular frame $ABCD$ in which the wire AB is movable. Dip the frame in soap solution. A film is formed which pulls the wire AB inward due to surface tension with a force,

$$F = 2\,\sigma \times l$$

Here the factor 2 is taken because the soap film has two free surfaces.

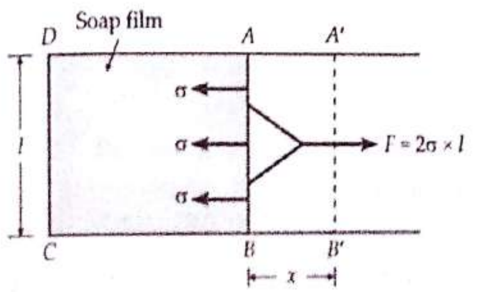


Fig. 10.51 Surface energy.

Suppose AB is moved out through distance x to the position $A'B'$. Then

$$\text{Work done} = \text{Force} \times \text{distance}$$
$$= 2\sigma \times l \times x$$

$$\text{Increase in surface area of film} = 2lx$$

$$\therefore \quad \text{Surface energy} = \frac{\text{Work done}}{\text{Increase in surface area}} = \frac{2\sigma lx}{2lx} = \sigma$$

Thus surface energy of liquid is numerically equal to its surface tension.

10.46 EXPERIMENTAL MEASUREMENT OF SURFACE TENSION

62. *Describe a simple experiment for measuring the surface tension of a liquid.*

Measurement of surface tension. As shown in Fig. 10.52, suspend a rectangular glass plate from one arm of a sensitive balance. Place a beaker containing some liquid below it. The plate is balanced by weights on the other side, with its lower edge just above water.

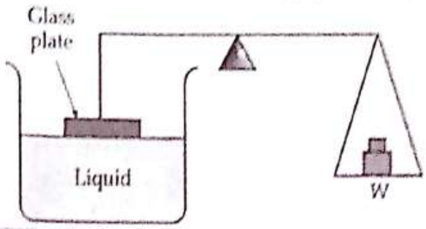


Fig. 10.52 Measurement of surface tension.

The beaker is raised slightly till the liquid just touches the glass plate and pulls it down a little because of surface tension. Weights are added on the other side till the glass plate just leaves the water surface. If the additional weight required is W, then the surface tension of the liquid-air interface will be

$$\sigma_{la} = \frac{W}{2l} = \frac{mg}{2l}$$

where m is the extra mass added and l is the length of the plate edge.

Examples based on Surface Tension and Surface Energy

FORMULAE USED

1. Surface tension $= \dfrac{\text{Force}}{\text{Length}}$ or $\sigma = \dfrac{F}{l}$
2. Increase in surface energy or work done,

   W = Surface tension × increase in area of the liquid surface.

UNITS USED

The unit of surface tension is Nm^{-1} and that of increase in surface energy or work done is joule.

EXAMPLE 72. *A wire ring of 3 cm radius is rested on the surface of a liquid and then raised. The pull required is 3.03 g more before the film breaks than it is afterwards. Find the surface tension of the liquid.*

Scanned by CamScanner

Solution. The additional pull F of 3.03 g wt is equal to the force of surface tension.

$$F = 3.03 \text{ g wt} = 3.03 \times 981 \text{ dyne}$$

As the liquid touches the ring both along the inner and outer circumference, so force on the ring due to surface tension,

$$F = 2 \times 2\pi r \times \sigma = 4\pi r \sigma$$

$$\therefore \quad 4\pi r\sigma = 3.03 \times 981$$

$$\sigma = \frac{3.03 \times 981}{4\pi r} = \frac{3.03 \times 981}{4 \times 3.14 \times 3}$$

$$= \mathbf{78.84 \text{ dyne cm}^{-1}}.$$

<u>EXAMPLE 73.</u> *Calculate the work done in blowing a soap bubble from a radius of 2 cm to 3 cm. The surface tension of the soap solution is 30 dyne cm^{-1}.* **[Delhi 11]**

Solution. Here $r_1 = 2$ cm, $r_2 = 3$ cm, $\sigma = 30$ dyne cm^{-1}

Increase in surface area

$$= 2 \times 4\pi(r_2^2 - r_1^2) = 8\pi(3^2 - 2^2) = 40\,\pi \text{ cm}^2$$

Work done $= \sigma \times$ Increase in surface area

$$= 30 \times 40 \times 3.142 = \mathbf{3770.4 \text{ erg.}}$$

<u>EXAMPLE 74.</u> *The surface tension of a soap solution is 0.03 Nm^{-1}. How much work is done to produce a soap bubble of radius 0.05 m?*

Solution. Work done = Total surface area × surface tension

$$= 2 \times 4\pi r^2 \times \sigma = 2 \times 4 \times 3.14 \times (0.05)^2 \times 0.03$$

$$= \mathbf{1.884 \times 10^{-3} \text{ J.}}$$

<u>EXAMPLE 75.</u> *A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ.* **[Central Schools 09]**

Solution. Radius of larger drop $= D/2$

Let radius of each small drop $= r$

Now volume of 27 small drops

= Volume of the larger drop

$$27 \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \quad \text{or} \quad r = \frac{D}{6}$$

Initial surface area of larger drop

$$= 4\pi R^2 = 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$$

Final surface area of 27 small drops $= 27 \times 4\pi r^2$

$$= 27 \times 4\pi\left(\frac{D}{6}\right)^2 = 3\pi D^2$$

$\therefore$ Increase in surface area $= 3\pi D^2 - \pi D^2 = 2\pi D^2$

Change in energy = Increase in surface area × Surface tension

$$= \mathbf{2\pi D^2 \sigma.}$$

<u>EXAMPLE 76.</u> *A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expended. Surface tension of mercury $= 32 \times 10^{-2}$ Nm^{-1}.* **[Roorkee]**

Solution. Volume of 10^6 droplets

= Volume of larger drop

$$10^6 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$r = 10^{-2} R = 10^{-2} \times 1.0 = 10^{-2} \text{ cm} = 10^{-4} \text{ m}$$

Surface area of larger drop

$$= 4\pi R^2 = 4\pi \times (10^{-2})^2 = 4\pi \times 10^{-4} \text{ m}^2$$

Surface area of 10^6 droplets

$$= 4\pi r^2 \times 10^6 = 4\pi \times (10^{-4})^2 \times 10^6$$

$$= 4\pi \times 10^{-2} \text{ m}^2$$

$\therefore$ Increase in surface area

$$= 4\pi \times 10^{-4}(100 - 1) = 4\pi \times 99 \times 10^{-4} \text{ m}^2$$

$\therefore$ Work done in spraying a spherical drop of mercury

= Surface tension × increase in surface area

$$= 32 \times 10^{-2} \times 4\pi \times 99 \times 10^{-4} = \mathbf{3.98 \times 10^{-2} \text{ J.}}$$

<u>EXAMPLE 77.</u> *A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1}.* **[Central Schools 05 ; Delhi 13]**

Solution. Volume of 1000 droplets = Volume of larger drop

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$r = \frac{R}{10} = \frac{2 \times 10^{-3} \text{ m}}{10} = 2 \times 10^{-4} \text{ m}$$

Surface area of larger drop

$$= 4\pi R^2 = 4\pi \times (2 \times 10^{-3})^2 = 16\,\pi \times 10^{-6} \text{ m}^2$$

Surface area of 1000 droplets

$$= 4\pi r^2 \times 1000 = 4\pi \times (2 \times 10^{-4})^2 \times 1000$$

$$= 16\pi \times 10^{-5} \text{ m}^2$$

$\therefore$ Increase in surface area

$$= 16\pi \times 10^{-6}(10 - 1) = 144\pi \times 10^{-6} \text{ m}^2$$

The resultant increase in surface energy

= Surface tension × increase in surface area

$$= 0.07 \times 144 \times \frac{22}{7} \times 10^{-6} = \mathbf{3168 \times 10^{-8} \text{ J.}}$$

<u>EXAMPLE 78.</u> *Two soap bubbles in vacuum having radii 3 cm and 4 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new bubble?*

Solution. Surface energy of first bubble

= Surface tension × surface area

$$= 2 \times 4\pi r_1^2\, \sigma = 8\pi r_1^2\, \sigma$$

Scanned by CamScanner

Similarly, surface energy of second bubble

$$= 8\pi r_2^2 \sigma$$

Let r be the radius of the coalesced bubble. Then,

surface energy of coalesced bubble $= 8\pi r^2 \sigma$

By the conservation of energy,

$$8\pi r^2 \sigma = 8\pi r_1^2 \sigma + 8\pi r_2^2 \sigma = 8\pi (r_1^2 + r_2^2) \sigma$$

or $r^2 = r_1^2 + r_2^2 = 3^2 + 4^2 = 25$

or $r = 5$ **cm.**

EXAMPLE 79. *If 500 erg of work is done in blowing a soap bubble to a radius r, what additional work is required to be done to blow it to a radius equal to 3r ?*

Solution. Work done in blowing the soap bubble from radius 0 to r is

$$W = \sigma \times 2 \times 4\pi r^2 \quad ...(i)$$

Additional work required to increase the radius from r to $3r$ will be

$$W' = \sigma \times \text{Increase in surface area}$$

$$= \sigma \times 2 \times 4\pi [(3r)^2 - r^2]$$

$$W' = \sigma \times 2 \times 4\pi \times 8r^2 \quad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{W'}{W} = 8$$

or $W' = 8W = 8 \times 500$ erg $=$ **4000 erg.**

EXAMPLE 80. *Soapy water drips from a capillary. When the drop breaks away, the diameter of its neck is 1 mm. The mass of the drop is 0.0129 g. Find the surface tension of soapy water.*

Solution. When the drop breaks away from the capillary, weight of drop

= Force of surface tension acting on the capillary

or $mg = \pi D \times \sigma$,

where D = diameter of the drop

or $$\sigma = \frac{mg}{\pi D} = \frac{1.29 \times 10^{-5} \times 9.8}{3.14 \times 1 \times 10^{-3}}$$

$$= \mathbf{4.03 \times 10^{-2}\ Nm^{-1}}.$$

EXAMPLE 81. *A glass plate of length 10 cm, breadth 4 cm and thickness 0.4 cm, weighs 20 g in air. It is held vertically with long side horizontal and half the plate immersed in water. What will be its apparent weight ? Surface tension of water = 70 dyne cm^{-1}.*

Solution. Here $l = 10$ cm, $b = 4$ cm,

$t = 0.4$ cm, $m = 20$ g, $\sigma = 70$ dyne cm^{-1}

Various forces acting on the plate are

(*i*) Weight of the plate acting vertically downwards,

$= mg = 20 \times 980$ dyne $= 20$ g f

(*ii*) Force due to surface tension acting vertically downwards,

$F = \sigma \times$ Length of plate in contact with water

$= \sigma \times 2$ (length + thickness)

$= 70 \times 2\ (10 + 0.4) = 70 \times 20.8$ dyne

$$= \frac{70 \times 20.8}{980} \text{ g f} = 1.4857 \text{ g f}$$

(*iii*) Upwards thrust due to liquid

= Weight of the liquid displaced

= Volume of liquid displaced × density × g

$= (l \times b/2 \times t) \times \rho \times g$

$= (10 \times 4/2 \times 0.4)\, 1 \times 980$ dyne

$$= \frac{8 \times 980}{980} \text{ g f} = 8 \text{ g f}.$$

$\therefore$ Apparent weight $= 20 + 1.4857 - 8 =$ **13.4857 g f.**

EXAMPLE 82. *If a number of little droplets of water of surface tension σ, all of the same radius r combine to form a single drop of radius R and the energy released is converted into kinetic energy, find the velocity acquired by the bigger drop.*

Solution. Volume of bigger drop

= Volume of n smaller drops

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \text{ or } n = \frac{R^3}{r^3}$$

Mass of bigger drop,

$$m = \text{Volume} \times \text{density}$$

$$= \frac{4}{3}\pi R^3 \times 1 = \frac{4}{3}\pi R^3$$

Energy released,

W = S.T. × Decrease in surface area

$$= \sigma \times 4\pi (nr^2 - R^2) = 4\pi\sigma \left(\frac{R^3}{r^3} r^2 - R^2 \right)$$

$$= 4\pi R^3 \sigma \left(\frac{1}{r} - \frac{1}{R} \right) = 3 \times \frac{4}{3}\pi R^3 \sigma \left(\frac{R-r}{rR} \right)$$

$$= 3m\sigma \left(\frac{R-r}{rR} \right)$$

But K.E. produced = W

$$\therefore \quad \frac{1}{2} mv^2 = 3m\sigma \left(\frac{R-r}{rR} \right) \quad \text{or} \quad v = \sqrt{\frac{6\sigma (R-r)}{rR}}.$$

EXAMPLE 83. *If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by*

$$\Delta\theta = \frac{3\sigma}{J} \left(\frac{1}{r} - \frac{1}{R} \right)$$

where σ is the surface tension of water and J is the mechanical equivalent of heat.

Scanned by CamScanner

Solution. Let n be the number of little droplets which coalesce to form single drop. Then

Volume of n little droplets

= Volume of single drop

or $\quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $nr^3 = R^3$

Decrease in surface area $= n \times 4\pi r^2 - 4\pi R^2$

$$= 4\pi[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right]$$

$$= 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right] \quad [\because nr^3 = R^3]$$

Energy evolved,

W = Surface tension × decrease in surface area

$$= 4\pi\sigma R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Heat produced,

$$Q = \frac{W}{J} = \frac{4\pi\sigma R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

But $\quad Q = ms\Delta\theta$

= Volume of single drop × density of water × specific heat of water × $\Delta\theta$

$$= \frac{4}{3}\pi R^3 \times 1 \times 1 \times \Delta\theta$$

Hence

$$\frac{4}{3}\pi R^3\Delta\theta = \frac{4\pi\sigma R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

or

$$\Delta\theta = \frac{3\sigma}{J}\left[\frac{1}{r} - \frac{1}{R}\right].$$

Problems For Practice

1. A soap film is formed on a rectangular frame of length 7 cm dipping in soap solution. The frame hangs from the arm of a balance. An extra weight of 0.38 g is to be placed in the opposite pan to balance the pull on the frame. Calculate the surface tension of the soap solution. Given $g = 980\ \text{cms}^{-2}$.
   (Ans. 26.6 dyne cm^{-1})
2. A soap film is on a rectangular wire ring of size 5 cm × 4 cm. If the size of the film is changed to 5 cm × 5 cm, then calculate the work done in this process. The surface tension of soap film is $5 \times 10^{-2}\ \text{Nm}^{-1}$.
   (Ans. 5×10^{-5} J)
3. A soap bubble is blown to a diameter of 7 cm. If 36,960 erg of work is done in blowing it further, find the new radius if the surface tension of soap solution is 40 dyne cm^{-1}.
   (Ans. 7 cm)
4. A soap bubble of radius $1/\sqrt{\pi}$ cm is expanded to radius $2/\sqrt{\pi}$ cm. Calculate the work done. Surface tension of soap solution = 30 dyne cm^{-1}.
   (Ans. 720 erg)
5. What amount of energy will be liberated if 10^{14} droplets of water, each of diameter 10^{-8} cm, coalesce to form a bigger drop ? Surface tension of water = 0.072 Nm^{-1}.
   (Ans. 2.035×10^{-14} J)
6. Calculate the force required to take away a flat plate of radius 5 cm from the surface of water. Given, surface tension of water = 72 dyne cm^{-1}.
   (Ans. 2260.8 dyne)
7. A thin wire is bent in the form of a ring of diameter 3.0 cm. The ring is placed horizontally on the surface of soap solution and then raised up slowly. How much upward force is necessary to break the vertical film formed between the ring and the solution ? Surface tension of a soap solution $= 3.0 \times 10^{-2}\ \text{Nm}^{-1}$.
   (Ans. 5.65×10^{-3} N.)
8. The length of a needle floating on water is 2.5 cm. How much minimum force, in addition to the weight of the needle, will be needed to lift the needle above the surface of water ? Surface tension of water $= 7.2 \times 10^{-4}\ \text{Ncm}^{-1}$.
   (Ans. 3.6×10^{-3} N)
9. A rectangular plate of dimensions 6 cm × 4 cm and thickness 2 mm is placed with its largest face flat on the surface of water.
   (i) What is the downward force on the plate due to surface tension ? Surface tension of water $= 7.0 \times 10^{-2}\ \text{Nm}^{-1}$
   (ii) If the plate is placed vertical so that the longest side just touches the water surface, find the downward force on the plate.
   (Ans. 1.4×10^{-2} N, 8.68×10^{-3} N)

Hints

1. As soap film has two free surfaces, so

$$F = \sigma \times 2l \quad \text{or} \quad 0.38\text{g f} = \sigma \times 2 \times 7$$

or $\quad \sigma = \frac{0.38 \times 980}{14} = $ **26.6 dyne cm^{-1}.**

3. Initial radius, $r = \frac{7}{2}$ cm = 3.5 cm

Initial surface area of soap bubble

$$= 2 \times 4\pi r^2 = 8\pi \times (3.5)^2\ \text{cm}^2.$$

Let new radius of the soap bubble = R cm

Then final surface area $= 2 \times 4\pi R^2 = 8\pi R^2\ \text{cm}^2$

Increase in surface area $= 8\pi[R^2 - (3.5)^2]\ \text{cm}^2$

Now Work done = Surface tension × Increase in surface area

Scanned by CamScanner

$\therefore \quad 36960 = 40 \times 8\pi [R^2 - (3.5)^2]$

or $\quad R^2 - 12.25 = \dfrac{36960}{40 \times 8 \times 3.14} = 36.75$

or $\quad R^2 = 36.75 + 12.25 = 49$

or $\quad R = $ **7 cm.**

4. Work done = S.T. × Increase in surface area

$$= 30 \times 2 \times 4\pi \left[\left(\frac{2}{\sqrt{\pi}}\right)^2 - \left(\frac{1}{\sqrt{\pi}}\right)^2 \right] = \textbf{720 erg.}$$

5. Radius of a droplet,

$$r = \frac{10^{-8}}{2} = 0.5 \times 10^{-8} \text{ m}$$

Volume of bigger drop = Volume of 1000 droplets

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

or $\quad R = 10\,r = 10 \times 0.5 \times 10^{-8} = 5 \times 10^{-8}$ m

Decrease in surface area

$= 1000 \times 4\pi r^2 - 4\pi R^2 = 4\pi [1000\,r^2 - R^2]$

$= 4 \times 3.14[1000 \times (0.5 \times 10^{-8})^2 - (5 \times 10^{-8})^2]$

$= 4 \times 3.14 \times 225 \times 10^{-16}\ \text{m}^2 = 2826 \times 10^{-16}\ \text{m}^2$

Energy liberated

= Surface tension × Decrease in surface area

$= 0.072 \times 2826 \times 10^{-16} = \mathbf{2.035 \times 10^{-14}}$ **J.**

6. Required force = $2\pi r$ × Surface tension

$= 2 \times 3.14 \times 5 \times 72$

= **2260.8 dyne.**

7. $F = 2 \times 2\pi r \times \sigma$

$= 4 \times 3.14 \times 1.5 \times 10^{-2} \times 3.0 \times 10^{-2}$

$= \mathbf{5.65 \times 10^{-3}}$ **N.**

8. $F = \sigma \times 2l$

$= 7.2 \times 10^{-4} \times 2 \times 2.5$

$= \mathbf{3.6 \times 10^{-3}}$ **N.**

9. (*i*) $F = \sigma \times 2(l + b)$

$= 7 \times 10^{-2} \times 2 \times (0.06 + 0.04)$

$= \mathbf{1.4 \times 10^{-2}}$ **N.**

(*ii*) $F = \sigma \times 2(l + t)$

$= 7 \times 10^{-2} \times 2 \times (0.06 + 0.002)$

$= \mathbf{8.68 \times 10^{-3}}$ **N.**

10.47 PRESSURE DIFFERENCE ACROSS A CURVED LIQUID SURFACE

63. *Show that a pressure difference exists between the two sides of a curved liquid surface.*

Pressure difference across a curved liquid surface. When the free surface of a liquid is curved, there is a difference of pressure between the liquid side and the vapour side of the surface. We consider the three possible liquid surfaces :

(*i*) As shown in Fig. 10.53(*a*), if the surface is plane, the molecule *A* on the surface is attracted equally in all directions. The resultant force due to surface tension is zero. Pressure on both sides of the surface is same *i.e.,*

$$P_L = P_V\ .$$

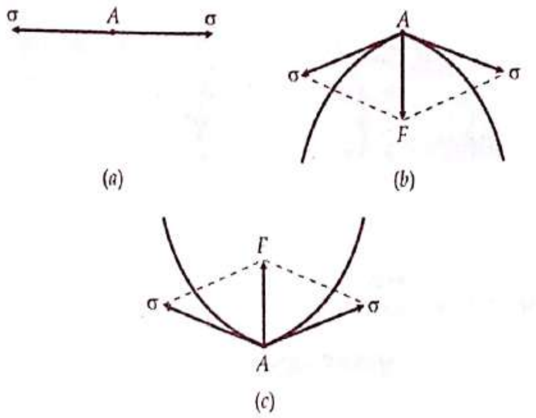


Fig. 10.53 Excess pressure across a curved surface.

(*ii*) As shown in Fig. 10.53(*b*), if the surface is convex, there is a resultant downward force *F* on molecule *A*. For the surface to remain in equilibrium, the pressure on the liquid side must be greater than the pressure on the vapour side *i.e.,*

$$P_L > P_V\ .$$

(*iii*) As shown in Fig. 10.53(*c*), if the surface is concave, there is a resultant upward force *F* due to surface tension on the molecule *A*. For the surface to remain in equilibrium, the pressure on the vapour side must be greater than the pressure on the liquid side *i.e.,*

$$P_V > P_L\ .$$

Thus we find that *whenever a liquid surface is curved, the pressure on its concave side is greater than the pressure on the convex side.*

10.48 EXCESS PRESSURE INSIDE A LIQUID DROP

64. *Derive an expression for the excess pressure inside a liquid drop.*

Excess pressure inside a liquid drop. Consider a spherical liquid drop of radius *R*. Let σ be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure *p* inside the drop over that on

Scanned by CamScanner

outside. This excess pressure acts normally outwards. Let the radius of the drop increase from R to $R + dR$ under the excess pressure p.

Initial surface area $= 4\pi R^2$

Final surface area

$$= 4\pi (R + dR)^2 = 4\pi (R^2 + 2R\,dR + dR^2)$$

$$= 4\pi R^2 + 8\pi R\,dR$$

dR^2 is neglected as it is small.

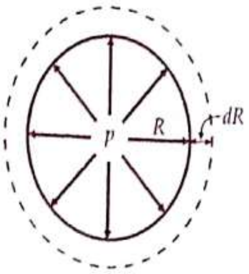


Fig. 10.54 Excess pressure inside a liquid drop.

Increase in surface area

$$= 4\pi R^2 + 8\pi R\,dR - 4\pi R^2 = 8\pi R\,dR$$

Work done in enlarging the drop

= Increase in surface energy

= Increase in surface area × Surface tension

$= 8\pi R\,dR\,\sigma$

But work done = Force × Distance

= Pressure × Area × Distance

$= p \times 4\pi R^2 \times dR$

Hence, $p \times 4\pi R^2 \times dR = 8\pi R\,dR\,\sigma$

Excess pressure,

$$p = \frac{2\sigma}{R}.$$

65. *Derive an expression for the excess pressure inside a soap bubble.*

Excess pressure inside a soap bubble. Proceeding as in the case of a liquid drop in the above question, we obtain

Increase in surface area $= 8\pi R\,dR$

But a soap bubble has air both inside and outside, so it has two free surfaces.

∴ Effective increase in surface area

$= 2 \times 8\pi R\,dR = 16\pi R\,dR$

Work done in enlarging the soap bubble

= Increase in surface energy

= Increase in surface area × Surface tension

$= 16\pi R\,dR\,\sigma$

But, Work done = Force × Distance

$= p \times 4\pi R^2 \times dR$

Hence

$$p \times 4\pi R^2 \times dR = 16\pi R\,dR\,\sigma$$

or $$p = \frac{4\sigma}{R}.$$

66. *Write an expression for the excess pressure inside an air bubble.*

Excess pressure inside an air bubble inside a liquid. An air bubble inside a liquid is similar to a liquid drop in air. It has only one free spherical surface. Hence excess pressure is given by

$$p = \frac{2\sigma}{R}$$

For Your Knowledge

- The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside the tiny fog droplets that they are rigid enough to behave like solids and resist fairly large deforming forces.
- When an ice-skater slides over the surface of smooth ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates. The tiny water droplets act as rigid ball-bearings and help the skaters to run along smoothly.
- When an air bubble of radius R lies at a depth h below the free surface of a liquid of density ρ and surface tension σ, the excess pressure inside the bubble will be

$$p = \frac{2\sigma}{R} + h\rho g$$

Examples based on
Excess Pressure in Drops & Bubbles

FORMULAE USED

1. Excess pressure inside a liquid drop,
   $p = \frac{2\sigma}{R}$ (with one free surface)
2. Excess pressure inside a soap bubble,
   $p = \frac{4\sigma}{R}$ (with two free surfaces)
3. Excess pressure in an air bubble,
   $p = \frac{2\sigma}{R}$ (with one free surface)

UNITS USED

Surface tension σ is in Nm^{-1}, pressure p in Nm^{-2} or Pa and radius R in metre

EXAMPLE 84. *What should be the pressure inside a small air bubble of 0.1 mm radius, situated just below the surface? Surface tension of water $= 7.2 \times 10^{-2}$ Nm^{-1} and atmospheric pressure $= 1.013 \times 10^5$ Nm^{-2}.* **[Chandigarh 04]**

Scanned by CamScanner

Solution. Here $R = 0.1 \text{ mm} = 0.1 \times 10^{-3}$ m,

$\sigma = 7.2 \times 10^{-2}\ \text{Nm}^{-1}$

Excess pressure,

$$p = \frac{2\sigma}{R} = \frac{2 \times 7.2 \times 10^{-2}}{0.1 \times 10^{-3}} = 1.44 \times 10^3\ \text{Nm}^{-2}$$

Pressure inside the bubble

= Atmospheric pressure + Excess pressure

$= 1.013 \times 10^5 + 1.44 \times 10^3 = \mathbf{1.027 \times 10^5\ Nm^{-2}}$.

EXAMPLE 85. *The excess pressure inside a soap bubble of radius 6 mm is balanced by 2 mm column of oil of specific gravity 0.8. Find the surface tension of soap solution.*

Solution. Here $R = 6 \text{ mm} = 6 \times 10^{-3}$ m,

$h = 2 \text{ mm} = 2 \times 10^{-3}$ m, $\rho = 0.8 \times 10^3\ \text{kgm}^{-3}$

Excess pressure inside soap bubble

= Pressure exerted by 2 mm oil column

or $\frac{4\sigma}{R} = h\rho g \quad \therefore \quad \sigma = \frac{1}{4} hR\rho g$

$$= \frac{1}{4} \times 2 \times 10^{-3} \times 6 \times 10^{-3} \times 0.8 \times 10^3 \times 9.8$$

$= \mathbf{2.35 \times 10^{-2} Nm^{-1}}$.

EXAMPLE 86. *Two soap bubbles have radii in the ratio 2 : 3. Compare the excess of pressure inside these bubbles. Also compare the works done in blowing these bubbles.*

Solution. If R_1 and R_2 are the radii of the two bubbles, then $\frac{R_1}{R_2} = \frac{2}{3}$

Let σ be the surface tension of the soap solution.

Excess pressure inside the bubble of radius R_1,

$$p_1 = \frac{4\sigma}{R_1}$$

Excess pressure inside the bubble of radius R_2,

$$p_2 = \frac{4\sigma}{R_2}$$

$$\therefore \quad \frac{p_1}{p_2} = \frac{4\sigma}{R_1} \times \frac{R_2}{4\sigma} = \frac{R_2}{R_1} = \frac{3}{2} = \mathbf{3 : 2}$$

Work done in blowing up the two soap bubbles is

$$W_1 = 2 \times 4\pi R_1^2 \times \sigma$$

and $W_2 = 2 \times 4\pi R_2^2 \times \sigma$

$$\therefore \quad \frac{W_1}{W_2} = \frac{R_1^2}{R_2^2} = \left(\frac{2}{3}\right)^2 = \mathbf{4 : 9}.$$

EXAMPLE 87. *A small hollow sphere having a small hole in it is immersed into water to a depth of 20 cm before any water penetrates into it. If the surface tension of water is 73 dyne cm^{-1}, find the radius of the hole.*

Solution. Here $\sigma = 73$ dyne cm^{-1}, $h = 20$ cm

At equilibrium position,

$$\frac{2\sigma}{R} = h\rho g$$

$$\therefore \quad R = \frac{2\sigma}{h\rho g} = \frac{2 \times 73}{20 \times 1 \times 980} = \mathbf{0.007449\ cm.}$$

EXAMPLE 88. *A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower end 2 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end? Given density of mercury = 13600 kg m^{-3} and surface tension of mercury = 35×10^{-3} Nm^{-1}.*

Solution.

Here $R = \frac{1}{2} \text{ mm} = 0.5 \text{ mm} = 5 \times 10^{-4}$ m,

$\sigma = 35 \times 10^{-3}\ \text{Nm}^{-1}$,

$\rho = 13600\ \text{kgm}^{-3}$, $h = 2 \text{ cm} = 0.02$ m

Pressure of air

$$= h\rho g + \frac{2\sigma}{R} = 0.02 \times 13600 \times 9.8 + \frac{2 \times 35 \times 10^{-3}}{5 \times 10^{-4}}$$

$= 2665.5 + 140 = \mathbf{2805.6\ Nm^{-2}}$.

EXAMPLE 89. *The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at the temperature of the experiment is 7.30×10^{-2} Nm^{-1}. 1 atmospheric pressure = 1.01×10^5 Pa, density of water = 1000 kgm^{-3}, g = 9.80 ms^{-2}. Also calculate the excess pressure.* **[NCERT]**

Solution. Here $R = \frac{2.00}{2} = 1.00 \text{ mm} = 1.00 \times 10^{-3}$ m,

$h = 8.00 \times 10^{-2}$ m, $\sigma = 7.30 \times 10^{-2}\ \text{Nm}^{-1}$,

$P = 1.01 \times 10^5$ Pa, $\rho = 1000\ \text{kgm}^{-3}$, $g = 9.80\ \text{ms}^{-2}$

Excess pressure inside the bubble is

$$p = \frac{2\sigma}{r} = \frac{2 \times 7.30 \times 10^{-2}}{1.00 \times 10^{-3}} = \mathbf{146\ Pa.}$$

Pressure outside the bubble

= Atmospheric pressure

+ Pressure due to 8.00 cm of water column

$= P + h\rho g$

$= 1.01 \times 10^5 + 8.00 \times 10^{-2} \times 1000 \times 9.80$

$= 1.01 \times 10^5 + 0.00784 \times 10^5 = 1.01784 \times 10^5$ Pa

Pressure inside the bubble

= Pressure outside the bubble + Excess pressure

$= 1.01784 \times 10^5 + 146$

$= \mathbf{1.0193 \times 10^5\ Pa \approx 1.02 \times 10^5\ Pa.}$

Scanned by CamScanner

PROBLEMS FOR PRACTICE

1. What would be the gauge pressure inside an air bubble of 0.2 mm radius situated just below the surface of water ? Surface tension of water is $0.07\ Nm^{-1}$ (**Ans.** $700\ Nm^{-2}$)

2. The pressure of air in a soap bubble of 0.7 cm diameter is 8 mm of water above the atmospheric pressure. Calculate the surface tension of soap solution. Take $g = 9.8\ ms^{-2}$. (**Ans.** $6.86 \times 10^{-2}\ Nm^{-1}$)

3. Calculate the total pressure inside a spherical bubble of radius 0.2 mm formed inside water at a depth of 10 cm. Surface tension of water at a depth of 30 cm is 70 dyne cm^{-1}, barometric pressure is 76 cm, density of mercury is 13.6 g cm^{-3} and $g = 980\ cms^{-2}$. (**Ans.** 1029728 dyne cm^{-2})

4. Calculate the total pressure inside a spherical air bubble of radius 0.1 mm at a depth of 10 cm below the surface of a liquid of density 1.1 g cm^{-3} and surface tension 50 dyne cm^{-1}. Height of Hg barometer = 76 cm. (**Ans.** 1.0337×10^6 dyne cm^{-2})

5. Find the difference in excess pressure on the inside and outside of a rain drop if its diameter changes from 0.03 cm to 0.0002 cm by evaporation. Surface tension of water is 72 dyne cm^{-1}. (**Ans.** 1430400 dyne cm^{-2})

6. What is the pressure inside a vapour bubble of radius 10^{-3} m formed in boiling water ? Surface tension of water at 100 is 0.059 Nm^{-1} and 1 atm = 101325 Nm^{-2}. (**Ans.** 101443 Nm^{-2})

7. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 Nm^{-1} and density 10^3 kgm^{-3}. The bubble is at a depth of 10.0 cm below the free surface of a liquid. By what amount is the pressure inside the bubble greater than the atmospheric pressure ? (**Ans.** 1130 Nm^{-2})

8. An ancient building has a dome of 5 m radius and uniform but small thickness. The surface tension of its masionary structure is about 500 Nm^{-1}. Treated as hemisphere, find the maximum load that dome can support. (**Ans.** 31420 N)

HINTS

1. Here $\sigma = 0.07\ Nm^{-1}$, $R = 0.2\ mm = 2 \times 10^{-4}$ m

Gauge pressure, $p = \frac{2\sigma}{R} = \frac{2 \times 0.07}{2 \times 10^{-4}} = 700\ Nm^{-2}$.

2. Excess pressure in soap bubble

= Pressure exerted by 8 mm column of water

or $\frac{4\sigma}{R} = h\rho g$ or $\sigma = \frac{1}{4} h R \rho g$

$\therefore \sigma = \frac{1}{4} \times 8 \times 10^{-3} \times 0.35 \times 10^{-2} \times 1000 \times 9.8$

$= \mathbf{6.86 \times 10^{-2}\ Nm^{-1}}$.

3. Total pressure

= Atmospheric pressure + Pressure due to liquid column of 10 cm + Excess pressure

$= h\rho g + h'\rho' g + \frac{2\sigma}{R}$

$= 76 \times 13.6 \times 980 + 10 \times 1 \times 980 + \frac{2 \times 70}{0.02}$

$= 1012928 + 9800 + 7000$

$= \mathbf{1029728}$ **dyne** $\mathbf{cm^{-2}}$.

5. Here $R_1 = \frac{0.03}{2} = 0.015$ cm,

$R_2 = \frac{0.0002}{2} = 0.0001$ cm

$\therefore p_2 - p_1 = \frac{2\sigma}{R_2} - \frac{2\sigma}{R_1} = 2 \times 72 \times \left[\frac{1}{0.0001} - \frac{1}{0.015}\right]$

$= \mathbf{1430400}$ **dyne** $\mathbf{cm^{-2}}$.

6. Pressure inside the bubble

= Excess pressure + Atmospheric pressure

$= \frac{2 \times 0.059}{10^{-3}} + 101325 = 118 + 101325$

$= \mathbf{101443\ Nm^{-2}}$.

7. Pressure inside the air bubble greater than atmospheric pressure

$= \frac{2\sigma}{R} + h\rho g = \frac{2 \times 0.075}{1 \times 10^{-3}} + 0.10 \times 10^3 \times 9.8$

$= \mathbf{1130\ Nm^{-2}}$.

8. Excess pressure inside the dome, $p = \frac{4\sigma}{R}$

Maximum load that the dome can support is

$F = p \times 4\pi R^2 = \frac{4\sigma}{R} \times 4\pi R^2 = 4\pi R\sigma$

$= 4 \times 3.142 \times 5 \times 500 = \mathbf{31420\ N}$.

10.49 ANGLE OF CONTACT

67. *Define the term angle of contact. On what factors does it depend ?*

Angle of contact. The liquid surface is usually curved when it is in contact with a solid. The particular shape that it takes depends on the relative strengths of cohesive and adhesive forces. If

Adhesive force > Cohesive force : Liquid wets the solid surface and has concave meniscus

Adhesive force < Cohesive force : Liquid does not wet the solid surface and has a convex meniscus

Adhesive force = Cohesive force : Liquid surface is plane

***Angle of contact** is defined as the angle θ between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.*

The value of angle of contact d
following factors :

(*i*) Nature of the solid and the liq
(*ii*) Cleanliness of the surface in c
(*iii*) Medium above the free surfa
(*iv*) Temperature of the liquid.

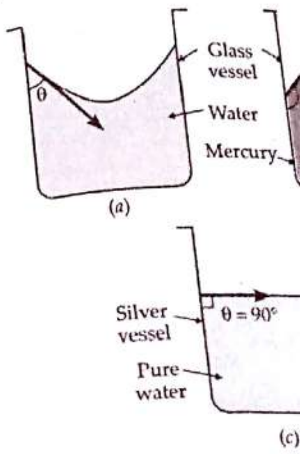


Fig. 10.55 Defining angle o
(*b*) Convex (c)

For those liquids which
the angle of contact is acut
not wet the walls of the ve
obtuse.

The angle of contact fo
for mercury and glass it is
silver, angle of contact is

10.50 SHAPE OF L A NARROW

68. *Explain what d meniscus in a narrow t*

Shape of liquid
Consider a molecule (
contact with the solid

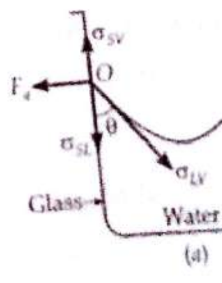


Fig. 10.56 Forces
bound

Scanned by CamScanner

The value of angle of contact depends on the following factors :

(*i*) Nature of the solid and the liquid in contact.

(*ii*) Cleanliness of the surface in contact.

(*iii*) Medium above the free surface of the liquid.

(*iv*) Temperature of the liquid.

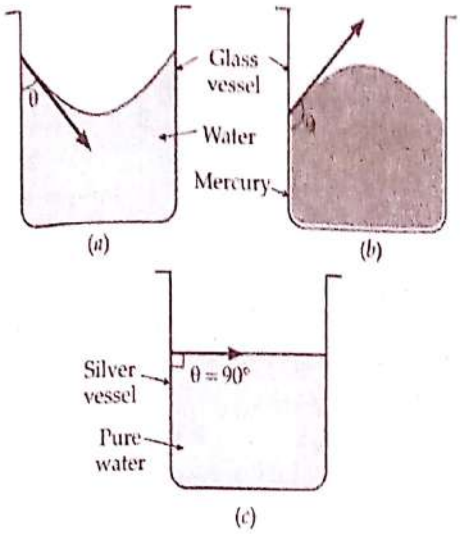


Fig. 10.55 Defining angle of contact. (*a*) Concave (*b*) Convex (*c*) Plane, menisci.

For those liquids which wet the walls of the vessel, the angle of contact is acute. For the liquids which do not wet the walls of the vessel, the angle of contact is obtuse.

The angle of contact for water and glass is about 8°, for mercury and glass it is 138° and for pure water and silver, angle of contact is 90°.

10.50 SHAPE OF LIQUID MENISCUS IN A NARROW TUBE

68. *Explain what determines the shape of liquid meniscus in a narrow tube.*

Shape of liquid meniscus in a narrow tube. Consider a molecule *O* on the surface of the liquid in contact with the solid wall of the vessel. The various forces acting at the boundary of the three surfaces are as follows :

(*i*) Surface tension σ_{LV} of the liquid-vapour surface acting tangentially to the liquid surface.

(*ii*) Surface tension σ_{SV} of the solid-vapour surface acting parallel to the walls of the vessel.

(*iii*) Surface tension σ_{SL} of the solid-liquid surface acting parallel to the wall of the vessel directed into the liquid.

(*iv*) Adhesive force F_a between the molecules of the vessel and the liquid acting normal to the wall of the container.

For equilibrium, no forces should act on molecule *O* in any direction. Let θ be the angle of contact. Then the components of σ_{LV} parallel and perpendicular to water surface are $\sigma_{LV}\sin\theta$ and $\sigma_{LV}\cos\theta$ respectively. For equilibrium, we must have

$$F_a = \sigma_{LV}\cos\theta$$

and $$\sigma_{SV} = \sigma_{SL} + \sigma_{LV}\cos\theta$$

or $$\cos\theta = \frac{\sigma_{SV} - \sigma_{SL}}{\sigma_{LV}}$$

The following three cases are possible :

(*i*) If $\sigma_{SV} > \sigma_{SL}$, cos θ is positive and $\theta < 90°$ *i.e.*, angle of contact is acute. The liquid meniscus is concave upwards. This happens in the case of water taken in a glass vessel [Fig. 10.56(*a*)].

(*ii*) If $\sigma_{SV} < \sigma_{SL}$, cos θ is negative and $\theta > 90°$ *i.e.*, angle of contact is obtuse. The liquid meniscus is convex, upwards. This happens in the case of mercury taken in a glass vessel [Fig. 10.56(*b*)].

(*iii*) When $\sigma_{SV} = \sigma_{SL}$, $\cos\theta = 0$ and $\theta = 90°$. The liquid meniscus is plane. This happens in the case of pure water taken in a silver vessel.

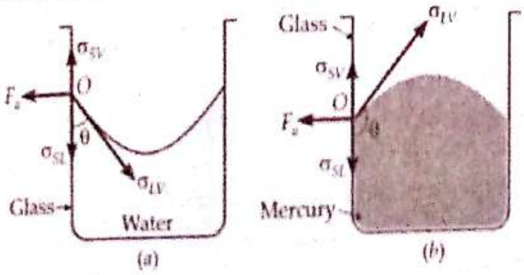


Fig. 10.56 Forces of surface tension at the boundaries of three phases.

10.51 CAPILLARITY

69. *What do you understand by the term capillarity? Give some examples of capillarity from daily life.*

Capillarity. The Latin word *capilla means hair. A tube of very fine (hair-like) bore is called a capillary tube.*

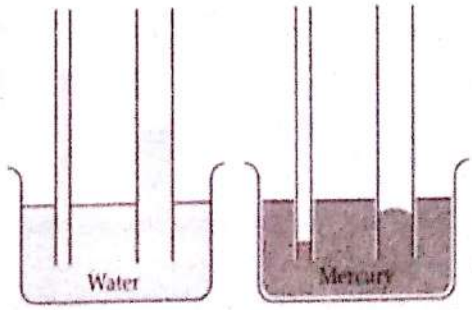


Fig. 10.57 Capillarity.

Scanned by CamScanner

When a capillary tube of glass open at both ends is dipped in liquid which wets its walls (*e.g.*, water, alcohol), the liquid rises in the tube. But when the capillary tube is dipped in a liquid which does not wet its walls (*e.g.*, mercury), the level of liquid is depressed in the tube.

The phenomenon of rise or fall of a liquid in a capillary tube in comparison to the surrounding is called capillarity.

Some examples of capillarity from daily life :

(*i*) A blotting paper soaks ink by capillary action. The pores of blotting paper act as capillaries.

(*ii*) Oil rises in the long narrow spaces between the threads of a wick, the narrow spaces act as capillary tubes.

(*iii*) We use towels made of a coarse cloth for drying our skin after taking bath.

(*iv*) Sap rises from the roots of a plant to its leaves and branches due to capillarity action.

(*v*) The tip of the nib of a pen is split to provide capillary action for the ink to rise.

10.52 RISE OF LIQUID IN A CAPILLARY TUBE : ASCENT FORMULA

70. *Derive an expression for the rise of liquid in a capillary tube and show that the height of the liquid column supported is inversely proportional to the radius of the tube.*

Ascent formula. Consider a capillary tube of radius r dipped in a liquid of surface tension σ and density ρ. Suppose the liquid wets the sides of the tube. Then its meniscus will be concave. The shape of the meniscus of water will be nearly spherical if the capillary tube is of sufficiently narrow bore.

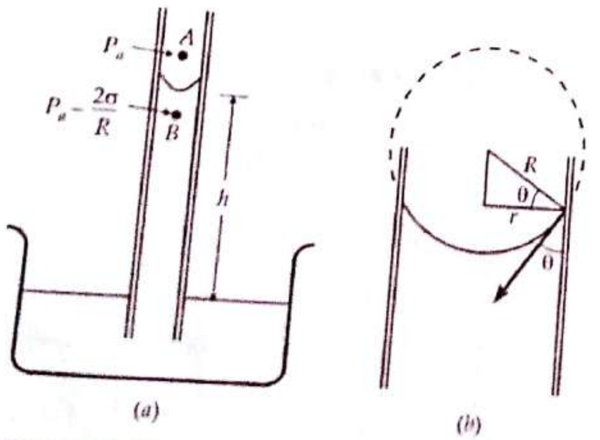


Fig. 10.58 (*a*) Rise of liquid in a capillary tube. (*b*) Enlarged view.

As the pressure is greater on the concave side of a liquid surface, so excess of pressure at a point A just above the meniscus compared to point B just below the meniscus is

$$p = \frac{2\sigma}{R}$$

where R = radius of curvature of the concave meniscus. If θ is the angle of contact, then from the right angled triangle shown in Fig. 10.58(*b*), we have

$$\frac{r}{R} = \cos\theta$$

or $$R = \frac{r}{\cos\theta}$$

$\therefore$ $$p = \frac{2\sigma\cos\theta}{r}$$

Due to this excess pressure p, the liquid rises in the capillary tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure p. Therefore, at equilibrium we have

$$p = h\rho g$$

or $$\frac{2\sigma\cos\theta}{r} = h\rho g$$

or $$h = \frac{2\sigma\cos\theta}{r\rho g}$$

This is the ***ascent formula*** for the rise of liquid in a capillary tube. If we take into account the volume of the liquid contained in the meniscus, then the above formula gets modified as

$$h = \frac{2\sigma\cos\theta}{r\rho g} - \frac{r}{3}$$

However, the factor $r/3$ can be neglected for a narrow tube.

The ascent formula shows that the height h to which a liquid rises in the capillary tube is

(*i*) *inversely proportional to the radius of the tube.*

(*ii*) *inversely proportional to the density of the liquid.*

(*iii*) *directly proportional to the surface tension of the liquid.*

Hence a liquid rises more in a narrower tube than in wider tube.

10.53 RISE OF LIQUID IN A CAPILLARY TUBE OF INSUFFICIENT HEIGHT

71. *Explain what happens when the length of a capillary tube is less than the height upto which the liquid may rise in it.*

Rise of liquid in a tube of insufficient height. The height to which a liquid rises in a capillary tube is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

Scanned by CamScanner

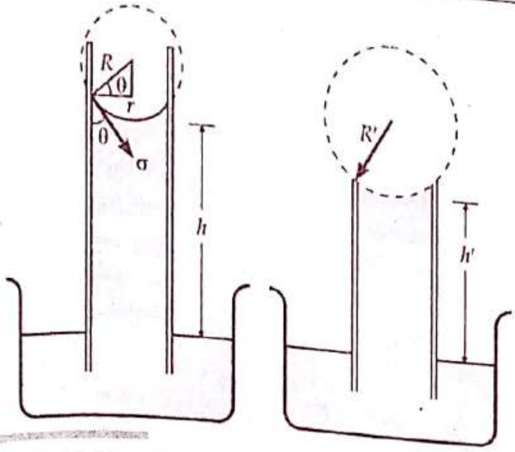


Fig. 10.59 Rise of liquid in a tube of insufficient height.

The radius r of the capillary tube and radius of curvature R of the liquid meniscus are related by $r = R\cos\theta$. Therefore,

$$h = \frac{2\sigma\cos\theta}{R\cos\theta\,\rho g} = \frac{2\sigma}{R\rho g}$$

As σ, ρ and g are constants, so

$$hR = \frac{2\sigma}{\rho g} = \text{a constant}$$

$$\therefore \quad hR = h'R'$$

where R' is the radius of curvature of the new meniscus at a height h'.

As $h' < h$, so $R' > R$

Hence in a capillary tube of insufficient height, the liquid rises to the top and spreads out to a new radius of curvature R' given by

$$R' = \frac{hR}{h'}$$

But the liquid will not overflow.

Examples based on Capillarity : Ascent Formula

FORMULAE USED

1. When a capillary tube of radius r is dipped in a liquid of density ρ and surface tension σ, the liquid rises or falls through a distance,

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

where θ is the angle of contact.

UNITS USED

Radius r is in metre, density ρ in kgm^{-3}, surface tension σ in Nm^{-1} and height h in metre.

EXAMPLE 90. *Calculate the height to which water will rise in capillary tube of 1.5 mm diameter. Surface tension of water is 7.4×10^{-3} Nm^{-1}.*

Solution.

Here $r = \frac{1.5}{2} = 0.75\text{ mm} = 0.75 \times 10^{-3}$ m,

$\sigma = 7.4 \times 10^{-3}\ Nm^{-1}$.

For water, $\rho = 10^3\ kg\ m^{-3}$,

Angle of contact $\theta = 0°$

$$\therefore \quad h = \frac{2\sigma\cos\theta}{r\rho g} = 2 \times \frac{7.4 \times 10^{-3} \times \cos 0°}{0.75 \times 10^{-3} \times 10^3 \times 9.8}$$

$$= \mathbf{0.002014\ m.}$$

EXAMPLE 91. *A liquid rises to a height of 7.0 cm in a capillary tube of radius 0.1 mm. The density of the liquid is 0.8×10^3 $kg\ m^{-3}$. If the angle of contact between the liquid and the surface of the tube be zero, calculate the surface tension of the liquid. Given $g = 10\ ms^{-2}$.*

Solution. Here $h = 7.0\text{ cm} = 7.0 \times 10^{-2}$ m,

$r = 0.1\text{ mm} = 10^{-4}$ m, $\rho = 0.8 \times 10^3\ kgm^{-3}$, $\theta = 0°$

$g = 10\ ms^{-2}$

$$\sigma = \frac{hr\rho g}{2\cos\theta} = \frac{7.0 \times 10^{-2} \times 10^{-4} \times 0.8 \times 10^3 \times 10}{2 \times \cos 0°}$$

$$= \mathbf{2.8 \times 10^{-2}\ Nm^{-1}.}$$

EXAMPLE 92. *Water rises up in a glass capillary upto a height of 9.0 cm, while mercury falls down by 3.4 cm in the same capillary. Assume angles of contact for water-glass and mercury-glass as 0° and 135° respectively. Determine the ratio of surface tensions of mercury and water. Take $\cos 135° = -0.71$.*

Solution. *For water :* $h_1 = 9\text{ cm} = 0.09$ m,

$$\rho_1 = 10^3\ kgm^{-3},\ \theta_1 = 0°$$

For mercury : $h_2 = -3.4\text{ cm} = -0.034$ m,

$$\rho_2 = 13.6 \times 10^3\ kgm^{-3},\ \theta_2 = 135°$$

Let σ_w and σ_m be the surface tensions of water and mercury respectively. Then

$$\sigma_w = \frac{h_1 r \rho_1 g}{2\cos\theta_1} \quad \text{and} \quad \sigma_m = \frac{h_2 r \rho_2 g}{2\cos\theta_2}$$

$$\frac{\sigma_m}{\sigma_w} = \frac{h_2\rho_2\cos\theta_1}{h_1\rho_1\cos\theta_2}$$

$$= \frac{-0.034 \times 13.6 \times 10^3 \times \cos 0°}{0.09 \times 10^3 \times \cos 135°}$$

$$= \frac{-0.034 \times 13.6 \times 10^3 \times 1}{0.09 \times 10^3 \times (-0.71)} = \mathbf{7.2 : 1.}$$

EXAMPLE 93. *Water rises in a capillary tube to a height of 2.0 cm. In another capillary whose radius is one-third of it, how much the water will rise ? If the first capillary is inclined at an angle of 60° with the vertical, then what will be the position of water in the tube ?*

Scanned by CamScanner

Solution. Ascent of a liquid in a capillary tube is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

$\therefore$ For a given liquid,

$$hr = \frac{2\sigma\cos\theta}{\rho g} = \text{constant}$$

$[\because \sigma, \theta, \rho, g \text{ are constants}]$

or $\quad h'r' = hr$

For a capillary tube of radius $r/3$, we have

$$h' = \frac{hr}{r'} = \frac{2.0 \text{ cm} \times r}{r/3} = \mathbf{6.0 \text{ cm}}.$$

When the first capillary is inclined at an angle of 60° to the vertical, the vertical height h (=2.0 cm) of the liquid will remain the same. Thus if the length of water in the capillary be l cm, then from Fig. 10.60, we have

$$l = \frac{h}{\cos 60°} = \frac{2.0 \text{ cm}}{0.5} = \mathbf{4.0 \text{ cm}}.$$

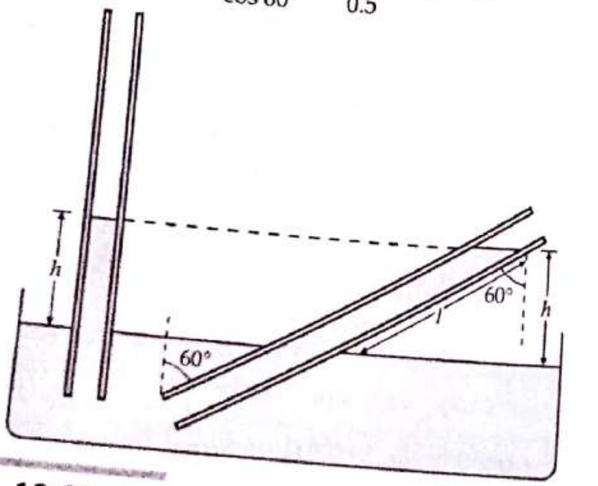


Fig. 10.60

Example 94. If a 5 cm long capillary tube with 0.1 mm internal diameter opens at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary (ii) water will rise up to the upper end of capillary (iii) water will overflow out of the upper end of capillary. Explain your answer.

Solution.

Radius, $r = \frac{0.1}{2}$ mm = 0.05 mm = 0.005 cm

Surface tension, $\sigma = 75$ dyne cm^{-1}

density, $\rho = 1$ gcm^{-3} ; angle of contact, $\theta = 0°$

Let h be the height to which water rises in the capillary tube. Then

$$h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 75 \times \cos 0°}{0.005 \times 1 \times 981} = \mathbf{30.58 \text{ cm}}.$$

Given length of capillary tube, $h' = 5$ cm

(*i*) As $h > \frac{h'}{2}$, so the first possibility is ruled out.

(*ii*) As the tube is of insufficient length, so the water will rise upto the upper end of the tube.

(*iii*) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary. The liquid meniscus will adjust its radius of curvature R' in such a way that

$$R'h' = Rh \qquad \left[\because hR = \frac{2\sigma}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length.

$$\therefore \quad R' = \frac{Rh}{h'} = \frac{rh}{h'} = \frac{0.005 \times 30.58}{5} = \mathbf{0.0306 \text{ cm}}.$$

$$\left[\because R = \frac{r}{\cos\theta} = \frac{r}{\cos 0°} = r\right]$$

Example 95. A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs ? Surface tension of water is 0.07 Nm^{-1}. Assume that the angle of contact between water and glass is 0°.

Solution. Let P_A, P_B, P_C and P_D be the pressures at points A, B, C and D respectively. The pressure on the concave side of the liquid surface is greater than that on the other side by $2\sigma/R$.

As angle of contact θ is 0°, so

$$R\cos 0° = r \quad \text{or} \quad R = r$$

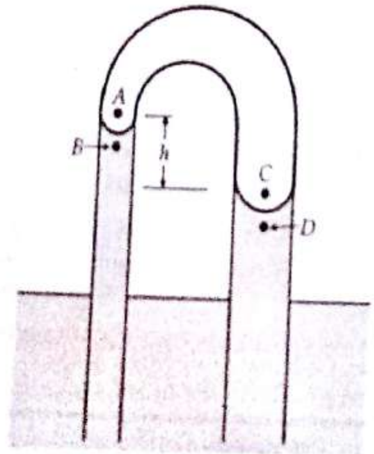


Fig. 10.61

$\therefore \quad P_A = P_B + \frac{2\sigma}{r_1}$ and $P_C = P_D + \frac{2\sigma}{r_2}$

where r_1 and r_2 are the radii of the two limbs.

But $\quad P_A = P_C$

$$\therefore \quad P_B + \frac{2\sigma}{r_1} = P_D + \frac{2\sigma}{r_2}$$

or $\quad P_D - P_B = 2\sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

or $\quad h\rho g = 2\sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

or $\quad h = \frac{2\sigma}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Given $\quad \sigma = 0.07\ \text{Nm}^{-1}, \ \rho = 1000\ \text{kgm}^{-3},$

$r_1 = 1.5 \times 10^{-3}\ \text{m}, r_2 = 3 \times 10^{-3}\ \text{m}$

$$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right)\text{m}$$

$= 4.76 \times 10^{-3}\ \text{m} = \mathbf{4.76\ mm.}$

Problems For Practice

1. The radius of a capillary tube is 0.025 mm. It is held vertically in a liquid whose density is $0.8 \times 10^3\ \text{kg m}^{-3}$, surface tension is $3.0 \times 10^{-2}\ \text{Nm}^{-1}$ and for which the cosine of the angle of contact is 0.3. Determine the height upto which the liquid will rise in the tube. Given $g = 10\ \text{ms}^{-2}$. **(Ans. 9 cm)**

2. A capillary tube of inner diameter 0.5 mm is dipped in a liquid of specific gravity 13.6, surface tension 545 dyne cm^{-1} and angle of contact 130°. Find the depression or elevation in the tube. **(Ans. – 2.1 cm)**

3. Calculate the diameter of a capillary tube in which mercury is depressed by 1.21 cm. Given surface tension for mercury is $540 \times 10^{-3}\ \text{Nm}^{-1}$, the angle of contact with glass is 140° and density of mercury is $13.6 \times 10^3\ \text{kg m}^{-3}$. **(Ans. 1026×10^{-3} m)**

4. The tube of mercury barometer is 5 mm in diameter. How much error does the surface tension cause in the reading ? S.T. of mercury $= 540 \times 10^{-3}\ \text{Nm}^{-1}$. Angle of contact = 135°. **(Ans. -0.2293×10^{-2} m)**

5. Water rises to a height of 9 cm in a certain capillary tube. If in the same tube, level of Hg is depressed by 3 cm, compare the surface tension of water and mercury. Specific gravity of Hg is 13.6, the angle of contact for water is zero and that for Hg is 135°. **(Ans. 0.152)**

6. A capillary tube whose inside radius is 0.5 mm is dipped in water of surface tension 75 dyne cm^{-1}. To what height is the water raised by the capillary action above the normal level ? What is the weight of water raised ? **(Ans. 3.061 cm, 23.55 dyne or 0.024 g wt)**

7. A U-tube is made up of capillaries of bore 1 mm and 2 mm respectively. The tube is held vertically and partially filled with a liquid of surface tension 49 dyne cm^{-1} and zero contact angle. Calculate the density of the liquid, if the difference in the levels of the meniscus is 1.25 cm. Take $g = 980\ \text{cms}^{-2}$. **(Ans. $0.8\ \text{g cm}^{-3}$)**

Hints

3. Here $\quad h = -1.21\ \text{cm} = -1.21 \times 10^{-2}\ \text{m}, \theta = 140°$

$\sigma = 540 \times 10^{-3}\ \text{Nm}^{-1},$

$\cos 140° = \cos(180° - 40°) = -\cos 40° = -0.7660$

Diameter,

$$2r = \frac{4\sigma\cos\theta}{h\rho g} = \frac{4 \times 540 \times 10^{-3} \times (-0.7660)}{-1.21 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8}$$

$= \mathbf{1.026 \times 10^{-3}\ m.}$

4. Error in the barometer reading

= Depression of mercury level due to surface tension

$$= \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 540 \times 10^{-3} \times \cos 135°}{2.5 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$= \mathbf{-0.2293 \times 10^{-2}\ m.}$

5. $$\frac{\sigma_w}{\sigma_m} = \frac{h_1\rho_1\cos\theta_2}{h_2\rho_2\cos\theta_1} = \frac{10 \times 1 \times \cos 135°}{-3.42 \times 13.6 \times \cos 0°}$$

$$= \frac{10 \times (-0.7071)}{-3.42 \times 13.6} = \mathbf{0.152.}$$

6. Here $r = 0.5\ \text{mm} = 0.05\ \text{cm}, \sigma = 75\ \text{dyne cm}^{-1},$

$\rho = 1\ \text{g cm}^{-3}, \theta = 0°, \ g = 980\ \text{dyne cm}^{-1}.$

$$h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 75 \times 1}{0.05 \times 1 \times 980} = \mathbf{3.061\ cm}$$

Weight of water raised,

$W = mg = \text{Volume} \times \text{Density} \times g = \pi r^2 h\rho g$

$= 3.14 \times (0.05)^2 \times 3.061 \times 980 = \mathbf{23.55\ dyne}$

$= \mathbf{0.024\ g\ wt.}$

7. Here $\quad h_1 = \frac{2\sigma\cos\theta}{r_1\rho g}$ and $h_2 = \frac{2\sigma\cos\theta}{r_2\rho g}$

$$\therefore \quad h_1 - h_2 = \frac{2\sigma\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\rho = \frac{2\sigma\cos\theta}{(h_1 - h_2)g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Scanned by CamScanner

Now $r_1 = \frac{1}{2}$ mm = 0.05 cm,

$r_2 = \frac{2}{2}$ mm = 0.1 cm,

$\sigma = 49$ dyne cm^{-1},

$h_1 - h_2 = 1.25$ cm, $\theta = 0°$, $g = 980$ cm s^{-2}

$$\therefore \quad \rho = \frac{2 \times 49 \times \cos 0°}{1.25 \times 980}\left[\frac{1}{0.05} - \frac{1}{0.1}\right]$$

$$= \frac{2 \times 49 \times 1}{1.25 \times 980} \times 10 = \mathbf{0.8\ g\ cm^{-3}}.$$

10.54 FACTORS AFFECTING SURFACE TENSION

72. *Describe the various factors which affect the surface tension of a liquid.*

Factors affecting surface tension of a liquid :

(i) **Effect of contamination.** If water surface has dust, grease or oil, the surface tension of water reduces. A small piece of camphor put in clear water dances vigorously due to decrease of surface tension of water.

(ii) **Effect of solute.** *(a)* A highly soluble substance like sodium chloride increases the surface tension of water. *(b)* A sparingly soluble substance like phenol or soap reduces the surface tension of water.

(iii) **Effect of temperature.** The surface tension of liquids decreases with increase of temperature. The surface tension of a liquid becomes zero at a particular temperature, called *critical temperature* of that liquid.

For small temperature differences, surface tension decreases almost linearly as

$$\sigma_t = \sigma_0 (1 - \alpha t)$$

where σ_t and σ_0 are the surface tensions at t°C and 0°C respectively, and α is the temperature coefficient of surface tension.

10.55 DETERGENTS AND SURFACE TENSION

73. *Describe the cleansing action of detergents.*

Cleansing action of detergents. Oil stains and grease on dirty clothes cannot be removed by simply washing the clothes with water because water does not wet them. By adding detergent or soap to water, the greasy dirt can be easily removed. The cleansing action of detergents can be explained as follows :

(i) Soap or detergent molecules have the shape of a hairpin.

(ii) When detergent is dissolved in water, the heads of its hairpin shape molecules get attracted to water surface.

(iii) When clothes with greasy stains are dipped in water containing detergent, the pointed ends of detergent molecules get attached to the molecules of grease. So a water-grease interface is formed. Thus surface tension is greatly reduced. The greasy dirt is held suspended.

(iv) When the clothes are rinsed in water, the greasy dirt is washed away by running water.

So when detergent is added to water, the surface tension of water is reduced, its area of contact with grease is increased and hence its cleansing ability is increased.

Very Short Answer Conceptual Problems

Problem 1. We can cut an apple easily with a sharp knife as compared to with a blunt knife. Explain why ? *[Himachal 09]*

Solution. The area of a sharp edge is much less than the area of a blunt edge. For the same total force, the effective force per unit area (or pressure) is more for the sharp edge than the blunt one. Hence a sharp knife cuts easily than a blunt knife.

Problem 2. Why are the bags and suitcases provided ith broad handles ?

Solution. Broad handles have large area. So the force unit area or the pressure exerted on the hand will be ll while carrying the bag or the suitcase.

roblem 3. Railway tracks are laid on large sized len sleepers. Why ?

Solution. This spreads force due to the weight of the train on a larger area and hence reduces the pressure considerably. This, in turn, prevents yielding of the ground under the weight of the train.

Problem 4. It is painful to walk barefooted on the ground covered with edged pebbles. Why ?

Solution. While walking, when entire weight of our body gets supported on the sharp edge of any pebble, it will exert a large pressure on our feet due to reaction. This causes considerable pain on our feet.

Problem 5. It is difficult for a man to walk on sand. Why ?

Solution. This is because sand yields under the weight of the man. This difficulty can be overcome by placing wooden plank on the sand. Then the weight of the man will act on larger area and hence pressure exerted on sand decreases. The sand does not yield.

Scanned by CamScanner

Problem 6. Water does not come out of a dropper unless its rubber bulb is pressed hard. Why ?

Solution. Water is held inside the dropper against the atmospheric pressure. When the rubber bulb is pressed, pressure on water becomes greater than the atmospheric pressure and so water comes out.

Problem 7. Why two holes are made to empty an oil tin ?

Solution. When oil comes out through a tin with one hole, the pressure inside the tin becomes less than the atmospheric pressure, soon the oil stops flowing out. When two holes are made in the tin, air keeps on entering the tin through the other hole and maintains pressure inside.

Problem 8. Why is a slight blow on a cork of bottle filled with a liquid sufficient to break the bottle ?

Solution. The blow on the cork exerts a pressure ($= f/a$) on the liquid. According to Pascal's law, this pressure is transmitted to the entire bottle through the liquid. Since the surface area (A) of the bottle is large, the large force ($= fA/a$) so developed is sufficient to break the bottle.

Problem 9. What is the force on a man due to atmospheric pressure ? Why one does not feel it ?

Solution. Atmospheric pressure on the surface of the earth is about 1 kg f per square centimetre. The surface area of the body of average man is about $2 m^2$ and hence, a total force equal to about 2×10^5 N, acts on the body of a man due to atmospheric pressure. But one does not feel this enormous force because his blood exerts a pressure slightly greater than the atmospheric pressure.

Problem 10. How does the boiling point of a liquid vary with pressure ?

Solution. The boiling point of a liquid increases with pressure. For example, if the pressure is more than the atmospheric pressure, water boils at a temperature higher than 100°C.

Problem 11. Why the boiling point of a liquid varies with pressure ?

Solution. At the boiling point, vapour pressure of the liquid is equal to the atmospheric pressure. Hence when the atmospheric pressure on the surface of the liquid increases, the liquid boils at higher temperature to generate greater vapour pressure.

Problem 12. Why the food is cooked faster in the pressure cooker ? Why it becomes difficult to cook food at the mountains ?

Solution. The pressure inside the pressure cooker is very high. This raises the boiling point of water and the temperature inside the cooker is higher than 100°C which results in faster cooking of food. At the mountains, the pressure is less, so the boiling point of water is less than 100°C. This makes the cooking of food difficult.

Problem 13. Why the passengers are advised to remove the ink from their pens while going up in an aeroplane ?

Solution. We know that atmospheric pressure decreases with height. Since ink inside the pen is filled at the atmospheric pressure existing on the surface of earth, it tends to come out to equalise the pressure. This can spoil the clothes of the passengers, so they are advised to remove the ink from the pen.

Problem 14. Why is it difficult to stop bleeding from a cut in human body at high altitudes ?

Solution. The atmospheric pressure is low at high altitudes. Due to greater pressure difference in blood pressure and the atmospheric pressure, it is difficult to stop bleeding from a cut in the body.

Problem 15. Why are straws used to suck soft drinks ?

Solution. When we suck through the straw, the pressure inside the straw becomes less than the atmospheric pressure. Due to the pressure difference, the soft drink rises in the straw and we are able to take the soft drink easily.

Problem 16. If a drop of water vapour is introduced in a mercury barometer, how will the barometric height change ?

Solution. Due to pressure exerted by water vapour, the barometric height decreases.

Problem 17. Why water is not used in barometers ?

Solution. Water is not used in barometer due to following reasons :

(*i*) It sticks to the walls of the barometer tube.

(*ii*) It has low density. A barometer with water will require a tube length of about 11 m which is unmanageable.

Problem 18. Why is mercury used in barometers ?

Solution. Mercury is used in barometers due to the following reasons :

(*i*) It does not stick to the walls of the barometric tube.

(*ii*) Its density is high, so the length of the tube used is conveniently small.

(*iii*) Its vapour pressure is quite small.

Problem 19. Why is the reading of a mercury barometer always less than actual pressure ?

Solution. Due to property of surface tension, mercury in the tube gets depressed. Consequently, the observed height of mercury in the barometer tube is less than its actual height. Thus, the reading of mercury barometer is always less than the actual pressure.

Problem 20. How can we check whether the barometric tube contains air or not ?

Solution. If on raising or lowering the barometric tube in a trough of mercury, the height of mercury column changes, then air is present in the barometric tube.

Scanned by CamScanner

Problem 21. The dams of water reservoir are made thick near the bottom. Why ?

Solution. Pressure exerted by a liquid column of height $h = h\rho g$. As the value of h is greatest near the bottom, so pressure exerted by water is greatest near the bottom. So dams are made thick near the bottom.

Problem 22. Why an air bubble in water rises from bottom to top and grows in size ?

Solution. The fluids move from higher pressure to lower pressure and a fluid pressure increases with depth. Hence pressure at the top is less than that at the bottom and so the air bubble will rise from bottom to top. When bubble moves from bottom to top, pressure decreases and in accordance with Boyle's law volume V will increase *i.e.*, bubble will grow in size.

Problem 23. A beaker containing a liquid is kept inside a big closed jar. If the air inside the jar is continuously pumped out, how will the pressure change inside the liquid near the bottom ?

Solution. The total pressure at a point near the bottom is equal to the sum of the pressure due to liquid column and due to air inside the jar. When air of the jar is pumped out, the pressure of air inside the jar decreases. So pressure near the bottom of the liquid also decreases.

Problem 24. Why does a siphon fail to work in vacuum ?

Solution. As siphon works on account of atmospheric pressure, hence it fails to work in vacuum.

Problem 25. A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. What will be the possible air pressure inside the elevator ?

Solution. The net acceleration of the elevator accelerating upwards $= g + a$

$\therefore$ Pressure inside the elevator

$$= h\rho\,(g + a) = \frac{76 \times 13.6 \times (g + a)}{13.6 \times g} \text{ cm of Hg}$$

Clearly, this pressure will be greater than 76 cm of Hg.

Problem 26. A barometer accelerating downwards reads 76 cm of Hg. What will be the possible air pressure inside the jar ?

Solution. The net acceleration of the elevator accelerating downwards $= g - a$

$\therefore$ Pressure inside the elevator

$$= h\rho\,(g - a) = \frac{76 \times 13.6 \times (g - a)}{13.6 \times g} \text{ cm of Hg}$$

Clearly, this pressure will be less than 76 cm of Hg.

Problem 27. In a mercury barometer, at sea level, the normal pressure of the air (one atmosphere) acting on the mercury in the dish supports a 76 cm column of mercury in a closed tube. If you go up in the air, until the density has fallen to half its sea level value, what height of mercury column would you expect ?

Solution. Pressure exerted by a gas is directly proportional to its density. When we go high up in air at a point where the density of air falls to half its sea level value, the pressure also reduces to half its sea-level value. Hence the height of mercury column is also halved *i.e.*, it becomes 38 cm.

Problem 28. A liquid cannot withstand a shear stress. How does this imply that the surface of a liquid at rest must be level, *i.e.*, normal to the gravitational force ?

Solution. If the free surface of the liquid is not perpendicular to the gravitational force, then there will be a component of force along the surface. The liquid will not be at rest in that case.

Problem 29. A wooden block is on the bottom of a tank when water is poured in. The contact between the block and the tank is so good that no water gets between them. Is there a buoyant force on the block ?

Solution. Since there is no water under the block to exert an upward force on it, therefore, there is no buoyant force.

Problem 30. A piece of iron sinks in water, but a ship made of iron floats in water. Why ?

Solution. The weight of water displaced by iron piece is less than its own weight, so it sinks. On the other hand, the ship displaces water more than its own weight, so it floats.

Problem 31. A man is sitting in a boat, which is floating in a pond. If the man drinks some water from the pond, will the level of water in the pond fall ?

Solution. No. When the man drinks water, say m kg, he displaces m kg of water and hence the level tends to increase. But m kg of water has already gone inside his stomach. So the level remains the same.

Problem 32. An ice cube floats in a glass of water filled to the brim. What happens when the ice melts ?

Solution. The water level remains unchanged. The ice cube displaces a weight of water equal to its own weight. When the ice cube melts, the volume of water produced equals the volume of water it displaced when frozen.

Problem 33. An ice piece with an air bubble in it is floating in a vessel containing water. What will happen to the level of water when the ice melts completely ?

Solution. The level of water does not change. The mass of air bubble is negligible. So the volume of water produced during melting is equal to the volume of water displaced by ice piece with air bubble.

Problem 34. Ice floats in water with about nine-tenths of its volume submerged. What is the fractional volume submerged for an iceberg floating on a fresh water lake of a (hypothetical) planet whose gravity is ten times that of earth ?

Solution. The fractional volume submerged does not depend upon the value of acceleration due to gravity. So, on the new planet, the ice will float in water with nearly nine-tenths of its volume submerged.

Scanned by CamScanner

Problem 35. Does the Archimedes' principle hold in a vessel in free fall ?

Solution. No. The vessel in free fall is in a state of weightlessness *i.e.*, the value of *g* is zero. The buoyant force does not exist. Hence Archimedes' principle does not hold good.

Problem 36. What is the fractional volume submerged of an ice cube in a pail of water placed in an enclosure which is falling freely under gravity ?

Solution. Since the pail of water is falling freely, therefore, it will be in state of weightlessness. Both the weight of the ice cube and the upthrust would be zero. So, the ice cube can float with any value of fractional volume submerged in water.

Problem 37. A piece of cork is floating in water contained in a beaker. What is the apparent weight of the cork piece ?

Solution. The weight of the cork piece acting vertically downwards is balanced by the upthrust due to water. So the apparent weight of the floating cork piece is zero.

Problem 38. One small and one big piece of cork are pushed below the surface of water. Which has greater tendency to rise swiftly ?

Solution. The upthrust on a piece of cork is equal to the weight of water displaced by it. So upthrust is greater on the bigger piece of cork and it has greater tendency to rise swiftly.

Problem 39. Why is it easier to swim in sea water than in river water ?

Solution. The sea water has many salts dissolved in it. So the density of sea water is greater than that of river water. Consequently, the sea water exerts greater upthrust on the swimmer than the river water. Hence it is easier to swim in sea water than in river water.

Problem 40. Two bodies of equal weight and volume and having the same shape, except that one has an opening at the bottom and the other is sealed, are immersed to the same depth in water. Is less work required to immerse one than the other ? If so, which one and why ?

Solution. More work is required in case of the body having hole at its bottom. As the liquid enters the hole, more work is required in compressing the air, less work is required in case of the sealed body.

Problem 41. A wooden cylinder floats in a vessel with its axis vertical. How will the level of water in the vessel change if the cylinder floats with its axis horizontal ?

Solution. The level of water will not change because in both cases, the cylinder displaces equal volume of water.

Problem 42. A block of ice is floating in a liquid of specific gravity 1.2 contained in a beaker. When the ice melts completely will the level in the beaker change ?

Solution. The level of liquid in the beaker will rise. This is because the density of water formed by melting of ice is less than the density of liquid in the beaker. Consequently, the volume of water formed by melting of ice will be more than the volume of the portion of the liquid displaced by ice while floating.

Problem 43. A boy is carrying a fish in one hand and a bucket full of water in the other hand. He then places the fish in the bucket and thinks that in accordance with Archimedes' principle he is now carrying less weight as weight of fish will reduce due to upthrust. Is he right ?

Solution. No. When he places the fish in the water in the bucket, the weight of fish is reduced due to upthrust but the weight of water is increased by the same amount so that the total weight carried by the boy remains the same.

Problem 44. A solid body floats on mercury with a part of its volume below the surface. Will the fractional volume of the body immersed in mercury increase or decrease, if a layer of water poured on the top of mercury covers the body completely ?

Solution. In the first case, the weight of the body is balanced by the weight of mercury displaced. But when water is poured to cover the body completely, the weight of (mercury + water) displaced by the body is equal to the weight of the body. The weight of mercury displaced is now less than that in the first case. So the fractional volume of the body inside the mercury will decrease.

Problem 45. A bucket of water is suspended from a spring balance. Does the reading of balance change (*a*) When a piece of stone suspended from a string is immersed in water without touching the bucket ? (*b*) When a piece of iron or cork is put in water in the bucket ?

Solution. (*a*) Yes, the reading of balance will increase but the increase in weight will be equal to the loss in weight of stone and not the weight of stone.

(*b*) Yes, the reading of balance will increase but increase in weight will be equal to weight of iron or cork piece.

Problem 46. Why a sinking ship often turns over as it becomes immersed in water ?

Solution. When the ship is floating, the meta-centre of the ship is above the centre of gravity. While sinking, the ship takes in water. As a result, the centre of gravity is raised above the meta-centre. The ship turns over due to the couple formed by the weight and the buoyant force.

Problem 47. A soft plastic bag weighs the same when empty as when filled with air at atmospheric pressure. Why ?

Solution. The weight of air displaced by the bag is same as the weight of air inside it. The increase in weight due to the filled air gets cancelled by the upthrust of air. So weight remains same when air is filled in the bag.

Scanned by CamScanner

Problem 48. Stirred liquid comes to rest after some time. Why ?

Solution. Different layers of a stirred liquid destroy the relative motion among themselves due to internal viscous force.

Problem 49. What is the reason that a constant driving force is always required for the maintenance of the flow of oil through the pipe lines in the oil refineries ?

Solution. Due to viscous force between the layers of oil, the motion of liquids gets retarded after a certain distance. Hence, a constant driving force is required for maintaining the flow of oil through the pipe lines.

Problem 50. What is the effect of temperature on the viscosity of liquids and gases ?

Solution. The viscosity of liquids decreases with rise in temperature while that of gases increases with an increase in temperature.

Problem 51. Hotter liquids move faster than colder ones. Why ? *[Central Schools 12]*

Solution. The viscosity or the internal force of friction of a liquid decreases with the increase in temperature. Hence hotter liquids move faster than colder ones.

Problem 52. Why oils of different viscosity are used in different seasons ?

Solution. Due to the rise of temperature in summer, the viscosity of oil decreases. Hence more viscous oils are used in summer than in winter.

Problem 53. Why machine parts are jammed in the winter ?

Solution. In winter season, the coefficient of viscosity of the lubricating oil gets increased due to fall in temperature. As a result of this, oil becomes more thick and therefore various machine parts are jammed.

Problem 54. One flask contains glycerine and other contains water. Both are stirred vigorously and kept on the table. Which liquid will come rest to earlier than the other ?

Solution. As the coefficient of viscosity of glycerine is greater than that of water, so the relative motion between different layers of glycerine is more strongly opposed compared to water. Hence glycerine comes to rest earlier than water.

Problem 55. Why should the lubricant oil be of high viscosity ?

Solution. Lubricant oil is used to reduce the dry friction between various machine parts. Due to high viscosity, lubricant oil sticks to the machine parts and cannot be squeezed out during the working of the machine.

Problem 56. Why does an object entering the earth's atmosphere at high velocity catch fire ?

Solution. When an object enters the earth's atmosphere at high velocity, its motion is strongly opposed by the viscous drag of air. As a result, the kinetic energy of the object gets converted into heat energy. The heat produced may be so large that the object catches fire.

Problem 57. The velocity of water in a river is less near the bank and large in the middle. Why ?

Solution. Due to large adhesive force between the water stream and the bank of the river, the velocity of water is very small near the bank. It increases towards the middle of the river due to decrease in the adhesive force.

Problem 58. Which fall faster – big rain drops or small rain drops ?

Solution. The terminal velocity of rain drops

$$v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)}{\eta}g \quad i.e., \; v_t \propto r^2.$$

Hence big drops fall faster.

Problem 59. Why rain drops falling under gravity do not gain very high velocity ?

Solution. When the rain drops fall freely under gravity, their motion is opposed by the viscous drag of air. They attain terminal velocity when the viscous force due to air becomes equal to the weight of the drops.

Terminal velocity, $v \propto r^2$

As radii of rain drops are small, so they do not acquire very high velocity.

Problem 60. Why dust generally settles down in a closed room ?

Solution. Dust particles are spheres of small radii. After acquiring the terminal velocity, they start falling through air with uniform speed. As the terminal velocity for small dust particles will be very small, they will settle down in a closed room after some time.

Problem 61. Why do clouds seen floating in the sky ?

Solution. We know that terminal velocity of a spherical body falling through a viscous medium is directly proportional to the square of its radius. Hence terminal velocity of a small drop of water is very small. The small drops of water acquire this terminal velocity much before reaching the earth, and are seen to float in the sky in the form of clouds.

Problem 62. Explain why parachute is invariably used, while jumping from an aeroplane.

Solution. A parachute experiences a large viscous force due to its huge structure. Hence, it descends through the air with a very small velocity and the person using the parachute does not get hurt.

Problem 63. Fog particles appear suspended in the atmosphere. Give reason. *[Delhi 11]*

Solution. Fog particles have very small sizes. So their terminal velocity ($v_t \propto r^2$) through air is very small. They appear suspended in the atmosphere.

Scanned by CamScanner

Problem 64. The sides of a horizontal pipe carrying dirty water get dirty. Why ?

Solution. While flowing through the horizontal pipe, the velocity of dirty water is maximum along the axis of the pipe and minimum near the walls of the pipe. The viscous force opposes the relative motion between adjacent layers. Due to this, the dirt particles move outwards and get struck on the walls of the pipe. This makes the pipe dirty.

Problem 65. Why two streamlines cannot cross each other ? *[Himachal 04]*

Solution. If two streamlines cross each other, there will be two directions of flow at the point of intersection which is impossible.

Problem 66. What happens when the velocity of a liquid becomes greater than its critical velocity ?

Solution. The flow of liquid changes from streamline to turbulent.

Problem 67. What happens to the external energy maintaining the flow of a liquid when the flow becomes turbulent ?

Solution. In turbulent flow of a liquid, most of the external energy maintaining the flow is used in setting up the eddies in the liquid.

Problem 68. Why does the velocity increase when water flowing in a broader pipe enters a narrow pipe ?

Solution. This is due to equation of continuity :

$$a_1 v_1 = a_2 v_2$$

As $a_2 < a_1$, so $v_2 > v_1$.

Problem 69. Why still water runs deep ?

Solution. In case of a deep water, the cross-sectional area, through which it flows, is quite large. Hence according to equation of continuity, its velocity is small and therefore it appears to be still.

Problem 70. Why it is dangerous to stand near the edge of the platform when a fast train is crossing it ?

Solution. When a fast train crosses the platform, the air dragged along with the train also moves with a high velocity. In accordance with Bernouilli's equation, the pressure in the region of high velocity gets decreased. If a person stands near the edge of the platform, he may be pushed towards the train due to high pressures outside.

Problem 71. Why two boats moving in parallel directions close to each other get attracted ? *[Delhi 11]*

Solution. When the two boats come closer to each other, the velocity of water between the narrow gap increases and so pressure decreases in accordance with Bernouilli's equation. The pressure on the outer surfaces of both the row boats becomes greater than the pressure in the gap. Therefore, the two boats are pulled towards each other.

Problem 72. Why does the speed of a liquid increase and its pressure decrease, when the liquid passes through a narrow constriction in a pipe ?

Solution. When the liquid passes through a narrow constriction, its velocity increases ($v \propto 1/a$) in accordance with the equation of continuity. The Bernouilli's equation for the horizontal flow is

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

As the velocity is high at the narrow constriction, so the pressure is low.

Problem 73. Why does a flag flutter, when strong winds are blowing on a certain day ?

Solution. When strong winds blow over the top of the flag, the kinetic energy of the wind at the top is more and hence pressure decreases. Due to difference in pressure above and below the flag, the flag flutters.

Problem 74. On which principle does a carburettor work ?

Solution. Carburettor is that chamber of an internal combustion engine in which air is mixed with petrol vapours. Air is allowed to enter through a nozzle with high velocity. The pressure is lowered at that point and petrol is sucked up into the chamber. The petrol vaporises quickly. The mixed vapours of petrol and air are then fed into the cylinder of internal combustion engine.

Problem 75. Roofs of the huts are blown up during stormy days. Why ? *[Delhi 01 ; Central Schools 09]*

Solution. The velocity of the wind is high above the roof. So the pressure is low in accordance with Bernouilli's principle. But the pressure below the roof is the atmospheric pressure which is high. So the roof is blown up.

Problem 76. When air is blown between two balls suspended close to each other, they are attracted towards each other. Why ? *[Delhi 10]*

Solution. When air is blown between the two balls, the velocity is increased and hence pressure is decreased (Bernolli's principle). On the outer sides of the balls, the pressure is high and hence the two balls get attracted.

Problem 77. An aeroplane runs for some distance on the runway before taking off. Why ?

Solution. Generally, air does not strike an aeroplane with a large velocity. To get the lift, the aeroplane runs for some distance on the runway before taking off. Due to its special shape, the velocity of air above the plane increases and hence pressure decreases. The aeroplane gets an uplift.

Problem 78. The accumulation of snow on the aeroplane may reduce the lift. Explain.

Solution. Due to the accumulation of snow on the wings of the aeroplane, the shape of the wings no longer remains that of the aerofoil. This reduces the path

Scanned by CamScanner

difference and hence the velocity difference between the layer of air on the two sides of the wings. Hence the pressure difference on the two sides of the wings is reduced. This reduces the uplift on the aeroplane.

Problem 79. Why bullets are given cylindrical shape ?

Solution. The magnus effect is absent if the spinning cylinder is moving linearly in the direction parallel to spin axis. That is why the bullets are made cylindrical instead of spherical. They do not deviate from the linear path.

Problem 80. If air is blown very fast into the vertical hole of a spool of thread, a card laid flat against the other end does not fall. Why ?

Solution. Refer to Fig. 10.62. Due to large velocity of air between the spool and the card, the pressure between them reduces below the atmospheric pressure. The card is held by the higher atmospheric pressure below it.

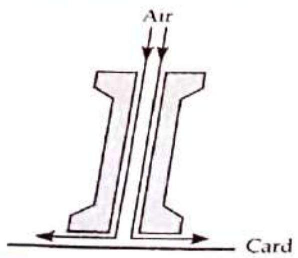


Fig. 10.62

Problem 81. Explain why we cannot remove a filter paper from a funnel by blowing air into its narrow end.

Solution. Refer to Fig. 10.63. When air is blown into the narrow end of the funnel, the velocity of air in the region between the filter paper and the curved wall of the funnel increases. This decreases the pressure. The filter paper gets more firmly held with the wall of the funnel. So it is not possible to remove the filter paper from the funnel by blowing air into its narrow end.

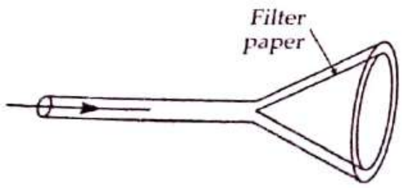


Fig. 10.63

Problem 82. According to Bernoulli's theorem, the pressure of water in a horizontal pipe of uniform diameter should remain constant. But actually it goes on decreasing with the increase in length of the pipe. Why ?

Solution. Bernoulli's theorem is valid only for non-viscous liquids. But water is a viscous liquid. A part of the pressure energy of water is used in doing work against the viscous force. So the pressure of water decreases.

Problem 83. What is the effect on the equilibrium of a physical balance when air is blown below one pan ?

Solution. Due to increase in velocity of air below the pan, the pressure decreases. So the pan goes down.

Problem 84. In case of an emergency, a vacuum brake is used to stop the train. How does this brake work ?

Solution. Steam at high pressure is allowed to enter the cylinder of the vacuum brake. Due to high velocity, the pressure decreases in accordance with Bernoulli's principle. The reduction of pressure lifts up the piston. This in turn lifts up the brake.

Problem 85. Is Bernoulli's theorem valid for viscous liquid ?

Solution. No, it should be modified to take into account the work done against viscous drag.

Problem 86. Two cylindrical vessels placed on a horizontal table contain water and mercury respectively up to the same heights. There is a small hole in the walls of each of the vessels at half the height of liquids in the vessels. Find out the ratio of the velocities of efflux of water and mercury from the holes. Which of the two jets of liquid will fall at a greater distance on the table from the vessel ? Relative density of mercury with respect to water = 13.6.

Solution. The velocity of efflux ($v = \sqrt{2gh}$) is independent of density of liquid. So, both the jets of liquid will fall at the same distance.

Problem 87. If a small ping pong ball is placed in a vertical jet of water or air, it will rise to a certain height above the nozzle and stay at that level. Explain.

Solution. Due to the high velocity of the jet of water, the pressure between the ball and the jet decreases. The greater (atmospheric) pressure on the other side of the ball pushes it against the jet and the ball remains suspended. The high velocity of water takes the ball upwards along with it and makes it to spin.

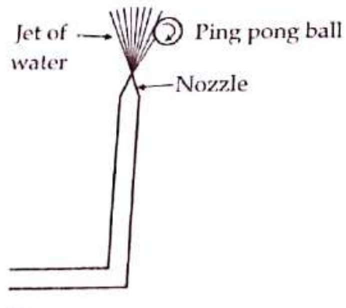


Fig. 10.64 A ping pong ball supported on a jet of water.

Problem 88. Bernoulli's theorem holds for incompressible, non-viscous fluids. How is this relationship changed when the viscosity of the fluid is not negligible ?

Solution. If the viscosity of the fluid is not negligible, a part of the mechanical energy of the fluid is spent in

Scanned by CamScanner

doing work against forces of viscosity. So the total energy : $p + \rho gh + \frac{1}{2}\rho v^2$ of the fluid goes on decreasing along the direction of flow of the fluid.

Problem 89. Why are the cars and aeroplanes given streamline shape ?

Solution. Cars and aeroplanes are given streamline shape to minimise the backward drag of atmosphere.

Problem 90. Why a glass rod coated with wax does not become wet when dipped in water ?

Solution. The reason is that force of adhesion between water and wax molecules is smaller than force of cohesion between water molecules.

Problem 91. Mercury does not cling to glass. Why ? *[Himachal 04]*

Solution. The force of adhesion between mercury and glass is smaller than the force of cohesion between the mercury molecules.

Problem 92. Why is it not possible to separate two pieces of paper joined by glue or gum ?

Solution. The adhesive force between the molecules of glue and paper is much more than the cohesive force between the glue molecules. As a result, the two papers stick together with a large force and cannot be separated.

Problem 93. Why does the free surface a liquid behave like an elastic stretched membrane ?

Solution. The liquid molecules on the surface experience a downward force. So they have greater potential energy. In order to have minimum energy, the free surface tends to contract to minimum area and hence behaves like an elastic stretched membrane.

Problem 94. Why does mercury collect itself into drops when placed on a clean glass plate ?

Solution. The adhesive force between mercury and glass molecules is less than the cohesive force between mercury molecules. So instead of sticking to glass, mercury collects itself into drops.

Problem 95. How dental plate clinges to the roof of the mouth ?

Solution. The dental plate is made exactly parallel to the roof of the mouth at all the points. A small amount of sliva gets enclosed between dental plate and the roof of the mouth. Due to the force of adhesion between the sliva and dental plate, the plate clinges to the mouth.

Problem 96. What shape does a liquid take when it weighs nothing ?

Solution. When a liquid weighs nothing, surface tension is the only force acting on it. Due to surface tension, the liquid surface tends to occupy minimum surface area. For a given volume, the surface area of a sphere is minimum, so the liquid assumes spherical shape.

Problem 97. One can form a fairly large vertical film of soap solution but not of pure water. Why ?

Solution. This is because the surface tension of soap solution is much smaller than that of pure water. When we try to make a vertical film of pure water, it breaks due to the high surface tension of water.

Problem 98. Small insects can move about on the surface of water. Why ? *[Central Schools 08]*

Solution. Due to surface tension, the free surface of water behaves like stretched elastic membrane which is able to support the small weight of insects and they can move on the surface of water.

Problem 99. Why ends of a glass tube become rounded on heating ?

Solution. When glass is heated, it melts to a liquid. The surface of this liquid tends to have a minimum area. Now, for a given volume, the area of the surface of a sphere is minimum. This is why the ends of a glass tube become rounded on heating.

Problem 100. Antiseptics have low surface tension. Why ? *[Himachal 03]*

Solution. Low surface tension helps the antiseptic to spread over large area over the wounds.

Problem 101. Why hot soup tastes better than cold soup ?

Solution. Hot soup has comparatively less surface tension than cold soup. So it spreads over larger area of the tongue and tastes better than cold soup.

Problem 102. Oil spreads over the surface of water whereas water does not spread over the surface of oil. Why ?

Solution. The surface tension of the water is more than that of oil. Therefore, when oil is poured over water, the greater value of surface tension of water pulls oil in all directions, and as such it spreads on the water. On the other hand, when water is poured over oil, it does not spread over it because the surface tension of oil being less than that of water, it is not able to pull water over it.

Problem 103. Why does a stream of water from a faucet become narrow as it falls ?

Solution. This is due to surface tension. The molecules of water come close to each other due to surface tension and surface area tends to be minimum making the stream narrow.

Problem 104. Why do the hair of a shaving brush cling together when taking out of water ?

Solution. When the brush is taken out of water, thin water film is formed at the tips of the hair. It contracts due to surface tension and so the hair cling together.

Problem 105. A needle floats on the surface of pure water but goes down when detergent is added to water. Why ?

Solution. Due to addition of detergent, the surface tension and hence the reaction of surface tension decreases. Hence the needle sinks due to smaller upward reaction.

Scanned by CamScanner

Problem 106. Why it becomes easier to spray the water in which some soap is dissolved ?

Solution. When soap is dissolved in water, the surface tension of water decreases and so less energy is needed for spraying the water *i.e.*, spraying of water in which soap is dissolved becomes easier.

Problem 107. The clothes are better cleaned with hot water than with cold water. Why ? [Delhi 1996]

Solution. Surface tension decreases with the increase of temperature. Lesser the surface tension, more is the wetting (and hence the washing) power of water.

Problem 108. How does soap help us to remove dirt better in washing clothes ? [Delhi 1999]

Solution. With the addition of soap, the surface tension of water decreases. The decrease in surface tension results in greater wetting and hence washing power.

Problem 109. A oil drop on a hot cup of soup spreads over when the temperature of the soup falls. Why ?

Solution. The surface tension of hot water is less than that of oil and hence oil drop does not spread over hot water. When water is cooled, its surface tension decreases. At low temperature, the surface tension of water becomes greater than that of oil and hence oil drop starts spreading over it.

Problem 110. Glass marbles are made by heating the end of a glass rod until drops of molten glass fall. Explain.

Solution. When drops of molten glass fall freely, they are in state of weightlessness. As only the force of surface tension acts on them, so they acquire spherical shape which on solidification become glass marbles.

Problem 111. Oil is sprinkled on sea waves to calm them. Why ?

Solution. When oil is sprinkled, the breeze spreads the oil on the sea-water in its own direction. The surface tension of sea-water (without oil) is greater than oily water. Hence the water without oil pulls the oily water against the direction of breeze, and the sea waves become calm.

Problem 112. A tiny liquid drop is spherical but a larger drop has oval shape. Why ?

Solution. In the case of a tiny drop, the force of surface tension is large compared to its weight, so the drop has spherical shape. A large drop has oval shape because the force of gravity (weight) exceeds the force of surface tension.

Problem 113. Why does a small piece of camphor dance about on the water surface ?

Solution. Due to its irregular shape, the camphor piece dissolves more rapidly at some points than at others. Where it dissolves, the surface tension of water is reduced. As the force of surface tension reduces by different amounts at different points of the camphor piece, a resultant force acts on it which makes it dance about on water surface.

Problem 114. The addition of flux to tin makes soldering easy. Why ?

Solution. The addition of flux reduces surface tension of the molten tin. This helps it spread easily over the area of soldering.

Problem 115. The paints and lubricating oils have low surface tension. Why ?

Solution. The paints and lubricating oils having low surface tension can spread over a large surface area.

Problem 116. Why are the droplets of mercury when brought in contact pulled together to from a bigger drop ? Also state with reason whether the temperature of bigger drop will be the same, or more, or less than the temperature of the smaller drops ?

Solution. Due to surface tension, liquid drops tend to have minimum surface area. When mercury droplets are brought in contact they form one drop thereby decreasing the surface area. Due to decrease in surface, surface energy is lost by the bigger drop which appears as heat. So its temperature increases.

Problem 117. The angle of contact for a solid and a liquid is less than 90°. Will the liquid wet the solid ? Will the liquid rise in the capillary made of that solid ?

Solution. The liquid will wet the solid and will rise in the capillary tube made of that solid.

Problem 118. Write down formula for excess pressure inside (*i*) a liquid drop (*ii*) a soap bubble.

Solution. (*i*) For a liquid drop : $p = \frac{2\sigma}{R}$.

(*ii*) For a soap bubble : $p = \frac{4\sigma}{R}$,

where σ is surface tension.

Problem 119. Why excess pressure in a soap bubble is twice the excess pressure of a liquid drop of the same radius ?

Solution. A soap bubble has two free surfaces, one internal and another external ; whereas liquid drop has only one outer free surface.

Problem 120. Two soap bubbles of unequal sizes are blown at the ends of a capillary tube. Which one will grow at the expense of the other and what does it show ?

Solution. The bigger one will grow at the expense of the smaller one. This is because excess pressure is inversely proportional to radius and air flows from higher pressure to lower pressure.

Problem 121. What is the importance of (*i*) wetting agents used by dyers, and (*ii*) water proofing agents ?

Solution. (*i*) They are added to decrease the angle of contact between the fabric and the dye so that the dye may penetrate well. (*ii*) They are used to increase the angle of contact between the fabric and water to prevent the water from penetrating the cloth.

Problem 122. Teflon is coated on the surface of non-sticking pans. Why ?

Solution. When the surface of a pan is coated with teflon, the angle of contact between the pan and the oil used for the frying purpose becomes obtuse. Thus the frying pan becomes non-sticking.

Problem 123. What makes water-proof rain coat water-proof ?

Solution. The angle of contact between water and the material of the raincoat is obtuse. So the rainy water does not wet the raincoat *i.e.*, the raincoat is water-proof.

Problem 124. How does the ploughing of fields help in preservation of moisture in the soil ? [Chandigarh 08]

Solution. This is done to break the tiny capillaries through which water can rise and finally evaporate. Th ploughing of field helps the soil to retain the moisture.

Problem 125. How does the cotton wick in oil-filled lamp keep on burning ?

Solution. The narrow spaces between the thread the wick serve as capillary tubes through which oil k on rising due to capillary action.

Problem 126. Why sand is drier than clay ?

Solution. Due to narrow pores in clay, water the clay due to capillary action and keeps it Practically no pores or capillaries exist in sand ar water cannot rise in sand. So sand is drier than

Problem 127. Why undergarments are usu of cotton ?

Solution. Cotton threads have large capillaries between them. These capillaries he sweat from the surface of the body due to capill

Problem 128. A piece of chalk immer emits bubbles in all directions. Why ? [

Solution. A chalk piece has a lar capillaries. As it is immersed in water, w capillary action. The air present in the c out in the form of bubbles in all directio

Problem 129. Why new earthen cooler than the old one ?

Solution. Due to capillary action, the pores of the new earthen pot. Thi heat from the pot to evaporate and so gets cooled. In old pots, most of blocked. So the cooling is not so eff

Problem 130. Why is the tip of t

Solution. The split in the tip capillary tube. The ink rises up in action. Hence we are able to writ

Problem 131. Water rises in a mercury falls in the same tube

Solution. In a capillary tube given by

Scanned by CamScanner

Problem 122. Teflon is coated on the surface of non-sticking pans. Why ?

Solution. When the surface of a pan is coated with teflon, the angle of contact between the pan and the oil used for the frying purpose becomes obtuse. Thus the frying pan becomes non-sticking.

Problem 123. What makes water-proof rain coat water proof ?

Solution. The angle of contact between water and the material of the raincoat is obtuse. So the rainy water does not wet the raincoat *i.e.*, the raincoat is water-proof.

Problem 124. How does the ploughing of fields help in preservation of moisture in the soil ? *[Chandigarh 08]*

Solution. This is done to break the tiny capillaries through which water can rise and finally evaporate. The ploughing of field helps the soil to retain the moisture.

Problem 125. How does the cotton wick in an oil filled lamp keep on burning ?

Solution. The narrow spaces between the threads of the wick serve as capillary tubes through which oil keeps on rising due to capillary action.

Problem 126. Why sand is drier than clay ?

Solution. Due to narrow pores in clay, water rises in the clay due to capillary action and keeps it damp. Practically no pores or capillaries exist in sand and hence water cannot rise in sand. So sand is drier than clay.

Problem 127. Why undergarments are usually made of cotton ?

Solution. Cotton threads have large number of capillaries between them. These capillaries help to absorb sweat from the surface of the body due to capillarity action.

Problem 128. A piece of chalk immersed in water emits bubbles in all directions. Why ? *[Chandigarh 04]*

Solution. A chalk piece has a large number of capillaries. As it is immersed in water, water rises due to capillary action. The air present in the chalk is expelled out in the form of bubbles in all directions.

Problem 129. Why new earthen pots keep water cooler than the old one ?

Solution. Due to capillary action, water oozes out of the pores of the new earthen pot. This water takes latent heat from the pot to evaporate and so the water in the pot gets cooled. In old pots, most of the capillaries are blocked. So the cooling is not so effective.

Problem 130. Why is the tip of the nib of a pen split ?

Solution. The split in the tip of the nib acts as a capillary tube. The ink rises up in the nib due to capillary action. Hence we are able to write with the pen.

Problem 131. Water rises in a capillary tube, whereas mercury falls in the same tube. Why ?

Solution. In a capillary tube, a liquid rises to a height h given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

For water, θ is acute, $\cos\theta$ is positive and hence h is positive. So water rises in the capillary tube. For mercury, θ is obtuse, $\cos\theta$ is negative and hence h is negative. So mercury gets depressed in the capillary tube.

Problem 132. Why is it difficult to make mercury enter a fine thermometer tube ?

Solution. The meniscus of mercury in a glass tube is convex. Since there exists an excess pressure ($p = 2\sigma/r$) on the concave side of the curved liquid surface, the pressure below the meniscus is greater than the atmospheric pressure. This excess pressure has a very large value for a tube of fine bore. As the pressure of mercury inside the tube is greater than that outside it, mercury instead of entering into the tube tends to flow out.

Problem 133. Water gets depressed in a glass tube whose inner surface is coated with wax. Why ?

Solution. The angle of contact between water and wax is obtuse. So $\cos\theta$ is negative and hence capillary rise : $h = 2\sigma\cos\theta / r\rho g$ is also negative. That is why water gets depressed.

Problem 134. If a capillary tube is immersed at first in cold water and then in hot water, the height of capillary is smaller in the second case. Why ?

Solution. The height upto which a liquid rises in a capillary tube is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

The surface tension (σ) of hot water is less than that of cold water. Moreover, capillary tube expands in hot water, so its radius r increases. So capillary rise h is smaller in hot water than in cold water.

Problem 135. If a capillary tube is put in water in a state of weightlessness how will the rise of water in a capillary tube be different to one observed under normal conditions ?

Solution. In normal conditions, when the force of surface tension (due to which water rises in capillary) becomes equal to the weight of the water column raised in the tube, water stops rising. In the state of weightlessness, the effective weight of water column raised is zero. Hence water will rise upto the other end of the capillary, however long the capillary is. Water will not overflow, its surface will become flat (its radius of curvature will become infinity).

Problem 136. How is the rise of liquid affected, if the top of the capillary tube is closed ?

Solution. As the liquid rises in the capillary tube, the air gets compressed between the top end of the tube and

Scanned by CamScanner

the liquid meniscus. The compressed air opposes the rise of liquid due to surface tension. The liquid rises till the two opposing forces just balance each other. Hence if the top end of the capillary tube is closed, the liquid rises to a smaller height.

Problem 137. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, what will be the length of water column in the capillary tube ? *[AIEEE 05]*

Solution. In the freely falling elevator, the entire arrangement is in a state of weightlessness, *i.e.*, $g = 0$. So water will rise in the tube $\left(h = \frac{2\sigma \cos \theta}{r\rho g}\right)$ to fill the entire 20 cm length of the tube.

Problem 138. Spherical balls of radii R are falling in a viscous fluid of viscosity η with a velocity v. How does the retarding viscous force acting on a spherical ball depend on R and v ? *[AIEEE 04]*

Solution. $F = 6\pi\eta R v$ *i.e.*, retarding viscous force is directly proportional to both R and v.

Problem 139. Why do air bubbles in a liquid move in upward direction ? *[Himachal 07]*

Solution. The density of air bubble is less than that of liquid. Initially, the resultant of upthrust and the viscous force is greater than the weight of the air bubble. So, the air bubble experiences a net upward force. Then bubble soon attains a terminal velocity in the upward direction.

Problem 140. Explain why some oils spread on water, when others float as drops. *[Central Schools 09]*

Solution. If the surface tension of oil is less than that of water, then it spreads on water. If the surface tension of oil is more than that of water, then it floats as drops on water.

Problem 141. What happens when a capillary tube of insufficient length is dipped in a liquid ?

Solution. When a capillary tube of insufficient length is dipped in a liquid, the liquid rises to the top. The radius of curvature of the concave meniscus increases till the pressure on its concave side becomes equal to the pressure exerted by the liquid column of insufficient length. But the liquid does not overflow.

Short Answer Conceptual Problems

Problem 1. Explain why :

(*i*) A balloon filled with helium does not rise in air indefinitely but halts after a certain height (Neglect winds).

(*ii*) The force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air. *[Delhi 06]*

Solution. (*i*) A balloon filled with helium goes on rising in air so long as the weight of the air displaced by it (*i.e.*, upthrust) is greater than the weight of filled balloon. We know that the density of air decreases with height. Therefore, the balloon halts after attaining a height at which density of air is such that the weight of air displaced just equals the weight of filled balloon.

(*ii*) Water exerts much more upthrust on the limbs of man than air. So the net weight of limbs in water is much less than that in air. Hence the force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air.

Problem 2. What height of water column produces the same pressure as a 760 mm high column of Hg ? *[Delhi 02]*

Solution. Pressure exerted by h height of water column = Pressure exerted by 760 mm of Hg column

$\therefore\ h \times 1000 \times 9.8 = 0.760 \times 13.6 \times 10^3 \times 9.8 \qquad [P = h\rho g]$

or $$h = \frac{0.760 \times 13.6 \times 10^3}{1000} = \mathbf{10.336\ m}.$$

Problem 3. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0. After some time, the ball falls with a constant velocity. Calculate the viscous force on the ball.

Solution. Volume of the ball, $V = \frac{m}{\rho}$

Mass of the liquid displaced, $m' = V\rho_0 = \frac{m}{\rho} \cdot \rho_0$

When the body falls with a constant velocity,

Viscous force = Effective weight of the ball

$$F = \text{Weight of the ball} - \text{Upthrust}$$
$$= mg - m'g$$

or $$F = mg - \frac{m\rho_0}{\rho} \cdot g = mg\left(1 - \frac{\rho_0}{\rho}\right).$$

Problem 4. A tank filled with fresh water has hole in its bottom and water is flowing out of it. If the size of the hole is increased what will be the change in :

(*a*) Volume of water flowing out per second ?

(*b*) Velocity of the outcoming water ?

(*c*) If in the above tank, the fresh water is replaced by sea water, will the velocity of outcoming water change ?

Solution. (*a*) The volume of water flowing out per sec will increase as its volume depends directly on the size of hole.

(*b*) The velocity of outflow of water remains unchanged because it depends upon the height of water level and is independent of the size of the hole.

(*c*) No, though the density of sea water is more than that of fresh water, but the velocity of outflow of water is independent of the density of water.

Problem 5. In a bottle of narrow neck, water is poured with the help of an inclined glass rod. Why ?

Solution. If water is directly poured into a bottle of narrow neck the stream of water blocks the neck due to the pressure of inside air and the strong force of adhesion between glass and water. If a small part of the glass rod is placed inside the bottle and water is poured along the length of the rod outside the bottle, water molecules cling to the glass rod and the force of gravity pulls down these molecules into the bottle.

Problem 6. The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. What is the ratio between the volume of the first and the second bubble ? *[Chandigarh 03]*

Solution. Given : $P_1 = 3P_2$

or $$\frac{4\sigma}{r_1} = \frac{3 \times 4\sigma}{r_2}$$

or $$r_2 = 3r_1$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = 1:27.$$

Problem 7. The viscous force '*F*' acting on a body of radius '*r*' moving with a velocity '*v*' in a medium of coefficient of viscosity 'η' is given by $F = 6\pi\eta rv$. Check the correctness of the formula. *[Chandigarh 03]*

Solution. $[F] = [MLT^{-2}]$

$$[6\pi\eta rv] = [ML^{-1}T^{-1}][L][T^{-1}] = [MLT^{-2}]$$

∴ Dimensions of LHS = Dimensions of RHS.

Hence the given formula for viscous force *F* is dimensionally correct.

Problem 8. Two soap bubbles of different diameters are in contact with a certain portion common to both the bubbles. What will be the shape of the common boundary as seen from inside the smaller bubble ? Support your answer with a neat diagram. Give reason for your answer. *[Delhi 04]*

Solution. When seen from inside the smaller bubble, the shape of the common boundary will appear concave, as shown in Fig. 10.65.

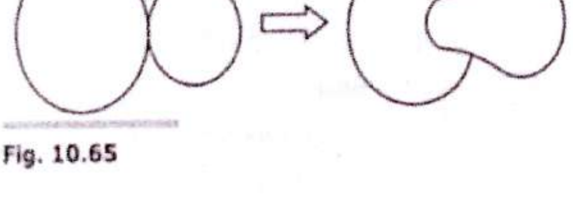


Fig. 10.65

Reasons. (*i*) For a curved liquid film, the pressure is greater on its concave side.

(*ii*) Pressure inside the smaller bubble is more than that inside the larger drop, because $p \propto 1/r$.

Problem 9. A big size balloon of mass *M* is held stationary in air with the help of a small block of mass *M*/2 tied to it by a light string such that both float in mid air. Describe the motion of the balloon and the block when the string is cut. Support your answer with calculations. *[Delhi 04]*

Solution. The situation is shown in Fig. 10.66.

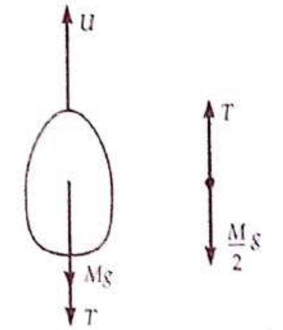


Fig. 10.66

When the balloon is held stationary in air, the forces acting on it get balanced.

Upthrust = Weight of balloon + Tension in string

$$U = Mg + T \quad ...(1)$$

For the small block of mass *M* / 2 floating stationary in air,

$$T = \frac{M}{2}g \quad ...(2)$$

From (1) and (2), $U = Mg + \frac{M}{2}g = \frac{3}{2}Mg$

When the string is cut, $T = 0$. The small block will begin to fall freely. The balloon will rise up with an acceleration *a* such that

$$U - Mg = Ma$$

or $$\frac{3}{2}Mg - Mg = Ma$$

or $$a = \frac{g}{2}, \text{ in the upward direction.}$$

Problem 10. A tarnado consists of rapidly whirling air vortex. Why is the pressure always much lower in the centre than at the outside ? How does this condition account for the destructive power of tarnado ?

Solution. The angular velocity of air in a tarnado increases as it goes towards the centre. As the air moves towards the centre, its moment of inertia (*I*) decreases and to conserve angular momentum ($L = I\omega$), the angular velocity ω increases. Because of the enormous increase in velocity of inner layers, the air pressure at the centre reduces greatly in accordance with Bernoulli's theorem. This sudden reduction in pressure may prove highly disastrous both for life and property in the vicinity of a tarnado.

Scanned by CamScanner

HOTS

Problems on Higher Order Thinking Sk

Problem 1. *A piece of ice with a stone frozen in it floats on water taken in a beaker. Will the level of water increase or decrease or remain the same when ice melts completely ?*

Solution. Let M be the mass of ice piece and m that of stone. As this combination of mass $(M+m)$ floats in water, so the mass of water displaced is $(M+m)$. If ρ is the density of water, then the volume of water displaced is

$$V = \frac{M+m}{\rho} \quad ...(i)$$

When ice melts, we get extra water of mass M and volume M/ρ. The stone sinks and displaces water equal to its own volume m/d, where d is the density of stone. Thus, total the volume of extra water (obtained by melting of ice) and water displaced by stone is

$$V' = \frac{M}{\rho} + \frac{m}{d} \quad ...(ii)$$

As $d > \rho$, so $\frac{1}{d} < \frac{1}{\rho}$ and $\frac{m}{d} < \frac{m}{\rho}$

$\therefore$ From (*i*) and (*ii*), it is obvious that $V' < V$

Thus the level of water in the beaker will come down.

Problem 2. *An ice block with a cork piece embedded inside floats in water. What will happen to the level of water when ice melts ?*

Solution. Let M be the mass of ice and m that of cork.

If ρ is the density of water, then the volume of water displaced by ice + cork is $V = \frac{M+m}{\rho}$

When ice melts, volume of extra water formed $= \frac{M}{\rho}$

As the cork floats, volume of water displaced by it

$$= \frac{m}{\rho}$$

Total volume of water, $V' = \frac{M}{\rho} + \frac{m}{\rho} = \frac{M+m}{\rho}$

Clearly, $V' = V$

Thus the level of water will not change.

Problem 3. *A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level ?* **[IIT]**

Solution. Let M be the mass of the boat and m that of stones. Then total mass $= M + m$. Let ρ be the density of water.

Volume of water displaced by boat and stones together,

$$V = \frac{M+m}{\rho}$$

After the stones are unloaded into water, volume of water displaced by boat alone,

$$V_1 = \frac{M}{\rho}$$

If σ is the density of stones, then volume of water displaced by stones,

$$V_2 = \frac{m}{\sigma}$$

Total volume of water displaced,

$$V' = V_1 + V_2 = \frac{M}{\rho} + \frac{m}{\sigma}$$

As $\sigma > \rho$, so $V > V'$ or $V' < V$

i.e. Volume of water displaced in second case < Volume of water displaced in first case.

Hence the level of water in the water tank will fall down.

Problem 4. *To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force, with which the liquid presses the side of the vessel equal to the force exerted by the liquid on the bottom of the vessel ?* **[IIT]**

Solution. Let h be the height of the liquid column of density ρ taken in the cylindrical vessel of radius r.

Force exerted by the liquid on the bottom of the vessel

$=$ Total weight of the liquid column

$= mg = \pi r^2 h\rho g$

Area of the sides of the vessel $= 2\pi rh$

Average pressure exerted by the liquid on the sides of the vessel

$$= \frac{\text{Pressure at the top} + \text{Pressure at the bottom}}{2}$$

$$= \frac{0 + h\rho g}{2} = \frac{1}{2} h\rho g$$

Force exerted by the liquid on the sides of the vessel

$$= \text{Pressure} \times \text{Area} = \frac{1}{2} h\rho g \times 2\pi rh$$

As the above two forces are given to be equal, so

$$\frac{1}{2} h\rho g \times 2\pi rh = \pi r^2 h\rho g \quad \text{or} \quad h = r$$

i.e., Height of liquid column

= Radius of the cylindrical vessel.

Problem 5. *A block of wood is floating on water at with a certain volume V above the water level. The te rature of water is slowly raised from 0°C to 20°C. Ho the volume V change with the rise in temperature ?*

Solution. Let V' be the volume of the block of and W be the weight of the block. Then by the floatation,

Weight of water displaced = Weight of blo

$$(V' - V)\rho_t g = W$$

where ρ_t is the density of water t°C.

As wood has negligible coefficient of exp V' may be taken constant.

$$\therefore \quad V = V' - \frac{W}{\rho_t g}$$

As the temperature is gradually inc 0°C, ρ_t increases and so V increases upto 4 density of water becomes maximum. A decreases and so V decreases continuou

Problem 6. *A ball floats on the surfa container exposed to the atmosphere. Will immersed at its initial depth or will it sink if the container is shifted to the moon ?*

Solution. The gravity on moon is a that on the earth. But gravity has eq weight of the body and the upthrust the floating body is not affected. O of the floating body is balanced b both water and air.

$$\therefore \quad W = mg = V_w \rho_w g + $$

$$\text{or} \quad m = V_w \rho_w + V_a \rho_a$$

But the moon has no atmosp

$$W = mg = V'_w \rho_w g$$

$$\text{or} \quad m = V'_w \rho_w$$

From (i) and (ii), we note t

$$V'_w = V_w + \frac{V_a \rho_a}{\rho_w}$$

Clearly, $V'_w > V_w$

That is, the volume of b the moon is greater than th sink slightly more in water

Problem 7. *A balloon f with air is weighed so th barely floats in water, as s in Fig. 10.67. Explain w sinks to the bottom whe submerged more by a distance.*

Scanned by CamScanner

Problem 5. *A block of wood is floating on water at 0°C with a certain volume V above the water level. The temperature of water is slowly raised from 0°C to 20°C. How will the volume V change with the rise in temperature ?*

Solution. Let V' be the volume of the block of wood and W be the weight of the block. Then by the law of floatation,

Weight of water displaced = Weight of block

$$(V' - V)\rho_t g = W$$

where ρ_t is the density of water t °C.

As wood has negligible coefficient of expansion, so V' may be taken constant.

$$\therefore \quad V = V' - \frac{W}{\rho_t g}$$

As the temperature is gradually increased from 0°C, ρ_t increases and so V increases upto 4°C, when the density of water becomes maximum. Above 4°C, ρ_t decreases and so V decreases continuously.

Problem 6. *A ball floats on the surface of water in a container exposed to the atmosphere. Will the ball remain immersed at its initial depth or will it sink or rise somewhat if the container is shifted to the moon ?*

Solution. The gravity on moon is about one-sixth of that on the earth. But gravity has equal effect both on weight of the body and the upthrust. So equilibrium of the floating body is not affected. On the earth, weight of the floating body is balanced by upthrust due to both water and air.

$$\therefore \quad W = mg = V_w \rho_w g + V_a \rho_a g$$

$$\text{or} \quad m = V_w \rho_w + V_a \rho_a \quad \ldots(i)$$

But the moon has no atmosphere. So

$$W = mg = V'_w \rho_w g$$

$$\text{or} \quad m = V'_w \rho_w \quad \ldots(ii)$$

From (*i*) and (*ii*), we note that

$$V'_w = V_w + \frac{V_a \rho_a}{\rho_w}$$

Clearly, $V'_w > V_w$

That is, the volume of ball immersed in water on the moon is greater than that on earth. Hence ball will sink slightly more in water when taken to the moon.

Problem 7. *A balloon filled with air is weighed so that it barely floats in water, as shown in Fig. 10.67. Explain why it sinks to the bottom when it is submerged more by a small distance.*

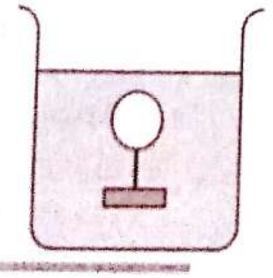


Fig. 10.67

Solution. When the balloon is submerged more by a small distance, the air inside it is slightly compressed. So the buoyant force on it decreases. Now the downward force (weight of sinker + weight of air) exceeds the buoyant force, so it sinks to the bottom.

Problem 8. *A beaker containing water is placed on a spring balance. A stone of weight W is hung and lowered into the water without touching the sides and bottom of the beaker. Explain how the reading will change.*

Solution. Like other forces, buoyant force is also exerted equally on the bodies in contact. When water exerts buoyant force B on the stone in the upward direction, the stone also exerts an equal downward force B on the water. Now the weight W of the vessel + water and the force B on water act downward. So the reaction of the spring scale is

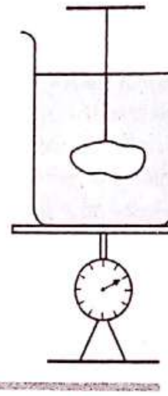


Fig. 10.68

$$R = W + B$$

Hence the reading of the spring scale will increase by an amount equal to the buoyant force.

Problem 9. *When sewing, why does a person often wet the end of a thread before trying to put it through the eye of a needle ?*

Solution. When the end of a thread is made wet with water, a thin film of water is formed over its fibres. The fibres of the thread cling together due to surface tension of water. The area of cross-section of the thread decreases and it becomes easier to put it through the eye of the needle.

Problem 10. *A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density = 13.6 g cm^{-3}). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. What is the density of material of sphere ?* **[IIT 88]**

Solution. Let V be the volume of sphere and ρ its density. Let ρ_0 and ρ_m be the density of oil and mercury respectively.

By the law of floatation,

Weight of sphere = Weight of oil displaced + Weight of mercury displaced

$$\text{or} \quad V\rho g = \frac{V}{2}\rho_0 g + \frac{V}{2}\rho_m g$$

$$\text{or} \quad \rho = \frac{\rho_0 + \rho_m}{2}$$

$$= \frac{0.8 + 13.6}{2} = 7.2 \text{ g cm}^{-3}.$$

Scanned by CamScanner

Problem 11. *An iceberg weighs 400 tonnes. The specific gravity of iceberg is 0.92 and the specific gravity of water is 1.02. What percentage of iceberg is below the water surface ?*

[Roorkee 86]

Solution. Fraction of the iceberg below the water surface will be

$$\frac{\text{Volume of the immersed part}}{\text{Total volume of the iceberg}} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

$$= \frac{0.92}{1.02} = 0.902 = \mathbf{90.2\%}$$

Problem 12. *A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. How much is the force on the bottom of the cylinder by the liquid ?*

[IIT Screening 01]

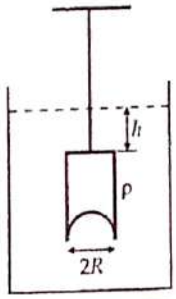


Fig. 10.69

Solution. (Upward) force on the bottom – (downward) force on the top

= Buoyant force on the cylinder

= Weight of liquid displaced

or $F - (h\rho g)\,\pi R^2 = V\rho g$

or $F = V\rho g + (h\rho g)\,\pi R^2 = \rho g\,\mathbf{(V + \pi R^2 h)}$.

Problem 13. *A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R ?*

[IIT Screening 2K]

Solution. The volume of water flowing out per second

= velocity × area of cross-section of the hole

= vA.

Equating the rates of flow,

$$v_1 A_1 = v_2 A_2$$

But $v_1 = \sqrt{2gy},\ A_1 = L^2,\ v_2 = \sqrt{2g \times 4y},\ A_2 = \pi R^2$

$\therefore\ \sqrt{2gy} \times L^2 = \sqrt{2g \times 4y} \times \pi R^2$

or $L^2 = 2\pi R^2$ or $R = \dfrac{\mathbf{L}}{\mathbf{\sqrt{2\pi}}}$.

Problem 14. *A bubble having surface tension T and radius R is formed on a ring of radius b (b << R). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring.* [Fig. 10.70] **[IIT Mains 03]**

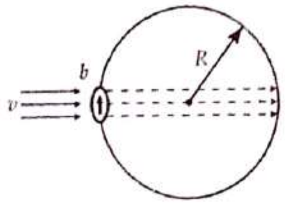


Fig. 10.70

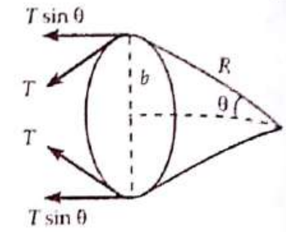


Fig. 10.71

Solution. The bubble will separate from the ring when

$$2\pi b \times 2T \sin\theta = \rho A v^2$$

or $$4\pi bT \times \frac{b}{R} = \rho \times \pi b^2 \times v^2$$

or $$R = \frac{\mathbf{4T}}{\boldsymbol{\rho v^2}}.$$

Problem 15. *A cylindrical vessel of radius 3 cm has at the bottom a horizontal capillary tube of length 20 cm and internal radius 0.4 mm. If the vessel is filled with water, find the time taken by it to empty one half of its contents. Given that the viscosity of water is 0.01 poise.* **[IIT]**

Solution. Let h be the height of water level in the vessel at any instant t and dh be the fall in level in small time dt. Then the rate of flow of water will be

$$Q = -A\frac{dh}{dt} = \frac{\pi p r^4}{8\eta l} \quad \text{[Here } A = \text{Area of the vessel]}$$

or $$-A\frac{dh}{dt} = \frac{\pi h\rho g r^4}{8\eta l} \quad [\because\ p = h\rho g]$$

$\therefore$ $$dt = -\frac{8\eta\, l\, A}{\pi\rho g r^4}\,\frac{dh}{h}$$

Let H be the initial height of the water in the vessel and T be the time taken by the vessel to become half-empty *i.e.,* water level falls to $H/2$. Then

$$\int_0^T dt = -\frac{8\eta l A}{\pi\rho g r^4}\int_H^{H/2}\frac{dh}{h}$$

or $$T = -\frac{8l\eta A}{\pi\rho g r^4}\left[\log_e\frac{H}{2} - \log_e H\right] = \frac{8l\eta A}{\pi\rho g r^4}\log_e 2$$

But $R = 5$ cm, $r = 0.04$ cm, $l = 20$ cm, $\eta = 0.01$ poise, $A = \pi R^2$

$$\therefore\ T = \frac{8 \times 20 \times 0.01 \times \pi \times (5)^2}{\pi \times 1 \times 981 \times (0.04)^4} \times 2.303 \times 0.3010$$

$= \mathbf{11041\ s.}$

Scanned by CamScanner

Problem 16. *A soap bubble of radius 4 cm and surface tension 30 dyne cm^{-1} is blown at the end of a tube of length 10 cm and internal radius 0.20 cm. If the viscosity of air is 189×10^{-4} poise, find the time taken by the bubble to be reduced to a radius of 2 cm.*

Solution. Let R be the radius of the bubble at any instant. Its volume is

$$V = \frac{4}{3}\pi R^3$$

$\therefore$ Rate of flow of air

$$= \frac{dV}{dt} = \frac{4}{3}\pi \times 3R^2 \frac{dR}{dt} = 4\pi R^2 \frac{dR}{dt}$$

But $\dfrac{dV}{dt} = \dfrac{\pi p r^4}{8\eta l}$ and for a soap bubble, $p = \dfrac{4\sigma}{R}$

$$\therefore \quad \frac{dV}{dt} = \frac{\pi r^4}{8\eta l} \cdot \frac{4\sigma}{R} = 4\pi R^2 \frac{dR}{dt} \quad \text{or} \quad dt = \frac{8l\eta}{\sigma r^4} R^3\, dR$$

Time taken by the bubble when its radius changes from R_1 to R_2 is

$$t = \int dt = \frac{8l\eta}{\sigma r^4} \int_{R_2}^{R_1} R^3\, dR = \frac{8l\eta}{\sigma r^4}\left(\frac{R_1^4 - R_2^4}{4}\right)$$

But $R_1 = 4$ cm, $R_2 = 2$ cm, $\sigma = 30$ dyne cm^{-1}

$l = 10$ cm, $r = 0.2$ cm, $\eta = 1.85 \times 10^{-4}$ poise

$$\therefore \quad t = \frac{8 \times 10 \times 1.85 \times 10^{-4}}{30 \times (0.2)^4} \times \left(\frac{4^4 - 2^2}{4}\right) = \mathbf{296\ s.}$$

Problem 17. *A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg m^{-3} enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given coefficient of viscosity of water $= 1.0 \times 10^{-3}$ Nsm^{-2}, $g = 10$ ms^{-2} and density of water $= 1.0 \times 10^3$ kg m^{-3}.* **[Roorkee 90]**

Solution. The velocity attained by the sphere after falling freely from height h is

$$v = \sqrt{2gh} \quad \quad ...(i)$$

After entering water, the velocity of the sphere does not change. So v is also the terminal velocity of the sphere. Hence

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$$

But $\rho = 10^4$ kgm^{-3}, $\rho' = 10^3$ kgm^{-3}, $r = 10^{-3}$ m,
$g = 10$ ms^{-2}, $\eta = 10^{-3}$ Nsm^{-2}

$$\therefore \quad v = \frac{2}{9} \times \frac{(10^{-3})^2 \times (10^4 - 10^3) \times 10}{10^{-3}} = 20 \text{ ms}^{-1}$$

From (*i*), $h = \dfrac{v^2}{2g} = \dfrac{20 \times 20}{2 \times 10} = \mathbf{20\ m.}$

Problem 18. *Water stands at a height H in a tank whose side walls are vertical. A hole is made in one of the walls at a depth h below the water surface. (i) Find at what distance from the foot of the wall does the emerging stream of water strike the floor ? (ii) For what value of h, this range is maximum ? (iii) Can a hole be made at another depth so that the second stream has the same range ?* **[Roorkee 88]**

Solution. (*i*) According to the Bernoulli's theorem,

Energy per unit volume at free surface = Energy per unit volume at the hole

$$\therefore \quad p + \rho g H + 0 = p + \rho g (H - h) + \frac{1}{2}\rho v^2$$

or $$\frac{1}{2}\rho v^2 = \rho g h \quad \text{or} \quad v = \sqrt{2gh}$$

This is the velocity with which the water comes out of the hole.

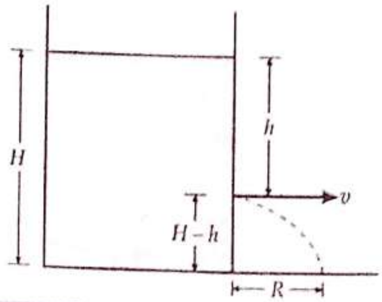


Fig. 10.72

Let t be the time taken by water to fall through height $(H - h)$. Then

$$(H - h) = 0 + \frac{1}{2}gt^2$$

[Initial velocity in the vertical downward direction = 0]

$$\therefore \quad t = \sqrt{\frac{2(H - h)}{g}}$$

The distance from the foot of the wall at which the liquid falls,

$$R = v \times t = \sqrt{\frac{2(H-h)}{g}} \times \sqrt{2gh} = 2\sqrt{h(H-h)}.$$

(*ii*) For R to be maximum, $\dfrac{dR}{dh} = 0$

or $$\frac{d}{dh}[2(hH - h^2)^{1/2}] = 0 \quad [\because R = 2\sqrt{h(H-h)}]$$

or $$2 \times \frac{1}{2}(hH - h^2)^{-1/2} \times (H - 2h) = 0$$

or $$\frac{H - 2h}{\sqrt{hH - h}} = 0$$

or $$H - 2h = 0 \quad \text{or} \quad h = H/2$$

Hence R will be maximum when the hole is made in the middle of the wall.

Scanned by CamScanner

(iii) Let x be the depth below the free surface at which range R is same.

$$\therefore \quad 2\sqrt{h(H-h)} = 2\sqrt{x(H-x)}$$

or $$hH - h^2 = xH - x^2$$

or $$x^2 - Hx + (hH - h^2) = 0$$

or $$x = \frac{H \pm \sqrt{H^2 - 4(hH - h^2)}}{2}$$

$$= \frac{H \pm \sqrt{(H-2h)^2}}{2} = h \text{ or } (H-h)$$

Hence another hole for which the range R is same must be at depth $(H-h)$.

Problem 19 *A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10 cm², the water velocity is 1 ms⁻¹ and the pressure is 2000 Pa. What is the pressure at another point where the cross-sectional area is 5 cm² ?* [IIT 94]

Solution. According to the equation of continuity,

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad 10 \text{ cm}^2 \times 1 \text{ ms}^{-1} = 5 \text{ cm}^2 \times v_2$$

$$\therefore \quad v_2 = 2 \text{ ms}^{-1}$$

Using Bernoulli's theorem for horizontal flow,

$$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$

or $$P_2 + \frac{1}{2} \times 10^3 \times 2^2 = 2000 + \frac{1}{2} \times 10^3 \times 1^2$$

or $$P_2 = 2000 + 500 - 2000 = \mathbf{500 \text{ Pa.}}$$

Problem 20. *A liquid is kept in a cylindrical vessel which is being rotated about its axis. The liquid rises at the sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 rps, find the difference in the heights of the liquid at the centre of the vessel and at its sides.* **[Roorkee 87]**

Solution. According to Bernoulli's theorem, we have

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

When the liquid rotates, the velocity at the sides is higher so the pressure is lower. Since the pressure on a given horizontal level must be same, the liquid rises at the sides to height h to compensate for this drop in pressure.

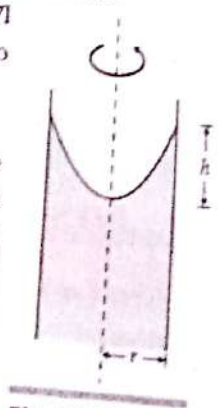


Fig. 10.73

$$\therefore \frac{1}{2}\rho v^2 = h\rho g$$

or $$h = \frac{v^2}{2g} = \frac{(2\pi r v)^2}{2g} = \frac{2\pi^2 r^2 v^2}{g} \quad [\because v = \omega r = 2\pi v r]$$

But $r = 0.05$ m, $v = 2$ rps

$$\therefore \quad h = \frac{2 \times 9.87 \times (0.05)^2 \times 2^2}{9.8} = \mathbf{0.02 \text{ m.}}$$

Problem 21. *Calculate the rate of flow of glycerine of density 1.25 × 10³ kgm⁻³ through the conical section of a pipe if the radii of its ends are 0.1 m and 0.04 m and the pressure-drop across its length is 10 Nm⁻².* [Roorkee 91]

Solution. Using Bernoulli's theorem for horizontal flow,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or $$v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$= \frac{2 \times 10 \text{ Nm}^{-2}}{1.25 \times 10^3 \text{ kgm}^{-3}} = 16 \times 10^{-3}$$

According to equation of continuity, $a_1 v_1 = a_2 v_2$

$$\therefore \quad \frac{v_1}{v_2} = \frac{a_2}{a_1} = \frac{\pi r_2^2}{\pi r_1^2} = \frac{r_2^2}{r_1^2} = \frac{(0.04)^2}{(0.1)^2} = 16 \times 10^{-2}$$

or $$v_1 = 16 \times 10^{-2} v_2$$

Hence $$v_2^2 - (16 \times 10^{-2} v_2)^2 = 16 \times 10^{-3}$$

or $$v_2^2 \left[1 - \frac{256}{10000}\right] = 16 \times 10^{-3}$$

or $$v_2^2 \times \frac{9744}{10000} = 16 \times 10^{-3}$$

or $$v_2^2 = \frac{160}{9744} \quad \text{or} \quad v_2 = 0.13 \text{ ms}^{-1}$$

Rate flow of glycerine

$$= a_2 v_2 = 3.14 \times (0.04)^2 \times 0.13 = \mathbf{6.53 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}}.$$

Problem 22. *Water from a tap emerges vertically downward with an initial speed of 1.0 ms⁻¹. The cross-sectional area of the tap is 10⁻⁴ m². Assume that the pressure is constant throughout the stream of water, and that the flow is steady. What is the cross-sectional area of the stream 0.15 m below the tap ?* **[IIT 98]**

Solution. Here $v_1 = 1.0 \text{ ms}^{-1}$, $a_1 = 10^{-4} \text{ m}^2$,

$$h_1 - h_2 = 0.15 \text{ m}, \quad v_2 = ?, \quad a_2 = ?$$

According to Bernoulli's theorem,

$$P + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$[\because P_1 = P_2 = P \text{ (say)}]$$

or $$\frac{1}{2} v_1^2 + g h_1 = \frac{1}{2} v_2^2 + g h_2$$

or $$v_2^2 = v_1^2 + 2g(h_1 - h_2) = (1.0)^2 + 2 \times 10 \times 0.15 = 4$$

or $$v_2 = 2 \text{ ms}^{-1}$$

By equation of continuity, $a_1 v_1 = a_2 v_2$

$$\therefore \quad a_2 = \frac{a_1 v_1}{v_2} = \frac{10^{-4} \times 1}{2} = \mathbf{5 \times 10^{-5} \text{ m}^2}.$$

Scanned by CamScanner

Guidelines to NCERT Exercises

10.1. *Explain why :*

(*a*) *The blood pressure in humans is greater at the feet than at the brain.* **[Himachal 08 ; Delhi 98]**

(*b*) *Atmospheric pressure at a height of about 6 km decreases to nearly half its value at the sea level, though the height of the atmosphere is more than 100 km.*

(*c*) *Hydrostatic pressure is a scalar quantity even though pressure is force divided by area, and force is a vector.* **[Delhi 06]**

Ans. (*a*) The height of the blood column is quite large at feet than at the brain. Consequently, the blood pressure in humans is greater at the feet than at the brain.

(*b*) The density of air does not decrease linearly with height. The density decreases rapidly upto a height of about 6 km and above 6 km, it decreases rather very slowly. For this reason, the atmospheric pressure at a height of about 6 km decreases to nearly half its value at the sea level.

(*c*) Hydrostatic pressure is transmitted equally in all directions, no definite direction is associated with it. Hence it is a scalar quantity.

10.2. *Explain why :*

(*i*) *The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.*

(*ii*) *Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets the glass while mercury does not).*

(*iii*) *Surface tension of a liquid is independent of the area of the surface.*

(*iv*) *Detergents should have small angles of contact.*

(*v*) *A drop of liquid under no external forces is always spherical in shape.* **[Central Schools 08]**

Ans. (*i*) The cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules. As a result, meniscus of mercury is convex and hence angle of contact is obtuse. On the other hand, the adhesive force between water and glass molecules is greater than the cohesive force between water molecules. So the meniscus of water is concave and hence angle of contact is acute.

(*ii*) The adhesive force between water and glass molecules is greater than the cohesive force between water molecules. So water tends to spread on a clean glass surface. On the other hand, the cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules. So mercury tends to form drops.

(*iii*) Surface tension is defined as the force acting per unit length of a line imagined tangential to a liquid surface at rest. It depends on the nature of the liquid and its temperature and is independent of the area of the liquid surface.

(*iv*) The detergents should have small angle of contact so that they have low surface tension and hence greater ability to wet a surface. Moreover, if θ is small, $\cos\theta$ will be large and hence detergent will penetrate $\left(h = \frac{2\sigma\cos\theta}{r\rho g}\right)$ more in the narrow spaces in the cloth and will easily remove the dirt.

(*v*) Due to surface tension, the free surface of a liquid tends to acquire minimum surface area. In the absence of any external force, a liquid drop becomes spherical in shape because for a given volume, a sphere has minimum surface area.

10.3. *Fill in the blanks using the word(s) from the list appended with each statement :*

(*i*) *Surface tension of liquids generally ______ with temperatures (increases/decreases).*

(*ii*) *Viscosity of gases ______ with temperature, whereas viscosity of liquids ______ with temperature (increases/ decreases).*

(*iii*) *For solids with elastic modulus of rigidity, the shearing force is proportional to ______ while for fluids it is proportional to ______*
(shear strain/rate of shear strain).

(*iv*) *For a fluid in steady flow, the increase in flow speed at a constriction follows from ______, while the decrease of pressure there follows from ______*
(conservation of mass/Bernoulli's principle).

(*v*) *For the model of a plane in a wind tunnel, turbulence occurs at a ______ speed than the critical speed for turbulence for an actual plane (greater/smaller).*

Ans. (*i*) decreases (*ii*) increases, decreases (*iii*) shear strain, rate of shear strain (*iv*) conservation of mass, Bernoulli's principle (*v*) greater.

10.4. *Explain why*

(*i*) *To keep a piece of paper horizontal, you should blow over, not under, it.*

(*ii*) *When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.*

(*iii*) *The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.*

Scanned by CamScanner

(*iv*) *A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.*

(*v*) *A spinning cricket ball in air does not follow a parabolic trajectory.*

Ans. (*i*) When we blow over the paper, the velocity of air increases and hence pressure of air decreases in accordance with Bernoulli's principle. But the pressure below the paper (atmospheric pressure) is high which keeps the paper horizontal.

(*ii*) According to the equation of continuity, velocity is inversely proportional to the area of cross-section. Since area between two fingers is very small as compared to the area of water tap above it, hence fast jets of water gush out through openings between the fingers.

(*iii*) According to Bernoulli's principle,

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

Thus the total energy of the injectable medicine depends upon second power of the velocity and first power of the pressure. It means that total energy of the injectable medicine has greater dependence on its velocity. Therefore, a doctor adjusts the flow rate of the medicine with the help of the size of the needle of the syringe ($a_1 v_1 = a_2 v_2$) rather than the thumb pressure, while administering an injection.

(*iv*) Due to small area of cross-section of the hole, the fluid flows out of the vessel with a large speed and hence with a large linear momentum. As no external force acts on the system, in order to conserve the linear momentum, the vessel acquires a backward momentum and hence a backward thrust acts on the vessel.

(*v*) Refer answer to Q. 52 on page 10.39.

10.5. *A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?*

Ans. Here $m = 50$ kg, $D = 1.0$ cm $= 10^{-2}$ m, $g = 9.8$ ms^{-2}

$$P = \frac{F}{A} = \frac{mg}{\pi (D/2)^2} = \frac{4\,mg}{\pi D^2}$$

$$= \frac{4 \times 50 \times 9.8}{(22/7) \times (10^{-2})^2} = \mathbf{6.2 \times 10^6\ Pa}.$$

10.6. *Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984. Determine the height of the wine column for normal atmospheric pressure.*

Ans. Pressure exerted by h height of wine column

= Pressure exerted by 76 cm of Hg column

or $\quad h \times 984 \times 9.8 = 0.76 \times 13.6 \times 10^3 \times 9.8$

$$\therefore \quad h = \frac{0.76 \times 13.6 \times 10^3}{984} = \mathbf{10.5\ m}.$$

10.7 *A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in Bombay High ? Take the depth of the sea to be roughly 3 km, and ignore ocean currents.*

Ans. Here $\quad h = 3$ km $= 3000$ m,

ρ(water) $= 1000$ kgm^{-3}

Pressure due to sea water,

$$P = h\rho g = 3000 \times 1000 \times 9.8 = 2.94 \times 10^7\ \text{Pa}.$$

This pressure is much less than the stress of 10^9 Pa which the structure can withstand, hence the structure is suitable for the required purpose.

10.8. *A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm². What maximum pressure would the smaller piston have to bear ?*

Ans. Area of cross-section of larger piston,

$$A = 425\ \text{cm}^2 = 425 \times 10^{-4}\ \text{m}^2$$

Load on larger piston,

$$F = mg = 3000 \times 9.8\ \text{N}$$

Pressure on larger piston,

$$P = \frac{F}{A} = \frac{3000 \times 9.8}{425 \times 10^{-4}} = 6.92 \times 10^5\ \text{Nm}^{-2}$$

As the liquids transmit pressure equally in all directions, therefore the pressure that the smaller piston would have to bear $= \mathbf{6.92 \times 10^5\ Nm^{-2}}$.

10.9 *A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?*

Ans. Refer to Fig. 10.74. As the mercury columns in the two arms of the U-tube are at the same level, therefore

Pressure due to water column

= Pressure due to spirit column

$$h_w \rho_w g = h_s \rho_s g \quad \text{or} \quad h_w \rho_w = h_s \rho_s$$

But

$h_w = 10$ cm,

$\rho_w = 1$ g cm^{-3}

$h_s = 12.5$ cm

$\therefore \quad 10 \times 1 = 12.5 \times \rho_s$

or $\quad \rho_s = \frac{10}{12.5}$

$= 0.8$ g cm^{-3}

Specific gravity of spirit

$$= \frac{\rho_s}{\rho_w} = \frac{0.8\ \text{g cm}^{-3}}{1\ \text{g cm}^{-3}} = \mathbf{0.8}.$$

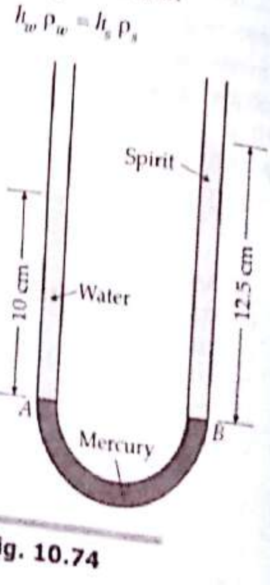


Fig. 10.74

10.10. *In previous exercise, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? Specific gravity of mercury = 13.6.*

Scanned by CamScanner

Ans. Refer to Fig. 10.75. Pressure on mercury level in one arm due to water,

$$P_1 = h_w \rho_w g$$
$$= (10 + 15) \times 1 \times g$$
$$= 25g$$

Pressure on mercury level in another arm due to spirit,

$$P_2 = h_s \rho_s g$$
$$= (12.5 + 15) \times 0.8 \times g$$
$$= 22g$$

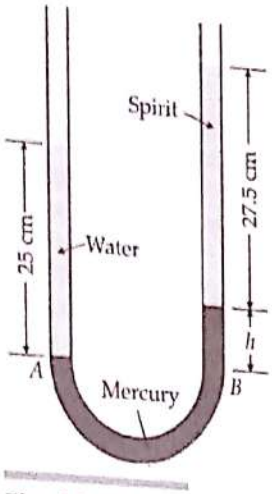


Fig. 10.75

As the pressure in water arm is more, the mercury will rise in spirit arm. If this pressure difference corresponds to height difference h in the two arms, then

$$P_1 - P_2 = h\rho g$$

$$25g - 22g = h \times 13.6 \times g \quad \text{or} \quad h = \frac{3}{13.6} = \mathbf{0.221\ cm.}$$

Thus mercury rises in the arm containing spirit ; the difference in the levels of mercury in the two columns is 0.221 cm.

10.11. *Can Bernoulli's equation be used to describe the flow of water through a rapid in a river ? Explain.*

Ans. No, Bernoulli's equation cannot be used to describe the rapid flow of water in a river because rapid flow is turbulent and not streamlined. The Bernoulli's equation is applicable to streamline flow of a fluid.

10.12. *Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation ? Explain.*

Ans. No, it does not matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation. However, this will be valid only if the atmospheric pressures at the two places of consideration are not appreciably different.

10.13. *Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kgs^{-1}, what is the pressure difference between the two ends of the tube ? Density of glycerine = 1.3×10^3 $kg\,m^{-3}$ and viscosity of glycerine = 0.83 Nsm^{-2}.*

Ans. Here $l = 1.5$ m, $r = 1.0$ cm $= 1.0 \times 10^{-2}$ m,

$$\rho = 1.3 \times 10^3 \text{ kg m}^{-3}, \eta = 0.83 \text{ Nsm}^{-2}$$

Mass collected per second $= 4.0 \times 10^{-3}$ kg

∴ Volume of the liquid flowing per second,

$$Q = \frac{\text{Mass collected per second}}{\text{Density}}$$

$$= \frac{4.0 \times 10^{-3}}{1.3 \times 10^3} \text{ m}^3\text{s}^{-1}$$

But $Q = \dfrac{\pi p r^4}{8 l \eta}$

$$\therefore \quad p = \frac{8 l \eta Q}{\pi r^4} = \frac{8 \times 1.5 \times 0.83}{3.14 \times (1.0 \times 10^{-2})^4} \times \frac{4.0 \times 10^{-3}}{1.3 \times 10^3}$$

$$= \mathbf{9.8 \times 10^2\ Pa.}$$

10.14. *In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 ms^{-1} and 63 ms^{-1} respectively. What is the lift of the wing if its area is 2.5 m^2 ? Density of air = 1.3 kg m^{-3}.*

Ans. Here $\rho = 1.3$ kg m^{-3}, $v_1 = 70$ ms^{-1}, $v_2 = 63$ ms^{-1}

Let p_1 and p_2 be the pressures on the upper and lower surfaces of the wing. Applying Bernoulli's theorem,

$$\frac{p_1}{\rho} + \frac{1}{2}v_1^2 = \frac{p_2}{\rho} + \frac{1}{2}v_2^2$$

or

$$p_2 - p_1 = \frac{1}{2}(v_1^2 - v_2^2)\rho$$

$$= \frac{1}{2}(70^2 - 63^2) \times 1.3 = 605.15 \text{ Nm}^{-2}$$

Lift of the wing = Net upward pressure × Area of the wing

$$= (p_2 - p_1)\,A = 605.15 \times 2.5 \text{ N}$$
$$= \mathbf{1512.9\ N.}$$

10.15. *Figs. 10.76(a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?*

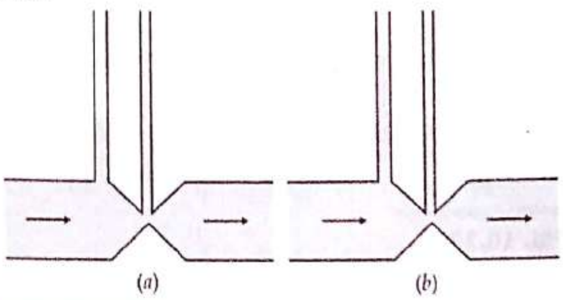


Fig. 10.76

Ans. Fig. 10.76(*a*) is incorrect. At the constriction, the area of cross-section is small and so the liquid velocity is large. Consequently, the liquid pressure must be small (Bernoulli's principle).

10.16. *The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2, one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes ?*

Ans. Here $a_1 = 8.0$ cm^2 $= 8 \times 10^{-4}$ m^2,

$$v = 1.5 \text{ m min}^{-1} = \frac{1.5}{60} \text{ ms}^{-1}$$

$$\text{Radius of a hole} = \frac{d}{2} = \frac{1.0 \text{ mm}}{2} = 0.5 \times 10^{-3} \text{ m}$$

∴ Cross-section of a hole $= \pi \times (0.5 \times 10^{-3})^2$ m^2

Scanned by CamScanner

Total cross-section of 40 holes,

$$a_2 = \pi \times (0.5 \times 10^{-3})^2 \times 40 \text{ m}^2$$

If v_2 is the speed of ejection of the liquid through the holes, then

$$a_1 v_1 = a_2 v_2 \qquad \text{(Equation of continuity)}$$

or

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{8 \times 10^{-4} \times 1.5}{\pi \times (0.5 \times 10^{-3})^2 \times 40 \times 60}$$

$$= \mathbf{0.637 \text{ ms}^{-1}}$$

10.17. *A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film ?* **[Central Schools 14]**

Ans. Here $F = 1.5 \times 10^{-2}$ N, $l = 30$ cm $= 0.3$ m

As the soap film has two free surfaces, so the force F acts over twice the length of the slider. Hence

$$\sigma = \frac{F}{2l} = \frac{1.5 \times 10^{-2}}{2 \times 0.30} = \mathbf{2.5 \times 10^{-2} \text{Nm}^{-1}}.$$

10.18. *Fig. 10.77(a) shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2}$ N. What is the weight supported by a film of the same liquid at the same temperature in Figs. 10.77(b) and (c) ? Explain your answer physically.*

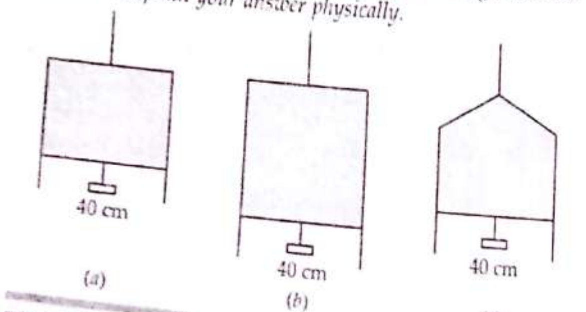


Fig. 10.77

Ans. The weight supported both in (b) and (c) is 4.5×10^{-2} N *i.e.*, same as in case (a). The weight supported $= 2\sigma l$. As σ and l are same in all cases, so the weight supported is same.

10.19. *What is the pressure inside a drop of mercury of radius 3.00 mm of room temperature ? Surface tension of mercury at that temperature (20 °C) is 4.65×10^{-1} Nm^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.* **[Central Schools 14]**

Ans. Here $R = 3.00$ mm $= 3.00 \times 10^{-3}$ m,

$$\sigma = 4.65 \times 10^{-1} \text{ Nm}^{-1}, \ P_0 = 1.01 \times 10^5 \text{ Pa}$$

Excess pressure inside the drop is

$$p = \frac{2\sigma}{R}$$

$$= \frac{2 \times 4.65 \times 10^{-1}}{3.00 \times 10^{-3}} = 310 \text{ Nm}^{-2}$$

Total pressure inside the drop,

$P =$ Atmospheric pressure + Excess pressure

$= 1.01 \times 10^5 + 310$

$= 101000 + 310 = 101310 \text{ Nm}^{-2} = \mathbf{1.013 \times 10^5 \text{ Pa}}.$

10.20. *What is the excess pressure inside a bubble of soap solution of radius 5.00 mm ? Given that the surface tension of soap solution at the temperature (20° C) is 2.50×10^{-2} Nm^{-1}. If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? (1 atm = 1.01×10^5 Pa).*

Ans. Here $r = 5.00$ mm $= 5.00 \times 10^{-3}$ m,

$$\sigma = 2.50 \times 10^{-2} \text{ Nm}^{-1}, \ h = 40 \text{ cm} = 0.4 \text{ m},$$

$$P_0 = 1.01 \times 10^5 \text{ Pa}$$

Excess pressure inside a soap bubble,

$$p = \frac{4\sigma}{R} = \frac{4 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}} = \mathbf{20 \text{ Pa}}.$$

Excess pressure inside an air bubble under soap solution

$$p' = \frac{2\sigma}{R} = \frac{2 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}} = 10 \text{ Pa}$$

Density of soap solution,

$$\rho = \text{R.D.} \times 1000 = 1.20 \times 1000 = 1200 \text{ kgm}^{-3}$$

Total pressure inside the air bubble

= Atmospheric pressure + Pressure due to 40 cm soap solution + Excess pressure

$= 1.01 \times 10^5 + h\rho g + p'$

$= 1.01 \times 10^5 + 0.40 \times 1200 \times 9.8 + 10$

$= 101000 + 4704 + 10 = \mathbf{105714 \text{ Pa}}.$

10.21. *A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door closed.*

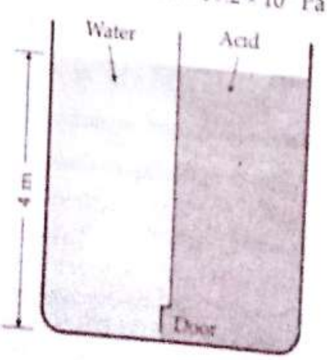


Fig. 10.78

Ans. *For compartment containing water :*

Height of water column, $h = 4.0$ m

Density of water, $\rho = 10^3$ kgm^{-3}

Pressure due to water at the door at the bottom,

$$P_w = h\rho g = 4.0 \times 10^3 \times 9.8 = 39.2 \times 10^3 \text{ Pa}$$

For compartment containing acid :

Height of acid column = 4.0 m

Density of acid,

$$\rho = 1.7 \times 10^3 \text{ kg m}^{-3}$$

Pressure due to acid at the door at the bottom,

$$P_a' = h\rho g = 4.0 \times 1.7 \times 10^3 \times 9.8$$
$$= 66.64 \times 10^3 \text{ Pa}$$

$$\therefore \quad P_a' - P_w' = 66.64 \times 10^3 - 39.2 \times 10^3$$
$$= 27.44 \times 10^3 \text{ Pa}$$

Area of the door, $A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$

Force on the door due to difference of pressure on its two sides

$$= (P_a' - P_w') \times A$$
$$= 27.44 \times 10^3 \times 20 \times 10^{-4} = 54.88 \text{ N}$$

Thus, to keep the door closed, a force of 54.88 N must be applied on it from the water side.

10.22. *A manometer reads the pressure of a gas in an enclosure as shown in Fig. 10.79(a). When some of the gas is removed by a pump, the manometer reads as in Fig. 10.79(b).*

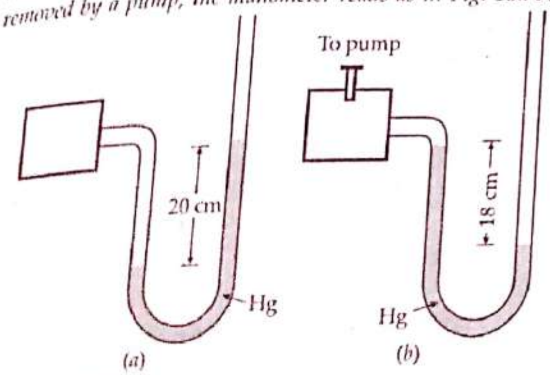


Fig. 10.79

The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

(*i*) *Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b) in units of cm of mercury.*

(*ii*) *How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer ? (Ignore the small change in volume of the gas).*

Ans. Here atmospheric pressure,

$$P = 76 \text{ cm of Hg}$$

(*i*) *In case (a)*: Pressure head, $h = +\,20$ cm of Hg

Absolute pressure $= P + h = 76 + 20 =$ **96 cm of Hg**

Gauge pressure $= h =$ **20 cm of Hg**

In case (b) : Pressure head, $h = -\,18$ cm of Hg

Absolute pressure

$$= P + h = 76 - 18 = \textbf{58 cm of Hg}$$

Gauge pressure $= h =$ **− 18 cm of Hg.**

(*ii*) As $\quad h_1 \rho_1 g = h_2 \rho_2 g$

$$h_1 \times 13.6 \times g = 13.6 \times 1 \times g$$

or $\quad h_1 = 1 \text{ cm}$

Therefore, as 13.6 cm of water is poured in right limb, it will displace mercury level by 1 cm in the left limb, so that difference of levels in the two limbs will become **19 cm.**

10.23. *Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height.*

(*i*) *Is the force exerted by the water on the base of the vessel the same in the two cases ?*

(*ii*) *If so, why do the vessels filled with water to that same height give different readings on a weighing scale ?*

Ans. (*i*) As the two vessels have been filled with water upto the same common height, the pressures exerted on the bases of the two vessels are equal. Moreover, the two vessels have the same base area, so forces exerted on the bases of the two vessels will also be equal.

(*ii*) Water exerts force on the sides of the vessel also. This force has a nonzero vertical component when the sides of the vessel are not perfectly normal to the base. This net vertical component of the force exerted by water on the sides of the vessel is greater for the first vessel than the second. Hence the vessels weigh different even when the force on the base is same in the two cases.

10.24. *During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ? The density of whole blood = 1.06×10^3 kg m^{-3}.*

Ans. Let h be the height of container at which its blood exerts pressure equal to gauge pressure in vein. Then

$$h\rho g = P_g$$

or

$$h = \frac{P_g}{\rho g}$$
$$= \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.1925 \text{ m}$$

The blood will just enter the vein if the blood container is kept at height slightly greater than 0.1925 m *i.e.*, at 0.2 m.

10.25. *In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) How does the pressure change as the fluid moves along the tube if dissipative forces are present ? (b) Do the dissipative forces become more important as the fluid velocity increases ? Discuss qualitatively.*

Ans. (*a*) If dissipative forces are present, some of the pressure energy of the fluid is spent in doing work against these forces, so the fluid pressure decreases with the increase in length of the tube.

(*b*) Yes, the dissipative forces become more important when the fluid velocity increases.

Scanned by CamScanner

10.26. *(a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar ? (b) What is the corresponding flow rate ? Take viscosity of blood to be 2.084×10^{-3} Pa s and density of blood $= 1.06 \times 10^3$ kg m^{-3}.* **[Delhi 06]**

Ans. (a) Here $\rho = 1.06 \times 10^3$ kg m^{-3},

$D = 2r = 4 \times 10^{-3}$ m, $\eta = 2.084 \times 10^{-3}$ Pa s

The maximum value of Reynold's number for laminar flow is 2000. Hence the maximum average velocity for laminar flow or critical velocity is given by

$$v_c = \frac{R_e\,\eta}{\rho D} = \frac{2000 \times 2.084}{1.06 \times 10^3 \times 4 \times 10^{-3}} = \mathbf{0.98\ ms^{-1}}.$$

(b) Volume of blood flowing per second,

$$Q = av_c = \pi r^2 v_c = \frac{22}{7} \times (2 \times 10^{-3})^2 \times 0.98$$

$$= 1.23 \times 10^{-5}\,\text{m}^3\text{s}^{-1}.$$

10.27. *A plane is in level flight at constant speed and each of its two wings has an area of 25 m². If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. Take air density to be 1 kg m^{-3} and g = 9.81 ms^{-2}.*

Ans. Here $v_1 = 180$ km h^{-1}

$$= 180 \times \frac{5}{18} = 50\ \text{ms}^{-1}$$

$$v_2 = 234\ \text{km h}^{-1} = 234 \times \frac{5}{18} = 65\ \text{ms}^{-1}$$

Area of the wings,

$A = 2 \times 25 = 50$ m², $\rho = 1$ kg m^{-3}.

For a plane in the level flight, Bernoulli's equation is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or $$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$= \frac{1}{2} \times 1 \times (65^2 - 50^2)$$

$$= 862.5\ \text{Nm}^{-2}$$

Upward force on the plane $= (p_1 - p_2) \times A$

$= 862.5 \times 50 = 43125$ N

In level flight, the upward force balances the weight of the plane, so

$mg = 43125$ N

Mass of the plane, $m = \frac{43125}{g} = \frac{43125}{9.81} = \mathbf{4396\ kg}$

10.28. *In Millikan's oil drop experiment, what is the terminal speed of a drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m^{-3} ? Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Nsm^{-2}. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.*

Ans. Here $r = 2.0 \times 10^{-5}$ m, $\rho = 1.2 \times 10^3$ kg m^{-3},

$\eta = 1.8 \times 10^{-5}$ Nsm^{-2}

If the buoyancy of the drop due to air is neglected, then the terminal speed is given by

$$v = \frac{2}{9} \cdot \frac{r^2 \rho g}{\eta}$$

$$= \frac{2}{9} \times \frac{(2.0 \times 10^{-5})^2 \times 1.2 \times 10^3 \times 9.8}{1.8 \times 10^{-5}}$$

$$= 5.8 \times 10^{-2}\ \text{ms}^{-1} = \mathbf{5.8\ cms^{-1}}$$

Viscous force,

$$F = 6\pi\eta r v$$

$$= 6 \times \frac{22}{7} \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.8 \times 10^{-2}$$

$$= \mathbf{3.9 \times 10^{-10}\ N}.$$

10.29. *Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-1}. Density of mercury = 13.6×10^3 kgm^{-3}.*

Ans. Here $\theta = 140°$, $r = 1.00$ mm $= 1.00 \times 10^{-3}$ m,

$\sigma = 0.465$ Nm^{-1}, $\rho = 13.6 \times 10^3$ kgm^{-3}.

$\cos 140° = \cos(180° - 40°) = -\cos 40° = -0.7660$

$$\therefore\quad h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 0.465 \times \cos 140°}{1.00 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= \frac{2 \times 0.465 \times (-0.7660)}{13.6 \times 9.8}$$

$$= -5.34 \times 10^{-3}\ \text{m} = \mathbf{-5.34\ mm}.$$

The negative sign indicates that the mercury level is depressed in the capillary tube.

10.30. *The narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube ? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-1}. Take the angle of contact to be zero, and density of water to be 1.0×10^3 kg m^{-3}. Take g = 9.8 ms^{-2}.*

Ans. Here $r_1 = \frac{3.0}{2} = 1.5$ mm $= 1.5 \times 10^{-3}$ m,

$r_2 = \frac{6.0}{2} = 3.0$ mm $= 3.0 \times 10^{-3}$ m

$\sigma = 7.3 \times 10^{-2}$ Nm^{-1}, $\theta = 0°$

$\rho = 1.0 \times 10^3$ kg m^{-3}, $g = 9.8$ ms^{-2}

Let h_1 and h_2 be the heights to which water rises in the two tubes. Then

$$h_1 = \frac{2\sigma\cos\theta}{r_1 \rho g} \quad \text{and} \quad h_2 = \frac{2\sigma\cos\theta}{r_2 \rho g}$$

Therefore,

$$h_1 - h_2 = \frac{2\sigma\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2 \times 7.3 \times 10^{-2} \cos 0°}{10^3 \times 9.8}\left[\ldots\right.$$

$$= \frac{14.6 \times 10^{-2}}{10^3 \times 9.8 \times 10^{-3}}\left[\frac{1}{1.5} - \ldots\right.$$

$$= \frac{14.6 \times 10^{-2}}{9.8} \times \frac{1}{3} = 0.5290\ldots$$

$= \mathbf{5.290\ mm.}$

10.31 *(a) It is known that densit... height y as* $\rho = \rho_0 e^{-y/y_0}$

where $\rho_0 = 1.25$ kg m^{-3} is the densit... constant. This density variation is ... pheres. Obtain this law assuming ... atmosphere remains a constant (is... assume that the value of g remains ...

(b) A large He balloon of volu... pay load of 400 kg. Assume that the... radius as it rises. How high does i...

Take $y_0 = 8000$ m and ρ_{He} = ...

Ans. (a) Let ρ be the densi... above the sea level. The rate o... height must be proportional to... That is,

$$-\frac{d\rho}{dy} \propto \rho \quad \text{or} \ \ldots$$

Here k is a constant of p... shows that the density of air d... height. Now

$$\frac{d\rho}{\rho} = -k\,dy$$

Text Based ...

1. Define the term flu...
2. What is the modul...
3. Is thrust a scalar o...
4. State Pascal's law...
5. State the principle...
6. Name two practic...
7. What is atmosph...

Scanned by CamScanner

Therefore,

$$h_1 - h_2 = \frac{2\sigma\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2\times 7.3\times 10^{-2}\cos 0^\circ}{10^3\times 9.8}\left[\frac{1}{1.5\times 10^{-3}} - \frac{1}{3\times 10^{-2}}\right]$$

$$= \frac{14.6\times 10^{-2}}{10^3\times 9.8\times 10^{-3}}\left[\frac{1}{1.5} - \frac{1}{3}\right]$$

$$= \frac{14.6\times 10^{-2}}{9.8}\times\frac{1}{3} = 0.5290\times 10^{-2}\text{ m}$$

= 5.290 mm.

10.31 (*a*) *It is known that density ρ of air decreases with height y as*

$$\rho = \rho_0 e^{-y/y_0}$$

where $\rho_0 = 1.25\ kg\,m^{-3}$ *is the density at sea level and* y_0 *is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.*

(*b*) *A large He balloon of volume* $1425\ m^3$ *is used to lift a pay load of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise ?*

Take $y_0 = 8000\ m$ *and* $\rho_{He} = 0.18\ kg\,m^{-3}$.

Ans. (*a*) Let ρ be the density of the air at a height y above the sea level. The rate of decrease of density with height must be proportional to the density at that height. That is,

$$-\frac{d\rho}{dy} \propto \rho \quad\text{or}\quad \frac{d\rho}{dy} = -k\rho$$

Here k is a constant of proportionality and –ve sign shows that the density of air decreases with the increase in height. Now

$$\frac{d\rho}{\rho} = -k\,dy$$

As height changes from 0 to y, density changes from ρ_0 to ρ. Integrating the above equation within these limits, we get

$$\int_{\rho_0}^{\rho}\frac{1}{\rho}d\rho = -k\int_0^y dy$$

or $$[\log_e \rho]_{\rho_0}^{\rho} = -k\,[y]_0^y$$

or $$\log_e \rho - \log_e \rho_0 = -k(y-0)$$

or $$\log_e\frac{\rho}{\rho_0} = -ky$$

or $$\frac{\rho}{\rho_0} = e^{-ky}$$

or $$\rho = \rho_0 e^{-ky}$$

Taking constant k equal to $1/y_0$, we get

$$\rho = \rho_0 e^{-y/y_0}$$

(*b*) The balloon will rise upto a height, where its density becomes equal to the density of air at that height.

Volume of balloon, $V = 1425\text{ m}^3$

Mass of He gas in the balloon $= 1425\times 0.18 = 256.5$ kg

Total mass of the balloon including pay-load

$$M = 400 + 256.5 = 656.5\text{ kg}$$

Density of the balloon,

$$\rho = \frac{M}{V} = \frac{656.5}{1425} = 0.46\text{kg m}^{-3}$$

Given $y_0 = 8000$ m, $\rho_0 = 1.25$ kg m^{-3}, $\rho = 0.46$ kg m^{-3}

As $$\rho = \rho_0 e^{-y/y_0}$$

∴ $$0.46 = 1.25 e^{-y/8000}$$

or $$e^{y/8000} = \frac{1.25}{0.46} = 2.72$$

or $$y = 8000\log_e 2.72$$

$$= 8000\times 1 = 8000\text{ m} = \mathbf{8\ km}$$

Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. Define the term fluid.
2. What is the modulus of rigidity of a fluid ?
3. Is thrust a scalar or vector quantity ?
4. State Pascal's law. **[Central Schools 07]**
5. State the principle of working of hydraulic press. **[Central Schools 13]**
6. Name two practical applications of Pascal's law.
7. What is atmospheric pressure ?
8. Does the atmospheric pressure vary with the height above the earth's surface ?
9. What is one torr of pressure ? **[Himachal 07C]**
10. What is meant by one bar of pressure ?
11. Write the relation between torr and millibar.
12. Name the factors on which the atmospheric pressure at a place depends.
13. Name the scientist who first experimentally measured the atmospheric pressure.

Scanned by CamScanner

14. Express 1 atmosphere in terms of Nm^{-2} and bar.
15. What is a manometer ?
16. If water is used instead of mercury in a barometer, what will be the height of water column ?
17. What does gradual fall of barometric height indicate ?
18. What does sudden fall in barometric height indicate ?
19. What is indicated by gradual increase of atmospheric pressure ?
20. Which practical unit of pressure is used in meteorological science ?
21. What will happen if water is used in place of mercury in a barometer tube ? [Meghalaya 96]
22. Define buoyancy.
23. In which case a body will weigh maximum (i) in air (ii) vacuum or (iii) in water ?
24. How much should be systolic blood pressure for a normal human being ?
25. How much should be diastolic blood pressure for a normal human being ?
26. State the law of floatation.
27. A body is just floating in a liquid (of equal density). What happens to the body if it is slightly pressed and released ?
28. In case of stable equilibrium, should the meta-centre be above or below the centre of gravity of the body ?
29. What is the internal force of friction of a fluid known as ?
30. Define viscosity.
31. What is reciprocal of viscosity known as ?
32. Define coefficient of viscosity of a liquid. [Manipur 97]
33. Name the CGS and SI units of the coefficient of viscosity.
34. State the dimensional formula of the coefficient of viscosity.
35. Define poise. [Meghalaya 98]
36. Define poiseuille or decapoise.
37. Define kinematic viscosity.
38. How does the viscosity of gases depend on temperature ?
39. Water flows through a pipe. Which of its layers moves fastest ?
0. State Stokes' law.
1. Mention two applications of Stokes' law.
. Out of solid friction and viscous force, which is independent of velocity ?
What is the nature of graph between terminal velocity of a spherical body and the square of its radius ?
44. Two balls A and B have radii in the ratio 1 : 2. What will be the ratio of their terminal velocities in a liquid ?
45. What is the net weight of a body when it falls with terminal velocity through a viscous medium ?
46. What do mean by a streamline and a tube of flow ?
47. When does the streamline flow become turbulent ?
48. What is laminar flow of a liquid ?
49. What is Reynold's number ?
50. What is the range of Reynold's number for the laminar flow of a liquid ?
51. Write the dimensional formula of Reynold's number.
52. Which of the following values of the Reynold's number can be true for turbulent flow :
(i) 200 (ii) 500 (iii) 1500 (iv) 3000 ?
53. What is critical velocity of a liquid ?
54. Draw a graph between the velocity of a small sphere dropped from rest into a viscous liquid and time. Also indicate the terminal velocity as v_t on the graph.
55. Which fundamental law forms the basis of equation of continuity ?
56. What will be the nature of graph between the velocity of fluid flow (v) and area of cross-section (a) of the pipe ?
57. What are the different forms of energy possessed by a fluid in a streamline flow ?
58. What is that fundamental principle on which Bernoulli's theorem is based ?
59. What is an ideal fluid ? [Delhi 13]
60. Write the expressions for pressure head, gravitational head and velocity head.
61. What should be the properties of a liquid to satisfy Bernoulli's theorem ?
62. State Torricelli's theorem.
63. State Bernoulli's theorem. [Central Schools 05]
64. What is a Pitot tube ? State the principle on which it is based.
65. Define force of cohesion.
66. Define surface tension. State its SI unit. [Central Schools 05]
67. Name a physical quantity that has same dimensions as surface tension.
68. What is meant by the term molecular range ?
69. Define sphere of influence of a liquid molecule.
70. Which part of the liquid is responsible for the phenomenon of surface tension ?
71. How does surface tension change with temperature ?
72. Write down the following liquids in the order of increasing surface tension :

Water, mercury, soap solution.

73.
74.
75.
76.
77.
78.
79.
80.
81.

Scanned by CamScanner

73. What is the value of surface tension at the critical temperature ?

74. What is the effect of solute on the surface tension of a liquid ?

75. Define angle of contact. [Delhi 13, 14]

76. Will the angle of contact be acute or obtuse for liquids which wet the walls of the container ?

77. Will the angle of contact be acute or obtuse for liquids which do not wet the walls of the container ?

78. What are the factors on which the angle of contact depends ?

79. How does the angle of contact of a liquid depend on temperature ?

80. What happens to the surface tension when some impurity is mixed in liquid ? [Himachal 01]

81. What is capillarity ? [Delhi 95]

82. Name the material in whose capillary water will descend instead of rising.

83. A lead sphere acquires a terminal velocity v when falls in a viscous liquid. What will be the terminal velocity attained by another lead sphere of radius three times in the same liquid ? [Delhi 96]

84. Water rises to a height of 20 mm in a capillary. If the radius of the capillary is made $\frac{1}{3}$rd of its previous value, to what height will the water now rise in the tube ? [Delhi 96]

85. By which phenomenon, the water rises from roots to leaves of plants ? [Himachal 04]

86. State conditions of equilibrium of floating bodies. [Central Schools 07]

87. Give relationship between poise and decapose. [Himachal 07]

Answers

1. A fluid is a substance that flows. So the term fluid refers to both liquids and gases.
2. Zero. A fluid has no definite shape of its own.
3. Vector quantity.
4. Pascal's law states that a change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.
5. See the statement of Pascal law in the above answer.
6. Hydraulic press and hydraulic brakes.
7. Atmospheric pressure at any point is equal to weight of air contained in a column of unit cross-sectional area and extending upto top of atmosphere.
8. The atmospheric pressure decreases with the height above the earth's surface.
9. 1 torr = pressure exerted by 1 mm of Hg column $= 10^{-3} \times 13.6 \times 10^{3} \times 9.8 = 133.8 \text{ Nm}^{-2}$.
10. $1 \text{ bar} = 10^{5} \text{ Nm}^{-2}$.
11. $1 \text{ torr} = 133.8 \text{ Nm}^{-2} = \frac{133.8}{10^{5}} \text{ bar} = 1.338$ millibar.
12. Atmospheric pressure at a place depends on (*i*) height of atmosphere (*ii*) density of atmosphere and (*iii*) acceleration due to gravity.
13. Torricellie.
14. $1 \text{ atm} = 1.013 \times 10^{5} \text{ Nm}^{-2} = 1.013 \times 10^{5}$ Pa $= 1.013$ bar.
15. Manometer is a device used to measure the pressure of a gas enclosed in a vessel.
16. 10.34 m.
17. The atmospheric pressure falls when the water vapours increase in air. This indicates the possibility of rain.
18. It indicates the possibility of storm.
19. It indicates dry weather.
20. Atmospheric pressure (atm). $1 \text{ atm} = 1.013 \times 10^{5} \text{ Nm}^{-2}$.
21. If water is used as barometric substance, it would require a tube about 11 m long. It is difficult to hold such a tube in a vertical position.
22. When a body is immersed in a fluid, it experiences a thrust. This effect is called buoyancy.
23. In vacuum.
24. Nearly 120 mm of Hg.
25. Nearly 80 mm of Hg.
26. According to the law of floatation, a body will float in a liquid if weight of the liquid displaced by the body is atleast equal to or greater than the weight of the body.
27. The body sinks to the bottom of the liquid.
28. Meta-centre should be above the centre of gravity.
29. Viscosity.
30. Viscosity is the property of the fluid by virtue of which it opposes the relative motion between its different layers.
31. Fluidity.
32. The coefficient of viscosity of a liquid is defined as the tangential viscous force required to maintain a unit velocity gradient between two liquid layers each of unit area.

Scanned by CamScanner

33. The CGS unit of the coefficient of viscosity is poise and the SI unit is decapoise.

34. The dimensional formula of the coefficient of viscosity is $[ML^{-1}T^{-1}]$.

35. The coefficient of viscosity of a liquid is said to be 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cms^{-1} between two layers of the liquid 1 cm apart.

36. The coefficient of viscosity of a liquid is said to be 1 poiseuille or decapoise if tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

37. The ratio of coefficient of viscosity η and the density ρ of a liquid is called its kinematic viscosity.

$$\text{Kinematic viscosity} = \frac{\eta}{\rho}$$

38. The viscosity of gases increases with the increase of temperature. For a gas, $\eta \propto \sqrt{T}$

39. The axial layer of water moves fastest.

40. According to Stroke's law the viscous drag on a small body of radius (r) moving with a uniform velocity (v) through a viscous medium of viscosity (η) is given by $F = 6\pi \eta r v$.

41. Stokes' law can be used to find (*i*) radius of an oil drop and (*ii*) coefficient of viscosity of a liquid.

42. Solid friction.

43. Straight line.

44. As $v \propto r^2$, so the ratio of the terminal velocities of A and B will be 4 : 1.

45. Zero, because the weight of the body acting vertically downwards is balanced by the viscous force and the upthrust due to the medium.

46. A streamline may be defined as the straight or curved path, such that the tangent to it at any point gives the direction of flow of the liquid at that point. A group of streamlines is said to form a tube of flow.

47. The streamline flow becomes the turbulent flow when the velocity of the liquid exceeds the *critical velocity*.

48. The type of flow of a liquid in which its layers slide over one another without mixing is called the laminar flow.

49. Reynold's number is dimensionless combination of four factors which decides the nature of flow of a viscous liquid through a pipe. It is given by

$$R_e = \frac{\rho V D}{\eta}.$$

50. For laminar flow, R_e lies between 0 and 2000.

51. The dimensional formula of Reynold's number is $[M^0L^0T^0]$.

52. 3000.

53. Critical velocity is the maximum velocity of liquid above which the flow of a liquid changes from streamline to turbulent.

54. See Fig. 10.27 on page 10.29.

55. Law of conservation of mass.

56. Rectangular hyperbola.

57. (*i*) Kinetic energy (*ii*) Potential energy (*iii*) Pressure energy.

58. Law of conservation of energy.

59. An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.

60. Pressure head $= \frac{P}{\rho g}$, gravitational head $= h$ and velocity head $= \frac{v^2}{2g}$.

61. The liquid should be incompressible and non-viscous and the flow of the liquid should be streamlined and irrotational.

62. Torricelli's theorem states that the velocity with which a liquid flows out through an orifice at a certain depth below the free surface is the same as that attained by another body which falls freely through the same height.

This velocity of efflux is given by $v = \sqrt{2gh}$.

63. Refer to point 36 of Glimpses.

64. It is a device which is used to measure the velocity of the flow at any depth in a flowing liquid. It is based on Bernouilli's theorem.

65. The force of attraction between the molecules of the same substance is called the force of cohesion.

66. The property of a liquid by virtue of which its free surface at rest behaves like a stretched membrane and tends to have the minimum surface area is called surface tension. Its SI unit is Nm^{-1}.

67. Spring constant.

68. The maximum distance upto which a molecule can exert measurable force of attraction on other molecules is called molecular range ($= 10^{-9}$ m).

69. A sphere drawn around a molecule as centre and radius equal to the molecular range is called the sphere of influence of the molecule.

70. The liquid in surface film is responsible for surface tension.

71. The surface tension of a liquid decreases with the rise of temperature.

72. Soap solution, water, mercury.

73. Zero.

74. If the solute (*e.g.*, salt in water) is highly soluble, the surface tension of liquid increases. If the solute (*e.g.*, soap in water) is less soluble, the surface tension decreases.

Scanned by CamScanner

75. Angle of contact is defined as the angle between the tangent to the liquid surface at the point of contact to the solid surface inside the liquid.

76. Acute.

77. Obtuse.

78. The angle of contact depends on (*i*) Nature of the solid and liquid in contact. (*ii*) Cleanliness of the surface in contact. (*iii*) Medium above the free surface of the liquid. (*iv*) Temperature of the liquid.

79. The angle of contact of a liquid increases with the increase of temperature.

80. If the liquid surface has dust, grease or oil, the surface tension of the liquid decreases.

81. The phenomenon of rise or fall of a liquid in a capillary with respect to the surroundings is called capillarity.

82. Paraffin wax.

83. The terminal of second sphere will be 9 *v* because $v_t \propto r^2$.

84. As $h \propto \frac{1}{R}$

$$\therefore \quad \frac{h'}{h} = \frac{R}{R'}$$

or $$h' = \frac{R}{R'} \cdot h = \frac{R}{\frac{1}{3}R} h$$

$$= 3 \times 20 \text{ mm} = 60 \text{ mm}.$$

85. Capillary action.

86. Refer answer to Q 28 on page 10.15.

87. 1 decapoise = 10 poise.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Define the terms thrust and pressure. Give their S.I. units.
2. Show that a liquid at rest exerts force perpendicular to the surface of the container at every point.
3. With the help of a suitable diagram, describe a method for measuring fluid pressure at any point inside a fluid.
4. State Pascal's law. How will you experimentally verify this law ?
5. State Pascal's law. Explain the working of hydraulic lift. **[Himachal 06, 07 ; Chandigarh 08]**
6. Show that the pressure exerted by a liquid column is proportional to its height. **[Manipur 98]**
7. Explain hydrostatic paradox with a suitable example.
8. What is atmospheric pressure ? How is atmospheric pressure measured with the help of a mercury barometer ?
9. What is meant by torr ? Calculate the height of the atmosphere. **[Himachal 07]**
10. Describe how an open tube manometer is used to measure the pressure of a gas. Distinguish between absolute pressure and gauge pressure.
11. Pressure of a gas in a closed cylinder is expressed in the following way : $P = P_a + h\rho g$.

    Identify the expressions for :

    (*a*) Absolute pressure of the gas.

    (*b*) Gauge pressure of the gas.

    [Central Schools 08, 09]
12. Define buoyancy and centre of buoyancy.
13. State and prove Archimedes' principle. **[Himachal 06, 07 ; Chandigarh 04]**
14. Explain the laws of floation with all possibilities of a body in a liquid. **[Himachal 06]**
15. State the conditions for the equilibrium of floating bodies. Also discuss the stability of a floating body.
16. Define viscosity. Describe the cause of viscosity.
17. Define coefficient of viscosity. State and define its S.I. unit.
18. Write two factors affecting viscosity. Which one is more viscous : pure water or saline water ? **[Delhi 12]**
19. State Poiseuille's formula. Deduce it on the basis of dimensional considerations.
20. State Stokes' Law. Given the numerical constant in Stokes' law as 6π, obtain this law from the definition of viscosity. **[Central Schools 03]**
21. Explain how does a body attain a terminal velocity when it is dropped from rest in a viscous medium.
22. State Stokes' law. Derive an expression for the terminal velocity of a sphere falling through a viscous fluid. **[Central Schools 14 ; Himachal 05]**
23. By using Stokes' law, derive an expression of terminal velocity. On what factors does it depend ? **[Chandigarh 08]**
24. Give an example for a force proportional to velocity. Prove that terminal velocity of a solid object moving in viscous medium is directly proportional to its size and inversely proportional to the viscosity of the medium. **[Central Schools 03]**
25. Distinguish between streamline and turbulent flows.
26. What do you mean by a streamline and tube of flow ? Give two properties of streamlines.
27. What is laminar flow of a liquid ? Draw velocity profiles for the laminar flow of viscous and non-viscous liquids.
28. What is Reynold's number ? What is its importance ? **[Central Schools 12]**

Scanned by CamScanner

29. Show that the Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.
30. State and prove the equation of continuity. [Himachal 05]
31. State and derive Torricelli's law of efflux.
32. Derive an expression for rate of flow of fluid as measured by venturimeter. [Delhi 14]
33. If a ball is thrown and given a spin, then the path of the ball is more curved than in a usual spin free path. Explain.
34. On the basis of Bernoulli's principle, explain the lift of an aircraft wing.
35. Define angle of contact and surface energy. [Delhi 95]
36. Define surface tension. Derive a relation between surface tension and surface energy. What is the unit of surface tension ? [Delhi 98 ; Himachal 05]
37. How is the surface tension of a liquid explained on the basis of intermolecular forces ? From where the energy comes when a liquid rises against gravity in a capillary tube ? [Manipur 97 ; Himachal 05]
38. Small mercury drops are spherical and larger ones tend to flattened. Explain.
39. Describe a simple experiment for measuring the surface tension of a liquid.
40. Show that a pressure difference exists between the two sides of a curved liquid surface.
41. Derive an expression for the excess pressure inside a liquid drop. [Himachal 05 ; Delhi 14]
42. Derive an expression for the excess of pressure inside a soap bubble. [Delhi 13 ; Central Schools 14]
43. Derive excess of pressure inside an air bubble. [Chandigarh 08]
44. State Stokes' law. Derive this law by the method of dimensions. [Delhi 99]
45. What is capillarity ? Derive an expression for the height to which a liquid rises in a capillary tube of radius r. [Delhi 14 ; Central Schools 14]
46. Describe the cleansing action of detergents.
47. Define terminal velocity. Derive an expression for it. [Himachal 07]
48. A liquid is in streamlined flow through a pipe of non-uniform cross-section. Prove the sum of its kinetic energy, pressure energy and potential energy per unit volume remains constant. [Delhi 11, 13]

Answers

1. Refer answer to Q. 4 on page 10.1 and Q. 6 on page 10.2.
2. Refer answer to Q. 5 on page 10.2.
3. Refer answer to Q. 7 on page 10.2.
4. Refer answer to Q. 12 on page 10.4.
5. Refer answer to Q. 13 on page 10.5.
6. Refer answer to Q. 15 on page 10.7.
7. Refer answer to Q. 17 on page 10.8.
8. Refer answer to Q. 18 on page 10.8 and Q. 19 on page 10.9.
9. Refer answer to Q. 21 on page 10.9.
10. Refer answer to Q. 20 on page 10.9.
11. (*a*) Total pressure of the gas is the absolute pressure. It is $P = P_a + h\rho g$.

    (*b*) The difference between the absolute pressure and the atomospheric pressure is the gauge pressure. It is : $P_g = P - P_a = h\rho g$.
12. Refer answer to Q. 24 on page 10.14.
13. Refer answer to Q. 25 on page 10.14.
14. Refer answer to Q. 26 on page 10.14.
15. Refer answer to Q. 28 on page 10.15.
16. Refer answer to Q. 29 on page 10.20.
17. Refer answer to Q. 31 on page 10.21.
18. The viscosity of a liquid depends on its nature and temperature. Saline water is more viscous than pure water.
19. Refer answer to Q. 35 on page 10.24.
20. Refer answer to Q. 37 on page 10.28.
21. Refer answer to Q. 38 on page 10.28.
22. Refer answers to Q. 37 and Q. 38 on page 10.28.
23. Refer answer to Q. 38 on page 10.28.
24. The backward viscous force acting on a small spherical body moving through a viscous medium is proportional to its velocity.

    For expression of terminal velocity, refer answer to Q. 38 on page 10.28.
25. Refer answer to Q. 39 on page 10.32.
26. Refer answer to Q. 39 on page 10.32.
27. Refer answer to Q. 40 on page 10.33.
28. Refer answer to Q. 42 on page 10.33.
29. Refer answer to Q. 43 on page 10.34.
30. Refer answer to Q. 45 on page 10.35.
31. Refer answer to Q. 49 on page 10.38.
32. Refer answer to Q. 50 on page 10.38.
33. Refer answer to Q. 52 on page 10.39.
34. Refer answer to Q. 53 on page 10.39.
35. Refer to points 46 and 48 of Glimpses.
36. Refer answer to Q. 58 on page 10.47 and Q. 61 on page 10.49.

Scanned by CamScanner

37. Refer answer to Q. 59 on page 10.47. The surface energy of the liquid is used during its capillary rise.
38. Refer answer to Q. 60 (iv) on page 10.48.
39. Refer answer to Q. 62 on page 10.49.
40. Refer answer to Q. 63 on page 10.53.
41. Refer answer to Q. 64 on page 10.53.
42. Refer answer to Q. 65 on page 10.54.
43. Refer answer to Q.66 on page 10.54.
44. Refer answer to Q.37 on page 10.28.
45. Refer answer to Q. 69 on page 10.57 and Q. 70 on page 10.58.
46. Refer answer to Q. 73 on page 10.62.
47. Refer answer to Q. 38 on page 10.28.
48. Refer answer to Q. 38 on page 10.28.

Type C : Long Answer Questions

5 Marks Each

1. State and prove the Pascal's law of transmission of fluid pressure. **[Himachal 05C, 09C]**
2. Give the principle and explain the working of hydraulic brakes with a suitable diagram. **[Himachal 09C]**
3. State Pascal's law. Discuss its two practical applications. **[Himachal 05]**
4. Discuss the variation of fluid pressure with depth. Hence explain how is Pascal's law affected in the presence of gravity.
5. Define coefficient of viscosity and give its SI unit. On what factors does the terminal velocity of a spherical ball falling through a viscous liquid depend ? Derive the formula

$$v_t = \frac{2r^2 g}{9\eta}(\rho - \rho')$$

where the symbols have their usual meanings. **[Delhi 02, 03C]**

6. Define terminal velocity. Show that the terminal velocity v of a sphere of radius r, density ρ falling vertically through a viscous fluid of density σ and coefficient of viscosity η is given by $v = \frac{2}{9}\frac{(\rho - \sigma)r^2 g}{\eta}$.

Use this formula to explain the observed rise of air bubbles in a liquid. **[Himachal 06 ; Delhi 14]**

7. (a) Define streamline.
   (b) Write any two properties of streamlines.
   (c) Draw streamlines for a clockwise spinning sphere.
   (d) Derive equation of continuity. **[Central Schools 08]**
8. What is meant by the term coefficient of viscosity ? State Stokes' law. Define terminal velocity and find an expression for the terminal velocity in case of a sphere falling through a viscous liquid such as glycerine. **[Himachal 05C ; Delhi 03]**
9. State and prove Bernoulli's principle for the flow of non-viscous, incompressible liquid in streamlined flow. Give its limitations. **[Himachal 05 ; Delhi 09, 12]**
10. State Bernoulli's theorem. With the help of suitable diagram, establish Bernoulli's equation for liquid flow. Explain the lifting of aeroplane by it. **[Central Schools 12, 14]**
11. (i) State and prove Bernoulli's theorem.
    (ii) A cylindrical vessel of uniform cross-section contains liquid upto the height 'H'. At a depth '$h = H/2$ below the free surface of the liquid there is an orifice. Using Bernoulli's theorem, find the velocity of efflux of liquid. **[Delhi 05]**
12. What is meant by Streamline flow ? State the equation of Continuity. Write the properties of an ideal fluid. Establish Bernoulli's equation as applied to an ideal fuel. **[Delhi 03]**
13. State Bernoulli's theorem. Prove that the total energy possessed by a flowing ideal liquid is conserved, stating assumptions used. **[Himachal 04]**
14. (a) State Bernoulli's equation.
    (b) Name the physical quantity corresponding to each term of this equation.
    (c) What type of liquid flow obeys this equation ?
    (d) Show that this equation is same as the equation due to Pascal's law in the presence of gravity if a liquid or gas is at rest. **[Central Schools 08]**
15. (a) Explain why sometimes the light roofs of thatched houses are blown off during a storm.
    (b) Derive Stokes' law dimensionally. **[Central Schools 09]**
16. Define surface tension and surface energy. Write units and dimensions of surface tension. Also prove that surface energy is numerically equal to the surface tension. **[Delhi 08]**
17. Show that there is always an excess pressure on the concave side of the meniscus of a liquid. Obtain expression for the excess pressure inside (i) a liquid drop, (ii) liquid bubble (iii) air bubble inside a liquid. **[Himachal 04]**
18. (a) How do the insects run on the surface of water ?
    (b) Derive an expression for excess pressure inside a soap bubble.
    (c) Define the term surface energy. Write down its dimensional formula and units. **[Central Schools 08]**
19. (i) What is the phenomenon of capillarity ? Derive an expression for the rise of liquid in a capillary tube.

Scanned by CamScanner

(ii) What will happen if the length of the capillary tube is smaller than the height to which the liquid rises ? Explain briefly. [Delhi 97, 05]

20. (a) Derive an expression for the rise of liquid in capillary tube of uniform diameter and sufficient length.

(b) A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ. [Central Schools 09]

21. What is capillarity ? Derive an expression for the height to which the liquid rises in a capillary tube of radius r with angle of contact θ. Give two examples of capillarity from daily life. [Central Schools 07, 12]

22. Explain why

(a) A balloon filled with liquid helium does not rise in air indefinitely but halts after a certain height. [Central Schools 07]

(b) The angle of contact of mercury with glass is obtuse while that of water with glass is acute.

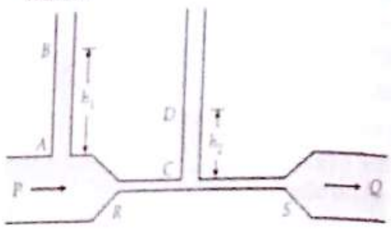
(c) A drop of liquid under no external force is always spherical in shape. [Delhi 0…]

23. (a) Explain the principle and working of Hydraulic lift with the help of a schematic diagram.

(b) To keep a piece of paper horizontal, you should blow over, not under it. Why ? [Central Schools 07]

24 (a) State Pascal's law of fluid pressure with a suitable diagram, explain how is Pascal's law applied in a hydraulic lift ?

(b) As shown in the figure, water flows from P to Q. Explain why is height h_1 of column AB of water greater than height h_2 of column CD of water ? [Delhi 10]



Answers

1. Refer answer to Q. 11 on page 10.4.
2. Refer answer to Q. 14 on page 10.5.
3. Refer answer to Q. 11 on page 10.4 and Q. 13 and Q. 14 on page 10.5.
4. Refer answer to Q. 16 on page 10.7.
5. Refer answer to Q. 38 on page 10.28.
6. Refer answer to Q. 38 on page 10.28.
7. (a) Refer answer to Q.39 on page 10.32.
   (b) Refer answer to Q.39 on page 10.32.
   (c) See Fig. 10.37 (a) on page 10.39.
   (d) Refer answer to Q.45 on page 10.35.
8. Refer answer to Q. 36 and Q. 38 on page 10.28.
9. Refer answer to Q. 48 on page 10.36.
10. Refer answer to Q. 48 on page 10.36 and Q. 53 on page 10.39.
11. (i) Refer answer to Q. 48 on page 10.36.
    (ii) Applying Bernoulli's theorem,

$$\frac{P}{\rho} + 0 + gH = \frac{P}{\rho} + \frac{1}{2}v^2 + g(H-h)$$

or $gH = \frac{1}{2}v^2 + g(H - H/2)$ or $v = \sqrt{gH} = \sqrt{2gh}$.

12. Refer to points 28, 33 and 35 of Glimpses.
13. Refer answer to Q. 48 on page 10.36.
14. (a) Refer answer to Q. 48 on page 10.36.

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

(b) P = Pressure energy per unit volume.

$\frac{1}{2}\rho v^2$ = Kinetic energy per unit volume

ρgh = Potential energy per unit volume

(c) The liquid flow must be streamlined and irrotational.

(d) When the liquid is at rest, $v = 0$.

$P + \rho gh$ = constant. This is Pascal's law.

15. (a) Refer to solution of Problem 75 on page 10.67.
    (b) Refer answer to Q.36 on page 10.28.
16. Refer answer to Q. 58 on page 10.47 and Q. 61 on page 10.49.
17. Refer answers to Q. 64, 65 and 66 on page 10.53 and 10.54.
18. (a) Refer to the solution of Problem 98 on page 10.69.
    (b) Refer answer to Q.65 on page 10.54.
    (c) Refer answer to Q.61 on page 10.49.
19. Refer answer to Q. 70 and Q. 71 on page 10.58.
20. (a) Refer answer to Q.70 on page 10.58.
    (b) Refer to the solution of Ex. 75 on page 10.58.
21. Refer answer to Q. 69 on page 10.57 and Q. 70 on page 10.58.
22. Refer to solutions of problems 1(i) on page 10.72 and NCERT Exercise 10.2(i) and (v) on page 10.79.
23. (a) Refer answer to Q.13 on page 10.5.
    (b) Refer answer to NCERT Exercise 10.4(i) on page 10.79.
24. (a) Refer answer to Q. 13 on page 10.5.
    (b) Cross-sectional area of part RS if the tube less than the remaining part, so speed of flow is large. By Bernoulli's theorem, liquid pressure must be small in part RS i.e., $h_1 > h_2$.

Scanned by CamScanner

Competition Section

Mechanical Properties of Fluids

GLIMPSES

1. **Fluid.** A fluid is a substance that can flow. The term fluid refers to both liquids and gases.

2. **Fluid statics.** The branch of physics that deals with the study of fluids at rest is called *fluid statics* or *hydrostatics*.

3. **Fluid dynamics.** The branch of physics that deals with the study of fluids in motion is called *fluid dynamics* or *hydrodynamics*.

4. **Thrust.** The total force exerted by a liquid on any surface in contact with it is called thrust. A liquid always exerts force perpendicular to the surface of the container at every point.

5. **Pressure.** The thrust exerted by a liquid per unit area of the surface in contact with it is known as pressure.

$$\text{Pressure} = \frac{\text{Thrust}}{\text{Area}} \quad \text{or} \quad P = \frac{F}{A}$$

Pressure is a scalar quantity.

6. **Units and dimensions of pressure.** The CGS unit of pressure is dyne cm^{-2} and its SI unit is Nm^{-2} which is also called pascal (Pa). The dimensional formula of pressure is $[ML^{-1}T^{-2}]$.

7. **Density.** The density of any material is defined as its mass per unit volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad \rho = \frac{M}{V}$$

Density is a positive scalar quantity.

8. **Units and dimensions of density.** The SI unit of density is kg m^{-3} and the CGS unit is g cm^{-3}. The dimensional formula of density is $[ML^{-3}]$.

9. **Specific gravity.** The relative density or *specific gravity* of a substance is defined as the ratio of the density of the substance to the density of water at 4°C.

$$\text{Specific gravity} = \frac{\text{Density of substance}}{\text{Density of water at } 4^\circ C}$$

Specific gravity is a dimensionless positive scalar quantity.

10. **Pascal's law.** It states that a change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel. Or, the pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.

11. **Hydraulic lift.** It is an application of Pascal's law. It is used to lift heavy objects.

According to Pascal's law,

Pressure applied on smaller piston

= Pressure transmitted to larger piston

$$p = \frac{f}{a} = \frac{F}{A} \quad \text{or} \quad F = P \times A = \frac{f}{a} \times A$$

As $A > a$, so $F > f$.

Thus hydraulic lift acts as a force multiplier.

12. **Hydraulic brakes.** The hydraulic brakes used in automobiles are based on Pascal's law of transmission of pressure in a liquid.

13. **Pressure exerted by a liquid.** A liquid column of height h and density ρ exerts a pressure given by

$$P = h \rho g$$

14. **Effect of gravity on fluid pressure.** The pressure in a fluid varies with depth h according to the expression

$$P = P_a + h \rho g$$

where ρ is the fluid density, assumed uniform.

15. **Hydrostatic paradox.** The pressure exerted by a liquid column depends only on the height of the liquid column and not on the shape of the containing vessel.

Scanned by CamScanner

16. **Atmospheric pressure.** The pressure exerted by the atmosphere is called atmospheric pressure. At sea-level, we have

Atmospheric pressure

= Pressure exerted by 0.76 m of Hg

$= h\rho g = 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5 \text{ Nm}^{-2}$

Other units used for atmospheric pressure are as follows :

$$1 \text{ atm} = 1.013 \times 10^6 \text{ dyne cm}^{-2}$$
$$= 1.013 \times 10^5 \text{ Nm}^{-2} \text{ (or Pa)}$$
$$1 \text{ bar} = 10^6 \text{ dyne cm}^{-2} = 10^5 \text{ Nm}^{-2}$$
$$1 \text{ millibar (m bar)} = 10^{-3} \text{ bar} = 10^3 \text{ dyne cm}^{-2}$$
$$= 10^2 \text{ Nm}^{-2}$$
$$1 \text{ torr} = 1 \text{ mm Hg}$$
$$1 \text{ atm} = 101.3 \text{ kPa} = 1.013 \text{ bar} = 760 \text{ torr}$$

17. **Absolute pressure and gauge pressure.** The total or actual pressure P at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (absolute pressure) at a point and the atmospheric pressure. Thus

$$P_g = P - P_a \quad \text{or} \quad P = P_a + P_g$$

Absolute pressure = Atmospheric pressure + Gauge pressure.

18. **Buoyancy and centre of buoyancy.** The upward force acting on a body immersed in a fluid is called upthrust or buoyant force and the phenomenon is called buoyancy. The force of buoyancy acts though the centre of gravity of the displaced fluid which is called centre of buoyancy.

19. **Archemedes' principle.** It states that when a body is immersed partly or wholly in a fluid, it loses some weight. The loss in weight is equal to the weight of the fluid displaced.

Apparent weight of a body in a fluid

= True weight – Weight of fluid displaced

$$W_{app} = W - U = V\sigma g - V\rho g$$
$$= V\sigma g\left(1 - \frac{\rho}{\sigma}\right) = W\left(1 - \frac{\rho}{\sigma}\right)$$

where $W = V\sigma g$ is the weight of the body and σ its density.

20. **Law of floatation.** A body will float in a liquid if weight of the liquid displaced by the body is atleast equal to or greater than the weight of the body.

When a body just floats,

Weight of the body = Weight of liquid displaced

$$V\sigma g = V'\rho g \quad \text{or} \quad \frac{V'}{V} = \frac{\sigma}{\rho}$$

or $\dfrac{\text{Volume of the immersed part}}{\text{Total volume of the body}} = \dfrac{\text{Density of the body}}{\text{Density of liquid}}$

21. **Viscosity.** It is the property of a fluid due to which an opposing force comes into play whenever there is relative motion between its different layers.

22. **Newton's formula for viscous force.** The viscous drag between two parallel layers each of area A and having velocity gradient dv/dx is given by

$$F = -\eta A \frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid.

23. **Coefficient of viscosity.** It may be defined as the tangential viscous force required to maintain a unit velocity gradient between two liquid layers each of unit area. Its dimensional formula is $[ML^{-1}T^{-1}]$.

24. **Units of η.** The CGS unit of η is *poise*. The coefficient of viscosity of a liquid is 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cm s^{-1} between two layers of the liquid 1 cm apart.

$$1 \text{ poise} = 1 \text{ dyne s cm}^{-2} = 1 \text{ g cm}^{-1}\text{s}^{-1}.$$

The SI unit of η is *decapoise*. The coefficient of viscosity of a liquid is 1 decapoise if a tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

$$1 \text{ poisuille} = 1 \text{ decapoise} = 1 \text{ Nsm}^{-2}$$
$$= 1 \text{ Pa s} = 10 \text{ poise}.$$

25. **Poiseuille's formula.** The volume of a liquid flowing per second through a horizontal capillary tube of length l, radius r under a pressure difference p across its two ends is given by

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8\eta l}.$$

26. **Stokes' law.** It states that the backward dragging force of viscosity acting on a spherical body of radius r moving with velocity v through a fluid of viscosity η is $\quad F = 6\pi\eta r v.$

27. **Terminal velocity.** It is the maximum constant velocity attained by a spherical body while falling through a viscous medium. The terminal velocity of a spherical body of density ρ and radius r moving through a fluid of density ρ' and viscosity η is given by $\quad v = \dfrac{2}{9}\dfrac{r^2}{\eta}(\rho - \rho')g$

28. **Streamline flow and turbulent flow.** It is the flow of liquid in which each particle of the liquid passing through a point travels along the same

path and with the same velocity as the preceding particle passing through the same point.

A liquid possesses streamline motion only when its velocity is less than a certain limiting value, called *critical velocity*. When the velocity of the liquid becomes greater than the critical velocity, the particles follow zig-zag path, such a disordered or irregular motion is called turbulent flow.

29. Tube of flow. A tube of flow is a bundle of streamlines having the same velocity of fluid element over any cross-section perpendicular to the direction of flow.

30. Laminar flow. The steady flow in which liquid moves in the form of layers is called laminar flow. The velocity of the layer varies from maximum at axis to zero for the layer at the wall of the tube.

31. Critical velocity. The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent. It is given by

$$v_c = \frac{k\eta}{\rho D}$$

32. Reynold's number. It is a dimensionless number which determines the nature of the flow of the liquid. For a liquid of viscosity η, density ρ and flowing through a pipe of diameter D, Reynold's number is given by

$$R_e = \frac{\rho\, v\, D}{\eta}$$

If $R_e < 2000$, the flow is laminar.

If $R_e > 3000$, the flow is turbulent.

If $2000 < R_e < 3000$, the flow is unstable. It may change from laminar to turbulent and vice-versa.

$$\text{Reynold's number} = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}}.$$

33. Ideal fluid. An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.

34. Rate of flow. The volume of a liquid flowing per second through a pipe of cross-section a with velocity v is given by

$$Q = \frac{V}{t} = av.$$

35. Equation of continuity. If there is no source or sink of the fluid along the length of the pipe, the mass of the fluid crossing any section of the pipe per second is always constant.

$$m = a_1 v_1 \rho_1 = a_2 v_2 \rho_2$$

It is called equation of continuity. For an incompressible liquid, $\rho_1 = \rho_2$, then

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad av = \text{constant.}$$

Thus during the streamlined flow of a non-viscous and incompressible fluid through a pipe of varying cross-section, the product of area of cross-section and the normal fluid velocity remains constant throughout the fluid flow.

36. Bernoulli's principle. It states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined, irrotational flow remains constant along a streamline. Thus

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

or
$$\frac{P}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$

The terms $\frac{P}{\rho g}$, h and $\frac{v^2}{2g}$ are called pressure head, gravitational head and velocity head respectively. For the horizontal flow of a liquid (h = constant), Bernoulli's equation takes the form

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

or
$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

It indicates that velocity increases where pressure decreases and vice-versa.

37. Torricelli's Law. It states that the velocity of efflux *i.e.* the velocity with which the liquid flows out of an orifice (a narrow hole) is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid. Hence the velocity of efflux of a liquid through an orifice at depth h from the liquid surface will be

$$v = \sqrt{2gh}.$$

38. Venturimeter. It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle. It is also called flow meter or venturi tube. Volume of the liquid flowing out per second is given by

$$Q = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}.$$

39. Magnus effect. The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards the greater pressure side, is called magnus effect.

40. Aerofoil. It is a solid object shaped to provide an upward vertical force as it moves horizontally through air.

41. Cohesive and adhesive forces. The force of attraction between the molecules of the same substance is called cohesive force while the force of attraction between the molecules of two different substances is called adhesive force.

Scanned by CamScanner

42. Molecular range. It is the maximum distance upto which a molecule can exert some measurable attraction on other molecules. The order of molecular range is 10^{-9} m in solids and liquids.

43. Sphere of influence. It is a sphere drawn with molecule as centre and molecular range as radius.

44. Surface film. A thin film of liquid near its surface and having thickness equal to the molecular range for that liquid is called surface film. The molecules present in the surface film possess additional potential energy.

45. Surface Tension. It is the property of a liquid by virtue of which, it behaves like an elastic stretched membrane with a tendency to contact so as to occupy a minimum surface area. It is measured as the force per unit length on an imaginary line drawn on the surface of liquid.

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}} \quad \text{or} \quad \sigma = \frac{F}{l}$$

Its SI unit is Nm^{-1} and CGS unit is dyne cm^{-1}.

46. Surface energy. The additional potential energy per unit area of the surface film as compared to the molecules in the interior is called the surface energy.

Surface energy

$$= \frac{\text{Work done in increasing the surface area}}{\text{Increase in surface area}}$$

Surface energy of a liquid is numerically equal to surface tension of the liquid.

47. Excess pressure inside a drop and bubble. There is excess of pressure on the concave side of a curved surface.

$$\text{Excess pressure inside a liquid drop} = \frac{2\sigma}{R}$$

(One free surface)

$$\text{Excess pressure inside a liquid bubble} = \frac{4\sigma}{R}$$

(Two free surfaces)

$$\text{Excess pressure inside an air bubble} = \frac{2\sigma}{R}$$

(One free surface)

where R is radius of the liquid drop, liquid bubble or the air bubble.

48. Angle of contact. The angle, which the tangent to the free surface of liquid at the point of contact makes with the wall of the containing vessel inside the liquid, is called angle of contact. For the liquids having concave meniscus, the angle of contact is acute and for the liquids having convex meniscus, the angle of contact is obtuse. The liquids, for which the angle of contact is acute, show a rise in the capillary tube; while those for which the angle of contact is obtuse, show a fall.

49. Capillarity. A tube of very fine bore is called a capillary tube. The phenomenon of rise or fall of a liquid in capillary tube is known as capillarity.

50. Ascent formula. When a capillary tube of radius r is dipped in a liquid of density ρ and surface tension σ, the liquid rises or falls through a distance,

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

51. Rise of liquid in a tube of insufficient height. The radius r of the capillary tube and radius of curvature R of the liquid meniscus are related by $r = R\cos\theta$. Therefore

$$h = \frac{2\sigma\cos\theta}{R\cos\theta \cdot \rho g} = \frac{2\sigma}{R\rho g}$$

As, σ, ρ and g are constants, so

$$hR = \frac{2\sigma}{\rho g} = \text{a constant}$$

If $h' < h$, then the radius of curvature R increases to R' such that $hR = h'R'$. The liquid rises and spreads out to a new radius $R' = hR/h'$. But the liquid does not overflow.

IIT Entrance Exam

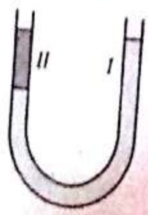
Multiple Choice Questions with One Correct Answer

1. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is

(a) same everywhere
(b) lower in the front side
(c) lower in the rear side
(d) lower in the upper side

[IIT 99]

2. A U-tube of uniform cross- section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be



(a) 1.12
(b) 1.1
(c) 1.05
(d) 1.0

[IIT 83]

Scanned by CamScanner

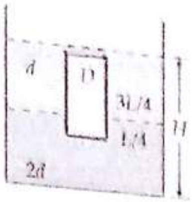
3. A man is sitting in a boat, which is floating on a pond. If the man drinks some water from the pond, the level of water in the pond

(*a*) decreases (*b*) increases

(*c*) remains unchanged

(*d*) may increase or decrease depending on the weight of the man. [IIT 80]

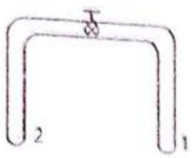
4. A homogeneous solid cylinder of length $L (L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then density of solid is given by



(*a*) $\frac{5}{4}d$ (*b*) $\frac{4}{5}d$

(*c*) $4d$ (*d*) $\frac{d}{5}$ [IIT 95]

5. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemi- spherical soap bubble as shown in figure. Just after opening the valve,



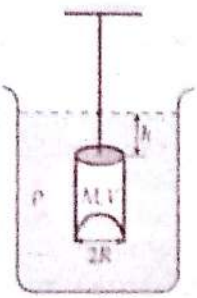
(*a*) air from end 1 flows towards end 2. No change in the volume of the soap bubbles.

(*b*) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases.

(*c*) no changes occur

(*d*) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases. [IIT 08]

6. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is

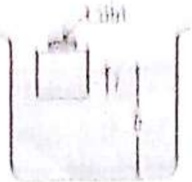


(*a*) Mg (*b*) $Mg - V\rho g$

(*c*) $Mg + \pi R^2 h\rho g$

(*d*) $\rho g(V + \pi R^2 h)$ [IIT 01]

7. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance l and h are shown here. After some time the coin falls into the water. Then



(*a*) l decreases and h increases

(*b*) l increases and h decreases

(*c*) both l and h increase

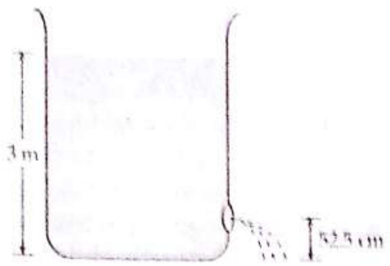
(*d*) both l and h decrease. [IIT 02; DPMT 06]

8. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then R is equal to

(*a*) $\frac{L}{\sqrt{2\pi}}$ (*b*) $2\pi L$

(*c*) L (*d*) $\frac{L}{2\pi}$ [IIT 2K]

9. Water is filled in a container upto height 3 m. A small hole of area a is punched in the wall of the container at a height 52.5 cm from the bottom. The



cross-sectional area of the container is A. If $a/A = 0.1$, then v^2 (where v is the velocity of water coming out of the hole) is

(*a*) $50\ m^2s^{-2}$ (*b*) $51\ m^2s^{-2}$

(*c*) $48\ m^2s^{-2}$ (*d*) $51.5\ m^2s^{-2}$ [IIT 05]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

10. A body floats in a liquid contained in a beaker. The whole system as shown in figure falls freely under gravity. The upthrust on the body is

(*a*) zero

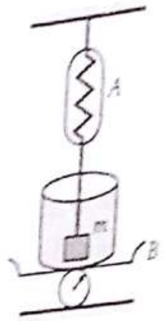
(*b*) equal to the weight of the liquid displaced

Scanned by CamScanner

(c) equal to the weight of the body in air

(d) equal to the weight of the immersed portion of the body. [IIT 82]

11. The spring balance *A* reads 2 kg with a block *m* suspended from it. A balance *B* reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation



(a) the balance *A* will read more than 2 kg

(b) the balance *B* will read more than 5 kg

(c) the balance *A* will read less than 2 kg and *B* will read more than 5 kg

(d) the balances *A* and *B* will read 2 kg and 5 kg respectively. [IIT 85]

12. A vessel contains oil (density $= 0.8\ g\ cm^{-3}$) over mercury (density $= 13.6\ g\ cm^{-3}$). A homogenous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material (in $g\ cm^{-3}$) is

(a) 3.3 (b) 6.4

(c) 7.2 (d) 2.8 [IIT 88]

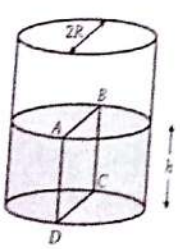
13. Water is filled up to a height *h* in a beaker of radius *R* as shown in the figure. The density of water is ρ, the surface tension of water is *T* and the atmospheric pressure is P_0. Consider a vertical section *ABCD* of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude

(a) $|2P_0Rh + \pi R^2 \rho gh - 2RT|$

(b) $|2P_0Rh + R\rho gh^2 - 2RT|$

(c) $|P_0 \pi R^2 + R\rho gh^2 - 2RT|$

(d) $|P_0 \pi R^2 + R\rho gh^2 + 2RT|$ [IIT 07]

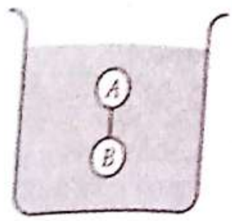


14. Water from a tap emerges vertically downwards with an initial speed of $1.0\ ms^{-1}$. The cross-sectional area of the tap is $10^{-4} m^2$. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is

(a) $5.0 \times 10^{-4} m^2$ (b) $1.0 \times 10^{-5} m^2$

(c) $5.0 \times 10^{-5} m^2$ (d) $2.0 \times 10^{-5} m^2$ [IIT 98]

15. Two solid spheres *A* and *B* of equal volumes of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density



They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if

(a) $d_A < d_F$ (b) $d_B > d_F$

(c) $d_A > d_F$ (d) $d_A + d_B = 2d_F$ [IIT 2...]

Reasoning Type

16. **Statement – 1.** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement – 2. In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

(a) Statement – 1 is true, Statement – 2 is true. Statement – 2 is a correct explanation for Statement – 1.

(b) Statement – 1 is true, Statement – 2 is true. Statement – 2 is not a correct explanation for Statement – 1.

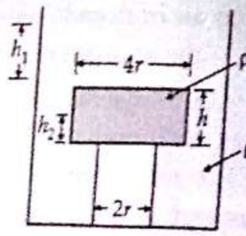
(c) Statement – 1 is true, Statement – 2 is false.

(d) Statement – 1 is false, Statement – 2 is true. [IIT 08]

Comprehension Based Questions

Paragraph for Question Nos. 17 to 19

A cylinder tank has a hole of diameter 2*r* in its bottom. The hole is covered with a wooden cylindrical block of diameter 4*r*, height *h* and density ρ/3.



Situation 1. Initially, the tank is filled with water of density ρ to a height such that the height of water above the top of the block is h_1 (measured from the top of the block).

Situation 2. The water is removed from the tank to a height h_2 (measured from the bottom of the block), as shown in the figure. The height h_2 is smaller than *h* (height of the block) and thus the block is exposed to the atmosphere.

Scanned by CamScanner

Read the passage given above and answer the following questions

17. Find the minimum value of height h_1 (in situation 1), for which the block just starts to move up.

(a) $\frac{2h}{3}$ (b) $\frac{5h}{4}$

(c) $\frac{5h}{3}$ (d) $\frac{5h}{2}$ [IIT 06]

18. Find the height of the water level h_2 (in situation 2), for which the block remains in its original position without the application of any external force.

(a) $\frac{h}{3}$ (b) $\frac{4h}{9}$

(c) $\frac{2h}{3}$ (d) h. [IIT 06]

19. In situation 2, if h_2 is further decreased, then

(a) cylinder will not move up and remains at its original position

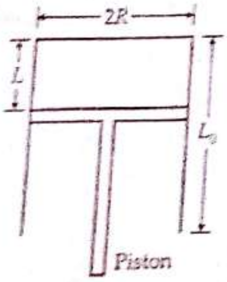
(b) for $h_2 = \frac{h}{3}$, cylinder again starts moving up

(c) for $h_2 = \frac{h}{4}$, cylinder again starts moving up

(d) for $h_2 = \frac{h}{5}$, cylinder again starts moving up. [IIT 06]

PARAGRAPH FOR Q.20 TO Q.22

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



Read the passage given above and answer the following questions

20. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be

(a) P_0 (b) $P_0/2$

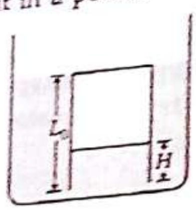
(c) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$ (d) $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$ [IIT 07]

21. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(a) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$ (b) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$

(c) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$ (d) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$ [IIT 07]

22. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(a) $\rho g(L_0 - H)^2 + P_0(L_0 - H) + L_0P_0 = 0$

(b) $\rho g(L_0 - H)^2 - P_0(L_0 - H) - L_0P_0 = 0$

(c) $\rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$

(d) $\rho g(L_0 - H)^2 - P_0(L_0 - H) + L_0P_0 = 0$ [IIT 07]

COMPREHENSION BASED QUESTIONS

PARAGRAPH FOR QUESTIONS 23 TO 25

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

23. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$ is)

(a) $2\pi rT$ (b) $2\pi RT$

(c) $\frac{2\pi r^2T}{R}$ (d) $\frac{2\pi R^2T}{r}$ [IIT 2010]

24. If $r = 5\times10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m

(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m [IIT 2010]

Scanned by CamScanner

25. After the drop detaches, its surface energy is

(*a*) 1.4×10^{-6} J (*b*) 2.7×10^{-6} J

(*c*) 5.4×10^{-6} J (*d*) 8.1×10^{-6} J

[IIT 2010]

Integer Answer Type

26. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m², density of water = 1000 kg/m³ and $g = 10$ m/s². Neglect any effect of surface tension.]

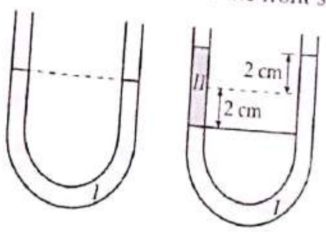
27. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m². The radii of bubbles A and B are 2 cm and 4 cm respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

[IIT ...]

Answers and Explanations

1. (*b*) As the compartment has an acceleration in the forward direction, the gas molecules experience an acceleration in the backward direction. This lowers the pressure in the compartment in the front side.

2. (*b*)



Pressure exerted by 4 cm column of liquid II
= Pressure exerted by 4 cm column of liquid I

$\rho \times g \times 4 = 1.1 \times g \times 4$ or $\rho = 1.1$

$\therefore$ Specific gravity of liquid II = 1.1.

3. (*c*) The level of water remains unchanged. Refer to the solution of Problem 31 on page 10.64.

4. (*a*) Weight of cylinder

= upthrust due to upper liquid
+ upthrust due to lower liquid.

$$\frac{A}{5} \cdot L . D . g = \frac{A}{5} \cdot \frac{3L}{4} \cdot d \cdot g + \frac{A}{5} \cdot \frac{L}{4} \cdot 2d \cdot g$$

$$D = \frac{3}{4} \cdot d + \frac{1}{4} \cdot 2d = \frac{5}{4} d.$$

5. (*b*) Excess pressures inside the soap bubbles at ends 1 and 2 respectively are

$$p_1 = \frac{2\sigma}{r_1}, \quad p_2 = \frac{2\sigma}{r_2}.$$

As $r_1 < r_2$, so, $p_1 > p_2$.

Hence air will flow from end 1 towards end 2.

6. (*d*) $F_{\text{bottom}} - F_{\text{upper surface}}$

= Upthrust

= Weight of liquid displaced

$$F_{\text{bottom}} - \pi R^2 (h \rho g) = V \rho g$$

$$F_{\text{bottom}} = \rho g (V + \pi R^2 h).$$

7. (*d*) l decreases as the block moves up. h will also decrease as when the coin falls into water it will displace water equal to its own volume (V_1) while previously it was displacing water (of volume V_2) equal to its weight. As density of coin is more than that of water, so $V_1 < V_2$.

8. (*a*) Velocity of efflux at depth h, $v = \sqrt{2gh}$

As the volumes of water flowing out per second from both the holes are equal,

$$\therefore \quad a_1 v_1 = a_2 v_2$$

or $$L^2 \sqrt{2g . y} = \pi R^2 \sqrt{2g . 4y}$$

or $$R = \frac{L}{\sqrt{2\pi}}.$$

9. (*b*) Here $h = 3 - 0.525 = 2.475$ m

If v is velocity of effux, then

$$v^2 = \frac{2gh}{1 - \left(\frac{a}{A}\right)^2} = \frac{2 \times 10 \times 2.475}{1 - (0.1)^2} = \frac{49.50}{0.99}$$

or $$v^2 = 50 \text{ m}^2\text{s}^{-2}.$$

10. (*a*) For a freely falling system, $g = 0$

Upthrust = Weight of liquid displaced
= Mass of liquid displaced × g
= Mass × 0 = 0.

Scanned by CamScanner

11. (*b*), (*c*) Liquid will exert an upthrust on mass *m*. By Newton's third law, an equal force will be exerted on the liquid in the downward direction. Hence the balance *A* will read less than 2 kg while the balance *B* will read more than 5 kg.

12. (*c*) Let ρ be the density of the material of the sphere. Then

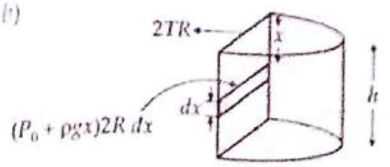
Weight of sphere

= Upthrust due to oil + Upthrust due to Hg

$$V\rho g = \frac{V}{2}\cdot\rho_{oil}g + \frac{V}{2}\cdot\rho_{Hg}g$$

$$\rho = \frac{\rho_{oil}+\rho_{Hg}}{2} = \frac{0.8+13.6}{2} = 7.2\text{ g cm}^{-3}.$$

13. (*b*)



Consider the half cylinder of water on one side of the vertical section *ABCD*. The forces on this half cylinder due to another cylinder are shown. Consider a strip of width *dx* at depth *x* from the top. Force on this strip is $(P_0+\rho xg)2R\,dx$. Total force on one half cylinder due to another half cylinder,

$$F=\int_0^h (P_0+\rho gx)2R\,dx - 2RT = 2P_0Rh + R\rho gh^2 - 2RT$$

14. (*c*) Velocity *v* of water stream 0.15 m below the top is given by

$$v^2 = u^2+2as = 1^2+2\times10\times0.15 = 4$$

$$v = 2\text{ ms}^{-1}$$

From equation of continuity,

$$a_2 = \frac{a_1v_1}{v_2} = \frac{10^{-4}\times1.0}{2} = 5.0\times10^{-5}\text{ m}^2.$$

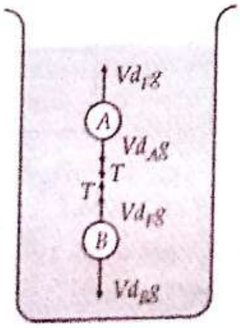
15. (*a*), (*b*), (*d*) *For equilibrium of A*

$$T + Vd_Ag = Vd_Fg$$

$$T = V(d_F-d_A)g \qquad \text{...(i)}$$

For $T>0,\ d_F>d_A \Rightarrow d_A<d_F$

∴ Opinion (*a*) is correct



From equilibrium of B

$$T + Vd_Fg = Vd_Bg$$

$$T = V(d_B-d_F)g \qquad \text{...(ii)}$$

For $T>0$; $d_B>d_F$

∴ Option (*b*) is correct

From (*i*) and (*ii*),

$$V(d_F-d_A)g = V(d_B-d_F)g$$

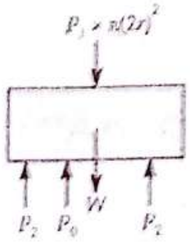
$$\therefore\quad d_F-d_A = d_B-d_F \quad\text{or}\quad 2d_F = d_A+d_B$$

⇒ Opinion (*d*) is correct

16. (*a*) Volume rate of flow, $Q = Av$ = constant

As speed of upstream decreases, its area of cross-section increases. As speed of downstream increases, its area of cross-section decreases.

17. (*c*) Consider the equilibrium of the wooden block.



Forces acting in the downward direction are :

(*i*) Weight of wooden block,

$$W = \pi(2r)^2\times h\times\frac{\rho}{3}\times g = \frac{4}{3}\pi r^2h\rho g$$

(*ii*) Force due to pressure P_1, exerted by liquid column of height *h*, above the block,

$$F_1 = P_1\times\pi(2r)^2 = (P_0+h_1\rho g)\times\pi\times4r^2$$

Resultant force acting in the upward direction due to liquid pressure P_2 from below the block and due to atmospheric pressure,

$$F_2 = P_2\times\pi[(2r)^2-(r)^2] + P_0\times\pi(r)^2$$

$$= [P_0+(h_1+h)\rho g]\times\pi\times3r^2 + \pi r^2P_0$$

The block will just begin to move up when

$$F_2 = F_1 + W$$

or $$[P_0+(h_1+h)\rho g]\times\pi\times3r^2+\pi r^2P_0 = (P_0+h_1\rho g)\times\pi\times4r^2 - \frac{4}{3}\pi r^2h\rho g$$

or $$h_1 = \frac{5h}{3}.$$

18. (*b*) Now in situation 2,

Downward force on block,

$$= W + \text{Force due to }P_0 = \frac{\pi\times4r^2\rho gh}{3} + P_0\times\pi(4r^2)$$

Scanned by CamScanner

Upward force on block,

$$= \text{Due to liquid pressure} + \text{Due to } P_0$$

$$= (h_2\rho g + P_0)\pi(4r^2 - r^2) + P_0 \times \pi r^2$$

For equilibrium,

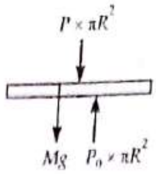
$$\frac{4\pi r^2 \rho g h}{3} + 4P_0\pi r^2 = (h_2\rho g + P_0)\times 3\pi r^2 + P_0 \times \pi r^2$$

$$\therefore \quad h_2 = \frac{4h}{9}.$$

19. (a) As the height h_2 of water level is further decreased, the upward force on the block decreases. The total downward force remains the same. The difference is compensated by the normal reaction of the tank wall. Thus the block does not move up and remains at its equilibrium position.

20. (a) As the hole remains open, the pressure in the cylinder becomes equal to P_0.

21. (d) Let the pressure of the enclosed air be P, then the force on the piston will be as shown in the figure.



For equilibrium of the piston,

$$Mg + P \times \pi R^2 = P_0 \times \pi R^2$$

As the walls are conducting, the expansion is isothermal. We use

$$P_1V_1 = P_2V_2$$

or $$P_0(2L \times \pi R^2) = P(x \times \pi R^2)$$

or $$P_0 \times \frac{2L}{x} = P$$

$$\therefore Mg + P_0 \times \frac{2L}{x} \times \pi R^2 = P_0 \times \pi R^2$$

or $$x = \left(\frac{P_0 \pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$$

22. (c) As the cylinder is placed in water, the air enclosed in the cylinder is isothermally compressed.

$$\therefore \quad P_0 + (\pi R^2 l_0) = P[\pi R^2(l_0 - H)] \quad ...(i)$$

Pressure at the bottom is $P_0 + \rho g l_0$

$$\therefore \quad P + \rho g h = P_0 + \rho g l_0$$

or $$P = P_0 + \rho g(l_0 - H) \quad ...(ii)$$

From (i) and (ii), we get

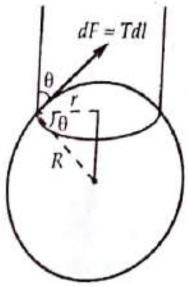
$$\rho g(l_0 - H)^2 + P_0(l_0 - H) - l_0 P_0 = 0$$

23. (c) Vertical force = Force due to surface tension

$$= \int dF = \int T\, dl \cos\theta$$

$$= T \times 2\pi r \times \frac{r}{R}$$

$$= \frac{2\pi r^2 T}{R}$$



$\therefore$ Option (c) is correct.

24. (a) The drop detaches from the dropper when

Vertical force due to surface tension

= Weight of the drop

or $$\frac{2\pi r^2 T}{R} = \frac{4}{3}\pi R^3 \rho g$$

or $$R = \left(\frac{3r^2T}{2\rho g}\right)^{1/4} = \left(\frac{3 \times (5 \times 10^{-4})^2 \times 0.11}{2 \times 10^3 \times 10}\right)^{1/4}$$

$$R = (4.125 \times 10^{-12})^{1/4}$$

$$\log R = \frac{1}{4}\log 4.125 - 3 = \frac{1}{4} \times 0.6154 - 3$$

$$= \bar{3}.1538$$

or $$R = \mathbf{1.425 \times 10^{-3}\ m}$$

$\therefore$ Option (a) is correct.

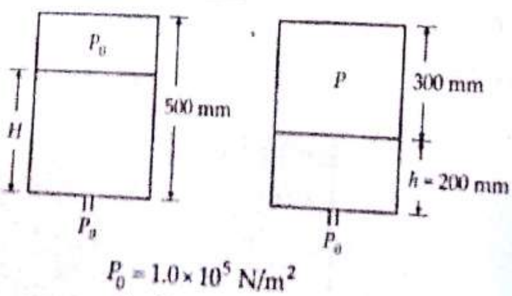
25. (b) Surface energy

$$= 4\pi R^2 T = 4 \times 3.14 \times (1.425 \times 10^{-3})^2 \times 0.11\ \text{J}$$

$$= \mathbf{2.7 \times 10^{-6}\ J}$$

26.

0	0	0	.6



$$P_0 = 1.0 \times 10^5\ \text{N/m}^2$$

$$P = P_0 - \rho g h = 1.0 \times 10^5 - 1000 \times 10 \times 0.2$$

$$= 98000\ \text{N/m}^2$$

Using Boyle's law,

$$P_1V_1 = P_2V_2$$

$$P_0A(500 - H) = P.A \times 300$$

$$10^5(500 - H) = 980000 \times$$

$$500 - H = 294$$

$$H = 500 - 29$$

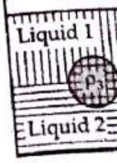
$\therefore$ Fall in height $= 206 - 200 = 6$

27.

0	0	0	6

SECTION

1. A jar is filled with two nor having densities ρ_1 and ρ_2 resp



A solid ball, made of a dropped in the jar. It com position shown in the figure true for ρ_1, ρ_2 and ρ_3?

(a) $\rho_1 < \rho_3 < \rho_2$ (b)

(c) $\rho_1 > \rho_3 > \rho_2$ (d)

2. Spherical balls of radi fluid of viscosity η with viscous force acting on the

(a) directly proportio proportional to v

(b) directly proportio velocity v

(c) inversely proport velocity v

(d) inversely proport tional to velocity

3. If the terminal spe $= 19.5\ \text{kg/m}^3$) is 0.2 m/ $= 1.5\ \text{kg/m}^3$), find the silver (density 10.5 kg same liquid

(a) 0.2 m/s

(c) 0.133 m/s

Scanned by CamScanner

Using Boyle's law,

$$P_1V_1 = P_2V_2$$

$$P_0A(500 - H) = P.A \times 300$$

$$10^5(500 - H) = 980000 \times 300$$

$$500 - H = 294$$

$$H = 500 - 294 = 206 \text{ mm}$$

$\therefore$ Fall in height $= 206 - 200 = \mathbf{6}$ **mm.**

27.

0	0	0	6

$$P_A = P_0 + \frac{4\sigma}{\sigma_A} = 8 + \frac{4 \times 0.04}{2 \times 10^{-2}} = 8 + 8 = 16 \text{ N/m}^2$$

$$P_B = P_0 + \frac{4\sigma}{\sigma_B} = 8 + \frac{4 \times 0.04}{4 \times 10^{-2}} = 8 + 4 = 12 \text{ N/m}^2$$

As $\quad PV = nRT$

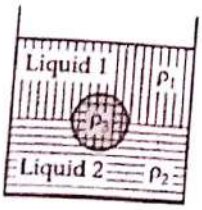
$\therefore \quad n \propto PV$

$$\frac{n_B}{n_A} = \frac{P_BV_B}{P_AV_A} = \frac{P_B}{P_A}\left(\frac{r_B}{r_A}\right)^3$$

$$= \frac{12}{16}\left(\frac{4}{2}\right)^3 = \mathbf{6.}$$

AIEEE

1. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively.



A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3 ?

(*a*) $\rho_1 < \rho_3 < \rho_2$ (*b*) $\rho_3 < \rho_1 < \rho_2$

(*c*) $\rho_1 > \rho_3 > \rho_2$ (*d*) $\rho_1 < \rho_2 < \rho_3$ [AIEEE 08]

2. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is

(*a*) directly proportional to R but inversely proportional to v

(*b*) directly proportional to both radius R and velocity v

(*c*) inversely proportional to both radius R and velocity v

(*d*) inversely proportional to R but directly proportional to velocity v. [AIEEE 04]

3. If the terminal speed of a sphere of gold (density $= 19.5$ kg/m^3) is 0.2 m/s in a viscous liquid (density $= 1.5$ kg/m^3), find the terminal speed of a sphere of silver (density 10.5 kg/m^3) of the same size in the same liquid

(*a*) 0.2 m/s (*b*) 0.4 m/s

(*c*) 0.133 m/s (*d*) 0.1 m/s. [AIEEE 06]

4. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2 (\rho_2 < \rho_1)$. Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{\text{viscous}} = -kv^2 (k > 0)$. The terminal speed of the ball is

(*a*) $\frac{Vg(\rho_1 - \rho_2)}{k}$ (*b*) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

(*c*) $\frac{Vg\rho_1}{k}$ (*d*) $\sqrt{\frac{Vg\rho_1}{k}}$ [AIEEE 08]

5. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is

(*a*) 10 (*b*) 20

(*c*) 25.5 (*d*) 5 [AIEEE 02]

6. If two soap bubbles of different radii are connected by a tube,

(*a*) air flows from the bigger bubble to the smaller bubble till the sizes become equal

(*b*) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

(*c*) air flows from the smaller bubble to the bigger

(*d*) there is no flow of air. [AIEEE 04]

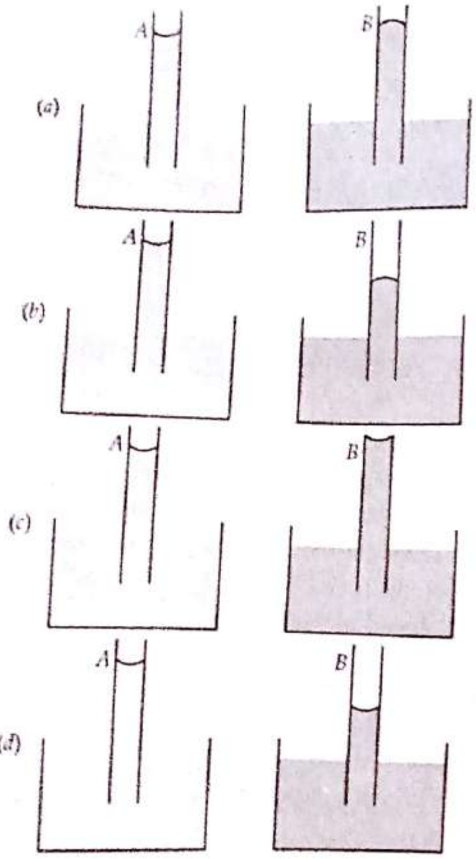
7. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be

(*a*) 4 cm (*b*) 20 cm

(*c*) 8 cm (*d*) 10 cm [AIEEE 05]

Scanned by CamScanner

8. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes ? [AIEEE 08]

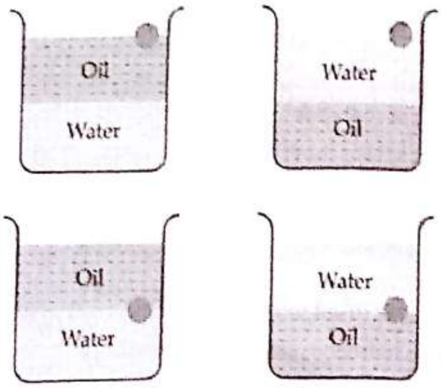


9. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

(*a*) 5.0×10^{-3} m (*b*) 7.5×10^{-3} m

(*c*) 9.6×10^{-3} m (*d*) 3.6×10^{-3} m [AIEEE 2011]

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium positions ?



[AIEEE 2010]

11. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(*a*) $\pi \frac{v^2}{g}$ (*b*) $\pi \frac{v^4}{g}$

(*c*) $\frac{\pi}{2} \frac{v^4}{g^2}$ (*d*) $\pi \frac{v^2}{g^2}$ [AIEEE 2011]

12. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1})

(*a*) 4 πmJ (*b*) 0.2 πmJ

(*c*) 2 πmJ (*d*) 0.4 πmJ [AIEEE 2011]

Answers and Explanations

1. (*a*) As liquid 1 floats over liqud 2, so $\rho_1 < \rho_2$. As solid ball of density ρ_3 sinkes in liquid 1 and floats over liquid 2, so

$$\rho_1 < \rho_3 < \rho_2$$

2. (*b*) $F = 6\pi\eta Rv$,

i.e., retarding viscous force is directly proportional to both R and v.

3. (*d*) $$v = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$$

$$\therefore \frac{V_G}{V_S} = \frac{\rho_G - \sigma}{\rho_S - \sigma} = \frac{19.5 - 1.5}{10.5 - 1.5} = \frac{18}{9}$$

$$V_S = \frac{1}{2} V_G = \frac{1}{2} \times 0.2 = \mathbf{0.1\ ms^{-1}}.$$

4. (*b*) When the ball attains terminal velocity,

Weight of ball = Buoyant force + Viscous force

$$V\rho_1 g = V\rho_2 g + kv^2$$

$$v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}.$$

Scanned by CamScanner

5. (b) Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{2\times 10\times 20} = 20\ \text{ms}^{-1}$$

6. (c) The excess pressure inside a soap bubble is inversely proportional to its radius. So, the pressure is more inside the smaller bubble than the bigger bubble. When these two bubbles are connected by a tube, air will flow from the smaller bubble to the bigger bubble.

7. (b) In the freely falling elevator, the entire arrangement is in a state of weightlessness *i.e.*, $g = 0$. So, water will rise $\left(h = \frac{2\sigma\cos\theta}{r\rho g}\right)$ to fill the entire 20 cm length of the tube.

8. (d) As both pure water and soap solution wet the walls of the tube, so the meniscus in both tubes must be concave upwards,

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

The surface tension σ of soap solution is less than that of pure water, so the height of capillary rise should be less for soap solution and more for pure water. Hence option (d) is correct.

9. (d) $$v^2 - u^2 = 2gh$$

$$v_b^2 - (0.4)^2 = 2\times 9.8\times 2\times 10^{-1}$$

$$\Rightarrow \quad v_b = 2\ \text{ms}^{-1}$$

By Equation of continuity,

$$(a\times v)_{top} = (a\times v)_{bottom}$$

$$\pi\left(\frac{8\times 10^{-3}}{2}\right)^2\times 0.4 = \pi\times\left(\frac{d}{2}\right)^2\times 2$$

$$d = 8\times 10^{-3}\times\sqrt{0.2}$$

$$= 3.6\times 10^{-3}\ \text{m}.$$

10. (c) $\rho > \rho_{oil}$, ball must sink in oil

As $\rho < \rho_{water}$, ball must float in water.

In equilibrium, the ball will stay at the interface of water and oil.

11. (b) Maximum range $= \frac{u^2}{g}$ *i.e.*, $\frac{v^2}{g}$ (= radius of the circle)

$$\text{Area covered} = \pi\left(\frac{v^2}{g}\right)^2 = \pi\frac{v^4}{g^2}$$

12. (d) W = Increase in surface area × S.T.

$$= 2\times 4\pi(r_2^2 - r_1^2)\sigma$$

$$= 8\pi[(5\times 10^{-2})^2 - (3\times 10^{-2})^2]\times 0.03\ \text{J}$$

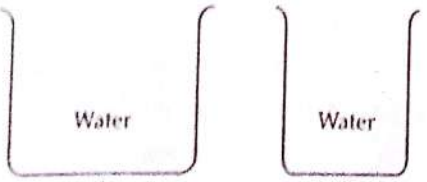
$$= 8\pi\times 16\times 10^{-4}\times 0.03\ \text{J}$$

$$= 384\times 10^{-4}\ \text{J} = 0.384\ \pi\,\text{mJ} \approx \mathbf{0.4\pi\,mJ}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. From the following figures, the correct observation is



A B

(a) the pressure on the bottom of tank A is greater than that at the bottom of B

(b) the pressure on the bottom of the tank A is smaller than that at the bottom of B

(c) the pressure depends on the shape of the container

(d) the pressure on the bottoms of A and B is the same. [IPUEE 06]

2. A wooden block is taken to the bottom of a deep, calm lake of water and then released. It rises up with a

(a) constant acceleration

(b) decreasing acceleration

(c) constant velocity

(d) decreasing velocity. [IPUEE 05]

3. If there were no gravity, which of the following will not be there for a fluid ?

(a) Viscosity

(b) Surface tension

(c) Pressure

(d) Archimedes' upward thrust. [IPUEE 07]

4. A bubble is at the bottom of the lake of depth h. As the bubble comes to sea level, its radius increases three times. If atmospheric pressure is equal to l metre of water column, then h is equal to

(a) $26l$ (b) l

(c) $25l$ (d) $30l$ [DCE 07]

Scanned by CamScanner

5. Radius of one arm of hydraulic lift is four times of radius of other arm. What force should be applied on narrow arm to lift 100 kg ?

(*a*) 26.5 N (*b*) 62.5 N

(*c*) 6.25 N (*d*) 8.3 N [DCE 07]

6. A liquid *X* of density 3.36 g/cm^3 is poured in the right arm of a *U*-tube, which contains Hg. Another liquid *Y* is poured in left arm with height 8 cm, upper levels of *X* and *Y* are same. What is density of *Y* ?

(*a*) 0.8 g/cm^3 (*b*) 1.2 g/cm^3

(*c*) 1.4 g/cm^3 (*d*) 1.6 g/cm^3 [DCE 06]

7. The unit of coefficient of viscosity is

(*a*) Nm/s (*b*) Nm^2/s

(*c*) N/(m^{2}s^{-1}) (*d*) Nms^2 [IPUEE 99]

8. An object is moving through the liquid. The viscous damping force acting on it is proprotional to the velocity. Then dimensions of constant of proportionality are

(*a*) [ML^{-1}T^{-1}] (*b*) [MLT^{-1}]

(*c*) [M^{0}LT^{-1}] (*d*) [ML^{0}T^{-1}] [IPUEE 04]

9. The rate of flow of liquids in a tube of radius *r*, length *l*, whose ends are maintained at a pressure difference *P* is

$$V = \frac{\pi Q p r^4}{\eta l},$$

where η is coefficient of viscosity and *Q* is

(*a*) 8 (*b*) 1/8

(*c*) 16 (*d*) 1/16 [DCE 02]

10. Motion of a liquid in a tube is best described by

(*a*) Bernoulli' theorem

(*b*) Poiseuille's equation

(*c*) Stokes' law

(*d*) Archimedes' principle [IPUEE 99, 01]

11. Which one is not a dimensional number ?

(*a*) Acceleration due to gravity

(*b*) Surface tension of water

(*c*) Velocity of light

(*d*) Reynold's number [DCE 97]

12. Critical velocity of the liquid

(*a*) decreases when radius decreases

(*b*) increases when radius increases

(*c*) decreases when density increases

(*d*) increases when density increases [DCE 05]

COMPETITION

13. A steel ball is dropped in oil, then

(*a*) the ball attains constant velocity after some time

(*b*) the ball stops

(*c*) the speed of ball will keep on increasing

(*d*) none of the above. [DCE 0..]

14. A sphere of mass *m* and radius *r* is falling in the column of a viscous fluid. Terminal velocity attained by falling object is proportional to

(*a*) r^2 (*b*) $1/r$

(*c*) r (*d*) $-1/r^2$ [DCE 2K, 03]

15. The ratio of the terminal velocities of two drops of radii *R* and *R*/2 is

(*a*) 2 (*b*) 1

(*c*) 1/2 (*d*) 4 [IPUEE 97]

16. The radii of two drops are in the ratio of 3 : 2, their terminal velocities are in the ratio

(*a*) 9 : 4 (*b*) 2 : 3

(*c*) 3 : 2 (*d*) 2 : 9 [DCE 98]

17. Bernoulli's equation is an example of conservation of

(*a*) energy (*b*) momentum

(*c*) angular momentum (*d*) mass [DCE 03]

18. An aeroplane gets its upward lift due to a phenomenon described by the

(*a*) Archimedes' principle

(*b*) Bernoulli's principle

(*c*) Buoyancy principle

(*d*) Pascal law. [IPUEE 05]

19. The rate of flow of liquid through an orifice of a tank does not depend upon

(*a*) the size of orifice (*b*) density of liquid

(*c*) the height of fluid column

(*d*) acceleration due to gravity. [DCE 03]

20. The velocity of efflux of a liquid through an orifice in the bottom of the tank does not depend upon

(*a*) size of orifice (*b*) height of liquid

(*c*) acceleration due to gravity

(*d*) none of the above. [DCE 2K]

21. A rectangular vessel when full of water, takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water ?

(*a*) 9 min (*b*) 7 min

(*c*) 5 min (*d*) 3 min [IPUEE 07]

SECTION

22. The SI unit of surf

(*a*) dyne/cm

(*c*) N/m

23. The water droplets

(*a*) gravity

(*c*) surface tenstion

24. One large soap into 27 bubbles having s surface energy is

(*a*) $2\pi TD^2$

(*c*) πTD^2

25. Two drops of bigger drop. What is drop to smaller one ?

(*a*) $2^{1/2}:1$

(*c*) $2^{2/3}:1$

26. 8 mercury dr drop, the energy cha

(*a*) 1

(*c*) 4

27. If a mercury its total energy,

(*a*) remains sam

(*c*) becomes hal

28. There is a s bubble at other en

A

(*a*) smaller wi

(*b*) bigger wil

(*c*) remain in

(*d*) none of th

1. (*d*) P = hρ area of cross- s of A and B.

2. (*a*) The w

Scanned by CamScanner

22. The SI unit of surface tension is

(a) dyne/cm (b) N/m^2

(c) N/m (d) Nm [DCE 2K, 03]

23. The water droplets in free fall are spherical due to

(a) gravity (b) viscosity

(c) surface tenstion (d) intermolecular attraction [DCE 99]

24. One large soap bubble of diameter D breakes into 27 bubbles having surface tension T. The change in surface energy is

(a) $2\pi TD^2$ (b) $4\pi TD^2$

(c) πTD^2 (d) $8\pi TD^2$ [DCE 05]

25. Two drops of equal radius coalesce to form a bigger drop. What is ratio of surface energy of bigger drop to smaller one ?

(a) $2^{1/3}:1$ (b) 1 : 1

(c) $2^{2/3}:1$ (d) none of these. [DCE 06]

26. 8 mercury drops coalesce to form 1 mercury drop, the energy changes by a factor of

(a) 1 (b) 2

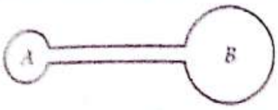
(c) 4 (d) 6 [DCE 2K]

27. If a mercury drop is divided into 8 equal parts, its total energy.

(a) remains same (b) becomes twice

(c) becomes half (d) becomes 4 times [DCE 97]

28. There is a small bubble at one end and bigger bubble at other end of a rod. What will happen ?



(a) smaller will grow until they collapse

(b) bigger will grow until they collapse

(c) remain in equilibrium

(d) none of the above. [DCE 07]

29. If a liquid does not wet glass, its angle of contact is

(a) zero (b) acute

(c) obtuse (d) right angle [IPUEE 06]

30. In a capillary tube experiment, a vertical, 30 cm long capillary tube is dipped in water. The water rises upto a height of 10 cm due to capillary action. If this experiment is conducted in a freely falling elevator, the length of the water column becomes

(a) 10 cm (b) 20 cm

(c) 30 cm (d) zero. [IPUEE 05]

31. Two capillaries of lengths L and $2L$ and of radii R and $2R$ respectively are connected in series. The net rate of flow of fluid through them will (Given, rate of the flow through single capillary, $X = \pi PR^4/8\eta l$) be

(a) $\frac{8}{9}X$ (b) $\frac{9}{8}X$

(c) $\frac{5}{7}X$ (d) $\frac{7}{5}X$ [DCE 05]

32. The rate of flow of water in a capillary tube of length l and radius r is V. The rate of flow in another capillary tube of length $2l$ and radius $2r$ for same pressure difference would be

(a) $16\,V$ (b) $9\,V$

(c) $8\,V$ (d) $2\,V$ [DCE 09]

33. The water flows from a tap of diameter 1.25 cm with a rate of $5\times10^{-5}\ m^3s^{-1}$. The density and coefficient of viscosity of water are $10^3\ kg\ m^{-3}$ and 10^{-3} Pas, respectively. The flow of water is

(a) steady with Reynold's number 5100

(b) tubulent with Reynold's number 5100

(c) steady with Reynold's number 3900

(d) turubulent with Reynold's number 3900 [DCE 09]

Answers and Explanations

1. (d) $P = h\rho g$. As pressure does not depend on the area of cross- section, its value is same on the bottoms of A and B.

2. (a) The wooden block rises up with an acceleration,

$$a = \frac{\text{Upthrust} - \text{Weight of block}}{\text{Mass of block}}$$

3. (d) Archimedes' upward thrust will be absent for a fluid, if there were no gravity.

4. (a) By Boyle's law,

$$P_1V_1 = P_2V_2$$

$$(h+l)\times\frac{4}{3}\pi r^3 = l\times\frac{4}{3}\times\pi(3r)^3$$

$$h + l = 27l \quad \therefore \quad h = 26l$$

Scanned by CamScanner

5. (b) By Pascal's law, $\frac{F}{A} = \frac{f}{a}$

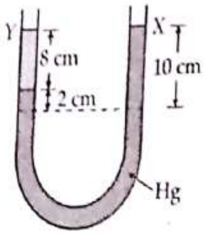
$$f = \frac{Fa}{A} = \frac{100g \times \pi r^2}{\pi(4r)^2} = 6.25g = 6.25 \times 10 = 62.5 \text{ N.}$$

6. (a) As $P_Y = P_X$

$$8 \times \rho_Y \times g + 2 \times \rho_{Hg} \times g = 10 \times \rho_X \times g$$

$$8\rho_Y + 2 \times 13.6 = 10 \times 3.36$$

$$\rho_Y = \frac{33.6 - 27.2}{8} = 0.8 \text{ g/cm}^3.$$



7. (c) $\eta = \frac{F}{A} \cdot \frac{dx}{dv}$

S.I. unit of $\eta = \frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}}{\text{ms}^{-1}} = \text{N/(m}^2\text{s}^{-1})$.

8. (d) Viscous force ∝ Velocity

$$F \propto v \quad \text{or} \quad F = kv$$

$$\therefore \quad [k] = \frac{[F]}{[v]} = \frac{[\text{MLT}^{-2}]}{[\text{LT}^{-1}]} = [\text{ML}^0\text{T}^{-1}].$$

9. (b) By Poiseuille's formula, $V = \frac{\pi p r^4}{8\eta l}$

Given : $V = \frac{\pi Q p r^4}{\eta . l}$. On comparing, $Q = \frac{1}{8}$.

10. (b) Poiseuille's formula gives the volume of a liquid flowing out per second through a horizontal capillary tube of length l, radius r, under a pressure difference p applied across its ends.

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8\eta l}.$$

11. (d) Reynold's number is a dimensionless number,

$$R_e = \frac{\rho v D}{n}$$

12. (c) $v_c = \frac{R_e \eta}{\rho D}$

Critically velocity decreases when density ρ increases or diameter D increases.

13. (a) The ball attains constant velocity after falling through some distance in oil when the weight of ball gets balanced by upthrust and the upward viscous force.

14. (a) $v = \frac{2}{9} r^2 \frac{(\rho - \sigma)g}{\eta}$

$$v \propto r^2.$$

15. (d) $\frac{v_1}{v_2} = \left(\frac{R}{R/2}\right)^2 = 4.$

16. (a) $\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{2}\right)^2 = 9 : 4.$

17. (a) Bernoulli's equation is based on conservation of energy.

18. (b) An aeroplane gets dynamic upward lift in accordance with Bernoulli's principle.

19. (d) The rate of flow of liquid through an orfice depends on size of orfice, atomising surface area, liquid characteristic. It does not depend on acceleration due to gravity.

20. (a) Velocity of efflux, $v = \sqrt{2gh}$

Clearly, it does not depend on the size of the orfice.

21. (b) If A_0 is the area of the orfice at the bottom (at depth H below the free surface) and A that of the vessel, then time t taken in emptying the tank will be

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}} \quad i.e., \quad t \propto \sqrt{H}$$

$$\therefore \quad \frac{t_2}{t_1} = \sqrt{\frac{H/2}{H}} = \frac{1}{\sqrt{2}}$$

$$t_2 = \frac{1}{\sqrt{2}} \times t_1 = 0.7 \times 10 = 7 \text{ min.}$$

22. (c) S.I. unit of surface tension = Nm^{-1}.

23. (c) Freely falling water droplets assume spherical shape due to surface tension of water.

24. (b) Volume of 27 small bubbles

= Volume of larger bubble

$$27 \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$\therefore \quad r = \frac{D}{6}$$

Increase in surface area,

$$= 2\left[27 \times 4\pi\left(\frac{D}{6}\right)^2 - 4\pi\left(\frac{D}{2}\right)^2\right] = 4\pi D^2$$

Increase in surface energy,

= Increase in surface area × surface tension

= $4\pi D^2 T$.

25. (*d*) Volume of bigger drop

= Volume of two smaller drops

$$\frac{4}{3}\pi R'^3 = 2 \times \frac{4}{3}\pi R^3$$

$$R' = 2^{1/3} R$$

∴ Initial surface energy,

$$U_1 = 8\pi R^2 \sigma$$

Final surface energy,

$$U_2 = 4\pi R'^2 \sigma = 4\pi \times 2^{2/3} R^2 \sigma$$

∴ $$\frac{U_2}{U_1} = 2^{-1/3} : 1.$$

26. (*b*) $$8 \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3$$

$$R' = 2R$$

∴ $$U_1 = 8 \times 4\pi R^2 \sigma = 32\pi R^2 \sigma$$

$$U_2 = 4\pi R'^2 \sigma = 4\pi (2R)^2 \sigma = 16\pi R^2$$

$$\frac{U_2}{U_1} = \frac{1}{2}$$

Energy decreases by a factor of 2.

27. (*b*) As obtained in the above problem,

$$\frac{U_1}{U_2} = 2.$$

28. (*b*) $$p = \frac{2\sigma}{R}$$

Excess pressure is more in small bubble than in the bigger bubble. Air will flow from smaller to bigger bubble which will grow more until both collapse.

29. (*b*) The angle of contact is obtuse for a liquid which does not wet glass.

30. (*c*) In the freely falling elevator, $g = 0$.

Water will rise $\left(h = \frac{2\sigma\cos\theta}{r\rho g}\right)$ to fill the entire 30 cm length of the tube.

31. (*a*) Volume of liquid flowing per second,

$$Q = \frac{\pi p r^4}{8\eta l}$$

Fluid resistance $= \frac{8\eta l}{\pi r^4}$

When two capillaries are joined in series, their equivalent fluid resistance is

$$R_{eq} = R_1 + R_2$$

$$= \frac{8\eta L}{\pi R^4} + \frac{8\eta(2L)}{\pi(2R)^4} = \frac{9}{8}\left(\frac{8\eta L}{\pi R^4}\right)$$

Net rate of flow,

$$= \frac{p}{R_{eq}} = \frac{\pi p R^4}{8\eta L} \times \frac{8}{9} = \frac{8}{9}X \qquad \left[\because X = \frac{\pi p R^4}{8\eta L}\right]$$

32. (*c*) Flow rate for first capillary tube,

$$V = \frac{\pi p r^4}{8\eta l}$$

Flow rate for second capillary tube,

$$V' = \frac{\pi p (2r)^4}{8\eta(2l)} = 8 \cdot \frac{\pi p r^4}{8\eta l} = 8V.$$

33. (*b*) Flow rate, $Q = aV = \pi\left(\frac{D}{2}\right)^2 V$

Speed of flow, $v = \frac{4Q}{\pi D^2}$

Reynold's number,

$$R_e = \frac{\rho v D}{\eta} = \frac{4\rho Q}{\pi\eta D} = \frac{4 \times 10^3 \times 5 \times 10^{-5}}{3.14 \times 10^{-3} \times 125 \times 10^{-2}} = 5100$$

For $R_e > 3000$, the flow is turbulent.

AIIMS Entrance Exam

1. The most characteristic property of a liquid is

(*a*) elasticity (*b*) fluidity
(*c*) formlessness (*d*) volume conservation [AIIMS 82]

2. Which of the following physical quantities do not have the same dimensions ?

(*a*) pressure and stress
(*b*) tension and surface tension
(*c*) strain and angle
(*d*) energy and work. [AIIMS 07]

3. A body is floating in a liquid. The upthrust on the body is

(*a*) zero
(*b*) equal to the weight of liquid displaced
(*c*) less than the weight of liquid displaced
(*d*) weight of body–weight of liquid displaced. [AIIMS 07]

4. A body is just floating in a liquid (their densities are equal). If the body is slightly pressed down and released, it will

Scanned by CamScanner

(*a*) start oscillating (*b*) sink to the bottom
(*c*) come back to the same position immediately
(*d*) come back to the same position slowly. [AIIMS 80]

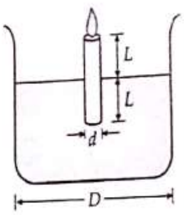
5. When a large bubble rises from the bottom of a lake to the surface, its radius doubles. The atmospheric pressure is equal to that of a column of water of height *H*. The depth of the lake is

(*a*) *H* (*b*) 2 *H*
(*c*) 7 *H* (*d*) 8 *H* [AIIMS 82, 97]

6. By sucking through a straw, a student can reduce the pressure in his lungs to 750 mm of mercury (density = 13.6 g cm^{-3}). Using the straw, he can drink water from a glass upto a maximum depth of

(*a*) 10 cm (*b*) 75 cm
(*c*) 13.6 cm (*d*) 1.36 cm [AIIMS 06]

7. A candle of diameter *d* is floating on a liquid in a cylindrical container of diameter $D (D >> d)$ as shown in figure. If it is burning at the rate of 2 cm h^{-1}, then the top of the candle will



(*a*) remain at the same height
(*b*) fall at the rate of 1 cm h^{-1}
(*c*) fall at the rate of 2 cm h^{-1}
(*d*) go up at the rate of 1 cm h^{-1}. [AIIMS 05]

8. A small ball of density ρ is dropped from a height *h* into a liquid of density $\sigma (\sigma > \rho)$. Neglecting damping forces, the maximum depth to which the body sinks is

(*a*) $\frac{h\sigma}{\rho - \sigma}$ (*b*) $\frac{h\rho}{\rho - \sigma}$
(*c*) $\frac{h(\sigma - \rho)}{\rho}$ (*d*) $\frac{h(\sigma - \rho)}{\sigma}$ [AIIMS 2K]

9. A vertical *U*-tube contains mercury in both its arms. A glycerine (density 1.3 g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8 g cm^{-3} is poured into the other arm until the upper surfaces of oil and glycerine are at the same level. The length of the oil column is (density of mercury = 1.3 g cm^{-3}).

(*a*) 8.5 cm (*b*) 9.6 cm
(*c*) 10.7 cm (*d*) 11.8 cm [AIIMS 2K]

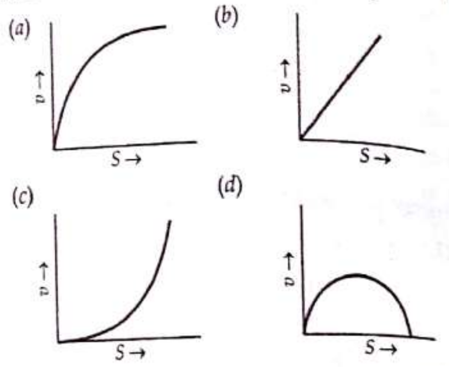
10. Under a constant pressure head, the rate of flow of orderly volume flow of liquid through a capillary tube is *V*. If the length of the capillary is doubled and the diameter of the bore is halved, the rate of flow would become

(*a*) $V/4$ (*b*) $16V$
(*c*) $V/8$ (*d*) $V/32$ [AIIMS]

11. A sphere of mass *M* and radius *R* is falling in a viscous fluid. The terminal velocity attained by the falling object will be proportional to

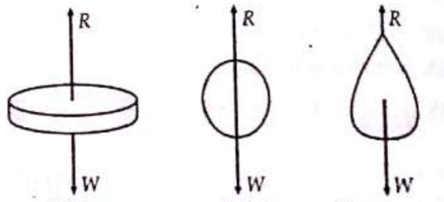
(*a*) R^2 (*b*) R
(*c*) $1/R$ (*d*) $1/R^2$ [AIIMS]

12. A lead shot of 1 mm diameter falls through a long column of glycerine. The variation of its velocity *v* with distance covered (*S*) is represented by



[AIIMS 03]

13. When a body falls in air, the resistance of air depends on a greater extent on the shape of the body. Three different shapes are given



1. Disc 2. Ball Cigar shaped

Identify the combination of air resistances, which truly represents the physical situation (The cross-sectional areas are the same).

(*a*) 1 < 2 < 3 (*b*) 2 < 3 < 1
(*c*) 3 < 2 < 1 (*d*) 3 < 1 < 2 [AIIMS 06 ; IPUEE 06]

14. Bernoulli's equation is a consequence of conservation of :

(*a*) energy (*b*) linear momentum
(*c*) angular momentum (*d*) mass. [AIIMS 03]

15. Scent sprayer is based on
(*a*) Charles' law (*b*) Boyle's law
(*c*) Archimedes' principle
(*d*) Bernoulli's theorem. [AIIMS 02]

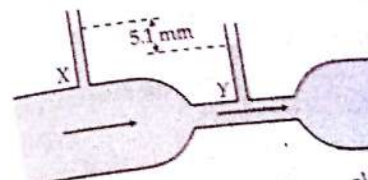
16. Bernoulli's principle is based on the law of conservation of
(*a*) energy (*b*) mass
(*c*) linear momentum (*d*) angular momentum. [AIIMS 01]

17. In old age arteries carrying blood in the human body become narrow resulting in an increase in the blood pressure. This follows from
(*a*) Pascal's law (*b*) Stoke's law
(*c*) Bernoulli's principle
(*d*) Archimedes' principle [AIIMS 0

18. In incompressible fluid flows steadily through cylindrical pipe which has radius 2 R at point *A* and at a point *B* further along the flow direction. If velocity at *A* is *v*, then that at B is
(*a*) $v/2$ (*b*) v
(*c*) $2v$ (*d*) $4v$ [AII

19. Dynamic lift is related to
(*a*) Bernoulli's theorem (*b*) Archimedes' pr
(*c*) Equation of continuity
(*d*) Pascal's law

20. Figure shows a venturimeter, throu water is flowing. The speed of water at X i The speed of water at Y (taking $g = 1{,}000$ c



(*a*) 23 cm s^{-1} (*b*) 32 cm s^{-1}
(*c*) 101 cm s^{-1} (*d*) 1,024 cm

21. The property utilised in the ma shots is
(*a*) specific weight of liquid lead
(*b*) specific gravity of liquid lea
(*c*) compressibility of liquid le
(*d*) surface of tension of liquid

22. The rain drops are in sph
(*a*) viscosity (*b*) sur
(*c*) thrust on drop (*d*) re

23. increa 10 cm

Scanned by CamScanner

15. Scent sprayer is based on
(*a*) Charles' law (*b*) Boyle's law
(*c*) Archimedes' principle
(*d*) Bernouli's theorem. [AIIMS 02]

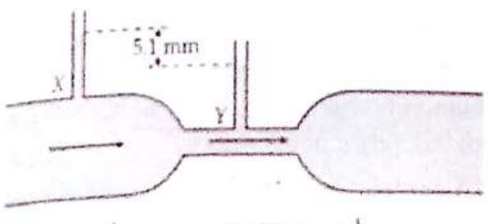
16. Bernouli's principle is based on the law of conservation of
(*a*) energy (*b*) mass
(*c*) linear momentum (*d*) angular momentum. [AIIMS 01]

17. In old age arteries carrying blood in the human body become narrow resulting in an increase in the blood pressure. This follows from
(*a*) Pascal's law (*b*) Stoke's law
(*c*) Bernoulli's principle
(*d*) Archimedes' principle [AIIMS 07]

18. In incompressible fluid flows steadily through a cylindrical pipe which has radius $2R$ at point A and R at a point B further along the flow direction. If the velocity at A is v, then that at B is
(*a*) $v/2$ (*b*) v
(*c*) $2v$ (*d*) $4v$ [AIIMS 2K]

19. Dynamic lift is related to
(*a*) Bernoulli's theorem (*b*) Archimedes' principle
(*c*) Equation of continuity
(*d*) Pascal's law [AIIMS 2K]

20. Figure shows a venturimeter, through which water is flowing. The speed of water at X is 2 cm s^{-1}. The speed of water at Y (taking $g = 1{,}000$ cm s^{-2}) is



(*a*) 23 cm s^{-1} (*b*) 32 cm s^{-1}
(*c*) 101 cm s^{-1} (*d*) 1,024 cm s^{-1} [AIIMS 87]

21. The property utilised in the manufacture of lead shots is
(*a*) specific weight of liquid lead
(*b*) specific gravity of liquid lead
(*c*) compressibility of liquid lead
(*d*) surface of tension of liquid lead [AIIMS 02]

22. The rain drops are in spherical shape due to
(*a*) viscosity (*b*) surface tension
(*c*) thrust on drop (*d*) residual pressure. [AIIMS 98]

23. Work of 3.0×10^{-4} J is required to be done in increasing the size of a soap film from 10 cm × 6 cm to 10 cm × 11 cm. The surface tension of the soap film is
(*a*) 5×10^{-2} Nm^{-1} (*b*) 3×10^{-2} Nm^{-1}
(*c*) 1.5×10^{-2} Nm^{-1} (*d*) 12×10^{-2} Nm^{-1} [AIIMS 2K, 07]

24. Two small drops of mercury, each of radius R coalesce to form a single large drop. The ratio of the total surface energies before and after the change is
(*a*) $1 : 2^{1/3}$ (*b*) $2^{1/3} : 1$
(*c*) $2 : 1$ (*d*) $1 : 2$ [DCE 03 ; AIIMS 03]

25. The potential energy possessed by a soap bubble, having surface tension equal to 0.04 Nm^{-1} of diameter 1 cm, is
(*a*) $2\pi \times 10^{-6}$ J (*b*) $4\pi \times 10^{-6}$ J
(*c*) $6\pi \times 10^{-6}$ J (*d*) $8\pi \times 10^{-6}$ J [AIIMS 94]

26. The radius of a soap bubble is r and the surface tension of soap solution is T. Keeping the temperature constant, the extra energy needed to double the radius of the soap bubble by blowing is
(*a*) $32\pi r^2 T$ (*b*) $24\pi r^2 T$
(*c*) $16\pi r^2 T$ (*d*) $8\pi r^2 T$ [AIIMS 94]

27. When the temperature is increased, the angle of contact of a liquid
(*a*) increases (*b*) decreases
(*c*) remains the same
(*d*) fisrt increases and then decreases. [AIIMS 80]

28. Extra pressure inside a soap bubble of radius(r) is proportional to
(*a*) r (*b*) $1/r$
(*c*) r^2 (*d*) $1/r^2$ [AIIMS 96]

29. The surface tension of soap solution is 25×10^{-3} Nm^{-1}. The excess pressure inside a soap bubble of diameter 1 cm is
(*a*) 10 Pa (*b*) 20 Pa
(*c*) 5 Pa (*d*) none of these. [AIIMS 87]

30. A spherical drop of water has 1 mm radius. If the surface tension of water is 70×10^{-3} Nm^{-1}, then difference of pressure between inside and outside of the spherical drop is
(*a*) 35 Nm^{-2} (*b*) 70 Nm^{-2}
(*c*) 140 Nm^{-2} (*d*) zero [AIIMS 01]

31. At critical temperature, the surface tension of a liquid
(*a*) is zero (*b*) is infinity
(*c*) is same as that any other temperature
(*d*) cannot be determined [AIIMS 80]

Scanned by CamScanner

32. The surface tension of liquid decreases with a rise in

(*a*) temperature of the liquid

(*b*) viscosity of the liquid

(*c*) diameter of container

(*d*) thickness of container. [AIIMS 94]

33. Two spherical soap bubbles of radii a and b in vacuum coalesce under isothermal conditions. The resulting bubble has a radius given by

(*a*) $\frac{(a+b)}{2}$ (*b*) $\frac{ab}{a+b}$

(*c*) $\sqrt{a^2+b^2}$ (*d*) $a+b$ [AIIMS 2010]

34. Neglecting the density of air, the terminal velocity obtained by a raindrop of radius 0.3 mm falling through air of viscosity 1.8×10^{-5} N s m^{-2} will be

(*a*) 10.9 m s^{-1} (*b*) 7.48 m s^{-1}

(*c*) 3.7 m s^{-1} (*d*) 12.8 m s^{-1} [AIIMS 2009]

35. A liquid is kept in a cylindrical vessel which is being rotated about a vertical axis through the centre of the circular base. If the radius of the vessel is r and angular velocity of rotation is ω, then the difference in the heights of the liquid at the centre of the vessel and the edge is

(*a*) $\frac{r\omega}{2g}$ (*b*) $\frac{r^2\omega^2}{2g}$

(*c*) $\sqrt{2gr\omega}$ (*d*) $\frac{\omega^2}{2gr^2}$ [AIIMS 2010]

36. A capillary tube of radius r is immersed in water and water rises in it to a height h. The mass of water in the capillary tube is 5 g. Another capillary tube of radius $2r$ is immersed in water. The mass of water that will rise in this tube is

(*a*) 2.5 g (*b*) 5.0 g

(*c*) 10 g (*d*) 20 g [AIIMS 2010]

ASSERTIONS AND REASONS

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(*a*) If both assertion and reason are true and reason is the correct explanation of the assertion.

(*b*) If both assertion and reason are true but reason is not correct explanation of the assertion.

(*c*) If assertion is true, but reason is false.

(*d*) If both assertion and reason are false.

37. **Assertion.** The size of a hydrogen balloon increases as it rises in air.

Reason. The material of the balloon can be easily stretched. [AIIMS 07]

38. **Assertion.** A hydrogen filled balloon stops rising after it has attained a certain height in the sky.

Reason. The atmospheric pressure decreases with height and becomes zero when maximum height is attained. [AIIMS 99]

39. **Assertion.** In taking into account the fact that any object, which floats must have an average density less than that of water, during World War I, a number of cargo vessels were made of concrete.

Reason. Concrete cargo vessels were filled with air. [AIIMS 07]

40. **Assertion.** The machine parts are jammed in winter.

Reason. The viscosity of the lubricants used in the machines increases at low temperature. [AIIMS 95]

41. **Assertion.** For Reynold's number $R_e > 2000$, the flow of fluid is turbulent.

Reason. Inertial forces are dominant compared to the viscoucs forces at such high Reynold's numbers. [AIIMS 05]

42. **Assertion.** The shape of an automobile is so designed that its front resembles the streamline pattern of the fluid through which it moves.

Reason. The resistance offered by the fluid is maximum. [AIIMS 94]

43. **Assertion.** A thin stainless steel needle can lay floating on a still water surface.

Reason. Any object floats, when the buoyancy force balances the weight of the object. [AIIMS 06]

44. **Assertion.** A needle placed carefully on the surface of water may float, whereas a ball of the same material will always sink.

Reason. The buoyancy of an object depends both on the material and shape of the object. [AIIMS 95]

45. **Assertion.** Smaller drops of liquid resist deforming forces better than the larger drops.

Reason. Excess pressure inside a drop is directly proportional to its surface area. [AIIMS 04]

46. **Assertion.** Bubble of soap is larger than that of water.

Reason. Surface tension of soap bubble is less than that of water. [AIIMS 97]

Scanned by CamScanner

Answers and Explanations

1. (a) Volume conservation or incompressibility is an important property of a liquid.

2. (b) $[\text{Tension}] = [\text{Force}] = [MLT^{-2}]$

$$[\text{Surface tension}] = \frac{[\text{Force}]}{[\text{Length}]} = [MT^{-2}].$$

3. (b) Upthrust on the body is always equal to the weight of liquid displaced.

4. (b) The body will sink to the bottom due to the applied force.

5. (c) According to Boyle's law,

$$P_1V_1 = P_2V_2$$

$$H \times \frac{4}{3}\pi(2r)^3 = P_2 \times \frac{4}{3}\pi r^3$$

$$8H = P_2$$

Depth of lake $= 8H - H = 7H$.

6. (c) Pressure difference between lungs and atmosphere

$$= 760 \text{ mm} - 750 \text{ mm}$$
$$= 10 \text{ mm of Hg}$$
$$= 1 \text{ cm of Hg}.$$

Suppose the student can suck water from depth h. Then

$$P = hdg = 1 \text{ cm of Hg}$$

$$h \times 1 \times g = 1 \times 13.6 \times g$$

$$h = \mathbf{13.6\ cm}.$$

7. (b) Initial weight of candle

= weight of liquid of displaced

$v_C \rho_C g$ = mass of liquid displaced × g

$$\pi\left(\frac{d}{2}\right)^2 2L\rho_C g = \pi\left(\frac{d}{2}\right)^2 L\rho_L g$$

$$2\rho_C = \rho_L$$

When 2 cm of candle burns out, its length becomes $(2L-2)$ cm. Suppose its length decreases by x outside the liquid. Then

$$\pi\left(\frac{d}{2}\right)^2 (2L-2)\rho_C g = \pi\left(\frac{d}{2}\right)^2 (L-x)\rho_L g$$

$$(2L-2)\rho_C = (L-x)2\rho_C$$

$$\therefore \quad x = 1 \text{ cm}$$

Hence the top of the candle comes down at half the rate of burning.

8. (b) Velocity of ball at the surface $= \sqrt{2gh}$

Inside the liquid,

$$g_{\text{apparent}} = \frac{V\rho g - V\sigma g}{V\rho} = \left(\frac{\rho - \sigma}{\rho}\right)g$$

Now $v^2 - u^2 = -2g_{app} \cdot h'$

$$0 - 2gh = -2\left(\frac{\rho - \sigma}{\rho}\right)gh'$$

or $$h' = \frac{h\rho}{\rho - \sigma}.$$

9. (b) Refer to the solution of Problem 6 on page 10.13.

