Day 19

Current Electricity

Day 19

Outlines ...

Electric Current

***Electric current** is defined as the time rate of flow of electric charge through a cross-section of the conductor.*

If a charge Δq passes through the area in time interval Δt at uniform rate, then current i is defined as instantaneous value of current is given by

$$I_{\text{ins}} = \lim_{\Delta t \to 0} \left(\frac{\Delta q}{\Delta t} \right) = \frac{dq}{dt}$$

SI unit of electric current is Ampere (A).

(i) Conventional direction of flow of current is taken to be the direction of flow of positive charge or opposite to the direction of flow of negative charge.

(ii) Electric current is a scalar as it does not follow the vector law of addition.

(iii) If n-particles, each carrying a charge q, pass through a given cross-section in time t, then current $I = \frac{nq}{t}$.

As an example, 1 A current is equivalent to flow of 6.25×10^{18} electron/s

Current Density

For flow of charge through a cross-section, current density **J** at a point is defined as a vector of magnitude equal to current per unit area around that point and directed in the direction of flow of positive charge.

Thus, $$\mathbf{J} = \frac{dI}{ds}\,\hat{n}$$

or $$I = \int \mathbf{J} \cdot d\mathbf{s}$$

Drift Velocity

Drift velocity is the average uniform velocity acquired by conduction electrons inside a metallic conductor on application of an external electric field.

Due to this drift of electrons there is a net transfer of charge across a cross-section resulting in an electric current flow.

The drift velocity is given by the relation

$$\mathbf{v}_d = -\frac{e\mathbf{E}}{m}\tau$$

where, τ known as relaxation time, is the mean value of time between two successive collisions of an electron with ions in the conductor.

Drift velocity per unit electric field is called the **mobility** of the electrons. Thus, **mobility**

$$\mu = \left|\frac{v_d}{E}\right| = \frac{e}{m}\tau$$

In terms of drift speed, electric current flowing through a conductor is expressed as $I = nAev_d$

where, A = cross-section area of conductor and
n = number of conduction electrons per unit volume
v_d = drift velocity of electrons
e = charge of 1 electron.

Ohm's Law

According to Ohm's law, physical conditions (temperature, mechanical strain, etc.) remaining unchanged, the current flowing through a conductor is directly proportional to the potential difference across its ends.

Thus, $$I \propto V \text{ or } V = IR$$

where, R is a constant known as the **electrical resistance** of given conductor.

Electrical Resistance

The obstraction offered by any conductor in the path of flow of current is called **electrical resistance** (R). It is equal to the ratio of the potential difference (V) applied across the ends of the conductor and the electric current (I) flowing through it.

$$R = \frac{V}{I}$$

Its unit is Ohm (Ω).

In terms of free electron density n and relaxation time τ, *the resistance of a conductor is given by*

$$R = \frac{m}{ne^2\tau}\frac{l}{A}$$

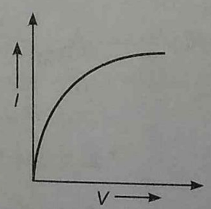
Resistance of Different Materials

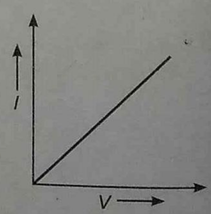
Resistance offered by the conductors is minimum while resistance offered by an insulator is maximum. Semiconductors have resistance which is intermediate to conductor and insulator.

V-I Characteristics of Ohmic and Non-ohmic Conductors

Substances obeying Ohm's law are called **ohmic resistors**, *e.g.*, metals and their alloys. *V-I* graph for an ohmic resistor is an inclined straight line.

Substances which do not obey Ohm's law are called **non-ohmic resistors**. *e.g.*, electrolytes, gases, thermionic tubes, transistors, rectifiers, *etc.*, *V-I* graph for them is non-linear.





Electrical Resistivity

The resistivity of the material of a given conductor is defined as the resistance offered by a piece of that substance having unit length and having unit cross-section area.

SI unit of resistivity is Ω m and its dimensional formula is $[ML^3T^{-3}A^{-2}]$.

In terms of free electron density n and relaxation time τ resistivity

$$\rho = \frac{m}{ne^2\tau}$$

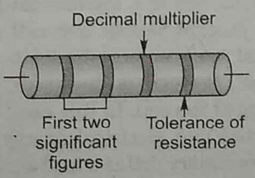
Resistivity is a characteristic of material, *i.e.*, it doesn't depend upon dimensions of the material. Although it depends on temperature.

Colour Code for Resistors

The value of resistance used in electric and electronic circuit vary over a wide range.

Such high resistances used are usually carbon resistances and the values of such resistances are marked on them according to a **colour code**.

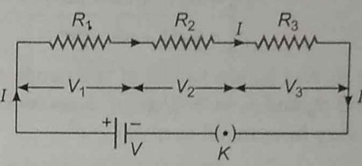
Colour	Figure
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9



Colour	Tolerance
Gold	5%
Silver	10%
No colour	20%

Series and Parallel Combinations of Resistors

- **Series Grouping** In series grouping of resistances same current I flows through all the resistances.



The potential difference applied across the combination is distributed across various resistors in the direct ratio of their resistances,

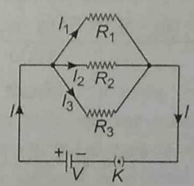
i.e., $V = V_1 + V_2 + V_3 + \ldots$

and $V_1 : V_2 : V_3 \ldots = R_1 : R_2 : R_3 \ldots$

Total equivalent resistance in series grouping is equal to the sum of individual resistances. Thus,

$$R_s = R_1 + R_2 + R_3 + \ldots$$

- **Parallel Grouping** In parallel grouping same potential difference V appears across each resistance.



The current is distributed among various resistors in the inverse ratio of their resistances. Thus,

$$I = I_1 + I_2 + I_3 + \ldots$$

and $I_1 : I_2 : I_3 \ldots = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$

Equivalent resistance in parallel grouping R_p is given by $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

- *If n identical resistances are first connected in series and then in parallel, the ratio of the equivalent resistance* $\frac{R_s}{R_p} = \frac{n^2}{1}$ *or* $\frac{R_p}{R_s} = \frac{1}{n^2}$
- *If there are n resistors of different values, then we can have 2^n different possible combinations.*
- *If a skeleton cube is made with 12 equal resistances, each having a resistance R, then the net resistance across (a) the diagonal of cube = 5/6 R (b) the diagonal of a face = 3/4 R and (c) along a side = 7/12 R*

Temperature Dependence of Resistance

Resistance and resistivity of metallic conductors increases with increase in temperature. The relation is written as

$$R_\theta = R_0(1 + \alpha\theta + \beta\theta^2)$$

and $$\rho_\theta = \rho_0(1 + \alpha\theta + \beta\theta^2)$$

where R_0 and ρ_0 are values of resistance and resistivity at $0°C$ and R_θ and ρ_θ at $\theta°C$. α and β are two constants whose value vary from metal to metal.

Electric Energy and Power

The total work done (or energy supplied) by the source of emf in maintaining the electric current in the circuit for a given time is called **electric energy** consumed in the circuit.

SI unit of electric energy is Joule but another unit is watt-hour.

The bigger unit of electric energy is kilowatt-hour (kWh). It is known as **Board of Trade Unit** (BTU).

The electric energy consumed in kWh is given by

$$W = \frac{V \text{ (in volt)} \times I \text{ (in amp)} \times t \text{ (in hour)}}{1000}$$

Whenever the electric current is passed through a conductor, it becomes hot after short time. This indicates that the electric energy is being converted into heat energy. This effect is known as **heating effect** of current or **Joule heating effect.**

$$H = W = I^2Rt \text{ joule} = \frac{I^2Rt}{4.18} \text{ cal}$$

The rate at which work is done by the source of emf in maintaining the effect of current in a circuit is called electric power of the circuit

$$P = VI$$

where, V is the potential difference across the conductor, I is the current flowing through the conductor.
Other expressions for power

$$P = I^2R$$

$$\Rightarrow \quad P = \frac{V^2}{R}$$

SI unit of electric power is Watt.
Another inportant unit is House Power (HP),
1 HP = 746 Watt.

Electric Cell

An electric cell is a device which maintains a continuous flow of charge (or electric current) in a circuit by a chemical reaction. In an electric cell, there are two rods of different metals called electrodes.

Types of Electric Cell

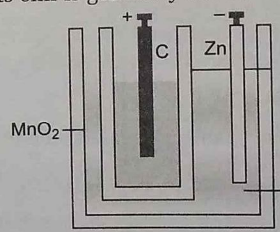
There are two basic types of electric cell

1. **Primary Cell**

   A cell is called primary, if it used only for discharge. The current leaves the cell at the positive (+) terminal goes through the external circuit and enters the cell at the negative (–) terminal, Examples of primary cells are daniell cell, laclanche cell and dry cell.

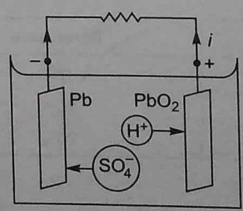
2. **Dry cell**

   The most popular cell is dry cell. It is a special type of laclanche cell in which both NH_4Cl are MnO_2 are prepared in the form of a paste. The paste is contained in a zinc container which is negative electrode. The internal resistance of a dry cell is very small generally 0.1 Ω. Its emf is generally 1.5 V.

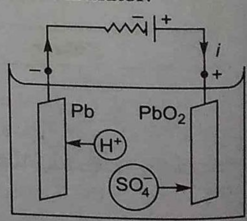


3. **Secondary Cell**

   In a secondary cell, the current pass in both directions When current leaves the cell at the positive (+ve) terminal and enters the cell at the negative (–ve) terminal, the cell is discharge. This is called normal working of the cell. In this case, the chemical energy is converted into electrical energy. The most commonly used secondary cell is a lead accumulator.



(a) Discharging



(b) Charging

A lead commulator consists of electrodes made of PbO_2 and Pb immered is dil. H_2SO_4. Discharging process, the SO_4^{2-} ions move towards Pb electrode give negative charge and H^+ ions move to the PbO_2 electrode, given up positive charge.

$$PbO_2 + 2H \longrightarrow PbO + H_2O$$

The PbO so formed reacts with dil. H_2SO_4 to get $PbSO_4$ and H_2O

i.e., $$PbO + H_2SO_4 \xrightarrow{\text{dil}} PbSO_4 + H_2O$$

Therefore, $PbSO_4$ is formed at the both electrodes. In charging process, a current forced from the positive to the negative inside the cell. The H^+ ions move towards the negative electrode and react with the $PbSO_4$.

$$PbSO_4 + 2H \longrightarrow Pb + H_2SO_4$$

At the positive electrode, the reactions

$$PbSO_4 + SO_4 + 2H_2O \longrightarrow PbO_2 + 2H_2SO_4$$

Electromotive Force (EMF) of Cell

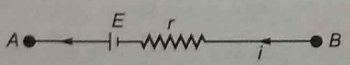
The potential difference between the two poles of the cell in an open circuit is called the electromotive force (emf) of the cell. It is denoted by E. Its S.I. unit is volt (V) or joule coloumb^{-1} (JC^{-1})

Internal Resistance

The potential difference across a real source in a circuit is not equal to the emf of the cell. The reason is that charge moving through the electrolyte of the cell encounters resistance. We call this the internal resistance of the source, denoted by r. As the current moves through r, it experiences associated drop in potential equal to ir. Thus, when a current is drawn through a source, the potential difference between the terminal of the source is

$$V = E - ir$$

This can also be shown as below



$$V_A - E + ir = V_B$$

or $$V_A - V_B = E - ir$$

Following three special cases are possible

(i) If the current flows in opposite direction (as in case of charging of battery), then $V = E + ir$

(ii) $V = E$, if the current through the cell is zero.

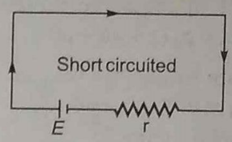
(iii) $V = 0$, if the cell is short circuited.

This is because current in the circuit

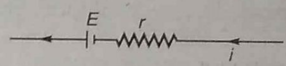
$$i = \frac{E}{r} \quad \text{or} \quad E = ir$$

$$\therefore \quad E - ir = 0$$

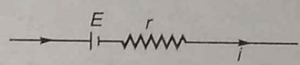
or $$V = 0$$



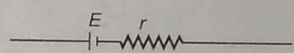
Thus, we can summarise it was follows



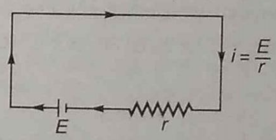
$$V = E - ir \text{ or } V < E$$



$$V = E + ir \text{ or } V > E$$



$$V = E, \text{ if } i = 0$$



$V = 0$ is short circuited

Potential Difference *and* Emf of a Cell

Emf of a cell is the terminal potential difference of cell when it is in an **open circuit,** *i.e.*, it is not supplying any current to the external circuit. However, when it is supplying a current to an external resistance, the voltage across the terminals of cell is called the **terminal voltage** or **terminal potential difference.**

If E be the emf and r the internal resistance of a cell and a resistance R is joined with it, then current in the circuit $I = \dfrac{E}{R + r}$ and terminal potential difference

$$V = IR = \frac{ER}{(R + r)} \text{ or } V = E - Ir$$

Internal resistance of cell

$$r = \left(\frac{E - V}{V}\right)R = R\left(\frac{E}{V} - 1\right)$$

Terminal voltage is more than EMF of cell when cell is charged and it is given by $V = E + Ir$

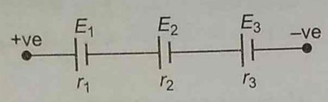
Combination of Cells in Series *and* in Parallel

A group of cells is called a battery. *Two common grouping of cells are*

1. Series Grouping

In series grouping if all the cells are joined so as to supply current in the same direction, then resultant emf

$$E_{eq} = E_1 + E_2 + E_3 + \ldots$$



However, if one or more cells are joined so as to supply current in reverse direction, then emf of that/those cells is taken as negative while calculating the equivalent emf.

The equivalent internal resistance of cell

$$r_{eq} = r_1 + r_2 + r_3 + \ldots$$

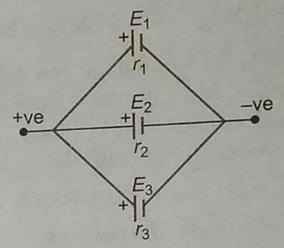
If n cells, each of emf E and internal resistance r, are joined in series, then

$$E_{eq} = nE \quad \text{and} \quad r_{eq} = nr$$

- *If in series grouping of n identical cells 'm' cells are connected with wrong polarity, then net emf of combination is (n – 2m) E but total internal resistance is still nr.*
- *In series grouping of cells their emf's are additive or subtractive whole their internal resistances are always additive.*

2. Parallel Grouping

In parallel grouping if positive terminals of all cells have been joined at one point and all negative terminals at another point, then $\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots$



The equivalent emf of the parallel grouping is given by

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \ldots$$

If n cells, each of emf E and internal resistance r, all joined in parallel, then $\quad r_{eq} = \frac{r}{n}$

But $\quad E_{eq} = E$

Kirchhoff's Laws *and* their Applications

*Sometimes complex electric circuits cannot be reduced to simple series parallel combination. For analysing such circuits **Kirchhoff gave two laws,** which are as follows*

1. Junction Law

The algebraic sum of the currents flowing into any junction is zero. Thus, $\sum_{\text{junction}} I = 0$

Sign Convention

Current carrying into the junction is taken as positive while current going out is taken as negative.

2. Loop Law

The algebraic sum of the potential differences in any closed loop is equal to zero.

Thus, $\sum_{\text{Closed loop}} \Delta V = 0$

$\Rightarrow \quad \Sigma E + RI = 0$

or $\quad \Sigma E = -\Sigma RI$

Sign Convention

(i) In a loop when we traverse through a source in the direction from negative terminal to positive terminal, emf is considered positive.

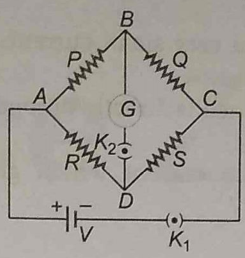
And from positive to negative terminal, its is taken negative.

(ii) When we traverse a resistor in the assumed direction of current IR is taken as negative and in its reverse direction, IR is taken as positive.

- *Kirchhoff's first law is based on conservation of electric charge.*
- *Kirchhoff's second law is based on the conservation of energy.*

Wheatstone's Bridge

It is a sensitive arrangement to determine the value of an unknown resistance. The bridge is said to be balanced if on switching the keys K_1 and K_2 there is no deflection in galvanometer. It is possible when $V_B = V_D$. In balanced condition

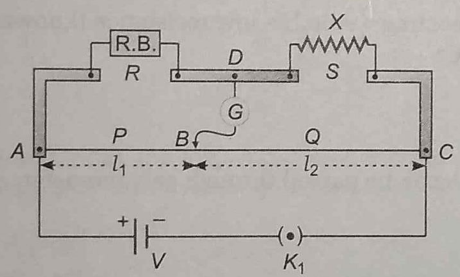


$$\frac{P}{Q} = \frac{R}{S}$$

In balanced bridge Q and R can be interchanged without affecting the balance condition. Similarly, P and S can be interchanged. Moreover, cell and galvanometer may also be **interchanged**. If an electric circuit resembles a balanced Wheatstone bridge, then resistance of branch BD may be ignored (or removed from the circuit) as no current is flowing through it.

Meter Bridge

A meter bridge is a practical arrangement to realise Wheatstone's bridge.

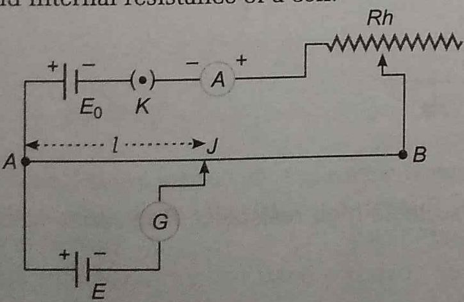


If by sliding the tapping point a null point is obtained on bridge wire at point B, then in balanced condition

$$\frac{R}{X} = \frac{P}{Q} = \frac{l_1}{l_2} \Rightarrow X = R\frac{l_2}{l_1}$$

Potentiometer

Potentiometer is an instrument which can be used for different electric measurements. It is commonly used to find emf of a given cell and to find internal resistance of a cell.



Here, AB is a long, uniform resistance wire (length AB may be ranging from 1 m to 10 m). E_0 is a battery whose emf is known supplying a constant current I for flow through the potentiometer wire. If R be the total resistance of potentiometer wire and L its total length, then potential gradient, *i.e.*, fall in potential per unit length along the potentiometer will be

$$k = \frac{V}{L} = \frac{IR}{L}$$

$$= \frac{E_0 R}{(R_0 + R)\,L}$$

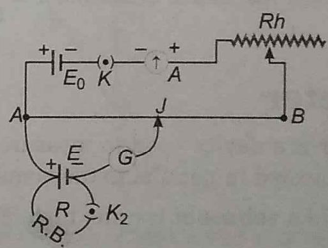
where, E_0 = emf of battery,

R_0 = resistance inserted by means of rheostat Rh.

Applications of Potentiometer

The several applications of potentiometer are given below

- **Determination of Emf of a Cell** If with a cell of emf E on sliding the contact point we obtain zero deflection in galvanometer G when contact point is at J at a length l from the end where positive terminal of cells have been joined, then fall in potential along length l is just balancing the emf of cell. Thus, we have $E = kl$
- **Comparison of Emfs of Two Cells** If with a given potentiometer arrangement we obtain balancing lengths l_1 and l_2 for cells of emfs E_1 and E_2, then
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$
- **Determination of Internal Resistance of a Cell** The arrangement is shown in figure. If the cell E is in open circuit and balancing length is l_1, then



$$E = kl_1$$

But if by inserting key K_2 circuit of cell is closed, then potential difference V is balanced by a length l_2 of potentiometer, where $V = kl_2$

⇒ Internal resistance of cell

$$r = \frac{E - V}{V} R$$

$$= \frac{l_1 - l_2}{l_2} R$$

Galvanometer

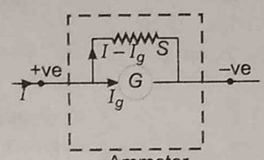
It is a sensitive instrument used to detect and measure very small currents even of the order of few micro ampere.

In a common moving coil galvanometer deflection obtained is directly proportional to the current passed, *i.e.*, $I \propto \theta$.

Figure of merit of a galvanometer is defined as the current which gives one division deflection in galvanometer.

Ammeter

An ammeter is a device used to measure current directly in ampere or its submultiples. An ammeter is always connected in series with the element, current through which is to be measured.



Ammeter

Resistance of an ammeter is extremely small. For an ideal ammeter its resistance is zero.

A galvanometer may be converted into an ammeter of rating I by connecting a suitable low resistance (known as shunt S) in parallel with the galvanometer. Value of shunt resistance

$$S = \frac{GI_g}{I - I_g}$$

where I_g = maximum safe current (full scale deflection current) which can be passed through galvanometer, I = range of ammeter, G = resistance of galvanometer.

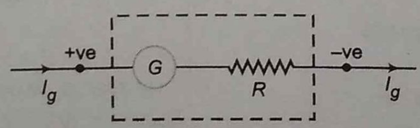
If $I = nI_g$, then shunt $S = \frac{G}{(n-1)}$

The equivalent resistance of ammeter $= \frac{GS}{G+S}$

Voltmeter

A voltmeter is a device used to measure potential difference across a circuit element in volts. A voltmeter is always connected in parallel to the element.

Resistance of a voltmeter is quite high. For an ideal voltmeter its resistance is taken as infinite.



Voltmeter

A galvanometer may be converted into a voltmeter by connecting a suitable high resistance R in series with galvanometer.

Value of series resistance $R = \frac{V}{I_g} - G$, where, V = range of voltmeter.

The equivalent resistance of voltmeter $= G + R$.

Practice Zone DAY 19

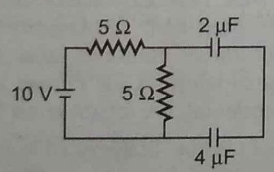
1. Two wires of the same material but of different diameters carry the same current I. If the ratio of their diameters is 2:1, then the corresponding ratio of their mean drift velocities will be
   (a) 4 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 4

2. There are two concentric spheres of radius a and b respectively. If the space between them is filled with medium of resistivity ρ, then the resistance of the intergap between the two spheres will be
   (a) $\frac{\rho}{4\pi(b+a)}$ (b) $\frac{\rho}{4\pi}\left(\frac{1}{b}-\frac{1}{a}\right)$
   (c) $\frac{\rho}{4\pi}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$ (d) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

3. A given resistor has the following colour scheme of the various strips on it brown, black, green and silver. Its value in ohm is
   (a) $1.0\times 10^4 \pm 10\%$ (b) $1.0\times 10^5 \pm 10\%$
   (c) $1.0\times 10^6 \pm 10\%$ (d) $1.0\times 10^7 \pm 10\%$

4. A galvanometer of resistance 15 Ω gives full scale deflection for a current of 2 mA. Calculate the shunt resistance needed to convert it into an ammeter of range 5 A.
   (a) 0.178 Ω (b) 0.002 Ω (c) 0.006 Ω (d) 5 Ω

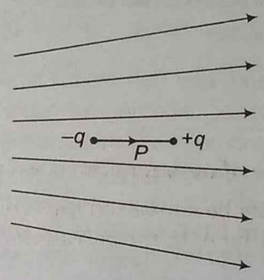
5. The charge on the 4 μF capacitor in the steady state, is



   (a) $\frac{20}{3}$ μC (b) $\frac{10}{3}$ μC (c) $\frac{5}{6}$ μC (d) 10μC

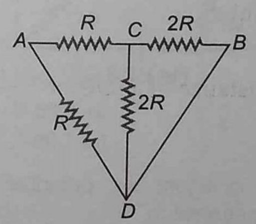
6. A point charge $+q$, is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of plane is **[NCERT Exemplar]**
   (a) directed perpendicular to the plane and away from the plane
   (b) directed perpendicular to the plane but towards the plane
   (c) directed radially away from point charge
   (d) directed radially towards the point charge

7. Figure shows electric field lines in which an electric dipole P is placed as shown. Which of the following statements is correct? **[NCERT Exemplar]**



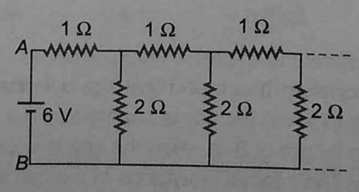
   (a) The dipole will not experience any force
   (b) The dipole will experience a force towards right
   (c) The dipole will experience a force towards left
   (d) The dipole will experience a force upwards

8. The effective resistance between points A and B is



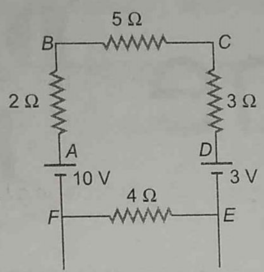
   (a) R (b) $R/3$
   (c) $2R/3$ (d) $3R/5$

9. An infinite ladder network of resistances is constructed with 1 Ω and 2 Ω resistance as shown in figure. The 6 V battery between A and B has negligible internal resistance then effective resistance between A and B is



   (a) 3 Ω (b) 2 Ω
   (c) 6/5 Ω (d) 5/6 Ω

10. In the circuit shown in figure. The point F is grounded. Which of the following is wrong statement?

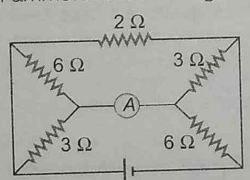


(a) D is at 5 V
(b) E is at zero potential
(c) The current in the circuit will be 0.5 A
(d) None of the above

11. The 80 Ω galvanometer deflects full scale for a potential of 20 mV. A voltmeter deflecting full scale of 5V is to made using this galvanometer. We must connect

(a) a resistance of 19.92 kΩ parallel to the galvanometer
(b) a resistance of 19.92 kΩ in series with the galvanometer
(c) a resistance of 20 kΩ parallel to the galvanometer
(d) a resistance of 20 kΩ in series with galvanometer

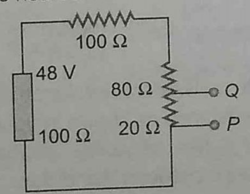
12. The reading of ammeter shown in figure is



(14 V 0.8 Ω)

(a) 6.56 A
(b) 3.28 A
(c) 2.18 A
(d) 1.09 A

13. In the circuit in figure the potential difference across P and Q will be nearest to



(a) 9.6 V
(b) 6.6 V
(c) 4 V
(d) 3.2 V

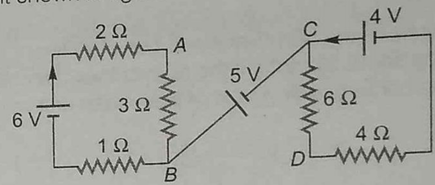
14. In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a 4 Ω resistance; and is at a length of 3 m when the cell is shunted by a 8 Ω resistance. The internal resistance of the cell is, then

(a) 12 Ω
(b) 8 Ω
(c) 16 Ω
(d) 1 Ω

15. A voltmeter having resistance of 1800 Ω is employed to measure the potential difference across 200 Ω resistance which is connected to DC power supply of 50V and internal resistance 20 Ω. What is the percentage change in potential difference across 200 Ω resistance as a result of connecting voltmeter across it?

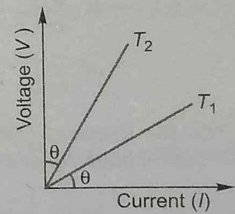
(a) 1%
(b) 5%
(c) 10%
(d) 20%

16. What is the potential difference between points A and D of circuit shown in figure?



(a) 5 V
(b) 9 V
(c) 10.4 V
(d) 11.4 V

17. The V-I graph for a conductor at temperatures T_1 and T_2 are as shown in the figure, The term $T_2 - T_1$ is proportional to



(a) $\cos 2\theta$
(b) $\sin 2\theta$
(c) $\cot 2\theta$
(d) $\tan 2\theta$

18. A 6 V battery is connected to the terminals of a three metre long wire of uniform thickness and resistance of 100 Ω. The difference of potential between two points on the wire separated by a distance of 50 cm will be

(a) 2 V
(b) 3 V
(c) 1 V
(d) 15 V

Directions *(Q. Nos. 19 to 22) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

19. Statement I As temperature decreases, the relaxation time of a conducting material increases.

Statement II Number of collisions per unit time of electrons with lattice ions increases as the temperature increases.

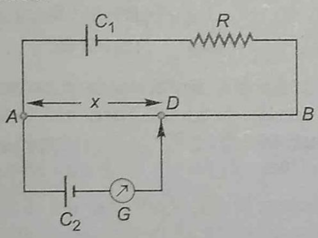
20. **Statement I** Potential difference across the terminals of a battery can be greater than its emf.

Statement II When current is taken from battery, $V = E - Ir$ (Symbols have their usual meaning).

21. **Statement I** In a meter bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

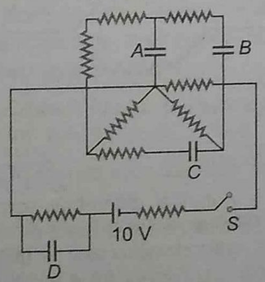
Statement II Resistance of a metal increases with increase in temperature.

22. **Statement I** In the potentiometer circuit shown in figure E_1 and E_2 are the emfs of cells C_1 and C_2 respectively with $E_1 > E_2$. Cell C_1 has negligible internal resistance. For a given resistor R, the balance length is x. If the diameter of the potentiometer wire AB is increased, the balance length x will decrease.



Statement II At the balance point, the potential difference between AD due to cell $C_1 = E_2$, the emf of cell C_2.

Directions *(Q. Nos. 23 to 25) In the given circuit, initially switch is open and all the capacitors are uncharged. Each capacitor has capacitance 2 μF and resistor has resistance 1 Ω.*



23. Find the current supplied by battery just after switch is closed.

(a) zero (b) 8 A
(c) 2 A (d) 17 A

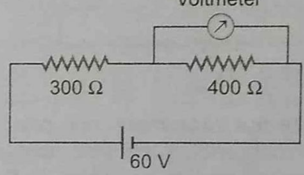
24. Find the current supplied by battery after a long time.

(a) zero (b) 8 A
(c) 4 A (d) 2 A

25. Find the heat dissipated in circuit in a long time.

(a) 56 μJ
(b) 40 μJ
(c) 10 μJ
(d) None of the above

Directions *(Q. Nos. 26 to 28) Two resistances 300 Ω and 400 Ω are connected to a 60 V power supply as shown in figure. A voltmeter connected across the 400 Ω resistor reads 30 V.*



26. The resistance of the voltmeter is

(a) 600 Ω
(b) 800 Ω
(c) 1000 Ω
(d) 1200 Ω

27. When the same voltmeter is connected across the 300 Ω resistor. The current in the circuit is

(a) $\frac{3}{32}$ A (b) $\frac{3}{16}$ A
(c) $\frac{3}{8}$ A (d) $\frac{3}{4}$ A

28. When the voltmeter is connected across the 300 Ω resistor, it will read

(a) 40 V (b) 22.5 V
(c) 37.5 V (d) 25 V

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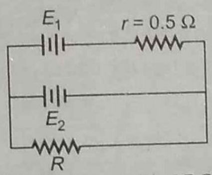
29. A letter A is constructed of a uniform wire with resistance $1.0\ \Omega$ per cm. The sides of the letter are 20 cm and the cross piece in the middle is 10 cm long. The apex angle is $60°$. The resistance between the ends of the legs is close to **[JEE Main Online 2013]**

(a) $50.0\ \Omega$ (b) $10\ \Omega$
(c) $36.7\ \Omega$ (d) $26.7\ \Omega$

30. A DC source of emf $E_1 = 100$ V and internal resistance $r = 0.5\ \Omega$, a storage battery of emf $E_2 = 90$ V and an external resistance R are connected as shown in figure. For what value of R no current will pass through the battery? **[JEE Main Online 2013]**

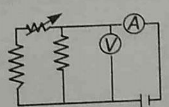


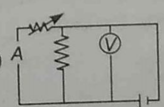
(a) $5.5\ \Omega$ (b) $3.5\ \Omega$ (c) $4.5\ \Omega$ (d) $2.5\ \Omega$

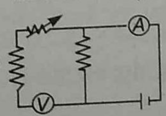
31. In a metre bridge experiment, null point is obtained at 40 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then the new position of the null points from the same end, if one decides to balance a resistance of $3X$ against Y, will be close to **[JEE Main Online 2013]**

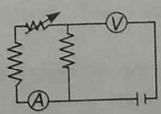
(a) 80 cm (b) 75 cm
(c) 67 cm (d) 50 cm

32. Correct set up to verify Ohm's law is **[JEE Main Online 2013]**

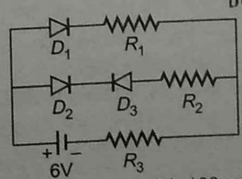
(a) 

(b) 

(c) 

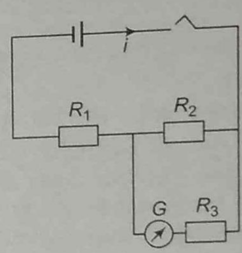
(d) 

33. Figure shows a circuit in which three identical diodes are used. Each diode has forward resistance of $20\ \Omega$ and infinite backward resistance. Resistors $R_1 = R_2 = R_3 = 50\ \Omega$. Battery voltage is 6 V. The current through R_3 is **[JEE Main Online 2013]**



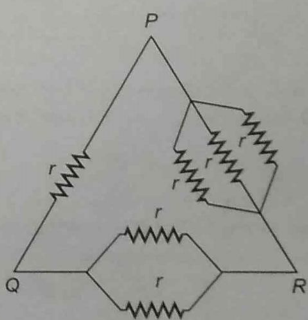
(a) 50 mA (b) 100 mA
(c) 60 mA (d) 25 mA

34. To find the resistance of a galvanometer by the half deflection method the following circuit is used with resistances $R_1 = 9970\ \Omega$, $R_2 = 30\ \Omega$ and $R_3 = 0$. The deflection in the galvanometer is d. With $R_3 = 107\ \Omega$ the deflection changed to $\frac{d}{2}$. The galvanometer resistance is approximately **[JEE Main Online 2013]**



(a) $107\ \Omega$ (b) $137\ \Omega$
(c) $107/2\ \Omega$ (d) $77\ \Omega$

35. Six equal resistances are connected between points P, Q and R as shown in figure. Then net resistance will be maximum between **[JEE Main Online 2013]**



(a) P and R (b) P and Q
(c) Q and R (d) Any two points

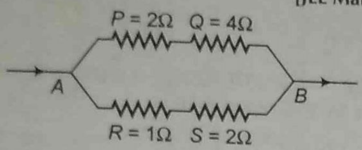
36. To establish an instantaneous current of 2 A through a $1\ \mu F$ capacitor ; the potential difference across the capacitor plates should be changed at the rate of **[JEE Main Online 2013]**

(a) 2×10^4 V/s (b) 4×10^6 V/s
(c) 2×10^6 V/s (d) 4×10^4 V/s

37. A shunt of resistance 1Ω is connected across a galvanometer of $120\ \Omega$ resistance. A current of 5.5 A gives full scale deflection in the galvanometer. The current that will give full scale deflection in the absence of the shunt is nearly **[JEE Main Online 2013]**

(a) 5.5 A (b) 0.5 A
(c) 0.004 A (d) 0.045 A

38. Which of the four resistances P, Q, R and S generate the greatest amount of heat when a current flows from A to B? **[JEE Main Online 2013]**



(a) Q (b) S
(c) P (d) R

39. Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse? **[AIEEE 2012]**

(a) Both (b) 100 W
(c) 25 W (d) Neither

40. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is **[AIEEE 2012]**

(a) 6% (b) zero (c) 1% (d) 3%

41. If a wire is stretched to make it 0.1% longer, its resistance will **[AIEEE 2012]**

(a) increase by 0.2% (b) decrease by 0.2%
(c) decrease by 0.05 % (d) increase by 0.05%

42. The current in the primary circuit of potentiometer is 0.2A. The specific resistance and cross-section of the potentiometer wire are $4 \times 10^{-7}\ \Omega$ m and 8×10^{-7} m^2 respectively. The potential gradient will be equal to **[AIEEE 2011]**

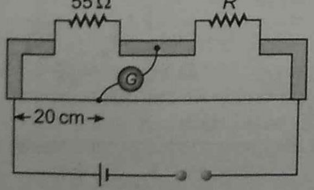
(a) 0.2 V/m (b) 1 V/m
(c) 0.3 V/m (d) 0.1 V/m

43. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly **[AIEEE 2010]**

(a) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$ (b) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

(c) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ (d) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

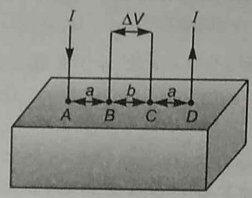
44. Shown in the figure adjacent is a meter bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is **[AIEEE 2008]**



(a) 13.75 Ω (b) 220 Ω (c) 110 Ω (d) 55 Ω

Directions *(Q. Nos. 45 and 46) Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps*

(i) *Take current I entering from A and assume it to spread over a hemi-spherical surface in the block*



(ii) *Calculate field E(r) at distance r from A by using Ohm's law E = ρJ, where J is the current per unit area at r.*

(iii) *From the r dependence of E(r), obtain the potential V(r) at r.*

(iv) *Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.* **[AIEEE 2008]**

45. ΔV measured between B and C is

(a) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$ (b) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$

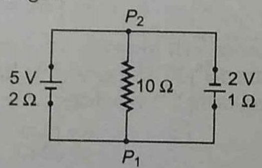
(c) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$ (d) $\frac{\rho I}{2\pi(a-b)}$

46. For current entering at A, the electric field at a distance r from A is

(a) $\frac{\rho I}{8\pi r^2}$ (b) $\frac{\rho I}{r^2}$

(c) $\frac{\rho I}{2\pi r^2}$ (d) $\frac{\rho I}{4\pi r^2}$

47. A 5 V battery with internal resistance 2 Ω and a 2V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. **[AIEEE 2008]**



The current in the 10 Ω resistor is

(a) 0.27 A, P_2 to P_1 (b) 0.03 A, P_1 to P_2
(c) 0.03 A, P_2 to P_1 (d) 0.27 A, P_1 to P_2

48. The resistance of a wire is 5 Ω at 50°C and 6 Ω at 100°C. The resistance of the wire at 0°C will be **[AIEEE 2007]**

(a) 2 Ω (b) 1 Ω
(c) 4 Ω (d) 3 Ω

49. A material B has twice the specific resistance of A. A circular wire made of B has twice the diameter of a wire made of A. Then, for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be **[AIEEE 2006]**

(a) 1 (b) 1/2
(c) 1/4 (d) 2/1

50. An electric bulb is rated 220 V-100 W. The power consumed by it when operated on 110 V will be **[AIEEE 2006]**

(a) 75 W (b) 40 W (c) 25 W (d) 50 W

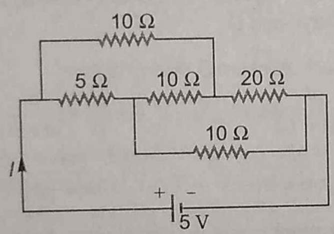
51. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200 Ω at a temperature of **[AIEEE 2006]**

(a) 300°C (b) 400°C (c) 500°C (d) 200°C

52. The Kirchhoff's first law ($\Sigma I = 0$) and second law ($\Sigma IR = \Sigma E$) where the symbols have their usual meanings, are respectively based on **[AIEEE 2006]**

(a) conservation of charge, conservation of momentum
(b) conservation of energy, conservation of charge
(c) conservation of momentum, conservation of charge
(d) conservation of charge, conservation of energy

53. The current I drawn from the 5 V source will be **[AIEEE 2006]**



(a) 0.33 A (b) 0.5 A
(c) 0.67 A (d) 0.17 A

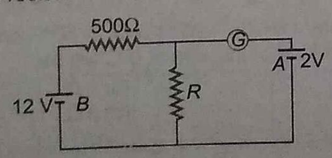
54. A moving coil galvanometer of resistance 100 Ω is used as an ammeter using a resistance 0.1 Ω. The maximum deflection current in the galvanometer is 100 μA. Find the current in the circuit, so that the ammeter shows maximum deflection. **[AIEEE 2005]**

(a) 100.1 mA (b) 1000.1 mA
(c) 10.01 mA (d) 1.01 mA

55. A rigid container with thermally insulated walls contains a coil of resistance 100 Ω, carrying current 1 A. Change in internal energy after 5 min will be **[AIEEE 2005]**

(a) zero (b) 10 kJ
(c) 20 kJ (d) 30 kJ

56. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be **[AIEEE 2005]**



(a) 200 Ω (b) 100 Ω
(c) 500 Ω (d) 1000 Ω

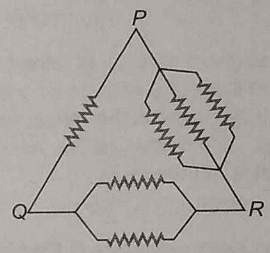
57. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be **[AIEEE 2005]**

(a) doubled (b) four times
(c) one-fourth (d) halved

58. An energy source will supply a constant current into the load, if its internal resistance is **[AIEEE 2005]**

(a) equal to the resistance of the load
(b) very large as compared to the load resistance
(c) zero
(d) non-zero but less than the resistance of the load

59. Six equal resistances are connected between points P, Q and R as shown in the figure. Then, the net resistance will be maximum between **[AIEEE 2004]**



(a) P and Q (b) Q and R
(c) P and R (d) any two points

60. The resistance of the series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then the minimum possible value of n is **[AIEEE 2004]**

(a) 4 (b) 3
(c) 2 (d) 1

61. The thermistors are usually made of **[AIEEE 2004]**

(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity

62. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is **[AIEEE 2004]**

(a) 50 s (b) 100 s
(c) 150 s (d) 200 s

63. The length of a wire of a potentiometer is 100 cm, and the emf of its cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at $l = 30$ cm from the positive end, the emf of the battery is **[AIEEE 2003]**

(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{100-0.5}$
(c) $\frac{30(E-0.5I)}{100}$, (d) $\frac{30E}{100}$

where I is the current in the potentiometer wire.

Answers

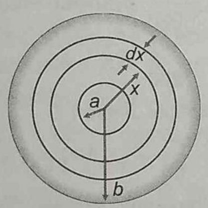
1. (d)	2. (d)	3. (c)	4. (c)	5. (a)	6. (a)	7. (c)	8. (c)	9. (b)	10. (b)
11. (b)	12. (c)	13. (d)	14. (b)	15. (a)	16. (c)	17. (c)	18. (c)	19. (a)	20. (b)
21. (d)	22. (d)	23. (b)	24. (c)	25. (a)	26. (d)	27. (a)	28. (b)	29. (d)	30. (c)
31. (c)	32. (a)	33. (a)	34. (d)	35. (b)	36. (c)	37. (d)	38. (b)	39. (c)	40. (a)
41. (a)	42. (d)	43. (d)	44. (b)	45. (c)	46. (c)	47. (c)	48. (c)	49. (d)	50. (c)
51. (b)	52. (d)	53. (b)	54. (a)	55. (d)	56. (b)	57. (a)	58. (c)	59. (a)	60. (a)
61. (c)	62. (c)	63. (c)							

Hints & Solutions

1. $v_d = \frac{I}{nAe} = \frac{I \times 4}{n\pi D^2 e}$ i.e., $v_d \propto \frac{1}{D^2}$

$$\frac{v_{d_1}}{v_{d_2}} = \frac{D_2^2}{D_1^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

2. Consider a concentric spherical shell of radius x and thickness dx as shown in figure. Its resistance, dR is



$$dR = \frac{\rho\, dx}{4\pi x^2} \qquad \left(\because R = \frac{\rho l}{A}\right)$$

$\therefore$ Total resistance,

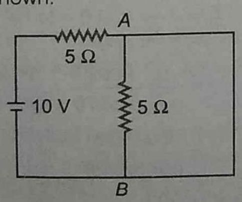
$$R = \frac{\rho}{4\pi}\int_a^b \frac{dx}{x^2} = \frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$

3. Numbers attached for brown, black, green and silver are 1, 0, 5, ± 10%. Therefore, the resistance of given resistor

$$= 10 \times 10^5\ \Omega \pm 10\%$$
$$= 1.0 \times 10^6\ \Omega \pm 10\%$$

4. $S = \frac{I_g G}{I - I_g} = \frac{2 \times 10^{-3} \times 15}{5 - 2 \times 10^{-3}} = 0.006\,\Omega$

5. In the steady state, the capacitors are fully charged and acts as open circuit, so the equivalent circuit in steady state would be as shown.



Steady state current $I = \frac{10}{5+5} = 1\text{A}$

So, potential drop across AB is $V = 5\text{V}$

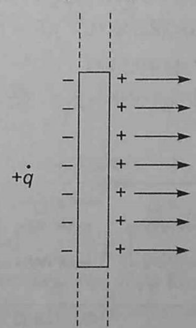
Sum of potential difference across $2\,\mu\text{F}$ and $4\,\mu\text{F}$ capacitors is 5V.

As capacitors are in series, charges on them would be same, let us say it is q.

From KVL, $\frac{q}{2} + \frac{q}{4} = 5$

$\Rightarrow$ $q = \frac{20}{3}\,\mu\text{C}$

6. Due to the charge $(+q)$, some negative charge would develop on the plane towards the $+q$ and equal positive charge would develop on opposite side of plane which would get uniformly distributed. So, field would be perpendicular and away from plane.



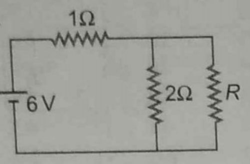
7. Spacing between electric field lines increases from left to right. Therefore, E on left is greater than E on right of dipole. So, force on $-q$ is more as compared to force on $+q$. So, dipole would experience a net force towards the left.

8. Here points B and D are common. So, $2R$ in arm DC and $2R$ in arm CB are in parallel between C and B. Their effective resistance $= \frac{2R \times 2R}{2R + 2R} = R$

Now, the effective resistance between A and B is

$$R_{\text{eff}} = \frac{R \times (R + R)}{R + (R + R)} = \frac{2}{3}R$$

9. Let R be the resistance of infinite ladder. The addition or subtraction of one step in the ladder will not affect the total resistance of network. Therefore, equivalent circuit will be as shown in figure.



Total resistance $= 1 + \dfrac{2 \times R}{R+2} = R$

or $\quad R + 2 + 2R = R^2 + 2R \;\Rightarrow\; R^2 - R - 2 = 0$

On solving, we get $R = 2\,\Omega$

10. Effective emf of circuit $= 10 - 3 = 7$ V

Total resistance of circuit $= 2 + 5 + 3 + 4 = 14\,\Omega$

Current $\quad I = 7/14 = 0.5$ A

Potential difference between A and $D = 0.5 \times 10 = 5$ A

Potential at $D = 10 - 5 = 5$ V

Potential at $E = 5 - 3 = 2$ V

Hence, E cannot be at zero potential, as there is potential drop at E.

