Day 12

Waves

Day 12

Outlines ...

- Wave Motion
- Speed of Waves
- Sound Waves
- Principle of Superposition of Waves
- Doppler's Effect in Sound

Wave Motion

*When a large number of particles vibrates simultaneously in a medium, then disturbance propagates in the medium. The motion of disturbance is called **wave motion**. Energy or momentum is transferred to the neighbouring particles of the medium as **wave** proceeds. There are two types of wave motion as given below.*

Longitudinal Waves

When particles of the medium vibrate parallel to the direction of propagation of wave, then wave is called **longitudinal wave**. These waves propagate in the form of compressions and rarefactions.

They involve changes in pressure and volume. The medium of propagation must possess elasticity of volume. They are set up in solids, liquids and gases.

Transverse Waves

These waves travel in the form of crest and trough set up alternatively. The medium must possess the elasticity of shape. There is no change in density of medium. These waves can be set up in solids, on surface of liquids but never in gases. Transverse waves undergo polarisation as against longitudinal waves, which do not get polarised. Some of the important terms of the wave motion are described below.

Wave Number It is the number of waves travelled in per unit length. It is measured in $(\text{metre})^{-1}$. *i.e.,* $Y = \frac{1}{\lambda}$

Particle Velocity It is the velocity of the particle executing in simple harmonic motion. *i.e.,* $v = \frac{dy}{dt}$, where y denotes displacement at any instant.

Wave Velocity The velocity of transverse wave motion is given by

$$v = \frac{\text{Distance travelled by wave}}{\text{Time taken}}$$

i.e.,
$$v = \frac{\lambda}{T} = \left(\frac{1}{T}\right)\lambda \quad \text{or} \quad v = \gamma\lambda$$

Particle velocity changes with time but the wave velocity is constant. Acceleration of wave is zero but acceleration of particle is not zero.

Differential Equation of Wave Motion $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$

Speed of Waves

In general, the speed of an object refers to how fast an object is moving and is usually expressed as the distance travelled per time of travel. In case of wave, the speed is the distance travelled by a given point on the wave in a given interval of time. Although the speed of transverse and longitudinal waves are described below.

1. Speed of Transverse Wave

The expression for speed of transverse waves in a solid and in case of a stretched string can be obtained theoretically given as

(i) In solids, $v = \sqrt{\frac{\eta}{d}}$

where η is the modulus of rigidity and d is the density of the medium.

(ii) In a stretched string, $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mg}{\pi r^2 d}}$

where, T = the tension in the string,
m = the mass per unit length of the string,
M = mass suspended from the string,
r = radius of the string and
d = density of the material of the string.

2. Speed of Longitudinal Wave (or Sound Wave)

According to Newton formula, speed of sound in a gas is

(i) Newton's Formula

$v = \sqrt{\frac{B}{D}}$, where B denotes bulk modulus of elasticity and D denotes density of medium.

(ii) Laplace's Correction

For gases, E = coefficient of adiabatic elasticity.
$E = \gamma p$
Here p is the pressure of the gas.

$v = \sqrt{\frac{\gamma p}{D}}$, γ denotes adiabatic constant $= \frac{C_p}{C_V}$

(iii) Effect of Temperature on Velocity

With rise in temperature then velocity of sound increases as $v = \sqrt{\frac{\gamma RT}{M}}$; $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$ *i.e.,* $v \propto \sqrt{T}$

(iv) Effect of Pressure for Gases Medium

$\frac{p}{D}$ remains constant. Pressure has no effect on the velocity of sound, provide temperature remains constant.

(v) Effect of Humidity

When humidity in air increases, its density decreases and so velocity of sound increases.

For solids, $v = \sqrt{\frac{Y}{D}}$. For liquids, $v = \sqrt{\frac{K}{D}}$

where, Y = Young's modulus of elasticity
K = bulk modulus of elasticity.

- *Although* $\rho_{solid} > \rho_{liquid} > \rho_{gases}$
  but $B_{solid} >> B_{liquid} >> B_{gases}$.
  As a result, it is observed that speed of sound is maximum in solids, lesser in liquids and least in gases. As an example
  $v_{steel} = 5941\ \text{ms}^{-1}$, $v_{water} = 1482\ \text{ms}^{-1}$ *(at 20°C)*
  and $v_{air} = 332\ \text{ms}^{-1}$ *(at 0°C)*
- *In gases, speed of sound* $v_{sound} = \sqrt{\frac{\gamma p}{\rho}}$ *and rms speed of gas molecules* $v_{rms} = \sqrt{\frac{3p}{\rho}}$. *Hence, we conclude that for a gas* $\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}$

Sound Waves

Mechanical waves in air having a frequency ranging from 20 Hz to 20 kHz are known as ***audio waves*** *or sound waves. Waves having frequencies less than 20 Hz are known as the infrasonic waves. Waves in air having frequencies greater than 20 kHz are known as* ***ultrasonic waves.***

Sound waves do not undergo polarisation. Transverse waves only get polarised.

(i) Velocity of sound in air $\approx 332\ \text{ms}^{-1}$

(ii) Velocity of sound in water $\approx 1400\ \text{ms}^{-1}$

(iii) Velocity of sound in steel $\approx 5000\ \text{ms}^{-1}$

Sound exhibits reflection, refraction, interference and diffraction but not polarisation.

A person can emit 5 syllables in 1s. Each syllable is produced in 0.2s.

Relation between Phase Difference, Path Difference and Time Difference

Phase difference $(\phi) = \frac{2\pi}{\lambda} \times$ path difference (x)

$\Rightarrow \quad \phi = \frac{2\pi x}{\lambda}$

$\Rightarrow \quad x = \frac{\phi\lambda}{2\pi}$

Phase difference $(\phi) = \frac{2\pi}{T} \times$ time difference (t)

$\Rightarrow \quad \phi = \frac{2\pi t}{T}$

$\Rightarrow \quad t = \frac{T\phi}{2\pi}$

Time difference $(t) = \frac{T}{\lambda} \times$ path difference (x)

$\Rightarrow \quad t = \frac{Tx}{\lambda}$

$\Rightarrow \quad x = \frac{\lambda t}{T}$

(i) For a wave, velocity

v = frequency $(n) \times$ wavelength (λ)

$\Rightarrow \quad v = n\lambda$

(ii) Angular speed,

$$\omega = 2\pi n = \frac{2\pi}{T} = \frac{2\pi v}{\lambda}$$

Displacement Relation for a Progressive or Harmonic Wave

The equation of a plane progressive or simple harmonic wave travelling along positive direction of x-axis is

$$y = a \sin(\omega t - kx)$$

$\Rightarrow \quad y = a \sin \frac{2\pi}{\lambda}(vt - x)$

$\Rightarrow \quad y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

If maximum value of $y = a$ *i.e.*, a is maximum amplitude, then

(i) $dy/dt =$ velocity of particle

$$\frac{dy}{dt} = \left(\frac{2\pi v}{\lambda}\right) a \cos \cdot \frac{2\pi}{\lambda}(vt - x)$$

(ii) $\left(\frac{dy}{dt}\right)_{max} = \frac{2\pi va}{\lambda}$

$$= 2\pi na = \omega a$$

Acceleration of particle

$$= \frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dt^2} = -\omega^2 a \sin \frac{2\pi}{\lambda}(vt - x)$$

Maximum value of

$$\frac{d^2y}{dt^2} = -\omega^2 a$$

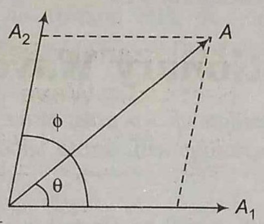
Principle of Superposition of Waves

When two or more than two waves of similar type propagate in a medium simultaneously, then resultant displacement of any particle of the medium is equal to the vector sum of displacements produced by individual waves separately. This principle is called ***principle of superposition of waves****.*

$$y = y_1 + y_2 + y_3 + \ldots$$

Interference of Waves

When two waves of same frequency (or same wavelength) travelling along same path superimpose each other, there occurs redistribution of energy in the medium. If at a given position (x being constant) displacement due to two waves be



$$y_1 = A_1 \sin \omega t$$

and $$y_2 = A_2 \sin (\omega t + \phi)$$

Then, resultant displacement

$$y = y_1 + y_2 = A \sin (\omega t + \phi)$$

where, $$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

and $$\tan \theta = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$$

At those points where phase difference $\phi = 0°$ or $2n\pi$, *i.e.*, an integer multiple of 2π ($n = 1, 2, 3, \ldots$),

$$A = A_1 + A_2 = A_{max}$$

This is known as **constructive interference**.

At those points where $\phi = (2n - 1)\pi$, *i.e.*, phase difference is an odd multiple of π and hence,

$$A = A_1 \sim A_2 = A_{min}$$

This is known as **destructive interference**.

If I_1 and I_2 are intensities of the interfering waves and ϕ is the phase difference, then resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2 \text{ for } \phi = 2\pi n$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2}$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 \text{ for } \phi = (2n + 1)\pi$$

If P is power of a sound source, then intensity follows inverse square law of distance (d),

$$I = \frac{P}{4\pi d^2}$$

Reflection *and* Transmission of Waves

When sound waves are incident on a boundary separating two media, a part of it is reflected back into the initial medium while the remaining is partly absorbed and partly transmitted into the second medium.

Characteristics

(i) In case of reflection and transmission of sound, the frequency of the wave remains unchanged, *i.e.*, $\omega_i = \omega_r = \omega_t = \omega$.

(ii) The incident ray, the reflected ray, normal and the refracted ray are always in the same plane.

(iii) In case of reflection of sound,
angle of incidence = angle of reflection

(iv) In case of refraction of sound,
$$\frac{\sin i}{\sin r'} = \frac{v_i}{v_t}$$

(v) In case of reflection from a denser medium or rigid support or fixed end there is inversion of the reflected displacement wave, *i.e.*, if the incident wave is $y = A_i \sin (\omega t - kx)$ the reflected wave will be
$$y = -A_r \sin (\omega t + kx)$$
$$= A_r \sin (\omega t + kx + \pi)$$
i.e., in case of reflection from a denser medium, displacement wave changes in phase by π while in case of reflection from a rarer medium, no inversion of wave or phase change occurs. The transmitted wave is never inverted.

(vi) On reflection, the amplitude and intensity of wave may decrease.

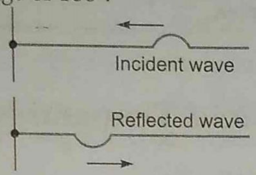
(vii) When a transverse wave is reflected from a denser medium, the trough is reflected as crest and *vice-versa*.

(viii) When a transverse wave is reflected from a rarer medium, crest and trough do not invert after reflection.

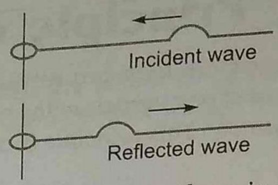
(ix) When a longitudinal wave is reflected from a denser medium, the compression and rarefaction do not invert after reflection.

(x) When a longitudinal wave is reflected from a rarer medium, compression is reflected as rarefaction and *vice-versa*.

(xi) Waves on reflection from a fixed end undergo a phase change of 180°.



(xii) While a wave reflected from a free end is reflected without a change in phase.



(xiii) In case of pressure wave, there is no phase change when reflected from a denser medium or fixed end. The concept of rarer or denser medium for a wave is through speed (and not density of medium). For example, water is rarer for sound and denser for light than air, as for sound $v_w > v_a$, while for light $v_w < v_a$.

Standing *or* Stationary Waves

Standing or stationary wave is formed due to superposition of two progressive waves of same nature, same frequency (or same wavelength), same amplitude travelling with same speed in a bounded medium in mutually opposite directions.

If the incident wave be represented as $y_1 + A\sin(\omega t - kx)$ and the reflected wave as $y_2 = A\sin(\omega t + kx)$, then

$$y = y_1 + y_2 = A\sin(\omega t - kx) + A\sin(\omega t + kx) \Rightarrow y = 2A\cos kx \sin \omega t$$

The resultant wave does not represent a progressive wave.

Standing Waves in String

Consider a string of length L stretched under tension T between two fixed points (*i.e.*, clamped at its ends). Transverse wave is set up on the string whose speed is given by $v = \sqrt{T/\mu}$, where μ is the mass per unit length of the string. Let only one antinode A is formed at the centre and string vibrates in one segment only, then $L = \frac{\lambda_1}{2}$ or $\lambda_1 = 2L$

(i) Frequency of vibration in fundamental mode

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

It is known as the **fundamental frequency** or **first harmonic.**

(ii) If string vibrates in two segments, then

$$L = \lambda_2 \text{ and } \nu_2 = \frac{v}{\lambda_2} = \frac{1}{L}\sqrt{\frac{T}{\mu}} = 2\nu_1$$

(iii) It is known as **first overtone** or **second harmonic.** Similarly, if the string vibrates in three segments, then $L = \frac{3\lambda_3}{2}$ and $\nu_3 = \frac{v}{\lambda_3} = 3\nu_1$

It is called **second overtone** or third harmonic.

(iv) In general, if a string vibrates in p segments [*i.e.*, have $(p+1)$ nodes and p antinodes], then $\nu_{pth} = \frac{p}{2L}\sqrt{\frac{T}{\mu}} = p\nu_1$ and it is known as pth harmonic or $(p-1)$th overtone.

Standing Waves in Organ Pipes

(Air Columns)

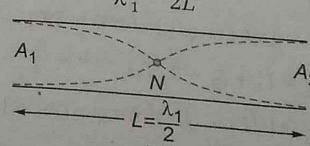
Organ pipes are those cylindrical pipes which are used for producing musical (longitudinal) sounds. The standing waves in both organ pipes (*i.e.*, open organ pipe and closed organ pipe) are described below.

1. Open Organ Pipe

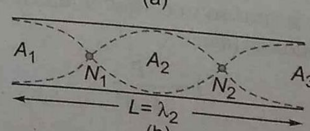
(i) As shown in figure antinodes are formed at both the open ends of pipe and one or more nodes are formed in between symmetrically. The number of nodes is 1 less than the number of antinodes.

(ii) In fundamental mode (1st harmonic) only one node is formed at the centre of pipe and so $L = \frac{\lambda_1}{2}$.

$$\therefore \quad \nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$



(a)



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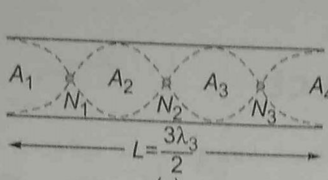


(c)

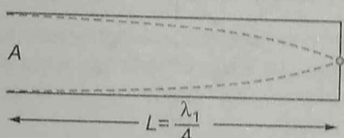
(iii) All harmonics are present in open pipe with their frequencies in the ratio 1 : 2 : 3 : 4 .... and ratio of overtones = 2 : 3 : 4 : 5...

Position of nodes from one end $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$

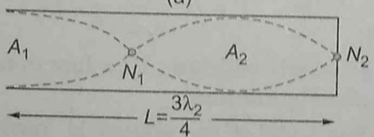
Position of antinodes from one end $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots$

2. **Closed Organ Pipe**

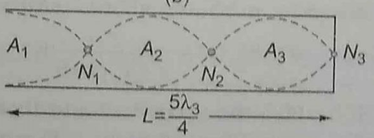
(i) Longitudinal stationary waves are formed such that open end of pipe behaves as an antinode and a node is formed at the closed end.



(a)



(b)



(c)

(ii) In fundamental mode,

$$L = \lambda_1/4 \text{ or } \lambda_1 = 4L$$

∴ Fundamental (1^{st} harmonic) frequency

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

(iii) For a pipe of length L, the fundamental frequency in a closed pipe is half of that in open pipe. Thus, $\nu_{open} = 2 \times \nu_{closed}$ for fundamental mode.

> ▸▸ *Standing wave is an example of interference. Node means destructive interference and antinode means constructive interference.*
>
> ▸▸ *Due to persistance of vision these waves appear in the form of loops. All the particles in a loop are in the same phase. But the particles in adjacent loops differ in phase by π.*
>
> ▸▸ *Stationary waves may be transverse or longitudinal.*
>
> ▸▸ *As in stationary waves nodes are permanently at rest, so energy cannot be transmitted across them.*

Beats

When two sound waves of nearly equal (but never equal) or slightly different frequencies and equal or nearly equal amplitudes travelling along the same direction superimpose at a given point, the resultant sound intensity alternately rises and falls. This alternate rise and fall of sound at a given position is called **beats.**

- One beat is said to be formed when starting from minimum sound the intensity once rises to its maximum value and then falls back to the minimum level.
- Number of beats formed per second is called the frequency of beats. If two sound waves of frequencies ν_1 and ν_2 superimpose, then frequency of beats $= (\nu_1 \sim \nu_2)$, *i.e.*, either $(\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$.
- For formation of distinct beats then difference between the frequencies of two superposing notes should be less than 10 Hz.

> ▸▸ *If source of sound is linear, then at a distance d from the line source,* $I = \frac{P}{2\pi / d}$
>
> ▸▸ *Our perception of loudness is better co-related with the second level measured in decibel (dB) and defined as follows* $\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$, *where* $I_0 = 10^{-12}\, Wm^2$ *at 1kHz.*

We can find true frequency of tuning fork B from a pair of tuning forks A and B, in which frequency of A is known, where x is the beats per second.

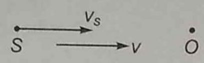
When B is loaded (its frequency decreases)	When B is filled (its frequency increases)
(i) If x increases, then $\nu_B = \nu_A - x$	(i) If x increases, then $\nu_B = \nu_A + x$
(ii) If x decreases, then $\nu_B = \nu_A + x$	(ii) If x decreases, then $\nu_B = \nu_A - x$
(iii) If x remains same, then $\nu_B = \nu_A + x$	(iii) If x remains same, then $\nu_B = \nu_A - x$
(iv) If x becomes zero, then $\nu_B = \nu_A + x$	(iv) If x becomes zero, then $\nu_B = \nu_A - x$

Doppler's Effect

The phenomena of apparent change in frequency of source due to a relative motion between the source and observer is called Doppler's effect.

(i) **When Source is Moving and Observer is at Rest** When source is moving with velocity v_s, towards an observer at rest, then apparent frequency

$$n' = n\left(\frac{v}{v - v_s}\right)$$

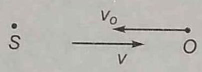


If source is moving away from observer, then

$$n' = n\left(\frac{v}{v + v_s}\right)$$

(ii) **When Source is at Rest and Observer is Moving** When observer is moving with velocity v_o, towards a source at rest, then apparent frequency.

$$n' = n\left(\frac{v + v_o}{v}\right)$$



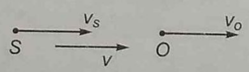
When observer is moving away from source, then

$$n' = n\left(\frac{v - v_o}{v}\right)$$

(iii) **When Source and Observer Both are Moving**

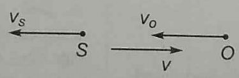
(a) When both are moving in same direction along the direction of propagation of sound, then

$$n' = n\left(\frac{v - v_o}{v - v_s}\right)$$



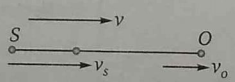
(b) When both are moving in same direction opposite to the direction of propagation of sound, then

$$n' = n\left(\frac{v + v_o}{v + v_s}\right)$$



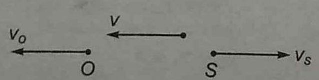
(c) When both are moving towards each other, then

$$n' = n\left(\frac{v + v_o}{v - v_s}\right)$$



(d) When both are moving in opposite direction, away from each other, then

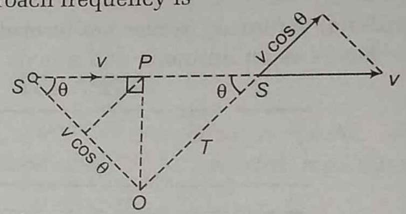
$$n' = n\left(\frac{v - v_o}{v + v_s}\right)$$



Transverse Doppler's Effect

(i) The Doppler's effect in sound does not take place in the transverse direction.

(ii) As shown in figure, the position of a source is S and of observer is O. The component of velocity of source towards the observer is $v \cos\theta$. For this situation, the approach frequency is



$$f' = \frac{v}{v - v_s \cos\theta} \times f$$

f' which will now be a function of θ so, it will no more be constant.

Similarly, if the source is moving away from the observer as shown above, with velocity component $v_s \cos\theta$ then, $f' = \frac{v}{v + v_s \cos\theta} \times f$

(iii) If $\theta = 90°$, the $v_s \cos\theta = 0$ and there is no shift in the frequency. Thus, at point P, Doppler's effect does not occur.

Effect of Wind

If wind is also blowing with a velocity w in the direction of sound, then its velocity is added to the velocity of sound. Hence, in this condition the apparent frequency is given by

$$n' = n\left(\frac{v + w - v_o}{v + w - v_s}\right)$$

Applications of Doppler's Effect

The measurement of Doppler shift has been used

- by police to check over speeding of vehicles.
- at airports to guide the aircraft.
- to study heart beats and blood flow in different parts of the body.
- by astrophysicist to measure the velocities of plants and stars.

Practice Zone

DAY 12

1. A stone is dropped into a lake from a tower 500 m high. The sound of the splash will be heard by the man approximately after (Take $g = 10\,\text{ms}^{-2}$)

(a) 11.5 s (b) 21 s (c) 10 s (d) 14 s

2. A sound wave of wavelength λ is travelling in a medium with a speed of v m/s enter into another medium where its speed is $2v$ m/s. Wavelength of sound waves in the second medium is **[NCERT Exemplar]**

(a) λ (b) $\frac{\lambda}{2}$ (c) 2λ (d) 4λ

3. A wave equation is given by $y = 4\sin\left[\pi\left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6}\right)\right]$

where x is in cm and t in sec. Which of the following is true?

(a) $\lambda = 18\,\text{cm}$ (b) $v = 4\,\text{ms}^{-1}$ (c) $a = 0.4\,\text{m}$ (d) $f = 50\,\text{Hz}$

4. Sound of frequency f passes through a Quinck's tube, adjusted for intensity I_m. What should be the length to which the tube should be moved to reduce intensity to 50% (speed of sound is v)?

(a) $\frac{v}{2f}$ (b) $\frac{v}{4f}$

(c) $\frac{v}{8f}$ (d) $\frac{v}{16f}$

5. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 15 cm during winter. Repeating the same experiment during summer, he measures the column length to be x cm for the second resonance. Then,

(a) $x > 45$ (b) $45 > x > 30$

(c) $30 > x > 15$ (d) $15 > x$

6. Which of the following is not true for progressive wave

$$y = 4\sin 2\pi\left[\frac{t}{0.02} - \frac{x}{100}\right]$$

where y and x are in cm and t in second.

(a) Its amplitude is 4 cm

(b) Its wavelength is 100 cm

(c) Its frequency is 50 Hz

(d) Its propagation speed is $50 \times 10^{-2}\,\text{cms}^{-1}$

7. A wave equation which gives the displacement along Y-direction is given by $y = 0.001\sin[100t + x]$, where x and y are in metre and t is time in second. This represents a wave

(a) of frequency $\frac{100}{\pi}$ Hz

(b) of wavelength 1 m

(c) travelling with a velocity of $\frac{50}{\pi}\,\text{ms}^{-1}$ in the positive X-direction

(d) travelling with a velocity of $100\,\text{ms}^{-1}$ in the negative X-direction

8. A sound wave is passing through air column in the form of compression and rarefaction. In consecutive compression and rarefactions, **[NCERT Exemplar]**

(a) density remains constant

(b) Boyle's law is obeyed

(c) Bulk modulus of air oscillates

(d) there is no transfer of heat

9. Two sounds of wavelength 5 m and 6 m travelling in a medium produce 10 beats per second. The speed of sound in the medium is

(a) $300\,\text{ms}^{-1}$ (b) $320\,\text{ms}^{-1}$

(c) $350\,\text{ms}^{-1}$ (d) $1200\,\text{ms}^{-1}$

10. Motion of two particles is given by

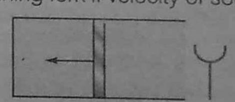
$$y_1 = 0.25\sin(310t),\ y_2 = 0.25\sin(316t)$$

Find beat frequency.

(a) 3 (b) $3/\pi$

c) $6/\pi$ (d) 6

11. A piston fitted in cylindrical pipe is pulled as shown in the figure. A tuning fork is sounded at open end and loudest sound is heard at open length 13 cm, 41 cm and 69 cm, the frequency of tuning fork if velocity of sound is $350\,\text{ms}^{-1}$, is



(a) 1250 Hz (b) 625 Hz

(c) 417 Hz (d) 715 Hz

12. Two tuning forks P and Q when set vibrating, give 4 beat/s. If a prong of the fork P is filled, the beats are reduced to $2\ s^{-1}$, what is the frequency of P, if Q is 250 Hz?
(a) 246 Hz (b) 250 Hz
(c) 254 Hz (d) 252 Hz

13. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q at distances of 2 m and 3 m respectively from the source. The ratio of the intensities of the waves at P and Q is
(a) 9:4 (b) 2:3 (c) 3:2 (d) 4:9

14. A racing car moving towards a cliff sounds its horn. The driver observes that the sound reflected from the cliff has a pitch one octave higher than the actual sound of the horn. If v is the velocity of sound, the velocity of the car is
(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{2}$ (c) $\frac{v}{3}$ (d) $\frac{v}{4}$

15. In order to double the frequency of the fundamental note emitted by a stretched string, the length is reduced to $\frac{3}{4}$th of the original length and the tension is changed. The factor, by which the tension is to be changed, is
(a) $\frac{3}{8}$ (b) $\frac{2}{3}$ (c) $\frac{8}{9}$ (d) $\frac{9}{4}$

16. A vehicle with a horn of frequency n is moving with a velocity of $30\ ms^{-1}$ in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $(n + n_1)$. If sound velocity in air be $330\ ms^{-1}$, then
(a) $n_1 = 10n$ (b) $n_1 = 0$
(c) $n_1 = \frac{n}{11}$ (d) $n_1 = -\frac{n}{11}$

17. Two trains are moving towards each other at speeds of $20\ ms^{-1}$ and $15\ ms^{-1}$ relative to the ground. The first train sounds a whistle of frequency 600 Hz, the frequency of the whistle heard by a passenger in the second train before the train meet is
(Speed of sound in air = $340\ ms^{-1}$)
(a) 600 Hz (b) 585 Hz (c) 645 Hz (d) 666 Hz

18. The driver of a car travelling with speed $30\ ms^{-1}$ towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is $330\ ms^{-1}$, the frequency of the reflected sound as heard by the driver is
(a) 720 Hz (b) 555.5 Hz (c) 550 Hz (d) 760 Hz

19. 16 tuning forks are arranged in the order of increasing frequencies. Any two successive forks give 8 beat/s, when sounded together. If the frequency of the last fork is twice the first, then the frequency of the first fork is
(a) 120 Hz (b) 160 Hz (c) 180 Hz (d) 220 Hz

20. A train of sound waves is propagated along an organ pipe and gets reflected from an open end. If the displacement amplitude of the waves (incident and reflected) are 0.002 cm, the frequency is 1000 Hz and wavelength is 40 cm. Then, the displacement amplitude of vibration at a point at distance 10 cm from the open end, inside the pipe is
(a) 0.002 cm (b) 0.003 cm (c) 0.001 cm (d) 0.000 cm

Directions *(Q. Nos. 21 to 26) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below .*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

21. Statement I A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of the same length.
Statement II The same tuning fork will not be in resonance with open pipe of same length due to end correction of pipe.

22. Statement I In a sound wave, a displacement node is a pressure antinode and *vice-versa*.
Statement II Displacement node is a point of minimum displacement.

23. Statement I Velocity of particles while crossing mean position (in stationary waves) varies from maximum at antinodes to zero at nodes.
Statement II Amplitude of vibration at antinodes is maximum and at nodes, the amplitude is zero and all particles between two successive nodes cross the mean position together.

24. Statement I We can recognised our friends by listening their voices.
Statement II The quality of sound produced by different persons are different.

25. Statement I The basic of Laplace correction was that, exchange of heat between the region of compression and rarefaction in air is not possible.
Statement II Air is a bad conduction of heat and velocity of sound in air is large.

26. Statement I If two waves of same amplitude, produce a resultant wave of same amplitude, then the phase difference between them will be 120°.
Statement II The resultant amplitude of two waves is equal to sum of amplitude of two waves.

Directions *(Q. Nos. 27 to 29) Two plane harmonic sound waves are expressed by the equations,*

$$y_1(x, t) = A\cos(0.5\pi x - 100\pi t)$$
$$y_2(x, t) = A\cos(0.46\pi x - 92\pi t)$$

(All parameters are in MKS)

27. How many times does an observer hear maximum intensity in 1 s?

(a) 4 (b) 10 (c) 6 (d) 8

28. What is the speed of the sound?

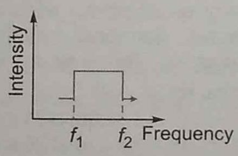
(a) $200\ \text{ms}^{-1}$ (b) $180\ \text{ms}^{-1}$ (c) $192\ \text{ms}^{-1}$ (d) $96\ \text{ms}^{-1}$

29. At $x = 0$, how many times the amplitude of $y_1 + y_2$ is zero in 1 s?

(a) 192 (b) 48 (c) 100 (d) 96

Directions *(Q. Nos. 30 to 32) Two trains A and B are moving with speeds 20 ms^{-1} and 30 ms^{-1} respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.*

Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency-lowest frequency) is thus 320 Hz. The speed of sound in air is 340 ms^{-1}.



30. The speed of sound of the whistle is

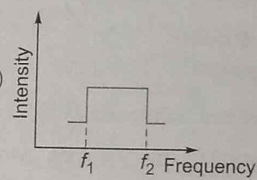
(a) $340\ \text{ms}^{-1}$ for passengers in *A* and $310\ \text{ms}^{-1}$ for passengers in *B*

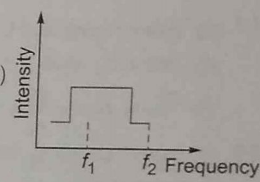
(b) $360\ \text{ms}^{-1}$ for passengers in *A* and $310\ \text{ms}^{-1}$ for passengers in *B*

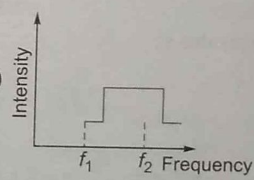
(c) $310\ \text{ms}^{-1}$ for passengers in *A* and $360\ \text{ms}^{-1}$ for passengers in *B*

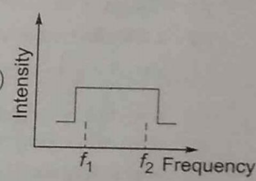
(d) $340\ \text{ms}^{-1}$ for passengers in both the trains

31. The distribution of the sound intensity of the whistle as observed by the passengers in train *A* is best represented by

(a)


(b)


(c)


(d)


32. The spread of frequency as observed by the passengers in train *B* is

(a) 310 Hz (b) 330 Hz
(c) 350 Hz (d) 290 Hz

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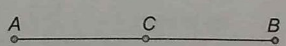
33. *A* and *B* are two sources generating sound waves. *A* listener is situated at *C*. The frequency of the source at *A* is 500 Hz. *A* now, moves towards *C* with a speed 4m/s. The number of beats heard at *C* is 6. When *A* moves away from *C* with speed 4 m/s, the number of beats heard at *C* is 18. The speed of sound is 340m/s. The frequency of the source at *B* is **[JEE Main Online 2013]**



(a) 500 Hz (b) 506 Hz
(c) 512 Hz (d) 494 Hz

34. An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km, it blows a whistle whose echo is heard by the driver after 5 s. If the speed of sound in air is 330 m/s, then the speed of the engine is **[JEE Main Online 2013]**

(a) 32 m/s (b) 27.5 m/s
(c) 60 m/s (d) 30 m/s

35. When two sound waves travel in the same direction in a medium the displacement of a particle located at *X* at time *t* is given by **[JEE Main Online 2013]**

$$y_1 = 0.05\cos(0.50\pi x - 100\pi t)$$
$$y_2 = 0.05\cos(0.46\pi x - 92\pi t)$$

where y_1, y_2 and x are in metres and t in seconds. The speed of sound in the medium is

(a) 92 m/s (b) 200 m/s (c) 100 m/s (d) 332 m/s

36. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1:3:4? **[JEE Main Online 2013]**

(a) At 36 cm and 84 cm from one end
(b) At 24 cm and 72 cm from one end
(c) At 48 cm and 96 cm from one end
(d) At 72 cm and 96 cm from one end

37. A cylindrical tube, open at both ends, has a fundamental frequency f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now **[AIEEE 2012]**

(a) f (b) $\frac{f}{2}$ (c) $\frac{3f}{4}$ (d) $2f$

38. The transverse displacement $y(x,t)$ of a wave on a string is given by $y(x,t)=e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$. This represents a **[AIEEE 2011]**

(a) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$

(b) standing wave of frequency $\sqrt{b}$

(c) standing wave of frequency $\frac{1}{\sqrt{b}}$

(d) wave moving in $+x$ direction with speed $\sqrt{a/b}$

39. A travelling wave represented by $y=A\sin(\omega t-kx)$ is superimposed on another wave represented by $y=A\sin(\omega t+kx)$. The resultant is **[AIEEE 2011]**

(a) a standing wave having nodes at $x=\left(n+\frac{1}{2}\right)\frac{\lambda}{2}$, $n=0,1,2$

(b) a wave travelling along $+x$ direction

(c) a wave travelling along $-x$ direction

(d) a standing wave having nodes at $x=\frac{n\lambda}{2}$, $n=0,1,2$.

40. Statement I Two longitudinal waves given by equation $y_1(x,t)=2a\sin(\omega t-kx)$ and $y_2(x,t)=a\sin(2\omega t-2kx)$ will have equal intensity.

Statement II Intensity of waves of given frequency in same medium is proportional to square of amplitude only. **[AIEEE 2011]**

(a) Statement I is false, Statement II is true

(b) Statement I is true, Statement II is false

(c) Statement I is true, Statement II true; Statement II is the correct explanation of Statement I

(d) Statement I is true, Statement II is true, Statement II is not correct explanation of Statement I

41. The equation of a wave on a string of linear mass density $0.04\ \text{kg m}^{-1}$ is given by **[AIEEE 2010]**

$$y=0.02\,(\text{m})\sin\left[2\pi\left(\frac{t}{0.04\,(\text{s})}-\frac{x}{0.50\,(\text{m})}\right)\right].$$

The tension in the string is

(a) 4.0 N (b) 12.5 N (c) 0.5 N (d) 6.25 N

42 A motor cycle start from rest and accelerates along a straight path at $2\ \text{ms}^{-2}$. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (speed of sound = $330\ \text{ms}^{-1}$). **[AIEEE 2009]**

(a) 49 m (b) 98 m (c) 147 m (d) 196 m

43. The function $\sin^2(\omega t)$ represents **[AIEEE 2005]**

(a) a simple harmonic motion with a period of $\frac{\pi}{\omega}$

(b) a periodic, simple harmonic motion with a period $\frac{2\pi}{\omega}$

(c) a periodic, but not simple harmonic motion with a period $\frac{\pi}{\omega}$

(d) a periodic, but not simple harmonic motion with a period of $\frac{2\pi}{\omega}$

44. A wave travelling along the x-axis is described by the equation $y(x,t)=0.005\cos(\alpha x-\beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are **[AIEEE 2008]**

(a) $\alpha=25.00\,\pi, \beta=\pi$ (b) $\alpha=\frac{0.08}{\pi}, \beta=\frac{2.0}{\pi}$

(c) $\alpha=\frac{0.04}{\pi}, \beta=\frac{1.0}{\pi}$ (d) $\alpha=12.50\,\pi, \beta=\frac{\pi}{2.0}$

45. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of **[AIEEE 2007]**

(a) 1000 (b) 10000

(c) 10 (d) 100

46. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x=2\times10^{-2}\cos\pi t$ m. The time at which the maximum speed first occurs, is **[AIEEE 2007]**

(a) 0.5 s (b) 0.75 s

(c) 0.125 s (d) 0.25 s

47. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed $v\ \text{ms}^{-1}$. The velocity of sound in air is $300\ \text{ms}^{-1}$. If the person can hear frequencies up to a maximum of 10000 Hz, the maximum value of v up to which he can hear the whistle is **[AIEEE 2006]**

(a) $15\sqrt{2}\ \text{ms}^{-1}$ (b) $15/\sqrt{2}\ \text{ms}^{-1}$

(c) $15\ \text{ms}^{-1}$ (d) $30\ \text{ms}^{-1}$

48. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option. **[AIEEE 2005]**

(a) $n=3, f_2=\frac{3}{4}f_1$ (b) $n=3, f_2=\frac{5}{4}f_1$

(c) $n=5, f_2=\frac{5}{4}f_1$ (d) $n=5, f_2=\frac{3}{4}f_1$

49. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency? **[AIEEE 2005]**

(a) Zero (b) 0.5%

(c) 5% (d) 20%

50. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 ms^{-1} and in air it is 300 ms^{-1}. The frequency of sound recorded by an observer who is standing in air is **[AIEEE 2004]**
(a) 200 Hz
(b) 3000 Hz
(c) 120 Hz
(d) 600 Hz

51. The displacement y of a particle in a medium can be expressed as

$$y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right) \text{m},$$

where t is in second and x in metre. The speed of the wave is **[AIEEE 2004]**
(a) 2000 ms^{-1} (b) 5 ms^{-1} (c) 20 ms^{-1} (d) 5π ms^{-1}

Answers

1. (a)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (d)
21. (c)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (a)	**28.** (a)	**29.** (c)	**30.** (b)
31. (a)	**32.** (a)	**33.** (c)	**34.** (d)	**35.** (b)	**36.** (d)	**37.** (a)	**38.** (a)	**39.** (a)	**40.** (b)
41. (d)	**42.** (b)	**43.** (a)	**44.** (a)	**45.** (d)	**46.** (a)	**47.** (c)	**48.** (c)	**49.** (d)	**50.** (d)
51. (b)									

Hints & Solutions

1. Time taken by stone to reach the lake

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500}{10}} = 10 \text{ s}$$

and time taken by sound of splash to reach the tower

$$t_2 = \frac{h}{v} = \frac{500}{332} \approx 1.5 \text{ s}$$

$\therefore$ Total time $t = t_1 + t_2 = 10 + 1.5 = 11.5$ s

2. In the first medium, frequency $\gamma r = \frac{c}{\lambda} = \frac{v'}{\lambda}$

It remains the same in second medium, *i.e.*, $v' = v$

$$\frac{v'}{\lambda'} = \frac{2\nu}{\lambda'} = \frac{\nu}{\lambda} \Rightarrow \lambda' = 2\lambda$$

3. The given equation be written as

$$y = 4 \sin\left[\pi\left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6}\right)\right] \quad \text{...(i)}$$

The standard wave equation can be written as

$$y = a \sin(\omega t - kx + \phi)$$

$$y = a \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \phi\right) \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we get

Amplitude $a = 4$ cm

Frequency $f = \frac{1}{T} = \frac{1}{10}$ Hz $= 0.1$ Hz

Wavelength $\lambda = 2 \times 9 = 18$ cm

Velocity $v = f\lambda = 0.1 \times 18 = 1.8 \text{ cms}^{-1}$

4. $I_m = 4ka^2$, $I = 2ka$

$I' = k(a^2 + a^2 + 2a^2 \cos^2\theta)$

$\Rightarrow \phi = \frac{\pi}{2}$

Path difference $= \frac{\lambda}{4} = 2x$

$$\Rightarrow \quad \frac{v}{4f} = 2x \Rightarrow x = \frac{v}{8f}$$

5. Speed increases slightly with temperature

$\Rightarrow \lambda$ increases $\Rightarrow$ for second resonance $x > 45$.

6. From the given wave equation, we find that

$$A = 4 \text{ cm}, \omega = \frac{2\pi}{0.02} \text{ s}^{-1} = 100\pi \text{s}^{-1}$$

and $k = \frac{2\pi}{100} \text{ cm}^{-1}$

$\therefore$ $\lambda = \frac{2\pi}{k} = 100$ cm

$$\nu = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

and $v = \frac{\omega}{k} = \frac{100\pi}{\frac{2\pi}{100}} = 50 \times 10^2 \text{ cms}^{-1}$

Hence, answer of wave speed v is wrong, *i.e.*, option (d) is correct.

7. Comparing given equation with standard form of wave equation, we get

$A = 0.001$ m, $\omega = 100 \text{ s}^{-1}$, and $k = 1 \text{ m}^{-1}$

$\therefore$ $\nu = \frac{100}{2\pi} = \frac{50}{\pi}$ Hz

$$\lambda = \frac{2\pi}{k} = 2\pi \text{ m}$$

and $v = \frac{\omega}{k} = \frac{100 \text{ s}^{-1}}{1 \text{ m}^{-1}} = 100 \text{ ms}^{-1}$

Moreover, as the wave equations of the form $y = A \sin(\omega t + kx)$, the wave is travelling along negative X-direction.

8. There is no transfer of heat from compression to rarefaction as air is a bad conductor of heat and time of compression rarefaction is too small.

9. Beat frequency $v_b = \frac{30}{3} = 10/s$

$$v_b = v_1 - v_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$$

$$v = \frac{\lambda_1 \lambda_2 v_b}{(\lambda_2 - \lambda_1)} = 300 \text{ m/s}$$

10. $y_1 = 0.25 \sin(310t)$ ...(i)

and $y_2 = 0.25 \sin(316t)$ ...(ii)

We have,

$$\omega_1 = 310$$

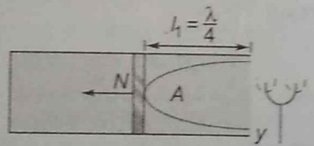
$\Rightarrow$ $f_1 = \frac{310}{2\pi}$ unit

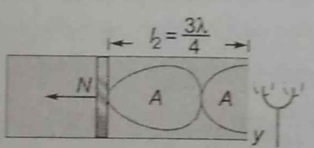
and $\omega_2 = 316 \Rightarrow f_2 = \frac{316}{2\pi}$ unit

Hence, beat frequency $= f_2 - f_1$

$$= \frac{316}{2\pi} - \frac{310}{2\pi} = \frac{3}{\pi} \text{ unit}$$

11. In a closed organ pipe in which length of air-column can be increased or decreased, the first resonance occurs at $\lambda/4$ and second resonance occurs at $3\lambda/4$.





Thus, at first resonance

$$\frac{\lambda}{4} = 13 \quad \text{...(i)}$$

and at second resonance

$$\frac{3\lambda}{4} = 41 \quad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii), we have

$$\frac{3\lambda}{4} - \frac{\lambda}{4} = 41 - 13$$

$\Rightarrow$ $\frac{\lambda}{2} = 28$

$\therefore$ $\lambda = 56 \text{ cm}$

Hence, frequency of tuning fork,

$$n = \frac{v}{\lambda} = \frac{350}{56 \times 10^{-2}}$$

$$= 625 \text{ Hz}$$

12. There are four beats between P and Q, therefore the possible frequencies of P are 246 or 254 (i.e., 250 ± 4) Hz.

When the prong of P is filled, its frequency becomes greater than the original frequency.

If we assume that the original frequency of P is 254, then on filing its frequency will be greater than 254. The beats between P and Q will be more than 4. But it is given that the beats are reduced to 2, therefore, 254 is not possible. Therefore, the required frequency must be 246 Hz.

(This is true, because on filing the frequency may increase to 248, giving 2 beats with Q of frequency 250 Hz).

13. It is given that

$$r_P = 2\text{m and } r_Q = 3\text{m}$$

$$\frac{I_P}{I_Q} = \left(\frac{r_Q}{r_P}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$\Rightarrow$ $I_P : I_Q = 9 : 4$

14. Let n be the actual frequency of sound of horn. If v_s be the velocity of car, then frequency of sound striking the cliff (source is moving towards listener)

$$n' = \frac{v \times n}{v - v_s} \quad \text{...(i)}$$

The frequency of sound heard on reflection

$$n'' = \frac{(v + v_s)n'}{v} = \frac{(v + v_s)}{v} \times \frac{v \times n}{(v - v_s)}$$

or $\frac{n''}{n} = \frac{v + v_s}{v - v_s} = 2$

$$v + v_s = 2v - 2v_s \quad \therefore 3v_s = v$$

or $v_s = \frac{v}{3}$

15. From law of string, other factors remaining unchanged,

$$\frac{v_1}{v_2} = \frac{l_2}{l_1} \sqrt{\frac{T_1}{T_2}}$$

Now, $v_2 = 2v_1, l_2 = \frac{3}{4} l_1$, hence, we have

$$\frac{1}{2} = \frac{3}{4} \times \sqrt{\frac{T_1}{T_2}}$$

$\Rightarrow$ $\frac{2}{3} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{T_2}{T_1} = \frac{9}{4}$ or $T_2 = \frac{9}{4} T_1$

16. Since, vehicle having siren is moving in a perpendicular direction, hence there will be no Doppler shift in frequency and $n_1 = 0$.

17. Here, $v = 340 \text{ ms}^{-1}$, v_s = velocity of 1st train $= 20 \text{ ms}^{-1}$, v_o = velocity of 2nd train $= 15 \text{ ms}^{-1}$ and $v_0 = 600$ Hz. As S and O are approaching each other, hence

$$v = \left[\frac{v + v_o}{v - v_s}\right] v_0 = \frac{340 + 15}{340 - 20} \times 600 = 666 \text{ Hz}$$

18. As $v = \left[\frac{v + v_c}{v - v_c}\right] v_0 = \left[\frac{330 + 30}{330 - 30}\right] \times 600 = \frac{360}{300} \times 600 = 720$ Hz

19. As forks have been arranged in ascending order of frequencies, hence if frequency of Ist fork be n, then

$$n_2 = n + 8$$
$$n_3 = n + 2 \times 8 = n + 16$$

and
$$n_{16} = n + 15 \times 8$$
$$= n + 120 = 2n$$
$$\Rightarrow \quad n = 120 \text{ Hz}$$

20. The equation of stationary wave for open organ pipe can be written as

$y = 2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi ft}{v}\right)$, where $x = 0$ is the open end from where wave gets reflected.

Amplitude of stationary wave is, $A_s = 2A\cos\left(\frac{2\pi x}{\lambda}\right)$

For $x = 0.1$ m, $A_s = 2 \times 0.002 \cos\left[\frac{2\pi \times 0.1}{0.4}\right] = 0$

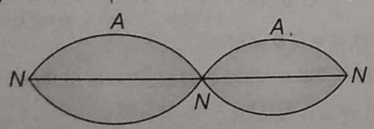
21. If a closed pipe of length L is in resonance with a tuning fork of frequency ν, then $\nu = \frac{v}{4L}$

An open pipe of same length L produces vibrations of frequency $\frac{v}{2L}$. Obviously, it cannot be in resonance with the given tuning fork of frequency $\nu\left(= \frac{v}{4L}\right)$.

22. At the point where a compression and a rarefaction meet, the displacement is minimum and it is called displacement node. At this point, the pressure difference is maximum *i.e.*, at the same time, it is a pressure antinode.

On the other hand, at the mid-point of a compression or a rarefaction, the displacement variation is maximum *i.e.*, such a point is displacement antinode. However such a point is pressure node, as pressure variation is minimum at such a point.

23. Stationary wave is represented as shown in figure.



It is quite clear from figure that at nodes the amplitude is zero and velocity of particle is also zero and at antinodes the amplitude is maximum. So that the velocity of particle is also maximum and all particles cross mean position between two successive nodes.

24. Sounds coming from the different sources can be recognised by virtue of their quality which is characteristics of sound. That is why we recognise the voices of our friends.

25. According to Laplace, the changes in pressure and volume of a gas, when sound waves propagated through it, are not isothermal but adiabatic. A gas is a bad conductor of heat. It does not allow the free exchange of heat between compressed layer, rarified layer and its surrounding.