10. (d) According to Poiseuille's formula

Rate of flow,

$$V = \frac{\pi p r^4}{8\eta l}$$

$$V' = \frac{\pi p (r/2)^4}{8\eta \times 2l} = \frac{1}{32}\frac{\pi p r^4}{8\eta l} = \frac{V}{32}.$$

11. (a) $V = \frac{2}{9}\frac{R^2}{\eta}(\rho - \sigma)g$

$$\therefore \quad V \propto R^2.$$

12. (a) Initially, the velocity increases due to the action of gravity. But the graph is not linear due to the upthrust exerted by glycerine. Then due to viscosity of glyserine, the lead shot attains a constant terminal velocity.

13. (c) $3 < 2 < 1$. The steamlined shaped body experiences less air resistance.

14. (a) Bernoulli's equation is based on conservation of energy.

15. (c) Scent sprayer is based on Bernoulli's principle.

16. (a) Bernoull's principle is based on the law of conservation of energy.

17. (c) Blood pressure in narrow arteries increases in accordance with Bernoulli's principle.

18. (d) Using equation of continuity,

$$a_1v_1 = a_2v_2$$

$$\pi(2R)^2 v = \pi R^2 v_2$$

$$v_2 = 4v.$$

19. (a) Dynamic lift is in accordance with Bernoulli's equation.

Scanned by CamScanner

20. (*b*) Applying Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$h\rho g = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$v_2^2 - v_1^2 = 2hg = 2 \times 0.51 \times 1000 = 1020.$$

$$v_2^2 = 1020 + v_1^2 = 1020 + 4 = 1024.$$

$$\therefore \quad v_2 = \sqrt{1024} = \mathbf{32\ cm\ s^{-1}}.$$

21. (*d*) Due to surface tension, spherical lead shots are formed.

22. (*b*) Rain drops are spherical due to surface tension.

23. (*b*) Increase in surface area of soap film

$$= 2(10 \times 11 - 6 \times 10)\ \text{cm}^2$$

$$= 100\ \text{cm}^2 = 10^{-2}\ \text{m}^2$$

$$\text{Surface tension} = \frac{\text{Work done}}{\text{Increase in area}}$$

$$= \frac{3 \times 10^{-4}}{10^{-2}} = \mathbf{3 \times 10^{-2}\ Nm^{-1}}.$$

24. (*b*) Volume of large drop

= Volume of two small drops

$$\frac{4}{3}\pi R'^3 = 2 \times \frac{4}{3}\pi R^3$$

$$\therefore \quad R' = 2^{1/3} R$$

Initial surface energy,

$$U_1 = 8\pi R^2 \sigma$$

Final surface energy,

$$U_2 = 4\pi R'^2 \sigma = 4\pi \times 2^{2/3} R^2 \sigma$$

$$\therefore \quad \frac{U_1}{U_2} = \frac{2}{2^{2/3}} = \mathbf{2^{1/3}}.$$

25. (*d*) $U = 2 \times 4\pi R^2 \sigma$

$$= 8\pi(0.5 \times 10^{-2})^2 \times 0.04\ \text{J}$$

$$= \mathbf{8\pi \times 10^{-6}\ J}.$$

26. (*b*) Required energy = Increase in surface area × surface tension

$$= 2 \times 4\pi[(2r)^2 - r^2] \times T$$

$$= \mathbf{24\ \pi r^2 T}.$$

27. (*a*) The angle of contact of a liquid increases with the increase of temperature.

28. (*b*) Excess pressure inside a soap bubble,

$$P = \frac{4\sigma}{r}$$

i.e., $$P \propto \frac{1}{r}$$

29. (*b*) $$P = \frac{4 \times 25 \times 10^{-3}}{0.5 \times 10^{-2}} = \mathbf{20\ Pa}.$$

30. (*c*) Excess pressure inside a water drop,

$$P = \frac{2\sigma}{r}$$

$$= \frac{2 \times 70 \times 10^{-3}}{1 \times 10^{-3}} = \mathbf{140\ Nm^{-2}}.$$

31. (*a*) At critical temperature, the surface tension of a liquid becomes zero.

32. (*a*) The surface tension of a liquid decreases with the increase of temperature.

33. (*c*) Surface energy of smaller bubbles

= Surface energy of resulting bubble

$$8\pi a^2\sigma + 8\pi b^2\sigma = 8\pi r^2\sigma$$

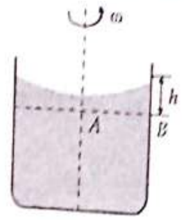
$$\Rightarrow \quad r^2 = a^2 + b^2$$

$$\Rightarrow \quad r = \sqrt{a^2 + b^2}$$

34. (*a*) $$v_t = \frac{2r^2\rho g}{9\eta}$$

$$= \frac{2 \times (0.3 \times 10^{-3})^2 \times 10^3 \times 9.8}{9 \times 1.8 \times 10^{-5}} = \mathbf{10.9\ ms^{-1}}$$

35. (*b*) When the liquid rotates, the velocity at the sides is higher, so the pressure is lower. The liquid



rises at the sides to height *h* to compensate for this drop in pressure. By Bernoulli's theorem,

$$p_A + \frac{1}{2}\rho v_A^2 = p_B + \frac{1}{2}\rho v_B^2$$

$$p_A - p_B = \frac{1}{2}\rho(v_B^2 - v_A^2)$$

$$h\rho g = \frac{1}{2}\rho(r^2\omega^2 - 0^2)$$

$$\therefore \quad h = \frac{r^2\omega^2}{2g}$$

36. (c) $m = \pi r^2 h\rho$

$$= \pi r^2 \left(\frac{2\sigma\cos\theta}{r\rho g}\right)\rho = \frac{2\pi r\sigma\cos\theta}{g}$$

$\Rightarrow$ $m \propto r$

$\therefore$ $\frac{m'}{m} = \frac{2r}{r} = 2$

or $m' = 2m = 2 \times 5\,g = 10\,g$

37. (b) Both the assertion and reason are true but the reason is not a correct explanation of the assertion.

38. (c) The assertion is true but the reason is false. When the atmospheric pressure becomes equal to the pressure inside the balloon, the balloon stops rising.

39. (a) Both the assertion and reason are true.

40. (a) Both the assertion and reason are true.

41. (a) Both the assertion and reason are true.

42. (d) Both the assertion and reason are false.

43. (c) The assertion is true but the reason is false. The needle floats when the upward tension on the needle balances its weight.

44. (c) The assertion is true but the reason is false. The needle floats because the force of surface tension on it is enough to balance its weight. In case of ball, the resultant of the force of surface tension and buyancy is enough to balance the weight of the ball.

45. (c) The assertion is true but the reason is false. The excess pressure inside a liquid drop,

$$p = \frac{2\sigma}{R}$$

i.e., $$p \propto \frac{1}{R}$$

The excess pressure is large in a small drop due to which it can resist the deforming forces.

46. (a) Both the assertion and reason are true. When soap is added, surface tension of water decreases.

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. A hydraulic lift is designed to lift cars of maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 4.25×10^{-2} m^2. What maximum pressure would the smaller piston have to bear ?

(a) 6.92×10^5 N/m^2
(b) 9.63×10^9 N/m^2
(c) 7.82×10^7 N/m^2
(d) 13.76×10^{11} N/m^2 [DPMT 92]

2. When equal volumes of two metals are melted together, the specific gravity of alloy is 4. When equal masses of the same metals are melted together the specific gravity of the alloy is 3. Calculate the specific gravity of each metal.

(a) 2, 6 (b) 3, 5
(c) 4, 2 (d) 3, 4 [DPMT 06]

3. A body of mass 15 kg is dropped into the water. If the apparent weight of the body is 107 N, then the applied thrust will be

(a) 40 N (b) 80 N
(c) 60 N (d) 100 N [DPMT 99]

4. A square wire frame of size L is dipped in a liquid. On taking out, a membrance is formed. If the surface tension of liquid is T, force acting on the frame will be

(a) $2TL$ (b) $4TL$
(c) $8TL$ (d) $10TL$ [DPMT 04]

5. Calculate the force required to separate the glass plate of area 10^{-2} m^2 with a film of water 0.05 mm thick (surface tension of water is 70×10^{-3} N/m).

(a) 25 N (b) 20 N
(c) 14 N (d) 28 N [VMMC 05]

6. A vertical tank with depth H is full with water. A hole is made on one side of the walls at a depth h below the water surface. At what distance from the foot of the wall does the emerging stream of water strike the foot ?

(a) $\sqrt{h(H-h)}$
(b) $\sqrt{h/(H-h)}$
(c) $2(H-h)\sqrt{h/(H-h)}$
(d) $\sqrt{2h/(H-h)}$ [DPMT 2011]

Scanned by CamScanner

Answers and Explanations

1. (*a*) $p = \frac{f}{a} = \frac{F}{A} = \frac{3000 \times 9.8}{4.25 \times 10^{-12}}$

$= 6.92 \times 10^{5}\ Nm^{-2}$.

2. (*a*) When equal volumes are mixed,

$$\rho = \frac{V\rho_1 + V\rho_2}{V + V}$$

$$= \frac{\rho_1 + \rho_2}{2} = 4$$

When equal masses are mixed,

$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = 3$$

On solving, $\rho_1 = 2$, $\rho_2 = 6$.

3. (*a*) Apparent weight = Weight − Upthrust

∴ Upthrust = Weight − apparent weight

$= 15 \times 9.8 - 107 = 147 - 107 = 40$ **N**.

4. (*c*) The membrane has two free surfaces.

Total force acting on the frame

= Surface tension × perimeter × 2

$= T \times 4L \times 2 = 8TL$.

5. (*d*) Force required to separate the plates,

$$F = \frac{2TA}{x} = \frac{2 \times 70 \times 10^{-3} \times 10^{-2}}{0.05 \times 10^{-3}}\ N$$

= 28 N.

6. (*c*) Refer to the solution of problem 18(*i*) on page 10.77.

Scanned by CamScanner

CHAPTER

11

THERMAL PROPERTIES OF MATTER

11.1 HEAT

1. *Explain the concept of heat.*

Heat. *Heat is a form of energy which produces in us the sensation of hotness or coldness.* For example, if we touch a piece of ice, heat flows from our body towards ice and we feel cold. Similarly, when we stand near a fire, heat from the fire flows towards our body and we feel hot.

(*a*) ***Caloric theory of heat.*** According to this theory, *heat is an invisible, weightless and odourless fluid called* **caloric**. When some caloric is added to a body, its temperature rises and when some caloric is removed from a body, its temperature falls. However, this theory failed to explain the production of heat by friction. So this was replaced by dynamic theory of heat.

(*b*) ***Dynamic theory of heat.*** According to this theory, all substances (solids, liquids and gases) are made of molecules. These molecules are in a state of continuous random motion.

Depending on temperature and nature of the substance, the molecules may possess *three* types of motion :

(*i*) *Translatory motion.* That is, the motion in a straight line which is common in gases.

(*ii*) *Vibratory motion.* That is, the to and fro motion of the molecules about their mean positions. This is common in liquids and gases.

(*iii*) *Rotatory motion.* That is, the rotation of the molecules about their axis. This occurs usually at high temperature.

When a body is heated, all these molecular motions become fast. The kinetic energy of a molecule due to each type of motion increases. So we can regard *heat as an energy of molecular motion which is equal to the sum total of the kinetic energy possessed by the molecules of a body by virtue of their translational, vibrational and rotational motions.*

2. *What is meant by the statement that heat is the energy in transit ?*

Heat is the energy in transit. The energy associated with the configuration and random motion of the molecules in a body is called internal energy. The part of this internal energy that is transferred from one body to another due to temperature difference between them is called heat. Clearly, the word 'heat' is meaningful only as long as the energy is being transferred. The expressions like 'heat in a body' or 'heat of a body' are meaningless. So we define *heat as the energy in transit that flows from one body to another due to temperature difference between them.* Once heat is transferred to a body, it becomes a part of its internal energy.

11.2 UNITS OF HEAT

3. *What are the CGS and SI units of heat ? How are they related to one another ?*

CGS unit of heat. The CGS unit of heat is ***calorie*** (cal). *One calorie is defined as the heat energy required to*

Scanned by CamScanner

raise the temperature of one gram of water through 1°C (from 14.5 to 15.5°C).

SI unit of heat. Like all other forms of energy, the SI unit of heat is joule (J).

1 calorie = 4.186 joule

11.3 JOULE'S MECHANICAL EQUIVALENT OF HEAT

4. *State Joule's law of equivalence between work and heat. Hence define mechanical equivalent of heat.*

Joule's mechanical equivalent of heat. From experiments, Joule established a relation between the work done and heat produced. He showed that whenever a given amount of work (W) is converted into heat, always the same amount of heat (Q) is produced, thus

$$W \propto Q \quad \text{or} \quad W = JQ$$

or

$$J = \frac{W}{Q}$$

If $\quad Q = 1$, then $J = W$

The proportionality constant J is called *Joule's mechanical equivalent of heat. It may be defined as the amount of work that must be done to produce a unit quantity of heat.*

$$J = 4.186 \text{ J cal}^{-1} = 4.186 \times 10^7 \text{ erg cal}^{-1}.$$

Note *J* is not a physical quantity. It just a conversion factor.

11.4 TEMPERATURE

5. *Explain the concept of temperature.*

Temperature. *Temperature is the degree of hotness or coldness of a body.* When two bodies are placed in contact, the heat flows from the body at higher temperature to the body at lower temperature. Thus *temperature may be defined as the thermal state of a body which decides the direction of flow of heat energy from one body to another when they are placed in thermal contact with each other.*

Kinetic interpretation of temperature. *The temperature of a body is the measure of the average kinetic energy of its molecules.* When a body is heated, its molecules move faster. Their average K.E. increases. This increases the temperature of the body.

In thermodynamics the concept of temperature follows from the zeroth law of thermodynamics. It shall be discussed in the next chapter.

11.5 HEAT VS. TEMPERATURE

6. *Give some points of differences between heat and temperature.*

	Heat	Temperature
1.	Heat is a form of energy which produces in us the sensation of hotness or coldness.	Temperature is the degree of hotness or coldness of a body.
2.	It is a cause, when some heat is supplied to a body, its temperature increases.	It is an effect.
3.	It represents the total kinetic energy of the molecules of a body.	It represents the average kinetic energy possessed by the molecules of a body.
4.	Heat flows from high temperature side to low temperature side irrespective of the amounts of heat possessed by the bodies in contact.	Temperature decides the direction of flow of heat from one body to another.
5.	It is measured in cal, kcal or joule.	It is measured in °C, °F or K.

11.6 THERMOMETRY

7. *Define the term thermometry.*

Thermometry. *The branch of physics that deals with the measurement of temperature is called thermometry.*

8. *What is a thermometer? What is its principle?*

Thermometer. *Any device used to measure the temperature of a body is called a thermometer.* It is named so because *thermo* is a Latin word which means *heat* and *meter* means a measuring device.

Principle of a thermometer. *A thermometer makes use of some measurable property (called thermometric property) of a substance which changes linearly with temperature.*

The thermometric properties of different substances and the corresponding thermometers are as follows:

(*i*) Length of a liquid column in a capillary (Mercury thermometer).

(*ii*) Pressure of a gas at constant volume (Constant volume gas thermometer).

(*iii*) Volume of a gas at constant pressure (Constant pressure gas thermometer).

(*iv*) Electrical resistance of a metal wire (Platinum resistance thermometer).

(*v*) Thermoelectrical e.m.f. (Thermoelectric or thermocouple thermometer).

(*vi*) Radiated power (Pyrometers).

9. *What are two fixed points on a temperature scale?*

Fixed points on a temperature scale. To construct a temperature scale, two fixed points (two well-defined thermodynamic states) are chosen and are assigned two arbitrary numbers for their temperature. One number fixes the origin of the scale and the other fixes the size of the unit of the scale. The temperature at which pure ice melts at standard pressure (ice-liquid water equilibrium state) is usually chosen as the *lower fixed point*. The temperature at which pure water boils at atmospheric pressure (liquid water-vapour equilibrium state) is chosen as the *upper fixed point*.

10. *Describe the different types of temperature scales commonly used. Write the relation between temperatures on different scales.*

Thermometric scales. The range between the two fixed temperatures is called *fundamental interval*, which when divided into a suitable number of equal divisions forms a thermometric scale. The commonly used temperature scales are as follows :

(*i*) ***The Celsius scale.*** On this scale, the lower fixed point (ice point) is taken as 0°C and the upper fixed point (steam point) as 100°C. The interval between the two fixed points is divided into hundred equal parts (hence the name centigrade) and each part is called 1°C.

(*ii*) ***The Fahrenheit scale.*** On this scale, the lower fixed point is taken as 32°F and the upper fixed point as 212°F. The interval between them is divided into 180 equal parts and each part represents 1°F.

(*iii*) ***The Reaumer scale.*** On this scale, the lower fixed point is taken as 0°R and the upper fixed point as 80°R. The interval between them is divided into 80 equal parts and each part represents 1°R.

(*iv*) ***The Kelvin scale.*** On this scale, the lower fixed point is taken as 273.15 K and the upper fixed point as 373.15 K. The interval between the two fixed points is divided into 100 equal parts. The SI unit of temperature is kelvin (K).

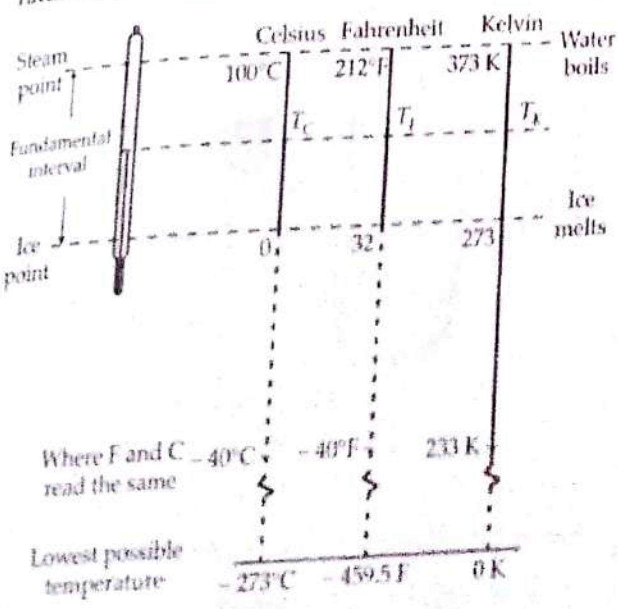


Fig. 11.1 Different temperature scales.

As the fundamental intervals of both Celsius and Kelvin scales have 100 divisions each, so the size of one division on Celsius scale is equal to the size of one division on Kelvin scale.

Conversion of temperature from one scale to another. To convert the temperature from one scale to another, the following relation is used :

$$\frac{\text{Temperature on one scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}} = \frac{\text{Temperature on other scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}}$$

As shown in the above figure, if the temperature of body is measured as T_C, T_F, T_R and T_K on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T_K - 273.15}{373.15 - 273.15}$$

or

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80} = \frac{T_K - 273.15}{100}.$$

11.7 ABSOLUTE SCALE OF TEMPERATURE

11. *What do you mean by absolute zero and absolute scale of temperature ?*

Absolute zero and absolute scale of temperature. According to Charles' law, if V_t and V_0 are the volumes at t°C and 0°C respectively of a given mass of a gas at constant pressure P, then

$$V_t = V_0\left(1 + \frac{t}{273.15}\right)$$

Clearly, the volume of the gas below 0°C will be less than V_0. For example, volume of the gas at $-t$°C is

$$V_{-t} = V_0\left(1 - \frac{t}{273.15}\right)$$

The decrease of temperature results in the decrease in volume of the gas. This has been shown graphically in Fig. 11.2 by plotting the volume of a given mass of a gas against temperature at constant pressure.

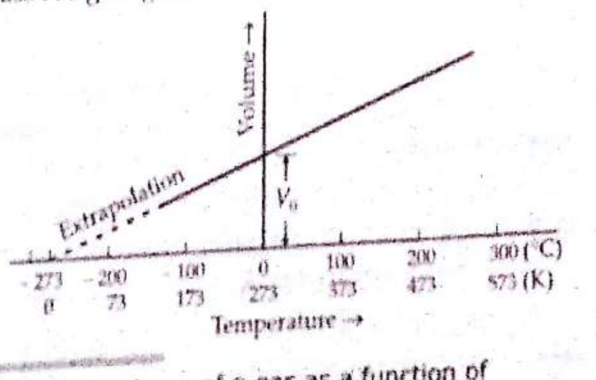


Fig. 11.2 Volume of a gas as a function of temperature.

The graph is a straight line. If we extrapolate the straight line, it meets the temperature axis at $-273.15°C$. Thus a gas occupies zero or no volume at $-273.15°C$. Clearly, a temperature below $-273.15°C$ is impossible because then the volume of the gas would be negative which is meaningless.

Moreover, according to the kinetic theory of gases, all molecular motion stops at $-273.15°C$. Hence *the lowest temperature of $-273.15°C$ at which a gas is supposed to have zero volume (and zero pressure) and at which entire molecular motion stops is called the absolute zero of temperature.* In practice, all gases condense to liquids and solids before this temperature is reached.

Lord Kelvin suggested new scale of temperature starting with -273.15 as its zero. This scale of temperature is known as Kelvin scale or absolute scale. The size of degree on Kelvin scale is same as that on Celsius scale. Therefore,

$$T(K) = t(°C) + 273.15$$

Thus ice point (°C) on absolute scale is 273.15 K and the steam point (100°C) is 373.15 K. The absolute scale of temperature is also called thermodynamic scale of temperature.

11.8 TRIPLE POINT OF WATER

12. *What is meant by triple point of water ? What is the advantage of taking triple point of water as the fixed point for a temperature scale ?*

Triple point of water. *The triple point of water is the state at which the three phases of water namely ice, liquid water and water vapour are equally stable and co-exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column.* Thus for water,

$$P_{tr} = 0.46 \text{ cm of Hg}$$

$$T_{tr} = 273.16 \text{ K or } 0.01°C$$

In modern thermometry, the triple point of water is chosen to be one of the fixed points. As it is characterised by a unique temperature and pressure, so it is preferred over the conventional fixed points namely the melting point of ice and boiling point of water. The melting point of ice and boiling point of water both change with pressure. Moreover, the presence of impurities changes their values. But the triple point of water is independent of the external factors.

In the absolute Kelvin scale, the triple point of water is assigned the value 273.16 K. The absolute zero is taken as the other fixed point on this scale.

11.9 CONSTANT VOLUME GAS THERMOMETER*

13. *Describe the construction and working of a constant volume gas thermometer. Give its advantages and disadvantages.*

Constant volume gas thermometer. A constant volume gas thermometer is an ideal thermometer because the increase of volume or pressure of a gas with temperature is independent of the nature of the gas.

Principle. *If the volume is kept constant, the pressure of a given mass of a gas increases linearly with the temperature.* This is called Gay Lussac's law and is illustrated graphically in Fig. 11.3.

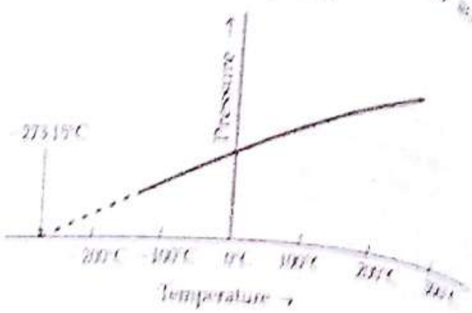


Fig. 11.3 Pressure versus temperature of a low density gas kept at constant volume.

Construction. Fig. 11.4 shows a schematic diagram of a constant volume gas thermometer. A fixed mass of gas is enclosed in a bulb *A* made of glass, quartz or platinum depending on the temperature range required to be measured. The bulb *A* is connected to a capillary *BC* which is connected to a manometer (*D*) containing mercury. The end *D* of the manometer is open to atmosphere. The mercury of the manometer is connected to a mercury reservoir through a rubber tube. The volume of the gas is kept constant by raising or lowering the mercury reservoir *F* until the mercury level in the left part of the manometer coincides with *C*. A vertical scale *E* measures the difference *h* of mercury levels in the manometer tube. The pressure of the gas is then the atmospheric pressure plus the pressure due to the mercury column of height *h*.

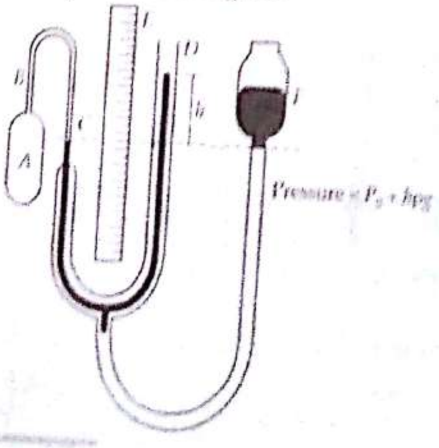


Fig. 11.4 Constant volume gas thermometer.

Working. The bulb is first immersed in the system (e.g., boiling oil) whose temperature is to be determined.

The pressure P of the gas is measured. Next, the bulb is surrounded by water at the triple point, and the level of mercury is again brought to C in the left side of the manometer. The pressure P_{tr} of the gas is measured. According to Gay Lussac's law :

$$\frac{P_{tr}}{T_{tr}} = \frac{P}{T} \quad \text{or} \quad T = T_{tr}\left(\frac{P}{P_{tr}}\right)$$

But $T_{tr} = 273.16$ K

$$\therefore \quad T = 273.16\left(\frac{P}{P_{tr}}\right)$$

Advantages :

(*i*) As the expansion of the gas is large, so the gas thermometers are very sensitive.

(*ii*) The gases expand uniformly and regularly over a wide range of temperature.

(*iii*) The expansion coefficient of all gases is nearly the same, so thermometers using different gases give same reading.

(*iv*) Gas thermometers can be used for a wide range of temperature. With helium gas low temperature of about -270°C and using nitrogen gas high temperature of about 1600°C can be measured.

Disadvantages :

(*i*) The correction has to be applied against the increase in the volume of the glass bulb.

(*ii*) The gas in the bulb is not at the same temperature as the gas in the capillary tube. Correction has to be applied for this purpose.

(*iii*) The gas is not ideal.

(*iv*) The thermometer is large and hence inconvenient to use.

(*v*) It is not a direct reading thermometer.

(*vi*) It requires a great deal of time to know the unknown temperature.

11.10 IDEAL GAS TEMPERATURE*

14. *What do you mean by ideal gas temperature ? Does it depend on nature of the gas ?*

Ideal gas temperature. If P and P_{tr} are the pressures of a gas (at constant volume) at constant temperature T and at the triple point (273.16 K) respectively, then from Gay Lussac's law, we have

$$T = 273.16\left(\frac{P}{P_{tr}}\right)$$

The temperature defined by the above equation depends slightly on the nature of the gas and its pressure. But at low pressures and high temperatures the real gases approach the ideal gas behaviour. If the pressure in the bulb of the gas thermometer is taken to be smaller and smaller (*i.e.*, in the limit $P_{tr} \to 0$), all the different gas thermometers give the same value of temperature for a given system. So we define a temperature by the equation

$$T = \lim_{P_{tr} \to 0} 273.16\left(\frac{P}{P_{tr}}\right)$$

The temperature defined by the above equation is known as *ideal gas temperature* on the Kelvin scale and is independent of the nature of the gas. However, it depends on the properties of the gases in general. In thermodynamics, it is possible to define a truly universal *absolute temperature scale* which does not depend on any property of the thermometric substance and in which the unit of temperature is Kelvin (K). The ideal gas thermometer scale is found to be identical with this absolute scale.

11.11 OTHER THERMOMETERS

15. *What is a liquid thermometer ? What are the advantages of using mercury as a thermometric substance over other liquids ?*

Liquid thermometer. Its working is based on the fact that *liquids expand uniformly and regularly on being heated.* Mercury thermometer is the most common liquid thermometer. It consists of a glass capillary tube of uniform bore having a bulb at its one end. The bulb is filled with mercury and the tube is sealed at the top after taking out air from it. The ice point and steam point are respectively marked as its lower and upper fixed points respectively. The use of capillary tube results in an easily observable rise in the level of mercury even for of a small rise in temperature.

If l_0, l_{100} and l_t be the lengths of mercury column at 0°C, 100°C and an unknown temperature t°C respectively, then the unknown temperature measured by the mercury thermometer is

$$t = \frac{l_t - l_0}{l_{100} - l_0} \times 100°\text{C}$$

Advantages of using mercury as a thermometric substance as compared to other liquids :

(*i*) Mercury has a uniform coefficient of expansion over a wide range of temperature.

(*ii*) Mercury is opaque and bright, so its level can be seen easily in a glass tube.

(*iii*) It does not stick to the walls of the glass tube.

(*iv*) It is a good conductor of heat and so attains the temperature of the hot body quickly.

Scanned by CamScanner

(*v*) It has low specific heat, it absorbs very small amount of heat from the body whose temperature is to be measured.

(*vi*) The range of mercury thermometer is quite large because of its low freezing point (−39°C) and high boiling point (357°C).

(*vii*) Mercury is non volatile.

16. *Describe the working principle of a platinum resistance thermometer.*

Platinum resistance thermometer. The electric resistance of a metal wire increases linearly with temperature as $R_t = R_0(1 + \alpha t)$

where α is the temperature coefficient of resistance.

So electric resistance may be used as a thermometric property to define a temperature scale. A platinum wire is often used in this thermometer because platinum has high melting point and its α is constant and large. If R_0 and R_{100} denote the resistance of a platinum wire at ice point and steam point respectively, then temperature t_R of a body for which the corresponding resistance is R_t, is given by

$$t_R = \frac{R_t - R_0}{R_{100} - R_0} \times 100°C$$

Platinum resistance thermometer can be used in the temperature range −170°C to 200°C. For measuring lower temperatures, a *germanium resistance thermometer* is used. Germanium is a semiconductor whose resistance increases with the decrease in temperature.

17. *Briefly describe the working principle of a thermoelectric thermometer.*

Thermoelectric thermometer. This thermometer is based on **Seebeck effect**. According to this effect, *when wires of two different metals (Bi − Sb or Cu − Fe) are joined to form a closed circuit called thermocouple and their junctions are maintained at different temperatures, an e.m.f. is produced and a current flows in the circuit.* If one junction is at 0°C (cold junction) and the other at t°C (hot junction), then the thermoelectric e.m.f. is given by

$$\mathcal{E} = at + bt^2$$

where a and b are constants for the given pair of metals. To measure an unknown temperature, each individual thermoelectric thermometer is first calibrated by drawing a curve between the temperature of hot junction and the e.m.f. generated.

For the linear part of the thermo e.m.f., the unknown temperature is given by

$$t_\varepsilon = \frac{\mathcal{E}_t - \mathcal{E}_0}{\mathcal{E}_{100} - \mathcal{E}_0} \times 100 \text{ degrees}$$

The normal range of a thermoelectric thermometer is −200°C to 1600°C.

Examples based on Measurement of Temperature

Formulae Used

1. If T_C, T_F, T_R and T are the temperatures of a body on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T - 273.15}{100}$$

or $$\frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_R}{4} = \frac{T - 273.15}{5}$$

2. $T_C = \frac{5}{9}(T_F - 32)$, $T_F = \frac{9}{5}T_C + 32$

3. $T = T_C + 273.15$, $T_C = T - 273.15$

4. $T_F = \frac{9}{5}(T - 273.15) + 32 = \frac{9}{5}T - 459.67$

or $T = \frac{5}{9}T_F + 255.37$

5. For a constant volume air thermometer,

$$T = T_0 \times \frac{P}{P_0}$$

In terms of triple point of water, $T = T_{tr} \times \frac{P}{P_{tr}}$

6. For a platinum resistance thermometer, resistance of platinum at t°C, $R = R_0(1 + \alpha t)$

Temperature coefficient of resistance, $\alpha = \frac{R - R_0}{R_0 \times t}$

Units Used

Pressure is in pascal (Pa), resistance in ohm (Ω) and temperature coefficient of resistance (α) in °C^{-1}.

Example 1. *A faulty thermometer has its fixed points marked as 5° and 95°. Temperature of a body as measured by the faulty thermometer is 59°. Find the correct temperature of the body on Celsius scale.*

Solution. $\frac{T_C - 0}{100 - 0}$

$$= \frac{\text{Temp. on faulty scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}}$$

or $\frac{T_C - 0}{100} = \frac{59 - 5}{95 - 5} = \frac{54}{90}$ or $T_C = $ **60°C.**

Example 2. *A thermometer has wrong calibration. It reads the melting point of ice −10°C. It reads 60°C in place of 50°. Calculate the temperature of boiling point of water on this scale.*

Solution. Let

θ_1 = Lower fixed point on faulty thermometer

θ_2 = Reading on faulty thermometer

n = number of divisions between upper and lower fixed points

Scanned by CamScanner

Now $\frac{C}{100} = \frac{\theta_2 - \theta_1}{n}$ ...(1)

In first case,

$\theta_2 = -10°C, \quad C = 0°C$

$\therefore \quad 0 = \frac{-10 - \theta_1}{n}$ or $\theta_1 = -10°C$

In second case,

$\theta_2 = 60°C, \quad C = 50°C$

$\therefore \quad \frac{50}{100} = \frac{60 - \theta_1}{n}$ or $\frac{1}{2} = \frac{60 - (-10)}{n} = \frac{70}{n}$

or $n = 140$

As boiling point of water on Celsius scale is 100°C, so putting $C = 100$ in equation (1), we get

$$\frac{100}{100} = \frac{\theta_2 - (-10)}{140} \quad \text{or} \quad \theta_2 = 130°C$$

$\therefore$ Boiling point of water on faulty thermometer $= 130°C$.

EXAMPLE 3. *At what temperature, do the readings of Celcius and Fahrenheit scales coincide ?* **[Himachal 07]**

Solution. Let $T_C = T_F = x$

As $\frac{T_C - 0}{100} = \frac{T_F - 32}{180}$

$\therefore \quad \frac{x}{100} = \frac{x - 32}{180}$

or $180x = 100x - 3200$ or $80x = -3200$

or $x = -40°$

Hence $-40°C$ and $-40°F$ are identical temperatures.

EXAMPLE 4. *A constant volume gas thermometer using helium records a pressure of 20.0 kPa at the triple-point of water, and pressure of 14.3 kPa at the temperature of 'dry ice' (solid CO_2). What is the temperature of 'dry ice' ?*

Solution. Here $T_{tr} = 273.16$ K, $P_{tr} = 20.0$ k Pa, $P = 14.3$ k Pa

Temperature of dry ice,

$$T = \frac{P}{P_{tr}} \times T_{tr} = \frac{14.3 \times 273.16}{20} = \mathbf{195.30\ K.}$$

EXAMPLE 5. *A constant volume thermometer using helium gas records a pressure of 1.75×10^4 Pa at normal freezing point of water, and a pressure of 2.39×10^4 Pa at normal boiling point of water. Obtain from these observations the temperature of absolute zero on the Celsius scale.* **[NCERT]**

Solution. Let T_1 and T_2 be the absolute temperatures corresponding to normal freezing and normal boiling points (*i.e.*, under 1 atmospheric pressure) of water.

For a constant volume thermometer,

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{2.39 \times 10^4 \text{ Pa}}{1.75 \times 10^4 \text{ Pa}} = \frac{239}{175}$$

Now the temperature t_c on Celsius scale and temperature T on the Kelvin scale are related as

$$T = t_c + T_0$$

where T_0 is a constant. As the normal freezing point and normal boiling point of water are 0°C and 100°C respectively, so from the above equation, we get

$$T_1 = 0 + T_0 \quad \text{and} \quad T_2 = 100 + T_0$$

$\therefore \quad \frac{T_0 + 100}{T_0} = \frac{239}{175}$ or $\frac{100}{T_0} = \frac{239}{175} - 1 = \frac{64}{175}$

or $T_0 = \frac{100 \times 175}{64} = 273$

Hence the absolute zero ($T = 0$) on the Celsius scale will be

$$t_c = T - T_0 = 0 - 273 = -273.$$

The exact value of absolute zero is -273.15.

EXAMPLE 6. *A plantinum wire has resistance of 10 Ω at 0°C and 20 Ω at 273°C. Find the value of coefficient of resistance.* **[NCERT]**

Solution. Here $R_0 = 10\,\Omega, \quad R = 20\,\Omega,$

$\theta = 273° - 0° = 273°C$

Temperature coefficient of resistance,

$$\alpha = \frac{R - R_0}{R_0 \times \theta} = \frac{20 - 10}{10 \times 273} = \frac{1}{273}\ °C^{-1}.$$

✕ PROBLEMS FOR PRACTICE

1. A faulty thermometer reads 5°C in melting ice and 99°C in steam. Find the correct temperature in °F when the faulty thermometer reads 52°C. **(Ans. 122°F)**
2. An ungraduated thermometer of uniform bore is attached to a centimeter scale and is found to read 10.3 cm in melting ice, 26.8 cm in boiling water and 6.5 cm in freezing mixture. Calculate the temperature of the freezing mixture. **(Ans. −23.03°C)**
3. Normal temperature of the human body is 98.4°F. Find the temperature on Celsius and absolute scale. **(Ans. 36.88°F, 310.03 K)**
4. What is the triple point of water on a Fahrenheit scale ? What is the absolute zero on this scale ? **(Ans. 32.018°F, −459°F)**
5. When a thermometer is taken from the melting ice to a warm liquid, the mercury level rises to 2/5th of

Scanned by CamScanner

the distance between the lower and the upper fixed points. Find the temperature of liquid in °C and K. **(Ans. 40°C, 313.15 K)**

6. The pressure of air in the bulb of constant volume air thermometer is 75 cm of mercury at 0°C, 100 cm at 100°C and 80 cm at the room temperature. Calculate the room temperature. **(Ans. 20°C)**

7. At what temperature is the Fahrenheit scale reading equal to twice of Celsius scale reading ? **(Ans. 160° C or 320° F)**

8. At what temperature is the Fahrenheit scale reading equal to half of Celsius scale reading ? **(Ans. – 24.6°C)**

9. A constant volume gas thermometer using sulphur records a pressure of 2×10^4 Pa at the triple point of water and 2.87×10^4 Pa at temperature of melting sulphur. Calculate the melting point of sulphur. **(Ans. 190.4 K)**

10. The resistance of a resistance thermometer at 19°C is 3.50 Ω and at 99°C is 3.66 Ω. At what temperature will its resistance be 4.30 Ω ? **(Ans. 419°C)**

Hints

1. As the lowest and the highest points on the faulty thermometer are 5°C and 99°C, so

$$\frac{T_C - 5}{99 - 5} = \frac{T_F - 32}{180} \quad \text{or} \quad \frac{52 - 5}{94} = \frac{T_F - 32}{180}$$

or $T_F = \mathbf{122°\,F}$.

2. Temperature reading of melting ice = 10.3 cm

Temperature reading of boiling water = 26.8 cm

Temperature reading of freezing mixture = 6.5 cm

Let T_C be the temperature of freezing mixture on Celsium scale. Then

$$\frac{6.5 - 10.3}{26.8 - 10.3} = \frac{T_C - 0}{100 - 0}$$

On solving, $T_C = \mathbf{-23.03°C}$

3. $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(98.4 - 32) = \frac{5}{9} \times 66.4 = \mathbf{36.88°C}$.

$T = T_C + 273.15 = 36.88 + 273.15 = \mathbf{310.03\ K}$.

5. Let the distance between the lower and upper fixed points be 1 unit. Then

$$\frac{\frac{2}{5} - 0}{1 - 0} = \frac{T_C - 0}{100}$$

or $T_C = \mathbf{40°C}$

and $T = 40 + 273.15 = \mathbf{313.15\ K}$

6. $$\frac{T_C - 0}{100 - 0} = \frac{P - P_0}{P_{100} - P_0} = \frac{80 - 75}{100 - 75} \quad \therefore T_C = \mathbf{20°C}$$

7. $T_F = \frac{9}{5} T_C + 32$ and $T_F = 2\,T_C$

$\therefore 2\,T_C = \frac{9}{5} T_C + 32$ or $T_C = \mathbf{160°C}$ and $T_F = 320°F$

8. $T_F = \frac{9}{5} T_C + 32$ and $T_F = \frac{1}{2} T_C$

$\therefore \frac{1}{2} T_C = \frac{9}{5} T_C + 32$ or $T_C = -\frac{320}{13} = \mathbf{-24.6°C}$

10. Let R_0, R_1 and R_2 be the resistances at 0°, θ_1°C and θ_2°C and α be the temperature coefficient of resistance. Then

$$R_1 = R_0(1 + \alpha\,\theta_1), \quad R_2 = R_0(1 + \alpha\,\theta_2)$$

On dividing and solving, $\alpha = \dfrac{R_2 - R_1}{R_1\theta_2 - R_2\theta_1}$

But $\theta_1 = 19°C$, $R_1 = 3.50\,\Omega$, $\theta_2 = 99°C$, $R_2 = 3.66\,\Omega$

$$\therefore \quad \alpha = \frac{3.66 - 3.50}{3.50 \times 99 - 3.66 \times 19} = \frac{0.16}{276.96} \quad ...(i)$$

Suppose the resistance becomes 4.30 Ω at t °C. Then

$$\alpha = \frac{4.30 - 3.50}{3.50 \times t - 4.30 \times 19} = \frac{0.80}{3.50t - 81.70} \quad ...(ii)$$

From (i) and (ii),

$$\frac{0.16}{276.96} = \frac{0.80}{3.50t - 81.70}$$

or $3.50t - 81.70 = 5 \times 276.96$

or $3.50t = 1384.80 + 81.70 = 1466.50$

$\therefore t = \mathbf{419°C}$.

11.12 THERMAL EXPANSION

18. *What is meant by thermal expansion of a body ? What are the different types of thermal expansion ?*

Thermal expansion. Almost all solids, liquids and gases expand on heating. *The increase in the size of a body when it is heated is called thermal expansion.*

Different types of thermal expansion :

(i) ***Linear expansion.*** *It is the increase in the length of a metal rod on heating.*

(ii) ***Superficial expansion.*** *It is the increase in the surface area of a metal sheet on heating.*

(iii) ***Cubical expansion.*** *It is the increase in the volume of block on heating.*

19. *Why do solids expand on heating ?*

Cause of thermal expansion. All solids consist of atoms and molecules. At any given temperature, these atoms and molecules are held at equilibrium distance by forces of attraction. When a solid is heated, the amplitude of vibration of its atoms and molecules increases. The average interatomic separation increases. This results in the thermal expansion of the solid.

Scanned by CamScanner

20. *Define coefficient of linear expansion. Write an expression for it. Give its units.*

Coefficient of linear expansion. Suppose a solid rod of length l is heated through a temperature ΔT and its final (increased) length is l'. It is found from experiments that

(*i*) Increase in length ∝ rise in temperature

i.e., $l' - l \propto \Delta T$

(*ii*) Increase in length ∝ original length

i.e., $l' - l \propto l$

Combining the above two factors, we get

$$l' - l \propto l\,\Delta T \quad \text{or} \quad l' - l = \alpha\, l\,\Delta T$$

The proportionality constant α is called ***coefficient of linear expansion***. Its value depends on the nature of the solid. Clearly,

$$l' = l[1 + \alpha\,\Delta T] \quad \text{and} \quad \alpha = \frac{l' - l}{l\,\Delta T} = \frac{\Delta l}{l\,\Delta T}$$

or
$$\alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temperature}}$$

Hence *the coefficient of linear expansion of the material of a solid rod is defined as the increase in length per unit original length per degree rise in its temperature.*

The unit of α is $°C^{-1}$ or K^{-1}.

Table 11.1 *Coefficients of linear expansion for some substances*

Materials	α (10^{-5} K)
Aluminium	2.5
Brass	1.8
Iron	1.2
Copper	1.7
Silver	1.9
Gold	1.4
Glass (pyrex)	0.32
Lead	0.29

Table 11.1 gives the average values of α for some materials in the temperature range 0 – 100°C. Clearly, copper expands 5 times more than glass for the same rise in temperature. Generally, metals expand more and have comparatively high values of α.

21. *Define coefficient of superficial expansion and give its units.*

Coefficient of superficial expansion. Suppose a metal sheet of initial surface area S is heated through temperature ΔT and its final surface area becomes S'.

Then $S' - S \propto \Delta T$ and $S' - S \propto S$

$S' - S \propto S\,\Delta T$ or $S' - S = \beta\, S\,\Delta T$

The proportionality constant β is called *coefficient of superficial expansion* and its value depends on the nature of the material. Clearly,

$$S' = S[1 + \beta\,\Delta T] \quad \text{and} \quad \beta = \frac{S' - S}{S\,\Delta T} = \frac{\Delta S}{S\,\Delta T}$$

or
$$\beta = \frac{\text{Increase in surface area}}{\text{Original surface area} \times \text{Rise in temperature}}$$

Hence the *coefficient of superficial expansion of a metal sheet is defined as the increase in its surface area per unit original surface area per degree rise in its temperature.*

The unit of β is $°C^{-1}$ or K^{-1}.

22. *Define coefficient of cubical expansion. Write an expression for it. Give its units.*

Coefficient of cubical expansion. Suppose a solid block of initial volume V is heated through a temperature ΔT and its final volume is V'.

Then $V' - V \propto \Delta T$

and $V' - V \propto V$ or $V' - V = \gamma\, V\,\Delta T$

The proportionality constant γ is called the **coefficient of cubical expansion** which depends on the nature of the material of the solid. Clearly,

$$V' = V[1 + \gamma\,\Delta T] \quad \text{and} \quad \gamma = \frac{V' - V}{V\,\Delta T} = \frac{\Delta V}{V\,\Delta T}$$

or
$$\gamma = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Hence *the coefficient of cubical expansion of a substance is defined as the increase in volume per unit original volume per degree rise in its temperature.*

The unit of γ is $°C^{-1}$ or K^{-1}.

23. *How does the coefficient of cubical expansion of a substance vary with temperature ? Draw γ versus T curve for copper.*

Variation of γ with temperature. For a given substance, γ varies with temperature. Fig. 11.5 shows

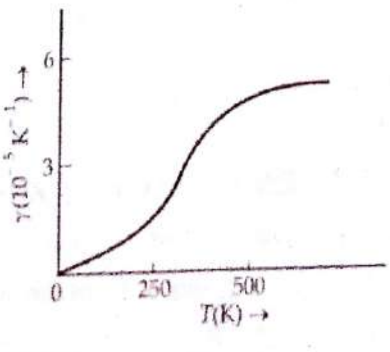


Fig. 11.5 Variation of γ of copper with temperature.

Scanned by CamScanner

the variation of the coefficient of cubical expansion of copper with temperature. The value of γ first increases with temperature and then becomes constant at a high temperature (above 500 K).

Table 11.2 *Coefficients of volume expansion for some substances*

Materials	$\gamma\ (K^{-1})$	*Materials*	$\gamma\ (K^{-1})$
Aluminium	7×10^{-5}	Hard rubber	2.4×10^{-4}
Brass	6×10^{-5}	Invar	2×10^{-6}
Iron	3.55×10^{-5}	Mercury	18.2×10^{-5}
Paraffin	58.8×10^{-5}	Water	20.7×10^{-5}
Ordinary Glass	2.5×10^{-5}	Ethyl alcohol	110×10^{-5}
Pyrex Glass	1×10^{-5}		

Table 11.2 gives the average values of γ for some common substances in the temperature range $0-100°C$. It can be noted that solids and liquids have small values of γ. The materials pyrex glass and invar (an alloy of iron and nickel) have still smaller values of γ. Ethyl alcohol has a higher value of γ than mercury and expands more than mercury for the same rise of temperature.

11.13 COEFFICIENT OF CUBICAL EXPANSION OF AN IDEAL GAS

24. *Show that the coefficient of cubical expansion of an ideal gas at constant pressure is equal to the reciprocal of its absolute temperature.*

Coefficient of cubical expansion of an ideal gas at constant pressure. For an ideal gas,

$$PV = nRT \qquad ...(i)$$

At constant pressure,

$$P\,\Delta V = nR\,\Delta T \qquad ...(ii)$$

[$\because$ n and R are constants]

Dividing (*ii*) by (*i*), we get

$$\frac{\Delta V}{V}=\frac{\Delta T}{T} \quad \text{or} \quad \frac{\Delta V}{V\,\Delta T}=\frac{1}{T} \quad \text{or} \quad \gamma=\frac{1}{T}$$

Hence for an ideal gas, the coefficient of volume expansion decreases with the increase in temperature.

11.14 RELATION BETWEEN α, β AND γ

25. *Derive the relation between α, β and γ.*

Relation between α, β and γ. Consider a cube of side l. Its original volume is

$$V = l^3$$

Suppose the cube is heated so that its temperature increases by ΔT. Its each side will become

$$l' = l + \Delta l = l + \alpha\, l\, \Delta T = l(1+\alpha\,\Delta T)$$

The new volume of the cube will be

$$V' = l'^3 = l^3(1+\alpha\,\Delta T)^3$$
$$= V(1+3\alpha\,\Delta T + 3\alpha^2\,\Delta T^2 + \alpha^3\,\Delta T^3)$$

As α is small, so the terms containing α^2 and α^3 can be neglected. Then

$$V' = V(1+3\alpha\,\Delta T)$$

By the definition of the coefficient of cubical expansion,

$$\gamma=\frac{\Delta V}{V\,\Delta T}=\frac{V'-V}{V\,\Delta T}=\frac{V(1+3\alpha\,\Delta T)-V}{V\,\Delta T}=3\alpha$$

Similarly, it can be proved that

$$\beta = 2\alpha$$

Hence $\quad \dfrac{\alpha}{1}=\dfrac{\beta}{2}=\dfrac{\gamma}{3}.$

For Your Knowledge

- The three coefficients of expansion α, β and γ are not constant for a given solid. Their values depend on the temperature range.
- For most of the solids, the value of α lies between 10^{-6} to $10^{-5}\ K^{-1}$ in the temperature range 0 to 100°C. The value of α is more for ionic solids than that for non-ionic solids.
- The coefficient of linear expansion of a solid rod is independent of the geometrical shape of its cross-section.
- The coefficient of volume expansion of solids and liquids is rather small, particularly very small for pyrex glass ($1\times10^{-5}\ K^{-1}$) and invar (Fe-Ni alloy with $\gamma = 2\times10^{-6}\ K^{-1}$).
- For an ideal gas γ varies inversely with temperature *i.e.*, $\gamma = 1/T$. At 0°C or 273 K, $\gamma = 1/273 = 3.7\times10^{-3}\ K^{-1}$, which is much larger than that for solids and liquids.
- Water contracts on heating between 0°C and 4°C. This is called ***anomalous expansion of water***. It has the minimum volume and hence the maximum density ($1000\ kg\ m^{-3}$) at 4°C. Silver iodide also contracts on heating between 80°C to 140°C.

11.15 MOLECULAR EXPLANATION OF THERMAL EXPANSION

26. *Explain thermal expansion of solids on the basis of the potential energy curve.*

Molecular explanation of thermal expansion. As shown in Fig. 11.6, the graph between the potential energy $U(r)$ of two neighbouring atoms in a crystalline solid and their interatomic separation r is an *asymmetric parabola*. The potential energy curve is asymmetric about its minimum because the attractive part of the potential energy rises slowly compared to the repulsive part.

Scanned by CamScanner

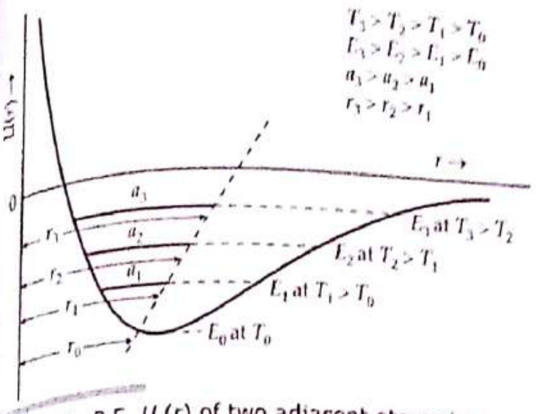


Fig. 11.6 P.E. $U(r)$ of two adjacent atoms in a crystalline solid versus interatomic separation r.

At the temperature $T_0 = 0$ K, the atoms remain at the equilibrium separation r_0 and their oscillation energy E_0 is minimum. As the temperature increases, the energy of the atoms increases and they start vibrating about their equilibrium positions with the interatomic separation oscillating between its minimum and maximum values : r_{min} and r_{max}. The average interatomic separation becomes

$$r = \frac{r_{min} + r_{max}}{2}$$

Clearly, as the temperature increases, the amplitude of vibration of the atoms increases. Due to the asymmetry of the P.E. curve, the equilibrium position shifts to the right on the curve (as shown by the dashed inclined line), *i.e.*, the average interatomic separation increases. It is thus *in consequence of this increase in the average interatomic separation with temperature that a solid expands when heated.*

11.16 PRACTICAL APPLICATIONS OF THERMAL EXPANSION

27. *Mention some applications of thermal expansion in daily life.*

Practical applications of thermal expansion :

(i) ***A small gap is left between the iron rails of railway tracks.*** The two rails are joined by fish plates. If no gap is left between the rails, the rails may bend due to expansion in summer and the train may get derailed.

(ii) ***Space is left between the girders used for supporting bridges.*** This allows their expansion during summer. Moreover, the ends of the girders are placed on metal rollers to allow the expansion and contraction to take place easily with the change of season.

(iii) ***The iron ring to be put on the rim of a cart wheel is always of slightly smaller diameter than that of the wheel.*** When the iron ring is heated to become red hot, it expands and slips on to the wheel easily. When it is cooled, it contracts and grips the wheel firmly.

(iv) ***Clock pendulums are made of invar.*** Invar is an alloy. It has extremely small temperature coefficient of expansion. So the length of invar pendulum does not change with the change of season and the clock gives almost correct time.

(v) ***A glass stopper jammed in the neck of a glass bottle can be removed by warming the neck of the bottle.*** When the neck of the bottle is slightly warmed, its mouth becomes slightly wider. The stopper becomes loose and comes out easily.

(vi) ***Only platinum wire is used for fusing into glass.*** This is because the coefficient of thermal expansion of platinum is almost the same as that of glass.

11.17 EXPANSION OF A LIQUID

28. *What do you mean by coefficients of apparent and real expansion of a liquid ? How are they related ?*

Expansion of a liquid. When a liquid is heated, its container also expands. The observed expansion of the liquid is called *apparent expansion* which is different from the *real expansion* of the liquid.

Coefficient of apparent expansion. *It is defined as the apparent increase in volume per unit original volume for 1°C rise in temperature.* The coefficient of apparent expansion of the liquid is given by

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Coefficient of real expansion. *It is defined as the real increase in volume per unit original volume for 1°C rise in temperature.* The coefficient of real expansion of the liquid is given by

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

It can be proved that $\gamma_r = \gamma_a + \gamma_g$

where γ_g is coefficient of cubical expansion of glass (material) of the container.

11.18 VARIATION OF DENSITY WITH TEMPERATURE

29. *How does the density of a solid or a liquid vary with temperature ? Show that its variation with temperature is given by $\rho' = \rho(1 - \gamma\,\Delta T)$, where γ is the coefficient of cubical expansion.*

Variation of density with temperature. When a given mass of a solid or a liquid is heated, its volume increases and hence density decreases. If V and V' are the volumes and ρ and ρ' are the densities of a given mass M at temperatures T and $T + \Delta T$ respectively, then

$$V' = V(1 + \gamma\,\Delta T)$$

or $$\frac{M}{\rho'} = \frac{M}{\rho}(1 + \gamma\,\Delta T) \quad \text{or} \quad \rho' = \rho(1 + \gamma\,\Delta T)^{-1}$$

Scanned by CamScanner

Expanding by Binomial theorem and neglecting the terms containing higher powers of $\gamma \Delta T$, we get

$$\rho' = \rho(1 - \gamma \Delta T)$$

Clearly the density of a solid or a liquid decreases with the increase in temperature.

11.19 ANOMALOUS EXPANSION OF WATER

30. *Discuss anomalous expansion of water. Give its practical importance.*

Anomalous expansion of water. Almost all liquids expand on being heated but water behaves in a peculiar manner. When water at 0°C is heated, its volume decreases and therefore, its density increases, until its temperature reaches 4°C. Above 4°C, the volume increases, and therefore the density decreases. ***Thus water at 4°C has the maximum density.***

Fig. 11.7(*a*) shows the variation of volume of 1 kg of water as the temperature increases from 0°C to 100°C. Fig. 11.7(*b*) shows the variation of density of water with temperature from 0° to 10°C.

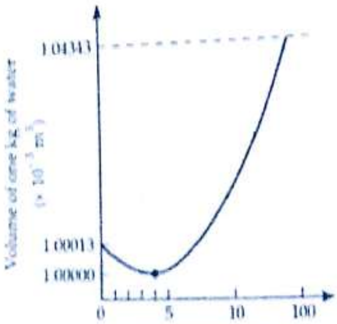


Fig. 11.7 (*a*) Thermal expansion of water.

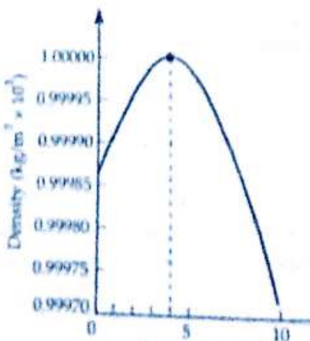


Fig. 11.7 (*b*) Variation of density of water with temperature

Practical importance of anomalous expansion of water. The anomalous expansion of water has a favourable effect on aquatic life. Since the density of water is maximum at 4°C, water at the bottom of lakes remains at 4°C even if it freezes at the top surface. This allows marine animals to remain alive and move freely near the bottom. If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy the entire aquatic animal and plant life.

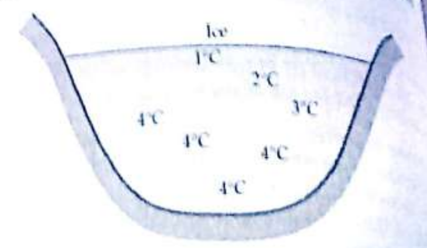


Fig. 11.8 Anomalous expansion of water helps aquatic life.

Examples based on Thermal Expansion

FORMULAE USED

1. Change in length, $l' - l = l\alpha(T' - T)$ or $\Delta l = l\alpha\Delta T$
2. Coefficient of linear expansion, $\alpha = \dfrac{\Delta l}{l\Delta T}$
3. Final length, $l' = l(1 + \alpha\Delta T)$
4. Change in surface area, $S' - S = S\beta(t' - t)$
   or $\Delta S = S\beta\Delta T$
5. Coefficient of superficial expansion, $\beta = \dfrac{\Delta S}{S\Delta T}$
6. Final surface area, $S' = S(1 + \beta\Delta T)$
7. Change in volume, $V' - V = V\gamma(t' - t)$
   or $\Delta V = V\gamma\Delta T$
8. Coefficient of cubical expansion, $\gamma = \dfrac{\Delta V}{V\Delta T}$
9. Final volume $V' = V(1 + \gamma\Delta T)$
10. Relation between α, β and γ
    $\dfrac{\alpha}{1} = \dfrac{\beta}{2} = \dfrac{\gamma}{3}$ ∴ $\beta = 2\alpha$ and $\gamma = 3\alpha$
11. Final density, $\rho' = \rho(1 - \gamma\Delta T)$

UNITS USED

Lengths l, l' and Δl are in cm. Surface areas S, S' and ΔS are in cm^2. Volumes V, V' and ΔV are in cm^3. Temperatures t, t' and ΔT are in °C and coefficients α, β and γ are in °C^{-1}.

EXAMPLE 7. *Show that the coefficient of area expansions, $(\Delta A/A)/\Delta T$ of a rectangular sheet of the solid is twice its linear expansivity, α.* [NCERT]

Solution. As shown in Fig. 11.9, consider a rectangular sheet of the solid material of length a and breadth b. Suppose its temperature increases by ΔT.

Increase in length a, $\Delta a = \alpha a\Delta T$

Increase in breadth b, $\Delta b = \alpha b\Delta T$

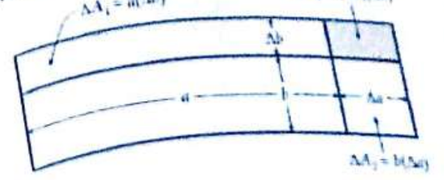


Fig. 11.9

Total increase in area of the sheet is

$$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$$

$$\Delta A = a\Delta b + b\Delta a + (\Delta a)(\Delta b)$$

$$= \alpha ab\,\Delta T + b\alpha\, a\Delta T + (\alpha)^2\, ab\,(\Delta T)^2$$

$$= \alpha ab\,\Delta T\,(2 + \alpha\Delta T) = \alpha A\Delta T\,(2 + \alpha\Delta$$

Since $\alpha = 10^{-5}$ K^{-1}, the product $\alpha\Delta T$ for fractio temperature is small in comparison with 2 and ma neglected. Therefore,

$$\beta = \left(\frac{\Delta A}{A}\right)\frac{1}{\Delta T} = 2\alpha.$$

EXAMPLE 8. *Railway lines are laid with gaps to expansion. If the gap between steel rails 66 m long at 10°C, then at what temperature will the lines ju Coefficient of linear expansion of steel = 11 × 10*

Solution. As the rails expand in both dir the gap between two rails is filled by the e half length of each rail. Equivalently, we expansion of one rail in one direction to f

∴ $l = 66$ m, $\Delta l = 3.63 \times 10^{-2}$ m

$\alpha = 11 \times 10^{-6}$ °C^{-1}

As $\Delta l = l\alpha\Delta T$

∴ $\Delta T = \dfrac{\Delta l}{l\alpha} = \dfrac{3.63 \times 10^{-2}}{66 \times 11 \times 10^{-6}}$

Final temperature

= Initial temperature + ΔT =

EXAMPLE 9. *A blacksmith fixes iron r wooden wheel of a bullock cart. The di the iron ring are 5.243 m and 5.231 m To what temperature should the ring rim of the wheel ?*

Scanned by CamScanner

EXAMPLE 7. *Show that the coefficient of area expansions, $(\Delta A/A)/\Delta T$ of a rectangular sheet of the solid is twice its linear expansivity, α.* **[NCERT]**

Solution. As shown in Fig. 11.9, consider a rectangular sheet of the solid material of length a and breadth b. Suppose its temperature increases by ΔT.

Increase in length a, $\Delta a = \alpha a \Delta T$

Increase in breadth b, $\Delta b = \alpha b \Delta T$

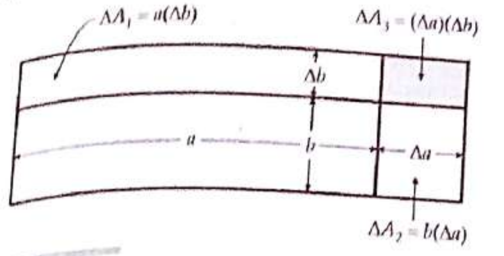


Fig. 11.9

Total increase in area of the sheet is

$$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$$

$$\Delta A = a\Delta b + b\Delta a + (\Delta a)(\Delta b)$$

$$= \alpha ab\, \Delta T + b\alpha\, a\Delta T + (\alpha)^2\, ab\,(\Delta T)^2$$

$$= \alpha ab\, \Delta T\,(2 + \alpha\Delta T) = \alpha A \Delta T\,(2 + \alpha\Delta T)$$

Since $\alpha = 10^{-5}\ K^{-1}$, the product $\alpha\Delta T$ for fractional temperature is small in comparison with 2 and may be neglected. Therefore,

$$\beta = \left(\frac{\Delta A}{A}\right)\frac{1}{\Delta T} = 2\alpha.$$

EXAMPLE 8. *Railway lines are laid with gaps to allow for expansion. If the gap between steel rails 66 m long be 3.63 cm at 10°C then at what temperature will the lines just touch? Coefficient of linear expansion of steel $= 11 \times 10^{-6}\ °C^{-1}$.*

Solution. As the rails expand in both directions, so the gap between two rails is filled by the expansion of half length of each rail. Equivalently, we can take the expansion of one rail in one direction to fill the gap.

$$l = 66\ \text{m},\ \Delta l = 3.63 \times 10^{-2}\ \text{m},$$

$$\alpha = 11 \times 10^{-6}\ °C^{-1}$$

As $\quad \Delta l = l\,\alpha\,\Delta T$

$$\therefore \quad \Delta T = \frac{\Delta l}{l\,\alpha} = \frac{3.63 \times 10^{-2}}{66 \times 11 \times 10^{-6}} = 50°C$$

Final temperature

= Initial temperature + ΔT = 10 + 50 = **60°C.**

EXAMPLE 9. *A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameters of the rim and the iron ring are 5.243 m and 5.231 m respectively at 27°C. To what temperature should the ring be heated so as to fit the rim of the wheel?* **[NCERT]**

Solution. Here $T_1 = 27°C$, $l_1 = 5.231$ m, $l_2 = 5.243$ m

As $\quad l_2 - l_1 = \alpha l_1\,(T_2 - T_1)$

$$\therefore \quad T_2 - T_1 = \frac{l_2 - l_1}{\alpha l_1} = \frac{5.243 - 5.231}{1.20 \times 10^{-5} \times 5.231} \approx 191°C$$

or $\quad T_2 = 191 + T_1 = 191 + 27 = \mathbf{218°C}.$

EXAMPLE 10. *A clock with an iron pendulum keeps correct time at 20°C. How much will it lose or gain if temperature changes to 40°C? Coefficient of cubical expansion of iron $= 36 \times 10^{-6}\ °C^{-1}$.* **[IIT]**

Solution. Time period of simple pendulum,

$$T_{20} = 2\ \text{s}$$

Let T_{40} be the time period at 40°C. If l_0, l_{20}, l_{40} be the lengths of the pendulum at 0°C, 20°C and 40°C respectively, then

$$l_{20} = l_0\,(1 + 20\,\alpha)$$

$$l_{40} = l_0\,(1 + 40\,\alpha)$$

$$T_{20} = 2\pi\sqrt{\frac{l_{20}}{g}} = 2\pi\sqrt{\frac{l_0\,(1 + 20\alpha)}{g}}$$

$$T_{40} = 2\pi\sqrt{\frac{l_{40}}{g}} = 2\pi\sqrt{\frac{l_0\,(1 + 40\,\alpha)}{g}}$$

$$\therefore \quad \frac{T_{40}}{T_{20}} = \sqrt{\frac{1 + 40\,\alpha}{1 + 20\,\alpha}} = (1 + 40\,\alpha)^{1/2}\,(1 + 20\,\alpha)^{-1/2}$$

$$= (1 + \tfrac{1}{2} \times 40\,\alpha)(1 - \tfrac{1}{2} \times 20\,\alpha)$$

[Using Binomial theorem]

$$= (1 + 20\,\alpha)(1 - 10\,\alpha) = 1 + 10\,\alpha$$

Fractional loss in time

$$= \frac{T_{40} - T_{20}}{T_{20}} = 10\,\alpha$$

$$= 10 \times 1.2 \times 10^{-5} = 1.2 \times 10^{-4}$$

As the temperature increases, time period also increases. The clock runs slow.

Time lost in 24 hours

$$= 1.2 \times 10^{-4} \times 24 \times 3600 = \mathbf{10.368\ s.}$$

EXAMPLE 11. *A metal ball 0.1 m in radius is heated from 273 to 348 K. Calculate the increase in surface area of the ball. Given coefficient of superficial expansion $= 0.000034\ K^{-1}$.*

Solution. Here $\quad r_{273} = 0.1$ m,

$$\Delta T = 348 - 273 = 75\ K, \quad \beta = 0.000034\ K^{-1}$$

$$S_{273} = 4\pi r_{273}^2 = 4\pi(0.1)^2 = \frac{4\pi}{100}\ \text{m}^2$$

Increase in surface area,

$$\Delta S = S_{273}\,\beta\,\Delta T = \frac{4\pi}{100} \times 0.000034 \times 75$$

$$= \mathbf{3.206 \times 10^{-4}\ m^2}.$$

Scanned by CamScanner

EXAMPLE 12. On heating a glass block of 10,000 cm^3, from 25°C to 40°C, its volume increases by 4 cm^3. Calculate coefficient of linear expansion of glass.

Solution. Here $V = 10{,}000\ cm^3$,

$\Delta T = (40 - 25) = 15°C,\ \Delta V = 4\ cm^3$

The coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V . \Delta T} = \frac{4}{10{,}000 \times 15} = 26.67 \times 10^{-6}\ °C^{-1}$$

∴ Coefficient of linear expansion,

$$\alpha = \frac{\gamma}{3} = \frac{26.67 \times 10^{-6}}{3} = 8.89 \times 10^{-6}\ °C^{-1}$$

EXAMPLE 13. If the volume of a block of metal changes by 0.12% when it is heated through 20°C, what is the coefficient of linear expansion of metal ?

Solution. Here $\frac{\Delta V}{V} = 0.12\% = \frac{0.12}{100}$, $\Delta T = 20°C$

Now $\gamma = \frac{\Delta V}{V . \Delta T} = \frac{0.12}{100 \times 20} = 6.0 \times 10^{-5}\ °C^{-1}$

∴ $\alpha = \gamma / 3 = 2.0 \times 10^{-5}\ °C^{-1}$.

EXAMPLE 14. Density ρ, mass m and volume V are related as ρ = m / V. Prove that

$$\gamma = -\frac{1}{\rho}\frac{d\rho}{dT}.$$

Solution. Given $\rho = \frac{m}{V} = mV^{-1}$

Differentiating both sides w.r.t. temperature T, we get

$$\frac{d\rho}{dT} = -mV^{-2}\frac{dV}{dT} \qquad [\because m = \text{constant}]$$

$$= -\frac{m}{V} \cdot \frac{dV}{V\,dT} = -\rho\gamma \qquad \left[\because \frac{dV}{V . dT} = \gamma\right]$$

∴ $\gamma = -\frac{1}{\rho}\frac{d\rho}{dT}$.

✻ PROBLEMS FOR PRACTICE

1. How much the temperature of a brass rod should be increased so as to increase its length by 1% ? Given that α for brass = 0.00002 $°C^{-1}$. **(Ans. 500°C)**
2. A steel scale measures the length of a copper rod as 80 cm when both are at 20°C, the calibration temperature of the scale. What would the scale read for the length of the rod when both are at 40°C ? Given α for steel = $1.1 \times 10^{-5}\ °C^{-1}$ and α for copper = $1.7 \times 10^{-5}\ °C^{-1}$. **(Ans. 80.0096 cm)**
3. A steel metre scale is to be ruled so that the millimetre intervals are accurate to within about 5×10^{-5} mm at a certain temperature. What is the maximum temperature variation allowable during the ruling ? Given α for steel = $1.1 \times 10^{-5}\ °C^{-1}$. **(Ans. 4.5°C)**
4. A brass rod at 30°C is observed to be 1 metre long when measured by a steel scale which is correct at 0°C. Find the correct length of the rod at 0°C. Given α for steel = 0.000012 per °C and α for brass = 0.000019 per °C. **(Ans. 99.98 cm)**
5. A cylinder of diameter 1.0 cm at 30°C is to be slid into a hole in a steel plate. The hole has a diameter of 0.99970 cm at 30°C. To what temperature must the plate be heated ? For steel, $\alpha = 1.1 \times 10^{-5}\ °C^{-1}$. **(Ans. 57.3°C)**
6. What should be the lengths of steel and copper rods at 0°C that the length of steel rod is 5 cm longer than copper at all temperatures ? Given α for copper = $1.7 \times 10^{-5}\ °C^{-1}$ and α for steel = $1.1 \times 10^{-5}\ °C^{-1}$. [IIT] **(Ans. 9.17cm, 14.17 cm)**
7. A clock having a brass pendulum beats seconds at 30°C. How many seconds will it lose or gain per day when temperature falls to 10°C ? Given α for brass = $1.9 \times 10^{-5}\ °C^{-1}$. **(Ans. gain of 16.42 s)**
8. A steel wire 2 mm in diameter is stretched between two clamps, when its temperature is 40°C. Calculate the tension in the wire when its temperature falls to 30°C.

   Given α for steel = $1.1 \times 10^{-6}\ °C^{-1}$ and Y for steel = 21×10^{11} dyne cm^{-2}. **(Ans. 7.26×10^6 dyne)**
9. Calculate the force required to prevent a steel wire of 1 mm^2 cross-section from contracting when it cools from 60°C to 15°C, if Young's modulus for steel is $2 \times 10^{11}\ Nm^{-2}$ and its coefficient of linear expansion is 0.000011 $°C^{-1}$. **(Ans. 99 N)**
10. The design of some physical apparatus requires that there be a constant difference in length at any temperature between iron and copper cylinders laid side by side. What should be the length of the cylinders at 0°C for the difference in length to be 10 cm at all temperatures ? Given α for iron = $1.1 \times 10^{-5}\ °C^{-1}$ and for copper = $1.7 \times 10^{-5}\ °C^{-1}$.

    (Ans. Length of iron cylinder = 28.33 cm,
    Length of copper cylinder = 18.33 cm.)
11. An iron sphere has a radius of 10 cm at a temperature of 0°C. Calculate the change in the volume of the sphere, if it is heated to 100°C. Coefficient of linear expansion of iron = $11 \times 10^{-6}\ °C^{-1}$. **(Ans. 13.8 cm^3)**
12. The volume of a metal sphere is increased by 1% of its original volume when it is heated from 320 K to 522 K. Calculate the coefficients of linear, superficial and cubical expansion of the metal.

    (Ans. $1.67 \times 10^{-5}\ °C^{-1}$, $3.34 \times 10^{-5}\ °C^{-1}$, $5 \times 10^{-5}\ °C^{-1}$)
13. The density of mercury is 13.6 g cm^{-3} at 0°C and its coefficient of cubical expansion is $1.82 \times 10^{-4}\ °C^{-1}$. Calculate the density of mercury at 50°C. **(Ans. 13.48 g cm^{-3})**
14. Suppose that one early morning when the temperature is 10°C, a driver of an automobile gets his gasoline tank which is made of steel, filled with 75 litre of gasoline, which is also at 10°C. During the day, the temperature rises to 30°C. How much gasoline will overflow ?

    Given α for steel = $1.2 \times 10^{-5}\ °C^{-1}$ and γ for gasoline = $9.5 \times 10^{-4}\ °C^{-1}$. **(Ans. 1.37 litre)**
15. A one litre flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in this flask ? Given α for glass = $9 \times 10^{-6}\ °C^{-1}$ and γ for mercury = $1.8 \times 10^{-4}\ °C^{-1}$. [IIT] **(Ans. 150 cm^3)**

✻ HINTS

2. 1 cm length of steel scale at 40°C
   $= 1 + 1.1 \times 10^{-5} \times (40 - 20) = 1.00022$ cm
   Length of copper rod at 40°C
   $= 80 + 80 \times 1.7 \times 10^{-5} \times (40 - 20) = 80.0272$ cm
   Number of divisions on steel scale at 40°
   $= \frac{80.0272}{1.00022} =$ **80.0096 cm.**
3. $\Delta T = \frac{\Delta l}{\alpha\, l} = \frac{5 \times 10^{-5}\ mm}{1.1 \times 10^{-5}\ °C^{-1} \times 1.0\ mm} =$ **4.5°C.**
4. As the scale is correct at 0°C, so each division of scale is 1 cm. At 30°C, each cm division becomes $(1 + 0.000012 \times 30)$ cm = 1.00036 cm.
   ∴ True length of steel scale at 30°C
   $= 100 \times 1.00036 = 100.036$ cm
   Length of brass rod at 30°C = 100.036 cm
   If l be the length of brass rod at 0°C, then
   $l(1 + 0.000019 \times 30) = 100.036$
   or $l = \frac{100.036}{1.00057} =$ **99.98 cm.**
6. Let l be the length of copper rod at 0°C, then length of steel rod at 0°C must be $l + 5$
   ∴ $l \times 1.7 \times 10^{-5} \times \Delta T = (l + 5) \times 1.1 \times 10^{-5} \times \Delta T$
   $1.7\,l = 1.1\,l + 5.5$ or $0.6\,l = 5.5$
   ∴ Length of copper rod, $l =$ **9.17 cm**
   Length of steel rod = 9.17 + 5 = **14.17 cm**

7. Here $T_{30} = 2$ s
   $l_{30} = l_0(1 + 30\alpha)$
   $l_{10} = l_0(1 + 10\alpha)$

522 K. Calculate the coefficients of linear, superficial and cubical expansion of the metal.

(**Ans.** $1.67 \times 10^{-5}\,°C^{-1}$, $3.34 \times 10^{-5}\,°C^{-1}$, $5 \times 10^{-5}\,°C^{-1}$)

13. The density of mercury is $13.6\ g\ cm^{-3}$ at 0°C and its coefficient of cubical expansion is $1.82 \times 10^{-4}\,°C^{-1}$. Calculate the density of mercury at 50°C.

(**Ans.** $13.48\ g\ cm^{-3}$)

14. Suppose that one early morning when the temperature is 10°C, a driver of an automobile gets his gasoline tank which is made of steel, filled with 75 litre of gasoline, which is also at 10°C. During the day, the temperature rises to 30°C. How much gasoline will overflow ?

Given α for steel $= 1.2 \times 10^{-5}\,°C^{-1}$ and γ for gasoline $= 9.5 \times 10^{-4}\,°C^{-1}$. (**Ans.** 1.37 litre)

15. A one litre flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in this flask ? Given α for glass $= 9 \times 10^{-6}\,°C^{-1}$ and γ for mercury $= 1.8 \times 10^{-4}\,°C^{-1}$. [IIT] (**Ans.** $150\ cm^3$)

HINTS

2. 1 cm length of steel scale at 40°C

$$= 1 + 1.1 \times 10^{-5} \times (40 - 20) = 1.00022\ cm$$

Length of copper rod at 40°C

$$= 80 + 80 \times 1.7 \times 10^{-5} \times (40 - 20) = 80.0272\ cm$$

Number of divisions on steel scale at 40°

$$= \frac{80.0272}{1.00022} = \mathbf{80.0096\ cm.}$$

3. $$\Delta T = \frac{\Delta l}{\alpha\, l} = \frac{5 \times 10^{-5}\ mm}{1.1 \times 10^{-5}\,°C^{-1} \times 1.0\ mm} = \mathbf{4.5\,°C.}$$

4. As the scale is correct at 0°C, so each division of scale is 1 cm. At 30°C, each cm division becomes $(1 + 0.000012 \times 30)\ cm = 1.00036\ cm$

True length of steel scale at 30°C

$$= 100 \times 1.00036 = 100.036\ cm$$

Length of brass rod at 30°C = 100.036 cm

If l be the length of brass rod at 0°C, then

$$l\,(1 + 0.000019 \times 30) = 100.036$$

or $$l = \frac{100.036}{1.00057} = \mathbf{99.98\ cm}$$

6. Let l be the length of copper rod at 0°C, then length of steel rod at 0°C must be $l + 5$

$$\therefore\ l \times 1.7 \times 10^{-5} \times \Delta T = (l + 5) \times 1.1 \times 10^{-5} \times \Delta T$$

$$1.7\,l = 1.1\,l + 5.5 \quad \text{or} \quad 0.6\,l = 5.5$$

$\therefore$ Length of copper rod, $l = \mathbf{9.17\ cm}$

Length of steel rod $= 9.17 + 5 = \mathbf{14.17\ cm.}$

7. Here $T_{30} = 2\,s$

$$l_{30} = l_0\,(1 + 30\,\alpha)$$

$$l_{10} = l_0\,(1 + 10\,\alpha)$$

$$T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} = 2\pi\sqrt{\frac{l_0\,(1 + \alpha \times 10)}{g}}$$

$$T_{30} = 2\pi\sqrt{\frac{l_{30}}{g}} = 2\pi\sqrt{\frac{l_0\,(1 + 30\,\alpha)}{g}}$$

$$\therefore\ \frac{T_{10}}{T_{30}} = \sqrt{\frac{1 + 10\,\alpha}{1 + 30\,\alpha}} = (1 + 10\,\alpha)^{1/2}\,(1 + 30\,\alpha)^{-1/2}$$

$$= \left(1 + \frac{1}{2}\,10\alpha\right)\left[1 + 30\left(-\frac{1}{2}\right)\alpha\right]$$

[Using Binomial theorem]

$$= (1 + 5\alpha)(1 - 15\alpha) = (1 - 10\alpha)$$

$$\therefore\ T_{10} = T_{30}\,(1 - 10\alpha)$$

or $$T_{10} = 2\,(1 - 10 \times 1.9 \times 10^{-5})\,s$$

$$= (2 - 3.8 \times 10^{-4})\,s$$

As $T_{10} < T_{30}$, the clock gains in time when the temperature falls to 10°C.

For 2 seconds, gain in time $= T_{30} - T_{10} = 3.8 \times 10^{-4}\,s$

For 1 day, gain in time

$$= \frac{3.8 \times 10^{-4} \times 24 \times 3600}{2}\,s = \mathbf{16.42\ s.}$$

8. $\Delta l = l\,\alpha\,\Delta T$

$$= l \times 1.1 \times 10^{-5} \times (40 - 30) = 1.1\,l \times 10^{-4}\ cm$$

As $$Y = \frac{F \times l}{A \times \Delta l} = \frac{F \times l}{\pi r^2 \times \Delta l}$$

$\therefore$ Tension, $$F = \frac{Y \times \pi r^2 \times \Delta l}{l}$$

$$= \frac{21 \times 10^{11} \times 22 \times (0.1)^2 \times 1.1\,l \times 10^{-4}}{7 \times l}$$

$$= \mathbf{7.26 \times 10^{6}\ dyne.}$$

10. Let l_1 and l_2 be lengths of iron and copper cylinders respectively and α_1, α_2 be their coefficients of linear expansion

Increase in length of iron rod $= l_1\,\alpha_1\,t$

Increase in length of copper rod $= l_2\,\alpha_2\,t$

According to the problem,

$$l_1 - l_2 = 10\ cm \qquad ...(i)$$

and $(l_1 + l_1\,\alpha_1\,t) - (l_2 + l_2\,\alpha_2\,t) = 10$

or $$(l_1 - l_2) + l_1\,\alpha_1\,t - l_2\,\alpha_2\,t = 10 \qquad ...(ii)$$

From (i) and (ii), $l_1\,\alpha_1\,t = l_2\,\alpha_2\,t$

or $$\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1} = \frac{1.7 \times 10^{-5}}{1.1 \times 10^{-5}} = \frac{17}{11} \quad \text{or} \quad l_1 = \frac{17}{11}\,l_2$$

From (i), $$\frac{17}{11}\,l_2 - l_2 = 10 \quad \text{or} \quad \frac{6}{11}\,l_2 = 10$$

Scanned by CamScanner

or $\qquad l_c = \frac{110}{6} = 18.33\ \text{cm}$

and $\qquad l_i = 10 + 18.33 = 28.33\ \text{cm}$

Length of iron cylinder $= 28.33$ cm

Length of copper cylinder $= 18.33$ cm.

13. Here $\quad V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (10)^3\ \text{cm}^3$

$$\gamma = 3\alpha = 3 \times 11 \times 10^{-6}\ {}^\circ\text{C}^{-1}$$

$$\Delta T = 100 - 0 = 100^\circ\text{C}$$

Change in volume,

$$\Delta V = V\gamma\Delta T = \frac{4}{3} \times 3.14 \times (10)^3 \times 3 \times 11 \times 10^{-6} \times 100$$

$$= 13.8\ \text{cm}^3$$

14. Change in volume of gasoline,

$$\Delta V_g = \gamma_g V \Delta T = 9.5 \times 10^{-4} \times V \times 20$$
$$= 190 \times 10^{-4}\ V$$

Change in volume of steel tank,

$$\Delta V_s = \gamma_s V \Delta T = 3\alpha_s V \Delta T$$
$$= 3 \times 12 \times 10^{-6} \times V \times 20 = 7.2 \times 10^{-4}\ V$$

Volume of gasoline that overflows,

$$\Delta V_g - \Delta V_s = (190 \times 10^{-4} - 7.2 \times 10^{-4})\ V$$
$$= 182.8 \times 10^{-4} \times 75 = 1.37\ \text{litre.}$$

15. Volume of glass flask = Volume of mercury + Volume of air

or $\qquad V = V_m + V_a$

As V_a remains constant, so $\Delta V = \Delta V_m$

or $\quad \gamma_g V \Delta T = \gamma_m V_m \Delta T$

or $\qquad V_m = \frac{\gamma_g V}{\gamma_m} = \frac{3\alpha_g V}{\gamma_m}$

$$= \frac{3 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}} \times 1000\ \text{cm}^3 = 150\ \text{cm}^3.$$

11.20 SPECIFIC HEAT

31. *Define the terms specific heat and molar specific heat. Give their CGS and SI units.*

Specific heat. *The specific heat of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance through one degree.* It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is needed to raise the temperature of M mass of a substance through ΔT, then specific heat is given by

$$c = \frac{\Delta Q}{M \times \Delta T}$$

The CGS unit of specific heat is cal $g^{-1}\ {}^\circ C^{-1}$ and the SI unit is J $kg^{-1}\ K^{-1}$.

Clearly, the amount of heat required to raise the temperature of M mass of a substance through ΔT is

$$\Delta Q = Mc\,\Delta T.$$

Molar specific heat. *The molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one mole of the substance through one degree.* It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is required to raise the temperature of n moles of a substance through ΔT, then molar specific heat is given by

$$C = \frac{\Delta Q}{n\,\Delta T}$$

The CGS unit of molar specific heat is cal $mol^{-1}\ {}^\circ C^{-1}$ and SI unit is J $mol^{-1}\ K^{-1}$.

Therefore, the amount of heat required to raise the temperature of n moles of a substance through ΔT is

$$\Delta Q = nC\,\Delta T.$$

32. *Define the terms heat capacity and water equivalent. Give their CGS and SI units.*

Heat capacity or thermal capacity. *The heat capacity of a body is defined as the amount of heat required to raise its temperature through one degree.*

By definition, the amount of heat required to raise the temperature of unit mass of a body is equal to specific heat c. So heat required for m mass is $m \times c$.

$\therefore$ Heat capacity = Mass × Specific heat

or $\qquad S = mc$

The CGS unit of heat capacity is cal $^\circ C^{-1}$ and the SI unit is JK^{-1}.

Water equivalent. *The water equivalent of a body is defined as the mass of water which requires the same amount of heat as is required by the given body for the same rise of temperature.*

Water equivalent = Mass × Specific heat

or $\qquad w = mc$

The CGS unit of water equivalent is g and the SI unit is kg.

11.21 CALORIMETRY

33. *What is calorimetry ? State the principle of calorimetry.*

Calorimetry. *The branch of physics that deals with the measurement of heat is called calorimetry.*

Principle of calorimetry or the law of mixtures. Whenever two bodies at different temperatures are placed in contact with one another, heat flows from the body at higher temperature to the body at lower temperature till the two bodies acquire the same temperature.

Scanned by CamScanner

The principle of calorimetry states that the heat gained by the cold body must be equal to the heat lost by the hot body, provided there is no exchange of heat with the surroundings.

Heat gained = Heat lost

This principle is a consequence of the law of conservation of energy and useful for solving problems relating to calorimetry.

34. *Briefly describe the construction of a calorimeter.*

Calorimeter. *It is a device used for measuring the quantities of heat.* It consists of a cylindrical vessel of copper provided with a stirrer. The vessel is kept inside a wooden jacket. The space between the calorimeter and the jacket is packed with a heat insulating material like glass wool, etc. Thus the calorimeter gets thermally isolated from the surroundings. The loss of heat due to radiation is further reduced by polishing the outer surface of the calorimeter and the inner surface of the jacket. The lid is provided with holes for inserting a thermometer and a stirrer into the calorimeter.

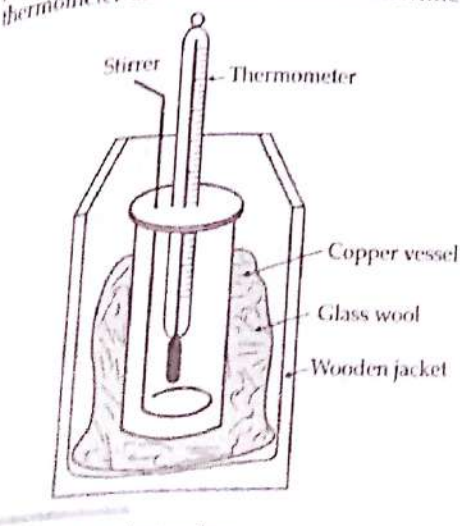


Fig. 11.10 A calorimeter

When bodies at different temperatures are mixed together in the calorimeter, heat is exchanged between the bodies as well as with the calorimeter. If there is no loss of heat to the surroundings, then according to the principle of calorimetry,

Heat gained by cold bodies = Heat lost by hot bodies.

This equation can be used to determine the specific heat and latent heat of different substances.

11.22 CHANGE OF STATE

35. *What do you mean by change of state of a substance ?*

Change of state. Matter exists in *three* states : *solid, liquid* and *gas*. Any state of a substance can be changed into another by heating or cooling it. *The transition of a substance from one state to another is called a change of state.*

The common changes of states are as follows :

1. Melting of a solid :

$$\text{Solid} \xrightarrow{\text{Heat}} \text{Liquid}$$

2. Vaporization of a liquid :

$$\text{Liquid} \xrightarrow{\text{Heat}} \text{Vapour}$$

3. Condensation of vapour :

$$\text{Vapour} \xrightarrow{\text{Cool}} \text{Liquid}$$

4. Freezing of a liquid :

$$\text{Liquid} \xrightarrow{\text{Cool}} \text{Solid}$$

36. *By a simple experiment, show that temperature of a substance remains constant during its change of state.*

Effect of heat on the change of state. Take some ice cubes in a beaker. Note the temperature of ice. It will be 0°C. As shown in Fig. 11.11, start heating it slowly on a constant heat source. Note the temperature after every minute. Continuously stir the mixture of water and ice.

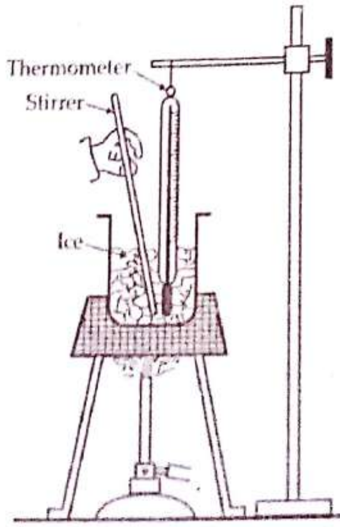


Fig. 11.11 Effect of heat on ice.

Plot a graph between temperature recorded and time. We obtain a curve of the shape shown in Fig. 11.12. It is found that the temperature does not change as long as there is any ice left in the beaker. Here the heat supplied is being used in changing the state from solid (ice) to liquid (water). *The change of state from solid to liquid is called* ***melting*** *and from liquid to solid is called* ***fusion***. It is seen that the temperature

Scanned by CamScanner

remains constant until the entire amount of the solid substance melts. Thus *both the solid and liquid states of the substance coexist in thermal equilibrium during the change of state from solid to liquid.*

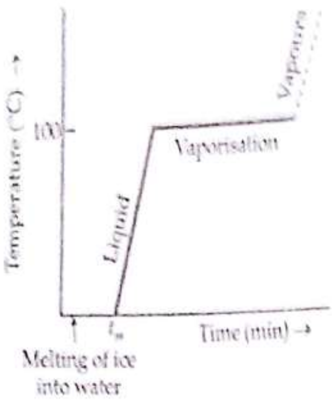


Fig. 11.12 A plot of temperature versus time showing the changes in the state of ice on heating (not to scale).

After the whole of ice gets melted into water and as we continue heating the beaker, we note that the temperature begins to increase till it reaches nearly 100°C when it again becomes steady. The heat supplied is now being used to change state of water from liquid to vapour. *The change of state from liquid to vapour is called* ***vaporisation***. It is noted that the temperature remains constant until the entire amount of the liquid is converted into vapour. Thus *both the liquid and the vapour states of the substance coexist in thermal equilibrium.*

Melting point. *The temperature at which the solid and the liquid states of a substance coexist in thermal equilibrium with each other is called its melting point.* It is a characteristic of the substance but also depends on pressure. *The melting point of a substance at standard atmospheric pressure is called its* ***normal melting point.***

Boiling point. *The temperature at which the liquid and vapour states of a substance coexist in thermal equilibrium with each other is called its boiling point. The boiling point of a substance at standard atmospheric pressure is called its* ***normal boiling point.***

Sublimation. Some substances, on being heated, pass from the solid state to liquid state directly. *The process of transition of a substance from the solid state to the vapour state without passing through the liquid state is called* ***sublimation***, and the substance is said to sublime. Substances like dry ice (solid CO_2), iodine, naphthalene and camphor undergo sublimation when heated. During the sublimation process, the solid and vapour states of a substance coexist in thermal equilibrium with each other.

11.23 EFFECT OF PRESSURE ON MELTING AND BOILING POINTS

37. *What is the effect of pressure on the melting point of a substance ? What is regelation ? Give an experiment to illustrate it.*

Effect of pressure on melting point. Under a given pressure, a pure substance melts at definite temperature. However, the melting point changes with the change in pressure. We know that paraffin wax expands on melting. An increase in pressure will make its expansion difficult. We will have to heat it more to melt it. That is, the increase in pressure increases the melting point of wax. On the other hand, ice contracts on melting. The increase in pressure will help in its contraction. So we expect a decrease in the melting point of ice as the pressure on it is increased. We can generalise these observations as follows :

The melting point of those substances which expand on melting (e.g., paraffin wax, phosphorus, sulphur, etc.) increases with the increase in pressure while the melting point of those substances which contract on melting (e.g., ice, cast iron, bismuth etc.) decreases with increase in pressure.

Effect of pressure on freezing point of ice : Regelation. When two pieces of ice are pressed against one another for few seconds and then released, they get frozen at the surface of contact. As the pressure is increased, the melting point of ice is lowered and ice melts. When pressure is released, the water so formed (at a temperature <0°C) immediately freezes again. This phenomenon of refreezing is called regelation.

The phenomenon in which ice melts when pressure is increased and again freezes when pressure is removed is called ***regelation*** (re = again ; gelare = freeze) :

We can demonstrate the phenomenon of regelation through the following simple experiment.

Take a slab of ice and support it on two wooden blocks. Take a metallic wire and attach two heavy weights, say 5 kg each, at its ends. Put the wire over the slab, as shown in Fig. 11.13. It is seen that the wire

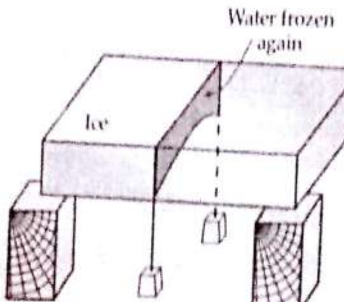


Fig. 11.13 A wire cuts its way through an ice slab without cutting it into two pieces.

Scanned by CamScanner

gradually cuts its way through the ice without cutting it into two pieces. Just below the wire, ice melts at a lower temperature due to the increase in pressure. When the wire has passed, water above the wire freezes again. Thus the wire passes through the slab and the slab does not split. *This phenomenon of refreezing is called regelation.*

Practical applications of regelation :

1. By pressing snow in our hand, we can transform it into a snow-ball. When snow is pressed, its crystals melt. As the pressure is released, water refreezes forming a snow-ball.
2. When the wheels of cart pass over snow, ice melts due to increase in pressure exerted by the wheels. When pressure is released, water so formed refreezes on the wheels. That is why wheels are covered with ice.
3. Skating is possible due to the formation of water layer below the skates. Water is formed due to the increase of pressure and it acts as a lubricant.
4. The ice of a glacier, pressed against the sides of its valley melts, and in this way adopts itself to the shape of the valley.

38. *What is the effect of pressure on the boiling point of a liquid ? Explain it with the help of a simple experiment.*

Effect of pressure on the boiling point of a liquid. *The boiling point of a liquid increases with the increase in pressure.* At 1 atmospheric pressure, the boiling point of water is 100°C. At 2 atmospheric pressure, the boiling point of water is 128°C.

To study the effect of pressure on boiling point, take a round bottom flask and fill it more than half with water. Support it over a burner and fix a thermometer and a steam outlet through the cork fitted in the neck of the flask, as shown in Fig. 11.14.

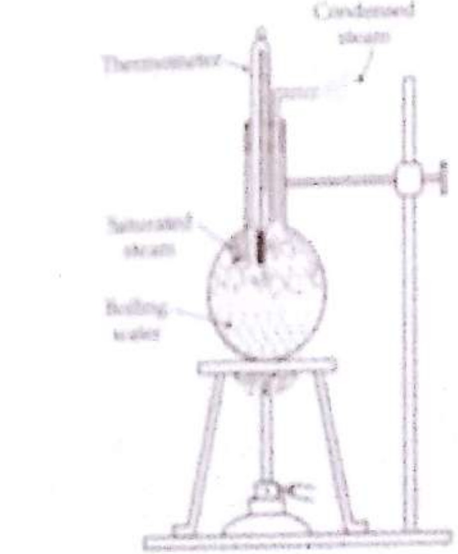


Fig. 11.14 Boiling process

Heat the flask over the burner slowly. As the temperature rises, small bubbles of water begin to form at the bottom which rise through water and escape from the surface. At a temperature of about 70° – 80°C, bubbles of steam begin to form at the bottom. These bubbles of steam rise to the upper cold layers of water, where they condense and disappear producing of a peculiar noise called *singing of the vessel*.

As the temperature of the entire mass of water becomes 100°C, the bubbles of steam begin to escape more rapidly and water gets violently agitated and *boiling is said to occur*. As the steam comes out of the flask, it condenses as tiny droplets of water, giving a foggy appearance.

If the steam outlet is partly closed, the pressure inside the flask increases and boiling stops. Water now has to be heated to a higher temperature to make it boil again. Thus *the boiling point increases with the increase in pressure.*

Remove the burner and allow the water to cool to a temperature below 100°C. If the pressure is reduced by removing the partial covering of the outlet, water begins to boil again. Thus *the boiling point decreases with the decrease in pressure.*

39. *Describe a simple experiment to demonstrate the boiling of water at a temperature much lower than 100°C.*

Franklin's experiment. Take a round-bottom flask and fill it about half with water. Heat it so that water begins to boil. Continue boiling for some time so that air of the flask escapes to the atmosphere and the empty space is filled with steam. Remove the burner. Close the flask with airtight cork. Hold the flask in the clamp of a stand in an inverted position, as shown in Fig. 11.15. Allow the water to cool for some time so that boiling stops altogether. Pour cold water on the flask. Due to the condensation of steam, the pressure over

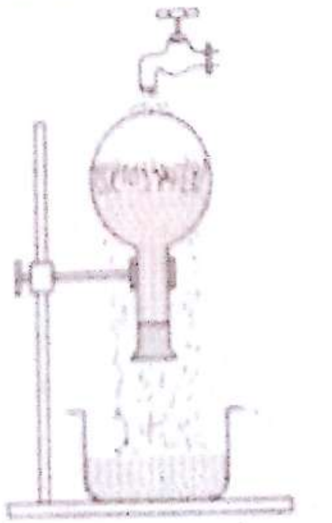


Fig. 11.15 Water boils below 100°C under reduced pressure.

Scanned by CamScanner

the water surface decreases. Water begins to boil again now at a lower temperature. This shows that the boiling point decreases with the decrease in pressure.

Practical Applications :

1. Cooking is difficult at mountains. The atmospheric pressure at mountains is much lower than that at plains, so water starts boiling at a temperature much lower than 100°C. Hence cooking is difficult.
2. The pressure inside a pressure cooker is increased much above the atmospheric pressure by not allowing the steam to escape. This increases the boiling point. Hence the vegetables are cooked inside a pressure cooker in a shorter time.

11.24 LATENT HEAT

40. What is latent heat ? Give its units. With the help of a suitable graph, explain the terms latent heat of fusion and latent heat of vaporisation.

Latent Heat. When a solid changes into liquid or a liquid changes into gas, it absorbs heat. But this heat does not show up as an increase in temperature. This heat, used to change the state, is hidden or *latent* (lying hidden), and is therefore called latent heat.

The amount of heat required to change the state of unit mass of a substance at constant temperature and pressure is called latent heat of the substance.

If *m* mass of a substance undergoes a change from one state to another, then the amount of heat required for the process is

$$Q = mL$$

where *L* is the latent heat of the substance and is a characteristic of the substance. Its value also depends on the pressure. Clearly,

$$L = \frac{Q}{m}$$

SI unit of latent heat = J kg^{-1}

CGS unit of latent heat = cal g^{-1}

Latent heat of fusion. *The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion or latent heat of melting. It is denoted by* L_f.

Latent heat of vaporisation. *The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point is called latent heat of vaporization or latent heat of boiling. It is denoted by* L_v.

Fig. 11.16 shows the plot of the heat supplied vs. rise of temperature. Here we take 1 kg of ice at −20°C. As we start heating, the temperature of ice increases until it reaches its melting point (0°C). At this temperature, the addition of more heat does not increase the temperature but causes the ice to melt, or changes its state. As is obvious from the graph, the latent heat of fusion of ice is,

$$L_f = 3.3 \times 10^5 \text{ J kg}^{-1}$$

That is, 3.3×10^5 J of heat is needed to melt 1 kg of ice into water at 0°C.

Once the entire ice melts, the addition of more heat causes the temperature of water to rise. The temperature keeps on increasing till it reaches 100°C. The water starts boiling. The addition of more heat to the boiling water causes vaporisation, without increase in temperature. Obviously, the latent heat of vaporisation of water is,

$$L_v = 22.6 \times 10^5 \text{ J}$$

That is, 22.6×10^5 J of heat is needed to convert 1 kg of water to steam at 100°C. Additional heat causes the temperature of the steam to rise.

By conservation of energy, when 1 kg of steam condenses to water, it gives up 22.6×10^5 J of heat. That is why steam burns are more serious than burns from boiling water, even though both are at 100°C.

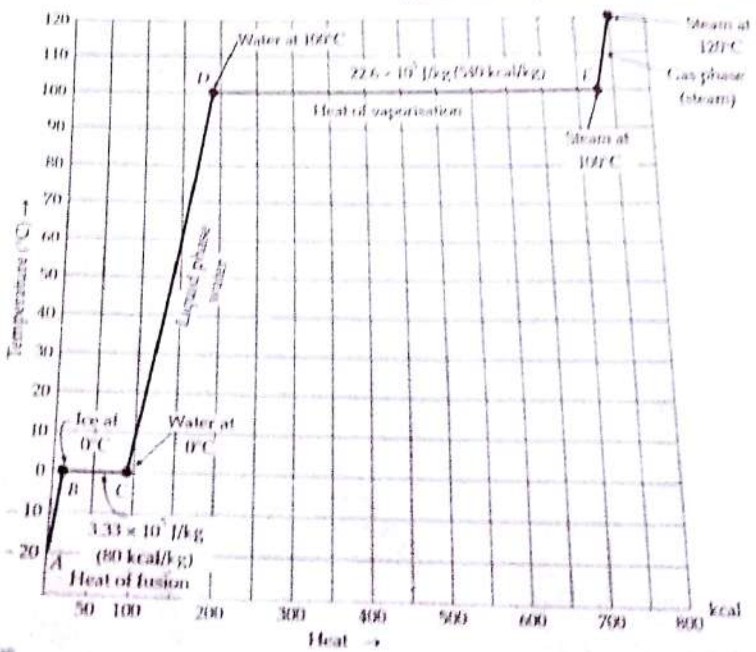


Fig. 11.16 Temperature vs. heat for 1 kg of water at atmospheric pressure.

Table 11.3 Temperatures of the change of state and latent heats for various substances at 1 atm pressure

Substance	Melting point (°C)	L_f (10^5 J kg^{-1})	Boiling point (°C)	L_v (10^5 J kg^{-1})
Ethyl alcohol	−114	1.0	78	8.5
Gold	1063	0.645	2660	15.8
Lead	328	0.25	1744	[illegible]
Mercury	−39	0.12	357	[illegible]
Nitrogen	−210	0.26	−196	[illegible]
Oxygen	−219	0.14	−183	[illegible]
Water	0	3.33	100	[illegible]

Examples based on Specific Heat and Latent H

FORMULAE USED

1. Heat gained or lost, $Q = mc\Delta T$
2. According to the principle of calorim
   Heat gained = Heat lost
3. Water equivalent, $w = mc$ (gram)
4. Heat capacity = mc (cal $°C^{-1}$)
5. Latent heat of vaporisation or f

UNITS USED

In CGS system, heat Q is in c specific heat c in cal g^{-1} $°C^{-1}$ heat Q is in joule, mass m in J kg^{-1} K^{-1} and ΔT in K.

changes its state. As is obvious from the graph, the latent heat of fusion of ice is,

$$L_f = 3.3 \times 10^5 \text{ J kg}^{-1}$$

That is, 3.3×10^5 J of heat is needed to melt 1 kg of ice into water at 0°C.

Once the entire ice melts, the addition of more heat causes the temperature of water to rise. The temperature keeps on increasing till it reaches 100°C. The water starts boiling. The addition of more heat to the boiling water causes vaporisation, without increase in temperature. Obviously, the latent heat of vaporisation of water is,

$$L_v = 22.6 \times 10^5 \text{ J.}$$

That is, 22.6×10^5 J of heat is needed to convert 1 kg of water to steam at 100°C. Additional heat causes the temperature of the steam to rise.

By conservation of energy, when 1 kg of steam condenses to water, it gives up 22.6×10^5 J of heat. That is why steam burns are more serious than burns from boiling water, even though both are at 100°C.

Table 11.3 *Temperatures of the change of state and latent heats for various substances at 1 atm pressure*

Substance	Melting point (°C)	L_f (10^5 J kg^{-1})	Boiling point (°C)	L_v (10^5 J kg^{-1})
Ethyl alcohol	– 114	1.0	78	8.5
Gold	1063	0.645	2660	15.8
Lead	328	0.25	1744	8.67
Mercury	– 39	0.12	357	2.7
Nitrogen	– 210	0.26	– 196	2.0
Oxygen	– 219	0.14	– 183	2.1
Water	0	3.33	100	22.6

Examples based on Specific Heat and Latent Heat

FORMULAE USED

1. Heat gained or lost, $Q = mc\,\Delta T$
2. According to the principle of calorimetry,

   Heat gained = Heat lost
3. Water equivalent, $w = mc$ (gram)
4. Heat capacity $= mc$ (cal °C^{-1})
5. Latent heat of vaporisation or fusion, $Q = mL$.

UNITS USED

In CGS system, heat Q is in cal, mass m in gram, specific heat c in cal g^{-1} °C^{-1} and ΔT in °C. In SI, heat Q is in joule, mass m in kg, specific heat c in J kg^{-1} K^{-1} and ΔT in K.

EXAMPLE 15. *A sphere of aluminium of 0.047 kg is placed for sufficient time in a vessel containing boiling water, so that the sphere is at 100°C. It is then immediately transferred to 0.14 kg copper calorimeter containing 0.25 kg of water at 20°C. The temperature of water rises and attains a steady state at 23°C. Calculate the specific heat capacity of aluminium.* [NCERT]

Solution. Mass of aluminium sphere,

$$m = 0.047 \text{ kg}$$

Fall in temperature of aluminium sphere,

$$\Delta T = 100 - 23 = 77°\text{C}$$

Let specific heat of aluminium $= c_{Al}$

Heat lost by the aluminium sphere

$$= mc_{Al}\,\Delta T = 0.047 \text{ kg} \times c_{Al} \times 77°\text{C}$$

Mass of water, $m_1 = 0.25$ kg

Mass of calorimeter, $m_2 = 0.14$ kg

Initial temperature of water and calorimeter = 20°C

Final temperature of the mixture = 23°C

Rise in temperature of mixture,

$$\Delta T' = 23 - 20 = 3°\text{C}$$

Specific heat of water,

$$c_w = 4.18 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$

Specific heat of copper,

$$c_{Cu} = 0.386 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$

Heat gained by water and calorimeter

$$= m_1 c_w \,\Delta T' + m_2 c_{Cu}\,\Delta T'$$
$$= (0.25 \times 4.18 \times 10^3 + 0.14 \times 0.386 \times 10^3) \times 3 \text{ J}$$

In the steady state,

Heat lost = Heat gained

or $$0.047 \times c_{Al} \times 77 = (0.25 \times 4.18 \times 10^3 + 0.14 \times 0.386 \times 10^3) \times 3$$

or $$c_{Al} = 0.911 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} = \mathbf{0.911 \text{ kJ kg}^{-1} \text{ K}^{-1}}.$$

EXAMPLE 16. *A thermally isolated vessel contains 100 g of water at 0°C when air above the water is pumped out, some of the water freezes and some evaporates at 0°C itself. Calculate the mass of ice formed, if no water is left in the vessel. Latent heat of vaporisation of water at 0°C = 2.10×10^6 J kg^{-1} and latent heat of fusion of ice = 3.36×10^5 J kg^{-1}.*

Solution. Latent heat of vaporisation of water at 0°C,

$$L_1 = 2.10 \times 10^6 \text{ J kg}^{-1} = 2.10 \times 10^3 \text{ Jg}^{-1}$$

Latent heat of fusion of ice,

$$L_2 = 3.36 \times 10^5 \text{ J kg}^{-1} = 3.36 \times 10^2 \text{ Jg}^{-1}$$

Let mass of ice formed = m gram

Scanned by CamScanner

Then mass of water evaporated

$= (100 - m)$ gram

As no water is left in the vessel,

Heat gained by water in evaporation

= Heat lost by water in freezing

or $\quad (100 - m)\, L_1 = mL_2$

or $\quad (100 - m) \times 2.10 \times 10^3 = m \times 3.36 \times 10^2$

or $\quad m = 86.2$ g.

Example 17. *When 0.15 kg of ice of 0°C is mixed with 0.30 kg of water at 50°C in a container, the resulting temperature is 6.7°C. Calculate the heat of fusion of ice. ($c_{water} = 4186\ J\ kg^{-1}\ K^{-1}$)*

Solution. Heat lost by 0.30 kg water when its temp. falls from 50°C to 6.7°C

$$= mc_{water}\, \Delta T = 0.30 \times 4186 \times (50 - 6.7)\ J$$
$$= 54376.14\ J$$

Heat required to melt 0.15 kg ice into water at 0°C

$$= mL_f = 0.15 \times L_f\ J$$

Heat required to raise temperature of 0.15 kg water from 0°C to 6.7°C

$$= mc_{water}\, \Delta T$$
$$= 0.15 \times 4186 \times (6.7 - 0) = 4206.93\ J$$

By the principle of calorimetry,

Heat gained = Heat lost

$$0.15 \times L_f + 4206.93 = 54376.14$$

or $\quad L_f = \mathbf{3.34 \times 10^5\ J\ kg^{-1}}$.

Example 18. *Calculate the heat required to convert 3 kg of ice at −12°C kept in a calorimeter to steam at 100°C at atmospheric pressure.*

Given :

specific heat capacity of ice = 2100 $J\ kg\ K^{-1}$,

specific heat capacity of water = 4186 $J\ kg^{-1}\ K^{-1}$,

latent heat of fusion of ice = $3.35 \times 10^5\ J\ kg^{-1}$

and latent heat of steam = $2.256 \times 10^6\ J\ kg^{-1}$. **[NCERT]**

Solution. Mass of the ice,

$$m = 3\ kg$$

Specific heat capacity of ice,

$$c_{ice} = 2100\ J kg^{-1}\ K^{-1}$$

Specific heat capacity of water,

$$c_{water} = 4186\ J kg^{-1}\ K^{-1}$$

Latent heat of fusion of ice,

$$L_{f,\ ice} = 3.35 \times 10^5\ J kg^{-1}$$

Latent heat of steam,

$$L_{steam} = 2.256 \times 10^6\ J kg^{-1}$$

Heat required to convert ice at −12°C to ice at 0°C,

$$Q_1 = mc_{ice}\, \Delta T_1$$
$$= (3\ kg)(2100\ J kg^{-1}\ K^{-1})[0 - (-12)]°C$$
$$= 75600\ J$$

Heat required to melt ice at 0°C to water at 0°C,

$$Q_2 = mL_{f\ ice} = (3\ kg)(3.35 \times 10^5\ J kg^{-1})$$
$$= 1005000\ J$$

Heat required to convert water at 0°C to water at 100°C,

$$Q_3 = mc_w\, \Delta T_2 = (3\ kg)(4186\ J kg^{-1}\ K^{-1})(100°C)$$
$$= 1255800\ J$$

Heat required to convert water at 100°C to steam at 100°C,

$$Q_4 = m\, L_{steam} = (3\ kg)(2.256 \times 10^6\ J kg^{-1})$$
$$= 6768000\ J$$

Total heat required to convert 3 kg of ice at −12°C to steam at 100°C,

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$
$$= 75600\ J + 1005000\ J + 1255800\ J + 6768000\ J$$
$$= \mathbf{9.1 \times 10^6\ J.}$$

Problems for Practice

1. A geyser heats water flowing at the rate of 3 kg per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is its rate of consumption of fuel if the heat of combustion is 4×10^4 J/g ? Given specific heat of water is 4.20×10^3 J / kg / K. **[Central Schools 13]** **(Ans.** 15.75 g min^{-1}**)**

2. A normal diet furnishes 2000 kcal to a 60 kg person in a day. If this energy was used to heat the person with no losses to the surroundings, how much would the person's temperature increase ? The specific heat of the human body = 0.83 cal g^{-1} °C^{-1}. **(Ans.** 40.16°C**)**

3. 0.75 gram of petroleum was burnt in a bomb calorimeter which contains 2 kg of water and has a water equivalent 500 gram. The rise in temperature was 3°C. Determine the calorific value of petroleum. **(Ans.** 10^4 cal g^{-1}**)**

4. The heat of combustion of ethane gas is 373 kcal per mole. Assuming that 60% of the heat is useful, how many litres of ethane measured at S.T.P. must be burnt to convert 50 kg of water at 10°C to steam at 100°C ? One mole of a gas occupies 22.4 litre at S.T.P. **(Ans.** 3131.5 litres**)**

5. **A refrigerator converts 50 gram of water at 15°C into ice at 0°C in one hour. Calculate the quantity of**

heat removed per minute. Take specific heat of water $= 1 \text{ cal g}^{-1} \text{ °C}^{-1}$ and latent heat of ice $= 80 \text{ cal g}^{-1}$. **(Ans.** 79.2 cal min^{-1}**)**

6. How many grams of ice at – 14°C are needed to cool 200 gram of water from 25°C to 10°C ? Take specific heat of ice = 0.5 cal g^{-1} °C^{-1} and latent heat of ice = 80 cal g^{-1}. **[MNREC 89] (Ans. 31 g)**

7. An electric heater of power 100 W raises the temperature of 5 kg of a liquid from 25°C to 31°C in 2 minutes. Calculate the specific heat of the liquid. **(Ans.** 400 J kg^{-1} °C^{-1}**)**

8. A piece of iron of mass 100 g is kept inside a furnace for a long time and then put in a calorimeter of water equivalent 10 g containing 240 g of water at 20°C. The mixture attains an equilibrium temperature of 60°C. Find the temperature of ice. Given specific heat of iron = 470 J kg^{-1} °C^{-1}. **(Ans. 953.6°C)**

9. When 0.45 kg of ice of 0°C mixed with 0.9 kg of water at 55°C in a container, the resulting temperature is 10°C. Calculate the heat of fusion of ice. (c_{water} = 4186 J kg^{-1} K^{-1}) **[Delhi 09]** **(Ans.** 334400 J kg^{-1}**)**

10. Calculate the heat required to convert 0.6 kg of ice at –20°C, kept in a calorimeter to steam at 100°C at atmospheric pressure. Given the specific heat capacity of ice = 2100 J kg^{-1} K^{-1}, specific heat capacity of water is 4186 J kg^{-1} K^{-1}, latent heat of ice = 3.35×10^5 J kg^{-1}, and latent heat of steam = 2.256×10^6 J kg^{-1}. **[Delhi 08] (Ans.** 1.8×10^8 J**)**

HINTS

1. $Q = mc\Delta T = 3 \text{ kg min}^{-1} \times 4.20 \times 10^3 \text{ J kg}^{-1}\text{K}^{-1} \times 50 \text{ K}$
$= 63 \times 10^4 \text{ J min}^{-1}$

Rate of consumption of fuel

$$= \frac{63 \times 10^4 \text{ J min}^{-1}}{4 \times 10^4 \text{ J g}^{-1}} = \mathbf{15.75 \text{ g min}^{-1}}.$$

2. Here $m = 60 \text{ kg} = 60 \times 10^3 \text{ g}$, $c = 0.83 \text{ cal g}^{-1} \text{ °C}^{-1}$

$Q = 2000 \text{ kcal} = 2 \times 10^6 \text{ cal}$

$$\Delta T = \frac{Q}{mc} = \frac{2 \times 10^6}{60 \times 10^3 \times 0.83} = \mathbf{40.16°C}.$$

3. Total heat gained by water and calorimeter,

$Q = (m + w)\, c\, \Delta T = (2000 + 500) \times 1 \times 3 = 7500 \text{ cal}$

Calorific value of petroleum $= \frac{7500}{0.75} = \mathbf{10^4 \text{ cal g}^{-1}}$.

4. Heat required to convert 50 kg of water at 10°C into steam at 100°C

$= mc\,\Delta T + mL$

$= 50 \times 10^3 \times (100 - 10) + \frac{50 \times 2.25 \times 10^6}{4.2}$

$= (4.5 + 26.79) \times 10^6 = 31.29 \times 10^6 \text{ cal}$

As only 60% of the heat is useful, so total heat produced is

$$Q = \frac{100}{60} \times 31.29 \times 10^6 = 52.15 \times 10^6 \text{ cal}$$

Heat of combustion of ethane

$= 373 \text{ kcal mole}^{-1} = 373 \times 10^3 \text{ cal mole}^{-1}$

Number of moles of ethane needed to be burnt

$$= \frac{52.15 \times 10^6}{373 \times 10^3} = 139.8 \text{ mole}$$

Volume of ethane needed to be burnt

$= 22.4 \times 139.8 = \mathbf{3131.5 \text{ litres}}.$

5. Total heat removed in converting 50 g of water at 15°C into ice at 0°C

$= mc\,\Delta T + mL = m(c\,\Delta T + L)$

$= 50(1 \times 15 + 80) = 4750 \text{ cal}$

Rate of removal of heat

$$= \frac{4750 \text{ cal}}{60 \text{ min}} = \mathbf{79.2 \text{ cal min}^{-1}}.$$

6. Heat lost by water in cooling from 25°C to 10°C,

$Q = mc\,\Delta T = 200 \times 1 \times (25 - 10) = 3000 \text{ cal}$

Heat gained by ice at – 14°C to change into water at 10°C,

$Q = (mc\,\Delta T)_{ice} + mL + (mc\,\Delta T)_{water}$

$= m \times 0.5 \times 14 + m \times 80 + m \times 1 \times 10$

$= 97\,m \text{ cal}$

By principle of calorimetry, $97\,m = 3000$

$\therefore$ Mass of ice $= 3000/97 = \mathbf{31 \text{ g}}.$

8. Here $m_1 = 100 \text{ g}$, $w = 10 \text{ g}$, $m_2 = 240 \text{ g}$, $T_1 = 20°C$,

Final temperature, $T = 60°C$,

Temperature of furnace $= T_2$ (say)

Heat lost by iron piece

= Heat gained by water and calorimeter

$m_1 c(T_2 - T) = (m_2 + w) \times 1 \times (T - T_1)$

$$\frac{100 \times 470 \times (T_2 - 60)}{4.2 \times 10^3} = (240 + 10) \times 1 \times (60 - 20)$$

or $$T_2 - 60 = \frac{250 \times 40 \times 4.2 \times 10^3}{100 \times 470} = 893.6$$

or $$T_2 = 893.6 + 60 = \mathbf{953.6°C}.$$

9. Proceed as in Example 17.

10. Proceed as in Example 18.

11.25 MODES OF TRANSFER OF HEAT : INTRODUCTION

41. *What are the three modes of transfer of heat ?*

Modes of transfer of heat. Heat can be transferred from one place to another by three different methods. These are (*i*) *conduction*, (*ii*) *convection and* (*iii*) *radiation*.

Scanned by CamScanner

Solids are usually heated by the process of conduction. Liquid and gases are heated by the process of convection. The process of radiation requires no intervening medium. We receive heat from the sun by the process of radiation. Conduction and convection are slow processes while radiation is a very fast process.

11.26 THERMAL CONDUCTION

42. *What is thermal conduction ? Briefly explain the molecular mechanism of thermal conduction.*

Thermal conduction. *It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.*

Molecular mechanism of thermal conduction. Solids are heated through conduction. When one end of a metal rod is heated, the molecules at the hot end vibrate with greater amplitude. So they have greater average kinetic energy. As these molecules collide with the neighbouring molecules of lesser kinetic energy, the energy is shared between them. The kinetic energy of the neighbouring molecules increases. This energy transfer takes place from one layer to the next, without the molecules leaving their average location. This way, heat is passed to the colder end of the rod.

11.27 STEADY STATE AND TEMPERATURE GRADIENT

43. *What do you mean by variable state and steady state in thermal conduction ? Define temperature gradient.*

Variable state, steady state and temperature gradient. Consider a metal rod heated at one end A. Heat flows from the hot end A to the cold end B by conduction.

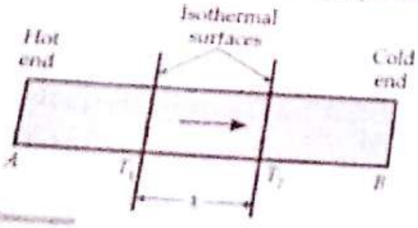


Fig. 11.17 Steady state heat flow by conduction.

In the process of conduction, each cross-section of the rod receives heat from the adjacent cross-section of the hotter side. A part of this heat is absorbed by the cross-section itself whose temperature increases, another part is lost into the atmosphere by convection and radiation from the sides of cross-section and the rest is conducted to the next cross section. *In this state the temperature of every cross-section of the rod goes on increasing with time.* The rod is said to be in the ***variable state*** of heat conduction.

Suppose the sides of the rod are covered with insulating material so that no heat is lost from the rod to the surroundings. After some time, a steady state is reached when the temperature of every cross-section of the rod becomes constant. In this state, no heat is absorbed by the rod. *This state of the rod when temperature of every cross-section of the rod becomes constant and there is no further absorption of heat in any part is called* ***steady state***. During steady state,

(*i*) the temperatures of two different parts of the rod are different, but the temperature of each part remains constant.

(*ii*) every transverse section of the rod is an *isothermal surface*.

(*iii*) the temperature decreases as we move away from the hot end.

(*iv*) the quantity of heat flowing per second through every cross-section is constant.

The rate of change of temperature with distance in the direction of flow of heat is called ***temperature gradient***.

If T_1 and T_2 are the temperatures of two isothermal surfaces separated by distance x, then

$$\text{Temperature gradient} = \frac{T_1 - T_2}{x}.$$

11.28 THERMAL CONDUCTIVITY

44. *State the factors on which the conduction of heat through a substance depends. Obtain an expression for the heat conducted and hence define coefficient of thermal conductivity and give its units and dimensions.*

Thermal conductivity. As shown in Fig. 11.18, consider a block of a material of cross-sectional area A and thickness x. Suppose its opposite faces are at temperatures T_1 and T_2, with $T_1 > T_2$.

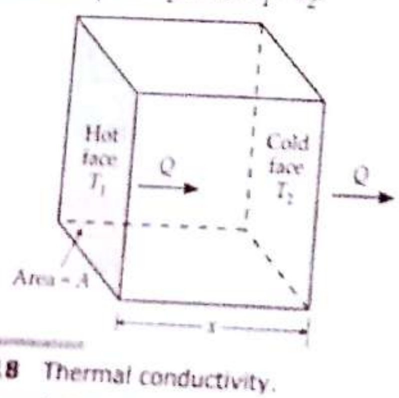


Fig. 11.18 Thermal conductivity.

It is found that the amount of heat Q that flows from hot to cold face during the steady state

(*i*) is directly proportional to the cross-sectional area A,

(ii) is directly proportional to the temperature difference $(T_1 - T_2)$ between the opposite faces,

(iii) is directly proportional to time t for which the heat flows,

(iv) is inversely proportional to thickness x of the block, and

(v) depends on the nature of the material of the block.

$$\therefore \quad Q \propto \frac{A(T_1 - T_2)\,t}{x}$$

or

$$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

The proportionality constant K is called ***coefficient of thermal conductivity*** of the given material. Its value depends on the nature of the material.

If $A = 1,\ T_1 - T_2 = 1,\ t = 1,\ x = 1$, then $Q = K$

Hence, *the coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.*

If the area of cross-section is not uniform or if the steady state condition is not reached, then we consider a thin layer of the material normal to the direction of heat flow. If A be the area of the cross-section at a place, dx be a small thickness along the direction of heat flow and dT be the temperature difference across this thickness dx, then the rate of flow of heat or heat current H will be

$$H = \frac{dQ}{dt} = -KA\frac{dT}{dx}$$

The quantity dT/dx is called the *temperature gradient*. The negative sign indicates that dT/dx is negative in the direction of flow of heat *i.e.*, temperature decreases along the positive x-direction. Thus the negative sign in the above equation ensures that K is positive.

Units and dimensions of K. The numerical value of K is

$$K = \frac{Q.x}{A(T_1 - T_2)\,t}$$

$\therefore$ SI unit of K

$$= \frac{\text{J . m}}{\text{m}^2\text{ . K . s}}$$

$$= \mathbf{J\,s^{-1}\,m^{-1}\,K^{-1}\ or\ W\,m^{-1}\,K^{-1}}.$$

CGS unit of $K = \mathbf{cal\ s^{-1}\ cm^{-1}\ {}^\circ C^{-1}}$.

Dimensions of K

$$= \frac{[ML^2T^{-2}]\,.\,[L]}{[L^2]\,.\,[K]\,[T]} = [MLT^{-3}K^{-1}]$$

Table 11.4 gives the thermal conductivities of some common materials. We can note that solids are better conductors than liquids and liquids are better conductors than gases. Moreover, metals are much better thermal conductors than the non-metals. This is because metals have a large number of free electrons which carry heat from hotter to colder regions very fastly. For most of the materials the value of K increases slightly with temperature.

Table 11.4 Thermal conductivities of some common materials

Material	Thermal conductivity in $J\,s^{-1}\,m^{-1}\,K^{-1}$
METALS	
Silver	406
Copper	385
Aluminum	205
Brass	109
Steel	50.2
Lead	34.7
Mercury	8.3
NON-METALS	
Insulating brick	0.15
Concrete	0.8
Body fat	0.20
Felt	0.04
Glass	0.8
Ice	1.6
Rock Wool	0.04
Wood	0.12 – 0.04
Water	0.8
GASES	
Air	0.024
Argon	0.016
Hydrogen	0.14

11.29 HEAT CURRENT AND THERMAL RESISTANCE

45. *Define heat current and thermal resistance. Write mathematical expressions for them in terms of thermal conductivity K.*

Heat current and thermal resistance. We know that charge flows in a circuit due to potential difference between its two points. The flow of charge per unit

Scanned by CamScanner

time is called electric current. Similarly, heat flows in a conductor due to temperature difference between its two points. *The flow of heat per unit time is called heat current*, denoted by H. Thus

$$H = \frac{Q}{t}$$

Its SI unit is Js^{-1} or watt (W).

From Ohm's law, electric resistance is given by

$$R = \frac{V}{I}$$

That is electric resistance is the ratio of the potential difference and the electric current. Similarly, *the ratio of the temperature difference between the ends of a conductors to the heat current through it is called thermal resistance*, denoted by R_H. Thus

$$R_H = \frac{\Delta T}{H}$$

As $\quad Q = KA \frac{\Delta T}{\Delta x} t$

$\therefore \quad H = \frac{Q}{t} = KA \frac{\Delta T}{\Delta x}$

and $\quad R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$

Hence *greater the coefficient of thermal conductivity of a material, smaller is the thermal resistance of rod of that material.*

Units and dimensions of R_H. As $R_H = \Delta T / H$, so

SI unit of $R_H = \frac{K}{Js^{-1}} = \frac{K}{W} = KW^{-1}$.

Dimensions of R_H

$$= \frac{[K]}{[ML^2T^{-2}].[T^{-1}]} = [M^{-1}L^{-2}T^3K].$$

11.30 APPLICATIONS OF CONDUCTIVITY IN DAILY LIFE

46. *Describe some applications of conductivity in daily life.*

Some applications of conductivity in daily life :

(*i*) ***In winter, a metallic handle appears colder than the wooden door.*** In winter, the human body is at a higher temperature than the surrounding objects. As we touch the metallic handle (good conductor), heat flows from our body to the handle and feels cold. But no heat flows from our body to the wooden door (bad conductor), so it does not feel that cold as the metallic handle.

(*ii*) ***Cooking utensils are provided with wooden handles.*** Wood is a bad conductor of heat. A wooden handle does not allow heat to be conducted from the hot utensil to the hand. So we can easily hold the hot utensils with the help of wooden handles.

(*iii*) ***A new quilt is warmer than an old quilt.*** A new quilt contains more air in its pores as compared to the old quilt. As air is bad conductor of heat, it does not allow heat to be conducted away from our body to the surroundings and we feel warmer in it.

(*iv*) ***Birds swell their feathers in winter.*** By doing so, the birds enclose air between their feathers and the body. Air is poor conductor of heat. It prevents the loss of heat from the bodies of the birds to the surroundings and as such they do not feel cold in winter.

(*v*) ***Ice is packed in saw dust.*** Saw dust and air trapped inside it are poor conductors of heat. This prevents the conduction of heat from the surroundings to the ice which may otherwise melt the ice.

(*vi*) ***Eskimos make double wall houses of the blocks of ice.*** Air trapped between the two walls of ice does not allow the heat to be conducted away from the inside of the house to the colder surroundings.

(*vii*) ***When a wire gauge is placed over the burning Bunsen's burner, the flame does not go beyond the gauge.*** Copper is a very good conductor of heat. The copper gauge absorbs most of the heat. Therefore, the temperature of the gas above the gauge is not high enough to ignite the gas.

(*viii*) ***A refrigerator is provided with insulated walls.*** Generally, fibre glass is used as an insulating material. This is done to minimise the chances of heat flowing into the refrigerator from outside.

Examples based on Thermal Conductivity

FORMULAE USED

1. The amount of heat that flows in time t across the opposite faces of a slab of thickness x and cross-section A,

$$Q = \frac{KA(T_1 - T_2)t}{x}$$

where T_1 and T_2 are the temperatures of hot and cold faces and K is the coefficient of thermal conductivity of the material of the slab.

2. Rate of flow of heat,

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

Here dT/dx is the rate of fall of temperature with distance and is called temperature gradient.

UNITS USED

SI unit of K is $Js^{-1}m^{-1}K^{-1}$ and CGS unit is $cal\,s^{-1}cm^{-1}\,°C^{-1}$.

Scanned by CamScanner

EXAMPLE 19. Calculate the rate of loss of heat through a glass window of area 1000 cm² and thickness 0.4 cm when temperature inside is 37°C and outside is – 5°C.

Coefficient of thermal conductivity of glass is 2.2×10^{-3} cal s^{-1} cm^{-1} K^{-1}. **[Delhi 02]**

Solution. Here $A = 1000\ \text{cm}^2$, $x = 0.4$ cm,

$$T_1 - T_2 = 37 - (-5) = 42\,°\text{C}$$

$$K = 2.2 \times 10^{-3}\ \text{cal s}^{-1}\ \text{cm}^{-1}\ \text{K}^{-1}$$

Rate of loss of heat,

$$H = \frac{Q}{t} = \frac{KA(T_1 - T_2)}{x}$$

$$= \frac{2.2 \times 10^{-3} \times 1000 \times 42}{0.4} = \mathbf{231\ cal\ s^{-1}}.$$

EXAMPLE 20. Steam at 100°C is passed into a copper cylinder 10 mm thick and of 200 cm² area. Water at 100°C collects at the rate of 150 g min^{-1}. Find the temperature of the outer surface, if the conductivity of copper is 0.8 cal s^{-1} cm^{-1} °C^{-1} and the latent heat of steam is 540 cal g^{-1}.

Solution. Here $T_1 = 100°\text{C}$, $x = 10\ \text{mm} = 1\ \text{cm}$,

$A = 200\ \text{cm}^2$, $m = 150$ g, $t = 1\ \text{min} = 60$ s,

$K = 0.8\ \text{cal s}^{-1}\ \text{cm}^{-1}\ °\text{C}^{-1}$, $L = 540\ \text{cal g}^{-1}$

Heat drawn from steam,

$$Q = mL = 150 \times 540\ \text{cal}$$

As $$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

$$\therefore\ 150 \times 540 = \frac{0.8 \times 200\,(100 - T_2) \times 60}{1}$$

or $$100 - T_2 = \frac{150 \times 540}{0.8 \times 200 \times 60} = 8.44\,°\text{C}$$

$$\therefore\ T_2 = 100 - 8.44 = \mathbf{91.56\,°C}.$$

EXAMPLE 21. A metal rod of length 20 cm and diameter 2 cm is covered with non-conducting substance. One of its ends is maintained at 100°C, while the other end is put in ice at 0°C. It is found that 25 g of ice melts in 5 minutes. Calculate the coefficient of thermal conductivity of the metal. Given latent heat of ice = 80 cal g^{-1}.

Solution. Here length of rod, $x = 20$ cm,

$T_1 = 100°\text{C}$, $T_2 = 0°\text{C}$, $t = 5\ \text{min} = 300$ s,

Diameter of rod, $d = 2$ cm

Area of cross-section, $A = \frac{\pi d^2}{4} = \frac{\pi \times (2)^2}{4} = \pi\ \text{cm}^2$

Mass of ice melted, $m = 25$ g

Latent heat of ice, $L = 80\ \text{cal g}^{-1}$

Heat used in melting ice,

$$Q = mL = 25 \times 80 = 2000\ \text{cal}$$

As $$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

$$\therefore\ K = \frac{Qx}{A(T_1 - T_2)\,t} = \frac{2000 \times 20}{\pi \times (100 - 0) \times 300}$$

$$= \frac{2000 \times 20}{3.142 \times 100 \times 300}$$

$$= \mathbf{0.424\ cal\ s^{-1}\ cm^{-1}\ °C^{-1}}.$$

EXAMPLE 22. A layer of ice 2 cm thick is formed on a pond. The temperature of air is – 20°C. Calculate how long it will take for the thickness of ice to increase by 1 mm. Density of ice = 1 g cm^{-3}. Latent heat of ice = 80 cal g^{-1}. Conductivity of ice = 0.008 cal s^{-1} cm^{-1} °C^{-1}. **[Delhi 98]**

Solution. Let surface area of the ice layer = A cm²

Thickness of the ice layer to be formed

$$= 1\ \text{mm} = 0.1\ \text{cm}$$

∴ Volume of the ice to be formed,

$$V = A \times 0.1 = 0.1\,A\ \text{cm}^3$$

Mass of the ice to be formed,

$$m = \text{Volume} \times \text{Density} = 0.1\,A \times 1 = 0.1\,A\ \text{gram}$$

In order to increase the thickness of ice layer, heat has to be conducted from the pond to the air through ice layer already formed. If Q is the amount of heat conducted from the pond to the air, to form $0.1\,A$ gram of ice, then

$$Q = mL = 0.1\,A\ \text{g} \times 80\ \text{cal g}^{-1} = 8\,A\ \text{cal}$$

As thickness of layer through which heat is being conducted increases continuously, so we calculate mean thickness x of the layer.

$$x = \frac{\text{Initial thickness + Final thickness}}{2}$$

$$= \frac{2\ \text{cm} + 2.1\ \text{cm}}{2} = 2.05\ \text{cm}$$

Temperature difference on two sides of ice layer,

$$T_1 - T_2 = 0 - (-20) = 20\,°\text{C}$$

Let t be the time taken by the ice layer to increase thickness by 1 mm. Then

$$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

or $$t = \frac{Qx}{KA(T_1 - T_2)} = \frac{8\,A \times 2.05}{0.008 \times A \times 20} = \mathbf{102.5\ s.}$$

EXAMPLE 23. Two metal cubes A and B of same size are arranged as shown in Fig. 11.19. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The co-efficients of thermal conductivity of A and B are 300W/m°C and 200 W/m°C, respectively. After steady state is reached, what will be the temperature T of the interface ?

[IIT 96 ; Delhi 14]

Scanned by CamScanner

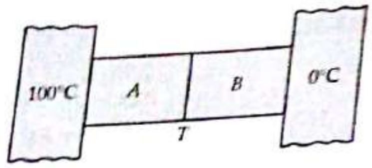


Fig. 11.19

Solution. In the steady-state,

Rate of flow of heat through cube A
= Rate of flow of heat through cube B

$$\frac{K_1 A (100 - T)}{x} = \frac{K_2 A (T - 0)}{x}$$

or
$$\frac{300\, A (100 - T)}{x} = \frac{200\, A (T - 0)}{x}$$

or
$$300 - 3T = 2T \quad \text{or} \quad 5T = 300$$

$\therefore$
$$T = \mathbf{60°C.}$$

EXAMPLE 24. *Three bars of equal lengths and equal area of cross-section are connected in series. Their thermal conductivities are in the ratio of 2 : 4 : 3. If the open ends of the first and the last bars are at temperatures 200°C and 18°C respectively in the steady state, calculate the temperatures of both the junctions.*

Solution. Let θ_1 and θ_2 be the temperatures of junctions B and C respectively.

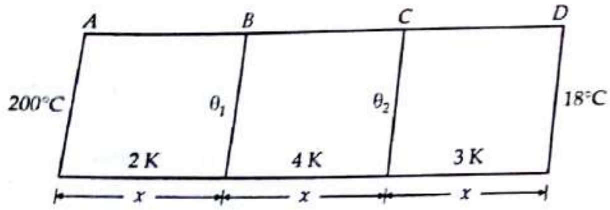


Fig. 11.20

In the steady state, the rate of flow of heat through each bar will be same.

$$\therefore \quad \frac{Q}{t} = \frac{2K \times A (200 - \theta_1)}{x}$$

$$= \frac{4K \times A (\theta_1 - \theta_2)}{x} = \frac{3K \times A (\theta_2 - 18)}{x}$$

or $\quad 2 (200 - \theta_1) = 4 (\theta_1 - \theta_2) = 3 (\theta_2 - 18)$

On solving, we get : $\theta_1 = \mathbf{116°C}$ and $\theta_2 = \mathbf{74°C}$.

EXAMPLE 25. *One end of a copper rod of uniform cross-section and of length 1.5 m is kept in contact with ice and the other end with water at 100°C. At what point along its length should a temperature of 200°C be maintained so that in steady state, the mass of ice melted be equal to that of the steam produced in the same interval of time ? Assume that the whole system is insulated from the surroundings. Latent heat of fusion of ice = 80 cal g^{-1}. Latent heat of vaporisation of water = 540 cal g^{-1}.*

Solution. Let the temperature of 200°C be maintained at a distance x from the end at 0°C. Heat will flow from this point towards both ice and water.

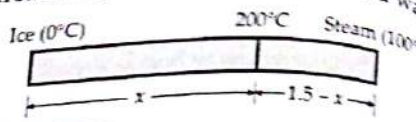


Fig. 11.21

If m is the mass of ice melted = mass of steam produced, then

$$m \times L_{ice} = \frac{KA (200 - 0)}{x} \quad ...(i)$$

$$m \times L_{steam} = \frac{KA (200 - 100)}{1.5 - x} \quad ...(ii)$$

Dividing (i) by (ii), we get

$$\frac{L_{ice}}{L_{steam}} = \frac{200}{x} \times \frac{1.5 - x}{100}$$

or
$$\frac{80}{540} = \frac{2 (1.5 - x)}{x}$$

On solving, $\quad x = \mathbf{1.396\ m.}$

EXAMPLE 26. *What is the temperature of the steel-copper junction in the steady state of the system shown in Fig. 11.22 ? Length of the steel rod = 15.0 cm, length of the copper rod = 10.0 cm, temperature of the furnace = 300°C, temperature of the other end = 0°C. The area of cross-section of the steel rod is twice that of the copper rod. Thermal conductivity of steel = 50.2 Js^{-1} m^{-1} °C^{-1} and of copper = 385 Js^{-1} m^{-1} °C^{-1}.*
[NCERT; Delhi 10; Central School 14]

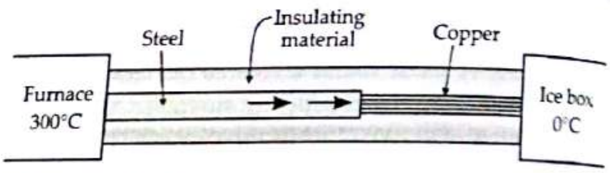


Fig. 11.22

Solution. Here $K_1 = 50.2$ Js^{-1} m^{-1} °C^{-1},
$K_2 = 385$ Js^{-1} m^{-1} °C^{-1}, $A_1 = 2\,A_2$,
$x_1 = 15.0$ cm, $\quad x_2 = 10.0$ cm

Let T be the temperature of the steel-copper junction in the steady state of the system. Then in the steady state,

Rate of heat flowing into the system
= Rate of heat flowing out of the system.

or
$$\frac{K_1 A_1 (300 - T)}{x_1} = \frac{K_2 A_2 (T - 0)}{x_2}$$

Scanned by CamScanner

or
$$\frac{300-T}{T} = \frac{K_2}{K_1} \times \frac{A_2}{A_1} \times \frac{x_1}{x_2}$$

$$= \frac{385}{50.2} \times \frac{1}{2} \times \frac{15}{10} = 5.75$$

or
$$T = \mathbf{44.4^\circ C}.$$

EXAMPLE 27. *An electric heater is used in a room of total wall area $137\ m^2$ to maintain a temperature of $+20^\circ C$ inside it, when the outside temperature is $-10^\circ C$. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 Watt/m/°C respectively.* **[IIT 86]**

Solution. Equivalent thermal conductivity of the series combination of three walls is

$$K = \frac{d_1 + d_2 + d_3}{\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}} = \frac{0.025 + 0.01 + 0.25}{\frac{0.025}{0.125} + \frac{0.01}{1.5} + \frac{0.25}{1.0}}$$

$$= \frac{0.285 \times 300}{137}\ Wm^{-1}\,^\circ C^{-1}$$

Rate of flow of heat is

$$\frac{Q}{t} = K A \frac{\theta_1 - \theta_2}{(d_1 + d_2 + d_3)}$$

$$= \frac{0.285 \times 300}{137} \times 137 \times \frac{20 - (-10)}{0.285} = \mathbf{9000\ W}.$$

This should be equal to the power of the heater.

EXAMPLE 28. *An iron bar ($L_1 = 0.1\ m$, $A_1 = 0.02\ m^2$, $K_1 = 79\ W\ m^{-1}\ K^{-1}$) and a brass bar ($L_2 = 0.1\ m$, $A_2 = 0.02\ m^2$, $K_2 = 109\ W\ m^{-1}\ K^{-1}$) are soldered end to end as shown in Fig. 11.23. The free ends of the iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expressions for and hence compute (i) the temperature of the junction of the two bars, (ii) the equivalent thermal conductivity of the compound bar, and (iii) the heat current through the compound bar.* **[NCERT]**

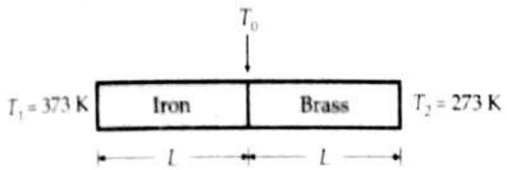


Fig. 11.23

Solution. Here

$L_1 = L_2 = L = 0.1$ m, $A_1 = A_2 = A = 0.02\ m^2$,
$K_1 = 79\ W\ m^{-1}\ K^{-1}$, $K_2 = 109\ W\ m^{-1}\ K^{-1}$,
$T_1 = 373$ K and $T_2 = 273$ K

In the steady state,

Heat current through iron bar = Heat current through brass bar

or
$$H_1 = H_2 = H \quad \text{(say)}$$

or
$$\frac{K_1 A_1 (T_1 - T_0)}{L_1} = \frac{K_2 A_2 (T_0 - T_2)}{L_2}$$

or
$$\frac{K_1 A (T_1 - T_0)}{L} = \frac{K_2 A (T_0 - T_2)}{L}$$

or
$$K_1 (T_1 - T_0) = K_2 (T_0 - T_2)$$

Thus the junction temperature T_0 of the two bars is

$$T_0 = \frac{(K_1 T_1 + K_2 T_2)}{(K_1 + K_2)}.$$

Using this value of T_0, the heat current through either bar will be

$$H = \frac{K_1 A (T_1 - T_0)}{L} = \frac{K_1 A}{L}\left(T_1 - \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2}\right)$$

$$= \frac{K_1 K_2 A}{K_1 + K_2} \cdot \frac{T_1 - T_2}{L}$$

Thus, the heat current H' through the compound bar of length $L_1 + L_2 = 2L$ and the equivalent thermal conductivity K', of the compound bar are given by

$$H' = H$$

or
$$\frac{K' A (T_1 - T_2)}{2L} = \frac{K_1 K_2 A}{K_1 + K_2} \cdot \frac{T_1 - T_2}{L}$$

or
$$K' = \frac{2 K_1 K_2}{K_1 + K_2}.$$

(i) $T_0 = \dfrac{(K_1 T_1 + K_2 T_2)}{(K_1 + K_2)}$

$$= \frac{(79\ Wm^{-1}\ K^{-1})(373\ K) + (109\ Wm^{-1}\ K^{-1})(273\ K)}{79\ Wm^{-1}\ K^{-1} + 109\ Wm^{-1}\ K^{-1}}$$

$= \mathbf{315\ K}$.

(ii) $K' = \dfrac{2 K_1 K_2}{K_1 + K_2}$

$$= \frac{2 \times (79\ Wm^{-1}\ K^{-1}) \times (109\ Wm^{-1}\ K^{-1})}{79\ Wm^{-1}\ K^{-1} + 109\ Wm^{-1}\ K^{-1}}$$

$= \mathbf{91.6\ m^{-1}\ K^{-1}}$.

(iii) $H' = H = \dfrac{K' A (T_1 - T_2)}{2L}$

$$= \frac{(91.6\ Wm^{-1}\ K^{-1}) \times (0.02\ m^2) \times (373\ K - 273\ K)}{2 \times (0.1\ m)}$$

$= \mathbf{916.1\ W}$

Scanned by CamScanner

✗ PROBLEMS FOR PRACTICE

1. Heat is flowing through a rod of length 25.0 cm having cross-sectional area $8.80\ cm^2$. The coefficient of thermal conductivity for the material of the rod is $K = 9.2 \times 10^{-2}\ kcal\ s^{-1}\ m^{-1}\ °C^{-1}$. The temperatures of the ends of the rod are 125°C and 0°C in the steady state. Calculate (*i*) temperature gradient in the rod (*ii*) temperature of a point at a distance of 10.0 cm from the hot end and (*iii*) rate of flow of heat.
[**Ans.** (*i*) $-5°C\ cm^{-1}$ (*ii*) 75°C (*iii*) $4.048 \times 10^{-2}\ kcal\ s^{-1}$]

2. Calculate the difference in temperatures between two sides of an iron plate 20 mm thick, when heat is conducted at the rate of $6 \times 10^5\ cal/min/m^2$. K for metal is $0.2\ cal\ s^{-1}\ cm^{-1}\ °C^{-1}$ (**Ans.** 10°C)

3. A flat-bottom kettle placed on a stove is being used to boil water and the thermal conductivity of the material is $0.5\ cal\ s^{-1}\ cm^{-1}\ °C^{-1}$. If the amount of steam being produced in the kettle is at the rate $10\ g\ min^{-1}$, calculate the difference of temperature between the inner and outer surfaces of the bottom. The latent heat of steam is $540\ cal\ g^{-1}$. (**Ans.** 0.2°C)

4. An iron boiler is 1 cm thick and has a heating area of $2\ m^2$. The two surfaces of the boiler are at 234°C and 100°C respectively. If the latent heat of steam is $536\ kcal\ kg^{-1}$ and thermal conductivity of iron is $1.6 \times 10^{-2}\ kcal\ s^{-1}\ m^{-1}\ °C^{-1}$, then how much water will be evaporated into steam per minute?
(**Ans.** 48 kg)

5. One end of a 0.25 m long metal bar is in steam and the other is in contact with ice. If 12 g of ice melts per minute, what is the thermal conductivity of the metal? Given cross-section of the bar $= 5 \times 10^{-4}\ m^2$ and latent heat of ice is $80\ cal\ g^{-1}$.
(**Ans.** $80\ cal\ s^{-1}\ m^{-1}\ °C^{-1}$)

6. A layer of ice 0.15 m thick has formed on the surface of a deep pond. If the temperature of upper surface of ice is constant and equal to that of the air which is −12°C, determine the time it will take to increase the thickness of ice layer by 0.2 mm. Take latent heat of ice $= 80\ cal\ g^{-1}$, density of ice $= 0.91\ g\ cm^{-3}$ and thermal conductivity of ice $= 0.5\ cal\ s^{-1}\ m^{-1}\ K^{-1}$.
(**Ans.** 364.2 s)

7. Water is boiled in a rectangular steel tank of thickness 2 cm by a constant temperature furnace. Due to vaporisation, water level falls at a steady rate of 1 cm in 9 minutes. Calculate the temperature of the furnace. Given K for steel $= 0.2\ cal\ s^{-1}\ m^{-1}\ °C^{-1}$.
(**Ans.** 110 °C)

8. Estimate the rate at which ice would melt in a wooden box 2.0 cm thick and of inside measurements $200\ cm \times 120\ cm \times 120\ cm$ assuming that the external temperature is 30°C and coefficient of thermal conductivity of wood $0.0004\ cal\ s^{-1}\ cm^{-1}\ °C$.
(**Ans.** 9.36 g s^{-1})

9. Steam at 373 K is passed through a tube of radius 50 cm and length 3 m. If thickness of the tube be 2 mm and conductivity of its material be $2 \times 10^4\ cal\ s^{-1}\ cm^{-1}\ K^{-1}$, calculate the rate of loss of heat in $J s^{-1}$. The outside temperature is 282 K.
(**Ans.** $36026\ J s^{-1}$)

10. The thermal conductivity of copper is four times that of brass. Two rods of copper and brass of same length and cross-section are joined end to end. The free end of copper rod is at 0°C and that of brass rod at 100°C. Calculate the temperature of junction at equilibrium. Neglect radiation losses. (**Ans.** 20°C)

11. The temperature difference between the two ends of a bar 1.0 m long is 50°C and that for the other bar 1.25 m long 75°C. Both the bars have same area of cross-section. If the rates of conduction of heat in the two bars are the same, find the ratio of the coefficients of thermal conductivity of the materials of the two bars.
(**Ans.** 6 : 5)

12. The ratio of the areas of cross-section of two rods of different materials is 1 : 2, and the ratio of the thermal conductivities of their materials is 4 : 3. On keeping equal temperature-difference between the ends of these rods, the rates of conduction of heat are equal. Determine the ratio of the lengths of the rods.
(**Ans.** 2 : 3)

13. In Fig. 11.24, two bars of the same metal are connected. The length of the first bar is half of that of the second, but the cross-sectional area is double. What is the temperature of the junction of the bars?
(**Ans.** 20°C)

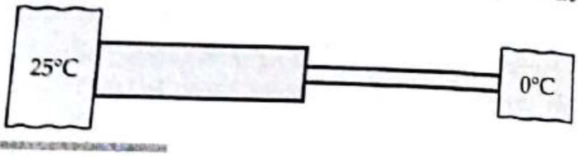


Fig. 11.24

14. A room at 20°C is heated by a heater of resistance 20 ohm connected to 200 V mains. The temperature is uniform throughout the room and heat is transmitted through a glass window of area $1\ m^2$ and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is $0.2\ cal\ s^{-1}\ m^{-1}\ °C^{-1}$ and $J = 4.2\ J\ cal^{-1}$. (**Ans.** 15.2°C)

✗ HINTS

1. (*i*) Temperature gradient,

$$\frac{dT}{dx} = \frac{T_2 - T_1}{x} = \frac{0 - 125}{25.0} = -5°C\ cm^{-1}.$$

Scanned by CamScanner

(ii) Change in temperature at a point distant 10.0 cm from the hot end

$= \text{Temp. gradient} \times \text{distance} = -5 \times 10.0 = -50°C$

Temperature at this point $= 125 - 50 =$ **75°C.**

Rate of flow of heat, $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x}$

$= \frac{9.2 \times 10^{-2} \times 8.80 \times 10^{-4} \times 125}{0.25}$

$= \mathbf{4.048 \times 10^{-2}\ kcal\ s^{-1}}$.

2. Here $x = 20\ mm = 2\ cm$, $Q = 6 \times 10^5$ cal,

$t = 1\ min = 60\ s$, $A = 1\ m^2 = 10^4\ cm^2$,

$K = 0.2\ cal\ s^{-1}\ cm^{-1}\ K^{-1}$

As $Q = \frac{KA(T_1 - T_2)t}{x}$

$\therefore T_1 - T_2 = \frac{Qx}{KAt} = \frac{6 \times 10^5 \times 2}{0.2 \times 10^4 \times 60} = \mathbf{10°C}.$

3. Rate of flow of heat from the stove into the kettle is

$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x} = \left(\frac{m}{t}\right) L$

$\therefore T_1 - T_2 = \frac{m}{t} L \times \frac{x}{KA} = \frac{10}{60} \times 540 \times \frac{0.3}{0.5 \times 270} = \mathbf{0.2°C}.$

4. Here $x = 1\ cm = 0.01\ m$, $A = 2\ m^2$,

$T_1 - T_2 = 234 - 100 = 134°C$

$K = 1.6 \times 10^{-2}\ kcal\ s^{-1}\ m^{-1}\ °C^{-1}$, $t = 1\ min = 60\ s$

$m = \frac{KA(T_1 - T_2)t}{xL} = \frac{1.6 \times 10^{-2} \times 2 \times 134 \times 60}{0.01}$

$=$ **48 kg.**

5. Here $x = 0.25\ m$, $T_1 - T_2 = 100 - 0 = 100°C$,

$t = 1\ min = 60\ s$, $A = 5 \times 10^{-4}\ m^2$

$Q = mL = 12 \times 80 = 960\ cal$

But $Q = \frac{KA(T_1 - T_2)t}{x}$

or $960 = \frac{K \times 5 \times 10^{-4} \times 100 \times 60}{0.25}$

$K = \frac{960 \times 0.25}{5 \times 10^{-2} \times 60} = \mathbf{80\ cal\ s^{-1}\ m^{-1}\ °C^{-1}}.$

6. Let A be the area of the upper face of ice layer.

Increase in thickness = 0.2 mm = 0.02 cm.

Mass of ice of to be frozen, $m = A \times 0.02 \times 0.91\ g$

As latent heat of ice, $L = 80\ cal\ g^{-1}$

$\therefore Q = mL = A \times 0.02 \times 0.91 \times 80\ cal$

Average thickness through which heat is to pass,

$x = \frac{15 + (15 + 0.02)}{2} = 15.01\ cm$

and $K = 0.5\ cal\ s^{-1}\ m^{-1}\ K^{-1}$

$= 0.5 \times 10^{-2}\ cal\ s^{-1}\ cm^{-1}\ K^{-1}$

As $Q = \frac{KA(T_1 - T_2)t}{x}$

$t = \frac{Qx}{KA(T_1 - T_2)}$

$= \frac{A \times 0.02 \times 0.91 \times 80 \times 15.01}{0.5 \times 10^{-2} \times A \times [0 - (-12)]} = \mathbf{364.2\ s}.$

7. Let area of the bottom of the tank $= A\ cm^2$

Volume of water that vaporises in 9 min (or 540 s)

$= A \times 1\ cm^3$

Mass of water that vaporises in 540 s

$= A\ cm^2 \times 1\ g\ cm^{-3} = A\ g$

$Q = mL = A \times 540\ cal$

But $Q = \frac{KA(T_1 - T_2)t}{x}$

or $T_1 - T_2 = \frac{Qx}{KAt} = \frac{A \times 540 \times 2}{0.2 \times A \times 540} = 10$

$\therefore T_1 = T_2 + 10 = 100 + 10 = \mathbf{110°C}.$

8. A = Area of all the six faces

$= (2 \times 120 \times 120 + 4 \times 200 \times 120) = 124800\ cm^2$

$x = 2\ cm$, $T_1 - T_2 = 30 - 0 = 30°C$,

$K = 0.004\ cal\ s^{-1}\ cm^{-1}\ °C^{-1}$

$Q = mL = \frac{KA(T_1 - T_2)t}{x}$

Rate of melting of ice,

$\frac{m}{t} = \frac{KA(T_1 - T_2)}{xL} = \frac{0.0004 \times 124800 \times 30}{2 \times 80}$

$= \mathbf{9.36\ gs^{-1}}.$

9. Here $T_1 - T_2 = 373 - 282 = 91\ K$, $x = 2\ mm = 0.2\ cm$

$K = 2 \times 10^{-4}\ cal\ s^{-1}\ cm^{-1}\ K^{-1}$

$A = 2\pi rl = 2\pi \times 50 \times 300 = 3\pi \times 10^4\ cm^2$

$[\because l = 3\ m = 300\ cm]$

$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x} = \frac{2 \times 10^{-4} \times 10^4 \times 91}{0.2}\ cal\ s^{-1}$

$= 8577.7\ cal\ s^{-1} = \mathbf{36026\ Js^{-1}}.$ $[\because 1\ cal = 4.2\ J]$

14. Rate of production of heat by the heater is,

$\frac{V^2}{R} = \frac{(200\ V)^2}{20\ \Omega} = 2000\ Js^{-1} = \frac{2000}{4.2}\ cal\ s^{-1}$

Rate of loss of heat through the window,

$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x} = \frac{0.2 \times 1 \times (20 - T_2)}{0.2 \times 10^{-2}}$

As the temperature of the room is uniform and constant, so

$\frac{0.2 \times 1 \times (20 - T_2)}{0.2 \times 10^{-2}} = \frac{2000}{0.2}$ or $20 - T_2 = \frac{20}{4.2} = 4.8$

$T_2 = 20 - 4.8 = \mathbf{15.2°C}.$

Scanned by CamScanner

11.31 CONVECTION

47. *What is thermal convection ? Briefly explain how are convection currents set up in water ? Distinguish between natural and forced convections.*

Convection. *It is the process by which heat flows from the region of higher temperature to the region of lower temperature by the actual movement of the material particles.*

Fluids (liquids and gases) are heated mainly by the process of convection in which buoyancy and gravity play an important role. As shown in Fig. 11.25, when a fluid is heated from below, the hot portion at the bottom expands and becomes less dense. Because of buoyancy, this lighter portion rises up. The denser colder fluid takes its place by moving downwards. Thus *convection current* is set up in the fluid. The actual movement of a liquid can be seen by colouring the liquid with potassium permanganate crystals placed at the bottom of the vessel.

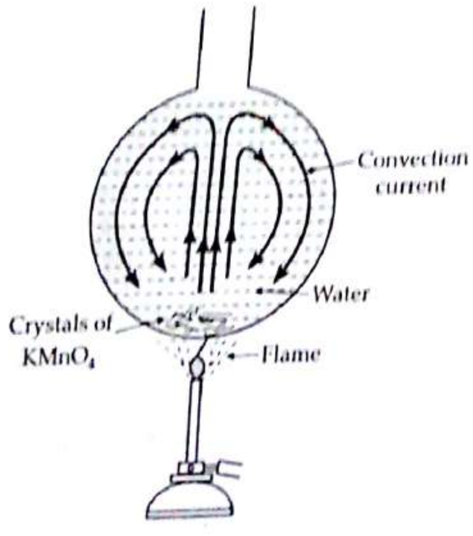


Fig. 11.25 Convection currents in water.

Natural convection. *If the material moves due to difference in density, the process of heat transfer is called natural or free convection.* Natural convection arises due to unequal heating (of fluid) and gravity. Here the more heated and less dense parts of the fluid rise and are replaced by the cooler parts. Natural convection is responsible for the origin of different types of winds in the atmosphere.

Forced convection. *If the heated material is forced to move by an agency like a pump or a blower, the process of heat transfer is called forced convection.* Air-conditioning, central heating systems and heating a liquid by brisk stirring are examples of forced convection.

11.32 PHENOMENA BASED ON THERMAL CONVECTION

48. *Describe some of the phenomena which are based on thermal convection.*

Phenomena based on thermal convection :

(*i*) ***In regulating the temperature of human body.*** In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection. This maintains a remarkably constant body temperature.

(*ii*) ***In maintaining comfortable room temperature in cold countries.*** In cold countries during winter, the outside temperature is much below 0°C while the room temperature is comfortably maintained around 20°C. However, the inside air close to the glass window is cooler than 20°C while the outside close to the window is warmer than the chilling temperature of the atmosphere. Thus heat is continuously transferred from the room to the outside by convection of air inside the room, conduction across the glass pane and again convection of air outside. But this heat loss is compensated by the heating system provided in the room.

(*iii*) ***In the formation of trade winds.*** Natural convection plays an important role in the formation of trade winds. The surface of the earth and hence the air above it near the equator gets strongly heated by the sun. The heated air expands and rises upwards. The colder air from polar region rushes in towards the equator. This produces northward wind in northern hemisphere and southward in southern hemisphere. Due to rotation of the earth about its axis from west to east, the air close to the equator has an eastward speed of 1600 kmh^{-1}, while it is zero close to the poles. As a result, the actual direction of the wind in the northern hemisphere is north east and in the southern hemisphere, south west. These winds are called *trade winds*. In ancient times, the traders used these winds to find the direction of motion of their sailing vessels.

(*iv*) ***Land and sea breezes.*** These are local convection currents. Specific heat of water is higher than that of soil. So land and hence air above it is heated faster in summer during day time than air above the sea. The air above land expands and rises and its place is taken up by the colder air from sea to land and is called sea breeze. At night the land gets cooled faster than water. So colder air flows from land to sea and is called land breeze.

(*v*) **Monsoons.** Water has much more specific heat than soil or rock. In summer, the land mass of the Indian subcontinent gets much hotter than the Indian Ocean. This sets up convention current with hot air

Scanned by CamScanner

from the land rising and moving towards the Indian Ocean, while the moisture-laden air from the Ocean moves towards the land. When obstructed by mountains, the moist air rushes upwards to great height and gets cooled. The moisture condenses and causes wide-spread rains in India. In winter, the landmass is cooler than the ocean. Winds blow from the land to ocean. These winds take up moisture as they pass the Bay of Bengal and cause rainfall in Tamilnadu and Srilanka.

11.33 RADIATION

49. *What is radiation ? Give examples.*

Radiation. *It is the process by which heat is transmitted from one place to another without heating the intervening medium.*

When we stand near a fire, we feel warmth because of the heat we receive by the process of radiation. The heat from the sun reaches the earth by the process of radiation, covering millions of kilometers of the empty space or vacuum.

For Your Knowledge

▲ The word *radiation* is used in two meanings. It refers to the process by which the energy is emitted by a body, is transmitted in space and falls on another body. It also refers to the energy itself which is being transmitted in space.

11.34 PREVOST'S THEORY OF HEAT EXCHANGE*

50. *Write the main features of Prevost's theory of heat exchange. How does this theory lead to the fact that good absorbers are good radiators ?*

Prevost's theory of heat exchange. Much before the nature of heat radiation was understood, the Swiss physicist *Pierre Prevost* in 1792 aptly described radiation as a mode of heat transfer. This theory is known as the *theory of heat exchange*. The salient features of this theory are

(*i*) *All bodies at temperatures above 0 K emit heat to the surroundings and gain heat from the surroundings at all times.*

(*ii*) *The amount of heat radiated per second depends on the nature of the emitting surface, its surface area and its temperature.*

(*iii*) *The rate of heat radiated by a body increases with the increase of its temperature and is unaffected by the presence of surrounding bodies.*

(*iv*) *There is a continuous exchange of heat between a body and its surroundings.*

(*v*) *The rise or fall in temperature of a body is the net result of the exchange of heat between the body and the surroundings.*

(*vi*) *The exchange of heat between a body and its surrounding continues till a dynamic thermal equilibrium is established between them and their temperatures become equal.*

When we stand near a fire, we feel the sensation of warmth, because our body is receiving more energy from the fire than it is losing by its own radiation. Similarly, when we stand near a huge block of ice, we feel a sensation of cold because our body loses more energy by radiation than it receives from the ice which is at a temperature lower than that of our body.

When the temperature of a body is equal to that of its surroundings, it radiates heat to the surroundings at the same rate at which it absorbs. The body is then in the state of *dynamic equilibrium*. In this state, if a body absorbs a large fraction of the total heat falling upon it, it must radiate the same amount of heat back to the surroundings, otherwise its temperature will change. This shows that *a body which is a good absorber is also a good radiator of heat and vice-versa.*

11.35 NATURE AND PROPERTIES OF THERMAL RADIATION

51. *What are electromagnetic waves ? In what respect is the thermal radiation different from light ?*

Electromagnetic waves. *These are the waves constituted by oscillating electric and magnetic fields. The oscillations of the two fields are mutually perpendicular to each other as well as to the direction of propagation of the waves.*

Every body at any temperature emits electromagnetic waves. These waves can have different wavelengths. Light is an electromagnetic wave. Visible light wavelengths extend from 4000 Å (violet) to 7500 Å (red). Beyond the red of the electromagnetic spectrum are the *infrared waves* having wavelengths from 1 μm to 100 μm which produce heating effect. These radiations emitted by hot bodies are also called *thermal radiations*. Much beyond the red end are the *medium wavelength radiowaves* (200 m to 500 m) and *short wavelength radiowaves* (20 m to 50 m). All electromagnetic waves travel through vacuum with a speed of $3 \times 10^8 \text{ ms}^{-1}$.

The atoms or molecules of a substance can be excited to higher energy states by thermal collisions or by some other means. When such atoms or molecules de-excite to lower energy states, electromagnetic radiations are emitted.

Scanned by CamScanner

52. *What is thermal radiation ? Give its important properties.*

Thermal radiation. *The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation or radiant energy.* All bodies having temperature above 0 K emit thermal radiation continuously. For example, the radiation emitted by red-hot iron or light from a filament lamp is thermal radiation.

Properties of thermal radiation :

(*i*) These are electromagnetic waves having wavelength range from 1 μm to 100 μm. These are also called infrared waves.

(*ii*) Like light, thermal radiations travel in straight lines.

(*iii*) These radiations obey the laws of reflection and refraction like light does.

(*iv*) They show the phenomena of interference, diffraction and polarisation.

(*v*) Thermal radiations produce heat when they are absorbed by a body.

11.36 NEWTON'S LAW OF COOLING

53. *State Newton's law of cooling. Express it mathematically. How can this law be verified experimentally ?*

Newton's law of cooling. The rate at which a body loses heat by radiation depends on (*i*) the temperature of the body, (*ii*) the temperature of the surrounding medium, and (*iii*) the nature and extent of the exposed surface.

Newton's law of cooling *states that the rate of cooling (or rate of loss of heat) of a body is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small.*

This is in accordance with Newton's law of cooling that a hot water bucket cools fast initially until it gets lukewarm after which it stays so for a longer time.

Mathematical expressions for Newton's law of cooling. Consider a hot body at temperature T. Let T_0 be the temperature of its surroundings. According to Newton's law of cooling,

Rate of loss of heat ∝ Temperature difference between the body and its surroundings

or $$-\frac{dQ}{dt} \propto (T - T_0)$$

or $$-\frac{dQ}{dt} = k(T - T_0) \quad ...(1)$$

where k is a proportionality constant depending upon the area and nature of the surface of the body.

Let m be the mass and c the specific heat of the body at temperature T. If the temperature of the body falls by small amount dT in time dt, then the amount of heat lost is

$$dQ = mc\,dT$$

∴ Rate of loss of heat is given by

$$\frac{dQ}{dt} = mc\frac{dT}{dt}$$

Combining the above equations, we get

$$-mc\frac{dT}{dt} = k(T - T_0)$$

or $$\frac{dT}{dt} = -\frac{k}{mc}(T - T_0) = -K(T - T_0) \quad ...(2)$$

where $K = k/mc$ is another constant. The negative sign indicates that as the time passes, the temperature of the body decreases. The above equation can be written as

$$\frac{dT}{T - T_0} = -K\,dt$$

On integrating both sides, we get

$$\int \frac{1}{T - T_0}\,dT = -K\int dt$$

or $$\log_e (T - T_0) = -Kt + c \quad ...(3)$$

or $$T - T_0 = e^{-Kt + c}$$

or $$T = T_0 + e^c e^{-Kt}$$

or $$T = T_0 + C e^{-Kt} \quad ...(4)$$

where c is a constant of integration and $C = e^c$. Equations (1), (2), (3) and (4) are the different mathematical representations for Newton's law of cooling. Using equation (4), one can calculate the time of cooling of a body through a particular range of temperature.

If we plot a graph by taking different values of temperature difference $\Delta T = T - T_0$ along y-axis and the corresponding values of t along x-axis, we get a curve of the form shown in Fig. 11.26. It clearly shows that the rate of cooling is higher initially and then decreases as the temperature of the body falls.

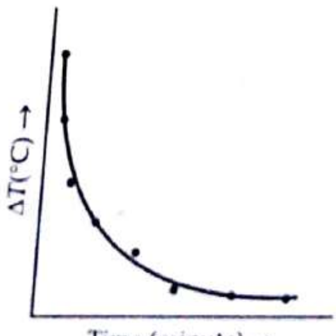


Fig. 11.26 Curve showing cooling of hot water with time.

Scanned by CamScanner

Moreover, the equation (3) is of the form $y = mx + c$. So if we plot a graph, by taking $\log_e (T - T_0)$ along y-axis and time t along x-axis, we must get a straight line, as shown in Fig. 11.27. It has a negative slope equal to $-K$ and intercept on y-axis equal to c.

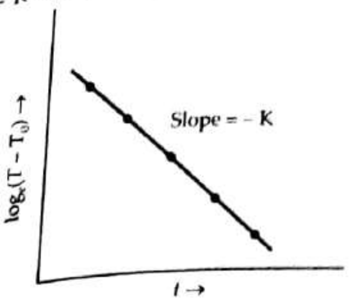


Fig. 11.27 Straight line graph between $\log_e (T - T_0)$ and time t.

In both of the above situations, Newton's law of cooling stands verified.

Experimental verification of Newton's law of cooling. The experimental set-up used for verifying Newton's law of cooling is shown in Fig. 11.28. The set-up consists of a double walled vessel (V) containing water in between the two walls. A copper calorimeter (C) containing hot water is placed inside the double walled vessel. Two thermometers through the corks are used to note the temperatures T of hot water in calorimeter and T_0 of water in between the double walls respectively.

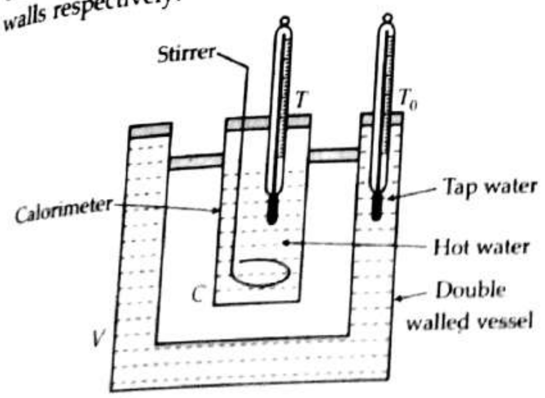


Fig. 11.28 Experimental set-up for verifying Newton's law of cooling.

Temperature of hot water in the calorimeter is noted after fixed intervals of time, say after every one minute of stirring the water gently with a stirrer. Continue noting its temperature till it attains a temperature about 5°C above that of surroundings. Plot a graph between $\log_e (T - T_0)$ and time (t). The nature of the graph is observed to be a straight line, having a negative slope, as shown in Fig. 11.27. This verifies newton's law of cooling.

Examples based on Newton's Law of Cooling

FORMULAE USED

Newton's law of cooling. If the temperature difference between body and surroundings is small, then Rate of loss of heat ∝ Temperature difference from the body

∴ Rate of loss of heat from the body is

$$mc\frac{(T_1 - T_2)}{t} = k(T - T_0) = k\left(\frac{T_1 + T_2}{2} - T_0\right)$$

Here temperature of the body falls from T_1 to T_2 in time-interval t.

UNITS USED

Temperatures T_1, T_2 and T_0 are in °C or K.

EXAMPLE 29. A body cools in 7 minutes from 60°C to 40°C. What will be its temperature after the next 7 minutes? The temperature of the surroundings is 10°C. Assume that Newton's law of cooling holds good throughout the process. **[REC 92]**

Solution. *In first case.* $T_1 = 60°$, $T_2 = 40°C$,

$T_0 = 10°C$, $t = 7$ min $= 420$ s

According to Newton's law of cooling,

$$mc\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$$

$$\therefore \quad mc\frac{60 - 40}{420} = K\left(\frac{60 + 40}{2} - 10\right)$$

$$\text{or} \quad mc\frac{20}{420} = K \times 40 \quad \ldots(i)$$

In second case. $T_1 = 40°C$, $T_2 = ?$, $T_0 = 10°C$,

$t = 7$ min $= 420$ s

$$\therefore \quad mc\frac{40 - T_2}{420} = K\left(\frac{40 + T_2}{2} - 10\right) \quad \ldots(ii)$$

Dividing equation (ii) by (i), we get :

$$\frac{20}{40 - T_2} = \frac{40}{\frac{40 + T_2}{2} - 10}$$

On solving, we get : $T_2 = $ **28°C.**

✗ PROBLEM FOR PRACTICE

1. A pan filled with hot food cools from 94°C to 86°C in 2 minutes when the room temperature is at 20°C. How long will it take to cool from 71°C to 69°C? **[NCERT]** (**Ans.** 42 s)

✗ HINT

1. Proceed as in NCERT Exercise 11.22 on page 11.62.

Scanned by CamScanner

11.37 REFLECTANCE, ABSORPTANCE AND TRANSMITTANCE

54. *Define the terms reflectance, absorptance and transmittance. How are they related ?*

Reflectance, absorptance and transmittance. When thermal radiations fall on a body, they are partly reflected, absorbed and transmitted. Let Q be the amount of radiant energy incident on a body. Suppose the part R is reflected, A is absorbed and T is transmitted. Then

$$R + A + T = Q$$

On dividing both sides by Q, we get

$$\frac{R}{Q} + \frac{A}{Q} + \frac{T}{Q} = 1$$

or

$$r + a + t = 1$$

i.e. Reflectance + Absorptance + Transmittance = 1.

Reflectance. *It is defined as the ratio of the amount of thermal energy reflected by a body in a certain time to the total amount of thermal energy falling upon the body in the same time.*

∴ Reflectance, $r = \frac{R}{Q}$.

Absorptance. *It is defined as the ratio of the amount of thermal energy absorbed by a body in a certain time to the total amount of thermal energy incident upon the body in the same time.*

∴ Absorptance, $a = \frac{A}{Q}$.

Transmittance. *It is defined as the ratio of the amount of thermal energy transmitted by a body in a certain time to the total amount of thermal radiation incident on it in the same time.*

∴ Transmittance, $t = \frac{T}{Q}$.

The reflectance, absorptance and transmittance of a body depend upon

(*i*) Nature of the surface of the body.

(*ii*) Wavelength of incident radiation.

However, these quantities do not depend on the nature of the material of the body. Hence for radiations of different wavelength, a given body may have different values of reflectance, absorptance and transmittance. It is more useful to define these quantities for a given wavelength λ. Then these quantities are called ***monochromatic reflectance*** **(r_λ)**, ***monochromatic absorptance*** **(a_λ)** *and* ***monochromic transmittance*** **(t_λ)** which are related as $r_\lambda + a_\lambda + t_\lambda = 1$.

Special cases. (*i*) If a body does not transmit radiation, $t = 0$, then $r + a = 1$.

Clearly, if r is more, a is less and vice versa. That is, *good reflectors are bad absorbers and bad reflectors are good absorbers of heat.*

(*ii*) If a body neither reflects nor transmits any radiation, $r = 0$ and $t = 0$, then $a = 1$.

Such a body which neither reflects nor transmits, but absorbs whole of the heat radiation incident on it is called a black body.

11.38 ABSORPTIVE AND EMISSIVE POWERS

55. *Define the terms absorptive power, emissive power and emissivity.*

Absorptive power. *The absorptive power of a body for a given wavelength λ is defined as the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength λ.* It is denoted by a_λ. A perfect black body absorbs all the heat radiations incident upon it. So its absorptive power is unity.

If the radiant energy dQ in wavelength range λ and $\lambda + d\lambda$ is incident on a body of absorptive power a_λ, then amount of radiant energy absorbed by the body $= a_\lambda \, dQ$.

The absorptive power is a dimensionless quantity.

Emissive power. The amount of heat energy radiated by a body per second depends upon (*i*) the area of its surface, (*ii*) the temperature of its surface and (*iii*) the nature of its surface. The strength of emission is measured by a quantity called emissive power. *The emissive power of a body at a given temperature and for a given wavelength λ is defined as the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range around the wavelength λ.*

If a heat radiation of wavelength range λ to $\lambda + d\lambda$ is incident on the surface of a body of emissive power e_λ, then the amount of radiant energy emitted per second per unit area $= e_\lambda \, d\lambda$.

The SI unit of emissive power is $\text{J s}^{-1}\,\text{m}^{-2}$ or Wm^{-2}.

Emissivity. *The emissivity of a body is defined as the ratio of the heat energy radiated per unit time per unit area by the given body to the amount of heat energy radiated per unit time per unit area by a perfect black body of the same temperature i.e.,* it is the ratio of the emissive power (e) of a body to the emissive power (E) of a black body at the same temperature. It is denoted by ε.

Thus $\varepsilon = \frac{e}{E}$

Scanned by CamScanner

It is dimensionless quantity. Its value lies between 0 and 1. The emissivity of a perfect black body is 1. The emissivities of polished copper, polished aluminium and lamp black are 0.018, 0.05 and 0.95 respectively.

11.39 BLACK BODY

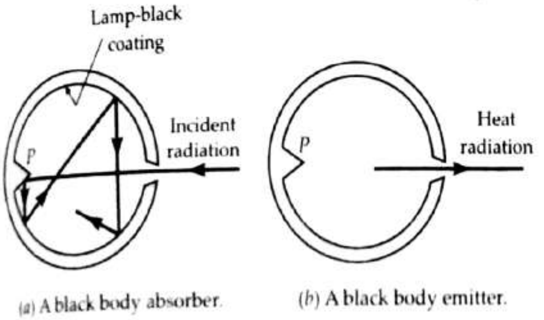
56. *What is a black body ? How can it be realised in practice ?*

Black body. *A black body is one which neither reflects nor transmits but absorbs whole of the heat radiation incident on it.* The absorptive power of a perfect black body is unity.

When a black body is heated to a high temperature, it emits radiations of all possible wavelengths within a certain wavelength range. The radiations emitted by a black body are called ***full*** or ***black body radiations***.

In practice, a surface coated with lamp black or platinum black absorbs 95 to 97% of the incident radiation. But on heating, it does not emit full radiation spectrum. So it acts as a black body only for absorption of heat radiation. It is observed that if a hollow cavity is heated, the radiation coming out from its inner surface through a small opening is a full radiation spectrum. Such a radiation is called **cavity radiation**. Hence the small opening of a heated hollow cavity acts as a perfect black body both for absorption and emission of heat radiation.

Fery's black body. Fery's black body consists of a hollow double walled metal sphere coated inside with lamp black and nickel polished from outside. Heat radiations entering the sphere through the small opening are completely absorbed due to multiple reflections. The conical projection opposite the opening prevents direct reflection.



(*a*) A black body absorber. (*b*) A black body emitter.

Fig. 11.29

To use it as a source of heat radiation, the enclosure is heated in a suitable bath to maintain its temperature constant. The radiations coming out from the small hole are black body radiations. The *wavelength range of emitted radiation is independent of the material of the body and depends only on the temperature of the black body.*

11.40 KIRCHHOFF'S LAW

57. *State and explain the Kirchhoff's law of heat radiation.*

Kirchhoff's law. *Kirchhoff 's law of heat radiation states that at any given temperature, the ratio of the emissive power to the absorptive power corresponding to the certain wavelength is constant for all bodies and this constant is equal to emissive power of the perfect black body at the same temperature and corresponding to the same wavelength.*

If e_λ and a_λ are the emissive and the absorptive powers of a body corresponding to wavelength λ,

then $$\frac{e_\lambda}{a_\lambda} = E_\lambda \text{ (constant)} \quad \text{...(i)}$$

where E_λ is the emissive power of perfect black body at the same temperature and corresponding to the same wavelength. Thus, if a_λ is large, then e_λ will also be large *i.e.*, if a body absorbs a radiation of certain wavelength strongly, then it will also strongly emit the radiation of that wavelength.

As the emissivity ε of a body is defined as the ratio of its emissive power to that of the emissive power of a black body at the same temperature, so

$$\frac{e_\lambda}{E_\lambda} = \varepsilon \quad \text{...(ii)}$$

From equations (*i*) and (*ii*), we get

$$a_\lambda = \varepsilon$$

Thus, *the absorptive power of a body is equal to its emissivity*. This is another form of ***Kirchhoff's law***.

Hence *a good absorber is a good emitter*. Since a good absorber is a poor reflector, so the ability of a body to emit radiation is related oppositely to its ability to reflect. That is, *a good emitter is a poor reflector*.

11.41 APPLICATIONS OF KIRCHHOFF'S LAW

58. *Describe some experimental observations to which Kirchhoff's law of heat radiation is applicable.*

Applications of Kirchhoff's law. According to Kirchhoff's law, a body strongly absorbs a radiation of certain wavelength, it must emit strongly the radiation of same wavelength. It is clear from the following observations :

(*i*) Take a piece of china having some dark paintings engraved on it. Heat it in a furnace to about 1000°C and then examine in a dark room immediately. The dark paintings will appear much brighter than white china. This is because the dark paintings are better absorbers and, therefore, also better emitters.

(*ii*) A green glass heated in a furnace when taken out in dark glows with red light. Green glass (when

Scanned by CamScanner

cold) is a good absorber of red light and a good reflector of green light. When heated, it becomes a good emitter of red light in accordance with Kirchhoff's law.

(*iii*) If a polished metal ball with a spot of platinum black on it is heated in a furnace to about 1200 K and then taken out into a dark room, the black spot appears brighter than the polished surface. This is because the black spot is a better absorber and hence, by Kirchhoff's law, a better emitter of radiation.

(*iv*) A Dewar flask or thermos bottle consists of a double-walled glass vessel with its inner and outer walls coated with silver. Radiation from the inner walls is reflected back into the contents of the bottle. Similarly, the outer wall reflects back any incoming radiation. The space between the walls is evacuated to reduce losses due to conduction and convection. The device helps in keeping hot contents hot and cold contents cold for a long time.

11.42 GREENHOUSE EFFECT

59. *What is Greenhouse effect for the atmosphere of the earth and what is its importance ?*

Greenhouse effect. *This is the phenomenon which keeps the earth's surface warm at night.*

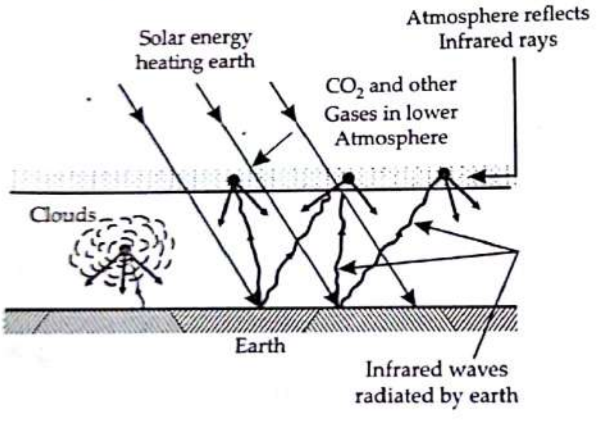


Fig. Greenhouse Effect

The radiation from the sun heats up the earth. Due to its lower temperature, the earth re-radiates it mostly in the infrared region. These infrared radiations cannot pass through the lower atmosphere, they get reflected back by gas molecules. Low lying clouds also reflect them back to the earth. These radiations heat up the objects on the earth's surface and so keep the earth's surface warm at night.

11.43 STEFAN-BOLTZMANN LAW

60. *State and explain Stefan-Boltzmann law of black body radiation.*

Stefan-Boltzmann law. *This law states that the heat energy emitted by a perfect black body per second per unit area is directly proportional to the fourth power of the absolute temperature of its surface.* Thus

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4.$$

If H is the rate of radiant energy emitted by a black body of surface area A, then the Stefan-Boltzmann law takes the form

$$H = EA = \sigma T^4 A$$

Here σ is a universal constant called Stefan-Boltzmann constant. The above relation was first deduced experimentally by Stefan and later proved theoretically by Boltzmann and hence is known as Stefan-Boltzmann law.

In SI units, $\sigma = 5.67 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}
$= 5.67 \times 10^{-8}$ W m^{-2} K^{-4}

In CGS units, $\sigma = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}.

If a black body is in an enclosure at temperature T_0, then the rate at which the black body absorbs radiation from the enclosure is σT_0^4. Therefore, the net loss of energy by the black body per unit time per unit area is

$$E = \sigma (T^4 - T_0^4)$$

If the body is not a perfect black body and has emissivity ε, then above relations get modified as follows:

$$E = \varepsilon \sigma T^4$$

$$E = \varepsilon \sigma (T^4 - T_0^4).$$

61. *Derive Newton's law of cooling from Stefan's law.*

Derivation of Newton's law of cooling from Stefan's law. Suppose a body of surface area A at an absolute temperature T is kept in an enclosure at lower temperature T_0. According to Stefan-Boltzmann law, the net rate of loss of heat from the body due to radiation is

$$H = \varepsilon\sigma A(T^4 - T_0^4) = \varepsilon\sigma A(T - T_0)(T + T_0)(T^2 + T_0^2)$$

$$= \varepsilon\sigma A(T - T_0)(T^3 + T^2 T_0 + T T_0^2 + T_0^3)$$

As $T - T_0$ is small, T can be taken approximately equal to T_0.

$$\therefore \quad H = \varepsilon\sigma A(T - T_0)(T_0^3 + T_0^3 + T_0^3 + T_0^3)$$

$$= 4\varepsilon\sigma A T_0^3 (T - T_0)$$

If c is the specific heat of the body and m its mass, then the rate of fall of temperature will be

$$-\frac{dT}{dt} = \frac{H}{mc} = \frac{4\varepsilon\sigma A T_0^3}{mc}(T - T_0)$$

or $$\frac{dT}{dt} = -kA(T - T_0)$$

This prove the Newton's law of cooling.

Scanned by CamScanner

Examples based on Stefan's Law

FORMULAE USED

1. *Stefan's law.* Energy emitted per second per unit area by a black body at absolute temperature T,
$$E = \sigma T^4, \quad \text{where } \sigma = \text{Stefan's constant.}$$
2. *Stefan-Boltzmann law.* When a black body at temperature T is placed in an enclosure at temperature T_0, the net heat energy radiated per second per unit area, $E = \sigma(T^4 - T_0^4)$
3. Energy radiated by a surface of emissivity ε, area A in time t,
   (*i*) $E = \varepsilon \sigma T^4 \times A \times t$ (*Stefan's law*)
   (*ii*) $E = \varepsilon \sigma (T^4 - T_0^4) \times A \times t$ (*Stefan-Boltzmann law*)

UNITS USED

Here E is in $Jm^{-2} s^{-1}$ or Wm^{-2} and T in kelvin.

CONSTANT USED

Stefan's constant,
(*i*) $\sigma = 5.67 \times 10^{-8}\ Js^{-1}\ m^{-2}\ K^{-4}$
$= 5.67 \times 10^{-8}\ Wm^{-2}\ K^4$
(*ii*) $\sigma = 5.67 \times 10^{-5}$ erg $s^{-1}\ cm^{-2}\ K^{-4}$

EXAMPLE 30. *Calculate the temperature (in K) at which a perfect black body radiates energy at the rate of 5.67 W cm^{-2}. Given $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$.* **[Delhi 05]**

Solution. Here $E = 5.67\ \text{W cm}^{-2} = 5.67 \times 10^4\ \text{Wm}^{-2}$
$$\sigma = 5.67\ \text{Wm}^{-2}\ \text{K}^{-4}$$
According to Stefan's law, $E = \sigma T^4$
$$\therefore \quad T = \left(\frac{E}{\sigma}\right)^{1/4} = \left(\frac{5.67 \times 10^4}{5.67 \times 10^{-8}}\right)^{1/4}$$
$$= (10^{12})^{1/4} = 10^3 = \mathbf{1000\ K.}$$

EXAMPLE 31. *Luminosity of Rigel star in Orion constellation is 17,000 times that of our sun. If the surface temperature of the sun is 6000 K, calculate the temperature of the star.*

Solution. Let E_1 and E_2 be the luminosities and T_1 and T_2 be the absolute temperatures of the star and sun respectively. According to Stefan's law,
$$E = \sigma T^4$$
$$\therefore \quad \frac{E_1}{E_2} = \frac{\sigma T_1^4}{\sigma T_2^4} = \frac{T_1^4}{T_2^4} \quad \text{or} \quad T_1 = \left(\frac{E_1}{E_2}\right)^{1/4} T_2$$
But $\quad \frac{E_1}{E_2} = 17{,}000, \quad T_2 = 6000\ \text{K}$
$$\therefore \quad T_1 = (17000)^{1/4} \times 6000$$
$$= 11.42 \times 6000 = \mathbf{68520\ K.}$$

EXAMPLE 32. *Due to the change in mains voltage, the temperature of an electric bulb rises from 3000 K to 4000 K. What is the percentage rise in electric power consumed ?*

Solution. Electric power consumed in first case,
$$P_1 = \sigma T_1^4 = \sigma (3000)^4$$
Electric power consumed in second case,
$$P_2 = \sigma T_2^4 = \sigma (4000)^4$$
$$\therefore \quad \frac{P_2}{P_1} = \frac{(4000)^4}{(3000)^4} = \frac{256}{81}$$
$$\frac{P_2 - P_1}{P_1} = \frac{P_2}{P_1} - 1 = \frac{256}{81} - 1 = \frac{175}{81}$$
$\therefore$ Percentage rise in power
$$= \left(\frac{P_2 - P_1}{P_1}\right) \times 100 = \frac{175}{81} \times 100 = \mathbf{216.}$$

EXAMPLE 33. *Consider the sun to be a perfect sphere of radius 6.8×10^8 m. Calculate the energy radiated by the sun in one minute. Surface temperature of the sun = 6200 K. Stefan's constant = $5.67 \times 10^{-8}\ Jm^{-2}\ s^{-1}\ K^{-4}$.*

Solution. Radius of the sun, $r = 6.8 \times 10^8$ m

Surface area of the sun,
$$A = 4\pi r^2 = 4 \times 3.142 \times (6.8 \times 10^8)^2$$
$$= 5.81 \times 10^{18}\ \text{m}^2$$
Temperature, $T = 6200$ K; Time $t = 1$ min $= 60$ s

Total energy radiated by the sun in 1 min,
$$E = \sigma T^4 \times A \times t$$
$$= 5.6 \times 10^{-8} \times (6200)^4 \times 5.81 \times 10^{18} \times 60$$
$$= \mathbf{2.92 \times 10^{28}\ J.}$$

EXAMPLE 34. *At what temperature will the filament of 100 W lamp operate if it is supposed to be perfectly black body of area 1 cm^2 ? Given $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}.*

Solution. Power of lamp = 100 W = 100 Js^{-1}

$\therefore$ Rate of emission of energy,
$$E = 100 \times 10^7\ \text{erg s}^{-1}$$
Area, $A = 1\ \text{cm}^2$, Temperature, $T = ?$

As $\quad E = \sigma T^4 \times A$
$$\therefore \quad T^4 = \frac{E}{\sigma A} = \frac{100 \times 10^7}{5.67 \times 10^{-5} \times 1} = \frac{100 \times 10^{12}}{5.67}$$
or $\quad T = \left(\frac{100}{5.67}\right)^{1/4} \times 10^3 = 2.049 \times 10^3 = \mathbf{2049\ K.}$

EXAMPLE 35. *Suppose the surface area of a person's body is 1.8 m^2 and the room temperature is 20°C. The skin temperature is 27°C and the emissivity of the skin is about 0.97 for the relevant region of electromagnetic radiation. Estimate the rate of heat radiation from the body of the person.*

Scanned by CamScanner

Solution. Here $A = 1.8\ m^2$, $\varepsilon = 0.97$,

$T = 27 + 273 = 300$ K, $T_0 = 20 + 273 = 293$ K,

$\sigma = 5.67 \times 10^{-8}\ Jm^{-2}s^{-1}K^{-4}$

The rate of heat loss

$$H = Q/t = \varepsilon \sigma A(T^4 - T_0^4)$$
$$= 0.97 \times 5.67 \times 10^{-8}[(300)^4 - (293)^4]$$
$$= 72.3\ Js^{-1} = \mathbf{72.3\ W}.$$

EXAMPLE 36. *A thin brass rectangular sheet of sides 15.0 cm and 12.0 cm is heated in a furnace to 600°C and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.0250 ? Neglect heat loss due to convection. (Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$).* **[NCERT]**

Solution. As the energy is radiated from both surfaces of the sheet, so

$$A = 2 \times 15.0 \times 12.0 \times 10^{-4} m^2 = 3.60 \times 10^{-2}\ m^2$$

$T = 600 + 273 = 873$ K, $\varepsilon = 0.250$,

$\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$.

The rate of heat loss by the sheet,

$$H = Q/t = \varepsilon \sigma AT^4$$
$$= 0.250 \times 5.67 \times 10^{-8} \times 3.60 \times 10^{-2} \times (873)^4$$
$$= 296\ Js^{-1} = \mathbf{296\ W}.$$

EXAMPLE 37. *A spherical body with radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, what would be the power radiated ?* **[IIT 97]**

Solution. Power radiated,

$$E = A\sigma T^4 = 4\pi r^2 \sigma T^4$$

When radius is halved and temperature is doubled, power radiated becomes

$$E' = 4\pi(r/2)^2 \sigma (2T)^4 = 4 \times 4\pi r^2 \sigma T^4 = 4E$$
$$= 4 \times 450 = \mathbf{1800\ W}.$$

EXAMPLE 38. *Calculate the maximum amount of heat which may be lost per second by radiation by a sphere 14 cm in diameter at a temperature of 227°C, when placed in an enclosure at 27°C.*
Given Stefan's constant = $5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$.

Solution. Temperature of sphere,

$$T = 227 + 273 = 500\ K$$

Temperature of surroundings,

$$T_0 = 27 + 273 = 300\ K$$

Radius of sphere, $r = 7$ cm $= 0.07$ m

Area of sphere,

$$A = 4\pi r^2 = 4 \times \frac{22}{7} \times (0.07)^2 = 6.16 \times 10^{-2}\ m^2$$

According to Stefan-Boltzmann law, the net heat lost per second by the sphere,

$$E = \sigma(T^4 - T_0^4) \times A$$
$$= 5.7 \times 10^{-8}(500^4 - 300^4) \times 6.16 \times 10^{-2}\ Js^{-1}$$
$$= 5.7 \times 6.16 \times 10^{-10} \times 100^4(5^4 - 3^4)\ Js^{-1}$$
$$= 5.7 \times 6.16 \times 10^{-2} \times 544\ Js^{-1}$$
$$= \frac{5.7 \times 6.16 \times 10^{-2} \times 544}{4.2} = \mathbf{45.48\ cal\ s^{-1}}.$$

EXAMPLE 39. *Two bodies A and B are kept in evacuated vessels maintained at a temperature of 27°C. The temperature of A is 527°C and that of B is 127°. Compare the rates at which heat is lost from A and B.*

Solution. Here $T_0 = 27 + 273 = 300$ K

$$T_1 = 527 + 273 = 800\ K$$
$$T_2 = 127 + 273 = 400\ K$$

According to Stefan-Boltzmann law, the net rates of loss heat by the bodies A and B will be

$$E_A = \sigma(T_1^4 - T_0^4)$$

and

$$E_B = \sigma(T_2^4 - T_0^4)$$

$$\therefore \quad \frac{E_A}{E_B} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4} = \frac{(800)^4 - (300)^4}{(400)^4 - (300)^4} = \mathbf{23}.$$

EXAMPLE 40. *How much faster does a cup of coffee cool off by one degree from 100°C than from 30°C in a room at 20°C ? Assume the coffee to act as a black body.*

Solution. According to Stefan-Boltzmann law, the rate of loss of heat by the body at temperature T is given by

$$E = \sigma(T^4 - T_0^4)$$

In first case : $T = 10 + 273 = 373$ K,

$$T_0 = 20 + 273 = 293\ K$$

$$\therefore \quad E_1 = \sigma[(373)^4 - (293)^4]. \quad ...(i)$$

In second case : $T = 30 + 273 = 303$ K,

$$T_0 = 20 + 273 = 293\ K$$

$$\therefore \quad E_2 = \sigma[(303)^4 - (293)^4] \quad ...(ii)$$

Dividing equation (*i*) by (*ii*), we get

$$\frac{E_1}{E_2} = \frac{(373)^4 - (293)^4}{(303)^4 - (293)^4}$$
$$= \frac{[1.93568 - 0.73701] \times 10^{10}}{[0.84289 - 0.73701] \times 10^{10}}$$
$$= \frac{1.19867}{0.10588} = \mathbf{11.32}.$$

Thus the coffee cools off by one degree from 100°C about 11.32 times faster than from 30°C.

Scanned by CamScanner

PROBLEMS FOR PRACTICE

1. A small hole is made in a hollow sphere whose walls are at 273°C. Find the total energy radiated per second per cm^2. Given Stefan's constant $= 5.7 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}. **(Ans.** 5.61×10^7 erg)

2. How much energy is radiated per minute from the filament of an incandescent lamp at 3000 K, if the surface area is 10^{-4} m^2 and its emissivity is 0.4 ? Stefan's constant $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}.
(Ans. 11022.5 J)

3. A full radiator at 0°C radiates energy at the rate of 3.2×10^4 erg cm^{-2} s^{-1}. Find (*i*) Stefan's constant and (*ii*) the amount of heat radiated per second by a sphere of radius 4 cm and at a temperature of 1000°C. [**Ans.** (*i*) 5.76×10^{-5} erg cm^{-2} s^{-1} K^{-4} (*ii*) 3.042×10^{10} erg s^{-1})]

4. To what temperature must a black body be raised in order to double total radiation, if original temperature is 727°C ? **(Ans.** 916°C)

5. The temperature of a body is increased from 27°C to 127°C. By what factor would the radiation emitted by it increase ? **[IIT 90]** **(Ans.** 256/81)

6. A black body initially at 27°C is heated to 327°C. How many times is total heat emitted at the higher temperature than that emitted at lower temperature ? What is the wavelength of the maximum energy radiation at the higher temperature ? Wien's constant $= 2.898 \times 10^{-3}$ mK. **(Ans.** 16, 48300 Å)

7. An electric bulb with tungsten filament having an area of 0.25 cm^2 is raised to a temperature of 3000 K, when a current passes through it. Calculate the electrical energy being consumed in watt, if the emissivity of the filament is 0.35. Stefan's constant, $\sigma = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}. If due to fall in main voltage the filament temperature falls to 2500 K, what will be wattage of the bulb ?
(Ans. 49.19 W, 19.38 W)

8. A sphere of radius 10 cm is hung inside an oven whose walls are at a temperature of 1000 K. Calculate total heat energy incident per second (in Js^{-1}) on the sphere. Given $\sigma = 5.67 \times 10^{-8}$ SI units.
(Ans. 7128 Js^{-1})

9. A body which has surface area of 5.0 cm^2 and a temperature of 727°C radiates 300 J of energy each minute. What is its emissivity ? Stefan's constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}. **(Ans.** 0.18)

10. An iron ball having a surface area of 200 cm^2 and at a temperature of 527°C is placed in an enclosure at 27°C. If the surface emissivity of iron be 0.4, at what rate is heat being lost by radiation by the ball ?
(Ans. 45.59 cal s^{-1})

HINTS

1. Here $T = 723 + 273 = 996$ K,
$\sigma = 5.7 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
Total energy radiated per second per cm^2,
$E = \sigma T^4 = 5.7 \times 10^{-5} \times (996)^4 = \mathbf{5.61 \times 10^7}$ **erg.**

2. Here $t = 1$ min $= 60$ s, $T = 3000$ K, $A = 10^{-4}$ m^2,
$\varepsilon = 0.4$
Total energy radiated from the filament per minute,
$E = \varepsilon \times \sigma T^4 \times A \times t$
$= 0.4 \times 5.67 \times 10^{-8} \times (3000)^4 \times 10^{-4} \times 60$ J
$=$ **11022.5 J.**

3. Here $T = 0 + 273 = 273$ K
Energy radiated per second per unit area,
$E = 3.2 \times 10^5$ erg cm^{-2} s^{-1}
(*i*) $\sigma = \frac{E}{T^4} = \frac{3.2 \times 10^5}{(273)^4}$
$= \mathbf{5.76 \times 10^{-5}}$ **erg** $\mathbf{cm^{-2}\ s^{-1}\ K^{-4}}$.
(*ii*) Radius of sphere, $r = 4$ cm
∴ Surface area of sphere,
$A = 4\pi r^2 = 4 \times 3.142 \times (4)^2$ cm^2
Also, $T = 1000 + 273 = 1273$ K
Energy radiated per second, $E = \sigma T^4 \times A$
$= 5.76 \times 10^{-5} \times (1273)^4 \times 4 \times 3.142 \times 4^2$
$= \mathbf{3.042 \times 10^{10}}$ **erg** $\mathbf{s^{-1}}$.

4. Here $T_1 = 727 + 273 = 1000$ K, $E_2 / E_1 = 2$
As $\frac{E_2}{E_1} = \frac{T_2^4}{T_1^4}$
∴ $2 = \frac{T_2^4}{(1000)^4}$
or $T_2 = (2)^{1/4} \times 1000 = 1.189 \times 1000$
$= 1189$ K $=$ **916°C.**

5. $\frac{E_2}{E_1} = \frac{T_2^4}{T_1^4} = \frac{(127 + 273)^4}{(27 + 273)^4} = \frac{(400)^4}{(300)^4} = \mathbf{\frac{256}{81}}$.

6. $T_1 = 273 + 27 = 300$ K, $T_2 = 327 + 273 = 600$ K,
$b = 2.889 \times 10^{-3}$ mK
$\frac{E_2}{E_1} = \left[\frac{T_2}{T_1}\right]^4 = \left[\frac{600}{300}\right]^4 = \mathbf{16}.$
Also $\lambda_m = \frac{b}{T} = \frac{2.898 \times 10^{-3}}{600}$
$= 0.483 \times 10^{-5}$ m $=$ **48300 Å**

7. (*i*) Here $A = 0.25$ cm^2 $= 0.25 \times 10^{-4}$ m^2, $T = 3000$ K,
$\varepsilon = 0.35$
Energy consumed per second $= \varepsilon \times \sigma T^4 \times A$
$= 0.35 \times 5.67 \times 10^{-8} \times (3000)^4 = 0.25 \times 10^{-4}$
$= 40.19$ Js^{-1} $=$ **40.19 W.**

Scanned by CamScanner

(ii) Here $T = 2500$ K

Energy consumed per second $= \varepsilon \times \sigma T^4 \times A$

$= 0.35 \times 5.67 \times 10^{-8} \times (2500)^4 \times 0.25 \times 10^{-4}$

$= 19.38\ \text{Js}^{-1} = \mathbf{19.38\ W}$.

8. Here $r = 10$ cm $= 0.10$ m, $T = 1000$ K

$\sigma = 5.67 \times 10^{-8}$ SI units

Heat energy incident/sec/area $= \sigma T^4$

Total heat energy incident/sec

$= \sigma T^4 \times$ area of sphere

or $E = \sigma T^4 \times 4\pi r^2$

$= 5.67 \times 10^{-8} \times (10^3)^4 \times 4 \times \frac{22}{7} (0.01)^2$

$= \mathbf{7128\ J\ s^{-1}}$.

9. Here $A = 5.0\ \text{cm}^2 = 5.0 \times 10^{-4}\ \text{m}^2$,

$T = 727 + 273 = 1000$ K.

Energy radiated per sec, $E = \frac{300}{60} = 5\ \text{Js}^{-1}$

As $E = \varepsilon(\sigma T^4) \times A$

$\therefore \quad \varepsilon = \frac{E}{\sigma T^4 \times A}$

$= \frac{5}{5.67 \times 10^{-8} \times (1000)^4 \times 5.0 \times 10^{-4}} = \mathbf{0.18}$.

10. $E = \varepsilon \times \sigma (T^4 - T_0^4) A$

$= 0.4 \times 5.7 \times 10^{-5} \times (800^4 - 300^4) \times 200\ \text{erg s}^{-1}$

$= \frac{0.4 \times 5.7 \times (4096 - 81) \times 2 \times 10^5}{4.2 \times 10^7} = \mathbf{45.59\ cal\ s^{-1}}$.

11.44 WIEN'S DISPLACEMENT LAW

62. ***State and illustrate Wien's displacement law. Give its importance.***

Wien's displacement law. The total energy radiated by a black body is not uniformly distributed over all the wavelengths but is maximum for a particular wavelength λ_m. The value of λ_m decreases with the increase of temperature.

Wien's displacement law states that the wavelength (λ_m) corresponding to which the energy emitted by a black body is maximum is inversely proportional to its absolute temperature (T). Mathematically,

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

where b is Wien's constant. Its value is 2.9×10^{-3} mK.

Illustration. When an iron piece is heated in a hot ame, its colour first becomes dull red, then reddish llow and finally white. This observation is an cordance with Wien's law because with increasing perature, the emission of energy is maximum esponding to smaller wavelength.

Importance. Wien's law can be used to estimate the surface temperatures of the moon, sun and other stars. Light from the moon shows a maximum of intensity at $\lambda_m = 14\ \mu$m. By applying Wien's law, the temperature of the surface of the moon turns out to be 200 K. Similarly, solar radiation shows a maximum at $\lambda_m = 4753$ Å. This corresponds to a surface temperature of 6060 K.

Examples based on Wien's Displacement Law

FORMULAE USED

1. *Wien's displacement law* : The wavelength λ_m corresponding to maximum energy emission by a black body at absolute temperature T is given by

$$\lambda_m = \frac{b}{T}$$

where b = Wien's constant = 0.002898 mK

UNITS USED

Wavelength λ_m is in metre, temperature T in kelvin.

EXAMPLE 41. *Wavelength corresponding to E_{max} for the moon is 14 microns. Estimate the surface temperature of the moon, if $b = 2.884 \times 10^{-3}$ mK.*

Solution. Here $\lambda_m = 14$ microns $= 14 \times 10^{-6}$ m,

$b = 2.884 \times 10^{-3}$ mK

By Wien's law,

$$T = \frac{b}{\lambda_m} = \frac{2.884 \times 10^{-3}}{14 \times 10^{-6}} = \mathbf{206\ K.}$$

EXAMPLE 42. *The surface temperature of a hot body is 1227°C. Find the wavelength at which it radiates maximum energy. Given Wien's constant = 0.2898 cm K.*

Solution. Here $T = 1227 + 273 = 1500$ K,

$b = 0.2898$ cm K

By Wien's law,

$$\lambda_m = \frac{b}{T} = \frac{0.2898}{1500}$$

$= 19320 \times 10^{-8}$ cm $= \mathbf{19320}$ **Å**.

EXAMPLE 43. *The spectral energy distribution of the sun has a maximum at 4753 Å. If the temperature of the sun is 6050 K, what is the temperature of a star for which this maximum is at 9506 Å ?*

Solution. Here $\lambda_m = 4753$ Å, $T = 6050$ K

$\lambda'_m = 9506$ Å, $T' = ?$

By Wien's law, $\lambda_m T = \lambda'_m T'$

$$T' = \frac{\lambda_m T}{\lambda'_m} = \frac{4753 \times 6050}{9506} = \mathbf{3025\ K.}$$

Scanned by CamScanner

EXAMPLE 44. *An indirectly heated filament is radiating maximum energy of wavelength 2.16×10^{-5} cm. Find the net amount of heat energy lost per second per unit area, the temperature of surrounding air is 13°C. Given $b = 0.288$ cm K, $\sigma = 5.77 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}.*

Solution. Here $b = 0.288$ cm K, $\lambda_m = 2.16 \times 10^{-5}$ cm

By Wien's law, $\lambda_m T = b$

or $$T = \frac{b}{\lambda_m} = \frac{0.288}{2.16 \times 10^{-5}} = 13333.3 \text{ K}$$

Also, $\sigma = 5.77 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}

Temperature of surrounding,

$$T_0 = 13 + 273 = 286 \text{ K}$$

∴ The net amount of heat energy lost per second per unit area,

$$E = \sigma(T^4 - T_0^4) = 5.77 \times 10^{-5}[(13333.3)^4 - (286)^4]$$
$$= 5.77 \times 10^{-5}[3.161 \times 10^{16} - 6.69 \times 10^9]$$
$$= 5.77 \times 10^{-5} \times 3.161 \times 10^{16}$$
$$= \mathbf{1.824 \times 10^{12} \text{ erg s}^{-1} \text{ cm}^{-2}}.$$

PROBLEMS FOR PRACTICE

1. The sun radiates maximum energy at wavelength 4753 Å. Estimate the surface temperature of the sun, if $b = 2.888 \times 10^{-3}$ mK. (**Ans.** 6076 K)
2. The temperature of an ordinary electric bulb is around 3000 K. At what wavelength will it radiate maximum energy ? Will this wavelength be within visible region ? Given $b = 0.288$ cm K. (**Ans.** 9600 Å, No)
3. A furnace is at a temperature of 2000 K. At what wavelength will it radiate maximum intensity ? Is it in the visible region ? (**Ans.** 14400 Å, No)

11.45 DISTRIBUTION OF ENERGY IN THE BLACK BODY SPECTRUM

63. *Explain the distribution of energy in the spectrum of a black body. What conclusions can be drawn from it ?*

Energy distribution in a black body spectrum. When a black body is heated, it emits heat radiations of different wavelengths. When these wavelengths are arranged in the increasing order, we get black body spectrum. Radiation of a particular wavelength has a definite energy at a particular temperature. Fig. 11.30 shows the experimental curves drawn between the wavelength λ and intensity of radiation E_λ (energy per second per unit area) emitted by a black body maintained at different constant temperatures.

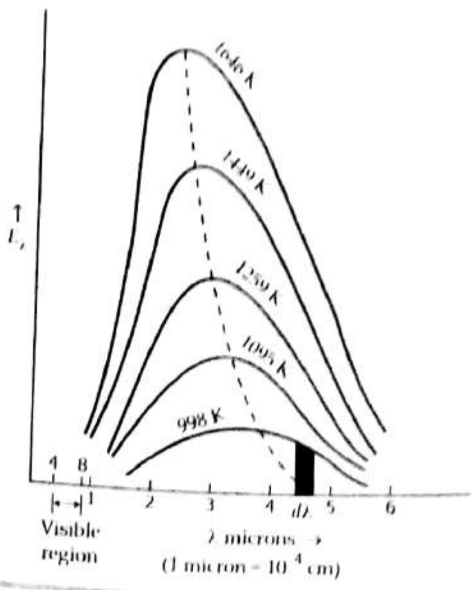


Fig. 11.30 Energy distribution in a black body spectrum

Conclusions drawn from the black body spectrum :

(*i*) At each temperature, a black body emits continuous heat radiation spectrum. The energy is not distributed equally amongst all wavelengths.

(*ii*) The energy associated with a radiation of particular wavelength increases with the increase in temperature.

(*iii*) As wavelength increases, the energy emitted increases, reaches a maximum for a particular wavelength λ_m and then decreases.

(*iv*) The wavelength (λ_m) of maximum emission shifts towards the lower wavelength side as the temperature of the black body increases.

$$\lambda_m T = \text{a constant}$$

This is ***Wien's displacement law***.

(*v*) Area under a curve represents the total energy (E) emitted by a perfect black body per second per unit area over the complete wavelength range at that temperature. This area is found to increase with fourth power of absolute temperature. Thus $E \propto T^4$.

This is ***Stefan-Boltzmann law*** of heat radiation.

The black body radiation curves are *universal*. They depend only on the temperature and not on the size, shape or material of the black body.

11.46 PHASES AND PHASE DIAGRAMS*

64. *Draw isotherms for water at different temperatures both above and below its critical temperature. What important conclusions can be drawn from them? Define the three critical constants.*

Scanned by CamScanner

Isotherms of water. *A graph drawn between the pressure and volume of a system at constant temperature is called an isotherm.* Fig. 11.31 shows few isotherms for water-steam system in the temperature range 350°–390°.

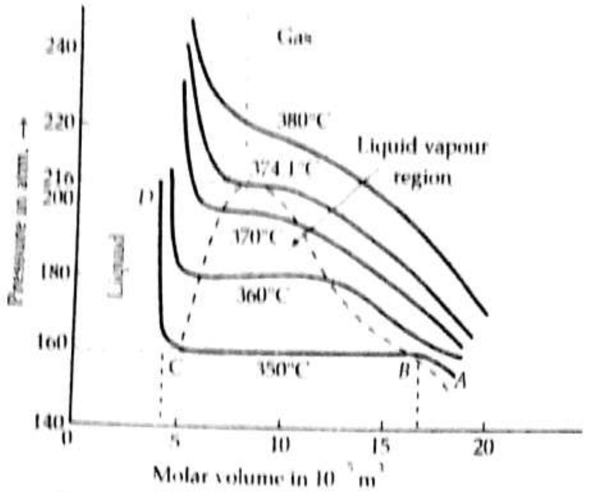


Fig. 11.31 Water–steam phase diagram for one mole of H_2O near the critical point C.

Consider the isotherm $ABCD$ at 350°C. AB represents the vapour phase (steam) which is compressible. This means when pressure is increased from A to B, the volume is decreased. Here steam is at 350°C and this is possible only at high pressure of about 160 atmosphere. From B to C, the pressure remains constant. At B the substance is in vapour state and at C it is in liquid state. Thus along BC liquid and vapour coexist in equilibrium.

Let V_l and V_g be the molar volumes of water in liquid and gaseous phases respectively. If V is the total volume of the system, then the fractions of the volume in liquid and gaseous phases will be

$$x_l = \frac{V_g - V}{V_g - V_l} \quad \text{and} \quad x_g = 1 - x_l$$

When pressure becomes more than 163 atm, the substance is in the liquid state (water). Along CD the pressure is increased. There is almost no change in volume. This shows that the liquids are incompressible.

As the temperature is increased, the volume difference $V_l - V_g$ decreases and at a temperature 374.1°C and pressure 216 atm, $V_l - V_g = 0$. For this isotherm there is no horizontal portion. This temperature is called the critical temperature (T_c) of water. This means that water can exist as liquid till 374.1°C only, there is only one phase *i.e.* vapour. This means if a gas is above critical temperature whatever pressure is applied we cannot liquify it.

Critical constants. *The critical temperature, T_c is the temperature below which a gas can be liquified by the application of pressure. The pressure required is called critical pressure P_c and the volume occupied by unit mass of the gas at critical temperature and critical pressure is called critical volume V_c.*

65. ***Draw a labelled P-T diagram of water. Explain its behaviour, when both pressure P and temperature T are above and below the triple point. Give the importance of triple point.***

Pressure-temperature phase diagram for water. Fig. 11.32 shows the P-T phase diagram for water. It consists of the following *three* curves :

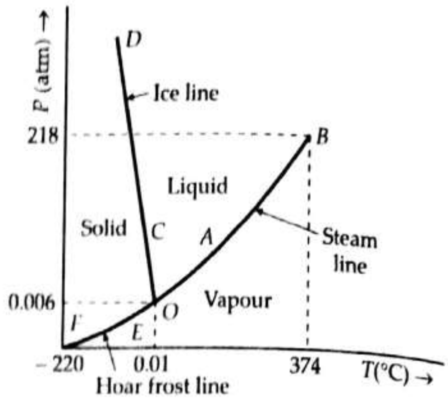


Fig. 11.32 Pressure-temperature phase diagram for water.

(*i*) **Vaporisation curve (Steam line *AB*).** It is a graph between pressure and the boiling point of the substance in the liquid state. Each point on this curve fixes a set of pressure and temperature at which the liquid and the gaseous phases can co-exist. If the pressure is increased, the vapour will at once condense into liquid but if the pressure is decreased, the liquid will evaporate. So, all points above the vaporisation curve correspond to liquid phase and below it to vapour phase.

(*ii*) **Fusion curve (Ice line *CD*).** *It is a graph between the pressure and the melting point of the substance in the solid state.* Each point on this curve gives the value of the pressure and the temperature at which the solid and liquid phases can co-exist. If pressure is increased, the solid would melt into liquid but if the pressure is decreased liquid will turn into solid. So all the points above the fusion curve correspond to liquid phase and those below it to solid phase.

(*iii*) **Sublimation curve (Hoar frost line *EF*).** *It is a graph between pressure and temperature at which a solid directly changes to vapour state.* Each point on this curve gives the values of pressure and temperature at which

Scanned by CamScanner

the solid and vapour phases can co-exist. If pressure is increased, the vapour changes to solid phase and if the pressure is decreased, the solid changes to vapour state. So all the points above this curve correspond to solid phase while those below it correspond to vapour state.

Conclusions. (*i*) In the space above the steam line and on the right of ice line, water exists in liquid phase as water.

(*ii*) In the space below the steam line and on the right of hoar frost line, water exists in gaseous phase as steam.

(*iii*) In the space above the hoar-frost line and on the left of ice-line, water exists in solid phase as ice.

Triple point. *It is a unique point on P-T diagram at which all the three phases of a substance can co-exist in equilibrium with each other.* The three curves *AB*, *CD* and *EF* on being extended meet at point *O* which represents the triple point. The values of pressure and temperature corresponding to this point for water are 0.46 cm of Hg and 273.16 K.

The negative slope of ice line for water indicates that melting point of ice decreases with the increase in pressure. The triple point of such substances is above its melting point at normal pressure.

66. *Draw a labelled P-T diagram for CO_2. Explain, its behaviour, when both pressure and temperature are above and below the triple point. Give importance of triple point.*

Pressure-temperature phase diagram for CO_2.

Figure 11.33 shows the *P-T* phase diagram for CO_2. It consists of the following *three* curves :

(*i*) **Vaporisation curve (*AB*).** Above *AB* the substance is in liquid phase and below it in vapour phase. Along *AB*, we get a set of values for *P* and *T* for which the liquid and the vapour co-exist. This graph shows the variation of boiling point with pressure. For all substances boiling point increases with pressure.

(*ii*) **Fusion curve (*CD*).** Above *CD* the substance is in the solid state and below it in the liquid state. Along *CD* solid and liquid co-exist *i.e.* the substance melts and the temperature is called melting point. The graph shows that the melting point of the substance increases with increase in pressure.

(*iii*) **Sublimation curve (*EF*).** Above *EF* the substance is in solid state and below in vapour state. Along *EF*, the solid and the vapour co-exist. Here the solid changes directly to vapour state and process is called sublimation.

The three curves meet at *O*. This is called the *triple point* of CO_2. The values of pressure and temperature corresponding to triple point for CO_2 are 5.11 atm and 216.4 K.

Here all the three curves have positive slopes. So the triple point of CO_2 is below its melting point at the normal pressure.

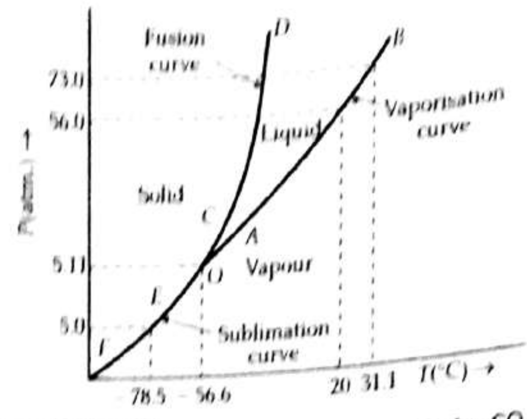


Fig. 11.33 Pressure-temperature diagram for CO_2.

Very Short Answer Conceptual Problems

Problem 1. Is temperature a macroscopic or microscopic concept ?

Solution. Temperature is macroscopic concept. It is related to the average kinetic energy of a large number of molecules forming a system. It is not possible to define the temperature for a single molecule.

Problem 2. What is dynamical theory of heat ?

Solution. According to dynamical theory, the amount of heat possessed by a body is equal to the total kinetic energy of its molecules and its temperature is proportional to the average kinetic energy of its molecules.

Problem 3. Two thermometers are constructed in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which of the two will respond quickly to temperature changes ?

Solution. A cylindrical bulb has a greater surface area than a spherical bulb of the same volume. Hence the thermometer with elongated bulb will respond to temperature changes more quickly than the one with a spherical bulb.

Problem 4. Why a clinical thermometer should not be sterilized by boiling ?

Solution. The range of clinical thermometer is usually from 95°F to 110°F and the boiling point of water is 212°F. So on sterilization by boiling, the capillary of thermometer will burst due to thermal expansion of mercury in the capillary.

Scanned by CamScanner

Problem 5. Why should a thermometer bulb have a small heat capacity ?

Solution. The thermometer bulb having small heat capacity will absorb less heat from the body whose temperature is to be measured. Hence the temperature of that body will practically remain unchanged.

Problem 6. What do you mean by triple point of water ? Why it is unique ?

Solution. It is the temperature at which the three phases of water – ice, liquid water and water vapour are equally stable and exist simultaneously. It is unique because it occurs at specific temperature (= 273.16 K) and a specific pressure of about 0.46 cm of Hg column.

Problem 7. Why are gas thermometers are more sensitive than mercury thermometers ?

Solution. The coefficient of expansion of a gas is very large as compared to the coefficient of expansion of mercury. For the same temperature range, a gas would undergo a much larger change in volume as compared to mercury.

Problem 8. Can the temperature of a body be negative on the kelvin scale ?

Solution. No. This is because the absolute zero on the kelvin scale is the minimum possible temperature.

Problem 9. Mercury boils at 357°C. How can then a mercury thermometer be used to measure temperature upto 500°C ?

Solution. The space above the mercury is filled with nitrogen. This increases the boiling point of mercury and raises the upper limit of the mercury thermometer.

Problem 10. Why the temperatures above 1200°C cannot be measured accurately by a platinum resistance thermometer ?

Solution. This is because platinum begins to evaporate above 1200°C.

Problem 11. Why is a constant volume gas thermometer preferred as a standard thermometer than a constant pressure gas thermometer ?

Solution. This is because the changes in pressure can be measured with greater accuracy than changes in volume.

Problem 12. A mercury thermometer is transferred from melting ice to a hot liquid. The mercury rises 0.9 of the distance between lower and upper fixed points. What is the temperature of the liquid in °C ? What in °F ?

Solution. $t_C = \frac{l_t - l_0}{l_{100} - l_0} \times 100°C = \frac{0.9 - 0}{1.0 - 0} \times 100 = \mathbf{90°C}$.

$$t_F = \frac{9}{5} t_C + 32 = \frac{9}{5} \times 90 + 32 = \mathbf{194°F}.$$

Problem 13. The readings of air thermometer at 0°C and 100°C are 50 cm and 75 cm of mercury column respectively. What is the temperature at which its reading is 80 cm of Hg column ?

Solution. $t_C = \frac{P_t - P_0}{P_{100} - P_0} \times 100 = \frac{80 - 50}{75 - 50} \times 100 = \mathbf{120°C}$.

Problem 14. Two bodies at different temperatures T_1 and T_2, if brought in thermal contact do not necessarily settle at the mean temperature $(T_1 + T_2)/2$. Why ?

Solution. The two bodies may have different masses and different materials *i.e.*, they may have different thermal capacities. In case the two bodies have equal thermal capacities they would settle at the mean temperature $(T_1 + T_2)/2$.

Problem 15. A body at higher temperature contains more heat, comment.

Solution. The statement is not always true. The heat content of a body depends upon its mass, specific heat and temperature.

Problem 16. Two hollow glass balls are connected by a tube, which has a pellet of mercury in the middle. Can the temperature of the surrounding air be determined from the position of the drop ?

Solution. Yes. If the tube is held vertically, the position of the pellet will change with any change in the temperature of the surrounding air. This can be used as a thermometer.

Problem 17. Do all solids expand on heating ? If not, give an example.

Solution. No. Camphor contracts on heating.

Problem 18. Why does a solid expand on heating ?

Solution. The average distance between the positions of equilibrium of the atoms of a solid increases with an increase in temperature which results in the thermal expansion of a solid.

Problem 19. Is the temperature coefficient always positive ?

Solution. No. Temperature coefficient α is positive for metals and alloys and negative for semiconductors and insulators.

Problem 20. The diameters of steel rods A and B having the same length are 2 cm and 4 cm respectively. They are heated through 100°C. What is the ratio of increase of length of A to that of B ?

Solution. 1 : 1. This is because the increase in length does not depend on the diameter of the steel rod.

Problem 21. The difference between lengths of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. Is this possible ?

Solution. Yes. This is possible when the lengths of the rods are in inverse ratio of their coefficients of linear expansion. For the difference in the lengths of the two rods to remain same,

$$\Delta l_1 = \Delta l_2 \quad \text{or} \quad l_1 \alpha_1 \Delta T = l_2 \alpha_2 \Delta T \quad \text{or} \quad \frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}.$$

Problem 22. Why a small gap is left between the iron rails of railway tracks ?

Solution. If no gap is left between the iron rails, the rails may bend due to expansion in summer and the train may get derailed.

Scanned by CamScanner

Problem 23. Why are loops provided in long metal pipes used for carrying oil and any other liquid over long distances ?

Solution. The loops in metal pipes are provided in order to avoid the strain that would develop in the pipes when the temperature changes and hence the pressure of liquids in them changes.

Problem 24. Pendulum clocks generally run fast in winter and slow in summer. Why ?

Solution. The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad i.e., \quad T \propto \sqrt{l}.$$

In winter, l decreases with the fall in temperature, so T decreases and clocks run fast. In summer l increases with the increase in temperature, so T increases and clock runs slow.

Problem 25. Why is invar used in making a clock pendulum ?

Solution. Invar, which is an alloy of nickel and steel, has extremely small temperature coefficient of expansion and so the length of a pendulum made of invar does not change appreciably during summer and winter seasons and hence the clock gives almost correct time.

Problem 26. Why must telephone or power lines necessarily slag a little ?

Solution. The sag is allowed for contraction in winter. If no sag is allowed, the wire may snap in extremely cold weather.

Problem 27. A brass disc fits snugly in a hole in a steel plate. Should we heat or cool the system to loosen the disc from the hole ?

Solution. α for brass is greater than that for steel. On cooling, the disc shrinks to a greater extent than the hole, and hence the disc would get loosened.

Problem 28. A tightened glass stopper can be taken out easily by pouring hot water around the neck of the bottle. Why ?

Solution. The neck expands but not the stopper due to poor conductivity of glass. Thus the stopper can be taken out easily.

Problem 29. A long cylindrical vessel having linear coefficient of expansion α is filled with a liquid up to a certain level. On heating, it is observed that the length of the liquid in the cylinder remains the same. What is the volume coefficient of expansion of the liquid ?

Solution. Since the level of liquid remains the same, therefore, the volume coefficient of expansion of the liquid is the same as the volume coefficient of expansion of the cylinder. So, the volume coefficient of expansion of the liquid is 3α.

Problem 30. Thick bottomed drinking glasses frequently crack if hot water is poured into them. Why ?

Solution. Glass is a bad conductor of heat. It does not pass down the heat quickly to the lower surface. Different layers of the bottom are at different temperatures and expand differently. This causes breakage of the glass at the bottom.

Problem 31. Two identical rectangular strips of copper, and the other of steel are riveted to form a bimetallic strip. What will happen on heating ?

Solution. Since α for copper is more than α for steel, hence on heating, the bi metallic strip will bend in such a way that the copper strip remains on outer or convex side.

Problem 32. Why iron rims are heated red hot before being put on the cart wheels ?

Solution. The iron ring to be put on the rim of a cart wheel is always of slightly smaller diameter than that of the wheel. When the iron ring is heated to become red hot, it expands and slips on to the wheel easily. When it is cooled, it contracts and grips the wheel firmly.

Problem 33. In riveting boiler plates, red hot rivets are used. Why ?

Solution. Red hot rivets are used so that on cooling, the grip becomes firm and steam-tight.

Problem 34. How does the diameter of the opening in the cast iron plate of a kitchen stove change, when the stove is heated ?

Solution. The opening in the stove is circular. But its diameter undergoes linear expansion on heating.

The increase in diameter is given by : $\Delta D = \alpha \; D \Delta T$.

Problem 35. A metal ball is heated through a certain temperature. Out of mass, radius, surface area and volume, which will undergo largest percentage increase and which one the least ?

Solution. The mass of the ball will not change. Its volume ($\frac{4}{3}\pi r^3$) will undergo largest percentage increase while the percentage increase in radius will be minimum.

Problem 36. Explain why a beaker filled with water at 4°C overflows if the temperature is decreased or increased ?

Solution. It is because of the anomalous expansion of water. Water has a maximum density at 4°C. Therefore, water expands whether it is heated above 4° C or cooled below 4°C.

Problem 37. A block of wood is floating on water at 0°C with a certain volume V above the level of water. The temperature of water is gradually increased from 0°C to 8°C. How does the volume V change with the change of temperature ?

Solution. The density of water increases from 0° C to 4°C and decreases from 4°C to 8°C. So, V will increase till the temperature of water reaches 4°C and then it will go on decreasing.

Problem 38. Is J a physical quantity ?

Solution. No. J is not a physical quantity. It is a conversion factor.

Scanned by CamScanner

Problem 39. Why is J called a conversion factor ?

Solution. Because it helps us to convert work measured in terms of joules into heat expressed in calories or *vice-versa*.

Problem 40. A thermos bottle containing water is vigorously shaken. What will be the effect on the temperature of water ?

Solution. The temperature of the water will increase slightly. This is because some of the work done against opposing viscous force will be converted into heat.

Problem 41. When we rub our hands, they are warmed but only to a certain maximum temperature. Why ?

Solution. The work done in rubbing is converted into heat. But after some time when the temperature of the hands is raised, the total heat produced goes into the atmosphere.

Problem 42. There is a slight temperature difference between the water fall at the top and the bottom. Why ?

Solution. The potential energy of water at the top of the fall gets converted into heat kinetic energy at the bottom of the fall. When water hits the ground, a part of its kinetic energy gets converted into heat which increases its temperature slightly.

Problem 43. Why do the brake drums of a car get heated, when the car moves down a hill at a constant speed ?

Solution. As the car moves down a hill at constant speed, its kinetic energy does not change. But its gravitational potential energy constantly decreases, which gets converted into heat and so the brake drums get heated.

Problem 44. Can a given amount of mechanical energy be completely converted into heat ?

Solution. Yes, a given amount of mechanical work can be completely converted into heat. This is because whole of the mechanical energy can be absorbed by the molecules of a system in the form of their kinetic energy which gets converted into heat.

Problem 45. A match stick can be lighted by rubbing it against a rough surface. Why ?

Solution. When the match stick is rubbed against a rough surface, work is done against friction. This work done appears as heat and lights up the match stick.

Problem 46. If an electric fan be switched in a closed room, will the air of the room be cooled ? If not, why do we feel cold ?

Solution. The air will not be cooled. In fact, it will get heated up due to the increase in the speed of its molecules. We feel cold due to faster evaporation of sweat.

Problem 47. Give an example of a system in which no heat is transferred to or from a system but the temperature of the system changes.

Solution. When a paddle-wheel arrangement is worked while dipping in water, the temperature of water increases without any addition of heat.

Problem 48. What is the difference between the specific heat and the molar specific heat ?

Solution. The specific heat is the heat capacity per unit mass whereas the molar specific heat is the heat capacity per mole.

Problem 49. Why water is preferred to any other liquid in the hot water bottles ?

Solution. Water is preferred to any other liquid in the hot water bottles because due to high specific heat it does not cool fast. Also for the given mass of water, the amount of heat contained is higher and it can provide more warmth as compared to any other liquid.

Problem 50. The coolant used in a nuclear reactor should have high specific heat. Why ? *[Delhi 2011]*

Solution. The purpose of a coolant is to absorb maximum heat with least rise in its own temperature. This is possible only if specific heat is high because $Q = mc\Delta T$. For a given value of m and Q, the rise in temperature ΔT will be small if c is large. This will prevent different parts of the nuclear reactor from getting too hot.

Problem 51. Why juice bottles are placed under water in the cold countries ?

Solution. This is done so to prevent the freezing of juice. Water has to release comparatively large amount of heat to lower its temperature to the same extent than juice and hence the chances of freezing are reduced.

Problem 52. Why is water used as an effective coolant?

Solution. The specific heat of water is very high. When it runs over hot parts of an engine or machinery, it absorbs a large amount of heat. This helps in maintaining the temperature of the engine low.

Problem 53. What kind of thermal conductivity and specific heat requirements would you specify for cooking utensils ? *[Central Schools 2004]*

Solution. A cooking utensil should have (*i*) *high conductivity* so that it can conduct heat through itself and transfer it to the contents quickly. (*ii*) *low specific heat* so that it immediately attains the temperature of the source.

Problem 54. Why do the metal utensils have wooden handles ?

Solution. Wood is a bad conductor of heat. Wooden handle does not allow heat to be conducted from the hot utensil to the hand. So we can easily hold the hot utensil with its help.

Problem 55. Why birds are often seen to swell their feathers in winter ? *[Himachal 07]*

Solution. When the birds swell their feathers, they are able to enclose air in the feathers. Air, being a poor conductor of heat, prevents the loss of heat from the bodies of the birds to the surroundings and as such they do not feel cold in winter.

Scanned by CamScanner

Problem 56. Why an ice box is constructed with a double wall ?

Solution. An ice-box is made of double wall, and the space between the walls is filled with some non-conducting material to provide heat insulation, so that the loss of heat can be minimised.

Problem 57. Why are two thin blankets are warmer than a single blanket of double the thickness ?

Solution. The air enclosed between two blankets prevents the transfer of heat from our body to outside. Thus it provides a better insulation than a single blanket of double thickness.

Problem 58. A squirred wraps its bushy tail round its body during its winter sleep. Why ?

Solution. The bushy tail provides its body a non-conducting blanket. So the loss of heat by conduction is minimised.

Problem 59. Calorimeters are made of metals not glass. Why ?

Solution. This is because metals are good conductors of heat and have low specific heat capacity.

Problem 60. When we step barefoot into an office with a marble floor, we feel cold. Why ?

Solution. This is because marble is a better conductor of heat than concrete. When we walk barefooted on a marble floor, heat flows our body through the feet and we feel cold.

Problem 61. Why we can easily boil water in a paper cup ?

Solution. This is because heat is easily conducted through the paper to the water, and as such the temperature attained is not sufficient for the paper to be charred.

Problem 62. A piece of paper wrapped tightly on a wooden rod is observed to get charred quickly when held over a flame as compared to a similar piece of paper when wrapped on a brass rod. Explain why.

Solution. Brass is a good conductor of heat. It quickly conducts away the heat. So, the paper does not alter its ignition point easily. On the other hand, wood is a bad conductor of heat and is unable to conduct away the heat. So, the paper quickly reaches its ignition point and is charred.

Problem 63. A piece of wire gauze is placed over the Bunsen burner. If the gas is turned on below the gauge, will the flame go above the gauge ?

Solution. Copper is a very good conductor of heat. The copper gauze absorbs most of the heat. So the temp. of the gas above the gauze does not reach its ignition temp.

Problem 64. Woolen clothes are worn in winter. Why ?

Solution. Woolen fibres enclose a large amount of air in them. Both wool and air are bad conductors of heat. The small coefficient of thermal conductivity prevents the loss of heat from our body due to conduction. So we feel warm in woolen clothes.

Problem 65. Why do we use copper gauze in Davy's safety lamp ?

Solution. In Davy's safety lamp used in mines, a copper gauze is placed around the flame of the lamp. Since the copper gauze is good conductor of heat, it absorbs heat of the flame. This keeps the temperature outside the copper gauze less than the ignition temperature and so the marsh gas does not catch fire.

Problem 66. Place a safety pin on a sheet of paper. Hold the sheet over a burning candle, until the paper becomes yellow and charr. On removing the pin, its white trace is observed on the paper. Why ?

Solution. The safety pin is made of steel which is good conductor of heat. So the safety pin takes heat from the paper under it and transfers it away to the surroundings. The portion of the paper under the safety pin remains comparatively colder than the remaining part.

Problem 67. Stainless steel cooking pans are preferred with extra copper bottom. Why ? *[Himachal 07]*

Solution. The thermal conductivity of copper is much larger than that of steel. The copper bottom allows more heat to flow into the pan and hence helps in cooking the food faster.

Problem 68. If a drop of water falls on a very hot iron, it does not evaporate for a long time. Give reason.

Solution. When a drop of water falls on a very hot iron, it gets insulated from the iron by a layer of poor conducting water vapour. As the heat is conducted very slowly through this layer, it takes quite long for the drop to evaporate. But if the drop of water falls on iron which is not very hot, then it comes in direct contact with iron and evaporates immediately.

Problem 69. Explain why a brass tumbler feels much colder than a wooden tray on a chilly day.

[Central Schools 13]

Solution. Both the brass tumbler and the wooden tray are at the same temperature. But brass is much better conductor of heat than wood. When we touch the brass tumbler, heat readily flows out from our hand to the tumbler and it feels colder. But this is not the case with the wooden tray.

Problem 70. Usually a good conductor of heat is a good conductor of electricity also. Give reason.

Solution. Electrons contribute largely both towards the flow of electricity and the flow of heat. A good conductor contains a large number of free electrons. So it is both a good conductor of heat and electricity.

Problem 71. Why do electrons in insulators not contribute towards its thermal conductivity ?

Solution. Insulators do not have free electrons inside them. So electrons have no contribution towards their thermal conductivity.

Problem 72. Why felt rather than air is employed for thermal insulation ?

Solution. Though air is a bad conductor of it, it transfers heat easily by convection. Felt traps air between its fibres and convection currents cannot be set up in it. This makes felt a better thermal isosulator than air.

Scanned by CamScanner

Problem 73. If air is poor conductor of heat, why do we not feel warm without clothes ?

Solution. Although air is poor conductor of heat, it carries away heat from body due to convection when we are without clothes. Hence we feel cold.

Problem 74. Why small holes are provided at the bottom of the chimney of the lamp ?

Solution. The hot air and burnt gases rise upwards through the chimney. Fresh air enters through the holes provided at the bottom. In the absence of these holes, convection currents will not be set up and the lamp would go off.

Problem 75. Why rooms are provided with the ventilators near the roof ?

Solution. It is done so to remove the harmful impure air, and to replace it by the cool fresh air. The air we breath out is warm and so it is lighter. It rises upwards and can go out through the ventilator provided near the roof. The cold fresh air from outside enters the room though the doors and windows. Thus the convection current is set up in the air.

Problem 76. Why it is much hotter above a fire than by its side ?

Solution. Heat carried away from a fire sideways mainly by radiation. Above the fire, heat is carried by both radiation and convection of air. But convection carries much more heat than radiation. So it is much hotter above a fire than by its sides.

Problem 77. Why snow is a better heat insulator than ice ?

Solution. When the temperature of the atmosphere reaches below 0°C, the water vapours present in air freeze directly in the form of minute particles of ice. Many particles combine and take cotton-like shape which is called snow. Snow contains a large number of air pockets which prevent the formation of convection currents. Hence snow acts as a good heat insulator than ice.

Problem 78. Can we boil water inside an earth satellite ?

Solution. No. The process of transfer of heat by convection is based on the fact that a liquid becomes lighter on becoming hot and rises up. In condition of weightlessness, this is not possible. So transfer of heat by convection is not possible in a satellite.

Problem 79. Water is heated from below. Why ?

Solution. When water is heated, its density decreases and it rises up. Cooler liquid of the upper part takes its place and so convection currents are set up and water gets heated up. If heated from the top, it will conduct very small amount of heat to the bottom because water is poor conductor of heat.

Problem 80. Suppose you want to cool your drink. Should you keep ice cubes floating on the top or should you arrange to keep the ice cubes at the bottom ?

Solution. Ice cubes should be kept floating in the drink. The liquid will then cool by convection. If the ice cubes are placed at the bottom, no convection currents are set up and liquid is not cooled. Also it cannot be cooled by conduction because liquid is a poor conductor of heat.

Problem 81. Why are the cooling coils fitted near the ceiling of a refrigerator ?

Solution. As the air gets cooled in the upper part of the refrigerator, it becomes denser and goes down. The warmer air of the lower part moves up. Thus convection currents are set up. This quickly cools up the entire inside of the refrigerator.

Problem 82. After some time of the switching on an electric heater, the temperature of the heater becomes constant although current remains continuously flowing in it, why so ?

Solution. When the steady state is reached, the rate of loss of heat by conduction, convection and radiation becomes equal to the rate of production of heat in the heater due to the flow of current.

Problem 83. The earth constantly receives heat radiation from the sun and gets warmed up. Why does the earth not get as hot as the sun ?

Solution. Because the earth is located at a very large distance from the sun, hence it receives only a small fraction of the heat radiation emitted by the sun. Further, due to loss of heat from the surface of earth due to convection and radiation also, the earth does not become as hot as the sun.

Problem 84. Why do animals curl into a ball, when they feel very cold ? *[Himachal 04, 07]*

Solution. The total energy radiated by a body depends on its surface area. Thus when the animals feel very cold, they curl their bodies into a ball so as to decrease the surface area of their bodies which in turn helps to reduce the amount of heat lost by them.

Problem 85. Two thermos flasks are of the same height and same capacity. One has a circular cross-section while the other has a square cross-section. Which of the two is better ?

Solution. As both flasks have same height and capacity, the area of the cylindrical wall will be less than that of the square wall. Hence the thermos flask of circular cross-section will transmit less heat as compared to the thermos flask of square cross-section and will be better.

Problem 86. Why a body with large reflectivity is a poor emitter ?

Solution. A body whose reflectivity is large would naturally absorb less heat. So, a body with large reflectivity is a poor emitter.

Problem 87. Why does a piece of red glass when heated and taken out glow with green light ?

Solution. At low temperature, the red glass absorbs green colour strongly. But at higher temperatures, it emits green colour strongly.

Scanned by CamScanner

Problem 88. Two stars radiate maximum energy at wavelengths 3.6×10^{-7} m and 4.8×10^{-7} m respectively. What is the ratio of their temperatures ?

Solution. Here $\lambda_m = 3.6 \times 10^{-7}$ m,

$\lambda'_m = 4.8 \times 10^{-7}$ m

By Wien's law, $\lambda_m T = \lambda'_m T'$

$$\therefore \frac{T}{T'} = \frac{\lambda'_m}{\lambda_m} = \frac{4.8 \times 10^{-7}}{3.6 \times 10^{-7}} = \mathbf{4:3}.$$

Problem 89. If all the objects radiate electromagnetic energy, why do not the objects around us in everyday life become colder and colder ?

Solution. According to the Prevost theory of heat exchanges, all the objects (above 0 K) not only radiate electromagnetic energy but also absorb at the same rate from their surroundings. Thus they do not become colder.

Problem 90. Is it necessary that all black coloured objects should be considered black bodies ?

Solution. No. A polished black surface is not a black body because it reflects radiation incident on it. On the other hand, the sun, which is a dazzling white body, is a black body.

Problem 91. Why are clear nights colder than cloudy nights ? *[Himachal 07C]*

Solution. Clouds are opaque to heat radiations. So on a cloudy night, radiations from the earth's surface fail to escape. But on a clear night, the surface of the earth is cooled due to excessive radiation. So a clear night is colder than a cloudy night.

Problem 92. White clothes are more comfortable in summer while colourful clothes are more comfortable in winter. Why ? *[Himachal 07]*

Solution. White clothes absorb very little heat radiation and hence they are comfortable in summer. Coloured clothes absorb almost whole of the incident radiation and keep the body warm in winter.

Problem 93. Explain why cooking utensils are often blackened at the bottom and polished at the top.

Solution. Black surfaces are good absorbers of heat radiations. The bottom of the cooking utensils is blackened so that it absorbs maximum heat radiations. Polished white surfaces are bad absorbers and hence bad emitters of heat radiations. By polishing the upper parts of the cooking utensils, the loss of heat by radiation is minimised.

Problem 94. Gasoline tanks are generally painted with aluminium paint. Why ?

Solution. The shining aluminium paint is a bad absorber of heat. So the tank painted with aluminium paint on the outside is prevented from getting excessively heated in the sun.

Problem 95. A hole in the cavity of a radiator is a black body. Why ?

Solution. A hole in the cavity of a radiator does not reflect any radiation and absorbs all the radiation incident on it. So it is a black body.

Problem 96. Why is there the word displacement in Wien's displacement law ?

Solution. As the temperature is increased, the wavelength having maximum intensity is displaced towards the shorter wavelength region. Hence the word displacement is used.

Problem 97. Black body radiation is white. Comment.

Solution. True. A black body absorbs radiations of all wavelengths. When heated to a suitable temperature, it emits radiations of all wavelengths. Hence a black body radiation is white.

Problem 98. Which object will cool faster when kept in open air, the one at 300° C or the one at 100° C? Why ? *[Central Schools 03]*

Solution. The object at 300°C will cool faster than the object at 100°C. This is in accordance with Newton's law of cooling.

Rate cooling of an object ∝ Temperature between the object and its surroundings

Problem 99. In what respect is the thermal radiation different from light ?

Solution. Thermal radiations are electromagnetic waves having wavelength range from 1 μm to 100 μm. When they are absorbed by a body, they produce heat. Light radiations are electromagnetic waves having wavelength range from 4000 Å to 7500 Å. They produce sensation of vision.

Problem 100. What is critical temperature ?

Solution. It is the temperature of a substance in the gaseous state below which the gas can be liquified by pressure only, and above which the gas cannot be liquified. This implies that gas is simply a vapour below its critical temperature.

Problem 101. Can a gas be liquified at any temperature by the increase of pressure alone ?

Solution. No. A gas can be liquified by pressure alone, only when its temperature is below its critical temperature.

Problem 102. What is the effect of pressure on melting point of a solid ?

Solution. The melting point of a solid may increase or decrease depending on the nature of solid. For solids such as ice which contracts on melting, it is lowered while for solids such as sulphur and wax which expand on melting it increases.

Problem 103. How does the boiling point of water change with pressure ?

Solution. The boiling point of water increases with the increase in pressure.

Scanned by CamScanner

Problem 104. What is the temperature above which steam will not condense to water even if it is compressed (isothermally) to very large pressure ?

Solution. Above the critical temperature (374.1°C for water), steam will not condense to water.

Problem 105. What is the significance of negative slope of ice line of water ?

Solution. It indicates that the melting point of ice decreases with increase in pressure (on ice line). This is because volume of water formed on melting is less than the volume of ice (before melting).

Problem 106. Are the relative amounts of ice, water and vapour fixed at the triple point water ?

Solution. At the triple point of water, the temperature and pressure are fixed. However, the relative amounts of the three phases are not unique. The relative amounts of the three phases can be varied by adding or taking out heat from the system.

Problem 107. What happens if water vapour at a pressure of 0.004 atm is cooled to 0°C ?

Solution. The pressure corresponding to triple point is 610 Pa which is equal to nearly 0.006 atm. It follows from the *P-T* phase diagram of water that at a pressure lower than this pressure, water vapour condenses directly to ice without passing through the liquid phase.

Problem 108. Water exists in liquid phase at 30°C at 1 atmospheric pressure. How would you convert this water to vapour form without increasing its temperature ?

Solution. Water at 30°C can be converted into vapour by reducing its pressure until it equals the vapour pressure of water at 30°C. This route is along a vertical line on the *P–T* diagram, with $T = 30°C$.

Problem 109. What are the critical temperature and pressure for CO_2 ? What is their significance ?

Solution. The critical temperature and pressure of CO_2 are 31.1°C and 73.0 atm respectively. Above this temperature, CO_2 will not liquify even if compressed to high pressures.

Problem 110. Ice of 0° is converted into steam at 100°C. State the isothermal changes in the process. *[Central Schools 14]*

Solution. The isothermal changes are

(*i*) Conversion of ice at 0°C into water at 0°C.

(*ii*) Conversion of water at 100°C into steam at 100°C.

Problem 111. Explain why a new quilt is warmer than an old one. *[Himachal 05]*

Solution. Refer answer to Q. 46(*iii*) on page 11.26.

Short Answer Conceptual Problems

Problem 1. Give reasons why water is considered unsuitable for use in thermometers.

Solution. Water is considered unsuitable for use in thermometers due to following reasons :

(*i*) The expansion of water with temperature is non-uniform.

(*ii*) Due to its large specific heat and low thermal conductivity, a water thermometer does not respond to changes in temperature quickly.

(*iii*) Water is invisible, sticks to glass and has high rate of evaporation.

(*iv*) Its temperature range is small from 0°C to 100°C

Problem 2. Give four reasons why is mercury used in hermometers.

Solution. The reasons for using mercury in a ermometer are

(*i*) Mercury has a uniform expansion over a wide range of temperature.

(*ii*) Mercury is opaque and bright, so it can be easily seen in a glass tube.

(*iii*) It does not stick to the walls of the glass tube.

(*iv*) It is a good conductor of heat and has low thermal capacity.

Problem 3. Give reasons why is a platinum wire used in a resistance thermometer.

Solution. The reasons for using a platinum wire in a resistance thermometer are : (*i*) The resistance of a platinum wire increases *uniformly* with the rise in temperature (from 200°C to 1200°C). (*ii*) It does not react chemically with other substances. (*iii*) Its melting point is quite high (1800°C).

Problem 4. Give some merits of gas thermometers over those of mercury thermometers.

Solution. Some merits of gas thermometers are (*i*) A gas thermometer is more sensitive than a mercury thermometer. (*ii*) The working of a gas thermometer is independent of the nature of the gas used. (*iii*) A gas thermometer can measure very low and very high temperatures.

Problem 5. Name the suitable thermometers to measure the following temperatures : – 80° C, 60° C, 250° C, 780° C, 2000° C.

Solution. Gas thermometer for – 80°C.

Mercury thermometer for 60°C.

Platinum resistance thermometer for 250°C and 780°C.

Total radiation pyrometer for 2000°C.

Scanned by CamScanner

Problem 6. Suggest suitable methods for measuring the temperature of

(i) surface of the sun, (ii) surface of the earth,

(iii) an insect, and (iv) liquid helium.

Solution. (i) By using total radiation pyrometer.

(ii) By using a thermoelectric thermometer by embedding its hot junction in the earth.

(iii) By using a thermoelectric thermometer by touching its hot junction with the insect.

(iv) By using a magnetic thermometer which is based on Curie's law : the susceptibility of a peramagnetic material varies inversely with its absolute temperature.

Problem 7. Two large holes are cut in a metal sheet. If the sheet is heated, how will the diameters of the holes change ?

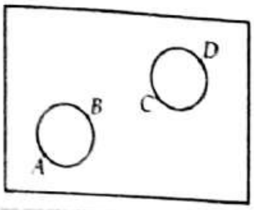


Fig. 11.34

Solution. When a body is heated, the distance between any of its two points increases. Hence the diameters AB and CD of the two holes will increase.

Problem 8. In problem 7, will the distance between the two holes increase or decrease on heating ?

Solution. When the metal sheet is heated, it expands a whole. Therefore, the holes will increase in diameter as well as move outwards. The distance BC between the two holes increases.

Problem 9. There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if (i) they are heated to the same temperature (ii) same amount of heat is given to each of them ?

Solution. (i) As thermal expansion of isotropic solids is similar to true photographic enlargement, the expansion of a cavity is same as if it were a solid body of the same material i.e., $\Delta V = \gamma V \Delta T$. As here V, γ and ΔT are same for both solid and hollow spheres, so the expansions of both will be equal.

(ii) If same amount of heat is given to the two spheres, then due to lesser mass, rise in temperature of hollow sphere will be more (as $\Delta T = Q/Mc$) and hence the expansion will be more as $\Delta V = \gamma V \Delta T$.

Problem 10. Two bodies of specific heats c_1 and c_2 having same heat capacities are combined to form a single composite body. What is the specific heat of the composite body ?

Solution. As the heat capacities are equal, so $m_1 c_1 = m_2 c_2$.

Let c be the specific heat of the composite body. Then

$$(m_1 + m_2)c = m_1 c_1 + m_2 c_2 = m_1 c_1 + m_1 c_1 = 2m_1 c_1$$

or
$$c = \frac{2m_1 c_1}{m_1 + m_2} = \frac{2m_1 c_1}{m_1 + m_1 \frac{c_1}{c_2}} = \frac{2c_1 c_2}{c_1 + c_2}$$

Problem 11. Two rods A and B are of equal length. Each rod has its ends at temperatures T_1 and T_2. What is the condition that will ensure equal rates of flow of heat through the rods A and B ? *[IIT]*

Solution. Let x be the length of each rod. The rates of flow of heat through the rods A and B will be equal if

$$\frac{K_1 A_1 (T_1 - T_2)}{x} = \frac{K_2 A_2 (T_1 - T_2)}{x}$$

or
$$K_1 A_1 = K_2 A_2 \quad \text{or} \quad \frac{A_1}{A_2} = \frac{K_2}{K_1}$$

Hence for equal rates of flow of heat, the areas of cross-section of the two rods should be inversely proportional to their coefficients of thermal conductivity.

Problem 12. Two vessels of different materials are identical in size and wall-thickness. They are filled with equal quantities of ice at 0° C. If the ice melts completely in 10 and 25 minutes respectively, compare the coefficients of thermal conductivity of the materials of the vessels.

Solution. Let K_1 and K_2 be the coefficients of thermal conductivity of the materials and t_1 and t_2 be the times in which ice melts in the two vessels.

As the same quantity of ice melts in the two vessels, the quantity of heat flowed into the vessels must be same.

$$\therefore \quad Q = \frac{K_1 A (T_1 - T_2) t_1}{x} = \frac{K_2 A (T_1 - T_2) t_2}{x}$$

or
$$K_1 t_1 = K_2 t_2 \quad \therefore \quad \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{25 \text{ min}}{10 \text{ min}} = \mathbf{5 : 2.}$$

Problem 13. Two vessels A and B of different materials but having identical shape, size and wall-thickness are filled with ice and kept at the same place Ice melts at the rate of 100 g min^{-1} and 150 g min^{-1} in A and B respectively. Assuming that heat enters the vessels through the walls only, calculate the ratio of thermal conductivities of their materials.

Solution. Let m_1 and m_2 be the masses of ice melted in same time t (= 1 min) in vessels A and B respectively. Then the amounts of heat flowed into the two vessels will be

$$Q_1 = \frac{K_1 A (T_1 - T_2) t}{x} = m_1 L \qquad ...(i)$$

$$Q_2 = \frac{K_2 A (T_1 - T_2) t}{x} = m_2 L \qquad ...(ii)$$

where L is latent heat of ice. Dividing (i) by (ii), we get

$$\frac{K_1}{K_2} = \frac{m_1}{m_2} = \frac{100 \text{ g}}{150 \text{ g}} = \frac{2}{3} = \mathbf{2 : 3.}$$

Problem 14. Water in a closed tube is heated with one arm placed vertically above an arc lamp. Water will begin to circulate along the tube in a counterclockwise direction. Is this true or false ?

Fig. 11.35

Solution. ***False.*** Water will circulate in the clockwise direction. The molecules immediately above the arc receive heat by conduction. They rise up and get replaced by cold molecules from the right side. This will make the water circulate in clockwise direction.

Problem 15. A sphere, a cube and a thin circular plate, all made of the same material and having the same mass are initially heated to a temperature of 200°C. Which of these objects will cool fastest and which one slowest when left in air at room temperature ? Give reasons.

Solution. The thin circular plate has the largest surface area. The sphere has the smallest surface area. Thus the plate will radiate maximum heat while the sphere will radiate minimum heat. Hence the plate will cool fastest and the sphere will cool slowest.

Problem 16. There are two rods of the same metal, same length, same area of cross-section, but one of square cross-section and the other of circular cross-section. One end of each is kept immersed in steam. After the steady state is reached, the other ends of the rods are touched. Which one will be hotter ? Give reason.

Solution. The surface area of the rod of circular cross-section will be smaller, because for a given area a circle has least perimeter. So the loss of heat by radiation will be small. Hence the other end of the circular rod will be hotter than the other end of the square rod.

Problem 17. A solid sphere of copper of radius *R* and a hollow sphere of the same material of inner radius *r* and outer radius *R* are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster ?

Solution. Rate of loss of heat by any sphere,

$$m \times c \times \left(-\frac{dT}{dt}\right) = \sigma A\,(T^4 - T_0^4)$$

Now σ, A, $(T^4 - T_0^4)$ are same for both the spheres, so the rate of cooling,

$$-\frac{dT}{dt} \propto \frac{1}{m}$$

Since the hollow sphere has less mass, its rate of cooling will be faster.

Problem 18. On a hot day, a car is left in sunlight with all the windows closed. After some time, it is found that the inside of the car is considerably warmer than the air outside. Explain, why.

Solution. Glass transmits about 50% of heat radiation coming from a hot source like the sun but does not allow the radiation from moderately hot bodies to pass through it. Due to this, when a car is left in the sun, heat radiation from the sun gets into the car but as the temperature inside the car is moderate, they do not pass back through windows. Hence, inside of the car becomes considerably warmer.

Problem 19. How does tea in a thermos flask remain hot for a long time ?

Solution. The air between the two walls of the thermos flask is evacuated. This prevents heat loss due to conduction and convection. The loss of heat due to radiation is minimised by silvering the inside surface of the double wall. As the loss of heat due to the three processes is minimised, the tea remains hot for a long time.

Problem 20. Distinguish between conduction, convection and radiation.

Solution.

	Conduction	*Convection*	*Radiation*
1.	Material medium is required.	Material medium is required.	No material medium is required.
2.	It is due to temperature difference. Heat flows from high temperature region to low temperature region.	It is due to difference in density. Heat flows from low density region to high density region.	It occurs from all bodies at temperatures above 0 K.
3.	It occurs in solids through molecular collisions, without actual flow of matter.	It occurs in fluids by actual flow of matter.	It can take place at large distances and does not heat the intervening medium.
4.	It is a slow process.	It is also a slow process.	It propagates at the speed of light.
5.	It does not obey the laws of reflection and refraction.	It does not obey the laws of reflection and refraction.	It obeys the laws of reflection and refraction.

Problem 21. A blackened platinum wire, when gradually heated, first appears dull red, then blue and finally white. Explain why.

Solution. According to Wien's displacement law, when blackened platinum wire is gradually heated, it first emits radiations of longer wavelengths, so it appears red. At higher temperatures, it emits blue radiations more strongly than red and appears blue. At very high temperatures, it emits all radiations strongly and appears white.

Problem 22. In a coal fire, the pockets formed by coals appear brighter than the coals themselves. Is the temperature of such a pocket higher than the surface temperature of a glowing coal ?

Solution. The temperature of pockets formed by coals are not appreciably different from the surface temperatures of glowing coals. However, the pockets formed by coals act as cavities. The radiations from these cavities are black body radiations and so have maximum intensity. Hence the pockets appear brighter than the glowing coals.

Scanned by CamScanner

Problem 23. Answer the following questions :

(a) A vessel with a movable piston maintained at a constant temperature by a thermostat contains a certain amount of liquid in equilibrium with its vapour. Does this vapour obey Boyle's law ? In other words, what happens when the volume of vapour is decreased ? Does the vapour pressure increase ?

(b) What is meant by 'superheated water' and 'supercooled vapour' ? Do these states of water lie on its P V T surface ? Give some practical applications of these states of water.

Solution. (a) No, the vapour in equilibrium with its liquid does not obey Boyle's law. When the volume of the vapour is decreased by applying pressure, some of the vapours condense into liquid, maintaining the same pressure of the vapour at the given temperature *i.e.*, vapour pressure does not increase when the volume of vapours is decreased.

(b) **Superheated water.** Water in liquid phase having temperature above the boiling point of water at the given pressure is called superheated water. It is highly unstable stage.

Supercooled vapour. Water in vapour phase having temperature below its boiling point at the given pressure is called supercooled vapour. It is also highly unstable stage.

As the above states of water are not equilibrium states, so they do not lie on P V T surface of water.

Applications. These unstable states of water are used in bubble chamber and cloud chamber for detecting high speed charged particles.

Problem 24. A fat man is used to consuming about 3000 kcal worth of food everyday. His food contains 50 g of butter plus a plate of sweets everyday, besides items which provide him with other nutrients (proteins, vitamins, minerals, etc.) in addition to fats and carbohydrates. The caloric value of 10 g of butter is 60 kcal and that of a plate of sweets is of average 700 kcal. What dietary strategy should he adopt to cut down his calories to about 2100 kcal per day ? Assume the man cannot resist eating the full plate of sweets once it is offered to him !

Solution. The man intends to cut down $3000 - 2100 = 900$ kcal. But avoiding sweets completely, he will cut down 700 kcal. To cut down another 200 kcal, he should cut down butter by $\frac{10}{60} \times 200 = 33$ g per day. He should not cut down consumption of food, that provides him with vitamins and other vital nutrients.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *2 kg of ice at −20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal / kg/°C and 0.5 kcal / kg/°C, while the latent heat of fusion of ice is 80 kcal / kg.* **[IIT Screening 03]**

Solution. Suppose m kg of ice melts into water. As the net heat change is zero, so

$$m_{ice}\, C_{ice}\,[0-(-20)] + mL + m_{water}\, C_{water}\,[0-20] = 0$$

or

$$2 \times 1 \times 20 + m \times 80 + 5 \times 1 \times (-20) = 0$$

$$m = 1 \text{ kg}$$

Final mass of water remaining in the beaker

$= 5 + 1 = $ **6 kg**.

Problem 2. *Two rods, one of aluminum and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminum and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperatures are raised by t°C, then find the ratio $l_1/(l_1 + l_2)$.* **[IIT Screening 03]**

Solution. As the lengths of the two rods increase by the same amount, so

$$l_1 \alpha_a t = l_2 \alpha_s t \quad \text{or} \quad \frac{l_2}{l_1} = \frac{\alpha_a}{\alpha_s} \quad \text{or} \quad \frac{l_2 + l_1}{l_1} = \frac{\alpha_a + \alpha_s}{\alpha_s}$$

$$\therefore \quad \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}.$$

Problem 3. *Three rods made of the same material and having the same cross-section have been joined as shown in Fig. 11.35. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. What will be the temperature of the junction of the three rods ?* **[IIT Screening 01]**

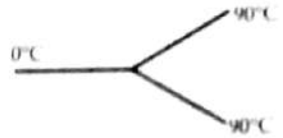


Fig. 11.36

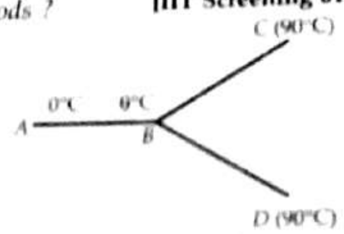


Fig. 11.37

Solution. Let R be the thermal resistance of each rod and θ be the temperature of the junction as shown in Fig. 11.36.

Then

Heat current in CB + Heat current in DB = Heat current in BA

or $$\frac{90-\theta}{R}+\frac{90-\theta}{R}=\frac{\theta-0}{R}$$

or $$180-2\theta=\theta \quad \text{or} \quad \theta = 60°C.$$

Problem 4. *When a block of iron floats in mercury at 0°C, a fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then find the ratio k_1/k_2.* **[IIT Screening 01]**

Solution. Let ρ_{Fe} and ρ_{Hg} denote the densities of iron and mercury at 0°C, and m = mass of the block.

$\therefore$ Volume of the block $=\dfrac{m}{\rho_{Fe}}$,

Volume of displaced mercury $=\dfrac{k_1 m}{\rho_{Fe}}$

Now,

Weight of mercury displaced = Weight of the block

or $$\left(\frac{k_1 m}{\rho_{Fe}}\right)\rho_{Hg}\, g = mg \quad \text{or} \quad k_1=\frac{\rho_{Fe}}{\rho_{Hg}}.$$

Also, $$\rho_t=\frac{\rho_0}{1+\gamma t}$$

$$\therefore \quad k_2=\frac{\rho_{Fe}}{1+\gamma_{Fe}\times 60}\times\frac{1+\gamma_{Hg}\times 60}{\rho_{Hg}}=k_1\,\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$$

or $$\frac{k_1}{k_2}=\frac{\mathbf{1+60\gamma_{Fe}}}{\mathbf{1+60\gamma_{Hg}}}.$$

Problem 5. *An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation S = A + BT, where A = 100 cal / kg-K and B = 2 × 10⁻² cal / kg-K². If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion of water = 8 × 10⁴ cal / kg. Specific heat of water = 10³ cal / kg - K).* **[IIT Mains 01]**

Solution. Heat gained by the ice cube,

$$Q_1 = mL + mC\,dT = 0.1\times 8\times 10^4 + 0.1\times 10^3\times 27$$
$$= 10700 \text{ cal}$$

Heat lost by the container,

$$Q_2=-\int_{500}^{300} m_c\,(A+BT)\,dT=-m_c\left[AT+\frac{BT^2}{2}\right]_{500}^{300}$$

$$=-m_c\left[100\,(300-500)+\frac{2\times10^{-2}}{2}(300^2-500^2)\right]$$

$$=21600\, m_c$$

But $Q_2 = Q_1$

$$\therefore \quad m_c=\frac{10700}{21600}=0.495 \text{ kg} \approx \mathbf{0.5\ kg}.$$

Problem 6. *Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity K = 0.149 J / (m-°C-sec), thickness t = 5 mm, emissivity = 0.6. Temperature of the top of the lid is maintained at T_l = 127°C. If the ambient temperature T_a = 27°C, calculate*

(a) rate of heat loss per unit area due to radiation from the lid.

(b) temperature of the oil.

(Given $\sigma=\dfrac{17}{3}\times10^{-8}$) **[IIT Mains 03]**

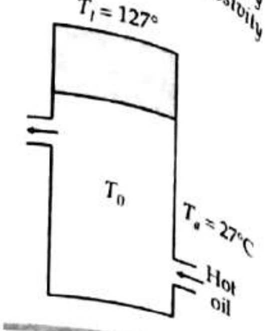


Fig. 11.38

Solution. (*a*) The rate of loss of heat per unit area from the lid,

$$\frac{dQ}{dt}=\sigma\,\varepsilon\,[T_l^{\,4}-T_a^{\,4}]$$

$$=\frac{17}{3}\times10^{-8}\times0.6\,[400^4-300^4] = \mathbf{595\ Wm^{-2}}.$$

(*b*) Let T_0 be the temperature of hot oil.

Then $$\frac{KA\,(T_0-T_l)}{t}=595\text{ A}$$

or $$\frac{0.149\text{ A}\,(T_0-400)}{5\times10^{-3}}=595\text{ A}$$

or $$T_0-400=\frac{595\times5\times10^{-3}}{0.149}=19.97\approx20$$

$$\therefore \quad T_0 = \mathbf{420\ K}$$

Problem 7. *The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2 K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). What is the rate of flow of heat through the slab in a steady state ?* **[AIEEE 04]**

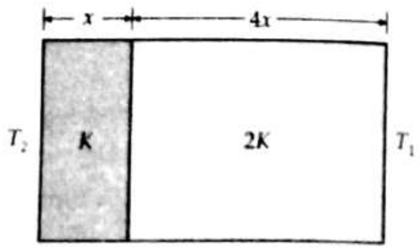


Fig. 11.39

Solution. Let T be the temperature of the interface. In the steady state, the rates of flow of heat through the two slabs will be equal, *i.e.*,

Scanned by CamScanner

$$H_1 = H_2$$

$$\frac{KA(T_2 - T)}{x} = \frac{2\,KA(T - T_1)}{4x}$$

or $$T_2 - T = \frac{T - T_1}{2} \quad \text{or} \quad T = \frac{T_1 + 2T_2}{3}$$

$\therefore$ $$Q_1 = \frac{KA}{x}\left[T_2 - \frac{T_1 + 2T_2}{3}\right] = \frac{KA\,(T_2 - T_1)}{3x}.$$

Problem 8. *Fig. 11.40 shows a system of two concentric spherical shells of radii r_1 and r_2 and kept at temperatures T_1 and T_2. Find the radial rate of flow of heat through a substance of thermal conductivity κ filled in the space between the two shells.* **[AIEEE 05]**

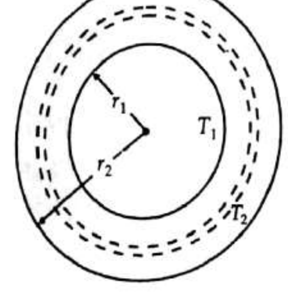


Fig. 11.40

Solution. Consider a thin concentric shell of radius r and thickness dr. The radial rate of flow of heat through this elementary shell will be

$$H = -\kappa A\frac{dT}{dr} = -\kappa\,4\pi r^2\frac{dT}{dr}$$

or $$H\frac{dr}{r^2} = -4\pi\kappa\,dT$$

Integrating both sides between the limits of radii and temperatures of the two shells, we get

$$H\int_{r_1}^{r_2} r^{-2}\,dr = -4\pi\kappa\int_{T_1}^{T_2} dT$$

or $$H\left[\frac{r^{-1}}{-1}\right]_{r_1}^{r_2} = -4\pi\kappa\,[T]_{T_1}^{T_2}$$

or $$H\left[-\frac{1}{r}\right]_{r_1}^{r_2} = 4\pi\kappa\,[-T]_{T_1}^{T_2}$$

or $$H\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi\kappa\,(T_1 - T_2)$$

or $$H = \frac{4\pi\kappa\,r_1 r_2\,(T_1 - T_2)}{(r_2 - r_1)}.$$

Problem 9. *A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses. (For the steel wire : Young's modulus = 2.1×10^{11} Pa ; Density = 7860 kg / m^3 ; Specific heat = 420 J / kg - K).* **[IIT Mains 01]**

Solution. The elongated wire has only elastic potential energy which ultimately gets converted into heat energy and brings the change in the temperature of the wire.

Elastic potential energy

$$= \frac{1}{2}\frac{(\text{Stress})^2}{Y} \times \text{Volume}$$

$$= \frac{1}{2Y}\left(\frac{Mg}{A}\right)^2 . AL = \frac{(Mg)^2 L}{2YA}$$

Heat produced $= ms\Delta T = (AL\rho)\,s\,\Delta T$

$$\therefore \quad (AL\rho)\,s\,\Delta T = \frac{(Mg)^2 L}{2Y\,A}$$

or $$\Delta T = \frac{(Mg)^2}{2\rho Y\,s\,A^2} = \frac{(Mg)^2}{2\rho Y\,s\,(\pi r^2)^2}$$

$$= \frac{(100)^2 \times (10)^2}{2 \times 7860 \times 2.1 \times 10^{11} \times 420 \times (3.14 \times 4 \times 10^{-6})^2}$$

$$= \mathbf{0.00457\ K}$$

Problem 10. *A sphere of diameter 7 cm and mass 266.5 g floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature of 35°C. If the density of the liquid at 0°C is 1.527 g cm^{-3}, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.* **[IIT]**

Solution. Volume of sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{539}{3}\ \text{cm}^3$$

Mass of sphere = 266.5 g

Density of sphere

$$= \frac{\text{Mass}}{\text{Volume}} = \frac{266.5 \times 3}{539} = 1.483\ \text{g cm}^{-3}$$

Density of liquid at 0°C = 1.527 g cm^{-3}

As the temperature of the bath is raised, the liquid expands and its density decreases. When the sphere just floats,

Density of liquid at 35°C = Density of the body

$\therefore \rho_t = 1.483$ g cm^{-3}, $\rho_0 = 1.527$ g cm^{-3}, $\Delta T = 35°C$

$$\gamma = \frac{\rho_0 - \rho_t}{\rho_0\,\Delta T} = \frac{1.527 - 1.483}{1.527 \times 35}$$

$$= \mathbf{0.000084\,°C^{-1}}.$$

Scanned by CamScanner

Guidelines to NCERT Exercises

11.1. *The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.* **[Delhi 2011]**

Ans. For neon : Triple point, $T = 24.57$ K

$\therefore \quad T_C = T(K) - 273.15$

$$= 24.57 - 273.15 = -\mathbf{248.58^\circ C}.$$

$$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}\times(-248.58) + 32 = -\mathbf{415.44^\circ F}.$$

For carbon dioxide : Triple point, $T = 216.55$ K

$\therefore \quad T_C = T(K) - 273.15$

$$= 216.55 - 273.15 = -\mathbf{56.6^\circ C}.$$

$$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}\times(-56.6) + 32 = -\mathbf{69.88^\circ C}.$$

11.2. *Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is relation between T_A and T_B ?*

Ans. $\frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7} \quad$ or $\quad T_A = \frac{4}{7}T_B$.

11.3. *The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law :*

$$R = R_0[1 + 5\times10^{-3}(T - T_0)]$$

The resistance is 101.6 Ω at the triple point of water, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω ?

Ans. When $\quad T = 273$ K, $\; R = 101.6\,\Omega$

$$\therefore \quad 101.6 = R_0[1 + 5\times10^{-3}(273 - T_0)] \quad ...(i)$$

When $\quad T = 600.5$ K, $\; R = 165.5\,\Omega$

$$\therefore \quad 165.5 = R_0[1 + 5\times10^{-3}(600.5 - T_0)] \quad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{165.5}{101.6} = \frac{1 + 5\times10^{-3}(600.5 - T_0)}{1 + 5\times10^{-3}(273 - T_0)}$$

On solving, $T_0 = -49.3$ K

Substituting in (*i*), we get

$$101.6 = R_0[1 + 5\times10^{-3}(273 + 49.3)]$$

or $$R_0 = \frac{101.6}{1 + 5\times10^{-3}\times322.3} = 38.9\,\Omega$$

For $R = 123.4\,\Omega$, we have

$$123.4 = 38.9[1 + 5\times10^{-3}(T + 49.3)]$$

On solving, $T = \mathbf{384.8\ K}$.

11.4. *Answer the following :*

(a) The triple-point of water is a standard fixed point in modern thermometry. Why ? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale) ?

(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0°C and 100°C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale ?

(c) The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by $t_c = T - 273.15$. Why do we have 273.15 in this relation, and not 273.16 ?

(d) What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale ?

Ans. (*a*) The melting point of ice as well as the boiling point of water change with change in pressure. The presence of impurities also changes the melting and boiling points. However, the triple point of water has a unique temperature and is independent of external factors.

(*b*) The other fixed point on Kelvin scale is absolute zero, which is the temperature at which the volume and pressure of any gas become zero.

(*c*) As the triple point of water on Celsius is 0.01°C (and not 0°C) and on Kelvin scale 273.16 and the size of degree on the two scales is same, so

$$t_c - 0.01 = T - 273.16 \quad \therefore \quad t_c = T - 273.15.$$

(*d*) One degree on Fahrenheit scale

$$= \frac{180}{100} = \frac{9}{5} \text{ divisions on Celsius scale.}$$

But one Celcius scale division is equal to one division on Kelvin scale.

$\therefore$ Triple point on Kelvin scale (whose size of a degree is equal to that of the Fahrenheit scale)

$$= 273.16\times\frac{9}{5} = \mathbf{491.69}.$$

11.5. *Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made :*

Temperature	*Pressure thermometer A*	*Pressure thermometer B*
Triple-point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

(*a*) *What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B ?*

(*b*) *What do you think is the reason for slightly different answers from A and B ?*

Ans. (*i*) *For pressure thermometer A :*

$$T_{tr} = 273\text{ K},\ P_{tr} = 1.250 \times 10^3\text{ Pa},\quad P = 1.797 \times 10^5\text{ Pa}$$

Normal freezing point of sulphur,

$$T = \frac{P}{P_{tr}} \times T_{tr} = \frac{1.795 \times 10^5 \times 273}{1.250 \times 10^5} = \mathbf{392.46\ K}$$

(*ii*) *For pressure thermometer B :*

$$T_{tr} = 273\text{ K},\ P_{tr} = 0.200 \times 10^5\text{ Pa},\quad P = 0.287 \times 10^5\text{ Pa}$$

$$\therefore\quad T = \frac{0.287 \times 10^5 \times 273}{0.200 \times 10^5} = \mathbf{391.75\ K}$$

(*b*) The slight difference is due to the fact that oxygen and hydrogen do not behave strictly as ideal gases.

11.6. *A steel tape 1 m long is correctly calibrated for a temperature of 27.0°C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0°C. What is the actual length of the steel rod on that day ? What is the length of the same steel rod on a day when the temperature is 27.0°C ? Coefficient of linear-expansion of steel = $1.20 \times 10^{-5}\ °C^{-1}$?*

Ans. Here $t_1 = 27°C$, $l_1 = 63$ cm,

$$t_2 = 45°C,\ \alpha = 1.20 \times 10^{-5}\ °C^{-1}$$

Length of the rod on the hot day is

$$l_2 = l_1\,[1 + \alpha\,(t_2 - t_1)]$$

$$= 63\,[1 + 1.20 \times 10^{-5}\,(45 - 27)]$$

$$= \mathbf{63.0136\ cm}$$

As the steel tape has been calibrated for a temperature of 27°C, so length of the steel rod at 27°C = **63 cm.**

11.7. *A large steel wheel is to be fitted on to a shaft of the same material. At 27°C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice' (solid carbon dioxide). At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range.*

$\alpha_{steel} = 120 \times 10^{-5}\ K^{-1}$.