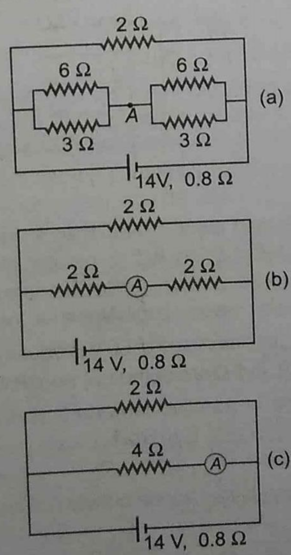
11. The current through galvanometer producing full scale deflection is $\quad I = \dfrac{V}{R} = \dfrac{20 \times 10^{-3}}{80} = 2.5 \times 10^{-4}$ A

To convert galvanometer into a voltmeter, a high resistance is connected in series with the galvanometer

$\therefore \quad 5 = (2.5 \times 10^{-4})(R + 80)$

$\therefore \quad R = 19.92\ \text{k}\Omega$

12. The equivalent circuit of the given circuit will be reduced to as shown in figure.



Total resistance of the circuit $= \dfrac{2 \times 4}{2+4} + 0.8$

$= \dfrac{8}{6} + 0.8 = \dfrac{12.8}{6}\,\Omega$

Main current in the circuit $= \dfrac{14}{(12.8/6)} = \dfrac{84}{12.8}$ A

Reading of ammeter $= \dfrac{84}{12.8} \times \dfrac{2}{6} = 2.18$ A

13. Total resistance of circuit $= 100 + 100 + 80 + 20 = 300\Omega$

Current $I = \dfrac{48}{300} = 0.16$ A

Potential difference across P

and $\quad Q = 20 \times 0.16 = 3.2$ V

14. As, $\left(\dfrac{l-2}{2}\right)4 = \left(\dfrac{l-3}{3}\right)8 \;\Rightarrow\; l = 6$

Therefore, $\quad r = \left(\dfrac{l-2}{2}\right)4 = 8\Omega$

15. $V_1 = E - Ir = 50 - \dfrac{50}{220} \times 20 = 50 - 4.5 = 45.5$ V

Now, $\quad V_2 = 50 - \dfrac{50}{200} \times 20 = 45$

% change in potential difference

$= \dfrac{(45.5 - 45)}{50} \times 100 = 1\%$

16. Let I_1 and I_2 be the currents drawn from cells of emf 6 V and 4 V in the circuits. Then,

$I_1 = \dfrac{6}{2+3+1} = 1$ A

and $\quad I_2 = \dfrac{4}{6+4} = 0.4$ A

$V_A - V_B = 1 \times 3 = 3\,\text{V}; V_B - V_C = 5$ V

and $\quad V_C - V_D = 0.4 \times 6 = 2.4$ V

$\therefore \quad V_A - V_D = (V_A - V_B) + (V_B - V_C) + (V_C - V_D)$

$= 3 + 5 + 2.4 = 10.4$ V

17. $R_1 = \tan\theta = R_0(1 + \alpha T_1)$

and $\quad R_2 = \cot\theta = R_0(1 + \alpha T_2)$

$\cot\theta - \tan\theta = R_0(1 + \alpha T_2)$

$- R_0(1 + \alpha T_1) = R_0\alpha(T_2 - T_1)$

or $\quad T_2 - T_1 = \dfrac{1}{\alpha R_0}(\cot\theta - \tan\theta)$

$= \dfrac{1}{\alpha R_0}\left(\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}\right)$

$= \dfrac{2\cos 2\theta}{\alpha R_0 \sin 2\theta}$

$= \dfrac{2}{\alpha R_0}\cot 2\theta$

18. Potential gradient along the wire, $K = \frac{6}{300}$ V/cm

Potential difference across 50 cm length is

$$V = k \times 50 = \frac{6}{300} \times 50 = 1 \text{ volt}$$

Alternative

Here, $R = 100 = \frac{\rho l}{A} = \frac{3}{A}$

or $\frac{\rho}{A} = \frac{100}{3}$

The resistance of 50 cm wire is

$$R_1 = \frac{\rho l'}{A}$$

$$= \frac{100}{3} \times 0.50 = \frac{50}{3}\ \Omega$$

Current in the wire $I = \frac{6}{100}$ A

Potential difference across the given portion of wire is

$$V_1 = IR_1 = \frac{6}{100} \times \frac{50}{3} = 1 \text{ volt}$$

19. As temperature increases, the number of collisions per unit time increases and *vice-versa*.

So, as temperature decreases, $\tau_{increases}$.

20. When the battery is undergoing charging processes then,

$$V = E + Ir > E$$

So, Statement I is correct.

Statement II is also correct but not explaining Statement I.

21. With increase in temperature, the value of unknown resistance will increase.

In balanced Wheatstone bridge condition,

$$\frac{R}{X} = \frac{l_1}{l_2}$$

Here, R = value of standard resistance,

X = value of unknown resistance.

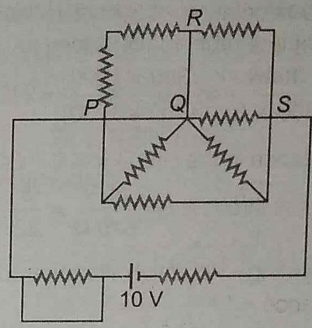
To take null point at same point or $\frac{l_1}{l_2}$ to remain unchanged, $\frac{R}{X}$ should also remain unchanged.

Therefore, if X is increasing R, should also increase.

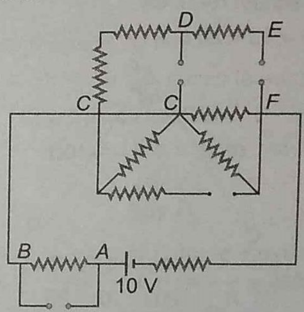
22. The correct choice is (d). If the diameter of wire AB is increased, its resistance will decrease. Hence, the potential difference between A and B due to cell C_1 will decrease. Therefore, the null point will be obtained at a higher value of x.

23. Just after the switch is closed, the capacitor would be treated as short circuit and the circuit can be redrawn as if P, Q, R are at the same potential.

Therefore, all resistors connected between these points are uneffective. After redrawing the circuit, $I = 8$ A. $R_{eq} = 1 + 0.25 = 1.25\ \Omega$



24. When steady state is reached, capacitor would be treated as an open circuit.



In this case, solve the circuit, I comes out to be 4 A.

25. Potential difference across $AB, V_{AB} = 4$ V

So, charge on $D, q_d = 8\ \mu C$

Potential difference across $CD = 0$

So, charge on $A, q_A = 0$

Charge on $B, q_B = 4\ \mu C$

Charge on $C, q_C = 4\ \mu C$

So, ΔH = work done by battery $- \Sigma U$

$= 8 \times 10\ \mu J - 24\ \mu J$

$= 56\ \mu J$

26. Potential difference across the 400 Ω resistance $= 30$ V. Therefore, potential difference across the 300 Ω resistance $= 60 - 30 = 30$ V. Let R be the resistance of the voltmeter. As the voltmeter is in parallel with the 400 Ω resistance, their combined resistance is

$$R' = \frac{400R}{(400 + R)}$$

As the potential difference of 60 V is equally, shared between the 300 Ω and 400 Ω resistance, R' should be equal to 300 Ω. Thus,

$$300 = \frac{400R}{(400 + R)}$$

Which gives $R = 1200\ \Omega$.

27. When the voltmeter is connected across the 300 Ω resistance, their combined resistance is

$$R'' = \frac{300R}{(300+R)} = \frac{300\times1200}{(300+1200)} = 240\ \Omega$$

∴ Total resistance in the circuit = 400 + 240 = 640 Ω

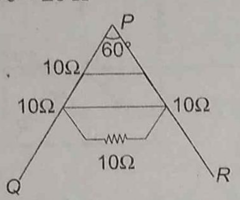
∴ Current in the circuit is $I = \frac{60\ \text{V}}{640\ \Omega} = \frac{3}{32}$ A

28. Voltmeter reading = Potential difference across 240 Ω resistance

$$= \frac{3}{32}\times 240 = 22.5\ \text{V}$$

29. We have, in series, required

$R_1 + R_2 = 10 + 10 = 20\ \Omega$



and in parallel, $= \frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{10} = \frac{3}{20}$

$$\Rightarrow \quad R_{eq} = \frac{20}{3}$$

Therefore, $R_{eq} = \frac{20}{3} + 10 + 10$

$$= \frac{20+30+30}{3}$$

$$= \frac{80}{3} = 26.66 = 26.7\ \Omega$$

30. Given, $E_1 = 100$ V, $r = 0.5\ \Omega$, $E_2 = 90$ V

External resistance = R

For no current pass through the battery

$$\frac{100}{R+r} = \frac{90}{R}$$

$$\Rightarrow \quad \frac{10}{R+\frac{1}{2}\Omega} = \frac{9}{R}$$

$$\Rightarrow \quad 10R = 9R + 4.5\ \Omega$$

∴ $R = 4.5\ \Omega$

31. As, $\frac{x}{y} = \frac{40}{100-40} = \frac{40}{60}$

$$\frac{3x}{y} = 3\left(\frac{40}{60}\right) = \frac{120}{60} = \frac{2}{1}$$

Now, the total length = 100

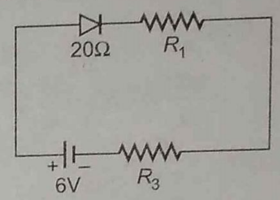
∴ Required length $= \frac{100}{3}\times 2 = 67$ cm

32. Ohm's law states that the current (I) flowing through a conductor is directly proportional to the potential difference (V) across the ends of the conductor, provided physical conditions of the conductor such as temperature, mechanical strain etc., kept constant,

i.e., $I \propto V$ or $V \propto I$

or $V = RI$ (where R is constant)

33. As diode is conducting in forward bias condition and now conducting in reverse bias condition. Diode D_1 is in forward bias, and diode D_2 is in forward bias but D_3 is reverse bias. So the figure can be drawn as



Here, 20 Ω, RΩ and R_3Ω are in series.

Equivalent resistance = 50 + 50 + 20 = 120 Ω

∴ $I = \frac{V}{R} = \frac{6}{120} = \frac{1}{20} \Rightarrow I = 50$ mA

34. As at initial condition the deflection is d while $R_3 = 0$,

then equivalent resistance of R_2 and R_3

$$= R_2 + R_3 = R_2 = 30\ \Omega$$

Now, when $R_3 = 107\ \Omega$ and $R_2 = 30\ \Omega$

Then, equivalent resistance should be $\frac{30}{2} = 15\ \Omega$

It is only when equivalent resistant resistance and R_3 and R_g will be parallel to R_2 giving resistance 15 Ω

Let $R_3 - R_g$ = equivalent = 30 Ω = R

∴ $\frac{1}{R_2} + \frac{1}{R} = \frac{1}{30} + \frac{1}{30} = \frac{1}{15}$

∴ $R_{eq} = 15\ \Omega$

Thus, R_g must will be 77 Ω in order to maintain

$R_3 - R_g = 30 \quad \Rightarrow \quad 107 - R_g = 30 = R_g = 77\ \Omega$

35. By solving this, we get net resistance will be max,

$$R_{PQ} = \frac{5}{11}r,\ R_{QR} = \frac{4}{11}r \text{ and } R_{PR} = \frac{3}{11}r$$

$$R_{PQ} > R_{QR} > R_{PR}$$

Therefore R_{PQ} is maximum.

36. As $Q = CV$

Now, differenating with respect to time (t)

$$\frac{dQ}{dt} = C\frac{dV}{dt}$$

$$2 = 1\times10^{-6}\frac{dV}{dt}$$

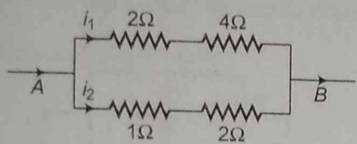
∴ $\frac{dV}{dt} = \frac{2}{1\times10^{-6}} = 2\times10^{6}$ V/s

37. As, $i_g = \dfrac{R_s}{R_g + R_s} i$

$$= \frac{1R}{120R + 1R} \times 5.5\text{ A}$$

$$= \frac{1}{121} \times 5.5 = 0.045\text{A}$$

38. We know that $i \propto \dfrac{i}{R}$, $i_2 = \dfrac{6}{3+6} i = \dfrac{2}{3} i$



or $$i_1 = \frac{3}{6+3} i = \frac{1}{3} i$$

Power rate in 2 Ω of upper series

$$= 2 \times \left(\frac{1}{3} i\right)^2 = \frac{2}{3} i^2$$

Power rate in 4 Ω of upper series

$$= 4 \times \left(\frac{1}{3} i\right)^2 = \frac{4}{9} i^2$$

Power rate of 1 Ω in lower series

$$= 1 \times \left(\frac{2}{3} i\right)^2 = \frac{4}{9} i^2$$

Power rate of 2 Ω in lower series

$$= 2 \times \left(\frac{2}{3} i\right)^2 = \frac{8}{9} i^2$$

$\therefore$ $P \to 2/3 Q \to 4/9, R \to 4/9, S \to 8/9$

39. Resistance of bulb is given by $R = \dfrac{V^2}{P}$.

As the rated power of 25 W is less than 100 W, it implies that 25 W bulb has higher resistance. As in series connection, current through both the bulbs is same so heating in 25 W bulb is more than that of 100 W bulb. So, 25 W bulb will get fused.

40. From Ohm's law,

$$R = \frac{V}{i} \Rightarrow \ln R = \ln V - \ln i$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta i}{i} = 3\% + 3\% = 6\%$$

41. $R = \dfrac{Sl}{A} = \dfrac{Sl^2}{V}$ (V = volume)

$$\therefore \quad \frac{\Delta R}{R} = 2\frac{\Delta l}{l} = +\,0.2\%$$

42. Potential gradient of a potentiometer,

$$K = \frac{I\rho}{A} = \frac{0.2 \times 4 \times 10^{-7}}{8 \times 10^{-7}} = 0.1\text{ V/m}$$

43. Let, R_0 be the initial resistance of both conductors

$\therefore$ At temperature θ their resistances will be,

$$R_1 = R_0(1 + \alpha_1\theta)$$

and $$R_2 = R_0(1 + \alpha_2\theta)$$

For series combination,

$$R_s = R_1 + R_2$$

$$R_{s0}(1 + \alpha_s\theta) = R_0(1 + \alpha_1\theta) + R_0(1 + \alpha_2\theta)$$

Where, $$R_{s0} = R_0 + R_0 = 2R_0$$

$$\therefore \quad 2R_0(1 + \alpha_s\theta) = 2R_0 + R_0\,\theta(\alpha_1 + \alpha_2)$$

or $$\alpha_s = \frac{\alpha_1 + \alpha_2}{2}$$

For parallel combination, $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

$$R_{p0}(1 + \alpha_p\theta) = \frac{R_0(1 + \alpha_1\theta)R_0(1 + \alpha_2\theta)}{R_0(1 + \alpha_1\theta) + R_0(1 + \alpha_2\theta)}$$

Where, $$R_{p0} = \frac{R_0 R_0}{R_0 + R_0} = \frac{R_0}{2}$$

$$\therefore \quad \frac{R_0}{2}(1 + \alpha_p\theta) = \frac{R_0^2(1 + \alpha_1\theta + \alpha_2\theta + \alpha_1\alpha_2\theta^2)}{R_0(2 + \alpha_1\theta + \alpha_2\theta)}$$

as α_1 and α_2 are small quantities.

$\therefore$ α_1, α_2 is negligible. So, neglect $\alpha_1, \alpha_2, \theta^2$

or $$\alpha_p = \frac{\alpha_1 + \alpha_2}{2 + (\alpha_1 + \alpha_2)\theta} = \frac{\alpha_1 + \alpha_2}{2}\left[1 - \left(\frac{\alpha_1 + \alpha_2}{2}\right)\theta\right]$$

[Binomial expansion]

as $(\alpha_1 + \alpha_2)^2$ is negligible

$$\therefore \quad \alpha_p = \frac{\alpha_1 + \alpha_2}{2}$$

44. From balanced Wheatstone's bridge concept,

$$\frac{55}{R} = \frac{20}{80} \Rightarrow R = 220\ \Omega$$

45. Electric field at a distance r from A is,

$$E = \rho . J = \rho \times \frac{I}{2\pi r^2}$$

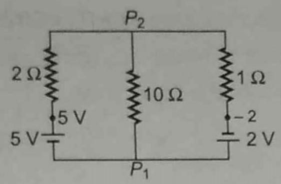
$$\Rightarrow \quad \int dV = -\int E dr$$

$$\Rightarrow \quad V_C - V_B = \Delta V = -\int_a^{a+b} \frac{\rho I}{2\pi} \times \frac{dr}{r^2}$$

$$\Rightarrow \quad \Delta V = \frac{\rho I}{2\pi}\left[\frac{1}{a} - \frac{1}{a+b}\right]$$

46. From $E = \rho J = \dfrac{\rho \times I}{2\pi r^2}$ $\left[\because J = \dfrac{I}{\text{Area of hemisphere}} = \dfrac{I}{2\pi r^2}\right]$

47. Let potential of P_1 is 0 V and potential of P_2 is V_0. Now, apply KCL at P_2.



$$\frac{V_0-5}{2}+\frac{V_0-0}{10}+\frac{V_0-(-2)}{1}=0 \Rightarrow V_0=\frac{5}{16}$$

So, current through 10 Ω resistor is $\frac{V_0}{10}$ from P_2 to P_1.

$$\Rightarrow i=\frac{5/16}{10}=\frac{5}{160}=0.03$$

48. From $R_t = R_0(1+\alpha t)$

$\therefore \quad 5 = R_0(1+50\alpha) \quad \ldots(i)$

and $\quad 6 = R_0(1+100\alpha) \quad \ldots(ii)$

$\therefore \quad \frac{5}{6}=\frac{1+50\alpha}{1+100\alpha}$

or $\quad \alpha=\frac{1}{200}$

Putting value of α in Eq. (i), we get

$$5 = R_0\left(1+50\times\frac{1}{200}\right)$$

or $\quad R_0 = 4\,\Omega$

49. Resistance of $A = R_A = \frac{\rho_A l_A}{A_A}=\frac{\rho_A l_A}{\pi r_A^2}$

Resistance of $B = R_B = \frac{\rho_B l_B}{A_B}=\frac{\rho_B l_B}{\pi r_B^2}$

From given information, $\rho_B = 2\rho_A$

$r_B = 2r_A$ and $R_A = R_B$

$\therefore \quad \frac{\rho_A l_A}{\pi r_A^2}=\frac{\rho_B l_B}{\pi r_B^2}$

or $\quad \frac{\rho_A l_A}{\pi r_A^2}=\frac{2\rho_A \times l_B}{\pi(2r_A)^2}$

or $\quad \frac{l_B}{l_A}=\frac{2}{1}=2:1$

50. Resistance of electric bulb $R=\frac{V^2}{P}$, where subscripts denote for rated parameters.

$$R=\frac{(220)^2}{100}$$

Power consumed at 110 V,

$$P_{\text{consumed}}=\frac{V^2}{R}=\frac{(110)^2}{(220)^2/100}=25\text{ W}$$

51. Let resistance of bulb filament is R_0 at 0°C, then from expression

$$R_\theta = R_0[1+\alpha\,\Delta\theta]$$

We have, $\quad 100 = R_0[1+0.005\times100]$

and $\quad 200 = R_0[1+0.005\times x]$

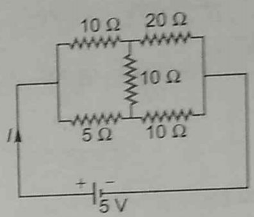
where x is temperature in °C at which resistance become 200 Ω.

Dividing the above two equations

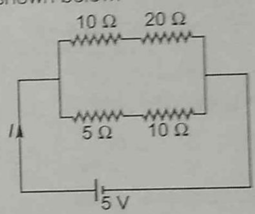
$$\frac{200}{100}=\frac{1+0.005x}{1+0.005\times100} \Rightarrow x = 400°\text{C}$$

52. According to Kirchoff's Law, it is based on coservation of charge and conservation of energy.

53. The given circuit can be redrawn as



which is a balanced Wheatstone's bridge and hence, no current flows in the middle resistor, so equivalent circuit would be as shown below.



10 Ω and 20 Ω resistances are in series

$\therefore \quad R' = 10\,\Omega + 20\,\Omega = 30\,\Omega$

Similarly, 5 Ω and 10 Ω are in series, $R'' = 15\,\Omega$

Now, R' and R'' are in parallel

$\therefore \quad R=\frac{15\times30}{15+30}=10\ \Omega$

So, $\quad I=\frac{V}{R}=\frac{5}{10}=0.5\text{ A}$

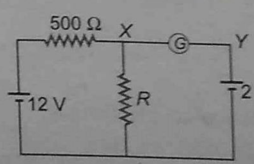
54. $V_{ab} = I_g G = (I - I_g)S \;\therefore\; I=\left(1+\frac{G}{S}\right)I_g$

Substituting the values, we get $I = 100.1$ mA

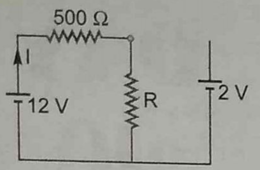
55. $W = 0$. Therefore, from first law of thermodynamics,

$$\Delta U = \Delta Q = I^2Rt = (1)^2(100)(5\times60)\text{ J} = 30\text{ kJ}$$

56. The galvanometer shows zero deflection *i.e.*, current through *XY* is zero



As a result potential drop across R is 2 V. Circuit can be redrawn as



$$I = \frac{12}{500 + R}$$

Voltage across R, $V = IR$

$$\Rightarrow \quad 2 = \frac{12}{500 + R} \times R$$

$$\text{or} \quad 1000 + 2R = 12R$$

$$\text{or} \quad R = 100\ \Omega$$

57. $H_1 = \frac{V^2}{R} t$ and $H_2 = \frac{V^2}{(R/2)} t$

$$\therefore \quad \frac{H_2}{H_1} = 2 \text{ or } H_2 = 2H_1$$

58. $$I = \frac{E}{R + r} = \frac{E}{R} = \text{constant}$$

where, R = external resistance

r = internal resistance = 0

59. $R_{PQ} = \frac{5}{11} r, R_{QR} = \frac{4}{11} r$

and $\quad R_{PR} = \frac{3}{11} r$

$\therefore \quad R_{PQ}$ is maximum.

60. Let resistances are R_1 and R_2. Then,

$$S = R_1 + R_2$$

$$\text{and} \quad P = \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore \quad (R_1 + R_2) = \frac{n \times R_1 R_2}{R_1 + R_2} \quad [\text{from } S = nP]$$

$$\text{or} \quad (R_1 + R_2)^2 = nR_1 R_2$$

$$\Rightarrow \quad n = \left[\frac{R_1^2 + R_2^2 + 2R_1R_2}{R_1R_2}\right]$$

$$= \left[\frac{R_1}{R_2} + \frac{R_2}{R_1} + 2\right]$$

We know,

Arithmetic Mean ≥ Geometric Mean

$$\frac{\frac{R_1}{R_2} + \frac{R_2}{R_1}}{2} \geq \sqrt{\frac{R_1}{R_2} \times \frac{R_2}{R_1}}$$

$$\Rightarrow \quad \frac{R_1}{R_2} + \frac{R_2}{R_1} \geq 2$$

So, n (minimum value) $= 2 + 2 = 4$

61. The thermistors are usually made of metaloxide with high temperature coefficient of resistivity.

62. Let time taken in boiling the water by the heater is t second.

Then. $\quad Q = ms\,\Delta T$

$$\Rightarrow \quad \frac{Pt}{J} = ms\,\Delta T$$

$$\therefore \quad \frac{836}{4.2} t = 1 \times 1000\,(40 - 10)$$

$$\text{or} \quad \frac{836}{4.2} t = 1000 \times 30$$

$$\text{or} \quad t = \frac{1000 \times 30 \times 4.2}{836} = 150 \text{ s}$$

63. $$V \propto l$$

$$\therefore \quad \frac{V}{E} = \frac{l}{L}$$

where, l = balance point distance

L = length of potentiometer wire.

$$\text{or} \quad V = \frac{l}{L} E$$

$$\text{or} \quad V = \frac{30 \times E}{100} = \frac{30}{100} E$$

Unit Test 4

DAY 20

(Electrostatics & Current Electricity)

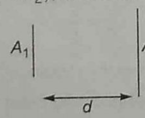
1. Two concentric spheres of radii r_1 and r_2 carry charges q_1 and q_2, respectively. If the surface charge density (σ) is the same for both the spheres, the electric potential at the common centre will be

(a) $\frac{\sigma}{\varepsilon_0} \times \frac{r_1}{r_2}$ (b) $\frac{\sigma}{\varepsilon_0} \times \frac{r_2}{r_1}$ (c) $\frac{\sigma}{\varepsilon_0}(r_1 - r_2)$ (d) $\frac{\sigma}{\varepsilon_0}(r_1 + r_2)$

2. Two point charges of $\pm 10\mu C$ are placed 5.00 mm apart, forming an electric dipole. Compute electric field at a point on the axis of the dipole 15 cm away from the centre on a line passing through the centre dipole.
$(1/4\pi\varepsilon_0 = 9.0 \times 10^9 \text{ Nm}^2\text{C}^{-2})$

(a) $1.66 \times 10^5 \text{NC}^{-1}$ (b) $3.66 \times 10^5 \text{NC}^{-1}$
(c) $2.66 \times 10^{-5} \text{NC}^{-1}$ (d) $2.66 \times 10^5 \text{NC}^{-1}$

3. The capacitance of a capacitor, is of the plates having the areas A_1 and $A_2 (A_1 < A_2)$, at a distance d, is having the areas



(a) $\frac{\varepsilon_0 (A_1 + A_2)}{2d}$ (b) $\frac{\varepsilon_0 A_2}{d}$ (c) $\frac{\varepsilon_0 \sqrt{A_1 A_2}}{d}$ (d) $\frac{\varepsilon_0 A_1}{d}$

4. An oil drop having 12 excess electrons is held stationary under a uniform electric field of $2.55 \times 10^4 \text{NC}^{-1}$. The density of the oil is 1.26 g-cm^{-3}. Estimate the radius of the drop. (Given, $g = 9.81 \text{ms}^{-2}, e = 1.6 \times 10^{-19}\text{C}$)

(a) 6.61×10^{-4}mm (b) 9.81×10^{-4}mm
(c) 6.61×10^{-2}mm (d) 9.81×10^{-2}mm

5. A charge $(-q)$ and another charge $(+Q)$ are kept at two points A and B respectively. Keeping the charge $(+Q)$ fixed at B, the charge $(-q)$ at A is moved to another point C such that ABC forms an equilateral triangle of side l. The net work done in moving the change $(-q)$ is

(a) $\frac{1}{4\pi\varepsilon_0} \frac{Qq}{l}$ (b) $\frac{1}{4\pi\varepsilon_0} \frac{Qq}{l^2}$
(c) $\frac{1}{4\pi\varepsilon_0} Qql$ (d) zero

6. The electric dipole moment of an electron and proton 4.30 nm apart is

(a) 6.88×10^{-28} C-m (b) 5.88×10^{-28} C-m
(c) 6.88×10^{28} C-m (d) 5.88×10^{28} C-m

7. At what distance along the central axis of a uniformly charged plastic disk of radius R is the magnitude of the electric field equal to one-half the magnitude of the field at the centre of the surface of the disk?

(a) $\frac{R}{\sqrt{2}}$ (b) $\frac{R}{\sqrt{3}}$
(c) $\sqrt{2}R$ (d) $\sqrt{3}R$

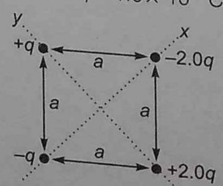
8. Work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 μF is

(a) 16×10^{-32} J (b) 3.1×10^{-28} J
(c) 64×10^{-32} J (d) 32×10^{-32} J

9. A drop, having a mass of 4.8×10^{-10}g and a charge of 2.4×10^{-18}C is suspended between two charged horizontal plates at a distance 1.0 cm apart. Find the potential difference between the plates. If polarity of the plates be changed, then calculate the instantaneous acceleration of the drop.

(a) 1.96×10^6V, 18.6 ms^{-2}
(b) 1.86×10^4V, 18.6 ms^{-2}
(c) 1.96×10^4V, 19.6 ms^{-2}
(d) 2.96×10^4V, 17.6 ms^{-2}

10. What is the direction of the electric field at the centre of the square of figure, if $q = 1.0 \times 10^{-8}$C and a = 5.0 cm?



(a) 30° with x-axis (b) 45° with x-axis
(c) 60° with x-axis (d) 90° with x-axis

11. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is

(a) 4 Ω (b) 2 Ω
(c) 1 Ω (d) 0.5 Ω

12. The resistance of a wire at 20°C is 20 Ω and at 500°C is 60 Ω. At which temperature its resistance will be 25 Ω?

(a) 50°C
(b) 60°C
(c) 70°C
(d) 80°C

13. A charged cloud system produces an electric field in the air near earth's surface. A particle of charge -2×10^{-9}C is acted on by a downward electrostatic force of 3×10^{-6}N when placed in this field. The ratio of the magnitude of the electrostatic force to the magnitude of the gravitational force in the case of proton is

(a) 1.6×10^{-19} (b) 1.5×10^{-10}
(c) 1.6×10^{19} (d) 1.5×10^{10}

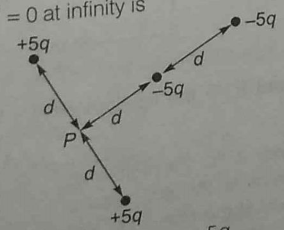
14. An infinite non-conducting sheet has a surface charge density $\sigma = 0.10\,\mu$C m^{-2} on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?

(a) 5.8×10^{-3}m
(b) 6.8×10^{-3}m
(c) 7.8×10^{-3}m
(d) 8.8×10^{-3}m

15. An electron is released from rest in a uniform electric field of magnitude 2.00×10^{4} NC^{-1}. Acceleration of the electron is (ignore gravitation)

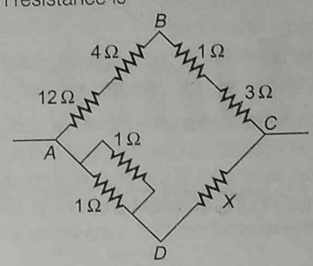
(a) 2.51×10^{15}ms^{-2}
(b) 2.51×10^{-15}ms^{-2}
(c) 3.51×10^{15}ms^{-2}
(d) 3.51×10^{-15}ms^{-2}

16. In figure, the net potential at point P due to the four point charges, if $V = 0$ at infinity is



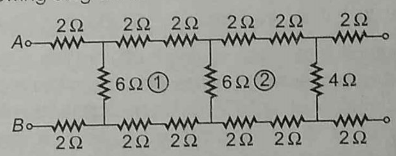
(a) $\frac{3q}{8\pi\varepsilon_0}$ (b) $\frac{5q}{8\pi\varepsilon_0}$
(c) $\frac{7q}{8\pi\varepsilon_0}$ (d) $\frac{9q}{8\pi\varepsilon_0}$

17. In the combination of resistances shown in the figure, the potential difference between B and D is zero, when unknown resistance is



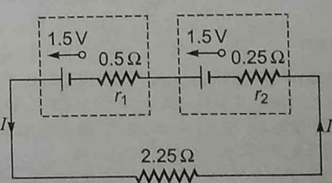
(a) 0.125 Ω
(b) 2 Ω
(c) 3 Ω
(d) for finding the value of X, the emf of cell is required

18. The equivalent resistance between points A and B in the following diagram is



(a) 2 Ω (b) 8 Ω
(c) 9 Ω (d) 10 Ω

19. Two cells connected in series have electromotive force of 1.5 V each. Their internal resistance are 0.5 Ω and 0.25 Ω respectively. This combination is connected to a resistance of 225 Ω. Potential difference across the terminals of each cell



(a) 1 V, 0.25 V (b) 1 V, 1.25 V
(c) 1.5 V, 2.25 V (d) 1.5 V, 2.56 V

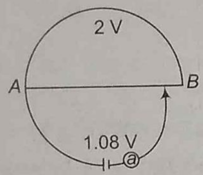
20. The belt of an electrostatic generator is 50 cm wide and travels 30 cm/s. The belt carries charge into the sphere at rate corresponding to 10^{-4}A. What is the surface density of charge on the belt?

(a) 6.7×10^{-5} C-m^{-2}
(b) 6.7×10^{-4}C-m^{-2}
(c) 6.7×10^{-7} C-m^{-2}
(d) 6.7×10^{-8} C-m^{-2}

21. A particular 12 V car battery can send a total charge of 84 A-h through a circuit, from one terminal to other. If this entire charge undergoes a potential difference of 12 V, how much energy is involved?

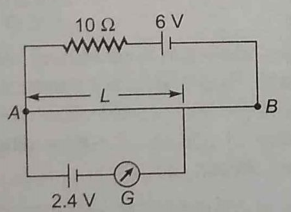
(a) 1.6×10^6 J (b) 2.6×10^6 J
(c) 3.6×10^6 J (d) 4.6×10^6 J

22. *AB* is uniform resistance wire of length 1 m. A 2 V accumulator, a Denial cell of 1.08 V and a galvanometer *G* are connected as shown. If the sliding contact is adjusted for null deflection then the potential gradient in *AB* and the balancing length, measured from end *A* are respectively.



(a) 0.02 V/cm, 54 cm (b) 0.0308 V/cm, 46 cm
(c) 0.0092 V/cm, 49.6 cm (d) 0.02 V/cm, 50.4 cm

23. A potentiometer wire of length 200 cm has a resistance of 20 Ω. It is connected in series with a resistance of 10 Ω and an accumulator of emf 6 V having negligible internal resistance. A source of 2.4 V is balanced against a length *L* of the potentiometer wire. Find the value of *L*.



(a) 100 cm (b) 120 cm (c) 110 cm (d) 140 cm

24. A voltmeter of resistance 1000 Ω gives full scale deflection when a current of 100 mA flow through it. The shunt resistance required across it to enable it to be used as an ammeter reading 1A at full scale deflection is

(a) 10000 Ω (b) 9000 Ω
(c) 222 Ω (d) 111 Ω

25. The resistance of a galvanometer is 50 Ω and the current required to give full scale deflection is 100 μA . In order to convert it into an ammeter, reading upto 10A, it is necessary to put a resistance of

(a) 5×10^{-3} Ω in parallel (b) 5×10^{-4} Ω in parallel
(c) 10^5 Ω in series (d) 99950 Ω in series

26. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt the resistance in ohms needed to be connected in series with the coil will be

(a) 99995 (b) 9995 (c) 10^3 (d) 10^5

27. Masses of the three wires of same material are in the ratio of 1 : 2 : 3 and their lengths in the ratio of 3 : 2 : 1. Electrical resistance of these wires will be in the ratio of

(a) 1 : 1 : 1 (b) 1 : 2 : 3
(c) 9 : 4 : 1 (d) 27 : 6 : 1

Directions *(Q. Nos. 28 to 35) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

28. Statement I *A* and *B* are two conducting spheres of same radius. *A* being solid and *B* hollow. Both are charged to the same potential. Then,

Charge on *A* = charge on *B*.

Statement II Potentials on both are same.

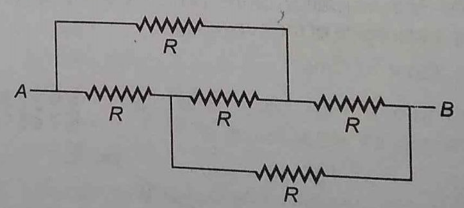
29. Statement I A small metal ball is suspended in a uniform magnetic field with the help of an insulated thread. When a high energy X-ray beam falls on the ball, then the ball will be deflected in the direction of electric field.

Statement II The ball will oscillate in the field.

30. Statement I The circuits containing capacitor be handed cautiously, even when there is no current.

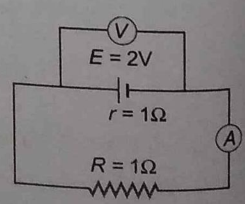
Statement II A dielectric differs from an insulator.

31. Statement I In the following circuit, the net resistance between points *A* and *B* is *R*



Statement II All the resistances are in parallel to each other.

32. Statement I In the following circuit, emf is 2V and internal resistance of the cell is 1Ω and $R = 1\Omega$, then reading of the voltmeter is 1V.



Statement II $V = E - Ir$, where $E = 2\text{ V}, I = \frac{2}{2} = 1\text{A}$ and $R = 10\ \Omega$

33. Statement I The power delivered to a light bulb is more just after it is switched ON and the glow of the filament is increasing, as compared to when the bulb is glowing steadily, *i.e.*, after sometime of switching ON.

Statement II As temperature increases, resistance of conductor increases.

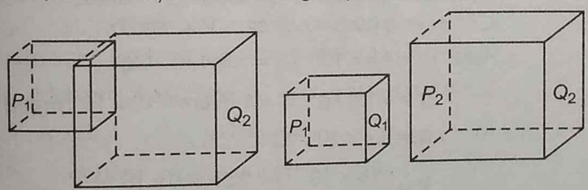
34. Statement I When a wire is stretched so that its diameter is halved, its resistance becomes 16 times.

Statement II Resistance of wire decreases with increase in length.

35. Statement I A potentiometer is preferred over that of a voltmeter for measurement of emf of a cell.

Statement II Potentiometer is preferred as it does not draw any current from the cell.

Directions *(Q. Nos. 36 and 37) Consider two conducting cubical blocks of sides a and 2a of identical material, of resistivity ρ. Potential difference V is individually applied between opposite faces P_1Q_1 of the smaller cube and also P_2Q_2 of the bigger cube. Same V is applied between P_1Q_2 the opposite faces of their combination as shown. The calculation of change in current $(I_1 - I_2)$ in the two cases(I_1 is current in smaller cube and I_2 that through the combination) is done by the following steps.*



(i) Assume current to enter normally and calculate the resistance of each cube using relation $R = \frac{\rho l}{A}$.

(ii) Find resistance of the combination by direct summation of resistance of the cubes.

(iii) Current I_1 and I_2 are found for smaller cube and also the combination applying Ohm's law $I = \frac{V}{R}$.

(iv) Result $(I_1 - I_2)$ is calculated.

36. The ratio of the electric field effective across P_1Q_1 and P_2Q_2 under the influence of the same applied voltage V should be

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

37. The difference in currents $(I_1 - I_2)$ is

(a) $\frac{V}{\rho}a - \frac{1}{2}\frac{V}{\rho}a$ (b) $\frac{V}{\rho}a - \frac{3}{2}\frac{V}{\rho}a$

(c) $\frac{1}{2}\frac{V}{\rho}a - \frac{2}{3}\frac{V}{\rho}a$ (d) $\frac{V}{\rho}a - \frac{2}{3}\frac{V}{\rho}a$

Directions *(Q. Nos. 38 to 40) Power supplied by an ideal battery (internal resistance = 0) is EI. Here, E is its emf and I is the current drawn from it. In series, current through different resistance is same and in parallel, potential difference is same.*

Power consumed by a resistance is $P = I^2R = \frac{V^2}{R} = VI$

In the circuit shown, total power supplied by an ideal battery is 80 W. If $R = 10\ \Omega$. Then,

38. Emf of the battery is

(a) 20 V (b) 10 V
(c) 40 V (d) 60 V

39. Ratio of power developed across R, $2R$ and $6R$ will be

(a) 2 : 4 : 3 (b) 1 : 2 : 6
(c) 2 : 4 : 6 (d) 1 : 2 : 3

40. If the resistance R is removed from the circuit. Then,

(a) power consumed by $2R$ will increase
(b) power consumed by $6R$ will increase
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

Hints & Solutions

1. (*d*) Electric potential of the common centre, is

$$V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2}$$

$$V = \frac{\sigma}{\varepsilon_0} \times r_1 + \frac{\sigma}{\varepsilon_0} \times r_2$$

$$= \frac{\sigma}{\varepsilon_0}[r_1 + r_2] \qquad \left[\because \begin{array}{l} q_1 = 4\pi r_1^2 \times \sigma \\ q_2 = 4\pi r_2^2 \times \sigma \end{array}\right]$$

2. (*d*) The magnitude of the electric dipole moment is

$$p = q \times 2l = (10 \times 10^{-6}) \times (5.00 \times 10^{-3})$$

$$= 50 \times 10^{-9}\text{cm}$$

The field is required at point far away from the centre of the dipole.

The electric field at an axial point (end-on position) distant $r(= 15\text{ cm})$ from the centre of the dipole is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$= (9.0 \times 10^9)\frac{2 \times (50 \times 10^{-9})}{(15 \times 10^{-2})^3}$$

$$= 2.66 \times 10^5\text{NC}^{-1}$$

3. (*d*) Electric field between the two plates is perpendicular to the surface of the conductor as shown, therefore, the effective area of the capacitor is A_1.

$$\therefore \qquad C = \frac{\varepsilon_0 A_1}{d}$$

4. (*b*) The(negative) charge on the drop is

$$q = 12e = 12 \times (1.6 \times 10^{-19})$$
$$= 1.92 \times 10^{-18} C$$

The density of oil is

$$\rho = 1.26 \text{ g-cm}^{-3} = 1.26 \times 10^{3} \text{ kg-m}^{-3}$$

The mass of the drop is

$$m = \text{volume} \times \text{density} = \frac{4}{3}\pi r^3 \rho,$$

where, r is the radius of the drop in metre.

The electric field is

$$E = 2.55 \times 10^4 \text{N C}^{-1}$$

The field E is directed vertically downwards so that the negatively charged drop experiences an upward electric force qE which balances the downward gravity force mg on the drop.

Thus,

$$qE = mg = \frac{4}{3}\pi r^3 \rho g$$

or
$$r = \left(\frac{3qE}{4\pi\rho g}\right)^{1/3}$$

Substituting the given values, we get

$$r = \left(\frac{3 \times (1.92 \times 10^{-18}) \times (2.55 \times 10^4)}{4 \times 3.14 \times (1.26 \times 10^3) \times 9.81}\right)^{1/3} \text{m}$$
$$= (0.946 \times 10^{-18})^{1/3} \text{m} = 0.981 \times 10^{-6} \text{ m}$$
$$= 9.81 \times 10^{-4} \text{ mm}$$

5. (*d*) Net work done = Final PE – Initial PE

$$= \frac{Qq}{4\pi\varepsilon_0 l} - \frac{Qq}{4\pi\varepsilon_0 l} = \text{zero}$$

6. (*a*) Magnitude of a dipole moment is

$$p = qd$$

or
$$= (1.60 \times 10^{-19})(4.30 \times 10^{-9})$$
$$= 6.88 \times 10^{-28} \text{ Cm}$$

7. (*b*) At a point on the axis of a uniformaly charged disk at a distance x above the centre of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

But
$$E_c = \frac{\sigma}{2\varepsilon_0} \text{ such that } \frac{E}{E_c} = \frac{1}{2}$$

Then,
$$1 - \frac{x}{\sqrt{x^2 + R^2}} = \frac{1}{2}$$

or
$$\frac{x}{\sqrt{x^2 + R^2}} = \frac{1}{2}$$

Squaring both side, we get

$$\frac{x^2}{x^2 + R^2} = \frac{1}{4}$$

or
$$x^2 = \frac{x^2}{4} + \frac{R^2}{4}$$

Thus,
$$x^2 = \frac{R^2}{3}$$
$$x = \frac{R}{\sqrt{3}}$$

8. (*c*) Here, $q = 8 \times 10^{-18}$ C, $C = 100\,\mu\text{F} = 10^{-4}$ F

$$V = \frac{q}{C} = \frac{8 \times 10^{-18}}{10^{-4}} = 8 \times 10^{-14} \text{ volt}$$

$$\text{Energy stored} = \frac{1}{2}qV = \frac{1}{2} \times 8 \times 10^{-18} \times 10^{-14}$$
$$= 32 \times 10^{-32}$$

$$\text{Work done} = 2 \times \text{Energy stored}$$
$$= 64 \times 10^{-32} \text{ J}$$

∵ Half energy is always wasted in the form of heat/ electromagnetic waves while charging the capacitor

9. (*c*) Let m and q be the mass and the charge of the drop and E the intensity of electric field between the plates. Since, the drop is in equilibrium, the electric force qE acting on it balances its weight mg. That is,

$$qE = mg$$

If the potential difference between the plates is V and the distance between them is d, then $E = V/d$.

∴ $q(V/d) = mg$ or $V = mgd/q$

Here, $m = 4.8 \times 10^{-10}$ g $= 4.8 \times 10^{-13}$kg,

$g = 9.8\text{N kg}^{-1}, d = 1.0$ cm $= 1.0 \times 10^{-2}$m and

$q = 2.4 \times 10^{-18}$C

∴
$$V = \frac{(4.8 \times 10^{-13}) \times 9.8 \times (1.0 \times 10^{-2})}{2.4 \times 10^{-18}}$$
$$= 1.96 \times 10^4 \text{ V}$$

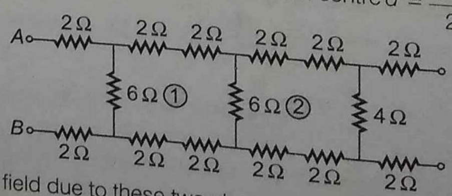
On changing the polarity of the plates, the electric force qE will also be directed downwards. Then, the acceleration of the drop is

$$a = \frac{qE + mg}{m}$$

But $qE = mg$

∴ $a = 2g = 19.6 \text{ ms}^{-2}$

10. (*b*) Since each charge distance from centre $d = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$



Net field due to these two charges is

$$E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2/2}$$

$$= \frac{(9\times 10^9)(1.0\times 10^{-8})}{\frac{(0.050)^2}{2}}$$

$$= 7.19\times 10^4 NC^{-1}$$

and $E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2/2}$

$$= 7.19\times 10^4 N/C$$

The magnitude of the field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{2\,(7.19\times 10^4)^2}$$

$$= 1.02\times 10^5 NC^{-1}$$

Angle made with the x-axis is

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1) = 45°$$

It is upward in the diagram, from the centre of the square towards the centre of the upper side.