26. The resultant amplitude of two waves is given by

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta}$$

Here, $a_1 = a_2 = A = a$

$1/2 = 1 + \cos\theta$ or $\cos\theta = -1/2$ or $\theta = 120°$

27. In one second number of maximas is called the beat frequency.

Hence, $f_b = f_1 - f_2 = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4$ Hz

28. Speed of wave $v = \frac{\omega}{k}$

or $v = \frac{100\pi}{0.5\pi}$ or $\frac{92\pi}{0.46\pi} = 200 \text{ ms}^{-1}$

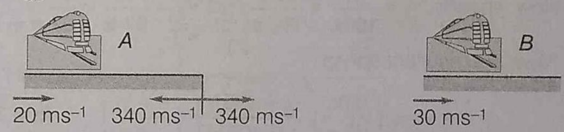
29. At $x = 0$, $y = y_1 + y_2 = 2A\cos 96\pi t \cos 4\pi t$

Frequency of cos $(96\pi t)$ function is 48 Hz and that of cos $(4\pi t)$ function is 2 Hz.

In one second, cos function becomes zero at $2f$ times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will becomes zero 100 times in 1 s.

30. $v_{SA} = 340 + 20 = 360 \text{ ms}^{-1}$

$v_{SB} = 340 - 30 = 310 \text{ ms}^{-1}$



31. For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20 ms^{-1}. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities. Therefore, the correct option is (a).

32. For the passengers in train B, observer is receding with velocity 30 ms^{-1} and source is approaching with velocity 20 ms^{-1}.

$$\therefore \quad f_1' = 800\left(\frac{340 - 30}{340 - 20}\right) = 775 \text{ Hz}$$

and
$$f_2' = 1120\left(\frac{340 - 30}{340 - 20}\right) = 1085 \text{ Hz}$$

$\therefore$ Spread of frequency $= f_2' - f_1' = 310$ Hz

33. Here, frequency of source = 500 Hz

Speed of source $A = 4$ m/s $= u$

Then, source is moving towards stationary observer,

$$\nu' = \frac{v}{v - u}\nu_0 \qquad \text{(where } v = \text{speed of sound)}$$
$$= \frac{340}{340 - 4} \times 500$$
$$\Rightarrow \quad \nu' = \frac{340}{336} \times 500 \text{ Hz} = 506 \text{ Hz}$$

Now, when source is reciding from the observer

$$\nu' = \frac{v}{v + u}\nu_0$$
$$\Rightarrow \quad = \frac{340}{344} \times 500 \text{ Hz} \qquad \therefore \nu' = 949 \text{ Hz}$$

According to question

Let freqency of source B is Z Hz

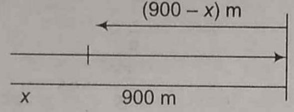
$\therefore \quad Z = 506 \pm 6 \Rightarrow Z = 500$ or 512

and $\quad Z = 494 \pm 18 \Rightarrow Z = 512$ or 476

Thus, required frequency = 512 Hz

34. We have $\dfrac{900 + 900 - x}{10800 - x} = 330 \times 5 = 1650$

$\therefore \quad x = 150$ m



$\therefore$ Distance $= \dfrac{150}{5} = 30$ m/s

35. $y_1 = 0.05 \cos (0.50\,\pi\, x - 100\pi\, t)$

and $y_2 = 0.05 \cos (0.46\,\pi\, x - 92\,\pi t)$

Comparing these two equations with $y = A \sin (k\,x - \omega t)$

We have, $\omega_1 = 100\,\pi$ and $\omega_2 = 92\,\pi$

Now, speeds, $v_1 = \dfrac{A}{100\pi} = \dfrac{0.05}{100\,\pi}$ and $v_2 = \dfrac{A}{92\,\pi} = \dfrac{0.05}{92\,\pi}$

Now, the resultant speed,

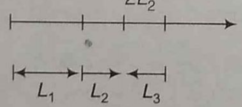
$$v = \sqrt{\left(\frac{0.05}{92\,\pi}\right)^2 + \left(\frac{0.05}{100\,\pi}\right)^2} \qquad [\pi = 3.14]$$

$= 200$ m/s

36. $\nu_1 = \dfrac{1}{2L_1}\sqrt{\dfrac{F}{\mu}}$

Let length of three segments be L_1, L_2 and L_3

$$\nu_2 = \frac{1}{2L_2}\sqrt{f/\mu}$$



$$\nu_3 = \frac{1}{2L_3}\sqrt{f/\mu}$$

So, that $\quad \nu_1 L_1 = \nu_2 L_2 = \nu_3 L_3$

As $\quad \nu_1 : \nu_2 : \nu_3 = 1 : 3 : 4$

$$\nu_1 = \frac{\nu_2 L_2}{L_1} \text{ and } \nu_2 = 3\,\nu_1,\ \nu_3 = 4\nu_1$$

$$L_2 = \frac{\nu_1}{\nu_2} L_1 = \frac{1}{3} L_1 = \frac{L_1}{3} \text{ and } L_3 = \frac{\nu_1}{\nu_3} L_1 = \frac{\nu_1}{4\nu} L_1 = \frac{L_1}{4}$$

$$L_1 + L_2 + L_3 = 114$$

Now, $L_1\left(1 + \dfrac{1}{3} + \dfrac{1}{4}\right) = 114 \Rightarrow L_1\left(\dfrac{12 + 4 + 3}{12}\right) = 114$

$$L_1\left(\frac{19}{12}\right) = 114 \Rightarrow L_1 = \frac{(114 \times 12)}{19} = 72$$

$$L_2 = \frac{L_1}{3} = \frac{72}{3} = 24 \text{ cm}$$

distance from one end

$= (72 + 24)$ cm $= 96$ cm

37. Initially for open organ pipe, fundamental frequency,

$$\nu_0 = \frac{v}{2l} = f \text{ (given)}$$

But when it is half dipped in water, then it becomes closed organ pipe of length $\dfrac{l}{2}$. In this case, fundamental frequency,

$$\nu_c = \frac{v}{4l'} = \frac{v}{4\dfrac{l}{2}} = \frac{v}{2l} = f$$

38. $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)} = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

It is a function of type

$$y = f(\omega t + kx)$$

$\therefore$ $y(x, t)$ represents wave travelling along $-x$ direction.

Speed of wave $= \dfrac{\omega}{k} = \dfrac{\sqrt{b}}{\sqrt{a}} = \sqrt{\dfrac{b}{a}}$

39. $y = y_1 + y_2 = A \sin (\omega t - kx) + A \sin(\omega t + kx)$

$y = 2A \sin\omega t \cos kx$

Clearly, it is equation of standing wave for position of nodes $y = 0$.

i.e., $\quad x = (2n + 1)\dfrac{\lambda}{4}$

$\Rightarrow \quad \left(n + \dfrac{1}{2}\right)\dfrac{\lambda}{2}$

40. $I = \dfrac{1}{2}\rho\omega^2 A^2 v$

Here, ρ = density of medium,

A = amplitude,

ω = angular frequency and

v = velocity of wave

$\therefore$ Intensity depend upon amplitude, frequency as well as velocity of wave

Also, $\quad I_1 = I_2$

41. $T = \mu v^2 = \mu \dfrac{\omega^2}{k^2}$

$$= \frac{0.04\,(2\pi / 0.04)^2}{(2\pi / 0.50)^2} = 6.25 \text{ N}$$

42. Motor cycle, $u = 0$, $a = 2\text{ms}^{-2}$

Observer is in the motion and source is at rest

$\Rightarrow \quad n' = n\dfrac{v - v_0}{v + v_s}$

$\Rightarrow \quad \dfrac{94}{100} n = n\dfrac{330 - v_0}{330}$

$\Rightarrow \quad 330 - v_0 = \dfrac{330 \times 94}{100}$

$\Rightarrow \quad v_0 = 330 - \dfrac{94 \times 33}{10}$

$$= \frac{33 \times 6}{10} \text{ms}^{-1}$$

$$s = \frac{v^2 - u^2}{2a} = \frac{9 \times 33 \times 33}{100}$$

$$= \frac{9 \times 1089}{100} = 98 \text{ m}$$

43. As, $y = \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{\cos 2\omega t}{2}$

Velocity, $\frac{dy}{dt} = \frac{1}{2} \times 2\omega \sin 2\omega t$

Acceleration, $\frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t = 4\omega^2 \left(\frac{1}{2} - y\right)$

As acceleration $\propto$ displacement and negative sign shows that it is direction towards mean position.

$\therefore$ Motion is simple harmonic and its period $= \frac{\pi}{\omega}$.

44. $y(x, t) = 0.005 \cos (\alpha x - \beta t)$

$$\frac{2\pi}{\lambda} = \alpha \text{ and } \frac{2\pi}{T} = \beta$$

So, $\alpha = \frac{2\pi}{0.08} = 25\pi$ and $\beta = \frac{2\pi}{2} = \pi$

45. Let intensity of sound be I and I'.

Loudness of sound initially

$$\beta_1 = 10 \log \left(\frac{I}{I_0}\right)$$

Later, $\beta_2 = 10 \log \left(\frac{I'}{I_0}\right)$

Given, $\beta_1 - \beta_2 = 20$

$\therefore$ $20 = 10 \log \left(\frac{I}{I'}\right)$ or $I' = \frac{I}{100}$

Therefore, intensity decreases by a factor of 100.

46. $x = (2 \times 10^{-2}) \cos \pi t$

Here, $a = 2 \times 10^{-2}$ m $= 2$ cm

At $t = 0$, $x = 2$ cm, *i.e.*, the object is at positive extreme, so to acquire maximum speed (*i.e.*, to reach mean position) it takes $\frac{1}{4}$th of time period.

$\therefore$ Required time $= \frac{T}{4}$

where, $\omega = \frac{2\pi}{T} = \pi \Rightarrow T = 2$ s

So, required time $= \frac{T}{4} = \frac{2}{4} = 0.5$ s

47. $10000 = f_{app.} = \left(\frac{300}{300 - v}\right) \times 9500$

or $v = 15 \text{ ms}^{-1}$

48. $f_1 = \frac{v}{l}$ (2nd harmonic of open pipe)

$f_2 = n\left(\frac{v}{4l}\right)$ (nth harmonic of closed pipe)

Here, n is odd and $f_2 > f_1$

It is possible when $n = 5$

because with $n = 5$

$$f_2 = \frac{5}{4}\left(\frac{v}{l}\right) = \frac{5}{4} f_1$$

49. $v_o = \frac{v}{5} \Rightarrow v_o = \frac{320}{5} = 64 \text{ ms}^{-1}$

When observer moves towards the stationary source, then

$$n' = \left(\frac{v + v_o}{v}\right) n$$

$\Rightarrow$ $n' = \left(\frac{320 + 64}{320}\right) n$

or $n' = \left(\frac{384}{320}\right) n$

or $\frac{n'}{n} = \frac{384}{320}$

Hence, percentage increase

$$\left(\frac{n' - n}{n}\right) = \left(\frac{384 - 320}{320} \times 100\right) \%$$

$$= \left(\frac{64}{320} \times 100\right) \% = 20\%$$

50. The frequency is a characteristic of source. It is independent of the medium. Hence, the correct option is (d).

51. As given $y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)$ ...(i)

Comparing it with

$y = a \sin (\omega t + kx + \phi)$ ...(ii)

We obtain, $\omega = 100 \text{ rad s}^{-1}$, $k = 20 \text{ m}^{-1}$

$\therefore$ $v = \frac{\omega}{k} = \frac{100}{20} = 5 \text{ ms}^{-1}$

Unit Test 2

(Waves and Oscillations)

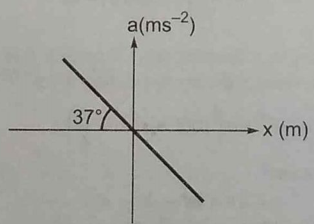
DAY 13

1. A mass m attached to a spring of spring constant k is stretched a distance x_0 from its equilibrium position and released with no initial velocity. The maximum speed attained by the mass in its subsequent motion and the time at which this speed would be attained are respectively,

(a) $\sqrt{\frac{k}{m}}x_0, \pi\sqrt{\frac{m}{k}}$ (b) $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \frac{\pi}{2}\sqrt{\frac{m}{k}}$

(c) $\sqrt{\frac{k}{m}}x_0, \frac{\pi}{2}\sqrt{\frac{m}{k}}$ (d) $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \pi\sqrt{\frac{m}{k}}$

2. The acceleration-displacement graph of a particle executing SHM is shown in the figure. The time period of SHM is



(a) $\frac{4\pi}{\sqrt{3}}$s

(b) $\frac{2\pi}{\sqrt{3}}$s

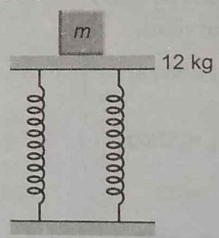
(c) The given graph doesn't represent SHM

(d) Information is insufficient

3. A spring balance has a scale that can read from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates harmonically with a time period of 0.6s. The mass of the body is [Take, $g = 10\,\text{ms}^{-2}$]

(a) 10 kg (b) 25 kg
(c) 18 kg (d) 22.8 kg

4. A plank of mass 12 kg is supported by two identical springs as shown in the figure. The plank always remains horizontal. When the plank is pressed down and released, it performs SHM with a time period of 3 s. When a block of mass m is attached to the plank, the time period changes to 6 s. The mass of the block is



(a) 48 kg (b) 36 kg
(c) 24 kg (d) 12 kg

5. For a particle executing SHM, determine the ratio of average acceleration of the particle between extreme position and the equilibrium position w.r.t. the maximum acceleration

(a) $\frac{4}{\pi}$ (b) $\frac{2}{\pi}$
(c) $\frac{1}{\pi}$ (d) $\frac{1}{2\pi}$

6. Two springs are made to oscillate simple harmonically when the same mass is suspended, individually. The time periods obtained are T_1 and T_2. If both the springs are connected in series and then made to oscillate when suspended by the same mass, the resulting time will be

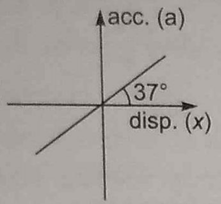
(a) $T_1 + T_2$ (b) $\frac{T_1T_2}{T_1 + T_2}$

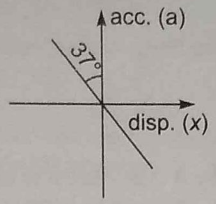
(c) $\sqrt{T_1^2 + T_2^2}$ (d) $\frac{T_1 + T_2}{2}$

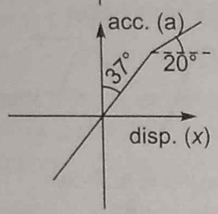
7. Find the time period of oscillations of a torsional pendulum, if the torsional constant of wire is $10\pi^2$ in SI units. The moment of inertia of the rigid body is 10 kg-m^2 about the axis of rotation

(a) 1 s (b) 2 s
(c) 4 s (d) $\frac{1}{2}$ s

8. Acceleration-displacement graph for four particles are shown, identify the one which represents SHM for all the values of displacements

(a)


(b)


(c)


(d) None of these

9. For a particle executing SHM, the velocity is plotted against displacement. The curve will be a/an

(a) straight line (b) parabola
(c) circle (d) ellipse

10. A simple pendulum of frequency n falls freely under gravity from certain height from the ground level. Its frequency of oscillation will

(a) remain unchanged
(b) be greater than n
(c) be less than n
(d) become zero

11. A simple pendulum is suspended from the ceiling of a stationary tram car. Now the car starts accelerating, the time period of a simple pendulum is the least when

[Take magnitude of acceleration to be same in all the cases]

(a) car is accelerating up
(b) car is accelerating down
(c) car is accelerating horizontally
(d) car is stationary

12. A particle executes SHM about O with an amplitude A and time period T. The magnitude of its acceleration, $\frac{T}{8}$ s after the particle reaches the extreme position, would be

(a) $\frac{4\pi^2 A}{\sqrt{2}T^2}$ (b) $\frac{4\pi^2 A}{T^2}$
(c) $\frac{2\pi^2 A}{\sqrt{2}T^2}$ (d) None of these

13. A string of length 1.5 m with its two ends clamped, is vibrating in the fundamental mode. The amplitude at the centre of the string is 4 mm. The minimum distance between two points having amplitude 2 mm, is

(a) 1 m (b) 75 cm
(c) 60 cm (d) 50 cm

14. A spring of negligible mass having a force constant k extends by an amount y when a mass m is hung from it. The mass is pulled down a little and then released. The system begins to execute SHM of amplitude A and angular frequency ω. The total energy of the mass-spring system will be

(a) $\frac{m\omega^2 A^2}{2}$
(b) $\frac{m\omega^2 A^2}{2} - \frac{ky^2}{2}$
(c) $\frac{ky^2}{2}$
(d) $\frac{m\omega^2 A^2}{2} + \frac{ky^2}{2}$

15. Two separated sources emit sinusoidal travelling waves that have the same wavelength λ and are in phase at their respective sources. One travels a distance l_1 to get to the observation point while the other travels a distance l_2. The amplitude is minimum at the observation point, if $l_1 - l_2$ is an

(a) odd multiple of λ (b) even multiple of λ
(c) odd multiple of $\frac{\lambda}{2}$ (d) odd multiple of $\frac{\lambda}{4}$

16. A standing wave can be produced by combining

(a) two longitudinal travelling waves
(b) two transverse travelling waves
(c) two sinusoidal travelling waves travelling in opposite direction
(d) All of the above

17. Two identical strings A and B, have nearly the same tension. When they both vibrate in their fundamental resonant modes, there is a beat frequency of 3 Hz. When string B is tightened slightly, to increase the tension, the beat frequency becomes 6 Hz. This means

(a) that before tightening A had a higher frequency than B, but after tightening, B has a higher frequency than A
(b) that before tightening B has higher frequency than A, but after tightening A has higher frequency than B
(c) that before and after tightening A has higher frequency than B
(d) that before and after tightening B has higher frequency than A

18. A string fixed at both ends having a fundamental frequency of 240 Hz, is vibrated with the help of a tuning fork, having frequency 480 Hz, then

(a) string will vibrate with a frequency of 240 Hz
(b) string will vibrate in resonance with the tuning fork
(c) string will vibrate with a frequency of 480 Hz, but is not in resonance with the tuning fork
(d) string is in resonance with the tuning fork and hence vibrates with a frequency of 240 Hz

19. A train is passing by a platform at a constant speed of 40 ms^{-1}. The horn of the train has a frequency of 320 Hz. Find the overall change in frequency detected by a person standing on the platform, *i.e.*, when the train approaching and then precedes from him. (Take, velocity of sound in air as 320 ms^{-1})

(a) 216.4 Hz (b) 81.3 Hz
(c) 365.7 Hz (d) 284.4 Hz

20. A string of length 0.4 m and mass 10^{-2} kg is clamped at one end. The tension in the string is 1.6N. Identical wave pulses are generated at the free end, after a time interval, Δt. The minimum value of Δt, so that a constructive interference takes place between successive pulses is

(a) 0.1s
(b) 0.05 s
(c) 02 s
(d) Constructive interference cannot take place

21. A string vibrates according to the equation $Y = 5\sin\left(\frac{2\pi x}{3}\right) \times \cos 20\pi t$, where x and y are in cm and t in second. The distance between two adjacent nodes is

(a) 3 cm (b) 4.5 cm
(c) 6 cm (d) 1.5 cm

22. A point source of sound is placed in a non-absorbing medium. Two points *A* and *B* are at the distance of 1 m and 2 m, respectively from the source. The ratio of amplitudes of waves at *A* to *B* is

(a) 1 : 1 (b) 1 : 4
(c) 1 : 2 (d) 2 : 1

23. Two canoes are 10 m apart on a lake. Each bobs up and down with a period of 4.0 s. When one canoe is at its highest point, the other canoe is at its lowest point. Both canoes are always within a single cycle of the waves. Determine the speed of the wave

(a) 2.5 ms^{-1} (b) 5 ms^{-1}
(c) 40 ms^{-1} (d) 4 ms^{-1}

24. The mathematical form of three travelling waves are given by

$$y_1 = (2\text{ cm})\sin(3x - 6t),$$
$$y_2 = (3\text{ cm})\sin(4x - 12t),$$
and $$y_3 = (4\text{ cm})\sin(5x - 11t)$$

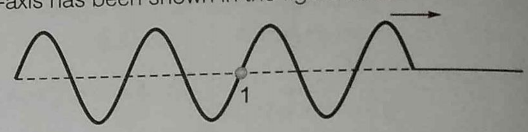
Of these waves,

(a) wave 1 has greatest wave speed and wave has maximum transverse string speed
(b) wave 2 has greatest wave speed and wave 1 has greatest maximum transverse string speed
(c) wave 3 has greatest wave speed and wave 1 has maximum transverse string speed
(d) wave 2 has greatest wave speed and wave 3 has maximum transverse string speed

25. If the maximum speed of a particle carrying a travelling wave is v_0, then find the speed of a particle when the displacement is half that of the maximum value

(a) $\frac{v_0}{2}$ (b) $\frac{\sqrt{3}v_0}{4}$
(c) $\frac{\sqrt{3}v_0}{2}$ (d) v_0

26. A transverse wave on a string travelling along positive *x*-axis has been shown in the figure below



The mathematical form of the wave is shown

$$y = (3.0\text{cm})\sin\left[2\pi \times 0.1t - \frac{2\pi}{100}x\right]$$

where t is in seconds and x is in cm. Find total distance travelled by the particle at the (1), in 10 min 15 s, measured from the instant shown in the figure and direction of the motion of the particle at the end of this time.

(a) 6 cm, in upward direction
(b) 6 cm, in downward direction
(c) 738 cm, in upward direction
(d) 732 cm, in upward direction

Directions *(Q. Nos. 27 to 33) Each of these questions contains two statements Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

27. **Statement I** Waves on a string can be longitudinal in nature.

Statement II The string cannot be compressed or rarified.

28. **Statement I** A wave of frequency 500 Hz is propagating with a velocity of 350 m/s. Distance between two particles with 60° phase difference is 12 cm.

Statement II $x = \frac{\lambda}{2\pi}\phi$

29. **Statement I** When a wave goes from one medium to other, average power transmitted by the wave may change.

Statement II Due to a change in the medium, amplitude, speed, wavelength and frequency of the wave may change.

30. **Statement I** A particle performs a simple harmonic motion with amplitude A and angular frequency ω. To change the angular frequency of simple harmonic motion to 3ω, and amplitude to $A/2$, we have to supply an extra energy of $\frac{5}{4}m\omega^2A^2$, where m is the mass of the particle executing simple harmonic motion.

Statement II Angular frequency of the simple harmonic motion is independent of the amplitude of oscillation.

31. **Statement I** Time period of spring pendulum is the same whether in an accelerated or in an inertial frame of reference.

Statement II Mass of the bob of the spring pendulum and the spring constant of spring are independent of the acceleration of the frame of reference.

32. **Statement I** The total energy of a particle executing simple harmonic motion, can be negative.

Statement II Potential energy of a system can be negative.

33. **Statement I** A circular metal hoop is suspended on the edge, by a hook. The hoop can oscillate from one side to the other in the plane of the hoop, or it can oscillate back and forth in a direction perpendicular to the plane of the hoop. The time period of oscillation would be more when oscillations are carried out in the plane of hoop.

Statement II Time period of physical pendulum is more if the moment of inertia of the rigid body about the corresponding axis passing through the pivoted point, is more.

Directions (Q. Nos. 34 to 36) *A block of mass m is connected to a spring of spring constant k and is at rest in equilibrium as shown in* **Fig.** *(a). Now, the block is displaced by a height h below its equilibrium position and imparted a speed v_0 in a downward direction as shown in* **Fig.** *(b). As a result of the jerk, the block executes simple harmonic motion about its equilibrium position.*

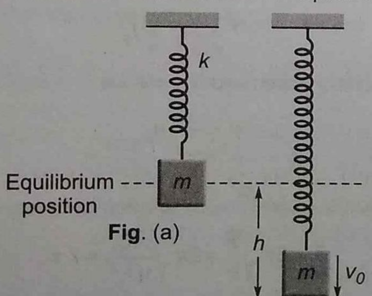


Fig. (a) Fig. (b)

34. The amplitude of oscillation is

(a) h (b) $\sqrt{\frac{mv_0^2}{k}+h^2}$

(c) $\sqrt{\frac{m}{k}}v_0+h$ (d) None of the above

35. The equation for the simple harmonic motion is

(a) $y=-A\sin[\omega t+\delta]$ (b) $y=-A\cos[\omega t+\delta]$

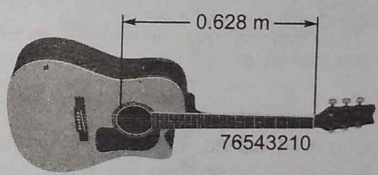
(c) $y=A\sin\left[\omega t+\delta+\frac{\pi}{2}\right]$ (d) $y=A\sin\left[\omega t+\delta+\frac{\pi}{4}\right]$

36. Find the time taken by the block to cross the mean position for the first time.

(a) $\frac{\delta}{\omega}$ (b) $\frac{\frac{\pi}{2}-\delta}{\omega}$ (c) $\frac{\pi-\delta}{\omega}$ (d) $\frac{\pi-\delta}{2\omega}$

where, A is the amplitude of oscillation, $S=\sin^{-1}\left[\frac{k}{A}\right]$, $\omega=\sqrt{\frac{k}{m}}$

Directions (Q. Nos. 37 to 40) *When the string is pressed against any fret i, the fundamental frequency of the shortened string is larger by a factor of $\sqrt[12]{2}$, than it is when the string is pushed against the fret $(i-1)$. For the sake of convenience, the frets have been numbered as shown in the figure*



Let us consider a particular guitar string having linear density 0.046 kg/m, which is stretched under a tension of 324.57, which can be varied using the knob system of the guitar. This string, when vibrated across the whole length, produces standing wave of musical note E having frequency 164.8 Hz.

37. Which fret does the guitar player press, so to produce a musical note E which has to be sounded two octave higher pitch from its fundamental note?

(a) 26th (b) 24th (c) 25th (d) 13th

38. Find the spacing between 5th and 6th fret.

$[2^{5/12}=1.335, 2^{4/12}=1.26, 2^{1/12}=1.059]$

(a) 2.04 cm (b) 1.56 cm (c) 1.13 cm (d) 3 cm

39. The E string of guitar can be worked as D string which produces musical note D having fundamental frequency $f=146.8$ Hz by using a device D-tuner. This device allows the E string to be used as D string, by extending the length of the string. Find the amount of the extended length.

(a) 4 cm (b) 3.1 cm (c) 2.4 cm (d) 5.85 cm

40. The G string has a fundamental frequency of 196 Hz. If E string is to be used as G string, then approximately which fret has to be pressed?

(a) 5 (b) 4 (c) 3 (d) 2

Answer *with* Solutions

1. (*c*) At the mean position, the speed will be maximum.

$$\frac{kx_0^2}{2} = \frac{mv^2}{2}$$

$$\Rightarrow \quad v = v_{max} = \sqrt{\frac{k}{m}}\, x_0$$

and this is attained at $t = \frac{T}{4}$.

Time period of motion is,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

So, the required time is,

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

2. (*a*) $\frac{da}{dx} = -\tan 37° = -\frac{3}{4}$

$$\Rightarrow \quad a = -\frac{3}{4}x$$

On comparing with $a = -\omega^2 x$, we get

$$\omega^2 = \frac{3}{4}$$

$$\Rightarrow \quad \frac{2\pi}{T} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad T = \frac{4\pi}{\sqrt{3}}\text{ s}$$

3. (*d*) The scale can read a maximum of 50 kg, for a length of 20 cm. Let spring constant be k then,

$$kx_0 = mg$$

[for $m = 50$ kg, $x_0 = 20$ cm]

$$\Rightarrow \quad k \times 0.2 = 50 \times 10$$

$$\Rightarrow \quad k = 2500\ \text{Nm}^{-1}$$

Let mass of the body be m_0, then from

$$T = 2\pi\sqrt{\frac{m_0}{k}}$$

$$\Rightarrow \quad 0.6 = 2\pi\sqrt{\frac{m_0}{2500}}$$

$$\Rightarrow \quad m_0 = 22.8\text{ kg}$$

4. (*b*) Let spring constant of each spring be k, then the equivalent spring constant of the two spring system in parallel, is $2k$.

Without mass, $T = 3\text{ s} = 2\pi\sqrt{\frac{12}{2k}}$

With mass, $T_1 = 6\text{s} = 2\pi\sqrt{\frac{12+m}{2k}}$

$$\frac{T}{T_1} = \frac{1}{2} = \sqrt{\frac{12}{12+m}}$$

$$\Rightarrow \quad m = 36\text{ kg}$$

5. (*b*) Let the equation of SHM be,

$$x = A\sin\omega t$$

Average acceleration between extreme position and the equilibrium position, *i.e.*, from time $t = 0$ to $t = \frac{T}{4}$

$$l_m = \frac{\int_0^{T/4} \omega^2 A\sin\omega t\, dt}{\frac{T}{4}}$$

Maximum acceleration $(a_{max}) = \omega^2 A$

Then, the required ratio is,

$$= \frac{\int_0^{T/4} \omega^2 A\sin\omega t\, dt}{\frac{T}{4}\times\omega^2 A} = \frac{2}{\pi}$$

6. (*c*) Let the spring constants of the two springs be k_1 and k_2 respectively, then,

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}}$$

and $$T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

$$\Rightarrow \quad k_1 = \frac{4\pi^2 m}{T_1^2}$$

and $$k_2 = \frac{4\pi^2 m}{T_2^2}$$

When the two springs are connected in series, then

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

where, $$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

$$\Rightarrow \quad T = \sqrt{T_1^2 + T_2^2}$$

7. (*b*) For torisional pendulum, $\tau = -k\theta$

$$\alpha = -\frac{k}{I}\theta$$

$$\Rightarrow \quad \omega^2 = \frac{k}{I}$$

$$T = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{10}{10\pi^2}} = 2\text{ s}$$

8. (*b*) For a particle to execute SHM, $a = -\omega^2 x$

So, $\frac{da}{dx} = -\omega^2$, where ω^2 is positive quantity. This means for a particle to execute SHM, the acceleration-displacement curve should be a straight line having a negative slope, which is shown option (b).

9. (*d*) From $v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \quad v^2 = \omega^2 (A^2 - x^2)$

$\Rightarrow \quad \dfrac{v^2}{\omega^2} + \dfrac{x^2}{1^2} = A^2$

which is a standard equation of ellipse.

10. (*d*) For freely falling case, the effective g is zero, so that frequency of oscillation will be zero.

11. (*a*) When car is accelerating up, $g_{eff} = g + a$

When car is accelerating down, $g_{eff} = g - a$

When car is accelerating horizontally, $g_{eff} = \sqrt{g^2 + a^2}$

12. (*a*) Let at $t = 0$,the particle be at the extreme position, then the equation of SHM can be written as

$$x = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T} \times t\right)$$

At $t = \dfrac{T}{8}$, $x = A\cos\dfrac{\pi}{4} = \dfrac{A}{\sqrt{2}}$

Acceleration $= -\omega^2 x = -\left(\dfrac{2\pi}{T}\right)^2 \times \dfrac{A}{\sqrt{2}}$

Magnitude of acceleration $= \dfrac{4\pi^2 A}{\sqrt{2}\, T^2}$

13. (*d*) $A_s = 2A\sin kx$

$$2\text{ mm} = 4\text{ mm}\sin kx$$

$\Rightarrow \quad kx = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$\Rightarrow \quad x_2 - x_1 = \left[\dfrac{5\pi}{6} - \dfrac{\pi}{6}\right] \times \dfrac{1}{k} = \dfrac{\lambda}{3}$

As the string is vibrating in fundamental mode, $L = \dfrac{\lambda}{2}$

$\Rightarrow \quad \lambda = 2L = 3\text{ m}$

So, required separation between two points,

$$x_2 - x_1 = 1\text{ m}$$

14. (*d*) From initial equilibrium position,

$$ky = mg$$

When block is at distance x below mean position
Kinetic energy of the block,

$$K = \frac{m\omega^2 A^2}{2}\cos^2(\omega t - \phi) \qquad \text{[From SHM theory]}$$

Elastic potential energy of spring-block-earth system,

$$U_e = \frac{k(y + x)^2}{2}$$

where, $x = A\sin(\omega t + \phi)$

Gravitational potential energy of spring-block-earth system is, $U_g = -mgx$ taking mean position of reference position for gravitation potential energy.

Total energy,

$$E = K + U_e + U_g = \frac{m\omega^2 A^2}{2} + \frac{ky^2}{2}$$

15. (*c*) For destructive interference, path difference has to be equal to an odd multiple of $\dfrac{\lambda}{2}$.

16. (*c*) Sound waves in an organ pipe (which are standing in nature) is an example of superposition of two longitudinal travelling waves. Standing waves on a string is an example of superposition of two transverse travelling waves on a string travelling in opposite directions.

17. (*d*) Let the fundamental frequencies of *A* and *B*, before tightening of *B* are f_1 and f_2, respectively. Then either $f_1 - f_2 = 3$ or $f_2 - f_1 = 3$.

As tension in *B* increases (due to tightening), its frequency increases and the beat frequency also increases.

If $f_1 - f_2 = 3$, then according to given condition, when f_2 increases $f_1 - f_2$ the decreases so the frequencies of strings are related by $f_2 - f_1 = 3$.

i.e., before tightening, $f_2 > f_1$

After tightening,

$$f_2' - f_1 = 6, \text{ i.e., } f_2' > f_1$$

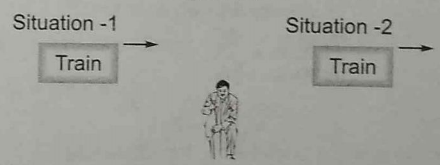
18. (*b*) If one of the natural frequencies of the string matches with the source frequency, then resonance condition arises and string starts vibrating with the source frequency.

19. (*b*) For situation 1,

$$f_{ap1} = \frac{v - 0}{v - v_s} \times f$$

$$= \frac{320}{320 - 40} \times 320$$

$$= 365.7\text{ Hz}$$



For situation 2,

$$f_{ap2} = \frac{v}{v - (-v_s)} \times f$$

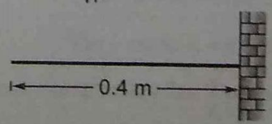
$$= 284.4\text{ Hz}$$

$$\Delta f = f_{ap1} - f_{ap2}$$

$$= 81.3\text{ Hz}$$

20. (*c*) Velocity of wave on the string

$$= \sqrt{\frac{T}{\mu}} = 8\text{ ms}^{-1}$$



The pulse gets inverted after reflection from the fixed end, so for constructive interference to take place between successive pulses, the first pulse has to undergo two reflections from the fixed end.

So, $\Delta t = \frac{2 \times 0.4 + 2 \times 0.4}{8}$

$= 0.2$ s

21. (*a*) The node and antinodes are formed in a standing wave pattern as a result of the interference of two waves. Distance between two nodes is half of wavelength (λ).

So, standard equation of standing wave is

$$y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda} \quad \ldots\text{(i)}$$

where, a is amplitude, λ is wavelength, v is velocity and t is time.

Given equation

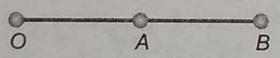
$$y = 5\sin\frac{2\pi x}{3}\cos 20\,\pi t \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{2\pi x}{\lambda} = \frac{2\pi x}{3}$$

$\Rightarrow \quad \lambda = 3$ cm

22. (*d*) Let the power of source be P and let it be placed at Q.



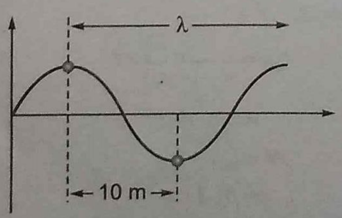
Then, intensity at A and at B would be given by

$$I_A = \frac{P}{4\pi \times 1^2}$$

and $$I_B = \frac{P}{4\pi \times 2^2}$$

$$\Rightarrow \quad \frac{(\text{Amp.})_A}{(\text{Amp.})_B} = \sqrt{\frac{I_A}{I_B}} = \sqrt{\frac{2^2}{1^2}} = 2:1$$

23. (*b*)



Frequency of the wave $= \frac{1}{4}$ Hz

Wavelength of the wave $= \lambda = 2 \times 10 = 20$ m

Velocity of the wave $= f\lambda = 5\ \text{ms}^{-1}$

24. (*d*) For the wave, $y = A\sin(kx - \omega t)$,

the wave speed is $\frac{\omega}{k}$ and the maximum transverse string speed is $A\omega$.

25. (*c*) For the wave $y = A\sin(\omega t - kx)$,

$$v_0 = A\omega$$

where A is the maximum displacement.

For the given condition,

$$\frac{A}{2} = A\sin(\omega t - kx) \Rightarrow \sin(\omega t - kx) = \frac{1}{2}$$

and $$\frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx) = A\omega\frac{\sqrt{3}}{2} = \frac{\sqrt{3}v_0}{2}$$

26. (*c*) We have, $T = \frac{1}{0.1}$ s $= 10$ s

In one complete cycle, particle travels a distance, 4 times the amplitude.

So, in a time interval of 10 min 15 s *i.e.*, 615 s *i.e.*, 61 full + 1 half-cycles, the distance travelled

$= (4 \times 3) \times 61 + (2 \times 3) \times 1 = 732 + 6 = 738$ cm

At this instant, the particle is moving in an upward direction.

27. (*d*) For longitudinal wave, the medium has to compress and rarify while the string cannot be compressed or rarified.

28. (*a*) $\lambda = \frac{v}{n} = \frac{350}{500} = 0.7$ m

$$\phi = 60° = 60 \times \frac{\pi}{180°} = \frac{\pi}{3}\ \text{radian}$$

As, $x = \frac{\lambda}{2\pi}\phi \Rightarrow x = \frac{0.7}{2\pi} \times \left[\frac{60\pi}{180}\right] = 0.12\ \text{m} = 12\ \text{cm}$

29. (*c*) $P_{av} = \frac{\rho v\omega^2 A^2}{2}$

When medium changes, v, ρ and A can change but frequency remains the same.

30. (*d*) Angular frequency of the simple harmonic motion depends upon the mass of the particle and the force constant, and is independent of amplitude of oscillation.

31. (*a*) The time period of a spring pendulum is given by, $T = 2\pi\sqrt{\frac{m}{k}}$ and hence is not affected by the acceleration of the frame of reference.

32. (*a*) Total energy of the particle performing simple harmonic motion is, $E = K + U = K_{max} + U_{min}$. K is always positive, while U could be positive, negative or zero. If U_{min} is negative and its value is greater than K_{max}, then E would be negative.

33. (*a*) When the hoop oscillates in its plane, moment of inertia is $I_1 = mR^2 + mR^2$ *i.e.*, $I_1 = 2mR^2$.

While when the hoop oscillates in a direction perpendicular to the plane of the hoop, moment of inertia is

$$I_2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$$

The time period of physical pendulum is,

$T = 2\pi\sqrt{\frac{I}{mgd}}$. Here, d is a same in both the cases.

34. (*b*) The angular frequency of simple harmonic motion is given by,

$$\omega = \sqrt{\frac{k}{m}}$$

The velocity of the block, when it is at a displacement of h from the mean position, is given by, $v = \omega\sqrt{A^2 - h^2}$, where A is the amplitude of oscillation.

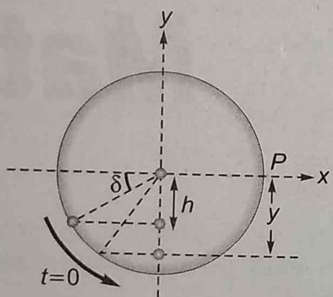
From the given initial condition,

$$v_0 = \sqrt{\frac{k}{m}}\sqrt{A^2 - h^2}$$

$$\Rightarrow \quad A^2 = \frac{mv_0^2}{k} + h^2$$

$$\Rightarrow \quad A = \sqrt{\frac{mv_0^2}{k} + h^2}$$

35. (*a*) To have the equilibrium in a simple harmonic motion, it is best to represent the simple harmonic motion as an uniform circular motion.



At $t = 0$, let the particle be making an angle δ with the negative, x-axis as shown, then

$$\sin\delta = \frac{h}{A} \Rightarrow \delta = \sin^{-1}\left(\frac{h}{A}\right)$$

At time t, $y = -A\sin(\omega t + \delta)$

So, the equation of simple harmonic motion is,

$$y = -\sqrt{\frac{mv_0^2}{k} + h^2}\left[\sin\left\{\sqrt{\frac{k}{m}}t + \sin^{-1}\left(\frac{h}{A}\right)\right\}\right]$$

36. (*c*) To compute the time taken by the block to cross the mean position for the first time, we can make use of the circular motion representation.

$$t = \frac{\pi - \delta}{\omega} = \frac{\pi - \sin^{-1}\left(\frac{h}{A}\right)}{\sqrt{\frac{k}{m}}}$$

37. (*c*) Let $x = 2^{1/12}$

So, for nth fret, frequency becomes $f = x^{(n-1)} f_0$

where f_0 is fundamental frequency for the 1st fret marked as zero.

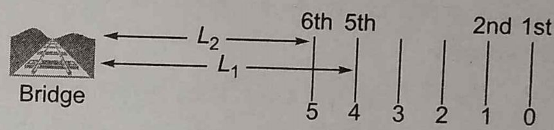
We have, $\quad f = 2^{(n-1)/12} f_0$

but $\quad f = 2^2 f_0$

So, $\quad \frac{n-1}{12} = 2$

$\Rightarrow \quad n = 25$

38. (*c*) $\frac{v}{2L_1} = x^4 f_0 \Rightarrow L_1 = \frac{v}{2x^4 f_0}$



$$\frac{v}{2L_2} = x^5 f_0$$

$$L_2 = \frac{v}{2x^5 f_0}$$

$$= 1.13\text{ cm}$$

$$\Rightarrow \quad L_1 - L_2 = \frac{v}{2f_0}\left[\frac{1}{x^4} - \frac{1}{x^5}\right]$$

$$= \frac{v}{2f_0}\left[\frac{1}{(2)^{4/12}} - \frac{1}{(2)^{5/12}}\right]$$

$$= 1.13\text{ cm}$$

[Hint for jth fret the length will be $= [(0.628)/(b)^j]$]

and for $(j+1)^{\text{th}}$ fret it will be $= [0.628/(b)^{j+1}]$

where b is frequency factor.

39. (*b*) $\quad f_D = 1.468 = \frac{v}{2L_D}$

$$f_E = 164.8 = \frac{v}{2L_E}$$

$$\Delta L = L_D - L_E = 3.1\text{ cm}$$

40. (*b*) $\quad x^n f_0 = f_G$

$\Rightarrow \quad x^n = \frac{196}{164.8}$

$\Rightarrow \quad n = 3$, *i.e.*, 4 th fret.

Day 14

Properties of Matter

Day 14 Outlines ...

- Elastic Behaviour
- Stress
- Strain
- Hooke's Law
- Thrust and Pressure
- Pascal's Law
- Archimedes' Principle
- Viscosity
- Stokes' Law
- Terminal Velocity
- Streamline and Turbulent Flow
- Reynolds' Number
- Equation of Continuity
- Bernoulli's Theorem
- Surface Tension
- Surface Energy
- Angle of Contact
- Excess Pressure Over a Liquid Film.
- Capillary Rise or Capillarity

Elastic Behaviour

Elasticity is the property of body by virtue of which a body regains or tends to regain its original configuration (shape as well as size), when the external deforming forces acting on it, is removed.

If a body completely regains its original configuration on removal of external deforming forces, it is called a **perfectly elastic body.**

If a body has no tendency to regain its original configuration and tends to maintain its deformed state even after the removal of the deforming force, the body is called a **plastic** (or **non-elastic**) **body.**

Stress

The internal restoring force per unit area of cross-section of the deformed body is called ***stress****. Thus,*

$$\text{Stress, } \sigma = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

Stress is numerically equal to the deforming force per unit area of cross-section.

SI unit of stress is Nm^{-2} or pascal (Pa).

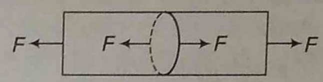
Stress are of two types normal stress or longitudinal stress and tangential stress or shearing stress.

Also the longitudinal stress can be of two types.

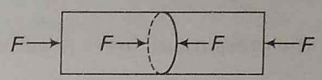
(i) If a solid in the form of a wire/rod is stretched by applying a force normally, then the stress is called **tensile stress** (first type of longitudinal stress).

(ii) If the solid wire/rod is compressed, then the stress is called **compressive stress.** (Other type of longitudinal stress)

(iii) For a bulk solid or liquid or gas if the deforming force is applied normal to the surface such that magnitude of force on any small area is proportional to the area, the stress is called **volumetric stress.**



Tensile stress



Compressive stress

Strain

Strain is the ratio of change in configuration to the original configuration of the body. Being the ratio of two similar quantities, strain is a unitless and dimensionless quantity.

(i) For a wire or rod, **linear strain** is defined as the ratio of change in length to the original length.

$$\therefore \quad \text{Longitudinal strain} = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

(ii) When the deforming force causes a change in volume, the strain is called **volumetric strain**.

$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

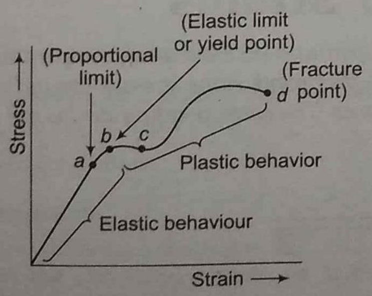
(iii) When the deforming force, applied tangentially to a surface, produces a change in shape of the body, the strain developed is called **shearing strain** or **shear**.

$$\text{Shearing strain} = \phi$$

Stress-Strain Relationship

Whenever a material is stressed, there are basically two different regimes, elastic and the non-elastic. The latter one is difficult to describe while in the former one stress is directly proportional to the strain.

If large deformation takes place between points can'd material is ductile and for small deformation between c and d material is said to brittle.



Hooke's Law

According to the Hooke's law, for any body, within the elastic limit, stress developed is directly proportional to the strain produced.

$$\text{stress} \propto \text{strain}$$
$$\text{stress} = E \times \text{strain}$$

The ratio of stress to strain, within the elastic limit, is called the coefficient (or modulus) of elasticity for the given material. *Depending on the type of stress applied and resulting strain, we have the following three of elasticity given as*

$$E = \frac{\text{stress}}{\text{strain}}$$

1. Young's Modulus

Young's modulus of elasticity (Y) is defined as the ratio of normal stress (either tensile or compressive) to the longitudinal strain within a elastic limit.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

2. Bulk Modulus

It is defined as the ratio of the normal stress to the volumetric strain.

Coefficient of volume elasticity

$$B = \frac{F/A}{\Delta V/V} = -\frac{pV}{\Delta V}$$

where, $p = \frac{F}{A}$ = the pressure or stress negative sign signifies that for an increase in pressure, the volume will decrease.

Reciprocal of bulk modulus is called **compressibility**.

3. Modulus of Rigidity (Shear modulus)

It is defined as the ratio of tangential stress to shearing stress.

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$= \frac{F/A}{\phi}$$

$$= \frac{F}{A\phi} = \frac{FL}{Ax}$$

Poisson's Ratio

For a long bar, the Poisson's ratio is defined as the ratio of lateral strain to longitudinal strain.

$\therefore$ Poisson's ratio $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$$= \frac{\Delta D/D}{\Delta L/L} = \frac{\Delta r/r}{\Delta L/L}$$

Poisson's ratio is a unitless and dimensionless term. Its value depends on the nature of the material. Theoretically, value of σ must lie between -1 and $+0.5$ but for most metallic solids $0 < \sigma < 0.5$.

