DAY 2

KINEMATICS

OUTLINES

1. Concept of Kinematics
2. Frame of Reference
3. Motion in a Straight Line
4. Distance & Displacement
5. Speed & Velocity
6. Acceleration
7. Graphs
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Concept of Kinematics

An object can have uniform motion, even when the number of forces are acting on it. Such forces are said to be in equilibrium. Kinematics is the branch of mechanics which describes the motion of points, bodies and systems of bodies without consideration of the cause of motion.

Frame of Reference

A frame of reference refers to a coordinate system or set of axes within which to measure the position, orientation and other physical properties of objects in it or it may refer to an observational reference frame tied to the state of motion of an observer. *There are two types of frame of reference*

(i) **Inertial frame of reference** A frame with respect to which a free body has the acceleration only when a net unbalanced force is acting on it.

(ii) **Non-inertial frame of reference** A frame with respect to which an stationary body has some acceleration. In case of non-inertial frame pseudo force, centrifugal forces etc., are taken into the account. *The earth is rotating but it is assumed as an inertial frame,*

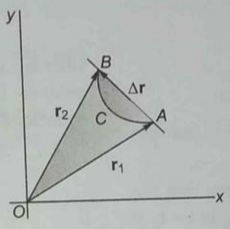
Motion in a Straight Line

The position x of a particle on the x-axis, locates the particle with respect to the origin, or the zero point, of the axis. The position is either positive or negative, according to which side of the origin, the particle is on, or zero if the particle is at the origin. The positive direction on an axis is the direction of increasing positive numbers, the opposite direction is the negative direction.

Distance *and* Displacement

Distance is the total length of the path travelled by the particle in a given interval of time.

Displacement is a vector joining the initial position of the particle to its final position in a given interval of time. Mathematically, it is equal to the change in position vectors *i.e.*, $\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$



Suppose a body is at point $A(x_1, y_1, z_1)$ at $t = t_1$.

It reaches at point $B(x_2, y_2, z_2)$ at $t = t_2$ through path ACB with respect to the frame shown in figure. The actual length of curved path ACB is the distance travelled by the body in time,

$$\Delta t = t_2 - t_1.$$

If we connect point A (initial position) and point B (final position) by a straight line, then the length of the straight line AB gives the magnitude of displacement of body in time interval, $\Delta t = t_2 - t_1$.

The direction of displacement is directed from A to B through the straight line AB and the magnitude of displacement is

$$|\mathbf{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Distance *Vs* Displacement

- Distance is a scalar quantity and displacement is a vector quantity.
- For motion between two points displacement is single valued while distance depends on actual path and so can have many values.
- Path length or distance is a positive scalar quantity which does not decrease with time and can never be zero for a moving body. Displacement of a body can be zero.
- Magnitude of displacement can never be greater than distance.
- When a body returns to its initial position, its displacement is zero but distance or path length is non-zero.
- In general, magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without any change in direction.

Speed *and* Velocity

*The **average speed** is defined as the total distance travelled by a body in a particular time interval Δt divided by the time interval.*

- Average Speed

$$v_{av} = \frac{\text{Total distance travelled in time } \Delta t}{\Delta t}$$

- The **average velocity** is defined as the total displacement $\Delta x = x_2 - x_1$ of the body in a particular time interval Δt, divided by the time interval. $\mathbf{v}_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$
- The algebraic sign of $\mathbf{v}_{av}$ indicates the direction of motion ($\mathbf{v}_{av}$ is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions. Slope of the displacement-time graph give the average velocity.
- The average velocity between two points in a time interval can be obtained from a position *versus* time graph by calculating the slope of the straight line joining the coordinates of the two points.

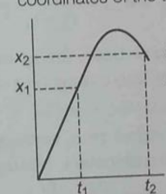


Fig. (a)

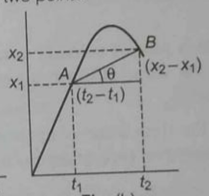


Fig. (b)

- The graph [shown in figure], describes the motion of a particle moving along x-axis (along a straight line).
- Suppose we wish to calculate the average velocity between $t = t_1$ and $t = t_2$. The slope of chord AB [shown in **Fig.** (b)] gives the average velocity.