Ans. Here $l_1 = 8.70$ cm, $l_2 = 8.69$ cm,

$$T_1 = 27 + 273 = 300\text{ K},\quad T_2 = ?$$

As $\quad l_2 - l_1 = \alpha\, l_1\,(T_2 - T_1)$

$$\therefore\quad T_2 - T_1 = \frac{l_2 - l_1}{\alpha\, l_1}$$

or $\quad T_2 - 300 = \dfrac{8.69 - 8.70}{1.20 \times 10^{-5} \times 8.70} = -95.8$

or $\quad T_2 = 300 - 95.8 = 204.2\text{ K} = \mathbf{-68.8°C.}$

11.8. *A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0°C. What is the change in the diameter of the hole when the sheet is heated to 227°C ? Coefficient of linear expansion of copper = $1.70 \times 10^{-5}\ °C^{-1}$.*

Ans. When the copper sheet is heated, the diameter of its hole increases in the same manner as the length of a rod.

$$\therefore\quad l = 4.24\text{ cm},\ \alpha = 1.70 \times 10^{-5}\ °C^{-1},$$

$$\Delta T = 227 - 27 = 200°C$$

Increase in diameter,

$$\Delta l = l\,\alpha\,\Delta T = 4.24 \times 1.70 \times 10^{-5} \times 200\text{ cm}$$

$$= \mathbf{1.44 \times 10^{-2}\ cm.}$$

11.9. *A brass wire 1.8 m long at 27°C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of −39°C, what is the tension developed in the wire, if its diameter is 2.0 mm ? Coefficient of linear expansion of brass = $2.0 \times 10^{-5}\ °C^{-1}$, Young's modulus of brass = 0.91×10^{11} Pa.* **[Central Schools 14]**

Ans. Here $l = 1.8$ m, $t_1 = 27°C$, $t_2 = -39°C$

$$r = \frac{2.0}{2} = 1.0\text{ mm} = 1.0 \times 10^{-3}\text{ m}$$

$$\alpha = 2.0 \times 10^{-5}\ °C^{-1},\ Y = 0.91 \times 10^{11}\text{ Pa}$$

As $\quad \Delta l = l\,\alpha\,(t_2 - t_1)$

$\therefore$ Strain, $\dfrac{\Delta l}{l} = \alpha\,(t_2 - t_1)$

Stress = Strain × Young's modulus

$$= \alpha\,(t_2 - t_1) \times Y$$

$$= 2.0 \times 10^{-5} \times (-39 - 27) \times 0.91 \times 10^{11}$$

$$= 1.2 \times 10^8\ \text{Nm}^{-2} \quad \text{[Numerically]}$$

$\therefore$ Tension developed in the wire

= Stress × Area of cross-section

$= \text{Stress} \times \pi r^2$

$= 1.2 \times 10^8 \times 3.14 \times (1.0 \times 10^{-3})^2$

$= \mathbf{3.77 \times 10^2\ N.}$

11.10. *A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C, if the original lengths are at 40.0°C ? Is there a 'thermal stress' developed at the junction ? The ends of the rod are free to expand. Coefficient of linear expansion of brass = $2.0 \times 10^{-5}\ °C^{-1}$ and that of steel = $1.2 \times 10^{-5}\ °C^{-1}$.*

Ans. *For brass rod :*

$$l = 50\text{ cm},\ t_1 = 40°C,\ t_2 = 250°C,$$

$$\alpha = 2.0 \times 10^{-5}\ °C^{-1}$$

Change in length of brass rod is

$$\Delta l = \alpha l\,(t_2 - t_1)$$

$= 2.0 \times 10^{-5} \times 50 \times (250 - 40) = 0.$[illegible] cm

Scanned by CamScanner

For steel rod :

$$l = 50\,\text{cm},\ t_1 = 40°\text{C},\ t_2 = 250°\text{C},$$

$$\alpha = 1.2 \times 10^{-5}\,°\text{C}^{-1}$$

Change in length of steel rod is

$$\Delta l' = \alpha l\,(t_2 - t_1)$$

$$= 1.2 \times 10^{-5} \times 50 \times (250 - 40) = 0.13\ \text{cm}$$

Change in length of the combined rod at 250°C

$$= \Delta l + \Delta l' = 0.21 + 0.13 = \mathbf{0.34\ cm}.$$

As the rods expand freely, so no thermal stress is developed at the junction.

11.11. *The coefficient of volume expansion of glycerine is* $49 \times 10^{-5}\,°C^{-1}$*. What is the fractional change in its density for a* 30°C *rise in temperature ?*

Ans. Let M be the mass of glycerine, ρ_0 its density at 0°C, ρ_t its density at t °C. Then

$$\gamma = \frac{V_t - V_0}{V_0\,\Delta T} = \frac{\frac{M}{\rho_t} - \frac{M}{\rho_0}}{(M/\rho_0)\,\Delta T} = \frac{\frac{1}{\rho_t} - \frac{1}{\rho_0}}{(1/\rho_0)\,\Delta T} = \frac{\rho_0 - \rho_t}{\rho_0\,\Delta T}$$

∴ Fractional change in density,

$$\frac{\rho_0 - \rho_t}{\rho_0} = \gamma\,\Delta T = 49 \times 10^{-5} \times 30 = \mathbf{0.0147}.$$

11.12. *A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium* $= 0.91\ Jg^{-1}\,°C^{-1}$.

Ans.

Power $P = 10\ \text{kW} = 10 \times 10^3\ \text{W}$

Time $t = 2.5\ \text{min} = 2.5 \times 60\ \text{s}$

Total energy used

$$= Pt = 10 \times 10^3 \times 2.5 \times 60 = 1.5 \times 10^6\ \text{J}$$

Energy absorbed by aluminium block,

$$Q = 50\%\ \text{of the total energy}$$

$$= \frac{1.5 \times 10^6}{2} = 0.75 \times 10^6\ \text{J}$$

Also, $m = 8.0\ \text{kg} = 8.0 \times 10^3\ \text{g},\ c = 0.91\ \text{Jg}^{-1}\,°\text{C}^{-1}$

As $Q = mc\,\Delta T$

$$\therefore \quad \Delta T = \frac{Q}{mc} = \frac{0.75 \times 10^6}{8.0 \times 10^3 \times 0.91} = \mathbf{103.02°C}.$$

11.13. *A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper* $= 0.39\ Jg^{-1}\,°C^{-1}$*, and heat of fusion of water* $= 335\ Jg^{-1}$*).*

Ans. Mass of copper block,

$$M = 2.5\ \text{kg} = 2.5 \times 10^3\ \text{g}$$

Specific heat of copper, $c = 0.39\ \text{J g}^{-1}\,°\text{C}^{-1}$

Fall in temperature, $\Delta T = 500 - 0 = 500°\text{C}$

Heat lost by copper block

$$= mc\,\Delta T = 2.5 \times 10^3 \times 0.39 \times 500\ \text{J}$$

Let mass of ice melted = M gram

Heat of fusion of ice, $L = 335\ \text{Jg}^{-1}$

Heat gained by ice = $ML = M \times 335$ J

∴ Heat gained = Heat lost

$$\therefore \quad M \times 335 = 2.5 \times 10^3 \times 0.39 \times 500$$

or

$$M = \frac{2.5 \times 10^3 \times 0.39 \times 500}{335}$$

$$= 1455.2\ \text{g} = \mathbf{1.455\ kg}.$$

11.14. *In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm³ of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal.*

Ans. Mass of metal block,

$$m = 0.20\ \text{kg} = 200\ \text{g}$$

Fall in temperature of metal block,

$$\Delta T = 150 - 40 = 110°\text{C}$$

Let specific heat of metal block = c cal g^{-1} °C^{-1}

∴ Heat lost by metal block

$$= mc\,\Delta T = 200 \times c \times 110\ \text{cal}$$

Volume of water in calorimeter = 150 cm^3

Mass of water, $m' = 150$ g

Water equivalent of calorimeter,

$$w = 0.025\ \text{kg} = 25\ \text{g}$$

Specific heat of water,

$$c' = 1\ \text{cal g}^{-1}\,°\text{C}^{-1}$$

∴ Heat gained by water and calorimeter

$$= (m' + w)\,c'\,\Delta T'$$

$$= (150 + 25) \times 1 \times (40 - 27)\ \text{cal} = 175 \times 13\ \text{cal}$$

By principle of calorimetry,

Heat lost = Heat gained

$$\therefore \quad 200 \times c \times 110 = 175 \times 13$$

or

$$c = \frac{175 \times 13}{200 \times 110} = \mathbf{0.1\ cal\ g^{-1}\,°C^{-1}}.$$

11.15. *Given below are observations on molar specific heats at room temperature of some common gases.*

Gas	Molar specific heat (C_V) (cal mol^{-1} K^{-1})
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

Scanned by CamScanner

The measured molar specific heats of these gases are markedly different from those for monoatomic gases. [Typically, molar specific heat of a monoatomic gas is 2.92 cal/mol K]. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine ?

Ans. A monoatomic gas has three degrees of freedom, while a diatomic gas possesses five degrees of freedom. Therefore, molar specific heat of a diatomic gas (at constant volume),

$$C_V = \frac{f}{2} R = \frac{5}{2} R = \frac{5}{2} \times \frac{8.31}{4.2} = 5 \text{ cal mol}^{-1} \text{ K}^{-1}$$

In the given table, all the gases are diatomic gases and for all of them (except chlorine), the value of C_V is about 5 cal mol^{-1} K^{-1}.

The slightly higher value of C_V for chlorine is due to the fact that even at room temperature, a chlorine gas molecule possesses the vibrational mode of motion also.

11.16. *Answer the following questions based on the P-T phase diagram of carbon dioxide as shown in Fig. 11.41.*

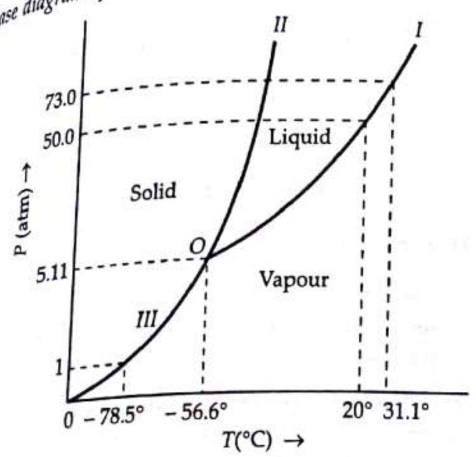


Fig. 11.41

(i) *At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?*

(ii) *What is the effect of decrease of pressure on the fusion and boiling point of CO_2 ?*

(iii) *What are the critical temperature and pressure for CO_2 ? What is their significance ?*

(iv) *Is CO_2 solid, liquid or gas at (a) $-70°C$ under 1 atm., (b) $-60°C$ under 10 atm., (c) $15°C$ under 56 atm ?*

Ans. (*i*) The solid, liquid and vapour phases of CO_2 co-exist in equilibrium at its triple point *O* for which

$$P_{tr} = \mathbf{5.11 \text{ atm}} \quad \text{and} \quad T_{tr} = \mathbf{-56.6°C}$$

(*ii*) The vaporisation curve *I* and fusion curve *II* show that both the boiling point and fusion point of CO_2 decrease with decrease of pressure.

(*iii*) For CO_2 , $P_c = \mathbf{73.0 \text{ atm}}$ and $T_c = \mathbf{31.1°C}$.

Above its critical temperature, CO_2 gas cannot be liquefied, however large pressure may be applied.

(*iv*)

(*a*) $-70°C$ *under 1 atm*. This point lies in ***vapour*** region. Therefore, at – 70°C under 1 atm, CO_2 is ***vapour***.

(*b*) $-60°C$ *under 10 atm*. This point lies in ***solid*** region. Therefore, CO_2 is ***solid*** at – 60°C under 10 atm.

(*c*) $15°C$ *under 56 atm*. This point lies in ***liquid*** region. Therefore, CO_2 is ***liquid*** at 15°C under 56 atm.

11.17. *Answer the following questions based on the P-T phase diagram of CO_2 :*

(i) *CO_2 at 1 atm pressure and temperature $-60°C$ is compressed isothermally. Does it go through the liquid phase ?*

(ii) *What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure ?*

(iii) *Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm. pressure and temperature $-65°C$ as it is heated up to room temperature at constant pressure.*

(iv) *CO_2 is heated to a temperature $70°C$ and compressed isothermally. What changes in its properties do you expect to observe ?*

Ans. (*i*) No. When CO_2 at 1 atm pressure and at – 60°C is compressed isothermally, it changes directly from vapour phase to solid phase without going through the liquid phase. This can be checked by drawing a vertical line at – 60°C which intersects the sublimation curve III.

(*ii*) CO_2 at 4 atm pressure and at temperature (say 25°C) is vapour. If it is cooled at constant temperature, it condenses directly into solid without going through liquid phase. This can be checked by drawing a horizontal line at $P = 4$ atm which intersects the sublimation curve III.

(*iii*) CO_2 at 10 atm pressure and at – 65°C is solid. As CO_2 is heated at constant pressure, it will go to ***liquid phase and then to the vapour phase***. It is because, the horizontal line through the initial point intersects both the fusion and the vaporisation curves. The fusion and boiling points can be known from the points, where the horizontal line at 10 atm (initial point) intersects the respective curves.

(*iv*) When the carbon dioxide is heated to 70°C (which is greater than its critical temperature), it will not exhibit any clear phase transition to the liquid phase. At this state, it will deviate more and more from ideal gas behaviour, as its pressure increases.

11.18. *A child running a temperature of 101° F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the*

Scanned by CamScanner

fever is brought down to 98° F in 20 min, what is the average rate of extra evaporation caused by the drug ? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.

Ans. Mass of child, $M = 30\text{ kg} = 30 \times 10^3\text{ g}$

Fall in temperature,

$$\Delta T = 101 - 98 = 3°F = 3 \times \frac{5}{9} = \frac{5}{3}\,°C$$

Specific heat of human body,

$$c = \text{specific heat of water} = 1\,\text{cal g}^{-1}\,°C^{-1}$$

Heat lost by child in the form of evaporation of sweat,

$$Q = Mc\,\Delta T = 30 \times 10^3 \times 1 \times \frac{5}{3} = 50{,}000\text{ cal}$$

If M' gram of sweat evaporates from the body of the child, then heat gained by sweat

$$Q = M'L = M' \times 580\text{ cal} \qquad [\because L = 580\text{ cal g}^{-1}]$$

$\therefore$ Heat gained = Heat lost

$\therefore$ $M' \times 580 = 50000$

or $$M' = \frac{50000}{580} = 86.2\text{ g}$$

Time taken by sweat to evaporate = 20 min

$\therefore$ Rate of evaporation of sweat $= \frac{86.2}{20} = \mathbf{4.31\ g\ min^{-1}}$.

11.19. *A 'thermocole' cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice are put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45° C and coefficient of thermal conductivity of thermocole = 0.01 $Js^{-1}\,m^{-1}\,°C^{-1}$. Given heat of fusion of water = 335×10^3 $J\,kg^{-1}$.*

Ans. Here $A = 6 \times \text{side}^2 = 6 \times 30 \times 30$,

$= 5400\text{ cm}^2 = 0.54\text{ m}^2$,

$x = 5\text{ cm} = 0.05\text{ m}$

$t = 6\text{ h} = 6 \times 3600\text{ s}$,

$T_1 - T_2 = 45 - 0 = 45°C$,

$K = 0.01\,\text{Js}^{-1}\,\text{m}^{-1}\,°C^{-1}$,

$L = 335 \times 10^3\text{ J kg}^{-1}$.

Total heat entering the box through all the six faces,

$$Q = \frac{KA(T_1 - T_2)t}{x} = \frac{0.01 \times 0.54 \times 45 \times 6 \times 3600}{0.05}$$

$= 104976\text{ J}$

Let m kg of ice melt due to this heat. Then

$Q = mL$

or $$m = \frac{Q}{L} = \frac{104976\text{ J}}{335 \times 10^3\text{ J kg}^{-1}} = 0.313\text{ kg}$$

Mass of ice left after six hours

$= 4 - 0.313 = \mathbf{3.687\ kg}$.

11.20. *A brass boiler has a base area of 0.15 m² and thickness 1.0 cm. It boils water at the rate of 6.0 kg min⁻¹ when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 $Js^{-1}\,m^{-1}\,°C^{-1}$ and heat of vaporisation of water = 2256 Jg^{-1}.*

Ans. Here $A = 0.15\text{ m}^2$, $x = 1.0\text{ cm} = 0.01\text{ m}$,

$K = 109\,\text{Js}^{-1}\,\text{m}^{-1}\,°C^{-1}$, $L = 2256\text{ Jg}^{-1}$,

$T_2 = 100°C$, $t = 1\text{ min} = 60\text{ s}$

Let T_1 be the temperature of the part of the flame in contact with boiler. Then amount of heat that flows into water in 1 min,

$$Q = \frac{KA(T_1 - T_2)t}{x} = \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01}\text{ J}$$

Mass of water boiled per min = 6 kg = 6000 g

Heat used to boil water,

$$Q = mL = 6000\text{ g} \times 2256\text{ J g}^{-1} = 6000 \times 2256\text{ J}$$

$$\therefore \quad \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} = 6000 \times 2256$$

or $$T_1 - 100 = \frac{6000 \times 2256 \times 0.01}{109 \times 0.15 \times 60} = 138°C$$

or $$T_1 = 138 + 100 = \mathbf{238°C}.$$

11.21. *Explain why :*

(*a*) *a body with large reflectivity is a poor emitter.*

(*b*) *a brass tumbler feels much colder than a wooden tray on a chilly day.*

(*c*) *an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.*

(*d*) *the earth without its atmosphere would be inhospitably cold.*

(*e*) *heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.*

Ans. (*a*) A body with large reflectivity is a poor absorber of heat. According to Kirchhoff's law, a poor absorber of heat is a poor emitter. Hence a body with large reflectivity is a poor emitter.

(*b*) Brass is a good conductor of heat. When a brass tumbler is touched, heat quickly flows from human body to tumbler. Consequently, the tumbler appears colder. Wood is a bad conductor. So, heat does not flow from the human body to the tray in this case. Thus, it appears comparatively hotter.

Scanned by CamScanner

(c) Let T be the temperature of the hot iron in the furnace

Heat radiated per second per unit area,

$$E = \sigma T^4$$

When the body is placed in the open at temperature T_0, the heat radiated/second/area, $E' = \sigma(T_4 - T_0^4)$.

Clearly $E' < E$. So the optical pyrometer gives too low a value for the temperature in the open.

(d) Heat radiated out by earth is reflected back by the atmosphere. In the absence of atmosphere, at night all heat would escape from the earth's surface and thereby earth's surface would be inhospitably cold. Also atmosphere helps in maintaining the temperature through convection current.

(e) Though steam and boiling water are at the same temperature but each unit mass of steam contains a larger amount of additional heat called the latent heat. For example, each gram of steam has 540 calories of more heat than each gram of boiling water. Hence steam loses more heat than boiling water.

11.22. *A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C, the temperature of the surrounding is 20°C.*

Ans. According to Newton's law of cooling, when the temperature difference is not large, rate of loss of heat is proportional to the temperature difference between the body and the surroundings.

$$mc\frac{T_1 - T_2}{t} = K(T - T_0)$$

where $T = \frac{T_1 + T_2}{2}$ = average of the initial and final temperatures of the body and T_0 is the temperature of the surroundings.

Here $T_1 = 80°C$, $T_2 = 50°$, $T_0 = 20°C$,

$t = 5 \text{ min} = 300 \text{ s}$

$$T = \frac{T_1 + T_2}{2} = \frac{80 + 50}{2} = 65°C$$

$$\therefore \quad mc\frac{80 - 50}{300} = K(65 - 20) \quad ...(i)$$

If the liquid takes t seconds to cool from 60°C to 30°C, then

$$T = \frac{60 + 30}{2} = 45°C$$

$$\therefore \quad mc\frac{60 - 30}{t} = K(45 - 20) \quad ...(ii)$$

Dividing equation (i) by (ii), we get :

$$\frac{30}{300} \times \frac{t}{30} = \frac{45}{25}$$

or $$t = \frac{45}{25} \times 300 = 540 \text{ s} = \mathbf{9 \text{ min.}}$$

Text Based Exercises

Type A : Very Short Answer Questions **1 Mark Each**

1. What is heat ?
2. What are the SI and CGS units of heat ? How are they related ?
3. What physical changes may be observed, if an object is heated ?
4. Define temperature.
5. What is thermometry ?
6. State the principle of a thermometer.
7. Name the various thermometric properties.
8. What is the normal temperature of a human body ?
9. The temperature of a gas is increased by 10°C. What is the corresponding change on the Kelvin scale ?
10. At what temperature will wood and iron appear equally hot or equally cold ?
11. What is the value of 0°F on the Kelvin scale ?
12. What is the temperature at which Celsius and Fahrenheit scales give the same reading ?
13. Which thermometer should be used to measure the temperature of – 150°C or 750°C ?
14. What are the values of the triple point of water on the Centigrade, Fahrenheit and absolute scales of temperature ?
15. Taking absolute zero as the basis of the scale of temperature, what are the values of melting point and boiling point of water ?
16. Name the gas law that forms the principle of a constant volume air thermometer.
17. Which one is more sensitive – a mercury thermometer or a gas thermometer ?
18. What is the minimum possible temperature ? Is there also a maximum possible temperature ?
19. What is pyrometry ?
20. When two bodies are said to be in thermal equilibrium ?
21. Of metals and alloys, which have a greater value of temperature coefficient of expansion ?
22. What are the units of α, β and γ ?
23. How are the three coefficients of expansion related to each other ?

Scanned by CamScanner

24. A liquid of cubical expansivity γ is heated in a vessel having linear expansivity $\gamma/3$. What would be the effect on the level of the liquid ?

25. A long metal rod is bent to form a ring with a small gap. If this rod is heated, will the gap increase or decrease ?

26. Is the coefficient of expansion constant for a given solid ?

27. A metal disc has a hole in it. What happens to the size of the hole, when the disc is heated ?

28. Define Joule's mechanical equivalent of heat. Give its value.

29. Why does a rifle bullet get heated on striking a target ?

30. If a piece of iron is hammered, it becomes warmer. Why ?

31. A spark is produced when two stones are struck against each other. Why ?

32. Tea gets cooled, when sugar is added to it. Why ?

33. Write the SI unit of specific heat.

34. Which substance has the highest specific heat ?

35. What is the specific heat of water in the SI units ?

36. Write the values of the specific heats of water at 20°C and ice.

37. Which has the maximum and which has the minimum specific heat amongst the following :
Carbon, Silver, Aluminium, Tungsten ?

38. What is the relation between heat capacity and water equivalent of a body ?

39. Name three modes of transmission of heat energy from one place to another.

40. Which is the only way of heat transfer through a solid ?

41. How does the heat energy from the sun reach the earth ?

42. Out of the three modes of transmission of heat, which one is fastest ?

43. The temperature gradient in a rod 0.5 m long is 40°C per metre. The temperature of the hotter end is 30°C. What is the temperature of its colder end ?

44. Write the SI unit of the coefficient of thermal conductivity.

45. Define coefficient of thermal conductivity. **[Delhi 98, 02]**

46. What is the dimensional formula of the coefficient of thermal conductivity ?

47. Arrange the metals Cu, Al and Ag in the order of their increasing thermal conductivity.

48. Why does the temperature of every part of a metallic rod become constant in steady state ?

49. What is thermal conductivity of a perfect heat conductor and a perfect heat insulator ?

50. Can you imagine a condition that on heating a rod at one end the temperature of the whole rod becomes the same ?

51. Write the unit and dimensional formula of thermal resistance.

52. In which methods of heat transfer, gravity does not play any part ?

53. In which methods of heat transfer, mass of the substance heated does not play any part ?

54. Do the thermal radiations obey the laws of reflection and refraction ?

55. Can thermal radiations pass through vacuum ?

56. What is the nature of heat radiations ?

57. What is the absorptivity of a perfect black body ?

58. If a hot body is kept in surroundings which are cooler than the body, only the hot body radiates heat, while the cooler surroundings do not ? Is it true or false ?

59. What is meant by saying that thermal radiations inside a constant temperature enclosure are isotropic ?

60. If the temperature of a black body is increased from 300 K to 900 K, by what factor the rate of emission will increase ?

61. The temperature of a body is 0°C. Is it radiating ?

62. At what temperature will a body stop radiating ?

63. Name the electrical analogies of temperature and temperature gradient.

64. What is sublimation ?

65. How much is the critical temperature of water ?

66. Why do the fusion line, vaporisation line and sublimation line of a substance, meet necessarily at a single point ?

67. At what temperature and pressure, can the solid, liquid and vapour phases of water co-exist in equilibrium ?

68. At what temperature and pressure, can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?

69. What is the effect of decrease of pressure on the fusion and boiling points of CO_2 ?

70. Is CO_2 solid, liquid or gas at (*a*) – 70°C under 1 atm (*b*) – 60°C under 10 atm (*c*) – 15°C under 56 atm ?

71. CO_2 at 1 atm pressure and temperature – 60°C is compressed isothermally. Does it go through liquid phase ?

72. What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure ?

73. Draw the graph showing cooling of hot water with time. **[Delhi 09]**

Scanned by CamScanner

Answers

1. Heat is a form of energy which produces in us the sensation of warmth.
2. SI unit of heat is joule and CGS unit of heat is calorie.

$$1 \text{ calorie} = 4.18 \text{ joule}$$

3. When an object is heated, the physical changes such as expansion, contraction, change of states, change of electrical properties, etc. are observed.
4. Temperature is the degree of hotness of a body. It is a condition which determines the direction of flow of heat, when the two bodies are placed in contact with each other.
5. The branch of physics that deals with the measurement of temperature is called the thermometery.
6. The working of a thermometer is based on the fact that some physical property of substance changes linearly with temperature.
7. Some of the thermometric properties are length, volume, pressure, electrical resistance, thermo-electric e.m.f., radiated power, etc.
8. $98.4°F$ or $37°C$.
9. 10 K.
10. When both wood and iron are at the temperature of the human body, they appear equally hot or equally cold.
11. $0°F = 255.23$ K.
12. $-40°C = -40°F$.
13. Radiation pyrometer.
14. $0.01°C$, $32.018°F$, 273.16 K
15. 273.15 K and 373.15 K.
16. Charle's law of pressure.
17. A gas thermometer.
18. 0 K is the minimum possible temperature. There is no limit to maximum temperature.
19. The branch of physics that deals with the measurement of high temperature is called pyrometry.
20. Two bodies are said to be in thermal equilibrium if they have the same temperature.
21. The value of coefficient of linear expansion (α) is greater for metals than for alloys.
22. All have same units, K^{-1} or $°C^{-1}$.
23. $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$.
24. There will be no change in the level of the liquid.
25. The gap will increase on heating.
26. No. The coefficient of expansion of a solid changes with temperature.
27. The size of the hole increases.
28. Joule's mechanical equivalent of heat may be defined as the amount of work that must be done to produce a unit quantity of heat. Its value is 4.18 J cal^{-1}.
29. The kinetic energy of the bullet gets converted into heat energy, which in turn heats up the bullet.
30. When the hammer strikes the piece of iron, its kinetic energy gets converted into heat.
31. The work done in striking the two stones against each other is converted into heat which produces spark.
32. When sugar is added to tea, its heat gets shared by sugar. So the temperature of the tea decreases.
33. SI unit of specific heat = $\text{J kg}^{-1} \text{K}^{-1}$.
34. Water.
35. $4186 \text{ J kg}^{-1} \text{K}^{-1}$.
36. Specific heat of water at $20°C = 1 \text{ cal g}^{-1} °C^{-1}$ and specific heat of ice = $0.5 \text{ cal g}^{-1} °C^{-1}$.
37. Aluminium has the maximum and tungsten the minimum specific heat.
38. The heat capacity and water equivalent of a body are numerically equal.
39. Conduction, convection and radiation.
40. Conduction.
41. By radiation.
42. Radiation.
43. Temperature of colder end

$$= 30 - 0.5 \times 40 = \mathbf{10°C}.$$

44. SI unit of $K = \text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ or $\text{W m}^{-1} \text{K}^{-1}$.
45. The coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of 1 degree.
46. $[MLT^{-3}K^{-1}]$.
47. Al < Cu < Ag.
48. In steady state, no part of the rod absorbs heat.
49. (*i*) For a perfect heat conductor, $K = \infty$.

    (*ii*) For a perfect heat insulator, $K = 0$.
50. Yes, this is possible if the rod is a perfect conductor *i.e.*, $K = \infty$.
51. SI unit of thermal resistance = KW^{-1}.

    Dimensional formula of thermal resistance is $[M^{-1}L^{-2}T^{3} K]$.

Scanned by CamScanner

52. Conduction and radiation.
53. Conduction and radiation.
54. Yes, thermal radiations obey the laws of reflection and refraction.
55. Yes, thermal radiations can pass through vacuum.
56. Heat radiations are electromagnetic waves having wavelength range from 1 μm to 100 μm. These are also called infrared waves.
57. One.
58. False. According to Prevost theorem of heat exchanges, both the hot body and the surroundings emit and absorb radiations.
59. This means that the intensities of different wavelengths inside a hollow enclosure are same in all directions.
60. 81 times because $E \propto T^4$.
61. Yes, a body radiates heat at 0°C.
62. A body stops radiating at 0 K.
63. Potential and potential gradient respectively.
64. When a substance changes from solid state to gaseous state directly without undergoing the liquid state, the process is called sublimation.
65. 374.1°C
66. This is because if these curves do not meet at a single point, they would enclose some finite area. In this region solid, liquid and vapour phases of the substance would exist simultaneously, which is not possible.
67. At the temperature of 263.16 K and a pressure of 0.46 cm of Hg.
68. At the temperature of − 50.6°C and under a pressure of 5.11 atmosphere.
69. Both the boiling point and freezing point of CO_2 decrease if pressure decreases.
70. (i) Vapour (ii) Solid (iii) Liquid.
71. No. Vapour condenses directly into solid.
72. It condenses to solid directly without passing through the liquid phase.
73. See Fig. 11.26 on page 11.34.

Type B : Short Answer Questions

2 or 3 Marks Each

1. Distinguish clearly between heat and temperature.
2. What is meant by the statement that heat is an energy in transit ?
3. Why is mercury used in a thermometer ?
4. State Joule's law of equivalence between work and heat. Hence define mechanical equivalent of heat. Give its value.
5. Starting from Charle's law, develop the concept of absolute zero and absolute scale of temperature.
6. What is meant by triple point of water ? What is the advantage of taking triple point of water as the fixed point for a temperature scale ?
7. Define ideal gas temperature. Does it depend on the nature of the gas ?
8. What is a liquid thermometer ? Briefly describe its working principle.
9. Describe the working principle of a platinum resistance thermometer.
10. How does the coefficient of cubical expansion of a substance vary with temperature ? Draw γ versus T curve for copper.
11. Prove that the coefficient of cubical expansion of an ideal gas at constant pressure is equal to the reciprocal of its absolute temperature.
12. What do you mean by coefficients of apparent and real expansion of a liquid ? How are they related ?
13. How does the density of a solid or liquid change with temperature ? Show that its variation with temperature is given by $\rho' = \rho(1 - \gamma\,\Delta T)$, where γ is the coefficient of cubical expansion.
14. Define the terms specific heat and molar specific heat. Give their SI units.
15. Define the terms heat capacity and water equivalent. Give their SI units.
16. Define latent heat of fusion of ice and latent heat of vaporisation of steam. [Meghalaya 98]
17. Define thermal conduction. Briefly explain its molecular mechanism.
18. Briefly explain, how does a metallic rod heated at one end attain steady state ? Write the important features of steady state.
19. Explain the following in reference to thermal conduction in a rod :
    (i) Steady state,
    (ii) Isothermal surface,
    (iii) Temperature gradient.
20. Define terms heat current and thermal resistance. Write their SI units.
21. Define thermal resistance. On what factors does it depend ? Deduce its dimensions.

22. Define thermal ... convection curre...
23. Distinguish betw... tions. Give one ...
24. Briefly explain ... breezes.
25. Write the main ... exchange. How ... good absorbers ...
26. What are ther... important prop...
27. Define the term...
    (i) absorptive ...
    (ii) emissive p...
    (iii) emissivity.
28. What is a blac... practice ?
29. Explain why ... global warmin...
30. State Stefan-B... units of Stefan...
31. Define therm... are these mo... radiation ?

Answers

1. Refer answe...
2. Refer answe...
3. Refer answe...
4. Refer answe...
5. Refer answ...
6. Refer answ...
7. Refer answ...
8. Refer answ...
9. Refer answ...
10. Refer answ...
11. Refer answ...
12. Refer ansv...
13. Refer ansv...
14. Refer ans...
15. Refer ans...
16. Refer to p...
17. Refer ans...
18. Refer ans...
19. Refer an...
20. Refer an...

Scanned by CamScanner

22. Define thermal convection. Briefly explain how convection currents are set up in water.
23. Distinguish between natural and forced convections. Give one example of each.
24. Briefly explain the formation of land and sea breezes.
25. Write the main features of Prevost theory of heat exchange. How does this theory lead to the fact that good absorbers are good radiators ?
26. What are thermal radiations ? Give their two important properties.
27. Define the terms
    (i) absorptive power,
    (ii) emissive power and
    (iii) emissivity. Write their SI units, if any.
28. What is a black body ? How can it be realised in practice ?
29. Explain why Greenhouse effect is responsible for global warming. [Central Schools 13]
30. State Stefan-Boltzmann law. Write the CGS and SI units of Stefan-Boltzmann constant.
31. Define thermal conduction and convection. How are these modes of transfer different from thermal radiation ? [Delhi 03C]
32. Define thermal radiation. State Prevost's theory of heat exchange. [Delhi 03C]
33. What is meant by coefficient of linear expansion and coefficient of cubical expansion ? Derive relationship between them. [Himachal 03]
34. Define coefficient of thermal conductivity. Write its SI unit. [Himachal 03, Delhi 02]
35. Name the three modes of transfer of heat from one object to other. Also cite one example for each one of them. [Himachal 07 ; Central Schools 08]
36. Define Newton's law of cooling, write the expression. [Central Schools 08, 12, 14]
37. State and prove Kirchoff's law of radiation. Hence show that a good absorber is also a good emitter of radiation. [Delhi 2006]
38. State Stefan's Law and Newton's Law of cooling. How do you deduce the later from the former ? [Central School 05 ; Chandigarh 04]
39. Discuss briefly energy distribution of black body radiation. Hence deduce Wien's displacement law and Stefan's law. [Chandigarh 03]
40. Draw energy distribution curves for a black body at two different temperature T_1 and $T_2 (T_1 > T_2)$. Write any two conclusions that can be drawn from these curves. [Delhi 2004]

Answers

1. Refer answer to Q. 6 on page 11.2.
2. Refer answer to Q. 2 on page 11.1.
3. Refer answer to Q. 15 on page 11.5.
4. Refer answer to Q. 4 on page 11.2.
5. Refer answer to Q. 11 on page 11.3.
6. Refer answer to Q. 12 on page 11.4.
7. Refer answer to Q. 14 on page 11.5.
8. Refer answer to Q. 15 on page 11.5.
9. Refer answer to Q. 16 on page 11.6.
10. Refer answer to Q. 23 on page 11.9.
11. Refer answer to Q. 24 on page 11.10.
12. Refer answer to Q. 28 on page 11.11.
13. Refer answer to Q. 29 on page 11.11.
14. Refer answer to Q. 31 on page 11.16.
15. Refer answer to Q. 32 on page 11.16.
16. Refer to points 26 and 27 of Glimpses.
17. Refer answer to Q. 2 on page 11.1.
18. Refer answer to Q. 3 on page 11.2.
19. Refer answer to Q. 42 on page 11.24.
20. Refer answer to Q. 45 on page 11.25.
21. Refer answer to Q. 45 on page 11.25.
22. Refer answer to Q. 47 on page 11.32.
23. Refer answer to Q. 47 on page 11.32.
24. Refer answer to Q. 48(*iv*) on page 11.32.
25. Refer answer to Q. 34 on page 11.17.
26. Refer answer to Q. 52 on page 11.34.
27. Refer answer to Q. 55 on page 11.36.
28. Refer answer to Q. 56 on page 11.37.
29. Refer answer to Q. 59 on page 11.38.
30. Refer answer to Q. 60 on page 11.38.
31. Refer to points 30, 35 and 36 of Glimpses.
32. Refer to points 33 and 37 of Glimpses.
33. Refer answer to Q. 22 on page 11.9.
34. Refer to point 33 of Glimpses.
35. Refer to points 30, 35 and 36 of Glimpses.
36. Refer to point 45 of Glimpses.
37. Refer answer to Q. 57 on page 11.37.
38. Refer answer to Q. 61 on page 11.38.
39. Refer answer to Q. 63 on page 11.43.
40. Refer answer to Q. 63 on page 11.43.

Scanned by CamScanner

Type C : Long Answer Questions

5 Marks Each

1. Describe the principle, construction and working of a constant volume gas thermometer. Give its two advantages over mercury thermometer.

2. What is meant by coefficient of linear expansion, superficial expansion and cubical expansion ? Derive the relationship between them. [Himachal 04, 05C, 07C]

3. Thermal expansion is a consequence of atomic vibrations and asymmetry of potential energy function. Explain.

4. Draw a labelled P-T diagram of water. Explain its behaviour, when both pressure P and temperature T are above and below the triple point. Give the importance of triple point.

5. Draw a P-T diagram for CO_2. Explain its behaviour, when both pressure and temperature are above and below the triple point. Give the importance of triple point.

6. On what factors does the rate of heat conduction in a metallic rod in the steady state depend ? Write the necessary expression and hence define the coefficient of thermal conductivity. Write its units and dimensions also.

7. What is meant by Steady state heat flow by conduction in case of a thick copper bar with its two ends maintained at two different temperatures ? On what factors does the amount of heat flowing through the bar depend ? [Delhi 03]

8. State and explain the three modes of transference of heat. Explain how the loss of heat due to these modes is minimised in a thermos flask. [Himachal 05; Delhi ...]

9. What are convection currents ? What role do they play in relation to trade winds and monsoons ?

10. State and prove the Kirchhoff's law of heat radiation. Why does a glass heated in a furnace when taken out in dark glow with red light ?

11. State : (*i*) Stefan's law and (*ii*) Wein's displacement law.

    How you will derive Newton's law of cooling from Stefan's law ? [Himachal 0?C]

12. State Newton's law of cooling. Deduce the relations :

$$\log_e(T - T_0) = -kt + c$$

and

$$\Delta t = T - T_0 = Ce^{-kt}$$

where the symbols have their usual meanings. Represent Newton's law of cooling graphically by using each of the above equations.

13. (*a*) Explain the terms specific heat and heat capacity.

    (*b*) State Newton's law of cooling. Derive mathematical expression for it. [Delhi 10]

14. State Newton's law of cooling. Express it mathematically. How can this law be verified experimentally ? [Delhi 12]

Answers

1. Refer answer to Q. 13 on page 11.4.
2. Refer to points 14, 15 and 16 of Glimpses and answer to Q. 25 on page 11.10.
3. Refer answer to Q. 26 on page 11.10.
4. Refer answer to Q. 65 on page 11.44.
5. Refer answer to Q. 66 on page 11.45.
6. Refer answer to Q. 44 on page 11.24.
7. Refer answer to Q. 43 and Q. 44 on page 11.24.
8. Refer to points 30, 35 and 36 of Glimpses. Refer answer to Q. 58(*iv*) on page 11.38.
9. Refer answer to Q. 48(*iii*) and (*v*) on page 11.32.
10. Refer answer to Q. 57 on page 11.37 and Q. 58(*ii*) on page 11.37.
11. Refer answer to Q. 60 on page 11.38 and Q. 61 on page 11.38.
12. Refer answer to Q. 53 on page 11.34.
13. (*a*) Refer to points 21 and 23 of Glimpses.
    (*b*) Refer answer to Q. 61 on page 11.38.
14. Refer answer to Q. 53 on page 11.34.

Scanned by CamScanner

Competition Section

Thermal Properties of Matter

GLIMPSES

1. **Heat.** Heat is a form of energy which produces in us the sensation of hotness or coldness. According to *dynamic theory*, heat may be regarded as the energy of molecular motion which is equal to the sum total of the kinetic energy possessed by the molecules by virtue of their translational, vibrational and rotational motions.

2. **Units of heat.** *Calorie* (cal) is the C.G.S. unit of heat. One calorie is defined as the heat energy required to raise the temperature of one gram of water from 14.5°C to 15.5°C. Like all other forms of energy, the S.I. unit of heat is joule.

   1 calorie = 4.186 joule.

3. **Joule's mechanical equivalent of heat.** Whenever a given amount of work (W) is converted into heat, always the same amount of heat Q is produced.

   Thus $W \propto Q$

   or $W = JQ$

   or $J = \frac{W}{Q}$.

   The proportionality constant J is called Joule's mechanical equivalent of heat. It may be defined as the amount of work that must be done to produce a unit quantity of heat.

   $$J = 4.2 \times 10^7 \text{ erg cal}^{-1} = 4.2 \text{ J cal}^{-1}.$$

4. **Temperature.** It is the degree of hotness of a body. The temperature of a body gives a measure of the average kinetic energy of its molecules.

5. **Thermometer.** It is a device used to measure the temperature of a body. It makes use of some measurable property (called thermometric property) of a substance which changes linearly with temperature.

6. **Different temperature scales.**

Temperature scale	Lower fixed point (Melting point of ice)	Upper fixed point (Boiling point of water)
1. Celsius	0°C	100°C
2. Fahrenheit	32°F	212°F
3. Reaumer	0°R	80°R
4. Kelvin	273.15 K	373.15 K

7. **Relations between different temperature scales.** If T_C, T_F, T_R and T are the temperatures of a body on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

   (*i*) $\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T - 273.15}{373.15 - 273.15}$

   or $\frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_R}{4} = \frac{T - 273.15}{5}$

   (*ii*) $T_C = \frac{5}{9}(T_F - 32)$, $\quad T_F = \frac{9}{5} T_C + 32$

   (*iii*) $T = T_C + 273.15$, $\quad T_C = T - 273.15$

   (*iv*) $T_F = \frac{9}{5}(T - 273.15) + 32 = \frac{9}{5} T - 459.67$,

   $T = \frac{5}{9} T_F + 255.37$.

8. **Absolute scale of temperature.** The lowest possible temperature of − 273.15°C at which a gas is supposed to have zero volume (and zero pressure) and at which entire molecular motion stops is called absolute zero of temperature. The temperature scale which starts with − 273.15°C as its zero is called Kelvin scale or absolute scale. The size of degree on Kelvin scale is same as that on Celcius scale.

   $$T\,(\text{K}) = t\,°(\text{C}) + 273.15.$$

Scanned by CamScanner

9. **Triple point of water.** The triple point of water is the state at which the three phases of water namely ice, liquid water and water vapour are equally stable and co-exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column.

10. **Constant volume air thermometer.** It is used to measure the pressure of a definite mass of air at different temperatures, the volume of air remaining constant. It is based on the pressure law,

$$\frac{P}{T} = \frac{P_0}{T_0} \quad \text{or} \quad T = T_0 \times \frac{P}{P_0}$$

In terms of triple point of water, $T = T_{tr} \times \frac{P}{P_{tr}}$.

11. **Ideal gas temperature scale.** The ideal gas temperature on the Kelvin scale is defined by the equation

$$T = \lim_{P_{tr} \to 0} 273.16\left(\frac{P}{P_{tr}}\right)$$

It is independent of the nature of the gas.

12. **Platinum resistance thermometer.** It is based on the fact that resistance of a platinum wire varies with temperature. If R_0 and R are the resistances at 0°C and t °C respectively, then

$$R = R_0\,(1 + \alpha t)$$

Here α is the temperature coefficient of resistance of platinum and is defined as the increase in resistance per unit resistance at 0°C for 1°C rise in temperature.

$$\alpha = \frac{R - R_0}{R_0 \times t}$$

Unit of α is °C^{-1}.

If R_0 and R_{100} are the resistances of a platinum wire at ice point and steam point respectively, then the temperature t_R of a body for which the corresponding resistance is R_t is given by

$$t_R = \frac{R_t - R_0}{R_{100} - R_0} \times 100°\text{C}.$$

13. **Thermoelectric thermometer.** It uses thermoelectric e.m.f. (ε) as thermometric property. For the linear part of the thermo e.m.f., the unknown temperature is given by

$$t_\varepsilon = \frac{\varepsilon_t - \varepsilon_0}{\varepsilon_{100} - \varepsilon_0} \times 100 \text{ degrees}$$

14. **Linear expansion.** When a solid rod of initial length l is heated through a temperature ΔT, its final (increased) length is given by

$$l' = l\,(1 + \alpha \Delta T)$$

where α is coefficient of linear expansion. It is given by

$$\alpha = \frac{l' - l}{l \times \Delta T}$$

The coefficient of linear expansion of the material of a solid rod is defined as the increase in length per unit length per degree rise in temperature.

15. **Superficial expansion.** When a solid sheet of initial surface area S is heated through a temperature ΔT, its final (increased) surface area is given by

$$S' = S\,(1 + \beta \Delta T)$$

where β is coefficient of superficial expansion. It is given by

$$\beta = \frac{S' - S}{S \times \Delta T}$$

It is defined as the increase in surface area per unit surface area per degree rise in temperature.

16. **Cubical expansion.** When a solid of initial volume V is heated through a temperature ΔT, its final (increased) volume is given by $V' = V\,(1 + \gamma \Delta T)$ where γ is coefficient of cubical expansion. It is given by

$$\gamma = \frac{V' - V}{V \times \Delta T}$$

It is defined as the increase in volume per unit volume per degree rise in temperature.

The coefficient of cubical expansion of an ideal gas is equal to the reciprocal of its absolute temperature.

$$\gamma = \frac{1}{T}.$$

17. **Relation between α, β and γ.** The three coefficients of thermal expansion are related as

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3} \quad \text{or} \quad \beta = 2\alpha \quad \text{and} \quad \gamma = 3\alpha.$$

The units of α, β and γ are same *viz.* °C^{-1} or K^{-1}.

18. **Coefficient of apparent expansion of a liquid.** It is defined as the apparent increase in volume per unit original volume per degree rise in temperature.

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}.$$

19. **Coefficient of real expansion of a liquid.** It is defined as the real increase in volume per unit original volume per degree rise in temperature.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

If γ_g is the coefficient of cubical expansion of glass (material of the container), then

$$\gamma_r = \gamma_a + \gamma_g.$$

20. **Variation of density with temperature.** The density of a solid or liquid decreases with the increase of temperature.

$$\rho' = \rho\,(1 - \gamma \Delta T)$$

21.
22.
23.
24.
25.
26.
27.
28.
29.

Scanned by CamScanner

21. **Specific heat.** It may be defined as the amount of heat required to raise the temperature of unit mass of a substance through one degree. If Q heat is needed to raise the temperature of m mass of a substance through ΔT, then specific heat is

$$c = \frac{Q}{m\Delta T}$$

The cgs unit of specific heat is cal $g^{-1}\,{}^\circ C^{-1}$ and the SI unit is $Jkg^{-1}\,K^{-1}$. Clearly,

Heat gained or lost, $Q = mc\,\Delta T$.

22. **Molar specific heat.** *It is defined as the amount of heat required to raise the temperature of 1 mole of the substance through one degree.* If Q heat is needed to raise the temperature of n moles of a substance through ΔT, then molar specific heat is

$$C = \frac{Q}{n\Delta T}$$

The cgs unit of molar specific heat is cal $mol^{-1}\,{}^\circ C^{-1}$ and the SI unit is J $mol^{-1}\,K^{-1}$.

23. **Heat capacity or thermal capacity.** *It is defined as the amount of heat required to raise the temperature of a body through one degree.*

Heat capacity = Mass × Specific heat = mc

The cgs unit of heat capacity is cal${}^\circ C^{-1}$ and the S.I. unit is JK^{-1}.

24. **Water equivalent.** The water equivalent of a body is defined as the mass of water which requires the same amount of heat as is required by the given body for the same rise of temperature.

w = Mass × specific heat = mc

The cgs unit of water equivalent is g and the SI unit is kg.

25. **Latent heat.** The amount of heat required to change the state of unit mass of a substance at a constant temperature is called its latent heat. It is denoted by L.

26. **Latent heat of fusion.** The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion.

27. **Latent heat of vaporisation.** The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point is called latent heat of vaporisation.

28. **Principle of calorimetry.** When two bodies at different temperatures are placed in contact with each other, the heat lost by the hot body is equal to the heat gained by the cold body. This is the principle of calorimetry or the principle of mixtures.

Heat gained = Heat lost

29. **Transfer of heat.** The three modes of transfer of heat are conduction, convection and radiation.

30. **Conduction.** It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.

31. **Steady state.** The state of the rod when temperature of every cross-section of the rod becomes constant and there is no further absorption of heat in any part is called steady state.

32. **Factors on which conduction of heat depends.** When two opposite faces of a slab each of area of cross-section A and separated by a distance x are maintained at temperatures T_1 and T_2 ($T_1 > T_2$), then amount of heat that flows in time t,

$$Q = \frac{KA\,(T_1 - T_2)\,t}{x}$$

where K is coefficient of thermal conductivity of the material of the slab between its two faces. The rate of flow of heat through the slab is

$$\frac{dQ}{dt} = -\,KA\,\frac{dT}{dx}$$

Here dT/dx is the rate of fall of temperature with distance and is called temperature gradient.

33. **Coefficient of thermal conductivity.** *It may be defined as the quantity of heat energy that flows in unit time between the opposite faces of a cube of unit side, the faces being kept at one degree difference of temperature.* Its cgs unit is cal $s^{-1}cm^{-1}\,{}^\circ C^{-1}$ and SI unit is $Js^{-1}m^{-1}K^{-1}$ or $Wm^{-1}K^{-1}$.

Dimensional formula of K is $[MLT^{-3}K^{-1}]$.

Thermal conductivities of metals are much greater than those for metals. Gases are poor thermal conductors.

34. **Heat current and resistance.** The flow of heat per unit time in conduction is called *heat current*. The ratio of the temperature difference between the ends of a conductor to the heat current through it is called *thermal resistance*.

Heat current, $H = \frac{Q}{t} = KA\,\frac{\Delta T}{\Delta x}$.

Thermal resistance, $R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$.

SI unit of $R_H = KW^{-1}$.

Dimensions of $R_H = [M^{-1}L^{-2}T^{-3}K]$.

35. **Convection.** It is the process by which heat is transmitted through a substance from one point to another due to the bodily motion of the heated particles of the substance. Fluids are mainly heated through convection. *Natural convection* arises due to unequal heating and gravity, when more heated and less dense parts rise and are replaced by the cooler parts of the fluid. In *forced convection*, a material is forced to move by an agency like a pump or a blower.

Scanned by CamScanner

36. **Radiation.** It is the process by which heat is transmitted from one place to another without heating the intervening medium.

37. **Prevost's theory of heat exchanges.** All bodies emit radiations irrespective of their temperatures. They emit radiations to the surroundings and receive radiations from the surroundings. In the equilibrium state, the exchange of energy between a body and its surroundings occurs in equal amounts.

38. **Thermal radiation.** *The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation or radiant energy.* Thermal radiations are electromagnetic waves of long wavelength ranging from 1 μm to 100 μm.

39. **Absorptive power or absorptivity (a_λ).** The absorptive power of a body for a given wavelength λ is defined as the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength λ. It is a dimensionless quantity.

40. **Emissive power (e_λ).** The emissive power of a body at a given temperature and for a given wavelength λ is defined as the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range around the wavelength λ. Its SI unit is $J\,s^{-1}\,m^{-2}$ or $W\,m^{-2}$.

41. **Emissivity (ε).** It is the ratio of the emissive power of a body (e) to the emissive power (E) of a black body at the same temperature. It is given by

$$\varepsilon = \frac{e}{E}$$

It is a dimensionless quantity having value between 0 and 1. The emissivity of a black body is 1.

42. **Black body.** *A black body is one which neither reflects nor transmits but absorbs whole of the heat radiation incident on it. The absorptive power of a black body is unity.*

43. **Kirchhoff's law.** It states that at any given temperature, the ratio of the emissive power (e_λ) to the absorptive power (a_λ) corresponding to certain wavelength is constant for all bodies and this constant is equal to the emissive power of the perfect black body (E_λ) at the same temperature and corresponding to the same wavelength. That is

$$\frac{e_\lambda}{a_\lambda} = E_\lambda \text{ (constant)}$$

Hence a good absorber is a good emitter.

44. **Stefan Boltzmann Law.** It states that the total amount of energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the body.

Thus $\quad E \propto T^4 \quad$ or $\quad E = \sigma T^4$

where σ is called *Stefan's constant*. Its value is

$$\sigma = 5.67 \times 10^{-5} \text{ erg s}^{-1}\text{ cm}^{-2}\text{ K}^{-4} \quad \text{(in CGS system)}$$

or

$$\sigma = 5.67 \times 10^{-8} \text{ Js}^{-1}\text{m}^{-2}\text{K}^{-4} = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{ K}^{-4} \quad \text{(in SI)}$$

If a perfect black body at temperature T is placed in an enclosure at temperature T_0, then net amount of energy radiated per second per unit area by the black body is

$$E = \sigma (T^4 - T_0^4)$$

If the body and the enclosure are not perfect black bodies and have relative emissivity ε, then

$$E = \varepsilon\sigma (T^4 - T_0^4)$$

45. **Newton's law of cooling.** For small temperature difference between a body and its surroundings, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed. This is known as Newton's law of cooling.

$$\frac{dT}{dt} = -kA\,(T - T_0).$$

46. **Wien's displacement law.** It states that the wavelength (λ_m) corresponding to which the energy emitted by a perfect black body is maximum is inversely proportional to the absolute temperature (T) of the black body *i.e.*,

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m = \frac{b}{T}$$

where b is a constant of proportionality and is called *Wien's constant*. Its value is

$$b = 2.9 \times 10^{-3} \text{ mK}$$

47. **Solar constant.** It is defined as the amount of solar radiant energy that a unit area of a perfect black body on the earth would receive per second in the absence of the atmosphere, with its surface perpendicular to the direction of the sun rays. Its value is 1340 W m^{-2}.

48. **Surface temperature of the sun.** If S_0 is solar constant, R_s the solar radius and R_0 is the mean distance of the earth from the sun, then surface temperature of the sun will be

$$T = \left[\frac{R_0^2\, S_0}{R_s^2\, \sigma}\right]^{1/4}.$$

Scanned by CamScanner

IIT Entrance Exam

Single Choice Questions with One Correct Answer

1. A constant volume air thermometer works on
(a) Archimedes' principle
(b) Boyle's law
(c) Pascal's law
(d) Gay Lussac's law. [IIT 80]

2. The temperature coefficient of resistance of a wire is 0.00125°C^{-1}. At 300 K, its resistance is 1Ω. The resistance of the wire will be 2Ω at
(a) 1,154 K (b) 1,100 K
(c) 1,400 K (d) 1,127 K [IIT 80]

3. Two rods, one of aluminium and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length l_1+l_2. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperatures are raised by t°C then find the ratio $l_1/(l_1+l_2)$
(a) α_s/α_a (b) α_a/α_s
(c) $\alpha_s/(\alpha_a+\alpha_s)$ (d) $\alpha_a/(\alpha_a+\alpha_s)$ [IIT 03]

4. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 50°C. The co-efficient of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that
(a) $W_1>W_2$ (b) $W_1=W_2$
(c) $W_1<W_2$ (d) all of these. [IIT 80]

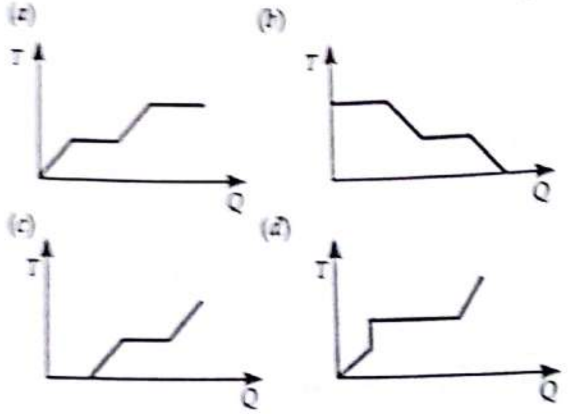
5. When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60°C a fraction k_2 is seen to be submerged. If the co-efficient of volume expansion of iron is γ_{Fe} and that of mercury of γ_{Hg}, then the ratio k_1/k_2 can be expressed as
(a) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (b) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$
(c) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$ [IIT 01]

6. Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1°C and it is defined under which of the following conditions ?
(a) From 14.5°C to 15.5°C at 760 mm of Hg
(b) From 98.5°C to 99.5°C at 760 mm of Hg
(c) From 13.5°C to 14.5°C at 76 mm of Hg
(d) From 3.5°C to 4.5°C at 76 mm of Hg [IIT 05]

7. Compared to burn due to air at 100°C, a burn due to steam at 100°C is
(a) more dangerous (b) less dangerous
(c) equally dangerous
(d) none of the above. [IIT 88]

8. 540 g of ice at 0°C is mixed with 540 g of water at 80 °C. The final temperature of mixture is
(a) 0°C (b) 40°C
(c) 80°C (d) less than 0°C [IIT 88]

9. A block of ice at − 10°C is slowly heated and converted to steam at 100 °C. Which of the following curves represents the phenomenon qualitatively ?



[IIT 2K]

10. 2 kg of ice at − 20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal kg^{-1}°C^{-1} and 0.5 kcal kg^{-1}, while the latent heat of fusion of ice is 80 kcal kg^{-1},
(a) 7 kg (b) 6 kg
(c) 4 kg (d) 2 kg [IIT 03]

11. Water of volume 2 litres in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at rate of 160 Js^{-1}. In how much time, temperature will rise from 27°C to 77°C ? Given that the specific heat of water is 4.2 kJ kg^{-1}.
(a) 8 min 20 s (b) 6 min 2s
(c) 7 min (d) 14 min [IIT 05]

Scanned by CamScanner

12. In which of the following processes, convection does not take place primarily ?

(a) sea and land breeze

(b) boiling of water

(c) warming of glass of the bulb due to filament

(d) heating air around a furnace [IIT 05]

13. A wall has two layers A and B, each made of a different material. Both the layers have the same thickness. The thermal conductivity of the material of A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36°C. The temperature difference across the layer A

(a) 6°C (b) 12°C

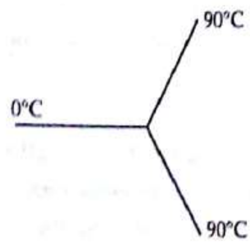
(c) 18°C (d) 24°C [IIT 80]

14. Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC, right-angled at B. The points A and B are maintained at temperatures T and $(\sqrt{2})T$ respectively. In the steady state, the temperature of the point C is T_C. Assuming that only heat conduction takes place, T_C / T is

(a) $\frac{1}{2(\sqrt{2}-1)}$ (b) $\frac{3}{\sqrt{2}+1}$

(c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{2}+1}$ [IIT 95]

15. Three rods made of same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be



(a) 45°C (b) 60°C

(c) 30°C (d) 20°C [IIT 01]

16. Two identical rods are connected between two containers. One of them is at 100°C and another is at 0°C. If rods are connected in parallel then the rate of melting of ice is q_1 g/sec. If they are connected in series then the rate is q_2. The ratio q_2 / q_1 is

(a) 2 (b) 4

(c) 1/2 (d) 1/4.

17. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{1}$ (d) $\left(\frac{1}{3}\right)^{1/3}$

18. A spherical body of area A and emissivity $e = 0.6$ is kept inside a perfectly black body. Energy radiated per second by the body at temperature T is

(a) $0.4\ \sigma AT^4$ (b) $0.8\ \sigma AT^4$

(c) $0.6\ \sigma AT^4$ (d) $1.0\ \sigma AT^4$

19. An ideal black body at room temperature is thrown into a furnace. It is observed that

(a) initially, it is the darkest body and at later times the brightest

(b) it is the darkest body at all times

(c) it cannot be distinguished at all times

(d) initially, it is the darkest body and at later times it cannot be distinguished. [IIT 02]

20. A spherical black body with a radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(a) 225 (b) 450

(c) 900 (d) 1800 [IIT 97]

21. The earth receives at its surface radiation from the sun at the rate of 1,400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7.0×10^8 m. Treating sun as a black body, it follows from the above data that its surface temperature is

(a) 5,801 K (b) 10^6 K

(c) 50.1 K (d) 5,801°C [IIT 89]

22. Two spheres of same material have radii 1 m and 4 m and temperatures 4,000 K and 2,000 K respectively. The energy radiated per second by the first sphere is

(a) greater than that by the second

(b) less that that by the second

(c) equal in both cases

(d) the information is incomplete to draw any conclusion. [IIT 88]

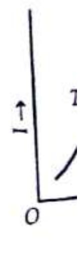
COMPETITION SECTION

23. Variation of ra
filament of tungsten lar

of its wavelength is
following options is
(a) Sun – T_3, tungs
(b) Sun – T_2, tungs
(c) Sun – T_3, tungs
(d) Sun – T_1, tung

24. The intensity
its maximum value
emitted by the Nor
350 nm. If these star
ratio of the surface
North Star is
(a) 1.46
(c) 1.21

25. The plots
three black bodies
tively are as show

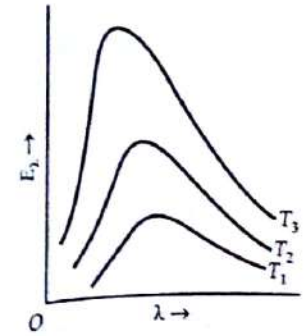


Their tempera
(a) $T_1 > T_2 > T_3$
(c) $T_2 > T_3 > T_1$

26. A sphere,
made of the sam
are initially heat
of these will coo
(a) sphere
(c) plate

Scanned by CamScanner

23. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function



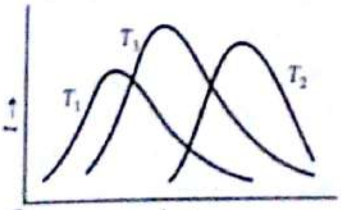
of its wavelength is shown in figure. Which of the following options is the correct match ?

(a) Sun – T_3, tungsten filament – T_1, welding arc – T_2
(b) Sun – T_2, tungsten filament – T_1, welding arc – T_3
(c) Sun – T_3, tungsten filament – T_2, welding arc – T_1
(d) Sun – T_1, tungsten filament – T_2, welding arc – T_3 [IIT 05]

24. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperatures of the sun and the North Star is

(a) 1.46 (b) 0.69
(c) 1.21 (d) 0.83 [IIT 97]

25. The plots of intensity versus wavelength for three black bodies at temperature T_1, T_2 and T_3 respectively are as shown.



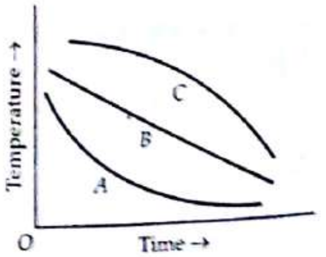
Their temperatures are such that

(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$
(c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$ [IIT 2K]

26. A sphere, a cube and a thin circular plate, all made of the same material and having the same mass, are initially heated to a temperature of 3,000°C. Which of these will cool fastest ?

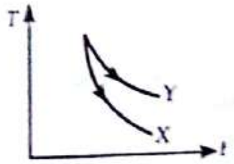
(a) sphere (b) cube
(c) plate (d) none of these. [IIT 82]

27. A block of steel heated to 100°C is left in a room to cool. Which of the curves shown in the figure, represents the correct behaviour ?



(a) A (b) B
(c) C (d) none. [IIT 80]

28. The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies X and Y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies.



(a) $e_X > e_Y$ and $a_X < a_Y$
(b) $e_X < e_Y$ and $a_X > a_Y$
(c) $e_X > e_Y$ and $a_X > a_Y$
(d) $e_X < e_Y$ and $a_X < a_Y$. [IIT 03]

29. Three discs A, B and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The powers radiated by them are Q_A, Q_B and Q_C respectively.

(a) Q_A is maximum (b) Q_B is maximum
(c) Q_C is maximum (d) $Q_A = Q_B = Q_C$ [IIT 04]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

30. A bimetallic strip is formed out of two identical strips, one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then, R is

(a) proportional to ΔT
(b) inversely proportional to ΔT
(c) proportional to $|\alpha_B - \alpha_C|$
(d) inversely proportional to $|\alpha_B - \alpha_C|$ [IIT 99]

31. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid

Scanned by CamScanner

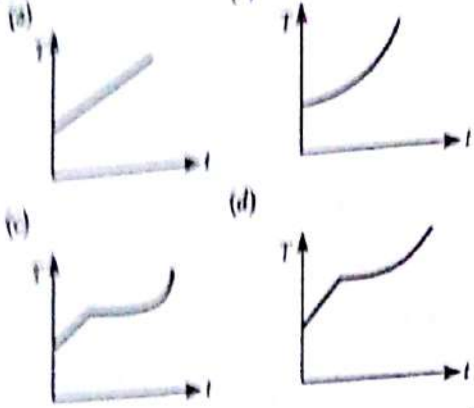
...walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2:3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to

(a) 2 : 3 (b) 1 : 1
(c) 3 : 2 (d) 4 : 9 [IIT 89]

32. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of steam condensed (in kg) is

(a) 0.130 (b) 0.065
(c) 0.260 (d) 0.195. [IIT 86]

33. If liquefied oxygen at 1 atmospheric pressure is heated from 50 K to 300 K by supplying heat at constant rate, the graph of temperature vs time will be



[IIT 04]

34. A cylinder of radius R, made of a material of thermal conductivity K_1 is surrounded by a cylindrical sheet of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $K_1 + K_2$ (b) $\dfrac{K_1 + 3K_2}{4}$

(c) $\dfrac{K_1 K_2}{K_1 + K_2}$ (d) $\dfrac{3K_1 + K_2}{4}$ [IIT 88]

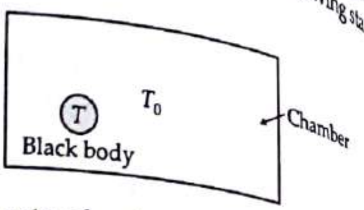
35. A black body is at a temperature of 2,880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1,499 nm and 1,500 nm is U_3. The Wien's constant, $b = 2.88 \times 10^6$ nm K. Then,

(a) $U_1 = 0$ (b) $U_3 = 0$
(c) $U_1 > U_2$ (d) $U_2 > U_1$

36. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K, then

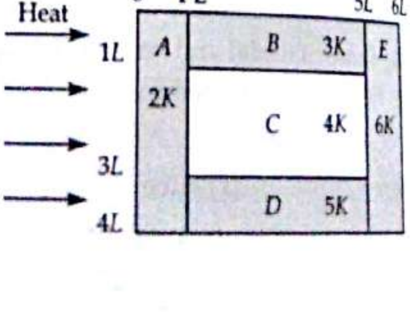
(a) the temperature of B is 1934 K
(b) the temperature of B is 11604 K
(c) the temperature of B is 2901 K
(d) $\lambda_B = 1.5$ μm.

37. Initially a black body at absolute temperature T is kept inside a closed chamber at absolute temperature T_0. Now the chamber is slightly opened to allow sun rays to enter. It is observed that temperatures T and T_0 remain constant. Which of the following statements is/are true ?



(a) the rate of emission of energy from the black body remains the same
(b) the rate of emission of energy from the black body increases
(c) the rate of absorption of energy by the black body increases
(d) the energy radiated by the black body equals the energy absorbed by it.

38. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat Q flows only from left to right through the blocks.



SECTION

Then in steady state
(a) heat flow through
(b) heat flow through
(c) temperature diffe
(d) heat flow throug
heat flow throug

INTEGER ANSW

39. Steel wire of leng
the ceiling and then a m
The wire is cooled dowr
original length L. The

1. (d) A constant v
Gay Lussac's law,

$P \propto T$

2. (d) Let R_0 = Resi

$1\,\Omega = R_0($

$2\,\Omega = R_0($

$\frac{2}{1} = \frac{1+}{1+}$

∴ $\theta = 854$

3. (c) Refer to
page 11.54

4. (a) $W_{app} =$

Upthrust, $F =$

At higher tempe

$F' = V$

$\frac{F'}{F} = \frac{V}{V}$

As $\gamma_S < \gamma$

or $W_2 > W$

5. (a) Refer to the

6. (a) One calor
raise the temperat
15.5°C at 760 mm

7. (a) Compare
due to steam at 1
additional heat (eq

8. (a) Heat gai

Scanned by CamScanner

then in steady state
(a) heat flow through A and E slabs are same
(b) heat flow through slab E is maximum
(c) temperature difference across slab E is smallest
(d) heat flow through C = heat flow through B + heat flow through D. [IIT 2011]

✗ Integer Answer Type

39. Steel wire of length L at 40°C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length L. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m² and radius of the wire is 1 mm. Assume that L >> diameter of the wire. What is the value of m in kg nearly? [IIT 2011]

40. A piece of ice (heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at −5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally, when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, find the value of m. [IIT 2010]

Answers and Explanations

1. (d) A constant volume thermometer is based on Gay Lussac's law.

$$p \propto T \qquad \text{(At constant V)}$$

2. (d) Let R_0 = Resistance at 0°C, then

$$1\,\Omega = R_0(1 + 27\alpha)$$

$$2\,\Omega = R_0(1 + \theta\alpha)$$

$$\frac{2}{1} = \frac{1+\theta\alpha}{1+27\alpha} = \frac{1+0.00125\theta}{1+0.00125\times 27}$$

$$\theta = 854°\text{C} = \mathbf{1127\ K}$$

3. (c) Refer to the solution of Problem 2 on page 11.54.

4. (a) $\quad W_{app} = W_{actual} - \text{Upthrust}$

Upthrust, $F = V_S \rho_L g$

At higher temperature,

$$F' = V_S' \rho_L' g$$

$$\frac{F'}{F} = \frac{V_S'}{V_S} \cdot \frac{\rho_L'}{\rho_L} = \frac{1+\gamma_S \Delta\theta}{1+\gamma_L \Delta\theta}$$

As $\quad \gamma_S < \gamma_L \quad \therefore \quad F' < F$

or $\quad W_2 > W_1 \quad$ or $\quad W_1 < W_2$

5. (a) Refer to the solution of Problem 4 on page 11.55.

6. (a) One calorie is defined as the heat required to raise the temperature of 1 g of water from 14.5°C to 15.5°C at 760 mm of Hg.

7. (a) Compared to burn due to air at 100°C, a burn due to steam at 100°C is more dangerous due to the additional heat (equal to latent heat) possessed by steam.

8. (a) Heat gained by ice

= Heat lost by water at 80°C

$$540 \times 80 + 540 \times 1 \times \theta = 540 \times 1 \times (80 - \theta)$$

or $\quad \theta = 0°\text{C}$

9. (a) The graph must show two regions of rising temperatures (−10°C to 0°C and 0°C to 100°C) and two regions of constant temperature (for change of state from ice to water and then from water to steam) alternately.

10. (b) Refer to the solution of Problem 1 on page 11.54.

11. (a) Energy gained by water per second

= Energy supplied − Energy lost

= 1000 J − 160 J = 840 J

Total heat required to raise the temperature of water from 27°C to 77°C

$$= mc\Delta T$$

Required time,

$$t = \frac{mc\Delta T}{\text{Energy gained per second}}$$

$$= \frac{2 \times 4.2 \times 10^3 \times (77 - 27)}{840}$$

$$= 500\ \text{s} = \mathbf{8\ min\ 20\ s}.$$

12. (c) Warming is due to conduction and radiation.

13. (b) Let θ be the temperature of the interface.

$$\left(\frac{\Delta Q}{\Delta t}\right)_A = \left(\frac{\Delta Q}{\Delta t}\right)_B$$

$$K_1 A \cdot \frac{\Delta T_1}{\Delta x} = K_2 A \frac{\Delta T_2}{\Delta x}$$

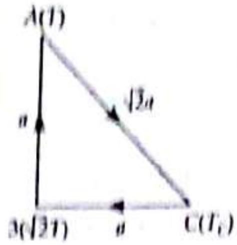
$$2K_2 \cdot \frac{(36-\theta)}{\Delta x} = K_2 \frac{(\theta - 0)}{\Delta x}$$

$$72 - 2\theta = \theta \quad \text{or} \quad \theta = \mathbf{24°C}$$

Temp. difference across layer A = 36 − 24 = **12°C**.

Scanned by CamScanner

14. (*d*) B is at higher temperature than A. For steady state, heat flows from B to A, A to C and then C to B



$$\left(\frac{\Delta Q}{\Delta t}\right)_{AC} = KA\frac{T - T_C}{\sqrt{2}a}$$

$$\left(\frac{\Delta Q}{\Delta t}\right)_{CB} = KA\frac{T_C - \sqrt{2}T}{a}$$

Equating two rates for steady state,

$$KA\frac{T - T_C}{\sqrt{2}a} = KA\frac{T_C - \sqrt{2}T}{a}$$

$$T - T_C = \sqrt{2}\,T_C - 2T$$

$$3T = (\sqrt{2}+1)\,T_C$$

or $$\frac{T_C}{T} = \frac{3}{\sqrt{2}+1}.$$

15. (*b*) Refer to the solution of Problem 3 page 11.54.

16. (*d*) In parallel,

$$K_p = K_1 + K_2 = K + K = 2K$$

In series,

$$K_s = \frac{K_1K_2}{K_1+K_2} = \frac{K\cdot K}{K+K} = \frac{K}{2}$$

For other factors to be same,

$$\frac{q_2}{q_1} = \frac{K_s}{K_p} = \frac{K/2}{2K} = \frac{1}{4}.$$

17. (*d*) Rate of loss of heat by an object,

$$\frac{\Delta Q}{\Delta T} = e\sigma AT^4$$

But $$\Delta Q = mc\Delta T$$

$\therefore$ $$\frac{\Delta T}{\Delta t} = \frac{e\sigma AT^4}{mC}$$

For a sphere,

$$m = \frac{4}{3}\pi r^3\rho$$

$\therefore$ $$A = \pi r^2 = \pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}$$

Hence $$\frac{\Delta T}{\Delta t} = \frac{e\sigma T^4}{mc}\left[\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right] = k\left(\frac{1}{m}\right)^{1/3}$$

Ratio of the rates of cooling of S_1 and S_2,

$$\frac{\left(\frac{\Delta T}{\Delta t}\right)_1}{\left(\frac{\Delta T}{\Delta t}\right)_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}.$$

18. (*c*) According to Stefan's law, energy radiated per second by a body of emissivity e at a temperature T,

$$E = e\sigma AT^4$$
$$= 0.6\sigma AT^4$$

19. (*a*) Initially at lower temperature, it absorbs all the radiations incident upon it. So, it is the darkest body.

At later times, when it attains the temperature of the furnace, the black body radiates maximum energy. It appears brightest of all bodies.

20. (*d*) For a spherical black body at temperature T, the power radiated is

$$P = \sigma(4\pi r^2)T^4$$

$\therefore$ $$\frac{P_2}{P_1} = \left(\frac{r_2}{r_1}\right)^2\left(\frac{T_2}{T_1}\right)^4 = \left(\frac{6}{12}\right)^2\left(\frac{2}{1}\right)^4 = 4.$$

$$P_2 = 4P_1 = 4\times 450 = \mathbf{1800\ W}.$$

21. (*a*) Energy radiated by the sun per second,

$$E = \sigma T^4 \times 4\pi R^2$$

This energy spreads over a sphere of radius r, where r is the distance of earth from the sun.

$\therefore$ E = Energy per second per unit area incident on earth $\times 4\pi r^2$

Hence $$\sigma T^4 \times 4\pi R^2 = 1400\times 4\pi r^2$$

$$T^4 = \frac{1400\times r^2}{\sigma R^2} = \frac{1400\times(1.5\times10^{11})^2}{5.67\times10^{-8}\times(7.0\times10^{8})^2}$$

$$T = \left(\frac{14\times 2.25}{5.67\times 49}\right)^{1/4}\times 10^4 = \mathbf{5802.7\ K}.$$

22. (*c*) $$\frac{E_1}{E_2} = \left(\frac{r_1}{r_2}\right)^2\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{1}{4}\right)^2\left(\frac{4000}{2000}\right)^4 = 1$$

$\therefore$ $$E_1 = E_2$$

23. (*a*) In accordance with Wien's displacement law,

$$\lambda^3_{max} < \lambda^2_{max} < \lambda^1_{max}$$

$\therefore$ $$T_3 > T_2 > T_1$$

Hence the correct match is option (*a*) :

Sun – T_3, tungsten filament – T_1, welding arc – T_2.

24. (*b*) By Wien's law,

$$\lambda_m^S T^S = \lambda_m^N T^N$$

$\therefore$ $$\frac{T^S}{T^N} = \frac{\lambda_m^N}{\lambda_m^S} = \frac{350}{510} = \mathbf{0.69}.$$

25. (*b*) Clearly, $\lambda_m^1 < \lambda_m^3 < \lambda_m^2$

$\therefore$ By Wien's law, $T_1 > T_3 > T_2$

26. (c) For the same mass and hence same volume, the plate has a maximum surface area. So, the plate will cool fastest.

27. (a) According to Newton's law of cooling, rate of cooling is proportional to the temperature difference between the body and its surroundings. Initially, the black body cools fast and then slowly when its temperature approaches room temperature.

28. (c) Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ Emissivity (*e*)

From the given graphs, $\left(-\frac{dT}{dt}\right)_X > \left(-\frac{dT}{dt}\right)_Y$

$\therefore \quad e_X > e_Y$

But emissivity (*e*) $\propto$ absorptive power (*a*)

$\therefore \quad a_X > a_Y$

29. (b) Stefan's law, $\quad Q \propto AT^4$

Wien's law, $\quad \lambda_m T = \text{constant}$

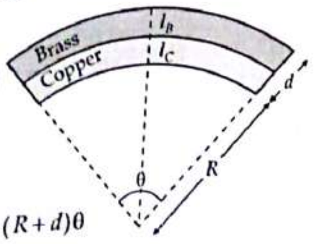
$$\therefore \quad Q \propto \frac{A}{(\lambda_m)^4} \propto \frac{r^2}{\lambda_m^4}$$

$$Q_A : Q_B : Q_C = \frac{2^2}{3^4} : \frac{4^2}{4^4} : \frac{6^2}{5^4} = \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

$$= 0.05 : 0.0625 : 0.0576$$

Hence Q_B is maximum.

30. (b), (d)



Let l_0 be the initial length of each strip.

After heating, the lengths will be

$$l_B = l_0(1 + \alpha_B \Delta T) = (R + d)\theta$$

$$l_C = l_0(1 + \alpha_C \Delta T) = R\theta$$

$$\therefore \quad \frac{R+d}{R} = (1 + \alpha_B \Delta T)(1 + \alpha_C \Delta T)^{-1}$$

$$= (1 + \alpha_B \Delta T)(1 - \alpha_C \Delta T)$$

$$1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_C)\Delta T$$

[Using binomial expansion and neglecting higher power terms]

$$R = \frac{d}{(\alpha_B - \alpha_C)\Delta T}$$

$$\therefore \quad R \propto \frac{1}{\Delta T} \quad \text{and} \quad R \propto \frac{1}{|\alpha_B - \alpha_C|}$$

Hence options (*b*) and (*d*) are correct.

31. (c) Coefficient of linear expansion,

$$\alpha = \frac{\Delta l}{l \Delta T}$$

$$\therefore \quad \frac{\Delta l}{l} = \alpha \Delta T$$

Young's modulus,

$$Y = \frac{\text{Stress}}{\Delta l / l}$$

Thermal stress,

$$= Y \frac{\Delta l}{l} = Y\alpha\Delta T$$

Thermal stress in first rod
= Thermal stress in second rod

$$Y_1 \alpha_1 \Delta T = Y_2 \alpha_2 \Delta T$$

$$\therefore \quad \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

$$= \mathbf{3 : 2}.$$

32. (a) Heat lost by steam
= Heat gained by water + calorimeter

$$mL + mc(100 - 80) = (1.1 + 0.02)c(80 - 15)$$

$$m[540 + 1 \times 20] = 1.12 \times 1 \times 65$$

$$m = \frac{1.12 \times 65}{560} = \mathbf{0.13\ kg}.$$

33. (c) Temperature of liquid oxygen first increases in same phase. Then liquid oxygen changes into gaseous phase, during which temperature remains constant. On further heating, the temperature of gaseous oxygen begins to increase.

34. (b) Rate of flow of heat in the combined system

= Rate of flow of heat through cross-ection of inner cylinder + Rate of flow of heat through cross-section of outer shell

$$\text{or} \quad \frac{KA(\theta_1 - \theta_2)}{l} = \frac{K_1 A_1(\theta_1 - \theta_2)}{l} + \frac{K_2 A_2(\theta_1 - \theta_2)}{l}$$

$$\text{or} \quad KA = K_1 A_1 + K_2 A_2$$

$$\text{or} \quad K\pi(2R)^2 = K_1(\pi R^2) + K_2\pi[(2R)^2 - R^2]$$

$$\text{or} \quad \pi R^2 (K \times 4) = \pi R^2 [K_1 + 3K_2]$$

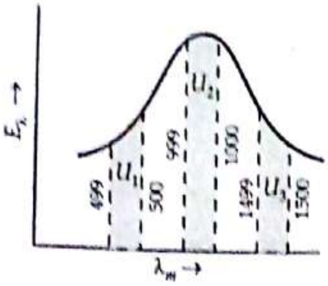
$$\text{or} \quad K = \frac{\mathbf{K_1 + 3K_2}}{\mathbf{4}}.$$

Scanned by CamScanner

35. (*d*) According to Wien's displacement law,

$$\lambda_m T = b$$

$$\therefore \quad \lambda_m = \frac{b}{T} = \frac{2.88\times10^6\,\text{nm K}}{2880\,\text{K}} = 1000\ \text{nm}.$$



Energy radiation is maximum at $\lambda_m = 1000$ nm.

$$\therefore \quad U_2 > U_1 \ \text{ and } \ U_2 > U_3$$

Also $\quad U_1 \neq 0 \ \text{ and } \ U_3 \neq 0.$

Hence option (*d*) is correct.

36. (*d*) As the powers radiated and surface areas are same for both *A* and *B*

$$e_A \sigma T_A^4 A = e_B \sigma T_B^4 A$$

$$\frac{T_A}{T_B} = \left(\frac{e_B}{e_A}\right)^{1/4} = \left(\frac{0.81}{0.01}\right)^{1/4} = 3$$

$$T_B = \frac{T_A}{3} = \frac{5802}{3} = 1934\ \text{K}.$$

According to Wien's law,

$$\lambda_A T_A = \lambda_B T_B$$

$$\lambda_A = \frac{T_B}{T_A}\lambda_B = \frac{1}{3}\lambda_B$$

But $\lambda_B - \lambda_A = 1\ \mu\text{m}$

or $\quad \lambda_B - \dfrac{\lambda_B}{3} = 1\ \mu\text{m}$

$\therefore \quad \lambda_B = $ **1.5 μm**.

37. (*a*), (*d*) From Stefan's law, rate of emission of energy $\propto (T - T_0)^4$.

As T and T_0 remain constant, the rate of emission of energy remains constant.

Hence option (*a*) is correct.

As the temperature of the black body remains constant, the energy radiated by it equals the energy absorbed by it.

Hence option (*d*) is correct.

38. (*a*), (*b*), (*c*), (*d*)

As heat flows from left to right, so $Q_A = Q_E$

Heat through slab *E* is maximum and equal to that through slab *A*. Heat flows through different slabs in time *t* are

$$Q_A = 2\,KA\frac{\Delta T_1}{L}t = 2\frac{KAt}{L}\Delta T_1,$$

$$Q_B = 3\,K\frac{A}{4}\frac{\Delta T_2}{4L}t = \frac{3}{16}\frac{KAt}{L}\Delta T_2,$$

$$Q_C = 4\,K\frac{A}{2}\frac{\Delta T_2}{4L}t = \frac{1}{2}\frac{KAt}{L}\Delta T_2,$$

$$Q_D = 5\,K\frac{A}{4}\frac{\Delta T_2}{4L}t = \frac{5}{16}\frac{KAt}{L}\Delta T_2,$$

$$Q_E = 6\,KA\frac{\Delta T_3}{L}t = 6\frac{KAt}{L}\Delta T_3$$

Clearly, $\quad Q_C = Q_B + Q_D$

Also, $Q_E = Q_A \Rightarrow \Delta T_3 < \Delta T_1$ *i.e.*, temperature difference across slab *E* is smallest.

39.

0	0	0	3

We know that

$$\Delta l = l\alpha\Delta T$$

$$\therefore \quad Y = \frac{mg/A}{\Delta l/l} = \frac{mgl}{A\Delta l} = \frac{mgl}{Al\alpha\Delta T}$$

or

$$m = \frac{YA\alpha\Delta T}{g}$$

$$= \frac{10^{11}\times\pi(10^{-3})^2\times10^{-5}\times10}{10}$$

$$= \pi \approx 3.$$

40.

0	0	0	8

Heat required for melting of 1 gram of ice,

$$Q = mL = \frac{1}{1000}\times3.36\times10^5 = 336\ \text{J}$$

Heat used for raising temperature of ice from −5°C to 0 °C

$$\Delta Q = 420 - 336 = 84\ \text{J}$$

But $\quad \Delta Q = mc\Delta T$

$\therefore \quad 84 = m\times2100\times5$

or $\quad m = 0.008\ \text{kg} = $ **8 g**.

Scanned by CamScanner

COMPETITION

$Q_A = Q_E$

d equal to that
ferent slabs in

emperature

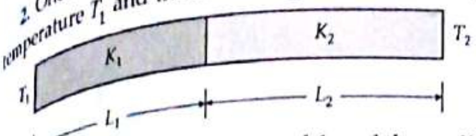
n −5°C

AIEEE

1. Heat given to a body, which raises its temperature by 1°C is

(*a*) water equivalent (*b*) temperature gradient
(*c*) thermal capacity (*d*) specific heat. [AIEEE 02]

2. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths L_1 and L_2 and the coefficients of thermal conductivity K_1 and K_2 respectively.

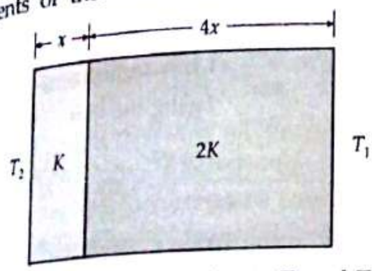


The temperature at the interface of the two sections is

(*a*) $\dfrac{K_2L_2T_1 + K_1L_1T_2}{K_1L_1 + K_2L_2}$ (*b*) $\dfrac{K_2L_1T_1 + K_1L_2T_2}{K_2L_1 + K_1L_2}$

(*c*) $\dfrac{K_1L_2T_1 + K_2L_1T_2}{K_1L_2 + K_2L_1}$ (*d*) $\dfrac{K_1L_1T_1 + K_2L_2T_2}{K_1L_1 + K_2L_2}$ [AIEEE 07]

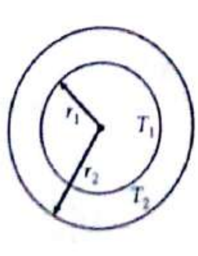
3. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and



thickness x and $4x$, respectively are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab in a steady state is $\left(\dfrac{A(T_2 - T_1)K}{x}\right)f$, where f equals to

(*a*) 1 (*b*) 1/2
(*c*) 2/3 (*d*) 1/3 [AIEEE 04]

4. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



(*a*) $(r_2 - r_1)/r_1r_2$ (*b*) $\log_e(r_2/r_1)$
(*c*) $r_1r_2/(r_2 - r_1)$ (*d*) $(r_2 - r_1)$ [AIEEE 05]

5. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings and n is equal to

(*a*) two (*b*) three
(*c*) four (*d*) one [AIEEE 03]

6. A pressure cooker reduces cooking time for food, because

(*a*) heat is more evenly distributed in the cooking space
(*b*) the higher pressure inside the cooker crushes the food material
(*c*) cooking involves chemical changes helped by a rise in temperature
(*d*) boiling point of water involved in cooking is increased. [AIEEE 03]

7. Which of the following is more close to a black body ?

(*a*) Blackboard paint (*b*) Green leaves
(*c*) Black holes (*d*) Red roses [AIEEE 02]

8. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously, will be

(*a*) 4 (*b*) 16
(*c*) 32 (*d*) 64 [AIEEE 04]

9. Two spheres of the same material have radii 1 m and 4 m and temperatures 4,000 K and 2,000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is

(*a*) 1 : 1 (*b*) 1 : 9
(*c*) 4 : 1 (*d*) 16 : 1 [AIEEE 02]

10. Assuming the sun to be a spherical body of radius R at a temperature of TK, evaluate the total radiant power, incident on earth, at a distance r from the sun.

(*a*) $\dfrac{r_0^2R^2\sigma T^4}{4\pi r^2}$ (*b*) $\dfrac{4\pi r_0^2R^2\sigma T^4}{r^2}$

(*c*) $\dfrac{\pi r_0^2R^2\sigma T^4}{r^2}$ (*d*) $\dfrac{R^2\sigma T^4}{r^2}$

Here, r_0 is the radius of the earth and σ is Stefan's constant. [AIEEE 06]

Scanned by CamScanner

11. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by

(*a*) Planck's law of radiation

(*b*) Stefan's law of radiation

(*c*) Rayleigh Jeans law

(*d*) Wien's law. [AIEEE 03]

12. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is

(*a*) $\frac{E}{c}$ (*b*) $\frac{2E}{c}$

(*c*) Ec (*d*) $\frac{E}{c^2}$ [AIEEE 04]

Answers and Explanations

1. (*c*) Heat required to raise the temperature of a body through 1°C is called its thermal capacity.

2. (*c*) Let T be the temperature of the interface. In the steady state, $Q_1 = Q_2$

or $$\frac{K_1A(T_1-T)}{l_1} = \frac{K_2A(T-T_2)}{l_2}$$

or $$K_1l_2(T_1-T) = K_2l_1(T-T_2)$$

ro $$T = \frac{K_1L_2T_1 + K_2L_1T_2}{K_1L_2 + K_2L_1}$$

3. (*d*) Refer to the solution of Problem 7 on page 11.56.

4. (*c*) Refer to the solution of Problem 8 on page 11.56.

5. (*d*) According to Newton's law of cooling,

Rate of cooling ∝ Temp. difference between body and its surroundings.

∴ $$n = 1.$$

6. (*d*) As the pressure inside the pressure cooker increases, the boiling point of water increases.

7. (*a*) Blackboard paint is close to a black body. A good absorber is a good emitter but a black hole does not emit all radiations.

8. (*d*) Energy radiated by the sun per second,

$$E = \sigma AT^4 = \sigma \times 4\pi R^2 \times T^4$$

When its radius and temperature change to $2R$ and $2T$ respectively,

$$E' = \sigma \times 4\pi(2R)^2 \times (2T)^4$$

∴ $$\frac{E'}{E} = 64.$$

9. (*a*) Using stefan's law,

$$\frac{E_1}{E_2} = \frac{\sigma \times 4\pi R_1^2 T_1^4}{\sigma \times 4\pi R_2^2 T_2^4} = \frac{R_1^2T_1^4}{R_2^2T_2^4} = \left(\frac{1}{4}\right)^2\left(\frac{4000}{2000}\right)^4 = 1$$

10. (*c*) From Stefan's law, radiant power of the sun,

$$E = \sigma AT^4 = \sigma \times 4\pi R^2 \times T^4$$

This radiant power spreads over a sphere of radius r. Hence radiant power incident on the earth.

$$E' = \frac{E}{4\pi r^2} \times \text{Cross-section area of the earth facing the sun}$$

$$= \frac{\sigma \times 4\pi R^2 \times T^4}{4\pi r^2} \times \pi r_0^2 = \frac{\pi r_0^2 R^2 \sigma T^4}{r^2}.$$

11. (*d*) By using Wien's law, the wavelength radiated by the earth can be determined.

12. (*b*) If the wave is totally reflected, the momentum transferred is $2E/c$.

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. On a hilly region, water boils at 95°C. The temperature expressed in Fahrenheit is

(*a*) 100°F (*b*) 203°F

(*c*) 150°F (*d*) 203°F [DCE 07]

2. If boiling point of water is 95°F, what will be reading at Celsius scale ?

(*a*) 7°C (*b*) 65°C

(*c*) 63°C (*d*) 35°C [DCE 02]

3. 50 g of ice at 0°C is mixed with 50 g of water at 80°C, final temperature of mixture of will be

(*a*) 0°C (*b*) 40°C

(*c*) 60°C (*d*) 4°C [DCE 02]

4. Which one of the following processes depends on gravity ?

(*a*) conduction (*b*) convection

(*c*) radiation (*d*) none of these. [DCE 2K]

Scanned by CamScanner

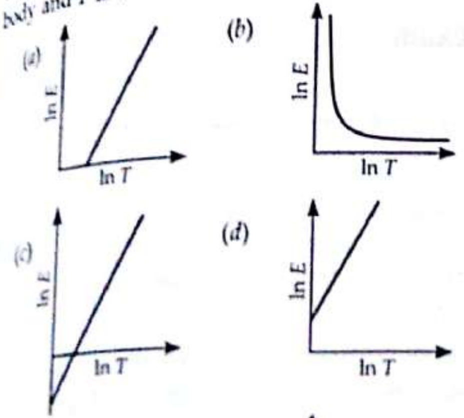
5. A composite rod made of copper ($\alpha = 1.8 \times 10^{-5} K^{-1}$) and steel ($\alpha = 1.2 \times 10^{-5} K^{-1}$) is heated. Then
(a) it bends with steel on concave side
(b) it bends with copper on concave side
(c) it does not expand
(d) data is insufficient. [DCE 05]

6. Heat current is maximum in which of the following (rods are of identical dimension) ?
(a) | Cu | (b) | Steel | Cu |
(c) | Cu | Steel | (d) | Steel | [IPUEE 05]

7. The sprinkling of water reduces slightly the temperature of a closed room because
(a) temperature of water is less than that of the room
(b) specific heat of water is high
(c) water has large latent heat of vaporisation
(d) water is a bad conductor of heat [IPUEE 07]

8. Two rods of lengths d_1 and d_2 and coefficients of thermal conductivities K_1 and K_2 are kept in contact with each other end to end. The equivalent thermal conductivity is
(a) $K_1d_1 + K_2d_2$ (b) $K_1 + K_2$
(c) $\frac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (d) $\frac{d_1 + d_2}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$ [DCE 08]

9. Which of the following graphs correctly represents the relation between E and T, where E is the amount of radiation emitted per unit time from unit area of body and T is the absolute temperature ? [DCE 02]



10. A black body at a temperature of 227°C radiates heat at the rate of 20 cal $m^{-2}s^{-1}$. When its temperature rises to 727°C, the rate of heat radiated will be
(a) 40 cal $m^{-2}s^{-1}$ (b) 160 cal $m^{-2}s^{-1}$
(c) 320 cal $m^{-2}s^{-1}$ (d) 640 cal $m^{-2}s^{-1}$ [DCE 03]

11. Temperature of a black body increases from 327°C to 927°C, the initial energy possessed is 2 kJ. What is its final energy ?
(a) 32 kJ (b) 320 kJ
(c) 1200 kJ (d) none of these. [DCE 01]

12. In determining the temperature of a distant star, one makes use of
(a) Kirchhoff's law (b) Stefan's law
(c) Wien's displacement law
(d) none of the above. [DCE 2K, 03]

13. The wavelength of radiation emitted by a body depends upon
(a) the nature of the surface
(b) the area of the surface
(c) the temperature of the surface
(d) all of the above factors. [DCE 06]

14. Temperatures of two stars are in ratio 3 : 2. If wavelength of maximum intensity of first body is 4000 Å, what is corresponding wavelength of second body ?
(a) 9000 Å (b) 6000 Å
(c) 2000 Å (d) 8000 Å [DCE 07]

15. A piece of blue glass heated to a high temperature and a piece of red glass at room temperature, are taken inside a dimly lit room, then
(a) the blue piece will look blue and red will look as usual
(b) red looks brighter red and blue looks ordinary blue
(c) blue shines like brighter red compared to the red piece
(d) both the pieces will look equally red. [IPUEE 06]

16. The unit of Stefan's constant is
(a) $Wm^{-2}K^{-1}$ (b) WmK^{-4}
(c) $Wm^{-2}K^{-4}$ (d) $Nm^{-2}K^{-4}$ [IPUEE 07]

Answers and Explanations

1. (d) $\frac{F-32}{9} = \frac{C}{5} = \frac{95}{5}$

$\therefore \quad F = 171 + 32 = 203°F.$

2. (d) $C = \frac{5}{9}(F-32)$

$= \frac{5}{9}(95-32) = 35°C$

Scanned by CamScanner

3. (*a*) Heat required to melt 50 g ice

$$= mL = 50 \times 80 = 4000 \text{ cal}$$

Heat given out by water in cooling from 80°C to 0°C

$$= mc\Delta T = 50 \times 1 \times 80 = 4000 \text{ cal}$$

Heat given by water is just sufficient to melt the whole ice. So the final temperature is 0°C.

4. (*b*) Convection occurs due to the presence of gravity.

5. (*a*) $\alpha_{copper} > \alpha_{steel}$

Copper expands more than steel. So rod bends with copper on convex side and steel on concave side.

6. (*a*) Heat current, $\frac{dQ}{dt} \propto K$

As the thermal conductivity of copper is maximum, heat current will maximum in option (*a*).

7. (*c*) Water has a large latent heat of vaporisation. When it is sprinked over a large area, its evaporation occurs which, in turn, causes cooling.

8. (*d*) Same amount of heat flows through the two rods in series combination.

$$Q = \frac{A(T_1 - T_2)t}{\frac{d_1}{K_1} + \frac{d_2}{K_1}} = \frac{A(T_1 - T_2)t}{\frac{d_1 + d_2}{K}}$$

$$\therefore \quad \frac{d_1 + d_2}{K} = \frac{d_1}{K_1} + \frac{d_2}{K_2}$$

or
$$K = \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}.$$

9. (*c*) According to Stefan's law,

$$E = \sigma T^4$$

$$\therefore \quad \ln E = 4 \ln T + \ln \sigma$$

This shows that graph between ln E and ln T must be straight line with $m = 4$. Also, value of σ $(= 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4})$ is quite small, y-intercept $(= \ln \sigma)$ must be on negative side. Hence the correct option is (*c*).

10. (*c*) $\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{273 + 727}{273 + 227}\right)^4 = (2)^4 = 16.$

$$E_2 = 16 E_1 = 16 \times 20 = 320 \text{ cal m}^{-2} \text{ s}^{-1}$$

11. (*a*) $\frac{E_2}{E_1} = \left(\frac{273 + 927}{273 + 327}\right)^4 = (2)^4 = 16.$

$$E_2 = 16 E_1 = 16 \times 2 = 32 \text{ kJ}.$$

12. (*c*) Wien's displacement law can be used to determine the temperature of distant star.

13. (*c*) According to Wien's displacement law,

$$\lambda_m \propto \frac{1}{T}$$

14. (*b*) $\frac{\lambda'_m}{\lambda_m} = \frac{T}{T'} = \frac{3}{2}$

$$\lambda'_m = \frac{3}{2}\lambda_m = \frac{3}{2} \times 4000 = 6000 \text{ Å}.$$

15. (*c*) According to Stefan's law, $E \propto T^4$

As the temperature of blue glass is more than that of red glass, so it will appear brighter than red glass.

16. (*c*) S.I. unit of Stefan's constant $= \text{Wm}^{-2}\text{K}^{-4}$.

AIIMS Entrance Exam

1. We plot a graph, having temperature in °C on x-axis and in °F on y-axis. If the graph is straight line, then it

(*a*) passes through origin

(*b*) intercepts the positive x-axis

(*c*) intercepts the positive y-axis

(*d*) intercepts the negative axis of both x- and y-axis [AIIMS 97]

2. At a common temperature, a block of wood and a block of metal feel equally cold or hot. The temperatures of block and wood are

(*a*) equal to the temperature of the body

(*b*) less than the temperature of the body

(*c*) greater than temperature of the body

(*d*) either (*b*) or (*c*) [AIIMS 99]

3. A quantity of heat required to change the unit mass of a solid substance, from solid state to liquid state, while the temperature remains constant, is known as

(*a*) latent heat (*b*) sublimation

(*c*) hoar frost (*d*) latent heat of fusion [AIIMS 98]

4. When a solid is converted into a gas, directly by heating, then this process is known as

(*a*) boiling (*b*) sublimation

(*c*) vaporization (*d*) condensation [AIIMS 99]

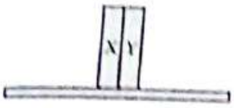
Scanned by CamScanner

5. A constant pressure air thermometer gave a reading of 47.5 units of volume when immersed in ice cold water, and 67 units in a boiling liquid. The boiling point of the liquid is

(a) 100°C (b) 112°C

(c) 125°C (d) 135°C [AIIMS 94]

6. A bimetallic strip consists of metals X and Y. It is mounted rigidly at the base as shown below :



The metal X has a higher coefficient of expansion compared to that for metal Y. When the bimetallic strip is placed in a cold bath

(a) it will bend towards the right

(b) it will bend towards the left

(c) it will not bend but shrink

(d) it will neither bend nor shrink [AIIMS 06]

7. The density of a substance at 0°C is 10 gcm^{-3} and at 100°C its density is 9.7 gcm^{-3}. The coefficient of linear expansion of the substance is

(a) 10^{-4} (b) 10^{-2}

(c) 10^{-3} (d) 10^{2} [AIIMS 02]

8. Calorimeters are made of which of the following ?

(a) glass (b) metal

(c) wood (d) either (a) or (c) [AIIMS 96]

9. Hailstone of 0°C falls from a height of 1 km on an insulating surface converting whole of its kinetic energy into heat. What part of it will melt ? ($g = 10\,ms^{-2}$)

(a) 1/33 (b) 1/8

(c) $(1/33)\times10^{-4}$ (d) all of it [AIIMS 2K]

10. The bulb of one thermometer is spherical, while that of other is cylindrical. If both of them have equal amount of mercury, which one will respond quickly to the temperature ?

(a) spherical (b) cylindrical

(c) elliptical (d) both (a) and (c) [AIIMS 94]

11. On a cold morning, a metal surface will feel colder to touch than a wooden surface, because

(a) metal has high specific heat

(b) metal has high thermal conductivity

(c) metal has low specific heat

(d) metal has low thermal conductivity [AIIMS 98]

12. Woolen clothes keep the body warm because wool

(a) is a bad conductor

(b) increases the temperature of body

(c) decreases the temperature

(d) all of these [AIIMS 98]

13. Heat travels through vacuum by

(a) conduction (b) convection

(c) radiation (d) both (a) and (b) [AIIMS 98]

14. Ratio of the amount of heat radiation transmitted through the body to the amount of heat radiation incident on it, is known as

(a) conductance (b) inductance

(c) transmittance (d) absortance [AIIMS 96]

15. Three objects coloured black, gray and white can withstand hostile conditions upto 2,800°C. These objects are thrown into a furnace, where each of them attains a temperature of 2,000°C. Which object will glow brightest ?

(a) the white object

(b) the black object

(c) all glow with equal brightness

(d) gray object [AIIMS 06]

16. If the amount of heat energy received per unit area from sun is measured on earth, mars and jupiter, it will be

(a) the same for all

(b) in decreasing order jupiter, mars, earth

(c) in increasing order jupiter, mars, earth

(d) in decreasing order mars, earth, jupiter [AIIMS 80]

17. A black body is at a temperature 300 K. It emits energy at a rate, which is proportional to

(a) 300 (b) 300^2

(c) 300^3 (d) 300^4 [AIIMS 02]

18. If temperature of a black body increases from 7°C to 287°C, then the rate of energy radiation increases by

(a) $(287/7)^4$ (b) 16

(c) 4 (d) 2 [AIIMS 97]

19. A black body is heated from 27°C to 127°C. The ratio of their energies of radiations emitted will be

(a) 3 : 4 (b) 9 : 16

(c) 27 : 64 (d) 81 : 256 [AIIMS 01]

Scanned by CamScanner

20. A black body, at a temperature of 227°C, radiates heat at a rate of 20 cal $m^{-2}s^{-1}$. When its temperature is raised to 727°C, heat radiated by it (in cal $m^{-2}s^{-1}$) will be closest to

(a) 40 (b) 160

(c) 320 (d) 640 [AIIMS 03]

21. A metal rod at a temperature of 150°C, radiates energy at a rate of 20 W. If its temperature is increased to 300°C, then it will radiate at the rate of

(a) 17.5 W (b) 37.2 W

(c) 40.8 W (d) 68.3 W [AIIMS 95]

22. Surface temperatures of stars A and B are 727°C and 327°C respectively. What is the ratio $H_A : H_B$ for the heat radiated per second by the two stars ?

(a) 5 : 3 (b) 25 : 9

(c) 625 : 81 (d) 125 : 27 [AIIMS 2K]

23. A black body at a high temperature T K radiates energy at the rate of E Wm^{-2}. When the temperature falls to $T/2$ K, the radiated energy will be

(a) $E/4$ (b) $E/2$

(c) $2E$ (d) $E/16$ [AIIMS 2K]

24. Suppose the sun expands so that its radius becomes 100 times its present radius and its surface temperature becomes half of its present value. The total energy emitted by it then will increase by a factor of

(a) 10^4 (b) 625

(c) 256 (d) 16 [AIIMS 04]

25. The sun radiates energy in all directions. The average radiation received on the earth's surface from the sun per second is 1.4 $kW m^{-2}$. The average earth-sun distance is 1.5×10^{11} m. The mass lost by the sun per day (1 day = 86,400 s) is

(a) 4.4×10^9 kg (b) 7.6×10^{14} kg

(c) 3.8×10^{12} kg (d) 3.8×10^{14} kg [AIIMS 2K]

26. Energy from the sun is received on earth at the rate of 2 cal $cm^{-2}min^{-1}$. If average wavelength of solar light be taken as 5,500 Å, then how many photons are received on the earth per cm^2 per min ?

($h = 6.6 \times 10^{-34}$ Js, 1 cal = 4.2 J)

(a) 1.5×10^{13} (b) 2.9×10^{13}

(c) 2.3×10^{13} (d) 1.75×10^{19} [AIIMS 07]

27. According to Wien's displacement law

(a) λT = constant (b) $\lambda \propto 1/T$

(c) λ/T = constant (d) both (a) and (b) [AIIMS 02]

28. For an enclosure maintained at 1,000 K, the maximum radiation occurs at wavelength λ_m. If the temperature is raised to 2,000 K, the peak will shift to

(a) $\lambda_m/2$ (b) $3\lambda_m/2$

(c) $5\lambda_m/2$ (d) $7\lambda_m/2$ [AIIMS 94]

29. The sun emits a light with maximum wavelength 510 nm, while another star X emits a light with maximum wavelength of 350 nm. What is the ratio of surface temperature of the sun and the star X?

(a) 1.45 (b) 0.68

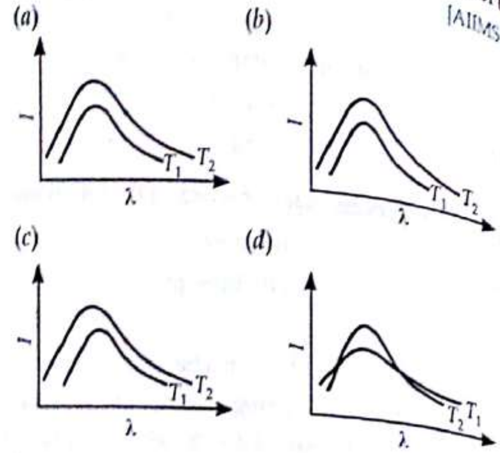
(c) 0.46 (d) 2.1 [AIIMS 2K]

30. On increasing the temperature of a substance gradually, its colour becomes

(a) red (b) green

(c) yellow (d) white [DPMT 94 ; AIIMS 96]

31. Shown below are the black body radiation curves at temperatures T_1 and T_2 ($(T_2 > T_1)$). Which of the following plots is correct ? [AIIMS 03]



32. The latent heat of vaporisation of a substance is always

(a) greater than its latent heat of fusion

(b) greater than its latent heat of sublimation

(c) equal to its latent heat of sublimation

(d) less than its latent heat of fusion [AIIMS 2010]

33. Black holes in orbit around a normal star are detected from the earth due to the frictional heating of infalling gas into the black hole, which can reach temperatures greater than 10^6 K. Assuming that the infalling gas can be modelled as a black body radiator, then the wavelength of maximum power lies

(a) in the visible region (b) in the X-ray region

(c) in the microwave region

(d) in the gamma-ray region of electromagnetic spectrum [AIIMS 2009]

Scanned by CamScanner

ASSERTIONS AND REASONS

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(*a*) If both assertion and reason are true and reason is the correct explanation of the assertion.

(*b*) If both assertion and reason are true but reason is not correct explanation of the assertion.

(*c*) If assertion is true, but reason is false.

(*d*) If both assertion and reason are false.

34. **Assertion.** Good conductors of heat are also good conductors of electricity and vice-versa.

Reason. Mainly electrons are responsible for these conductors. [AIIMS 2K]

35. **Assertion.** At room temperature water does not sublimate from ice to steam.

Reason. The critical point of water is much above the room temperature. [AIIMS 94]

36. **Assertion.** Water kept in an open vessel will quickly evaporate on the surface of the moon.

Reason. The temperature at the surface of the moon is much higher than the boiling point of water. [AIIMS 95]

37. **Assertion.** In a pressure cooker the water is brought to boil. The cooker is then removed from the stove. Now on removing the lid of the pressure cooker, the water starts boiling again.

Reason. The impurities in water bring down its boiling point. [AIIMS 04]

38. **Assertion.** It is hotter over the top of a fire than at the same distance on the sides.

Reason. Air surrounding the fire conducts more heat upwards. [AIIMS 03]

39. **Assertion.** Air at some distance above the fire is hotter than the same distance below it..

Reason. Air surrounding the fire carries heat upwards. [AIIMS 2K]

40. **Assertion.** Woollen clothes keep the body warm in winter.

Reason. Air is a bad conductor of heat. [AIIMS 02]

41. **Assertion.** The earth without its atmosphere would be inhospitably cold.

Reason. All heat would escape in the absence of atmosphere. [AIIMS 02]

42. **Assertion.** While measuring the thermal conductivity of a liquid experimentally, the upper layer is kept hot and lower layer is kept cold.

Reason. This avoids heating of the liquid by convection. [AIIMS 07]

43. **Assertion.** A body that is a good radiator is also a good absorber of radiation at a given wavelength.

Reason. According to Kirchhoff's law the absorptivity of a body is equal to its emissivity at a given wavelength. [AIIMS 05]

44. **Assertion.** Temperature near the sea coast are moderate.

Reason. Water has a high thermal conductivity. [AIIMS 03]

45. **Assertion.** Perspiration from human body helps in cooling the body.

Reason. A thin layer of water on the skin enhances its emissivity. [AIIMS 06]

46. **Assertion.** A hallow metallic closed container maintained at a uniform temperature can act as a source of black body radiation.

Reason. All metals act as black bodies. [AIIMS 96]

47. **Assertion.** Blue star is at higher temperature than red star.

Reason. Wien's displacement law states that $T \propto 1/\lambda_m$. [AIIMS 02]

48. **Assertion.** For higher temperatures, the peak emission wavelength of a black body shifts to lower wavelengths.

Reason. Peak emission wavelength of a black body is proportional to the fourth-power of temperature. [AIIMS 05]

49. **Assertion.** The radiation from the sun's surface varies as the fourth power of its absolute temperature.

Reason. Sun is not a black body. [AIIMS 99]

50. **Assertion.** Liquid molecules have greater potential energy at the melting point.

Reason. Intermolecular spacing between molecules increases at melting point. [AIIMS 2009]

51. **Assertion.** Water kept in an open vessel will quickly evaporate on the surface of the moon.

Reason. The temperature at the surface of the moon is much higher than boiling point of water. [AIIMS 2010]

52. **Assertion.** When hot water is poured in a beaker of thick glass, the beaker cracks.

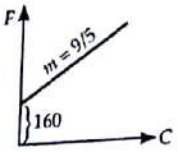
Reason. Outer surface of the beaker expands suddenly. [AIIMS 2009]

Scanned by CamScanner

Answers and Explanations

1. (*c*) As $\frac{C}{100} = \frac{F-32}{180}$

$\therefore \quad F = \frac{9}{5}C + 160$



Thus the graph between C and F is a straight with positive intercept (= 160) on Y-axis as shown in the figure.

2. (*a*) The temperatures of the block and wood are equal to the temperature of the body as both feel equally hot or cold.

3. (*d*) The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion.

4. (*b*) The direct conversion of a solid into gas is called sublimation.

5. (*b*) From Charle's law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \text{or} \quad \frac{47.5}{0+273} = \frac{67}{T_2}$$

$$T_2 = \frac{67 \times 273}{47.5} = 385 \text{ K} = \mathbf{112°C}$$

6. (*b*) When the bimetallic strip is kept in a cold bath, the length of strip X decreases more rapidly than Y, so the bimetallic strip bends towards the left.

7. (*a*) $\gamma = \frac{\rho_0 - \rho}{\rho_0 \Delta T} = \frac{10 - 9.7}{10 \times 100} = 3 \times 10^{-4}\ °\text{C}$

$$\alpha = \frac{1}{3}\gamma = \mathbf{10^{-4}\ °C^{-1}}.$$

8. (*b*) In order that the heat exchange is quick, the calorimeters are made of metals.

9. (*a*) $mL = Mgh$

$$\frac{m}{M} = \frac{gh}{L} = \frac{10 \times 1000}{80 \times 4.2} = \frac{10000}{336} \simeq \mathbf{\frac{1}{33}}.$$

10. (*b*) As the surface area of cylindrical bulb is larger than a spherical bulb, heat will be transmitted quickly through a cylindrical bulb and it will respond quickly to the temperature.

11. (*b*) Due to higher thermal conductivity of metal than wood, heat begins to flow readily from our body to the metal surface and so we feel colder.

12. (*a*) Woolen fibres enclose a large amount of air in them. Both wool and air are bad conductors of heat and do not allow loss of heat from our body due to conduction.

13. (*c*) Heat travels through vacuum due to radiation.

14. (*c*) The ratio of the amount of heat transmitted through an object to the amount of heat incident on it is called transmittance.

15. (*b*) Black objects are good absorbers of heat. So they are also good emitters (Kirchhoff's law).

16. (*c*) Heat energy received on a planet varies inversely as the square of the distance from the sun. So Jupiter < Mars < Earth

17. (*d*) According to Stefan's law,

$$E \propto T^4.$$

18. (*b*) $\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{273+287}{273+7}\right)^4$

$$= \left(\frac{560}{280}\right)^4 = \left(\frac{2}{1}\right)^4 = \mathbf{16}.$$

19. (*d*) $\frac{E_1}{E_2} = \left(\frac{273+27}{273+127}\right)^4 = \left(\frac{300}{400}\right)^4$

$$= \frac{81}{256} = \mathbf{81 : 256}.$$

20. (*c*) $\frac{E_2}{E_1} = \left(\frac{273+727}{273+227}\right)^4 = \left(\frac{1000}{500}\right)^4 = 16$

$$E_2 = 16\,E_1 = 16 \times 20$$

$$= \mathbf{320\ cal\ m^{-2}\ s^{-1}}.$$

21. (*d*) $\frac{E_2}{E_1} = \left(\frac{273+300}{273+150}\right)^4 = \left(\frac{573}{423}\right)^4$

$$E_2 = \left(\frac{573}{423}\right)^4 \times 20 = \mathbf{68.3\ W}.$$

22. (*c*) $\frac{H_A}{H_B} = \left(\frac{T_A}{T_B}\right)^4 = \left(\frac{727+273}{327+273}\right)^4 = \mathbf{\frac{625}{81}}$

23. (*d*) $\frac{E'}{E} = \left(\frac{T/2}{T}\right)^4 = \frac{1}{16}$

$$E' = \mathbf{\frac{E}{16}}.$$

24. (*b*) $E \propto 4\pi R^2 T^4$

$$E' \propto 4\pi(100\,R)^2 (T/2)^4$$

$$\frac{E'}{E} = \frac{(100)^2}{2^4} = 625$$

or $\quad E' = \mathbf{625\ E}$

Scanned by CamScanner

25. (*d*) Total energy radiated by the sun

$$= 1.4 \times 10^3 \times 4\pi \times (1.5 \times 10^{11})^2 \ \text{J s}^{-1}$$
$$= 39.5 \times 10^{25} \ \text{J s}^{-1}$$
$$= 39.5 \times 86400 \times 10^{25} \ \text{J day}^{-1}$$
$$= 3.4 \times 10^{31} \ \text{J day}^{-1}$$
$$\Delta m = \frac{E}{c^2} = \frac{3.4 \times 10^{31}}{9 \times 10^{16}} = \mathbf{3.8 \times 10^{14}} \ \text{kg}.$$

26. (*c*) Energy received from the sun

$$= 2 \ \text{cal cm}^{-2} \ \text{min}^{-1}$$
$$= 2 \times 4.2 = 8.4 \ \text{J cm}^{-2} \ \text{min}^{-1}$$

Energy of one photon from the sun,

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5500 \times 10^{-10}}$$
$$= 3.6 \times 10^{-19} \ \text{J}$$

$\therefore$ Number of photons reaching the earth per cm^2 per minute,

$$n = \frac{8.4}{3.6 \times 10^{-19}} = \mathbf{2.3 \times 10^{19}}.$$

27. (*d*) Wien's law :

$$\lambda_m T = \text{constant}$$

or

$$\lambda_m \propto \frac{1}{T}$$

28. (*a*) $\dfrac{\lambda'_m}{\lambda_m} = \dfrac{T}{T'} = \dfrac{1000}{2000} = \dfrac{1}{2}$

$$\lambda'_m = \frac{\lambda_m}{2}$$

29. (*b*) $\dfrac{T_{sun}}{T_{star}} = \dfrac{\lambda_m\,(\text{star})}{\lambda_m\,(\text{sun})} = \dfrac{350}{510} = \mathbf{0.68}.$

30. (*c*) By Wien's law,

$$\lambda_m \propto \frac{1}{T}$$

On increasing temperature, wavelength decreases. As yellow colour has minimum wavelength, the substance gradually becomes yellow.

31. (*c*) From Wien's law,

$$T_2 > T_1$$

$\therefore \quad \lambda_m(2) < \lambda_m(1)$

This is true for option (*c*) only.

32. (*a*) The latent heat of vaporisation is always greater than the latent heat of fusion for any substance.

33. (*b*) $\lambda_m T = 2.898 \times 10^{-3} \ \text{mK}$

$$\lambda_m = \frac{2.9 \times 10^{-3} \ \text{mK}}{10^6 \ \text{K}} = 2.9 \times 10^{-9} \ \text{m} = \mathbf{2.9 \ nm}$$

This wavelength lies in the *X*-ray region of the electromagnetic spectrum.

34. (*a*) Both the assertion and reason are true. Good conductors have a large number of free electrons which make them good conductors of both heat and electricity.

35. (*a*) Both the assertion and reason are true.

36. (*c*) The assertion is true but the reason is false. Evaporation occurs at all temperatures. Due to low gravity on moon, the escape velocity is small.

37. (*c*) The assertion is true but the reason is false. When the lid is removed, the pressure decreases. This decreases the boiling point of water and water begins to boil.

38. (*c*) The assertion is true but the reason is false. Above the fire, heat is carried by convection of air.

39. (*a*) Both the assertion and reason are true.

40. (*a*) Both the assertion and reason are true.

41. (*a*) Both the assertion and reason are true.

42. (*a*) Both the assertion and reason are true.

43. (*a*) Both the assertion and reason are true.

44. (*b*) Both the assertion and reason are true but the reason is not a correct explanation of the assertion.

45. (*c*) The assertion is true but the reason is false.

46. (*c*) The assertion is true but the reason is false.

47. (*a*) Both the assertion and reason are true.

48. (*c*) The assertion is true but the reason is false. According to Wien's law, the peak emission wavelength of a body is inversely proportional to its absolute temperature.

49. (*c*) The assertion is true but the reason is false. At a high temperature of 6000 K, the sun acts like a black body emitting complete radiation. It follows from Stefan's law that : $E \propto T^4$

50. (*c*) When a liquid changes into solid, the work done against the intermolecular attraction is stored as potential energy.

51. (*a*) As there is no atmosphere on the moon, the surface temperature on the moon is very high due to which water quickly evaporates.

52. (*c*) Glass is bad conductor of heat. Heat does not pass easily from inner to outer surface. This causes unequal expansion and cracks the beaker.

Scanned by CamScanner

CBSE PMT Prelims and Final Exams

1. According to kinetic theory of gases, at absolute zero of temperature

(*a*) water freezes (*b*) liquid helium freezes

(*c*) molecular motion stops

(*d*) liquid hydrogen freezes [CBSE PMT 90]

2. Mercury thermometer can be used to measure temperature upto

(*a*) 260°C (*b*) 100°C

(*c*) 360°C (*d*) 500°C [CBSE PMT 92]

3. For measuring temperatures in the range of 2,000 to 2,500°C, we should employ

(*a*) barometer (*b*) pyrometer

(*c*) gas thermometer

(*d*) platinum-rhodium thermometer [CBSE PMT 95]

4. A Centigrade and Fahrenheit thermometers are dipped in boiling water. The water temperature is lowered, until the Fahrenheit thermometer registers 140°. What is the fall in temperature registered by the centigrade thermometer ?

(*a*) 80° (*b*) 60°

(*c*) 360° (*d*) 30°

[AFMC 04 ; AIIMS 98 ; CBSE PMT 90]

5. The coefficient of linear expansion of brass and steel are α_1 and α_2 respectively. If we take a brass rod of length l_1 and steel rod of length l_2 at 0°C, the difference in their lengths (l_1 and l_2) will remain the same at all temperatures, if

(*a*) $\alpha_1 l_1 = \alpha_2 l_2$ (*b*) $\alpha_1 l_2 = \alpha_2 l_1$

(*c*) $\alpha_1^2 l_1 = \alpha_2^2 l_2$ (*d*) $\alpha_1 l_2^2 = \alpha_2^2 l_1$

[CBSE PMT 99]

6. The thermal capacity of 40 g of aluminium (specific heat = 0.2 cal°g^{-1}C^{-1}) is

(*a*) 40 cal °C^{-1} (*b*) 160 cal °C^{-1}

(*c*) 200 cal °C^{-1} (*d*) 8 cal °C^{-1} [CBSE PMT 90]

7. 80 g of water at 30°C are poured on a large block of ice at 0°C. The mass of ice that melts is

(*a*) 30 g (*b*) 80 g

(*c*) 150 g (*d*) 1,600 g [CBSE PMT 90]

8. If 1 g of steam is mixed with 1 g of ice, the resultant temperature of the mixture is

(*a*) 270° (*b*) 230°

(*c*) 100° (*d*) 50° [CBSE PMT 99]

9. 10 g of ice cubes at 0°C are released in a tumbler (water equivalent 55 g) at 40°C. Assuming that negligible heat is taken from the surroundings, the temperature of water in the tumbler becomes nearly ($L = 80$ cal / g)

(*a*) 31°C (*b*) 22°C

(*c*) 19°C (*d*) 15°C

10. On a new scale of temperature (which is linear) called the *W* scale, freezing and boiling points of water are respectively 39° W and 239° W. What will be the temperature on the new scale corresponding to a temperature of 39° C on the celcius scale ?

(*a*) 200° W (*b*) 139° W

(*c*) 78° W (*d*) 117° W

11. The two ends of a rod of length L and a uniform cross-sectional area A are kept at two temperatures T_1 and T_2 ($T_1 > T_2$). The rate of heat transfer, $\frac{dQ}{dt}$, through the rod in a steady state is given by

(*a*) $\frac{dQ}{dt} = \frac{K(T_1 - T_2)}{LA}$ (*b*) $\frac{dQ}{dt} = KLA(T_1 - T_2)$

(*c*) $\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$ (*d*) $\frac{dQ}{dt} = \frac{KL(T_1 - T_2)}{A}$

12. The presence of gravitational field is required for the heat transfer by

(*a*) stirring liquids (*b*) conduction

(*c*) natural convection

(*d*) radiation

13. Consider two rods of same length and different specific heats (c_1, c_2), thermal conductivities (K_1, K_2) and area of cross-sections (A_1, A_2) and both having temperatures (T_1, T_2) at their ends. If their rate of loss of heat due to conduction is equal, then

(*a*) $K_1 A_1 = K_2 A_2$ (*b*) $K_1 A_1 / c_1 = K_2 A_2 / c_2$

(*c*) $K_2 A_1 = K_1 A_2$ (*d*) $K_2 A_1 / c_2 = K_1 A_2 / c_1$

14. Which of the following circular rods (radius r and length l) each made of the same material and whose ends are maintained at the same temperature will conduct most heat ?

(*a*) $r = 2r_0, l = 2l_0$ (*b*) $r = 2r_0, l = l_0$

(*c*) $r = r_0, l = 2l_0$ (*d*) $r = r_0, l = l_0$

SECTION

15. A cylin... at its end. The... dimensions (l... temperature ... will be

(*a*) $Q_2 = 2$... (*c*) $Q_2 = Q_1$...

16. Heat is... the same mate... ratio 1 : 2 and... temperature ... then the ratio ... them per unit...

(*a*) 1 : 1 (*c*) 1 : 4

17. Consi... different mate... conductivities... thermal cond...

(*a*) $\frac{2K}{3}$ (*c*) $\sqrt{2}K$

18. A beak... cools from 8... 70°C in t_2 min... then

(*a*) $t_1 = t_2$ (*c*) $t_1 < t_2$

19. Which... black body ?

(*a*) platin... (*b*) black... (*c*) cavity... (*d*) a lum...

20. Unit ...

(*a*) W m^2... (*c*) W m...

21. A bla... emits energy...

(*a*) 500 (*c*) (500)...

Scanned by CamScanner

15. A cylindrical rod having temperatures T_1 and T_2 at its end. The rate of flow of heat Q_1 cal s^{-1}. If all the linear dimensions (length and radius) are doubled keeping temperature constant, then the rate of flow of heat Q_2 will be

(a) $Q_2 = 2Q_1$ (b) $Q_2 = Q_1/2$
(c) $Q_2 = Q_1/4$ (d) $Q_2 = 4Q_1$ [CBSE PMT 01]

16. Heat is flowing through the cylindrical rods of the same material. The diameters of the rods are in the ratio 1 : 2 and their lengths are in the ratio 2 : 1. If the temperature difference between their ends is the same, then the ratio of the amount of heat conducted through them per unit time will be

(a) 1 : 1 (b) 2 : 1
(c) 1 : 4 (d) 1 : 8 [CBSE PMT 95]

17. Consider a compound slab consisting of two different materials having equal thickness and thermal conductivities K and $2K$ respectively. The equivalent thermal conductivity of the slab is

(a) $\frac{2K}{3}$ (b) $\frac{4K}{3}$
(c) $\sqrt{2}K$ (d) $3K$ [CBSE PMT 03]

18. A beaker full of hot water is kept in a room. If it cools from 80°C to 75°C in t_1 minutes, from 75°C to 70°C in t_2 minutes and from 70°C to 65°C in t_3 minutes, then

(a) $t_1 = t_2 = t_3$ (b) $t_1 < t_2 = t_3$
(c) $t_1 < t_2 < t_3$ (d) $t_1 > t_2 > t_3$ [CBSE PMT 95]

19. Which of the following is best close to an ideal black body ?

(a) platinum black
(b) black lamp
(c) cavity maintained at constant temperature
(d) a lump of charcoal heated to high temperature [CBSE PMT 02]

20. Unit of Stefan's constant is

(a) $W\,m^2\,K^4$ (b) $W\,m^2\,K^{-4}$
(c) $W\,m^{-2}\,K^{-1}$ (d) $W\,m^{-2}\,K^{-4}$ [CBSE PMT 02]

21. A black body is at a temperature of 500 K. It emits energy at a rate, which is proportional to

(a) 500 (b) $(500)^2$
(c) $(500)^3$ (d) $(500)^4$ [CBSE PMT 97]

22. A black body is at 727° C. It emits energy at a rate, which is proportional to

(a) $(727)^2$ (b) $(1000)^2$
(c) $(727)^4$ (d) $(1000)^4$ [CBSE PMT 07]

23. A black body at 227°C radiates heat at the rate of 7 cal $cm^{-2}\,s^{-1}$. At a temperature of 727°C, the rate of heat radiated in the same units will be

(a) 50 (b) 112
(c) 80 (d) 60 [CBSE PMT 09]

24. For a black body at temperature of 727°C, its radiating power is 60 W and temperature of surroundings is 227°C. If temperature of black body is changed 1,227°C, then its radiating power will be

(a) 120 W (b) 240 W
(c) 304 W (d) 320 W [CBSE PMT 02]

25. The radiant energy from the sun, incident normally at the surface of earth, is 20 kcal $m^{-2}\,min^{-1}$. What would have been the radiant energy incident normally on the earth, if the sun had a temperature twice of the present one ?

(a) 40 kcal $m^{-2}min^{-1}$ (b) 80 kcal $m^{-2}min^{-1}$
(c) 160 kcal $m^{-2}min^{-1}$ (d) 320 kcal $m^{-2}min^{-1}$ [CBSE PMT 98]

26. If the temperature of the sun is doubled, the rate of energy received on earth will be increased by a factor of

(a) 2 (b) 4
(c) 8 (d) 16 [CBSE PMT 93]

27. Assuming the sun to have a spherical outer surface of radius R, radiating like a black body at temperature t°C, the power received by a unit surface (normal to the incident rays) at a distance r from the centre of the sun is

(a) $\frac{4\pi R^2 \sigma t^4}{r^2}$ (b) $\frac{R^2\sigma(t+273)^4}{4\pi r^2}$
(c) $\frac{16\pi R^2 \sigma t^4}{r^2}$ (d) $\frac{R^2\sigma(t+273)^4}{r^2}$ [CBSE PMT 07]

28. Which of the following statement is true about the radiation emitted by human body ?

(a) the radiation emitted lies in the ultraviolet region and hence is not visible
(b) the radiation is emitted during the summers and absorbed during the winters
(c) the radiation is emitted only during the day
(d) the radiation emitted is in the infrared region. [CBSE PMT 03]

Scanned by CamScanner

29. If λ_m denotes the wavelength at which the radiative emission from a black body at a temperature T K is maximum, then

(a) λ_m is independent of temperature
(b) $\lambda_m \propto T^4$
(c) $\lambda_m \propto T$ (d) $\lambda_m \propto 1/T$ [CBSE PMT 04]

30. The Wien's displacement law expresses relation between

(a) wavelength corresponding to maximum energy and absolute temperature
(b) radiated energy and wavelength
(c) temperature and emissive power
(d) colour of light and temperature [CBSE PMT 02]

31. A black body emits radiation of maximum intensity of wavelength λ at 2,000 K. Its corresponding wavelength at 3,000 K will be

(a) $16\lambda/81$ (b) $81\lambda/16$
(c) $2\lambda/3$ (d) $4\lambda/3$ [CBSE PMT 06]

32. A black body at 1,227°C emits radiations with maximum intensity at a wavelength of 5000 Å. If the temperature of the body is increased by 1,000 °C, the maximum intensity will be observed at

(a) 3000 Å (b) 4000 Å
(c) 5000 Å (d) 6000 Å [CBSE PMT 06]

33. Two containers A and B are partly filled with water and closed. The volume of A is twice that of B and it contains half the amount of water in B. If both are at the same temperature, the water vapour in the containers will have pressure in the ratio of

(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 4 : 1

34. A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t. The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod, when placed in thermal contact with the two reservoirs in time t? [CBSE PMT 10]

(a) $\frac{Q}{4}$ (b) $\frac{Q}{16}$
(c) $2Q$ (d) $\frac{Q}{2}$ [CBSE Pre 2010]

35. The total radiant energy per unit area, normal to the direction of incidence, received at a distance R from the centre of a star of radius r, whose outer surface radiates as a black body at a temperature T K is given by

(a) $\frac{\sigma r^2 T^4}{R^2}$ (b) $\frac{\sigma r^2 T^4}{4\pi r^2}$
(c) $\frac{\sigma r^4 T^4}{r^4}$ (d) $\frac{4\pi\sigma r^2 T^4}{R^2}$

where σ is stefan's constant. [CBSE Pre 2010]

Answers and Explanations

1. (c) According to kinetic theory of gases, at absolute zero of temperature molecular motion stops.

2. (c) Mercury thermometer can measure temperatures ranging from −37°C to 357°C.

3. (b) Pyrometers can be used to measure temperature in the range of 2000 to 2500 °C.

4. (c) $\frac{C}{5} = \frac{F-32}{9} = \frac{140-32}{9} = \frac{108}{9} = 12$

$C = 60°$

On centigrade scale, the boiling point of water is 100°C

∴ Fall in temperature registered by the centrigrade thermometer = 100 − 60 = **40°**.

5. (a) As the difference between the lengths of the rods remains same at all temperatures,

$$\Delta l_1 = \Delta l_2$$

or $$\alpha_1 l_1 \Delta T = \alpha_2 l_2 \Delta T$$

or $$\alpha_1 l_1 = \alpha_2 l_2$$

6. (d) Thermal capacity = mc

$= 40\text{ g} \times 0.2\text{ cal g}^{-1}{}^\circ\text{C}^{-1}$

$= \mathbf{8\ cal\ {}^\circ C^{-1}}$.

7. (a) Heat used in melting m gram of ice

= Heat lost by 80 g water

$mL = 80 \times 1 \times 30$

$m = \frac{80 \times 1 \times 30}{80} = \mathbf{30\ g}$ $[L = 80 \text{ cal g}^{-1}]$

8. (c) Heat required to change 1 g of ice at 0°C first into water at 0°C and then into water at 100°C,

$Q = mL + mc\Delta T$

$= 1 \times 80 + 1 \times 1 \times 100 = 180$ cal

Scanned by CamScanner

Now 1 g of steam carries 540 cal of heat, when 1 g of steam is mixed with 1 g of ice, entire steam will not condense. Finally, we will have a mixture of water and steam at 100°C

9. (*b*) Let T be the final temperature.

Heat gained by ice = Heat lost by water

$$mL + mc(T-0) = Mc(40-T)$$

$$10\times 80 + 10\times 1\times T = 55\times 1\times(40-1)$$

$$T = 21.5°C \approx \mathbf{22°C}$$

10. (*d*) $\dfrac{W-39}{239-39} = \dfrac{C}{100} = \dfrac{39}{100}$

$$W = 117°W.$$

11. (*c*) $\dfrac{dQ}{dt} = \dfrac{KA(T_1-T_2)}{L}$

12. (*c*) Gravitational field is needed for the transfer of heat by natural convection.

13. (*a*) Given : $\dfrac{Q_1}{t} = \dfrac{Q_2}{t}$

or

$$\frac{K_1A_1(T_1-T_2)}{l} = \frac{K_2A_2(T_1-T_2)}{l}$$

or

$$K_1A_1 = K_2A_2.$$

14. (*b*) $H = \dfrac{KA(T_1-T_2)}{l}$

$$= \frac{K\times \pi r^2 \times (T_1-T_2)}{l} \propto \frac{r^2}{l}$$

In (*a*), $H \propto \dfrac{(2r_0)^2}{2l_0}$ or $H \propto \dfrac{2r_0^2}{l_0}$

In (*b*), $H \propto \dfrac{(2r_0)^2}{l_0}$ or $H \propto \dfrac{4r_0^2}{l_0}$

In (*c*), $H \propto \dfrac{r_0^2}{2l_0}$

In (*d*), $H \propto \dfrac{r_0^2}{l_0}$

Clearly, rate of flow of heat is maximum in option (*b*).

15. (*a*) For constant $(T_1 - T_2)$,

$$\frac{Q_2}{Q_1} = \left(\frac{r_2}{r_1}\right)^2 \frac{l_1}{l_2} = \left(\frac{2r_1}{r_1}\right)^2 \cdot \frac{l_1}{2l_1} = 2$$

or $Q_2 = \mathbf{2Q_1}$.

16. (*d*) For rods of same material and constant (T_1-T_2),

$$\frac{Q_1}{Q_2} = \left(\frac{d_1}{d_2}\right)^2 \frac{l_2}{l_1} = \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{1}{8} = \mathbf{1:8}.$$

17. (*b*) Equivalent thermal conductivity for a series combination of 2 slabs,

$$K_{eq} = \frac{d_1+d_2}{\dfrac{d_1}{K_1}+\dfrac{d_2}{K_1}} = \frac{d+d}{\dfrac{d}{K}+\dfrac{d}{2K}}$$

$$= \frac{2d}{\dfrac{3}{2}\dfrac{d}{K}} = \frac{4}{3}K.$$

18. (*c*) According to Newton's law of cooling,

Rate of cooling ∝ Temperature difference of the body and its surroundings.

As the temperature of the body approaches the room temperature, rate of cooling decreases.

∴ $t_1 < t_2 < t_3$

19. (*c*) A cavity maintained at a constant temperature acts as an ideal black body.

20. (*d*) $\sigma = \dfrac{E}{T^4} = \dfrac{\text{Js}^{-1}\text{m}^{-2}}{\text{K}^4} = \mathbf{Wm^{-2}K^{-4}}.$

21. (*d*) According to Stefan's law, $E \propto (500)^4$.

22. (*d*) $E \propto (273+727)^4$

or $E \propto (1000)^4$

23. (*b*) $\dfrac{E_2}{E_1} = \left(\dfrac{273+727}{273+227}\right)^4 = \left(\dfrac{1000}{500}\right)^4 = 16$

$$E_2 = 16E_1 = 16\times 7 = \mathbf{112\ cal\ cm^{-2}\ s^{-1}}.$$

24. (*d*) According to Stefan's Boltzmann's law,

$$E = \sigma(T^4 - T_0^4),\quad E' = \sigma(T'^4 - T_0^4)$$

∴ $\dfrac{E'}{E} = \dfrac{T'^4 - T_0^4}{T^4 - T_0^4}$

$$= \frac{(1227+273)^4 - (227+273)^4}{(727+273)^4 - (227+273)^4}$$

$$E' = \frac{1500^4 - 500^4}{1000^4 - 500^4}\times 60 \qquad [E = 60\text{ W}]$$

$$= \mathbf{320\ W}.$$

25. (*d*) $\dfrac{E'}{E} = \left(\dfrac{T'}{T}\right)^4$

$$E' = \left(\frac{2T}{T}\right)^4 \times E$$

$$= 16\times 20 = \mathbf{320\ kcal\ m^{-2}\ min^{-1}}.$$

Scanned by CamScanner

26. (*d*) As seen above,

$$E' = 16\,E.$$

27. (*d*) From Stefan's law, energy radiated by the sun per second,

$$E = \sigma A T^4 = \sigma \times 4\pi R^2 \times T^4$$

Power received per unit area at distance *r* from the sun,

$$I = \frac{E}{4\pi r^2} = \frac{\sigma \times 4\pi R^2 \times T^4}{4\pi r^2}$$

$$= \frac{\sigma R^2 (t+273)^4}{r^2}.$$

28. (*d*) Due to its low temperature, the human body emits radiation in the infrared region.

29. (*d*) According to Wien's law,

$$\lambda_m \propto \frac{1}{T}$$

30. (*a*) Wien's displacement law gives relation between wavelength corresponding to maximum energy emitted by a black body and its absolute temperature.

31. (*c*) By Wien's law,

$$\lambda'_m T' = \lambda_m T$$

$$\therefore \quad \lambda'_m = \frac{\lambda T}{T'} = \frac{\lambda \times 2000}{3000} = \frac{2\lambda}{3}.$$

32. (*a*) $\lambda'_m = \frac{\lambda T}{T'} = \frac{5000 \times (1227 + 273)}{(2227 + 273)}$

$$= \frac{5000 \times 1500}{2500} = 3000\ \text{Å}.$$

33. (*b*) Vapour pressure does not depend on the amount of the substance. It depends on temperature only.

34. (*b*) $A' = A/4$ and $l' = 4l$

$$\frac{Q'}{Q} = \frac{A'}{A} \cdot \frac{l}{l'} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$\therefore \quad Q' = \frac{Q}{16}$$

35. (*a*) $\frac{1}{A}\frac{dE}{dt} = \frac{4\pi r^2}{4\pi R^2}\sigma T^4 = \sigma \frac{\rho^2}{R^2} T^4$

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. At which temperature, the centrigrade and Feherenheit scales are equal ?
   (*a*) 40° (*b*) −40°
   (*c*) 37° (*d*) −80° [DPMT 98, 07]

2. The resistance of tungsten filament at 150°C is 133 Ω. What will be its resistance at 500°C ? The temperature coefficient of resistance of tungsten is 0.0045 per °C
   (*a*) 366 Ω (*b*) 69 Ω
   (*c*) 266 Ω (*d*) 109 Ω [DPMT 04]

3. When a body is heated, then maximum rise will be in its
   (*a*) length (*b*) surface area
   (*c*) volume (*d*) density [DPMT 05]

4. The volume of a gas at 20°C is 100 cm^3 at normal pressure. If it is heated to 100°C, its volume becomes 125 cm^3 at the same pressure, then volume coefficients of the gas (at normal pressure) is
   (*a*) 0.0033 /°C (*b*) 0.0025 /°C
   (*c*) 0.0030 /°C (*d*) 0.0021 /°C [DPMT 91]

5. Density of substance at 0°C is 10 g/cc and 100°C its density is 9.7 g/cc. The coefficient of linear expansion of the substance is
   (*a*) 10^{-1} (*b*) 10^{-2}
   (*c*) 10^{-3} (*d*) 10^{-4} [DPMT 99]

6. The coefficient of volumetric expansion of mercury is 18×10^{-5} /°C. The thermometer bulb has a volume of 10^{-6} m^3 and cross-section of the stem is 0.002 cm^2. Assuming that bulb is filled with mercury at 0°C, the increase in length of the mercury column at 100°C will be
   (*a*) 9 cm (*b*) 9 mm
   (*c*) 18 cm (*d*) 18 mm [DPMT 9?]

7. Coefficient of real expansion of mercury is 0.18×10^{-3}/°C. If the density of mercury at 0°C is 13.6 g/cc, its density at 473 K will be
   (*a*) 13.65 g/cc (*b*) 13.22 g/cc
   (*c*) 13.51 g/cc (*d*) 13.11 g/cc. [DPMT ?]

8. Amount of heat required to raise the temperature of a body through 1 K is called its
   (*a*) specific heat (*b*) thermal capacity
   (*c*) water equivalent (*d*) entropy [DPMT ?]

Scanned by CamScanner

9. Water falls from a height 500 m. What is the rise in the temperature of water at the bottom, if the whole energy remains in water ? (Specific heat of water = 4200 J/kg°C)

(*a*) 1.16°C (*b*) 0.24°C

(*c*) 0.40°C (*d*) 0.19°C [DPMT 97]

10. In an energy recycling process, X g of steam at 100°C becomes water at 100°C which converts Y g of ice at 0°C into water at 100°C. The ratio of X and Y will be

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$

(*c*) 3 (*d*) 2 [DPMT 06]

11. If the temperature of a rod is increased by 10°C, its length increases by 1%. What is the percentage change in volume of the body of the same material for 10°C increase in temperature ?

(*a*) 9% (*b*) 1%

(*c*) 5% (*d*) 3% [DPMT 08]

12. In order that the heat flows from one part of a solid to another part, what is required ?

(*a*) uniform density (*b*) temperature gradient

(*c*) density gradient (*d*) uniform temperature [DPMT 97]

13. A body A of mass 0.5 kg and specific heat 0.85 is at a temperature of 60°C. Another body B of mass 0.3 kg and specific heat 0.9 is at a temperature of 90°C. When they are connected to a conducting rod, heat will flow from

(*a*) A to B (*b*) heat can't flow

(*c*) B to A (*d*) first (*a*) and then (*c*) [DPMT 94]

14. The ratio of thermal conductivity of two rods of different material is 5 : 4. The two rods of same area of cross-section and same thermal resistance will have the lengths in the ratio

(*a*) 4 : 5 (*b*) 9 : 1

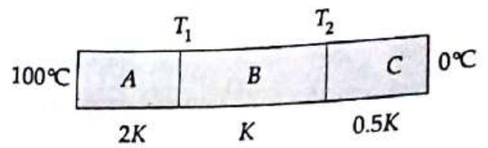
(*c*) 1 : 9 (*d*) 5 : 4 [DPMT 05]

15. Two rods having thermal conductivity in the ratio of 5 : 3 having equal lengths and equal cross-sectional area are joined end to end. If the temperature of the free end of the first rod is 100°C and free end of the second rod is 20°C, then the temperature of the junction is

(*a*) 50°C (*b*) 70°C

(*c*) 60°C (*d*) 90°C [DPMT 97, 03]

16. Three identical rods A, B and C of equal lengths and equal diameters are joined in series as shown in figure. Their thermal conductivities are $2K$, K and $K/2$



respectively. Calculate the temperature at two junction points.

(*a*) 85.7, 57.1°C (*b*) 80.85, 50.3°C

(*c*) 77.3, 48.3°C (*d*) 75.8, 49.3°C [DPMT 06]

17. Three rods of equal length of thermal conductivities k, $2k$ and $3k$ are symmetrically joined to a point. If temperatures of ends are 0°C, 50°C and 100°C respectively, what is the temperature of the junction ?

(*a*) 20°C (*b*) $\frac{100}{3}$°C

(*c*) $\frac{200}{3}$°C (*d*) none of these. [DPMT 08]

18. A body cools from 80°C to 64°C in 5 minutes and same body cools from 80°C to 52°C in 10 minutes. What is the temperature of surroundings ?

(*a*) 24°C (*b*) 28°C

(*c*) 22°C (*d*) 25°C [DPMT 08]

19. If σ is Stefan's constant and b is Wien's constant, then the dimensions of σb^4 are

(*a*) $[M^0L^0T^0]$ (*b*) $[ML^{-1}T]$

(*c*) $[ML^6T^{-3}]$ (*d*) $[M^1L^4T^{-3}]$ [DPMT 08]

20. Mud houses are cooler in summer and warmer in winter because

(*a*) mud is a good conductor of heat

(*b*) mud is a superconductor of heat

(*c*) mud is a bad conductor of heat

(*d*) none of these. [DPMT 05]

21. A perfectly black body is one whose

(*a*) absorptive power is 1

(*b*) absorptive power is 0

(*c*) emissive power is 1

(*d*) absorptive power is 0.5. [DPMT 98]

22. The emissive power of a black body is proportional (T = absolute temperature) to

(*a*) $E \propto T^0$ (*b*) $E \propto T^2$

(*c*) $E \propto T^4$ (*d*) $E \propto T^2$ [DPMT 80]

Scanned by CamScanner

23. Two identical metallic balls, whose temperatures are 200°C and 400°C respectively, are placed in an enclosure at 27°C. The ratio of heat-loss of the balls will be

(a) 1 : 2
(b) 1 : 4
(c) $\frac{(473)^4-(300)^4}{(673)^4-(300)^4}$
(d) $\frac{(200)^4-(27)^4}{(400)^4-(27)^4}$ [VMMC 07]

24. A body at a temperature of 727°C and having surface area 5 cm^2, radiates 300 J of energy each minute. The emissivity (*Given* : Boltzmann constant = 5.67×10^{-8} $Wm^{-2}K^{-4}$) is

(a) e = 0.18
(b) e = 0.02
(c) e = 0.2
(d) e = 0.15 [VMMC 06]

25. If a body is heated from 27°C to 927°C, then the ratio of their energies of radiations emitted will be

(a) 1 : 4
(b) 1 : 64
(c) 1 : 16
(d) 1 : 256 [DPMT 93, 98, 03]

26. A black body radiates energy at the rate of 1×10^5 J/sm^2 at a temperature of 227°C. The temperature to which it must be heated so that it radiates energy at rate of 1×10^9 J/sm^2

(a) 5000 K
(b) 5000°C
(c) 500 K
(d) 500°C [DPMT 04]

27. The room temperature is +20°C when outside temperature is −20°C and room temperature is +10°C when outside temperature is −40°C. Find the temperature of the radiator heating the room.

(a) 30°C
(b) 45°C
(c) 60°C
(d) 80°C [DPMT 06]

28. Rate of cooling at 600 K, if surrounding temperature is 300 K is *R*.

The rate of cooling at 900 K is

(a) 16/3 *R*
(b) 2 *R*
(c) 3 *R*
(d) 2 /3 *R* [DPMT 02]

29. The surface temperature of the sun is *T* K and the solar constant for a planet is *S*. The sun subtends an angle θ at the planet. Then

(a) $S \propto T^4$
(b) $S \propto T^2$
(c) $S \propto \theta^2$
(d) $S \propto \theta$

30. If wavelength at 4500 K is λ_m, the wavelength at 1500 K is [DPMT 00]

(a) $3\lambda_m$
(b) $\lambda_m/3$
(c) $9\lambda_m$
(d) $\lambda_m/9$

31. A black body at 200 K is found to emit maximum energy at a wavelength 14 μm. When its temperature is raised to 1000 K, then wavelength at which maximum energy emitted is [DPMT 02]

(a) 14 mm
(b) 7 μm
(c) 2.8 μm
(d) 28 mm

32. The triple point of water is [VMMC 05]

(a) 273.16°C
(b) 273.16 K
(c) 273.16°F
(d) 0.15 K

33. During constant temperature, we feel colder on the day when the relative humidity is [VMMC 07]

(a) 85%
(b) 40%
(c) 60%
(d) 25% [DPMT 93]

Assertions and Reasons

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion.
(c) If assertion is true, but reason is false.
(d) If both assertion and reason are false.

34. **Assertion.** In a pressure cooker the water is brought to boil. The cooker is then removed from the stove. Now on removing the lid of the pressure cooker, the water starts boiling again.

Reason. The impurities in water bring down its boiling point. [AIIMS 04, VMMC 04]

Answers and Explanations

1. (b) $\frac{C}{5}=\frac{F-32}{9}$ or $\frac{C}{5}=\frac{C-32}{9}$

$4C=-160$

$C=F=-40°$.

2. (c) $R_t=R_0(1+\alpha t)$

$$\frac{R_{500}}{R_{150}}=\frac{1+0.0045\times500}{1+0.0045\times150}=\frac{3.25}{1.675}\simeq2$$

$R_{500}=2\times R_{150}=2\times133=266\Omega$.

Scanned by CamScanner

3. (c) When a body is heated, the maximum change is in its volume.

4. (a) $V_t = V_0(1+\gamma t)$

$$\frac{V_{100}}{V_{20}} = \frac{1+100\gamma}{1+20\gamma} = \frac{125}{100} = \frac{3}{4}$$

$$\gamma = \frac{1}{300} = 0.0033°\text{C}^{-1}.$$

5. (d) $\gamma = \frac{\rho_0 - \rho_t}{\rho_0 \times t} = \frac{10-9.7}{10\times 100} = 3\times 10^{-4}$

$$\therefore \quad \alpha = \frac{\gamma}{3} = 10^{-4}\,°\text{C}^{-1}.$$

6. (a) Let h be the length of mercury column at 100°C. Then

Expansion in mercury = Volume of stem

$$\gamma V \Delta T = Ah$$

$$h = \frac{\gamma V \Delta T}{A} = \frac{18\times 10^{-5}\times 10^{-6}\times 100}{0.002\times 10^{-4}}\ \text{m}$$

$$= 9\times 10^{-2}\ \text{m}$$

$$= \mathbf{9\ cm}.$$

7. (d) $\rho_t = \rho_0(1-\gamma\Delta T)$

$$= 13.6(1-0.18\times 10^{-3}\times 200)$$

$$= \mathbf{13.11\ g/cc}.$$

8. (b) The amount of heat required to raise the temperature of a body through 1 K is called its thermal capacity.

9. (a) $mc\Delta T = mgh$

$$\therefore \quad \Delta T = \frac{gh}{c} = \frac{10\times 500}{4200} = \frac{50}{42}\,°\text{C} = \mathbf{1.10°C}$$

10. (a) Energy lost by X g of steam

= Energy gained by Y g of ice

$$X\times 540 = Y\times 80 + Y\times 1\times 100$$

$$\therefore \quad \frac{X}{Y} = \frac{180}{540} = \frac{1}{3}.$$

11. (d) $\frac{\Delta l}{l}\times 100 = 100\alpha\Delta T$

$$\frac{\Delta V}{V}\times 100 = 100\gamma\Delta T = 100\times 3\alpha\times\Delta T$$

Hence for the same rise of temperature, increase in volume is 3 times the increase in length.

12. (b) As heat flows from higher temperature to lower temperature, so a temperature gradient is required.

13. (c) Heat always flows from a body at higher temperature to a body at lower temperature.

14. (d) Thermal resistance $= \frac{d}{KA}$

$$\therefore \quad \frac{d_1}{K_1 A} = \frac{d_2}{K_2 A} \quad \text{or} \quad \frac{d_1}{d_2} = \frac{K_1}{K_2} = \frac{5}{4} = \mathbf{5:4}.$$

15. (b) Let T be temperature of the junction. In the steady state,

$$\frac{Q_1}{t} = \frac{Q_2}{t}$$

$$\frac{K_1 A(100-T)}{d} = \frac{K_2 A(T-T_2)}{d}$$

or $\quad 5(100-T) = 3(T-20)$

or $\quad 8T = 560 \quad \text{or} \quad T = \mathbf{70°C}$

16. (a) In the steady state,

$$\frac{Q_A}{t} = \frac{Q_B}{t} = \frac{Q_C}{t}$$

$$\frac{2KA(100-T_1)}{d} = \frac{KA(T_1-T_2)}{d} = \frac{KA(T_2-0)}{2d}$$

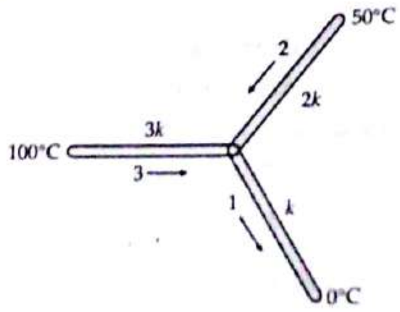
or $\quad 2(100-T_1) = T_1 - T_2 = \frac{T_2}{2}$

$$T_1 = \frac{3}{2}T_2 \quad \text{or} \quad T_2 = \frac{2}{3}T$$

$$2(100-T_1) = \frac{T_1}{3}$$

$$\therefore \quad T_1 = \mathbf{85.7°C},\ T_2 = \mathbf{57.1°C}.$$

17. (c) Let T be the temperature of the junction



In steady state, rate of flow of heat through rod 1 = sum of rates of flow of heat through rods 2 and 3

$$\frac{Q_1}{t} = \frac{Q_2}{t} + \frac{Q_3}{t}$$

$$\frac{kA(T-0)}{d} = \frac{2kA(50-T)}{d} + \frac{3kA(100-T)}{d}$$

$$T = 2(50-T) + 3(100-T)$$

$$\therefore \quad T = \frac{200}{3}\,°\text{C}$$

Scanned by CamScanner

18. (*a*) By Newton's law of cooling,

$$\frac{\theta_1-\theta_2}{t}=k\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$

where θ_0 is the temperature of the surroundings.

$$\therefore \quad \frac{80-64}{5}=k\left(\frac{80+64}{2}-\theta_0\right) \quad ...(i)$$

$$\frac{80-52}{10}=k\left(\frac{80+52}{2}-\theta_0\right) \quad ...(ii)$$

Solving (*i*) and (*ii*), θ_0 = **24°C**.

19. (*d*) $\sigma=\frac{E}{T^4}=\frac{\text{Energy / time / area}}{T^4}$

$b=\lambda_m \cdot T$

$$\therefore \quad \sigma b^4=\frac{\text{Energy}}{\text{Time}\times\text{Area}}\cdot(\lambda_m)^4$$

$$[\sigma b^4]=\frac{[ML^2T^{-2}]}{[TL^2]}\times[L]^4=[ML^4T^{-3}]$$

20. (*c*) Mud is a bad conductor of heat. It does not allow heat to come in from outside in summer and it does not allow heat to go out from the house in winter.

21. (*a*) A black body is one which absorbs all the radiation incident upon it. Its absortive power is unity.

22. (*c*) For a black body, $E\propto T^4$.

23. (*c*) From Stefan Boltzmann's law,

$$\frac{E_1}{E_2}=\frac{T_1^4-T_0^4}{T_2^4-T_0^4}=\frac{(273+200)^4-(273+27)^4}{(273+400)^4-(273+27)^4}$$

$$=\frac{(473)^4-(300)^4}{(673)^4-(300)^4}.$$

24. (*a*) Here $A=5\text{ cm}^2=5\times10^{-4}\text{ m}^2$,

$T=727+273=1000$ K.

Energy radiated per second $=\frac{300}{60}=5\text{ Js}^{-1}$

$$E=e(\sigma T^4)\times A \quad\Rightarrow\quad e=\frac{E}{\sigma T^4\times A}$$

$$=\frac{5}{5.67\times10^{-8}\times(1000)^4\times5\times10^{-4}}=\mathbf{0.18}.$$

25. (*b*) $\frac{E_1}{E_2}=\left(\frac{T_1}{T_2}\right)^4=\left(\frac{273+27}{273+927}\right)^4=\frac{1}{64}$.

26. (*a*) From Stefan's law,

$$T_2=\left(\frac{E_2}{E_1}\right)^{1/4}T_1=\left(\frac{10^9}{10^5}\right)^4\times(273+227)$$

$=10\times500=$ **5000 K**.

27. (*c*) Let T be the temperature of the radiator. According to Newton's law of cooling,

$K_1(T-T_{in})=K_2(T_{in}-T_{out})$

In first case,

$K_1(T-20)=K_2[20-(-20)]$

In second case,

$K_2(T-10)=K_2[10-(-40)]$

$$\therefore \quad \frac{T-20}{T-10}=\frac{40}{50}$$

or $10T=600$ or $T=60°$C

28. (*b*) Rate of cooling $\propto(T_2-T_1)$

$$\frac{E_2}{E_1}=\frac{900-300}{600-300}=2$$

$E_2=2E_1=$ **2*R***.

29. (*a*) Power radiated by the sun

$P=\sigma AT^4=\sigma(4\pi R^2)T^4$

Energy received/area/s at the planet,

$$S=\frac{P}{4\pi r^2}=\frac{\sigma(4\pi R^2)T^4}{4\pi r^2}$$

$$=\frac{1}{4}\sigma T^4\left(\frac{2R}{r}\right)^2$$

But $\frac{2R}{r}=\theta$

= angle subtended by the sun at the planet

$$\therefore \quad S=\frac{1}{4}\sigma T^4\theta^2$$

Clearly, $S\propto T^4$.

30. (*c*) $\lambda'_m T'=\lambda_m T$

$\lambda'_m\times1500=\lambda_m\times4500$

or $\lambda'_m=3\lambda_m$.

31. (*c*) By Wien's law,

$200\times14=1000\times T'$

$T'=$ **2.8 μm**.

32. (*b*) Triple point of water corresponds to a temperature of 273.16 K and a pressure of 0.46 cm of Hg.

33. (*d*) If relative humidity is low, we feel colder because of rapid evaporation of sweat from our body.

34. (*c*) The assertion is true but the reason is false. When the lid is removed, pressure decreases. This decreases the boiling point of water which begins to boil.

Scanned by CamScanner

CHAPTER

12

THERMODYNAMICS

12.1 THERMODYNAMICS AND RELATED TERMS

1. *Define the term thermodynamics. How does it differ from kinetic theory of gases and mechanics ?*

Thermodynamics. *Thermodynamics is the branch of science that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy.* It mainly deals with the transformation of heat into mechanical work and vice versa. Its great success lies in the fact that it explains the bulk properties of matter in terms of few macroscopic variables such as pressure, volume, temperature, mass and composition which are easily observable and directly measurable.

Thermodynamics versus kinetic theory of gases. Thermodynamics is a macroscopic science. It deals with bulk systems without going into the molecular constitution of matter. On the other hand, kinetic theory of gases deals with the molecular distribution of velocities in any gas.

Thermodynamics versus mechanics. Mechanics deals with motion of a system as a whole (in fact with its centre of mass) under the action of several forces and torques. Thermodynamics is not concerned with the motion of a system as a whole. It is concerned with the internal macroscopic state of the body. When a bullet is fired, its mechanical state changes due to the increase in the K.E. of its centre of mass but its temperature is not affected. But when the bullet pierces a wood and stops, its kinetic energy changes into heat increasing the temperature of the bullet and surrounding wooden layers. Thus temperature is related to the internal disordered motion of the molecules of the bullet, not to the motion of the bullet as a whole.

2. *Define the terms thermodynamic system, surroundings, thermodynamic variable and equation of state.*

(*i*) **Thermodynamic system.** *An assembly of a very large number of particles having a certain value of pressure, volume and temperature is called a thermodynamic system.*

(*ii*) **Surroundings.** *Everything outside the system which can have a direct effect on the system is called its surroundings.*

(*iii*) **Thermodynamic variables.** The quantities like pressure (P), volume (V), and temperature (T) which help us to study the behaviour of a thermodynamic system are called thermodynamic variables.

(*iv*) **Equation of state.** *The mathematical relation between the pressure, volume and temperature of a thermodynamic system is called its equation of state.* For example, the equation of state for n moles of an ideal gas can be written as

$$PV = nRT.$$

12.2 THERMAL EQUILIBRIUM

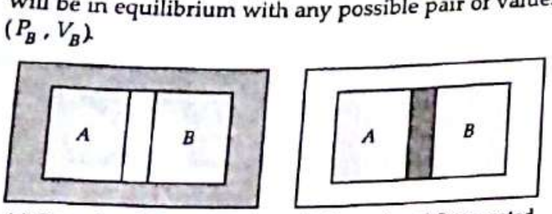
3. *Define thermal equilibrium. How is it attained ?*

Thermal equilibrium. *Two systems are said to be in thermal equilibrium with each other if they have the same temperature.*

Consider two gases A and B contained in two different vessels. Let the pressure and volume of the

Scanned by CamScanner

gases by (P_A, V_A) and (P_B, V_B) respectively. As shown in Fig. 12.1(*a*), if the two vessels are separated by an *adiabatic wall* (an insulating wall that does not allow the flow of heat), then any possible pair of values (P_A, V_A) will be in equilibrium with any possible pair of values (P_B, V_B).



(*a*) Gases *A* and *B* separated by an adiabatic wall.

(*b*) Gases *A* and *B* separated by a diathermic wall.

Fig. 12.1

As shown in Fig. 12.1(*b*), if *A* and *B* are now separated by a *diathermic wall* (a conducting wall that allows heat to flow through it), then the pressure and volume variables of the two gases change to (P'_A, V'_A) and (P'_B, V'_B) such that the new states of *A* and *B* are in equilibrium with each other. There is no more flow of heat. The two systems attain equal temperature and we say they are in equilibrium with each other.

12.3 THERMODYNAMIC EQUILIBRIUM

4. *When is a system said to be in the state of thermodynamic equilibrium ?*

Thermodynamic equilibrium. *A system is said to be in the state of thermodynamic equilibrium if the macroscopic variables describing the thermodynamic state of the system do not change with time.* Consider a gas inside a closed rigid container completely insulated from the surroundings. If the pressure, volume, temperature, mass and composition of the gas do not change with time, then it is in a state of thermodynamic equilibrium.

A system in the state of thermodynamic equilibrium possesses the following equilibria simultaneously :

(*i*) ***Mechanical equilibrium.*** There is no unbalanced force in its interior or between the system and the surroundings.

(*ii*) ***Thermal equilibrium.*** All parts of the system and the surroundings are at the same temperature.

(*iii*) ***Chemical equilibrium.*** The system does not undergo any spontaneous change in its internal structure due to chemical reaction, diffusion, etc.

12.4 ZEROTH LAW OF THERMODYNAMICS

5. *State and explain zeroth law of thermodynamics. How does it lead to the concept of temperature ?*

Zeroth law of thermodynamics. *It states that if two systems A and B are separately in thermal equilibrium with a third system C, then A and B are also in thermal equilibrium with each other.* Fig. 12.2 shows two systems *A* and *B* separated by an adiabatic wall (through which heat does not flow). They are separated from system *C* by a diathermic wall (through which heat can flow). The systems *A* and *C* and *B* and *C* will reach thermal equilibrium separately. If adiabatic wall between *A* and *B* is removed, there will be no exchange of heat showing that *A* and *B* are already in thermal equilibrium.

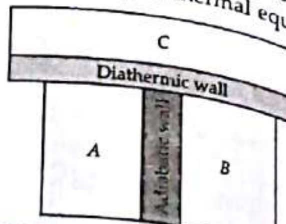


Fig. 12.2 Zeroth law of thermodynamics

Concept of temperature. Zeroth law of thermodynamics implies that *temperature is a physical quantity which has the same value for all systems which are in thermal equilibrium with each other.* Temperature of a system determines whether it is in thermal equilibrium or not with another system. Thus if *A* and *B* are separately in thermal equilibrium *C*, $T_A = T_C$ and $T_B = T_C$. This shows that $T_A = T_B$ *i.e.*, the systems *A* and *B* are also in thermal equilibrium.

As the modern concept of temperature follows from the zeroth law of thermodynamics, so this law may be stated in a more fundamental way as follows :

There exists a scalar quantity called temperature which is a property of all thermodynamic systems such that temperature equality is a necessary and sufficient condition for thermal equilibrium.

For Your Knowledge

- The temperature which was first defined as the *degree of hotness* and later on the *condition determining the flow of heat*, is now regarded as *one of the seven fundamental quantities* like mass, length, time, etc.
- The zeroth law of thermodynamics was formulated by *R.H. Fowler* in 1931 long after the First and Second laws of thermodynamics were stated. But as this law leads to the concept of the fundamental quantity temperature, so this law was called the zeroth law.

12.5 HEAT, INTERNAL ENERGY AND WORK

6. *What do you mean by internal energy of a system ? Is it a state variable ? What is the nature of the internal energy of an ideal gas ?*

Internal energy. *The internal energy of a system is the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest.*

The molecules of a real gas exert mutual force of attraction on one another. Hence they possess intermolecular potential energy. If the volume of the gas increases, work is done by the gas against intermolecular attraction and so its potential energy increases. Thus *intermolecular potential energy of a real gas is a function of its volume.*

The molecules of a gas are always in a state of random motion. The motion may be translational, rotational and vibrational. Hence the molecules possess kinetic energy. As the temperature increases, the average kinetic energy of the gas molecules also increases. Thus *the internal kinetic energy of a gas is a function of its temperature.*

The internal energy does not include the over-all kinetic energy of the system as a whole. It includes only the (disordered) energy associated with the random motion of the molecules of the system. We donate it by *U*.

Internal energy of a system is a thermodynamic state variable. That is its value depends only on the state of existence of the system and not on the path along which that state has been brought about. Thus the internal energy of a given mass of a gas depends only on its state described by the specific values of pressure, volume and temperature.

Internal energy of an ideal gas is purely kinetic in nature. In an ideal gas, there are no molecular forces of attraction. So the gas does not possess intermolecular potential energy. Its internal energy is just the sum of kinetic energies associated with various random (translational, rotational and vibrational) motions of its molecules. Thus the internal energy of an ideal gas is wholly kinetic in nature and depends only on its temperature.

7. *Describe the two ways of changing the internal energy of a system. Are heat and work state variables ?*

Heat and work as two distinct modes of energy transfer. As shown in Fig. 12.3, consider a fixed mass of gas in a cylinder provided with a movable piston. The internal energy of the gas can be increased in *two* ways :

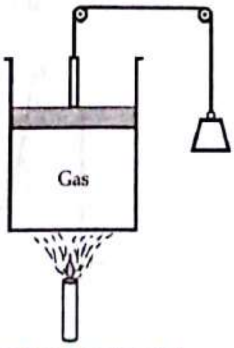


Fig. 12.3 Heat and work as two different modes of energy transfer to a system.

(*i*) Place the cylinder over a hot body. Heat energy will flow from the hotter body to the gas due to temperature difference. This increases the internal energy of the gas.

(*ii*) Push the piston down by raising some weight attached to it. Work is done on the gas. This also increases the internal energy of the gas.

No, heat and work are not state variables.

8. *Give difference between heat and work.*

Difference between heat and work.

(*i*) *Heat is a mode of energy transfer due to temperature difference between the system and the surroundings. Work is the mode of energy transfer brought about by means that do not involve temperature difference* such as moving the piston of a cylinder containing the gas, by raising or lowering the weight connected to it.

(*ii*) When heat is supplied to a gas, its molecules move faster in all directions at random. So *heat is a mode of energy transfer that produces random motion*, when a piston compresses a gas to do work on it, it forces the molecules to move in the direction of piston's motion. So *work may be regarded as the mode of energy transfer that produces organised motion.*

9. *State the sign conventions used in the measurement of heat, work and internal energy.*

Sign conventions used :

(*i*) *Heat absorbed by a system is* ***positive****. Heat given out by a system is* ***negative****.*

(*ii*) *Work done by a system is* ***positive****. Work done on a system is* ***negative****.*

(*iii*) *The increase in internal energy of a system is* ***positive****. The decrease in internal energy of a system is* ***negative****.*

For Your Knowledge

▲ The thermodynamic state of a system is characterised by its internal energy, not heat. That is, internal energy is a state variable. For this reason, the statement like 'a gas in given state has a certain amount of energy' is meaningful.

▲ In thermodynamics, ***heat and work are not state variables***. These are the modes of energy transfer to a system resulting in the change in its internal energy. Thus the statement like 'a gas in a given state has certain amount of heat or work' is meaningless, on the other hand, the statement like 'a certain amount of heat is supplied to the system or a certain amount of work is done by the system' is meaningful.

Scanned by CamScanner

12.6 INDICATOR DIAGRAM

10. *What is an indicator diagram ? What is its importance ?*

Indicator diagram. The state of a thermodynamic system can be completely described if only two thermodynamical variables are known because the third variable gets automatically fixed by the equation of the state of the system. *A graphical representation of the state of system with the help of two thermodynamical variables is called an indicator diagram.* A graph drawn between the pressure and volume of a gas under thermodynamic operation is called *P-V* diagram. Such diagrams are drawn with the help of a device called *indicator* which records the changes in volume and pressure accompanying the movement of the piston in the cylinder.

Fig. 12.4(*a*) shows *P-V* diagram for a system undergoing expansion from the state $A(P_1, V_1)$ to $B(P_2, V_2)$, while Fig. 12.4(*b*) shows the *P-V* diagram for a system undergoing compression from the state $A(P_1, V_1)$ to $B(P_2, V_2)$.

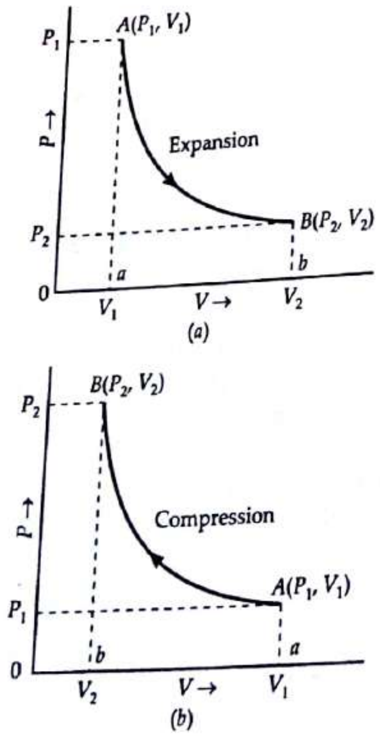


Fig.12.4 Indicator diagrams.

Importance of *P-V* diagram. The area under the *P-V* diagram is numerically equal to the work done by a system or on the system.

12.7 WORK DONE DURING EXPANSION

11. *Describe an analytical method for determining the work done during the expansion of gas.*

Analytical method for the work done during expansion. Consider a gas contained in a cylinder of cross-sectional area *A* and provided with a frictionless movable piston. Let *P* be the pressure of the gas.

Force exerted by the gas on the piston,

$$F = P \times A$$

Suppose the gas expands a little and pushes out the piston through a small distance dx. The work done by the gas is

$$dW = F dx = PA\, dx = P\, dV$$

where $dV = A dx$, is the change in volume of the gas.

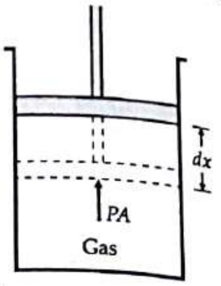


Fig.12.5 Work done during expansion.

The total work done by the gas when its volume increases from V_1 to V_2 will be

$$W = \int dW = \int_{V_1}^{V_2} P dV.$$

12. *What is non-cyclic process ? Show that the area under the P-V diagram gives the work done by a system in a non-cyclic process.*

Non-cyclic process. *A non-cyclic process is one in which the system does not return to its initial state.*

Indicator diagram method for the work done during expansion. In Fig. 12.6, the points *A* and *B* represent the *initial state* (P_1, V_1) and final state (P_2, V_2) respectively of a system on a *P-V* diagram. At any point *a*, let *P* and *V* be the pressure and volume

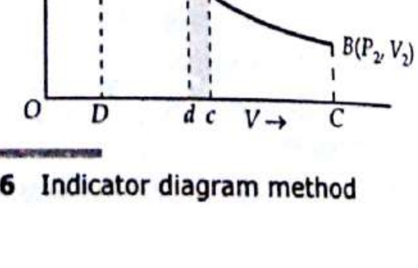


Fig. 12.6 Indicator diagram method

Scanned by CamScanner

respectively. Suppose that the volume increases from V to $V + dV$ corresponding to a point b on the indicator diagram such that the pressure remains constant. Then

$$ad = bc = P \quad \text{and} \quad cd = dV$$

The small work done when the system changes from state a to state b,

$$dW = PdV = ad \times cd$$
$$= \text{area of shaded strip } abcd$$

The total work done by the gas during the expansion from the initial state $A(P_1, V_1)$ to the final state $B(P_2, V_2)$ can be obtained by adding the areas of all such strips formed between AD and BC under the P-V diagram. Clearly, then the total work done will be

$$W = \text{area } ABCDA$$

or $$W = \int_{V_1}^{V_2} P\,dV = \text{area under } P\text{-}V \text{ diagram}$$

Hence the work done by a system is numerically equal to the area enclosed between the P-V diagram and the volume- axis.

During *expansion*, the area under the P-V diagram is traced in the *clockwise direction*. Work is done by the gas and is taken positive.

During *compression*, the area under the P-V diagram is traced in the *anticlockwise direction*. Work is done on the gas and is taken *negative*.

12.8 WORK DONE DURING A CYCLIC PROCESS

13. *What is cyclic process ? Prove that the net work done during a cyclic process is numerically equal to the area of the loop representing the cycle.*

Cyclic process. *Any process in which the system returns to its initial state after undergoing a series of changes is known as a cyclic process.*

Work done during a cyclic process. Suppose a gas expands from the initial state A to the final state B after undergoing a series of changes of pressure and volume, along the path AXB.

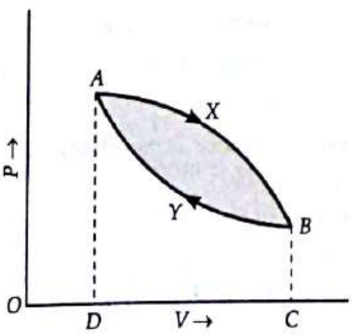


Fig. 12.7 *P-V* diagram for a cyclic process

Work done by the gas during the expansion is

$$W_1 = + \text{ area } AXBCDA$$

Now the gas is subjected to a different series of changes of pressure and volume so that it returns to its initial state A via the path BYA.

Work done on the gas during the compression is

$$W_2 = - \text{ area } BYADCB$$

During compression, work is done on the gas which is taken negative.

The net work done during the cyclic process is

$$W = W_1 + W_2$$
$$= \text{area } AXBCDA - \text{area } BYADCB$$
$$= + \text{ area } AXBYA$$

Conclusion. For a cyclic process :

(*i*) Work done per cycle is numerically equal to the area of the loop representing the cycle.

(*ii*) If the closed loop is traced in the *clockwise direction*, the expansion curve lies above the compression curve ($W_1 > W_2$), the area of the loop is *positive*, indicating that the *net work is done by the system.*

(*iii*) If the closed loop is traced in the anticlockwise direction, the expansion curve lies below the compression curve, ($W_1 < W_2$), the area of the loop is *negative*, indicating that *the net work is done on the system.*

Examples based on Work Done during a Cyclic Process

CONCEPTS USED

1. Work done during the expansion or compression of a gas is equal to the area enclosed between the *P-V* curve and the volume axis.
2. Work done per cycle
   = Area of the loop representing the cycle.
3. If the loop is traced *clockwise*, the work done is *positive* and work is done by the system.
4. If the loop is traced *anticlockwise*, the work done is *negative* and work is done on the system.

UNITS USED

When pressure P is in Nm^{-2} and volume V in m^3, the work done W is in joule.

EXAMPLE 1. *One mole of an ideal gas undergoes a cyclic change ABCD. From the given diagram (Fig. 12.8), calculate the net work done in the process.*

$$1 \text{ atm} = 10^6 \text{ dyne cm}^{-2}.$$

Solution. The work done in a cyclic process is equal to the area of the loop.

Scanned by CamScanner

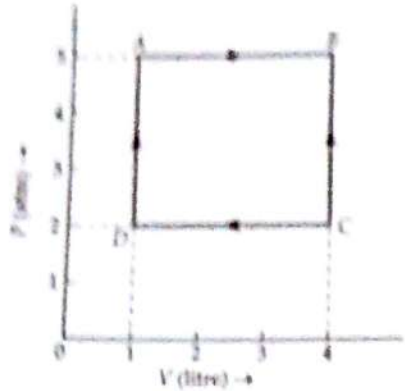


Fig. 12.8

As the loop *ABCD* is traced in the clockwise direction, the work done is positive.

∴ $W = +\text{ Area } ABCD = DC \times AD$

Now $DC = 4 - 1 = 3 \text{ litre} = 3 \times 10^3 \text{ cm}^3$

$AD = 5 - 2 = 3 \text{ atm} = 3 \times 10^6 \text{ dyne cm}^{-2}$

∴ $W = 3 \times 10^3 \times 3 \times 10^6 = \mathbf{9 \times 10^9 \text{ erg}}$.

EXAMPLE 2. *One mole of an ideal gas undergoes a cyclic change ABCD where the (P, V) co-ordinates are A(5, 1) B(5, 3) C(2, 3) and D(2, 1). P is in atmosphere and V is in litre. Calculate work done along AB, BC, CD and DA and also net work done in the process. Given 1 atmosphere* $= 1.01 \times 10^5 \text{ Nm}^{-2}$.

Solution. (*i*) Work done along *AB*

$$W_1 = \text{area } ABFEA = EA \times EF$$
$$= (5 \times 1.01 \times 10^5) \times 2 \times 10^{-3} = \mathbf{1010 \text{ J}}.$$

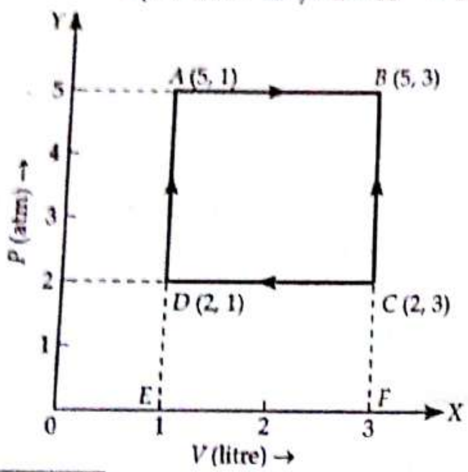


Fig. 12.9

(*ii*) Work done along *BC*, $W_2 = PdV = 0$.

(*iii*) Work done along *CD*

$$W_3 = -\text{ area } CDEFC = -EF \times CF$$
$$= -(2 \times 1.01 \times 10^5) \times 2 \times 10^{-3} = \mathbf{-404 \text{ J}}.$$

(*iv*) Work done along *DA*, $W_4 = PdV = 0$.

Net work done in the process

$$W = W_1 + W_2 + W_3 + W_4$$
$$= 1010 + 0 - 404 + 0 = \mathbf{606 \text{ J}}.$$

EXAMPLE 3. *The P-V diagram (Fig. 12.10), for a cycle process is a triangle ABC drawn in order. The coordinates of A, B, C are (4, 1), (2, 4) and (2, 1). The coordinates are in the order P, V. Pressure is in* Nm^{-2} *and volume is in litre. Calculate the work done during the process from A to B, B to C and C to A. Also calculate the work done in the complete cycle.*

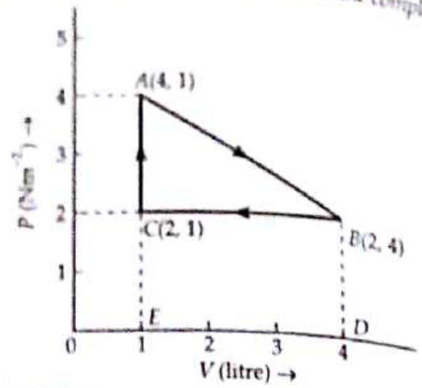


Fig. 12.10

Solution. (*i*) *Work done during the process from A to B (expansion) is*

$$W_{AB} = +\text{ Area of trapezium } ABDEA$$
$$= \text{Area of } \Delta ABC + \text{Area of rectangle } BDEC$$
$$= \frac{1}{2} BC \times AC + DE \times CE$$
$$= \frac{1}{2}(4-1) \text{ litre} \times (4-2) \text{ Nm}^{-2} + (4-1) \text{ litre} \times (2-0) \text{ Nm}^{-2}$$
$$= \frac{1}{2} \times 3 \times 10^{-3} \text{ m}^3 \times 2 \text{ Nm}^{-2} + 3 \times 10^{-3} \text{ m}^3 \times 2 \text{ Nm}^{-2}$$
$$= \mathbf{9 \times 10^{-3} \text{ J}}.$$

As the gas is expanding, so the work done is positive.

(*ii*) *Work done during the process from B to C (compression) is*

$$W_{BC} = -BCDE = -DE \times CE$$
$$= -3 \times 10^{-3} \times 2 = \mathbf{-6 \times 10^{-3} \text{ J}}.$$

As the gas is compressed, so the work done is negative.

(*iii*) *Work done during the process from C to A is*

$$W_{CA} = 0$$

This is because there is no change in volume in going from *C* to *A*.

(*iv*) *Work done in the complete cycle,*

$$W = +\text{ Area } ABC = W_{AB} + W_{BC} + W_{CA}$$
$$= \mathbf{3 \times 10^{-3} \text{ J}}.$$

Here *W* is positive because the cycle *ABCA* is traced in the clockwise direction.

Scanned by CamScanner

PROBLEMS FOR PRACTICE

1. An ideal monoatomic gas is taken round the cycle ABCDA, where co-ordinates of A, B, C, D on P-V diagram are $A(p, V)$, $B(2p, V)$, $C(2p, 2V)$ and $D(p, 2V)$. Calculate work done during the cycle. (Ans. pV)

2. Calculate net work done by the gas whose thermodynamical behaviour is represented by right angled triangle ABC on P-V diagram. The P-V co-ordinates are A (20, 6), B (10, 12) and C (10, 6), where P is in Nm^{-2} and V is in m^3. (Ans. 30 J)

12.9 FIRST LAW OF THERMODYNAMICS

14. *State and explain first law of thermodynamics.*

First law of thermodynamics. It is simply the law of conservation of energy applied to any system in which energy transfer (in the form of heat or work) from or to the surroundings is taken into consideration. According to the first law of thermodynamics, *if some heat is supplied to a system which is capable of doing work, then the quantity of heat absorbed by the system will be equal to the sum of the increase in its internal energy and the external work done by the system on the surroundings.*

Let

ΔQ = Heat supplied to the system by the surroundings

ΔW = Work done by the system on the surroundings

ΔU = Change in internal energy of the system

Then according to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

As shown in Fig. 12.11, suppose the system is a gas contained in a cylinder provided with a movable piston. Then the gas does work in moving the piston. The work done by the system against a constant pressure P is

ΔW = Force × Distance

= Pressure × Area × Distance

$= PA\,dx$

or $\Delta W = P\,\Delta V$

where $\Delta V = A dx$ = the change in the volume of the gas.

So the first law of thermodynamics takes the form,

$$\Delta Q = \Delta U + P\,\Delta V$$

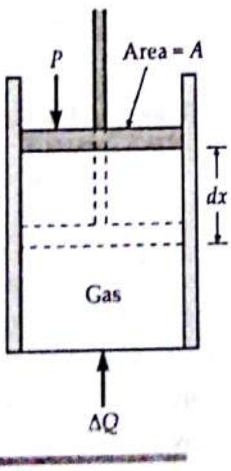


Fig. 12.11 Absorption of heat by a gas in a cylinder

Examples based on First Law of Thermodynamics

FORMULAE USED

1. According to first law of thermodynamics $dQ = dU + dW = dU + PdV$
2. For change of state, $dQ = mL$
3. For rise in temperature, $dQ = mC\,\Delta T$
4. Change in internal energy, $dU = U_f - U_i$

UNITS USED

dQ, dU and dW are all in joule. Pressure P is in Nm^{-2} and the change in volume dV in m^3.

EXAMPLE 4. *1 g of water at 100°C is converted into steam at the same temperature. If the volume of steam is 1671 cm^3, find the change in the internal energy of the system. Latent heat of steam = 2256 Jg^{-1}. Given 1 atmospheric pressure $= 1.013 \times 10^5\ Nm^{-2}$.*

Solution. Mass of water, $m = 1\ g = 10^{-3}$ kg

Latent heat of steam

$$L = 2256\ Jg^{-1} = 2256 \times 10^3\ J\,kg^{-1}$$

Atmospheric pressure, $P = 1.013 \times 10^5\ Nm^{-2}$

Volume of steam, $V_s = 1671\ cm^3 = 1671 \times 10^{-6}\ m^3$

Volume of water, $V_w = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{10^{-3}}{10^3} = 10^{-6}\ m^3$

According to first law of thermodynamics,

$$dQ = dU + PdV$$

or $$mL = dU + P(V_s - V_w)$$

∴ Change in internal energy is

$$dU = mL - P(V_s - V_w)$$
$$= 10^{-3} \times 2256 \times 10^3 - 1.013 \times 10^5 (1671 \times 10^{-6} - 10^{-6})$$
$$= 2256 - 1.013 \times 10^5 \times 10^{-6} \times 1670$$
$$= 2256 - 0.1013 \times 1670$$
$$= 2256 - 169.171 = \mathbf{2086.829\ J.}$$

EXAMPLE 5. *The volume of steam produced by 1 g of water at 100°C is 1650 cm^3. Calculate the change in internal energy during the change of state. Given $J = 4.2 \times 10^7$ erg cal^{-1}, $g = 981\ cm\,s^{-2}$. Latent heat of steam = 540 cal g^{-1}.*

Solution. Here $J = 4.2 \times 10^7$ erg cal^{-1}

Latent heat of steam, $L = 540$ cal g^{-1}

Mass of water = 1 g

Temperature of water = 100°C

Initial volume, $V_1 = 1\ cm^3$

Final volume, $V_2 = 1650\ cm^3$

Scanned by CamScanner

∴ Change in volume,

$$dV = V_2 - V_1 = 1650 - 1 = 1649 \text{ cm}^3$$

When 1 g of water at 100°C is changed to steam at 100°C, temperature remains constant. So the heat supplied is

$$dQ = mL = 1 \times 540 = 540 \text{ cal} = 540 \times 4.2 \times 10^7 \text{ erg}$$

Pressure, $P = 1 \text{ atm} = 76 \times 13.6 \times 981 \text{ dyne cm}^{-2}$.

From first law of thermodynamics,

$$dU = dQ - PdV$$
$$= 540 \times 4.2 \times 10^7 - 76 \times 13.6 \times 981 \times 1649$$
$$= 22.68 \times 10^9 - 1.67 \times 10^9$$
$$= 21.01 \times 10^9 = \mathbf{2.1 \times 10^{10}} \textbf{ erg.}$$

Example 6. *1.0 m^3 of water is converted into 1671 m^3 of steam at atmospheric pressure and 100°C temperature. The latent heat of vaporisation of water is 2.3×10^6 J kg^{-1}. If 2.0 kg of water be converted into steam at atmospheric pressure and 100°C temperature, then how much will be the increase in its internal energy ? Density of water 1.0×10^3 kg m^{-3}, atmospheric pressure $= 1.01 \times 10^5$ Nm^{-2}.*

Solution. Heat gained by 2.0 kg of water in changing into steam,

$$Q = mL = 2.0 \text{ kg} \times 2.3 \times 10^6 \text{ J kg}^{-1} = 4.6 \times 10^6 \text{ J}$$

Volume of 2.0 kg of water

$$= \frac{\text{Mass}}{\text{Density}} = \frac{2.0 \text{ kg}}{10^3 \text{ kg m}^{-3}} = 2.0 \times 10^{-3} \text{ m}^3$$

Volume of steam formed by 1.0 m^3 of water

$$= 1671 \text{ m}^3$$

∴ Volume of steam formed by 2.0×10^{-3} m^3 of water

$$= 1671 \times 2.0 \times 10^{-3} = 3342 \times 10^{-3} \text{ m}^3$$

When 2.0 kg of water changes into steam, the increase in volume is

$$dV = 3342 \times 10^{-3} - 2.0 \times 10^{-3} = 3340 \times 10^{-3} \text{ m}^3.$$

The external work done against the atmospheric pressure,

$$dW = PdV = 1.01 \times 10^5 \times 3340 \times 10^{-3}$$
$$= 0.337 \times 10^6 \text{ J}$$

From first law of thermodynamics, the increase in internal energy is

$$dU = Q - dW = 4.6 \times 10^6 - 0.337 \times 10^6$$
$$= \mathbf{4.263 \times 10^6 \text{ J}}.$$

The positive value of dU indicates that the internal energy of ice increases on melting.

Example 7. *At 0°C and normal atmospheric pressure, the volume of 1 g of water increases from 1 cm^3 to 1.091 cm^3 on freezing. What will be the change in its internal energy ? Normal atmospheric pressure is 1.013×10^5 Nm^{-2} and latent heat of melting of ice $= 80$ cal g^{-1}.*

Solution. Heat lost by water on freezing,

$$Q = -mL = -1 \text{ g} \times 80 \text{ cal g}^{-1} = -80 \text{ cal}$$

The negative sign shows that heat is given out by water.

During freezing water expands against the atmospheric pressure. Hence external work done by the water is

$$dW = P\,dV$$
$$= 1.013 \times 10^5 \text{ Nm}^{-2} \times [(1.091 - 1) \times 10^{-6} \text{ m}^3]$$
$$= 0.0092 \text{ joule} = \frac{0.0092}{4.18} \text{ cal} = 0.0022 \text{ cal}$$

As this work is done by ice, it is taken as positive.

According to first law of thermodynamics, the change in the internal energy will be

$$dU = Q - dW$$
$$= -80 \text{ cal} - 0.0022 \text{ cal} = -80.0022 \text{ cal.}$$

The negative sign shows the internal energy of water decreases on freezing.

Example 8. *5 moles of oxygen are heated at constant volume from 10°C to 20°C. What will be the change in the internal energy of the gas ? The gram molecular specific heat of oxygen at constant pressure, $C_p = 8$ cal $mole^{-1}$ °C^{-1} and $R = 8.36$ J $mole^{-1}$ °C^{-1}*

Solution. Here $C_p = 8$ cal mole^{-1} °C^{-1},

$$R = 8.36 \text{ J mole}^{-1} \,°\text{C}^{-1}$$
$$= \frac{8.36}{4.18} = 2 \text{ cal mole}^{-1} \,°\text{C}^{-1}$$

As $C_p - C_v = R$

∴ $C_v = C_p - R = 8 - 2 = 6$ cal mole^{-1} °C^{-1}

Heat gained by 5 moles of oxygen when heated from 10°C to 20°C,

$$Q = nC_v\,\Delta T = 5 \times 6 \times (20 - 10) = 300 \text{ cal.}$$

As the gas is heated at constant volume, the external work done,

$$dW = PdV = P \times 0 = 0$$

From first law of thermodynamics, the increase in internal energy is

$$dU = Q - dW = 300 - 0 = \textbf{300 cal.}$$

Example 9. *A metal of mass 1 kg at constant atmospheric pressure and at initial temperature 20°C is given a heat of 20000 J. Find (i) change in temperature, (ii) work done and (iii) change in internal energy. Given*

specific heat, $c = 400$ Jkg^{-1} °C^{-1},

coefficient of cubical expansion, $\gamma = 9 \times 10^{-5}$ °C^{-1}

density, $\rho = 9000$ kg m^{-3}

atmospheric pressure, $P = 10^5$ Nm^{-2}. **[IIT 05]**

Scanned by CamScanner

Solution. (*i*) As $\Delta Q = mc\Delta T$

Rise in temperature,

$$\Delta T = \frac{\Delta Q}{mc} = \frac{20000\ \text{J}}{1\ \text{kg} \times 400\ \text{Jkg}^{-1}\,{}^\circ\text{C}^{-1}} = 50^\circ\text{C}.$$

(*ii*) Density, $\rho = \frac{M}{V}$

Volume, $V = \frac{M}{\rho} = \frac{1\ \text{kg}}{9{,}000\ \text{kg m}^{-3}} = \frac{1}{9000}\ \text{m}^3$

Change in volume,

$$\Delta V = \gamma V \Delta T = 9 \times 10^{-5} \times \frac{1}{9000} \times 50$$

$$= 5 \times 10^{-7}\ \text{m}^3$$

Work done,

$$W = P\Delta V = 10^5 \times 5 \times 10^{-7} = \mathbf{0.05\ J.}$$

(*iii*) The change in internal energy,

$$\Delta U = \Delta Q - \Delta W$$

$$= 20000 - 0.05 = \mathbf{19999.95\ J.}$$

dheeraj koty

✘ Problems For Practice

1. Calculate the change in internal energy of a block of copper of mass 200 g when it is heated from 25°C to 75°C. Take specific heat of copper = 0.1 cal $g^{-1}\,{}^\circ C^{-1}$, and assume the change in volume as negligible. **(Ans. 4200 J)**

2. 1 kg of water at 373 K is converted into steam at the same temperature. The volume of 1 cm^3 of water becomes 1671 cm^3 on boiling. Calculate change in the internal energy of the system, if heat of vaporisation is 540 cal g^{-1}. Given standard atmospheric pressure = $1.013 \times 10^5\ Nm^{-2}$. **(Ans. 499.84 kcal)**

3. Calculate the change in internal energy when 5 g of air is heated from 0° to 4°C at constant volume. The specific heat of air at constant volume is 0.172 cal $g^{-1}\,{}^\circ C^{-1}$. **(Ans. 14.4 J)**

4. A volume of 10 m^3 of a liquid is supplied with 100 kcal of heat and expands at a constant pressure of 10 atm to a final volume of 10.2 m^3. Calculate the work done and the change in internal energy. **(Ans. 48 kcal, 52 kcal)**

5. The internal energy of a monoatomic gas is 1.5 nRT. One mole of helium is kept in a cylinder of cross-section 8.5 cm^2. The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. If the temperature rises through 2°C, find the distance moved by the piston. Atmospheric pressure = 100 kPa. **(Ans. 20 cm)**

✘ Hints

1. $dQ = mC\,\Delta T = 200 \times 0.1 \times (75 - 25) = 100$ cal

$dW = PdV = P \times 0 = 0$

$dU = dQ - dW = 1000 - 0 = 1000$ cal

$= 1000 \times 4.2 = \mathbf{4200\ J.}$

2. Here $m = 1\ \text{kg} = 10^3$ g

Initial volume, $V_1 = 10^3\ \text{cm}^3$;

Final volume, $V_2 = 1671 \times 10^3\ \text{cm}^3$

$P = 1\ \text{atom} = 1.013 \times 10^6\ \text{dyne cm}^{-2}$

$dQ = mL = 10^3 \times 540\ \text{cal} = 540$ kcal

$dW = PdV = P(V_2 - V_1)$

$= 1.013 \times 10^6\,(1671 - 1)$ erg

$= \frac{1.013 \times 10^6 \times 1670}{4.2 \times 10^7}$ cal

$= 40.16 \times 10^3\ \text{cal} = 40.16$ kcal.

From first law of thermodynamics,

$dU = dQ - dW = 540 - 40.16$

$= \mathbf{499.84\ kcal.}$

3. At constant volume, $dW = PdV = P \times 0 = 0$

$dQ = mC_v\,\Delta T = 5 \times 0.172 \times 4$ cal

$= 5 \times 0.172 \times 4 \times 4.2\ \text{J} = 14.4$ J

$\therefore\ dU = dQ - dW = 14.4 - 0 = \mathbf{14.4\ J}$

4. $dW = PdV = 10 \times 1.013 \times 10^5 \times (10.2 - 10) = 2 \times 10^5$ J

$= \frac{2 \times 10^5\ \text{J}}{4186\ \text{J kcal}^{-1}} = \mathbf{48\ kcal}$

$dQ = 100$ kcal

$\therefore\ dU = dQ - dW = 100 - 48 = \mathbf{52\ kcal.}$

5. $dU = 1.5\,nR\,dT = 1.5 \times 1 \times 8.3 \times 2 = 24.9$ J [$\because n = 1$]

$dQ = 42$ J

$dW = dQ - dU = 42 - 24.9 = 17.1$ J

$P = 100\ \text{kPa} = 10^5$ Pa,

$A = 8.5\ \text{cm}^2 = 8.5 \times 10^{-4}\ \text{m}^2$

As $dW = PdV = PA\,dx$

$$\therefore\ dx = \frac{dW}{PA} = \frac{17.1}{1 \times 10^5 \times 8.5 \times 10^{-4}}$$

$= 0.2\ \text{m} = \mathbf{20\ cm.}$

12.10 SPECIFIC HEATS OF A GAS

15. ***Why does a gas not possess a unique or single specific heat ? Discuss the limits of the specific heat of a gas.***

Specific heats of a gas. When a gas is heated, its volume and pressure change with the increase in temperature. So the amount of heat required to raise the temperature of 1 gram of gas through 1°C is not fixed. That is a gas does not possess a unique or single

Scanned by CamScanner

If now the heat ΔQ is absorbed at constant pressure, then

$$C_P = \left(\frac{\Delta Q}{\Delta T}\right)_P = \left(\frac{\Delta U}{\Delta T}\right)_P + P\left(\frac{\Delta V}{\Delta T}\right)_P$$

$$= \left(\frac{\Delta U}{\Delta T}\right) + P\left(\frac{\Delta V}{\Delta T}\right)_P$$

Again, we have dropped the subscript P from the first term because U of an ideal gas depends only on T.

Clearly

$$C_P - C_V = P\left(\frac{\Delta V}{\Delta T}\right)_P$$

But for one mole of an ideal gas, $PV = RT$

Differentiating both sides w.r.t. T for constant pressure P,

$$\frac{\Delta(PV)}{\Delta T} = \frac{\Delta(RT)}{\Delta T}$$

or

$$P\left(\frac{\Delta V}{\Delta T}\right)_P = R$$

Hence

$$C_P - C_V = R$$

This is the required relation between C_P and C_V. It is also known as *Mayer's formula*.

For Your Knowledge

- *For one mole of a gas :*
  - $C_P - C_V = R$. (When C_P, C_V are in units of work)
  - $C_P - C_V = \frac{R}{J}$ (When C_P, C_V are in units of heat)

  where R is universal gas constant for one mole of a gas.
- The above relations will remain same even if we consider any number of moles of a gas because in that case both sides will get multiplied by the same number.
- As R is always positive, it follows that $C_P > C_V$.
- *For one gram of a gas :*
  - $c_P - c_V = r$.

    (When c_P, c_V are in units of work)
  - $c_P - c_V = \frac{r}{J}$.

    (When c_P, c_V are in units of heat)

  where $r = \frac{R}{M}$ = gas constant for 1 g of a gas.
- Heat lost or gained by n moles of a gas,
  - $Q = nC_P\,\Delta T$ (At constant pressure)
  - $Q = nC_V\,\Delta T$ (At constant volume)
- The ratio of the two principal specific heats is represented by γ.

  $\therefore \quad \gamma = \frac{C_P}{C_V}$
- The value of γ depends on the atomicity of the gas.

Examples based on Relation between Two Specific Heats of a Gas

Formulae Used

1. For one mole of a gas.

   (i) $C_P - C_V = R$ (When C_P, C_V are in units of work)

   (ii) $C_P - C_V = \frac{R}{J}$ (When C_P, C_V are in units of heat)

2. For 1 g of a gas

   (i) $c_P - c_V = r$ (when c_P, c_V are in units of work)

   (ii) $c_P - c_V = \frac{r}{J}$ (when c_P, c_V are in units of heat)

   where $r = \frac{R}{M}$ = gas constant for 1 g of a gas

3. Heat lost or gained by a gas.

   (i) $Q = nC_P\,\Delta T$ (At constant pressure)

   (ii) $Q = nC_V\,\Delta T$ (At constant volume)

   where n = Number of moles of gas

   $= \frac{\text{Mass of gas}}{\text{Molecular mass}}$.

Constants Used

1. $J = 4.18\ \text{J cal}^{-1}$
2. $R = 8.31\ \text{J mol}^{-1}\ \text{K}^{-1} = 1.98\ \text{cal mol}^{-1}\ \text{K}^{-1}$.

Example 10. *Calculate the specific heat at constant volume for a gas. Given specific heat at constant pressure is 6.85 cal mol^{-1} K^{-1}, $R = 8.31$ J mol^{-1} K^{-1} and $J = 4.18$ J cal^{-1}.*

Solution. Here $C_P = 6.85\ \text{cal mol}^{-1}\ \text{K}^{-1}$,

$R = 8.31\ \text{J mol}^{-1}\ \text{K}^{-1}$, $J = 4.18\ \text{J cal}^{-1}$

As $C_P - C_V = \frac{R}{J}$

$$\therefore \quad C_V = C_P - \frac{R}{J} = 6.85 - \frac{8.31}{4.18}$$

$$= 6.85 - 1.988 = \mathbf{4.862\ cal\ mol^{-1}\ K^{-1}}.$$

Example 11. *Calculate the difference between the two principal specific heats of 1 g of helium gas at S.T.P. Given atomic weight of helium = 4 and $J = 4.186$ J cal^{-1} and $R = 8.31$ J mol^{-1} K^{-1}.*

Solution. For 1 g of helium gas,

$$c_P - c_V = \frac{r}{J} = \frac{R}{MJ}$$

$$= \frac{8.31}{4 \times 4.186} = \mathbf{0.496\ cal\ g^{-1}\ K^{-1}}.$$

Example 12. *The difference between two specific heats of a gas is 5000 J kg^{-1} K^{-1} and the ratio of specific heats is 1.6. Find the two specific heats.*

Scanned by CamScanner

Solution. Here $c_p - c_V = 5000 \text{ J kg}^{-1} \text{ K}^{-1}$

and $\frac{c_p}{c_V} = 1.6$ or $c_p = 1.6\, c_V$

$\therefore 1.6\, c_V - c_V = 5000$ or $0.6\, c_V = 5000$

or $c_V = \frac{5000}{0.6} = \mathbf{8333.33 \text{ J kg}^{-1} \text{ K}^{-1}}$

$c_p = c_V + 5000 \text{ J kg}^{-1} \text{ K}^{-1}$

$= \mathbf{13333.33 \text{ J kg}^{-1} \text{ K}^{-1}}$.

Example 13. *Specific heat of argon at constant pressure is 0.125 cal g^{-1} K^{-1} and at constant volume is 0.075 cal g^{-1} K^{-1}. Calculate the density of argon at STP. Given $J = 4.18 \times 10^7$ erg cal^{-1} and normal pressure $= 1.01 \times 10^6$ dyne cm^{-2}.*

Solution. Here $c_p = 0.125 \text{ cal g}^{-1} \text{ K}^{-1}$,

$c_V = 0.075 \text{ cal g}^{-1} \text{ K}^{-1}$,

$J = 4.18 \times 10^7 \text{ erg cal}^{-1}$

Standard pressure, $P = 1.01 \times 10^6 \text{ dyne cm}^{-2}$

For 1 g of a gas, $c_p - c_V = \frac{r}{J}$

$\therefore \quad r = J(c_p - c_V) = 4.18 \times 10^7 \times (0.125 - 0.075)$

$= 4.18 \times 10^7 \times 0.050 = 2.09 \times 10^7 \text{ erg g}^{-1} \text{ K}^{-1}$

But for 1 g of a gas, $PV = rT$

$\therefore$ Volume of 1 g of argon at S.T.P. is

$$V = \frac{rT}{P} = \frac{2.09 \times 10^7 \times 273}{1.01 \times 10^6} = 564.92 \text{ cm}^3$$

Density of argon,

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{1}{564.92} = \mathbf{1.77 \times 10^{-3} \text{ g cm}^{-3}}.$$

Example 14. *Calculate the value of c_V for air, given that $c_P = 0.23$ calorie g^{-1} K^{-1}. Density of the air at S.T.P. is 1.293 g $litre^{-1}$ and $J = 4.2 \times 10^7$ erg $calorie^{-1}$.*

Solution. Sp. heat of air at constant pressure,

$c_V = 0.23 \text{ cal g}^{-1} \text{ K}^{-1}$

Density of air at S.T.P. $= 1.293 \text{ g litre}^{-1}$

$\therefore$ Volume of 1 g of air at S.T.P., $V = \frac{1000}{1.293} \text{ cm}^3$

Normal pressure, $P = 76 \times 13.6 \times 980 \text{ dyne cm}^{-2}$

Normal temperature, $T = 273$ K,

$J = 4.2 \times 10^7 \text{ erg cal}^{-1}$

$\therefore$ Gas constant for 1 g of air,

$$r = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 1000}{1.293 \times 273}$$

$= 2.87 \times 10^6 \text{ erg g}^{-1} \text{ K}^{-1}$

Also for 1 g of a gas, $c_P - c_V = \frac{r}{J}$

$$\therefore \quad c_V = c_p - \frac{r}{J} = 0.23 - \frac{2.87 \times 10^6}{4.2 \times 10^7}$$

$= 0.23 - 0.068 = \mathbf{0.162 \text{ cal g}^{-1} \text{ K}^{-1}}$

Example 15. *For air, specific heat at constant pressure is 0.237 cal g^{-1} $°C^{-1}$ and specific heat at constant volume is 0.169 cal g^{-1} $°C^{-1}$, density of air $= 0.001293$ g cm^{-3} at S.T.P. Calculate the value of J.*

Solution. Here $c_p = 0.237 \text{ cal g}^{-1} \text{ °C}^{-1}$,

$c_V = 0.169 \text{ cal g}^{-1} \text{ °C}^{-1}$

Density of air at S.T.P.,

$\rho = 0.001293 \text{ g cm}^{-3}$

$\therefore$ Volume of 1 g air at S.T.P.,

$$V = \frac{1}{0.001293} \text{ cm}^3$$

Normal pressure, $P = 76 \times 13.6 \times 980 \text{ dyne cm}^{-2}$

Normal temperature, $T = 273$ K

Gas constant for 1 g of gas,

$$r = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 1}{273 \times 0.001293}$$

$= 2.87 \times 10^6 \text{ erg g}^{-1} \text{ K}^{-1}$

$$\therefore \quad J = \frac{r}{c_p - c_V} = \frac{2.869 \times 10^6}{0.237 - 0.169}$$

$$= \frac{2.87 \times 10^6}{0.06} = \mathbf{4.219 \times 10^7 \text{ erg cal}^{-1}}.$$

Example 16. *An ideal gas has a specific heat at a constant pressure, $C_p = (5/2)R$. The gas is kept in a closed vessel of volume 0.0083 m^3 at a temperature of 300 K and a pressure of 1.6×10^6 Nm^{-2}. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.*

Solution. Here $P = 1.6 \times 10^6 \text{ Nm}^{-2}$,

$V = 0.0083 \text{ m}^3, \quad T = 300 \text{ K}$

As $\quad PV = nRT$

$$\therefore \quad n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3}$$

Now $\quad C_V = C_p - R = \frac{5}{2}R - R = \frac{3}{2}R$

When 2.49×10^4 J of heat energy is supplied to the gas, suppose its temperature increases by ΔT. Then

$$n\, C_V\, \Delta T = Q$$

or $\quad \frac{16}{3} \times \frac{5}{2} R \times \Delta T = 2.49 \times 10^4$

or $\quad \Delta T = \frac{2.49 \times 10^4 \times 6}{80\, R} = \frac{2.49 \times 10^4 \times 6}{80 \times 8.3} = 375 \text{ K}$

Scanned by CamScanner

Final temperature,

$$T' = T + 375 = 300 + 375 = 675\ \text{K}$$

As the gas is heated at constant volume, so

$$\frac{P'}{P} = \frac{T'}{T}$$

$$P' = \frac{T'}{T} \times P = \frac{675 \times 1.6 \times 10^6}{300}$$

$$= 3.6 \times 10^6\ \text{Nm}^{-2}.$$

PROBLEMS FOR PRACTICE

1. Calculate the difference between two specific heats of 1 g of nitrogen. Given molecular weight of nitrogen = 28 and $J = 4.2 \times 10^7$ erg cal^{-1}. **(Ans.** 0.0706 cal g^{-1}K^{-1}**)**

2. Calculate the gas constant for 1 g of a gas. Given that $c_p = 0.245$ cal g^{-1}°C^{-1}, $c_V = 0.165$ cal g^{-1}°C^{-1} and $J = 4.2 \times 10^7$ erg cal^{-1}. **(Ans.** 3.36×10^6 erg g^{-1}°C^{-1}**)**

3. Calculate difference in specific heats for 1 g of air at S.T.P. Given density of air at S.T.P. is 1.293 g litre^{-1}, $J = 4.2 \times 10^7$ erg cal^{-1}. **(Ans.** 0.068 cal g^{-1} K^{-1}**)**

4. The specific heats of air at constant pressure and constant volume are 0.237 cal g^{-1}°C^{-1} and 0.169 cal g^{-1}°C^{-1} respectively and $J = 4.2$ J cal^{-1}. Calculate density of air. **(Ans.** 1.293×10^{-3} g cm^{-3}**)**

5. Calculate the ratio of specific heats for nitrogen, given that specific heat at constant pressure is 0.236 cal g^{-1}K^{-1} and density at S.T.P is 0.001234 g cm^{-3}. **(Ans.** 1.434**)**

6. For hydrogen gas, $c_p = 3.409$ cal g^{-1}°C^{-1}, $c_V = 2.409$ cal g^{-1}°C^{-1} and molecular weight of hydrogen = 2. Calculate the value of J. **(Ans.** 4.156×10^7 erg cal^{-1}**)**

7. The specific heat of argon at constant pressure is 0.127 and ratio of specific heats is 1.667. Calculate the value of J. One litre of argon weighs 1.786 g at N.T.P. **(Ans.** 4.07×10^7 erg cal^{-1}**)**

8. One mole of oxygen is heated at a constant pressure from 0°C. What must be the quantity of heat that should be supplied to the gas for the volume to be doubled ? The specific heat of oxygen under these conditions is 0.218 cal g^{-1} K^{-1}. **(Ans.** 1904 cal**)**

HINTS

1. Here $J = 4.2 \times 10^7$ erg cal^{-1} = 4.2 J cal^{-1}

$$\therefore\ c_p - c_V = \frac{r}{J} = \frac{R}{MJ} = \frac{8.3}{28 \times 4.2}$$

$$= \mathbf{0.0706\ cal\ g^{-1}\ K^{-1}}.$$

2. $r = J(c_p - c_V) = 4.2 \times 10^7 \times (0.245 - 0.165)$

$$= 3.36 \times 10^6\ \text{erg g}^{-1}\,°\text{C}^{-1}.$$

3. Here $\rho = 1.293$ g litre^{-1} $= 1.293 \times 10^{-3}$ g cm^{-3}

Now $c_p - c_V = \frac{r}{J} = \frac{Pv}{TJ} = \frac{P}{\rho T J}$

[v = volume of 1 g gas $= \frac{1}{\rho}$]

$$= \frac{1.013 \times 10^6}{1.293 \times 10^{-3} \times 273 \times 4.2 \times 10^7} = 0.068\ \text{cal g}^{-1}\ \text{K}^{-1}.$$

4. $$\rho = \frac{P}{TJ(c_p - c_V)} = \frac{1.01 \times 10^6}{273 \times 4.2 \times 10^7 \times (0.237 - 0.169)}$$

$$= 1.293 \times 10^{-3}\ \text{g cm}^{-3}.$$

5. For 1 g of nitrogen,

$$c_p - c_V = \frac{r}{J} = \frac{Pv}{TJ} = \frac{P}{\rho T J}$$

$$= \frac{1.01 \times 10^6}{0.001234 \times 234 \times 4.2 \times 10^7}$$

$$= 0.0714\ \text{cal g}^{-1}\ \text{K}^{-1}$$

$$c_V = c_p - 0.0714 = 0.236 - 0.0714 = 0.1646$$

$$\therefore\ \gamma = \frac{c_p}{c_V} = \frac{0.236}{0.1646} = 1.434$$

7. $$c_V = \frac{c_p}{\gamma} = \frac{0.127}{1.667} = 0.076$$

$$\therefore\ c_p - c_V = 0.127 - 0.076 = 0.051$$

For 1 g of gas, $c_p - c_V = \frac{P}{\rho T J}$

$$\therefore\ J = \frac{P}{\rho T}(c_p - c_V) = \frac{1.01 \times 10^6}{1.786 \times 10^{-3} \times 273 \times 0.051}$$

$$= \mathbf{4.07 \times 10^7\ erg\ cal^{-1}}.$$

8. Initial temperature, $T_1 = 0 + 273 = 273$ K

At constant pressure, $\frac{V_2}{V_1} = \frac{T_2}{T_1}$ or $\frac{2V_1}{V_1} = \frac{T_2}{273}$

$\therefore\ T_2 = 2 \times 273 = 546$ K

Rise in temperature,

$$\Delta T = T_2 - T_1 = 546 - 273 = 273\ \text{K}$$

Mass of 1 mole of oxygen, $m = 32$ g

Heat required,

$$Q = mc_p\,\Delta T = 32 \times 0.218 \times 273 = \mathbf{1904\ cal}.$$

12.13 THERMODYNAMIC PROCESSES

18. ***What is a thermodynamic process ? Mention its different types.***

Thermodynamic process. *A thermodynamic process is said to occur if the thermodynamic variables of a system undergo a change with time.*

Scanned by CamScanner

Different types of thermodynamic processes are as follows :

(*i*) **Isothermal process.** *It is a thermodynamic process which occurs at a constant temperature.*

(*ii*) **Isobaric process.** *It is a thermodynamic process which occurs at a constant pressure.*

(*iii*) **Isochoric process.** *It is a thermodynamic process which occurs at a constant volume.*

(*iv*) **Adiabatic process.** *It is a thermodynamic process in which there is no exchange of heat energy between system and surroundings.*

Table 12.1 *Some typical thermodynamic processes*

Type of process	*Special feature*
Isothermal	Constant temperature
Isobaric	Constant pressure
Isochoric	Constant volume
Adiabatic	No heat flow between the system and surroundings ($\Delta Q = 0$)

19. *What is a quasi-static process ? Briefly explain.*

Quasi-static process. *A quasi-static process is an infinitely slow process such that the system remains in thermal and chemical equilibrium with the surroundings.* In a quasi-static (meaning nearly static) process, at every stage, the temperature and pressure of the surroundings differ only infinitesimally from those of the system.

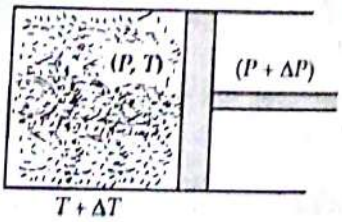


Fig. 12.12 A quasi-static process.

As illustrated in Fig. 12.12, suppose we wish to take a gas from the state (P, T) to another state (P', T') via a quasi-static process. For this we first change the external pressure by a very small amount ΔP, allow the system to equalise its pressure with that of the surroundings and continue the process infinitely slowly until the pressure of the gas becomes P'. Now to change the temperature, we create a very small temperature difference ΔT between the system and surrounding reservoir and continue the process by choosing reservoirs of progressively different temperatures from T to T'. The system finally attains the temperature T'.

Practically, the processes that are sufficiently slow and do not involve accelerated motion of the piston, large temperature gradients, etc., are nearly quasi-static processes.

12.14 ISOTHERMAL PROCESS

20. *What is an isothermal process ? Give an example. What are the essential conditions for an isothermal process to take place ? Write the equation for an isothermal process.*

Isothermal process. *An isothermal process is one in which the pressure and volume of the system change but temperature remains constant.*

As shown in Fig. 12.13, consider an ideal gas enclosed in a cylinder provided with a piston and having conducting walls. If the gas is slowly compressed, the heat produced due to the work done on the gas is transferred to the surroundings so that temperature of the gas remains constant. Similarly, when the gas is allowed to expand slowly, its temperature tends to fall but some heat from the surroundings is conducted to the gas, keeping the temperature constant.

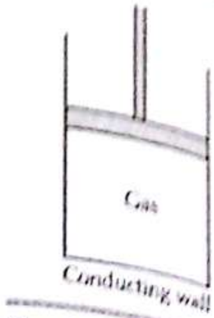


Fig. 12.13 Apparatus for isothermal process.

Essential conditions for an isothermal process to take place :

(*i*) The walls of the container must be perfectly conducting to allow free exchange of heat between the system and the surroundings.

(*ii*) The process of compression or expansion should be very slow, so as to provide sufficient time for the exchange of heat.

Equation of isothermal process. The ideal gas equation for n moles of a gas is

$$PV = nRT$$

For a fixed mass (n fixed) of a gas undergoing an isothermal process (T fixed), the above equation gives

$$PV = \text{constant}$$

This equation is the **equation of state** of an isothermal process. It is a nothing but Boyle's law according to which the pressure of a given mass of a gas varies inversely as its volume.

12.15 FIRST LAW OF THERMODYNAMICS APPLIED TO ISOTHERMAL PROCESS

21. *Discuss the application of the first law of thermodynamics to an isothermal process.*

Scanned by CamScanner

First law of thermodynamics applied to an isothermal process. The internal energy of an ideal gas depends only on its temperature. As temperature remains constant in an isothermal process, there is no change in internal energy of the gas *i.e.*, $\Delta U = 0$.

Applying first law of thermodynamics to an isothermal process,

$$\Delta Q = \Delta U + P\Delta V = 0 + P\Delta V \quad \text{or} \quad \Delta Q = P\Delta V$$

(*i*) **When a gas expands isothermally,** ΔV and hence $P\Delta V$ is positive and so ΔQ will also be positive. Therefore, when a gas expands isothermally, *an amount of heat equivalent to the work done by the gas has to be supplied from an external source.*

(*ii*) **When a gas is compressed isothermally,** ΔV and hence $P\Delta V$ is negative and ΔQ will also be negative. Therefore, *when a gas is compressed isothermally, an amount of heat equivalent to the work done on the gas has to be removed from the gas.*

(*iii*) In an isothermal expansion or compression, the internal energy of the gas remains unchanged.

12.16 WORK DONE IN AN ISOTHERMAL PROCESS

22. *Derive an expression for the work done during the isothermal expansion of an ideal gas.*

Work done in an isothermal expansion. Consider n moles of an ideal gas contained in a cylinder having conducting walls and provided with frictionless and movable piston, as shown in Fig. 12.14. Let P be the pressure of the gas.

Work done by the gas when the piston moves up through a small distance dx is given by

$$dW = PA\,dx = PdV$$

where A is the cross-sectional area of the piston and $dV = Adx$, is the small increase in the volume of the gas. Suppose the gas expands isothermally from initial state (P_1, V_1) to the final state (P_2, V_2). The total amount of work done will be

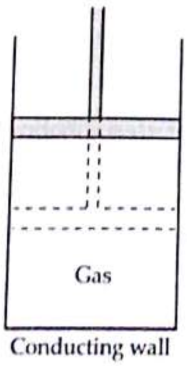


Fig. 12.14 Isothermal expansion

$$W_{iso} = \int_{V_1}^{V_2} P\,dV$$

For n moles of a gas, $PV = nRT$ or $P = \frac{nRT}{V}$

$$\therefore W_{iso} = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV = nRT\int_{V_1}^{V_2} \frac{1}{V}\,dV = nRT\,[\ln V]_{V_1}^{V_2}$$

$$= nRT\,[\ln V_2 - \ln V_1] = nRT \ln \frac{V_2}{V_1}$$

or $$W_{iso} = 2.303\,nRT \log \frac{V_2}{V_1} = 2.303\,nRT \log \frac{P_1}{P_2}$$

This is the expression for the work done during the isothermal expansion of n moles of an ideal gas.

12.17 ADIABATIC PROCESS

23. *What is an adiabatic process ? Give an example. What are the essential conditions for an adiabatic process to occur ?*

Adiabatic process. *An adiabatic process is one in which the pressure, volume and temperature of the system change but there is no exchange of heat between the system and surroundings* (a = not, dia = through, bates = heat, so the Greek word adiabatic means heat not passing through).

Consider a gas enclosed in a cylinder having perfectly insulated walls. Suppose the gas is allowed to expand *very quickly*. Work is done by the gas during its expansion, so its internal energy decreases. As the heat cannot enter the system from the surroundings, so the temperature of the gas falls.

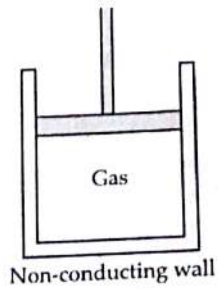


Fig. 12.15 Apparatus for adiabatic process.

Similarly, when the gas is suddenly compressed, work is done on the gas. This increases the internal energy of the gas. As heat cannot escape to the surroundings, the temperature of the gas increases.

Essential conditions for an adiabatic process to take place :

(*i*) The walls of the container must be perfectly insulated so that there cannot be any exchange of heat between the gas and the surroundings.

(*ii*) The process of compression or expansion should be sudden, so that heat does not get time to get exchanged with the surroundings.

12.18 ADIABATIC RELATIONS BETWEEN *P*, *V* AND *T*

24. *Applying first law of thermodynamics, obtain an adiabatic relation between pressure and volume. Hence write relation between P and T and also between V and T.*

Scanned by CamScanner

(i) Adiabatic relation between P and V. According to the first law of thermodynamics,

$$dQ = dU + dW$$

For one mole of a gas,

$$dU = C_V \, dT \text{ and } dW = P\,dV$$

Also, for an adiabatic process, $dQ = 0$

$$\therefore \quad C_V \, dT + P\,dV = 0$$

According to the ideal gas equation,

$$PV = RT$$

Differentiating both sides, we get

$$P\,dV + V\,dP = R\,dT$$

or $$dT = \frac{P dV + V dP}{R}$$

$$\therefore \quad C_V \cdot \frac{P\,dV + V\,dP}{R} + P dV = 0$$

or $$C_V \, P dV + C_V \, V dP + RP\,dV = 0$$

or $$C_V \, V dP + (C_V + R)\,P\,dV = 0$$

or $$C_V \, V\,dP + C_P \, P\,dV = 0 \qquad [\because C_P = C_V + R]$$

Dividing both sides by $C_V \, PV$, we get

$$\frac{dP}{P} + \frac{C_P}{C_V}\frac{dV}{V} = 0$$

or $$\frac{dP}{P} + \gamma \frac{dV}{V} = 0 \qquad \left[\because \gamma = \frac{C_P}{C_V}\right]$$

Integrating both sides, we get

$$\int \frac{dP}{P} + \gamma \int \frac{dV}{V} = C$$

where C is constant of integration

$$\therefore \quad \log_e P + \gamma \log_e V = C$$

or $$\log_e PV^{\gamma} = C \text{ or } PV^{\gamma} = e^C$$

or $$PV^{\gamma} = K$$

where K is another constant. This is the adiabatic relation between pressure P and volume V of an ideal gas.

(ii) Adiabatic relation between P and T. For one mole of a gas $PV = RT$, therefore

$$V = \frac{RT}{P}$$

Putting in $PV^{\gamma} = K$, we get

$$P\left(\frac{RT}{P}\right)^{\gamma} = K$$

or $$P^{1-\gamma} T^{\gamma} = \frac{K}{R^{\gamma}} = \text{another constant}$$

i.e., $$P^{1-\gamma} T^{\gamma} = \text{constant}$$

This is the adiabatic relation between pressure P and temperature T of an ideal gas.

(iii) Adiabatic relation between V and T. Again, for one mole of a gas $PV = RT$, therefore

$$P = \frac{RT}{V}$$

Putting in $PV^{\gamma} = K$, we get

$$\frac{RT}{V} \cdot V^{\gamma} = K \text{ or } TV^{\gamma - 1} = \frac{K}{R} = \text{another constant}$$

i.e., $$TV^{\gamma - 1} = \text{constant}$$

This is the adiabatic relation between volume V and temperature T of an ideal gas.

12.19 WORK DONE IN AN ADIABATIC PROCESS

25. *Derive an expression for the work done during the adiabatic expansion of an ideal gas.*

Work done in an adiabatic expansion. Consider n moles of an ideal gas contained in a cylinder having insulating walls and provided with frictionless and insulating piston. Let P be the pressure of the gas. When the piston moves up through a small distance dx, the work done by the gas will be

$$dW = PA\,dx = P\,dV$$

where A is the cross-sectional area of the piston and $dV = A\,dx$ is the increase in the volume of the gas.

Suppose the gas expands adiabatically and changes from the initial state (P_1, V_1, T_1) to the final state (P_2, V_2, T_2). The total work done by the gas will be

$$W_{adia} = \int_{V_1}^{V_2} P\,dV.$$

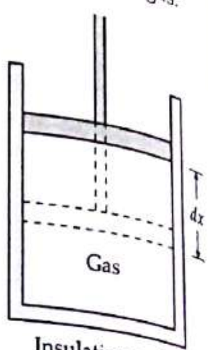


Fig. 12.16 Adiabatic expansion.

For an adiabatic change

$$PV^{\gamma} = K \text{ or } P = KV^{-\gamma}$$

$$\therefore \quad W_{adia} = \int_{V_1}^{V_2} KV^{-\gamma}\,dV$$

$$= K\int_{V_1}^{V_2} V^{-\gamma}\,dV = K\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}[V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{1}{\gamma - 1}[KV_1^{1-\gamma} - KV_2^{1-\gamma}]$$

But $K = P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$\therefore \quad W_{adia} = \frac{1}{\gamma - 1}[P_1 V_1^{\gamma} V_1^{1-\gamma} - P_2 V_2^{\gamma} V_2^{1-\gamma}]$$

or $$W_{adia} = \frac{1}{\gamma - 1}[P_1 V_1 - P_2 V_2]$$

Scanned by CamScanner

Also $P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$

$$W_{adia} = \frac{1}{\gamma - 1}[nRT_1 - nRT_2]$$

or

$$W_{adia} = \frac{nR}{\gamma - 1}(T_1 - T_2)$$

This equation gives the work done during the adiabatic expansion of n moles of an ideal gas.

Clearly, when work is done by the gas during its adiabatic expansion, $W_{adia} > 0$ and $T_2 < T_1$, *i.e.*, temperature of the gas decreases. When work is done on the gas during its adiabatic compression, $W_{adia} < 0$ and $T_2 > T_1$, *i.e.*, temperature of the gas rises.

12.20 FIRST LAW OF THERMODYNAMICS APPLIED TO ISOCHORIC PROCESS

26. *Apply first law of thermodynamics to show that the change in the internal energy of a system is equal to the heat given to or taken from the system in an isochoric process.*

First law of thermodynamics applied to an isochoric process. In an isochoric process, the volume V of the system remains constant. As there is no change in volume ($\Delta V = 0$), no work is done on or by the system ($W = P\,\Delta V = 0$). According to the first law of thermodynamics,

$$Q = \Delta U + W = \Delta U + 0 = \Delta U$$

Hence in an isochoric process, the entire heat given to or taken from the system goes to change its internal energy and temperature of the system. The change in temperature can be determined from the equation

$$Q = nC_V\,\Delta T.$$

12.21 FIRST LAW OF THERMODYNAMICS APPLIED TO A CYCLIC PROCESS

27. *Apply first law of thermodynamics to show that the entire heat absorbed by a system is equal to the work done by the system in a cyclic process.*

First law of thermodynamics applied to a cyclic process. In a cyclic process, the system returns to its initial state after undergoing a series of changes. As internal energy is a state function, so $\Delta U = 0$ for a cyclic process. From first law of thermodynamics,

$$Q = \Delta U + W = 0 + W = W$$

Hence in a cyclic process, the total heat absorbed by a system is equal to the work done by the system.

12.22 FIRST LAW OF THERMODYNAMIC APPLIED TO ISOBARIC PROCESS

28. *Discuss the application of first law of thermodynamics to an isobaric process.*

First law of thermodynamics applied to an isobaric process. A thermodynamic process which occurs at a constant pressure is called an isobaric process. For example, freezing of water, formation of steam, etc.

Suppose the pressure P of a gas remains constant and its volume changes from V_1 to V_2, then the work done by the gas is

$$W = \int_{V_1}^{V_2} P\,dV = P\int_{V_1}^{V_2} dV = P(V_2 - V_1)$$

$$= nR(T_2 - T_1)$$

As the temperature of the gas changes, so its internal energy also changes. Hence *in an isobaric process, the absorbed heat goes partly to increase internal energy and partly to do work.*

29. *Apply the first law of thermodynamics to determine the change in internal energy during the boiling process.*

Boiling process. Suppose m mass of a liquid is heated at the temperature of its boiling point so that it changes into vapour at a pressure P. Let V_i be the volume of the liquid and V_f that of vapour. The work done by the liquid during its expansion at temperature T is given by

$$\Delta W = P\,\Delta V = P(V_f - V_i)$$

Let L be the heat of vaporisation of the liquid. It represents the heat needed per unit mass to change from liquid to vapour phase at constant temperature and pressure. Then the amount of heat required for vaporisation of m mass of liquid will be

$$\Delta Q = mL.$$

Let U_i and U_f be the initial and final values of internal energy. According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

or $$mL = U_f - U_i + P(V_f - V_i)$$

or $$U_f - U_i = mL - P(V_f - V_i)$$

Knowing m, L, P, V_i and V_f, the change in internal energy can be determined.

30. *Apply the first law of thermodynamics to determine the change in internal energy during the melting process.*

Melting process. Suppose m mass of a solid is heated at its melting point. When the solid melts, the change in its volume ΔV is negligibly small. So work done by the solid is

$$\Delta W = P\,\Delta V = P \times 0 = 0.$$

Let L be the latent heat of fusion. It represents the heat needed per unit mass to change from solid to liquid phase at constant temperature and pressure.

Scanned by CamScanner

Then the amount of heat required for fusion of m mass of a solid will be

$$\Delta Q = mL$$

Let U_i and U_f be the initial and final values of internal energy. According to the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \quad \text{or} \quad mL = U_f - U_i + 0$$

$\therefore \quad U_f - U_i = mL$

Hence the internal energy of a system increases by mL during the melting process.

Examples based on Isothermal and Adiabatic Processes

FORMULAE USED

1. Equation for isothermal process.
$$PV = \text{constant} \quad \text{or} \quad P_1V_1 = P_2V_2$$
2. Equations for adiabatic processes.
(i) $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ (ii) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
(iii) $\dfrac{P_1^{\gamma-1}}{T_1^{\gamma}} = \dfrac{P_2^{\gamma-1}}{T_2^{\gamma}}$, where $\gamma = C_P/C_V$
3. Work done when 1 mole of a gas expands isothermally,
(i) $W_{iso} = 2.303\, RT \log \dfrac{V_2}{V_1}$
(ii) $W_{iso} = 2.303\, RT \log \dfrac{P_1}{P_2}$
4. Work done when 1 mole of a gas expands adiabatically and its temperature falls from T_1 to T_2,
(i) $W_{adi} = \dfrac{R}{\gamma - 1}[T_1 - T_2]$
(ii) $W_{adi} = \dfrac{1}{\gamma - 1}[P_1V_1 - P_2V_2]$

UNITS USED

All pressures are in Nm^{-2}, volumes in m^3, temperatures in K, work done W in joule.

Example 17. The compression ratio of a certain diesel engine is 15. This means that air in the cylinder is compressed to 1/15 of its initial volume. If the initial pressure is 1.0×10^5 Pa and the initial temperature is 300 K, find the final pressure and temperature after compression. Air is mostly a mixture of oxygen and nitrogen and $\gamma = 1.4$.

Solution. Here $\dfrac{V_1}{V_2} = 15$, $P_1 = 1.0 \times 10^5$ Pa,

$T_1 = 300$ K, $P_2 = ?$, $T_2 = ?$

For an adiabatic process, $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\therefore \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$= 1.0 \times 10^5 \times (15)^{1.4} = 44.3 \times 10^5 \text{ Pa.}$$

Again, for an adiabatic process,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\therefore \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 \times (15)^{0.4} = 886 \text{ K.}$$

Example 18. If, at 50°C and 75 cm of mercury pressure, a definite mass of a gas is compressed (i) slowly, (ii) suddenly, then what will be the final pressure and temperature of the gas in each case if the final volume is one-fourth of the initial volume ? ($\gamma = 1.5$).

Solution. Here $V_2 = \frac{1}{4}V_1$, $P_1 = 75$ cm of Hg,

$$T_1 = 50 + 273 = 323 \text{ K}$$

(i) When the gas is compressed suddenly, the process is *isothermal.*

$$\therefore \quad P_1V_1 = P_2V_2 \quad \text{or} \quad 75 \times V_1 = P_2 \times \tfrac{1}{4}V_1$$

or $\quad P_2 = 75 \times 4 = 300$ **cm of Hg.**

As the process is isothermal, so $T_2 = 50°C$.

(ii) When the gas is compressed suddenly, the process is *adiabatic.*

$$\therefore \quad P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

or $$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 75\left(\frac{V_1}{V_1/4}\right)^{1.5} = 75 \times 4^{1.5}$$

$$= 75 \times 4 \times 4^{1/2} = 75 \times 4 \times 2 = 600 \text{ cm of Hg.}$$

Also, $\quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 323 \times (4)^{0.5} = 323 \times 2$$

$$= 646 \text{ K} = 373°\text{C.}$$

Example 19. A tyre pumped to a pressure of 3.375 atmosphere and at 27°C suddenly bursts. Calculate the temperature of escaping air. Given $\gamma = 1.5$.

Solution. Here $P_1 = 3.375$ atm, $P_2 = 1$ atm, $\gamma = 1.4$

$$T_1 = 27 + 277 = 300 \text{ K}, T_2 = ?$$

When tyre bursts suddenly, its air expands adiabatically. So

$$\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}} \quad \text{or} \quad \left(\frac{T_1}{T_2}\right)^{\gamma} = \left(\frac{P_1}{P_2}\right)^{\gamma-1}$$

Scanned by CamScanner

$$\left(\frac{300}{T_2}\right)^{1.5} = \left(\frac{3.375}{1}\right)^{1.5-1}$$

or $$\left(\frac{300}{T_2}\right)^{3/2} = \left(\frac{3.375}{1}\right)^{1/2}$$

or $$\frac{300}{T_2} = (3.375)^{1/3} = 1.5$$

or $$T_2 = \frac{300}{1.5} = 200 \text{ K} = \mathbf{-73°C.}$$

EXAMPLE 20. *Calculate the fall in temperature of helium initially at 15°C, when it is suddenly expanded to 8 times its volume. Given $\gamma = 5/3$.*

Solution. Here $T_1 = 15 + 273 = 288$ K,

$$V_2 = 8\,V_1, \quad \gamma = 5/3, \quad T_2 = ?$$

As the expansion of helium is sudden, so the process is adiabatic. For such a process, we have

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 288\left(\frac{1}{8}\right)^{5/3-1}$$

$$= 288 \times \left(\frac{1}{8}\right)^{2/3} = 288 \times \frac{1}{4} = 72 \text{ K}$$

Fall in temperature of helium

$$= T_1 - T_2 = 288 - 72 = \mathbf{216 \text{ K or } 216°C.}$$

EXAMPLE 21. *Two different adiabatic parts for the same gas intersect two isothermals at T_1 and T_2 as shown in P-V diagram, Fig. 12.17. How does the ratio (V_a / V_d) compare with the ratio (V_b / V_c)?*

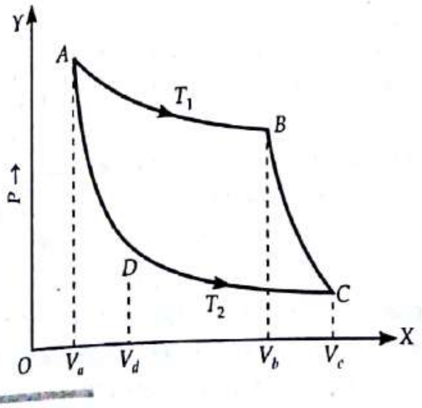


Fig. 12.17

Solution. For adiabatic curve *BC*, we have

$$T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1} \quad ...(i)$$

For adiabatic curve *AD*, we have

$$T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1} \quad ...(ii)$$

Dividing (*ii*) by (*i*), we get :

$$\left(\frac{V_a}{V_b}\right)^{\gamma-1} = \left(\frac{V_d}{V_c}\right)^{\gamma-1} \quad \text{or} \quad \frac{V_a}{V_b} = \frac{V_d}{V_c}$$

$\therefore$ $$\frac{V_a}{V_d} = \frac{V_b}{V_c},$$ *i.e.*, the two ratios are equal.

EXAMPLE 22. *Two samples of a gas initially at same temperature and pressure are compressed from a volume V to $V/2$. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater ?*

Solution. Here $V_1 = V$, $V_2 = V/2$

$\therefore$ $V_1 / V_2 = 2$

In isothermal compression, $P_1 V_1 = P_2 V_2$

$\therefore$ $$P_2 = P_1 \times \frac{V_1}{V_2} = P_1 \times 2 = 2P_1$$

In adiabatic compression,

$$P_2' = P_1'\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1 \times (2)^{\gamma}$$

As $\gamma > 1$, so $2^{\gamma} > 2$ and hence $P_2' > P_2$ *i.e.*, the pressure is greater in the sample compressed adiabatically.

EXAMPLE 23. *Three moles of an ideal gas kept at a constant temperature of 300 K are compressed from a volume of 4 litre to 1 litre. Calculate the work done in the process. Given $R = 8.31$ J mol^{-1} K^{-1}.*

Solution. Here $n = 3$, $T = 300$ K, $V_1 = 4$ litre,

$$V_2 = 1 \text{ litre,}$$

Work done in isothermal process is given by

$$W = 2.303\, nRT \log_{10} \frac{V_2}{V_1}$$

$$= 2.303 \times 3 \times 8.31 \times 300 \log_{10} \frac{1}{4}$$

$$= 2.303 \times 3 \times 8.31 \times 300 \times (-2 \log_{10} 2)$$

$$= -2.303 \times 3 \times 8.31 \times 300 \times 2 \times 0.3010$$

$$= \mathbf{-1.037 \times 10^4 \text{ J.}}$$

EXAMPLE 24. *A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rose from 27°C to 97°C. Calculate the work done and heat produced in the gas. Given $\gamma = 1.5$.*

Solution. Here $T_1 = 27 + 273 = 300$ K,

$$T_2 = 97 + 273 = 370 \text{ K,}$$

Work done in adiabatic compression of the gas is given by

$$W = \frac{R}{1-\gamma}(T_2 - T_1) = \frac{8.3 \times (370 - 300)}{1 - 1.5} = -1162 \text{ J}$$

Heat produced, $$H = \frac{W}{J} = \frac{1162}{4.2} = \mathbf{276.67 \text{ cal.}}$$

Scanned by CamScanner

Example 25. One gram molecule of an ideal gas at S.T.P. is subjected to a reversible adiabatic expansion to double its volume. Find the change in internal energy in the process. Given $\gamma = 1.4$.

Solution. Here $T_1 = 273$ K, $V_2 = 2V_1$

or $$V_2 / V_1 = 2, \quad \gamma = 1.4$$

For an adiabatic change,

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

$$\therefore \quad T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = 273 \times \left(\frac{1}{2}\right)^{1.4 - 1}$$

$$= \frac{273}{2^{0.4}} = \frac{273}{1.319} = 207 \text{ K}$$

For an adiabatic process, $dQ = 0$

From first law of thermodynamics, change in internal energy is given by

$$dU = dQ - dW = 0 - dW$$

$$= -\frac{R}{\gamma - 1}(T_2 - T_1) = \frac{R}{\gamma - 1}(T_1 - T_2)$$

$$= \frac{8.3 \times (273 - 207)}{1.4 - 1} = \frac{8.3 \times 66}{0.4} = \mathbf{1369.5 \ J.}$$

Example 26. A sample of gas ($\gamma = 1.5$) is compressed adiabatically from a volume of 1600 cm^3 to 400 cm^3. If the initial pressure is 150 kPa, what is the final pressure and how much work is done on the gas in the process ?

Solution. Here $\gamma = 1.5$, $V_1 = 1600 \text{ cm}^3$,

$$V_2 = 400 \text{ cm}^3, \quad P_1 = 150 \text{ kPa}, \quad P_2 = ?$$

For an adiabatic process,

$$P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$$

$$\therefore \quad P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 150 \left(\frac{1600}{400}\right)^{1.5} = \mathbf{1200 \ kPa}.$$

Work done in the adiabatic compression,

$$W_{adi} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$= \frac{150 \times 10^3 \times 1600 \times 10^{-6} - 1200 \times 10^3 \times 400 \times 10^{-6}}{1.5 - 1}$$

$$= \frac{240 - 480}{0.5} = \mathbf{-480 \ J.}$$

Problems For Practice

1. A certain gas at atmospheric pressure is compressed adiabatically so that its volume becomes half of its original volume. Calculate the resulting pressure in Nm^{-2}. Given γ for air = 1.4. **(Ans.** $2.674 \times 10^5 \text{ Nm}^{-2}$)

2. A gas is suddenly compressed to one fourth of the original volume. Calculate the rise in temperature, the original temperature being 27°C and $\gamma = 1.5$. **(Ans.** 300°C)

3. A tyre pumped to a pressure of 6 atmosphere suddenly bursts. Room temperature is 15°C. Calculate the temperature of escaping air. $\gamma = 1.4$. **(Ans.** – 94.4°C)

4. 200 cm^3 of a gas is compressed to 100 cm^3 at the atmospheric pressure (10^6 dyne cm^{-2}). Find the resultant pressure if the change is (*i*) slow (*ii*) sudden. Given $\gamma = 1.4$. **[Ans.** (*i*) 2 atm (*ii*) 2.636 atm]

5. A quantity of air at 27°C and atmospheric pressure is suddenly compressed to half its original volume. Find the final (*i*) pressure and (*ii*) temperature. Given γ for air = 1.42. **[Ans.** (*i*) 2.675 atm (*ii*) 128.3°C]

6. A quantity of air at normal temperature is compressed (*i*) slowly (*ii*) suddenly to one third of its volume. Find the rise in temperature, if any in each case, $\gamma = 1.4$. **[Ans.** (*i*) Zero (*ii*) 150.6°C]

7. A quantity of air is kept in a container having walls which are slightly conducting. The initial temperature and volume are 27°C and 800 cm^3 respectively. Find the rise in temperature if the gas is compressed to 200 cm^3 (*i*) in a short time and (*ii*) in a long time. Given $\gamma = 1.4$. **[Ans.** (*i*) 222 K (*ii*) Zero]

8. Dry air at 10 atm pressure and 15°C is suddenly compressed to atmospheric pressure. Find the new temperature. Given $\gamma = 1.41$, log 2.88 = 0.4594, log 1.474 = 0.1686. **(Ans.** – 125.1°C)

9. Calculate the rise in temperature when a gas, for which $\gamma = 1.5$ is compressed to 27 times its original pressure, assuming the initial temperature to be 27°C. **(Ans.** 599.9°C)

10. 1000 cm^3 of argon at 27°C is adiabatically compressed so that the temperature is 127°C. Calculate the resulting volume. Given $\gamma = 5/3$. **(Ans.** 649.4 cm^3)

11. Find the final volume of a gram molecule of a gas after an isothermal expansion at 127°C, if the original volume is 400 cm^3. Given the amount of work done by a gram molecule of a gas during expansion is 2302.6 joule, $R = 8.3$ joule $mole^{-1}$ K^{-1}. **(Ans.** 800 cm^3)

12. Calculate work done to compress isothermally 1 g of hydrogen gas at N.T.P. to half its initial volume. Find the amount of heat evolved and change in internal energy. Given $R = 8.31$ J mol^{-1} K^{-1}. **(Ans.** – 786.2 J, 187.2 cal., zero)

Scanned by CamScanner

13. A sample of hydrogen of mass 6 g is allowed to expand isothermally at 27°C, till its volume is doubled.
(i) How many moles of H_2 do we have?
(ii) What is the final temperature of the H_2?
(iii) Calculate work done during expansion? Given $R = 8.31\ J\ mol^{-1}\ K^{-1}$.

[Ans. (i) 3 moles (ii) 27°C (iii) 5184 J]

14. 80 g of oxygen at N.T.P. is compressed adiabatically to a pressure of 5 atmosphere. Calculate the work done on the gas, if $\gamma = 1.4$ and $R = 8.31\ J\ mol^{-1}\ K^{-1}$.

(Ans. − 5167 J)

HINTS

1. $P_1 = 1\,atm = 1.013 \times 10^5\ Nm^{-2}$, $V_1 / V_2 = 2$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma} = 1.013 \times 10^5 \times (2)^{1.4}$$

$$= 1.013 \times 10^5 \times 2.64 = \mathbf{2.694 \times 10^5\ Nm^{-2}}.$$

2. $T_1 = 27 + 273 = 300\ K$, $V_1 / V_2 = 4$, $\gamma = 1.5$, $T_2 = ?$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1} = 300\,(4)^{1.5-1}$$

$$= 300\,(4)^{1/2} = 300 \times 2 = 600\ K$$

or $\quad T_2 = 600 - 273 = 327°C$

Rise in temperature = 327 − 27 = **300°C**

3. Here $P_1 = 6$ atm, $T_1 = 15 + 273 = 288\ K$, $\gamma = 1.4$, $P_2 = 1\,atm$

As $\quad \dfrac{P_1^{\gamma-1}}{T_1^{\gamma}} = \dfrac{P_2^{\gamma-1}}{T_2^{\gamma}}$

$$\therefore \quad T_2^{\gamma} = T_1^{\gamma} \left(\frac{P_2}{P_1} \right)^{\gamma - 1} = (298)^{1.4} \left(\frac{1}{6} \right)^{1.4-1}$$

or $\quad T_1^{1.4} = (298)^{1.4} \left(\dfrac{1}{6} \right)^{0.4}$

Taking log of both sides, we get

$$1.4 \log T_2 = 1.4 \log 298 - 0.4 \log 6$$

$$= 1.4\,(2.4742) - 0.4\,(0.7782) = 3.1526$$

or $\quad \log T_2 = \dfrac{3.1526}{1.4} = 2.2519$

$\therefore \quad T_2 = \text{Antilog}\,(2.2519) = 178.6\ K$

$= 178.6 - 273 = \mathbf{-\,94.4°C}.$

4. Here $V_1 = 200\ cm^3$, $V_2 = 100\ cm^3$, $P_1 = 1\,atm$

(i) When the change is slow, it is isothermal change. So we have

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{1 \times 200}{100} = \mathbf{2\ atm.}$$

(ii) When the change is sudden, it is adiabatic change. So we have

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma} = 1 \times \left(\frac{200}{100} \right)^{1.4} = (2)^{1.4} = \mathbf{2.638\ atm.}$$

5. Here $T_1 = 27 + 273 = 300\ K$, $P_1 = 1\,atm$, $V_2 = V_1 / 2$, $\gamma = 1.42$

(i) As $P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma} = 1\,(2)^{1.42} = 2.675\ atm$$

(ii) As $T_2 V_2^{\gamma - 1} = T_1 V_1^{\gamma - 1}$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1}$$

$$= 300\,(2)^{1.42 - 1} = 401.3\ K$$

or $\quad T_2 = 401.3 - 273 = 128.3°C.$

6. Here $V_2 = V_1 / 3$, $T_1 = 273\ K$, $\gamma = 1.4$, $T_2 - T_1 = ?$

(i) When the air is compressed slowly, temperature remains constant. So

$$T_2 - T_1 = 0$$

(ii) When the air is compressed suddenly, the change is adiabatic.

$$\therefore \quad T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1}$$

$$= 273\,(3)^{1.4-1} = 273\,(3)^{0.4} = 423.6\ K$$

Rise in temperature

$$= 423.6 - 273 = 150.6\ K = \mathbf{150.6°C.}$$

7. Here $T_1 = 27 + 273 = 300\ K$, $V_1 = 800\ cm^3$, $V_2 = 200\ cm^3$

(i) When the gas is compressed in a short time, the process is adiabatic.

$$\therefore \quad T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1}$$

$$= 300 \left(\frac{800}{200} \right)^{1.4-1} = 300\,(4)^{0.4} = 522\ K$$

Rise in temperature = 522 − 300 = **222 K.**

(ii) When the gas is compressed in long time, the process is isothermal.

$\therefore$ Rise in temperature = **0**.

8. Here $T_1 = 15 + 273 = 288\ K$, $P_1 / P_2 = 10$

For adiabatic process, $\left(\dfrac{T_1}{T_2} \right)^{\gamma} = \left(\dfrac{P_1}{P_2} \right)^{\gamma - 1}$

$$\therefore \quad \left(\frac{288}{T_2} \right)^{1.41} = (10)^{1.41-1} = (10)^{0.41}$$

Taking log of both sides

$$1.41\,(\log 288 - \log T_2) = 0.41 \log 10 = 0.41 \times 1$$

$$\log T_2 = \frac{1.41 \log 288 - 0.41}{1.41}$$

$$= \frac{1.41 \times 2.4594 - 0.41}{1.41} = 2.1701$$

T_2 = Antilog (2.1701) = **147.9 K or −125.1°C.**

Scanned by CamScanner

9. Here $T_1 = 27 + 273 = 300\text{ K}$, $P_1/P_2 = 1/27$, $\gamma = 1.5$

As $\left(\frac{T_1}{T_2}\right)^{\gamma} = \left(\frac{P_1}{P_2}\right)^{\gamma-1}$

$\therefore \left(\frac{300}{T_2}\right)^{1.5} = \left(\frac{1}{27}\right)^{1.5-1}$

Taking log of both sides,

$1.5(\log 300 - \log T_2) = 0.5(\log 1 - \log 27)$

$= -0.5 \log 27$

or $\log T_2 = \frac{1.5 \log 300 + 0.5 \log 27}{1.5}$

$= \frac{1.5 \times 2.4771 + 0.5 \times 1.4314}{1.5} = 2.9542$

$\therefore T_2 = \text{Antilog}(2.9542)$

$= 899.9\text{ K}$ or $626.9°\text{C}$

Rise in temperature $= 626.9 - 27 =$ **599.9°C.**

10. Here $V_1 = 1000\text{ cm}^3$, $T_1 = 27 + 273 = 300\text{ K}$,

$T_2 = 127 + 273 = 400\text{ K}$, $\gamma = 5/3$

As $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ or $\frac{300}{400} = \left(\frac{V_2}{100}\right)^{5/3-1}$

or $\left(\frac{V_2}{100}\right)^{2/3} = \frac{3}{4}$ or $\frac{V_2}{100} = \left(\frac{3}{4}\right)^{3/2}$

or $V_2 = 100 \times \left(\frac{3}{4}\right)^{3/2} = \mathbf{649.4\text{ cm}^3}$.

11. Here $T_1 = 127 + 273 = 400\text{ K}$, $V_1 = 400\text{ cm}^3$,

$W = 2306.6\text{ J}$, $R = 8.3\text{ J mol}^{-1}\text{ K}^{-1}$, $V_2 = ?$

As $W = 2.3026\, RT \log_{10}\left(\frac{V_2}{V_1}\right)$

$\therefore 2302.6 = 2.3026 \times 8.3 \times 400 \log_{10}\left(\frac{V_2}{V_1}\right)$

or $\log_{10}\left(\frac{V_2}{V_1}\right) = \frac{2302.6}{2.3026 \times 8.3 \times 400}$

$= 0.3012 = \log_{10} 2$

Hence $\frac{V_2}{V_1} = 2$, $V_2 = 2V_1 = 2 \times 400 = \mathbf{800\text{ cm}^3}$.

12. $W = 2.303\left(\frac{R}{M}\right) T \log_{10}\left(\frac{V_2}{V_1}\right)$

$= 2.303 \times \frac{8.31}{2} \times 273 \log\left(\frac{1}{2}\right) = \mathbf{-786.2\text{ J}}$.

Amount of heat evolved $= \frac{786.2}{4.2} =$ **187.2 cal.**

As the change is isothermal, temperature remains constant. Internal energy of the gas also remains constant.

$\therefore$ Change in internal energy = 0.

13. Here $T = 27 + 273 = 300\text{ K}$, $V_2/V_1 = 2$,
mass $m = 6\text{ g}$

(*i*) No. of moles of hydrogen

$= \frac{\text{Mass of hydrogen}}{\text{Molecular mass}} = \frac{6}{2} = 3.$

(*ii*) As the expansion is isothermal, so final temperature = **27°C.**

(*iii*) $W_{iso} = 2.303\, nRT \log \frac{V_2}{V_1}$

$= 2.303 \times 3 \times 8.31 \times 300 \log 2$

$= 2.303 \times 3 \times 8.31 \times 300 \times 0.3010 = 5184\text{ J}$.

14. Here $T_1 = 273\text{ K}$, $P_1 = 1\text{ atm}$, $P_2 = 5\text{ atm}$,
no. of moles of oxygen $= 50/32$

$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 273\left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}}$

$= 273 \times (5)^{2/7} = 273 \times 1.584 = 432.37\text{ K}$

$W = \frac{nR}{\gamma - 1}[T_1 - T_2]$

$= \frac{50}{32} \times \frac{8.31}{1.4-1}(273 - 432.37) = -5167\text{ J}.$

12.23 HEAT ENGINE

31. *What is a heat engine ? Explain its working principle. Define its efficiency.*

Heat engine. *It is a device which converts continuously heat energy into mechanical energy in a cylic process.*

As shown in Fig. 12.18, a heat engine has the following essential parts :

(*i*) **Source.** It is a heat reservoir at higher temperature T_1. It is supposed to have infinite thermal capacity so that any amount of heat can be drawn from it without changing its temperature.

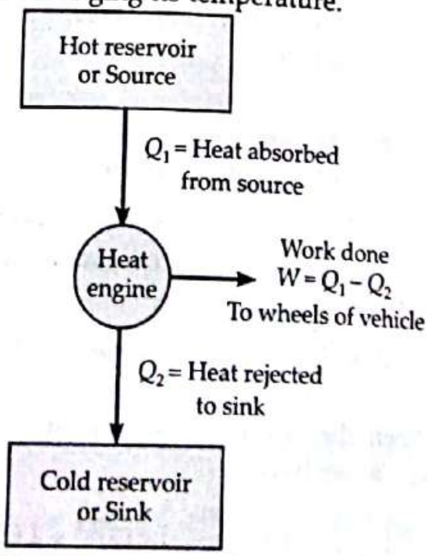


Fig. 12.18 Heat engine.

Scanned by CamScanner

(b) **Sink.** It is a heat reservoir at a lower temperature T_2. It has also infinite thermal capacity so that any amount of heat can be added to it without changing its temperature.

(c) **Working substance.** Working substance is any material (solid, liquid or gas) which performs mechanical work when heat is supplied to it.

For example a mixture of fuel vapour and air is used in a gasoline or diesel engine or steam in a steam engine.

Working. In every cycle of operation, the working substance absorbs a definite amount of heat Q_1 from the source at higher temperature T_1, converts a part of this heat energy into mechanical work W and rejects the remaining heat Q_2 to the sink at lower temperature T_2. The work done W in a cycle is transferred to the environment by some arrangement e.g. the working substance may be in a cylinder with a moving piston that transfers mechanical energy to the wheels of a vehicle via a shaft.

Efficiency of a heat engine. *The efficiency of a heat engine is defined as the ratio of the net work done by the engine in one cycle to the amount of heat absorbed by the working substance from the source.*

As the working substance returns to its initial state after completing one cycle, there is no change in its internal energy. Hence by first law of thermodynamics,

Net heat absorbed in a cycle = Work done

$$Q_1 - Q_2 = W$$

The efficiency of heat engine is given by

$$\eta = \frac{\text{Work done by engine (output)}}{\text{Heat absorbed from the source (input)}}$$

$$= \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \quad \text{or} \quad \eta = 1 - \frac{Q_2}{Q_1}.$$

Efficiency of a heat engine is always less than unity. Clearly, when $Q_2 = 0$, $\eta = 1$ or 100%. But any working substance working in a cycle cannot convert all the heat extracted from the source into work. It has to reject some amount of heat to the sink. That is why the efficiency of a heat engine is always less than unity. The efficiency of a steam engine varies from 12 to 16%. The maximum efficiency of a petrol engine is 26% and that of a diesel engine is 40%.

32. *What are the two basic types of heat engines ? Give examples.*

Types of heat engines. The heat engines are of two types :

(*i*) **External combustion engine.** In such a heat engine, the heat needed for the working substance is produced by burning the fuel outside the cylinder and piston arrangement of the engine. A steam engine is an external combustion engine.

(*ii*) **Internal combustion engine.** In such a heat engine, the heat needed for the engine is produced by burning the fuel inside the main cylinder. The petrol and diesel engines are internal combustion engines.

12.24 LIMITATIONS OF THE FIRST LAW OF THERMODYNAMICS

33. *State the limitation of the first law of thermodynamics.*

Limitations of the first law of thermodynamics. These are as follows :

(*i*) ***It does not indicate the direction of transfer of heat.*** Heat always flows from a hot body to a cold one. First law does not give any reason as to why heat cannot flow from a cold body to a hot one.

(*ii*) ***It does not tell anything about the conditions under which heat can be converted into mechanical work.*** First law explains the stopping of a revolving wheel due to conversion of its kinetic energy into heat due to friction. But it fails to explain as to why the heat energy cannot be converted into kinetic energy of rotation of the wheel and put it back into rotation.

(*iii*) ***It does not indicate the extent to which heat energy can be converted into mechanical work continuously.*** No heat engine can convert all the heat extracted from the source into mechanical work continuously without rejecting a part of it to the surrounding. First law has no explanation for this fact.

12.25 SECOND LAW OF THERMODYNAMICS

34. *State and explain second law of thermodynamics. What is its significance ?*

Second law of thermodynamics. There are many processes in which energy is conserved and yet they are never observed. The principle which disallows such phenomena (as discussed in limitations of first law of thermodynamics) consistent with the first law of thermodynamics is called second law of thermodynamics. It can be stated in a number of ways as follows :

(*i*) **Kelvin-Planck statement.** *It is impossible to construct an engine, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.*

This is applicable to a heat engine. It indicates that a working substance, operating in a cycle, cannot convert all the heat extracted from the source into mechanical work. It must reject some heat to the sink at a lower temperature.

Scanned by CamScanner

(ii) **Clausius statement.** *It is impossible for a self-acting machine, unaided by any external agency, to transfer heat from a body to another at higher temperature.*

This is applicable to a refrigerator. The working substance can absorb heat from a cold body only if work is done on it. The work is done by an electric compressor. If no external work is done, the refrigerator will not work.

Significance of second law. The second law of thermodynamics puts a fundamental limit to the efficiency of a heat engine and the coefficient of performance of a refrigerator.

(i) According to second law, *the efficiency of a heat engine can never be unity.* This in turn, implies that the heat released to the cold reservoir can never be made zero.

(ii) According to second law, *the coefficient of performance of a refrigerator can never be infinite.* This implies that the external work (W) can never be zero.

35. *State the limitations of the second law of thermodynamics.*

Limitations of the second law of thermodynamics :

(i) The second law of thermodynamics cannot be proved directly. But its validity has not been contradicted by any machine designed so far.

(ii) It is applicable to a cyclic process in which the system returns to its original state after a complete cycle of changes.

(iii) It makes no pridictions as to what will happen under certain conditions but simply states what will happen under a given set of conditions.

12.26 REVERSIBLE AND IRREVERSIBLE PROCESSES

36. *What do you understand by reversible and irreversible processes ? Give examples. What are the necessary conditions for a process to be reversible ?*

(a) ***Reversible process.*** *Any process which can be made to proceed in the reverse direction by variation in its conditions such that any change occurring in any part of the direct process is exactly reversed in the corresponding part of reverse process is called a reversible process.* Thus if some work is done by the system in the direct process, an equal amount of work must be done on the system in the reverse process. If some heat is absorbed by the system in the direct process, it must release an equal amount of heat to the surroundings in the reverse process. At the end of the reversible process, both the system and surroundings must return to their initial states.

Necessary conditions for a reversible process :

(i) The process must be quasi-static. For this, the process must be carried out infinitesimally slowly so that the system remains in thermal and mechanical equilibrium with the surroundings throughout.

(ii) The dissipative forces such as viscosity, friction, inelasticity, etc. should be absent.

Examples. (i) ***An infinitesimally slow compression and expansion of an ideal gas at constant temperature.*** Consider some ideal gas in a cylinder having conducting walls and provided with a frictionless piston. Increase the pressure of the gas very slowly. The volume of the gas decreases isothermally. The temperature remains constant because heat is continuously rejected by the gas to the surroundings. Now decrease the pressure of the gas very slowly. The volume of the gas increases isothermally. The temperature remains constant because the gas continuously absorbs heat from the surroundings. The process is illustrated in Fig. 12.19.

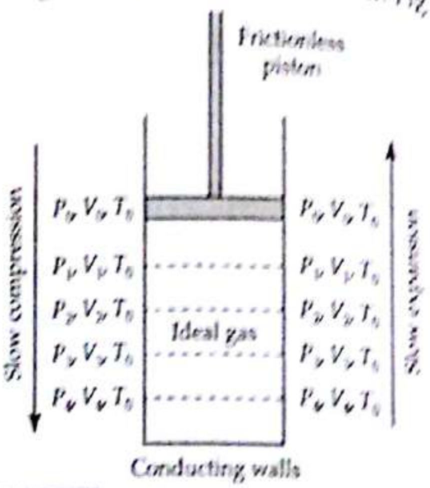


Fig. 12.19 A reversible process

(ii) The process of gradual compression and extension of an elastic spring is approximately reversible.

(iii) A working substance taken along the complete Carnot's cycle.

(iv) The process of electrolysis is reversible if the resistance offered by the electrolyte is negligibly small.

A complete reversible process is an idealised concept as it can never be realised because dissipative forces cannot be completely eliminated.

(b) ***Irreversible process.*** *Any process which cannot be retraced in the reverse direction exactly is called an irreversible process.* Most of the processes occurring in the nature are irreversible processes.

Examples. (i) Diffusion of gases.

(ii) Dissolution of salt in water.

(iii) Rusting of iron.

(iv) Sudden expansion or contraction of a gas.

Scanned by CamScanner

17. *Why is the concept of a reversible process a basic concept of thermodynamics ?*

Reversibility – a basic concept of thermodynamics. Thermodynamics mainly deals with the efficiency with which a heat engine converts heat into work. According to the second law of thermodynamics no heat engine can have a efficiency of 100%. But then what is the maximum efficiency possible for heat engine working between two temperatures T_1 and T_2. It is found that an idealised heat engine based on a reversible cyclic process can have the highest possible efficiency. All practical engines (involving irreversible processes) have efficiency lower than this limiting efficiency. Hence the reversibility is an important concept.

12.27 CARNOT ENGINE

18. *Describe the operation of a Carnot's engine. Calculate the efficiency of a Carnot's engine and explain why the efficiency of an irreversible engine is small.*

Carnot engine. *It is an ideal reversible heat engine that operates between two temperatures T_1 (source) and T_2 (sink).* It was first conceived by a French engineer, *Sadi Carnot* in 1824. It operates through a series of two isothermal and two adiabatic processes called Carnot cycle. It is a theoretical heat engine with which the efficiency of practical engines is compared.

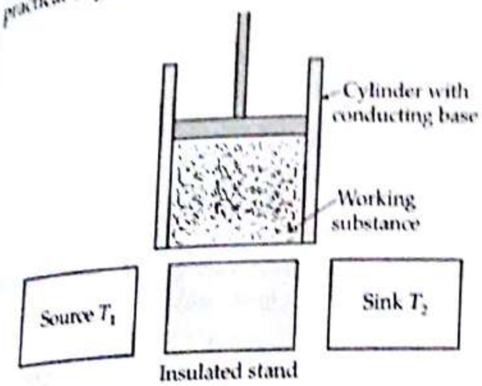


Fig. 12.20 Carnot engine.

Construction. As shown in Fig. 12.20, a Carnot engine has the following main parts :

(*i*) **Cylinder.** This main part of the engine has conducting base and insulating walls. It is fitted with an insulating and frictionless piston.

(*ii*) **Source.** It is a heat reservoir at a higher temperature T_1 from which the engine draws heat. It is supposed that the source has an infinite thermal capacity and so any amount of heat can be drawn from it without changing its temperature.

(*iii*) **Sink.** It is heat reservoir at a lower temperature T_2 to which any amount of heat can be rejected by the engine. It has also infinite thermal capacity and so any amount of heat can be added to it without changing its temperature.

(*iv*) **Working substance.** The working substance is an ideal gas contained in the cylinder.

(*v*) **Insulating stand.** When the base of the cylinder is attached to the insulating stand, the working substance gets isolated from the surroundings.

Carnot cycle. The working substance is carried through a reversible cycle of the following *four* steps :

Step 1. Isothermal expansion. Place the cylinder on the source so that the gas acquires the temperature T_1 of the source. The gas is allowed to expand by slow outward motion of the piston. The temperature of the gas falls. As the gas absorbs the required amount of heat from the source, it expands isothermally.

If Q_1 heat is absorbed from the source and W_1 work is done by the gas in isothermal expansion which takes its state from (P_1, V_1, T_1) to (P_2, V_2, T_1), then

$$W_1 = Q_1 = nRT_1 \ln\left(\frac{V_2}{V_1}\right) = \text{area } ABMKA.$$

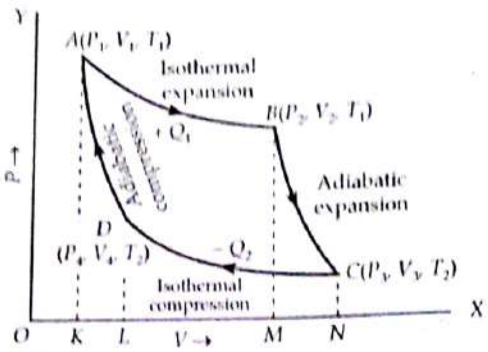


Fig. 12.21 Carnot cycle.

Step 2. Adiabatic expansion. The gas is now placed on the insulating stand and allowed to expand slowly till its temperature falls to T_2.

If W_2 work is done by the gas in the adiabatic expansion which takes its state from (P_2, V_2, T_1) to (P_3, V_3, T_2), then

$$W_2 = \frac{nR(T_1 - T_2)}{\gamma - 1} = \text{area } BCNMB.$$

Step 3. Isothermal compression. The gas is now placed in thermal contact with the sink at temperature T_2. The gas is slowly compressed so that as heat is produced, it easily flows to the sink. The temperature of the gas remains constant at T_2.

Scanned by CamScanner

If Q_2 heat is released by the gas to the sink and W_3 work is done on the gas by the surroundings in the isothermal compression which takes its state from (P_3, V_3, T_2) to (P_4, V_4, T_2), then

$$W_3 = Q_2 = nRT_2 \ln\left(\frac{V_3}{V_4}\right) = \text{area } CNLDC$$

Step 4. Adiabatic compression. The cylinder is again placed on the insulating stand. The gas is further compressed slowly till it returns to its initial state (P_1, V_1, T_1).

If W_4 is the work done in the adiabatic compression from (P_4, V_4, T_2) to (P_1, V_1, T_1), then

$$W_4 = \frac{nR(T_1 - T_2)}{\gamma - 1} = \text{area } DAKLD$$

Net work done by the gas per cycle.

Total work done by the gas $= W_1 + W_2$ (in steps 1 and 2)

Total work done on the gas $= W_3 + W_4$ (in steps 3 and 4)

$\therefore$ *Net work done by the gas in one complete cycle,*

$$W = W_1 + W_2 - (W_3 + W_4)$$

But $\quad W_2 = W_4$

$\therefore \quad W = W_1 - W_3 = Q_1 - Q_2$

Also, $\quad W =$ area $ABMKA$ + area $BCNMB$ − area $CNLDC$ − area $DAKLD$

or $\quad W =$ area $ABCDA$.

Hence *in a Carnot engine, the mechanical work done by the gas per cycle is numerically equal to the area of the Carnot cycle.*

Efficiency of Carnot engine. *It is defined as the ratio of the net work done per cycle by the engine to the amount of heat absorbed per cycle by the working substance from the source.*

$$\therefore \quad \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

or

$$\eta = 1 - \frac{nRT_2 \ln(V_3/V_4)}{nRT_1 \ln(V_2/V_1)}$$

Now step 2 is an adiabatic process, therefore

$$T_1 V_2^{\gamma - 1} = T_2 V_3^{\gamma - 1} \quad \ldots(i)$$

Similarly, step 4 is an adiabatic compression, therefore

$$T_1 V_1^{\gamma - 1} = T_2 V_4^{\gamma - 1} \quad \ldots(ii)$$

On dividing (*i*) by (*ii*), we get

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

or

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Hence

$$\eta = 1 - \frac{T_2}{T_1}.$$

For Your Knowledge

- The efficiency of a Carnot engine
  - depends upon the temperatures of the source and the sink.
  - is independent of the nature of the working substance.
  - is the same for all reversible engines working between the same two temperatures.
  - is directly proportional to the temperature difference $(T_1 - T_2)$.
  - is always less than 100% because $Q_2 < Q_1$.
- The efficiency of a Carnot engine will be unity or 100% if $T_1 = \infty$ or $T_2 = 0$ K. As 0 K or infinite temperature cannot be realised, hence a Carnot engine working on reversible cycle cannot have 100% efficiency.
- If $T_1 = T_2$, then $\eta = 0$. This means that the conversion of heat into mechanical work is impossible without having the source and sink at different temperatures.
- As $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

  $\therefore$ If $T_2 = 0$ K, then $Q_2 = 0$.

  Since $T_2 = 0$ K cannot be realised, so $Q_2 = 0$ is also not possible. This means that it is not possible to convert whole of the heat energy absorbed from the source into mechanical work continuously, without rejecting a part of it to the sink.

39. *Can a Carnot engine be realised in practice ?*

Non-practicability of Carnot engine. Carnot engine is an ideal engine. It cannot be realised in practice due to the following reasons :

(*i*) It is difficult to realise source and sink of infinite thermal capacity.

(*ii*) The working substance should be an ideal gas. But no real gas fulfills the ideal gas behaviour.

(*iii*) The cylinder cannot be provided perfect frictionless piston.

(*iv*) It is difficult to attain the conditions of reversibility because the processes of expansion and compression have to be carried out very slowly.

Scanned by CamScanner

Examples based on Carnot Engine

FORMULAE USED

1. Efficiency of a heat engine,

$$\eta = \frac{\text{Work output}}{\text{Heat input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

2. Efficiency of a Carnot's engine (an ideal heat engine),

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

where Q_1 = heat extracted from the source
Q_2 = heat rejected to the sink
T_1 = temperature of the source
T_2 = temperature of the sink

UNITS USED

Heats Q_1 and Q_2 are in joule and temperatures T_1 and T_2 are in kelvin. Efficiency η has no units.

EXAMPLE 27. *What is the efficiency of a Carnot engine working between ice point and steam point ?*

Solution. Temperature of source,

$$T_1 = \text{Steam point} = 373 \text{ K}$$

Temperature of sink, T_2 = Ice point = 273 K

$$\text{Efficiency, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{373} = 0.268 = \mathbf{26.8\%}.$$

EXAMPLE 28. *One of the most efficient engines ever developed operated between 2100 K and 700 K. Its actual efficiency is 40%. What percentage of its maximum possible efficiency is this ?*

Solution. $T_1 = 2100$ K, $T_2 = 700$ K, $\eta_{actual} = 40\%$

Maximum possible efficiency is

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{700}{2100} = 0.666 = 66.6\%.$$

Actual efficiency as the percentage of the maximum possible efficiency is

$$\frac{\eta_{actual}}{\eta_{max}} \times 100 = \frac{40}{66.6} \times 100 \simeq \mathbf{60\%}.$$

EXAMPLE 29. *In a heat engine, the temperature of the source and sink are 500 K and 375 K. If the engine consumes 25×10^5 J per cycle, find (i) the efficiency of the engine, (ii) work done per cycle, and (iii) heat rejected to the sink per cycle.* **[Central Schools 12]**

Solution. Here $T_1 = 500$ K, $T_2 = 375$ K, $Q_1 = 25 \times 10^5$ J

$$(i)\ \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{375}{500} = 0.25 = \mathbf{25\%}.$$

$$(ii)\ W = \eta Q_1 = 0.25 \times 25 \times 10^5 = \mathbf{6.25 \times 10^5\ J}.$$

$$(iii)\ Q_2 = Q_1 - W = (25 - 6.25) \times 10^5 = \mathbf{18.75 \times 10^5\ J}.$$

EXAMPLE 30. *A carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. Find the work done by the engine.* **[AIEEE 03]**

Solution. Here $T_1 = 627 + 273 = 900$ K,

$$T_2 = 27 + 273 = 300 \text{ K}, \quad Q_1 = 3 \times 10^6 \text{ cal}$$

As $$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\therefore \quad Q_2 = \frac{T_2}{T_1} \times Q_1 = \frac{3 \times 10^6 \times 300}{900} = 10^6 \text{ cal}$$

Work done by the engine,

$$W = Q_1 - Q_2 = 3 \times 10^6 - 10^6 \text{ cal} = 2 \times 10^6 \text{ cal}$$

$$= 2 \times 10^6 \times 4.2 \text{ J} = \mathbf{8.4 \times 10^6\ J}.$$

EXAMPLE 31. *The efficiency of a Carnot cycle is 1/6. If on reducing the temperature of the sink by 65°C, the efficiency becomes 1/3, find the initial and final temperatures between which the cycle is working.*

Solution. Given $\eta_1 = \frac{1}{6}$, $\eta_2 = \frac{1}{3}$

If the temperatures of the source and the sink between which the cycle is working are T_1 and T_2, then the efficiency in the first case will be

$$\eta_1 = 1 - \frac{T_2}{T_1} \quad \text{or} \quad \frac{1}{6} = 1 - \frac{T_2}{T_1} \qquad \ldots(1)$$

In the second case, the temperature of the sink is reduced by 65°C. Hence

$$\eta_2 = 1 - \frac{T_2 - 65}{T_1} \quad \text{or} \quad \frac{1}{3} = 1 - \frac{T_2 - 65}{T_1} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$T_1 = 390 \text{ K} = \mathbf{117°C}, \quad T_2 = 325 \text{ K} = \mathbf{52°C}.$$

EXAMPLE 32. *A reversible engine converts one fifth of heat which it absorbs from source into work. When the temperature of the sink is reduced by 70°, its efficiency is doubled. Calculate the temperature of the source and the sink.*

Solution. Here $W = \frac{1}{5} Q_1$

$$Q_2 = Q_1 - W = Q_1 - \frac{1}{5} Q_1 = \frac{4}{5} Q_1$$

Hence $$\frac{Q_2}{Q_1} = \frac{4}{5} \quad \text{and} \quad \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{4}{5} \qquad \ldots(1)$$

$$\text{Efficiency, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{4}{5} = \frac{1}{5}$$

Scanned by CamScanner

On reducing the temperature of the sink by 70°, the efficiency is doubled.

$$\therefore \quad 2\eta = 1 - \frac{T_2 - 70}{T_1}$$

or $$2 \times \frac{1}{5} = 1 - \frac{T_2}{T_1} + \frac{70}{T_1} = 1 - \frac{4}{5} + \frac{70}{T_1}$$

[Using equation (1)]

or $$\frac{2}{5} = \frac{1}{5} + \frac{70}{T_1} \quad \text{or} \quad \frac{70}{T_1} = \frac{1}{5}$$

or $$T_1 = \mathbf{350\ K}.$$

and $$T_2 = \frac{4}{5} T_1 = \frac{4}{5} \times 350 = \mathbf{280\ K}.$$

Example 33. *A Carnot engine has the same efficiency (i) between 100 K and 500 K and (ii) between T K and 900 K. Calculate the temperature T K of the sink.*

Solution. (*i*) Here $T_1 = 500$ K, $T_2 = 100$ K

$$\therefore \text{ Efficiency, } \quad \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{500} = \frac{400}{500} = \frac{4}{5}$$

(*ii*) Here $T_1' = 900$ K, $T_2' = T$ K

$$\therefore \text{Efficiency, } \quad \eta' = 1 - \frac{T_2'}{T_1'} = 1 - \frac{T}{900}$$

But $\eta' = \eta \quad \therefore \quad 1 - \frac{T}{900} = \frac{4}{5} \quad$ or $\quad T = \mathbf{180\ K}.$

Example 34. *A Carnot engine whose heat sink is at 27°C has an efficiency of 40%. By how many degrees should the temperature of the source be changed to increase the efficiency by 10% of the original efficiency ?*

Solution. Here $T_2 = 27 + 273 = 300$ K, $\eta = 40\%$,

$$T_1 = ?$$

As $$\eta = 1 - \frac{T_2}{T_1}$$

$$\therefore \quad \frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{60}{100} = \frac{3}{5}$$

or $$T_1 = \frac{5}{3} \times T_2 = \frac{5}{3} \times 300 = 500 \text{ K}$$

Increase in efficiency = 10% of 40 = 4%.