Work Done (or Potential Energy) in a Stretched Wire

Work is done against the internal restoring forces, while stretching a wire. This work is stored as elastic potential energy. The work done is given by,

(i) Work done $W = \frac{1}{2} \times$ stretching force $\times$ elongation

$$= \frac{1}{2} F\Delta L = \frac{1}{2}\frac{YA}{L}(\Delta L)^2$$

= Energy stored in the wire (U)

(ii) Energy stored per unit volume (or energy density)

$$= \frac{U}{V} = \frac{1}{2}\frac{F\Delta L}{AL} = \frac{1}{2}\text{stress} \times \text{strain}$$

$$= \frac{1}{2Y}(\text{stress})^2 = \frac{Y}{2}(\text{strain})^2$$

Inter-relations between Elastic Constants

Y = Young's modulus, η = Rigidity modulus,
K = Bulk modulus, σ = Poisson's ratio

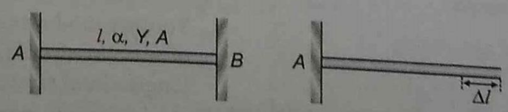
- $Y = 2\eta(1+\sigma)$
- $Y = 3K(1-2\sigma)$
- $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$ or $Y = \frac{9K\eta}{\eta + 3K}$
- $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$

» *Volume elasticity of a gas under an isothermal condition is equal to the pressure exerted by the gas $B_{iso} = p$*

» *Adiabatic elasticity of a gas $B_{adia} = \gamma p$, where γ is the ratio of the two principal specific heats of the gas.*

Thermal Stresses and Strains

When a body is allowed to expand or contract with increasing temperature or decreasing temperature, no stresses are induced in the body. But if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are called thermal stresses or temperature stresses. The corresponding strains are called thermal strains or temperature strains.



A body having linear dimensions is shown in above figure. Let the temperature of the rod is increased by an amount t. The length of the rod would increase by an amount Δl, if it were not fixed at two supports. Here,

$$\Delta l = l\alpha t$$

But since the rod is fixed at the supports a compressive strain will be produced in the rod. Because at the increased temperature, the natural length of the rod is $l + \Delta l$, while being fixed at two supports its actual length is l. Hence, thermal strain

$$\varepsilon = \frac{\Delta l}{l} = \frac{l\alpha t}{l} = \alpha t$$

or $\varepsilon = \alpha t$

Therefore, thermal stress

$$S = Y\varepsilon \quad (\because \text{Stress} = Y \times \text{strain})$$

or $S = Y\alpha t$

Fluid Statics

The substances which flow are called fluids. Fluids include both liquids and gases. The science of fluids at rest is called **fluid statics** in this part of fluid mechanics fluid mass is stationary w.r.t. container, containing the fluid. Fluid statics includes hydrostatic pressure, floatation, Pascal's law and Archimedes' principle, while **hydrodynamics** includes continuity equation and Bernoulli's principle and Torricelli's theorem.

Thrust *and* Pressure

The normal force exerted by a fluid at rest on a given surface in contact with it, is called the **thrust of the fluid.**

The thrust exerted by a fluid at rest per unit surface area of contact surface is called the **fluid pressure.**

$$\therefore \text{ Pressure } p = \frac{\text{Normal force (thrust) } F}{\text{Surface area } A}$$

Pressure is a scalar and its SI unit is Nm^{-2} or pascal (Pa), where, $1 \text{ Pa} = 1 \text{ N m}^{-2}$.

Pressure due to Fluid Column

Hydrostatic pressure at a point a depth h below the fluid surface, is given by $p = h\rho g$

where, ρ = density of fluid.

Gauge Pressure

The pressure difference between the real hydrostatic pressure and the atmospheric pressure is known as the gauge pressure.

$\therefore$ Gauge pressure

= real pressure (p) – atmospheric pressure (p_0)

Pascal's Law

According to Pascal's law of transmission of pressure, the increase in pressure at any one point of the enclosed liquid in equilibrium or at rest, is transmitted equally to all other points of the liquid and also to the walls of the container walls. Hydraulic lift, hydraulic press, hydraulic brakes etc., are based on the Pascal's law.

Archimedes' Principle *and* Buoyancy

Whenever a body is partly or wholly immersed in a fluid at rest, there is a decrease in its weight and this decrease in weight is equal to the weight of the fluid displaced by the immersed part of the body. Infact, when a body is immersed in a fluid, it experiences an upthrust due to the fluid and as a result the apparent weight of the body is reduced.

$\therefore$ Apparent weight of the body

= weight of the body – upthrust due to fluid

= weight of the body – weight of the fluid displaced

e.g., For a floating body, the volume of a body $(V - V_s)$ remaining outside the liquid will be given by

$$V_0 = V - V_s = V - V\frac{\rho}{\sigma} = V\left(1 - \frac{\rho}{\sigma}\right)$$

Buoyant Force *or* Buoyancy

(i) It is an upward force acting on the body immersed in a liquid.

(ii) It is equal to the weight of liquid displaced by the immersed part of the body.

(iii) The buoyant force acts at the centre of buoyancy which is the centre of gravity of the liquid displaced by the body when immersed in the liquid.

(iv) The line joining the centre of gravity and centre of buoyancy is called central line.

(v) Metacentre, is a point where the vertical line passing through the centre of buoyancy intersects the central line.

Laws of Floatation

When a body of density ρ_B and volume V is immersed in a liquid of density σ, the forces acting on the body are

(i) The weight of body $W = mg = V\rho_B g$ acting vertically downwards through the centre of gravity of the body.

(ii) The upthrust $F = V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.*, centre of buoyancy.

So, the following three cases are possible.

Case I The density of body is greater than that of liquid ($i.e., \rho_B > \sigma$). In this case, as weight will be more than upthrust, the body will sink. As shown in Fig. (a)

Case II The density of body is equal to the density of liquid ($i.e., \rho_B = \sigma$). In this case, $W = F$. so, the body will float fully submerged in neutral equilibrium anywhere in the liquid as shown in Fig. (b).

Case III The density of body is lesser than that of liquid ($i.e., \rho_B < \sigma$). In this case, $W < F$,, so the body will move upwards and in equilibrium will float partially immersed in the liquid such that

$$W = V_{\text{in}}\sigma g$$

[V_{in} is the volume of body in the liquid]

or $V\rho_B g = V_{\text{in}}\rho g$ [as $W = mg = \rho_R V_g$]

or $V\rho_B = V_{\text{in}}\sigma$ ...(i)

Viscosity

Viscosity is the property of a fluid due to which it opposes the relative motion between its different layers.

If there are two fluid layers each of surface area A and having a velocity gradient **dv**/dr, then the viscous force acting between the layers, tangentially to the surface of contact in a direction opposite to the relative motion, is given by $\mathbf{F} = -\eta A \frac{d\mathbf{v}}{dr}$. Here, the constant η is called the coefficient of viscosity of the given fluid. SI unit of coefficient of viscosity is Ns m^{-2} or Pa-s.

Stokes' Law

Stokes proved that for a small spherical body of radius r moving with a constant speed v called terminal velocity through a fluid having coefficient of viscosity η, the viscous force F is given by $F = 6\pi\eta r v$ It is known as the **Stokes' law.**

Terminal Velocity

If a small spherical body is dropped in a fluid, then initially it is accelerated under the action of gravity. However, with an increase in speed, the viscous force increases and soon it balances the weight of the body. Now, the body moves with a constant velocity, called the **terminal velocity**. Terminal velocity v_t is given by $v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

where, r = radius of the falling body,
ρ = density of the falling body and
σ = density of the fluid.

Streamline *and* Turbulent Flow

Flow of a fluid is said to be **streamlined** if each element of the fluid passing through a particular point travels along the same path, with exactly the same velocity as that of the preceding element. A special case of streamline flow is **laminar flow**, in which a fluid has a steady flow in the form of parallel layers and these do not mix with one another.

A **turbulent flow** is the one in which the motion of the fluid particles is disordered or irregular. In such a flow, most of the energy used up in maintaining the flow, is spent in causing eddies in the fluid and only a small part of the energy is used for the actual forward flow.

For a fluid, the **critical velocity** is that limiting velocity of the fluid flow upto which the flow is streamlined and beyond which the flow becomes turbulent. Value of critical velocity for the flow of liquid of density ρ and coefficient of viscosity η, flowing through a horizontal tube of radius r is given by

$$v_c \propto \frac{\eta}{\rho r}$$

Reynolds' Number (N_R)

It is a unitless and dimensionless number given by

$$N_R = \frac{\rho v r}{\eta}$$

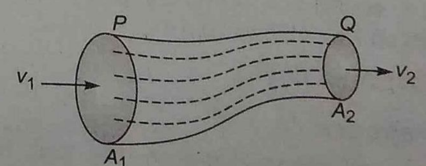
A smaller value of Reynolds' number (generally $N_R \leq 1000$) indicates a streamline flow but a higher value ($N_R \geq 1500$) indicates that the flow is turbulent and between 1000 to 1500, the flow is unstable.

Equation of Continuity

Let us consider the streamline flow of an ideal, non-viscous fluid through a tube of variable cross-section. Let at the two sections, the cross-sectional areas be A_1 and A_2 respectively and the fluid flow velocities are v_1 and v_2, then according to the equation of continuity

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

where ρ_1 and ρ_2 are the respective densities of the fluid. Equation of continuity is based on the conservation of mass.



If the fluid which is flowing, is incompressible, then $\rho_1 = \rho_2$
So, equation of continuity is simplified as

$$A_1 v_1 = A_2 v_2$$

Bernoulli's Theorem

According to the ***Bernoulli's theorem*** *for steady flow of an incompressible, non-viscous fluid through a tube/pipe, the total energy (i.e., the sum of kinetic energy, potential energy and pressure energy) per unit volume (or per unit mass too) remains constant at all points of flow provided that there is no source or sink of the fluid along the flow.*

Mathematically, we have

$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

- *If a liquid is filled in a vessel up to a height H and a small orifice O is made at a height h, then from Bernoulli's theorem it can be shown that velocity of efflux v of the liquid from the vessel is* $v = \sqrt{2g(H-h)}$
- *The flowing fluid describes a parabolic path and hits the base level at a horizontal distance (called the range)* $R = 2\sqrt{h(H-h)}$. *The range is maximum, when* $h = \frac{H}{2}$ *and in that case* $R_{max} = H$.

Energy of a Flowing Liquid

There are three types of energies in a flowing liquid.

- **Pressure Energy**

  If p is the pressure on the area A of a fluid, and the liquid moves through a distance l due to this pressure, then

  Pressure energy of liquid

  $= \text{work done}$

  $= \text{force} \times \text{displacement}$

  $= pAl$

  The volume of the liquid is Al.

  Hence, pressure energy per unit volume of liquid

  $$= \frac{pAl}{Al} = p$$

- **Kinetic Energy**

  If a liquid of mass m and volume V is flowing with velocity v, then the kinetic energy is

  $$= \frac{1}{2}mv^2$$

  ∴ Kinetic energy per unit volume of liquid

  $$= \frac{1}{2}\left(\frac{m}{V}\right)v^2$$

  $$= \frac{1}{2}\rho v^2$$

  Here, ρ is the density of liquid.

- **Potential Energy**

  If a liquid of mass m is at a height h from the reference line ($h = 0$), then its potential energy is mgh.

  ∴ Potential energy per unit volume of the liquid

  $$= \left(\frac{m}{V}\right)gh$$

  $$= \rho g h$$

Applications Based on the Bernoulli's Principle

1. The action of carburetor, paintgun, scent sprayer, atomiser and insect sprayer is based on the Bernoulli's principle.
2. The action of the Bunsen's burner, gas burner, oil stove and exhaust pump is also based on the Bernoulli's principle.
3. Motion of a spinning ball (Magnus effect) is based on Bernoulli's theorem.
4. Blowing of roofs by wind storms, attraction between two parallel moving boats moving close to each other, fluttering of a flag etc., are also based on Bernoulli's theorem.

Limitations of Bernolli's Theorem

1. When a fluid is at rest, *i.e.*, its velocity is zero everywhere, Bernoulli's equation becomes $p_1 + \rho g h_1 = p_2 + \rho g h_2 \Rightarrow (p_1 - p_2) = \rho g (h_2 - h_1)$
2. Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids.
3. Bernoulli's equation applies to fluids which must be incompressible, as the elastic energy of the fluid is also not taken into consideration.
4. Bernoulli's equation does not hold for steady or turbulent flows, because in that situation velocity and pressure are constantly flctuating in time.
5. Bernoulli's equation for flowing liquid is $p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

   Dividing this equation by ρg, we have $\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$

   In this expression, $\frac{v^2}{2g}$ is velocity head and $\frac{p}{\rho g}$ is pressure head.
6. Bernoulli's theorem for unit mass of liquid $\frac{p}{\rho} + \frac{1}{2}v^2 = \text{constant}$

Surface Tension

Surface tension is the property of a liquid due to which its free surface behaves like a stretched elastic membrane and tends to have the least possible surface area.

Surface tension $S = \frac{F}{l}$

SI unit of surface tension is Nm^{-1} or Jm^{-2}. It is a scalar and its dimensional formula is $[MT^{-2}]$.

Surface Energy

Surface energy of a liquid is the potential energy of the molecules of a surface film of the liquid by virtue of its position.

When the surface area of a liquid is increased, work is done against the cohesive force of molecules and this work is stored in the form of additional surface energy.

Increase in surface potential energy

$$\Delta U = \text{Work done } (\Delta W) = S\Delta A$$

where, ΔA is the increase in surface area of the liquid.

1. Work done in Blowing a Liquid Drop

If a liquid drop is blown up from a radius r_1 to r_2, then work done in the process,

$$W = S(A_2 - A_1) = S \times 4\pi(r_2^2 - r_1^2)$$

2. Work done in Blowing a Soap Bubble

As a soap bubble has two free surfaces, hence, work done in blowing a soap bubble so as to increase its radius from r_1 to r_2, is given by

$$W = S \times 8\pi(r_2^2 - r_1^2) \quad \text{(equating the volume)}$$

3. Work done in Splitting a Bigger Drop into *n* Smaller Droplets

If a liquid drop of radius R is split up into n smaller droplets, all of the same size, then radius of each droplet

$$r = R(n)^{-1/3} \quad \text{(equating the volume)}$$

and work done

$$W = S \times 4\pi(nr^2 - R^2)$$
$$= S \times 4\pi R^2 (n^{1/3} - 1)$$

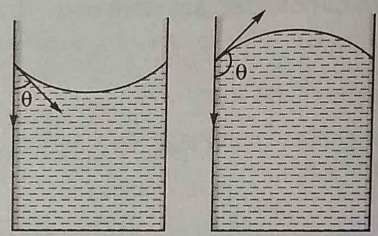
4. Coalescence of Drops

If n small liquid drops of radius r each, combine together so as to form a single bigger drop of radius $R = n^{1/3} r$, then in the process, energy is released. Release of energy is given by

$$\Delta U = S \times 4\pi(nr^2 - R^2) = S \times 4\pi r^2 n(1 - n^{-1/3})$$

Angle of Contact

- Angle of contact for a given liquid-solid combination is defined as the angle subtended between the tangents to the liquid surface and the solid surface, inside the liquid, the tangents are drawn at the point of contact.
- Value of the angle of contact depends on the nature of liquid and solid both. For a liquid having concave meniscus, angle of contact θ is acute ($\theta < 90°$) but for a convex meniscus, the angle of contact is obtuse ($\theta > 90°$).



- Value of angle of contact θ decreases with an increase in temperature.

Excess Pressure Over a Liquid Film

If a free liquid surface film is plane, then pressure on the liquid and the vapour sides of the film are the same, otherwise there is always some pressure difference. *Following cases arise.*

- For a spherical liquid drop of radius r, the excess pressure inside the drop

$$p = \frac{2S}{r}$$

where, S = surface tension of the liquid.

- For an air bubble in a liquid, excess pressure

$$p = \frac{2S}{r}$$

- For a soap bubble in air, excess pressure

$$p = \frac{4S}{r}$$

Capillary Rise *or* Capillarity

Capillarity is the phenomenon of rise or fall of a liquid in a capillary tube as compared to that in a surrounding liquid.

The height h up to which a liquid will rise in a capillary tube is given by

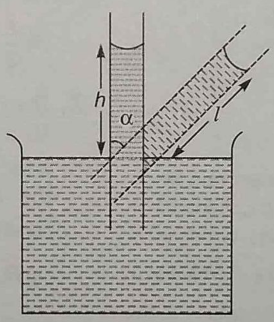
$$h = \frac{2S\cos\theta}{r\rho g} = \frac{2S}{r\rho g}$$

where, r = radius of the capillary tube and

$R = \frac{r}{\cos\theta}$ = radius of liquid meniscus.

(i) The rise in capillary tube $h \propto \frac{1}{r}$.

(ii) If a capillary tube, dipped in a liquid is tilted at an angle α from the vertical, the vertical height h of the liquid column remains the same. However, the length of the liquid column (l) in the capillary tube increases to



$$l = \frac{h}{\cos\alpha}.$$

(iii) If the capillary tube is of insufficient length, the liquid rises up to the upper end of the tube and then the radius of its meniscus changes from R to R' such that $hR = h'R'$, where, h' = insufficient length of the tube.

(iv) After connection due to the weight of liquid contained in the meniscus, the formula for the height is given by $h = \frac{2s}{\rho rg} - \frac{r}{3}$

Practice Zone

DAY 14

1. A wooden block of mass m and density ρ is tied to a string, the other end of the string is fixed to the bottom of a tank. The tank is filled with a liquid of density σ with $\sigma > \rho$. The tension in the string will be

(a) $\left(\frac{\sigma-\rho}{\sigma}\right)mg$ (b) $\left(\frac{\sigma-\rho}{\rho}\right)mg$

(c) $\frac{\rho mg}{\sigma}$ (d) $\frac{\sigma mg}{\rho}$

2. A solid sphere falls with a terminal velocity of 32 m/s in air. If it is allowed to fall in vacuum, then

(a) the terminal velocity will be 32 m/s

(b) the terminal velocity will be less than 32 m/s

(c) the terminal velocity will be greater than 32 m/s

(d) there will be no terminal velocity

3. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%. (The bulk modulus of rubber is 9.8×10^8 N/m^2; and the density of sea water is 10^3 kg/m^3.) **[NCERT Exemplar]**

(a) 100 m (b) 60 m

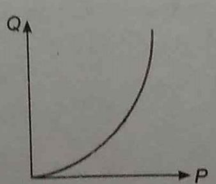
(c) 75 m (d) 65 m

4. While measuring surface tension of water using capillary rise method, height of the lower meniscus from free surface of water is 3 cm while inner radius of capillary tube is found to be 0.5 cm. Then, compute tension of water using this data. Take contact angle between glass and water as 0 and $g = 9.81$ m/s^2

(a) 0.72 N/m (b) 0.77 N/m

(c) 1.67 N/m (d) None of these

5. The graph shows the behaviour of a length of wire in the region for which the substance obeys Hooke's law. P and Q represent



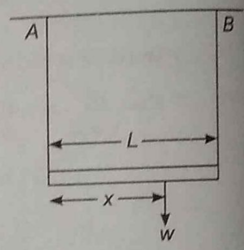
(a) P = applied force, Q = extension

(b) P = extension, Q = applied force

(c) P = extension, Q = stored elastic energy

(d) P = stored elastic energy, Q = extension

6. A light rod of length L is suspended from a support horizontally by means of two vertical wires A and B of equal lengths as shown in the figure. Cross-section area of A is half that of B and Young's modulus of A is double than that of B. A weight w is hung on the rod as shown. The value of x so that the stress in A is same as that in B, is



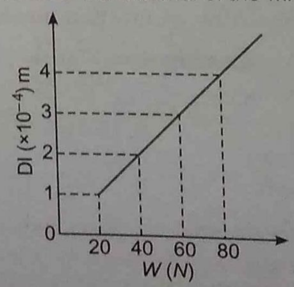
(a) $\frac{L}{3}$ (b) $\frac{L}{2}$ (c) $\frac{2L}{3}$ (d) $\frac{3L}{4}$

7. The Young's modulus of brass and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}. A brass wire and a steel wire of the same length extend by 1 mm, each under the same force. If radii of brass and steel wires are R_B and R_S respectively, then

(a) $R_S = \sqrt{2}\, R_B$ (b) $R_S = \frac{R_B}{\sqrt{2}}$

(c) $R_S = 4R_B$ (d) $R_S = \frac{R_B}{2}$

8. The adjacent graph shows the extension Δl of a wire of length 1 m, suspended from the top of a roof at one end and with a load w connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire.



(a) 2×10^{11} Nm^{-2} (b) 2×10^{-11} Nm^{-2}

(c) 3×10^{-12} Nm^{-2} (d) 2×10^{-13} Nm^{-2}

9. A rubber cord catapult has a cross-sectional area of $25\,mm^2$ and the initial length of rubber cord is 10 cm. It is stretched by 5 cm and then released to project a missile of mass 5 g. Taking, $Y_{rubber} = 5 \times 10^8\ Nm^{-2}$, velocity of the projected missile is

(a) $20\ ms^{-1}$ (b) $100\ ms^{-1}$ (c) $250\ ms^{-1}$ (d) $200\ ms^{-1}$

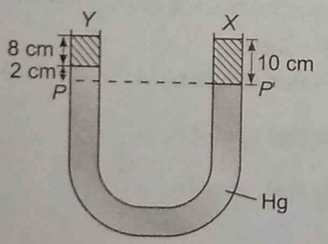
10. Two narrow bores of diameter 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} N/m. Take the angle of contact to be zero and density of water to be $1.0 \times 10^3\ kg/m^3$. ($g = 9.8\ m/s^2$). [NCERT Exemplar]

(a) 6 mm (b) 2 mm (c) 5 mm (d) 3 mm

11. An ice-berg of density 900 kgm^{-3} is floating in water of density 1000 kgm^{-3}. The percentage of volume of ice-berg outside the water is

(a) 20% (b) 35% (c) 10% (d) 11%

12. A liquid X of density 3.36 g/cm^3 is poured in a U-tube, which contains Hg. Another liquid Y is poured in the left arm with height 8 cm, upper levels of X and Y are same. What is density of Y?



(a) 0.8 g/cc (b) 1.2 g/cc (c) 1.4 g/cc (d) 1.6 g/cc

13. A wire of mass m, and length l is suspended from a ceiling. Due to its own weight it elongates, consider cross-section area of wire as A and Young's modulus of material of wire as Y. The elongation in the wire is

(a) $\frac{2mg}{3YA}$ (b) $\frac{mg\,l}{YA}$

(c) $\frac{mgl}{2YA}$ (d) Cannot be calculated

14. A raindrop of radius 0.2 cm is falling through air with a terminal velocity of 8.7 m/s. The viscosity of air in SI units is [Take, $\rho_{water} = 1000\ kg/m^2$ and $\rho_{air} = 1\ kg/m^3$].

(a) 10^{-4} poise (b) 1×10^{-3} poise

(c) 8.6×10^{-3} poise (d) 1.02×10^{-3} poise

15. A bubble is at the bottom of a lake of depth h. As the bubble comes to the sea level, its radius increases three times. If atmospheric pressure is equal to l metre of a water column, then h is equal to

(a) $26\,l$ (b) l (c) $25\,l$ (d) $30\,l$

16. At what speed, the velocity head of water is equal to pressure head of 40 cm of Hg?

(a) $10.3\ ms^{-1}$ (b) $2.8\ ms^{-1}$

(c) $5.6\ ms^{-1}$ (d) $8.4\ ms^{-1}$

17. The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The original length of the wire is

(a) $\frac{l_1 + l_2}{2}$ (b) $\frac{l_1T_2 + l_2T_1}{T_1 + T_2}$

(c) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (d) $\sqrt{T_1T_2l_1l_2}$

18. If work done in stretching a wire by 1mm is 2 J. The work necessary for stretching another wire of same material but with double the radius and half the length, by 1mm distance, is

(a) 16 J
(b) 4 J
(c) 1/4 J
(d) 8 J

19. The length of an elastic string is 1m, when the longitudinal tension is 4N and the length is b metres, when the tension is 5N. The length of the string (in metre) when the longitudinal tension is 9N is

(a) $2b - \frac{a}{2}$

(b) $5b - 4a$

(c) $4a - 3b$

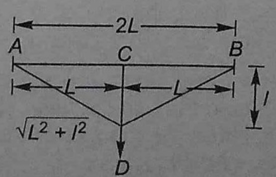
(d) $a - b$

20. A long metal rod of length l and relative density σ, is held vertically with its lower end just touching the surface of water. The speed of the rod when it just sinks in water, is given by

(a) $\sqrt{2gl\sigma}$ (b) $\sqrt{2gl(2\sigma - 1)}$

(c) $\sqrt{2gl\left(1 - \frac{l}{2\sigma}\right)}$ (d) $\sqrt{2gl}$

21. A wire of length $2L$ and radius r is stretched and clamped between A and B. If the Young's modulus of the material of the wire be Y, tension in the wire, when stretched in the position ADB will be



(a) $\pi r^2 YLt$ (b) $\pi r^2 Yl^2/2L^2$

(c) $2\pi r^2 YL^2/l^2$ (d) None of these

Directions *(Q. Nos. 22 to 27) Each of these questions contains two statements Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

22. **Statement I** The stream of water flowing at high speed from a garden hosepipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement II In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

23. **Statement I** Finer the capillary, greater is the height to which the liquid rises in the tube.

Statement II This is in accordance with the ascent formula.

24. **Statement I** A small drop of mercury is spherical but bigger drops are oval in shape.

Statement II Surface tension of liquid decreases with an increase in temperature.

25. **Statement I** Aeroplanes are made to run on the runway before take off, so that they acquire the necessary lift.

Statement II This is as per the Bernoulli's theorem.

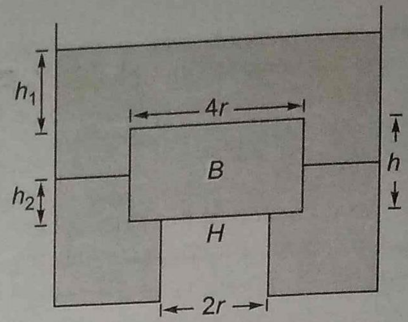
26. **Statement I** In taking into account the fact that any object which floats must have an average density less than that of water, during world war-I, a number of cargo vessels were made of concrete.

Statement II Concrete cargo vessels were filled with air.

27. **Statement I** The bridges are declared unsafe after a long use.

Statement II Elastic strength of bridges decreases with time.

Directions *(Q. Nos. 28 to 30) A tank has a cylindrical hole H of diameter 2r at its bottom as shown in the figure. A cylindrical block B of diameter 4 r and height h is placed on the hole H to prevent the flow of liquid through the hole. The liquid in the tank stands at a height h_1 above the top face of the block. The density of liquid is ρ and that of the block is ρ/3.*



28. If the liquid is gradually taken out of the tank, the height h_1 of the liquid surface above the top face of the block for which the block just begins to rise is

(a) $\frac{2h}{9}$ (b) $\frac{3h}{4}$ (c) $\frac{5h}{3}$ (d) $2h$

29. If the liquid level is further lowered so that it stands at a depth h_2 above the bottom face of the block as shown in the figure, then the maximum value of h_2, so that the block does not move is

(a) $\frac{4h}{9}$ (b) $\frac{h}{3}$
(c) $\frac{2h}{3}$ (d) $\frac{5h}{9}$

30. If the liquid level is lowered below h_2, then

(a) the block will never rise
(b) the block will start rising if $h_2 = \frac{h}{3}$
(c) the block will start rising if $h_2 = \frac{h}{4}$
(d) the block will start rising if $h_2 = \frac{h}{5}$

Directions *(Q. Nos. 31 to 33) A thin rod of negligible mass and cross-sectional area 4×10^{-6} m^2, suspended vertically from one end, has a length of 0.5 m at 100°C. The rod is cooled to 0°C. Young's modulus $=10^{11}$ Nm^{-2}, coefficient of linear expansion $=10^{-5}$ °C^{-1} and g = 10 ms^{-2}.*

31. Whenever the rod is cooled its length in this process energy is

(a) lost by heat emission (b) lost as potential energy
(c) lost as kinetic energy (d) None of these

32. What mass must be attached at the lower end of the rod so that the rod is prevented from contracting on cooling?

(a) 40 kg (b) 30 kg
(c) 20 kg (d) 10 kg

33. The total energy stored in the rod is

(a) 0.1 J (b) 0.2 J
(c) 0.3 J (d) 0.4 J

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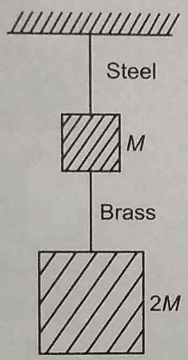
34. The ratio of the coefficient of volume expansion of a glass container to that of a viscous liquid kept inside the container is 1 : 4. What fraction of the inner volume of the container should the liquid occupy so that the volume of the remaining vacant space will be same at all temperature? **[JEE Main Online 2013]**

(a) 2 : 5 (b) 1 : 4
(c) 1 : 64 (d) 1 : 8

35. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are *a*, *b* and *c* respectively, then the corresponding ratio of increase in their lengths is **[JEE Main Online 2013]**



(a) $\frac{3c}{2ab^2}$ (b) $\frac{2a^2c}{b}$
(c) $\frac{3a}{2b^2c}$ (d) $\frac{2ac}{b^2}$

36. A uniform wire (Young's modulus $2\times10^{11}\ Nm^{-2}$) is subjected to longitudinal tensile stress of $5\times10^{7}\ Nm^{-2}$. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to **[JEE Main Online 2013]**

(a) 1.0×10^{-4} (b) 1.5×10^{-4}
(c) 0.25×10^{-4} (d) 5×10^{-4}

37. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's modulii of copper and steel are respectively $1.0\times10^{11}\ Nm^{-2}$ and $2.0\times10^{11}\ Nm^{-2}$, the total extension of the composite wire is **[JEE Main Online 2013]**

(a) 1.75 mm (b) 2.0 mm
(c) 1.50 mm (d) 1.25 mm

38. A uniform cylinder of length *L* and mass *M* having cross-sectional area *A* is suspended,with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a downward push and released, it starts oscillating vertically with a small amplitude. The time period *T* of the oscillations of the cylinder will be **[JEE Main Online 2013]**

(a) Smaller than $2\pi\left[\frac{M}{(k+A\sigma g)}\right]^{1/2}$ (b) $2\pi\sqrt{\frac{M}{k}}$

(c) Larger than $2\pi\left[\frac{M}{(k+A\sigma g)}\right]^{1/2}$ (d) $2\pi\left[\frac{M}{(k+A\sigma g)}\right]^{1/2}$

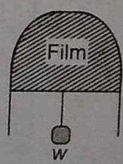
39. A uniform cylinder of length *L* and mass *M* having cross-sectional area *A* is suspended, with its length veritcal from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extenstion x_0 of the spring when it is in equilibrium is **[JEE Main 2013]**

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(\frac{1-LA\sigma}{M}\right)$
(c) $\frac{Mg}{k}\left(\frac{1-LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(\frac{1+LA\sigma}{M}\right)$

40. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is *T*, density of liquid is ρ and *L* is its latent heat of vaporization **[JEE Main 2013]**

(a) $\rho L/T$ (b) $\sqrt{T/\rho L}$
(c) $T/\rho L$ (d) $2T/\rho L$

41. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2}N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is **[AIEEE 2012]**



(a) $0.0125\ Nm^{-1}$ (b) $0.1\ Nm^{-1}$
(c) $0.05\ Nm^{-1}$ (d) $0.025\ Nm^{-1}$

42. Work done in increasing the size of a soap bubble from radius of 3 cm to 5 cm is nearly (surface tension of soap solution = $0.03\ Nm^{-1}$) **[AIEEE 2011]**

(a) 0.2 π mJ (b) 2 π mJ
(c) 0.4 π mJ (d) 4 π mJ

43. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to **[AIEEE 2011]**

(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m
(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

44. Two mercury drops (each of radius *r*) merge to form a bigger drop. The surface energy of the bigger drop, if *T* is the surface tension, is **[AIEEE 2011]**

(a) $2^{5/3}\pi^2T$ (b) $4\pi^2T$
(c) $2\pi r^2T$ (d) $2^{8/3}\pi r^2T$

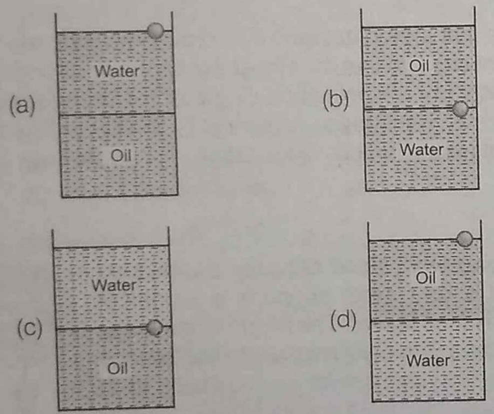
45. If a ball of steel (density $\rho = 7.8 \text{ g cm}^{-3}$) attains a terminal velocity of 10 cms^{-1} when falling in a tank of water (coefficient of viscosity $\eta_{\text{water}} = 8.5 \times 10^{-4}$ Pa-s) then its terminal velocity in glycerine $(\rho = 12 \text{gcm}^{-3}, \ \eta = 13.2 \text{ Pa-s})$ would be nearly **[AIEEE 2011]**

(a) $1.6 \times 10^{-5} \text{ cms}^{-1}$ (b) $6.25 \times 10^{-4} \text{ cms}^{-1}$
(c) $6.45 \times 10^{-4} \text{ cms}^{-1}$ (d) $1.5 \times 10^{-5} \text{ cms}^{-1}$

46. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t°C, the linear stress developed in it is **[AIEEE 2011]**

(a) $\frac{\alpha t}{Y}$ (b) $\frac{Y}{\alpha t}$
(c) $Y\alpha t$ (d) $\frac{1}{Y\alpha t}$

47. A ball is made of a material of density ρ where $\rho_{\text{oil}} < \rho < \rho_{\text{water}}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position? **[AIEEE 2010]**



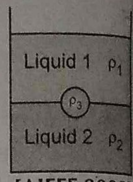
48. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? **[AIEEE 2009]**

(a) F (b) $4F$
(c) $6F$ (d) $9F$

49. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2 (\rho_2 < \rho_1)$. [Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{\text{viscous}} = -kv^2 \ (k > 0)$]. The terminal speed of the ball is **[AIEEE 2008]**

(a) $\sqrt{\frac{V_g(\rho_1 - \rho_2)}{k}}$ (b) $\frac{V_g \rho_1}{k}$
(c) $\sqrt{\frac{V_g \rho_1}{k}}$ (d) $\frac{V_g(\rho_1 - \rho_2)}{k}$

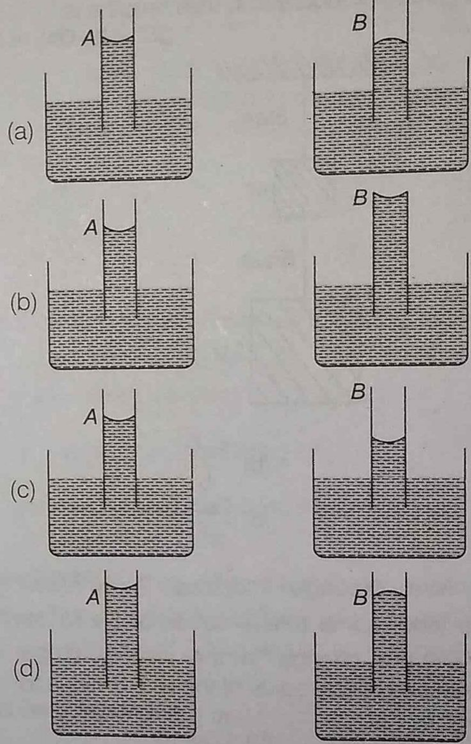
50. A jar filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3?



[AIEEE 2008]

(a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 < \rho_3 < \rho_2$
(c) $\rho_1 < \rho_2 < \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$

51. A capillary tube (A) is dropped in water. Another identical tube (B) is dipped in a soap water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[AIEEE 2008]**



52. If the terminal speed of a sphere of gold (density $= 19.5 \text{ kgm}^{-3}$) is 0.2 ms^{-1} in a viscous liquid (density $= 1.5 \text{ kgm}^{-3}$), find the terminal speed of a sphere of silver (density $= 10.5 \text{ kgm}^{-3}$) of the same size in the same liquid. **[AIEEE 2006]**

(a) 0.4 ms^{-1} (b) 0.133 ms^{-1}
(c) 0.1 ms^{-1} (d) 0.2 ms^{-1}

53. A wire elongates by l mm when a load w is hung from it. If the wire goes over a pulley and two weights w each are hung at the two ends, the elongation of the wire will be (in mm) **[AIEEE 2006]**

(a) l (b) $2l$
(c) zero (d) $\frac{l}{2}$

54. The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10%, keeping the temperature constant. The bulk modulus of the medium is **[AIEEE 2005]**

(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa
(c) 51.2×10^5 Pa (d) 1.55×10^5 Pa

55. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be **[AIEEE 2005]**

(a) 8 cm (b) 10 cm (c) 4 cm (d) 20 cm

56. If S is the stress and Y is the Young's modulus of the material of the wire, the energy stored in the wire per unit volume is **[AIEEE 2005]**

(a) $2S^2Y$ (b) $\frac{S^2}{2Y}$ (c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$

57. Spherical balls having the same radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on each spherical ball is **[AIEEE 2004]**

(a) directly proportional to the radius R but inversely proportional to the velocity v
(b) directly proportional to both the radius R and the velocity v
(c) inversely proportional to both the radius R and the velocity v
(d) inversely proportional to the radius R but directly proportional to the velocity v

58. A wire suspended vertically from one of its ends, is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1mm. Then, the elastic energy stored in the wire is **[AIEEE 2003]**

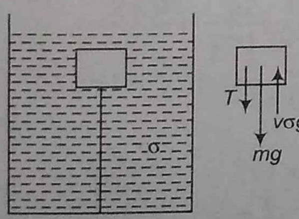
(a) 0.2 J (b) 10 J
(c) 20 J (d) 0.1 J

Answers

1. (b)	2. (d)	3. (a)	4. (b)	5. (c)	6. (c)	7. (b)	8. (a)	9. (c)	10. (c)
11. (c)	12. (a)	13. (c)	14. (b)	15. (a)	16. (b)	17. (c)	18. (a)	19. (b)	20. (c)
21. (c)	22. (d)	23. (a)	24. (a)	25. (a)	26. (b)	27. (a)	28. (c)	29. (a)	30. (a)
31. (a)	32. (a)	33. (a)	34. (b)	35. (c)	36. (c)	37. (d)	38. (d)	39. (c)	40. (d)
41. (d)	42. (c)	43. (c)	44. (d)	45. (b)	46. (c)	47. (b)	48. (d)	49. (a)	50. (d)
51. (c)	52. (c)	53. (d)	54. (d)	55. (a)	56. (b)	57. (b)	58. (d)		

Hints & Solutions

1. From free body diagram of the wooden block,
$V\sigma g = mg + T$ [V is the volume of block]



$$T = V\sigma g - mg$$
$$T = \frac{m}{\rho}\sigma g - mg = mg\left(\frac{\sigma - \rho}{\rho}\right)$$

2. Terminal velocity is attained by an object due to some dragging force acting opposite to its motion. When sphere is allowed to fall in air, the air friction force causes the ball to acquire terminal velocity, out in vacuum no dragging force is present and the ball is falling down with acceleration g, so no terminal velocity is attained.

3. Bulk modulus of rubber $(K) = 9.8 \times 10^8$ N/m^2
Density of sea water $(\rho) = 10^3$ kg/m^3

Percentage decrease in volume, $\left(\frac{\Delta V}{V} \times 100\right) = 0.1$

or $\frac{\Delta V}{V} = \frac{0.1}{100}$

or $\frac{\Delta V}{V} = \frac{1}{1000}$

Let the rubber ball be taken up to depth h.
∴ Change in pressure $(p) = h\rho g$
∴ Bulk modulus $(K) = \frac{p}{(\Delta V/V)} = \frac{h\rho g}{(\Delta V/V)}$

or $h = \frac{K \times (\Delta V/V)}{\rho g}$

$$= \frac{9.8 \times 10^8 \times \frac{1}{1000}}{10^3 \times 9.8} = 100 \text{ m}$$

4. As, $T = \dfrac{r\left(h + \dfrac{r}{3}\right)\rho g}{2\cos\theta}$

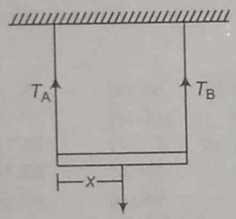
$$= \frac{0.5\times 10^{-2}\left[3 + \dfrac{0.5}{3}\right]\times 10^{-2}\times 10^{3}\times 9.81}{2} = 0.77\ \text{N/m}$$

5. If Hooke's law is being obeyed then force extension graph is a straight line. Stored elastic energy extension $\left(U = \dfrac{1}{2}Fx = \dfrac{1}{2}\dfrac{YA}{L}x^2\right)$ should be a parabolic curve symmetric about the U axis. Hence, in the graph P represents extension and Q the stored elastic energy.

6. Let tension in wire A and B be T_A and T_B, respectively,

$$T_A + T_B = W$$
$$T_A \times x = T_B\,(L - x)$$

Solving the above equations,



$$T_A = \frac{W\,(L-x)}{L},\ T_B = \frac{Wx}{L}$$

Stress in $A = \dfrac{T_A}{A_A}$, where A_A is cross-section area of wire A.

Stress in $B = \dfrac{T_B}{A_B}$, where A_B is cross-section area of wire B.

It is given, $A_A = \dfrac{A_B}{2}$, $\dfrac{T_A}{A_A} = \dfrac{T_B}{A_B}$

which gives $x = \dfrac{2L}{3}$

7. $\Delta L = \dfrac{FL}{YA} = \dfrac{FL}{Y\pi R^2}$

As F, L and ΔL are the same, hence

$$YR^2 = \text{a constant}$$

$\therefore \quad 2.0\times 10^{11} R_S^2 = 1.0\times 10^{11} R_B^2$

$\Rightarrow \quad R_S = \dfrac{R_B}{\sqrt{2}}$

8. (Δl - w) graph is a straight line, where

$$\frac{w}{\Delta l} = \frac{(80-20)}{(4-1)\times 10^{-4}} = 2\times 10^{5}\ \text{Nm}^{-2}$$

Moreover, $L = 1$ m and $A = 10^{-6}\,\text{m}^2$

Hence, $Y = \dfrac{FL}{A\Delta l} = \dfrac{wL}{A\Delta l} = \dfrac{2\times 10^{5}\times 1}{10^{-6}} = 2\times 10^{11}\ \text{Nm}^{-2}$

9. Elastic potential energy stored in the catapult $\left(U = \dfrac{1}{2}F\Delta L = \dfrac{1}{2}\dfrac{YA}{L}\Delta L^2\right)$ is converted into the kinetic energy of the projectile $\left(K = \dfrac{1}{2}mv^2\right)$

$$\Rightarrow \quad v = \sqrt{\frac{YA\Delta L^2}{Lm}} = \left[\frac{(5\times 10^{8})\times(25\times 10^{-6})\times(5\times 10^{-2})^2}{(10\times 10^{-2})\times(5\times 10^{-3})}\right]^{1/2}$$

$$= 250\ \text{ms}^{-1}$$

10. Given, surface tension of water $(S) = 7.3\times 10^{-2}$ N/m

Density of water $(\rho) = 1.0\times 10^{3}$ kg/m^3

Acceleration due to gravity $g = 9.8$ m/s^2

Angle of contact $\theta = 0°$

Diameter of one side, $2r_1 = 3.0$ mm

$\therefore \quad r_1 = 1.5\,\text{mm} = 1.5\times 10^{-3}$ m

Diameter of other side

$$2r_2 = 6.0\ \text{mm}$$
$$r_2 = 3.0\ \text{mm} = 3.0\times 10^{-3}\ \text{m}$$

Height of water column rises in first and second tubes

$$h_1 = \frac{2S\cos\theta}{r_1\rho g}$$
$$h_2 = \frac{2S\cos\theta}{r_2\rho g}$$

$\therefore$ Difference in levels of water rises in both tubes

$$\Delta h = h_1 - h_2$$
$$= \frac{2S\cos\theta}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
$$= \frac{2\times 7.3\times 10^{-2}\times\cos 0°}{1.0\times 10^{3}\times 9.8}\left[\frac{1}{1.5\times 10^{-3}} - \frac{1}{3.0\times 10^{-3}}\right]$$
$$= \frac{14.6}{9.8}\times 10^{-2}\left[\frac{2-1}{3}\right]$$
$$= \frac{14.6}{9.8\times 2}\times 10^{-2}$$
$$= \frac{14.6}{9.8\times 2}\times 10^{-2}$$
$$= 0.497\times 10^{-2}\ \text{m}$$
$$= 4.97\times 10^{-3}\ \text{m}$$
$$= 4.9\ \text{mm}$$

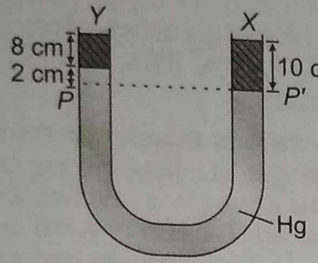
11. $V\times 900\times g = (V - V_{\text{out}})\times 1000\times g$

$$\Rightarrow \quad \frac{V_{\text{out}}}{V} = \frac{1}{10} = \frac{1}{10}\times 100\% = 10\%$$

12. As shown in the figure, in the two arms of the tube the pressure remains the same on the surface PP'.

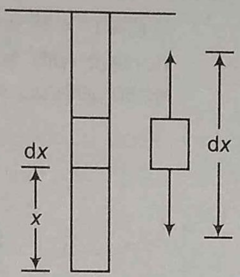
Hence, $8\times\rho_Y\times g + 2\times\rho_{Hg}\times g = 10\times\rho_Y\times g$

$\therefore \quad 8\rho_Y + 2\times 13.6 = 10\times 3.36$



or $\rho_Y = \dfrac{33.6 - 27.2}{8} = 0.8 \text{g/cc}$

13. Consider an element of wire of width dx at a distance x from bottom end of wire.



The force experienced by this element is due to the gravitational force of portion of wire lower to it.

So, $Y = \dfrac{T/A}{\dfrac{\Delta(dx)}{dx}}$, where $\Delta(dx)$ is the elongation in this element.

Now, $\Delta(dx) = \dfrac{T}{YA} dx = \dfrac{mg}{YAl} \times x dx$

Total elongation,

$$\Delta l = \int \Delta(dx)$$

$$= \int_0^l \frac{mg \times x dx}{YAl} = \frac{mgl}{2YA}.$$

14. We have,

$$6\pi\eta r v = \frac{4}{3}\pi r^3 g\rho - \frac{4}{3}\pi r^3 \sigma g$$

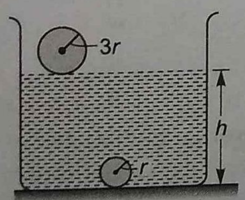
where, $\rho \to \rho_{water}$ and $\sigma \to \rho_{air}$

$$\Rightarrow \quad \eta = \frac{2gr^2(\rho - \sigma)}{9v}$$

$$= \frac{2 \times 9.81 \times (0.2 \times 10^{-2})^2 \times 999}{9 \times 8.7}$$

$$= 1 \times 10^{-3} \text{ poise}$$

15.