11. (*b*) Here, $r = \frac{l_1 - l_2}{l_2}\times 2\,\Omega$

where, $l_1 = 240$ cm, $l_2 = 120$ cm

$$= \frac{240-120}{120}\times 2 = \frac{120}{120}\times 2 = 2\,\Omega$$

12. (*d*) Use $R_t = R_0(1+\alpha t)$

Here, $20 = R_0(1+20\,\alpha)$

$60 = R_0(1+500\,\alpha)$

Here, $R_t = 25\,\Omega$

Solving, we find $t = 80°C$

13. (*d*) We have $F = qE$

Thus, $E = \frac{F}{q} = \frac{3\times 10^{-6}}{2\times 10^{-9}} = 1.5\times 10^3 NC^{-1}$

Magnitude of the electrostatic force on a proton is

$$F_e = eE = (1.60\times 10^{-19})(1.5\times 10^3)$$

$$= 2.4\times 10^{-16} N$$

Magnitude of the gravitational force on the proton is

$$F_g = mg = (1.67\times 10^{-27})(9.8)$$

$$= 1.64\times 10^{-26} N$$

The ratio of the force is

$$\frac{F_e}{F_g} = \frac{2.4\times 10^{-16}}{1.64\times 10^{-26}} = 1.5\times 10^{10}$$

14. (*d*) Electric field $E = \frac{\sigma}{2\varepsilon_0}$

and electric potential

$$V = V_s - \int_0^x E\,dx$$

$$= V_s - Ex$$

Here two surfaces are separated by Δx, then their potentials differ in magnitude by

$$\Delta V = E\Delta x = \left(\frac{\sigma}{2\varepsilon_0}\right)\Delta x$$

Thus, $\Delta x = \frac{2\varepsilon_0 \Delta V}{\sigma}$

$$= \frac{2(8.85\times 10^{-12})(50)}{0.10\times 10^{-6}}$$

$$= 8.8\times 10^{-3} m$$

15. (*c*) $F = eE$

By Newton's second law,

$$a = \frac{F}{m} = \frac{eE}{m}$$

$$= \frac{(1.60\times 10^{-19})(2.00\times 10^4)}{9.11\times 10^{-31}}$$

$$= 3.51\times 10^{15} ms^{-2}$$

16. (*b*) Net potential at point P,

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{5}{2d} - \frac{5}{d} + \frac{5}{d} + \frac{5}{d}\right] = \frac{5q}{8\pi\varepsilon_0}$$

17. (*a*) As the potential difference between point B and point D is zero. So, bridge is balance bridge.

For balance bridge, $\frac{R_{AB}}{R_{AD}} = \frac{R_{BC}}{R_{CD}}$

$$\frac{12+4}{0.5} = \frac{4}{x}$$

$$x = \frac{1}{8} = 0.125\Omega$$

18. (*b*) The resistances 2Ω and 2Ω at the last terminals are outside the circuit and so they may be ignored. Now, in loop 2, the resistances $(2\Omega + 2\Omega)$, 4Ω and $(2\Omega + 2\Omega)$ are in series. Their equivalent resistance is $12\,\Omega$, which is in parallel with 6Ω. The equivalent resistance is

$$R' = \frac{6\times 12}{6+12} = 4\Omega$$

Similarly, in loop 1, the resistances $(2\Omega + 2\Omega)$, $R'(= 4\Omega)$ and $(2\Omega + 2\Omega)$ are in series and these are in parallel with 6Ω. Hence, their equivalent resistance is

$$R'' = 4\Omega$$

Lastly, between the points A and B, the resistances 2Ω, $R'' = (4\Omega)$ and 2Ω are in series. Hence, their equivalent resistance is

$$2\Omega + 4\Omega + 2\Omega = 8\Omega$$

19. (*b*) The arrangement is shown in the figure. The effective emf in the circuit is

$$E = 1.5 + 1.5 = 3.0 V$$

and the total resistance is

$$R = 0.5 + 025 + 2.25 = 3.0\,\Omega$$

Hence, the current in the circuit is

$$I = \frac{E}{R} = \frac{3.0}{3.0} = 1.0\text{A}$$

Potential difference across the terminals of the first cell is

$$V_1 = E - Ir_1 = 1.5 - (1.0)\times(0.5) = 1.0\text{ V}$$

Potential difference across the terminals of the second cell is

$$V_2 = E - Ir_2 = 1.5 - (1.0)\times(0.25) = 1.25\text{ V}$$

20. (*b*) $J = \frac{I}{A} = \frac{10^{-4}}{(0.30\times 0.500)} = 6.7\times 10^{-4}\text{ C-m}^{-2}$.

21. (*c*) An ampere is coulomb per second, so

$$84\text{ A-h} = 84\times 3600 = 3.0\times 10^5\text{C}$$

The change in potential energy is

$$\Delta U = q\Delta V = 3.0\times 10^5\times 12 = 3.6\times 10^6\text{ J}$$

22. (*a*) Potential difference per cm $= \frac{2\text{ V}}{100\text{ cm}} = 0.02\text{ V/cm}$

Balancing length $= \frac{100}{2}\times 1.08 = 54\text{ cm}$

23. (*b*) The current in the potentiometer wire *AB* is

$$I = \frac{6}{20+10} = 0.2\text{ A}$$

The potential difference across the potentiometer wire is

V = current × resistance = $0.2\times 20 = 4\text{V}$

The length of the wire is $l = 200$ cm. So, the potential gradient along the wire is

$$k = \frac{V}{l} = \frac{4}{200} = 0.02\text{ Vcm}^{-1}$$

The emf 2.4V is balanced against a length L of the potentiometer wire. That is

$$2.4 = kL$$

or

$$L = \frac{2.4}{k} = \frac{2.4}{0.02} = 120\text{ cm}$$

24. (*d*) By using $\frac{I}{I_g} = 1 + \frac{G}{S}$

$$\frac{I}{100\times 10^{-3}} = 1 + \frac{1000}{S}$$

$\Rightarrow$ $$S = \frac{1000}{9} = 111\Omega$$

25. (*b*) Resistance in parallel

$$S = \frac{GI_g}{I - I_g} = \frac{50\times 100\times 10^{-6}}{(10 - 100\times 10^{-6})}$$

$\Rightarrow$ $$S = 5\times 10^{-4}\,\Omega$$

26. (*b*) Voltage sensitivity $= \frac{\text{Current sensitivity}}{\text{Resistance of galvanometer } G}$

$\Rightarrow$ $$G = \frac{10}{2} = 5\,\Omega$$

Here, I_g = Full scale deflection current

$$= \frac{150}{10} = 15\text{ mA}$$

V = voltage to be measured

$$= 150\times 1 = 150\text{ V}$$

Hence,

$$R = \frac{V}{I_g} - G = \frac{150}{15\times 10^{-3}} - 5 = 9995\,\Omega$$

27. (*d*) Mass, M = Volume × Density = $Al\times d$

or $$A = \frac{M}{ld}$$

Resistance $$R = \frac{\rho l}{A} = \frac{\rho l}{\left(\frac{M}{ld}\right)} = \frac{\rho l^2 d}{M}$$

So, $$R \propto \frac{l^2}{M}$$

Thus, $$R_1 : R_2 : R_3 = \frac{l_1^2}{M_1} : \frac{l_2^2}{M_2} : \frac{l_3^2}{M_3} = \frac{3^2}{1} : \frac{2^2}{2} : \frac{1^2}{3} = 27 : 6 : 1.$$

28. (*a*) Let Q_A and Q_B be the charges on the solid and the hollow conducting spheres respectively and R be the radius of each sphere. When charge is given to a solid conducting sphere, it appears on the outer surface.

For the calculation of electric field or potential due to a sphere (whether hollow or solid) at a point on or outside the sphere, the charge behaves as if it is concentrated at the centre of the sphere.

If V_A and V_B are potentials of two spheres, then

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{R}$$

or $$V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{R}$$

Since, $$V_A = V_B$$

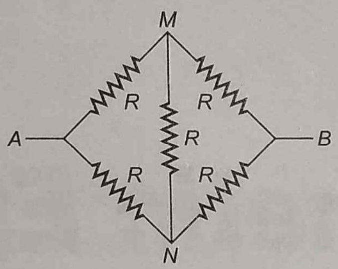
$\Rightarrow$ $$Q_A = Q_B$$

29. (*c*) We know that, when high energy X-rays beam falls on metallic ball, then ball will emit the photoelectrons, so ball will acquire positive charge, because of it the ball will be deflected by the electric and magnetic field present in X-rays. But it will not oscillate.

30. (*b*) A capacitor does not discharge itself. In case the capacitor is connected in a circuit containing a source of high voltage, the capacitor charges itself to a very high potential. So, if a person handles it without discharging, he may get a severe shock.

Dielectrics and insulators cannot conduct electricity but in case of a dielectric, when an external field is applied, induced charges appear on the faces of the dielectric. In other words, the dielectric have the property of transmitting electric effects without conducting.

31. (*c*) The equivalent circuit is represented as,



This is balanced Wheatstone bridge hence, resistance in branch *MN* is not taken into consideration. Hence, the equivalent resistance between points *A* and *B* is given by

$$\frac{1}{R_{AB}} = \frac{1}{(R+R)} + \frac{1}{(R+R)}$$

or $$\frac{1}{R_{AB}} = \frac{2}{2R} = \frac{1}{R}$$

$\therefore$ $$R_{AB} = R$$

32. (*a*) Here, $E = 2\text{V}, I = \frac{2}{2} = 1\text{A}$

and $r = 1\Omega$

Therefore, $V = E - Ir = 2 - 1 \times 1 = 1\text{V}$

33. (*a*) Since power delivered is $P = \frac{V^2}{R}$

and initially resistance of filament would be less which would go up with time due to increase in temperature, until it attains the steady state. So, power drawn by bulb initially is more.

34. (*c*) The resistance of a wire is

$$R = \rho\frac{l}{A}, \rho \text{ being specific resistance}$$

or $$R \propto \frac{Al}{A^2} \quad \text{or} \quad R \propto \frac{1}{r^4} \qquad (\because A = \pi r^2)$$

Hence, when diameter is halved the resistance of the wire is

$$R \propto \frac{1}{\left(\frac{r}{2}\right)^4} = 16R \qquad \ldots\text{(i)}$$

Hence, its resistance will become 16 times.

Again from Eq. (i), we get

$$R \propto \frac{l}{A} \quad \text{or} \quad R \propto \frac{l^2}{Al}$$

or $$R \propto l^2$$

Therefore, on increasing the length resistance increases.

35. (*a*) When a voltmeter is connected across the two poles of a cell, it draws a small current from the cell. So, it measures terminal potential difference between the two poles of the cell, which is always less than the emf of the cell.

On the other hand, when a potentiometer is used for the measurement of emf of a cell it does not draw any current from the cell. Hence, it accurately measures the emf of a cell. Thus, a potentiometer is preferred over a voltmeter.

36. (*b*) Field across small cube $= \frac{V}{a}$

Field across cube of side $2a = \frac{V}{2a}$

Ratio of fields $= \frac{V}{a} : \frac{V}{2a} = 2:1$

37. (*d*) $R = \frac{\rho l}{A}$ for small cube $R_1 = \rho\frac{a}{a^2} = \frac{\rho}{a}$

For the other cube $R_2 = \frac{\rho \cdot 2a}{(2a)^2} = \frac{\rho}{2a}$

Combination resistance $*R_S = R_1 + R_2 = \frac{3}{2}\frac{\rho}{a}$

$$I_1 = \frac{V}{R_1} = \frac{aV}{\rho} \Rightarrow I_2 = \frac{V}{R_2} = \frac{2aV}{3\rho}]$$

$\therefore$ $$(I_1 - I_2)) = \frac{aV}{\rho} - \frac{2aV}{3\rho}$$

38. (*c*) $$R_{\text{net}} = \frac{(3R)(6R)}{(3R)+(6R)} = 2R = 20\,\Omega$$

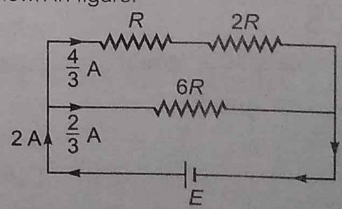
Now, $$P = I^2 R_{\text{net}}$$

$\therefore$ $$I^2 = \frac{80}{20} = 4\text{A}^2 \Rightarrow I = 2\text{A}$$

Further $$P = EI$$

$\therefore$ $$E = \frac{P}{I} = 40\text{V}$$

39. (*a*) Current 2A from the battery will be distributed in the circuit as shown in figure.



Now, from $P = I^2R$, we have

$$P_1 : P_2 : P_6 = \left(\frac{4}{3}\right)^2 R : \left(\frac{4}{3}\right)^2 (2R) : \left(\frac{2}{3}\right)^2 (6R)$$

$$= 16 : 32 : 24 = 2 : 4 : 3$$

40. (*a*) Potential difference across $6R$ in still $E = 40\text{V}$. Therefore, power consumed by $6R$ will remain constant. Potential difference of 40 V was earlier in R and $2R$ and now it is only across $2R$. Or potential difference across $2R$ has increased. Therefore, power consumed by $2R$ will increase.

Day 21

Magnetic Effect of Current

Day 21 Outlines ...

- Concept of Magnetic Field
- Biot-Savart Law and its Applications
- Ampere's Circuital Law
- Cyclotron
- Moving Coil Galvanometer

Concept of Magnetic Field

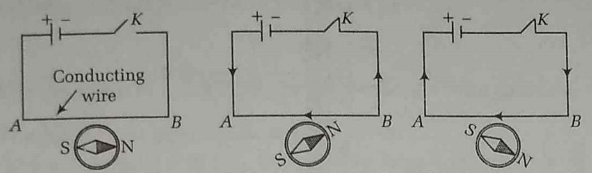
In 1820, Oersted observed that a compass needle suffers, a deflection when brought near a current-carrying wire. This means that, electric current, that is electric charge in motion, gives rise to magnetism.

On this basis the magnetic properties of a substance are attributed to the electronic motions (orbital and spin) in the atoms of the substance. Due to these motions, each atom is equivalent to a tiny current loop and produces magnetic field.

In the unmagnetised state of the substance, the current loops are oriented at random so that the magnetic fields mutually cancel. When the substance is magnetised by some process, all current loops are aligned with their planes parallel to one another and currents circulating in the same direction. Hence, a resultant magnetic field is produced.

Oersted's Experiment

Oersted found experimentally that a magnetic field is established around a current carrying conductor just as it occurs around a magnet.

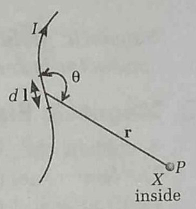


No current, key is open | When key is pressed needle suffers deflection. | When key is pressed it suffers deflection in opposite direction.

Biot-Savart's Law *and* its Applications

According to **Biot-Savart's** law, the magnetic field $d\mathbf{B}$ at a point P due to a current element $I\,d\mathbf{l}$ is given by

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I(d\mathbf{l}\times\mathbf{r})}{r^3}$$



θ is the angle between $d\mathbf{l}$ & $\mathbf{r}$.

Here, $\frac{\mu_0}{4\pi}$ is a proportionality constant having a value of $10^{-7}\,\text{TmA}^{-1}$ in free space.

The term μ_0 is known as the magnetic permeability of free space, where $\mu_0 = 4\pi\times10^{-7}\,\text{TmA}^{-1}$. Dimensional formula for μ_0 is $[\text{MLT}^{-2}\text{A}^{-2}]$.

Direction of magnetic field produced due to a current carrying straight wire can be obtained by the following rules.

(i) According to **right hand thumb rule** grasp the wire carrying current in your right hand with your extended thumb pointing in the direction of current. Then, the fingers will curl around in the direction of the magnetic field.

(ii) According to **Maxwell's cork screw rule**, if we imagine a right handed screw placed along the current carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force.

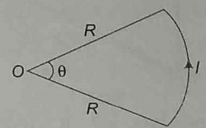
Magnetic Field due to Circular Current Loop

If there is a circular coil of radius R and N number of turns, carrying a current I through the turns, then magnetic field at the centre of coil is given by

$$B = \frac{\mu_0 NI}{2R}$$

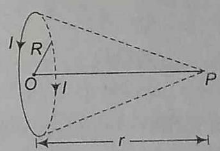
If there is a circular arc of wire subtending an angle θ at the centre of arc, then the magnetic field at the centre point

$$B = \frac{\mu_0 I}{2R}\left(\frac{\theta}{2\pi}\right)$$



At a point situated at a distance r from centre of a current carrying circular coil along its axial line. The magnetic field is

$$B = \frac{\mu_0 NIR^2}{2(R^2 + r^2)^{3/2}}$$

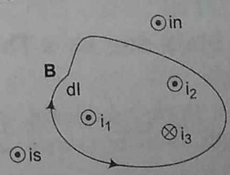


If $r >> R$, then at a point along the axial line $B = \frac{\mu_0 NIR^2}{2r^3}$

Ampere's Circuital Law

According to Ampere's circuital law, the line integral of the magnetic field **B** around any closed path is equal to μ_0 times the net current I threading through the area enclosed by the closed path. Mathematically $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 \Sigma I$

Now consider the diagram below



Here $\Sigma I = i_1 + i_2 - i_3$

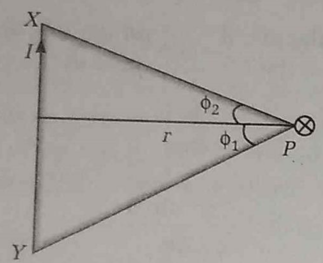
Hence, $\oint \mathbf{B}.dl = \mu_0\cdot(i_1 + i_2 - i_3)$

Applications of Ampere's law

The followings are the few important applications of apere's law may be given as

1. Magnetic field due to Straight Current Carriyng Wire

The magnetic field due to a current carrying wire of finite length at a point P situated at a normal distance r is



From Ampere's law, $\int B \cdot dl = \mu_0 \Sigma I$

$$B = \frac{\mu_0 I}{4\pi r}(\sin \phi_1 + \sin \phi_2)$$

If point P lies symmetrically on the perpendicular bisector of wire XY, then $\phi_1 = \phi_2 = \phi$ (say) and hence

$$B = \frac{\mu_0 I}{4\pi r} \cdot 2 \sin \phi$$

$$= \frac{\mu_0 I \sin \phi}{2\pi r}$$

For a wire of infinite length $\phi_1 = \phi_2 = 90°$ and hence

$$B = \frac{\mu_0 I}{2\pi r}.$$

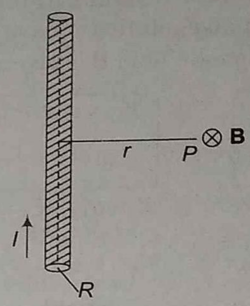
When the wire XY is of infinite length but the point P lies near the end X or Y, then $\phi_1 = 0°$ and $\phi_2 = 90°$ and hence $B = \frac{\mu_0 I}{4\pi r}$.

When point P lies on axial position of current carrying conductor then magnetic field at $P, B = 0$

When wire is of infinite length, then magnetic field near the end will be half, that of at the perpendicular bisector.

2. Magnetic Field due to a Thick (Cylindrical) Wire

Let us have a thick cylindrical wire of radius R and infinite length, which carries a current I. Then, magnetic field at a point outside the wire $B = \frac{\mu_0 I}{2\pi r}$, where r is the distance of given point from centre of wire and $r > R$.



Thick cylindrical wire

Magnetic field at a point inside the wire at a distance r from centre of wire ($r < R$) is

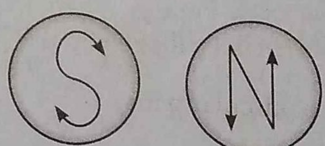
$$B = \frac{\mu_0 I}{2\pi} \cdot \frac{r}{R^2}$$

Magnetic field inside a hollow current carrying conductor is zero.

3. Magnetic Field due to a Solenoid

A current carrying solenoid behaves as a bar magnet. The face where current is flowing clockwise behaves as south pole and the face, where current is seen flowing anti-clockwise, behaves as north pole.

For such a solenoid, the magnetic field inside it is uniform and directed axially.



(a) (b)

For a solenoid coil of infinite length at a point on its axial line, the magnetic field $B = \mu_0 nI$

where n is number of turns per unit length.

At the end of solenoid $B = \frac{1}{2}\mu_0 nI$

AT the end field is half of at the centre this is called end effect.

Toroidal Solenoids

For a toroid (*i.e.*, a ring shaped closed solenoid) magnetic field at any point within the core of toroid $B = \mu_0 nI$, where $n = \frac{N}{2\pi R}$, R = radius of toroid.

Force on a Moving Charge in Uniform Magnetic Field

The magnetic force $\mathbf{F}_m$ acting on a charge q moving with a velocity $\mathbf{v}$ in magnetic field $\mathbf{B}$ is given by

$$\mathbf{F}_m = q\,[\mathbf{v} \times \mathbf{B}]$$

The direction of force $\mathbf{F}_m$ is given by rule of vector product.

The force F_m is zero, if either (a) $\mathbf{B} = 0$ or (b) $\mathbf{v} = 0$ or (c) $q = 0$, *i.e.*, the particle is neutral particle or $\theta = 0°$ or $180°$ *i.e.*, motion of charged particle is parallel or anti-parallel to the magnetic field.

The force $\mathbf{F}_m$ is maximum having a value $F_m = qvB$ when $\theta = 90°$ *i.e.*, charged particle is moving in a direction perpendicular to that of magnetic field.

Thus,magnetic field B may be defined by the relation

$$B = \frac{F_m}{qv\sin\theta}.$$

Thus, SI unit $1\ \text{T} = 1\,\text{NA}^{-1}\text{m}^{-2}$.

Motion of a Charged Particle in a Uniform Magnetic Field

1. (i) If a charge particle enters a uniform magnetic field B with a velocity v in a direction perpendicular to that of $B (i.e., \theta = 90°)$, then the charged particle experiences a force $F_m = qvB$. Under its influence the particle describes a circular path such that

   Radius of circular path $r = \dfrac{mv}{qB}$

   In general $r = \dfrac{mv}{qB} = \dfrac{p}{qB}$

   $$= \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2mqV}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

   where, $p = mv$ = momentum of charged particle, K = kinetic energy of charged particle and V = accelerating potential difference.

   (ii) The period of revolution of charged particle $T = \dfrac{2\pi m}{qB}$, the frequency of revolution $\nu = \dfrac{qB}{2\pi m}$ or angular frequency $\omega = \dfrac{qB}{m}$.

2. (i) If a charged particle is moving at an angle θ to the magnetic field (where θ is other than 0°, 90° or 180°), it describes a helical path, where radius of helical path $r = \dfrac{mv\sin\theta}{qB}$.

   (ii) Revolution period $T = \dfrac{2\pi m}{qB}$

   or frequency $\nu = \dfrac{qB}{2\pi m}$

   (iii) Moreover, pitch (the linear distance travelled during one complete revolution) of helical path is given by

   $$p = v\cos\theta \cdot T = \frac{2\pi mv\cos\theta}{qB}$$

3. If the direction of a $\mathbf{v}$ is parallel or anti-parallel to $\mathbf{B}, \theta = 0$ or $\theta = 180°$ and therefore $F = 0$. Hence, the trajectory of the particle is a straight line.

   If the velocity of the charged particle is not perpendicular to the field, we will break the velocity in parallel ($v_{||}$) and perpendicular ($v_{\perp}$) components.

   $$r = \frac{mv_{\perp}}{qB}$$

   $$\text{pitch } p = (v_{||})T$$

Cyclotron

It is a device used to accelerate positively charged particles e.g., proton, deuteron, α-particle and other heavy ions to high energy of 100 MeV or more. It utilises the principle that an electric field accelerates the charged particles and a normal magnetic field keeps them revolving in circular orbits or constant frequency.

For continuous acceleration the cyclotron frequency ν is arranged so that during the time ion completes a semicircular path in a dee, direction of electric field is just reversed. Cyclotron frequency $\nu = \dfrac{Bq}{2\pi m}$

Maximum energy gained by the charged particle $E_{max} = \left[\dfrac{q^2B^2}{2m}\right] r^2$, where r = maximum radius of the circular path followed by the positive ion.

Maximum energy obtained by the particle is in the form of kinetic energy.

Magnetic Force on a Current Carrying Conductor

If a current carrying conductor is placed in a magnetic field **B**, then a small current element $I\,d\mathbf{l}$ experiences a force given by $d\mathbf{F}_m = Id\mathbf{l} \times \mathbf{B}$

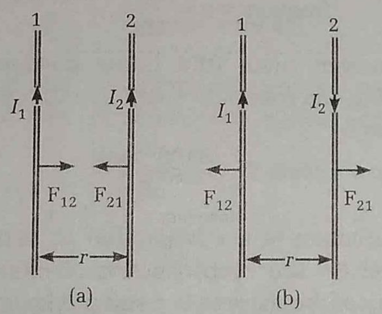
and the total force experienced by whole current carrying conductor will be

$$\mathbf{F}_m = \int d\mathbf{F}_m = \int I(d\mathbf{l} \times \mathbf{B})$$

The direction of force when current element $Id\mathbf{l}$ and **B** are perpendicular to each other can also be determined by applying Fleming's left hand rule or right hand thumb rule.

Force between Two Parallel Current Carrying Conductors

Two parallel current carrying conductors exert magnetic force on one another.



If two infinitely long parallel conductors carry currents I_1 and I_2 respectively and are separated by a distance r, then magnetic force experienced by length l of any one conductor due to the other current carrying conductor is

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} l$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}$$

Force per unit length

$$\frac{F}{l} = F_0 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}$$

- *If conductors carries current in same direction, then force between them will be attractive.*
- *If the conductor carries current in opposite direction, then force will be repulsive.*

Current Loop as a Magnetic Dipole

A current carrying loop (of any shape) behaves as a magnetic dipole whose magnetic moment is given by $\mathbf{M} = I\mathbf{A}$, where I is the current flowing through the loop and **A** the surface area of the loop.

If we have a current carrying coil having N turns, then magnetic moment **M** of dipole will be $\mathbf{M} = NI\mathbf{A}$

Magnetic moment of a current carrying coil is a vector and its direction is given by right hand thumb rule.

For given perimeter circular shape have maximum area. Hence, maximum magnetic moment.

*For any loop or coil **B** at centre due to current in loop and **M** are always parallel.*

Torque

The current carrying coil is equivalent to a magnetic dipole.

$\tau = NiAB \sin\theta$, where NiA is defined as the magnitude of the dipole moment of the coil (p_m).

$$\tau = p_m B \sin\theta$$

$$\tau = \mathbf{p}_m \times \mathbf{B}$$

Moving Coil Galvanometer (MCG)

MCG is used to measure the current upto nanoampere.

In MCG the coil is suspended between the pole pieces of a strong horseshoe magnet.

The pole pieces are made cylindrical and a soft iron cylindrical core is placed within the coil without touching it. This makes the field radial.

In the plane of the coil always remain parallel to the field. Therefore $\theta = 90°$ and the deflecting torque always has the maximum value

$$\tau_{def} = NBIA$$

A restoring torque is set up in the suspension fibre. If α is the angle of trust, the restoring torque is

$$\tau_{rest} = C\alpha$$

where C is the torional constant of the fibre.

When the coil is in equilibrium

$$I = k\alpha$$

where, $k = \frac{C}{NBA}$ is galvanometer constant.

Some Important Concepts Related to Moving Coil Galvanometer

Some of the important concept related to galvanometer, i.e., current sensitivity, voltage sensitivity and some of conversions used in galvanometer are given below.

Current Sensitivity

The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

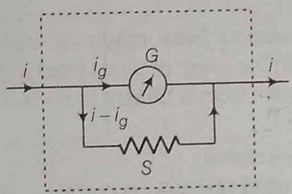
$$S_I = \frac{\alpha}{I} = \frac{NBA}{C}$$

Voltage Sensitivity

Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{S_I}{R} = \frac{NBA}{RC}$$

Conversion of Galvanometer *into* Ammeter



Ammeter

An ammeter is made by connecting a low resistance S in parallel with a pivoted type moving coil galvanometer G. S is known as shunt. Then from circuit

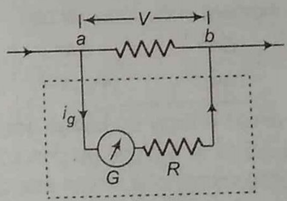
$$i_g \times G = (i - i_g) \times S$$

$$\frac{i_g}{i} = \frac{S}{S + G}$$

$$\Rightarrow \quad S = \left(\frac{i_g}{i - i_g}\right) G$$

So, $S << G$, only a small fraction of current goes through the galvonometer.

Conversion of Galvanometer into Voltmeter



Voltmeter

A voltmeter is made by connecting a resistor of high resistance R in series with a pivoted type moving coil galvanometer G.

From the circuit,

$$i_g = \frac{V}{G + R}$$

$$\Rightarrow \quad G + R = \frac{V}{i_g}$$

$$\Rightarrow \quad R = \frac{V}{i_g} - G$$

Practice Zone

DAY 21

1. The magnetic field normal to the plane of a wire of n turns and radius r which carries a current I is measured on the axis of the coil at a small distance is from the centre of the coil. This is smaller than the magnetic field at the centre by the fraction

(a) $(2/3)r^2/h^2$ (b) $(3/2)r^2/h^2$
(c) $(2/3)h^2/r^2$ (d) $(3/2)h^2/r^2$

2. Magnetic field at the centre of a circular loop of area A is B. The magnetic moment of the loop will be

(a) $\frac{BA^2}{\mu_0\pi}$ (b) $\frac{BA^{3/2}}{\mu_0\pi}$ (c) $\frac{BA^{3/2}}{\mu_0\pi^{1/2}}$ (d) $\frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$

3. A deutron of kinetic energy 50 keV is describing a circular orbit of radius 0.5 m, in a plane perpendicular to magnetic field B. The kinetic energy of a proton that describes circular orbit of radius 0.5 m in the same plane with the same magnetic field is

(a) 200 keV (b) 50 keV
(c) 100 keV (d) 25 keV

4. A magnetic field 4×10^{-3} kT exerts a force $(4\hat{i}+3\hat{j})\times10^{-10}$N on a particle having a charge 10^{-9}C and going on the x-y plane. The velocity of the particle is

(a) $-75\hat{i}+100\hat{j}$ (b) $-100\hat{i}+75\hat{j}$
(c) $25\hat{i}+2\hat{j}$ (d) $2\hat{i}+25\hat{i}$

5. A proton and an α-particle enters a uniform magnetic field perpendicularly with the same speed. If proton takes 25 μs to make 5 revolutions, then the periodic time for the α-particle would be

(a) 50 μs (b) 25 μs
(c) 10 μs (d) 5 μs

6. A uniform magnetic field $B=B_0\hat{j}$ exists in space. A particle of mass m and charge q is projected towards X-axis with speed v from a point $(a, 0, 0)$. The maximum value of v for which the particle does not hit the Y-Z plane is

(a) $\frac{Bqa}{m}$ (b) $\frac{Bqa}{2m}$
(c) $\frac{Bq}{am}$ (d) $\frac{Bq}{2am}$

7. Through two parallel wires A and B, 10 A and 2 A of currents are passed respectively in opposite directions. If the wire A is infinitely long the length of the wire B is 2 m, then force on the conductor B, which is situated at 10 cm distance from A, will be

(a) 8×10^{-7} N
(b) 8×10^{-5} N
(c) 4×10^{-7} N
(d) 4×10^{-5} N

8. A loosely wound helix made of stiff wire is mounted vertically with the lower end just touching a dish of mercury. When a current from a battery is started in the coil through the mercury

(a) the wire oscillates
(b) the wire continues making contact
(c) the wire breaks contact just as current is passed
(d) the mercury will expand by heating due to passes of current

9. Two parallel long wires A and B carry currents I_1 and $I_2\,(<I_1)$. When I_1 and I_2 are in the same direction, the magnetic field at a point mid way between the wires is 10 μT. If I_2 is reversed, the field becomes 30 μT. The ratio I_1/I_2 is

(a) 1 (b) 3
(c) 2 (d) 4

10. A cell is connected between two points of a uniformly thick circular conductor. i_1 and i_2 are the currents flowing in two parts of the circular conductor of radius a. The magnetic field at the centre of the loop will be

(a) zero (b) $\frac{\mu_0}{4\pi}(i_1-i_2)$
(c) $\frac{\mu_0}{2a}(i_1+i_2)$ (d) $\frac{\mu_0}{a}(i_1+i_2)$

11. A current carrying circular loop of radius R is placed in the $x-y$ plane with centre at the origin. Half of the loop with $x>0$ is now bent so that it now lies in the $y-z$ plane. **[NCERT Exemplar]**

(a) The magnitude of magnetic moment now diminishes
(b) The magnetic moment does not change
(c) The magnitude of B at (0.0. z), $z>>R$ increases
(d) The magnitude of B at(0.0. z), $z>>R$ is unchanged

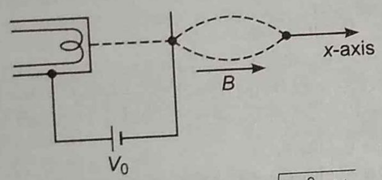
12. A long solenoid of length L has a mean diameter D. It has n layers of windings of N turns each. If it carries a current I, the magnetic field at its centre will be

(a) proportional to D
(b) inversely proportional to D
(c) independent of D
(d) proportional to L

13. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is

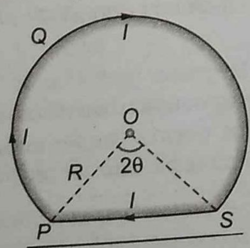
(a) $\frac{\mu_0 NI}{b}$
(b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$
(d) $\frac{\mu_0 I^N}{2(b-a)}\ln\frac{b}{a}$

14. Electrons emitted with negligible speed from an electron gun are accelerated through a potential difference V_0 along the x-axis. These electrons emerge from a narrow hole into a uniform magnetic field of strength B directed along x-axis. Some electrons emerging at slightly divergent angles as shown. These paraxial electrons are refocussed on the x-axis at a distance.



(a) $\frac{8\pi^2 mV_0}{3eB}$
(b) $\sqrt{\frac{8\pi^2 mV_0}{eB^2}}$
(c) $\sqrt{\frac{2\pi^2 mV_0}{eB^2}}$
(d) $\sqrt{\frac{4\pi^2 mV_0}{eB^2}}$

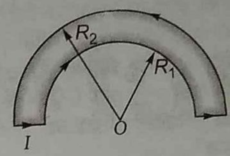
15. A current I flows through a closed loop shown in figure.



The magnetic field induction at the centre O is

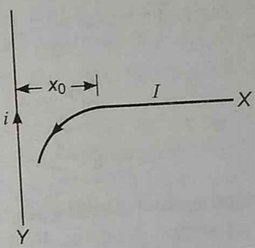
(a) $\frac{\mu_0 I}{4\pi R}\theta$
(b) $\frac{\mu_0 I}{4\pi R}(\theta + \sin\theta)$
(c) $\frac{\mu_0 I}{4\pi R}(\pi - \theta + \sin\theta)$
(d) $\frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta)$

16. The magnetic induction at the centre O in the figure shown is



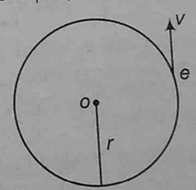
(a) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
(b) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
(c) $\frac{\mu_0 I}{4}(R_1 - R_2)$
(d) $\frac{\mu_0 I}{4}(R_1 + R_2)$

17. A long straight wire carries a current i. A particle of charge $+q$ and mass m is projected with a speed v from a distance x_0 as shown. The minimum separation between the wire and particle is



(a) $x_0 e^{\frac{-2\pi mv}{\mu_0 qi}}$
(b) $x_0 e^{\frac{-mv2\pi x_0}{\mu_0 qi}}$
(c) $x_0 e^{\frac{-2\pi mv}{qi}}$
(d) zero

18. An electron moves in a circular orbit with a uniform speed v. It produces a magnetic field B at the centre of the circle. The radius of the circle is proportional to

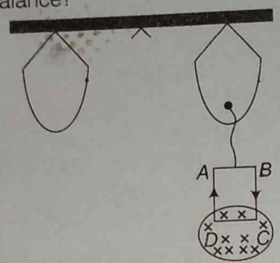


(a) $\frac{B}{v}$
(b) $\frac{v}{R}$
(c) $\sqrt{\frac{v}{B}}$
(d) $\sqrt{\frac{B}{v}}$

19. The cyclotron frequency of an electron gyrating in a magnetic field of 1 T is approximately

(a) 28 MHz
(b) 280 MHz
(c) 2.8 GHz
(d) 28 GHz

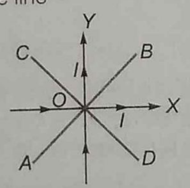
20. A 100 turn rectangular coil *ABCD* (in *XY*-plane) is hung from one arm of a balance (shown in figure). A mass 500 g is added to the other arm to balance the weight of the coil. A current of 4.9 A passes through the coil and a constant magnetic field of 0.2 T acting inward (in *XZ*-plane) is switched on such that only arm *CD* of length 1 cm lies in the field. How much additional mass *m* must be added to regain the balance? **[NCERT Exemplar]**



21. Two long conductors separated by a distance *d* carry current I_1 and I_2 in the same direction. They exert a force *F* on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to 3*d*. The new value of the force between them is

(a) $-2F$
(b) $\frac{F}{3}$
(c) $\frac{2F}{3}$
(d) $-\frac{F}{3}$

22. Two very thin metallic wires placed along *X* and *Y*-axes carry equal currents as shown in figure. *AB* and *CD* are lines at 45° with the axes with origin of axes at *O*. The magnetic field will be zero on the line



(a) *AB*
(b) *CD*
(c) segment *OB* only of line *AB*
(d) segment *OC* only of line *CD*

23. A current carrying loop is free to turn in a uniform magnetic field. The loop will then come into equilibrium when its plane is inclined at

(a) 0° to the direction of the field
(b) 45° to the direction of the field
(c) 90° to the direction of the field
(d) 135° to the direction of the field

Directions *(Q. Nos. 24 to 29) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choice, only one of which is the correct answer. you have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

24. Statement I If a charged particle is projected in a region where **B** is perpendicular to velocity of projection, then the net force acting on the particle is independent of its mass.

Statement II The particle is performing uniform circular motion, and net force acting on it is $\frac{mv^2}{r}$.

25. Statement I A uniformly moving charged particle in a magnetic field, may follow a path along magnetic field lines.

Statement II The direction of magnetic force experienced by a charged particle is perpendicular to its velocity and **B**.

26. Statement I The magnetic force experienced by a moving charged particle in a magnetic field is invariant in nature just like any other force.

Statement II Magnetic force experienced by a charged particle is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$, where **v** is the velocity of charge particle w.r.t. frame of reference in which we are taking **F**.

27. Statement I Cyclotron is a device which is used to accelerate the positive ion.

Statement II Cyclotron frequency depends upon the velocity.

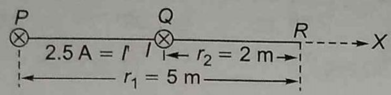
28. Statement I Magnetic field due to a infinite straight conductor varies inversely as the distance from it.

Statement II The lines of electric force due to a straight current carrying conductor are concentric circles.

29. Statement I If a proton and α-particle enter a uniform magnetic field perpendicularly with the same speed, the time period of revolution of α-particle will be double than that of proton.

Statement II Time period of charged particle is given by $T = \frac{2\pi m}{Bq}$.

Directions (Q. Nos. 30 to 32) *Two long parallel wires carrying currents 2.5 A and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of the paper. The points P and Q are located at a distance of 5 m and 2 m, respectively, from a collinear point R (see figure).*



An electron moving with a velocity of $4 \times 10^5 ms^{-1}$ along the positive X-direction experiences a force of magnitude 3.2×10^{-20}N at the point R.

30. The magnitude of magnetic field at point R is
(a) 2.5×10^{-7} T (b) 5.0×10^{-7} T
(c) 5.0×10^{-6} T (d) 2.5×10^{-6} T

31. The magnitude of magnetic field at point R due to current $I' = 2.5$A in wire P is
(a) 1×10^{-7} T (b) 2×10^{-7} T
(c) 3×10^{-7} T (d) 4×10^{-7} T

32. The current I in wire Q is
(a) 1 A (b) 2 A
(c) 3 A (d) 4 A

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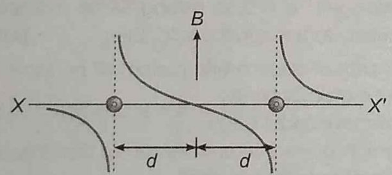
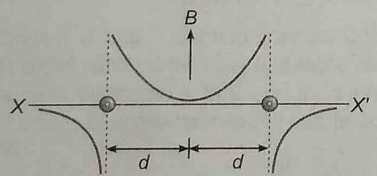
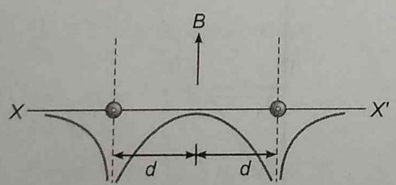
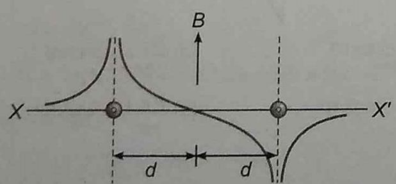
33. A current I flows in an infinity long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is **[AIEEE 2011]**
(a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$ (c) $\frac{\mu_0 I}{4\pi R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

34. Two long parallel wires are at a distance $2d$ apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by **[AIEEE 2010]**

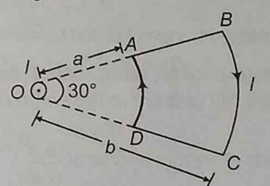
(a)


(b)


(c)


(d)


Directions (Q. Nos. 35 and 36) *A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by the straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.*



35. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is **[AIEEE 2009]**
(a) zero
(b) $\frac{\mu_0 I(b-a)}{24ab}$
(c) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$
(d) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

36. Due to the presence of the current I_1 at the origin, **[AIEEE 2009]**
(a) the forces on AB and DC are zero
(b) the forces on AD and BC are zero
(c) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$
(d) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$

37. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by **[AIEEE 2007]**

(a) $\frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{d}\right)^{1/2}$ (b) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)^{1/2}$

(c) $\frac{\mu_0}{2\pi d}(I_1+I_2)$ (d) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)$

38. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields **E** and **B** with a velocity **v** perpendicular to both **E** and **B** and comes out without any change in magnitude or direction of **v**. Then **[AIEEE 2007]**

(a) $\mathbf{v}=\mathbf{E}\times\frac{\mathbf{B}}{B^2}$ (b) $\mathbf{v}=\mathbf{B}\times\frac{\mathbf{E}}{B^2}$ (c) $\mathbf{v}=\mathbf{E}\times\frac{\mathbf{B}}{E^2}$ (d) $\mathbf{v}=\mathbf{B}\times\frac{\mathbf{E}}{E^2}$

39. A charged particle moves through a magnetic field perpendicular to its direction. Then **[AIEEE 2007]**

(a) the momentum changes but the kinetic energy is constant
(b) both momentum and kinetic energy of the particle are not constant
(c) both momentum and kinetic energy of the particle are constant
(d) kinetic energy changes but the momentum is constant

40. Two thin, long, parallel wires, separated by a distance d carry a current of I ampere in the same direction. They will **[AIEEE 2005]**

(a) attract each other with a force of $\frac{\mu_0 I^2}{(2\pi d)}$

(b) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d)}$

(c) attract each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

(d) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

41. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 A and 4 A are the currents flowing in each coil respectively. The magnetic induction in Wb m^{-2} at the centre of the coils will be ($\mu_0 = 4\pi\times10^{-7}$ Wb A^{-1}m^{-1}) **[AIEEE 2005]**

(a) 12×10^{-5} (b) 10^{-5} (c) 5×10^{-5} (d) 7×10^{-5}

42. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is **[AIEEE 2004]**

(a) $-2F$ (b) $\frac{F}{3}$ (c) $-\frac{2F}{3}$ (d) $-\frac{F}{3}$

43. A particle of mass M and charge Q moving with velocity **v** describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is **[AIEEE 2003]**

(a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) zero

(c) $BQ2\pi R$ (d) $BQv2\pi R$

44. A particle of mass m and charge q moves with a constant velocity v along the positive X-direction. It enters a region containing a uniform magnetic field B directed along the negative Z-direction, extending from $x=a$ to $x=b$. The minimum value of v required so that the particle can just enter the region $x>b$ is **[AIEEE 2002]**

(a) $\frac{qbB}{m}$ (b) $\frac{q(b-a)B}{m}$

(c) $\frac{qaB}{m}$ (d) $\frac{q(b+a)B}{2m}$

45. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields, B_A and B_B produced by them, will be **[AIEEE 2002]**

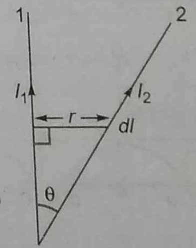
(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) 4

46. If an electron and a proton having same momenta enter perpendicularly to a magnetic field, then **[AIEEE 2002]**

(a) curved path of electron and proton will be same (ignoring the sense of revolution)
(b) they will move undeflected
(c) curved path of electron is more curved than that of proton
(d) path of proton is more curved

47. Wires 1 and 2 carrying currents I_1 and I_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance r from wire 1 (as shown in figure) due to the magnetic field of wire 1? **[AIEEE 2002]**



(a) $\frac{\mu_0}{2\pi r}I_1I_2\,dl\tan\theta$ (b) $\frac{\mu_0}{2\pi r}I_1I_2\,dl\sin\theta$

(c) $\frac{\mu_0}{2\pi r}I_1I_2\,dl\cos\theta$ (d) $\frac{\mu_0}{4\pi r}I_1I_2\,dl\sin\theta$

Answers

1. (d)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (a)
11. (a)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (b)
21. (c)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (d)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (b)
31. (a)	**32.** (d)	**33.** (d)	**34.** (a)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (a)
41. (c)	**42.** (c)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (a)	**47.** (c)			

Hints & Solutions

1. $B_1 = \frac{\mu_0}{4\pi}\frac{2\pi ni}{r}$ and $B_2 = \frac{\mu_0}{4\pi}\frac{2\pi nir^2}{(r^2+h^2)^{3/2}}$

So, $$\frac{B_2}{B_1} = \left(1+\frac{h^2}{r^2}\right)^{-\frac{3}{2}}$$

Fractional decrease in the magnetic field will be

$$= \frac{B_1 - B_2}{B_1} = \left(1-\frac{B_2}{B_1}\right) = \left[1-\left(1+\frac{h^2}{r^2}\right)\right]$$

$$= 1-\left(1-\frac{3}{2}\cdot\frac{h^2}{r^2}\right) = \frac{3}{2}\frac{h^2}{r^2}$$

2. As, $B = \frac{\mu_0}{2\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r} \Rightarrow I = \frac{2Br}{\mu_0}$

Also $$A = \pi r^2 \text{ or } r = \left(\frac{A}{\pi}\right)^{1/2}$$

Magnetic moment, $M = IA = \frac{2Br}{\mu_0}A$

$$= \frac{2BA}{\mu_0}\times\left(\frac{A}{\pi}\right)^{1/2}$$

$$= \frac{2BA^{3/2}}{\mu_0 \pi^{1/2}}$$

3. As, $r = \frac{\sqrt{2mE}}{Bq} = \frac{\sqrt{2m_1E_1}}{Bq}$

or $E_1 = \frac{mE}{m_1} = \frac{(2m_1)}{m_1}\times 50\text{ keV} = 100\text{ keV}$

4. From Lorentz force

$$F = q(v \times B)$$

Given, $F = (4\hat{i} + 3\hat{j})\times 10^{-10}\text{ N}, q = 10^{-9}\text{C}$

$$B = 4\times 10^{-3}\,\hat{k}\text{T}$$

$\therefore \quad (4\hat{i} + 3\hat{j})\times 10^{-10} = 10^{-9}(a\hat{i} + b\hat{j})\times(4\times 10^{-3})$

Solving, we get

$$a = -75, b = 100$$

$\Rightarrow \quad v = -75\,\hat{i} + 100\hat{j}$

5. Time taken by to make one revolution $= \frac{25}{5} = 5\,\mu\text{s}$

As $$T = \frac{2\pi m}{qB};\text{ so } \frac{T_2}{T_1} = \frac{m_2}{m_1}\times\frac{q_1}{q_2}$$

or $$T_2 = T_1\frac{m_2q_1}{m_1q_2} = \frac{5\times 4m_1}{m_1} = \frac{q}{2q} = 10\,\mu\text{s}$$

6. Here, the particle is projected in a direction perpendicular to the uniform magnetic field, hence it will describe a circular path. The particle will not hit the *Y-Z* plane, if the radius of the circular path is smaller than *a*. For maximum value of *v*, the radius of circular path is just equal to *a*. Hence,

$$\frac{mv}{Bq} = a \text{ or } v = \frac{Bqa}{m}$$

7. As, $F = \frac{\mu_0}{2\pi}\frac{2i_1i_2}{r}l = \frac{10^{-7}\times 2\times 10\times 2}{0.1}\times 2 = 8\times 10^{-5}\text{N}$

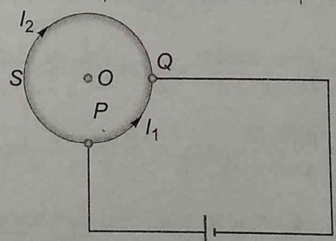
9. $$\frac{\mu_0}{4\pi}\frac{2I_1}{r} - \frac{\mu_0}{4\pi}\frac{2I_2}{r} = 10\mu\text{T}$$

$$\frac{\mu_0 2I_1}{4\pi r} + \frac{\mu_0}{4\pi}\frac{2I_2}{r} = 30\mu\text{T}$$

On solving, $I_1 = 20\text{ A}$ and $I_2 = 10\text{ A}$

So, $I_1 / I_2 = 2$

10. The resistance of the portion *PRQ* will be $R_1 = l_1\rho$



Resistance of the portion *PSQ* will be $R_2 = l_2\rho$

Potential difference across *P* and $Q = I_1R_1 = I_2R_2$

$$I_1l_1\rho = I_2l_2\rho \text{ or } I_1l_1 = I_2l_2 \quad \text{...(i)}$$

Magnetic field induction at the centre *O* due to currents through circular conductors *PRQ* and *PSQ* will be $= B_1 - B_2$

$$= \frac{\mu_0}{4\pi}\frac{I_1l_1 \sin 90°}{r^2} - \frac{\mu_0}{4\pi}\frac{I_2l_2 \sin 90°}{r^2} = 0$$

11. For a circular loop of radius R, carrying current I in x-y plane, the magnetic moment $M = I \times \pi R^2$. It acts perpendicular to the loop along z-direction. When half of the current loop is bent in y-z plane, then magnetic moment due to half current loop in x-y plane, $M_1 = I(\pi R^2/2)$ acting along z-direction. Magnetic moment due to half current loop in y-z plane, $M_2 = I(\pi R^2/2)$ along x-direction.