$\therefore$ New efficiency, $\eta' = 40 + 4 = 44\%$

Let new temperature of the source $= T_1'$ K. Then

$$\eta' = 1 - \frac{T_2}{T_1'} \quad \text{or} \quad \frac{44}{100} = 1 - \frac{300}{T_1'}$$

or $$\frac{300}{T_1'} = 1 - \frac{44}{100} = \frac{56}{100}$$

$$\therefore \quad T_1' = \frac{100 \times 300}{56} = 535.7 \text{ K}$$

Increase in temperature of the source

$$= 535.7 - 500 = \mathbf{35.7\ K \text{ or } 35.7°C}.$$

Example 35. *An ideal engine operates by taking in steam from a boiler at a temperature of 327°C and rejecting heat to the sink at a temperature of 27°C. The engine runs at 500 rpm and the heat taken is 600 kcal in each revolution. Calculate (i) the Carnot efficiency of the engine (ii) the work done in each cycle (iii) the heat rejected in each revolution and (iv) the power output of this engine.*

Solution. Here $T_1 = 327 + 273 = 600$ K,

$T_2 = 273 + 27 = 300$ K, $Q_1 = 600$ kcal

(*i*) Efficiency, $$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 0.5 = 50\%.$$

(*ii*) As $$\eta = \frac{W}{Q_1}$$

$$\therefore \quad W = \eta Q_1 = 0.5 \times 600 = 300 \text{ kcal}$$
$$= 300 \times 4.2 \times 10^3 \text{ J} = \mathbf{1.26 \times 10^6\ J}.$$

(*iii*) Heat rejected,

$$Q_2 = Q_1 - W = 600 - 300 = \mathbf{300\ kcal}.$$

(*iv*) Power output,

$$P = \frac{W}{t} = \frac{500 \times 1.26 \times 10^6 \text{ J}}{60 \text{ s}} = 1.05 \times 10^7 \text{ W}$$
$$= \mathbf{1.05 \times 10^4\ kW}.$$

Example 36. *Two Carnot engines A and B are operated in series. The first one A receives heat 800 K and rejects to a reservoir at temperature T K. The second engine B receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 300 K. Calculate the temperature T K for the following cases :*

(*i*) *When the outputs of the two engines are equal.*

(*ii*) *When the efficiencies of the two engines are equal.*

Solution. *For engine A* : $T_1 = 800$ K, $T_2 = T$ K

Efficiency, $$\eta_A = 1 - \frac{T_2}{T_1} = 1 - \frac{T}{800}$$

Also, $$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{T}{800}$$

Work output,

$$W_A = Q_1 - Q_2 = \eta_A \times Q_1 \quad \left[\because \eta_A = 1 - \frac{Q_2}{Q_1}\right]$$

or $$W_A = \left(1 - \frac{T}{800}\right) Q_1$$

For engine B : $T_1' = T$ K, $T_2' = 300$ K

Efficiency, $$\eta_B = 1 - \frac{T_2'}{T_1'} = 1 - \frac{300}{T}$$

Work output,

$$W_B = Q_1' - Q_2' = \eta_B \times Q_1' = \left(1 - \frac{300}{T}\right) Q_1'$$

Scanned by CamScanner

Since the engine B absorbs the heat rejected by the engine A so

$$Q_1' = Q_2$$

$$W_B = \left(1 - \frac{300}{T}\right) Q_2$$

Case (i). When outputs of the two engines are equal,

$$W_A = W_B$$

$$\left(1 - \frac{T}{800}\right) Q_1 = \left(1 - \frac{300}{T}\right) Q_2$$

or $$1 - \frac{T}{800} = \left(1 - \frac{300}{T}\right) \frac{Q_2}{Q_1} = \left(1 - \frac{300}{T}\right) \frac{T}{800}$$

On solving, we get : $T = 550$ **K.**

Case (ii). When the efficiencies are equal, $\eta_A = \eta_B$

$$1 - \frac{T}{800} = 1 - \frac{300}{T} \quad \text{or} \quad T^2 = 24 \times 10^4$$

or $T = 489.9$ **K.**

EXAMPLE 37. *Five moles of an ideal gas are taken in a Carnot engine working between 100°C and 30°C. The useful work done in one cycle is 420 joule. Calculate the ratio of the volume of the gas at the end and beginning of the isothermal expansion. $R = 8.4$ J mole^{-1} K^{-1}.* **[NCERT]**

Solution. Here $T_1 = 100 + 273 = 373$ K

$$T_2 = 30 + 273 = 303 \text{ K}$$

Amount of ideal gas = 5 mole

Useful work done in one cycle,

$$W = Q_1 - Q_2 = 420 \text{ J} \qquad ...(i)$$

Now, $$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} = \frac{373}{303} \quad \text{or} \quad Q_1 = \frac{373}{303} Q_2 \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{373}{303} Q_2 - Q_2 = 420 \quad \text{or} \quad \frac{70}{303} Q_2 = 420$$

or $$Q_2 = \frac{420 \times 303}{70} = 1818 \text{ J.}$$

From equation (*i*),

$$Q_1 = Q_2 + 420 = 1818 + 420 = 2238 \text{ J.}$$

When the gas is carried through a Carnot cycle, the heat absorbed Q_1 during the isothermal expansion is equal to the work done by the gas. If V_1 and V_2 are the volumes of the gas at the beginning and at the end of the isothermal expansion, then

$$Q_1 = 2.303\, nRT_1 \log_{10} \frac{V_2}{V_1}$$

or $$2238 = 2.303 \times 5 \times 8.4 \times 373 \log_{10} \frac{V_2}{V_1}$$

or $$\log_{10} \frac{V_2}{V_1} = \frac{2238}{2.303 \times 5 \times 8.4 \times 373} = 0.0620$$

Hence $$\frac{V_2}{V_1} = \mathbf{1.153.}$$

EXAMPLE 38. *A Carnot cycle is performed by air initially at 327°C. Each stage represents a compression or expansion in the ratio 1 : 6. Calculate (i) the lowest temperature and (ii) efficiency of the cycle. Given $\gamma = 1.4$.*

Solution. Here $T_1 = 327 + 273 = 600$ K,

$$V_1 / V_2 = 1/6, \quad \gamma = 1.4$$

(*i*) As $T_2 V_2^{\gamma - 1} = T_1 V_1^{\gamma - 1}$

$$\therefore \quad T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \times T_1$$

$$= \left(\frac{1}{6}\right)^{1.4 - 1} \times 600 = \mathbf{293\ K} \quad \text{or} \quad \mathbf{20°C.}$$

(*ii*) Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

$$= 1 - \frac{293}{600} = 0.512 = \mathbf{51.2\%.}$$

PROBLEMS FOR PRACTICE

1. Efficiency of an engine is 0.4, when temperature of its sink (cold body) is 300 K. What is the temperature of the hot body ? Find its efficiency, if the temperature of the hot body is kept unchanged, while temperature of the sink (cold body) is lowered by 50 K. (Treat it as reversible engine).
   (Ans. 500 K, 0.5)

2. A Carnot engine absorbs 1000 J of heat from a reservoir at 127°C and rejects 600 J of heat during each cycle. Calculate (*i*) efficiency of the engine (*ii*) temperature of the sink and (*iii*) amount of the useful work done during each cycle. *[Central Schools 14]*
   [**Ans.** (*i*) 40% (*ii*) – 33°C (*iii*) 400 J]

3. A reversible heat engine operates with an efficiency of 50%. If during each cycle it rejects 150 cal to a reservoir of heat at 30°C, then (*i*) what is the temperature of the other reservoir and (*ii*) how much work does it carry out per cycle ?
   [**Ans.** (*i*) 333° C (*ii*) 630 J]

4. A reversible engine converts one sixth of heat input into work. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. Find the temperature of the source and the sink.
   (Ans. 372 K, 310 K)

5. Calculate the difference in efficiencies of a Carnot engine working between (*i*) 400 K and 350 K and (*ii*) between 350 K and 300 K. **(Ans.** 1.8%)

Scanned by CamScanner

6. A Carnot engine operates between 227°C and 127°C. If it absorbs 60×10^4 calorie at higher temperature, how much work per cycle can the engine perform ? (Ans. 5.04×10^5 J)

7. A perfect Carnot engine utilises an ideal gas as the working substance. The source temperature is 227°C and the sink temperature is 127°C. Find the efficiency of this engine, and find the heat received from the source and the heat released to the sink when 10,000 J of external work is done.

(Ans. 20%, 5×10^4 J, 4×10^4 J)

8. A Carnot engine takes in heat from a reservoir of heat at 427°C and gives out heat to the sink at 77°C. How many calorie per second must it take from the reservoir in order to produce useful mechanical work at the rate of 357 W ? (Ans. 170 cal s^{-1})

9. Two Carnot engines A and B are operated in series. The first one A receives heat at 900 K and rejects to a reservoir at temperature T K. The second engine B receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 400 K. Calculate the temperature T for the situation when (*i*) the efficiencies of the two engines are equal (*ii*) the work outputs of the two engines are equal.

[Ans. (*i*) 600 K (*ii*) 650 K]

HINTS

1. *Case* (*i*) $\eta = 0.4$, $T_2 = 300$ K, $T_1 = ?$

As $\eta = 1 - \dfrac{T_2}{T_1}$ $\therefore$ $0.4 = - \dfrac{300}{T_1}$ or $T_1 = \mathbf{500\ K}$.

Case (*ii*) $T_1 = 500$ K, $T_2 = 300 - 50 = 250$ K

$\therefore$ $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{250}{500} = \mathbf{0.5}$.

2. (*i*) $Q_1 = 1000$ J, $Q_2 = 600$ J, $T_1 = 127 + 273 = 400$ K

$\therefore$ $\eta = 1 - \dfrac{Q_2}{Q_1} = 1 - \dfrac{600}{1000} = 0.4 = \mathbf{40\%}$.

$T_2 = \dfrac{Q_2}{Q_1} \times T_1 = \dfrac{600}{1000} \times 400 = 240\ \text{K} = \mathbf{-33°C}$.

(*iii*) $W = Q_1 - Q_2 = 1000 - 600 = \mathbf{400\ J}$.

3. Here $Q_2 = 150$ cal, $T_2 = 30 + 273 = 303$ K,

$\eta = 50\% = 1/2$

(*i*) As $\eta = 1 - \dfrac{T_2}{T_1}$ $\therefore$ $\dfrac{1}{2} = 1 - \dfrac{303}{T_1}$

or $T_1 = 606\ \text{K} = \mathbf{333°C}$.

(*ii*) $Q_1 = \dfrac{T_1}{T_2} \times Q_2 = \dfrac{606}{303} \times 150 = 300$ cal

$\therefore$ $W = Q_1 - Q_2 = 300 - 150 = 150$ cal

$= 150 \times 4.2 = \mathbf{630\ J}$.

4. Here $\eta = 1 - \dfrac{T_2}{T_1} = \dfrac{1}{6}$ or $\dfrac{T_2}{T_1} = \dfrac{5}{6}$

When the temperature of the sink is reduced by 62°C or 62 K, efficiency is doubled.

$\therefore$ $T_2' = T_2 - 62$, $\eta' = \dfrac{1}{6} \times 2 = \dfrac{1}{3}$

But $\eta' = 1 - \dfrac{T_2'}{T_1}$ or $\dfrac{1}{3} = 1 - \dfrac{T_2 - 62}{T_1} = 1 - \dfrac{T_2}{T_1} + \dfrac{62}{T_1}$

or $\dfrac{62}{T_1} = \dfrac{T_2}{T_1} - \dfrac{2}{3} = \dfrac{5}{6} - \dfrac{2}{3} = \dfrac{1}{6}$ or $T_1 = 372$ K

and $T_2 = \dfrac{5}{6} \times T_1 = \dfrac{5}{6} \times 372 = \mathbf{310\ K}$.

5. $\eta = 1 - \dfrac{T_2}{T_1} = \left(1 - \dfrac{350}{400}\right) \times 100 = 12.5\%$.

$\eta' = 1 - \dfrac{T_2'}{T_1'} = \left(1 - \dfrac{300}{350}\right) \times 100 = 14.3\%$.

Difference in efficiencies

$= \eta' - \eta = 14.3 - 12.5 = 1.8\%$.

6. Here $T_1 = 227 + 273 = 500$ K, $T_2 = 127 + 273 = 400$ K

As $\eta = \dfrac{W}{Q_1} = 1 - \dfrac{T_2}{T_1}$ $\therefore$ $\dfrac{W}{60 \times 10^4} = 1 - \dfrac{400}{500} = \dfrac{1}{5}$

or $W = 12 \times 10^4$ cal

$= 12 \times 10^4 \times 4.2\ \text{J} = \mathbf{5.04 \times 10^5\ J}$.

8. Use $\eta = 1 - \dfrac{T_2}{T_1}$ and $\eta = \dfrac{W}{Q_1}$.

9. Here $\eta_A = \dfrac{W_A}{Q_1} = \dfrac{T_1 - T_2}{T_1}$ and $\eta_B = \dfrac{W_B}{Q_1'} = \dfrac{T_1' - T_2'}{T_1'}$

Take (*i*) $W_A = W_B$ and (*ii*) $\eta_A = \eta_B$.

12.28 CARNOT THEOREM

40. ***State Carnot theorem. Prove that the efficiency of a reversible heat engine is maximum.***

Carnot theorem. *It states that (i) no engine working between two given temperatures can have efficiency greater than that of the Carnot engine working between the same two temperatures and (ii) the efficiency of the Carnot engine is independent of the nature of the working substance.*

Proof. As shown in Fig. 12.22, consider two engines – an irreversible engine I and a reversible engine R. The two engines are so coupled that as I runs forwards, it drives R backwards. So R works as a refrigerator driven by I.

The engine I absorbs heat Q_1 from the source, performs work W and rejects heat Q_2 to the sink.

$\therefore$ Efficiency of engine I, $\eta_I = \dfrac{W}{Q_1} = \dfrac{Q_1 - Q_2}{Q_1}$

Scanned by CamScanner

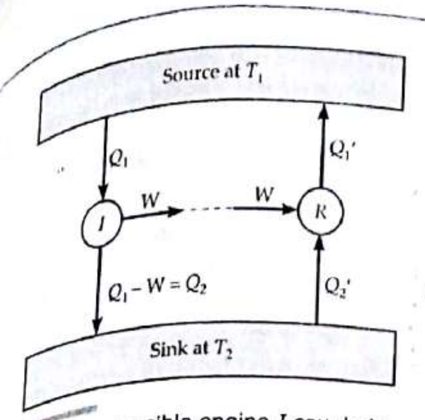


Fig. 12.22 An irreversible engine I coupled to a reversible engine R.

The engine R absorbs heat Q_2' from the sink, work W is done on it and it rejects heat Q_1' to the source.

$\therefore$ Efficiency of engine R, $\eta_R = \dfrac{W}{Q_1'} = \dfrac{Q_1' - Q_2'}{Q_1'}$

Suppose the engine I is more efficient than R. Then

$$\eta_I > \eta_R \quad \text{or} \quad \frac{W}{Q_1} > \frac{W}{Q_1'}$$

$$\therefore \quad Q_1 < Q_1' \quad \textit{i.e.,} \quad Q_1' - Q_1 \text{ is positive.}$$

The source loses heat Q_1 to I and gains Q_1' from R.

$\therefore$ Net heat gained by the source per cycle

$$= Q_1' - Q_1 .$$

The sink gains heat $(Q_1 - W)$ from I and loses Q_2' to R.

$\therefore$ Net heat lost by the sink per cycle

$$= Q_2' - (Q_1 - W) = Q_2' - Q_1 + W$$
$$= Q_2' - Q_1 + (Q_1' - Q_2') = Q_1' - Q_1$$

The compound engine IR is a self-acting machine which transfers heat $(Q_1' - Q_1)$ from the sink at lower temperature T_2 to the source at higher temperature T_1, without any work being done by any external agency. This is against the second law of thermodynamics. So our assumption that I is more efficient than R is wrong. *Hence no engine can have efficiency greater than that of the Carnot engine.* Similarly, we can prove that a reversible engine with one working substance cannot be more efficient than the one using another working substance.

12.29 REFRIGERATOR OR HEAT PUMP

41. *Describe the working of a refrigerator as a heat pump. Derive an expression for its coefficient of performance.*

Refrigerator. *A refrigerator is a Carnot's heat engine working in the reverse direction.*

In a refrigerator, the working substance absorbs an amount of heat Q_2 from the cold reservoir at temperature T_2. An amount of work W is done on it by some external agency (a compressor pump driven by an electric motor) and rejects a larger quantity of heat Q_1 to the source at temperature T_1, as shown in Fig. 12.23.

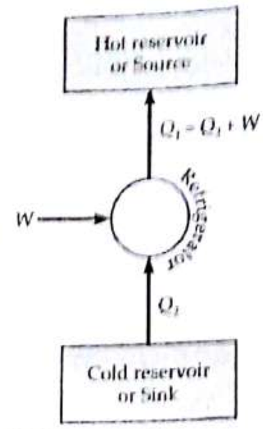


Fig. 12.23 Refrigerator as a reversed heat engine.

In domestic refrigerators, food and ice constitute the cold reservoir and the surroundings act as hot reservoir. Work is done by an electric motor and freon (CCl_2F_2) is used as a working substance.

The working substance is carried through a cycle of the following four steps :

(*i*) The gas is allowed to expand suddenly (adiabatically) from high to low pressure. This cools it and converts it into a vapour-liquid mixture.

(*ii*) The cold fluid is allowed to absorb heat Q_2 isothermally from the cold reservoir. This converts the mixture into vapour.

(*iii*) Then the vapour is adiabatically compressed till it heats up to the temperature of the surroundings.

(*iv*) Finally the vapour is compressed isothermally in contact with the surroundings. The vapour releases heat Q_1 $(= Q_2 + W)$ to the surroundings and returns to the initial state. Here W is the work done on the gas per cycle.

Coefficient of performance. *It may be defined as the ratio of the amount of heat removed (Q_2) per cycle to the mechanical work (W) required to be done on it.*

$$\therefore \quad \beta = \frac{Q_2}{W}$$

By first law of thermodynamics (energy conservation), the heat released to the hot reservoir is

$$Q_1 = Q_2 + W$$

$$\therefore \quad W = Q_1 - Q_2 \quad \text{and} \quad \beta = \frac{Q_2}{Q_1 - Q_2}$$

or

$$\beta = \frac{1}{\left(\dfrac{Q_1}{Q_2} - 1\right)} = \frac{1}{\dfrac{T_1}{T_2} - 1} \qquad \left[\because \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right]$$

or

$$\beta = \frac{T_2}{T_1 - T_2}.$$

Scanned by CamScanner

For Your Knowledge

- The coefficient of performance represents the efficiency of a refrigerator. Higher the value of β, more efficient is the refrigerator.
- In a heat engine, heat cannot be fully converted into work. Likewise a refrigerator cannot work without some external work done on the system. Hence *the coefficient of performance cannot be infinite*. Practical refrigerators have a coefficient of performance close to ten.
- Lesser the temperature difference $(T_1 - T_2)$ between the atmosphere and the cooling chamber, higher is the coefficient of performance of the refrigerator.
- **Why is defrosting necessary ?** As the refrigerator works, T_2 decreases due to formation of too much ice and T_1 remains almost constant. The temperature difference $(T_1 - T_2)$ increases. This decreases the value of β. Defrosting increases T_2 and hence improves the coefficient of performance.

Examples based on Refrigerators

FORMULAE USED

Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

where Q_2 = heat drawn per cycle from sink

W = work done per cycle on refrigerator

UNITS USED

Q_1, Q_2 and W are in joule, temperatures T_1 and T_2 in K.

EXAMPLE 39. *In a refrigerator, heat from inside at 277 K is transferred to a room at 300 K. How many joules of heat shall be delivered to the room for each joule of electrical energy consumed ideally ?*

Solution. Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$\therefore \quad Q_2 = W \frac{T_2}{T_1 - T_2}$$

But W = Energy consumed by the refrigerator = 1 J, $T_1 = 300$ K, $T_2 = 277$ K

$$\therefore \quad Q_2 = 1 \times \frac{277}{300 - 277} = \frac{277}{23} = 12 \text{ J}$$

Heat rejected by the refrigerator,

$$Q_1 = W + Q_2 = 1 + 12 = \mathbf{13\ J.}$$

EXAMPLE 40. *(i) Calculate the least amount of work that must be done to freeze one gram of water at 0°C by means of a refrigeration machine. The temperature of the surroundings is 27°C. (ii) How much heat is passed on to the surroundings in the process ?*

Solution. Here $T_1 = 27°\text{C} = 300$ K,

$T_2 = 0°\text{C} = 273$ K

(i) To freeze one gram of water at 0°C, 80 cal of heat must be transferred from water at 0°C to the surroundings at 27°C.

$$\therefore \quad Q_2 = 80 \text{ cal}$$

Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

or $$\frac{80 \text{ cal}}{W} = \frac{273}{300 - 273}$$

$$\therefore \quad W = \frac{80 \times 27}{273} = \mathbf{7.91\ cal.}$$

(ii) Heat transferred to the surroundings,

$$Q_1 = Q_2 + W = 80 + 7.91 = \mathbf{87.91\ cal.}$$

EXAMPLE 41. *How much energy in watt hour may be required to convert 2 kg of water into ice at 0°C, assuming that the refrigerator is ideal ? Given temperature of freezer is −15°C, room temperature is 25°C and initial temperature of water is 25°C.*

Solution. Here $T_1 = 25 + 273 = 298$ K,

$T_2 = -15 + 273 = 258$ K

Specific heat of water, $c = 4.2 \times 10^3$ J kg^{-1}

Latent heat of ice, $L = 3.36 \times 10^5$ J kg^{-1}

Amount of heat required to be removed from 2 kg of water at 25°C to change it into ice at 0°C,

$$Q_2 = Mc(\theta_2 - \theta_1) + ML$$
$$= 2 \times 4.2 \times 10^3 (25 - 0) + 2 \times 3.36 \times 10^5$$
$$= 2.1 \times 10^5 + 6.72 \times 10^5$$
$$= 8.82 \times 10^5 \text{ J}$$

Heat rejected to the surroundings,

$$Q_1 = \frac{T_1}{T_2} \times Q_2 = \frac{298}{259} \times 8.82 \times 10^5 = 10.15 \times 10^5 \text{ J}$$

Energy supplied to convert water into ice,

$$W = Q_1 - Q_2 = (10.15 - 8.82) \times 10^5 = 1.33 \times 10^5 \text{ J}$$

$$= \frac{1.33 \times 10^5}{3600} = \mathbf{36.96\ Wh} \qquad [\because 1 \text{ Wh} = 3600 \text{ J}]$$

EXAMPLE 42. *Ice in a cold storage melts at the rate of 2 kg per hour when the external temperature is 20°C. Find the minimum power output of the motor used to drive the refrigerator which just prevents the ice from melting. Take latent heat of ice = 80 cal g^{-1} and J = 4.2 J cal^{-1}.*

Scanned by CamScanner

Solution. Mass of ice that melts per second

$$= \frac{2 \times 1000}{3600} = \frac{5}{9} \text{ g}$$

To prevent melting of ice, heat needed to be drawn out per second,

$$Q_2 = mL = \frac{5}{9} \times 80 = \frac{400}{9} \text{ cal}$$

Also $T_2 = 273$ K, $T_1 = 20 + 273 = 293$ K

Coefficient of performance, $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore W = \frac{Q_2(T_1 - T_2)}{T_2} = \frac{400 \times (293 - 273)}{9 \times 273}$$

$$= \frac{400 \times 20}{9 \times 273} \text{ cal s}^{-1} = \frac{8000 \times 4.2}{9 \times 273} \text{ Js}^{-1}$$

$$= 13.67 \text{ Js}^{-1}$$

$\therefore$ Minimum power of the motor used = **13.67 W.**

EXAMPLE 43. *A Carnot engine having a perfect gas as the working substance is driven backward and is used for freezing water already at 0°C. If the engine is driven by a 500 W electric motor having an efficiency of 60%, how long will it take to freeze 15 kg of water. Take 15°C and 0°C as the working temperatures of the engine and assume there are no heat losses in the refrigerating system. Latent heat of ice $= 333 \times 10^3$ J kg^{-1}.*

Solution. Here $T_1 = 273$ K, $T_2 = 15 + 273 = 288$ K,

$$L = 333 \times 10^3 \text{ J kg}^{-1}$$

Efficiency of electric motor = 60%

Useful power of the engine

$$= 60\% \text{ of } 500 \text{ W} = 300 \text{ W}$$

or useful work done by the engine, $W = 300 \text{ Js}^{-1}$

Coefficient of performance of the refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$\therefore$ Heat drawn from water at 0°C to freeze it into ice,

$$Q_2 = \frac{T_2}{T_1 - T_2} \times W$$

$$= \frac{273}{285 - 273} \times 300 = 5460 \text{ Js}^{-1}$$

Total heat needed to be drawn from 15 kg water to freeze into ice,

$$Q = mL = 15 \times 333 \times 10^3 \text{ J}$$

Total time taken in freezing water into ice

$$= \frac{Q}{Q_2} = \frac{15 \times 333 \times 10^3}{5640} = \mathbf{914.8 \text{ s.}}$$

PROBLEMS FOR PRACTICE

1. Refrigerator *A* works between − 10°C and 27°C, while refrigerator *B* works between − 27°C and 17°C, both removing heat equal to 2000 J from the freezer. Which of the two is the better refrigerator? **(Ans.** Refrigerator *A***)**

2. Assuming that a domestic refrigerator can be regarded as a reversible engine working between the temperature of melting ice and that of the atmosphere (17°C), calculate the energy which must be supplied to freeze one kilogram of water already at 0°C. **(Ans.** 2.092×10^4 J**)**

3. A refrigerator whose coefficient of performance is 5 extracts heat from the cooling compartment at the rate of 250 J/cycle. How much electric energy is spent per cycle? How much heat per cycle is discharged to the room? **(Ans.** 50 J, 300 J**)**

4. A refrigerator has to transfer an average of 263 J of heat per second from temperature − 10°C to 25°C. Calculate the average power consumed, assuming no energy losses in the process. **[Central Schools 04]** **(Ans.** 35 W**)**

5. A refrigerator freezes 5 kg of water at 0°C into ice at 0°C in a time interval of 20 minutes. Assume that the room temperature is 20°C. Calculate the minimum power needed to accomplish it. **(Ans.** 102.5 W**)**

HINTS

2. Here $T_2 = 273$ K, $T_1 = 17 + 273 = 290$ K

Heat required to be removed to freeze 1 kg of water at 0°C

$$Q_2 = mL = 1 \times 80{,}000 \text{ cal} = 80{,}000 \times 4.2 \text{ J}$$

As $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore W = \frac{Q_2(T_1 - T_2)}{T_1}$$

$$= \frac{80000 \times 4.2 \times (290 - 273)}{273} = \mathbf{2.092 \times 10^4 \text{ J.}}$$

3. Here $Q_2 = 250$ J / cycle, $\beta = 5$

As $\beta = \frac{Q_2}{W}$

$$\therefore W = \frac{Q_2}{\beta} = \frac{250}{5} = \mathbf{50 \text{ J / cycle}}$$

Heat discharged to the room per cycle,

$$Q_1 = Q_2 + W = 250 + 50 = \mathbf{300 \text{ J.}}$$

4. Here $T_1 = 25 + 273 = 298$ K,

$T_2 = -10 + 273 = 263$ K,

$Q_2 = 263$ J

Scanned by CamScanner

As $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$\therefore \quad Q_1 = \frac{T_1}{T_2} \times Q_2 = \frac{298}{263} \times 263 = 298 \text{ J s}^{-1}$

Average power consumed,

$W = Q_1 - Q_2 = 298 - 263 = 35 \text{ J s}^{-1} = \mathbf{35\ W}.$

5. $Q_2 = mL = 5 \times 80 = 400$ kcal

$T_1 = 20 + 273 = 293$ K, $T_2 = 273$ K

As $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$\therefore \quad W = \frac{Q_2 (T_1 - T_2)}{T_2} = \frac{400 \times (293 - 273)}{273}$

$= 29.3 \text{ kcal} = 29.3 \times 4.2 \times 10^3 \text{ J} = 123 \times 10^3 \text{ J}$

$t = 20 \text{ min} = 1200 \text{ s}$

$P = \frac{W}{t} = \frac{123 \times 10^3}{1200} = \mathbf{102.5\ W}.$

Very Short Answer Conceptual Problems

Problem 1. What is a cyclic process ? What is the change in internal energy of a system after it completes one cycle of such a process ?

Solution. Any process in which the system returns to its initial state after undergoing a series of changes is known as a cyclic process. The change in internal energy after complete cycle is zero because the system returns to its initial state.

Problem 2. State the first law of thermodynamics. *[Delhi 1999]*

Solution. According to the first law of thermodynamics, the amount of heat ΔQ absorbed by a system capable of doing mechanical work is equal to the sum of the increase in internal energy ΔU of the system and the external work ΔW done by the system. Mathematically,

$$\Delta Q = \Delta U + \Delta W = \Delta U + P\,\Delta V.$$

Problem 3. How does the internal energy of an ideal gas differ from that of real gas ?

Solution. The internal energy of an ideal gas consists of only the kinetic energy of the particles. But for real gases it consists of both the kinetic as well as potential energies.

Problem 4. Is the internal energy of a gas a function of pressure ? Comment.

Solution. The internal energy of an ideal gas depends only on the temperature of the gas, while that of a real gas depends on the temperature and volume, which in turn is dependent on pressure.

Problem 5. Out of a solid, liquid and gas of the same mass and at the same temperature, which one has the greatest internal energy ? Which one least ? Justify.

Solution. The gas has greatest internal energy because the potential energy (which is negative) of the molecules is very small. On the other hand, the (negative) potential energy of the molecules of a solid is very large ; hence the internal energy of solid is least.

Problem 6. When is the heat supplied to a system equal to the increase in its internal energy ?

Solution. According to the first law of thermodynamics,

$$\Delta Q = \Delta U + P\,\Delta V.$$

If the heat is supplied in such a manner that the volume does not change (for isochoric change, $\Delta V = 0$). Then whole of the heat energy supplied to the system will increase internal energy only.

Problem 7. A gas does work during isothermal expansion. What is the source of mechanical energy so produced ?

Solution. By first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$. But for an isothermal process, $\Delta U = 0$, so $\Delta W = \Delta Q$. Thus the energy required for doing mechanical work during an isothermal process is obtained as heat by the gas from the surroundings.

Problem 8. A gas does work during adiabatic expansion. What is the source of mechanical energy so produced ?

Solution. By first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$. But for an adiabatic process, $\Delta Q = 0$, so $\Delta W = -\Delta U$. Thus the source of energy required for doing mechanical work during adiabatic expansion is the internal energy of the gas itself.

Problem 9. The temperature of a gas rises during an adiabatic compression, although no heat is given to the gas from outside. Why ?

Solution. By first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$. For an adiabatic compression, $\Delta Q = 0$, so $\Delta U = -\Delta W$. That is work is done on the gas which increases its internal energy. Hence temperature of the gas rises.

Problem 10. An ideal gas is compressed at constant temperature. Will its internal energy increase or decrease ?

Solution. It will remain same because the internal energy of a gas depends only on its temperature.

Problem 11. Cooling is produced when a gas at high pressure suddenly expands. Why ?

Solution. During its expansion, the gas does work against high pressure. This decreases the internal energy and hence the temperature of the gas.

Scanned by CamScanner

Problem 12. When a gas is suddenly compressed, its temperature rises. Why ?

Solution. Sudden compression of a gas is an adiabatic process. The work done in compressing the gas increases the internal energy of the gas. Hence the temperature of the gas rises.

Problem 13. If an inflated tyre bursts, the air escaping out is cooled, why ?

Solution. When the tyre bursts, there is an adiabatic expansion of air because the pressure of the air inside is sufficiently greater than the atmospheric pressure. During the expansion, the air does some work against the surroundings, therefore, its internal energy decreases, and as such temperature falls.

Problem 14. Is it possible that there is change in temperature of a body without giving heat to it or taking heat from it ?

Solution. Yes, for example, during an adiabatic compression temperature increases and in an adiabatic expansion temperature decreases, although no heat is given to or taken from the system in these changes.

Problem 15. Is it possible that there is no increase in the temperature of a body despite being heated ?

Solution. Yes, for example, during a change of state (from solid to liquid or from liquid to gas), the system takes heat, but there is no rise in temperature. Internal energy of the system increases in each case.

Problem 16. Can the temperature of a gas be increased by keeping its pressure and volume constant ?

Solution. No, it cannot be increased.

Problem 17. Why does air pressure in a car tyre increase during driving ? *[Delhi 09]*

Solution. During driving, as a result of the friction between the tyre and the road, the temperature of the tyre and hence that of air inside it, increases. Since the volume of air in the tyre remains constant, pressure of the air increases due to increase of temperature (Charle's law).

Problem 18. Whose molecules : liquid water at 0°C or ice at 0°C have greater potential energy ? Give reason.

Solution. Water molecules possess more potential energy. The heat given to melt the ice at 0°C is used up in increasing the potential energy of water molecules formed at 0°C.

Problem 19. Why mechanical energy can be completely converted into heat energy but the whole of the heat energy cannot be converted into mechanical energy ?

Solution. The whole of the mechanical energy can be absorbed by the molecules of the system in the form of their kinetic energy. This kinetic energy gets converted into heat. But the whole of the heat energy cannot be converted into work as a part of it is always retained by the system as its internal energy.

Problem 20. During adiabatic changes, the volume of a gas is found to depend inversely on the square of its absolute temperature. Find how its pressure will depend on the absolute temperature.

Solution. Given $V \propto \frac{1}{T^2}$

or $$V = \frac{\text{constant}}{T^2}$$

But $$\frac{PV}{T} = \text{constant}$$

$$\therefore \quad \frac{P}{T} \cdot \frac{\text{constant}}{T^2} = \text{constant} \quad \text{or} \quad P \propto T^3.$$

Problem 21. Does the mass of a body change when it is heated or cooled ?

Solution. When a body is heated or cooled, it absorbs or releases energy and so its mass increases or decreases in accordance with the Einstein's relation : $E = mc^2$. As the speed of light c is very large, so the change in mass is extremely small.

Problem 22. When air of the atmosphere rises up, it cools. Why ?

Solution. When the air rises up, it expands due to the decrease in atmospheric pressure. It does work at the expense of its internal energy. So its temperature falls.

Problem 23. Is the equation $PV = RT$ valid for isothermal and adiabatic processes ?

Solution. Yes, the equation $PV = RT$ is valid for all types of the thermodynamically processes.

Problem 24. If $PV = RT$ is valid for all types of thermodynamically processes, what do the relations PV = a constant and PV^γ = a constant, signify ?

Solution. The relations PV = a constant and PV^γ = a constant, are the relations between pressure and volume in the isothermal and adiabatic processes respectively. They are derived from the equation $PV = RT$.

Problem 25. First law of thermodynamics does not forbid flow of heat from lower temperature to higher temperature. Comment.

Solution. First law of thermodynamics simply tells about the conversion of mechanical energy into heat energy and vice-versa. It does not put any condition as to why heat cannot flow from lower temperature to higher temperature.

Problem 26. Can two isothermal curves intersect ?

Solution. No. If two isotherms intersect, then this would mean that the pressure and volume of a gas are the same at two different temperatures. This is not possible.

Problem 27. What is the significance of the area of closed curve on a P-V diagram ?

Solution. The area gives the work done in a cyclic process.

Scanned by CamScanner

Problem 28. Can the work done during a cyclic process be zero ?

Solution. Yes. The work done in a cyclic process is zero, if the process reverses exactly under similar conditions such that it retraces the same P-V diagram. In such a case, the area of the curve on a P-V diagram is zero and hence the work done is zero.

Problem 29. How many specific heats does a gas possess ?

Solution. A gas can possess infinite number of specific heats, depending upon the conditions of temperature and pressure. However, generally we consider only two specific heats, one at constant pressure and the other at constant volume.

Problem 30. Why a gas has two principal specific heat capacities ? *[Delhi 97, 05]*

Or

Gases have two specific heats, but the solids and liquids possess only one specific heat. Why ?

Solution. When the gases are heated, there occurs an appreciable change in their volume. So energy is required for expansion and we have two specific heats one at constant pressure and the other at constant volume. In case of solids and liquids, expansion is negligible, so they have only one specific heat *i.e.*, at constant volume.

Problem 31. Compare the formula $C_P - C_V = R$ for an ideal gas with the thermodynamic relation

$$\Delta U = \Delta Q - P\,\Delta V.$$

Solution. The equation $\Delta U = \Delta Q - P\,\Delta V$ can be written as

$$\Delta Q = \Delta U + P\,\Delta V$$

This equation implies that when the heat ΔQ is given to an ideal gas, a part of it is used in increasing the internal energy (ΔU) and the remaining in doing work of expansion ($P\,\Delta V$).

Similarly, the formula $C_P - C_V = R$ can be written as

$$C_P\,\Delta T = C_V\,\Delta T + R\,\Delta T$$

This equation implies that when the heat $C_P\,\Delta T$ is given to an ideal gas at constant pressure, its one part is used in increasing the temperature or internal energy ($C_V\,\Delta T$) and the other part in doing work against external pressure ($R\,\Delta T$).

Problem 32. A gas has two principal specific heats. Which one is greater and why ? *[Delhi 97]*

Solution. Refer answer to Q. 16 on page 12.10.

Problem 33. Of what physical significance is the difference between the two principal specific heat capacities and their ratio ? *[Delhi 97]*

Solution. (*i*) The difference between the two principal specific heats is equal to the amount of heat equivalent to the work performed by the gas during expansion at constant pressure.

(*ii*) Knowing the specific heat ratio of a gas, we can determine the number of degrees of freedom and the atomicity of any gas.

$$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

Problem 34. Can specific heat of a gas be negative ?

Solution. Yes ; negative specific heat will imply that with rise in temperature (ΔT is +ve) heat will be released (Q is –ve). This actually happens in case of saturated vapour *i.e.*, specific heat of saturated vapour is negative.

Problem 35. What is the specific heat of a gas in an isothermal process ? *[Chandigarh 04 ; Central Schools 10]*

Solution. As $c = \dfrac{\Delta Q}{m\,\Delta T}$

For an isothermal process, $\Delta T = 0$, so $c = \infty$.

Problem 36. What is the specific heat of a gas in an adiabatic process ? *[Chandigarh 04 ; Central Schools 10]*

Solution. As $c = \dfrac{\Delta Q}{m\Delta T}$.

For an adiabatic process, $\Delta Q = 0$, so $c = 0$.

Problem 37. A liquid is being converted into steam at its boiling point. What will be the specific heat of the liquid at this time ?

Solution. During vaporisation, the temperature of the liquid remains constant ($\Delta T = 0$). Hence the specific heat,

$$c = \frac{\Delta Q}{m\,\Delta T} = \infty.$$

Problem 38. By what methods can the internal energy of an ideal gas be changed ? Give examples.

Solution. The internal energy of an ideal gas can be changed by heating or cooling the gas in a closed vessel, by adiabatic compression or expansion of the gas.

Problem 39. Heat equivalent to 50 joule is supplied to a thermodynamic system and 10 joule work is done on the system. What is the change in the internal energy of the system in the process ?

Solution. Here $\Delta Q = +50$ J, $\Delta W = -10$ J

$$\therefore \quad \Delta U = \Delta Q - \Delta W = 50 - (-10) = +60 \text{ J}.$$

Thus internal energy of the system increases by 60 J.

Problem 40. 400 J of work is done on a gas to reduce its volume by compression adiabatically. What is the change in internal energy of the gas ?

Solution. For an adiabatic process,

$$\Delta Q = 0$$

As work is done on the gas, so

$$\Delta W = -400 \text{ J}$$

Change in internal energy,

$$\Delta U = \Delta Q - \Delta W = 0 - (-400)$$

$$= +400 \text{ J}.$$

Scanned by CamScanner

Problem 41. An ideal gas is taken around the cycle ABCDA as shown in the P-V diagram. What is the work done during the cycle ? [IIT 93]

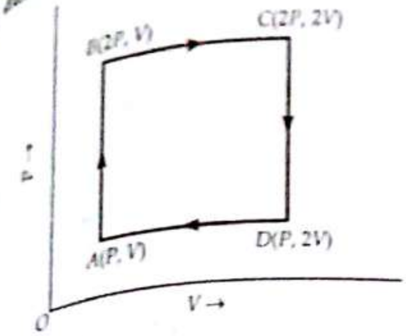


Fig. 12.24

Solution. Work done in the cyclic process

$= \text{Area of the loop } ABCD$

$= (2P - P) \times (2V - V) = PV.$

Problem 42. What is difference between heat and work ?

Solution. Heat and work are two different modes of energy transfer to a system. Heat is the energy transfer that occurs due to temperature difference between the system and the surroundings. Work in the energy transfer that is brought about by other means, such as moving the piston of a cylinder containing the gas, by raising or lowering some weight connected to it.

Problem 43. Why is the conversion of heat into work not possible without sink at lower temperature ?

Solution. For converting heat energy into work continuously, a part of the heat energy absorbed from the source has to be rejected. The heat energy can be rejected only if there is a body, whose temperature is less than that of the source. This body at low temperature is called sink.

Problem 44. Explain why it is impossible to design a heat engine with 100% efficiency.

Or

Even Carnot engine cannot have 100% efficiency. Explain, why ? [AIEEE 02]

Solution. Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

The efficiency will be 100% or 1, if $T_2 = 0$ K. Since, temperature equal to 0 K cannot be realised, a heat engine with 100% efficiency cannot be designed.

Problem 45. Why can a ship not use the internal energy of sea water to operate its engine ?

Solution. For the operation of a heat engine we require a sink at temperature lower than the source and of sufficiently high thermal capacity, which is not possible in the sea.

Problem 46. In a Carnot engine, the temperature of the sink is increased. What will happen to its efficiency ?

Solution. Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

If the temperature (T_2) of the sink is increased, the efficiency of the Carnot engine will decrease.

Problem 47. What is meant by reversible engine ? Explain why the efficiency of a reversible engine is maximum ?

Solution. The engine in which the process can be retraced at any stage of its operation by reversing the boundary conditions is called reversible engine. Its efficiency is maximum because in such a device no dissipation of energy takes place against friction, etc.

Problem 48. If you are asked to increase the efficiency of a Carnot engine by increasing the temperature of the source or by decreasing the temperature of the sink by 10 K, which method would you prefer and why ?

Solution. Let T_1 and T_2 be initial temperatures of source and sink respectively. When temperature of the source is increased by 10 K, the efficiency becomes

$$\eta_1 = 1 - \frac{T_2}{T_1 + 10}$$

When the temperature of the sink is decreased by 10 K, the efficiency becomes

$$\eta_2 = 1 - \frac{T_2 - 10}{T_1}$$

It can be easily seen that

$$\eta_2 - \eta_1 > 0 \text{ or } \eta_2 > \eta_1 .$$

Hence the efficiency of a Carnot engine can be increased by a greater amount by decreasing the temperature of the sink by 10 K than by increasing the temperature of the source by 10 K.

Problem 49. The temperature of the surface of the sun is approximately 6000 K. If we take a big lens and focus the sun rays, can we produce a temperature of 8000 K ?

Solution. No. According to second law of thermodynamics, heat by itself cannot flow from a body at lower temperature to a body at higher temperature. This can only be accomplished with the help of an external agency.

Problem 50. Is the efficiency of a heat engine more in hilly areas than in the plains ?

Solution. In hilly areas, the temperature of the surroundings is lower than that in plains, so the ratio T_2 / T_1 is less in hilly areas than that in plains.

But $\eta = 1 - \frac{T_2}{T_1}$

Hence efficiency η is more in hilly areas than in plains.

Scanned by CamScanner

Problem 51. Is it theoretically possible to devise a heat engine which will create no thermal pollution ?

Solution. No. According to the second law of thermodynamics, whole of the heat cannot be converted into work. So a part of the heat that is not converted into work is exhausted by the engine to the atmosphere (sink). Hence thermal pollution will always occur.

Problem 52. Can a kitchen be cooled by leaving the door of an electric refrigerator open ?

Or

If a door of a working refrigerator is kept open for a long time in a closed room, will it make the room warm or cool ? *[Himachal 06, 07C]*

Solution. No. A kitchen cannot be cooled by leaving the door of a refrigerator open, rather it will get slightly heated. When the door of the refrigerator is kept open, refrigerator now extracts heat from the kitchen room (acting as cooling chamber). Work is done on it by the electric motor and the total energy is rejected to the room (now acting as surroundings). Thus the work done by the motor gets added to the room, so it gets heated.

Problem 53. How a refrigerator can be used as a heat pump to heat a house in winter ?

Solution. When the outside environment is colder than the inside of a room, we leave a refrigerator open with its radiator (backside) facing the room. The refrigerator pumps in heat from the environment to the room. This heats up the room.

Problem 54. Milk is poured into a cup of tea and is mixed with a spoon. Is this an example of a reversible process ? Give reason.

Solution. No. When milk is poured into a cup of tea and mixed, some work is done which gets converted into heat. Heat produced cannot be converted back into work which will separate milk from tea. Hence the mixing of milk with tea is an irreversible process.

Problem 55. A system goes from state A to B via two processes I and II, as shown in Fig. 12.25. How are ΔU_1 and ΔU_2 (the changes in internal energies in the processes I and II) related to each other ? *[AIEEE 05]*

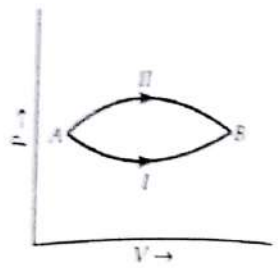


Fig. 12.25

Solution. Since the initial and final states are same for both processes I and II and the change in internal energy is path independent, therefore,

$$\Delta U_1 = \Delta U_2$$

Short Answer Conceptual Problems

Problem 1. A thermos flask contains coffee. It is vigorously shaken. Consider the coffee as the system. (i) Has any heat been added to it ? (ii) Has any work been done on it ? (iii) Has its internal energy changed ? (iv) Does its temperature rise ?

Solution. (i) No. As the thermos flask is insulated, heat has not been added to the coffee ($\Delta Q = 0$).

(ii) Yes. Some work is done by the man in shaking the coffee against the forces of viscosity i.e., ΔW is negative.

(iii) By first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$. As $\Delta Q = 0$ and ΔW is negative, so ΔU is positive i.e., internal energy of the coffee increases.

(iv) Because of the increase in internal energy of the coffee, the temperature of the coffee will also increase.

Problem 2. When a bore is made with a small drill in a hard board, the drill becomes very hot. But when the bore is made in a soft board, the drill does not become so hot. Why ?

Solution. When a bore is made in a hard board, the friction between the drill and the board is quite large. A large amount of work has to be done in making the bore which produces a large amount of heat and the drill becomes too hot to touch. In case of the soft board, the friction is less, small work has to be done in making the bore. Hence heat produced is also small.

Problem 3. What is an isothermal process ? What are the essential conditions for an isothermal process to take place ? *[Delhi 10]*

Solution. A process in which temperature remains constant is called isothermal process. The essential conditions for an isothermal process to take place are :

(i) The walls of the container must be perfectly conducting to allow free exchange of heat between the gas and the surroundings.

(ii) The process of compression or expansion should be slow, so as to provide sufficient time for the exchange of heat.

Problem 4. What is an adiabatic process ? What are the essential conditions for an adiabatic process to take place ?

Solution. A process in which a thermally insulated system neither loses nor gains heat from the surroundings is called adiabatic process.

Scanned by CamScanner

The essential conditions for an adiabatic process to take place are :

(i) The walls of the container must be perfectly nonconducting to prevent any exchange of heat between the gas and the surroundings.

(ii) The process of compression or expansion should be rapid, so that there is time for the exchange of heat.

Problem 5. Explain why the internal energy of a compressed gas is less than that of a rarefied gas at the same temperature.

Solution. When a real gas is compressed, its molecules come closer. Their mutual interactions become stronger. This increases the magnitude of potential energy. But potential energy is negative. Hence the total energy of the compressed gas decreases. However, the kinetic energy is same for both the compressed and rarefied gases as both are at the same temperatures.

Problem 6. When you whistle out air on to your palm held close to your mouth, the air feels cold ; but when you blow out air from your mouth, keeping it wide open, the air feels hot. Why ?

Solution. During whistling, we blow out air through a small opening between the lips. This is an adiabatic expansion, so there is a fall in temperature. But when we keep our mouth wide open, hot air of the mouth blows on to the palm which feels hot.

Problem 7. Two bodies of specific heats c_1 and c_2 having same heat capacities are combined to form a single composite body. What is the specific heat of the composite body ?

Solution. As the heat capacities are equal, so $m_1c_1 = m_2c_2$.

Let c be the specific heat of the composite body. Then

$$(m_1 + m_2)\,c = m_1c_1 + m_2c_2 = m_1c_1 + m_1c_1 = 2m_1c_1$$

or $$c = \frac{2m_1c_1}{m_1 + m_2} = \frac{2m_1c_1}{m_1 + m_1\dfrac{c_1}{c_2}} = \frac{2c_1c_2}{c_1 + c_2}.$$

Problem 8. A gas expands in such a manner that its pressure and volume comply with the condition PV^2 = constant. Will the gas cool or get heated on expansion ?

Solution. For a perfect gas, $\dfrac{PV}{T} = \text{constant}\ (K)$

Given $PV^2 = \text{constant}\ (K')$

$$\therefore \quad P = \frac{K'}{V^2}$$

$$\therefore \quad \frac{K'}{V^2}\cdot\frac{V}{T} = K \quad \text{or} \quad VT = \frac{K'}{K} = \text{constant}$$

Hence $V \propto \dfrac{1}{T}$

That is the expansion of the gas will result in the decrease of temperature.

Problem 9. What happens to the change in internal energy of a gas during (i) isothermal expansion and (ii) adiabatic expansion ? [Delhi 08 ; Central Schools 14]

Solution. (i) **Isothermal expansion.** Temperature remains constant during an isothermal change. As internal energy is a function of temperature only, so it will remain constant during an isothermal change.

As $\Delta T = 0$, so $\Delta U = C_V\,\Delta T = 0$.

(ii) **Adiabatic expansion.** For an adiabatic change, $\Delta Q = 0$, so from first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W = 0 \quad \text{or} \quad \Delta W = -\Delta U$$

During expansion, work is done by a gas *i.e.*, ΔW is positive. So ΔU must be negative. Hence internal energy of a gas decreases during an adiabatic expansion.

Problem 10. Show that the slope of an adiabatic curve at any point is γ times the slope of an isothermal curve at the corresponding point. [Chandigarh 08]

Solution. For an *isothermal change*, $PV = K$

Differentiating both sides, we get

$$P\,.\,dV + V\,.\,dP = 0 \quad \text{or} \quad V\,.\,dP = -PdV$$

$$\therefore \text{ Slope of an isothermal curve, } \left(\frac{dP}{dV}\right)_{iso} = -\frac{P}{V}$$

For an *adiabatic change*, $PV^\gamma = K'$

Differentiating both sides, we get

$$P\,.\,\gamma V^{\gamma-1}\,.\,dV + V^\gamma\,.\,dP = 0$$

or $$\gamma\,PdV + VdP = 0 \quad \text{or} \quad VdP = -\gamma\,PdV$$

$$\therefore \text{ Slope of an adiabatic curve, } \left(\frac{dP}{dV}\right)_{adia} = -\frac{\gamma P}{V}$$

Clearly, slope of an adiabatic curve $= \gamma \times$ slope of an isothermal curve.

As $\gamma > 1$, so *an adiabatic P-V curve is steeper than the corresponding isothermal P-V curve.*

Problem 11. The volume versus temperature T graphs for a certain amount of a perfect gas at two pressures P_1 and P_2 are shown in Fig. 12.26. Which one is greater — P_1 or P_2 ?

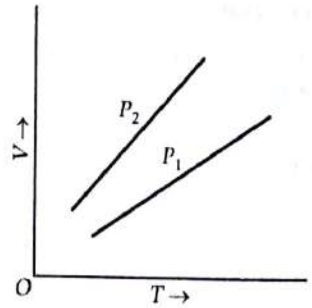


Fig. 12.26

Solution. For a perfect gas,

$$PV = nRT$$

$$\therefore \quad V = \frac{nR}{P} T$$

$\therefore$ Slope of V-T graph with T-axis $= \frac{nR}{P}$

For a given amount of gas, slope $\propto \frac{1}{P}$

Hence $P_1 > P_2$.

Problem 12. In Figs. 12.27(a), (b) and (c) given below, identify the isothermal and adiabatic process in each diagram.

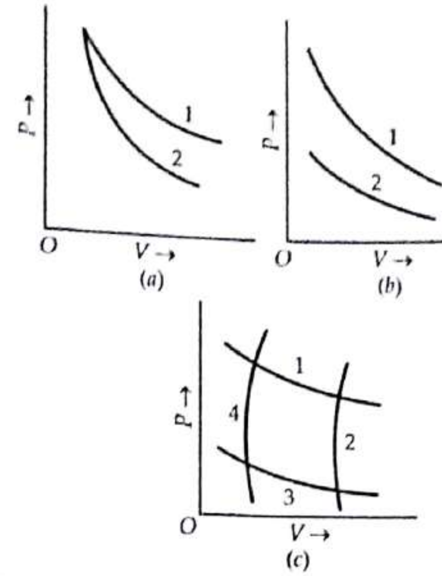


Fig. 12.27

Solution. For the same values of pressure and volume, the slope of an adiabatic curve is greater than the slope of an isothermal curve.

Using this fact, we find

(a) 1 – isothermal 2 – adiabatic

(b) 1 – adiabatic 2 – isothermal

(c) 1, 3 – isothermal 2, 4 – adiabatic.

Problem 13. No real engine can have an efficiency greater than that of a Carnot engine working between the same two temperatures. Give reason.

Solution. A Carnot engine is an ideal engine satisfying the following conditions :

(i) There is no friction between the walls of the cylinder and the piston.

(ii) The working substance is an ideal gas. That is the gas molecules have point sizes and have no attractive forces between them.

Real engines Cannot fulfil these conditions.

Hence no heat engine working between the same two temperatures can have efficiency greater than that of a Carnot engine.

Problem 14. Can a Carnot engine be realised in practice ?

Solution. No. A Carnot engine should consist of

(i) a source of infinite thermal capacity,

(ii) a sink of infinite thermal capacity,

(iii) an ideal gas as its work substance.

Moreover, the working substance should be contained in cylinder provided with frictionless, non-conducting movable piston. It is not possible to achieve all these conditions. Hence a Carnot engine cannot be realised in practice.

Problem 15. The volume of an ideal gas is V at pressure P. On increasing the pressure by ΔP, the change in volume of the gas is ΔV_1 under isothermal conditions and ΔV_2 under adiabatic conditions. Under which of the two conditions, will the change in volume be more ? Given reason.

Solution. Under isothermal conditions, the bulk modulus of elasticity is

$$K_{iso} = \frac{\Delta P}{\Delta V_1 / V} = P \quad ...(i)$$

Under adiabatic conditions, the bulk modulus of elasticity is

$$K_{adia} = \frac{\Delta P}{\Delta V_2 / V} = \gamma P \quad ...(ii)$$

Dividing (ii) by (i), we get

$$\frac{\Delta V_1}{\Delta V_2} = \gamma$$

As $\gamma > 1$, so $\Delta V_1 > \Delta V_2$.

Problem 16. Discuss whether the following phenomena are reversible :

(i) Water fall

(ii) Rusting of iron

(iii) Electrolysis.

Solution. (i) Water fall. It is not a reversible process. During fall of the water, the major part of its potential energy is converted into kinetic energy of the water. However, on striking the ground, a part of it is converted into heat and sound. It is not possible to convert the heat and the sound produced alongwith the K.E. of water into potential energy so that the water may rise back to its initial height. Therefore, water fall is not a reversible process.

(ii) Rusting of iron. During rusting, iron gets oxidised by the oxygen of the air. Since it is a chemical change, it is not a reversible process.

(iii) Electrolysis. It is a reversible process, provided the electrolyte does not offer any resistance to the flow of current. If we reverse the direction of current, the direction of motion of ions is also reversed.

Scanned by CamScanner

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *A certain amount of gas occupies volume V_0 at pressure P_0 and temperature T_0. It is allowed to expand (i) isobarically, (ii) adiabatically and (iii) isothermally. In which case the work done is maximum and in which case it is minimum? Explain.*

Solution. During expansion, work is done by the gas. The horizontal line AB_1 represents the isobaric expansion (P = constant).

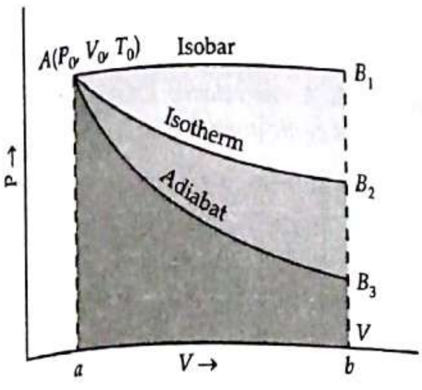


Fig. 12.28

Since an adiabatic is steeper than an isothermal, so the adiabatic expansion curve AB_3 lies below the isothermal expansion curve AB_2.

Work done = Area between the *P-V* curve and the *V*-axis

As the area under curve AB_1 is maximum, so *maximum work is done in isobaric expansion*.

Again, the area under curve AB_3 is minimum, so *minimum work is done* in adiabatic expansion.

Problem 2. *Two samples of a gas initially at the same temperature and pressure are compressed from a volume V to V/2, one isothermally, the other adiabatically. In which sample is the final pressure greater?*

Solution. Fig. 12.29 shows the *P-V* diagrams for the two samples of gas compressed from volume *V* to *V*/2 isothermally and adiabatically. As an adiabatic is steeper than an isotherm, so the adiabatic compression curve *AC* lies above the isothermal compression curve *AB*.

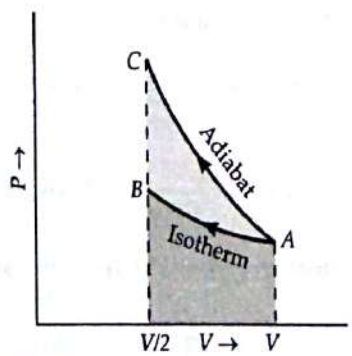


Fig. 12.29

Clearly, the final pressure represented by point *C* is greater than that represented by point *B*. Hence the pressure of the sample compressed adiabatically will be greater.

Problem 3. *Two gases have the same initial pressure P_0, volume V_0 and temperature T_0. They expand to the same volume, one adiabatically and the other isothermally.*

(i) In which case is the final pressure greater?

(ii) In which case is the work done greater?

(iii) In which case is the final temperature greater?

Solution. Fig. 12.30 shows the *P-V* diagrams for two gases expanded from volume V_0 to *V*. As an adiabatic is steeper than an isotherm, so the adiabatic expansion curve *AB* lies below the isothermal expansion curve *AC*.

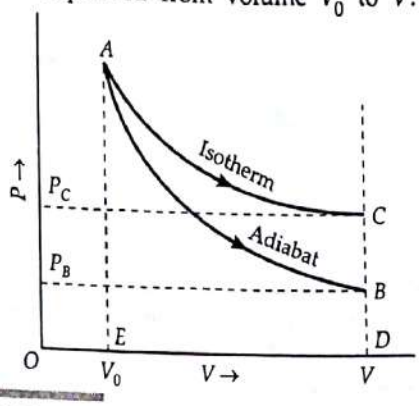


Fig. 12.30

(*i*) P_B and P_C are the final pressures for adiabatic and isothermal expansions respectively. Clearly, $P_C > P_B$. Hence the final pressure is greater for the isothermal expansion.

(*ii*) Work done in adiabatic expansion

= area *ABDE*

Work done in isothermal expansion

= area *ACDE*

As area *ACDE* > area *ABDE*

So more work is done in the isothermal expansion.

(*iii*) In isothermal expansion, temperature remains constant T_0. In adiabatic expansion temperature decreases below T_0. So the final temperature is greater for the isothermal expansion.

Scanned by CamScanner

Problem 4. *The work of 146 J is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7°C. Identify the atomicity of the gas. Given R = 8.3 J mol^{-1} K^{-1}* **[AIEEE 06]**

Solution. By first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

For an adiabatic process, $\Delta Q = 0$

$$\therefore \quad \Delta U = \Delta W = -(-146 \text{ kJ}) = +146 \text{ kJ}$$

As $\quad \Delta U = nC_V \, \Delta T$

$$\therefore \quad C_V = \frac{\Delta U}{n\Delta T} = \frac{146 \times 10^3}{1 \times 10^3 \times 7} \quad [\because \Delta T = 7°C = 7 \text{ K}]$$

$$= 20.8 \text{ J mol}^{-1}\text{K}^{-1}$$

For a diatomic gas,

$$C_V = \frac{5}{2} R = \frac{5}{2} \times 8.3 = 20.8 \text{ J mol}^{-1} \text{ K}^{-1}.$$

Hence the gas is diatomic.

Problem 5. *In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. What happens to the temperature of the gas ?* **[IIT Screening 01]**

Solution. As $dQ = dU + dW$

But $\quad dW = 0$ and $dQ < 0$, so $dU < 0$

For an ideal gas, $U \propto T$, so $dT < 0$ *i.e.*, temperature of the gas decreases.

Problem 6. *An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in Fig. 12.31. If the net heat supplied to the gas in the cycle is 5 J, what is the work done by the gas in the process $C \rightarrow A$?* **[IIT Screening 02]**

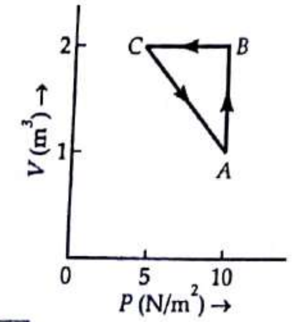


Fig. 12.31

Solution. For the cyclic process,

$$\Delta Q = \Delta W$$

or $\quad \Delta Q = W_{AB} + W_{BC} + W_{CA}$

or $\quad 5 = 10\,(2 - 1) + 0 + W_{CA}$

or $\quad W_{CA} = 5 - 10 = -5$ **J.**

Problem 7. *A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively then, what is the ratio T_1/T_2 ?* **[IIT 2k]**

Solution. For an adiabatic process,

$$TV^{\gamma - 1} = \text{constant}$$

For a monoatomic gas, $\gamma = 5/3$

$$\therefore \quad TV^{2/3} = \text{constant}$$

Let A = the area of cross-section of the cylinder. Then

$$T_1 (L_1 A)^{2/3} = T_2 (L_2 A)^{2/3}$$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}.$$

Problem 8. *A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in*

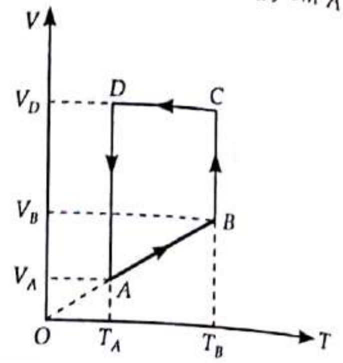


Fig. 12.32

Fig. 12.32. The volume ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature T_A at A is 27°C, calculate,

(a) the temperature of the gas at point B,

(b) heat absorbed or released by the gas in each process,

(c) the total work done by the gas during the complete cycle.

Express your answer in terms of the gas constant R. **[IIT Mains 01]**

Solution. (*a*) The process $A \rightarrow B$ is isobaric.

$$\therefore \quad V/T = \text{constant}$$

Hence $\quad \dfrac{V_A}{T_A} = \dfrac{V_B}{T_B}$

or $\quad T_B = \dfrac{V_B}{V_A} . T_A = 2 \times 300 = 600 \text{ K} = 327°\text{C}$

(*b*) The heat absorbed in the process $A \rightarrow B$ is given by

$$Q_1 = n\, C_p \, dT$$

$$= 2\,(5R/2)(600 - 300) = 1500\, R.$$

The process $B \to C$ is isothermal. The heat absorbed is given by

$$Q_2 = W_2 = nRT_B \log(V_D / V_B)$$
$$= 2 \times R \times 600 \log_e 2 = 1200\ R \times 0.693$$
$$= 831.6\ R$$

The process $C \to D$ is isochoric. The heat released is given by

$$Q_3 = n\,C_v\,dT = 2 \times \frac{3}{2}\ R \times (300 - 600)$$
$$= -900\ R.$$

The process $D \to A$ is isothermal. The heat released is given by

$$Q_4 = nRT_A \log_e \frac{V_A}{V_D} = 2\ R \times 300 \log_e \frac{1}{4}$$
$$= -1200\ R \log_e 2 = -\mathbf{831.6}\ \boldsymbol{R}.$$

(c) The total work done by the gas during the complete cycle,

$$W = Q_1 + Q_2 + Q_3 + Q_4$$
$$= 1500\ R + 831.6\ R - 900\ R - 831.6\ R$$
$$= \mathbf{600\ R}.$$

Problem 9. *An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J and $Q_4 = 3645$ J respectively. The corresponding works involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4 respectively. Find (i) W_4 and (ii) efficiency of the cycle.* **[IIT 94]**

Solution. (*i*) By the first law of thermodynamics,

$$dQ = dU + dW$$

But for a cyclic process, $dU = 0$

$$dQ = dW$$

$\therefore$

$$Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$$

or $5960 - 5585 - 2980 + 3645 = 2200 - 825 - 1100 + W_4$

or $W_4 = (5960 + 3645 + 825 + 1100) - (5585 + 2980 + 2200)$

$$= 11530 - 10765 = \mathbf{765\ J}.$$

(*ii*) Efficiency,

$$\eta = \frac{\text{Total work done}}{\text{Heat absorbed}} = \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4}$$
$$= \frac{2200 - 825 - 1100 + 765}{5960 + 3645}$$
$$= \frac{1040}{9605} = 0.1083 = \mathbf{10.83\%}.$$

Problem 10. *Let the temperatures T_1 and T_2 of the two heat reservoirs in an ideal carnot engine be 1500°C and 500°C respectively. Which of these, increasing T_1 by 100°C or decreasing T_2 by 100°C, would result in a greater improvement in the efficiency of the engine?* **[Roorkee 80]**

Solution. Efficiency of Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$$

(*i*) When T_1 is increased from 1500°C to 1600°C or $1600 + 273 = 1873$ K and T_2 remains constant *i.e.*, 500°C or $500 + 273 = 773$ K, we have

$$\eta_1 = \frac{1873 - 773}{1873} = \frac{1100}{1873} = \frac{1100 \times 100}{1873}\% = \mathbf{58.73\%}.$$

(*ii*) When T_1 remains constant *i.e.*, 1500°C or $1500 + 273 = 1773$ K and T_2 is decreased by 100°C from 500°C to 400° or $400 + 273 = 673$ K, we have

$$\eta_2 = \frac{1773 - 673}{1773} = \frac{1100}{1773} = \frac{1100 \times 100}{1773}\% = \mathbf{62.04\%}.$$

Thus, $\eta_2 > \eta_1$.

Hence efficiency will increase if T_2 is decreased from 500°C to 400°C.

Problem 11. *An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to T/2. (i) How many degrees of freedom do the gas molecules have? (ii) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V.* **[IIT 90]**

Solution. (*i*) *For an adiabatic expression* :

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

$\therefore$ $$TV^{\gamma - 1} = \frac{T}{2} \times (5.66\ V)^{\gamma - 1}$$

or $$(5.66)^{\gamma - 1} = 2$$

Taking logarithms, $(\gamma - 1) \log 5.66 = \log 2$

or $$\gamma - 1 = \frac{\log 2}{\log 5.66} = \frac{0.3010}{0.7528} = 0.4$$

or $$\gamma = 1.4$$

Let f be the degrees of freedom of the gas. Then

$$\gamma = 1 + \frac{2}{f} \quad \text{or} \quad 1.4 = 1 + \frac{2}{f}$$

or $$\frac{2}{f} = 1.4 - 1 = 0.4 \quad \text{or} \quad f = \frac{2}{0.4} = \mathbf{5}.$$

(*ii*) Work done by the gas in adiabatic expansion from temperature $T_1 (= T)$ to $T_2 (= T/2)$ is

$$W = \frac{R}{\gamma - 1}(T_1 - T_2) = \frac{R}{0.4}\left(T - \frac{T}{2}\right) = \frac{5}{4}\ RT = \mathbf{\frac{5}{4}\ PV}.$$

Scanned by CamScanner

Problem 12. *Two different adiabatic paths for the same gas intersect two isothermals at T_1 and T_2 as shown in P-V diagram of Fig. 12.33. How does the ratio V_a/V_d compare with the ratio V_b/V_c?* **[Roorkee 83]**

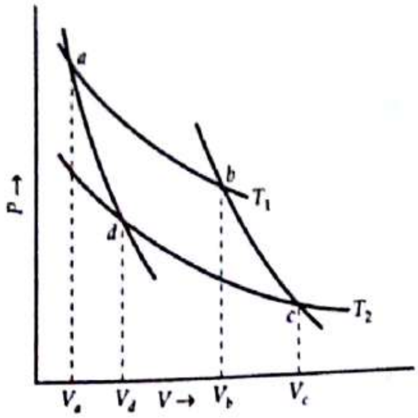


Fig. 12.33

Solution. Let P_a, P_b, P_c and P_d be the pressures at *a*, *b*, *c* and *d* respectively. As *a* and *b* are on the same isothermal, so

$$\therefore \qquad P_aV_a = P_bV_b \qquad ...(i)$$

Also, *c* and *d* are on the same isothermal, so

$$P_cV_c = P_dV_d \qquad ...(ii)$$

Now, *a* and *d* are on the same adiabatic and similarly *b* and *c* are on the same adiabatic, so

$$P_dV_d^{\gamma} = P_aV_a^{\gamma} \qquad ...(iii)$$

$$\text{and} \qquad P_bV_b^{\gamma} = P_cV_c^{\gamma} \qquad ...(iv)$$

Multiplying equations (*i*), (*ii*), (*iii*) and (*iv*), we get

$$P_aV_a \cdot P_cV_c \cdot P_dV_d^{\gamma} \cdot P_bV_b^{\gamma} = P_bV_b \cdot P_dV_d \cdot P_aV_a^{\gamma} \cdot P_cV_c^{\gamma}$$

$$\text{or} \qquad V_aV_cV_d^{\gamma}V_b^{\gamma} = V_bV_dV_a^{\gamma}V_c^{\gamma}$$

$$\text{or} \qquad \frac{V_aV_d^{\gamma}}{V_a^{\gamma}V_d} = \frac{V_bV_c^{\gamma}}{V_cV_b^{\gamma}}$$

$$\text{or} \qquad \left(\frac{V_d}{V_a}\right)^{\gamma-1} = \left(\frac{V_c}{V_b}\right)^{\gamma-1}$$

$$\text{or} \qquad \frac{V_d}{V_a} = \frac{V_c}{V_b} \quad \text{or} \quad \frac{V_a}{V_d} = \frac{V_b}{V_c}$$

Problem 13. *In Fig. 12.34, an ideal gas changes its state from state A to C by two paths ABC and AC.*

(*i*) *Find the path along which work done is the least.*

(*ii*) *The internal energy of gas at A is 10 J and amount of heat supplied to change its state to C through the path AC is 200 J. Calculate the internal energy at C.*

(*iii*) *The internal energy of gas at state B is 20 J. Find the amount of heat supplied to the gas to go from A to B.* **[Roorkee 98]**

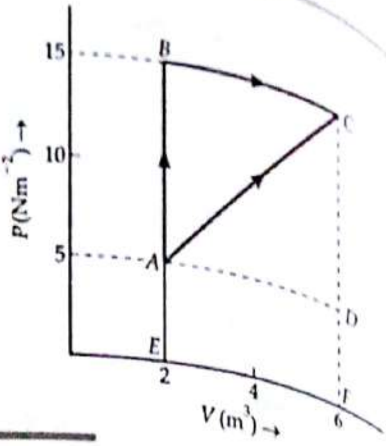


Fig. 12.34

Solution. (*i*) Along path *ABC*,

$$W_{AB} = PdV = 0$$

$$W_{BC} = P_B(V_C - V_B) = 15(6-2) = 60 \text{ J} \qquad [\because dV = 0]$$

$$\therefore \qquad W_{ABC} = W_{AB} + W_{BC} = 0 + 60 \text{ J} = 60 \text{ J}$$

Along path *AC*,

$$W_{AC} = \text{Area under the curve } AC$$

$$= \text{Area of } \Delta ACD + \text{Area of rectangle } ADFE$$

$$= \frac{1}{2} AD \times DC + AD \times FD$$

$$= \frac{1}{2} \times 4 \times 10 + 4 \times 5 = 20 + 20 = 40 \text{ J}$$

Clearly, work done is the least along path *AC*.

(*ii*) Here $U_A = 10$ J, $\Delta Q_{AC} = 200$ J

By the first law of thermodynamics,

$$\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC} = U_C - U_A + \Delta W_{AC}$$

$$U_C = \Delta Q_{AC} + U_A - \Delta W_{AC}$$

$$= 200 \text{ J} + 10 \text{ J} - 40 \text{ J} = \mathbf{170 \text{ J}}.$$

(*iii*) Here $U_B = 20$ J. By the first law of thermodynamics,

$$\Delta Q_{AB} = \Delta U_{AB} + \Delta W_{AB} = U_B - U_A + \Delta W_{AB}$$

$$= 20 \text{ J} - 10 \text{ J} + 0 = \mathbf{10 \text{ J}}.$$

Problem 14. *A sample of 2 kg of monoatomic helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC (Fig. 12.35). Given molecular mass of Helium = 4, R = 8.3 J mol^{-1} K^{-1}.*

(*i*) *What is the temperature of Helium in each of the states A, B, C and D?*

(*ii*) *How much is the heat involved in each of the processes ABC and ADC?* **[IIT 97]**

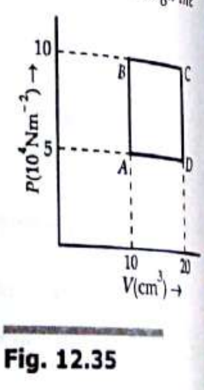


Fig. 12.35

Scanned by CamScanner

Solution. (*i*) Number of moles of He

$$n = \frac{\text{Mass of He}}{\text{Molecular mass}} = \frac{2000}{4} = 500$$

As $P_A V_A = n R T_A$

$$T_A = \frac{P_A V_A}{nR} = \frac{5 \times 10^4 \times 10}{500 \times 8.3} = \mathbf{120.5\ K.}$$

For the isochoric process (V = constant) AB,

$$\frac{T_B}{T_A} = \frac{P_B}{P_A} \quad \therefore \quad T_B = \frac{P_B}{P_A} \times T_A = \frac{10}{5} \times 120.5 = \mathbf{241\ K.}$$

For the isobaric process (P = constant) BC,

$$\frac{T_C}{T_B} = \frac{V_C}{V_B} \quad \therefore \quad T_C = \frac{V_C}{V_B} \times T_B = \frac{20}{10} \times 241 = \mathbf{482\ K.}$$

For the isochoric process DC,

$$\frac{T_D}{T_C} = \frac{P_D}{P_C} \quad \therefore \quad T_D = \frac{P_D}{P_C} \times T_C = \frac{5}{10} \times 482 = \mathbf{241\ K.}$$

(*ii*) Heat involved in the process ABC,

$$Q_{ABC} = \Delta U_{AC} + W_{ABC} = nC_V\,(T_C - T_A) + W_{BC}$$

$$= nC_V\,(T_C - T_A) + P_B\,(V_C - V_B)$$

$$= 500 \times \frac{3}{2} \times 8.3 \times (482 - 120.5) + 10 \times 10^4 \times (20 - 10) \quad [\because C_V = 3/2\ R]$$

$$= 2.25 \times 10^6 + 10^6 = 3.25 \times 10^6 \text{ J} = \mathbf{3.25\ MJ.}$$

Heat involved in the process ADC,

$$Q_{ADC} = \Delta U_{AC} + W_{ADC} = \Delta U_{AC} + W_{AD}$$

$$= 2.25 \times 10^6 + P_A\,(V_D - V_A)$$

$$= 2.25 \times 10^6 + 5 \times 10^4 \times (20 - 10)$$

$$= 2.75 \times 10^6 \text{ J} = \mathbf{2.75\ MJ.}$$

Problem 15. *One mole of an ideal monoatomic gas is taken round the cyclic process ABCA as shown in Fig. 12.36. Calculate*

(*i*) *the work done by the gas,*

(*ii*) *the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB,*

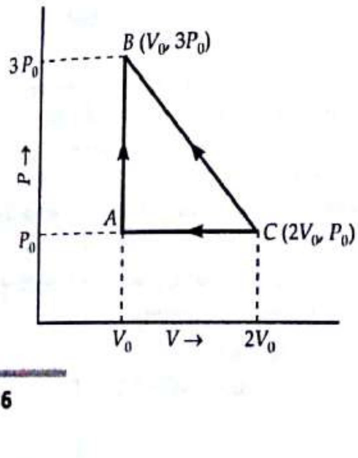


Fig. 12.36

(*iii*) *the net heat absorbed by the gas in the path BC, and*

(*iv*) *the maximum temperature attained by the gas during the cycle.* **[IIT 98]**

Solution. (*i*) Work done = Area of closed curve $ABCA$ ($\Delta\ ABC$)

$$dW = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}\, V_0 \times 2\,P_0 = P_0 V_0.$$

(*ii*) Heat rejected by the gas in the path CA during Isobaric compression process,

$$dQ_{CA} = nC_p \Delta T = 1 \times (5R/2) \times (T_A - T_C)$$

$$T_C = \frac{2\,P_0 V_0}{1 \times R},\quad T_A = \frac{P_0 V_0}{1 \times R}$$

$$dQ_{CA} = \frac{5R}{2}\left[\frac{P_0 V_0}{R} - \frac{2\,P_0 V_0}{R}\right] = -\frac{5}{2}\,P_0 V_0$$

Heat absorbed by the gas in the path AB of Isobaric process,

$$dQ_{AB} = nC_v\, dT = 1 \times (3R/2)(T_B - T_A)$$

$$= \frac{3R}{2}\left[\frac{3P_0 V_0}{1 \times R} - \frac{P_0 V_0}{1 \times 2}\right] = 3\,P_0 V_0$$

(*iii*) As $dU = 0$ in cyclic process, hence $dQ = dW$

$$dQ_{AB} + dQ_{CA} + dQ_{BC} = dW$$

or $\quad 3P_0 V_0 - \frac{5}{2}P_0 V_0 + dQ_{BC} = P_0 V_0 \quad$ or $\quad dQ_{BC} = \frac{P_0 V_0}{2}$

As net heat is absorbed by the gas during path BC, temperature will reach maximum between B and C.

(*iv*) Equation for line BC: $\quad P = -\left[\frac{2\,P_0}{V_0}\right] V + 5\,P_0$

As $\quad PV = RT \quad$ or $\quad P = \frac{RT}{V} \quad$ [For one mole]

$$\therefore \quad RT = -\frac{2\,P_0}{V_0}\, V^2 + 5\,P_0 V \qquad \text{...(1)}$$

$$\frac{dT}{dV} = 0 \text{ for maximum}$$

$$\therefore \quad -\frac{2\,P_0}{V_0} \times 2\,V + 5\,P_0 = 0 \quad \text{or} \quad V = \frac{5}{4}\,V_0 \qquad \text{...(2)}$$

From equations (1) and (2), we get

$$RT_{\max} = -\frac{2\,P_0}{V_0} \times \left(\frac{5V_0}{4}\right)^2 + 5\,P_0\left(\frac{5V_0}{4}\right)$$

$$= -2\,P_0 V_0 \times \frac{25}{16} + \frac{25\,P_0 V_0}{4} = \frac{25}{8}\,P_0 V_0$$

$$\therefore \quad T_{\max} = \frac{25}{8}\,\frac{P_0 V_0}{R}.$$

Scanned by CamScanner

Guidelines to NCERT Exercises

12.1. *A geyser heats water flowing at the rate of 3.0 litres per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0×10^4 J/g ?*

Ans. Volume of water heated = 3.0 litre min^{-1}

Mass of water heated = 3000 g min^{-1}

Rise in temperature, $\Delta T = 77 - 27 = 50°C$

Specific heat of water, $c = 4.2 \text{ J g}^{-1}$

Heat absorbed by water,

$$Q = mc\,\Delta T = 3000 \times 4.2 \times 50$$
$$= 63 \times 10^4 \text{ J min}^{-1}$$

Heat of combustion = $4.0 \times 10^4 \text{ J g}^{-1}$

Rate of combustion of fuel

$$= \frac{63 \times 10^4 \text{ J min}^{-1}}{4.0 \times 10^4 \text{ J g}^{-1}} = \mathbf{15.75 \text{ g min}^{-1}}.$$

12.2. *What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen at room temperature to raise its temperature by 45°C at constant pressure ? Given molecular weight of N_2 is 28 and $R = 8.3$ J mol^{-1} K^{-1} and C_V (diatomic gases) $= \frac{7}{2}R$.*

Ans. Number of moles of gas,

$$n = \frac{\text{Mass of gas in grams}}{\text{Molecular mass}}$$
$$= \frac{2.0 \times 10^{-2} \times 10^3}{28} = \frac{20}{28}$$
$$R = 8.3 \text{ J mol}^{-1}\text{ K}^{-1}, \Delta T = 45°C$$

Molar sp. heat of N_2 at constant pressure,

$$C_P = \frac{7}{2}R = \frac{7}{2} \times 8.3 \text{ J mol}^{-1}\text{ K}^{-1}$$

Heat supplied to the gas,

$$Q = nC_P\,\Delta T = \frac{20}{28} \times \frac{7}{2} \times 8.3 \times 45 = \mathbf{933.75 \text{ J}}.$$

12.3. *Explain why*

(a) Two bodies at different temperatures T_1 and T_2 if brought in thermal contact do not necessarily settle to mean temperature $(T_1 + T_2)/2$.

(b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat. **[Delhi 11]**

(c) Air pressure in a car tyre increases during driving. **[Delhi 09, 11]**

(d) The climate of a harbour town is more temperate than that of a town in a desert at the same latitude.

Solution. (*a*) The two bodies may have different masses and different materials *i.e.*, they may have different thermal capacities. In case the two bodies have equal thermal capacities, they would settle at the mean temperature $(T_1 + T_2)/2$.

(*b*) The purpose of a coolant is to absorb maximum heat with least rise in its own temperature. This is possible only if specific heat is high because $Q = mc\,\Delta T$. For a given value of m and Q, the rise in temperature ΔT will be small if c is large. This will prevent different parts of the nuclear reactor from getting too hot.

(*c*) Due to the friction between the tyres and the road, the tyres get heated. The temperature of air inside the tyres increases. Consequently, the air pressure in the tyres increases slightly.

(*d*) The relative humidity of a harbour town is more than that of a desert town. Due to high specific heat of water, the variations in the temperature of humid air are less. Hence the climate of a harbour town is without the extremes of hot or cold.

12.4. *A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume ?*

Solution. The piston and the walls of the cylinder are insulated. No heat exchange occurs. For this adiabatic process,

$$P_1V_1^\gamma = P_2V_2^\gamma \quad \text{or} \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

But $\quad V_2 = V_1/2$

For a diatomic gas, $\gamma = 1.4$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_1/2}\right)^{1.4} = 2^{1.4} = \mathbf{2.64}$$

12.5. *In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case ? (Take 1 cal = 4.19 J)*

Solution. Work done on a system is negative and heat absorbed by a system is positive.

For the adiabatic process,

$$Q = 0 \quad \text{and} \quad W = -22.3 \text{ J}$$

Scanned by CamScanner

$$Q = \Delta U + W$$

As
$$0 = \Delta U - 22.3\text{ J} \quad \text{or} \quad \Delta U = 22.3\text{ J}$$

In the second case,

$$Q = 9.35\text{ cal} = 9.35 \times 4.19\text{ J} = 39.2\text{ J}$$

$$W = Q - \Delta U = 39.2 - 22.3 = \mathbf{+16.9\ J.}$$

The positive sign for W indicates that the work is done by the system.

12.6. Two cylinders A and B of equal capacity are connected to each other via a stopcock. The cylinder A contains a gas at standard temperature and pressure, while the cylinder B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened.

Answer the following :

(i) *What is the final pressure of the gas in A and B ?*

(ii) *What is the change in internal energy of the gas ?*

(iii) *What is the change in temperature of the gas ?*

(iv) *Do the intermediate states of the system (before settling to final equilibrium state) lie on its P-V-T surface ?* **[Delhi 09]**

Solution. (i) When the stopcock is suddenly opened, the volume available to the gas at 1 atm becomes twice the original volume and hence pressure becomes half the original volume (Boyle's law). Hence the pressure of the gas in each of the cylinders A and B is **0.5 atm**.

(ii) As the system is thermally insulated, so $\Delta Q = 0$. Also, the gas expands against zero pressure, so $\Delta W = 0$. Hence by first law of thermodynamics, $\Delta U = 0$ *i.e.*, there is no change in the internal energy of the gas.

(iii) As there is no change in the internal energy of the gas, so the temperature of the gas remains unchanged.

(iv) No. The free expansion of the gas is very rapid and hence cannot be controlled. The intermediate states are non-equilibrium states and do not satisfy the gas equation. In due course, the gas returns to equilibrium state which lies on P-V-T surface.

12.7. A steam engine delivers 5.4×10^8 J of work per minute and services 3.6×10^9 J of heat per minute from its boiler. What is the efficiency of the engine ? How much heat is wasted per minute ? **[Delhi 09 ; Central Schools 13]**

Solution.

Work output, $W = 5.4 \times 10^8$ J

Heat input, $Q_1 = 3.6 \times 10^9$ J

Efficiency, $$\eta = \frac{W}{Q_1} = \frac{5.4 \times 10^8}{3.6 \times 10^9} = 0.15 = \mathbf{15\%}$$

Heat wasted per minute,

$$Q_2 = Q_1 - W = 3.6 \times 10^9 - 5.4 \times 10^8$$
$$= 30.6 \times 10^8\text{ J} \simeq \mathbf{3.1 \times 10^9\ J.}$$

12.8. An electric heater supplies heat to a system at a rate of 100 W. If system performs work at a rate of 75 joules per second, at what rate is the internal energy increasing ?

Solution. According to first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad \frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$$

or $$100\text{ W} = \frac{\Delta U}{\Delta t} + 75\text{ W}$$

or $$\frac{\Delta U}{\Delta t} = \mathbf{25\ W.}$$

12.9. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in Fig. 12.37.

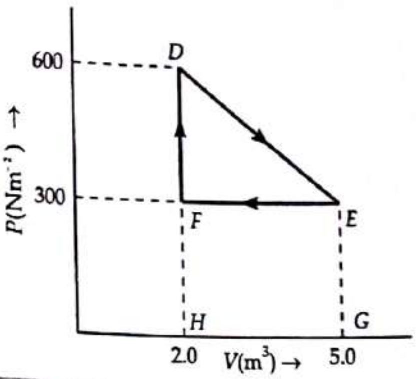


Fig. 12.37

Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F.

Solution. Total work done by the gas from D to E to F,

$$W = W_{DE} + W_{EF}$$
= Area of trapezium $DEGHD$ − Area of rectangle $EFHG$
= Area of triangle DEF

$$= \frac{1}{2} DF \times FE$$

$$= \frac{1}{2}(600 - 300)\text{ Nm}^{-2} \times (5.0 - 2.0)\text{ m}^3$$

$$= \mathbf{450\ J.}$$

12.10. A refrigerator is to maintain eatables kept inside at 9°C. If room temperature is 36°C, calculate the coefficient of performance.

Solution. Here $T_1 = 273 + 36 = 309$ K,

$$T_2 = 273 + 9 = 282\text{ K}$$

Coefficient of performance,

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{282}{309 - 282} = \frac{282}{27} = \mathbf{10.4.}$$

Scanned by CamScanner

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. What is an isobaric process ?
2. What is an isochoric process ?
3. What is meant by free expansion ?
4. Write the relation among heat energy, work done and change in internal energy.
5. Is it possible to increase the temperature of a gas without giving it heat ? [Himachal 03]
6. An ideal gas is compressed at constant temperature. Will its internal energy increase or decrease ?
7. Can the temperature of an isolated system remain constant ?
8. Is the super heating of steam an isobaric process or an isothermal process, and why ?
9. If no external energy is supplied to an expanding gas, will the gas do any work ? If yes, then what will be the source of energy ?
10. Name the thermodynamical variables defined by
    (*i*) Zeroth law and
    (*ii*) first law of thermodynamics.
11. Is it possible to convert internal energy into work ?
12. A sample of an ideal gas in a cylinder is compressed adiabatically to one-third of its volume. Will the final pressure be more or less than three times the initial pressure ?
13. What is the specific heat of a gas
    (*i*) in an isothermal process and
    (*ii*) in an adiabatic process ?
    [Chandigarh 04 ; Central Schools 10]
14. Can the specific heat of a gas be infinity ?
15. Out of the parameters : temperature, pressure, work and volume, which parameter does not characterise the thermodynamic state of matter ? [AIEEE 04]
16. State two limitations of the first law of thermodynamics.
17. What is a heat engine ?
18. Which law forbids the complete conversion of heat energy into mechanical work ?
19. What type of process is Carnot's cycle ?
20. Can a Carnot engine be realised in practice ?
21. Name the hot reservoir in the steam engine and petrol engine.
22. Name the sink in case of a steam engine.
23. Name the working substance in the steam engine and petrol engine.
24. What is a heat pump ? Give an example.
25. Is rusting of iron a reversible process ?
26. What is the nature of P-V diagram for isobaric and isochoric processes ?
27. How is the efficiency of a Carnot engine affected by the nature of the working substance ?
28. State two essential requirements of an ideal heat engine.
29. Can we increase the coefficient of performance of a refrigerator by increasing the amount of working substance ?
30. Refrigerator transfers heat from a cold body to a hot body. Does this not violate the second law of thermodynamics ?
31. Is coefficient of performance of a refrigerator a constant quantity ?
32. Heat cannot flow itself from a body at lower temperature to a body at higher temperature is a statement or consequence of which law of thermodynamics ?
    [AIEEE 03]
33. On what factors, the efficiency of a Carnot engine depends ? [Himachal 04]
34. Heat is supplied to a system, but its internal energy does not increase. What is the process involved ? [Delhi 12]

Answers

1. A thermodynamic process which occurs at a constant pressure is called an isobaric process.
2. A thermodynamic process which occurs at a constant volume is called an isochoric process.
3. The expansion of a gas against zero external pressure is known as its *free expansion*.
4. $\Delta Q = \Delta U + \Delta W$.
5. Yes, it happens during an adiabatic process.
6. It will remain same because the internal energy of an ideal gas depends only upon the temperature.
7. Yes. The temperature of a system remains constant when no physical or chemical change takes place inside the system.
8. Isobaric, because during heating the temperature of the steam does not remain constant.
9. Yes, the gas will do work at the expense of its internal energy.
10. (*i*) Zeroth law of thermodynamics defines temperature. (*ii*) First law of thermodynamics defines internal energy.
11. Yes. For example, in an explosion of a bomb, chemical energy (which is a form of internal energy) is converted into kinetic energy.
12. The change in pressure will be more than three times the initial pressure.
13. (*i*) Infinity (*ii*) Zero.
14. Yes.
15. Work.
16. (*i*) First law does not indicate the direction of heat transfer.
    (*ii*) It does not indicate as to why the whole of heat energy cannot be converted into work continuously.
17. Heat engine is a device for converting heat into mechanical work.
18. Second law of thermodynamics
19. Carnot's cycle is a reversible cyclic process.
20. No, Carnot engine is an ideal heat engine.
21. Boiler is the hot reservoir in a steam-engine. The source of heat is the combustion of petrol vapours and air in case of a petrol engine.
22. Atmosphere is the sink in a steam-engine.
23. In the steam-engine, the working substance is the steam. In the case of a petrol-engine, the hot gas obtained from the combustion of the air-petrol mixture is the working substance.
24. Heat pump is a device which uses mechanical work to remove heat. A refrigerator is a heat pump.
25. No. Rusting of iron is an irreversible process.
26. (*i*) For an isobaric process, the P-V diagram is a straight line parallel to the volume-axis.
    (*ii*) For an isochoric process, the P-V diagram is a straight line parallel to the pressure-axis.
27. The efficiency of a Carnot engine is independent of the nature of the working substance.
28. (*i*) An ideal heat engine should have a source of infinite thermal capacity.
    (*ii*) It should have a sink of infinite thermal capacity.
29. No.
30. No. External work is done by the compressor of the refrigerator.
31. No. As the inside temperature of the refrigerator decreases, its coefficient of performance decreases.
32. Second law of thermodynamics.
33. Temperatures of source of heat and sink.
34. Isothermal expansion.

Type B : Short Answer Questions

2 or 3 Marks Each

1. Define the terms thermodynamic system, surroundings, thermodynamic variables and equation of state.
2. Describe when a system is said to be in a state of thermodynamic equilibrium.
3. State Zeroth law of thermodynamics. How does it lead to the concept of temperature ?
4. Define internal energy of a system. Is it a state variable or not ? What is the nature of the internal energy of an ideal gas ?

Scanned by CamScanner

5. Distinguish between the terms heat and work.
6. What is an indicator diagram ? What does the area between *P-V* curve and volume-axis signify ?
7. What is a cyclic process ? Prove that the net work done during a cyclic process is numerically equal to the area of the loop representing the cycle.
8. State and explain first law of thermodynamics. Discuss its use in isothermal and adiabatic processes. **[Himachal 05]**
9. What are the limitations of the first law of thermodynamics ? **[Central Schools 05 ; Chandigarh 07]**
10. Establish relation between two specific heats of a gas. Which is greater and why ? **[Himachal 05 ; Central Schools 04]**
11. State first law of thermodynamics. Derive the relation $C_p - C_v = R$ for an ideal gas. **[Delhi 13]**
12. (*i*) Why a gas has two principal specific heat capacities ?
    (*ii*) Which one is greater and why ?
    (*iii*) Of what significance is the difference between these two specific heat capacities and their ratio ? **[Delhi 97, 05]**
13. State first law of thermodynamics. On its basis establish the relation between two molar specific heats for a gas. **[Delhi 06, 08 ; Central Schools 14]**
14. Define *two* principal specific heats of a gas. Which is greater and why ? **[Himachal 05 ; Delhi 03]**
15. Derive the relation between specific heats of a gas at constant pressure and at constant volume, when the amount of gas is one gram molecule. **[Chandigarh 07, 08]]**
16. State the first law of thermodynamics. Apply this to derive an expression for the change in internal energy during boiling process. **[Delhi 1998]**
17. Compare between isothermal and adiabatic processes. **[Central Schools 05]**
18. Give two statements for the second law of thermodynamics. **[Delhi 99 ; Himachal 05, 06C]**
19. What do you understand by reversible process and irreversible process ? Give an example of each. **[Himachal 05C, 09 ; Central Schools 07, 12]**
20. What is an isothermal process ? Derive an expression for work done during an isothermal process. **[Himachal 05, 06, 09]**
21. What is an adiabatic process ? Derive expression for the work done during such a process. **[Himachal 05, 07, 09C ; Delhi 05 ; Central Schools 09]**
22. Apply first law of thermodynamics to
    (*i*) an isochoric process,
    (*ii*) a cyclic process and
    (*iii*) an isobaric process.
    State what happens to heat absorbed in each case.
23. Draw a neat *P-V* diagram showing cycle of operations for an ideal heat engine. Also list the four stages of operations in proper order. **[Central Schools 07]**
24. What is Carnot's engine ? Derive an expression for the efficiency of a Carnot's engine. On what factors does it depend ? **[Chandigarh 07 ; Central Schools 08]**
25. Give two characteristics of Carnot engine as compared to other engines, which are sometimes known as Carnot's theorem. **[Central Schools 08]**
26. With the help of a block diagram, explain the working principle of a refrigerator and obtain an expression for its coefficient of performance. **[Himachal 01, 05, 06 ; Delhi 13 ; Central Schools 04]**
27. Define the co-efficient of performance of a heat pump and obtain an expression for it in terms of temperature T_1 of the source and T_2 of the sink. **[Central School 07]**
28. State second law of thermodynamics. Write a difference between heat engine and refrigerator. **[Delhi 12]**

Answers

1. Refer answer to Q. 2 on page 12.1.
2. Refer answer to Q. 4 on page 12.2.
3. Refer answer to Q. 5 on page 12.2.
4. Refer answer to Q. 6 on page 12.2.
5. Refer answer to Q. 8 on page 12.3.
6. Refer answer to Q. 10 on page 12.4.
7. Refer answer to Q. 13 on page 12.5.
8. Refer answer to Q. 14 on page 12.7, Q. 21 on page 12.14 and Problem 29 on page 12.39.
9. Refer answer to Q. 33 on page 12.23.
10. Refer answer to Q. 16 and Q. 17 on page 12.10.
11. Refer answer to Q. 17 on page 12.10.

12. Refer answer to Q. 16 on page 12.10. The value of γ indicates the atomicity of a gas.
13. Refer answer to Q. 17 on page 12.10.
14. Refer answer to Q. 16 on page 12.10.
15. Refer answer to Q. 17 on page 12.10.
16. Refer answer to Q. 29 on page 12.17.
17. Refer answer to Q. 20 on page 12.14 and Q. 23 on page 12.15.
18. Refer answer to Q. 34 on page 12.23.
19. Refer answer to Q.36 on page 12.24.
20. Refer answer to Q.22 on page 12.15.
21. Refer answer to Q.25 on page 12.16.
22. Refer answer to Q. 26, Q. 27 and Q. 28 on page 12.17.
23. See Fig. 12.21 on page 12.25.
24. Refer answer to Q. 38 on page 12.25
25. See statement of Carnot theorem in the answer of Q. 40 on page 12.30.
26. Refer answer to Q. 41 on page 12.31.
27. Refer answer to Q. 41 on page 12.31.
28. Refer to point 28 of Glimpses and answer to Q. 7(*i*) on page 12.52.

Type C : Long Answer Questions

5 Marks Each

1. Define C_p and C_v. Why is $C_p > C_v$? For an ideal gas, prove that $C_p - C_v = R$. **[Himachal 04, 05C, 06, 07C]**

2. State mathematically, first law of thermodynamics and use it to find the expression for work done during adiabatic expansion. Write two limitations of first law of thermodynamics. **[Chandigarh 07]**

3. Derive an expression for work done during an adiabatic process. Show that slope in adiabatic process is γ times the slope in isothermal process. **[Chandigarh 08]**

4. What is an isothermal process ? State two essential conditions for such a process to take place. Show analytically that work done by one mole of an ideal gas during isothermal expansion from volume V_1 to volume V_2 is given by

$$W = RT\log_e \frac{V_2}{V_1}$$

What is the change in internal energy of a gas, which is compressed isothermally ? **[Himachal 06 ; Delhi 14]**

5. What is an adiabatic process ? Show that its gas equation is

$$TV^{\gamma-1} = \text{constant}$$

where γ is specific heat constant. **[Himachal 07C]**

6. Define an adiabatic process and state two essential conditions for such a process to take place. Show analytically that work done by one mole of an ideal gas during adiabatic expansion from temperature T_1 to T_2 is given by

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

[Himachal 06, 09]

7. (*i*) State second law of thermodynamics. How is a heat engine different from a refrigerator ?
   (*ii*) Show that for a Carnot engine, efficiency

$$\eta = 1 - \frac{T_2}{T_1} \quad (T_2 < T_1)$$

where T_1 is the temperature of the source and T_2 is the temperature of the sink. **[Delhi 05]**

8. Give Kelvin-Planck statement for second law of thermodynamics. Define Carnot engine. Draw *P-V* diagram for Carnot cycle. List down and mark the sequence of processes involved in Carnot cycle. **[Central Schools 2003]**

9. What is meant by a thermodynamic reversible process ? Write two characteristics of a reversible process.
   State in brief the meaning of Carnot cycle. Write an expression for the efficiency of a Carnot engine. What is the advantage of Carnot engine in terms of efficiency ? **[Delhi 03C]**

10. Explain the construction and various operations for Carnot's heat engine working between two temperatures. Hence derive from it the efficiency of the engine. **[Himachal 03, 07C]**

11. (*a*) Draw *P-V* diagram for Carnot cycle.
    (*b*) Write the name of thermodynamic process carried out by each part of the cycle.
    (*c*) Label and shade the area corresponding to net work done by the engine in one cycle. **[Central Schools 08]**

12. Write Kelvin-Planck and Clausius statements for second law of thermodynamics. Define coefficient of efficiency and coefficient of performance. Show the heat flow in case of an engine and refrigerator using schematic diagram. **[Central Schools 08, 09]**

Scanned by CamScanner

Answers

1. Refer answer to Q.16 and Q.17 on page 12.10.
2. Refer answer to Q.14 on page 12.7, Q.25 on page 12.16 and Q.33 on page 12.23.
3. Refer answer to Q. 25 on page 12.16 and refer to solution of Problem 10 on page 12.39.
4. Refer answer to Q. 20 on page 12.14, Q.21 and Q.22 on page 12.15.
5. Refer answer to Q.24 on page 12.15.
6. Refer answer to Q.23 on page 12.15 and Q.25 on page 12.16.
7. (*i*) Refer answer to Q. 34 on page 12.23. A heat engine is a device in which the working substance undergoes a cyclic process to convert heat into work. The working substance extracts some heat Q_1 from the source at a high temperature, converts a part of it into work W and rejects the rest Q_2 to the sink at lower temperature. A refrigerator is a heat engine working in the reverse direction. Here the working substance extracts heat Q_1 from the sink at a lower temperature. Then work W is done on it and a higher amount of heat $Q_1 = Q_2 + W$ is rejected to the source at a higher temperature.

   (*ii*) Refer answer to Q. 38 on page 12.25.
8. Refer answer to Q. 34 on page 12.23 and Q. 38 on page 12.25.
9. Refer answer to Q. 36 on page 12.24 and Q. 38 on page 12.25.
10. Refer answer to Q. 38 on page 12.25.
11. See Fig. 12.21 on page 12.25. The area of the Carnot cycle gives the net work done by the engine per cycle.
12. Refer to points 27, 28 and 34 of Glimpses. See Fig. 12.18 on page 12.22 and Fig. 12.23 on page 12.31.

Scanned by CamScanner

Competition Section

Thermodynamics

GLIMPSES

1. **Thermodynamics.** It is the branch of science that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy.

2. **Thermodynamic system.** An assembly of a very large number of particles having a certain value of pressure, volume and temperature is called a thermodynamic system.

3. **Surroundings.** Everything outside the system which can have a direct effect on the system is called its surroundings.

4. **Thermodynamic variables.** The quantities like pressure (P), volume (V) and temperature (T) which help us to study the behaviour of a thermodynamic system are called thermodynamic variables.

5. **Equation of state.** The mathematical relation between the pressure, volume and temperature of a thermodynamic system is called its equation of state. For example, the equation of state of n moles of an ideal gas can be written as

$$PV = nRT$$

6. **Thermal equilibrium.** Two systems are in thermal equilibrium with each other if they have the same temperature.

7. **Thermodynamic equilibrium.** A system is said to be in the state of thermodynamic equilibrium if the macroscopic variables describing the thermodynamic state of the system do not change with time. A system in a state of thermodynamic equilibrium possesses mechanical, thermal and chemical equilibria simultaneously.

8. **State variables.** The macroscopic quantities which are used to describe the equilibrium states of a thermodynamic system are called state variables. The value of a state variable depends only on the particular state, not on the path used to attain that state. Pressure (P), volume (V), temperature (T) and mass (m) are state variables. Heat (Q) and work (W) are not state variables.

9. **Zeroth law of thermodynamics.** If two systems A and B are in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other. According to this law, temperature is a physical quantity which has the same value for all systems which are in thermal equilibrium with each other.

10. **Internal energy.** The internal energy of a system is the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest. It does not include the over-all kinetic energy of the system. It is a state variable denoted by U.

11. **Quasi-static process.** A quasi-static process is an infinitely slow process such that system remains in thermal and mechanical equilibrium with the surroundings throughout. In such a process, the pressure and temperature of the surroundings can differ from those of the system only infinitesimally.

12. **Isothermal process.** A process in which temperature remains constant is called an isothermal process. For such a process,

$$PV = \text{constant} \quad \text{or} \quad P_1V_1 = P_2V_2$$

13. **Adiabatic process.** A process in which thermally insulated system neither loses nor gains heat from the surroundings is called adiabatic process.

Equations of state for adiabatic processes are :

(i) $PV^{\gamma} = \text{constant}$ or $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

(ii) $TV^{\gamma-1} = \text{constant}$ or $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

(iii) $\dfrac{P^{\gamma-1}}{T^{\gamma}} = \text{constant}$ or $\dfrac{P_1^{\gamma-1}}{T_1^{\gamma}} = \dfrac{P_2^{\gamma-1}}{T_2^{\gamma}}$

where $\gamma = C_p / C_v$.

Scanned by CamScanner

14. **Isobaric process.** A process in which volume remains constant is called isobaric process. For such a process

$$\frac{V}{T} = \text{constant} \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

15. **Isochoric process.** A process in which volume remains constant is called isochoric process. For such a process,

$$\frac{P}{T} = \text{constant} \quad \text{or} \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}.$$

16. **Indicator diagram.** A graphical representation of the state of a system with the help of two thermodynamical variables is called indicator diagram of the system. The graph between pressure P and volume V is called P-V diagram.

17. **Work done during the expansion of a gas.** When the volume of a gas changes from V_1 to V_2, the work done is

$$W = \int_{V_1}^{V_2} P\,dV$$

= Area enclosed between the P-V curve and the volume axis.

18. **Work done during a cyclic process.** From the P-V diagram,

Work done per cycle = Area of the loop representing the cycle

(*i*) If the loop is traced *clockwise*, the work done is *positive* and work is done by the system.

(*ii*) If the loop is traced *anticlockwise*, the work done is *negative* and work is done on the system.

19. **First law of thermodynamics.** It states that if heat dQ is given to a system, a part of it is used in increasing the internal energy by an amount dU and the remaining energy is used in doing the external work dW. It is just a restatement of the law of conservation of energy. Thus

$$dQ = dU + dW \quad \text{or} \quad dQ = dU + PdV.$$

Sign conventions used.

(*i*) Heat absorbed by a system is positive and heat given out by a system is negative.

(*ii*) Increase in internal energy of a system is positive and decrease in internal energy of a system is negative.

(*iii*) Work done by a system is positive and work done on a system is negative.

20. **Work done in an isothermal process.** Work done when 1 mole of a gas expands isothermally,

$$W_{iso} = 2.303\,RT \log \frac{V_2}{V_1} = 2.303\,RT \log \frac{P_1}{P_2}.$$

21. **Work done in an adiabatic process.** Work done when 1 mole of a gas expands adiabatically and its temperature falls from T_1 to T_2,

$$W_{adia} = \frac{R}{\gamma - 1}[T_1 - T_2] = \frac{1}{\gamma - 1}[P_1V_1 - P_2V_2]$$

22. **Dulong and Petit's law.** Near the room temperature, the molar specific heat of most of the solids at constant volume is equal to 3R or $6\ \text{cal mol}^{-1}\ \text{K}^{-1}$ or $25\ \text{J mol}^{-1}\ \text{K}^{-1}$. This statement is known as Dulong and Petit's law.

23. **Molar specific heat of a gas at constant volume (C_v).** *It is defined as the amount of heat required to raise the temperature of 1 mole of the gas through 1°C at constant volume.*

If c_v is specific heat of gas for 1 g at constant volume and M is its molecular weight, then molar specific heat at constant volume,

$$C_v = Mc_v$$

24. **Molar specific heat of a gas at constant pressure (C_p).** It is defined as the amount of heat required to raise the temperature of 1 mole of the gas through 1°C at constant pressure. Thus,

$$C_p = Mc_p$$

25. **Relation between two specific heats of a gas.** Specific heat of a gas at constant pressure is greater than the specific heat at constant volume.

For one mole of a gas :

(*i*) $C_p - C_v = R$ (when C_p, C_v are in units of work)

(*ii*) $C_p - C_v = \frac{R}{J}$ (when C_p, C_v are in units of heat)

where R is universal gas constant for 1 mole of a gas. For 1 g of a gas :

(*i*) $c_p - c_v = r$ (when c_p, c_v are in units of work)

(*ii*) $c_p - c_v = \frac{r}{J}$ (when c_p, c_v are in units of heat)

where $r = \frac{R}{M}$ = gas constant for 1 g of a gas.

Clearly, heat lost or gained by n moles of a gas,

(*i*) $Q = nC_p\,\Delta T$ (At constant pressure)

(*ii*) $Q = nC_v\,\Delta T$ (At constant volume)

where n = number of moles of gas

$$= \frac{\text{Mass of gas}}{\text{Molecular mass}}.$$

26. **Heat engine.** It is a device which converts continuously heat energy into mechanical energy in a cyclic process. It essentially consists of (*i*) a source of heat (*ii*) a sink of heat and (*iii*) a working substance.

Scanned by CamScanner

27. **Efficiency of a heat engine.** It is the ratio of useful work done (W) by the engine per cycle to the heat energy (Q_1) absorbed from the source per cycle.

$$\eta = \frac{\text{Work output}}{\text{Heat input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

where Q_2 is the heat rejected to the sink.

28. **Second law of thermodynamics.**

(*i*) **Kelvin-Planck statement.** It is impossible to construct an engine, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.

(*ii*) **Clausius statement.** *It is impossible for a self acting machine, unaided by an external agency, to transfer heat from a body to another at higher temperature.*

29. **Reversible process.** *A process which can be made to proceed in the reverse direction by variation in its conditions so that any change occurring in any part of the direct process is exactly reversed in the corresponding part of the reverse process is called a reversible process.*

30. **Irreversible process.** *A process which cannot be made to proceed in the reverse direction is called an irreversible process.*

31. **Carnot Engine.** It is an ideal heat engine which is based on Carnot's reversible cycle. Its working consists of four steps *viz.* Isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression. The efficiency of Carnot engine is given by

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

where T_1 and T_2 are the temperatures of source and sink respectively.

32. **Carnot theorem.** It states that

(*i*) no engine working between two given temperatures can have efficiency greater than that of the Carnot engine working between the same two temperatures and

(*ii*) the efficiency of the Carnot engine is independent of the nature of the working substance.

33. **Refrigerator.** It is a heat engine working in the reverse direction. Here a working substance absorbs heat Q_2 from the sink at temperature T_2. An external agency does work W on the working substance. A larger amount of heat Q_1 is rejected to source at a higher temperature T_2.

$$Q_1 = Q_2 + W.$$

34. **Coefficient of performance.** It is defined as the ratio of the amount of heat (Q_2) removed per cycle from the contents of the refrigerator to the work done (W) by the external agency to remove it.

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}.$$

IIT Entrance Exam

✓ Multiple Choice Questions with One Correct Answer

1. In a given process of an ideal gas, $dW = 0$ and $\partial Q < 0$. Then for the gas

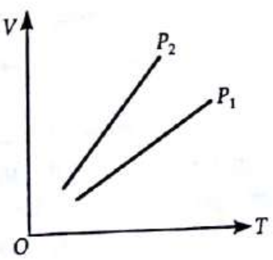
(*a*) the temperature will decrease

(*b*) the volume will increase

(*c*) the pressure will remain constant

(*d*) the temperature will increase. [IIT 01]

2. The volume (V) versus temperature (T) graphs for a certain amount of a perfect gas at two pressures P_1 and P_2 are shown in the figure. It follows from the graphs that



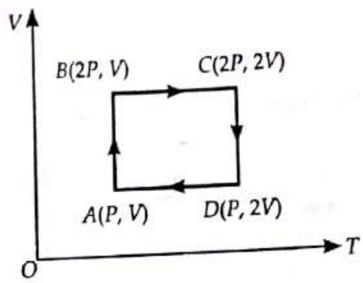
(*a*) $P_1 > P_2$

(*b*) $P_1 < P_2$

(*c*) $P_1 = P_2$

(*d*) information is insufficient to draw any conclusion. [IIT 88]

3. An ideal monoatomic gas is taken round the cycle $ABCDA$ as shown in the P-V diagram.

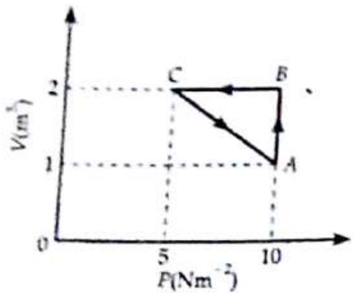


The work done during the cycle is

(*a*) PV (*b*) $2\ PV$

(*c*) $PV/2$ (*d*) zero. [IIT 83]

4. An ideal gas is taken through the cycle $A \to B \to C \to A$ as shown in the figure. If the net heat

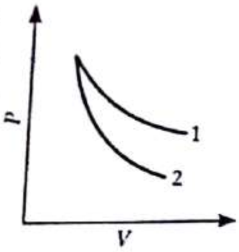


supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is

(a) – 5 J (b) – 10 J

(c) – 15 J (d) – 20 J [IIT 02]

5. P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to



(a) He and O_2

(b) O_2 and He

(c) He and Ar (d) O_2 and N_2 [IIT 01]

6. An ideal gas initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in complete process, which of the following is true ?

(a) $W > 0$; $P_3 > P_1$ (b) $W < 0$; $P_3 > P_1$

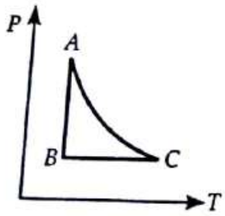
(c) $W > 0$; $P_3 < P_1$ (d) $W < 0$; $P_3 < P_1$ [IIT 04]

7. A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

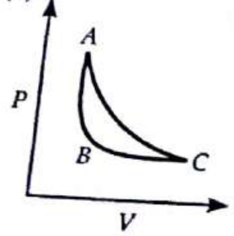
(a) $(L_1/L_2)^{2/3}$ (b) L_1/L_2

(c) L_2/L_1 (d) $(L_2/L_1)^{2/3}$ [IIT 2K]

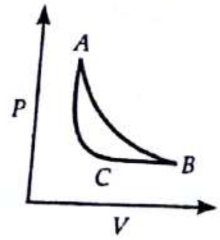
8. The PT diagram for an ideal gas is shown in the figure, where AC is an adiabatic process. Find the corresponding PV diagram.



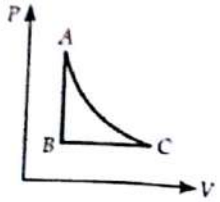
(a)



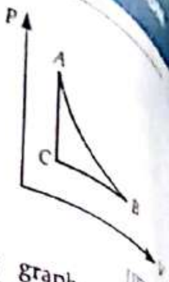
(b)



(c)

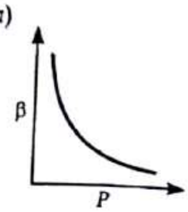


(d)

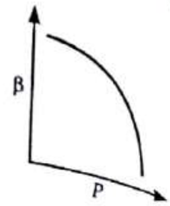


9. Which of the following graphs correctly represents the variation of $\beta = -\dfrac{dV/dP}{V}$ with P for an ideal gas at constant temperature ? [IIT 02]

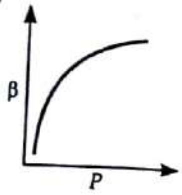
(a)



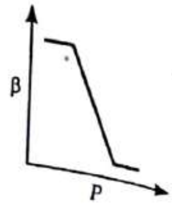
(b)



(c)

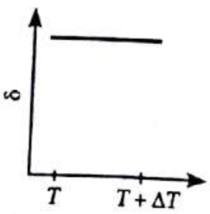


(d)

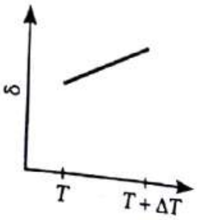


10. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V / V\Delta T$ varies with temperature as [IIT 2K]

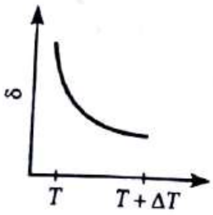
(a)



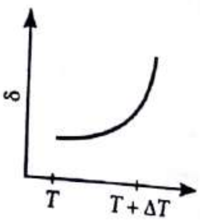
(b)



(c)



(d)



11. One mole of a monoatomic gas is heated at a constant pressure of 1 atmosphere from 0 K to 100 K. If the gas constant $R = 8.32$ J/mol K, the change in internal energy of the gas is approximately

(a) 2.3 J (b) 46 J

(c) 8.67×10^3 J (d) 1.25×10^3 J [IIT 98]

Scanned by CamScanner

12. An ideal gas heat engine is operating between 27°C and 127°C. It absorbs 10^4 J of heat at the higher temperature. The amount of heat converted into work is

(a) 2000 J (b) 4000 J

(c) 8000 J (d) 5600 J [IIT 98]

13. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is

(a) $\frac{9}{8} RT_1$ (b) $\frac{3}{2} RT_1$

(c) $\frac{15}{8} RT_1$ (d) $\frac{9}{2} RT_1$ [IIT 2011]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

14. An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure $P/2$, volume $2V$) along a straight line path in the P-V diagram. Select the correct statement(s) from the following :

(a) the work done by the gas in the process A to B exceeds the work that would be done by it, if the system were taken from A to B along an isotherm

(b) in the T-V diagram, the path AB becomes a part of a parabola

(c) in the P-T diagram, the path AB becomes a part of a hyperbola

(d) in going from A to B, the temperature T of the gas first increases to a maximum value and then decreases. [IIT 93]

15. During the melting of a slab of ice at 273 K at atmospheric pressure,

(a) positive work is done by the ice-water system on the atmosphere

(b) positive work is done on the ice-water system by the atmosphere

(c) the internal energy of the ice-water system increases

(d) the internal energy of the ice-water system decreases. [IIT 98]

16. 70 calories are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required (in calories) to raise the temperature of the same gas through the same range (30° to 35°C) at constant volume is

(a) 30 (b) 50

(c) 70 (d) 90 [IIT 85]

17. For an ideal gas

(a) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_V(T_2 - T_1)$, where C_V is the molar specific heat at constant volume and n, the number of moles of the gas

(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process

(c) no heat is added or removed in an adiabatic process

(d) the internal energy does not change in an isothermal process. [IIT 89]

18. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is

(a) 18 K (b) 30 K [IIT 98]

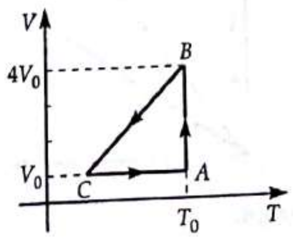
(c) 50 K (d) 42 K

19. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that is B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are found to be P and $1.5P$ respectively. Then

(a) $2m_A = 3m_B$ (b) $3m_A = 2m_B$ [IIT 98]

(c) $4m_A = 9m_B$ (d) $9m_A = 4m_B$

20. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$ as shown in the figure. Its pressure at A is P_0. Choose the correct option(s), from the following :



(a) Internal energies at A and B are the same

(b) Work done by the gas in process AB is $P_0V_0 \ln 4$

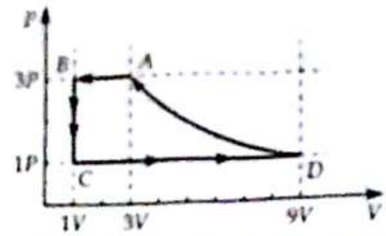
(c) Pressure at C is $\frac{P_0}{4}$

(d) Temperature at C is $\frac{T_0}{4}$ [IIT 2010]

Scanned by CamScanner

MATRIX MATCH TYPE

21. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column I		Column II	
(a)	Process A-B	(p)	Internal energy decreases
(b)	Process B-C	(q)	Internal energy decreases
(c)	Process C-D	(r)	Heat is lost
(d)	Process D-A	(s)	Heat is gained
		(t)	Work is done on the gas

[IIT 2011]

INTEGER ANSWER TYPE

22. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in kelvin) and the final temperature is aT_i, find the value of a.

23. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ?

[IIT 2010]

Answers and Explanations

1. (a) By first law of thermodynamics,

$$dQ = dU + dW$$

As $dW = 0$ and $dQ < 0$, so $dU < 0$

But for an ideal gas,

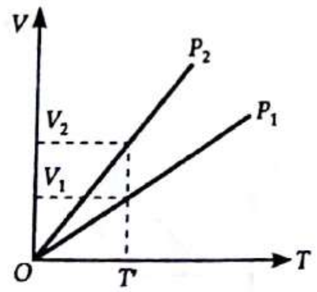
$$U \propto T$$

$\therefore$ $dT < 0$

Hence temperature of the gas decreases.

2. (a) At a given temperature T',

$$P_1V_1 = P_2V_2$$



From the graph,

$$V_1 < V_2$$

$\therefore$ $P_1 > P_2$.

3. (a) Work done in the cyclic process

$= $ Area of the loop ABCD

$= (2P - P) \times (2V - V) = PV.$

4. (a) For the cyclic process,

$$\Delta Q = \Delta W = W_{AB} + W_{BC} + W_{CA}$$
$$5 = 10(2-1) + 0 + W_{CA}$$
$$W_{CA} = 5 - 10 = -5 \text{ J}.$$

5. (b) Slope of an adiabatic curve,

$$\frac{dP}{dV} = -\gamma \frac{P}{V}$$

$\therefore$ slope $\propto \gamma$ [with a –ve sign]

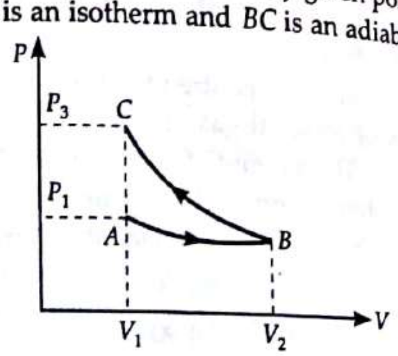
From the given graph,

slope of plot 2 > slope of plot 1

$\therefore$ $\gamma_2 > \gamma_1$

Hence plot 1 should correspond to diatomic O_2 (small $\gamma = 1.4$) and plot 2 should correspond to nonoatomic He (large $\gamma = 1.67$).

6. (b) The slope of an adiabatic curve is γ times the slope of an isothermal curve at any given point. In the figure, AB is an isotherm and BC is an adiabat.



As volume increases from A to B, W_{AB} = + ve.

As volume decreases from B to C, W_{BC} = − ve.

Area under P-V graph gives work done.

Clearly, $|W_{BC}| > |W_{AB}|$

$$W_{AB} + W_{BC} = W < 0$$

Also, from the graph, $P_3 > P_1$

Hence option (*b*) is correct.

7. (*d*) For an adiabatic expansion,

$$TV^{\gamma-1} = \text{constant}$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

For a monoatomic gas, $\gamma = \frac{5}{3}$.

If A is the area of cross-section of the piston, then

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{AL_2}{AL_1}\right)^{\frac{5}{3}-1} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

8. (*b*) From the given P-T graph, it is obvious that

(*i*) AC is an adiabatic process (given)

(*ii*) AB is an isothermal process, as T = constant.

(*iii*) BC is an isobaric process, as P = constant.

In all the four options, BC is an isobaric process.

For AB to be an isothermal process (PV = constant), P-V graph must be a rectangular hyperbola. This is satisfied in options (*b*) and (*d*).

In option (*d*), AC is not an isothermal process but it is an isochroic process (V = constant).

Slope of adiabat AC > Slope of isotherm AB.

This is satisfied in (*b*). Hence only option (*b*) is correct.

9. (*a*) $\beta = -\frac{dV/dP}{V} = -\frac{dV/V}{dP} = \frac{1}{\text{Bulk modulus}}$

Under isothermal conditions,

Bulk modulus = Pressure P of the gas

$$\therefore \quad \beta = \frac{1}{P}$$

Hence the graph between β and P is a rectangular hyperbola as in option (*a*).

10. (*c*) For an ideal gas, $PV = nRT$

At constant pressure, $P\Delta V = nR\Delta T$

On dividing, $\frac{\Delta V}{V} = \frac{\Delta T}{T}$

$$\therefore \quad \delta = \frac{\Delta V}{V\Delta T} = \frac{1}{T}$$

Hence graph between δ and T is a rectangular hyperbola as in option (*c*).

11. (*d*) For a monoatomic gas

$$C_V = \frac{3}{2}R$$

$$\therefore \quad dU = C_V dT = \frac{3R}{2}dT$$

$$= \frac{3}{2} \times 8.32 \times (100 - 0) = \mathbf{1.25 \times 10^3\ J}$$

12. (*a*) $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273+127}{273+227} = 1 - \frac{400}{500} = \frac{1}{5}$

$$W = \eta Q_1 = \frac{1}{5} \times 10^4 = \mathbf{2000\ J}$$

13. (*a*) For monoatomic He gas, $\gamma = 5/3$

$V_1 = 5.6$ litre, $T_1 = 273$ K, $P_1 = 1$ atm, $V_2 = 0.7$ litre

For adiabatic compression,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\therefore \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1}$$

$$= T_1(8)^{2/3} = 4T_1$$

$$n = \frac{5.6\ \text{litre}}{22.4\ \text{litre}} = \frac{1}{4}$$

$$W_{adia} = \frac{nR(T_2 - T_1)}{\gamma - 1} = \frac{(1/4)R(4T_1 - T_1)}{(5/3) - 1}$$

$$= \frac{9}{8}RT_1$$

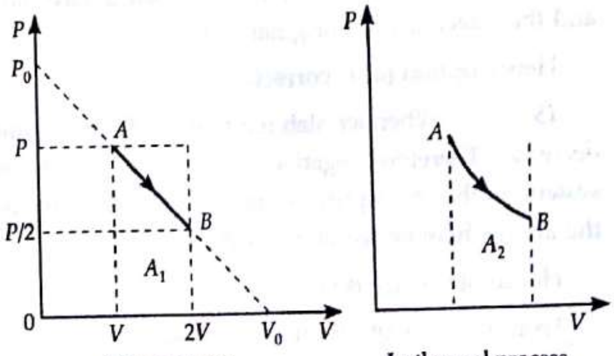
14. (*a*), (*b*), (*d*)

(*a*) Work done = Area under P-V graph

Area A_1 > Area A_2

$\therefore \quad W_{\text{given process}} > W_{\text{isothermal process}}$

Hence option (*a*) is correct.



Given process — Isothermal process

(*b*) If P_0 and V_0 are the intercepts on P and V axes, then equation of line AB will be

$$P = -\frac{P_0}{V_0}V + P_0$$

Scanned by CamScanner

or $$\frac{nRT}{V} = -\frac{P_0}{V_0}V + P_0$$

or $$T = \frac{1}{nR}\left(-\frac{P_0}{V_0}V^2 + P_0V\right)$$

As $T \propto V^2$,

the T-V graph will be a parabola. Hence option (*b*) is correct.

(*c*) Again, $P = -\frac{P_0}{V_0}\frac{nRT}{P} + P_0$

or $$\frac{V_0P^2}{nRP_0} = -T + \frac{V_0}{nR}P$$

or $$T = -\frac{V_0}{nRP_0}P^2 + \frac{V_0}{nR}P$$

As $T \propto P^2$,

so, P-T graph is a parabola not a hyperbola. Hence option (*c*) is incorrect.

(*d*) $$T = \frac{V_0}{nR}\left(-\frac{1}{P_0}P^2 + P\right)$$

$$\frac{dT}{dP} = \frac{V_0}{nR}\left(-\frac{2}{P_0}P + 1\right) = 0$$

or $$P = \frac{P_0}{2}$$

$$\frac{d^2T}{dP^2} = -\frac{2V_0}{nRP_0} < 0$$

$\therefore$ T has a maximum value.

$$T \propto PV$$

$$(PV)_A = (PV)_B \quad \text{or} \quad T_A = T_B$$

In going from A to B, T first increases to a maximum and then decreases to original value.

Hence option (*d*) is correct.

15. (*b*), (*c*) When ice slab melts at 273 K, its volume decreases. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done by the atmosphere on ice-water system.

Hence option (*b*) is correct.

From first law of thermodynamics,

$$dQ = dU + dW \quad \text{or} \quad dU = dQ - dW$$

dQ is +ve as ice absorbs heat during melting.

Also, dW is –ve. Hence dU will be +ve or internal energy of ice-water system increases.

Hence option (*c*) is correct.

16. (*b*) At constant pressure, $Q_1 = nC_p dT$

At constant volume, $Q_2 = nC_v dT$

$$\therefore \quad \frac{Q_2}{Q_1} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

or $$Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50 \text{ cal}.$$

17. (*a*), (*b*), (*c*), (*d*)

(*a*) $\Delta U = nC_V\Delta T = nC_V(T_2 - T_1)$, for all processes

(*b*) In an adiabatic process, $\Delta Q = 0$

$\therefore \quad \Delta U = -\Delta W$ or $|\Delta U| = |\Delta W|$

(*c*) In an isothermal process, $\Delta T = 0$

$\therefore \quad \Delta U = nC_V\Delta T = 0$

(*d*) In an adiabatic process, $\Delta Q = 0$

Hence all the options are correct.

18. (*d*) As the piston of A is free to move, heat is supplied at constant pressure.

$$\therefore \quad dQ = nC_p\, dT_A$$

As the piston of B is A held fixed, heat is supplied at constant volume

$$dQ_B = nC_V\, dT_B$$

But $$dQ_A = dQ_B$$

or $$nC_P dT_A = nC_V dT_B$$

or $$dT_B = \left(\frac{C_P}{C_V}\right)dT_A = \gamma dT_A$$

$$= 1.4 \times 30 \text{ K} \quad [\text{For a diatomic gas, } \gamma = 1.4]$$

$$= 42 \text{ K}.$$

19. (*b*) $$\Delta P = P_i - P_f = \frac{nRT}{V} - \frac{nRT}{2V} = \frac{nRT}{2V} = \frac{mRT}{2MV}$$

$$\therefore \quad \Delta P \propto m$$

$$\frac{m_A}{m_B} = \frac{\Delta P_A}{\Delta P_B} = \frac{P}{1.5A} = \frac{2}{3}$$

or $$3m_A = 2m_B.$$

20. (*a*), (*b*), (*c*), (*d*)

Process $A \to B$ is isothermal, so $U_A = U_B$

$$W_{AB} = nRT\ln\frac{V_2}{V_1} = RT_0\ln\frac{4V_0}{V_0}$$

$$= P_0V_0\ln 4 \quad [P_0V_0 = RT_0 \text{ for } n = 1]$$

$$\frac{P_0V_0}{T_0}(\text{at } A) = \frac{P_B 4V_0}{T_0}(\text{at } B)$$

$$\Rightarrow \quad P_B = \frac{P_0}{4}$$

Scanned by CamScanner

Process $B \to C$ is isobaric, so $P_C = P_B = \frac{P_0}{4}$

Process $C \to A$ is isochoric, $\frac{P}{T}$ = constant

$$T_C = \frac{T_0}{4} \qquad [\frac{P_0}{T_0}(\text{at } A) = \frac{P_0/4}{T_0/4}(\text{at } C)]$$

21. $(a) \to p, r, t; (b) \to p, r; (c) \to q, s; (d) \to r, t$

(a) Process $A \to B$ is isobaric compression

∴ Volume decreases ⇒ Temperature decreases

Work is done on the gas,

$$W = P(3V - V) = 2PV$$

Heat is lost and internal energy decreases.

Hence, $(a) \to p, r, t$ are correct matching.

(b) Process $B \to C$ is isochoric process

∴ Pressure decreases ⇒ Temperature decreases.

Heat is lost and internal energy decreases.

Hence, $(b) \to p, r$ are correct matching

(c) Process $C \to D$ is isobaric expansion.

∴ Volume increases ⇒ Temperature increases

Heat is gained and internal energy increases.

Hence, $(c) \to q, r$ are correct matching.

(d) Process $D \to A$ is polytropic with $T_A = T_D$ because

$$T_A = \frac{(3P)(3V)}{nR} = \frac{9PV}{nR}$$

$$T_D = \frac{PV}{nR} = \frac{(P)(9V)}{nR} = \frac{9PV}{nR}$$

$$T_A = T_D \Rightarrow \Delta U = 0$$

$$\therefore \quad \Delta Q = \Delta U + W = W$$

⇒ Work done on the gas is lost as heat.

Hence, $(d) \to r, t$ are correct matching.

22.

0	0	0	4

For an adiabatic process, $TV^{\gamma-1}$ = constant

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\therefore \quad T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

$$= T_i\left(\frac{V_i}{V_i/32}\right)^{\frac{7}{5}-1}$$

[For a diatomic gas, $\gamma = \frac{7}{5}$]

$$= T_i(2^5)^{2/5} = 4T_i$$

Hence, $a = 4$

23.

0	0	0	9

As per Wein's displacement law,

$$\frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A} = \frac{1500}{1500} = 3$$

As per Stefan's law,

$$\frac{P_A}{P_B} = \frac{A_A T_A^4}{A_B T_B^4} = \frac{r_A^2}{r_B^2}\left(\frac{T_A}{T_B}\right)^4 = \frac{(6)^2}{(18)^2} \times (3)^4 = 9$$

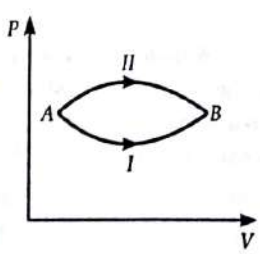
AIEEE

1. Which of the following parameters does not characterize the thermodynamic state of matter ?

(a) Temperature (b) Pressure

(c) Work (d) Volume [AIEEE 03]

2. A system goes from A to B via two processes I and II as shown in the figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then



(a) $\Delta U_1 = \Delta U_2$

(b) $\Delta U_1 > \Delta U_2$

(c) $\Delta U_1 < \Delta U_2$

(d) relation between ΔU_1 and ΔU_2 cannot be determined. [AIEEE 05]

3. The internal energy change, when a system goes from state A and B is 40 kJ $mole^{-1}$. If the system goes from A to B by a reversible path and returns to state A by an irreversible path, what would be the net change in internal energy ?

(a) 40 kJ (b) > 40 kJ

(c) < 40 kJ (d) zero.

[AIEEE 80]

Scanned by CamScanner

4. Which of the following is incorrect regarding the first law of thermodynamics ?

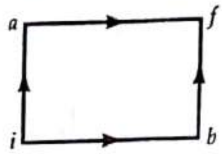
(*a*) It introduces the concept of the internal energy

(*b*) It introduces the concept of the entropy

(*c*) It is not applicable to any cyclic process

(*d*) It is a restatement of the principle of conservation of energy. [AIEEE 80]

5. When a system is taken from the initial state *i* to final state *f* along the path *iaf*, it is found that $Q = 50$ cal and $W = 20$ cal. If along the path *ibf*, $Q = 36$ cal, then W along the path *ibf* is



(*a*) 6 cal (*b*) 16 cal

(*c*) 66 cal (*d*) 14 cal [AIEEE 07]

6. Which of the following statements is correct for any thermodynamic system ?

(*a*) The internal energy changes in all processes

(*b*) Internal energy and entopy are state functions

(*c*) The change in entropy can never be zero

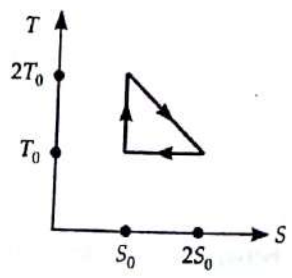
(*d*) The work done in an adiabatic process is always zero. [AIEEE 04]

7. If c_p and c_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(*a*) $c_p - c_v = R/28$ (*b*) $c_p - c_v = R/14$

(*c*) $c_p - c_v = R$ (*d*) $c_p - c_v = 28R$ [AIEEE 07]

8. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



(*a*) 1/3 (*b*) 1/2

(*c*) 2/3 (*d*) 1/4 [AIEEE 05]

9. "Heat cannot itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of

(*a*) second law of thermodynamics

(*b*) conservation of momentum

(*c*) conservation of mass

(*d*) first law of thermodynamics. [AIEEE 03]

10. Even Carnot engine cannot give 100% efficiency, because we cannot

(*a*) prevent radiation

(*b*) find ideal sources

(*c*) reach absolute zero temperature

(*d*) eliminate friction. [AIEEE 02]

11. Which statement is incorrect ?

(*a*) all reversible cycles have same efficiency

(*b*) reversible cycle has more efficiency than an irreversible one.

(*c*) Carnot cycle is a reversible one.

(*d*) Carnot cycle has the maximum efficiency of all the cycles. [AIEEE 02]

12. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is

(*a*) zero (*b*) 8.4×10^6 J

(*c*) 4.2×10^6 J (*d*) 16.8×10^6 J [AIEEE 03]

13. A Carnot engine, having an efficiency of 1/10 as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

(*a*) 1 J (*b*) 90 J

(*c*) 99 J (*d*) 100 J [AIEEE 07]

14. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K)

(*a*) 4.2 kJ (*b*) 8.4 kJ

(*c*) 84 kJ (*d*) 2.1 kJ [AIEEE 2011]

15. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively

(*a*) 372 K and 310 K (*b*) 372 K and 330 K

(*c*) 330 K and 268 K (*d*) 310 K and 248 K [AIEEE 2011]

16. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the has increased from V to $32\ V$, the efficiency of the engine is

(*a*) 0.25 (*b*) 0.5

(*c*) 0.75 (*d*) 0.99 [AIEEE 2010]

Scanned by CamScanner

Answers and Explanations

1. (*c*) Work is not a state variable. It is a mode of transfer of energy from system to surrounding or vice versa.

2. (*a*) Since the initial and final states are the same and the change in internal energy is path independent, therefore, $\Delta U_1 = \Delta U_2$

3. (*d*) As the system returns to the initial state *A*, the change in internal energy is zero.

4. (*b*) The first law of thermodynamics does not introduce the concept of entropy.

5. (*a*) Change in internal energy is path independent,

$$\Delta U_{iaf} = \Delta U_{ibf}$$
$$Q - W = Q' - W'$$
$$50 - 20 = 36 - W'$$
$$W' = 36 - 30 = \mathbf{6\ cal.}$$

6. (*b*) Internal energy and entropy are state functions.

7. (*a*) For 1 g of a gas,

$$C_p - C_v = r = \frac{R}{M}$$

For N_2, $M = 28$

$$\therefore \quad C_p - C_v = \frac{R}{28}.$$

8. (*a*) $$\eta = \frac{\Delta W}{Q_{BC}} = \frac{\text{Area of } \Delta ABC}{\text{Area under } BC}$$

$$= \frac{\frac{1}{2} S_0 \times T_0}{S_0 T_0 + \frac{1}{2} S_0 T_0} = \frac{\frac{1}{2} S_0 T_0}{\frac{3}{2} S_0 T_0} = \frac{1}{3}.$$

9. (*a*) It is a consequence of the second law of thermodynamics.

10. (*a*) The efficiency of a Carnot engine will be 100% when its sink is at 0 K. But the temperature 0 K cannot be realised in practice, so the efficiency is never 100%.

11. (*a*) Work done per cycle

= Area of the loop representing the cycle

As different reversible cycles may have different loop areas, their efficiencies will also be different.

12. (*b*) Refer to the solution of Example 30 on page 12.27.

13. (*b*) Coefficient of performance,

$$\beta = \frac{1 - \eta}{\eta} = \frac{1 - (1/10)}{1/10} = 9$$

But $$\beta = \frac{Q_2}{W}$$

$$\therefore \quad Q_2 = \beta W = 9 \times 10 = \mathbf{90\ J.}$$

14. (*b*) $$\Delta U = mc\Delta T = \frac{100}{1000} \times 4184 \times 20$$
$$= 8368\ \text{J} \simeq \mathbf{8.4\ kJ}$$

15. (*a*) Here $$\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{5}{6}$$

When T_2 is lowered by 62 K,

$$\frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1}$$

or $$\frac{62}{T_1} = \frac{T_2}{T_1} - \frac{2}{3} = \frac{5}{6} - \frac{2}{3} = \frac{1}{6} \text{ or } T_1 = 372\ \text{K}$$

and $$T_2 = \frac{5}{6} \times T_1 = \frac{5}{6} \times 372 = \mathbf{310\ K}$$

16. (*c*) For adiabatic expansion,

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

For a diatomic gas, $\gamma = \frac{7}{5}$, so

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = (32)^{\frac{7}{5} - 1} = (32)^{2/5} = 4$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = \mathbf{0.75}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. First law of thermodynamics corresponds to
(*a*) conservation of energy
(*b*) heat flow from hotter to cooler body
(*c*) law of conservation of angular momentum
(*d*) Newton's law of cooling. [DCE 2K]

2. Which one of the following is not a state function?
(*a*) temperature (*b*) entropy
(*c*) pressure
(*d*) work [DCE 01]

Scanned by CamScanner

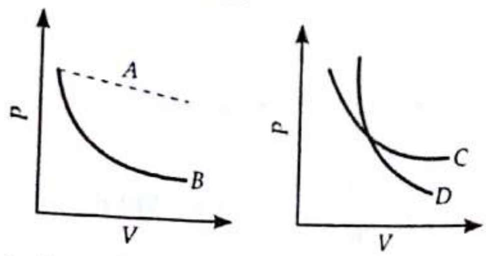
3. Which is an intensive property ?
(a) volume (b) mass
(c) refractive index (d) weight. [DCE 97]

4. Which one is correct ?
(a) In an isobaric process, $\Delta P = 0$
(b) In an isochoric process, $\Delta W = 0$
(c) In an isothermal process, $\Delta T = 0$
(d) In an isothermal process, $\Delta Q = 0$ [DCE 01]

5. In an adiabatic system which is true ?
(a) $P^{\gamma}T^{\gamma} - 1$ = constant
(b) $P^{\gamma}T^{1-\gamma}$ = constant
(c) PT^{γ} = constant
(d) $P^{1-\gamma}T^{\gamma}$ = constant. [DCE 01]

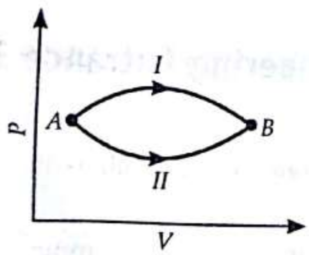
6. In an adiabatic process, the state of a gas is changed from P_1, V_1, T_1 to P_2, V_2, T_2. Which of the following relations is correct ?
(a) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ (b) $P_1V_1^{\gamma-1} = P_2V_2^{\gamma-1}$
(c) $T_1P_1^{\gamma} = T_2P_2^{\gamma}$ (d) $T_1V_1^{\gamma} = T_2V_2^{\gamma}$ [IPUEE 05]

7. In the following figure, four curves A, B, C and D are shown. The curves are



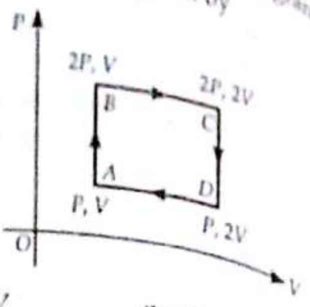
(a) isothermal for A and D while adiabatic for B and C
(b) adiabatic for A and C while isothermal for B and D
(c) isothermal for A and B while adiabatic for C and D
(d) isothermal for A and C while adiabatic for B and D. [IPUEE 03]

8. A gas at state A changes to state B through path I and II shown in figure. The changes in internal energy are ΔU_1 and ΔU_2 respectively. Then



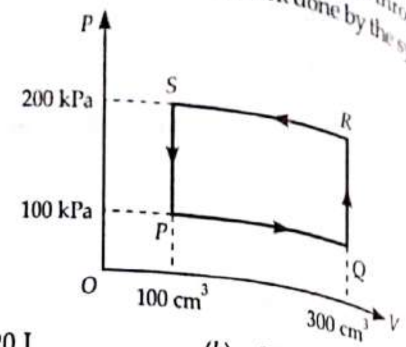
(a) $\Delta U_1 > \Delta U_2$ (b) $\Delta U_1 < \Delta U_2$
(c) $\Delta U_1 = \Delta U_2$ (d) $\Delta U_1 = \Delta U_2 = 0$ [IPUEE 05]

9. An ideal monoatomic gas is taken around the cycle ABCDA as shown in the PV diagram. The work done during the cycle is given by



(a) 1/2 PV (b) PV
(c) 2 PV (d) 4 PV

10. A thermodynamic system is taken through the cycle PQRSP process. The net work done by the system is



(a) 20 J (b) – 20 J
(c) 400 J (d) – 374 J

11. Universal gas constant is
(a) $\dfrac{C_P}{C_V}$ (b) $C_P - C_V$
(c) $C_P + C_V$ (d) $\dfrac{C_V}{C_P}$

12. 1 g of water at atmospheric pressure has volume of 1 cm^3 and when boiled it becomes 1681 cm^3 of steam. The heat of vaporisation of water is 540 cal/g. Then the change in its internal energy in this process is
(a) 540 cal (b) 500 cal
(c) 1681 cal (d) none of these. [IPUEE 99]

13. Calculate the work done if temperature is changed from 0°C to 200°C at one atmosphere ($R = 2\ \text{cal K}^{-1}$).
(a) 100 cal (b) 200 cal
(c) 400 cal (d) 800 cal. [DCE 97]

14. One mole of an ideal gas at an initial temperature of T K does 6R joules of work adiabatically. If the ratio of specific heats of this gas at

...constant pressure and at constant volume is 5/3, the final temperature of gas will be

(a) $(T+2.4)$K (b) $(T-2.4)$K

(c) $(T+4)$K (d) $(T-4)$K [DCE 06]

15. A container having 1 mole of a gas at a temperature 27°C has a movable piston which maintains a constant pressure of 1 atm in the container. The gas is compressed until temperature becomes 127°C. The work done (C_P for gas is 7.03 J/mol-K) is

(a) 703 J (b) 814 J

(c) 121 J (d) 2035 J [DCE 05]

16. A gas is suddenly compressed to 1/4th of its original volume at normal temperature. The increase in its temperature ($\gamma = 1.5$) is

(a) 273 K (b) 573 K

(c) 373 K (d) 473 K [DCE 04]

17. In an adiabatic process the pressure is increased by 2/3%. If $C_P/C_V = 3/2$, then the volume decreases by about

(a) $\frac{4}{9}$% (b) $\frac{2}{3}$% (c) 4% (d) $\frac{9}{4}$% [DCE 05]

18. The efficiency of a Carnot engine when source temperature is T_1 and sink temperature is T_2 will be

(a) $\frac{T_1-T_2}{T_1}$ (b) $\frac{T_2-T_1}{T_2}$

(c) $\frac{T_1-T_2}{T_2}$ (d) $\frac{T_1}{T_2}$ [DCE 2K]

19. A Carnot engine takes heat from a reservoir at 627°C and rejects heat to a sink at 27°C. Its efficiency will be

(a) 3/5 (b) 1/3

(c) 2/3 (d) 200/209 [IPUEE 07]

20. The temperature of reservoir of Carnot's engine operating with an efficiency of 70% is 1000 K. The temperature of source is

(a) 300 K (b) 400 K

(c) 500 K (d) 700 K [DCE 03]

21. The temperature of sink of carnot engine is 27°C. Efficiency of engine is 25%. Then

(a) 227°C (b) 327°C

(c) 127°C (d) 27°C [DCE 02]

22. A Carnot engine has efficiency 1/5. Efficiency becomes 1/3 when temperature of sink is decreased by 50 K. What is the temperature of sink ?

(a) 325 K (b) 375 K

(c) 300 K (d) 350 K [DCE 06]

23. The efficiency of Carnot engine is 0.6. It rejects 20 J of heat to sink.

The work done by the engine is

(a) 20 J (b) 30 J

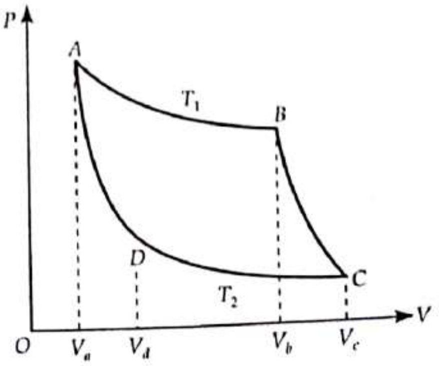
(c) 33.3 J (d) 50 J [IPUEE 05]

24. In a Carnot engine, when $T_2 = 0°C$ and $T_1 = 200°C$, its efficiency is η_1 and when $T_1 = 0°C$ and $T_2 = -200°C$, its efficiency is η_2, then what is η_1/η_2 ?

(a) 0.577 (b) 0.733

(c) 0.638 (d) cannot be calculated. [DCE 04]

25. The P-V diagram shows that two adiabatic parts of the same gas intersect two isothermals at T_1 and T_2. How the ratios (V_a/V_d) and (V_b/V_c) are related to each other ?



(a) $\left(\frac{V_a}{V_d}\right) = 2\left(\frac{V_b}{V_c}\right)$ (b) $\left(\frac{V_a}{V_d}\right) = \left(\frac{V_b}{V_c}\right)$

(c) $\left(\frac{V_a}{V_d}\right) = \left(\frac{V_b}{V_d}\right)$ (d) $\left(\frac{V_a}{V_d}\right) = \left(\frac{V_b}{V_c}\right)^2$

[DCE 08]

26. The freezer in a refrigerator is located at the top section so that

(a) the entire chamber of the refrigerator is cooled quickly due to convection

(b) the motor is not heated

(c) the heat gained from the environment is high

(d) the heat gained from the environment is low. [IPUEE 07]

Scanned by CamScanner

Answers and Explanations

1. (*a*) First law of thermodynamics corresponds to the conservation of energy.

2. (*d*) Work is not a thermodynamic state function.

3. (*c*) An intensive property is that which does not depend on the quantity of matter (or mass) of the system. Refractive index is an intensive property. Volume, mass and weight are extensive properties.

4. (*d*) For an isothermal process, $\Delta Q \neq 0$. In fact for an adiabatic process, $\Delta Q = 0$.

5. (*d*) For an adiabatic process, $p^{1-\gamma}T^{\gamma}$ = constant.

6. (*a*) Adiabatic relation between T and V is

$$TV^{\gamma-1} = \text{constant}$$

$$\therefore \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

7. (*d*) For the same values of pressure and volume, the slope of an adiabatic curve is greater than the slope of an isothermal curve. Hence curves are isothermal for A and C and adiabatic for B and D.

8. (*c*) Since the initial and final states are same for both processes I and II and the change in internal energy is path independent, therefore,

$$\Delta U_1 = \Delta U_2.$$

9. (*b*) Work done = Area of the loop $ABCDA$

$$= (2P - P)(2V - V) = \mathbf{PV}.$$

10. (*b*) Work done = – Area of the loop $PQRSP$

$$= -(200-100)\times 10^3 \times (300-100)\times 10^{-6} = \mathbf{-20\ J}.$$

Work done is –ve because the loop is traced anti-clockwise.

11. (*b*) $C_P - C_V = R$

12. (*b*)

$$Q = mL = 1\times 540 = 540 \text{ cal}$$

$$P = 1 \text{ atm} = 10^5 \text{ Nm}^{-2}$$

$$\Delta V = 1681 - 1 = 1680 \text{ cm}^3$$

$$= 1680\times 10^{-6} \text{ m}^3$$

$$W = P\Delta V = 10^5 \times 1680 \times 10^{-6}$$

$$= 168 \text{ J} = \frac{168}{4.2} \text{ cal} = 40 \text{ cal}.$$

$$\Delta U = Q - W = 540 - 40 = \mathbf{500\ cal}.$$

13. (*c*)

$$W = P\Delta V = R\Delta T$$

$$= 2 \text{ cal K}^{-1} \times 200 \text{ K}$$

$$= \mathbf{400\ cal}.$$

14. (*d*) Refer to the solution of Problem 22 on page 12.71.

15. (*a*)

$$W = nC_V\Delta T = 1\times 7.03 \text{ Jmol}^{-1}\text{K}^{-1}\times 100 \text{ K}$$

$$= \mathbf{703\ J}.$$

16. (*a*) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V/4}{V}\right)^{1.5-1}$$

$$= \left(\frac{1}{4}\right)^{0.5} = \frac{1}{2}$$

or

$$T_2 = 2T_1$$

$$\Delta T = T_2 - T_1$$

$$= 2T_1 - T_1 = T_1$$

$$= \mathbf{273\ K}.$$

17. (*a*) For an adiabatic process,

$$PV^{\gamma} = \text{constant}$$

$$\therefore \quad \frac{\Delta P}{P} + \gamma\frac{\Delta V}{V} = 0$$

or

$$\frac{\Delta V}{V}\times 100 = -\frac{1}{\gamma}\frac{\Delta P}{P}\times 100$$

$$= -\frac{2}{3}\times\frac{2}{3} = -\frac{4}{9}\%.$$

The negative sign indicates that the volume decreases when pressure increases.

18. (*a*) Efficiency of a Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$$

19. (*c*)

$$\eta = 1 - \frac{(273+27)}{(273+627)}$$

$$= 1 - \frac{1}{3} = \frac{2}{3}.$$

20. (*a*)

$$0.70 = 1 - \frac{T_2}{1000}$$

$$T_2 = 0.30\times 1000$$

$$= \mathbf{300\ K}.$$

21. (*c*)

$$\frac{25}{100} = 1 - \frac{300}{T_1}$$

or

$$\frac{300}{T_1} = 1 - \frac{1}{4} = \frac{3}{4}.$$

$$T_1 = \frac{300\times 4}{3} = 400 \text{ K}$$

$$= \mathbf{127°C}$$

Scanned by CamScanner

22. (b) $\eta = 1 - \frac{T_2}{T_1} = \frac{1}{5}$ or $\frac{T_2}{T_1} = \frac{4}{5}$

$$\eta' = 1 - \frac{T_2 - 50}{T_1} = \frac{1}{3}$$

or $1 - \frac{T_2}{T_1} + \frac{50}{T_1} = \frac{1}{3}$

$$\frac{50}{T_1} = \frac{1}{3} - 1 + \frac{4}{5} = \frac{2}{15}$$

or $T_1 = \frac{15 \times 50}{2} = 375$ K.

23. (b) $\eta = 1 - \frac{Q_2}{Q_1}$ or $0.6 = 1 - \frac{20}{Q_1}$

$\frac{20}{Q_1} = 0.4$ or $Q_1 = \frac{20}{0.4} = 50$ J.

or $W = Q_1 - Q_2 = 50 - 20 = 30$ J.

24. (b) $\eta_1 = 1 - \frac{273+0}{273+200} = 1 - \frac{273}{473} = \frac{200}{473}$

$$\eta_2 = 1 - \frac{273-200}{273+0} = 1 - \frac{73}{273} = \frac{200}{273}$$

$\therefore$ $\frac{\eta_1}{\eta_2} = \frac{200}{473} \times \frac{273}{200} = 0.577.$

25. (b) BC is adiabatic,

$$T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1}$$

AD is adiabatic,

$$T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1}$$

$\therefore$ $\left(\frac{V_a}{V_b}\right)^{\gamma-1} = \left(\frac{V_d}{V_c}\right)^{\gamma-1}$

or $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

26. (a) The upper cold denser air goes down while the lower warmer air goes up. The convection currents so set up quickly cool the chamber of the refrigerator.

AIIMS Entrance Exam

1. Which of the following is not a thermodynamics co-ordinate ?

(a) P (b) T

(c) V (d) R [AIIMS 01]

2. Which of the following is path dependent ?

(a) U (b) $P\,dV$

(c) P (d) V [AIIMS 01]

3. The volume of a gas expands by 0.25 m^3 at a constant pressure of 10^3 Nm^{-1}.

The work done is equal to

(a) 2.5 erg (b) 250 J

(c) 250 W (d) 250 N [AIIMS 96]

4. The latent heat of vaporisation of water is 2,240 J. If the work done in the process of vaporisation of 1 g is 168 J, then increase in internal energy is

(a) 2,408 J (b) 2,240 J

(c) 2,072 J (d) 1,904 J [AIIMS 02]

5. If the amount of heat given to a system is 35 J and the amount of work done by the system is – 15 J and the amount of work done by the system is – 15 J, then the change in the internal energy of the system is

(a) – 50 J (b) 20 J

(c) 30 J (d) 50 J [AIIMS 2K]

6. If C_P and C_V are the specific heats for a gas at constant pressure and at constant volume respectively, then the relation $C_P - C_V = R$ is exact for

(a) ideal and real gases at all pressures

(b) ideal gas at all pressures and real gas at moderate pressure

(c) ideal gas and nearly true for real gases at high pressure

(d) ideal gas and nearly true for real gases at moderate pressure.

[AIIMS 94]

7. In an adiabatic process, the quantity which remains constant is

(a) volume (b) pressure

(c) temperature (d) total heat of the system.

[AIIMS 99]

8. The gas law $\left(\frac{PV}{T} = \text{constant}\right)$ is true for

(a) isothermal change only

(b) adiabatic change only

(c) both isothermal and adiabatic changes

(d) neither isothermal nor adiabatic changes.

[AIIMS 89]

Scanned by CamScanner

9. A perfect gas is contained in a cylinder kept in vacuum. If the cylinder suddenly bursts, then the temperature of the gas

(*a*) becomes zero K (*b*) is decreased

(*c*) is increased (*d*) remains unchanged. [AIIMS 96]

10. The increase in internal energy of a system is equal to the work done on the system. Which process does the system undergo ?

(*a*) isochoric (*b*) adiabatic

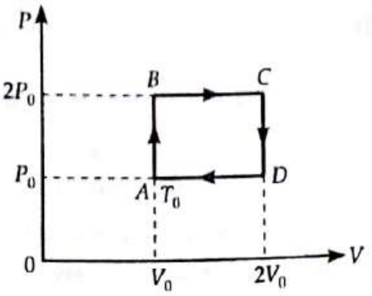
(*c*) isobaric (*d*) isothermal. [AIIMS 2K]

11. A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. The amount of heat energy supplied to the engine from source per cycle is

(*a*) 1,200 J (*b*) 1,600 J

(*c*) 2,400 J (*d*) 3,200 J [AIIMS 2K]

12. *N* moles of a monoatomic gas it carried round the reversible rectangular cycle *ABCDA* as shown in the diagram.



The temperature at *A* is T_0. The thermodynamic efficiency of the cycle is

(*a*) 15% (*b*) 20%

(*c*) 25% (*d*) 50%. [AIIMS 04]

13. Heat capacity of a substance is infinite. It means

(*a*) heat is given out

(*b*) heat is taken out

(*c*) no change in temperature, whether heat is taken in or given out

(*d*) all of these. [AIIMS 97]

14. If the temperature of the source is increased, the efficiency of a Carnot engine [AIIMS 92]

(*a*) increases (*b*) decreases

(*c*) remains constant

(*d*) first increases and then remains constant.

15. A reversible engine converts one-sixth of the heat input into work. When the temperature of the sink is reduced by 62°C, the efficiency of the engine is doubled. The temperatures of the source and sink are

(*a*) 99°C, 37°C (*b*) 80°C, 37°C

(*c*) 95°C, 37°C (*d*) 90°C, 37°C [AIIMS 99]

ASSERTIONS AND REASONS

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(*a*) If both assertion and reason are true and reason is the correct explanation of the assertion.

(*b*) If both assertion and reason are true but reason is not correct explanation of the assertion.

(*c*) If assertion is true, but reason is false.

(*d*) If both assertion and reason are false.

16. **Assertion.** The melting of solid causes an increase in the internal energy.

Reason. Latent heat is the heat required to melt a unit mass of the solid. [AIIMS ...]

17. **Assertion.** The isothermal curves intersect each other at a certain point.

Reason. The isothermal changes take place slowly, so the isothermal curves have very little slope. [AIIMS 01]

18. **Assertion.** Heat energy is completely transformed into work during the isothermal expansion of a gas.

Reason. During an isothermal process, the change in internal energy of a gas due to decrease in pressure is nullified by the change due to increase in volume. [AIIMS 98]

19. **Assertion.** The temperature of a gas does not change, when it undergoes an adiabatic expansion.

Reason. During an adiabatic process, heat energy is exchanged between a system and its surroundings. [AIIMS 98]

20. **Assertion.** Air quickly leaking out of a balloon becomes cooler.

Reason. The leaking air undergoes adiabatic expansion. [AIIMS 05]

21. **Assertion.** In adiabatic compression, the internal energy and temperature of the system get decreased.

Reason. The adiabatic compression is a slow process. [AIIMS 01]

Scanned by CamScanner

22. **Assertion.** When a bottle of cold carbonated drink is opened, a slight fog forms around the opening.

Reason. Adiabatic expansion of the gas causes lowering of temperature and condensation of water vapours. [AIIMS 03]

23. **Assertion.** Reversible systems are difficult to find in real world.

Reason. Most processes are dissipative nature. [AIIMS 05]

24. **Assertion.** Thermodynamic processes in nature are irreversible.

Reason. Dissipative effects cannot be eliminated. [AIIMS 04 , VMMC 05]

25. **Assertion.** It is not possible for a system, unaided by an external agency to transfer heat from a body at a lower temperature to another at a higher temperature.

Reason. It is not possible to violate the second law of thermodynamics. [AIIMS 94]

26. **Assertion.** The temperature of the surface of the sun is approximately 6,000 K. If we take a big lens and focus the sun rays, we can produce a temperature of 8,000 K.

Reason. This higher temperature can be produced according to second law of thermodynamics. [AIIMS 07]

27. **Assertion.** The Carnot cycle is useful in understanding the performance of heat engines.

Reason. The Carnot cycle provides a way of determining the maximum possible efficiency achievable with reservoirs of given temperatures. [AIIMS 06]

28. **Assertion.** When a glass of hot milk placed in a room is allowed to cool, its entropy decreases.

Reason. Allowing hot object to cool does not violate the second law of thermodynamics. [AIIMS 06]

29. **Assertion.** In an isolated system, the entropy increases.

Reason. The processes in an isolated system are adiabatic. [AIIMS 06]

30. **Assertion.** For an isothermal process in an ideal gas, the heat absorbed by the gas is entirely used in the work done by the gas.

Reason. During a process taking place in a system, the temperature remains constant, then the process is isothermal. [AIIMS 2009]

Answers and Explanations

1. (*d*) P, T and V are thermodynamic variables while the gas constant R is not a thermodynamics coordinate.

2. (*b*) The work done dV is path dependent.

3. (*b*) $W = P\Delta V = 10^3 \times 0.25 = \mathbf{250\ J}$.

4. (*c*) From first law of thermodynamics,

$$dQ = mL = dU + dW$$

$$dU = mL - dW = 1 \times 2240 - 168$$

$$= \mathbf{2072\ J.}$$

5. (*d*) $\Delta Q = \Delta U + \Delta W$

$$35 = \Delta U - 15$$

$$\Delta U = 35 + 15 = \mathbf{50\ J.}$$

6. (*d*) $C_P - C_V = R$

holds for ideal gas and for real gases at moderate pressure.

7. (*d*) In an adjabatic process, no heat is exchanged between system and surroundings. Hence total heat of the system remains constant.

8. (*c*) The gas law $\left(\frac{PV}{T} = \text{constant}\right)$ holds for both isothermal and adiabatic changes.

9. (*d*) No work is done during the free expansion of a gas. There is no change in internal energy or temperature of the gas.

10. (*b*) As work done on a system is negative, so

$$\Delta U = -\Delta W \quad \text{or} \quad \Delta U + \Delta W = 0 \quad \text{or} \quad \Delta Q = 0$$

Hence the system undergoes an adiabatic process.

11. (*b*) $\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = \frac{1}{2}$

$$Q_1 = 2W = 2 \times 800$$

$$= \mathbf{1600\ J.}$$

12. (*d*) At lower temperature T_0 corresponding to point A,

$$P_0V_0 = \eta R T_0 \quad \text{or} \quad T_0 = \frac{P_0V_0}{nR}$$

At higher temperature T' corresponding to point B,

$$2\,P_0V_0 = nRT'$$

Scanned by CamScanner

$\therefore \quad T' = \frac{2P_0V_0}{nR} = 2T_0$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{T_0}{2T_0} = 0.5 = 50\%.$$

13. (*c*) Whatever amount of heat is taken in or given out by a substance of infinite heat capacity, there is no change in its temperature.

14. (*a*) Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

When the temperature T_1 of the source increases, the efficiency η increases.

15. (*a*) Refer to Problem 15 on page 12.63.

$T_1 = 372 - 273 = 99°C$ and $T_2 = 310 - 273 = 37°C$

16. (*a*) Both the assertion and reason are true.

17. (*d*) Both the assertion and reason are true.

18. (*b*) The assertion is true but the reason is false. For an isothermal change,

$$\Delta U = 0, \quad \text{so, } \Delta Q = \Delta W.$$

19. (*d*) Both the assertion and reason are false. Temperature decreases during adiabatic expansion while it increases during adiabatic compression.

20. (*a*) Both the assertion and reason are true.

21. (*d*) Both the assertion and reason are false. During adiabatic compression, work is done on the system. Both the internal energy and temperature of the system increase.

22. (*a*) Both the assertion and reason are true.

23. (*a*) Both the assertion and reason are true. The energy consumed in doing work against dissipative forces cannot be recovered.

24. (*a*) Both the assertion and reason are true.

25. (*a*) Both the assertion and reason are true.

26. (*d*) Both the assertion and reason are true.

27. (*a*) Both the assertion and reason are false.

28. (*b*) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. As the milk cools, its temperature decreases. The decrease in temperature takes the milk to a lesser disorderly state. As the entropy is a measure of disorder, the entropy of the milk decreases.

29. (*b*) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. The entropy of an isolated system increases in accordance with the second law of thermodynamics.

30. (*b*) For an isothermal change, $\Delta U = 0$, so $\Delta Q = W$.

CBSE PMT Prelims and Final Exams

1. The first law of thermodynamics confirms the law of

(*a*) conservation of momentum of molecules

(*b*) conservation of energy

(*c*) flow of heat in a particular direction

(*d*) conservation of heat energy and mechanical energy. [CBSE PMT 92]

2. A gas expands under constant pressure P from volume V_1 to V_2. The work done by the gas is

(*a*) $P(V_2 - V_1)$ (*b*) $P(V_1 - V_2)$

(*c*) $P(V_1^\gamma - V_2^\gamma)$ (*d*) $P\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$ [CBSE PMT 90]

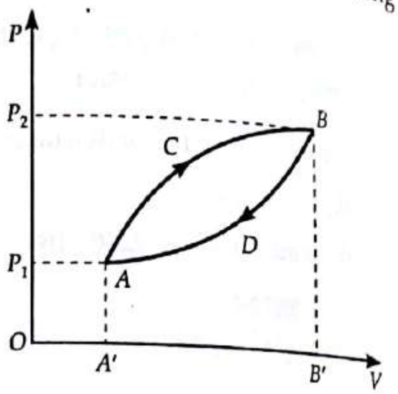
3. A sample of gas expands from volume V_1 to V_2. The amount of work done by the gas is greatest, when the expansion is

(*a*) isothermal (*b*) isobaric

(*c*) adiabatic (*d*) equal in all cases.

[AFMC 07 ; AIIMS 98 ; CBSE PMT 97]

4. A thermodynamic system is taken from state A to B along ACB and is brought back to A along BDA as shown in the given P-V diagram. The net work done during the complete cycle is given by the area



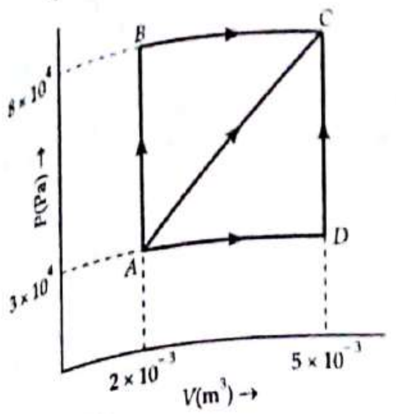
(*a*) $P_1ACBP_2P_1$ (*b*) $ACBB'A'A$

(*c*) $ACBDA$ (*d*) $ADBB'A'A$

[CBSE PMT 92]

5. A thermodynamic process is shown in the figure. The pressures and volumes corresponding to some points in the figure are

$P_A = 3\times10^4$ Pa, $V_A = 2\times10^{-3}$ m^3,
$P_B = 8\times10^4$ Pa, $V_D = 5\times10^{-3}$ m^3.



In process AB, 600 J of heat is added to the system and in process BC, 200 J of heat is added to the system. The change in internal energy of the system in process AC would be

(a) 560 J (b) 800 J
(c) 600 J (d) 640 J [CBSE PMT 92]

6. We consider a thermodynamic system. If ΔU represents the increase in its internal energy and W, the work done by the system ; which of the following statements is true ?

(a) $\Delta U = -W$ in an isothermal process
(b) $\Delta U = W$ in an isothermal process
(c) $\Delta U = -W$ in an adiabatic process
(d) $\Delta U = W$ in an adiabale process. [CBSE PMT 98]

7. The internal energy change in a system that has absorbed 2 kcal of heat and done 500 J of work is

(a) 6400 J (b) 5400 J
(c) 7900 J (d) 8900 J [CBSE PMT 09]

8. In thermodynamic processes which of the following statements is not true ?

(a) in an isochoric process pressure remains constant.
(b) in an isothermal process the temperature remains constant
(c) in an adiabatic process PV^γ = constant
(d) in an adiabatic process the system is insulated from the surroundings. [CBSE PMT 09]

9. Which of the following is not thermodynamical function ?

(a) Enthalpy (b) Work done
(c) Gibb's energy (d) Internal energy.
[CBSE PMT 93]

10. An ideal gas A and a real gas B have their volumes increased from V to $2V$ under isothermal conditions.

The increase in internal energy

(a) will be same in both A and B
(b) will be zero in both the gases
(c) of B will be more than that of A
(d) of A will be more than that of B
[CBSE PMT 93]

11. 110 J of heat is added to a gaseous system, whose internal energy increases by 40 J. Then the amount of external work done is

(a) 150 J (b) 70 J
(c) 110 J (d) 40 J
[AFMC 99 ; CBSE PMT 93]

12. In an isothermal change of an ideal gas, $\Delta U = 0$. The change in heat energy ΔQ is equal to

(a) 0.5 ΔW (b) ΔW
(c) 1.5 ΔW (d) 2 ΔW [CBSE PMT 98]

13. For hydrogen gas $C_P - C_V = a$ and for oxygen gas $C_P - C_V = b$, so the relation between a and b is given by

(a) $a = 16b$ (b) $16b = a$
(c) $a = 4b$ (d) $a = b$ [CBSE PMT 91]

14. The molar specific heat at constant pressure of an ideal gas is $7R/2$. The ratio of specific heat at constant pressure to that at constant volume is

(a) $\frac{9}{7}$ (b) $\frac{8}{7}$
(c) $\frac{7}{5}$ (d) $\frac{5}{7}$ [CBSE PMT 06]

15. For a certain gas, the ratio of specific heats is given to be $\gamma = 1.5$. For this gas

(a) $C_v = 3R/J$ (b) $C_p = 3R/J$
(c) $C_v = 5R/J$ (d) $C_p = 5R/J$.
[CBSE PMT 90]

16. If the ratio of specific heat of a gas at constant pressure to that at constant volume is γ, the change in internal energy of a mass of gas, when the volume changes from V to $2V$ at constant pressure P, is

(a) $\frac{PV}{(\gamma-1)}$ (b) PV
(c) $\frac{R}{(\gamma-1)}$ (d) $\frac{\gamma PV}{(\gamma-1)}$ [CBSE PMT 98]

Scanned by CamScanner

17. One mole of an ideal gas requires 207 J heat to raise the temperature by 10 K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10 K, the heat required is

(Given the gas constant $R = 8.3$ J/mole K)

(*a*) 198.7 J (*b*) 29 J

(*c*) 215.3 J (*d*) 124 J. [CBSE PMT 90]

18. The P-T relation for an adiabatic expansion is

(*a*) $P^{\gamma}T^{\gamma-1}$ = constant

(*b*) $P^{\gamma-1}T^{\gamma}$ = constant

(*c*) $P^{\gamma}T^{1-\gamma}$ = constant

(*d*) $P^{1-\gamma}T^{\gamma}$ = constant. [CBSE PMT 92, 96]

19. An ideal gas at 27°C is compressed adiabatically to 8/27 of its initial volume. If $\gamma = 5/3$, then the rise in temperature is

(*a*) 275 K (*b*) 375 K

(*c*) 475 K (*d*) 175 K [CBSE PMT 84, 99]

20. A diatomic gas initially at 18°C is compressed adiabatically to 1/8th of its original volume. The temperature after compression will be

(*a*) 18°C (*b*) 887.4°C

(*c*) 395.5°C (*d*) 144°C [CBSE PMT 96]

21. If in adiabatic change, the pressure P and temperature T of a monoatomic gas are related by the relation $P \propto T^c$, where c is equal to

(*a*) 5/2 (*b*) 5/3

(*c*) 2/5 (*d*) 3/5 [CBSE PMT 94]

22. One mole of an ideal gas at an initial temperature of T K does 6 R joules of work adiabatically. If the ratio of specific heats of this gas at constant pressure and at constant volume is 5/3, the final temperature of gas will be

(*a*) $(T + 2.4)$ K (*b*) $(T - 2.4)$ K

(*c*) $(T + 4)$ K (*d*) $(T - 4)$ K [CBSE PMT 04]

23. Which of the following processes is reversible ?

(*a*) transfer of heat by conduction

(*b*) transfer of heat by radiation

(*c*) isothermal compression

(*d*) electrical heating of a nichrome wire.

[CBSE PMT 05]

24. The efficiency of a Carnot engine operating with reservoir temperatures kept at 100°C and −23°C will be

(*a*) $\frac{100+23}{100}$ (*b*) $\frac{100-23}{100}$

(*c*) $\frac{373+250}{373}$ (*d*) $\frac{373-250}{373}$

25. A scientist says that the efficiency of his heat engine, which works at source temperature 127°C and sink temperature 27°C, is 26%. Then [CBSE PMT [illegible]]

(*a*) it is impossible

(*b*) it is possible but less probable

(*c*) it is quite probable

(*d*) data are incomplete.

26. An ideal gas heat engine operates in Carnot cycle between 227°C and 127°C. It absorbs 6×10^4 cal of heat at higher temperature. The amount of heat converted into work is [CBSE PMT [illegible]]

(*a*) 1.2×10^4 cal (*b*) 2.4×10^4 cal

(*c*) 4.8×10^4 cal (*d*) 6×10^4 cal

27. An ideal heat engine operates in a Carnot cycle between 227°C and 127°C. It absorbs 6 kJ of heat at the higher temperature. The amount of heat converted into work is [CBSE PMT [illegible]]

(*a*) 1.2 kJ (*b*) 1.6 kJ

(*c*) 3.5 kJ (*d*) 4.8 kJ

28. An ideal Carnot engine, whose efficiency is 40% receives heat at 500 K. If its efficiency is 50%, then the intake temperature for the same exhaust temperature is [CBSE PMT [illegible]]

(*a*) 600 K (*b*) 700 K

(*c*) 800 K (*d*) 900 K

29. A reversible engine converts one-sixth of the heat input into work. When the temperature of the sink is reduced by 62°C ; the efficiency of the engine is doubled. The temperatures of the source and sink are [CBSE PMT [illegible]]

(*a*) 80°C, 37°C (*b*) 90°C, 37°C

(*c*) 95°C, 37°C (*d*) 99°C, 37°C

[CBSE PMT 2K]

30. A engine has an efficiency of 1/6. When the temperature of sink is reduced by 62°C, its efficiency is doubled. The temperature of source is

(*a*) 124°C (*b*) 37°C

(*c*) 62°C (*d*) 99°C [CBSE PMT 07]

31. A Carnot engine, whose sink is at 300 K, has an efficiency of 40%. By how much should the temperature of source be increased, so as to increase its efficiency by 50% of original efficiency ?

(*a*) 380 K (*b*) 275 K

(*c*) 325 K (*d*) 250 K [CBSE PMT 06]

Scanned by CamScanner

32. The efficiency of Carnot engine is 50% and temperature of sink is 500 K. If temperature of source is kept constant and its efficiency is raised to 60%, then the required temperature of the sink will be

(a) 100 K (b) 400 K

(c) 500 K (d) 600 K [CBSE PMT 02]

33. During isothermal exansion, a confined ideal gas does + 150 J of work against its surroundings. This implies that

(a) 150 J of heat has been removed from the gas

(b) 300 J of heat has been added to the gas

(c) no heat is transferred because the process is isothermal

(d) 150 J of heat has been added to the gas [CBSE Pre 2011]

34. A mass of diatomic gas ($\gamma = 1.4$) at a pressure of 2 atmospheres is compressed adiabatically so that its temperature rises from 27°C to 927°C. The pressure of the gas in the final state is

(a) 256 atm (b) 8 atm

(c) 28 atm (d) 68.7 atm [CBSE Final 2011]

35. When 1 kg of ice at 0 °C melts to water at 0°C, the resulting change in its entropy, taking latent heat of ice to be 80 cal/°C, is

(a) 273 cal/K (b) 8×10^4 cal/K

(c) 80 cal/K (d) 293 cal/K [CBSE Pre 2011]

Answers and Explanations

1. (b) The first law of thermodynamics confirms the law of conservation of energy.

2. (a) Work done during expansion,

$$W = P\Delta V = P(V_2 - V_1)$$

3. (b) The amount of work done by a gas is greatest when its expansion is isobaric. For explanation refer to the solution of Problem 1 on page 12.41.

4. (c) Net work done during the complete cycle = Area of the loop $ACBDA$.

5. (a) $\Delta Q = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800$ J.

In process AB, there is no change in volume, so

$$W_{AB} = 0$$

$$W_{BC} = P_B \times (V_C - V_B) = 8 \times 10^4 (5 \times 10^{-3} - 2 \times 10^{-3}) = 240 \text{ J}.$$

$$\Delta W = W_{AB} + W_{BC} = 240 \text{ J}$$

By first law of thermodynamics,

$$\Delta U = \Delta Q - \Delta W = 800 - 240 = \mathbf{560\ J}.$$

6. (c) In an adiabatic process,

$$\Delta Q = 0$$

$$\therefore \quad \Delta U = \Delta Q - \Delta W = 0 - W = -W.$$

7. (c) As $Q = \Delta U + W$

$$\therefore \quad \Delta U = Q - W = 2 \times 4.2 \times 1000 - 500 = 8400 - 500 = \mathbf{7900\ J}.$$

8. (a) In an isochoric process volume remains constant.

9. (b) Work done is not a thermodynamical function.

10. (b) Under isothermal conditions, the internal energy remains unchanged.

11. (b) $\Delta Q = +110$ J, $\Delta U = +40$ J

$$\Delta W = \Delta Q - \Delta U = 110 - 40 = \mathbf{70\ J}.$$

12. (b) $\Delta Q = \Delta U + \Delta W = 0 + \Delta W = \Delta W$.

13. (d) For all gases,

$$C_P - C_V = R.$$

$$\therefore \quad a = b = R.$$

14. (c) $C_P = \dfrac{7R}{2}$

$$C_V = C_P - R = \frac{7R}{2} - R = \frac{5R}{2}$$

$$\therefore \quad r = \frac{C_P}{C_V} = \frac{7R/2}{5R/2} = \frac{7}{5}.$$

15. (b) Given : $r = \dfrac{C_P}{C_V} = 1.5$ or $C_P = 1.5\,C_V$

Now $C_P - C_V = \dfrac{R}{J}$

$$\therefore \quad 1.5C_V - C_V = \frac{R}{J}$$

or $$C_V = \frac{2R}{J}$$

and $$C_P = 1.5 \times \frac{2R}{J} = \frac{3R}{J}.$$

Hence the correct option is (b).

Scanned by CamScanner

16. (a) As $\frac{C_P}{C_V} = \gamma$

$\therefore \quad \frac{C_P - C_V}{C_V} = \gamma - 1$

or $\quad C_V = \frac{C_P - C_V}{\gamma - 1} = \frac{R}{\gamma - 1}$

Now $\quad \Delta U = nC_V dT = \frac{nRdT}{\gamma - 1}$

$= \frac{nPdV}{\gamma - 1} = \frac{nP(2V - V)}{\gamma - 1} = \frac{nPV}{\gamma - 1}$

As $n = 1$,

$\therefore \quad \Delta U = \frac{PV}{\gamma - 1}$.

17. (d) $C_p = \frac{207}{10} = 20.7$ J/mole K

$C_V = C_p - R = 20.7 - 8.3 = 12.4$ J/mole K.

Heat required for 10 K rise of temperature at constant volume

$= 12.4 \times 10 = \mathbf{124\ J}.$

18. (d) The P-T relation for an adiabatic expansion is

$P^{1-\gamma} T^{\gamma}$ = constant.

19. (b) $T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$

$300 \times V_1^{(5/3)-1} = T_2 \left(\frac{8V_1}{27} \right)^{(5/3)-1}$

$T_2 = 300 \times \left(\frac{27}{8} \right)^{2/3} = 300 \times \frac{9}{4} = 675$ K

Rise in temperature $= 675 - 300 = \mathbf{375\ K}.$

20. (c) $T_1 = 273 + 18 = 291$ K, $V_2 = (1/8)V_1$

Now $\quad T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$

$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1} = 291 \times (8)^{1.4-1}$

$= 291 \times (8)^{0.4} = 291 \times 2.297$

$= 668.4\ \text{K} = \mathbf{395.4°C}$

21. (a) For an adiabatic change,

$P \propto T^{\frac{\gamma}{\gamma - 1}}$

Given $\quad P \propto T^c$

$\therefore \quad c = \frac{\gamma}{\gamma - 1}$

For a momentomic gas, $\gamma = \frac{5}{3}$

$\therefore \quad c = \frac{5/3}{5/3 - 1} = \frac{5}{3} \times \frac{3}{2} = \frac{5}{2}$.

22. (d) Here $T_1 = T$, $W_{adia} = 6R$, $\gamma = \frac{5}{3}$

For one mole of a gas,

$W_{adia} = \frac{R(T_1 - T_2)}{\gamma - 1}$

$\therefore \quad 6R = \frac{R(T - T_2)}{\frac{5}{3} - 1}$

or $\quad T - T_2 = 4$

or $\quad T_2 = (T - 4)$ K.

23. Isothermal compression is a reversible process as in a Carnot's engine.

24. (d) $\eta = \frac{T_1 - T_2}{T_1} = \frac{(273 + 100) - (273 - 23)}{(273 + 100)}$

$= \frac{373 - 250}{373}$.

25. (c) Efficiency of ideal Carnot engine is maximum. It is

$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{400} = \frac{1}{4} = 25\%.$

Hence efficiency of 26% with real engine is impossible.

26. (a) $\frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{273 + 127}{273 + 227} = \frac{400}{500}$

$Q_2 = \frac{4}{5} \times 6 \times 10^4 = 4.8 \times 10^4$ cal

$W = Q_1 - Q_2 = (6 - 4.8) \times 10^4$

$= \mathbf{1.2 \times 10^4\ cal.}$

27. (a) $Q_2 = \frac{4}{5} \times 6$ kJ $= 4.8$ kJ

$W = Q_1 - Q_2 = 6 - 4.8 = 1.2$ kJ.

28. (a) $\eta = 1 - \frac{T_2}{500} = 50\% = 0.4$

$\therefore \quad T_2 = 0.6 \times 500 = 300$ K

$\eta' = 1 - \frac{300}{T_1'} = 50\% = 0.5$

$\therefore \quad T_1' = \frac{300}{0.5} = \mathbf{600\ K.}$

29. (d) $\eta = \frac{W}{Q} = 1 - \frac{T_2}{T_1}$

$\therefore \quad \frac{1}{6} = 1 - \frac{T_2}{T_1}$

or $\quad \frac{T_2}{T_1} = \frac{5}{6}$

Scanned by CamScanner

Again, $\eta' = \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} = 1 - \frac{5}{6} + \frac{62}{T_1}$

$T_1 = 62 \times 6 = 372\ \text{K} = 99°\text{C}$ and $T_2 = 37°$

30. (d) Refer to the solution of above problem.

31. (d) $\eta = 1 - \frac{T_2}{T_1}$

$0.4 = 1 - \frac{300}{T_1}$

or $T_1 = \frac{300}{0.6} = 500\ \text{K}$

Increase in temperature of the source

$= T_1' - T_1 = 750 - 500 = 250\ \text{K}.$

32. (b) $\eta = 1 - \frac{T_2}{T_1}$

$0.5 = 1 - \frac{500}{T_1}$

or

Again, $\eta' = 1 - \frac{T_2'}{T_1}$

or $0.6 = 1 - \frac{T_2'}{1000}$

$T_2' = 0.4 \times 1000 = 400\ \text{K}$

33. (d) For an isothermal process, $\Delta U = 0$

$\Delta Q = \Delta U + W = 0 + 150 = +150\ \text{J}$

$\Rightarrow$ 150 J of heat has been added to the gas.

34. (a) For an adiabatic compression,

$P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$

$P_2^{1-\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma} P_1^{1-\gamma}$

$P_2^{-0.4} = 2^{-3.2}$

$P_2^{4} = 2^{32}$

$P_2 = 2^{8} = \mathbf{256\ atm.}$

35. (d) $\Delta S = \frac{\Delta Q}{T} = \frac{80 \times 1000}{273} = \mathbf{293\ cal/K}$

Delhi PMT and VMMC Entrance Exam

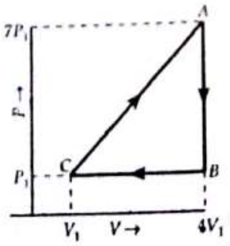
(Similar Questions)

1. First law of thermodynamics is a special case of
(a) Boyle's law
(b) Charle's law
(c) law of conservation of mass
(d) law of conservation energy [DPMT 99]

2. Which of the following is not a state function ?
(a) work done at constant pressure
(b) enthalpy
(c) work done by conservative force
(d) work done by non-conservative force. [DPMT 07]

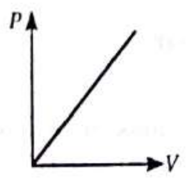
3. Air is expanded from 50 litres to 150 litres at 2 atmospheric pressure. The external work done is
($1\ \text{atmosphere} = 1 \times 10^5\ \text{N/m}^2$)
(a) 2×10^{-8} J
(b) 2×10^{4} J
(c) 200 J
(d) 2000 J [VMMC 05]

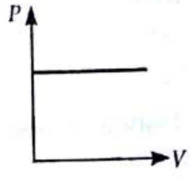
4. In the cyclic process shown in figure, the work done by the gas in one cycle is

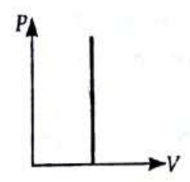


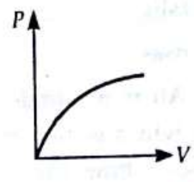
(a) $28\ P_1V_1$ (b) $14\ P_1V_1$
(c) $18\ P_1V_1$ (d) $9\ P_1V_1$ [DPMT 03]

5. Which of the following graphs between pressure and volume correctly shows isochoric changes.

(a)


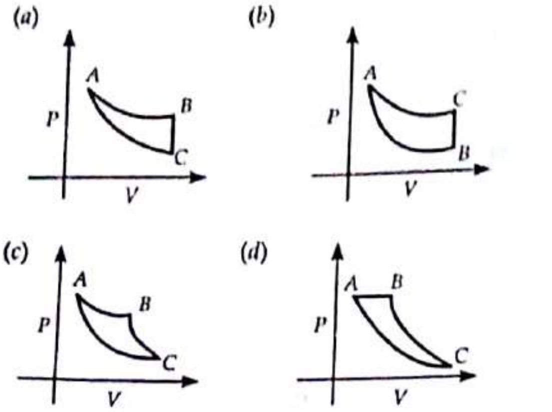
(b)


(c)


(d)


[DPMT 04]

6. AB = isothermal, BC = isochoric, AC = adiabatic. Which of the graphs correctly represents them



[DPMT 02]

7. The heat of 100 J is added to a gaseous system whose internal energy is 40 J, then the amount of external work done will be

(a) 70 J (b) 140 J

(c) 40 J (d) none of these. [DPMT 03]

8. In a thermodynamics process, pressure of a fixed mass of a gas is changed in such a manner that the gas molecules give out 30 J of heat and 10 J of work is done on the gas. If the initial internal energy of the gas was 40 J, then the final internal energy will be

(a) 30 J (b) 20 J

(c) 60 J (d) 40 J [DPMT 04]

9. If C_P and C_V are molar heats at constant pressure and constant volume respectively and R is gas constant for 1 mole, then the correct relation is

(a) $C_P - C_V = R$ (b) $C_P - C_V < R$

(c) $C_P - C_V > R$ (d) $C_P - C_V = 0$ [DPMT 07]

10. The change in internal energy, when a gas is cooled from 927° to 27°, is

(a) 100% (b) 300%

(c) 200% (d) 75% [DPMT 91]

11. During adiabatic compression of a gas, its temperature

(a) falls (b) remains constant

(c) rises (d) becomes zero [DPMT 91]

12. Air in a cylinder is suddenly compressed by a piston, which is then maintained at the same position. After some time, the

(a) pressure will increase

(b) pressure remains the same

(c) pressure will decrease

(d) pressure may increase or decrease. [DPMT 99]

13. 1 g mole of an ideal gas at STP is subjected to a reversible adiabatic expansion to double its volume. Find the change in internal energy ($\gamma = 1.4$).

(a) 1169.5 J (b) 769.5 J

(c) 1369.5 J (d) 969.5 J

[DPMT 96]

14. An ideal gas is made to go through a cyclic thermodynamical process in four steps. The amounts of heat involved are $Q_1 = 600$ J, $Q_2 = -400$ J, $Q_3 = -300$ J and $Q_4 = 200$ J respectively. The corresponding work involved are $W_1 = 300$ J, $W_2 = -200$ J, $W_3 = -150$ J and W_4. What is the value of W_4 ?

(a) – 50 J (b) 100 J

(c) 150 J (d) 50 J

[DPMT 09]

15. Two cylinders of equal size are filled with equal amount of ideal diatomic gas at room temperature. Both the cylinders are fitted with pistons. In cylinder A the piston is free to move, while in cylinder B the piston is fixed. When same amount of heat is added to both the cylinders, the temperature of the gas in cylinder A raises by 20 K. What will be the rise temperature of the gas in cylinder B?

(a) 28 K (b) 20 K

(c) 15 K (d) 10 K. [DPMT 09]

16. A system, after passing through different states returns back to its original state, is called

(a) cyclic process (b) isobaric process

(c) isothermal process (d) adiabatic process

[DPMT 92]

17. The efficiency of Carnot's engine operating between reservoirs, maintained at temperatures 27°C and –123°C is

(a) 50% (b) 24%

(c) 0.75% (d) 0.4%. [DPMT 03]

18. The temperature of sink of a Carnot engine is 27°C. If the efficiency of engine be 25%, then the temperature of source must be

(a) 27°C (b) 127°C

(c) 227°C (d) 327°C [VMMC 07]

19. In a Carnot engine the temperature of source is 1000 K and the efficiency of engine is 70%. What is the temperature of the sink ?

(a) 600 K (b) 300 K

(c) 450 K (d) 700 K [DPMT 2K]

COMPETITION SECTION

20. A Carnot eng... between 800 K to 500 K ...
(a) 1000 K
(c) 960 K
21. The door of a d... while the switch is on
(a) get heated
(b) get neither heat...
(c) get cooled
(d) both (a) and (c)
22. The inside ... refrigerator are 273 K ...

1. (d) The first la... case of the law of co...
2. (d) Work done ... a state function. The ... different paths be... system.
3. (b) $W = P \times \Delta ...$
$= 2 \times 10 ...$
$= 2 \times 10 ...$
4. (d) Work do...

The negative ... anticlockwise.
5. (c) Volume ... process.
6. (a) Adiab... isothermal curve ... line i.e., V = cons...
7. (a) $\Delta W = \Delta ...$
$= 1 ...$
8. (c) $\Delta Q ...$
30 ...
$U_f ...$
9. (a) $C_P - C ...$

20. A Carnot engine has the same efficiency between 800 K to 500 K and x K to 600 K. The value of x is

(a) 1000 K
(b) 846 K
(c) 960 K
(d) 754 K
[DPMT 92]

21. The door of a domestic refrigerator is kept open while the switch is on. Then the room will

(a) get heated
(b) get neither heated nor cooled
(c) get cooled
(d) both (a) and (c)
[DPMT 91]

22. The inside and outside temperatures of a refrigerator are 273 K and 303 K respectively. Assuming that refrigerator cycle is reversible, for every joule of work done, the heat delivered to the surrounding will be

(a) 10 J
(b) 20 J
(c) 30 J
(d) 50 J
[VMMC 06]

23. A Carnot engine has efficiency 25%. It operates between reservoirs of constant temperatures with temperature difference of 80°C. What is the temperature of the low-temperature reservoir ?

(a) –25°C
(b) 25°C
(c) –33°C
(d) 33°C
[DPMT 2011]

Answers and Explanations

1. (d) The first law of thermodynamics is a special case of the law of conservation of energy.

2. (d) Work done by a non-conservative force is not a state function. The work done will be different along different paths between two given states of the system.

3. (b) $W = P \times \Delta V$

$$= 2 \times 10^5 \times (150 - 50) \times 10^{-3}$$

$$= \mathbf{2 \times 10^4\ J.}$$

4. (d) Work done = Area $ABCA$

$$= -\frac{1}{2}(7P_1 - P_1)(4V_1 - V_1)$$

$$= -9P_1V_1$$

The negative sign shows that the loop is traced anticlockwise.

5. (c) Volume remains constant in an isochoric process.

6. (a) Adabatic curve (AC) is steeper than isothermal curve (AB). Isochoric curve AB is a vertical line *i.e.*, V = constant.

7. (a) $\Delta W = \Delta Q - \Delta U$

$$= 110 - 40 = 70\ \text{J}.$$

8. (c) $\Delta Q = (U_f - U_i) + \Delta W$

$$30 = U_f - 40 + 10$$

$$U_f = 60\ \text{J}.$$

9. (a) $C_P - C_V = R$

10. (d) $U = nC_VT$ *i.e.,* $U \propto T$

$$\frac{\Delta U}{U} \times 100 = \frac{\Delta T}{T} \times 100$$

$$= \frac{1200 - 300}{1200} \times 100 = \frac{9}{12} \times 100 = \mathbf{75\%}.$$

11. (c) The work done on the gas during adiabatic process increases its internal energy and hence its temperature rises.

12. (c) Sudden compression increases inside temperature. After some time, heat flows out. This decreases temperature of air. As V is constant, so pressure inside decreases ($P \propto T$).

13. (c) For adiabatic expansion,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} T_1 = \left(\frac{1}{2}\right)^{1.4-1} \times 273 = \frac{273}{(2)^{0.4}} = 207\ \text{K}$$

$$\Delta U = \frac{R(T_1 - T_2)}{\gamma - 1} = \frac{8.31(273 - 207)}{1.4 - 1} = \mathbf{1369.5\ J}$$

14. (c) $Q = Q_1 + Q_2 + Q_3 + Q_4$

$$= 600 - 400 - 300 + 200 = 100\ \text{J}$$

$$W = W_1 + W_2 + W_3 + W_4$$

$$= 300 - 200 - 150 + 200 = -50 + W_4$$

As $Q = \Delta U + W$

$\therefore$ $100 = 0 + (-50 + W_4)$

or $W_4 = \mathbf{150\ J}$.

Scanned by CamScanner

15. (*a*) For gas in cylinder *A*, $Q = nC_P \Delta T_1$

For gas in cylinder *B*, $Q = nC_V \Delta T_2$

$$\therefore \quad \Delta T_2 = \frac{C_P}{C_V} \Delta T_1 = \frac{7}{5} \times 20 = \mathbf{28\ K.}$$

16. (*a*) A process in which the system returns to its initial state after undergoing a series of changes is called a cyclic process.

17. (*a*)
$$\eta = 1 - \frac{T_2}{T_1}$$
$$= 1 - \frac{(273 - 123)}{(273 + 27)} = 1 - \frac{150}{300} = \frac{1}{2} = \mathbf{50\%}.$$

18. (*b*)
$$\eta = 1 - \frac{T_2}{T_1}$$

or
$$0.25 = 1 - \frac{273 + 27}{T_1}$$
$$T_1 = \frac{300}{0.75} = 400\ \text{K} = \mathbf{127°C}$$

19. (*b*)
$$0.70 = 1 - \frac{T_2}{1000}$$
$$T_2 = 0.30 \times 1000 = \mathbf{300\ K.}$$

20. (*a*)
$$\eta = \frac{T_1 - T_2}{T_1}$$
$$\therefore \quad \frac{800 - 500}{500} = \frac{x - 600}{x}$$
$$3x = 8x - 4800$$
$$x = \mathbf{960\ K.}$$

21. (*a*) Temperature of the room will rise because refrigerator takes less heat from the room and rejects more heat.

22. (*a*)
$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2} = \frac{273}{303 - 273}$$
$$Q_2 = \frac{273}{30} W = \frac{273}{30} \times 1\ \text{J} \approx 9\ \text{J}.$$
$$Q_1 = Q_2 + W = 9 + 1 = \mathbf{10\ J.}$$

23. (*c*)
$$\eta = 1 - \frac{T_2}{T_2 + 80} = \frac{25}{100}$$

or
$$\frac{80}{T_2 + 80} = \frac{1}{4}$$
$$\Rightarrow \quad T_2 = 240\ \text{K} = -33°\text{C}$$

Scanned by CamScanner

CHAPTER 13

KINETIC THEORY OF GASES

13.1 BOYLE'S LAW

1. *State and explain Boyle's law.*

Boyle's law. It in a fundamental gas law, discovered by *Robert Boyle* is 1662. *It states that the volume of a given mass of a gas is inversely proportional to its pressure, provided the temperature remains constant.* Mathematically

$$V \propto \frac{1}{P} \quad \text{or} \quad V = \frac{K}{P} \quad \text{or} \quad PV = K$$

where K is a constant. Its value depends on (*i*) mass of the gas, (*ii*) its temperature and (*iii*) the units in which P and V are measured.

If P_1 and V_1 are the initial values of pressure and volume and P_2 and V_2 are their final values, then according to Boyle's law, $P_1V_1 = P_2V_2$

Fig. 13.1 shows graph between P and V and Fig. 13.2 shows the graph between P and $1/V$ for a given mass of a gas at a constant temperature T.

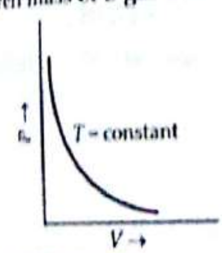


Fig. 13.1 P versus V graph

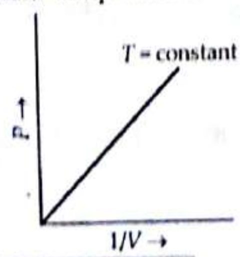


Fig. 13.2 P versus $1/V$ graph.

13.2 CHARLES' LAW

2. *State and explain Charles' law.*

Charles' law. This law gives relationship between volume and temperature of a gas at constant pressure. It was discovered by *Alexander Charles* in 1787. *It states that if the pressure remains constant, then the volume of a given mass of a gas increases or decreases by* $\frac{1}{273.15}$ *of its volume at 0°C for each 1°C rise or fall of temperature.*

Let V_0 be the volume of the given mass of a gas at 0°C. According to Charles' law, its volume at 1°C is

$$V_1 = V_0 + \frac{V_0}{273.15} = V_0\left(1 + \frac{1}{273.15}\right)$$

Volume of the gas at 2°C,

$$V_2 = V_0\left(1 + \frac{2}{273.15}\right)$$

∴ Volume of the gas at t°C,

$$V_t = V_0\left(1 + \frac{t}{273.15}\right) = V_0\left(\frac{273.15 + t}{273.15}\right)$$

If T_0 and T are temperatures on Kelvin scale corresponding to 0°C and t°C, then

$$T_0 = 273.15 + 0 = 273.15$$

and $$T = 273.15 + t$$

Scanned by CamScanner

$$\therefore \quad V_t = V_0 \frac{T}{T_0} \quad \text{or} \quad \frac{V_t}{T} = \frac{V_0}{T_0}$$

or $$\frac{V}{T} = \text{constant} \quad i.e., \; V \propto T$$

So Charles' law can be stated in another way. *Pressure remaining constant, the volume of a given mass of a gas is directly proportional to its absolute temperature.*

Fig. 13.3 shows the straight line graph between V and T for a given mass of a gas at constant pressure.

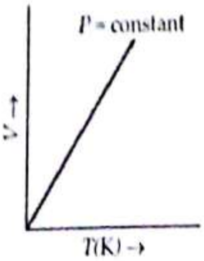


Fig. 13.3 V versus T graph

13.3 GAY LUSSAC'S LAW

3. *State and explain Gay Lussac's law.*

Gay Lussac's law. This law gives relationship between pressure and temperature of a gas. It was discovered by *Joseph Gay Lussac. It states that if the volume remains constant, the pressure of a given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its pressure at 0°C for each 1°C rise or fall of temperature.*

If P_0 and P_t are the pressures of a given mass of gas at 0°C and t°C respectively, then according to Gay Lussac's law,

$$P_t = P_0\left(1 + \frac{t}{273.15}\right) = P_0\left(\frac{273.15 + t}{273.15}\right)$$

or $$P_t = P_0 \frac{T}{T_0}$$

where $T_0\,(\text{K}) = 273.15$ and $T\,(\text{K}) = 273.15 + t$

$$\therefore \quad \frac{P_t}{P_0} = \frac{T}{T_0}$$

or $$\frac{P}{T} = \text{constant} \quad \text{or} \quad P \propto T.$$

So Gay Lussac's law can be stated in another way. *Volume remaining constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature.*

13.4 PERFECT GAS EQUATION

4. *State and derive the perfect or ideal gas equation.*

Ideal/perfect gas equation. This equation gives the relation between pressure P, volume V and absolute temperature T of a gas. The equation is

$$PV = nRT$$

where n is the number of moles of the gas and R is the universal gas constant.

Derivation. According to Boyle's law, for a given mass of a gas at constant temperature,

$$V \propto \frac{1}{P}$$

According to Charles' law, for a given mass of a gas at constant pressure,

$$V \propto T$$

Combining the above two laws,

$$V \propto \frac{T}{P} \quad \text{or} \quad V = \text{constant} \times \frac{T}{P} \quad \text{or} \quad \frac{PV}{T} = \text{constant}$$

For one mole of a gas, the constant has same value for all gases and is called *universal gas constant*, denoted by R. So the above equation becomes

$$PV = RT$$

For n moles of a gas,

$$PV = nRT$$

This is *perfect or ideal gas equation.*

If v is the volume of 1 gram mass of the gas and M_0 is the molecular mass, then the number of moles is

$$n = \frac{\text{Mass of the gas (in g)}}{\text{Molecular mass}} = \frac{1}{M_0}$$

$$\therefore \quad Pv = \frac{1}{M_0} RT \quad \text{or} \quad Pv = rT$$

This is perfect gas equation for 1 gram of the gas. Here $r = R / M_0$ is the gas constant for one gram of the gas.

We define another fundamental constant of nature, called *Boltzmann's constant* (k_B). *It is the gas constant per molecule.*

$$\therefore \quad k_B = \frac{R}{N_A} \quad \text{or} \quad R = k_B N_A$$

As number of moles,

$$n = \frac{\text{No. of molecules}}{\text{Avogadro's number}} = \frac{N}{N_A}$$

$$\therefore \quad PV = nRT = \frac{N}{N_A} \cdot k_B N_A \cdot T$$

or $$PV = k_B NT$$

13.5 UNIVERSAL GAS CONSTANT

5. *Define universal gas constant. Give its SI and CGS units.*

Universal gas constant. From ideal gas equation,

$$R = \frac{PV}{nT} = \frac{\text{pressure} \times \text{volume}}{\text{number of moles} \times \text{temperature}}$$

$$= \frac{\text{work done}}{\text{number of moles} \times \text{temperature}}$$

Scanned by CamScanner

Clearly, the *universal gas constant represents the work done by (or on) a gas per mole per kelvin.*

SI unit of $R = \frac{J}{mole \times K} = \mathbf{J\ mole^{-1}\ K^{-1}}$.

CGS unit of $R = cal\ mole^{-1}\ {}^\circ C^{-1}$.

6. Determine the numerical values of R and k_B.

Numerical value of R. Consider one mole of a gas at S.T.P. Then $R = \frac{P_0 V_0}{T_0}$

Standard pressure,

$$P_0 = 0.76 \text{ m of Hg column}$$
$$= 0.76 \times 13.6 \times 10^3 \times 9.8 \text{ Nm}^{-2}$$

Standard temperature, $T_0 = 273.15$ K

Volume of one mole of gas at S.T.P. is

$$V_0 = 22.4 \text{ litre} = 22.4 \times 10^{-3} \text{ m}^3$$

$$\therefore \quad R = \frac{0.76 \times 13.6 \times 10^3 \times 9.8 \times 22.4 \times 10^{-3}}{273.15}$$

or $\quad R = \mathbf{8.31\ J\ mole^{-1}\ K^{-1}}$.

In the CGS system,

$$R = \frac{8.31}{4.2} \text{ cal mole}^{-1}{}^\circ\text{C}^{-1} = \mathbf{1.98\ cal\ mole^{-1}{}^\circ C^{-1}}.$$

Numerical value of k_B. We know that

$$k_B = \frac{R}{N_A} = \frac{8.31 \text{ J mole}^{-1} \text{ K}^{-1}}{6.02 \times 10^{23} \text{ mole}^{-1}} = \mathbf{1.38 \times 10^{-23}\ JK^{-1}}.$$

13.6 IDEAL GASES

7. What is an ideal gas ? Why do the real gases show deviations from ideal behaviour ? Show these deviations graphically.

Ideal gas. *A gas which obeys the ideal gas equation : $PV = nRT$, at all temperatures and pressures is called an ideal gas or perfect gas.*

While deriving the ideal gas equation, the following two assumptions are used :

(*i*) The size of the gas molecules is negligibly small.

(*ii*) There is no force of attraction amongst the molecules of the gas.

However, no real or actual gas fulfills the above conditions. Hence the behaviour of a real gas differs from that of an ideal gas. *At low pressures and high temperatures,* the above assumptions are valid and some real gases like hydrogen, oxygen, nitrogen, helium, etc., almost behave like an ideal gas.

Deviations from ideal behaviour. (*i*) Fig. 13.4 shows the graph of PV/nT against pressure P for three different temperatures. For an ideal gas $PV/nT = R = 8.314$ J mol^{-1} K^{-1}. Clearly, departures from ideal gas behaviour become less at low pressures and high temperatures.

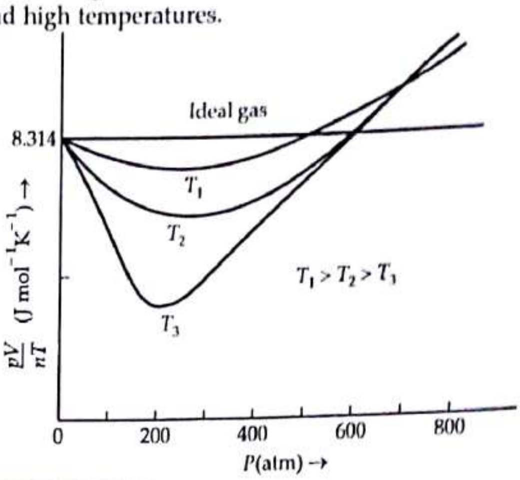


Fig. 13.4 Departures from ideal gas behaviour at three different temperatures

(*ii*) Fig. 13.5 shows the comparison between the experimental *P-V* curves and the theoretical curves predicted by Boyle's law.

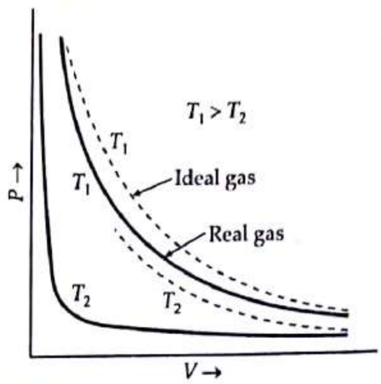


Fig. 13.5 Experimental *P-V* curves (solid lines) compared with Boyle's law (dotted lines)

(*iii*) Fig. 13.6 shows the comparison between experimental *T-V* curves and the theoretical curves predicted by Charles' law.

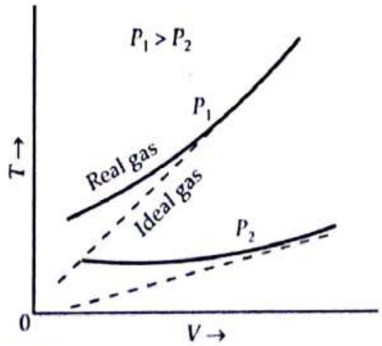


Fig. 13.6 Experimental *T-V* curves (solid lines) compared with Charles' law (dotted lines)

Scanned by CamScanner

In the above cases, we note the behaviour of a real gas approaches the ideal gas behaviour for low pressures and high temperatures.

Examples based on Gas Laws and Ideal Gas Equation

FORMULAE USED

1. *Boyle's law* : At constant temperature, PV = constant or $P_1V_1 = P_2V_2$
2. *Charles' law* : At constant pressure,
$$V \propto T \quad \text{or} \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$$
3. *Gay Lussac's law* : At constant volume,
$$P \propto T \quad \text{or} \quad \frac{P_2}{P_1} = \frac{T_2}{T_1}$$
4. Perfect gas equation is $PV = nRT$
$$\text{or} \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$
5. Boltzmann's constant, $k_B = \frac{R}{N}$

UNITS USED

Pressure P is in Nm^{-2} or Pa, Volume V in m^3 and temperature T in Kelvin (K).

CONSTANTS USED

$R = 8.31 \text{ J mole}^{-1} \text{ K}^{-1}$

$k_B = 1.38 \times 10^{-23} \text{ J molecule}^{-1} \text{ K}^{-1}$.

EXAMPLE 1. *A narrow uniform glass tube 80 cm long and open at both ends is half immersed in mercury. Then, the top of the tube is closed and it is taken out of mercury. A column of mercury 22 cm long then remains in the tube. What is the atmospheric pressure ?*

Solution. Let A cm^2 be the area of cross-section of the tube and P be the atmospheric pressure. When half of the tube is immersed in mercury [Fig. 13.7(*a*)],

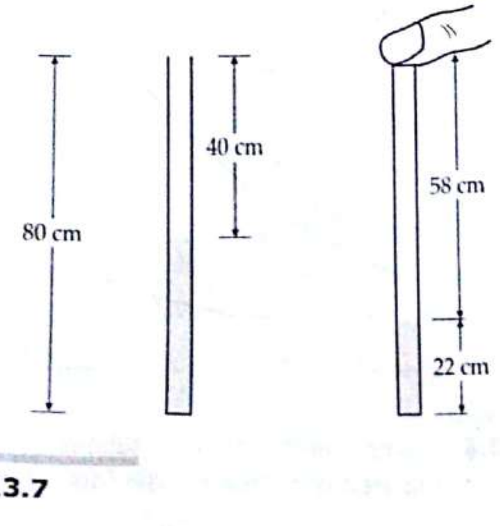


Fig. 13.7

Pressure of enclosed air, $P_1 = P$

Volume, $V_1 = A \times 40$ cm^3

When the tube is taken out of mercury,

$$P_2 = (P - 22) \text{ cm of Hg},$$
$$V_2 = A \times 58 \text{ cm}^3$$

If temperature remains constant, then from Boyle's law, we have

$$P_1 V_1 = P_2 V_2$$
$$P \times A \times 40 = (P - 22) \times A \times 58$$

or $\quad 18P = 22 \times 58$ or $P = $ **70.9 cm.**

EXAMPLE 2. *A gas at 27°C in a cylinder has a volume of 4 litre and pressure 100 Nm^{-2}. (i) Gas is first compressed at constant temperature so that the pressure is 150 Nm^{-2}. Calculate the change in volume (ii) It is then heated at constant volume so that temperature becomes 127°C. Calculate the new pressure.*

Solution. (*i*) Here $V_1 = 4$ litre, $P_1 = 100$ Nm^{-2},

$P_2 = 150$ Nm^{-2}, $V_2 = ?$

Using Boyle's law for constant temperature,

$$P_1V_1 = P_2V_2$$

$$\therefore \quad V_2 = \frac{P_1 V_1}{P_2} = \frac{100 \times 4}{150} = 2.667 \text{ litre}$$

$\therefore$ Change in Volume

$$= V_1 - V_2 = 4 - 2.667 = \mathbf{1.333 \text{ litres.}}$$

(*ii*) Here $T_1 = 27°C = 127 + 273 = 400$ K,

$P_1 = 150$ Nm^{-2}, $T_2 = 127°C = 127 + 273 = 400$ K, $P_2 = ?$

Using Gay Lussac's law for constant volume,

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

or $$P_2 = \frac{T_1}{T_2} \times P_1 = \frac{400 \times 150}{300} = 200 \text{ Nm}^{-2}.$$

EXAMPLE 3. *As an air bubble rises from the bottom of a lake to the surface, its volume is doubled. Find the depth of the lake. Take atmospheric pressure as to be 76 cm of Hg.*

Solution. On reaching the surface of lake, volume of the air bubble becomes double. By Boyle's law (assuming T = constant), its pressure becomes half. As pressure on the lake is 1 atm, so pressure inside the bubble, at the depth of the lake = 2 atm. Here 2 atm = 1 atm + Pressure due to water column of height h

$\therefore$ Pressure due to water column of height h

$$= 1 \text{ atm} = 76 \text{ cm of Hg}$$

or $$h \times 1 \times g = 76 \times 13.6 \times g$$

$$\therefore \quad h = 76 \times 13.6 = 1033.6 \text{ cm} \simeq 10.34 \text{ m}.$$

EXAMPLE 4. Using the ideal gas equation, determine the value of R. Given that one gram molecule of a gas at S.T.P. occupies 22.4 litres.

Solution. Here $P = 1 \text{ atm} = 1.013 \times 10^5$ Pa,

$T = 273$ K, $V = 22.4 \text{ litre} = 22.4 \times 10^{-3} \text{ m}^3$

For 1 mole of a gas, $PV = RT$

$$\therefore \quad R = \frac{PV}{T} = \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273}$$

$$= \mathbf{8.31 \ J \ mol^{-1} K^{-1}}.$$

EXAMPLE 5. A balloon partially filled with Helium has a volume of 30 m^3, at the earth's surface, where pressure is 76 cm of Hg and temperature is 27°C. what will be the increase in volume of gas if balloon rises to a height, where pressure is 7.6 cm of Hg and temperature is (−54°C) ? **[Chandigarh 08]**

Solution. Here $V_1 = 30 \text{ m}^3$,

$P_1 = 76 \text{ cm of Hg}, \ T_1 = 273 + 27 = 300 \text{ K}$

$P_2 = 7.6 \text{ cm of Hg}, \ T_2 = 273 - 54 = 219 \text{ K}$

As $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{76 \times 30 \times 219}{300 \times 7.6} = 219 \text{ m}^3$$

$\therefore$ Increase in volume of gas

$$= V_2 - V_1 = 219 - 30 = \mathbf{189 \ m^3}$$

EXAMPLE 6. A vessel contains two non-reacting gases : neon (monoatomic) and oxygen (diatomic). The ratio of their partial pressures is 3 : 2. Estimate the ratio of (i) number of molecules and (ii) mass density of neon and oxygen in the vessel. Atomic number of Ne = 20.2, molecular mass of O_2 = 32.0. **[NCERT]**

Solution. As V and T are same for the two gases, we can write

$$P_1 V = n_1 RT \text{ and } P_1 V = n_2 RT$$

or $$\frac{P_1}{P_2} = \frac{n_1}{n_2}$$

But $$\frac{P_1}{P_2} = \frac{3}{2} \quad \therefore \quad \frac{n_1}{n_2} = \frac{3}{2}$$

If N_1 and N_2 are the number of molecules of the two gases and N is the Avogadro's number, then

$$\frac{n_1}{n_2} = \frac{N_1/N}{N_2/N} = \frac{3}{2} \quad \therefore \quad \frac{N_1}{N_2} = \mathbf{1.5.}$$

Now $$n_1 = \frac{m_1}{M_1} \quad \text{and} \quad n_2 = \frac{m_2}{M_2}$$

$$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{m_1/V}{m_2/V} = \frac{m_1}{m_2} = \frac{n_1 M_1}{n_2 M_2}$$

$$= \frac{3}{2} \times \frac{20.2}{32} = \mathbf{0.947.}$$

EXAMPLE 7. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at 27°C temperature and 1.0×10^5 Nm^{-2} pressure. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal. ($R = 8.314$ J mole^{-1} K^{-1}). Total mass of the mixture = 28 g. **[IIT 94]**

Solution. Let mass of neon gas = m g

Then mass of argon gas = $(28 - m)$ g

Number of moles of neon, $n_1 = \dfrac{m}{20}$

Number of moles of argon, $n_2 = \dfrac{(28 - m)}{40}$

Now $(P_1 + P_2)V = (n_1 + n_2)RT$

$$\therefore 1 \times 10^5 \times 0.02 = \left(\frac{m}{20} + \frac{28 - m}{40}\right) \times 8.314 \times (27 + 273)$$

or $$2 \times 10^3 = 2494.2\left(\frac{2m + 28 - m}{40}\right) = 62.355\,(m + 28)$$

or $$m + 28 = 32.07 \text{ or } m = 4.07 \text{ g}$$

$\therefore$ Mass of neon $= 4.07$ g

Mass of argon $= 28 - 4.07 = \mathbf{23.93 \ g.}$

✕ PROBLEMS FOR PRACTICE

1. Air is filled in a bottle and it is corked at 35°C. If the cork can come out at 3 atmospheric pressure, then upto what temperature should the bottle be heated in order to remove the cork ? **(Ans. 651°C)**
2. A narrow uniform glass tube contains air enclosed by 15 cm long thread of mercury. When the tube is vertical with the open end upper most, the air column is 30 cm long. When the tube is inverted, the length of the air column becomes 45 cm. Calculate the atmospheric pressure. **(Ans. 75 cm of Hg)**
3. An open glass tube is immersed in mercury so that a length of 8 cm of the tube projects above the mercury. The tube is then closed and raised through 44 cm. What length of the tube will be occupied by the air after it has been raised ? Given 1 atm = 76 cm of Hg. **(Ans. 15.4 cm)**
4. An empty barometer tube 1 m long is lowered vertically (mouth downwards) into a tank of water. What will be the depth above the water level in the tube, when the water has risen 20 cm inside the tube ? Take 1 atm = 10.4 m column of water. **(Ans. 2.6 m)**
5. When a gas filled in a closed vessel is heated through 1°C, its pressure increases by 0.4%. What is the initial temperature of the gas ? **(Ans. 250 K)**

Scanned by CamScanner

6. Molecular weight of oxygen is 32. At S.T.P., volume of 11 g of oxygen is 700 cm^3. Find the value of gas constant R. (Ans. 8.31 J mol^{-1} K^{-1})

7. A 3000 cm^3 tank contains O_2 gas at 20°C and a gauge pressure of 2.5×10^6 Pa. Find the mass of oxygen in the tank. (Ans. 102.56 g)

8. A vessel of volume 8.0×10^{-3} m^3 contains an ideal gas at 300 K and 200 kPa. The gas is allowed to leak till the pressure falls to 125 kPa. Calculate the amount of gas leaked assuming that the temperature remains constant. (Ans. 0.24 mole)

9. A vessel of volume of 2000 cm^3 contains 0.1 mole of O_2 and 0.2 mole of CO_2. If temperature of the mixture is 300 K, find the pressure exerted by it. (Ans. 3.74×10^5 Pa)

10. A vessel of volume, $V = 5.0$ litres contains 1.4 g of nitrogen at temperature, $T = 1800$ K. Find the pressure of the gas if 30% of its molecules are dissociated into atoms at this temperature. (Ans. 1.94×10^5 Nm^{-2})

✖ HINTS

2. Let P be the atmospheric pressure. When the open end of the tube is up, [Fig. 13.8(a)],

$$P_1 = P + 15$$

When the tube inverted [Fig. 13.8(b)],

$$P_2 = P - 15$$

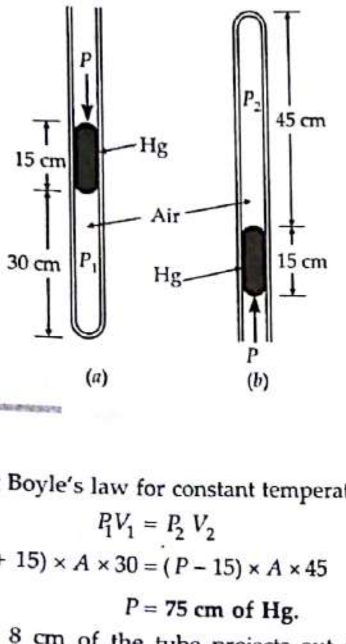


Fig. 13.8

Using Boyle's law for constant temperature,

$$P_1V_1 = P_2V_2$$

$$(P + 15) \times A \times 30 = (P - 15) \times A \times 45$$

or $P =$ **75 cm of Hg.**

3. When 8 cm of the tube projects out of mercury [Fig. 13.9(a)], for the air in the tube, we have

$$P_1 = P = 76 \text{ cm of Hg}, \; V_1 = A \times 8 \text{ cm}^3.$$

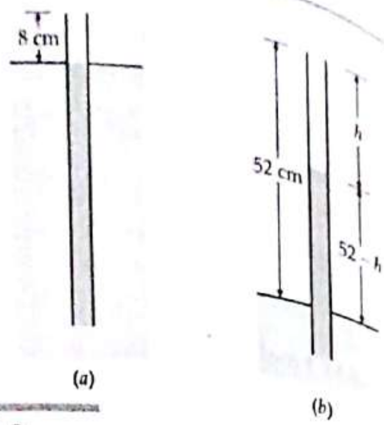


Fig. 13.9

When the tube is raised through 44 cm, total length of the tube outside the mercury becomes 52 cm. If h be the length of air column in the tube, then

$$P_2 = [P - (52 - h)] = (24 + h) \text{ cm of Hg}$$

$$V_2 = A \times h \text{ cm}^3$$

where A = area of cross-section of the tube.

Using Boyle's law for constant temperature,

$$P_1V_1 = P_2V_2$$

$$76 \times A \times 8 = (24 + h) \times A \times h$$

On solving, $h =$ **15.4 cm.**

4. Let A cm^2 be the area of cross-section of the barometer tube. Then

$$P_1 = 10.4 \text{ m of water}, \; V_1 = 100 \times A \text{ cm}^3$$

$$P_2 = ? \; V_2 = (100 - 20)A = 80 A \text{ cm}^3$$

At constant temperature,

$$P_1V_1 = P_2V_2$$

$$P_2 = \frac{P_1V_1}{V_2} = \frac{10.4 \times 100 \times A}{80 A} = 13 \text{ m of water}$$

Depth of water level $= P_2 - P_1 = 13 - 10.4 = 2.6$ m.

5. $P_1 = P, \; P_2 = P + \frac{0.4}{100}P; \; T_1 = T, \; T_2 = T + 1$

At constant volume,

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\therefore \quad \frac{P(1 + 0.004)}{P} = \frac{T + 1}{T}$$

On solving, $T =$ **250 K.**

6. $P = 1 \text{ atm} = 1.013 \times 10^5$ Pa, $T = 273$ K, $M = 32$

Volume of 1 g of oxygen = 700 cm^3

Volume of 1 mole (32 g) of oxygen

$$= 700 \times 32 \text{ cm}^3 = 700 \times 32 \times 10^{-6} \text{ m}^3$$

For 1 mole of a gas, $PV = RT$

$$\therefore \quad R = \frac{PV}{T} = \frac{1.013 \times 10^5 \times 700 \times 32 \times 10^{-6}}{273}$$

$$= \mathbf{8.31 \; mol^{-1} \; K^{-1}}.$$

Scanned by CamScanner

7. Here $P = 3000 \times 10^{-6}\ m^3$, $T = 20 + 273 = 293$ K,
$R = 8.31\ J\ mol^{-1}\ K^{-1}$.
$P = 2.5 \times 10^6$ Pa + 1 atm $= 25 \times 10^5 + 1.013 \times 10^5$
$= 26.013 \times 10^5$ Pa.

As $PV = nRT$

$$n = \frac{PV}{RT} = \frac{26.013 \times 10^5 \times 3 \times 10^{-3}}{8.31 \times 293} = 3.205$$

∴ Mass of oxygen $= 3.205 \times 32 =$ **102.5 g.**

8. $$n_1 - n_2 = \frac{P_1V}{RT} - \frac{P_2V}{RT} = \frac{(P_1 - P_2)V}{RT}$$

$$= \frac{(200 - 125) \times 10^3 \times 80 \times 10^{-3}}{8.31 \times 300} = \mathbf{0.24\ mole.}$$

9. $$P = P_1 + P_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = \frac{(n_1 + n_2) RT}{V}$$

$$= \frac{(0.1 + 0.2) \times 8.31 \times 300}{2000 \times 10^{-6}} = \mathbf{3.74 \times 10^5\ Pa.}$$

10. Mass of molecular nitrogen $= 1.4 \times \frac{70}{100} = 0.98$ g

Mass of atomic nitrogen $= 1.4 \times \frac{30}{100} = 0.42$ g

No. of moles of molecular nitrogen,

$$n_1 = \frac{0.98}{28} = 0.035$$

No. of moles of atomic nitrogen,

$$n_2 = \frac{0.02}{14} = 0.03$$

$$P = P_1 + P_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V}$$

$$= \frac{(n_1 + n_2) RT}{V}$$

$$= \frac{(0.035 + 0.03) \times 8.31 \times 1800}{5 \times 10^{-3}}$$

$$= \mathbf{1.94 \times 10^5\ Nm^{-2}.}$$

13.7 KINETIC THEORY OF AN IDEAL GAS

8. *State the assumptions on which kinetic theory of gas is based.*

Kinetic theory of gases. All matter is made of molecules. The molecules of a gas are in state of rapid and continuous motion. Their velocity depends on temperature. Using this molecular motion, various properties of a gas like pressure, temperature, energy, etc. can be explained. Hence this theory is called *kinetic theory of gases* which was developed by *Claussius* and *Maxwell*.

Assumptions. 1. *All gases consist of molecules. The molecules are rigid, elastic spheres identical in all respects for a given gas and different for different gases.*

2. *The size of a molecule is negligible compared with the average distance between the molecules.*

3. *The molecules are in a state of continuous random motion*, moving in all directions with all possible velocities.

4. *During the random motion, the molecules collide with one another and with the walls of the vessel.* During collision, their velocities are changed in magnitude and direction.

5. *The collisions are perfectly elastic and there are no forces of attraction or repulsion between the molecules.* Thus all internal energy of the gas is kinetic.

6. *Between two collisions a molecule moves in a straight path with a uniform velocity.* The average distance covered by a molecule between two successive collisions is called *mean free path*.

7. *The collisions are almost instantaneous i.e.,* the time during which a collision lasts is negligible compared to the time of the free path between the molecules.

8. Inspite of the molecular collisions, *the density remains uniform throughout the gas.*

13.8 HOW DOES A GAS EXERT PRESSURE ?

9. *On the basis of kinetic theory of gases, explain how does a gas exert pressure.*

Pressure exerted by a gas. According to kinetic theory, the molecules of a gas are in a state of continuous random motion. They collide with one another and also with the walls of the vessel. Whenever a molecule collides with the wall, it returns with a changed momentum and an equal momentum is transferred to the wall (conservation of momentum). According to Newton's second law of motion, the rate of transfer of momentum to the wall is equal to the force exerted on the wall. Since a large number of molecules collide with the wall, a steady force is exerted on the wall. The force exerted per unit area of the wall is the pressure of the gas. Hence *a gas exerts pressure due to the continuous collisions of its molecules with the walls of the vessel.*

13.9 EXPRESSION FOR PRESSURE EXERTED BY A GAS

10. *On the basis of kinetic theory, derive an expression for the pressure exerted by an ideal gas.*

Expression for pressure exerted by a gas. Consider an ideal gas enclosed in a cubical vessel. Suppose the sides of the cube are parallel to the co-ordinate axes, as shown in Fig. 13.10. Let n be the number of gas molecules per unit volume and m be the mass of each molecule. A molecule moving with velocity (v_x, v_y, v_z) hits the planar wall (perpendicular to x-axis) of area A.

Scanned by CamScanner

As the collision is elastic, the molecule rebounds with the same velocity. The y-and z-components of velocity do not change while the x-component reverses sign. So the velocity after the collision is $(-v_x, v_y, v_z)$.

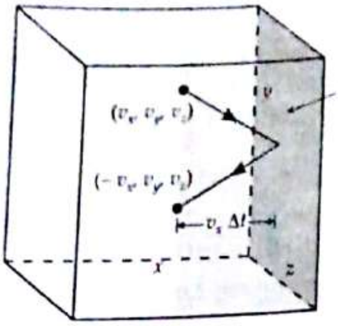


Fig. 13.10 Elastic collision of a gas molecule with the wall of the vessel.

The change in momentum of the molecule

$$= -mv_x - mv_x = -2\,mv_x.$$

By the conservation of momentum, the momentum imparted to the wall in each collision $= 2\,mv_x$.

In small time interval Δt, all those molecules which lie within distance $v_x\,\Delta t$ from the wall of area A will hit this wall. That is, the molecules which lie in the volume $Av_x\,\Delta t$ only will hit the wall in time Δt. On the average, half of such molecules are moving towards the wall and other half away from the wall.

$\therefore$ Number of molecules hitting wall of area A in time Δt

$$= \frac{1}{2} Av_x\,\Delta t \times \text{number of molecules per unit volume}$$

$$= \frac{1}{2} Av_x\,\Delta t\, n$$

Total momentum transferred to the wall in time Δt is

$$\Delta p = 2\,mv_x \times \frac{1}{2} Av_x\,\Delta t\, n = nmv_x^2\,A\,\Delta t$$

Force exerted on the wall of area A

$$= \frac{\Delta p}{\Delta t} = nmv_x^2\,A$$

$$\text{Pressure on the wall} = \frac{\text{Force}}{\text{Area}} = \frac{nmv_x^2\,A}{A}$$

or $\quad P = nmv_x^2$

As the molecules move with different velocities, so we replace v_x^2 by its average value $\overline{v_x^2}$ in the above equation.

$$\therefore \quad P = nm\,\overline{v_x^2}$$

Again, the gas is isotropic. So the molecular velocities are almost equally distributed in different directions. By symmetry,

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}) = \frac{1}{3}\overline{v^2}$$

where $\overline{v^2}$ is the mean square velocity of the molecules.

Hence $\quad P = \frac{1}{3} nm\,\overline{v^2}$

$$\text{Density of gas, } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{nm}{1} = nm \quad (1)$$

$$\therefore \quad P = \frac{1}{3}\rho\,\overline{v^2} \quad (2)$$

According to Pascal's law, a gas transmits pressure equally in all directions. So equations (1) and (2) give the pressure exerted by the gas in any direction.

11. *Show that the pressure exerted by a gas is two-thirds of the average kinetic energy per unit volume of the gas molecules.*

Relation between pressure and K.E. per unit volume. According to the kinetic theory of gases, the pressure exerted by a gas of density ρ and r.m.s. velocity v is given by

$$P = \frac{1}{3}\rho\, v^2$$

Mass per unit volume of the gas = Volume × density

$$= 1 \times \rho = \rho$$

Average K.E. of translation per unit volume of the gas,

$$E' = \frac{1}{2}\rho\, v^2 \qquad \therefore \quad \frac{P}{E'} = \frac{\frac{1}{3}\rho\, v^2}{\frac{1}{2}\rho\, v^2} = \frac{2}{3}$$

or $\quad P = \frac{2}{3} E' = \frac{2}{3} \times$ Average K.E. per unit volume.

Hence *the pressure exerted by a gas is equal to two-thirds of average kinetic energy of translation per unit volume of the gas.*

13.10 KINETIC INTERPRETATION OF TEMPERATURE

12. *Show that the average K.E. of a gas molecule is directly proportional to the temperature of the gas. Hence give the kinetic interpretation of temperature.*

Average K.E. per molecule of a gas : Kinetic interpretation of temperature. Consider one mole of a gas. Let P, V, T and M be the pressure, volume, temperature and molecular mass of the gas respectively.

$$\text{Density, } \rho = \frac{M}{V}$$

According to kinetic theory, the pressure exerted by the gas is

$$P = \frac{1}{3}\rho\,\overline{v^2} = \frac{1}{3}\frac{M}{V}\overline{v^2} \quad \text{or} \quad PV = \frac{1}{3}M\overline{v^2} = \frac{2}{3}\cdot\frac{1}{2}M\overline{v^2}$$

But $\frac{1}{2}M\overline{v^2}$ is the average kinetic energy E of one mole of the gas.

$$\therefore \quad PV = \frac{2}{3}E$$

The ideal gas equation for one mole of a gas is

$$PV = RT$$

Scanned by CamScanner

$$\frac{2}{3}E = RT \quad \text{or} \quad E = \frac{3}{2}RT$$

The above equation gives the *mean kinetic energy of one mole of the gas*. If N is the Avogadro's number, then the mean kinetic energy per molecule is given by

$$\bar{E} = \frac{E}{N} = \frac{3}{2}\frac{R}{N}T \quad \text{or} \quad \bar{E} = \frac{3}{2}k_B T$$

where $k_B = R/N$ is the gas constant per molecule and is called *Boltzmann's constant*. Thus *the mean kinetic energy per molecule is proportional to the absolute temperature of the gas. It is independent of the pressure, volume and the nature of the ideal gas*. Clearly,

$$E = \frac{1}{2}M\overline{v^2} = \frac{3}{2}RT \quad \text{or} \quad \overline{v^2} = \frac{3RT}{M}$$

The square root of $\overline{v^2}$ is known as root mean square velocity and is given by

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{M}} \quad i.e., \quad v_{rms} \propto \sqrt{T}.$$

Thus faster the motion of the molecules of a gas, higher will be their kinetic energy and hence higher will be the temperature of the gas. Hence the temperature of a gas is the measure of the average kinetic energy of its molecules. This is what we mean by the kinetic interpretation of temperature.

Also, at $T = 0$, $v_{rms} = 0$.

So we can define **absolute zero** *as that temperature at which all molecular motion stops.*

Examples based on Kinetic Theory of Gases & Kinetic Interpretation of Temperature

FORMULAE USED

1. Pressure exerted by a gas, $P = \frac{1}{3}\frac{M}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$
2. $v_{rms} = \sqrt{\frac{3P}{\rho}}$
3. Mean K.E. per molecule of a gas,
$$\bar{E} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T$$
4. Mean K.E. per mole of a gas,
$$E = \frac{1}{2}M v_{rms}^2 = \frac{3}{2}RT = \frac{3}{2}k_B NT$$
3. K.E. of 1 g of a gas $= \frac{1}{2}v_{rms}^2 = \frac{3}{2}\frac{R}{M}T$
5. Avogadro's number $= \frac{\text{Molecular mass}}{\text{Mass of 1 molecule}}$

   or $N = \frac{M}{m}$
6. No. of moles, $n = \frac{\text{Mass of gas}}{\text{Molecular mass}}$.

UNITS USED

Kinetic energies $\bar{E}$ and E are in joule, pressure P in Nm^{-2} and density ρ in $kg\ m^{-3}$.

EXAMPLE 8. *Calculate the r.m.s. velocity of air molecules at S.T.P. Given density of air at S.T.P. is 1.296 $kg\ m^{-3}$.*

Solution. Here $P = 1$ atm $= 1.013 \times 10^5\ Nm^{-2}$,

$$\rho = 1.296\ kgm^{-3}$$

∴ Root mean square velocity of air molecules at S.T.P.,

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.013 \times 10^5}{1.296}} = \mathbf{482.24\ ms^{-1}}.$$

EXAMPLE 9. *A vessel is filled with a gas at a pressure of 76 cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.*

Solution. According to kinetic theory of gases,

$$PV = \frac{1}{3}Mv_{rms}^2$$

At constant temperature, v_{rms}^2 is constant. As V is also constant, so $P \propto M$

When the mass of the gas increases by 50%, pressure also increases by 50%.

∴ Final pressure $= 76 + \frac{50}{100} \times 76 =$ **114 cm of Hg.**

EXAMPLE 10. *Calculate the kinetic energy of one mole of argon at 127°C. Given, Boltzmann's constant, $k_B = 1.381 \times 10^{-23}$ J molecule^{-1} K^{-1}. Avogadro's number, $N = 6.02 \times 10^{23}$ mol^{-1}.*

Solution. Here $T = 127 + 273 = 400$ K,

$$N = 6.02 \times 10^{23}\ \text{mol}^{-1},$$
$$k_B = 1.381 \times 10^{-23}\ \text{J mol}^{-1}\ \text{K}^{-1}$$

Kinetic energy of one mole of gas,

$$E = \frac{3}{2}RT = \frac{3}{2}k_B NT$$
$$= \frac{3}{2} \times 1.381 \times 10^{-23} \times 6.02 \times 10^{23} \times 400$$
$$= \mathbf{4988.2\ J.}$$

EXAMPLE 11. *Calculate the kinetic energy per molecule and also r.m.s. velocity of a gas at 127°C. Given $k_B = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1} and mass per molecule of the gas $= 6.4 \times 10^{-27}$ kg.*

Solution. Here $T = 127 + 273 = 400$ K,

$$m = 6.4 \times 10^{-27}\ \text{kg}$$

(*i*) K.E. per molecule

$$= \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_B T$$
$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 400$$
$$= \mathbf{8.28 \times 10^{-21}\ J.}$$

Scanned by CamScanner

(ii) Now $\frac{1}{2} mv_{rms}^2 = 8.28 \times 10^{-21}$ J

$$\therefore \quad v_{rms} = \sqrt{\frac{2 \times 8.28 \times 10^{-21}}{m}}$$

$$= \sqrt{\frac{2 \times 8.28 \times 10^{-21}}{6.4 \times 10^{-27}}} = \mathbf{1.608 \times 10^3\ ms^{-1}}.$$

EXAMPLE 12. *Calculate the number of molecules in $2 \times 10^{-6} m^3$ of a perfect gas at 27°C and at a pressure of 0.01 m of mercury. Mean kinetic-energy of a molecule at 27°C $= 4 \times 10^{-11}$ J and $g = 9.8\ ms^{-2}$.* **[Chandigarh 07]**

Solution. Here $V = 2 \times 10^{-6} m^3$

$$P = h\rho g = 0.01 \times 13.6 \times 10^3 \times 9.8 = 13.6 \times 98\ Nm^{-2}$$

Total K.E. of the gas molecules,

$$\frac{1}{2} Mv_{rms}^2 = \frac{3}{2} PV = \frac{3}{2} \times 13.6 \times 98 \times 2 \times 10^{-6}\ J$$

∴ No. of molecules in the given volume

$$= \frac{\text{Total K.E. of the gas molecules}}{\text{K.E. per molecule}}$$

$$= \frac{3 \times 13.6 \times 98 \times 10^{-6}}{4 \times 10^{-11}} = 9996 \times 10^4 \simeq \mathbf{10^8}.$$

EXAMPLE 13. *(a) Calculate (i) the root mean square speed and (ii) the mean kinetic energy of one gram molecule of hydrogen at S.T.P. Given that the density of hydrogen at S.T.P. is 0.09 kg m^{-3} and $R = 8.31$ J $mole^{-1} K^{-1}$.*

(b) Given that the mass of a molecule of hydrogen is 3.34×10^{-27} kg, calculate Avogadro's number.

(c) Calculate Boltzmann's constant.

Solution. (a) (i) Here $\rho = 0.09\ kgm^{-3}$,

$$P = 1\ atm = 1.013 \times 10^5\ Nm^{-2}$$

As $\quad P = \frac{1}{3} \rho v_{rms}^2$

$$\therefore \quad v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.013 \times 10^5}{0.09}} = \mathbf{1837.5\ ms^{-1}}.$$

(ii) At S.T.P., volume of one gram molecule of
rogen = 22.4 litre

$$= 22.4 \times 10^{-3}\ m^3$$

lass of one gram molecule of hydrogen,

$$M = \text{Volume} \times \text{density} = 22.4 \times 10^{-3}\ m^3$$

ass of one gram molecule of hydrogen,

$$M = \text{Volume} \times \text{density}$$

$$= 22.4 \times 10^{-3} \times 0.09 = 2.016 \times 10^{-3}\ kg$$

n K.E. of one gram molecule of hydrogen at S.T.P.

$$= \frac{1}{2} Mv_{rms}^2 = \frac{1}{2} \times 2.016 \times 10^{-3} \times (1837.5)^2$$

$$= \mathbf{3.4 \times 10^3\ J}.$$

(b) Avogadro's number,

$$N = \frac{\text{Molecular mass of hydrogen}}{\text{Mass of 1 molecule of hydrogen}}$$

$$= \frac{M}{m} = \frac{2.016 \times 10^{-3}}{3.34 \times 10^{-27}} = \mathbf{6.03 \times 10^{23}}.$$

(c) Boltzmann's constant,

$$k_B = \frac{R}{N} = \frac{8.31}{6.03 \times 10^{23}} = \mathbf{1.38 \times 10^{-23}\ JK^{-1}}.$$

EXAMPLE 14. *At what temperature will the average velocity of oxygen molecules be sufficient so as to escape from the earth ? Escape velocity of earth is 11.0 kms^{-1} and mass of one molecule of oxygen is 5.34×10^{-26} kg. Boltzmann constant $= 1.38 \times 10^{-23}$ J $molecule^{-1} K^{-1}$.*

Solution. For the molecule to just escape from the earth,

Average K.E. of the molecule at temperature T
= Escape energy of the molecule

or $\quad \frac{3}{2} k_B T = \frac{1}{2} mv_e^2$

$$\therefore \quad T = \frac{mv_e^2}{3k_B} = \frac{5.34 \times 10^{-26} \times (11.0 \times 10^3)^2}{3 \times 1.38 \times 10^{-23}}$$

$$= \mathbf{1.56 \times 10^5\ K}.$$

EXAMPLE 15. *A vessel A contains hydrogen and another vessel B whose volume is twice of A contains same mass of oxygen at the same temperature. Compare (i) average kinetic energies of hydrogen and oxygen molecules (ii) root mean square speeds of the molecules and (iii) pressures of gases in A and B. Molecular weights of hydrogen and oxygen are 2 and 32 respectively.*

Solution. (i) For all gases at the same temperature, average K.E. per molecule is same and is

$$\bar{E} = \frac{3}{2} k_B T$$

As the gases in both vessels are at the same temperature, so the ratio of their average K.E. per molecule = **1 : 1.**

(ii) As $\quad v_{rms}^2 = \sqrt{\frac{3RT}{M}}$

$$\therefore \quad \frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{32}{2}} = \mathbf{4 : 1}.$$

(iii) According to kinetic theory, $\quad P = \frac{1}{3} \frac{M}{V} v_{rms}^2$

where M is the mass and V is the volume of the gas. Masses of both gases are equal. So the ratio of their pressures is

$$\frac{P_H}{P_O} = \left[\frac{v_H}{v_O}\right]^2 \times \frac{V_O}{V_H} = \frac{16}{1} \times \frac{2}{1} = \mathbf{32 : 1}.$$

EXAMPLE 16. *A flask c... ratio of 2 : 1 by mass. T... 27°C. Obtain the ratio of (i) average kinetic en... (ii) root mean square... two gases. Atomic... mass of chlorine...*

Solution. (i) The ... molecule of any gas ... temperature and not ... As both argon ... temperature in the fla... molecule of the two g...

(ii) If m is mass ... molecular mass, then

$\frac{1}{2} mv_{rms}^2 = \frac{3}{2}$ ...

= co...

$\therefore \quad \frac{(v_{rms}^2)_{Ar}}{(v_{rms}^2)_{Cl_2}} =$ ...

or $\quad \frac{(v_{rms})_{Ar}}{(v_{rms})_{Cl_2}} =$ ...

EXAMPLE 17. *Two p... and T_2 are mixed. ... temperature of the m... m_1 and m_2 and the n... and n_2 respectively.*

Solution. Acc... average kinetic e... Before mixing th... energy of all the r...

After mixing, ... gases

where T is the te... no energy loss, ...

$\frac{3}{2} k_B (n$ ...

∴

PROBLEM...

1. Calculate ... density 1...

Scanned by CamScanner

Example 16. *A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27°C. Obtain the ratio of*

(i) average kinetic energy per molecule, and

(ii) root mean square speed (v_{rms}) of the molecules of the two gases. Atomic mass of argon = 39.9 u ; Molecular mass of chlorine = 70.9 u **[NCERT]**

Solution. (i) The average kinetic energy per molecule of any gas is $\frac{3}{2}k_B T$. It depends only on temperature and not on the nature of the gas.

As both argon and chlorine have the same temperature in the flask, the ratio of average K.E. per molecule of the two gases is **1 : 1**.

(ii) If m is mass of single molecule and M the molecular mass, then

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$$

= constant at a given temperature

$$\therefore \quad \frac{(v_{rms}^2)_{Ar}}{(v_{rms}^2)_{Cl_2}} = \frac{(m)_{Cl_2}}{(m)_{Ar}} = \frac{M_{Cl_2}}{M_{Ar}} = \frac{70.9}{39.9} = 1.777$$

$$\text{or} \quad \frac{(v_{rms})_{Ar}}{(v_{rms})_{Cl_2}} = \sqrt{1.777} = \mathbf{1.333}.$$

Example 17. *Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if the masses of the molecules are m_1 and m_2 and the number of molecules in the gases are n_1 and n_2 respectively.* **[Roorkee 89]**

Solution. According to the kinetic theory, the average kinetic energy of a gas molecule $= \frac{3}{2}k_B T$. Before mixing the two gases, the average kinetic energy of all the molecules of the gas

$$= \frac{3}{2} k_B n_1 T_1 + \frac{3}{2} k_B n_2 T_2$$

After mixing, the mean kinetic energy of both the gases

$$= \frac{3}{2} k_B (n_1 + n_2) T$$

where T is the temperature of the mixture. If there is no energy loss, then

$$\frac{3}{2} k_B (n_1 + n_2) T = \frac{3}{2} k_B n_1 T_1 + \frac{3}{2} k_B n_2 T_2.$$

$$\therefore \quad T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}.$$

Problems for Practice

1. Calculate the root mean square velocity of a gas of density 1.5 g litre^{-1} at a pressure of 2×10^6 Nm^{-2}. **(Ans.** 2×10^3 ms^{-1}**)**
2. The r.m.s. velocity of the molecules of a gas at S.T.P. is 485.6 ms^{-1}. Calculate the density of the gas. **(Ans.** 1.289 kg m^{-3}**)**
3. Calculate the value of Boltzmann constant k_B, given $R = 8.3 \times 10^3$ J/kg-mol-K and Avogadro's number, $N = 6.03 \times 10^{26}$/kg-mol. **(Ans.** 1.376×10^{-23} J molecule^{-1} K^{-1}**)**
4. Kinetic energy of oxygen molecule at 0°C is 5.64×10^{-21} J. Calculate the value of Avogadro's number. Given $R = 8.31$ J mole^{-1} K^{-1}. **(Ans.** 6.06×10^{23}**)**
5. Calculate the total K.E. of 1 g of nitrogen at 300 K. Molecule weight of nitrogen = 28. **(Ans.** 133.4 J**)**
6. Calculate for hydrogen at 27°C (*i*) kinetic energy of one gram-molecule of the gas (*ii*) kinetic energy of one gram gas and (*iii*) root mean square velocity of the molecules. Molecular weight of hydrogen = 2. **[Ans.** (*i*) 3.74×10^3 J (*ii*) 1.87×10^3 J (*iii*) 1.93×10^3 ms^{-1}**]**
7. At what temperature the average value of the kinetic energy of the molecule of a gas will be 1/3 of the average value of kinetic energy at 27 ? **(Ans.** – 173°C**)**
8. If the temperature of air is increased from 27°C to 227°C, in what ratio will the average kinetic energies of its molecules be increased ? **(Ans.** It increases 5/3 times**)**

Hints

1. $\rho = 1.5 \text{ g litre}^{-1} = 1.5 \text{ kg m}^{-3}$, $P = 2 \times 10^6 \text{ Nm}^{-2}$

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 2 \times 10^6}{1.5}} = \mathbf{2 \times 10^3 \ ms^{-1}}.$$

2. $$\rho = \frac{3P}{v_{rms}^2} = \frac{3 \times 1.013 \times 10^5}{485.6 \times 485.6} = \mathbf{1.289 \ kgm^{-3}}.$$

6. (*i*) $\frac{3}{2} RT = \mathbf{3.74 \times 10^3 \ J}.$

(*ii*) $\frac{3}{2} \frac{RT}{M} = \mathbf{1.87 \times 10^3 \ J}.$

(*iii*) $v_{rms} = \sqrt{\frac{3RT}{M}} = \mathbf{1.93 \times 10^3 \ ms^{-1}}.$

13.11 DERIVATION OF THE GAS LAWS

13. *Derive Boyle's law on the basis of kinetic theory of gases.*

Boyle's law. *It states that the volume (V) of a given mass of a gas is inversely proportional to its pressure (P) provided the temperature (T) remains unchanged, i.e.,*

$$V \propto 1/P \quad \text{or} \quad PV = \text{constant} \qquad \text{(At constant } T\text{)}$$

Scanned by CamScanner

Derivation. According to kinetic theory, pressure exerted by a gas is

$$P = \frac{1}{3}\rho\overline{v^2} = \frac{1}{3}\frac{M}{V}\overline{v^2} \quad \therefore \quad PV = \frac{1}{3}M\overline{v^2}$$

But at constant temperature, total kinetic energy of gas $\frac{1}{2}M\overline{v^2}$ or $\overline{v^2}$ will be constant.

$\therefore$ At a constant temperature, PV = constant

This proves the Boyle's law.

14. *Derive Charles' law on the basis of kinetic theory of gases.*

Charles' law. *It states that at constant pressure, the volume of a given mass of a gas is directly proportional to its absolute temperature, i.e.,*

$$V \propto T \qquad \text{(At constant } P\text{)}$$

Derivation. According to kinetic theory of gases,

Pressure of a gas, $P = \frac{1}{3}\rho\overline{v^2} = \frac{1}{3}\frac{M}{V}\overline{v^2}$

or $$V = \frac{1}{3}\cdot\frac{M}{P}\overline{v^2}$$

For a given mass of a gas and at constant pressure P, we have

$$V \propto \overline{v^2}$$

According to kinetic theory, $\overline{v^2} \propto T$

$\therefore \quad V \propto T$

This proves the Charles' law.

15. *Derive Gay Lussac's law on the basis of kinetic theory of gases.*

Gay Lussac's law. *It states that at constant volume, the pressure exerted by a given mass of a gas is directly proportional to its absolute temperature, i.e.,*

$$P \propto T \qquad \text{(At constant } V\text{)}$$

Derivation. According to kinetic theory of gases,

$$P = \frac{1}{3}\frac{M}{V}\overline{v^2}$$

For a given mass and at constant volume V, we have

$$P \propto \overline{v^2}$$

But $\overline{v^2} \propto T \quad \therefore \quad P \propto T$

This proves Gay Lussac's law, also called Regnault's law.

16. *Derive perfect gas equation on the basis of kinetic theory of gases.*

Derivation of perfect gas equation. The pressure exerted by a gas is given by

$$P = \frac{1}{3}\frac{M}{V}\overline{v^2} \quad \therefore \quad PV = \frac{1}{3}M\overline{v^2}$$

But $\overline{v^2} \propto T$

$\therefore \quad PV \propto T \quad$ or $\quad PV = RT,$

where R is called the gas constant for one mole of gas. The above equation is called the perfect gas equation.

17. *Derive Avogadro's law on the basis of kinetic theory of gases.*

Avogadro's law. *It states that equal volumes of all gases under similar conditions of temperature and pressure contain equal number of molecules.*

Derivation. Consider equal volume (say V each) of two gases A and B at the same temperature T and pressure P.

According to kinetic theory, pressure exerted by a gas is

$$P = \frac{1}{3}\frac{M}{V}\overline{v^2} = \frac{1}{3}\frac{mn}{V}\overline{v^2}$$

As pressures exerted by two gases are equal

i.e., $$P_1 = P_2$$

$$\therefore \quad \frac{1}{3}\frac{m_1 n_1 \overline{v_1^2}}{V} = \frac{1}{3}\frac{m_2 n_2 \overline{v_2^2}}{V}$$

or $$m_1 n_1 \overline{v_1^2} = m_2 n_2 \overline{v_2^2} \qquad ...(1)$$

Again, at a given temperature the kinetic energy per molecule of each and every gas is constant and is independent of the nature of gas or the mass of gas molecule. Therefore, for the given gases, we have

$$\frac{1}{2}m_1\overline{v_1^2} = \frac{1}{2}m_2\overline{v_2^2}$$

or $$m_1\overline{v_1^2} = m_2\overline{v_2^2} \qquad ...(2)$$

Dividing (1) by (2), we get : $n_1 = n_2$

$\therefore$ Number of molecules in gas A = Number of molecules in gas B.

This proves Avogadro's law.

18. *Deduce Graham's law of diffusion from kinetic theory of gases using expression for pressure.*

Graham's law of Diffusion. *It states that rate of diffusion of a gas is inversely proportional to the square root of its density.*

Derivation. Let us consider two gases A and B diffusing into one another. Let ρ_1 and ρ_2 be their densities and v_1 and v_2 be their respective r.m.s. velocities.

Pressure exerted by gas A, $P_1 = \frac{1}{3}\rho_1 v_1^2$

and Pressure exerted by gas B, $P_2 = \frac{1}{3}\rho_2 v_2^2$

Scanned by CamScanner

When steady state of diffusion is reached,

$$P_1 = P_2$$

$$\frac{1}{3}\rho_1 v_1^2 = \frac{1}{3}\rho_2 v_2^2$$

$$\frac{v_1^2}{v_2^2} = \frac{\rho_2}{\rho_1}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}.$$

We know that diffusion is a direct consequence of molecular motion and rate of diffusion of a gas is directly proportional to its r.m.s. velocity. Thus if r_1 and r_2 be the rates of diffusion of gases A and B respectively, then

$$\frac{r_1}{r_2} = \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}.$$

19. *Derive Dalton's law of partial pressures on the basis of kinetic theory of gases.*

Dalton's law of partial pressure. *It states that the total pressure exerted by a mixture of non-reacting gases occupying a given volume is equal to the sum of the partial pressures which each gas would exert if it alone occupied the same volume at the given temperature.*

Derivation. Consider a mixture of gases occupying a volume V. Let $m_1, m_2, m_3, ...$ be the molecular masses of the gases ; $n_1, n_2, n_3, ...$ the number of their molecules ; $P_1, P_2, P_3, ...$ the pressures exerted by individual gases and $v_1, v_2, v_3, ...$ be the r.m.s. velocities of the molecules of various gases. According to kinetic theory

$$P_1 = \frac{1}{3}\frac{m_1 n_1}{V} v_1^2, \quad P_2 = \frac{1}{3}\frac{m_2 n_2}{V} v_2^2, \quad P_3 = \frac{1}{3}\frac{m_3 n_3}{V} v_3^2, ...$$

Adding, we get

$$P_1 + P_2 + P_3 + ... = \frac{1}{3}\frac{m_1 n_1}{V} v_1^2 + \frac{1}{3}\frac{m_2 n_2}{V} v_2^2 + \frac{1}{3}\frac{m_3 n_3}{V} v_3^2 + ...$$

As the temperature of all the gases in the mixture is the same, therefore

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{3} m_3 v_3^2 = ... = \frac{1}{2} mv^2 \text{ (say)}$$

or

$$m_1 v_1^2 = m_2 v_2^2 = m_3 v_3^2 = ... = mv^2$$

$$\therefore \; P_1 + P_2 + P_3 + ... = \frac{1}{3V}(n_1 + n_2 + n_3 + ...)\, mv^2$$

$$= \frac{1}{3}\frac{mn}{V} v^2$$

where $n = n_1 + n_2 + n_3 + ...$, is the total number of molecules in the mixture.

But $\frac{1}{3}\frac{mn}{V} v^2 = P$, the total pressure exerted by the mixture

$$\therefore \quad P = P_1 + P_2 + P_3 + ...$$

This proves the Dalton's law of partial pressures.

13.12 MAXWELL'S SPEED DISTRIBUTION*

20. *Discuss Maxwell's distribution of molecular speeds for a gas. Also define the most probable speed.*

Maxwell's distribution of molecular speeds. In any gas, the molecules randomly collide against each other. So the velocity of any individual gas molecule changes continuously. At any instant, the speeds of the molecules vary over a wide range. However, the velocities distribution remains fixed in a steady state. *James Clerk Maxwell* was the first to derive a mathematical relation for the most probable distribution of speeds among the molecules of a gas.

Maxwell's law of speed distribution in a gas at temperature T is

$$dN_v = 4\pi\, N a^3\, e^{-bv^2}\, v^2\, dv = n_v\, dv$$

where $a = \sqrt{\frac{m}{2\pi k_B T}}$, $b = \frac{m}{2k_B T}$

N = the total number of gas molecules

dN_v = the number of molecules having speeds between v and $v + dv$

The graph of n_v versus v is known as Maxwellian speed distribution and is shown in Fig. 13.11.

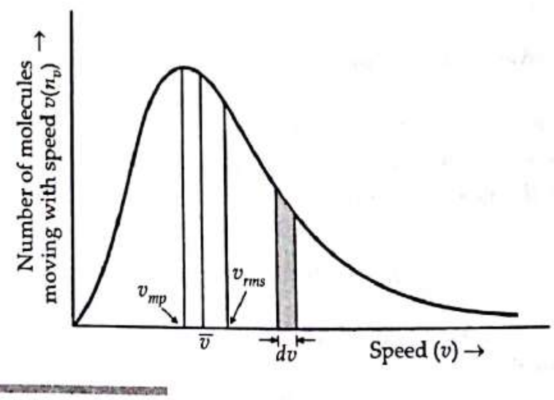


Fig. 13.11 Maxwell's distribution of molecular speeds.

The important features of the speed distribution curve are as follows :

(*i*) At any temperature, the speed of molecules varies from zero to infinity.

(*ii*) The area of the shaded region gives the number of molecules whose velocities lie in between v and $v + dv$.

Scanned by CamScanner

(iii) *The speed possessed by the largest fraction of molecules at a given temperature is called most probable speed* (v_{mp}). It corresponds to the maximum of the curve.

(iv) *The distribution is not symmetric about the most probable speed. Instead, it is skewed.* The area under the curve to the right of the maximum is greater than that to the left. This is because the lowest speed is zero whereas there is no limit to the upper speed that a molecule can have.

(v) The total area under the speed distribution curve gives the total number of molecules in the given sample of the gas.

13.13 AVERAGE, ROOT MEAN SQUARE AND MOST PROBABLE SPEEDS

22. *Define average, root mean square and most probable speeds. Express these speeds in terms of temperature of the gas.*

Average speed. *It is defined as the arithmetic mean of the speeds of the molecules of a gas at a given temperature.* If $v_1, v_2, v_3, ..., v_n$ are the speeds of the n gas molecules, then the average speed $\bar{v}$ is given by

$$\bar{v} = \frac{v_1 + v_2 + v_3 + ... + v_n}{n}$$

By using Maxwell speed distribution law, it can be shown that

$$\bar{v} = \sqrt{\frac{8\, k_B\, T}{\pi\, m}} = \sqrt{\frac{8\, RT}{\pi M}} = \sqrt{\frac{8\, PV}{\pi M}}$$

$$[k_B = R / N,\ mN = M,\ RT = PV]$$

where m is the mass of a single molecule and M is the molecular mass of the gas.

Root mean square speed. *It is defined as the square root of the mean of the squares of the speeds of the individual molecules of a gas.* If $v_1, v_2, v_3, ..., v_n$ are the speeds of the n gas molecules, then the root mean square speed for the gas is given by

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + ... + v_n^2}{n}}$$

From Maxwell's speed distribution law, it can be seen that

$$v_{rms} = \sqrt{\frac{3\, k_B\, T}{m}} = \sqrt{\frac{3\, RT}{M}} = \sqrt{\frac{3\, PV}{M}}$$

Clearly, $v_{rms} \propto \sqrt{T}$

Thus, *the root mean square speed of the gas molecules is directly proportional to the square root of the temperature of the gas.*

At a given temperature, $v_{rms} \propto \frac{1}{\sqrt{M}}$

Hence for lighter gases, the r.m.s. speed is comparatively high, as it depends inversely upon the square root of its molecular mass. The r.m.s. speed of hydrogen molecules is four times that of oxygen molecules at the same temperature.

Most probable speed. *It is defined as the speed possessed by the maximum number of molecules in a gas sample at a given temperature.*

From Maxwell's speed distribution law, it can be seen that

$$v_{mp} = \sqrt{\frac{2\, k_B\, T}{m}} = \sqrt{\frac{2\, RT}{M}} = \sqrt{\frac{2\, PV}{M}}$$

Relations between $\bar{v}$, v_{rms} and v_{mp}. As the Maxwell's speed distribution curve is not symmetric, so $\bar{v}$, v_{rms} and v_{mp} are not same. Clearly,

$$\bar{v} = \sqrt{\frac{8\, k_B\, T}{\pi\, m}} = \sqrt{\frac{8}{3\pi}}\, v_{rms} = 0.92\, v_{rms}$$

$$v_{mp} = \sqrt{\frac{2\, k_B\, T}{m}} = \sqrt{\frac{2}{3}}\, v_{rms} = 0.816\, v_{rms}$$

Also, $$v_{rms} = \sqrt{3}\sqrt{\frac{k_B\, T}{m}} = 1.73\sqrt{\frac{k_B\, T}{m}}$$

$$\bar{v} = \sqrt{\frac{8}{\pi}}\sqrt{\frac{k_B\, T}{m}} = 1.60\sqrt{\frac{k_B\, T}{m}}$$

$$v_{mp} = \sqrt{2}\sqrt{\frac{k_B\, T}{m}} = 1.41\sqrt{\frac{k_B\, T}{m}}$$

Ratio $v_{rms} : \bar{v} : v_{mp} = 1.73 : 1.60 : 1.41$

Clearly, $v_{rms} > \bar{v} > v_{mp}$.

Examples based on Average, R.M.S. & Most Probable Speeds

FORMULAE USED

1. Average speed, $\bar{v} = \frac{v_1 + v_2 + v_3 + ... + v_n}{n}$
2. $\bar{v} = \sqrt{\frac{8k_B\, T}{\pi\, m}} = \sqrt{\frac{8RT}{\pi\, M}} = \sqrt{\frac{8PV}{\pi\, M}}$
3. R.M.S. speed, $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + ... + v_n^2}{n}}$
4. $v_{rms} = \sqrt{\frac{3k_B\, T}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$
5. Most probable speed,

$$v_{mp} = \sqrt{\frac{2k_B\, T}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

UNITS USED

All speeds $\bar{v}$, v_{rms} and v_{mp} are in ms^{-1}.

Scanned by CamScanner

EXAMPLE 18. Four molecules of a gas have speeds 2, 4, 6 and 8 kms^{-1} respectively. Calculate their average speed and root mean square speed. **[Central Schools 12]**

Solution. Average speed,

$$\bar{v} = \frac{v_1 + v_2 + v_3 + v_4}{4} = \frac{2+4+6+8}{4} = \mathbf{5\ kms^{-1}}.$$

Root mean square speed,

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^4}{4}} = \sqrt{\frac{2^2 + 4^4 + 6^2 + 8^2}{4}}$$

$$= \sqrt{\frac{120}{4}} = \sqrt{30} = \mathbf{5.48\ kms^{-1}}.$$

EXAMPLE 19. If three gas molecules have velocities of 0.5, 1 and 2 kms^{-1} respectively, calculate the ratio of their root mean square speed and the average speed.

Solution. Root mean square speed,

$$v_{rms} = \sqrt{\frac{(0.5)^2 + 1^2 + 2^2}{3}} = \sqrt{\frac{5.25}{3}} = \sqrt{1.75}$$

$$= \mathbf{1.3229\ kms^{-1}}$$

Average speed,

$$\bar{v} = \frac{0.5 + 1 + 2}{3} = \frac{3.5}{3} = 1.1666\ kms^{-1}$$

Ratio, $\frac{v_{rms}}{\bar{v}} = \frac{1.3229}{1.1666} = \mathbf{1.13.}$

EXAMPLE 20. Calculate the r.m.s. velocity of oxygen molecules at S.T.P. The molecular weight of oxygen is 32. **[Himachal 05C]**

Solution. Here $P = 1\ atm = 1.013 \times 10^5\ Nm^{-2}$,

$M = 32\ g = 32 \times 10^{-3}\ kg$,

V = Molar volume at S.T.P.

$= 22.4\ litre = 22.4 \times 10^{-3}\ m^3$

Root mean square velocity of oxygen molecules at S.T.P.,

$$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3 \times 1.013 \times 10^5 \times 22.4 \times 10^{-3}}{32 \times 10^{-3}}}$$

$$= \mathbf{461.23\ ms^{-1}}.$$

EXAMPLE 21. The r.m.s. velocity of hydrogen at S.T.P. is u ms^{-1}. If the gas is heated at constant pressure till its volume is three fold, what will be its final temperature and the r.m.s. velocity?

Solution. Here $v_1 = u\ ms^{-1}$, $T_1 = 273\ K$,

$V_1 = V$ (say), $V_2 = 3\ V$

Using Charles' law for constant pressure,

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

or $$T_2 = \frac{V_2}{V_1} \times T_1 = \frac{3V}{V} \times 273 = \mathbf{819\ K.}$$

As $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \quad \therefore \quad \frac{v_2}{u} = \sqrt{\frac{819}{273}} = \sqrt{3}$$

or $v_2 = \mathbf{\sqrt{3}\ u\ ms^{-1}}$.

EXAMPLE 22. The r.m.s. speed of oxygen molecules at a certain temperature T is v. If the temperature is doubled and the oxygen gas dissociates into atomic oxygen, what is the changed r.m.s. speed?

Solution. The r.m.s. speed of molecular oxygen at temperature T is

$$v = \sqrt{\frac{3RT}{M}}$$

The r.m.s. speed of atomic oxygen at temperature $2T$ will be

$$v' = \sqrt{\frac{3R \times 2T}{M/2}} = 2\sqrt{\frac{3RT}{M}} = \mathbf{2\ v.}$$

Thus the changed r.m.s. speed is **2 v.**

EXAMPLE 23. At what temperature is the r.m.s. velocity of hydrogen molecule equal to that of an oxygen molecule at 47°C? **[AIEEE 02]**

Solution. $v_{rms} = \sqrt{\frac{3RT}{M}}$

Now r.m.s. velocity of H_2 molecule

= r.m.s velocity of O_2 molecule

or $$\sqrt{\frac{3R \times T}{2}} = \sqrt{\frac{3R \times (47 + 273)}{32}}$$

or $$T = \frac{2 \times 320}{32} = \mathbf{20\ K.}$$

EXAMPLE 24. Calculate the temperature at which r.m.s. velocity of gas molecules is double its value at 27°C, pressure of the gas remaining the same. **[Central Schools 07]**

Solution. Let t°C be the temperature at which the r.m.s. velocity (v_t) of the gas molecules is double its value at 27°C (v_{27}).

As $v \propto \sqrt{T}$ $\quad \therefore \quad \frac{v_t}{v_{27}} = \sqrt{\frac{273 + t}{273 + 27}}$

But $v_t = 2v_{27}$ or $\frac{v_t}{v_{27}} = 2$

$$\therefore \sqrt{\frac{273 + t}{300}} = 2 \quad \text{or} \quad \frac{237 + t}{300} = 4$$

or $\mathbf{t = 927°C.}$

EXAMPLE 25. Calculate the temperature at which r.m.s. velocity of a gas is half its value at 0°C, pressure remaining constant. **[Chandigarh 04]**

Solution. Let v be the r.m.s. velocity at 0°C. Let t be the temperature at which r.m.s. velocity becomes $v/2$.

Scanned by CamScanner

Then $v_1 = v, \quad T_1 = 273 + 0 = 273 \text{ K}$

$v_2 = v/2, \quad T_2 = (273 + t) \text{ K}$

As $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\therefore \quad \frac{v/2}{v} = \sqrt{\frac{273+t}{273}} \quad \text{or} \quad \frac{1}{2} = \sqrt{1 + \frac{t}{273}}$$

or $$\frac{t}{273} = \frac{1}{4} - 1 = -\frac{3}{4}$$

or $$t = -\frac{3}{4} \times 273 = -\mathbf{204.75\,°C}$$

<u>EXAMPLE 26.</u> *Uranium has two isotopes of masses 235 and 238 units. If both are present in Uranium hexafluoride gas, which would have the larger average speed ? If atomic mass of fluorine is 19 units, estimate the percentage difference in speeds at any temperature.*

How is the above concept of difference in speeds utilised in the enrichment of uranium needed for nuclear fission ? **[NCERT]**

Solution. At a fixed temperature, the average energy $= \frac{1}{2} m v_{rms}^2$ is constant. So smaller the mass of a molecule, faster will be its speed. Clearly,

$$\text{Speed of molecule} \propto \frac{1}{\sqrt{\text{Molecular mass}}}$$

Molecular mass of ^{235}U hexafluoride

$$= 235 + 6 \times 19 = 349$$

Molecular mass of ^{238}U hexafluoride

$$= 238 + 6 \times 19 = 352$$

$$\therefore \quad \frac{v_{349}}{v_{352}} = \left(\frac{352}{349}\right)^{1/2} = 1.0044$$

Percentage difference in speeds,

$$\frac{\Delta v}{v} \times 100 = 0.0044 \times 100 = \mathbf{0.44\ \%}$$

^{235}U is the isotope needed for nuclear fission. To separate it from the more abundant isotope ^{238}U, the mixture is surrounded by a porous cylinder. The porous cylinder must be thick and narrow, so that the molecule wanders through individually, colliding with the walls of the long pore, as shown in Fig. 13.12. The faster molecule will leak out more than the slower one and so there is more of the lighter molecule (enrichment) outside the porous cylinder. The method is not very efficient and has to be repeated several times for sufficient enrichment.

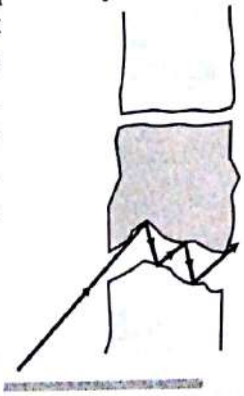


Fig. 13.12 A molecule going through a porous wall.

✗ PROBLEMS FOR PRACTICE

1. The velocities of 10 particles in ms^{-1} are ... 4, 5, 5, 6, 9. Calculate their (*i*) average speed and (*ii*) r.m.s. speed. [**Ans.** (*i*) 4.2 ms^{-1} (*ii*) ...]

2. The velocities of ten molecules of any gas are $v, 0, 2v, 4v, 3v, 2v, v, 3v, 5v, v$. Calculate the ... mean square velocity. (Ans. ...)

3. Calculate the rms velocity of the molecules of ammonia at S.T.P. Given molecular weight of ammonia = 17. (**Ans.** 632.8 ...)

4. Show that the rms velocity of O_2 is $\sqrt{2}$ times that of SO_2. Atomic weight of sulphur is 32 and that of oxygen is 16.

5. Calculate the temperature at which rms velocity of SO_2 is the same as that of oxygen at 27 °C. (**Ans.** 327 °C)

6. Estimate the temperature at which the oxygen molecules will have the same rms velocity as the hydrogen molecules at 150°C. Molecular weight of oxygen is 32 and that of hydrogen is 2. (**Ans.** 6495 ...)

7. If the root mean square velocity of the molecules of hydrogen at S.T.P. is 1.84 kms^{-1}, calculate the rms velocity of oxygen molecules at S.T.P. Molecular weights of hydrogen and oxygen are 2 and 32 respectively. (**Ans.** 0.46 kms^{-1})

8. The density of CO_2 gas at 0°C and at a pressure of $1.0 \times 10^5\ Nm^{-2}$ is $1.98\ kg\ m^{-3}$. Find the root mean square velocity of its molecules at 0°C and 30°C assuming pressure to be constant. (**Ans.** 389 ms^{-1}, 410 ms^{-1})

✗ HINTS

3. $$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3 \times 1.013 \times 10^5 \times 22.4 \times 10^{-3}}{17 \times 10^{-3}}}$$
$$= \mathbf{632.8\ ms^{-1}}.$$

4. Molecular weight of $O_2 = 32$

Molecular weight of $SO_2 = 64$

As $$v \propto \frac{1}{\sqrt{M}}$$

$$\frac{v_{O_2}}{v_{SO_2}} = \sqrt{\frac{64}{32}} = \sqrt{2} \quad \text{or} \quad v_{O_2} = \sqrt{2}\, v_{SO_2}.$$

5. For O_2, $$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3R \times 300}{32}}$$

For SO_2, $$v'_{rms} = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3RT'}{64}}$$

As $v' = v \quad \therefore$ $$\sqrt{\frac{3RT'}{64}} = \sqrt{\frac{3R \times 300}{32}}$$

or $$T' = 600 \text{ K} = 600 - 273 = \mathbf{327°C}.$$

Scanned by CamScanner

$$\frac{v_{O_2}}{v_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

$$\therefore \quad v_{O_2} = \frac{1}{4} v_{H_2} = \frac{1}{4} \times 1.84 = \mathbf{0.46\ kms^{-1}}.$$

5. Root mean square velocity at 0°C is

$$v_0 = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.0 \times 10^5}{1.98}} = \mathbf{389\ ms^{-1}}.$$

As $v \propto \sqrt{T}$

$$\therefore \quad \frac{v_{30}}{v_0} = \sqrt{\frac{273+30}{273+0}} = \sqrt{\frac{303}{273}} = 1.053$$

or $\quad v_{30} = v_0 \times 1.053 = 389 \times 1.053 = \mathbf{410\ ms^{-1}}.$

13.14 DEGREES OF FREEDOM

23. *What do you mean by degrees of freedom ? Show that the number of degrees of freedom of a system consisting of N particles and having k independent relations between them is (3N − k).*

Degrees of freedom. *The degrees of freedom of a dynamical system are defined as the total number of co-ordinates or independent quantities required to describe completely the position and configuration (arrangement of constituent atoms in space) of the system.*

The degrees of freedom of a system may also be defined as the total number of independent ways in which the particles of the system can absorb energy.

Consider a system with just one particle.

(*i*) If the particle moves along a straight line, we need just one (x-) coordinate to specify its position. So it has one translational degree of freedom.

(*ii*) If the particle moves along a plane, we need two (x-, y-) co-ordinates to specify its position. So it has two translational degrees of freedom.

(*iii*) If the particle moves in space, we need three (x-, y-, z-) coordinates to specify its position. So it has three translational degrees of freedom.

Consider now a system of two particles. Each particle has three degrees of freedom, so that the system has six degrees of freedom. If the two particles remain at fixed distance from each other, then there is one definite relation between them. As a result, the number of coordinates required to describe the configuration of the system reduces by one. So the system has (6 − 1) = 5 degrees of freedom.

In general, we can say that *the number of degrees of freedom of a system is equal to the total number of coordinates required to specify the positions of the constituent particles of the system minus the number of independent relations existing between the particles.*

If N = number of particles in the system,

k = number of independent relations between the particles,

then the number of degrees of freedom of the system is

$$f = 3N - k$$

Degrees of freedom of a rigid body. A rigid body of finite size can have both translatory and rotatory motions. Just like translatory motion, the rotatory motion can be resolved into mutually perpendicular components. Hence *a rigid body has six degrees of freedom*, three for translatory motion and three for rotatory motion.

24. *Find the degrees of freedom of monoatomic, diatomic and triatomic gas molecules.*

(*a*) **Degrees of freedom of a monoatomic gas.** The molecule of a monoatomic gas like He, Ne, Ar, etc. consists of a single atom (a point-mass). It is capable of translatory motion only. So it has three degrees of freedom.