From Boyle's law,

$$pV = \text{constant}$$

$$\therefore \quad p_1V_1 = p_2V_2$$

Here, $p_1 = (h + l), V_1 = \dfrac{4}{3}\pi r^3$

$p_2 = l, V_2 = \dfrac{4}{3}\pi(3r)^3$

$$\therefore \quad (h + l)\frac{4}{3}\pi r^3 = l \times \frac{4}{3}\pi(3r)^3 \Rightarrow h + l = 27l$$

$$\therefore \quad h = 26l$$

16. Velocity head $= \dfrac{v^2}{2g}$ and pressure head $= \dfrac{p}{\rho g}$

As velocity of water is equal to the pressure head of 40 cm of Hg column, hence

$$\frac{v^2}{2g} = \frac{h\rho g}{\rho g} \Rightarrow v^2 = 2hg$$

$$\Rightarrow \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.4} = 2.8 \text{ ms}^{-1}$$

17. Let l = original length of the wire,

$$\Delta l_1 = l_1 - l$$

Similarly, the change in length of the second wire is

$$\Delta l_2 = l_2 - l$$

Now, $Y = \dfrac{T_1}{A} \times \dfrac{l}{\Delta l_1} = \dfrac{T_2}{A} \times \dfrac{l}{\Delta l_2}$

$$\Rightarrow \quad \frac{T_1}{\Delta l_1} = \frac{T_2}{\Delta l_2}$$

$$\Rightarrow \quad \frac{T_1}{l_1 - l} = \frac{T_2}{l_2 - l}$$

$$\Rightarrow \quad T_1l_2 - T_1l = T_2l_1 - T_2l$$

$$\Rightarrow \quad l = \frac{T_2l_1 - T_1l_2}{T_2 - T_1}$$

18. As work done $= \dfrac{1}{2} Y \times (\text{strain})^2 \times \text{volume}$

Thus, $W = \dfrac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$$= \frac{1}{2} \times \frac{\text{stress}}{\text{strain}} \times (\text{strain})^2 \times \text{volume}$$

$$= \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume}$$

$$\Rightarrow \quad 2 = \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 \times AL = \frac{YA(\Delta L)^2}{2L}$$

Now work done, $W' = \dfrac{Y(4A)(\Delta L)^2}{2(L/2)} = 8\left(\dfrac{YA(\Delta L)^2}{2L}\right) = 8 \times 2 = 16 \text{J}$

19. If L is the initial length, then the increase in length by a tension F is given by $l = \dfrac{FL}{\pi r^2 Y}$

Hence, $a = L + l = L + \dfrac{4L}{\pi r^2 Y} = L + 4C$

and $b = L + \dfrac{5L}{\pi r^2 Y} = L + 5C$

where, $C = \dfrac{L}{\pi r^2 Y}$

Thus, on solving for L and C, we get

$$L = 5a - 4b \text{ and } C = b - a$$

Hence, for $F = 9\text{N}$, we get

$$x = L + \frac{9L}{\pi r^2 Y} = L + 9C$$

$$= (5a - 4b) + 9(b - a) = 5b - 4a$$

20. Let the densities of metal and water be ρ and ρ_0 respectively and let x be the length of the rod immersed in water at an instant of time t. Then, acceleration at that instant = apparent weight divided by the mass of the rod, *i.e.*,

$$\frac{dv}{dt}=\frac{\pi r^2 l\rho g-\pi r^2 x\rho_0 g}{\pi r^2 l\rho}=g-\frac{gx\rho_0}{l\rho}=g\left(1-\frac{x}{\sigma l}\right)$$

or $\dfrac{dv}{dx}\dfrac{dx}{dt}=g\left(1-\dfrac{x}{\sigma l}\right)$ or $v\dfrac{dv}{dx}=g\left(1-\dfrac{x}{\sigma l}\right)$

On integrating, we get

$$\frac{v^2}{2}=g\left|x-\frac{x^2}{2\sigma l}\right|_0^l=g\left(l-\frac{l}{2\sigma}\right)$$

$$\Rightarrow \quad v=\sqrt{2gl\left(1-\frac{l}{2\sigma}\right)}$$

21. As, $Y=\dfrac{\text{Stress}}{\text{Strain}}=\dfrac{F}{\pi r^2\times \text{strain}}$. $\Rightarrow$ $F=T=Y\pi r^2\times\text{strain}$

Now, $\text{strain}=\dfrac{\sqrt{L^2+l^2}-L}{L}=\left(1+\dfrac{l^2}{L^2}\right)^{1/2}-1$

$$\approx 1+\frac{1}{2}\frac{l^2}{L^2}-1=\frac{l^2}{2L^2} \text{ (by Binomial expansion)}$$

$$\therefore \quad T=\frac{Y\pi r^2\cdot l^2}{2L^2}$$

22. From the continuity equation, $Av=$ constant

or $A\propto\dfrac{1}{v}$

At lower heights, speed will be more. Therefore, area of cross-section will be less.

23. According to the ascent formula,

$$h=\frac{2\cos\theta}{r\rho g} \Rightarrow h\propto\frac{1}{r}$$

From the relation, for small radius is less, height to which the liquid rise will be greater.

24. In a small drop, the force due to the surface tension is very large as compared to its weight and hence, it is spherical in shape. A big drop becomes oval in shape due to its large weight. The surface tension of the liquid decreases with an increase of temperature.

25. Usually the air does not strike the wings of the aeroplane with a large velocity. To get the lift, the aeroplane runs for some distances on the runway before taking off. Due to the special shape of wings, the velocity of the layers of air above the wings increases (because layers come near each other) and hence, pressure decreases. Due to this, the aeroplane gets an uplift.

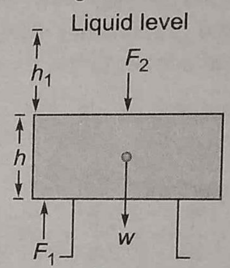
26. The density of concrete ofcourse, is more than that of water and a block of concrete will sink like a stone, if dropped into water. Concrete cargo were filled with air and as such, average density of cargo vessels

$$=\frac{\text{Mass of concrete + Mass of air}}{\text{Volume of concrete + Volume of air}}$$

It follows that the average density of cargo vessels must b less than that of water. As a result, the concrete carg vessels did not sink.

27. When a huge traffic is passes over the bridge, the strai undergoes changes for a large number of times every da When the bridge is used for a long time, the elastic strengt is lost continuously. Because of it, the strain in the bridg becomes very large for a particular stress and hence bridg may collapse.

28. Area of hole $=\pi(2r)^2=4\pi r^2=A$. The area of the bloc $=\pi(4r)^2=16\pi r^2=4A$. The area of the lower face of block ir contact with liquid $=4A-A=3A$. The block starts rising when upthrust = weight.



The upthrust $U=F_1-F_2=\rho g(h_1+h)3A-\rho gh_1\times 4A$

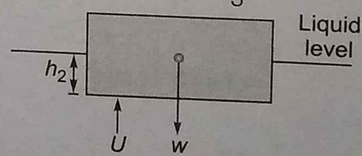
Weight of the block, $w=\dfrac{\rho}{3}gh\times 4A$

Now, $U=w$

i.e., $\rho g(h_1+h)3A-4\rho g\,h_1A=\dfrac{4}{3}\rho g\,hA$

Which gives, $h_1=\dfrac{5h}{3}$

29. The block will not move if the upthrust U equals the weight w of the block *i.e.*, if $\rho gh_2\times 3A=\dfrac{\rho}{3}gh\times 4A$



Which gives, $h_2=\dfrac{4h}{9}$

30. If the liquid level is lowered below h_2, the upthrust will become less than the weight of the block. Therefore, the block will not rise.

31. Energy loss is only due to heat emission.

32. If the rod is to be prevented from contracting, the mass m attached at the lower end must increase its length by an amount $\Delta L=5\times10^{-4}$m.

Now, $Y=\dfrac{mgL}{A\Delta L}$

or $m=\dfrac{YA\Delta L}{gL}=\dfrac{10^{11}\times4\times10^{-6}\times5\times10^{-4}}{10\times0.5}=40$ kg

33. As, $U = \frac{1}{2} F \times \Delta L = \frac{1}{2} mg\Delta L = \frac{1}{2} \times 40 \times 10 \times 5 \times 10^{-4} = 0.1$ J

34. The coefficient of volume expansion is 1 : 4 and volume expansion constant is given by

$$\frac{\Delta V}{V} = \gamma \Delta t \quad \text{or} \quad \frac{\frac{\Delta V}{V}}{\Delta t} = \gamma$$

Rate of change fractional change with temperature constant.

$$\therefore \quad \frac{\gamma_g}{\gamma_e} = \frac{1}{4} = \frac{\frac{\Delta V_g}{V_g} / \Delta t}{\frac{\Delta V_e}{V_e} / \Delta t} = \frac{1}{4}$$

35. For steel wire

As change in length $(\Delta l_1) = \frac{Mgl}{\pi r_1^2 y_1}$ ...(i)

and for beam wire,

Change in length $(\Delta l_2) = \frac{2Mg\,al}{\pi r_2^2\, y_2}$ ...(ii)

Dividing Eqs. (i) by (ii), we get, r

Corresponding ratio of increase in their lengths $= \frac{3a^2}{2b^2 r}$

36. Given, $Y = 2 \times 10^{11}$ N/m^2

Stress $= 5 \times 10^7$ N / m^2

As, $\frac{\text{stress}}{\text{strain}} = Y \Rightarrow \text{Strain} = \frac{5 \times 10^7}{2 \times 10^{11}} = 2.5 \times 10^{-4}$

It is symmetric strain.

Now, strain of 2.5×10^{-4} is equivalent.

As, $\frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right) \quad \therefore \frac{2.5 \times 10^{-4}}{3} = \frac{\Delta r}{r} = 0.75 \times 10^{-4}$

$\therefore$ Fraction decrease in radius

$= (1.00 - 0.75)\, 10^{-4} = 0.25 \times 10^{-4}$

37. Here, $Y_c = 1 \times 10^{11}$ N/m^2

$Y_s = 2 \times 10^{11}$ N/m^2

$l_c = 1.0$ m, $l_2 = 0.5$ m and $\Delta l_c = 1 \times 10^{-3}$ m

As, $(\text{strain})_c = \frac{\text{stress}}{Y_c}$

$\Rightarrow 1 \times 10^{-3} = \frac{\text{stress}}{1 \times 10^{11}} \Rightarrow \text{stress} = 10^8$ N/m^2

Now, $Y_s = \frac{\text{stress}}{\text{strain}}$

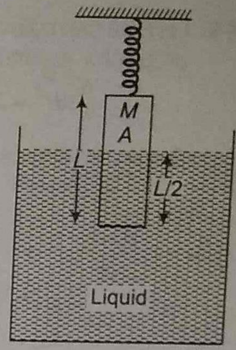
$\Rightarrow \quad \text{strain} = \frac{10^8}{2 \times 10^{11}} = 0.5 \times 10^{-3}$

or $\quad \frac{\Delta l_s}{1/2} = 0.5 \times 10^{-3} \Rightarrow \Delta l_s = 0.25 \times 10^{-3}$

$\therefore \quad \Delta l = \Delta l_c + \Delta l_s = 1 + 0.25 = 1.25$ mm

38. Weight of displaced liquid

$= \frac{L}{2} \times A \times \rho g$



Now, at equilibrium

$\frac{L}{2} A\sigma g + kx = Mg$ ...(i)

Further after downward push, suppose cylinder is dipressed through a depth h.

$$f = Mg - \left[\left(\frac{L}{2} + h\right) A\sigma g + k(x + h)\right]$$

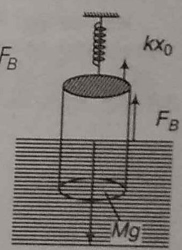
$= \frac{L}{2} A\sigma g + kx - \frac{L}{2} A\sigma g - hA\sigma g - kx - kh$ [From Eq. (i)]

$= -h(A\sigma g + k) = -k_1 h$

Now the time period, $T = \frac{1}{2\pi}\sqrt{\frac{M}{K_1}} = \frac{1}{2\pi}\sqrt{\frac{M}{(A\sigma g + k)}}$

39. In equilibrium, upward force = downward force

$kx_0 + F_B = mg$



Here, kx_0 is restoring force of spring and F_B is buoyancy force.

$kx_0 + \sigma \frac{L}{2} Ag = Mg$

$$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(\frac{1 - \sigma LA}{2M}\right)$$

40. When radius is decreased by dr.

Decrease in surface energy = Heat required for vaporisation $(4\pi r dr) \times T \times 2 = 4\pi r^2\, dr\rho \Rightarrow r = \frac{2T}{\rho L}$

41. At equilibrium, weight of the given block is balanced by force due to surface tension, *i.e.*, $2LS = w$

or $\quad S = \frac{w}{2L} = \frac{1.5 \times 10^{-2}\ \text{N}}{2 \times 0.3\ \text{m}} = 0.025$ Nm^{-1}

42. Work done = Change in surface energy

$\Rightarrow \quad W = 2T \times 4\pi\,(R_2^2 - R_1^2)$

$= 2 \times 0.03 \times 4\pi\,[(5)^2 - (3)^2] \times 10^{-4}$ J $= 0.4\,\pi$ mJ

43. From Bernoulli's theorem,

$\rho gh = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$

$\Rightarrow gh = \frac{1}{2} v_1^2 \left[\left(\frac{v_2}{v_1}\right)^2 - 1\right] = \frac{1}{2} v_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] \quad (\because A_1 v_1 = A_2 v_2)$

$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = 1 + \frac{2hg}{v_1^2} \Rightarrow \left(\frac{D_1}{D_2}\right)^2 = 1 + \frac{2gh}{v_1^2}$

$$\Rightarrow D_2 = \frac{D_1}{\left(1 + \frac{2gh}{v_1^2}\right)^{1/4}} = \frac{8 \times 10^{-3}}{\left(1 + \frac{2 \times 10 \times 0.2}{(0.4)^2}\right)^{1/4}} = 3.6 \times 10^{-3}\ \text{m}$$

44. Let R be the radius of the bigger drop, then volume of bigger drop $= 2 \times$ volume of small drop

$$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$$

Surface energy of bigger drop,

$$E = 4\pi R^2 T = 4 \times 2^{2/3}\pi r^2 T = 2^{8/3}\pi r^2 T$$

45. $v \propto \frac{\rho - \rho_0}{\eta}$ $\quad\therefore\quad$ $\frac{v_2}{v_1} = \frac{\rho - \rho_{02}}{\rho - \rho_{01}} \times \frac{\eta_1}{\eta_2}$

$$= \frac{7.8 - 1.2}{7.8 - 1} \times \frac{8.5 \times 10^{-4} \times 10}{132} = 6.25 \times 10^{-4}\ \text{cms}^{-1}$$

46. Change in length $\Delta L = \alpha L\,\Delta T = \frac{FL}{AY}$

$$\Rightarrow \qquad \text{Stress} = \frac{F}{A} = Y\alpha\Delta T = Y\alpha t \qquad (\text{as}\,\Delta T = t)$$

47. $\rho_{oil} < \rho < \rho_{water}$

Oil is the least dense of them, so it should settle at the top with water at the base. Now, the ball is denser than oil but less denser than water. So, it will sink through oil but will not sink in water. So, it will stay at the oil-water interface.

48. As, $A_1 l_1 = A_2 l_2 \Rightarrow l_2 = \frac{A_2 l_1}{A_1} = \frac{A \times l_1}{3A} = \frac{l_1}{3} \Rightarrow \frac{l_1}{l_2} = 3$

$$\Delta x_1 = \frac{F_1}{A_Y} \times l_1 \qquad \ldots\text{(i)}$$

$$\Delta x_2 = \frac{F_2}{3A_Y} l_2 \qquad \ldots\text{(ii)}$$

Here, $\quad \Delta x_1 = \Delta x_2$

$$\frac{F_2}{3A_Y} l_2 = \frac{F_1}{A_Y} l_1$$

$$F_2 = 3F_1 \times \frac{l_1}{l_2} = 3F_1 \times 3 = 9F$$

49. We have $\rho_1 V_g - \rho_2 V_g = kv_T^2 \Rightarrow v_T = \sqrt{\frac{V_g(\rho_1 - \rho_2)}{k}}$

50. As liquid 1 floats above liquid 2

$$\rho_1 < \rho_2$$

The ball is unable to sink into liquid 2

$$\rho_3 < \rho_2$$

The ball is unable to rise over liquid 1

$$\rho_1 < \rho_3$$

Thus, $\quad \rho_1 < \rho_3 < \rho_2$

51. Capillary rise, $h = \frac{2T\cos\theta}{\rho g r}$

As soap solution has lower T, h will be low.

52. As, terminal velocity, $v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$

where, ρ = density of substance of the body,
σ = density of the liquid.

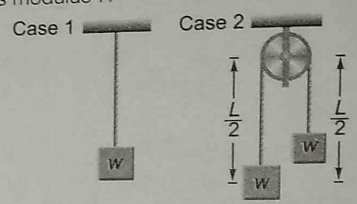
From the given data,

$$\frac{v_T(\text{Ag})}{v_T\ (\text{Gold})} = \frac{r_{Ag} - s_l}{r_{Gold} - s_l}$$

$$\Rightarrow \qquad v_T\ (\text{Ag}) = \frac{10.5 - 1.5}{19.5 - 1.5} \times 0.2$$

$$= \frac{9}{18} \times 0.2 = 0.1\ \text{ms}^{-1}$$

53. Let us consider the length of the wire be L, the cross-sectional area be A, and the material of wire has Young's modulus Y.



Then for the 1st case, $Y = \frac{w/A}{l/L}$

For 2nd case, $Y = \frac{w/A}{2l'/L} \Rightarrow l' = \frac{l}{2}$

So, the total elongation of both the sides $= 2l' = l$

54. Bulk modulus $B = \frac{-dp}{(dV/V)}$

$$\therefore \qquad B = \frac{(1.165 - 1.01) \times 10^5}{(10/100)} = 1.55 \times 10^5\ \text{Pa}$$

55. Freely falling body or system has the acceleration equal to the gravitational acceleration.

56. Energy stored in the wire $= \frac{1}{2}$ stress $\times$ strain $\times$ volume

and Young's modulus $= \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain} = \frac{S}{Y}$

$$\therefore \frac{\text{Energy stored in the wire}}{\text{Volume}} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

57. $\qquad F = 6\pi\eta R v$

where, $\quad R$ = radius of ball

v = velocity of ball

and $\quad \eta$ = coefficient of viscosity

$\therefore \qquad F \propto R$ and $F \propto v$

or in words, retarding force is directly proportional to both R and v.

58. As, $U = \frac{1}{2} F\Delta l$

$$= \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1\ \text{J}$$

Day 15

Heat and Thermodynamics

Day 15

Outlines ...

- Heat
- Therometry
- Thermal Expansion
- Specific Heat Capacity
- Thermal Equilibrium
- Zeroth Law of Thermodynamics
- First Law of Thermodynamics
- Second Law of Thermodynamics
- Different Thermodynamical Processes
- Carnot Engine and its Efficiency
- Refrigerator
- Equations of State of a Perfect Gas

Heat

Heat *is the energy, which is transferred between system and surroundings due to the temperature difference. In other words heat is a form of energy that flows from one body to another because of temperature difference between them.*

The conventional unit of heat is called the calorie. It is defined as the amount of heat required to raise the temperature of 1 gm of water through 1°C. I cal = 4.186 J.

Temperature

Temperature is the property of a state of matter by virtue of which we predict its **hotness** or **coldness**, relative to some body.

(i) When energy is given to a body and its state is not changing, it means its temperature is rising.

(ii) The devices which are used to measure the temperature are termed as **thermometers**, while the science related to measurement of temperature is termed as thermometry.

(iii) For construction of a thermometer two fixed reference points are chosen : ice point (equilibrium tempera4ture of a mixture of ice and water at one atmosphere pressure), and steam point (equilibrium temperature of mixture of steam and water at one atmospheric pressure).

(iv) Three most common scales are Celsius scale or Centigrade scale, Fahrenheit scale and Kelvin scale (Absolute scale).

Scale	Ice point / Lower reference point	Steam point / Upper reference point	Unit
Celsius	0	100	°C
Fahrenheit	32	212	°F
Kelvin	273.15	373.15	K

(v) °C and C° conventionally have different meanings, °C represents simply temperature on Celsius scale while C° represents temperature difference or change in temperature (this is valid for any other scale also).

(vi) Kelvin scale is the absolute scale and minimum temperature which can exist is 0 K, but practically it is not possible to acheive 0 K, on the other hand there is no upper limit on the temperature.

(vii) Relation between C, F and K scales is

$$\frac{C}{5}=\frac{F-32}{9}=\frac{K-273.15}{5}$$

(viii) In general,

$$\frac{\text{Temperature of } X - \text{Ice point of } X}{\text{Steam point of } X - \text{Ice point of } X} = \frac{\text{Temperature of } Y - \text{Ice point of } Y}{\text{Steam point of } Y - \text{Ice point of } Y}$$

- *For practical purposes, in Kelvin scale the term 273.15 is treated as 273.*
- *The Celsius and Kelvin scales have different zero points but same size of the degree. Hence, any temperature difference measured on the Kelvin scale is the same as on the Celsius scale, i.e.,* $\Delta T\,\text{K} = \Delta T\,°\text{C}$

Thermometry

The branch dealing with measurement of temperature is called thermometry and the devices used to measure temperature are called thermometers.

To establish the measurement of temperature that property of a substance is used which changes linearly with temperature. For example, at changing temperature, change in pressure of a gas at constant volume, change in electric resistance of a metallic wire etc. Such property of a substance is called thermometric property. Let thermometric properties at temperatures 0°C (ice point), 100°C (steam point) and t°C (unknown temperature) are X_0, X_{100}, and X_t respectively. Then

$$\frac{X_t - X_0}{t} = \frac{X_{100}-X_0}{100} \quad \text{or} \quad \frac{X_t - X_0}{X_{100}-X_0} = \frac{t}{100}$$

Thus, $$t = \left(\frac{X_t - X_0}{X_{100}-X_0}\right)\times 100\,°\text{C}$$

Triple Point

Triple point is a state in which ice, water and water vapour can stay together in equilibrium. It refers to temperature at the equilibrium.

∴ The temperature scale by the equation

$$T = \lim_{p_{tr}\to 0} \frac{p}{p_{tr}} \times 273.16\,\text{K}$$

where, p = pressure

p_{tr} = pressure at equilibrium and K stands for kelvin scale of temperature.

Now-a-days in modern technology instead of two fixed points only one reference point is chosen, which is triple point of water (temperature at which ice, water and water vapour co-exist) and has been assigned arbitrarily a value 273.16 K. So, if values of thermometric properties at 0 K, 273.16 K and T K are 0, X_{tr} and X respectively, then

$$\frac{T}{T_{tr}} = \frac{X}{X_{tr}} \quad \text{or} \quad T = \frac{X}{X_{tr}}T_{tr} = \left[\frac{X}{X_{tr}}\times 273.16\right]\text{K}$$

Different Thermometers

Constant-Volume Gas Thermometer

If p_0, p_{100}, p_{tr} and p_t are the pressures of gas at temperatures 0°C, 100°C, triple point of water and unknown temperature (t°C) respectively keeping the volume constant, then

$$t = \left(\frac{p-p_0}{p_{100}-p_0}\times 100\right)°\text{C} \quad \text{or} \quad T = \left(273.16\,\frac{p}{p_{tr}}\right)\text{K}$$

Platinum Resistance Thermometer

If R_0, R_{100}, R_{tr} and R_t are the resistances of a platinum wire at temperatures 0°C, 100°C, triple point of water and unknown temperature (t°C) respectively, then

$$t = \left(\frac{R_t - T_0}{R_{100} - R_0} \times 100\right)°C$$

or $$T = \left(\frac{R_T}{R_{tr}} \times T_{tr}\right)K = \left(\frac{R_t}{R_{tr}} \times 273.16\right)K$$

Mercury Thermometer

In this thermometer, the length of a mercury column from some fixed point is taken as thermometric property. Thus,

$$t = \left(\frac{l_t - l_0}{l_{100} - l_0}\right) \times 100°C \quad \text{or} \quad T = \left(\frac{l_t}{l_{tr}} \times 273.16\right)K$$

Ranges of Different Thermometers

Thermometer	Lower Limit	Upper Limit
Mercury thermometer	– 30°C	300°C
Gas thermometer	– 268°C	1500°C
Platinum resistance thermometer	– 200°C	1200°C
Thermo-couple thermometer	– 200°C	1600°C
Radiation thermometer	800°C	– 6000°C

Saturated *and* Unsaturated Vapour Pressure

When a space actually, contains the maximum possible amount of vapour, the vapour is called saturated. If the amount is less than the maximum possible, the vapour is called unsaturated.

Dew Point

The temperature at which the saturation vapour is equal to the present vapour pressure is called dew point.

If the temperature is decreased below the dew point, source of the vapour condenses.

Humidity *and* Relative Humidity

The amount of water vapour present in a unit volume of air is called the **absolute humidity** of air. It is denoted by gm^{-3}.

The ratio of the amount of water vapour required to saturated the volume at same temperature is called relative humidity. Relative humidity is generally expressed as a percentage.

Thermal Expansion

When we supply energy to a body, then its temperature may rise. This develops a thermal stress in the body which results in an increase in the intermolecular separation and hence, the body expands which we call as thermal expansion. As the intermolecular forces are weakest in gases, expansion is maximum in gases.

As for liquids and gases, length and area have no meaning, only cubical expansion is defined for liquids and gases while for solids all the three-linear, superficial (areal) and cubical (volume) expansions are possible.

1. Linear Expansion

Thermal expansion along a single dimension of a solid body is defined as the linear expansion.

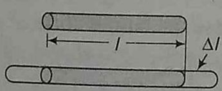
If a rod is having length l_0 at temperature T, then elongation in length of rod due to rise in temperature by ΔT is, $\Delta l = l_0 \alpha \, \Delta T$

where, α is the coefficient of linear expansion whose value depends on the nature of the material.

$$l_f = l_0 + l_0 \alpha \Delta T = l_0(1 + \alpha \, \Delta T)$$

If temperature increases, then the rod expands and if temperature decreases, the rod contracts.

The above relation $l = l_0(1 + \alpha \, \Delta T)$ is not only valid for length of the rod, it is valid for any two points on the object (provided material is present).



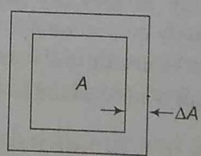
Linear Expansion $\frac{\Delta l}{l} = a_1 \Delta T$

2. Superficial Expansion *or* Areal Expansion

Superficial expansion is also valid only for solids.

$A_f = A_0(1 + \beta \Delta T)$, where A_0 is the area of the body at temperature T, β is the coefficient of superficial expansion and A_f is the area of the body when temperature has been changed by ΔT.

For isotropic materials, expansion or any other properties are same in all three directions and hence, for isotropic materials, $\beta = 2\alpha$.



Area Expansion

$$\frac{\Delta A}{A} = 2a_1 \Delta T$$

3. Volume or Cubical Expansion

As temperature of a body increases, the expansion takes place. Till now we have discussed in terms of linear and real expansion but in actual, expansion takes place uniformly in all directions, which leads to volume or cubical expansion.

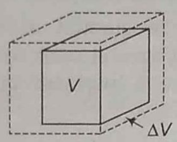
Cubical expansion is seen in all the three states of matter.

$$V = V_0(1 + \gamma \Delta T),$$

where symbols have their usual meanings. γ is the coefficient of cubical expansion.

For isotropic solids, $\gamma = 3\alpha$. For liquids and gases $\gamma = 3\alpha$ is not valid as α is not defined for liquids and gases.

$\gamma_{gas} > \gamma_{liquid} > \gamma_{solid}$ as molecules in gases are more mobile.



Volume Expansion

$$\frac{\Delta V}{V} = 3a_1 \Delta T$$

As temperature increases,

density decreases according to relation,

$$\rho = \frac{\rho_0}{1 + \gamma \Delta T}$$

or $$\rho = \rho_0(1 - \gamma \Delta T) \quad \text{[valid for small } \Delta T]$$

Apparent Expansion of Liquid

If in a beaker (container) a liquid is fully filled and if the temperature of the system increases, then because of the fact that $\gamma_{liquid} > \gamma_{solid}$, the expansion in liquid is more than the expansion in solid and thus the liquid overflows from the container. This is termed as apparent expansion of liquid.

Consider a vessel of volume V_0 fully filled with a liquid of coefficient of cubical expansion γ_l. If temperature of the system is increased by ΔT, then

$\Delta V_l = V_0 \gamma_l \Delta T$, $\Delta V_c = V_0 \gamma_c \Delta T$, where subscript c denotes the container.

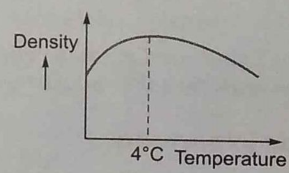
The volume of overflowing liquid is,

$$\Delta V = V_0(\gamma_l - \gamma_c)\, \Delta T = V_0\, \gamma_{lc} \Delta T$$

where, $\gamma_{lc} = \gamma_l - \gamma_c$ is termed as the apparent coefficient of cubical expansion.

Anomalous/Exceptional Behaviour of Water

As the temperature of water increases from 0 to 4°C, the density of water increases and as temperature increases beyond 4°C, the density decreases. The variation in the density in the water with temperature is shown in the figure below.



Specific Heat Capacity

The specific heat of a substance is the quantity of heat in calorie required to raise the temperature of 1 g of that substance by 1°C. Its unit is cal $g^{-1}\,C^{-1}$. The heat lost by a body or gained from a body depends upon the difference in the temperature. When we supply (or withdraw) heat to (or from) a body two things may occur, its temperature may change or phase may change.

The quantity of heat ΔQ required to change the temperature of a body of mass m by ΔT, is approximately proportional to the product of m and ΔT *i.e.*, $\Delta Q \propto m\Delta T$ or $\Delta Q = ms\,\Delta T$, where s is the specific heat capacity of the material.

The product of mass of the body and specific heat capacity is termed as heat capacity.

$s = \frac{\Delta Q}{\Delta T}$ *i.e.*, heat capacity is defined as the amount of heat required to raise the temperature of a body by 1°C.

- *Specific heat capacity can have any value from 0 to ∞.*
- *For some substances under particular situations the specific heat capacity can have negative values also.*

Molar Heat Capacity

The amount of heat required to change the temperature of a unit mole of substance by 1°C is termed as its molar heat capacity, $C = \frac{\Delta Q}{n \Delta T}$

Generally, for gases, two molar heat capacities are very common—molar heat capacity at constant pressure (C_p) and molar heat capacity at constant volume (C_V).

Calorimetry

Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body and provided no heat is allowed to escape to the surrounding. A device in which heat measurement can be made is called a calorimeter. 1 calorie is the quantity of heat required to raise the temperature of 1 g of water by 1°C.

Water Equivalent

It is the quantity of water whose thermal capacity is same as the heat capacity of the body. It is denoted by W.

$W = ms =$ Heat capacity of the body

Principle of Calorimetry

When two bodies at different temperatures are placed in contact with each other or mixed with each other (liquid-in-liquid, solid-in-liquid), the heat will pass from the body at higher temperature to the body at lower temperature until both bodies reach a common temperature.

This state is called as thermal equilibrium.

At this state,

Heat lost by one body = Heat gained by the other body

Let two bodies of masses m_1 and m_2, specific heats s_1 and s_2 and at temperatures θ_1 and θ_2 are brought in contact with each other.

Assuming $\theta_1 > \theta_2$, heat will flow from body 1 to body 2. If θ is the common temperature of two bodies at the state of thermal equilibrium, then (assuming no heat is gained or lost from or to the surroundings)

Heat lost by body 1 = Heat gained by body 2

$$m_1 s_1 (\theta_1 - \theta) = m_2 s_2 (\theta - \theta_2) \qquad (\theta_2 < \theta < \theta_1)$$

Change of State

When any substances changes from one state to another state at a constant temperature, then this process is called change of state. When we supply heat (energy) to a body and its temperature doesn't change, then the energy consumed by the body is used up in changing its phase.

For any given pressure, a phase change takes place at a definite temperature, usually accompanied by absorption or emission of heat and a change of volume and density. Heat is consumed during melting and boiling while released during freezing and condensation.

Latent Heat

The heat required to change the state of a system is proportional to the mass of the system *i.e.*, $Q \propto m$

$Q = mL$, where L is the **latent heat**.

Melting of Solids *or* Freezing of Liquids

When we supply heat to a solid, due to an increase in intermolecular separation, the bonds may break. The instant the first bond breaks, melting starts and from then onwards whatever heat is supplied to the body is used up in breaking the bonds and the temperature remains constant. When all the bonds break, the melting process is said to be complete.

Latent heat for melting is termed as **latent heat of fusion** L_f.

$Q = mL_f$, where Q is the amount of heat required to melt a solid body of mass m.

Vaporization of Liquids

When liquid phase is converted to gaseous phase at boiling point. The process is termed as **vaporization**.

Latent heat in the vaporization process is termed as **latent heat of vaporization** L_V.

$Q = mL_V$, where Q is the amount of heat required to vaporize a liquid of mass m.

It is a reversible process just like melting. Reverse of the vaporization process is termed as condensation.

Sublimation, Supercooling *and* Superheating

A substance can sometimes change directly from solid to gaseous phase, this process is termed as **sublimation**. Corresponding latent heat is termed as **latent heat of sublimation**, L_s. The reverse process can also occur.

Very pure water can be cooled several degrees below the freezing temperature without freezing, the resulting unstable state is described as **supercooled**. When this supercooled water is disturbed (either by dropping dust particles etc), it crystallizes within a second or less.

A liquid can sometimes be **superheated** above its normal boiling temperature. Any small disturbance such as agitation causes local boiling with bubble formation.

Water Equivalent of a Substance

Water equivalent of certain amount of substance is defined as the amount of water, which when replaced by the substance requires the same amount of heat for the same rise in temperature.

$$m_w = \frac{mS}{S_w},$$

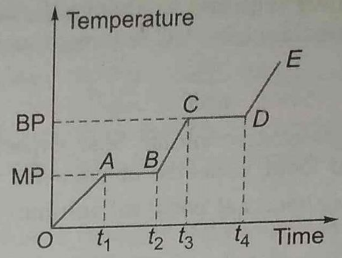
where, m_w = water equivalent of substance whose mass is m,

S = specific heat capacity of substance and

S_w = specific heat capacity of water.

Heating Curve

If we supply energy to a body in solid state (temperature< melting point) at a constant rate, then the curve drawn between temperature and time is termed as the heating curve.



OA represents heating of the solid,

$$S_{\text{solid}} \propto \frac{1}{\text{Slope of } OA}$$

AB represents melting of the solid, length of

$$AB \propto L_f$$

BC represents heating of the liquid,

$$S_{\text{liquid}} \propto \frac{1}{\text{Slope of } BC}$$

CD represents boiling (vaporization) of the liquid, length of $CD \propto L_V$

DE represents heating of the gaseous phase,

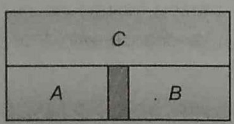
$$S_{\text{gas}} \propto \frac{1}{\text{Slope of } DE}$$

Thermal Equilibrium

When there is no exchange of heat between two objects placed in contact, then both are called in thermal equilibrium. Thus, the temperature remains constant throughout all the portions. Thermodynamics is a branch of science which deals with transformation of heat energy into other forms of energy and vice-versa.

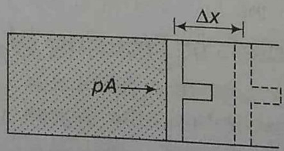
Zeroth Law of Thermodynamics

If two objects A and B are separately in thermal equilibrium with another object C, then objects A and B will also be in thermal equilibrium.



Work

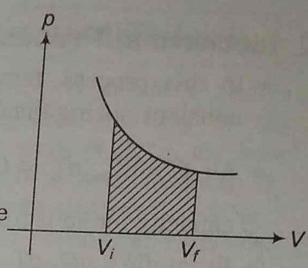
Consider a system whose volume can change (a gas, liquid or solid) in a cylinder with movable piston. Suppose the cylinder has a cross-sectional area A and pressure exerted by system on the piston face is p. The work done by the system on the surroundings for small displacement dx is $dW = pAdx$. as shown in figure here.



$$W = \int dW = \int_{V_i}^{V_f} p\,dV$$

i.e., work done in a finite change of volume from V_i to V_f.

- Area under p-V curve (indicator diagram) gives the work done. In the given figure
- p-V diagram work done will be $W = \int dw = \int_{V_i}^{V_f} p\,dV$



- Work done by the system depends on the initial and final states.
- If volume of the system increases then work is done by the system and it is taken as positive work done.
- If volume of the system decreases then work is done on the system and it is taken as negative work done.

Internal Energy

Internal energy of a system is defined as the sum of the total kinetic energy of all its constituent particles and sum of all the potential energies of interaction among these particles.

- Internal energy of an ideal gas depends only on temperature and not on pressure and volume.
- For non-ideal gases, internal energy depends not only on the temperature but also on the pressure.

First Law of Thermodynamics

When a system changes for a given initial state to a given final state, both the work W and heat Q depend on the nature of process.

First law of thermodynamics is the extension of energy conservation for a thermodynamic process.

According to this law,

$$\Delta Q = \Delta U + W$$

where,

ΔQ is the heat supplied to the system,

ΔU is the increase in internal energy,

W is the work done by the system on the surroundings. *i.e.,* energy supplied to the system is equal to sum of increase in internal energy and work done by the system on the surroundings.

Second Law of Thermodynamics

The first law of thermodynamics tells us that in a thermodynamic process taking place energy will be conserved. However, it does not tell us whether a given process in which energy is conserved will actually take place or not. So, there must be a law of nature other than 1st law, which decides whether a given process, allowed by 1st law, will actually take place or not. This law is the **second law of thermodynamics.** The following two forms are worth mentioning about this law. It defines the direction of flow of heat. *Two statements of this law are as follows*

(i) **Kelvin-Planck statement** It is impossible to construct an engine, which will not produce any other effect other than extracting heat from a reservoir and performing an equivalent amount of work.

(ii) **Clausius statement** It is impossible for a self-acting machine, unaided by any external agency, to transfer heat from/to another at higher temperature.

Different Thermodynamical Processes

Thermodynamic process can be precisely categorized as cyclic process and non-cyclic process. The cyclic process is the one in which the initial and final state are identical i.e., system returns to its initial states after occurence of process. The non cyclic process is the one in which the initial and final states are different i.e., the occurence of process is accompanied by the state change. ***Now we are going to discuss different types of thermodynamics processes.***

1. Reversible and Irreversible Processes

Any process which can be made to proceed in the reverse direction so that it passes through exactly the same states in all respects as in the direct process is called a reversible process. ***e.g,*** slow expansion or compression of an ideal gas.

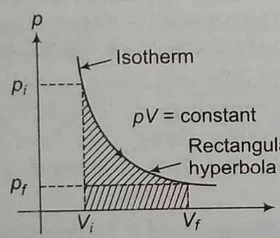
A process that cannot be made to proceed in the reverse direction is called an irreversible process. *e.g.,* rusting of iron, sudden expansion or compression of a gas etc.

2. **Isothermal Process**

- In this process, temperature of the system is kept constant during the change of state.
- As $Q = nC_{iso}dT \Rightarrow C_{iso} = \frac{Q}{ndT} = \infty$ *i.e.*, molar heat capacity for an isothermal process is infinity.
- From $dU = nC_V dT$ as $dT = 0$, so $dU = 0$ *i.e.*, internal energy is constant.
- Gas equation is pV = constant.
- From first law of thermodynamics, $dQ = dW$ *i.e.*, heat given to the system is equal to the work done by system on the surroundings.

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{p_i}{p_f}\right)$$

- After differentiating pV = constant, we have $\frac{dp}{dV} = -\frac{p}{V}$ (slope of p-V curve) and $\frac{-dp}{dV/V} = p$, *i.e.*, bulk modulus of a gas in isothermal process, $B = p$
- p-V curve is a rectangular hyperbola.

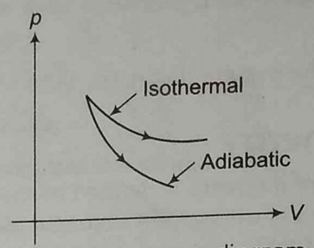


3. **Adiabatic Process**

- In this process, no heat exchange takes place between the system and the surroundings, *i.e.*, $dQ = 0$ or Q = constant
- From $dQ = nCdT$, $C_{adi} = 0$ as $dQ = 0$ *i.e.*, molar heat capacity for an adiabatic process is zero.
- From first law, $dU = -dW$ *i.e.*, work done by the system is equal to decrease in internal energy. When a system expands adiabatically, work done is positive and hence internal energy decreases *i.e.*, the system cools down and *vice-versa*.
- Gas equation is pV^{γ} = constant. Gas equation can be written in many other ways as $TV^{\gamma-1}$ = constant or $p^{1-\gamma}T^{\gamma}$ = constant. where $\gamma = \frac{C_p}{C_V}$
- Work done in an adiabatic process is,

$$W = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$$

- Using $pV = nRT$ and pV^{γ} = constant, we can have $\frac{dp}{dV} = -\frac{\gamma p}{V}$ *i.e.*, slope of p-V curve in an adiabatic process is γ times the slope of p-V curve in isothermal process.



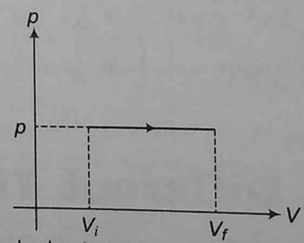
Adiabatic process diagram

4. **Isobaric Process**

- This is the process in which pressure is kept constant.
- Molar heat capacity of the process is C_p and $dQ = nC_pdT$
- $dU = nC_V dT$
- From the first law of thermodynamics,

$$dQ = dU + dW; \; dW = pdV = nRdT$$
$$\Rightarrow \quad W = p(V_f - V_i) = nR(T_f - T_i)$$

- Gas equation is $\frac{V}{T}$ = constant.
- p-V curve is a straight line parallel to the volume axis.

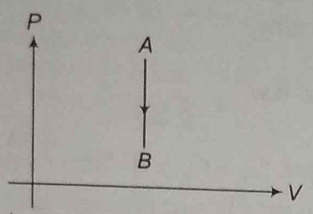


Isobaric process diagram

5. **Isochoric Process**

- This is the process in which volume is kept constant.
- $dQ = nC_V dT$, molar heat capacity for isochoric proces is C_V.
- Volume is constant, so $dW = 0$
- From the first law of thermodynamics, $dQ = dU$. As heat is supplied to the system, internal energy increases and hence the temperature increases.

- Gas equation is $\frac{p}{T}$ = constant.
- p-V curve is a straight line parallel to the pressure axis.



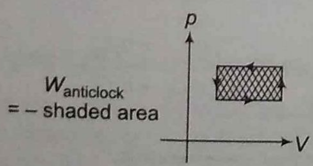
Isochoric process diagram

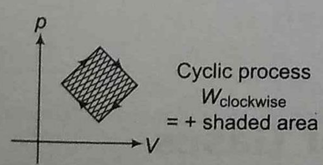
6. Cyclic Process

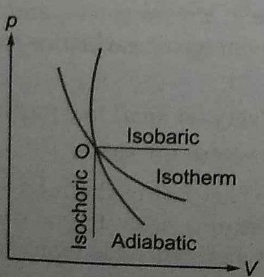
- In a cyclic process, the system returns to its initial state.
- As initial and final states are the same, so $dU = 0$ for a cyclic process.
  In a cyclic process as $du = 0 \Rightarrow dQ = dW$ (from first law of thermodynamic)
- Efficiency of the cycle is given by,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

- Work done by the cycle can be computed from area enclosed by cycle on the p - V curve.
- If the cycle is clockwise, work is done by the system and if the cycle is ant-iclockwise, work is done on the system.



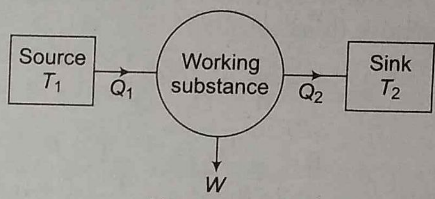




Carnot Engine and its Efficiency

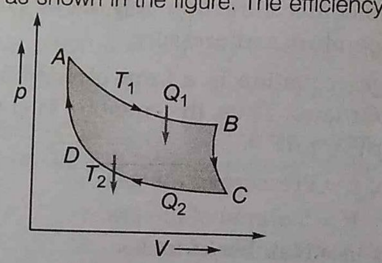
- **Heat engine** is a device in which a system undergoes a cyclic process resulting in the conversion of heat into work. If Q_1 is the heat absorbed from the heat source (hotter reservoir), Q_2 is the heat released to the sink (colder reservoir) and the work output of the engine in one cycle is W, then the efficiency of the engine η is given by

$$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$



Heat engine

- **Carnot engine** is a theoretical, ideal heat engine working in a reversible cyclic process operating between two temperatures T_1(heat source) and T_2(heat sink). The Carnot's cycle consists of two isothermal processes connected by two adiabatic processes as shown in the figure. The efficiency of a



Carnot's cycle is given by $\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

After doing the calculations for different processes we can show that $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

Thus, efficiency does not depend on the nature or quantity of the working substance.

Like pressure, volume, temperature, internal energy etc. We have another thermodynamic variable known as entropy we define the change in the entropy as $\Delta S = \frac{\Delta Q}{T}$, where ΔS = change in entropy, ΔQ = heat given, T = temperature.

> *The heat engine in which, heat is produced by burning the fuel in a chamber outside the main body of the engine is called external combustion engine.*
>
> *The heat engine in which heat is produced by burning the fuel inside the main body of the engine is called internal combustion engine.*

Refrigerator

Refrigerator is a device which takes heat from a cold body, work is done on it and the work done together with the heat absorbed is rejected to the source. An ideal refrigerator can be regarded as Carnot's ideal heat engine working in the reverse direction. here diagram will be reverse of heat engine.

Coefficient of Performance of a Refrigerator (β)

It is defined as the ratio of quantity of heat removed per cycle (Q_2) to the work done on the working substance per cycle to remove this heat.

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

or $$\beta = \frac{T_2}{T_1 - T_2} = \frac{1-\eta}{\eta}$$

Equations of State of a Perfect Gas

In practice, the gases do not obey the gas laws at all values of temperature and pressure. It is because of the intermolecular forces between the gas molecules.

An ideal gas is one whose molecules are free from intermolecular attraction and obeys gas laws at all values of temperature and pressure.

Ideal gas equation is a form of combined effect of above first four laws. Thus, the equation is given by

$$pV = nRT$$

where, p = Pressure of the gas
V = Volume of the gas
n = Number of moles
R = Universal gas constant
T = Absolute temperature of the gas

This equation contains four laws

(i) Boyle's law $V \propto \frac{1}{p}$ (T = constant)

(ii) Charles' law
$V \propto T$ (p = constant)

For an ideal gas, $C_p - C_V = R$ (Mayer's relation).

(iii) Gay Lussac's law or pressure law
$p \propto T$ (V = constant)

(iv) Avogadro's law
It is states that, at same temperature and pressure equal volumes of all gases contain equal number of molecules.

Work Done on Compressing a Gas

Work done $W = p \cdot \Delta V$, where p = pressure of the gas and ΔV = change in volume of the gas.

When two ideal gases having molar masses M_1 and M_2 are mixed, then thermodynamic variables/parameters for mixture would be given by

- M (molar mass) $= \dfrac{n_1 m_1 + n_2 m_2}{n_1 + n_2}$,
- C_V (of the mixture) $= \dfrac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$,
- C_p (of the mixture) $= \dfrac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$
- γ (of the mixture) $= \dfrac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$, or is given by $\dfrac{n_1 + n_2}{\gamma - 1} = \dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}$

where symbols have their usual meanings.

Kinetic Theory of Gases

The kinetic theory of gases correlates the macroscopic properties of gases e.g., pressure, temperature etc., to the microscopic properties of gas molecules e.g., speed, momentum, kinetic energy of molecules etc. The kinetic theory of gases is based on the following assumptions

1. Molecules of a gas are small hard spheres and these molecules are very far apart in comparision to their sizes.
2. The total volume of the molecules is negligible as compared to the size of the gas.
3. The molecules collides elastically with each other.
4. The molecules exert no force on each other except during collisions.