  Mathematically, $v_{av} = \tan\theta = \frac{x_2 - x_1}{t_2 - t_1}$
- When a body travels equal distance with speeds v_1 and v_2, the average speed (v) is the harmonic mean of the two speeds.

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$

- When a body travels for equal time with speeds v_1 and v_2, the average speed v is the arithmetic mean of the two speeds.

$$3v = \frac{v_1 + v_2}{2}$$

Instantaneous Velocity

The **instantaneous velocity** (or simply velocity) v of a moving particle is $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$. The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of x *versus* t.

Instantaneous Speed

Instantaneous speed is simply the magnitude of the velocity. Instantaneous speed at an instant is equal to the magnitude of the instantaneous velocity at that instant of time.

Acceleration

Acceleration of an object is defined as rate of change of velocity. It is a vector quantity having unit m/s^2 or ms^{-2}. Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform while negative acceleration (retardation) means velocity is decreasing with time.

Average and Instantaneous Acceleration

Average acceleration is defined as the change in velocity (Δv) divided by the time interval (Δt).

Let us consider the motion of a particle. Suppose that the particle has velocity $\mathbf{v}_1$ at $t = t_1$ and at a later time $t = t_2$ it has velocity $\mathbf{v}_2$. Thus, the average acceleration during the time interval $\Delta t = t_2 - t_1$, is $\mathbf{a}_{av} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t}$

On a plot of velocity *versus* time, the average acceleration is the slope of the straight line connecting the points (v_2, t_2) and (v_1, t_1).

If the time interval approaches zero, average acceleration is known as **instantaneous acceleration.** Mathematically,

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

When a body is moving with a constant acceleration, then acceleration time graph is a straight line.

Negative acceleration is known as retardation. It indicates that velocity is decreasing with respect to time.

For one dimensional motion, the angle between acceleration and velocity is either 0° or 180°. If it is 0°, then acceleration is said to be positive. Acceleration is said to be negative, if angle between acceleration and velocity is 180° and it is taken as retardation of the body.

If a body moves with, constant velocity, then instantaneous velocity is equal to average velocity. The instantaneous speed is equal to modulus of instantaneous velocity and the acceleration becomes zero in this case.

Kinematics Equations of Motion

For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x, time taken t, initial velocity u, final velocity v and acceleration a, are related by a set of simple equations called kinematic equations of motion

$$v = u + at$$

$$x = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2ax,$$

At time $t = 0$, the object is at the origin.

If the particle starts the motion from $x = x_0$, x in above equations is replaced by $(x - x_0)$.

Free Fall Acceleration

An important example of a straight-line motion with constant acceleration, is that of an object rising or falling freely near the earth's surface. The constant acceleration equation describes this motion, but we make two changes in notation, (1) we refer the motion to the vertical y-axis with $+y$ vertically up, and (2) we replace a with g, where g is the magnitude of the free fall acceleration. Near earth's surface, $g = 9.8\,\text{ms}^{-2}$ $(= 32\,\text{fts}^{-2})$

- *If an object is thrown upwards with velocity u from top of a building and another object is thrown downwards with same velocity from the same point, then both reach the ground with the same velocity, if air resistance is neglected.*
- *A ball is dropped from a building of height h and it reaches after t second on earth. From the same building if two balls are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 second respectively, then $t = \sqrt{t_1 t_2}$, if the air resistance is neglected.*
- *A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of l m each will then be in the ratio of the difference in the square roots of the integers*
  i.e., $\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), \ldots (\sqrt{4} - \sqrt{3}), \ldots$

Uniformly Accelerated Motion

A motion, in which change in velocity in each unit of time is constant, is called an uniformly accelerated motion. So, for an uniformly accelerated motion, acceleration is constant. For an uniformly accelerated motion ($\mathbf{a}$ = constant), equations of motion are as follows

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t \quad \text{...(i)}$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 \quad \text{...(ii)}$$

and

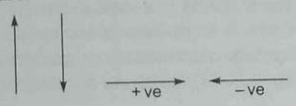
$$\mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2\,\mathbf{a} \cdot \mathbf{s} \quad \text{...(iii)}$$

$$v^2 = u^2 + 2as$$

where, $\mathbf{u}$ = initial velocity, $\mathbf{v}$ = velocity at time t,
$\mathbf{s}$ = displacement of particle at time t.

If motion is described in one dimension, so vector sign ($\rightarrow$) need not be used.