Here $N = 1, k = 0$, so $f = 3 \times 1 - 0 = 3$.

(*b*) **Degrees of freedom of a diatomic gas.** The molecule of a diatomic gas like N_2, O_2, H_2, CO, etc. consists of two atoms *A* and *B*, a fixed distance apart. Corresponding to the translatory motion, the molecule has 3 degrees of freedom. The molecule has two additional degrees of freedom due to rotational motion, about two mutually perpendicular axes passing through its centre of mass. As the atoms are point-masses, so rotation is not possible about the line *AB*. The rotatory motion contributes 2 degrees of freedom, so that the total degrees of freedom is 5.

Here $N = 2, k = 1$, so $f = 3 \times 2 - 1 = 5$.

At high temperature (≈5000 K), a diatomic molecule has two additional degrees of freedom due to vibrational motion. Each vibrational motion has both kinetic and potential energies. So one degree of freedom of vibrational motion is taken as two.

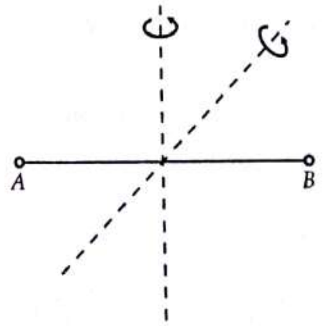


Fig. 13.13 Rotational motion of a diatomic molecule about two independent axes.

Degrees of freedom of a triatomic gas. Triatomic gas molecules are of two types :

(*a*) In a *non-linear molecule* like H_2O, SO_2, etc., the three atoms are located at the vertices of a triangle [Fig. 13.14(*a*)]. The molecule has 3 degrees of freedom due to translatory motion and 3 degrees of freedom due to rotational motion about three mutually perpendicular axes through its centre of mass. At ordinary temperature, vibrational motion may be ignored.

Here $N = 3$, $k = 3$,

so $f = 3 \times 3 - 3 = 6$.

(*b*) In a *linear molecule* such as CO_2, CS_2, HCN, etc., the three atoms are arranged along a straight line [Fig. 13.14(*b*)]. The number of independent relations between them is only 2.

$\therefore \quad f = 3N - k = 3 \times 3 - 2 = 7.$

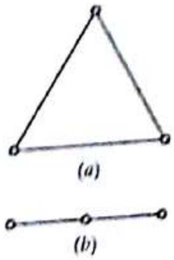


Fig. 13.14 Two types of triatomic molecules.

13.15 LAW OF EQUIPARTITION OF ENERGY

25. *State and prove the law of equipartition of energy.*

Law of equipartition of energy. *It states that in any dynamical system in thermal equilibrium, the energy is equally distributed amongst its various degrees of freedom and the energy associated with each degree of freedom per molecule is* $\frac{1}{2} k_B T$, *where* k_B *is Boltzmann's constant and* T *is the absolute temperature of the system.*

Proof. Consider one mole of a monoatomic gas in thermal equilibrium at temperature T. A monoatomic gas molecule can be taken as a point mass. So each such molecule has 3 degrees of freedom due to translatory motion. According to the kinetic theory of gases, the average translational kinetic energy of a gas molecule is given by

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} k_B T$$

where $\overline{v^2}$ is the mean square velocity of a gas molecule of mass m.

If $\overline{v_x^2}$, $\overline{v_y^2}$ and $\overline{v_z^2}$ are the components of mean square velocity of the gas molecules along the three coordinate axes, then

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

$$\therefore \quad \frac{1}{2} m\overline{v_x^2} + \frac{1}{2} m\overline{v_y^2} + \frac{1}{2} m\overline{v_z^2} = \frac{3}{2} k_B T$$

As the molecular motion is random, there is no preferred direction of motion. So the average kinetic energy of each molecule along each of the three axes is the same.

$$\therefore \quad \frac{1}{2} m\overline{v_x^2} = \frac{1}{2} m\overline{v_y^2} = \frac{1}{2} m\overline{v_z^2}$$

Combining the above two equations, we get

$$\frac{1}{2} m\overline{v_x^2} = \frac{1}{2} m\overline{v_y^2} = \frac{1}{2} m\overline{v_z^2} = \frac{1}{2} k_B T$$

Thus *the average kinetic energy per molecule per degree of freedom is* $\frac{1}{2} k_B T$. This result was first deduced by Boltzmann and is called the *law of equipartition of energy*.

For Your Knowledge

▲ The law of equipartition of energy holds good for all degrees of freedom whether translational, rotational or vibrational.

▲ Each square term in the total energy expression of a molecule contributes towards one degree of freedom.

▲ A monoatomic gas molecule has only translational kinetic energy,

$$\varepsilon_t = \frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2$$

So a monoatomic gas molecule has only three (translational) degrees of freedom.

▲ In addition to translational kinetic energy, a diatomic molecule has two rotational kinetic energies.

$$\varepsilon_t + \varepsilon_r = \frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2$$

Here the line joining the two atoms has been taken as X-axis about which there is no rotation. So the degrees of freedom of a diatomic molecule is 5, it does not vibrate.

▲ Diatomic molecule like CO has a mode of vibration even at moderate temperatures. Its atoms vibrate along the interatomic axis and contribute a vibrational energy term ε_v to the total energy.

$$\varepsilon = \varepsilon_t + \varepsilon_r + \varepsilon_v$$

$$= \frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2 + \frac{1}{2} I_y \omega_y^2 + \frac{1}{2} I_z \omega_z^2 + \frac{1}{2} m\dot{\eta}^2 + \frac{1}{2} k\eta^2$$

where k is the force constant of the oscillator, η the vibrational coordinate and $\dot{\eta} = \frac{dx}{dt}$.

So a diatomic molecule has 7 degree of freedom if it vibrates.

▲ Each translational and rotational degree of freedom corresponds to one mode of absorption of energy and has energy $\frac{1}{2} k_B T$. Each vibrational frequency has two modes of energy (kinetic and potential) with corresponding energy equal to $2 \times \frac{1}{2} k_B T = k_B T$.

13.16 SPECIFIC HEATS OF MONOATOMIC, DIATOMIC AND POLYATOMIC GASES

26. *Using the law of equipartition of energy, determine the values of C_P, C_V and γ for (i) monoatomic, (ii) diatomic and (iii) triatomic gases.*

(*i*) **Specific heats of monoatomic gas.** In case of a monoatomic gases, like He, Ar, etc., a molecule has three translational degree of freedom. According to the law of equipartition of energy, average energy associated with each degree of freedom per molecule $= \frac{1}{2} k_B T$.

$\therefore$ Average energy associated with three degrees of freedom per molecule $= \frac{3}{2} k_B T$

Let R = gas constant per mole of a gas

N_A = Avogadro's number *i.e.*, the number of atoms in one mole of the gas.

Then the total internal energy of one mole of a monoatomic gas,

$$U = \frac{3}{2} k_B T \times N_A = \frac{3}{2} RT \quad [\because k_B N_A = R]$$

The molar specific heat at constant volume will be

$$C_V \text{ (monoatomic)} = \frac{dU}{dT} = \frac{d}{dT}\left(\frac{3}{2} RT\right) = \frac{3}{2} R$$

The molar specific heat at constant pressure,

$$C_P \text{ (monoatomic)} = C_V + R = \frac{3}{2} R + R = \frac{5}{2} R$$

Specific heat ratio, $\gamma = \frac{C_P}{C_V} = \frac{(5/2) R}{(3/2) R} = \frac{5}{3} = \mathbf{1.67.}$

(*ii*) **Specific heats of diatomic gas.** (*a*) Diatomic molecules such as N_2, O_2, etc., behave as *rigid rotator* at moderate temperatures. Such molecules have 5 degrees of freedom : 3 translational and 2 rotational. According to the law of equipartition of energy, the total energy of a mole of such a gas is

$$U = \frac{5}{2} k_B T \times N_A = \frac{5}{2} RT$$

$$\therefore C_V \text{ (rigid diatomic)} = \frac{dU}{dT} = \frac{5}{2} R$$

$$C_P \text{ (rigid diatomic)} = C_V + R = \frac{7}{2} R$$

$$\gamma \text{ (rigid diatomic)} = \frac{(7/2) R}{(5/2) R} = \frac{7}{5} = \mathbf{1.4.}$$

(*b*) If the diatomic molecule is not rigid but has also a vibrational mode, then each molecule has an additional energy equal to $2 \times (\frac{1}{2}) k_B T = k_B T$, because a vibrational frequency has both kinetic and potential energy modes.

$$\therefore U = \left(\frac{5}{2} k_B T + k_B T\right) N_A = \frac{7}{2} k_B N_A T = \frac{7}{2} RT$$

$$C_V \text{ (diatomic with vibrational mode)} = \frac{dU}{dT} = \frac{7}{2} R$$

$$C_P \text{ (diatomic with vibrational mode)} = C_V + R = \frac{9}{2} R$$

γ (diatomic with vibrational mode)

$$= \frac{(9/2) R}{(7/2) R} = \frac{9}{7} = \mathbf{1.28.}$$

(*iii*) **Specific heats of triatomic gas.** (*a*) *A non-linear* triatomic gas molecule has *six* degrees of freedom.

$$\therefore \quad U = \frac{6}{2} k_B T \times N_A = 3RT$$

$$C_V = \frac{dU}{dT} = 3R$$

$$C_P = C_V + R = 4R$$

$$\gamma = \frac{C_P}{C_V} = \frac{4}{3} = \mathbf{1.33.}$$

(*b*) *A linear* triatomic molecule has *seven* degrees of freedom.

$$\therefore \quad U = \frac{7}{2} k_B T \times N_A = \frac{7}{2} RT$$

$$C_V = \frac{dU}{dT} = \frac{7}{2} R$$

$$C_P = C_V + R = \frac{9}{2} R$$

$$\gamma = \frac{C_P}{C_V} = \frac{(9/2) R}{(7/2) R} = \frac{9}{7} = \mathbf{1.28.}$$

27. *Using the law of equipartition of energy, obtain a relation between the degrees of freedom f and the specific heat ratio γ of a polyatomic gas.*

Specific heats of a polyatomic gas. Consider one mole of a perfect polyatomic gas at absolute temperature T. Suppose the total degrees of freedom of each molecule be f. According to the law of equipartition of energy,

$$\text{average energy of each molecule} = \frac{f}{2} k_B T$$

$\therefore$ Internal energy of one mole of the gas,

$$U = \frac{f}{2} k_B T \times N_A = \frac{f}{2} RT$$

$$C_V = \frac{dU}{dT} = \frac{f}{2} R$$

$$C_P = C_V + R = \frac{f}{2} R + R = \left(\frac{f}{2} + 1\right) R$$

$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{f}{2} + 1\right) R}{\frac{f}{2} R} \quad \text{or} \quad \gamma = 1 + \frac{2}{f}$$

Scanned by CamScanner

For Your Knowledge

- Generally, a polyatomic atomic gas has 3 translational, 3 rotational degrees of freedom and a certain number (f') of vibrational modes. Therefore, the internal energy of one mole of such a gas is

$$U = \left(\frac{3}{2}k_B T + \frac{3}{2}k_B T + f' k_B T\right) N_A$$

$$= (3 + f') k_B N_A T = (3 + f') RT$$

$$\therefore \quad C_V = \frac{dU}{dT} = (3 + f')R$$

$$C_P = C_V + R = (4 + f')R \text{ and } \gamma = \frac{C_P}{C_V} = \frac{4 + f'}{3 + f'}.$$

Examples based on Degrees of Freedom, Specific Heats of Monoatomic Diatomic and Polyatomic Gases

FORMULAE USED

1. Energy associated with each degree of freedom per molecule $= \frac{1}{2} k_B T$
2. For a gas of *polyatomic molecules* having f degrees of freedom,

   Energy associated with 1 mole of gas, $U = \frac{f}{2} RT$

   $$C_V = \frac{f}{2} R, \quad C_P = \left(1 + \frac{f}{2}\right) R, \quad \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$
3. For *monoatomic gas* $f = 3$ so

   $$U = \frac{3}{2} RT, \quad C_V = \frac{3}{2} R, \quad C_P = \frac{5}{2} R, \quad \gamma = 1.66$$
4. For a *diatomic gas*, $f = 5$

   $$U = \frac{5}{2} RT, \quad C_V = \frac{5}{2} R, \quad C_P = \frac{7}{2} R, \gamma = 1.4$$
5. For a *triatomic gas of non-linear molecules* $f = 6$, so

   $$U = 3\,RT, \quad C_V = 3R, \quad C_P = 4R, \quad \gamma = 1.33$$
6. For a *triatomic gas of linear molecules* $f = 7$, so

   $$U = \frac{7}{2} RT, \quad C_V = \frac{7}{2} R, \quad C_P = \frac{9}{2} R, \quad \gamma = 1.28$$

UNITS USED

C_V, C_P are in J mol^{-1} K^{-1} and c_V, c_P are in J kg^{-1} K^{-1}.

EXAMPLE 27. *Calculate the total number of degrees of freedom possessed by the molecules in 1 cm^3 of H_2 gas at N.T.P.*

Solution. Number of H_2 molecules in 22.4 litres or 22400 cm^3 at N.T.P.

$$= 6.02 \times 10^{23}$$

$\therefore$ Number of H_2 molecules in 1 cm^3 at N.T.P.

$$= \frac{6.02 \times 10^{23}}{22400} = 2.6875 \times 10^{19}$$

Number of degrees of freedom associated with each H_2 (a diatomic) molecule

$$= 5$$

$\therefore$ Total number of degrees of freedom associated with 1 cm^3 of gas

$$= 2.6875 \times 10^{19} \times 5 = \mathbf{1.34375 \times 10^{20}}.$$

EXAMPLE 28. *Calculate the internal energy of 1 g of oxygen at N.T.P.*

Solution. Oxygen is a diatomic gas.

$\therefore$ Energy associated with 1 mole of oxygen,

$$U = \frac{5}{2} RT$$

Hence internal energy of 1 g of oxygen,

$$u = \frac{U}{M} = \frac{1}{32} \times \frac{5}{2} RT$$

$$= \frac{5}{64} \times 8.31 \times 273 = \mathbf{177.2\ J}.$$

EXAMPLE 29. *Hydrogen is heated in a vessel to a temperature of 10,000 K. Let each molecule possess an average energy E_1. A few molecules escape into the atmosphere at 300 K. Due to collisions, their energy changes to E_2. Calculate ratio E_1 / E_2.*

Solution. Number of degrees of freedom of H_2 at 10,000 K = 7

Number of degrees of freedom of H_2 at 300 K = 5

$$\therefore \quad \frac{E_1}{E_2} = \frac{\frac{7}{2} k_B T_1}{\frac{5}{2} k_B T_2} = \frac{7}{5} \times \frac{T_1}{T_2} = \frac{7}{5} \times \frac{10{,}000}{300} = \frac{\mathbf{140}}{\mathbf{3}}.$$

EXAMPLE 30. *Calculate the molecular K.E. of 1 gram of Helium (Molecular weight 4) at 127°C. Given $R = 8.31\ J\ mol^{-1}\ K^{-1}$.* **[Delhi 05]**

Solution. Here $T = 127 + 273 = 400$ K

Helium is a monoatomic gas.

$\therefore$ Average K.E. per mole of helium $= \frac{3}{2} RT$

Average K.E. of 1 gram of helium $= \frac{3}{2} \frac{RT}{M}$

$$= \frac{3 \times 8.31 \times 400}{2 \times 4} = \mathbf{12.465\ J}.$$

EXAMPLE 31. *How many degrees of freedom are associated with 2 g of He at N.T.P. ? Calculate the amount of heat energy required to raise the temperature of this amount from 27°C to 127°C. Given Boltzmann's constant $= 1.38 \times 10^{-16}$ erg molecule^{-1} K^{-1} and Avogadro's number $= 6.02 \times 10^{23}$.*

Scanned by CamScanner

Solution. Molecular weight of helium = 4

Number of molecules in 4 g of helium

$$= 6.02 \times 10^{23}$$

Number of molecules in 2 g of helium

$$= \frac{1}{2} \times 6.02 \times 10^{23} = 3.01 \times 10^{23}$$

As helium is a monoatomic gas, it has 3 degrees of freedom

Total degrees of freedom of 3.01×10^{23} molecules

$$= 3 \times 3.01 \times 10^{23} = 9.03 \times 10^{23}$$

Boltzmann's constant,

$$k_B = 1.38 \times 10^{-16} \text{ erg molecule}^{-1} \text{ K}^{-1}.$$
$$= 1.38 \times 10^{-23} \text{ J molecule}^{-1} \text{ K}^{-1}$$

Energy associated with 1 degree of freedom per molecule $= \frac{1}{2} k_B T$

$\therefore$ Energy associated with 2 g of He at 300 K,

$$E_1 = 9.03 \times 10^{23} \times \frac{1}{2} \times 1.38 \times 10^{-23} \times 300 = 1869.2 \text{ J}$$

Energy associated with 2 g of He at 400 K,

$$E_2 = 9.03 \times 10^{23} \times \frac{1}{2} \times 1.38 \times 10^{-23} \times 400 = 2492.3 \text{ J}$$

Here energy required to raise the temperature from 27°C to 127°C

$$= E_2 - E_1 = 2492.3 - 1869.2 = \mathbf{623.1\ J.}$$

EXAMPLE 32. Calculate the limiting ratio of the internal energy possessed by helium and hydrogen gases at 10,000 K.

Solution. *For helium gas* : As helium is monoatomic gas, the number of degrees of freedom of helium molecule = 3

$\therefore$ The internal energy of helium molecule at 10,000 K,

$$U_{He} = 3 \times \frac{1}{2} k_B T$$
$$= 3 \times \frac{1}{2} \times k_B \times 10000 = 1.5 \times 10^4 k_B$$

For hydrogen gas : At 10,000 K, no. of degrees of freedom of a H_2 molecule = 7

$\therefore$ The internal energy of hydrogen molecule at 10,000 K,

$$U_H = 7 \times \frac{1}{2} k_B T$$
$$= 7 \times \frac{1}{2} k_B \times 10000 = 3.5 \times 10^4 k_B$$

Hence $\frac{U_{He}}{U_H} = \frac{1.5 \times 10^4 k_B}{3.5 \times 10^4 k_B} = \mathbf{3 : 7.}$

EXAMPLE 33. A cylinder of fixed capacity 44.8 litres contains helium gas at standard pressure and temperature. What is the amount of heat needed to raise the temperature of the gas by 15.0°C ? R = 8.31 J mol⁻¹ K⁻¹. **[NCERT]**

Solution. Volume of 1 mole of He at S.T.P.

$$= 22.4 \text{ litres}$$

Total volume of He at S.T.P. = 44.8 litres

$\therefore$ No. of moles of He, $n = \frac{44.8}{22.4} = 2$

Molar specific heat of He (monoatomic gas) at constant volume,

$$C_V = \frac{3}{2} R = \frac{3}{2} \times 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$
$$\Delta T = 15°C = 15 \text{ K}$$

Heat required

$$Q = nC_V \, \Delta T = 2 \times \frac{3}{2} \times 8.31 \times 15 = \mathbf{373.95\ J.}$$

EXAMPLE 34. One mole of a monoatomic gas is mixed with three moles of a diatomic gas. What is the molecular specific heat of the mixture at constant volume ? **[Roorkee 93 ; Delhi 06]**

Take $R = 8.31$ J mol⁻¹ K⁻¹.

Solution. For monoatomic gas, $C_V = \frac{3}{2} R$, $n = 1$ mole

For diatomic gas, $C'_V = \frac{5}{2} R$, $n' = 3$ mole

From conservation of energy, the molecular specific heat of the mixture is

$$C''_V = \frac{n(C_V) + n'(C'_V)}{(n + n')} = \frac{1 \times \frac{3}{2} R + 3 \times \frac{5}{2} R}{(1+3)} = \frac{9}{4} R$$

or $C''_V = \frac{9}{4} \times 8.31 = \mathbf{18.7\ J\ mole^{-1}\ K^{-1}.}$

EXAMPLE 35. One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture ? Here γ denotes the ratio of specific heat at constant pressure to that at constant volume.

[AIEEE 02, 04]

Solution. $\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$ or $C_V = \frac{R}{\gamma - 1}$

For monoatomic gas, $C_V = \frac{R}{5/3 - 1} = \frac{3}{2} R$

For diatomic gas $C'_V = \frac{R}{7/5 - 1} = \frac{5}{2} R$

$$C_V \text{ (mixture)} = \frac{n\,C_V + n'\,C'_V}{n + n'} = \frac{1 \times \frac{3}{2} R + 1 \times \frac{5}{2} R}{1 + 1} = 2R$$

$$\gamma \text{ (mixture)} = 1 + \frac{R}{C_V \text{ (mixture)}} = 1 + \frac{R}{2R} = \mathbf{1.5.}$$

Scanned by CamScanner

EXAMPLE 36. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. Find the ratio C_P / C_V of the mixture. **[AIEEE 05]**

Solution. Number of moles of helium $= \frac{16}{4} = 4$

Number of moles of oxygen $= \frac{16}{32} = \frac{1}{2}$

For the monoatomic helium gas, degrees of freedom $f = 3$, so

$$C_V = \frac{f}{2} R = \frac{3}{2} R$$

For the diatomic oxygen gas, $f = 5$, so

$$C_V' = \frac{f}{2} R = \frac{5}{2} R$$

$$\therefore C_V \text{ (mixture)} = \frac{nC_V + n' C_V'}{n + n'}$$

$$= \frac{4 \times \frac{3}{2} R + \frac{1}{2} \times \frac{5}{2} R}{4 + \frac{1}{2}} = \frac{29}{18} R$$

$$\gamma \text{ (mixture)} = \frac{C_P}{C_V} = 1 + \frac{R}{C_V \text{ (mixture)}}$$

$$= 1 + \frac{R}{\frac{29}{18} R} = 1 + \frac{18}{29} = \frac{47}{29} = \mathbf{1.62.}$$

EXAMPLE 37. A gaseous mixture enclosed in a vessel contains 1 gram mole of a gas A (with $\gamma = 5/3$) and another gas B (with $\gamma = 7/5$) at a temperature T. The gases A and B do not react with each other and assumed to be ideal. Find the number of gram moles of B if γ for the gaseous mixture is 19/13. **[IIT 95]**

Solution. Let the mixture contain n moles of gas B.

As $C_P - C_V = R$ and $\gamma = \frac{C_P}{C_V}$

$$\therefore \quad C_V = \frac{R}{\gamma - 1}$$

For gas A, $C_V = \frac{R}{5/3 - 1} = \frac{3}{2} R$

For gas B, $C_V = \frac{R}{7/2 - 1} = \frac{5}{2} R$

For the mixture, $C_V = \frac{R}{19/13 - 1} = \frac{13}{6} R$

By conservation of energy, molar sp. heat of the mixture is

$$C_V = \frac{n_A (C_V)_A + n_B (C_V)_B}{n_A + n_B}$$

$$\therefore \quad \frac{13}{6} R = \frac{1 \times \frac{3}{2} R + n \times \frac{5}{2} R}{1 + n} = \frac{(3 + 5n) R}{2(1 + n)}$$

$n = 2.$

✖ PROBLEMS FOR PRACTICE

1. Calculate the total number of degrees of freedom for a mole of diatomic gas at N.T.P. **(Ans. 30.1×10^{23})**
2. Calculate the number of degrees of freedom in 10 cm^3 of O_2 at N.T.P. **(Ans. 1.344×10^{21})**
3. Calculate the number of degrees of freedom in 15 cm^3 of nitrogen at N.T.P. **(Ans. 2.015×10^{21})**
4. The specific heat of argon at constant volume is $0.075 \text{ kcal kg}^{-1}\text{K}^{-1}$. Calculate its atomic weight. Given $R = 2 \text{ cal mol}^{-1}\text{K}^{-1}$ **(Ans. 40)**
5. A certain gas possesses 3 degrees of freedom corresponding to the translational motion and 2 degrees of freedom corresponding to rotational motion. (*i*) What is the kinetic energy of translational motion of one such molecule of gas at 300 K? (*ii*) If the temperature is raised by 1°C, what energy must be supplied to the one molecule of the gas? Given Avogadro's number, $N = 6.023 \times 10^{23} \text{ mol}^{-1}$ and Boltzmann'a constant, $k_B = 1.38 \times 10^{-23} \text{ J molecule}^{-1}\text{K}^{-1}$. **[Ans. (*i*) 6.21×10^{-21} J (*ii*) 20.78 J]**

✖ HINTS

1. No. of degrees of freedom of a diatomic molecule at 273 K = 5

   $\therefore$ No. of degrees of freedom of 1 mole or 6.02×10^{23} molecules at N.T.P.

   $$= 5 \times 6.02 \times 10^{23} = \mathbf{30.1 \times 10^{23}}.$$

3. No. of nitrogen molecules in 22400 cm^3 of gas at N.T.P. $= 6.02 \times 10^{23}$

   $\therefore$ No. of nitrogen molecules in 15 cm^3 of gas at N.T.P.

   $$= \frac{6.02 \times 10^{23} \times 15}{22400} = 4.2 \times 10^{20}$$

   No. of degrees of freedom of nitrogen (diatomic) molecule at 273 K = 5

   $\therefore$ Total degrees of freedom of 15 cm^3 of gas

   $$= 4.03 \times 10^{20} \times 5 = \mathbf{2.015 \times 10^{21}}.$$

4. Argon is a monoatomic gas, so its molar specific heat at constant volume is

   $$C_V = \frac{3}{2} R = \frac{3}{2} \times 2 = \text{cal mol}^{-1}\text{ K}^{-1}$$

   Given $c_V = 0.075 \text{ kcal kg}^{-1}\text{ K}^{-1} = 0.075 \text{ cal g}^{-1}\text{ K}^{-1}$

   As $C_V = Mc_V$

   $$\therefore \quad M = \frac{C_V}{c_V} = \frac{3}{0.075} = \mathbf{40.}$$

Scanned by CamScanner

(i) Translational K.E. of a gas molecule at 300 K

$$= 3 \times \frac{1}{2} k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$= 6.21 \times 10^{-21} \text{ J}.$$

(ii) Energy required to raise the temperature of one mole of gas by 1°C or 1 K

$$= 5 \times \frac{1}{2} k_B \Delta T \times N$$

$$= 5 \times \frac{1}{2} \times 1.38 \times 10^{-23} \times 1 \times 6.023 \times 10^{23}$$

$$= 20.78 \text{ J}.$$

13.17 SPECIFIC HEAT OF SOLIDS

28. *State and prove Dulong and Petit's law. What does it signify ?*

Specific heat of solids : Dulong and Petit's law. *Near the room temperature the molar specific heat of most of the solids at constant volume is equal to 3R or 6 cal mol⁻¹ K⁻¹ or 25 J mol⁻¹ K⁻¹. This statement is known as Dulong and Petit's law.*

Proof. In a solid, the atoms vibrate about their mean positions. During vibration, the kinetic energy (E_k) of an atom changes continuously into potential energy (E_p) and vice-versa. So the average values of E_k and E_p are equal in a solid. Since an atom can vibrate along three mutually perpendicular directions, it has three degrees of freedom. Applying the law of equipartition of energy, we get

$$E_k = 3 \times \frac{1}{2} k_B T = \frac{3}{2} k_B T$$

$$E_p = 3 \times \frac{1}{2} k_B T = \frac{3}{2} k_B T$$

∴ Average vibrational energy per atom

$$= E_k + E_p = 3 k_B T$$

The total vibrational energy or the internal energy of one mole of atoms of the solid is given by

$$U = N_A \times 3 k_B T = 3RT \qquad [\because R = k_B N_A]$$

Since for a solid ΔV is negligible, so

$$\Delta Q = \Delta U + P \Delta V \approx \Delta U$$

Therefore, $C_V = \dfrac{\Delta Q}{\Delta T} = \dfrac{\Delta U}{\Delta T} = 3R$

This proves the Dulong and Petit's law.

Significance of Dulong and Petit's law. This law signifies the fact that the heat energy required to raise the temperature of a sample of metal depends on the total number of atoms present in the sample and not on the masses of individual atoms of the metal. Since one mole of every metal contains an Avogadro's number of atoms, so the molar specific heat of all metals is same near the room temperature.

29. *How does the specific heat of a solid vary with temperature ? What is Debye temperature ?*

Variation of specific heat of a solid with temperature. Fig. 13.15 shows the variation of molar specific heat (C_V) of a solid as a function of temperature. Clearly, at higher temperatures the molar specific heat (C_V) of all solids is close to the Dulong and Petit's value 3R. At lower temperatures, the molar specific heat decreases rapidly with temperature, tending to become zero at 0 K. Physically, this is related to the fact that the number of degrees of freedom of a molecule decreases as we go to low temperatures. In fact, some of the modes of motion get frozen.

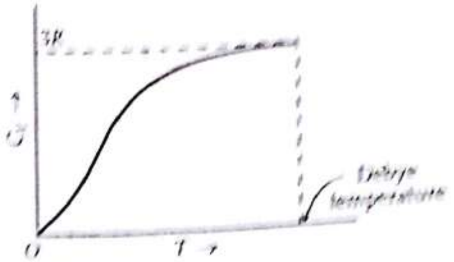


Fig. 13.15 Variation of C_V of a solid with temperature.

The behaviour of specific heats at low temperatures could not be explained on the basis of classical physics according to which the modes of motion remain unchanged and only the amplitude of motion decreases at low temperatures. This behaviour was first explained by Einstein in 1905 by using quantum theory. It was explained more satisfactorily by Debye in 1915.

Quantum mechanics requires a minimum, non-zero amount of energy before a degree of freedom comes into play. That is why vibrational degrees of freedom become active only under some situations.

The temperature at which the molar specific heat of a solid at constant volume becomes equal to 3 R is called ***Debye temperature.***

Table 13.1 Specific heats and molar specific heats of some solids at room temperature and atmospheric pressure

Substance	Specific heat (J kg⁻¹ K⁻¹)	Molar specific heat (J mol⁻¹ K⁻¹)
Aluminium	900.0	24.4
Carbon	506.5	6.1
Copper	386.4	24.5
Lead	127.7	26.5
Silver	236.1	25.5
Tungsten	134.4	24.9

Scanned by CamScanner

The experimental values of molar specific heat agree well with the predicted value 25 $J\,mol^{-1}\,K^{-1}$. Here carbon is an exception. The deviation from the predicted value is more at lower temperatures.

13.18 SPECIFIC HEAT OF WATER

30. *Using the law of equipartition of energy, predict the specific heat of water. How does the specific heat of water vary with temperature ? Define one calorie.*

Specific heat of water. We may treat water like a solid. By the law of equipartition of energy, the average vibrational energy per atom is $3\,k_BT$. Now a water molecule has three atoms : two hydrogen and one oxygen.

$\therefore$ Average vibrational energy per water molecule

$$= 3 \times 3k_BT = 9\,k_BT$$

The total vibrational or the internal energy of one mole of water molecules,

$$U = N_A \times 9\,k_B\,T = 9RT \qquad [\because R = k_BN_A]$$

Neglecting ΔV, like for a solid, we get

$$\Delta Q = \Delta U + P\Delta V = \Delta U$$

$$\therefore \qquad C_V = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 9R$$

This predicted value is found to be in good agreement with the observed value. The specific heat of water is nearly 75 $J\,mol^{-1}\,K^{-1} \approx 9R$.

Variation of specific heat of water with temperature. Fig. 13.16 shows the variation of specific heat of water with temperature in the temperature range 0°C to 100°C. Water shows peculiar behaviour, its specific heat first decreases and then increases with temperature. For this reason, we have to specify the unit temperature interval for defining calorie.

One calorie is defined as the amount of heat required to raise the temperature of 1 g of water from 14.5°C to 15.5°C.

$$1 \text{ calorie} = 4.186 \text{ J}.$$

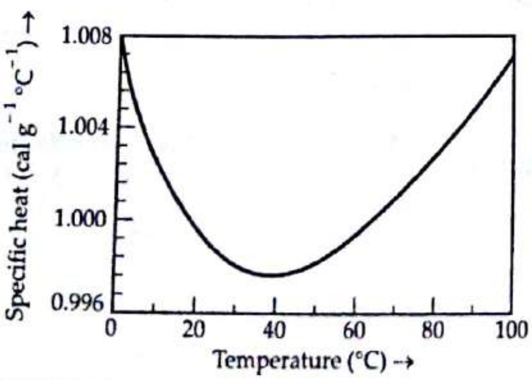


Fig. 13.16 Variation of specific heat of water with temperature

13.19 MEAN FREE PATH

31. *What is meant by the mean free path of a gas molecule ? Derive an expression for it. On which factors does the mean free path depend ?*

Mean free path. The molecules of a gas are in state of continuous, rapid and random motion. As the molecules have a finite though small size, so they collide against one another frequently. *Between two successive collisions, a molecule moves along a straight line path with uniform velocity.* This path is called free path. But after every collision, velocity of each molecule changes both in magnitude and direction. Hence each molecule follows a series of straight line zig-zag paths, as shown in Fig. 13.17.

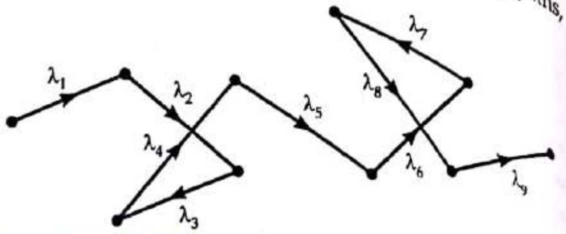


Fig. 13.17 Zig-zag path of a gas molecule.

The **mean free path** *of a gas molecule may be defined as the average distance travelled by the molecule between two successive collisions.*

As shown in Fig. 13.17, if a molecule covers free paths $\lambda_1, \lambda_2, \lambda_3, ...$, after successive collisions, then its mean free path is given by

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ...}{\text{Total number of collisions}}$$

Expression for mean free path. In order to derive the expression for the mean free path, we make use of the following assumptions :

(*i*) Each molecule of the gas is a sphere of diameter d.

(*ii*) All molecules of the gas except the molecule A under consideration are at rest.

As shown in Fig. 13.18, suppose the molecule A has average speed $\bar{v}$. It will collide with all those molecules whose centres lie within a distance d from its path. In

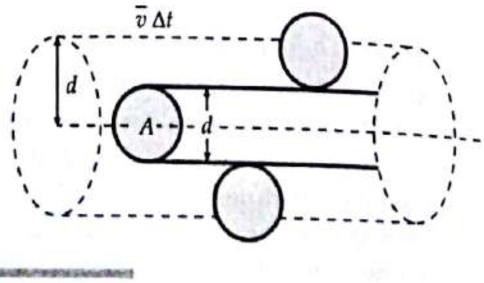


Fig. 13.18 Volume swept by molecule A in time Δt in which any molecule can collide with it.

Scanned by CamScanner

time Δt, it will obviously collide with all those molecules in the cylinder of volume $\pi d^2 \bar{v}\, \Delta t$. Let n be the number of molecules per unit volume.

Number of collisions suffered by the molecule A in time Δt

= Volume of the cylinder swept by molecule A in time Δt × number of molecules per unit volume

$= \pi d^2 \bar{v}\, \Delta t \times n$

Mean free path of a gas molecule,

$$\bar{\lambda} = \frac{\text{Distance covered in time } \Delta t}{\text{Number of collisions suffered in time } \Delta t}$$

$$= \frac{\bar{v}\, \Delta t}{\pi d^2 \bar{v}\, \Delta t\, n} = \frac{1}{n \pi d^2}$$

In the above derivation, we have assumed the other molecules to be at rest. Taking into consideration the motion of all the gas molecules, the mean free path comes out to be

$$\bar{\lambda} = \frac{1}{\sqrt{2}\, n \pi d^2}$$

If m is the mass of each gas molecule, then the density of the gas is

$$\rho = mn \quad \text{or} \quad n = \frac{\rho}{m}$$

$$\therefore \quad \bar{\lambda} = \frac{m}{\sqrt{2}\, \pi d^2 \rho}$$

For one mole of a gas,

$$PV = RT \quad \text{or} \quad P = \frac{RT}{V} = \frac{N}{V} \times \frac{R}{N} \times T = nk_B T$$

$$\text{or} \quad n = \frac{P}{k_B T} \quad \therefore \quad \bar{\lambda} = \frac{k_B T}{\sqrt{2} \pi d^2 P}$$

Factors on which the mean free path depends. It is obvious from the above expressions for $\bar{\lambda}$ that

(i) $\bar{\lambda} \propto m$, *i.e., the mean free path is directly proportional to the mass of the gas molecule.*

(ii) $\bar{\lambda} \propto \frac{1}{\rho}$, *i.e., the mean free path is inversely proportional to the density of the gas.*

(iii) $\bar{\lambda} \propto \frac{1}{d^2}$, *i.e., the mean free path is inversely proportional to the square of the molecular diameter.*

(iv) $\bar{\lambda} \propto T$, *i.e., the mean free path is directly proportional to the absolute temperature of the gas.*

(v) $\bar{\lambda} \propto \frac{1}{P}$, *i.e., the mean free path is inversely proportional to the pressure of the gas.*

13.20 AVOGADRO'S NUMBER

32. *What is Avogadro's number ? Give its significance.*

Avogadro's number. It is the number of atoms present in one gram atom of an element or the number of molecules present in one gram molecule of the substance. In general, *it is the number of particles present in one mole of the substance.* Its most accepted value is

$$N_A = 6.0225 \times 10^{23}\ \text{mole}^{-1}.$$

Importance of Avogadro's number.

(i) To *calculate the actual weight of one atom of an element.*

Weight of one atom of an element

$$= \frac{\text{Atomic weight (in gram)}}{\text{Avogadro's number}}$$

(ii) To *calculate the actual weight of one molecule of a substance.*

Weight of one molecule of a substance

$$= \frac{\text{Molecular weight (in gram)}}{\text{Avogadro's number}}$$

(iii) To *calculate the number of atoms present in given amount of an element.*

Number of atoms present in m gram of an element $= \frac{\text{Avogadro's number}}{\text{Atomic weight}} \times m$

(iv) To *calculate the number of molecules present in given amount of a substance.*

Number of molecules present in m gram of a substance $= \frac{\text{Avogadro's number}}{\text{Molecular weight}} \times m$

(v) To *calculate the number of molecules present in given volume of the gas.* At S.T.P., the 22.4 litres of every gas contain an Avogadro's number of molecules.

$\therefore$ Number of molecules present in V litres of a gas at S.T.P. $= \frac{\text{Avogadro's number}}{22.4} \times V$.

EXAMPLE 38. *The density of water is 1000 kg m^{-3}. The density of water vapour at 100°C and 1 atm pressure is 0.6 kg m^{-3}. The volume of a molecule multiplied by the total number gives, what is called molecular volume. Estimate the ratio (or fraction) of the molecular volume to the total volume occupied by the water vapour under the above conditions of temperature and pressure.* **[NCERT]**

Solution. For a given mass of water molecules,

$$\text{Volume} \propto \frac{1}{\text{Density}}$$

Scanned by CamScanner

Therefore, the ratio (or fraction) of the molecular volume to the total volume of water vapour

$$= \frac{\text{Density of water vapour}}{\text{Density of water}} = \frac{0.6 \text{ kg m}^{-3}}{1000 \text{ kg m}^{-3}}$$

$$= \mathbf{6 \times 10^{-4}}.$$

EXAMPLE 39. Estimate the volume of a water molecule. Given density of water is 1000 kg m^{-3} and Avogadro's number = 6×10^{23} $mole^{-1}$. **[NCERT]**

Solution. Molecular mass of water = 18

∴ Number of molecules in 18 g or 0.018 kg of water

$$= 6 \times 10^{23}$$

Mass of 1 molecule of water

$$= \frac{0.018}{6 \times 10^{23}} = 3 \times 10^{-26}$$

In the liquid phase, the molecules of water are closely packed. The density of water molecules may be taken equal to the density of bulk water = 1000 kg m^{-3}.

∴ Volume of a water molecule

$$= \frac{\text{Mass}}{\text{Density}} = \frac{3 \times 10^{-26}}{1000} = \mathbf{3 \times 10^{-29} \ m^3}$$

EXAMPLE 40. What is the average distance between atoms (interatomic distance) in water ? Use the data given in Examples 38 and 39. **[NCERT]**

Solution. Volume of water in vapour state

$$= \frac{1}{6 \times 10^{-4}} \times \text{Volume of water in liquid state}$$

$$= 1.67 \times 10^3 \times \text{Volume of water in liquid state.}$$

This is also the increase in amount of volume available for each molecule of water. When volume V increases by 10^3 times, the radius increases by $V^{1/3}$ or 10 times.

But volume of a water molecule,

$$\frac{4}{3}\pi r^3 = 3 \times 10^{-29} \text{ m}^3$$

$$\therefore \quad r = \left(\frac{9}{4\pi} \times 10^{-29}\right)^{1/3} = 2 \times 10^{-10} \text{ m} = 2 \text{ Å}$$

Increased radius = 10 × 2 = 20 Å

Average distance = 2 × Increased radius = 40 Å.

13.21 BROWNIAN MOTION*

33. *What is Brownian motion ? How can it be accounted for ? What are the factors which affect Brownian motion ?*

Brownian motion. In 1827, a Scottish botanist *Robert Brown*, while examining pollen grains of a flower suspended in water under a microscope, noticed that they continuously moved about in a zig-zag, random motion. This irregular motion of the suspended particles is called Brownian motion. It provided a direct evidence for the existence of molecules and their motion.

Cause of Brownian motion. Any object suspended in a fluid is continuously bombarded by the fluid molecules from all directions. If the object is sufficiently small but still visible under a microscope (such as pollen grains which are about 10^{-5} m in diameter), the impact of molecules from all sides gives rise to an unbalanced force in a certain direction. As soon as the particle moves a little, the magnitude and direction of the unbalanced force change making the particle move in a new direction. The suspended object thus moves about in a zig-zag manner and tumbles about randomly.

Factors affecting the Brownian motion. The Brownian motion increases

(*i*) with the decrease in size of the suspended particles.

(*ii*) with the increase in temperature of the fluid.

(*iii*) with the decrease in density of the fluid.

(*iv*) with the decrease in viscosity of the fluid.

Very Short Answer Conceptual Problems

Problem 1. Mention the different ways of increasing the number of molecular collisions per unit time in a gas.

Solution. The number of collisions per unit time can be increased by

(*i*) increasing the temperature of the gas,

(*ii*) increasing the number of molecules, and

(*iii*) decreasing the volume of the gas.

Problem 2. What is an ideal gas ? Give its main characteristics.

Solution. An ideal gas is one which obeys the gas laws at all values of temperature and pressure. Its main characteristics are

(*i*) The size of the molecules is negligibly small.

(*ii*) There is no force of attraction or repulsion amongst its molecules.

Scanned by CamScanner

Problem 3. Fig. 13.19 shows the variation of the product *PV* with respect to the pressure (*P*) of given masses of three gases, *A*, *B* and *C*. The temperature is kept constant. State with proper arguments which of these gases is ideal. *[Central Schools 14]*

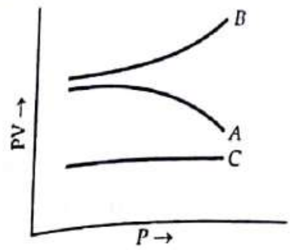


Fig. 13.19

Solution. Gas *C* is ideal, because *PV* is constant for it. That is, gas *C* obeys Boyle's law at all pressures.

Problem 4. Why do the gases at low temperature and high pressure show large deviations from ideal behaviour ?

Solution. At low temperature and high pressure, the intermolecular attractions become appreciable. Moreover, the volume occupied by the gas molecules cannot be neglected in comparison to the volume of the gas. Hence the real gases show large deviations from ideal gas behaviour.

Problem 5. On reducing the volume of a gas at constant temperature, the pressure of the gas increases. Explain it on the basis of kinetic theory.

Solution. On reducing the volume, the number of molecules per unit volume increases. Hence a large number of molecules collide with the walls of the vessel per second and a larger momentum is transferred to the wall per second. This increases the pressure of the gas.

Problem 6. When an automobile travels for a long distance, the air pressure in the tyres increases slightly. Why ?

Solution. Due to the friction between the tyres and the road, the tyres get heated. The temperature of air inside the tyres increases. Consequently, the air pressure in the tyres increases slightly.

Problem 7. When a gas is heated, its temperature increases. Explain it on the basis of kinetic theory of gases.

Solution. When a gas is heated, the root mean square velocity of its molecules increases. As $v_{rms} \propto \sqrt{T}$, so the temperature of the gas increases.

Problem 8. In the upper part of the atmosphere the kinetic temperature of air is of the order of 1000 K, even then one feels severe cold there. Why ?

Solution. As we go up in the atmosphere, the number of air molecules per unit volume decreases. The quantity of heat per unit volume or the heat density is low. But the translational kinetic energy per molecule is quite large. As the kinetic temperature is the measure of translational kinetic energy, so the kinetic temperature is quite high in the upper atmosphere but one feels severe cold there due to low heat density.

Problem 9. In the kinetic theory of gases, why do we not take into account the changes in gravitational potential energy of the molecule ?

Solution. The changes in gravitational potential energy are negligibly small as compared to the mean kinetic energy of molecules.

Problem 10. What type of motion is associated with the molecules of a gas ?

Solution. Brownian motion. In this motion any particular molecule will follow a zig-zag path due to the collisions with the other molecules or with the walls of the container.

Problem 11. Although the root-mean-square speed of gas molecules is of the order of the speed of sound in that gas, yet on opening a bottle of ammonia in one corner of a room its smell takes time in reaching the other corner. Why ?

Solution. The molecules of ammonia have random motion. They frequently collide with one another. Consequently, their net speed in any particular direction is low. So gas takes several seconds to go from one corner to the other corner of the room.

Problem 12. On which factors does the average kinetic energy of gas molecules depend : Nature of the gas, temperature, volume ?

Solution. The average K.E. of a gas molecule depends only on the absolute temperature of the gas and is directly proportional to it.

Problem 13. What do you mean by the r.m.s. speed of the molecules of a gas ? Is r.m.s. speed same as the average speed ?

Solution. The r.m.s. speed of the molecules of a gas is defined as the square root of the mean of the squared velocities of the molecules of a gas. No, r.m.s. speed is different from the average speed. For example,

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}}$$

and

$$\bar{v} = \frac{v_1 + v_2 + v_3}{3}.$$

Problem 14. The ratio of vapour densities of two gases at the same temperature is 8 : 9. Compare the r.m.s. velocities of their molecules.

Solution. $$\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{M_2}{M_1}}$$

$$= \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{9}{8}} = 3 : 2\sqrt{2}.$$

Scanned by CamScanner

Problem 15. What is the average velocity of the molecules of an ideal gas ?

Solution. The average velocity of the molecules of an ideal gas is zero, because the molecules possess all sorts of velocities in all possible directions so their vector sum and hence average is zero.

Problem 16. Given a sample of 1 cm^3 of hydrogen and 1 cm^3 of oxygen both at S.T.P. Which sample has a larger number of molecules ?

Solution. Both the samples contain the same number of molecules in accordance with Avogadro's law.

Problem 17. At what temperature does all molecular motion cease ? Explain. *[Himachal 06, 07C]*

Or

Molecular motion ceases at zero kelvin. Explain. *[Himachal 09]*

Solution. All molecular motion ceases at absolute zero or at 0 K. According to the kinetic interpretation of temperature,

$$\bar{E} = \frac{3}{2} k_B T \quad \text{or} \quad T = \frac{2}{3} \frac{\bar{E}}{k_B}$$

or absolute temperature $\propto$ average K.E. of molecules

$\therefore$ The temperature = 0 K, average K.E. = 0.

Thus at 0 K, the velocity of molecules becomes zero.

Problem 18. Why temperature less than absolute zero is not possible ?

Solution. According to the kinetic interpretation of temperature,

Absolute temperature $\propto$ average K.E. of molecules

As the heat is removed, the temperature falls and velocity of molecules decreases. At absolute zero, the molecular motion eases *i.e.*, the kinetic energy becomes zero. As kinetic energy cannot be negative, so no further decrease in kinetic energy is possible. Hence temperature cannot be decreased below 0 K.

Problem 19. For an ideal gas, the internal energy can only be translational kinetic energy. Explain.

Solution. In an ideal gas, the molecules can be considered as point masses with no intermolecular forces between them. So there can neither be internal potential energy nor the internal energy due to rotation or vibration. The molecules can have only translational kinetic energy.

Problem 20. At a given temperature, equal masses of monoatomic and diatomic gases are supplied equal quantities of heat. Which of the two gases will suffer a larger temperature rise ?

Solution. The temperature of the monoatomic gas will rise by a large value. In case of the monoatomic gas, the heat supplied is used entirely to increase the translational K.E. of the molecules. In case of the diatomic gas, the heat supplied is used to increase the translational, rotational and sometimes even the vibrational kinetic energy of the molecules. It is only the translational K.E. which increases the temperature.

Problem 21. Deduce the dimensional formula for R used in the ideal gas equation $PV = nRT$.

Solution. $R = \frac{PV}{nT} = \frac{FV}{nAT}$

$$[R] = \frac{\text{MLT}^{-2} \cdot \text{L}^3}{\text{mol} \cdot \text{L}^2 \cdot \text{K}} = [\text{ML}^2\text{T}^{-2}\text{K}^{-1}\text{mol}^{-1}]$$

Problem 22. A box contains equal number of molecules of hydrogen and oxygen. If there is a fine hole in the box, which gas will leak rapidly ? Why ?

Solution. As $v_{rms} \propto \frac{1}{\sqrt{M}}$

Hence hydrogen gas will leak more rapidly because of its smaller molecular mass.

Problem 23. The mass of a molecule of krypton is 2.25 times the mass of a hydrogen molecule. A mixture of equal masses of these gases is enclosed in a vessel. Calculate at any temperature the ratio of the root mean square velocities of krypton and hydrogen gases.

Solution. $$\frac{(v_{rms})_{Kr}}{(v_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{Kr}}} = \sqrt{\frac{1}{2.25}} = \frac{10}{15} = 2:3$$

Problem 24. A sample of an ideal gas occupies a volume V at a pressure P and absolute temperature T. The mass of each molecule is m. If k_B is the Boltzmann's constant, then write the expression for the density of the gas.

Solution. According to kinetic theory of gases,

$$P = \frac{1}{3} \rho \overline{v^2} = \frac{2}{3} \frac{\rho}{m} \cdot \frac{1}{2} m \overline{v^2} = \frac{2}{3} \cdot \frac{\rho}{m} \cdot \frac{3}{2} k_B T$$

$\therefore$ Density, $\rho = \frac{Pm}{k_B T}$.

Problem 25. The volume of vessel A is twice the volume of another vessel B, and both of them are filled with the same gas. If the gas in A is at twice the temperature and twice the pressure in comparison to the gas in B, what is the ratio of the gas molecules in A and B ?

Solution. As $PV = nRT$

$\therefore$ $n_B = \frac{PV}{RT}$ and $n_A = \frac{2P \cdot 2V}{R \cdot 2T} = 2\frac{PV}{RT}$ and $n_A : n_B = 2:1$.

Problem 26. Two gases, each at temperature T, volume V and pressure P are mixed such that the temperature and volume of the mixture are T and V respectively. What would be the pressure of the mixture ? Justify your answer on the basis of kinetic theory.

Solution. From kinetic theory, $P = \frac{1}{3} \frac{M}{V} \overline{v^2}$

But $\quad \overline{v^2} \propto T \quad \therefore \quad P \propto \frac{MT}{V}$

As both T and V remain unchanged but mass M is doubled, so the pressure of mixture gets doubled *i.e.*, it is equal to $2P$.

Problem 27. The total translational kinetic energy of molecules of a gas having volume V and pressure P is 500 J. What will be the total translational kinetic energy of the molecules of the same gas occupying the same volume V but exerting a pressure $2P$?

Solution. If E is the translational K.E. per unit volume, then

$$P = \frac{2}{3}E \quad \text{or} \quad E = \frac{3P}{2}$$

Total translational K.E. of volume V is

$$EV = \frac{3PV}{2} = 500 \text{ J}$$

When pressure becomes $2P$, total translational K.E.

$$= 2 \times 500 = \mathbf{1000\ J}.$$

Problem 28. Write the equation of state for 16 g of O_2.

Solution. Number of moles in 32 g of O_2 = 1

$\therefore$ Number of moles in 16 g of $O_2 = \frac{1}{32} \times 16 = \frac{1}{2}$

As $PV = nRT$ and $n = 1/2$, so $PV = \frac{1}{2}RT$.

Problem 29. When a gas filled in a closed vessel is heated through 1°C, its pressure increases by 0.4%. What is the initial temperature of the gas ?

Solution. $P' = P + \frac{0.4}{100}P, \quad T' = T + 1$

By Gay Lussac's law, $\frac{P}{T} = \frac{\left(P + \frac{0.4}{100}P\right)}{T + 1}$

On solving, $T = \mathbf{250\ K}$

Problem 30. A gas in a closed vessel is at the pressure P_0. If the masses of all the molecules be made half and their speeds be made double, then find the resultant pressure.

Solution. $P_0 = \frac{1}{3}\frac{mN}{V}v^2$

$\therefore \quad P' = \frac{1}{3}\frac{(m/2)}{V}(2v)^2 = \frac{2}{3}\frac{mN}{V}v^2 = \mathbf{2P_0}$.

Problem 31. What is evaporation ?

Solution. All the molecules do not have the same velocity. The molecules which possess large velocities are able to overcome the molecular attraction and escape the liquid surface, which is known as evaporation.

Problem 32. Why does evaporation cause cooling ? *[Himachal 05C]*

Solution. The evaporation of a liquid from its surface occurs because some molecules acquire velocities sufficient enough to escape from the attractive force at the surface. Because escaping molecules have higher kinetic energy, hence the average kinetic energy of the molecules left behind decreases. As average kinetic energy is directly related with temperature, hence evaporation causes cooling.

Problem 33. Cooking gas containers are kept in a lorry moving with uniform speed. What will be the effect on temperature of the gas molecules ? *[AIEEE 02]*

Solution. As the lorry is moving with a uniform speed, there will be no change in the translational motion or K.E. of the gas molecules. Hence the temperature of the gas will remain same.

Problem 34. Should the specific heat of monoatomic gas be less than, equal to or greater than that of a diatomic gas at room temperature ? Justify your answer.

Solution. The specific heat of a gas at constant volume is equal to $\frac{f}{2}R$.

For monoatomic gases, $f = 3$, so $C_V = \frac{3}{2}R$.

For diatomic gases, $f = 5$, so $C_V = \frac{5}{2}R$.

Hence the specific heat for monoatomic gas is less than that for a diatomic gas.

Problem 35. What is Debye temperature ?

Solution. The temperature above which the molar specific heat of a solid becomes equal to 6 cal mol^{-1} K^{-1} is called Debye temperature.

Problem 36. Equal masses of helium and oxygen gases are given equal quantities of heat. Which gas will undergo a greater temperature rise ?

Solution. Helium is a monoatomic gas, while oxygen is diatomic. Therefore the heat given to helium will be totally used up in increasing the translational kinetic energy of its molecules ; whereas the heat given to oxygen will be used up in increasing the translational kinetic energy of its molecules and also in increasing the kinetic energy of rotation and vibration. Hence, there will be a greater rise in the temperature of helium.

Short Answer Conceptual Problems

Problem 1. A gas is filled in a cylinder fitted with a piston at a constant temperature. Explain on the basis of kinetic theory :

(*i*) The pressure of the gas increases by raising the temperature.

(*ii*) On pulling the piston out, the pressure of the gas decreases.

Solution. (*i*) On raising the temperature, the average speed of the gas molecules increases. As a result, the molecules collide more frequently with the walls of the

Scanned by CamScanner

vessel and also greater momentum is transferred to the wall in each collision. Both of these factors increase the pressure of the gas.

(ii) When the piston is pulled out, the volume of the gas increases. The molecules have to traverse a larger distance to collide with the walls of the cylinder. So a lesser number of molecules collides with the walls per second. Moreover, the collisions now occur on a larger area of the walls. Both of these factors decrease the pressure of the gas.

Problem 2. There are N molecules of a gas in a container. If the number of molecules is increased to 2N, what will be (i) pressure of the gas, (ii) total energy of the gas and (iii) r.m.s. speed of the gas ?

Solution. $P = \frac{1}{3} mn \overline{v^2} = \frac{1}{3} \frac{mN}{V} \overline{v^2}$ $\left[n = \frac{N}{V} \right]$

(i) As $P \propto N$, so the pressure of the gas is *doubled* when the number of molecules is increased from N to 2N.

(ii) Average K.E. per molecule, $\frac{1}{2} m\overline{v^2} = \frac{3}{2} k_B T$

Total energy of N molecules $= \frac{1}{2} mN\overline{v^2} = \frac{3}{2} k_B NT.$

When the number of molecules is increased from N to 2N, total energy of the gas is **doubled**, though the average K.E. per molecule remains same.

(iii) The r.m.s. speed remains same because it depends only upon temperature.

Problem 3. Although the velocity of air molecules is nearly 0.5 km s^{-1}, yet the smell of scent spreads at a much slower rate. Why ?

Solution. The air molecules travel along a zig-zag path due to frequent collisions. As a result, their displacement per unit time is very small. Hence the smell of scent spreads very slowly.

Problem 4. When do the real gases obey more correctly the gas equation : $PV = nRT$?

Solution. An ideal gas is one whose molecules have zero volume and no mutual force between them. At low pressure, the volume of a gas is large and so the volume occupied by the molecules is negligible in comparison to the volume of the gas. At high temperature, the molecules have large velocities and so the intermolecular force has no influence on their motion. Hence at low pressure and high temperature, the behaviour of real gases approach the ideal gas behaviour.

Problem 5. Explain qualitatively how the extent of Brownian motion is affected by the

***(a)* size of the Brownian particle,**

***(b)* density of the medium,**

***(c)* temperature of the medium,**

***(d)* viscosity of the medium ?** [*NCERT ; Delhi 13*]

Solution. The effect of the various factors on the Brownian motion is as follows :

Factors	Effects
(a) Decrease in the size of the Brownian particle.	Increase of Brownian motion.
(b) Decrease in the density of the medium.	Increase of Brownian motion.
(c) Increase in temperature of the medium.	Increase of Brownian motion.
(d) Increase in viscosity of the medium.	Decrease of Brownian motion.

Problem 6. For Brownian motion of particles in suspensions in liquids, what should be the typical size of suspended particles ? Why should not the size of the particles be too small (say of atomic dimensions 10^{-10} m) or too large (say of the order of 1 m) ? [NCERT]

Solution. The typical size of the suspended particles should be 10^{-5} m. If the size of the suspended particle is too small ($\approx 10^{-10}$ m), the net momentum imparted to the suspended particle due to bombardment of neighbouring fluid particles would be zero and hence no Brownian motion is possible. If the size of the suspended particle is too large (≈ 1 m), the particle will not move due to its large inertia, even when an unbalanced force is acting on it.

Problem 7. What is the simplest evidence in nature that you can think of to suggest that atoms are not point particles ? [NCERT]

Solution. Brownian motion is the simplest evidence in nature to suggest that atoms are not point particles but have finite size. The particles suspended in a fluid move in the direction of the unbalanced force due to the unequal bombardment caused by the atoms or molecules of the fluid in which they are suspended. This shows that the atoms have finite size.

Problem 8. In an experiment, the specific heats of some inert gases (at ordinary temperatures) are measured to be as follows :

Gas	Atomic mass (u)	Specific heat (cal g^{-1} K^{-1})
Helium	4.00	0.748
Neon	20.18	0.147
Argon	39.94	0.0760
Krypton	83.80	0.0358
Xenon	131.3	0.0226

Try to discover a regularity in the data and explain it on the basis of kinetic energy. [NCERT]

Solution. Molar specific heat

= Atomic mass × specific heat

Gas	Molar specific h
Helium	4.00 × 0.748 = 2.992 cal mol
Neon	20.18 × 0.147 = 2.966 cal m
Argon	39.94 × 0.0760 = 3.035 cal
Krypton	83.80 × 0.0358 = 3.000 cal
Xenon	131.3 × 0.0226 = 2.967 cal

Thus the molar specific heat of
3 cal mol^{-1} K^{-1}. It should be so on
theory of gases according to whic
gas.

$C_V = \frac{3}{2} R = \frac{3}{2} \times 2$ cal m

$= 3$ cal mol^{-1} K^{-1}

Problem 9. What is meant by
gas ? The molar specific heat o
$\frac{5}{2}$ R in the temperature range of
lower temperatures, molar sp
decreases to the value typical o
At higher temperatures, it ten
do you think is happening ?

Solution. The molar speci
the amount of heat required
mole of a gas through 1°C.

The molar specific heat
given by $C_V = \frac{f}{2}$ R, where
gas. In the range of about 2
as H_2 gas possesses 5 d
ponding to translationa
rotational modes of moti

C_V

But at lower tempera
excited and hydroger
3 degrees of freedom cor

HOTS

Problem 1. *Two*
gases are placed on
nitrogen at tempera
helium at temperat
thermal contact wit
until the gases rea
heat capacity of th
the gases in term

Scanned by CamScanner

Gas	Molar specific heat
Helium	$4.00 \times 0.748 = 2.992\ \text{cal mol}^{-1}\ \text{K}^{-1}$
Neon	$20.18 \times 0.147 = 2.966\ \text{cal mol}^{-1}\ \text{K}^{-1}$
Argon	$39.94 \times 0.0760 = 3.035\ \text{cal mol}^{-1}\ \text{K}^{-1}$
Krypton	$83.80 \times 0.0358 = 3.000\ \text{cal mol}^{-1}\ \text{K}^{-1}$
Xenon	$131.3 \times 0.0226 = 2.967\ \text{cal mol}^{-1}\ \text{K}^{-1}$

Thus the molar specific heat of each gas is nearly $3\ \text{cal mol}^{-1}\ \text{K}^{-1}$. It should be so on the basis of kinetic theory of gases according to which for a monoatomic gas,

$$C_V = \frac{3}{2} R = \frac{3}{2} \times 2\ \text{cal mol}^{-1}\ \text{K}^{-1} = 3\ \text{cal mol}^{-1}\ \text{K}^{-1}.$$

Problem 9. What is meant by molar specific heat of a gas ? The molar specific heat of hydrogen H_2 is about $\frac{5}{2}R$ in the temperature range of about 250 K to 750 K. At lower temperatures, molar specific heat of hydrogen decreases to the value typical of monoatomic gases : $\frac{3}{2}R$. At higher temperatures, it tends to the value $\frac{7}{2}R$. What do you think is happening ? *[NCERT ; Delhi 03]*

Solution. The molar specific heat of a gas is defined as the amount of heat required to raise the temperature of 1 mole of a gas through 1°C.

The molar specific heat of a gas at constant volume is given by $C_V = \frac{f}{2}R$, where f is degrees of freedom of the gas. In the range of about 250 to 750 K, a diatomic gas such as H_2 gas possesses 5 degrees of freedom (3 corresponding to translational and 2 corresponding to rotational modes of motion of the gas). Hence

$$C_V = \frac{5}{2} R \quad \text{(between 250 to 750 K)}$$

But at lower temperatures, the rotational motion is not excited and hydrogen gas molecule possesses only 3 degrees of freedom corresponding to translational motion.

Hence at lower temperatures,

$$C_V = \frac{3}{2} R$$

At higher temperatures, hydrogen gas molecule has 2 additional degrees of freedom corresponding to vibrational mode of the motion, so that total degrees of freedom become $f = 5 + 2 = 7$.

Hence at higher temperatures,

$$C_V = \frac{7}{2} R$$

Problem 10. (*a*) When a molecule (or an elastic ball) hits a (massive) wall, it rebounds with the same speed. When a ball hits a massive bat held firmly, the same thing happens. However, when the bat is moving towards the ball, the ball rebounds with a different speed. Does the ball move faster or slower ?

(*b*) When gas in a cylinder is compressed by pushing in a piston, its temperature rises. Guess at an explanation of this in terms of kinetic theory using (*a*) above.

(*c*) What happens when a compressed gas pushes a piston out and expands ? What would you observe ?

(*d*) Sachin Tendulkar uses a heavy cricket bat while playing. Does it help him in any way ? *[NCERT]*

Solution. (*a*) Let the speed of the ball be u relative to the wicket behind the bat. If the bat is moving towards the ball with a speed V relative to the wicket, then the relative speed of the ball to bat is $V + u$ towards the bat. When the ball rebounds (after hitting the massive bat) its speed, relative to bat, $V + (V + u) = 2V + u$, moving away from the wicket. So the ball speeds up after the collision with the bat. For a molecule this would imply an increase in temperature.

(*b*) When a gas in cylinder is compressed by pushing in a piston, the speed of the molecules or their kinetic energy increases. This increases the temperature of the gas.

(*c*) When a compressed gas pushes a piston out, the speed of the molecules or their kinetic energy decreases. This decreases the temperature of the gas.

(*d*) When the ball is hit with a heavy bat, the rebound speed of the ball further increases. It helps to score better.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. Ignore the heat capacity of the boxes. Find the final temperature T_f of the gases in terms of T_0.* **[AIEEE 06]**

Solution. Nitrogen is a diatomic gas while helium is a monoatomic gas. There is no net change in internal energy of the system.

$$\Delta U = 0$$

or $$1 \times \frac{5}{2} R (T_0 - T_f) + 1 \times \frac{3}{2} R \left(\frac{7}{3} T_0 - T_f \right) = 0$$

or $$12 T_0 - 8 T_f = 0 \quad \text{or} \quad T_f = \frac{3}{2} T_0$$

Scanned by CamScanner

Problem 2. *A vessel is filled with a mixture of two different gases. However, the number of molecules per unit volume of the two gases in the mixture are the same. (i) Will the mean K.E. per molecule of both the gases be equal ? (ii) Will the root mean square velocities of the molecules be equal. (iii) Will the pressure be equal ? Give reasons.*

Solution. (*i*) **Yes.** Mean K.E. per molecule,

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} k_B T$$

As the temperature of both the gases in the mixture is the same, so mean K.E. per molecule of both the gases will be equal.

(*ii*) **No.** As $v_{rms} = \sqrt{\frac{3 k_B T}{m}}$

Due to different masses of the molecules, the r.m.s. velocities for the two gases will not be equal.

(*iii*) **No.** As $P = \frac{1}{3} mn\overline{v^2}$

Due to different masses of the molecules, the pressures exerted by the two gases will not be equal.

Problem 3. *Two vessels of the same size are at the same temperature. One of them contains 1 g of H_2 gas, and the other contains 1 g to N_2 gas. (i) Which of the vessels contains more molecules ? (ii) Which of the vessels is under greater pressure and why ? (iii) In which vessel is the average molecular speed greater ? How many times greater ?*

Solution. (*i*) According to Avogadro's hypothesis,

Number of molecules in 2 g of H_2 $= N_A$

$\therefore$ Number of molecules in 1 g of H_2 $= N_A/2$

Number of molecules in 28 g of $N_2 = N_A$

$\therefore$ Number of molecules in 1 g of N_2 $= N_A/28$

$$\frac{\text{Number of molecules of } H_2}{\text{Number of molecules of } N_2} = \frac{N_A/2}{N_A/28} = 14$$

So hydrogen containing vessel contains more molecules.

(*ii*) The average K.E. per molecule at a given temperature is independent of the molecular mass.

$$P = \frac{2}{3} \times n \times \overline{K.E.} = \frac{2}{3} \times \frac{N}{V} \times \overline{K.E.}$$

$$\frac{\text{Pressure exerted by } H_2}{\text{Pressure exerted by } N_2} = \frac{\text{No. of molecules of } H_2}{\text{No. of molecules of } N_2}$$

$$= \frac{N_A/2}{N_A/28} = 14$$

$\therefore$ Pressure exerted by hydrogen is 14 times the pressure exerted by N_2.

(*iii*) $$\frac{(v_{rms})_{H_2}}{(v_{rms})_{N_2}} = \sqrt{\frac{M_{N_2}}{M_{H_2}}} = \sqrt{\frac{28}{2}} = \sqrt{14} = 3.74$$

The r.m.s. speed of H_2 is 3.74 times the r.m.s. speed of N_2.

Problem 4. *Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volumes (V_1, V_2) and pressures (P_1, P_2) respectively. If the valve joining the two vessels is opened, what will be the temperature inside the vessel at equilibrium ?* [AIEEE]

Solution. As $PV = nRT$ $\therefore$ $n = \frac{PV}{RT}$

For first vessel, $n_1 = \frac{P_1V_1}{RT_1}$

For second vessel, $n_2 = \frac{P_2V_2}{RT_2}$

For the combined vessel, $n = \frac{P(V_1 + V_2)}{RT}$

But $n = n_1 + n_2$

$$\therefore \quad \frac{P(V_1 + V_2)}{RT} = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}$$

or $$T = \frac{T_1T_2\, P(V_1 + V_2)}{P_1V_1T_2 + P_2V_2T_1}$$

Using Boyle's law,

$$P(V_1 + V_2) = P_1V_1 + P_2V_2$$

Hence $$T = \frac{T_1T_2\,(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}.$$

Problem 5. *An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.* [IIT 03]

Solution. Suppose the container has n moles of the monoatomic gas. Then the loss in kinetic energy of the gas

$$\Delta E = \frac{1}{2}(mn)\, v_0^2$$

If the temperature of the gas changes by ΔT, then heat gained by the gas,

$$\Delta Q = \frac{3}{2} nR\, \Delta T$$

Now $\Delta Q = \Delta E$

or $$\frac{3}{2} nR\, \Delta T = \frac{1}{2}(mn)\, v_0^2 \quad \text{or} \quad \Delta T = \frac{mv_0^2}{3R}.$$

Problem 6. *A cubical box of side 1 metre contains helium gas (atomic weight 4) at a pressure of 100 N/m². During an observation time of 1 second, an atom travelling*

Scanned by CamScanner

with the root-mean-squre speed parallel to one of the edges of the cube was found to make 500 hits with a particular wall, without any collision with other atoms.

Take $R = \frac{25}{3}$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K.

(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box.

[IIT Mains 02]

Solution. (*a*) Time interval between two successive collisions,

$$t = \frac{2l}{v_{rms}}$$

$$\frac{1}{500} = \frac{2 \times 1}{v_{rms}} \qquad \left[\because\ t = \frac{1}{500}\ \text{s},\ l = 1\ \text{m}\right]$$

or $\quad v_{rms} = 1000\ \text{ms}^{-1}$.

But $\quad v_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore \quad 1000 = \sqrt{\frac{3 \times (25/3)T}{4}}$

On solving, we get $T = \mathbf{160\ K}$.

(*b*) The average kinetic energy per atom of the monoatomic gas,

$$\overline{\text{K.E.}} = \frac{3}{2} k_B\ T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 160$$

$$= \mathbf{3.312 \times 10^{-21}\ J.}$$

(*c*) From kinetic gas equation,

$$P = \frac{1}{3} \frac{m}{V} v_{rms}^2$$

where m is the mass of the gas.

$\therefore \quad 100 = \frac{1}{3} \times \frac{m}{1} \times (1000)^2$ or $m = \mathbf{3 \times 10^{-4}\ kg}$.

Problem 7. *The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square velocity of the gas molecules is V, then what will be the root mean square velocity at 480 K ?* **[IIT 96]**

Solution. Here $T_1 = 120$ K, $T_2 = 480$ K, $C_1 = V$, $C_2 = ?$

As $\quad C \propto \sqrt{T}$

$\therefore \quad \frac{C_2}{C_1} = \sqrt{\frac{T_2}{T_1}}$ or $\frac{C_2}{V} = \sqrt{\frac{480}{120}} = 2$

or $\quad C_2 = \mathbf{2V}$.

Problem 8. *You have the following group of particles, n_i represents the number of molecules with speed v_i :*

n_i	2	4	8	6	3
$v_i\ (ms^{-1})$	1.0	2.0	3.0	4.0	5.0

Calculate (i) the average speed (ii) the r.m.s. speed and (iii) the most probable speed. **[IIT]**

Solution. (*i*)

$$v_{av} = \frac{n_1 v_1 + n_2 v_2 + n_3 v_3 + n_4 v_4 + n_5 v_5}{n_1 + n_2 + n_3 + n_4 + n_5}$$

$$= \frac{2 \times 1 + 4 \times 2 + 8 \times 3 + 6 \times 4 + 3 \times 5}{2 + 4 + 8 + 6 + 3}$$

$$= \mathbf{3.17\ ms^{-1}.}$$

(*ii*) $$v_{rms} = \sqrt{\frac{n_1 v_1^2 + n_2 v_2^2 + n_3 v_3^2 + n_4 v_4^2 + n_5 v_5^2}{n_1 + n_2 + n_3 + n_4 + n_5}}$$

$$= \sqrt{\frac{2 \times 1^2 + 4 \times 2^2 + 8 \times 3^2 + 6 \times 4^2 + 3 \times 5^2}{2 + 4 + 8 + 6 + 3}}$$

$$= \sqrt{\frac{261}{23}} = \mathbf{3.37\ ms^{-1}.}$$

(*iii*) Most probable speed = Speed possessed by maximum number of particles

$$= \mathbf{3.0\ ms^{-1}.}$$

Problem 9. *In a certain region of space there are only 5 molecules per cm^3 on an average. The temperature there is 3 K. What is the pressure of this gas ?* **[Roorkee 88]**
($k_B = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1}).

Solution. Let μ be the number of molecules in the gas. Then

$$PV = \mu k_B T \quad \text{or} \quad P = \frac{\mu\ k_B T}{V}$$

But $\quad \frac{\mu}{V} = 5\ \text{cm}^{-3} = 5 \times 10^{-6}\ \text{m}^{-3}$

$\therefore \quad P = 5 \times 10^{-6} \times 1.38 \times 10^{-23} \times 3$

$$= \mathbf{20.7 \times 10^{-17}\ Nm^{-2}.}$$

Problem 10. *Two glass bulbs of equal volumes are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other in a water-bath maintained at 62°C What is the new value of the pressure inside the bulbs ? The volume of the connecting tube is negligible.* **[IIT 85]**

Solution. According to the gas equation, $PV = nRT$, so long as the mass of a system remains constant, the sum of the terms pV/T for different parts of the system remains unchanged.

$$\Sigma \frac{PV}{T} = \text{constant}$$

Let V be the volume of each bulb. Initially for the two bulbs, we have

$$\Sigma \frac{PV}{T} = \frac{76\ V}{273} + \frac{76\ V}{273} = \frac{2 \times 76 \times V}{273} \qquad ...(i)$$

Scanned by CamScanner

When one bulb is placed at melting ice and another maintained at 62 °C we have

$$\Sigma \frac{PV}{T} = \frac{P \times V}{273} + \frac{P \times V}{335} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\frac{2 \times 76 \times V}{273} = \frac{PV}{273} + \frac{PV}{335}$$

or $$P = \frac{2 \times 76}{273} \times \frac{273 \times 335}{608} = \mathbf{83.75\ cm\ of\ Hg.}$$

Problem 11. *A vessel of volume $2 \times 10^{-2}\ m^3$ contains a mixture of hydrogen and helium at 47 °C temperature and $4.15 \times 10^5\ Nm^{-2}$ pressure. The mass of the mixture is 10^{-2} kg. Calculate the masses of hydrogen and helium in the given mixture.* **[Roorkee 94]**

Solution. Mass of mixture $= 10^{-2}$ kg $= 10$ g

Let mass of hydrogen in the mixture $= m$ g

Then mass of helium in the mixture $= (10 - m)$ g

Number of moles of hydrogen, $n_1 = \frac{m}{2}$

Number of moles of helium, $n_2 = \frac{(10 - m)}{4}$

Also, $V = 2 \times 10^{-2}\ m^3$, $T = 47 + 273 = 320$ K, $R = 8.3$ J mole^{-1} K^{-1}

Let P_1 and P_2 be the pressures exerted by hydrogen and helium respectively.

Then $P_1 + P_2 = 4.15 \times 10^5\ Nm^{-2}$

Now $P_1 V = n_1 RT$ and $P_2 V = n_2 RT$

$\therefore \quad (P_1 + P_2) V = (n_1 + n_2) RT$

or $$(4.15 \times 10^5) \times 2 \times 10^{-2} = \left(\frac{m}{2} + \frac{10 - m}{4}\right) \times 8.3 \times 320$$

or $$8.3 \times 10^3 = 2656 \left[\frac{2m + 10 - m}{4}\right] = 664\,(m + 10)$$

$\therefore \quad (m + 10) = 12.5$ or $m = 2.5$ g

Mass of hydrogen $= 2.5$ g $= \mathbf{2.5 \times 10^{-3}}$ **kg.**

Mass of helium $= 10 - 2.5 = 7.5$ g $= \mathbf{7.5 \times 10^{-3}}$ **kg.**

Problem 12. *A thin tube, sealed at both ends, is 100 cm long. It lies horizontally, the middle 10 cm containing mercury and the two equal ends containing air at standard atmospheric pressure. If the tube is now turned to a vertical position, by what amount will the mercury be displaced?* **[Roorkee 89, IIT]**

Solution. When the tube is horizontal, the length of the air column on the either side of the mercury thread will be 45 cm [Fig. 13.20(*a*)]. Initially for each half, we have

$$P = 76 \text{ cm of mercury}, V = 45 \text{ cm}$$

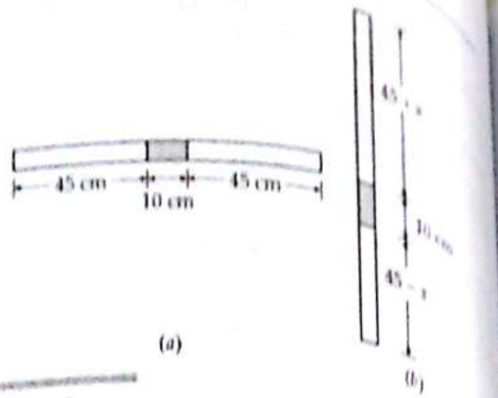


Fig. 13.20

When the tube is held vertically [Fig. 13.20(*b*)], suppose that mercury thread moves through a distance *x*. Let P_1 and P_2 be pressure of the air in the upper half and lower half respectively. The volume of air in the two halves will be

$$V_1 = 45 + x \quad \text{and} \quad V_2 = 45 - x$$

Using Boyle's law for upper half,

$$PV = P_1 V_1 \quad \text{or} \quad 76 \times 45 = P_1 \times (45 + x)$$

and for lower half,

$$PV = P_2 V_2 \quad \text{or} \quad 76 \times 45 = P_2 \times (45 - x)$$

Now $P_2 = P_1 + 10$ cm of Hg

or $$\frac{76 \times 45}{45 - x} = \frac{76 \times 45}{45 + x} + 10$$

or $$76 \times 45 \left[\frac{1}{45 - x} - \frac{1}{45 + x}\right] = 10$$

or $$\frac{76 \times 45 \times 2x}{45^2 - x^2} = 10 \quad \text{or} \quad \frac{6840\,x}{2025 - x^2} = 10$$

$$x^2 + 684\,x - 2025 = 0$$

$$\therefore \quad x = \frac{-684 \pm \sqrt{(684)^2 + 8100}}{2} = \frac{-684 \pm 6899}{2}$$

$$x = -686.95 \text{ cm or } 2.95 \text{ cm}$$

As *x* cannot be negative, so $x = \mathbf{2.95}$ **cm.**

Problem 13. *A thin tube of uniform cross-section is sealed at both ends. When it lies horizontally, the middle 5 cm length contains mercury and the two equal ends contain air*

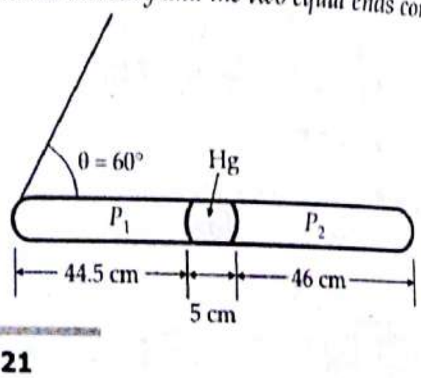


Fig. 13.21

...the same pressure P. When the tube is held at an angle of ... with the vertical, then the lengths of the air columns above and below the mercury column are 46 cm and 44.5 cm respectively. Calculate the pressure P in cm of mercury. The temperature of the system is kept at 30°C. **[IIT 86]**

Solution. Let A be the area of cross-section of the tube. When the tube is horizontal, the 5 cm column of Hg is in the middle, so length of air column on either side at pressure P

$$= \frac{46 + 44.5}{2} = 45.25 \text{ cm}$$

When the tube is held at 60° with the vertical, the lengths of air columns at the bottom and the top are 44.5 cm and 46 cm respectively. If P_1 and P_2 are their pressures, then

$$P_1 - P_2 = 5 \cos 60° = 5 \times \frac{1}{2} = \frac{5}{2} \text{ cm of Hg}$$

Using Boyle's law for constant temperature,

$$PV = P_1V_1 = P_2V_2$$

$$P \times A \times 45.25 = P_1 \times A \times 44.5 = P_2 \times A \times 46$$

$$\therefore \quad \frac{P \times 45.25}{44.5} - \frac{P \times 45.25}{46} = \frac{5}{2}$$

or $$P = \frac{5 \times 44.5 \times 46}{2 \times 45.25 \times 1.5} = \mathbf{75.4 \text{ cm.}}$$

Problem 14. *Calculate the root mean square speed of smoke particles each of mass 5×10^{-17} kg in their Brownian motion in air at N.T.P.*
Given $k_B = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1}. **[Roorkee 82]**

Solution. According to kinetic theory, average K.E. of a gas molecule,

$$\frac{1}{2} mv_{rms}^2 = \frac{3}{2} k_B T$$

$$\therefore \quad v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 273}{5 \times 10^{-17}}}$$

$$= \mathbf{1.5 \times 10^{-2} \text{ ms}^{-1}}.$$

Problem 15. *N molecules each of mass m of gas A and 2 N molecules each of mass 2 m of gas B are contained in the same vessel which is maintained at a temperature T. If the mean square velocity of the molecules of B type is denoted by v^2 and the mean square of the x component of the velocity of A is denoted by ω^2, then what is the value of ω^2 / v^2?* **[PMT 1990]**

Solution. The mean square velocity of the gas molecules is given by

$$C^2 = 3kT / m$$

For gas A: $\quad C_A^2 = 3kT / m$ and

For gas B: $\quad C_B^2 = 3kT / 2m = v^2 \qquad ...(i)$

The molecule A has equal probability of motion in all directions, therefore

$$C_x^2 = C_y^2 = C_z^2 = \omega^2 \text{ (given)}$$

$$\therefore \quad C_A^2 = C_x^2 + C_y^2 + C_z^2 = 3C_x^2 = 3\omega^2$$

or $$\omega^2 = \frac{C_A^2}{3} = \frac{1}{3}\left(\frac{3kT}{m}\right) = \frac{kT}{m} \qquad ...(ii)$$

Dividing (ii) by (i), we get : $\quad \frac{\omega^2}{v^2} = \frac{kT/m}{3kT/2m} = \mathbf{\frac{2}{3}}.$

Guidelines to NCERT Exercises

13.1. *Estimate the fraction of the molecular volume to the actual volume occupied by oxygen gas at STP. Take the radius of an oxygen molecule to be roughly 3 Å.*

Ans. Radius of oxygen molecule

$$= 3 \text{ Å} = 3 \times 10^{-10} \text{ m}$$

Volume of an oxygen molecule

$$= \frac{4}{3} \pi (3 \times 10^{-10})^3 \text{ m}^3$$

Number of molecules in 1 mole of oxygen

$$= 6 \times 10^{23}$$

Volume of 6×10^{23} molecules

$$= \frac{4}{3} \pi (3 \times 10^{-10})^3 \times 6 \times 10^{23}$$

$$= 0.78 \times 10^{-5} \text{ m}^3$$

Actual volume occupied by 1 mole of oxygen gas at STP

$$= 22.4 \text{ litre} = 22.4 \times 10^{-3} \text{ m}^3$$

Hence fraction of the molecular volume to the actual volume

$$= \frac{6.78 \times 10^{-5}}{22.4 \times 10^{-3}}$$

$$\approx \mathbf{3 \times 10^{-3}}.$$

13.2. *Molar volume is the volume occupied by 1 mole of any (ideal) gas at standard temperature and pressure (0° C, 1 atmospheric pressure). Show that it is 22.4 litres.*

Ans. $\quad T = 273 \text{ K}, \; n = 1 \text{ mol}$

$$P = 1 \text{atm} = 1.013 \times 10^5 \text{ Pa}$$

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

Scanned by CamScanner

As $\quad PV = nRT$

$\therefore \quad V = \frac{nRT}{P} = \frac{1 \times 8.31 \times 273}{1.013 \times 10^5}$

$= 22.4 \times 10^{-3}\ m^3$

$=$ **22.4 litres.**

13.3. *Fig. 13.22 shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.*

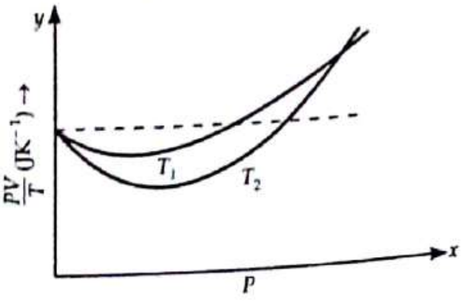


Fig. 13.22

(a) What does the dotted plot signify ?

(b) Which is true : $T_1 > T_2$ or $T_1 < T_2$? **[Delhi 12]**

(c) What is the value of PV/T where the curves meet on the y-axis ? **[Central Schools 14]**

(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis ? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot) ?

(Molecular mass of $H_2 = 2.02\ u$ of $O_2 = 32.0\ u$ $R = 8.31\ J\ mol^{-1}\ K^{-1}$).

Ans. (*a*) As the dotted line is parallel to *P*-axis, it shows that PV/T remains same when the pressure is increased. It thus indicates the ideal behaviour of the gas.

(*b*) The curve at temperature T_1 is more close to the dotted line than the curve at temperature T_2. Since the behaviour of a real gas approaches the ideal gas behaviour at high temperature, so $T_1 > T_2$.

(*c*) Number of moles in 1.00×10^{-3} kg or 1 g of oxygen,

$$n = \frac{\text{Mass of oxygen in grams}}{\text{Molecular mass}}$$

$$= \frac{1}{32} \text{ mole}$$

As $\quad PV = nRT$

$$\therefore \quad \frac{PV}{T} = nR = \frac{1}{32} \times 8.31$$

$$= \mathbf{0.26\ JK^{-1}}.$$

(*d*) No. This is because PV/T depends upon the volume of the gas and volumes of equal masses of different gases are different.

If we wish to have the curve of hydrogen gas to start from the same point, the PV/T of hydrogen should be equal to PV/T of oxygen.

From part (*c*),

$$\frac{PV}{T} = nR = 0.26$$

$$\therefore \quad n = \frac{0.26}{R} = \frac{0.26}{8.31} = \frac{1}{32} \text{ mole}$$

Mass of 1 mole of H_2 = 2.02 g

$\therefore$ Mass of 1/32 mole of $H_2 = 2.02 \times \frac{1}{32} = 0.063$ g

$= \mathbf{6.3 \times 10^{-5}}$ **kg**

13.4. *An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder, $R = 8.3\ J\ mol^{-1}\ K^{-1}$, molecule weight of oxygen = 32.*

Ans. Initial volume,

$$V_1 = 30 \text{ litres} = 30 \times 10^{-3}\ m^3$$

Initial pressure,

$$P_1 = 15 \text{ atm} = 15 \times 1.013 \times 10^5\ Nm^{-2}$$

Initial temperature,

$$T_1 = 27 + 273 = 300\ K$$

Initial number of moles,

$$n_1 = \frac{P_1 V_1}{RT_1} = \frac{15 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 300} = 18.3$$

Final pressure,

$$P_2 = 11 \text{ atm} = 11 \times 1.013 \times 10^5\ Nm^{-2}$$

Final volume,

$$V_2 = 30 \text{ litres} = 30 \times 10^{-3}\ m^3$$

Final temperature,

$$T_2 = 17 + 273 = 290\ K$$

Final number of moles,

$$n_2 = \frac{P_2 V_2}{RT_2} = \frac{11 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 290} = 13.8$$

Number of moles of oxygen taken out

$= 18.3 - 13.9 = 4.4$

Mass of gas taken out of cylinder

$= 4.4 \times 32\ g = 140.8\ g \simeq$ **0.141 kg.**

13.5. *An air bubble of volume 1.0 cm³ rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow, when it reaches the surface, which is at a temperature of 35°C ?*

Given 1 atm = 1.01×10^5 Pa.

Ans. Temperature at 40 m depth,

$$T_1 = 12 + 273 = 285\ K$$

Scanned by CamScanner

Volume of the air bubble at 40 m depth,

$V_1 = 1.0\ cm^3 = 1.0 \times 10^{-6}\ m^3$

Pressure at 40 m depth, $P_1 = 1\ atm + h\rho g$

$= 1.01 \times 10^5 + 40 \times 10^3 \times 9.8 = 4.93 \times 10^5$ Pa

Temperature at the surface of water,

$T_2 = 35 + 273 = 308$ K

Pressure at the surface of the lake,

$P_2 = 1\ atm = 1.01 \times 10^5$ Pa

Let V_2 be volume of air bubble at the surface of the lake.

As $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$$

$$= \frac{4.93 \times 10^5 \times 1.0 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5}$$

$$= \mathbf{5.275 \times 10^{-6}\ m^3}.$$

13.6. *Estimate the total number of molecules inclusive of oxygen, nitrogen, water vapour and other constituents in a room of capacity 25.0 m^3 at a temperature of 27° C and 1 atmospheric pressure.*

Given $k_B = 1.38 \times 10^{-23}\ JK^{-1}$.

Ans. As Boltzmann's constant,

$$k_B = \frac{R}{N} \quad \therefore \quad R = k_B N$$

Now $PV = nRT = nk_B NT$

$\therefore$ The number of molecules in the room

$$= nN = \frac{PV}{Tk_B}$$

$$= \frac{1.013 \times 10^5 \times 25.0}{300 \times 1.38 \times 10^{-23}}$$

$$= \mathbf{6.117 \times 10^{26}}.$$

13.7. *Estimate the average energy of a helium atom at (i) room temperature (27° C) (ii) the temperature on the surface of the sun (6000 K) and (iii) the temperature of 10^7 K.*
Given $k_B = 1.38 \times 10^{-23}$ *J molecule*$^{-1}$ K^{-1}.

Ans. Here $k_B = 1.38 \times 10^{-23}\ J\ mol^{-1}\ K^{-1}$

Average kinetic energy per molecule is given by

$$\bar{E} = \frac{3}{2} k_B T$$

(i) $T = 27 + 273 = 300$ K

$$\therefore \quad \bar{E} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = \mathbf{6.21 \times 10^{-21}\ J}.$$

(ii) $T = 6000$ K

$$\bar{E} = \frac{3}{2} \times 1.3 \times 10^{-23} \times 6000 = \mathbf{1.242 \times 10^{-19}\ J}.$$

(iii) $T = 10^7$ K

$$\bar{E} = \frac{3}{2} k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7$$

$$= \mathbf{2.07 \times 10^{-16}\ J}.$$

13.8. *Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and the third contains uranium hexaflouoride (polyatomic). (i) Do the vessels contain equal number of respective molecules ? (ii) Is the root mean squared speed of molecules same in the three cases ? If not, in which case is v_{rms} the largest ?* **[Delhi 2010]**

Ans. (i) Yes, the vessels contain equal number of respective molecules. This is in accordance with Avogadro's hypothesis that equal volumes of all gases under similar conditions of temperature and pressure contain equal number of molecules.

(ii) $v_{rms} = \sqrt{\frac{3RT}{M}}$ i.e., $v_{rms} \propto \frac{1}{\sqrt{M}}$

As the molecular masses of the three gases are different, so the r.m.s. speeds of the molecules will be different in the three cases. Moreover as the value of M is smallest for neon, so v_{rms} is largest for neon.

13.9. *At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the r.m.s. speed of a helium gas atom at – 20° C ? Atomic mass of argon = 39.9 u and that of helium = 4.0 u.*

Ans. Root mean square speed for argon at temperature T,

$$v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$$

Root mean square speed for helium at temperature –20 is

$$v' = \sqrt{\frac{3R \times 253}{4.0}}$$

As $v = v'$, so we have

$$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4.0}}$$

$$\frac{T}{39.9} = \frac{253}{4.0} \quad \text{or} \quad T = \frac{253 \times 39.9}{4.0} = \mathbf{2523.7\ K.}$$

13.10. *Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17° C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of N_2 = 28.0 u).*

Ans. Here $T = 273 + 17 = 290$ K,

$$d = 2 \times 1.0 \times 10^{-10}\ m$$

$$P = 2.0\ atm = 2 \times 1.013 \times 10^5\ Pa$$

$$k_B = 1.37 \times 10^{-23}\ JK^{-1}$$

Scanned by CamScanner

Mean free path,

$$\bar{\lambda} = \frac{k_B T}{\sqrt{2}\pi d^2 P}$$

$$= \frac{1.37 \times 10^{-23} \times 290}{1.414 \times 3.14 \times (2 \times 10^{-10})^2 \times 2 \times 1.013 \times 10^5}$$

$$\simeq 1.1 \times 10^{-7} \text{ m.}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 290}{0.028}} = 5.1 \times 10^2 \text{ ms}^{-1}.$$

Collision frequency

$$f = \frac{v_{rms}}{\bar{\lambda}} = \frac{5.1 \times 10^2}{1.1 \times 10^{-7}} = 4.6 \times 10^9 \text{ s}^{-1}$$

Time taken for the collision

$$t = \frac{d}{v_{rms}} = \frac{2.0 \times 10^{-10}}{5.1 \times 10^2} \simeq 4 \times 10^{-13} \text{ s}$$

Time taken between two successive collisions

$$T = \frac{1}{f} = \frac{1}{4.6 \times 10^9 \text{s}^{-1}} = 2 \times 10^{-10} \text{ s}$$

$$\therefore \quad \frac{T}{t} = \frac{2 \times 10^{-10} \text{s}}{4 \times 10^{-13} \text{s}} = 500$$

Obviously, the time taken between two successive collisions is 500 times the time taken for a collision. Hence a molecule in a gas moves essentially free for most of the time.

13.11. *A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom ?*

Ans. As shown in Fig. 13.23(*a*), when the tube is held horizontally, 76 cm of Hg thread traps 15 cm of air and a length of 9 cm is left at the open end.

Area of cross-section of tube = A cm^2

Pressure of air enclosed, $P_1 = 1\text{atm} = 76$ cm of Hg

Volume, $V_1 = A \times l = A \times 15 \text{cm}^3$.

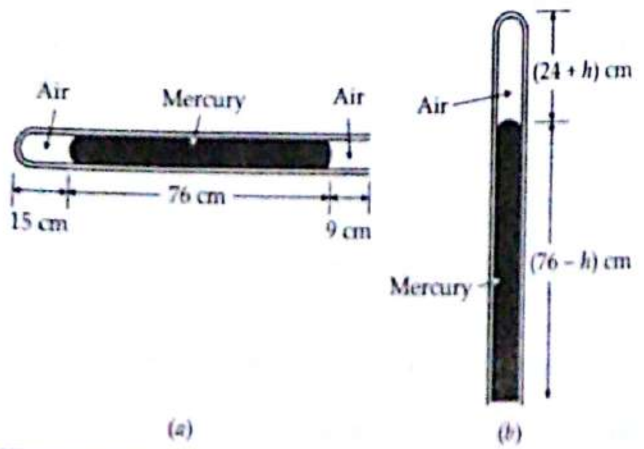


Fig. 13.23

As shown in Fig. 13.23(*b*), when the tube is held vertically, mercury thread reaches the open end and length of air column becomes 15 + 9 = 24 cm. Let h cm of Hg flow out to balance the atmospheric pressure. Length of mercury thread decreases to $(76 - h)$ cm and that air column increases to $(24 + h)$ cm. Let P_2 be the pressure exerted by air column. Then

$$P_2 + (76 - h) = 1\text{atm} = 76 \text{ cm of Hg}$$

$$P_2 = 76 - 76 + h = h \text{ cm of Hg}$$

$$V_2 = A \times (24 + h) \text{cm}^3.$$

If temperature remains constant, then from Boyle's law,

$$P_1 V_1 = P_2 V_2$$

$$76 \times A \times 15 = h \times A \times (24 + h)$$

or $\quad h^2 + 24h - 1140 = 0$

or
$$h = \frac{-24 \pm \sqrt{576 + 4560}}{2}$$

$$= \frac{-24 \pm \sqrt{5126}}{2} = \frac{-24 \pm 71.6}{2}$$

$$= 23.8 \text{ cm or } -47.8 \text{ cm}$$

Since, h cannot be negative (more mercury cannot flow into the tube), $h = 23.8$ cm. Therefore, when the tube is held vertically, the mercury thread will **decrease in length by 23.8 cm.**

13.12. *From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 cm^{3}s^{-1}. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 cm^{3}s^{-1}. Identify the gas.*

Ans. According to Graham's law of diffusion,

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

Here

r_1 = diffusion rate of hydrogen = 28.7 cm^{3}s^{-1}

r_2 = diffusion rate of unknown gas = 7.2 cm^{3}s^{-1}

M_1 = molecular mass of hydrogen = 2

M_2 = molecular mass of unknown gas = ?

$$\text{Now } M_2 = \left(\frac{r_1}{r_2}\right)^2 M_1 = \left(\frac{28.7}{7.2}\right)^2 \times 2 = 31.78 \approx 32$$

Obviously, the unknown gas is *oxygen*.

13.13. *A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres,*

$$n_2 = n_1 \exp[-mg(h_2 - h_1)/k_B T]$$

where n_2, n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column

$$n_2 = n_1 \exp[-mg N_A(\rho - \rho')(h_2 - h_1)/\rho RT]$$

Scanned by CamScanner

where ρ is the density of the suspended particle, and ρ' that of surrounding medium. [N_A is Avogadro's number, and R the universal gas constant]

Ans. According to the law of atmospheres,

$$n_2 = n_1 \exp\left[-\frac{mg(h_2 - h_1)}{k_B T}\right] \quad ...(i)$$

When the suspension is in sedimentation equilibrium in a liquid column, the weight mg of the suspended particle needs to be replaced by its apparent weight.

Now the apparent weight of the suspended particle

$$mg' = \text{Actual weight} - \text{Upthrust}$$

$$= mg - V\rho' g$$

$$= mg - \frac{m}{\rho}\rho' g = mg\left(\frac{\rho - \rho'}{\rho}\right)$$

Also, $k_B = \dfrac{R}{N_A}$

Replacing mg by mg' and putting the value of k_B in equation (i), we get

$$n_2 = n_1 \exp\left[-\frac{mg'(h_2 - h_1)}{k_B T}\right]$$

or

$$n_2 = n_1 \exp\left[-\frac{mgN_A(\rho - \rho')(h_2 - h_1)}{\rho RT}\right]$$

13.14. *Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms :*

Substance	Atomic Mass (u)	Density (10^3 kg m^{-3})
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

Ans. If r is the atomic radius, then

volume of each atom, $v = \dfrac{4}{3}\pi r^3$

Volume of one mole of atoms,

$$V = N_A v = N_A \times \frac{4}{3}\pi r^3$$

If M is the atomic mass and ρ the density of the substance, then

$$V = \frac{M}{\rho}$$

$$\therefore \quad N_A \times \frac{4}{3}\pi r^3 = \frac{M}{\rho} \quad \text{or} \quad r = \left(\frac{3M}{4\pi\rho N_A}\right)^{1/3}$$

For carbon : $M = 12.01 \times 10^{-3}$ kg, $\rho = 2.22 \times 10^3$ kg m^{-3}

$$\therefore \quad r = \left(\frac{3 \times 12.01 \times 10^{-3}}{4\pi \times 2.22 \times 10^3 \times 6.023 \times 10^{23}}\right)^{1/3}$$

$$= 1.29 \times 10^{-10} \text{m} = \mathbf{1.29\ Å}$$

For gold : $M = 197 \times 10^{-3}$ kg, $\rho = 19.32 \times 10^3$ kg m^{-3}

$$\therefore \quad r = \left(\frac{3 \times 197 \times 10^{-3}}{4\pi \times 19.32 \times 10^3 \times 6.023 \times 10^{23}}\right)^{1/3}$$

$$= 1.59 \times 10^{-10} \text{m} = \mathbf{1.59\ Å}$$

For lithium : $M = 6.94 \times 10^{-3}$ kg, $\rho = 0.53 \times 10^3$ kg m^{-3}

$$\therefore \quad r = \left(\frac{3 \times 6.94 \times 10^{-3}}{4\pi \times 0.53 \times 10^3 \times 6.023 \times 10^{23}}\right)^{1/3}$$

$$= 1.73 \times 10^{-10} \text{m} = \mathbf{1.73\ Å}$$

For fluorine : $M = 19 \times 10^{-3}$ kg, $\rho = 1.14 \times 10^3$ kg m^{-3}

$$\therefore \quad r = \left(\frac{3 \times 19 \times 10^{-3}}{4\pi \times 1.14 \times 10^3 \times 6.023 \times 10^{23}}\right)^{1/3}$$

$$= 1.88 \times 10^{-10} \text{m} = \mathbf{1.88\ Å}$$

Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. Name the phenomena which give direct experimental evidence in support of the molecular motion.
2. What is an equation of state ?
3. At which temperature does all molecular motion cease ?
4. What is the nature of graph between pressure (P) and volume (V) for a given mass of a gas at a fixed temperature ?

Scanned by CamScanner

5. What is the nature of graph of P versus $(1/V)$ for a given mass of gas at constant temperature ?

6. What is the nature of graph of PV versus P for a given mass of gas at constant temperature ?

7. What is the lowest temperature attainable according to Charles' law ?

8. Who proposed a model for a gas for the kinetic theory of gases ?

9. What do you mean by mean free path of a gas molecule ?

10. Write the equation of state for an ideal gas.

11. What does the universal gas constant R signify ?

12. What is the value of R in SI units ?

13. Does the average K.E. per molecule of the gas depend upon the mass of the molecule ?

14. On which factor, does the average K.E. of translation per molecule of a gas depend ?

15. Define absolute zero, according to kinetic interpretation of temperature. [Delhi 08]

16. What is Boltzmann's constant ? Give its value.

17. How is the average K.E. of a gas molecule related to the temperature of the gas ?

18. The absolute temperature of a gas is increased four times its original value. What will be the change in r.m.s. velocity of its molecules ? [Delhi 12]

19. A molecule of mass m normally strikes a wall with velocity u and retraces its path after striking the wall. What is the change of momentum of the molecule ?

20. At a constant temperature, what is the relation between the pressure P and density ρ of a gas ?

21. A gas enclosed in a vessel has pressure P, volume V and absolute temperature T. Write the formula for the number of molecules N of the gas.

22. Write the value of gas constant in CGS system for 1 g of helium.

23. The velocities of three molecules are $3v$, $4v$ and $5v$. Determine the root mean square velocity.

24. What will be the ratio of the root mean square speeds of the molecules of an ideal gas at 270 K and 30 K ?

25. A mixture of helium and hydrogen gases is filled in a vessel at 30°C. Compare the root mean square velocities of the molecules of these gases at this temperature. Atomic weight of hydrogen is 1.

26. A container with porous walls is filled with a mixture of two gases. It is placed in evacuated space. The lighter of the two gases will escape soon. Write the relevant relation which we use for explanation.

27. How much volume does one mole of a gas occupy at normal temperature and pressure ? [Himachal 95]

28. Write the relation between the pressure and kinetic energy per unit volume of a gas.

29. What is the mean translational kinetic energy of a perfect gas molecule at temperature T ?

30. Two vessels A and B of the same volume are filled with the same gas at the same temperature. The pressure of the gas in vessel B is twice the pressure of the gas in A. What is the ratio of the number of molecules in B and A ?

31. Two cylinders contain helium at 2 atmosphere and argon at 1 atmosphere respectively. If both the gases are filled in one of the cylinders, then what would be the pressure ?

32. What is the translational K.E. per unit volume of a gas whose pressure is P ?

33. A liquid is frozen at absolute zero of temperature. What happens to its molecular motion ?

34. Water solidifies into ice at 273 K. What happens to the kinetic energy of water molecules ?

35. Two gases having same volume V, temperature T and pressure P are mixed. If the mixture is also at the temperature T, and has volume V, what will be the final pressure ? [Delhi 96]

36. Define Avogadro's number and give its value. [Himachal 02]

37. Write the relation between the ratio of the specific heats of a gas and degrees of freedom.

38. Is molar specific heat of a solid a constant quantity ?

39. What is the mean translationl kinetic energy of a perfect gas molecule at temperature T ? [Central Schools 08]

40. Name two factors on which the degrees of freedom of a gas depend. [Central Schools 08]

Answers

1. The phenomena of diffusion of gases and Brownian motion provide direct experimental evidence in support of molecular motion.

2. An equation which relates the pressure, volume and temperature of a system is called an equation of state.

Scanned by CamScanner

3. At zero kelvin.
4. It is a rectangular hyperbola.
5. It is a straight line passing through the origin.
6. It is a straight line parallel to the pressure axis.
7. $-273.15°C$.
8. R. Claussius and J.C. Maxwell.
9. The mean free path of a gas molecule is the average distance traversed by the molecule between its two successive collisions with other molecules.
10. $PV = nRT$.
11. The universal gas constant R signifies the work done by (or on) a gas per mole per kelvin.
12. $R = 8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$.
13. No.
14. Temperature.
15. Absolute zero is the temperature at which all molecular motion ceases.
16. Boltzmann's constant is defined as the gas constant per molecule.

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23}\ \text{JK}^{-1}.$$

17. Average K.E. of a gas molecule,

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} k_B T.$$

18. $v_{rms} \propto \sqrt{T}$

$\therefore \quad v'_{rms} \propto \sqrt{4T}$

$$\frac{v'_{rms}}{v_{rms}} = 2$$

or $\quad v'_{rms} = 2\, v_{rms}$

Change in r.m.s. velocity of molecules

$$= v'_{rms} - v_{rms} = v_{rms}$$

Thus the change in r.m.s. velocity will be equal to its initial value.

19. $2\, mu$
20. At constant temperature,

$$\frac{P}{\rho} = \text{constant}.$$

21. $N = \dfrac{PV}{k_B\, T}$.
22. $r = 2.078 \times 10^7\ \text{erg g}^{-1}\ °\text{C}^{-1}$.
23. $v_{rms} = \sqrt{\dfrac{(3v)^2 + (4v)^2 + (5v)^2}{3}}$

$$= \sqrt{\frac{50}{3}}\, v = \mathbf{4.08}\, v.$$

24. $\dfrac{v_{rms}}{v'_{rms}} = \sqrt{\dfrac{T}{T'}} = \sqrt{\dfrac{270}{30}} = \mathbf{3:1}$.
25. $\dfrac{(v_{rms})_{He}}{(v_{rms})_{H_2}} = \sqrt{\dfrac{M_{H_2}}{M_{He}}}$

$$= \sqrt{\frac{2}{4}} = \mathbf{1 : \sqrt{2}}.$$

26. The ratio of the rates of diffusion of two gases

$$\frac{r_1}{r_2} = \frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{M_2}{M_1}}.$$

Hence lighter gas diffuses more rapidly.

27. 22.4 litre or $22.4 \times 10^{-3}\ \text{m}^3$.
28. $P = \dfrac{2}{3} E$.
29. $\overline{K.E.} = \dfrac{3}{2} k_B\, T$.
30. $P = \dfrac{1}{3} \dfrac{mN}{V} \overline{v^2}$

For constant m, V and T, $P \propto n$

$$\therefore \quad \frac{n_B}{n_A} = \frac{P_B}{P_A} = \frac{2P}{P} = \mathbf{2:1}.$$

31. According to Dalton's law of partial pressures,

$$P = P_1 + P_2 = 2 + 1 = 3 \text{ atmosphere}.$$

32. $E = \dfrac{3}{2} P$.
33. The molecular motion ceases at absolute zero.
34. The kinetic energy of the water molecules gets partly converted into the binding energy of the ice.
35. $2P$. Refer to solution of Problem 26 on page 13.28.
36. It is the number of particles present in one mole of a substance.

$$N_A = 6.0225 \times 10^{23}\ \text{mole}^{-1}.$$

37. $\gamma = \dfrac{C_P}{C_V} = 1 + \dfrac{2}{f}$.
38. Yes. Near the room temperature, its value is $3\ \text{cal mol}^{-1}\ \text{K}^{-1}$.
39. A perfect gas molecules has only translational K.E.

$$\therefore \quad E = \frac{3}{2} k_B T$$

40. Degrees of freedom of a gas depend on
    (*i*) Atomicity of the gas molecules.
    (*ii*) Shape of the molecules.
    (*iii*) Temperature of the gas.

Scanned by CamScanner

Type B : Short Answer Questions

2 or 3 Marks Each

1. Write any *four* fundamental postulates of the kinetic theory of an ideal gas. [Himachal 08, 09 ; Delhi 03]
2. What is an ideal gas ? Why do the real gases show deviation from ideal behaviour.
3. Draw *P-V* curves showing deviations from ideal behaviour for a given mass of a gas for two different temperatures.
4. On the basis of kinetic theory of gases, explain how does a gas exert pressure.
5. Show that the pressure exerted by a gas is two-thirds of the average kinetic energy per unit volume of the gas molecules.

Or

Derive the relation between pressure and mean kinetic energy of a gas. [Himachal 05]

6. Calculate the total random kinetic energy for one mole of a gas at constant volume. [Delhi 97]
7. Discuss the kinetic interpretation of temperature. Hence define absolute zero of temperature [Himachal 05, 07C, 09C]
8. Show that the average kinetic energy of a gas molecule is directly proportional to the absolute temperature of the gas. Hence give the kinetic interpretation of temperature. [Delhi 14 ; Central Schools 14]
9. Explain the concept of absolute zero of temperature on the basis of kinetic theory of gases. [Delhi 09]
10. Derive Boyle's law on the basis of kinetic theory of gases.
11. Derive Charles' law on the basis of kinetic theory of gases.
12. Deduce Gay Lussac's law on the basis of kinetic theory of gases.
13. Derive perfect gas equation from kinetic theory of gases.
4. State Avogadro's law. Deduce it on the basis of kinetic theory of gases.

Deduce Grahm's law of diffusion from kinetic theory of gases using expression for pressure.

State Dalton's law of partial pressures. Deduce it from the kinetic theory of gases.

)raw a curve showing Maxwell's distribution of olecular speeds of a gas at a given temperature. dicate on this curve

(*i*) most probable velocity,

(*ii*) average velocity and

(*iii*) root mean square velocity.

18. Define average, root mean square and most probable speeds. Arrange them in the decreasing order of their values.
19. What do you mean by degrees of freedom ? Show that the number of degrees of freedom of a system consisting of *N*-particles and having *k* independent relation between them is $(3N - k)$.
20. Define degrees of freedom. Calculate the degrees of freedom of monoatomic, diatomic and triatomic gas molecules. [Himachal 07C]
21. State the law of equipartition of energy. Determine the values of γ for diatomic gas N_2 at moderate temperature. [Delhi 11]
22. State the law of equipartition of energy and prove that for a diatomic gas at high temperature, the ratio of the two specific heats is $9/7$.
23. Using the law of equipartition of energy, show that for an ideal gas having f degrees of freedom,
$$\gamma = 1 + \frac{2}{f}.$$
24. State the law of equipartition of energy and using this find the relation for the total internal energy of a mole of monoatomic gases. [Delhi 08]
25. What is Dulong and Petit's law ? Illustrate it graphically also. [Delhi 95]
26. What is Brownian motion ? How can it be accounted for ?
27. State four factors on which Brownian motion depends.
28. Derive the expression for pressure exerted by an ideal gas using kinetic theory of gases. Hence define root mean square velocity. [Himachal 07C ; Central Schools 09 ; Delhi 12]
29. Using the expression for pressure exerted by a gas, deduce Avogadro's law and Graham's law of diffusion. [Chandigarh 08]
30. (*i*) Define Absolute zero.

(*ii*) Deduce the dimensional formula for *R*, using ideal gas equation
$$PV = nRT.$$
(*iii*) Find the degree of freedom of a monoatomic gas. [Central Schools 07 ; Delhi 10]

Scanned by CamScanner

31. State the number of degrees of freedom possessed by a monoatomic molecule in space. Also give the expression for total energy possessed by it at a given temperature. Hence give the total energy of the atom at 300 K. **[Centrl Schools 08]**

32. State the law of equipartition of energy. How much kinetic energy is associated with each molecule of a (*i*) monoatomic, (*ii*) diatomic ideal gas, at T kilvin temperature. **[Central Schools 13]**

33. Drive the relation between the ratio of two specific heats of gas and degree of freedom. **[Central Schools 09]**

34. State the law of equipartition of energy of a dynamic system and use it to find the value of the ratio of two specific heats of monoatomic and diatomic gas molecules. **[Himachal 09C]**

Answers

1. Refer answer to Q. 8 on page 13.7.
2. Refer answer to Q. 7 on page 13.3
3. See Fig. 13.5 on page 13.3
4. Refer answer to Q. 9 on page 13.7.
5. Refer answer to Q. 11 on page 13.8.
6. Refer answer to Q. 12 on page 13.8.
7. Refer answer to Q. 12 on page 13.8.
8. Refer answer to Q. 12 on page 13.8.
9. Refer answer to Q. 12 on page 13.8.
10. Refer answer to Q. 13 on page 13.11.
11. Refer answer to Q. 14 on page 13.12.
12. Refer answer to Q. 15 on page 13.12.
13. Refer answer to Q. 16 on page 13.12.
14. Refer answer to Q. 17 on page 13.12.
15. Refer answer to Q.18 on page 13.12.
16. Refer answer to Q. 19 on page 13.13.
17. See Fig. 13.11 on page 13.13.
18. Refer answer to Q.22 on page 13.14.
19. Refer answer to Q.23 on page 13.17.
20. Refer answer to Q. 24 on page 13.17.
21. Refer answer to Q. 26(*ii*)(*a*) on page 13.19.
22. Refer answer to Q. 26(*ii*)(*b*) on page 13.19.
23. Refer answer to Q. 27 on page 13.19.
24. Refer answer to Q. 25 on page 13.18 and Q. 26(*i*) on page 13.19.
25. Refer answer to Q. 28 on page 13.23 and see Fig. 13.15.
26. Refer answer to Q. 33 on page 13.26.
27. Refer answer to Q. 33 on page 13.26.
28. Refer answer to Q. 10 on page 13.17.
29. Refer answer to Q. 17 and Q. 18 on page 13.12.
30. (*i*) Absólute zero is defined as that temperature at which all molecular motion stops.
    (*ii*) Refer to the solution of Problem 22 on page 13.28.
    (*iii*) Refer answer to Q. 24(*a*) on page 13.17.
31. No. of degrees of freedom of a monoatomic molecule = 3

    Total energy possessed by a monoatomic molecule

    $$\overline{E} = \frac{3}{2} k_B T$$

    At temperature $T = 300$ K,

    $$\overline{E} = \frac{3}{2} \times 138 \times 10^{23} \times 300 = 621 \times 10^{23} \text{ J.}$$

32. (*i*) K.E. of each monoatomic gas molecule is $\frac{3}{2} k_B T$.
    (*ii*) K.E. of each diatomic gas molecule is $\frac{5}{2} k_B T$.
33. Refer answer to Q. 27 on page 13.19.
34. Refer answer to Q. 26(*i*) and (*ii*) on page 13.19.

Type C : Long Answer Questions **5 Marks Each**

1. What are the basic assumptions of kinetic theory of gases ? On their basis derive an expression for the pressure exerted by an ideal gas. **[Himachal 05, 08C ; Delhi 13]**

2. State the postulates of kinetic theory of gases. Derive an expression for the pressure exerted by an ideal gas. Molar volume is the volume occupied by 1 mole of any (ideal) gas at standard temperature and pressure (STP = 1 atm pressure, 0°C). Show that it is 22.4 litres. **[Delhi 06]**

Scanned by CamScanner

3. *Prove that the pressure exerted by a gas is $P = \frac{1}{3}\rho c^2$, where ρ is the density and c is the root mean square velocity.* **[Himachal 04 ; Central Schools 09, 12]**

4. *What is meant by degrees of freedom ? State the law of equipartition of energy. Hence calculate the* **values of molar specific heats at constant volume** *and pressure for monoatomic, diatomic and triatomic gases.* **[Himachal 02]**

5. **State the law of equipartition energy of a dynamic system and use it to find the values of average energy and the ratio of the specific heats of (a) monoatomic, (b) diatomic and (c) triatomic molecules.** **[Delhi 14 ; Central Schools [illegible]]**

6. **What is meant by mean free path of a molecule ? Derive an expression for it. On which factors does it depend ?** **[Himachal [illegible]]**

Answers

1. *Refer answer to Q. 8 and Q. 10 on page 13.7.*
2. *Refer answer to Q. 8 and Q. 10 on page 13.7 and see answer of NCERT Exercise 13.2 on page 13.35.*
3. *Refer answer to Q. 10 on page 13.7.*
4. **Refer to points 19, 20 and 21 of Glimpses.**
5. **Refer to points 20 and 21 of Glimpses.**
6. **Refer answer to Q. 31 on page 13.24.**

Scanned by CamScanner

Competition Section

Kinetic Theory of Gases

GLIMPSES

1. **Boyle's law.** It states that at constant temperature, the volume of a given mass of a gas is inversely proportional to its pressure.

$$V \propto 1/P \quad \text{or} \quad PV = \text{constant}$$

or $P_1V_1 = P_2V_2$ (For constant m and T)

2. **Charles' law.** It states that if the pressure remains constant, then the volume of a given mass of a gas increases or decreases by $1/273.15$ of its volume at 0°C for each 1°C rise or fall of temperature. Mathematically,

$$V_t = V_0\left(1+\frac{t}{273.15}\right) = V_0\left(\frac{273.15+t}{273.15}\right)$$

If we put $273.15 + t = T$ and $273.15 = T_0$, then

$$\frac{V_t}{T} = \frac{V_0}{T_0} \quad \text{or} \quad \frac{V}{T} = \text{constant}$$

So Charles' law can also be stated as follows :

At constant pressure, the volume of a given mass of a gas is directly proportional to its absolute temperature.

3. **Gay Lussac's law.** It states that if the volume remains constant, then the pressure of the given mass of a gas increases or decreases by $1/273.15$ of its pressure at 0°C for each 1°C rise or fall of temperature. Mathematically,

$$P_t = P_0\left(1+\frac{t}{273.15}\right) = P_0\left(\frac{273.15+t}{273.15}\right)$$

or $$\frac{P_t}{P_0} = \frac{T}{T_0}$$

or $$\frac{P}{T} = \text{constant}$$

So Gay Lussac's law can also be stated as follows :

At constant volume, the pressure of a given mass of a gas is directly proportional to its absolute temperature.

4. **Ideal gas equation.** For n moles of a gas,

$$PV = nRT \quad \text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

For 1 mole of a gas,

$$PV = RT$$

5. **Universal gas constant.** It signifies the work done by the gas per mole per kelvin.

$$R = 8.31 \text{ J mol}^{-1}\text{ K}^{-1}.$$

6. **Boltzmann's constant.** It is the gas constant per molecule of a gas. If N is Avogadro's number, then

$$k_B = \frac{R}{N} = 1.38 \times 10^{-23} \text{ J molecule}^{-1}\text{K}^{-1}.$$

7. **Ideal or perfect gas.** A gas which obeys gas laws strictly is an ideal or perfect gas. The molecules of such a gas are of point size and there is no force of attraction between them.

The actual real gases obey the ideal gas equation only approximately at low pressures and high temperatures.

8. **Assumptions of kinetic theory of gases :**

(*i*) All gases consist of molecules. The molecules are rigid, elastic spheres identical in all respects for a given gas and different for different gases.

(*ii*) The size of a molecule is negligible compared with the average distance between two molecules.

(*iii*) The molecules are in a state of continuous random motion, moving in all directions with all possible velocities.

(*iv*) During the random motion, the molecules collide with one another and with the walls of the vessel.

Scanned by CamScanner

(v) The collisions are perfectly elastic and there are no forces of attraction or repulsion between them.

(vi) Between two collisions, a molecule moves in a straight path with a uniform velocity.

(vii) The collisions are almost instantaneous.

(viii) The molecular density remains uniform throughout the gas.

9. **Pressure exerted by a gas.** According to kinetic theory of gases, the pressure exerted by a gas of mass M and volume V or density ρ is given by

$$P = \frac{1}{3}\frac{M}{V}\overline{v^2} = \frac{1}{3}\rho\,\overline{v^2} = \frac{1}{3}mn\,\overline{v^2}$$

Here n is the number of molecules per unit volume, m the mass of each molecule and $\overline{v^2}$ is the mean of square speed.

10. **Average kinetic energy of a gas.** Let M be the molecular mass and V the molar volume of a gas. Let m be the mass of each molecule. Then

(i) Mean K.E. per mole of a gas,

$$E = \frac{1}{2}M\overline{v^2} = \frac{3}{2}PV = \frac{3}{2}RT = \frac{3}{2}k_B N_A T$$

(ii) Mean K.E. per molecule of a gas,

$$\bar{E} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T$$

This is the *kinetic interpretation of temperature.* That is, the temperature of a gas is a measure of the average kinetic energy of a molecule.

(iii) K.E. of 1 gram of gas $= \frac{1}{2}\overline{v^2} = \frac{3}{2}\frac{RT}{M}$.

11. **Avogadro's law.** It states that equal volume of all gases under similar conditions of temperature and pressure contain equal number of molecules.

12. **Avogadro's number.** It is the number of particles present in one mole of a substance. Its most accepted value is

$$N_A = 6.0225 \times 10^{23}\ \text{mole}^{-1}.$$

13. **Graham's law of diffusion.** *It states that the rate of diffusion of a gas is inversely proportional to the square root of its density.*

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

14. **Dalton's law of partial pressures.** It states that the total pressure exerted by a mixture of non-reacting gases occupying a given volume is equal to the sum of the partial pressures which gas would exert if it alone occupied the same volume at the given temperature.

$$P = P_1 + P_2 + P_3 + \ldots$$

15. **Average speed.** *It is defined as the arithmetic mean of the speeds of the molecules of a gas at a given temperature.*

$$\bar{v} = \frac{v_1 + v_2 + v_3 + \ldots + v_n}{n}$$

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8PV}{\pi M}}.$$

16. **Root mean square speed.** It is defined as the square root of the mean of the squares of the speeds of the individual molecules of a gas.

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_n^2}{n}}$$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$$

17. **Most probable speed.** It is defined as the speed possessed by the maximum number of molecules in a gas sample at a given temperature.

$$v_{mp} = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

18. **Relations between $\bar{v}$, v_{rms} and v_{mp}.**

$$\bar{v} = 0.92\,v_{rms}$$

$$v_{mp} = 0.816\,v_{rms}$$

$$v_{rms} : \bar{v} : v_{mp} = 1.73 : 1.60 : 1.41$$

Clearly, $v_{rms} > \bar{v} > v_{mp}$.

19. **Degrees of freedom.** The degrees of freedom of a dynamical system are defined as the total number of coordinates or independent quantities required to describe completely the position and configuration of the system.

If N = number of particles in the system
k = number of independent relations between the particles

then the number of degrees of freedom of the system is

$$f = 2N - k.$$

A monoatomic molecule gas has 3 degrees of freedom, a diatomic gas molecule has 5 degrees of freedom. At high temperature, a diatomic molecule has 7 degrees of freedom.

20. **Law of equipartition of energy.** It states that in any dynamical system in thermal equilibrium, the energy of the system is equally divided amongst its various degrees of freedom and the energy associated with each degree of freedom is $\frac{1}{2}k_BT$,

where k_B is Boltzmann's constant and T is the absolute temperature of the system.

Scanned by CamScanner

21. **Internal energies and specific heats of mono-atomic, diatomic and polyatomic gases.** The law of equipartition of energy leads to the following results :

(*i*) For a gas of polyatomic molecules having f degrees of freedom,

Energy associated with 1 mole of gas,

$$U = \frac{f}{2} RT \qquad C_v = \frac{f}{2} R$$

$$C_p = \left(1 + \frac{f}{2}\right) R \qquad \gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

(*ii*) For a monoatomic gas, $f = 3$, so

$$U = \frac{3}{2} RT, \qquad C_v = \frac{3}{2} R,$$

$$C_p = \frac{5}{2} R, \qquad \gamma = 1.66.$$

(*iii*) For a diatomic gas with no vibrational mode, $f = 5$, so

$$U = \frac{5}{2} RT, \qquad C_v = \frac{5}{2} R,$$

$$C_p = \frac{7}{2} R, \qquad \gamma = 1.4.$$

(*iv*) For a diatomic gas with vibrational mode, $f = 7$, so

$$U = \frac{7}{2} RT, \qquad C_v = \frac{7}{2} R,$$

$$C_p = \frac{9}{2} R, \qquad \gamma = \frac{9}{7} = 1.28.$$

(*v*) For a triatomic gas of non-linear molecules, $f = 6$, so

$$U = 3RT, \qquad C_v = 3R,$$

$$C_p = 4R, \qquad \gamma = 1.33$$

(*vi*) For a triatomic gas of linear molecules, $f = 7$, so

$$U = \frac{7}{2} RT, \qquad C_v = \frac{7}{2} R,$$

$$C_p = \frac{9}{2} R, \qquad \gamma = 1.28.$$

22. **Mean free path.** It is the average distance covered by a molecule between two successive collisions. It is given by

$$\bar{\lambda} = \frac{1}{\sqrt{2}\, n\pi d^2}$$

where n is the number density and d is the diameter of the molecule.

23. **Brownian motion.** It provides a direct evidence for the existence of molecules and their motion. The zig-zag motion of gas molecules is Brownian motion because it occurs due to random collision of molecules. But this motion cannot be seen. However, the zig-zag motion of pollen grains ($\approx 10^{-5}$ m) can be seen under a microscope.

IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

1. A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (relative molar mass 4) at a temperature $2T$ has a pressure of

(*a*) $P/8$ (*b*) P
(*c*) $2P$ (*d*) $8P$ [IIT 97]

2. The average translational kinetic energy of O_2 molecules (relative molar mass 32) at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 molecules (relative molar mass 28) in eV at the same temperature is

(*a*) 0.0015 (*b*) 0.003
(*c*) 0.048 (*d*) 0.768 [IIT 97]

3. The average translational energy and the r.m.s. speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 ms^{-1} respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)

(*a*) 12.42×10^{-21} J, 968 ms^{-1}
(*b*) 8.78×10^{-21} J, 684 ms^{-1}
(*c*) 6.21×10^{-21} J, 968 ms^{-1}
(*d*) 12.42×10^{-21} J, 684 ms^{-1} [IIT 97]

4. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(*a*) 4RT (*b*) 9RT
(*c*) 11RT (*d*) 15RT [IIT 99]

5. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is

(*a*) $\sqrt{2/7}$ (*b*) $\sqrt{1/7}$
(*c*) $(\sqrt{3})/5$ (*d*) $(\sqrt{6})/5$ [IIT 99]

Scanned by CamScanner

6. From the following statements, concerning ideal gas at any given temperature T, select the correct one(s) :

(a) The co-efficient of volume expansion at constant pressure is the same for all ideal gases

(b) The average translational kinetic energy per molecule of oxygen gas is $3kT$, k being Boltzmann constant

(c) The mean-free path of molecules increases with increases in the pressure

(d) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different. [IIT 95]

7. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(a) $\frac{1}{T}$ (b) $\frac{2}{T}$

(c) $\frac{3}{T}$ (d) $\frac{4}{T}$ [IIT 08]

8. A real gas behaves like an ideal gas if its

(a) Pressure and temperature are both high

(b) Pressure is high and temperature is low

(c) Pressure and temperature are both low

(d) Pressure is low and temperature is high [IIT 2010]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

9. At room temperature, the r.m.s. speed of the molecules of a certain diatomic gas is found to be 1,920 ms^{-1}. The gas is

(a) H_2 (b) F_2

(c) O_2 (d) Cl_2 [IIT 84]

10. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the r.m.s. velocity of the gas molecules is v; at 480 K, it becomes

(a) $4v$ (b) $2v$

(c) $v/2$ (d) $v/4$ [IIT 96]

11. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of the molecule is m. Then

(a) no molecule can have a speed greater than $\sqrt{2}\, v_{rms}$

(b) no molecule can have a speed less than $v_p/\sqrt{2}$

(c) $v_p < \bar{v} < v_{rms}$

(d) the average kinetic energy of a molecule is $\frac{3}{4}mv_p^2$ [IIT 98]

12. A vessel contains a mixture of 1 mole of oxygen and 2 moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is

(a) 1 : 1 (b) 1 : 2

(c) 2 : 1

(d) depends on the moment of inertia of the two molecules. [IIT 98]

13. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied, which increases the internal energy of the gas is

(a) 2/5 (b) 3/5

(c) 3/7 (d) 5/7. [IIT 90]

14. If one mole of monoatomic gas $(\gamma = 5/3)$ is mixed with one mole of a diatomic gas $(\gamma = 7/5)$, the value of adiabatic exponent γ for mixture is

(a) 1.35 (b) 1.40

(c) 1.50 (d) 1.75. [IIT 88]

15. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is

(a) $\frac{2}{3}P$ (b) P

(c) $\frac{3}{2}P$ (d) $2P$. [IIT 98]

16. Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1, that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is

(a) $\frac{v_1 + v_2}{2}$ (b) v_1

(c) $(v_1 . v_2)^{1/2}$ (d) $\sqrt{\frac{3kT}{M}}$

where M is the mass of an oxygen molecule.

17. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively.

Scanned by CamScanner

Then

(a) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(b) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(c) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(d) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas [IIT 09]

Answers and Explanations

1. (c) $PV = nRT$ or $P = \frac{nRT}{V}$

For same n, R and V, $P \propto T$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \quad \text{or} \quad \frac{P_2}{P} = \frac{2T}{T} \quad \text{or} \quad P_2 = 2P.$$

2. (c) Average translational K.E. of a molecule

$$= \frac{3}{2}kT$$

It depends on temperature and not on molecular mass. Hence average translational K.E. for N_2 will also be 0.048 eV.

3. (d) Average translational K.E. $= \frac{3}{2}kT \propto T$

$$\text{r.m.s. speed} = \sqrt{\frac{3RT}{M}} \propto \sqrt{T}$$

When temperature increases from 300 K to 600 K, average translational K.E. increases 2 times and r.m.s. speed increases $\sqrt{2}$ or 1.414 times.

$\therefore$ Average translational K.E.

$$= 2 \times 6.21 \times 10^{-21} = 12.42 \times 10^{-21}\ \text{J}$$

r.m.s. speed $= 1.414 \times 484 = \mathbf{684\ ms^{-1}}$.

4. (c) For two moles of diatomic nitrogen with no vibrational mode,

$$U_1 = 2 \times \frac{5}{2}RT = 5RT$$

For four moles of monoatomic argon,

$$U_2 = 4 \times \frac{3}{2}RT = 6RT \quad \therefore \quad U = U_1 + U_2 = \mathbf{11\,RT}.$$

5. (c) $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v(N_2)}{v(He)} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \cdot \frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3} \times \frac{4}{28}} = \frac{\sqrt{3}}{5}.$$

6. (a) Coefficient of cubical expansion of an ideal gas, $\gamma = \frac{1}{T}$.

At a given T, γ is same for all ideal gases.

(b) The average translational K.E. per molecule is $(3/2)\,kT$ and not $3kT$.

(c) With increase of pressure, volume decreases. The collisions become more frequent. The mean free path decreases.

(d) The average K.E. does not depend on the nature of the gas, so each component of the gasesous mixture has the same average translational kinetic energy.

Hence only option (a) is correct.

7. (c) Given $PT^2 = C$

But $PV = nRT$ or $P = \frac{nRT}{V}$

$$\therefore \quad \frac{nRT^3}{V} = C \quad \text{or} \quad V = \frac{nRT^3}{C}$$

Hence $\frac{dV}{dT} = \frac{3nRT^2}{C} = \frac{3V}{T}$ [Put $T^3 = \frac{CV}{nR}$]

or $\gamma = \frac{dV/dT}{V} = \frac{3}{T}$.

8. (d) Real gas behaves like an ideal gas if its pressure is low and temperature is high. This ensures large intermolecular separation and no interaction.

9. (a) $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore \quad M = \frac{3RT}{(v_{rms})^2} = \frac{3 \times 8.3 \times 300}{(1920)^2} = 2$$

Hence the gas is H_2.

10. (b) $\frac{v_{rms(2)}}{v_{rms(1)}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{480}{120}} = 2.$

$$v_{rms(2)} = 2v_{rms(1)} = 2v.$$

11. (c), (d) $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\bar{v} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{2.5\,RT}{M}}$$

$$v_p = \sqrt{\frac{2RT}{M}}$$

Clearly, $v_p < \bar{v} < v_{rms}$

$$v_{rms} = \sqrt{\frac{3}{2}}\,v_p$$

Scanned by CamScanner

∴ Average K.E. of a gas molecule

$$=\frac{1}{2}mv_{rms}^2=\frac{1}{2}m\left(\sqrt{\frac{3}{2}}v_p\right)^2=\frac{3}{4}mv_p^2$$

Hence options (*c*) and (*d*) are correct.

12. (*a*) Both gases are diatomic. Both have two rotational degrees of freedom at 300 K.

Average rotational K.E. per molecule of each gas will be same

$$=2\times\frac{1}{2}kT=1\,kT.$$

Hence the required ratio is **1 : 1**.

13. (*d*) Required fraction,

$$\frac{\Delta U}{\Delta Q}=\frac{nC_V\,dT}{nC_p\,dT}=\frac{C_V}{C_P}=\frac{1}{\gamma}$$

For a diatomic gas, $\gamma=7/5$

$$\therefore \quad \frac{\Delta U}{\Delta Q}=\frac{5}{7}.$$

14. (*c*) Refer to the solution of Example 36 on page 13.21.

15. (*a*) At a given temperature, $PV=\text{constant}$

$$\therefore \quad P\Delta V+V\Delta P=0$$

or $$P=-\frac{V\Delta P}{\Delta V}=-\frac{\Delta P}{\Delta V/V}=\text{Isothermal bulk modulus}$$

16. (*b*) $$\bar{v}=\sqrt{\frac{8RT}{\pi M}}\propto\sqrt{T}\quad\text{(for a given gas)}$$

As vessels *A* and *C* have the same temperature,

Average speed of O_2 in C

= Average speed of O_2 in $A=v_1$.

17. (*b*), (*d*) $C_P-C_V=R$ for all ideal gases.

Hence option (*a*) is incorrect.

$$(C_P+C_V)_{\text{Diatomic}}=\frac{7R}{2}+\frac{5R}{2}=6R$$

$$(C_P+C_V)_{\text{Monoatomic}}=\frac{5R}{2}+\frac{3R}{2}=4R$$

∴ (C_P+C_V) is larger for a diatomic gas.

Hence option (*b*) is correct.

$$\left(\frac{C_P}{C_V}\right)_{\text{Diatomic}}=\frac{7}{5}=1.4$$

$$\left(\frac{C_P}{C_V}\right)_{\text{Monoatomic}}=\frac{5}{3}=1.67$$

∴ C_P/C_V is smaller for a diatomic gas.

Hence option (*c*) is incorrect.

$$(C_PC_V)_{\text{Diatomic}}=\frac{35}{4}R^2$$

$$(C_PC_V)_{\text{Monoatomic}}=\frac{15}{2}R^2$$

∴ Product C_PC_V is larger for a diatomic gas.

Hence option (*d*) is correct.

AIEEE

1. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volumes (V_1, V_2) and pressures (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(*a*) T_1+T_2 (*b*) $\frac{T_1+T_2}{2}$

(*c*) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (*d*) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$

[AIEEE 04]

2. An insulated container of gas has two chambers separated by an insulation partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(*a*) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$ (*b*) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$

(*c*) $\frac{P_1V_1T_1+P_2V_2T_2}{P_1V_1+P_2V_2}$ (*d*) $\frac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$

[AIEEE

3. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will

(*a*) increase (*b*) decrease

(*c*) remain the same

(*d*) decreases for some, while increases for others

[AIEEE

4. At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(*a*) –73 K (*b*) 3 K

(*c*) 20 K (*d*) 80 K

[AIEEE

Scanned by CamScanner

5. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p / C_v for the gas is

(a) 4/3 (b) 2
(c) 5/3 (d) 3/2 [AIEEE 03]

6. One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture ? γ denotes the ratio of specific heat at constant pressure to that at constant volume.

(a) 3/2 (b) 23/15
(c) 35/23 (d) 4/3 [AIEEE 04]

7. 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of gas with $\gamma = 5/3$, then value of γ of the resulting mixture is

(a) 7/5 (b) 2/5
(c) 3/2 (d) 12/7 [AIEEE 02 ; AFMC 01]

8. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio of the two specific heats of the mixture is

(a) 1.4 (b) 1.54
(c) 1.59 (d) 1.62 [AIEEE 05]

9. The work of 146 kJ is performed in order to compress one kilomole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is

(a) a mixture of monoatomic and diamtomic
(b) monoatomic (c) diatomic
(d) triatomic [AIEEE 06]

10. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion ?

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×0^4 J (d) 7×10^4 J [AIEEE 09]

11. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is

(a) $T_f = \frac{5}{2}T_0$ (b) $T_f = \frac{3}{7}T_0$
(c) $T_f = \frac{7}{3}T_0$ (d) $T_f = \frac{3}{2}T_0$. [AIEEE 06]

12. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is

(a) $\dfrac{(T_1 + T_2 + T_3)}{3}$ (b) $\dfrac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

(c) $\dfrac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(d) $\dfrac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ [AIEEE 2011]

13. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(a) $\dfrac{(\gamma-1)}{2(\gamma+2)R} Mv^2$ K (b) $\dfrac{(\gamma-1)}{2\gamma R} Mv^2$ K

(c) $\dfrac{\gamma Mv^2}{2R}$ K (d) $\dfrac{(\gamma-1)}{2R} Mv^2$ K [AIEEE 2011]

Answers and Explanations

1. (a) Refer to the solution of Problem 4 on page 13.32.

2. (b) Refer to the solution of Problem 4 on page 13.32.

3. (c) The centre of mass of the gas molecules moves with uniform speed along with the lorry. As these is no change in relative motion, the translational kinetic energy and hence the temperature of the gas molecules will remain same.

4. (c) Refer to the solution of Example 24 on page 13.15.

5. (d) $P \propto T^3$ or $P = \text{constant} \times T^3$

or $PT^{-3} = \text{constant}$.

For an adiabatic process,

$$PT^{\frac{\gamma}{1-\gamma}} = \text{constant}$$

$$\therefore \quad \gamma = \frac{3}{2}.$$

6. (a) Refer to the solution of Example 36 on page 13.21.

7. (c) Refer to the solution of Example 36 on page 13.21.

8. (d) Refer to the solution Example 37 on page 13.22.

9. (c) Refer to the solution of Problem 4 on page 12.42.

Scanned by CamScanner

10. (*b*) Thermal energy corresponds to internal energy

Mass $= 1$ kg

Density $= 4$ kg m^{-3}

Volume $= \dfrac{\text{Mass}}{\text{Density}} = \dfrac{1}{4}$ m^3

Pressure $= 8 \times 10^4$ Nm^{-2}

Internal energy of the diatomic gas

$$E = \frac{5}{2}PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4}\ \text{J}$$

$$= 5 \times 10^4\ \text{J}.$$

11. (*d*) Refer to the solution of Problem 1 on page 13.31.

12. (*b*) $$P_1V_1 + P_2V_2 + P_3V_3 = PV$$

$$n_1k_BT_1 + n_2k_BT_2 + n_3k_BT_3 = (n_1 + n_2 + n_3)k_BT_{mix}$$

$$T_{mix} = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

13. (*d*) Loss in K.E. of the gas $= \frac{1}{2}mv^2$

Heat gained by gas $= nC_v\Delta T$

$$\therefore \quad \frac{1}{2}mv^2 = nC_v\Delta T = \frac{m}{M} \cdot \frac{R}{\gamma - 1}\Delta T$$

or $$\Delta T = \frac{(\gamma - 1)}{2R}Mv^2\ \text{K}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

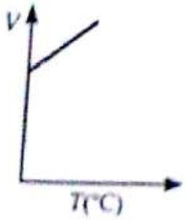
1. An absolute zero is the temperature at which
(*a*) water solidifies (*b*) all gases become liquid
(*c*) rms velocity become zero
(*d*) none of the above. [DCE 99]

2. At Boyle's temperature
(*a*) Joule's effect is positive
(*b*) *b* of van der Waals' equation is zero
(*c*) gas obeys Boyle's law
(*d*) none of the above. [DCE 97]

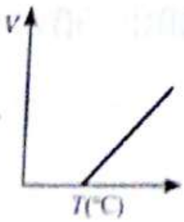
3. The equation of state for 5 g of oxygen at a pressure P and temperature T, when occupying a volume V, will be
(*a*) $PV = (5/32)RT$ (*b*) $PV = 5RT$
(*c*) $PV = (5/2)RT$ (*d*) $PV = (5/16)RT$ [DCE 06]

4. Volume-temperature graph at atmospheric pressure for a monoatomic gas (V in m^3, T in °C) is

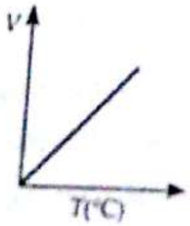
(*a*)



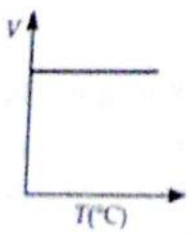
(*b*)



(*c*)



(*d*)



[DCE 05]

5. Surface of the lake is at 2°C and depth of the lake is 20 m. Find the temperature of the bottom of the lake
(*a*) 2°C (*b*) 3°C
(*c*) 4°C (*d*) none of these.
[IPUEE 00]

6. The degree of freedom in case of an monoatomic gas is
(*a*) 1 (*b*) 3
(*c*) 5 (*d*) none of these. [DCE 99]

7. If a gas has n degrees of freedom, ratio of specific heats of gas is
(*a*) $\dfrac{1+n}{2}$ (*b*) $1 + \dfrac{1}{n}$
(*c*) $1 + \dfrac{n}{2}$ (*d*) $1 + \dfrac{2}{n}$
[DCE 00]

8. One mole of monoatomic gas and three moles of diatomic gas are put together in a container. The molar specific heat (in JK^{-1}mol^{-1}) at constant volume ($R = 8.3$ JK^{-1}mol^{-1}) is
(*a*) 18.7 (*b*) 18.9
(*c*) 19.2 (*d*) none of these.
[DCE 04]

9. A monoatomic gas is suddenly compressed to (1/8)th of its initial volume adiabatically. The ratio of its final pressure to the initial pressure is (Given : the ratio of the specific heats of the given gas to be 5/3)
(*a*) 32 (*b*) 40/3
(*c*) 24/5 (*d*) 8 [DPMT 96 ; IPUEE 07]

Scanned by CamScanner

10. Two gases are at absolute temperatures 300 K and 350 K respectively. Ratio of average kinetic energy of their molecules is

(a) 7 : 6
(b) 6 : 7
(c) 36 : 49
(d) 49 : 36 [DCE 02]

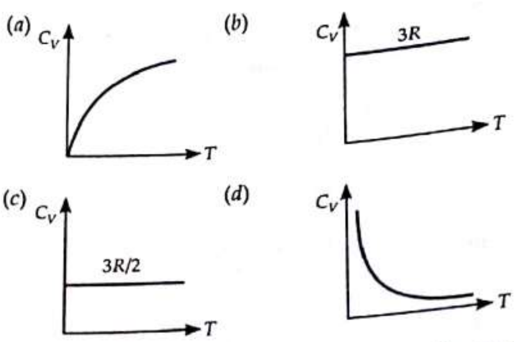
11. The gas having average speed four times as that of SO_2 (molecular mass 64) is

(a) He (molecular mass 64)
(b) O_2 (molecular mass 4)
(c) H_2 (molecular mass 32)
(d) CH_4 (molecular mass 16) [DCE 05]

12. The temperature of a given mass is increased from 27°C to 327°C. The rms velocity of the molecules increases

(a) $\sqrt{2}$ times
(b) 2 times
(c) $2\sqrt{2}$ times
(d) 4 times. [DCE 2K, 03]

13. Graph of specific heat at constant volume for a monoatomic gas is [DCE 08]



14. The mean kinetic energy of one mole of gas per degree of freedom (on the basis of kinetic theory of gases) is

(a) $\frac{1}{2}kT$ (b) $\frac{3}{2}kT$
(c) $\frac{3}{2}RT$ (d) $\frac{1}{2}RT$ [DCE 09]

15. The ratio of the vapour densities of two gases at a given temperature is 9 : 8. The ratio of the rms velocities of their molecules is

(a) $3 : 2\sqrt{2}$ (b) $2\sqrt{2} : 3$
(c) 9 : 8 (d) 8 : 9 [DCE 09]

Answers and Explanations

1. (c) According to kinetic theory of gases, absolute zero is that temperature at which all molecular motion ceases. Hence at $T = 0\,\text{K}$, $v_{rms} = 0$.

2. (c) At Boyle's temperature, a gas obeys Boyle's law to a high degree of accuracy. Above and below this temperature, the gas deviates from Boyle's law.

3. (a) Number of moles in 5 g of oxygen $= \frac{5}{32}$

As $PV = nRT \quad \therefore \quad PV = \frac{5}{32}RT.$

4. (c) At constant pressure, $V \propto T$. Hence the correct option is (c).

5. (d) $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\frac{1 \times 10^5}{273 + 2} = \frac{h\rho g + 1 \times 10^5}{T_2}$$

$$\frac{1 \times 10^5}{275} = \frac{20 \times 10^3 \times 10 + 1 \times 10^5}{T_2}$$

$$T_2 = 275 \times 3 = 825\text{ K} = 552\,°\text{C}$$

6. (b) A monoatomic gas has 3 degrees of freedom due to translatory motion.

7. (d) $\gamma = 1 + \frac{2}{n}$

8. (a) $C_V = \frac{R}{\gamma - 1}$

For monoatomic gas, $C_V = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2}R$

For diatomic gas, $C_V = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2}R$

$$C_V\,(\text{mixture}) = \frac{nC_V + n'C_V'}{n + n'}$$

$$= \frac{1 \times \frac{3}{2}R + 3 \times \frac{5}{2}R}{1 + 3}$$

$$= \frac{9}{4}R = \frac{9}{4} \times 8.3\ \text{JK}^{-1}\text{mol}^{-1}$$

$$= \mathbf{18.7\ JK^{-1}mol^{-1}}.$$

Scanned by CamScanner

9. (*a*) In an adiabatic process,

PV^{γ} = constant

$$\therefore \quad \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} = \left(\frac{1}{8}\right)^{5/3} = \left(\frac{1}{2^3}\right)^{5/3} = \frac{1}{32}$$

$$\therefore \quad \frac{P_2}{P_1} = 32.$$

10. (*b*) $\frac{E_1}{E_2} = \frac{T_1}{T_2} = \frac{300}{350} = \frac{6}{7} = 6:7.$

11. (*a*) $v_{rms} = \sqrt{\frac{3RT}{M}}$ *i.e.,* $v_{rms} \propto \frac{1}{\sqrt{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}} \quad \text{or} \quad \frac{v}{4v} = \sqrt{\frac{M_2}{64}}$$

$$\therefore \quad M_2 = 4$$

Hence the gas is He which has molecular mass 4.

12. (*a*) $v_{rms} \propto \sqrt{T}$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{273+327}{273+27}} = \sqrt{\frac{600}{300}} = \sqrt{2}$$

$$v_2 = \sqrt{2}\, v_1.$$

13. (*c*) For a monotomic gas,

$$C_V = \frac{R}{\gamma - 1} = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2}R.$$

Hence the correct option is (*c*).

14. (*d*) Mean kinetic energy per mole of a gas per degree of freedom = (1/2) *RT*.

15. (*b*) At a given temperature, $v_{rms} \propto \frac{1}{\sqrt{\rho}}$

$$\therefore \quad \frac{v_{rms}(1)}{v_{rms}(2)} = \sqrt{\frac{8}{9}} = 2\sqrt{2} : 2$$

AIIMS Entrance Exam

1. For Boyle's law to hold good, the gas should be

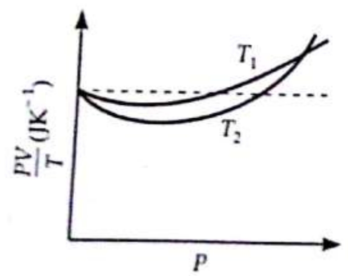
(*a*) perfect and of constant mass and temperature

(*b*) real and of constant mass and temperature

(*c*) perfect and at constant temperature but variable mass

(*d*) real and at constant temperature but variable mass. [AIIMS 94]

2. The figure shows the plot of $\frac{PV}{nT}$ versus P for oxygen gas at two different temperatures.



Read the following statements concerning the above curves :

(*i*) The dotted line corresponds to ideal gas behaviour.

(*ii*) $T_1 > T_2$

(*iii*) The value of $\frac{PV}{nT}$ at the point, where the curves meet on the *Y*-axis is the same for all gases.

Which of the above statements is true ?

(*a*) (*i*) only (*b*) (*i*) and (*ii*) only

(*c*) all the above (*d*) none of the above.

[AIIMS (?)]

3. A gas behaves as an ideal gas at

(*a*) low pressure and high temperature

(*b*) low pressure and low temperature

(*c*) high pressure and low temperature

(*d*) high pressure and high temperature. [AIIMS 94]

4. A gas in a container *A* is in thermal equilibrium with another gas of the same mass in container *B*. If we denote the corresponding pressures and volumes by the suffixes *A* and *B*, then which of the following statements is most likely to be true :

(*a*) $P_A = P_B,\ V_A \neq V_B$ (*b*) $P_A \neq P_B,\ V_A = V_B$

(*c*) $P_A / V_A = P_B / V_B$ (*d*) $P_A V_A = P_B V_B$

[AIIMS 9?]

5. An ideal gas is heated from 27°C to 627°C at constant pressure. If initial volume was 4 m^3, then the final volume of the gas will be

(*a*) 2 m^3 (*b*) 4 m^3

(*c*) 6 m^3 (*d*) 12 m^3

[AIIMS 9?]

6. In kinetic theory of gass a molecule of mass *m* of an ideal gas collides with a wall of vessel with velocity *v*. The change in the linear momentum of the molecule is

(*a*) 2 *mv* (*b*) *mv*

(*c*) – *mv* (*d*) zero.

[AIIMS 97]

Scanned by CamScanner

7. The temperature of a gas is held constant, while its volume is decreased. The pressure exerted by the gas on the walls of the container increases, because its molecules

(a) strike the walls with higher velocities
(b) strike the walls with large force
(c) strike the walls more frequently
(d) are in contact with the walls for a shorter time. [AIIMS 96]

8. In a vessel, the gas is at a pressure P. If the mass of all the molecules is halved and their speed is doubled, then the resultant pressure will be

(a) $4P$ (b) $2P$
(c) P (d) $P/2$ [AIIMS 94]

9. We have a jar filled with gas characterised by parameters P, V and T and another jar B filled with a gas with parameters $2P$, $V/4$, $2T$; where the symbols have their usual meanings. The ratio of the number of molecules of jar A to those of jar B is

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1 [AIIMS 82]

10. The absolute zero is the temperature, at which

(a) all substances exist in solid state
(b) water freezes
(c) molecular motion ceases
(d) none of these. [AIIMS 98]

11. The average kinetic energy of a gas molecule at 27°C is 6.21×10^{-21} J. The average kinetic energy of a gas molecule at 227°C will be

(a) 52.2×10^{-21} J (b) 5.22×10^{-21} J
(c) 10.35×10^{-21} J (d) 11.35×10^{-21} J [AIIMS 99]

12. When we heat a gas-sample from 27°C to 327°C, then the initial average kinetic energy of the molecules was E. What will be the average kinetic energy ?

(a) $327\ E$ (b) $300\ E$
(c) $2\ E$ (d) $\sqrt{2}\ E$ [AIIMS 95]

13. v_{rms}, v_{av} and v_{mp} are root mean square, average and most probable speeds of molecules of a gas obeying Maxwellian velocity distribution. Which of the following statements is correct ?

(a) $v_{rms} < v_{av} < v_{mp}$ (b) $v_{rms} > v_{av} > v_{mp}$
(c) $v_{mp} < v_{rms} < v_{av}$ (d) $v_{mp} > v_{rms} > v_{av}$ [AIIMS 04]

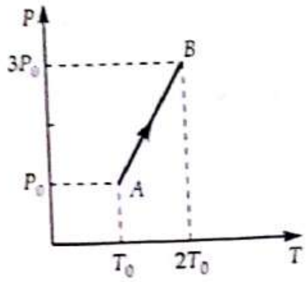
14. A bulb contains one mole of hydrogen mixed with one mole of oxygen at temperature T. The ratio of r.m.s. values of velocity of hydrogen molecules to that of oxygen molecules is

(a) 1 : 16 (b) 1 : 4
(c) 4 : 1 (d) 16 : 1 [AIIMS 94]

15. In an adiabatic change, the pressure and temperature of a monatomic gas are related as $P \propto T^c$, where c equals

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$
(c) $\frac{3}{5}$ (d) $\frac{5}{3}$ [AIIMS 07]

16. Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_0. Density at point B will be



(a) $\frac{3}{4}\rho_0$ (b) $\frac{3}{2}\rho_0$
(c) $\frac{4}{3}\rho_0$ (d) $2\rho_0$ [AIIMS 2010]

ASSERTIONS AND REASONS

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not correct explanation of the assertion.
(c) If assertion is true, but reason is false.
(d) If both assertion and reason are false.

17. **Assertion.** Air pressure in a car tyre increases during driving.

Reason. Absolute zero temperature is not zero energy temperature. [AIIMS 07]

18. **Assertion.** For an ideal gas at constant temperature, the product of the pressure and volume is a constant.

Reason. The mean square velocity of the molecules is inversely proportional to mass. [AIIMS 98]

Scanned by CamScanner

19. **Assertion.** The root mean square and most probable speeds of the molecules in a gas are the same.

Reason. The Maxwell distribution for the speed to molecules in a gas is symmetrical. [AIIMS 06]

20. **Assertion.** The ratio C_p / C_v is more for helium gas than that for hydrogen gas.

Reason. Atomic mass of helium is more than that of hydrogen. [AIIMS 96]

21. **Assertion.** The ration C_p / C_v for diatomic gas is more than that for a monoatomic gas.

Reason. The molecules of a monoatomic gas have more degrees of freedom than those of a diatomic gas. [AIIMS ...]

22. **Assertion.** The melting point of ice decreases with increase of pressure.

Reason. Ice contracts on melting. [AIIMS ...]

23. **Assertion.** In pressure-temperature (P-T) phase diagram of water, the slope of the melting curve is found to be negative.

Reason. Ice contracts on melting to water. [AIIMS ...]

Answers and Explanations

1. (*a*) Boyle's law is valid for a constant mass of a perfect gas at a given temperature.

2. (*c*) All three given statements are true.

3. (*a*) A gas behaves as an ideal gas at low pressure and high temperature when the attractive forces between the molecules become negligible.

4. (*d*) As the two gases are in thermal equilibrium, Boyle's law will be obeyed *i.e.*, $P_A V_A = P_B V_B$.

5. (*d*) Using Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = \frac{T_2}{T_1} \times V_1 = \left(\frac{273+627}{273+27}\right) \times 4 = \frac{900}{300} \times 4 = \mathbf{12\ m^3}.$$

6. (*a*) Change in linear momentum,

$$= mv - (-mv) = \mathbf{2mv}.$$

7. (*c*) When the volume is decreased, the gas molecules strike the walls more frequently.

8. (*c*) According to kinetic theory of gases, the pressure exerted by a gas,

$$P = \frac{1}{3} \times \frac{mn}{V} v^2 \quad i.e., \quad P \propto mv^2$$

$$\therefore \quad P' \propto \left(\frac{m}{2}\right)(2v)^2$$

$$\frac{P'}{P} = 2 \quad \text{or} \quad P' = 2P.$$

9. (*d*) $n_1 r = \frac{PV}{T}$

and $n_2 r = \frac{(2P)(V/2)}{2T} = \frac{1}{4}\frac{PV}{T} = \frac{1}{4} n_1 r$

$$\therefore \quad \frac{n_1}{n_2} = \frac{4}{1} = \mathbf{4:1}.$$

10. (*c*) The absolute zero is the temperature at which all molecular motion ceases.

11. (*c*) $\frac{E_2}{E_1} = \frac{T_2}{T_1} = \frac{273+227}{273+27}$

$$E_2 = \frac{500}{300} E_1 = \frac{5}{3} \times 6.25 \times 10^{-21}\ \text{J}$$

$$= \mathbf{10.35 \times 10^{-21}\ J}.$$

12. (*c*) $\frac{E'}{E} = \frac{273+327}{273+27} = \frac{600}{300}$

$$E' = 2E.$$

13. (*b*) $v_{rms} > v_{av} > v_{mp}$

Refer to the solution of Problem 3 on page 13.4...

14. (*c*) $v_{rms} = \sqrt{\frac{RT}{m}}$

For constant temperature,

$$\frac{v_{rms}(H_2)}{v_{rms}(O_2)} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = \mathbf{4:1}.$$

15. (*b*) For an adiabatic change, $P \propto T^{\frac{\gamma}{\gamma-1}}$

Given : $P \propto T^C$

$$\therefore \quad C = \frac{\gamma}{\gamma - 1}$$

For a monoatomic gas,

$$\gamma = \frac{5}{3} \quad \therefore \quad C = \frac{5/3}{5/3-1} = \frac{5}{3} \times \frac{3}{2} = \frac{5}{2}$$

16. (*b*) $PV = nRT = \frac{m}{M} RT$

or $\frac{PM}{RT} = \frac{m}{V} = \rho$

Scanned by CamScanner

$$\rho \propto \frac{P}{T}$$

$$\left(\frac{P}{T}\right)_A = \frac{P_0}{T_0} \text{ and } \left(\frac{P}{T}\right)_B = \frac{3}{2}\frac{P_0}{T_0}$$

$$\left(\frac{P}{T}\right)_B = \frac{3}{2}\left(\frac{P}{T}\right)_A$$

$$\rho_B = \frac{3}{2}\rho_A = \frac{3}{2}\rho_0$$

17. (*b*) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. Air pressure in a car tyre increases due to rise in temperature during driving.

18. (*b*) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. According to Boyle's law, PV = constant, at a given temperature.

19. (*d*) Both the assertion and reason are false. The two speeds are different from each other. Also, the Maxwell distribution for the speed to molecules in a gas is asymmetrical.

20. (*a*) Both the assertion and reason are true but the reason is not a correct explanation of the assertion.

For monoatomic He, $\gamma = \frac{C_P}{C_V} = \frac{5}{3}$

For diatomic H_2, $\gamma = \frac{C_P}{C_V} = \frac{7}{5}$.

21. (*a*) Both the assertion and reason are true.

For monoatomic gas, $\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f} = 1 + \frac{2}{3} = \frac{5}{3}$

For a diatomic gas, $\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{5} = \frac{7}{5}$.

22. (*a*) Both the assertion and reason are true.

23. (*b*) Both assertion and reason are true but the reason is not a correct explanation of the assertion. The negative slope of the melting curve indicates that the melting point of ice decreases with the increase of pressure.

CBSE PMT Prelims and Final Exams

1. At constant volume temperature is increased, then
(*a*) collisions on walls will be less
(*b*) number of collisions per unit time will increase
(*c*) collisions will be in straight lines
(*d*) collisions will not change. [CBSE PMT 89]

2. Three containers of the same volume contain three different gases. The masses of the molecules are m_1, m_2 and m_3 and the number of molecules in their respective containers are N_1, N_2 and N_3. The gas pressures in the containers are P_1, P_2 and P_3 respectively. All the gases are now mixed and put in one of these containers. The pressure P of the mixture will be
(*a*) $P < (P_1 + P_2 + P_3)$ (*b*) $P = \frac{P_1 + P_2 + P_3}{3}$
(*c*) $P = P_1 + P_2 + P_3$ (*d*) $P > (P_1 + P_2 + P_3)$ [CBSE PMT 91]

3. The relation $PV = RT$ can describe behaviour of a real gas at
(*a*) high temperature and high density
(*b*) high temperature and low density
(*c*) low temperature and low density
(*d*) low temperature and high density. [CBSE PMT 90]

4. The equation of a state for 5 g of oxygen at a pressure P and temperature T, when occupying a volume V, will be
(*a*) $PV = 5RT/32$ (*b*) $PV = 5RT/16$
(*c*) $PV = 5RT/2$ (*d*) $PV = 5RT$ [CBSE PMT 94, 04]

5. Relation between pressure P and average kinetic energy E per unit volume of a gas is
(*a*) $P = 2E/3$ (*b*) $P = E/3$
(*c*) $P = 3E/2$ (*d*) $P = 3E$. [CBSE PMT 93]

6. At 0 K, which of the following properties of a gas will be zero ?
(*a*) kinetic energy (*b*) potential energy
(*c*) vibrational energy (*d*) density. [CBSE PMT 96]

7. The root mean square velocity of a gas molecule of mass m at a given temperature is proportional to
(*a*) m^0 (*b*) m
(*c*) $\sqrt{m}$ (*d*) $m^{-1/2}$ [CBSE PMT 90]

8. The temperature of a gas is raised from 27°C to 927°C. The r.m.s. molecular speed
(*a*) gets halved

Scanned by CamScanner

(b) gets doubled

(c) gets $\sqrt{927/27}$ times the earlier value

(d) remains unchanged. [CBSE PMT 94 ; AIIMS 2K]

9. An ant is walking on the horizontal surface. The number of degrees of freedom of ant will be

(a) 1 (b) 2

(c) 3 (d) 6 [CBSE PMT 93]

10. The number of degrees of freedom for a diatomic gas molecule is

(a) 2 (b) 3

(c) 5 (d) 6 [CBSE PMT 93]

11. The degree of freedom of a triatomic gas is

(a) 1 (b) 2

(c) 6 (d) 8 [CBSE PMT 99]

12. A polyatomic gas with n degrees of freedom has a mean energy per molecule given by

(a) $\frac{nkT}{N}$ (b) $\frac{nkT}{2N}$

(c) $\frac{nkT}{2}$ (d) $\frac{3kT}{2}$ [CBSE PMT 92]

13. Temperature of oxygen kept in a vessel is raised by 1°C at constant volume. Heat supplied to the gas may be taken partly as translational and partly rotational kinetic energies. Their respective shares are

(a) 60%, 40% (b) 50%, 50%

(c) 100%, zero (d) 40%, 60%. [CBSE PMT 9

14. If for a gas $\frac{R}{C_V} = 0.67$, this gas is made up of molecules, which are

(a) diatomic

(b) mixture of diatomic and polyatomic

(c) monoatomic (d) polyatomic. [CBSE PMT 92]

15. If γ is the ratio of specific heats of a perfect gas, then the number of degrees of freedom of a molecule of the gas is

(a) $\frac{25(\gamma-1)}{2}$ (b) $\frac{9(\gamma-1)}{2}$

(c) $\frac{3\gamma-1}{2\gamma-1}$ (d) $\frac{2}{\gamma-1}$ [CBSE PMT 2K]

16. The value of critical temperature in terms of van der Walls' constants a and b is given by

(a) $T_c = \frac{a}{2Rb}$ (b) $T_c = \frac{a}{27bR}$

(c) $T_c = \frac{8a}{27Rb}$ (d) $\frac{27a}{8Rb}$ [CBSE PMT 99]

Answers and Explanations

1. (b) As the temperature increases, the average molecular velocity increases. This increases collision frequency.

2. (c) According to Dalton's law of partial pressures,

$$P = P_1 + P_2 + P_3$$

3. (b) At high temperature and low density, real gas behaves like an ideal gas. Then the intermolecular attractions and actual volume of gas molecules become negligible.

4. (a) Number of moles present in 5 g of oxygen,

$$n = \frac{5}{32}. \quad \text{As} \quad PV = nRT \quad \therefore \quad PV = \frac{5RT}{32}$$

5. (a) $P = \frac{2E}{3}$

6. (a) At 0 K, all molecular motion stops, so kinetic energy becomes zero.

7. (d) $v_{rms} = \sqrt{\frac{3kT}{m}}$ i.e. $v_{rms} \propto m^{-1/2}$.

8. (b) $v_{rms} \propto \sqrt{T}$

$$\frac{v_{rms}(927°C)}{v_{rms}(27°C)} = \sqrt{\frac{927+273}{27+273}} = \sqrt{\frac{1200}{300}} = 2$$

$$v_{rms}(927°C) = 2v_{rms}(27°).$$

9. (b) As the ant can move on a plane, it has 2 degrees of freedom.

10. (c) A diatomic molecule has 3 degrees of freedom due to translatory motion and 2 degrees of freedom due to rotatory motion.

11. (c) A triatomic (non-linear) gas molecule has 3 degrees of freedom due to translatory motion and 3 degrees of freedom due to rotatory motion.

12. (c) Energy associated with each degree of freedom $= \frac{1}{2}kT$

Energy associated with n degrees of freedom

$$= \frac{1}{2}nkT.$$

13. (*a*) A diatomic oxygen molecule has 3 degrees of freedom due to translatory motion and 2 degrees of freedom due to rotatory motion. Their associated kinetic energies will be in the ratio 3 : 2 or 60% and 40%.

14. (*c*) $C_V = \frac{f}{2}R = \frac{1}{0.67}R \quad \therefore \quad f = \frac{2}{0.67} = 3$

Hence the gas is monoatomic.

15. (*d*) $\gamma = 1 + \frac{2}{f}$

$\therefore \quad f = \frac{2}{\gamma - 1}$

16. (*c*) $T_c = \frac{8a}{27Rb}$.

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. Gas exerts pressure on the walls of the container because

(*a*) gas has weight
(*b*) gas molecules have momentum
(*c*) gas molecules collide with each other
(*d*) gas molecules collide with the walls of the container.

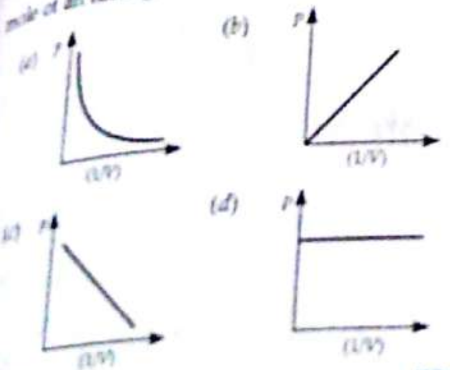
[DPMT 06]

2. At 0 K what happens ?

(*a*) efficiency of engines becomes infinite
(*b*) all liquids freeze
(*c*) molecular motion ceases
(*d*) none of these.

[DPMT 2K]

3. The graph of pressure P and $\left(\frac{1}{\text{volume}, (V)}\right)$ of 1 mole of an ideal gas at constant temperature is



[VMMC 07]

4. Two gases of equal masses are in thermal equilibrium. If P_a, P_b and V_a, V_b are their respective pressures and volumes, then which relation is true ?

(*a*) $2P_aV_a = P_bV_b$ (*b*) $P_a \neq P_b$, $V_a = V_b$
(*c*) $P_a / V_a = P_b / V_b$ (*d*) $P_aV_a = P_bV_b$

[VMMC 05]

5. Two gases A and B having same pressure P, volume V and temperature T are mixed. If mixture has volume and temperature as V and T respectively, then pressure of mixture is

(*a*) P (*b*) $3P$
(*c*) $2P$ (*d*) $4P$

[DPMT 99 ; VMMC 04]

6. The slope at any point on the curve in PV graph for a gas is given involving the relation

(*a*) $\frac{dP}{P} = -\frac{dV}{V}$ (*b*) $\frac{dP}{V} = -\frac{dV}{P}$
(*c*) $\frac{dP}{P} = \frac{dV}{V}$ (*d*) $\frac{dP}{V} = \frac{dV}{P}$

[DPMT 05]

7. If a given mass of gas occupies a volume of 10 cc at 1 atmospheric pressure and temperature 100°C, what will be its volume at 4 atmospheric pressure, the temperature being the same ?

(*a*) 100 cc (*b*) 400 cc
(*c*) 104 cc (*d*) 2.5 cc.

[VMMC 05]

8. A gas at one atmosphere and having volume 100 ml is mixed with another gas of equal moles at 0.5 atm and having volume 50 ml in flask of one litre, what is the final pressure ?

(*a*) 0.5 atm (*b*) 1 atm
(*c*) 0.75 atm (*d*) 0.125 atm

[DPMT 08]

9. If an ideal gas has volume V at 27°C and it is heated at a constant pressure, so that its volume becomes 1.5 V, then the value of final temperature is

(*a*) 327 K (*b*) 177°C
(*c*) 873 K (*d*) 600°C

[DPMT 95]

10. A cylinder contains 10 kg of gas at a pressure of 10^7 N/m^2. The quantity of gas taken out of the cylinder, if final pressure is 2.5×10^6 N/m^2 is

(*a*) zero (*b*) 9.5 kg
(*c*) 7.5 kg (*d*) 14.2 kg

[DPMT 99, 03]

Scanned by CamScanner

11. The r.m.s. of speed of a group of 7 gas molecules having speed (6, 4, 2, 0, − 2, − 4, − 6) m/s is

(a) 1.5 m/s (b) 3.4 m/s

(c) 9 m/s (d) 4 m/s. [DPMT 04, 05]

12. The kinetic energy of one g-molecule of a gas, at normal temperature and pressure, is
$[R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}]$

(a) 0.56×10^4 J (b) 2.7×10^2 J

(c) 1.3×10^2 J (d) 3.4×10^3 J

[DPMT 94, 97]

13. If γ be the ratio of specific heats of a perfect gas, the number of degrees of freedom of a molecule of a gas is

(a) $\frac{25}{2}(\gamma - 1)$ (b) $\frac{3\gamma - 1}{2\gamma - 1}$

(c) $\frac{2}{\gamma - 1}$ (d) $\frac{9}{2}(\gamma - 1)$ [DPMT 02]

14. A diatomic gas initially at 18°C is compressed adiabatically to one eighth of its original volume. The temperature after compression will be

(a) 18°C (b) 395.4°C

(c) 887.4°C (d) 114°C. [DPMT 94, 01]

15. Average velocity of a gas becomes 4 times, then what will be the effect on rms velocity at the same temperature ?

(a) 1.4 times (b) 4 times

(c) 3 times (d) 2 times. [DPMT 07]

16. The r.m.s. velocity at a temperature is 2 times the r.m.s. velocity at 300 K. What is this temperature ?

(a) 900 K (b) 2400 K

(c) 600 K (d) 1200 K.

[DPMT 06]

17. At room temperature (27°C) the rms speed of the molecules of a certain diatomic gas is found to be 1920 ms^{-1}. The gas is

(a) H_2 (b) F_2

(c) O_2 (d) Cl_2.

[DPMT 06]

18. The temperature of H_2 at which the rms velocity of its molecules is seven times the rms velocity of the molecules of nitrogen at 300 K is

(a) 2100 K (b) 1700 K

(c) 1350 K (d) 1050 K

[VMMC 06]

19. The temperature is changed from 27°C to 327°C. Find ratio of K.E. of molecules at two temperatures.

(a) 3 : 2 (b) 2 : 3

(c) 1 : 2 (d) 2 : 1

[VMMC 02]

20. At which of the following temperature would the molecules of gas have twice the average kinetic energy they have at 20°C ?

(a) 40°C (b) 80°C

(c) 586°C (d) 313°C

[VMMC 03]

21. The mean free path of collision of gas molecules varies with its diameter (d) of the molecules as

(a) d^{-1} (b) d^{-2}

(c) d^{-3} (d) d^{-4}

[DPMT 2011]

Answers and Explanations

1. (d) Gas exerts pressure due to collision of its molecules with the walls of the container.

2. (c) At 0 K, molecular motion ceases.

3. (b) At constant temperature $P \propto 1/V$.

Hence the correct option is (b).

4. (d) For gases in thermal equilibrium,

Boyle's law holds good.

$\therefore \quad P_a V_a = P_b V_b$.

5. (c) According to Dalton's law of partial pressures,

$$P' = P_1 + P_2 = P + P = 2P.$$

6. (a) PV = constant

$\therefore \quad P dV + V dP = 0$

Slope of P-V graph is

$$\frac{dP}{dV} = -\frac{P}{V}$$

Hence $\frac{dP}{P} = -\frac{dV}{V}$.

7. (d) From Boyle's law,

$$P_1 V_1 = P_2 V_2$$

$\therefore \quad 1 \times 10 = 4 \times V_2$

$$V_2 = \frac{10}{4} = 2.5 \text{ cc.}$$

8. (d) Total number of moles is conserved.

$$\therefore \quad \frac{P_1 V_1}{RT} + \frac{P_2 V_2}{RT} = \frac{PV}{RT}$$

Scanned by CamScanner

$$\frac{1\times 100}{RT}+\frac{0.5\times 50}{RT}=\frac{P\times 1000}{RT}$$

$P = 0.125$ **atm.**

9. (*b*) At constant pressure,

$$\frac{V_2}{V_1}=\frac{T_2}{T_1}$$

$$\frac{1.5V}{V}=\frac{T_2}{273+27}$$

$$T_2 = 300\times 1.5 = 450\text{ K}$$

$$= \mathbf{177°C.}$$

10. (*c*) $\frac{P_1}{P_2}=\frac{\rho_1}{\rho_2}=\frac{m_1}{m_2}$

$$m_2=\frac{P_2}{P_1}\times m_1=\frac{2.5\times 10^6}{10^7}\times 10$$

$$= 2.5\text{ kg}$$

Mass of gas taken out $= 10 - 2.5 =$ **7.5 kg.**

11. (*d*) $v_{rms}=\sqrt{\frac{v_1^2+v_2^2+...+v_7^2}{7}}$

$$=\sqrt{\frac{6^2+4^2+2^2+0^2+(-2)^2+(-4)^2+(-6)^2}{7}}$$

$= \mathbf{4}$ **m/s.**

12. (*d*) K.E. of 1 g mole of a gas at temperature T

$$=\frac{3}{2}RT=\frac{3}{2}\times 8.31\times 273$$

$$= \mathbf{3.4\times 10^3\text{ J.}}$$

13. (*c*) As $\gamma = 1+\frac{2}{f}$

$\therefore$ $f=\frac{2}{\gamma-1}$

14. (*b*) $\frac{T_1}{T_2}=\left(\frac{V_2}{V_1}\right)^{\gamma-1}=\left(\frac{1}{8}\right)^{(7/5)-1}$

$$T_2 = T_1(8)^{2/5}$$

$$= 291(2^3)^{2/5} = 291\times 2^{6/5}$$

$$= 668.6\text{ K} = \mathbf{395.6°C}$$

15. (*b*) $\bar{v} = 0.92\, v_{rms}$

$$v_{rms}=\frac{1}{0.92}\bar{v}$$

When $\bar{v}$ becomes 4 times,

$$v'_{rms}=\frac{4}{0.92}\bar{v}=4v_{rms}$$

16. (*d*) $v_{rms} \propto \sqrt{T}$

$$\frac{v_2}{v_1}=\sqrt{\frac{T_2}{T_1}}$$

or $2=\sqrt{\frac{T_2}{300}}$

$\therefore$ $T_2 = 4\times 300$

$$= \mathbf{1200\text{ K.}}$$

17. (*a*) $v_{rms}=\sqrt{\frac{3RT}{M}}$

$\therefore$ $M=\frac{3RT}{(v_{rms})^2}$

$$=\frac{3\times 8.3\times 300}{(1920)^2}=2\times 10^{-3}\text{ kg} = 2\text{ kg}$$

Hence the given gas is H_2 for which $M = 2$.

18. (*d*) $v_{rms}=\sqrt{\frac{3RT}{M}}$

$\therefore$ $\frac{v_1}{v_2}=\sqrt{\frac{T_1}{T_2}\cdot\frac{M_2}{M_1}}$

$$\frac{V_{H_2}}{V_{N_2}}=\sqrt{\frac{T_{H_2}}{300}\times\frac{28}{2}}=7$$

$$T_{H_2}=\frac{49\times 600}{28} = \mathbf{1050\text{ K}}.$$

19. (*c*) $\frac{E_1}{E_2}=\frac{T_1}{T_2}=\frac{273+27}{273+327}=\frac{300}{600}=\mathbf{\frac{1}{2}}$

20. (*d*) $\frac{E_1}{2E_1}=\frac{273+20}{T_2}$

$$T_2 = 2\times 293 = 586\text{ K} = \mathbf{313°C}$$

21. (*b*) Mean free path,

$$\bar{\lambda}=\frac{1}{\sqrt{2}n\pi d^2}\quad i.e.,\ \bar{\lambda}\propto d^{-2}$$

Scanned by CamScanner

CHAPTER

14

OSCILLATIONS

14.1 PERIODIC MOTION

1. *What is periodic motion ? Give some of its examples.*

Periodic motion. *Any motion that repeats itself over and over again at regular intervals of time is called periodic or harmonic motion.* The smallest interval of time after which the motion is repeated is called its time period. The time period is denoted by T and its SI unit is second.

Examples of periodic motion :

(*i*) The motion of any planet around the sun in an elliptical orbit is periodic. The period of revolution of Mercury is 87.97 days.

(*ii*) The motion of the moon around the earth is periodic. Its time period is 27.3 days.

(*iii*) The motion of Halley's comet around the sun is periodic. It appears on the earth after every 76 years.

(*iv*) The motion of the hands of a clock is periodic.

(*v*) The heart beats of a human being are periodic. The periodic time is about 0.8 second for a normal person.

14.2 OSCILLATORY OR HARMONIC MOTION

2. *What is oscillatory motion ? Give some of its examples.*

Oscillatory motion. *If a body moves back and forth repeatedly about its mean position, its motion is said to be oscillatory or vibratory or harmonic motion.* Such a motion repeats itself over and over again about a mean position such that it remains confined within well defined limits (known as extreme positions) on either side of the mean position.

Examples of oscillatory motion :

(*i*) The swinging motion of the pendulum of a wall clock.

(*ii*) The oscillations of a mass suspended from a spring.

(*iii*) The motion of the piston of an automobile engine.

(*iv*) The vibrations of the string of a guitar.

(*v*) When a freely suspended bar magnet is displaced from its equilibrium position along north-south line and released, it executes oscillatory motion.

14.3 PERIODIC MOTION VS. OSCILLATORY MOTION

3. *Every oscillatory motion is necessarily periodic but every periodic motion need not be oscillatory. Justify.*

Distinction between periodic and oscillatory motions. Every oscillatory motion is necessarily periodic because it is repeated at regular intervals of

Scanned by CamScanner

time. In addition, it is bounded about one mean position. But every periodic motion need not be oscillatory. For example, the earth completes one revolution around the sun in 1 year but it is not a to and fro motion about any mean position. Hence its motion is periodic but not oscillatory.

14.4 PERIODIC FUNCTIONS AND FOURIER ANALYSIS

4. *With suitable examples, explain the meaning of a periodic function. Construct two infinite sets of periodic functions with period T. Hence state Fourier theorem.*

Periodic function. *Any function that repeats itself at regular intervals of its argument is called a periodic function.* Consider the function $f(\theta)$ satisfying the property,

$$f(\theta + T) = f(\theta)$$

This indicates that the value of the function f remains same when the argument is increased or decreased by an integral multiple of T for all values of θ. A function f satisfying this property is said to be periodic having a period T. For example, trigonometric functions like $\sin\theta$ and $\cos\theta$ are periodic with a period of 2π radians, because

$$\sin(\theta + 2\pi) = \sin\theta$$

$$\cos(\theta + 2\pi) = \cos\theta$$

If the independent variable θ stands for some dimensional quantity such as time t, then we can construct periodic functions with period T as follows:

$$f_1(t) = \sin\frac{2\pi t}{T} \quad \text{and} \quad g_1(t) = \cos\frac{2\pi t}{T}$$

We can check the periodicity by replacing t by $t + T$. Thus

$$f_1(t+T) = \sin\frac{2\pi}{T}(t+T) = \sin\left(\frac{2\pi t}{T} + 2\pi\right)$$

$$= \sin\frac{2\pi t}{T} = f_1(t)$$

Similarly, $g_1(t+T) = g_1(t)$

It can be easily seen that functions with period T/n, where $n = 1, 2, 3, ...$ also repeat their values after a time T. Hence it is possible to construct two infinite sets of periodic functions such as

$$f_n(t) = \sin\frac{2\pi nt}{T} \qquad n = 1, 2, 3, 4, ...$$

$$g_n(t) = \cos\frac{2\pi nt}{T} \qquad n = 0, 1, 2, 3, 4, ...$$

In the set of cosine functions we have included the constant function $g_0(t) = 1$.

The constant function 1 is periodic for any value of T and hence does not alter the periodicity of $g_n(t)$.

Fourier theorem. *This theorem states that any arbitrary function F (t) with period T can be expressed as the unique combination of sine and cosine functions $f_n(t)$ and $g_n(t)$ with suitable coefficients.* Mathematically, it can be expressed as

$$F(t) = b_0 + b_1\cos\frac{2\pi t}{T} + b_2\cos\frac{4\pi t}{T} + b_3\cos\frac{6\pi t}{T} + ...$$

$$+ a_1\sin\frac{2\pi t}{T} + a_2\sin\frac{4\pi t}{T} + a_3\sin\frac{6\pi t}{T} + ...$$

$$= b_0 + b_1\cos\omega t + b_2\cos 2\omega t + b_3\cos 3\omega t + ...$$

$$+ a_1\sin\omega t + a_2\sin 2\omega t + a_3\sin 3\omega t + ...$$

or $$F(t) = b_0 + \sum_n b_n\cos n\omega t + \sum_n a_n\sin n\omega t$$

where $\omega = 2\pi/T$.

The coefficients $b_0, b_1, b_2, ..., a_1, a_2, a_3, ...$ are called ***Fourier coefficients***. These coefficients can be determined uniquely by a mathematical method called ***Fourier analysis***. Suppose all the Fourier coefficients except a_1 and b_1 are zero, then

$$F(t) = a_1\sin\frac{2\pi t}{T} + b_1\cos\frac{2\pi t}{T}$$

This equation is a special periodic motion called ***simple harmonic motion*** (S.H.M.).

14.5 PERIODIC, HARMONIC AND NON-HARMONIC FUNCTIONS

5. *Distinguish between periodic, harmonic and non-harmonic functions. Give examples of each.*

Periodic, harmonic and non-harmonic functions. *Any function that repeats itself at regular intervals of its argument is called a periodic function.* The following sine and cosine functions are periodic with period T:

$$f(t) = \sin\omega t = \sin\frac{2\pi t}{T}$$

and $$g(t) = \cos\omega t = \cos\frac{2\pi t}{T}$$

Fig. 14.1. shows how these functions vary with time t.

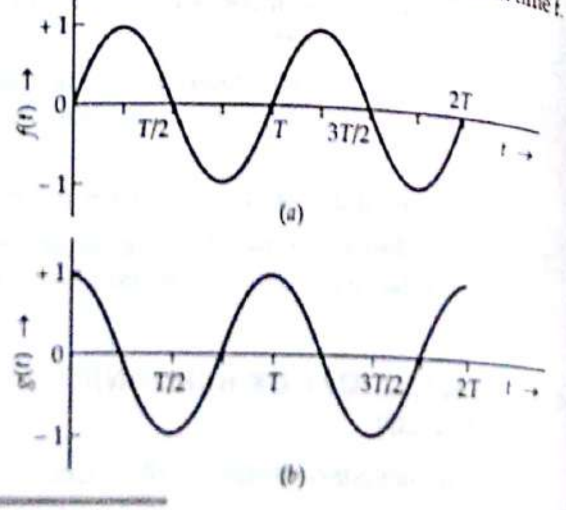


Fig. 14.1 Periodic functions which are harmonic.

Obviously, these functions vary between a maximum value + 1 and minimum value – 1 passing through zero in between. *The periodic functions which can be represented by a sine or cosine curve are called* **harmonic functions.**

All harmonic functions are necessarily periodic but all periodic functions are not harmonic. The periodic functions which cannot be represented by single sine or cosine function are called non-harmonic functions. Fig. 14.2 shows some periodic functions which repeat themselves in a period T but are not harmonic.

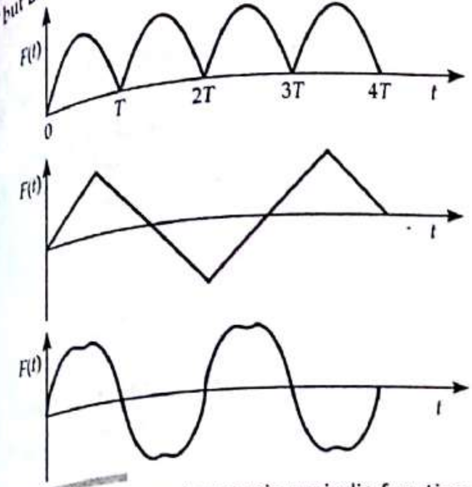


Fig. 14.2 Some non-harmonic periodic functions.

Any non-harmonic periodic function can be constructed from two or more harmonic functions.

One such function is : $F(t) = a_1 \sin \omega t + a_2 \sin 2\omega t$

It can be easily checked that the functions $\tan \omega t$ and $\cot \omega t$ are periodic with period $T = \pi/\omega$ while $\sec \omega t$ and $\operatorname{cosec} \omega t$ are periodic with period $T = 2\pi/\omega$. Thus

$$\tan\left\{\omega\left(t+\frac{\pi}{\omega}\right)\right\} = \tan(\omega t + \pi) = \tan \omega t$$

$$\sec\left\{\omega\left(t+\frac{2\pi}{\omega}\right)\right\} = \sec(\omega t + 2\pi) = \sec \omega t$$

But such functions take values between zero and infinity. So these functions cannot be used to represent displacement functions in periodic motions because displacement always takes a finite value in any physical situation.

Examples Based on Periodic and Harmonic Functions

Concepts Used

1. A function which can be represented by a single sine or cosine function is a harmonic function otherwise non-harmonic.
2. A periodic function can be expressed as the sum of sine and cosine functions of different time periods with suitable coefficients.

Example 1. *On an average a human heart is found to beat 75 times in a minute. Calculate its beat frequency and period.* **[NCERT]**

Solution. Beat frequency of the heart,

$$\nu = \frac{\text{No. of beats}}{\text{Time taken}} = \frac{75}{1 \text{ min}}$$

$$= \frac{75}{60 \text{ s}} = 1.25 \text{ s}^{-1} = \mathbf{1.25 \text{ Hz}}.$$

Beat period, $T = \frac{1}{\nu} = \frac{1}{1.25 \text{ s}^{-1}} = \mathbf{0.8 \text{ s}}.$

Example 2. *Which of the following functions of time represent (a) periodic and (n) non-periodic motion ? Give the period for each case of periodic motion.* [ω *is any positive constant*]. **[NCERT]**

(*i*) $\sin \omega t + \cos \omega t$ (*ii*) $\sin \omega t + \cos 2\omega t + \sin 4\omega t$

(*iii*) $e^{-\omega t}$ (*iv*) $\log(\omega t)$.

Solution. (*i*) Here $x(t) = \sin \omega t + \cos \omega t$

$$= \sqrt{2}\left[\sin \omega t \cos \frac{\pi}{4} + \cos \omega t \sin \frac{\pi}{4}\right]$$

$$= \sqrt{2} \sin(\omega t + \pi/4)$$

Moreover,

$$x\left(t+\frac{2\pi}{\omega}\right) = \sqrt{2} \sin[\omega(t + 2\pi/\omega) + \pi/4]$$

$$= \sqrt{2} \sin\left(\omega t + 2\pi + \frac{\pi}{4}\right)$$

$$= \sqrt{2} \sin\left(\omega t + \frac{\pi}{4}\right) = x(t).$$

Hence $\sin \omega t + \cos \omega t$ is a ***periodic function*** with time period equal to $2\pi/\omega$

(*ii*) Here $x(t) = \sin \omega t + \cos 2\omega t + \sin 4\omega t$

$\sin \omega t$ is a periodic function with period

$$= 2\pi/\omega = T$$

$\cos 2\omega t$ is a periodic function with period

$$= 2\pi/2\omega = \pi/\omega = T/2$$

$\sin 4\omega t$ is a periodic function with period

$$= 2\pi/4\omega = \pi/2\omega = T/4$$

Clearly, the entire function $x(t)$ repeats after a minimum time $T = 2\pi/\omega$. Hence the given function is ***periodic***.

(*iii*) The function $e^{-\omega t}$ decreases monotonically to zero as $t \to \infty$. It is an exponential function with a negative exponent of e, where $e = 2.71828$. It never repeats its value. So it is ***non-periodic***.

(*iv*) The function $\log(\omega t)$ increases monotonically with time. As $t \to \infty$, $\log(\omega t) \to \infty$. It never repeats its value. So it is ***non-periodic***.

Scanned by CamScanner

✖ PROBLEMS FOR PRACTICE

Which of the following functions of time represent (a) simple harmonic motion, (b) periodic but not simple harmonic and (c) non-periodic motion ? Find the period of each periodic motion. Here ω is a positive real constant.

1. $\sin \omega t + \cos \omega t$. (**Ans.** Simple harmonic)
2. $\sin \pi t + 2\cos 2\pi t + 3\sin 3\pi t$. (**Ans.** Periodic but not simple harmonic)
3. $\cos(2\omega t + \pi/3)$. (**Ans.** Simple harmonic)
4. $\sin^2 \omega t$. (**Ans.** Periodic but not simple harmonic)
5. $\cos \omega t + 2\sin^2 \omega t$. (**Ans.** Periodic but not simple harmonic)

✖ HINTS

1. $\sin \omega t + \cos \omega t = \sqrt{2}\sin(\omega t + \pi/4)$, $T = 2\pi/\omega$
2. Each term represents S.H.M.

   Period of $\sin \pi t$, $T = \frac{2\pi}{\pi} = 2s$

   Period of $2\cos 2\pi t = \frac{2\pi}{2\pi} = 1s = T/2$

   Period of $3\sin 3\pi t = \frac{2\pi}{3\pi} = \frac{2}{3}s = T/3$

   The sum is not simple harmonic but periodic with $T = 2s$.
3. $\cos(2\omega t + \pi/3)$ represents S.H.M. with

   $$T = 2\pi/2\omega = \pi/\omega$$
4. $\sin^2 \omega t = 1/2 - (1/2)\cos 2\omega t$.

   The function does not represent S.H.M. but is periodic with $T = 2\pi/2\omega = \pi/\omega$
5. $\cos \omega t + 2\sin^2 \omega t = \cos \omega t + 1 - \cos 2\omega t$

   $= 1 + \cos \omega t - \cos 2\omega t$

   $\cos \omega t$ represents S.H.M. with $T = 2\pi/\omega$

   $\cos 2\omega t$ represents S.H.M. with period

   $= 2\pi/2\omega = \pi/\omega = T/2$

   The combined function does not represent S.H.M. but is periodic with $T = 2\pi/\omega$

14.6 SIMPLE HARMONIC MOTION

6. *What is meant by simple harmonic motion ? Give some examples.*

Simple harmonic motion. *A particle is said to execute simple harmonic motion if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from the mean position and is always directed towards the mean position.*

If the displacement of the oscillating body from the mean position is small, then

Restoring force ∝ Displacement

$$F \propto x \quad \text{or} \quad F = -kx$$

This equation defines S.H.M. Here k is a positive constant called ***force constant*** or ***spring factor*** and is *defined as the restoring force produced per unit displacement*. The SI unit of k is Nm^{-1}. The negative sign in the above equation shows that the restoring force F always acts in the opposite direction of the displacement x.

Now, according to Newton's second law of motion,

$$F = ma$$

$$\therefore \quad ma = -kx$$

or $$a = -\frac{k}{m}x \quad i.e., \quad a \propto x$$

Hence simple harmonic motion may also be defined as follows :

A particle is said to possess simple harmonic motion if it moves to and fro about a mean position under an acceleration which is directly proportional to its displacement from the mean position and is always directed towards that position.

Examples of simple harmonic motion :

(*i*) Oscillations of a loaded spring.

(*ii*) Vibrations of a tuning fork.

(*iii*) Vibrations of the balance wheel of a watch.

(*iv*) Oscillations of a freely suspended magnet in a uniform magnetic field.

7. *State some important features of simple harmonic motion.*

Some important features of S.H.M.

(*i*) The motion of the particle is periodic.

(*ii*) It is the oscillatory motion of simplest kind in which the particle oscillates back and forth about its mean position with constant amplitude and fixed frequency.

(*iii*) Restoring force acting on the particle is proportional to its displacement from the mean position.

(*iv*) The force acting on the particle always opposes the increase in its displacement.

(*v*) A simple harmonic motion can always be expressed in terms of a single harmonic function of sine or cosine.

14.7 DIFFERENTIAL EQUATION FOR S.H.M.

8. *Write down the differential equation for S.H.M. Give its solution. Hence obtain expression for time period of S.H.M.*

Differential equation of S.H.M. In S.H.M. the restoring force acting on the particle is proportional to its displacement. Thus

$$F = -kx.$$

The negative sign shows that F and x are oppositely directed. Here k is spring factor or force constant.

Scanned by CamScanner

By Newton's second law,

$$F = m\frac{d^2x}{dt^2}$$

where m is the mass of the particle and $\frac{d^2x}{dt^2}$ is its acceleration.

$$\therefore \quad m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Put $\frac{k}{m} = \omega^2$, then $\frac{d^2x}{dt^2} = -\omega^2 x$

or

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad ...(1)$$

This is the ***differential equation of S.H.M.*** Here ω is the angular frequency. Clearly, x should be such a function whose second derivative is equal to the function itself multiplied with a negative constant. So a possible solution of equation (1) may be of the form

$$x = A\cos(\omega t + \phi_0)$$

Then

$$\frac{dx}{dt} = -\omega A\sin(\omega t + \phi_0)$$

and

$$\frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi_0) = -\omega^2 x$$

or

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

which is same as equation (1). Hence the solution of the equation (1) is

$$x = A\cos(\omega t + \phi_0) \quad ...(2)$$

It gives displacement of the harmonic oscillator at any instant t.

Here A is the *amplitude* of the oscillating particle.

$\phi = \omega t + \phi_0$, is the phase of the oscillating particle.

ϕ_0 is the initial *phase* (at $t = 0$) or epoch.

Time period of S.H.M. If we replace t by $t + \frac{2\pi}{\omega}$ in equation (2), we get

$$x = A\cos\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \phi_0\right]$$

$$= A\cos(\omega t + 2\pi + \phi_0) = A\cos(\omega t + \phi_0)$$

i.e., the motion repeats after time interval $\frac{2\pi}{\omega}$. Hence $\frac{2\pi}{\omega}$ is the time period of S.H.M.

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \qquad \left[\because \ \omega^2 = \frac{k}{m}\right]$$

or

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

In general, m is called inertia factor and k the spring factor.

14.8 SOME IMPORTANT TERMS CONNECTED WITH S.H.M.

9. *Define the terms harmonic oscillator, displacement, amplitude, cycle, time period, frequency, angular frequency, phase and epoch with reference to oscillatory motion.*

Some important terms connected with S.H.M.

(*i*) **Harmonic oscillator.** *A particle executing simple harmonic motion is called harmonic oscillator.*

(*ii*) **Displacement.** *The distance of the oscillating particle from its mean position at any instant is called its displacement.* It is denoted by x.

There can be other kind of displacement variables. These can be voltage variations in time across a capacitor in an a.c. circuit, pressure variations in time in the propagation of a sound wave, the changing electric and magnetic fields in the propagation of a light wave, etc.

(*iii*) **Amplitude.** *The maximum displacement of the oscillating particle on either side of its mean position is called its amplitude.* It is denoted by A. Thus $x_{max} = \pm A$.

(*iv*) **Oscillation or cycle.** *One complete back and forth motion of a particle starting and ending at the same point is called a cycle or oscillation or vibration.*

(*v*) **Time period.** *The time taken by a particle to complete one oscillation is called its time period. Or, it is the smallest time interval after which the oscillatory motion repeats.* It is denoted by T.

(*vi*) **Frequency.** *It is defined as the number of oscillations completed per unit time by a particle.* It is denoted by ν (nu). Frequency is equal to the reciprocal of time period. That is,

$$\nu = \frac{1}{T}$$

Clearly, the unit of frequency is $(\text{second})^{-1}$ or s^{-1}. It is also expressed as *cycles per second* (cps) or *hertz* (Hz).

SI unit of frequency $= s^{-1}$ = cps = Hz.

(*vii*) **Angular frequency.** *It is the quantity obtained by multiplying frequency ν by a factor of 2π.* It is denoted by ω

Thus, $\omega = 2\pi\nu = \frac{2\pi}{T}$

SI unit of angular frequency = rad s^{-1}.

(*viii*) **Phase.** *The phase of a vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant.* It is equal to the argument of sine or cosine function occurring in the displacement equation of the S.H.M. Suppose a simple harmonic equation is represented by

$$x = A\cos(\omega t + \phi_0)$$

Then phase of the particle is : $\phi = \omega t + \phi_0$

Scanned by CamScanner

Clearly, the phase ϕ is a function of time t. It is usually expressed either as the fraction of the time period T or fraction of angle 2π that has elapsed since the vibrating particle last passed its mean position in the positive direction.

$\phi = \omega t + \phi_0$	0	$\pi/2$	π	$3\pi/2$	2π
$x = A\cos(\omega t + \phi_0)$	$+A$	0	$-A$	0	$+A$

Thus the phase ϕ gives an idea about the position and the direction of motion of the oscillating particle.

(*ix*) **Initial phase or epoch.** *The phase of a vibrating particle corresponding to time t = 0 is called initial phase or epoch.*

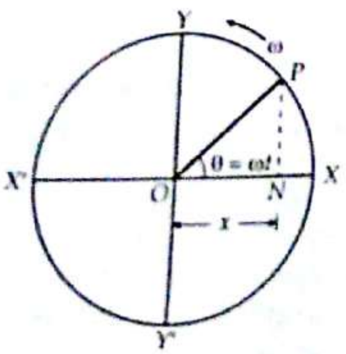
At $t = 0$, $\quad \phi = \phi_0$

The constant ϕ_0 is called initial phase or epoch. It tells about the initial state of motion of the vibrating particle.

14.9 UNIFORM CIRCULAR MOTION AND S.H.M.

10. *Show that simple harmonic motion may be regarded as the projection of uniform circular motion along a diameter of the circle. Hence derive an expression for the displacement of a particle in S.H.M.*

Relation between S.H.M. and uniform circular motion. As shown in Fig. 14.3, consider a particle P moving along a circle of radius A with uniform angular velocity ω. Let N be the foot of the perpendicular drawn from the point P to the diameter XX'. Then N is called the *projection* of P on the diameter XX'. As P moves along the circle from X to Y, Y to X', X' to Y' and Y' to X; N moves from X to O, O to X', X' to O and O to X. Thus, as P revolves along the circumference of the circle, N moves to and fro about the point O along the diameter XX'. The motion of N about O is said to be simple harmonic. Hence ***simple harmonic motion*** *may be defined as the projection of uniform circular motion upon a diameter of a circle.* The particle P is called *reference particle or generating particle* and the circle along which the particle P revolves is called *circle of reference.*



14.3 Reference circle.

Displacement in simple harmonic motion. As shown in Fig. 14.4, consider a particle moving in anticlockwise direction with uniform angular velocity ω along a circle of radius A and centre O. Suppose at time $t = 0$, the reference particle is at point A such that $\angle XOA = \phi_0$. At any time t, suppose the particle reaches the point P such that $\angle AOP = \omega t$. Draw $PN \perp XX'$.

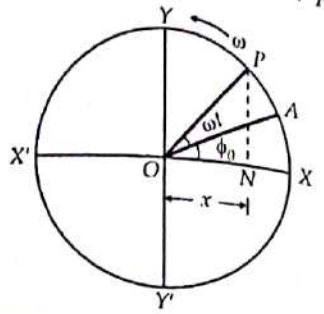


Fig. 14.4 Displacement in S.H.M., epoch $(+\phi_0)$

Clearly, displacement of projection N from centre O at any instant t is $\quad x = ON$.

In right-angled ΔONP,

$$\angle PON = \omega t + \phi_0$$

$$\therefore \quad \frac{ON}{OP} = \cos(\omega t + \phi_0)$$

or $$\frac{x}{A} = \cos(\omega t + \phi_0)$$

or $$x = A\cos(\omega t + \phi_0).$$

This equation gives displacement of a particle in S.H.M. at any instant t. The quantity $\omega t + \phi_0$ is called phase of the particle and ϕ_0 is called *initial phase or phase constant or epoch* of the particle. The quantity A is called *amplitude* of the motion. It is a positive constant whose value depends on how the motion is initially started. Thus

Phase

x = A cos(ωt + φ0)

⇑ Displacement ⇑ Amplitude ⇑ Angular frequency ⇑ Initial phase

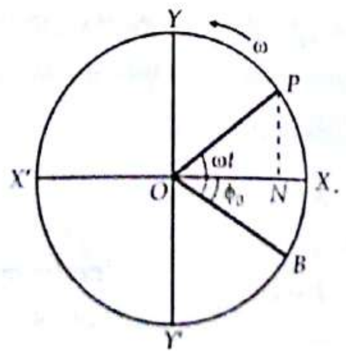


Fig. 14.5 Epoch $(-\phi_0)$

Scanned by CamScanner

As shown in Fig. 14.5, if the reference particle starts from the point P such that $\angle BOX = \phi_0$ and $\angle P'OP = \omega t$, then

$$\angle PON = \omega t - \phi_0$$

$$x = A \cos(\omega t - \phi_0)$$

Here $-\phi_0$ is the initial phase of the S.H.M.

11. *Show that a linear combination of sine and cosine functions like*

$$x(t) = a \sin \omega t + b \cos \omega t$$

represents a simple harmonic motion. Determine its amplitude and phase constant.

General expression for S.H.M. We are given

$$x = a \sin \omega t + b \cos \omega t \qquad ...(1)$$

Differentiating w.r.t. time t, we get

$$\frac{dx}{dt} = \omega a \cos \omega t - \omega b \sin \omega t$$

Again, differentiating w.r.t. time t, we get

$$\frac{d^2x}{dt^2} = -\omega^2 a \sin \omega t - \omega^2 b \cos \omega t$$

$$= -\omega^2 (a \sin \omega t + b \cos \omega t)$$

or $$\frac{d^2x}{dt^2} = -\omega^2 x$$

i.e., acceleration $\propto$ displacement

Hence the equation (1) represents S.H.M.

To determine its amplitude and phase constant, we put

$$a = A \cos \phi \qquad ...(2)$$

and $$b = A \sin \phi \qquad ...(3)$$

Then $$x = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

$$= A(\sin \omega t \cos \phi + \cos \omega t \sin \phi)$$

or $$x = A \sin(\omega t + \phi)$$

This again shows that equation (1) represents S.H.M. of amplitude A and phase constant ϕ.

Squaring and adding (2) and (3), we get

$$a^2 + b^2 = A^2(\cos^2 \phi + \sin^2 \phi) = A^2 \times 1$$

$\therefore$ Amplitude, $A = \sqrt{a^2 + b^2}$

Dividing (3) by (2), we get : $\frac{b}{a} = \frac{A \sin \phi}{A \cos \phi} = \tan \phi$

$\therefore$ Phase constant, $\phi = \tan^{-1} \frac{b}{a}$.

14.10 VELOCITY IN S.H.M.

12. *Deduce an expression for the velocity of a particle executing S.H.M. When is the particle velocity (i) maximum and (ii) minimum ?*

Expression for the velocity of a particle executing S.H.M. As shown in Fig. 14.6, consider a particle P moving with uniform angular speed ω in a circle of radius A. Its velocity vector $\vec{v}$ is directed along the tangent and the magnitude of this velocity vector is

$$v = \text{Angular velocity} \times \text{radius} = \omega A$$

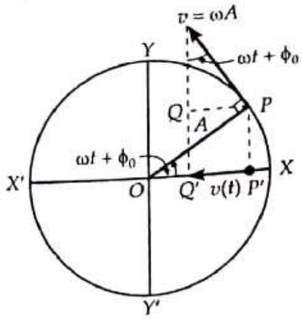


Fig. 14.6 Velocity of a particle in S.H.M.

Draw PP' and QQ' perpendiculars to the diameter XX'. The motion of P' is simple harmonic. Clearly, the instantaneous velocity of a particle executing S.H.M. will be

$$v(t) = \text{Velocity of the particle } P' \text{ at any instant } t$$

$$= \text{Projection of the velocity } v \text{ of the reference particle } P$$

$$= P'Q' = PQ = -v \sin(\omega t + \phi_0)$$

or $$v(t) = -\omega A \sin(\omega t + \phi_0).$$

The negative sign shows that the velocity of P' is directed towards left *i.e.*, in the negative X-direction.

Moreover,

$$v(t) = -\omega A \sqrt{1 - \cos^2(\omega t + \phi_0)} = -\omega A \sqrt{1 - \frac{x^2}{A^2}}$$

or $$v(t) = -\omega \sqrt{A^2 - x^2} \qquad [\because x = A \cos(\omega t + \phi_0)]$$

Special cases. (*i*) *When the particle is at the mean position*, then $x = 0$, so

$$v(t) = -\omega \sqrt{A^2 - 0^2} = -\omega A$$

This is the maximum velocity which a particle in S.H.M. can execute and is called velocity amplitude, denoted by v_{max}

$\therefore$ $$v_{max} = \omega A = \frac{2\pi}{T} A$$

(*ii*) *When the particle is at the extreme position*, then $x = \pm A$, so

$$v = -\omega \sqrt{A^2 - A^2} = 0.$$

Thus the velocity of a particle in S.H.M. is zero at either of its extreme positions.

Scanned by CamScanner

14.11 ACCELERATION OF A PARTICLE IN S.H.M.

13. *Show that the acceleration of a particle in S.H.M. is proportional to its displacement from the mean position. Hence write the expression for the time period of S.H.M.*

Expression for the acceleration of a particle executing S.H.M. As shown in Fig. 14.7, consider a particle P moving with uniform angular speed ω in a circle of radius A. The particle has the centripetal acceleration $\vec{a}_c$ acting radially towards the centre O. The magnitude of this acceleration is $a_c = \omega^2 A$.

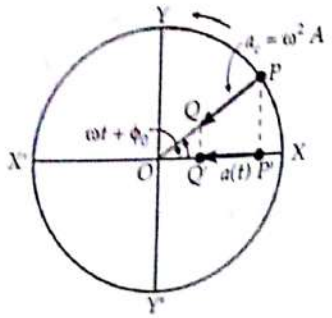


Fig. 14.7 Acceleration of a particle in S.H.M.

Draw PP' and QQ' perpendiculars to the diameter XX'. The motion of P' is simple harmonic. Clearly, the instantaneous acceleration of a particle executing S.H.M. will be

$a(t)$ = Acceleration of particle P' at any instant t

= Projection of the acceleration a_c of the reference particle P

= Projection of PQ on diameter XX'

$= P'Q' = -a_c \cos(\omega t + \phi_0)$

or $\quad a(t) = -\omega^2 A \cos(\omega t + \phi_0) = -\omega^2 x$

This equation expresses the acceleration of a particle executing S.H.M. It shows that *the acceleration of a particle in S.H.M. is proportional to its displacement from the mean position and acts in the opposite direction of the displacement.*

Special cases. (*i*) *When the particle is at the mean position*, then $x = 0$, so, acceleration $= -\omega^2 (0) = 0$.

Hence *the acceleration of a particle in S.H.M. is zero at the mean position.*

(*ii*) *When the particle is at the extreme position*, then $x = A$, so, acceleration $= -\omega^2 A$

This is the maximum value of acceleration which a particle in S.H.M. can possess and is called **acceleration amplitude**, denoted by a_{max}.

$$\therefore \quad a_{max} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A$$

14.12 PHASE RELATIONSHIP BETWEEN DISPLACEMENT, VELOCITY AND ACCELERATION

14. *Draw displacement-time, velocity-time and acceleration-time graphs for a particle executing simple harmonic motion. Discuss their phase relationship.*

Inter-relationship between particle displacement, velocity and acceleration in S.H.M. If a particle executing S.H.M. passes through its positive extreme position ($x = +A$) at time $t = 0$, then its displacement equation can be written as

$$x(t) = A \cos \omega t$$

Velocity, $\quad v(t) = \dfrac{dx}{dt} = -\omega A \sin \omega t$

$$= \omega A \cos\left(\omega t + \frac{\pi}{2}\right)$$

Acceleration, $\quad a(t) = \dfrac{dv}{dt} = -\omega^2 A \cos \omega t$

$$= \omega^2 A \cos(\omega t + \pi).$$

Using the above relations, we determine the values of displacement, velocity and acceleration at various instant t for one complete cycle as illustrated below.

Time, t	*0*	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T
Phase angle, $\omega t = \frac{2\pi}{T} t$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement, $x(t)$	$+A$ max.	0 min.	$-A$ max.	0 min	$+A$ max
Velocity, $v(t)$	0 min.	$-\omega A$ max.	0 min.	$+\omega A$ max	0 min
Acceleration, $a(t)$	$-\omega^2 A$ max.	0 min.	$+\omega^2 A$ max.	0 min	$-\omega^2 A$ max

In Fig. 14.8, we have plotted separately the x versus t, v versus t and a versus t curves for a simple harmonic motion.

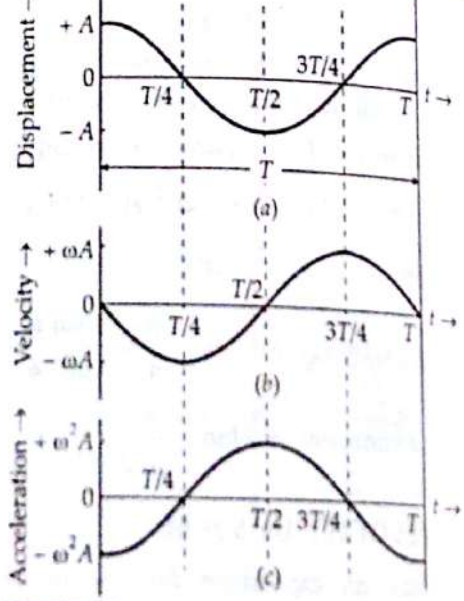


Fig. 14.8 Relation between velocity, displacement and acceleration in S.H.M.

Scanned by CamScanner

Conclusions. From the above graphs, we can draw the following conclusions about simple harmonic motion :

(*i*) Displacement, velocity and acceleration, all vary harmonically with time.

(*ii*) The velocity amplitude is ω times ; and acceleration amplitude is ω^2 times the displacement amplitude *A*.

(*iii*) Clearly, the velocity curve lies shifted to the left of the displacement curve by an interval of $T/4$. Thus *the particle velocity is ahead of its displacement by a phase angle of $\pi/2$ rad.* This means that whichever value displacement attains at any instant, velocity attains a similar value a $T/4$ time (a quarter of cycle) earlier. When the particle velocity is maximum, the displacement is minimum and vice versa.

(*iv*) Clearly, the acceleration curve lies shifted to the left of the displacement curve by an interval of $T/2$. Thus *the particle acceleration is ahead of its displacement by a phase angle of π rad.* Or, acceleration is ahead of velocity in phase by $\pi/2$ rad. When acceleration has maximum positive value, displacement has maximum negative value and vice versa. When the displacement is zero, the acceleration is also zero.

Examples based on Displacement, Velocity, Acceleration and Time Period of SHM

FORMULAE USED

1. Displacement, $x = A\cos(\omega t + \phi_0)$
   where A = amplitude, ω = angular frequency and ϕ_0 = initial phase of particle in SHM.
2. Velocity, $v = \dfrac{dx}{dt} = -\omega A \sin(\omega t + \phi_0)$
   $= -\omega\sqrt{A^2 - x^2}$
   Maximum velocity, $v_{max} = \omega A$.
3. Acceleration, $a = \dfrac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi_0) = -\omega^2 x$
   Maximum acceleration, $a_{max} = \omega^2 A$
4. Restoring force, $F = -kx = -m\omega^2 x$
   where k = force constant and $\omega^2 = k/m$.
5. Angular frequency, $\omega = 2\pi\nu = 2\pi/T$.
6. Time period, $T = 2\pi\sqrt{\dfrac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\dfrac{x}{a}}$.
7. Time period, $T = 2\pi\sqrt{\dfrac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\dfrac{m}{k}}$.

UNITS USED

Displacement x and amplitude A are in m or cm, force constant k in Nm^{-1}, frequency ν in Hz, angular frequency ω in rad s^{-1}.

EXAMPLE 3. *The following figures depict two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated on the figures. Obtain the simple harmonic motions of the x-projection of the radius vector of the rotating particle P in each case.* **[NCERT]**

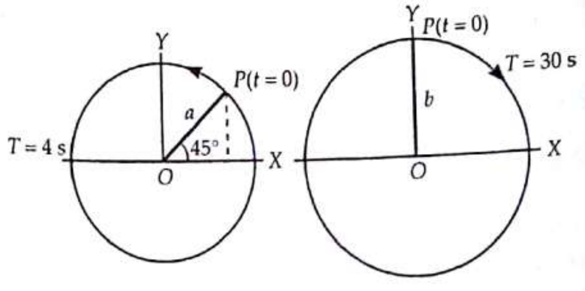


Fig. 14.9

Solution. (*a*) As shown in Fig. 14.10(*a*), suppose the particle moves in the anticlockwise sense from *P* to *P'* in time *t*.

Angle swept by the radius vector,

$$\theta = \omega t = \frac{2\pi}{T}t = \frac{2\pi}{4}t \qquad [\because T = 4 \text{ s}]$$

N is the foot of perpendicular drawn from *P'* on the *x*-axis.

Displacement,

$$ON = OP'\cos(\theta + \pi/4)$$

or $$x(t) = a\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$$

This represents S.H.M. of amplitude *a*, period 4 s and an initial phase $= \pi/4$ rad.

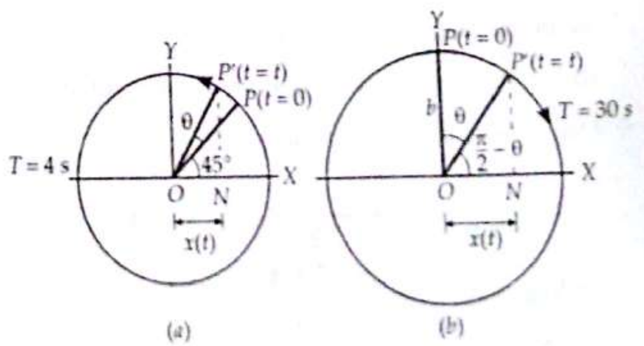


Fig. 14.10

(*b*) As shown in Fig. 14.10(*b*), suppose the particle moves in the clockwise sense from *P* to *P'* in time *t*.

Angle swept by the radius vector,

$$\theta = \omega t = \frac{2\pi}{T}t = \frac{2\pi}{30}t \qquad [\because T = 30 \text{ s}]$$

Scanned by CamScanner

Displacement,

$$ON = OP' \cos\left(\frac{\pi}{2} - \theta\right)$$

or $$x(t) = b\cos\left(\frac{\pi}{2} - \frac{2\pi}{30}t\right)$$

or $$x(t) = b\cos\left(\frac{2\pi}{30}t - \frac{\pi}{2}\right) \quad [\because \cos(-\theta) = \cos\theta]$$

This represents S.H.M. of amplitude b, period 30 s and an initial phase $= -\pi/2$ rad.

EXAMPLE 4. *A simple harmonic motion is represented by*

$$x = 10 \sin(20\,t + 0.5)$$

Write down its amplitude, angular frequency, frequency, time period and initial phase, if displacement is measured in metres and time in seconds. **[Himachal 09C]**

Solution. Given $x = 10 \sin(20\,t + 0.5)$

Standard equation for displacement in SHM is

$$x = A\sin(\omega t + \phi_0)$$

Comparing the above two equations, we get

(*i*) Amplitude, $A = \mathbf{10\ m}$. [$\because$ A and x have same units]

(*ii*) Angular frequency, $\omega = \mathbf{20\ rad\ s^{-1}}$.

(*iii*) Frequency, $\nu = \frac{\omega}{2\pi} = \frac{20}{2\pi} = \frac{10}{\pi} = \mathbf{3.18\ Hz}$.

(*iv*) Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{20} = \frac{\pi}{10} = \mathbf{0.314\ s}$.

(*v*) Initial phase, $\phi_0 = \mathbf{0.5\ rad}$.

EXAMPLE 5. *A particle executes SHM with a time period of 2 s and amplitude 5 cm. Find (i) displacement (ii) velocity and (iii) acceleration, after 1/3 second ; starting from the mean position.*

Solution. Here $T = 2$ s, $A = 5$ cm, $t = 1/3$ s

(*i*) For the particle starting from mean position,

Displacement, $x = A\sin\omega t = A\sin\frac{2\pi}{T}t$

$$= 5\sin\frac{2\pi}{2}\times\frac{1}{3} = 5\sin\frac{\pi}{3} = 5\times\frac{\sqrt{3}}{2} = \mathbf{4.33\ cm.}$$

(*ii*) Velocity, $v = \frac{dx}{dt} = \frac{2\pi A}{T}\cos\frac{2\pi}{T}t$

$$= \frac{2\pi\times 5}{2}\cos\frac{\pi}{3} = 5\times 3.14\times 0.5 = \mathbf{7.85\ cm\ s^{-1}.}$$

(*iii*) Acceleration, $a = \frac{dv}{dt} = \frac{4\pi^2 A}{T^2}\sin\frac{2\pi}{T}t$

$$= \frac{4\times 9.87\times 5}{4}\sin\frac{\pi}{3}$$

$$= 9.87\times 5\times\frac{\sqrt{3}}{2} = \mathbf{42.77\ cm\ s^{-2}.}$$

Example 6. *A body oscillates with SHM according to the equation :*

$$x(t) = 5\cos(2\pi t + \pi/4),$$

where t is in sec. and x in metres. Calculate

(*a*) *Displacement at $t = 0$* (*b*) *Time period*

(*c*) *Initial velocity* **[Central Schools 08]**

Solution. Given $x(t) = 5\cos(2\pi t + \pi/4)$

We compare with standard equation,

$$x(t) = A\cos(\omega t + \phi_0)$$

(*a*) Displacement at $t = 0$,

$$x(0) = 5\cos\frac{\pi}{4} = 5\times\frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}}\ \text{m}.$$

(*b*) Clearly, $\omega = 2\pi$ or $\frac{2\pi}{T} = 2\pi$

$\therefore$ Time period, $T = \mathbf{1\ s}$.

(*c*) Velocity, $v = \frac{dx}{dt} = -5\sin\left(2\pi t + \frac{\pi}{4}\right)\times 2\pi$

Initial velocity at $t = 0$,

$$v = -10\pi\sin\frac{\pi}{4} = -\frac{10\pi}{\sqrt{2}}\ \text{m/s}.$$

EXAMPLE 7. *A body oscillates with SHM according to the equation,*

$$x = (5.0\ m)\cos[(2\pi\ rad\ s^{-1})\,t + \pi/4].$$

At $t = 1.5$ s, calculate (a) displacement, (b) speed and (c) acceleration of the body. **[NCERT]**

Solution. Here $\omega = 2\pi$ rad s^{-1}, $T = 2\pi/\omega = 1$ s,

$$t = 1.5\ \text{s}$$

(*a*) Displacement,

$$x = 5.0\cos(2\pi\times 1.5 + \pi/4) = 5.0\cos(3\pi + \pi/4)$$
$$= -5.0\cos\pi/4 = -5.0\times 0.707 = -3.535\ \text{m}$$

(*b*) Velocity,

$$v = \frac{dx}{dt} = \frac{d}{dt}[5.0\cos(2\pi t + \pi/4)]$$
$$= -5.0\times 2\pi\sin(2\pi t + \pi/4)$$
$$= -5.0\times 2\pi\sin(2\pi\times 1.5 + \pi/4)$$
$$= +5.0\times 2\pi\sin\pi/4 = 5.0\times 2\times\frac{22}{7}\times 0.707$$
$$= \mathbf{22.22\ m\ s^{-1}}.$$

(*c*) Acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}[-10\pi\sin(2\pi t + \pi/4)]$$
$$= -20\pi^2\cos(2\pi t + \pi/4)$$
$$= -4\pi^2[5.0\cos(2\pi\times 1.5 + \pi/4)]$$
$$= -4\times 9.87\times(-3.535) \quad \text{[Using (a)]}$$
$$= \mathbf{139.56\ m\ s^{-2}}.$$

Scanned by CamScanner

EXAMPLE 8. The equation of a simple harmonic motion is given by $y = 6 \sin 10\pi t + 8 \cos 10\pi t$, where y is in cm and t in sec. Determine the amplitude, period and initial phase.

Solution. Given $y = 6 \sin 10\pi t + 8 \cos 10\pi t$ ...(1)

The general equation of SHM is

$$y = a \sin(\omega t + \phi) = a \sin \omega t \cos \phi + a \cos \omega t \sin \phi$$

$$= (a \cos \phi) \sin \omega t + (a \sin \phi) \cos \omega t \quad ...(2)$$

Comparing equations (1) and (2), we get

$$a \cos \phi = 6 \quad ...(3)$$

$$a \sin \phi = 8 \quad ...(4)$$

and

$$\omega t = 10\pi t \text{ or } \omega = 10\pi$$

∴ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = \mathbf{0.2\ s.}$

Squaring and adding (3) and (4), we get

$$a^2(\cos^2 \phi + \sin^2 \phi) = 6^2 + 8^2$$

$$= 36 + 64 = 100 \text{ or } a^2 = 100$$

∴ Amplitude, $a = \mathbf{10\ cm}$

Dividing (4) by (3), we get

$$\tan \phi = \frac{8}{6} = 1.3333$$

∴ Initial phase, $\phi = \tan^{-1}(1.3333) = \mathbf{53°8'}$.

EXAMPLE 9. A particle executes S.H.M. of amplitude 25 cm and time period 3 s. What is the minimum time required for the particle to move between two points 12.5 cm on either side of the mean position ?

Solution. When the particle starts from mean position, its displacement at instant t is given by

$$y = A \sin \omega t$$

Given $A = 25$ cm, $T = 3$ s, $y = 12.5$ cm

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3} \text{ rad s}^{-1}$$

∴ $$12.5 = 25 \sin \frac{2\pi}{3} t$$

or $$\sin \frac{2\pi}{3} t = \frac{12.5}{25} = \frac{1}{2}$$

∴ $$\frac{2\pi t}{3} = \frac{\pi}{6} \text{ or } t = \frac{1}{4} \text{ s}$$

∴ Time taken by the particle to move between two points 12.5 cm on either side of mean position is given by

$$2t = 2 \times \frac{1}{4} = \frac{1}{2} \text{ s} = \mathbf{0.5\ s.}$$

EXAMPLE 10. The shortest distance travelled by a particle executing SHM from mean position in 2 s is equal to $(\sqrt{3}/2)$ times its amplitude. Determine its time period.

Solution. Here $t = 2$ s, $y = (\sqrt{3}/2)$ A, $T = ?$

As $$y = A \sin \omega t = A \sin \frac{2\pi}{T} t$$

∴ $$\frac{\sqrt{3}}{2} A = A \sin \frac{2\pi \times 2}{T}$$

or $$\sin \frac{4\pi}{T} = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \quad \therefore \quad \frac{4\pi}{T} = \frac{\pi}{3}$$

or $$T = \mathbf{12\ s.}$$

EXAMPLE 11. The time-period of a simple pendulum is 2 s and it can go to and fro from equilibrium position at a maximum distance of 5 cm. If at the start of the motion the pendulum is in the position of maximum displacement towards the right of the equilibrium position, then write the displacement equation of the pendulum.

Solution. The displacement in SHM is given by

$$y = A \sin(\omega t + \phi_0)$$

Given $T = 2$ s, $A = 5$ cm

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad s}^{-1}$$

∴ $$y = 5 \sin(\pi t + \phi_0)$$

At time $t = 0$, displacement $y = 5$ cm. Therefore,

$$5 = 5 \sin(\pi \times 0 + \phi_0)$$

or $$\sin \phi_0 = 1 \quad \therefore \quad \phi_0 = \pi/2$$

Hence displacement equation for the pendulum is

$$y = 5 \sin\left(\pi t + \frac{\pi}{2}\right) = \mathbf{5 \cos \pi t}.$$

EXAMPLE 12. A particle executes S.H.M. of time period 10 seconds. The displacement of particle at any instant is given by : $x = 10 \sin \omega t$ (in cm). Find (i) the velocity of body 2 s after it passes through mean position (ii) the acceleration 2 s after it passes the mean position. **[Central Schools 04]**

Solution. Here $T = 10$ s, $x = 10 \sin \omega t$ cm

(i) Velocity, $v = \frac{dx}{dt} = 10\,\omega \cos \omega t \text{ cm s}^{-1}$

$$= 10\left(\frac{2\pi}{T}\right) \cos \frac{2\pi}{T} t \text{ cm s}^{-1}$$

∴ Velocity of the body 2s after it passes through the mean position,

$$v = 10\left(\frac{2\pi}{10}\right) \cos\left(\frac{2\pi}{10} \times 2\right) \text{cm s}^{-1}$$

$$= 2\pi \cos 72° = 2 \times 3.14 \times 0.309 = \mathbf{1.94\ cm\ s^{-1}}.$$

(ii) Acceleration of the body 2s after it passes through the mean position,

$$a = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 \times 10 \sin\left(\frac{2\pi}{T} t\right)$$

$$= -\frac{4\pi^2}{(10)^2} \times 10 \sin 72°$$

$$= -\frac{4 \times 9.87 \times 0.951}{10} = \mathbf{-3.75\ cm\ s^{-2}}.$$

Scanned by CamScanner

EXAMPLE 13. *For a particle in SHM, the displacement x of the particle as a function of time t is given as*

$$x = A \sin(2\pi t)$$

Here x is in cm and t is in seconds.

Let the time taken by the particle to travel from $x = 0$ to $x = A/2$ be t_1 and the time taken to travel from $x = A/2$ to $x = A$ be t_2. Find t_1 / t_2 **[Delhi 04]**

Solution. Here $x = 0$ at $t = 0$.

Also $\omega = \frac{2\pi}{T} = 2\pi \quad \therefore \quad T = 1\text{ s}$

At $t = t_1$, $x = A/2$. Therefore,

$$\frac{A}{2} = A\sin(2\pi t_1) \quad \text{or} \quad \frac{1}{2} = \sin(2\pi t_1)$$

$$\therefore \quad 2\pi t_1 = \frac{\pi}{6} \quad \text{or} \quad t_1 = \frac{1}{12}\text{ s}$$

Time taken from $x = 0$ to $x = A$ is $\frac{T}{4} = \frac{1}{4}$ s

or $\quad t_1 + t_2 = \frac{T}{4} = \frac{1}{4}$ s

or $\quad t_2 = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$ s

Hence $\quad \frac{t_1}{t_2} = \frac{1/12}{1/6} = \mathbf{\frac{1}{2}}$.

EXAMPLE 14. *In a HCl molecule, we may treat Cl to be of infinite mass and H alone oscillating. If the oscillation of HCl molecule shows frequency $9 \times 10^{13}\ s^{-1}$, deduce the force constant. The Avogadro number $= 6 \times 10^{26}$ per kg-mole.*

Solution. Frequency, $\nu = 9 \times 10^{13}\ \text{s}^{-1}$

Mass of a H-atom, $m = \frac{M}{N} = \frac{1}{6 \times 10^{26}}$ kg

As $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad$ or $\quad \nu^2 = \frac{1}{4\pi^2} \cdot \frac{k}{m}$

$$\therefore \quad k = 4\pi^2 m\nu^2$$

$$= 4\left(\frac{22}{7}\right)^2 \times \frac{1}{6 \times 10^{26}} \times (9 \times 10^{13})^2$$

$= \mathbf{533.4\ Nm^{-1}}$.

EXAMPLE 15. *A particle is moving with SHM in a straight line. When the distance of the particle from the equilibrium position has values x_1 and x_2, the corresponding values of velocities are u_1 and u_2. Show that the time period of oscillation is given by*

$$T = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}.$$

Solution. When $\quad x = x_1, \quad v = u_1$

When $\quad x = x_2, \quad v = u_2$

As $v = \omega\sqrt{A^2 - x^2}$

$\therefore \quad u_1 = \omega\sqrt{A^2 - x_1^2} \quad$ or $\quad u_1^2 = \omega^2(A^2 - x_1^2)$

and $\quad u_2 = \omega\sqrt{A^2 - x_2^2} \quad$ or $\quad u_2^2 = \omega^2(A^2 - x_2^2) \quad$ ...(1)

Subtracting (2) from (1), we get ...(2)

$$u_1^2 - u_2^2 = \omega^2(A^2 - x_1^2) - \omega^2(A^2 - x_2^2) = \omega^2(x_2^2 - x_1^1)$$

or $\quad \omega = \left[\frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}\right]^{1/2}$

$$T = \frac{2\pi}{\omega} = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}.$$

EXAMPLE 16. *If the distance y of a point moving on a straight line measured from a fixed origin on it and velocity v are connected by the relation $4v^2 = 25 - y^2$, then show that the motion is simple harmonic and find its time period.*

Solution. Given $4v^2 = 25 - y^2$

or $\quad v = \frac{1}{2}\sqrt{25 - y^2}$

Also velocity in SHM, $v = \omega\sqrt{A^2 - y^2}$

Comparing the above two equations, we find that the given equation represents SHM of amplitude $A = 5$ and $\omega = 1/2$ rad s^{-1}.

Time-period, $\quad T = \frac{2\pi}{\omega} = \frac{2\pi \times 2}{1} = \mathbf{4\pi\ s}$.

EXAMPLE 17. *A particle executing SHM along a straight line has a velocity of 4 ms^{-1} when at a distance 3 m from the mean position and 3 ms^{-1} when at a distance of 4 m from it. Find the time it takes to travel 2.5 m from the positive extremity of its oscillation.*

Solution. When $\quad y_1 = 3$ m, $\quad v_1 = 4$ ms^{-1}

When $\quad y_2 = 4$ m, $\quad v_2 = 3$ ms^{-1}

As $\quad v = \omega\sqrt{A^2 - y^2} \quad \therefore \quad 4 = \omega\sqrt{A^2 - 3^2}$

or $\quad 16 = \omega^2(A^2 - 9) \quad$ ...(1)

and $\quad 3 = \omega\sqrt{A^2 - 4^2}$ or $9 = \omega^2(A^2 - 16) \quad$ ...(2)

Dividing (1) by (2), we get

$$\frac{16}{9} = \frac{A^2 - 9}{A^2 - 16} \quad \text{or} \quad 16A^2 - 256 = 9A^2 - 81$$

or $\quad 7A^2 = 256 - 81 = 175 \quad$ or $\quad A^2 = 25$

$\therefore \quad A = \sqrt{25} = 5$ m

From (1), $\quad 4 = \omega\sqrt{5^2 - 3^2} = \omega \times 4$

or $\quad \omega = 1$ rad s^{-1}

Scanned by CamScanner

When the particle is 2.5 m from the positive extreme position, its displacement from the mean position is

$$y = 5 - 2.5 = 2.5 \text{ m}$$

When the time is noted from the extreme position, we can write

$$y = A \cos \omega t$$

$$2.5 = 5 \cos (1 \times t)$$

$$\therefore \quad \cos t = \frac{2.5}{5} = \frac{1}{2} = \cos \frac{\pi}{3}$$

or

Hence $\quad t = \frac{\pi}{3} = \frac{3.142}{3} = \mathbf{1.047\ s.}$

EXAMPLE 18. A particle executing linear SHM has a maximum velocity of 40 cm s^{-1} and a maximum acceleration of 50 cm s^{-2}. Find its amplitude and the period of oscillation.

Solution. Maximum velocity,

$$v_{max} = \omega A = 40 \text{ cm s}^{-1}$$

Maximum acceleration,

$$a_{max} = \omega^2 A = 50 \text{ cm s}^{-2}$$

$$\therefore \quad \frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \frac{50}{40}$$

or $\quad \omega = \frac{5}{4} \text{ rad s}^{-1}.$

Amplitude, $A = \frac{v_{max}}{\omega} = \frac{40 \times 4}{5} = \mathbf{32\ cm.}$

Period of oscillation,

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.142 \times 4}{5} = \mathbf{5.03\ s.}$$

EXAMPLE 19. The vertical motion of a huge piston in a machine is approximately simple harmonic with a frequency of 0.50 s^{-1}. A block of 10 kg is placed on the piston. What is the maximum amplitude of the piston's SHM for the block and the piston to remain together ?

Solution. Here $\nu = 0.5 \text{ s}^{-1}$, $g = 9.8 \text{ m s}^{-2}$

The maximum acceleration in SHM is given by

$$a_{max} \doteq \omega^2 A = (2\pi\nu)^2 A = 4\pi^2 \nu^2 A$$

The block will remain in contact with the piston if

$$a_{max} \le g \quad \text{or} \quad 4\pi^2 \nu^2 A \le g$$

Hence the maximum amplitude of the piston will be

$$A_{max} = \frac{g}{4\pi^2 \nu^2} = \frac{9.8}{4\pi^2 (0.5)^2} = \mathbf{0.99\ m.}$$

EXAMPLE 20. A block of mass one kg is fastened to a spring with a spring constant 50 Nm^{-1}. The block is pulled to a distance x = 10 cm from its equilibrium position at x = 0 on a frictionless surface from rest at t = 0. Write the expression for its x (t) and v (t). **[Central Schools 03]**

Solution. Here $m = 1$ kg, $k = 50 \text{ Nm}^{-1}$,

$$A = 10 \text{ cm} = 0.10 \text{ m}.$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{1}} = 7.07 \text{ rad s}^{-1}.$$

As the motion starts from the mean position, so the displacement equation can be written as

$$x(t) = A \sin \omega t \quad \text{or} \quad x(t) = 0.10 \sin 7.07\, t$$

and $\quad v(t) = \frac{dx}{dt} = 0.10 \times 7.07 \cos 7.07\, t$

or $\quad v(t) = \mathbf{0.707 \cos 7.07\, t\ ms^{-1}}.$

EXAMPLE 21. A person normally weighing 60 kg stands on a platform which oscillates up and down harmonically at a frequency of 2.0 s^{-1} and an amplitude 5.0 cm. If a machine on the platform gives the person's weight against time, deduce the maximum and minimum readings it will show. Take g = 10 ms^{-2}.

Solution. The platform vibrates between the positions A and B about the mean position O, as shown in Fig. 14.11.

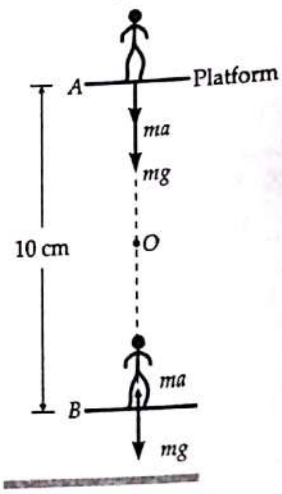


Fig. 14.11

Given $A = 5.0$ cm,

$m = 60$ kg, $\nu = 2$ Hz

At A and B, the acceleration is maximum and is directed towards the mean position.

It is given by

$$a_{max} = \omega^2 A$$
$$= 4\pi^2 \nu^2 A$$
$$= 4 \times 9.87 \times (2)^2 \times 0.05 = 7.9 \text{ ms}^{-2}$$

At A, both the weight mg and the restoring force F are directed towards O. Therefore, the weight at A is maximum and is given by

$$W_1 = (mg + F) = (mg + ma_{max}) = m(g + a_{max})$$
$$= 60\,(10 + 7.9) = 60 \times 17.9 = 1074 \text{ N}$$
$$= \frac{1074}{g} = \frac{1074}{10} = \mathbf{107.4\ kg\ f.}$$

At B, mg and F are opposed to each other so that the weight is minimum. It is given by

$$W_2 = (mg - F) = (mg - ma_{max}) = m(g - a_{max})$$
$$= 60\,(10 - 7.9) = (60 \times 2.1) \text{ N} = 126 \text{ N}$$
$$= \frac{126}{10} = \mathbf{12.6\ kg\ f.}$$

Scanned by CamScanner

EXAMPLE 22. *A body of mass 0.1 kg is executing SHM according to the equation*

$$y = 0.5\cos\left(100\,t + \frac{3\pi}{4}\right) \text{ metre}$$

Find (i) the frequency of oscillation (ii) initial phase (iii) maximum velocity (iv) maximum acceleration and (v) total energy.

Solution. Given $x = 0.5\cos\left(100\,t + \frac{3\pi}{4}\right)$ metre

For any SHM, $x = A\cos(\omega t + \phi_0)$

Comparing the above two equations, we get

$$A = 0.5 \text{ m}, \quad \omega = 100 \text{ rad s}^{-1}, \quad \phi_0 = \frac{3\pi}{4} \text{ rad}$$

(i) Frequency, $\nu = \frac{\omega}{2\pi} = \frac{100}{2\pi} = \frac{50}{\pi}$ **Hz.**

(ii) Initial phase, $\phi_0 = \frac{3\pi}{4}$ **rad.**

(iii) $v_{max} = \omega A = 100 \times 0.5 = 50 \text{ ms}^{-1}$.

(iv) $a_{max} = \omega^2 A = (100)^2 \times 0.5 = 5000 \text{ ms}^{-2}$.

(v) Total energy

$$= \frac{1}{2} m v_{max}^2 = \frac{1}{2} \times 0.1 \times (50)^2 = 125 \text{ J.}$$

PROBLEMS FOR PRACTICE

1. A simple harmonic oscillation is represented by the equation, $y = 0.40\sin(440\,t + 0.61)$
   Here y and t are in m and s respectively. What are the values of (i) amplitude (ii) angular frequency (iii) frequency of oscillations (iv) time period of oscillations and (v) initial phase ?
   [Ans. (i) 0.40 m (ii) 440 rad s^{-1} (iii) 70 Hz (iv) 0.0143 s (v) 0.61 rad]

2. The periodic time of a body executing SHM is 2 s. After how much time interval from $t = 0$, will its displacement be half of its amplitude ?
   (**Ans.** 1/6 s)

3. A particle executes SHM represented by the equation : $10y = 0.1\sin 50\,\pi\,t$, where the displacement y is in metre and time t in second. Find the amplitude and frequency of the particle.
   (**Ans.** $A = 0.01$ m, $\nu = 25$ Hz)

4. The displacement of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(1000\,t)$. Find the independent constituent SHM's. **[IIT 93]**
   [Ans. $\sin(1001\,t)$, $\sin(1000\,t)$, $\sin(999t)$]

5. A particle executing SHM completes 1200 oscillations per minute and passes through the mean position with a velocity of 31.4 ms^{-1}. Determine the maximum displacement of the particle from the mean position. Also obtain the displacement equation of the particle if its displacement be zero at the instant $t = 0$.
   [Ans. $A = 0.025$ m, $y = 0.025\sin(40\pi t)$ metre]

6. The acceleration of a particle performing SHM is 12 cm s^{-2} at a distance of 3 cm from the mean position. Calculate its time-period. (**Ans.** 3.142 s)

7. In a pendulum, the amplitude is 0.05 m and a period of 2 s. Compute the maximum velocity.
   (**Ans.** 0.1571 ms^{-1})

8. In what time after its motion begins, will a particle oscillating according to the equation, $y = 7\sin 0.5\,\pi t$, move from the mean position to maximum displacement ? **[Himachal 08C]**
   (**Ans.** 1s)

9. A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the magnitude of its acceleration is equal to its velocity. Find the time period, maximum velocity and maximum acceleration of SHM.
   (**Ans.** 3.63 s, 3.464 cms^{-1}, 6 cms^{-2})

10. The velocity of a particle describing SHM is 16 cm s^{-1} at a distance of 8 cm from mean position and 8 cm s^{-1} at a distance of 12 cm from mean position. Calculate the amplitude of the motion.
    (**Ans.** 13.06 cm)

11. A particle is executing SHM. If u_1 and u_2 are the speeds of the particle at distances x_1 and x_2 from the equilibrium position, show that the frequency of oscillation,
    $$f = \frac{1}{2\pi}\left(\frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}\right)^{1/2}.$$
    [Central Schools 14]

12. If a particle executes SHM of time period 4 s and amplitude 2 cm, find its maximum velocity and that at half its full displacement. Also find the acceleration at the turning points and when the displacement is 0.75 cm. (**Ans.** 3.14 cm s^{-1}, 2.72 cm s^{-1}, 4.93 cm s^{-2}, 1.85 cm s^{-2})

13. Show that if a particle is moving in SHM, its velocity at a distance $\sqrt{3}/2$ of its amplitude from the central position is half its velocity in central position.
    [Chandigarh 03 ; Central Schools 09]

14. A particle executes SHM of period 12 s. Two seconds after it passes through the centre of oscillation, the velocity is found to be 3.142 cm s^{-1}. Find the amplitude and the length of the path.
    (**Ans.** 12 cm, 24 cm)

15. A block lying on a horizontal table executes SHM of period 1 second, horizontally. What is the maximum

Scanned by CamScanner

amplitude for which the block does not slide ? Coefficient of friction between block and surface is 0.4. $\pi^2 = 10$. (**Ans.** 9.8 cm)

16. A horizontal platform moves up and down simple harmonically, the total vertical movement being 10 cm. What is the shortest period permissible, if objects resting on the platform are to remain in contact with it throughout the motion ? Take $g = 980\ cm\ s^{-2}$. (**Ans.** 0.449 s)

17. In a gasoline engine, the motion of the piston is simple harmonic. The piston has a mass of 2 kg and stroke (twice the amplitude) of 10 cm. Find maximum acceleration and the maximum unbalanced force on the piston, if it is making 50 complete vibrations each minute. (**Ans.** $1.371\ ms^{-2}$, 2.742 N)

18. A man stands on a weighing machine placed on a horizontal platform. The machine reads 50 kg. By means of a suitable mechanism the platform is made to execute harmonic vibrations up and down with a frequency of 2 vibrations per second. What will be the effect on the reading of the weighing machine ? The amplitude of vibration of the platform is 5 cm. Take $g = 10\ ms^{-2}$.

(**Ans.** Max. reading = 89.5 kg f,
Min. reading = 10.5 kg f)

✗ HINTS

1. Comparing $y = 0.40 \sin(440t + 0.61)$ with $y = A\sin(\omega t + \phi_0)$, we get

(i) Amplitude, A = **0.40 m.**

(ii) Angular frequency, ω = **440 rad s^{-1}.**

(iii) Frequency, $\nu = \frac{\omega}{2\pi} = \frac{440 \times 7}{2 \times 22}$ = **70 Hz.**

(iv) Time period, $T = \frac{1}{\nu} = \frac{1}{70}$ = **0.0143 s.**

(v) Initial phase, ϕ_0 = **0.61 rad.**

2. Here $T = 2\,s$, $y = A/2$, $t = ?$

As $y = A\sin\omega t = A\sin\frac{2\pi}{T}t$

$\therefore \quad \frac{A}{2} = A\sin\frac{2\pi}{2}t = A\sin\pi t$

or $\sin\pi t = \frac{1}{2} = \sin\frac{\pi}{6}$ or $\pi t = \frac{\pi}{6}$

$\therefore \quad t = 1/6\ s.$

4. $y = 4\cos^2(t/2)\sin(1000t)$

$= 2(1 + \cos t)\sin(1000t)$ $\quad [\because 1 + \cos 2\theta = 2\cos^2\theta]$

$= 2\sin(1000t) + 2\sin(1000t)\cos t$

$= 2\sin(1000t) + [\sin(1000t + t) + \sin(1000t - t)]$

$[\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)]$

$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$

Thus motion y is composed of three independent SHMs which are $\sin(1000t)$, $\sin(1001t)$ and $\sin(999t)$.

5. Here $\nu = \frac{1200}{60} = 20\ Hz$, $v_{max} = 3.14\ ms^{-1}$

But $v_{max} = \omega A = 2\pi\nu A$

$\therefore \quad A = \frac{v_{max}}{2\pi\nu} = \frac{3.14}{2 \times 3.14 \times 20}$ = **0.025 m.**

As displacement is zero at $t = 0$, so we can write

$y = A\sin\omega t = A\sin(2\pi\nu t)$ = **0.025 sin (40π t).**

6. Here $a = 12\ cm\ s^{-2}$, $y = 3\ cm$

$\omega = \sqrt{\frac{a}{y}} = \sqrt{\frac{12}{3}} = 2\ rad\ s^{-1}$

Time period, $T = \frac{2\pi}{\omega} = \frac{2 \times 3.142}{2}$ = **3.142 s.**

7. $v_{max} = \omega A = \frac{2\pi}{T}A = \frac{2\pi}{2} \times 0.05$

$= 3.142 \times 0.05$ = **0.1571 ms^{-1}.**

8. Given $y = 7\sin 0.5\pi t$

On comparing with the standard equation, $y = a\sin\omega t$, we get : $a = 7$, $\omega = 0.5\pi$

Let t be the time taken by the particle in moving from mean position to maximum displacement.

Then $7 = 7\sin 0.5\pi t$ or $\sin 0.5\pi t = 1 = \sin\frac{\pi}{2}$

$\therefore \quad 0.5\pi t = \frac{\pi}{2}$ or $t = \frac{1}{2 \times 0.5}$ = **1 s.**

9. Here $A = 2\ cm$. When displacement $y = 1\ cm$,

magnitude of velocity = magnitude of acceleration

or $\quad \omega\sqrt{A^2 - y^2} = \omega^2 y$

or $\quad A^2 - y^2 = \omega^2 y^2$

or $\quad 2^2 - 1^2 = \omega^2 \times 1^2$ or $\omega = \sqrt{3}\ rad\ s^{-1}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$ = **3.63 s.**

$v_{max} = \omega A = \sqrt{3} \times 2 = 1.732 \times 2$ = **3.464 cm s^{-1}.**

$a_{max} = \omega^2 A = 3 \times 2$ = **6 cm s^{-2}.**

10. As $\quad v = \omega\sqrt{A^2 - y^2}$

$\therefore$ In first case : $\quad 16 = \omega\sqrt{A^2 - 8^2}$ ...(1)

In second case : $8 = \omega\sqrt{A^2 - 12^2}$ ...(2)

Dividing (1) by (2),

$\frac{16}{8} = \frac{\omega\sqrt{A^2 - 8^2}}{\omega\sqrt{A^2 - 12^2}}$ or $2 = \sqrt{\frac{A^2 - 64}{A^2 - 144}}$

On solving $\quad 3A^2 = 512$ or $A^2 = 170.6$

or $\quad$ **A = 13.06 cm.**

Scanned by CamScanner

12. Here $T = 4$ s, $A = 2$ cm.

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{4} = 1.57 \text{ rad s}^{-1}$$

$$v_{max} = \omega A = 1.57 \times 2 = \mathbf{3.14 \text{ cm s}^{-1}}.$$

At $y = A/2 = 1$ cm,

$$v = \omega\sqrt{A^2 - y^2} = 1.57\sqrt{2^2 - 1^2} = \mathbf{2.72 \text{ cm s}^{-1}}.$$

At the turning points, acceleration is maximum

$\therefore$ $a_{max} = \omega^2 A = (1.57)^2 \times 2 = \mathbf{4.93 \text{ cm s}^{-2}}.$

At $y = 0.75$ cm,

$$a = \omega^2 y = (1.57)^2 \times 0.75 = \mathbf{1.85 \text{ cm s}^{-2}}.$$

13. Here $y = \sqrt{3}/2\, A$

$\therefore$ $v = \omega\sqrt{A^2 - y^2} = \omega\sqrt{A^2 - 3/4\, A^2}$

$$= \frac{1}{2}\omega A = \frac{1}{2} v_{max}.$$

14. Let $y = A \sin \omega t$

Then $v = \frac{dy}{dt} = \omega A \cos \omega t = \frac{2\pi}{T} A \cos \frac{2\pi}{T} t$

$\therefore$ $3.142 = \frac{2 \times 3.142}{12} A \cos \frac{2\pi}{12} \times 2$

or $A = \mathbf{12 \text{ cm}}$ and length of path $= 2A = \mathbf{24 \text{ cm}}$.

15. $a_{max} = \omega^2 A = \mu g$

$\therefore$ $A = \frac{\mu g}{\omega^2} = \frac{\mu g T^2}{4\pi^2}$

$$= \frac{0.4 \times 9.8 \times (1)^2}{4 \times 10} = 0.098 \text{ m} = \mathbf{9.8 \text{ cm}}.$$

16. Take $a_{max} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A = g.$

17. Length of stroke $= 2A = 10$ cm.

18. Here $m = 50$ kg, $\nu = 2$ Hz,

$A = 5 \text{ cm} = 0.05 \text{ m} = 4 \times 9.87 \times 2^2 \times 0.05 = 7.9 \text{ ms}^{-2}$.

$$a_{max} = \omega^2 A = 4\pi^2 \nu^2 A$$

Max. force on the man

$= m(g + a_{max}) = 50(10 + 7.9) = 895.0 \text{ N} = \mathbf{89.5 \text{ kg f}}.$

Min. force on the man

$= m(g - a_{max}) = 50(10 - 7.9) = 105.0 \text{ N} = \mathbf{10.5 \text{ kg f}}.$

14.13 ENERGY IN S.H.M. : KINETIC AND POTENTIAL ENERGIES

15. *Derive expressions for the kinetic and potential energies of a simple harmonic oscillator. Hence show that the total energy is conserved in S.H.M. In which positions of the oscillator, is the energy wholly kinetic or wholly potential ?*

Total energy in S.H.M. The energy of a harmonic oscillator is partly kinetic and partly potential. When a body is displaced from its equilibrium position by doing work upon it, it acquires potential energy. When the body is released, it begins to move back with a velocity, thus acquiring kinetic energy.

(*i*) **Kinetic energy.** At any instant, the displacement of a particle executing S.H.M. is given by

$$x = A \cos(\omega t + \phi_0)$$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi_0)$

Hence kinetic energy of the particle at any displacement x is given by

$$K = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi_0)$$

But $A^2 \sin^2(\omega t + \phi_0) = A^2 [1 - \cos^2(\omega t + \phi_0)]$

$$= A^2 - A^2 \cos^2(\omega t + \phi_0) = A^2 - x^2$$

$\therefore$ $K = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi_0)$

or $K = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k(A^2 - x^2)$

(*ii*) **Potential energy.** When the displacement of a particle from its equilibrium position is x, the restoring force acting on it is

$$F = -kx$$

If we displace the particle further through a small distance dx, then work done against the restoring force is given by

$$dW = -F\, dx = +k\, x\, dx$$

The total work done in moving the particle from mean position ($x = 0$) to displacement x is given by

$$W = \int dW = \int_0^x k\, x\, dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2} kx^2$$

This work done against the restoring force is stored as the potential energy of the particle. Hence potential energy of a particle at displacement x is given by

$$U = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi_0)$$

(*iii*) **Total energy.** At any displacement x, the total energy of a harmonic oscillatory is given by

$$E = K + U = \frac{1}{2} k(A^2 - x^2) + \frac{1}{2} kx^2$$

or $E = \frac{1}{2} k A^2 = \frac{1}{2} m\omega^2 A^2 = 2\pi^2 m\nu^2 A^2$

$[\because \omega = 2\pi\nu]$

Thus the total mechanical energy of a harmonic oscillator is independent of time or displacement. Hence *in the absence of any frictional force, the total energy of a harmonic oscillator is conserved.*

Scanned by CamScanner

Obviously, the total energy of particle in S.H.M. is

(*i*) directly proportional to the mass m of the particle,

(*ii*) directly proportional to the square of its frequency ν, and

(*iii*) directly proportional to the square of its vibrational amplitude A.

Graphical representation. *At the mean position,* $x = 0$

Kinetic energy, $K = \frac{1}{2} k (A^2 - 0^2) = \frac{1}{2} kA^2$

Potential energy, $U = \frac{1}{2} k (0^2) = 0$

Hence at the mean position, the energy is all kinetic.

At the extreme positions, $x = \pm A$

Kinetic energy, $K = \frac{1}{2} k (A^2 - A^2) = 0$

Potential energy, $U = \frac{1}{2} kA^2$

Hence at the two extreme positions, the energy is all potential.

Figure 14.12 shows the variations of kinetic energy K, potential energy U and total energy E with displacement x. The graphs for K and U are parabolic while that for E is a straight line parallel to the displacement axis. At $x = 0$, the energy is all kinetic and for $x = \pm A$, the energy is all potential.

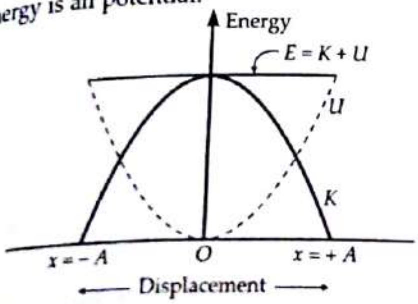


Fig. 14.12 *K, U* and *E* as functions of displacement *x* for a harmonic oscillator.

Figure 14.13 shows the variations of energies K, U and E of a harmonic oscillator with time t. Clearly, twice in each cycle, both kinetic and potential energies assume their peak values. Both of these energies are periodic functions of time, the time period of each being $T/2$.

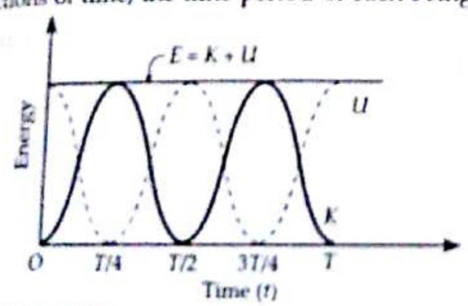


Fig. 14.13 *K, U* and *E* as functions of time *t* for a harmonic oscillator.

Examples based on Energy of S.H.M.

Formulae Used

1. P.E. at displacement y from the mean position,

$$E_p = \frac{1}{2} ky^2 = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

2. K.E. at displacement y from the mean position,

$$E_k = \frac{1}{2} k (A^2 - y^2) = \frac{1}{2} m\omega^2 (A^2 - y^2)$$
$$= \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

3. Total energy at any point,

$$E = \frac{1}{2} kA^2 = \frac{1}{2} m\omega^2 A^2 = 2\pi^2 m A^2 \nu^2$$

Units Used

Energies E_p, E_k and E are in joule, displacement in metre, force constant k in Nm^{-1} and angular frequency ω in rad s^{-1}.

Example 23. *A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 N m^{-1}. The block is pulled to a distance $x = 10$ cm from its equilibrium position at $x = 0$ on a frictionless surface from rest at $t = 0$. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.* **[NCERT]**

Solution. Here $m = 1$ kg, $k = 50$ N m^{-1},

$A = 10$ cm $= 0.10$ m, $y = 5$ cm $= 0.05$ m

Kinetic energy,

$$E_k = \frac{1}{2} k (A^2 - y^2) = \frac{1}{2} \times 50 [(0.10)^2 - (0.05)^2]$$
$$= \mathbf{0.1875\ J.}$$

Potential energy,

$$E_p = \frac{1}{2} ky^2 = \frac{1}{2} \times 50 \times (0.05)^2 = \mathbf{0.0625\ J.}$$

Total energy,

$$E = E_k + E_p = 0.1875 + 0.0625 = \mathbf{0.25\ J.}$$

Example 24. *A body executes SHM of time period 8 s. If its mass be 0.1 kg, its velocity 1 second after it passes through its mean position be 4 ms^{-1}, find its (i) kinetic energy (ii) potential energy and (iii) total energy.*

Solution. Here $m = 0.1$ kg, $T = 8$ s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4} \text{ rad s}^{-1}$$

When $t = 1$ s, $v = 4$ ms^{-1}

But $v = \omega A \cos \omega t$

$\therefore$ $4 = \frac{\pi}{4} \times A \cos\left(\frac{\pi}{4} \times 1\right) = \frac{\pi}{4} \times A \times \frac{1}{\sqrt{2}}$

or $A = \frac{16\sqrt{2}}{\pi}$ m

Scanned by CamScanner

Total energy,

$$E = \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} \times 0.1 \times \left(\frac{\pi}{4}\right)^2 \times \left(\frac{16\sqrt{2}}{\pi}\right)^2 = \mathbf{1.6\ J.}$$

Kinetic energy,

$$E_k = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.1 \times (4)^2 = \mathbf{0.8\ J.}$$

Potential energy,

$$E_p = E - E_k = 1.6 - 0.8 = \mathbf{0.8\ J.}$$

EXAMPLE 25. *A spring of force constant 800 Nm^{-1} has an extension of 5 cm. What is the work done in increasing the extension from 5 cm to 15 cm?* **[AIEEE 02]**

Solution. Here $k = 800\ Nm^{-1}$, $x_1 = 5\ cm = 0.05\ m$,

$$x_2 = 15\ cm = 0.15\ m$$

W = Increase in P.E. of the spring

$$= \frac{1}{2} k (x_2^2 - x_1^2)$$

$$= \frac{1}{2} \times 800 [(0.15)^2 - (0.05)^2]\ J$$

$$= \mathbf{8\ J.}$$

EXAMPLE 26. *A particle of mass 10 g is describing SHM along a straight line with a period of 2 s and amplitude of 10 cm. What is the kinetic energy when it is (i) 2 cm (ii) 5 cm from its equilibrium position? How do you account for the difference between its two values?*

Solution. Velocity at displacement y is

$$v = \omega\sqrt{A^2 - y^2}$$

Given $A = 10$ cm, $T = 2$ s

Angular frequency,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad s}^{-1}$$

(*i*) When $y = 2$ cm,

$$v = \pi\sqrt{100 - 4} = \pi\sqrt{96} \text{ cm s}^{-1}$$

$$\therefore \quad K.E. = \frac{1}{2} mv^2 = \frac{1}{2} \times 10 \times \pi^2 \times 96$$

$$= \mathbf{480\ \pi^2\ erg.}$$

(*ii*) When $y = 5$ cm,

$$v = \pi\sqrt{100 - 25} = \pi\sqrt{75} \text{ cm s}^{-1}$$

$$\therefore \quad K.E. = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 10 \times \pi^2 \times 75 = \mathbf{375\ \pi^2\ erg.}$$

The K.E. decreases when the particle moves from $y = 2$ cm to $y = 5$ cm. This is due to the increase in the potential energy of the particle.

EXAMPLE 27. *At a time when the displacement is half the amplitude, what fraction of the total energy is kinetic and what fraction is potential in S.H.M.?*

Solution.

Displacement $= \frac{1}{2}$ amplitude or $y = \frac{1}{2} A$

Total energy of SHM, $E = \frac{1}{2} m\omega^2 A^2$

Kinetic energy of SHM, $E_k = \frac{1}{2} m\omega^2 (A^2 - y^2)$

$$= \frac{1}{2} m\omega^2 \left[A^2 - \left(\frac{A}{2}\right)^2\right]$$

$$= \frac{1}{2} m\omega^2 \left(\frac{3A^2}{4}\right) = \frac{3}{4} \cdot \frac{1}{2} m\omega^2 A^2 = \frac{3}{4} E$$

Potential energy of SHM,

$$E_p = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 \left(\frac{A}{2}\right)^2$$

$$= \frac{1}{4} \cdot \frac{1}{2} m\omega^2 A^2 = \frac{1}{4} E.$$

EXAMPLE 28. *A particle is executing SHM of amplitude A. At what displacement from the mean position, is the energy half kinetic and half potential?*

Solution. As $E_K = E_P$

$$\therefore \quad \frac{1}{2} m\omega^2 (A^2 - y^2) = \frac{1}{2} m\omega^2 y^2$$

or $\quad A^2 - y^2 = y^2 \quad$ or $\quad 2y^2 = A^2$

or $\quad y^2 = \frac{A^2}{2} \quad$ or $\quad y = \pm \frac{A}{\sqrt{2}}$

Thus the energy will be half kinetic and half potential at displacement $\frac{A}{\sqrt{2}}$ on either side of the mean position.

EXAMPLE 29. *A particle executes simple harmonic motion of amplitude A. (i) At what distance from the mean position is its kinetic energy equal to its potential energy? (ii) At what points is its speed half the maximum speed?*

Solution. The potential energy and kinetic energy of a particle at a displacement y are given by

$$E_p = \frac{1}{2} ky^2$$

and $\quad E_k = \frac{1}{2} k (A^2 - y^2)$

where A is the amplitude

and $\quad k$ is the force constant.

Scanned by CamScanner

(i) As $E_k = E_p$

$$\frac{1}{2}k(A^2 - y^2) = \frac{1}{2}ky^2 \quad \text{or} \quad 2y^2 = A^2$$

$$y = \pm\frac{A}{\sqrt{2}} = \pm 0.71\,A$$

or **= 0.71 times the amplitude on either side of mean position.**

(ii) Here, $v = \frac{1}{2}v_{max}$

In general, kinetic energy

$$= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{1}{2}v_{max}\right)^2 = \frac{1}{4}\cdot\frac{1}{2}mv_{max}^2$$

$$= \frac{1}{4} \times \text{Maximum kinetic energy}$$

or $$E_k = \frac{1}{4}\times(E_k)_{max} \qquad ...(2)$$

From equation (1),

$$E_k = \frac{1}{2}k(A^2 - y^2)$$

$$\therefore \quad (E_k)_{max} = \frac{1}{2}kA^2 \qquad [\text{Put } y = 0]$$

Putting these values in equation (2), we get

$$\frac{1}{2}k(A^2 - y^2) = \frac{1}{4}\times\frac{1}{2}kA^2$$

$$4y^2 = 3A^2$$

or $$y = \pm\frac{\sqrt{3}}{2}A = \pm\,\mathbf{0.86\,A}$$

or **= 0.86 times the amplitude on either side of mean position.**

✗ Problems For Practice

1. A bob of simple pendulum of mass 1 g is oscillating with a frequency 5 vibrations per second and its amplitude is 3 cm. Find the kinetic energy of the bob in the lowest position. **(Ans.** 4441.5 erg**)**

2. A body weighing 10 g has a velocity of 6 cm s^{-1} after one second of its starting from mean position. If the time period is 6 seconds, find the kinetic energy, potential energy and the total energy.

   (Ans. 180 erg, 540 erg, 720 erg**)**

3. A particle executes SHM of period 8 seconds. After what time of its passing through the mean position will the energy be half kinetic and half potential ?

   [Chandigarh 08]

   (Ans. 1 s**)**

4. The total energy of a particle executing SHM of period 2π seconds is 1.024×10^{-3} J. The displacement of the particle at $\pi/4$ s is $0.08\sqrt{2}$ m. Calculate the amplitude of motion and mass of the particle. **(Ans.** 0.16 m , 0.08 kg**)**

5. A particle which is attached to a spring oscillates horizontally with simple harmonic motion with a frequency of $1/\pi$ Hz and total energy of 10 J. If the maximum speed of the particle is 0.4 ms^{-1}, what is the force constant of the spring ? What will be the maximum potential energy of the spring during the motion ? **(Ans.** k = 500 Nm^{-1}, U_{max} = 10 J**)**

6. The length of a weightless spring increases by 2 cm when a weight of 1.0 kg is suspended from it. The weight is pulled down by 10 cm and released. Determine the period of oscillation of the spring and its kinetic energy of oscillation. Take g = 10 ms^{-2}. **(Ans.** 0.28 s, 2.5 J**)**

✗ Hints

1. At the lowest or the mean position, energy of the bob is entirely kinetic and maximum.

$$(E_k)_{max} = \frac{1}{2}m\omega^2 A^2$$

$$= \frac{1}{2}m(2\pi\nu)^2 A^2 = 2\pi^2 m\nu^2 A^2$$

$$= 2\times9.87\times1\times5^2\times3^2 = \mathbf{4441.5\ erg.}$$

2. Here m = 10 g , T = 6 s,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}\ \text{rad s}^{-1}$$

When t = 1 s, v = 6 cm s^{-1}

As $v = A\omega\cos\omega t$

$$\therefore \quad 6 = A\times\frac{\pi}{3}\cos\frac{\pi}{3}\times1 = A\times\frac{\pi}{3}\cos 60^\circ$$

$$= A\times\frac{\pi}{3}\times\frac{1}{2} = \frac{\pi A}{6} \quad \text{or} \quad A = \frac{36}{\pi}\ \text{cm}$$

Total energy, $$E = \frac{1}{2}mA^2\omega^2$$

$$= \frac{1}{2}\times10\times\left(\frac{36}{\pi}\right)^2\times\left(\frac{\pi}{3}\right)^2 = \mathbf{720\ erg.}$$

Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}\times10\times6^2 = \mathbf{180\ erg.}$

$\therefore$ Potential energy

= Total energy − Kinetic energy

= 720 − 180 = **540 erg.**

3. As P.E. = K.E.

$$\therefore \quad \frac{1}{2}ky^2 = \frac{1}{2}k(A^2 - y^2)$$

or $$y^2 = A^2 - y^2 \quad \text{or} \quad y = \frac{A}{\sqrt{2}}$$

Scanned by CamScanner

Now $y = A \sin \omega t = A \sin \frac{2\pi}{T} t$

$\therefore \quad \frac{A}{\sqrt{2}} = A \sin \frac{2\pi}{8} t$

or $\sin \frac{\pi t}{4} = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$

or $\frac{\pi t}{4} = \frac{\pi}{4}$ or $t = \mathbf{1\ s}$.

4. When $t = \frac{\pi}{4}$ s, $y = 0.08\sqrt{2}$ m

As $y = A \sin \omega t = A \sin \frac{2\pi}{T} t$

$\therefore \quad 0.08\sqrt{2} = A \sin \frac{2\pi}{2\pi} \times \frac{\pi}{4} = A \sin \frac{\pi}{4}$

or $0.08\sqrt{2} = A \times \frac{1}{\sqrt{2}}$

$\therefore \quad A = 0.08\sqrt{2} \times \sqrt{2} = \mathbf{0.16\ m}$.

Total energy $= \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 A^2$

$\therefore \quad 1.024 \times 10^{-3} = \frac{1}{2} m \left(\frac{2\pi}{2\pi}\right)^2 \times (0.16)^2$

or $m = \frac{2 \times 1.024 \times 10^{-3}}{(0.16)^2} = \mathbf{0.08\ kg}$.

5. Here $\nu = 1/\pi$ Hz, $E = 10$ J, $v_{max} = 0.4\ \text{ms}^{-1}$

Now $v_{max} = \omega A = 2\pi \nu A$

$\therefore \quad A = \frac{v_{max}}{2\pi\nu} = \frac{0.4 \times \pi}{2\pi \times 1} = 0.2$ m

As $E = \frac{1}{2} k A^2$

$\therefore \quad k = \frac{2E}{A^2} = \frac{2 \times 10}{(0.2)^2} = \mathbf{500\ Nm^{-1}}$.

$(E_p)_{max} = E = \mathbf{10\ J}$.

6. Here $F = mg = 1.0 \times 10$ N, $y = 2$ cm $= 0.02$ m

$\therefore \quad k = \frac{F}{y} = \frac{1.0 \times 10}{0.02} = 500\ \text{Nm}^{-1}$

$T = 2\pi \sqrt{\frac{m}{k}} = 2 \times 3.14 \times \sqrt{\frac{1.0}{500}} = \mathbf{0.28\ s}$.

E_1 = Work done in pulling the spring through 10 cm or 0.1 m

$= \frac{1}{2} k x^2 = \frac{1}{2} \times 500 \times (0.1)^2 = \mathbf{2.5\ J}$.

14.14 OSCILLATIONS DUE TO A SPRING

16. *Derive an expression for the time-period of the horizontal oscillations of a massless loaded spring.*

Horizontal oscillations of a body on a spring. Consider a massless spring lying on a frictionless horizontal table. Its one end is attached to a rigid support and the other end to a body of mass m. If the body is pulled towards right through a small distance x and released, it starts oscillating back and forth about its equilibrium position under the action of the restoring force of elasticity,

$$F = -kx$$

where k is the force constant (restoring force per unit compression or extension) of the spring. The negative sign indicates that the force is directed oppositely to x.

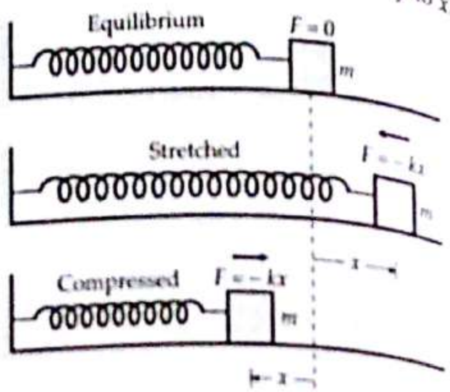


Fig. 14.14 Horizontal oscillations of a loaded spring.

If d^2x/dt^2 is the acceleration of the body, then

$$m \frac{d^2x}{dt^2} = -kx$$

or $$\frac{d^2x}{dt^2} = -\frac{k}{m} x = -\omega^2 x$$

This shows that the acceleration is proportional to displacement x and acts opposite to it. Hence the body executes simple harmonic motion. Its time period is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}}$$

or $$T = 2\pi \sqrt{\frac{m}{k}}$$

Frequency of oscillation will be

$$\nu = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Clearly, the time period T will be small or frequency ν large if the spring is stiff (high k) and attached body is light (small m).

17. *Deduce an expression for the time-period of the vertical oscillations of a massless loaded spring. Does it depend on acceleration due to gravity ?*

Vertical oscillations of a body on a spring. If a spring is suspended vertically from a rigid support and a body of mass m is attached to its lower end, the spring gets stretched to a distance d due to the weight mg of the body. Because of the elasticity of the spring, a restoring force equal to kd begins to act in the upward direction. Here k is the spring factor of the spring. In the position of equilibrium,

$$mg = kd$$

If the body be pulled vertically downwards through a small distance x from its equilibrium position and then released, it begins to oscillate up and down about this position. The weight mg of the body acts vertically downwards while the restoring force $k(d+x)$ due to elasticity acts vertically upwards. Therefore, the resultant force on the body is

$$F = mg - k(d+x)$$

$$= kd - kd - kx \quad [\because mg = kd]$$

or

$$F = -kx$$

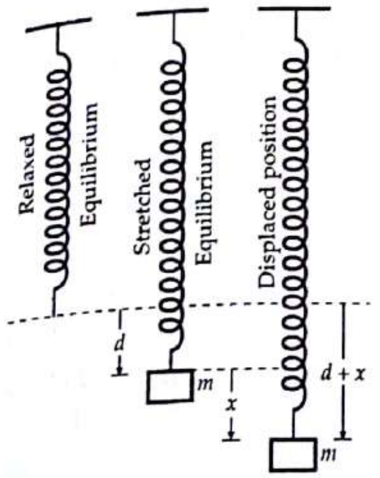


Fig. 14.15

If d^2x/dt^2 is the acceleration of the body, then

$$m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

Thus acceleration is proportional to displacement x and is directed opposite to it. Hence the body executes S.H.M. and its time period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Obviously, *the force of gravity has no effect on the force constant k and hence the time period of the oscillating mass.*

14.15 OSCILLATIONS OF LOADED SPRING COMBINATION

18. *Fig. 14.16 shows four different spring arrangements. If the mass of each arrangement is displaced from its equilibrium position and released, what is the resulting frequency of vibration in each case ? Neglect the mass of each spring.* **[NCERT]**

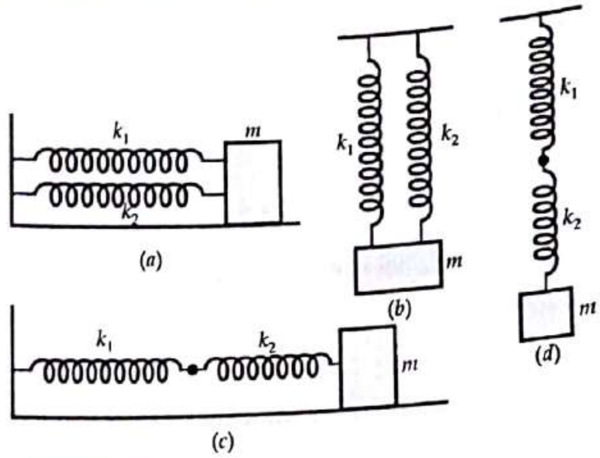


Fig. 14.16

Springs connected in parallel. Figs. 14.16(*a*) and (*b*) show two springs connected in parallel. Let k_1 and k_2 be their spring constants. Let y be the extension produced in each spring. Restoring forces produced in the two springs will be

$$F_1 = -k_1 y \quad \text{and} \quad F_2 = -k_2 y$$

The total restoring force is

$$F = F_1 + F_2 = -(k_1 + k_2)y \quad ...(1)$$

Let k_p be the force constant of the parallel combination. Then

$$F = -k_p y \quad ...(2)$$

From (1) and (2), $\quad k_p = k_1 + k_2$

Frequency of vibration of the parallel combination is

$$\nu_p = \frac{1}{2\pi}\sqrt{\frac{k_p}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

Springs connected in series. Figs. 14.16(*c*) and (*d*) represent two springs connected in series. Let x_1 and x_2 be the extensions produced in the two springs. The restoring force F acting in the two springs is same.

$$\therefore \quad F = -k_1 x_1 = -k_2 x_2$$

or

$$x_1 = -\frac{F}{k_1} \quad \text{and} \quad x_2 = -\frac{F}{k_2}$$

Total extension, $x = x_1 + x_2 = -\frac{F}{k_1} - \frac{F}{k_2}$

$$= -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

or

$$F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right)x \quad ...(3)$$

Let k_s be the force constant of the series combination. Then

$$F = -k_s x \quad ...(4)$$

Scanned by CamScanner

From (3) and (4), $k_s = \dfrac{k_1 k_2}{k_1 + k_2}$

Frequency of oscillation of the series combination is

$$\nu_s = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}.$$

Examples based on Oscillations of a Loaded Spring

FORMULAE USED

1. Spring factor or force constant, $k = \dfrac{F}{y}$
2. Period of oscillation of a mass m suspended from massless spring of force constant k,
$$T = 2\pi\sqrt{\frac{m}{k}}$$
3. For two springs of springs factors k_1 and k_2 connected in parallel, effective spring factor,
$$k = k_1 + k_2 \quad \therefore \quad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$
4. For two springs connected in series, effective spring factor k is given by
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{or} \quad k = \frac{k_1 k_2}{k_1 + k_2}$$
$$\therefore \quad T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$
5. When length of a spring is made n times, its spring factor becomes $1/n$ times and hence time period increases $\sqrt{n}$ times.
6. When a spring is cut into n equal pieces, spring factor of each part becomes nk.
$$\therefore \quad T = 2\pi\sqrt{\frac{m}{nk}}$$

UNITS USED

Spring factors k, k_1, k_2 are in Nm^{-1}, mass m is in kg and time period T in second.

EXAMPLE 30. *The pan attached to a spring balance has a mass of 1 kg. A weight of 2 kg when placed on the pan stretches the spring by 10 cm. What is the frequency with which the empty pan will oscillate ?*

Solution. Applied force,

$$F = 2 \text{ kg wt} = 2 \times 9.8 \text{ N} = 19.6 \text{ N}$$

Displacement, $\quad y = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Force constant, $\quad k = \dfrac{F}{y} = \dfrac{19.6}{0.1} = 196 \text{ Nm}^{-1}$

For the empty pan, $m = 1$ kg

Hence the frequency of oscillation of the empty pan will be

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{196}{1}}$$
$$= \frac{1}{2\pi} \times 14 = \frac{7}{\pi} \text{ Hz}.$$

EXAMPLE 31. *A spring compressed by 0.2 m develops a restoring force of 25 N. A body of mass 5 kg is placed on it. Deduce :*

(i) force constant of the spring

(ii) the depression of the spring under the weight of the body and

(iii) the period of oscillation, if the body is disturbed. Take $g = 10 \text{ N kg}^{-1}$.

Solution. (i) Here $y = 0.2$ m, $F = 25$ N

$\therefore$ Force constant,

$$k = \frac{F}{y} = \frac{25}{0.2} = 125 \text{ Nm}^{-1}.$$

(ii) Here $F = 5 \text{ kg wt} = 5 \times 10 \text{ N} = 50 \text{ N}$

$\therefore$ Displacement,

$$y = \frac{F}{k} = \frac{50}{125} = 0.4 \text{ m}.$$

(iii) Here $m = 5$ kg, $k = 125 \text{ Nm}^{-1}$

Time period,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5}{125}} = \frac{2\pi}{5} \text{ s}.$$

EXAMPLE 32. *A 0.2 kg of mass hangs at the end of a spring. When 0.02 kg more mass is added to the end of the spring, it stretches 7 cm more. If the 0.02 kg mass is removed, what will be the period of vibration of the system ?*

[Central Schools

Solution. When 0.02 kg mass is added, the spring stretches by 7 cm.