Concept of Pressure

The pressure exerted by a gas is the result of collisions of gas molecules with the walls of the container. The total change in momentum per second is the force exerted on the walls by the gas. The pressure of the gas is equal to the force exerted per unit area of the walls by the gas.

Pressure of an ideal gas

$$p = \frac{1}{3}\frac{mn}{V} v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$$

where, m = mass of molecule of gas

n = number of molecules

V = volume of the gas

ρ = density of the gas

v_{rms} = root mean square speed of the molecules.

Kinetic Energy *and* Temperature

Average kinetic energy of a gas molecule

$$K = \frac{3}{2}kT \; (k \propto T)$$

where k = Boltzmann's constant,

T = Absolute temperature of the gas

Average kinetic energy of one mole of an ideal gas $K = \frac{3}{2}RT$

where, R = gas constant.

RMS Speed of Gas Molecules

The square root the mean of the squares of the speed of gas molecules is called their root mean square speed (v_{rms}).

$$v_{rms} = \sqrt{\frac{3RT}{m'}} \quad (v_{rms} \propto \sqrt{T})$$

where, m' = Molecular weight.

Degree of Freedom

Degree of freedom represents the number of independent possible ways in which the system can have energy (due to its motion or configuration).

A system can possess energy due to translational, rotational or vibrational motion or due to vibrational configuration or any combination of these.

(i) For an ideal monoatomic gas, $f = 3$ due to translational motion in three directions.

(ii) For an ideal diatomic gas at room temperature $f = 5$ due to 3 translational and 2 rotational. At high temperature, $f = 7$ because of 2 additional ways due to vibration (one kinetic energy and other potential energy).

(iii) For an ideal polyatomic gas at room temperature, $f = 6$ due to 3 translational and 3 rotational. At high temperature, the vibrational mode will also come into existence. For a polyatomic gas, number of vibrational terms can be greater than two.

(iv) For solids, $f = 6$ due to vibrational motion in all 3 directions.

Law of Equipartition of Energy

Total internal energy of an ideal gas distributes equally in all degrees of freedom and energy per degree of freedom of one mole of gas is $\frac{1}{2}RT$, where T is the absolute temperature of the gas. If f is the degree of freedom and n is number of moles of the gas internal energy of the gas $U = \frac{n}{2}fRT$

Mean Free Path

The average distance travelled by a molecule between two successive collisions is called mean free path. It is denoted by λ.

Avogadro's Number

According to Avogadro's hypothesis, gram atomic masses of all elements contain the same number of atoms and this number is called Avogadro's number (N_A) and its value is 6.02×10^{23}.

Practice Zone

DAY 15

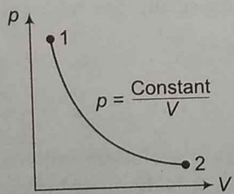
1. A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 be the lengths of the gas column before and after expansion respectively, then $\frac{T_1}{T_2}$ is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\left(\frac{L_1}{L_2}\right)$ (c) $\left(\frac{L_2}{L_1}\right)$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

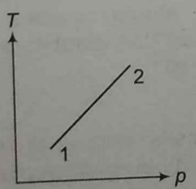
2. One mole of an ideal monoatomic gas is heated at a constant pressure of 1 atmosphere from 0°C to 100°C. Work done by the gas is

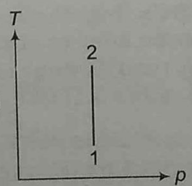
(a) 8.31×10^{3} J (b) 8.31×10^{-3} J
(c) 8.31×10^{-2} J (d) 8.31×10^{2} J

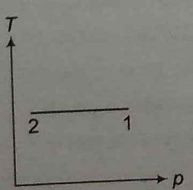
3. Consider *p*-*V* diagram for an ideal gas shown if figure **[NCERT Exemplar]**

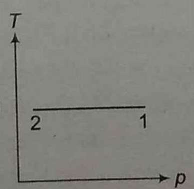


Out of the following diagrams which represents the *T*-*p* diagram?

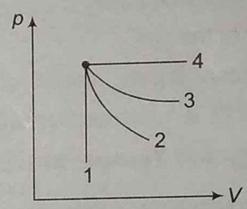
(a)


(b)


(c)


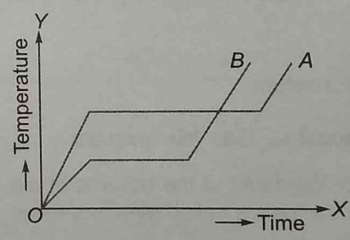
(d)


4. An ideal gas undergoes four different processes from the same initial state. Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1, 2, 3 and 4 which one is adiabatic. **[NCERT Exemplar]**



(a) 4 (b) 3 (c) 2 (d) 1

5. Equal masses of two liquids *A* and *B* contained in vessels of negligible heat capacity are supplied heat at the same rate. The temperature-time graphs for the two liquids are shown in the figure. If *S* represents specific heat and *L* represents latent heat of liquid, then

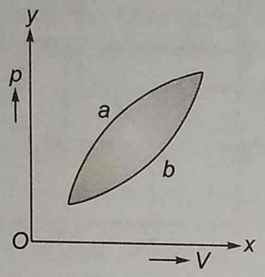


(a) $S_A > S_B; L_A < L_B$ (b) $S_A > S_B; L_A > L_B$
(c) $S_A < S_B; L_A < L_B$ (d) $S_A < S_B; L_A > L_B$

6. Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory $pV = \frac{2}{3}E$, E is **[NCERT Exemplar]**

(a) the total energy per unit volume.
(b) only the translational part of energy because rotational energy is very small compared to the translational energy.
(c) only the translational part of the energy because during collisions with the wall, pressure related to change in linear momentum.
(d) the translational part of the energy because rotational energies of molecules can be of either sign and its average over all the molecules is zero.

7. Figure shows two processes *a* and *b* for a given sample of a gas. If $\Delta Q_1, \Delta Q_2$ are the amounts of heat absorbed by the system in the two cases and ΔU_1, ΔU_2 are changes in internal energies respectively, then



(a) $\Delta Q_1 = \Delta Q_2; \Delta U_1 = \Delta U_2$ (b) $\Delta Q_1 > \Delta Q_2; \Delta U_1 > \Delta U_2$
(c) $\Delta Q_1 < \Delta Q_2; \Delta U_1 < \Delta U_2$ (d) $\Delta Q_1 > \Delta Q_2; \Delta U_1 = \Delta U_2$

8. A refrigerator works between the temperature of melting ice and room temperature (17°C). The amount of energy in kWh that must be supplied to freeze 1kg of water at 0°C is

(a) 1.4 (b) 1.8 (c) 0.058 (d) 2.5

9. Three designs are proposed for an engine operating between 500 K and 300 K. For 1 kcal of heat input, design *A* claims to produce 3000 J of work, design *B* claims to produce 2000 J of work and design *C* claims to produce 1680 J of work. Which of the designs is possible?

(a) *A* only (b) *B* only
(c) All (d) *C* only

10. A gas expands with temperature according to the relation $V = kT^{2/3}$. Calculate the work done when the temperature changes by 60 K ?

(a) 10 *R* (b) 30 *R*
(c) 40 *R* (d) 20 *R*

11. A gas undergoes a process in which its pressure *p* and volume *V* are related as Vp^n = constant, the bulk modulus for the gas in the process is

(a) np (b) $p^{1/n}$
(c) $\frac{p}{n}$ (d) p^n

12. An ideal Carnot engine whose efficiency is 40%, receives heat at 500 K. If the efficiency is to be 50%, the intake temperature for the same exhaust temperature is

(a) 600 K (b) 900 K
(c) 700 K (d) 800 K

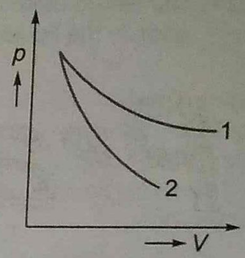
13. The pressure inside a tyre is 4 atm at 27°C. If the tyre bursts suddenly, its final temperature will be

(a) $300(4)^{7/2}$ (b) $300(4)^{2/7}$
(c) $300(2)^{7/2}$ (d) $300(4)^{-2/7}$

14. *p-V* plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to

(a) He and O_2 (b) O_2 and He
(c) He and Ar (d) O_2 and N_2

15. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is



(a) 4.2×10^6 J
(b) 8.4×10^6 J
(c) 16.8×10^6 J
(d) 3×10^6 J

16. Two moles of helium are mixed with *n* moles of hydrogen. The root mean square speed of the gas molecules in the mixture is $\sqrt{2}$ times the speed of sound in the mixture. Then value of *n* is

(a) 1 (b) 3/2
(c) 2 (d) 3

17. *p-V* diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be

(a) 4 *R*
(b) 3 *R*
(c) 4 *R*/3
(d) 2.5 *R*

18. An ideal gas is taken from the state *A* (pressure *p*, volume *V*) to the state *B* (pressure *p*/2, volume 2*V*), a long a straight line path in the *p-V* diagram. Select the correct statement from the following.

(a) The work done by the gas in the process *A* to *B*, exceeds the work that would be done by it if system were taken along the isotherm
(b) In the *T-V* diagram, the path *AB* becomes a part of a hyperbola
(c) In the *p-T* diagram, the path *AB* becomes a part of a hyperbola
(d) In going from *A* to *B*, the temperature *T* of the gas first decreases to a minimum value and then increases

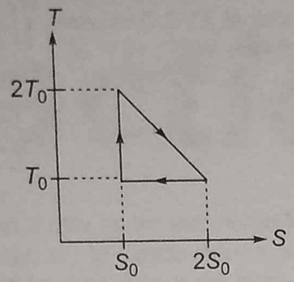
19. The temperatures of inside and outside of a refrigerator are 273 K and 303 K, respectively. Assuming that the refrigerator cycle is reversible, for every joule of work done, the heat delivered to the surrounding will be nearly

(a) 9 J
(b) 20 J
(c) 10 J
(d) 30 J

20. A vessel of volume *V* contains a mixture of 1mole of hydrogen and 1 mole of oxygen (both considered as ideal). Let $f_1(\upsilon)\,d\upsilon$, denote the fraction of molecules with speed between υ and $(\upsilon + d\upsilon)$ with $f_2(\upsilon)\,d\upsilon$, similarly for oxygen. Then, **[NCERT Exemplar]**

(a) $f_1(\upsilon) + f_2(\upsilon) = f(\upsilon)$ obeys the Maxwell's distribution law
(b) $f_1(\upsilon), f_2(\upsilon)$ will obey the Maxwell's distribution law
(c) Neither $f_1(\upsilon)$, nor $f_2(\upsilon)$ will obey the Maxwell's distribution law
(d) $f_2(\upsilon)$ and $f_1(\upsilon)$ will be the same

21. The temperature-entropy diagram of a reversible engine is given in the figure. Its efficiency is



(a) 1/4 (b) 1/2
(c) 1/3 (d) 2/3

Directions *(Q. Nos. 22 to 25) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

22. Statement I In equation $dQ = ms\,dT$, dQ represents a change in the amount of heat contained in the body due to a change in the temperature dT.

Statement II Heat is energy in transit.

23. Statement I When 1 g of water at 100°C is converted to steam at 100°C, the internal energy of the system does not change.

Statement II From $dU = nC_V\,dT$, if temperature of the system remains constant, then $dU = 0$, *i.e.*, internal energy remains constant.

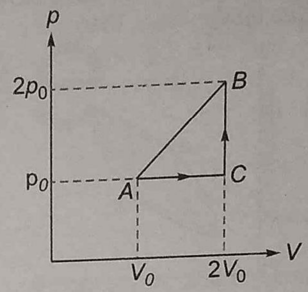
24. Statement I In an isothermal process (quasi- static), the heat exchange between the system and surroundings takes place even though the gas has the same temperature as that of the surrounding.

Statement II There is an infinitesimal difference in temperature between the system and the surroundings.

25. Statement I A special type of thermometer (used to measure very high temperatures and calibrated for an ideal black body) measures a value lower than the actual value of the temperature of a red hot iron piece kept in open.

Statement II As the iron piece is kept in open, it loses its heat.

Directions *(Q. Nos. 26 and 27) 2 mol of an ideal monoatomic gas is taken from a state A to a state B, along the path ACB. Temperature of B is T_0.*



26. For the process $A \rightarrow C \rightarrow B$, the heat absorbed by the gas is

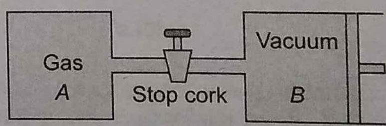
(a) $\frac{9RT_0}{4}$ (b) $\frac{RT_0}{2}$
(c) $\frac{11RT_0}{4}$ (d) $\frac{16RT_0}{3}$

27. If the gas is taken from A to B directly along the straight line path AB, then the molar heat capacity of the gas for this process, is

(a) $\frac{3R}{2}$ (b) $\frac{5R}{2}$ (c) $8R$ (d) $2R$

Directions *(Q. Nos. 28 to 30) An ideal diatomic gas is confined in a cylinder A of volume V_0, this cylinder is connected to another cylinder B with the help of a tube of negligible volume. The cylinder B is fitted with a movable piston which can be adjusted from outside. Initially, the piston is adjusted so that the volume of B is the same as volume of A i.e., V_0. B is evacuated and the stopcork is opened so that the gas expands and occupies the volume $2V_0$. [System is thermally isolated from the surroundings].*

Now with the stop-cork open, the piston is slowly moved to compress the gas back to cylinder A at temperature T. For this



28. During this free expansion, the internal energy of the system

(a) increases (b) decreases
(c) remains constant (d) nothing can be said

29. Work done on the gas is [for n moles of gas]

(a) $nRT \ln 2$ (b) $-nRT \ln 2$
(c) nRT (d) $-nRT$

30. The heat absorbed by the gas is

(a) $nRT \ln 2$ (b) $-nRT \ln 2$
(c) nRT (d) $-nRT$

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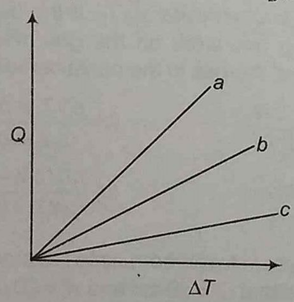
31. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two statements.

Statement I The internal energy of a perfect gas is entirely kinetic and depends only on absolute temperature of the gas and not on its pressure or volume.

Statement II A perfect gas is heated keeping pressure constant and later at constant volume. For the same amount of heat the temperature of the gas at constant pressure is lower than that at constant volume. **[JEE Main Online 2013]**

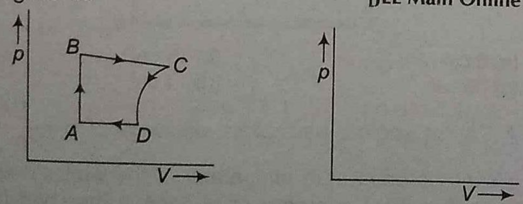
(a) Statement I is true, Statement II is true but Statement II is correct the explanation of Statement I
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I
(d) Statement I is false, Statement II is true

32. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic (P) gas. The initial state of all the gases are the same and the scale for the axes coincide, ignoring vibrational degrees of freedom, the lines a, b and c respectively correspond to **[JEE Main Online 2013]**



(a) P, M and D (b) M, D and P (c) P, D and M (d) D, M and P

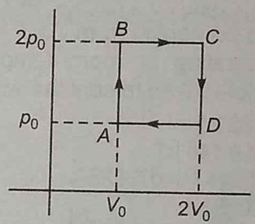
33. A certain amount of gas is taken through a cyclic process ($A\,B\,C\,D\,A$) that has two isobars, one isochore and one isothermal. The cycle can be represented on a p-V indicator diagram as **[JEE Main Online 2013]**



34. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be **[AIEEE 2012]**

(a) efficiency of Carnot engine cannot be made larger than 50%
(b) 1200 K
(c) 750 K
(d) 600 K

35. Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly (assume the gas to be close to ideal gas). **[AIEEE 2012]**



(a) 15.4% (b) 9.1%
(c) 10.5%2 (d) 12.5%

36. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and its suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by **[AIEEE 2011]**

(a) $\dfrac{(\gamma - 1)}{2\gamma R} Mv^2 K$ (b) $\dfrac{\gamma Mv^2}{2R} K$

(c) $\dfrac{(\gamma - 1)}{2R} Mv^2 K$ (d) $\dfrac{(\gamma - 1)}{2(\gamma + 1)R} Mv^2 K$

37. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K). **[AIEEE 2011]**

(a) 8.4 kJ
(b) 84 kJ
(c) 2.1 kJ
(d) 4.2 kJ

38. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is **[AIEEE 2011]**

(a) $\dfrac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$ (b) $\dfrac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(c) $\dfrac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (d) $\dfrac{(T_1 + T_2 + T_3)}{3}$

39. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively **[AIEEE 2011]**

(a) 372 K and 330 K (b) 330 K and 268 K
(c) 310 K and 248 K (d) 372 K and 310 K

40. An aluminium sphere of 20 cm diameter is heated from 0°C to 100°C. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{Al} = 23 \times 10^{-6}$ /°C) **[AIEEE 2011]**

(a) 28.9 cc (b) 2.89 cc
(c) 9.28 cc (d) 49.8 cc

41. The specific heat capacity of a metal at low temperature (T) is given as $C_p(\text{kJK}^{-1}\text{ kg}^{-1}) = 32\left(\frac{T}{400}\right)^3$. A 100 g vessel of this metal is to be cooled from 20 K and 4 K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is **[AIEEE 2011]**

(a) equal to 0.002 kJ
(b) greater than 0.148 kJ
(c) between 0.148 kJ and 0.028 kJ
(d) less than 0.028 kJ

42. A diatomic ideal gas is used in a car engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to $32V$, the efficiency of the engine is **[AIEEE 2010]**

(a) 0.5 (b) 0.75
(c) 0.99 (d) 0.25

43. One kg of a diatomic gas is at a pressure of 8×10^4 Nm^{-2}. The density of the gas is 4 kgm^{-3}. What is the energy of the gas due to its thermal motion? **[AIEEE 2009]**

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J

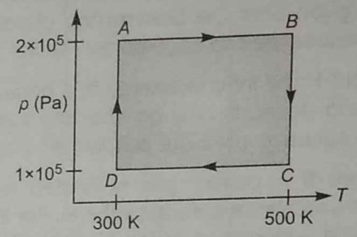
44. This question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2009]**

Statement I The temperature dependence of resistance is usually given as $R = R_0(1+\alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5\times 10^{-3}$ /°C.

Statement II $R = R_i(1+\alpha\Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) << R_0$.

(a) Statement I is true, Statement II is false
(b) Statement I is true; Statement II is true; Statement II is the correct explanation of Statement I
(c) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(d) Statement I is false, Statement II is true

Directions *Questions number 42, 43 and 44 are based on the following paragraph.*
Two moles of helium gas are taken over the cycle ABCDA, as shown in the p-T diagram.



45. Assume the gas to be ideal the work done on the gas in taking it from A to B is **[AIEEE 2009]**

(a) 200 R (b) 300 R (c) 400 R (d) 500 R

46. The work done on the gas in taking it from D to A is **[AIEEE 2009]**

(a) −414 R (b) +414 R
(c) −690 R (d) +690 R

47. The net work on the gas in the cycle $ABCDA$ is **[AIEEE 2009]**

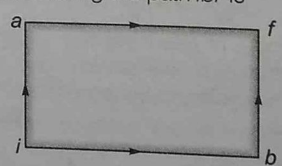
(a) zero (b) 276 R (c) 1076 R (d) 1904 R

48. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure p_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be **[AIEEE 2008]**

(a) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_2 + p_2V_2T_1}$ (b) $\frac{p_1V_1T_1 + p_2V_2T_2}{p_1V_1 + p_2V_2}$

(c) $\frac{p_1V_1T_2 + p_2V_2T_1}{p_1V_1 + p_2V_2}$ (d) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_1 + p_2V_2T_2}$

49. When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path ibf, Q = 36 cal. W along the path ibf is **[AIEEE 2007]**



(a) 6 cal (b) 16 cal
(c) 66 cal (d) 14 cal

50. A Carnot engine, having an efficiency of $\eta = \frac{1}{10}$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at a lower temperature is **[AIEEE 2007]**

(a) 99 J (b) 90 J
(c) 1 J (d) 100 J

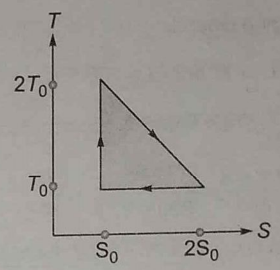
51. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is ($R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$) **[AIEEE 2006]**

(a) diatomic
(b) triatomic
(c) a mixture of monoatomic and diatomic
(d) monoatomic

52. Water of volume 2 L is filled in a container and is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at the rate of 160 J/s. In how much time the temperature will rise from 27°C to 77°C ?
[Specific heat of water is 4.2 kJ/kg] **[AIEEE 2005]**

(a) 8 min 20 s (b) 6 min 2 s
(c) 7 min (d) 14 min

53. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is **[AIEEE 2005]**



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

54. Which of the following statements is correct for any thermodynamic system? **[AIEEE 2004]**

(a) The internal energy changes in all the processes
(b) Internal energy and entropy are state functions
(c) The change in entropy can never be zero
(d) The work done in an adiabatic process is always zero

Answers

1. (d)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (c)
11. (c)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (b)
21. (c)	**22.** (d)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (b)
31. (a)	**32.** (c)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (c)	**37.** (a)	**38.** (a)	**39.** (d)	**40.** (a)
41. (c)	**42.** (b)	**43.** (b)	**44.** (d)	**45.** (c)	**46.** (b)	**47.** (b)	**48.** (a)	**49.** (a)	**50.** (b)
51. (a)	**52.** (a)	**53.** (c)	**54.** (b)						

Hints & Solutions

1. For an adiabatic process,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

or $$T_1L_1^{\gamma-1} = T_2L_2^{\gamma-1} \quad (\because A = \text{constant})$$

or $$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\gamma-1}$$

$\because \gamma = \frac{5}{3}$ for a monoatomic gas,

$\therefore \gamma - 1 = \frac{2}{3}$

Thus, $$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

2. $dW = dQ - dU$

$= C_p (T_2 - T_1) - C_V (T_2 - T_1)$
$= R [T_2 - T_1]$
$= 8.31 \times 100$
$= 8.31 \times 10^2$ J

3. In the diagram, T is constant and $p_1 > p_2$. This situation is represented by curve (iii). In the solution figure. In which $p_1 > p_2$ and straight line parallel to pressure axis represents constant temperature.

4. As it is clear from the figure, for curve 1, V is constant. It represent isochoric process. Curve 4. $p \to$ constant $\Rightarrow$ isobaric process. Slope of curve $3 \to 2 \to$ adiabatic process.

5. As temperature of A rises faster than the temperature of B, therefore, specific heat of A is less than that of B *i.e.*, $s_A < s_B$. Horizontal portions of graphs represent conversion of liquid into vapours. The horizontal portion is larger for liquid A, therefore $L_A > L_B$.

6. In the relation $pV = \frac{2}{3}E$, E is only the translational part of energy of molecules. This is because during collision of molecules with the walls, pressure exerted relates to change in linear momentum of gas molecules.

7. As initial and final states in the two processes are same, therefore, $\Delta U_1 = \Delta U_2$. As area under curve $a >$ area under curve b, therefore

8. $T_2 = 0°C = 273\,K,$

$T_1 = 17°C = 17 + 273 = 290\,K$

Coefficient of performance $= \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\frac{80 \times 1000 \times 4.2}{W} = \frac{273}{290 - 273} = \frac{273}{17}$$

$\therefore$ $$W = \frac{80 \times 1000 \times 4.2 \times 17}{273}\,J$$

or $$W = \frac{33.6 \times 17 \times 10^4}{273 \times 3.6 \times 10^5}\,kWh = 0.058\,kWh$$

9. Maximum value of efficiency, $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = \frac{2}{5}$

As $$\eta = \frac{W}{Q_1}$$

$$W = \eta Q_1 = \frac{2}{5} \times 1000\text{ cal} = 400 \times 4.2\text{ J} = 1680\text{ J}$$

As no engine can produce more than 1680 J, designs *A* and *B* are not possible. Only design *C* is possible

10. $$dW = pdV = \frac{RT}{V}dV \quad \ldots(i)$$

As, $$V = kT^{2/3}, dV = k\frac{2}{3}T^{-1/3}\,dT$$

$$\frac{dV}{V} = \frac{k\frac{2}{3}T^{-1/3}\,dT}{kT^{2/3}} = \frac{2}{3}\frac{dT}{T}$$

From Eq. (i), $W = \int_{T_1}^{T_2} RT\frac{dV}{V} = \int_{T_1}^{T_2} RT\frac{2}{3}\frac{dT}{T}$

$$W = \frac{2}{3}R(T_2 - T_1) = \frac{2}{3}R \times 60 = 40R$$

11. $Vp^n = \text{constant} = (V + \Delta V)(p + \Delta p)^n$

$$\approx Vp^n\left(1 + \frac{\Delta V}{V}\right)\left(1 + n\frac{\Delta p}{p}\right)$$

$$1 = 1 + \frac{\Delta V}{V} + n\frac{\Delta p}{p} + \frac{n\,\Delta V}{V}\cdot\frac{\Delta p}{p}$$

$\frac{\Delta V}{V} \approx -n\frac{\Delta p}{p}$ (neglecting the term $\frac{n\Delta V}{V}\cdot\frac{\Delta p}{p}$)

Bulk modulus of the gas

$\therefore$ $$k = \frac{-\Delta p}{\Delta V/V} = \frac{p}{n}$$

12. As $\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = 1 - \eta$

$$\frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$$

$$T_2 = 300\,K$$

Now, $$\frac{T_2}{T_1'} = 1 - \eta' = 1 - \frac{50}{100} = \frac{1}{2}$$

$$T_1' = 2T_2 = 2 \times 300 = 600\,K$$

13. In an adiabatic process,

$$p_2^{(1-\gamma)}\,T_2^{\gamma} = p_1^{(1-\gamma)}\,T_1^{\gamma}$$

$$T_2 = T_1\left(\frac{p_1}{p_2}\right)^{(1-\gamma)/\gamma} = 300\left(\frac{4}{1}\right)^{\frac{(1-7/5)}{7/5}} = 300\,(4)^{-2/7}$$

14. As is clear from the figure.

Slope of curve 2 > Slope of curve 1

$$(\gamma p)_2 > (\gamma p)_1 \Rightarrow \gamma_2 > \gamma_1 \Rightarrow \gamma_{He} > \gamma_{O_2}$$

Adiabatic curve 2 corresponds to helium and adiabatic curve 1 corresponds to oxygen.

15. Here, $T_1 = 627°C = 627 + 273 = 900\,K$

$T_2 = 27°C = 27 + 273 = 300\,K, Q_1 = 3 \times 10^6$ cal

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{300}{900} = \frac{1}{3}$$

$$Q_2 = \frac{Q_1}{3},\; W = Q_1 - Q_2 = Q_1 - \frac{Q_1}{3}$$

$$= \frac{2}{3}Q_1 = \frac{2}{3} \times 3 \times 10^6\text{ cal}$$

$$W = 2 \times 10^6\text{ cal} = 8.4 \times 10^6\text{ J}$$

16. $v_{rms} = \sqrt{\frac{3RT}{M}}, v_{sound} = \sqrt{\frac{\gamma RT}{M}}, v_{rms} = \sqrt{2}\,v_{sound}$

Solving it, we get $\sqrt{3} = \sqrt{2\gamma}$

$\therefore$ $\gamma = \frac{3}{2}$ for the mixture.

As, $$\gamma = \frac{C_p}{C_V} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2} \times \frac{n_1 + n_2}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$$\gamma = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

For helium, $C_{p_1} = \frac{5}{2}R, C_{V_1} = \frac{3}{2}R$

For hydrogen, $C_{p_2} = \frac{7}{2}R, C_{V_2} = \frac{5}{2}R$

$\therefore$ $$\frac{3}{2} = \frac{2\left(\frac{5}{2}R\right) + n\left(\frac{7}{2}R\right)}{2\left(\frac{3}{2}R\right) + n\left(\frac{5}{2}R\right)} = \frac{10 + 7n}{6 + 5n}$$

or $$20 + 14n = 18 + 15n \Rightarrow n = 2$$

17. As *p*-*V* diagram is a straight line passing through origin, therefore, $p \propto V$ or pV^{-1} = constant

In the process, pV^x = constant, molar heat capacity is given by $$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$$

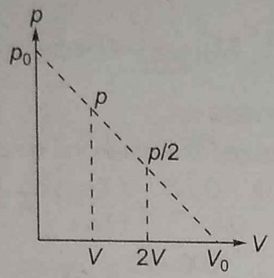
where, $x = -1$ here and $\gamma = 1.4$ for diatomic gas

$$C = \frac{R}{1.4 - 1} + \frac{R}{1 - (-1)} = \frac{5}{2}R + \frac{R}{2}$$

$$C = 3R$$

18. Work done $= \frac{1}{2}\left(p + \frac{p}{2}\right)V = \frac{3}{4}pV = 0.75\,pV$

Work done during isothermal process



$$= RT \times 2.3026\log_{10}\left(\frac{2V}{V}\right) = 0.693\,pV$$

Thus, statement (a) is correct.

19. Coefficient of performance of refrigerator is, $K = \frac{T_i}{T_0 - T_i} = \frac{Q}{W}$

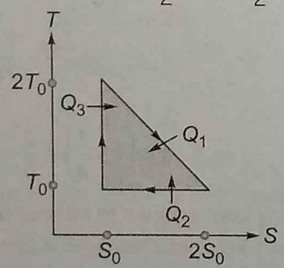
Here, $T_i = 273$ K, $T_0 = 303$ K, $W = 1$ J

So, $Q = 9.1 \text{ J} \simeq 9\text{J}$

20. As the vessel contains 1 mole of hydrogen and 1 mole of oxygen, therefore as per Maxwell's law of speed distribution, $f_1(v)$ and $f_2(v)$ will obey the law separately.

So, change in temperature $= (402 - 27)°\text{C} = 375°\text{C}$

21. We have, $Q_1 = T_0 S_0 + \frac{1}{2}T_0 S_0 = \frac{3}{2}T_0 S_0$



$Q_2 = T_0 S_0, Q_3 = 0$

$$\Rightarrow \quad \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$$

22. Heat is defined as the energy in transit due to temperature difference. dQ represents the heat transferred to the body to change its temperature by dT.

24. Both the statements are correct and Statement II is the correct explanation of Statement I.

25. Since, the thermometer is caliberated with an ideal black body, the body that emits or absorbs all the radiations falling on it, shows a lower value of temperature. This is because iron is not a black body *i.e.*, does not absorb/emit all radiation falling on it.

26. Let temperature at A is T_A, then from $pV = nRT$,

$$p_0V_0 = 2RT_A$$

At B, $\quad 4p_0V_0 = 2R \times T_0$

$$\Rightarrow \quad p_0V_0 = \frac{RT_0}{2} \Rightarrow T_A = \frac{T_0}{4}$$

For $A \to C \to B$,

$$dU = nC_VT = 2 \times \frac{3R}{2}\left(T_0 - \frac{T_0}{4}\right) = \frac{9RT_0}{4}$$

$$dW = dW_{A\to C} + dW_{C\to B}$$

$$= p_0 \times (2V_0 - V_0) + 0 = p_0V_0 = \frac{RT_0}{2}$$

So, $\quad dQ = \frac{9RT_0}{4} + \frac{RT_0}{2} = \frac{11RT_0}{4}$

27. For direct path, $A \to B$

$$dQ = nCdT = nC_VdT + \int p\,dV$$

$$\Rightarrow \quad nCdT = \frac{9RT_0}{4} + \frac{1}{2}[3p_0 \times V_0] = \frac{9RT_0}{4} + \frac{3RT_0}{4}$$

$$\Rightarrow \quad 2 \times C \times \frac{3T_0}{4} = \frac{12RT_0}{4} \Rightarrow C = 2R$$

28. For present case, $dQ = 0, dW = 0$, so $dU = 0$

29. During compression, process is slow so it is isothermal at a constant temperature T.

So, work done, $W = nRT\ln\frac{V_f}{V_i} = -nRT\ln 2$

This is the work done by the system on the surroundings, so work done by the surroundings on the system is $nRT\ln 2$.

30. From the 1st law of thermodynamics,

$dQ = dU + dW \Rightarrow dU = 0$ for the constant temperature process $\Rightarrow dQ = dW \Rightarrow dQ = -nRT\ln 2$

Negative sign shows that heat is released by the system.

31. The external energy depends upon absolute temperature of gas. Also Statement II is correct but both the statements are independently true.

32. As the prism one is constant for isobaric process, so only volume of gases would be change. As the change in temperature *i.e.*, ΔT. For monoatomic gas, the volume will change more rapidly with the change of temperature as compared to diatomic gases and triatomic gases because the molecular structure will absorbed some heat. The heat absorbed the molecular structure of gas will be lowest in case of monoatomic gases, as there is molecular structure for the monoatomic gases, which is made up of a single atom only.

33. For a cyclic process initial and final points are same. In isobaric process pressure is constant and in isochoric process volume is constant while in isothermal process, there is no change of temperature.

34. Efficiency, $\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

Now, $\quad 0.4 = 1 - \frac{T_{\text{sink}}}{500\text{ K}}$

$\Rightarrow \quad T_{sink} = 0.6 \times 500\text{ K} = 300\text{ K}$

Thus, $\quad 0.6 = 1 - \dfrac{300\text{ K}}{T'_{source}}$

$\Rightarrow \quad T'_{source} = \dfrac{300\text{ K}}{0.4} = 750\text{ K}$

35. Efficiency of a process is defined as the ratio of work done to energy supplied. Here,

$$\eta = \frac{\Delta W}{\Delta Q} = \frac{\text{Area under } p-V \text{ diagram}}{\Delta Q_{AB} + \Delta Q_{BC}}$$

$$\therefore \quad \eta = \frac{p_0V_0}{nC_V\Delta T_1 + nC_p\Delta T_2} = \frac{p_0V_0}{\frac{3}{2}nR(T_B - T_A) + \frac{5}{2}nR(T_C - T_D)}$$

$$= \frac{p_0V_0}{\frac{3}{2}(2p_0V_0 - p_0V_0) + \frac{5}{4}(4p_0V_0 - 2p_0V_0)}$$

$$= \frac{p_0V_0}{\frac{3}{2}p_0V_0 + \frac{5}{2}\cdot 2p_0V_0} = \frac{1}{6.5} = 15.4\%$$

36. As no heat is lost,

Loss of kinetic energy = Gain of internal energy of gas

$$\frac{1}{2}mv^2 = n\,C_V\Delta T \Rightarrow \frac{1}{2}mv^2 = \frac{m}{M}\cdot\frac{R}{\gamma - 1}\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma - 1)}{2R}\text{ K}$$

37. As work done = 0

$$\Delta U = mC\Delta T = 100 \times 10^{-3} \times 4184 \times (50 - 30) = 8.4\text{ kJ}$$

38. $\dfrac{F}{2}n_1kT_1 + \dfrac{F}{2}n_2kT_2 + \dfrac{F}{2}n_3kT_3$

$$= \frac{F}{2}(n_1 + n_2 + n_3)kT$$

$$\Rightarrow \quad T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

39. $\eta_1 = 1 - \dfrac{T_2}{T_1} \Rightarrow \dfrac{1}{6} = 1 - \dfrac{T_2}{T_1}$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{5}{6} \quad \text{...(i)}$$

$$\eta_2 = 1 - \frac{T_2 - 62}{T_1}$$

$$\Rightarrow \quad \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$T_1 = 372$ K and $T_2 = 310$ K

40. Cubical expansion, we get

$$\Delta V = \gamma V\Delta T = 3\alpha V\Delta T$$

$$= 3 \times 23 \times 10^{-6} \times \left(\frac{4}{3}\pi(10)^3\right) \times 100 \left(r = \frac{d}{2} = 10\right)\text{ cm}$$

$$= 28.9\text{ cc}$$

41. Heat required to change the temperature of vessel by a small amount dT $\quad -dQ = mC_pdT$

Total heat required

$$-Q = m\int_{20}^{4} 32\left(\frac{T}{400}\right)^3 dT = \frac{100 \times 10^{-3} \times 32}{(400)^3}\left[\frac{T^4}{4}\right]_{20}^{4}$$

$\Rightarrow \quad Q = 0.001996$ kJ

Work done required to maintain the temperature of sink to T_2

$$W = Q_1 - Q_2 = \frac{Q_1 - Q_2}{Q_2}Q_2 = \left(\frac{T_1}{T_2} - 1\right)Q_2 \Rightarrow W = \left(\frac{T_1 - T_2}{T_2}\right)Q_2$$

For $T_2 = 20$ K, $W_1 = \dfrac{300 - 20}{20} \times 0.001996 = 0.028$ kJ

For $T_2 = 4$ K, $\quad W_2 = \dfrac{300 - 4}{4} \times 0.001996 = 0.148$ kJ

As temperature is changing from 20 K to 4 K, work done required will be more than W_1 but less than W_2.

42. The efficiency of cycle, $\eta = 1 - \dfrac{T_2}{T_1}$

For adiabatic process, $TV^{\gamma - 1}$ = constant

For diatomic gas $\quad \gamma = \dfrac{7}{5}$

$$T_1V_1^{\gamma - 1} = T_2V_2^{\gamma - 1}$$

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

$$T_1 = T_2(32)^{\frac{7}{5} - 1} = T_2(2^5)^{2/5} = T_2 \times 4$$

$$T_1 = 4T_2$$

$$\therefore \quad \eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$$

43. Thermal energy corresponds to internal energy

Mass = 1 kg and Density = 4 kgm^{-3}

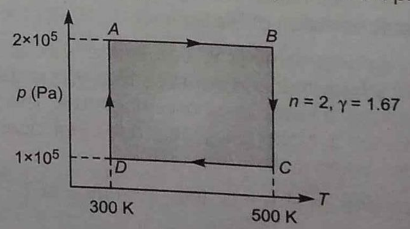
$\Rightarrow$ Volume $= \dfrac{\text{mass}}{\text{density}} = \dfrac{1}{4}\text{ m}^3$

Pressure = 8×10^4 N m^{-2}

$\therefore$ Internal energy $= \dfrac{5}{2}p \times V = 5 \times 10^4$ J

44. Statement I is false but Statement II is true.

45. $W_{AB} = \Delta Q - \Delta U = nC_pdT - nC\,vdt$ (at constant pressure)



$$= n(C_p - C_V)\,dt = nR\,dt$$

$$= 2 \times R \times (500 - 300) = 400\,R$$

46. At constant temperature (isothermal process)

$$W_{DA} = nRT \ln\left(\frac{p_1}{p_2}\right)$$

$$= 2.303 \times 2R \times 300 \log\left(\frac{10^5}{2 \times 10^5}\right)$$

$$= 2.303 \times 600\, R \log\left(\frac{1}{2}\right)$$

$$= -0.693 \times 600R$$

$$= -414\, R$$

So, work done on the gas is $+414\,R$

47. Net work done in a cycle

$$= W_{AB} + W_{BC} + W_{CB} + W_{BA}$$

$$= 400R + 2 \times 2.303 \times 500R\, \ln 2 - 400R - 414R$$

$$= 1000R \times \ln 2 - 600R \times \ln 2$$

$$= 400R \times \ln 2 = 276\, R$$

48. $U = U_1 = U_2$

$$T = \frac{(p_1V_1 + p_2V_2)\, T_1T_2}{(p_1V_1T_2 + p_2V_2T_1)}$$

49. From the first law of thermodynamics,

$$Q = \Delta U + W$$

For path *iaf*,

$$50 = \Delta U + 20$$

$\therefore$ $dU = U_f - U_i = 30$ cal

For path *ibf*,

$$Q = \Delta U + W$$

or $W = Q - \Delta U = 6$ cal

50. For the Carnot engine used as a refrigerator

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

It is given that, $\eta = \frac{1}{10}$

$\Rightarrow$ $\eta = 1 - \frac{T_2}{T_1}$

or $\frac{T_2}{T_1} = \frac{9}{10}$

So, $Q_2 = 90$ J (as $W = 10$ J)

51. For an adiabatic process,

$$dQ = 0$$

So, $dU = -\Delta W$

$\Rightarrow$ $nC_V\, dT = +146 \times 10^3$ J

$\Rightarrow$ $\frac{nf\,R}{2} \times 7 = 146 \times 10^3$

[$f \rightarrow$ degree of freedom]

$\Rightarrow$ $\frac{10^3 \times f \times 8.3 \times 7}{2} = 146 \times 10^3$

or $f = 5.02 = 5$

So, it is a diatomic gas.

52. Energy gained by water (in 1 s)

$= $ energy supplied – energy lost

$= 1000$ J $- 160$ J

$= 840$ J

Total heat required to raise the temperature of water from 27°C to 77°C is $ms\Delta\theta$

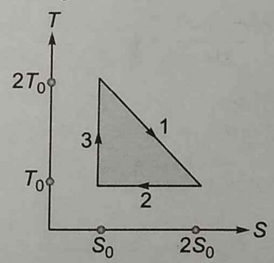
Hence, the required time

$$t = \frac{ms\Delta\theta}{\text{rate by which energy is gained by water}}$$

$$= \frac{(2)(4.2 \times 10^3)(50)}{840}$$

$= 500$ s $= 8$ min 20 s

53. According to the figure



$$Q_1 = T_0S_0 + \frac{1}{2}T_0S_0 = \frac{3}{2}T_0S_0$$

$$Q_2 = T_0(2S_0 - S_0) = T_0S_0$$

$$Q_3 = 0$$

$$\therefore \quad \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$$

54. Internal energy does not change in isothermal process. ΔS can be zero for an adiabatic process. Work done in an adiabatic process may be non-zero.

Day 16

Transfer of Heat

Modes of Heat Transfer

The heat can be transferred within the body and from one body to the other body, through the following modes

(i) Conduction (ii) Convection (iii) Radiation

Conduction

Conduction of heat is that mode of heat transfer in which molecules of the body transfer heat from a place at higher temperature to a place at a lower temperature without actually moving the body. Particles of the body simply oscillate but do not move from their place.

Thermal Conductivity

The amount of heat transmitted through a conductor is given by

$$\Delta Q = \frac{KA\Delta T\Delta t}{l}$$ or in differential use we can write

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

where $\frac{dT}{dx}$ = temperature gradient

where, A = area of cross-section,
ΔT = temperature difference,
Δt = time elapsed and
K = thermal conductivity, l = length,

The rate of transmission of heat by conduction is given by

$$H = \frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{l}$$

The unit of thermal conductivity is $Wm^{-1}K^{-1}$.

We can write $\frac{dQ}{dt} = mc\frac{dT}{dt} = \pm L\frac{dm}{dt}$

Thermal Resistance

$$|H| = \left|\frac{\Delta Q}{\Delta t}\right| = \frac{KA}{l}\cdot\Delta T = \frac{\Delta T}{l/KA}$$

The term $\frac{l}{KA}$ is generally called the **thermal resistance.**

According to the Wiedmann-Franz law, $\frac{K}{\sigma T}$ = constant = L

(called Lorenz number)

In a series combination of two metal rods, thermal conductivity

$$K_s = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}$$

or $$K_s = \frac{2K_1K_2}{K_1 + K_2} \quad (\text{if } l_1 = l_2)$$

If temperature of the interface of the series combination be T, then

$$T = \frac{K_1T_1 + K_2T_2}{K_1 + K_2}$$

In a parallel combination of two metal rods, thermal conductivity

$$K_p = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

or $$K_p = \frac{K_1 + K_2}{2} \quad (\text{if } A_1 = A_2)$$

Formation *and* Growth of Ice on a Lake

Time required for the thickness of the layer of ice to increase from d_1 to d_2 will be $t = \frac{\rho L_f}{2KT}(d_2^2 - d_1^2)$

where, ρ = density of ice,
L_f = latent heat of fusion of ice and
K = thermal conductivity of ice.

Convection

Convection is the mode of heat transfer by actual motion of the matter (or particles of matter). Convection is not possible in solids and can take place in fluids only. THe convection is of two types as given below

Natural Convection

In natural convection gravity plays an important role. When a fluid is heated, the hot part expands and becomes less dense. Consequently it rises and the upper colder part replaces it. This again gets hot, rises up and is replaced by the colder part of the fluid.

Forced Convection

In a forced convection the material is forced to move up by a pump or by some other physical means. Common examples of forced convection are human circulatory system, cooling system of an automobile engine and forced air heating system in offices, etc.

Radiation

In radiation, heat is transferred from one body to other or to the surroundings even in the absence of any medium in the intervening space. Heat energy of the sun is transmitted to earth through radiations.

Heat Transfer through Radiation

Radiation is only a mode of transfer of energy by transverse electromagnetic waves. While studying heat radiations (Radiant energy) we are concerned with thermal radiations which form the infrared region of electromagnetic waves.

All bodies emit heat to the surroundings at all temperatures and at all times. When the temperature of a body remains constant, it emits as much heat to the surroundings as it gains from them. The body is then in a state of dynamic (thermal) equilibrium.

Absorption, Reflection and Transmission

When radiations are incident on a surface, then three things happen—a part of the radiation is absorbed, some is reflected back, and remaining is transmitted.

$$Q_{\text{incident}} = Q_{\text{absorbed}} + Q_{\text{reflected}} + Q_{\text{transmitted}}$$

where, Q represents the energy of thermal radiation.

Absorptivity or absorptive power, $a = \dfrac{Q_{\text{absorbed}}}{Q_{\text{incident}}}$

Reflectivity, $r = \dfrac{Q_{\text{reflected}}}{Q_{\text{incident}}}$ Transmissivity, $t = \dfrac{Q_{\text{transmitted}}}{Q_{\text{incident}}}$

$$\frac{Q_s}{Q} + \frac{Q_v}{Q} + \frac{Q_t}{Q} = a + r + t = 1$$

For a perfect black body, $a = 1, r = t = 0$

For a perfect reflector, $a = t = 0, r = 1$

For a perfect transmitter, $a = r = 0, t = 1$

Some Common Terms and Points

The thermal radiation emitted by a body comprises of all the wavelengths; intensities of radiation corresponding to different wavelengths are different.

Absorptive power (α) It is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same interval of time

$$\alpha = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As a perfectly black body absorbs all radiations incident on it, the absorptive power of a perfectly black body is maximum and unity.

Spectral Absorptive Power (a_λ) The spectral absorptive power is the ratio of radiant energy absorbed by a surface to the radiant energy incident on it for a particular wavelength λ. It may have different values for different wavelengths for a given surface. The spectral absorptive power a_λ is related to absorptive power a through the relation $a = \int_0^\infty a_\lambda d\lambda$

Emissive power (e) For a given surface it is defined as the radiant energy emitted per second per unit area of the surface. It is the total amount of energy radiated by a body per second per unit area of surface

$$e = \frac{1}{A}\frac{\Delta Q}{\Delta t}$$

Spectral emissive power (e_λ) It is emissive power for a particular wavelength λ. Thus, $e = \int_0^\infty e_\lambda d\lambda$

Emissivity (ε) Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body (e) to the total emissive power of a perfect black body (E) at that temperature,

i.e., $\varepsilon = \dfrac{e}{E}$

Heat Transfer through Radiation

Radiation is that mode of heat transfer which needs no medium. Radiant energy travels in the form of electromagnetic waves with a speed of $3 \times 10^8\ \text{ms}^{-1}$ in free space.

When radiant energy Q is incident on a body, a part of it Q_a is absorbed, another part Q_r is reflected back and yet another part Q_t is transmitted such that

$$Q = Q_a + Q_r + Q_t$$

or $$\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1$$

$$a + r + t = 0$$

where,

$a = \dfrac{Q_a}{Q}$ = absorbing power or absorptance,

$r = \dfrac{Q_r}{Q}$ = reflecting power or reflectance

and $t = \dfrac{Q_t}{Q}$ = transmitting power or transmittance.