Effective magnetic moment due to entire bent current loop,

$$M' = \sqrt{M_1^2 + M_2^2} = \sqrt{(I\pi R^2/2)^2 + (I\pi R^2/2)^2} = \frac{I\pi R^2}{2}\sqrt{2} < M$$

i.e., **magnetic moment diminishes.**

The magnitude of **B** at a point on the axis of loop, distance z from the center of current loop in x-y plane is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I R^2}{(R^2 + z^2)^{3/2}}$$

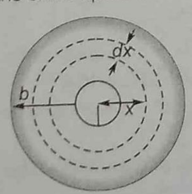
The magnitude of **B** at a point distance z from the centre of bent current loop, whose half part is in x-y plane and half part is in y-x plane, is

$$B = \sqrt{\left[\frac{\mu_0}{4\pi}\frac{\pi I R^2}{(R^2 + z^2)^{3/2}}\right]^2 + \left[\frac{\mu_0}{4\pi}\frac{\pi I R^2}{(R^2 + z^2)^{3/2}}\right]^2}$$

$$= \frac{\mu_0}{4\pi}\frac{\pi I R^2}{(R^2 + z^2)^{3/2}}\sqrt{2} < B$$

12. $B = \mu_0 \frac{(N)}{L} I$ which is independent of diameter D.

13. Refer to the figure, number of turns in dx, $n = \frac{N \cdot dx}{b - a}$. The magnetic field induction at the centre O due to current I through the entire spiral is



$$B = \int_a^b \frac{\mu_0}{4\pi}\frac{2\pi n I dx}{x} = \frac{\mu_0}{4\pi}2\pi\int_a^b\left(\frac{N}{b-a}\right)\frac{dx}{x} I$$

$$= \frac{\mu_0}{4\pi}\frac{NI}{(b-a)}(\log_e x)_a^b = \frac{\mu_0}{2}\frac{NI}{(b-a)}\log_e\frac{b}{a}$$

14. The electrons will be refocussed after distance equal to pitch.

$$\text{Pitch} = V_x T = V\frac{2\pi m}{eB} = \sqrt{\frac{2eV_0}{m}} \cdot \frac{2\pi m}{eB} = \sqrt{\frac{8\pi^2 m V_0}{eB^2}}$$

$$\because \quad eV_0 = \frac{1}{2}mv^2$$

15. $B_0 = B_{POS} + B_{SP}$

$$= \frac{\mu_0}{4\pi}\frac{I}{R}(2\pi - 2\theta) + \frac{\mu_0}{4\pi}\frac{1}{R\cos\theta}(\sin\theta + \sin\theta)$$

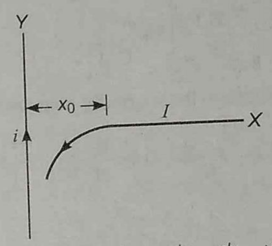
$$= \frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta)$$

16. Magnetic field due to straight parts of wire at point $O = 0$. Field due to a semicircular current loop of radius R_1, $B_1 = \frac{\mu_0 I}{2R_1}\left(\frac{\pi}{2\pi}\right) = \frac{\mu_0 I}{4R_1}$ into the plane of paper. Field due to semicircular current loop of radius R_2, $B_2 = \frac{\mu_0 I}{4R_2}$ outside the plane of paper.

$$\therefore \quad \text{Net field} \quad B = B_1 - B_2 = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

17. $F = q(\hat{i}v_x + \hat{j}v_y) \times \left[\frac{\mu_0 i}{2\pi x}k\right] = \hat{j}q\,v_x\frac{\mu_0 i}{2\pi x} - \hat{i}q\,v_y\frac{\mu_0 i}{2\pi x}$

$$\therefore \quad a_x = \frac{F_x}{m} = -\frac{\mu_0 iqv_y}{2\pi xm}$$



Also, $\quad a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \frac{dx}{dt} = \frac{v_x\,dv_x}{dx}$

Since, $\quad v_x^2 + v_y^2 = v^2$

or $\quad 2v_x\,dv_x + 2v_y dv_y = 0$

$\Rightarrow \quad v_x dv_x = -v_y dv_y$

Hence, $\quad \frac{v_x dv_x}{dx} = -\frac{v_y dv_y}{dx} = -\frac{\mu_0 iqv_y}{2\pi xm}$

$$\Rightarrow \quad \frac{dx}{x} = \frac{dv_y 2\pi m}{\mu_0 qi}$$

Initially, $x = x_0$ and $v_y = 0$

At minimum separation $v_x = 0$, $v_y = v$

Thus, $\quad \int_{x_0}^{x}\frac{dx}{x} = \frac{2\pi m}{\mu_0 qi}\int_0^{-v} dv_y$

$$\Rightarrow \quad \log\frac{x}{x_0} = -\frac{2\pi mv}{\mu_0 qi}$$

$$\Rightarrow \quad x = x_0 e^{-\frac{2\pi mv}{\mu_0 qi}}$$

18. Equivalent current $I = \frac{ev}{2\pi r}$

Hence, magnetic field at centre of circle

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r} \cdot \frac{ev}{2\pi r} = \frac{\mu_0 ev}{4\pi r^2}$$

$$\Rightarrow \quad r = \sqrt{\frac{\mu_0 ev}{4\pi B}} \Rightarrow r \propto \sqrt{\frac{v}{B}}$$

19. Cyclotron frequency $\nu = \frac{Bq}{2\pi m} = \frac{1 \times 1.6 \times 10^{-19}}{2\pi \times 9.1 \times 10^{-31}}$

$= 2.8 \times 10^{10}\ \text{Hz} = 28\ \text{GHz}$

20. Let the mass of coil $= M$

Mass added other arm $= 500\ \text{g} = 500 \times 10^{-3}$ kg

Mass of coil = Mass in other arm (for balancing)

$Mg = 500 \times 10^{-3}$ kg

$M = 0.5$ kg

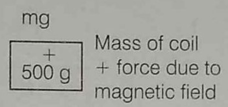
When the current is switched on,

Current $I = 4.9$ A

Magnetic field $B = 0.2$ T (XZ-plane)

Length of arm $CD = 1$ cm

Mass added to balance $= m$



Let F be the force due to magnetic field.

The direction of magnetic field is inward in XZ-plane, the length vector is left, so by using the Fleming's left hand rule, the direction of force is downwards in the plane of paper

$F = I(l \times B) = 4.9\,(0.01 \times 0.2 \sin 90°)$

$F = 4.9 \times 0.01 \times 0.2$ ...(i)

For balancing, mass of coil $\times g$ + force due to magnetic field $= 500 \times 10^{-3} g + m \times g$

$0.5 \times 9.8 + 4.9 \times 0.01 \times 0.2 = 500 \times 10^{-3} \times 9.8 + m \times 9.8$

$9.8\,(0.5 + 0.001) = 9.8\,(0.5 + m)$ [From Eq. (i)]

$m = 0.001\ \text{kg} = 1$ kg

Thus, 1 g mass must be added to regain the balance.

21. Initial force $F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{d}$ (attractive)

and final force $F' = \frac{\mu_0}{4\pi} \cdot \frac{2\,(2I_1)I_2}{(3d)}$ (repulsive) $\Rightarrow$ $F' = \frac{2}{3}F$

22. Along the line AB, magnetic field due to two wires is equal but in mutually opposite directions (as per right hand thumb rule). Hence, net magnetic field will be zero along the line AB.

23. In equilibrium state the plane of the loop is 90° to the direction of the field. In this position angle between magnetic field **B** and normal to the plane of coil is zero and hence torque $\tau = NAIB \sin 0° = 0$.

24. In this case, the charged particle performs uniform circular motion and magnetic force is providing the necessary centripetal force

i.e., $\frac{mv^2}{r} = qvB$

$\frac{mv^2}{r}$ is not the force acting on charged particle it is simply equal to net force acting on the particle.

25. Statement I is false, as **B** is perpendicular to **F**, so particle cannot follow magnetic field lines (tangent to which gives the direction of magnetic field).

26. Statement I is false and Statement II is true.

Magnetic force may have different value in different frames of reference, that's why it is not invariant in nature.

27. Cyclotron frequency is given by $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

It is obvious that cyclotron frequency does not depend upon velocity of charged particle.

28. The magnetic field at a point due to current flowing through an infinitely long conductor is given by $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$

where a is the distance of that point from conductor. Now according to right hand thumb rule it follows that magnetic field is in the form of concentric circles, whose centres lie on the straight conductor.

29. As, $T = \frac{2\pi m}{qB} \Rightarrow T \propto m$ and $m \propto 2\pi m_p$

30. The magnitude of the force experienced by a particle of charge q moving with a velocity v in a magnetic field B is given by

$F = qvB\sin\theta$

where θ is the angle between v and B.

Given $F = 3.2 \times 10^{-20}$ N, $v = 4 \times 10^5\ \text{ms}^{-1}$ and $\theta = 90°$.

For electron, $q = 1.6 \times 10^{-19}$ C.

Using these value, we get $B = 5 \times 10^{-7}$ T

31. The magnetic field at point R due to current I' in wire P is

$B_1 = \frac{\mu_0 I'}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 2} = 1 \times 10^{-7}$ T

32. The magnetic field at point R due to current I in wire Q is

$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times I}{2\pi \times 2} = I \times 10^{-7}$ T

Both fields B_1 and B_2 will be in the downward direction, parallel and colliner. Hence, the resultant magnetic field at point R is

$B = B_1 + B_2 = (1 + I) \times 10^{-7}$ T

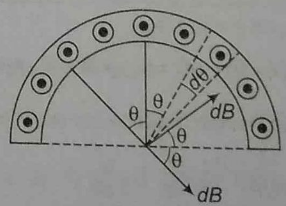
Now, $B = 5 \times 10^{-7}$ T. Therefore, $(1 + I) \times 10^{-7} = 5 \times 10^{-7}$

or $1 + I = 5$ or $I = 4$ A

33. Consider the wire to be made up of large number of thin wires of infinite length. Consider such wire of thickness dl subtending an angle $d\theta$ at centre.

Current through this wire, $dI = \frac{d\theta}{\pi} I$

$\therefore$ Magnetic field at centre due to this portion,



$dB = \frac{\mu_0}{4\pi} \cdot \frac{2dI}{R} = \frac{\mu_0 I}{2\pi^2 R} d\theta$

Net magnetic field at the centre

$B = \int_{-\pi/2}^{\pi/2} dB\cos\theta = \frac{\mu_0 I}{2\pi^2 R} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta = \frac{\mu_0 I}{\pi^2 R}$

34. The magnetic field in between because of each will be in opposite direction

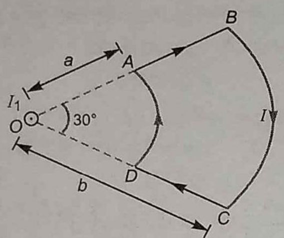
$$B_{\text{in between}} = \frac{\mu_0 i}{2\pi x}\hat{j} - \frac{\mu_0 i}{2\pi(2d-x)}(-\hat{j}) = \frac{\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{2d-x}\right](\hat{j})$$

at $x = d$, $B_{\text{in between}} = 0$

For $x < d$, $B_{\text{in between}} = (\hat{j})$ and For $x > d$, $B_{\text{in between}} = (-\hat{j})$

Towards x, net magnetic field will add up and direction will be $(-\hat{j})$ and Towards x', net magnetic field will add up and direction will be $(\hat{j})$.

35. Net magnetic field due to loop $ABCD$ at O is

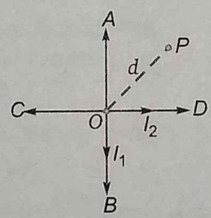


$$B = B_{AB} + B_{BC} + B_{CD} + B_{DA}$$

$$= 0 + \frac{\mu_0 I}{4\pi a} \times \frac{\pi}{6} + 0 - \frac{\mu_0 I}{\pi b} \times \frac{\pi}{6} = \frac{\mu_0 I}{24a} - \frac{\mu_0 I}{24b} = \frac{\mu_0 I}{24ab}(b-a)$$

36. The forces on AD and BC are zero because magnetic field due to a straight wire on AD and BC is parallel to elementary length of the loop.

37. The magnetic field inductions at a point P, at a distance d from O in a direction perpendicular to the plane of the wires due to currents through AOB and COD are perpendicular to each other, is



Hence, $B = \sqrt{B_1^2 + B_2^2}$

$$= \left[\left(\frac{\mu_0}{4\pi}\frac{2I_1}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}\frac{2I_2}{d}\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{\mu_0}{2\pi d}\sqrt{(I_1^2 + I_2^2)}$$

38. As $\mathbf{v}$ of charged particle is remaining constant, it means force acting on charged particle is zero.

So, $\quad q(\mathbf{v} \times \mathbf{B}) = q\mathbf{E} \Rightarrow \mathbf{v} \times \mathbf{B} = \mathbf{E} \Rightarrow \mathbf{v} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$

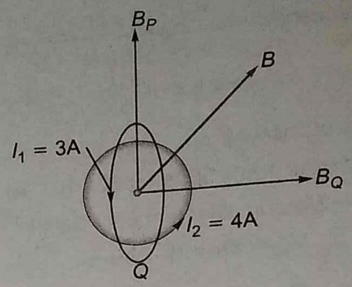
40. The force per unit length between the two wires is

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2I^2}{d} = \frac{\mu_0 I^2}{2\pi d}$$

The force will be attractive as current directions in both are same.

41.
$$B_p = \frac{\mu_0 I_2}{2R}$$

$$= \frac{4\pi \times 10^{-7} \times 4}{2 \times 0.02\pi} = 4 \times 10^{-5}\ \text{Wbm}^{-2}$$



$$B_Q = \frac{\mu_0 I_1}{2R} = \frac{4\pi \times 10^{-7} \times 3}{2 \times 0.02\pi} = 3 \times 10^{-5}\ \text{Wbm}^{-2}$$

$$\therefore \quad B = \sqrt{B_P^2 + B_Q^2} = \sqrt{(4 \times 10^{-5})^2 + (3 \times 10^{-5})^2}$$

$$= 5 \times 10^{-5}\ \text{Wbm}^{-2}$$

42. Force acting between two current carrying conductors

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l \qquad \text{...(i)}$$

where, d = distance between the conductors

l = length of each conductor.

Again $\quad F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{3d} \cdot l = -\frac{\mu_0}{2\pi}\frac{2I_1 I_2}{3d} \cdot l \qquad \text{...(ii)}$

Thus, from Eqs. (i) and (ii), we have

$$\frac{F'}{F} = -\frac{2}{3} \Rightarrow F' = -\frac{2}{3}F$$

43. As, force $(F) = qv \times B = qvB \sin 90° = qvB$

$\therefore$ Work done $= \mathbf{F} \cdot \mathbf{d} = F.d \cos 90° = 0$

44. If $(b-a) \geq r$

(r = radius of circular path of particle)

The particle cannot enter the region $x \geq b$.

So, to enter in the region $x \geq b$ or $r \geq (b-a)$

or $\quad \frac{mv}{Bq} \geq (b-a) \quad$ or $\quad v \geq \frac{q(b-a)B}{m}$

$$\therefore \quad v_{\min} = \frac{q(b-a)B}{m}$$

45. Magnetic field in circular coil A is $B_A = \frac{\mu_0 NI}{2R}$

where R is radius and I is current flowing in coil.

Similarly, $B_B = \frac{\mu_0 N(2I)}{2 \cdot (2R)} = \frac{\mu_0 NI}{2R}$

$$\therefore \quad \frac{B_A}{B_B} = 1$$

47. The component $dl \cos\theta$ of element dl is parallel to the length the wire 1. Hence, force on this elemental component

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}(dl \cos\theta) = \frac{\mu_0 I_1 I_2\, dl \cos\theta}{2\pi r}$$

Day 22

Magnetism

Current Loop as a Magnetic Dipole

A current loop is equivalent to a magnetic dipole. If $A(=\pi a^2)$ be the area of the loop, then the magnitude of its dipole moment is

$$p_m = iA = i\pi a^2$$

where a is radius of coil; i is current flowing through it

$$i = \frac{p_m}{\pi a^2} \quad \ldots\text{(i)}$$

Magnetic field at the centre of a circular current loop is given by

$$B = \frac{\mu_0 i}{2a} \quad \ldots\text{(ii)}$$

Putting the value of i from Eq. (i) in Eq. (ii), we get

$$B = \frac{\mu_0}{2\pi}\frac{p_m}{a^3}$$

This is the expression for the magnetic field at the centre of the current loop in terms of its dipole moment.

Instead of circular loop, if there is a circular coil having n turn, its dipole moment would be $P_m = niA = ni\pi a^2$

Bar Magnet

In magnetism, isolated magnetic poles, similar to isolated electric charges in electricity, have yet not been found to exist.

The simplest magnetic structure is the **magnetic dipole** characterised by a **magnetic dipole moment M**.

A bar magnet exhibits two important properties, namely

(i)the attractive property, and (ii) the directive property.

A bar magnet may be viewed as a combination of two magnetic poles, north pole and south pole, separated by some distance. The distance is known as the magnetic length of the given bar magnet. If m is the pole strength and $2\mathbf{l}$ the magnetic length of the bar magnet,then its magnetic moment is $\mathbf{M} = m(2\mathbf{l})$. Magnetic moment is a vector whose direction is from S pole towards N pole. SI unit of magnetic pole strength (m), is **ampere metre** (Am) and of magnetic dipole moment (**M**) is **ampere metre²** (Am^2).

If a bar magnet is broken, the fragments are independent magnetic dipoles and not isolated magnetic poles.

Bar Magnet as an Equivalent Solenoid

The resemblance of magnetic field lines for a bar magnet may be as a large number of circulating current in analogy with a solenoid.

The magnetic field (axial) at a point at a distance r is given by

$$\mathbf{B} = \frac{\mu_0 n I l a^2}{r^3}$$

and magnetic moment of solenoid is

$$\mathbf{M} = n(2l) I \pi a^2$$

- *Even a single electron moving in its orbit behaves as a magnetic dipole and has a definite magnetic moment.*
- *Bohr magnetron is the magnetic moment due to the orbital motion of an electron revolving in the inner most orbit ($n = 1$). Its value is*

$$m_B = \frac{eh}{4pm_e} = 9.27 \times 10^{-24}\ \mathrm{A\text{-}m^2}$$

Magnetic Field due to a Bar Magnet

A bar magnet has a magnetic field around it. Magnetic field is mathematically measured by a vector term **B**, *whose SI unit is 1 tesla (1 T).*

The magnetic field in free space, at a point at a distance r from the given bar magnet (or magnetic dipole)

(i) **Along its axial line**

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}r}{(r^2 - l^2)^2}$$

and the direction of **B** is the same as the direction of **M**. For a short dipole (or for a far away point on the axis) when $r >> l$, the above relation is simplified as (neglecting,)

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{r^3}$$

(ii) **Along the equatorial line** of a magnetic dipole,the magnetic field B in free space is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{(r^2 + l^2)^{3/2}}$$

If $r >> l$, the relation is modified as $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3}$

However, along the equatorial line, the direction of **B** is opposite to that of **M**.

In general, in a direction making an angle θ from with the magnetic axis, the magnetic field is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3} \sqrt{(3\cos^2\theta + 1)}$$

In these relations, μ_0 is a constant having a value of $4\pi \times 10^{-7}\ \mathrm{T\,mA^{-1}}$ and it is known as the **magnetic permeability of free space.**

For solenoid $\mathbf{B} = \mu_0 n i$, where n is number of turns of solenoid and i the current through it.

Magnetic Field Lines

A magnetic field line is a smooth curve in space, tangent on which, at any point, gives the direction of the magnetic field at that point.

Magnetic field lines starts from north pole of a magnet and end at the south pole. However, within the magnet, they move from the south pole to the north pole and form closed loops.

No two magnetic field lines can ever intersect each other.

Relative closeness of magnetic field lines at a given place gives the idea of magnetic field strength at that place. If field lines are close to one another, it shows a stronger field and vice-versa.

Magnetic Dipole in a Magnetic Field

A magnetic dipole when placed in an uniform magnetic field, does not experience any net force. However, it experiences a torque given by

$$\tau = \mathbf{M} \times \mathbf{B}$$

or $$\tau = MB \sin\theta$$

where θ is the angle from the magnetic field, along which the dipole has been placed.

Work done in rotating a magnetic dipole in an uniform magnetic field from an initial orientation θ_1 to the final orientation θ_2, is given by

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Potential energy of a magnetic dipole placed in a uniform magnetic field, is given by

$$U_B = -\mathbf{M} \cdot \mathbf{B} = -MB\cos\theta$$

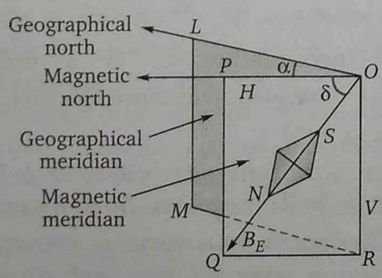
Thus, potential energy $U_B = -MB$ = minimum when dipole is parallel to B and $U_B = MB$ = maximum when dipole is anti-parallel to B.

The magnetic compass (needle) of magnetic moment M and moment of inertia I and allowing it to oscillate in the magnetic field. Then, its time-period is $T = 2\pi\sqrt{I/MB}$

- *Behaviour of a magnetic dipole in a magnetic field, is similar to the behaviour of an electric dipole in an electric field. However, the constant $\frac{1}{4\pi\varepsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$.*
- *If a magnetic dipole is in the form of a wire or a thin rod, when bent, its magnetic dipole moment **M** changes because the separation between its poles has changed.*

The Earth's Magnetism

Magnetic Elements of Earth



- **Angle of Declination (α)**

  At a given place, the acute angle between the magnetic meridian and the geographical meridian is called the angle of declination (or magnetic declination) α at that place.

- **Angle of Inclination or Dip (δ)**

  The angle of dip δ at a place is the angle which the direction of the earth's total magnetic field B_E subtends with the horizontal direction.

- **Horizontal Component of the Earth's Magnetic Field (B_H)**

  As earth's magnetic field, in general, is inclined at an angle δ with the horizontal direction, it may be resolved into horizontal component B_H and a vertical component B_V, where

  $$B_H = B_E \cos\delta$$

  and $$B_V = B_E \sin\delta$$

  ⇒ $$B_E = \sqrt{B_H^2 + B_V^2}$$

  and $$\tan\delta = \frac{B_V}{B_H}$$

Variation of Magnetic Elements of the Earth

At the magnetic equator, angle of dip is zero. Value of the angle of dip gradually increases, initially rapidly and then slowly, on going from equator to magnetic poles. At the magnetic poles, value of the dip angle is 90°. At the magnetic equator, $B_H = B_E \cos 0° = B_E$ and at poles, $B_H = B_E \cos 90° = 0$. Similarly, at the magnetic equator, $B_V = B_E \sin 0° = 0$ and at the poles, $B_V = B_E \sin 90° = B_E$. *Magnetic elements of the earth at a place change with time also.*

Magnetic Maps

Magnetic maps show the variation of magnetic elements from place to place. Some important lines drawn on magnetic maps are

1. Isoclinic Lines

These are the lines joining points of equal dip or inclination. The line joining places of zero dip, is called an **aclinic line** or the **magnetic equator**.

2. Isogonic Lines

These are the lines joining places of equal declination. The line joining places of zero declination, is called an **agonic line.**

3. Isodynamic Lines

These are the lines joining places having the same value of the horizontal component of the earth's magnetic field.

Neutral Points

A neutral point is a point at which the resultant magnetic field is zero. *Following two cases are of special interest.*

- When a bar magnet is placed along the magnetic meridian with its north pole pointing towards geographic north, two neutral points are obtained on either side of the magnet along its equatorial line. If r be the distance of the neutral point, then $\frac{\mu_0}{4\pi}\frac{M}{r^3} = B_H$
- When a bar magnet is placed along the magnetic meridian, with its north pole pointing towards the geographic south, two neutral points are obtained on either side of the magnet along its axial line. Hence, we have $\frac{\mu_0}{4\pi}\frac{2M}{r^3} = B_H$.

Tangent Galvanometer

It is an instrument to measure electric current. The essential parts are a vertical coil of conducting wire and a small compass needle pivoted at centre of coil. The deflection, θ of needle is given by,

$$\tan\theta = \frac{B}{B_H}$$

$$\Rightarrow \quad B_H \tan\theta = \frac{\mu_0 in}{2r} \text{ or, } i = \frac{2r B_N}{\mu_0 n}\tan\theta = K\tan\theta$$

Important Terms Used in Magnetism

1. Magnetic Induction or Magnetic Flux Density (B)

Whenever a piece of magnetic substance is placed in an external magnetising field, the substance becomes magnetised. If $\mathbf{B}_0$ is the magnetic field in free space, then on placing the given magnetic substance at that place, the magnetic field changes from $\mathbf{B}_0$ to $\mathbf{B}$, where $\mathbf{B} = \mu_r \mathbf{B}_0$

$\oint \mathbf{B}\cdot d\mathbf{s}$ is magnetic flux which is equal to $\mu_0 m_{\text{inside}}$, where m_{inside} is the net pole strength inside a close surface.

For a dipole, Gauss's law for magnetism is $\oint \mathbf{B}\cdot d\mathbf{s} = 0$ for a closed surface.

2. Magnetic Permeability (μ)

It is the degree or extent to which the magnetic lines of induction may pass through a given distance.

Magnetic permeability of free space μ_0 has a value of $4\pi \times 10^{-7}$ TmA^{-1}. However, for a material substance, **absolute permeability** (μ) has a value different then μ_0.

For any magnetic substance $\frac{\mu}{\mu_0} = \frac{\mathbf{B}}{B_0} = \mu_r$ = relative magnetic permeability of that substance.

Relative magnetic permeability μ_r is a unitless and dimensionless term. For magnetic substance like iron,

$$\mu_r > 1000$$

Relative permeability of a diamagnetic substance is less than 1 ($\mu_r < 1$), *but it can never be negative. Thus,* $0 \le \mu_r < 1$, *for a diamagnetic material.*

3. Intensity of Magnetisation (I)

Intensity of magnetisation of a substance is defined as the magnetic moment induced in the substance per unit volume, when placed in the magnetising field.

Thus, $$\mathbf{I} = \frac{\mathbf{M}}{V}$$

It is a vector and its SI unit is Am^{-1}.

4. **Intensity of Magnetising Field or Magnetic Intensity** (H)

It is a measure of the capability of external magnetising field to magnetise the given substance and is mathematically defined as

$$\mathbf{H} = \frac{\mathbf{B}_0}{\mu_0} \quad \text{or} \quad \mathbf{H} = \frac{\mathbf{B}}{\mu} \quad \text{or} \quad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}$$

Magnetic intensity H is a vector and its SI unit is Am^{-1}.

5. **Magnetic Susceptibility** (χ_m)

Magnetic susceptibility of a substance is the ratio of the intensity of magnetisation I induced in the substance to the magnetic intensity H. Thus, $\chi_m = \frac{I}{H}$. It is a scalar term and has no units or dimensions.

Relation (i) $\mathbf{IB} = \mu_0(\mathbf{H} + \mathbf{I})$ and (ii) $\mu_r = 1 + \chi_m$

Magnetic Behaviour of Materials

Diamagnetic Materials

These are materials which show a very small decrease in magnetic flux when placed in a strong magnetising field. Hydrogen, water, copper, zinc, antimony, bismuth, etc.,are examples of diamagnetic materials.

- In a diamagnetic material, the net magnetic moment (sum of that due to orbital motion and spin motion of electrons) of an atom is zero. The external magnetic field **B** distorts the electron orbit and thus, induces a small magnetic moment in the opposite direction.
- Diamagnetic materials are feebly repelled in an external magnetic field and thus, have a tendency to shift from the stronger to weaker regions of the magnetic field.
- The relative permeability of any diamagnetic substance is slightly less than 1 (*i.e.*, $\mu_r < 1$) and susceptibility has a small negative value.
- Diamagnetism is an intrinsic property and does not vary with magnetic field **B** or temperature.

Paramagnetic Materials

These are the materials which show a small increase in the magnetic flux when placed in a magnetising field.

Oxygen, air, platinum, aluminium, etc., are examples of paramagnetic materials.

- In a paramagnetic material, the net magnetic moment of every atom is non-zero.
- Paramagnetic materials are feebly attracted in an external magnetic field and thus have a tendency to shift from the weaker to the stronger regions of magnetic field.
- The relative permeability μ_r, of a paramagnetic material is slightly greater that one ($\mu_r > 1$). Magnetic susceptibility χ_m of paramagnetic materials is positive.
- Paramagnetism is temperature dependent. According to the **Curie's law**, the magnetic susceptibility of a paramagnetic substance is inversely proportional to its temperature T.

Mathematically,

$$\chi_m = \frac{CB}{T},$$

where B is the external magnetic field and C is a constant, known as the **Curie constant**.

Ferromagnetic Materials

These are the materials which are strongly attracted by a magnetic field and can themselves be magnetised even in a weak magnetising field. Iron, steel,nickel and cobalt are ferromagnetic.

- These materials show a large increase in the magnetic flux, when placed in a magnetic field. Thus, for them $\mu_r >> 1$. Accordingly χ_m is positive and large.
- Ferromagnetic materials exhibit all properties exhibited by paramagnetic substances and by a much larger measure.
- Magnetic susceptibility of ferromagnetic materials decreases steadily with a rise in temperature. Above a certain temperature T_c (known as **Curie temperature**), the substance loses its ferromagnetic character and begins to behave as a paramagnetic substance.
- Above the Curie temperature T_c, the magnetic susceptibility of a ferromagnetic material varies as

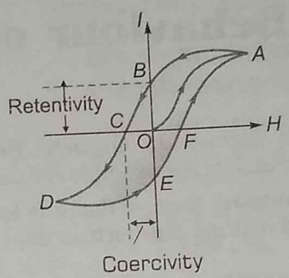
$$\chi_m \propto \frac{1}{(T - T_c)} \quad \text{or} \quad \chi_m = \frac{C}{(T - T_c)}$$

where C is a constant. It is known as the **Curie-Weiss** law.

Hysteresis Curve

A great deal of information can be learned about the magnetic properties of a material by studying its hysteresis loop. A hysteresis loop shows the relationship between the induced magnetic flux density (**B**) *and the magnetizing force* (**H**). It is often referred as **B–H** *loop.*

A ferromagnetic material can be easily magnetised by placing it in an external magnetising field H.



Initially as **H** is increased, the intensity of magnetisation **I** developed in the material increases non-linearly along the curve *OA* and reaches a maximum, known as saturated magnetism. Now on reducing **H**, **I** follows the path *AB*. Point *B* corresponds to **H** = 0 but **I** has a finite positive value. This value of **I** is called **remanence** or **retentivity** or **residual magnetisation.**

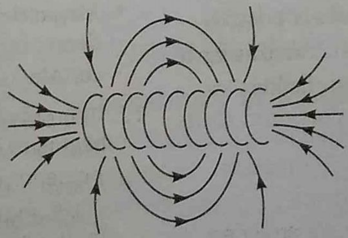
If direction of **H** is reversed and is gradually increased, *H* curve is along *BC*. Point *C* represents the situation when **I** is finally reduced to zero. The reversed value of **I**, represented by *OC*, is known as the **coercivity** of the material.

On increasing **H** in the reverse direction further, graph *CD* is obtained. Now, if **H** is taken back from its negative saturation value to its original positive saturation value a similar **I** - **H** curve represented by *DEFA* is traced.

The whole graph *ABCDEFA* is a closed loop and known as **hysteresis loop.**

Electromagnet

When a soft-iron road is placed in the solenoid, the magnetism of the solenoid increases hundreds of times. Then, the solenoid is called an electromagnet. It is a temporary magnet



Practice Zone DAY 22

1. Consider the plane S formed by the dipole axis and the axis of the earth. Let P be point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. The declination and dip angles at P and Q are
(a) $0°$ and $11.3°$ (b) $0°$ and $0°$
(c) $11\text{-}3°$ and $6.5°$ (d) $11.3°$ and $11.3°$

2. The current on the winding of a toroid is 2A. It has 400 turns and mean circumferential length is 40 cm. With the help of search coil and charge measuring instrument the magnetic field is found to be 1T. The susceptibility is
(a) 100 (b) 290 (c) 398 (d) 397

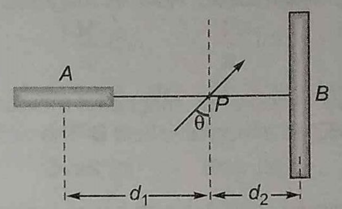
3. The magnetic needle of a tangent galvanometer is deflected at an angle 30° with respect to the magnet. The horizontal component of the earth's magnetic field is 0.34×10^{-4} T along the plane of the coil. The magnetic intensity is
(a) 1.96×10^{-4} T (b) 1.96×10^{4} T
(c) 1.96×10^{-5} T (d) 1.96×10^{5} T

4. A magnet of length 14 cm and magnetic moment M, is broken into two parts of length 6 cm and 8 cm respectively. They are put at right angles to each other, with opposite poles together. The magnetic moment of the combination is
(a) $M/10$ (b) M (c) $M/1.4$ (d) $2.8\,M$

5. If the areas under the I-H hysteresis loop and B-H hysteresis loop are denoted by A_1 and A_2, then
(a) $A_2 = \mu_0 A_1$ (b) $A_2 = A_1$ (c) $A_2 = \frac{A_1}{\mu_0}$ (d) $A_2 = \mu_0^2 A_1$

6. A bar magnet of length 10 cm and having pole strength equal to 10^{-3} Wb, is kept in a magnetic field having magnetic induction B equal to $4\pi \times 10^{-3}$ T. It makes an angle of 30° with the direction of magnetic induction. The value of the torque acting on the magnet is
(a) 0.5 N-m (b) $2\pi \times 10^{-5}$ N-m
(c) $\pi \times 10^{-5}$ N-m (d) 0.5×10^{2} N-m

7. Two magnets A and B are identical and these are arranged as shown in the figure. Their lengths are negligible in comparision to the separation between them. A magnetic needle is placed between the magnets at point P and it gets deflected by an angle θ. The ratio of distances d_1 and d_2, will be



(a) $(2\cot\theta)^{1/3}$ (b) $(2\tan\theta)^{1/3}$ (c) $(2\cot\theta)^{-1/3}$ (d) $(2\tan\theta)^{-1/3}$

8. A bar magnet has pole strength 3.6 A-m and length 12 cm. Its area of cross-section is $0.9\ \text{cm}^2$. The magnetic field B at the centre of the bar magnet is
(a) 6×10^{-3} T (b) 5×10^{-2} T
(c) 2.5×10^{-2} T (d) 2.5×10^{-8} T

9. The magnetic susceptibility of a paramagnetic substance at –73°C is 0.0060, then its value at –173°C will be
(a) 0.0030 (b) 0.0120 (c) 0.0180 (d) 0.0045

10. A deflection magnetometer is adjusted in the usual way. When a magnet is introduced, the deflection observed is θ, and the period of oscillation of the needle in the magnetometer is T. When the magnet is removed, the period of oscillation is T_0. The relation between T and T_0 is
(a) $T^2 = T_0^2 \cos\theta$ (b) $T = T_0 \cos\theta$
(c) $T = \frac{T_0}{\cos\theta}$ (d) $T^2 = \frac{T_0^2}{\cos\theta}$

11. Two magnets are suspended by a given wire, one by one. In order to deflect the first magnet through 45°, the wire has to be twisted through 540° whereas for the second magnet, the wire requires a twist of 360° for producing the same deflection. Then, the ratio of magnetic moments of the two is
(a) 11/7 (b) 3/2 (c) 4/3 (d) 7/6

12. The plane of a dip circle is set in the geographic meridian and the apparent dip is δ_1. It is then set in a vertical plane perpendicular to the geographic meridian. The apparent dip angle is δ_2. The declination θ at the place is
(a) $\theta = \tan^{-1}(\tan\delta_1 \tan\delta_2)$
(b) $\theta = \tan^{-1}(\tan\delta_1 + \tan\delta_2)$
(c) $\theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$
(d) $\theta = \tan^{-1}(\tan\delta_1 - \tan\delta_2)$

13. A bar magnet suspended by a suspension fibre, is placed in the magnetic meridian with no twist in the suspension fibre. On turning the upper end of the suspension fibre by an angle of 120° from the meridian, the magnet is deflected by an angle of 30° from the meridian. Then, the angle by which the upper end of the suspension fibre has to be twisted, so as to deflect the magnet through 90° from the meridian is

(a) 270° (b) 240° (c) 330° (d) 180°

14. An iron rod of length L and magnetic moment M is bent in the form of a semicircle. Now, its magnetic moment will be

(a) M (b) $\frac{2M}{\pi}$ (c) $\frac{M}{\pi}$ (d) $M\pi$

15. A magnetic dipole of magnetic moment M is rotated through 360° in a magnetic field **B**. The work done will be

(a) MB (b) $2MB$ (c) $2\pi MB$ (d) zero

16. A short magnet oscillates with a time period 0.1 s at a place where horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire 20 cm east of the magnet. The new time period of oscillator

(a) 0.1 s (b) 0.089 s
(c) 0.076 s (d) 0.057 s

17. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 T, experiences a torque of magnitude 0.032 J. The magnetic moment of the bar magnet will be

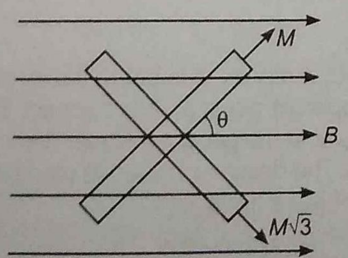
(a) $0.23\ JT^{-1}$ (b) $0.40\ JT^{-1}$
(c) $0.80\ JT^{-1}$ (d) zero

18. There are two current carrying planer coils made each from wire of length LG is circular coil (radius R) and C_2 is square (side a). These are so constructed that they have same frequency of oscillation when they are placed in the same uniform magnetic field **B** and carry the same current. **[NCERT Exemplar]**

The value of a in terms of R is

(a) $3R$ (b) $\sqrt{3}R$ (c) $\sqrt{2}R$ (d) $2R$

19. M and $M\sqrt{3}$ are the magnetic dipole moments of the two magnets, which are joined to form a cross figure. The inclination of the system with the field, if their combination is suspended freely in a uniform external magnetic field B is



(a) $\theta = 30°$ (b) $\theta = 45°$
(c) $\theta = 60°$ (d) $\theta = 15°$

20. Two bar magnets of the same mass, same length and breadth but having magnetic moments M and $2M$, respectively are joined together pole to pole and suspended by a string. The time period of the assembly in a magnetic field of strength H is 3 seconds. If now the polarity of one of the magnets is reversed and the combination is again made to oscillate in the same field, the time of oscillation is

(a) 3 s (b) $3\sqrt{3}$ s (c) $3/\sqrt{3}$ s (d) 6 s

21. A bar magnet 8 cm long, is placed in the magnetic meridian with the N pole, pointing towards the geographical north. Two neutral points, separated by a distance of 6 cm are obtained on the equatorial axis of the magnet. If $B_H = 3.2 \times 10^{-5}$ T, then the pole strength of the magnet is

(a) 5 A-cm^2 (b) 10 A-cm^2
(c) 2.5 A-cm^2 (d) 20 A-cm^2

22. A coil of 50 turns and area 1.25×10^{-3} m^2 is pivoted about a vertical diameter in an uniform horizontal magnetic field and carries a current of 2 A. When the coil is held with its plane in the N-S direction, it experiences a couple of 0.04 Nm, and when its plane is along the east-west direction, it experiences a couple of 0.03 Nm. The magnetic induction is

(a) 0.2 T (b) 0.3 T (c) 0.5 T (d) 0.4 T

Directions *(Q. Nos. 23 to 25) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

23. **Statement I** A current carrying loop is free to rotate. It is placed in a uniform magnetic field. It attains equilibrium when its plane is perpendicular to the magnetic field.

Statement II The torque on the coil is zero when its plane is perpendicular to the magnetic field.

24. **Statement I** A proton and an alpha particle having the same kinetic energy are moving in circular paths, in an uniform magnetic field. The radii of their circular paths will be equal.

Statement II Any two charged particles having equal kinetic energies and entering a region of uniform magnetic field **B**, in a direction perpendicular to **B**, will describe circular trajectories of equal radii.

25. **Statement I** Cyclotron is a device which is used to accelerate the positive ion.

Statement II Cyclotron frequency depends upon the velocity.

Directions *(Q. Nos. 26 to 28) The force experienced by a particle of charge q moving with a velocity* **v**, *in a uniform magnetic field B, is given by,* $\mathbf{F} = \mathbf{v} \times \mathbf{B}$, *which is perpendicular to both* **v** *and* **B**. *Since,* **F** *is perpendicular to* **v**, *no work is done on the charged particle moving in an uniform magnetic field. If* **v** *is perpendicular to* **B** *the force cannot change the kinetic energy of the particle. Hence, the magnitude of* **v** *cannot change, only the direction of motion changes continuously. These are exactly the conditions for uniform circular motion. Thus, if* **v** *is perpendicular to* **B**, *the charged particle follows a circular path whose radius is given by*

$$r = \frac{mv}{qB}$$

The frequency of revolution of the particle along the circular path is given by

$$\nu = \frac{qB}{2\pi m}$$

26. A proton and an alpha particle are projected perpendicular to an uniform magnetic field with equal velocities. The mass of an alpha particle is 4 times that of a proton and its charge is twice that of a proton. If r_p and r_α are the radii of their circular paths, then the ratio r_p/r_α is

(a) $1/\sqrt{2}$ (b) $\frac{1}{4}$ (c) 1 (d) $\sqrt{2}$

27. In Q. 26, what is the ratio r_p/r_α, if the two particles have equal kinetic energies before entering the region of the magnetic field?

(a) $\frac{1}{2}$ (b) 1 (c) $\sqrt{2}$ (d) 4

28. In Q. 26, what is the ratio r_p/r_α, if the two particles have equal linear momenta before entering the region of the magnetic field?

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

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29. The earth's magnetic field lines resemble that of a dipole at the centre of the earth. If the magnetic moment of this dipole is close to 8×10^{22} Am^2, the value of earth's magnetic field near the equator is close to

(radius of the earth $= 6.4 \times 10^6$ m) **[JEE Main Online 2013]**

(a) 0.6 gauss (b) 1.2 gauss
(c) 1.8 gauss (d) 0.32 gauss

30. A particle of charge 16×10^{-16} C moving with velocity 10 ms^{-1} along *x*-axis enters a region where magnetic field of induction **B** is along the *y*-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative *z*-axis. If the charged particle continues moving along *x*-axis, the magnitude of **B** is **[JEE Main Online 2013]**

(a) 16×10^3 Wb m^{-2} (b) 2×10^3 Wb m^{-2}
(c) 1×10^3 Wb m^{-2} (d) 4×10^3 Wb m^{-2}

31. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their *N* poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point *O* of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is 3.6×10^{-5} Wb/m^2) **[JEE Main 2013]**

(a) 3.6×10^{-5} Wb/m^2
(b) 2.56×10^{-4} Wb/m^2
(c) 3.50×10^{-4} Wb/m^2
(d) 5.80×10^{-4} Wb/m^2

32. Relative permittivity of a material are ε_r and μ_r respectively. Which of the following value of these quantities are allowed for a diamagnetic material? **[AIEEE 2008]**

(a) $\varepsilon_r = 0.5, \mu_r = 1.5$ (b) $\varepsilon_r = 1.5, \mu_r = 0.5$
(c) $\varepsilon_r = 0.5, \mu_r = 0.5$ (d) $\varepsilon_r = 1.5, \mu_r = 1.5$

33. Needles N_1, N_2 and N_3 are made up of ferromagnetic, paramagnetic and diamagnetic substances respectively. A magnet when brought close to them will **[AIEEE 2006]**

(a) attract N_1 and N_2 strongly but repel N_3
(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(c) attract N_1 strongly, but repel N_2 and N_3 weakly
(d) attract all three of them

34. A magnetic needle is kept in a non-uniform magnetic field. It experiences **[AIEEE 2005]**

(a) a torque but not a force
(b) neither a force nor a torque
(c) a force and a torque
(d) a force but not a torque

35. The materials suitable for making electromagnets should have **[AIEEE 2004]**

(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity

36. A magnetic needle lying parallel to a magnetic field requires *W* units of work to turn it through 60°. The torque needed to maintain the needle in this position will be **[AIEEE 2003]**

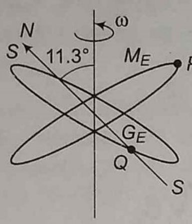
(a) $\sqrt{3}W$ (b) W (c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$

Answers

1. (a)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)
11. (a)	**12.** (c)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (b)
21. (a)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (c)
31. (b)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (c)	**36.** (a)				

Hints & Solutions

1. P is in the plane S, needle is in north, so the declination is zero.



P is also on the magnetic equator so the angle of dip = 0, because the value of angle of dip at equator is zero Q is also on the magnetic equator, thus the angle of dip is zero. As earth tilted on its axis by 11.3°, thus the declination at Q is 11.3°.

