BRAIN MAP

OSCILLATIONS

The world is filled with oscillations in which objects move back and forth repeatedly. We cannot even say 'vibration' properly without the tip of the tongue oscillating.

Simple Harmonic Motion

Differential Equations of SHM

- For linear SHM, $\frac{d^2y}{dt^2} + \omega^2 y = 0$
- For angular SHM, $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

Angular SHM

- Angular displacement, $\theta = \theta_0 \sin(\omega t + \delta)$
- Torque, $\tau = -c\theta$
- Angular velocity, $\omega = \sqrt{c/I}$
- Angular acceleration, $\alpha = -\frac{c\theta}{I}$
- Time period of oscillation, $T = 2\pi\sqrt{\frac{I}{c}}$
- Frequency of oscillation, $\upsilon = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{c}{I}}$

Energy in Angular SHM

- Potential energy, $U = \frac{1}{2}c\theta^2 = \frac{1}{2}I\omega^2\theta^2$
- Kinetic energy, $K = \frac{1}{2}I\omega^2$
- Total energy, $E = \frac{1}{2}I\omega^2\theta_0^2$

Characteristics of Linear SHM

- General equation of dispacement of particle executing linear SHM
  $y = A\sin(\omega t + \phi)$
- Time period of SHM, $T = \frac{1}{\upsilon} = \frac{2\pi}{\omega}$
- Velocity, $v = \omega\sqrt{A^2 - y^2}$
- Acceleration, $a = -\omega^2 y$

Maximum Velocity and Maximum Acceleration in SHM

- At mean position velocity is maximum
  $v_{max} = A\omega$
- Acceleration is maximum at extreme position
  $a_{max} = -\omega^2 A$

Energy in Linear SHM

- Kinetic energy
  $K = \frac{1}{2}m\omega^2(A^2 - y^2) = \frac{1}{2}m\omega^2A^2\cos^2\omega t$
- Potential energy
  $U = \frac{1}{2}m\omega^2y^2 = \frac{1}{2}m\omega^2A^2\sin^2\omega t$
- Total energy
  $E = \frac{1}{2}m\omega^2A^2 = \frac{2\pi^2 mA^2}{T^2}$
- At mean position kinetic energy is maximum, at extreme position potential energy is maximum and total energy is constant at every position during simple harmonic motion.

Dynamics of SHM

- $\vec{F} = -m\omega^2\vec{x}$ or $\vec{F} = -k\vec{x}$
  where, force constant $k = m\omega^2$
- Angular velocity, $\omega = \sqrt{\frac{k}{m}}$
- Time period, $T = 2\pi\sqrt{\frac{m}{k}}$
- Frequency, $\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Simple Pendulum

- Time period $T = 2\pi\sqrt{\frac{l}{g}}$
- If the length of simple pendulum is very large, $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$
- If a simple pendulum oscillates in a non viscous liquid of density σ then its time period $T = 2\pi\left[\frac{l}{\left(1 - \frac{\sigma}{\rho}\right)g}\right]^{1/2}$ where ρ is the density of suspended mass.
- When a pendulum kept in a car which is sliding down then
  $T = 2\pi\sqrt{\frac{l}{g\cos\theta}}$ where θ is the angle of inclination.

Physical Pendulum

- Time period of physical pendulum $T = 2\pi\sqrt{\frac{I}{mgd}}$
  where d is the distance from centre of gravity of rigid body to pivoted point.
- Acceleration due to gravity, $g = \frac{8\pi^2 L}{3T^2}$
  $(\because\ d = \frac{L}{2},\ I = \frac{1}{3}mL^2)$

SHM in Spring

- Equation of motion $\frac{d^2y}{dt^2} = \frac{-ky}{m} = -\omega^2 y$
- If the spring is not light but has a definite mass then $T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$
- Two bodies of masses m_1 and m_2 are attached through a light spring of spring constant k, the time period of oscillation $T = 2\pi\sqrt{\frac{\mu}{k}}$ where $\mu = \frac{m_1 m_2}{m_1 + m_2}$
- If the mass m attached to a spring oscillates in a non viscous liquid of density σ then $T = 2\pi\left[\frac{m}{k}\left(1 - \frac{\sigma}{\rho}\right)\right]^{1/2}$ where ρ is the density of suspended mass.
- The force of gravity has no effect on force constant k and time period of oscillating mass.

Oscillations of Loaded Spring Combinations

- For two springs of spring factors k_1 and k_2 connected in parallel, effective spring factor $k = k_1 + k_2$ and $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$
- For springs connected in series, $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$ or $k = \frac{k_1k_2}{k_1 + k_2}$ and $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$
- When length of a spring made n times, its spring factor becomes $1/n$ times and hence time period increases $\sqrt{n}$ times.
- When a spring is cut into n equal pieces, spring factor of each part becomes nk and $T = 2\pi\sqrt{\frac{m}{nk}}$

Time Period of Different SHMs

- A plank of mass m and area of cross section A is floating in a liquid of density ρ when depressed, it starts oscillating then $T = 2\pi\sqrt{\frac{m}{\rho Ag}}$
- In case of water oscillating in U-tube, then $T = 2\pi\sqrt{\frac{h}{g}}$
  where h is the height of liquid column in each limb
- A ball of mass m is made to oscillate in the neck of an air chamber having volume V and neck area a then $T = 2\pi\sqrt{\frac{mV}{Ba^2}}$
  where B = bulk modulus of elasticity in air.
- A small ball of radius r is rolling down in a hemispherical bowl of radius R, then $T = 2\pi\sqrt{\frac{R - r}{g}}$
  where R is the radius of bowl and r is the radius of ball.
- For a body executing SHM in a tunnel dug along any chord of earth $T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6$ min.
- Time period of torsional pendulum $T = 2\pi\sqrt{\frac{I}{C}}$ where $C = \frac{\pi\eta r^4}{2l}$
  where, η = modulus of elasticity of wire, r = radius of wire, l = length of wire

Damped and Forced Oscillations

- The differential equation of damped harmonic oscillator is given by
  $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$
- The displacement of the damped oscillator at any instant t is given by
  $x(t) = Ae^{-bt/2m}\sin(\omega' t + \phi)$
- Angular frequency of the damped oscillation
  $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
  where b is damping constant.
- Mechanical energy of the damped osillator $E(t) = \frac{1}{2}kA^2e^{-bt/m}$
- The differential equation of forced damped harmonic oscillator is given by
  $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\sin\omega_d t$
  where ω_d is the angular frequency of the external force.
- The displacement of the forced damped harmonic oscillator at any instatn t is given by
  $x(t) = A\sin(\omega_d t - \phi)$
  and $\phi = \tan^{-1}\left[\frac{b\omega_d}{m(\omega^2 - \omega_d^2)}\right]$
- Amplitude of forced oscillations when driving frequency is far from natural frequency,
  $A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$
- When driving frequency is close to natural frequency, *i.e.*, at resonance, $A = \frac{F_0}{\omega_d b}$