As $\quad mg = kx$

$$\therefore \quad k = \frac{mg}{x} = \frac{0.02 \times 10}{7 \times 10^{-2}} = \frac{20}{7} \text{ Nm}^{-1}$$

When 0.02 kg mass is removed, the period of vibration will be

$$T = 2\pi\sqrt{\frac{m'}{k}} = 2\pi\sqrt{\frac{0.2}{20/7}}$$
$$= 2\pi\sqrt{\frac{7}{100}} = \frac{2\pi \times 2.645}{10} = 1.66 \text{ s}.$$

EXAMPLE 33. *A body of mass 12 kg is suspended by a coil spring of natural length 50 cm and force constant $2.0 \times 10^3 \text{ Nm}^{-1}$. What is the stretched length of the spring ?*

Scanned by CamScanner

...the body is pulled down further stretching the spring to a length of 59 cm and then released, what is the frequency of oscillation of the suspended mass ? (Neglect the mass of the spring)

Solution. Here $m = 12$ kg, $k = 2.0 \times 10^3$ Nm^{-1}

Natural length, $l = 50$ cm

Extension produced in the spring due to 12 kg mass,

$$y = \frac{F}{k} = \frac{mg}{k} = \frac{12 \times 9.8}{2.0 \times 10^3} = 0.0588 \text{ m} = 5.88 \text{ cm}$$

Stretched length of the spring

$$= l + y = 50 + 5.88 = \mathbf{55.88\ cm.}$$

When the loaded spring is further stretched, its frequency of oscillation does not change and is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{2 \times 10^3}{12}} = \mathbf{2.06\ Hz.}$$

EXAMPLE 34. An impulsive force gives an initial velocity of -1.0 ms^{-1} to the mass in the unstretched spring position [see Fig. 14.17(a)]. What is the amplitude of motion ? Give x as a function of time t for the oscillating mass. Given m = 3 kg and k = 1200 Nm^{-1}.

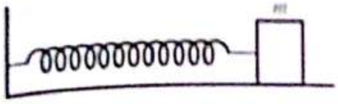


Fig. 14.17 (a)

Solution. Here initial velocity in unstretched position,

$$v = -1.0 \text{ ms}^{-1}$$

Clearly, $v_{max} = 1.0$ ms^{-1}

Also, $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20 \text{ rad s}^{-1}$$

Amplitude, $$A = \frac{v_{max}}{\omega} = \frac{1.0}{20} = \frac{1}{20} \text{ m} = \mathbf{5\ cm.}$$

As the motion starts from the unstretched position, the expression for the displacement can be written as

$$x = A \sin \omega t = 5 \sin 20\,t$$

As initial impulse is negative, the displacement is towards negative X-axis. So

$$x = \mathbf{-5 \sin 20\,t.}$$

EXAMPLE 35. A 5 kg collar is attached to a spring of force constant 500 Nm^{-1}. It slides without friction on a horizontal rod as shown in Fig. 14.17(b). The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate

(i) the period of oscillation, (ii) the maximum speed, and (iii) the maximum acceleration of the collar.

[NCERT ; Delhi 03C]

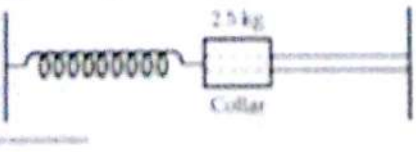


Fig. 14.17 (b)

Solution. Here $m = 5$ kg, $k = 500$ Nm^{-1},

$$A = 10.0 \text{ cm} = 0.10 \text{ m}$$

(i) Period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5}{500}}$$

$$= 2 \times 3.14 \times \frac{1}{10} \text{ s} = \mathbf{0.628\ s.}$$

(ii) The maximum speed of the collar,

$$v_{max} = \omega A = \sqrt{\frac{k}{m}}\,A = \sqrt{\frac{500}{5}} \times 0.10$$

$$= \mathbf{1.0\ m\ s^{-1}}.$$

(iii) The maximum acceleration of the collar,

$$a_{max} = \omega^2 A = \frac{k}{m}\,A = \frac{500}{5} \times 0.10 = \mathbf{10\ m\ s^{-2}}.$$

EXAMPLE 36. A small trolley of mass 2.0 kg resting on a horizontal turn table is connected by a light spring to the centre of the table. When the turn table is set into rotation at a speed of 300 rpm, the length of the stretched spring is 40 cm. If the original length of the spring is 35 cm, determine the force constant of the spring.

Solution. Mass of trolley, $m = 2.0$ kg

Frequency of rotation of turn table,

$$\nu = \frac{300}{60} = 5 \text{ rps}$$

Extension produced in the string,

$$y = 40 - 35 = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

When the turn-table is set into rotation, the tension (restoring force) in spring is equal to the centripetal force. Thus

Restoring force = Centripetal force

$$F = ky = mr\omega^2 = mr(2\pi\nu)^2$$

or $$k = \frac{4\pi^2\nu^2\,mr}{y}$$

[r = length of stretched spring = 40 cm]

$$= \frac{4 \times 9.87 \times 5^2 \times 2.0 \times 40 \times 10^{-2}}{5 \times 10^{-2}}$$

$$= \mathbf{15795\ Nm^{-1}}.$$

EXAMPLE 37. Two masses $m_1 = 1.0$ kg and $m_2 = 0.5$ kg are suspended together by a massless spring of force constant, $k = 12.5$ Nm^{-1}. When they are in equilibrium position, m_1 is gently removed. Calculate the angular frequency and the amplitude of oscillation of m_2. Given g = 10 ms^{-2}.

Scanned by CamScanner

Solution. Let y be the extension in the length of the spring when both m_1 and m_2 are suspended. Then

$$F = (m_1 + m_2)g = ky$$

or $$y = \frac{(m_1 + m_2)g}{k}$$

Let the extension be reduced to y' when m_1 is removed, then

$$m_2 g = ky'$$

or $$y' = \frac{m_2 g}{k}$$

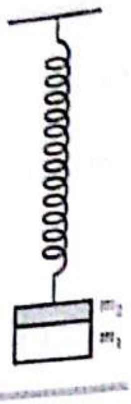


Fig. 14.18

$$\therefore \quad y - y' = \frac{(m_1 + m_2)g}{k} - \frac{m_2 g}{k} = \frac{m_1 g}{k}$$

This will be the amplitude of oscillation of m_2.

$\therefore$ Amplitude, $A = \frac{m_1 g}{k} = \frac{1.0 \times 10}{12.5} = 0.8$ **m.**

Angular frequency,

$$\omega = \sqrt{\frac{k}{m_2}} = \sqrt{\frac{12.5}{0.5}} = 5 \text{ rad s}^{-1}.$$

EXAMPLE 38. *Two identical springs, each of spring factor k, may be connected in the following ways. Deduce the spring factor of the oscillation of the body in each case.*

Solution. For each spring,

$$F = -ky \qquad ...(1)$$

where F = restoring force, k = spring factor, and y = displacement of the spring.

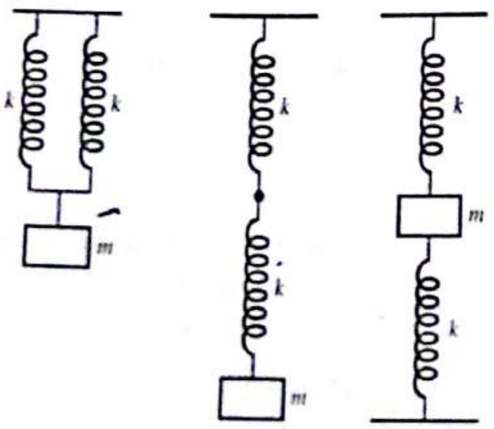


Fig. 14.19

(*i*) In Fig. 14.19(*a*), let the mass m produce a displacement y in each spring and F be the restoring force in each spring. If k_1 be the spring factor of the combined system, then

$$2F = -k_1 y$$

or $$F = -\frac{k_1}{2} y \qquad ...(2)$$

Comparing (1) and (2), we get

$$\frac{k_1}{2} = k \quad \text{or} \quad k_1 = 2k.$$

(*ii*) In Fig. 14.19(*b*), as the length of the spring is doubled, the mass m will produce double the displacement $(2y)$. If k_2 be the spring factor of the combined system, then

$$F = -k_2(2y) = -2k_2 y \qquad ...(3)$$

Comparing (1) and (3),

$$2k_2 = k \quad \text{or} \quad k_2 = \frac{k}{2}.$$

(*iii*) In Fig. 14.19(*c*), the mass m stretches the upper spring and compresses the lower spring, each giving rise to a restoring force F in the same direction. If k_3 be the spring factor of the combined system, then

$$2F = -k_3 y$$

or $$F = -\frac{k_3}{2} y \qquad ...(4)$$

Comparing (1) and (4),

$$\frac{k_3}{2} = k \quad \text{or} \quad k_3 = 2k.$$

EXAMPLE 39. *Two identical springs, each of force constant k are connected in (a) series (b) parallel, and they support a mass m. Calculate the ratio of the time periods of the mass in the two systems.* **[Central Schools 07]**

Solution. (*a*) For series combination, the effective force constant is

$$k_s = \frac{k \times k}{k + k} = \frac{k}{2}$$

$$\therefore \quad T_s = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m}{k/2}}$$

(*b*) For parallel combination, the effective force constant is

$$k_p = k + k = 2k$$

$$\therefore \quad T_p = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{2k}}$$

Required ratio of the time periods,

$$\frac{T_s}{T_p} = \sqrt{\frac{2k}{k/2}} = 2$$

EXAMPLE 40. A tray of mass 12 kg is supported by two identical springs as shown in Fig. 14.20. When the tray is pressed down slightly and released, it executes SHM with a time period of 1.5 s. What is the force constant of each spring? When a block of mass M is placed on the tray, the period of SHM changes to 3.0 s. What is the mass of the block?

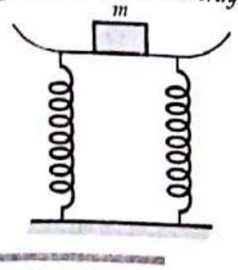


Fig. 14.20

Solution. Let k be the force constant of each spring. As the two springs are connected in parallel, so the force constant of the combination is

$$k' = k + k = 2k$$

Now $$T = 2\pi\sqrt{\frac{m}{k'}}$$

or $$k' = \frac{4\pi^2 m}{T^2} = \frac{4 \times (3.14)^2 \times 12}{(1.5)^2}$$

$$= 210.34 \text{ Nm}^{-1}$$

∴ $$k = k'/2 = \mathbf{105.17 \text{ Nm}^{-1}}.$$

When a block of mass M is placed in the tray, the period of oscillation becomes

$$T' = 2\pi\sqrt{\frac{M+m}{k'}}$$

Hence $$\frac{T'}{T} = \sqrt{\frac{M+m}{m}} \quad \text{or} \quad \frac{3.0}{1.5} = \sqrt{\frac{M+12}{12}}$$

or $$\sqrt{\frac{M+12}{12}} = 2 \quad \text{or} \quad \frac{M+12}{12} = 4$$

∴ $$M = 48 - 12 = \mathbf{36 \text{ kg}}.$$

EXAMPLE 41. The identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown below.

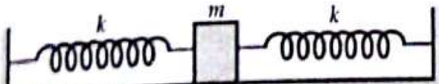


Fig. 14.21

Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations. **[NCERT ; Delhi 13]**

Solution. As shown in Fig. 14.22, suppose the mass m is displaced by a small distance x to the right side of the equilibrium position O. Then the left spring gets elongated by length x and the right spring gets compressed by the same length x.

Force exerted by the left spring,

$$F_1 = -kx, \text{ towards left}$$

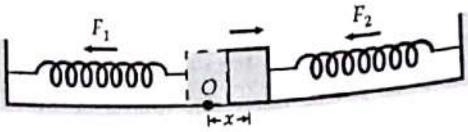


Fig. 14.22

Force exerted by the right spring,

$$F_2 = -kx, \text{ towards left}$$

The net force acting on mass m is

$$F = F_1 + F_2 = -2kx$$

Thus the force acting on the mass m is proportional to its displacement x and is directed towards its mean position. Hence the motion of the mass m is simple harmonic. Force constant is

$$k' = 2k$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}.$$

EXAMPLE 42. A trolley of mass 3.0 kg is connected to two identical springs each of force constant 600 Nm^{-1}, as shown in Fig. 14.23. If the trolley is displaced from its equilibrium position by 5.0 cm and released, what is (i) the period of ensuing oscillations, (ii) the maximum speed of the trolley ? (iii) How much is the total energy dissipated as heat by the time the trolley comes to rest due to damping forces ?

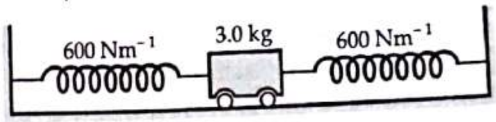


Fig. 14.23

Solution. When the trolley is displaced from the mean position, it stretches one spring and compresses the other by the same amount. The restoring forces developed in the two springs are in the same direction. If the trolley is displaced through distance y, then total restoring force is

$$F = F_1 + F_2 = -ky - ky = -2ky$$

If k' is the force constant of the combination, then

$$F = -k'y$$

Clearly, $k' = 2k = 2 \times 600 = 1200 \text{ Nm}^{-1}$

Also, $m = 3.0$ kg

amplitude, $A = 5.0 \times 10^{-2}$ m

(*i*) Period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k'}}$$

$$= 2 \times 3.14\sqrt{\frac{3.0}{1200}} = \mathbf{0.314 \text{ s}}.$$

Scanned by CamScanner

(ii) Maximum speed

$$v_{max} = \omega A = \sqrt{\frac{k}{m}} \times A$$

$$= \sqrt{\frac{1200}{3.0}} \times 5.0 \times 10^{-2} = \mathbf{1.0\ ms^{-1}}.$$

(iii) Total energy dissipated as heat

= Initial maximum K.E. of the trolley

$$= \frac{1}{2} m v_{max}^2 = \frac{1}{2} \times 3.0 \times (1.0)^2 = \mathbf{1.5\ J}.$$

✖ PROBLEMS FOR PRACTICE

1. A spring compressed by 0.1 m develops a restoring force of 10 N. A body of mass 4 kg is placed on it. Deduce (i) the force constant of the spring (ii) the depression of the spring under the weight of the body and (iii) the period of oscillation, if the body is disturbed.

[**Ans.** (i) 100 Nm^{-1} (ii) 0.4 m (iii) 1.26 s]

2. The period of oscillation of a mass m suspended by an ideal spring is 2 s. If an additional mass of 2 kg be suspended, the time period is increased by 1 s. Find the value of m. (**Ans.** 1.6 kg)

3. An uncalibrated spring balance is found to have a period of oscillation of 0.314 s, when a 1 kg weight is suspended from it ? How does the spring elongate, when a 1 kg weight is suspended from it ? [Take π = 3.14] (**Ans.** 2.45 cm)

4. The frequency of oscillations of a mass m suspended by a spring is v_1. If the length of the spring is cut to one-half, the same mass oscillates with frequency v_2. Determine the value of v_2 / v_1. [Chandigarh 03]

(**Ans.** $\sqrt{2}$)

5. The periodic time of a mass suspended by a spring (force constant k) is T. If the spring is cut in three equal pieces, what will be the force constant of each part ? If the same mass be suspended from one piece, what will be the periodic time ?

(**Ans.** $3k$, $T/\sqrt{3}$)

6. The time period of a body suspended by a spring be T. What will be the new period, if the spring is cut into two equal parts and when (i) the body is suspended from one part (ii) the body is suspended from both the parts connected in parallel.

[**Ans.** (i) $T/\sqrt{2}$ (ii) $T/2$]

7. Two identical springs have the same force constant of 147 Nm^{-1}. What elongation will be produced in each spring in each case shown in Fig. 14.24 ? Take g = 9.8 ms^{-2}.

[**Ans.** (a) 1/6 m (b) 1/3 m, 1/3 m (c) 1/3 m]

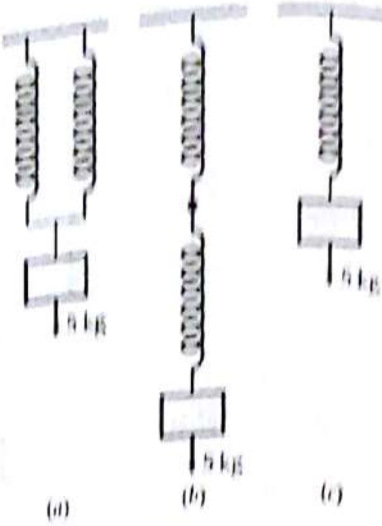


Fig. 14.24

Fig. 14.25

8. Three springs are connected to a mass m as shown in Fig. 14.25. When mass m oscillates, what is the effective spring constant and time period of vibration ? Given k = 2 Nm^{-1} and m = 80 g.

(**Ans.** 8 Nm^{-1}, 0.628 s)

9. Two springs are joined and connected to a mass m as shown in Fig. 14.26. If the force constants of the two springs are k_1 and k_2, show that frequency of oscillation of mass m is

$$v = \frac{1}{2\pi} \sqrt{\frac{k_1 k_2}{(k_1 + k_2) m}}.$$

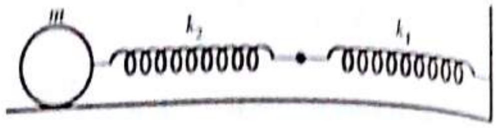


Fig. 14.26

✖ HINTS

1. (i) $k = \frac{F}{y} = \frac{10}{0.1} = \mathbf{100\ Nm^{-1}}$.

(ii) $y = \frac{mg}{k} = \frac{4 \times 10}{100} = \mathbf{0.4\ m}$.

(iii) $T = 2\pi \sqrt{\frac{m}{k}} = 2 \times \frac{22}{7} \times \sqrt{\frac{4}{100}} = \mathbf{1.26\ s}$.

2. As $T = 2\pi \sqrt{\frac{m}{k}}$

∴ In first case,

$$2 = 2\pi \sqrt{\frac{m}{k}} \quad \text{or} \quad 4 = 4\pi^2 \times \frac{m}{k} \qquad ...(1)$$

In second case,

$$3 = 2\pi \sqrt{\frac{m+2}{k}} \quad \text{or} \quad 9 = 4\pi^2 \times \frac{m+2}{k} \qquad ...(2)$$

Dividing (2) by (1), we get

$$\frac{9}{4} = \frac{m+2}{m} \quad \text{or} \quad m = \frac{8}{5} = 1.6 \text{ kg}.$$

4. Let k be the force constant of the full spring. Then frequency of oscillation of mass m will be

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

When the spring is cut to one half of its length, its force constant is doubled $(2k)$. Frequency of oscillation of mass m will be

$$\nu_2 = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \quad \therefore \quad \nu_2/\nu_1 = \sqrt{2}.$$

5. Time period of mass m when suspended from the full spring is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

When the spring is cut into three equal parts, the force constant of each part becomes $3k$. Time period of mass m when suspended from one such piece will be

$$T' = 2\pi\sqrt{\frac{m}{3k}} = \frac{T}{\sqrt{3}}.$$

6. For full spring, $T = 2\pi\sqrt{\frac{m}{k}}$.

If the spring is cut into two equal parts, then the force constant of each part becomes $2k$.

(i) When the body is suspended from one part, its period of oscillation is

$$T' = 2\pi\sqrt{\frac{m}{2k}} = \frac{T}{\sqrt{2}}.$$

(ii) For the two parts connected in parallel, force constant

$$= 2k + 2k = 4k.$$

The period of oscillation becomes

$$T'' = 2\pi\sqrt{\frac{m}{4k}} = \frac{T}{2}.$$

7. Here $k = 147 \text{ Nm}^{-1}$. In Fig. 14.24(a), the effective spring constant,

$$K = k + k = 2k = 2 \times 147 = 294 \text{ Nm}^{-1}$$

$\therefore$ Elongation in the spring,

$$y_1 = \frac{mg}{K} = \frac{5 \times 9.8}{294} = \frac{1}{6} \text{ m}.$$

In Fig. 14.24(b), the effective spring constant,

$$K = \frac{k \times k}{k + k} = \frac{k}{2} = \frac{147}{2} \text{ Nm}^{-1}$$

$\therefore$ Total elongation in the spring,

$$y_2 = \frac{5 \times 9.8 \times 2}{147} = \frac{2}{3} \text{ m}$$

$\therefore$ Elongation in each spring $= \frac{1}{3}$ m.

In Fig. 14.24(c), the effective spring constant,

$$K = 147 \text{ Nm}^{-1}$$

$\therefore$ Elongation in the spring, $y_3 = \frac{5 \times 9.8}{147} = \frac{1}{3}$ m.

8. The given arrangement is equivalent to the three springs connected in parallel. The effective spring constant is

$$K = k + 2k + k = 4k = 4 \times 2 = 8 \text{ Nm}^{-1}.$$

Time period,

$$T = 2\pi\sqrt{\frac{m}{4k}} = 2 \times \frac{22}{7} \times \sqrt{\frac{0.08}{8}} = 0.628 \text{ s}.$$

9. Let a force F applied on the body produce displacements x_1 and x_2 in the two springs. Then

$$F = -k_1 x_1 = -k_2 x_2$$

$$\therefore \quad x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2}$$

Total extension,

$$x = x_1 + x_2 = -F\left[\frac{1}{k_1} + \frac{1}{k_2}\right] = -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

or $$F = -\frac{k_1 k_2}{k_1 + k_2}\, x$$

Clearly, force constant of the system, $k = \frac{k_1 k_2}{k_1 + k_2}$

Frequency, $$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)\, m}}.$$

14.16 SIMPLE PENDULUM

19. *Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period. Does it depend on the mass of the bob ?*

Simple pendulum. *An ideal simple pendulum consists of a point-mass suspended by a flexible, inelastic and weightless string from a rigid support of infinite mass.* In practice, we can neither have a point-mass nor a weightless string.

In practice, a simple pendulum is obtained by suspending a small metal bob by a long and fine cotton thread from a rigid support.

Expression for time period. In the equilibrium position, the bob of a simple pendulum lies vertically below the point of suspension. If the bob is slightly displaced on either side and released, it begins to oscillate about the mean position.

Suppose at any instant during oscillation, the bob lies at position A when its displacement is $OA = x$ and the thread makes angle θ with the vertical. The forces acting on the bob are

(i) Weight mg of the bob acting vertically downwards.

(ii) Tension T along the string.

Scanned by CamScanner

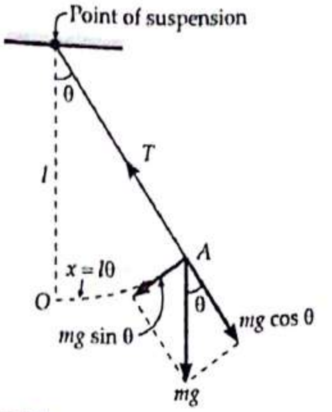


Fig. 14.27 Force acting on the bob of a pendulum.

The force mg has two rectangular components (*i*) the component $mg\cos\theta$ acting along the thread is balanced by the tension T in the thread and (*ii*) the tangential component $mg\sin\theta$ is the net force acting on the bob and tends to bring it back to the mean position. Thus, the restoring force is

$$F = -mg\sin\theta = -mg\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - .....\right)$$

$$= -mg\,\theta\left(1 - \frac{\theta^2}{6} + \frac{\theta^4}{120} - .....\right)$$

where θ is in radians. Clearly, oscillations are not simple harmonic because the restoring force F is not proportional to the angular displacement θ.

However, if θ is so small that its higher powers can be neglected, then

$$F = -mg\,\theta$$

If l is the length of the simple pendulum, then

$$\theta\,(\text{rad}) = \frac{\text{arc}}{\text{radius}} = \frac{x}{l}$$

$$\therefore \quad F = -mg\frac{x}{l}$$

or $$ma = -\frac{mg}{l}x$$

or $$a = -\frac{g}{l}x = -\omega^2 x$$

Thus, the acceleration of the bob is proportional to its displacement x and is directed opposite to it. Hence for small oscillations, the motion of the bob is simple harmonic. Its time period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g/l}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Obviously, the time period of a simple pendulum depends on its length l and acceleration due to gravity g but is independent of the mass m of the bob.

Examples based on Oscillations of a Simple Pendulum

FORMULAE USED

1. Time period, $T = 2\pi\sqrt{\frac{l}{g}}$
2. Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

UNITS USED

Length l of the pendulum is in metre and acceleration due to gravity g in ms^{-2}.

EXAMPLE 43. *What is the length of a simple pendulum, which ticks seconds ?* **[NCERT ; Delhi 09 ; Central Schools 14]**

Solution. The simple pendulum which ticks seconds is a second pendulum whose time period is 2 s. Thus

$$T = 2\text{ s}, \quad g = 9.8\text{ ms}^{-2}$$

As $$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad T^2 = 4\pi^2\frac{l}{g}$$

$$\therefore \quad l = \frac{T^2 g}{4\pi^2} = \frac{(2)^2 \times 9.8}{4 \times 9.87} = \mathbf{0.992\ m}.$$

EXAMPLE 44. *A pendulum clock shows accurate time. If the length increases by 0.1%, deduce the error in time per day.* **[Delhi 99]**

Solution. Correct number of seconds per day,

$$\nu = 24 \times 60 \times 60 = 86400.$$

Let error introduced per day = x seconds

Then incorrect number of seconds per day,

$$\nu' = 86400 + x$$

If l is the original length of the pendulum, then its new length will be

$$l' = l + 0.1\%\text{ of } l = l + \frac{0.1 \times l}{100} = (1 + 0.001)\,l$$

Now frequency, $$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad i.e., \quad \nu \propto \frac{1}{\sqrt{l}}$$

$$\therefore \quad \frac{\nu'}{\nu} = \sqrt{\frac{l}{l'}} \quad \text{or} \quad \frac{86400 + x}{86400} = \sqrt{\frac{l}{(1 + 0.001)\,l}}$$

or $$1 + \frac{x}{86400} = (1 + 0.001)^{-1/2}$$

$$= 1 - \frac{1}{2} \times 0.001 = 1 - 0.0005$$

or $$\frac{x}{86400} = -0.0005$$

or $$x = -0.0005 \times 86400 = \mathbf{-43.2\ s}.$$

The negative sign shows that the clock will run slow and it will lose 43.2 seconds per day.

Scanned by CamScanner

EXAMPLE 45. *Two pendulums of lengths 100 cm and 110.25 cm start oscillating in phase. After how many oscillations will they again be in same phase ?*

Solution. The two pendulums will be in same phase again when large pendulum completes ν oscillations and small pendulum completes (ν + 1) oscillations.

For larger pendulum,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{g}{110.25}}$$

For smaller pendulum, $\nu + 1 = \frac{1}{2\pi}\sqrt{\frac{g}{100}}$

$$\frac{\nu+1}{\nu} = \sqrt{\frac{110.25}{100}}$$

$$= \sqrt{\frac{100 + 10.25}{100}} = \left(1 + \frac{10.25}{100}\right)^{1/2}$$

or

$$1 + \frac{1}{\nu} = 1 + \frac{1}{2} \times \frac{10.25}{100} = 1 + 0.05$$

or

$$\nu = \frac{1}{0.05} = 20$$

Thus the two pendulums will be in same phase when the larger pendulum completes 20 oscillations or smaller pendulum completes 21 oscillations.

EXAMPLE 46. *A second's pendulum is taken in a carriage. Find the period of oscillation when the carriage moves with an acceleration of 4 ms^{-2} (i) vertically upwards (ii) vertically downwards, and (iii) in a horizontal direction.*

Solution. Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

For second's pendulum, $T = 2$ s

$$\therefore \quad 2 = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad 1 = \pi\sqrt{\frac{l}{g}}$$

or

$$1 = \pi^2 \frac{l}{g} \quad \therefore \quad l = \frac{g}{\pi^2} = \frac{9.8}{\pi^2}.$$

(i) When the carriage moves up with an acceleration $a = 4$ ms^{-2}, the time period is

$$T_1 = 2\pi\sqrt{\frac{l}{g+a}} = 2\pi\sqrt{\frac{9.8}{\pi^2(9.8+4)}}$$

$$= \frac{2\pi}{\pi}\sqrt{\frac{9.8}{13.8}} = 2 \times 0.843 = \mathbf{1.69\ s}.$$

(ii) When the carriage moves down with an acceleration $a = 4$ ms^{-2}, the time period is

$$T_2 = 2\pi\sqrt{\frac{l}{g-a}} = 2\pi\sqrt{\frac{9.8}{\pi^2(9.8-4)}}$$

$$= 2\sqrt{\frac{9.8}{5.8}} = 2 \times 1.299 = \mathbf{2.59\ s}.$$

(iii) When the carriage moves horizontally, both g and a are at right angle to each other, hence the net acceleration is

$$a' = \sqrt{g^2 + a^2} = \sqrt{(9.8)^2 + (4)^4}$$

$$= \sqrt{96.04 + 16} = \sqrt{112.04} = 10.58 \text{ ms}^{-2}$$

Time period will be

$$T_3 = 2\pi\sqrt{\frac{l}{a'}} = 2\pi\sqrt{\frac{9.8}{\pi^2 \times 10.58}}$$

$$= 2 \times 0.96 = \mathbf{1.92\ s}.$$

EXAMPLE 47. *The bottom of a dip on a road has a radius of curvature R. A rickshaw of mass M left a little away from the bottom oscillates about the dip. Deduce an expression for the period of oscillation.* **[Chandigarh 02]**

Solution. As shown in Fig. 14.28, let the rickshaw of mass M be at position A at any instant and $\angle AOB = \theta$.

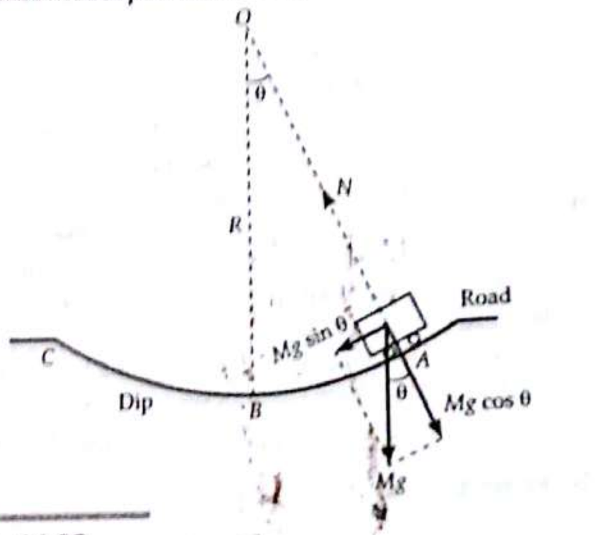


Fig. 14.28

Forces acting on the rickshaw at position A are

(i) Weight Mg acting vertically downwards.

(ii) The normal reaction N of the road.

The weight Mg can be resolved into two rectangular components :

(i) $Mg \cos\theta$ perpendicular to the road. It balances the normal reaction N.

(ii) $Mg \sin\theta$ tangential to the road. It is the only unbalanced force acting on the rickshaw which acts towards the mean position B. Hence the restoring force is

$$F = -Mg \sin\theta$$

For small θ, $\quad \sin\theta \simeq \theta = \frac{\text{Arc}}{\text{Radius}} = \frac{AB}{R} = \frac{y}{R}$

$$\therefore \quad F = -\frac{Mg}{R}y \quad i.e., \quad F \propto y$$

Scanned by CamScanner

Hence the motion of the rickshaw is simple harmonic with force constant,

$$k = \frac{Mg}{R}$$

Time period,

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M}{Mg/R}} = 2\pi\sqrt{\frac{R}{g}}.$$

PROBLEMS FOR PRACTICE

1. The time taken by a simple pendulum to perform 100 vibrations is 8 minutes 9 seconds in Bombay and 8 minutes 20 seconds in Pune. Calculate the ratio of acceleration due to gravity in Bombay and Pune. **(Ans. 1.0455)**
2. If the length of a pendulum is decreased by 2%, find the gain or loss in time per day. **(Ans. Gain of 864 s)**
3. If the length of a second's pendulum is increased by 1%, how many seconds will it lose or gain in a day ? **(Ans. Loss of 432 s)**
4. If the length of a simple pendulum is increased by 45%, what is the percentage increase in its time period ? **(Ans. 22.5%)**
5. What will be the time period of second's pendulum if its length is doubled ? **(Ans. 2.828 s)**
6. If the acceleration due to gravity on moon is one-sixth of that on the earth, what will be the length of a second pendulum there ? Take $g = 9.8\ ms^{-2}$. **(Ans. 16.5 cm)**

HINTS

1. Let g_1 and g_2 be the values of acceleration due to gravity in Bombay and Pune and T_1 and T_2 be the values of the time-periods at the respective places. Then

$$T_1 = \frac{8 \text{ min } 9 \text{ s}}{100} = \frac{489}{100} \text{ s} = 4.89 \text{ s}$$

$$T_2 = \frac{8 \text{ min } 20 \text{ s}}{100} = \frac{500 \text{ s}}{100} = 5 \text{ s}$$

As $\quad \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$

$$\therefore \quad \frac{g_1}{g_2} = \frac{T_2^2}{T_1^2} = \frac{(5)^2}{(4.89)^2} = \mathbf{1.0455}.$$

2. As $v \propto 1/\sqrt{l}$, so the number of seconds gained per day on decreasing the length by 2%

$$= \frac{1}{2}\frac{\Delta l}{l} \times 86400 = \frac{1}{2} \times \frac{2}{100} \times 86400 = \mathbf{864\ s}.$$

4. As $T \propto \sqrt{l}$, so the percentage increase in time period on increasing the length by 45%

$$= \frac{1}{2}\frac{\Delta l}{l} \times 100 = \frac{1}{2} \times \frac{45}{100} \times 100 = 22.5\%.$$

6. On the moon, $g_m = \frac{g}{6} = \frac{9.8}{6}\ ms^{-2}, \quad T = 2\ s.$

As $\quad T = 2\pi\sqrt{\frac{l}{g_m}}$

$$\therefore \quad l = \frac{T^2 g_m}{4\pi^2} = \frac{2^2 \times 9.8}{4 \times 9.87 \times 6} = 0.165\ m = 16.5\ cm.$$

14.17 OTHER EXAMPLES OF S.H.M.

20. *One end of a U-tube containing mercury is connected to a suction pump and the other end is connected to the atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid in the U-tube executes SHM.* [NCERT]

Oscillations of a liquid column in a U-tube. Initially, suppose the U-tube of cross-section A contains liquid of density ρ upto height h. Then mass of the liquid in the U-tube is

$$m = \text{Volume} \times \text{density} = A \times 2h \times \rho$$

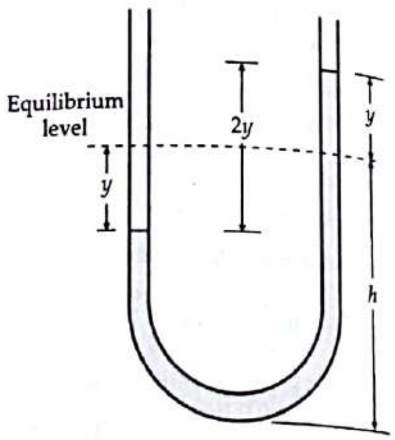


Fig. 14.29 Oscillations of a liquid column in a U-tube.

If the liquid in one arm is depressed by distance y, it rises by the same amount in the other arm. If left to itself, the liquid begins to oscillate under the restoring force,

$$F = \text{Weight of liquid column of height } 2y$$

$$F = -A \times 2y \times \rho \times g = -2A\rho g y$$

i.e., $\quad F \propto y$

Thus the force on the liquid is proportional to displacement and acts in its opposite direction. Hence the liquid in the U-tube executes SHM with force constant,

$$k = 2A\rho g$$

Scanned by CamScanner

The time-period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{A \times 2h \times \rho}{2\,A\rho g}} = 2\pi\sqrt{\frac{h}{g}}.$$

If l is the length of the liquid column, then

$$l = 2h \quad \text{and} \quad T = 2\pi\sqrt{\frac{l}{2g}}.$$

21. *If the earth were a homogeneous sphere of radius R and a straight hole bored in it through its centre, show that a body dropped into the hole will execute SHM and find its time period.*

Oscillations of a body dropped in a tunnel along the diameter of the earth. As shown in Fig. 14.30, consider earth to be a sphere of radius R and centre O. A straight tunnel is dug along the diameter of the earth. Let g be the value of acceleration due to gravity at the surface of the earth.

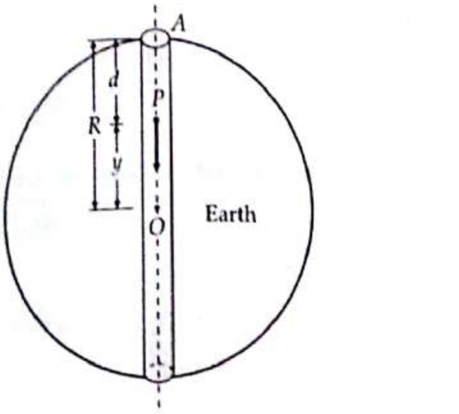


Fig. 14.30 A body dropped in a tunnel along the diameter of the earth.

Suppose a body of mass m is dropped into the tunnel and it is at point P *i.e.*, at a depth d below the surface of the earth at any instant. If g' is acceleration due to gravity at P, then

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right)$$

If y is distance of the body from the centre of the earth (displacement from mean position), then

$$R - d = y \quad \therefore \quad g' = g\,\frac{y}{R}$$

Force acting on the body at point P is

$$F = -mg' = -\frac{mg}{R}\,y \qquad i.e., \qquad F \propto y$$

Negative sign shows that the force F acts in the opposite direction of displacement *i.e.*, it acts towards the mean position O. Thus the body will execute SHM with force constant,

$$k = \frac{mg}{R}$$

The period of oscillation of the body will be

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/R}} = 2\pi\sqrt{\frac{R}{g}}.$$

22. *A cylindrical piece of cork of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$, where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).* **[NCERT]**

Oscillations of a floating cylinder. In equilibrium, weight of the cork is balanced by the upthrust of the liquid.

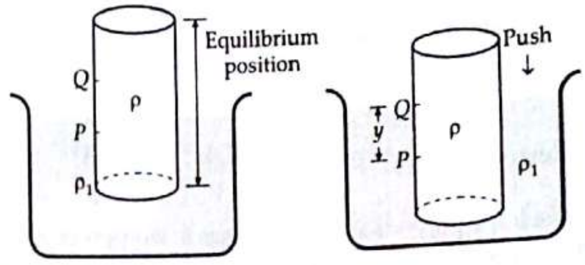


Fig. 14.31 Oscillations of a floating cylinder.

Let the cork be slightly depressed through distance y from the equilibrium position and left to itself. It begins to oscillate under the restoring force,

$$F = \text{Net upward force}$$
$$= \text{Weight of liquid column of height } y$$

or $\quad F = -A\,y\,\rho_1\,g = -A\rho_1\,g\,y \quad i.e., \quad F \propto y.$

Negative sign shows that F and y are in opposite directions. Hence the cork executes SHM with force constant, $\quad k = A\rho_1\,g$

Also, mass of cork $= A\rho\,h$

$\therefore$ Period of oscillation of the cork is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{A\rho\,h}{A\rho_1\,g}} = 2\pi\sqrt{\frac{\rho\,h}{\rho_1\,g}}.$$

23. *An air chamber of volume V has a neck of area of cross-section A into which a ball of mass m can move without friction. Show that when the ball is pressed down through some distance and released, the ball executes SHM. Obtain the formula for the time period of this SHM, assuming pressure-volume variations of the air to be (i) isothermal and (ii) adiabatic.* **[NCERT]**

Oscillations of a ball in the neck of an air chamber. Fig. 14.32 shows an air chamber of volume V, having a

Scanned by CamScanner

neck of area of cross-section A and a ball of mass m fitting smoothly in the neck. If the ball be pressed down a little and released, it starts oscillating up and down about the equilibrium position.

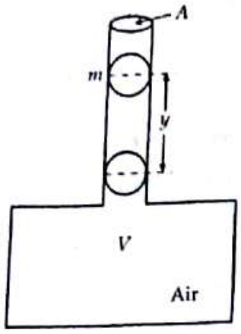


Fig. 14.32

If the ball be depressed by distance y, then the decrease in volume of air in the chamber is $\Delta V = Ay$.

$\therefore$ Volume strain $= \dfrac{\Delta V}{V} = \dfrac{Ay}{V}$

If pressure P is applied to the ball, then hydrostatic stress $= P$

$\therefore$ Bulk modulus of elasticity of air,

$$E = -\frac{P}{\Delta V / V} = -\frac{P}{Ay/V} \quad \text{or} \quad P = -\frac{EA}{V} y$$

Restoring force, $F = PA = -\dfrac{EAy}{V} A = -\dfrac{EA^2}{V} y$

Thus F is proportional to y and acts in its opposite direction. Hence the ball executes SHM with force constant,

$$k = \frac{EA^2}{V}$$

Period of oscillation of the ball is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{EA^2/V}} = 2\pi\sqrt{\frac{mV}{EA^2}}$$

(*i*) If the P-V variations are *isothermal*, then $E = P$,

$$\therefore \quad T = 2\pi\sqrt{\frac{mV}{PA^2}}.$$

(*ii*) If the P-V variations are adiabatic, then $E = \gamma P$

$$\therefore \quad T = 2\pi\sqrt{\frac{mV}{\gamma PA^2}}.$$

24. *Show that the angular oscillations of a balance wheel of a watch are simple harmonic. Hence derive an* xpression for its period of* scillation.*

Oscillations of the alance-wheel of a watch. a watch, a balance-wheel ntrols the movement of hands. An axle passing ough its centre is held ween two diamond nts. A hair-spring con- s its oscillations.

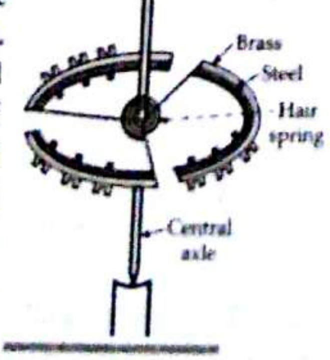


Fig. 14.33

For an angular displacement θ, the hair-spring develops a restoring torque $C\theta$, which tends to bring back the wheel into its equilibrium position. Here C is the restoring torque produced per unit angular displacement. Now

Torque = Moment of inertia × angular acceleration

$$\therefore \quad C\theta = -I \times \frac{d^2\theta}{dt^2}$$

$$\text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta = -\omega^2\theta$$

where I is the moment of inertia of the wheel about its axis of rotation. Clearly, angular acceleration $\dfrac{d^2\theta}{dt^2}$ is proportional to angular displacement θ and acts in its opposite direction. Hence oscillations of the balance-wheel are simple harmonic.

Angular frequency, $\omega = \sqrt{\dfrac{C}{I}}$

Period of oscillation, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{C/I}} = 2\pi\sqrt{\dfrac{I}{C}}$.

Examples based on Other Examples of SHM

Formulae Used

1. For a liquid of density ρ contained in a U-tube upto height h, $T = 2\pi\sqrt{\dfrac{h}{g}}$
2. For a body dropped in a tunnel along the diameter of the earth,
   $T = 2\pi\sqrt{\dfrac{R}{g}}$, where R = radius of the earth
3. For a cylinder of density ρ floating with length h submerged in a liquid of density ρ, $T = 2\pi\sqrt{\dfrac{\rho h}{\sigma g}}$
4. For a ball of mass m oscillating in the neck of air chamber of volume V, $T = 2\pi\sqrt{\dfrac{mV}{EA^2}}$
   where A = area of cross-section of the neck,
   E = bulk modulus of elasticity of air
5. For a balance-wheel of a watch of moment of inertia I and torsional constant C, $T = 2\pi\sqrt{\dfrac{I}{C}}$

Units Used

Here h and R are in metre, densities ρ and σ in kg m^{-3}, bulk modulus E in Nm^{-2}, moment of inertia I in kg m^2, torsional constant C in Nm rad^{-1}.

Scanned by CamScanner

EXAMPLE 48. *A vertical U-tube of uniform cross-section contains water upto a height of 2.45 cm. If the water on one side is depressed and then released, its up and down motion in tube is simple harmonic. Calculate its time period. Given $g = 980\ cm\ s^{-2}$.*

Solution. Here $h = 2.45$ cm, $g = 980$ cm s^{-2}

$$\therefore \quad T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{2.45}{980}} = 2\pi \times 0.05 = \mathbf{0.314\ s.}$$

EXAMPLE 49. *A test tube weighing 10 g and external diameter 2 cm is floated vertically in water by placing 10 g of mercury at its bottom. The tube is depressed in water a little and then released. Find the time of oscillation. Take $g = 10\ ms^{-2}$.*

Solution. Total mass of test tube and mercury,

$$m = 10 + 10 = 20\ \text{g} = 0.02\ \text{kg}$$

Area of cross-section of the test-tube,

$$A = \pi r^2 = \frac{22}{7} \times \left(\frac{1}{100}\right)^2 = \frac{22}{7} \times 10^{-4}\ \text{m}^2$$

Density of water, $\rho = 10^3$ kg m^{-3}

Let the tube be depressed in water by a little distance y and then released.

Spring factor,

$$k = \frac{F}{y} = \frac{Ay \cdot \rho \cdot g}{y} = A\rho g$$

$$= \frac{22}{7} \times 10^{-4} \times 10^3 \times 10 = \frac{22}{7}\ \text{Nm}^{-1}.$$

Inertia factor, $m = 0.02$ kg

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2 \times \frac{22}{7} \times \sqrt{\frac{0.02 \times 7}{22}} = \mathbf{0.5\ s.}$$

EXAMPLE 50. *A cylindrical wooden block of cross-section 15.0 cm² and mass 230 g is floated over water with an extra weight of 50 g attached to its bottom. The cylinder floats vertically. From the state of equilibrium, it is slightly depressed and released. If the specific gravity of wood is 0.30 and $g = 9.8\ ms^{-2}$, deduce the frequency of the block. $A = 15.0\ cm^2 = 15 \times 10^{-4}\ m^2$.*

Solution. Area of cross-section of the block,

$$A = 15.0\ \text{cm}^2 = 15 \times 10^{-4}\ \text{m}^2$$

Total mass of the block,

$$m = 230 + 50 = 280\ \text{g} = 0.28\ \text{kg}$$

Density of water,

$$\sigma = 10^3\ \text{kg m}^{-3}$$

Density of wood,

$$\rho = 0.30 \times 10^3\ \text{kg m}^{-3} = 300\ \text{kg m}^{-3}$$

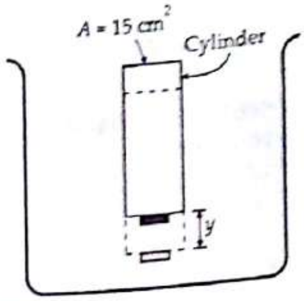


Fig. 14.34

Let the cylinder be depressed through a small distance y. Then

Restoring force = Weight of water displaced

or $\quad F = Ay\sigma g$

Force constant,

$$k = \frac{F}{y} = A\sigma g = 15 \times 10^{-4} \times 10^3 \times 9.8 = \mathbf{14.7\ Nm^{-1}}$$

$$\text{Frequency, } \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{14.7}{0.28}}$$

$$= \frac{7}{44} \times \sqrt{52.5} = \mathbf{1.15\ Hz.}$$

EXAMPLE 51. *The balance wheel of a watch has a moment of inertia of $2 \times 10^{-8}\ kg\ m^2$ and the torsional constant of its hair spring is $9.8 \times 10^{-6}\ Nm\ rad^{-1}$. Calculate its frequency.*

Solution. Here $I = 2 \times 10^{-8}$ kg m^2,

$$C = 9.8 \times 10^{-6}\ \text{Nm rad}^{-1}$$

Frequency,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{7 \times \sqrt{10}}{2 \times 3.14} = \frac{7 \times 3.17}{2 \times 3.14} = \mathbf{3.53\ Hz.}$$

EXAMPLE 52. *A sphere is hung with a wire. 30° rotation of the sphere about the wire generates a restoring torque of 4.6 Nm. If the moment of inertia of the sphere is 0.082 kg m², deduce the frequency of angular oscillations.*

Solution. Here $\theta = 30° = \frac{\pi}{6}$ rad, $\tau = 4.6$ Nm,

$$I = 0.082\ \text{kg m}^2$$

Restoring torque per unit angular displacement,

$$C = \frac{\tau}{\theta} = \frac{4.6}{\pi/6}$$

$$= \frac{4.6 \times 6 \times 7}{22} = 8.78\ \text{Nm rad}^{-1}$$

$\therefore$ Frequency,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

$$= \frac{7}{2 \times 22}\sqrt{\frac{8.78}{0.082}} = \mathbf{1.65\ Hz.}$$

Scanned by CamScanner

✖ PROBLEMS FOR PRACTICE

1. If the earth were a homogeneous sphere and a straight hole was bored in it through its centre, show that a body dropped in the hole will execute SHM and calculate the time period of its vibration. Radius of earth is 6.4×10^5 m and $g = 9.8\ \text{ms}^{-2}$. **(Ans.** 5077.6 s**)**
2. A weighted glass tube is floating in a liquid with 20 cm of its length immersed. It is pushed down through a certain distance and then released. Show that up and down motion executed by the glass tube is SHM and find the time period of vibration. Given, $g = 980\ \text{cm s}^{-2}$. **(Ans.** 0.898 s**)**
3. A sphere is hung with a wire. 60° rotation of the sphere about the wire produces a restoring torque of 4.1 Nm. If the moment of inertia of the sphere is $0.082\ \text{kg m}^2$, find the frequency of angular oscillations. **(Ans. 1.1 Hz)**
4. A lactometer whose mass is 0.2 kg is floating vertically in a liquid of relative density 0.9. Area of cross-section of the marked portion of lactometer is $0.5 \times 10^{-4}\ \text{m}^2$. If it is dipped down in the liquid slightly and released, what type of motion will it execute ? What will be its time-period ?

   (Ans. Motion is simple harmonic, 4.2 s**)**

✖ HINTS

1. Here $R = 6.4 \times 10^5$ m, $g = 9.8\ \text{ms}^{-2}$

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4 \times 10^6}{9.8}} = 2\pi \times 808.1 = \mathbf{5077.6\ s.}$$

2. Here $h = 20$ cm, $g = 980\ \text{cm s}^{-2}$

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{20}{980}} = 2\pi \times 0.143 = \mathbf{0.898\ s.}$$

3. Restoring torque, $\tau = 4.1$ Nm

Angular displacement, $\theta = 60° = \frac{\pi}{3}$ rad

Torsion constant, $C = \frac{\tau}{\theta} = \frac{4.1}{\pi/3} = \frac{4.1 \times 3}{\pi}\ \text{Nm rad}^{-1}$

Moment of inertia, $I = 0.082\ \text{kg m}^2$

Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{3 \times 4.1}{\pi \times 0.082}} = \mathbf{1.1\ Hz.}$

4. When the lactometer is depressed through distance y,

F = upthrust of the liquid $= -A y \rho \times g = -A \rho g y$

As $F \propto y$, so motion of lactometer is SHM with

$k = A \rho y$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{A\rho g}}$$

$$= 2 \times 3.14\sqrt{\frac{0.2}{0.5 \times 10^{-4} \times 0.9 \times 10^3 \times 9.8}} = \mathbf{4.2\ s.}$$

14.18 FREE, DAMPED AND MAINTAINED OSCILLATIONS

25. *What are free, damped and maintained oscillations ? Give examples.*

(a) Free oscillations. *If a body, capable of oscillation, is slightly displaced from its position of equilibrium and left to itself, it starts oscillating with a frequency of its own. Such oscillations are called free oscillations.* The frequency with which a body oscillates freely is called natural frequency and is given by

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Some important features of free oscillations are

(*i*) In the absence of dissipative forces, such a body vibrates with a constant amplitude and fixed frequency, as shown in Fig. 14.35. Such oscillations are also called **undamped oscillations**.

(*ii*) The amplitude of oscillation depends on the energy supplied initially to the oscillator.

(*iii*) The natural frequency of an oscillator depends on its mass, dimensions and restoring force i.e., on its inertial and elastic properties (m and k).

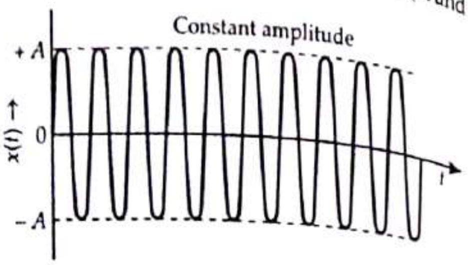


Fig. 14.35 Free or undamped oscillations.

Examples. (*i*) The vibrations of the prongs of tunning fork struck against a rubber pad.

(*ii*) The vibrations of the string of a sitar when pulled aside and released.

(*iii*) The oscillations of the bob of a pendulum when displaced from its mean position and released.

(b) Damped oscillations. *The oscillations in which the amplitude decreases gradually with the passage of time are called damped oscillations.*

In actual practice, most of the oscillations occur in viscous media, such as air, water, etc. A part of the energy of the oscillating system is lost in the form of heat, in overcoming these resistive forces. As a result, the amplitude of such oscillations decreases exponentially with time, as shown in Fig. 14.36. Eventually, these oscillations die out.

In an oscillatory motion, friction produces three effects :

(i) It changes the simple harmonic motion into periodic motion.

(ii) It decreases the amplitude of oscillation.

(iii) It slightly reduces the frequency of oscillation.

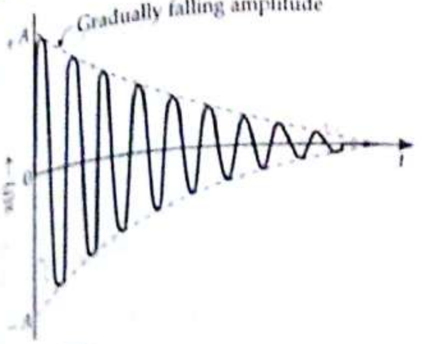


Fig. 14.36 Damped oscillations.

Examples. (i) As shown in Fig. 14.37, consider a block of mass m that oscillates vertically on a spring with spring constant k. The block is connected to a vane through a rod. The vane is submerged in a liquid. As the block oscillates up and down, the vane also oscillates in a similar manner inside the liquid. The liquid exerts an opposing force of viscosity on the vane. The energy of the oscillating system is lost in the liquid as heat. The amplitude of oscillation decreases continuously with time.

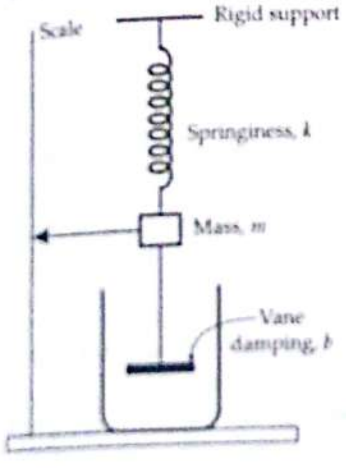


Fig. 14.37 A damped simple harmonic oscillator.

(ii) The oscillations of a swing in air.

(iii) The oscillations of the bob of a pendulum in a fluid.

(c) **Maintained oscillations.** *If to an oscillating system, energy is continuously supplied from outside at the same rate at which the energy is lost by it, then its amplitude can be maintained constant. Such oscillations are called maintained oscillations.* Here, the system oscillates with its own natural frequency.

Examples. (i) The oscillations of the balance wheel of a watch in which the main spring provides the required energy.

(ii) An electrically maintained tuning fork.

(iii) A child's swing in which energy is continuously fed to maintain constant amplitude.

For Your Knowledge

▲ **Differential equation for damped oscillators and its Solution.** In a real oscillator, the damping force is proportional to the velocity v of the oscillator.

$$F_d = -bv$$

where b is damping constant which depends on the characteristics of the fluid and the body that oscillates in it. The negative sign indicates that the damping force opposes the motion.

∴ Total restoring force $= -kx - bv$

or $$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \qquad \left[v = \frac{dx}{dt}\right]$$

or $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

This is the differential equation for damped S.H.M. The solution of the equation is

$$x(t) = a\,e^{-bt/2m}\cos(\omega' t + \phi)$$

The amplitude of the damped S.H.M. is

$$a' = ae^{-bt/2m}$$

where a is amplitude of undamped S.H.M. Clearly, a' decreases exponentially with time.

The angular frequency of the damped oscillator is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Time period, $$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

Clearly, damping increases the time period (due to the presence of the term $b^2/4m^2$ in the denominator).

The mechanical energy of the damped oscillator at any instant t will be

$$E(t) = \frac{1}{2}ka'^2 = \frac{1}{2}ka^2e^{-bt/m}$$

Obviously, the total energy decreases exponentially with time.

As damping constant, $b = F/v$

SI unit of $$b = \frac{N}{ms^{-1}} = \frac{kg\ ms^{-2}}{ms^{-1}} = kg\ s^{-1}$$

CGS unit of $b = g\ s^{-1}$.

Scanned by CamScanner

EXAMPLE 53. *For the damped oscillator shown in earlier Fig. 14.37, the mass m of the block is 200 g, k = 90 Nm⁻¹ and the damping constant b is 40 g s⁻¹. Calculate (a) the period of oscillation, (b) time taken for its amplitude of vibrations to drop to half of its initial value and (c) the time taken for its mechanical energy to drop to half its initial value.* **[NCERT]**

Solution. (*a*) Here $\sqrt{km} = \sqrt{90 \times 0.200} = 4.24 \text{ kg s}^{-1}$

Damping constant, $b = 40 \text{ g s}^{-1}$

As the damping constant, $b << \sqrt{km}$, is small, so the time period T is given by

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2 \text{ kg}}{90 \text{ Nm}^{-1}}} = \mathbf{0.3 \text{ s}}.$$

(*b*) The time, $T_{1/2}$ for the amplitude to drop to half of its initial value is given by

$$\frac{A}{2} = A\,e^{-(bT_{1/2})/2m}$$

$$\therefore \quad T_{1/2} = -\frac{\ln(1/2)}{b/2m} = \frac{0.693}{40} \times 2 \times 200 \text{ s} = \mathbf{6.93 \text{ s}}.$$

(*c*) The time, $t_{1/2}$ for the mechanical energy to drop to half its initial value is given by

$$E(t_{1/2}) = E(0)\,e^{-(bt_{1/2})/m}$$

or $$E(t_{1/2})/E(0) = \exp(-bt_{1/2}/m)$$

or $$1/2 = \exp(-bt_{1/2}/m)$$

$$\ln(1/2) = -(bt_{1/2}/m)$$

or $$t_{1/2} = \frac{0.693}{40 \text{ g s}^{-1}} \times 200 \text{ g} = \mathbf{3.4 \text{ s}}.$$

14.19 FORCED AND RESONANT OSCILLATIONS

26. ***Distinguish between forced and resonant oscillations. Give an experimental illustration in support of your answer. Give examples.***

Forced oscillations. *When a body oscillates under the influence of an external periodic force, not with its own natural frequency but with the frequency of the external periodic force, its oscillations are said to be forced oscillations.* The external agent which exerts the periodic force is called the *driver* and the oscillating system under consideration is called the *driven body*.

Examples. (*i*) When the stem of a vibrating tuning fork is pressed against a table, a loud sound is heard. This is because the particles of table are forced to vibrate with the frequency of the tuning fork.

(*ii*) When the free end of the string of a simple pendulum is held in hand and the pendulum is made to oscillate by giving jerks by the hand, the pendulum executes forced oscillations. Its frequency is same as that of the periodic force exerted by the hand.

(*iii*) The sound boards of all stringed musical instruments like sitar, violin, etc. execute forced oscillations and the frequency of oscillation is equal to the natural frequency of the vibrating string.

Suppose an external periodic force of frequency ν is applied to an oscillator of natural frequency ν_0. Initially, the body tries to vibrate with its own natural frequency, while the applied force tries to drive the body with its own frequency. But soon the natural vibrations of the body die out and finally the body vibrate a with a constant amplitude and with the frequency of the driving force. In this steady state, the rate of loss of energy through friction equals the rate at which energy is fed to the oscillator by the driver.

Fig. 14.38 shows the variation of the amplitude of forced oscillations as the frequency of the driver varies from zero to a large value. Clearly, the amplitude of forced oscillations is very small for $\nu << \nu_0$ and $\nu >> \nu_0$. But when $\nu \approx \nu_0$, the amplitude of the forced oscillations becomes very large. In this condition, the oscillator responds most favourably to the driving force and draws maximum energy from it. The case $\nu = \nu_0$ is called *resonance* and the oscillations are called *resonant oscillations*.

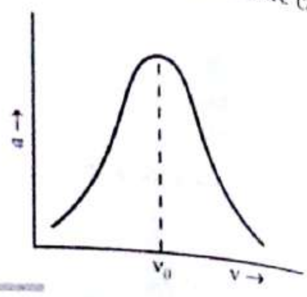


Fig. 14.38 Amplitude *a* of a forced oscillator as a function of the frequency ν of the driver.

Resonant oscillations and resonance. *It is a particular case of forced oscillations in which the frequency of the driving force is equal to the natural frequency of the oscillator itself and the amplitude of oscillations is very large. Such oscillations are called resonant oscillations and the phenomenon is called resonance.*

Examples. (*i*) An aircraft passing near a building shatters its window panes, if the natural frequency of the window matches the frequency of the sound waves sent by the aircraft's engine.

(*ii*) The air-column in a reasonance tube produces loud sound when its frequency matches the frequency of the tuning fork.

(*iii*) A glass tumbler or a piece of china-ware on a shelf is set into resonant vibrations when some note is sung or played.

Scanned by CamScanner

Experimental illustration. As shown in Fig. 14.39, suspend four pendulums A, B, C and D from an elastic string PQ. Set the pendulum A into oscillation. It executes free oscillations. The energy from this pendulum is transferred to other pendulums through the elastic string. Initially, the motions of B, C and D are irregular. But soon all these pendulums start oscillating with the frequency of A. The oscillations of B, C and D are forced oscillations. But pendulums B and D have small amplitudes. This is because the frequency of B is much larger than that of A (due to shorter length) and the frequency of D is much smaller than that of A (due to larger length). The pendulum C which has same length as the pendulum A (and hence the same frequency) oscillates with largest amplitude. Hence the oscillations of C are resonant oscillations.

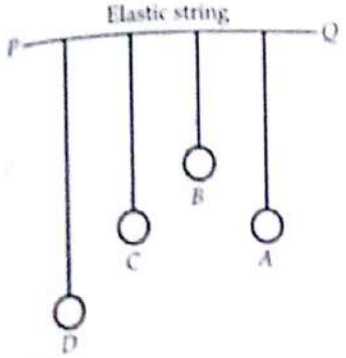


Fig. 14.39 Illustrating free, forced and resonant oscillations.

27. *Briefly explain the principle underlying the tuning of a radio receiver.*

Principle of tuning of a radio receiver. Tuning of the radio receiver is based on the principle of resonance. Waves from all stations are present around the antenna. When we tune our radio to a particular station, we produce a frequency of the radio circuit which matches with the frequency of that station. When this condition of resonance is achieved, the radio receives and responds selectively to the incoming waves from that station and thus gets tuned to that station.

14.20 COUPLED OSCILLATIONS

28. *What are coupled oscillations ? Give examples.*

Coupled oscillations. *A system of two or more oscillators linked together in such a way that there is mutual exchange of energy between them is called a coupled oscillator. The oscillations of such a system are called coupled oscillations.*

Examples. (*i*) Two masses attached to each other by three springs between two rigid supports. The middle spring provides the coupling between the driver and the driven system [Fig. 14.40(*a*)].

(*ii*) Two simple pendulums coupled by a spring [Fig. 14.40(*b*)].

(*iii*) Two LC-circuits placed close to each other. The circuits are linked by each other through the magnetic lines of force [Fig. 14.40(*c*)].

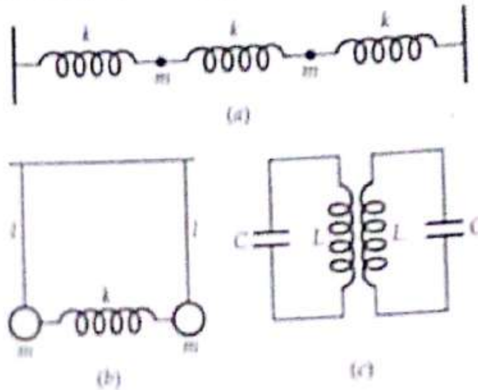


Fig. 14.40 Coupled oscillators.

When two identical oscillators are coupled together, the general motion of such a system is complex. It is periodic but not simple harmonic. It can be viewed as the superposition of two independent simple harmonic motions, called normal modes having angular frequencies ω_1 and ω_2. The constituent oscillators execute fast oscillations of average angular frequency, $\omega_{av} = (\omega_1 + \omega_2)/2$. The amplitude of either oscillator varies with an angular frequency $(\omega_1 - \omega_2)$. This phenomenon of variation of amplitudes is known as *beats* and the frequency $(\omega_1 - \omega_2)$ is called *beat frequency*.

Very Short Answer Conceptual Problems

Problem 1. Can a motion be periodic and not oscillatory ?

Solution. Yes. For example, uniform circular motion is periodic but not oscillatory.

Problem 2. Can a motion be oscillatory but not simple harmonic ? If your answer is yes, give an example and if not, explain why.

Solution. Yes, when a ball is dropped from a height on a perfectly elastic surface, the motion is oscillatory but not simple harmonic as restoring force $F = mg$ = constant and not $F \propto -x$, which is an essential condition for S.H.M.

Problem 3. Every simple harmonic motion is periodic motion, but every periodic motion need not be simple harmonic motion. Do you agree ? Give one example to justify your statement.

Scanned by CamScanner

Solution. Yes, every periodic motion need not be simple harmonic motion. For example, the motion of the earth round the sun is a periodic motion, but not simple harmonic motion as the back and forth motion is not taking place.

Problem 4. The rotation of the earth about its axis is periodic but not simple harmonic. Justify.

Solution. The earth takes 24 hours to complete its rotation about its axis, but the concept of to and fro motion is absent, and hence the rotation of the earth is periodic and not simple harmonic.

Problem 5. What is the basic condition for the motion of a particle to be S.H.M.? *[Delhi 02]*

Solution. The motion of a particle will necessarily be simple harmonic if the restoring force acting on it is proportional to its displacement from the mean position *i.e.*, $F = -kx$.

Problem 6. Which of the following conditions is not sufficient for simple harmonic motion and why ?

(*i*) acceleration $\propto$ displacement,

(*ii*) restoring force $\propto$ displacement.

Solution. Condition (*i*) is not sufficient because it does not mention the direction of acceleration. In S.H.M. the acceleration is always in a direction opposite to that of the displacement.

Problem 7. Are the functions $\tan \omega t$ and $\cot \omega t$ periodic ? Are they harmonic ?

Solution. Both $\tan \omega t$ and $\cot \omega t$ are periodic functions each with period $T = \pi / \omega$, because

$$\tan\left[\omega\left(t + \frac{\pi}{\omega}\right)\right] = \tan(\omega t + \pi) = \tan \omega t$$

and

$$\cot\left[\omega\left(t + \frac{\pi}{\omega}\right)\right] = \cot(\omega t + \pi) = \cot \omega t$$

But these functions are not harmonic because they can take any value between 0 and ∞.

Problem 8. What provides the restoring force for simple harmonic oscillations in the following cases :

(*i*) Simple pendulum (*ii*) Spring

(*iii*) Column of Hg in U-tube ?

Solution. (*i*) Gravity (*ii*) Elasticity (*iii*) Weight of difference column.

Problem 9. When are the displacement and velocity in the same direction in S.H.M. ?

Solution. When a particle moves from mean position to extreme position, its displacement and velocity are in the same direction.

Problem 10. When are the velocity and acceleration in the same direction in S.H.M. ?

Solution. When a particle moves from extreme position to mean position, its velocity and acceleration are in the same direction.

Problem 11. Can displacement and acceleration be in the same direction in S.H.M. ?

Solution. No. In S.H.M., acceleration is always in the opposite direction of displacement.

Problem 12. The relation between the acceleration and displacement x of a particle executing S.H.M. is $a = -(p/q)\,y$, where p and q are constants. What will be the time period T of the particle ?

Solution. Here $a = -\frac{p}{q}y = -\omega^2 y$, where $\omega = \sqrt{\frac{p}{q}}$

$$\therefore \text{ Time period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{q}{p}}.$$

Problem 13. The maximum acceleration of a simple harmonic oscillator is a_0 and the maximum velocity is v_0. What is the displacement amplitude ? *[Delhi 99]*

Solution. Let A be the displacement amplitude and ω be the angular frequency of S.H.M. Then

Maximum velocity, $v_0 = \omega A \quad \therefore \quad \omega = v_0 / A$

Maximum acceleration, $a_0 = \omega^2 A = \left(\frac{v_0}{A}\right)^2 A = \frac{v_0^2}{A}$

$$\therefore \text{ Displacement amplitude, } A = \frac{v_0^2}{a_0}.$$

Problem 14. The time period of an oscillating body is given by $T = 2p\sqrt{m/adg}$. What would be the force equation for the body ?

Solution. On comparing the given equation $T = 2\pi\sqrt{m/adg}$ with the standard equation $T = 2\pi\sqrt{m/k}$, we get $k = adg$, which gives the force equation $F = -adg\,(y)$.

Problem 15. Two simple pendulums of unequal length meet each other at mean position while oscillating. What is their phase difference ?

Solution. If both pendulums are moving in the same direction, then $\phi = 0°$ and if they are moving in opposite directions, then $\phi = 180°$ or π radian.

Problem 16. Velocity and displacement of a body executing S.H.M. are out of phase by $\pi/2$. How ?

Solution. Displacement, $x = a\cos \omega t$

Velocity, $v = \frac{dx}{dt} = -\omega a \sin \omega t = \omega a \cos(\omega t + \pi/2)$

Clearly, velocity leads the displacement by $\pi/2$ rad.

Problem 17. A particle executes S.H.M. of amplitude A. At what positions of its displacement (x), will its (*i*) velocity be zero and maximum and (*ii*) acceleration be zero and maximum ?

Solution. (*i*) Zero velocity at $x = \pm A$, maximum velocity at $x = 0$.

(*ii*) Zero acceleration at $x = 0$, maximum acceleration at $x = A$.

Scanned by CamScanner

Problem 18. At what points along the path of a simple pendulum is the tension in the string (*i*) maximum and (*ii*) minimum ?

Solution. (*i*) The tension is maximum at the mean position and is equal to mg, where m is the mass of the bob.

(*ii*) The tension is minimum at either extreme position and is equal to $mg \cos \theta$, where θ is the angle through which the string gets displaced to reach the extreme position.

Problem 19. Is the statement "the bob of a simple pendulum moves faster at the lowest position for larger amplitude" true ? Justify your answer.

Solution. We know that velocity of a simple pendulum is maximum at the lowest position (mean position) and is given by

$$v_{max} = \omega A.$$

i.e. for larger amplitude (A), the bob of simple pendulum would move faster.

Problem 20. Can we use a pendulum watch in an artificial satellite ?

Solution. No. In an artificial satellite, a body is in a state of weightlessness, *i.e.* $g = 0$.

$$T = 2\pi \sqrt{\frac{l}{g}} = \infty$$

Inside the satellite, the pendulum does not oscillate. Hence a pendulum watch cannot be used in an artificial satellite.

Problem 21. A girl is swinging in the sitting position. How will the period of the swing change if she stands up ? [*AIEEE 02 ; Central Schools 09*]

Solution. The girl and the swing together constitute a pendulum of time period,

$$T = 2\pi \sqrt{\frac{l}{g}}$$

As the girl stands up, her C.G. is raised. The distance between the point of suspension and the C.G. decreases *i.e.*, length l decreases. Hence the time period T decreases.

Problem 22. Will a pendulum clock lose or gain time when taken to the top of a mountain ? [*Himachal 04*]

Solution. On the top of the mountain, the value of g is less than that on the surface of the earth. The decrease in the value of g increases the time period of the pendulum on the top of the mountain. So the pendulum clock loses time.

Problem 23. What will be the period of oscillation, if the length of a second's pendulum is halved ?

Solution. $\frac{l_1}{l_2} = \frac{T_1^2}{T_2^2}$ or $\frac{l}{l/2} = \frac{2 \times 2}{T_2^2}$

or $T_2^2 = 2$ or $T_2 = \sqrt{2}$ s.

Problem 24. The length of a second's pendulum on the surface of earth is 1 m. What will be the length of a second's pendulum on the surface of moon ?

Solution. $T = 2\pi \sqrt{\frac{l}{g}}$

In both the cases, T is same so that

$$l \propto g$$

On the moon, the value of acceleration due to gravity is one-sixth of that on the surface of earth. So the length of second's pendulum is $\frac{1}{6}$ m.

Problem 25. The bob of a simple pendulum is made of wood. What will be the effect on the time period if the wooden bob is replaced by an identical bob of iron ?

Solution. There will be no effect because the time period does not depend upon the nature of material of the bob.

Problem 26. If a hollow pipe passes across the centre of gravity of the earth, then what changes would take place in the velocity and acceleration of a ball dropped in the pipe ?

Solution. The ball will execute S.H.M. to and fro about the centre of the earth. At the centre, the velocity of the ball will be maximum (acceleration zero) and at the earth's surface the velocity will be zero (acceleration maximum).

Problem 27. The bob of a simple pendulum of length l is negatively charged. A positively-charged metal plate is placed just below the bob and the pendulum is made to oscillate. What will be the effect on the time-period of the pendulum ?

Solution. The positively charged metal plate attracts the negatively charged bob. This increases the effective value of g. Hence the time period will decrease.

Problem 28. A simple pendulum of length l and with a bob of mass m is moving along a circular arc of angle θ in a vertical plane. A sphere of mass m is placed at the end of the circle. What momentum will be given to the sphere by the moving bob ?

Solution. Zero. This is because the velocity of the bob at the end of the arc will be zero.

Problem 29. A body moves along a straight line OAB simple harmonically. It has at zero velocity at the points A and B which are at distances a and b respectively from O and has velocity v when half way between them. Find the period of S.H.M.

Solution. Clearly, C is the mean position of S.H.M., as shown in Fig. 14.41

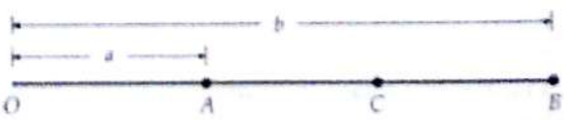


Fig. 14.41

The amplitude of S.H.M. is

$$A = AC = CB = \frac{AB}{2} = \frac{b-a}{2}$$

The velocity at the mean position C will be

$$v = \omega A = \frac{2\pi}{T} \cdot \frac{b-a}{2}$$

$$\therefore \quad T = \frac{\pi(b-a)}{v}.$$

Problem 30. When a 2.0 kg body is suspended by a spring, the spring is stretched. If the body is pulled down slightly and released, it oscillates up and down. What force is applied on the body by the spring when it passes through the mean position? (g = 9.8 newton/kg).

Solution. There is no acceleration in the body at the mean position, hence the resultant force applied by the spring will be exactly equal to the weight of the body *i.e.,* 2 × 9.8 or 19.6 newton.

Problem 31. A spring having a force constant k is divided into three equal parts. What would be the force constant for each individual part ?

Solution. Force constant of the spring $k = \frac{F}{x}$, where F is the restoring force. When the spring is divided into three parts, the displacement for the same force reduces to $x/3$, therefore, the force constant for each individual part is

$$k' = \frac{F}{x/3} = 3\left(\frac{F}{x}\right) = 3k$$

Problem 32. How would the time period of a spring mass system change, when it is made to oscillate horizontally, and then vertically ? *[Himachal 04]*

Solution. Time period will remain the same for both the cases.

Problem 33. Alcohol in a U-tube executes S.H.M. of time period T. Now, alcohol is replaced by water up to the same height in the U-tube. What will be the effect on the time period ?

Solution. The time period T remains same. This is because the period of oscillation of a liquid in a U-tube does not depend on the density of the liquid.

Problem 34. There are two springs, one delicate and another stiffer one. Which spring will have a greater frequency of oscillation for a given load ?

Solution. Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Force constant k is larger for the stiffer spring, so its frequency of oscillation will be greater than that of delicate spring.

Problem 35. What is the ratio between the potential energy and the total energy of a particle executing S.H.M., when its displacement is half of its amplitude ?

Solution. $\frac{\text{Potential energy}}{\text{Total energy}} = \frac{\frac{1}{2} m\omega^2 y^2}{\frac{1}{2} m\omega^2 a^2}$

$$= \frac{y^2}{a^2} = \frac{(a/2)^2}{a^2} = \frac{1}{4} = 1:4$$

Problem 36. What fraction of the total energy is kinetic when the displacement of a simple harmonic oscillator is half of its amplitude ?

Solution. $\frac{\text{Kinetic energy}}{\text{Total energy}} = \frac{\frac{1}{2} m\omega^2 \left(A^2 - \frac{A^2}{4}\right)}{\frac{1}{2} m\omega^2 A^2} = \frac{3}{4}$

Problem 37. Why is restoring force necessary for a body to execute S.H.M. ?

Solution. A body in S.H.M. oscillates about its mean position. At the mean position, it possesses kinetic energy because of which it moves from mean position to extreme position. Then the body can return to the mean position only if it is acted upon by a restoring force.

Problem 38. What would happen to the motion of the oscillating system if the sign of the force term in the equation $F = -kx$ is changed ?

Solution. The force will not be the restoring nature. The back and forth nature of the motion is lost. The body will continue to move in a particular sense.

Problem 39. What determines the natural frequency of a body ?

Solution. Natural frequency of a body depends upon (*i*) elastic properties of the material of the body and (*ii*) dimensions of the body.

Problem 40. Why does the amplitude of an oscillating pendulum go on decreasing ?

Solution. Due to frictional resistance between air and bob, the amplitude of oscillations of the pendulum gradually decreases and finally the bob stops.

Problem 41. Why are army troops not allowed to march in steps while crossing a bridge ? *[Himachal 05]*

Solution. Army troops are not allowed to march in steps while crossing a bridge because it is quite likely that the frequency of the foot steps may match with the natural frequency of the bridge, and due to resonance the bridge may pick up large amplitude and break.

Problem 42. A passing aeroplane sometimes causes the rattling of the windows of a house. Why ?

Solution. When the frequency of the sound waves from the engine of an aeroplane matches with the natural frequency of a window, resonance takes place which causes the rattling of window.

Problem 43. How can earthquakes cause disaster sometimes ? *[Himachal 05C]*

Solution. The resonance may cause disaster during the earthquake, if the frequency of oscillations present within the earth per chance coincides with the natural frequency of some building, which may start vibrating with large amplitude due to resonance and may get damaged.

Problem 44. Sometimes a wine glass is broken by the powerful voice of a celebrated singer. Why ?

Solution. When the natural frequency of the wine glass becomes equal to that of the singer's voice, the resulting resonance due to the powerful voice of the singer may break the glass.

Problem 45. Glass windows may be broken by a far away explosion. Explain why. *[Himachal 05 ; Central Schools 08]*

Solution. A distant explosion sends out sound waves of large amplitude in all directions. As these sound waves strike the glass windows, they set them into forced oscillations. Since glass is brittle, so the glass windows break as soon as they start oscillating due to forced oscillations.

Problem 46. The body of a bus begins to rattle sometimes, when the bus picks up a certain speed. Why ? *[Himachal 05]*

Solution. At a particular speed, the frequency of the engine of the bus becomes equal to the natural frequency of the body of the bus. The frame of the bus begins to vibrate strongly due to resonance.

Problem 47. What will be the change in time period of a loaded spring, when taken to moon ? *[Himachal 03]*

Solution. Time period of a loaded spring,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

As T is independent of g, it will not be affected when the loaded spring is taken to the moon.

Problem 48. A spring of force constant k is cut into two pieces, such that one piece is double the length of the other. What is the force constant of the longer piece of the spring ? *[IIT 99]*

Solution. Force constant,

$$k = \frac{F}{x}$$

The length of longer part is $2x/3$. So its force constant is

$$k' = \frac{F}{2x/3} = \frac{3}{2}\frac{F}{x} = \frac{3}{2}k.$$

Problem 49. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. What is relation between ω_1 and ω_2 ? *[AIEEE 04]*

Solution. Only in case of resonance, both amplitude and energy of oscillation are maximum. In the condition of resonance,

$$\omega_1 = \omega_2.$$

Problem 50. The maximum velocity of a particle, executing simple harmonic motion with an amplitude of 7 mm, is 4.4 m s^{-1}. What is the period of oscillation ? *[AIEEE 06]*

Solution. $v_{max} = \omega A = \frac{2\pi}{T} A$

$$\therefore \quad T = \frac{2\pi A}{v_{max}}$$

$$= \frac{2 \times 22 \times 7 \times 10^{-3}}{7 \times 4.4} = 0.01 \text{ s}.$$

Short Answer Conceptual Problems

Problem 1. Justify the following statements :

(i) The motion of an artificial satellite around the earth cannot be taken as S.H.M.

(ii) The time period of a simple pendulum will get doubled if its length is increased four times. *[Himachal 06]*

Solution. (*i*) The motion of an artificial satellite around the earth is periodic as it repeats after a regular interval of time. But it cannot be taken as S.H.M. because it is not a to-and-fro motion about any mean position.

(*ii*) Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad i.e., \quad T \propto \sqrt{l}.$$

Clearly, if the length is increased four times, the time period gets doubled.

Problem 2. (i) What is meant by simple harmonic motion (S.H.M.) ?

(ii) At what points is the energy entirely kinetic and potential in S.H.M. ?

(iii) What is the total distance travelled by a body executing S.H.M. in a time equal to its time period, if its amplitude is A ? *[Delhi 09]*

Solution. (*i*) Refer to point 5 of Glimpses.

(*ii*) The energy is entirely kinetic at mean position *i.e.*, at $y = 0$. The energy is entirely potential at extreme positions, *i.e.*,

$$y = \pm A.$$

(*iii*) Total distance travelled in time period T

$$= 2A + 2A = 4A.$$

Scanned by CamScanner

Problem 3. A simple pendulum consisting of an inextensible length l and mass m is oscillating in a stationary lift. The lift then accelerates upwards with a constant acceleration of 4.5 m/s^2. Write expression for the time period of simple pendulum in two cases. Does the time period increase, decrease or remain the same, when lift is accelerated upwards? *[Central Schools 08]*

Solution. When the lift is stationary,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(*ii*) When the lift accelerates upwards with an acceleration of 4.5 m/s^2,

$$T' = 2\pi\sqrt{\frac{l}{g+a}} = 2\pi\sqrt{\frac{l}{g+4.5}}$$

Clearly, the time period decreases when the lift accelerates upwards.

Problem 4. What is meant by restoring force? Give one example.

Solution. The force which tends to bring a vibrating body from its displaced position to the equilibrium position is called restoring force. When the bob of a simple pendulum is displaced through an θ from the vertical, a restoring force equal to $mg \sin \theta$ due to gravity acts on it.

Problem 5. Two particles execute simple harmonic motions of the same amplitude and frequency along the same straight line. They cross one another when going in opposite directions. What is the phase difference between them when their displacements are half of their amplitudes?

Solution. The general equation for S.H.M. is

$$y = A \sin(\omega t + \phi_0)$$

As the displacement is half of the amplitude ($y = A/2$), so

$$A/2 = A \sin(\omega t + \phi_0)$$

or $\quad \sin(\omega t + \phi_0) = \frac{1}{2}$

$\therefore \quad \omega t + \phi_0 = 30°$ or $150°$.

As the two particles are going in opposite directions, the phase of one is 30° and that of the other 150°.

Hence the phase difference between the two particles $= 150 - 30 = 120°$.

Problem 6. A simple pendulum is hung in a stationary lift and its periodic time is T. What will be the effect on its periodic time T if

(*i*) the lift goes up with uniform velocity v,

(*ii*) the lift goes up with uniform acceleration a, and

(*iii*) the lift comes down with uniform acceleration a?

[Central Schools 14]

Solution. (*i*) When the lift goes up [Fig. 14.42(*a*)] with uniform velocity v, tension in the string, $T = mg$.

The value of g remains unaffected.

The period T remains same as that in stationary ... *i.e.*,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

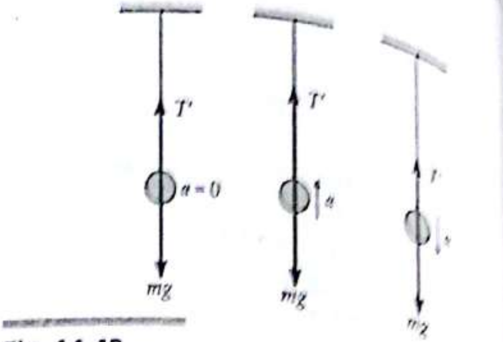


Fig. 14.42

(*ii*) When the lift goes up with acceleration [Fig. 14.42(*b*)], the net upward force on the bob is

$$T' - mg = ma$$

$$\therefore \quad T' = m(g + a)$$

The effective value of g is $(g + a)$ and time period,

$$T_1 = 2\pi\sqrt{\frac{l}{g+a}}$$

Clearly, $T_1 < T$ *i.e.*, time period decreases.

(*iii*) When lift comes down with acceleration [Fig. 14.42(*c*)], the net downward force on the bob is

$$mg - T' = ma \quad \therefore \quad T' = m(g - a)$$

The effective value of g becomes $(g - a)$ and time period is

$$T_2 = 2\pi\sqrt{\frac{l}{g-a}}$$

Clearly, $T_2 > T$ *i.e.*, time period increases.

Problem 7. The bob of a vibrating pendulum is made of ice. How will the time period change when the ice starts melting?

Solution. If the ice bob is of very small size, the position of its C.G. will remain same as the ice melts. Hence its time period will remain same.

If the size of the ice bob is large, then

$$T = 2\pi\sqrt{\frac{\frac{2r^2}{5l} + l}{g}}$$

As ice melts, the radius r and hence the time period will decrease. The pendulum will oscillate faster.

Problem 8. The amplitude of a simple harmonic oscillator is doubled. How does this affect (*i*) periodic time, (*ii*) maximum velocity, (*iii*) maximum acceleration and (*iv*) maximum energy? *[Chandigarh ...]*

Scanned by CamScanner

Solution.

(i) $T = 2\pi\sqrt{\dfrac{1}{\text{Acceleration per unit displacement}}}$

As the acceleration per unit displacement is a constant quantity, T is not affected on changing the amplitude.

(ii) $v_{max} = \omega A$

When amplitude is doubled, maximum velocity is also doubled.

(iii) $a_{max} = \omega^2 A$

When amplitude is doubled, the maximum acceleration is also doubled.

(iv) $E = 2\pi^2 m\nu^2 A^2$ *i.e.*, $E \propto A^2$

When amplitude is doubled, the energy of the oscillator becomes four times.

Problem 9. You have a light spring, a metre scale and a known mass. How will you find the time period of oscillation of mass without the use of a clock ?

Solution. Suspend the known mass m form the spring and note the extension l of the spring with the metre scale. If k is the force constant of the spring, then in equilibrium

$$kl = mg \quad \text{or} \quad \frac{m}{k} = \frac{l}{g}$$

Time period of the loaded spring, $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{l}{g}}$

So by knowing the value of extension l, time period T can be determined.

Problem 10. A man is standing on a platform which oscillates up and down simple harmonically. How will the weight of the man change as recorded by a weighing machine on the platform ?

Solution. As the platform moves from the mean position to the upper extreme position or from upper extreme position to mean position, the acceleration of the oscillating system acts vertically downwards and hence weight of the man will decrease.

On the other hand, as the platform moves from mean position to lower extreme position and then back to mean position, the acceleration acts vertically upwards. Hence weight of the man increases.

Problem 11. The frequency of oscillations of a mass m suspended by a spring is ν_1. If the length of the spring is cut to one-half, the same mass oscillates with frequency ν_2. Determine the value of ν_2 / ν_1.

[Chandigarh 03]

Solution. Let k be the force constant of the full spring. Then frequency of oscillation of mass m will be

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

When the spring is cut to one-half of its length, its force constant is doubled ($2k$).

Frequency of oscillation of mass m will be

$$\nu_2 = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

$$\therefore \quad \nu_2 / \nu_1 = \sqrt{2}.$$

Problem 12. All trigonometric functions are periodic, but only sine or cosine functions are used to define S.H.M. Why ? [Central Schools 03]

Solution. All trigonometric functions are periodic. The sine and cosine functions can take value between − 1 and + 1 only. So they can be used to represent a bounded motion like S.H.M. But the functions such as tangent, cotangent, secant and cosecant can take value between 0 and ∞ (both positive and negative). So those functions cannot be used to represent bounded motion like S.H.M.

Problem 13. A simple harmonic motion is represented by $\dfrac{d^2x}{dt^2} + \alpha x = 0$. What is its time period ? [AIEEE 05]

Solution. Clearly, $\dfrac{d^2x}{dt^2} = -\alpha x$ or $a = -\alpha x$

Time period, $\quad T = 2\pi\sqrt{\dfrac{x}{a}} = 2\pi\sqrt{\dfrac{x}{\alpha x}} = \dfrac{2\pi}{\sqrt{\alpha}}$.

Problem 14. Does the function $y = \sin^2 \omega t$ represent a periodic or a simple harmonic motion ? What is the period of the motion ? [AIEEE 05]

Solution. Displacement, $\quad y = \sin^2 \omega t$

Velocity, $\quad v = \dfrac{dy}{dt} = 2\sin\omega t \times \cos\omega t \times \omega$

$$= \omega \sin 2\omega t$$

Acceleration, $a = \dfrac{dv}{dt} = \omega \times \cos 2\omega t \times 2\omega$

$$= 2\omega^2 \cos 2\omega t$$

As the acceleration a is not proportional to displacement y, the given function does not represent SHM. It represents a periodic motion of angular frequency 2ω.

∴ Time period,

$$T = \frac{2\pi}{\text{Angular frequency}} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}.$$

Problem 15. The length of a simple pendulum executing SHM is increased by 21%. What is the percentage increase in the time period of the pendulum of increased length. [AIEEE 03]

Solution. Time period,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad i.e., \quad T \propto l^{1/2}$$

The percentage increase in time period is given by

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100$$

$$= \frac{1}{2} \times 21\% = 10.5\%$$

Scanned by CamScanner

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *Two simple harmonic motions are represented by the equations :*

$x_1 = 5 \sin (2\pi t + \pi/4)$, $x_2 = 5\sqrt{2}\ (\sin 2\pi t + \cos 2\pi t)$

What is the ratio of their amplitudes ? **[Roorkee 96]**

Solution. $x_1 = 5 \sin (2\pi t + \pi/4) \quad \therefore \quad A_1 = 5$

$$x_2 = 5\sqrt{2}\ (\sin 2\pi t + \cos 2\pi t)$$

$$= 10 \sin (\sin 2\pi t \cos \pi/4 + \cos 2\pi t \sin \pi/4)$$

or $\quad x_2 = 10 \sin (2\pi t + \pi/4)$

$\therefore \quad A_2 = 10$

Hence $\dfrac{A_1}{A_2} = \dfrac{5}{10} = \mathbf{1:2.}$

Problem 2. *The bob of a simple pendulum is a hollow sphere filled with water. How will the period of oscillation change if the water begins to drain out of the hollow sphere from a fine hole at its bottom ?*

Or

The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscilalting body gets suddenly unplugged. How would the time period of oscillation of the pendulum change till water is coming out ? **[AIEEE 05]**

Solution. Time period, $T = 2\pi \sqrt{\dfrac{l}{g}}$

As water flows out of the sphere, the time period first increases and then decreases. Initially when the sphere is completely filled with water, its C.G. lies at its centre. As water flows out, the C.G. begins to shift below the centre of the sphere. The effective length of the pendulum increases and hence its time period increases.

When the sphere becomes more than half empty, its C.G. begins to rise up. The effective length of the pendulum increases and time period T decreases.

When the entire water is drained out of the sphere, the C.G. is once again shifted to centre of the sphere and the time period T attains its initial value.

Problem 3. *The period of vibration of a mass m suspended by a spring is T. The spring is cut into n equal parts and the body is again suspended by one of the pieces. Find the time period of oscillation of the mass.* **[AIEEE 02]**

Solution. The force constant is inversely proportional to the length. If k is the force constant of the original spring, then the force constant of each part will be nk.

$\therefore \quad T = 2\pi \sqrt{\dfrac{m}{k}}$ and $T' = 2\pi \sqrt{\dfrac{m}{nk}}$

Hence $T' = \dfrac{T}{\sqrt{n}}$.

Problem 4. *Two simple harmonic motions are represented by the equations :*

$y_1 = 0.1 \sin (100\pi t + \pi/3)$ and $y_2 = 0.1 \cos \pi t$

What is the phase difference of the velocity of the particle 1 with respect to the velocity of particle 2 ? **[AIEEE 05]**

Solution. Velocity of particle 1,

$$v_1 = \frac{dy_1}{dt} = 0.1 \cos (100\pi t + \pi/3) \times 100\pi$$

$$= 10\pi \cos (100\pi t + \pi/3)$$

Velocity of particle 2,

$$v_2 = \frac{dy_2}{dt} = 0.1(-\sin \pi t) \times \pi = -0.1\pi \sin \pi t$$

$$= 0.1 \cos (\pi t + \pi/2)$$

Phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

$$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}.$$

Problem 5. *A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force,*

$$f(t) \propto \cos \omega t \qquad (\omega \neq \omega_0)$$

is applied to the oscillator. How does the time displacement of oscillator vary ? **[AIEEE 04]**

Solution. With natural angular frequency ω_0, the acceleration of the particle at displacement y is

$$a_0 = -\omega_0^2 y$$

The external force $F(t) \propto \cos \omega t$ has an angular frequency ω. The acceleration produced by this force at displacement y is

$$a' = \omega^2 y$$

The net acceleration of the particle at displacement y is

$$a = a_0 + a' = -\omega_0^2 y + \omega^2 y = -(\omega_0^2 - \omega^2) y$$

The resultant force on the particle at displacement y is

$$F = ma = -m(\omega_0^2 - \omega^2) y \quad \text{or} \quad y = -\frac{F}{m(\omega_0^2 - \omega^2)}$$

Clearly, $\quad y \propto \dfrac{1}{m(\omega_0^2 - \omega^2)}$

Scanned by CamScanner

Problem 6. *A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$ ($K = 1\ ms^{-2}$), where y is the vertical displacement. The time period now becomes T_2. What is the ratio T_1^2/T_2^2? Given $g = 10\ ms^{-2}$* **[IIT 05]**

Solution. In first case,

$$T_1 = 2\pi\sqrt{\frac{l}{g}} \quad ...(1)$$

In second case, displacement $y = Kt^2$

Upward velocity, $v = \frac{dy}{dt} = 2Kt$

Upward acceleration, $a = 2K = 2 \times 1ms^{-2} = 2\ ms^{-2}$

$$\therefore \quad T_2 = 2\pi\sqrt{\frac{l}{g+a}} = 2\pi\sqrt{\frac{l}{g+2}} \quad ...(2)$$

Hence $$\frac{T_1^2}{T_2^2} = \frac{4\pi^2 l}{g} \times \frac{g+2}{4\pi^2 l}$$

$$= \frac{g+2}{g} = \frac{10+2}{10} = \frac{6}{5}.$$

Problem 7. *The bob of simple pendulum executes SHM in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $\frac{4000}{3}\ kg\ m^{-3}$, find the relationship between t and t_0?* **[AIEEE 04]**

Solution. In air, $t_0 = 2\pi\sqrt{\frac{l}{g}}$

Let V be the volume of the bob. Then

Apparent weight of bob in water

= Weight of bob in air – Upthrust

or $$V\rho g' = V\rho g - V\sigma g$$

or $$g' = \left(1 - \frac{\sigma}{\rho}\right)g$$

Density of bob, $\rho = \frac{4000}{3}\ kg\ m^{-3}$

Density of water, $\sigma = 1000\ kg\ m^{-3}$

$$\therefore \quad g' = \left(1 - \frac{1000 \times 3}{4000}\right)g = \frac{g}{4}$$

Time period of the pendulum in water,

$$t = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g/4}} = 2 \times 2\pi\sqrt{\frac{l}{g}} = \mathbf{2t_0}.$$

Problem 8. *A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes 5T/3. What is the ratio m/M?* **[AIEEE 03]**

Solution. With mass M, the time period of the spring is

$$T = 2\pi\sqrt{\frac{M}{k}}$$

With mass $M + m$, the time period becomes

$$\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$$

or $$\frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M+m}{k}}$$

or $$\frac{25}{9}M = M + m \quad \text{or} \quad \frac{16}{9}M = m$$

or $$\frac{m}{M} = \frac{16}{9}.$$

Problem 9. *Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically, such that their maximum velocities are equal, then find the ratio of the amplitude of M to that of N.* **[AIEEE 03]**

Solution. The maximum velocity of body in SHM is given by

$$v_{max} = A\omega = A\sqrt{\frac{k}{m}} \qquad \left[\because \omega^2 = \frac{k}{m}\right]$$

Given $$v_{max}(M) = v_{max}(N)$$

or $$A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}} \qquad [m_M = m_N = m]$$

or $$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}.$$

Problem 10. *A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. What is the period of oscillation when the two springs are connected in series?* **[AIEEE 04]**

Solution. If a force F applied to the series combination produces displacements t_1 and t_2 in the two springs, then

$$F = -k_1x_1 = -k_2x_2$$

$$\therefore \quad x_1 = -\frac{F}{k_1} \quad \text{and} \quad x_2 = -\frac{F}{k_2}$$

Total extension,

$$x = x_1 + x_2 = -F\left[\frac{1}{k_1} + \frac{1}{k_2}\right] = -F\left[\frac{k_1 + k_2}{k_1k_2}\right]$$

or $$F = -\frac{k_1k_2}{k_1 + k_2}x$$

$\therefore$ Force constant of the series combination,

$$k_s = \frac{k_1k_2}{k_1 + k_2}$$

Scanned by CamScanner

Period of oscillation for the series combination,

$$T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}} = 2\pi\sqrt{\frac{m}{k_1}+\frac{m}{k_2}}$$

or $$T^2 = 4\pi^2\left(\frac{m}{k_1}+\frac{m}{k_2}\right) = \left(2\pi\sqrt{\frac{m}{k_1}}\right)^2 + \left(2\pi\sqrt{\frac{m}{k_2}}\right)^2$$

or $$T^2 = t_1^2 + t_2^2.$$

Problem 11. *A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then how are T_1 and T_2 related ?* **[IIT Screening 01]**

Solution. The displacement equation for S.H.M. is $x = A \sin \omega t$

At $t = T_1$, $x = A/2$

$\therefore$ $$\frac{A}{2} = A \sin \omega T_1 \quad \text{or} \quad \frac{1}{2} = \sin \omega T_1$$

or $$\omega T_1 = \frac{\pi}{6} \quad \text{or} \quad T_1 = \frac{\pi}{6\omega}$$

At $t = T_1 + T_2$, $x = A$

$\therefore$ $$A = A \sin \omega(T_1+T_2) \quad \text{or} \quad 1 = \sin \omega(T_1+T_2)$$

or $$\omega(T_1+T_2) = \frac{\pi}{2} \quad \text{or} \quad T_1+T_2 = \frac{\pi}{2\omega}$$

$\therefore$ $$T_2 = \frac{\pi}{2\omega} - T_1 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega} = \mathbf{2T_1}.$$

Problem 12. *Two simple harmonic motions are represented by the equations :*

$$y_1 = 10 \sin\frac{\pi}{4}(12t+1), \quad y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

Find the ratio of their amplitudes. What are time periods of the two motions ? **[IIT 86 ; MNREC 90]**

Solution. $y_1 = 10 \sin \frac{\pi}{4}(12t+1)$

$$= 10 \sin\left(3\pi t + \frac{\pi}{4}\right) \quad ...(1)$$

$$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

$$= 10\left(\sin 3\pi t \times \frac{1}{2} + \cos 3\pi t \times \frac{\sqrt{3}}{2}\right)$$

$$= 10\left(\sin 3\pi t \cos\frac{\pi}{3} + \cos 3\pi t \sin\frac{\pi}{3}\right)$$

or $$y_2 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right) \quad ...(2)$$

The general equation for SHM is

$$y = A\sin(\omega t + \phi_0) = A \sin\left(\frac{2\pi}{T}t + \phi_0\right) \quad ...(3)$$

Comparing equations (1) and (2) with (3), we get

$$A_1 = 10, \quad A_2 = 10, \quad \frac{2\pi}{T_1} = \frac{2\pi}{T_2} = 3\pi$$

$\therefore$ $$\frac{A_1}{A_2} = \mathbf{1:1} \; ; \; T_1 = T_2 = \frac{2}{3}\ \mathbf{s}$$

Problem 13. *A point particle of mass 0.1 kg is executing SHM of amplitude 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Obtain the equation of motion of the particle if the initial phase of oscillation is 45°.* **[Roorkee 91]**

Solution. Here $m = 0.1$ kg, $A = 0.1$ m, $E = 8 \times 10^{-3}$ J,

$\phi_0 = 45° = \frac{\pi}{4}$ rad

K.E. at the mean position $= (E_k)_{max} = \frac{1}{2} m\omega^2 A^2$

$$\therefore \; 8 \times 10^{-3} = \frac{1}{2} \times 0.1 \times \omega^2 \times (0.1)^2$$

or $$\omega^2 = 16 \quad \text{or} \quad \omega = 4 \text{ rad s}^{-1}$$

The equation of motion for the particle is

$$y = A \sin \omega t = \mathbf{0.1 \sin (4\,t + \pi/4)}.$$

Problem 14. *A simple harmonic motion has an amplitude A and time period T. What is the time taken to travel from $x = A$ to $x = A/2$?* **[REC 92]**

Solution. Displacement from mean position

$$= A - \frac{A}{2} = \frac{A}{2}$$

When the motion starts from the positive extreme position,

$$y = A\cos\omega t \quad \therefore \quad \frac{A}{2} = A\cos\frac{2\pi}{T}t$$

or $$\cos\frac{2\pi}{T}t = \frac{1}{2} = \cos\frac{\pi}{3} \quad \text{or} \quad \frac{2\pi}{T}t = \frac{\pi}{3}$$

$\therefore$ $$t = \frac{\mathbf{T}}{\mathbf{6}}.$$

Problem 15. *A block is resting on a piston which is moving vertically with simple harmonic motion of period 1.0 second. At what amplitude of motion will the block and piston separate ? What is the maximum velocity of the piston at this amplitude ?* **[Roorkee 85]**

Solution. The block and piston will just separate when

$$a_{max} = g \quad \text{or} \quad \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A = g$$

$\therefore$ $$A = \frac{gT^2}{4\pi^2} = \frac{9.8 \times (1.0)^2}{4 \times 9.87} = 0.248 \text{ m}$$

Maximum velocity of the block,

$$v_{max} = \omega A = \frac{2\pi}{T}A = \frac{2 \times 3.142}{1.0} \times 0.248 = 1.56 \text{ ms}^{-1}.$$

Scanned by CamScanner

Problem 16. *A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface. Take $g = 10\ ms^{-1}$.* **[Roorkee 96]**

Solution. Maximum acceleration of the block,

$$a_{max} = \omega^2 A$$

Maximum force on the block, $F = ma_{max} = m\omega^2 A$

Frictional force on the block $= \mu\ mg$

The block will not slip on the surface of the table if

$$m\omega^2 A = \mu\ mg$$

$\therefore$ Amplitude,

$$A = \frac{\mu g}{\omega^2} = \frac{0.72 \times 10}{(2\pi v)^2} = \frac{0.72 \times 10}{(2 \times 3.14 \times 3)^2} = \mathbf{0.02\ m.}$$

Problem 17. *Springs of spring constants k, 2k, 4k, 8k, ... are connected in series. A mass m kg is attached to the lower end of the last spring and the system is allowed to vibrate. What is the time period of oscillations ?*

Given m = 40 g and k = 2.0 N cm^{-1}

Solution. Here $m = 40\ g = 0.04\ kg$,

$$k = 2.0\ N\ cm^{-1} = 2.0 \times 100\ Nm^{-1}$$

The effective spring constant k of the series combination is given by

$$\frac{1}{k'} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \ldots = \frac{1}{k}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right]$$

$$= \frac{1}{k}\left[\frac{1}{1 - 1/2}\right] = \frac{2}{k} \qquad \left[\text{Sum of finite G.P.} = \frac{a}{1-r}\right]$$

or $k' = k/2$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{2m}{k}}$$

$$= 2 \times \frac{22}{7} \times \sqrt{\frac{2 \times 0.04}{2.0 \times 100}}$$

$$= \mathbf{0.126\ s.}$$

Problem 18. *A uniform spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2, where $l_1 = nl_2$ and n is an integer. What are the corresponding force constants k_1 and k_2 in terms of n and k ? What is the ratio k_1 / k_2 ?*

Solution. Here $l = l_1 + l_2$ and $l_1 = nl_2$ or $\frac{l_1}{l_2} = n$

As $\quad k = \frac{mg}{l}$

$$\therefore \quad k_1 = \frac{mg}{l_1} \text{ and } k_2 = \frac{mg}{l_2}$$

Hence $\frac{k_1}{k} = \frac{mg}{l_1} \times \frac{l}{mg} = \frac{l}{l_1} = \frac{l_1 + l_2}{l_1} = 1 + \frac{l_2}{l_1} = 1 + \frac{1}{n}$

or $\quad k_1 = \left(\frac{n+1}{n}\right)k.$

Also $\frac{k_2}{k} = \frac{mg}{l_2} \times \frac{l}{mg} = \frac{l}{l_2} = \frac{l_1 + l_2}{l_2} = \frac{l_1}{l_2} + 1 = n + 1$

or $\quad k_2 = (n + 1)\ k.$

Clearly, $\frac{k_1}{k_2} = \frac{1}{n}.$

Problem 19. *A horizontal spring block system of mass M executes simple harmonic motion. When the block is passing through its equilibrium position, an object of mass m is put on it and the two move together. Find the new amplitude and frequency of vibration.* **[Roorkee 88]**

Solution. Original frequency,

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

Let A = Initial amplitude of oscillation

v = Velocity of mass M when passing through mean position

Maximum K.E. = Total energy

or $$\frac{1}{2}Mv^2 = \frac{1}{2}kA^2$$

$$\therefore \quad v = \sqrt{\frac{k}{M}}\ A$$

When mass m is put on the system, total mass $= (M + m)$. If v' is the velocity of the combination in equilibrium position, then by the conservation of linear momentum,

$$Mv = (M + m)\ v' \quad \text{or} \quad v' = \frac{Mv}{M + m}$$

If A' is the new amplitude, then

$$\frac{1}{2}(M + m)\ v'^2 = \frac{1}{2}kA'^2$$

or $$A' = \sqrt{\frac{M + m}{k}}\ .\ v' = \sqrt{\frac{M + m}{k}} \times \frac{Mv}{M + m}$$

$$= \sqrt{\frac{M + m}{k}} \times \frac{M}{M + m} \times \sqrt{\frac{k}{M}}\ A = \sqrt{\frac{M}{M + m}}\ .\ \mathbf{A}$$

New frequency, $\quad v' = \frac{1}{2\pi}\sqrt{\frac{k}{M + m}}.$

Problem 20. *The bob of pendulum of length l is pulled aside from its equilibrium position through an angle θ and then released. Find the speed v with which the bob passes through the equilibrium position.* **[Kurukshetra CEE 96]**

Scanned by CamScanner

Solution. The situation is shown in Fig. 14.43.

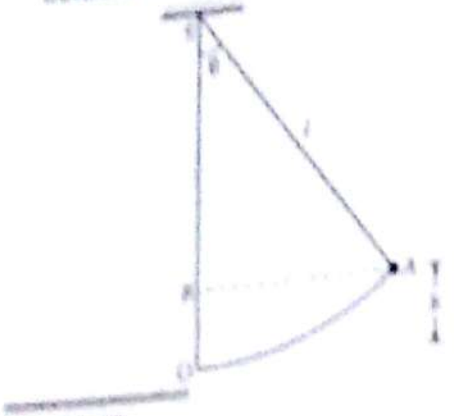


Fig. 14.43

Clearly, $h = OB = OS - BS = l - l\cos\theta$

$= l(1 - \cos\theta)$

Let v and v' be the velocities of the bob at positions O and A respectively. Then by the conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mgh$$

or $$v' = \sqrt{v^2 - 2gh}$$

$$= \sqrt{v^2 - 2gl(1 - \cos\theta)}$$

Guidelines to NCERT Exercises

14.1. *Which of the following examples represent periodic motion?*

(i) *A swimmer completing one (return) trip from one bank of a river to the other and back.*

(ii) *A freely suspended bar magnet displaced from its N - S direction and released.*

(iii) *A hydrogen molecule rotating about its center of mass.*

(iv) *An arrow released from a bow.* (v) *Halley's comet.*

Ans. (i) ***Not periodic.*** Because the motion of the swimmer is not repeated over and over again after any fixed time interval.

(ii) ***Periodic.*** As the magnet is released from its displaced position, it oscillates about the N - S direction with a definite time period.

(iii) ***Periodic.*** The motion of the hydrogen molecule rotating about its centre of mass repeats after a fixed time interval.

(iv) ***Not periodic.*** The motion of the arrow does not repeat itself after a fixed time interval.

(v) ***Periodic.*** Halley's comet appears after every 76 years.

14.2. *Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?*

(i) *The rotation of earth about its axis.*

(ii) *Motion of an oscillating mercury column in a U-tube.*

(iii) *Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.*

(iv) *General vibrations of a polyatomic molecule about its equilibrium position.*

Ans. (i) ***Periodic but not simple harmonic.*** The motion of the earth about its axis repeats after every 24 hours but it is not a to and fro motion.

(ii) ***Simple harmonic.*** The restoring force is proportional to the displacement of the mercury column from the equilibrium level.

(iii) ***Simple harmonic.*** The motion of the ball bearing is to and fro about the lower most point and the restoring force is proportional to its displacement from the mean...

(iv) ***Periodic but not simple harmonic.*** A polyatomic molecule has a number of natural frequencies. In general, its vibration is a superposition of SHM's of a number of different frequencies. This superposition is periodic but not simple harmonic.

14.3. *Fig. 14.44 depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?*

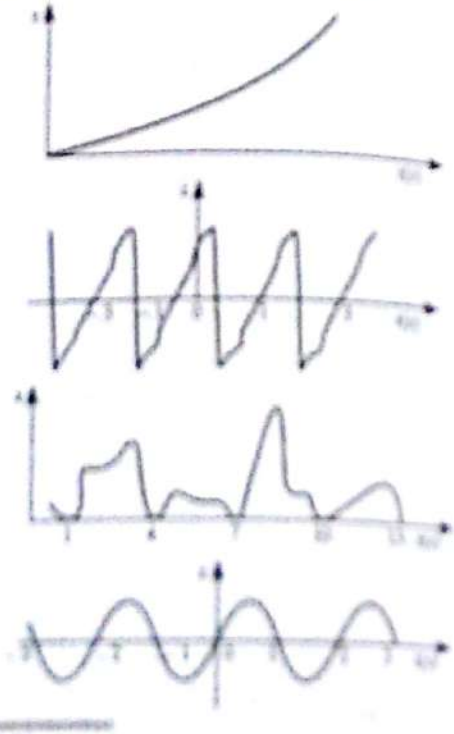


Fig. 14.44

Ans. (*i*) Plot (*a*) does not represent periodic motion because the motion is not repeated after a fixed interval.

(*ii*) Plot (*b*) represents periodic motion with $T = 2s$.

(*iii*) Plot (*c*) does not represent periodic motion. The repetition of merely one position is not enough for the motion to be periodic. The entire motion during one period must be repeated successively.

(*iv*) Plot (*d*) represents periodic motion with $T = 2s$.

14.4. *Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion ? Give period for each case of periodic motion (ω is any positive constant)*

(*i*) $\sin \omega t - \cos \omega t$ (*ii*) $\sin^3 \omega t$

(*iii*) $3\cos(\pi/4 - 2\omega t)$ (*iv*) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$

(*v*) $\exp(-\omega^2 t^2)$ (*vi*) $1 + \omega t + \omega^2 t^2$.

Ans.

(*i*) Here $x(t) = \sin \omega t - \cos \omega t$

$$= \sqrt{2}\,(\sin \omega t \cos \pi/4 - \cos \omega t \sin \pi/4)$$

$$= \sqrt{2} \sin(\omega t - \pi/4)$$

Moreover,

$$x(t + 2\pi/\omega) = \sqrt{2}\sin[\omega(t + 2\pi/\omega) - \pi/4]$$

$$= \sqrt{2}\sin(\omega t + 2\pi - \pi/4)$$

$$= \sqrt{2}\sin(\omega t - \pi/4) = x(t)$$

Hence the given function represents a ***simple harmonic motion*** with $T = 2\pi/\omega$ and phase angle $= -\pi/4$ or $7\pi/4$.

(*ii*) $x(t) = \sin^3 \omega t = \frac{1}{4}(3\sin \omega t - \sin 3\omega t)$

$$[\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta]$$

It represents two separate simple harmonic motions but their combination does not represent SHM.

$$\text{Period of } \frac{3}{4}\sin \omega t = \frac{2\pi}{\omega} = T$$

$$\text{Period of } \frac{1}{4}\sin 3\omega t = \frac{2\pi}{3\omega} = \frac{T}{3}$$

Thus the minimum time after which the combined function repeats is $T = 2\pi/\omega$. Hence the given function is ***periodic but not simple harmonic***.

(*iii*) Here $x(t) = 3\cos(\pi/4 - 2\omega t)$

$$= 3\cos[-(2\omega t - \pi/4)]$$

$$= 3\cos(2\omega t - \pi/4)$$

$$[\because \cos(-\theta) = \cos\theta]$$

It represents S.H.M. with period $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

(*iv*) $x(t) = \cos \omega t + \cos 3\omega t + \cos 5\omega t$

$\cos \omega t$ represents S.H.M. with period $= \frac{2\pi}{\omega} = T$

$\cos 3\omega t$ represents S.H.M. with period $= \frac{2\pi}{3\omega} = \frac{T}{3}$

$\cos 5\omega t$ represents S.H.M. with period $= \frac{2\pi}{5\omega} = \frac{T}{5}$

The minimum time after which the combined function repeats its value is T. The given function is ***periodic but not simple harmonic***.

(*v*) $x(t) = \exp(-\omega^2 t^2) = e^{-\omega^2 t^2}$

It is an exponential function. It decreases monotonically to zero as $t \to \infty$. It never repeats its value. It is a ***non-periodic function***.

(*vi*) $x(t) = 1 + \omega t + \omega^2 t^2$

As t increases, $x(t)$ increases monotonically. Again, as $t \to \infty$, $x(t) \to \infty$. The function never repeats its value. So $x(t)$ is ***non-periodic***.

14.5. *A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is*

(*a*) *at the end A,*   (*b*) *at the end B,*

(*c*) *at the mid-point of AB going towards A,*

(*d*) *at 2 cm away from B going towards A,*

(*e*) *at 3 cm away from A going towards B, and*

(*f*) *at 4 cm away from A going towards A.*

Ans.

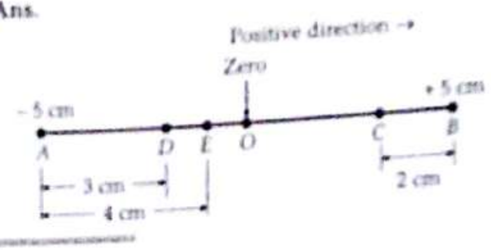


Fig. 14.45

Position	*Velocity*	*Acceleration*	*Force*
(*a*) At *A*	0 (at extreme position)	+ ve (acts from *A* to *O*)	+ ve (acts from *A* to *O*)
(*b*) At *B*	0 (at extreme position)	− ve (acts from *B* to *O*)	− ve (acts from *B* to *O*)
(*c*) At midpoint *O*, going towards *A*	− ve and maximum (acts from *O* to *A*)	0 (at mid-point)	0 (at mid-point)
(*d*) At *C*, going towards *A*	− ve (acts from *C* to *O*)	− ve (acts from *C* to *O*)	− ve (acts from *C* to *O*)
(*e*) At *D*, going towards *B*	+ ve (acts from *D* to *O*)	+ ve (acts from *D* to *O*)	+ ve (acts from *D* to *O*)
(*f*) At *E*, going towards *A*	− ve (acts from *E* to *A*)	+ ve (acts from *E* to *O*)	+ ve (acts from *E* to *O*)

14.6. *Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion ?*

(*a*) $a = 0.7x$ (*b*) $a = -200x^2$ (*c*) $a = -10x$ (*d*) $a = 100x^3$.

Ans. Only (*c*) represents S.H.M. because here $a \propto x$ and a acts in the opposition direction of x.

14.7. *(a) A particle in SHM is described by the displacement function*

$$x(t) = A\cos(\omega t + \phi),\ \omega = \frac{2\pi}{T}.$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm s^{-1}, what are its amplitude and initial phase angle ? The angular frequency of the particle is π s^{-1}.

(b) A particle in SHM is described by the displacement function

$$x(t) = B\sin(\omega t + \alpha),\ \omega = \frac{2\pi}{T}.$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm s^{-1}, what are its amplitude and initial phase angle ? The angular frequency of the particle is π s^{-1}.

Ans. (*a*) At $t = 0$, $x = 1$ cm and $v = \pi$ cm s^{-1}. Also, $\omega = \pi\ s^{-1}$

In SHM, displacement at any time t is given by

$$x = A\cos(\omega t + \phi)$$

Since, at $t = 0$, $x = 1$, therefore

$$1 = A\cos(\omega \times 0 + \phi)$$

or $A\cos\phi = 1$ ...(*i*)

Now velocity,

$$v = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)]$$

$$= -A\omega\sin(\omega t + \phi)$$

Again at $t = 0$, $v = \pi$ cm s^{-1}, so we have

$$\pi = -A(\pi)\sin(\omega \times 0 + \phi)$$

or $A\sin\phi = -1$ ...(*ii*)

Squaring and adding equations (*i*) and (*ii*), we get

$$A^2\cos^2\phi + A^2\sin^2\phi = 1^2 + (-1)^2$$

or $A^2(\cos^2\phi + \sin^2\phi) = 2$ or $A^2(1) = 2$

∴ $A = \sqrt{2}$ **cm.**

Dividing equation (*ii*) by (*i*), we get

$$\frac{A\sin\phi}{A\cos\phi} = \frac{-1}{1} \text{ or } \tan\phi = -1$$

or $\phi = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$.

(*b*) At $t = 0$, $x = 1$ cm and $v = \pi$ cm s^{-1}.

Also, $\omega = \pi\ s^{-1}$

Given $x = B\sin(\omega t + \alpha)$

Since, at $t = 0$, $x = 1$, therefore

$$1 = B\sin(\omega \times 0 + \alpha)$$

or $B\sin\alpha = 1$ ...(*i*)

Now velocity,

$$v = \frac{dx}{dt} = \frac{d}{dt}[B\sin(\omega t + \alpha)] = -B\omega\cos(\omega t + \alpha)$$

Again, at $t = 0$, $v = \pi$ cm s^{-1}, so we have

$$\pi = B(\pi)\cos(\omega \times 0 + \alpha)$$

or $B\cos\alpha = 1$ ...(*ii*)

Squaring and adding equations (*i*) and (*ii*), we get

$$B^2\sin^2\alpha + B^2\cos^2\alpha = 1^2 + 1^2 \text{ or } B^2 = 2$$

or $B = \sqrt{2}$ **cm.**

Dividing equation (*i*) by (*ii*), we get

$$\frac{B\sin\alpha}{B\cos\alpha} = \frac{1}{1} \text{ or } \tan\alpha = 1$$

or $\alpha = \frac{\pi}{4}$ or $\frac{5\pi}{4}$.

14.8. *A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this spring, when displaced and released, oscillates with a period of 0.60 s. What is the weight of the body ?*

Ans. The 20 cm length of the scale reads upto 50 kg, so

$$F = mg = 50 \times 9.8 \text{ N},\quad y = 20 \text{ cm} = 0.20 \text{ m}$$

Force constant, $k = \frac{F}{y} = \frac{50 \times 9.8}{0.20} = 2450$ Nm^{-1}

Suppose the spring oscillates with time period of 0.60 s when loaded with a mass of M kg. Then

$$T = 2\pi\sqrt{\frac{M}{k}}$$

or $T^2 = 4\pi^2\frac{M}{k}$

∴ $M = \frac{T^2k}{4\pi^2} = \frac{(0.60)^2 \times 2450}{4 \times (3.14)^2} = 22.36$ kg

Weight $= Mg = 22.36 \times 9.8 =$ **219.13 N.**

14.9. *A spring of force constant 1200 Nm^{-1} is mounted horizontally on a horizontal table. A mass of 3.0 kg is attached to the free end of the spring, pulled sideways to a distance of 2.0 cm and released. (i) What is the frequency of oscillation of the mass ? (ii) What is the maximum acceleration of the mass ? (iii) What is the maximum speed of the mass ?*

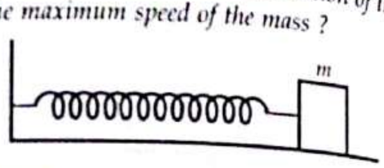


Fig. 14.46

Ans. Here $k = 1200$ Nm^{-1}, $m = 3.0$ kg,

$$A = 2.0 \text{ cm} = 2.0 \times 10^{-2} \text{ m}$$

(*i*) Frequency of oscillation of the mass,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3.0}}$$

$$= \frac{1}{2 \times 3.14} \times 20 = \mathbf{3.2\ s^{-1}}.$$

(*ii*) Angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3.0}} = 20\ s^{-1}.$$

Scanned by CamScanner

∴ Maximum acceleration of the mass

$= \omega^2 A = (20)^2 \times 2.0 \times 10^{-2}$

$= \mathbf{8.0\ ms^{-2}}$.

(*iii*) Maximum speed of the mass

$= \omega A = 20 \times 2.0 \times 10^{-2}$

$= \mathbf{0.40\ ms^{-1}}$.

14.10. *In Exercise 14.9, let us take the position of the mass, when the spring is unstretched, as x = 0, and the direction from left to right as the positive direction of X-axis. Give x as a function of time t for the oscillating mass, if at the moment we start the stop watch (t = 0), the mass is (i) at the mean position (ii) at the maximum stretched position (iii) at the maximum compressed position.*

In what do these different functions of SHM differ ? Frequency, amplitude or initial phase ?

Ans. When the mass starts motion from mean position, the displacement of SHM is given by

$$x = A \sin \omega t$$

And, when the mass starts motion from extreme position, the displacement of SHM is given by

$$x = \pm A \cos \omega t$$

From above exercise, we have

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20 \text{ rad s}^{-1}$$

(*i*) At $t = 0$, when the mass is at *mean position*

∴ Displacement is given by

$$x = A \sin \omega t = \mathbf{2 \sin 20t}.$$

(*ii*) At $t = 0$, when the mass is at the *maximum stretched position*. The motion starts from positive extreme position, thus

$$x = + A \cos \omega t = \mathbf{2 \cos 20t}.$$

(*iii*) At $t = 0$, when the mass is at the *maximum compressed position*. The mass starts its motion from negative extreme position, thus

$$x = - A \cos \omega t = \mathbf{-2 \cos 20t}.$$

14.11. *Fig. 14.47 corresponds to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e., clockwise or anti-clockwise) are indicated on each figure.*

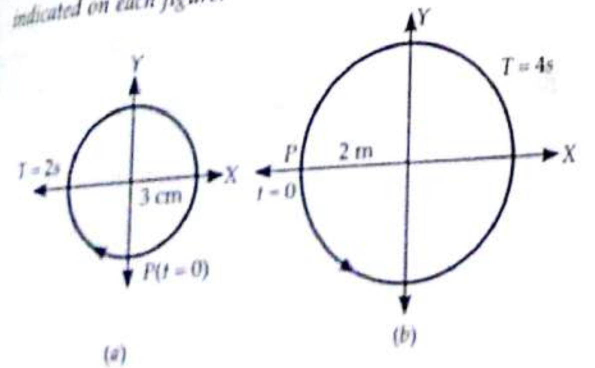


Fig. 14.47

Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Ans.

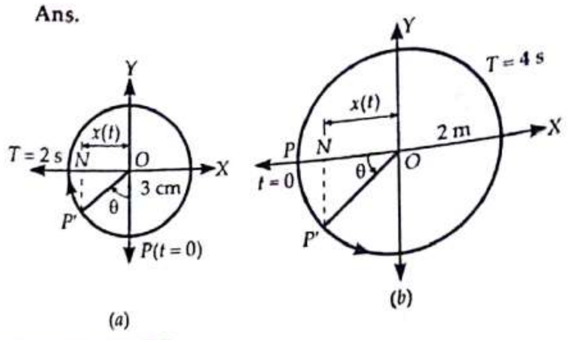


Fig. 14.48

(*a*) As shown in Fig. 14.48(*a*), suppose the particle moves from P to P' in time t.

Angle swept by the radius vector,

$$\theta = \omega t = \frac{2\pi}{T} t = \frac{2\pi}{2} t = \pi t \text{ rad}$$

Displacement,

$$ON = OP' \cos\left(\frac{\pi}{2} - \theta\right) = OP' \sin \theta$$

or $\quad -x(t) = 3 \sin \theta$

[Displacement being to the left O]

or $\quad x(t) = -3 \sin \pi t$

(*b*) As shown in Fig. 14.48(*b*), suppose the particle moves from P to P' in time t.

Angle swept by the radius vector,

$$\theta = \omega t = \frac{2\pi}{T} t = \frac{2\pi}{4} t = \frac{\pi t}{2} \text{ rad}$$

Displacement,

$$ON = OP' \cos \theta$$

or $\quad -x(t) = 2 \cos \dfrac{\pi t}{2} \qquad \left[\because OP' = 2\text{m}, \theta = \dfrac{\pi t}{2}\right]$

or $\quad x(t) = -2 \cos \dfrac{\pi t}{2}$.

14.12. *Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case (x is in cm and t is in s).*

(*i*) $x = -2 \sin(3t + \pi/3)$ (*ii*) $x = \cos(\pi/6 - t)$

(*iii*) $x = 3 \sin(2\pi t + \pi/4)$ (*iv*) $x = 2 \cos \pi t$.

Ans. (*i*) $x = -2 \sin(3t + \pi/3)$

$= 2 \cos(3t + \pi/3 + \pi/2)$

or $\quad x = 2 \cos(3t + 5\pi/6) \quad [-\sin\theta = \cos(\pi/2 + \theta)]$

Scanned by CamScanner

Comparing with $x = A\cos(\omega t + \phi_0)$, it follows that

$A = 2$ cm, $\omega = 3$ rad s^{-1}, $\phi_0 = 5\pi/6$ rad

The reference circle is shown in Fig. 14.49(*a*).

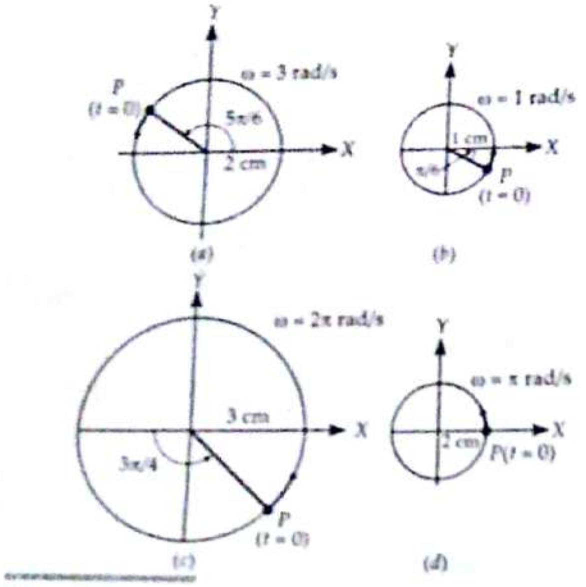


Fig. 14.49

(*ii*) $x = \cos(\pi/6 - t) = \cos[-(t - \pi/6)]$

or $\quad x = \cos(t - \pi/6) \qquad [\because \cos(-\theta) = \cos\theta]$

Comparing with $x = A\cos(\omega t + \phi_0)$, it follows that

$A = 1$ cm, $\omega = 1$ rad s^{-1}, $\phi_0 = -\pi/6$ rad.

The reference circle is shown in Fig. 14.49(*b*).

(*iii*) $x = 3\sin(2\pi t + \pi/4) = -3\cos\left(2\pi t + \frac{\pi}{4} + \frac{\pi}{2}\right)$

or $\quad x = -3\cos(2\pi t + 3\pi/4)$

The negative sign shows that the motion starts on the negative side of *x*-axis.

Here $A = 3$ cm, $\omega = 2\pi$ rad s^{-1}, $\phi_0 = 3\pi/4$ rad

The reference circle is shown in Fig. 14.49(*c*).

(*iv*) $x = 2\cos\pi t$

Comparing with $x = A\cos(\omega t + \phi_0)$, it follows that

$A = 2$ cm, $\omega = \pi$ rad s^{-1}, $\phi_0 = 0$

The reference circle is shown in Fig. 14.49(*d*).

14.13. *Fig. 14.50(a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. The spring is stretched by a force F at its free end. Fig. 14.50(b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.50(b) is stretched by the same force F.*

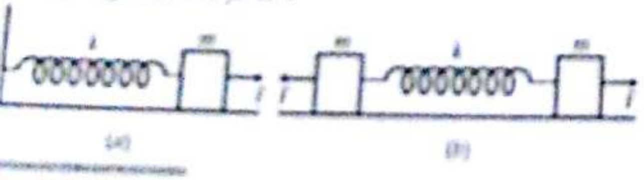


Fig. 14.50

(*i*) *What is the maximum extension of the spring in the two cases ? (ii) If the mass in (a) and the two masses in (b) are released free, what is the period of oscillation in each case ?*

Ans. (*i*) *Maximum extension of the spring.* In case (*b*), the force at either end of the spring is *F* and they act in opposite directions. In case (*a*), the force of reaction at the clamped end is also *F*, so both systems are identical. The maximum extension in each case is given by

$$y = \frac{F}{k}.$$

(*ii*) In case (*a*), the period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In case (*b*), the spring can be considered to be divided into two equal halves and its centre can be regarded to be fixed as it does not move. Let k' be the force constant of each half and x' be the extension produced in each half. Then

$$x' = \frac{F}{k'}$$

Total extension, $x = 2x'$

or $$\frac{F}{k} = 2 \cdot \frac{F}{k'}$$

$\therefore \quad k' = 2k$

Hence the period of oscillation in case (*b*) is

$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

14.14. *The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rev/min, what is its maximum speed ?*

Ans. Here $\quad A = \frac{1}{2}$ m, $\omega = 200$ rev / min

$$v_{max} = \omega A = 200 \times \frac{1}{2} = \mathbf{100\ m / min}$$

14.15. *The acceleration due to gravity on the surface of the moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the moon if its time period on the earth is 3.5 s ? Given g on earth = 9.8 ms^{-2}.* [Delhi 06]

Ans. *For the moon* : $g_m = 1.7$ ms^{-2}, $T_m = ?$

For the earth : $g_e = 9.8$ ms^{-2}, $T_e = 3.5$ s

But $\quad T_e = 2\pi\sqrt{\frac{l}{g_e}}$ and $T_m = 2\pi\sqrt{\frac{l}{g_m}}$

$$\therefore \quad \frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}}$$

or $$T_m = \sqrt{\frac{g_e}{g_m}} \times T_e$$

$$= \sqrt{\frac{9.8}{1.7}} \times 3.5 = \mathbf{8.4\ s.}$$

Scanned by CamScanner

14.16. *Answer the following questions :*

(a) *Time period of a particle in SHM depends on the force constant k and mass m of the particle :*

$T = 2\pi\sqrt{\frac{m}{k}}$. *A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum ?* **[Delhi 12]**

(b) *The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{l/g}$. Think of a qualitative argument to appreciate this result.*

(c) *A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall ?*

(d) *What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity ?*

Ans. (a) For simple pendulum, force constant

$$k = \frac{mg}{l} \quad i.e., \quad k \propto m$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/l}} = 2\pi\sqrt{\frac{l}{g}}$$

Thus m cancels out. Hence time period of a simple pendulum is independent of mass.

(b) The acceleration of the bob of a simple pendulum is given by $\quad a = -g \sin\theta.$

If θ is small, then $\sin\theta \simeq \theta$ and $a = -g\theta$

If θ is large, then $\sin\theta < \theta$, so that there is effective decrease in the value of g for large angles. Hence the time period, $T = 2\pi\sqrt{l/g}$ increases.

(c) Yes, the wrist watch will give correct time because the working of a wrist watch depends on its spring action (i.e., the P.E. stored in the wound spring) and is independent of the gravity.

(d) Inside a cabin falling freely under gravity, $g = 0$. Hence the frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ of a simple pendulum mounted in the cabin will be zero.

14.17. *A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period ?*

Ans. The bob of the pendulum has two accelerations :

(i) Centripetal acceleration, $a_c = \frac{v^2}{R}$, acting horizontally

(ii) Acceleration due to gravity $= g$, acting vertically downwards.

The effective acceleration due to gravity,

$$g' = \sqrt{g^2 + a_c^2} = \sqrt{g^2 + \frac{v^4}{R^2}}$$

$\therefore$ Time period,

$$T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + v^4/R^2}}}.$$

14.18. *A cylindrical piece of cork of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$, where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).*

Ans. Refer answer to Q. 22 on page 14.31.

14.19. *One end of a U-tube containing mercury is connected to a suction pump and the other end is connected to the atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid in the U-tube executes SHM, with time period $T = 2\pi\sqrt{l/g}$, where l is the length of the liquid column is the U-tube.* **[Delhi 13]**

Ans. Refer answer to Q. 20 on page 14.30.

14.20. *An air chamber of volume V has a neck of area of cross-section A into which a ball of mass m can move without friction. Show that when the ball is pressed down through some distance and released, the ball executes SHM. Obtain the formula for the time period of this SHM, assuming pressure- volume variations of the air to be (i) isothermal and (ii) adiabatic.*

Ans. Refer answer to Q. 23 on page 14.31.

14.21. *You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.*

Ans. (a) Here $m = 3000$ kg, $x = 0.15$ m

If k is the spring constant of each spring, then the spring constant of the four springs connected in parallel will be $4k$.

$$\therefore \quad 4kx = mg$$

or $$k = \frac{mg}{4x} = \frac{3000 \times 10}{4 \times 0.15} = \mathbf{5 \times 10^4\ Nm^{-1}}.$$

(b) As $A' = Ae^{-bt/2m}$

$$\therefore \quad \frac{A}{2} = Ae^{-bt/2m} \quad \text{or} \quad 2 = e^{bt/2m}$$

or $$\log_e 2 = \frac{bt}{2m}\log_e e = \frac{bt}{2m} \quad \text{or} \quad b = \frac{2m \log_e 2}{t}$$

But $$t = 2\pi\sqrt{\frac{m}{4k}} = 2 \times \frac{22}{7} \times \sqrt{\frac{3000}{4 \times 5 \times 10^4}} = \frac{44}{70}\sqrt{\frac{3}{2}}\ \text{s}$$

Hence $$b = \frac{2 \times 750 \times 0.693}{\frac{44}{70}\sqrt{\frac{3}{2}}} = \mathbf{1350.4\ kg\ s^{-1}}.$$

Scanned by CamScanner

14.22. *Show that for a particle in linear SHM, the average kinetic energy over a period of oscillation equals the average potential energy over the same period.* **[Delhi 14]**

Ans. Suppose a particle of mass m executes SHM of period T. The displacement of the particle at any instant t is given by $y = A \sin \omega t$

$\therefore$ Velocity, $v = \frac{dy}{dt} = \omega A \cos \omega t$.

Kinetic energy, $E_k = \frac{1}{2} mv^2 = \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t$.

Potential energy, $E_p = \frac{1}{2} m \omega^2 y^2 = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t$.

$\therefore$ Average K.E. over a period of oscillation,

$$E_{k_{av}} = \frac{1}{T}\int_0^T E_k \, dt = \frac{1}{T}\int_0^T \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t \, dt$$

$$= \frac{1}{2T} m \omega^2 A^2 \int_0^T \frac{(1 + \cos 2\omega t)}{2} dt$$

$$= \frac{1}{4T} m \omega^2 A^2 \left[t + \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{1}{4T} m \omega^2 A^2 (T) = \frac{1}{4} m \omega^2 A^2 \quad ...(1)$$

Average P.E. over a period of oscillation,

$$E_{P_{av}} = \frac{1}{T}\int_0^T E_p \, dt = \frac{1}{T}\int_0^T \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t \, dt$$

$$= \frac{1}{2T} m \omega^2 A^2 \int_0^T \frac{(1 - \cos 2\omega t)}{2} dt$$

$$= \frac{1}{4T} m \omega^2 A^2 \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{1}{4T} m \omega^2 A^2 (T) = \frac{1}{4} m \omega^2 A^2 \quad ...(2)$$

Clearly, from equations (1) and (2), $E_{k_{av}} = E_{P_{av}}$.

14.23. *A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillation is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire.*

Ans. Period of torsional oscillations is given by

$$T = 2\pi \sqrt{\frac{I}{C}} \quad \text{or} \quad T^2 = \frac{4\pi^2 I}{C}$$

$\therefore$ Torsional spring constant, $C = \frac{4\pi^2 I}{T^2}$

But $I = \frac{1}{2} MR^2$, $M = 10$ kg, $R = 15$ cm $= 0.15$ m,

$T = 1.5$ s

$$\therefore C = \frac{4\pi^2 \times \frac{1}{2} MR^2}{T^2}$$

$$= \frac{2 \times (3.14)^2 \times 10 \times (0.15)^2}{(1.5)^2} = \mathbf{2.0 \ Nm \ rad^{-1}}$$

14.24. *A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm, (b) 3 cm, (c) 0 cm.*

Ans. Here $A = 5$ cm, $T = 0.2$ s

Velocity and acceleration at any displacement x are given by

$$v = \omega\sqrt{A^2 - x^2} = \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$a = -\omega^2 x = -\frac{4\pi^2}{T^2} x$$

(a) When $x = 5$ cm,

$$v = \frac{2\pi}{0.2}\sqrt{5^2 - 5^2} = 0.$$

$$a = -\frac{4\pi^2}{(0.2)^2} \times 5 \text{ cm s}^{-2}$$

$$= -500\,\pi^2 \text{ cm s}^{-2} = -5\pi^2 \text{ ms}^{-2}.$$

(b) When $x = 3$ cm,

$$v = \frac{2\pi}{0.2}\sqrt{5^2 - 3^2} \text{ cm s}^{-2}$$

$$= 40\,\pi \text{ cm s}^{-2} = 0.40\,\pi \text{ ms}^{-1}.$$

$$a = -\frac{4\pi^2}{(0.2)^2} \times 3 \text{ cm s}^{-2}$$

$$= -300\,\pi^2 \text{ cm s}^{-2} = -3\pi^2 \text{ ms}^{-2}.$$

(c) When $x = 0$ cm,

$$v = \frac{2\pi}{0.2}\sqrt{5^2 - 0^2} \text{ cm s}^{-1} = 50\,\pi \text{ cm s}^{-1}$$

$$= \mathbf{0.50\,\pi \ m \ s^{-1}}.$$

$$a = -\frac{4\pi^2}{(0.2)^2} \times 0 = 0.$$

14.25. *A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_o and pushed towards the centre with a velocity v_o at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0.*

Ans. By conservation of energy,

(K.E. + P.E.) at distance x_0

= Total energy at the extreme position

or $\quad \frac{1}{2} mv_0^2 + \frac{1}{2} kx_o^2 = \frac{1}{2} kA^2$

or $\quad \frac{m}{k} v_o^2 + x_o^2 = A^2$

or $\quad \frac{v_o^2}{\omega^2} + x_o^2 = A^2$

$$\therefore \quad A = \sqrt{\frac{v_o^2}{\omega^2} + x_o^2}.$$

Scanned by CamScanner

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. What is periodic motion ? **[Himachal 04]**
2. What is oscillatory motion ? **[Himachal 04, 05C]**
3. What are harmonic functions ?
4. What is the period of each of the functions sec ωt and cosec ωt ?
5. Justify that sin θ and cos θ are periodic functions.
6. Define force constant. Give its SI unit.
7. Write the values of oscillation—amplitude and frequency from the equation $y = A \sin \omega t$ of S.H.M.
8. Write the relation between acceleration, displacement and frequency of a particle executing S.H.M.
9. The equation of motion of a particle executing S.H.M. is $a = -bx$, where a is the acceleration of the particle, x is the displacement from the mean position and b is a constant. What is the time period of the particle ?
10. Write the relation between time period T, displacement x and acceleration a of a particle in S.H.M.
11. Is spring constant a dimensional or non-dimensional constant ?
12. What is meant by phase of an oscillating particle ?
13. What is initial phase or epoch. Give a unit for its measurement.
14. Two simple pendulums of same length are crossing at their mean positions, what is phase difference between them ?
15. What is the phase relationship between particle displacement, velocity and acceleration in S.H.M. ?
16. What is phase difference between the displacement and acceleration of a particle executing S.H.M. ?
17. What is a second's pendulum ? What is its length ? **[Himachal 95C]**
18. A simple pendulum moves from one end to the other in 1/4 second. What is its frequency ?
19. Write the values of amplitude and angular frequency for the following simple harmonic motion.

$$y = 0.2 \sin(99t + 0.36)$$

20. Write the displacement equation representing the following conditions obtained in a simple harmonic motion :

    Amplitude = 0.01 m,

    Frequency = 600 Hz,

    Initial phase = $\pi / 6$. **[Delhi 06]**

21. How will the time period of a simple pendulum change if its length is doubled ? **[Delhi 98]**
22. What would be the effect on the time period, if the amplitude of a simple pendulum increases ?
23. How will a simple pendulum behave if it is taken to the moon ?
24. A pendulum clock is thrown out of an aeroplane. How will it behave during its free fall in air ?
25. If on going up a hill, the value of g decreases by 10%, then what change must be made in the length of a pendulum clock in order to obtain accurate time ?
26. Which quantity is conserved during the oscillation of a simple pendulum ?
27. A girl is sitting in a swing. Another girl sits by her side. What will be the effect on the periodic time of the swing ?
28. What is the frequency of a second pendulum in an elevator rising up with an acceleration equal to $g/2$?
29. Two identical springs of force constant k each are connected in series. What will be the equivalent spring constant ?
30. Two identical springs of force constant k each are connected in parallel. What will be the equivalent spring constant ?
31. The time period of a body executing S.H.M. is 0.05 s and the amplitude of vibration is 4 cm. What is the maximum velocity of the body ?
32. A particle executes S.H.M. of 2 cm. At the extreme position, the force is 4 N. What is the force at a point midway between mean and extreme positions ?
33. The potential energy of a particle in S.H.M. varies periodically. If ν is the frequency of oscillation of the particle, then what is the frequency of variation of potential energy ?

Scanned by CamScanner

34. When will the motion of a simple pendulum be simple harmonic ?

35. When is the potential energy and kinetic energy of a harmonic oscillator maximum ? What are these maximum values ?

36. On what factors does the energy of a harmonic oscillator depend ?

37. What would be the time period of a simple pendulum at the centre of the earth ?

38. Can an ideal simple pendulum be realised in practice ? Is the motion of a simple pendulum-linear simple harmonic or angular simple harmonic ?

39. A simple harmonic motion of acceleration a and displacement x is represented by

$$a + 4\pi^2 x = 0.$$

What is the time period of S.H.M ?

40. State force law for a simple harmonic motion. **[Delhi 03]**

41. Give the general expression for displacement of a particle undergoing S.H.M. **[Central Schools 03]**

42. What are the two basic characteristics of an oscillating system ? **[Delhi 97]**

43. What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity ? **[Delhi 97]**

44. The amplitudes of oscillations of two simple pendulums similar in all respects are 2 cm and 5 cm respectively. Find the ratio of their energies of oscillations. **[Delhi 96]**

45. Define periodic time. Give its SI unit. **[Delhi 96]**

46. What is the main difference between forced oscillations and resonance ? **[Delhi 02]**

47. What is meant by SHM ? **[Himachal 05]**

48. What is meant by the displacement of a particle executing SHM ? **[Himachal 05]**

49. Define amplitude of SHM. **[Himachal 05]**

50. Define force constant and give its dimensional formula. **[Himachal 03]**

51. List any two characteristics of simple harmonic motion. **[Delhi 04]**

52. What is the time period of second's pendulum ? **[Himachal 03, 04]**

53. A pendulum is making one oscillation in every two seconds. What is the frequency of oscillation ? **[Delhi 04]**

54. A simple pendulum is inside a space craft. What should be its time period of vibration ? **[Central Schools 05]**

55. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion ? **[Central Schools 08]**

56. A simple pendulum is described by $a = -16x$; where a is acceleration, x is displacement in metre. What is the time period ? **[Delhi 13]**

57. The angular velocity and amplitude of a simple pendulum are ω and A. At a displacement x from the mean position, the kinetic energy is K and potential energy is U. Find the ratio of K to U. **[Central Schools 14]**

Answers

1. The motion which repeats itself over and over again after a fixed interval of time is called a periodic motion.

2. The motion which repeats itself over and over again about a mean position such that it remains confined within well defined limits (known as extreme positions) on either side of the mean position is called oscillatory motion.

3. The functions which can be represented by a sine or cosine curve are called harmonic functions.

4. Period of sec ωt or cosec ωt = $2\pi/\omega$

5. Both sin θ and cos θ are the periodic functions of θ because,

$$\sin(\theta + 2\pi n) = \sin\theta \text{ and } \cos(\theta + 2\pi n) = \cos\theta,$$

where $n = 1, 2, 3...$

6. The restoring force produced per unit displacement of an oscillating body is called force constant or spring factor (k). Its SI unit is Nm^{-1}.

7. Amplitude = A,

frequency $= \omega / 2\pi$.

8. Acceleration,

$$a = -\omega^2 y = -4\pi^2 v^2 y.$$

9. Here $a = -bx = -\omega^2 x$,

where $\omega = \sqrt{b}$.

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{b}}.$$

10. $T = 2\pi\sqrt{\frac{x}{a}}$.

Scanned by CamScanner

11. Spring constant,

$$k = \frac{F}{x} = \frac{\text{Restoring force}}{\text{Displacement}} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$$

Hence spring constant is a *dimensional constant*.

12. The phase of an oscillating particle at any instant gives the state of the particle as regards to its position and the direction of motion at that instant.

13. The phase of a vibrating particle corresponding to the time $t = 0$ is called initial phase or epoch. It is measured in radian.

14. 180° or π radian.

15. In S.H.M., the particle velocity leads the displacement in phase by $\pi/2$ rad and acceleration leads the velocity in phase by $\pi/2$ rad.

16. 180° or π radian.

17. A simple pendulum whose time period is 2 seconds is called a second's pendulum. Its length is 99.3 cm.

18. 2 Hz

19. Amplitude $A = 0.2$ m, angular frequency $\omega = 99$ Hz.

20. $y = a\sin(2\pi\nu t + \phi_0) = 0.01\sin\left(1200\,\pi t + \frac{\pi}{6}\right)$.

21. As $T = 2\pi\sqrt{\frac{l}{g}}$, so when the length is doubled, the time period will be increased by $\sqrt{2}$ times.

22. The time period of the simple pendulum will remain the same, because time period is independent of its amplitude.

23. On the moon, the simple pendulum will oscillate $\sqrt{6}$ times slower than that it does on the surface of the earth because the value of g on the moon is 1/6th of that on the earth.

24. During its free fall in air, the pendulum clock is in a state of weightlessness *i.e.*, $g = 0$. Hence

$$T = 2\pi\sqrt{\frac{l}{g}} = \infty$$

The pendulum clock will not oscillate at all.

25. The length of the pendulum clock should be decreased by 10%.

26. Total mechanical energy of the bob is conserved.

27. The periodic time remains unchanged because the length of the pendulum does not change when the second girl sits besides the first girl and T is independent of the mass of oscillating bob.

28. Frequency of a second pendulum, $\nu = \frac{1}{2}\,s^{-1}$

The effective value of g in the elevator,

$$g' = g + a = g + g/2 = 3g/2$$

As $\quad \nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ *i.e.*, $\nu \propto \sqrt{g}$

$$\therefore \quad \frac{\nu'}{\nu} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{3}{2}} = 1.225$$

Hence $\nu' = 1.225\nu = 1.225 \times \frac{1}{2}$

$$= \mathbf{0.612\ Hz}$$

29. $k_s = \frac{k \times k}{k + k} = \frac{k}{2}$.

30. $k_p = k + k = 2k$.

31. $v_{max} = \frac{2\pi}{T}A = \frac{2\pi}{0.05} \times \frac{4}{100}$
$= \mathbf{1.6\pi\ ms^{-1}}$.

32. 2 N, because $F \propto x$.

33. 2ν.

34. When the displacement of the bob from the mean position is so small that $\sin\theta \approx \theta$, the oscillations of the pendulum will be simple harmonic.

35. Potential energy of a harmonic oscillator is maximum at extreme position and minimum at extreme position, while kinetic energy is maximum at mean position.
Max. value of K.E. = Max. value of P.E. = $\frac{1}{2}m\omega^2 A^2$.

36. The energy of a harmonic oscillator depends on its (*i*) mass m (*ii*) frequency ν and (*iii*) amplitude A.

$$E = 2\pi^2 m\nu^2 A^2$$

37. At the centre of the earth, $g = 0$, so

$$T = 2\pi\sqrt{l/g} = \infty.$$

38. No. The motion of simple pendulum is angular simple harmonic.

39. $a = -4\pi^2 x = -\omega^2 x$, where $\omega = 2\pi$

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = \mathbf{1\,s}$$

40. The force acting in S.H.M. is proportional to the displacement and is always directed towards the mean position. Hence the force law for S.H.M. is

$$F = -kx.$$

41. $x(t) = A\cos(\omega t + \phi_0)$ or $x(t) = a\cos\omega t + b\sin\omega t$.

42. The oscillations of a system result from its two basic characteristics, namely, *elasticity* and *inertia*.

43. In a freely falling cabin, $g = 0$, therefore

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = 0$$

Scanned by CamScanner

44. $\frac{E_1}{E_2} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{2}{5}\right)^2 = 4 : 25$

45. The smallest interval of time after which a motion repeats itself over and over again is called its periodic time. The SI unit of periodic time is second (s).

46. In forced oscillations the frequency of the external periodic force is different from the natural frequency of the oscillator while these two frequencies are equal in resonant oscillations.

47. Refer to point 5 of Glimpses.

48. Refer to point 7 of Glimpses.

49. Refer to point 8 of Glimpses.

50. The restoring force produced per unit displacement of an oscillating body is called force constant. Its dimensional formula is $[ML^0T^{-2}]$

51. **Characteristics of SHM :**

(i) It is the simplest kind of oscillatory motion of constant amplitude and fixed frequency.

(ii) Restoring force is proportional to the displacement of the particle from its mean position.

52. 2 s.

53. $\nu = 1/2$ cps.

54. Inside a spacecraft, $g = 0$.

Therefore,

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{0}} = \infty.$$

55. A periodic motion repeats after a definite time interval T. So

$$y(t) = y(t + T) = y(t + 2T), \text{ etc.}$$

56. $a = -16x = -\omega^2 x$

$\Rightarrow \quad \omega^2 = 16$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}\text{ s.}$$

57. $$\frac{K}{U} = \frac{\frac{1}{2}m\omega^2(A^2 - x^2)}{\frac{1}{2}m\omega^2 x^2} = \frac{A^2 - x^2}{x^2}$$

Type B : Short Answer Questions

2 or 3 Marks Each

1. Giving examples of each type, distinguish between periodic, harmonic and non-harmonic functions.

2. What is simple harmonic motion ? Write its any four properties.. **[Central School 13]**

3. Write down the differential equation for S.H.M. Give its solution. Hence obtain expression for the time period of S.H.M.

4. Prove that the displacement equation

$$x(t) = a\cos\omega t + b\sin\omega t$$

represents a simple harmonic motion. Determine its amplitude and phase constant.

[Central Schools 05]

5. Write expression for the particle velocity and acceleration during simple harmonic motion as function of time **[Delhi 03C]**

6. Derive an expression for the instantaneous velocity and acceleration of a particle executing S.H.M.

[Himachal 01, 04, 05]

7. Derive an expression for the instantaneous velocity of the particle executing S.H.M. Find the position at which the particle velocity is (i) maximum and (ii) minimum. **[Central Schools 14]**

8. What is S.H.M. ? Show that the acceleration of a particle in S.H.M. is proportional to its displacement. Also write expression for the time-period in terms of acceleration. **[Central Schools**

9. Show that in simple harmonic motion (S.H.M.) the acceleration is directly proportional to its displacement at the given instant. **[Delhi**

10. The relation between the acceleration a and displacement x of a particle executing SHM is

$$a = -\left(\frac{p}{q}\right)y;$$

where p and q are constants.

What will be the time period T of the particle ?

11. Find an expression for the total energy of a particle executing S.H.M.

[Delhi 02 ; Himachal 05 ; Central Schools

12. Show that the total energy of a body executing S.H.M. is constant. **[Central Schools**

13. Show that the total energy of a particle executing simple harmonic motion is directly proportional to the square of amplitude and frequency. **[Himachal**

Scanned by CamScanner

14. A body is executing simple harmonic motion. At what distance from its mean position, its energy is half kinetic and half potential ? [Delhi 96]

15. Show that the horizontal oscillations of a massless loaded spring are simple harmonic. Deduce an expression for its time period.

16. Show that when a body is suspended from a spring and is pulled down a little and released, it executes S.H.M. Also find an expression for its time period. Does it depend on acceleration due to gravity ? [Himachal 05 C]

17. What is an ideal simple pendulum ? Derive an expression for its time period. [Himachal 05C ; Chandigarh 07]

18. What is a simple pendulum ? Show that motion executed by the bob of the pendulum is S.H.M. Derive an expression for its time period. [Himachal 06 ; Chandigarh 08 ; Central Schools 12]

19. Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period. Does it depend on the mass of the bob ? [Himachal 04 ; Delhi 08, 11]

20. Prove that if a liquid taken in a U-tube is disturbed from the state of equilibrium, it will oscillate harmonically. Find expressions for the angular frequency and time period.

21. A ball of mass m fits smoothly in the cylindrical neck of an air chamber of volume V. The neck area is A. Show that the oscillations of the ball in the neck of the air chamber are simple harmonic. Calculate the time-period.

22. Show that the angular oscillations of the balance-wheel of a watch are simple harmonic. Hence deduce an expression for the time-period of its oscillations.

23. A cylindrical piece of cork of base area A and height h floats in a liquid of density ρ_l. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a time period $T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$, where ρ is the density of the cork. [Delhi 03]

24. What are free and damped oscillations ? Show them graphically. [Central Schools 14]

25. With the help of examples, differentiate between free oscillations and forced oscillations. [Delhi 03]

26. Briefly explain the principle underlying the tuning of a radio receiver.

27. What are coupled oscillations ? Give examples.

28. Show that in a S.H.M. the phase difference between displacement and velocity is $\pi/2$, and between displacement and acceleration it is π. [Delhi 06]

29. Draw the graphical representation of simple harmonic motion, showing the
    (a) displacement-time curve.
    (b) velocity-time curve and
    (c) acceleration-time curve. [Chandigarh 07]

Answers

1. Refer answer to Q. 5 on page 14.2.
2. Refer answer to Q. 6 and Q. 7 on page 14.4.
3. Refer answer to Q. 8 on page 14.4.
4. Refer answer to Q. 11 on page 14.7.
5. Refer answer to Q. 14 on page 14.8.
6. Refer to solution of Q. 12 on page 14.7 and Q. 13 on page 14.8.
7. Refer answer to Q. 12 on page 14.7.
8. Displacement, $x = A\cos\omega t$

Velocity, $v = \frac{dx}{dt} = -\omega A\sin\omega t$

Acceleration,

$$a = \frac{dv}{dt} = -\omega^2 A\cos\omega t = -\omega^2 x$$

$\therefore$ Therefore, acceleration $\propto$ displacement.

Magnitude of acceleration in S.H.M. is

$$a = \omega^2 x$$

or $$\omega^2 = a/x$$

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{a/x}}$$

$$= 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

9. Refer answer to the above question.

10. $a = -\left(\frac{p}{q}\right)y = -\omega^2 y$

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{p/q}} = 2\pi\sqrt{\frac{q}{p}}$$

11. Refer answer to Q. 15 on page 14.16.
12. Refer answer to Q. 15 on page 14.16.
13. Refer answer to Q. 15 on page 14.16.

Scanned by CamScanner

14. Refer to the solution of Example 28 on page 14.18.
15. Refer answer to Q. 16 on page 14.20.
16. Refer answer to Q. 17 on page 14.20.
17. Refer answer to Q. 19 on page 14.27.
18. Refer answer to Q. 19 on page 14.27.
19. Refer answer to Q. 19 on page 19.27.
20. Refer answer to Q. 20 on page 14.30.
21. Refer answer to Q. 23 on page 14.31.
22. Refer answer to Q. 24 on page 14.32.
23. Refer answer to Q. 22 on page 14.31.
24. Refer answer to Q. 25 on page 14.34.
25. Refer answer to Q. 26 on page 14.36.
26. Refer answer to Q. 27 on page 14.37.
27. Refer answer to Q. 28 on page 14.37.
28. Refer answer to Q. 14 on page 14.8.
29. Refer answer to Q. 14 on page 14.8.

Type C : Long Answer Questions

5 Marks Each

1. With suitable examples, explain the meaning of a periodic function. Construct two infinite sets of periodic functions with period *T*. Hence state Fourier theorem.
2. Define the terms harmonic oscillator, displacement, amplitude, cycle, time period, frequency, angular frequency, phase and epoch with reference to an oscillatory system.
3. Show that simple harmonic motion may be regarded as the projection of uniform circular motion along a diameter of the circle. Hence derive an expression for the displacement of a particle in S.H.M.
4. Explain the relation in phase between displacement, velocity and acceleration in SHM, graphically as well as theoretically. **[Chandigarh 04]**
5. Derive expressions for the kinetic and potential energies of a harmonic oscillator. Hence show that total energy is conserved in S.H.M. **[Delhi 12]**
6. Find the total energy of the particle executing S.H.M. and show graphically the variation of P.E. and K.E. with time in S.H.M. What is the frequency of these energies with respect to the frequency of the particle executing S.H.M ? **[Delhi 05, 14]**
7. Show that for a particle in linear S.H.M., the average kinetic energy over a period of oscillation is equal to the average potential energy over the same period. At what distance from the mean position is the kinetic energy in simple harmonic oscillator equal potential energy ? **[Delhi 06, 14]**
8. What is a spring factor ? Derive the expression for resultant spring constant when two springs having constants k_1 and k_2 are connected in (*i*) parallel, and (*ii*) in series. **[Chandigarh 04 ; Central Schools 05]**

Answers

1. Refer answer to Q. 4 on page 14.2.
2. Refer answer to Q. 9 on page 14.4.
3. Refer answer to Q. 10 on page 14.6.
4. Refer answer to Q. 14 on page 14.8.
5. Refer answer to Q. 15 on page 14.16.
6. Refer answer to Q. 15 on page 14.16.
7. Refer to the solution on NCERT Exercise 14.22 on page 14.54 and Example 28 on page 14.18.
8. Refer answer to Q. 18 on page 14.21.

Scanned by CamScanner

Competition Section

Oscillations

GLIMPSES

1. **Periodic motion.** A motion which repeats itself over and over again after a regular interval of time is called a periodic motion.
2. **Oscillatory motion.** A motion in which a body moves back and forth repeatedly about a fixed point (called mean position) is called oscillatory or vibratory motion.
3. **Periodic function.** Any function that repeats its value at regular intervals of its argument is called a periodic function. The following sine and cosine functions are periodic with period T.

$$f(t) = \sin \frac{2\pi t}{T} \quad \text{and} \quad g(t) = \cos \frac{2\pi t}{T}$$

The periodic functions which can be represented by a sine or cosine curve are called *harmonic functions*. All harmonic functions are necessarily periodic but all periodic functions are not harmonic.

The periodic functions which cannot be represented by single sine or cosine function are called *non-harmonic functions*.

4. **Fourier theorem.** Two infinite sets of periodic functions with period T are

$$f_n(t) = \sin \frac{2\pi n t}{T}, \quad n = 1, 2, 3, 4, \ldots$$

$$g_n(t) = \cos \frac{2\pi n t}{T}, \quad n = 0, 1, 2, 3, \ldots$$

Fourier theorem states that any periodic function $F(t)$ with period T can be expressed as the unique combination of sine and cosine functions $f_n(t)$ and $g_n(t)$ with suitable coefficients. Mathematically,

$$F(t) = b_0 + \sum_n b_n \cos n\omega t + \sum_n a_n \sin n\omega t$$

where $\omega = 2\pi / T$. The coefficients $b_0, b_1, b_2, \ldots$; $a_1, a_2, a_3, \ldots$ are called *Fourier coefficients*. The special case of Fourier theorem in which only a_1 and b_1 are non-zero represents *simple harmonic motion* (S.H.M.).

$$F(t) = a_1 \sin \frac{2\pi t}{T} + b_1 \cos \frac{2\pi t}{T}.$$

5. **Simple harmonic motion.** A particle is said to execute simple harmonic motion if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from the mean position and is always directed towards the mean position. If the displacement of the oscillating particle from the mean position is small, then

Restoring force $\propto$ Displacement

or $\quad F \propto x$

or $\quad F = -kx$

where k is a positive constant called *force constant* or *spring factor* and is defined as the restoring force produced per unit displacement. The negative sign shows that the restoring force always acts in the opposite direction of displacement x. The above equation defines SHM.

6. **Oscillation or cycle.** One complete back and forth motion of a particle is called cycle or vibration or oscillation.
7. **Displacement.** It is the distance of the oscillating particle from the mean position at any instant. It is denoted by x.
8. **Amplitude (A).** The maximum displacement of the oscillating particle on either side of its mean position is called its amplitude. Thus $x_{max} = \pm A$.
9. **Time period.** It is the time taken by a particle to complete one oscillation about its mean position. It is denoted by T.
10. **Frequency.** It is the number of oscillations completed per second by a particle about its mean position. It is denoted by ν and is equal to the reciprocal of time period. Thus $\nu = \frac{1}{T}$

Frequency is measured in hertz.

1 hertz = 1 Hz = 1 oscillation per second = $1\,s^{-1}$.

11. **Angular frequency**. It is the quantity obtained by multiplying frequency ν by a factor of 2π. It is denoted by ω.

Thus $\omega = 2\pi\nu = \frac{2\pi}{T}$.

SI unit of ω = rad s^{-1}.

12. **Phase.** The phase of vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant. It is denoted by ϕ.

13. **Initial phase or epoch.** The phase of a vibrating particle corresponding to time $t = 0$ is called initial phase or epoch. It is denoted by ϕ_0.

14. **Phase difference.** The phase difference between two vibrating particles tells the lack of harmony in the vibrating states of the two particles at any instant.

15. **Relation between SHM and uniform circular motion.** Simple harmonic motion is the projection of uniform circular motion upon a diameter of a circle. This circle is called the *reference circle* and the particle which revolves along it is called *reference particle* or *generating particle*.

16. **Displacement in SHM.** In a simple harmonic motion, the displacement of a particle from its equilibrium position at any instant t is given by

$$x(t) = A\cos(\omega t + \phi_0)$$

Here A is *amplitude* of the displacement, the quantity $(\omega t + \phi_0)$ is the *phase* of the motion and ϕ_0 is the initial phase.

When the time is measured from the mean position,

$$x(t) = A\sin\omega t$$

When the time is measured from the extreme position,

$$x(t) = A\cos\omega t$$

The angular frequency ω, frequency ν and time period T of the motion are given by

$$\omega = \sqrt{\frac{a}{x}} \quad \text{or} \quad \omega = \sqrt{\frac{k}{m}}$$

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{a}{x}} \quad \text{or} \quad \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{a}}$$

or $$T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{k}}.$$

17. **Velocity in SHM.** It is the rate of change of displacement of the particle at any instant. It is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi_0)]$$

$$= -\omega A\sin(\omega t + \phi_0) = -\omega\sqrt{A^2 - x^2}$$

The maximum value of velocity is called *velocity amplitude* v_m of the motion.

Thus $v_m = \omega A = \frac{2\pi}{T}A$

At the mean position, particle velocity $= v_m = \omega A$

At the extreme position, particle velocity $= 0$.

18. **Acceleration in SHM.** It is the rate of change of velocity of the particle at any instant. It is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}[-\omega A\sin(\omega t + \phi_0)]$$

$$= -\omega^2 A\cos(\omega t + \phi_0) = -\omega^2 x$$

i.e., $a \propto x$

The maximum value of acceleration of particle is called acceleration amplitude a_m. Thus

$$a_m = \omega^2 A$$

At the mean position, particle acceleration $= 0$

At the extreme position, particle acceleration,

$$a_m = \omega^2 A.$$

19. **Phase relationship between displacement, velocity and acceleration.** In SHM, the particle velocity is ahead of displacement by π/2 rad while acceleration is ahead of displacement by π rad.

20. **Energy of SHM.** If a particle of mass m executes SHM, then at a displacement x from mean position, the particle possesses potential and kinetic energy. At any displacement x,

Potential energy, $U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}kx^2$

Kinetic energy, $K = \frac{1}{2}m\omega^2(A^2 - x^2)$

$= \frac{1}{2}k(A^2 - x^2)$

Total energy,

$$E = U + K = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 m\nu^2 A^2.$$

If there is no friction, the total mechanical energy, $E = K + U$, of the system always remains constant even though K and U change.

21. **Motion of a massless loaded spring.** When a mass m is attached to a massless spring and pulled downwards, it executes SHM. If l is extension in the spring on attaching mass m and k is its force constant, then time period of SHM executed by the spring

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{m}{k}}$$

22. Spring cut i… spring const… constant of … period when… each part is … $T =$

23. Springs co… spring cons… then the sp… given by $\frac{1}{k}$ …

24. Springs c… spring c… parallel, t… is

25. Simple … point m… sible an… support… distance… point o… (l). Wh… positio… Time p…

26. Secon… pendu… length…

27. Moti… densi… is de…

28. Mot… the … a tu… exe… tim…

29. M… bo… ve…

22. **Spring cut into parts.** If we divide the spring of spring constant *k* into *n* equal parts, the spring constant of each part becomes *nk*. Hence the time period when the same mass *m* is suspended from each part is

$$T = 2\pi\sqrt{\frac{m}{nk}}$$

23. **Springs connected in series.** If two springs of spring constants k_1 and k_2 are connected in series, then the spring constant *k* of the combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{or} \quad k = \frac{k_1 k_2}{k_1 + k_2}$$

$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

24. **Springs connected in parallel.** If two springs of spring constants k_1 and k_2 are connected in parallel, then the spring constant *k* of combination is

$$k = k_1 + k_2 \qquad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

25. **Simple pendulum.** A simple pendulum is a heavy point mass suspended by a weightless, inextensible and a perfectly flexible string from a rigid support about which it can vibrate freely. The distance between the point of suspension and the point of oscillation is called *length of the pendulum* (*l*). When the metallic bob is displaced from mean position, it executes SHM.

Time period, $T = 2\pi\sqrt{\frac{l}{g}}$

26. **Second's pendulum.** A second's pendulum is a pendulum whose time period is two seconds. Its length is 99.3 cm.

27. **Motion of a liquid in a U-tube.** When a liquid of density ρ and contained in a U-tube upto height *h* is depressed, it executes SHM of time period,

$$T = 2\pi\sqrt{\frac{h}{g}}$$

28. **Motion of a body dropped in a tunnel dug along the diameter of earth.** When a body is dropped in a tunnel dug along the diameter of the earth, it executes SHM. If *R* is radius of the earth, then its time period is

$$T = 2\pi\sqrt{\frac{R}{g}}$$

29. **Motion of a body floating in a liquid.** When a body made of material of density ρ and total vertical length *l* floats in a liquid of density ρ, such that its length *h* is submerged in the liquid. If pressed into the liquid

$$T = 2\pi\sqrt{\frac{\rho l}{\sigma g}} = 2\pi\sqrt{\frac{h}{g}}$$

30. **Free oscillations.** If a body, capable of oscillation, is slightly displaced from its position of equilibrium and then released, it starts oscillating with a frequency of its own. Such oscillations are called free oscillations. The frequency with which a body oscillates is called *natural frequency* and is given by

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here a body continues to oscillate with constant amplitude and fixed frequency.

31. **Damped oscillations.** The oscillations in which amplitude decreases gradually with the passage of time are called damped oscillations.
The energy of a real oscillator decreases because a part of its mechanical energy is used in doing work against the frictional forces and is lost as heat. If the damping force is given by $F_d = -bv$, where *v* is the velocity of the oscillator and *b* is a damping constant, then the displacement of the oscillator is given by,

$$x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)$$

where ω′, the angular frequency of the damped oscillator, is given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

If the damping constant is small then $\omega' \approx \omega$, where ω is the angular frequency of the undamped oscillator. The mechanical energy *E* of the oscillator is given by

$$E(t) = \frac{1}{2}kA^2 e^{-bt/m}.$$

32. **Forced oscillations.** When a body oscillates under the influence of an external periodic force, not with its own natural frequency but the frequency of the external periodic force, its oscillations are said to be forced oscillations.

33. **Resonant oscillations.** It is a particular case of forced oscillations in which the frequency of the driving force is equal to the natural frequency of the oscillator itself and the amplitude of oscillations is greatest. Such oscillations are called resonant oscillations and phenomenon is called *resonance*.

34. **Coupled oscillations.** A system of two or more oscillators linked together in such a way that there is mutual exchange of energy between them is called a coupled oscillator. The oscillations of such a system are called coupled oscillations.

Scanned by CamScanner

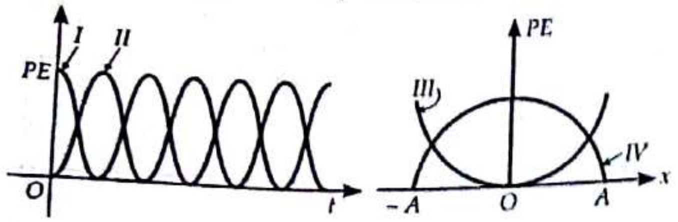
IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

1. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then

(*a*) $T_1 < T_2$ (*b*) $T_1 > T_2$
(*c*) $T_1 = T_2$ (*d*) $T_1 = 2T_2$ [IIT 01]

2. For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x



(*i*) (*ii*)

(*a*) I, III (*b*) II, IV
(*c*) II, III (*d*) I, IV [IIT 03]

3. A particle free to move along the x-axis has potential energy given by $U(x) = k[1 - \exp(-x)^2]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then

(*a*) at points away from the origin, the particle is in unstable equilibrium
(*b*) for any finite nonzero value of x, there is a force directed away from the origin
(*c*) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin
(*d*) for small displacements from $x = 0$, the motion is simple harmonic. [IIT 99]

4. A spring of force constant k is cut into two pieces, such that one piece is double the length of the other. Then, the long piece will have a force constant of

(*a*) $\frac{2}{3}k$ (*b*) $\frac{3}{2}k$
(*c*) $3k$ (*d*) $6k$ [IIT 99]

5. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is

(*a*) $\frac{k_1}{k_2}$ (*b*) $\sqrt{k_1/k_2}$
(*c*) $\frac{k_2}{k_1}$ (*d*) $\sqrt{\frac{k_2}{k_1}}$

6. An object of mass 0.2 kg executes simple harmonic motion along the x-axis with a frequency of $(25/\pi)$ Hz. At the position $x = 0.04$, the object has kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is

(*a*) 6 cm (*b*) 4 cm
(*c*) 8 cm (*d*) 2 cm

7. A simple pendulum has a time period T_1 when on the earth's surface; and T_2, when taken to a height R above the earth's surface (R is the radius of the earth). The value of T_2/T_1 is

(*a*) 1 (*b*) $\sqrt{2}$
(*c*) 4 (*d*) 2

8. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by

(*a*) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (*b*) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$
(*c*) $2\pi\sqrt{\frac{L}{g}}$ (*d*) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

9. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$, $(K = 1\,\text{m/s}^2)$ where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ $(g = 10\,\text{m/s}^2)$ is

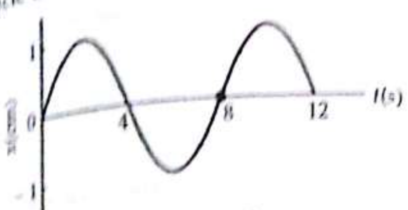
(*a*) $\frac{5}{6}$ (*b*) $\frac{6}{5}$
(*c*) 1 (*d*) $\frac{4}{5}$

10. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector is $\vec{a}$ correctly shown in

Scanned by CamScanner

[IIT 02]

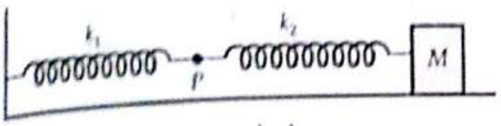
11. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ is



(a) $\frac{\sqrt{3}}{32}\pi^2$ cm/s² (b) $\frac{-\pi^2}{32}$ cm/s²

(c) $\frac{\pi^2}{32}$ cm/s² (d) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s² [IIT 09]

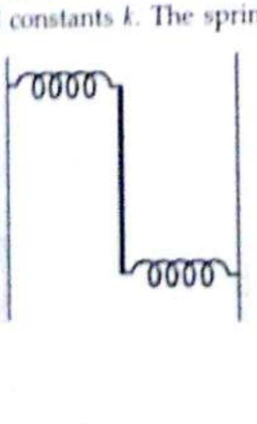
12. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

(c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$ [IIT 09]

13. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal constants k. The springs are fixed to



rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is

(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$ [IIT 09]

14. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is

(a) $\frac{1}{2\pi}\left(\frac{k - A\rho g}{M}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{M}\right)^{1/2}$

(c) $\frac{1}{2\pi}\left(\frac{k + \rho g L}{M}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{k + \rho g L}{M}\right)^{1/2}$ [IIT 90]

15. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

(a) $2\pi(m/k)^{1/2}$ (b) $2\pi\sqrt{\frac{m(YA + kL)}{YAk}}$

(c) $2\pi(mYA/kL)^{1/2}$ (d) $2\pi(mL/YA)^{1/2}$ [IIT 93]

16. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$

(c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$. [IIT 92]

Scanned by CamScanner

17. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$

(c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$ [IIT 2011]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

18. The function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents simple harmonic motion for which of the option (s) ?

(a) for all values of A, B and C $(C \neq 0)$

(b) $A = B$, $C = 2B$

(c) $A = -B$, $C = 2B$

(d) $A = B$, $C = 0$. [IIT 06]

19. Three simple harmonic motions in the same direction having the same amplitude a and same period are suspended. It each differs in phase from the next by 45°, then

(a) the resultant amplitude is $(1 + \sqrt{2})a$

(b) the phase of the resultant relative to the first is 90°

(c) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(d) the resulting motion is not simple harmonic. [IIT 99]

20. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is

(a) $\frac{f}{2}$ (b) f

(c) $2f$ (d) $4f$ [IIT 87]

21. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its

(a) maximum potential energy is 100 J

(b) maximum kinetic energy is 100 J

(c) maximum potential energy is 160 J

(d) maximum potential energy is zero. [IIT 89]

22. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is

(a) proportional to $1/\sqrt{a}$

(b) independent of a

(c) proportional to $\sqrt{a}$

(d) proportional to $a^{3/2}$. [IIT 98]

23. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\theta(|\theta| < \phi)$, the tension in the string and the velocity of the bob are T and v respectively. The following relations hold good under the above conditions :

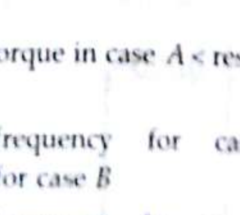
(a) $T\cos\theta = Mg$

(b) $T - Mg\cos\theta = \frac{Mv^2}{L}$

(c) The magnitude of the tangenial acceleration of the bob $|a_T| = g\sin\theta$

(d) $T = Mg\cos\theta$. [IIT 86]

24. A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius $R(< L)$ is attached at its centre to the free end of the rod. Consider two ways the disc is attached : (case A) The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement (s) is/are true ?



(a) restoring torque in case A = restoring torque in case B

(b) restoring torque in case A < restoring torque in case B

(c) angular frequency for case A > angular frequency for case B

(d) angular frequency for case A < angular frequency for case B. [IIT 2011]

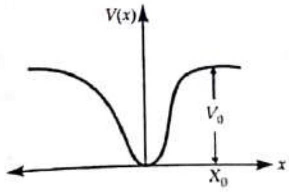
Scanned by CamScanner

SECTION

COMPREHENSION BASED QUESTIONS

PARAGRAPH FOR QUESTIONS 25 TO 27

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4 (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



25. If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$ (b) $E > 0$
(c) $V_0 > E > 0$ (d) $E > V_0$ [IIT 2010]

26. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$ [IIT 2010]

27. The acceleration of this particle for $|x| > X_0$ is

(a) proportional to V_0

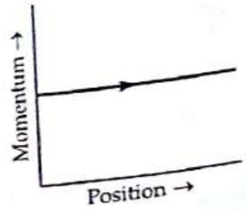
(b) proportional to $\frac{V_0}{mX_0}$

(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$

(d) zero [IIT 2010]

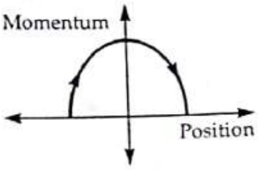
PARAGRAPH FOR QUESTION 28 TO 30

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such system, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

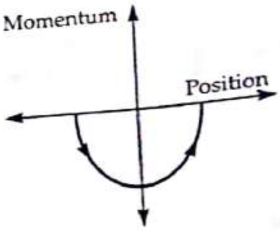


28. The phase space diagram for a ball thrown vertically up from ground is

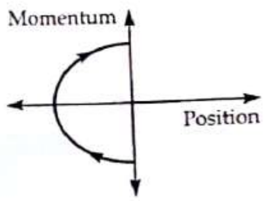
(a)



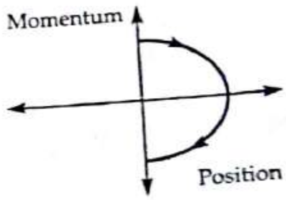
(b)



(c)

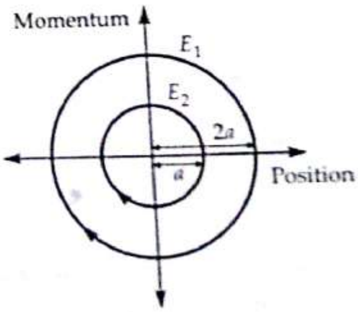


(d)



[IIT 2011]

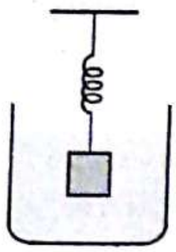
29. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



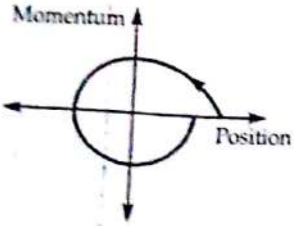
(a) $E_1 = \sqrt{2}\, E_2$ (b) $E_1 = 2\, E_2$
(c) $E_1 = 4\, E_2$ (d) $E_1 = 16\, E_2$ [IIT 2011]

Scanned by CamScanner

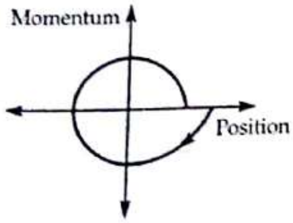
30. Consider the spring mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



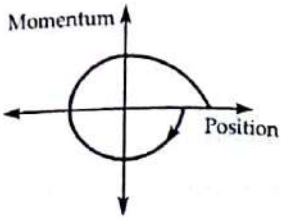
(a)



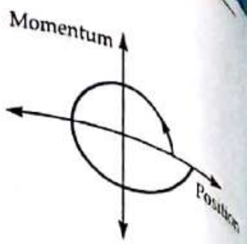
(b)



(c)



(d)



INTEGER TYPE ANSWER

[IIT 2010]

31. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is $4.9 \times 10^{-7}\ m^2$. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9\ Nm^{-2}$, find the value of n. [IIT 2010]

MATCH – MATRIX TYPE

32. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situations in column I with the characteristics in column II. [IIT 07]

Column I	Column II
(a) The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(b) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(c) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the objects keeps on decreasing.
(d) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e / R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

Scanned by CamScanner

Answers and Explanations

1. (a) $x = A\sin\omega t$

For $x = \dfrac{A}{2}$, $\quad \sin\omega T_1 = \dfrac{1}{2}$

$$\omega T_1 = \frac{\pi}{6} \quad \text{or} \quad T_1 = \frac{\pi}{6\omega}$$

For $x = A$, $\quad \sin\omega(T_1 + T_2) = 1$

$$\omega(T_1 + T_2) = \frac{\pi}{2}$$

$$T_1 + T_2 = \frac{\pi}{2\omega}$$

$$T_2 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega} = 2T_1.$$

2. (a) At $t = 0$, $x = A\cos 0 = A$. The particle is at extreme position and its P.E. must be maximum. Hence the correct options are I and III.

3. (d) $U(x) = k(1 - e^{-x^2})$

$$F = -\frac{dU}{dx} = -2kxe^{-x^2} = -2kx(1 - x^2 + \ldots)$$

For small x, $F = -2\,kx$.

This shows that the force is directed towards the origin and for smaller x, $F \propto x$. Hence the motion is simple harmonic.

4. (b) Force constant, $k = \dfrac{F}{x}$

The length of the long piece is $2x/3$.

So, its force constant is

$$k' = \frac{F}{2x/3} = \frac{3}{2}\frac{F}{x} = \frac{3}{2}k.$$

5. (d) $v_{max}(A) = v_{max}(B)$

$$\omega_1 A_1 = \omega_2 A_2$$

$$\sqrt{\frac{k_1}{m}}A_1 = \sqrt{\frac{k_2}{m}}A_2$$

$$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}.$$

6. (a) Total energy,

$$E = 2\pi^2 m\nu^2 A^2$$

$$0.5 + 0.4 = 2\pi^2 \times 0.2 \times \left(\frac{25}{\pi}\right)^2 A^2$$

$$A^2 = \frac{0.9}{0.4 \times (25)^2}$$

$$A = \frac{3}{2 \times 25} = \frac{3}{50}\ \text{m} = 6\ \text{cm}.$$

7. (d) $\dfrac{g'}{g} = \left(\dfrac{R}{R+R}\right)^2 = \dfrac{1}{4}$

As $\quad T \propto \dfrac{1}{\sqrt{g}}$

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{4}{1}} = 2.$$

8. (a) The effective value of g will be equal to the component of g normal to the inclined plane which is

$$g' = g\cos\alpha$$

$$\therefore \quad T = 2\pi\sqrt{\frac{L}{g'}} = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

9. (b) $\quad y = kt^2$

Velocity $\quad = \dfrac{dy}{dt} = 2kt$

Acceleration $= \dfrac{d^2y}{dt^2} = 2k = 2 \times 1 = 2\ \text{ms}^{-2}$

$$\therefore \quad g_2 = g_1 + 2 = 10 + 2 = 12\ \text{ms}^{-2}$$

As $\quad T = 2\pi\sqrt{\dfrac{l}{g}}$ or $T \propto \dfrac{1}{\sqrt{g}}$

$$\therefore \quad \frac{T_1^2}{T_2} = \frac{g_2}{g_1} = \frac{12}{10} = \frac{6}{5}.$$

10. (c) When the displacement of the bob is less than maximum, there will two component accelerations of the bob :

Transverse component $= \vec{a_T}$

Centripetal or radial component $= \vec{a_C}$

The resultant acceleration $\vec{a}$ will be along the diagonal of the parallelogram.

11. (d) From the x-t graph,

$$T = 8\ \text{s}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}\ \text{rad}, \quad t = \frac{4}{3}\ \text{s}$$

$$\omega t = \frac{\pi}{4} \times \frac{4}{3} = \frac{\pi}{3}\ \text{rad}$$

$$a = -\omega^2 A\sin\omega t$$

$$= -\left(\frac{\pi}{4}\right)^2 \times 1 \times \sin\frac{\pi}{3}$$

$$= -\frac{\sqrt{3}}{32}\pi^2\ \text{cm/s}^2.$$

Scanned by CamScanner

12. (*d*) Here $x_1 + x_2 = A$

As internal forces in the two springs are the same,

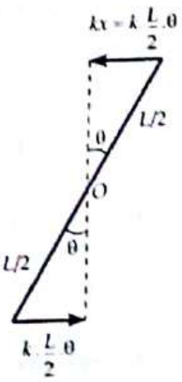
$$k_1 x_1 = k_2 x_2$$

or $$k_1 x_1 = k_2 (A - x_1)$$

or $$x_1 = \frac{k_2 A}{k_1 + k_2}.$$

13. (*c*) Restoring torque about O,

$$\tau = -2\left(k.\frac{L}{2}.\theta\right)\frac{L}{2} = -\frac{kL^2\theta}{2}$$



$$I = \frac{ML^2}{12}$$

Angular acceleration,

$$\alpha = \frac{\tau}{I} = \frac{-\frac{kL^2\theta}{2}}{\frac{ML^2}{12}}$$

or $$\alpha = -\frac{6k}{M}\theta = -\omega^2\theta$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}.$$

14. (*b*) Suppose the cylinder is given a downward push through a small distance *y*. Then

Restoring upward force set up in the spring $= -ky$

Additional upward force of buoyancy $= -Ay\rho g$

Total upward restoring force,

$$F = -(ky + Ay\rho g) = -(k + A\rho g)y$$

Clearly, $F \propto y$ and it acts towards equilibrium position. Hence motion of the cylinder is simple harmonic. Here spring factor $= k + A\rho g$

Inertia factor = Mass of cylinder = *M*

∴ Frequency of oscillation of the cylinder,

$$v = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{M}{k + A\rho g}}.$$

15. (*b*) Young's modulus of wire,

$$Y = \frac{F}{A}.\frac{L}{\Delta L}$$

∴ Stretching force, $$F = YA\frac{\Delta L}{L}$$

Force constant of wire, $$k_1 = \frac{F}{\Delta L} = \frac{YA}{L}$$

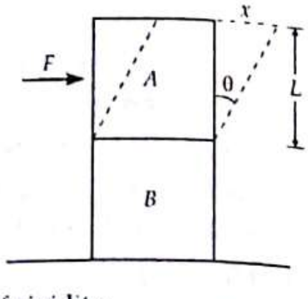
As the wire and the spring are connected in series, their effective force constant is

$$K = \frac{kk_1}{k + k_1} = \frac{k(YA/L)}{k + (YA/L)} = \frac{kYA}{kL + YA}$$

∴ Time period,

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m(kL + YA)}{kYA}}.$$

16. (*d*) When the force *F* is applied, the upper face of block *A* gets displaced through distance *x*.



Modulus of rigidity,

$$\eta = -\frac{F/A}{\theta}$$

$$= -\frac{F}{A\theta} = -\frac{F}{L^2\left(\frac{x}{L}\right)} = -\frac{F}{Lx}$$

Restoring force,

$$F = -\eta Lx \quad i.e., \; F \propto x$$

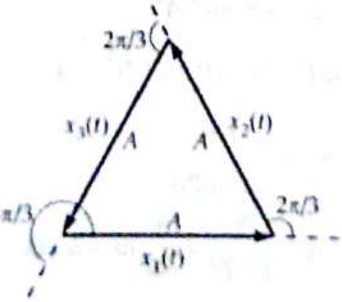
Hence the motion of *A* is simple harmonic with

$$k = \eta L$$

Time period of oscillation,

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M}{\eta L}}.$$

17. (*b*) Displacements $x_1(t)$ and $x_2(t)$ have amplitude *A* each, and phase difference $\frac{2\pi}{3}$. The third displacement $x_3(t)$ brings the mass to complete rest For this, $x_3(t)$ must have amplitude *A* and phase difference $4\pi/3$ with $x_1(t)$ as shown in the figure.



Scanned by CamScanner

18. (a), (b), (c)

$x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$

$x = A\left(\frac{1-\cos 2\omega t}{2}\right) + B\left(\frac{1+\cos 2\omega t}{2}\right) + \frac{C\sin 2\omega t}{2}$

(a) For $A = 0$, $B = 0$, $x = \frac{C}{2}\sin 2\omega t$

This represents SHM. Hence option (a) is correct.

(b) For $A = B$, $C = 2B$, $x = B + B\sin 2\omega t$

This represents SHM of amplitude B.

Hence option (b) is correct.

(c) For $A = -B$, $C = 2B$

$$x = B\cos 2\omega t + B\sin 2\omega t$$

$$= \sqrt{2}B\sin\left(2\omega t + \frac{\pi}{4}\right)$$

This represents SHM. Hence option (c) is correct.

(d) For $A = B$, $C = 0$, $x = A$.

This does not represent SHM.

19. (a), (c) Using the principle of superposition,

$y = y_1 + y_2 + y_3$

$= a\sin(\omega t + 45°) + a\sin\omega t + a\sin(\omega t - 45°)$

$= a[\sin(\omega t + 45°) + \sin(\omega t - 45°)] + a\sin\omega t$

$= 2a\sin\omega t\cos 45° + a\sin\omega t$

$= \sqrt{2}a\sin\omega t + a\sin\omega t$

$y = (1+\sqrt{2})a\sin\omega t$

(a) Amplitude of resultant motion $= (1+\sqrt{2})a$.

Hence option (a) is correct.

(b) Option (b) is incorrect because the phase of resultant motion relative to the first is 45° not 90°.

(c) $E \propto (\text{amplitude})^2$

$$\therefore \frac{E_{\text{resultant}}}{E_{\text{single}}} = \frac{(1+\sqrt{2})^2a^2}{a^2}$$

$$= 3 + 2\sqrt{2}$$

$$E_{\text{resultant}} = (3+2\sqrt{2})E_{\text{single}}$$

Hence option (c) is correct.

(d) Option (d) is incorrect as the resultant motion is simple harmonic.

20. (c) In one oscillation, the energy of an oscillator becomes twice kinetic and twice potential.

$\therefore$ Frequency of oscillation of K.E. $= 2f$.

21. (b), (c) Energy of oscillation of the particle,

$$= \frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2$$

$$= 100\text{ J}$$

Different energies at mean and extreme positions are shown below :

$x = 0$	$x = \pm A$
$K = 100$ J = Maximum	$K = 0$ J
$U = 60$ J = Minimum	$U = 160$ J = Maximum
$E = 160$ J = Constant	$E = 160$ J = Constant.

22. (a) $U(x) = k|x|^3$

$$\therefore \quad [k] = \frac{[U]}{[x^3]} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

$$T \propto (\text{mass})^x(\text{amplitude})^y(k)^z$$

$$\therefore \quad [M^0L^0T] = [M]^x[L]^y[ML^{-1}T^{-2}]^z$$

$$= [M^{x+z}L^{y-z}T^{-2z}]$$

Equating the powers, we get

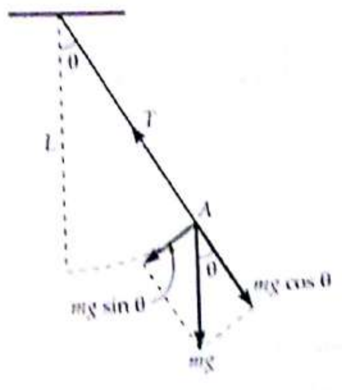
$-2z = 1$ or $z = -1/2$

$y - z = 0$ or $y = z = -1/2$

$$\therefore \quad T \propto (\text{amplitude})^{-1/2}$$

or $$T \propto \frac{1}{\sqrt{a}}.$$

23. (b), (c) As shown in the figure, $T - Mg\sin\theta$ provides the centripetal force.



$$T - Mg\sin\theta = \frac{Mv^2}{L}$$

Also, $$A_T = \frac{Mg\sin\theta}{M}$$

$$= g\sin\theta$$

Hence options (b) and (c) are correct.

Scanned by CamScanner

24. (*a*). (*d*) We use $\tau = I\alpha$

For case A : $\tau_A = I_A \alpha_A$

or $mg\left(\frac{l}{2}\sin\theta\right) + Mg(l\sin\theta) = \left(\frac{ml^2}{3} + \frac{MR^2}{2} + Ml^2\right)\alpha_A$

For case B : $\tau_B = I_B \alpha_B$

or $mg\left(\frac{l}{2}\sin\theta\right) + Mg(l\sin\theta) = \left(\frac{ml^2}{3} + Ml^2\right)\alpha_B$

As $\tau_A = \tau_B$ and $I_A > I_B$, so

$\alpha_A < \alpha_B \Rightarrow \omega_A < \omega_B$

25. (*c*) For motion to be periodic, it must reverse its path *i.e.*, K.E. should become zero for a finite value of x.

Now $E = K + U \Rightarrow K = E - U$

Given $U_{max} = V_0 \therefore K_{min} = E - V_0$

The particle will escape if $K_{min} > 0$

$\therefore E - V_0 < 0 \Rightarrow E < V_0$

Also, $E = K + U > 0$

26. (*b*) As $V = \alpha x^4$

$\Rightarrow [\alpha] = \frac{ML^2T^{-2}}{L^4} = ML^{-2}T^{-2}$

Hence, $\left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = \frac{1}{L}\left[\frac{M}{ML^{-2}T^{-2}}\right]^{1/2}$

$= M^0L^0T$

27. (*d*) For $|x| > X_0$, potential energy $= V_0$ (constant)

$\therefore F = -\frac{dV_0}{dx} = 0$

Hence acceleration is zero for $|x| > X_0$

28. (*d*) The momentum is initially positive as the ball moves up, becomes zero at the highest position and then becomes negative as the ball moves down.

29. (*c*) $\frac{E_1}{E_2} = \left(\frac{2a}{a}\right)^2 = 4 \Rightarrow E_1 = 4E_2$

30. (*b*) Amplitude of the mass oscillating in water should decrease with time. So options (*c*) and (*d*) are ruled out.

When the position of the mass is at one extreme end in the positive side (the topmost point), the momentum is zero. As the mass moves towards the mean position the momentum increases in the negative direction.

$\therefore$ Only option (*b*) is correct.

31.

0	0	0	4

Young's modulus,

$$Y = \frac{F}{A}\cdot\frac{l}{\Delta l}$$

Elongation, $\Delta l = \frac{Fl}{AY} = \frac{F}{\frac{AY}{L}} = \frac{F}{k}$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{YA}{ml}}$$

$\therefore Y = \frac{\omega^2 ml}{A} = \frac{140\times140\times0.1\times1}{4.9\times10^{-7}} = 4\times10^9$

$= n\times10^9\,Nm^{-2} \Rightarrow n = 4$

32. $a \to p$; $b \to q, r$; $c \to p$; $d \to r, q$

AIEEE

1. The function $\sin^2 \omega t$ represents

(*a*) a periodic but not simple harmonic motion with a period $2\pi/\omega$

(*b*) a periodic, but not simple harmonic motion with a period π/ω

(*c*) a simple harmonic motion with a period $2\pi/\omega$

(*d*) a simple harmonic motion with a period π/ω

[AIEEE 05]

2. Two simple harmonic motions are represented by the equations

$$y_1 = 0.1\sin(100\pi t + \pi/3)$$

and $$y_2 = 0.1\cos\pi t$$

The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(*a*) $-\pi/6$ (*b*) $\pi/3$

(*c*) $-\pi/3$ (*d*) $\pi/6$

[AIEEE 05]

Scanned by CamScanner

3. The displacement of an object attached to a spring and executing S.H.M. is given by

$$x = 2 \times 10^{-2} \cos \pi t \text{ (in m)}$$

The time at which the maximum speed first occurs is

(*a*) 0.5 s (*b*) 0.75 s
(*c*) 0.125 s (*d*) 0.25 s [AIEEE 07]

4. A point mass oscillates along the *X*-axis according to the relation

$$x = x_0 \cos(\omega t - \pi/4)$$

If acceleration of the particle is written as

$$a = a_0 \cos(\omega t + \delta),$$

then

(*a*) $a_0 = x_0\omega^2$; $\delta = -\pi/4$
(*b*) $a_0 = x_0\omega^2$; $\delta = \pi/4$
(*c*) $a_0 = x_0\omega^2$; $\delta = -\pi/4$
(*d*) $a_0 = x_0\omega^2$; $\delta = 3\pi/4$. [AIEEE 07]

5. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m s^{-1}. The period of oscillation is

(*a*) 0.01 s (*b*) 0.1 s
(*c*) 10 s (*d*) 100 s [AIEEE 06]

6. If a simple harmonic motion is represented by

$$\frac{d^2x}{dt^2} + \alpha x = 0,$$

its time period is

(*a*) $2\pi/\alpha$ (*b*) $2\pi/\sqrt{\alpha}$
(*c*) $2\pi\alpha$ (*d*) $2\pi\sqrt{\alpha}$ [AIEEE 05]

7. If *x*, *v* and *a* denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period *T*, then which of the following does not change with time ?

(*a*) $a^2T^2 + 4\pi^2v^2$ (*b*) aT/x
(*c*) $aT + 2\pi v$ (*d*) aT/v [AIEEE 09]

8. A coin is placed on a horizontal platform, which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

(*a*) for an amplitude of g^2/ω^2.
(*b*) for an amplitude of g/ω^2
(*c*) at the highest position of the platform
(*d*) at the mean position of the platform. [AIEEE 06]

9. If a spring has time period *T* and is cut into *n* equal parts, then the time period of each part will be

(*a*) $T\sqrt{n}$ (*b*) $T/\sqrt{n}$
(*c*) nT (*d*) T [AIEEE 02]

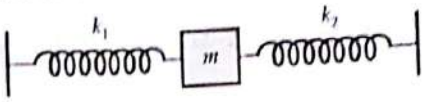
10. A mass *M* is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes S.H.M. of time period *T*. If the mass is increased by *m* the time becomes $5T/3$. Then the ratio of m/M is

(*a*) 3/5 (*b*) 25/9
(*c*) 16/9 (*d*) 5/3. [AIEEE 03]

11. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is *T*, then

(*a*) $T = t_1 + t_2$ (*b*) $T^2 = t_1^2 + t_2^2$
(*c*) $T^{-1} = t_1^{-1} + t_2^{-1}$ (*d*) $T^{-2} = t_1^{-2} + t_2^{-2}$ [AIEEE 04]

12. Two springs of force constant k_1 and k_2 are connected to a mass *m* as shown in the figure.



The frequency of oscillation of the mass is ν. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes

(*a*) ν/2 (*b*) ν/4
(*c*) 4ν (*d*) 2ν [AIEEE 07]

13. Two bodies *M* and *N* of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically, such that their maximum velocities are equal, then the ratio of the amplitude of *M* to that of *N* is

(*a*) k_1/k_2 (*b*) $\sqrt{k_1/k_2}$
(*c*) k_2/k_1 (*d*) $\sqrt{k_2/k_1}$ [AIEEE 03 ; IIT 88]

14. A child swinging on a swing in sitting position stands up. Then the time period of the swing will

(*a*) increase
(*b*) decrease
(*c*) remain the same
(*d*) increase, if the child is long and decrease, if the child is short. [DPMT 04 ; AIEEE 02]

15. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is

(a) 50% (b) 21%

(c) 30% (d) 10%. [AIEEE 03 ; AFMC 01 ; AIIMS 01]

16. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

(a) first increase and then decrease to the original value

(b) first decreases and then increase to the original value

(c) remain unchanged

(d) increase towards a saturation value. [AIEEE 05]

17. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is 4000/3 kg m^{-3}, what relationship between t and t_0 is true ?

(a) $t = t_0$ (b) $t = t_0/2$

(c) $t = 2t_0$ (d) $t = 4t_0$ [AIEEE 04]

18. The total energy of a particle executing simple harmonic motion is

(a) $\propto x$ (b) $\propto x^2$

(c) independent of x (d) $\propto x^{1/2}$ [AIEEE 04]

19. In a simple harmonic oscillator, at the mean position

(a) kinetic energy is minimum, potential energy is maximum

(b) both kinetic and potential energies are maximum

(c) kinetic energy is maximum, potential energy is minimum

(d) both kinetic and potential energies are minimum. [AIEEE 02]

20. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statements is true ?

(a) K.E. is maximum, when $x = 0$

(b) T.E. is zero, when $x = 0$

(c) K.E. is maximum, when x is maximum

(d) P.E. is maximum, when $x = 0$. [AIEEE 03]

21. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 8 J (b) 16 J

(c) 24 J (d) 32 J.

22. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work done to stretch it further by another 5 cm is

(a) 6.25 N m (b) 12.50 N m

(c) 18.75 N m (d) 25.00 N m

23. A particle of mass m executes S.H.M. with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $\pi^2 ma^2\nu^2$ (b) $\frac{1}{4}\pi^2 ma^2\nu^2$

(c) $4\pi^2 ma^2\nu^2$ (d) $2\pi^2 ma^2\nu^2$ [AIEEE]

24. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy ?

(a) 1/12 s (b) 1/6 s

(c) 1/4 s (d) 1/3 s. [AIEEE]

25. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos \omega t (\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to

(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$ [AIEEE]

26. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. Then

(a) $\omega_1 = \omega_2$

(b) $\omega_1 > \omega_2$

(c) $\omega_1 < \omega_2$, when damping is small and $\omega_1 > \omega_2$ when damping is large

(d) $\omega_1 < \omega_2$. [AIEEE]

27. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance [illegible] $(x_0 + A)$. If the maximum separation between them is $(x_0 + A)$, the phase difference between their motions is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$ [AIEEE 2011]

28. A mass M, attached to a horizontal spring, executes SHM with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is

(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$

(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$ [AIEEE 2011]

Answers and Explanations

1. (*b*) $\sin^2 \omega t = \frac{1}{2} - \left(\frac{1}{2}\right)\cos 2\omega t$

The function does not represent SHM but it is periodic with

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

Refer to the solution of Problem 14 on page 14.43.

2. (*a*) Refer to the solution of Problem 4 on page 14.44.

3. (*a*) Displacement, $x = 2 \times 10^{-2} \cos \pi t$

Velocity, $v = \frac{dx}{dt} = -2 \times 10^{-2} \pi \sin \pi t$

Velocity becomes maximum when

$$\sin \pi t = 1$$

or $$\pi t = \frac{\pi}{2}$$

or $$t = \mathbf{0.5\ s.}$$

4. (*d*) $x = x_0 \cos(\omega t - \pi/4)$

$$v = \frac{dx}{dt} = -x_0 \omega \sin(\omega t - \pi/4)$$

$$a = \frac{dv}{dt} = -x_0 \omega^2 \cos(\omega t - \pi/4)$$

$$= x_0 \omega^2 \cos[\pi + (\omega t - \pi/4)]$$

or $$a = x_0 \omega^2 \cos(\omega t + 3\pi/4)$$

Given : $a = a_0 \cos(\omega t + \delta)$

On comparing,

$$a_0 = x_0 \omega^2,\ \delta = \frac{3\pi}{4}.$$

5. (*a*) $v_{max} = \omega A = \frac{2\pi}{T} A$

$$\therefore \quad T = \frac{2\pi A}{v_{max}} = \frac{2 \times 22 \times 7 \times 10^{-3}}{7 \times 4.4} = \mathbf{0.01\ s.}$$

6. (*b*) Refer to the solution of Problem 13 on page 14.43.

7. (*b*) $\frac{aT}{x} = \frac{\omega^2 x T}{x} = \frac{4\pi^2}{T^2} \times T$

$$= \frac{4\pi^2}{T} = \text{constant.}$$

8. (*b*) The coin will remain in contact with the platform if a_{max} does not exceed g *i.e.*, a_{max} is at the most equal to g.

$$\therefore \quad a_{max} = g$$

or $$a\omega^2 = g$$

or $$a = g/\omega^2$$

9. (*b*) $T' = T/\sqrt{n}$.

Refer to the solution of Problem 3 on page 14.44.

10. (*c*) Refer to the solution of Problem 8 on Page 14.45.

11. (*b*) Refer to the solution of Problem 10 on Page 14.45.

12. (*d*) Initial frequency,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{M}}$$

When both k_1 and k_2 are made four times their original values,

$$\nu' = \frac{1}{2\pi}\sqrt{\frac{4(k_1 + k_2)}{M}} = 2\nu.$$

13. (*d*) $\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$.

Refer to the solution of Problem 9 on Page 14.45.

Scanned by CamScanner

14. (*b*) Refer to the solution of Problem 21 on Page 14.39.

15. (*d*) Refer to the solution of Problem 15 on Page 14.43.

16. (*a*) Refer to the solution of Problem 2 on Page 14.44.

17. (*c*) $t = 2t_0$. Refer to the solution of Problem 7 on Page 14.45.

18. (*c*) $E = 2\pi^2 m v^2 A^2$

Clearly, the total energy is independent of displacement x.

19. (*c*) At the mean position, the kinetic energy is maximum and potential energy is minimum.

20. (*a*) The K.E. of a simple harmonic oscillator is maximum when $x = 0$.

21. (*a*) At $x_1 = 5$ cm,

$$U_1 = \frac{1}{2}kx_1^2$$

$$= \frac{1}{2} \times 800 \times (0.05)^2 = 1 \text{ J}$$

At $x_2 = 15$ cm,

$$U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2} \times 800 \times (0.15)^2 = 9 \text{ J}.$$

$$\therefore \quad W = U_2 - U_1 = 9 - 1$$

$$= \mathbf{8\ J}.$$

22. (*c*) $W = \frac{1}{2}k(x_2^2 - x_1^2)$

$$= \frac{1}{2} \times 5 \times 10^3 \times [(0.10)^2 - (0.05)^2]$$

$$= \mathbf{18.75\ Nm}.$$

23. (*a*) K.E. of a simple harmonic oscillator at any instant t,

$$K = \frac{1}{2}ma^2\omega^2 \sin^2 \omega t$$

The average value of $\sin^2 \omega t$ over a cycle is $\frac{1}{2}$.

$$\therefore \quad K_{av} = \frac{1}{2}ma^2\omega^2 \times \frac{1}{2}$$

$$= \frac{1}{4}ma^2(2\pi v)^2 = \pi^2 m a^2 v^2.$$

24. (*b*) K.E. = 75% of total energy

$$\frac{1}{2}k(a^2 - y^2) = \frac{75}{100} \times \frac{1}{2}ka^2$$

$$a^2 - y^2 = \frac{3}{4}a^2$$

$$y^2 = \frac{1}{4}a^2$$

or $$y = \frac{a}{2}$$

For a body starting from mean position,

$$y = a \sin \omega t$$

or $$\frac{a}{2} = a \sin \omega t$$

$$-\sin \omega t = \frac{1}{2}$$

or $$\omega t = \frac{\pi}{6}$$

or $$\frac{2\pi}{T}t = \frac{\pi}{6}$$

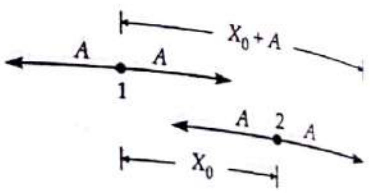
or $$t = \frac{T}{12} = \frac{2}{12} = \frac{1}{6} \text{ s}.$$

25. (*b*) $y \propto \frac{1}{m(\omega_0^2 - \omega^2)}$.

Refer to the solution of Problem 5 on Page 14.44.

26. (*a*) Only in case of resonance, both the amplitude and energy of oscillation are maximum. Hence, $\omega_1 = \omega_2$.

27. (*a*) When the maximum separation is $X_0 + A$ one particle is at mean position and the other is at the extreme position. So phase difference $= \pi/2$



28. (*d*) Energy of the simple harmonic oscillator is constant.

$$\therefore \quad \frac{1}{2}M\omega A_1^2 = \frac{1}{2}(M + m)\omega^2 A_2^2$$

$$\Rightarrow \quad \frac{A_1^2}{A_2^2} = \frac{M + m}{M}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \left(\frac{M + m}{M}\right)^{1/2}$$

Scanned by CamScanner

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. A particle executing simple harmonic motion along y-axis has its motion described by the equation $y = A\sin(\omega t) + B$. The amplitude of the simple harmonic motion is

(a) A (b) B
(c) $A + B$ (d) $\sqrt{A+B}$ [IPUEE 05]

2. What is the maximum acceleration of the particle executing the SHM, $y = 2\sin\left[\frac{\pi t}{2} + \phi\right]$, where y is in cm.

(a) $\frac{\pi}{2}$ cm/s^2 (b) $\frac{\pi^2}{2}$ cm/s^2
(c) $\frac{\pi}{4}$ cm/s^2 (d) $\frac{\pi}{4}$ cm/s^2 [DCE 03]

3. When the maximum K.E. of a simple pendulum is K then what is its displacement (in terms of amplitude a) when its K.E. is $K/2$?

(a) $a/\sqrt{2}$ (b) $a/2$
(c) $a/\sqrt{3}$ (d) $a/3$ [DCE 07]

4. The displacement of an SHM doing oscillation when K.E. = P.E. (amplitude = 4 cm) is

(a) $2\sqrt{2}$ cm (b) 2 cm
(c) $\frac{1}{\sqrt{2}}$ cm (d) $\sqrt{2}$ cm [IPUEE 2K]

5. A particle is executing SHM at mid point of mean position and extremity. What is the potential energy in terms of total energy (E) ?

(a) $\frac{E}{4}$ (b) $\frac{E}{16}$
(c) $\frac{E}{2}$ (d) $\frac{E}{8}$ [DCE 06]

6. If the frequency of oscillations of a particle doing SHM is n, the frequency of kinetic energy is

(a) $2n$ (b) n
(c) $n/2$ (d) none of these. [DCE 97]

7. Natural length of spring is 60 cm and its spring constant is 4000 N/m. A mass of 20 kg is hung from it. The extension produced in the spring (take $g = 9.8\,\text{m/s}^2$) is

(a) 4.9 cm (b) 0.49 cm
(c) 9.4 cm (d) 0.94 cm [DCE 04]

8. A spring (spring constant = k) is cut into 4 equal parts and two parts are connected in parallel. What is the effective spring constant ?

(a) $4k$ (b) $16k$
(c) $8k$ (d) $6k$. [DCE 07]

9. A mass m is suspended from a spring. Its frequency of oscillation is f. The spring is cut into two halves and the same mass is suspended from one of the two pieces of the spring. The frequency of oscillation of mass will be

(a) $\sqrt{2}\,f$ (b) $f/2$
(c) f (d) $2f$. [DCE 02, 06]

10. Two simple motions are represented by

$$y_1 = 5(\sin 2\pi t + \sqrt{3}\cos 2\pi t)$$

$$y_2 = 5\sin(2\pi t + \pi/4)$$

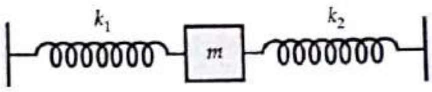
The ratio of the amplitude of two SHM's is

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : $\sqrt{3}$. [DCE 09]

11. If a particle is oscillating on the same horizontal plane on the ground,

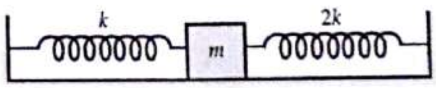
(a) it has only kinetic energy but no potential energy
(b) it has only potential energy but no kinetic energy
(c) it has both kinetic energy and potential energy.
(d) none of these. [DCE 08]

12. If both spring constants k_1 and k_2 are increased to $4k_1$ and $4k_2$ respectively, then new frequency in terms of original frequency f is



(a) f (b) $2f$
(c) $f/2$ (d) $4f$ [DCE 98]

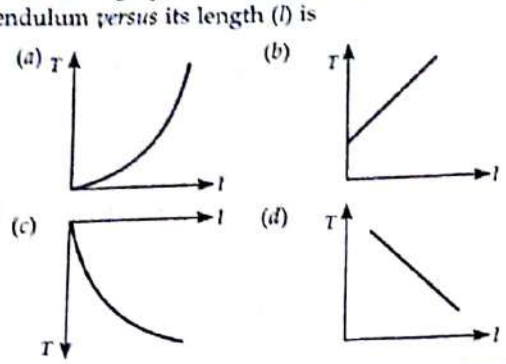
13. Frequency of oscillation is proportional to



(a) $\sqrt{\frac{3k}{4}}$ (b) $\sqrt{\frac{k}{m}}$
(c) $\sqrt{\frac{2k}{m}}$ (d) $\sqrt{\frac{m}{3k}}$ [DCE 2K]

Scanned by CamScanner

14. The graph of time period (*T*) of a simple pendulum *versus* its length (*l*) is



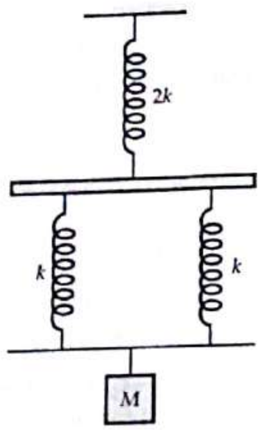
[DCE 98]

15. If the length of simple pendulum is tripled, what will be its new time period in terms of original period *T* ?

(*a*) 0.7 *T* (*b*) 1.73 *T*

(*c*) *T*/2 (*d*) *T* [DCE 08]

16. The spring constants of three springs connected to a mass *M* are shown in figure. When mass *M* oscillates, what are the effective spring constant and the time period of vibration ?



(*a*) $4k,\ 2\pi\sqrt{\frac{M}{4k}}$

(*b*) $3k,\ 2\pi\sqrt{\frac{M}{3k}}$

(*c*) $2k,\ 2\pi\sqrt{\frac{M}{2k}}$

(*d*) none of these. [DCE 08]

17. The ratio of frequencies of two pendulums are 2 : 3, then their lengths are in ratio

(*a*) $\sqrt{2/3}$ (*b*) $\sqrt{3/2}$

(*c*) 4/9 (*d*) 9/4. [DCE 05]

18. What is time period of a pendulum hanged in a satellite ? (*T* is time period on earth)

(*a*) zero (*b*) *T*

(*c*) infinite (*d*) $T/\sqrt{6}$. [IPUEE 06]

19. A simple pendulum is attached to the roof of a lift. If time period of oscillation, when the lift is stationary is *T*, then frequency of oscillation when the lift falls freely, will be

(*a*) zero (*b*) *T*

(*c*) $1/T$ (*d*) none of these. [DCE 02]

20. There is a simple pendulum hanging from the ceiling of a lift. When the lift is standstill, the time period of the pendulum is *T*. If the resultant acceleration becomes $g/4$, then the new time period of the pendulum is

(*a*) 0.8 *T* (*b*) 0.25 *T*

(*c*) 2 *T* (*d*) 4 *T* [DCE 04]

21. The mass and the radius of a planet are twice that of earth. Then, period of oscillation of a second pendulum on that planet will be

(*a*) $\frac{1}{\sqrt{2}}$ s (*b*) $2\sqrt{2}$ s

(*c*) 2 s (*d*) $\frac{1}{2}$ s. [DCE 02]

22. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical ?

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90° [IPUEE 04]

23. The length of the second's pendulum is decreased by 0.3 cm when it is shifted to Chennai from London. If the acceleration due to gravity at London is 981 cm/s², the acceleration due to gravity at Chennai (assume $\pi^2 = 10$) is

(*a*) 981 cm/s² (*b*) 978 cm/s²

(*c*) 984 cm/s² (*d*) 975 cm/s². [DCE 05]

24. A pendulum suspended from the roof of an elevator at rest has a time period T_1. When the elevator moves up with an acceleration *a* its time period becomes T_2, when the elevator moves down with an acceleration *a*, its time period becomes T_3, then

(*a*) $T_1 = \sqrt{T_2 T_3}$ (*b*) $T_1 = \sqrt{T_2^2 + T_3^2}$

(*c*) $T_1 = \frac{T_2 T_3 \sqrt{2}}{\sqrt{T_2^2 + T_3^2}}$ (*d*) none of these. [DCE 08]

25. If a simple pendulum of length '*L*' has maximum angular displacement. Then the maximum kinetic energy of bob of mass '*M*' is

(*a*) $\frac{1}{2}\frac{ML}{g}$ (*b*) $\frac{Mg}{2L}$

(*c*) $MgL(1-\cos\alpha)$ (*d*) $MgL\sin\alpha/2$ [DCE 09]

26. In case of a forced vibration, the resonance wave becomes very sharp when the

(*a*) applied periodic force is small

(*b*) quality factor is small

(*c*) damping force is small

(*d*) restoring force is small. [DCE 01]

SECTION

1. (*a*) For $y = A\sin($

$y = 2\sin($

2. (*b*) $y = A\sin($

$A = 2$ cm,

∴ $a_{max} = \omega^2 A =$

3. (*a*) Here K_{max}

Let $\frac{1}{2}ky^2 = \frac{K}{2}$

4. (*a*) When K.E

5. (*a*) At $y = \frac{A}{2}$

$P.E = \frac{1}{2}k$

6. (*c*) In one os twice becomes ki of K.E. = 2*n*.

7. (*a*) In equili

8. (*c*) Spring When two parts

9. (*a*) $f =$

When sprin of each half is

10. (*c*) y_1

11. (*c*) Th energies.

Scanned by CamScanner

Answers and Explanations

1. (a) For $y = A\sin(\omega t) + B$ the amplitude is A

2. (b) $y = 2\sin\left(\frac{\pi t}{2} + \phi\right)$

$y = A\sin(\omega t + \phi)$

$A = 2$ cm, $\omega = \frac{\pi}{2}$ rad/s

∴ $a_{max} = \omega^2 A = \frac{\pi^2}{2}$ cm/s^2.

3. (d) Here $K_{max} = \frac{1}{2}ka^2 = K$

Let $\frac{1}{2}ky^2 = \frac{K}{2} = \frac{1}{2} \times \frac{1}{2}ka^2$ or $y = \frac{a}{\sqrt{2}}$.

4. (d) When K.E. = P.E.,

$$y = \frac{a}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \mathbf{2\sqrt{2}\ cm.}$$

5. (d) At $y = \frac{A}{2}$,

$$\text{P.E} = \frac{1}{2}ky^2 = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}\cdot\frac{1}{2}kA^2 = \frac{1}{4}E.$$

6. (c) In one oscillation, the energy of the oscillation twice becomes kinetic and twice potential. Frequency of K.E. $= 2n$.

7. (d) In equilibrium, $kx = mg$

∴ $x = \frac{mg}{k} = \frac{20 \times 9.8}{9000} = 0.049\text{ m} = \mathbf{4.9\ cm.}$

8. (c) Spring constant of each one-fourth part $= 4k$. When two parts are connected in parallel,

$$k_p = 4k + 4k = \mathbf{8k}.$$

9. (d) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

When spring is cut into two halves, spring constant of each half is $2k$.

∴ $f' = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \sqrt{2}f.$

10. (c) $y_1 = 5 \times 2[\sin 2\pi t \cos \pi/3 + \sin \pi/3 \cos 2\pi t]$

$= 10\sin(2\pi t + \pi/3)$

$y_2 = 5\sin(2\pi t + \pi/4)$

∴ $\frac{A_1}{A_2} = \frac{10}{5} = \mathbf{2 : 1}.$

11. (c) The particle has both kinetic and potential energies.

12. (b) $F = F_1 + F_2 = -k_1x - k_2x = -(k_1 + k_2)x$

$k_{eff} = k_1 + k_2$

∴ $f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

When spring constants are increased four times,

$$f' = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}} = \mathbf{2f}.$$

13. (a) $k_{eff} = k_1 + k_2 = k + 2k = 3k$

$f = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

∴ $f \propto \sqrt{\frac{3k}{m}}$.

14. (a) For a simple pendulum,

$T \propto \sqrt{l}$ ∴ $T^2 \propto l$

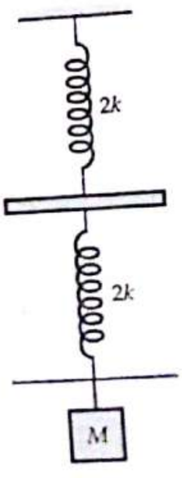
Graph between T and l with be parabola symmetric about T-axis. Hence option (a) correct.

15. (b) $T \propto \sqrt{l}$

and $T' \propto \sqrt{3l}$

∴ $T' = \sqrt{3l} = \mathbf{1.732\ T}.$

16. (d) In lower part, there are two springs (each of force constant k) connected in parallel.



∴ $k_p = k + k = 2k$

Now we have two springs (each of force constant $2k$) connected in series.

∴ $k_s = \frac{k_1k_2}{k_1 + k_2} = \frac{2k \times 2k}{2k + 2k} = \mathbf{k}$

Hence $\mathbf{T = 2\pi\sqrt{\frac{M}{k}}}$.

17. (d) $\frac{v_1}{v_2} = \sqrt{\frac{l_2}{l_1}}$

or $\frac{l_1}{l_2} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{3}{2}\right)^2 = \mathbf{\frac{9}{4}}.$

18. (c) In a satellite, $g = 0$

∴ $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{0}} = \infty$

Scanned by CamScanner

19. (*a*) In a freely falling lift,

$$g = 0$$

$$\therefore \quad \nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{0}{l}} = 0$$

20. (*c*) $T = 2\pi\sqrt{\frac{l}{g}}$

$$T' = 2\pi\sqrt{\frac{l}{g/4}} = 2T.$$

21. (*b*) On earth,

$$g = \frac{GM}{R^2}$$

On planet,

$$g' = \frac{G \times 2M}{(2R)^2} = \frac{g}{2}$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{g/2}} = \sqrt{2}$$

$$T' = \sqrt{2} \times T = \sqrt{2} \times 2 = 2\sqrt{2}\ \text{s}.$$

22. (*d*) In mean position,

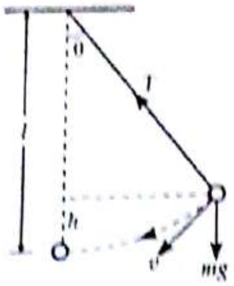
$$T = 3mg$$

Net force,

$$T - mg = \frac{mv^2}{l}$$

or $\quad 3mg - mg = \frac{mv^2}{l}$

$$\therefore \quad v = \sqrt{2gl}$$



Let maximum angular displacement with vertical be θ. Then

$$\frac{1}{2}mv^2 = mgh = mg(l - l\cos\theta)$$

or $\quad \frac{1}{2}m \times 2gl = mgl - mgl\cos\theta$

or $\quad \cos\theta = 0 \quad$ or $\quad \theta = 90°.$

23. (*b*) $\quad T = 2\pi\sqrt{\frac{l}{g}} \quad$ or $\quad l = \frac{gT^2}{4\pi^2}$

For second's pendulum, $T = 2$ s

$$\therefore \quad l = \frac{4g}{4\pi^2} = \frac{g}{\pi^2}$$

For London, $\quad l = \frac{g}{\pi^2} = \frac{981}{10} = 98.1$ cm

For Chennai, $\quad l - 0.3 = \frac{g'}{\pi^2} = \frac{g'}{10}$

or $\quad g' = 10l - 3$

$$= 10 \times 98.1 - 3 = 978\ \text{cm/s}^2$$

24. (*c*) $\quad T_1 = 2\pi\sqrt{\frac{l}{g}} \quad$ or $\quad \frac{4\pi^2 l}{T_1^2} = g$

$$T_2 = 2\pi\sqrt{\frac{l}{g + a}} \quad \text{or} \quad \frac{4\pi^2 l}{T_2^2} = g + a$$

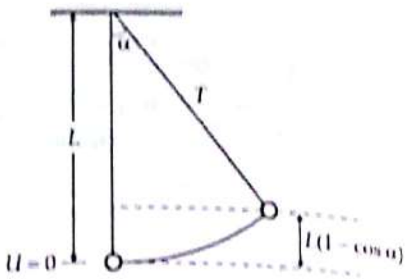
$$T_3 = 2\pi\sqrt{\frac{l}{g - a}} \quad \text{or} \quad \frac{4\pi^2 l}{T_3^2} = g - a$$

$$\therefore \quad \frac{4\pi^2 l}{T_2^2} + \frac{4\pi^2 l}{T_3^2} = 2g = 2\,\frac{4\pi^2 l}{T_1^2}$$

or $\quad T_1 = \frac{\sqrt{2}\,T_2 T_3}{\sqrt{T_2^2 + T_3^2}}.$

25. (*c*) $K_{max} = U_{max}$

$$= Mgh_{max} = Mgl(1 - \cos\alpha)$$



26. (*c*) Lasser the damping force, more sharp is the resonance peak.

AIIMS Entrance Exam

1. Which of the following functions represents a simple harmonic oscillation ?

(*a*) $\sin\omega t - \cos\omega t$

(*b*) $\sin\omega t + \sin 2\omega t$

(*c*) $\sin\omega t - \sin 2\omega t$

(*d*) $\sin^2\omega t.$ [AIIMS 05]

2. A particle executes simple harmonic motion with an amplitude *a*. The period of oscillation is *T*. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(*a*) $T/12$ (*b*) $T/8$

(*c*) $T/4$ (*d*) $T/2$ [AIIMS 07]

3. The periodic time of a body executing S.H.M. is 1 s. After how much interval from time $t = 0$, its displacement will be half of its amplitude ?

(a) 1/2 s (b) 1/3 s

(c) 1/4 s (d) 1/6 s [AIIMS 95]

4. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is

(a) 0.5π (b) π

(c) 0.707π (d) 0.61 m. [AIIMS 07]

5. A particle executes simple harmonic motion with an angular velocity of 3.5 rad s^{-1} and maximum velocity acceleration 7.5 m s^{-2} respectively. The amplitude of oscillations is

(a) 0.28 m (b) 0.36 m

(c) 0.707 m (d) zero. [AIIMS 99]

6. If a simple pendulum oscillates with an amplitude of 50 mm and time period of 2 s, then its maximum velocity is

(a) 0.10 ms^{-1} (b) 0.16 ms^{-1}

(c) 0.24 ms^{-1} (d) 0.32 ms^{-1} [AIIMS 98]

7. For a particle executing simple harmonic motion, which of the following statements is not correct ?

(a) total energy of the particle always remains the same

(b) restoring force is always directed towards a fixed point

(c) restoring force is maximum at the extreme positions

(d) acceleration of the particle is maximum at the equilibrium position. [AIIMS 99]

8. A spring 40 mm long is stretched by the application of a force. If 10 N force is required to stretch the spring through 1 mm, then work done in stretching the spring through 40 mm is

(a) 84 J (b) 48 J

(c) 24 J (d) 8 J. [AIIMS 98]

9. If a spring of mass 30 kg has spring constant of 15 Nm^{-1}, then its time period is

(a) 2π s (b) $2\sqrt{2}\,\pi$ s

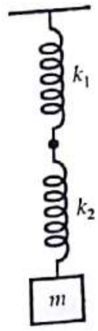
(c) $2\sqrt{2\pi}$ s (d) $2\sqrt{2}$ s [AIIMS 96]

10. If the period of oscillation of mass m suspended from a spring is 2 s, then the period of mass $4m$ will be

(a) 1 s (b) 4 s

(c) 8 s (d) 16 s. [AIIMS 98]

11. The frequency of oscillation of the springs shown in the figure will be

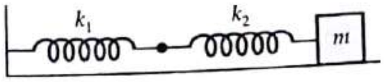


(a) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{(k_1+k_2)m}{k_1k_2}}$

(d) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)m}}$ [AIIMS 01]

12. Two springs are connected to a block of mass m placed on a frictionless surface as shown below :

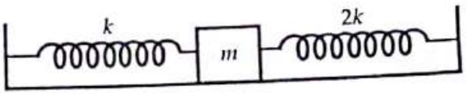


If both the springs have a spring constant k, the frequency of oscillation of the block is

(a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$ [AIIMS 04]

13. Two springs of force constants k and $2k$ are connected to a mass m as shown below :

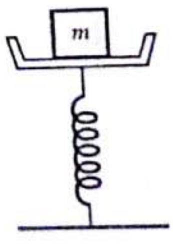


The frequency of oscillation of the mass is

(a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$. [AIIMS 03]

14. A mass of 2 kg is put on a flat pan attached to a vertical spring fixed on the ground shown in the figure. The mass of pan and the spring is negligible. When pressed slightly and released, the mass executes S.H.M. The spring constant of the spring is 200 Nm^{-1}. What should be the minimum amplitude of the motion, so that the mass gets detached from the pan ?



Take $g = 10$ ms^{-2}.

(a) 4 cm (b) 8 cm

(c) 10 cm

(d) Any value less than 12 cm. [AIIMS 07]

Scanned by CamScanner

15. A horizontal platform with an object placed on it is executing S.H.M. in the vertical direction. The amplitude of oscillation is 3.92×10^{-3} m. What must be the least period of these oscillations, so that the object is not detached from the platform ?

(*a*) 0.1256 s (*b*) 0.1356 s

(*c*) 0.1456 s (*d*) 0.1556 s. [AIIMS 99]

16. If the metal bob of a simple pendulum is replaced by wooden bob, then its time period will

(*a*) increase (*b*) decrease

(*c*) remain the same (*d*) first (*a*) then (*b*)

[AIIMS 98]

17. The time period of a simple pendulum on a satellite, orbiting around the earth, is

(*a*) infinite (*b*) zero

(*c*) 84.6 min (*d*) 24 hours [AIIMS 97]

18. A simple pendulum has a time period *T*. The pendulum is completely immersed in a non-viscous liquid, whose density is 1/10th of that of the material of the bob. The time period of the pendulum immersed in the liquid is

(*a*) T (*b*) $T/10$

(*c*) $\sqrt{\frac{9}{10}}\,T$ (*d*) $\sqrt{\frac{10}{9}}\,T$. [AIIMS 2K]

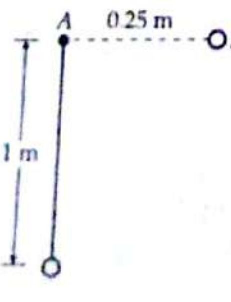
19. A lightly damped oscillator with a frequency ν is set in motion by a harmonic driving force of frequency ν′. When ν′ < ν, then response of the oscillator is controlled by

(*a*) spring constant (*b*) inertia of the mass

(*c*) oscillator frequency

(*d*) damping coefficient. [AIIMS 96]

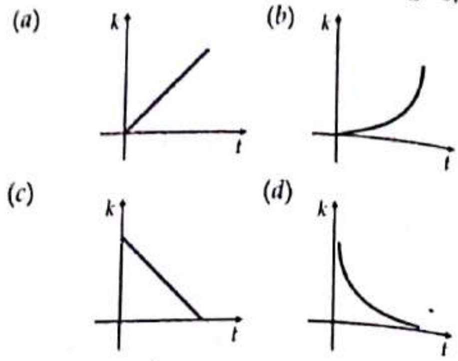
20. A simple pendulum has a bob suspended by inextensible thread of length 1 metre from a point *A* of suspension. At the extreme position of oscillation, the thread is suddenly caught by the peg at a point *B* distant 0.25 m from *A* and the bob begins to oscillate in the new condition. The change in frequency of oscillation of the pendulum is approximately ($g = 10\,\text{ms}^{-2}$) given by



(*a*) $\frac{\sqrt{10}}{2}$ Hz (*b*) $\frac{1}{4\sqrt{10}}$ Hz

(*c*) $\frac{\sqrt{10}}{3}$ Hz (*d*) $\frac{1}{\sqrt{10}}$ Hz. [AIIMS 08]

21. Which of the following graphs depicts spring constant *k* versus length *l* of the spring correctly ?



[AIEEE 2009]

22. Let T_1 and T_2 be the time periods of springs *A* and *B* when mass *M* is suspended from one end of each spring. If both springs are taken in series combination, the time period is *T*, then

(*a*) $T = T_1 + T_2$ (*b*) $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

(*c*) $T^2 = T_1^2 + T_2^2$ (*d*) $\frac{1}{T^2} = \frac{1}{T_1^1} + \frac{1}{T_2^2}$

[AIEEE 2010]

23. A ball is suspended by a thread of length *L* at the point *O* on a wall which is inclined to the vertical by angle α. The thread with the ball is displaced by a small angle β away from the vertical and also away from the wall. If the ball is released, the period of oscillation of the pendulum when β > α will be

(*a*) $\sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\frac{\alpha}{\beta}\right]$ (*b*) $\sqrt{\frac{L}{g}}\left[\pi - 2\sin^{-1}\frac{\beta}{\alpha}\right]$

(*c*) $\sqrt{\frac{L}{g}}\left[2\sin^{-1}\frac{\alpha}{\beta} - \pi\right]$

(*d*) $\sqrt{\frac{L}{g}}\left[2\sin^{-1}\frac{\alpha}{\beta} + \pi\right]$

[AIIMS 2009]

24. A particle executes simple harmonic motion of period *T* and amplitude *l* along a rod *AB* of length 2*l*. The rod *AB* itself executes simple harmonic motion of

Scanned by CamScanner

the same period and amplitude in a direction perpendicular to its length. Initially, both the particle and the rod are in their mean positions. The path traced out by the particle will be

(a) a circle of radius l

(b) a straight line inclined at $\frac{\pi}{4}$ to the rod

(c) an ellipse

(d) a figure of eight. [AIIMS 2009]

✓ Assertions and Reasons

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both assertion and reason are false.

25. Assertion. In S.H.M., the motion is to and fro and periodic.

Reason. Velocity of the particle,

$$v = \omega\sqrt{a^2 - x^2}$$

where x is displacement. [AIIMS 02]

26. Assertion. In simple harmonic motion, the velocity is maximum, when the acceleration is minimum.

Reason. Displacement and velocity in simple harmonic motion differ in phase by $\pi/2$ [AIIMS 02]

27. Assertion. The amplitude of an oscillating pendulum decreases gradually with time.

Reason. The frequency of the pendulum decreases with time. [AIIMS 03]

28. Assertion. The time period of a pendulum on a satellite orbiting the earth is infinity.

Reason. The time period of a pendulum is inversely proportional to $\sqrt{g}$. [AIIMS 02]

29. Assertion. Water in a U-tube executes S.H.M. The time period for mercury filled upto the same height in the U-tube be greater than that in case of water.

Reason. The amplitude of an oscillating pendulum goes on increasing. [AIIMS 07]

30. Assertion. Resonance is a special case of forced vibration in which the nature and frequency of vibration of the body is the same as the impressed frequency and the amplitude of forced vibration, is maximum.

Reason. The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force. [AIIMS 94]

31. Assertion. The bob of a simple pendulum is a ball full of water, if a fine hole is made in the bottom of the ball, the time period first increases and then decreases.

Reason. As water flows out of the bob, the weight of bob decreases. [AIIMS 2011]

Answers and Explanations

1. (a) $y = \sin\omega t - \cos\omega t$

$$\frac{dy}{dt} = \omega\cos\omega t + \omega\sin\omega t$$

$$\frac{d^2y}{dt^2} = -\omega^2\sin\omega t + \omega^2\cos\omega t$$

$$= -\omega^2(\sin\omega t - \cos\omega t)$$

or $a = -\omega^2 y$ i.e., $a \propto y$

This satisfies the condition of SHM.

2. (a) $y = a\sin\omega t$

$$\frac{a}{2} = a\sin\frac{2\pi}{T}t$$

$$\sin\frac{2\pi}{T}t = \sin\frac{\pi}{6}$$

$$\frac{2\pi}{T}t = \frac{\pi}{6} \quad \text{or} \quad t = \frac{T}{12}.$$

3. (b) As seen in the above problem,

$$t = \frac{T}{12} = \frac{4}{12} = \mathbf{\frac{1}{3}\ s}.$$

4. (a) In SHM, instantaneous acceleration is ahead of instantaneous velocity in phase by $\pi/2$ rad.

5. (d) $a_{max} = \omega^2 A$

$$A = \frac{a_{max}}{\omega^2} = \frac{7.5}{3.5 \times 3.5} = \mathbf{0.61\ m}.$$

Scanned by CamScanner

6. (*b*) $v_{max} = \omega A = \frac{2\pi}{T} A$

$$= \frac{2 \times 3.14 \times 0.05}{2} = \mathbf{0.16\ ms^{-1}}$$

7. (*d*) At the equilibrium position, the velocity of a particle in SHM is maximum and consequently its acceleration is minimum.

8. (*d*) $k = \frac{F}{x} = \frac{10\text{ N}}{1\text{ mm}} = \frac{10\text{ N}}{10^{-3}\text{ m}} = 10^4\text{ Nm}^{-1}$.

Work done in stretching the spring through 40 mm,

$$W = \frac{1}{2}kx^2 = \frac{1}{2} \times 10^4 \times 0.04 \times 0.04 = \mathbf{8\ J}$$

9. (*b*) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{30}{15}} = \mathbf{2\pi\sqrt{2}\ s}$

10. (*b*) $\frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4m}{m}} = 2$

$$T_2 = 2T_1 = 2 \times 2 = \mathbf{4\ s}.$$

11. (*d*) For series combination,

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}.$$

12. (*d*) Here $k_s = \frac{k \times k}{k + k} = \frac{k}{2}$

$$\therefore \quad \nu = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$$

13. (*a*) When mass m is displaced slighly, restoring force is set up in both springs in the same direction.

$$F = F_1 + F_2 = -kx - 2kx = -3kx$$

Also, $\quad F = -k_{eff}\, x$

$$\therefore \quad k_{eff} = 3k$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}.$$

14. (*c*) Refer to the solution of Problem 31 on page 14.85.

15. (*a*) For object not to get detected from the platform,

$$a_{max} \le g \quad \text{or} \quad \omega^2 A \le g$$

or $\quad \left(\frac{2\pi}{T}\right)^2 A \le g$

$$T_{min} = 2\pi\sqrt{\frac{A}{g}} = 2\pi\sqrt{\frac{3.92 \times 10^{-3}}{9.8}}$$

$$= 2\pi \times 2 \times 10^{-2} = \mathbf{0.1256\ s}$$

16. (*c*) Time period, $T = 2\pi\sqrt{\frac{l}{g}}$ is independent of the mass of the bob.

17. (*a*) In a satellite, $g = 0$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{0}} = \infty.$$

18. (*d*) In air, $T = 2\pi\sqrt{\frac{l}{g}}$

Let ρ be the density of the bob material. When the bob is immersed in a non-viscous liquid of density $\rho_0 = \rho/10$, time period becomes

$$T' = 2\pi\sqrt{\frac{l}{\left(1 - \frac{\rho_0}{\rho}\right)g}} = 2\pi\sqrt{\frac{l}{\frac{9}{10}g}}$$

$$= \sqrt{\frac{10}{9}}\, \mathbf{T}.$$

19. (*a*) Frequency ν' of driving force < frequency ν of damped oscillator.

The vibrations are nearly in phase with the driving force and response of the oscillator is controlled by spring constant.

20. (*b*) Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

As $l = 1$ m, $\quad \nu = \frac{\sqrt{g}}{2\pi}$

When the thread is caught by the peg at B, the length of the pendulum becomes

$$l' = 1 - 0.25 = 0.75\text{ m}.$$

New frequency,

$$\nu' = \frac{1}{2\pi}\sqrt{\frac{g}{0.75}} = \frac{1}{2\pi}\sqrt{\frac{4g}{3}}$$

Change in frequency,

$$\nu' - \nu = \frac{1}{2\pi}\left[\sqrt{\frac{4g}{3}} - \sqrt{g}\right]$$

$$= \frac{1}{2\pi}\left[\frac{\sqrt{4} - \sqrt{3}}{\sqrt{3}}\right]\sqrt{g}$$

$$= \frac{(2 - \sqrt{3})\sqrt{g}}{2\sqrt{3}\,\pi} = \frac{1}{40}\sqrt{g} = \frac{1}{40}\sqrt{10}$$

$$= \mathbf{\frac{1}{4\sqrt{10}}\ Hz}$$

21. (*d*) As $k \propto \frac{1}{l}$, so option (*d*) is correct.

Scanned by CamScanner

22. (c) $T_1 = 2\pi\sqrt{\frac{M}{k_1}}$

and $T_2 = 2\pi\sqrt{\frac{M}{k_2}}$

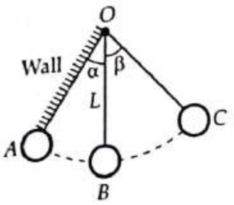
For series combination,

$$T = 2\pi\sqrt{\frac{M}{k_s}} = 2\pi\sqrt{\frac{M(k_1+k_2)}{k_1 k_2}}$$

or $$T^2 = \frac{4\pi^2 M}{k_1} + \frac{4\pi^2 M}{k_2} = T_1^2 + T_2^2$$

23. (a) Here $\beta > \alpha$. Time taken by the pendulum to move from B to C and C to B

$$t_1 = \frac{1}{2}T = \frac{1}{2}\times 2\pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{L}{g}}$$



For a particle in SHM when $\alpha < \beta$,

$$\alpha = \beta \sin \omega t$$

$$\therefore \quad t = \frac{1}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right) = \sqrt{\frac{L}{g}}\sin^{-1}\frac{\alpha}{\beta}$$

Time taken by the pendulum to move from B to A and back to B,

$$t_2 = 2t = 2\sqrt{\frac{L}{g}}\sin^{-1}\frac{\alpha}{\beta}$$

$\therefore$ Time period of motion,

$$T = t_1 + t_2 = \sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\frac{\alpha}{\beta}\right]$$

24. (b) SHM of the particle is given by

$$x = l\sin\omega t$$

As the rod executes SHM perpendicular to its own length, so its equation is

$$y = l\cos(\omega t + \phi_0)$$

Now both the particle and the rod pass through the mean position initially, so $x = y = 0$ at $t = 0$. Hence $\phi_0 = \pi/2$

$$y = l\cos\left(\omega t + \frac{\pi}{2}\right) = -l\sin\omega t$$

Clearly, $y = -x$

This is the equation of a straight line inclined at $\pi/4$ to the length of the rod.

25. (b) Both the assertion and reason are true but the reason is not a correct explanation. SHM is a periodic motion in which acceleration $\propto$ displacement from mean position and acceleration acts in the opposite direction of displacement.

26. (b) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. In fact, the phase difference between velocity and acceleration is $\pi/2$.

27. (c) The assertion is true but the reason is false. Due to friction of air, the amplitude gradually decreases with time.

28. (b) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. In a satellite, $g = 0$ and hence time period is infinity.

29. (d) Both the assertion and reason are false. Period of oscillation of a liquid in a U-tube is independent of the density of the liquid. The amplitude of the oscillating pendulum decreases due to air friction.

30. (c) The assertion is true but the reason is false. The amplitude of forced vibrations increases when the frequency of the impressed force approaches the natural frequency of the driven body.

31. (b) Refer to the solution of Problem 2 on page 14.44.

CBSE PMT Prelims and Final Exams

1. The circular motion of a particle with constant speed is

(a) periodic but not simple harmonic

(b) simple harmonic but not periodic

(c) periodic and simple harmonic

(d) neither periodic not simple harmonic.

[CBSE PMT 05]

2. Which of the following is simple harmonic motion ?

(a) Particle moving in a circle with uniform speed

(b) Wave moving through a string fixed at both ends

(c) Earth spinning about its axis

(d) Ball bouncing between two rigid vertical walls.

[CBSE PMT 94]

Scanned by CamScanner

3. A particle is executes S.H.M. along x-axis. The force acting on it is given by

(a) $A\cos(kx)$ (b) Ae^{-kx}

(c) Akx (d) $-Akx$

[CBSE PMT 88, 94]

4. Which one of the following represents simple harmonic motion ?

(a) Acceleration $= kx$

(b) Acceleration $= k_0x + k_1x^2$

(c) Acceleration $= -k(x+a)$

(d) Acceleration $= k(x+a)$

where k, k_0, k_1 are all positive. [CBSE PMT 09]

5. A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is

(a) 4 Hz (b) 3 Hz

(c) 2 Hz (d) 1 Hz [CBSE PMT 04]

6. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(a) $T/8$ (b) $T/12$

(c) $T/2$ (d) $T/4$. [CBSE PMT 07]

7. A simple harmonic oscillator has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = A/2$ is

(a) $T/6$ (b) $T/4$

(c) $T/3$ (d) $T/2$ [CBSE PMT 92]

8. If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 m/s^2 at any time, the angular frequency of the oscillator is equal to

(a) 10 rad/s (b) 0.1 rad/s

(c) 100 rad/s (d) 1 rad/s [CBSE PMT 92]

9. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is

(a) π (b) $0.707\ \pi$

(c) zero (d) $0.5\ \pi$ [CBSE PMT 07]

10. Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion ?

(a) when v is maximum, a is maximum

(b) value of a is zero, whatever may be the value of v

(c) when v is zero, a is zero

(d) when v is maximum, a is zero.

[CBSE PMT

11. A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a and time period T. The speed of the pendulum at $x = a/2$ will be

(a) $\frac{\pi a\sqrt{3}}{T}$ (b) $\frac{\pi a\sqrt{3}}{2T}$

(c) $\frac{\pi a}{T}$ (d) $\frac{3\pi^2 a}{T}$

[CBSE PMT

12. A body is executing simple harmonic motion. When the displacements from the mean position are 4 cm and 5 cm, the corresponding velocities of the body are 10 cm/sec and 8 cm/sec. Then the time period of the body is

(a) 2π sec (b) $\pi/2$ sec

(c) π sec (d) $(3\pi/2)$ sec.

[CBSE PMT

13. The total energy of particle performing SHM depends on

(a) k, a, m (b) k, a

(c) k, a, x (d) k, x.

[CBSE PMT

14. Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing simple harmonic motion is

(a) $\pm a/2$ (b) $+a$

(c) $\pm a$ (d) -1.

[CBSE PMT

15. The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively

(a) $K_0/2$ and K_0 (b) K_0 and $2K_0$

(c) K_0 and K_0 (d) 0 and $2K_0$

[CBSE PMT 07]

16. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its

(a) P.E. is 160 J (b) P.E. is zero

(c) P.E. is 100 J (d) P.E. is 120 J.

[CBSE PMT

17. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is

(a) $\frac{2}{3}E$ (b) $\frac{1}{8}E$

(c) $\frac{1}{4}E$ (d) $\frac{1}{2}E$ [CBSE PMT

Scanned by CamScanner

18. In a simple harmonic motion, when the displacement is one half the amplitude, what fraction of the total energy is kinetic ?

(a) 1/2 (b) 3/4

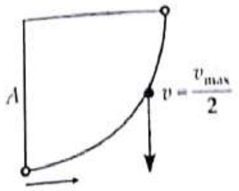
(c) zero (d) 1/4. [CBSE PMT 95]

19. A body executes simple harmonic motion with an amplitude A. At what displacement from the mean position is the potential energy of the body is one-fourth of its total energy ?

(a) $A/4$ (b) $A/2$

(c) $3A/4$ (d) some other fraction of A. [CBSE PMT 93]

20. A particle starts with S.H.M. from the mean position as shown in the figure. Its amplitude is A and

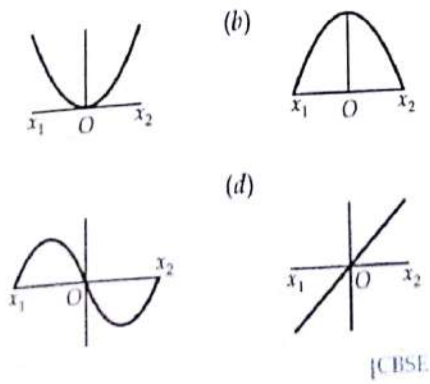


its time period is T. At one time, its speed is half that of the maximum speed. What is this displacement ?

(a) $\frac{2A}{\sqrt{3}}$ (b) $\frac{3A}{\sqrt{2}}$

(c) $\frac{\sqrt{2}A}{3}$ (d) $\frac{\sqrt{3}A}{2}$. [CBSE PMT 96]

21. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being O. Its potential energy is plotted. It will be as given below in the graph



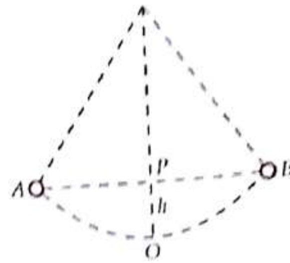
[CBSE PMT 03]

22. The angular velocity and the amplitude of a simple pendulum are ω and a respectively. At a displacement x from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is

(a) $\frac{(a^2 - x^2\omega^2)}{x^2\omega^2}$ (b) $\frac{x^2\omega^2}{(a^2 - x^2\omega^2)}$

(c) $\frac{(a^2 - \omega^2)}{x^2}$ (d) $\frac{x^2}{(a^2 - x^2)}$ [CBSE PMT 91]

23. A simple pendulum with a bob of mass m oscillates from A to B and back to A such that OP is h. If the acceleration due to gravity is g, then the velocity of the bob as it passes through B is



(a) mgh (b) $\sqrt{2gh}$

(c) zero (d) 2gh. [AIEEE 95]

24. The bob of simple pendulum having length l, is displaced from mean position to an angular position θ with respect to vertical. If it is released, then velocity of bob at equilibrium position

(a) $\sqrt{2gl(1-\cos\theta)}$ (b) $\sqrt{2gl(1+\cos\theta)}$

(c) $\sqrt{2gl\cos\theta}$ (d) $\sqrt{2gl}$. [CBSE PMT 2K]

25. A loaded vertical spring executes S.H.M. with a time period of 4 sec. The difference between the kinetic energy and potential energy of this system varies with a period of

(a) 2 sec (b) 1 sec

(c) 8 sec (d) 4 sec. [CBSE PMT 94]

26. A mass m is vertically suspended from a spring of negligible mass ; the system oscillates with a frequency n. What will be the frequency of the system, if a mass 4m is suspended from the same spring ?

(a) $\frac{n}{2}$ (b) 4n

(c) $\frac{n}{4}$ (d) 2n [CBSE PMT 98]

27. A body of mass 5 kg hangs from a spring and oscillates with a time period of 2π seconds. If the body is removed, the length of the spring will decrease by

(a) g/k metres (b) k/g metres

(c) 2π metres (d) g metres. [CBSE PMT 94]

28. The time period of mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be

(a) T/4 (b) T

(c) T/2 (d) 2T [CBSE PMT 03]

Scanned by CamScanner

29. Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by

(*a*) $\sqrt{k_1k_2}$ (*b*) $(k_1+k_2)/2$

(*c*) k_1+k_2 (*d*) $k_1k_1/(k_1+k_2)$

[CBSE PMT 04]

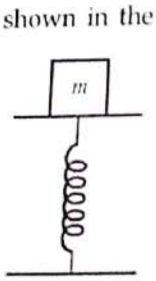
30. A mass m is suspended from the two coupled springs connected in series. The force constants for springs are k_1 and k_2. The time period of the suspended mass will be

(*a*) $T=2\pi\sqrt{\dfrac{m}{k_1-k_2}}$ (*b*) $T=2\pi\sqrt{\dfrac{mk_1k_2}{k_1+k_2}}$

(*c*) $T=2\pi\sqrt{\dfrac{m}{k_1+k_2}}$ (*d*) $T=2\pi\sqrt{\dfrac{m(k_1+k_2)}{k_1k_2}}$.

[CBSE PMT 90]

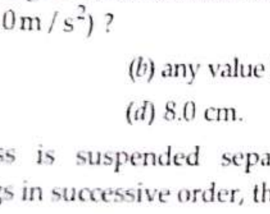
31. A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N/m. What should be the minimum amplitude of the motion so that the mass gets detached from the pan (take $g=10\,\text{m/s}^2$) ?



(*a*) 10.0 cm (*b*) any value less than 12.0 cm

(*c*) 4.0 cm (*d*) 8.0 cm. [CBSE PMT 07]

32. A mass is suspended separately by two different springs in successive order, then time periods are t_1 and t_2 respectively. If it is connected by both springs as shown in figure, then time period is t_0, the correct relation is



(*a*) $t_0^2=t_1^2+t_2^2$ (*b*) $t_0^{-2}=t_1^{-2}+t_2^{-2}$

(*c*) $t_0^{-1}=t_1^{-1}+t_2^{-1}$ (*d*) $t_0=t_1+t_2$ [CBSE PMT 02]

33. Time period of a simple pendulum is 2 sec. If its length is increased by 4 times, then its period becomes

(*a*) 8 sec (*b*) 12 sec

(*c*) 16 sec (*d*) 4 sec. [CBSE PMT 99]

34. If the length of a simple pendulum is increased by 2%, then the time period

(*a*) increases by 1% (*b*) decreases by 1%

(*c*) increases by 2% (*d*) decreases by 2%.

[CBSE PMT 97]

35. Two masses M_A and M_B are hung from two strings of length l_A and l_B respectively. They are executing SHM with frequency relation $f_A=2f_B$, then relation

(*a*) $l_A=\dfrac{l_B}{4}$, does not depend on mass

(*b*) $l_A=4l_B$, does not depend on mass

(*c*) $l_A=2l_B$ and $M_A=2M_B$

(*d*) $l_A=\dfrac{l_B}{2}$ and $M_A=\dfrac{M_B}{2}$.

[CBSE PMT 2K]

36. A second's pendulum is mounted in a rocket. Its period of oscillation will decrease when rocket is

(*a*) moving down with uniform acceleration

(*b*) moving around the earth in geostationary orbit

(*c*) moving up with uniform velocity

(*d*) moving up with uniform acceleration.

[CBSE PMT 94]

37. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T=2\pi\sqrt{(l/g)}$, where g is equal to

(*a*) g (*b*) $g-a$

(*c*) $g+a$ (*d*) $\sqrt{(g^2+a^2)}$

[CBSE PMT 91]

38. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T, then

(*a*) $T\propto\dfrac{1}{\sqrt{m}}$ (*b*) $T\propto\sqrt{\rho}$

(*c*) $T\propto\dfrac{1}{\sqrt{A}}$ (*d*) $T\propto\dfrac{1}{\rho}$ [CBSE PMT 06]

39. In case of a forced vibration, the resonance peak becomes very sharp when the

(*a*) damping force is small

(*b*) restoring force is small

(*c*) applied periodic force is small

(*d*) quality factor is small. [CBSE PMT 03]

40. A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force $F \sin \omega t$. If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle maximum for $\omega = \omega_2$, then

(*a*) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$ (*b*) $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$
(*c*) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$
(*d*) $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$ [CBSE PMT 89, 98]

41. When an oscillator completes 100 oscillations, its amplitude reduces to 1/3 of initial value. What will be its amplitude, when it completes 200 oscillations ?

(*a*) 1/8 (*b*) 2/3
(*c*) 1/6 (*d*) 1/9. [CBSE PMT 02]

42. Two simple pendulums of lengths 5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed .... oscillations.

(*a*) 2 (*b*) 1
(*c*) 5 (*d*) 3 [CBSE PMT 98]

43. Two simple pendulums of time periods 2.0 s and 2.1 s are made to vibrate simultaneously. They are phase initially. After how may vibrations, they are in the same phase ?

(*a*) 21 (*b*) 25
(*c*) 30 (*d*) 35 [CBSE PMT 08]

44. Two SHM's with same amplitude and time period, when acting together in perpendicular directions with a phase difference of $\pi/2$, give rise to

(*a*) straight motion (*b*) elliptical motion
(*c*) circular motion (*d*) none of these.
[CBSE PMT 97]

45. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along [CBSE PMT 90 ; AIIMS 94]

(*a*) circle (*b*) figure of eight
(*c*) straight line (*d*) ellipse

46. The displacement of a particle along the *x*-axis is given by $x = a \sin^2 \omega t$. The motion of the particle corresponds to

(*a*) simple harmonic motion of frequency ω/π
(*b*) simple harmonic motion of frequency $3\omega/2\pi$
(*c*) non-simple harmonic motion
(*d*) simple harmonic motion of frequency $\omega/2\pi$
[CBSE Pre 2010]

47. Out of the following functions representing motion of a particle which represents SHM

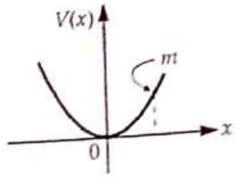
(*A*) $y = \sin \omega t - \cos \omega t$ (*B*) $y = \sin^3 \omega t$
(*C*) $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$ (*D*) $y = 1 + \omega t + \omega^2 t^2$

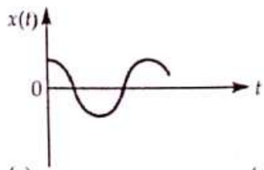
(*a*) Only (*A*)
(*b*) Only (*D*) does not represent SHM
(*c*) Only (*A*) and (*C*)
(*d*) Only (*A*) and (*B*) [CBSE Pre 2011]

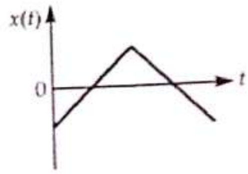
48. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitude. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is

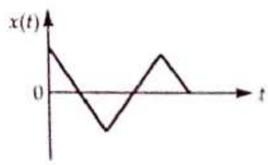
(*a*) π (*b*) $\frac{\pi}{6}$
(*c*) 0 (*d*) $\frac{2\pi}{3}$ [CBSE Pre 2011]

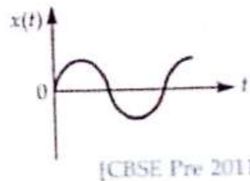
49. A particle of mass *m* is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small, which graph correctly depicts the position of the particle as a function of time ?



(*a*)


(*b*)


(*c*)


(*d*)


[CBSE Pre 2011]

50. The period of oscillation of a mass *M* suspended from a spring of negligible mass is *T*. If along with it another mass *M* is also suspended, the period of oscillation will now be :

(*a*) T (*b*) $T/\sqrt{2}$
(*c*) $2T$ (*d*) $\sqrt{2}T$ [CBSE Pre 2010]

Scanned by CamScanner

Answers and Explanations

1. (*a*) Uniform circular motion is a periodic motion but not simple harmonic.

2. (*b*) Wave moving through a string fixed at both ends has simple harmonic nature.

3. (*d*) $F = -Akx$ implies that the force is proportional displacement and acts in its opposite direction. So it represents SHM.

4. (*c*) Acceleration $= -kX$, $X = x + a$.

Thus the acceleration is proportional to displacement and acts in its opposite direction. Hence option (*c*) represents SHM.

5. (*d*) $v_{max} = 2\pi\nu A$

$$31.4 = 2 \times 3.14 \times \nu \times 5$$

$\therefore$ $\nu = \mathbf{1\ Hz}$.

6. (*b*) $y = a\sin\omega t$

or $\frac{a}{2} = a\sin\frac{2\pi}{T}t$

$$\sin\frac{2\pi}{T}t = \sin\frac{\pi}{6}$$

$$\frac{2\pi}{T}t = \frac{\pi}{6}$$

or $T = \frac{\mathbf{T}}{\mathbf{12}}$.

7. (*a*) As the oscillator starts from $x = A$, we can take

$$x = a\cos\omega t$$

$$\frac{a}{2} = a\cos\frac{2\pi}{T}t$$

$$\cos\frac{2\pi}{T}t = \frac{1}{2} = \cos\frac{\pi}{6}$$

$$\frac{2\pi}{T}t = \frac{\pi}{6} \quad \text{or} \quad t = \frac{T}{6}.$$

8. (*a*) $a = -\omega^2 y$

$\therefore$ $\omega^2 = \frac{a}{y} = \frac{2.0}{0.02} = 100$

$\omega = \mathbf{10\ rad/s}$.

9. (*d*) The phase difference between the instantaneous velocity and acceleration of a particle executing SHM is $\pi/2$.

10. (*d*) In SHM, acceleration is ahead of velocity in phase by $\pi/2$ rad. So, when velocity is maximum, acceleration is zero and vice versa.

11. (*a*) Speed, $v = \omega\sqrt{a^2 - x^2}$, $x = a/2$

$\therefore$ $v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \frac{2\pi}{T}\cdot\frac{\sqrt{3}a}{2} = \frac{\pi a\sqrt{3}}{T}$.

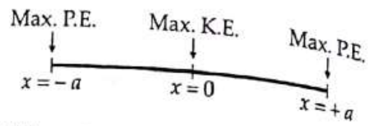
12. (*c*) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}} = 2\pi\sqrt{\frac{5^2 - 4^2}{10^2 - 8^2}}$

$$= 2\pi\sqrt{\frac{9}{36}} = 2\pi \times \frac{3}{6} = \boldsymbol{\pi}\ \mathbf{sec.}$$

13. (*b*) $E = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}ka^2$

Clearly, E depends on k and a.

14. (*c*) In SHM, potential energy is maximum at either extreme position and kinetic energy is maximum at the mean position.



15. (*c*) When K.E. is maximum, P.E. is zero and vice-versa. But

$$\text{K.E.} + \text{P.E.} = \text{Total energy.}$$

$$\text{Max. P.E.} = \text{Max. K.E.} = \text{Total energy} = K_0.$$

16. (*c*) $\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = \mathbf{100\ J}$.

17. (*c*) Total energy $E = \frac{1}{2}ka^2$

At $x = \frac{a}{2}$, the potential energy is

$$\text{P.E} = \frac{1}{2}k\left(\frac{a}{2}\right)^2 = \frac{1}{4}\cdot\frac{1}{2}ka^2 = \frac{\mathbf{E}}{\mathbf{4}}.$$

18. (*b*) At $x = \frac{a}{2}$, the kinetic energy is

$$\text{K.E} = \frac{1}{2}k\left[a^2 - \left(\frac{a}{2}\right)^2\right]$$

$$= \frac{3}{4}\cdot\frac{1}{2}ka^2 = \frac{\mathbf{3}}{\mathbf{4}}\mathbf{E}.$$

19. (*b*) $\text{P.E} = \frac{1}{4} \times \text{Total energy}$

$$\frac{1}{2}kx^2 = \frac{1}{4} \times \frac{1}{2}kA^2$$

$$x = \frac{\mathbf{A}}{\mathbf{2}}.$$

Scanned by CamScanner

20. (*d*) $v = \frac{v_{max}}{2}$

$\omega\sqrt{A^2 - x^2} = \frac{1}{2}\omega A \qquad [v_{max} = \omega A]$

or $A^2 - x^2 = \frac{1}{4}A$

or $x = \frac{\sqrt{3}A}{2}$.

21. (*a*) $P.E = \frac{1}{2}kx^2$ *i.e.*, $P.E \propto x^2$

Thus graph of P.E. versus *x* is a parabola. Morever, P.E. is zero at mean position and maximum at extreme position. Hence the correct option is (*a*).

22. (*c*) Kinetic energy,

$T = \frac{1}{2}m\omega^2(a^2 - x^2)$

Potential energy,

$V = \frac{1}{2}m\omega^2x^2 \quad \therefore \quad \frac{T}{V} = \frac{(a^2 - x^2)}{x^2}$

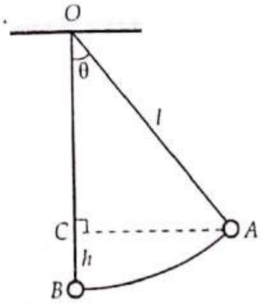
23. (*c*) At the extreme position *B*, the kinetic energy of the pendulum is zero. Hence velocity of the bob is zero at position *B*.

24. (*a*) $OC = OA\cos\theta$

$= l\cos\theta$

$h = BC = OB - OC$

$= l(1 - \cos\theta)$



K.E at *B* = P.E at *A*

$\frac{1}{2}mv^2 = mgh$

$v^2 = 2gh = 2gl(1 - \cos\theta)$

$v = \sqrt{2gl(1 - \cos\theta)}$.

25. (*a*) Here $T = 4$ s. In one oscillation, both K.E. and P.E. become twice maximum and twice minimum. Hence the difference between K.E. and P.E. varies with a period of 2 s.

26. (*a*) Here $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

$n' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}} = \frac{n}{2}$.

27. (*d*) Here $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi$ or $\frac{m}{k} = 1$

In equilibrium,

$mg = kd$

or $d = \frac{mg}{k} = 1 \times g = g$ **metres**.

28. (*c*) $T = 2\pi\sqrt{\frac{m}{k}}$, $k = \frac{F}{x}$

When the spring is cut into four equal parts,

$k' = \frac{F}{x/4} = 4\left(\frac{F}{x}\right) = 4k$

$\therefore \quad T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{4k}} = \frac{T}{2}$.

29. (*d*) $k_s = \frac{k_1k_2}{k_1 + k_2}$

For derivation refer answer to Q.18 on Page 14.21.

30. (*d*) $T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$.

31. (*a*) The mass will get detached from the pan when

$a_{max} \geq g$

or $\omega^2 A \geq g$

or $A \geq \frac{g}{\omega^2}$

or $A \geq \frac{gm}{k} \qquad \left[\omega^2 = \frac{k}{m}\right]$

∴ Minimum value of required amplitude

$= \frac{gm}{k} = \frac{10 \times 2}{200} = 0.10 \text{ m} = \mathbf{10.0\ cm}$.

32. (*b*) Here $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$

or $t_1^2 = \frac{4\pi^2m}{k_1}$ or $k_1 = \frac{4\pi^2m}{t_1^2}$

Similarly, $k_2 = \frac{4\pi^2m}{t_2^2}$ and $k_p = \frac{4\pi^2m}{t_0^2}$

But $k_p = k_1 + k_2$

or $\frac{4\pi^2m}{t_0^2} = \frac{4\pi^2m}{t_1^2} + \frac{4\pi^2m}{t_2^2}$

or $\frac{1}{t_0^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$.

33. (*d*) $\frac{T'}{T} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{4l}{l}} = 2$

$\therefore \quad T' = 2T = 2 \times 2 = \mathbf{4\ s}$.

34. (*a*) $T \propto \sqrt{l}$

Percentage increase in time period,

$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100$

$= \frac{1}{2} \times 2\% = \mathbf{1\%}$.

Scanned by CamScanner

35. (*a*) Given $f_A = 2 f_B$

or $$\frac{1}{2\pi}\sqrt{\frac{g}{l_A}} = 2 \times \frac{1}{2\pi}\sqrt{\frac{g}{l_B}}$$

$\therefore$ $$l_A = \frac{l_B}{4}$$

which does not depend on mass.

36. (*d*) When at rest, $T = 2\pi\sqrt{\frac{l}{g}}$

When the rocket moves up with uniform acceleration,

$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$

Clearly, $T' < T$.

37. (*d*) As g and a are acting along perpendicular directions, the effective value of acceleration due to gravity is

$$g' = \sqrt{g^2 + a^2}.$$

38. (*c*) When the block is depressed through distance y,

$$F = \text{Upthrust of the liquid}$$
$$= -Ay\rho \times g = -A\rho g y$$

As $F \propto y$, motion of the block is SHM with

$$k = A\rho g$$

$\therefore$ $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Clearly, $T \propto \frac{1}{\sqrt{A}}$.

39. (*a*) When the damping force is small, the resonance peak is high and narrow.

40. (*a*) In a driven harmonic oscillator, the energy is maximum at $\omega_2 = \omega_0$ and amplitude is maximum at frequency, $\omega_1 < \omega_0$ in the presence of a damping of force. Therefore, $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$.

41. (*d*) In damped oscillations, the amplitude at any instant t is given by

$$a = a_0 e^{-bt}$$

where a_0 = initial amplitude and b = damping constant.

At $t = 100T$, $a = \frac{a_0}{3}$

$\therefore$ $$\frac{a_0}{3} = a_0 e^{-b \times 100T} \quad \text{or} \quad e^{-100bT} = \frac{1}{3}$$

At $t = 200T$,

$$a = a_0 e^{-b \times 200T} = a_0 (e^{-100bT})^2$$
$$= a_0\left(\frac{1}{3}\right)^2 = \frac{a_0}{9}.$$

42. (*b*) $$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{5}{20}} = \frac{1}{2}$$

$$T_1 = 2T_2$$

When the longer pendulum completes 2 oscillations, the shorter pendulum completes one oscillation and both are again in same phase.

43. (*a*) The two pendulums will be in same phase again when they complete $(\nu + 1)$ and ν oscillations respectively.

$\therefore$ $$(\nu+1) \times T_1 = \nu \times T_2$$
$$(\nu+1)2 = \nu \times 2.1 \quad \text{or} \quad \nu = 20$$

Thus the two pendulums should respectively complete 21 and 20 oscillations for to be in same phase again.

44. (*c*) Let $x = a \sin \omega t$

and $$y = a\sin(\omega t + \pi/2) = a\cos\omega t$$

$\therefore$ $$\frac{x}{y} = \tan\omega t$$

or $$\frac{x}{y} = \frac{x}{\sqrt{a^2 - x^2}}$$

or $$y^2 = a^2 - x^2 \quad \text{or} \quad x^2 + y^2 = a^2$$

This is the equation of a circle.

45. (*c*) Let $x = a\sin\omega t$

and $$y = b\sin(\omega t + \pi) = -b\sin\omega t$$

$\therefore$ $$\frac{x}{a} = -\frac{y}{b} \quad \text{or} \quad y = -\frac{b}{a}x$$

This is the equation of a straight line.

46. (*c*) $$x = a\sin^2\omega t$$

$$= \frac{a}{2}(1 - \cos 2\omega t) = \frac{a}{2} - \frac{a}{2}\cos 2\omega t$$

The function does not represent SHM but is periodic with

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

47. (*c*) (*D*) is algebraic function and (*B*) is not harmonic being the product of $\sin\omega t$ (thrice). But (*A*) and (*C*) are simple harmonic functions. Hence (*A*) and (*C*) represent SHM. (For detailed explanation, refer answer to NCERT Exercise 14.4 on page 14.49.)

48. (*d*) Refer to the solution of Problem 5 on page 14.42

49. (*a*) The particle executes SHM.

At $t = 0$, $v = 0 \Rightarrow x = x_{max}$

50. (*d*) As $T \propto M$

$\therefore$ $$\frac{T_2}{T_1} = \sqrt{\frac{M+M}{M}} \Rightarrow T_2 = \sqrt{2}T_1 = \sqrt{2}T.$$

Scanned by CamScanner

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. A simple harmonic motion is represented by $f(t) = 17\sin(20t + 0.5)$. The amplitude of the SHM is

(a) $A = 30$ (b) $A = 17$
(c) $A = 20$ (d) $A = 5$. [DPMT 98]

2. A particle in SHM is described by the displacement function $x(t) = A\cos(\omega t + \theta)$. If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is π cm/s, what is its amplitude ? The angular frequency of the particle is π rad s^{-1}.

(a) 1 cm (b) $\sqrt{2}$ cm
(c) 2 cm (d) 2.5 cm. [DPMT 04]

3. A particle executes SHM, its time period is 16 s. If it passes through the centre of oscillation, then its velocity is 2 m/s at time 2 s.

The amplitude will be

(a) 7.2 m (b) 4 cm
(c) 6 cm (d) 0.72 m [DPMT 06]

4. A particle executes SHM of amplitude A. If T_1 and T_2 are the times taken by the particle to transverse from 0 to $\frac{A}{2}$ and from $\frac{A}{2}$ to A respectively, then $\frac{T_1}{T_2}$ will be equal to

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 2 [VMMC 07]

5. Velocity of a body moving in simple harmonic motion is

(a) $\omega\sqrt{a^2 + y^2}$ (b) $\omega^2\sqrt{a^2 + y^2}$
(c) $\omega\sqrt{a^2 - y^2}$ (d) $\omega^2\sqrt{a^2 - y^2}$ [DPMT 91]

6. A body is executing the S.H.M. with an angular frequency of 2 rad/sec. Velocity of the body at 20 m displacement, when amplitude of motion is 60 m, is

(a) 90 m/s (b) 118 m/s
(c) 113 m/s (d) 131 m/s [DPMT 92]

7. A particle is executing SHM of amplitude 10 cm. Its time period of oscillation is π seconds. The velocity of the particle when it is 2 cm from extreme position is

(a) 10 cms^{-1} (b) 12 cms^{-1}
(c) $16\sqrt{16}$ cms^{-1} (d) none of these. [DPMT 06]

8. A particle has displacement y given by $y = 3\sin(5\pi t + \pi)$, where y is in metre and t is in second. What are frequency and period of motion ?

(a) 0.4 Hz, 2.5 s (b) 2.5 Hz, 0.4 s
(c) 2.5 Hz, 2.5 s (d) 0.4 Hz, 0.4 s. [DPMT 08]

9. The motion of a particle executing SHM in one dimension is described by :

$$x = -0.5\sin(2t + \pi/4)$$

where x is in metres and t in seconds. The frequency of oscillation in Hz is

(a) 2 (b) π
(c) $\frac{\pi}{2}$ (d) $\frac{1}{\pi}$ [DPMT 09]

10. The magnitude of acceleration of particle executing SHM at the position of maximum displacement is

(a) zero (b) minimum
(c) maximum (d) none of these. [DPMT 05]

11. The maximum velocity and maximum acceleration of body moving in a simple harmonic motion are 2 m/s and 4 m/s^2 respectively. Then the angular velocity will be

(a) 4 rad/sec (b) 3 rad/sec
(c) 2 rad/sec (d) 8 rad/sec.
[DPMT 94 ; VMMC 04]

12. A particle executing SHM has amplitude 0.01 and frequency 60 Hz. The maximum acceleration of the particle is

(a) $144\ \pi^2$ m/s^2 (b) $80\ \pi^2$ m/s^2
(c) $120\ \pi^2$ m/s^2 (d) $60\ \pi^2$ m/s^2 [DPMT 98]

13. If $x = R\sin\omega t + R\omega t$ and $y = R\cos(\omega t) + R$ (where ω and R are constants), what are x and y components of acceleration when y is minimum ?

(a) $0, R\omega^2$ (b) $R\omega^2, 0$
(c) $0, -R\omega^2$ (d) $-R\omega^2, 0$ [DPMT 02]

14. A simple harmonic oscillator has amplitude A, angular velocity ω and mass m. Then average kinetic energy in one time period will be

(a) $\frac{1}{4}m\omega^2A^2$ (b) $\frac{1}{2}m\omega^2A^2$
(c) $m\omega^2A^2$ (d) 0 [VMMC 06]

Scanned by CamScanner

15. Average value of K.E. and P.E. over entire time period is

(*a*) $0, 1/2\, m\omega^2 A^2$

(*b*) $1/2\, m\omega^2 A^2, 0$

(*c*) $1/2\, m\omega^2 A^2,\ 1/2\, m\omega^2 A^2$

(*d*) $1/4\, m\omega^2 A^2,\ 1/4\, m\omega^2 A^2$. [DPMT 02]

16. A particle is having potential energy 1/3 of the maximum value at a distance of 4 cm from mean position. Amplitude of motion is

(*a*) $4\sqrt{3}$ (*b*) $\frac{6}{\sqrt{2}}$

(*c*) $\frac{2}{\sqrt{6}}$ (*d*) $2\sqrt{6}$ [VMMC 02]

17. The potential energy of a particle doing SHM is 2.5 J when displacement is half of amplitude. Then the total energy is

(*a*) 5 J (*b*) 10 J

(*c*) 15 J (*d*) 20 J. [VMMC 03]

18. A particle executes simple harmonic motion of amplitude *A*. At what distance from the mean position is its kinetic energy equal to its potential energy ?

(*a*) 0.51*A* (*b*) 0.71*A*

(*c*) 0.61*A* (*d*) 0.81*A*

[DPMT 96, 01]

19. Frequency of variation of kinetic energy of a simple harmonic motion of frequency *n* is

(*a*) $2n$ (*b*) n

(*c*) $n/2$ (*d*) $3n$ [DPMT 2K]

20. Two springs of spring constants k_1 and k_2 have equal maximum velocities. When executing simple harmonic motion, the ratio of their amplitudes (masses are equal) will be

(*a*) $\left(\frac{k_2}{k_1}\right)^{1/2}$ (*b*) $\left(\frac{k_1}{k_2}\right)^{1/2}$

(*c*) $\frac{k_2}{k_1}$ (*d*) $k_1 k_2$

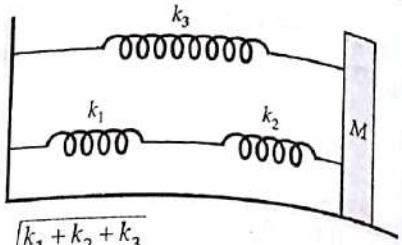
[DPMT 96 ; VMMC 04]

21. When two springs of spring force constant k and $2k$ are connected in series then force constant becomes k_s, if they are connected in parallel, then force constant becomes k_p. Ratio k_s / k_p is

(*a*) 2/9 (*b*) 1/2

(*c*) 2/1 (*d*) 1/3. [DPMT 07]

22. If the mass *M* in the figure is slightly displaced and then released, the frequency of the motion of the block is



(*a*) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2+k_3}{M}}$

(*b*) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2+k_2k_3+k_3k_1}{(k_2+k_3)M}}$

(*c*) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2+k_2k_3+k_3k_1}{M(k_1+k_2)}}$

(*d*) none of these.

[DPMT 06]

23. To show that a simple pendulum executes simple harmonic motion, it is necessary to assure that

(*a*) length of the pendulum is small

(*b*) amplitude of oscillation is small

(*c*) mass of the pendulum is small

(*d*) acceleration due to gravity is small. [DPMT 93]

24. Time period of a simple pendulum will be double, if we

(*a*) decrease the length 2 times

(*b*) decrease the length 4 times

(*c*) increase the length 2 times

(*d*) increase the length 4 times.

[VMMC 04]

25. The time-period of a simple pendulum is 2 sec. If its length is increased by 4 times, then its period becomes

(*a*) 16 sec (*b*) 8 sec

(*c*) 12 sec (*d*) 4 sec.