2. As, $n = \frac{400}{2\pi R} = \frac{400}{40 \times 10^{-2}} = 1000$

$$\mu = ni = 1000 \times 2 = 2000$$
$$B = \mu_0 \mu_r \mu$$
$$\Rightarrow \mu_0 \mu_r = \frac{1}{2000} = 5 \times 10^{-4}$$
$$\Rightarrow \mu_r = \frac{5 \times 10^{-4}}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = 398$$
$$\Rightarrow \chi = \mu_r - 1 = 397$$

3. As,
$$B = B_H \tan\theta$$
$$B = 0.34 \times 10^{-4} \tan 30°$$
$$= 0.34 \times 10^{-4} \times \frac{1}{\sqrt{3}}$$
$$B = 1.96 \times 10^{-5} \text{ T}$$

4. Pole strength of the original magnet, $m = \frac{M}{14}$

Effective distance between the poles = AB
$$= \sqrt{6^2 + 8^2} = 10 \text{ cm}$$

∴ Magnetic moment of the combination
$$M' = m.2l = \frac{M}{14} \times 10 = \frac{M}{1.4}$$

5. As, $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{I})$

$$\Rightarrow \quad dB = \mu_0\, dH + \mu_0\, dI$$
or
$$\oint H dB = \mu_0 \oint H dH + \mu_0 \oint H\, dI$$
$$\oint H dH = 0$$
$$\oint H dB = \mu_0 \oint H \cdot dI$$

Area of the B-H loop $= \mu_0 \times$ area of I-H loop

i.e., $A_2 = \mu_0 A_1$

6. As, $\mu_0 m = 10^{-3}$ Wb
$$m = \frac{10^{-3}}{\mu_0}$$

Magnetic moment of the magnet,
$$M = m \times 2l$$
$$= \frac{10^{-3}}{\mu_0}(0.1)$$
$$= \frac{10^{-4}}{\mu_0}$$

Now,
$$\tau = MB \sin\theta$$
$$= \left(\frac{10^{-4}}{\mu_0}\right) \times 4\pi \times 10^{-3} \sin 30°$$
$$= 0.5 \text{ N-m}$$

7. $B = B_H \tan\theta$
$$\frac{\mu_0}{4\pi}\left(\frac{2M}{d_1^3}\right) = \left(\frac{\mu_0}{4\pi}\frac{M}{d_2^3}\right)\tan\theta$$
$$\frac{2}{d_1^3} = \frac{\tan\theta}{d_2^3}$$
$$\left(\frac{d_1}{d_2}\right)^3 = \frac{2}{\tan\theta} = 2\cot\theta$$
$$\frac{d_1}{d_2} = (2\cot\theta)^{1/3}$$

8. As, $I = \frac{m}{A} = \frac{3.6}{0.9 \times 10^{-4}} = 4 \times 10^4\ \text{Am}^{-1}$

$$H_N = \frac{m}{4\pi d^2} = \frac{3.6}{4\pi \times (6 \times 10^{-2})^2} = 79.6\ \text{Am}^{-1}$$

$$H = H_N + H_S$$

$\Rightarrow$ $H = H_N + H_S = 1592$ A/m, towards S pole

$$B = \mu_0 (H + I)$$
$$= 4\pi \times 10^{-7} (4 \times 10^4 - 1592)$$
$= 5 \times 10^{-2}$ T, towards N pole

9. As, $\chi_m \propto \frac{1}{T}$

$\Rightarrow$ $\frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$

or $\frac{\chi_2}{0.0060} = \frac{273 - 73}{273 - 173}$

$$= \frac{200}{100} = 2$$

or $\chi_2 = 2 \times 0.0060 = 0.0120$

10. Since, $F^2 = H^2$

and $T = 2\pi \sqrt{\frac{I}{M\sqrt{F^2 + H^2}}}$ ...(i)

When magnet is removed

$T_0 = 2\pi \sqrt{\frac{T}{MH}}$ ...(ii)

Also, $\frac{F}{H} = \tan\theta$

From Eqs. (i) and (ii), we have

$$\frac{T}{T_0} = \sqrt{\frac{H}{\sqrt{F^2 + H^2}}}$$

$$\frac{T^2}{T_0^2} = \cos\theta$$

$$T^2 = T_0^2 \cos\theta$$

11. As, $\tau = C\phi = MH \sin\theta$

Ist case, $\phi_1 = 540° - 45° = 495°$

IInd case, $\phi_2 = 360° - 45° = 315°$

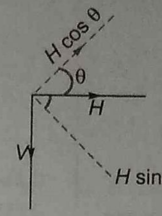
$C(495°) = M_1 H \sin 45°$ ...(i)

$C(315°) = M_2 H \sin 45°$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{M_1}{M_2} = \frac{495}{315} = \frac{11}{7}$$

12. $\tan\delta_1 = \frac{V}{H\cos\theta}$



$$\tan\delta_2 = \frac{V}{H\cos(90° - \theta)} = \frac{V}{H\sin\theta}$$

$$\frac{\tan\delta_1}{\tan\delta_2} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

or $\theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$

13. As, $\tau = C\phi = MH \sin\theta$

Ist case $\theta = 30°$, $\phi° = 120° - 30° = 90°$

IInd case $C(\phi - 90°) = MH \sin 90°$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\phi - 90°}{90°} = \frac{MH \sin 90°}{MH \sin 30°} = \frac{1}{1/2} = 2$$

or $\phi - 90° = 180°$

or $\phi = 180° + 90° = 270°$

14. When the bar magnet is straight, its pole strength is $m = \frac{M}{L}$

When bent in the form of a semicircle $NS = L \cdot \frac{2}{\pi}$, hence new magnetic moment of the magnet

$$M' = m(NS) = \frac{M}{L} \cdot L \cdot \frac{2}{\pi} = \frac{2M}{\pi}$$

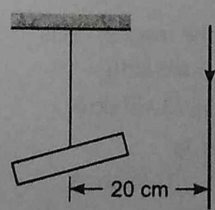
15. Work done $W = MB(\cos\theta_1 - \cos\theta_2)$

$$= MB[\cos\theta_1 - \cos(360° + \theta_1)]$$
$$= MB[\cos\theta_1 - \cos\theta_1] = 0$$

16. Given, $B_H = 24\mu\text{T}$

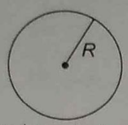
$$B_{\text{current}} = \frac{\mu_0 I}{2\pi d} = \frac{2 \times 10^{-7} \times 18}{0.2} = 18\mu\text{T}$$

Now, $T' = T\sqrt{\frac{24}{42}} = \frac{0.1 \times 2}{\sqrt{7}} = 0.076$ s



17. As, $M = \frac{\tau}{B \sin \theta} = \frac{0.032}{0.16 \times \sin 30^\circ} = 0.40\ \text{JT}^{-1}$

18.



C_1 = circular coil of radius R, length L, number of turns per unit length $n_1 = \frac{L}{2\pi R}$

Magnetic moment of

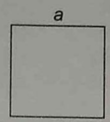
$$C_1 \Rightarrow m_1 = n_1 I A_1$$

$$m_1 = \frac{L \cdot I \cdot \pi R^2}{2\pi R},$$

$$m_1 = \frac{LIR}{2} \quad \ldots(i)$$

Moment of inertia of

$$C_1 \Rightarrow I_1 = \frac{MR^2}{2} \quad \ldots(iii)$$



C_2 = square of side a and perimeter L, number of turns per unit length $n_2 = \frac{L}{4a}$

Magnetic moment of

$$C_2 \Rightarrow m_2 = n_2 I A_2$$

$$m_2 = \frac{L}{4a} \cdot I \cdot a^2$$

$$m_2 = \frac{LIa}{4} \quad \ldots(ii)$$

Moment of inertia of

$$C_2 \Rightarrow I_2 = \frac{Ma^2}{12} \quad \ldots(iv)$$

Frequency of $C_1 \Rightarrow f_1 = 2\pi\sqrt{\frac{I_1}{m_1 B}}$

Frequency of $C_2 \Rightarrow f_2 = 2\pi\sqrt{\frac{I_2}{m_2 B}}$

According to question, $f_1 = f_2$

$$2\pi\sqrt{\frac{I_1}{m_1 B}} = 2\pi\sqrt{\frac{I_2}{m_2 B}}$$

$$\frac{I_1}{m_1} = \frac{I_2}{m_2}$$

or
$$\frac{m_2}{m_1} = \frac{I_2}{I_1}$$

Plugging the values by Eqs. (i), (ii), (iii) and (iv)

$$\frac{LIa \cdot 2}{4 \times LIR} = \frac{Ma^2 \cdot 2}{12 \cdot MR^2}$$

$$\frac{a}{2R} = \frac{a^2}{6R^2}$$

$$3R = a$$

Thus, the value of a is $3R$.

19. Torque (τ) acting on the magnet (1) is

$$\tau_1 = MB\sin\theta$$

$$\tau_2 = \sqrt{3}\, MB\sin\theta$$

For equilibrium, $\tau_1 = \tau_2$

$\therefore$ $$MB\sin\theta = \sqrt{3}\, MB\cos\theta$$

$\Rightarrow$ $$\tan\theta = \sqrt{3} = \tan 60^\circ$$

$\Rightarrow$ $$\theta = 60^\circ$$

20. At the pole, for the combination

$$M_1 = 2M + M = 3M,\ T_1 = 3\ \text{s}$$

When the polority of one is reversed, then

$$M_2 = 2M - M = M$$

Thus we have, from $\frac{T_2^2}{T_1^2} = \frac{M_1}{M_2}$ $\quad \left(\because T = 2\pi\sqrt{\frac{I}{MB}}\right)$

$\Rightarrow$ $$\frac{T_2^2}{T_1^2} = \frac{3M}{M} = 3$$

$\therefore$ $$T_2^2 = 3T_1^2 = 3 \times 9 = 27$$

$\therefore$ $$T_2 = \sqrt{27} = 3\sqrt{3}\ \text{s}$$

21. At the neutral point, $B = B_H$

$\Rightarrow$ $$\frac{\mu_0}{4\pi} \times \frac{M}{(r^2 + l^2)^{3/2}} = B_H$$

In CGS system, $\frac{M}{(r^2 + l^2)^{3/2}} = B_H$

$\Rightarrow$ $$\frac{m \times 2l}{(r^2 + l^2)^{3/2}} = B_H$$

$\Rightarrow$ $$\frac{m \times 2 \times 4}{(3^2 + 4^2)^{3/2}} = 0.32$$

$\Rightarrow$ $$\frac{8m}{(25)^{3/2}} = 0.32$$

$\Rightarrow$ $$m = \frac{125 \times 0.32}{8} = 5\ \text{A-cm}^2$$

22. Here, $N = 50$, $A = 1.25 \times 10^{-3}\ \text{m}^2$, $I = 2\text{A}$

$$M = NIA$$
$$= 50 \times 2 \times 1.25 \times 10^{-3}$$
$$= 0.125\ \text{A-m}^2$$

If the normal to the face of the coil makes an angle θ with the magnetic induction B, the torque

$$\tau = MB\cos\theta = 0.04 \quad \ldots(i)$$

Now, when the plane of the coil is turned through 90°, the torque becomes,

$$\tau = MB\sin\theta = 0.03 \quad \ldots(ii)$$

Squaring and adding Eqs. (i) and (ii), we get

$$\tau = 0.05 \Rightarrow MB = 0.05$$

$\Rightarrow$ $$B = \frac{0.05}{M} = \frac{0.05}{0.125} = 0.40\ \text{T}$$

23. The loop will rotate and come to rest when the torque acting on it becomes zero. The magnitude of the torque acting on a loop of area A and carrying a current I in a magnetic field B, is given by $\tau = BIA\sin\theta$, where θ is the angle between the direction of the magnetic field and the normal to the plane of the coil. It is clear that $\tau = 0$ when $\theta = 0$, *i.e.*, when the plane of the coil is perpendicular to the magnetic field.

24. The radius of the circular path is given by

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where, $K = \frac{1}{2}mv^2$

Since, K and B are the same for the two particles, $r \propto \frac{\sqrt{m}}{q}$.

Now, the charge of an alpha particle is twice that of a proton and its mass is four times the mass of a proton, $\sqrt{m}/q$ will be the same for both the particles. Hence, r will be the same for both the particles.

25. Statement I is true but Statement II is false.

26. As, $r_p = \frac{m_p V}{q_p B}$

and $r_\alpha = \frac{m_\alpha V}{q_\alpha B}$

$\therefore \quad r_p / r_\alpha = \frac{1}{4} \left(\because \frac{q_p}{q_\alpha} = \frac{1}{2} \text{ and } \frac{m_p}{m_\alpha} = \frac{1}{4} \right)$

27. Kinetic energy, $K = \frac{1}{2}mv^2$, which gives $v = \sqrt{\frac{2K}{m}}$.

Hence, $r = \frac{mv}{qB} = \frac{1}{qB}\sqrt{2mK}$

$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{2}$

28. Linear momentum $p = mv$. Hence,

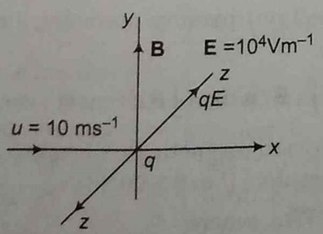
$$r = \frac{mv}{qB} = \frac{p}{qB} \Rightarrow \frac{r_p}{r_\alpha} = 2$$

Using this relation we find that the correct choice is (c).

29. As we know that $\frac{M}{B_H} = \frac{4\pi}{\mu_0}(d^2 + l^2)^{3/2}\tan\theta$

which gives B_H at equator $= 0.32$ G

30. As, $F = qvB$



For the particles, velocity to remain $qvB = qE$

$$B = \frac{E}{v} = \frac{10^4}{10} \times 1 \times 10^3 = 1 \times 10^3 \text{ Wb m}^{-2}$$

31. $B_{net} = B_1 + B_2 + B_H$

$$B_{net} = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H$$

$$= \frac{10^{-7}(1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4} \text{ Wb/m}^2$$

32. For diamagnetic material, $0 < \mu_r < 1$ and for any material $\varepsilon_r > 1$.

33. Ferromagnetic substances have strong tendency to get magnetised (induced magnetic moment) in the same direction as that of applied magnetic field, so magnet attracts N_1 strongly. Paramagnetic substances get weakly magnetised (magnetic moment induced is small) in the same direction as that of applied magnetic field, so magnet attracts N_2 weakly. Diamagnetic substances also get weakly magnetised when placed in an external magnetic field but in opposite direction and hence, N_3 is weakly repelled by magnet.

34. Magnetic needle is placed in non-uniform magnetic field. It experiences force and torque both due to unequal forces acting on poles.

35. Electromagnets are made of soft iron. The soft iron has high retentivity and low coercivity.

36. $W = MB(1 - \cos\theta)$

$\Rightarrow \quad W = MB(1 - \cos 60°) \quad (\because \theta = 60°)$

or $\quad W = \frac{MB}{2}$ or $MB = 2W$

Torque, $\tau = MB \sin 60°$

$$= \frac{MB\sqrt{3}}{2} = \frac{2W\sqrt{3}}{2} = W\sqrt{3}$$

Day 23

Electromagnetic Induction

Day 23

Outlines ...

- Concept of Electromagnetic Induction
- Magnetic Flux (ϕ_B)
- Motional Emf
- Rotational Emf
- Self Induction
- Mutual Induction
- Eddy Currents

Concept of Electromagnetic Induction

Electromagnetic induction is the phenomenon due to which an induced emf is set up in a conductor or in an electric circuit, on changing the magnetic flux linked with it.

Magnetic Flux (ϕ_B)

The magnetic flux linked with a given surface area is defined as the total number of magnetic field lines (lines of induction) passing normally through the given area.

Mathematically, $\phi_B = \int \mathbf{B} \cdot \mathbf{ds} = \int \mathbf{B} \cdot \hat{\mathbf{n}}\, dx = \int B\, ds \cos\theta$

Magnetic flux is a scalar quantity. Outward magnetic flux is taken as positive (*i.e.*, $\theta < 90°$) and inward flux is taken as negative (*i.e.*, $\theta > 90°$).

SI unit of magnetic flux is 1 weber (1 Wb), where

$$1\ \text{Wb} = 1\ \text{T} \times 1\ \text{m}^2 = 1\ \text{Tm}^2$$

Dimensional formula of magnetic flux is $[ML^2T^{-2}A^{-1}]$

Faraday's Law of Electromagnetic Induction

According to the Faraday's law, an emf is induced in a circuit whenever magnetic flux linked with that circuit changes (increases or decreases). The induced emf lasts so long as the change in magnetic flux continues.

The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the circuit, *i.e.*,

$$\text{Induced emf}\,|e| = \frac{d\phi_B}{dt}$$

For N turns,

$$|e| = N\frac{d\phi_B}{dt}$$

However, if we consider the direction of induced emf too, then

$$e = -N\frac{d\phi_B}{dt}$$

If the given electric circuit is a closed circuit having a total resistance R, then the induced current

$$I = \frac{e}{R} = -\frac{N}{R}\frac{d\phi_B}{dt}$$

and induced charge

$$dq = Idt = -\frac{N}{R}d\phi_B$$

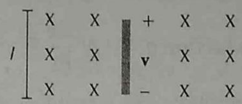
Lenz's Law

This law states that the direction of the induced current in a circuit is so as to oppose the cause of change in the magnetic flux. Lenz's law is strictly in accordance with the law of conservation of energy.

Motional Emf

Let a conducting rod of length l be moving with a uniform velocity $\mathbf{v}$ perpendicular to a uniform magnetic field $\mathbf{B}$, an induced emf is set up. The magnitude of the induced emf will be

$$|e| = B\,l\,v$$



If the rod is moving such that it makes an angle θ with the direction of the magnetic field, then $|e| = B\,l\,v \sin\theta$. Hence, for the motion parallel to $\mathbf{B}$, the induced emf is zero.

- *When a conducting rod moves horizontally, then an induced emf is set up between its ends due to the vertical component of the earth's magnetic field. However, at the magnetic equator, induced emf will be zero because $B_V = 0$.*
- *If during landing or taking off, the wings of an aeroplane are along the east-west direction, an induced emf is set up across the wings (due to the effect of B_H).*

Motional Emf in a Loop

If a conducting rod moves on two parallel conducting rails then an emf is induced whose magnitude is $|e| = B\,l\,v$ and the direction is given by the Fleming's right hand rule.

$$\text{Induced current}\,|I| = \frac{|e|}{R} = \frac{B\,l\,v}{R}$$

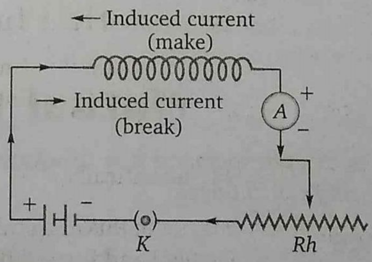
Rotational Emf

Let a conducting rod of length l rotate about an axis passing through one of its ends (that end may be fixed), with an angular velocity ω in a plane perpendicular to the magnetic field B, then an induced emf is set up between the ends of the rod, whose magnitude is given by

$$|e| = \frac{1}{2}Bl^2\omega$$

Self Induction

Self induction is the phenomenon due to which an induced emf is set up in a coil or a circuit whenever the current passing through it changes. The induced emf opposes the change that causes it and is thus known as back emf.



- *Inductance is the inherent property of electrical circuits and is known as the electrical inertia.*
- *An inductor is said to be an ideal inductor if its resistance is zero. A capacitor is said to be ideal if its resistance is infinite.*
- *An inductor does not oppose current but opposes changes (growth or decay of current) in the circuit.*

Self Inductance

Flux linked with the coil is $N\phi_B \propto I$ or $N\phi_B = LI$, where the constant L is known as the ***coefficient of self induction*** *or* ***self inductance*** *of the given coil. It may be defined as the magnetic flux linked with the coil, when a constant current of 1 A is passed through it.*

Induced emf due to self induction $e = -N\dfrac{d\phi}{dt} = -L\dfrac{dI}{dt}$

From this relation, self inductance of a coil/circuit is the magnitude of induced emf produced per unit rate of change of current in the circuit.

SI unit of inductance is **henry**. Inductance is 1 henry if on changing the current at a rate of 1 As^{-1}, an induced emf of 1 V is set up in the circuit.

Dimensional formula of self inductance (L) is $[ML^2T^{-2}A^{-2}]$

Method of Finding Self-Inductance of a Circuit

We use the equation, $L = N\phi_B / i$ to calculate the inductance of given circuit.

A good approach for calculating the self-inductance of a circuit consists of the following steps :

(a) Assume that there is a current i flowing through the circuit (we can call the circuit an inductor).

(b) Determine the magnetic field **B** produced by the current.

(c) Obtain the magnetic flux ϕ_B.

(d) With the flux known, the self-inductance can be found from $L = N\phi_B / i$.

To demonstrate this procedure we now calculate the self-inductance of two inductors.

Magnetic Potential Energy of an Inductor

1. In building a steady current in an electric circuit, some work is done by the emf of the source, against the self inductance of the coil. The work done $W = \dfrac{1}{2}LI^2$
2. The work done is stored as the magnetic potential energy of that inductor.

   Thus, $$U = \frac{1}{2}LI^2$$

Formulae for Self Inductance

1. For a circular coil of radius R and N turns, the self inductance
$$L = \frac{1}{2}\mu_0 \pi N^2 R$$
2. For a solenoid coil having length l, total number of turns N and cross-sectionalarea A
$$L = \frac{\mu_0 N^2 A}{l}$$
$$= \mu_0 n^2 Al \quad \left[\text{where } n = \frac{N}{l}\right]$$
3. For a toroid of radius R and number of turns N
$$L = \frac{1}{2}\mu_0 N^2 R$$
4. For a square coil of side a and number of turns N
$$L = \frac{2\sqrt{2}}{\pi}\mu_0 N^2 a$$

Mutual Induction

Mutual induction is the phenomenon due to which an emf is induced in a coil when the current flowing through a neighbouring coil changes.

Mutual Inductance

Mutual inductance of a pair of coils is defined as the magnetic flux linked with one coil when a constant current, of unit magnitude, flows through the other coil.

Mathematically, $N\phi_{B_2} = MI_1$

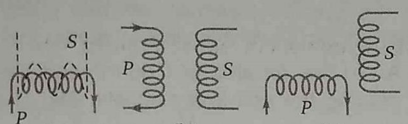
where M is known as the **mutual inductance** for the given pair of coils.

Induced emf due to mutual inductance
$$e_2 = -N\frac{d\phi_{B_2}}{dt} = -M\frac{dI_1}{dt}$$

Hence, mutual inductance for a pair of coils is numerically equal to the magnitude of induced emf in one coil when current in the other coil changes at a rate of 1 As^{-1}.

SI unit of mutual inductance M, is **henry**.

Mutual inductance of a pair of coils is maximum, when the two coils are wound on the same frame. However, mutual inductance is negligible when the two coils are oriented mutually perpendicular to each other (figure). In this context we define a term **coupling coefficient** k.



(a) (b) (c)

Coupling coefficient is given by

$$k = \frac{\text{magnetic flux linked with secondary coil}}{\text{magnetic flux developed in primary coil}}$$

It is observed that $0 \le k \le 1$.

For a pair of two magnetically coupled coils of self inductances L_1 and L_2 respectively, the mutual inductance

$$M_{12} = M_{21} = M = k\sqrt{L_1 L_2}$$

where k is the coupling coefficient.

Formulae for Mutual Inductance

1. Assuming the coupling coefficient $k = 1$ and medium to be a free space or air.

   Mutual inductance of a pair of concentric circular coils is

   $$M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}$$

   where, r = radius of the coil (of small radius)

   and R = radius of the coil (of larger radius).

2. For a pair of two solenoid coils, wound one over the other

   $$M = \frac{\mu_0 N_1 N_2 A}{l}$$

   For a pair of concentric coplanar square coils

   $$M = \frac{2\sqrt{2}\,\mu_0 N_1 N_2 a^2}{\pi b}$$

   where, a = side of the smaller coil and b = side of the larger coil.

For a given pair of coils, mutually coupled, then according to theorem of reciprocity.

$$\boldsymbol{M_{12} = M_{21} = M}$$

Combination of Inductors

- If two coils of self-inductances L_1 and L_2 are placed quite far apart and are arranged in series, then their equivalent inductance

  $$L_s = L_1 + L_2$$

- However, if the coils are placed quite close to each other so as to mutually affect each other, then

  $$L_s = L_1 + L_2 \pm 2M$$

- Here, M has been written with $\pm$ sign depending on the fact whether currents in the two coils are flowing in same sense or opposite sense.

- If two coils of self inductances L_1 and L_2 are connected in parallel, then equivalent inductance L_p is given by

  $$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$$

  $$\Rightarrow \quad L_p = \frac{L_1 L_2}{L_1 + L_2}$$

  In a parallel arrangement, effect of mutual inductance may be omitted because value of k in this arrangement is quite small.

Eddy Currents

Whenever a changing magnetic field is applied to a bulk piece of a conducting material, due to change in magnetic flux, circulating induced currents are induced in the material of the conductor. Such high magnitude of circulating currents induced in the body of bulk conductors are called the eddy currents.

The production of eddy currents in a metallic conductor leads to a loss of electric energy in the form of heat energy.

Eddy currents can be minimised by taking the metal (generally soft iron) core in the form of a combination of thin laminated sheets or by slotting process.

Electric brake, speedometer, electric induction furnace, AC induction motor, electrical energy meter and dead beat galvanometer make use of the concept of eddy currents.

Practice Zone

DAY 23

1. A helicopter rises vertically with a speed of 10 ms^{-1}. If helicopter has a length of 10 m and the horizontal component of the earth's magnetic field is 1.5×10^{-3} Wbm^{-2}, the emf induced between the tip of the nose and the tail of the helicopter, is

(a) 0.15 V (b) 125 V
(c) 130 V (d) 5 V

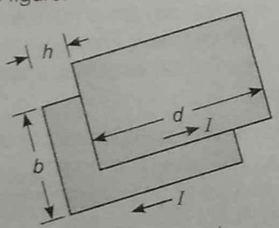
2. An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid. **[NCERT Exemplar]**

(a) 6.5 V (b) 7.4 V
(c) 8.2 V (d) 9.3 V

3. A horizontal straight wire, 10 m long, extending along the east and west direction, is falling at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wbm^{-2}. If the induced emf is 1.5×10^{-3}, the velocity of the wire is

(a) 5×10^{-4} ms^{-1} (b) 5×10^{2} ms^{-1}
(c) 50 ms^{-1} (d) 5 ms^{-1}

4. The inductance per unit length of a double tape line as shown in the figure.

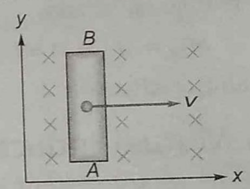


(a) $\frac{\mu_0 h}{b}$ (b) $\frac{b}{\mu_0 h}$
(c) $\frac{\mu_0 b}{h}$ (d) $\frac{hb}{\mu_0}$

5. A coil has an area of 0.05 m^2 and has 800 turns. After placing the coil in a magnetic field of strength 4×10^{-5} Wb m^{-2}, perpendicular to the field, the coil is rotated by 90° in 0.1 s. The average emf induced is

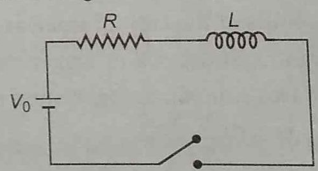
(a) zero (b) 0.016 V
(c) 0.01 V (d) 0.032 V

6. A conducting rod AB moves parallel to x-axis in a uniform magnetic field, pointing along the positive z-direction. The ends A and B are charged as shown in the figure



(a) negative (b) positive
(c) B is positive , A is negative (d) B is negative, A is positive

7. In series R-L circuit, switch is closed at $t = 0$. The change which passes through the battery in one time constant is



(a) $\frac{V_0 e}{R\tau}$ (b) $\frac{V_0 t}{Re}$
(c) $\frac{R\tau}{V_0 e}$ (d) $\frac{V_0 t}{Re}$

8. A short circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns are quadrupoled and the wire radius is halved, the electrical power dissipated in the coil, would be

(a) halved (b) the same
(c) doubled (d) quadrupoled

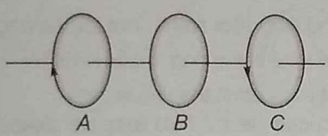
9. A long straight solenoid with cross-sectional radius a and number of turns per unit length n has a current varying with time as $I\ As^{-1}$. The magnitude of the eddy current as a function of distance r from the solenoid axis is

(a) $\frac{-n\mu_0 a^2 I}{2r}$ (b) $\frac{\mu_0 In}{2a}$ (c) $\frac{-na^2 I}{2\mu_0 r}$ (d) $\frac{\mu_0 I a}{2n}$

10. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If $R_1 > R_2$, the mutual inductance M between them will be directly proportional to

(a) $\frac{R_1}{R_2}$ (b) $\frac{R_2}{R_1}$ (c) $\frac{R_1^2}{R_2}$ (d) $\frac{R_2^2}{R_1}$

11. Three identical coils, A, B and C are placed with their planes parallel to one another. Coils A and C carry currents as shown in the figure. Coils B and C are fixed in position and coil A is moved towards B. Then, current induced in B is



(a) clockwise current
(b) anti-clockwise current
(c) no current is induced in B
(d) current is induced only when both the coils move

12. The magnetic field in a certain region is given by $B = (40\,\hat{i} - 18\hat{k})$ gauss. How much flux passes through a loop of area $50\ cm^2$, in this region, if the loop lies flat on the xy-plane?

(a) -900×10^{-9} Wb (b) 900×10^{-9} Wb
(c) Zero (d) 9 Wb

13. A metal rod of resistance 20 Ω is fixed along the diameter of a conducting ring of radius 0.1 m and lies on the x - y plane. There is a magnetic field $B = |50\hat{k}|$. The ring rotates with an angular velocity $\omega = 20\ rads^{-1}$ about its axis. An external resistance of 10 Ω is connected across the centre of the ring and the rim. The current through the external resistance is

(a) $\frac{1}{2}$ A (b) $\frac{1}{3}$ A (c) $\frac{1}{4}$ A (d) zero

14. An uniformly wound solenoid of inductance 1.8×10^{-4}H and resistance 6 Ω is broken into two identical parts. These identical coils are then connected in parallel across a 15 V battery of negligible resistance. The time constant of the circuit is

(a) 3×10^{-5}s (b) 6×10^{-5}s
(c) 1.5×10^{-5}s (d) 1.8×10^{-5}s

15. A uniformly wound solenoidal coil of self inductance 1.8×10^{-4} H and a resistance of 6 Ω is broken up into two identical coils. These identical coils are then connected in parallel across a 120 V battery of negligible resistance. The time constant of the current in the circuit and the steady state current through the battery is

(a) 3×10^{-5} s, 8 A
(b) 1.5×10^{-5} s, 8 A
(c) 0.75×10^{-4}s, 4 A
(d) 6×10^{-5} s, 2 A

16. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad/s in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed-loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. **[NCERT Exemplar]**

(a) 0.012 W (b) 0.1 W
(c) 0.018 W (d) 0.42 W

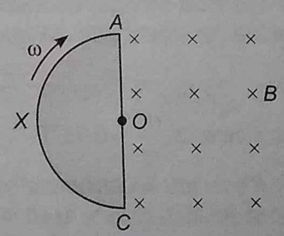
17. A coil is wound in a transformer of rectangular cross-section. If all the linear dimensions of the transformer are increased by a factor of 3 and the number of turns per unit length remains the same, the self inductance increases by a factor of

(a) 3 (b) 9
(c) 27 (d) 8

18. An inductor coil stores 32J of magnetic energy and dissipates it as heat at the rate of 320 W when a current of 4A is passed through it. The time constant of the circuit is

(a) 0.2 s (b) 0.1 s
(c) 0.3 s (d) 0.4 s

19. The magnetic field as shown in the figure is directed into the plane of paper. $AXCA$ is a semicircular conducting loop of radius a with centre O. The loop rotates clockwise with velocity ω about an axis fixed at O and perpendicular to the plane of the paper. The resistance of the loop is R. The induced current is



(a) $\frac{\omega r^2}{2R}$ (b) $-\frac{B\omega r^2}{2R}$
(c) $\frac{-2R}{B\omega r}$ (d) $\frac{2R}{\omega r^2}$

Directions *(Q. Nos. 20 to 24) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

20. **Statement I** The mutual inductance of two coils is doubled if the self-inductance of the primary or the secondary coil, is doubled.

Statement II Mutual inductance is proportional to the self-inductance of primary and secondary coils.

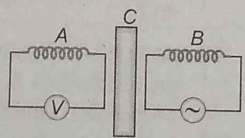
21. **Statement I** The energy stored in the inductor of 2 H, when a current of 10 A flows through it, is 100 J.

Statement II Energy stored in an inductor is directly proportional to its inductance.

22. **Statement I** An artificial satellite with a metal surface, is moving about the earth in a circular orbit. A current is induced when the plane of the orbit is inclined to the plane of the equator.

Statement II The satellite cuts the magnetic field of earth.

23. **Statement I** A coil A is connected to a voltmeter V and the other coil B is connected to an alternating current source. If a large copper sheet C is placed between the two coils, the induced emf in the coil A is reduced.

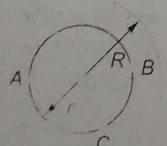


Statement II Copper sheet between the coils, has no effect on the induced emf in coil A.

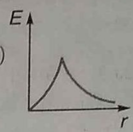
24. **Statement I** A direct current (constant in magnitude and direction) flows in a closed loop made of soft iron wire. This closed wire will acquire a circular shape.

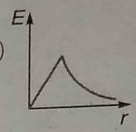
Statement II The loop changes its shape or moves in such a manner that the flux linked with the loop is maximum.

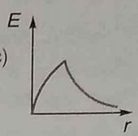
Directions *(Q. Nos. 25 to 28) Magnetic field in a cylindrical region increases at a rate of* $\frac{dB}{dt} = 0.05\ Ts^{-1}$. *The radius of the cylindrical region is* $R = 3$ cm. *A concentric non-conducting ring of radius r is placed in this region as shown.*

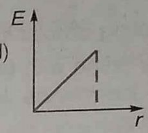


25. The magnitude of the induced electric field as a function of distance r, from the centre is best represented by

(a)


(b)


(c)


(d)


26. The direction of the induced electric field at the location of the ring, is

(a) tangential to the ring in a clockwise direction
(b) tangential to the ring in an anti-clockwise direction
(c) radially inwards
(d) radially outwards

27. Mark out the incorrect statement for the ring of radius $r = \frac{R}{2}$.

(a) Emf induced in the ring is 3.54×10^{-5} V
(b) Current induced in the ring is zero
(c) The induced electric field lines are closed curves.
(d) If a point charge is moved slowly along the circumference of the ring, then the work done by the induced electric field, on the charge in one complete trip, is zero

28. The emf induced in the ring between the points A and B, is

(a) zero (b) 7.08×10^{-5} V
(c) 3.54×10^{-5} V (d) 1.76×10^{-5} V

Directions *(Q. Nos. 29 and 30) A circular wire loop of radius a and resistance R is placed in an external magnetic field. The external field then decreases to zero, as a result of which a current is induced in the loop. However, this current doesn't become zero at the instant when B stops changing, the reason for this can be explained from the Lenz's law.*
Using a rough approximation that magnetic field has the same value at all points within the loop as that at the centre, answer the following questions

29. If current in the loop is i_0 at $t = 0$ (the instant when external magnetic field stops changing), then find the current in the loop, as a function of time, for $t > 0$?

(a) $i = 0$ always (b) $i = i_0 e^{-\frac{2RT}{\mu_0 \pi a}}$
(c) $i = \frac{\mu_0 i}{2a} \times R$ (d) Information insufficient

30. If loop has a resistance $R = 100\ \Omega$ and radius $a = 5$ cm, then determine the time (from $t = 0$) in which current in loop decreases to $10^{-3} i_0$?

(a) $\frac{3\pi^2 \ln 10}{10^{10}}$ s (b) $\frac{3\pi^2}{10^6}$ s
(c) 1 s (d) Question is irrelevant

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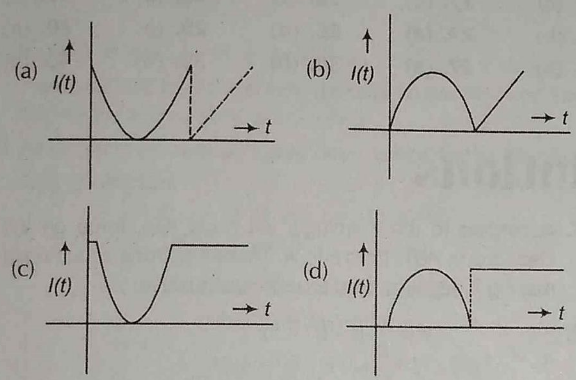
31. Two coils, x and y are kept in close vicinity o feach other. When a varying current, $I(t)$ flows through coil x, the induced emf $[V(t)]$ in coil Y, varies in the manner shown here.

The variation if $I(t)$, with time can then be represented by the graph labbelled as graph. **[JEE Main Online 2013]**



32. A current i is flowing in a straight conductor of length L. The magnetic induction at a point on its axis at a distance $\frac{L}{4}$ from its centre will be **[JEE Main Online 2013]**

(a) zero (b) $\frac{\mu_0 i}{2\pi L}$

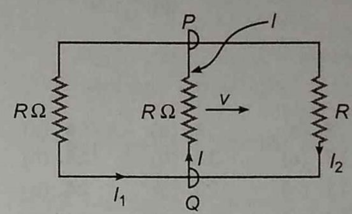
(c) $\frac{\mu_0 i}{\sqrt{2}L}$ (d) $\frac{4\mu_0 i}{\sqrt{5}\pi L}$

33. A coil is suspended in a uniform magnetic field with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil, it starts oscillating; it is very difficult to stop.

But if an aluminium plate is placed near to the coil, it stops. This is due to **[AIEEE 2012]**

(a) development of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

34. A rectangular loop has a sliding connector PQ of length l and resistance $R\,\Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are **[AIEEE 2010]**



(a) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$ (b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$ (d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

35. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. 2s after the connection is made, the current flowing (in ampere) in the circuit is **[AIEEE 2007]**

(a) $(1-e)$ (b) e (c) e^{-1} (d) $(1-e^{-1})$

36. The flux linked with a coil, at any instant t is given by

$$\phi = 10t^2 - 50t + 250$$

The induced emf at $t = 3$ s, is **[AIEEE 2006]**

(a) –190 V (b) –10 V (c) 10 V (d) 190 V

37. A long solenoid has 200 turns/cm and carries a current I. The magnetic field at its centre is 6.28×10^{-2} Wbm^{-2}. Another long solenoid has 100 turns/cm and it carries a current $I/3$. The value of the magnetic field at its centre is **[AIEEE 2006]**

(a) 1.05×10^{-2} Wb m^{-2} (b) 1.05×10^{-5} Wb m^{-2}

(c) 1.05×10^{-3} Wb m^{-2} (d) 1.05×10^{-4} Wb m^{-2}

38. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[AIEEE 2005]**

(a) 0.05 s (b) 0.1 s (c) 0.15 s (d) 0.3 s

39. A metal conductor of length 1 m rotates vertically about one of its ends, with an angular velocity of 5 rads^{-1}. If the horizontal component of the earth's magnetic field is 0.2×10^{-4} T, then the emf developed between the two ends of the conductor is **[AIEEE 2004]**

(a) 5 μV (b) 50 μV

(c) 5 mV (d) 50 mV

40. Two coils are placed close to each other. The mutual inductance of the pair of coils depend upon **[AIEEE 2003]**

(a) the rates at which the current is changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils

41. When the current changes from +2 A to −2 A in 0.05 s, an emf of 8 V is induced in the coil. The coefficient of self-induction of the coil is [AIEEE 2003]

(a) 0.2 H (b) 0.4 H
(c) 0.8 H (d) 0.1 H

42. If a current is passed through a spring, then the spring [AIEEE 2002]

(a) expands (b) gets compressed
(c) remains same (d) None of these

Answers

1. (a)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (d)
11. (b)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (c)
21. (b)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (a)
31. (c)	**32.** (d)	**33.** (d)	**34.** (b)	**35.** (d)	**36.** (b)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (b)
41. (d)	**42.** (b)								

Hints & Solutions

1. $e = Blv = B_H l v$

$= 1.5\times 10^{-3}\times 10\times 10$

$= 0.15$ V

2. Given, length of solenoid $l = 30$ cm $= 30\times 10^{-2}$ m

Area of cross-section $A = 25\,\text{cm}^2 = 25\times 10^{-4}\,\text{m}^2$

Number of turns $N = 500$

Current $I_1 = 2.5$ A, $I_2 = 0$

Brief time $dt = 10^{-3}$ s

Induced emf in the solenoid

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) \qquad (\because \phi = BA)$$

Magnetic field induction B at a point well inside the long solenoid carrying current I is

$B = \mu_0 n I$ $\left(\text{where, } n = \text{Number of turns per unit length} = \frac{N}{l}\right)$

$$e = NA\frac{dB}{dt} = A\frac{d}{dt}\left(\mu_0 \frac{N}{l} I\right) = A\frac{\mu_0 N}{l}\cdot\frac{dI}{dt}$$

$$e = 500\times 25\times 10^{-4}\times 4\times 3.14\times 10^{-7}\times\frac{500}{30\times 10^{-2}}\times\frac{2.5}{10^{-3}} = 6.5\text{ V}$$

3. From, $e = Blv$

$$v = \frac{e}{Bl} = \frac{1.5\times 10^{-3}}{0.3\times 10^{-4}\times 10} = 5\text{ ms}^{-1}$$

4. Neglecting end effects of magnetic field, we have

$$B = \frac{\mu_0 I}{b}$$

Flux ϕ per unit length of the plates is

$$\frac{\mu_0 I}{b}\times h\times l = \frac{\mu_0 h I}{b}$$

Also, $\phi = LI \Rightarrow L = \frac{\mu_0 h}{b}$

5. $e = \frac{d\phi}{dt} = \frac{NAdB}{dt} = 800\times 0.05\times\frac{4\times 10^{-5}}{0.1}$

$= 0.016$ V

6. According to the Fleming's left hand rule, force on the free electrons in AB is from B to A. These electrons accumulate at A, making it negative. End B becomes positive.

7. $i = \frac{V_0}{R}(1 - e^{-t/\tau})$

$$Q = \int i\,dt = \int_0^{\tau}\frac{V_0}{R}(1 - e^{-t/\tau})\,dt$$

$$\Rightarrow \quad Q = \frac{V_0}{R}\tau + \tau(e^{-1} - 1)\frac{V_0}{R} = \frac{V_0 t}{Re}$$

8. The magnitude of the induced voltage is proportional to the rate of change of magnetic flux which, in turns depends on the number of turns in the coil *i.e.*, $V \propto n$.

So, resistance of a wire is given by $R = \frac{\rho l}{\pi r^2}$ $[A = \pi r^2]$

i.e., $R \propto \frac{1}{r^2}$ [ρ is a resistivity of a wire]

$$\therefore \quad P = \frac{V^2}{R} \propto \frac{n^2}{1/r^2} \Rightarrow P = \frac{(nr)^2}{l}$$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2\times\left(\frac{r_2}{r_1}\right)^2\times\left(\frac{l_1}{l_2}\right)$$

or $\quad \frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2$

$$\Rightarrow \quad \frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^2\times\left(\frac{r_2}{r_1}\right)^4 \qquad \left[\text{Given, } \frac{n_2}{n_1} = 4 \text{ and } \frac{r_2}{r_1} = \frac{1}{2}\right]$$

So, $\quad \frac{P_2}{P_1} = (4)^2\times\left(\frac{1}{2}\right)^4 = 16\times\frac{1}{16} = 1$

9. $B = n\mu_0 i$ and $\oint E\cdot dl = -\frac{d\phi}{dt}$

For $r < a$, $(E(2\pi r) = -\pi r^2 n\mu_0 I$ or $E = -\frac{n\mu_0 I r}{2}$ (for $r < a$)

where, $I = \frac{dI}{dt}$. For $r > a$

$$E(2\pi r) = -\pi a^2 n\mu_0 I \Rightarrow E = \frac{-n\mu_0 a^2 I}{2r}$$

10. Magnetic field at the centre of the primary coil $B = \mu_0 I_1/2R_1$. Considering it to be uniform, the magnetic flux passing through coil is

$$\phi_2 = BA = \frac{\mu_0 I_1}{2R_1}(\pi R_2^2)$$

Now, $$M = \frac{\phi_2}{I_1} = \frac{\mu_0 \pi R_2^2}{2R_1}$$

$\therefore$ $$M \propto \frac{R_2^2}{R_1}$$

11. As coil A moves closer to B, field due to A intercepting B, increases. Induced current B must oppose this increase. Hence, the current in B must be anticlockwise.