Sign convention for **Fig.** (a) motion in the vertical direction, and **Fig.** (b) motion in the horizontal direction, is shown in figure.



Non-uniformly Accelerated Motion

When motion of a particle is not uniform *i.e.*, acceleration of particle is not constant or acceleration is a function of time, then following relations hold for one dimensional motion.

(i) $v = \frac{ds}{dt}$

(ii) $a = \frac{dv}{dt} = v\frac{dv}{ds}$

(iii) $ds = vdt$ and

(iv) $dv = adt$ or $v\,dv = ads$

Such problems can be solved either by differentiation or integration, by applying some boundary conditions.

» *If an object starts from rest and moves with a uniform acceleration, then displacement covered by it in time t is directly proportional to t^2, i.e., $s \propto t^2$. and If a body starts from rest and moves with a uniform acceleration, then displacement covered by it in successive seconds will be in the ratio of odd natural number, i.e.,* $s_{1st} : s_{2nd} : s_{3rd} = 1 : 3 : 5 \ldots$

» *When particle covers one-third distance at speed v_1, next one-third at speed v_2 and last one-third at speed v_3, then*

$$v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

Graphs

The theory of graphs can be generalised and summarised in following six points.

(i) A linear equation between x and y represents a straight line. *e.g.*, $y = 4x - 2$, $y = 5x + 3$, $3x = y - 2$.

(ii) $x \propto y$ or $y = kx$ represents a straight line passing through origin.

(iii) A quadratic equation in x and y represents a parabola in x-y graph. *e.g.*, $y = 3x^2 + 2$, $y^2 = 4x$, $x^2 = y - 2$.

(iv) If $m = dy/dx$ or y/x, then the value of z at any point on the x-y graph can be obtained by the slope of the graph at the point (Here m is the slope).

(v) If $m = \frac{dy}{dx}$ or $y(dx) = x(dy)$, then the value of x lies between x_1 and x_2 or value of y lies between y_1 and y_2 can be obtained by the area of graph between x_1 and x_2 or y_1 and y_2 respectively (Here, m is the slope).

(vi) $x \propto \frac{1}{y}$ represents a rectangular hyperbola in the x-y graph.

From the above six-points, we may conclude that in case of a one dimensional motion

(i) Slope of displacement-time graph gives velocity $\left(\text{as}, v = \frac{dx}{dt}\right)$.

(ii) Slope of velocity-time gives acceleration $\left(\text{as } a = \frac{dv}{dt}\right)$

(iii) Area under velocity-time graph gives displacement (as, $ds = v\,dt$).

(iv) Area under acceleration-time graph gives change in velocity (as, $dv = adt$).

(v) Displacement-time graph in an uniform motion, is a straight line passing through origin if the displacement is zero at time $t = 0$ (as, $s = vt$).

(vi) Velocity-time graph is a straight line passing through the origin, in a uniformly accelerated motion, if initial velocity $u = 0$.

And a straight line not passing through origin, if initial velocity $u \neq 0$ (as, $v = u + at$).

(vii) Displacement-time graph for an uniformly accelerated or retarded motion is a parabola $\left(\text{as}, s = ut \pm \frac{1}{2}at^2\right)$.

Relative Motion

If the velocity of two bodies are known w.r.t a common frame of reference, then the velocity of one of the two bodies can be measured w.r.t. the second body.

Let us consider, if the velocities of the two bodies A and B w.r.t. earth are $\mathbf{v}_A$ and $\mathbf{v}_B$, then the relative velocity of A w.r.t. B is

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

Similarly, we see that $\mathbf{v}_{AB} = -\mathbf{v}_{BA}$

Also, acceleration of A w.r.t. B is $\mathbf{a}_{AB} = \mathbf{a}_A - \mathbf{a}_B$

and $\mathbf{a}_{AB} = -\mathbf{a}_{BA}$

If a car moves from west to east (the direction of motion of the earth) $v_C = v_{CE} + v_E$
and if the car moves from east to west (opposite to the motion of the earth) $v_C = v_{CE} - v_E$
Where, v_C and v_E are the velocities of car and the earth respectively with respect to an inertial frame of reference.

Also, note that earth is assumed as the rotating frame which is a non-inertial one.