[DPMT 99]

26. If the frequency of oscillation of a pendulum in simple harmonic motion is *n*, then frequency of pendulum whose length becomes four times is

(*a*) n (*b*) $n/2$

(*c*) $2n$ (*d*) $4n$.

[DPMT 01]

27. On a planet, a freely falling body takes 2 sec when it is dropped from a height of 8 m. The time period of a simple pendulum of length 1 m on that planet is

(*a*) 1.42 sec (*b*) 3.14 sec

(*c*) 2.92 sec (*d*) 5.36 sec. [DPMT 93]

Scanned by CamScanner

28. A simple pendulum is executing simple harmonic motion with a time period T. If the length of the pendulum is increased by 21%, the increase in the time period of the pendulum of increased length is

(a) 10% (b) 30%

(c) 21% (d) 50% [DPMT 96]

29. The period of a simple pendulum measured inside a stationary lift is found to be T. If the lift starts accelerating upwards with acceleration of $g/3$, then the time period of the pendulum is

(a) $\frac{2T}{\sqrt{3}}$ (b) $\frac{T}{3}$

(c) $\frac{\sqrt{3}}{2}T$ (d) $\sqrt{3}T$. [DPMT 03]

30. A hollow spherical pendulum is filled with mercury has time period T. If mercury is thrown out completely, then the new time period

(a) increases (b) decreases

(c) same (d) none of these. [DPMT 07]

31. A hollow sphere filled with water forms the bob of a simple pendulum. A small hole at the bottom of the bob allows the water to slowly flow out as it is set into small oscillation and its period of oscillation is measured. The time-period will

(a) increase (b) remain constant

(c) decrease (d) first (a) then (c). [DPMT 94]

32. A rod of length l and mass m is capable of rotating freely about an axis passing through a hole at the end. The period of oscillation of this physical pendulum is

(a) $2\pi\sqrt{\frac{l}{3g}}$ (b) $2\pi\sqrt{\frac{2l}{3g}}$

(c) $2\pi\sqrt{\frac{m}{2k}}$ (d) $2\pi\sqrt{\frac{2l}{g}}$ [DPMT 06]

33. A simple pendulum is vibrating in an evacuated chamber. It will oscillate with

(a) constant amplitude (b) increasing amplitude

(c) decreasing amplitude [DPMT 91]

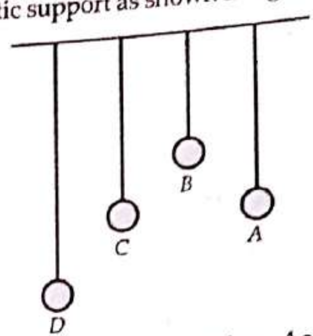
(d) none of these.

34. Two simple pendulums whose lengths are 100 cm and 121 cm are suspended side by side. Their bobs are pulled together and then released. After how many minimum oscillations of the longer pendulum, will the two be in phase again ?

(a) 11 (b) 10 [DPMT 05]

(c) 21 (d) 20.

35. Four pendulums A, B, C and D are hanged from the same elastic support as shown in figure. A and C are



of same length while B is smaller than A and D is larger than A. If A is given displacement, then at steady state

(a) D will vibrate with maximum amplitude

(b) C will vibrate with maximum amplitude

(c) B will vibrate with maximum amplitude

(d) all the four will oscillate with equal amplitude. [DPMT 02]

36. During the phenomenon of resonance

(a) the amplitude of oscillation becomes large

(b) the frequency of oscillation becomes large

(c) the time period of oscillation becomes large [DPMT 2011]

(d) all of these

Answers and Explanations

1. (b) $F(t) = 17\sin(20t + 0.5)$

Here $A = 17$

2. (b) $v = \omega\sqrt{A^2 - x^2}$

$\therefore$ $\pi = \pi\sqrt{A^2 - 1^2}$

$A^2 - 1 = 1$ or $A^2 = 2$

$\therefore$ $A = \sqrt{2}$ cm.

3. (a) Here $t = 2$ s, $v = 2$ ms^{-1}, $T = 16$ s

$$v = A\omega\cos\omega t$$

$$2 = A \times \frac{2\pi}{16}\cos\left(\frac{2\pi}{16} : 2\right)$$

$$A = \frac{16\sqrt{2}}{\pi} = 7.2 \text{ m.}$$

Scanned by CamScanner

4. (*b*) $\frac{T_1}{T_2} = \frac{1}{2}$

Refer to the solution of Problem 1 on page 14.67.

5. (*c*) In SHM,

$$v = \omega\sqrt{a^2 - y^2}.$$

6. (*c*) $v = \omega\sqrt{A^2 - y^2} = 2\sqrt{(60)^2 - (20)^2}$

$= 80\sqrt{2}$

$= \mathbf{113\ ms^{-1}}$.

7. (*b*) $v = \frac{2\pi}{T}\sqrt{A^2 - y^2}$

$= \frac{2\pi}{\pi}\sqrt{(10)^2 - 8^2}$ $[y = 10 - 2 = 8\text{ cm}]$

$= 2 \times 6 = \mathbf{12\ cm\ s^{-1}}$.

8. (*b*) $y = 3\sin(5\pi t + \pi)$

$y = A\sin(\omega t + \phi_0)$

$\omega = 2\pi\nu = 5\pi$

$\therefore$ $\nu = \mathbf{2.5\ Hz}$.

$T = \frac{1}{\nu} = \frac{1}{2.5} = \mathbf{0.4\ s}$.

9. (*d*) $x = -0.5\sin\left(2t + \frac{\pi}{4}\right)$

$x = A\sin(\omega t + \phi_0)$

$\therefore$ $\omega = 2\pi\nu = 2$

or $\nu = \frac{2}{2\pi} = \frac{1}{\pi}$ **Hz**.

10. (*c*) Acceleration in SHM is

$$a = \omega^2 y$$

At $y_{max} = A$, $a_{max} = \omega^2 A$

Acceleration is maximum at the position of maximum displacement.

11. (*c*) $v_{max} = \omega A$, $a_{max} = \omega^2 A$

$\therefore$ $\omega = \frac{a_{max}}{v_{max}} = \frac{4}{2}$

$= \mathbf{2\ rad/s}$.

12. (*a*) $a_{max} = \omega^2 A = 4\pi^2\nu^2 A$

$= 4\pi^2 \times 60 \times 60 \times 0.01$

$= \mathbf{144\ \pi^2\ m/s^2}$.

13. (*a*) $y = R\cos\omega t + R$

For y to be minimum,

$$\frac{dy}{dt} = -\omega R\sin\omega t = 0$$

$\sin\omega t = 0$

or $\omega t = 0, \pi, 2\pi, \ldots$

$$\frac{d^2y}{dT^2} = -\omega^2 R\cos\omega t$$

At $\omega t = \pi$,

$$\frac{d^2y}{dt^2} = +\omega^2 R > 0$$

Hence y is minimum at $\omega t = \pi$

$a_x = -\omega^2 R\sin\pi = 0$

$a_y = -\omega^2 R\cos\pi = \omega^2 \mathbf{R}$.

14. (*a*) Average K.E. over one time period

$$= \frac{1}{4}m\omega^2 A^2$$

Refer to the solution of NCERT Exercise 14.22.

15. (*d*) $E_{k_{av}} = E_{p_{av}} = \frac{1}{4}m\omega^2 A^2$

Refer to the Solution of NCERT Exercise 14.22.

16. (*a*) $E_p = \frac{1}{3}E$

$\frac{1}{2}ky^2 = \frac{1}{3}\cdot\frac{1}{2}kA^2$

$A = \sqrt{3}y = \sqrt{3} \times 4 = \mathbf{4\sqrt{3}\ cm}$.

17. (*b*) $E_p = \frac{1}{2}ky^2 = 2.5$ J

$\frac{1}{2}k\left(\frac{A}{2}\right)^2 = 2.5$

or $\frac{1}{4}\cdot\frac{1}{2}kA^2 = 2.5$

or $\frac{1}{4}E = 2.5$

$\therefore$ $E = 4 \times 2.5 = \mathbf{10\ J}$.

18. (*b*) $E_k = E_p$

or $\frac{1}{2}k(A^2 - y^2) = \frac{1}{2}ky^2$

or $y = \pm\frac{A}{\sqrt{2}}$

$= \pm\ \mathbf{0.71\ A}$.

Scanned by CamScanner

19. (*a*) Frequency of variation K.E. = **2*n***.

20. (*a*) For equal maximum velocities,

$$\omega_1 A_1 = \omega_2 A_2$$

$$\sqrt{\frac{k_1}{m}} \cdot A_1 = \sqrt{\frac{k_2}{m}} \cdot A_2$$

$$\frac{A_1}{A_2} = \left(\frac{k_2}{k_1}\right)^{1/2}.$$

21. (*a*) $k_s = \frac{k \times 2k}{k+2k} = \frac{2k}{3}$

$$k_p = k + 2k = 3k$$

$$\frac{k_s}{k_p} = \frac{2k/3}{3k} = \frac{2}{9}.$$

22. (*c*) $T = 2\pi\sqrt{\frac{M}{\frac{k_1 k_2}{k_1+k_2} + k_3}}$

$$T = 2\pi\sqrt{\frac{M(k_1+k_2)}{k_1k_2 + k_2k_3 + k_2k_1}}$$

or

$$\nu = \frac{1}{T}$$

or

$$= \frac{1}{2\pi}\sqrt{\frac{k_1k_2 + k_2k_3 + k_3k_1}{M(k_1+k_2)}}.$$

23. (*b*) Motion is simple harmonic only when amplitude of oscillation is small because only then $F \propto x$.

24. (*d*) $T = 2\pi\sqrt{\frac{l}{g}}$ or $T \propto \sqrt{l}$

When length is increased four times, time period gets doubled.

25. (*d*) As $T \propto \sqrt{l}$, $T' \propto \sqrt{4l}$

$$\frac{T'}{T} = 2$$

or

$$T' = 2T = 2 \times 2 = \mathbf{4\ s}.$$

26. (*b*) $n \propto \frac{1}{\sqrt{l}}$

$$\therefore \quad n' \propto \frac{1}{\sqrt{4l}}$$

$$\frac{n'}{n} = \sqrt{\frac{l}{4l}} = \frac{1}{2}$$

$$n' = \frac{n}{2}.$$

27. (*b*) $s = \frac{1}{2}gt^2$

or $8 = \frac{1}{2}g \times (2)^2$

or $g = 4\ \text{ms}^{-2}$

Also, $l = 1$ m

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

$$= 2\pi\sqrt{\frac{1}{4}} = \pi = \mathbf{3.14\ s}.$$

28. (*a*) As $T \propto \sqrt{l}$, the percentage increase in time period on increasing the length by 21%

$$= \frac{1}{2}\frac{\Delta l}{l} \times 100 = \frac{1}{2} \times 21 = \mathbf{10.5\%}.$$

29. (*c*) Inside the stationary lift,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

When the lift accelerates upwards,

$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$

$$= 2\pi\sqrt{\frac{l}{g + g/3}}$$

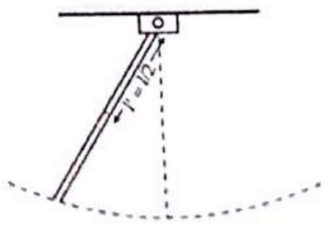
$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{g}{4g/3}} = \frac{\sqrt{3}}{2}$$

$$T' = \frac{\sqrt{3}}{2}T.$$

30. (*c*) Position of C.G. remains unaffected when mercury is thrown out. Hence effective length and time period remain same.

31. (*d*) Refer to the solution of Problem 2 on page 14.14.

32. (*b*)



$$T = 2\pi\sqrt{\frac{I}{mgl'}} = 2\pi\sqrt{\frac{\frac{1}{3}ml^2}{mg\frac{l}{2}}} = 2\pi\sqrt{\frac{2l}{3g}}.$$

Scanned by CamScanner

33. (*a*) In vacuum, there is no loss of energy due to resistive forces.

So, amplitude remains constant.

34. (*b*) $T \propto \sqrt{l}$

For the two pendulums in same phase

$$nT_1 = (n+1)T_2$$

$$n\sqrt{l_1} = (n+1)\sqrt{l_2}$$

$$n\sqrt{121} = (n+1)\sqrt{100}$$

$$n \times 11 = (n+1) \times 10$$

or $$n = \mathbf{10}.$$

35. (*b*) *C* will vibrate in resonance with *A*. Hence *C* will vibrate with maximum amplitude.

36. (*a*) The amplitude of oscillation is very large during the phenomenon of resonance.

Scanned by CamScanner

th A. Hence C

is very large

CHAPTER

15

WAVES

15.1 WAVE MOTION

1. *Define the term wave motion. Explain it with the help of a suitable example. Give important characteristics of wave motion.*

Wave motion. *Wave motion is a kind of disturbance which travels through a medium due to repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next.* In a wave, both information and energy propagate (in the form of signals) from one point to another but there is no motion of matter as a whole through a medium.

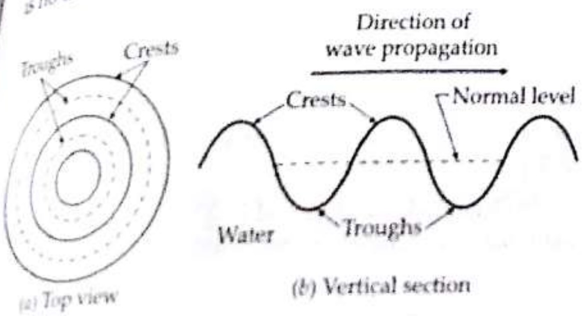


Fig. 15.1 Formation of waves over a water surface.

If we drop a pebble into a pond of still water, a circular pattern of alternate crests and troughs spreads out from the point where the pebble strikes the water surface. The kinetic energy of the pebble makes the particles oscillate which come in contact with it. These particles, in turn, transfer energy to the particles of next layer which also begin to oscillate. Energy is further transferred to the particles of next layer which also begin to oscillate and so on. In this way energy is transferred from one point to another. Further, if we throw a piece of paper or a cork on the water surface, it is found to oscillate up and down about the mean position and does not move forward with the wave. This shows that it is the disturbance or the wave which travels forward and not the particles of the medium.

Characteristics of wave motion. Some of the important characteristics of wave motion are as follows :

(*i*) In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.

(*ii*) The energy is transferred from one place to another without any actual transfer of the particles of the medium.

(*iii*) Each particle receives disturbance a little later than its preceding particle *i.e.*, there is a regular phase difference between one particle and the next.

(*iv*) The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.

Scanned by CamScanner

(v) The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.

(vi) For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

2. *What are the different types of waves we come across ? Give examples of each type.*

Different types of waves. The waves we come across are mainly of *three* types :

(i) **Mechanical waves.** *The waves which require a material medium for their propagation are called mechanical waves.* Such waves are also called elastic waves because their propagation depends on the elastic properties of the medium. These waves are governed by Newton's laws and can exist in a material mediums, such as water, air, rock etc.

Examples. Water waves, sound waves, seismic waves (waves produced during earthquake), etc.

(ii) **Electromagnetic waves.** *The waves which travel in the form of oscillating electric and magnetic fields are called electromagnetic waves. Such waves do not require any material medium for their propagation and are also called non-mechanical waves.* Light from the sun and distant stars reaches us through inter-stellar space, which is almost vacuum. All electromagnetic waves travel through vacuum at the same speed c, given by

$$c = 29{,}97{,}92{,}458 \text{ ms}^{-1} \quad \text{(speed of light)}$$

Examples. Visible and ultraviolet light, radiowaves, microwaves, X-rays, etc.

(iii) **Matter waves.** *The waves associated with microscopic particles, such as electrons, protons, neutrons, atoms, molecules, etc., when they are in motion, are called matter waves or de-Broglie waves.* These waves become important in the quantum mechanical description of matter.

Examples. Electron microscopes make use of the matter waves associated with fast moving electrons.

15.2 SPRING-MODEL FOR PROPAGATION OF A WAVE THROUGH AN ELASTIC MEDIUM

3. *How the propagation of a wave through an elastic medium can be explained on the basis of spring-model. Hence explain the propagation of a sound wave in (a) air and (b) solids.*

Spring-model for the propagation of a wave through an elastic medium. As shown in Fig. 15.2, consider a number of springs connected to one another. One end is fixed to a rigid support. The first spring is pulled to the left and released. It gets stretched. Due to elasticity, a restoring force is developed in the first spring. This force brings to the first spring back to its original size and stretches the second spring, and so on. Thus the disturbance moves from one end to the other, but each spring only executes small oscillations about its equilibrium position or length.

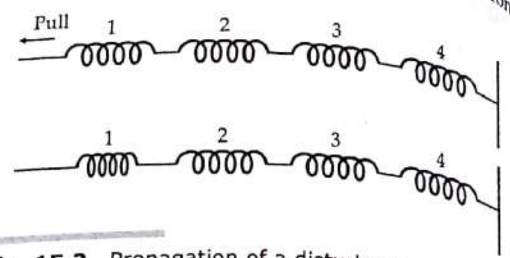


Fig. 15.2 Propagation of a disturbance through a combination of springs.

The above spring model can be used to explain the propagation of sound waves through air or any solid.

(a) **Propagation of sound waves through air.** A small region of air can be considered as a spring. It is connected to the neighbouring regions or springs. As sound wave travels through air, it compresses or expands a small region of air. This changes the density and pressure of the region.

According to Boyle's law,

Change in pressure (ΔP) $\propto$ change in density ($\Delta \rho$)

As the pressure is force per unit area, so a restoring force proportional to the change in density is developed, just like in an extended or compressed spring. If a region is compressed, its molecules tend to move out to the adjacent region, thereby increasing the density or creating *compression* in that region. The density of air in the first region decreases and is called *rarefaction*. But when a region is rarefied, the air of the surrounding air rushes in. This shifts the rarefaction to the adjacent region. Thus compressions and rarefactions move from one region to another. This makes possible the propagation of disturbance in air.

(b) **Propagation of sound in a solid.** In a crystalline solid, various atoms can be considered as end points, with springs connected between pairs of them. Each atom is in its state of equilibrium, as the forces exerted by the other atoms are cancelled out. When an elastic (sound) wave propagates, the atom is displaced from its equilibrium position and a restoring force is developed. The disturbance produced by the force travels to the next atom and so on. Thus the wave propagates through the solid.

Scanned by CamScanner

15.3 TRANSVERSE AND LONGITUDINAL WAVES

4. *What are transverse and longitudinal waves ? Explain with suitable examples.*

Types of wave motion. Depending on the relationship between the direction of oscillation of individual particles and the direction of wave propagation, the waves are classified into *two* categories : transverse waves and longitudinal waves.

Transverse waves. *These are the waves in which the individual particles of the medium oscillate perpendicular to the direction of wave propagation.* As shown in Fig. 15.3(*a*), consider a horizontal string with its one end fixed to a rigid support and other end held in the hand. If we give its free end a smart upward jerk, an upward kink or pulse is created there which travels along the string towards the fixed end. Each part of the string successively undergoes a disturbance about its mean position. As shown in Fig. 15.3(*b*), if we continuously give up and down jerks to the free end of the string, a number of sinusoidal waves begin to travel along the string.

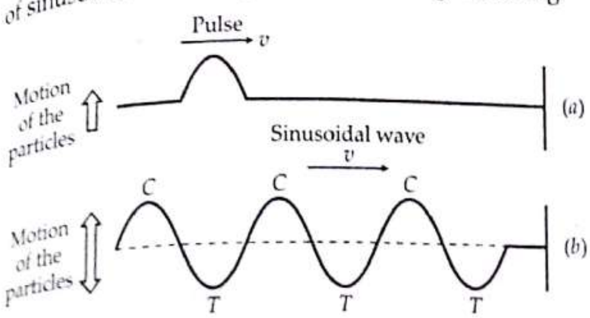


Fig. 15.3 (*a*) A single pulse, (*b*) A sinusoidal wave sent along a stretched string.

Each part of the string vibrates up and down while the wave travels along the string. So the waves in the string are transverse in nature.

The points (C, C, ...) of maximum displacement in the upward direction are called *crests*. The points (T, T, ...) of maximum displacement in the downward direction are called *troughs*. *One crest and one trough together form one wave.*

Longitudinal waves. *These are the waves in which the individual particles of the medium oscillate along the direction of wave propagation.*

As shown in Fig. 15.4, consider a long hollow cylinder *AB* closed at one end and having a movable piston at the other end. If we suddenly move the piston rapidly towards right, a small layer of air just near the piston-head is compressed and after being compressed, this layer moves towards right and compresses the next layer and soon the compression reaches the other end. Now if the piston is suddenly moves towards left, the layer adjacent to it is rarefied resulting in the fall of pressure. The air from the next layer moves in to restore pressure. Consequently the next layer is rarefied. In this way a pulse of rarefaction moves towards right.

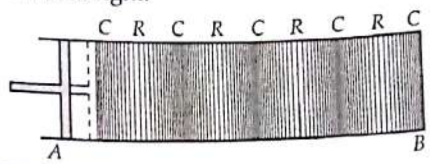


Fig. 15.4 A sound wave produced in a cylinder by moving a piston back and forth.

If we continuously push and pull the piston in a simple harmonic manner, a sinusoidal sound wave travels along the cylinder in the form of alternate compressions and rarefactions, marked C, R, C, R, etc. As the oscillations of an element of air are parallel to direction of wave propagation, the wave is a longitudinal wave. Hence sound waves produced in air are longitudinal waves.

5. *Mention the important properties which a medium must possess for the propagation of mechanical waves through it.*

Essential properties of a medium for the propagation of mechanical waves. Both transverse and longitudinal waves can propagate through those media which have the following properties :

(*i*) ***Elasticity.*** The medium must possess elasticity so that the particles can return to their mean positions after being disturbed.

(*ii*) ***Inertia.*** The medium must possess inertia or mass so that its particles can store kinetic energy.

(*iii*) ***Minimum friction.*** The frictional force amongst the particles of the medium should be negligibly small so that they continue oscillating for a sufficiently long time and the wave travels a sufficiently long distance through the medium.

6. *Through what type of media, can (i) transverse waves and (ii) longitudinal waves be transmitted ? Give reason.*

(*i*) **Media through which transverse waves can propagate.** Transverse waves travel in the form of crests and troughs. They involve changes in the shape of the medium. So they can be transmitted through media which have rigidity. As solids and strings can sustain shearing stress, so transverse waves can be formed in solids and strings, not in fluids.

Due to surface tension, the free surface of liquid tends to maintain its level. So transverse waves can be formed over liquid surfaces.

Scanned by CamScanner

(*ii*) **Media through which longitudinal waves can propagate.** Longitudinal waves travel in the form of compressions and rarefactions. They involve changes in volume and density of the medium. All media—solids, liquids and gases can sustain compressive stress, so longitudinal waves can be transmitted through all the three types of media.

15.4 SOME DEFINITIONS IN CONNECTION WITH WAVE MOTION

7. *In reference to a wave motion, define the terms (i) amplitude, (ii) time period, (iii) frequency, (iv) angular frequency, (v) wavelength, (vi) wave number, (vii) angular wave number and (viii) wave velocity.*

Some definitions in connection with wave motion. When a transverse or a longitudinal wave propagates through a medium, all the particles of the medium oscillate about the mean positions in the same manner but the phase of oscillation changes from one particle to the next.

(*i*) **Amplitude.** *It is the maximum displacement suffered by the particles of the medium about their mean positions.* It is denoted by *A*.

(*ii*) **Time period.** *The time period of a wave is the time in which a particle of medium completes one vibration to and fro about its mean position.* It is denoted by *T*.

(*iii*) **Frequency.** *The frequency of a wave is the number of waves produced per unit time in the given medium.* It is equal to the number of oscillations completed per unit time by any particle of the medium. It is equal to the reciprocal of the time period *T* of the particle and is denoted by ν. Thus

$$\nu = \frac{1}{T}$$

SI unit of ν is s^{-1} or hertz (Hz).

(*iv*) **Angular frequency.** *The rate of change of phase with time is called angular frequency of the wave.* It is clearly equal to $2\pi/T$, because the phase change in time *T* is 2π. It is denoted by ω. Thus

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

SI unit of ω = rad s^{-1}.

(*v*) **Wavelength.** *It is the distance covered by a wave during the time in which a particle of the medium completes one vibration to and fro about its mean position.* Or, *it is the distance between two nearest particles of the medium which are vibrating in the same phase.* It is denoted by λ.

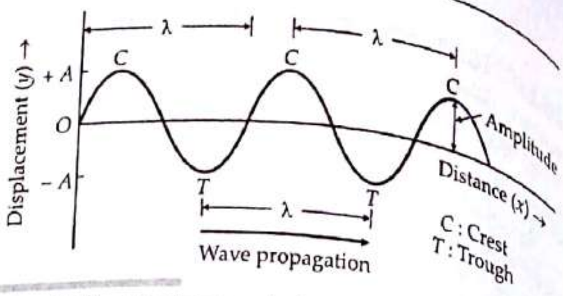


Fig. 15.5 Displacement-distance (*y*–*x*) graph for a transverse wave.

In a transverse wave, the distance between two successive crests or troughs is equal to the wavelength λ, as shown in Fig. 15.5. In a longitudinal wave, the distance between the centres of two nearest compressions or rarefactions is equal to wavelength λ.

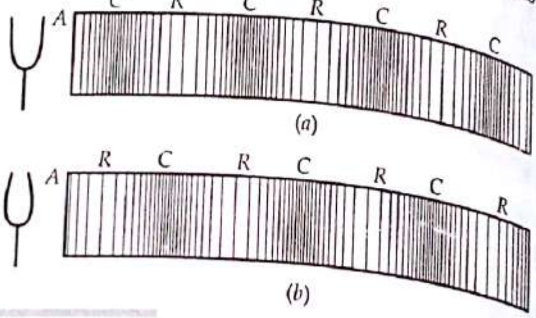


Fig. 15.6 Sound (longitudinal) waves from a tuning fork.

(*vi*) **Wave number.** *The number of waves present in a unit distance of the medium is called wave number.* It is equal to the reciprocal of wavelength λ. Thus

$$\text{Wave number, } \bar{\nu} = \frac{1}{\lambda}$$

SI unit of wave number = m^{-1}

(*vii*) **Angular wave number or propagation constant.** *The quantity* $2\pi/\lambda$ *is called angular wave number or propagation constant of a wave.* It represents the phase change per unit path difference. It is denoted by *k*. Thus

$$k = \frac{2\pi}{\lambda}$$

The SI unit of *k* is radian per metre or rad m^{-1}.

(*viii*) **Wave velocity or phase velocity.** *The distance covered by a wave per unit time in its direction of propagation is called its wave velocity or phase velocity.* It is denoted by *v*.

8. *Derive relation between wave velocity, frequency and wavelength of a wave.*

Relation between wave velocity, frequency and wavelength. We know that when a particle of the

Scanned by CamScanner

...medium completes one oscillation about its mean position in periodic time T, the wave travels a distance equal to its wavelength λ. Therefore,

$$\text{Wave velocity} = \frac{\text{Distance}}{\text{Time}}$$

$$v = \frac{\lambda}{T}$$

$$v = \nu\lambda \qquad [\because \nu = 1/T]$$

Wave velocity = Frequency × Wavelength

Examples based on Relation between Frequency, Wavelength and Wave Velocity

FORMULAE USED

1. Wave velocity = Frequency × Wavelength
   $v = \nu\lambda$
   or
2. $\text{Wave velocity} = \frac{\text{Wavelength}}{\text{Time period}}$ or $v = \frac{\lambda}{T}$
3. $\text{Wavelength} = \frac{\text{Wave velocity}}{\text{Frequency}}$ or $\lambda = \frac{v}{\nu}$

UNITS USED

Wavelength λ is in metre, frequency ν in Hz or s^{-1}, time period T in second and wave velocity v in ms^{-1}.

EXAMPLE 1. *How far does the sound travel in air when a tuning fork of frequency 256 Hz makes 64 vibrations? Velocity of sound in air = 320 ms^{-1}.* **[Delhi 05]**

Solution. Here $\nu = 256$ Hz,

$$v = 320 \text{ ms}^{-1}$$

Distance travelled by the wave in one vibration is equal to its wavelength.

$$\lambda = \frac{v}{\nu} = \frac{320}{256} = 125 \text{ m}$$

Distance travelled by the wave in 64 vibrations

$$= 125 \times 64 = \mathbf{80\ m.}$$

EXAMPLE 2. *A source of sound is placed at one end of an iron bar two kilometre long and two sounds are heard at the other end at an interval of 5.6 seconds. If the velocity of sound in air is 330 ms^{-1}, find the velocity of sound in iron.*

Solution. One sound is heard through air and another through iron.

Time taken by sound in air,

$$t = \frac{\text{Distance}}{\text{Velocity}}$$

$$= \frac{2000 \text{ m}}{330 \text{ ms}^{-1}} = 6.06 \text{ s}$$

As the interval between the two sounds is 5.6 s and sound travels faster in iron than in air, so the time taken by sound in iron is

$$t' = 6.06 - 5.6 = 0.46 \text{ s}$$

Velocity of sound in iron

$$= \frac{\text{Distance}}{\text{Time}} = \frac{2000 \text{ m}}{0.46 \text{ s}} = \mathbf{4348\ ms^{-1}}.$$

EXAMPLE 3. *Audible frequencies have a range 20 Hz to 20,000 Hz. Express this range in terms of (i) period T (ii) wavelength λ in air and (iii) angular frequency. Given velocity of sound in air is 330 ms^{-1}.*

Solution. Here $\nu_1 = 20$ Hz, $\nu_2 = 20{,}000$ Hz,

$$v = 330 \text{ ms}^{-1}$$

(*i*) $T_1 = \frac{1}{\nu_1} = \frac{1}{20} = 5 \times 10^{-2}$ s,

$$T_2 = \frac{1}{\nu_2} = \frac{1}{20{,}000} = 5 \times 10^{-5} \text{ s}$$

Thus the audible range in terms of period is from 5×10^{-2} s to 5×10^{-5} s.

(*ii*) $\lambda_1 = \frac{v}{\nu_1} = \frac{330}{20} = 16.5$ m

$$\lambda_2 = \frac{v}{\nu_2} = \frac{330}{20{,}000} = 0.0165 \text{ m}$$

Thus the audible range in terms of wavelength is from 16.5 m to 0.0165 m.

(*iii*) $\omega_1 = 2\pi\nu_1 = 2\pi \times 20 = 40\pi \text{ rad s}^{-1}$

$$\omega_2 = 2\pi\nu_2 = 2\pi \times 20{,}000 = 40{,}000\pi \text{ rad s}^{-1}$$

Thus the audible range in terms of angular frequency is from 4π rad s^{-1} to 40,000 π rad s^{-1}.

PROBLEMS FOR PRACTICE

1. A radio station broadcasts its programme at 219.3 metre wavelength. Determine the frequency of radio waves if velocity of radio waves be 3×10^8 ms^{-1}. **(Ans.** 1.368×10^6 Hz**)**
2. The audible frequency range of a human's ear is 20 Hz – 20 kHz. Convert this into the corresponding wavelength range. Take the speed of sound in air at ordinary temperatures to be 340 ms^{-1}. **(Ans.** 0.017 m to 17 m**)**
3. The speed of a wave in a medium is 960 ms^{-1}. If 3600 waves are passing through a point in the medium in 1 minute, then calculate the wavelength. **(Ans.** 16 m**)**
4. If a splash is heard 4.23 seconds after a stone is dropped into a well 78.4 m deep, find the speed of sound in air. **(Ans.** 340.87 ms^{-1}**)**

Scanned by CamScanner

5. A stone is dropped into a well and its splash is heard at the mouth of the well after an interval of 1.45 s. Find the depth of the well. Given that velocity of sound in air at room temperature is equal to 332 ms^{-1}. [MNREC 81]
(Ans. 9.9 m)

6. A body sends waves 100 mm long through medium A and 0.25 m long in medium B. If the velocity of waves in medium A is $80 cm s^{-1}$, calculate the velocity of waves in medium B. (Ans. 2 ms^{-1})

✕ HINTS

2. Here $\lambda_1 = \frac{v}{\nu_1} = \frac{340}{20} = 17$ **m.**

and $\lambda_2 = \frac{v}{\nu_2} = \frac{340}{20 \times 10^3} = 0.017$ **m.**

3. Speed of the wave,
$$v = 960 \text{ ms}^{-1}$$
Frequency of the wave,
$$\nu = 3600 \text{ min}^{-1} = \frac{3600}{60} = 60 s^{-1}$$
Wavelength,
$$\lambda = \frac{v}{\nu} = \frac{960}{60} = 16 \text{ m.}$$

4. For downward motion of the stone,
$u = 0$, $a = 9.8 \text{ ms}^{-2}$, $s = 74.8$ m, $t = ?$

As $s = ut + \frac{1}{2}at^2$

$\therefore$ $78.4 = 0 + \frac{1}{2} \times 9.8 t^2 = 4.9 t^2$

or $t^2 = \frac{78.4}{4.9} = 16$ or $t = 4$ s

Let t' be the time taken by the splash of sound to reach the top of the well. Then
$$t + t' = 4 + t' = 4.23 \text{ s} \quad \text{or} \quad t' = 4.23 - 4 = 0.23 \text{ s}$$
Speed of sound in air
$$= \frac{\text{Distance}}{\text{Time}} = \frac{78.4}{0.23} = 340.87 \text{ ms}^{-1}.$$

5. Let h be the depth of the well. Then time t_1 taken by the stone to fall into well under gravity is given by
$$h = 0 + \frac{1}{2}gt_1^2 \quad \text{or} \quad t_1 = \sqrt{\frac{2h}{g}}$$
Time taken for the splash to travel height h is given by
$$t_2 = \frac{h}{v}$$
where v = velocity of sound

But $t_1 + t_2 = 1.45$ s

$\therefore$ $\sqrt{\frac{2h}{g}} + \frac{h}{v} = 1.45$

or $\sqrt{\frac{2h}{9.8}} + \frac{h}{332} = 1.45$

On solving, $h = 9.9$ **m.**

6. Here $\lambda_A = 100$ mm $= 0.10$ m, $\lambda_B = 0.25$ m,
$v_A = 80 \text{ cm s}^{-1} = 0.80 \text{ ms}^{-1}$

As the frequency of the wave remains same in the two media, so
$$\nu = \frac{v_A}{\lambda_A} = \frac{v_B}{\lambda_B}$$
$$\therefore \quad v_B = \frac{\lambda_B}{\lambda_A} \times v_A = \frac{0.25}{0.10} \times 0.80 = 2 \text{ ms}^{-1}.$$

15.5 SPEED OF TRANSVERSE WAVES

9. *On the basis of dimensional considerations, write the formula for the speed of transverse waves (a) on a stretched string and (b) in a solid.*

(*a*) **Speed of a transverse wave on a stretched string.** The wave velocity through a medium depends on its inertial and elastic properties. So the speed of transverse wave through a stretched string is determined by two factors :

(*i*) Tension T in the string is a measure of elasticity in the string. Without tension no disturbance can propagate in the string.
Dimensions of T = [Force] = $[MLT^{-2}]$

(*ii*) Mass per unit length or *linear mass density m* of the string so that the string can store kinetic energy.
$$\text{Dimensions of } m = \frac{[\text{Mass}]}{[\text{Length}]} = [ML^{-1}]$$

$$\text{Now, dimensions of ratio } \frac{T}{m} = \frac{[MLT^{-2}]}{[ML^{-1}]} = [L^2T^{-2}]$$

As the speed v has the dimensions $[LT^{-1}]$, so we can express v in terms of T and m as
$$v = C\sqrt{\frac{T}{m}}$$

From detailed mathematical analysis or from experiments, the dimensionless constant $C = 1$. Hence the speed of transverse waves on a stretched string is given by
$$v = \sqrt{\frac{T}{m}}$$

Scanned by CamScanner

Clearly, the speed of a transverse wave along a stretched string depends only on the tension T and linear mass density m of the string. It does not depend on the frequency of the wave. The frequency of a wave depends on the source generating that wave.

(b) **Speed of transverse wave in a solid.** The speed of transverse wave through a solid is determined by two factors : (i) Elasticity of shape or modulus of rigidity η of the solid. (ii) Mass per unit volume or density ρ determines its inertia.

Now,

Dimensions of ratio $\frac{\eta}{\rho} = \frac{[ML^{-1}T^{-2}]}{[ML^{-3}]} = [L^2T^{-2}]$

Dimensions of speed $v = [LT^{-1}]$

So we can express v in terms of η and ρ as

$$v = C\sqrt{\frac{\eta}{\rho}}$$

The dimensionless constant C is found to be unity. Thus the speed of transverse wave in a solid is given by

$$v = \sqrt{\frac{\eta}{\rho}}.$$

Examples based on Velocity of Transverse Waves in Solids and Strings

Formulae Used

1. Velocity of transverse waves in a solid of modulus of rigidity η and density ρ,

$$v = \sqrt{\frac{\eta}{\rho}}$$

2. Velocity of transverse waves in a string of mass per unit length m and stretched under tension T,

$$v = \sqrt{\frac{T}{m}}$$

Units Used

Here η is Nm^{-2}, ρ in $kg\ m^{-3}$, tension T in N, linear mass density in $kg\ m^{-1}$ and velocity v in ms^{-1}.

EXAMPLE 4. *For aluminium the modulus of rigidity is $2.1 \times 10^{10}\ Nm^{-2}$ and density is $2.7 \times 10^3\ kg\ m^{-3}$. Find the speed of transverse waves in the medium.*

Solution. Here $\eta = 2.1 \times 10^{10}\ Nm^{-2}$,

$$\rho = 2.7 \times 10^3\ kg\ m^{-3}$$

Speed of transverse waves in aluminium is given by

$$v = \sqrt{\frac{\eta}{\rho}} = \sqrt{\frac{2.1 \times 10^{10}}{2.7 \times 10^3}} = \mathbf{2.79 \times 10^3\ ms^{-1}}.$$

EXAMPLE 5. *A steel wire 0.72 m long has a mass of 5.0×10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse waves on the wire ?* **[NCERT]**

Solution. Here $T = 60$ N, Mass $= 5.0 \times 10^{-3}$ kg, Length $= 0.72$ m

Mass per unit length,

$$m = \frac{5.0 \times 10^{-3}\ kg}{0.72\ m} = \mathbf{6.9 \times 10^{-3}\ kg\ m^{-1}}$$

The speed of the transverse wave on the wire,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{60\ N}{6.9 \times 10^{-3}\ kgm^{-1}}} = \mathbf{93\ ms^{-1}}.$$

EXAMPLE 6. *In a sonometer experiment, the density of the material of the wire used is $7.5 \times 10^3\ kg\ m^{-3}$. If the stress of the wire is $3.0 \times 10^8\ Nm^{-2}$, find out the speed of the transverse wave in the wire.*

Solution. Let A be the area of cross-section of the wire.

Tension in the wire,

$$T = \text{Stress} \times \text{area} = 3.0 \times 10^8 \times A \text{ newton}$$

Mass per unit length,

$$m = A \times 1 \times \rho = A \times 7.5 \times 10^3\ kg\ m^{-1}$$

Speed,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{3.0 \times 10^8 \times A}{A \times 7.5 \times 10^3}} = \mathbf{200\ ms^{-1}}.$$

EXAMPLE 7. *A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut with negligible tension. Find the speed of transverse waves in the wire at 10°C. ($\alpha = 1.7 \times 10^{-5}\ °C^{-1}$, $Y = 1.4 \times 10^{11}\ Nm^{-2}$ and $\rho = 9 \times 10^3\ kg\ m^{-3}$).*

Solution. When the temperature changes from 30°C to 10°C, then change in length of the wire is

$$\Delta l = \alpha\ l\ \Delta T = 1.7 \times 10^{-5} \times l \times (30 - 10)$$
$$= l \times 3.4 \times 10^{-4}\ m$$

Young's modulus,

$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

So the tension produced in the wire is

$$T = F = \frac{Y A \Delta l}{l} = \frac{1.4 \times 10^{11} \times A \times l \times 3.4 \times 10^{-4}}{l}$$
$$= 4.76 \times 10^7 \times A \text{ newton}$$

Mass per unit length,

$$m = A \times 1 \times \rho = A\rho = A \times 9 \times 10^3\ kg\ m^{-1}$$

Speed of transverse wave,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{4.76 \times 10^7 \times A}{A \times 9 \times 10^3}} = \mathbf{72\ ms^{-1}}.$$

Scanned by CamScanner

Problems For Practice

1. A steel wire 70 cm long has a mass of 7 kg. If the wire is under a tension of 100 N, what is the speed of transverse waves in the wire ? (Ans. 100 ms^{-1})

2. The speed of a transverse wave in a stretched string is 348 ms^{-1}, when the tension of the string is 3.6 kg wt. Calculate the speed of the transverse wave in the same string, if the tension in the string is changed to 4.9 kg wt. (**Ans.** 406 ms^{-1})

3. Calculate the velocity of transverse waves in a copper wire 1 mm^2 in cross-section, under the tension produced by 1 kg wt. The density of copper $= 8.93 \text{ kg m}^{-3}$. (**Ans.** 33.12 ms^{-1})

4. A wave-pulse is travelling on a string of linear mass density 10 g cm^{-1} under a tension of 1 kg wt. Calculate the time taken by the pulse to travel a distance of 50 cm on the string. Given $g = 10 \text{ ms}^{-2}$. (Ans. 0.05 s)

5. The diameter of an iron wire is 1.20 mm. If the speed of the transverse wave in the wire be 50.0 ms^{-1}, what is the tension in the wire ? The density of iron is $7.7 \times 10^3 \text{ kg m}^{-3}$. (**Ans.** 21.78 N)

Hints

2. $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \qquad \text{(For a given string)}$$

$$\therefore \quad v_2 = \sqrt{\frac{T_2}{T_1}} \times v_1 = \sqrt{\frac{4.9 \times g}{3.6 \times g}} \times 348$$

$$= \frac{7}{6} \times 348 = \mathbf{406 \text{ ms}^{-1}}.$$

3. Here $A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$, $\rho = 8.93 \times 10^3 \text{ kg m}^{-3}$,

$$T = 1 \text{ kg wt} = 9.8 \text{ N}$$

Mass per unit length of wire,

$$m = A \times 1 \times \rho = 10^{-6} \times 8.93 \times 10^3$$

$$= 8.93 \times 10^{-3} \text{ kg m}^{-1}$$

$$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{9.8}{8.93 \times 10^{-3}}}$$

$$= \mathbf{33.12 \text{ ms}^{-1}}.$$

4. Here $$m = \frac{1 \text{ g}}{1 \text{ cm}} = \frac{10^{-3} \text{ kg}}{10^{-2} \text{ m}} = 10^{-1} \text{ kg m}^{-1}$$

$$T = 1 \text{ kg wt} = 10 \text{ N}$$

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{10}{10^{-1}}} = 10 \text{ ms}^{-1}$$

Time taken to travel 50 cm $= \dfrac{50 \times 10^{-2} \text{ m}}{10 \text{ ms}^{-1}} = \mathbf{0.05 \text{ s}}.$

5. Mass per unit length,

$$m = A \times 1 \times \rho = \frac{\pi d^2 \rho}{4}$$

As $$v = \sqrt{\frac{T}{m}} \quad \text{or} \quad v^2 = \frac{T}{m}$$

$$\therefore \quad T = mv^2 = \frac{\pi d^2 \rho v^2}{4}$$

$$= \frac{22 \times (1.20 \times 10^{-3})^2 \times 7.7 \times 10^3 \, (50)^2}{7 \times 4}$$

$$= \mathbf{21.78 \text{ N}}.$$

15.6 SPEED OF A LONGITUDINAL WAVE

10. *Write expression for the speed of a longitudinal wave in (a) a liquid or gas, (b) a solid and (c) a long solid rod.*

(*a*) **Speed of a longitudinal wave in a liquid or gas.** In a longitudinal wave, the particles of the medium oscillate forward and backward in the direction of propagation of the wave. They cause compressions and rarefactions of small volume elements of fluid. So the speed of a longitudinal wave through a fluid is determined by two factors :

(*i*) The volume elasticity or bulk modulus κ of the fluid.

(*ii*) The density of the fluid which determines its inertia.

$\therefore$ Dimensions of the ratio $\dfrac{\kappa}{\rho}$

$$= \frac{[\text{ML}^{-1}\text{T}^{-2}]}{[\text{ML}^{-3}]} = [\text{L}^2\text{T}^{-2}]$$

Dimensions of speed $v = [\text{LT}^{-1}]$

So the speed v can be expressed in terms of κ and ρ as

$$v = C\sqrt{\frac{\kappa}{\rho}}$$

The dimensionless constant C is found to be unity. Hence the speed of a longitudinal wave in any fluid (liquid or gas) is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

Clearly, *the speed of a longitudinal wave through a fluid depends only on its bulk modulus κ and density ρ.*

(*b*) **Speed of a longitudinal wave in a solid.** The speed of a longitudinal wave through a solid of bulk modulus κ, modulus of rigidity η and density ρ is given by

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

Scanned by CamScanner

(*i*) **Speed of a longitudinal wave in a solid rod.** When a long solid rod is given blows at one end, longitudinal waves travel through it in the form of compressions and rarefactions. As the sidewise expansion of the rod is negligible, we need to consider only longitudinal strain. In this case, the relevant modulus of elasticity is the Young's modulus. Hence the speed of a longitudinal wave through a solid rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}.$$

15.7 SPEED OF SOUND : NEWTON'S FORMULA AND LAPLACE CORRECTION

11. *Write Newton's formula for the speed of sound in a gas. Why and what correction was applied by Laplace in this formula ?*

Newton's formula for the speed of sound in a gas. Newton gave the first theoretical expression for the speed of sound in a gas. He assumed that sound waves travel through a gas under isothermal conditions. He argued that the small amount of heat produced in a compression is rapidly conducted to the surrounding rarefactions where slight cooling is produced. Thus the temperature of gas remains constant. If κ_{iso} is the isothermal volume elasticity (bulk modulus of the gas at constant temperature), then the speed of sound in the gas will be

$$v = \sqrt{\frac{\kappa_{iso}}{\rho}}$$

For an isothermal change,

$$PV = \text{constant} \qquad \text{(Boyle's law)}$$

Differentiating both sides, we get

$$P\,dV + V dP = 0$$

or $$P dV = -V\,dP$$

or $$P = -\frac{VdP}{dV} = -\frac{dP}{dV/V}$$

$$= \frac{\text{Volume stress}}{\text{Volume strain}} = \kappa_{iso}$$

Hence the *Newton's formula* for the speed of sound in a gas is

$$v = \sqrt{\frac{P}{\rho}}$$

At STP, $P = 0.76$ m of Hg $= 0.76 \times 13.6 \times 10^3 \times 9.8$
$= 1.013 \times 10^5\ \text{Nm}^{-2}$

ρ = Density of air = $1.293\ \text{kg m}^{-3}$.

$\therefore$ Speed of sound in air at STP,

$$v = \sqrt{\frac{1.013 \times 10^5}{1.293}} = 280\ \text{ms}^{-1}.$$

This value is about 15% less than the experimental value ($331\ \text{ms}^{-1}$) of the speed of sound in air at STP. Hence Newton's formula is not acceptable.

Laplace's correction. In 1816, the French scientist *Laplace* pointed out that sound travels through a gas under adiabatic conditions not under isothermal conditions (as suggested by Newton). This is because of the following reasons :

(*i*) As sound travels through a gas, temperature rises in the regions of compressions and falls in the regions of rarefactions.

(*ii*) A gas is a poor conductor of heat.

(*iii*) The compressions and rarefactions are formed so rapidly that the heat generated in the regions of compressions does not get time to pass into the regions of rarefactions so as to equalise the temperature.

So when sound travels through a gas, the pressure-temperature does not remain constant. The pressure-volume variations are adiabatic. If κ_{adia} is the adiabatic bulk modulus of the gas, then the formula for the speed of sound in the gas would be

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}}$$

For an adiabatic change, $PV^{\gamma} = \text{constant}$

Differentiating both sides, we get

$$P(\gamma V^{\gamma - 1})\,dV + V^{\gamma}\,dP = 0$$

or $$\gamma\,PdV + VdP = 0$$

$$\gamma\,P = -\frac{dP}{dV/V} = \kappa_{adia}$$

where $\gamma = C_p / C_v$, is the ratio of two specific heats.

Hence the Laplace formula for the speed of sound in a gas is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

This modification of Newton's formula is known as *Laplace correction.*

For air $\gamma = 7/5$, so speed of sound in air at STP will be

$$v = \sqrt{\gamma}\,\sqrt{\frac{P}{\rho}} = \sqrt{\frac{7}{5}} \times 280 = 331.3\ \text{ms}^{-1}.$$

This value is in close agreement with the experimental value. Hence the Laplace correction is justified.

Scanned by CamScanner

Examples based on Velocity of Longitudinal Waves

FORMULAE USED

1. Velocity of longitudinal waves in a solid of bulk modulus κ, modulus of rigidity η and density ρ is given by
$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$
2. Velocity of longitudinal waves in a long rod of Young's modulus Y and density ρ is given by
$$v = \sqrt{\frac{Y}{\rho}}$$
3. Velocity of longitudinal waves in liquid of bulk modulus κ and density ρ is given by
$$v = \sqrt{\frac{\kappa}{\rho}}$$
4. *Newton's formula* for the velocity of sound in a gas is
$$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}$$
where P = pressure of a gas
5. *Laplace formula* for the velocity of sound in a gas is
$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}, \quad \text{where} \quad \gamma = \frac{C_p}{C_v}$$

UNITS USED

Modulii of elasticity κ, Y and η are in Nm^{-2}, pressure P in Nm^{-2} or Pa, velocity v in ms^{-1} and specific heats ratio γ has no units.

EXAMPLE 8. *For aluminium the bulk modulus and modulus of rigidity are 7.5×10^{10} Nm^{-2} and 2.1×10^{10} Nm^{-2}. Find the velocity of longitudinal waves in the medium. Density of aluminium is 2.7×10^3 kg m^{-3}.*

Solution. Here $\kappa = 7.5 \times 10^{10}$ Nm^{-2},

$\eta = 2.1 \times 10^{10}$ Nm^{-2}, $\rho = 2.7 \times 10^3$ kg m^{-3}

Velocity of longitudinal waves in aluminium is

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}} = \sqrt{\frac{7.5 \times 10^{10} + \frac{4}{3} \times 2.1 \times 10^{10}}{2.7 \times 10^3}}$$
$$= 6.18 \times 10^3 \text{ ms}^{-1}.$$

EXAMPLE 9. *For a steel rod, the Young's modulus of elasticity is 2.9×10^{11} Nm^{-2} and density is 8×10^3 kg m^{-3}. Find the velocity of the longitudinal waves in the steel rod.*

Solution. Here $Y = 2.9 \times 10^{11}$ Nm^{-2},

$\rho = 8 \times 10^3$ kg m^{-3}

Velocity of longitudinal waves in steel is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.9 \times 10^{11}}{8 \times 10^3}} = 6.02 \times 10^3 \text{ ms}^{-1}.$$

EXAMPLE 10. *At a pressure of 10^5 Nm^{-2}, the volume strain of water is 5×10^{-5}. Calculate the speed of sound in water. Density of water is 10^3 kg m^{-3}.*

Solution. Bulk modulus of water is

$$\kappa = \frac{\text{Normal stress (pressure)}}{\text{Volume strain}}$$
$$= \frac{10^5}{5 \times 10^{-5}} = 2 \times 10^9 \text{ Nm}^{-2}$$

Density, $\rho = 10^3$ kg m^{-3}

Speed of sound in water is

$$v = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{2 \times 10^9}{10^3}} = 1.414 \times 10^3 \text{ ms}^{-1}.$$

EXAMPLE 11. *Estimate the speed of sound in air at standard temperature and pressure by using (i) Newton's formula and (ii) Laplace formula. The mass of 1 mole of air $= 29.0 \times 10^{-3}$ kg. For air, $\gamma = 1.4$.* [NCERT]

Solution. Density of air,

$$\rho = \frac{\text{Mass of 1 mole of air}}{\text{Volume of 1 mole of air}}$$
$$= \frac{29.0 \times 10^{-3} \text{ kg}}{22.4 \text{ litre}}$$
$$= \frac{29.0 \times 10^{-3} \text{ kg}}{22.4 \times 10^{-3} \text{ m}^3} = 1.29 \text{ kg m}^{-3}$$

Standard pressure, $P = 1.01 \times 10^5$ Pa.

(*i*) According to Newton's formula, speed of sound in air at S.T.P. is

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.01 \times 10^5}{1.29}} = 280 \text{ ms}^{-1}.$$

(*ii*) According to Laplace formula, speed of sound in air at S.T.P. is

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.29}} = 331.5 \text{ ms}^{-1}.$$

PROBLEMS FOR PRACTICE

1. The speed of sound in a liquid is 1500 ms^{-1}. The density of the liquid is 1.0×10^3 kg m^{-3}. Determine the bulk modulus of elasticity of the liquid.

(**Ans.** 2.25×10^9 Nm^{-2})

Scanned by CamScanner

2. The longitudinal waves starting from a ship return from the bottom of the sea to the ship after 2.64 s. If the bulk modulus of water be 220 kg mm^{-2} and the density 1.1×10^3 kg m^{-3}, calculate the depth of the sea. Take $g = 9.8$ N kg^{-1}. **(Ans. 1848 m)**

3. At 10^5 Nm^{-2} atmospheric pressure the density of air is 1.29 kg m^{-3}. If $\gamma = 1.41$ for air, calculate the speed of sound in air. **(Ans. 330.6 ms^{-1})**

4. At normal temperature and pressure, 4 g of helium occupies a volume of 22.4 litre. Determine the speed of sound in helium. For helium, $\gamma = 1.67$ and 1 atmospheric pressure = 10^5 Nm^{-2}.

(Ans. 967 ms^{-1})

HINTS

2. Here $\kappa = 220 \text{ kg mm}^{-2} = 220 \times 10^6 \text{ kg m}^{-2}$

$$= 220 \times 9.8 \times 10^6 \text{ Nm}^{-2}$$

$$\rho = 1.1 \times 10^3 \text{ kg m}^{-3}$$

$$\therefore \quad v = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{220 \times 9.8 \times 10^6}{1.1 \times 10^3}} = 1400 \text{ ms}^{-1}$$

$$\text{Depth of the sea} = \frac{vt}{2}$$

$$= \frac{1400 \times 2.64}{2} = \mathbf{1848\ m.}$$

4. Density of helium,

$$\rho = \frac{4 \text{ g}}{22.4 \text{ litre}} = \frac{4 \times 10^{-3} \text{ kg}}{22.4 \times 10^{-3} \text{ m}^3} = \frac{4}{22.4} \text{ kg m}^{-3}$$

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.67 \times 10^5 \times 22.4}{4}} = \mathbf{967\ ms^{-1}}.$$

15.8 FACTORS AFFECTING SPEED OF SOUND IN A GAS

12. *Discuss the various factors which affect the speed of sound in a gas.*

Factors affecting the speed of sound in a gas. The factors such as density of a gas, its pressure, temperature, presence of moisture, etc., affect the speed of sound in a gaseous medium.

(*i*) **Effect of pressure.** The speed of sound in a gas is given by the Laplace formula,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

At constant temperature,

$$PV = \text{constant}$$

or

$$\frac{Pm}{\rho} = \text{constant} \qquad \left[\because \rho = \frac{m}{V} \text{ or } V = \frac{m}{\rho}\right]$$

Since m is a constant, so

$$\frac{P}{\rho} = \text{constant}$$

i.e., when pressure changes, density also changes in the same ratio so that the factor P/ρ remains unchanged. Hence pressure has no effect on the speed of sound in a gas.

(*ii*) **Effect of density.** Suppose two gases have the same pressure P and same value of γ (both are either monoatomic, diatomic or triatomic). If ρ_1 and ρ_2 are the densities of the two gases, then the speeds of sound in them will be

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \quad \text{and} \quad v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Hence *at constant pressure, the speed of sound in a gas is inversely proportional to the square root of its density*. For example, the density of oxygen is 16 times the density of hydrogen.

$$\therefore \quad \frac{v_H}{v_O} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = 4$$

or

$$v_H = 4v_O$$

i.e., the speed of sound in hydrogen is four times the speed of sound in oxygen.

(*iii*) **Effect of humidity.** The speed of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad i.e., \quad v \propto \frac{1}{\sqrt{\rho}}$$

As the density of water vapour (0.8 kgm^{-3} at STP) is less than that of dry air (1.293 kgm^{-3} at STP), so the presence of moisture in air decreases the density of air. Since, the speed of sound is inversely proportional to the square root of density, so sound travels faster in moist air than in dry air.

(*iv*) **Effect of temperature.** For one mole of a gas, $PV = RT$. If M is the molecular mass of the gas and ρ its density, then

$$\rho = \frac{M}{V} \quad \text{or} \quad V = \frac{M}{\rho}$$

$$\therefore \quad \frac{PM}{\rho} = RT \quad \text{or} \quad \frac{P}{\rho} = \frac{RT}{M}$$

$$\therefore \quad v = \sqrt{\frac{\gamma RT}{M}}$$

Clearly, $\quad v \propto \sqrt{T}$

Hence *the speed of sound in a gas is directly proportional to the square root of its absolute temperature.*

Scanned by CamScanner

Temperature coefficient for the speed of sound in air. *It is defined as the increase in the velocity of sound for 1°C (or 1 K) rise in temperature of the gas.*

As $v \propto \sqrt{T}$ and $T(K) = t°C + 273$

$\therefore$ At 0°C, speed $v_0 \propto \sqrt{0 + 273}$

At t°C, speed $v_t \propto \sqrt{t + 273}$

Hence $$\frac{v_t}{v_0} = \sqrt{\frac{t + 273}{0 + 273}}$$

or $$v_t = v_0\left[1 + \frac{t}{273}\right]^{1/2} \simeq v_0\left[1 + \frac{1}{2}\cdot\frac{t}{273}\right]$$

or $$v_t = v_0 + \frac{v_0 \times t}{546}$$

But speed of sound in air at 0°C,

$$v_0 = 332 \text{ ms}^{-1}$$

$$\therefore \quad v_t - v_0 = \frac{332 \times t}{546} = 0.61\, t$$

When $t = 1$°C, $v_t - v_0 = 0.61 \text{ ms}^{-1} = 61 \text{ cm s}^{-1}$

Hence the *velocity of sound in air increases* by 61 cm s^{-1} *for every* 1°C *rise of* temperature. This is known as temperature coefficient for sound in air.

(*v*) **Effect of wind.** As the sound is carried by air, so its velocity is affected by the wind velocity. Suppose the wind travels with velocity w at angle θ with direction of propagation of sound, as shown in Fig. 15.7. Clearly, the component of wind velocity in the direction of sound is $w \cos\theta$.

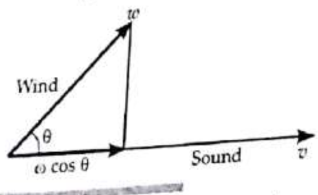


Fig. 15.7 Effect of wind.

$\therefore$ Resultant velocity of sound $= v + w\cos\theta$

When the wind blows in the direction of sound ($\theta = 0°$), resultant velocity $= v + w$.

When the wind blows in the opposite direction of sound ($\theta = 180°$), resultant velocity $= v - w$.

(*vi*) **Effect of frequency.** The speed of sound in air is independent of its frequency. Sound waves of different frequencies travel with the same speed in air, though their wavelengths in air are different. If the speed of sound were dependent on the frequency, we could not have enjoyed orchestra.

(*vii*) **Effect of amplitude.** To a large extent, the speed of sound is independent of the amplitude of the sound wave. But if the amplitude is very large, the compressions and rarefactions may cause large temperature variations which may affect the speed of sound.

Examples based on

Factors affecting Velocity of Sound through Gases

FORMULAE USED

1. *Effect of pressure.* There is no effect of pressure on velocity of sound.
2. *Effect of density* $v \propto \frac{1}{\sqrt{\rho}}$

   or $$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$
3. *Effect of temperature* $v \propto \sqrt{T}$

   or $$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

   Also $$v = \sqrt{\frac{\gamma RT}{M}},$$

   where M = molecular mass of the gas.
4. *Temperature coefficient of sound.* It is given by

   $$\alpha = \frac{v_t - v_0}{t}$$

   For air, $\alpha = 0.61 \text{ ms}^{-1}\,°\text{C}^{-1}$.

UNITS USED

Density ρ is in kg m^{-3}, pressure P in Nm^{-2}, temperature T in kelvin (K) and velocity v in ms^{-1}.

EXAMPLE 12. *At what temperature will the speed of sound be double its value at 273 K?* **[Central Schools 14]**

Or

Calculate the temperature at which the speed of sound will be two times its value at 0°C. **[Delhi 02]**

Solution. $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

Given $v_2 = 2v_1$, $T_1 = 273$ K

$$\therefore \quad \frac{2v_1}{v_1} = \sqrt{\frac{T_2}{273}} \quad \text{or} \quad \frac{T_2}{273} = 4$$

$$\therefore \quad T_2 = 4 \times 273 = \mathbf{1092\ K.}$$

EXAMPLE 13. *A tuning fork of frequency 220 Hz produces sound waves of wavelength 1.5 m in air at S.T.P. Calculate the increase in wavelength, when temperature of air is 27°C.*

Solution. Here $\nu = 220$ Hz, $T_0 = 273$ K,

$$T = 273 + 27 = 300 \text{ K}, \quad \lambda_0 = 1.5 \text{ m}$$

Speed of sound at S.T.P. is

$$v_0 = \nu\lambda_0 = 220 \times 1.5 = 330 \text{ ms}^{-1}$$

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{300}{273}}$$

Scanned by CamScanner

$$v = \sqrt{\frac{300}{273}}\, v_0 = \sqrt{\frac{300}{273}} \times 330 \text{ ms}^{-1}$$

$$v = \nu\lambda = 220 \times \lambda$$

But

$$220 \times \lambda = \sqrt{\frac{300}{273}} \times 330$$

$$\lambda = \sqrt{\frac{300}{273}} \times \frac{330}{220} = 1.57 \text{ m}$$

Increase in wavelength

$$= \lambda - \lambda_0 = 1.57 - 1.5 = \mathbf{0.07\ m.}$$

EXAMPLE 14. *Find the temperature at which sound travels in hydrogen with the same velocity as in oxygen at 1000°C. Density of oxygen is 16 times that of hydrogen.*

Solution. Let v_0, v_{1000} = velocity of sound in oxygen at 0°C and 1000°C

v_0', v_t' = velocities of sound in hydrogen at 0°C and t°C

For oxygen,

$$\frac{v_{1000}}{v_0} = \sqrt{\frac{273 + 1000}{273}} = \sqrt{\frac{1273}{273}}$$

$$v_{1000} = \sqrt{\frac{1273}{273}}\, v_0$$

Similarly, for hydrogen,

$$v_t' = \sqrt{\frac{273 + t}{273}}\, v_0'$$

Given $v_{1000} = v_t'$

$$\therefore \quad \sqrt{\frac{1273}{273}}\, v_0 = \sqrt{\frac{273 + t}{273}}\, v_0'.$$

or

$$\sqrt{\frac{273 + t}{1273}} = \frac{v_0}{v_0'} = \sqrt{\frac{\rho_H}{\rho_O}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

or

$$\frac{273 + t}{1273} = \frac{1}{16}$$

$$273 + t = \frac{1273}{16} = 79.56$$

or

$$t = 79.56 - 273 = \mathbf{-193.44°C.}$$

EXAMPLE 15. *Speed of sound in air is 332 ms^{-1} at S.T.P. What will be its value in hydrogen at S.T.P, if density of hydrogen at S.T.P. is $1/16^{th}$ that of air?* **[MNREC 95]**

Solution. Speed sound, $v = \sqrt{\frac{\gamma P}{\rho}}$

Taking γ and P same for air and hydrogen, we can write

$$\frac{v_H}{v_a} = \sqrt{\frac{\rho_a}{\rho_H}} = \sqrt{\frac{\rho_a}{(1/16)\rho_a}} = 4$$

$$\therefore \quad v_H = 4\, v_a = 4 \times 332 = \mathbf{1328\ ms^{-1}}.$$

EXAMPLE 16. *At normal temperature and pressure the speed of sound in air is 332 ms^{-1}. What will be the speed of sound in hydrogen at 546°C and 3 atmospheric pressure? Air is 16 times heavier than hydrogen.*

Solution. First we find out speed of sound in hydrogen at normal temperature and pressure.

$$\frac{v_H}{v_a} = \sqrt{\frac{\rho_a}{\rho_H}} = \sqrt{\frac{\rho_a}{(1/16)\rho_H}} = 4$$

$$\therefore \quad v_H = 4 \times 332 = 1328 \text{ ms}^{-1}$$

As the speed of sound is not affected by the change of pressure, we determine the effect of temperature alone. Let v_0 and v_{546} be the speeds of sound in hydrogen at 0°C and 546°C respectively. Then

$$\frac{v_{546}}{v_0} = \sqrt{\frac{273 + 546}{273 + 0}}$$

$$= \sqrt{\frac{819}{273}} = \sqrt{3} = 1.732$$

$$\therefore \quad v_{546} = 1.732\, v_0 = 1.732 \times 1328$$

$$= \mathbf{2300\ ms^{-1}}.$$

EXAMPLE 17. *Find the ratio of velocity of sound in hydrogen gas ($\gamma = 7/5$) to that in helium gas ($\gamma = 5/3$) at the same temperature. Given that molecular weights of hydrogen and helium are 2 and 4 respectively.* **[IIT 85]**

Solution. $v = \sqrt{\frac{\gamma RT}{M}}$

At constant temperature,

$$\frac{v_{\text{H}}}{v_{\text{He}}} = \sqrt{\frac{\gamma_{\text{H}}}{\gamma_{\text{He}}} \cdot \frac{M_{\text{He}}}{M_{\text{H}}}}$$

$$= \sqrt{\frac{7/5}{5/3} \cdot \frac{4}{2}} = \sqrt{\frac{42}{25}} = \mathbf{1.68.}$$

EXAMPLE 18. *The ratio of densities of oxygen and nitrogen is 16 : 14. At what temperature, the speed of sound in oxygen will be equal to its speed in nitrogen at 14°C?*

Solution. Speed of sound,

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma R (t + 273)}{M}}$$

Speed of sound in oxygen at t°C

$$= \sqrt{\frac{\gamma R (t + 273)}{M_O}}$$

Speed of sound in nitrogen at 14°C

$$= \sqrt{\frac{\gamma R (14 + 273)}{M_N}}$$

But these two speeds have to be equal. Also, γ is same for both gases.

Scanned by CamScanner

Hence

$$\sqrt{\frac{\gamma R(t+273)}{M_O}}=\sqrt{\frac{\gamma R(14+273)}{M_N}}$$

or $$\frac{M_O}{M_N}=\frac{t+273}{287}$$

Given $$\frac{M_O}{M_N}=\frac{16}{14}$$

$\therefore$ $$\frac{16}{14}=\frac{t+273}{287}$$

On solving, $t = 55°C$.

EXAMPLE 19. *A gas is a mixture of two parts by volume of hydrogen and one part by volume of nitrogen. If the velocity of sound in hydrogen at 0°C is 1300 ms^{-1}, find the velocity of sound in the gaseous mixture at 27°C.*

Solution. Both hydrogen and nitrogen are diatomic gases. So the value of γ can be taken same for hydrogen, nitrogen and the mixture of gases.

$$\text{Density of mixture} = \frac{\text{Total mass}}{\text{Total volume}}$$

or $$\rho_{mix}=\frac{2V\times\rho_H+V\times\rho_N}{2V+V}$$

$$=\frac{2V\times\rho_H+V\times 14\rho_H}{3V}=\frac{16\rho_H}{3}$$

$[\because \rho_N = 14\rho_H]$

Velocity of sound in the mixture at 0°C,

$$v_O=\sqrt{\frac{\gamma P}{\rho_{mix}}}=\sqrt{\frac{\gamma P\times 3}{16\rho_H}}$$

Velocity of sound in hydrogen at 0°C

$$v_H=\sqrt{\frac{\gamma P}{\rho_H}}$$

$\therefore$ $$\frac{v_O}{v_H}=\sqrt{\frac{\gamma P\times 3}{16\rho_H}}\times\sqrt{\frac{\rho_H}{\gamma P}}=\sqrt{\frac{3}{16}}=\frac{\sqrt{3}}{4}$$

or $$v_O=\frac{\sqrt{3}}{4}v_H=\frac{\sqrt{3}}{4}\times 1300=325\sqrt{3}\ \text{ms}^{-1}$$

Velocity of sound in the mixture at 27°C,

$$v_t=v_0\left(1+\frac{t}{546}\right)$$

$$=325\sqrt{3}\left(1+\frac{27}{546}\right)=\mathbf{591\ ms^{-1}}.$$

PROBLEMS FOR PRACTICE

1. Find the temperature at which the velocity of sound in air will be $1\frac{1}{2}$ times the velocity at 11°C. **(Ans. 366°C)**

2. The speed of sound in air is 332 ms^{-1} at 0°C. At what temperature will the speed become one half of that at 0°C? **[Himachal 08]** **(Ans. –204.75°C)**

3. What is the ratio of the velocity of sound in hydrogen ($\gamma = 7/5$) to that in helium gas ($\gamma = 5/3$) at the same temperature? **[IIT 85]** **(Ans. $\sqrt{42}/5$)**

4. An observer sets his watch by the sound of a signal fired from a tower yet he finds that his watch is slow by 5 s. Find the distance of the tower from the observer. The temperature of air during the observation is 20°C and the velocity of sound in air at 0°C is 332 ms^{-1}. **(Ans. 1720 m)**

5. A sound wave propagating in air has a frequency of 4000 Hz. Calculate the percentage change in wavelength when the wavefront, initially in a region where $T = 27°C$, enters a region where the temperature decreases to 10°C. **(Ans. 3%)**

6. At what temperature will the velocity of sound in hydrogen be the same as in oxygen at 100°C? Density of oxygen is 16 times the density of hydrogen. **(Ans. – 249.7°C)**

7. The speed of sound in dry air at S.T.P. is 332 ms^{-1}. Assuming air as composed of 4 parts of nitrogen and one part of oxygen, calculate velocity of sound in oxygen under similar conditions, when the densities of oxygen and nitrogen at S.T.P. are in the ratio of 16 : 14. **(Ans. 314.77 ms^{-1})**

HINTS

1. Given $v_t = \frac{3}{2}v_{11}$

or $$v_0\sqrt{\frac{273+t}{273}}=\frac{3}{2}v_0\sqrt{\frac{273+11}{273}}$$

On squaring, $$\frac{273+t}{273}=\frac{9}{4}\times\frac{284}{273}$$

On solving, $t = \mathbf{366°C}$.

3. Speed of sound, $v=\sqrt{\frac{\gamma RT}{M}}$

$\therefore$ $$\frac{v_H}{v_{He}}=\sqrt{\frac{\gamma_H}{\gamma_{He}}\cdot\frac{M_{He}}{M_H}}=\sqrt{\frac{7/5}{5/3}\cdot\frac{4}{2}}=\frac{\sqrt{42}}{5}.$$

4. $$v_{20}=v_0\sqrt{\frac{273+20}{273}}=332\sqrt{\frac{293}{273}}=340\ \text{ms}^{-1}$$

Distance of tower from the observer

$= 340\times 5 = \mathbf{1720\ m}$.

5. $$\frac{v_2}{v_1}=\sqrt{\frac{273+10}{273+27}}=\sqrt{\frac{283}{300}}=0.97$$

As frequency remains unchanged, so

$$\frac{\lambda_2}{\lambda_1}=\frac{\nu\lambda_2}{\nu\lambda_1}=\frac{v_2}{v_1}=0.97$$

Scanned by CamScanner

percentage change in wavelength,

$$\frac{\lambda_1 - \lambda_2}{\lambda_1} \times 100 = \left(1 - \frac{\lambda_2}{\lambda_1}\right) \times 100$$

$$= (1 - 0.97) \times 100 = 3\%.$$

6. Given : $(v_H)_t = (v_O)_{100}$

$$(v_H)_0 \cdot \sqrt{\frac{273 + t}{273}} = (v_O)_0 \sqrt{\frac{273 + 100}{273}}$$

$$\left(\frac{v_H}{v_O}\right)_0 = \sqrt{\frac{273 + 100}{273 + t}} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16}{1}} = \frac{4}{1}$$

or $\frac{373}{273 + t} = 16$

On solving, $t = -249.7^\circ C$.

7. Density of mixture $= \frac{\text{Total mass}}{\text{Total volume}}$

$$\rho_{mix} = \frac{4V \times \rho_N + V \times \rho_O}{4V + V}$$

$$= \frac{\rho_O\left(4\frac{\rho_N}{\rho_O} + 1\right)}{5} = \frac{\rho_O\left(4 \times \frac{14}{16} + 1\right)}{5}$$

$$= \frac{9}{10}\rho_O = 0.9\rho_O$$

$$\frac{v_O}{v_{mix}} = \sqrt{\frac{\rho_{mix}}{\rho_O}} = \sqrt{\frac{0.9\rho_O}{\rho_O}} = \sqrt{0.9} = 0.9487$$

$\therefore$ $v_O = 0.9487 \times v_{mix} = 0.9487 \times 332 = \mathbf{314.77\ ms^{-1}}$.

15.9 DISPLACEMENT RELATION FOR A PROGRESSIVE WAVE

13. *What is a progressive wave ? What is a plane progressive harmonic wave ? Establish the displacement relation for harmonic wave travelling along the positive direction of X-axis.*

Progressive wave. *A wave that travels from one point of the medium to another is called a progressive wave.* A progressive wave may be transverse or longitudinal.

Plane progressive harmonic wave. *If during the propagation of a wave through a medium, the particles of the medium vibrate simple harmonically about their mean positions, then the wave is said to be plane progressive harmonic wave.* In a harmonic progressive wave of given frequency, all particles have same amplitude but the phase of oscillation changes from one particle to the next.

Displacement relation for a progressive harmonic wave. Suppose a simple harmonic wave starts from the origin O and travels along the positive direction of X-axis with speed v. Let the time be measured from the instant when the particle at the origin O is passing through the mean position. Taking the initial phase of the particle to be zero, the displacement of the particle at the origin O $(x = 0)$ at any instant t is given by

$$y(0, t) = A \sin \omega t \quad ...(1)$$

where T is the periodic time and A the amplitude of the wave.

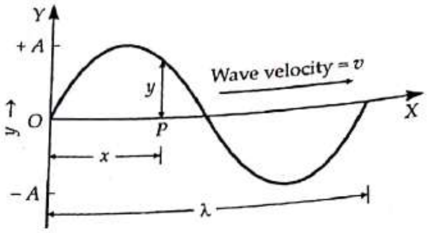


Fig. 15.8 A simple harmonic wave.

Consider a particle P on the X-axis at a distance x from O. The disturbance starting from the origin O will reach P in x/v seconds. This means the particle P will start vibrating x/v seconds later than the particle at O. Therefore,

Displacement of the particle at P at any instant t
= Displacement of the particle at O at a time x/v seconds earlier
= Displacement of the particle at O at time $(t - x/v)$.

Thus the displacement of the particle at P at any time t can be obtained by replacing t by $(t - x/v)$ in the equation (1). It is given by

$$y(x, t) = A \sin \omega\left(t - \frac{x}{v}\right) = A \sin\left(\omega t - \frac{\omega}{v}x\right)$$

But $\frac{\omega}{v} = \frac{2\pi\nu}{v} = \frac{2\pi}{\lambda} = k$

The quantity $k = 2\pi/\lambda$ is called *angular wave number* or *propagation constant*. Hence

$$y(x, t) = A \sin(\omega t - kx) \quad ...(2)$$

This equation represents a harmonic wave travelling along the positive direction of the X-axis. It can also be written in the following forms :

$$y(x, t) = A \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

or $$y(x, t) = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad ...(3)$$

$$= A \sin \frac{2\pi}{T}\left(t - \frac{x}{\lambda}T\right)$$

But $\frac{\lambda}{T} = v$

Scanned by CamScanner

$$\therefore \quad y(x,t) = A \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \quad ...(4)$$

Also $y(x,t) = A \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{T}t - x\right)$

$$\text{or} \quad y(x,t) = A \sin \frac{2\pi}{\lambda}(vt - x) \quad ...(5)$$

Equations (2), (3), (4) and (5) are the various forms of plane progressive wave. If the initial phase of the particle at O is ϕ_0, then the equation of wave motion will be

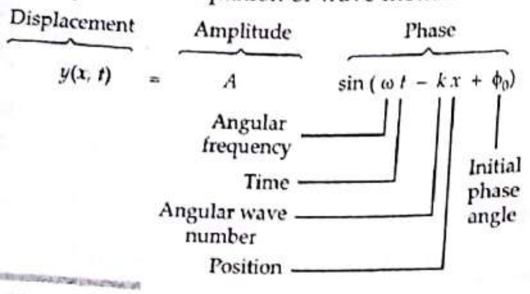


Fig. 15.9

A harmonic wave travelling along negative direction of X-axis can be written as

$$y(x,t) = A \sin(\omega t + kx + \phi_0)$$

15.10 PHASE AND PHASE DIFFERENCE

14. *What do you mean by phase of a wave ? Discuss the phase change with time and position.*

Phase of a wave. *The phase of a harmonic wave is a quantity that gives complete information of the wave at any time and at any position.* It is equal to the argument of the sine or cosine function representing the wave. Suppose a harmonic wave is given by

$$y(x,t) = A \sin(\omega t - kx + \phi_0) \quad ...(1)$$

Then the phase of the wave at position x and time t is given by

$$\phi = \omega t - kx + \phi_0 \quad ...(2)$$

Clearly, the phase of a wave is periodic both in time and space. At a given point (x = constant), the phase changes with time t and at a given instant (t = constant), it changes with distance x.

Phase change with time. Taking x as constant, if we differentiate equation (2) w.r.t., time t, we get

$$\frac{\Delta\phi}{\Delta t} = \omega$$

Thus the phase change at a given position (x = constant) in time Δt is given by

$$\Delta\phi = \omega\,\Delta t = \frac{2\pi}{T}\Delta t$$

Hence we can define the time period of a wave as the time in which the phase of a particle of the medium changes by 2π.

Phase change with position. Taking t as constant, if we differentiate equation (2) w.r.t. position x, we get

$$\frac{\Delta\phi}{\Delta x} = -k$$

Thus the phase difference, at any instant of time t, between two particles separated by distance Δx is given by

$$\Delta\phi = -k\,\Delta x = -\frac{2\pi}{\lambda}\Delta x$$

Hence *we can define the wavelength of a wave as the distance between two points (or particles) which have a phase difference of 2π at any given instant.* The negative sign indicates that farther the particle is located from the origin in the positive X-direction, the more it lags behind in phase.

15.11 PARTICLE VELOCITY AND ACCELERATION

15. *For a simple harmonic wave, deduce expressions for (a) particle velocity and (b) particle acceleration. Discuss their phase relationship with displacement.*

(*a*) **Particle velocity.** The particle velocity V is different from the wave velocity v. *It is the velocity with which the particles of the medium vibrate about their mean positions.*

The displacement relation for a harmonic wave travelling along positive X-direction is

$$y(x,t) = A \sin(\omega t - kx) \quad ...(1)$$

Differentiating (1) w.r.t. time t, and taking x constant, we get the particle velocity

$$V = \frac{dy}{dt} = \omega A \cos(\omega t - kx) \quad ...(2)$$

$$\text{or} \quad V = \omega A \sin[(\omega t - kx) + \pi/2] \quad ...(3)$$

It may be noted that

(*i*) While the wave velocity ($v = \nu\lambda$) remains constant, the particle velocity changes simple harmonically with time.

(*ii*) The particle velocity is ahead of displacement in phase by $\pi/2$ radian.

(*iii*) The maximum particle velocity or the *velocity amplitude* is

$$V_0 = \omega A = \frac{2\pi}{T}A$$

$= \frac{2\pi}{T}$ *times the displacement amplitude A.*

(*iv*) If we differentiate equation (1) w.r.t. position x, we get

$$\frac{dy}{dx} = -kA\cos(\omega t - kx) \quad ...(4)$$

Scanned by CamScanner

From equations (2) and (4), we get

$$\frac{V}{dy/dx} = \frac{\omega A \cos(\omega t - kx)}{-kA\cos(\omega t - kx)}$$

$$= -\frac{\omega}{k} = -\frac{2\pi\nu}{2\pi/\lambda} = -v \quad \text{or} \quad V = -v\frac{dy}{dx}.$$

Particle velocity at a point = – Wave velocity × slope of displacement curve at that point.

(*v*) **Particle acceleration.** If we differentiate equation (5) with respect to time *t*, we get the particle acceleration

$$a = \frac{dV}{dt} = -\omega^2 A \sin(\omega t - kx) = -\omega^2 y$$

$$a = \omega^2 A \sin[(\omega t - kx) + \pi]$$

It may be noted that

(*i*) The maximum value of particle acceleration or the *acceleration amplitude* is

$$a_0 = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A$$

$$= \left(\frac{2\pi}{T}\right)^2 \textit{times the displacement amplitude.}$$

(*ii*) The particle acceleration is ahead of the particle displacement in phase by π radian.

15.12 SPEED OF A TRAVELLING WAVE

16. *Define wave velocity or phase velocity. Deduce its relation with angular frequency ω and propagation constant k.*

Wave velocity or phase velocity. *The distance covered by a wave in the direction of its propagation per unit time is called the wave velocity.* It represents the velocity with which a disturbance is transferred from one particle to the next with the actual motion of the particles.

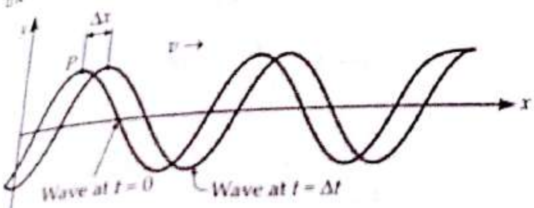


Fig. 15.10 Plot of a harmonic wave at $t = 0$ and $t = \Delta t$.

Fig. 15.10 shows two plots of the harmonic wave $y = A\sin(\omega t - kx)$ at two different instants of time t and $t + \Delta t$. During the small time interval Δt, the entire wave pattern moves through distance Δx in the positive X-direction. As the wave moves, each point of the moving waveform, such as point *P* marked on the peak retains its displacement *y*. This is possible only when the phase of the wave remains constant.

$\therefore \quad \omega t - kx = \text{constant.}$

Differentiating both sides w.r.t., time *t*, we get

$$\omega - k\frac{dx}{dt} = 0 \quad \text{or} \quad \frac{dx}{dt} = \frac{\omega}{k}$$

But $\frac{dx}{dt}$ = wave velocity, v

$$\therefore \quad v = \frac{\omega}{k} = \frac{\lambda}{T} = \nu\lambda \qquad \left[\because \omega = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}\right]$$

Examples based on Progressive Waves

FORMULAE USED

1. A plane progressive harmonic wave travelling along positive direction of X-axis can be represented by any of the following expressions :

   (*i*) $y = A\sin(\omega t - kx), \quad k = 2\pi/\lambda$

   (*ii*) $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

   (*iii*) $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

   where λ is the wavelength, v is the velocity, A the amplitude and x is the distance of observation point from the origin.

2. For a progressive wave travelling along –ve X-axis,

   $$y = A\sin(\omega t + kx)$$

   or $$y = A\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) = A\sin\frac{2\pi}{\lambda}(vt + x)$$

3. Phase, $\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi_0$,

   where ϕ_0 is the initial phase.

4. Phase change with time, $\Delta\phi = \frac{2\pi}{T}\Delta t$

5. Phase change with position, $\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$

6. Instantaneous particle velocity

   $$u = \frac{dy}{dt} = \frac{2\pi A}{T}\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

   Velocity amplitude, $u_0 = \frac{2\pi A}{T} = \omega A$

7. Instantaneous particle acceleration

   $$f = \frac{du}{dt} = -\frac{4\pi^2}{T^2} A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = -\omega^2 y$$

   Acceleration amplitude, $f_0 = \frac{4\pi^2}{T^2} A = \omega^2 A$

UNITS USED

Displacement *y* and amplitude *A* have same units m or cm. If λ and *x* are in m, wave velocity *v* is in ms^{-1}. Propagation constant $k = 2\pi/\lambda$ is in rad m^{-1}.

Scanned by CamScanner

EXAMPLE 20. *The displacement y of a particle in a medium can be expressed as*

$$y = 10^{-6} \sin (100\, t + 20x + \pi/4)$$

where t is in second and x in metre. What is the speed of the wave ? **[AIEEE 04]**

Solution. We compare the given wave equation with the standard wave equation,

$$y = A \sin (\omega t + kx + \phi)$$

We get $\omega = 100 \text{ rad s}^{-1}$ and $k = 20 \text{ rad m}^{-1}$.

Speed of the wave,

$$v = \frac{\omega}{k} = \frac{100}{20} = \mathbf{20\ ms^{-1}}.$$

EXAMPLE 21. *A harmonically moving transverse wave on a string has a maximum particle velocity and acceleration of 3 ms^{-1} and 90 ms^{-2} respectively. Velocity of the wave is 20 ms^{-1}. Find the waveform.* **[IIT 05]**

Solution. Here $v_{max} = \omega A = 3 \text{ ms}^{-1}$

$$a_{max} = \omega^2 A = 90 \text{ ms}^{-2}$$

Velocity of the wave,

$$v = \frac{\omega}{k} = 20 \text{ ms}^{-1}$$

Clearly, $A = \dfrac{\omega^2 A^2}{\omega^2 A} = \dfrac{(3)^2}{90} = 0.1 \text{ m}$

$$\omega = \frac{\omega^2 A}{\omega A} = \frac{90}{3} = 30 \text{ rad s}^{-1}$$

$$k = \frac{\omega}{\omega / k} = \frac{30}{20} = 1.5 \text{ rad m}^{-1}$$

The equation for the waveform is

$$y = A \sin (\omega t + kx) = \mathbf{0.1 \sin (30t + 1.5x)}.$$

EXAMPLE 22. *A wave travelling along a string is described by*

$$y(x, t) = 0.005 \sin (80.0\, x - 3.0\, t),$$

in which the numerical constants are in SI units (0.005 m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}). Calculate (a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave. Also calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s. **[NCERT ; Central Schools 05]**

Solution. Given $y(x, t) = 0.005 \sin (80.0\, x - 3.0\, t)$

The displacement equation for a harmonic wave is

$$y(x, t) = A \sin \left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t \right)$$

On comparing the above two equations, we get

$$A = 0.005 \text{ m}, \frac{2\pi}{\lambda} = 80.0 \text{ rad m}^{-1}, \frac{2\pi}{T} = 3.0 \text{ rad s}^{-1}.$$

(*a*) Amplitude, $A = \mathbf{0.005\ m.}$

(*b*) Wavelength,

$$\lambda = \frac{2\pi \text{ rad}}{80.0 \text{ rad m}^{-1}} = 7.85 \times 10^{-2} \text{ m} = \mathbf{7.85\ cm.}$$

(*c*) Time period,

$$T = \frac{2\pi \text{ rad}}{3.0 \text{ rad s}^{-1}} = \mathbf{2.09\ s.}$$

Frequency,

$$\nu = \frac{1}{T} = \frac{1}{2.09 \text{ s}} = \mathbf{0.48\ Hz.}$$

Displacement of the wave at a distance $x = 30.0$ cm and time $t = 20$ s,

$$y = (0.005 \text{ m}) \sin (80.0 \times 0.3 - 3.0 \times 20)$$
$$= (0.005 \text{ m}) \sin (-36 \text{ rad}) = \mathbf{zero.}$$

EXAMPLE 23. *The equation of a plane progressive wave is*

$$y = 10 \sin 2\pi (t - 0.005\, x)$$

where y and x are in cm and t in seconds. Calculate the amplitude, frequency, wavelength and velocity of the wave. **[Delhi 99]**

Solution. Given : $y = 10 \sin 2\pi (t - 0.005\, x)$ ...(1)

The standard equation for a harmonic wave is

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \quad ...(2)$$

Comparing equations (1) and (2), we get

$$A = 10, \ \frac{1}{T} = 1, \ \frac{1}{\lambda} = 0.005$$

(*i*) Amplitude, $A = \mathbf{10\ cm.}$

[y and A have same units]

(*ii*) Frequency, $\nu = \dfrac{1}{T} = \mathbf{1\ Hz.}$

(*iii*) Wavelength, $\lambda = \dfrac{1}{0.005} = \mathbf{200\ cm.}$

[x and λ have same units]

(*iv*) Velocity, $v = \nu \lambda = 1 \times 200 = \mathbf{200\ cm\ s^{-1}}$.

Example 24. *A wave travelling along a string is described by equation $y(x,t) = 0.05 \sin (40x - 5t)$ in which the numerical constants are in SI units (0.05 m, 40 rad m^{-1} and 5 rad s^{-1}). Calculate the displacement at distance 35 cm and time 10 sec.* **[Delhi 08]**

Solution. Given :

$$y(x, t) = 0.05 \sin (40x - 5t)$$

We compare with the standard equation,

$$y(x, t) = A \sin \left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} \right)$$

Scanned by CamScanner

$$A = 0.05 \text{ m}, \quad \frac{2\pi}{\lambda} = 40 \text{ rad m}^{-1}$$

$$\frac{2\pi}{T} = 5 \text{ rad s}^{-1}.$$

(a) Amplitude, $A = \mathbf{0.05\ m.}$

(b) Wavelength, $\lambda = \frac{2\pi \text{rad}}{40 \text{rad m}^{-1}} = \frac{\pi}{20} \text{ m} = \mathbf{15.7\ cm.}$

(c) Time period, $T = \frac{2\pi \text{rad}}{5 \text{rad s}^{-1}} = \frac{2\pi}{5} \text{ s} = \mathbf{1.26\ cm.}$

(d) Frequency, $\nu = \frac{1}{T} = \frac{5}{2\pi} = \mathbf{0.8\ Hz.}$

At $x = 35$ cm $= 0.35$ m and $t = 10$ s, the displacement

$$y = 0.05 \sin(40 \times 0.35 - 5 \times 10)$$
$$= 0.05 \sin(-36) \text{ m} = -0.05 \sin 36 \text{ m}.$$

Example 25. *A wave travelling along a string is given by*

$$y(x, t) = 0.005 \sin(80x - 3t)$$

where the numerical values are in SI units. Symbols have their usual meanings. Calculate :

(a) Frequency of the wave. (b) Velocity of the wave.

(c) Amplitude of particle velocity.

[Central Schools 08, 09]

Solution. Given : $y(x,t) = 0.005 \sin(80x - 3t)$

Also $y(x,t) = A \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$

$$A = 0.005 \text{ m},$$

$$\frac{2\pi}{\lambda} = 80 \text{ rad m}^{-1},$$

$$\frac{2\pi}{T} = 3 \text{ rad s}^{-1}.$$

(a) Frequency, $\nu = \frac{1}{T} = \frac{3}{2\pi} = \mathbf{0.48\ Hz.}$

(b) $\lambda = \frac{2\pi}{80}$ m

Wave velocity,

$$v = \nu\lambda = \frac{3}{2\pi} \times \frac{2\pi}{80} = \frac{3}{80} \text{ ms}^{-1} = \mathbf{7.5\ cms^{-1}}$$

(c) Amplitude of particle velocity

$$= \frac{2\pi}{T} A = 3 \times 0.005 = \mathbf{0.015\ ms^{-1}}.$$

Example 26. *A displacement wave is represented by*

$$y = 0.25 \times 10^{-3} \sin(500\,t - 0.025\,x)$$

where y, t and x are in cm, sec and metres respectively. Deduce (i) amplitude (ii) period (iii) angular frequency, and (iv) wavelength. Also deduce the amplitude of particle velocity and particle acceleration. **[Delhi 03C]**

Solution. Given

$$y = 0.25 \times 10^{-3} \sin(500t - 0.025x)$$

Comparing it with standard equation :

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right), \text{ we get}$$

(i) Amplitude,

$$A = \mathbf{0.25 \times 10^{-3}\ cm.}$$

(ii) $\frac{2\pi}{T} = 500$ or $T = \frac{2\pi}{500} = \frac{\pi}{250} = \mathbf{0.01257\ s.}$

(iii) Angular frequency,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} \times 250 = \mathbf{500\ rad\ s^{-1}.}$$

(iv) $\frac{2\pi}{\lambda} = 0.025$ or $\lambda = \frac{2\pi}{0.025} = \mathbf{251.2\ m.}$

(v) Velocity amplitude

$$= \omega A = 500 \times 0.25 \times 10^{-3} = \mathbf{0.125\ cm\ s^{-1}.}$$

(vi) Acceleration amplitude

$$= \omega^2 A = (500)^2 \times 0.25 \times 10^{-3} = \mathbf{62.5\ cm\ s^{-2}.}$$

Example 27. *The speed of a wave in a stretched string is $20\ ms^{-1}$ and its frequency is 50 Hz. Calculate the phase difference in radian between two points situated at a distance of 10 cm on the string.*

Solution. Here $v = 20 \text{ ms}^{-1}$, $\nu = 50$ Hz, $\Delta x = 10$ cm

Wavelength, $\lambda = \frac{v}{\nu} = \frac{20}{50} = 0.4 \text{ m} = 40 \text{ cm}$

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{40} \times 10 = \frac{\pi}{2}$ **rad.**

Example 28. *Write the equation of a progressive wave propagating along the positive x-direction, whose amplitude is 5 cm, frequency 250 Hz and velocity $500\ ms^{-1}$.*

Solution. Here $A = 5$ cm $= 0.05$ m, $\nu = 250$ Hz, $v = 500 \text{ ms}^{-1}$

Wavelength, $\lambda = \frac{v}{\nu} = \frac{500}{250} = 2$ m,

Period, $T = \frac{1}{\nu} = \frac{1}{250}$ s

The equation for the given wave can be written as

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = 0.05 \sin 2\pi\left(250t - \frac{x}{2}\right)$$

or $\quad y = \mathbf{0.05 \sin \pi (500\,t - x)}$ **metre.**

Example 29. *For the plane wave*

$$y = 2.5 \times 10^{-0.02x} \cos(800t - 0.82x + \pi/2),$$

write down

(i) the general expression for phase ϕ

(ii) the phase at $x = 0$, $t = 0$

Scanned by CamScanner

(*iii*) *the phase difference between the points separated by 20 cm along x-axis.*

(*iv*) *the change in phase at a given place 0.6 milli second and*

(*v*) *the amplitude at* $x = 100$ m.

Take units of y, t, x as 10^{-5} *cm, s and m respectively.*

Solution. (*i*) Phase, $\phi = \mathbf{800t - 0.82x + \frac{\pi}{2}}$.

(*ii*) At $x = 0, \quad t = 0, \quad \phi = \pi/2$ **rad.**

(*iii*) Here $\Delta x = 20$ cm $= 0.20$ m. Therefore

$$\Delta\phi = -0.82\ \Delta x = -0.82 \times 0.20 = \mathbf{-0.164\ rad.}$$

(*iv*) Here $\Delta t = 0.6$ ms $= 0.6 \times 10^{-3}$ s. Therefore,

$$\Delta\phi = 800\ \Delta t = 800 \times 0.6 \times 10^{-3} = \mathbf{0.48\ rad.}$$

(*v*) At $x = 100$ m, the amplitude is

$$A = 2.5 \times 10^{-0.02x}$$
$$= 2.5 \times 10^{-0.02 \times 100} \times 10^{-5}\ \text{cm}$$
$$= \mathbf{0.025 \times 10^{-5}\ cm.}$$

[$\because$ unit of A is 10^{-5} cm as that of y]

Example 30. *A simple harmonic wave train of amplitude 1 cm and frequency 100 vibrations is travelling in positive x-direction with velocity 15* ms^{-1}. *Calculate the displacement y, the particle velocity and particle acceleration at* $x = 180$ *cm from the origin at* $t = 5$ *s.*

Solution. Let the displacement of the wave be given by

$$y = A \sin \frac{2\pi}{\lambda}(vt - x) = A \sin \frac{2\pi\nu}{v}(vt - x) \qquad \left[\because \lambda = \frac{v}{\nu}\right]$$

But $A = 1$ cm, $\nu = 100$ Hz,

$v = 15\ ms^{-1} = 1500\ cm\ s^{-1}$,

$x = 180$ cm, $t = 5$ s

$$\therefore \quad y = 1 \sin \frac{2\pi \times 100}{1500}(1500 \times 5 - 180)$$
$$= \sin \frac{2\pi}{15} \times 7320 = \sin 2\pi \times 4800 = \mathbf{0.}$$

Particle velocity,

$$u = \frac{dy}{dt} = 2\pi A\nu \cos \frac{2\pi}{\lambda}(vt - x)$$

As calculated above,

$$\sin \frac{2\pi}{\lambda}(vt - x) = 0 \quad \text{at} \quad x = 180\ \text{cm} \quad \text{and} \quad t = 5\ \text{s}$$

$$\therefore \quad \cos \frac{2\pi}{\lambda}(vt - x) = 1$$

$\therefore$ Particle velocity,

$$u = 2\pi A\nu = 2\pi \times 1 \times 100 = \mathbf{200\pi\ cm\ s^{-1}.}$$

Particle acceleration,

$$f = -\frac{4\pi^2 v^2}{\lambda^2} y = \mathbf{0.} \qquad [\because y = 0]$$

Example 31. *A certain spring has a linear mass density of 0.25 kg* m^{-1} *and is stretched with a tension of 25 N. One end is given a sinusoidal motion with frequency 5 Hz and amplitude 0.01 m. At time* $t = 0$, *the other end has zero displacement and is moving in the positive y-direction.*

(*i*) *Find the wave speed, amplitude, angular frequency, period, wavelength and wave number.*

(*ii*) *Write a wave function representing the wave.*

(*iii*) *Find the position of the point at* $x = 0.25$ *m at time* $t = 0.1$ s.

Solution. Here $m = 0.25$ kg m^{-1},

Tension $\quad T = 25$ N, $\nu = 5$ Hz, $A = 0.01$ m

(*i*) Wave speed, $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{25}{0.25}} = \mathbf{10\ ms^{-1}}$

Amplitude, $\quad A = 0.01$ m

Angular frequency,

$$\omega = 2\pi\nu = 2 \times 3.14 \times 5 = \mathbf{31.4\ rad\ s^{-1}}.$$

Time period, $\quad T = \frac{1}{\nu} = \frac{1}{5} = \mathbf{0.2\ s.}$

Wavelength, $\quad \lambda = \frac{v}{\nu} = \frac{10}{5} = \mathbf{2\ m.}$

Wave number, $\quad k = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{2} = \mathbf{3.14\ m^{-1}}.$

(*ii*) The given wave can be represented by the wave function,

$$y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$
$$= 0.01 \sin 2\pi\left(\frac{t}{0.2} - \frac{x}{2}\right)$$
$$= 0.01 \sin (10\pi t - \pi x)$$
$$= \mathbf{0.01 \sin (31.4t - 3.14x).}$$

(*iii*) At $x = 0.25$ m and $t = 0.1$ s, the displacement is

$$y = 0.01 \sin 2\pi\left(\frac{0.1}{0.2} - \frac{0.25}{2}\right)$$
$$= 0.01 \sin 2\pi(0.5 - 0.125)$$
$$= 0.01 \sin (0.75\pi) = 0.01 \sin 135^\circ$$
$$= 0.01 \times 0.707 = \mathbf{0.00707\ m.}$$

PROBLEMS FOR PRACTICE

1. A wave on a string is described by $y(x, t) = 0.005 \sin (6.28x - 314t)$, in which all quantities are in SI units. Calculate its (*i*) amplitude and (*ii*) wavelength. **[Central Schools 03]**

[Ans. (*i*) 0.005 m (*ii*) 1 m**]**

Scanned by CamScanner

2. The equation of a transverse wave travelling along a coil spring is $y = 4.0 \sin \pi (0.010\, x - 2.0\, t)$ where y and x are in cm and t in s. Find the (*i*) amplitude (*ii*) wavelength (*iii*) initial phase at the origin (*iv*) speed and (*v*) frequency on the wave.
[Ans. (*i*) 4.0×10^{-2} m (*ii*) 2.0 m (*iii*) 0 (*iv*) 2.0 ms^{-1} (*v*) 1.0 s^{-1}]

3. The equation of transverse wave travelling in a rope is given by $y = 10 \sin \pi (0.01\, x - 2.00\, t)$ where y and x are in cm and t in seconds. Find the amplitude, frequency, velocity and wavelength of the wave. **[Delhi 97]**
(**Ans.** 10 cm, 1 Hz, 200 cm s^{-1}, 200 cm)

4. A simple harmonic wave is expressed by equation,
$$y = 7 \times 10^{-6} \sin \left(800\, \pi t - \frac{\pi}{42.5} x\right)$$
where y and x are in cm and t in seconds. Calculate the following : (*i*) amplitude (*ii*) frequency (*iii*) wave length (*iv*) wave velocity, and (*v*) phase difference between two particles separated by 17.0 cm. **[Delhi 05]**

5. For a travelling harmonic wave,
$$y = 2.0 \cos (10t - 0.0080\, x + 0.18)$$
where x and y are in cm and t is in seconds. What is the phase difference between two points separated by (*i*) a distance of 0.5 m and (*ii*) a time gap of 0.5 s?
[Ans. (*i*) – 0.4 rad (*ii*) 5 rad]

6. Find the displacement of an air particle 3.5 m from the origin of disturbance at $t = 0.05$ s, when a wave of amplitude 0.2 mm and frequency 500 Hz travels along it with a velocity 350 ms^{-1}. (**Ans.** 0)

7. A simple harmonic wave-train is travelling in a gas in the positive direction of the X-axis. Its amplitude is 2 cm, velocity 45 ms^{-1} and frequency 75 s^{-1}. Write down the equation of the wave. Find the displacement of the particle of the medium at a distance of 135 cm from the origin in the direction of the wave at the instant $t = 3$ s.
[Ans. $y = 2 \sin 2\pi \left(75\, t - \frac{x}{60}\right)$, – 2 cm]

8. The phase difference between the vibrations of two medium particles due to the transmission of a wave is $2\pi/3$. The distance between the particles is 15 cm. Determine the wavelength of the wave.
(**Ans.** 45 cm)

9. The distance between two points on a stretched string is 20 cm. The frequency of the progressive wave is 400 Hz and velocity 100 ms^{-1}. Find the phase difference between these two points.
(**Ans.** 1.6 π or 288°)

10. A sound-source of frequency 500 Hz is producing longitudinal waves in a spring. The distance between two consecutive rarefractions is 24 cm. If the amplitude of vibration of a particle of the spring is 3.0 cm and the wave is travelling in the negative x-direction, then write the equation for the wave. Assume that the source is at $x = 0$ and at this point the displacement is zero at the time $t = 0$.
[Ans. $y = 3.0 \sin 2\pi (25\, t + x/24)$]

✖ HINTS

1. Comparing $y = 0.005 \sin (6.28\, x - 314\, t)$
and $y = A \sin \left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right)$, we get $A = \mathbf{0.005\ m.}$
$$\frac{2\pi}{\lambda} = 6.28 \quad \text{or} \quad \lambda = \frac{2\pi}{6.28} = \frac{2 \times 3.14}{6.28} = \mathbf{1\ m.}$$

4. We compare the given equation with
$$y = A \sin \left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)$$
(*i*) $A = \mathbf{7 \times 10^{-6}\ cm.}$
(*ii*) $\frac{2\pi}{T} = 800\,\pi \quad \therefore \quad \nu = \frac{1}{T} = \mathbf{400\ Hz.}$
(*iii*) $\frac{2\pi}{\lambda} = \frac{\pi}{42.5} \quad \therefore \quad \lambda = 2 \times 42.5 = \mathbf{85\ cm.}$
(*iv*) $v = \nu\lambda = 400 \times 85 = 34000\ cm\ s^{-1} = \mathbf{340\ ms^{-1}.}$
(*v*) $\Delta\phi = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{85\ cm} \times 17.0\ cm = \mathbf{\frac{2\pi}{5}\ rad.}$

5. Given : $y = 2.0 \cos (10t - 0.0080\, x + 0.18)$
Comparing with $y = A \cos \left(\frac{2\pi t}{T} - \frac{2\pi}{\lambda} x + \phi_0\right)$,
we get,
$$A = 2.0\ cm, \quad \frac{2\pi}{T} = 10\, s^{-1}, \quad \frac{2\pi}{\lambda} = 0.0080\ cm^{-1}$$
(*i*) Here $\Delta x = 0.5\ m = 50$ cm. The phase difference is
$$\Delta\phi = -\frac{2\pi}{\lambda} \Delta x = -0.0080 \times 50 = \mathbf{-0.4\ rad.}$$
(*ii*) Here $\Delta t = 0.5$ s. The phase difference is
$$\Delta\phi = \frac{2\pi}{T} \Delta t = 10 \times 0.5 = \mathbf{5\ rad.}$$

6. Let $y = A \sin \left[\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right]$
But $A = 0.2$ mm, $T = \frac{1}{\nu} = \frac{1}{500}$ s,
$$\lambda = \frac{v}{\nu} = \frac{350}{500} = \frac{7}{10}\ m$$
$$\therefore \quad y = 0.2 \sin \left[1000\,\pi t - \frac{20\pi}{7} x\right]$$
At $x = 3.5$ m and $t = 0.05$ s,
$$y = 0.2 \sin \left[1000\pi \times 0.05 - \frac{20\pi}{7} \times 3.5\right]$$
$$= 0.2 \sin (50\pi - 10\pi) = 0.2 \sin (40\pi) = \mathbf{0.}$$

Scanned by CamScanner

7. Here $T = \frac{1}{\nu} = \frac{1}{75}$ s, $\lambda = \frac{v}{\nu} = \frac{45}{75} = 0.6$ m $= 60$ cm,

$A = 2$ cm

$\therefore \quad y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$= 2 \sin 2\pi \left(75t - \frac{x}{60}\right)$

At $t = 3s$ and $x = 135$ cm,

$y = 2 \sin 2\pi (225 - 2.25)$

$= 2 \sin (450\pi - 4.5\pi)$

$= 2 \sin (-4.5\pi)$

$= -2 \sin (4.5\pi) = -2 \sin (4\pi + \pi/2)$

$= -2 \sin \pi/2 = -2$ **cm.**

8. $\Delta\phi = \frac{2\pi}{\lambda} \Delta x \quad \therefore \quad \frac{2\pi}{3} = \frac{2\pi}{\lambda} \times 15$ or $\lambda = 45$ **cm.**

15.13 BOUNDARY EFFECTS

17. *Explain the phenomenon of reflection of waves by considering a wave pulse travelling along a string, whose one end is (i) fixed to a rigid support and (ii) tied to a ring which can freely slide up and down a vertical rod. What are the phase changes in each case ?*

Reflection of a wave from a rigid boundary. As shown in Fig. 15.11, consider a wave pulse travelling along a string (rarer medium) attached to a rigid support, such as a wall (denser medium). As the pulse reaches the wall, it exerts an upward force on the wall. By Newton's third law, the wall exerts an equal downward force on the string. This produces a reflected pulse in the downward direction, which travels in the reverse direction. Thus a crest is reflected as a trough.

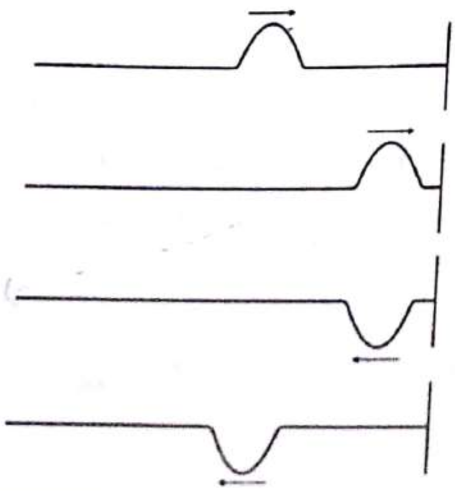


Fig. 15.11 Reflection of a pulse in a string from a rigid support.

Hence *when a travelling wave is reflected from a rigid boundary, it is reflected back with a phase reversal or phase difference of π radians.*

Reflection of a wave from an open boundary. As shown in Fig. 15.12, consider a wave pulse travelling along a string attached to a light ring, which slides without friction up and down a vertical rod. As the crest produced in the string at A reaches the end B, it meets little or no opposition there. The ring rises above its equilibrium position. As the ring moves up, it stretches the string and produces a reflected crest which travels back towards A. There is no phase reversal and crest is reflected as a crest.

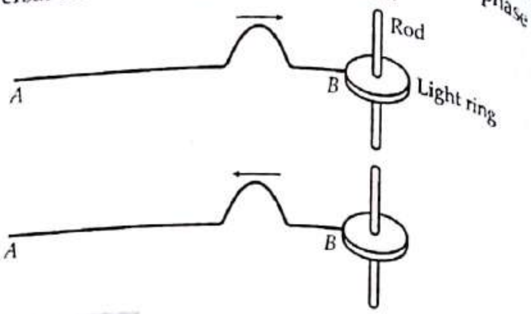


Fig. 15.12 Reflection of a wave pulse on a string from a free boundary.

Hence *when a travelling wave is reflected from a free or open boundary, it suffers no phase change.*

Suppose an incident wave is represented by

$$y_i(x, t) = A \sin(\omega t - kx)$$

For reflection at a rigid boundary, the reflected wave can be represented as

$$y_r(x, t) = -A \sin(\omega t + kx)$$

[Signs of both y and x change]

For reflection at an open boundary, the reflected wave can be represented as

$$y_r(x, t) = A \sin(\omega t + kx)$$

[Only sign of x changes]

Obviously, in case of reflection from a rigid boundary, the incident and reflected pulses meet in opposite phases at the end point and so cancel each other. Hence a ***node*** is formed at the boundary *i.e.*, the net displacement is zero.

In case of reflection from an open boundary, the incident and reflected pulses meet in same phase at the end point and reinforce (or get added to) each other. Hence an ***antinode*** is formed at the boundary *i.e.*, the displacement is maximum and is twice the amplitude of the either pulse.

17. *Considering the wave pulses travelling on stretched strings, discuss the phase change during the refraction of a wave.*

Refraction of a wave. Consider a combination of a thinner string A and a thicker string B kept under same tension. As $v = \sqrt{T/m}$, so a wavepulse travels

Scanned by CamScanner

on a thin string than that on a thick string *i.e.*, the thinner string *A* acts as rarer medium while the thicker string *B* acts as denser medium.

As shown in Fig. 15.13(*a*), when a wave pulse travels from thinner string to the thicker string, it is partly reflected and partly refracted at the interface. The reflected pulse travels faster while the refracted pulse travels slower. Also, the reflected pulse suffers a phase change of 180° while the refracted pulse does not suffer any phase change.

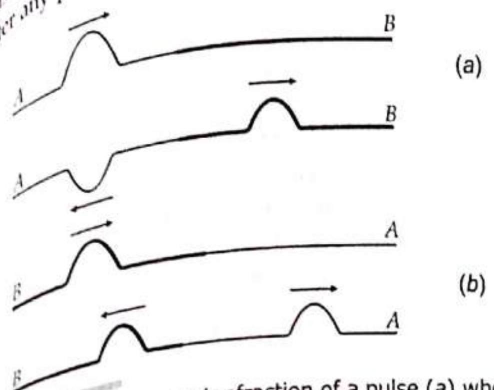


Fig. 15.13 Reflection and refraction of a pulse (*a*) when the second string is denser than the first and (*b*) when the second string is lighter than the first.

As shown in Fig. 15.13(*b*), when a pulse of crest travels from the thicker to the thinner string, both the transmitted and reflected pulses travel as crest *i.e.*, they do not suffer any phase change.

Hence a wave suffers no phase change during its refraction from one medium to another.

For Your Knowledge

- If a wave coming in a medium where its velocity is larger meets a medium where its velocity is smaller, it is reflected back with a reversal of phase or a phase change of π radians.
- If the wave initially comes from a medium where its velocity is smaller, the reflected wave does not suffer any phase change.
- The incident and reflected waves obey the usual laws of reflection. The frequency, wavelength and velocity of the reflected wave are same as those of incident wave.
- The wave transmitted into the second medium always goes without any change in phase.
- The incident and refracted rays obey the Snell's law of refraction.
- The wave velocity and wavelength of the refracted wave are different from those of the incident wave but their frequencies are equal. Hence $\nu = \frac{v_i}{\lambda_i} = \frac{v_r}{\lambda_r}$

Here the subscripts *i* and *r* stand for the incident and the refracted waves respectively.

15.14 PRINCIPLE OF SUPERPOSITION OF WAVES

19. *What is meant by the independent behaviour of waves ?*

Independent behaviour of waves. A wave preserves its individuality while travelling through space. So *when a number of waves travel through a region at the same time, each wave travels independently of the others i.e.*, as if all other waves were absent. That is why, with so many different musical instruments playing simultaneously in a full orchestra, we can still identify the note produced by an individual instrument. An important consequence of the independent behaviour of the waves is the principle of superposition of waves.

20. *State and explain the principle of superposition of waves.*

Principle of superposition of waves. When one wave reaches a particle of the medium, the particle suffers one displacement. When two waves simultaneously cross this particle, it suffers two displacements, one due to each wave. *The resultant displacement of the particle is equal to the algebraic sum of the individual displacements given to it by the two waves. This is the principle of superposition of waves.*

The principle of superposition of waves states that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves.

If $y_1, y_2, y_3, \ldots, y_n$ are the displacements due to waves acting separately, then according to the principle of superposition the resultant displacement, when all the waves act together is given by the algebraic sum

$$y = y_1 + y_2 + y_3 + \ldots + y_n$$
$$= f_1(vt - x) + f_2(vt - x) + \ldots + f_n(vt - x)$$
$$= \sum_{i=1}^{n} f_i(vt - x)$$

Explanation of the superposition principle. ***(i) Superposition of two identical pulses travelling towards each other.*** Fig. 15.14 shows two pulses moving towards each other with a speed of 1 ms^{-1}. The positions of the two pulses after every one second are shown in Figs. 15.14(*a*) to (*f*). They cross each other between $t = 2$ s and $t = 3$ s. When the two pulses overlap or superpose, the displacement of the resultant pulse is twice the displacement of either pulse *i.e.*, equal to the sum of the displacements of the two pulses. This is the case of *constructive interference*.

Scanned by CamScanner

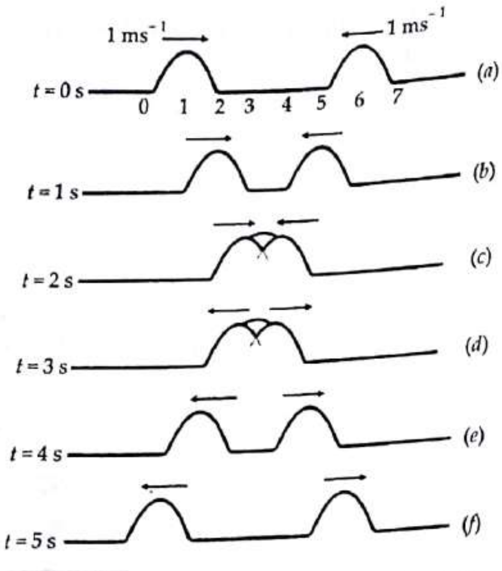


Fig. 15.14 Superposition of two identical pulses travelling in opposite directions.

(*ii*) ***Superposition of two pulses of equal and opposite shapes moving towards each other.*** Fig. 15.15 shows what happens when two equal and opposite pulses moving in opposite directions cross each other. In Fig. 15.15(*c*), we see that there is an instant when the string appears undisturbed. In this situation the positive pulse overlaps the negative pulse and seem to cancel each other. This is the case of *destructive interference*. This again shows that the resultant wave profile is the algebraic sum of individual waves.

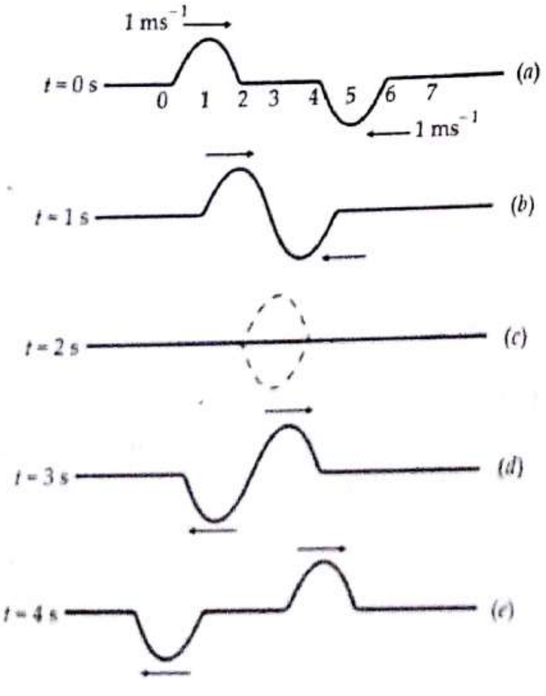


Fig. 15.15 Superposition of two equal and opposite pulses travelling in opposite directions.

Clearly, the two pulses continue to retain their individual shapes after crossing each other. However, at the instant they cross each other, the appearance of the wave profile is different from the shape of either individual pulse.

The superposition of *two* waves may lead to following *three* different effects :

(*i*) When two waves of the same frequency moving with the same speed in the same direction in a medium superpose on each other, they give rise to effect called ***interference of waves.***

(*ii*) When two waves of same frequency moving with the same speed in the opposite directions in a medium superpose on each other, they produce ***stationary waves.***

(*iii*) When two waves of slightly different frequencies moving with the same speed in the same direction in a medium superpose on each other, they produce ***beats.***

For Your Knowledge

- The principle of superposition holds not only for the mechanical waves but also for electromagnetic waves.
- In case of mechanical waves, the superposition principle does not hold if the amplitude of disturbance is so large that the ordinary linear laws of mechanical action no longer hold good. For example, the superposition principle fails in case of *shock waves* generated by a violent explosion.

15.15 STATIONARY WAVES

21. *What are stationary waves ? What is the necessary condition for the formation of stationary waves ? What are the two types of stationary waves ?*

Stationary waves. *When two identical waves of same amplitude and frequency travelling in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called stationary or standing wave.*

The resultant wave keeps on repeating itself in the same fixed position. Some particles of the medium remain permanently at rest *i.e.*, they have zero displacement. Their positions are called ***nodes***. Some other particles always suffer maximum displacement. Their positions are called ***antinodes***. The positions of nodes and antinodes do not change with time. That is why, such waves are called *stationary* or *standing waves*, to distinguish them from *progressive* or *travelling waves* which travel through the medium with a definite speed v. In such waves, there is no transfer of energy along the medium in either direction.

Necessary condition for the formation of stationary waves. A stationary wave cannot be formed from two independent waves travelling in a medium in opposite directions. In actual practice a stationary wave is produced when a progressive wave and its reflected wave are superposed. Hence *a stationary wave can be produced only in a finite medium which has its boundaries,* for example, a string of finite length or a rod, or a column of liquid or gas. The wave reflected from the boundary is of the same kind as the incident wave. The incident and reflected waves superpose each other continuously, giving rise to stationary waves.

Two types of stationary waves :

(i) **Transverse stationary waves.** *When two identical transverse waves travelling in opposite directions overlap, a transverse stationary wave is formed.* For example, transverse stationary waves are formed in a sonometer and Melde's experiment.

(ii) **Longitudinal stationary waves.** *When two identical longitudinal waves travelling in opposite directions overlap, a longitudinal stationary wave is formed.* For example, longitudinal stationary waves are formed in a resonance apparatus, organ pipes and Kundt's tube.

15.16 GRAPHICAL TREATMENT OF STATIONARY WAVES

22. *Explain graphically the formation of stationary waves and mark out clearly the positions of nodes and antinodes.*

Formation of stationary waves by graphical method. In Fig. 15.16, the full line curve represents a harmonic wave of time period T and wavelength λ travelling from left to right while the dashed curve represents an identical wave travelling from right to left. The resultant wave is obtained by taking the algebraic sum of the displacements of the two waves at every point and is shown by the thick line curve. The situations at different intervals of time are shown in Figs. 15.16(*i*) to (*v*).

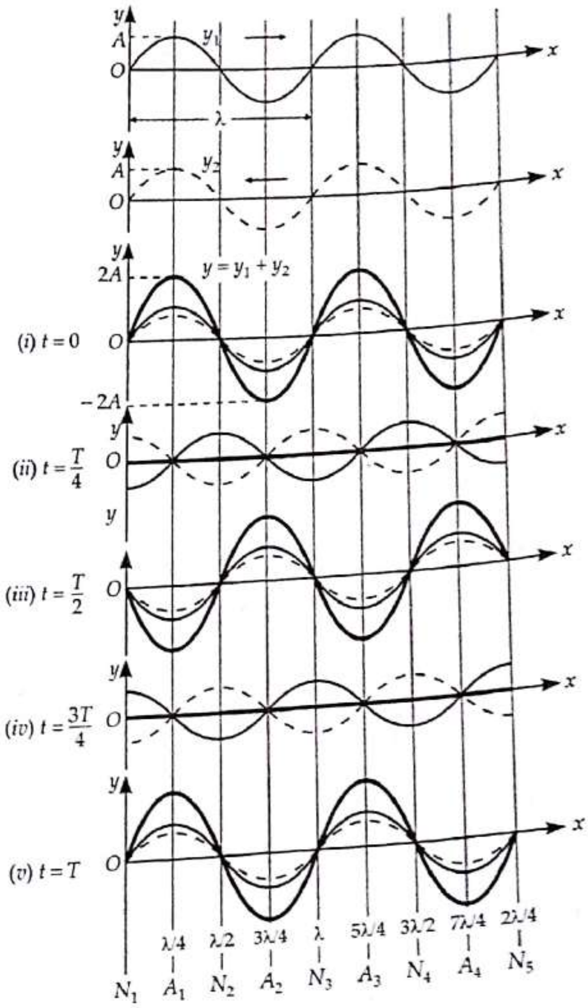


Fig. 15.16 Formation of stationary waves by graphical method.

(*i*) **At $t = 0$**, the two waves are in same phase *i.e.*, the crests and troughs of the two waves coincide respectively with each other. The amplitude of the resultant wave is twice of that due to each individual wave. All particles are at their positions of maximum displacement [Fig. 15.16(*i*)].

(*ii*) **At $t = T/4$**, each wave has advanced through a distance of $\lambda/4$ from the opposite direction. The two waves are in opposite phases. The resultant wave is the central straight line. All the particles of the medium are now passing through their mean positions [Fig. 15.16(*ii*)].

(*iii*) **At $t = T/2$**, each wave has advanced through a distance of $\lambda/2$ from the opposite direction. The two waves are again in same phase. The resultant wave is reciprocal of that at $t = 0$. All the particles are at their positions of maximum displacement but in the directions opposite to those at $t = 0$ [Fig. 15.16(*iii*)].

(*iv*) **At $t = 3T/4$**, each wave has advanced through a distance of $3\lambda/4$ from the opposite direction. The two waves are again in opposite phases. The resultant wave is the central straight line. All the particles are again passing through their mean positions, but their directions of motion are opposite to those at $t = T/4$ [Fig. 15.16(*iv*)].

(*v*) **At $t = T$**, each wave has advanced through a distance λ from opposite direction. The two waves are again in same phase. The resultant wave is similar to that at $t = 0$. This completes one cycle [Fig. 15.16(*v*)].

Scanned by CamScanner

The whole cycle continues to repeat again and again. The various segments of the string, on which stationary waves are formed, keep on vibrating up and down. The positions $N_1, N_2, N_3, ...$ where the amplitude of oscillation is zero are called *nodes*. The positions $A_1, A_2, A_3, ...$ where the amplitude of oscillation is maximum are called *antinodes*. Clearly, the separation between two successive nodes or antinodes is $\lambda/2$. The separation between a node and the next antinode is $\lambda/4$.

15.17 ANALYTICAL TREATMENT OF STATIONARY WAVES

23. *Obtain an expression for a stationary wave formed by two sinusoidal waves travelling along the same path in opposite directions and obtain the positions of nodes and antinodes.*

Analytical treatment of stationary waves. Consider two sinusoidal waves of equal amplitude and frequency travelling along a long string in opposite directions. The wave travelling along positive X-direction can be represented as

$$y_1 = A \sin(\omega t - kx)$$

The wave travelling along negative X-direction can be represented as

$$y_2 = A \sin(\omega t + kx)$$

According to the principle of superposition, the resultant wave is given by

$$y = y_1 + y_2$$
$$= A \sin(\omega t - kx) + A \sin(\omega t + kx)$$
$$= 2A \sin \omega t \cos kx$$
$$[\because \sin(A+B) + \sin(A-B) = 2 \sin A \cos B]$$

or $\quad y = (2A \cos kx) \sin \omega t$

This equation represents a stationary wave. It cannot represent a progressive wave because the argument of any of its trigonometric functions does not contain the combination $(\omega t \pm kx)$. The stationary wave has the same angular frequency ω but has amplitude

$$A' = 2A \cos kx$$

Obviously in case of a stationary wave, the amplitude of oscillation is not same for all the particles. It varies harmonically with the location x of the particle.

Changes with position x. The amplitude will be zero at points, where

$$\cos kx = 0$$

or $\quad kx = \left(n + \frac{1}{2}\right)\pi$, where $n = 0, 1, 2, 3, ...$

or $\quad \frac{2\pi x}{\lambda} = \left(n + \frac{1}{2}\right)\pi \qquad \left[\because k = \frac{2\pi}{\lambda}\right]$

or $\quad x = (2n+1)\frac{\lambda}{4}$

or $\quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$

These positions of zero amplitude are called nodes. Clearly, the separation between two consecutive nodes is $\lambda/2$.

The amplitude will have a maximum value of $2A$ at points, where

$$\cos kx = \pm 1$$

or $\quad kx = n\pi$, where $n = 0, 1, 2, 3, ...$

or $\quad \frac{2\pi}{\lambda} x = n\pi \quad$ or $\quad x = n\frac{\lambda}{2}$

or $\quad x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$

These positions of maximum amplitude are called antinodes. Clearly, the antinodes are separated by $\lambda/2$ and are located half way between pairs of nodes.

Changes with time t. At the instants $t = 0, T/2, 3T/2, ...$, we have

$$\sin \omega t = \sin \frac{2\pi}{T} t = 0$$

Thus at these instants the displacement y becomes zero at all the points. That is, *all the particles of the medium pass through their mean positions simultaneously twice in each cycle.*

At the instants $t = T/4, 3T/4, 5T/4, ...,$ we have

$$\sin \omega t = \sin \frac{2\pi}{T} t = \pm 1.$$

Thus at these instants the displacement y is maximum at all the points and becomes alternately positive and negative. That is, *all the particles of the medium pass through their positions of maximum displacements twice in each cycle.*

15.18 CHARACTERISTICS OF STATIONARY WAVES

24. *Mention some of the important characteristics of stationary waves.*

Characteristics of stationary waves :

(*i*) In a stationary wave, the disturbance does not advance forward. The conditions of crests and troughs merely appear and disappear in fixed positions to be followed by opposite conditions after every half the time period.

Scanned by CamScanner

(*iii*) All particles of the medium, except those at nodes, execute simple harmonic motions with the same time period about their mean positions.

(*iv*) During the formation of a stationary wave, the medium is broken into loops or segments between equally spaced points called *nodes* which remain permanently at rest and midway between them are points called *antinodes* where the displacement amplitude is maximum.

(*v*) The distance between two successive nodes or antinodes is $\lambda/2$.

(*v*) The amplitudes of the particles are different at different points. The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

(*vi*) The maximum velocity is different at different points. Its value is zero at the nodes and progressively increases towards the antinode. All the particles attain their maximum velocities simultaneously when they pass through their mean positions.

(*vii*) All the particles in a particular segment between two nodes vibrate in the same phase but the particles in two neighbouring segments vibrate in opposite phases, as shown in Fig. 15.17.

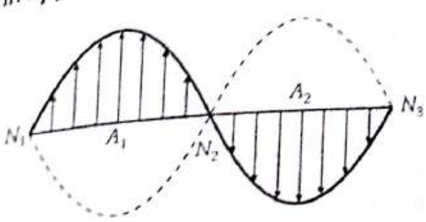


Fig. 15.17 Opposite phases of particles in consecutive segments.

(*viii*) Twice in each cycle, the energy becomes alternately wholly potential and wholly kinetic. It is wholly kinetic when the particles are at their positions of maximum displacements and wholly kinetic when the particles pass through their mean positions.

(*ix*) There is no transference of energy across any section of the medium because no energy can flow past a nodal point which remains permanently at rest.

(*x*) A stationary wave has the same wavelength and time period as the two component waves.

15.19 COMPARISON BETWEEN STATIONARY AND PROGRESSIVE WAVES

25. *Give important differences between progressive and stationary waves.*

Progressive waves	*Stationary waves*
1. The disturbance travels forward with a definite velocity.	The disturbance remains confined to the region where it is produced.
2. Each particle of the medium executes SHM about its mean position with the same amplitude.	Except nodes, all particles of the medium execute SHM with varying amplitude.
3. There is a continuous change of phase from one particle to the next.	All the particles between two successive nodes vibrate in the same phase, but the phase reverses for particles between next pair of nodes.
4. No particle of the medium is permanently at rest.	The particles of the medium at nodes are permanently at rest.
5. There is no instant when all the particles are at the mean positions together.	Twice during each cycle, all particles pass through their mean positions simultaneously.
6. There is flow of energy across every plane along the direction of propagation of the wave.	Energy of one region remains confined in that region.
7. The energy averaged over a wavelength is half kinetic and half potential.	Twice during each cycle, the energy becomes alternately wholly potential and wholly kinetic.
8. All the particles have same maximum velocity which they attain while passing their mean position one after the other.	All the particles attain their individual maximum velocities at the same time as they pass through their mean positions. This velocity varies from zero at nodes to maximum at antinodes.

Scanned by CamScanner

Examples based on Equation of Stationary Waves

FORMULAE USED

1. Let $y_1 = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (incident wave)

   $y_2 = \pm a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$ (reflected wave)

   Then stationary wave formed by the superposition is given by

   $$y = y_1 + y_2 = \pm 2a \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi t}{T}$$

   It involves the product of separate harmonic functions of time t and position x.
2. For (+) sign in the above equation, antinodes are formed at the positions $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$ and nodes are formed at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$
3. For (−) sign, antinodes are formed at the positions $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$ and nodes at $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$
4. The distance between two successive nodes or antinodes is $\lambda/2$ and that between a node and nearest antinode is $\lambda/4$.

UNITS USED

Units of y and a are same cm or m, units of x and λ are same cm or m.

EXAMPLE 32. *The constituent waves of a stationary wave have amplitude, frequency and velocity as 8 cm, 30 Hz and 180 cm s⁻¹ respectively. Write down the equation of the stationary wave.*

Solution. Here $a = 8$ cm, $\nu = 30$ Hz,

$$v = 180 \text{ cm s}^{-1}, \quad T = 1/\nu = 1/30 \text{ s}$$

$$\lambda = \frac{v}{\nu} = \frac{180}{30} = 6 \text{ cm}$$

Equation of stationary wave is

$$y = 2a \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi t}{T}$$

$$= 2 \times 8 \cos\frac{2\pi x}{6} \sin\frac{2\pi t}{1/30}$$

or $$y = 16 \cos\frac{\pi x}{3} \sin 60\pi t.$$

EXAMPLE 33. *Stationary waves are set up by the superposition of two waves given by*

$$y_1 = 0.05 \sin(5\pi t - x) \text{ and } y_2 = 0.05 \sin(5\pi t + x)$$

where x and y are in metres and t in seconds. Find the displacement of a particle situated at a distance x = 1 m.

Solution. According to principle of superposition, the resultant displacement is given by

$$y = y_1 + y_2$$

$$= 0.05 \sin(5\pi t - x) + 0.05 \sin(5\pi t + x)$$

$$= 0.05 \times 2 \sin\frac{5\pi t - x + 5\pi t + x}{2} \cos\frac{5\pi t + x - 5\pi t + x}{2}$$

$$= 0.1 \cos x \sin 5\pi t$$

∴ Amplitude, $A = 0.1 \cos x$

At $x = 1$ m,

$$A = 0.1 \cos 1 = 0.1 \cos\frac{180°}{\pi} = 0.1 \cos\frac{180°}{3.14}$$

$$= 0.1 \cos 57.3° = 0.1 \times 0.5402 = \mathbf{0.054 \text{ m}}.$$

PROBLEMS FOR PRACTICE

1. The distance between two consecutive nodes in a stationary wave is 25 cm. If the speed of the wave is 300 ms⁻¹, calculate the frequency. **(Ans. 600 Hz)**
2. The equation of a longitudinal stationary wave produced in a closed organ pipe is

   $$y = 6 \sin\frac{2\pi x}{6} \cos 160\pi t$$

   where x, y are in cm and t in seconds. Find (*i*) the frequency, amplitude and wavelength of the original progressive wave (*ii*) separation between two successive nodes and (*iii*) equation of the original progressive wave.

   [Ans. (*i*) $\nu = 80$ Hz, $a = 3$ cm, $\lambda = 6$ cm (*ii*) 3 cm (*iii*) $y = 3 \sin\left(\frac{\pi}{3}x - 160\pi t\right)$**]**
3. (*i*) Write the equation of a wave identical to the wave represented by the equation :

   $$y = 5 \sin \pi(4.0t - 0.02x)$$

   but moving in opposite direction.

   (*ii*) Write the equation of stationary wave produced by the composition of the above two waves and determine the distance between two nearest nodes. All the distances in the equation are in mm.

   [Ans. (*i*) $y = 5 \sin \pi(4.0t + 0.02x)$ (*ii*) $y = 10 \cos 0.02\pi x \sin 4.0\pi t$, 50 mm**]**

15.20 STATIONARY WAVES IN A STRING FIXED AT BOTH ENDS

26. *Give a qualitative discussion of the modes of vibrations of a stretched string fixed at both the ends.*

Normal modes of vibration of a stretched string: Qualitative discussion. Consider a string clamped to rigid supports at its ends. If the wire be plucked in the middle, traverse waves travel along it and get reflected from the ends. These identical waves travelling in opposite directions give rise to stationary waves. Due to boundary conditions, the string vibrates in one or

Scanned by CamScanner

more segments or loops with certain natural frequencies. These special patterns are called *normal modes*.

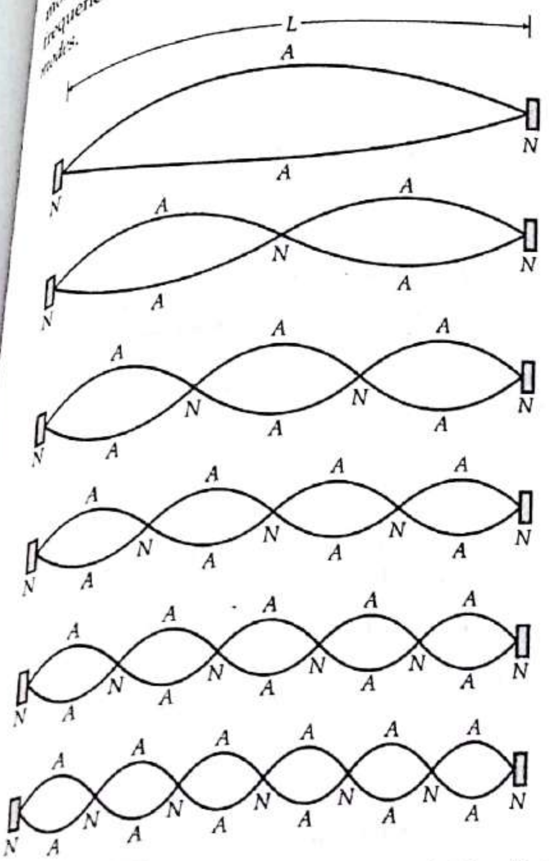


Fig. 15.18 Stationary waves in a stretched string fixed at both ends. Various modes of vibration are shown.

Consider a string of length L, stretched under tension T. Let m be the mass per unit length of string. The speed of the transverse wave on the string will be

$$v = \sqrt{\frac{T}{m}}$$

As the two ends of the string are fixed, they remain at rest. So there is a node N at each end. The different modes of vibration of stretched string fixed at both the ends are shown in Fig. 15.18.

First mode of vibration. If the string is plucked in the middle and released, it vibrates in one segment with nodes at its ends and an antinode in the middle.

Here length of string,

$$L = \frac{\lambda_1}{2} \quad \text{or} \quad \lambda_1 = 2L$$

∴ Frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu \text{ (say)}$$

This is the minimum frequency with which the string can vibrate and is called *fundamental note* or *first harmonic*

Second mode of vibration. If the string is pressed in the middle and plucked at one-fourth length, then the string vibrates in two segments.

Here $\quad L = 2 \cdot \frac{\lambda_2}{2} \quad$ or $\quad \lambda_2 = L$

∴ Frequency of vibration,

$$\nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{T}{m}} = 2\nu$$

This frequency is called *first overtone* or *second harmonic*.

Third mode of vibration. If string is pressed at one-third of its length from one end and plucked at one-sixth length, it will vibrate in three segments. Then

$$L = 3\,\frac{\lambda_3}{2} \quad \text{or} \quad \lambda_3 = \frac{2L}{3}$$

∴ Frequency of vibration,

$$\nu_3 = \frac{v}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu$$

This frequency is called *second overtone* or *third harmonic*. In general, if the string vibrates in p segments, then

$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\nu.$$

27. *Give an analytical treatment of stationary waves in a stretched string.*

Analytical treatment of stationary waves in a string fixed at both the ends. Consider a uniform string of length L stretched by a tension T along the x-axis, with its ends rigidly fixed at the end $x = 0$ and $x = L$. Suppose a transverse wave produced in the string travels along the string along positive x-direction and gets reflected at the fixed end $x = L$. The two waves can be represented as

$$y_1 = A \sin(\omega t - kx)$$

and $\quad y_2 = -A \sin(\omega t + kx)$

The negative sign before A is due to phase reversal of the reflected wave at the fixed end. By the principle of superposition, the resultant wave is given by

$$y = y_1 + y_2 = -A[\sin(\omega t + kx) - \sin(\omega t - kx)]$$

$$= -2A\cos\omega t \sin kx$$

$$[\sin(A + B) - \sin(A - B) = 2\cos A \sin B]$$

or $\quad y = -2A \sin kx \cos \omega t \quad$ ...(1)

If stationary waves are formed, then the ends $x = 0$ and $x = L$ must be nodes because they are kept fixed. So, we have the boundary conditions :

$$y = 0 \quad \text{at } x = 0 \quad \text{for all } t$$

and $$y = 0 \quad \text{at } x = L \quad \text{for all } t$$

The first boundary condition $(y = 0, x = 0)$ is satisfied automatically by equation (1). The second boundary condition $(y = 0, x = L)$ will be satisfied if

$$y = -2 \sin kL \cos \omega t = 0$$

This will be true for all values of t only if

$$\sin kL = 0 \quad \text{or} \quad kL = n\pi, \text{ where } n = 1, 2, 3, ...$$

or $$\frac{2\pi L}{\lambda} = n\pi$$

For each value of n, there is a corresponding value of λ, so we can write

$$\frac{2\pi L}{\lambda_n} = n\pi \quad \text{or} \quad \lambda_n = \frac{2L}{n}$$

The speed of transverse wave on a string of linear mass density m is given by

$$v = \sqrt{\frac{T}{m}}$$

So the frequency of vibration of the string is

$$\nu_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

For $n = 1$, $$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu \quad \text{(say)}$$

This is the lowest frequency with which the string can vibrate and is called *fundamental frequency* or *first harmonic*.

For $n = 2$, $$\nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu$$

(First overtone or second harmonic)

For $n = 3$, $$\nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu$$

(Second overtone or third harmonic)

For $n = 4$, $$\nu_4 = \frac{4}{2L}\sqrt{\frac{T}{m}} = 4\nu$$

(Third overtone or fourth harmonic)

Thus the various frequencies are in the ratio 1 : 2 : 3 : ... and hence form a harmonic series. These frequencies are called ***harmonics*** with the fundamental itself as the *first harmonic*. The higher harmonic are called ***overtones***. Thus second harmonic is first overtone, third harmonic is second overtone and so on. These are shown in Fig. 15.18.

Nodes. *These are the positions of zero amplitude.* In the n^{th} mode of vibration, there are $(n + 1)$ nodes, which are located from one end at distances

$$x = 0, \frac{L}{n}, \frac{2L}{n}, \ldots, L$$

Antinodes. *These are positions of maximum amplitude.* In the n^{th} mode of vibration, there are n antinodes, which are located at distances

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots, \frac{(2n-1)L}{2n}.$$

28. *State the laws of vibrations of stretched strings.*

Laws of transverse vibrations of a string. The fundamental frequency produced in a stretched string of length L under tension T and having mass per unit length m is given by

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

The above equation gives the following laws of vibrations of strings :

(*i*) **Law of length.** *The fundamental frequency of a vibrating string is inversely proportional to its length provided its tension and mass per unit length remain the same.*

$$\nu \propto \frac{1}{L} \qquad (T \text{ and } m \text{ are constants})$$

(*ii*) **Law of tension.** *The fundamental frequency of a vibrating string is proportional to the square root of its tension provided its length and the mass per unit length remain the same.*

$$\nu \propto \sqrt{T} \qquad (L \text{ and } m \text{ are constants})$$

(*iii*) **Law of mass.** *The fundamental frequency of a vibrating string is inversely proportional to the square root of its mass per unit length provided the length and tension remain the same.*

$$\nu \propto \frac{1}{\sqrt{m}} \qquad (L \text{ and } T \text{ are constants})$$

If ρ is the density of the string and D its diameter, then its mass per unit length will be

$$m = \text{Volume of unit length} \times \text{density}$$

$$= \pi\left(\frac{D^2}{4}\right)\rho$$

$$\therefore \quad \nu = \frac{1}{2L}\sqrt{\frac{T}{\pi D^2 \rho / 4}}$$

$$= \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

Scanned by CamScanner

Hence the law of mass may be expressed in the form of following two alternative laws :

(a) **Law of diameter.** *The fundamental frequency of a vibrating string is inversely proportional to its diameter provided its tension, length and density remain the same.*

$$\nu \propto \frac{1}{D} \qquad (T, L \text{ and } \rho \text{ are constants})$$

(b) **Law of density.** *The fundamental frequency of a vibrating string is inversely proportional to the square root of its density provided its tension, length and diameter remain constant.*

$$\nu \propto \frac{1}{\sqrt{\rho}} \qquad (T, L \text{ and } D \text{ are constants})$$

Examples based on Modes of Vibrations of Strings

FORMULAE USED

1. Fundamental frequency, $\nu = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{m}}$
2. When the stretched string vibrates in p loops,
$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\nu$$
3. For a string of diameter D and density ρ,
$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$
4. Law of length, $\nu \propto 1/L$ or νL = constant
or $\nu_1 L_1 = \nu_2 L_2$

UNITS USED

Tension T is in newton, linear mass density m in $N\,kg^{-1}$, length L in metre, density ρ in $kg\,m^{-3}$, frequency ν in Hz.

EXAMPLE 34. *A metal wire of linear mass density of $9.8\ gm^{-1}$ is stretched with a tension of 10 kg wt into between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a parmanent magnet and it vibrates in resonance, when carrying an alternating current of frequency ν. Find the frequency of the alternating source.* **[AIEEE 03]**

Solution. Here $m = 9.8\ gm^{-1} = 9.8 \times 10^{-3}\ kg\,m^{-1}$,

$L = 1$ m, $T = 10$ kg wt $= 10 \times 9.8$ N $= 98$ N

Frequency of vibration of the string

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 1}\sqrt{\frac{98}{9.8 \times 10^{-3}}} = 50\ \text{Hz}$$

As the string vibrates in resonance, the frequency of alternating current, n = **50 Hz.**

EXAMPLE 35. *Calculate the fundamental frequency of a sonometer wire of length = 20 cm, tension 25 N, cross-sectional area = $10^{-2}\ cm^2$ and density of the material of wire = $10^4\ kg\ m^{-3}$.*

Solution. Here $L = 20$ cm $= 0.20$ m, $T = 25$ N,

$A = 10^{-2}\ cm^2 = 10^{-6}\ m^2$, $\rho = 10^4\ kg\,m^{-3}$

Mass per unit length of the wire,

$$m = A \times 1 \times \rho = 10^{-6} \times 10^4\ kg\,m^{-1} = 10^{-2}\ kg\,m^{-1}$$

Fundamental frequency of the sonometer wire is

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.20}\sqrt{\frac{25}{10^{-2}}} = \frac{50}{2 \times 0.20} = \mathbf{125\ Hz.}$$

EXAMPLE 36. *The length of a sonometer wire is 0.75 m and density $9 \times 10^3\ kg\ m^{-3}$. It can bear a stress of $8.1 \times 10^8\ N/m^2$ without exceeding the elastic limit. What is the fundamental frequency that can be produced in the wire ?* **[IIT 90]**

Solution. Here $L = 0.75$ m, $\rho = 9 \times 10^3\ kg\,m^{-3}$,

Stress $= 8.1 \times 10^9\ Nm^{-2}$

Let a be the area of cross-section of the wire, then fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{\text{Stress} \times a}{a \times 1 \times \rho}} = \frac{1}{2L}\sqrt{\frac{\text{Stress}}{\rho}}$$

$$= \frac{1}{2 \times 0.75}\sqrt{\frac{8.1 \times 10^8}{9 \times 10^3}} = \mathbf{200\ Hz.}$$

EXAMPLE 37. *A stretched wire emits a fundamental note of 256 Hz. Keeping the stretching force constant and reducing the length of wire by 10 cm, the frequency becomes 320 Hz. Calculate the original length of the wire.*

Solution. Frequency of fundamental note,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

In first case : $256 = \frac{1}{2L}\sqrt{\frac{T}{m}}$

In second case : $320 = \frac{1}{2(L-10)}\sqrt{\frac{T}{m}}$

On dividing, we get

$$\frac{320}{256} = \frac{2L}{2(L-10)} \quad \text{or} \quad \frac{L}{L-10} = \frac{5}{4}$$

or L = **50 cm.**

EXAMPLE 38. *Find the fundamental note emitted by a string of length $10\sqrt{10}$ cm under tension of 31.4 kg. Radius of string is 0.55 mm and density = $9.8\ g\ cm^{-3}$.*

Scanned by CamScanner

Solution. Here $L = 10\sqrt{10}$ cm $= 0.1\sqrt{10}$ m,

$T = 3.14$ kg wt $= 3.14 \times 9.8$ N, $r = 0.5$ mm

or $D = 1$ mm $= 10^{-3}$ m,

$\rho = 9.8$ g cm^{-3} $= 9800$ kg m^{-3}

Fundamental note,

$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

$$= \frac{1}{0.1\sqrt{10} \times 10^{-3}}\sqrt{\frac{3.14 \times 9.8}{3.14 \times 9800}} = \textbf{100 Hz.}$$

EXAMPLE 39. *A rope 5 m long has a total mass of 245 g. It is stretched with a constant tension of 1 kg wt. If it is fixed at one end and shaken by hand at the other end, what frequency of shaking will make it break up into three vibrating segments ? Take $g = 980$ cm s^{-2}.*

Solution. Here $L = 5$ m $= 500$ cm,

$T = 1$ kg wt $= 1000 \times 980$ dyne, $p = 3$

$$m = \frac{245 \text{ g}}{500 \text{ cm}} = 0.49 \text{ g cm}^{-1}$$

When the rope vibrates in p segments, its frequency of vibration is

$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore \quad \nu_3 = \frac{3}{2 \times 500}\sqrt{\frac{1000 \times 980}{0.49}} = \textbf{4.24 Hz.}$$

EXAMPLE 40. *In an experiment it was found that the string vibrated in three loops when 8 g were placed on the scale pan. What mass must be placed on the pan to make the string vibrate in six loops ? Neglect the mass of the string and the scale pan.*

Solution. Frequency of vibration of a string vibrating in p loops is

$$\nu = \frac{p}{2L}\sqrt{\frac{T}{m}}$$

As the quantities ν, L and m remain constant, so

$T \times p^2 =$ constant or $T_1 \times p_1^2 = T_2 \times p_2^2$

or $$T_2 = \frac{p_1^2}{p_2^2} \times T_1$$

Given : $p_1 = 3$, $T_1 = 8$ g, $p_2 = 6$

$$\therefore \quad T_2 = \left(\frac{3}{6}\right)^2 \times 8 = \textbf{2 g.}$$

EXAMPLE 41. *A wire of length 108 cm produces a fundamental note of frequency 256 Hz, when stretched by a weight of 1 kg. By how much its length should be increased so that its pitch is raised by a major tone, if it is now stretched by a weight of 4 kg ?*

Solution. *In first case.*

$\nu = 256$ Hz, $L = 108$ cm, $T = 1 \times 1000 \times 980$ dyne

As $$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 256 = \frac{1}{2 \times 108}\sqrt{\frac{1 \times 1000 \times 980}{m}} \quad \text{...(1)}$$

In second case. Major tone,

$$\nu' = \frac{9}{8}\nu = \frac{9}{8} \times 256 = 288 \text{ Hz},$$

$$T = 4 \times 1000 \times 980 \text{ dyne}$$

Let the length of the wire be increased by x cm. Its new length will be

$$L' = L + x = (108 + x) \text{ cm}$$

Now $$\nu' = \frac{1}{2L'}\sqrt{\frac{T}{m}}$$

or $$288 = \frac{1}{2 \times (108 + x)}\sqrt{\frac{4 \times 1000 \times 980}{m}} \quad \text{...(2)}$$

Dividing (1) by (2), we get

$$\frac{256}{288} = \frac{108 + x}{108} \times \frac{1}{\sqrt{4}}$$

$$\frac{8}{9} = \frac{108 + x}{216}$$

On solving, $x = \textbf{84 cm.}$

EXAMPLE 42. *The length of a wire between the two ends of a sonometer is 105 cm. Where should the two bridges be placed so that the fundamental frequencies of the three segments are in the ratio of 1 : 3 : 15 ?*

Solution. Total length of the wire,

$$L = 105 \text{ cm}$$

$$\nu_1 : \nu_2 : \nu_3 = 1 : 3 : 15$$

Let L_1, L_2 and L_3 be the lengths of the three parts.

As $$\nu \propto \frac{1}{L}$$

$$\therefore \quad L_1 : L_2 : L_3 = \frac{1}{1} : \frac{1}{3} : \frac{1}{15} = 15 : 5 : 1$$

Sum of the ratios $= 15 + 5 + 1 = 21$

$$\therefore \quad L_1 = \frac{15}{21} \times 105 = 75 \text{ cm};$$

$$L_2 = \frac{5}{21} \times 105 = 25 \text{ cm};$$

$$L_3 = \frac{1}{21} \times 105 = 5 \text{ cm}$$

Hence the bridges should be placed at **75 cm** and $(75 + 25 =)$ **100 cm** from one end.

Scanned by CamScanner

EXAMPLE 43. The fundamental frequency of a sonometer wire increases by 5 Hz if its tension is increased by 21%. How will the frequency be affected if its length is increased by 10%?

Solution. Fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} \quad ...(1)$$

On increasing the tension by 21%, the new tension becomes 1.21 T. Therefore,

$$\nu + 5 = \frac{1}{2L}\sqrt{\frac{1.21\,T}{m}} \quad ...(2)$$

Dividing (2) by (1), we get

$$\frac{\nu+5}{\nu} = \sqrt{1.21} = 1.1 \quad \text{or} \quad \nu = 50 \text{ Hz}$$

On increasing the length by 10%, the new frequency becomes

$$\nu' = \frac{1}{2 \times 1.1L}\sqrt{\frac{T}{m}} = \frac{\nu}{1.1} = \frac{50}{1.1} = \mathbf{45.45\ Hz.}$$

EXAMPLE 44. A stone hangs in air from one end of a wire which is stretched over a sonometer. The wire is in unison with a certain tuning fork when the bridges of the sonometer are 45 cm apart. Now the stone hangs immersed in water at 4°C and the distance between the bridges has to be altered by 9 cm to re-establish unison of the wire with the same fork. Calculate the density of the stone.

Solution. Let V be the volume and ρ be the density of the stone. When the stone hangs in air, tension in the string is

$$T = V\rho g$$

Frequency of vibration,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{V\rho g}{m}} \quad ...(1)$$

When the stone is immersed in water, it loses in weight due to the upthrust of water. Tension in the string decreases to T'.

$$T' = \text{Apparent weight of stone}$$
$$= V\rho g - V\sigma g = V(\rho - \sigma)g$$

where σ is the density of water. The length of the wire has to be decreased to L' to bring it in unison with the tuning fork.

$$\therefore \quad \nu = \frac{1}{2L'}\sqrt{\frac{T'}{m}} = \frac{1}{2L'}\sqrt{\frac{V(\rho-\sigma)g}{m}} \quad ...(2)$$

From equations (1) and (2), we get

$$\frac{1}{2L}\sqrt{\frac{V\rho g}{m}} = \frac{1}{2L'}\sqrt{\frac{V(\rho-\sigma)g}{m}}$$

$$\text{or} \quad \frac{L'}{L} = \sqrt{\frac{\rho-\sigma}{\rho}}$$

Now $L = 45$ cm, $L' = 45 - 9 = 36$ cm, $\sigma = 1$ g m^{-3}

$$\therefore \quad \frac{36}{45} = \sqrt{\frac{\rho-1}{\rho}}$$

$$\frac{4}{5} = \sqrt{\frac{\rho-1}{\rho}}$$

$$\text{or} \quad \frac{\rho-1}{\rho} = \frac{16}{25} \quad \text{or} \quad 25\rho - 25 = 16\rho$$

$$\therefore \quad \rho = 25/9 = \mathbf{2.778\ g\ cm^{-3}}.$$

EXAMPLE 45. A wire having a linear mass density of 5.0×10^{-3} kg m^{-1} is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire. **[IIT]**

Solution. Suppose the wire resonates at 420 Hz in its pth harmonic and at 490 Hz in its $(p+1)$th harmonic.

$$\text{As} \quad \nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 420 = \frac{p}{2L}\sqrt{\frac{T}{m}} \quad ...(1)$$

$$\text{and} \quad 490 = \frac{p+1}{2L}\sqrt{\frac{T}{m}} \quad ...(2)$$

Dividing (2) by (1), we get

$$\frac{490}{420} = \frac{p+1}{p}$$

$$\text{or} \quad 1 + \frac{70}{420} = 1 + \frac{1}{p}$$

$$\text{or} \quad p = 6$$

Putting the value of p in (1), we get

$$420 = \frac{6}{2L}\sqrt{\frac{450}{5 \times 10^{-3}}} = \frac{3}{L}\sqrt{9 \times 10^4}$$

$$[\because m = 5.0 \times 10^{-3} \text{ kg m}^{-1}, T = 450 \text{ N}]$$

$$\text{or} \quad L = \frac{3 \times 3 \times 10^2}{420} = \frac{15}{7} = \mathbf{2.14\ m.}$$

PROBLEMS FOR PRACTICE

1. A sonometer wire is under a tension of 40 N and the length between the bridges is 50 cm. A metre long wire of the sonometer has a mass of 1.0 g. Determine its fundamental frequency. (**Ans.** 200 Hz)
2. A cord 80 cm long is stretched by a load of 8.0 kg f. The mass per unit length of the cord is 4.0×10^{-5} kg m^{-1}. Find (*i*) speed of the transverse wave in the cord and (*ii*) frequency of the fundamental and that of the second overtone.

   [**Ans.** (*i*) 1400 ms^{-1} (*ii*) 875 Hz, 2625 Hz]

Scanned by CamScanner

3. The length of a stretched wire is 1 m and its fundamental frequency is 300 Hz. What is the speed of the transverse wave in the wire ? (Ans. 600 ms^{-1})

4. The mass of a 1 m long steel wire is 20 g. The wire is stretched under a tension of 800 N. What are the frequencies of its fundamental mode of vibration and the next three higher modes ? [Roorkee 82] (Ans. 100, 200, 300, 400 Hz)

5. If the tension in the string is increased by 5 kg wt, the frequency of the fundamental tone increases in the ratio 2 : 3. What was the initial tension in the string ? (Ans. 4 kg wt)

6. A sonometer wire has a length of 114 cm between its two fixed ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ? [Roorkee 84] (Ans. At a distance of 72 cm, 96 cm from one end)

7. Two wires of the same material are stretched with the same force. Their diameters are 1.2 mm and 1.6 mm, while their lengths are 90 cm and 60 cm respectively. If the frequency of vibrations of first is 256 Hz, find that of the other. (Ans. 288 Hz)

8. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz ? (Ans. 60 cm)

9. The ratio of frequencies of two wires having same length and same tension and made of the same material is 2 : 3. If the diameter of one wire be 0.09 cm, then determine the diameter of the other. (Ans. 0.06 cm)

10. A 50 cm long wire is in unison with a tuning fork of frequency 256, when stretched by a load of density 9 g cm^{-3} hanging vertically. The load is then immersed in water. By how much the length of the wire be reduced to bring it again in unison with the same tuning fork ? (Ans. 2.867 cm)

11. A string vibrates with a frequency of 200 Hz. Its length is doubled and its tension is altered until it begins to vibrate with a frequency of 300 Hz. What is the ratio of new tension to the original tension ? (Ans. 9 : 1)

12. In Melde's experiment, a string vibrates in 3 loops when 8 grams were placed in the pan. What mass must be placed in the pan to make the string vibrate in 5 loops ? (Ans. 2.88 g)

HINTS

2. (i) $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{8 \times 9.8}{4 \times 10^{-5}}} = 1400\ ms^{-1}$.

(ii) Fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.80} \times 1400 = 875\ Hz.$$

Second overtone or third harmonic,

$$\nu_3 = 3\nu = 3 \times 875 = 2625\ Hz.$$

3. Fundamental frequency, $\nu = \frac{v}{2L}$

$\therefore \quad v = 2L\nu = 2 \times 1 \times 300 = 600\ ms^{-1}$.

5. $\sqrt{\frac{T_1}{T_2}} = \frac{\nu_1}{\nu_2} \quad \therefore \quad \frac{T_1}{T_1 + 5} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

or $\quad 9T_1 = 4T_1 + 20$

or $\quad T_1 = 4$ kg wt.

7. $\nu_1 = \frac{1}{L_1 D_1}\sqrt{\frac{T}{\pi \rho}}, \quad \nu_2 = \frac{1}{L_2 D_2}\sqrt{\frac{T}{\pi \rho}}$

$\therefore \quad \frac{\nu_2}{\nu_1} = \frac{L_1 D_1}{L_2 D_2} = \frac{90}{60} \times \frac{1.2}{1.6} = \frac{9}{8};$

$\nu_2 = \frac{9}{8}\nu_1 = \frac{9}{8} \times 256 = 288$ Hz.

8. Here $L_1 = 90$ cm, $\nu_1 = 124$ Hz, $\nu_2 = 186$ Hz, $L_2 = ?$

According to the law of length,

$\nu_2 L_2 = \nu_1 L_1$

$\therefore \quad L_2 = \frac{\nu_1 L_1}{\nu_2} = \frac{124 \times 90}{186} = 60$ cm.

10. As proved in Example 44,

$$\frac{L'}{L} = \sqrt{\frac{\rho - \sigma}{\rho}}$$

Here $L = 50$ cm, $\rho = 9\ g\ cm^{-3}$, $\sigma = 1\ g\ cm^{-3}$

$\therefore \quad \frac{L'}{50} = \sqrt{\frac{9-1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3} = \frac{2 \times 1.414}{3}$

or $\quad L' = \frac{141.4}{3} = 47.133$ cm

Decrease in length $= L - L' = 50 - 47.133 = 2.867$ cm.

11. In first case, $\quad 200 = \frac{1}{2L}\sqrt{\frac{T_1}{m}}$ ...(1)

In second case, $\quad 300 = \frac{1}{2 \times 2L}\sqrt{\frac{T_2}{m}}$ ...(2)

Dividing (2) by (1), we get

$\frac{300}{200} = \frac{1}{2}\sqrt{\frac{T_2}{T_1}} \quad$ or $\quad \frac{3}{1} = \sqrt{\frac{T_2}{T_1}}$

or $\quad \frac{T_2}{T_1} = \frac{9}{1} = 9 : 1.$

Scanned by CamScanner

In first case, $\nu = \frac{3}{2L}\sqrt{\frac{T_1}{m}}$

In second case, $\nu = \frac{5}{2L}\sqrt{\frac{T_2}{m}}$

$$\frac{3}{2L}\sqrt{\frac{T_1}{m}} = \frac{5}{2L}\sqrt{\frac{T_2}{m}}$$

or $\sqrt{\frac{T_2}{T_1}} = \frac{3}{5}$ or $\frac{T_2}{T_1} = \frac{9}{25}$

$$T_2 = \frac{9}{25} \times T_1 = \frac{9}{25} \times 8 = 2.88 \text{ g wt.}$$

15.21 STATIONARY WAVES IN ORGAN PIPES OR AIR COLUMNS

29. *What is an organ pipe ? How does it produce sound ?*

Organ pipe. *It is the simplest musical instrument in which sound is produced by setting an air column into vibrations.* Fig. 15.19 shows the section of a flute organ pipe.

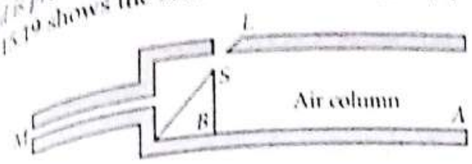


Fig. 15.19 Organ pipe.

When air is blown into the pipe through the *mouth piece* (*M*), it strikes against the slanting surface or *bevelled* (*B*). It is deflected upwards and issues out of the slit *S* in the form of a jet. This jet of air strikes against the sharp edge called the lip (*L*), setting it into vibrations. This produces a sound called *edge tone*, the frequency of which depends on the pressure of the air stream.

The sound waves travel down the pipe and get reflected at its closed or open end, producing longitudinal stationary waves. If the frequency of these waves is equal to the frequency of the edge tone, resonance occurs and a loud sound is produced.

If both the ends of the pipe are open, it is called an *open pipe*. If one end of the pipe is closed, it is called *closed pipe*.

30. *Describe the various modes of vibrations of an open organ pipe.*

Normal modes of vibration of an open organ pipe. Both the ends of an open organ pipe are open. The waves are reflected from these ends without change of type. However, the particles continue to move in the same direction even after the reflection of the waves. Consequently, the particles have the maximum displacements at the open ends. Hence antinodes are formed at the open ends. The various modes of vibration of an open organ pipe are shown in Fig. 15.20.

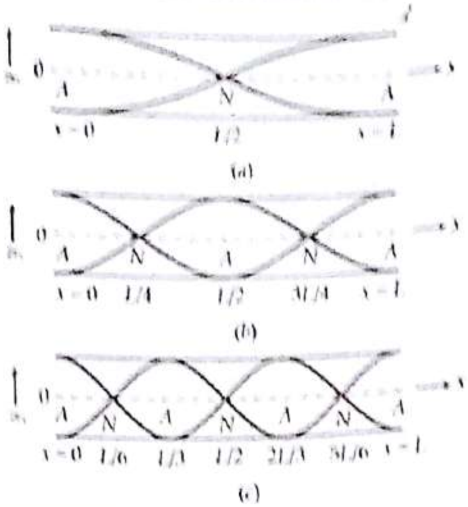


Fig. 15.20 Normal modes of vibration of an open pipe.

(*i*) ***First mode of vibration.*** In the simplest mode of vibration, there is one node in the middle and two antinodes at the ends of the pipe.

Here length of the pipe,

$$L = 2 \cdot \frac{\lambda_1}{4} = \frac{\lambda_1}{2}$$

$$\therefore \quad \lambda_1 = 2L$$

Frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu \quad \text{(say)}$$

This frequency is called *fundamental frequency* or *first harmonic*.

(*ii*) ***Second mode of vibration.*** Here antinodes at the open ends are separated by two nodes and one antinode.

$$L = 4 \cdot \frac{\lambda_2}{4} = \lambda_2$$

$$\text{Frequency,} \quad \nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{\gamma P}{\rho}} = 2\nu$$

This frequency is called *first overtone* or *second harmonic*.

(*iii*) ***Third mode of vibration.*** Here the antinodes at the open ends are separated by three nodes and two antinodes.

$$L = 6 \cdot \frac{\lambda_3}{4} \quad \text{or} \quad \lambda_3 = \frac{2L}{3}$$

$$\therefore \quad \text{Frequency,} \quad \nu_3 = \frac{v}{\lambda_3} = \frac{3}{2L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called *second overtone* or *third harmonic*.

Hence various frequencies of an open organ pipe are in the ratio 1 : 2 : 3 : 4 :...... These are called harmonics.

Scanned by CamScanner

31. *Prove analytically that in the case of an open organ pipe of length L, the frequencies of vibrating air column are given by* $\nu = n\,(v / 2L)$, *where n is an integer.*

Analytical treatment of stationary waves in an open organ pipe. Consider a cylindrical pipe of length L lying along the x-axis, with its open ends at $x = 0$ and $x = L$. The sound wave travelling along the pipe may be represented as

$$y_1 = A \sin(\omega t - kx)$$

The wave reflected from right open end may be represented as

$$y_2 = A \sin(\omega t + kx)$$

There is no phase reversal on reflection from the open end because it is a free or loose boundary. So the sign of A in the reflected wave is same as that in the incident wave.

By the principle of superposition, the resultant stationary wave is given by

$$y = y_1 + y_2 = A[\sin(\omega t - kx) + \sin(\omega t + kx)]$$

$$= 2A \sin \omega t \cos kx = (2A \cos kx) \sin \omega t$$

For all values of t, the resultant displacement is maximum (+ve or –ve) or antinodes are formed at the open ends *i.e.*, at $x = 0$ and $x = L$. This condition is satisfied if

$$\cos kL = \pm 1 \quad \text{or} \quad kL = n\pi,$$

where $n = 1, 2, 3, \ldots$

or $$\frac{2\pi}{\lambda} L = n\pi \quad \text{or} \quad \lambda_n = \frac{2L}{n}$$

The frequency of vibration is given by

$$\nu_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\gamma P}{\rho}}$$

For $n = 1$, $\nu_1 = \dfrac{1}{2L}\sqrt{\dfrac{\gamma P}{\rho}} = \nu$ (say)

This is the smallest frequency of the stationary waves produced in the open pipe. It is called *fundamental frequency* or *first harmonic*.

For $n = 2$, $\nu_2 = \dfrac{2v}{2L} = 2\nu$

(First overtone or second harmonic)

For $n = 3$, $\nu_3 = \dfrac{3v}{2L} = 3\nu$

(Second overtone or third harmonic)

and so on. The various modes of vibration of an open pipe are shown in Fig. 15.20.

32. *Give qualitative discussion of the different modes of vibration of a closed organ pipe.*

Normal modes of a closed organ pipe. In a closed organ pipe, one end of the pipe is open and the other end is closed. As the wave is reflected from the closed end, the direction of motion of the particles changes. The displacement is zero at the closed end *i.e.*, a node is formed at the closed end. The displacement of the particles is maximum at the open end, so an antinode is formed at the open end. The different modes of vibration of closed pipe are shown in Fig. 15.21.

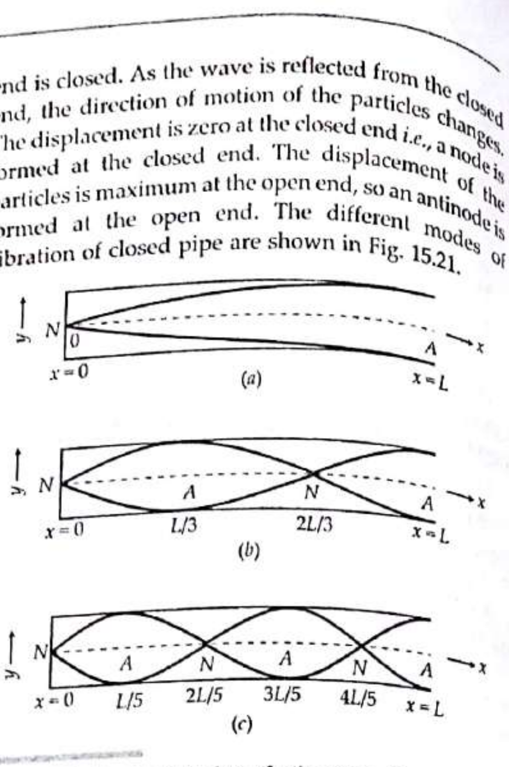


Fig. 15.21 Normal modes of vibration of closed pipe.

(*i*) ***First mode of vibration.*** In this simplest mode of vibration, there is only one node at the closed end and one antinode at the open end. If L is the length of the organ pipe, then

$$L = \frac{\lambda_1}{4} \quad \text{or} \quad \lambda_1 = 4L$$

Frequency,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

This frequency is called *first harmonic* or *fundamental frequency*.

(*ii*) ***Second mode of vibration.*** In this mode of vibration, there is one node and one antinode between a node at the closed end and an antinode at the open end.

$$\therefore \quad L = \frac{3\lambda_2}{4} \quad \text{or} \quad \lambda_2 = \frac{4L}{3}$$

Frequency,

$$\nu_2 = \frac{v}{\lambda_2} = \frac{3}{4L}\sqrt{\frac{\gamma P}{\rho}} = 3\nu$$

This frequency is called *first overtone* or *third harmonic*.

Scanned by CamScanner

(iii) *Third mode of vibration.* In this mode of vibration, there are two nodes and two antinodes in between a node at the closed end and an antinode at the open end.

$$L = \frac{5\lambda_3}{4} \quad \text{or} \quad \lambda_3 = \frac{4L}{5}$$

Frequency,

$$\nu_3 = \frac{v}{\lambda_3} = \frac{5}{4L}\sqrt{\frac{\gamma P}{\rho}} = 5\nu$$

Hence different frequencies produced in a closed organ pipe are in the ratio 1 : 3 : 5 : 7 : ...... *i.e.*, only odd harmonics are present in a closed organ pipe.

33. Prove analytically that in the case of a closed organ pipe of length L, the frequencies of the vibrating air column are given by $v = (2n+1)(v/4L)$, where n is an integer.

Analytical treatment of stationary waves in a closed organ pipe. Consider a cylindrical pipe of length L lying along the x-axis, with its closed end at $x = 0$ and open end at $x = L$. The sound wave sent along the pipe may be represented as

$$y_1 = A\sin(\omega t + kx)$$

The wave reflected from the closed end may be represented as

$$y_2 = -A\sin(\omega t - kx)$$

The negative sign before A is due to reversal of phase at the closed end.

By the principle of superposition, the resultant stationary wave is given by

$$y = y_1 + y_2 = A[\sin(\omega t + kx) - \sin(\omega t - kx)]$$
$$= 2A\cos\omega t\sin kx = (2A\sin kx)\cos\omega t.$$

Clearly, $y = 0$ at $x = 0$ *i.e.*, a node is formed at the closed. The resultant displacement at $x = L$ will be maximum (+ve or –ve) because the open end is a free or loose boundary. This condition is satisfied if

$$\sin kL = \pm 1$$

or $$kL = (2n-1)\frac{\pi}{2} \quad \text{where } n = 1, 2, 3, ...$$

or $$\frac{2\pi}{\lambda}L = (2n-1)\frac{\pi}{2} \quad \text{or} \quad \lambda_n = \frac{4L}{2n-1}$$

The corresponding frequency of vibration is given by

$$\nu_n = \frac{v}{\lambda_n} = \frac{(2n-1)v}{4L} = \frac{(2n-1)}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

For $n = 1$

$$\nu_1 = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

This is the smallest frequency of the stationary waves produced in the closed pipe. It is called *fundamental frequency* or *first harmonic*.

For $n = 2$, $\quad \nu_2 = \frac{3v}{4L} = 3\nu$

(First overtone or third harmonic)

For $n = 3$, $\quad \nu_3 = \frac{5v}{4L} = 5\nu$

(Second overtone or fifth harmonic)

and so on. The various modes of vibration of a closed pipe are shown in Fig. 15.21.

Examples based on Organ Pipes and Rods clamped in the Middle

Formulae Used

1. *In an organ pipe closed at one end*, only odd harmonics are present.

   Fundamental mode, $\nu_1 = \frac{v}{4L} = \nu$ (First harmonic)

   Second mode, $\nu_2 = 3\nu$
   (Third harmonic or first overtone)

   Third mode, $\nu_3 = 5\nu$
   (Fifth harmonic or second overtone)

   nth mode, $\nu_n = (2n-1)\nu$
   [$(2n-1)$th harmonic or $(n-1)$th overtone]

2. *In an organ pipe open at both ends.* Both odd and even harmonics are present.

   Fundamental mode, $\nu_1' = \frac{v}{2L} = \nu'$ (First harmonic)

   Second mode, $\nu_2' = 2\nu'$
   (Second harmonic or first overtone)

   Third mode $\nu_3' = 3\nu'$
   (Third harmonic or second overtone)

   nth mode $\nu_n' = n\nu$
   [nth harmonic or $(n-1)$th overtone]

   Clearly, $\nu_1' = 2\nu_1$

3. *Resonance tube.* If L_1 and L_2 are the first and second resonance lengths with a tuning fork of frequency ν, then the speed of sound,

   $v = 4\nu(L_1 + 0.3D)$,

   D = internal diameter of resonance tube

   or $\quad v = 2\nu(L_2 - L_1)$

   End correction $= 0.3D = \frac{L_2 - 3L_1}{2}$

Units Used

Velocity of sound v is in ms^{-1}, length of organ pipe L is in metre and frequency ν in Hz.

Scanned by CamScanner

EXAMPLE 46. *What should be minimum length of an open organ pipe for producing a note of 110 Hz ? The speed of sound is 330 ms^{-1}.*

Solution. Frequency, $\nu = 110$ Hz,

Speed of sound, $v = 330\ ms^{-1}$

Fundamental frequency of an open organ pipe,

$$\nu = \frac{v}{2L}$$

$$\therefore \quad L = \frac{v}{2\nu} = \frac{330}{2 \times 110} = \mathbf{1.5\ m.}$$

EXAMPLE 47. *The length of an organ pipe open at both ends is 0.5 m. Calculate the fundamental frequency of the pipe, if the velocity of sound in air be 350 ms^{-1}. If one end of the pipe is closed, then what will be the fundamental frequency ?*

Solution. Speed of sound, $v = 350\ ms^{-1}$,

Length of the pipe, $L = 0.5$ m

Fundamental frequency of the open pipe,

$$\nu' = \frac{v}{2L} = \frac{350}{2 \times 0.5} = \mathbf{350\ Hz.}$$

Fundamental frequency of the closed pipe,

$$\nu = \frac{v}{4L} = \frac{350}{4 \times 0.5} = \mathbf{175\ Hz.}$$

EXAMPLE 48. *A pipe 30.0 cm long is open at both ends. Which harmonic mode of the pipe is resonantly excited by a 1.1 kHz source ? Will resonance with the same source be observed if one end of the pipe is closed ? Take the speed of sound in air as 330 ms^{-1}.* **[NCERT]**

Solution. Length of the pipe,

$L = 30$ cm $= 0.30$ cm

Speed of sound, $v = 330\ ms^{-1}$

Fundamental frequency of the open pipe,

$$\nu_1 = \frac{v}{2L} = \frac{330}{2 \times 0.30} = 550 \text{ Hz}$$

Second harmonic,

$$\nu_2 = 2\nu_1 = 2 \times 550 = 1100 \text{ Hz}$$

Third harmonic,

$$\nu_3 = 3\nu_1 = 3 \times 550 = 1650 \text{ Hz}$$

Fourth harmonic,

$$\nu_4 = 4\nu_1 = 4 \times 550 = 2200 \text{ Hz, and so on.}$$

Clearly, a source of frequency 1.1 kHz (or 1100 Hz) will resonantly excite the *second harmonic* of the open pipe.

If one end of the pipe is closed, its fundamental frequency becomes

$$\nu_1' = \frac{v}{4L} = \frac{330}{4 \times 0.30} = 275 \text{ Hz}$$

As only odd harmonics are present in a closed pipe, so

Third harmonic,

$$\nu_3' = 3\nu_1' = 3 \times 275 = 825 \text{ Hz}$$

Fifth harmonic,

$$\nu_5' = 5\nu_1' = 5 \times 275 = 1375 \text{ Hz, and so on.}$$

As no frequency of the closed pipe matches with the source frequency of 1.1 kHz, so no resonance will be observed with the source, the moment one end of the pipe is closed.

EXAMPLE 49. *Find the ratio of the length of a closed pipe to that of an open pipe in order that the second overtone of the former is in unison with the fourth overtone of the latter.*

Solution. Fundamental frequency of a closed organ pipe, $\nu = \frac{v}{4L}$

Fundamental frequency of an open organ pipe,

$$\nu' = \frac{v}{2L'}$$

Second overtone of the closed pipe $= 5\nu = \frac{5v}{4L}$

Fourth overtone of the open pipe $= 5\nu' = \frac{5v}{2L'}$

As the two overtones are in unison, therefore

$$\frac{5v}{4L} = \frac{5v}{2L'} \quad \text{or} \quad \frac{L}{L'} = \frac{2}{4} = \frac{1}{2}$$

or $\quad L : L' = \mathbf{1 : 2.}$

EXAMPLE 50. *An open pipe is suddenly closed at one end with the result that the frequency of the third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. What is the fundamental frequency of the open pipe ?* **[IIT 96]**

Solution. Fundamental frequency of open pipe,

$$\nu_o = \frac{v}{2L}$$

Frequency of third harmonic of closed pipe,

$$\nu_c = \frac{3v}{4L}$$

$$\therefore \quad \frac{\nu_c}{\nu_o} = \frac{3}{2} \quad \text{or} \quad \nu_c = \frac{3}{2}\nu_o$$

Given $\quad \nu_c = \nu_o + 100$

$$\therefore \quad \frac{3}{2}\nu_o = \nu_o + 100 \quad \text{or} \quad \nu_o = \mathbf{200\ Hz.}$$

EXAMPLE 51. *The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Velocity of sound in air = 330 ms^{-1}.* **[IIT 97]**

Solution. Let lengths of the open and closed organ pipes be L_0 and L_c respectively.

Fundamental frequency of closed pipe,

$$v_c = \frac{v}{4L_c}$$

$$L_c = \frac{v}{4v_c}$$

But $v = 330\ \text{ms}^{-1}$, $v_c = 110$ Hz

$$L_c = \frac{330}{4 \times 110} = 0.75\ \text{m} = \mathbf{75\ cm.}$$

Frequency of first overtone of open organ pipe,

$$v_0 = 2 \times \frac{v}{2L_0} = \frac{v}{L_0}$$

Frequency of first overtone of closed pipe,

$$v_c' = 3 \times \frac{v}{4L_c} = 3 \times 110 = 330\ \text{Hz.}$$

But $v_0 - v_c' = 2.2$ Hz

$$\frac{v}{L_0} - 330 = 2.2 \quad \text{or} \quad \frac{v}{L_0} = 332.2$$

or

$$L_0 = \frac{v}{332.2} = \frac{330}{332.2} = 0.99\ \text{m} = \mathbf{99\ cm.}$$

EXAMPLE 52. A well with vertical sides and water at the bottom resonates at 7 Hz and at no other lower frequency. The air in the well has density 1.10 kg m^{-3} and bulk modulus of 1.33×10^5 Nm^{-2}. How deep is the well ?

Solution. For air $\rho = 1.10$ kg m^{-3},

$$\kappa = 1.33 \times 10^5\ \text{Nm}^{-2}$$

∴ Speed of sound in air,

$$v = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{1.33 \times 10^5}{1.10}} = 347.7\ \text{ms}^{-1}$$

A well can be regarded as a closed organ pipe. So its fundamental frequency,

$$v = \frac{v}{4L}$$

or

$$L = \frac{v}{4v} = \frac{347.7}{4 \times 7} = \mathbf{12.41\ ms^{-1}.}$$

EXAMPLE 53. A resonance air column resonates with a tuning fork of 512 Hz at length 17.4 cm. Neglecting the end correction, deduce the speed of sound in air.

Solution. Here $v = 512$ Hz, $L_1 = 17.4$ cm

When end correction is neglected, speed of sound in air is given by

$$v = 4v\,L_1 = 4 \times 512 \times 17.4 = 35635.2\ \text{cm s}^{-1}$$
$$= \mathbf{356.35\ ms^{-1}.}$$

EXAMPLE 54. A resonance tube is resonated with tuning fork of frequency 512 Hz. Two successive lengths of the resonated air-column are 16.0 cm and 51.0 cm. The experiment is performed at the room temperature of 40°C. Calculate the speed of sound at 0°C and the end correction.

Solution. Here $v = 512$ Hz, $L_1 = 16.0\ \text{cm} = 0.16$ m,

$$L_2 = 51.0\ \text{cm} = 0.51\ \text{m}$$

Speed of sound at 40°C,

$$v = 2v\,(L_2 - L_1) = 2 \times 512 \times (0.51 - 0.16)$$
$$= 358.4\ \text{ms}^{-1}$$

Speed of sound at 0°C is given by

$$v_0 = v - 0.61\,t = 358.4 - 0.61 \times 40$$
$$= 358.4 - 24.4 = \mathbf{334\ ms^{-1}.}$$

End correction

$$= \frac{L_2 - 3L_1}{2} = \frac{51.0 - 48.0}{2} = \mathbf{1.5\ cm.}$$

EXAMPLE 55. Determine the possible harmonics in the longitudinal vibration of a rod clamped in the middle.

Solution. Consider a rod of length L clamped in the middle. As shown in Fig. 15.22(*a*), it has one node in the middle and two antinodes at its free ends in the fundamental mode.

$$\therefore \quad L = 2 \cdot \frac{\lambda_1}{4} \quad \text{or} \quad \lambda_1 = 2L$$

Fundamental frequency or first harmonic,

$$v_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = v\ \text{(say)}$$

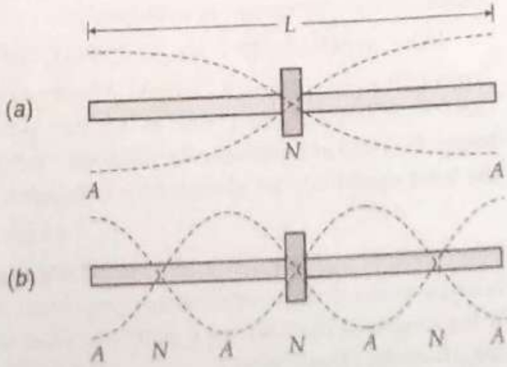


Fig. 15.22

In the second mode, there is an additional node and antinode on the two sides of the clamp, as shown in Fig. 15.22(*b*).

$$\therefore \quad L = 6 \cdot \frac{\lambda_2}{4} \quad \text{or} \quad \lambda_2 = \frac{2L}{3}$$

and

$$v_2 = \frac{v}{\lambda_2} = \frac{3v}{2L} = 3\,v$$

This is called *third harmonic or first overtone*.

Scanned by CamScanner

Similarly, for third mode,

$$\nu_3 = \frac{5v}{2L} = 5\nu$$

This is called *fifth harmonic* or *second overtone*.

Hence $\nu_1 : \nu_2 : \nu_3 : ... = 1 : 3 : 5 : , ...$

EXAMPLE 56. A brass rod 1 metre long is firmly clamped in the middle and one end is stroked by a resined cloth. What is the pitch of the note you will hear ? Young's modulus for brass $= 10^{12}$ dyn cm^{-2} and density $= 9$ g cm^{-3}.

Solution. Here $Y = 10^{12}$ dyn cm^{-2}, $\rho = 9$ g cm^{-3}

Speed of sound in the brass rod,

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{12}}{9}} = \frac{10^6}{3} \text{ cm s}^{-1}.$$

Length of rod,

$$L = 1 \text{ m} = 200 \text{ cm}$$

Fundamental note,

$$\nu = \frac{v}{\lambda} = \frac{v}{2L} = \frac{10^6}{3 \times 200} = \mathbf{1666.67\ Hz.}$$

✖ Problems For Practice

1. An open organ pipe produces a note of frequency 512 Hz at 15°C, calculate the length of the pipe. Velocity of sound at 0°C is 335 ms^{-1}. **(Ans. 0.336 m)**

2. Find the frequencies of the fundamental note and the first overtone in an open air column and a closed air column of length 34 cm. The velocity of sound at room temperature is 340 ms^{-1}. **[Ans. (*i*) 500 Hz, 1000 Hz (*ii*) 250 Hz, 750 Hz]**

3. Prove that a pipe of length 2 *L* open at both ends has same fundamental frequency as another pipe of length *L* closed at the other end. Also, state whether the total sound will be identical for two pipes. **(Ans. No)**

4. The fundamental frequency of a closed organ pipe is equal to the first overtone of an open organ pipe. If the length of the open pipe is 60 cm, what is the length of the closed pipe ? **(Ans. 15 cm)**

5. The fundamental tone produced by an organ pipe has a frequency of 110 Hz. Some other frequencies in the notes produced by this pipe are 220, 440, 550, 660 Hz. Is this pipe open at both ends or open at one end and closed at the other ? Calculate the effective length of the pipe. Speed of sound = 330 ms^{-1}. **(Ans. Pipe is open at both ends, 1.5 m)**

6. The fundamental frequency of an open organ pipe is 300 Hz. The frequency of the first overtone of another closed organ pipe is the same as the frequency of the first overtone of open pipe. What are the lengths of the pipes ? The speed of sound = 330 ms^{-1}. **(Ans. 55.0 cm, 41.25 cm)**

7. Find the ratio of length of a closed organ pipe to that of open pipe in order that the second overtone of the former is in unison with fourth overtone of the latter. **(Ans. 3 : 2)**

8. A tuning fork of frequency 341 Hz is vibrated just over a tube of length 1 m. Water is being poured gradually in the tube. What height of water column will be required for resonance ? The speed of sound in air is 341 ms^{-1}. **(Ans. 25 cm or 75 cm)**

9. A resonance air column shows resonance with a tuning fork of frequency 256 Hz at column lengths 33.4 cm and 101.8 cm. Find (*i*) end-correction and (*ii*) the speed of sound in air. **[Ans. (*i*) 0.8 cm (*ii*) 350.2 ms^{-1}]**

10. A metallic bar clamped at its middle point vibrates with a frequency ν when it is rubbed at one end. If its length is doubled, what will be its natural frequency of vibration ? **(Ans. ν / 2)**

✖ Hints

1. Velocity of sound at 15°C will be

$$v = v_0 + 0.61t = 335 + 0.61 \times 15 = 344.15 \text{ ms}^{-1}$$

Fundamental frequency of an open organ pipe,

$$\nu = \frac{v}{4L} \quad \therefore \quad L = \frac{v}{4\nu} = \frac{344.15}{4 \times 512} = 0.336 \text{ m.}$$

3. Fundamental frequency of closed pipe of length *L*,

$$\nu = \frac{v}{4L}$$

Fundamental frequency of an open pipe of length 2*L*,

$$\nu' = \frac{v}{2 \times 2L} = \frac{v}{4L}. \text{ Clearly, } \nu = \nu'$$

In an open organ pipe all harmonics are present whereas in closed organ pipe, only odd harmonics are present. Hence the quality of sound will be different for the two pipes.

4. Fundamental frequency of a closed organ pipe,

$$\nu_1 = \frac{v}{4L}$$

Fundamental frequency of an open organ pipe,

$$\nu_1' = \frac{v}{2L'}$$

Frequency of first overtone of open organ pipe,

$$\nu_2' = 2\nu_1' = \frac{v}{L'}$$

Given $\nu_1 = \nu_2'$

$$\therefore \quad \frac{v}{4L} = \frac{v}{L'} \quad \text{or} \quad L = \frac{L'}{4} = \frac{60}{4} = \mathbf{15\ cm.}$$

Scanned by CamScanner

End correction $= \frac{L_2 - 3L_1}{2} = \frac{101.8 - 3 \times 33.4}{2}$

$= \frac{1.6}{2} = 0.8$ cm.

Speed of sound,

$v = 2\nu (L_2 - L_1) = 2 \times 256 \times (1.018 - 0.334)$

$= 2 \times 256 \times 0.684 = 350.2\ ms^{-1}$.

Natural frequency, $\nu = \frac{v}{\lambda} = \frac{v}{2L}$

On doubling the length, frequency is halved.

15.22 BEATS

34. *What are beats ? What is the essential condition for the formation of beats ?*

Beats. When two sound waves of slightly different frequencies travelling along the same path in the same direction in a medium superpose upon each other, the intensity of the resultant sound at any point in the medium rises and falls (technically known as waxing and waning of sound) alternately with time. *These periodic variations in the intensity of sound caused by the superposition of two sound waves of slightly different frequencies are called beats.* One rise and one fall of intensity constitute one beat. The number of beats produced per second is called beat frequency.

Beat frequency = Difference in frequencies of the two superposing waves

$$\nu_{beat} = \nu_1 - \nu_2$$

Essential condition for the formation of beats. For beats to be audible, the difference in the frequency of the two sound waves should not exceed 10. If the difference is more than 10, we shall hear more than 10 beats per second. But due to persistence of hearing, our ear is not able to distinguish between two sounds as separate if the time interval between them is less than (1/10)th of a second. Hence beats heard will not be distinct if the number of beats produced per second is more than 10.

35. *Explain the formation of beats by graphical method.*

Formation of beats by graphical method. In Fig. 15.23(*a*), the full line curve is the displacement-time curve of a wave of frequency ν_1 and the dashed curve is for a wave of frequency ν_2. Here ν_1 is slightly greater than ν_2, so the first wave is slightly smaller than second.

At time t_1, the two waves meet in the same phase at a given point. They reinforce to produce maximum sound intensity. With the passage of time, the phase difference between the two waves increases and so the two curves gradually get more and more out of step.

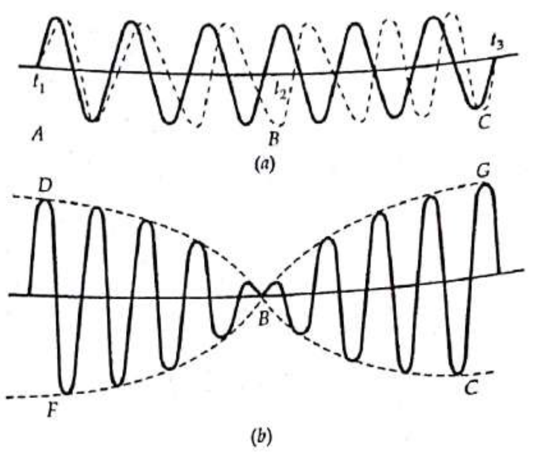


Fig. 15.23 Formation of beats graphically.

At time t_2, the two waves are in exactly opposite phases. This happens when one wave gains half a vibration over the other. Now they produce minimum sound intensity. Now the phase difference goes on decreasing with time. At time t_3 one wave gains one full vibration on the other and the two waves are again in same phase and produce maximum intensity, and so on. The resultant wave, obtained by the algebraic sum of the displacements of the two waves, is shown by full line curve in Fig. 15.23(*b*). The dashed envelopes above and below it show how the amplitude of the resultant wave varies with time. The time interval from t_1 to t_3 is *one beat period*, because during this interval only one beat is formed. Moreover, during this time interval, the first wave completes $(\nu + 1)$ oscillations while the second wave completes ν oscillations. Thus beat frequency is equal to the difference in frequencies of the two superposing waves.

36. *Explain the formation of beats analytically. Prove that the beat frequency is equal to the difference in frequencies of the two superposing waves.*

Analytical treatment of beats. Consider two harmonic waves of frequencies ν_1 and ν_2 (ν_1 being slightly greater than ν_2) and each of amplitude A travelling in a medium in the same direction. The displacements due to the two waves at a given observation point may be represented as

$$y_1 = A \sin \omega_1 t = A \sin 2\pi \nu_1 t$$

$$y_2 = A \sin \omega_2 t = A \sin 2\pi \nu_2 t$$

By the principle of superposition, the resultant displacement at the given point will be

$$y = y_1 + y_2 = A \sin 2\pi \nu_1 t + A \sin 2\pi \nu_2 t$$

$$= 2A \cos 2\pi \left(\frac{\nu_1 - \nu_2}{2}\right) t \,.\, \sin 2\pi \left(\frac{\nu_1 + \nu_2}{2}\right) t$$

Scanned by CamScanner

If we write

$$\nu_{mod} = \frac{\nu_1 - \nu_2}{2} \quad \text{and} \quad \nu_{av} = \frac{\nu_1 + \nu_2}{2}$$

then $\quad y = 2A \cos(2\pi \nu_{mod}\, t) \sin(2\pi \nu_{av}\, t)$

or $\quad y = R \sin(2\pi \nu_{av}\, t)$

where $R = 2A \cos(2\pi \nu_{mod}\, t)$ is the amplitude of the resultant wave. As ν_1 is slightly greater than ν_2, so $\nu_{mod} << \nu_{av}$ *i.e.*, R varies very slowly with time. Hence the above equation represents a wave of periodic rapid oscillation of average frequency ν_{av} 'modulated' by a slowly varying oscillation of frequency ν_{mod}.

The amplitude R of the resultant wave will be maximum, when

$$\cos 2\pi \nu_{mod}\, t = \pm 1$$

or $\quad 2\pi \nu_{mod}\, t = n\pi$

or $\quad \pi(\nu_1 - \nu_2)\, t = n\pi$

or $\quad t = \frac{n}{\nu_1 - \nu_2} = 0, \frac{1}{\nu_1 - \nu_2}, \frac{2}{\nu_1 - \nu_2}, \ldots$

∴ Time interval between two successive maxima

$$= \frac{1}{\nu_1 - \nu_2}$$

Similarly, the amplitude R will be minimum, when

$$\cos 2\pi \nu_{mod}\, t = 0$$

or $\quad 2\pi \nu_{mod}\, t = (2n+1)\pi/2$

or $\quad \pi(\nu_1 - \nu_2) t = (2n+1)\pi/2$

or $\quad t = \frac{(2n+1)}{2(\nu_1 - \nu_2)} = \frac{1}{\nu_1 - \nu_2}, \frac{3}{2(\nu_1 - \nu_2)}, \frac{5}{2(\nu_1 - \nu_2)}, \ldots$

∴ The time interval between successive minima

$$= \frac{1}{\nu_1 - \nu_2}$$

Clearly, both maxima and minima of intensity occur alternately. Technically, one maximum of intensity followed by a minimum is called a *beat*. Hence the time interval between two successive beats or the *beat period* is

$$t_{beat} = \frac{1}{\nu_1 - \nu_2}$$

The *number of beats produced per second is called* **beat** ***frequency.***

$$\nu_{beat} = \frac{1}{t_{beat}} \quad \text{or} \quad \nu_{beat} = \nu_1 - \nu_2$$

∴ Beat frequency = Difference between the frequencies of two superposing waves.

37. *How will you experimentally demonstrate the phenomenon of beats in sound ?*

Experimental demonstration of beats in sound. As shown in Fig. 15.24, mount two tuning forks A and B of exactly the same frequency on two sound boxes, placed with their open ends facing each other.

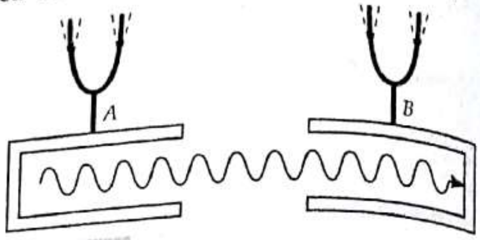


Fig. 15.24 Experimental demonstrations of beats.

Now stick a little wax to the prong of one of them to as to slightly reduce its frequency. Set the two tuning forks into vibrations. The two tuning forks will produce sound waves of different frequencies. The intensity of the resulting sound will increase and decreasing periodically with time. We will actually hear beats. By counting the number of beats heard in a given interval of time, we can calculate the beat frequency and hence can determine the difference $(\nu_1 - \nu_2)$ between the frequencies of the two tuning forks.

15.23 PRACTICAL APPLICATIONS OF BEATS

38. *Explain some practical applications of beats.*

Practical applications of beats. (*i*) **Determination of an unknown frequency.** Suppose ν_1 is the known frequency of tuning fork A and ν_2 is the unknown frequency of tuning fork B. When the two tuning forks are sounded together, suppose they produce b beats per second. Then

$$\nu_2 = \nu_1 + b \quad \text{or} \quad \nu_1 - b$$

The exact frequency may be determined by any of the following two methods :

(*a*) ***Loading method.*** Attach a little wax to the prong of the tuning fork B. Again, find the number of beats produced per second. If the frequency of B is greater than that of A *i.e.*, $(\nu_1 + b)$, then the attaching of a little wax lowers its frequency and reduces the difference in frequencies of A and B. This would decrease the beat frequency.

Hence, if *the beat frequency decreases on loading the prong of the tuning fork of the unknown frequency, then the unknown frequency is greater than the known frequency.* That is,

$$\nu_2 = \nu_1 + b$$

On the other hand, if the frequency of B is less than that of A *i.e.*, $(\nu_1 - b)$, then the attaching of a little wax further lowers its frequency and increases the difference in frequencies of A and B. This would increase the beat frequency.

Scanned by CamScanner

Hence, if the beat frequency increases on loading the prong of the tuning fork of the unknown frequency, then unknown frequency is less than the known frequency.

$$\nu_2 = \nu_1 - b$$

(b) By filing method. If a prong of the tuning fork B is filed, its frequency increases. Again, note the number of beats produced per second.

If on filing the prong of B, the beat frequency decreases, then $\nu_2 = \nu_1 - b$

If on filing the prong of B, the beat frequency increases, then $\nu_2 = \nu_1 + b$

(ii) **For tuning musical instruments.** Musicians use beat phenomenon in tuning their musical instruments. If an instrument is sounded against a standard frequency and tuned until the beats disappear, then the instrument is in tune with the standard frequency.

(iii) **For producing colourful effects in music.** Sometimes, a rapid succession of beats is knowingly produced in music. This produces an effect similar to that of human voice and is appreciated by the audience.

(iv) **For detection of marsh gas in mines.** Here we use two small organ pipes tuned to the same frequency. One pipe contains pure dry air and the other ordinary mine air.

If any marsh gas or methane appears in the mine, the density of the mine air in the second pipe decreases which slightly changes its frequency of vibration. When sounded with the first organ pipe, it gives rise to beats. The miners are thus warned well in advance of the explosive marsh gas.

(v) **Use in electronics.** It is difficult to make low frequency oscillators. In practice, two high frequency oscillators with a small difference in their frequencies are used. Their low beat frequency $(\nu_1 - \nu_2)$ serves the purpose of a low frequency oscillator.

Examples based on Beats Formation

FORMULAE USED

1. Beat frequency = Number of beats per second = Difference in frequencies of two sources

or $b = (\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$

2. $\nu_2 = \nu_1 \pm b$
3. If the prong of tuning fork is *filed*, its frequency increases. If the prong of a tuning fork is *loaded* with a little wax, its frequency decreases. These facts can be used to decide about + or − sign in the above equation.

UNITS USED

All frequencies are in Hz or s^{-1}.

EXAMPLE 57. *The points of the prongs of a tuning fork B originally in unison with a tuning fork A of frequency 384 are filed and the fork produces 3 beats per second, when sounded together with A. What is the pitch of B after filing ?*

Solution. Frequency of tuning fork $A = 384$ Hz.

As tuning fork B is in unison with A, so its original frequency = 384 Hz

Beat frequency $= 3\ s^{-1}$

Possible frequencies of B after filing

$$= 384 \pm 3 = 387 \text{ or } 381 \text{ Hz}$$

As the frequency increases on filing, so frequency of B after filing = **387 Hz.**

EXAMPLE 58. *A tuning fork arrangement (pair) produces 4 beats s^{-1} with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats s^{-1}. What is the frequency of the unknown fork ?* **[AIEEE 02]**

Solution. Unknown frequency

= Known frequency ± Beat frequency

$= 288 \pm 4 = 292$ or 284 Hz

On loading with wax, the frequency decreases, the beat frequency also decreases to 2.

∴ Unknown frequency = **292 cps** (higher one).

EXAMPLE 59. *A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.* **[IIT]**

Solution. Unknown frequency

= Known frequency ± Beat frequency

$= 310 \pm 4 = 314$ or 306 Hz

Out of these two possible frequencies, one must be the initial value and the other final value. As frequency increases on filing, therefore

initial unknown frequency = **306 Hz.**

EXAMPLE 60. *A fork of unknown frequency when sounded with one of frequency 288 Hz gives 4 beats per second and when loaded with a piece of wax again gives 4 beats per second. How do you account for this and what was the unknown frequency ?* **[IIT]**

Solution. Unknown frequency

= Known frequency ± Beat frequency

$= 288 \pm 4 = 292$ or 284 Hz.

As the beat frequency remains unchanged even on loading the tuning fork, of the two possible frequencies one must be initial frequency and the other the final one. But the frequency of a tuning fork decreases on loading, therefore

initial unknown frequency = **292 Hz.**

Scanned by CamScanner

EXAMPLE 61. *Two tuning forks A and B produce 4 beats/second. On loading B with wax, 6 beats/second were heard. If the quantity of wax is reduced, the number of beats per second again becomes 4. Find the frequency of B if the frequency of A is 256 Hz.*

Solution. Frequency of tuning fork $A = 256$ Hz

Beat frequency $= 4\ s^{-1}$

$\therefore$ Possible frequencies of B

$$= 256 \pm 4 = 260 \text{ or } 252 \text{ Hz.}$$

As some wax continues to remain attached finally with the tuning fork B and beats/second $= 4$, so the final frequency must be less than the initial frequency.

$\therefore$ Initial frequency of $B = $ **260 Hz.**

EXAMPLE 62. *A tuning fork produces 4 beats/s when sounded with a tuning fork of frequency 512 Hz. The same tuning fork when sounded with another tuning fork of frequency 514 Hz produces 6 beats/s. Find the frequency of the tuning fork.*

Solution. Let the frequency of the tuning fork $= \nu$

It produces 4 beats/s with a tuning fork of frequency 512 Hz

$$\therefore \quad \nu = 512 \pm 4 = 516 \text{ or } 508 \text{ Hz} \quad ...(1)$$

It also produces 6 beats/s with a tuning fork of frequency 514 Hz

$$\therefore \quad \nu = 514 \pm 6 = 520 \text{ or } 508 \text{ Hz} \quad ...(2)$$

Equations (1) and (2) show that the common frequency is 508.

$\therefore$ $\nu = $ **508 Hz.**

EXAMPLE 63. *A tuning fork of known frequency of 256 Hz makes 5 beat s^{-1} with the vibrating string of a piano. The beat frequency decreases to 2 beats s^{-1}, when the tension in the piano string is slightly increased. What was the frequency of the piano string before increasing the tension?* **[AIEEE 03]**

Solution. Frequency of a piano string

$$= 256 \pm 5 = 261 \text{ or } 251 \text{ Hz}$$

When the tension in the piano string is increased, its frequency increases ($\nu \propto \sqrt{T}$). If the original frequency is 251 Hz (the lower one), the beat frequency should decrease on increasing the tension. This is given to be the case as the beat frequency decreases from 5 beats s^{-1} to 2 beats s^{-1}.

$\therefore$ Original frequency of piano $=$ **251 Hz**.

EXAMPLE 64. *When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?* **[AIEEE 05]**

Solution. Frequency of fork 2

$$= \text{Frequency of fork 1} \pm \text{Beat frequency}$$
$$= 200 \pm 4 = 204 \text{ or } 196 \text{ Hz}$$

When some tape is attached to the prong of fork 2, its frequency decreases, but the beat frequency increases from 4 to 6.

$\therefore$ Frequency of fork 2 $=$ **196 Hz** (the lower one).

EXAMPLE 65. *A set of 24 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats per second with the preceding one and the last sounds the octave of the first, find the frequencies of the first and the last forks.*

Solution. Let frequency of first fork $= \nu$

Then frequency of second fork $= \nu + 4$

Frequency of third fork $= \nu + 2 \times 4$

Frequency of fourth fork $= \nu + 3 \times 4$

Frequency of 24th fork $= \nu + 23 \times 4$

But the frequency of the last is the octave of the first.

$$\therefore \quad 2\nu = \nu + 23 \times 4 \quad \text{or} \quad \nu = 92 \text{ Hz}$$

Frequency of the first fork

$= \nu = $ **92 Hz.**

Frequency of the last fork

$= 2\nu = $ **184 Hz.**

EXAMPLE 66. *In an experiment, it was found that a tuning fork and a sonometer wire gave 5 beats per second both when the length of the wire was 1 m and 1.05 m. Calculate the frequency of the fork.*

Solution. Here $L_1 = 1$ m, $L_2 = 1.05$ m,

Beat frequency $= 5\ s^{-1}$

As $\quad \nu \propto \dfrac{1}{L} \quad \therefore \quad \dfrac{\nu_1}{\nu_2} = \dfrac{L_2}{L_1}$

But $\quad L_1 < L_2 \quad \therefore \quad \nu_1 > \nu_2$

If ν is the frequency of the tuning fork, then

$$\nu_1 = \nu + 5 \quad \text{and} \quad \nu_2 = \nu - 5$$

Hence $\quad \dfrac{\nu + 5}{\nu - 5} = \dfrac{1.05}{1}$

or $\quad \nu + 5 = 1.05\,\nu - 5.25 \quad \text{or} \quad 0.05\,\nu = 10.25$

or $\quad \nu = \dfrac{10.25}{0.05} = $ **205 Hz.**

EXAMPLE 67. *A tuning fork of frequency 200 Hz is in unison with a sonometer wire. How many beats per second will be heard if the tension of the wire were increased by 2%?*

Solution. Here $\nu_1 = 200$ Hz

As the tension in the wire is increased by 2%, therefore if $T_1 = 100$ units, then $T_2 = 102$ units

Now $\quad \dfrac{\nu_2}{\nu_1} = \sqrt{\dfrac{T_2}{T_1}}$ ...

$\simeq 1 + \dfrac{1}{2}$ ...

$\nu_2 = 1.01$ ...

$\therefore$ Beat frequency

$= \nu_2$ ...

EXAMPLE 68. *The tw... movable knife differ... second when sounded... whole length of the w...*

Solution. Let ... parts of a sonomet... edge. Then

$L_1 + L_2 =$ ...

and $\quad L_1 - L_2 =$ ...

Adding and ...

Let ν_1 and ν... wire correspon... Then the numb...

According ...

or

From (1)

50 ...

or

or

$\therefore$

From ...

EXAMPLE ... material ... 50 cm ... frequen...

Sol...

wher...

Scanned by CamScanner

Now $\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{102}{100}} = \left(1 + \frac{2}{100}\right)^{1/2}$

$= 1 + \frac{1}{2} \times \frac{2}{100} = 1.01$

$\nu_2 = 1.01\,\nu_1 = 1.01 \times 200 = 202$ Hz

Beat frequency

$= \nu_2 - \nu_1 = 202 - 200 =$ **2 Hz.**

EXAMPLE 68. *The two parts of sonometer wire divided by a movable knife differ by 2 mm and produce one beat per second when sounded together. Find their frequencies if the whole length of the wire is one metre.*

Solution. Let L_1 and L_2 be the lengths of the two parts of a sonometer wire divided by a movable knife edge. Then

$L_1 + L_2 = 100$ cm

and $L_1 - L_2 = 2$ mm $= 0.2$ cm

Adding and subtracting, we get

$L_1 = 50.1$ cm and $L_2 = 49.9$ cm

Let ν_1 and ν_2 be the frequencies of the sonometer wire corresponding to lengths L_1 and L_2 respectively. Then the number of beats produced per second,

$$\nu_2 - \nu_1 = 1 \quad ...(1)$$

According to the law of length of a stretched string,

$$\frac{\nu_2}{\nu_1} = \frac{L_1}{L_2} = \frac{50.1}{49.9}$$

or $$\nu_2 = \frac{50.1}{49.9}\,\nu_1 \quad ...(2)$$

From (1) and (2), we get

$$\frac{50.1}{49.9}\,\nu_1 - \nu_1 = 1$$

or $$\frac{50.1\,\nu_1 - 49.9\,\nu_1}{49.9} = 1$$

or $$\frac{0.2\,\nu_1}{49.1} = 1$$

$\therefore$ $$\nu_1 = \frac{49.9}{0.2} = \textbf{249.5 Hz.}$$

From (1), $\nu_2 = \nu_1 + 1 = 249.5 + 1 =$ **250.5 Hz.**

EXAMPLE 69. *Two similar sonometer wires of the same material produce 2 beats per second. The length of one is 50 cm and that of the other is 50.1 cm. Calculate the frequencies of the two wires.*

Solution. Frequency, $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{k}{l}$,

where $k = \frac{1}{2}\sqrt{\frac{T}{m}}$ = a constant

Now $\nu_1 = \frac{k}{l_1}$ and $\nu_2 = \frac{k}{l_2}$

$\therefore$ $\nu_1 - \nu_2 = k\left(\frac{1}{l_1} - \frac{1}{l_2}\right)$

But $l_1 = 50$ cm, $l_2 = 50.1$ cm, $\nu_1 - \nu_2 = 2\ s^{-1}$

$\therefore$ $2 = k\left(\frac{1}{50} - \frac{1}{50.1}\right)$ or $k = 50100$

Hence $\nu_1 = \frac{k}{l_1} = \frac{50100}{50} =$ **1002 Hz**

and $\nu_2 = \frac{k}{l_2} = \frac{50100}{50.1} =$ **1000 Hz.**

EXAMPLE 70. *A tuning fork of unknown frequency vibrates in unison with a wire of certain length stretched under a tension of 5 kg f. It produces 6 beats per second with the same wire, when tension is changed to 4.5 kg f. Find the frequency of tuning fork.*

Solution. Fundamental frequency of a stretched string,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = k\sqrt{T},$$

where $k = \frac{1}{2L\sqrt{m}}$ = a constant

When $T = 5$ kg f $= 5 \times 9.8$ N, the wire is in unison with a tuning fork of frequency ν. Therefore,

$$\nu = k\sqrt{5 \times 9.8}$$

When the tension decreases, frequency of vibration decreases. When the tension reduces to 4.5 kg f or 4.5×9.8 N, it produces 6 beats per second. Clearly, the frequency of vibration is

$$\nu' = \nu - 6$$

Also $\nu' = k\sqrt{4.5 \times 9.8}$

$\therefore$ $k\sqrt{4.5 \times 9.8} = k\sqrt{5 \times 9.8} - 6$

or $6.64\,k = 7k - 6$

or $k = \frac{6}{0.36} = 16.67$

$\therefore$ $\nu = k\sqrt{5 \times 9.8}$

$= 16.67 \times 7 =$ **116.72 Hz.**

EXAMPLE 71. *Calculate the speed of sound in a gas in which two sound waves of wavelengths 1.00 m and 1.01 m produce 24 beats in 6 seconds.*

Solution. Let v be the sound in the gas. Then

$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{1.00}$ and $\nu_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$

Beat frequency,

$\nu_1 - \nu_2 = 24/6 = 4\ s^{-1}$

Scanned by CamScanner

$$\therefore \quad v\left[\frac{1}{1.00}-\frac{1}{1.01}\right]=4 \quad \text{or} \quad \frac{v\times 0.01}{1.01}=4$$

or $$v=\frac{4\times 1.01}{0.01}=\mathbf{404\ ms^{-1}}.$$

Example 72. Two air columns (of resonance tubes) 100 cm and 101 cm long give 17 beats in 20 seconds, when each is sounding its fundamental mode. Calculate the velocity of sound.

Solution. Here $L_1 = 100$ cm $= 1.00$ m,

$L_2 = 101$ cm $= 1.01$ m

Beat frequency $= \nu_1 - \nu_2$

$$\frac{17}{20}=\frac{v}{4L_1}-\frac{v}{4L_2}=\frac{v}{4}\left[\frac{1}{1.00}-\frac{1}{1.01}\right]=\frac{v\times 0.01}{4\times 1.01}$$

or $$v=\frac{17\times 4\times 1.01}{20\times 0.01}=\mathbf{343.4\ ms^{-1}}.$$

Example 73. Two tuning forks A and B give 5 beats per second. A resounds with a closed column of air 15 cm long and B with an open column 30.5 cm long. Calculate their frequencies. Neglect end correction.

Solution. Let ν_1 and ν_2 be the frequencies of the tuning forks A and B respectively.

As fork A resounds with a closed air column of length 15 cm or 0.15 m, so

$$\nu_1=\frac{v}{4L_1}=\frac{v}{4\times 0.15}=\frac{v}{0.60}$$

Again, fork B resounds with an open air column of length 30.5 cm or 0.305 m, so

$$\nu_2=\frac{v}{2L_2}=\frac{v}{2\times 0.305}=\frac{v}{0.61}$$

Beat frequency $= \nu_1 - \nu_2$

$$5=\frac{v}{0.60}-\frac{v}{0.61}=\frac{v\times 0.01}{0.60\times 0.61}$$

or $$v=\frac{5\times 0.60\times 0.61}{0.01}=183\ \text{ms}^{-1}$$

$$\therefore \quad \nu_1=\frac{183}{0.60}=\mathbf{305\ Hz}$$

and $$\nu_2=\frac{183}{0.61}=\mathbf{300\ Hz}.$$

Example 74. At 16°C, two open end organ pipes, when sounded together produce 34 beats in 2 seconds. How many beats per second will be produced, if the temperature rises to 51°C? Neglect the increase in length of the pipes.

Solution. Number of beats produced per second at 16°C $= 34/2 = 17$

Let L_1, L_2 = lengths of the two organ pipes,

ν_1, ν_2 = lowest frequencies emitted by the pipes at 16°C,

ν'_1, ν'_2 = lowest frequencies emitted by the pipes at 51°C,

v_{16}, v_{51} = velocities of sound at 16°C and 51°C respectively.

$$\therefore \quad \nu_1=\frac{v_{16}}{2L_1} \quad \text{and} \quad \nu_2=\frac{v_{16}}{2L_2}$$

Beat frequency at 16°C,

$$\nu_1-\nu_2=17$$

or $$\frac{v_{16}}{2L_1}-\frac{v_{16}}{2L_2}=17$$

$$v_{16}\left[\frac{1}{2L_1}-\frac{1}{2L_2}\right]=17 \quad \ldots(1)$$

Let the number of beats per second at 51°C $= b$

Then $$\nu'_1-\nu'_2=b$$

or $$\frac{v_{51}}{2L_1}-\frac{v_{51}}{2L_2}=b$$

$$v_{51}\left[\frac{1}{2L_1}-\frac{1}{2L_2}\right]=b \quad \ldots(2)$$

Dividing equation (2) by (1), we get

$$\frac{v_{51}}{v_{16}}=\frac{b}{17}$$

or $$\sqrt{\frac{51+273}{16+273}}=\frac{b}{17} \quad [\because v_t \propto \sqrt{t+273}]$$

or $$b=17\times\sqrt{\frac{324}{289}}$$

$$=17\times\frac{18}{17}=\mathbf{18\ s^{-1}}.$$

Example 75. A column of air and a tuning fork produce 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15°C. When the temperature falls to 10°C, the two produce 3 beats per second. Find the frequency of the fork.

Solution. Let frequency of the tuning fork $= \nu$

Frequency of air column at 15°C,

$$\nu_{15}=\nu+4$$

Frequency of air column at 10°C,

$$\nu_{10}=\nu+3$$

As $$v=\nu\lambda$$

$$\therefore \quad v_{15}=(\nu+4)\lambda \quad \text{and} \quad v_{10}=(\nu+3)\lambda$$

Scanned by CamScanner

Hence $\dfrac{\upsilon_{15}}{\upsilon_{10}} = \dfrac{\nu + 4}{\nu + 3}$ ...(1)

Also $\dfrac{\upsilon_{15}}{\upsilon_{10}} = \sqrt{\dfrac{273 + 15}{273 + 10}} = \sqrt{\dfrac{288}{283}}$ ...(2)

From (1) and (2), we have

$$\frac{\nu + 4}{\nu + 3} = \left(\frac{288}{283}\right)^{1/2} = \left(1 + \frac{5}{283}\right)^{1/2} \simeq 1 + \frac{1}{2} \times \frac{5}{283}$$

$$\frac{(\nu + 3) + 1}{\nu + 3} = 1 + \frac{5}{566} \quad \text{or} \quad 1 + \frac{1}{(\nu + 3)} = 1 + \frac{5}{566}$$

$$\frac{1}{(\nu + 3)} = \frac{5}{566} \quad \text{or} \quad (\nu + 3) = \frac{566}{5} = 113.2$$

$$\therefore \quad \nu = 110.2 \text{ Hz} \simeq \mathbf{110 \text{ Hz}}.$$

PROBLEMS FOR PRACTICE

1. When two tuning forks were sounded together, 20 beats were produced in 10 seconds. On loading one of the forks with wax, the number of beats increases. If the frequency of unloaded fork is 512 Hz, calculate the frequency of other. **(Ans. 510 Hz)**

2. A tunning fork of unknown frequency gives 4 beats per second when sounded with a fork of frequency 320 Hz. When loaded with little wax, it gives 3 beats per second. Find the unknown frequency. **[Central Schools 12]** **(Ans. 324 Hz)**

3. A tuning fork A makes 4 beats per second with a fork B of frequency 256 Hz. A is filed and the beats occur at shorter interval, find its original frequency. **[MNREC 85]** **(Ans. 260 Hz)**

4. A set of 25 tuning forks is arranged in order of decreasing frequency. Each fork gives 3 beats with succeeding one. The first fork is octave of the last. Calculate the frequency of the first and 16th fork. **(Ans. 144 Hz, 99 Hz)**

5. The string of a violin emits a note of 440 Hz at its correct tension. The string is bit taut and produces 4 beats per second with a tuning fork of frequency 440 Hz. Find the frequency of the note emitted by this taut string. **(Ans. 444 Hz)**

6. A tuning fork when vibrating along with a sonometer produces 6 beats per second when the length of the wire is either 20 cm or 21 cm. Find the frequency of the tuning fork. **(Ans. 246 Hz)**

7. A 70 cm long sonometer wire is in unison with a tuning fork. If the length of the wire is decreased by 1.0 cm, it produces 4 beats per second with the same tuning fork. Find the frequency of the tuning fork. **(Ans. 276 Hz)**

8. When two tuning forks are sounded together, 4 beats per second are heard. One of the forks is in unison with 0.96 m length of a sonometer wire and the other is in unison with 0.97 m length of the same wire. Calculate the frequency of each. **(Ans. 388 Hz, 384 Hz)**

9. Two perfectly identical wires are in unison. If the tension in one wire is increased by 1%, then on sounding them together, 3 beats are produced in 2 seconds. Calculate the frequency of each wire. **(Ans. 300 Hz)**

10. A and B are two wires whose fundamental frequencies are 256 and 382 Hz respectively. How many beats in two seconds will be heard by the third harmonic of A and second harmonic of B ? **(Ans. 8)**

11. In an experiment, it was found that a tuning fork and a sonometer wire gave 4 beats per second, both when the length of the wire was 1 m and 1.05 m. Calculate the frequency of the fork. **(Ans. 164 Hz)**

12. A tuning fork of frequency 300 Hz resonates with an air-column closed at one end at 27°C. How many beats will be heard in the vibrations of the fork and the air-column at 0°C ? End-correction is negligible. **(Ans. 14 s^{-1})**

HINTS

1. Beat frequency $= \dfrac{20}{10} = 2\,s^{-1}$

   Possible unknown frequencies $= 512 \pm 2$

   $= 514$ or 510 Hz

   As loading decreases the frequency, also the beat frequency increases, so unknown frequency = 510 Hz (lower one).

4. Let the frequency of last fork = ν

   Then frequency of first fork $= 2\nu$

   Frequency of second fork $= 2\nu - 3$

   Frequency of third fork $= 2\nu - 2 \times 3$

   Frequency of 25th fork $= 2\nu - 24 \times 3 = \nu$

   $\therefore \quad \nu = 72$

   $\therefore$ Frequency of first fork

   $= 2\nu = \mathbf{144\ Hz}$

   Frequency of 16th fork

   $= 144 - 15 \times 3 = \mathbf{99\ Hz.}$

5. When the string is taut, its frequency increases because $\nu \propto \sqrt{T}$.

   As the taut string produces 4 beats per second with a tuning fork of frequency 440, so its frequency $= 440 + 4 = \mathbf{444\ Hz.}$

Scanned by CamScanner

6. Here $\nu + 6 = \frac{1}{2 \times 20}\sqrt{\frac{T}{m}}$

and $\nu - 6 = \frac{1}{2 \times 21}\sqrt{\frac{T}{m}}$

$\therefore \quad \frac{\nu + 6}{\nu - 6} = \frac{21}{20}$ or $(\nu + 6) \times 20 = (\nu - 6)\, 21$

On solving, ν = **246 Hz.**

7. Let ν be the frequency of the tuning fork. Then

$$\nu = \frac{1}{2 \times 0.70}\sqrt{\frac{T}{m}}$$

When the length decreases to 69 cm or 0.69 m, frequency increases.

$\therefore \quad \nu + 4 = \frac{1}{2 \times 0.69}\sqrt{\frac{T}{m}}$

Hence $\frac{\nu}{\nu + 4} = \frac{0.69}{0.70}$

or $0.70\,\nu = 0.69\,(\nu + 4)$

or $\nu = \frac{4 \times 0.69}{0.01} = $ **276 Hz.**

8. Frequency of first fork,

$$\nu_1 = \frac{1}{2 \times 0.96}\sqrt{\frac{T}{m}}$$

Frequency of second fork,

$$\nu_2 = \frac{1}{2 \times 0.97}\sqrt{\frac{T}{m}}$$

$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{97}{96}$ or $96\nu_1 = 97\nu_2$

Clearly, $\nu_1 > \nu_2 \quad \therefore \; \nu_1 - \nu_2 = 4$ or $\nu_1 = \nu_2 + 4$

Hence $96(\nu_2 + 4) = 97\nu_2$

or ν_2 = **384 Hz.**

$\nu_1 = \nu_2 + 4 = $ **388 Hz.**

9. Here $\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$

and $\nu + \frac{3}{2} = \frac{1}{2L}\sqrt{\frac{1.01\,T}{m}}$

$\therefore \quad \frac{\nu + 1.5}{\nu} = \sqrt{1.01} = (1 + 0.01)^{1/2}$

$= 1 + \frac{1}{2} \times 0.01 = 1.005$

On solving ν = **300 Hz.**

10. Beat frequency

$= 3\nu_1 - 2\nu_2 = 3 \times 256 - 2 \times 382$

$= 768 - 764 = 4\,s^{-1}$

Number of beats produced in 2 seconds

$= 4 \times 2 =$ **8.**

11. As $\frac{\nu_1}{\nu_2} = \frac{l_2}{l_1}$

$\therefore \quad \frac{\nu + 4}{\nu - 4} = \frac{1.05}{1} = \frac{21}{20}$

or $20\nu + 80 = 21\nu - 84$

or ν = **164 Hz.**

12. Frequency of air column at 27°C,

$$\nu = \frac{v_{27}}{4L} = 300$$

Now $\frac{v_0}{v_{27}} = \sqrt{\frac{0 + 273}{27 + 273}} = \left(1 - \frac{27}{300}\right)^{1/2} = 0.954$

$\therefore \quad v_0 = 0.954 \times v_{27}$

Frequency of air column at 27°C,

$$\nu' = \frac{v}{4L} = \frac{0.954 \times v_{27}}{4L} = 0.954 \times 300 = 286$$

$\therefore$ Beats produced per second = 300 − 286 = **14.**

15.24 DOPPLER EFFECT

39. *What is Doppler effect ? Give an example.*

Doppler effect. Whenever there is a relative motion between the source of sound, the observer and the medium ; the frequency of sound as received by the observer is different from the frequency of sound emitted by the source. *The apparent change in the frequency of sound when the source, the observer and the medium are in relative motion is called Doppler effect.* For example, consider a man standing on a railway platform. When a train, blowing its whistle, approaches him, the pitch of the whistle appears to rise and it suddenly appears to drop as the engine moves away from him. Similar effect is observed when the source is at rest and the observer moves towards or away from the source.

Doppler effect is a wave phenomenon, it holds not only for sound waves but also for electromagnetic waves such as microwaves, radiowaves and visible light. However, Doppler effect is noticeable only when the relative velocity between the source and the observer is an appreciable fraction of the wave velocity.

For the waves which require a medium for their propagation, the apparent frequency depends on *three* factors : (*i*) velocity of the source, (*ii*) velocity of the observer and (*iii*) velocity of medium or wind.

40. *Derive an expression for the apparent frequency of sound as heard by a stationary observer in a still medium, when the source is moving towards the observer with a uniform velocity. Hence write the expression for the apparent frequency when the source moves away from the stationary observer.*

Scanned by CamScanner

Apparent frequency when the source moves towards the stationary observer. Consider a source S moving with speed v_s towards an observer O who is at rest with respect to the medium, as shown in Fig. 15.25. If ν is the frequency of vibration of the source, then it sends out sound (or compressional) waves with speed v at a regular interval of $T = 1/\nu$.

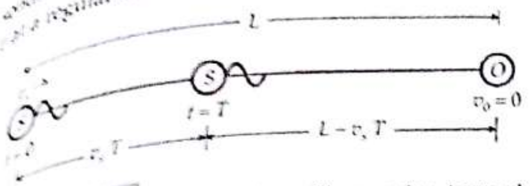


Fig. 15.25 A source S moving with speed v_s towards a stationary observer O.

At time $t = 0$, suppose the source is at distance L from the observer and emits a compression pulse. It reaches the observer at time,

$$t_1 = \frac{L}{v}$$

The source emits next compression pulse after a time T. In the mean time, the source has moved a distance $v_s T$ towards the observer and is now at distance $L - v_s T$ from the observer. The second compression pulse reaches the observer at time,

$$t_2 = T + \frac{L - v_s T}{v}$$

The time interval between two successive compression pulses or the period of the wave as detected by the observer is

$$T' = t_2 - t_1 = T + \frac{L - v_s T}{v} - \frac{L}{v}$$

$$= \left(1 - \frac{v_s}{v}\right) T = \frac{v - v_s}{v} T$$

The apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v}{v - v_s} \cdot \frac{1}{T}$$

or $$\nu' = \frac{v}{v - v_s} \nu \qquad \text{...(1)}$$

Clearly, $\nu' > \nu$. Hence *the pitch of sound appears to increase when the source moves towards the stationary observer.*

Apparent frequency when the source moves away from the stationary observer. If the source moves away from the observer with speed v_s, then the apparent frequency of sound can be obtained by replacing v_s by $-v_s$ in equation (1).

Thus $$\nu' = \frac{v}{v + v_s} \nu \qquad \text{...(2)}$$

Clearly, $\nu' < \nu$. Hence *the pitch of sound appears to decrease when the source of sound moves away from the stationary observer.*

41. *Derive an expression for the apparent frequency of the sound, when the observer moves towards a stationary source of sound. Hence write the expression for the apparent frequency when the observer moves away from the stationary source.*

Apparent frequency when the observer moves towards the stationary source. As shown in Fig. 15.26, now we consider the case when the source S remains stationary with respect to the medium and the observer O moves towards the source with speed v_0. If ν is the frequency of vibration of the source, then it sends out compression pulses at a regular interval of $T = 1/\nu$, which travel through the medium with speed v.

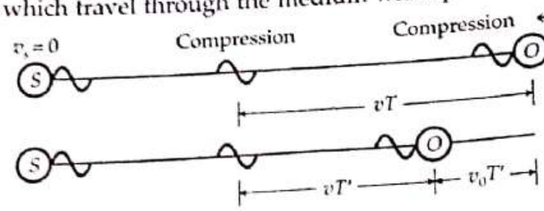


Fig. 15.26 An observer moving with speed v_0 towards the stationary source S.

At any instant, the separation between two compression pulses is $\lambda = vT$. So when the observer receives a compression pulse, the next compression pulse is a distance vT away from him. This second compression pulse moves towards the observer with speed v and also the observer moves towards it with speed v_0. Hence the observer will receive the second compression wave a time T' after receiving the first pulse, where

$$T' = \frac{vT}{v + v_0}$$

This is the period of the wave as detected by the observer. Hence the apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v + v_0}{v} \frac{1}{T}$$

or $$\nu' = \frac{v + v_0}{v} \nu \qquad \text{...(3)}$$

Clearly, $\nu' > \nu$. Thus *the pitch of sound appears to increase when the observer moves towards the source.*

Apparent frequency when the observer moves away from the stationary source. If the observer moves away from the stationary source with speed v_0,

Scanned by CamScanner

then the apparent frequency of sound can be obtained by replacing v_0 by $-v_0$ in equation (3). Thus

$$\nu' = \frac{v - v_0}{v}\nu \qquad ...(4)$$

Clearly $\nu' < \nu$. Thus *the pitch of sound appears to decrease when the observer moves away from the stationary source.*

42. *Obtain an expression for the observed frequency of the sound produced by a source when both observer and source are in motion and the medium at rest.*

Apparent frequency when both the source and observer are in motion. As shown in Fig. 15.27, consider the case when both the source and the observer are moving towards each other with speeds v_s and v_0 respectively. If ν is the frequency of the source, it sends out compression pulses through the medium at regular intervals of $T = 1/\nu$.

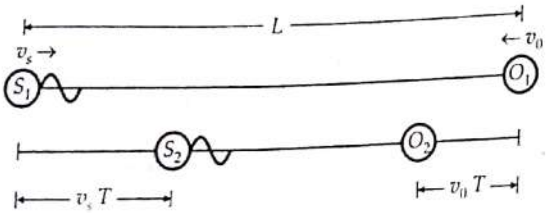


Fig. 15.27 A source and observer both moving towards each other.

At time $t = 0$, the observer is at O_1 and the source at S_1 and the distance between them is L when the source emits the first compression pulse. Since the observer is also moving towards the source, so the speed of the wave relative to the observer is $(v + v_0)$. Therefore, the observer will receive the first compression pulse at time,

$$t_1 = \frac{L}{v + v_0}$$

At time $t = T$, both the source and observer have moved towards each other covering distances $S_1 S_2 = v_s T$ and $O_1 O_2 = v_0 T$ respectively. The new distance between the source and the observer is

$$S_2 O_2 = L - (v_s + v_0)T$$

The second compression pulse will reach the observer at time,

$$t_2 = T + \frac{L - (v_s + v_0)T}{v + v_0}$$

The time interval between two successive compression pulses or the period of the wave as recorded by the observer is

$$T' = t_2 - t_1$$

$$= T + \frac{L - (v_s + v_0)T}{v + v_0} - \frac{L}{v + v_0}$$

$$= \left(1 - \frac{v_s + v_0}{v + v_0}\right)T = \left(\frac{v - v_s}{v + v_0}\right)T$$

Hence the apparent frequency of the sound as heard by the observer is

$$\nu' = \frac{1}{T'} = \frac{v + v_0}{v - v_s} \cdot \frac{1}{T}$$

or

$$\nu' = \frac{v + v_0}{v - v_s}\nu \qquad ...(5)$$

When the source moves towards the observer and the observer moves away from the source. In this case, the apparent frequency can be obtained by replacing v_0 by $-v_0$ in equation (5). Thus

$$\nu' = \frac{v - v_0}{v - v_s}\nu \qquad ...(6)$$

If the medium also moves with a velocity v_m in the direction of propagation of sound, then

$$\nu' = \frac{v + v_m - v_0}{v + v_m - v_s}\nu$$

If the medium moves with a velocity v_m in the opposite direction of sound, then

$$\nu' = \frac{v - v_m - v_0}{v - v_m - v_s}\nu.$$

43. *Doppler effect in sound is asymmetric. What do you mean by this statement ?*

Doppler effect in sound is asymmetric. Suppose a source of sound moves towards a stationary observer with the speed v'. Then the observed frequency will be

$$\nu' = \frac{v}{v - v'}\nu$$

Now if the observer moves towards the stationary source with the same speed v', then the observed frequency will be

$$\nu'' = \frac{v + v'}{v}\nu$$

Clearly, $\nu' \neq \nu''$. Thus the observed frequency is not same when the observer is stationary and the source moves towards it or when the source is stationary and the observer moves towards it with the same speed. For this reason, the *Doppler effect in sound is said to asymmetric. However, the Doppler effect in light is symmetric.* This is because sound or mechanical waves, in general, have a velocity relative to the medium through which they travel whereas light or electromagnetic waves travel quite independent of it.

Scanned by CamScanner

For Your Knowledge

- The observed frequency depends on the actual velocities of the source and the observer and not on their relative velocities.
- The motion of the source brings about a change in the wavelength of the sound waves and hence there is a change in the observed frequency.
- The motion of the observer merely changes the rate at which the sound waves are received by him. The observer intercepts more waves (when he approaches) or fewer waves (when he recedes) each second. The wavelength of the sound waves remains unaffected.
- The apparent frequency is larger than the actual frequency, if the separation between the source and the observer is decreasing and is smaller if the separation is increasing.
- **No Doppler effect is observed** *i.e.*, there is no shift in frequency in the following situations : (*i*) When both the source and the observer move in the same direction with the same speed. (*ii*) When either the source or the observer is at the centre of a circle and the other is moving along it with an uniform speed. (*iii*) When both the source and the observer are at rest and the wind alone is blowing.
- The Doppler formula for apparent frequency is applicable when $v_0 < v$ and $v_s < v$. It does not hold when the speed of the source or the observer becomes equal to greater than the speed of the wave.
- When the observer is at rest and the source moves with a supersonic speed (a speed greater than the speed of sound), the resultant wave motion is a conical wave called a **shock wave**. It is due to the shock wave that we hear a sudden and violent sound, called **sonic boom**, when a supersonic jet plane passes by.
- The ratio of the speed of the source to the speed of sound (v/v_s) is called **Mach number**. Shock waves are produced by objects moving with Mach number greater than one.

Examples based on Doppler Effect in Sound

FORMULAE USED

1. If v, v_0, v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency $\nu' = \frac{v + v_m - v_0}{v + v_m - v_s} \times \nu$
2. If the medium is at rest ($v_m = 0$), then $\nu' = \frac{v - v_0}{v - v_s} \times \nu$
3. All the velocities are taken *positive* in the *source to observer* ($S \rightarrow O$) direction and *negative* in the opposite ($O \rightarrow S$) direction.

UNITS USED

Velocities v, v_0, v_s and v_m are in ms^{-1} and frequencies ν and ν' in Hz.

EXAMPLE 76. *A source and an observer are approaching one another with the relative velocity 40 ms^{-1}. If the true source frequency is 1200 Hz, deduce the observed frequency under the following conditions :*

(*i*) *All velocity is in the source alone.*

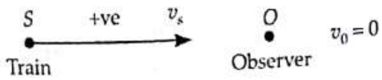
(*ii*) *All velocity is in the observer alone.*

(*iii*) *The source moves in air at 100 ms^{-1} towards the observer, but the observer also moves with the velocity v_0 in the same direction.*

Solution. Here $\nu = 1200$ Hz, $v = 340$ ms^{-1},

Relative velocity = 40 ms^{-1}

(*i*) Here source moves towards the stationary observer, $v_s = +40$ ms^{-1}, $v_0 = 0$

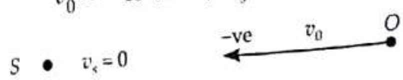


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 40} \times 1200$$

$$= \frac{340 \times 1200}{300} = \mathbf{1360\ Hz.}$$

(*ii*) Here observer moves towards the stationary source,

$$v_0 = -40\ ms^{-1},\ v_s = 0.$$

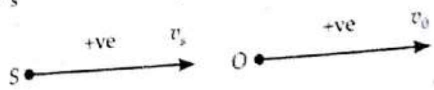


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 40}{340 - 0} \times 1200$$

$$= \frac{380}{340} \times 1200 = \mathbf{1341\ Hz.}$$

(*iii*) Here observer and source move in the same direction,

$$v_s = 100\ ms^{-1},\ v_0 = 100 - 40 = 60\ ms^{-1}$$



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 60}{340 - 100} \times 1200$$

$$= \frac{280}{240} \times 1200 = \mathbf{1400\ Hz.}$$

EXAMPLE 77. *A railway engine and a car are moving on parallel tracks in opposite directions with speed of 144 kmh^{-1} and 72 kmh^{-1}, respectively. The engine is continuously sounding a whistle of frequency 500 Hz. The velocity of sound is 340 ms^{-1}. Calculate the frequency of sound heard in the car when*

(*i*) *the car and the engine are approaching each other,*

(*ii*) *the two are moving away from each other.*

Solution. Here $v_s = 144 \text{ km h}^{-1}$

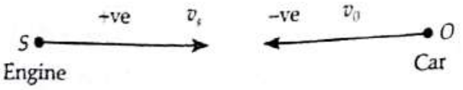
$$= \frac{144 \times 1000}{3600} = 40 \text{ ms}^{-1}$$

and $$v_0 = 72 \text{ km h}^{-1} = \frac{72 \times 1000}{3600} = 20 \text{ ms}^{-1}$$

$$\nu = 500 \text{ Hz}, \quad v = 340 \text{ ms}^{-1}$$

(*i*) When the car and the engine approach each other,

$$v_s = +40 \text{ ms}^{-1}, \quad v_0 = -20 \text{ ms}^{-1}$$

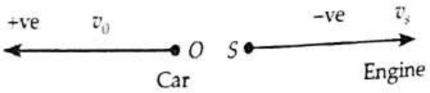


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 20}{340 - 40} \times 500$$

$$= \frac{360}{300} \times 500 = \mathbf{600\ Hz.}$$

(*ii*) When the car and the engine are moving away from each other,

$$v_s = -40 \text{ ms}^{-1}, \quad v_0 = +20 \text{ ms}^{-1}$$



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 20}{340 + 40} \times 500$$

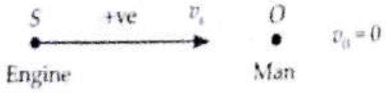
$$= \frac{320}{380} \times 500 = \mathbf{421\ Hz.}$$

EXAMPLE 78. *The sirens of two fire engines have a frequency of 600 Hz each. A man hears the sirens from the two engines, one approaching him with a speed of 36 km h^{-1} and the other going away from him at a speed of 54 km h^{-1}. What is the difference in frequency of two sirens heard by the man ? Take the speed of sound to be 340 ms^{-1}.* **[Punjab 91]**

Solution. Here $v = 340 \text{ ms}^{-1}$, $\nu = 600$ Hz

For the engine approaching the man :

$$v_s = +36 \text{ km h}^{-1} = 10 \text{ ms}^{-1}, \quad v_0 = 0$$

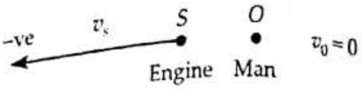


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 10} \times 600$$

$$= \mathbf{618.2\ Hz}$$

For the engine going away from the man :

$$v_s = -54 \text{ km h}^{-1} = -15 \text{ ms}^{-1}, \quad v_0 = 0$$



$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 + 15} \times 600 = \mathbf{576.6\ Hz}$$

Difference in frequencies

$$= \nu' - \nu'' = 618.2 - 576.6 = \mathbf{41.6\ Hz.}$$

EXAMPLE 79. *An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency ?* **[AIEEE 05]**

Solution. Here observer moves towards the stationary source.

$$\therefore \quad v_0 = -v/5, \quad v_s = 0$$

Apparent frequency

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{v + v/5}{v - 0} \times \nu$$

$$= \frac{6}{5}\nu = 1.2\,\nu$$

The percentage increase in apparent frequency,

$$\frac{\nu' - \nu}{\nu} \times 100 = \frac{1.2\,\nu - \nu}{\nu} \times 100 = \mathbf{20\%.}$$

EXAMPLE 80. *An observer standing on a railway crossing receives frequencies of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train. The speed of sound in air is 300 ms^{-1}.* **[IIT 05]**

Solution. When the train approaches the stationary observer, the apparent frequency is

$$\nu' = \frac{v}{v - v_s} \times \nu$$

or $$2.2 = \frac{300}{300 - v_s} \times \nu \qquad \text{...(1)}$$

When the train recedes from the stationary observer, the apparent frequency is

$$\nu'' = \frac{v}{v + v_s} \times \nu$$

or $$1.8 = \frac{300}{300 + v_s} \times \nu \qquad \text{...(2)}$$

Dividing (1) by (2), we get

$$\frac{2.2}{1.8} = \frac{300 + v_s}{300 - v_s} \quad \text{or} \quad \frac{11}{9} = \frac{300 + v_s}{300 - v_s}$$

or $3300 - 11\,v_s = 2700 + 9\,v_s$

or $20 v_s = 600$

or $v_s = \mathbf{30\ ms^{-1}}.$

Scanned by CamScanner

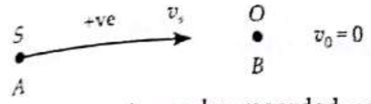
EXAMPLE 81. On a quiet day, two persons A and B, each sounding a note of frequency 580 Hz, are standing a few metres apart. Calculate the number of beats heard by each in one second when A moves towards B with a velocity of 4 ms^{-1}. (Speed of sound in air = 330 ms^{-1}.)

Solution. In one case, A can be regarded as a source of sound moving towards observer B.

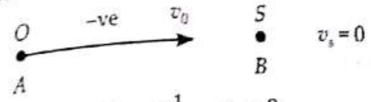
$$v_s = +4 \text{ ms}^{-1}, \quad v_0 = 0$$

$$\therefore \quad \nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{330 - 0}{330 - 4} \times 580$$

$$= \frac{330}{320} \times 580 = 587 \text{ Hz}$$



In another case, A can be regarded as observer moving towards stationary source B.



$$v_0 = -4 \text{ ms}^{-1}, \quad v_s = 0$$

$$\therefore \quad \nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{330 + 4}{330 - 0} \times 580$$

$$= \frac{334}{330} \times 580 = 587 \text{ Hz}$$

Number of beats heard per second by A

$$= \nu'' - \nu = 587 - 580 = \mathbf{7}.$$

Number of beats heard per second by B

$$= \nu' - \nu = 587 - 580 = \mathbf{7}.$$

EXAMPLE 82. Find the velocity of source of sound, when the frequency appears to be (i) double (ii) half the original frequency to a stationary observer. Velocity of sound = 330 ms^{-1}.

Solution. (*i*) Here $\nu' = 2\nu$, $v_0 = 0$, $v = 330$ ms^{-1}

As $\quad \nu' = \frac{v - v_0}{v - v_s} \times \nu \quad \therefore \quad 2\nu = \frac{v - 0}{v - v_s} \times \nu$

or $\quad v - v_s = v/2$

$\quad v_s = v/2 = 330/2 = \mathbf{165\ ms^{-1}}$.

Positive value of v_s shows that the source is moving towards the observer.

(*ii*) Here $\nu' = \nu/2$, $v_0 = 0$, $v_s = 330$ ms^{-1}

As $\quad \nu' = \frac{v - v_0}{v - v_s} \times \nu \quad \therefore \quad \frac{\nu}{2} = \frac{v - 0}{v - v_s} \times \nu$

or $\quad v - v_s = 2v$

or $\quad v_s = -v = \mathbf{-330\ ms^{-1}}$.

Negative value of v_s shows that the source is moving away from the observer.

EXAMPLE 83. A train stands at a platform blowing a whistle of frequency 400 Hz in still air.

(i) What is the frequency of the whistle heard by a man running (a) towards the engine at 10 ms^{-1}, (b) away from the engine at 10 ms^{-1}?

(ii) What is speed of sound in each case?

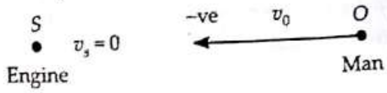
(iii) What is the wavelength of sound received by the running man in each case?

Take speed of sound in still air = 340 ms^{-1}

Solution. Here $\nu = 400$ Hz, $v = 340$ ms^{-1}

(*i*) (*a*) When the man runs towards the engine,

$$v_0 = -10 \text{ ms}^{-1}, \quad v_s = 0$$

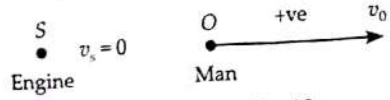


$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 10}{340 - 0} \times 400$$

$$= \frac{350}{340} \times 400 = \mathbf{411.8\ Hz.}$$

(*b*) When the man runs away from the engine,

$$v_0 = +10 \text{ ms}^{-1}, \quad v_s = 0$$



$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 10}{340 - 0} \times 400$$

$$= \frac{330}{340} \times 400 = \mathbf{388.2\ Hz.}$$

(*ii*) (*a*) When the man runs towards the engine, relative velocity of sound,

$$v' = v + v_0$$

$$= 340 + 10 = \mathbf{350\ ms^{-1}}.$$

(*b*) When the man runs away from the engine, relative velocity of sound,

$$v' = v - v_0$$

$$= 340 - 10 = \mathbf{330\ ms^{-1}}.$$

(*iii*) The wavelength of sound is not affected by the motion of the listener. Its value is

$$\lambda = \frac{v}{\nu} = \frac{340}{400} = \mathbf{0.85\ m.}$$

EXAMPLE 84. Consider a source moving towards an observer at the speed $v_s = 0.95\,v$. Deduce the observed frequency if the original frequency is 500 Hz. Think what would happen if $v_s > v$. (Jet planes moving faster than sound are so common). Here v is the velocity of sound.

Solution. Here the source is moving towards the observer at rest.

Scanned by CamScanner

$\therefore \quad v_s = +0.95\,v, \quad v_0 = 0, \quad \nu = 500$ Hz

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{v - 0}{v - 0.95\,v} \times 500 = \frac{500}{0.05}$$

$$= \mathbf{10{,}000\ Hz.}$$

If $v_s > v$, $v - v_s$ is negative and as such ν' is negative which has no physical meaning. Doppler formula is not applicable when the velocity of source exceeds the velocity of the wave.

EXAMPLE 85. *A machine gun is mounted on an armoured car. The gun can point (a) in the direction of motion or against the direction of motion of the car. The muzzle speed of the bullet equals the speed of sound in still air i.e., 340 ms^{-1}. If the car moves with a speed of 20 ms^{-1}, find out the sign and magnitude of the time difference between the bullet's arrival (t_b) and the arrival of the sound of firing (t_s) at a target 500 m away from the car at the instant of firing in each case.*

Solution. Speed of sound, $\quad v = 340\ \text{ms}^{-1}$

Muzzle speed of the bullet, $v_b = 340\ \text{ms}^{-1}$

Speed of the car, $\quad v_c = 20\ \text{ms}^{-1}$

Distance of the target $\quad = 500$ m

(*a*) *When the gun points in the direction of motion of the car.* Here the car is moving towards the target.

$\therefore$ Effective speed of the bullet

$$= v_b + v_c = 360\ \text{ms}^{-1}$$

Time taken by the bullet to hit the target,

$$t_b = \frac{500}{360} = 1.3889\ \text{s}$$

Time taken by sound to reach the target,

$$t_s = \frac{500}{340} = 1.4706\ \text{s}$$

$$\therefore \quad t_b - t_s = 1.3889 - 1.4706 = \mathbf{-0.0817\ s.}$$

(*b*) *When the gun points against the direction of motion of the car.* Here the car is moving away from the target.

$\therefore$ Effective speed of the bullet

$$= v_b - v_c = 340 - 20 = \mathbf{320\ ms^{-1}}$$

Time taken by the bullet to hit the target,

$$t_b' = \frac{500}{320} = 1.5625\ \text{s}$$

$$\therefore \quad t_b' - t_s = 1.5625 - 1.405 = \mathbf{0.0919\ s.}$$

EXAMPLE 86. *An observer is moving towards a wall at 2 ms^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the wall. Calculate the beat frequency between these two sounds, if the true frequency of the source is 680 Hz. Velocity of sound = 340 ms^{-1}.*

Solution. When the observer receives sound directly from the source, he is moving away from the source, so $\quad v_0 = +2\ \text{ms}^{-1}, \quad v_s = 0$

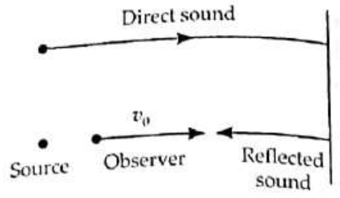


Fig. 15.28

Apparent frequency,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 2}{340 - 0} \times 680 = 676\ \text{Hz.}$$

When the observer receives reflected sound, he is moving towards the wall (source), so

$$v_0 = -2\ \text{ms}^{-1}, \quad v_s = 0$$

Apparent frequency,

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 2}{340 - 0} \times 680 = \mathbf{684\ Hz}$$

Beat frequency $= \nu'' - \nu' = 684 - 676 = \mathbf{8\ Hz.}$

EXAMPLE 87. *A rocket is moving at a speed of 200 ms^{-1} towards a stationary target. While moving, it emits a sound wave of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. Calculate (a) the frequency of the sound wave as detected by a detector attached to the target and (b) the frequency of the echo as detected by a detector attached to the rocket.*

[NCERT ; Chandigarh 07]

Solution. (*a*) Here the observer is at rest and the source is moving with a speed of 200 ms^{-1}. As the speed of the source is comparable to that of the sound wave, so the observed frequency is

$$\nu' = \nu\left(1 - \frac{v_s}{v}\right)^{-1} = 1000\left(1 - \frac{200}{330}\right)^{-1}$$

$$= \frac{1000 \times 330}{130} = \mathbf{2538.5\ Hz.}$$

(*b*) Now the target is the source (as it is the source of echo) and the rocket's detector is the observer who intercepts the echo of frequency ν'. Hence the frequency of the echo as detected by a detector attached to the rocket is

$$\nu'' = \frac{v + v_0}{v - v_s} \times \nu = \frac{330 + 200}{330 - 0} \times 2538.5$$

$$= \mathbf{4077\ Hz.}$$

Scanned by CamScanner

EXAMPLE 88. *A siren is fitted on a car going towards a vertical wall at a speed of 36 kmh^{-1}. A person standing on the ground, behind the car, listens to the siren sound coming directly from the source as well as that coming after reflection from the wall. Calculate the apparent frequency of the wave*

(a) coming directly from the siren to the person, and

(b) coming after reflection. Take the speed of sound to be 340 ms^{-1}. Actual frequency of siren = 500 Hz.

Solution. The situation is shown in Fig. 15.29.

Here $v_s = 36\ kmh^{-1} = 10\ ms^{-1}, v = 340\ ms^{-1}, \nu = 500\ Hz$

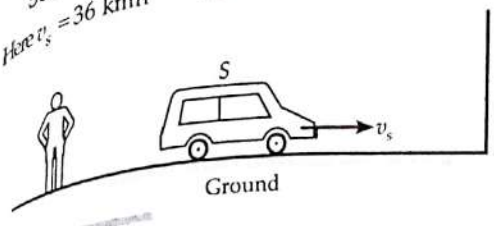


Fig. 15.29

(a) For the sound coming directly from the siren to the observer, source (siren) is moving away from the observer, so

$$v_s = -10\ ms^{-1}, \quad v_0 = 0$$

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 + 10} \times 500 = 485.7\ Hz$$

(b) For the sound received by the wall, source (siren) is moving towards the wall (observer), so

$$v_s = +10\ ms^{-1}, v_0 = 0$$

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 10} \times 500 = 515.2\ Hz$$

As the wall reflects the sound without changing the frequency, so apparent frequency of the wave after reflection = **515.2 Hz.**

EXAMPLE 89. *If the pitch of the sound of a source appears to drop by 10% to a moving person, then determine the velocity of motion of the person. Velocity of sound = 330 ms^{-1}.*

Solution. Apparent frequency heard by the person is given by

$$\nu' = \left(\frac{v - v_0}{v}\right)\nu \quad \text{or} \quad \frac{\nu'}{\nu} = \frac{v - v_0}{v}$$

But $\quad \frac{\nu'}{\nu} = \frac{90}{100} = \frac{9}{10}, \quad v = 330\ ms^{-1}$

$$\therefore \quad \frac{9}{10} = \frac{330 - v_0}{330}$$

or $\quad 330 - v_0 = \frac{9}{10} \times 330 = 297$

$$\therefore \quad v_0 = 330 - 297 = \mathbf{33\ ms^{-1}}.$$

As the pitch of the sound appears to decrease, so the person is moving away from the observer.

EXAMPLE 90. *Two aeroplanes A and B are approaching each other and their velocities are 108 kmh^{-1} and 144 kmh^{-1} respectively. The frequency of a note emitted by A as heard by the passengers in B is 1170 Hz. Calculate the frequency of the note heard by the passenger in A. Velocity of sound = 350 ms^{-1}.*

Solution. Here aeroplane *A* (source) and aeroplane *B* (observer) both approach each other, so

$$v_s = +108\ km\ h^{-1} = +30\ ms^{-1},$$

$$v_0 = -144\ km\ h^{-1} = -40\ ms^{-1},$$

$$\nu' = 1170\ Hz, \quad v = 350\ ms^{-1}, \quad \nu = ?$$

As $\quad \nu' = \frac{v - v_0}{v - v_s} \times \nu \quad \therefore \quad 1170 = \frac{350 + 40}{350 - 30} \times \nu$

$$\nu = \frac{1170 \times 320}{390} = \mathbf{960\ Hz.}$$

EXAMPLE 91. *A whistle of frequency 540 Hz rotates in a circle of radius 2 m at an angular speed of 15 rad s^{-1}. What is the lowest and highest frequency heard by a listener a long distance away at rest w.r.t. centre of the circle ? Can the apparent frequency be ever equal to the actual frequency ? Take v = 330 ms^{-1}.* **[MNREC 96 ; IIT 96]**

Solution. The situation is shown in Fig. 15.30.

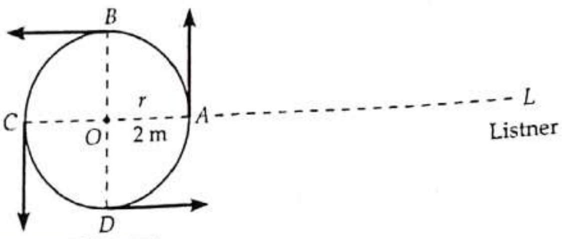


Fig. 15.30

Speed of source (whistle),

$$v_s = r\,\omega = 2 \times 15 = 30\ ms^{-1}$$

Actual frequency, ν = 540 Hz,

Speed of sound, $\quad v = 330\ ms^{-1}$

When the whistle (at position *B*) is moving away from the observer, the apparent frequency is lowest. It is given by

$$\nu' = \frac{v}{v + v_s} \times \nu = \frac{330}{330 + 30} \times 540 = 495\ Hz$$

When the whistle (at position *D*) is moving towards the observer, the apparent frequency is highest. It is given by

$$\nu'' = \frac{v}{v - v_s} \times \nu = \frac{330}{330 - 30} \times 540 = 594\ Hz$$

When the whistle is at *A* or *C*, its speed along $OL = v_s \cos 90° = 0$

∴ Apparent frequency = Actual frequency.

Scanned by CamScanner

✕ PROBLEMS FOR PRACTICE

1. A policeman blows a whistle with a frequency of 500 Hz. A car approaches him with a velocity of $15\ ms^{-1}$. Calculate the change in frequency as heard by the driver of the car as he passes the policeman. Speed of sound in air is $300\ ms^{-1}$. **(Ans. 50 Hz)**

2. Calculate the apparent frequency of the horn of a car approaching a stationary listener with a velocity of $12\ ms^{-1}$. The frequency of horn is 500 Hz. The speed of sound is $332\ ms^{-1}$. **(Ans. 519 Hz)**

3. A man standing near a railway line hears the whistle of an engine, which has a velocity of $20\ ms^{-1}$. What frequency does the man hear, when the engine is coming towards and going away from him, if the true frequency of the whistle is 1000 Hz ? Speed of sound in air = $340\ ms^{-1}$. **(Ans. 1062.5 Hz, 944.4 Hz)**

4. Two engines pass each other in opposite directions with a velocity of $60\ kmh^{-1}$ each. One of them is emitting a note of frequency 540. Calculate the frequencies heard in the other engine before and after they have passed each other. Given velocity of sound = $316.67\ ms^{-1}$. **(Ans. 600 Hz, 486 Hz)**

5. A train approaches a stationary observer, the velocity of train being 1/20 of the velocity of sound. A sharp blast is blown with the whistle of the engine at equal intervals of a second. Find the interval between the successive blasts as heard by the observer. **(Ans. 19/20 s)**

6. When an engine goes away from a stationary observer, the frequency of the engine appears $\frac{6}{7}$ times the real frequency. Calculate the speed of the engine. Speed of sound in air = $330\ ms^{-1}$. **(Ans. 55 ms^{-1})**

7. A motor car is approaching towards a crossing with a velocity of $75\ kmh^{-1}$. The frequency of sound of its horn as heard by a policeman standing on the crossing is 260 Hz. What is the real frequency of the horn ? Speed of sound = $332\ ms^{-1}$. **(Ans. 244 Hz)**

8. Two cars are approaching each other on a straight road and moving with a velocity of $30\ kmh^{-1}$. If the sound produced in a car is of frequency 500 Hz, what will be the frequency of sound as heard by the person sitting in the other car ? When the two cars have crossed each other and are moving away from each other, what will be the frequency of sound as heard by the same person ? Speed of sound = $330\ ms^{-1}$ **(Ans. 526 Hz, 475 Hz)**

9. A source emitting sound of frequency 1000 Hz is moving towards an observer at speed $v_s = 0.90\ v$ (where v is the velocity of sound). What frequency will be heard by the observer ? **(Ans. 10,000 Hz)**

10. A policeman on duty detects a drop of 15% in the pitch of the horn of a motor car as it crosses him. If the velocity of sound is $330\ ms^{-1}$, calculate the speed of the car. **[Central Schools 04 ; Chandigarh 0[illegible]]** **(Ans. 26.7 ms^{-1})**

11. The whistle of an engine moving at $30\ kmh^{-1}$ is heard by a motorist driving at $15\ kmh^{-1}$ and he estimated the pitch to be 500. What would be the actual pitch, if the two are approaching each other ? Velocity of sound = $1220\ kmh^{-1}$. **(Ans. 481.8 Hz)**

12. A car passing a check post gives sound of frequency 1000 cps. If the velocity of the car is $72\ kmh^{-1}$ and of sound is $350\ ms^{-1}$, find the change in apparent frequency as it crosses the post. **(Ans. 114.66 Hz)**

✕ HINTS

1. When the car approaches the policeman,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{300 - (-15)}{300 - 0} \times 500 = 525\ \text{Hz}$$

When the car moves away from the police man,

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{300 - 15}{300 - 0} \times 500 = 475\ \text{Hz}$$

Change in frequency $= \nu' - \nu'' = 525 - 475 = 50$ Hz.

3. Here $v_0 = 0,\quad v_s = 20\ ms^{-1}$,

$$\nu = 1000\ \text{Hz}, \quad v = 340\ ms^{-1}$$

(*i*) When the engine approaches the man,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 20} \times 1000 = \mathbf{1062.5\ Hz.}$$

(*ii*) When the engine goes away from the man,

$$\nu'' = \frac{340 - 0}{340 - (-20)} \times 1000 = \mathbf{944.4\ Hz.}$$

4. Here $v_s = v_0 = 60\ km\ h^{-1} = 16.67\ ms^{-1}$,

$$\nu = 540\ \text{Hz}, \quad v = 316.67\ ms^{-1}$$

(*i*) *Before the two engines cross each other.*

$$v_s = +\ 16.67\ ms^{-1}, \quad v_0 = -\ 16.67\ ms^{-1}$$

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{316.67 + 16.67}{316.67 - 16.67} \times 540$$

$$= \mathbf{600\ Hz.}$$

(*ii*) *After the engines cross each other,*

$$v_s = -\ 16.67\ ms^{-1}, \quad v_0 = +\ 16.67\ ms^{-1}$$

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu = \frac{316.67 - 16.67}{316.67 + 16.67} \times 540$$

$$= \mathbf{486\ Hz.}$$

5. Let v be the velocity of sound.

Velocity of train (source of sound), $v_s = v/20$

Since the blast is blown at regular intervals of one second, therefore, the actual frequency of blasts

$$\nu = 1\ s^{-1}$$

Scanned by CamScanner

The apparent frequency of blasts

$$\nu' = \frac{v}{v - v_s}\,\nu = \frac{v}{v - \frac{v}{20}} \times 1 = \frac{v}{\frac{19}{20}v} = \frac{20}{19}$$

The apparent interval between the two successive blasts,

$$t' = \frac{1}{\nu'} = \frac{1}{20/19} = \frac{19}{20}\ \textbf{s.}$$

6. Here $v_0 = 0$, $\nu'/\nu = 6/7$, $v = 330\ \text{ms}^{-1}$, $v_s = ?$

As $\nu' = \frac{v - v_0}{v - v_s} \times \nu$

∴ $\frac{\nu'}{\nu} = \frac{v - v_0}{v - v_s}$ or $\frac{6}{7} = \frac{330 - 0}{330 - v_s}$

or $v_s = -55\ \textbf{ms}^{-1}$.

Negative sign shows engine goes away from the observer.

7. Here $v_s = 75\ \text{km h}^{-1} = 20.8\ \text{ms}^{-1}$, $v_0 = 0$, $\nu' = 260\ \text{Hz}$, $\nu = ?$, $v = 332\ \text{ms}^{-1}$

As $\nu' = \frac{v - v_0}{v - v_s} \times \nu$ ∴ $260 = \frac{332 - 0}{332 - 20.8} \times \nu$

or $\nu = \frac{260 \times 311.2}{332} = \textbf{244 Hz.}$

8. Here $v_s = v_0 = 30\ \text{kmh}^{-1} = 25/3\ \text{ms}^{-1}$, $v = 330\ \text{ms}^{-1}$, $\nu = 500\ \text{Hz}$

(i) *When the two cars are approaching.*

$v_s = +25/3\ \text{ms}^{-1}$, $v_0 = -25/3\ \text{ms}^{-1}$

$$\nu' = \frac{330 + 25/3}{330 - 25/3} \times 500 = \frac{1015 \times 500}{965} = \textbf{526 Hz.}$$

(ii) *When the two cars are moving away from each other.*

$v_s = -25/3\ \text{ms}^{-1}$, $v_0 = +25/3\ \text{ms}^{-1}$

$$\nu'' = \frac{330 - 25/3}{330 + 25/3} \times 500 = \frac{965 \times 500}{1015} = \textbf{475 Hz.}$$

10. Before crossing, source is moving towards the stationary observer,

$$\nu' = \frac{v - 0}{v - v_s} \times \nu \qquad \text{...(1)}$$

After crossing, source is moving away from the stationary observer,

$$\nu'' = \frac{v - 0}{v + v_s} \times \nu \qquad \text{...(2)}$$

Dividing (2) by (1), $\frac{\nu''}{\nu'} = \frac{v - v_s}{v + v_s}$

As the pitch drops by 15%, so

$$\frac{\nu''}{\nu'} = \frac{85}{100} = \frac{17}{20}$$

Hence $\frac{17}{20} = \frac{330 - v_s}{330 + v_s}$

On solving, $v_s = \textbf{26.7 ms}^{-1}$.

15.25 CHARACTERISTICS OF MUSICAL SOUNDS

44. *Distinguish between music and noise.*

Music. The sound which has a pleasing sensation to the ears is called music. It is produced by regular and periodic vibrations, without any sudden change in loudness. Musical sound can be represented by a periodic wave function and can be splited into various harmonics.

Example. The sound produced by plucking the string of a sitar, by bowing the string of a voiline, sound from a tabla etc.

Noise. The sound which has non-pleasing or jarring effect on the ears is called noise. It is produced at irregular intervals and there is sudden change in loudness. The components of wave function have no definite regularities.

Example. The sound produced by an explosion, sound from a market, etc.

45. *Explain the characteristics of musical sounds. On what factors do they depend ?*

Characteristics of musical sounds. The *three* characteristics of musical sounds are (*i*) Loudness or intensity (*ii*) Pitch (*iii*) Quality or timbre.

(*i*) **Loudness**. *Loudness is the amount of energy crossing unit area around a point in one second.* Loudness depends on :

(*a*) Intensity which depends on amplitude and is proportional to the square of the amplitude.

(*b*) The surface area of the sounding body.

(*c*) Density of the medium.

(*d*) The presence of other resonant objects around the sounding body.

(*e*) The distance of the source from the listener.

(*f*) The motion of air.

(*ii*) **Pitch.** *Pitch is a sensation which helps a listener to distinguish between a high and a grave note. Pitch depends on frequency.*

The voice of a child or a lady is shriller than that of a man *i.e.*, the pitch of a lady's sound is higher than that of a man. The mosquito's sound is of high pitch and hence high frequency.

The pitch of a sound heard can change due to Doppler effect.

(*iii*) **Quality or timbre.** *Quality of sounds enables us to distinguish between two sounds of same pitch and loudness. It is due to the quality of sound that one can recognise one's friend without seeing him.* Quality of sound depends on the number of overtones present in it. It is due to the different overtones present in musical instruments that we are able to recognise them by their sounds.

Scanned by CamScanner

46. *What is meant by threshold of hearing ?*

Threshold of hearing. The lowest intensity of sound that can be perceived by the human ear is called threshold of hearing. For a sound of frequency 10 kHz, the threshold of hearing is 10^{-12} Wm^{-2}.

47. *What is relation between loudness and intensity ?*

Relation between loudness and intensity. According to Weber-Fechner law, the loudness of a sound of intensity I is given by

$$L = \log_{10} \frac{I}{I_0}$$, where I_0 is the threshold of hearing.

48. *What is a musical scale ?*

Musical Scale. *A series of notes whose fundamental frequencies have definite ratios and which produce a pleasing effect on the ear when sounded in succession constitute a musical scale.* The simplest musical scale called the diatonic scale has eight notes comprising an octave. The frequency ratio of the eighth and the first note is 2 : 1. Conventionally, the fundamental frequency of the first note is taken to be 256 Hz and that of the last 512 Hz.

The frequencies of the intermediate notes with their Indian names are given below :

Symbol	Indian Name	Frequency in the base 256 Hz
C	Sa	256
D	Re	288
E	Ga	320
F	Ma	341.3
G	Pa	384
A	Dha	426.7
B	Ni	480
C_1	Sa	512

15.26 ACOUSTICS OF BUILDINGS

49. *What is meant by reverberation ? What is its cause ?*

Reverberation. *The persistence of audible sound after the source has ceased to emit sound is called reverberation.* When sound is produced in a hall or an auditorium, the sound waves suffer multiple reflections from the walls, ceiling and other materials present in the hall. The intensity of the sound heard is the combined effect of direct waves and reflected waves. Due to this, the sound persists for some time even after the source has stopped producing sound. This is the cause of reverberation.

50. *Define reverberation time. What is the optimum reverberation time for hearing a speech in a hall ?*

Reverberation time. *It is defined as the time which sound takes to fall in intensity to one millionth* (10^{-6}) *part of its original intensity after it was stopped.* Optimum time of reverberation depends upon the size of the hall, absorption material and the nature of programme to be heard. Reverberation time should neither be too high nor too low for quality of sound.

Sabine formula for reverberation time of a hall is

$$T = \frac{0.16\ V}{\Sigma\ as}$$

Here V = Volume of the hall in cubic metre.

$\Sigma\ as = a_1 s_1 + a_2 s_2 + a_3 s_3 + ...$

= Total absorption of the hall

$s_1, s_2, s_3, ...$ = Areas of the various surfaces in m^2.

$a_1, a_2, a_3, ...$ = Absorption coefficients of the the various surfaces.

Usually a reverberation period varying between 0.5 to 1.5 seconds is quite good enough in hearing depending upon the nature of auditorium and programme.

51. *How can reverberation time be controlled ?*

Methods for controlling reverberation time. The reverberation time can be controlled by following methods :

(*i*) Covering walls and doors with absorbent materials like asbestos, perforated card board, etc.

(*ii*) Providing open windows in the space.

(*iii*) Providing heavy curtains with folds and folding or opening some of the curtains.

(*iv*) Decorating the walls with pictures and maps.

(*v*) By increasing the number of audience.

(*vi*) By the floors which help in absorbing sound.

52. *What do you mean by acoustics ? Mention four important acoustical requirements of a building.*

Acoustics. *The branch of science which deals with the methods of production, reception and propagation of sound is called acoustics.*

Acoustical requirements of a building. These are as follows :

(*i*) The reverberation time must have the optimum value *i.e.*, it should neither be too low nor too high.

(*ii*) The total quality of music should not be altered.

(*iii*) There should be no echoes.

(*iv*) There should not be any undesirable focusing of sound due to reflections from the walls etc., and also there should not be any silence zones in the hall.

Scanned by CamScanner

Very Short Answer Conceptual Problems

Problem 1. Is an oscillation a wave ? Give reason.

Solution. No, an oscillation is not a wave. The term wave implies the transfer of energy through successive vibrations of the particles of the medium. So the oscillations of a body do not constitute a wave.

Problem 2. A wave transmits momentum. Can it transfer angular momentum ?

Solution. A wave transmits momentum. It cannot transmit angular momentum. The transference of angular momentum means the action of a torque which causes rotatory motion.

Problem 3. Frequency is the most fundamental property of a wave. Why ?

Solution. When a wave travels from one medium to other, its wavelength as well as velocity may change. But frequency does not change. This is the reason that frequency is the fundamental property of a wave.

Problem 4. Which of the following is not a wave characteristic :

Reflection, refraction, interference, diffraction, polarisation, rectilinear propagation ?

Solution. Rectilinear propagation is not a wave characteristic.

Problem 5. How is energy transmitted in wave motion ?

Solution. The neighbouring oscillating parts of the medium are coupled together through elastic forces. During wave motion, a part of the medium is set into oscillation. This part hands over its motion to the next part of the medium and so on. This results in transmission of energy.

Problem 6. Name two important properties of a material medium responsible for the propagation of waves through it.

Solution. Properties of elasticity and inertia.

Problem 7. What is the source of electromagnetic waves ?

Solution. They are generated due to the changes of the electric and magnetic fields associated with the oscillating charges.

Problem 8. Why are the longitudinal waves also called pressure waves ?

Solution. Longitudinal waves travel in a medium as series of alternate compressions and rarefactions *i.e.*, they travel as variations in pressure and hence are called pressure waves.

Problem 9. What is a non-dispersive medium ? Give an example.

Solution. A medium in which the speed of a wave is independent of its frequency is called a non-dispersive medium. For example, air is a non-dispersive medium for sound waves.

Problem 10. Can transverse waves be produced in air ?

Solution. No. Transverse waves travel in the form of crests and troughs and so involve change in shape. So the transverse waves can be produced in a medium which has elasticity of shape. As air has no elasticity of shape, hence transverse waves cannot be produced in it.

Problem 11. Do displacement, particle velocity and pressure variation in a longitudinal wave vary with the same phase ?

Solution. No. The particle velocity is $\pi/2$ out of phase with the displacement and the pressure variation is out of by phase by π with the displacement.

Problem 12. How can we distinguish experimentally between longitudinal and transverse waves ?

Solution. We can distinguish between longitudinal and transverse waves by performing polarization experiments. Transverse waves can be polarised while longitudinal waves cannot be polarised.

Problem 13. What is the direction of oscillations of the particle of the medium through which (*i*) a transverse and (*ii*) a longitudinal wave is propagating ?

Solution. (*i*) When a transverse wave propagates through a medium, its particles oscillate perpendicular to the direction of propagation of the wave.

(*ii*) When a longitudinal wave propagates through a medium, its particles oscillate along the direction of propagation of the wave.

Problem 14. What is the difference between wave velocity and particle velocity ?

Solution. The wave velocity (or phase velocity) is constant for a given medium and is given by $v = \nu\lambda$ while the particle velocity changes harmonically with time. It is maximum at the mean position and zero at the extreme position.

Problem 15. We always see lightning before we hear thundering. Why ?

Solution. The speed of light ($3 \times 10^8 \text{ ms}^{-1}$) is much larger than the speed of sound ($\sim 340 \text{ ms}^{-1}$). Consequently, the flash of light reaches us much earlier than the sound of thunder.

Problem 16. What does cause the rolling sound of thunder ?

Solution. The multiple reflections of sound of lightning produce the rolling sound of thunder.

Problem 17. Two astronauts on the surface of the moon cannot talk to each other. Why ?

Solution. Sound waves require material medium for their propagation. As there is no atmosphere on the moon, hence the sound wave cannot propagate on the moon.

Scanned by CamScanner

Problem 18. Why explosions on other planets cannot be heard on the earth ?

Solution. Since no material medium is present in the space between the planets and the earth so the sound of explosions on other planets cannot propagate upto the earth.

Problem 19. When a stone is thrown on the surface of water, a wave travels out. From where does the energy come ?

Solution. The energy of the surface wave spreading on the surface of water comes from the kinetic energy of the stone shared by the water molecules on which it falls.

Problem 20. Why does sound travel faster in solids than in gases ? *[Himachal 03]*

Solution. The speed of sound through any medium of bulk modulus κ and density ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

The value of the ratio κ / ρ is much higher for solids than that for gases. That is why sound travels faster in solids than in gases.

Problem 21. How is it possible to detect the approaching of a distant train by placing the ear very close to the railway line ?

Solution. Sound waves travel much faster in solids than that in air. Moreover, due to high elasticity of solids, sound waves do not die out in solids as soon as in air. This makes possible to detect the sound of a distant approaching train by placing the ear very close to the railway line.

Problem 22. If a person places his ear to one end of a long iron pipeline, he can distinctly hear two sounds when a workman hammers the other end of the pipeline. How ?

Solution. Sound travels sixteen times faster in iron than in air. So the person hears two sounds, the first one travelling through the iron pipeline and the second travelling through air.

Problem 23. Ocean waves hitting a beach are always found to be nearly normal to the shore. Why ?

Solution. Ocean waves are transverse in nature and spread out in the from of concentric circles. When these waves reach the beach shore, their radius of curvature becomes so large that they can be treated as plane waves. Hence the ocean waves hit the beach nearly normal to the shore.

Problem 24. In which medium, do the sound waves travel faster, solids, liquids or gases ? Give reason.

Solution. Sound waves travel in solids with highest speed. This is because the coefficient of elasticity of solids is much greater than coefficient of elasticity of liquids and gases.

Problem 25. Sound travels faster on a rainy day than on a dry day. Why ?

Solution. The amount of water vapours present in the atmosphere is much higher on a rainy day than on a dry day. As the water vapours are lighter than dry air, hence density of wet air becomes less than that of dry air. Now, because the speed of sound is inversely proportional to the square root of the density, hence sound travels faster on a rainy day than on a dry day.

Problem 26. If the pressure of a gas at constant temperature is increased four times, how the velocity of sound in the gas will be affected ? *[Central Schools 11]*

Solution. $v = \sqrt{\frac{\gamma P}{\rho}}$, any increase in '$P$' produces corresponding increase in 'ρ' so that $\frac{P}{\rho}$ = constant. Hence the velocity of sound in a gas is independent of pressure.

Problem 27. Explain why sound travels faster in warm air than cool air ? *[Himachal 05C, 06]*

Solution. Velocity of sound is directly proportional to the square root of the temperature of the air

i.e., $$v \propto \sqrt{T}.$$

As temperature of the warm air is more than that of the cool air, so sound travels faster in warm air than in cool air.

Problem 28. What is the nature of thermal changes in air when sound propagates through air ?

Solution. The sound wave travels through air under adiabatic conditions.

Problem 29. What characteristics of a medium determine the speed of sound waves through it ?

Solution. The speed of sound waves in a medium is determined by (*i*) elasticity and (*ii*) density of the medium.

Problem 30. If we set our watch by the sound of a distant siren, will it go slow or fast ?

Solution. The speed of sound in air has a finite value of nearly 350 ms^{-1}. The sound of siren will take a finite time to reach us. Hence the watch set according to the sound of distant siren will go slow by as much time as that taken by sound to reach us.

Problem 31. What will be the velocity of sound in a perfectly rigid rod ? Give reason.

Solution. The velocity of sound in a perfectly rigid rod will be infinite. This is because the value of Young's modulus of elasticity is infinite for a perfectly rigid rod.

Problem 32. Sound is simultaneously produced at the ends of the two strings of the same length, one of rubber and the other of steel. In which string will the sound reach the other end earlier and why ?

Solution. Speed of sound in a string, $v = \sqrt{\frac{Y}{\rho}}$.

As the value of Y / ρ is larger for steel than for rubber, so sound will reach the other end earlier in the case of steel string.

Scanned by CamScanner

Problem 33. The speed of sound does not depend upon its frequency. Give an example in support of this statement.

Solution. If sounds are produced by different musical instruments simultaneously, then all these sounds are heard by the ear at the same time.

Problem 34. The speed of sound in moist air is greater than that in dry air, why ? Will the speed of sound in moist hydrogen be greater than that in dry hydrogen ?

Solution. The density of water vapour is less than that of air. So the density of air mixed with water vapour (moist air) is less than that of dry air. Hence the speed of sound in moist air is greater than that in dry air ($v \propto 1/\sqrt{\rho}$). However, the density of water vapour is more than that of hydrogen, so the density of moist hydrogen is more than that of dry hydrogen. Consequently, the speed of sound in moist hydrogen is less than that in dry hydrogen.

Problem 35. The velocity of sound in a tube containing air at 27°C and a pressure 76 cm of mercury is 330 ms^{-1}. What will be the velocity of sound when pressure is increased to 100 cm of mercury and the temperature is kept constant ?