12. As loop is in the xy-plane, only the z-component of the magnetic field, is effective.

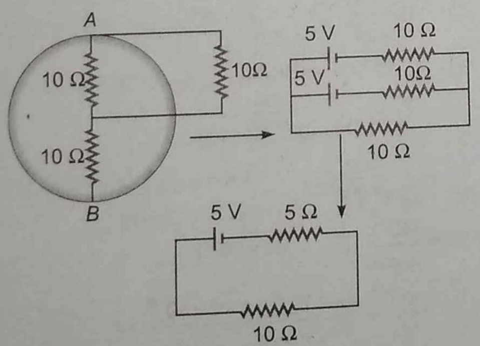
$$B = -18 \text{ gauss} = -18 \times 10^{-4}\text{ T}$$
$$A = 5 \times 10^{-4}\text{m}^2$$
$$\phi = BA\cos 0^\circ = -18 \times 10^{-4} \times 5 \times 10^{-4}$$
$$= -90 \times 10^{-8}\text{ Wb}$$
$$= -900 \times 10^{-9}\text{ Wb} = -900\text{ n Wb}$$

13. Here, resistance of rod = 20 Ω, $r = 0.1$ m, $B = 50$ T, acting along the z-axis and $\omega = 20$ rad s^{-1}.

Potential difference between the centre of the ring and the rim is

$$V = \frac{1}{2}B\omega r^2 = \frac{1}{2} \times 50 \times 20 \times (0.1)^2 = 5\text{ V}$$

The equivalent circuit of the arrangement is shown in figures.



$$\frac{1}{R_p} = \frac{1}{10} + \frac{1}{10} \Rightarrow \frac{2}{10} = \frac{1}{5}$$

$\Rightarrow$ $$R_p = 5\,\Omega$$

Current through the external resistance,

$$i = \frac{E}{R+r} = \frac{5}{10+5} = \frac{1}{3}\text{ A}$$

14. Inductance of each part,

$$L_1 = L_2 = \frac{L}{2} = 0.9 \times 10^{-4}\text{ H}$$

Resistance of each part,

$$R_1 = R_2 = \frac{R}{2} = 3\Omega$$

Time constant,

$$\tau = \frac{L_1 || L_2}{R_1 || R_2} = \frac{L_1L_2}{L_1 + L_2} \times \frac{R_1 + R_2}{R_1R_2}$$

$$\tau = \frac{1.8 \times 10^{-4} \times 0.9 \times 10^{-4}}{1.8 \times 10^{-4} + 0.9 \times 10^{-4}} \times \frac{6+3}{6 \times 3}$$

$$= \frac{1.62 \times 10^{-4}}{5.4} = 0.3 \times 10^{-4} = 3 \times 10^{-5}\text{s}$$

15. Since, the self-inductance in parallel is given by

$$\frac{1}{L_p} = \frac{1}{L} + \frac{1}{L} = \frac{2}{L} \Rightarrow L_p = \frac{L}{2}$$

and $$L = \frac{1.8 \times 10^{-14}}{2} = 0.9 \times 10^{-4}\text{H}$$

$\therefore$ $$L_p = 0.45 \times 10^{-4}\text{ H}$$

Resistance of each part, $r = 6/2 = 3\Omega$

Now, $$\frac{1}{r_p} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$\therefore$ $$r_p = \frac{3}{2}\,\Omega$$

So, the time constant of the circuit is given by $\tau = \frac{L_p}{r_p} = \frac{0.45 \times 10^{-4}}{3/2} = 3 \times 10^{-5}$ s and the steady current is,

$$I = \frac{V}{r_p} = \frac{12}{3/2} = 8\text{ A}$$

16. Average induced emf

$$e_{av} = \frac{1}{T}\int_0^{2\pi} e\,dt = \frac{1}{T}\int_0^{\pi} NBA\,\omega \sin\omega t\,dt$$

$$e_{av} = \frac{1}{T} \cdot NAB\,\omega\left[\frac{\cos \omega t}{\omega}\right]_0^{2\pi} = \frac{NBA}{T}[\cos 2\pi - \cos 0^\circ]$$

$$e_{av} = \frac{NBA}{T}[1-1] = 0$$

For full cycle average emf, $e_{av} = 0$

Average power loss due to heating

$$= \frac{E_0 I_0}{2} = \frac{0.603 \times 0.0603}{2} = 0.018\text{ W}$$

17. $L = \mu_0 N^2 \frac{A}{l} = \mu_0 \left(\frac{N}{b}\right)^2 Al$

Here, N/l remains unchanged. When the linear dimensions are increased by a factor 3, the area becomes 9 times and the length becomes 3 times. Therefore, the coefficient of self-inductance becomes $9 \times 3 = 27$ times.

18. $U = \frac{Li^2}{2}$ or $32 = \frac{L(4)^2}{2} = L = 4\text{H}$

$$P = i^2R,\ R = \frac{320}{(4)^2} = 20\,\Omega$$

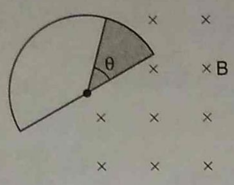
$\therefore$ $$\tau = \frac{L}{R} = \frac{4}{20} = 0.2\text{ s}$$

19. $A = \frac{\theta}{\pi}\frac{\pi r^2}{2} = \frac{\theta r^2}{2} = \frac{\omega t\, r^2}{2}$

Flux $\phi = BA = B\frac{\omega t\, r^2}{2}$

Emf $\varepsilon = -\frac{d\phi}{dt} = -\frac{B\omega r^2}{2}$

$i = \frac{-B\omega r^2}{2R}$



After half rotation $A(t) = \pi r^2 - \frac{\omega t r^2}{2}$ will give same current but in opposite direction.

20. If two coils of inductances L_1 and L_2 are joined together, then their mutual inductance is given by

$$M = k\sqrt{L_1 L_2} \quad \text{...(i)}$$

It is clear from the relation, if the self-inductance of the primary and the secondary coil is doubled, the mutual inductance of the coils, will also be doubled.

21. The energy stored in the inductor is given by

$$U = \frac{1}{2}Li_0^2 = \frac{1}{2}\times 2\times(10)^2 = 100\ \text{J}$$

It is obvious that energy stored in the inductor, is directly proportional to its inductance.

22. We know that for current to be induced in a satellite, it must cut the magnetic field of earth, hence its plane must be inclined to the plane of the equator.

23. In the absence of the copper sheet, induced emf will be produced in the coil *A* due to the mutual induction between the coils *A* and *B*. As a result, voltmeter will show deflection depending on the magnitude of the induced emf.

When the copper sheet is placed between the two coils, eddy currents will be setup in the coil. Since, the eddy currents have an opposing effect, the magnetic flux linked with *A* due to, eddy current will always be opposite to that due to the alternating current through *B*. Thus, induced current will be reduced.

24. Both the statements are correct and the statement II is the correct explanation of statement I. Out of the various shapes for a given perimeter, the circle has the maximum area, to link maximum flux.

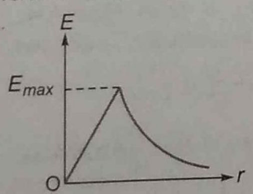
25. For $r \le R$, induced electric field is given by,

$$E\times 2\pi r = \frac{dB}{dt}\times\pi r^2 \Rightarrow E = \frac{0.05}{2} r\ \text{Vm}^{-1}$$

For $r \ge R$, induced electric field is given by,

$$E\times 2\pi r = \frac{dB}{dt}\times\pi R^2 \Rightarrow E = \frac{2.25\times 10^{-5}}{r}\ \text{Vm}^{-1}$$

So, graph of *E versus r* is best represented as



26. Direction of the induced electric field, is the same as that of the induced current, *i.e.*, tangential (due to symmetric situation and absence of any charged particle).

27. Emf induced,

$$e = \oint E\cdot dl = \frac{dB}{dt}\times\pi r^2$$

$$= 0.05\times\pi(R/2)^2$$

$$= 0.05\times 3.14\times\left(\frac{0.03}{2}\right)^2$$

$$= 3.54\times 10^{-5}\ \text{V}$$

The ring is non-conducting, so no current flows through it.

As electric fields (induced) are non-conservative and non-electrostatic in nature, the field lines are closed curves and work done by the electric field, along a closed loop, is non-zero.

$$W = q\int_0^{2\pi r}\mathbf{E}\cdot d\mathbf{r}$$

28. $$e_{AB} = \frac{0.05}{2}r\times\pi r$$

$$= \frac{0.05\times\pi}{2}\times(R/2)^2 = 1.76\times 10^{-5}\ \text{V}$$

29. When current in the loop is i, flux linked with the loop, due to its own magnetic field is

$$\phi = \frac{\mu_0 i}{2a}\times\pi a^2 = \frac{\mu_0\pi a}{2}i$$

Emf induced $\frac{-d\phi}{dt} = e = -\frac{\mu_0\pi a}{2}\frac{di}{dt}$

From $V = iR \Rightarrow\ i = \frac{e}{R} = -\frac{\mu_0\pi a}{2R}\frac{di}{dt}$

$$\Rightarrow \int_{i_0}^{i}\frac{di}{i} = -\int_0^t\frac{2R}{\mu_0\pi a}dt$$

$$\Rightarrow i = i_0 e^{-\frac{2Rt}{\mu_0\pi a}}$$

30. $10^{-3}\times i_0 = i_0\, e^{\frac{-2Rt}{\mu_0\pi a}}$

$$-3\ln 10 = \frac{-2Rt}{\mu_0\pi a}\quad[\mu_0 = 4\pi\times 10^{-7}]$$

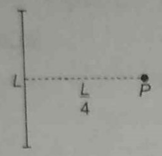
$$\Rightarrow \frac{3\ln 10\times 4\pi\times 10^{-7}\times\pi\times 5\times 10^{-2}}{2\times 100} = t$$

$$\Rightarrow \frac{3\ln 10\times\pi^2\times 10^{-8}}{10^2} = t$$

$$\Rightarrow t = \frac{3\pi^2\ln 10}{10^{10}}\ \text{s}$$

31. Firstly, the current decreases due to electrical inertia goes to zero, but due to back emf induced in the easily, the induced current in the coil decreases off a point when back emf is equal to the applied emf induced in the another coil. The value of emf of two current is zero. Then, current is regularly increased, after that time became it is continuously by supplied by the source (variable).

32.



The magnitude field induction at point P is given by

$$B = \frac{\mu_0 i\,(L)}{2\pi\left(\frac{L}{4}\right)\sqrt{L^2 + 4\left(\frac{L}{4}\right)^2}}$$

$$= \frac{\mu_0 i L}{2\pi \frac{L}{4}\sqrt{L^2 + \frac{4L^2}{16}}} = \frac{4\mu_0 iL}{2\pi L\sqrt{5L^2/4}}$$

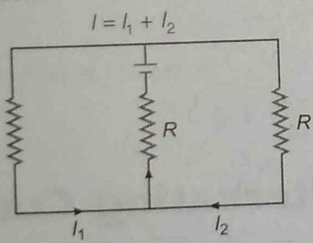
$$\Rightarrow \quad B = \frac{4\mu_0 i}{\pi\sqrt{5}\,L}$$

33. According to Lenz's law, electromagnetic induction takes place in the aluminium plate for which eddy current is developed. This causes loss in energy which results in damping of oscillatory motion of the coil.

34. A moving conductor is equivalent to a battery of emf

$= vBl$ (motion emf)

Equivalent circuit



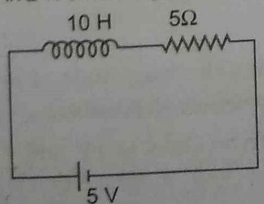
Applying Kirchhoff's law

$$I_1R + IR - vBl = 0 \quad \text{...(i)}$$
$$I_2R + IR - vBl = 0 \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2IR + IR = 2vBl$$
$$I = \frac{2vBl}{3R}$$
$$I_1 = I_2 = \left|\frac{Blv}{3R}\right|$$

35. Rise of current in L-R circuit is given by



$$I = I_0(1 - e^{-t/\tau})$$

where, $I_0 = \frac{E}{R} = \frac{5}{5} = 1\text{A}$

Now, $\tau = \frac{L}{R} = \frac{10}{5} = 2\text{ s}$

After 2s, *i.e.*, at $t = 2\text{s}$

Rise of current $I = (1 - e^{-1})$ A

36. Given, $\phi = 10t^2 - 50t + 250$

From the Faraday's law of electromagnetic induction,

$$e = -\frac{d\phi}{dt}$$
$$\therefore \quad e = -[10 \times 2t - 50]$$
$$\therefore \quad e|_{t=3\text{s}} = -[10 \times 6 - 50]$$
$$= -10\text{ V}$$

37. Magnetic field due to a long solenoid is given by

$$B = \mu_0 nI$$

From the given data,

$$6.28 \times 10^{-2} = \mu_0 \times 200 \times 10^2 \times I \quad \text{... (i)}$$

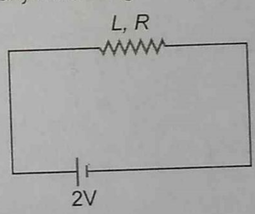
and

$$B = \mu_0 \times 100 \times 10^2 \times \left(\frac{I}{3}\right) \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$B \approx 1.05 \times 10^{-2}\text{ Wbm}^{-2}$$

38. The current at any instant is given by



$$\Rightarrow \quad I = I_0(1 - e^{-Rt/L})$$
$$\Rightarrow \quad \frac{I_0}{2} = I_0(1 - e^{-Rt/L}) \quad \text{or} \quad \frac{1}{2} = (1 - e^{-Rt/L})$$
$$\text{or} \quad e^{-Rt/L} = \frac{1}{2} \quad \text{or} \quad \frac{Rt}{L} = \ln 2$$
$$\therefore \quad t = \frac{L}{R}\ln 2 = \frac{300 \times 10^{-3}}{2} \times 0.693$$
$$= 150 \times 0.693 \times 10^{-3} = 0.10395\text{ s} = 0.1\text{ s}$$

39. The emf induced between the ends of the conductor

$$e = \frac{1}{2}B\omega L^2 = \frac{1}{2} \times 0.2 \times 10^{-4} \times 5 \times (1)^2 = 0.5 \times 10^{-4}\text{ V}$$
$$= 5 \times 10^{-5}\text{ V} = 50\,\mu\text{V}$$

40. Mutual inductance of the pair of coils depends on the distance between the two coils and on the geometry of the two coils.

41.

$$e = -L\frac{di}{dt} = -L\frac{(-2-2)}{0.05},\ 8 = L\frac{(4)}{0.05}$$
$$\therefore \quad L = \frac{8 \times 0.05}{4} = 0.1\text{ H}$$

42. Due to the flow of current, in the same direction in two adjacent sides, an attractive magnetic force will be produced due to which the spring gets compressed.

Day 24

Alternating Current

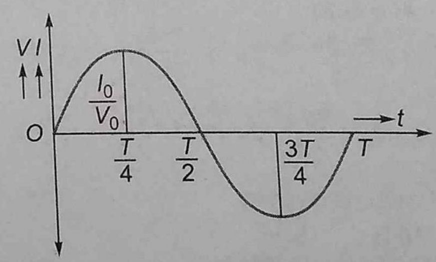
Day 24

Outlines ...

- Concept of Alternating Current
- Reactance and Impedance
- AC Circuit Elements
- Series AC Circuits
- Power in an AC Circuit
- Choke Coil
- AC Generator
- Transformer

Concept of Alternating Current

*An **alternating current** is the current (or voltage) whose magnitude keeps on changing continuously with time, between zero and a maximum value and the direction also reverses periodically.*



We can analyse from the given figure the magnitude of the current cyc change as positive to negative and negative to positive.

In positive half cycle we take positive values as $+V$ and $+I$ and in neg half cycle we take negative values $-V$ and $-I$, hence this is called altern current.

Peak and RMS Values of Alternating Current/Voltage

The common alternating current varying as a sine function of time, is given by

$$I = I_0 \sin \omega t = I_0 \sin 2\pi \nu t = I_0 \sin \frac{2\pi t}{T}$$

and $$V = V_0 \sin 2\pi \nu t = V_0 \sin \frac{2\pi t}{T}$$

Here, I_0 and V_0 are the **maximum or peak values** of current and voltage, ω the angular frequency, ν the frequency and T the period of the given AC.

In our domestic circuits, the frequency of AC is 50 Hz. Thus, it changes its direction after every $\frac{1}{100}$ s.

The average value of an AC, for one complete cycle, is zero.

Generally, **average** or **mean value** of an AC means the average value of the given AC over one half cycle *i.e.*, $t = 0$ to $\frac{T}{2}$

$$\therefore \quad I_{av} = \frac{\int_0^{T/2} I\, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637\, I_0$$

and $$V_{av} = 0.637 V_0$$

The **rms value** of AC is defined as

$$(I_{rms})^2 = \frac{\int_0^T I^2\, dt}{\int_0^T dt} = \frac{I_0^2}{2}$$

$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$

Similarly, $$V_{rms} = \frac{V_0}{\sqrt{2}}$$

- *An alternating current/voltage besides having a sinusoidal variation, may also be square wave or triangular wave or saw tooth wave type. However, AC varying only as a sine curve, is produced by an AC generator.*
- *Alternating current and voltage are rotating vectors called phasors. A phasor is not a real physical quantity with a direction in space like velocity. It is a geometric entity, which helps us to analyse AC circuits.*

Reactance *and* Impedance

The opposition offered by a pure inductor or capacitor or both to the flow of AC, through it, is called reactance X. Its unit is ohm (Ω) and dimensional formula is $[ML^2T^{-3}A^{-2}]$.

Reactance is of two types

(i) Inductive reactance,

$X_L = L\omega$, and

(ii) Capacitive reactance,

$X_C = \frac{1}{C\omega}$

Reciprocal of reactance is known as susceptance

Thus, $S = \frac{1}{X}$

Total opposition offered by an AC circuit to the flow of current through it, is called its impedance Z. Its unit is ohm and dimensional formula is $[ML^2T^{-3}A^{-2}]$. For an AC circuit

$$Z = \sqrt{X^2 + R^2}$$

$$= \sqrt{(X_L - X_C)^2 + R^2}$$

Reciprocal of impedance is known as **admittance.**

Thus, $Y = \frac{1}{Z}$

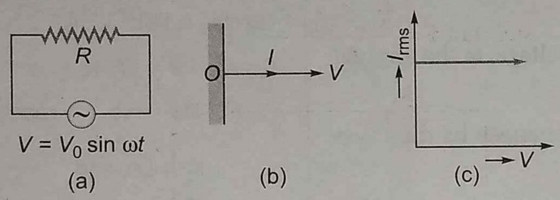
Its unit is Siemens (S).

AC Circuit Elements

The circuit consist of resistor, capacitor and inductor are callled pure resistive, pure inductive and pure capacitive circuit. There circuits with their respective phasor diagram are given below.

1. Pure Resistive Circuit

Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure resistance R. Then,



Current $I = \frac{V}{R}$ or $I_{rms} = \frac{V_{rms}}{R}$

Current and voltage are in the same phase *i.e.*, current is given by $I = I_0 \sin \omega t$.

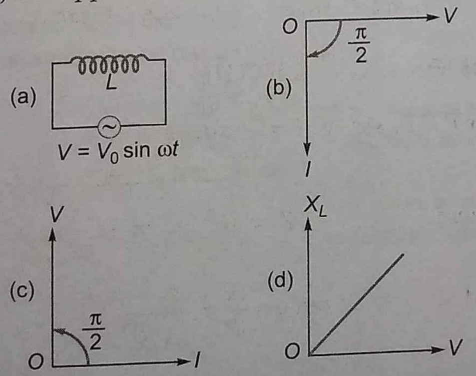
2. Pure Inductive Circuit

Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure inductance L. Then, the average power

$$= V_{rms} I_{rms} \cos \frac{\pi}{2} = 0$$

Such a current, for which average power as well as power factor is zero, is called as **wattless current**.

The inductance offers some opposition to the flow of AC. It is known as "inductive reactance" $X_L = 2\pi\nu L = L\omega$. Thus, a pure inductance does not oppose the flow of DC $(\omega = 0)$ but opposes the flow of AC.



Current flowing $I = \frac{V}{X_L}$

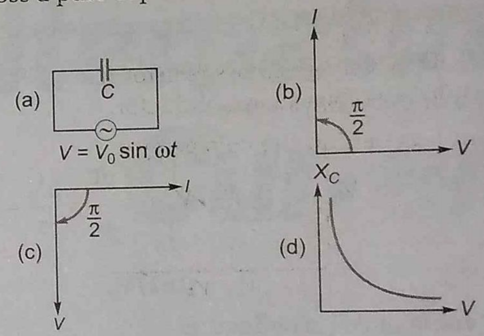
The current decreases with an increase in frequency.

The current lags behind the voltage by $\frac{\pi}{2}$ (or voltage leads the current by $\frac{\pi}{2}$) and is thus given by

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

3. Pure Capacitive Circuit

Let an alternating voltage $V = V_0 \sin \omega t$ be applied across a pure capacitance C. Then,



The capacitance offers some opposition to the flow of current but allows AC to pass through it. The opposition offered is known as the **capacitive reactance.**

$$X_C = \frac{1}{C\omega}\,\Omega = \frac{1}{C \times 2\pi\nu}\,\Omega$$

Current flowing $I = \frac{V}{X_C}$

The current increases with an increase in frequency.

The current leads the voltage by $\frac{\pi}{2}$ (or voltage lags behind the current by $\frac{\pi}{2}$) and is thus given by

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

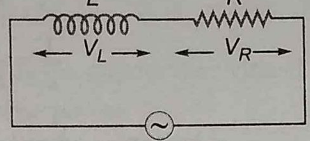
- *Out of the three elements in an AC circuit, the resistor is called the active element because it always opposes the flow of current under all conditions.*
- *For DC circuit, frequency $\nu = 0$ and hence, $X_L = 0$ but $X_C = \infty$*

Series AC Circuits

Some of the series AC circuits are given below.

1. Series L-R Circuit

The potential difference across a resistance in AC is in phase with current and it leads in phase by 90° with current across the inductor.



For $E = E_0 \sin \omega t$

$$I = \frac{E_0}{Z} \sin(\omega t + \phi)$$

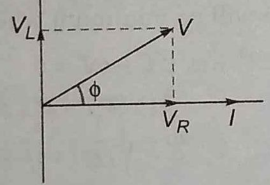
where, $Z = \sqrt{R^2 + (\omega L)^2}$

and $\tan \phi = \dfrac{\omega L}{R}$

$$V = \sqrt{V_R^2 + V_L^2}$$

where, V_R = voltage across resistor R.
V_L = voltage across inductor.

Current lags behind the voltage by ϕ.

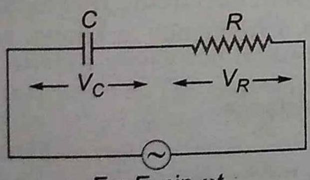


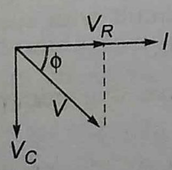
2. Series R-C Circuit

Potential difference across a capacitor in AC lags in phase by 90° with the current in the circuit.

For $E = E_0 \sin \omega t$,

$$I = \frac{E_0}{Z} \sin(\omega t - \phi)$$





where, $Z = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

and $\tan \phi = \dfrac{-1/\omega C}{R}$

Current leads the voltage by ϕ.

$$V^2 = V_R^2 + V_C^2$$

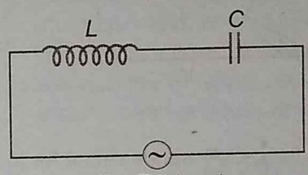
$$\Rightarrow \quad V = \sqrt{V_R^2 + V_C^2}$$

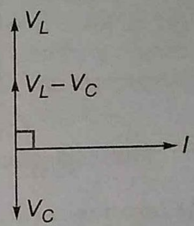
3. Series L-C Circuit

For $E = E_0 \sin \omega t$, $I = \dfrac{E_0}{Z} \sin(\omega t - \phi)$

where, $Z = X_L - X_C$

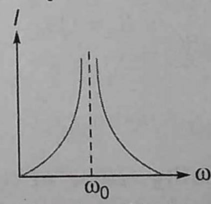
and $\tan \phi = \dfrac{X_L - X_C}{0} = \infty$



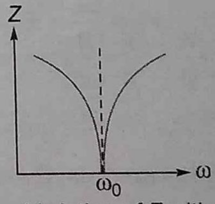


For $X_L > X_C$, $\phi = \dfrac{\pi}{2}$ and for $X_L < X_C$, $\phi = -\dfrac{\pi}{2}$

If $X_L = X_C$ *i.e.*, at $\omega = \dfrac{1}{\sqrt{LC}}$, $Z = 0$ and I_0 becomes infinity. This condition is termed as the resonant condition and this frequency is termed as natural frequency of the circuit.



Variation of I with ω in L-C circuit



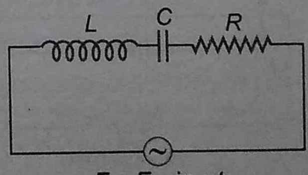
Variation of Z with ω in L-C circuit

4. Series L-C-R Circuit

For $E = E_0 \sin \omega t$, $I = \dfrac{E_0}{Z} \sin(\omega t - \phi)$

where, $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

and $\tan \phi = \dfrac{X}{R} = \dfrac{X_L - X_C}{R}$

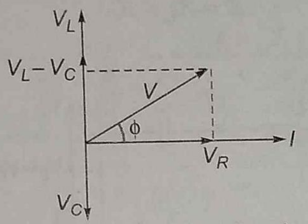


For $X_L > X_C$, current lags voltage

$X_L < X_C$, current leads voltage

$X_L = X_C$, current and voltage are in phase

If $X_L = X_C \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ *i.e.*, the natural frequency of the circuit is equal to the applied frequency, then the circuit is said to be in **resonance**.



Phasor diagram for *L-C-R* series circuit for $X_L > X_C$

At resonance, the current in the circuit is maximum and the impedance is minimum.

At resonance frequency, $\nu = \frac{1}{2\pi\sqrt{LC}}$

Quality Factor

At resonance, $I_0 = \frac{E_0}{R}$ and $V_L = I_0 X_L = V_C = I_0 X_C$

i.e., $$V_L = \frac{\omega L}{R} \times E_0 = \frac{1}{\omega RC} \times E_0 = QE_0$$

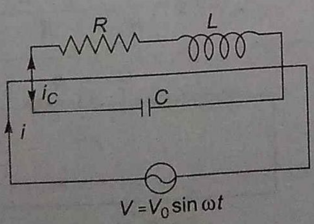
where, $$Q = \frac{\omega L}{R}$$

or $\frac{1}{\omega RC}$ is termed as the **Quality factor of the circuit**. It determines the sharpness of resonance. Higher the value of Q, sharper is the resonance.

Parallel Resonant Circuit

Figure shows a parallel resonant circuit in which resistor R and inductor L have been connected in series and this combination is connected in parallel with the capacitor C.

To this combination, an alternating source of $V = V_0 \sin \omega t$ is applied.



From the figure,

$$i = i_L + i_C$$

or $$\frac{V}{Z} = \frac{V}{R + j\omega L} + \frac{V}{-j/\omega C}$$

$$= \frac{V}{R + j\omega L} - \frac{\omega CV}{j}$$

$$= \frac{V}{R + j\omega L} + j(\omega C)V$$

$\therefore$ $$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as admittance (Y). Thus,

$$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$\therefore$ $$Y = \frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum when

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

or $$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$\therefore$ $$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

is known as resonance frequency. At resona frequency admittance is minimum of the impedanc maximum.

Thus, the parallel circuit does not allow this freque from the source to pass in the circuit. Due to this rea the circuit with such a frequency is known as reje circuit.

we have, dynamic resistance

$$Z_{max} = \frac{1}{Y_{max}} = \frac{L}{CR}$$

$\therefore$ Peak current through the supply $= \frac{V_0}{L/CR} = \frac{V_0 CR}{L}$

The peak current through capacitor $= \frac{V_0}{1/\omega C} = V_0 \omega C$

Hence, $$Q\text{-factor} = \frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R}$$

This is basically the measure of current magnificatic

Power in an AC Circuit

Let a voltage $V = V_0 \sin \omega t$ be applied across an AC and consequently a current $I = I_0 \sin(\omega t - \phi)$ flows through the circuit. Then,

(i) **Instantaneous power** $= VI$

$= V_0 I_0 \sin \omega t \sin(\omega t - \phi)$, and its value varies with time.

Here ϕ is known as **phase difference** between V and I

(ii) **Average power** over a full cycle of AC is

$$P_{av} = V_{rms} I_{rms} \cos\phi = \frac{1}{2} V_0 I_0 \cos\phi$$

The term $V_{rms} I_{rms}$ is known as the apparent or virtual power but $V_{rms}\ I_{rms} \cos\phi$ is called the **true power**.

(iii) The term $\cos\phi$ is known as the **power factor** of the given circuit.

Thus, $\cos\phi = \frac{R}{Z} = \text{power factor} = \frac{\text{true power}}{\text{apparent power}}$

(iv) For a pure resistive circuit, V and I are in phase ($\phi = 0°$) hence, $\cos\phi = 1$ and average power $= V_{rms} I_{rms}$

For a pure inductive or a pure capacitive circuit, current and voltage differ in phase by $\frac{\pi}{2}\left(i.e.,\ \phi = \frac{\pi}{2}\right)$.

Wattless Current

Average power is given by $P_{av} = E_{rms} I_{rms} \cos\phi$

The phase difference between E_{rms} and I_{rms} is ϕ. We can resolve I_{rms} into two components

$$I_{rms} \cos\phi \text{ and } I_{rms} \sin\phi$$

Here, the component $I_{rms} \cos\phi$ contributes towards power dissipation and the component $I_{rms} \sin\phi$ does not contribute towards power dissipation. Therefore, it is called **Wattless current.**

Choke Coil

Let us consider a choke coil (used in tube lights) of large inductance, L and low resistance R. The power factor for such a coil is given by,

$$\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{R}{\omega L} \qquad (\text{as } R << \omega L)$$

As $R << \omega L$, $\cos\phi$ is very small. Thus, the power absorbed by the coil $V_{rms}\ i_{rms} \cos\phi$ is very small. On account of its large impedance $Z = \sqrt{R^2 + \omega^2 L^2}$, the current passing through the coil is very small. Such a coil is used in AC circuits for the purpose of adjusting current to any required value without waste of energy. The only loss of energy is due to hystersis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current if placed instead of the choke coil.

AC Generator

An electric generator or dynamo is a device used to produce electrical energy at the expense of mechanical/thermal energy.

It works on the principle of electromagnetic induction : when a coil is rotated in a uniform magnetic field, an induced emf is set up between its ends. The induced emf is given by $e = e_0 \sin \omega t = NBA\omega \sin \omega t$. The direction of the induced emf is alternating in nature.

Transformer

It is a device which works in AC circuits only and is based on the principle of mutual induction.

Transformer is used to suitably increase or decrease the voltage in an AC circuit. Transformer which transforms strong AC at low voltage into a weaker current at high alternating voltage is called a **step up transformer**. A **step down transformer** transforms weak current at a higher alternating voltage into a strong current at a lower alternating voltage.

For an **ideal transformer** $\frac{e_s}{e_p} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = k$

where, k is known as the transformation ratio.

For a step up transformer, $k > 1$ but for a step down transformer, $k < 1$.

In a transformer, the input emf and the output emf, differ in phase by π radians.

The **efficiency of a transformer** is given by

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{V_s I_s}{V_p I_p}$$

For an ideal transformer, $\eta = 100\%$ or 1. However, for practical transformers, $\eta \approx 85\text{-}90\%$

Possible causes of energy loss in transformer are

- heating due to winding resistance,
- eddy current losses,
- magnetic flux leakage, and
- hysteresis loss. To minimize these losses, the transformer core is made up of a laminated soft iron strips.

Practice Zone

DAY 24

1. An L-C circuit is in a state of resonance. If $C = 0.1\mu F$ and $L = 0.25$ H, then neglecting the ohmic resistance of the circuit, find the frequency of oscillations.

(a) 1007 Hz (b) 100 Hz
(c) 109 Hz (d) 500 Hz

2. A transformer has turn ratio 2 and input power 3600 W. Load current is 20 A. Efficiency $\eta = 90\%$. The internal resistance is

(a) 1 Ω (b) 0.9 Ω
(c) 1.9 Ω (d) 3 Ω

3. An alternating voltage $V = 200\sqrt{2}\sin(100t)$ is connected to a $1\mu F$ capacitor through an AC ammeter. The reading of the ammeter shall be

(a) 10 mA (b) 20 mA
(c) 40 mA (d) 80 mA

4. In an AC circuit, the potential difference across an inductor and a resistor, joined in series, are 16 V and 20 V, respectively. The potential difference across the circuit is

(a) 20.0 V (b) 25.6 V
(c) 31.9 V (d) 53.5 V

5. A coil of 0.01 H inductance and 1 Ω resistance is connected to 200 V, 50 Hz AC supply. The impedance of the circuit and time lag between maximum alternating voltage and current would be **[NCERT Exemplar]**

(a) 3.3 kΩ and $\frac{1}{250}$ s (b) 3.9 kΩ and $\frac{1}{160}$ s
(c) 4.2 kΩ and $\frac{1}{100}$ s (d) 2.8 kΩ and $\frac{1}{120}$ s

6. Which of the following components of an L-C-R circuit, with an AC supply, dissipates energy?

(a) L (b) R
(c) C (d) All of these

7. In an AC circuit, V and I are given by

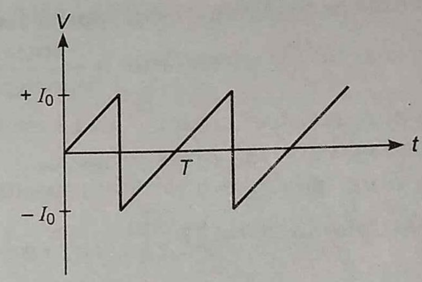
$$V = 100\sin(100t)\text{ V}$$

and
$$I = 100\sin\left(100t + \frac{\pi}{3}\right)\text{ A}$$

The power dissipated in the circuit is

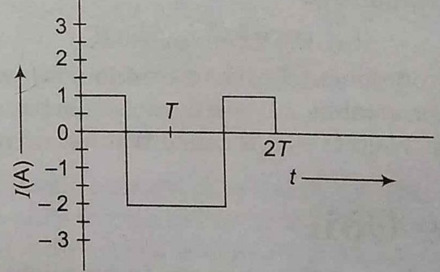
(a) 10^4 W (b) 2.5 kW
(c) 5 kW (d) 10 W

8. The average current in terms of I_0 for the waveform as shown is



(a) I_0 (b) $\frac{I_0}{3}$ (c) $\frac{I_0}{2}$ (d) $\frac{I_0}{4}$

9. The alternating current in a circuit is described by the graph shown in figure. The rms current obtained from the graph would be



(a) 1.4 A (b) 2.2 A
(c) 1.9 A (d) 2.6 A

10. A bulb is rated 55 W/110 V. It is to be connected to a 220 V/50 Hz with inductor in series. The value of inductance so that bulb gets correct voltage is

(a) 200 Ω (b) 110 Ω
(c) 50 Ω (d) 220 Ω

11. The power factor of an AC circuit having resistance R and inductance L, connected in series is

(a) $\frac{R}{\omega L}$ (b) $\frac{R}{(R^2 + \omega^2L^2)^{1/2}}$
(c) $\frac{\omega L}{R}$ (d) $\frac{R}{(R^2 - \omega^2L^2)^{1/2}}$

12. The bandwidth in a series L-C-R circuit is

(a) $\frac{LC}{2\sqrt{R^2C^2+4LC}}$ (b) $\frac{2LC}{\sqrt{R^2C^2+4LC}}$

(c) $\frac{\sqrt{R^2C^2+4LC}}{LC}$ (d) zero

13. In an L-C-R circuit, if V is the effective value of the applied voltage, V_R is the voltage across R, V_L is the effective voltage across L, V_C is the effective voltage across C, then

(a) $V = V_R + V_L + V_C$

(b) $V^2 = V_R^2 + V_L^2 + V_C^2$

(c) $V^2 = V_R^2 + (V_L - V_C)^2$

(d) $V^2 = V_L^2 + (V_R - V_C)^2$

14. An alternating voltage (in volts) given by

$$V = 200\sqrt{2}\sin(100t)$$

is connected to a 1 μF capacitor through an AC ammeter. The reading of the ammeter will be

(a) 10 mA (b) 20 mA
(c) 40 mA (d) 80 mA

15. An L-C circuit contains 10 mH inductor and a 25 μF capacitor. The ratio of the time periods for the energy to be completely magnetic, is

(a) 0, 1.57, 4.71

(b) 1.57, 3.14, 4.71

(c) 1.57, 4.71, 7.85

(d) None of the above

16. A capacitor of 5μF is charged to a potential difference of 200 V. If it is discharged through two resistors of 700 Ω and 300 Ω, what is the energy dissipated in each of the two resistors?

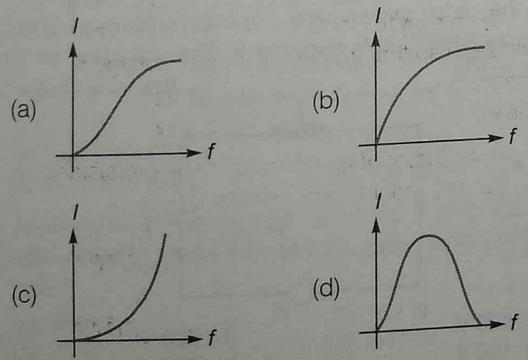
(a) 0.07 J, 0.03 J

(b) 0.03 J, 0.07 J

(c) 0.7 J, 0.3 J

(d) 0.3 J, 0.7 J

17. An AC circuit of variable frequency f is connected to an L-C-R series circuit. Which one of the graphs in the figure, represents the variation of current I in the circuit with frequency f?



18. An AC circuit consists of a 220 Ω resistance and a 0.7 H choke. The power absorbed from 220 V and 50 Hz source connected in this circuit if the resistance and choke are joined in series is

(a) 110 W (b) 50 W (c) 220 W (d) 440 W

Directions *(Q. Nos. 19 to 23) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

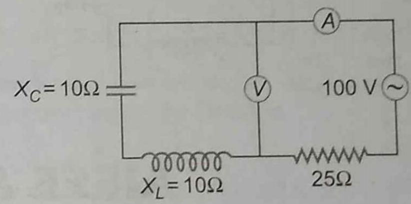
(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

19. **Statement I** For the AC circuit shown in the figure, the voltmeter reading is 0 V whatever be the value of ammeter reading.



Statement II Net resistance of the AC circuit is given by $X = X_L - X_C$.

20. **Statement I** In a series L-C-R circuit, the resonance can take place.

Statement II Resonance takes place if the inductive and capacitive reactances are equal and opposite.

21. **Statement I** In a series R-L-C circuit, the voltage across the resistor, inductor and capacitor are 8V, 16V and 10V respectively. The resultant emf in the circuit is 10V.

Statement II Resultant emf of the circuit is given by the relation $E = \sqrt{V_R^2 + (V_L - V_C)^2}$

22. **Statement I** A sinusoidal AC current flows through a resistance R. If the peak current is I_0, then the power dissipated is $\frac{RI_0^2}{2}$.

Statement II For a purely resistive circuit, the power factor, $\cos\phi = 1$.

23. **Statement I** The nature of the impedance of R-L-C circuit, at resonance is pure inductive.

Statement II The phase angle between E and I in a R-L-C circuit at resonance, is zero.

Directions (Q. Nos. 24 to 26) *An L-C-R series circuit with a 100 Ω resistance, is connected to an AC source of 200 V and angular frequency 300 rads^{-1}. When only the capacitance is removed, the current leads the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°.*

24. The impedance of the L-C-R circuit is

(a) 100 Ω (b) $100\sqrt{2}$ Ω (c) 200 Ω (d) $200\sqrt{2}$ Ω

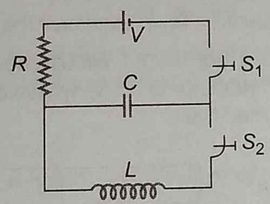
25. The current in the circuit is

(a) $\sqrt{2}$ A (b) 2 A (c) $2\sqrt{2}$ A (d) 1 A

26. The power dissipated in the circuit is

(a) 200 W (b) 400 W (c) 800 W (d) 100 W

Directions (Q. Nos. 27 and 28) *An L-C-R circuit, consists of an inductor, a capacitor and a resistor driven by a battery and connected by two switches S_1 and S_2, as shown in the figure.*



27. At time $t = 0$, the switch S_1 is closed and S_2 is left open. The maximum charge the capacitor plate can hold is q_0 and τ is the time constant of the R-C circuit. Then

(a) at time $t = \tau$, the charge on the capacitor plates is $q = q_0/2$

(b) at $t = 2\tau, q = q_0(1 - e^{-2})$

(c) at $t = 2\tau, q = q_0(1 - e^{-1})$

(d) work done by the battery is half the energy, dissipated in the resistor

28. At time $t = 0$, when the charge on the capacitor plates is q, switch S_1 is opened and S_2 is closed. The maximum charge the capacitor can hold, is q_0. Choose the correct equation

(a) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} + \frac{\pi}{2}\right)$

(b) $q = q_0 \cos\left(\frac{t}{\sqrt{LC}} - \frac{\pi}{2}\right)$

(c) $q = -LC\frac{d^2q}{dt^2}$

(d) $q = -\frac{1}{\sqrt{LC}}\frac{d^2q}{dt^2}$

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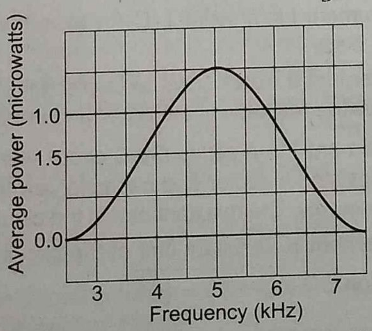
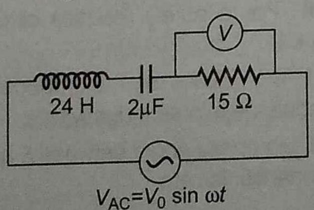
29. The plot given below is of the average power delivered to an L-R-C circuit versus frequency. The quality factor of the circuit is **[JEE Main Online 2013]**



(a) 5.0 (b) 2.0 (c) 2.5 (d) 0.4

30. An L-C-R circuit as shown in the figure is connected to a voltage source V_{AC} whose frequency can be varied. The frequency, at which the voltage across the resistor is maximum, is **[JEE Main Online 2013]**

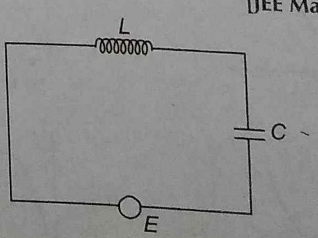


(a) 902 Hz (b) 143 Hz
(c) 23 Hz (d) 345 Hz

31. In a series L-C-R circuit, $C = 10^{-11}$ F, $L = 10^{-5}$ H and $R = 100$ Ω, when a constant DC voltage E is applied to the circuit, the capacitor acquires a charge 10^{-9} C. The DC source is replaced by a sinusoidal voltage source in which the peak voltage E_0 is equal to the constant DC voltage E. At resonance the peak value of the charge acquired by the capacitor will be **[JEE Main Online 2013]**

(a) 10^{-15} C (b) 10^{-6} C
(c) 10^{-10} C (d) 10^{-8} C

32. In the circuit shown here, the voltage across L and C are respectively 300 V and 400 V. The voltage E of the AC source is **[JEE Main Online 2013]**



(a) 400 V (b) 500 V
(c) 100 V (d) 700 V

33. When resonance is produced in a series *L-C-R* circuit, then which of the following is not correct? **[JEE Main Online 2013]**

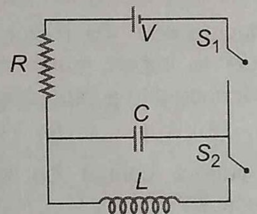
(a) Current in the circuit is in phase with the applied voltage
(b) Inductive and capacitive reactances are equal.
(c) If *R* is reduced, the voltage across capacitor will increase
(d) Impedance of the circuit is maximum

We know that, $\omega_0 = \frac{1}{\sqrt{LC}}$ ...(i)

34. The supply voltage to room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched ON. What is the decrease of voltage across the bulb, when a 240 W heater is switched ON in parallel to the bulb? **[JEE Main 2013]**

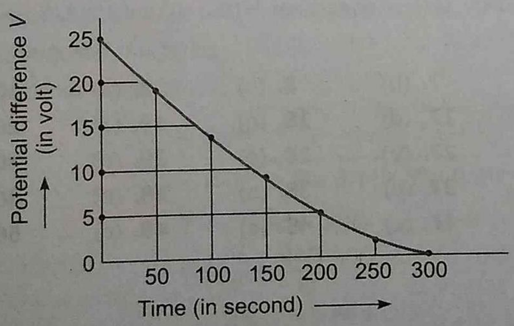
(a) zero (b) 2.9 V (c) 13.3 V (d) 10.04 V

35. In a *L-C-R* circuit as shown below both switches are open initially. Now switch S_1 and S_2, kept open. (*q* is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **[JEE Main 2013]**



(a) Work done by the battery is half of the energy dissipated in the resistor
(b) At $t = \tau, q = \frac{CV}{2}$
(c) At $t = 2\tau, q = CV(1 - e^{-2})$
(d) At $t = \frac{\tau}{2}, q = CV(1 - e^{-1})$

36. The figure shows an experimental plot discharging of a capacitor in an *R-C* circuit. The time constant τ of this circuit lies between **[AIEEE 2012]**



(a) 150 s and 200 s (b) 0 and 50 s
(c) 50 s and 100 s (d) 100 s and 150 s

37. Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse? **[AIEEE 2012]**

(a) Both (b) 100 W
(c) 25 W (d) Neither

38. A fully charged capacitor *C* with initial charge q_0 is connected to a coil of self inductance *L* at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is **[AIEEE 2011]**

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$ (c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

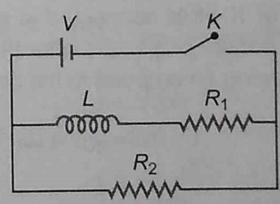
39. A resistor *R* and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of *R* to make the bulb light up 5 s after the switch has been closed ($\log_{10} 2.5 = 0.4$) **[AIEEE 2011]**

(a) $1.7 \times 10^5\,\Omega$ (b) $2.7 \times 10^6\,\Omega$
(c) $3.3 \times 10^7\,\Omega$ (d) $1.3 \times 10^4\,\Omega$

40. Let *C* be the capacitance of a capacitor discharging through a resistor *R*. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then, the ratio $\frac{t_1}{t_2}$ will be **[AIEEE 2010]**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2

41. In the circuit shown below, the key *K* is closed at $t = 0$. The current through the battery is **[AIEEE 2010]**

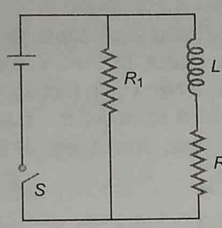


(a) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$
(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$
(d) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

42. In a series *L-C-R* circuit $R = 200\,\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the *L-C-R* circuit is **[AIEEE 2010]**

(a) 305 W (b) 210 W
(c) zero (d) 242 W

43. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 4\,\Omega$ and $R_2 = 2\,\Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[AIEEE 2010]**



(a) $6e^{-5t}$ V
(b) $\frac{12}{t}e^{-3t}$ V
(c) $6(1-e^{-t/0.2})$ V
(d) $12e^{-5t}$ V

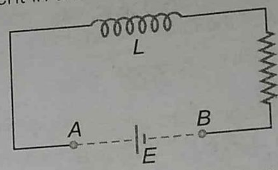
44. In an AC circuit, the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by **[AIEEE 2007]**

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$
(b) $P =$ zero
(c) $P = \frac{E_0 I_0}{2}$
(d) $P = \sqrt{2} E_0 I_0$

45. An ideal coil of 10 H, is connected in series with a resistance of 5 Ω and a battery of 5 V. 2 s after the connection is made, the current flowing (in ampere) in the circuit is **[AIEEE 2007]**

(a) $1-e$
(b) e
(c) e^{-1}
(d) $(1-e^{-1})$

46. An inductor ($L = 100$ mH), a resistor ($R = 100\,\Omega$) and a battery ($E = 100$ V), are initially connected in series as shown in the figure. After short circuiting the points A and B, the current in the circuit, 1 ms after the short **[AIEEE 2006]**



(a) $1/e$ A
(b) e A
(c) 0.1 A
(d) 1 A

47. In a series resonant L-C-R circuit, the voltage across R is 100 V and $R = 1\,\text{k}\Omega$ with $C = 2\,\mu$F. The resonant frequency ω, is 200 rads^{-1}. At resonance, the voltage across L is **[AIEEE 2006]**

(a) 2.5×10^{-2} V
(b) 40 V
(c) 250 V
(d) 4×10^{-3} V

48. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitor of capacitance **[AIEEE 2005]**

(a) 4 μF
(b) 8 μF
(c) 1 μF
(d) 2 μF

49. Alternating current cannot be measured by a DC ammeter, because **[AIEEE 2004]**

(a) AC cannot pass through a DC ammeter
(b) AC changes direction
(c) average value of current, for the complete cycle is zero
(d) DC ammeter gets damaged

50. The inductance between A and D is **[AIEEE 2002]**

(a) 3.66 H
(b) 9 H
(c) 0.66 H
(d) 1 H

Answers

1. (a)	2. (b)	3. (b)	4. (b)	5. (a)	6. (b)	7. (b)	8. (c)	9. (a)	10. (d)
11. (b)	12. (c)	13. (c)	14. (b)	15. (b)	16. (a)	17. (d)	18. (a)	19. (a)	20. (a)
21. (a)	22. (b)	23. (d)	24. (a)	25. (b)	26. (b)	27. (b)	28. (c)	29. (d)	30. (c)
31. (d)	32. (c)	33. (d)	34. (d)	35. (c)	36. (d)	37. (c)	38. (a)	39. (b)	40. (c)
41. (b)	42. (d)	43. (d)	44. (b)	45. (d)	46. (a)	47. (c)	48. (c)	49. (c)	50. (d)

Hints & Solutions

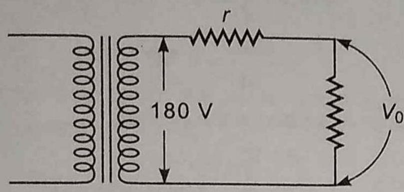
1. At the resonance, $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.14 \times \sqrt{0.25 \times 0.1 \times 10^{-6}}} = 1007\text{ Hz}$$

2. As, $V_2 = \frac{3600}{20} = 180\text{ V}$

$\therefore\ V_0 = V_2 \times \eta = 180 \times 0.9 = 162\text{ V}$

Now, $V_0 = V_2 - i_2 r$



$\therefore$ $$r = \frac{V_2 - V_0}{i} = \frac{18}{20} = 0.9\,\Omega$$

3. As $V = 200\sqrt{2}\sin(100t)$

Hence, $V_0 = 200\sqrt{2}\text{ V}$

or $V_{rms} = 200\text{ V}$ and $\omega = 100\text{ s}^{-1}$

Moreover, $C = 1\,\mu\text{F} = 10^{-6}\text{F}$

$\therefore$ $$I = \frac{V_{rms}}{X_C} = V_{rms} \times C \times \omega$$

$$= 200 \times 10^{-6} \times 100$$

$$= 2 \times 10^{-2}\text{ A} = 20\text{ mA}$$

4. As, $V = \sqrt{V_R^2 + V_L^2} = \sqrt{(20)^2 + (16)^2}$

$$= \sqrt{400 + 256} = \sqrt{656} = 25.6\text{ V}$$

5. Given, inductance $L = 0.01\text{H}$, resistance $R = 1\Omega$, voltage $V = 200\text{V}$ and frequency $f = 50\text{ Hz}$

Impedance of the circuit

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{R^2 + (2\pi fL)^2}$$

$$= \sqrt{1^2 + (2 \times 3.14 \times 50 \times 0.01)^2}$$

or $$Z = \sqrt{10.86} = 3.3\Omega$$

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi fL}{R}$$

$$= \frac{2 \times 3.14 \times 50 \times 0.01}{1} = 3.14$$

$$\phi = \tan^{-1}(3.14) = 72°$$

Phase difference $\phi = \frac{72 \times \pi}{180}$ rad

Time lag between alternating voltage and current

$$\Delta t = \frac{\phi}{\omega} = \frac{72\pi}{180 \times 2\pi \times 50} = \frac{1}{250}\text{ s}$$

6. In an AC circuit only resistor R dissipates energy. L and C do not dissipate energy because for both of them current is wattless ($\phi = 90°$).