- *Diathermanous A surface or a medium which transmits most of the radiation ($t = 1$) is called diathermanous, also the substances, which allow heat radiation to pass through them are called diathermanous, e.g., dry air, rock salt etc. where $0 \le \varepsilon \le 1$*
- *Adiathermanous A surface or a medium which does not transmit radiation at all ($t = 0$) is known as an opaque or a adiathermanous medium. Moreover the substances which absorb heat radiation and get themselves heated are called adiathermanous, e.g., water, wood and solid etc. where, $0 \le \varepsilon \le 1$ $\varepsilon = 0$; for completely reflecting surface $\varepsilon = 1$; for black body.*

Perfectly Black Body

A perfectly black body is the one which completely absorbs the radiations of all the wavelengths that are incident on it. Thus, absorbing power of a perfectly black body is 1 (i.e., $a = 1$).

No material body is a perfectly black body. However, lamp black and platinum black are nearly perfectly black bodies.

For scientific work we prepare black bodies by special techniques. Fery's black body and Wien's black body are commonly used in laboratories.

Kirchhoff's Law of Radiation

Kirchhoff's law of radiation states that the ratio of emissive power to absorptive power of a body, is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature. Mathematically,

$$\frac{e_1}{a_1} = \frac{e_2}{a_2} = \ldots = E(\text{Black body})$$

Kirchhoff's law implies that "a good absorber is a good emitter (or radiator) too".

Fraunhoffer's lines (dark lines observed in solar spectrum) can be easily explained on the basis of Kirchhoff's laws.

Stefan's Law

According to the Stefan's law, the emissive power of a perfectly black body (energy emitted by black body per unit surface area per unit time) is directly proportional to the fourth power of its absolute temperature.

Mathematically,

$$E \propto T^4$$

or

$$E = \sigma T^4$$

where σ is a constant known as the **Stefan's constant** and its value is $5.67 \times 10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}$.

For a body, whose emissivity is ε, Stefan's law is modified as $e = \varepsilon\sigma T^4$.

The total radiant energy Q emitted by a body of surface area A in time t, is given by

$$Q = Ate = At\varepsilon\sigma T^4$$

The radiant power (P), *i.e.*, energy radiated by a body per unit time is given by

$$P = \frac{Q}{t} = A\varepsilon\sigma T^4$$

- *Solar constant The amount of heat received from the sun by one square centimetre area of a surface placed normally to the sun rays at a mean distance of earth from sun, is known as solar constant. It is denoted by S.*

$$S = \left(\frac{r}{R}\right)^2 \sigma T^4$$

Here, r is the radius of sun and R the mean distance of earth from the sun.

- *If a body at temperature T is surrounded by another body at temperature T_0 (where, $T_0 < T$), then according to Stefan's law power*

$$P = \varepsilon\sigma A(T^4 - T_0^4)$$

Newton's Law of Cooling

According to the Newton's law of cooling, rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings, provided the temperature difference is small.

Mathematically,

$$\frac{dT}{dt} \propto (T - T_0)$$

or $$-\frac{dT}{dt} = k(T - T_0)$$

where k is a constant.

Newton's law of cooling is a special case of Stefan's law under the condition that the temperature difference is small enough.

If a body cools by radiation through a small temperature difference from T_1 to T_2 in a short time t when the surrounding temperature is T_0, then

$$\frac{dT}{dt} = \frac{T_1 - T_2}{t}$$

$$= k\left[\frac{T_1 + T_2}{2} - T_0\right]$$

In a generalised form, we have

$$\log\left(\frac{T_1 - T_0}{T_2 - T_0}\right) = kt$$

or we can write

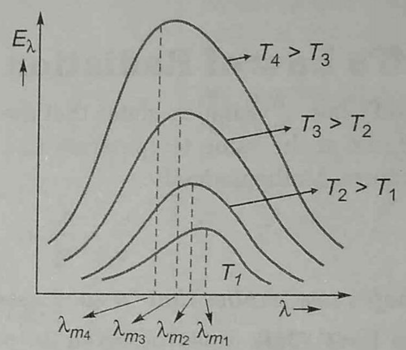
$$T = T_0 + (T_i - T_0)e^{-kt}$$

where, T_i = initial temperature,

T = temperature at time t.

Wein's Displacement Law

The black body spectrum is a continuous spectrum as shown in the figure. At a given temperature, initially the intensity of thermal radiation increases with an increase in wavelength and reaches a maximum value at a particular wavelength λ_m. On increasing the wavelength beyond λ_m, the intensity of radiation E_λ starts decreasing.



The total area under E_λ - λ curve gives the total intensity of radiation at that temperature.

The area, in accordance with the Stefan's law of radiation, is directly proportional to the fourth power of the temperature.

Emissive power $E = \int_0^\infty E_\lambda \, d\lambda$

$= \text{Area under } E_\lambda - \lambda \text{ graph}$

$= \sigma T^4$

From E_λ-λ graph, we find that as the temperature T of a black body increases, the wavelength λ_m corresponding to the maximum emission decreases such that

$$\lambda_m \propto \frac{1}{T}$$

or $$\lambda_m T = \text{constant} = b,$$

where b is known as the Wien's constant and its value is 2.89×10^{-3} mK.

Practice Zone DAY 16

1. A black body maintained at a certain temperature radiates heat energy at the rate Q Watt. If its surface is smoothened so as to lower its emissivity by 10%, what will be the increase in its rate of radiation at double the initial temperature?
   (a) $(0.9\times 2^4 - 1)$ Q Watt
   (b) 0.9×2^4 Q Watt
   (c) $(0.9\times 2)^4$Q Watt
   (d) $(0.9)^4 \times 2$Q Watt

2. A cylindrical rod is having temperatures T_1 and T_2 at its ends. The rate of flow of heat is Q_1. If all the linear dimensions are doubled keeping the temperature constant, then rate of flow of heat Q_2 will be
   (a) $4Q_1$ (b) $2Q_1$ (c) $\frac{Q_1}{4}$ (d) $\frac{Q_1}{2}$

3. Three objects coloured black, grey and white can withstand hostile conditions at 2800°C. These objects are thrown into furnace where each of them attains a temperature of 2000°C. Which object will have the brightest glow?
   (a) The white object
   (b) The black object
   (c) All glow with equal brightness
   (d) Grey object

4. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun.
   (a) $4\pi r_0^2 R^2 \sigma T^4 / r^2$
   (b) $\pi r_0^2 R^2 \sigma T^4 / r^2$
   (c) $r_0^2 R^2 \sigma T^4 / 4\pi r^2$
   (d) $R^2 \sigma T^4 / r^2$

   where r_0 is the radius of earth and σ is the Stefan's constant.

5. Three discs, A, B and C having radii 2 m, 4 m and 6 m respectively, are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C respectively
   (a) Q_A is maximum
   (b) Q_B is maximum
   (c) Q_C is maximum
   (d) $Q_A = Q_B = Q_C$

6. The energy emitted per second by a black body at 27°C is 10 J, the temperature of the black body is increased 327°C, the energy emitted per second will be
   (a) 80 J (b) 160 J (c) 2.15×10^5 J (d) 120 J

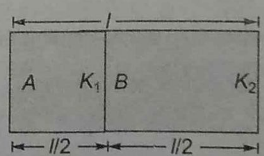
7. A sphere, a cube and a thin circular plate, all of same material and same mass are initially heated to same high temperature. **[NCERT Exemplar]**
   (a) Plate will cool fastest and cube the slowest
   (b) Sphere will cool fastest and cube the slowest
   (c) Plate will cool fastest and sphere the slowest
   (d) Cube will cool fastest and plate the slowest

8. Two black metallic spheres of radius 4 m at 2000 K and 1 m, at 4000 K will have the ratio of energy radiation as
   (a) 1 : 1 (b) 4 : 1 (c) 1 : 4 (d) 2 : 1

9. Temperatures of two stars are in the ratio 3 : 2. If wavelength for the maximum intensity of the first body is 4000 Å, what is the corresponding wavelength of the second body?
   (a) 9000 Å (b) 6000 Å (c) 2000 Å (d) 8000 Å

10. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly **[NCERT Exemplar]**
    (a) Its speed of rotation increases
    (b) Its speed of rotation decreases
    (c) Its speed of rotation remains same
    (d) Its speed increases because its moment of inertia increases

11. Two slabs A and B of different materials but with the same thickness are joined as shown in the figure. The thermal conductivities of A and B are K_1 and K_2 respectively. The thermal conductivity of the composite slab will be



    (a) $\frac{1}{2}(K_1 + K_2)$
    (b) $\sqrt{K_1 K_2}$
    (c) $(K_1 + K_2)$
    (d) $\frac{2K_1K_2}{(K_1 + K_2)}$

12. A metallic sphere cools from 50°C to 40°C in 300 s. If the room temperature is 20°C, then its temperature in the next 5 min will be
    (a) 38°C
    (b) 33.3°C
    (c) 30°C
    (d) 36°C

13. The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. The power radiated by the two black bodies will now increase by a factor of

(a) 64/27 (b) 256/81
(c) 4/3 (d) 16/9

14. The top of a lake is frozen. Air in contact with the surface of the lake is at – 15°C. Then the maximum temperature of the water in contact with the lower surface of ice will be

(a) 0°C
(b) 4°C
(c) – 7.5°C
(d) – 15°C

Directions (Q. Nos. 16 to 18) *Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II i/s false
(d) Statement I is false; Statement II is true

15. Statement I A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. The hollow sphere cools faster.

Statement II Rate of cooling follows the Stefan's law which is $E \propto T^4$.

16. Statement I A body that is a good radiator is also a good absorber of radiation at a given wavelength.

Statement II According to Kirchhoff's law, the absorptivity of a body is equal to its emissivity at a given wavelength.

17. Statement I For higher temperatures, the peak emission wavelength of a black body shifts towards the lower wavelength side.

Statement II Peak emission wavelength of a black body is proportional to the fourth-power of the temperature.

Directions (Q. Nos. 18 to 20) *Consider a spherical body A of radius R which is placed concentrically in a hollow enclosure H, of radius 4R as shown in the figure. The temperature of the bodies A and H are T_A and T_H, respectively.*

Emissivity, transmittivity and reflectivity of the two bodies A and H are (e_A, e_H), (t_A, t_H) and (r_A, r_H), respectively.

For answering the following questions, assume no absorption of the thermal energy by the space in between the body and the enclosure as well as outside the enclosure. All radiations that are emitted and absorbed, are normal to the surface. [Take, $\sigma \times 4\pi R^2 \times 300^4 = \beta\ Js^{-1}$]

18. The temperature of A (a perfect black body) is $T_A = 300$ K and temperature of H is $T_H = 0$ K. For H, take $e_H = 0.5$ and $t_H = 0.5$. For this situation mark out the correct statement(s).

(a) The rate at which A loses the energy is β Js^{-1}
(b) The rate at which the spherical surface containing P receives the energy is $\frac{\beta}{2}$ Js^{-1}
(c) The rate at which spherical surface containing Q receives the energy is β Js^{-1}
(d) All of the above

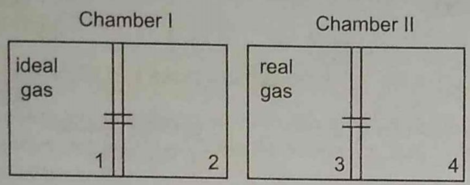
19. In the above question, if body A has $e_A = 0.5$, $r_A = 0.5$ and for H, $e_H = 0.5$, $r_H = 0.5$, then mark out the correct statement.

(a) The rate at which A loses the energy is $\frac{\beta}{2}$
(b) The rate at which the spherical surface containing P receives the energy is zero
(c) The rate at which the spherical surface containing Q receives the energy is β
(d) All of the above

20. Consider two cases, first one in which A is a perfect black body and the second in which A is a non-black body. In both the cases, temperature of body A is 300 K and H is at a temperature of 600 K. For H, $t = 0$ and $a \neq 1$. For this situation, mark out the correct statement.

(a) The bodies lose their distinctiveness inside the enclosure and both of them emit the same radiation as that of the black body
(b) The rate of heat loss by A in both cases is the same and is equal to β Js^{-1}
(c) The rates of heat loss by A in both the cases are different
(d) From this information we can calculate exact rate of heat loss by A in different cases

21.



There are two identical chambers, completely thermally insulated from surrounding. Both chambers have a partition wall dividing the chambers in two compartments. Compartment 1 is filled with an ideal gas and compartment 3 is filled with a real gas. Compartments 2 and 4 are vacuum. A small hole (orifice) is made in the partition walls and the gases are allowed to expand in vacuum.

Statement I No change in the temperature of the gas takes place when ideal gas expands in vacuum. However, the temperature of real gas goes down (cooling) when it expands in vacuum.

Statement II The internal energy of an ideal gas is only kinetic. The internal energy of a real gas is kinetic as well as potential. **[JEE Main Online 2013]**

(a) Statement I is false and Statement II is true
(b) Statement I and Statement II both are true. Statement II is the correct explanation of Statement I
(c) Statement I is true and Statement II is false
(d) Statement I and Statement II both are true, but Statement II is not the correct explanation of Statement I

22. 500 g of water and 100 g of ice at 0°C are in a calorimeter whose water equivalent is 40 g. 10 g of steam at 100°C is added to it. Then water in the calorimeter is
(Latent heat of ice = 80 cal/g, Latent heat of steam = 540 cal/g) **[JEE Main Online 2013]**

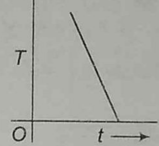
(a) 580 g
(b) 590 g
(c) 600 g
(d) 610 g

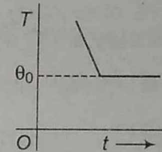
23. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30°C to 25°C.

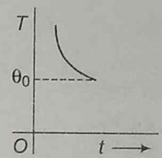
A mass of 100g of another liquid in an identical vessel with identical surroundings takes the same time to cool from 30°C to 25°C. The specific heat of the liquid is (The water equivalent of the vessel is 30 g.) **[JEE Main Online 2013]**

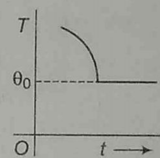
(a) 2.0 kcal/kg
(b) 7 kcal/kg
(c) 3 kcal/kg
(d) 0.5 kcal/kg

24. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time will be closed to **[JEE Main 2013]**

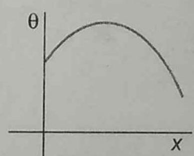
(a)


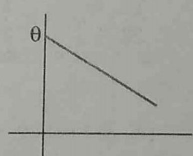
(b)


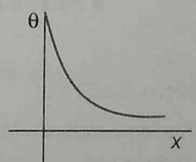
(c)


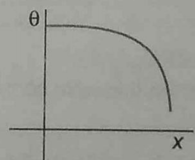
(d)


25. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figure? **[AIEEE 2009]**

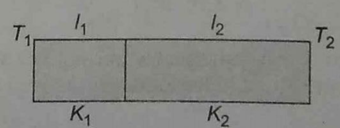
(a)


(b)


(c)


(d)


26. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is **[AIEEE 2007]**

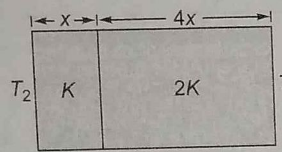


(a) $(K_2l_2T_1 + K_1l_1T_2)/(K_1l_1 + K_2l_2)$
(b) $(K_2l_1T_1 + K_1l_2T_2)/(K_2l_1 + K_1l_2)$
(c) $(K_1l_2T_1 + K_2l_1T_2)/(K_1l_2 + K_2l_1)$
(d) $(K_1l_1T_1 + K_2l_2T_2)/(K_1l_1 + K_2l_2)$

27. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously, will be **[AIEEE 2004]**

(a) 4 (b) 16 (c) 32 (d) 64

28. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equals to **[AIEEE 2004]**



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3

29. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings and n is equal to **[AIEEE 2003]**

(a) 2 (b) 3 (c) 4 (d) 1

30. Which of the following is more close to a black body? **[AIEEE 2002]**

(a) Black board paint (b) Green leaves
(c) Black holes (d) Red roses

31. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by $t°C$, the linear stress developed in it is **[AIEEE 2011]**

(a) $\frac{\alpha t}{Y}$ (b) $\frac{Y}{\alpha t}$
(c) $Y\alpha t$ (d) $\frac{1}{Y\alpha t}$

Answers

1. (a)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (b)
11. (b)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (c)
21. (c)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (c)	**28.** (d)	**29.** (d)	**30.** (a)
31. (c)									

Hints & Solutions

1. For black body,

Rate of radiation $Q = \sigma T^4$

After smoothing and doubling the temperature = Rate Q

$$= 0.9\, \sigma\, (2T)^4 = 0.9 \times 10^4\, Q$$

Charge $= (0.9 \times 2^4 - 1)\, Q$ Watt

2. Initially, $Q_1 = \frac{KA_1(T_1 - T_2)}{l_1}$ but on doubling all dimensions $l_2 = 2l_1$ and $A_2 = 4A_1$.

Hence, $Q_2 = \frac{K A_2(T_1 - T_2)}{l_2}$

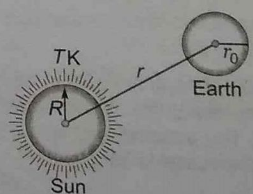
$$= \frac{K 4 A_1(T_1 - T_2)}{2l_1}$$

$$= 2\frac{K A_1(T_1 - T_2)}{l_1} = 2Q_1$$

3. An ideal black body absorbs all the radiations incident upon it and has an emissivity equal to 1. If a black body and an identical body are kept at the same temperature, then the black body will radiate the maximum power.

Hence, the black object at a temperature of 2000°C will have the brightest glow.

4. From Stefan's law, the rate at which energy is radiated by sun at its surface is



$$P = \sigma \times 4\pi R^2 \times T^4$$

[Sun is a perfectly black body as it emits radiations of all wavelengths and so for it, e = 1.]

The intensity of this power at the surface of the earth [under the assumption $r >> r_0$] is

$$I = \frac{P}{4\pi r^2} = \frac{\sigma \times 4\pi R^2 T^4}{4\pi r^2} = \frac{\sigma R^2 T^4}{r^2}$$

The area of the earth which receives this energy is only one-half of the total surface area of earth, whose projection would be πr_0^2.

∴ Total radiant power as received by the earth

$$= \pi r_0^2 \times I$$

$$= \frac{\pi r_0^2 \times \sigma R^2 T^4}{r^2}$$

$$= \frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$$

5. $Q \propto AT^4$ and $\lambda_m T$ = constant

Hence, $Q \propto \frac{A}{(\lambda_m)^4}$ or $Q \propto \frac{r^2}{(\lambda_m)^4}$

$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4} = \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

$$= 0.05 : 0.0625 : 0.0576$$

i.e., Q_B is maximum.

6. For a black body, rate at which energy is emitted by it is given by

$P = \sigma A T^4$, where T is in kelvin.

$$P_i = 10 \text{ J} = \sigma A \times (300)^4$$

$$P_f = \sigma A (600)^4 = 16 \times P_i = 160 \text{ J}$$

7. We know that the rate of loss of heat from a body is directly proportional to the surface area of the body. For a given mass of a material, the surface area of a circular plate is maximum and of sphere is least. Hence, plate will cool fastest and sphere the slowest.

8. $\frac{P_1}{P_2} = \frac{A_1}{A_2} \frac{T_1^4}{T_2^4} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{4}{1}\right)^2 \times \left(\frac{2000}{4000}\right)^4 = \frac{1}{1}$

9. According to Wien's displacement law,

$$\lambda_m T = \text{constant}$$

$$\therefore \quad \frac{(\lambda_m)_1}{(\lambda_m)_2} = \frac{T_2}{T_1}$$

Here, $\frac{T_1}{T_2} = \frac{3}{2}, (\lambda_m)_1 = 4000 \text{ Å} = 4000 \times 10^{-10}$ m

$$\therefore \quad (\lambda_m)_2 = \frac{4000 \times 10^{-10} \times 3}{2}$$

$$= 6000 \text{ Å}$$

10. When a metallic rod is heated it expands. Its moment of inertia (I) about a perpendicular bisector increases. According to law of conservation of angular momentum, its angular speed (ω) decreases, since $\omega \propto 1/I$. (According to law of conservation of angular momentum).

11. The thermal resistance of a slab of length l_1, area of cross section A and thermal conductivity K is given by

$$R = \frac{l}{KA}$$

For slab A $\quad R_1 = \frac{l/2}{K_1 A}$

For slab B $\quad R_2 = \frac{l/2}{K_2 A}$

Since, the slabs are joined in series, the thermal resistance of the composite slab is

$$R_C = R_1 + R_2$$

or $\quad \frac{l}{K_C A} = \frac{l/2}{K_1 A} + \frac{l/2}{K_2 A}$

or $\quad \frac{1}{K_C} = \frac{1}{2}\left[\frac{1}{K_1} + \frac{1}{K_2}\right]$

or $\quad K_C = \frac{2K_1 K_2}{(K_1 + K_2)}$

12. According to the Newton's law of cooling,

$$\frac{50 - 40}{300} = K\left[\frac{50 + 40}{2} - 20\right]$$

$$\Rightarrow \quad \frac{10}{300} = K\left[\frac{90}{2} - 20\right] = K \times 25$$

$$\Rightarrow \quad K = \frac{10}{300 \times 25} = \frac{1}{30 \times 25}$$

Similarly, $\quad \frac{40 - \theta}{300} = K\left[\frac{40 + \theta}{2} - 20\right]$

$$= K\left[20 + \frac{\theta}{2} - 20\right] = \frac{K\theta}{2}$$

$$= \frac{\theta}{2 \times 30 \times 25} = \frac{\theta}{1500}$$

$$\Rightarrow \quad 300\,\theta = 1500(40 - \theta) = 60000 - 1500\,\theta$$

$$\Rightarrow \quad 1800\,\theta = 60000$$

$$\Rightarrow \quad \theta = \frac{60000}{1800} = 33.3°\text{C}$$

13. We know that $\lambda_m T$ = constant and the power radiated by a black body is proportional to T^4 *i.e.*, $P \propto T^4$, Hence

$$P \propto (\lambda_m)^{-4}$$

$$\Rightarrow \quad \frac{P_2}{P_1} = \left(\frac{\lambda_{m_1}}{\lambda_{m_2}}\right)^4 = \left(\frac{\lambda_0}{3\lambda_0 4}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$

14. Water and ice can be in equilibrium at freezing temperature.

15. As external radii of both the spheres are equal, the surface areas of the two are also equal. Therefore, when the two spheres are heated to the same temperature, both radiate heat at the same rate.

Now, rate of loss of heat from a sphere $= Mc\dfrac{d\theta}{dt}$

Therefore, rate of cooling

$$\frac{d\theta}{dt} = \frac{\text{rate of loss of heat}}{Mc}$$

or $$\frac{d\theta}{dt} \propto \frac{1}{M}$$

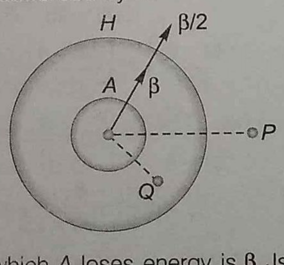
Since, mass of a hollow sphere is less, its rate of cooling will be fast.

16. According to the Kirchhoff's law, at a given wavelength, the absorptivity of a body is equal to its emissivity. Also a body which is a good radiator, is also a good absorber of radiation or a poor reflector.

17. As the temperature of the black body increases, two distinct behaviours are observed. The first effect is that the peak of the distribution shifts towards the shorter wavelength side. This shift is found to obey the following relationship called the Wien's displacement law, which is given by $\lambda_m T =$ constant.

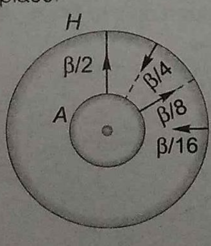
The second effect is that the total amount of energy, the black body emits per unit area per unit time increases with fourth power of the absolute temperature T.

18. The diagram shows the situation clearly. The rate at which energy is emitted by A is β Js^{-1}, while crossing the enclosure the rate at which the energy is transmitted out is $\dfrac{\beta}{2}$, while remaining is absorbed by H.



So, rate at which A loses energy is β Js^{-1} and the rate at which P and Q receive energy are $\beta/2$ Js^{-1} and β Js^{-1}, respectively. This energy is received on the area of sphere passing through P and Q.

19. Now, in this case, each of incidence, reflection and absorption take place.



The rate at which energy has been lost by A is,

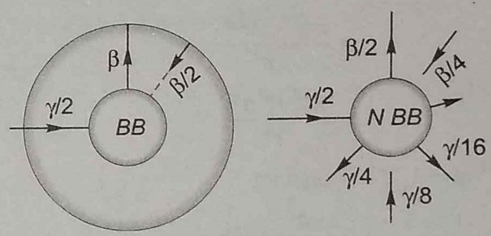
$$P = -[P_{\text{absorbed}} - P_{\text{emitted}}]$$
$$= -\left[\frac{\beta}{8} + \frac{\beta}{32} + \dots\right] + \left[\frac{\beta}{2} + \frac{\beta}{8} + \frac{\beta}{32} + \dots\right] = \frac{\beta}{2}$$

The rate at which energy is received by P is, $P_1 = 0$

The rate at which energy is received by Q is,

$$P_2 = \left(\frac{\beta}{2} + \frac{\beta}{8} + \dots\right) + \left(\frac{\beta}{4} + \frac{\beta}{16} + \dots\right)$$
$$= \frac{\beta}{2} \times \frac{4}{3} + \frac{\beta}{4} \times \frac{4}{3} = \beta$$

20.



If $\quad \beta = \sigma \times 4\pi R^2 \times (300)^4$,

then $\quad \sigma \times 4\pi(4R)^2 \times (600)^4 = 256\beta = \gamma$

Let $\quad a_H = e_H = 0.5$

and for A in 2nd case, $e_A = a_A = 0.5$

For 1st case, $\quad P_{\text{emitted}} = \beta$ Js^{-1}

$$P_{\text{absorbed}} = \frac{\gamma}{2} + \frac{\beta}{2}$$

Rate at which energy is lost, $P = \left(\beta - \dfrac{\gamma}{2} - \dfrac{\beta}{2}\right)$ Js^{-1}

For 2nd case,

$$P_{\text{emitted}} = \left(\frac{\beta}{2} + \frac{\beta}{8} + \frac{\beta}{32} + \dots\right) + \left(\frac{\gamma}{4} + \frac{\gamma}{16} + \dots\right) = \frac{2\beta}{3} + \frac{\gamma}{3}$$

$$P_{\text{absorbed}} = \left(\frac{\beta}{8} + \frac{\beta}{32} + \dots\right) + \left(\frac{\gamma}{4} + \frac{\gamma}{16} + \dots\right) = \frac{\beta}{6} + \frac{\gamma}{3}$$

Rate at which heat is lost, $P = \dfrac{\beta}{2}$

21. Intermolecular distance in ideal gases is assume to be large as compared to real one. Hence, the internal energy of an ideal gas and a real gas is only kinetic as well as potential. According to Newton's cooling law, option (c) is correct answer.

22. As latent heat of steam goes to melt the ice

$$540 \times 10 = m \times 80$$

$\therefore$ $$m = \frac{540}{8} = 67.5 \approx 68\text{ g}$$

Now amount of water $= 500 + 68 + 10$
$= 578$ g

23. As, $\Delta Q = ms\Delta\theta$ (for water)

$= 50 \times s \times 5$

$\Rightarrow \left(\frac{dQ}{dt}\right)_s$ = rate of cooling

$= \frac{250}{2 \times 60} = \frac{25}{26}$ $(\because S_w = 1 \text{ cal/g})$

Now other liquid $\left(\frac{dQ}{dt}\right)_l$ = rate of cooling

$= \frac{100 \times s \times 5}{2 \times 60} = \frac{50}{26} s$

Now, $\left(\frac{dQ}{dt}\right)_l = \left(\frac{dQ}{dt}\right)_s$

$\Rightarrow s = 0.5 \text{ cal/g} = 0.5 \text{ kcal/kg}$

24. According to Newton's cooling law, option (c) is correct answer.

25. We know that $\frac{dQ}{dt} = kA\frac{d\theta}{dx}$

In steady state flow of heat

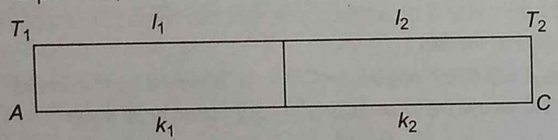
$$d\theta = \frac{dQ}{dt} \cdot \frac{1}{kA} dx$$

$\Rightarrow \theta_H - \theta = k'x \Rightarrow \theta = \theta_H - k'x$

Equation $\theta = \theta_H - k'x$ represents a straight line.

26. Let temperature at the Interface be T.

For part AB,



$$\frac{Q_1}{t} \propto \frac{(T_1 - T)k_1}{l_1}$$

For part BC, $\frac{Q_2}{t} \propto \frac{(T - T_2)}{l_2} \times k_2$

At equilibrium, $\frac{Q_1}{t} = \frac{Q_2}{t}$

$\Rightarrow \frac{(T_1 - T)k_1}{l_1} = \frac{(T - T_2)k_2}{l_2}$

$\Rightarrow T = \frac{T_1k_1l_2 + T_2k_2l_1}{k_1l_2 + k_2l_1}$

27. From Stefan's law, the energy radiated by the sun is given by $P = \sigma e AT^4$

In 1st case, $P_1 = \sigma e \times 4\pi R^2 \times T^4$

In 2nd case, $P_2 = \sigma e \times 4\pi(2R)^2 \times (2T)^4$

$= \sigma e \times 4\pi R^2 \times T^4 \times 64 = 64P_1$

The rate at which energy is received at the earth is

$$E = \frac{P}{4\pi R_{SE}^2} \times A_E$$

where, A_E = area of earth

R_{SE} = distance between the sun and earth

$\therefore$ In 1st case,

$$E_1 = \frac{P_1}{4\pi R_{SE}^2} \times A_E$$

$$E_2 = \frac{P_2}{4\pi R_{SE}^2} \times A_E = 64E_1$$

28. Let the temperature of common interface be $T°C$.

Rate of heat flow,

$$H = \frac{Q}{t} = \frac{KA\Delta T}{l}$$

$\therefore$ $H_1 = \left[\frac{Q}{t}\right]_1 = \frac{2KA(T - T_1)}{4x}$

and $H_2 = \left(\frac{Q}{t}\right)_2 = \frac{KA(T_2 - T)}{x}$

In steady state, the rate of heat flow should be the same in the whole system *i.e.*,

$$H_1 = H_2$$

$\Rightarrow \frac{2KA(T - T_1)}{4x} = \frac{KA(T_2 - T)}{x}$

or $\frac{T - T_1}{2} = T_2 - T$

or $T - T_1 = 2T_2 - 2T$

or $T = \frac{2T_2 + T_1}{3}$ ...(i)

Hence, heat flow from the composite slab is

$H = \frac{KA(T_2 - T)}{x}$

$= \frac{KA}{x}\left(T_2 - \frac{2T_2 + T_1}{3}\right) = \frac{KA}{3x}(T_2 - T_1)$ ...(ii)

Accordingly, $H = \left[\frac{A(T_2 - T_1)K}{x}\right]f$ ...(iii)

By comparing Eqs. (ii) and (iii), we get

$$f = \frac{1}{3}$$

29. According to Newton's law of cooling,

$$\frac{dQ}{dt} \propto \Delta\theta$$

But $\frac{dQ}{dt} \propto (\Delta\theta)^n$ (given)

$\therefore$ $n = 1$

Black board pain is more close to a black body.

30. $\Delta L = \alpha L \Delta T = \frac{FL}{AY}$

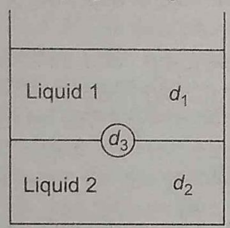
$\Rightarrow \text{Stress} = \frac{F}{A} = Y\alpha T$

Unit Test 3

DAY 17

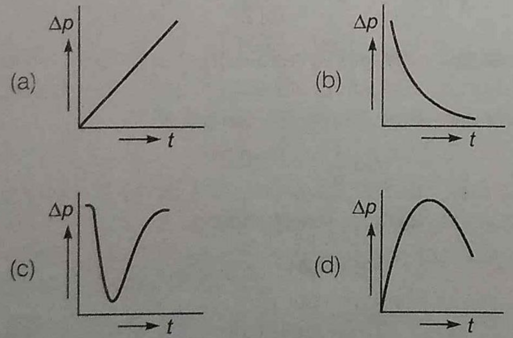
(General Properties of Matter)

1. A jar is filled with two non-mixing liquids 1 and 2 having densities d_1 and d_2 respectively. A solid ball, made of a material of density d_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for d_1, d_2 and d_3?



(a) $d_1 > d_3 > d_2$
(b) $d_1 < d_2 < d_3$
(c) $d_1 < d_3 < d_2$
(d) $d_3 < d_1 < d_2$

2. A soap bubble is very slowly blown on the end of a glass tube by a mechanical pump which supplies a fixed volume of air every minute whatever be the pressure against which it is pumping. The excess pressure Δp inside the bubble varies with time as shown by which of the graph?



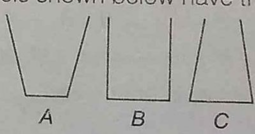
3. A diver is hunting for a fish with a water gun. He accidentally fires the gun so that bullet punctures the side of the ship. The hole is located at a depth of 10 m below the water surface. The speed with which water enter is the ship is

(a) 18 ms^{-1}
(b) 14 ms^{-1}
(c) 25 ms^{-1}
(d) Cannot be determined

4. A swimmer of mass m rests on top of a styrofoam slab, which has a thickness h and density ρ_S. The area of the slab if it floats in water with its upper surface just awash is [Take density of water to be ρ_w]

(a) $\dfrac{m}{h(\rho_s + \rho_w)}$
(b) $\dfrac{m}{h\rho_w}$
(c) $\dfrac{m}{h(\rho_s - \rho_w)}$
(d) $\dfrac{m}{h(\rho_w - \rho_s)}$

5. The three vessels shown below have the same base areas



Equal volume of water is poured into three, the force on the base of vessel

(a) A would be maximum
(b) B would be maximum
(c) C would be maximum
(d) equal in all three

6. Determine the energy stored in the surface of a soap bubble of radius 2.1 cm if its surface tension is 4.5×10^{-2} Nm^{-1}?

(a) 8 mJ
(b) 2.46 mJ
(c) 4.93×10^{-4} J
(d) None of these

7. An aeroplane has a mass of 1.60×10^4 kg and each wing has an area of 40 m^2. During level flight, the pressure on the wings's lower surface is 7×10^4 Pa. The pressure on the upper surface of the wing is

(Take $p_0 = 10^5$ Pa and assume the pressure difference is only on wings and not on body)

(a) 10^5 Pa
(b) 6.8×10^4 Pa
(c) 7×10^4 Pa
(d) 6.6×10^4 Pa

8. A steel wire 10 m, long and 10^{-5} m^2 in cross-sectional area elongates by 0.01 m under a tension of 2500 N. Young's modulus for steel from this data is computed as

(a) 2.5×10^7 Nm^{-2}
(b) 2.5×10^9 Nm^{-2}
(c) 2.5×10^{11} Nm^{-2}
(d) None of these

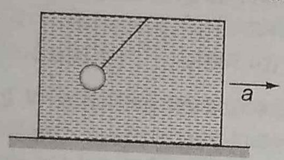
9. A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal sizes. The energy spent in this process is [Surface tension of mercury is equal to $32 \times 10^{-2}\ \text{Nm}^{-1}$]

(a) 3.98×10^{-4} J (b) 8.46×10^{-4} J
(c) 3.98×10^{-2} J (d) 8.46×10^{-2} J

10. The property of metals which allows them to be drawn readily into thin wires beyond the elastic limit without rupturing, is known as

(a) malleability (b) ductility (c) elasticity (d) hardness

11. A spherical body of volume V and density σ is suspended from a string, the other end of the string is connected to the roof of a sealed container filled with an ideal fluid of density ρ.



If the container accelerates towards right with a constant acceleration a, then the force exerted by the liquid on the body when it is in equilibrium w.r.t. fluid, is

(a) $V\rho\sqrt{a^2 + g^2} + V\sigma a$ (b) $V\sigma a$
(c) $\sqrt{[V\rho(g + a)]^2 + [V\sigma a]^2}$ (d) $V\rho\sqrt{g^2 + a^2}$

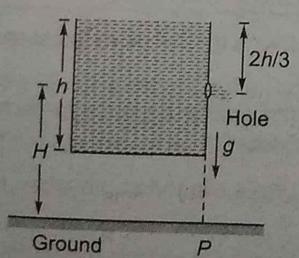
12. If a substance breaks down by a stress of $10^6\ \text{Nm}^{-2}$. If the density of the material of the wire is $3 \times 10^3\ \text{kgm}^{-2}$, then the length of the wire of the substance which will break under its own weight when suspended vertically is

(a) 66.6 m (b) 60.0 m
(c) 33.3 m (d) 30.0 m

13. A solid sphere of radius R, made up of a material of bulk modulus K, is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, the fractional change in the radius of the sphere is

(a) $\frac{Mg}{AK}$ (b) $\frac{Mg}{3AK}$ (c) $\frac{3Mg}{AK}$ (d) $\frac{Mg}{2\ AK}$

14. An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel, as shown in the figure.



The water which comes out from the hole at the instant when the hole is at height H above the ground, strikes the ground at a distance x from P. Which of the following is correct for the situation described?

(a) The value of x is $2\sqrt{\frac{2hH}{3}}$
(b) The value of x is $\sqrt{\frac{4hH}{3}}$
(c) The value of x cannot be computed from the information provided
(d) The question is irrelevant as no water comes out from the hole

15. An incompressible fluid flows through a horizontal pipe. At one point in the pipe the pressure in the fluid is p_1. At a point further away in the direction of flow, the pressure is $p_2 > p_1$. If A_1 and A_2 be the respective cross-section area, then

(a) $A_1 > A_2$ (b) $A_1 < A_2$
(c) $A_1 = A_2$ (d) Nothing can be said

16. A plate of face area 5 cm^2 and thickness 0.5 cm is fixed rigidly at the lower surface. A tangential force of 100 N is applied at the upper surface. The lateral displacement of the upper surface w.r.t. lower surface will be $[\eta = 5 \times 10^{10}\ \text{Nm}^{-2}]$

(a) 2×10^{-8} m (b) 2 μm
(c) 8×10^{-7} m (d) 10 μm

17. A wire of length L and radius r is fixed at one end. When a stretching force F is applied at the free end, the elongation in the wire is l. When another wire of the same material but of length $2L$ and radius $2r$, also fixed at one end is stretched by a force $2F$ applied at the free end, then elongation in the 2nd wire will be

(a) $l/2$ (b) l (c) $2l$ (d) $l/4$

18. A material has a Poisson's ratio 0.50. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(a) 0.6 (b) 0.4 (c) 0.2 (d) zero

19. On applying a stress of $x\ \text{Nm}^{-2}$, the length of wire of some material becomes double. Value of the Young's modulus for the material of the wire in Nm^{-2}, is [Assume Hooke's law to be valid] "Go for approx results"

(a) x (b) $2x$
(c) $x/2$ (d) Insufficient information

20. A slab consists of two parallel layers of copper and brass of the same thickness same area of cross-section and having thermal conductivities in the ratio 1 : 4. If the free face of brass is at 100°C and that of copper is at 0°C, the temperature of the interface is

(a) 80°C (b) 20°C
(c) 60°C (d) 40°C

21. A steel rod is 3.00 cm in diameter at 25°C. A brass ring has an interior diameter of 2.992 cm at 25°C. At what common temperature will the ring just slide onto the rod? $(\alpha_s = 11\times 10^{-6}\ °C^{-1}, \alpha_b = 19\times 10^{-6}\ °C^{-1})$

(a) 460°C (b) 260°C (c) 500°C (d) 360°C

22. Four moles of an ideal gas undergo a reversible isothermal expansion from volume V_1 to volume $V_2 = 2V_1$ at temperature $T = 400$ K. Find the entropy change of the gas.

(a) 8.22×10^3 J K^{-1} (b) 8.22×10^2 J K^{-1}
(c) 23.1 J K^{-1} (d) 10.00×10^3 J K^{-1}

23. Pressure p, volume V, and temperature T for a certain material are related by

$$p = \frac{AT - BT^2}{V}$$

where A and B are constants. Find an expression for the work done by the material if the temperature changes from T_1 to T_2 reduce while the pressure remains constant.

(a) $W = A(T_2 - T_1) - B(T_2^3 - T_1^3)$ (b) $W = A(T_2^2 - T_1^2) - B(T_2 - T_1)$
(c) $W = A(T_2 - T_1) - B\left(T_2 - \frac{1}{2}T_1\right)$ (d) $W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$

24. Oxygen gas having a volume of 1000 cm^3 at 40.0°C and 1.01×10^5 Pa expands until its volume is 1500 cm^3 and its pressure is 1.06×10^5Pa. Find the final temperature of the sample.

(a) 220°C (b) 220 K (c) 300°C (d) 300 K

25. A small electric immersion heater is used to heat 100 g of water for a cup of instant coffee. The heater is labelled "200 W," which means that it converts electrical energy to thermal energy at this rate. Calculate the time required to bring all this water from 23°C to 100°C, ignoring any heat losses. [c = 4190 J kg^{-1} K^{-1}]

(a) 100 s (b) 200 s (c) 190 s (d) 160 s

26. A chef, on finding his stove out of order, decides to boil the water for his wife's coffee by shaking it in a thermos flask. Suppose that he uses tap water at 15°C and that the water falls 30 cm in each shake, the chef making 30 shakes each minute. Neglecting any loss of thermal energy by the flask, how long must he shake the flask until the water reaches 100°C ?

(a) 2.25×10^3min (b) 3.97×10^3min
(c) 4.03×10^3min (d) 5.25×10^3min

27. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K, $2K$ and thickness x, $4x$, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab in a steady state is $\left[\frac{A(T_2 - T_1)K}{x}\right]f$ with f which is equal to

(a) 1 (b) 1/2 (c) 2/3 (d) 1/3

28. Compute the number of moles and in 1.00 cm^3 of an ideal gas at a pressure of 100 Pa and at a temperature of 220 K.

(a) 3.35×10^{-8}mol (b) 4.57×10^{-7}mol
(c) 5.47×10^{-8}mol (d) 2.75×10^{-8}mol

29. The temperature of the source of a Carnot's heat engine is 1000°C. Its efficiency could be 100% only if the temperature of the sink is

(a) 1000°C (b) 0°C
(c) equal to triple of water (d) − 273.16°C

30. 743 J of heat energy is added to raise the temperature of 5 mole of an ideal gas by 2 K at constant pressure. How much heat energy is required to raise the temperature of the same mass of the gas by 2K at constant volume? [Take R = 8.3 J/Kmol]

(a) 826 J (b) 743 J (c) 660 J (d) 620 J

31. A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by 75.0°C. What are the temperatures of the two reservoirs?

(a) 58°C, 10°C (b) 78°C, −5°C
(c) 68°C, −7°C (d) 50°C, 0°C

32. A hot metallic sphere of radius r radiates heat. Its rate of cooling is

(a) independent of r (b) proportional to r
(c) proportional to r^2 (d) proportional to $1/r$

Directions *(Q. Nos. 33 to 41) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

33. Statement I A ship floats higher in water on a high pressure day than on a low pressure day.

Statement II Floating of ship in the water is possible because of the buoyant force which is present due to the pressure difference.

34. Statement I More is the cohesive force, more is the surface tension.

Statement II More cohesive force leads to more shrinking of the liquid surface.

35. Statement I Water expands both when heated or cooled from 4°C.

Statement II Density of water is minimum at 4°C.

36. **Statement I** If the temperature of a star is doubled, then the rate of loss of heat from it becomes 16 times.

Statement II Specific heat varies with temperature.

37. **Statement I** As the temperature increases, the coefficient of viscosity of a liquid decreases.

Statement II Viscosity in liquids originate with the intermolecular cohesive forces. As the temperature of the liquid increases, due to an increase in kinetic energy of molecules, intermolecular force decreases.

38. **Statement I** While blowing a soap bubble, to increase the size of soap bubble we have to increase the air pressure within the soap bubble.

Statement II To increase the size of the soap bubble, more air has to be pushed into the bubble.

39. **Statement I** Coefficient of absorption of radiation of an ideal black body is 1.

Statement II An ideal black body emits radiation of all wavelengths.

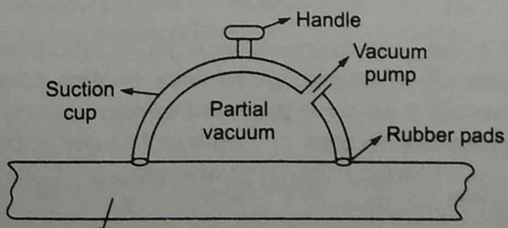
40. **Statement I** A solid and a hollow sphere of same diameter and of the same material when heated through the same temperature, expand by the same amount.

Statement II The change in volume is independent of the original mass but depends on the original volume.

41. **Statement I** When a bottle of cold carbonated drink is opened, a slight fog forms around the opening.

Statement II Adiabatic expansion of the gas causes lowering of temperature and condensation of water vapours.

Directions *(Q. Nos. 42 and 43) Figure shows an arrangement in which a piece of glass being lifted up by a suction cup. The suction cup is kept inverted on the glass slab and the air is removed from suction cup using vacuum pump and a partial vacuum is created inside the cup.*



The area of the glass covered by the cup is 0.025 m^2. The pressure inside the cup is reduced to 0.5×10^5 Nm^{-2}. [Take atmospheric pressure as 10^5 Nm^{-2}]

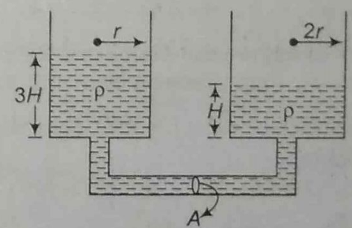
42. The suction cup is able to support the weight of glass slab because

(a) rubber pads are exerting an upward force on the slab
(b) of the pressure difference on the two sides of the slab area corresponding to suction cup area
(c) of the force exerted by the suction cup on the slab
(d) None of the above

43. The weight of the glass slab which can be supported by the given data is

(a) 1000 N
(b) 2500 N
(c) 1100 N
(d) Both (a) and (c)

Directions *(Q. Nos 44 to 46) Two cylindrical tanks of radii r and $2r$ with their bases at the same level contain a liquid of density ρ to heights H and $3H$, respectively as shown in figure. The tanks are linked through a pipe of very small cross-sectional area A. Due to pressure difference liquid starts flowing from narrower vessel to broader vessel to equalize the pressure. Based on above information, answer the following questions.*



44. The final common level of liquid in both vessels is

(a) $2H$ (b) $\frac{7H}{5}$
(c) $\frac{3H}{2}$ (d) $\frac{5H}{2}$

45. The time taken for the liquid levels to become equal from initial level is

(a) $\sqrt{\frac{5H\times\pi r^2}{gA}}$ (b) $\frac{5\pi r^2}{A}\sqrt{\frac{H}{g}}$
(c) $\frac{\pi r^2}{A}\times\sqrt{\frac{H}{g}}$ (d) $\frac{8\pi r^2}{5A}\times\sqrt{\frac{H}{g}}$

46. The work done by gravity in equalising these levels is

(a) $\pi r^2\times\rho g H^2$ (b) $\frac{41}{10}\times\pi r^2\rho g H^2$
(c) $\frac{8}{5}\times\pi r^2\times\rho g H^2$ (d) $\frac{15}{2}\times\pi r^2\rho g H^2$

Hints & Solutions

1. (c) d_3 floats in d_2 and sinks in $d_1 \Rightarrow d_1 < d_3 < d_2$

2. (b) $\Delta p = \frac{2T}{R}$

As the number of moles of air increases. Radius increases and Δp decreases.