Solution. At a given temperature, the velocity of sound in a gas is independent of pressure. Hence the velocity of sound in the tube will remain 330 ms^{-1}.

Problem 36. The shape of a pulse gets distorted as it passes through a dispersive medium. Why ?

Solution. When a pulse passes through a dispersive medium, the wavelength of the wave changes. Consequently, the shape of the pulse changes *i.e.*, it gets distorted.

Problem 37. If an explosion takes place at the bottom of a lake, will the shock waves in water be longitudinal or transverse ?

Solution. An explosion in a lake produces shock waves thereby creating enormous increase in pressure in the medium (water). A shock wave is thus a longitudinal wave travelling at a speed which is greater than that of a longitudinal wave of ordinary intensity.

Problem 38. Does the sound of a bomb explosion travel faster than the sound produced by a humming bee ?

Solution. The velocity of sound in a medium does not depend upon its loudness, pitch or quality. Thus the sound of bomb explosion and of a humming bee, even though having entirely different characteristics, travel with the same speed.

Problem 39. If a balloon is filled with CO_2 gas, then how will it behave for sound as a lens ? What happens if CO_2 gas is replaced H_2 gas ?

Solution. The velocity of sound in CO_2 is less than that in air. Hence a balloon filled with CO_2 will behave like a convex lens. But the velocity of sound in H_2 is greater than that in air, so a balloon filled with H_2 will behave like a concave lens.

Problem 40. Sound can be heard over longer distances on a rainy day. Why ? *[Himachal 06]*

Solution. On a rainy day, the air contains a larger amount of water vapour. This decreases the density of air. As a result, the sound travels faster in air and can be heard over longer distances.

Problem 41. Why sound can be heard more distinctly at a greater distance over water surface ?

Solution. Sound waves are almost totally reflected from the air-water interface because the critical angle for this interface is only 14°. Consequently, the listener receives more sound energy. Hence sound is heard more distinctly.

Problem 42. What is a periodic wave function ?

Solution. A wave function $y(x, t)$ of position and time which satisfies the following periodicity conditions is called periodic wave function :

(*i*) $y(x + m\lambda, t) = y(x, t)$ (*ii*) $y(x, t + nT) = y(x, t)$.

Problem 43. What is a harmonic wave function ?

Solution. A periodic wave function, whose functional form is sine or cosine is called harmonic wave function.

Problem 44. A heavy uniform rope is held vertically and is tensioned by clamping it to a rigid support at the lower end. A wave of certain frequency is set up at the lower end. Will the wave travel up the rope at the same speed ?

Solution. No. Due to the weight of the rope, the tension increases along the rope from the lower end to the upper end. Hence the wave will travel up the rope with an increasing velocity.

Problem 45. Sometimes, in a stringed instrument, a thick wire is wrapped by a thin wire. Why ?

Solution. This increases mass per unit length and hence helps in obtaining a desired low frequency which would otherwise require a string of inconveniently large length.

Problem 46. Why are stationary waves called so ? *[Himachal 04, 05, 05C]*

Solution. In a stationary wave, the particles of the medium vibrate about their mean positions, but disturbance does not travel in any direction.

Problem 47. When are stationary waves produced ?

Solution. Stationary waves are produced when two identical waves travelling in opposite directions through a medium superpose each other.

Problem 48. Under what condition does a sudden phase reversal of waves on reflection take place ?

Solution. On reflection from a denser medium, a wave suffers a sudden phase reversal.

Scanned by CamScanner

Problem 49. A light wave is reflected from a mirror and the incident and the reflected waves superpose to form stationary waves. Explain why nodes and antinodes are not observed, similar to that found in case of sound waves.

Solution. The wavelength of light is of the order of 10^{-7} metre, hence the distance between the nodes or antinodes will also be of the same order. It is not possible to resolve this distance by eye or by an ordinary optical instrument.

Problem 50. What is difference between a tone and a note ?

Solution. A sound of single frequency is called a tone. A combination of tones of different frequencies is called a note.

Problem 51. Why is the note produced by a open organ pipe sweeter than that produced by the closed organ pipe ?

Solution. The note produced by open organ pipe consists of both odd and even harmonics but the note produced by closed organ pipe consists of only the odd harmonics. Due to presence of larger number of overtones or harmonics, the note produced by the open organ pipe is sweeter.

Problem 52. Why are there so many holes in a flute ?

Solution. The flute is basically an open organ pipe. The location of the open end can be changed by keeping the one hole open and closing the other holes. Thus the frequency of the note produced by the flute can be changed.

Problem 53. Why does the pitch of a note produced by a wooden open end pipe becomes sharper when the temperature rises ?

Solution. With rise in temperature, the velocity of sound increases. The fundamental frequency of an open organ pipe is given by ($\nu_1 = v/2l$). Hence with an increase in the value of v, ν_1 increases and the pitch of the note becomes sharper.

Problem 54. When we start filling an empty bucket with water, the pitch of sound produced goes on changing. Why ?

Solution. A bucket may be regarded as a pipe closed at one end. It produces a note of frequency,

$$\nu = \frac{v}{4L}$$

where ν is the velocity of sound in air and L is the length of air column, which is equal to depth of water level from the open end. As the bucket is filled with water, the value of L decreases. Consequently, the frequency and hence the pitch of the sound produced goes on changing.

Problem 55. A vessel is placed below a water-tap. We can estimate the height of water level reached in the vessel from a distance simply by listening the sound. How ?

Solution. The frequency of the note produced by an air column is inversely proportional to its length. As the level of water in the vessel rises, the length of the air column above it decreases. It produces sound of decreasing frequency *i.e.*, the sound becomes shriller. From the extent of shrillness of sound, we can estimate the height upto which the vessel has been filled with water.

Problem 56. If oil of density higher than the density of water is used in a resonance tube, how will the frequency change ?

Solution. The frequency of vibration depends on the length of the air column. The liquid surface only causes the reflection of water. Hence frequency does not change if oil of density higher than that of water is used in the resonance tube.

Problem 57. A vibrating string is heated to a higher temperature. What happens to the pitch of the note produced by it ?

Solution. When a string is heated to a higher temperature, its length increases and its frequency of vibration decreases ($\nu \propto 1/L$). Hence the pitch of the note produced by it decreases.

Problem 58. Why are strings of different thicknesses and materials are used in a sitar or a violin ?

Solution. Fundamental frequency of a stretched string,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

When we use strings of different thicknesses and materials, they have different values of mass per unit length (m). So the strings will produce notes of different frequencies.

Problem 59. A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of same length. Why ?

Solution. As the tuning fork of frequency ν is in resonance with a closed pipe of length L, so

$$\nu = \frac{v}{4L}$$

But an open pipe of length L produces a frequency of $v/2L$. Hence it cannot be in resonance with the tuning fork of frequency ν ($= v/4L$).

Problem 60. Why does the pitch of a note produced by a wooden open end organ pipe become shaper when the temperature increases ?

Solution. The fundamental frequency of an open end organ pipe is given by

$$\nu = \frac{v}{2L}$$

As the temperature increases, the velocity of sound (v) increases. The frequency ν increases. The pitch of the note produced becomes sharper.

Scanned by CamScanner

Problem 61. Two organ pipes of same length open at both ends produce sound of different frequencies if their radii are different. Why ?

Solution. The fundamental frequency of an open organ pipe is given by

$$v = \frac{v}{2(L + 0.3\,D)}$$

Here D is the diameter of the tube and $0.3\,D$ is the *end correction*. Obviously, two organ pipes of same length but of different radii will produce sounds of different frequencies.

Problem 62. How does the frequency of a tunning fork change, when the temperature is increased ? *[AIEEE 02]*

Solution. As the temperature increases, the length of the prong of the tuning fork increases. This increases the wavelength of the stationary waves set up in the tuning fork. As frequency, $v \propto 1/\lambda$, so frequency of the tuning fork decreases.

Problem 63. How does the frequency of a vibrating wire change, when the attached load is immersed in water ?

Solution. When the load is immersed in water, its apparent weight decreases. So the tension in the string decreases. As $v \propto \sqrt{T}$, the frequency of the vibrating wire decreases.

Problem 64. What points of the stretched string between the fixed points must be plucked and touched to excite its second harmonic ?

Solution. In the second harmonic, there is a node at the centre and two antinodes at 1/4 and 3/4 points. To excite second harmonic, the wire should be plucked at its one-fourth length and touched at its centre.

Problem 65. What is the function of the wooden box in the sonometer ? Does it increase or decrease the duration of emission ?

Solution. The function of the wooden box in a sonometer is to increase the sound intensity by its forced vibrations. It decreases the duration of emission of sound energy.

Problem 66. Why is a sonometer box provided with holes ?

Solution. The holes in the walls of the sonometer box keep the inside air in contact with the outside air. When the sonometer wire vibrates, the vibrations are handed over from the bridges to the upper surface of the sound board and the air inside it. Consequently, the outside air also begins to vibrate and a loud sound is heard.

Problem 67. The beats are not heard, if the difference in frequencies of the two sounding notes is more than 10. Why ? *[Himachal 06]*

Solution. If the difference in frequencies of the two waves is more than 10, we shall hear more than 10 beats per second. Due to persistence of hearing, our ear is not able to distinguish between two sounds as separate if the time interval between them is less than (1/10)th of a second. Hence beats heard will not be distinct if the number of beats produced per second is more than 10.

Problem 68. Why do we not hear beats due to sound waves produced by the violins in the violin-section of an orchestra ?

Solution. All the violins in the violin section of an orchestra are tuned to the same frequency. Since there is no difference in the frequencies of these violins, no beats are heard.

Problem 69. As in sound, can beats be observed by two light-sources ?

Solution. No, to observe beats by two light-sources the phase difference between the sources should change regularly. In light sources, however this change occurs at random, because the light-source consists of a large number of atoms and each atom emits wave independently.

Problem 70. Is it necessary for beat production that the two waves must have exactly equal amplitudes ?

Solution. It is not at all necessary that the amplitudes of two waves producing beats should be equal. It is only when we wish to get zero sound at minima that the two amplitudes should be equal. However, the beats become more clear as the amplitudes of two waves approach each other.

Problem 71. If two sound waves of frequencies 500 Hz and 550 Hz superpose, will they produce beats ? Would you hear the beats ?

Solution. Yes, the sound waves will produce 50 beats every second. But due to persistence of hearing, we would not be able to hear these beats. We would hear a continuous sound of frequency 50 Hz, called beat tone.

Problem 72. Can we hear beats when sounds from two different sources are heard together ?

Solution. Yes. Though the two sources are not coherent, they can produce beats.

Problem 73. Does Doppler's effect apply to only sound waves ?

Solution. No, it applies to light waves also.

Problem 74. What physical change occurs when a source of sound moves and the listener is stationary ?

Solution. Wavelength of sound waves changes.

Problem 75. What physical change occurs when the source of sound is stationary but the listener moves ?

Solution. The number of sound waves received by the listener changes.

Scanned by CamScanner

Problem 76. Will there be Doppler effect, when the direction of motion of the source or observer is perpendicular to the direction of propagation of sound ?

Solution. No, there is no Doppler effect, when the source or observer moves perpendicular to the direction of propagation of sound.

Problem 77. A person riding on a merry-go-round emits a sound wave of a certain frequency. Will the person at the centre observe Doppler effect ?

Solution. No, because the source moves perpendicular to the line joining the source and the observer.

Problem 78. Will there be any Doppler effect, if both the sound and the listener are moving with the same velocity and in the same direction ?

Solution. The apparent frequency is given by

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu$$

As $v_0 = v_s$, therefore, $\nu' = \nu$ *i.e.* there is no Doppler effect.

Problem 79. What is an echo ? What should be the minimum distance between the source of sound and the reflector for hearing a distinct echo ?

Solution. *Echo is the phenomenon of repetition of sound due to its reflection from the surface of a large obstacle.*

If s be the distance between the source and reflector, v the velocity of sound and t be the total time taken by sound to reach the listener after the reflection, then

$$2s = vt \quad \text{or} \quad s = \frac{vt}{2} = \frac{340}{2} \times \frac{1}{10} = 17 \text{ m}.$$

Problem 80. Explain why we cannot hear an echo in a small room ?

Solution. For an echo of a simple sound to be heard, the minimum distance between the speaker and the walls should be 17 m. As the length of a room is generally less than 17 m, so we do not hear an echo.

Problem 81. What do you mean by reverberation ? What is reverberation time ?

Solution. *The phenomenon of persistence or prolongation of sound after the source has stopped emitting sound is called reverberation. The time for which the sound persists until it becomes inaudible is called the reverberation time.*

Problem 82. What is the difference between an echo and a reverberation ?

Solution. An echo is produced when sound reflected from a distant obstacle comes back after an interval of 1/10 second or more. In an echo, the original and reflected sounds are heard separately. Reverberation, on the other hand, consists of successive reflections which follow each other so quickly that they cannot produce separate echoes.

Problem 83. The reverberation time is larger for an empty hall than in a crowded hall. Why ?

Solution. In a crowded hall, the absorption of sound waves is much more than in an empty hall because reverberation depends upon the total absorbing material in the hall.

Problem 84. Thick and long curtains are preferred in a big hall. Why ?

Solution. A big hall has large reverberation time due to which different syllables are not heard distinctly. By making use of thick and long curtains, which have large absorption coefficient, reverberation time can be suitably decreased.

Problem 85. An organ pipe emits a fundamental note of frequency 128 Hz. On blowing into it more strongly it produces the first overtone of frequency 384 Hz. What is the type of pipe – closed or open ? *[Delhi 96]*

Solution. The organ pipe must be a closed organ pipe, because the frequency of the first overtone is 3 times the fundamental frequency.

Problem 86. What are infrasonics and ultrasonics ?

Solution. Frequencies below 20 Hz are called infrasonics. Frequencies above 20,000 Hz are called ultrasonics.

Problem 87. How do we identify our friend from his voice while sitting in a dark room ?

Solution. On the basis of quality of sound.

Problem 88. What determines the quality of sound ?

Solution. Quality of sound is determined by the number of harmonic components present in the sound.

Problem 89. A violin note and a sitar note may have the same frequency and yet we can distinguish between the two notes. Explain, why it is so.

Solution. This is due to the fact that overtones produced by the two sources may be different. In other words the quality of sound produced by two instruments of same fundamental frequency is different.

Problem 90. What do you understand by the fidelity of an instrument ?

Solution. The property of a device to reproduce the original sound in all its details is called its fidelity.

Problem 91. What is the factor on which pitch of a sound depends ?

Solution. The pitch of a sound depends on its frequency.

Problem 92. Where will a man hear a louder sound – at the node or at the antinode in case of a stationary wave ?

Solution. The sound is heard due to variation in pressure, which is given by

$$\Delta P = -\text{Elasticity} \times \text{Strain}$$

At the antinodes, amplitude is maximum but strain is minimum. At the nodes, the amplitude is minimum but strain is maximum. So variation in pressure is maximum at the nodes. Hence a loud sound is heard at node not at antinode.

Scanned by CamScanner

Short Answer Conceptual Problems

Problem 1. Given below are some examples of wave motion. State in each case, if the wave motion is transverse, longitudinal or a combination of both :

(i) Motion of a kink in a long coil spring produced by displacing one end of the spring sideways.

(ii) Waves produced in a cylinder containing a liquid by moving its piston back and forth.

(iii) Waves produced by a motor boat sailing in water.

(iv) Light waves travelling from the sun to the earth.

(v) Ultrasonic waves in air produced by a vibrating quartz crystal. *[NCERT]*

Solution. (*i*) ***Transverse wave motion***, because the vibrations of particles (kinks) of the spring are at right angles to the direction of wave propagation.

(*ii*) ***Longitudinal wave motion***, because the molecules of the liquid vibrate to and fro about their mean position along the direction of propagation of the wave.

(*iii*) ***Combination of longitudinal and transverse waves***, because the propeller of the molar boat cuts the water surface laterally and also pushes it in backward direction.

(*iv*) ***Transverse wave motion***, because the light waves are electromagnetic waves in which electric and magnetic fields vibrate in the direction at right angle to each other and also to the direction of propagation of the wave.

(*v*) ***Ultrasonic waves produced by a quartz crystal in air are longitudinal*** because the molecules of air vibrate to and fro about their mean positions along the direction of propagation of wave due to vibrations of quartz crystal.

Problem 2. Why is the sound produced in air not heard by a person deep inside the water ?

Solution. The speed of sound in water is nearly four times the speed of sound in air. From Snell's law of refraction,

$$\mu = \frac{\sin i}{\sin r} = \frac{v_a}{v_w} = \frac{1}{4} = 0.25$$

For refraction, $r_{max} = 90°$, so $(\sin i)_{max} = 0.25$. Hence $i_{max} = 14°$. Consequently, most of the sound produced in air and incident at $\angle i > 14°$ gets reflected back in air and very small amount is refracted into water. Hence a person deep inside water cannot hear the sound produced in air.

Problem 3. In summer, the sound of a siren is heard louder in the night than in the day to a person on the earth. Why ?

Solution. During the day, the temperature of the earth is maximum near the ground and it progressively decreases upwards. The velocity of sound is maximum near the ground and decreases upwards ($v \propto \sqrt{T}$). The vertical plane wavefronts produced by the siren continuously bend upwards, so the sound waves curl upwards during the day. The temperature conditions are reversed at night. The sound waves curl downwards, making the sound of siren louder on the earth.

Problem 4. If two waves of same frequency but of different amplitudes travelling in opposite directions through a medium superpose upon each other, will they form stationary wave ? Is energy transferred ? Are there any nodes ?

Solution. Yes, the given waves superpose to form stationary waves of the form shown in Fig. 15.31. No energy is transferred. There are no nodes but there are positions of minimum amplitude.

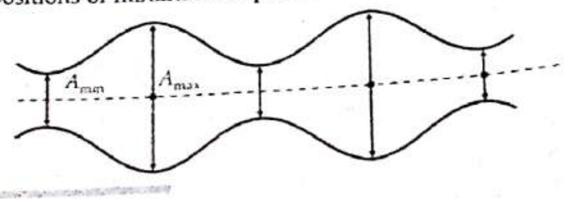


Fig. 15.31

Problem 5. What are overtones and harmonics ? The presence of which makes a sound musical ?

Or

Distinguish between harmonics and overtones. *[Himachal 06]*

Solution. A vibrating system such as a stretched string, an air column, etc., can vibrate with a number of frequencies. The mode with lowest frequency is called *fundamental mode* and others are called *overtones*. When the frequencies of overtones are exact multiples of the fundamental frequency, they are called *harmonics*. The fundamental itself is called *first harmonic*. The presence of the harmonics makes a sound musical.

Problem 6. All harmonics are overtones but all overtones are not harmonics. How ?

Solution. The overtones with frequencies which are integral multiples of the fundamental are called harmonics. Hence all harmonics are overtones. But overtones which are non-integral multiples of the fundamental are not harmonics.

Problem 7. The fundamental frequency of a source of sound is 200 Hz and the source produces all the harmonics. State, with reasons, with which of the following frequencies this source will resonate : 150, 200, 300 and 600 Hz ?

Solution. The source will produce harmonics of frequencies 200 Hz, 400 Hz, 600 Hz, 800 Hz, etc. Clearly, the source will resonate with frequencies of 200 Hz and 600 Hz.

Scanned by CamScanner

Problem 8. An organ pipe is in resonance with a tuning fork. What change will have to be done in the length L to maintain the resonance, if (*i*) the temperature increases, (*ii*) hydrogen is filled in place of air and (*iii*) pressure becomes higher ?

Solution. Suppose the organ pipe has both ends open. Then its fundamental frequency of vibration will be

$$\nu = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}}$$

(*i*) As the temperature increases, the velocity of sound increases, so to fix ν constant, the value of L must be increased.

(*ii*) If hydrogen is filled in the pipe in place of air, the value of density ρ decreases. To keep ν constant, the value of L must be increased.

(*iii*) The pressure has no effect on the velocity of sound. It does not affect resonance.

Problem 9. Fig. 15.32 shows two vibrating modes of an air column. Find the ratio of frequencies of the two modes.

Solution. Let L be the length of the air column.

For mode (*a*),

$L = 3\lambda/4$ or $\lambda = 4L/3$

$\therefore$ Frequency,

$$\nu_1 = \frac{v}{\lambda} = \frac{3v}{4L}$$

For mode (*b*),

$L = 5\lambda/4$ or $\lambda = 4L/5$

$\therefore$ Frequency,

$$\nu_2 = \frac{v}{\lambda} = \frac{5v}{4L}$$

Hence $\nu_1 : \nu_2 = 3 : 5$.

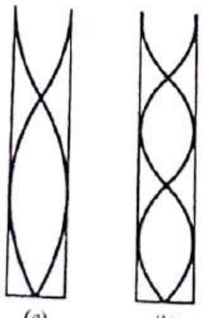


Fig. 15.32

Problem 10. Two progressive sound waves each of frequency 170 Hz and travelling in opposite directions in air superpose to produce stationary waves. The speed of sound in air is 340 ms^{-1}. What is the separation between (*i*) two successive nodes, (*ii*) two successive antinodes and (*iii*) a node and its nearest antinode ?

Solution. Wavelength of sound,

$$\lambda = \frac{v}{\nu} = \frac{340}{170} = 2.0 \text{ m}$$

(*i*) Distance between two successive nodes

$= \lambda/2 = \mathbf{1.0\ m}$.

(*ii*) Distance between two successive antinodes

$= \lambda/2 = \mathbf{1.0\ m}$.

(*iii*) Distance between a node and its nearest antinode

$= \lambda/4 = \mathbf{0.5\ m}$.

Problem 11. A sonometer wire resonates with a tuning fork. If the length of the wire between the bridges is made twice even then it can resonate with the same tuning fork. How ?

Solution. Initially, the fundamental frequency of the sonometer wire is equal to the frequency ν of the tuning fork. When the length of the wire is made twice, its fundamental frequency becomes $\nu/2$. But it now produces a first overtone of frequency ν, vibrating in two segments. Hence the wire can still vibrate in resonance with the tuning fork.

Problem 12. Why does a tuning fork have two prongs ? Would the tuning fork be of any use, if one of the prongs is cut off ?

Solution. When a tuning fork is set into vibrations, its two prongs vibrate in opposite phases. However, the centre of mass of the tuning fork, which lies at the midpoint of the bend, remains at rest. Hence by holding its stem in the hand, a tuning fork can be set into vibrations and no external force is required to maintain its vibrations.

If one of the prongs is cut off, the oscillations of the tuning fork will soon die out and can be maintained only with the help of an external periodic force.

Problem 13. Why is a tuning fork used as a standard oscillator ? On what factors does the pitch of a tuning fork depend ?

Solution. When a tuning fork is struck lightly against a rubber pad, it produces only fundamental tone. If it is struck forcefully, it produces overtones which soon die out. So a tuning fork can be used as a source of standard frequency.

Factors on which the pitch of a tuning fork depends:

(*i*) It is inversely proportional to the square of the length of its prongs. Thus

$$\nu \propto \frac{1}{l^2}$$

(*ii*) It is directly proportional to the thickness of the fork.

$$\nu \propto b$$

(*iii*) It is directly proportional to the square root of the Young's modulus of elasticity of its material.

$$\nu \propto \sqrt{Y}$$

(*iv*) It is inversely proportional to the square root of the density of its material.

$$\nu \propto \frac{1}{\sqrt{\rho}}$$

Hence low-frequency tuning forks are long and thin while high-frequency tuning forks are short and thick.

Problem 14. A sitar wire and a tabla, when sounded together, produce 5 beats per second. What can be concluded from this ? If the tabla membrane is tightened, will the beat rate increase or decrease ?

Scanned by CamScanner

Solution. If ν_1 and ν_2 are the frequencies of sitar wire and the tabla membrane, then

$$\nu_1 - \nu_2 = 5$$

If the tabla membrane is tightened *i.e.*, tension is increased, the frequency (ν_2) of the sound produced by the tabla will increase. If $\nu_1 > \nu_2$, the beat frequency will decrease. And if $\nu_1 < \nu_2$, the beat frequency will increase.

Problem 15. Doppler effect is asymmetric in sound whereas in case of light it is symmetric. Explain.

Solution. Sound waves require a material medium for their propagation. So the observed frequency of sound when the source moves towards the observer is different from the case when the observer moves towards the source with the same relative velocity. We say that the Doppler effect in sound is asymmetric. On the other hand, no material medium is required for propagation of light waves. So the apparent frequency is same whether the source moves towards the observer or the observer moves towards the source. We say that the Doppler effect in light is symmetric.

Problem 16. Distinguish between transverse and longitudinal waves. *[Himachal 09]*

Solution.

Transverse waves	Longitudinal waves
1. The vibrations of the particles of the medium are perpendicular to the direction of propagation of the wave.	The vibrations of the particles of the medium are parallel to the direction of propagation of the wave.
2. In transverse waves, alternate crests and troughs are formed.	In longitudinal waves, alternate zones of compression and rarefaction are formed.
3. These waves may be formed in solids and over liquid surfaces.	These waves may be formed in solids, liquids and gases.
4. These waves do not involve changes of pressure and density of the medium.	These waves involve changes of pressure and density of the medium.
5. These waves can be polarised.	These waves cannot be polarised.

Problem 17. What is red shift ? What does it indicate ?

Solution. The spectral lines received from the distant stars and galaxies are found to be shifted towards the higher wavelength side *i.e.*, towards the red end of the visible spectrum. This shift in wavelength is called red shift. This indicates that stars and galaxies are receding away from us or the universe is expanding.

Problem 18. An incident wave is represented by $y(x, t) = 20 \sin(2x - 4t)$. Write the expression for reflected wave :

(*i*) from a rigid boundary.

(*ii*) from an open boundary. *[Central Schools 03, 11]*

Solution. (*i*) The wave reflected from a rigid boundary is

$$y(x, t) = -20 \sin(-2x - 4t) = \mathbf{20 \sin(2x + 4t)}.$$

(*ii*) The wave reflected from an open boundary is

$$y(x, t) = 20 \sin(-2x - 4t) = \mathbf{-20 \sin(2x + 4t)}.$$

Problem 19. State the principle of superposition of waves. Distinguish between conditions for the production of stationary waves and beats. *[Delhi 03C]*

Solution. The *principle of superposition states* that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves. Mathematically,

$$y = y_1 + y_2 + y_3 + \dots + y_n$$

(*i*) When two waves of same frequency moving with the same speed in the opposite directions in medium superpose an each other, they produce *stationary waves*.

(*ii*) When two waves of slightly different frequencies moving with the same speed in the same direction in a medium superpose on each, they produce *beats*.

Problem 20. Differentiate between Stationary waves and Progressive waves. *[Himachal 05 ; Delhi 11]*

Solution.

Stationary Waves	Progresssive Waves
(*i*) The disturbance remains confined to a particular region, and there is no onward motion.	The disturbance travels forward, being handed over from one particle to the neighbouring particle.
(*ii*) There is no transfer of energy in the medium.	Energy is transferred in the medium along the waves.
(*iii*) The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.	The amplitude of vibration of each particle is same.
(*iv*) The particles of the medium at nodes are permanently at rest.	No particle of the medium is permanently at rest.

Scanned by CamScanner

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *The length of a string tied to two rigid supports is 40 cm. What is the maximum length (wavelength in cm) of the stationary wave produced on it?* **[AIEEE 02]**

Solution. When the string vibrates in one segment,

$$L = \frac{\lambda}{2}$$

or $$\lambda = 2L = 2 \times 40 \text{ cm}$$

$$= \mathbf{80 \text{ cm}}.$$

Problem 2. *Tube A has both ends open, while B has one end closed. Otherwise the two tubes are identical. What is the ratio of fundamental frequency of the tubes A and B?* **[AIEEE 02]**

Solution. The fundamental frequency for tube A with both ends open is

$$\nu_A = \frac{v}{2L}$$

The fundamental frequency for tube B with one end closed is

$$\nu_B = \frac{v}{4L}$$

$$\therefore \quad \frac{\nu_A}{\nu_B} = \frac{v/2L}{v/4L} = \mathbf{2}.$$

Problem 3. *An open pipe is in second harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Find the value of n. How are f_1 and f_2 related to each other?* **[IIT 05]**

Solution. With both ends of the pipe open, the frequency of second harmonic is

$$f_1 = 2 \cdot \frac{v}{2L} = \frac{v}{L}$$

With one end of the pipe closed, the frequency of n^{th} harmonic is

$$f_n = n\frac{v}{4L},$$

where n is an odd integer.

Clearly, f_n will be just greater than f, when $n = 5$.

Hence $$f_2 = 5\frac{v}{4L} = \frac{5}{4}f_1.$$

Problem 4. *A string is stretched between two fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. What is the lowest resonant frequency for this string?* **[AIEEE 06]**

Solution. Here $p\frac{v}{2l} = 315 \text{ Hz}$

and $$(p+1)\frac{v}{2l} = 420 \text{ Hz}$$

Hence the lowest resonant frequency,

$$\nu = \frac{v}{2l} = 420 - 315 = \mathbf{105 \text{ Hz}}.$$

Problem 5. *A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v_s ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, what is the maximum value of v_s upto which he can hear the whistle.* **[AIEEE 06]**

Solution. Here source moves towards the stationary observer.

The apparent frequency is

$$\nu' = \frac{v}{v - v_s} \times \nu$$

$$10{,}000 = \frac{300}{300 - v_s} \times 9500$$

or $$300 - v_s = 285$$

or $$v_s = 300 - 285 = \mathbf{15 \text{ ms}^{-1}}.$$

Problem 6. *Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in the Fig. 15.33. The speed of each pulse is 2 cm/s. After 2 seconds, what will be the nature of the total energy of the pulses?* **[IIT Screening 01]**

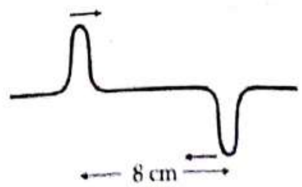


Fig. 15.33

Solution. After 2 seconds, the two pulses will be at the same location. They add up destructively, giving zero displacement at each point. The string becomes straight and so does not have any potential energy. Its total energy must be kinetic.

Problem 7. *Two monoatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. What is the ratio of the speed of sound in gas 1 to that in gas 2?* **[IIT Screening 2K]**

Scanned by CamScanner

Solution. The speed of sound in a gas of molecular mass M is

$$v = \sqrt{\frac{\gamma RT}{M}} \qquad i.e., \qquad v \propto \frac{1}{\sqrt{M}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}.$$

Problem 8. *Two vibrating strings of the same material but lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency ν_1 and the other with frequency ν_2. What is the ratio ν_1/ν_2 ?* **[IIT Screening 2K]**

Solution. The frequency of vibration of the fundamental mode,

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}} = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}$$

As T and l are the same for both wires, so

$$\nu \propto \frac{1}{lr}$$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{l_2 r_2}{l_1 r_1} = \frac{2L.r}{L.2r} = \mathbf{1}.$$

Problem 9. *The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1, and in another experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then how are E_1 and E_2 related ?* **[IIT Screening 2K]**

Solution. The equation of a stationary wave is

$$y = A \sin(kx) \sin(\omega t)$$

Here, for y_1, $k_1 = \pi/L$, $\omega_1 = \omega$

$$\therefore \quad v_1 = \frac{\omega_1}{k_1} = \frac{\omega L}{\pi}.$$

For y_2, $k_2 = 2\pi/L$, $\omega_2 = 2\omega$

$$\therefore \quad v_2 = \frac{\omega_2}{k_2} = \frac{\omega L}{\pi} = v_1.$$

Thus the wave velocities are the same in both cases. Also, they have the same amplitude. The frequency for y_2 is twice the frequency for y_1.

As energy $\propto$ (frequency)^2

$$\therefore \quad E_2 = \mathbf{4E_1}.$$

Problem 10. *A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges, find the value of M.* **[IIT Screening 02]**

Solution. $\nu_p = \frac{p}{2l}\sqrt{\frac{T}{m}}$

In the first case, $\quad \nu_5 = \frac{5}{2l}\sqrt{\frac{9g}{m}}$

In the second case, $\quad \nu_3 = \frac{3}{2l}\sqrt{\frac{Mg}{m}}$

As the two modes are in resonance with the same tunning fork, so

$$\frac{5}{2l}\sqrt{\frac{9g}{m}} = \frac{3}{2l}\sqrt{\frac{Mg}{m}}$$

$$15 = 3\sqrt{M} \quad \text{or} \quad M = \mathbf{25\ kg}.$$

Problem 11. *In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.* **[IIT Screening 03]**

Solution. End correction

$$= \frac{L_2 - 3L_1}{2} = \frac{0.35 - 3 \times 0.1}{2} = 0.025 \text{ m} = \mathbf{2.5\ cm}.$$

Problem 12. *A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then find the ratio f_1/f_2.* **[IIT Screening 2K ; Central Schools 11]**

Solution. For the stationary observer,

$$f' = \frac{v}{v - v_s} \times f$$

$$\therefore \quad f_1 = \frac{340}{340 - 34} \times f$$

and $\quad f_2 = \frac{340}{340 - 17} \times f$

Hence $\quad \frac{f_1}{f_2} = \frac{340 - 17}{340 - 34} = \mathbf{\frac{19}{18}}.$

Problem 13. *A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B, he records a frequency of 6.0 kHz while approaching the same siren. What is the ratio of the velocity of train B to that train A ?* **[IIT Screening 02]**

Solution. When the observer approaches the stationary source,

$$\nu' = \frac{v + v_0}{v}\nu$$

Scanned by CamScanner

For train A, $5.5 = \frac{v + v_A}{v} \times 5$ or $v_A = 0.1v$

For train B, $6.0 = \frac{v + v_B}{v} \times 5$ or $v_B = 0.2v$

$\therefore \quad \frac{v_B}{v_A} = 2.$

Problem 14. *A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observe any beats.* **[IIT Screening 03]**

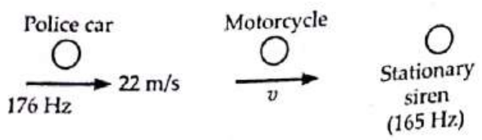


Fig. 15.34

Solution. Frequency of the police horn heard by the motorcyclist,

$$\nu_1 = \frac{330 - v}{330 - 22} \times 176$$

Frequency of the stationary siren heard by the motorcyclist,

$$\nu_2 = \frac{330 + v}{330} \times 165$$

For no beats,

$$\nu_1 = \nu_2$$

$$\frac{330 - v}{330 - 22} \times 176 = \frac{330 + v}{330} \times 165$$

or $\quad v = \mathbf{22\ ms^{-1}}.$

Problem 15. *A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air.* **[IIT Mains 03]**

Solution. In the fundamental mode of vibration,

$$(l + 0.3\,D) = \frac{\lambda}{4} = \frac{v}{4\nu}$$

$$\therefore \quad v = 4\nu\,(l + 0.3\,D)$$

$$= 4 \times 480\,(0.16 + 0.3 \times 0.05)\ \text{ms}^{-1}$$

$$= \mathbf{336\ ms^{-1}}.$$

Problem 16. *The length of a sonometer wire is 0.75 m and density 9×10^3 kg m^{-3}. It can bear a stress of 8.1×10^8 N/m^2 without exceeding the elastic limit. What is the fundamental frequency that can be produced in the wire?* **[IIT 90]**

Solution. Here $L = 0.75$ m, $\rho = 9 \times 10^3$ kg m^{-3},

Stress $= 8.1 \times 10^9$ Nm^{-2}

Let a be the area of cross-section of the wire, then

Fundamental frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{\text{Stress} \times a}{a \times 1 \times \rho}}$$

$$= \frac{1}{2L}\sqrt{\frac{\text{Stress}}{\rho}} = \frac{1}{2 \times 0.75}\sqrt{\frac{8.1 \times 10^8}{9 \times 10^3}} = \mathbf{200\ Hz.}$$

Problem 17. *A transverse sinusoidal wave of amplitude A, wavelength λ and frequency ν is travelling on a stretched string. The maximum speed of any point on the string is $\frac{v}{10}$, where v is the speed of propagation of the wave. If $A = 10^{-3}$ m and $v = 10$ ms^{-1}, then find the values of ν and λ.* **[IIT 98]**

Solution. Here $v_{max} = \frac{v}{10} = \frac{10}{10} = 1\ \text{ms}^{-1}$,

$$A = 10^{-3}\ \text{m}$$

Velocity amplitude or maximum velocity is given by

$$v_{max} = \omega A = 2\pi\nu A$$

$$\nu = \frac{v_{max}}{2\pi A} = \frac{1}{2\pi \times 10^{-3}} = \mathbf{\frac{10^3}{2\pi}\ Hz.}$$

Wavelength,

$$\lambda = \frac{v}{\nu} = \frac{10 \times 2\pi}{10^3} = \mathbf{2\pi \times 10^{-2}\ m.}$$

Problem 18. *An open pipe is suddenly closed at one end with the result that the frequency of the third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. What is the fundamental frequency of the open pipe?* **[IIT 96]**

Solution. Fundamental frequency of open pipe,

$$\nu_o = \frac{v}{2L}$$

Frequency of third harmonic of closed pipe,

$$\nu_c = \frac{3v}{4L}$$

$$\therefore \quad \frac{\nu_c}{\nu_o} = \frac{3}{2} \qquad \text{or} \qquad \nu_c = \frac{3}{2}\nu_o$$

Given $\nu_c = \nu_o + 100$

$$\therefore \quad \frac{3}{2}\nu_o = \nu_o + 100 \qquad \text{or} \qquad \nu_o = \mathbf{200\ Hz.}$$

Problem 19. *The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Velocity of sound in air = 330 ms^{-1}.* **[IIT 97]**

Scanned by CamScanner

Solution. Let lengths of the open and closed organ pipes be L_0 and L_c respectively.

Fundamental frequency of closed pipe,

$$\nu_c = \frac{v}{4L_c} \quad \therefore \quad L_c = \frac{v}{4\nu_c}$$

But $v = 330 \text{ ms}^{-1}, \quad \nu_c = 110 \text{ Hz}$

$$L_c = \frac{330}{4 \times 110} = 0.75 \text{ m} = \mathbf{75\ cm.}$$

∴ Frequency of first overtone of open organ pipe,

$$\nu_0 = 2 \times \frac{v}{2L_0} = \frac{v}{L_0}$$

Frequency of first overtone of closed pipe,

$$\nu_c = 3 \times \frac{v}{4L_c} = 3 \times 110 = 330 \text{ Hz.}$$

But $\nu_0 - \nu_c = 2.2$ Hz

$$\therefore \quad \frac{v}{L_0} - 330 = 2.2$$

or $$\frac{v}{L_0} = 332.2$$

or $$L_0 = \frac{v}{332.2} = \frac{330}{332.2} = 0.99 \text{ m} = \mathbf{99\ cm.}$$

Problem 20. *A metallic rod of length 1 m is rigidly clamped at its midpoint. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the midpoint. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the midpoint and those of the constituent waves in the rod.*

(Young's modulus $= 2 \times 10^{11}$ Nm^{-2}, density $= 8000$ kg m^{-3}). **[IIT 94]**

Solution. Velocity of longitudinal waves in the rod is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{8000}} = 5000 \text{ ms}^{-1}$$

As there are two nodes on either side of the midpoint, so

$$L = \frac{5\lambda}{2} = 1 \text{ m}$$

or $$\lambda = 0.4 \text{ m}$$

Frequency,

$$\nu = \frac{v}{\lambda} = \frac{5000}{0.4} = 12500 \text{ Hz}$$

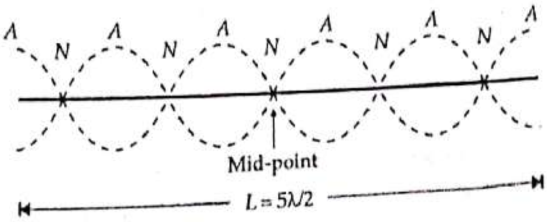


Fig. 15.35

∵ Amplitude of an antinode = 2 × Amplitude of a constituent wave

$$\therefore \quad 2A = 2 \times 10^{-6} \text{ m} \quad \text{or} \quad A = 1 \times 10^{-6} \text{ m}$$

Equation for a stationary wave is

$$y = 2A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$$

But $A = 1 \times 10^{-6}$ m, $\quad x = 2$ cm $= 0.02$ m,
$\lambda = 0.4$ m, $\quad 1/T = 12500$ Hz

$$\therefore \quad \mathbf{y = 2 \times 10^{-6} \cos \frac{\pi}{10} \sin 25000\pi t.}$$

This is the equation of motion at a point 2 cm from the mid-point.

The waves constituting the stationary waves are given by

$$y_{1,2} = A \sin 2\pi\left(\frac{t}{T} \mp \frac{x}{\lambda}\right)$$

or $$y_{1,2} = 1 \times 10^{-6} \sin 2\pi (12500\, t \mp 2.5\, x).$$

Guidelines to NCERT Exercises

15.1. *A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If a transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end ?*

Ans. Given $m = \frac{2.50}{20.0}$ kg m^{-1},

$$T = 200 \text{ N}$$

Speed of the transverse jerk is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{200 \times 20.0}{2.50}} = \sqrt{1600} = 40 \text{ ms}^{-1}.$$

∴ Time taken by the jerk to reach the other end

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{20}{40} = \mathbf{0.5\ s.}$$

Scanned by CamScanner

15.2. *A stone dropped from the top of a tower 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top ? Speed of sound in air = $340\ ms^{-1}$; $g = 9.8\ ms^{-2}$.*

Ans. Let t be the time taken by the stone to reach the water surface.

Here $s = 300$ m, $u = 0$, $a = g = 9.8\ \text{ms}^{-2}$

As $s = ut + \frac{1}{2}at^2$

$\therefore$ $300 = \frac{1}{2} \times 9.8 \times t^2$

or $t^2 = \frac{2 \times 300}{9.8} = 61.2$

$\therefore$ $t = \sqrt{61.2} = 7.82$ s.

Time taken by the splash to reach from water surface to the top,

$$t' = \frac{\text{Distance}}{\text{Speed}} = \frac{300}{340} = 0.88\ \text{s}$$

$\therefore$ Total time taken by the splash to be heard at the top

$= t + t' = 7.82 + 0.88 =$ **8.7 s.**

15.3. *A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C which is $343\ ms^{-1}$?*

Ans. Speed of a transverse wave in the steel wire is given by

$$v = \sqrt{\frac{T}{m}} \quad \text{or} \quad v^2 = \frac{T}{m}$$

or $T = v^2 m$

Given $v = 343\ \text{ms}^{-1}$, $m = \frac{2.10}{12.0}\ \text{kg m}^{-1}$

$\therefore$ $T = (343)^2 \times \frac{2.10}{12.0} = \mathbf{2.06 \times 10^4\ N.}$

15.4. *Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air*

(a) is independent of pressure,

(b) increases with temperature,

(c) increases with humidity.

Ans. Refer answer to Q. 12 on page 15.11.

15.5. *You have learnt that a travelling wave in one dimension is represented by a function $y = f(x, t)$ where x and t must appear in the combination $x - vt$ or $x + vt$, i.e., $y = f(x \pm vt)$. Is the converse true ? Examine if the following functions for y can possibly represent a travelling wave :*

(i) $(x - vt)^2$ *(ii)* $\log[(x + vt)/x_0]$

(iii) $\exp[-(x + vt)/x_0]$ *(iv)* $1/(x + vt)$

Ans. If $y = f(x \pm vt)$ represents a travelling wave, then the converse may not be true *i.e.*, every function of $x - vt$ or $x + vt$ may not always represent a travelling wave. The basic requirement for a function to represent a travelling wave is that it must be finite for all values of x and t.

The functions (*i*), (*ii*) and (*iv*) are not finite for all values of x and t, hence they cannot represent a travelling wave. Only function (*iii*) satisfies the condition to represent a travelling wave.

15.6. *A bat emits ultrasonic sound of frequency 100 kHz in air. If this sound meets a water surface, what is the wavelength of (i) the reflected sound, (ii) the transmitted sound ? Speed of sound in air = $340\ ms^{-1}$ and in water = $1486\ ms^{-1}$.*

[Delhi 06, 10]

Ans. Here $\nu = 100\ \text{kHz} = 10^5$ Hz,

$v_a = 340\ \text{ms}^{-1}$, $v_w = 1486\ \text{ms}^{-1}$

Frequency of both the reflected and transmitted sound remains unchanged.

(*i*) Wavelength of reflected sound,

$$\lambda_a = \frac{v_a}{\nu} = \frac{340}{10^5} = \mathbf{3.4 \times 10^{-3}\ m.}$$

(*ii*) Wavelength of transmitted sound,

$$\lambda_w = \frac{v_w}{\nu} = \frac{1486}{10^5} = \mathbf{1.49 \times 10^{-2}\ m.}$$

15.7. *A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is $1.7\ kms^{-1}$? The operating frequency of the scanner is 4.2 MHz.*

Ans. Here $v = 1.7\ \text{kms}^{-1} = 1.7 \times 10^3\ \text{ms}^{-1}$,

$\nu = 4.2\ \text{MHz} = 4.2 \times 10^6$ Hz

Wavelength,

$$\lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = \mathbf{4.047 \times 10^{-4}\ m.}$$

15.8. *A transverse harmonic wave on a string is described by*

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4),$$

where x, y are in cm and t in s. The positive direction of x is from left to right.

(i) Is this a travelling or a stationary wave ? If it is travelling, what are the speed and direction of its propagation ?

(ii) What are its amplitude and frequency ?

(iii) What is the initial phase at the origin ?

(iv) What is the least distance between two successive crests in the wave ?

Ans. Given :

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4) \quad ...(1)$$

The standard equation for a harmonic wave is

$$y(x, t) = A \sin\left(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x + \phi_0\right) \quad ...(2)$$

Scanned by CamScanner

Comparing equations (1) and (2), we get

$$A = 3.0, \quad \frac{2\pi}{T} = 36, \quad \frac{2\pi}{\lambda} = 0.018, \quad \phi_0 = \frac{\pi}{4}$$

(i) The given equation represents travelling wave propagating from right to left (as x term is +ve).

Speed of the wave,

$$v = \frac{\lambda}{T} = \frac{\lambda/2\pi}{T/2\pi} = \frac{1/0.018}{1/36}$$

$$= \frac{36}{0.018} = 2000 \text{ cm s}^{-1} = \mathbf{20\ ms^{-1}}.$$

(ii) Amplitude, $A = 3.0$ cm

Frequency, $\nu = \frac{1}{T} = \frac{36}{2\pi} = \frac{18}{3.14} = \mathbf{5.73\ s^{-1}}$.

(iii) Initial phase at the origin, $\phi = \frac{\pi}{4}$ **rad.**

(iv) Least distance between two successive crests is equal to wavelength.

$$\lambda = \frac{2\pi}{0.018} = 349.0 \text{ cm} = \mathbf{3.49\ m.}$$

15.9. *For the wave described in Exercise 15.8 displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shapes of these graphs ? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase ?*

Ans. The transverse harmonic wave on a string is given by

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$$

Displacement-time graph for x = 0 :

For $x = 0$, we have

$$y(0, t) = 3.0 \sin(36t + \pi/4)$$

$$a = 3 \text{ cm}, \ \phi_0 = \pi/4 \text{ and } \omega = 36 \text{ rad s}^{-1}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{36} = \frac{\pi}{18} \text{ s.}$$

The displacement at different instants of time will be as follows :

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{3}{\sqrt{2}}$	3	$\frac{3}{\sqrt{2}}$	0	$-\frac{3}{\sqrt{2}}$	−3	$-\frac{3}{\sqrt{2}}$	0	$\frac{3}{\sqrt{2}}$

The y-t graph will be as shown in Fig. 15.36.

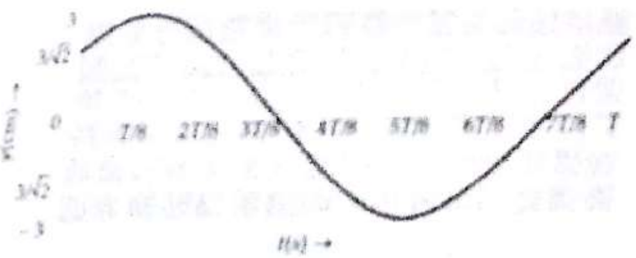


Fig. 15.36

Thus, the y-t graph is sinusoidal in nature.

Similarly, we can obtain y-t graphs for $x = 2$ cm and $x = 4$ cm. It is seen that all the three graphs are sinusoidal. They have same amplitude and frequency, but they differ in initial phases.

15.10. *For a travelling harmonic wave*

$$y = 2.0 \cos(10t - 0.0080x + 0.35)$$

where x and y are in cm and t in s. What is the phase difference between oscillatory motion at two points separated by a distance of (i) 4 m, (ii) 0.5 m, (iii) $\frac{\lambda}{2}$, (iv) $\frac{3\lambda}{4}$?

Ans. Given $y = 2.0 \cos(10t - 0.0080x + 0.35)$ ...(1)

The standard equation of travelling harmonic wave is

$$y = A \cos\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi\right] \quad ...(2)$$

Comparing equations (1) and (2), we get

$$\frac{2\pi}{\lambda} = 0.0080 \text{ or } \lambda = \frac{2\pi}{0.0080} \text{ cm}$$

$$= \frac{2\pi}{0.0080 \times 100} \text{ m} = \frac{2\pi}{0.80} \text{ m}$$

Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda} \times \Delta x$$

(i) When $\Delta x = 4$ m

$$\therefore \quad \Delta\phi = \frac{2\pi}{2\pi} \times 0.80 \times 4 = \mathbf{3.2\ rad.}$$

(ii) When $\Delta x = 0.5$ m

$$\therefore \quad \Delta\phi = \frac{2\pi}{2\pi} \times 0.80 \times 0.5 = \mathbf{0.40\ rad.}$$

(iii) When $\Delta x = \frac{\lambda}{2}$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \textbf{ rad.}$$

(iv) When $\Delta x = \frac{3\lambda}{4}$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2} \textbf{ rad.}$$

15.11. *The transverse displacement of a string (clamped at its two ends) is given by*

$$y(x, t) = 0.06 \sin \frac{2\pi}{3} x \cos(120\pi t)$$

where x, y are in m and t is in s. The length of the string is 1.5 m and its mass is 3.0 × 10^{-2} kg. Answer the following :

(a) Does the function represent a travelling or a stationary wave ?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave ?

(c) Determine the tension in the string.

Scanned by CamScanner

Ans. The equation for transverse displacement is

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\pi t \quad ...(i)$$

(*a*) The equation (*i*) represents a stationary wave as it involves the product of two separate harmonic functions of x and t.

(*b*) Stationary waves are formed by the superposition of two waves of same frequency travelling in opposite directions. Let the two waves be represented as

$$y_1 = A \sin \frac{2\pi}{\lambda}(vt - x)$$

and $$y_2 = -A \sin \frac{2\pi}{\lambda}(vt + x)$$

The resultant stationary wave is given by

$$y = y_1 + y_2$$

$$= A\left[\sin \frac{2\pi}{\lambda}(vt - x) - \sin \frac{2\pi}{\lambda}(vt + x)\right]$$

$$= -2A \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda} \quad ...(ii)$$

Comparing the equations (*i*) and (*ii*), we get

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \quad \text{or} \quad \lambda = 3 \text{ m}$$

and $$\frac{2\pi}{\lambda} v = 120\pi$$

or $$v = 60\lambda = 60 \times 3 = \mathbf{180 \text{ ms}^{-1}}.$$

Frequency,

$$\nu = \frac{v}{\lambda} = \frac{180}{3} = \mathbf{60 \text{ Hz}}.$$

(*c*) Velocity of transverse wave in a string is given by

$$v = \sqrt{\frac{T}{m}}$$

But $$m = \frac{3.0 \times 10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kg m}^{-1},$$

$$v = 180 \text{ ms}^{-1}$$

$$\therefore \quad T = v^2 m = (180)^2 \times 2 \times 10^{-2} = \mathbf{648 \text{ N}}.$$

15.12. *The transverse displacement of a string (clamped at its two ends) is given by*

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\pi t,$$

where x, y are in m and t is in s.

(*i*) *Do all the points on the string oscillate with the same* (*a*) *frequency,* (*b*) *phase,* (*c*) *amplitude ? Explain your answers.*

(*ii*) *What is the amplitude of a point 0.375 m away from one end ?*

Ans. (*i*) The transverse displacement is given by

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\pi t$$

(*a*) The time dependent harmonic function $\cos 120\pi t$ of the stationary wave represents its frequency. As this function does not depend on x, so frequency of oscillation of all points on the string is same.

(*b*) The phase of all the points on the string is same for the reasons similar to (*a*).

(*c*) The amplitude of stationary wave is given by

$$A = 0.06 \sin \frac{2\pi}{3} x \quad \text{[time independent part]}$$

As A depends on x, amplitude of all the points on the string is *not same*.

(*ii*) Now, amplitude at a point 0.375 m away from one end is given by

$$A = 0.06 \sin \frac{2\pi}{3} \times 0.375 = 0.06 \sin 0.7854$$

$$= 0.06 \times 0.707 = \mathbf{0.042 \text{ m}}.$$

15.13. *Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all :*

(*a*) $y = 2\cos(3x)\sin(10t)$

(*b*) $y = 2\sqrt{x - vt}$

(*c*) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

(*d*) $y = \cos x \sin t + \cos 2x \sin 2t$.

Ans. (*a*) As the function is the product of two separate harmonic functions of x and t, so it represents a stationary wave.

(*b*) It cannot represent any type of wave.

(*c*) Here $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

$$= 3\sin\theta + 4\cos\theta \quad [\theta = 5x - 0.5t]$$

If we put $A\cos\alpha = 3$ and $A\sin\alpha = 4$, then

$$y = a\sin(\theta + \alpha)$$

It represents a simple harmonic travelling wave of amplitude,

$$A = \sqrt{3^2 + 4^2} = 5 \quad \text{and} \quad \alpha = \tan^{-1}(4/3).$$

(*d*) It represents the superposition of two stationary waves, one represented by $\cos x \sin t$ and another by $\cos 2x \sin 2t$.

15.14. *A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear density is 4.0×10^{-2} kg m^{-1}. What is*

(*i*) *the speed of a transverse wave on the string, and*

(*ii*) *the tension in the string ?*

Ans. Length of the wire,

$$L = \frac{\text{Mass of the wire (M)}}{\text{Linear density (m)}} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875 \text{ m}$$

Given : $\nu = 45$ Hz, $L = 0.875$ m,

$$m = 4.0 \times 10^{-2} \text{ kg m}^{-1}$$

Scanned by CamScanner

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

or
$$45 = \frac{1}{2 \times 0.875}\sqrt{\frac{T}{4.0 \times 10^{-2}}}$$

Squaring,
$$(45)^2 = \frac{1}{(2 \times 0.875)^2} \times \frac{T}{4.0 \times 10^{-2}}$$

$$T = (45)^2 \times (2 \times 0.875)^2 \times 4.0 \times 10^{-2}$$

$$= \mathbf{248\ N.}$$

Speed of the transverse wave,

$$v = 2\nu L = 2 \times 45 \times 0.875 = \mathbf{78.75\ ms^{-1}}.$$

15.15. *A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz), when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. Ignore edge effect.*

Ans. The frequency of nth mode of vibration for a closed end pipe is given by

$$\nu_n = \frac{(2n-1)v}{4L}, \quad \text{where} \quad n = 1, 2, 3, \ldots$$

Suppose the lengths $L_1 = 25.5$ cm and $L_2 = 79.3$ cm of the resonance columns correspond to $n = n_1$ and $n = n_2$ respectively. Then

$$\nu = \frac{(2n_1 - 1)v}{4L_1} = \frac{(2n_2 - v)}{4L_2} \qquad \ldots(1)$$

$$340 = \frac{(2n_1 - 1)v}{4 \times 25.5} = \frac{(2n_2 - 1)v}{4 \times 79.3}$$

or
$$\frac{2n_1 - 1}{2n_2 - 1} = \frac{25.5}{79.3} = \frac{1}{3}$$

This is possible if $n_1 = 1$ and $n_2 = 2$. Thus the resonance length 25.5 cm corresponds to the fundamental note and 79.3 cm corresponds to first overtone or third harmonic. From equation (1), we get

$$v = \frac{4L_1 \nu}{2n_1 - 1} = \frac{4 \times 25.5 \times 340}{2 \times 1 - 1}$$

$$= 34680\ \text{cm s}^{-1} = \mathbf{346.8\ ms^{-1}}.$$

15.16. *A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel ?* **[Delhi 14]**

Ans. For the fundamental mode of vibration

$$\lambda = 2L = 2 \times 100 = 200\ \text{cm} = 2\ \text{m}$$

Frequency,
$$\nu = 2.53\ \text{kHz} = 2530\ \text{Hz}$$

Speed of sound,
$$v = \nu\lambda = 2530 \times 2 = 5060\ \text{ms}^{-1} = \mathbf{5.06\ km\ s^{-1}}$$

15.17. *A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will this same source be in resonance with the pipe if both ends are open ? (Speed of sound = 340 ms^{-1}).*

Ans. Length of the pipe $L = 20$ cm $= 0.20$ m,

Speed of sound $v = 340\ \text{ms}^{-1}$

Fundamental frequency of vibration of the closed organ pipe is

$$\nu = \frac{v}{4L} = \frac{340}{4 \times 0.20} = 425\ \text{Hz}$$

Hence the fundamental mode of the closed organ pipe may be reasonably excited by a source of frequency 430 Hz.

Fundamental frequency of vibration of the open organ pipe is

$$\nu' = \frac{v}{2L} = \frac{340}{2 \times 0.20} = 850\ \text{Hz}$$

Hence the same source of frequency 430 Hz will not be in resonance with open organ pipe.

15.18. *Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B ?* **[Delhi 14]**

Ans. Frequency of $A = 324$ Hz

Beat frequency $= 6$ Hz

$\therefore$ Possible frequencies of B

$$= 324 \pm 6 = 330 \text{ or } 318\ \text{Hz}$$

When the tension reduces, frequency of string A decreases ($\nu \propto \sqrt{T}$). If the original frequency of B is 318 Hz, the beat frequency should decrease on reducing the tension in A. This is given to be the case as beat frequency decreases from 6 Hz to 3 Hz.

$\therefore$ Frequency $B = \mathbf{318\ Hz.}$

15.19. *Explain why (or how) :*

(*a*) *in a sound wave, a displacement node is a pressure antinode and vice versa,*

(*b*) *bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",*

(*c*) *a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,*

(*d*) *solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and*

(*e*) *the shape of a pulse gets distorted during propagation in a dispersive medium.*

Ans. (*a*) At a displacement node (the point of zero displacement), the variation of pressure is maximum. Hence the displacement node is the pressure antinode and vice-versa.

Scanned by CamScanner

(*b*) Bats can produce and detect ultrasonic waves (sound waves of frequencies above 20 kHz). (*i*) From the interval of time between their producing the waves and receiving the echo after reflection from an object, they can estimate the distance of the object from them. (*ii*) From the intensity of the echo, they can estimate the nature and size of the object. (*iii*) Also, from the small time interval between the reception of the echo by their two ears, they can determine the direction of the object.

(*c*) The instruments produce different overtones (integral multiples of fundamental frequency). Hence the *quality* of sound produced by the two instruments of even same fundamental frequency is different.

(*d*) Solids have both volume and shear elasticity. So both longitudinal and transverse waves can propagate through them. On the other hand, gases have only volume elasticity and not shear elasticity. So only longitudinal waves can propagate through them.

(*e*) When a pulse passes through a dispersive medium, the wavelength of the wave changes. Consequently, the shape of the pulse changes *i.e.*, it gets distorted.

15.20. *A train standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air.*

(*i*) *What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of* $10\ ms^{-1}$, *(b) recedes from the platform with a speed of* $10\ ms^{-1}$ *?*

(*ii*) *What is the speed of sound in each case ?*

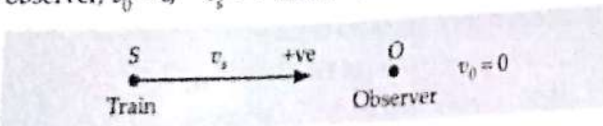
(Speed of sound in still air $= 340\ ms^{-1}$*).*

[Central Schools 08, 12]

Ans. Here $\nu = 400\ Hz$, $v_s = 10\ ms^{-1}$,

$$v_0 = 0, \quad v = 340\ ms^{-1}.$$

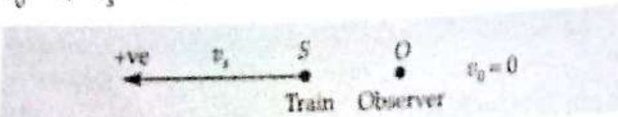
(*i*) (*a*) When the train approaches the stationary observer, $v_0 = 0$, $v_s = +10\ ms^{-1}$.



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 - 10} \times 400$$

$$= \mathbf{412.12\ Hz.}$$

(*b*) When the train recedes from the observer, $v_0 = 0$, $v_s = -10\ ms^{-1}$



$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - 0}{340 + 10} \times 400$$

$$= \mathbf{388.57\ Hz.}$$

(*ii*) Speed of sound in each case $= \mathbf{340\ ms^{-1}}$.

15.21. *A train standing in a station yard blows a whistle of frequency 400 Hz in still air. (a) A wind starts blowing in the direction from the yard to the station with a speed of* $10\ ms^{-1}$. *What are the frequency, wavelength and speed of the sound for an observer standing on the station platform ? (b) Is the situation exactly equivalent to the case, when the air is still and the observer runs towards the yard at a speed of* $10\ ms^{-1}$ *? Take speed of sound in still air* $= 340\ ms^{-1}$.

Ans. Here $\nu = 400\ Hz$, $v = 340\ ms^{-1}$,

$$v_m = 10\ ms^{-1},$$

(*a*) As wind is blowing in the direction of sound, therefore, for an observer standing on the platform,

Speed of sound,

$$v' = v + v_m = 340 + 10 = \mathbf{350\ ms^{-1}}.$$

As there is no relative motion between the source and the observer, the frequency of sound remains unchanged.

Frequency of sound,

$$\nu = 400\ Hz$$

Wavelength of sound,

$$\lambda' = \frac{v'}{\nu} = \frac{350}{400} = \mathbf{0.875\ m.}$$

(*b*) When the observer moves towards the stationary engine (source) in still air,

$$v_0 = -10\ ms^{-1}, \quad v_s = 0$$

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 + 10}{340 - 0} \times 400$$

$$= \frac{350}{340} \times 400 = \mathbf{411.8\ Hz.}$$

As wavelength of sound waves is not affected by motion of the observer, it remains unchanged.

Speed of sound relative to the observer,

$$v'' = 340 + 10 = \mathbf{350\ ms^{-1}}.$$

Thus the situations (*a*) and (*b*) are not equivalent.

15.22. *A travelling harmonic wave on a string is described by*

$$y = 7.5 \sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

(*i*) *What are the displacement and velocity of oscillation of a point at* $x = 1$ *cm, and* $t = 1$ *s ? Is this velocity equal to the velocity of wave propagation ?*

(*ii*) *Locate the points of the string, which have the same transverse displacement and velocity as the* $x = 1$ *cm point at* $t = 2$ *s, 5 s, 11 s.*

Ans. Given $y = 7.5 \sin(0.0050x + 12t + \pi/4)$ ...(1)

$$= 7.5 \sin\left[0.0050\left\{\frac{12}{0.0050}t + x\right\} + \pi/4\right]$$

The standard equation of a travelling wave is

$$y = A \sin\left[\frac{2\pi}{\lambda}(vt + x) + \pi/4\right] \quad ...(2)$$

Comparing equations (1) and (2), we get

$$A = 7.5\,\text{cm},\quad v = \frac{12}{0.0050}\,\text{cm s}^{-1}$$

and $$\frac{2\pi}{\lambda} = 0.0050\,\text{cm}^{-1}$$

(i) At $x = 1$ cm and $t = 1$ s, displacement is

$$y = 7.5\sin(0.0050 \times 1 + 12 \times 1 + \pi/4)$$
$$= 7.5\sin 12.79$$
$$= 7.5 \times 0.2222 = \mathbf{1.67\ cm.}$$

Velocity of oscillation of the particle is

$$u = \frac{dy}{dt} = \frac{d}{dt}[7.5\sin(0.0050x + 12t + \pi/4)]$$
$$= 7.5 \times 12\cos(0.0050x + 12t + \pi/4)$$
$$= 90\cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

At $x = 1$ cm and $t = 1$ s,

$$u = 90\cos(0.0050 \times 1 + 12 \times 1 + \pi/4)$$
$$= 90\cos 12.79$$
$$= 90 \times 0.9751 = \mathbf{87.76\ cm\ s^{-1}}.$$

Velocity of wave propagation is

$$v = \frac{12}{0.0050} = \mathbf{2400\ cm\ s^{-1}}$$

∴ Velocity of oscillation of a point is not equal to the velocity of wave.

(ii) As $\frac{2\pi}{\lambda} = 0.0050$

$$\therefore \quad \lambda = \frac{2\pi}{0.0050} = \mathbf{1256.64\ cm}$$

All points located at distance $n\lambda$ (where n is an integer) from the point $x = 1$ cm have the same transverse displacement and velocity.

15.23. *A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite : (i) frequency, (ii) wavelength, (iii) speed of propagation ? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz ?*

Ans. (a) A narrow sound pulse such as a short *pip* by a whistle does not have a definite wavelength or frequency. But being a sound wave, it has a definite speed (in a non-dispersive medium).

(b) The frequency of the note produced by the whistle is not equal to 1/20 or 0.05 Hz, it is only the frequency of pulse repetition.

15.24. *One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string $x = 0$ has zero transverse displacement ($y = 0$) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.*

Ans. Tension in the string,

$$T = 90 \times 9.8 = 882\ \text{N}$$

Mass per unit length of the string,

$$m = 8.0 \times 10^{-3}\ \text{kg m}^{-1}.$$

Frequency of the wave,

$$\nu = 250\ \text{Hz}$$

Amplitude of the wave,

$$A = 5.0\ \text{cm} = 0.05\ \text{m}$$

The velocity of the transverse wave along the string is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{882}{8 \times 10^{-3}}} = 3.32 \times 10^2\ \text{ms}^{-1}$$

Angular frequency,

$$\omega = 2\pi\nu = 2 \times 3.142 \times 256$$
$$= 16.1 \times 10^2\ \text{rad s}^{-1}$$

As $$v = \frac{\omega}{k}$$

$$\therefore \quad k = \frac{\omega}{v} = \frac{16.1 \times 10^2}{3.32 \times 10^2} = 4.84\ \text{rad m}^{-1}$$

As the wave propagates along positive X-axis, so the displacement equation is

$$y = A\sin(\omega t - kx)$$

or $$y = 0.05\sin(16.1 \times 10^2 t - 4.84x),\ x \text{ and } y \text{ are in m.}$$

15.25. *A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 kmh^{-1}. What is the frequency of sound reflected by the submarine ? Take the speed of sound in water to be 1450 ms^{-1}.*

Ans. Frequency, $\nu = 40$ kHz,

Speed of sound = 1450 ms^{-1}

Speed of enemy submarine = 360 km h^{-1} = 100 ms^{-1}

Firstly sound is observed by enemy submarine. Here observer (enemy submarine) is moving towards the source (SONAR), so $v_0 = -100$ ms^{-1}, $v_s = 0$. Frequency of sonar waves received by the enemy submarine,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{1450 + 100}{1450 - 0} \times 40$$
$$= \frac{1550}{1450} \times 40 = 42.76\ \text{kHz}$$

After the sound is reflected, enemy submarine acts as a source of sound of frequency ν'. This source moves with a speed of 100 ms^{-1} towards the observer (SONAR), so $v_s = +100$ ms^{-1}, $v_0 = 0$. Frequency of sound reflected by the enemy submarine,

$$\nu'' = \frac{v - v_0}{v - v_s} \times \nu' = \frac{1450 - 0}{1450 - 100} \times 42.76$$
$$= \mathbf{45.93\ kHz.}$$

Scanned by CamScanner

15.26. *Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, how far away does the earthquake occur?*

Ans. Suppose the earthquake occurs at a distance of x km from the seismograph.

Speed of S wave = 4.0 km s^{-1}

Time taken by the S wave to reach the seismograph

$$= \frac{x}{4} \text{ s}$$

Speed of P wave = 8.0 km s^{-1}

Time taken by the P wave to reach the seismograph

$$= \frac{x}{8} \text{ s}$$

But $\frac{x}{4} - \frac{x}{8} = 4 \times 60 \text{ s}$

$\therefore$ $x = $ **1920 km.**

15.27. *A bat is flitting about in a cave, navigating via ultrasonic bleeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?*

Ans. As the bat approaches the stationary flat wall surface, the apparent frequency is

$$\nu' = \frac{v}{v - v_s} \nu$$

The stationary flat surface (source) reflects the sound of frequency ν' to the bat (observer) moving towards the flat surface. So the apparent frequency is

$$\nu'' = \frac{v + v_0}{v} \times \nu' = \frac{v + v_0}{v} \times \frac{v}{v - v_s} \times \nu = \frac{v + v_0}{v - v_s} \times \nu$$

$$= \frac{v + 0.03v}{v - 0.03v} \times \nu$$

$$= \frac{1.03}{0.97} \times 40 \text{ kHz}$$

$$= \mathbf{42.47 \text{ kHz}}$$

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. Define the term wave motion.
2. What are mechanical waves ?
3. What do mechanical waves transfer : energy, matter, information or none ?
4. Which type of waves do not require a material medium for their propagation ?
5. Give some common examples of electromagnetic waves.
6. What is the speed of electromagnetic waves in vacuum ?
7. What are matter waves ?
8. What is the angle between particle velocity and wave velocity in (*i*) transverse wave (*ii*) longitudinal wave ?
9. Which type of elasticity a medium must possess for transverse wave motion to be possible ?
10. Amongst solids, liquids and gases, in which type of media, transverse wave motion is possible ?
11. Amongst solids, liquids and gases, in which type of media, longitudinal wave motion can be transmitted ? **[Delhi 05]**
12. What is the nature of the sound waves ?
13. Among reflection, refraction, diffraction, interference and polarization, which is the sole characteristic of transverse waves ?
14. When the wire of a sonometer is plucked, what is the nature of the waves in (*i*) the string (*ii*) in air ?
15. What is the minimum distance between two points in a wave having a phase difference 2π ? **[Delhi 06]**
16. What is the phase difference between two nearest crests (or troughs) ?
17. A harmonic wave travelling in a medium has a period T and wave-length λ. How are λ and T related ?
18. A harmonic wave travelling in a medium has a period T and wave-length λ. How far does the wave travel in time T ?
19. What is the relation between frequency and wave-length ?
20. The time period of a vibrating source producing sound is 0.01 s. If the velocity of sound is 340 ms^{-1}, calculate the wave-length.

Scanned by CamScanner

21. The density of oxygen is 16 times the density of hydrogen. What is the relation between the speeds of sound in two gases ? [Chandigarh 08]
22. What is the audible range of frequency ?
23. What is the wavelength range of audible sound and visible light ? [Delhi 05]
24. What is the effect of pressure on the speed of sound in air ?
25. What is the increase in the speed of sound in air when the temperature of air rises by 1°C ?
26. Can mechanical waves travel through vacuum ?
27. Draw a graph between the pressure of a gas and the speed of sound waves passing through that gas.
28. Does a vibrating body always produce sound ?
29. How far the consecutive nodes are separated from each other ?
30. What is the distance between a node and the nearest antinode ?
31. What is the phase difference between particles being on either side of a node ?
32. What will be the effect on the frequency of a sonometer wire if the tension is decreased by 2% ?
33. What is the minimum frequency with which a string of length L stretched under tension T can vibrate ?
34. Fundamental frequency of oscillation of a close pipe is 400 Hz. What will be the fundamental frequency of oscillation of an open pipe of same length ? [Delhi 04 ; Central Schools 14]
35. The fundamental frequency of an open organ pipe is 512 Hz. What will be its fundamental frequency if its one end is closed ?
36. The frequency of the fundamental note of a closed organ pipe and that of an open organ pipe are the same. What is the ratio of their lengths ?
37. The frequency of the first overtone of a closed organ pipe is same as that of the first overtone of an open organ pipe. What is the ratio between their lengths ?
38. An organ pipe produces a fundamental frequency of 128 Hz. When blown forcefully, it produces first overtone of 384 Hz. Is the pipe open or closed ?
39. How will the fundamental frequency of a closed organ pipe be affected if instead of air it is filled with a gas heavier than air ?
40. In an open organ pipe, third harmonic is 450 Hz. What is frequency of fifth harmonic ? [Delhi 11]
41. What is the main difference between a flute and a violin ?
42. Where to pluck and where to touch a stretched string to excite its first overtone ?
43. What is beat frequency ?
44. What is the essential condition for the formation of beats ?
45. Two sound sources produce 12 beats in 4 seconds. By how much do their frequencies differ ? [Delhi 99 ; Chandigarh 08]
46. What is the exact speed of light in vacuum ?
47. Give one use of beat phenomenon. [Central Schools 03]
48. State the principle of superposition of waves. [Delhi 03C]
49. State the factors on which the speed of a wave travelling along a stretched ideal string depends. [Delhi 13, 14]
50. What are harmonics ?
51. What is Doppler effect ?
52. Velocity of sound in air at NPT is 332 m/s. What will be the velocity, when pressure is doubled and temp. is kept constant ? [Central Schools 09]
53. Name two instruments based on superposition of waves. [Himachal 05C]
54. Two sounds of very close frequencies, say 256 Hz and 260 Hz are produced simultaneously. What is the frequency of resultant sound and also write the number of beats heard in one second ? [Central Schools 08]
55. The frequencies of two tuning forks A and B are 250 Hz and 255 Hz respectively. Both are sounded together. How many beats will be heard in 5 seconds ? [Delhi 12]

Answers

1. Wave motion is a form of disturbance which travels through the medium due to the repeated periodic motion of the particles of the medium about their mean position, the disturbance being handed over from one particle to the next.
2. The waves which require an elastic or material medium for their propagation are called mechanical waves.
3. Energy and information.
4. Electromagnetic waves.

Scanned by CamScanner

5. The common examples of electromagnetic waves are visible and ultraviolet light, radio waves, microwaves, X-rays, infra-red light, etc.
6. All electromagnetic waves travel through vacuum at the same speed, $c = 29, 97, 92, 458 \text{ ms}^{-1}$.
7. Matter waves are the waves associated with moving electrons, protons, neutrons and other fundamental particles, and even atoms and molecules.
8. Angle between particle velocity and wave velocity for transverse wave is $\pi/2$ That in case of longitudinal wave is either zero or π.
9. Elasticity of shape.
10. Solids.
11. In all the three types of media *i.e.*, solids, liquids and gases.
12. Sound waves travel through any medium in the term of longitudinal waves.
13. Polarization.
14. The waves produced in the string are transverse and that in the air are longitudinal.
15. One wavelength (λ).
16. The phase difference between two nearest crests is 2π rad.
17. Wave velocity,

$$v = \frac{\lambda}{T} \quad \text{or} \quad \lambda = vT.$$

18. By definition, the wave will travel distance λ in time T (time period).
19. Wave velocity = Frequency × wavelength

i.e., $v = \nu\lambda$.

20. $\lambda = vT = 340 \times 0.01 = \mathbf{3.4\ m}$.
21. $$\frac{v_H}{v_O} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = 4$$

or $v_H = 4v_O$.

22. The audible range of frequency is 20 Hz to 20 kHz.
23. (*i*) The wavelength range of audible sound in air is from 16.6×10^{-3} m to 16.6 m.

(*ii*) The wavelength range of visible light is from 4×10^{-7} m to 8×10^{-7} m.

24. The increase of pressure has no effect on the speed of sound in air.
25. The speed of sound increases by 0.61 ms^{-1} for every 1°C rise in temperature of the air.
26. No, mechanical waves cannot travel through vacuum.
27. As shown in Fig. 15.37, the graph drawn between the pressure of a gas and the speed of sound

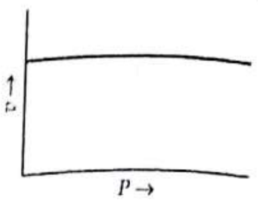


Fig. 15.37

through the gas is a straight line parallel to the pressure axis, because the speed of sound is independent of the pressure of the gas.

28. No. A vibrating body will produce sound only when its frequency of oscillation lies between 20 Hz to 20 kHz.
29. The separation between two consecutive nodes is $\lambda/2$.
30. The distance between a node and the nearest antinode is $\lambda/4$.
31. π rad.
32. As $\nu \propto \sqrt{T}$, so frequency decreases by 1% when tension is decreased by 2%.
33. Minimum frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}},$$

where m is the mass per unit length of the string.

34. 800 Hz.
35. 256 Hz.
36. 1 : 2.
37. 3 : 4.
38. Closed pipe, because the first overtone is 3 times the fundamental frequency.
39. The fundamental frequency will decrease, because

$$\nu = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

i.e., $\nu \propto \frac{1}{\sqrt{\rho}}$.

40. Third harmonic = 3ν = 450 Hz

∴ Fundamental frequency, ν = 150 Hz

Fifth harmonic = 5ν = **750 Hz**.

41. Flute is an organ pipe while violin is a musical instrument based on stretched string.
42. The wire should be plucked at its one-fourth length and touched at its centre.
43. The number of beats produced per second is called beat frequency.

Scanned by CamScanner

The difference in frequency of the two sound waves should not exceed 10.

14. Number of beats produced per second,

$$b = \frac{12}{4} = 3\,s^{-1}.$$

15. $\nu_1 - \nu_2 = b = 3.$

16. Speed of light in vacuum,

$$c = 29,97,92,458\,ms^{-1}.$$

17. The phenomenon of beats is used in tuning two musical instruments.

18. The principle of superposition of waves states that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves. Mathematically,

$$y_1 = y_1 + y_2 + y_3 + \ldots + y_n.$$

19. The speed of a wave travelling on a string depends on

(i) the tension (T) in string and

(ii) its mass per unit length (m)

$$v = \sqrt{\frac{T}{m}}.$$

50. Harmonics are the notes of frequencies which are integral multiples of the fundamental frequencies.

51. The apparent change in the frequency of sound when the source, observer and the medium are in relative motion, is called Doppler effect.

52. 332 m/s, because pressure has no effect on the velocity of sound in air.

53. (i) Sonometer

(ii) Organ pipe.

54. Frequency of resultant sound,

$$\nu_{av} = \frac{\nu_1 + \nu_2}{2}$$

$$= \frac{256 + 260}{2} = \mathbf{258\ Hz.}$$

Number of beats heard in one second

$$= 260 - 256 = \mathbf{4}.$$

55. Number of beats heard in 5 seconds

$$= (255 - 250) \times 5 = \mathbf{25}.$$

Type B : Short Answer Questions

2 or 3 Marks Each

1. Define the term wave motion. Give four important characteristics of wave motion. **[Chandigarh 03, 04]**

2. What are mechanical, electromagnetic and matter waves ? Give an example of each type.

3. What are transverse and longitudinal wave motions ? Give an example for each type.

4. Mention the important properties which a medium must possess for the propagation of mechanical waves through it.

5. Through what type of media, can (i) transverse waves and (ii) longitudinal waves be transmitted ? Explain.

6. Given below are some examples of wave motion. State in each case if the motion is transverse, longitudinal or a combination of both.
   (a) Motion of a kink in a long coil spring produced by displacing one end of the spring sideways.
   (b) Wave produced in a cylinder containing water by moving its piston back and forth.
   (c) Wave produced by a motorboat sailing in water.
   (d) Ultrasonic waves in air produced by a vibrating quartz crystal. **[Delhi 03]**

7. Derive a relation between wave velocity, frequency and wavelength.

8. On the basis of dimensional considerations, write the formula for the speed of transverse waves on a stretched string.

9. On the basis of dimensional considerations, write the formula for the speed of transverse waves in a solid.

10. Write Newton's formula for the speed of sound in air. What was wrong with this formula ? What correction was made by Laplace in this formula ? **[Chandigarh 03 ; Delhi 02, 03C, 06, 13]**

11. The speed of longitudinal waves 'v' in a given medium of density ρ is given by the formula

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Use this formula to explain why the speed of sound in air
   (a) is independent of pressure,
   (b) increases with temperature, and
   (c) increases with humidity. **[Delhi 03]**

Scanned by CamScanner

12. Discuss the effect of temperature on the velocity of sound in gases. **[Delhi 96 ; Himachal 05]**

13. Show that the speed of sound in air increases by $61\,cms^{-1}$ for every 1°C rise of temperature.

14. What is the effect of (*i*) frequency and (*ii*) amplitude, on the speed of sound in air ?

15. Why does sound travel faster in moist air than in dry air ?

16. What is a plane progressive harmonic wave ? Establish displacement relation for a harmonic wave travelling along the positive direction of *X*-axis.

17. What is wave motion ? Derive the equation of a harmonic wave. **[Delhi 98]**

18. What do you mean by phase of a wave ? Discuss the phase change with (*i*) time and (*ii*) position. Hence define time period and wavelength of a wave.

19. Considering suitable examples, discuss the phase change when a wave is reflected from (*i*) a rigid boundary and (*ii*) a free or open boundary.

20. State and illustrate the principle of superposition of waves.

21. What are stationary waves ? State the necessary condition for the formation of stationary waves.

22. Write any three characteristics of stationary waves. **[Central Schools 08]**

23. State the laws of vibrations of stretched strings.

24. Prove analytically that in the case of an open organ pipe of length *L*, the frequencies of vibrating air column are given by

$$\nu = n(v/2L),$$

where *n* is an integer.

25. Describe the various modes of vibration in case of a closed end organ pipe. **[Himachal 08C]**

26. Prove analytically that in the case of a closed organ pipe of length *L*, the frequencies of the vibrating air column are given by $\nu = (2n+1)(v/4L)$, where *n* is an integer.

27. Describe a simple experiment for showing the formation of beats.

28. Explain how the phenomenon of beats may be used for finding the unknown frequency of a tuning fork.

29. Explain Doppler effect in sound. Obtain an expression for the apparent frequency of sound when the source is moving towards the stationary observer with a uniform velocity. **[Chandigarh 03, 04 ; Himachal 04, 05 ; Central Schools 05]**

30. Derive an expression for the apparent frequency of the sound when the observer moves towards a stationary source of sound with a uniform velocity.

31. Bats have no eyes, still they travel during night. Explain, why ? **[Himachal 07C]**

32. Explain Doppler effect in sound. Obtain an expression for apparent frequency of sound when source and listener are approaching each other. **[Delhi 05 ; Central Schools 10]**

33. Distinguish between transverse and longitudinal waves. **[Himachal 05]**

34. What is the effect of pressure on the speed of sound ? Justify your answer. **[Delhi 05]**

35. What is the nature of sound waves in air ? How is the speed of sound waves in atmosphere affected by the (*i*) humidity (*ii*) temperature ? **[Delhi 04]**

36. Give any three differences between progressive waves and stationary waves. **[Himachal 09C ; Delhi 11]**

37. What is a progressive wave ? Derive an expression which represents a progessive wave. **[Himachal 2K, 05C]**

38. What is the difference between velocity of wave and velocity of a particle ? Obtain the relation for the velocity of wave in wave motion. **[Chandigarh 07]**

39. From the equation $y = r\sin\frac{2\pi}{\lambda}(vt - x)$, establish the relation between particle velocity, and wave velocity. **[Chandigarh 08]**

40. What are beats ? Prove that the number of beats produced per second by the two sound sources is equal to the difference between their frequencies. **[Central Schools 08, 09]**

41. Show that in open organ pipe, all harmonics are present. **[Central Schools 07]**

42. Diagrammatically show first two modes of vibrations in case of an open organ pipe and write the ratio of their frequencies. **[Central Schools 12]**

Answers

1. Refer answer to Q. 1 on page 15.1.
2. Refer answer to Q. 2 on page 15.2.
3. Refer answer to Q. 4 on page 15.3.
4. Refer answer to Q. 5 on page 15.3.

Scanned by CamScanner

5. Refer answer to Q. 6 on page 15.3.
6. Refer to solution of Problem 3 on page 15.65.
7. Refer answer to Q. 8 on page 15.4.
8. Refer answer to Q. 9(*a*) on page 15.6.
9. Refer answer to Q. 9(*b*) on page 15.7.
10. Refer answer to Q. 11 on page 15.11.
11. Refer answer to Q. 12 on page 15.11.
12. Refer answer to Q. 12(*iv*) on page 15.11.
13. Refer answer to Q. 12(*iv*) on page 15.11.
14. Refer answer to Q. 12(*vi*) and (*vii*) on page 15.12.
15. Refer answer to Q. 12(*iii*) on page 15.11.
16. Refer answer to Q. 13 on page 15.15.
17. Refer answer to Q. 1 on page 15.1 and Q. 13 on page 15.15.
18. Refer answer to Q. 14 on page 15.16.
19. Refer answer to Q. 17 on page 15.22.
20. Refer answer to Q. 20 on page 15.23.
21. Refer answer to Q. 21 on page 15.24.
22. Refer answer to Q. 24 on page 15.26.
23. Refer answer to Q. 28 on page 15.30.
24. Refer answer to Q. 31 on page 15.36.
25. Refer answer to Q. 32 on page 15.36.
26. Refer answer to Q. 33 on page 15.37.
27. Refer answer to Q. 37 on page 15.42.
28. Refer answer to Q. 38(*i*) on page 15.42.
29. Refer answer to Q. 41 on page 15.49.
30. Refer answer to Q. 42 on page 15.50.
31. Refer answer to NCERT Exercise 15.19(*b*) on page 15.75.
32. Refer answer to Q. 42 on page 15.50.
33. Refer to solution of Problem 16 on page 15.67.
34. Refer answer to Q. 12(*i*) on page 15.11.
35. Refer answer to Q. 12(*iii*) and (*iv*) on page 15.11.
36. Refer to solution to Problem 20 on page 15.67.
37. Refer answer to Q.13 on page 15.15.
38. Refer answer to Q.15(*a*) on page 15.16.
39. Refer answer to Q.15(*a*) on page 15.16.
40. Refer answer to Q.34 and Q.36 on page 15.41.
41. Refer answer to Q.30 on page 15.35.
42. See Fig. 15.20 on page 15.35. Ratio $v_1 : v_2 = 1 : 2$.

Type C : Long Answer Questions

5 Marks Each

1. On the basis of spring model, explain the propagation of a sound in (*i*) air and (*ii*) solids.
2. In reference to a wave motion, define the terms (*i*) amplitude, (*ii*) time period, (*iii*) frequency, (*iv*) angular frequency, (*v*) wavelength, (*vi*) wave number, (*vii*) angular wave number and (*viii*) wave velocity.
3. Write Newton's formula for the speed of sound in gases. Why and what correction was applied by Laplace in this formula ? Also deduce modified formula for speed of sound. **[Delhi 06, 12]**
4. Derive Newton's formula for speed of sound in an ideal gas. What is Laplace correction ? **[Delhi 09]**
5. For a simple harmonic wave, deduce expressions for (*i*) particle velocity and (*ii*) particle acceleration. Write their phase relation with displacement.
6. What are stationary waves ? Explain the formation of stationary waves graphically. **[Chandigarh 04]**
7. Obtain an expression for a stationary wave formed by two sinusoidal waves travelling along the same path in opposite directions and obtain the positions of nodes and antinodes. **[Delhi 10, 13]**
8. What are standing waves ? Derive an expression for the standing waves. Also define the terms node and antinode. **[Delhi 09]**
9. Discuss the formation of standing waves in a string fixed at both ends and the different modes of vibrations. **[Himachal 05C, 08C]**
10. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4. **[Delhi 06]**
11. What are beats ? Explain their formation analytically. Prove that the beat frequency is equal to the difference in frequencies of the two superposing waves. **[Himachal 03]**
12. (*a*) What are beats ? Prove that the number of beats per second is equal to the difference between the frequencies of the two superimposing waves.
    (*b*) Draw the fundamental modes of vibration of stationary waves in :
    (*i*) Closed pipe (*ii*) An open pipe. **[Central Schools 08]**

Scanned by CamScanner

13. Explain Doppler effect in sound. Obtain an expression for apparent frequency of sound when source and listeners are :

(*a*) approaching each other,

(*b*) moving away from each other.

[Delhi 12, 14]

14. (*a*) What is Doppler effect ?

(*b*) Derive an expression for the apparent frequency when a source moves towards a stationary observer.

(*c*) A policeman on duty detects a drop of 15% in the pitch of the horn of a motor-car as it crosses him. Calculate the speed of car, if the velocity of sound is 330 m/s. **[Central Schools 04, 07]**

Answers

1. Refer answer to Q. 3 on page 15.2.
2. Refer answer to Q. 7 on page 15.4.
3. Refer answer to Q. 11 on page 15.9.
4. Refer answer to Q.11 on page 15.9.
5. Refer answer to Q. 15 on page 15.16.
6. Refer answer to Q. 22 on page 15.25.
7. Refer answer to Q. 23 on page 15.26.
8. Refer answer to Q.26 on page 15.28.
9. Refer answer to Q. 26 on page 15.28.
10. Refer answer to Q. 27 on page 15.29.
11. Refer answer to Q. 34 and Q. 36 on page 15.41.
12. (*a*) Refer answer to Q.27 on page 15.29.
    (*b*) (*i*) see Fig. 15.21(*a*) (*ii*) see Fig. 15.20(*a*).
13. Refer answer to Q. 42 on page 15.50.
14. Refer answer to Q.41 on page 15.49 and see hint of Problem 10 on page 15.57.

Scanned by CamScanner

Competition Section

Waves

GLIMPSES

1. **Wave motion.** It is a kind of disturbance which travels through a medium due to the repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next. In a wave both information and energy propagate from one point to another but there is no motion of matter as a whole through a medium.

2. **Three basic types of waves.**
   (i) ***Mechanical waves.*** The waves which require a mechanical medium for their propagation are called mechanical waves or elastic waves. For their propagation, the medium must possess the properties of inertia and elasticity. For example, water waves, sound waves, etc.
   (ii) ***Electromagnetic waves.*** The waves which travel in the form of oscillating electric and magnetic fields are called electromagnetic waves. Such waves do not require any material medium for their propagation. For example, visible light, radio waves, etc.
   (iii) ***Matter waves.*** The waves associated with microscopic particles, such as electrons, protons, neutrons, atoms, molecules, etc., when they are in motion, are called matter waves or de-Broglie waves. Matter waves associated with fast moving electrons are used in electron microscopes.

3. **Spring-model for the propagation of a wave through an elastic medium.** Energy transfer takes place because of the coupling through elastic forces between neighbouring oscillating parts of the medium.

4. **Transverse waves.** These are the waves in which particles of the medium vibrate about their mean positions in a direction perpendicular to the direction of propagation of the disturbance. These waves can propagate in those media which have a shear modulus of elasticity *e.g.*, solids.

5. **Longitudinal waves.** These are the waves in which particles of the medium vibrate about their mean positions along the direction of propagation of the disturbance. These waves can propagate in those media having a bulk modulus of elasticity and are therefore possible in all media : solids, liquids and gases.

6. **Progressive wave.** A wave that moves from one point of medium to another is called a progressive wave.

7. **Amplitude (A).** It is the maximum displacement suffered by the particles of the medium from the mean position during the propagation of a wave.

8. **Time period (T).** It is the time in which a particle of the medium completes one vibration about its mean position.

9. **Frequency (ν).** It is the number of waves produced per second in a given medium.

10. **Wavelength (λ).** It is the distance covered by a wave during the time a particle of the medium completes one vibration about its mean position. It is the distance between two nearest particles of the medium which are vibrating in the same phase.

11. **Angular wave number or propagation constant.** It represents the phase change per unit distance (or per unit path difference). It is equal to $2\pi/\lambda$.

    Thus $$k = \frac{2\pi}{\lambda}$$

    The SI unit of k is radian per metre (rad m^{-1}).

12. **Wave velocity (v).** It is the distance travelled by a wave in one second.

13. **Relation between wave velocity, frequency and wavelength.**

    Wave velocity = Frequency × wavelength

    or $$v = \nu \lambda$$

14. Relation between wave velocity, time period and wavelength.

$$\text{Wave velocity} = \frac{\text{Wavelength}}{\text{Time period}}$$

or $$v = \frac{\lambda}{T} = \frac{\omega}{k}, \quad k = \frac{2\pi}{\lambda}.$$

15. Velocity of transverse waves.

(*i*) Velocity of transverse waves in a solid of modulus of rigidity η and density ρ is given by $v = \sqrt{\frac{\eta}{\rho}}$

(*ii*) Velocity of transverse waves in a string of mass per unit length m and stretched under tension T is given by $v = \sqrt{\frac{T}{m}}$

16. Velocity of longitudinal waves.

(*i*) Velocity of longitudinal waves in an extended solid (earth's crust) of bulk modulus κ, modulus of rigidity η and density ρ is given by

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

The factor $\kappa + (\frac{4}{3})\eta$ is called elongational elasticity.

(*ii*) Velocity of longitudinal waves in a long rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

(*iii*) Velocity of longitudinal waves in a liquid of bulk modulus κ and density ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

(*iv*) Velocity of longitudinal waves in a gaseous medium of bulk modulus κ and density ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

According to Newton, when sound travels through gas, the changes taking place in the medium are isothermal in nature. So *Newton's formula* for the speed of sound is

$$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}, \text{ where } P = \text{pressure of the gas.}$$

According to Laplace, when sound travels in a gas, the changes taking place in the medium are adiabatic. So *Laplace formula* for the speed of sound is

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

where $\gamma = \frac{C_p}{C_v}$ = specific heat ratio.

17. Factors affecting velocity of sound through gases.

(*i*) *Effect of pressure.* Pressure has no effect on the speed of sound in a gas.

(*ii*) *Effect of density.* $v \propto \frac{1}{\sqrt{\rho}}$ or $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

(*iii*) *Effect of temperature.* As $v = \sqrt{\frac{\gamma RT}{M}}$

$\therefore \quad v \propto \sqrt{T}$ or $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

Temperature coefficient of velocity of sound is given by

$$\alpha = \frac{v_t - v_0}{t}$$

For air, $\alpha = 0.61 \text{ ms}^{-1}\,{}^\circ\text{C}^{-1}$.

(*iv*) *Effect of humidity.* Sound travels faster in moist air.

(*v*) *Effect of wind.* If the wind blows with velocity w in a direction making an angle θ with the direction of sound, then the resultant velocity of sound will be $v' = v + w\cos\theta$

18. Wave equation. A plane progressive harmonic wave travelling along positive X-direction may be represented as

(*i*) $y = A\sin(\omega t - kx)$, where $k = 2\pi/\lambda$

(*ii*) $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

(*iii*) $y = A\sin\frac{2\pi}{T}(vt - x)$

where v is the wave-velocity, λ the wavelength, A the amplitude of the oscillating particles of the medium and y is the displacement of the particle located at position x at any instant t.

If the wave is travelling along *negative* X-direction, the minus sign is replaced by plus sign in the above equations. Thus

(*i*) $y = A\sin(\omega t + kx)$, where $k = 2\pi/\lambda$

(*ii*) $y = A\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$

(*iii*) $y = A\sin\frac{2\pi}{\lambda}(vt + x)$

19. Phase and phase difference. Phase is the argument of the sine or cosine function representing the wave. Thus phase,

$$\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

The relation between phase difference ($\Delta\phi$) and time interval Δt is

$$\Delta\phi = \frac{2\pi}{T}\Delta t$$

Scanned by CamScanner

Thus in time period T, the phase of a particle changes by 2π.

The relation between phase difference ($\Delta\phi$) and path difference (Δx) is

$$\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$$

The negative sign indicates that farther the particle is located from the origin in the positive X-direction, the more it lags behind in phase. Clearly, the phase difference between two particles located at separation λ is 2π.

20. **Principle of superposition of waves.** When a number of waves travel through a medium simultaneously, the resultant displacement at any point of the medium is equal to the vector sum of the displacements of the individual waves. If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \ldots, \vec{y}_n$ are the displacements of n waves superposing each other at a point, then the resultant displacement at that point will be

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots + \vec{y}_n .$$

21. **Reflection of a wave.** When a wave is reflected from a rigid boundary or a closed end, it is reflected back with a phase reversal or phase difference of π radians but reflection at an open boundary takes place without any phase change.

22. **Stationary waves.** When two progressive waves of equal amplitude and frequency, travelling in opposite directions along a straight line superpose each other, the resultant wave does not travel in either direction and is called a *stationary* or *standing wave*. At some points, the particles of the medium always remain at rest. These are called *nodes*. At some other points, the amplitude of oscillation is maximum. These are called *antinodes*.

Consider a plane progressive harmonic wave travelling along positive X-direction.

$$y_1 = A\sin(\omega t - kx) \qquad \text{(incident wave)}$$

If this wave is reflected from a *free end*, then

$$y_2 = A\sin(\omega t + kx) \qquad \text{(reflected wave)}$$

The stationary wave formed by the superposition of the incident and reflected waves will be

$$y = y_1 + y_2 = 2A\cos kx \sin\omega t$$

In this case, the points $x = 0, \lambda/2, 3\lambda/2, \ldots$ will be antinodes and the points $x = \lambda/4, 3\lambda/4, 5\lambda/4$, will be nodes.

If the wave is reflected from a *rigid end*, then

$$y_2 = -A\sin(\omega t + kx)$$

The equation of the stationary wave will be

$$y = -2A\sin kx\cos\omega t$$

In this case, the points $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$, will be nodes and the points $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$ will be antinodes.

Separation between two successive nodes or antinodes $= \lambda/2$

Separation between a node and nearest antinode $= \lambda/4$

23. **Modes of vibrations of strings.** On a stretched string, transverse stationary waves are formed due to superposition of direct and the reflected transverse waves.

For *fundamental mode* :

$$\lambda_1 = 2L, \quad \nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu \text{ (say)}$$

For *second mode* :

$$\lambda_2 = L, \quad \nu_2 = 2\nu$$

(*second harmonic* or *first overtone*)

For *third mode* :

$$\lambda_3 = 2L/3, \quad \nu_3 = 3\nu$$

(*third harmonic or second overtone*)

For *pth mode* : When the string vibrates in p loops,

$$\nu_p = \frac{p}{2L}\sqrt{\frac{T}{m}} = p\nu$$

[*pth harmonic* or $(p-1)$*th overtone*]

Fundamental frequency for a string of diameter D and density ρ,

$$\nu = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}.$$

24. **Laws of transverse vibrations of a stretched string.** The fundamental frequency of a vibrating is

(*i*) inversely proportional to its length.

$$\nu \propto \frac{1}{L} \qquad \text{(Law of length)}$$

(*ii*) directly proportional to the square root of its tension.

$$\nu \propto \sqrt{T} \qquad \text{(Law of tension)}$$

(*iii*) inversely proportional to its mass per unit length.

$$\nu \propto \frac{1}{\sqrt{m}} \qquad \text{(Law of mass)}$$

Combining all the factors, we get

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

where ρ is the density and D the diameter of the string.

25. **Organ pipe.** It is the simplest musical instrument in which sound is produced by setting an air column into vibrations. Longitudinal stationary waves are formed on account of superposition of incident and reflected longitudinal waves.

Scanned by CamScanner

26. **Modes of vibrations of closed organ pipes.** Longitudinal stationary waves are formed in an organ pipe closed at one end.

For *fundamental mode*

$$\lambda_1 = 4L, \quad \nu_1 = \frac{v}{\lambda_1} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

For *second mode* : $\lambda_2 = 4L/3, \quad \nu_2 = 3\nu$
(*third harmonic* or *first overtone*)

For *third mode* : $\lambda_3 = 4L/5, \ \nu_3 = 5\nu$
(*fifth harmonic* or *second overtone*)

For *pth mode* :
$\lambda_p = 4L/(2p-1), \quad \lambda_p = (2p-1)\nu$
[$(2p-1)$ *harmonic or* $(p-1)$*th overtone*]

Here $\nu_1 : \nu_2 : \nu_3 : \nu_4 \ldots = 1:3:5:7:\ldots$
(*only odd harmonics*)

27. **Modes of vibrations of open organ pipe.** Anti-nodes are formed at both ends, separated by one node in the fundamental mode.

For *fundamental mode.*

$$\lambda_1' = 2L, \ \nu_1' = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \nu \text{ (say)}$$

For *second mode* : $\lambda_2' = L, \quad \nu_2' = 2\nu$
(*second harmonic or first overtone*)

For *third mode* : $\lambda_3 = 2L/3, \quad \nu_3 = 3\nu$
(*third harmonic or second overtone*)

For *pth mode* $\lambda_p' = 2L/p, \quad \nu_p' = p\nu$
[*pth harmonic or* $(p-1)$ *overtone*]

Here $\nu_1 : \nu_2 : \nu_3 : \nu_4 \ldots = 1:2:3:4:\ldots$
(*Both odd and even harmonics*)

28. **Resonance tube.** It is an organ pipe closed at one end. If L_1 and L_2 be the first and second resonance lengths with a tuning fork of frequency ν, then the speed of sound in air is given by

$v = 4\nu(L_1 + 0.3\,D)$,
D = internal diameter of resonance tube

or $v = 2\nu(L_2 - L_1)$

End correction $= 0.3\,D = \dfrac{L_2 - 3L_1}{2}$.

29. **Vibrations in rods clamped in the middle.** Here antinodes are formed at the ends and a node in the middle.

Fundamental frequency, $\nu_1 = \dfrac{v}{2L} = \nu$

First overtone or third harmonic, $\nu_2 = \dfrac{3v}{2L} = 3\nu$

Second overtone or fifth harmonic, $\nu_3 = \dfrac{5v}{2L} = 5\nu$

30. **Beats.** The periodic variations in the intensity of sound due to the superposition of two sound waves of slightly different frequencies are called beats. One rise and one fall of intensity constitute one beat. The number of beats produced per second is called beat frequency.

Beat frequency, $\nu_{beat} = \nu_1 - \nu_2$

Beats may be used to determine the frequency of a tuning fork. It must be noted here that

(*i*) When the prong of a tuning fork is slightly loaded with wax, its frequency of vibration decreases.

(*ii*) When the prong of a tuning fork is filed slightly, its frequency of vibration increases.

31. **Doppler effect in sound.** The phenomenon of the change in apparent pitch of sound due to relative motion between the source of sound and the observer is called Dopper effect. If v, v_0, v_s and v_m are the velocities of sound, observer, source and medium (in the direction of sound) respectively, then the apparent frequency is given by

$$\nu' = \frac{v + v_m - v_0}{v + v_m - v_s} \times \nu$$

For the medium at rest ($v_m = 0$),

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu$$

Here the velocities are taken *positive* in the *source to observer* ($S \to O$) direction and *negative* in the opposite ($O \to S$) direction.

Special cases :

(*i*) When the source moves towards the stationary observer,

$$\nu' = \frac{v}{v - v_s} \times \nu \qquad (\nu' > \nu)$$

(*ii*) When the source moves away from the stationary observer,

$$\nu' = \frac{v}{v + v_s} \times \nu \qquad (\nu' < \nu)$$

(*iii*) When the observer moves towards the stationary source,

$$\nu' = \frac{v + v_0}{v} \times \nu \qquad (\nu' > \nu)$$

(*iv*) When the observer moves away from the stationary source,

$$\nu' = \frac{v - v_0}{v} \times \nu \qquad (\nu' < \nu)$$

(*v*) When both source and observer move towards each other,

$$\nu' = \frac{v + v_0}{v - v_s} \times \nu \qquad (\nu' > \nu)$$

Scanned by CamScanner

(vi) When both source and observer move away from each other,

$$v' = \frac{v - v_0}{v + v_s} \times \nu \qquad (\nu' < \nu)$$

(vii) When source moves towards observer and observer away from the source,

$$\nu' = \frac{v - v_0}{v - v_s} \times \nu$$

(viii) When source moves away from observer and observer towards the source,

$$\nu' = \frac{v + v_0}{v + v_s} \times \nu$$

32. **Musical sound.** A musical sound consists of quick, regular and periodic succession of compressions and rarefactions without a sudden change in amplitude. It produces pleasing effect on the ears.

33. **Noise.** A noise consists of slow, irregular and non periodic succession of compressions and rarefractions, that may have a sudden change in amplitude. It produces non-pleasing effect on the ears.

34. **Intensity of sound (I).** The intensity of sound at any point may be defined as the amount of sound energy passing per unit time per unit area around that point in a perpendicular direction.

35. **Zero level or threshold of hearing (I_0).** The lowest intensity of sound that can be perceived by the human ear is called threshold of hearing.

For a sound of frequency 1 kHz, it is found that the threshold of hearing is 10^{-12} Wm^{-2}.

36. **Characteristics of a musical sound.** These are

(i) ***Pitch.*** It is the characteristic of musical sound that helps the listener in distinguishing a shrill note from a grave (flat or dull) one. It depends on frequency.

(ii) ***Quality.*** It is the characteristic of the musical sound that distinguishes between two sounds of same pitch and loudness from one another. It depends on the number or intensity of overtones.

(iii) ***Loudness.*** The sensation of hearing which enables us to distinguish between a loud and a faint sound, is called loudness. It depends on intensity.

37. **Units of loudness.** The unit of loudness of a sound is ***bel***. The loudness of a sound is said to be 1 bel, if its intensity is 10 times that of the threshold of hearing.

The loudness of a sound of intensity I is given by

$$L = \log_{10} \frac{I}{I_0},$$

where I_0 is threshold of hearing. It is called *Weber-Fechner law*.

The practical and a smaller unit of loudness is ***decibel*** (dB).

$$1 \text{ decibel} = \frac{1}{10} \text{ bel}$$

In decibels, the loudness of a sound of intensity I is given by

$$L = 10 \log_{10} \frac{I}{I_0}$$

38. **Reverberation of sound.** It is the phenomenon of persistence of sound after the source has stopped producing sound. The time for which sound persists after the source has stopped producing sound is called *reverberation time* (T).

According to *Sabine* law, reverberation time of a hall is given by

$$T = \frac{0.16\, V}{\Sigma\, a_i\, s_i}$$

where V is volume of the hall and $\Sigma\, a_i\, s_i = a_1\, s_1 + a_2 s_2 + \ldots$ is the total absorption of the hall. Here $s_1, s_2 \ldots$ are the surface areas of materials with absorption coefficients $a_1, a_2, \ldots$ respectively.

39. **Interval between two notes.** The ratio of the frequencies of two notes is called interval between them. Two notes are said to be in *unison* if their frequencies are equal, that is interval between them is 1 : 1. Some other common intervals found useful in producing musical sound are as follows :

(a) *octave* (1 : 2) (b) *majortone* (8 : 9)
(c) *minortone* (9 : 10) (d) *semitone* (15 : 16)

The interval between any two notes is obtained by multiplying the various intervening intervals.

40. **Musical scale.** A series of notes arranged such that their fundamental frequencies have definite ratios is called a musical scale.

41. **Major diatonic scale.** The eight notes of major diatonic scale starting with 256 as keynote along with their Helmholtz and Indian notation are as below :

Helmholtz notation :	C	D	E	F	G	A	B	C
Indian notation :	Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
Frequency of notes with 256 as keynote :	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512
Intervals :	9/8	10/9	16/15	9/8	10/9	9/8	16/15	

Scanned by CamScanner

IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

1. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer, who is standing in air, is

(*a*) 200 Hz (*b*) 3000 Hz
(*c*) 120 Hz (*d*) 600 Hz. [IIT 04]

2. The ratio of the speed of sound in nitrogen gas to that in the helium gas at 300 K is

(*a*) $\sqrt{2/7}$ (*b*) $\sqrt{1/7}$
(*c*) $\sqrt{3}/5$ (*d*) $\sqrt{6}/5$ [IIT 99]

3. Two monatomic ideal gases 1 and 2 of molecular masses M_1 and M_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

(*a*) $\sqrt{\frac{M_1}{M_2}}$ (*b*) $\sqrt{\frac{M_2}{M_1}}$
(*c*) $\frac{M_1}{M_2}$ (*d*) $\frac{M_2}{M_1}$. [IIT 2K]

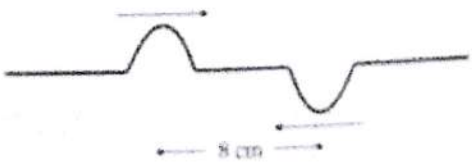
4. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be

(*a*) $1.22v$ (*b*) $0.61v$
(*c*) $1.50v$ (*d*) $0.75v$. [IIT 80]

5. A travelling wave in a stretched string is described by the equation $y = A\sin(kx - \omega t)$. The maximum particle velocity is

(*a*) $A\omega$ (*b*) ω / k
(*c*) $d\omega / dk$ (*d*) x / t. [IIT 97]

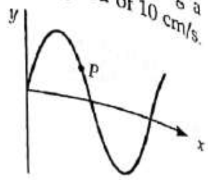
6. Two pulses in a stretched string whose centres are intially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is



2 cm/s. After 2 seconds, the total energy of the pulses will be

(*a*) zero (*b*) purely kinetic
(*c*) purely potential
(*d*) purely kinetic and partly potential. [IIT 2K]

7. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is



(*a*) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (*b*) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s
(*c*) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (*d*) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s [IIT 08]

8. A wave represented by the equation

$$y = a\cos(kx - \omega t)$$

is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is

(*a*) $a\cos(kx - \omega t)$ (*b*) $-a\cos(kx - \omega t)$
(*c*) $-a\cos(kx + \omega t)$ (*d*) $-a\sin(kx - \omega t)$ [IIT 88]

9. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, one of length L with frequency f_1 and the other with frequency f_2. The ratio f_1 / f_2 is given by

(*a*) 2 (*b*) 4
(*c*) 8 (*d*) 1 [IIT 2K]

10. A sonometer wire resonates with a given tuning form forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning form forming three antinodes for the same positions of the bridges. The value of M is

(*a*) 25 kg (*b*) 5 kg
(*c*) 12.5 kg (*d*) (1/25) kg [IIT 02]

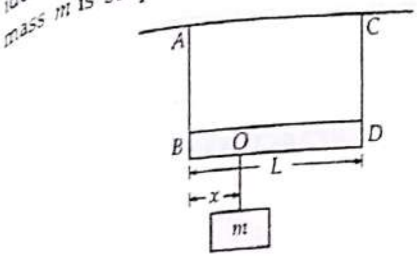
11. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its

volume is submerged. The new fundamental frequency in Hz is

(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$ [IIT 95]

12. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is



equal to x. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. x is

(a) $\frac{L}{5}$ (b) $\frac{4L}{5}$

(c) $\frac{3L}{5}$ (d) $\frac{L}{4}$. [IIT 06]

13. In the experiment to determine the speed of sound using a resonance column,

(a) prongs of the tuning fork are kept in a vertical plane

(b) prongs of the tuning fork are kept in a horizontal plane

(c) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air

(d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air. [IIT 07]

14. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option

(a) $n = 3, f_2 = \frac{3}{4}f_1$ (b) $n = 3, f_2 = \frac{5}{4}f_1$

(c) $n = 5, f_2 = \frac{3}{4}f_1$ (d) $n = 5, f_2 = \frac{5}{4}f_1$. [IIT 05]

15. In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is

(a) 51.2 cm/s (b) 102.4 m/s

(c) 204.8 cm/s (d) 153.6 cm/s. [IIT 96]

16. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is

(a) 200 Hz (b) 300 Hz

(c) 240 Hz (d) 480 Hz. [IIT 96]

17. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.

(a) 0.012 m (b) 0.025 m

(c) 0.05 m (d) 0.024 m. [IIT 03]

18. A pipe of length l_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal.

(a) $\frac{4}{3}l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (b) $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$

(c) $l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (d) $l_1\sqrt{\frac{\rho_1}{\rho_2}}$ [IIT 04]

19. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A\sin(\pi x/L)\sin\omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 2\omega t$ and energy is E_2. Then

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$

(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$ [IIT 01]

20. Two plane harmonic sound waves are expressed by the equations :

$$y_1(x,t) = A\cos(0.5\pi x - 100\pi t)$$

and $$y_2(x,t) = A\cos(0.46\pi x - 92\pi t)$$

All parameters are in mks system. How many times does an observer hear maximum intensity in one second ?

(a) 4 (b) 6

(c) 8 (d) 10 [IIT 06]

Scanned by CamScanner

21. A vibrating string of length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to 340 m/s, the frequency n of the tuning fork in Hz is

(*a*) 344 (*b*) 336

(*c*) 117.3 (*d*) 109.3 [IIT 08]

22. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is (speed of sound = 330 m/s)

(*a*) 409 (*b*) 429

(*c*) 517 (*d*) 500 [IIT 97]

23. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1/f_2 is

(*a*) $\frac{18}{19}$ (*b*) $\frac{1}{2}$

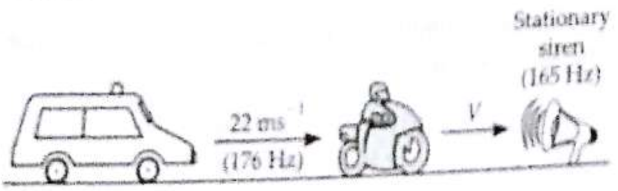
(*c*) 2 (*d*) $\frac{19}{18}$ [IIT 2K]

24. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train *A* records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train *B* he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train *B* to that train *A* is

(*a*) 242/252 (*b*) 2

(*c*) 5/6 (*d*) 11/6 [IIT 2K]

25. A police car moving at 22 ms^{-1}, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observe any beats.



(*a*) 33 ms^{-1} (*b*) 22 ms^{-1}

(*c*) 11 ms^{-1} (*d*) zero. [IIT 03]

26. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is

(*a*) 5 grams (*b*) 10 grams

(*c*) 20 grams (*d*) 40 grams [IIT 2010]

27. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(*a*) 8.50 kHz (*b*) 8.25 kHz

(*c*) 7.75 kHz (*d*) 7.50 kHz [IIT 2011]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

28. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4} \sin(60t + 2x)$, where x and y are in metre and t is time in second. This represents a wave

(*a*) travelling with a velocity of 30 m/s in the negative x-direction

(*b*) of wavelength π m

(*c*) of frequency $(30/\pi)$ hertz

(*d*) of amplitude 10^{-4} m travelling along the negative x-direction. [IIT 82]

29. A transverse wave is described by the equation

$$y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right).$$

The maximum particles velocity is equal to four times the wave velocity if

(*a*) $\lambda = \pi \frac{y_0}{4}$ (*b*) $\lambda = \pi \frac{y_0}{2}$

(*c*) $\lambda = \pi y_0$ (*d*) $\lambda = 2\pi y_0$. [IIT 84]

30. The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion are

(*a*) $\cos kx \sin \omega t$ (*b*) $k^2x^2 - \omega^2t^2$

(*c*) $\cos^2(kx + \omega t)$ (*d*) $\cos(k^2x^2 - \omega^2t^2)$ [IIT 87]

Scanned by CamScanner

31. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in metre and t is in second. The expression represents

(*a*) a wave travelling in the positive x-direction with a velocity 1.5 m/s

(*b*) a wave traelling in the negative x-direction with a velocity 1.5 m/s

(*c*) a wave travelling in the negative x-direction having a wavelength 0.2 m

(*d*) a wave travelling in the positive x-direction having a wavelength 0.2 m. [IIT 99]

32. In a wave motion $y = a\sin(kx - \omega t)$, y can represent

(*a*) electric field (*b*) magnetic field

(*c*) displacement (*d*) pressure. [IIT 99]

33. $y(x,t) = 0.8/[(4x+5t)^2+5]$ represents a moving pulse, where x and y are in metre and t in second. Then

(*a*) pulse is moving in $+x$ direction

(*b*) in 2 s it will travel a distance of 2.5 m

(*c*) its maximum displacement is 0.16 m

(*d*) it is a symmetric pulse. [IIT 99]

34. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10\,\text{ms}^{-1}$, then λ and f are given by

(*a*) $\lambda = 2\pi \times 10^{-2}$ m (*b*) $\lambda = 10^{-3}$ m

(*c*) $f = 10^3/2\pi$ Hz (*d*) $f = 10^4$ Hz. [IIT 98]

35. As a wave propagates,

(*a*) the wave intensity remains constant for a plane wave

(*b*) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

(*c*) the wave intensity decreases as the the inverse square of the distance from the source for a spherical wave

(*d*) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all time. [IIT 99]

36. Standing waves can be produced

(*a*) on a string clamped at both the ends

(*b*) on a string clamped at one end free at the other

(*c*) when incident wave gets reflected from a wall

(*d*) when two identical waves with a phase difference of π are moving in the same direction. [IIT 99]

37. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is

(*a*) 31.25 (*b*) 62.50

(*c*) 93.75 (*d*) 125. [IIT 85]

38. A tube closed at one end and containing air produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends, the fundamental frequency that can be excited is (in Hz)

(*a*) 1024 (*b*) 512

(*c*) 256 (*d*) 128. [IIT 80]

39. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then

(*a*) the intensity of the sound heard at the first resonance was more than that at the second resonance

(*b*) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(*c*) the amplitude of the vibration of the ends of the prongs is typically around 1 cm

(*d*) the length of air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air. [IIT 09]

40. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is

(*a*) 8/3 (*b*) 3/8

(*c*) 1/6 (*d*) 1/3. [IIT 88]

41. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency

(*a*) 80 Hz (*b*) 240 Hz
(*c*) 320 Hz (*d*) 400 Hz. [IIT 99]

42. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is

(*a*) 0.05 s (*b*) 0.10 s
(*c*) 0.20 s (*d*) 0.40 s. [IIT 98]

43. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tensions in the strings, then it could be said that while making the above changes in tension

(*a*) T_2 was decreased
(*b*) T_2 was increased
(*c*) T_1 was decreased
(*d*) T_1 was increased. [IIT 91]

44. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$

This expression may be considered to be a result of the superposition of

(*a*) two (*b*) three
(*c*) four (*d*) five.

independent harmonic motions. [IIT 92]

45. A wave disturbance in a medium is described by

$$y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x),$$

where x and y are in metre and t is in second.

(*a*) A node occurs at $x = 0.15$ m
(*b*) An antinode occurs at $x = 0.3$ m
(*c*) The speed of wave is 5 ms^{-1}
(*d*) The wavelength is 0.2 m. [IIT 95]

46. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c.

(*a*) The number of waves striking the surface per second is $f\frac{(c+v)}{c}$

(*b*) The wavelength of reflected wave is $\frac{c(c-v)}{f(c+v)}$

(*c*) The frequency of the reflected wave is $f\frac{(c+v)}{(c-v)}$

(*d*) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vf}{c-v}$. [IIT 95]

47. The (x, y) co-ordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x,y)$ denotes the displacement of the plate at the point (x,y) at some instant of time, the possible expressions for u is (are) (a = positive constant)

(*a*) $a\cos(\pi x/2L)\cos(\pi y/2L)$
(*b*) $a\sin(\pi x/L)\sin(\pi y/L)$
(*c*) $a\sin(\pi x/L)\sin(2\pi y/L)$
(*d*) $a\cos(2\pi x/L)\sin(\pi y/L)$. [IIT 98]

INTEGER ANSWER TYPE

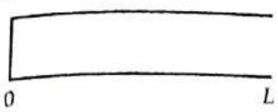
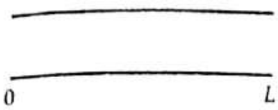
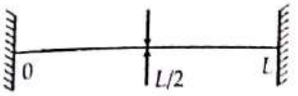
48. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, what is the amplitude of the resultant wave ? [IIT 2010]

49. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. [IIT 2010]

MATCH-MATRIX TYPE

50. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves. [IIT 2011]

Scanned by CamScanner

Column I		Column II	
(a)	Pipe closed at one end 	(p)	Longitudinal waves
(b)	Pipe open at both ends 	(q)	Transverse waves
(c)	Stretched wire clamped at both ends 	(r)	$\lambda_f = L$
(d)	Stretched wire clamped at both ends and at mid-point 	(s)	$\lambda_f = 2L$
		(t)	$\lambda_f = 4L$

Answers and Explanations

1. (d) The frequency of sound does not change during its refraction from water into air.

2. (c) $\frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \cdot \frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3} \cdot \frac{4}{28}} = \frac{\sqrt{3}}{5}$.

3. (b) As $v = \sqrt{\frac{3RT}{M}}$ i.e., $v \propto \frac{1}{\sqrt{M}}$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}.$$

4. (c) According to Hooke's law,

Tension $(T) \propto$ Extension (x)

Also, speed of sound,

$$v \propto \sqrt{T} \qquad \therefore \qquad v \propto \sqrt{x}$$

$$v' \propto \sqrt{1.5x}$$

or $$\frac{v'}{v} = \sqrt{1.5} = 1.22$$

or $$v' = 1.22\,v.$$

5. (a) $$y = A\sin(kx - \omega t)$$

$$u = \frac{dy}{dx} = -\omega A\cos(kx - \omega t)$$

$$\therefore \quad u_{max} = \omega A$$

6. (b) After 2 s, crest moves 4 cm towards right and trough moves 4 cm towards left. They superpose and cancel the displacement of each other. The string becomes straight. The energy becomes totally kinetic.

7. (a) Here $v = 10\ ms^{-1}$, $\lambda = 0.5\ m = 50\ cm$, $A = 10\ cm$, $y = 5\ cm$.

Particle velocity,

$$u = \omega\sqrt{A^2 - y^2} = \frac{2\pi v}{\lambda}\sqrt{A^2 - y^2}$$

$$= \frac{2\pi \times 10}{50}\sqrt{10^2 - 5^2}\ cms^{-1}$$

$$= 2\sqrt{3}\,\pi\ cms^{-1}$$

$$= \frac{\sqrt{3}\pi}{50}\ ms^{-1}$$ **(in +ve y-direction).**

8. (c) For the formation of a stationary wave, two identical waves travelling in opposite directions must superpose each other. At $x = 0$, resultant y should be zero for getting a node. Hence option (c) is correct.

$$y = y_1 + y_2 = a\cos(kx - \omega t) - a\cos(kx + \omega t)$$

$$= 2a\sin kx \sin \omega t$$

At $x = 0$, $y = 0$ i.e., a node is formed.

Scanned by CamScanner

9. (*d*) $f = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{T}{\pi R^2 \rho}} = \frac{1}{2lR}\sqrt{\frac{T}{\pi\rho}}$

As T and ρ are same for both strings,

$\therefore \quad f \propto \frac{1}{lR}$

For first string, $\quad f_1 \propto \frac{1}{L \times 2r}$

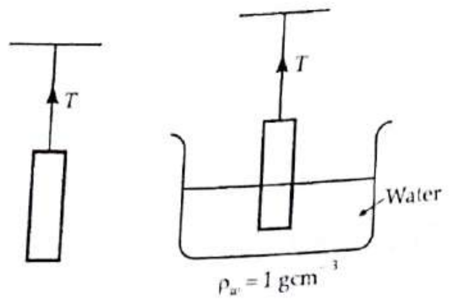
For second string, $\quad f_2 \propto \frac{1}{2L \times r}$

Hence $\quad f_1 = f_2 \quad$ or $\quad \frac{f_1}{f_2} = \mathbf{1}.$

10. (*a*) $\quad M = 25$ kg.

Refer to the solution of Problem 10 on page 15.69.

11. (*a*) The situation is shown in the figure.



In air, $\quad T = Mg = V\rho g$

$$\therefore \quad \nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{V\rho g}{m}}$$

When the object is half immersed in water

$$T' = Mg - \text{upthrust}$$

$$= V\rho g - \left(\frac{V}{2}\right)\rho_\omega g$$

$$= \left(\frac{V}{2}\right)g(2\rho - \rho_\omega)$$

New fundamental frequency,

$$\nu' = \frac{1}{2l}\sqrt{\frac{T'}{m}} = \frac{1}{2l}\sqrt{\frac{(V/2)g(2\rho - \rho_\omega)}{m}}$$

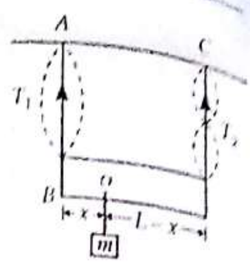
$$\therefore \quad \frac{\nu'}{\nu} = \sqrt{\frac{2\rho - \rho_\omega}{2\rho}}$$

or $$\nu' = \nu\left(\frac{2\rho - \rho_\omega}{2\rho}\right)^{1/2}$$

$$= 300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{ Hz}$$

12. (*a*) Frequency of first harmonic in AB

= Frequency of second harmonic in CD



$$\therefore \quad \frac{1}{2l}\sqrt{\frac{T_1}{m}} = \frac{1}{l}\sqrt{\frac{T_2}{m}}$$

or $$T_1 = 4T_2 \quad ...(i)$$

For translational equilibrium,

$$T_1 + T_2 = mg \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$T_1 = \frac{4mg}{5} \text{ and } T_2 = \frac{mg}{5}$$

For rotational equilibrium about O,

Torque due to T_1 about O

= Torque due to T_2 about O

$$T_1 x = T_2(L - x)$$

or $$\frac{4mg}{5}x = \frac{mg}{5}(L - x)$$

or $$4x = L - x \quad \text{or} \quad x = \frac{\mathbf{L}}{\mathbf{5}}.$$

13. (*a*) The prongs of vibrating tuning fork are kept in a vertical plane just above opening of the resonance tube.

14. (*d*) In case of open pipe, frequency of second harmonic is

$$f_1 = \frac{v}{\lambda} = \frac{v}{L} \qquad [\lambda = L]$$

In case of closed pipe, frequency of nth harmonic,

$$f_2 = \frac{nv}{4L}$$

where n is odd number.

Clearly, $\quad f_2 = \frac{n}{4}f_1$

If $n = 3$, $\quad f_2 < f_1$, $\quad$ which is not acceptable.

If $n = 5$, $\quad f_2 = \frac{5}{4}f_1$, $\quad$ which is acceptable.

Hence option (*d*) is correct.

15. (*c*) Speed of sound,

$$v = 2\nu(l_2 - l_1)$$

Maximum possible error in speed,

$$\Delta v = 2\nu(\Delta l_2 + \Delta l_1)$$

$$= 2 \times 512\,(0.1 + 0.1) = \mathbf{204.8\ cm\ s^{-1}}.$$

16. (*a*) $\quad \nu = 200$ Hz.

Refer to the solution of Problem 18 on page 15.70.

17. (*b*) End correction $= \frac{l_2 - 3l_1}{2}$

$$= \frac{0.35 - 3 \times 0.1}{2} = \mathbf{0.025\ m}.$$

Scanned by CamScanner

18. (*b*) f_c (first overtone) = f_0 (first over tone)

$$3\left(\frac{v_c}{4l_1}\right) = 2\left(\frac{v_0}{2l_0}\right)$$

$$l_0 = \frac{4}{3}\left(\frac{v_0}{v_c}\right)l_1$$

As

$$v \propto \frac{1}{\sqrt{\rho}} \quad \text{or} \quad \frac{v_0}{v_c} = \sqrt{\frac{\rho_1}{\rho_2}}$$

$$\therefore \quad l_2 = \frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}.$$

19. (*c*) $E_2 = 4E_1$, refer to the solution of Problem 9 on page 15.69.

20. (*a*) For first wave,

$$\omega_1 = 2\pi\nu_1 = 100\pi$$

$$\therefore \quad \nu_1 = 50 \text{ Hz}$$

For second wave,

$$\omega_2 = 2\pi\nu_2 = 92\pi$$

$$\therefore \quad \nu_2 = 46 \text{ Hz}$$

Beat frequency $= \nu_1 - \nu_2 = 4$ Hz.

Hence the intensity of sound becomes maximum 4 times in one second.

21. (*a*) If ν is the frequency of the string, then

$$n - 4 = \nu$$

Third harmonic of closed pipe,

$$\nu = \frac{3v}{4L}$$

where L = Length of the tube, and

v = velocity of sound in air

$$\therefore \quad n = \nu + 4 = \frac{3v}{4L} + 4 = \frac{3 \times 340}{4 \times 0.75} + 4$$

$$= \mathbf{344\ Hz}$$

22. (*d*) $\nu' = \dfrac{v}{v - v_s} \times v = \dfrac{330}{330 - 33} \times 450$

$$= \mathbf{500\ Hz}$$

23. (*d*) $\dfrac{f_1}{f_2} = \dfrac{19}{18}$

Refer to the solution of Problem 12 on page 15.69.

24. (*b*) $\dfrac{v_B}{v_A} = 2$

Refer to the solution of Problem 13 on page 15.69.

25. (*b*) $v = 22 \text{ ms}^{-1}$

Refer to the solution of Problem 14 on page 15.70.

26. (*b*) For hollow pipe, fundamental frequency is

$$f = \frac{v}{4l} = \frac{320}{4 \times 0.8}$$

For string in 2nd harmonic,

$$f = \frac{1}{l}\sqrt{\frac{T}{\mu}} = \frac{1}{l}\sqrt{\frac{Tl}{m}} = \frac{1}{0.5}\sqrt{\frac{50 \times 0.5}{m}}$$

Equating and solving, we get

$$m = 0.01 \text{ kg} = \mathbf{10\ g}$$

27. (*a*) Frequency received by the building

$$\nu' = \left(\frac{v}{v - v_c}\right)\nu$$

The wall (source) reflects this frequency, so frequency heard by the car driver is

$$\nu'' = \left(\frac{v + v_c}{v}\right)\nu' = \left(\frac{v + v_c}{v}\right)\left(\frac{v}{v - v_c}\right)\nu$$

$$= \left(\frac{v + v_c}{v - v_c}\right)\nu$$

$$= \left(\frac{320 + 10}{320 - 10}\right) \times 8 \text{ kHz}$$

$$[v_c = 36 \text{ kmh}^{-1} = 10 \text{ ms}^{-1}]$$

$$= \frac{33}{31} \times 8 = \mathbf{8.5\ kHz}$$

28. (*a*), (*b*), (*c*), (*d*)

$$y = 10^{-4}\sin(60t + 2x)$$

Standard equation of a wave travelling along –ve *x*-direction is

$$y = a\sin(\omega t + kx)$$

$$\therefore \quad a = 10^{-4} \text{ m}, \quad \omega = 60 \text{ rad s}^{-1}$$

$$\nu = \frac{\omega}{2\pi} = \frac{60}{2\pi} = \frac{30}{\pi} \text{ Hz}$$

$$k = \frac{2\pi}{\lambda} = 2 \text{ rad m}^{-1}$$

$$v = \frac{\omega}{k} = \frac{60}{2} = 30 \text{ ms}^{-1}$$

Clearly, all the given options are correct.

29. (*b*) $y = y_0 \sin 2\pi\left(ft - \dfrac{x}{\lambda}\right)$

$$u = \frac{dy}{dt} = 2\pi f y_0 \cos 2\pi\left(ft - \frac{x}{\lambda}\right)$$

$$u_{max} = 2\pi f y_0$$

Scanned by CamScanner

Wave velocity,

$$v = f\lambda$$

$$\therefore \quad 2\pi f y_0 = 4 \times f\lambda$$

or $$\lambda = \frac{\pi y_0}{2}.$$

30. (*a*) $y = \cos kx \sin \omega t$ is the only equation of standing wave. The conditions for a function of x and t to represent a wave is

$$\frac{\partial^2 y}{\partial x^2} = \text{constant} \times \frac{\partial^2 y}{\partial t^2}$$

Only first expression satisfies this condition.

31. (*b*), (*c*)

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

$$y = A\sin(kx + \omega t + \phi)$$

This is the equation of wave travelling along –ve x-direction. Hence options (*a*) and (*d*) are incorrect.

$$k = \frac{2\pi}{\lambda} = 10\,\pi, \quad \omega = 15\,\pi$$

$$\lambda = \frac{1}{5} = 0.2 \text{ m}$$

$$v = \frac{\omega}{k} = \frac{15\pi}{10\pi} = 1.5 \text{ ms}^{-1}$$

Hence options (*b*) and (*c*) are correct.

32. (*a*), (*b*), (*c*), (*d*) $y = a\sin(kx - \omega t)$

(*a*) y represents electric field in an electromagnetic wave.

(*b*) y represents magnetic field in an electromagnetic wave.

(*c*) y represents displacement in a sound wave.

(*d*) y represents pressure in a sound wave.

33. (*b*), (*c*), (*d*) A wave produced by a sudden disturbance of short duration is called a pulse. Its shape does not change during propagation. It can be expressed as

$$y = \frac{a}{b + (x \mp vt)^2}$$

–ve sign for propagation along +ve x-direction and +ve sign for propagation along –ve x-direction.

Given : $$y(x,t) = \frac{0.8}{(4x + 5t)^2 + 5}$$

$$= \frac{0.8}{5 + 16[x + (5/4)t]^2}$$

We can note that,

(*a*) The pulse is moving along –ve x-direction. Hence option (*a*) is incorrect.

(*b*) $vt = (5/4)t$ or $v = 1.25 \text{ ms}^{-1}$
Distance travelled in 2 s $= 1.25 \times 2 = $ **2.5 m.**
Hence option (*b*) is correct.

(*c*) Put $(4x + 5t)^2 = 0$, then

$$y_{max} = \frac{0.8}{5} = \mathbf{0.16 \text{ m.}}$$

Hence option (*c*) is correct.

(*d*) At $t = 0$,

$$y(x) = \frac{0.8}{16x^2 + 5}$$

Also, $$y(-x) = \frac{0.8}{16x^2 + 5}$$

i.e., $$y(x) = y(-x) \text{ at } t = 0.$$

Thus the given pulse is symmetric. Hence option (*d*) is correct.

34. (*a*), (*c*) $\lambda = 2\pi \times 10^{-2}$ m, $f = 10^3/2\pi$ Hz.

Refer to the solution of Problem 17 on page 15.70.

35. (*a*), (*c*), (*d*) For a spherical wave, $I \propto 1/r^2$. With the increase in distance from the source, though intensity decreases, the total energy transmitted remains the same.

36. (*a*), (*b*), (*c*) Standing waves can be formed by superposition of the identical waves (same frequency and wavelength) travelling in opposite directions. Hence options (*a*), (*b*) and (*c*) are correct. Option (*d*) is not correct as the waves are not travelling in opposite directions.

37. (*a*), (*c*) Fundamental frequency of a closed pipe,

$$\nu = \frac{v}{4L}$$

$$\therefore \quad L = \frac{v}{4\nu} = \frac{330}{4 \times 264} = 31.25 \text{ cm}$$

For first overtone,

Resonance length $= 3L = 93.75$ cm

For second overtone,

Resonance length $= 5L = 156.25$ cm

Options (*a*) and (*c*) are correct

38. (*a*) Fundamental frequency of a closed pipe,

$$\nu = \frac{v}{4L} = 512 \text{ Hz.}$$

Fundamental frequency of open pipe,

$$\nu' = \frac{v}{4L} = 2\nu = 2 \times 512 = \mathbf{1024 \text{ Hz.}}$$

Scanned by CamScanner

39. (a), (d) In case of second resonance, the same energy is shared by a larger number of particles as the air column is longer. Hence energy per particle is less resulting in lower intensity. Option (a) is correct.

In first resonance,

$$l + e = \frac{\lambda}{4}, \quad e = \text{end correction}$$

Due to end correction, l is slightly less than $\lambda/4$. Option (d) is correct.

The prongs of the tuning fork are kept in a vertical plane above the resonance tube.

40. (d) First harmonic of closed pipe

= Third harmonic of open pipe

$$\frac{v}{4L_1} = 3 \times \frac{v}{2L_2}$$

$$\therefore \quad \frac{L_1}{L_2} = \frac{1}{6}.$$

41. (a), (b), (d) For a closed pipe,

$$\nu = n\frac{v}{4L}, \quad n = 1, 3, 5, \ldots$$

$$\nu_1 = \frac{v}{4L} = \frac{320}{4 \times 1} = \mathbf{80\ Hz.}$$

$$\nu_3 = 3\nu_1 = \mathbf{240\ Hz}$$

$$\nu_5 = 5\nu_1 = \mathbf{400\ Hz.}$$

42. (b) Mass per unit length of the string,

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2}\ \text{kg m}^{-1}$$

Velocity of the wave in the string,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}} = 8\ \text{m/s}$$

For constructive interference between successive pulses, the minimum time interval is

$$\Delta t_{min} = \frac{2L}{v} = \frac{2 \times 0.4}{8} = \mathbf{0.10\ s.}$$

43. (b), (c) $\nu = \dfrac{1}{2L}\sqrt{\dfrac{T}{m}}$

As $T_1 > T_2$, so $\nu_1 > \nu_2$

Initially, $\nu_1 - \nu_2 = 6$ Hz.

When T_1 decreases, ν_1 decreases and it may be possible that $\nu_2 - \nu_1$ becomes equal to 6 Hz. Hence option (c) is correct and option (d) is incorrect.

When T_2 increases, ν_2 increases and it may be possible that $\nu_2 - \nu_1$ becomes equal to 6 Hz. Hence option (b) is correct and option (a) is incorrect.

44. (b) $\quad y = 4\cos^2\left(\dfrac{t}{2}\right)\sin(1000t)$

$$\cos t = 2\cos^2\left(\frac{t}{2}\right) - 1$$

$$\Rightarrow \quad \cos^2\left(\frac{t}{2}\right) = \frac{1 + \cos t}{2}$$

$$\therefore \quad y = 2(1 + \cos t)\sin(1000t)$$

$$= 2\sin(1000t) + 2\cos t \sin(1000t)$$

$$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$$

Clearly, y is the resultant of the superposition of three harmonic functions of angular frequencies 999, 1000 and 1001 rad/s.

45. (a), (b), (c), (d)

$$y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$$

Clearly, $\quad a = 0.02$ m

$$\omega = 50\,\pi\ \text{rad s}^{-1}, \quad k = 10\,\pi\ \text{rad m}^{-1}$$

$$v = \frac{\omega}{k} = \frac{50\,\pi}{10\,\pi} = 5\ \text{ms}^{-1}$$

$\therefore$ Option (c) is correct.

Displacement node occurs at

$$10\pi x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ etc.} \quad \text{or} \quad x = \frac{1}{20}, \frac{3}{20}$$

or $\quad x = 0.05$ m and 0.15 m

$\therefore$ Option (a) is correct.

Displacement antinode occurs at

$$10\pi x = 0, \pi, 2\pi, 3\pi, \text{etc.}$$

or $\quad x = 0, 0.1$ m, 0.2 m, 0.3 m

$\therefore$ Option (b) is correct.

$\lambda = 2 \times$ distance between two consecutive nodes or antinodes.

$= 2 \times 0.1 = 0.2$ m.

$\therefore$ Option (d) is correct.

46. (a), (b), (c) The number of waves encountered by the moving plane per unit time,

$$f' = \frac{\text{Distance travelled}}{\text{Wavelength}} = \frac{c + v}{\lambda}$$

$$= \frac{c}{\lambda}\left(1 + \frac{v}{c}\right) = f\left(\frac{c + v}{c}\right)$$

$\therefore$ Option (a) is correct.

The stationary observer intercepts the incident wave of frequency f' and receives the reflected wave of frequency f'' emitted by the moving platform.

$$f'' = \frac{f'}{1 - v/c} = \frac{f(1 + v/c)}{1 - v/c}$$

$$= \frac{f(c + v)}{(c - v)}$$

$\therefore$ Option (c) is correct.

Scanned by CamScanner

$$\lambda'' = \frac{c}{f''} = \frac{c(c-v)}{f(c+v)}$$

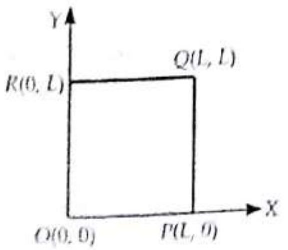
∴ Option (*b*) is correct.

Beat frequency = $f'' - f$

$$= \frac{f(1+v/c)}{(1-v/c)} - f = f\left(\frac{1+v/c}{1-v/c} - 1\right) = \frac{2vf}{c-v}$$

∴ Option (*d*) is incorrect.

47. (*b*), (*c*) As the edges of the square plate are clamped, displacement will be zero at the edges.



For option (a) :

$u(x, y) = 0$ at $x = L, y = L$

$u(x, y) \neq 0$ at $x = 0, y = 0$

For option (b) :

$u(x, y) = 0$ at $x = 0, y = 0$ $[\sin 0 = 0]$

$u(x, y) = 0$ at $x = L, y = L$ $[\sin \pi = 0]$

For option (c) :

$u(x, y) = 0$ at $y = 0, y = 0$ $[\sin 0 = 0]$

$u(x, y) = 0$ at $x = L, x = L$ $[\sin \pi = 0, \sin 2\pi = 0]$

For option (d) :

$u(x, y) = 0$ at $y = 0, y = L$ $[\sin 0 = 0, \sin \pi = 0]$

$u(x, y) \neq 0$ at $x = 0, x = L$ $[\cos 0 = 1, \cos 2\pi = 1]$

Hence only options (*b*) and (*c*) are correct.

48.

0	0	0	5

$$A = \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\left(-\frac{\pi}{2}\right)} = 5$$

49.

0	0	0	7

Frequency of sound received by car, $f' = \left(\frac{v + v_c}{v}\right) f_0$

Frequency of sound reflected by the car is

$$f = \left(\frac{v}{v - v_c}\right) f' = \left(\frac{v + v_c}{v - v_c}\right) f_0$$

As $v_c << v$, so $f = \left(\frac{1 + \frac{v_c}{v}}{1 - \frac{v_c}{v}}\right) f_0 = \left(1 - \frac{2v_c}{v}\right) f_0$

∴ $$\Delta f = \frac{-2 f_0}{v} \Delta v_c$$

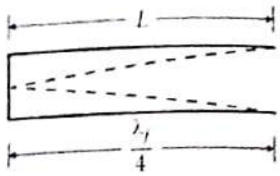
or $$\frac{1.2}{100} f_0 = \frac{-2 f_0}{v} \Delta v_c$$

Difference, $|\Delta v_c| = \frac{1.2}{100} \times \frac{300}{2}$ m/s

$$= \frac{1.2}{100} \times \frac{300}{2} \times \frac{18}{5} \approx \mathbf{7\ km/h}$$

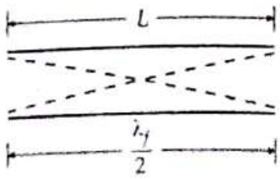
50. (*a*) → *p, t* ; (*b*) → *p, s* ; (*c*) → *q, s* ; (*d*) → *q, r*

(*a*) $\frac{\lambda_f}{4} = L \Rightarrow \lambda_f = 4L$ (Longitudinal wave)



∴ (*a*) → *p, t* are the correct matching

(*b*) $\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$ (Longitudinal wave)

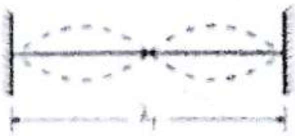


∴ (*b*) → *p, s* are the correct matching

(*c*) $\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$ (Transverse wave)

∴ (*c*) → *q, s* are the correct matching

(*d*) $\frac{\lambda_f}{2} + \frac{\lambda_f}{2} = L \Rightarrow \lambda_f = L$ (Transverse wave)



∴ (*d*) → *q, r* are the correct matching

Scanned by CamScanner

AIEEE

1. The displacement of a particle varies according to the relation

$$x = 4(\cos \pi t + \sin \pi t)$$

The amplitude of the particle is

(a) -4 (b) 4

(c) $4\sqrt{2}$ (d) 8 [AIEEE 03]

2. The displacement y of a particle in a medium can be expressed as

$$y = 10^{-6} \sin(100t + 20x + \pi/4)$$

where t is in second and x in metre. The speed of the wave is

(a) 2,000 ms^{-1} (b) 5 ms^{-1}

(c) 20 ms^{-1} (d) 5π ms^{-1} [AIEEE 04]

3. The displacement y of a wave travelling in the x-direction is given by

$$y = 10^{-4} \sin(600t - 2x + \pi/3)$$

where x is expressed in metres and t is seconds. The speed of the wave motion (in ms^{-1}) is

(a) 300 (b) 600

(c) 1200 (d) 200. [AIEEE 03]

4. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then, the equation of the unknown wave is

(a) $y = a\sin(\omega t + kx)$ (b) $y = -a\sin(\omega t + kx)$

(c) $y = a\sin(\omega t - kx)$ (d) $y = -a\sin(\omega t - kx)$

[AIEEE 02]

5. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 and 2.0 s, respectively, then α and β in appropriate units are

(a) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$ (b) $\alpha = 25.00\,\pi, \beta = \pi$

(c) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ (d) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$

[AIEEE 08]

6. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)

(a) 460 ms^{-1} (b) 500 ms^{-1}

(c) 650 ms^{-1} (d) 1420 ms^{-1}. [AIEEE 08]

7. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is

(a) 20 (b) 80

(c) 40 (d) 120. [AIEEE 02]

8. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then the lowest resonant frequency for this string is

(a) 1,050 Hz (b) 10.5 Hz

(c) 105 Hz (d) 1.05 Hz. [AIEEE 06]

9. A metal wire of linear mass density of 9.8 gm^{-1} is stretched with a tension of 10 kg wt between two rigid supports 1 m apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance, when carrying an alternating current of frequency ν. The frequency ν of the alternating source is

(a) 50 Hz (b) 100 Hz

(c) 200 Hz (d) 25 Hz. [AIEEE 03]

10. Tube A has both ends open, while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes A and B is

(a) 1 : 2 (b) 1 : 4

(c) 2 : 1 (d) 4 : 1 [AIEEE 02]

11. When temperature increases, the frequency of a tuning fork

(a) increases

(b) decreases

(c) increases or decreases depending on the material

(d) remains the same. [AIEEE 02]

12. A tuning fork arrangement (pair) produces 4 beats s^{-1} with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats s^{-1}. The frequency of the unknown fork is

(a) 286 cps (b) 292 cps

(c) 294 cps (d) 288 cps. [AIEEE 02]

13. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per

Scanned by CamScanner

second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork ?

(*a*) 200 Hz (*b*) 202 Hz
(*c*) 196 Hz (*d*) 204 Hz. [AIEEE 05]

14. A tuning fork of known frequency of 256 Hz makes 5 beats s^{-1} with the vibrating string of a piano. The beat frequency decreases to 2 beats s^{-1}, when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(*a*) $(256+2)$ Hz (*b*) $(256-2)$ Hz
(*c*) $(256-5)$ Hz (*d*) $(256+5)$ Hz. [AIEEE 03]

15. An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency ?

(*a*) zero (*b*) 0.5%
(*c*) 5% (*d*) 20%. [AIEEE 05]

16. A whistle producing sound waves of frequencies 9,500 Hz and above is approaching a stationary person with speed u ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of u upto which he can hear the whistle is

(*a*) $15\sqrt{2}$ ms^{-1} (*b*) $15/\sqrt{2}$ ms^{-1}
(*c*) 15 ms^{-1} (*d*) 30 ms^{-1}. [AIEEE 06]

17. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest ? (Speed of sound = 330 ms^{-1}).

(*a*) 49 m (*b*) 98 m
(*c*) 147 m (*d*) 196 m. [AIEEE 09]

18. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of

(*a*) 100 (*b*) 1000
(*c*) 10000 (*d*) 10 [AIEEE 07]

19. The transverse displacement $y(x,t)$ of a wave on a string is given by

$$y(x,t)=e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$$

This represents a

(*a*) wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$

(*b*) wave moving in $+x$ direction with speed $\sqrt{\frac{b}{a}}$

(*c*) standing wave of frequency $\sqrt{b}$

(*d*) standing wave of frequency $\frac{1}{\sqrt{b}}$

[AIEEE 2011]

20. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y=0.02(\text{m})\sin\left[2\pi\left(\frac{t}{0.04(\text{s})}-\frac{x}{0.50(\text{m})}\right)\right].$$

The tension in the string is

(*a*) 6.25 N (*b*) 4.0 N
(*c*) 12.5 N (*d*) 0.5 N [AIEEE 2010]

Answers and Explanations

1. (*c*) $x=4(\cos\pi t+\sin\pi t)$

$$=4\sqrt{2}\left(\sin\frac{\pi}{4}\cos\pi t+\cos\frac{\pi}{4}\sin\pi t\right)$$

$$=4\sqrt{2}\sin\left(\pi t+\frac{\pi}{4}\right)$$

Clearly, amplitude $=\mathbf{4\sqrt{2}}$.

2. (*b*) $y=10^{-6}\sin\left(100t+20x+\frac{\pi}{4}\right)$

Comparing with standard equation,

$$y=a\sin(\omega t+kx+\phi_0)$$

$$\omega=100\text{ rad s}^{-1},\ k=20\text{ rad m}^{-1}$$

$$v=\frac{\omega}{k}=\frac{100}{20}=\mathbf{5\ ms^{-1}}.$$

3. (*a*) $y=10^{-4}\sin\left(600t-2x+\frac{\pi}{3}\right)$

and $y=a\sin\left(\omega t-kx+\frac{\pi}{3}\right)$

$\therefore$ $\omega=600\text{ rad s}^{-1},\ k=2\text{ rad}^{-1}\text{m}$

$$v=\frac{\omega}{k}=\frac{600}{2}$$

$$=\mathbf{300\ ms^{-1}}.$$

Scanned by CamScanner

4. (*b*) For producing node, the superposing wave must travel in opposite direction (*x* term must have opposite sign) and its displacement must be negative. Hence the correct option is (*b*).

Moreover,

$$y = y_1 + y_2 = a\sin(\omega t - kx) - a\sin(\omega t + kx)$$
$$= -2a\cos\omega t \sin kx$$

At $x = 0$, $y = 0$ *i.e.*, a node is formed.

5. (*b*) $y(x,t) = 0.005\cos(\alpha x - \beta t)$

Comparing with standard equation,

$$y(x,t) = a\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

$$\alpha = \frac{2\pi}{\lambda} = \frac{2\pi}{0.08} = 25\pi$$

$$\beta = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \pi.$$

6. (*d*) $$\frac{v_{He}}{v_{O_2}} = \sqrt{\frac{\gamma_{He}}{\gamma_{O_2}} \cdot \frac{M_{O_2}}{M_{He}}}$$

$$= \sqrt{\frac{5/3}{7/5} \cdot \frac{32}{4}} = \sqrt{\frac{200}{21}}$$

$$= \sqrt{9.52} = 3.086$$

$$v_{He} = 3.086 \times v_{O_2}$$
$$= 3.086 \times 460$$
$$= \mathbf{1420\ ms^{-1}}.$$

7. (*b*) When the string vibrates in one segment,

$$L = \frac{\lambda}{2}$$

$$\therefore \quad \lambda = 2L = 2 \times 40 = \mathbf{80\ cm.}$$

8. (*c*) Refer to the solution of Problem 4 on Page 15.68.

9. (*a*) Here, $m = 9.8\ gm^{-1} = 9.8 \times 10^{-3}\ kg\ m^{-1}$

$T = 10$ kg wt $= 10 \times 9.8$ N $= 98$ N

$L = 1$ m

The fundamental frequency of vibration of the string,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$$= \frac{1}{2 \times 1}\sqrt{\frac{98}{9.8 \times 10^{-3}}}$$

$$= \mathbf{50\ Hz.}$$

10. (*c*) Refer to the solution of Problem 2 on Page 15.68.

11. (*b*) When temperature increases, the frequency of tuning fork decreases. Refer to the solution of Problem 62 on Page 15.63.

12. (*b*) Refer to the solution of Example 58 on Page15.43.

13. (*c*) Refer to the solution of Example 64 on Page 15.44.

14. (*c*) Refer to the solution of Example 63 on Page 15.44.

15. (*d*) Refer to the solution of Example 79 on Page 15.52.

16. (*c*) Refer to the solution of Problem 5 on Page 15.68.

17. (*b*) $$\nu' = \frac{v - v_0}{v} \times \nu$$

$$\frac{\nu'}{\nu} = \frac{v - v_0}{v}$$

$$0.94 = 1 - \frac{v_0}{v}$$

$$\frac{v_0}{v} = 0.06$$

$$v_0 = 0.06\ v = 0.06 \times 330 = 19.8\ ms^{-1}$$

Distance covered

$$= \frac{v_0^2}{2a} = \frac{19.8 \times 19.8}{2 \times 2} = \mathbf{98\ m.}$$

18. (*a*) $$L_1 = 10\log\frac{I_1}{I_0},\quad L_2 = 10\log\frac{I_2}{I_0}$$

$$\therefore \quad L_1 - L_2 = 10\log\frac{I_1}{I_2}$$

$$20\,dB = 10\log\frac{I_1}{I_2}$$

or $$\frac{I_1}{I_2} = 10^2$$

or $$I_2 = \frac{1}{100}I_1.$$

19. (*b*) $y(x,t) = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

$$v = \frac{\omega}{k} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$$

20. (*a*) $$v = \frac{\omega}{k} = \frac{\left(\frac{2\pi}{0.04}\right)}{\left(\frac{2\pi}{0.50}\right)} = \frac{0.50}{0.04} = 12.5\ m/s$$

As $$v = \sqrt{\frac{T}{\mu}}$$

$$\therefore \quad T = \mu v^2 = (12.5)^2 \times 0.04 = \mathbf{6.25\ N}$$

Scanned by CamScanner

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. The ratio of velocity of sound in hydrogen and oxygen at STP is

(a) 16 : 1 (b) 8 : 1
(c) 4 : 1 (d) 2 : 1 [IPUEE 06]

2. It takes 2.0 seconds for a sound wave to travel between two fixed points when the day temperature is 10°C. If the temperature rises to 30°C, the sound wave travels between the same fixed parts in

(a) 1.9 sec (b) 2.0 sec
(c) 2.1 sec (d) 2.2 sec. [IPUEE 05]

3. The disc of a siren containing 60 holes rotates at a constant speed of 360 rpm. The emitted sound is in unison with a tuning fork of frequency

(a) 10 Hz (b) 360 Hz
(c) 216 Hz (d) 60 Hz [IPUEE 06]

4. The speed (v) of ripples on the surface of water depends on surface tension (σ), density (ρ) and wavelength (λ). The square of speed (v) is proportional to

(a) $\frac{\sigma}{\rho\lambda}$ (b) $\frac{\rho}{\sigma\lambda}$
(c) $\frac{\lambda}{\sigma\rho}$ (d) $\rho\lambda\sigma$ [IPUEE 07]

5. The quantity which does not change, when sound enters from one medium to another

(a) wavelength (b) speed
(c) frequency (d) none of these. [IPUEE 99]

6. The equation of a simple harmonic wave is given by $y = 5\sin\frac{\pi}{2}(100t - x)$; where x and y are in metre and time is in second. The period of the wave in second will be

(a) 0.04 (b) 0.01
(c) 1 (d) 5. [IPUEE 07]

7. If wave $y = A\cos(\omega t + kx)$ is moving along x-axis, the shapes of pulse at $t = 0$ and $t = 2$ s

(a) are different (b) are same
(c) may not be same (d) none of these. [DCE 07]

8. The phase difference between two waves, represented by

$$y_1 = 10^{-6}\sin[100t + (x/50) + 0.5]\text{m}$$
$$y_2 = 10^{-6}\cos[100t + (x/50)]\text{m}$$

where x is expressed in metres and t is expressed in seconds, is approximately :

(a) 1.07 rad (b) 2.07 rad
(c) 0.5 rad (d) 1.5 rad [DCE 98]

9. $y_1 = 4\sin(\omega t + kx)$, $y_2 = -4\cos(\omega t + kx)$, the phase difference is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$
(c) π (d) zero. [IPUEE 04]

10. A wave equation is $y = 0.1\sin[100\pi t - kx]$ and wave velocity is 100 m/s, its number is equal to

(a) 1 m^{-1} (b) 2 m^{-1}
(c) $\pi\ \text{m}^{-1}$ (d) $2\pi\ \text{m}^{-1}$ [DCE 06]

11. A particle on the trough of a wave at any instant will come to the mean position after a time (T = time period)

(a) $\frac{T}{2}$ (b) $\frac{T}{4}$
(c) T (d) $2T$ [IPUEE 06]

12. Consider the three waves z_1, z_2 and z_3 as

$$z_1 = A\sin(kx - \omega t)$$
$$z_2 = A\sin(kx - \omega t)$$
$$z_3 = A\sin(ky - \omega t)$$

Which of the following represents a standing wave ?

(a) $z_1 + z_2$ (b) $z_2 + z_3$
(c) $z_3 + z_1$ (d) $z_1 + z_2 + z_3$ [DCE 04]

13. A string is tied on a sonometer, second end is hanging downward through a pulley with tension T. The velocity of the transverse wave produced is proportional to

(a) $\frac{1}{\sqrt{T}}$ (b) $\sqrt{T}$
(c) T (d) $\frac{1}{T}$ [DCE 97]

14. The fundamental frequency of a sonometer wire is n. If the tension is made 3 times and length and diameter are also increased 3 times, the new frequency will be

(a) $3n$ (b) $\frac{n}{3\sqrt{3}}$
(c) $\frac{n}{3}$ (d) $\sqrt{3}n$ [DCE 99]

Scanned by CamScanner

15. When the length of the vibrating segment of a sonometer wire is increased by 1%, the percentage change in its frequency is

(a) $\frac{100}{101}$ (b) $\frac{99}{100}$

(c) 1 (d) 2 [IPUEE 07]

16. In an experiment with sonometer a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension of the string remaining constant the frequency of the second tuning fork is

(a) 163.84 Hz (b) 400 Hz

(c) 320 Hz (d) 204.8 Hz. [IPUEE 06]

17. An organ pipe of length 'l' vibrates in its fundamental mode. The pressure variation is maximum

(a) at the two ends

(b) at the distance $l/2$ inside the ends

(c) at the distance $l/4$ inside the ends

(d) at the distance $l/8$ inside the ends. [DCE 09]

18. An open pipe resonates with a tuning fork of frequency 500 Hz. It is observed that two successive nodes are formed at distances 16 and 46 cm from the open end. The speed of sound in air in the pipe is

(a) 230 m/s (b) 300 m/s

(c) 320 m/s (d) 360 m/s. [IPUEE 05]

19. An organ pipe open at one end is vibrating in first overtone and is in resonance with another pipe open at both ends and vibrating in third harmonic. The ratio of length of two pipes is

(a) 1 : 2 (b) 4 : 1

(c) 8 : 3 (d) 3 : 8. [DCE 05]

20. The velocity of sound in open ended tube is 330 m/s, the frequency of wave is 1.1 kHz and length of tube is 30 cm, then number of harmonics that it will emit be

(a) 2 (b) 3

(c) 4 (d) 5. [DCE 98]

21. An organ pipe, open at both ends produces 5 beats per second when vibrated with a source of frequency 200 Hz. The second harmonic of the same pipe produces 10 beats per second with a source of frequency 420 Hz. The frequency of source is

(a) 195 Hz (b) 205 Hz

(c) 190 Hz (d) 210 Hz. [DCE 05]

22. Following two wave trains are approaching each other $y_1 = a\sin 200\pi t$, $y_2 = a\sin 208\pi t$. The number of beats heard per second is

(a) 8 (b) 4

(c) 1 (d) zero. [DCE 99]

23. Two waves of wavelengths 99 cm and 100 cm both travelling with velocity 396 m/s are made to interfere. The number of beats produced by them per second are

(a) 1 (b) 2

(c) 4 (d) 8 [DCE 01, 03]

24. Two waves are propagating with same amplitude and nearly same frequency in opposite direction, they result in

(a) beats (b) stationary wave

(c) resonance (d) wave packet. [DCE 2K]

25. A tuning fork A produces 4 beat/s with another tuning fork B of frequency 320 Hz. On filing one of the prongs of A, 4 beat/s are again heard when sounded with the same fork B. Then, the frequency of the fork A before filing is

(a) 328 Hz (b) 316 Hz

(c) 324 Hz (d) 320 Hz. [IPUEE 07]

26. A source emits a sound of frequency of 400 Hz, but the listener hears it's to be 390 Hz. Then

(a) the listener is moving towards the source

(b) the source is moving towards the listener

(c) the listener is moving away from the source

(d) the listener has a defective ear. [IPUEE 05]

27. A train is approaching with velocity 25 m/s towards a pedestrian standing on the track, frequency of horn of train is 1 kHz. Frequency heard by the pendestrain is ($v = 350$ m / s)

(a) 1077 Hz (b) 1167 Hz

(c) 985 Hz (d) 945 Hz. [DCE 06]

28. A person is standing on a railway platform and a train is approaching to him, what is maximum wavelength of sound he can hear ? Wavelength of whistle = 1 m, speed of sound in air = 330 m/s, speed of train = 36 km/h.

(a) 1 m (b) $\frac{32}{33}$ m

(c) $\frac{33}{32}$ m (d) $\frac{12}{13}$ m. [DCE 98]

Scanned by CamScanner

29. The apparent frequency of a note is 200 Hz, when a listener is moving with a velocity of 40 ms^{-1} towards a stationary source. When he moves away from the same source with the same speed, the apparent frequency of the same note is 160 Hz. The velocity of sound in air in m/s is

(*a*) 340 (*b*) 330

(*c*) 360 (*d*) 320. [IPUEE 06]

30. The source of sound generating a frequency of 3 kHz reaches an observer with a speed of 0.5 times the velocity of sound in air. The frequency heard by the observer is

(*a*) 1 kHz (*b*) 2 kHz

(*c*) 4 kHz (*d*) 6 kHz. [DCE 02]

31. A car sounding its horn at 480 Hz moves towards a high wall at a speed of 20 m/s, the frequency of the reflected sound heard by the man sitting in the car will be nearest to

(*a*) 480 Hz (*b*) 510 Hz

(*c*) 540 Hz (*d*) 570 Hz. [DCE 02]

32. A racing car moving towards a cliff sounds its horn. The driver observes that the sound reflected from the cliff has a pitch one octave higher than the actual sound of the horn. If v is the velocity of sound, the velocity of the car is

(*a*) $\frac{v}{\sqrt{2}}$ (*b*) $\frac{v}{2}$ (*c*) $\frac{v}{3}$ (*d*) $\frac{v}{4}$ [DCE 07]

33. If a particle travelling with a speed of 0.9 of the speed of sound and is emitting radiations of frequency of 1 kHz and moving towards the observer, what is the apparent frequency ?

(*a*) 1.1 kHz (*b*) 0.8 kHz

(*c*) 0.4 kHz (*d*) 10 kHz. [DCE 97]

34. When the source is moving towards the stationary observer, the apparent frequency is given by

(*a*) $n_1 = \frac{v + v_0}{v - v_s} n_0$ (*b*) $n_1 = \frac{vn}{(v + v_s)}$

(*c*) $n_1 = \frac{vn}{(v - v_s)}$ (*d*) $n_1 = \frac{(v + v_0)n}{v}$ [DCE 08]

35. Two trains, each moving with a velocity of 30 ms^{-1}, cross each other. One of the trains gives a whistle whose frequency is 600 Hz. If the speed of sound is 330 ms^{-1}, the apparent frequency for passengers sitting in the other train before crossing would be

(*a*) 600 Hz (*b*) 630 Hz

(*c*) 930 Hz (*d*) 720 Hz. [DCE 09]

36. The heavenly body is receding from earth, such that the fractional change in λ is 1, then its velocity is

(*a*) c (*b*) $\frac{3c}{5}$

(*c*) $\frac{c}{5}$ (*d*) $\frac{2c}{5}$. [IPUEE 2K]

37. The fractional change in wavelength of light coming from a star is 0.014%. What is its velocity ?

(*a*) 4.2×10^3 m/s (*b*) 3.8×10^8 m/s

(*c*) 3.5×10^3 m/s (*d*) 4.2×10^4 m/s [DCE 04]

38. The phenomenon by which stars recede from each other is explained by

(*a*) black hole theory (*b*) neutron star theory

(*c*) white dwarf (*d*) red shift. [DCE 2K, 03]

39. Speed of recession of galaxy is proportional to its distance :

(*a*) directly (*b*) inversely

(*c*) exponentially (*d*) none of these. [DCE 95]

40. The loudness and pitch of a sound note depends on

(*a*) intensity and frequency

(*b*) frequency and number of harmonics

(*c*) intensity and velocity

(*d*) frequency and velocity. [IPUEE 04]

41. Which is different from other by units ?

(*a*) Phase difference

(*b*) Mechanical equivalent

(*c*) Loudness of sound

(*d*) Poisson's ratio. [IPUEE 04]

Answers and Explanations

1. (*c*) $\frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{32}{2}}$

$= \mathbf{4 : 1}$

2. (*a*) $v \propto \sqrt{T}$ and $t \propto \frac{1}{v}$

∴ $t \propto \frac{1}{\sqrt{T}}$

$\frac{t_2}{t_1} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{273 + 10}{273 + 30}} = \sqrt{\frac{283}{303}}$

or $t_2 = \sqrt{\frac{283}{303}} \times 2\ \text{s} = \mathbf{1.9\ s}$.

3. (*b*) Frequency of revolution of disc,

$$= 360 \text{ rpm} = \frac{360}{60} \text{ rps} = 60 \text{ rps}$$

Frequency of emitted sound

$$= 6 \times \text{No. of holes} = 6 \times 60 = \mathbf{360\ Hz}.$$

4. (*a*) Let $v \propto \sigma^a \rho^b \lambda^c$. Then

$$[M^0L^1T^{-1}] \propto [MT^{-2}]^a [ML^{-3}]^b [L]^c$$

$$\propto [M]^{a+b} [L]^{-3b+c} [T]^{-2a}$$

Equating the powers of M, L and T on both sides, we get

$$a + b = 0, \quad -3b + c = 1, \quad -2a = -1$$

On solving,

$$a = \frac{1}{2}, \quad b = -\frac{1}{2}, \quad c = -\frac{1}{2}$$

$\therefore$ $$v \propto \sigma^{1/2} \rho^{-1/2} \lambda^{-1/2}$$

or $$v^2 \propto \frac{\sigma}{\rho\lambda}.$$

5. (*c*) Frequency remains unchanged when sound travels from one medium to another.

6. (*a*) $$y = 5 \sin \frac{\pi}{2}(100t - x)$$

$$y = A \sin(\omega t - kx)$$

$\therefore$ $$\omega = \frac{2\pi}{T} = \frac{\pi}{2} \times 100$$

or $$T = \frac{2}{50} = \mathbf{0.04\ s.}$$

7. (*b*) The shapes of y-x graphs remain same at $t = 0$ and $t = 2$ s.

8. (*a*) $$y_1 = 10^{-6} \sin[100t + (x/50) + 0.5] \text{ m}$$

$$y_2 = 10^{-6} \cos[100t + (x/50)] \text{m}$$

$$= 10^{-6} \sin[100t + (x/50) + (\pi/2)]$$

$$\Delta\phi = \frac{\pi}{2} - 0.05 = \frac{3.14}{2} - 0.5 = 1.57 - 0.5$$

$$= \mathbf{1.07\ rad}.$$

9. (*b*) $$y_1 = 4 \sin(\omega t + kx)$$

$$y_2 = -4 \cos(\omega t + kx)$$

$$= 4 \sin(\omega t + kx + 3\pi/2)$$

$\therefore$ $$\Delta\phi = \frac{3\pi}{2}.$$

10. (*c*) $$y = 0.1 \sin[100\pi t - kx]$$

$$y = A \sin[\omega t - kx]$$

$$\omega = 100\pi$$

Wave number $= \dfrac{\omega}{v} = \dfrac{100\pi}{100} = \pi \text{m}^{-1}$.

11. (*b*) Time taken by a particle to move from trough to the mean position $= T/4$.

12. (*a*) z_1 travels along +ve x-direction

z_2 travels along –ve x-direction

z_3 travels along +ve y-direction.

$\therefore$ $z_1 + z_2$ represents a standing wave *i.e.*, wave obtained by the superposition of two waves travelling along opposite directions.

13. (*b*) $v = \sqrt{\dfrac{T}{m}}$ $\quad \therefore \ v \propto \sqrt{T}$.

14. (*b*) $$n = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

$$n' = \frac{1}{3L \times 3D}\sqrt{\frac{3T}{\pi\rho}} = \frac{\sqrt{3}}{9} n = \frac{1}{3\sqrt{3}} \mathbf{n}.$$

15. (*c*) $$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

For constant T and m,

$$\frac{\Delta\nu}{\nu} \times 100 = \frac{\Delta L}{L} \times 100 = \mathbf{1\%}.$$

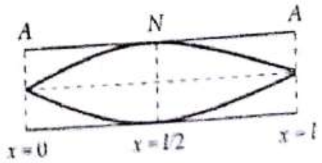
Frequency will decrease by **1%**.

16. (*b*) $$\nu \propto \frac{1}{L}$$

$\therefore$ $$\frac{\nu_2}{\nu_1} = \frac{L_1}{L_2}$$

$$\nu_2 = \frac{L_1}{L_2} \cdot \nu_1 = \frac{25}{16} \times 256 = \mathbf{400\ Hz}.$$

17. (*b*) The pressure variation is maximum at $x = \dfrac{l}{2}$ because the displacement node is pressure antinode.



18. (*b*) $$v = 2\nu(l_2 - l_1)$$

$$= 2 \times 500(46 - 16) \text{ cm/s}$$

$$= 30000 \text{ cm/s} = \mathbf{300\ m/s}.$$

19. (*a*) Fundamental frequency of a closed pipe,

$$\nu = \frac{v}{4L}$$

First overtone of a closed pipe,

$$= 3\nu = \frac{3v}{4L}$$

Scanned by CamScanner

Fundamental frequency of an open pipe,

$$\nu' = \frac{v}{2L'}$$

Third harmonic of an open pipe,

$$= 3\nu' = \frac{3v}{2L'}$$

As the two pipes are in resonance,

$$\frac{3v}{4L} = \frac{3v}{2L'} \quad \text{or} \quad \frac{L}{L'} = \frac{1}{2} = 1:2.$$

20. (*a*) For an open tube,

$$\nu_n = \frac{nv}{2L}$$

$$1.1 \times 10^3 = \frac{n \times 330}{2 \times 0.30} \quad \text{or} \quad n = 2.$$

21. (*b*) Fundamental frequency of open pipe,

$$f = 200 \pm 5 = 195 \text{ Hz} \quad \text{or} \quad 205 \text{ Hz}$$

Second harmonic of open pipe,

$$2f = 420 \pm 10 = 410 \text{ Hz} \quad \text{or} \quad 430 \text{ Hz}$$

or $$f = 205 \text{ Hz} \quad \text{or} \quad 215 \text{ Hz}.$$

The common frequency is **205 Hz**.

22. (*c*) $\omega_1 = 2\pi\nu_1 = 200\pi$ or $\nu_1 = 100$ Hz

$\omega_2 = 2\pi\nu_2 = 208\pi$ or $\nu_2 = 104$ Hz.

Beat frequency $= \nu_2 - \nu_1 =$ **4 Hz**.

23. (*c*) Beat frequency $= v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$

$$= 396\left[\frac{1}{0.99} - \frac{1}{1.00}\right] = \mathbf{4 \text{ Hz}}.$$

24. (*b*) Stationary waves are formed when two waves of same frequency travelling in opposite directions are superimposed.

25. (*b*) Frequency of $A = 320 \pm 4 = 324$ or 316 Hz. As frequency increases on filing, so frequency of $A = 316$ Hz (lower value).

26. (*c*) As apparent frequency < actual frequency, the listener is moving away from the source.

27. (*a*) $\nu' = \frac{v}{v - v_s} \times \nu = \frac{350}{350 - 25} \times 1000$ Hz

$$= \mathbf{1077 \text{ Hz}}.$$

28. (*b*) As $\nu' = \frac{v}{v - v_s} \times \nu$, $v_s = 36$ km/h $= 10$ m/s

$$\therefore \quad \lambda' = \frac{v - v_s}{v} \times \lambda$$

$$= \frac{330 - 10}{330} \times 1 = \frac{32}{33} \text{ m}.$$

29. (*c*) When the listener moves towards the source,

$$\nu' = \frac{v + v_0}{v} \times \nu$$

or $$200 = \frac{v + 40}{v} \times \nu \quad \ldots(i)$$

When the listener moves away from the source,

$$160 = \frac{v - 40}{v} \times \nu \quad \ldots(ii)$$

From (*i*) and (*ii*), we get

$$\frac{200}{160} = \frac{v + 40}{v - 40}$$

or $$v = 360 \text{ ms}^{-1}.$$

30. (*d*) $\nu' = \frac{v}{v - v_s} \times \nu = \frac{v}{v - 0.5v} \times 3 \text{ kHz} = \mathbf{6 \text{ kHz}}.$

31. (*c*) After reflection from the wall, the sound moves towards observer in the car.

$$\nu' = \frac{v + v_0}{v - v_s} \times \nu = \frac{340 + 20}{340 - 20} \times 480 = \mathbf{540 \text{ Hz}}.$$

32. (*c*) $\frac{\nu'}{\nu} = \frac{v + v_s}{v - v_s} = 2$

$$v + v_s = 2v - 2v_s$$

$$3v_s = v \quad \text{or} \quad v_s = \mathbf{v/3}.$$

33. (*d*) $\nu' = \frac{v}{v - v_s} \times \nu$

$$= \frac{v}{v - 0.9v} \times 1 \text{ kHz} = \mathbf{10 \text{ kHz}}.$$

34. (*c*) When the source moves towards the stationary observer, the apparent frequency is

$$n_1 = \frac{vn}{v - v_s}.$$

35. (*d*) $\nu' = \frac{v - v_0}{v - v_s} \times \nu$

$$= \frac{330 - (-30)}{330 - 30} \times 600$$

$$= \frac{360}{300} \times 600 = \mathbf{720 \text{ Hz}}.$$

36. (*a*) Doppler's shift in light is given by

$$\Delta\lambda = \frac{v}{c}\lambda$$

$$\frac{\Delta\lambda}{\lambda} = 1 = \frac{v}{c}$$

$$\therefore \quad v = c$$

Thus the speed of the heavenly body is equal to the speed of light.

Scanned by CamScanner

37. (*d*) $v = \frac{\Delta\lambda}{\lambda} \times c$

$= \frac{0.014}{100} \times 3 \times 10^8 \text{ ms}^{-1}$

$= 4.2 \times 10^4 \text{ ms}^{-1}$.

38. (*d*) Red shift confirms that stars are continuously receding away from each other.

39. (*a*) According to Hubble's law, the speed of recession (v) of a galaxy is directly proportional to its distance (r) from us.

$v = Hr$ where H is Hubble's constant.

40. (*a*) Loudness depends on intensity while pitch depends on frequency of sound.

41. (*d*) Of all the given quantities, Poisson's ratio has no units.

AIIMS Entrance Exam

1. The waves, in which the particles of the medium vibrate in a direction perpendicular to the direction of wave motion, is known as

(*a*) transverse waves

(*b*) longitudinal waves

(*c*) propagated waves

(*d*) none of these. [AIIMS 98]

2. The waves produced by a motor boat sailing in water are

(*a*) transverse (*b*) longitudinal

(*c*) longitudinal and transverse

(*d*) stationary. [AIIMS 04]

3. For a wave propagating in a medium, identify the property that is independent of the others.

(*a*) velocity

(*b*) wavelengths

(*c*) frequency

(*d*) all these depend on each other. [AIIMS 06]

4. A boat at anchor is rocked by waves, whose crests are 100 m apart and velocity is 25 ms^{-1}. The boat bounces up once in every

(*a*) 2,500 s (*b*) 75 s

(*c*) 4 s (*d*) 0.25 s [AIIMS 06]

5. Newton's formula for the velocity of sound in gases is

(*a*) $v = \sqrt{P/\rho}$ (*b*) $v = \sqrt{\rho/P}$

(*c*) $v = \sqrt{\rho/2P}$ (*d*) $v = \sqrt{2P/\rho}$ [AIIMS 98]

6. The velocities of sound at the same temperature in two monoatomic gases of densities of ρ_1 and ρ_2 are v_1 and v_2 respectively. If $\rho_1/\rho_2 = 4$, then the value of v_1/v_2 is

(*a*) 1/4 (*b*) 1/2

(*c*) 2 (*d*) 4 [AIIMS 02]

7. An earthquake generates both transverse (S) and longitudinal (P) sound waves in the earth. The speed of S waves is about 4.5 km s^{-1} and that of P waves is about 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4.0 min before the first S wave. The epicentre of the earthquake is located at a distance of about

(*a*) 25 km (*b*) 250 km

(*c*) 2,500 km (*d*) 5,000 km [AIIMS 03]

8. If equation of a sound wave is

$$y = 0.0015 \sin(62.8x + 314t)$$

then its wavelength will be

(*a*) 0.1 unit (*b*) 0.2 unit

(*c*) 0.3 unit (*d*) 2 unit. [AIIMS 02]

9. A transverse wave passes through a string with the equation

$$y = 10 \sin \pi(0.02x - 2t)$$

where x is in metres and t in seconds. The maximum velocity of the particles in wave motion is

(*a*) 63 ms^{-1} (*b*) 78 ms^{-1}

(*c*) 100 ms^{-1} (*d*) 121 ms^{-1} [AIIMS 2K]

10. The plane wave is described by the equation

$$y = 3\cos(x/4 - 10t - \pi/2),$$

where x and y are in meters and t in seconds. The maximum velocity of the particles of the medium due to this is

(*a*) 30 ms^{-1} (*b*) $\frac{3\pi}{2}$ ms^{-1}

(*c*) $\frac{3}{4}$ ms^{-1} (*d*) 40 ms^{-1}. [AIIMS 2K]

Scanned by CamScanner

11. The equation of wave is given by

$$y = 10\sin(2\pi t/30 + \alpha)$$

If the displacement is 5 cm at $t = 0$, then the total phase at $t = 7.5$ s will be

(a) $\pi/3$ rad (b) $\pi/2$ rad

(c) $2\pi/5$ rad (d) $2\pi/3$ rad [AIIMS 2K]

12. If two sound waves having a phase difference of 60°, then they will have a path difference of

(a) $\lambda/6$ (b) $\lambda/3$

(c) λ (d) 3λ. [AIIMS 01]

13. Energy is not carried by which of the following wave ?

(a) stationary (b) progressive

(c) transverse (d) electromagnetic. [AIIMS 99]

14. Standing waves are produced in 10 m long stretched string. If the string vibrates in 5 segments and wave velocity is 20 ms^{-1}, its frequency is

(a) 2 Hz (b) 4 Hz

(c) 5 Hz (d) 10 Hz. [AIIMS 98]

15. If vibrations of a string are to be increased by a factor 2, tension in the string must be made

(a) half (b) twice

(c) four times (d) eight times. [AIIMS 99]

16. The tension in piano wire is 10 N. What should be the tension in the wire to produce a note of double the frequency ?

(a) 5 N (b) 20 N

(c) 40 N (d) 80 N. [AIIMS 01, 95]

17. A string in a musical instrument is 50 cm long and its fundamental frequency is 800 Hz. If a frequency of 1,000 Hz is to be produced, then required length of string is

(a) 62.5 cm (b) 50 cm

(c) 40 cm (d) 37.5 cm. [AIIMS 02]

18. The frequency of a tuning fork is 256. It will not resonate with a fork of frequency

(a) 256 (b) 512

(c) 738 (d) 768. [AIIMS 94]

19. An organ pipe closed at one end has fundamental frequency of 1,500 Hz. The maximum number of overtones generated by this pipe, which a normal person can hear is

(a) 12 (b) 9

(c) 6 (d) 4. [AIIMS 04]

20. A tube closed at one end containing air produces fundamental note of frequency 512 Hz. If the tube is open at both the ends, the fundamental frequency will be

(a) 256 Hz (b) 768 Hz

(c) 1,024 Hz (d) 1,280 Hz. [AIIMS 95]

21. A closed organ pipe and an open pipe of the same length produce four beats per second, when sounded together. If the length of the closed pipe is increased, then the number of beats will

(a) increase (b) decrease

(c) remain the same (d) first (b) then (c).

[AIIMS 96]

22. A resonance air column of length 20 cm resonates with a tuning fork of frequency 450 Hz. Ignoring end correction, the velocity of sound in air is

(a) 720 ms^{-1} (b) 820 ms^{-1}

(c) 920 ms^{-1} (d) 360 ms^{-1}. [AIIMS 99]

23. A sings with a frequency ν and B sings with a frequency 1/8th that of A. If the energy remains the same and the amplitude of A is a, then the amplitude of B is

(a) a (b) 2a

(c) 8a (d) 16a. [AIIMS 01]

24. If fundamental frequency is 50 and next successive frequencies are 150 and 250, then it is

(a) a pipe closed at both ends

(b) a pipe closed at one end

(c) an open pipe

(d) a stretched string. [AIIMS 01]

25. A stone thrown into still water, creates a circular wave pattern moving radially outwards. If r is the distance measured from the centre of the pattern, the amplitude of the wave varies as

(a) $r^{-1/2}$ (b) r^{-1}

(c) $r^{-3/2}$ (d) r^{-2} [AIIMS 06]

26. A siren emitting sound of frequency 800 Hz is going away from a static listener with a speed of 30 ms^{-1}. Frequency of the sound to be heard by the listener is (Take velocity of sound as 330 ms^{-1})

(a) 733.3 Hz (b) 644.8 Hz

(c) 481.2 Hz (d) 286.5 Hz. [AIIMS 02]

27. An observer standing by the side of a road hears the siren of an ambulance, which is moving away from him. If the actual frequency of the siren is 2,000 Hz, then the frequency heard by the observer will be

(a) 1,990 Hz (b) 2,000 Hz

(c) 2,100 Hz (d) 4,000 Hz. [AIIMS 98]

Scanned by CamScanner

28. A source of frequency 240 Hz is moving towards an observer with a velocity of 20 ms^{-1}. The observer is now moving towards the source with a velocity of 20 ms^{-1}. Apparent frequency heard by observer, if velocity of sound is 340 ms^{-1}, is

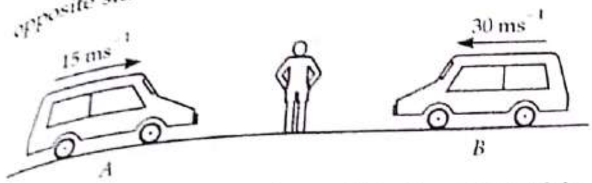
(*a*) 240 Hz

(*b*) 270 Hz

(*c*) 330 Hz

(*d*) 360 Hz. [AIIMS 96]

29. Two cars approach a stationary observer from opposite sides as shown in the figure.



The observer hears no beats. If the frequency of the horn of the car *B* is 504 Hz, the frequency of the horn of the car *A* will be

(*a*) 529.2 Hz

(*b*) 440.5 Hz

(*c*) 295.2 Hz

(*d*) none of these. [AIIMS 2K]

30. Five sinusoidal waves have the same frequency 500 Hz but their amplitudes are in the ratio $2:\frac{1}{2}:\frac{1}{2}:1:1$ and their phase angles $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and π respectively. The phase angle of resultant wave obtained by the superposition of these five waves is

(*a*) 30°   (*b*) 45°

(*c*) 60°   (*d*) 90° [AIEEE 2010]

31. A uniform string is vibrating with a fundamental frequency '*f*'. The new frequency, if radius and length both are doubled, would be

(*a*) 2 *f*   (*b*) 3 *f*

(*c*) $\frac{f}{4}$ (*d*) $\frac{f}{3}$ [AIIMS 2010]

32. The second overtone of an open pipe has the same frequency as the first overtone of a closed pipe 2 m long. The length of the open pipe is

(*a*) 8 m   (*b*) 4 m

(*c*) 2 m   (*d*) 1 m [AIIMS 2010]

ASSERTION AND REASONS

Directions. In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as

(*a*) If both assertion and reason are true and reason is the correct explanation of the assertion.

(*b*) If both assertion and reason are true but reason is not correct explanation of the assertion.

(*c*) If assertion is true, but reason is false.

(*d*) If both assertion and reason are false.

33. **Assertion.** Sound wave cannot propagate through vacuum but light can.

Reason. Sound wave cannot be polarised but light can. [AIIMS 97]

34. **Assertion.** Sound wave cannot propagate fastest in solids.

Reason. Sound wave can propagate slightly in vacuum. [AIIMS 97]

35. **Assertion.** Speed of wave = $\frac{\text{wavelength}}{\text{time period}}$

Reason. Wavelength is the distance between two nearest particles in phase. [AIIMS 02]

36. **Assertion.** Ocean waves hitting a beach are always found to be nearly normal to the shore.

Reason. Ocean waves hitting a beach are assumed to be plane waves. [AIIMS 07]

37. **Assertion.** When a beetle moves along the sand within a few tens of centrimetres of a sand scorpion, the scorpion immediately turns towards the beetle and dashes towards it.

Reason. When a beetle disturbs the sand, it sends pulses along the sand's surface. One set of pulses is longitudinal, while the other set is transverse. [AIIMS 03]

38. **Assertion.** A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of the same length.

Reason. The same tuning fork will not be in resonance with an open pipe of the same length due to end correction. [AIIMS 2K]

39. **Assertion.** When two vibrating tuning forks having frequencies 256 Hz and 512 Hz are held near each other, beats cannot be heared.

Reason. The principle of superposition is valid if the frequencies of the oscillators are nearly equal. [AIIMS 94]

Scanned by CamScanner

Answers and Explanations

1. (*a*) In transverse waves, particles of the medium vibrate in a direction perpendicular to the direction of propagation of the wave.

2. (*c*) Transverse waves are generated on the water surface. Inside water longitudinal waves are produced due to vibrations of the rudder.

3. (*c*) Wave velocity = frequency × wave length

Frequency remains unchanged while velocity and wavelength are interdependent.

4. (*c*) $\lambda = 100$ m, $v = 25$ ms^{-1}

$$T = \frac{\lambda}{v} = \frac{100}{25} = \mathbf{4\ s}.$$

5. (*a*) Newton's formula for the velocity of sound in gases is

$$v = \sqrt{\frac{P}{\rho}}.$$

6. (*b*) $$v = \sqrt{\frac{\gamma P}{\rho}} \quad i.e., \quad v \propto \frac{1}{\sqrt{\rho}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{1}{4}} = \mathbf{\frac{1}{2}}.$$

7. (*c*) For *S* waves, $v_1 = 4.5$ km/s

For *P* waves, $v_2 = 8.0$ km/s

Let the two waves take times t_1 and t_2 to travel distance *s* from the epicentre.

Then $t_1 - t_2 = 4$ min $= 240$ s

and $$s = v_1 t_1 = v_2 t_2$$

$$4.5 \times t_1 = 8 t_2 \quad \text{or} \quad t_2 = \frac{4.5}{8} t_1$$

$$\therefore \quad t_1 - \frac{4.5}{8} t_1 = 240$$

or $$\frac{3.5}{8} t_1 = 240$$

or $$t_1 = \frac{240 \times 8}{3.5} = 548.5 \text{ s}$$

Hence $$s = v_1 t_1 = 4.5 \times 548.5 = 2468.5 = \mathbf{2500\ km}.$$

8. (*a*) $$y = 0.0015 \sin(62.8x + 314t)$$

$$y = a \sin\left(\frac{2\pi}{\lambda} x + \frac{2\pi}{T} t\right)$$

$$\therefore \quad \frac{2\pi}{\lambda} = 62.8$$

$$\lambda = \frac{2 \times 3.14}{62.8} = \mathbf{0.1\ unit}.$$

9. (*a*) $$y = 10 \sin \pi(0.02x - 2t)$$

$$u = \frac{dy}{dt} = -20\pi \cos \pi(0.02x - 2t)$$

$$u_{max} = 20\pi = \mathbf{63\ ms^{-1}}.$$

10. (*a*) $$y = 3\cos(x/4 - 10t - \pi/2)$$

$$u = \frac{dy}{dt} = 30 \sin(x/4 - 10t - \pi/2)$$

$$u_{max} = \mathbf{30\ ms^{-1}}.$$

11. (*d*) $$y = 10 \sin\left(\frac{2\pi t}{300} + \alpha\right)$$

At $t = 0$, $5 = 10 \sin\alpha$ or $\sin\alpha = \frac{1}{2}$ or $\alpha = \frac{\pi}{6}$.

Total phase at $t = 7.5$ s,

$$\phi = \frac{2\pi \times 7.5}{300} + \frac{\pi}{6} = \frac{\pi}{2} + \frac{\pi}{6} = \mathbf{\frac{2\pi}{3}}.$$

12. (*a*) $\Delta\phi = 60° = \frac{\pi}{3}$

$$\Delta x = \frac{\lambda}{2\pi} \Delta\phi = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \mathbf{\frac{\lambda}{6}}.$$

13. (*a*) Stationary waves do not carry energy.

14. (*c*) $5 \times \frac{\lambda}{2} = 10$ or $\lambda = 4$ m

$$v = 20 \text{ ms}^{-1}$$

$$\nu = \frac{v}{\lambda} = \frac{20}{4} = \mathbf{5\ Hz}.$$

15. (*c*) $$\nu = \frac{1}{2\pi}\sqrt{\frac{T}{m}}$$

To double ν, the tension *T* must be made 4 times the original tension.

16. (*c*) As seen in the above problem,

$$T' = 4T = 4 \times 10 = \mathbf{40\ N}.$$

17. (*c*) $\nu \propto 1/l$

$$\frac{\nu'}{\nu} = \frac{l}{l'}$$

$$l' = \frac{\nu}{\nu'} . l = \frac{800}{1000} \times 50 = \mathbf{40\ cm}.$$

18. (*c*) Tuning fork of frequency 256 Hz will resonate with forks of integral multiple frequencies such as 256, 768, 1024 etc. It will not resonate with a fork of frequency 738 Hz as it is not an integral multiple of 256 Hz.

Scanned by CamScanner

19. (c) Human year can hear frequencies upto 20,000 Hz. For a closed organ pipe,

$$\nu = n \times \frac{v}{4L}$$

$$= n \times \text{ fundamental frequency}$$

$$20{,}000 = n \times 1500 \quad \text{or} \quad n = 13$$

The possible harmonics are : 1, 3, 5, 7, 9, 11, 13

Maximum number of overtones $= 7 - 1 = 6$.

20. (c) Fundamental frequency of a closed pipe,

$$\nu = \frac{v}{4L} = 512 \text{ Hz}$$

Fundamental frequency of open pipe,

$$\nu' = \frac{v}{2L} = 2\nu = 2 \times 512 = \mathbf{1024\ Hz.}$$

21. (a) $$\nu_{open} - \nu_{closed} = \frac{v}{2L_{open}} - \frac{v}{4L_{closed}}$$

$$= \text{Beat frequency}$$

Clearly, if the length of the closed pipe is increased, then the beat frequency will increase.

22. (d) $$\frac{\lambda}{4} = 20 \text{ cm}, \ \lambda = 80 \text{ cm} = 0.80 \text{ m},$$

$$\nu = 450 \text{ Hz}$$

$$v = \nu\lambda = 450 \times 0.80 = \mathbf{360\ ms^{-1}}.$$

23. (c) $$E_A \propto \nu^2 a^2$$

$$E_B \propto \left(\frac{1}{8}\nu\right)^2 a'^2$$

$$E_A = E_B$$

$$\therefore \quad \nu^2 a^2 = \frac{1}{64}\nu^2 a'^2$$

$$a' = 8a.$$

24. (b) For a pipe closed at one end, higher frequencies are odd multiples of the fundamental frequency.

25. (a) For a circular wave,

$$I \propto \frac{1}{r}$$

But $$A \propto \sqrt{I}$$

$$\therefore \quad A \propto \frac{1}{\sqrt{r}} \quad \text{or} \quad A \propto r^{-1/2}$$

26. (a) $$\nu' = \frac{v}{v + v_s} \times \nu = \frac{330}{330 + 30} \times 800 = \mathbf{733.3\ Hz.}$$

27. (a) $$\nu' = 1990$$

Apparent frequency decreases when the source moves away from the observer.

28. (b) $$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{340 - (-20)}{340 - 20} \times 240$$

$$= \frac{360}{320} \times 240 = \mathbf{270\ Hz.}$$

29. (a) Apparent frequency of car A

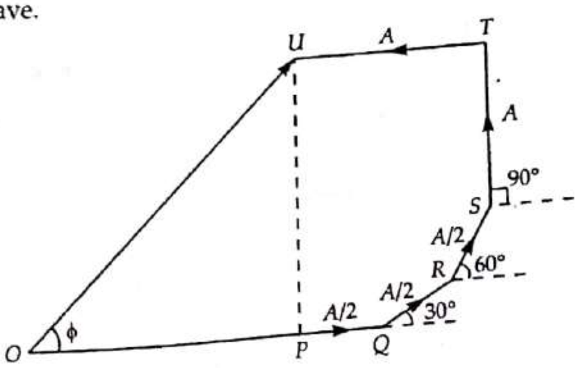
= Apparent frequency of car B.

$$\frac{v}{v - v_s} \times \nu = \frac{v}{v - v_s} \times \nu'$$

$$\frac{\nu}{\nu - 15} = \frac{504}{v - 30}$$

$$\nu = \frac{340 - 15}{340 - 30} \times 504 = \mathbf{529.2\ Hz.}$$

30. (b) The five waves are as shown in the figure. The resultant wave has phase angle ϕ *w.r.t.* the first wave.



Clearly
$$\tan\phi = \frac{UP}{OP} = \frac{2A}{2A} = 1$$

$$\therefore \quad \mathbf{\phi = 45°}$$

31. (c) Fundamental frequency,

$$f = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

When radius and length are both doubled,

$$f' = \frac{1}{2L \cdot 2D}\sqrt{\frac{T}{\pi\rho}} = \frac{1}{4}f$$

32. (b) Second overtone of open pipe

= First overtone of closed pipe

$$3 \cdot \frac{v}{2L'} = 3 \cdot \frac{v}{4L}$$

$$L' = 2L = 2 \times 2\text{m} = \mathbf{4\ m}$$

33. (b) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. Sound waves, being mechanical waves, cannot travel through vacuum. Light waves being electromagnetic waves, can travel through vacuum.

Scanned by CamScanner

34. (*c*) The assertion is true but the reason is false. Sound waves, being mechanical waves, cannot propagate through vacuum.

35. (*b*) Both the assertion and reason are true but the reason is not a correct explanation of the assertion. Speed of a wave is the distance travelled by the wave in the time during which a particle of the medium completes one vibration.

36. (*a*) Both assertion and reason are true. Ocean waves are transverse in nature, they hit the shore normally. As these waves spread out, their radius of curvature becomes so large that they may be considered as plane waves.

37. (*a*) Both the assertion and reason are true. A beetle's motion sends fast longitudinal and slower transverse waves along sand's surface. The sand scorpion intercepts the longitudinal waves first and senses the direction of the beetle by noting which are of the eight legs of the scorpion is disturbed first by the pulses. Then the scorpion dashes towards the beetle to catch it.

38. (*c*) The assertion is true but the reason is false.

For a closed pipe, $\nu = \frac{v}{4L}$

For an open pipe, $\nu = \frac{v}{2L}$

When both pipes are of same length, their fundamental frequencies will be different and they cannot be in resonance with the same tuning fork.

39. (*c*) The assertion is true but the reason is false. The beats can be heard only if the two sounds do not exceed 10 Hz.

CBSE PMT Prelims and Final Exams

1. With the propogation of a longitudinal wave through a material medium, the quantities tranmitted in the propogation direction are

(*a*) energy, momentum and mass

(*b*) energy (*c*) energy and mass

(*d*) energy and linear momentum. [CBSE PMT 92]

2. Which one of the following statements is true ?

(*a*) both light and sound waves can travel in vacuum

(*b*) both light and sound waves in air are transverse

(*c*) the sound waves in air are longitudinal while the light waves are transverse

(*d*) both light and sound waves in air are longitudinal. [CBSE PMT 06]

3. The velocity of sound in any gas depends upon

(*a*) wavelength of sound only

(*b*) density and elasticity of gas

(*c*) intensity of sound waves only

(*d*) amplitude and frequency of sound. [CBSE PMT 88]

4. A hospital uses an ultrasonic scanner to locate tumours in a tissue. The operating frequency of the scanner is 4.2 MHz. The speed of sound in a tissue is 1.7 km/s. The wavelength of sound in the tissue is close to

(*a*) 4×10^{-3} m (*b*) 8×10^{-3} m

(*c*) 4×10^{-4} m (*d*) 8×10^{-4} m [CBSE PMT 95]

5. A 5.5 metre length of string has a mass of 0.035 kg. If the tension in the string in 77 N, the speed of a wave on the string is

(*a*) 110 ms^{-1} (*b*) 165 ms^{-1}

(*c*) 77 ms^{-1} (*d*) 102 ms^{-1} [CBSE PMT 89]

6. The temperature at which the speed of sound becomes double as was at 27°C is

(*a*) 273°C (*b*) 0°C

(*c*) 927°C (*d*) 1027°C [CBSE PMT 93]

7. Velocity of sound waves in air is 330 m/s. For a particular sound wave in air, a path difference of 40 cm is equivalent to phase difference of 1.6 π. The frequency of this wave is

(*a*) 165 Hz (*b*) 150 Hz

(*c*) 660 Hz (*d*) 330 Hz [CBSE PMT 90]

8. Two sound waves having a phase difference of 60° have path difference of

(*a*) $\lambda/6$ (*b*) $\lambda/3$

(*c*) 2λ (*d*) $\lambda/2$ [CBSE PMT 96]

9. In a sinusoidal wave, the time required for a particular point to move from maximum displacement to zero displacement is 0.170 sec. The frequency of wave is

(*a*) 0.73 Hz (*b*) 0.36 Hz

(*c*) 1.47 Hz (*d*) 2.94 Hz. [CBSE PMT 98]

10. Which of the following represents a wave ?

(*a*) $y = A\sin(\omega t - kx)$

(*b*) $y = A\cos(at - bx + c)$

(*c*) $y = A\sin kx$

(*d*) $y = A\sin \omega t$. [CBSE PMT 94]

Scanned by CamScanner

11. A wave travelling in positive X-direction with $a = 0.2$ m, velocity $= 360$ m/s and $\lambda = 60$ m, then correct expression for the wave is

(a) $y = 0.2 \sin\left[2\pi\left(6t + \frac{x}{60}\right)\right]$

(b) $y = 0.2 \sin\left[\pi\left(6t + \frac{x}{60}\right)\right]$

(c) $y = 0.2 \sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$

(d) $y = 0.2 \sin\left[\pi\left(6t - \frac{x}{60}\right)\right]$. [CBSE PMT 02]

12. The frequency of sinusodial wave

$$y = 0.40 \cos[200t + 0.80x]$$

would be

(a) $1000\,\pi$ Hz (b) 2000 Hz

(c) 20 Hz (d) $\frac{1000}{\pi}$ Hz. [CBSE PMT 92]

13. The equation of a sound wave is

$$y = 0.0015 \sin(62.4 + 316t)$$

The wavelength of this wave is

(a) 0.3 unit (b) 0.2 unit

(c) 0.1 unit (d) cannot be calculated. [CBSE PMT 96]

14. A transverse wave propagating along x-axis is represented by $y(x,t) = 8.0 \sin(0.5\pi x - 4\pi t - \pi/4)$, where x is in metres and t is in seconds. The speed of the wave is

(a) 8 m/s (b) 4π m/s

(c) 0.5π m/s (d) $\pi/4$ m/s [CBSE PMT 06]

15. The equation of a wave is represented by $y = 10^{-4} \sin\left(100t - \frac{x}{10}\right)$ m, then the velocity of wave will be

(a) 100 m/s (b) 4 m/s

(c) 1000 m/s (d) 10 m/s. [CBSE PMT 01]

16. A wave in a string has an amplitude of 2 cm. The wave travels in the +ve direction of x-axis with a speed of 128 m/s and it is noted that 5 complete waves fit in 4 m length of the string. The equation describing the wave is

(a) $y = (0.02)$ m $\sin(15.7x - 2010t)$

(b) $y = (0.02)$ m $\sin(15.7x + 2010t)$

(c) $y = (0.02)$ m $\sin(7.85x - 1005t)$

(d) $y = (0.02)$ m $\sin(7.85x + 1005t)$. [CBSE PMT 09]

17. Equation of progressive wave is given by $y = 4\sin\left[\pi\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right]$ where y, x are in cm and t is in second. Then which of the following is correct ?

(a) $v = 5$ cm/s (b) $\lambda = 18$ cm

(c) $a = 0.04$ cm (d) $f = 50$ Hz. [CBSE PMT 88]

18. A transverse wave is represented by the equation $y = y_0 \sin\frac{2\pi}{\lambda}(vt - x)$. For what value of λ, is the maximum particle velocity equal to two times the wave velocity ?

(a) $\lambda = \frac{\pi y_0}{2}$ (b) $\lambda = \frac{\pi y_0}{3}$

(c) $\lambda = 2\pi y_0$ (d) $\lambda = \pi y_0$ [CBSE PMT 98]

19. The phase difference between two waves, represented by

$$y_1 = 10^{-6} \sin[100t + (x/50) + 0.5] \text{ m}$$
$$y_2 = 10^{-6} \cos[100t + (x/50)] \text{ m}$$

where x is expressed in metres and t is expressed in seconds, is approximately

(a) 1.07 radians (b) 2.07 radians

(c) 0.5 radian (d) 1.5 radians. [CBSE PMT 04]

20. Two waves have equations,

$$x_1 = a\sin(\omega t - kx + \phi_1)$$
$$x_2 = a\sin(\omega t - kx + \phi_2)$$

If in the resultant wave the frequency and amplitude remain equal to those of superimposing waves, the phase difference between them is

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$. [CBSE PMT 01]

21. The equations of two waves acting in perpendicular directions are given as

$$x = a\cos(\omega t + \delta) \quad \text{and} \quad y = a\cos(\omega t + \alpha)$$

where $\delta = \alpha + \frac{\pi}{2}$, the resultant wave represents

(a) a parabola (b) a circle

(c) an ellipse (d) a straight line. [CBSE PMT 2K]

22. A stationary wave is represented by

$$y = A\sin(100t)\cos(0.01x),$$

where y and A are in millimetres, t is in seconds and x is in metres. The velocity of the wave is

(a) 10^4 m/s (b) not derivable

(c) 1 m/s (d) 10^2 m/s. [CBSE PMT 94]

Scanned by CamScanner

23. A standing wave having 3 nodes and 2 antinodes is formed between two atoms having a distance 1.21 Å between them. The wavelength of the standing wave is

(a) 6.50 Å (b) 2.42 Å
(c) 1.21 Å (d) 3.63 Å [CBSE PMT 98]

24. Standing waves are produced in 10 m long stretched string. If the string vibrates in 5 segments and wave velocity is 20 m/s, the frequency is

(a) 5 Hz (b) 10 Hz
(c) 2 Hz (d) 4 Hz. [CBSE PMT 97]

25. A stretched string resonates with tuning fork of frequency 512 Hz when length of the string is 0.5 m. The length of the string required to vibrate resonantly with a tuning fork of frequency 256 Hz would be

(a) 0.25 m (b) 0.5 m
(c) 1 m (d) 2 m [CBSE PMT 93]

26. If the tension and diameter of a sonometer wire of fundamental frequency n are doubled and density is halved, then its fundamental frequency will become

(a) $\frac{n}{4}$ (b) $\sqrt{2}n$
(c) n (d) $\frac{n}{\sqrt{2}}$ [CBSE PMT 01]

27. A wave of frequency 100 Hz travels along a string towards its fixed end. When this wave travels back, after reflection, a node is formed at a distance of 10 cm from the fixed end. The speed of the wave (incident and reflected) is

(a) 20 m/s (b) 40 m/s
(c) 5 m/s (d) 10 m/s [CBSE PMT 94]

28. The length of a sonometer wire AB is 110 cm. Where should the two bridges be placed from A to divide the wire in 3 segments whose fundamental frequencies are in the ratio of 1 : 2 : 3 ?

(a) 60 cm and 90 cm (b) 30 cm and 60 cm
(c) 30 cm and 90 cm (d) 40 cm and 80 cm [CBSE PMT 95]

29. A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has

(a) three nodes and three antinodes
(b) three nodes and four antinodes
(c) four nodes and three antinodes
(d) four nodes and four antinodes [CBSE PMT 91]

30. A cylindrical tube, open at both ends has fundamental frequency f in air. The tube is dipped vertically in water, so that half of it is in water. The fundamental frequency of air column is now

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$
(c) $2f$ (d) f. [CBSE PMT 92]

31. A string is cut into three parts, having fundamental frequencies n_1, n_2, n_3 respectively. Then original fundamental frequency n is related by the expression as

(a) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$ (b) $n = n_1 \times n_2 \times n_3$
(c) $n = n_1 + n_2 + n_3$ (d) $n = \frac{n_1 + n_2 + n_3}{3}$ [CBSE PMT 2K]

32. For production of beats the two sources must have

(a) different frequencies and same amplitude
(b) different frequencies
(c) different frequencies, same amplitude and same phase
(d) different frequencies and same phase. [CBSE PMT 92]

33. A source of frequency ν gives 5 beats/second when sounded with a source of frequency 200 Hz. The second harmonic of frequency 2ν of source gives 10 beats/second when sounded with a source of frequency 420 Hz. The value of ν is

(a) 205 Hz (b) 195 Hz
(c) 200 Hz (d) 210 Hz. [CBSE PMT 94]

34. A source of sound gives 5 beats per second, when sounded with another source of frequency 100 second^{-1}. The second harmonic of the source, together with a source of frequency 205 sec^{-1} gives 5 beats per second. What is the frequency of the source ?

(a) 105 second^{-1} (b) 205 second^{-1}
(c) 95 second^{-1} (d) 100 second^{-1}. [CBSE PMT 95]

35. Two waves of lengths 50 cm and 51 cm produced 12 beats per sec. The velocity of sound is

(a) 340 m/s (b) 331 m/s
(c) 306 m/s (d) 360 m/s. [CBSE PMT 99]

Scanned by CamScanner

36. Two sound waves with wavelengths 5.0 m and 5.5 m respectively, each propagates in a gas with velocity 330 m/s. We expect the following number of beats per second

(*a*) 6 (*b*) 2

(*c*) 0 (*d*) 1 [CBSE PMT 06]

37. The wave described by

$$y = 0.25\sin(10\pi x - 2\pi t)$$

where x and y are in metres and t in seconds, is a wave travelling along the

(*a*) –ve x-direction with frequency 1 Hz

(*b*) –ve x-direction with frequency π Hz and wavelength $\lambda = 0.2$ m

(*c*) +ve x-direction with frequency 1 Hz and wavelength $\lambda = 0.2$ m

(*d*) –ve x-direction with amplitude 0.25 m and wavelength $\lambda = 0.2$ m. [CBSE PMT 08]

38. Each of the two strings of length 51.6 cm and 49.1 km are tensioned separately by 20 N force. Mass per unit length of both the strings is same and equal to 1 g/m. When both the strings vibrate simultaneously the number of beats is

(*a*) 3 (*b*) 6

(*c*) 7 (*d*) 8 [CBSE PMT 09]

39. The driver of a car travelling with speed 30 m/s towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by driver is

(*a*) 500 Hz (*b*) 550 Hz

(*c*) 555.5 Hz (*d*) 720 Hz. [CBSE PMT 09]

40. Two vibrating tuning forks produce waves given by $y_1 = 4\sin 500\pi t$ and $y_2 = 2\sin 506\pi t$. Number of beats produced per minute is

(*a*) 360 (*b*) 180

(*c*) 60 (*d*) 3. [CBSE PMT 06]

41. Two trains move towards each other with the same speed. The speed of sound is 340 m/s. If the height of the tone of the whistle of one of them heard on the other changes to 9/8 times, then the speed of each train should be

(*a*) 20 m/s (*b*) 2 m/s

(*c*) 200 m/s (*d*) 2000 m/s [CBSE PMT 80]

42. A car is moving towards a high cliff. The driver sounds a horn of frequency f. The reflected sound heard by the driver has frequency $2f$. If v is the velocity of sound, then the velocity of the car, in the same velocity units, will be

(*a*) $v/\sqrt{2}$ (*b*) $v/3$

(*c*) $v/4$ (*d*) $v/2$. [CBSE PMT 04]

43. An observer moves towards a stationary source of sound with a speed 1/5th of the speed of sound. The wavelength and frequency of the source emitted are λ and f respectively. The apparent frequency and wavelength recorded by the observer are respectively

(*a*) $1.2\ f$, $1.2\ \lambda$ (*b*) $1.2\ f$, λ

(*c*) f, $1.2\ \lambda$ (*d*) $0.8\ f$, $0.8\ \lambda$. [CBSE PMT 03]

44. A vehicle, with a horn of frequency n is moving with a velocity of 30 m/s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $n + n_1$. Then (if the sound velocity in air is 300 m/s)

(*a*) $n_1 = 0.1n$ (*b*) $n_1 = 0$

(*c*) $n_1 = 10n$ (*d*) $n_1 = -0.1n$ [CBSE PMT 98]

45. A whistle revolves in a circle with angular speed $\omega = 20$ rad/sec using a string of length 50 cm. If the frequency of sound from the whistle is 385 Hz, then what is the minimum frequency heard by an observer which is far away from the centre (velocity of sound = 340 m/s) ?

(*a*) 385 Hz (*b*) 374 Hz

(*c*) 394 Hz (*d*) 333 Hz. [CBSE PMT 02]

46. Two sound sources each emitting waves of wavelength λ are fixed a given distance apart and an observer moves from one source to another with velocity u. Then number of beats heard by him

(*a*) $\frac{2u}{\lambda}$ (*b*) $\frac{u}{\lambda}$

(*c*) $\sqrt{u\lambda}$ (*d*) $\frac{u}{2\lambda}$. [CBSE PMT 2K]

47. A star, which is emitting radiation at a wavelength of 5000 Å is approaching the earth with a velocity of 1.5×10^6 m/s. The change in wavelength of the radiation as received on the earth is

(*a*) 25 Å (*b*) 100 Å

(*c*) zero (*d*) 2.5 Å. [CBSE PMT 95]

48. If the amplitude of sound is doubled and the frequency reduced to one forth, the intensity of sound at the same point will be

(*a*) increasing by a factor of 2

(*b*) decreasing by a factor of 2

(*c*) decreasing by a factor of 4

(*d*) unchanged. [CBSE PMT 89]

Scanned by CamScanner

49. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distances of 2 m and 3 m respectively from the source. The ratio of the intensities of the waves at P and Q is

(*a*) 3 : 2 (*b*) 2 : 3

(*c*) 9 : 4 (*d*) 4 : 9. [CBSE PMT 06]

50. The time of reverberation of a room A is one second. What will be the time (in seconds) of reverberation of a room, having all the dimensions double of those of room A?

(*a*) 1 (*b*) 2

(*c*) 4 (*d*) 1/2. [CBSE PMT 06]

51. Wave has simple harmonic motion whose period is 4 seconds while another wave which also possesses simple harmonic motion has its period 3 sec. If both are combined, then the resultant wave will have the period equal to

(*a*) 4 sec (*b*) 5 sec

(*c*) 12 sec (*d*) 3 sec. [CBSE PMT 93]

52. Sound waves travel at 350 m/s through warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from war air

(*a*) decreases by a factor 10

(*b*) increases by a factor 20

(*c*) increases by a factor 10

(*d*) decreases by a factor 20 [CBSE Pre 2011]

53. Two waves are represented by the equations $y_1 = a\sin(\omega t + kx + 0.57)$m and $y_2 = a\cos(\omega t + kx)$m, where x is in metre and t in sec. The phase difference between them is

(*a*) 1.0 radian (*b*) 1.25 radian

(*c*) 1.57 radian (*d*) 0.57 radian

[CBSE Pre 2011]

54. A transverse wave is represented by $y = \sin(\omega t - kx)$. For what value of the wavelength is the wave velocity equal to the maximum particle velocity?

(*a*) $\frac{\pi A}{2}$ (*b*) πA

(*c*) $2\pi A$ (*d*) A [CBSE Pre 2010]

55. A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per sec when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(*a*) 510 Hz (*b*) 514 Hz

(*c*) 516 Hz (*d*) 508 Hz [CBSE Pre 2011]

56. Two identical piano wires, kept under the same tenstion T have a fundamental frequency of 600 Hz. The fractional increase in the tension of one of the wires which will lead to occurrence of 6 beats/s when both the wires oscillate together would be

(*a*) 0.04 (*b*) 0.01

(*c*) 0.02 (*d*) 0.03 [CBSE Final 2011]

Answers and Explanations

1. (*b*) In wave propagation, energy is transmitted in the direction of propagation of the wave.

2. (*c*) The sound waves in air are longitudinal while the light waves are transverse.

3. (*b*) The velocity of sound in a gas depends on density and elasticity of the gas.

4. (*c*) Here $v = 17 \text{ kms}^{-1} = 1.7 \times 10^3 \text{ ms}^{-1}$

$$\nu = 4.2 \text{ MHz} = 4.2 \times 10^6 \text{ Hz}$$

$$\lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6}$$

$$= \mathbf{4.047 \times 10^{-4} \text{ m}}.$$

5. (*a*) Here $m = \frac{0.035}{5.5} \text{ kg m}^{-1}$, $T = 77$ N

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{77 \times 5.5}{0.035}} = \mathbf{110 \text{ ms}^{-1}}.$$

6. (*c*) As $v \propto \sqrt{T}$

$$\therefore \quad \frac{v}{2v} = \sqrt{\frac{273 + 27}{T}}$$

or $$T = 1200 \text{ K}$$

$$= \mathbf{927°C}.$$

7. (*c*) Path difference,

$$\Delta x = \frac{\lambda}{2\pi} \Delta\phi$$

$$\therefore \quad \lambda = 2\pi \frac{\Delta x}{\Delta\phi} = \frac{2\pi \times 0.40}{1.6\pi} = 0.5 \text{ m}.$$

$$\nu = \frac{v}{\lambda} = \frac{330}{0.5}$$

$$= \mathbf{660 \text{ Hz}}.$$

8. (*a*) $\Delta x = \frac{\lambda}{2\pi} \Delta\phi = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$.

Scanned by CamScanner

9. (c) Time taken in moving from maximum displacement to zero displacement is

$$\frac{T}{4} = 0.170 \text{ s} \quad \text{or} \quad T = 0.68 \text{ s}$$

Frequency,

$$\nu = \frac{1}{T} = \frac{1}{0.68} = \mathbf{1.47\ Hz.}$$

10. (a), (b) Both (a) and (b) represent harmonic waves travelling along +ve direction of X-axis.

11. (c) Here $a = 0.2$ m, $v = 360\ \text{ms}^{-1}$, $\lambda = 60$ m

$$\nu = \frac{v}{\lambda} = \frac{360}{60} = 6 \text{ Hz} \quad \text{or} \quad T = \frac{1}{6} \text{ s}$$

Equation of a harmonic wave,

$$y = a\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$$= 0.2\sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$$

Hence the correct option is (c).

12. (d) Given : $y = 0.40\cos[2000t + 0.80x]$

Comparing with standard wave equation,

$$y = a\cos[\omega t + kx]$$

$$\omega = 2\pi\nu = 2000$$

$$\therefore \quad \nu = \frac{1000}{\pi} \text{ Hz}$$

13. (c) Given : $y = 0.0015\sin(62.4x + 316t)$

But $\quad y = a\sin(\omega x + kx)$

$$\therefore \quad k = \frac{2\pi}{\lambda} = 62.4$$

or $$\lambda = \frac{2\pi}{62.4} = \frac{2\times 3.14}{62.4}$$

$$= \mathbf{0.1\ unit.}$$

14. (a) $y(x,t) = 8.0\sin(0.5\pi x - 4\pi t - \pi/4)$

$$y(x,t) = a\sin(kx - \omega t + \phi)$$

$$\therefore \quad k = 0.5\,\pi, \quad \omega = 4\pi$$

$$v = \frac{\omega}{k} = \frac{4\pi}{0.5\pi} = \mathbf{8\ m/s.}$$

15. (c) Here $\omega = 100$, $k = \frac{1}{10}$

$$\therefore \quad v = \frac{\omega}{k} = \frac{100}{1/10} = \mathbf{1000\ m/s.}$$

16. (c) Here $a = 2$ cm $= 0.02$ m, $v = 128$ m/s

$$\lambda = 4/5 = 0.8 \text{ m}, \quad k = \frac{2\pi}{\lambda} = 7.85 \text{ rad/m}$$

$$\omega = vk = 7.85 \times 128 = 1004.8 = 1005 \text{ rad/s}$$

$$\therefore \quad y = a\sin(kx - \omega t)$$

$$= (0.02)\text{m}\ \sin(7.85x - 1005t)$$

17. (b) $y = 4\sin\left[\pi\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right]$

$$y = a\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right]$$

On comparison, $a = 4$ cm

$$\frac{2\pi}{T} = \frac{\pi}{5}$$

or $$f = \frac{1}{T} = \frac{1}{10} = 0.1 \text{ Hz}$$

$$\frac{2\pi}{\lambda} = \frac{\pi}{9} \quad \text{or} \quad \lambda = 18 \text{ cm}$$

$$v = f\lambda = 0.1 \times 18 = 1.8 \text{ cm/s.}$$

Hence only option (b) is correct.

18. (d) $\quad y = y_0 \sin\frac{2\pi}{\lambda}(vt - x)$

Particle velocity,

$$u = \frac{dy}{dx} = y_0\frac{2\pi v}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$$

Maximum particle velocity,

$$u_{max} = y_0\frac{2\pi v}{\lambda}$$

But $\quad u_{max} = 2v$

or $$y_0\frac{2\pi v}{\lambda} = 2v \quad \text{or} \quad \lambda = \pi y_0$$

19. (a) $\quad y_1 = 10^{-6}\sin[100t + (x/50) + 0.5]$ m

$$y_2 = 10^{-6}\cos[100t + (x/50)] \text{ m}$$

$$= 10^{-6}\sin[100t + (x/50) + \pi/2] \text{ m}$$

$$= 10^{-6}\sin[100t + (x/50) + 1.57] \text{ m}$$

$$\therefore \quad \Delta\phi = 1.57 - 0.5$$

$$= \mathbf{1.07\ rad.}$$

20. (b) Resultant amplitude,

$$2a(1 + \cos\phi) = a$$

$$\cos\phi = -\frac{1}{2}$$

$$\therefore \quad \phi = \frac{2\pi}{3}$$

Scanned by CamScanner

21. (*b*) $x = a\cos(\omega t + \delta)$

$y = a\cos(\omega t + \alpha)$

But $\delta = \alpha + \dfrac{\pi}{2}$

$\therefore$ $x = a\cos(\omega t + \alpha + \pi/2)$

$= -a\sin(\omega t + \alpha)$

Clearly, $x^2 + y^2 = a^2$.

This is the equation of a circle.

22. (*a*) $y = A\sin(100t)\cos(0.01x)$

$$y = A\sin\left(\frac{2\pi}{T}t\right)\cos\left(\frac{2\pi}{\lambda}x\right)$$

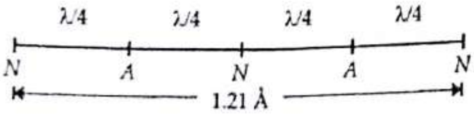
$\therefore$ $\dfrac{2\pi}{T} = 100$

or $T = \dfrac{\pi}{50}$.

$\dfrac{2\pi}{\lambda} = 0.01$ or $\lambda = 200\pi$

$$v = \frac{\lambda}{T} = \frac{200\pi}{\pi/50} = 10^4 \text{ m/s.}$$

23. (*c*)



Clearly, $4 \times \dfrac{\lambda}{4} = 1.21$ Å

or $\lambda = \mathbf{1.21}$ **Å**

24. (*a*) Frequency of standing waves in a stretched string,

$$\nu_p = \frac{p}{2l}\sqrt{\frac{T}{m}} = \frac{p}{2l}\cdot v$$

$$\nu_5 = \frac{5}{2\times 10}\times 20 = \mathbf{5\ Hz.}$$

25. (*c*) For a stretched string,

$$\nu \propto \frac{1}{l}$$

$$\frac{\nu_1}{\nu_2} = \frac{l_2}{l_1}$$

$$l_2 = \frac{\nu_1}{\nu_2}\times l_1 = \frac{512}{250}\times 0.5 = \mathbf{1.0\ m}$$

26. (*c*) $n = \dfrac{1}{LD}\sqrt{\dfrac{T}{\pi\rho}}$

$$n' = \frac{1}{L(2D)}\cdot\sqrt{\frac{2T}{\pi(\rho/2)}} = n.$$

27. (*a*) Since a node is formed at the fixed end, so

$\dfrac{\lambda}{2} = 10$ cm or $\lambda = 20$ cm

$\nu = 100$ Hz.

$v = \nu\lambda = 100 \times 0.20 = \mathbf{20\ m/s.}$

28. (*a*) $AB = 110$ cm

Frequency,

$\nu \propto \dfrac{1}{l}$ or $l \propto \dfrac{1}{\nu}$

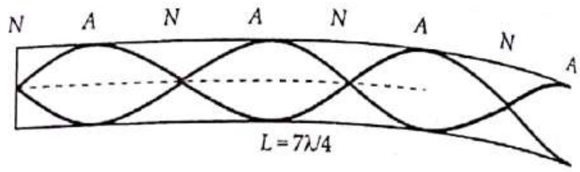
$\therefore$ $AC : CD : DB = \dfrac{1}{1} : \dfrac{1}{2} : \dfrac{1}{3} = 6 : 3 : 2$

$\therefore$ $AC = \dfrac{6}{11}\times 110 = \mathbf{60\ cm.}$

$CD = \dfrac{3}{11}\times 110 = \mathbf{30\ cm.}$

$AD = AC + CD = \mathbf{90\ cm.}$

29. (*d*) An third overtone, the length of the closed organ pipe is $7\lambda/4$. Air in the pipe will have four nodes and four antinodes as shown in the figure.



30. (*a*) Fundamental frequency of tube open at both ends,

$$f = \frac{v}{2L}$$

When the tube is half dipped in water, it behaves as tube of length $L/2$ closed at one end. Its fundamental frequency becomes

$$f' = \frac{v}{4L'} = \frac{v}{4\times(L/2)} = \frac{v}{2L} = f$$

31. (*a*) As $n \propto \dfrac{1}{l}$

and $l = l_1 + l_2 + l_3$

$\therefore$ $\dfrac{1}{n} = \dfrac{1}{n_1} + \dfrac{1}{n_2} + \dfrac{1}{n_3}$.

32. (*b*) For production of beats the sources must have different frequencies.

33. (*a*) In first case, $\nu = 200 \pm 5 = 195$ or 205 Hz.

In second case, $2\nu = 420 \pm 10$

or $\nu = 210 \pm 5 = 205$ or 215 Hz

The common value of frequency is $v = \mathbf{205\ Hz.}$

Scanned by CamScanner

34. (a) In first case, $\nu = 100 \pm 5 = 95$ or 105 Hz

In second case, $2\nu = 205 \pm 5 = 200$ or 210 Hz

$\nu = 100$ or 105 Hz

The common value of frequency is $\nu = \mathbf{105\ Hz}$.

35. (c) Beat frequency,

$$\nu_1 - \nu_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$$

$$12 = v\left[\frac{1}{50} - \frac{1}{51}\right] = \frac{v \times 1}{50 \times 51}$$

$$v = 12 \times 50 \times 51 \text{ cms}^{-1}$$

$$\therefore \quad = \mathbf{306\ ms^{-1}}.$$

36. (a) Beat frequency,

$$\nu_1 - \nu_2 = v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = 330\left[\frac{1}{5} - \frac{1}{5.5}\right]$$

$$= \frac{330 \times 0.5}{5 \times 5.5} = \mathbf{6\ Hz}.$$

37. (c) $y = 0.25 \sin(10\pi x - 2\pi t)$

$$= -0.25 \sin(2\pi t - 10\pi x)$$

$$y = a \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

$$\therefore \quad \frac{2\pi}{T} = 2\pi \quad \text{or} \quad T = 1 \text{ s}$$

$$\nu = \frac{1}{T} = 1 \text{ Hz}.$$

$$\frac{2\pi}{\lambda} = 10\pi \quad \text{or} \quad \lambda = 0.2 \text{ m}$$

Thus the wave is travelling along +ve x-axis with frequency 1 Hz and $\lambda = 0.2$ m.

38. (c) Frequency,

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$$\nu_2 - \nu_1 = \frac{1}{2}\sqrt{\frac{T}{m}}\left[\frac{1}{L_2} - \frac{1}{L_1}\right]$$

$$= \frac{1}{2}\sqrt{\frac{20}{10^{-3}}}\left[\frac{100}{49.1} - \frac{100}{51.6}\right]$$

$$= \frac{1}{2} \times \sqrt{2} \times 10^4\left[\frac{51.6 - 49.1}{49.1 \times 51.6}\right]$$

$$= \frac{1.414 \times 10^4 \times 2.5}{2 \times 50 \times 50}$$

$$\approx \mathbf{7\ beats/s}.$$

39. (d) $\nu' = \frac{v + v_0}{v - v_s} \times \nu = \frac{330 + 30}{330 - 30} \times 600 = \mathbf{720\ Hz}.$

40. (b) $y_1 = 4 \sin 500\pi t$, $y_2 = 2 \sin 506\pi t$

$$\omega_1 = 2\pi\nu_1 = 500\pi$$

$$\therefore \quad \nu_1 = 250 \text{ Hz}$$

$$\omega_2 = 2\pi\nu_2 = 506\pi$$

$$\therefore \quad \nu_2 = 253 \text{ Hz}$$

Beat frequency $= \nu_2 - \nu_1 = 3$ beats/s

Number of beats produced per minute

$$= 3 \times 60 = \mathbf{180}$$

41. (a) Her $\nu' = \frac{9}{8}\nu$

As the source and observer are moving towards each other with the same speed,

$$\nu' = \frac{v + v_0}{v - v_s}\nu = \frac{v + u}{v - u} \times v$$

$$\frac{9}{8}\nu = \frac{340 + u}{340 - u} \times \nu$$

On solving, $u = \mathbf{20\ ms^{-1}}$.

42. (b) At first the car is the source and cliff the observer,

$$\nu' = \frac{v}{v - v_s}\nu$$

At second the cliff is source of frequency ν' and observer is the car.

$$\nu'' = \frac{v + v_0}{v}\nu' \quad \text{or} \quad 2\nu = \frac{v + v_0}{v - v_s}.\nu$$

As $\quad v_0 = v_s$,

$$2(v - v_0) = v + v_0$$

$$v_0 = \frac{v}{3}.$$

43. (b) Apparent frequency,

$$f' = \frac{v + v_0}{v}f = \frac{v + (1/5)v}{v}f = \mathbf{1.2\ f}$$

Wavelength does not change due to the motion of the observer.

44. (b) As the vehicle moves in direction perpendicular to the line joining the observer and the vehicle, the component of the velocity along this direction zero. There is no Doppler effect and the frequency of sound remains unchanged.

$$n + n_1 = n \quad \text{or} \quad n_1 = 0$$

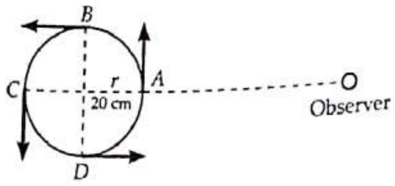
45. (b) Speed of source (whistle),

$$v_s = r\omega = 0.50 \times 20 = 10 \text{ m/s}$$

$$\nu = 385 \text{ Hz}, \quad v = 340 \text{ m/s}$$

Scanned by CamScanner

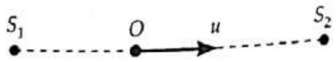
When the whistle (at position B) is moving away from the observer, the apparent frequency is minimum,



$$v' = \frac{v}{v+v_s} \times v$$

$$= \frac{340}{340+10} \times 385 = 374 \text{ Hz.}$$

46. (*a*)



Number of extra waves received from S_2 per second

$$= +\frac{u}{\lambda}$$

Number of lesser waves received from S_1 per second

$$= -\frac{u}{\lambda}$$

$\therefore$ Beat frequency $= \frac{u}{\lambda} - \left(-\frac{u}{\lambda}\right) = \frac{2u}{l}$.

47. (*a*) Here $\lambda = 5000\,\text{Å}$, $c = 3 \times 10^8 \text{ ms}^{-1}$,

$$v = 1.5 \times 10^6 \text{ ms}^{-1}$$

For the star moving towards the earth,

$$\Delta\lambda = -\frac{v}{c}.\lambda$$

$$= -\frac{1.5 \times 10^6}{3 \times 10^8} \times 5000\,\text{Å}$$

$$= -25\,\text{Å}.$$

48. (*c*) $I \propto v^2 A^2$

$\therefore$ $I' \propto \left(\frac{v}{4}\right)^2 (2A)^2$

$$I' = \frac{I}{4}$$

49. (*c*) $I \propto \frac{1}{r^2}$

$\therefore$ $\frac{I_1}{I_2} = \left(\frac{3}{2}\right)^2 = 9:4.$

50. (*b*) Reverberation time,

$$T = \frac{0.16V}{aS}$$

where V = volume, S = surface area and a = average absorption coefficient.

$\therefore$ $T \propto \frac{V}{S}$

$$\frac{T_2}{T_1} = \frac{V_2}{V_1} \cdot \frac{S_1}{S_2}$$

$$= \frac{8V}{V} \cdot \frac{S}{4S} = 2$$

$$T_2 = 2T_1 = 2 \times 1 = 2 \text{ s.}$$

51. (*c*) On superposition of the two waves, beats are produced.

Beat frequency $= \frac{1}{T_2} - \frac{1}{T_1} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$\therefore$ Time period = 12 s.

53. (*c*) $\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

$\therefore$ $\lambda_2 = \lambda_1 \cdot \frac{v_2}{v_1} = \lambda_1 \times 10$

53. (*a*) $y_1 = a\sin(\omega t + kx + 0.57)\text{m}$

$$y_2 = a\cos(\omega t + kx)\text{m}$$

$$= a\sin\left(\omega t + kx + \frac{\pi}{2}\right)\text{m}$$

$\therefore$ Phase difference $= \frac{\pi}{2} - 0.57$

$$= \frac{3.14}{2} - 0.57 = 1.57 - 0.57 = 1 \text{ rad}$$

54. (*c*) Maximum particle velocity = Wave velocity

$$\omega A = \frac{\omega}{k}$$

or $k = \frac{2\pi}{\lambda} = \frac{1}{A}$

$\therefore$ $\lambda = 2\pi A$

55. (*d*) Frequency of piano string = 512 ± 4 = 508 or 516 Hz. When tension in the piano string is increased, its frequency increases. As the beat frequency decreases, so frequency of the string < 512 Hz ⇒ 508 Hz.

56. (*c*) $v = K\sqrt{T}$

$\therefore$ $\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$

or $\frac{\Delta T}{T} = \frac{2\Delta v}{v}$

$$= \frac{2 \times 6}{600} = 0.2$$

Scanned by CamScanner

Delhi PMT and VMMC Entrance Examination

1. Light can travel in vacuum but not sound, because
(a) speed of sound is very slow than light
(b) light waves are electromagnetic in nature
(c) sound waves are electromagnetic in nature
(d) none of these. [DPMT 92]

2. Velocity of sound in air is
(a) faster in dry air than in moist air
(b) directly proportional to temperature
(c) directly proportional to pressure
(d) none of these. [DPMT 94]

3. If at same temperature and pressure, the desitities for two diatomic gases are respectively d_1 and d_2, then the ratio of velocities of sound in these gases will be
(a) $\sqrt{\frac{d_2}{d_1}}$ (b) $d_1 d_2$
(c) $\sqrt{\frac{d_1}{d_2}}$ (d) $\sqrt{d_1 d_2}$. [DPMT 96]

4. If the density of oxygen is 16 times that of hydrogen, what will be the corresponding ratio of their velocities of sound waves ?
(a) 1 : 4 (b) 16 : 1
(c) 4 : 1 (d) 1 : 16. [DPMT 93]

5. At what temperature, the speed of sound in air will become double of its value at 27°C ?
(a) 54°C (b) 627°C
(c) 327°C (d) 927°C
[DPMT 95, 03 ; VMMC 05]

6. The speed of a wave in a medium is 760 m/s. If 3600 waves are passing through a point in the medium in 2 min, then its wavelength is
(a) 13.8 m (b) 41.5 m
(c) 25.3 m (d) 57.2 m. [DPMT 96]

7. A tuning fork makes 256 vibrations per sec in air. When the velocity of sound is 330 m/s, then wavelength of the tone emitted is
(a) 0.56 m (b) 1.11 m
(c) 0.89 m (d) 1.29 m. [DPMT 94]

8. When sound travels from air to water, which parameter does not change ?
(a) wavelength (b) frequency
(c) velocity (d) all of these. [DPMT 2K, 07]

9. An underwater sonar source operating at a frequency of 60 kHz directs its beam towards the surface. If velocity of sound in air is 330 m/s, wavelength and frequency of the waves in air are
(a) 5.5 mm, 60 kHz (b) 330 m, 60 kHz
(c) 5.5 mm, 30 kHz (d) 5.5 mm, 80 kHz.
[DPMT 04]

10. If the equation of transverse wave is
$$y = 5\sin 2\pi\left(\frac{t}{0.04} - \frac{x}{40}\right)$$
where distance is in cm and time in sec, then the wavelength of wave will be
(a) 20 cm (b) 40 cm
(c) 60 cm (d) none of these.
[DPMT 95, 03]

11. A wave is expressed by the equation
$$y = 0.5 \sin \pi(0.01x - 3t)$$
where x, y are in metres and t in seconds. The speed of propagation will be
(a) 150 m/s (b) 300 m/s
(c) 350 m/s (d) 250 m/s. [VMMC 05]

12. If the wave equation is
$$y = 0.08 \sin \frac{2\pi}{\lambda}(150t - x) \text{ s}$$
Then velocity of the wave will be
(a) 150 units (b) $150\sqrt{2}$ units
(c) 300 units (d) $300\sqrt{2}$ units. [DPMT 99]

13. A transverse wave is expressed as : $y = y_0 \sin 2\pi ft$. For what value of λ, the maximum particle velocity is equal to 4 times the wave velocity ?
(a) $y_0\pi/2$ (b) $2y_0\pi$
(c) $y_0\pi$ (d) $y_0\pi/4$. [DPMT 05]

14. The phase difference between two points separated by 1 m in a wave of frequency 120 Hz is 90°. The wave velocity will be
(a) 720 m/s (b) 480 m/s
(c) 240 m/s (d) 180 m/s [VMMC 02]

15. In a transverse progressive wave of amplitude A, the maximum particle velocity is four times its wave velocity. The wavelength of the wave is
(a) $\frac{\pi A}{4}$ (b) πA
(c) $\frac{\pi A}{2}$ (d) $2\pi A$.
[DPMT 92]

Scanned by CamScanner

16. Two waves each of amplitude a and frequency f have a phase difference $\pi/2$. The amplitude and frequency of resultant wave due to their superposition will be

(a) $\frac{a}{\sqrt{2}}, \frac{f}{2}$ (b) $\frac{a}{\sqrt{2}}, f$

(c) $2a, \frac{f}{2}$ (d) $\sqrt{2}a, f$. [VMMC 03]

17. Two waves are represented by

$$y_1 = a\sin\left(\omega t + \frac{\pi}{6}\right)$$

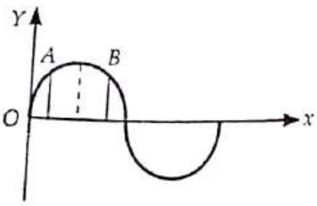
and $$y_2 = a\cos\omega t$$

What will be their resultant amplitude ?

(a) a (b) $\sqrt{2}a$

(c) $\sqrt{3}a$ (d) $2a$. [VMMC 03]

18. Standing wave is shown in the figure, point A and B reach at $y = 0$



(a) in same time

(b) time difference is $T/4$ (B is lagging)

(c) time difference is $T/2$ (B is lagging)

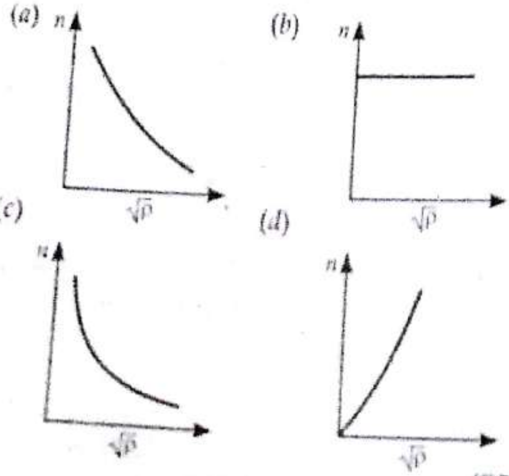
(d) time difference is $T/4$ (A is lagging) [DPMT 07]

19. The distance between the successive nodes is

(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$

(c) λ (d) 2λ [VMMC 07]

20. The correct graph between the frequency n and square root of density (ρ) of a wire, keeping its length, radius and tension constant, is



[DPMT 06]

21. A stretched string is vibrating according to equation, $y = 5\sin\left(\frac{\pi x}{2}\right)\cos 4\pi t$

where y and a are in cm and t in secs. The distance between two consecutive nodes on the strings is

(a) 2 cm (b) 4 cm

(c) 8 cm (d) 16 cm. [DPMT 04]

22. A wave of frequency 100 Hz travels along a string towards its fixed end. When this wave travels back, after reflection, a node is formed at a distance of 10 cm from the fixed end. The speed of the wave (incident and reflected) is

(a) 5 m/s (b) 20 m/s

(c) 10 m/s (d) 40 m/s. [DPMT 97]

23. A uniform rope of mass 0.1 kg and length 2.5 m hangs from ceiling. The speed of transverse wave in the rope at upper end and at a point 0.5 m distance from lower end will be

(a) 5 m/s, 2.24 m/s (b) 10 m/s, 3.23 m/s

(c) 7.5 m/s, 1.2 m/s (d) none of these. [VMMC 05]

24. Fundamental frequency of sonometer wire is n. If the length, tension and diameter of wire are tripled, the new fundamental frequency is

(a) $n/\sqrt{3}$ (b) $n/3$

(c) $n\sqrt{3}$ (d) $n/3\sqrt{3}$ [DPMT 02]

25. The tension in vibrating stretched piano wire is 10 N. To double the frequency, the tension in wire must be

(a) 5 N. (b) 20 N

(c) 40 N (d) 80 N. [VMMC 07]

26. The first overtone of a stretched wire of given length is 320 Hz. The first harmonic is

(a) 320 Hz (b) 160 Hz

(c) 480 Hz (d) 640 Hz. [DPMT 04]

27. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel ?

(a) 5.06 km/s (b) 7.06 km/s

(c) 6.06 km/s (d) 8.06 km/s. [DPMT 95]

28. The disc of siren has n holes and the frequency of its rotation is 300 r.p.m. It produces a note of wavelength 2.4 m, when the velocity of sound in air is 360 m/s. The value of n is

(a) 8 (b) 24

(c) 12 (d) 30. [DPMT 91]

Scanned by CamScanner

29. The frequency of tuning fork is 256 Hz. It will not resonate with a fork of frequency

(a) 768 Hz (b) 738 Hz

(c) 512 Hz (d) 256 Hz. [DPMT 92 ; VMMC 04]

30. Resonance is an example of

(a) tuning fork (b) free-vibration

(c) forced-vibration (d) damped vibration. [DPMT 97, 99]

31. A resonance air column of length 40 cm resonates with a tuning fork of frequency 450 Hz. Ignoring end correction, the velocity of sound in air will be

(a) 720 m/s (b) 920 m/s

(c) 820 m/s (d) 1020 m/s. [DPMT 02]

32. What is minimum length of a tube, open at both ends, that resonates with tuning fork of frequency 350 Hz ? [velocity of sound in air = 350 m/s]

(a) 50 cm (b) 100 cm

(c) 75 cm (d) 25 cm. [DPMT 04]

33. In a resonance tube the first resonance with a tuning fork occurs at 16 cm and second at 49 cm. If the velocity of sound is 330 m/s, the frequency of tuning fork is

(a) 500 (b) 300

(c) 330 (d) 165. [DPMT 02]

34. If the length of a closed organ pipe is 1 m and velocity of sound is 330 m/s, then the frequency for the second note is

(a) $4 \times \frac{330}{4}$ Hz (b) $2 \times \frac{330}{4}$ Hz

(c) $3 \times \frac{330}{4}$ Hz (d) $2 \times \frac{4}{330}$ Hz. [DPMT 97]

35. The frequency of vibrating air column in closed organ pipe is n. If its length be doubled and radius halved, its frequency will be nearly

(a) n (b) $n/2$

(c) $2n$ (d) $4n$. [VMMC 07]

36. For beats to be produced

(a) frequency of sources should be different and amplitude should be same

(b) frequency of sources should be same and amplitude should be different

(c) frequency of sources should be different and amplitude should be different

(d) frequency of sources should be same and amplitude should be same. [DPMT 07]

37. A source of sound gives 5 beats per second, when sounded with another source of frequency 100 sec^{-1}. The second harmonic of the source, together with a source of frequency 205 sec^{-1} gives 5 beats per second. What is frequency of the source ?

(a) 95 sec^{-1} (b) 105 sec^{-1}

(c) 100 sec^{-1} (d) 205 sec^{-1}. [DPMT 95]

38. Two closed organ pipes, when sounded simultaneously gave 4 beats per sec. If longer pipe has a length of 1 m, then length of shorter pipe is ($v = 300$ m/s)

(a) 80 cm (b) 94.9 cm

(c) 90 cm (d) 185.5 cm [DPMT 93]

39. 16 tuning forks are arranged in the order of increasing frequencies. Any two successive forks give 8 beats per sec when sounded together. If the last fork gives the octave of the first, then the frequency of the first fork is

(a) $n = 120$ (b) $n = 180$

(c) $n = 160$ (d) $n = 220$. [DPMT 91]

40. 50 tunning forks are arranged in increasing order of their frequencies such that each gives 4 beats/sec with its previous tuning fork. If the frequency of the last fork is octave of the first, then the frequency of the first tuning fork is

(a) 200 Hz (b) 204 Hz

(c) 196 Hz (d) none of these. [DPMT 05]

41. Two waves of wavelength 50 cm and 51 cm produce 12 beats per sec. The velocity of sound will be

(a) 340 m/s (b) 332 m/s

(c) 153 m/s (d) 306 m/s. [VMMC 04]

42. Number of beats between A and B is 5 Hz and between B and C is 3 Hz. Beat frequency between A and C may be

(a) 1 (b) 2

(c) 8 (d) none of these. [DPMT 08]

43. Doppler's effect in sound takes place when source and observer are

(a) stationary

(b) moving with same velocity

(c) in relative motion

(d) none of the above. [VMMC 07]

Scanned by CamScanner

44. The apparent frequency in Doppler's effect does not depend upon

(*a*) speed of the observer

(*b*) distance between observer and source

(*c*) speed of the source

(*d*) frequency of the source. [DPMT 05]

45. Source of sound and observer are moving towards each other, the observer will hear

(*a*) high frequency, high wavelength

(*b*) low frequency, low wavelength

(*c*) high frequency, low wavelength

(*d*) low frequency, high wavelength. [DPMT 07]

46. An observer is moving away from a sound source of frequency 100 Hz. If the observer is moving with a velocity of 49 m/sec and the speed of the sound in air is 330 m/sec, then the apparent frequency is

(*a*) 85 Hz (*b*) 100 Hz

(*c*) 91 Hz (*d*) 149 Hz. [DPMT 93]

47. A source and an observer move away from each other, with a velocity of 10 m/s with respect to ground. If observer finds the frequency of sound coming from the source as 1950 Hz, then original frequency of source is (velocity of sound in air = 340 m/s)

(*a*) 1950 Hz (*b*) 2132 Hz

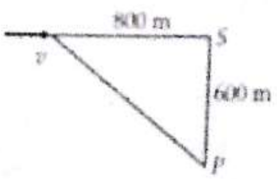
(*c*) 2068 Hz (*d*) 2486 Hz. [DPMT 94]

48. A source of frequency 1 kHz is moving towards a stationary observer with velocity of 0.9 times that of sound. What is the frequency heard by the observer ?

(*a*) 5 kHz (*b*) 15 kHz

(*c*) 10 kHz (*d*) 17 kHz. [DPMT 2K]

49. A person *P* is 600 m away from the station when train is approaching station with 72 km/h, it blows a whistle of frequency 800 Hz when 800 m away from the station. Find the frequency heard by the person. Speed of sound = 340 ms^{-1}.



(*a*) 800 Hz (*b*) 839.5 Hz

(*c*) 829.5 Hz (*d*) 843.5 Hz. [DPMT 06]

50. A body is moving forwards and backward. Change in frequency observed of source is 2%. What is velocity of the body ? (Speed of sound is 300 m/s)

(*a*) 6 m/s (*b*) 2 m/s

(*c*) 2.5 m/s (*d*) 3 m/s. [DPMT 08]

51. An astronaut is approaching the moon. He sends out a radio signal of frequency 5000 MHz and the frequency of echo is different from that of the original frequency by 100 kHz. His velocity of approach with respect to the moon is

(*a*) 2 km/s (*b*) 3 km/s

(*c*) 4 km/s (*d*) 5 km/s. [VMMC 06]

52. The pitch of a note depends upon its

(*a*) speed (*b*) amplitude

(*c*) frequency (*d*) wavelength. [DPMT 98]

53. Reverberation time can't be controlled by

(*a*) temperature

(*b*) size of window

(*c*) volume of room

(*d*) changing carpet. [DPMT 07]

54. Pressure variation of mechanical wave depends upon as

(*a*) $\propto$ intensity

(*b*) independent of intensity

(*c*) $\propto \dfrac{1}{\text{intensity}}$

(*d*) none of these. [DPMT 08]

55. The longitudinal wave can be observed in

(*a*) elastic media

(*b*) inelastic media

(*c*) both (*a*) and (*b*)

(*d*) none of these [DPMT 2011]

56. The two waves of the same frequency moving in the same direction give rise to

(*a*) beats

(*b*) interference

(*c*) stationary waves

(*d*) none of these [DPMT 2011]

Scanned by CamScanner

Answers and Explanations

1. (*a*) Light can travel in vacuum because light waves are electromagnetic in nature.

2. (*b*) Velocity of sound $\propto \sqrt{T}$. It is independent of pressure and increases with humidity.

3. (*a*) $v = \sqrt{\dfrac{\gamma P}{d}} \quad \therefore \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{d_2}{d_1}}$.

4. (*a*) $\dfrac{v_O}{v_H} = \sqrt{\dfrac{d_H}{d_O}} = \sqrt{\dfrac{1}{16}} = \dfrac{1}{4} = \mathbf{1:4}$.

5. (*d*) As $v \propto \sqrt{T}$

$$\therefore \quad \frac{v}{2v} = \sqrt{\frac{273+27}{T}}$$

$$T = 1200 \text{ K} = \mathbf{927\ ^\circ C}$$

or

6. (*c*) Number of waves crossing a point per second,

$$\nu = \frac{3600}{2 \times 60} = 30$$

$$\lambda = \frac{v}{\nu} = \frac{760}{30} = \mathbf{25.3\ m}.$$

7. (*d*) $\lambda = \dfrac{v}{\nu} = \dfrac{330}{256} = \mathbf{1.29\ m}$.

8. (*b*) Frequency remains unchanged when sound travels from air to water.

9. (*a*) Frequency in air will also be 60 kHz.

$$\lambda_a = \frac{v_a}{\nu} = \frac{330}{60 \times 10^3} = 5.5 \times 10^{-3} \text{ m}$$

$$= \mathbf{5.5\ mm}.$$

10. (*b*) $y = 5\sin 2\pi\left[\dfrac{t}{0.04} - \dfrac{x}{40}\right]$

$$y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$$

$$\therefore \quad \lambda = 40 \text{ cm}.$$

11. (*b*) $y = 0.5\sin\pi(0.01x - 3t) = 0.5\sin(0.01\pi x - 3\pi t)$

$$y = A\sin(\omega x - kt)$$

$$\omega = 0.01\ \pi, \quad k = 3\pi$$

$$v = \frac{\omega}{k} = \frac{3\pi}{0.01\pi} = \mathbf{300\ ms^{-1}}.$$

12. (*a*) $y = A\sin\dfrac{2\pi}{\lambda}(vt - x)$

$$y = 0.08\sin\frac{2\pi}{\lambda}(150t - x)$$

$$\therefore \quad v = \mathbf{150\ units}.$$

13. (*a*) Max. particle velocity = 4 × wave velocity

$$y_0\omega = 4v$$

or $\quad y_0 \times 2\pi\nu = 4\nu\lambda$

or $\quad \lambda = y_0\pi/2$

14. (*b*) $\Delta x = 1$ m, $\Delta\phi = 90^\circ = \dfrac{\pi}{2}$ rad, $\nu = 120$ Hz

As $\quad \Delta\phi = \dfrac{2\pi}{\lambda} \times \Delta x$

or $\quad \dfrac{\pi}{2} = \dfrac{2\pi}{\lambda} \times 1$

or $\quad \lambda = 4$ m

$$v = \nu\lambda = 120 \times 4 = \mathbf{480\ m/s}.$$

15. (*c*) Max. particle velocity = 4 × wave velocity

or $\quad \omega A = 4 \times v$

or $\quad 2\pi\nu A = 4 \times \nu\lambda$

$$\lambda = \frac{\pi A}{2}.$$

16. (*d*) $y_1 = a\sin 2\pi ft$

$$y_2 = a\sin\left(2\pi ft + \frac{\pi}{2}\right)$$

$$\therefore \quad y = y_1 + y_2$$

$$= 2a\sin\left(2\pi ft + \frac{\pi}{4}\right)\cos\frac{\pi}{4}$$

$$= \frac{2a}{\sqrt{2}}\sin\left(2\pi ft + \frac{\pi}{4}\right) = \sqrt{2}a\sin\left(2\pi ft + \frac{\pi}{4}\right)$$

Hence the resultant wave has amplitude $\sqrt{2}a$ and frequency f.

17. (*c*) $y_1 = a\sin\left(\omega t + \dfrac{\pi}{6}\right)$

$$y_2 = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$$

Phase difference,

$$\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$A = \sqrt{a^2 + a^2 + 2a \times a\cos\frac{\pi}{3}} = \sqrt{3}a.$$

18. (*a*) Both A and B have same amplitude. In a standing wave, all particles cross the mean position at the same time.

19. (*b*) The distance between two successive nodes is $\lambda/2$.

Scanned by CamScanner

20. (*c*) $\eta = \frac{1}{2l}\sqrt{\frac{T}{A\rho}}$

For l, T, A to constant, $n \propto \frac{1}{\sqrt{\rho}}$

Hence the correct graph between n and $\sqrt{\rho}$ is the one given in option (*c*).

21. (*a*) $y = 5\sin\left(\frac{\pi x}{2}\right)\cos 4\pi t$

$$y = a\sin kx \cos \omega t$$

$$\therefore \quad k = \frac{2\pi}{\lambda} = \frac{\pi}{2}$$

or $\lambda = \mathbf{4\ cm}$.

22. (*b*) Distance between two successive nodes

$$= \frac{\lambda}{2} = 10 \text{ cm}$$

or $\lambda = 20$ cm

$$v = \nu\lambda = 100 \times 20 = 2000 \text{ m/s} = \mathbf{20\ m/s}.$$

23. (*a*) Tension in the rope at a point x metre from the lower end,

$$T = \text{Weight of } x \text{ metre rope}$$

$$= mx \times g \quad [m = \text{mass per unit length}]$$

$$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{mxg}{m}} = \sqrt{xg}$$

At $x = 0.5$ m, $g = 10 \text{ m/s}^2$

$$v = \sqrt{0.5 \times 10} = \sqrt{15} = \mathbf{2.24\ m/s}$$

At the upper end,

$$x = \mathbf{2.5\ m}.$$

$$v = \sqrt{2.5 \times 10} = \mathbf{5\ m/s}.$$

24. (*d*) $n = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$

$$n' = \frac{1}{3L \times 3D}\sqrt{\frac{3T}{\pi\rho}} = \frac{1}{3\sqrt{3}}n.$$

25. (*d*) $\nu \propto \sqrt{T}$

$$\frac{\nu}{2\nu} = \sqrt{\frac{10}{T'}} \quad \text{or} \quad T' = \mathbf{40\ N}.$$

26. (*b*) First overtone $= 2\nu = 320$ Hz

$\therefore$ First harmonic $= \nu = \mathbf{160\ Hz}$.

27. (*a*) $v = 5.06$ km/s. Refer to the answer of NCERT exercise 15.16.

28. (*d*) Frequency of the note produced

$$= \frac{v}{\lambda} = \frac{360}{2.4} = 150 \text{ Hz}$$

If number of harmonic is n, then

$$n \times \frac{300}{60} = 150$$

$$n = \mathbf{30}.$$

29. (*b*) Tuning fork of 256 Hz will resonate with fork of frequencies,

$$1 \times 256,\ 2 \times 256,\ 3 \times 256, \text{ etc.}$$

or 256 Hz, 512 Hz, 768 Hz, etc.

Fork of 738 Hz will not resonate.

30. (*c*) Resonance is a special case of forced vibrations.

31. (*a*) Here $\lambda = 4l = 4 \times 40 \text{ cm} = 1.6$ m

$$v = \nu\lambda = 450 \times 1.6 = \mathbf{720\ m/s}.$$

32. (*a*) Fundamental frequency of open pipe

$$= \frac{v}{2L}$$

$$350 = \frac{350}{2L}$$

$$L = \frac{1}{2} \text{ m} = \mathbf{50\ cm}.$$

33. (*a*) $v = 2\nu(l_2 - l_1)$

$$330 = 2\nu(0.49 - 0.16)$$

$$\nu = \frac{330}{2 \times 0.33} = \mathbf{500\ Hz}.$$

34. (*c*) Frequency of second note of a closed pipe,

$$\nu = \frac{3v}{4L} = \frac{3 \times 330}{4 \times 1} \text{ Hz}$$

35. (*b*) For a closed organ pipe,

$$n = \frac{v}{4L}$$

$$n \propto \frac{1}{L}$$

$$\frac{n'}{n} = \frac{L}{2L} \quad \text{or} \quad n' = \frac{n}{2}$$

Radius has no effect on frequency.

36. (*a*) For beats formation, frequency of sources should be slightly different and amplitude should be same.

37. (*b*) Let the frequency of the source be ν. Then

$$\nu = 100 \pm 5 = 95 \text{ or } 105 \text{ Hz}$$

In second case

$$2\nu = 205 \pm 5 = 200 \text{ or } 210 \text{ Hz}.$$

or $\nu = 100$ or 105 Hz

Hence common $\nu = \mathbf{105\ Hz}$.

Scanned by CamScanner

38. (*b*) $\nu = \frac{v}{4L} = \frac{300}{4 \times 1} = 75$ Hz.

For shorter pipe,

$$\nu + 4 = \frac{300}{4L'}$$

$$75 + 4 = \frac{300}{4L'}$$

or $$L' = \frac{300}{4 \times 79} \text{ m} = \mathbf{94.9\ cm.}$$

39. (*a*) Let the frequency of first fork $= \nu$

Frequency of second fork $= \nu + 8$

Frequency of third fork $= \nu + 2 \times 8$

Frequency of 16th fork $= \nu + 15 \times 8$

But the frequency of the last is the octave of the first.

$$2\nu = \nu + 15 \times 8 \quad \text{or} \quad \nu = \mathbf{120\ Hz.}$$

40. (*c*) As in the above problem,

$$2\nu = \nu + 49 \times 4$$

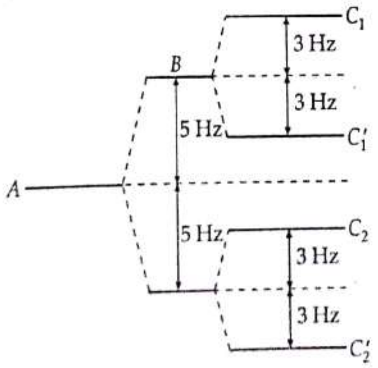
$$\nu = \mathbf{196\ Hz.}$$

41. (*d*) Beat frequency $= \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$

$$12 = v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = v\left[\frac{1}{0.50} - \frac{1}{0.51}\right]$$

$$v = \frac{12 \times 0.50 \times 0.51}{0.01} = \mathbf{306\ m/s.}$$

42. (*b*), (*c*)



$$A \sim C_1 = 8 \text{ Hz}$$

$$A \sim C_1' = 2 \text{ Hz}$$

Same holds for C_2 and C_2'

43. (*c*) Doppler's effect in sound takes place when source and observer are in relative motion.

44. (*b*) Apparent frequency in Dopper's effect depends on frequency of source, direction and velocity of source and observer. It does not depend on the distance between observer and source.

45. (*c*) When source and observer approach each other, frequency of sound increases while wavelength decreases.

46. (*a*) $\nu' = \frac{v - v_0}{v_0} \times \nu = \frac{330 - 49}{330} \times 100$ Hz

$$= \frac{28100}{330} = \mathbf{85\ Hz.}$$

47. (*c*) $\nu' = \frac{v - v_0}{v - v_s} \times \nu$

$$= \frac{340 - (-10)}{340 - 10} \times 1950 = \frac{350}{330} \times 1950$$

$$= 1.06 \times 1950 = \mathbf{2068\ Hz.}$$

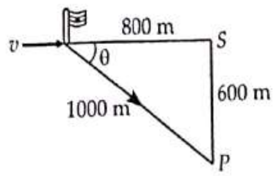
48. (*c*) $\nu' = \frac{v}{v - v_s} \times \nu = \frac{v}{v - 0.9v} \times 1$ kHz

$$= 10 \text{ kHz.}$$

49. (*b*) $\nu' = \frac{v}{v - v_s \cos\theta} \times \nu$

$$v_s = 72 \text{ kmh}^{-1} = 72 \times \frac{5}{18}$$

$$= 20 \text{ ms}^{-1}$$



$$\cos\theta = \frac{800}{1000} = 0.8$$

$$v_s \cos\theta = 20 \times 0.8 = 16 \text{ ms}^{-1}$$

$$\nu' = \frac{340}{340 - 16} \times 800 = \mathbf{839.5\ Hz.}$$

50. (*d*) When the source is moving forward towards the observer, the apparent frequency is

$$f_1 = \frac{v}{v - v_s} \times f$$

When source moves backwards,

$$f_2 = \frac{v}{v + v_s} \times f$$

$$f_2 - f_1 = fv\left[\frac{1}{v + v_s} - \frac{1}{v - v_s}\right]$$

$$= fv\left[\frac{-2v_s}{v^2 - v_s^2}\right]$$

Scanned by CamScanner

As $v_s << v$, so

$$\frac{f_2 - f_1}{f} = \left|\frac{2v_s}{v}\right| = \frac{2}{100}$$

$$v_s = \frac{v}{100} = \frac{300}{100} = 3 \text{ m/s.}$$

51. (*a*) $\frac{\Delta \nu}{\nu} = 2\frac{v}{c}$

$$v = \frac{\Delta \nu}{2\nu} c$$

$$= \frac{100 \times 10^3 \times 3 \times 10^8}{2 \times 5000 \times 10^6} = 3000 \text{ m/s}$$

$$= 3 \text{ km/s.}$$

52. (*d*) The pitch of a note depends on its frequency.

53. (*d*) Reverberation time is the time which sound takes to fall in intensity to one millionth (10^{-6}) part of its original intensity.

54. (*c*) Pressure amplitude vibration in a medium $\propto$ Amplitude of the wave (*A*)

Hence pressure variation $\propto$ intensity of the wave.

55. (*a*) Longitudinal waves can be formed only in elastic media.

56. (*b*) When two waves of the same frequency moving with the same speed in the same direction superpose on each other, they produce interference of waves.

Sample Value Based Questions

Chapter 9 : MECHANICAL PROPERTIES OF SOLIDS

Question 1

One day, two brothers Sonu and Monu decided to go market by Metro rail. They were waiting at the platform for the train. Suddenly the younger brother Monu noticed that the railway track had a special shape like I. *He asked his elder brother Sonu, who was a science student of class XI, why the track had such a particular shape. Sonu explained him the scientific reason behind this.*

Answer the following questions based on the above information :

(*a*) *What are the values displayed by Monu in his actions ?*

(*b*) *Why are girders given* I *shape ?*

Answer

(*a*) Keen observer, quest for scientific knowledge.

(*b*) I-shaped cross-section provides a large load bearing surface and enough depth to prevent buckling. Also it reduces the weight of the girder without sacrificing its strength.

Question 2

Ramesh and Suresh were going to their school in the school bus. On their way, they found that a bridge over Yamuna river had collapsed. They saw that a local bus had fallen into the river. Ramesh and Suresh were good swimmers. They immediately came out of their bus and jumped into the river. They did their best to save many lives.

(*a*) *What were the values displayed by Ramesh and Suresh ?*

(*b*) *Why are bridges declared unsafe after long use ?*

Answer

(*a*) Empathy, brave, compassion and concern for society.

(*b*) On account of long use, a bridge develops elastic fatigue and there appears a permanent change in its structure. This change may sometimes exceed elastic limit and the bridge may collapse.

Scanned by CamScanner

Chapter 10 : MECHANICAL PROPERTIES OF FLUIDS

Question 3

Rohit saw a street vendor blowing big bubbles from soap solution. He was joyful on seeing a pool of big soap bubbles in air. He asked his grandfather, who was a retired science teacher, could such bubbles be formed from ordinary water also ? His grandfather explained him that such large bubbles cannot be formed from ordinary water.

(a) *What were the values displayed by Rohit here ?*

(b) *Why are soap-bubbles easier to blow and long-lasting than water bubbles ?*

Answer

(a) Keen observer, quest for scientific knowledge.

(b) A soap solution has smaller surface tension than water. For the same radius, a soap bubble will have less excess pressure inside than a water bubble and will need less surface energy to balance it. Hence a soap bubble is easier to blow and long-lasting than water bubbles.

Question 4

Sunil's little sister was crying. He tried every trick to calm her but failed. Then he took a piece of camphor and placed it on water surface. On seeing the camphor piece dancing on water surface, the little sister stopped crying and began to enjoy it.

(a) *What are the values displayed by Sunil here ?*

(b) *Why a small piece of camphor dance about on the water surface ?*

Answer

(a) Sense of responsibility and concern for the little sister.

(b) Due to irregular shape, the camphor piece dissolves more rapidly at some points than at others. This reduces surface tension by different amounts at different points. An unbalanced force acts on the camphor piece and makes it dance on water surface.

Question 5

Bunty was fond of painting. He observed that the hairs of paint brush do not cling together when dry and even when dipped in water but form a fine tip when taken out of it. He was quite surprised to see it but found it very useful for doing neat and beautiful painting. He asked his elder brother Shunty, who was a science student of class XI, why do hairs of a paint brush cling together when taken out of water. His elder brother explained him the reason nicely.

(a) *What are the values displayed by Bunty here ?*

(b) *Why do the hairs of a paint brush cling together when taken out of water ?*

Answer

(a) Inquisitiveness, quest for learning scientific phenomena.

(b) When the brush is taken out of water, thin water film is formed at the tips of the hairs. It contracts due to surface tension and the hairs cling together.

Question 6

Vineet went to his native village during summer vacations. One day he went to the fields with his uncle. His uncle was making preparations for planting seeds for the next crop. He first ploughed the field and

Scanned by CamScanner

then planted seeds. Vineet asked his uncle why should we plough the land before planting the seeds. His uncle explained him the reason. After few days Vineet felt happy on seeing the seeds growing into healthy plants.

(a) *What are values displayed by Vineet ?*

(b) *How does the ploughing of fields help in preservation of moisture in the soil ?*

Answer

(a) Love for nature, quest for gaining knowledge of agriculture.

(b) Ploughing is done to break the tiny capillaries through which otherwise water can rise and finally evaporate. The ploughing of field helps the soil to retain moisture.

Question 7

Two sisters, Meenakshi and Sakshi, decided to go INA market by Metro rail. Both were waiting at the platform for the arrival of the rail. The elder sister Meenakshi advised her younger sister Sakshi to stand slightly away from the yellow line marked to the edge of the platform. Sakshi moved away from the yellow line. Meenakshi affectionately explained her younger sister the reason behind this action.

(a) *What are the values displayed by Meenakshi towards her younger sister ?*

(b) *Why it is dangerous to stand near the edge of the platform when a fast train is crossing it ?*

Answer

(a) Concern towards her sister's life, ability to apply scientific knowledge in daily life.

(b) The fast moving air between the person and the train produces a decrease in pressure as per Bernoulli's principle. When the person stands near the edge of the platform, he may be pushed towards the train due to high pressure outside.

Question 8

Rajeev and his friends were passing through an unauthorised colony. Suddenly the weather turned stormy. Strong dusty wind began to blow. Suddenly they saw that the roofs of some houses getting blown off even without damaging the other parts of those houses. There was a hue and cry and people were panicked. Rajeev and his friends immediately began to rescue these people from the storm and took them to safer places.

(a) *What are the values displayed by Rajeev and his friends in their actions ?*

(b) *Why are the roofs of some houses are blown off during the wind storm ?*

Answer

(a) Empathy, helping attitude, sense of social responsibility.

(b) The high velocity of the wind creates low pressure over the roof as per Bernoulli's principle. But the pressure below the roof is the atmospheric pressure which is high. So the roof is blown up.

Chapter 11 : THERMAL PROPERTIES OF MATTER

Question 9

Anuj went to his native village during summer vacations. One day he went to the local market alongwith his grandfather. He saw there a blacksmith heating an iron ring to red hot and then slipping it onto the cart wheel. He was very surprised to see all this. He asked his grandfather why is the iron rim heated red hot before being put on the cart wheel. His grandfather explained him the reason for this action.

Scanned by CamScanner

(a) *What are the values being displayed by Anuj here ?*

(b) *Why are iron rings heated red hot before being put on the cart wheels ?*

Answer

(a) Good observant, keen to learn scientific phenomena.

(b) The iron ring has a slightly smaller diameter than that of the wheel. When the iron ring is heated to become red hot, it expands and slips on to the wheel easily. When it is cooled, it contracts and grips the wheel firmly.

Question 10

In one of the demonstration experiments, one day the Physics teacher in class XI took a slab of ice and supported it on two wooden blocks. He took a metallic wire and attached two heavy weights at its ends. He placed the wire over the slab. It was seen that the wire gradually cut its way through the ice without cutting it into two pieces. The students were very happy to see this. The teacher asked his students can any one explain what is happening. Rakesh stood up and explained that just below the wire, ice melts at a lower temperature due to the increase in pressure. When the wire has passed, water above the wire freezes again. Thus the wire passes through the ice slab without cutting it into two pieces.

(a) *What are the values being displayed by Rakesh in his actions ?*

(b) *Name the phenomenon involved in the above demonstration and briefly explain this phenomenon.*

Answer

(a) Attentive, intelligent and ability to explain things scientifically.

(b) This phenomenon is called regelation in which ice melts when pressure is increased and again freezes when pressure is removed.

Chapter 12 : THERMODYNAMICS

Question 11

While teaching Thermodynamics to class XI, one day the Physics teacher called one of the best students, Ajay, to come near him. He told Ajay to whistle out air on to his palm holding close to his mouth. He asked Ajay how do you feel. Ajay replied the air feels cold. The teacher then asked Anuj to blow out air from his mouth by keeping it wide open. He again asked Anuj how do you now feel. Anuj replied that the air now feels hot. The teacher then asked Anuj to explain the difference in the two cases. Anuj explained the reason quite satisfactorily. The teacher praised Anuj for his correct answer.

(a) *What are the values displayed by Ajay ?*

(b) *What is the reason behind feeling air cold and hot in the two cases in the above demonstration ?*

Answer

(a) Good observer, intelligent, good knowledge of Physics.

(b) During whistling, we blow out air through a small opening between the lips. This is an adiabatic expansion, so there is fall in temperature. But when we keep our mouth wide open, hot air of the mouth blows on to the palm which feels hot.

Question 12

It was a hot summer day. Ramu and his mother were perspiring heavily. Feeling quite uneasy, Ramu's mother opened the door of the working refrigerator kept in the room in the hope that it will cool the room also. But it brought no relief instead it further warmed the room. Ramu's mother was not able to

Scanned by CamScanner

understand why it is happening so. Next day Ramu asked his Physics teacher why cannot we cool a room by leaving the door of a refrigerator open. The teacher explained the reason behind it.

(*a*) *What are the values displayed by Ramu in his actions ?*

(*b*) *Why can't a room be cooled by leaving the door of a working refrigerator open ?*

Answer

(*a*) Concern for his mother, quest for scientific knowledge.

(*b*) When the refrigerator is kept open, it extracts heat from the room. Work is done on it by the electric motor and the total energy is rejected to the same room. Thus the work done by the motor also gets added to the room, so it gets warmed.

Chapter 13 : KINETIC THEORY OF GASES

Question 13

One day Ramesh was pumping air into his cycle tyre. He noticed that both the volume and pressure of the air in the tyre were increasing simultaneously. He was a bit confused as he had learnt in his Physics class that pressure varies inversely with volume as per Boyle's law. Next day he talked to his Physics teacher about this phenomenon who explained him the correct reasoning.

(*a*) *What are the values displayed by Ramesh ?*

(*b*) *What in your opinion can be the correct explanation for the above observation ?*

Answer

(*a*) Good observant, highly interested in learning physical phenomena observed in daily life.

(*b*) Boyle's law is valid for a fixed mass of a gas. When we pump air into a cycle tyre, air molecules are pushed into the tyre and so the mass of the air in the tyre increases. Hence Boyles's law is not applicable under the given situation.

Question 14

Vijay went alongwith his mother to a marriage party. When they came back to their home, his mother felt giddy. She vomited up all she had eaten up at the party. Vinay immediately went to the kitchen and dissolved little eno powder in water and asked his mother to drink the solution. His mother felt relief. He then wiped the vomit from the floor. Inspite of doing all this, a foul smell was prevalent in the room. He then scented the entire room.

(*a*) *What are the values being displayed by Vinay in his actions ?*

(*b*) *Although the random speed of air molecules is about 500 m/s at room temperature, yet on opening a bottle of scent in one corner of a room its aroma takes time in reaching the other corner. Why ?*

Answer

(*a*) Highly concerned for the mother, responsible and have knowledge of curing general health related problems.

(*b*) During their random motion, the molecules collide frequently with one another and follow zig-zag paths. Their net speed in any particular direction is very small.

Scanned by CamScanner

Chapter 14 : OSCILLATIONS

Question 15

Once an NCC camp was organised near Jammu. The cadets were being trained for march-past. All of a sudden the cadets came near a bridge. On seeing this, their incharge immediately stopped them from marching on the bridge. The cadets were first surprised to see this but later on the incharge explained them the reason for doing so.

(a) What are the values being displayed by the incharge ?

(b) Why are army troops not allowed to march in steps while crossing a bridge ?

Answer

(a) Responsibility, concern for the lives of cadets and safety of the bridge.

(b) It is quite likely that the frequency of the marching steps may match with the natural frequency of the bridge, and due to resonance the bridge may pick up large amplitude and even collapse.

Question 16

The Physics teacher was teaching resonance to the students of class XI. He said that earthquakes cause vast devastation. In an earthquake, sometimes short and tall structures remain unaffected while the medium height structures fall down. He asked his students what may be the reason behind such disaster. Sunita stood up and correctly explained the reason. The teacher was happy with the answer given by Sunita.

(a) What are the values displayed by Sunita ?

(b) What in you opinion may be the reason for above said observation ?

Answer

(a) Intelligent, logical thinking, scientific temper.

(b) The natural frequencies of the short structures are higher and those of taller structures lower than the frequency of the seismic waves. The medium structures have frequencies equal to those of seismic waves. They vibrate resonantly and pick up large amplitudes and hence fall down.

Chapter 15 : Waves

Question 17

One morning Ashutosh went to the bathroom for taking a bath. He opened the tap to fill an empty bucket with water. As the bucket started filling with water, Ashutosh noticed that the pitch of sound produced went on increasing. He was surprised to observe this. He could not understand the reason. When he went to the school, he shared this problem with his teacher in the Physics period. On getting a solution of problem from the teacher, Ashutosh felt very happy and expressed gratitude to the teacher.

(a) What are the values being displayed by Ashutosh in his actions ?

(b) When we start filling an empty bucket with water, the pitch of sound goes no increasing. Why ?

Answer

(a) Keen observer, curiosity and quest for knowledge.

(b) As the bucket is filled with water, the length L of the air column above the water level goes on decreasing. This increases the frequency ($\nu = v/4L$) and hence the pitch of sound produced.

Scanned by CamScanner

Question 18

Sunita was waiting at the platform alongwith his uncle for Rajdhani express. Soon she saw the train coming towards the platform. She noticed that as the train was approaching her, the pitch of its whistle appeared to rise and it suddenly appeared to drop as the engine moved away from her. When the train stopped, Sunita and her uncle entered the train. Sunita discussed this observation with her uncle. His uncle, being a Physics teacher, explained her the reason for the change in pitch of the whistle.

(*a*) *What are the values being displayed by Sunita ?*

(*b*) *Name the effect involved in the above observation. State this effect briefly.*

Answer

(*a*) Good observant, enthusiastic and quest for knowledge.

(*b*) The effect involved is Doppler effect. This is the phenomenon of the change in apparent pitch of sound due to the relative motion between the source of sound and the observer.

Question 19

Manu was a good football player. But since last few days he was getting pain in his stomach. His parents took him to a doctor who examined him and asked him to get an ultrasound done to detect the exact cause. Manu was afraid of ultrasound scanner and refused to get it done. His parents made him understand that the scanner uses ultrasonic rays which go inside and detect any problem inside the body. So he got it done and the scanner showed that he has small tumour in his stomach and that has to be operated as early as possible. Doctor operated him off the tumour and after a month he became fine again. **[Central Schools 14]**

(*a*) *What are the values shown by Manu's parents ?*

(*b*) *On which principle does the ultrasonic scanner work ? If the ultrasound uses the operating frequency of 4.2 MHz, the speed of sound in the tissue is 1.7 kms^{-1}. What is the wavelength of the sound in tissue ?*

Answer

(*a*) Sense of responsibility and concern for their son.

(*b*) An ultrasonic scanner works on the principle of reflection of ultrasonic waves from a region where there is a change of tissue density.

$$\text{Wavelength, } \lambda = \frac{v}{\nu} = \frac{1.7\ \text{kms}^{-1}}{4.2\ \text{MHz}} = \frac{1.7 \times 10^3\ \text{ms}^{-1}}{4.2 \times 10^6\ \text{s}^{-1}} = \mathbf{333.33\ m}.$$

Question 20

One day Kapil had to go to his friend's house. He took his mobile and headphone to listen to music while walking. On the way he had to cross a railway line. He was so engrossed in the music that he did not hear the approaching train. Train driver was blowing the horn but Kapil did not hear it. Subhash in the nearby field who was seeing this ran towards Kapil and pushed him away just as train reached there. Kapil realised his mistake and thanked that person. **[Central Schools 14]**

(*a*) *What are the values shown by Subhash ?*

(*b*) *If a train emitting sound of frequency 500 Hz, is moving towards an observer with a speed of 30 ms^{-1}, what is the frequency as heard by the listener ?*

Answer

(*a*) Concern towards other's life, sense of social responsibility.

(*b*) $$\nu' = \frac{v - v_0}{v - v_s} \times \nu = \frac{330 - 0}{330 - 30} \times 500 = \mathbf{550\ Hz}$$

Scanned by CamScanner

New Simplified

PHYSICS

New Simplified Physics *is meant for anyone who wants to follow a self-study course.* It thoroughly covers the CBSE syllabus and NCERT guidelines prescribed for the students of 10+2 level. The book has been carefully designed to make the learning of physics as easy and straight forward as possible.

THE SALIENT FEATURES OF THE BOOK ARE :

- ✓ May be used both as textbook and help book.
- ✓ *Glimpses* given in each chapter provide an insight into the entire chapter.
- ✓ Easy and complete definitions of all terms.
- ✓ Explanations of all concepts in full depth and detail.
- ✓ Simplest and shortest derivations of all theoretical principles.
- ✓ Simple, lucid and easily understandable language.
- ✓ A large number of neat, explicit and well-labelled illustrative diagrams.
- ✓ A large number of *Numerical Examples and Problems* based on all topics are given under the appropriate articles. Useful hints and complete solutions of unsolved problems are given at the end of each exercise.
- ✓ All exercises of NCERT textbook fully solved.
- ✓ A large number of *Very Short Answer Conceptual Problems* and *Short Answer Conceptual Problems* given in each chapter.
- ✓ A Plenty of problems requiring Higher Order Thinking Skills (HOTS) given in each chapter.
- ✓ Additional information useful for competitive examinations given in *For Your Knowledge* boxes at various places in the book.
- ✓ Exhaustive *Text Based Exercises* including *Value Based Questions* alongwith useful hints given at the end of each chapter.
- ✓ All previous papers of common examinations conducted by different state boards and central schools have been fully solved.
- ✓ Objective questions from previous papers of JEE Adv., JEE Main, IPUCET, AIIMS, AIPMT, VMMC PMT, etc. given with detailed solutions.

DHANPAT RAI

Published by
DHANPAT RAI & CO. (P) LTD.
Educational & Technical Publis
4576/15, Agarwal Road, Darya Ganj, N
Ph.: 2324 7736, 37, 38 • E-mail: dhan

EBK 000
397761

Scanned by CamScanner