7. Here, $V_0 = 100\text{ V}$, $I_0 = 100\text{ A}$ and phase difference, $\phi = \frac{\pi}{3}$

$\therefore$ $$P = V_{rms} \times I_{rms} \times \cos\phi$$

$$= \frac{V_0 I_0}{2}\cos\phi = \frac{100 \times 100}{2} \times \frac{1}{2}$$

$$= 2500\text{ W} = 2.5\text{ kW}$$

8. As, $I = 2I_0 \frac{t}{T_0}$ where, $0 < t < \frac{T_0}{2}$

and $I = 2I_0\left(\frac{t}{T_0} - 1\right)$ where, $\frac{T_0}{2} < t < T_0$

$\therefore$ $$I_{av} = \frac{2}{T}\int_0^{T_0} I\,dt = \frac{2}{T_0}\left[\int_0^{\frac{T}{2}} \frac{2I_0 t}{T_0}\,dt\right] = \frac{2}{T_0^2}\left[\frac{2I_0 T_0^2}{2 \times 4}\right] = \frac{I_0}{2}$$

9. $$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}} = \sqrt{\frac{1^2 + 2^2 + 1^2}{3}} = \sqrt{\frac{6}{3}} = \sqrt{2} = 1.41$$ A

10. We have, $$110 = \frac{V_{app}\,R}{\sqrt{R^2 + L^2\omega^2}}$$

$\Rightarrow$ $$110 = \frac{220R}{\sqrt{R^2 + L^2\omega^2}} \Rightarrow 4R^2 = R^2 + L^2\omega^2$$

$\Rightarrow$ $$L\omega = \sqrt{3}R$$

$\Rightarrow$ $$L(100\pi) = 220\sqrt{3}$$

$\Rightarrow$ $$L = \frac{2.2\sqrt{3}}{3.14} = \frac{2.2 \times (1.732)}{3.14} = 12\text{ H}$$

$\Rightarrow$ $$R = \frac{110 \times 110}{55} = 220\,\Omega$$

11. Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{[R^2 + \omega^2 L^2]^{1/2}}$

12. At cut-off frequency $Z = \sqrt{2}R$

$$R^2 + \left(L\omega - \frac{1}{C\,\omega}\right)^2 = 2R^2$$

$\Rightarrow$ $$L\omega - \frac{1}{C\omega} = R$$

$\Rightarrow$ $$LC\omega^2 - RC\omega - L = 0$$

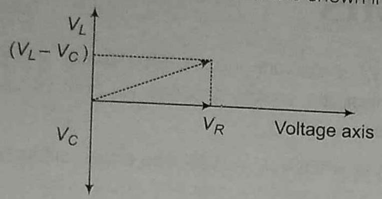
$\Rightarrow$ $$\omega = \frac{RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

$$\Delta\omega = \omega_{01} - \omega_{02}$$

$$= \frac{2\sqrt{R^2C^2 + 4LC}}{2LC}$$

$$= \frac{\sqrt{R^2C^2 + 4LC}}{LC}$$

13. Phaser diagram of R-L-C series circuit is shown in figure.



$$V^2 = V_R^2 + (V_L - V_c)^2$$

14. Given, $V = 200\sqrt{2}\sin(100t)$. Comparing this equation with $V = V_0 \sin \omega t$, we have

$$V_0 = 200\sqrt{2} \text{ V and } \omega = 100 \text{ rad s}^{-1}$$

The current in the capacitor is

$$I = \frac{V_{rms}}{Z_C} = V_{rms} \times \omega C \qquad \left(\because Z_C = \frac{1}{\omega C}\right)$$

$$= \frac{V_0}{\sqrt{2}} \times \omega C = \frac{200\sqrt{2}}{\sqrt{2}} \times 100 \times 1 \times 10^{-6}$$

$$= 20 \times 10^{-3} \text{ A} = 20 \text{ mA}$$

15. When resistance R of the circuit is negligible,

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{10^{-2} \times 25 \times 10^{-6}}}$$

$$= \frac{10^4}{10\pi} = \frac{10^3}{\pi}$$

Thus the time period, $T = \frac{1}{\nu} = \frac{\pi}{10^3} \text{ s} = \pi \text{ ms}$

Thus, for the energy to be completely magnetic,

$$t = \frac{T}{2}, T, \frac{3T}{2}, \ldots\ldots = \frac{\pi}{2}, \pi, \frac{3\pi}{2} \ldots\ldots \text{ ms}$$

$$= 1.57, 3.14, 4.71, \ldots \text{ms}$$

16. Current in the circuit is given by

$$I^2 [700 + 300] = \frac{1}{2} \times 5 \times 10^{-6} \times (200)^2$$

This gives $\quad I^2 = 0.1 \times 10^{-3}$

Hence, $\quad H_1 = I^2 R_1 = 0.07 \text{ J}$

and $\quad H_2 = I^2 R_2 = 0.03 \text{ J}$

17. The current in an L-C-R circuit is given by

$$I = \frac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}, \text{ where } \omega = 2\pi f$$

Thus, I increases with an increase in ω upto a value given by, $\omega = \omega_c$, *i.e.*, at $\omega = \omega_c$, we have

$$\omega L = \frac{1}{\omega C}$$

$\Rightarrow \omega_c = \frac{1}{\sqrt{LC}}$, where I is maximum. At $\omega > \omega_c$. I again starts decreasing with an increase in ω.

18. In series impedance of circuit is

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$= \sqrt{R^2 + (2\pi f L)^2}$$

$$= \sqrt{(220)^2 + (2 \times 3.14 \times 50 \times 0.7)^2}$$

$$= 311 \,\Omega$$

$$\therefore \quad i_{rms} = \frac{V_{rms}}{Z} = \frac{220}{311} = 0.707 \text{ A}$$

$$= 0.707 \text{ A}$$

and $\quad \cos\phi = \frac{R}{Z}$

$$= \frac{220}{311} = 0.707$$

Now, power absorbed in the circuit is

$$P = V_{rms} i_{rms} \cos\phi$$

$$= (220)(0.707)(0.707) \text{ W}$$

$$= 110.08 \text{ W}$$

19. The net reactance of the branch across which voltmeter is connected is zero and hence the voltmeter reading is zero.

20. Resonance in an L-C-R series circuit takes place when the inductive reactance and capacitive reactance are equal and opposite *i.e.*,

$$X_L = X_C \text{ or } \omega_0 L = \omega_0 C$$

or $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

or $\quad f_0 = \frac{1}{2\pi\sqrt{LC}}$

In other words, we can say that at resonant frequency $\left(f_0 = \frac{1}{2\pi\sqrt{LC}}\right)$, resonance can take place.

21. The resultant emf in the L-C-R circuit is given by

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$E = \sqrt{(8)^2 + (16 - 10)^2} = \sqrt{64 + 36}$$

$$E = 10 \text{ V}$$

22. Power dissipated is given by

$$P = E_{rms} I_{rms} \cos\phi$$

We know that for a purely resistive circuit, the power factor $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

Hence, $\quad P = E_{rms} \times I_{rms}$

$$= (RI_{rms}) \times I_{rms} = R(I_{rms})^2$$

$$= R\left(\frac{I_0}{\sqrt{2}}\right)^2$$

$$= \frac{RI_0^2}{2}$$

23. Since, at resonance, $\omega L = \frac{1}{\omega C}$

and impedance $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ or $Z = \sqrt{R^2} = R$

Hence, nature of impedance at resonance is resistive.

Also in a R-L-C circuit, phase angle between the emf and the alternating current I, is zero at resonance.

24. When capacitor is removed, the phase difference θ between the current and the voltage is given by

$$\tan\theta = \frac{\omega L}{R} \text{ or } \omega L = R\tan\theta$$
$$= 100\tan 60^\circ$$

When inductor is removed, the phase difference between the voltage and current is given by

$$\tan\phi = \frac{1}{RC\omega}$$

$\therefore$ $$\frac{1}{C\omega} = R\tan\phi = 100\tan 60^\circ$$

Impedance of the L-C-R circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
$$= \sqrt{R^2 + (100\tan 60^\circ - 100\tan 60^\circ)^2}$$
$$= R = 100\,\Omega$$

25. The current is given by $I = \frac{V}{R} = \frac{200}{100} = 2$ A

26. The power dissipated in the circuit is

$$P = I^2R = 4 \times 100 = 400\text{ W}$$

27. In an R-C circuit, the charge on the capacitor plates at a time t is given by $q = q_0(1 - e^{-t/\tau})$

where, $\tau = RC$ is the time constant. At $t = 2\tau$, we have

$$q = q_0(1 - e^{-2t/\tau})$$
$$= q_0(1 - e^{-2})$$

28. When S_2 is closed and S_1 is open, the charge oscillates in the L-C circuit at an angular frequency is given by

$$\omega = \frac{1}{\sqrt{LC}} \quad \text{...(i)}$$

Now, $q \neq 0$ at $t = 0$. Hence, options (a) and (b) are wrong. The charge q varies with time t as

$$q = q_0\cos(\omega t + \phi) \quad \text{...(ii)}$$

where, ϕ is not equal to $\pi/2$. Differentiating Eq. (ii) twice with respect to t, we get

$$\frac{d^2q}{dt^2} = -\omega^2 q_0\cos(\omega t + \phi) = -\omega^2 q$$
$$q = -\frac{1}{\omega^2}\frac{d^2q}{dt^2}$$
$$= -LC\frac{d^2q}{dt^2} \quad \text{[use Eq. (i)]}$$

29. As quality factor $Q = \frac{\omega_0}{B}$

where, ω_0 = resonant frequency

B = bandwidth

From the graph $B = 2.5$ kHz

$Q = 0.4$ (By observing the curve)

30. As voltage across resistance is maximum, therefore a power is maximum which is at the resonance frequency

At resonance

$$\frac{1}{\text{frequency}} = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2\pi}\frac{1}{\sqrt{24\times 2\times 10^{-6}}} = \frac{1}{2\pi}\frac{1000}{\sqrt{48}}$$
$$= \frac{1}{2\pi}\frac{1000}{6.93} = \frac{1000}{2\times 3.14\times 6.93}$$
$$= \frac{1000}{43.52} = 23\text{ Hz}$$

31. As energy spread in capacitor

$$= \frac{1}{2}CV^2$$

Now, when AC is connected to the circuit energy speed

$$= \frac{1}{2}LI^2$$

By equating the energies, we get

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2$$

$\Rightarrow$ $I = 10$ A

Now, $V = IR = 10 \times 100 = 1000$ V

Therefore, $Q = CV$

$= 10^{-11} \times 1000 = 10^{-8}$ C

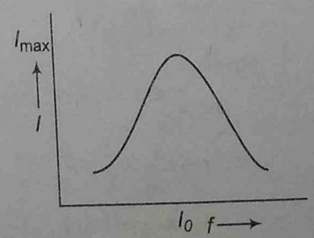
32. Since, reactances produced by inductor and capacitor in opposite direction. So, voltage in these elements are distributed at 180° *i.e.*, out of phase.

Net voltage = 400 V − 300 V = 100 V

33. If f_0 is the resonance frequency corresponding to resonant angular frequency ω_0,

then $f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$ ...(ii)

The Eqs. (i) or (ii) is called the condition for resonance in the L-C-R circuit. Figure shows the variation of current with frequency of the source. It follows that for $f = f_0$, the current in the circuit is maximum and hence, impedence of the circuit is minimum.



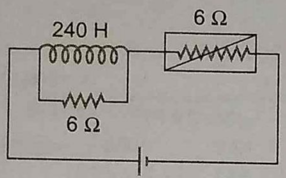
34. $P = \dfrac{V^2}{R} \Rightarrow R = \dfrac{120 \times 120}{60} = 240\ \Omega$

$$R_{eq} = 240 + 6 = 246\ \Omega$$

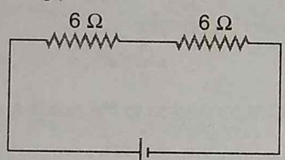
$$\Rightarrow \quad i_1 = \frac{V}{R_{eq}} = \frac{120}{246}$$

$$V_1 = \frac{240}{246} \times 120 = 117.073\ \text{V}$$

$$\Rightarrow \quad i_2 = \frac{V}{R_2} = \frac{120}{54}$$



$$V_2 = \frac{48}{54} \times 120 = 106.66\ \text{V}$$



$$V_1 - V_2 = 10.04\ \text{V}$$

35. For changing of capacitor $q = CV(1 - e^{-t/\tau})$ at $t = 2\tau$, $q = CV(1 - e^{-2})$

36. Time constant τ is the duration when the value of potential drops by 63% of its initial maximum value (*i.e.*, V_0/e). Here, 37% of 25 V = 9.25 V which lies between 100 s to 150 s in the graph.

37. As the rated power of 25 W is less than 100 W, it implies that 25 W bulb has higher resistance. As in series connection, current through both the bulbs is same but heating in 25 W bulb is more than that of 100 W bulb. So, 25 W bulb will get fused.

38. As initially charge is maximum,

$$q = q_0 \cos \omega t$$

$$\Rightarrow \quad i = \frac{dq}{dt} = -\omega q_0 \sin\omega t$$

Given, $\quad \dfrac{1}{2} L i^2 = \dfrac{q^2}{2C}$

$$\Rightarrow \quad \frac{1}{2} L (\omega q_0 \sin \omega t)^2 = \frac{(q_0 \cos\omega t)^2}{2C}$$

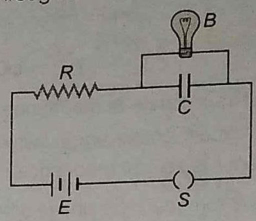
But $\quad \omega = \dfrac{1}{\sqrt{LC}}$

$$\Rightarrow \quad \tan \omega t = 1$$

$$\omega t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$$

39. Neon bulb is filled with gas, so its resistance is infinite, hence, no current flows through it.



Now, $\quad V_C = E(1 - e^{-t/RC})$

$$\Rightarrow \quad 120 = 200(1 - e^{-t/RC})$$

$$\Rightarrow \quad e^{-t/RC} = \frac{2}{5}$$

$$\Rightarrow \quad t = RC \ln 2.5$$

$$\Rightarrow \quad R = \frac{t}{C \ln 2.5} = \frac{t}{2.303\, C \log 2.5} = 2.7 \times 10^6\ \Omega$$

40. As $U = \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2C}(q_0 e^{-t/\tau})^2$

$$= \frac{q_0^2}{2C} e^{-2t/\tau} \text{ (where, } \tau = CR)$$

$$U = U_i e^{-2t/\tau}$$

$$\frac{1}{2} U_i = U_i e^{-2t_1/\tau}$$

$$\frac{1}{2} = e^{-2t_1/\tau}$$

$$\Rightarrow \quad t_1 = \frac{\tau}{2} \ln 2$$

Now, $\quad q = q_0 e^{-t/\tau}$

$$\frac{1}{4} q_0 = q_0 e^{-t_2/\tau}$$

$$t_2 = \tau \ln 4 = 2\tau \ln 2$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{1}{4}$$

41. At $t = 0$, inductor behaves like an infinite resistance

So, at $\quad t = 0,\ i = \dfrac{V}{R_2}$

and at $t = \infty$, inductor behaves like a conducting wire

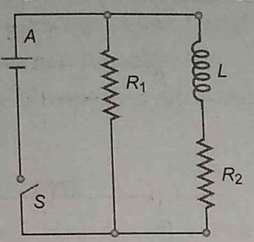
$$i = \frac{V}{R_{eq}} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

42. The given circuit is under resonance as $X_L = X_C$

Hence, power dissipated in the circuit is

$$P = \frac{V^2}{R} = 242\ \text{W}$$

43. $I_1 = \frac{E}{R_1} = \frac{12}{2} = 6\text{ A}$ and $E = L\frac{dI_2}{dt} + R_2 \times I_2$



Now, $I_2 = I_0(1 - e^{-\tau/\tau_c})$

$\Rightarrow \quad I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{ A}$

Also, $t_c = \frac{L}{R} = \frac{400 \times 10^{-3}}{2} = 0.2$

$\therefore \quad I_2 = 6(1 - e^{-t/0.2})$

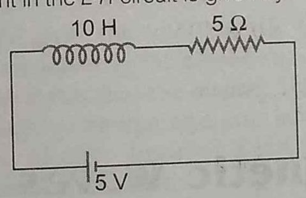
Potential drop across

$L = E - R_2L_2 = 12 - 2 \times 6(1 - e^{-bt})$

$= 12e^{-5t}$

44. For the given circuit, the voltage lags behind by $\frac{\pi}{2}$, so circuit is purely inductive and there is no power consumption in the circuit. The work done by the battery is stored as magnetic energy in the inductor.

45. Rise of current in the L-R circuit is given by



$I = I_0(1 - e^{-t/\tau})$

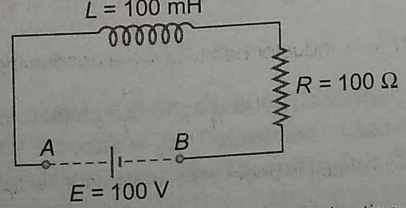
where, $I_0 = \frac{E}{R} = \frac{5}{5} = 1\text{ A}$

Now, $\tau = \frac{L}{R} = \frac{10}{5} = 2\text{ s}$

After 2 s, *i.e.*, at $t = 2$ s

Rise of current $I = (1 - e^{-1})$ A

46. This is a combined example of growth and decay of current in an L-R circuit.

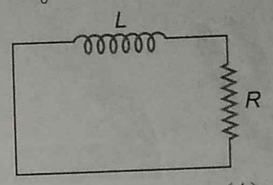


The current through circuit just before shorting the battery,

$I_0 = \frac{E}{R} = 1\text{A}$

[as inductor would be shorted in steady state]

After this, the decay of current starts in the circuit according to the equation $I = I_0e^{-t/\tau}$ where, $\tau = L/R$



$I = 1 \times e^{-(1\times10^{-3})/(100\times10^{-3}/100)} = \left(\frac{1}{e}\right)\text{A}$

47. At resonance $\omega L = \frac{1}{\omega C}$

Current flowing through the circuit, $I = \frac{V_R}{R} = \frac{100}{1000} = 0.1\text{ A}$

So, voltage across L is given by $V_L = IX_L = I\omega L$

but, $\omega L = \frac{1}{\omega C}$

$\therefore \quad V_L = \frac{1}{\omega C} = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250\text{ V}$

48. Given, $L = 10$ H, $f = 50$ Hz

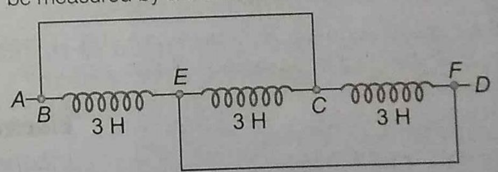
For maximum power $X_C = X_L$

or $\frac{1}{\omega C} = \omega L$ or $C = \frac{1}{\omega^2 L}$

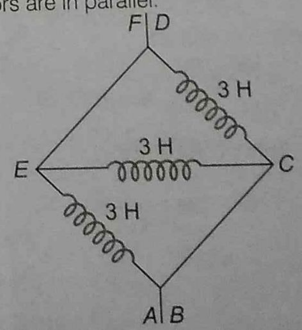
$\therefore \quad C = \frac{1}{4\pi^2 \times 50 \times 50 \times 10} = 0.1 \times 10^{-5}\text{ F} = 1\mu\text{F}$

49. The full cycle of the alternating current consists of two half cycles. For one half, the current is positive and for the second half, the current is negative. Therefore, for an AC cycle, the net value of current averages out to zero. While the DC ammeter, reads the average value. Hence, the alternating current cannot be measured by a DC ammeter.

50.



Here, inductors are in parallel.



$\therefore \quad \frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

or $\quad L = 1\text{H}$

Day 25

Electromagnetic Wave

Day 25

Outlines ...

- Electromagnetic Waves and their Characteristics
- Maxwell's Equations
- Transverse Nature of Electromagnetic Waves
- Spectrum of Electromagnetic Radiation

Electromagnetic Waves and their Characteristics

Electromagnetic waves *are those waves in which electric and magnetic fields vary sinusoidally in space and with time. The electric and magnetic fields are mutually perpendicular to each other and each field is perpendicular to the direction of propagation of the wave.*

(i) Maxwell's theory predicted that electromagnetic waves of all frequenci (and hence all wavelengths) propagate in vacuum, with a speed given

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where μ_0 is the magnetic permeability and ε_0 is the electric permittivity vacuum. Now, for the vacuum, $\mu_0 = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$ a $\varepsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}$. Substituting these values in the above relati we have

$$c = \frac{1}{[(4\pi \times 10^{-7})(8.85 \times 10^{-12})]^{1/2}} \simeq 3.0 \times 10^{8}\,\text{ms}^{-1}$$

(ii) All the electromagnetic waves are of the transverse nature whose speed depends upon the medium but their frequency does not depend on the medium.

(iii) Transverse waves can be polarised.

(iv) Energy is being transported with the electromagnetic waves.

Conduction Current

It is a current in the electric circuit which arises due to the flow of electrons in the connecting wires of the circuit, in a definite closed path. When a capacitor is connected to the battery, it starts storing the charge, due to conduction current. When the capacitor gets fully charged, the conduction current becomes zero in the circuit. Conduction current exists even if the flow of electrons is at uniform rate.

Maxwell's Displacement Current

It is that current which comes into play in the region, whenever the electric field and hence, the electric flux is changing with it.

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

The generalised form of the Ampere's law is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(i + i_d) = \mu_0\left(i + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

Mechanical *and* **Non-mechanical Waves**

Waves requiring a medium to propagate are called **mechanical waves** such as sound waves while the waves do not require a medium to propagate are called the **non-mechanical waves** such as electromagnetic waves.

Maxwell's Equations

Maxwell in 1862, gave the basic laws of electricity and magnetism in the form of four fundamental equations, which are known as Maxwell's equations. In the absence of any dielectric and magnetic material, the Maxwell's equations are based on experimental observations followed by all electromagnetic phenomena, may be stated in the integral form as below

1. **Gauss's law for electrostatics**

   (i) This law gives the total electric flux in terms of charge enclosed by the closed surface.

   (ii) This law states that the electric lines of force start from positive charge and end at negative charge *i.e.*, the electric lines of force do not form a continuous closed path.

   Mathematically, $\oint_s \mathbf{E} \cdot \mathbf{ds} = \frac{q}{\varepsilon_0}$

2. **Gauss's law for magnetism**

   (i) This law shows that the number of magnetic lines of force entering a closed surface is equal to number of magnetic lines of force leaving that closed surface.

   (ii) This law tells that the magnetic lines of force form a continuous closed path.

   (iii) This law also predicts that the isolated magnetic monopole does not exist.

   Mathematically, $\oint_s \mathbf{B} \cdot \mathbf{ds} = 0$

3. **Faraday's law of electromagnetic induction**

   (i) This law gives a relation between electric field and a changing magnetic flux.

   (ii) This law tells that the changing magnetic field is the source of electric field.

   Mathematically, $\oint \mathbf{E} \cdot \mathbf{dl} = -\frac{d\phi_B}{dt}$

4. **Ampere-Maxwell's law**

   (i) This law states that the magnetic field can be produced by a conduction current as well as by displacement current.

   (ii) This law also states that the conduction current and displacement current together have a property of continuity.

   (iii) At an instant, in a circuit, the conduction current is equal to displacement current.

   Mathematically,

$$\oint \mathbf{E} \cdot \mathbf{dl} = \mu_0\left(i_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

These equations are collectively called Maxwell's equations.

Properties of Electromagnetic Waves

If the electromagnetic wave is travelling along the positive direction of the x-axis, the electric field is oscillating parallel to the y-axis and the magnetic field is oscillating parallel to the z-axis

$$E = E_0 \sin(kx - \omega t)$$

$$B = B_0 \sin(kx - \omega t)$$

In this, E_0 and B_0 are the amplitudes of the fields.

Further, $c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ = speed of light in vacuum

The rate of flow of energy in an electromagnetic wave is described by the vector **S** called the Poynting vector, which is defined by the expression,

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

Its magnitude S is related to the rate at which energy is transported by a wave across a unit area at any instant.

Since, **E** and **B** are mutually perpendicular. Hence

$$|\mathbf{E} \times \mathbf{B}| = EB$$

Thus, $$S = \frac{EB}{\mu_0}$$

Further $B = E/c$, therefore, S can also be written as,

$$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$$

The SI unit of Poynting vector **S** is $Js^{-1}m^{-2}$ or Wm^{-2}.

The direction of poynting vector S of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

The time average of S over one cycle is known as the wave intensity. When the average is taken, we obtain an expression involving the time average of $\cos^2(kx - \omega t)$ which equals $\frac{1}{2}$. Thus,

$$I = S_{av} = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{cB_0^2}{2\mu_0}$$

The total average energy per unit volume is,

$$u = u_E + u_B = \frac{\varepsilon_0 E_0^2}{2} = \frac{B^2}{2\mu_0}$$

The radiation pressure p exerted on a perfectly absorbing surface, $p = \frac{S}{c}$

If the surface is a perfect reflector and incidence is normal, then the momentum transported to the surface in a time t is given by, $p = \frac{2U}{c}$ and the radiation pressure will be, $p = \frac{2S}{c}$.

Transverse Nature *of* Eelctromagnetic Waves

According to Maxwell electromagnetic waves conerst of time varying electric and magnetic fields, which are perpendicular to each other, as well as direction of wave propagation.

Let us consider a plane of EM wave travelling in the x-direction and a rectangular parallelopiped *OPQRSUVW* placed with its edges parallel to the three axes. The electric and magnetic fields sinusoidally with x and t only are independent of y and z.

If the rectangular parallelopiped does not enclosed any charge, then according to Gauss's law, the total electric flux across it must be zero *i.e.*, $\oint \mathbf{E} \cdot d\mathbf{s} = 0$.

or $$\int_{PQRS} \mathbf{E} \cdot d\mathbf{s} + \int_{OWVU} \mathbf{E} \cdot d\mathbf{s} + \int_{QWVR} \mathbf{E} \cdot d\mathbf{s}$$

$$+ \int_{OUSP} \mathbf{E} \cdot d\mathbf{s} + \int_{SRVU} \mathbf{E} \cdot d\mathbf{s} + \int_{OPQW} \mathbf{E} \cdot d\mathbf{s} = 0 \quad \text{...(i)}$$

Since electric field E does not depend on y and z.

$$\therefore \int_{QWVR} \mathbf{E} \cdot d\mathbf{s} + \int_{OUSP} \mathbf{E} \cdot d\mathbf{s} = 0 \quad \text{...(ii)}$$

and $$\int_{SRVU} \mathbf{E} \cdot d\mathbf{s} + \int_{OPQW} \mathbf{E} \cdot d\mathbf{s} = 0 \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\int_{PQRS} \mathbf{E} \cdot d\mathbf{s} + \int_{OWVU} \mathbf{E} \cdot d\mathbf{s} = 0 \quad \text{...(iv)}$$

When E_x and E_x' are the x-component of E on faces *PQRS* and *OWVU and s* is the area of each face, then

$$\int_{PQRS} \mathbf{E} \cdot d\mathbf{s} = E_x s \text{ and } \int_{OWVU} \mathbf{E} \cdot d\mathbf{s} = -E_x' s$$

From Eq. (iv), we get

$$E_x s - E_x' s = 0$$

i.e., $$E_x = E_x'$$

or $$E_x = E_x' = 0$$

The possibility, $E_x = E_x'$ predicts that electric field (E) remains the same at different points on x-axis *i.e.*, field is static. But static field cannot propagate a wave of finite wavelength then, $E_x = E_x' = 0$ *i.e.*, no component of E is parallel to the direction of wave propagation. In other words, the electric field is perpendicular to the direcction of propagation of EM wave.

By preceeding the same manner holds time to the magnetic field *i.e.*, the magnetic field also perpendicular to direction of wave propagation since both the electric and magnetic fields are perpendicular to the direction of wave propagation. Therefore, EM waves are transverse in nature.

Spectrum of Electromagnetic Radiation Waves

The array obtained on arranging all the electromagnetic waves in an order on the basis of their wavelength is called the electromagnetic spectrum. The spectrum of electromagnetic radiation has no upper or lower limits and all the regions overlap.

The Electromagnetic Spectrum

Name	Frequency Range (Hz)	Wavelength Range (m)	Source
Radiowaves	10^4 to 10^8	0.1 to 600	Oscillating electric circuits
Microwaves	10^9 to 10^{12}	10^{-3} to 0.3	Oscillating current in special vacuum tubes
Infrared	10^{11} to 5×10^{14}	10^{-6} to 5×10^{-3}	Outer electrons in atoms and molecules
Visible light	4×10^{14} to 7×10^{14}	4×10^{-7} to 8×10^{-7}	Outer electrons in atoms
Ultraviolet	10^{15} to 10^{17}	1.5×10^{-7} to 3.5×10^{-7}	Outer electrons in atoms
X-rays	10^{18} to 10^{20}	10^{-11} to 10^{-8}	Inner electrons in atoms and sudden deceleration of high energy free electrons
Gamma rays	10^{19} to 10^{24}	10^{-16} to 10^{-13}	Nuclei of atoms and sudden deceleration of high energy free electrons

- *The cosmic rays (wavelength range 10^{-13} m to 10^{-17} m), α-particles and β-particles present in space are not the electromagnetic waves, but they are the charged particles.*
- *Visible light is only a very small part of the total electromagnetic spectrum. Electromagnetic waves have a wide range of wavelength (and hence of frequencies). Although they are identical in nature, their interaction with matter or their physiological action on living bodies depends on their frequency.*
- *Infrared rays are thermal radiations which produce heat, X-rays and γ-rays are highly penetrating, to mention only a few of the effects.*

Various Electromagnetic Radiations

Gamma rays *They were discovered by Becquerel and Curie in 1896. Their wavelength is of the order of 10^{-14} to 10^{-10} m. The main sources are the natural and artificial radioactive substances. These rays affect the photographic plate. These rays are mainly used in the treatment of cancer disease.*

X-rays

They were discovered by Roentgen in 1895. Their wavelength is of the order of 10^{-12} m to 10^{-8} m. X-rays are produced when highly energetic cathode rays are stopped by a metal target of high melting point. They affect the photographic plate and can penetrate through the transparent materials.

They are mainly used in detecting the fracture of bones, hidden bullet, needle, costly material, etc., inside the body and also used in the study of crystal structure.

Ultraviolet Rays

They were discovered by Ritter in 1801. Their wavelength is of the order 10^{-9} m to 4×10^{-7} m. In the radiations received from sun, major part is that of the ultraviolet radiation. Its other sources are the electric discharge tube, carbon arc etc.

These radiations are mainly used in excitation of photoelectric effect and to kill the bacteria of many diseases.

Visible Light

This was first studied in 1666 by Newton. The radiations in the range of wavelength from 4×10^{-7} m to 7×10^{-7} m fall in the visible region.

The wavelength of the light of violet colour is the shortest and that of red colour is the longest. Visible light is obtained from the glowing bodies, while they are white hot. The light obtained from the electric bulbs, sodium lamp, fluorescent tube is the visible light.

Thermal *or* Infrared Waves

They were discovered by Herchell in 1800. Their wavelength is of the order of 7×10^{-7} m to 10^{-3} m. A body on being heated, emits out the infrared waves. These radiations have the maximum heating effect. The glass absorbs these radiations, therefore for the study of these radiations rock salt prism is used instead of a glass prism. These waves are mainly used for therapeutic purpose by the doctors because of their heating effect.

Microwaves

They were discovered by Hertz in 1888. Their wavelength is in the range of nearly 10^{-4} m to 1 m. These waves are produced by the spark discharge or magnetron valve. They are detected by the crystal or semiconductor detector. These waves are used mainly in radar and long distance communication.

Radiowaves

They were first discovered in 1895 by Marconi. Their wavelength is in the range of 0.1m to 10^5m. They can be obtained by the flow of high frequency alternating current in an electric conductor.

These waves are detected by the tank circuit in a radio receiver or transmitter.

Applications of Electromagnetic Spectrum

The different regions of the total electromagnetic spectrum have been put to the following uses

- Radiowaves are used in radar and radio broadcasting.
- Microwaves are used in long distance wireless communications *via* satellites.
- Infrared, visible and ultraviolet radiations are used to know the structure of molecules.
- Diffraction of X-rays by crystals gives the details of the structure of crystals.
- The bones are opaque to X-rays but flesh is transparent. X-ray pictures of a human body are used in medical diagnosis of fractures and cracks of bones.
- The γ-rays are used in the study of the structure of the nuclei of atoms.

Practice Zone DAY 25

1. The magnetic field in a plane electromagnetic wave is given by $B_y = (2 \times 10^{-7}\ \text{T})\ \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$. The frequency of the wave is

(a) 2.38×10^{10} Hz (b) 1.5×10^{11} Hz
(c) 2.38×10 Hz (d) 1.5×10 Hz

2. A plane electromagnetic wave propagating in the x-direction has a wavelength of 6.0 mm. The electric field is in the y-direction and its maximum magnitude is $33\,\text{Vm}^{-1}$. The equation for the electric field as function of x and t is

(a) $11 \sin\pi(t - x/c)$ (b) $33 \sin\pi \times 10^{11}(t - x/c)$
(c) $33 \sin\pi(t - x/c)$ (d) $11 \sin\pi \times 10^{11}(t - x/c)$

3. An electric field **E** and a magnetic field **B** exist in a region. If these fields are not perpendicular to each other, then the electromagnetic wave

(a) will not pass through the region
(b) will pass through the region
(c) may pass through the region
(d) Nothing is definite

4. A large parallel plate capacitor, whose plates have an area of $1\ \text{m}^2$ and are separated from each other by 1 mm, is being charged at a rate of $25\ \text{Vs}^{-1}$. If the dielectric between the plates has the dielectric constant 10, then the displacement current at this instant is

(a) $25\ \mu$A (b) $11\ \mu$A
(c) $2.2\ \mu$A (d) $1.1\ \mu$A

5. The magnetic field of a beam emerging from a filter facing a flood light is given by $B = 12 \times 10^8 \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$ T. What is the average intensity of the beam? **[NCERT Exemplar]**

(a) $1.7\ \text{W/m}^2$ (b) $2.3\ \text{W/m}^2$
(c) $2.7\ \text{W/m}^2$ (d) $3.2\ \text{W/m}^2$

6. A parallel plate capacitor with plate area A and separation between the plates d, is charged by a constant current i. Consider a plane surface of area $A/2$ parallel to the plates and drawn simultaneously between the plates. The displacement current through this area is

(a) i (b) $\frac{i}{2}$
(c) $\frac{i}{4}$ (d) $\frac{i}{8}$

7. An FM radio station antenna radiates a power of 10 kW at a wavelength of 3 m. Assume the radiated power is confined to and is uniform over a hemisphere with antenna at its centre. E_{max} at a distance of 10 km from antenna is

(a) $0.62\ \text{NC}^{-1}$ (b) $0.41\ \text{NC}^{-1}$
(c) $0.31\ \text{NC}^{-1}$ (d) $0.11\ \text{NC}^{-1}$

8. You are given a $2\ \mu$F parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates? **[NCERT Exemplar]**

(a) By applying a varying potential difference of 500V/s
(b) By applying a varying potential difference of 400V/s
(c) By applying a varying potential difference of 100V/s
(d) By applying a varying potential difference of 300V/s

9. Consider the following two statements regarding a linearly plane polarised electromagnetic wave.

(i) Electric field and the magnetic field have equal average values.
(ii) Electric energy and the magnetic energy have equal average values.

(a) (i) is true (b) (ii) is true
(c) Both are true (d) Both are false

10. A perfectly reflecting mirror has an area of $1\ \text{cm}^2$. Light energy is allowed to fall on it for 1 h at the rate of $10\ \text{W cm}^{-2}$. The force acting on the mirror is

(a) 6.7×10^{-8} N
(b) 2.3×10^{-4} N
(c) 10^{-3} N
(d) zero

11. Instantaneous displacement current of 1.0 A in the space between the parallel plates of a $1\ \mu$F capacitor, can be established by changing potential difference of

(a) $10^{-6}\ \text{Vs}^{-1}$ (b) $10^6\ \text{Vs}^{-1}$
(c) $10^{-8}\ \text{Vs}^{-1}$ (d) $10^8\ \text{Vs}^{-1}$

12. The magnetic field between the plates of radius 12 cm, separated by a distance of 4 mm of a parallel plate capacitor of capacitance 100 pF along the axis of plates having conduction current of 0.15 A, is

(a) zero (b) 1.5 T
(c) 15 T (d) 0.15 T

13. In an apparatus, the electric field was found to oscillate with an amplitude of 18 Vm^{-1}. The magnitude of the oscillating magnetic field will be

(a) 4×10^{-6} T (b) 6×10^{-8} T (c) 9×10^{-9} T (d) 11×10^{-11} T

14. A fire screen produces sensation of cooling as

(a) it allows both infrared and visible light but cuts off ultraviolet
(b) it allows infrared and cuts off shorter wavelengths
(c) it cuts off both visible light and infrared
(d) it allows only visible light and cuts off infrared

15. The radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is **[NCERT Exemplar]**

(a) I/C (b) $2I/C$ (c) $I/2C$ (d) I^2/C

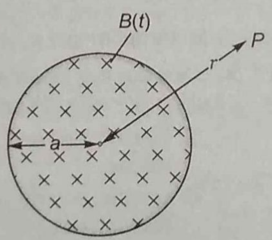
16. The magnetic field at a point between the plates of a capacitor at a perpendicular distance R from the axis of the capacitor plate radius R, having the displacement current I_D is given by

(a) $\frac{\mu I_D r}{2\pi R^2}$ (b) $\frac{\mu_0 I_D}{2\pi R}$ (c) $\frac{\mu_0 I_D}{\pi r^2}$ (d) zero

17. The speed of an electromagnetic wave is same for

(a) odd frequencies (b) even frequencies
(c) all frequencies (d) relatively high frequencies

18. A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in the figure. The magnitude of the induced electric field at a point P, a distance r from the centre of the circular region



(a) increases with r (b) decreases with r
(c) decreases as $\frac{1}{r^2}$ (d) zero

19. Assume that all the energy from a 1000 W lamp is radiated uniformly, then the amplitude of electric field of radiation at a distance of 2m from the lamp is

(a) 244.89 V/m (b) 17 V/m (c) 0 (d) 2.96 V/m

20. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$; $B = B_0 \sin(kx - \omega t)$ Which of the following equations is true?

(a) $E_0 k = B_0 \omega$ (b) $E_0 \omega = B_0 k$
(c) $E_0 B_0 = \omega k$ (d) None of these

Directions *(Q. Nos. 21 to 24) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

21. Statement I Electromagnetic waves exert radiation pressure.

Statement II Electromagnetic waves carry energy.

22. Statement I Light is a transverse wave but not an electromagnetic wave.

Statement II Maxwell showed that speed of electromagnetic waves is related to the permeability and the permittivity of the medium through which it travels.

23. Statement I For cooking in a microwave oven, food is always kept in a metal container.

Statement II The energy of microwave cannot be easily transferred to the food in a metal container.

24. Statement I X-rays astronomy is possible only from satellites orbiting the earth.

Statement II Efficiency of X-rays telescope is large as compared to any other telescope.

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25. This question has statement I and Statement II. Of the four choices given after the Statement, choose the one that best describes the two Statements.

Statement I Out of radio waves and microwaves, the radio waves undergo more diffraction.