3. (b) Applying the Bernoulli's theorem for any two convenient points, let us say we are applying just at the water surface and just inside the hole.

$$p_0 + 0 + 0 = p_0 + \frac{\rho v^2}{2} + \rho g(-h)$$

where v is required speed and water surface is taken as the reference level.

$\Rightarrow$ $v = \sqrt{2gh} = 14\,\text{ms}^{-1}$

4. (d) From equilibrium, $mg + Ah\rho_s \times g = Ah\rho_w \times g$

where A is the required cross-sectional area

$\Rightarrow$ $A = \frac{m}{h(\rho_w - \rho_s)}$

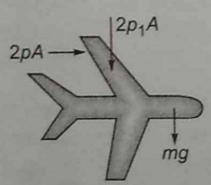
5. (c) Height of liquid in vessel A is minimum and maximum in C. So, from $p = p_0 + \rho gh$, force on base of C is maximum.

6. (c) Surface energy $U = S \times 2 \times 4\pi R^2$ [As there are 2 surfaces in soap bubble]

$$U = 4.5 \times 10^{-2} \times 8\pi \times (2.1 \times 10^{-2})^2 = 4.93 \times 10^{-4}\,\text{J}$$

7. (b) Let p_1 be the pressure on the upper wing surface, then for the vertical equilibrium of the plane

$$2p_1A + mg = 2pA$$



$\Rightarrow$ $p_1 = p - \frac{mg}{2A} = 7 \times 10^4 - \frac{1.6 \times 10^4 \times 10}{2 \times 40} = 6.8 \times 10^4$ Pa

8. (c) $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\Delta l/l} = \frac{2500 \times 10}{10^{-5} \times 0.01}$

$= 2.5 \times 10^{11}\,\text{Nm}^{-2}$

9. (c) Let r be the radius of one droplet.

Now, $\frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3$

$$r = \frac{R}{100} = \frac{1}{100}\text{cm} = 10^{-4}\text{m}$$

$A_i = 4\pi R^2$

$A_f = 10^6 \times 4\pi r^2$

Change in area,

$\Delta A = A_f - A_i = 4\pi \times 99 \times 10^{-4}\,\text{m}^2$

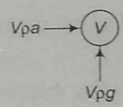
Increase in surface energy

$= S\Delta A = 32 \times 10^{-2} \times 4\pi \times 99 \times 10^{-4}\,\text{J} = 3.98 \times 10^{-2}\,\text{J}$

The increase in surface energy is at the expense of internal energy, so energy spent $= 3.98 \times 10^{-2}$ J

10. (b) The metal used beyond elastic limit without being ruptured are called ductile materials and this property is called ductility.

11. (d) The forces exerted by liquid on body is shown in **figure**, when body is in equilibrium w.r.t. fluid.



Forces exerted by liquid

So, required force, $F = \rho V\sqrt{a^2 + g^2}$

12. (c) $L = \frac{p}{dg} = \frac{10^6}{3 \times 10^3 \times 10} = \frac{100}{3} = 33.3$ m

13. (b) Change in pressure due to placing of mass on piston is,

$$\Delta p = \frac{Mg}{A}$$

From bulk modulus definition $K = \frac{-dp}{dV/V}$

$\Rightarrow$ $\left|\frac{dV}{V}\right| = \frac{\Delta p}{K} = \frac{Mg}{AK}$

From $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{V} = \frac{3dR}{R} \Rightarrow \frac{dR}{R} = \frac{1}{3}\frac{dV}{V} = \frac{Mg}{3AK}$$

14. (d) As vessel is falling freely under gravity, the pressure at all points within the liquid remains the same as the atmospheric pressure. If we apply Bernoulli's theorem just inside and outside the hole, then

$$p_{\text{inside}} + \frac{\rho v_{\text{inside}}^2}{2} + \rho g y = p_{\text{outside}} + \frac{\rho v_{\text{outside}}^2}{2} + \rho g y$$

$$v_{\text{inside}} = 0, p_{\text{inside}} = p_{\text{outside}} = p$$ [atmospheric pressure]

Therefore, $v_{\text{outside}} = 0$

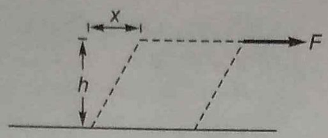
i.e., no water comes out.

15. (b) From the Bernoulli's theorem, $p_1 + \frac{\rho v_1^2}{2} = p_2 + \frac{\rho v_2^2}{2}$

From continuity equation, $A_1v_1 = A_2v_2$

Using $p_1 < p_2$ we can get $A_1 < A_2$

16. (*a*) $\eta = \dfrac{\frac{F}{A}}{\frac{x}{h}} = \dfrac{Fh}{Ax}$



$\Rightarrow \quad x = \dfrac{Fh}{A\eta} = \dfrac{100 \times 0.5 \times 10^{-2}}{5 \times 10^{-4} \times 5 \times 10^{10}} = 2 \times 10^{-8}\text{m}$

17. (*b*) $Y = \dfrac{F/A}{\Delta l/l}$

For the 1st wire, $Y = \dfrac{F \times L}{\pi r^2 \times l}$, For the 2nd wire, $Y = \dfrac{2F \times 2L}{\pi (2r)^2 \times l'}$

From above two equations, $l' = l$

18. (*b*) $\dfrac{dV}{V} = (1 + 2\sigma)\dfrac{dl}{l} = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3} \left[\because \ \sigma = 0.5 = \dfrac{1}{2}\right]$

$\therefore$ Percentage change in volume $= 4 \times 10^{-1} = 0.4\%$

19. (*a*) $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{x}{\frac{2l - l}{l}} = \dfrac{x}{1} = x$

In actual, the above expression is not exact for this much elongation.

20. (*a*) Temperature of the interface $\theta = \dfrac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

$\left(\because \ \dfrac{K_1}{K_2} = \dfrac{1}{4} \Rightarrow \text{ If } K_1 = K \text{ then } K_2 = 4K\right)$

$\Rightarrow \quad \theta = \dfrac{K \times 0 + 4K \times 100}{5K} = 80°\text{C}$

21. (*d*) If $D_s = D_b$. Then $D_{s0} + \alpha_s D_{s0}\Delta T = D_{b0} + \alpha_b D_{b0}\Delta T$

So, $\quad \Delta T = \dfrac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}}$

$= \dfrac{3.000 - 2.992}{(19 \times 10^{-6})(2.992) - (11 \times 10^{-6})(3.00)} = 335°\text{C}$

The temperature is $T = 25°\text{C} + 335°\text{C} = 360°\text{C}$

22. (*c*) We have $p = nRT/V$. The work done by the gas during the isothermal expansion is

$$W = \int_{V_1}^{V_2} p dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \frac{V_2}{V_1}$$

Substitute $V_2 = 2V_1$ to obtain

$$W = nRT \ln 2 = (4.00)(8.314)(400) \ln 2 = 9.22 \times 10^3 \text{ J}$$

Since, the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$

Thus, $\quad \Delta S = \dfrac{W}{T} = \dfrac{9.22 \times 10^3 \text{ J}}{400 \text{ K}} = 23.1 \text{ J K}^{-1}$

23. (*d*) Work $W = p(V_2 - V_1)$ at constant p

Initial volume, $\quad V_1 = (AT_1 - BT_1^2)/p$

Final volume is $V_2 = (AT_2 - BT_2^2)/p$

$\Rightarrow \quad W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$

24. (*a*) Using $pV = nRT$

$$T = \frac{pV}{nR} = \frac{(1.06 \times 10^5)(1500 \times 10^{-6})}{(3.88 \times 10^{-2})(8.31)} = 493 \text{ K} = 220°\text{C}$$

25. (*d*) Heat required to raise the temperature of water must be equal to the power output of the heater P, multiplied by time t

$t = \dfrac{Q}{P} = \dfrac{cm(T_f - T_i)}{P}$

$= \dfrac{(4190)\ (0.100)\ (100 - 23°)}{200} = 160 \text{ s}$

26. (*c*) Time taken,

$t = \dfrac{Q}{Rmgh} = \dfrac{cm(T_f - T_i)}{Rmgh} = \dfrac{c(T_f - T_i)}{Rgh}$

$= \dfrac{(4190)\ (100 - 15)}{(30)(9.8)(0.30)} = 4.03 \times 10^3 \text{min}$

27. (*d*) Equation of thermal conductivity of the given combination

$$K_{eq} = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} = \frac{x + 4x}{\frac{x}{K} + \frac{4x}{2K}} = \frac{5}{3}K$$

Hence, rate of flow of heat through the given combination is

$$\frac{\theta}{t} = \frac{K_{eq}A(T_2 - T_1)}{(x + 4x)} = \frac{5/3KA(T_2 - T_1)}{5x} = \frac{1/3KA(T_2 - T_1)}{x}$$

On comparing it with given equations, we get $f = \dfrac{1}{3}$

28. (*c*) Solve the ideal gas law $pV = nRT$ for n

$$n = \frac{pV}{RT} = \frac{(100)(1.0 \times 10^{-6})}{(8.31)(220)} = 5.47 \times 10^{-8} \text{mol}$$

29. (*d*) Efficiency of Carnot's heat engine = 100%

$\dfrac{T_1 - T_2}{T_1} \times 100 = 100$

$\Rightarrow \quad T_1 - T_2 = T_1$

$\Rightarrow \quad T_2 = 0 \text{ K} = -273.16° \text{ C}$

30. (*c*) For constant pressure process, $Q_1 = nC_p\Delta T = 743 \text{ J}$

For constant volume process,

$Q_2 = nC_V dT = n(C_p - R)dT = nC_p dT - nRdT$

$Q_2 = 743 - 5 \times 8.3 \times 2 = 660 \text{ J}$

31. (*c*) For an ideal engine, the efficiency is related to the reservoir temperatures by $\varepsilon = (T_H - T_C)/T_H$. Thus, $T_H = (T_H - T_C)/\varepsilon = (75 \text{ K}) / (0.22) = 341 \text{ K} (= 68°\text{C})$.

The temperature of the cold reservoir is

$T_C = T_H - 75 = 341\text{K} - 75\text{K} = 266\text{K}(= -7°\text{C})$.

32. (*d*) Rate of cooling $R_c = \dfrac{d\theta}{dt} = \dfrac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$

$\Rightarrow \quad \dfrac{d\theta}{dt} \propto \dfrac{A}{V} \propto \dfrac{r^2}{r^3} \Rightarrow \dfrac{d\theta}{dt} \propto \dfrac{1}{r}$

33. (*d*) The level of floating of a ship in water is unaffected by the atmospheric pressure. It depends on buoyant force which results from the pressure difference in the fluid. If the atmospheric pressure changes, the pressure at all points in water changes by the same amount keeping the pressure difference in water same.

34. (*b*) Surface tension can be understood as a property of a surface due to which it tries to acquire the minimum possible area. If cohesive force is more, the liquid is having capability to shrink its surface more *i.e.*, surface area of the liquid is less and hence, more is the surface tension.

35. (*c*) At 4°C, the volume of water is minimum. When it is cooled below 4°C or heated above 4°C, then it expands or its volume increases. As volume at 4°C is minimum, thus its density $\left(=\frac{\text{mass}}{\text{volume}}\right)$ will be maximum.

36. (*b*) From Stefan's law, $E = \sigma T^4$ or $E \propto T^4$

$$\therefore \quad \frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{T}{2T}\right)^4$$

$$\text{or} \quad \frac{E_1}{E_2} = \frac{1}{16} \quad \text{or} \quad E_2 = 16E_1 = 16 \quad \text{times}$$

Specific heat too varies with temperature. As a matter of fact, specific heat is zero at 0K for all the materials..

37. (*a*) Both the statements are correct and statement II is the correct explanation for statement I.

38. (*d*) Excess pressure inside a soap bubble is given by $p_i - p_0 = \frac{4S}{r}$ as $p_i - p_0 \propto \frac{1}{r}$, so as r increases, p_i decreases (p_0 constant), therefore, statement I is wrong. Statement II is correct as more air has to be forced in to cause the bubble to grow.

39. (*b*) We know that for ideal black body reflectance $(r) = 0$, transmittance $(t) = 0$, absorbtance $(a) = 1$. It is clear that, when heated, the black body will radiate all the energy, which is absorbed by it.

40. (*a*) The change in volume depends on original volume so, when heated through the same temperature, both will expand by the same amount.

41. (*a*) When a bottle of cold carbonated drink is opened, then adiabatic expansion of gas evolved, takes place. Due to this, the temperature of gas decreases. It condenses the water vapour which forms a slight fog around the opening.

42. (*b*) The concept behind the working of suction cup is "due to the pressure difference between the air below the glass and the partial vacuum produces a net upward force which is used to support the weight of slab".

Outside the suction cup, the pressure on top and below the slab are the same.

43. (*d*) The net upward force acting on slab, due to pressure difference is, $F_p = p_0 A - pA$

where, p_0 = atmospheric pressure

p = partial vacuum pressure

A = area covered by suction cup

$$\Rightarrow \quad F_p = 10^5 \times 0.025 - 0.5 \times 10^5 \times 0.025 = 1250\ \text{N}$$

So, weight that can be supported by given suction cup is ≤ 1250 N.

44. (*b*) Volume of liquid remains same, let H' be the final common level, [at common height, pressure difference would no longer exist and flow would stop.]

Then, $\pi r^2 \times 3H + 4\pi r^2 \times H = (\pi r^2 + 4\pi r^2)H'$

[Volume of liquid in pipe always remains same]

$$\Rightarrow \quad H' = \frac{7H}{5}$$

45. (*d*) Let difference in liquid levels at any time t be h, the velocity of flow through the pipe is $\sqrt{2gh}$.

[Using Bernoulli's principle].

Let in time dt, the liquid level in left cylinder falls by dh_1 and rises in right cylinder by dh_2, then $\pi r^2 dh_1 = 4\pi r^2 dh_2$

$$i.e., \quad dh_2 = \frac{dh_1}{4}$$

i.e., in time dt, the difference in liquid level decreases by

$$dh = dh_1 + dh_2 = \frac{5}{4} dh_1$$

$$dh_1 = \frac{4}{5} dh$$

Rate of change in volume through left cylinder

$$= -\pi r^2 \times \rho \times \frac{dh_1}{dt} = \rho \times A \times \sqrt{2gh}$$

$$\Rightarrow \quad -\pi r^2 \times \frac{4}{5}\frac{dh}{dt} = A\sqrt{2gh}$$

$$\int_{2H}^{0} \frac{dh}{\sqrt{2gh}} = -\int_0^t \frac{5A}{4\pi r^2} dt$$

$$\Rightarrow \quad t = \frac{8\pi r^2}{5A} \times \sqrt{\frac{H}{g}}$$

46. (*c*) Work done by gravity force $= -[U_f - U_i]$, where U_f and U_i are gravitational potential energies of the liquid in final and initial states, respectively,

$$U_i = (\pi r^2 \times 3H)\rho g \times \frac{3H}{2} + (4\pi r^2 \times H)\rho g \times \frac{H}{2}$$

$$V_f = (\pi r^2 \times H')\rho g \times \frac{H'}{2} + (4\pi r^2 \times H')\rho g \frac{H'}{2}$$

Substituting $H' = \frac{7H}{5}$ we get,

$$W = -(-U_f - U_i) = \frac{8}{5}\pi r^2 \rho g H^2$$

Day 18

Electrostatics

Day 18

Outlines ...

- Electric Charge
- Electric Field
- Electric Dipole
- Electric Flux (ϕ_E)
- Gauss's Law
- Electric Potential
- Conductors and Insulators
- Capacitor

Electric Charge

Electric charge is the property associated with matter due to which it produces/experiences electric and magnetic effects. Electric charge is a scalar and is additive in nature. Total charge of the system is

$$q = q_1 + q_2 + q_3 + \ldots\ldots + q_n.$$

Like charges repel but unlike charges attract each other.

Conservation of Charge

According to the principle of conservation of charge, we can neither create nor destroy electric charge. The charge can simply be transferred from one body to another.

Quantization of Charge

Electric charge is quantised. The minimum unit of charge, which may reside independently is the electronic charge e having a value of 1.60×10^{-19} C. Charge on any other body *i.e.*, $Q = \pm ne$, where n is any integer. SI unit of electric charge is coulomb (C).

Charge is invariant i.e., charge does not change with change in velocity.

Properties of Charges

1. Like charges repel while opposite charges attract each other.
2. As electron is a fundamental particle which cannot be sub-divided, hence smallest number of electron lost or gained as $n = 1$ and hence charge on a body, $q = ne = 1 \times e$. Hence, smallest charge in nature is electronic charge. Charge smaller than e does not exist and any amount of charge is an integral multiple of $e, q = ne$, where n = an integer. Thus, charge is quantized.
3. Charge is invariant *i.e.*, charge does not change with change in velocity.
   According to theory of relativity, the mass, time and length change with a charge in velocity but charge does not change.
4. A charged body attracts a lighter neutral body.
5. Electronic charge is additive *i.e.*, the total charge on a body is the algebraic sum of all the charges present in different parts of the body. For example, if a body has different charges as $+2q, +4q, -3q, -q$, then the total charge on the body is $+2q$.

Coulomb's Law of Forces between Two Point Charges

If q_1 and q_2 be two stationary point charges in free space separated by a distance r, then the force of attraction / repulsion between them is

$$F = \frac{K|q_1||q_2|}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_1||q_2|}{r^2} \qquad \left[K = \frac{1}{4\pi\varepsilon_0}\right]$$

$$= \frac{9 \times 10^9 \times |q_1||q_2|}{r^2} \qquad [K = 9 \times 10^9]$$

The term ε_0 is called the electric permittivity of free space having a value of 8.85×10^{-12} $C^2N^{-1}m^{-2}$. Its dimensional formula is $[M^{-1}L^{-3}T^4A^2]$.

If some dielectric medium is completely filled between the given charges, then the Coulomb's force between them becomes

$$F_m = \frac{1}{4\pi\varepsilon} \frac{q_1q_2}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1q_2}{r^2}$$

$$\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r \text{ or } K$$

$$= \frac{1}{4\pi K\varepsilon_0} \cdot \frac{q_1q_2}{r^2}$$

$$= \frac{F_{\text{Free space}}}{K} \qquad \left[F_{\text{Free space}} = \frac{1.q_1q_2}{4\pi r^2\varepsilon_0}\right]$$

Here, ε_0 = absolute electric permittivity of the given medium. K dielectric constant and ε_r is the relative permittivity of the given medium.

- *Force of two charges exert on each other is not changed by the presence of a third charge.*
- *Coulomb's force between two protons is 10^{36} times the gravitational force between them.*

Forces between Multiple Charges

When a number of point charges are present in a region then force acting between any two point charges remains unaffected by the presence of other charges and remains same as according to Coulomb's law. If four identical charges are placed at the four corners, then the force on any one charge due to the rest of the three charges is

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a^2} (2\sqrt{2} + 1)$$

Continuous Charge Distribution

- When charge is distributed along a line, straight or curved, then it is called linear charge distribution and its charge per unit length is called linear charge density (λ).
  $$\lambda = \frac{\Delta q}{\Delta l}$$
  Its unit is coulomb/metre (cm^{-1}).
- When charge is distributed over a surface area, then it is called surface charge distribution and charge over its per unit area is called surface charge density (σ). $\sigma = \frac{\Delta q}{\Delta S}$
  Its unit is coulomb-metre^{-2} (cm^{-2}).
- When charge is distributed over entire volume of a body, then it is called volume charge distribution and charge per unit volume is called volume charge density (ρ). $\rho = \frac{\Delta q}{\Delta V}$
  Its unit is coulomb-metre^{-3}(cm^{-3}).

Superposition Principle

If a number of discrete point charges are present then the net electric force on any one charge is equal to the vector sum of the electric force due to all other charges on that charge. If charges $q_0, q_1, q_2 \dots$ are present, then

$$\mathbf{F}_0 = \mathbf{F}_{01} + \mathbf{F}_{02} + \dots + \mathbf{F}_{0n}$$

Electric Field

The space surrounding an electric charge q in which another charge q_0 experiences a force of attraction or repulsion, is called the electric field of charge q. The charge q is called the ***source charge*** *and the charge q_0 is called the* ***test charge****. The test charge must be negligibly small so that it does not modify the electric field of the source charge.*

Electric field vector **E** (also known as the electric field intensity) at any point, is given by

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$$

where q_0 is a small positive test charge which experiences a force **F** at a given point.

SI unit of electric field is NC^{-1} and it is also known as Vm^{-1}. The dimensional formula for electric field is $[MLT^{-3}A^{-1}]$.

Electric Field Lines

An electric field line in an electric field is a smooth curve, tangent to which, at any point, gives the direction of the electric field at that point.

Motion of a Charged Particle in an Electric Field

Case I A charged particle is released from rest in an electric field E.

Then force on charged particle is given by $F = qE$

The acceleration produced by this force is given by

$$a = \frac{F}{m} = \frac{qE}{m}$$

Since, E is constant, the acceleration a is also constant. Hence, the particle is uniformly accelerated. Let the particle starts from rest, then velocity of charged particle after time t is given by $v = u + at$

$$v = \left(\frac{qE}{m}\right)t \qquad \left(u = 0 \text{ and } a = \frac{qE}{m}\right)$$

The distance travelled by the particle is given by

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}\frac{qE}{m}t^2$$

The kinetic energy gained by the particle

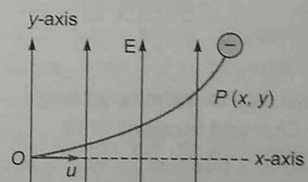
$$= \frac{1}{2}mv^2 = \frac{q^2E^2t^2}{2m}$$

Case II A charged particle enters the field in perpendicular direction.

Let a charged particle of mass m and charge q, enters the electric field along x-axis with speed u. The electric field E is along y–axis is given by

$$F_y = qE$$

and force along x–axis remains zero *i.e.*, $F_x = 0$



∴ Acceleration of the particle along y-axis is given by

$$a_y = \frac{F_y}{m} = \frac{qE}{m}$$

The initial velocity is zero along y-axis ($u_y = 0$).

∴ The deflection of charged particle along y-axis after time t is given by $y = u_y t + \frac{1}{2}a_y t^2 = \frac{qE}{2m}t^2$

Along x-axis there is no acceleration, so the distance covered by particle in time t along x-axis is given by $x = ut$

Eliminating t, we have

$$y = \left(\frac{qE}{2mu^2}x^2\right)$$

$$y \propto x^2$$

This shows that the path of charged particle in perpendicular field is a parabola.

Important properties of electric field lines are

(i) Electric field lines come out of a positive charge and go into the negative charge.

(ii) No two electric field lines intersect each other.

(iii) Electric field lines are continuous but they never form a closed loop.

(iv) Electric field lines cannot exist inside a conductor. Electric shielding is based on this property.

Electric Field due to a Point Charge

1. Electric Field at a Distance *r* from a Point Charge *q* is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

» *If q_1 and q_2 are two like point charges, separated by a distance r, a neutral point between them is obtained at a point distant r_1 from q_1, such that* $r_1 = \dfrac{r}{\left[1 + \sqrt{\dfrac{q_2}{q_1}}\right]}$

» *If q_1 and q_2 are two charges of opposite nature separated by a distance r, a neutral point is obtained in the extended line joining them, at a distance r_1 from q_1, such that*

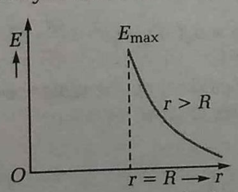
$$r_1 = \frac{r}{\left[\sqrt{\dfrac{q_2}{q_1}} - 1\right]}$$

2. Electric Field due to Inifinitely Long Uniformly Charged Straight Wire

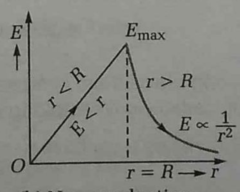
Electric field at a point situated at a normal distance r, from an infinitely long uniformly charged straight wire having a linear charge density λ, is $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

3. Electric Field due to a Charged Cylinder

(i) For a conducting charged cylinder of linear charge density λ and radius R, the electric field is given by



(a) Conducting cylinder



(b) Non-conducting cylinder

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}, \text{ for } r > R,$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}, \text{ for } r = R \quad \text{and} \quad E = 0, \text{ for } r < R$$

(ii) For a non-conducting charged cylinder, for $r \le R$,

$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$

and $$E = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ for } r > R$$

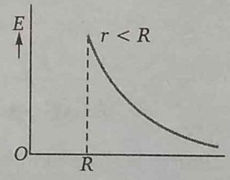
4. Electric Field due to a Uniformly Charged Infinite Plane Sheet

Electric field near a uniformly charged infinite plane sheet having surface charge density σ is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

5. Electric Field due to a Uniformly Charged Thin Spherical Shell

For a charged conducting sphere/ shell of radius R and total charge Q, the electric field is given by



Case I $E = 0$, for $r < R$

Case II $E = \dfrac{Q}{4\pi\varepsilon_0 R^2}$, for $r = R$

Case III and $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, for $r > R$

Continuous Charge Distribution

The continuous charge distribution may be one dimensional, two dimensional and three-dimensional.

- **Linear charge density** (λ) If charge is distributed along a line, *i.e.*, straight or curve is called linear charge distribution. The uniform charge distribution q over a length L of the straight rod.

  Then, the linear charge density, $\lambda = \dfrac{q}{L}$

  Its unit is coulomb metre^{-1} (cm^{-1}).

- **Surface charge density** (σ) If charge is distributed over a surface is called surface charge density, *i.e.*,

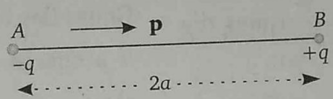
  $$\sigma = q/A$$

  Its unit is coulomb m^{-2} (Cm^{-2})

- **Volume charge density** (ρ) If charge is distributed over the volume of an object, is called volume charge density, *i.e.*, $\rho = \dfrac{q}{V}$. Its unit is coulomb metre^{-3} (Cm^{-3}).

Electric Dipole

An electric dipole consists of two equal and opposite charges separated by a small distance.



The dipole moment of a dipole is defined as the product of the magnitude of either charges and the distance between them. Therefore, dipole moment $\mathbf{p} = q(2\mathbf{a})$

Dipole moment is a vector whose direction is from negative to postive charge. Its SI unit is coulomb-metre C-m and has the dimensional formula is [LTA].

Electric Field due to a Dipole

1. At a point distant r from the centre of a dipole, along its axial line

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}r}{(r^2 - a^2)^2}$$

(direction of **E** is the same as that of **p**)

For a short dipole *i.e.*, $r >> a$.

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}}{r^3}$$

2. At a point distant r from the centre of a dipole, along its equatorial line

$$\mathbf{E} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p}}{(r^2 + a^2)^{3/2}}$$

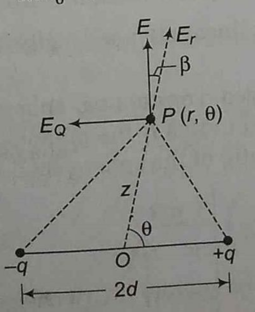
(direction of **E** is opposite to that of **p**)

For a short dipole

$$\mathbf{E} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p}}{r^3} \qquad [r >> a]$$

3. At a point distant r from the centre of a short dipole, along a line inclined at an angle θ with the dipole axis

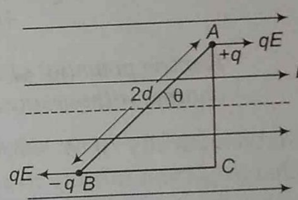
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}\sqrt{3\cos^2\theta + 1}$$



4. **E** subtends an angle β from **r** such that

$$\tan\beta = \frac{1}{2}\tan\theta$$

Torque on a Dipole in a Uniform Electric Field



When a dipole is placed in an external electric field, making an angle θ with the direction of the uniform electric field E, it experiences a torque given by $\tau = qE \times AC$

$$\tau = \mathbf{p} \times \mathbf{E} = pE\sin\theta$$

or $\quad qE \times 2d\sin\theta = (q \times 2d)\,E\sin\theta$

Work Done

If an electric dipole initially kept in an uniform electric field **E**, making an angle θ_1, is rotated so as to finally subtend an angle θ_2, then the work done for rotating the dipole is, $W = pE(\cos\theta_2 - \cos\theta_1)$

Potential Energy of a Dipole

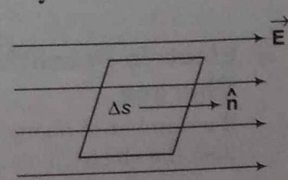
It is the amount of work done in rotating an electric dipole from a direction perpendicular to electric field to a particular direction. Hence, $U = -pE\cos\theta$ or $U = -\mathbf{p}\cdot\mathbf{E}$

Obviously potential energy of an electric dipole is a scalar quantity. It is measured in Joule.

Electric Flux (ϕ_E)

Electric flux is a measure of the flow of electric field through a surface. Mathematically, electric flux is the product of an area element **ds** and the normal component of **E**, integrated over a surface,

i.e., $\quad \phi_E = \int E\,ds\cos\theta = \int \mathbf{E}\cdot\mathbf{ds} = \int \mathbf{E}\cdot\hat{\mathbf{n}}\,ds$



where $\hat{\mathbf{n}}$ is the unit vector normal to area element ds.

Electric flux is a scalar quantity having SI unit as Nm^2C^{-1} or V-m. Its dimensional formula is $[ML^3T^{-3}A^{-1}]$.

Gauss's Law

According to Gauss's law in electrostatics, the total electric flux linked with a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by that surface. Thus,

$$\phi_E = \oint_S \mathbf{E} \cdot \mathbf{ds} = \frac{1}{\varepsilon_0} [Q_{enclosed}]$$

where $Q_{enclosed} \sum_{i=1}^{i=n} qi$ is the algebraic sum of all the charges inside the closed surface.

Any closed surface, real or hypothetical, for which the Gauss law is applied, is called a gaussian surface.

If a charge Q is placed at the centre of a cube, then the electric flux from the entire surface of the cube is $\phi = \frac{Q}{\varepsilon_0}$, and the electric flux from any one face of the cube is $\phi = \frac{Q}{6\varepsilon_0}$.

Electric Potential

Electric potential at a point in an electric field is defined as the amount of work done in bringing a unit positive charge, without any acceleration, from infinity to that point, along any arbitrary path.

Mathematically, if W work is to be done to bring a test charge q_0 from infinity to a point, then the potential of that point, is

$$V = \frac{W}{q_0}$$

SI unit of potential is volt, where $1\text{ V} = \frac{1\text{ J}}{1\text{ C}}$. Its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

Electric potential is a state function and does not depend on the path followed.

1. Electric Potential Due to a Point Charge

Potential due to a point charge Q, at a distance r is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$$

- *At the centre of the line joining two equal and opposite charges, $V = 0$ but $E \neq 0$.*
- *At the centre of the line joining two equal and like charges, $E = 0$ but $V \neq 0$.*
- *If four identical charges q each are placed at the four vertices of a square then the net electric field at the centre of the square is zero, but $V = \frac{\sqrt{2}q}{\pi\varepsilon_0 a}$*

2. Electric Potential Due to a System of Charges

If a number of charges $q_1, q_2, q_3 \ldots$ are present in space, then the electric potential at any point will be

$$V = V_1 + V_2 + V_3 + \ldots$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \left(\frac{q_i}{r_i}\right)$$

3. Electric Potential Due to an Electric Dipole

At any general point,

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$$

On the dipole axis, $\theta = 0°$

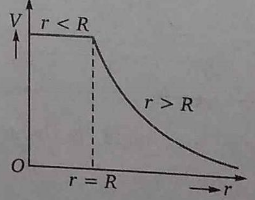
and $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$$

On the equitorial axis $\theta = 90°$ and $V = 0$

4. Electric Potential due to Some Common Charge Distributions

Potential at a point distant r from an infinitely long wire having linear charge density λ, is $V = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln r$

For a charged conducting sphere/shell having total charge Q and radius R, the potential at a point distant r from the centre of the sphere/shell is



(i) $V = \dfrac{Q}{4\pi\varepsilon_0 r}$, for $r > R$

(ii) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$, for $r = R$

(iii) $V = \dfrac{Q}{4\pi\varepsilon_0 R}$, for $r \leq R$

For a charged non-conducting (dielectric) sphere of radius R, the charge Q is uniformly distributed over the entire volume.

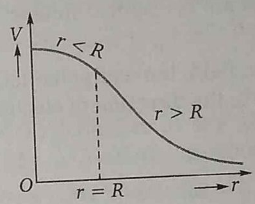
Hence, (i) $V = \dfrac{Q}{4\pi\varepsilon_0 r}$, for $r > R$

(ii) $V = \dfrac{Q}{4\pi\varepsilon_0 R}$, for $r = R$ and

(iii) $V = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{3R^2 - r^2}{2R^3}\right]$, for $r < R$

At the centre of the sphere ($r = 0$)

$$V = \frac{3Q}{8\pi\varepsilon_0 R} = \frac{3}{2}V_s \qquad \left[V_s = \frac{Q}{4\pi\varepsilon_0 R}\right]$$



Electric Potential Energy

The electric energy of a system of charges is the work that has been done in bringing those charges from infinity to near each other to form the system. For two point charges q_1 and q_2 separated by distance r_{12}, the potential energy is given by $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r_{12}}$

Electric potential energy is a scalar quantity. In the above formula, the values of q_1 and q_2 are used with proper signs.

If there are more than two charges in a system, then the electric energy is calculated for each pair and then all energies so obtained, are added algebraically.

For example, for a system of three charges q_1, q_2 and q_3 are placed at three corners of a triangle (figure), then the electric potential energy of the system will be given by

$$U = U_{12} + U_{23} + U_{31}$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{31}}\right]$$

In general, for a system of n charges, the electric potential energy is given by

$$U = \frac{1}{2}\Sigma \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}\ i \neq j$$

[$\frac{1}{2}$ *is used as each term in summation will appear twice*]

Relation between E and V

Because E is force per unit charge and V is work per unit charge. E and V are related in the same way as work and force. If $\Delta V = (V_B - V_A)$ is the increase in potential over a short displacement Δs, $\Delta V = -E\Delta s$

where the negative sign indicates that the work is done against the field. If α is the angle between E and Δs, we have

$$\Delta V = -E(\Delta s)\cos\alpha$$
$$= -E\Delta x \qquad \text{(along } \mathbf{E}\text{)}$$

where, $\Delta x = \Delta s \cos\alpha$ is the component of $\mathbf{\Delta s}$ along $\mathbf{E}$

Therefore, $E = -\dfrac{\Delta V}{\Delta x}$

For small changes, above expression may be written as

$$E = \frac{-dV}{dx}$$

Thus, the electric field intensity E is the negative gradient of potential.

This means that decrease in potential is along the direction of E. The SI unit of E is therefore, volt per metre (Vm^{-1})

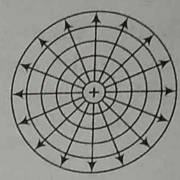
Equipotential Surface

For a given charge distribution, an equipotential surface is the locus of the all the points having the same electric potential.

For a point charge or a spherical charge distribution, equipotential surfaces are concentric spheres as shown in figure.

For a uniform electric field, the equipotential surfaces are planes perpendicular to the direction of electric field.

There is no component of electric field along an equipotential surface.



As a result, work done in moving a charge along an equipotential surface, is always zero.

i.e., $dW = \mathbf{E} \cdot d\mathbf{l}$

$dW = E\, dl \cos\theta = 0$

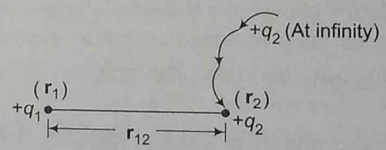
$\therefore$ $\cos\theta = 0$ or $\theta = 90°$. So, $\mathbf{E} \perp d\mathbf{l}$

- *Equipotential surface may be planar, solid etc. But equipotential surface can never be point size.*
- *Equipotential surface is single valued. So, equipotential surfaces never cross each other.*
- *Electric field is always perpendicular to equipotential surface.*

Electrostatic Potential Energy of a System of Two Point Charges in an Electrostatic Field

Electrostatic potential energy of a charge q, placed at a point in an electric field, is equal to work done in bringing the given charge from infinity to that point. Hence, electrostatic potential energy is $U = qV$, where V is the electric potential at that point.

For a system of two point charges q_1 and q_2 separated by a distance r, $W = U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{22}}$



Conductors and Insulators

***Conductors** are those materials through which electricity can pass through easily because they contain a large number of free electrons. e.g., metals like copper, silver, iron etc. **Insulators** are those materials through which electricity cannot pass through because they do not contain free electrons. e.g., rubber, ebonite, mica etc.*

Dielectrics and Polarisation

Dielectrics are insulating materials which transmit electric effect without actually conducting electricity.

For example : mica, glass, water etc.

When a dielectric is placed in an external electric field, the centres of positive and negative charges gets separated in non-polar dielectrics and get farther away in polar dielectrics, so the molecules of dielectric gain a permanent electric dipole moment. This process is called polarisation.

Electrical Capacitance

Capacitance of a conductor is the amount of charge needed in order to raise the potential of the conductor by unity. Mathematically,

$$\text{Capacitance } C = \frac{Q}{V}$$

Electrical capacitance is a scalar. SI unit of capacitance is 1 farad (1F), where $1\text{ F} = \frac{1\text{ C}}{1\text{ V}}$. Its dimensional formula is $[M^{-1}L^{-2}T^{4}A^{2}]$.

The electrostatic potential energy of a charged conductor having capacitance C, charge Q and a potential V, is given by

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Sharing of Charges

Let us have two charged conductors having charges Q_1 and Q_2 (or potentials V_1, V_2 and capacitances C_1, C_2 respectively). If these are joined together by means of a connecting wire, charge begins to flow from the higher potential to the lower potential side, till their potential is the same, which is called the common potential. In such a cases

Common potential $V = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

During sharing of charges, there is some loss of electrostatic energy, which in turn reappears as heat or light. The loss of electrostatic energy

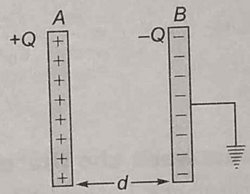
$$\Delta U = U_i - U_f = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

- When charges are shared between any two bodies, their potential become equal. The charges acquired are in the ratio of their capacities.
- No charge is really lost, but same loss of energy does occur.

Capacitor

A capacitor is a device which stores electrostatic energy. It consists of conductors of any shape and size carrying charges of equal magnitudes and opposite signs and separated by an insulating medium.

Net charge on a capacitor is zero. However, ordinarily we talk in terms of charge on either plate of a capacitor and that is finite and non zero.



We conclude that the capacitance of an insulated conductance is increased considerably by bringing near it an uncharged each conductor.

Combination of Capacitors

There are two common methods of grouping of capacitors

1. Series Grouping

In a series arrangement, the charge on each plate of each capacitor has the same magnitude, equal to the charge supplied by the battery.

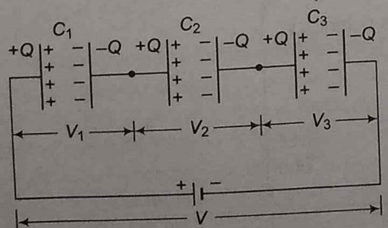
The potential difference is distributed inversely in the ratio of capacitors,

i.e., $V = V_1 + V_2 + V_3 + \ldots$

and $V_1 : V_2 : V_3 \ldots = \dfrac{1}{C_1} : \dfrac{1}{C_2} : \dfrac{1}{C_3} : \ldots$

The equivalent capacitance C_s is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots = \sum_{i=1}^{i=n} \frac{1}{C_i}$$



2. Parallel Grouping

In a parallel arrangement, the potential across each of the capacitor is, exactly same.

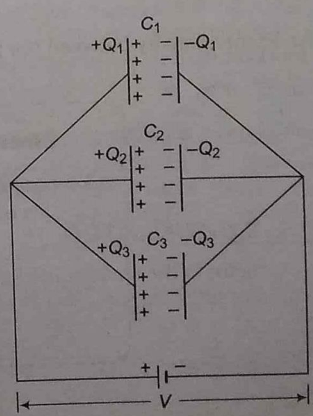
Charges on different capacitors are different. In fact, the charge is distributed in the ratio of capacitance,

i.e., $Q = Q_1 + Q_2 + Q_3 + \ldots$

and $Q_1 : Q_2 : Q_3 \ldots = C_1 : C_2 : C_3 \ldots$

The equivalent capacitance is given by

$$C_p = C_1 + C_2 + C_3 + \ldots = \sum_{i=1}^{i=n} \frac{1}{C_i}$$



If n identical conductor plates are arranged such that alternate plates are joined together then the combination is equivalent to $(n-1)$ capacitors all joined in parallel.

Hence, $$C = (n-1)\frac{\varepsilon_0 A}{d}$$

Capacitance of a Parallel Plate Capacitor

The parallel plate capacitor consists of two metal plates parallel to each other and separated by a distance that is very small as compared to the dimensions of the plates.

1. Capacitor without Dielectric Medium between the plates

If the magnitude of charge on each plate of a parallel plate capacitor be Q and the overlapping area of plates be A, then

(i) Electric field between the plates $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

(ii) Potential difference between the plates $V = E \cdot d = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{\varepsilon_0 A}$, where d = separation between the two plates.

(iii) Capacitance $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$

2. Capacitor with Dielectric Medium between the Plates

(i) If a dielectric medium of dielectric constant K is completely filled between the plates of a capacitor, then its capacitance becomes,

$$C' = \frac{K\varepsilon_0 A'}{d} = KC_0 \qquad \left[\text{where, } C_0 = \frac{\varepsilon_0 A'}{d}\right]$$

(ii) If a dielectric slab/sheet of thickness t (where $t < d$) is introduced between the plates of the capacitor, then

$$C' = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{K}\right)}$$

(iii) If a metallic slab/plate of thickness t (where $t < d$), is inserted between the plates of capacitor, then

$$C' = \frac{\varepsilon_0 A}{(d-t)}$$

(iv) Magnitude of the attractive force between the plates of a parallel plate capacitor is given by

$$F = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2A\varepsilon_0} = \frac{CV^2}{2d}$$

(v) The energy density between the plates of a capacitor $u = \frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$

Energy Stored in a Capacitor

If a capacitor of capacity C is charged to a potential V, the electrostatic energy stored in it is,

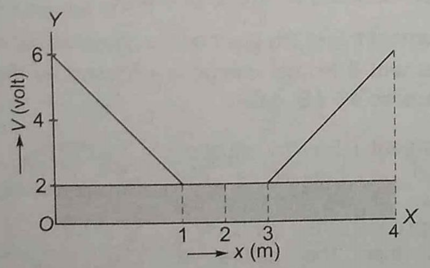
$$U = \frac{1}{2}CV^2$$
$$= \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Practice Zone DAY 18

1. In one gram of a solid, there are 5×10^{21} atoms. If one electron is removed from even one of the 0.01 %, the atoms of the charge gained by the solid would be
(a) +0.08 C (b) +0.8 C
(c) – 0.08 C (d) – 0.8 C

2. The electric potential V at any point (x, y, z) in space is given by $V = 4x^2$ volt. The electric field at (1, 0, 2) m in Vm^{-1} is
(a) 8, along the positive x-axis
(b) 8, along the negative x-axis
(c) 16, along the x-axis
(d) 16, along the z-axis

3. The variation of electric potential with distance from a fixed point is shown in the figure. What is the value of electric field at $x = 2$ m?



(a) Zero (b) 6/2 (c) 6/1 (d) 6/3

4. Charge Q is given by the displacement $\mathbf{r} = a\hat{i} + b\hat{j}$ in an electric field $\mathbf{E} = E_1\hat{i} + E_2\hat{j}$. The work done is
(a) $Q(E_1a + E_2b)$ (b) $Q\sqrt{(E_1a)^2 + (E_2b)^2}$
(c) $Q(E_1 + E_2)\sqrt{a^2 + b^2}$ (d) $Q(\sqrt{E_1^2 + E_2^2})\sqrt{a^2 + b^2}$

5. A large insulated sphere of radius r, charged with Q units of electricity, is placed in contact with a small insulated uncharged sphere of radius r' and is then separated. The charge on the smaller sphere will now be
(a) $Q(r + r')$ (b) $Q(r - r')$
(c) $\frac{Q}{r' + r}$ (d) $\frac{Qr'}{r' + r}$

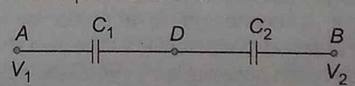
6. If the electric flux entering and leaving an enclosed surface respectively are ϕ_1 and ϕ_2, the electric charge inside the surface will be
(a) $\frac{\phi_2 - \phi_1}{\varepsilon_0}$ (b) $\frac{\phi_1 + \phi_2}{\varepsilon_0}$
(c) $\frac{\phi_1 - \phi_2}{\varepsilon_0}$ (d) $\varepsilon_0(\phi_1 + \phi_2)$

7. One plate of a capacitor is connected to a spring and area of both the plates is A. In steady state, separation between the plates is 0.8 d (spring was unstretched and the distance between the plates was d when the capacitor was uncharged). The force constant of the spring is approximately
(a) $\frac{6\varepsilon_0 E^2}{Ad^3}$ (b) $\frac{4\varepsilon_0 AE^2}{d^3}$
(c) $\frac{\varepsilon_0 AE^2}{2d^3}$ (d) $\frac{2\varepsilon_0 AE}{d^3}$

8. Capacitance of a capacitor becomes $\frac{4}{3}$ times its original value if a dielectric slab of thickness $t = d/2$ is inserted between the plates (d = separation between the plates). The dielectric constant of the slab is
(a) 6 (b) 8
(c) 2 (d) 4

9. An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density. **[NCERT Exemplar]**
(a) 10^{-3} C/m (b) 10^{-4} C/m
(c) 10^{-5} C/m (d) 10^{-7} C/m

10. Two condensers C_1 and C_2 in a circuit are joined as shown in the figure. The potential of point A is V_1 and that of B is V_2. The potential of point D will be

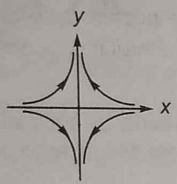


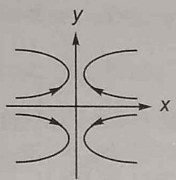
(a) $\frac{1}{2}(V_1 + V_2)$ (b) $\frac{C_1V_2 + C_2V_1}{C_1 + C_2}$
(c) $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ (d) $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

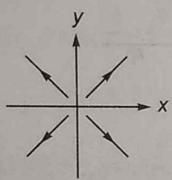
11. A cylinder of radius R and length L is placed in a uniform electric field E parallel to the axis of the cylinder, the total electric flux for the surface of the cylinder is

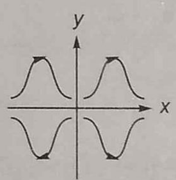
(a) $2\pi R^2 E$ (b) $\frac{\pi R^2}{E}$ (c) $\frac{\pi R^2 + \pi R^2}{E}$ (d) zero

12. The potential field depends on the x and y-coordinates as $V = (x^2 - y^2)$. The corresponding electric field lines in the x-y plane are as shown in the figure

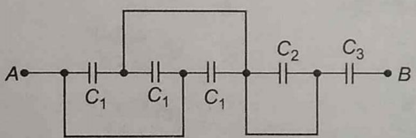
(a)


(b)


(c)


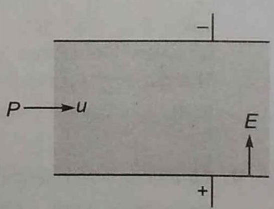
(d)


13. A circuit is shown in figure for which $C_1 = (3 \pm 0.011)\,\mu F$, $C_2 = (5 \pm 0.01)\,\mu F$ and $C_3 = (1 \pm 0.01)\,\mu F$. If C is the equivalent capacitance across AB, then C is given by



(a) $(0.9 \pm 0.114)\,\mu F$ (b) $(0.9 \pm 0.01)\,\mu F$
(c) $(0.9 \pm 0.023)\,\mu F$ (d) $(0.9 \pm 0.09)\,\mu F$

14. A positively charged particle P enters the region between two parallel plates with a velocity u, in a direction parallel to the plates. There is a uniform electric field in this region. P emerges from this region with a velocity v. Taking C as a constant, v will depend on u as

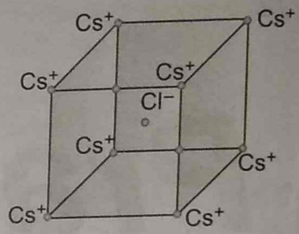


(a) $v = Cu$ (b) $v = \sqrt{u^2 + Cu}$
(c) $v = \sqrt{u^2 + \frac{C}{u}}$ (d) $v = \sqrt{u^2 + \frac{C}{u^2}}$

15. An uncharged parallel plate capacitor having a dielectric of constant K is connected to a similar air cored parallel capacitor charged to a potential V. The two share the charge and the common potential is V'. The dielectric constant K is

(a) $\frac{V' - V}{V' + V}$ (b) $\frac{V' - V}{V'}$
(c) $\frac{V' - V}{V}$ (d) $\frac{V - V'}{V'}$

16. In the basic CsCl crystal structure, $^{+}$Cs and $^{-}$Cl ions are arranged in a bcc configuration as shown below. The net electrostatic force exerted by the 88 Cs^+ on the the Cl^- ion is



(a) $\frac{1}{4\pi\varepsilon_0}\frac{4e^2}{3a^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{16e^2}{3a^2}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{32a^2}{3a^2}$ (d) zero

Directions *(Q. Nos. 17 to 22) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below :*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true; Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

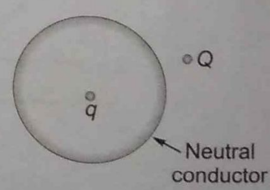
17. Statement I Due to the displacement of charges within a closed surface, the **E** at any point on the surface does not change.

Statement II The flux passing through a closed surface, is independent of the location of charge.

18. Statement I E at any point on the gaussian surface is zero.

Statement II No net charge is enclosed by the gaussian surface, so $\phi = \oint \mathbf{E} \cdot d\mathbf{s} = 0$.

19. Statement I For the situation shown in the figure that follows, if we displace the charge q within the conducting shell, then the nature of distribution of charge on the outer surface of the shell changes.



Statement II Any conducting shell divides the entire space into two regions (inside and outside the shell), which are independent of each other in terms of electric field.

20. Statement I An electric dipole is placed in a uniform electric field. Its equilibrium will be stable when dipole is set along the direction of electric field.

Statement II In stable equilibrium, energy of dipole should be least possible.

21. Statement I Electric field on the surface of a conductor is more at the sharp corners.

Statement II Surface charge density on the surface of the conductor is inversely proportional to the radius of curvature.

22. **Statement I** If electric potential is constant in a certain region of space, then the electric field must be zero in this region.

Statement II $E = -\frac{dV}{dr} r$

Directions (Q. Nos. 23 to 25) *Some cell walls in the human body have a layer of negative charge on the inside surface and a layer of positive charge, of equal magnitude, on the outside surface. Consider one such cell having the thickness of the cell wall as 10^{-10} m. The charge densities on the walls are $-\sigma$ and $+\sigma$ Cm^{-2}, respectively, and the relative permittivity of the cell wall material is 5. Take the volume of the cell wall as 10^{-15} m^3. Assume the cell wall to be plane.*

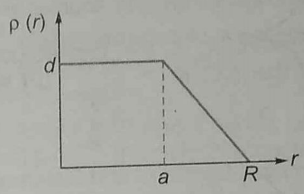
23. Which wall is at a higher potential?
(a) Inner
(b) Outer
(c) Both are at the same potential
(d) Cannot be determined from the given information

24. Determine the electric field intensity in between the cell walls.
(a) $\frac{\sigma}{5\varepsilon_0} NC^{-1}$ (b) $\frac{\sigma}{2\varepsilon_0} NC^{-1}$
(c) $\frac{\sigma}{4\varepsilon_0} \times 10^{-3} NC^{-1}$ (d) $\frac{\sigma}{2\varepsilon_0} \times 10^{-5} NC^{-1}$

25. The potential difference between the inside and the outside walls of the cell is
(a) $\frac{\sigma}{5\varepsilon_0} \times 10^{-5} NC^{-1}$ (b) $\frac{\sigma}{2\varepsilon_0} \times 10^{-5} NC^{-1}$
(c) $\frac{\sigma}{5\varepsilon_0} \times 10^{-10} NC^{-1}$ (d) $\frac{\sigma}{2\varepsilon_0} \times 10^{-10} NC^{-1}$

26. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N/C in Millikan's oil drop experiment. The density of the oil is $1.26 g/cm^3$. Estimate the radius of the drop. ($g = 9.81 m/s^2$; $e = 1.60 \times 10^{-19}$C). **[NCERT Exemplar]**
(a) 7.2×10^{-5} N (b) 6.3×10^{-4} N (c) 8×10^{-4} N (d) 9.81×10^{-7} N

Directions (Q. Nos. 27 to 29) *The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density ρ(r) [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in the figure. The electric field is only along the radial direction.*



27. The electric field at $r = R$ is
(a) independent of a (b) directly proportional to a
(c) directly proportional to a^2 (d) inversely proportional to a

28. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is
(a) $\frac{3Ze}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$
(c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

29. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies
(a) $a = 0$ (b) $a = \frac{R}{2}$
(c) $a = R$ (d) $a = \frac{2R}{3}$

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30. A parallel plate capacitor of area 60 cm^2 and separation 3 mm is charged initially to 90 μC. If the medium between the plate gets slightly conducting and the plate loses the charge initially at the rate of 2.5×10^{-8} C/s, then what is the magnetic field between the plates? **[JEE Main Online 2013]**
(a) 2.5×10^{-8} T (b) 2.0×10^{-7} T
(c) 1.63×10^{-11} T (d) Zero

31. Two small equal point charges of magnitude q are suspended from a common point on the ceiling by insulating massless strings of equal lengths. They come to equilibrium with each string making angle θ from the vertical. If the mass of each charge is m, then the electrostatic potential at the centre of line joining them will be $\left(\frac{1}{4\pi\varepsilon_0} = k\right)$. **[JEE Main Online 2013]**
(a) $2\sqrt{k\,mg\,\tan\theta}$ (b) $\sqrt{k\,mg\,\tan\theta}$
(c) $4\sqrt{k\,mg/\tan\theta}$ (d) $4\sqrt{k\,mg\,\tan\theta}$

32. A point charge of magnitude +1μC is fixed at (0, 0, 0). An isolated uncharged spherical conductor, is fixed with its centre at (4, 0, 0). The potential and the induced electric field at the centre of the sphere is **[JEE Main Online 2013]**
(a) 1.8×10^5 V and -5.625×10^6 V/m
(b) 0 V and 0 V/m
(c) 2.25×10^3 V and 5.625×10^2 V/m
(d) 2.25×10^5 V and 0 V/m

33. A uniform electric field **E** exists between the plates of a charged condenser. A charged particle enters the space between the plates and perpendicular to **E**. The path of the particle between the plates is a **[JEE Main Online 2013]**

(a) straight line (b) hyperbola (c) parabola (d) circle

34. A parallel plate capacitor having a separation between the plates d, plate area A and material with dielectric constant K has capacitance C_0. Now one-third of the material is replaced by another material with dielectric constant $2K$, so that effectively there are two capacitors one with area $\frac{1}{3}A$, dielectric constant $2K$ and another with area $\frac{2}{3}A$ and dielectric constant K. If the capacitance of this new capacitor is C then C/C_0 is **[JEE Main Online 2013]**

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

35. This question has Statement I and Statement II. Of the four choices given after the Statement, choose the one that best describes the two Statements.

Statement I No work is required to be done to move a test charge between any two points on an equipotential surface.

Statement II Electric lines of force at the equipotential surfaces are mutually perpendicular to each other. **[JEE Main Online 2013]**

(a) Statement I is true, Statement II is true, Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

36. The surface charge density of a thin charged disc of radius R is σ. The value of the electric field at the centre of the disc is $\frac{\sigma}{2\varepsilon_0}$. With respect to the field at the centre, the electric field along the axis at a distance R from the centre of the disc **[JEE Main Online 2013]**

(a) reduces by 71% (b) reduces by 29.3%
(c) reduces by 9.7% (d) reduces by 14.6%

37. Consider a finite insulated, uncharged conductor placed near a finite positively charged conductor. The uncharged body must have a potential **[JEE Main Online 2013]**

(a) less than the charged conductor and more than at infinity
(b) more than the charged conductor and less than at infinity
(c) more than the charged conductor and more than at infinity
(d) less than the charged conductor and less than at infinity

38. Two points dipoles of dipole moment $\mathbf{p}_1$ and $\mathbf{p}_2$ are at a distance x from each other an $\mathbf{p}_1 \| \mathbf{p}_2$. The force between the dipoles is **[JEE Main Online 2013]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{4p_3p_2}{x^4}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{3p_3p_2}{x^4}$
(c) $\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{x^4}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{3p_1p_2}{x^4}$

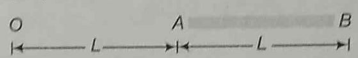
39. Two balls of same mass and carrying equal charge are hung from a fixed support of length l. At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, X between the balls is proportional **[JEE Main Online 2013]**

(a) l (b) l^2 (c) $l^{2/3}$ (d) $l^{1/3}$

40. Two capacitors C_1 and C_2 are changed to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zer. Then **[JEE Main 2013]**

(a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$
(c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

41. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance **[JEE Main 2013]**

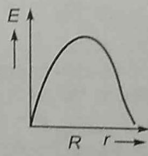


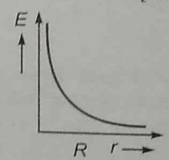
(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$ (c) $\frac{Q}{4\pi\varepsilon_0 L\ln 2}$ (d) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$

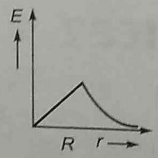
42. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement ($y < a$) along the y-axis, the net force acting on the particle is proportional to **[JEE Main 2013]**

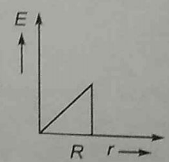
(a) y (b) $-y$
(c) $1/y$ (d) $-1/y$

43. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be **[AIEEE 2012]**

(a)


(b)


(c)


(d)


44. The question has statement 1 and statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniform positive charge density ρ. As a result of this uniform charge distribution, there is a finite value of electric potential, at the surface of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement I When a charge q is taken from the centre of the surface of the sphere its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement II The electric field at a distance $r(r < R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$. **[AIEEE 2012]**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement II
(d) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I

45. Two positive charges of magnitude q are placed at the ends of a side 1 of a square of side $2a$. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **[AIEEE 2011]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$ (b) zero
(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

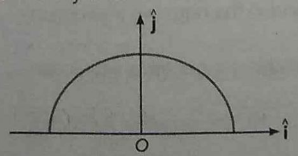
46. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where r is the distance from the centre a, b are constants. Then the charge density inside the ball is **[AIEEE 2011]**

(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0$
(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_0 r$

47. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d $(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then as a function of distance x between them, **[AIEEE 2011]**

(a) $v \propto x^{-1}$
(b) $v \propto x^{1/2}$
(c) $v \propto x$
(d) $v \propto x^{-1/2}$

48. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field **E** at the centre O is **[AIEEE 2010]**



(a) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$
(c) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

49. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4}-\frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by **[AIEEE 2010]**

(a) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (b) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$
(c) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

50. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 16 g cm^{-3}, the dielectric constant of the liquid is **[AIEEE 2010]**

(a) 4 (b) 3
(c) 2 (d) 1

51. The questions contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2009]**

Statement I For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement II The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement I is true, Statement II is false
(b) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(c) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(d) Statement I is false, Statement II is true

52. Let $p(r) = \frac{Q}{\pi R^4}r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is **[AIEEE 2009]**

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$
(c) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$

53. Two points P and Q are maintained at the potentials of 10 V and −4V respectively. The work done in moving 100 electrons from P to Q is **[AIEEE 2009]**

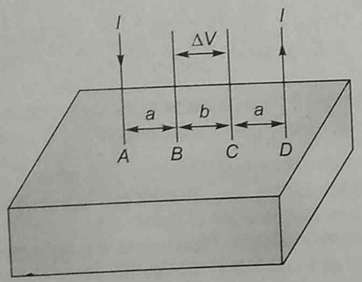
(a) -19×10^{-17} J
(b) 9.60×10^{-17} J
(c) -2.24×10^{-16} J
(d) 2.24×10^{-16} J

54. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the $\frac{Q}{q}$ equals **[AIEEE 2009]**

(a) $-2\sqrt{2}$
(b) -1
(c) 1
(d) $-\frac{1}{\sqrt{2}}$

55. Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps

(i) Take current I entering from A and assume it to spread over a hemispherical surface in the block.
(ii) Calculate field $E(r)$ at distance r from A by using Ohm's law $E = \rho j$, where j is the current per unit area at r.
(iii) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.
(iv) Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.



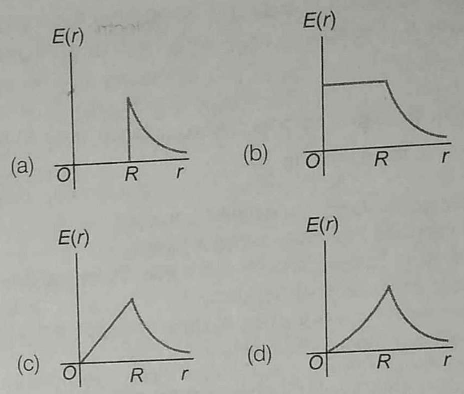
ΔV measured between B and C be **[AIEEE 2008]**

(a) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$
(b) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$
(c) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$
(d) $\frac{\rho I}{2\pi(a+b)}$

56. For current entering at A, the electric field at a distance r from A is **[AIEEE 2008]**

(a) $\frac{\rho I}{8\pi r^2}$
(b) $\frac{\rho I}{r^2}$
(c) $\frac{\rho I}{2\pi r^2}$
(d) $\frac{\rho I}{4\pi r^2}$

57. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graph most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell? **[AIEEE 2008]**



58. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral **[AIEEE 2007]**

(a) a potential difference appears between the two cylinders when a charge density is given to the inner cylinder
(b) a potential difference appears between the two cylinders when a charge density is given to the outer cylinder
(c) no potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders
(d) no potential difference appears between the two cylinders when the same charge density is given to both the cylinders

59. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then **[AIEEE 2007]**

(a) negative and distributed uniformly over the surface of the sphere
(b) negative and appears only at the point on the sphere, closest to the point charge
(c) negative and distributed non-uniformly over the entire surface of the sphere
(d) zero

60. The potential at a point x (measured in μm) due to some charge situated on the x-axis, is given by

$$V(x) = 20 / (x^2 - 4) \text{ volt}$$

The electric field E at $x = 4\,\mu$m is given by **[AIEEE 2007]**

(a) $\frac{5}{3}$ Vμm^{-1} and in the negetive x-direction
(b) $\frac{5}{3}$ Vμm^{-1} and in the negetive x-direction
(c) $\frac{10}{9}$ Vμm^{-1} and in the negetive x-direction
(d) $\frac{10}{9}$ Vμm^{-1} and in the positive x-direction

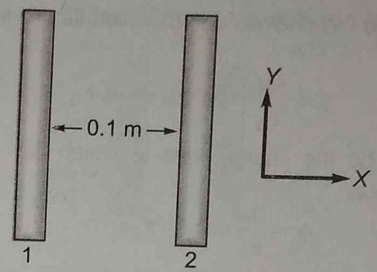
61. A parallel plate condenser with a dielectric of dielectric constant K placed between the plates, has a capacity C and is charged to a potential of V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[AIEEE 2007]**

(a) $\frac{1}{2}(K-1)CV^2$
(b) $CV^2(K-1)/K$
(c) $(K-1)CV^2$
(d) zero

62. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be **[AIEEE 2007]**

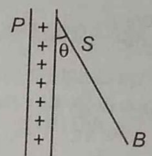
(a) 1 (b) 2
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

63. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (*i.e.*, plate 2 is at a higher potential). The plates are kept at a distance of $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its the speed when it hits plate 2? ($e = 1.6 \times 10^{-19}$ C, $m_0 = 9.11 \times 10^{-31}$ kg) **[AIEEE 2006]**



(a) 2.65×10^6 ms^{-1} (b) 7.02×10^{12} ms^{-1}
(c) 1.87×10^6 ms^{-1} (d) 32×10^{-19} ms^{-1}

64. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P. The surface charge density σ of the sheet is proportional to **[AIEEE 2005]**



(a) $\cos\theta$ (b) $\cot\theta$ (c) $\sin\theta$ (d) $\tan\theta$

65. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is **[AIEEE 2005]**

(a) $(n-1)C$ (b) $(n+1)C$
(c) C (d) nC

Answers

1. (a)	2. (b)	3. (a)	4. (a)	5. (d)	6. (d)	7. (b)	8. (c)	9. (d)	10. (b)
11. (d)	12. (a)	13. (c)	14. (d)	15. (d)	16. (d)	17. (d)	18. (d)	19. (a)	20. (a)
21. (a)	22. (a)	23. (b)	24. (a)	25. (c)	26. (d)	27. (a)	28. (b)	29. (c)	30. (d)
31. (c)	32. (c)	33. (c)	34. (b)	35. (b)	36. (a)	37. (a)	38. (c)	39. (d)	40. (b)
41. (d)	42. (a)	43. (c)	44. (a)	45. (a)	46. (c)	47. (d)	48. (c)	49. (b)	50. (c)
51. (d)	52. (d)	53. (d)	54. (a)	55. (c)	56. (c)	57. (a)	58. (a)	59. (d)	60. (d)
61. (d)	62. (d)	63. (a)	64. (d)	65. (a)					

Hints & Solutions

1. $n = \frac{0.01}{100} \times 5 \times 10^{21} = 5 \times 10^{17}$

$$q = -ne = -5 \times 10^{17}(-1.6 \times 10^{-19}) = +0.08 \text{ C}$$

2. $E = -\frac{dV}{dx} = -\frac{d}{dx}(4x^2) = -8x = -8(1) = -8 \text{ Vm}^{-1}$

Negative sign indicates **E** is along negative direction of x-axis.

3. As, $E = \frac{dV}{dr}$ and around $x = 2$ m, V = constant

$\therefore$ $dV = 0$ and $E = 0$

4. $W = \mathbf{F} \cdot \mathbf{r} = Q\mathbf{E} \cdot \mathbf{r}$

$$= Q(E_1 \hat{\mathbf{i}} + E_2 \hat{\mathbf{j}}) \cdot (a\hat{\mathbf{i}} + b\hat{\mathbf{j}})$$
$$= QE_1a + QE_2b$$
$$= Q(E_1a + E_2b)$$

5. Common potential, $V = \frac{Q + 0}{4\pi\varepsilon_0(r + r')}$

Charge on smaller sphere,

$$4\pi\varepsilon_0 r' \times V = \frac{Qr'}{r + r'}$$

6. Let $-q_1$ be the charge, due to which flux ϕ_1 is entering the surface.

$$\phi_1 = \frac{-q_1}{\varepsilon_0} \quad \text{or} \quad q_1 = -\phi_1 \varepsilon_0$$

Let $+q_2$ be the charge, due to which flux ϕ_2, is leaving the surface.

$$\phi_2 = \frac{q_2}{\varepsilon_0} \text{ or } q_2 = \varepsilon_0 \phi_2$$

Electric charge inside the surface

$$= q_2 - q_1 = \varepsilon_0 \phi_2 + \varepsilon_0 \phi_1 = \varepsilon_0(\phi_2 + \phi_1)$$

7. In equilibrium, electrostatic force of attraction between the plates = restoring forces in the string.

$$\frac{q^2}{2\varepsilon_0 A} = kx$$

$$\frac{(CE)^2}{2\varepsilon_0 A} = k(d - 0.8d) = 0.2kd$$

$$k = \frac{C^2E^2}{2\varepsilon_0 A(0.2d)}$$

Now, $$C = \frac{\varepsilon_0 A}{0.8d}$$

$$k = \frac{\varepsilon_0^2 A^2}{0.64d^2} \frac{E^2}{2\varepsilon_0 A(0.2d)} = \frac{4\varepsilon_0 AE^2}{d^3}$$

8. Original capacity, $C_0 = \frac{\varepsilon_0 A}{d}$

On introducing dielectric slab of thickness $d/2$, the capacity becomes

$$C = \frac{\varepsilon_0 A}{\left(d - \frac{d}{2}\right) + 2\frac{d}{2K}} = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$$

As $$C = \frac{4}{3}C_0$$

$$\therefore \quad \frac{\varepsilon_0 A}{\frac{d}{2}\left(1+\frac{1}{K}\right)} = \frac{4}{3}\frac{\varepsilon_0 A}{d}$$

$$1 + \frac{1}{K} = \frac{3}{2}$$

$$\Rightarrow \quad K = 2$$

9. Let λ be the linear charge density.

Given, distance $r = 2 \text{ cm} = 2 \times 10^{-2}$ m

Electric field $E = 9 \times 10^4$ N/C

Using the formula of electric field due to an infinite line charge.

Electric field due to infinite line charge, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$. Dividing and multiplying by 2 to get $\frac{1}{4\pi\varepsilon_0}$ because, we have the value of $\frac{1}{4\pi\varepsilon_0}$

$$E = \frac{2}{2} \times \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\lambda}{4\pi\varepsilon_0 r}$$

Putting the values, we get

$$9 \times 10^4 = \frac{2 \times 9 \times 10^9 \times \lambda}{2 \times 10^{-2}}$$

$$\lambda = \frac{9 \times 10^4 \times 2 \times 10^{-2}}{2 \times 9 \times 10^9} = 10^{-7} \text{ C/m}$$

Thus, the linear charge density is 10^{-7} C/m.

10. Let the potential of point D be V.

If q is charge on each condenser, then

$$V_1 - V = qC_1 \Rightarrow V - V_2 = qC_2$$

Divide $$\frac{V_1 - V}{V - V_2} = \frac{C_1}{C_2}$$

$$VC_1 - V_2C_1 = V_1C_2 - VC_2$$

$$V(C_1 + C_2) = C_1V_2 + C_2V_1;$$

$$V = \frac{C_1V_2 + C_2V_1}{C_1 + C_2}$$

11. As uniform electric field is parallel to the cylindrical axis

$$\int \mathbf{E} \cdot d\mathbf{S} = \int E\, dS \cos 90° = 0$$

Further flux entering the cylinder at one end = flux leaving the cylinder at other end.

Therefore, total electric flux is zero.

12. Here, $V = (x^2 - y^2)$

$$\mathbf{E} = -\frac{dV}{dr} = \left(-\frac{\partial V}{\partial x}\hat{\mathbf{i}} + -\frac{\partial V}{\partial y}\hat{\mathbf{j}}\right) = -2x\,\hat{\mathbf{i}} + 2y\,\hat{\mathbf{j}}$$

$$E = |\mathbf{E}| = \sqrt{(-2x^2 + (2y)^2} = 2\sqrt{x^2 + y^2}$$

At, $x = 0, \mathbf{E} = 2y\,\hat{\mathbf{j}}$

i.e., $\mathbf{E}$ is along positive direction, for points $y > 0$ and $\mathbf{E}$ is along negative direction, for points $y < 0$

Similarly, at $y = 0, \mathbf{E} = -2x\,\hat{\mathbf{i}}$

i.e., $\mathbf{E}$ is along positive x-direction for points $x > 0$ and $\mathbf{E}$ is along negative x-direction, for points $x < 0$.

13. The capacitor C_2 is shorted, so it is not playing any role in circuit and can be removed. The 3 capacitors each of C_1 are connected in parallel and this is connected to C_3 in series.

$$C_{eq} = \frac{3C_1C_3}{3C_1 + C_3} = C$$

$$= \frac{3 \times 3 \times 1}{3 \times 3 + 1} = 0.9\ \mu\text{F}$$

So, $$\frac{\Delta C}{C} = \frac{3\Delta C_1}{C_1} + \frac{\Delta C_3}{C_3} + \frac{3\Delta C_1 + \Delta C_3}{3C_1 + C_3}$$

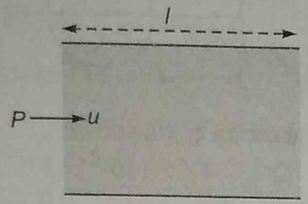
[For computation of errors worst has to be taken]

$$\Rightarrow \quad \frac{\Delta C}{0.9} = \frac{3 \times 0.011}{3} + \frac{0.01}{1} + \frac{(0.033 + 0.01)}{10}$$

$$\Rightarrow \quad \Delta C = \pm\ 0.023\ \mu\text{F}$$

14. $u_x = u_1, t = \frac{l}{u}$

$$a_y = \frac{Eq}{m} = a$$



$$v_y = at = \frac{al}{u}$$

$$v^2 = u^2 + \frac{a^2 l^2}{u^2} = u^2 + \frac{C}{u^2}$$

$$v = \sqrt{u^2 + \frac{C}{u^2}}$$

15. Initial charge $= CV$ and Final charge $= CV' + KCV'$

Since initial charge = final charge

$$K = \frac{V - V'}{V'} K = \frac{V - V'}{V'}$$

16. Force applied at the charge $-q$ by change on one corner $+q$

$$F = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q(-q)}{r^2}$$

By symmetry resultant force applied by eight charges on corners is zero.

17. When we displace the charges within the closed surface, **E** at every point on surface changes but flux crossing through closed surface does not change.

18. As q_{net} (enclosed charge) $= 0$, so $\phi = \oint \mathbf{E} \cdot \mathbf{ds} = 0$, but **E** is not necessarily zero as flux for some part can be positive while or other can be negative.

19. If we displaces charge q. So, charge will induced, then nature of distribution of charge on outer surface of shell changes. While shell are independent of each other in terms of electric field.

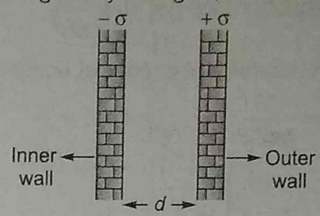
20. When dipole is aligned along the direction of electric field, torque on it, is zero and its electrical potential energy is minimum $(U = -pE)$. Hence, it is a stable equilibrium condition.

21. From $\sigma \propto \frac{1}{R}$ and electric field at conductor's surface, $E = \frac{\sigma}{\varepsilon_0}$.

We can say that Statement I is correct and Statement II is correct explanation for it.

22. Statement II is correct, if we take a conductor then potential inside it and on its surface is same but **E** over the surface is discontinuous. It is zero everywhere inside the conductor and non-zero outside the conductor. So it is not necessary that if $V =$ constant, then $\mathbf{E} = 0$.

23. Inner wall is negatively charged, so it is at a lower potential



24. Electric field between the cell walls

$$E = \frac{\sigma}{k\varepsilon_0} = \frac{\sigma}{5\varepsilon_0} \text{NC}^{-1}$$

25. $dV = -\mathbf{E} \cdot \mathbf{dr}$

$$V_{outer} - V_{inner} = \frac{\sigma}{5\varepsilon_0} \times 10^{-10} \text{ volt}$$

26. The electrostatic force on drop $= qE = neE$ $[\because q = ne]$

The gravitational force on the drop $= mg$ [where, $m =$ mass of the drop] $=$ Volume $\times$ Density $\times\, g$

($\because$ mas = volume $\times$ density)

$$= \frac{4}{3}\pi r^3 \times \rho \times g$$

As the drop is held stationary. So, the net force on the drop is zero.

$\therefore$ Electrostatic force = Gravitational force

$$neE = \frac{4}{3}\pi r^3 \rho g$$

$$r^3 = \frac{3neE}{4\pi\rho g} = \frac{3 \times 12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.8}$$

$$r^3 = 0.94 \times 10^{-18}$$

$$r = (0.94 \times 10^{-18})^{1/3} \times 9.81 \times 10^{-7} \text{ m}$$

Thus, the radius of the drop is 9.81×10^{-7} m.

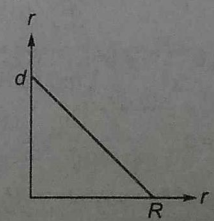
27. At $r = R$, from Gauss's law

$$E(4\pi R^2) = \frac{q_{net}}{\varepsilon_0} = \frac{Ze}{\varepsilon_0} \text{ or } E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze}{R^2}$$

E is independent of a.

28. For $a = 0$

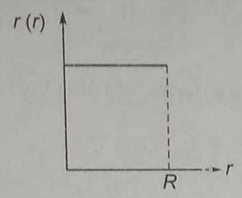
$$\rho(r) = \left(-\frac{d}{R} \cdot r + d\right)$$



Solving this equation, we get $d = \frac{3Ze}{\pi R^3}$

29. In case of solid sphere of charge of uniform volume density

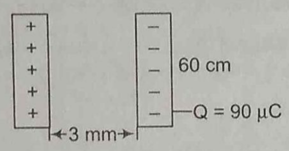
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^3} \cdot r \quad \text{or} \quad E \propto r$$



Thus, for E to be linearly dependent on r, volume charge density should be constant.

or $a = R$

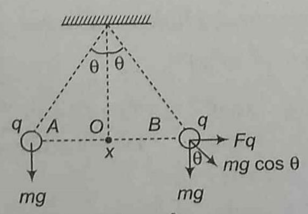
30.



Given, $\frac{dQ}{dt} = 2.5 \times 10^{-8}$ C/s

But in case of capacitor, there is no magnetic field inside the capacitor *i.e.*, zero.

31.



Force of attraction $= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{x^2}$

Potential at the mid-point O

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{x} + \frac{1}{4\pi\varepsilon_0} \frac{q}{x}$$

$$= \frac{1}{2\pi\varepsilon_0} \frac{q}{x} \quad \ldots\text{(i)}$$

From the figure $\tan\theta = \frac{\frac{1}{4\pi\varepsilon_0} \frac{q^2}{x^2}}{mg}$

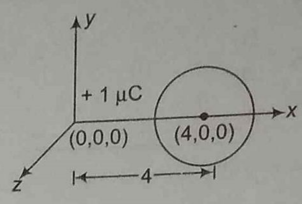
$\therefore \quad mg \tan\theta = k\frac{q^2}{x^2}, \; x = \sqrt{\frac{kq^2}{mg \tan\theta}}$

$$\Rightarrow \quad x = q\sqrt{\frac{k}{mg \tan\theta}} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

Potential $= 4\sqrt{4\,mg / \tan\theta}$

32.



Potential at the centre of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r} = \frac{1}{4\pi\varepsilon_0} \frac{1 \times 10^{-6}}{4}$$

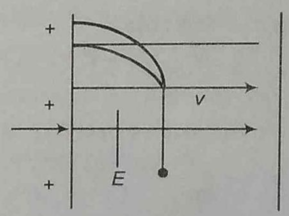
$$= \frac{9 \times 10^9 \times 10^{-6}}{4} = 2.25 \times 10^3 \text{ unit}$$

Now, electric field at the centre of sphere

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} = 9 \times 10^9 \frac{1 \times 10^{-6}}{4^2}$$

$$= 0.5625 \times 10^3 = 5.625 \times 10^2 \text{ units}$$

33.



As the speed of particle is far to the intensity vector, and there is no acceleration in the direction for to **E**, but there is a electric force exerting on the particle (charge) whenever, it motion is in electric field.

Hence, continuously a force exerting on the particle for to its velocity or speed. Hence, path is particle must be parabola. As in the projectile motion.

34. As capacitance, $C_0 = k\frac{\varepsilon_0 A}{d}$

Now equivalent capacitance

$$C = \frac{2K}{3} \frac{\varepsilon_0 A}{d} + \frac{2k}{3} \frac{\varepsilon_0 A}{d}$$

$$= \frac{4k}{3} \frac{\varepsilon_0 A}{d}$$

$$\therefore \quad \text{ratio} = \frac{c}{\varepsilon_0} = \frac{\frac{4k}{3} \frac{\varepsilon_0 A}{d}}{k\frac{\varepsilon_0 A}{d}} = \frac{4}{3}$$

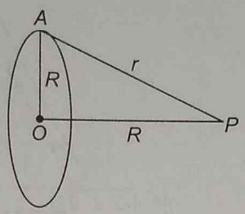
35. As, $W = q(V_A - V_B)$

At equipotential surface

$$V_A = V_B \text{ so, } W = 0$$

Now, we know that field lines makes an angle of 90° with the equipotential surfaces but these are parallel to one-another.

36. Given, $E_0 = \frac{\sigma}{2\varepsilon_0}$



Also, we know that

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{qR}{(R^2+R^2)^{3/2}}$$

but $q = 6A = 6\pi R^2$

$$\Rightarrow \quad E_p = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi R^2}{2\sqrt{2}R^3}$$

$$= \frac{\sigma}{8\sqrt{2}\,\varepsilon_0}$$

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi R^3}{2\sqrt{2}} = \frac{\sigma}{8\sqrt{2}\,\varepsilon_0}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{4\sqrt{2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[\frac{4\sqrt{2}-1}{4\sqrt{2}}\right]$$

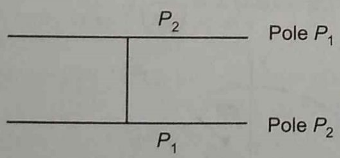
$$= \frac{\sigma}{2\varepsilon_0}\left[4-\frac{1}{\sqrt{2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[4-\frac{\sqrt{2}}{2}\right] = 0.82\frac{\sigma}{2\varepsilon_0}$$

Clearly, reduction is approx 71%.

37. The uncharged body must have a potential less than the charged conductor and more than at infinity.

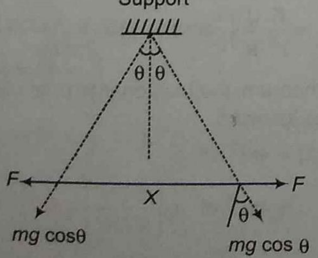
i.e., $V_\infty < V$ or $V > V_\infty$

38. We know that $U = -\frac{1}{4\pi\varepsilon_0}\frac{2P_1P_2}{r^3}$



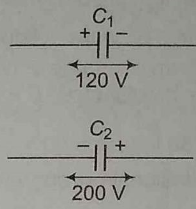
With the help of this relation, we find the force between dipole is $\frac{1}{4\pi\varepsilon_0}\frac{6P_1P_2}{x^3}$

39. $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}$



The resultant of components $mg\cos\theta$ and force of repulsion valances, the tension in the string for the equilibrium massive change.Thus we find the separation between the balls is proportional to $l^{1/3}$ where, l is length of string.

40.



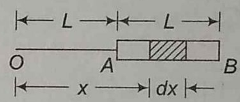
For potential to be made zero after connection, the charge of both capacitors are equal.

$$q_1 = q_2$$

$$\therefore \quad C_1V_1 = C_2V_2,\ 120\,C_1 = 200\,C_2$$

$$\Rightarrow \quad 3C_1 = 5C_2$$

41.



$$V = \int_L^{2L}\frac{kdq}{x} = \int_L^{2L}\frac{\left(\frac{q}{L}\right)dx}{x}$$

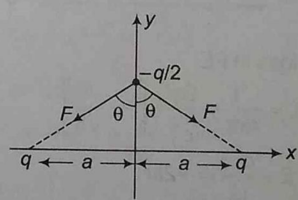
$$= \frac{q}{4\pi\varepsilon_0 L}\int_L^{2L}\left(\frac{1}{x}\right)dx$$

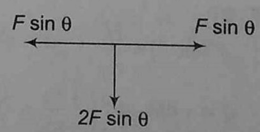
$$= \frac{q}{4\pi\varepsilon_0 L}[\log_e x]_L^{2L}$$

$$= \frac{q}{4\pi\varepsilon_0 L}[\log_e 2L - \log_e L]$$

$$= \frac{q}{4\pi\varepsilon_0 L}\left[\log_e\frac{2L}{L}\right] = \frac{q}{4\pi\varepsilon_0 L}\ln(2)$$

42.





$$F_{\text{net}} = 2F\cos\theta$$

$$F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2+a^2})^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}}$$

$$\Rightarrow \quad \frac{kq^2y}{a^3} \propto y$$

43. Electric field inside the uniformly charged sphere varies linearly, $E = \frac{kQ}{R^3} \cdot r, (r \le R)$, while outside the sphere, it varies as inverse square of distance, $E = \frac{kQ}{r^2}$; $(r \ge R)$ which is correctly represented in option (c).

44. Statement 1 is dimensionally wrong while from Gauss's law,

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0}$$

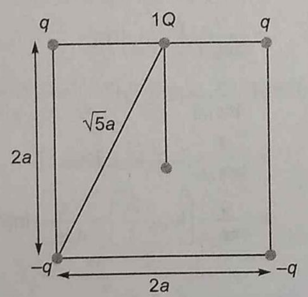
Given statement 2 is correct.

45. $U_i = \frac{2kqQ}{a} + \frac{2k(-q)Q}{\sqrt{5}a}$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

$U_f = 0$

By conservation of energy,



Gain in KE = loss in PE

$$K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

46. Electric field, $E = \frac{d\phi}{dt} = -2ar$

By Gauss's theorem,

$$E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad q = -8\pi\varepsilon_0 a r^3$$

$$\rho = \frac{dq}{dV} = \frac{dq}{dr} \times \frac{dr}{dV}$$

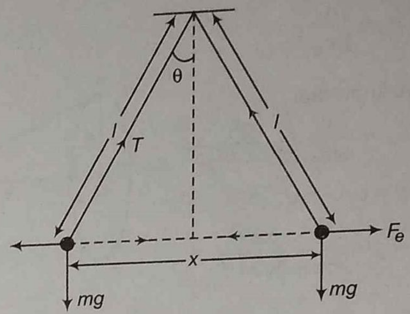
$$= (-24\,\pi\varepsilon_0 a r^2)\left(\frac{1}{4\pi r^2}\right)$$

$$= -6\varepsilon_0 a$$

47. At any instant,

$$T\cos\theta = mg \quad \ldots\text{(i)}$$

$$T\sin\theta = F_e = \frac{ka^2}{x^2} \quad \ldots\text{(ii)}$$



From Eqs. (i) and (ii), we get

$$\frac{ka^2}{x^2} = mg\tan\theta \quad (a = \text{magnitude of charge})$$

$$\Rightarrow \quad q^2 = \frac{mg}{k} \cdot \frac{x}{2l} \cdot x^2 \quad \left(\because \tan\theta = \frac{x}{2l}\right)$$

$$\Rightarrow \quad q^2 = \frac{mg}{2kl}x^3 \quad \ldots\text{(iii)}$$

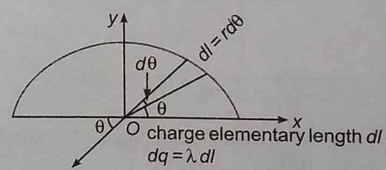
$$\Rightarrow \quad 2q\frac{dq}{dt} = \frac{3mg}{2kl}x^2\frac{dx}{dt}$$

$$\Rightarrow \quad 2\left(\frac{mg}{2kl}x^3\right)^{1/2}\frac{dq}{dt} = \frac{3mg}{2kl}x^2 v \quad \left[\because q = \left(\frac{mg}{2kl}x^3\right)^{1/2}\right]$$

$$\Rightarrow \quad vx^{1/2} = \text{constant}$$

$$\Rightarrow \quad v \propto x^{-1/2}$$

48. Linear charge density $\lambda = \left(\frac{q}{\pi r}\right)$



Net field at O, $E = \int dE\sin\theta\,(-\hat{j}) = \int \frac{K \cdot dq}{r^2}\sin\theta\,(-\hat{j})$

$$E = \frac{K}{r^2}\int\frac{qr}{\pi r}d\theta\sin\theta\,(-\hat{j}) \quad (\because dl = rd\theta$$

$$= \frac{K}{r^2}\frac{q}{\pi}\int_0^\pi \sin\theta\,(-\hat{j}) = \frac{q}{2\pi^2\varepsilon_0 r^2}(-\hat{j})$$

49. Apply shell theorem, the total charge upto distance r can be calculated as followed

$$dq = 4\pi r^2 \cdot dr \cdot \rho$$

$$= 4\pi r^2 \cdot dr \cdot \rho_0\left[\frac{5}{4} - \frac{r}{R}\right]$$

$$= 4\pi\rho_0\left[\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right]$$

$$\int dq = q = 4\pi\rho_0\int_0^r\left(\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right)$$

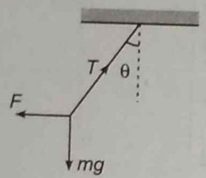
$$= 4\pi\rho_0\left[\frac{5}{4}\frac{r^3}{3} - \frac{1}{R}\frac{r^4}{4}\right]$$

$$E = \frac{kq}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cdot 4\pi\rho_0\left[\frac{5}{4}\left(\frac{r^3}{3}\right) - \frac{r^4}{4R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

50. From FBD of sphere, using Lami's theorem



$$\frac{F}{mg} = \tan\theta \qquad \text{...(i)}$$

When suspended in liquid, as θ remains same,

$$\therefore \quad \frac{F'}{mg\left(1 - \frac{\rho}{d}\right)} = \tan\theta \qquad \text{...(ii)}$$

Using Eqs. (i) and (ii), we get

$$\frac{F}{mg} = \frac{F'}{mg\left(1 - \frac{\rho}{d}\right)} \quad \text{where, } F' = \frac{F}{K}$$

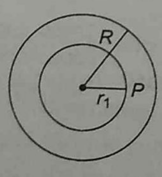
$$\therefore \quad \frac{F}{mg} = \frac{F}{mgK\left(1 - \frac{\rho}{d}\right)}$$

or $$K = \frac{1}{1 - \frac{\rho}{d}} = \frac{1}{\left(1 - \frac{0.8}{1.6}\right)} = 2$$

51. Work done by conservative force does not depend on the path. Electrostatic force is a conservative force.

52. In figure, dotted sphere or radius r_1 is the Gaussian surface.

According to Gauss' theorem,



$$\oint \mathbf{E}.d\mathbf{s} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad Eds\cos 0° = \frac{1}{\varepsilon_0}\rho(r) \times dv$$

$$E \times (4\pi r_1^2) \times 1 = \frac{Qr_1}{\varepsilon_0 \pi R^4} \times \left[\frac{4}{3}\pi r_1^3\right]$$

or $$E = \frac{Q.4\pi r_1^4}{3\varepsilon_0.\pi R^4(4\pi r_1^2)}$$

$$= \frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$$

53. $W = QdV = Q(V_q - V_p)$

$$= -100 \times (1.6 \times 10^{-19}) \times (-4 - 10)$$

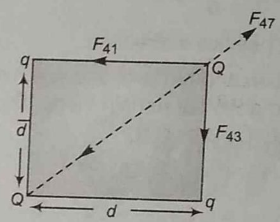
$$= +100 \times 1.6 \times 10^{-19} \times 14$$

$$= +2.24 \times 10^{-16}\text{ J}$$

54. Three forces F_{41}, F_{42} and F_{43} acting on Q as shown resultant of $F_{41} + F_{43}$

$$= \sqrt{2}F_{each}$$

$$= \sqrt{2}\,\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}$$



Resultant on Q becomes zero only when q charges are of negative nature.

$$F_{42} = \frac{1}{4\pi\varepsilon_0}\frac{Q \times Q}{(\sqrt{2}d)^2}$$

$$\Rightarrow \quad \sqrt{2}\,\frac{dQ}{d^2} = \frac{Q \times Q}{2d^2}$$

$$\Rightarrow \quad \sqrt{2} \times q = \frac{Q \times Q}{2}$$

$$\therefore \quad q = \frac{Q}{2\sqrt{2}}$$

or $$\frac{Q}{q} = -2\sqrt{2}$$

55. Choosing A as origin,

$$E = \rho j = \rho\frac{I}{2\pi r^2}$$

$$\Rightarrow \int dv = -\int Edr$$

$$V_C - V_B = \frac{\rho I}{2\pi}\int_a^{(a+b)}\frac{1}{r^2}dr = \frac{\rho I}{2\pi}\left[\frac{1}{(a+b)} - \frac{1}{a}\right]$$

$$V_B - V_C = \frac{\rho I}{2\pi}\left[\frac{1}{a} - \frac{1}{(a+b)}\right]$$

56. Electric field at a distance r from A is

From, $E = \rho j = \dfrac{\rho I}{2\pi r^2}$

57. $$E(r) = \begin{cases} 0 & ;\text{ if } r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & ;\text{ if } r \geq R \end{cases}$$

58. There will be an electric field between two cylinders (using Gauss's theorem). This electric field will produce a potential difference.

59. Charge will be induced in the conducting sphere, but net charge on it will be zero.

60. $\mathbf{E} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$

$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x} = -\frac{d}{dx}\left[\frac{20}{x^2-4}\right]$$

$$= \frac{40x}{(x^2-4)^2}$$

$$\Rightarrow \quad E_x \text{ at } x = 4\,\mu\text{m}$$

$$= \frac{10}{9}\,\text{V}\mu\text{m}^{-1}$$

and is along positive x-direction.

61. On introduction and removal and again on introduction, the capacity and potential remain same. So, net work done by the system in this process

$$W = U_f - U_i$$

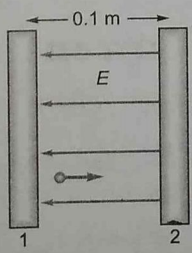
$$= \frac{1}{2}CV^2 - \frac{1}{2}CV^2 = 0$$

62. Ratio of energy stored in the capacitor and the work done by the battery

$$= \frac{\frac{1}{2}qV}{qV} = \frac{1}{2}$$

63. Since $V_2 > V_1$, so electric field will point from plate 2 to plate 1.

The electron will experience an electric force, opposite to the direction of electric field, and hence move towards the plate 2.



Use work-energy theorem to find speed of electron when it strikes the plate 2.

$$\frac{m_e v^2}{2} - 0 = e(V_2 - V_1)$$

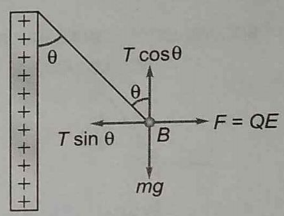
where v is the required speed.

$$\therefore \quad \frac{9.11\times 10^{-31}}{2}v^2 = 1.6\times 10^{-19}\times 20$$

$$\text{or} \quad v = \sqrt{\frac{1.6\times 10^{-19}\times 40}{9.11\times 10^{-31}}}$$

$$= 2.65\times 10^6\ \text{ms}^{-1}$$

64. Electric field due to a charged conducting sheet of surface charge density σ is given by $E = \frac{\sigma}{\varepsilon_0\varepsilon_r}$



where ε_0 is the permittivity in vacuum and ε_r the relative permittivity of medium.

Here, electrostatic force on B

$$QE = \frac{Q\sigma}{\varepsilon_0\varepsilon_r}$$

FBD of B is shown in figure.

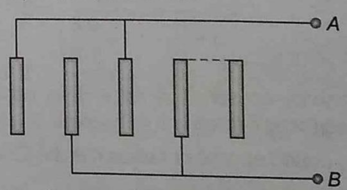
In equilibrium, $\quad T\cos\theta = mg$

and $\quad T\sin\theta = \frac{Q\sigma}{\varepsilon_0\varepsilon_r}$

Thus, $\quad \tan\theta = \frac{Q\sigma}{\varepsilon_0\varepsilon_r mg}$

$$\Rightarrow \quad \tan\theta \propto \sigma$$

65. Each plate is taking part in the formation of two capacitors except the plates at the ends.



These capacitors are in parallel and n plates form $(n-1)$ capacitors.

Thus, equivalent capacitance between A and B

$$= (n-1)C$$