Statement II Radio waves have greater frequency compared to microwaves. **[JEE Main Online 2013]**

(a) Statement I is true, Statement II is true and Statement II is the correct explanation of Statement I
(b) Statement I is false, Statement II is true
(c) Statement I is true, Statement II is false
(d) Statement I is true, Statement II is true but Statement II is not the correct explanation of Statement II

26. In a transverse wave the distance between a crest and neighbouring through at the same instant is 4.0 cm and the distance between a crest and trough at the same place is 1.0 cm. The next crest appears at the same place after a time interval of 0.4 s. The maximum speed of the vibrating particles in the medium is **[JEE Main Online 2013]**

(a) $\frac{3\pi}{2}$ cm/s (b) 5π cm/s (c) $\frac{\pi}{2}$ cm/s (d) 2π cm/s

27. Select the correct statement from the following **[JEE Main Online 2013]**

(a) Electromagnetic waves cannot travel in vacuum
(b) Electromagnetic waves are longitudinal waves
(c) Electromagnetic waves are produced by charges moving with uniform velocity
(d) Electromagnetic waves carry both energy and momentum as they propagate through space

28. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is **[JEE Main 2013]**

(a) 3 V/m (b) 6 V/m (c) 9 V/m (d) 12 V/m

29. An electromagnetic wave in vacuum has the electric and magnetic fields **E** and **B**, which are always perpendicular to each other. The direction of polarisation is given by **X** and that of wave propagation by **k**. Then, **[AIEEE 2012]**

(a) **X** || **B** and **K** || **B**×**E** (b) **X** || **E** and **K** || **E**×**B**
(c) **X** || **B** and **K** || **E**×**B** (d) **X** || **B** and **K** || **B**×**E**

30. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\ \text{cm}^2$ and length $= 20$ cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu = 4\pi \times 10^{-7}\ \text{Tm A}^{-1}$) **[AIEEE 2008]**

(a) $2.4\pi \times 10^{-5}$ H (b) $4.8\pi \times 10^{-4}$ H
(c) $4.8\pi \times 10^{-5}$ H (d) $2.4\pi \times 10^{-4}$ H

31. The rms value of the electric field of light coming from the sun is 720 NC^{-1}. The average total energy density of the electromagnetic wave is **[AIEEE 2006]**

(a) $4.58 \times 10^{-6}\ \text{Jm}^{-3}$ (b) $6.37 \times 10^{-9}\ \text{Jm}^{-3}$
(c) $81.35 \times 10^{-12}\ \text{Jm}^{-3}$ (d) $3.3 \times 10^{-3}\ \text{Jm}^{-3}$

32. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$. Then, **[AIEEE 2004]**

(a) wavelength is doubled and the frequency remains unchanged
(b) wavelength is doubled and the frequency becomes half
(c) wavelength is halved and the frequency remains unchanged
(d) wavelength and frequency both remain unchanged

33. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is **[AIEEE 2004]**

(a) $\frac{E}{c}$ (b) $\frac{2E}{c}$ (c) Ec (d) $\frac{E}{c^2}$

34. Which of the following radiations has the least wavelength? **[AIEEE 2003]**

(a) γ-rays (b) β-rays
(c) α-rays (d) X-rays

35. Which of the following are not electromagnetic waves? **[AIEEE 2002]**

(a) Cosmic-rays (b) γ-rays
(c) Ultraviolet-rays (d) X-rays

Answers

1. (a)	2. (b)	3. (c)	4. (c)	5. (a)	6. (b)	7. (d)	8. (a)	9. (c)	10. (a)
11. (b)	12. (a)	13. (b)	14. (d)	15. (a)	16. (b)	17. (d)	18. (b)	19. (a)	20. (a)
21. (b)	22. (d)	23. (d)	24. (c)	25. (c)	26. (b)	27. (d)	28. (b)	29. (b)	30. (a)
31. (a)	32. (c)	33. (b)	34. (a)	35. (c)					

Hints & Solutions

1. Comparing the given equation with standard equation of electromagnetic wave, $B = B_0 \sin(kx + \omega t)$ we have

$$\omega = 2\pi f = 1.5 \times 10^{11}$$

$$\therefore \quad f = \frac{1.5 \times 10^{11}}{2\pi} = 2.38 \times 10^{10}\ \text{Hz}$$

2. As, $\omega = 2\pi\nu = \frac{2\pi c}{\lambda} = \frac{2\pi \times 3 \times 10^8}{6 \times 10^{-3}}$

$$= \pi \times 10^{11}\ \text{rads}^{-1}$$

The equation for the electric field, along y-axis, of the electromagnetic wave is

$$E_y = E_0 \sin\omega\left(t - \frac{x}{c}\right) = 33 \sin\pi \times 10^{11}(t - x/c)$$

3. The electromagnetic wave being packets of energy, moving with speed of light, may pass through the region.

4. As, $C = \frac{\varepsilon_0 KA}{d} = \frac{(8.85 \times 10^{-12}) \times 10 \times 1}{10^{-3}} = 8.85 \times 10^{-8}$ F

$$\therefore i = \frac{d}{dt}(CV) = C\frac{dV}{dt} = 8.85 \times 10^{-8} \times 25 = 2.2 \times 10^{-6} = 2.2\ \mu\text{A}$$

5. Magnetic field $\mathbf{B} = B_0 \sin\omega t$

Given, equation $B = 12 \times 10^8 \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$ T

On comparing this equation with standard equation, we get

$$B_0 = 12 \times 10^{-8}$$

The average intensity of the beam, $I_{avg} = \frac{1}{2}\frac{B_0^2}{\mu_0} \cdot c$

$$= \frac{1}{2} \times \frac{(12 \times 10^{-8})^2 \times 3 \times 10^{-8}}{4\pi \times 10^{-7}} = 1.71\ \text{W/m}^2$$

6. Charge on the capacitor plates, at time t is, $q = it$

Electric field between the plates at this instant,

$$E = \frac{q}{A\varepsilon_0} = \frac{it}{A\varepsilon_0}$$

Electric flux through the given area

$$\phi_E = \left(\frac{A}{2}\right)E = \frac{it}{2\varepsilon_0}$$

Therefore, displacement current

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{it}{2\varepsilon_0}\right) = \frac{i}{2}\frac{\varepsilon_0}{\varepsilon_0} = \frac{i}{2}$$

7. As, $I = \frac{\rho}{2\pi r^2} = \frac{10^4}{2\pi(10 \times 10^3)^2} = \frac{10^{-4}}{2\pi}$

and $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$\Rightarrow$ $E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$

$= \sqrt{\frac{10^{-4}}{\pi \times 8.85 \times 10^{-12} \times 3 \times 10^8}}$

or, $E_0 = 0.11\ \text{NC}^{-1}$

8. Given, capacitance of capacitor $C = 2\mu\text{F}$,

Displacement current $I_d = 1\text{mA}$

Charge $q = CV$

$I_d dt = CdV$ $\quad [\because q = it]$

or $I_d = C\frac{dV}{dt}$

$1 \times 10^{-3} = 2 \times 10^{-6} \times \frac{dV}{dt}$

or $\frac{dV}{dt} = \frac{1}{2} \times 10^{+3} = 500\ \text{V/s}$

Clearly, by applying a varying potential difference of 500 V/s, we would produce a displacement current of desired value.

10. Let E = energy falling on the surface per second = 10 J

Momentum of photons $p = \frac{h}{\lambda} = \frac{h}{c/f}$

$$\frac{h\nu}{c} = \frac{E}{c}$$

On reflection, change in momentum per second = force

$$= 2p = \frac{2E}{c} = \frac{2 \times 10}{3 \times 10^8} = 6.7 \times 10^{-8}\ \text{N}$$

11. As, $i_d = C\left(\frac{V}{t}\right)$ $\quad \left(\because i = \frac{dQ}{dt}\right)$

or $\frac{V}{t} = \frac{i_d}{C} = \frac{1.0}{10^{-6}} = 10^6\ \text{Vs}^{-1}$

12. As $B \propto r$, since the point is on the axis, where $r = 0$, so $B = 0$.

13. Here, $E_0 = 18\ \text{Vm}^{-1}$,

$\therefore$ $B_0 = \frac{E_0}{c} = \frac{18}{3 \times 10^8} = 6 \times 10^{-8}\ \text{T}$

14. Infrared radiation produces heating. A fire screen does not allow the infrared radiation but allows the visible light, therefore, we can see the fire.

15. Pressure $= \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

Force is the rate of change of momentum

i.e., $F = \frac{dp}{dt}$

Energy in time dt, $U = p \cdot C$ or $p = \frac{U}{C}$

$\therefore$ Pressure $= \frac{1}{A} \cdot \frac{U}{C \cdot dt}$

Pressure $= \frac{I}{C}$ $\quad \left[\because I = \text{Intensity} = \frac{U}{A \cdot dt}\right]$

16. According to the Ampere-Maxwell's law, for a closed surface

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_D$$

as $B(2\pi R) = \mu I_D \Rightarrow B = \frac{\mu I_D}{2\pi R}$

17. The speed of the electromagnetic wave is same for all frequencies.

18. A time varying magnetic field produces an electric field. The magnitude of the electric field at a distance r from the centre of a circular region of radius a, where a time varying field B exists, is given by

$$E = \frac{a^2}{2r}\frac{dB}{dt}$$

At $r = a$ $\quad E = \left(\frac{a}{2}\right)\frac{dB}{dt}$,

This is the value of E at the edge of the circular region. For $r > a$, E decreases with r.

19. Poynting vector

$$S = E \times H = EH\sin 90^\circ = EH$$

Energy of lamp $= \frac{1000\ \text{W}}{\pi r^2} = \frac{1000}{\pi \times 2^2}\ \text{Jm}^{-2}\ \text{s}^{-1}$

S represents energy flow per unit area per second, we have,

$$EH = \frac{1000}{\pi \times 2^2} = 79.56$$

$$\frac{E}{H} = 377$$

$$EH \times \frac{E}{H} = 79.56 \times 377 = 29996.8$$

$\Rightarrow$ $E = \sqrt{29996.8} = 173.19\ \text{V/m}$

Amplitude of electric field of radiation is

$$E_0 = E\sqrt{2} = 244.89\ \text{V/m}$$

20. We know $E_0 = cB_0$, where c is the velocity of light.

$\therefore$ $c = \nu\lambda = \frac{\omega}{2\pi}\lambda = \frac{\omega}{k}$

Thus, $E_0 = cB_0 = \frac{\omega}{k}B_0$ or $E_0 k = B_0 \omega$

21. Electromagnetic waves have linear momentum as well as energy. From this we conclude that we can exert radiation pressure by making a beam of electromagnetic radiation fall on an object. Let us assume that object is free to move and that the radiation is entirely absorbed in the object during time interval Δt. The object gains an energy ΔU from the radiation.

Maxwell showed that the object also gains linear momentum, the magnitude Δp of the change in momentum of the object is related to the energy change ΔU as

$$\Delta p = \frac{\Delta U}{c} \quad \text{(total absorption)}$$

22. In free space or vacuum, the speed of electromagnetic waves is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \dots\text{(i)}$$

Here, $\mu_0 = 4\pi \times 10^{-7}\ \text{Ns}^2\text{C}^{-2}$ is permeability (constant) of free space.

$\varepsilon_0 = 8.85418 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$ is the permittivity of free space.

Substituting the values in Eq. (i), we have

$$c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85418 \times 10^{-12}}}$$

$$= 2.99792 \times 10^8\ \text{ms}^{-1}$$

This is same as the speed of light in vacuum.

From this we conclude that light is an electromagnetic wave.

23. The atoms of the metallic container are set into forced vibrations by microwaves. Hence, energy of the microwaves is not efficiently transferred to the metallic container. Therefore, the food in the metallic containers cannot be cooked in a microwave oven. Normally in a microwave oven, the energy of waves is converted into the kinetic energy of the molecules. This results in raising the temperature of any food.

24. The earth's atmosphere is transparent to visible light and radio waves, but absorbs X-rays. Thus, X-ray telescope cannot be used on the surface of the earth.

25. The frequency of radiowaves less than the frequency of microwaves.

$\because$ Frequency of radiowaves $= 3 \times 10^8$ Hz

and frequency of microwaves $= 10^{10}$ Hz

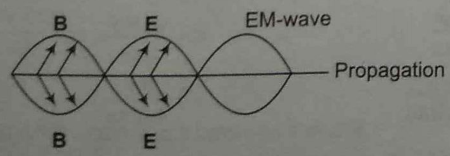
$\therefore \quad v_{\text{radiowaves}} < v_{\text{microwaves}}$

26. Given, $\frac{\lambda}{4} = 4\ \text{cm} \therefore \lambda = 16$ cm and $T = 0.4$ s

As, $f\lambda \times T = 2\pi \Rightarrow f = \frac{2\pi}{16 \times 0.4} = \frac{5\pi}{16} s^{-1}$

Now, $v = f\lambda = \frac{5\pi}{8} \times 16 = 5\pi$ cm/s

27.



As electromagnetic waves contains both electric field and magnetic field. It carry both energy and momentum according to de-Broglie, wave particle duality of radiations.

28. $\mathbf{E} = \mathbf{B} \times c$. Keep value of electric field

$$|\mathbf{E}| = |\mathbf{B}||c| = 20 \times 10^{-9} \times 3 \times 10^8 = 6\ \text{V/m}$$

29. In electromagnetic wave, the direction of porpagation of wave, electric field and magnetic field are mutually perpendicular, *i.e.*, wave propagates perpendicular to **E** and **B** or along $\mathbf{E} \times \mathbf{B}$. While polarization of wave takes place parallel to electric field vector.

30. As, $M = \frac{\mu_0 N_1 N_2 A}{l} = 2.4\pi \times 10^{-4}$ H

31. Total average energy $= \varepsilon_0 E_{\text{rms}}^2 \quad \left(\because E_{\text{rms}} = \frac{E_0}{\sqrt{2}}\right)$

$$= 8.85 \times 10^{-12} \times (720)^2$$

$$= 4.58 \times 10^{-6}\ \text{Jm}^{-3}$$

32. In vacuum, $\varepsilon_0 = 1$

In medium, $\varepsilon = 4$

$\therefore$ Refractive index, $\mu = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\frac{4}{1}} = 2$

Wavelength, $\quad \lambda' = \frac{\lambda}{\mu} = \frac{\lambda}{2}$

and wave velocity, $v = \frac{c}{\mu} = \frac{c}{2}$

Hence, it is clear that wavelength and velocity become half but frequency remains unchanged when the wave passes through any medium.

33. Initial momentum of the surface,

$$p_i = -\frac{E}{c}$$

where, c = velocity of light (constant).

Since, the surface is perfectly reflecting, so the same momentum will be reflected completely. Final momentum

$$p_y = -\frac{E}{c}$$

$\therefore$ Change in momentum

$$\Delta p = p_f - p_i = -\frac{E}{c} - \frac{E}{c}$$

$$= -\frac{2E}{c}$$

Thus, momentum transferred to the surface is

$$\Delta p' = |\Delta p| = \frac{2E}{c}$$

34. γ has least wavelength and maximum frequency.

35. Cosmic rays are flow of high energy changed particles and is not a part of electromagnetic wave.

Unit Test 5

DAY 26

(Magnetostatics, EMI & AC, EM Wave)

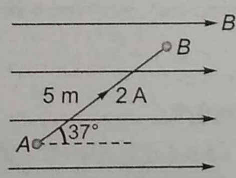
1. A solenoid of some length and radius 2 cm has a layer of winding. A 2cm long wire of mass 5 g lies inside the solenoid along the axis of solenoid. The wire is connected to some external circuit so that a current of 5 A flows through the wire. The value of current to be in the winding so that magnetic force supports the wire weight is(Take $g = 10\,\text{ms}^{-2}$)

(a) zero
(b) $\simeq 400$A
(c) $\simeq 32000$A
(d) Not possible

2. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.1 T normal to the plane of the coil. The coil carries a current of 5 A. The coil is made up of copper wire of cross-sectional area $10^{-5}\,\text{m}^2$ and the number of free electrons per unit volume of copper is 10^{29}. The average force experienced by an electron in the coil due to magnetic field is

(a) 5×10^{-25}N
(b) zero
(c) 8×10^{-24}N
(d) None of these

3. A wire *AB* of length 5 m carrying a current of 2A is placed in a region of uniform magnetic field $B = 0.5$ T as shown in figure. The magnetic force experienced by wire is



(a) 5 N
(b) 4 N
(c) 3 N
(d) 8 N

4. A coil of metal wire is kept stationary in a non-uniform magnetic field, then

(a) an emf and current both induced in the coil
(b) a current but no emf is induced in the coil
(c) an emf but no current is induced in the coil
(d) Neither emf nor current is induced in the coil

5. A thin circular ring of area *A* is held perpendicular to a uniform magnetic field of induction *B*. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is *R*. When the ring is suddenly squeesed to zero area, the charge flowing through the galvanometer is

(a) $\frac{BR}{A}$
(b) $\frac{AB}{R}$
(c) ABR
(d) $\frac{B^2A}{R^2}$

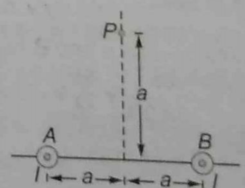
6. A particle having a mass of 0.5 g carries a charge of 2.5×10^{-8} C. The particle is given an initial horizontal velocity of $6\times 10^4\,\text{ms}^{-1}$. The minimum magnitude of the magnetic field that is required so that, particle will keep moving in a horizontal direction is [Take $g = 10\,\text{ms}^{-2}$]

(a) 8.46 T
(b) 3.33 T
(c) 8.64×10^{-2} T
(d) None of these

7. A bar magnet of pole strength *m* and magnetic moment *M* is divided into two equal pieces by cutting it along its length. Then,

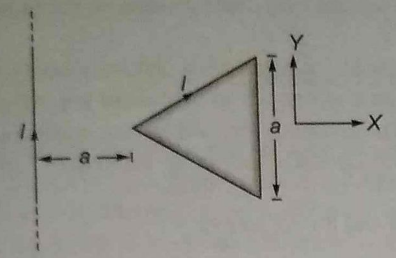
(a) *m* is halved and *M* is doubled
(b) *m* and *M* both are halved
(c) *m* is halved but *M* remains the same
(d) *m* remains the same but *M* is halved

8. Two infinite long current carrying wires *A* and *B* are placed as shown in figure. Each wire carries same current *I*. The resultant magnetic field intensity at point *P* is



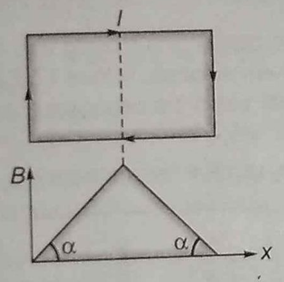
(a) $\frac{\mu_0 I}{2\pi a}$
(b) $\frac{\sqrt{2}\mu_0 I}{2\pi a}$
(c) $\frac{\mu_0 I}{2\sqrt{2}\pi a}$
(d) $\frac{\mu_0 I}{4\sqrt{2}\pi a}$

9. An equilateral triangular loop is kept near to a current carrying long wire as shown in figure. Under the action of magnetic force alone, the loop



(a) must move along X-axis
(b) must move in XY-plane and not along X or Y-axis
(c) loop does not move
(d) must move along Y-axis

10. A current carrying loop is placed in the non-uniform magnetic field whose variation in space is shown in figure. Direction of magnetic field is into the plane of paper. The magnetic force experienced by loop is



(a) non-zero
(b) zero
(c) can't say anything
(d) None of these

11. A magnetic field
(a) always exerts a force on a charged particle
(b) never exerts a force on a charged particle
(c) exerts a force if the charged particle is moving along the magnetic field lines
(d) exerts a force if the charged particle is moving inclined to the magnetic field lines

12. An electron is launched with velocity **v** in a uniform magnetic field **B**. The angle θ between **v** and **B** lies between 0 and $\frac{\pi}{2}$. Its velocity vector **v** returns to its initial value in a time interval of

(a) $\frac{2\pi m}{eB}$
(b) $\frac{2 \times 2\pi m}{eB}$
(c) $\frac{\pi m}{eB}$
(d) depends upon angle between **v** and **B**

13. An electric current I enters and leaves a uniform circular wire of radius a through diametrically opposite points. A charged particle q moving along the axis of the circular wire passes through its centre at speed v. The magnetic force acting on the particle when it passes through the centre has a magnitude

(a) $qv \times \frac{\mu_0 I}{2a}$ (b) $qv \times \frac{\mu_0 I}{2\pi a}$ (c) $qv \times \frac{\mu_0 I}{a}$ (d) zero

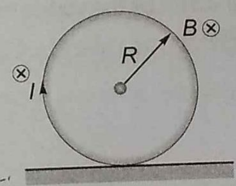
14. A wire in the form of a circular loop of radius r lies with its plane normal to a magnetic field B. If the wire is pulled to take a square shape in the same plane in time t, the emf induced in the loop is given by

(a) $\frac{\pi B r^2}{t}\left(1-\frac{\pi}{10}\right)$
(b) $\frac{\pi B r^2}{t}\left(1-\frac{\pi}{8}\right)$
(c) $\frac{\pi B r^2}{t}\left(1-\frac{\pi}{6}\right)$
(d) $\frac{\pi B r^2}{t}\left(1-\frac{\pi}{4}\right)$

15. The mutual inductance between two planar concentric rings of radii r_1 and r_2 (with $r_1 > r_2$) placed in air is given by

(a) $\frac{\mu_0 \pi r_2^2}{2r_1}$
(b) $\frac{\mu_0 \pi r_1^2}{2r_2}$
(c) $\frac{\mu_0 \pi (r_1 + r_2)^2}{2r_1}$
(d) $\frac{\mu_0 \pi (r_1 + r_2)^2}{2r_2}$

16. A conducting loop is placed in a magnetic field (uniform) as shown in the figure. For this situation mark the correct statement.



(a) The force of compression experienced by loop is IRB
(b) The force of compression experienced by loop is $2\pi IRB$
(c) The force of expansion experienced by loop is IRB
(d) The force of expansion experienced by loop is $2\pi IRB$

17. Mark the correct statement.
(a) Ideal inductor does not dissipate power in an AC circuit
(b) Ideal inductor dissipates maximum power in an AC circuit
(c) In an inductor, current lags behind the voltage by π
(d) In inductor, current leads voltage by π

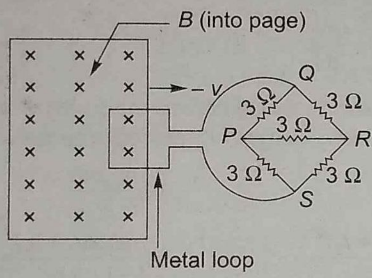
18. Two very long straight parallel wires carry steady currents I and $-I$, respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the wires in the plane of the wires. Its instantaneous velocity v is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(a) $\frac{\mu_0 Iqv}{2\pi d}$ (b) $\frac{\mu_0 Iqv}{\pi d}$ (c) $\frac{2\mu_0 Iqv}{\pi d}$ (d) zero

19. An aeroplane is moving north horizontally, with a speed of 200 ms^{-1}, at a place where the vertical component of the earth's magnetic field is 0.5×10^{-4}T. What is the induced emf set up between the tips of the wings if they are 10 m apart?

(a) 0.01 V (b) 0.1 V
(c) 1 V (d) 10 V

20. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v in a uniform magnetic field $B = 2$ T as shown in figure. The magnetic field is perpendicular to the plane of the loop and directed into the paper. The loop is connected to a network of resistors, each equal to 3 Ω. What should be the speed of the loop so as to have a steady current of 1 mA in the loop?



(a) 1 cm s^{-1} (b) 2 cm s^{-1}
(c) 3 cm s^{-1} (d) 4 cm s^{-1}

21. At a certain place, the horizontal component of earth's magnetic field is $\sqrt{3}$ times the vertical component. The angle of dip at that place is

(a) 30°
(b) 45°
(c) 60°
(d) 90°

22. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude of $\sqrt{2} \times 10^{-2}$T. The dipole attains stable equilibrium at an angle of 30° with this field. What is the magnitude of the other fields?

(a) 0.01 T (b) 0.02 T
(c) 0.03 T (d) 0.04 T

23. Mark the correct statement.

(a) For long parallel conductors carrying steady currents, the Biot-Savart law and Lorentz force yield result in accordance with Newton's third law
(b) For long parallel conductors carrying steady currents, the Biot-Savart law and Lorentz force, Newton's third law does not hold good
(c) For long parallel conductors carrying time varying currents, the Biot-Savart law and Lorentz force, Newton's third law hold good
(d) Both (a) and (c) are correct

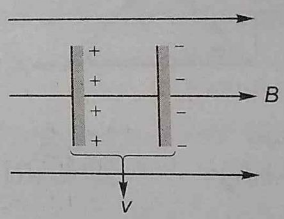
24. A current carrying circular loop lies on a smooth horizontal plane. Then,

(a) it is possible to establish a uniform magnetic field in the region so that loop starts rotating about its perpendicular axis.
(b) it is possible to establish a uniform magnetic field in the region so that loop will tip over about any of the point
(c) it is not possible that loop will tip over about any of the point whatever be the direction of established magnetic field (uniform)
(d) Both (a) and (b) are correct

25. A charge of 4 μC is placed on a small conducting sphere that is located at the end of thin insulating rod of length 0.5 m. The rod rotates in horizontal plane with a constant angular velocity of 100 $rads^{-1}$ about a vertical axis that passes through its other end. The magnetic moment of the rotating charge is

(a) zero
(b) 0.5×10^{-4} Am^2
(c) 1.25×10^{-4} Am^2
(d) magnetic moment is not defined for this case

26. A parallel plate capacitor is moving with a velocity of 25 ms^{-1} through a uniform magnetic field of 1.5 T as shown in figure. If the electric field within the capacitor plates is 175 NC^{-1} and plate area is 25×10^{-7} m^2, then the magnetic force experienced by positive charge plate is



(a) 1.45×10^{-13} N (b) zero
(c) 8.67×10^{-15} N (d) 3.87×10^{-15} N

27. In an AC circuit, the potential difference V and current I are given respectively by

$$V = 100 \sin (100t) V$$

and $$I = 100 \sin \left(100t + \frac{\pi}{3}\right) \text{mA}$$

The power dissipated in the circuit will be

(a) 10^4 W (b) 10 W
(c) 2.5 W (d) 5 W

28. An inductor L, a capacitor of 20 μF and a resistor of 10 Ω are connected in series with an AC source of frequency 50 Hz. If the current is in phase with the voltage, then the inductance of the inductor is

(a) 2.00 H b) 0.51 H
(c) 1.5 H (d) 0.99 H

29. An AC circuit having an inductor and a resistor in series draws a power of 560 W from an AC source marked 210 V-60 Hz. The power factor of the circuit is 0.8, the impedance of the circuit and the inductance of the inductor is

(a) 65 Ω, 0.2 H
(b) 64 Ω, 1.0 H
(c) 63 Ω, 0.1 H
(d) 50 Ω, 1.5 H

30. The resonant frequency and Q-factor of a series *LCR* circuit with $L = 3.0$ H, $C = 27\,\mu$F and $R = 7.4\,\Omega$. How will you improve the sharpness of resonance of the circuit by reducing its full width at half maximum by a factor of 2?

(a) Resistance of circuit should be increased
(b) Resistance of the circuit remain same
(c) Resistance of circuit should be increased by 3.7 Ω
(d) Resistance of the circuit should be reduced to 3.7 Ω

31. About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation

(i) at a distance of 1 m from the bulb?
(ii) at a distance of 10 m?

Assume that the radiation is emitted isotropically and neglect reflection.

(a) 0.2 Wm^{-2}, 0.002 Wm^{-2}
(b) 0.6 Wm^{-2}, 0.006 Wm^{-2}
(c) 0.3 Wm^{-2}, 0.003 Wm^{-2}
(d) 0.4 Wm^{-2}, 0.004 Wm^{-2}

32. A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. Frequency of the electromagnetic waves produced by the oscillator is

(a) 10 Hz
(b) 10^5 Hz
(c) 10^9 Hz
(d) 10^{10} Hz

33. The amplitude of the magnetic field part of a electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave?

(a) 140 NC^{-1}
(b) 153 NC^{-1}
(c) 163 NC^{-1}
(d) 133 NC^{-1}

Directions *(Q. Nos. 34 to 42) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

34. **Statement I** Flux of magnetic field **B** through a closed surface is equal to zero *i.e.*, $\oint \mathbf{B} \cdot d\mathbf{S} = 0$

Statement II Magnetic field lines are closed curves, they do not have any beginning or end.

35. **Statement I** A flexible wire loop of irregular shape carrying current when placed in a uniform external magnetic field acquires circular shape.

Statement II Any current carrying loop when placed in external magnetic field tries to acquires minimum energy and hence maximum magnetic flux, and for a given perimeter circular shape is having greatest area.

36. **Statement I** A hanging spring is attached to a battery and switch. On closing the switch a current suddenly flows in the spring, as a result spring compresses.

Statement II When two current carrying coils are placed close to each other in same plane, then they attract each other if sense of current is same in both.

37. **Statement I** A uniformly moving charged particle in a magnetic field, may follow a path along magnetic field lines.

Statement II The direction of magnetic force experienced by a charged particle is perpendicular to its velocity and **B**.

38. **Statement I** No current is induced in a metal loop if it is rotated in an electric field.

Statement II The electric flux through the loop does not change with time.

39. **Statement I** The power factor of an inductor is zero.

Statement II In the inductor, the emf and current differ in phase by $\frac{\pi}{2}$.

40. **Statement I** In a series *R-L- C* circuit the voltage across-resistor, inductor and capacitor are 8 V, 16 V and 10 V respectively. The resultant emf in the circuit is 10 V.

Statement II Resultant emf of the circuit is given by the relation $E = \sqrt{V_R^2 + (V_L - V_C)^2}$.

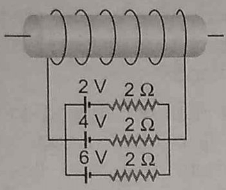
41. **Statement I** X-rays travel faster than light waves in vacuum.

Statement II The energy of X-rays photon is greater than the light photon.

42. **Statement I** An electron moving in the positive *x*-direction enters a region where uniform electric and magnetic fields exist perpendicular to each other. The electric field is in the negative *y*-direction. If the electron travels undeflected in this region, the direction of the magnetic field is along the negative *z*-axis.

Statement II If a charged particle moves in a direction perpendicular to a magnetic field, the direction of the force acting on it is given by Fleming's left hand rule.

Directions *(Q. Nos. 43 to 45) A copper wire having resistance of 0.05 Ωm^{-1} is used to wind a 2500 turns solenoid of radius 2 cm and length 2 m. The turns of solenoid are closely spaced. The combination of three cells are connected across the ends of solenoid as shown in figure.*



Neglect the resistance of connecting wires and magnetic field inside the solenoid due to current carrying wires of circuit.

43. The magnetic field at the centre of solenoid is

(a) 6×10^{-2} T (b) 3.77×10^{-4} T
(c) 4.68×10^{-2} T (d) 3.26×10^{-1} T

44. If a positive charge particle is performing circular motion (normal to plane of motion is along the axis of solenoid) inside the solenoid, then as seen from left the charge particle rotates in

(a) clockwise direction
(b) anti-clockwise direction
(c) first clockwise then anti-clockwise
(d) first anti-clockwise then clockwise

45. If the charge particle is moving in a circle of radius 1 cm whose centre is lying on axis of solenoid, then the speed with which particle moves is (Take charge/mass of charge particle as α)

(a) $6 \times 10^{-4}\ \alpha\ ms^{-1}$
(b) $7.54 \times 10^{-6}\ \alpha\ ms^{-1}$
(c) $4.68 \times 10^{-4}\ \alpha\ ms^{-1}$
(d) $3.26 \times 10^{-3}\ \alpha\ ms^{-1}$

Answer *with* Solutions

1. (*d*) Whatever be the current through solenoid winding, the direction of magnetic field is along the axis of solenoid and hence the magnetic force experienced by wire is zero and hence its weight cannot be supported by magnetic force.

2. (*a*) Drift speed of electron,

$$v_d = \frac{I}{neA} = \frac{5}{10^{29} \times 1.6 \times 10^{-19} \times 10^{-5}} = 3.125 \times 10^{-5}\ ms^{-1}$$

Average magnetic force experienced by each electron is,

$$F = qvB$$
$$= 1.6 \times 10^{-19} \times 3.125 \times 10^{-5} \times 0.1$$
$$= 0.5 \times 10^{-24}\ N = 5 \times 10^{-25}\ N$$

3. (*c*) $F = I\,(l \times B) = 2\left[5 \times \frac{1}{2} \times \sin 37^\circ\right] = 3\,N$

4. (*d*) If a coil is not moved in a magnetic field, the magnetic flux does not change. Hence, no emf or current is induced in the coil.

5. (*b*) Induced charge $q = \frac{\text{change of flux}}{\text{resistance}} = \frac{\phi_f - \phi_i}{R}$.

But final area $= 0$, therefore, $\phi_f = 0$. Numerically, $\phi_i = BA$. Therefore, $q = BA/R$.

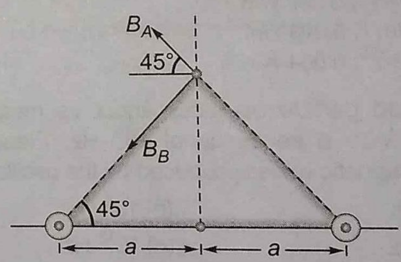
6. (*b*) $qvB = mg$

[for minimum value of B angle between **v** and **B** has to be $\frac{\pi}{2}$]

$$\Rightarrow \quad B = \frac{0.5 \times 10^{-3} \times 10}{2.5 \times 10^{-8} \times 6 \times 10^{4}} = 3.33\ T$$

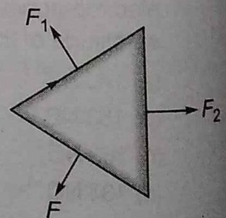
7. (*b*)

8. (*a*) $B_A = B_B = \frac{\mu_0 I}{2\pi \times \sqrt{2}\,a}$



So, $B = \sqrt{B_A^2 + B_B^2} = \frac{\mu_0 I}{2\,\pi a}$

9. (*a*) The net force acting on loop would be along X-axis (to determine whether it is along positive or negative, X-axis calculation has to be carried out) as shown in figure.



10. (*b*) Each and every pair of loop elements located symmetrically w.r.t. central line experiences zero net force. So, total magnetic force experienced by loop is zero.

11. (*d*) $F = q\,(\mathbf{v} \times \mathbf{B})$. If v is parallel to B, $\mathbf{v} \times \mathbf{B} = 0$. Hence, $F = 0$. Thus, the correct option is (d).

12. (*a*) Time interval in which its **v** returns to its initial value is same as time period of the particle, to execute the circle.

Since, it doesn't depend upon θ. So, time required $= \frac{2\pi m}{eB}$.

13. (*d*) Magnetic force at centre due to the given configuration is zero because the angle between net **B** and **v** is zero.

14. (*d*) Induced emf $(e) = \frac{\text{magnetic field} \times \text{change in area}}{\text{time}}$

$$= \frac{B\Delta A}{t}$$

Since, the circumference of the circular loop $= 2\pi r$,

The side of the square loop $= \frac{2\pi r}{4} = \frac{\pi r}{2}$

Therefore,

$$\Delta A = \pi r^2 - \left(\frac{\pi r}{2}\right)^2$$

$$= \pi r^2 \left(1 - \frac{\pi}{4}\right)$$

$$\therefore \quad e = \frac{B(\pi r^2)}{t}\left(1 - \frac{\pi}{4}\right)$$

15. (*a*) Magnetic field due to the larger coil at its centre is

$$B = \frac{\mu_0 I}{2r_1}$$

where I is the current in the larger coil. Flux through the inner coil is

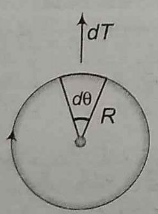
$$\phi = B \times \pi r_2^2 = \frac{\mu_0 I}{2r_1} \times \pi r_2^2$$

But $\quad \phi = MI$

Therefore, $\quad M = \frac{\mu_0 \pi r_2^2}{2r_1}$

16. (*d*) By right hand thumb rule force of expansion would act on it whose magnitude is geven by $dT = I\,(\mathbf{dl} \times \mathbf{B})$

$$\Rightarrow \quad T = IB\int dl = 2\pi\, IRB$$



17. (*a*) The instantaneous voltage and current in an AC circuit containing an ideal inductance only are

$$E = E_0 \sin\omega t$$

$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$P_{\text{ins}} = Ei = E_0 i_0 \sin\omega t \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -E_0 i_0 \sin\omega t \cos\omega t$$

$$= -\frac{1}{2} E_0 i_0 \sin 2\omega t$$

Average power for one complete cycle is

$$P = \frac{1}{T}\left(-\frac{1}{2}E_0 i_0\right)\int_0^T \sin 2\omega t\, dt = 0$$

18. (*d*) Since **B** and v are anti-parallel to each other, angle between the two is zero.

$$F = q\,\mathrm{v} \times \mathrm{B}$$

$$= q\,v\,B\sin\theta$$

For $\theta = 0$, $\sin 0 = 0$

$$\Rightarrow \quad F = 0$$

19. (*b*) $e = Blv = 0.5 \times 10^{-4} \times 10 \times 200 = 0.1\text{ V}$

20. (*b*) The network *PQRS* is a balanced Wheatston's bridge. Hence, the resistance of 3 Ω between *P* and *R* is ineffective. The net resistance of the network, therefore, is 3Ω. Total resistance $R = 3\Omega + 1\Omega = 4\Omega$. Now, induced emf is $e = Blv = 2 \times 0.1 \times v = 0.2v$.

$\therefore$ Induced current $I = \frac{e}{R} = \frac{0.2v}{4}$

Given, $\quad I = 1 \times 10^{-3}\text{ A}$

Hence, $\quad 1 \times 10^{-3} = \frac{0.2v}{4}$

Which gives $v = 2 \times 10^{-2}\text{ms}^{-1} = 2\text{ cms}^{-1}$

21. (*a*) $B_H = B\cos\theta$ and $B_V = B\sin\theta$

Hence, $\quad \frac{B_V}{B_H} = \tan\theta$

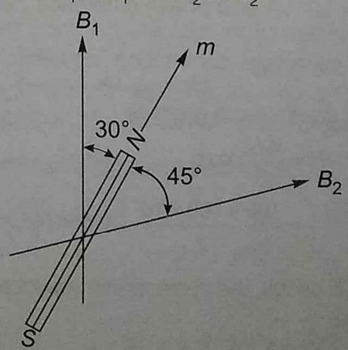
Given, $\quad \frac{B_V}{B_H} = \frac{1}{\sqrt{3}}$

Therefore, $\quad \tan\theta = \frac{1}{\sqrt{3}}$ *i.e.*, $\theta = 30°$

22. (*a*) Refer to figure. Let θ_1 (= 30°) be the angle between the magnetic moment vector **m** and the field vector $\mathbf{B}_1$ $(= 1.5 \times 10^{-2}\text{ T})$. Then, as shown in figure, the angle between *m* and the other field $\mathbf{B}_2$ will be $\theta_2 = 75° - 30° = 45°$.

The field $\mathbf{B}_1$ exerts a torque $\tau_1 = \mathbf{m} \times \mathbf{B}_1$ on the dipole and the field $\mathbf{B}_2$ exerts a torque $\tau_2 = \mathbf{m} \times \mathbf{B}_2$, where **m** is the magnetic moment of the dipole. Since, the dipole is in stable equilibrium, the net torque $\tau\ (= \tau_1 + \tau_2)$ must be zero, *i.e.*, the two torques must be equal and opposite. In terms of magnitudes, we have

$$mB_1 \sin\theta_1 = mB_2 \sin\theta_2$$

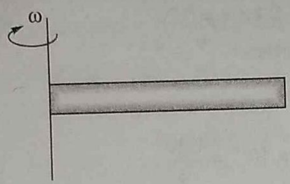


or $\quad B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{\sqrt{2} \times 10^{-2} \times \sin 30°}{\sin 45°} = 0.01\text{T}$

23. (*a*) In general, Newton's third law is not valid for electromagnetic phenomena but for long parallel conductors and steady currents we can prove that Biot-Savart law and Lorentz force give the result in accordance with Newton's third law.

24. (*b*) As loop is placed in horizontal plane, so area vector is along vertical direction. From $\tau = I\,(\mathbf{A} \times \mathbf{B})$, as **A** is in vertical direction, τ would be the plane of loop only so Ist option is wrong because for rotation of loop about its own axis τ must be along vertical direction.

25. (*b*) A moving charge along a circle is equivalent to a current carrying ring, whose current is given by,



$$I = \frac{q}{T} = \frac{q}{2\pi/\omega}$$

$$\Rightarrow \quad I = \frac{4 \times 10^{-6}}{2\pi} \times 100 = 0.64 \times 10^{-4}\,\text{A}$$

Magnetic moment of rotating charge is $M = IA$

$$\Rightarrow \quad M = 0.64 \times 10^{-4} \times \pi \times (0.5)^2$$
$$= 0.5 \times 10^{-4}\ \text{Am}^2$$

26. (*a*) Electric field in between the capacitor plates is given by

$$E = \frac{q}{\varepsilon_0 A}$$

where q is the charge on capacitor.

$$q = \varepsilon_0 A \times E$$
$$= 8.85 \times 10^{-12} \times 25 \times 10^{-7} \times 175$$
$$= 3.87 \times 10^{-15}\ \text{C}$$

Magnetic force experienced by positive plate is,

$F_m = qvB = 1.45 \times 10^{-13}$ N in a direction out of plane of paper.

27. (*c*) Voltage amplitude $V_0 = 100$ volt, current amplitude $I_0 = 100$ mA $= 100 \times 10^{-3}$ A and phase difference between I and V is

$$\phi = \frac{\pi}{3} = 60^\circ$$

Now, power dissipated is given by

$$P = \frac{V_0 I_0}{2} \cos\phi$$
$$= \frac{100 \times 100 \times 10^{-3}}{2} \times \cos 60^\circ = 2.5\ \text{W}$$

28. (*b*) In an *LCR* circuit, the current and the voltage are in phase ($\phi = 0$), when

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0$$

or $\quad \omega L = \dfrac{1}{\omega C}$

or $\quad L = \dfrac{1}{\omega^2 C}$

Here, $\omega = 2\pi f = 2 \times 3.14 \times 50\ \text{s}^{-1}$
$= 314\ \text{s}^{-1}$ and $C = 20\ \mu\text{F} = 20 \times 10^{-6}$ F

$$\therefore \quad L = \frac{1}{(314\ \text{s}^{-1})^2 \times (20 \times 10^{-6}\ \text{F})} = 0.51\ \text{H}$$

29. (*c*) The average power over a complete cycle is

$$P = E_{\text{rms}} \times I_{\text{rms}} \times \cos\phi$$

where $\cos\phi$ is the power factor

$$\therefore \quad I_{\text{rms}} = \frac{P}{E_{\text{rms}} \times \cos\phi} = \frac{560}{210 \times 0.8} = \frac{10}{3}\ \text{A}$$

The impedance of the circuit is

$$Z = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{210\ \text{V}}{(10/3)\ \text{A}} = 63\,\Omega$$

The power is consumed in R only. Therefore,

$$P = (I_{\text{rms}})^2 R$$

or $\quad R = \dfrac{P}{(I_{\text{rms}})^2}$

$$= \frac{560}{\left(\frac{10}{3}\right)^2} = 50.4\ \Omega$$

Now, the impedance of an *L-R* circuit is

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\therefore \quad (\omega L)^2 = Z^2 - R^2 = (63)^2 - (50.4)^2$$
$$= (63 + 50.4) \times (63 - 50.4)$$
$$= 113.4\ \Omega \times 12.6\,\Omega$$
$$= 1428.84\,\Omega^2$$

or $\quad \omega L = \sqrt{1428.84} = 37.8\ \Omega$

$$\therefore \quad L = \frac{37.8}{2\pi f}$$
$$= \frac{37.8}{2 \times 3.14 \times 60} = 0.1\ \text{H}$$

30. (*d*) Given, $L = 3.0$ H, $C = 27\mu\text{F} = 27 \times 10^{-6}$F and $R = 7.4\ \Omega$

The resonant frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.0 \times (27 \times 10^{-6})}} = 111\text{rads}^{-1}$$

The *Q*-factor of the circuit is given by

$$Q = \frac{\omega_0 L}{R} = \frac{111 \times 3.0}{7.4} = 45$$

The "bandwidth" (the difference of half-power frequencies given by $\quad \omega_2 - \omega_1 = R/L$

Smaller the value of $(\omega_2 - \omega_1)$, sharper is the resonance. T reduce $(\omega_2 - \omega_1)$ by a factor of 2, the resistance R should b halved that is, the resistance of the circuit should be reduced 3.7 Ω.

31. (*d*) Power converted into visible radiation,

$$P = \frac{5}{100} \times 100 = 5\,W$$

$$\text{Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$$

(i) $\text{Intensity} = \frac{5}{4 \times 3.14 \times 1 \times 1} = 0.4\ Wm^{-2}$

(ii) $\text{Intensity} = \frac{5}{4 \times 3.14 \times 10 \times 10} = 0.004\ Wm^{-2}$

32. (*c*) The frequency of electromagnetic waves produced by the oscillator is equal to the frequency of the oscillating particle *i.e.*, 10^9 Hz.

33. (*b*) $\frac{E_0}{B_0} = c$ or $E_0 = cB_0 = 3 \times 10^8 \times 510 \times 10^{-9}\ NC^{-1}$

$$= 153\ NC^{-1}$$

34. (*a*) Magnetic field lines are closed curves and hence the number of field lines entering the closed surface is equal to number of field lines leaving the surface or in other words we can say $\oint B \cdot dS = 0$

35. (*a*) $U = -M \cdot B = -[NiA \cdot B]$

So, to acquire stable equilibrium position current carrying loop must have minimum energy and hence maximum flux. If the loop wire is flexible it changes its shape so that flux linked with it become maximum and hence acquire circular shape, because for given perimeter, circle has maximum area.

36. (*a*) When switch is closed in situation described in Statement I, then a current exists in coils (turns) constituting the spring which is in same sense in all coils, and as a current carrying coil behaves as a magnetic dipole they attract each other (as sense of current is same) and hence spring compresses.

37. (*b*) Force experienced by moving charge particle in magnetic field is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. If $\mathbf{v}$ is initially parallel to $\mathbf{B}$, then $\mathbf{F}$ = zero. So, then only the particle may follow the path along field line. Also direction of $\mathbf{F}$ is obviously perpendicular to $\mathbf{v}$ and $\mathbf{B}$. $[\because \vec{F} \propto (\vec{v} \times \vec{B})]$

38. (*b*) A current is induced in a loop only if magnetic flux linked with the coil changes.

39. (*a*) Power factor is defined as cosine of phase difference between emf and current *i.e.*, $\cos\phi$.

The phase difference between emf and current is 90°. Thus, power factor

$$= \cos\phi = \cos 90° = 0$$

40. (*a*) The resultant emf in the *L-C-R* circuit is given by

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$E = \sqrt{(8)^2 + (16-10)^2} = \sqrt{64 + 36}$$

$$E = 10\,V$$

41. (*d*) Since, X-rays are electromagnetic waves, we know that electromagnetic wave travels with same velocity of light in vacuum.

Now, from the formula $E = \frac{hc}{\lambda}$ ... (i)

The wavelength of X-rays are small than light waves and energy is inversely proportional to the wavelength. Hence, the energy of X-rays photon will be greater than light waves.

42. (*b*) Because electron has a negative charge, an electric field in the negative *y*-direction will deflect it in the positive *y*-direction. It will travel undeflected if the magnetic field imparts an equal deflection in the negative *y*-direction. Since, the magnetic force is perpendicular to the magnetic field and the charge of electron is negative, the direction of the magnetic field (according to Fleming's left hand rule) should be along the negative *z*-direction.

43. (*b*) Resistance of solenoid winding is, $R = (0.05) \times l$

where l is length of copper wire used to wound the solenoid winding.

$$l = 2\pi r \times 2500, \text{ where } r = 2\,cm$$

$$\Rightarrow \quad R = 0.05 \times 2\pi \times \frac{2}{100} \times 2500 = 5\pi\,\Omega$$

Equivalent emf of battery,

$$E = \frac{\Sigma E_i / r_i}{\Sigma 1/r_i} = \frac{1+2+3}{3/2} = 4\,V$$

Current in solenoid winding is,

$$I = \frac{E}{R + \frac{2}{3}} = \frac{4}{5\pi + \frac{2}{3}} = 0.24\,A$$

Magnetic field intensity inside the solenoid is,

$$B = \mu_0 n I$$

where, $n = \frac{2500}{2} = 1250$

So, $B = 3.77 \times 10^{-4}\,T$

44. (*b*) As seen from left the current in solenoid winding is in clockwise direction and hence magnetic field inside the solenoid is from left to right. So, the charge particle will move in circular path in anti-clockwise direction as seen from left.

45. (*b*) From Newton's dynamics equation,

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow \quad v = \frac{q}{m} \times Br$$

$$= \alpha \times 3.77 \times 10^{-4} \times 2 \times 10^{-2}$$

$$= 7.54 \times 10^{-6} \alpha\ ms^{-1}$$