Motion Under Gravity

The most familiar example of motion with constant acceleration on a straight line is motion in a vertical direction, near the surface of earth. If air resistance is neglected, the acceleration of such type of particle is gravitational acceleration which is nearly constant for a height negligible with respect to the radius of earth. The magnitude of gravitational acceleration, which is nearly constant for a height negligible with respect to the radius of earth. The magnitude of gravitational acceleration near the surface of earth is $g = 9.8\,ms^{-2} = 32\,fts^{-2}$.

Cases of Motion Under Gravity

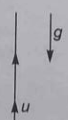
Case I If particle is moving upwards

In this case, applicable kinematics relations are

$$v = u - gt \qquad \text{... (i)}$$

$$h = ut - \frac{1}{2}gt^2 \qquad \text{... (ii)}$$

$$v^2 = u^2 - 2gh \qquad \text{... (iii)}$$



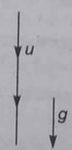
Here, h is the vertical height of the particle in upward direction.

For maximum height attained by a projectile

$h = h_{max}$ $v = 0$ *i.e.*, $(0)^2 = u^2 - 2gh_{max}$

$$\therefore \qquad h_{max} = \frac{u^2}{2g}$$

Case II If particle is moving vertically downwards.



In this is case,

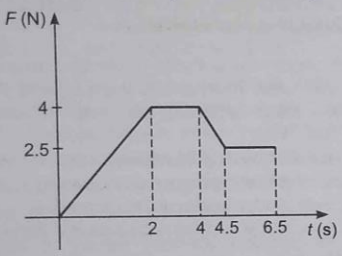
$$v = u + gt \qquad \text{... (i)}$$

$$h = ut + \frac{1}{2}gt^2 \qquad \text{... (ii)}$$

$$v^2 = u^2 + 2gh \qquad \text{... (iii)}$$

Here, h is the vertical height of particle in downward direction.

1. A body of 2 kg has an initial speed of 5 ms^{-1}. A force acts on it for 4s in the direction of motion. The force-time graph is shown below. The final speed of the body is



(a) 14.26 ms^{-1} (b) 18.51 ms^{-1}
(c) 10.28 ms^{-1} (d) zero

2. When two bodies move uniformly towards each other the distance between them decreases by 8 ms^{-1}. If both bodies move in the same direction with different speeds, the distance between them increases by 2 ms^{-1}. The speeds of two bodies will be

(a) 4 ms^{-1} and 3 ms^{-1}
(b) 4 ms^{-1} and 2 ms^{-1}
(c) 5 ms^{-1} and 3 ms^{-1}
(d) 7 ms^{-1} and 3 ms^{-1}

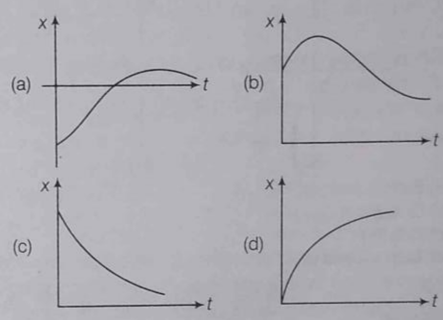
3. An aeroplane moves 400 m towards north, 300 m towards west and then 1200 m vertically upwards. Then, its displacement from the initial position is

(a) 1300 m (b) 1400 m
(c) 1500 m (d) 1600 m

4. The displacement of a particle is given by $x = (t-2)^2$ where, x is in metres and t in seconds. The distance covered by the particle in first 4 seconds is **[NCERT Exemplar]**

(a) 4 m
(b) 8 m
(c) 12 m
(d) 16 m

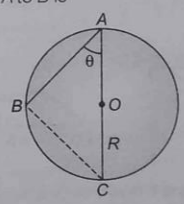
5. Among the four graphs there is only one graph for which average velocity over the time interval $(0, T)$ can vanish for a suitably chosen T. Which one is it? **[NCERT Exemplar]**



6. A body sliding down on a smooth inclined plane slides down 1/4 th distance in 2s. It will slide down the complete plane in

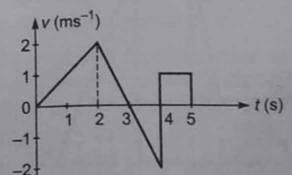
(a) 4 s (b) 5 s (c) 2 s (d) 3 s

7. A frictionless wire AB is fixed on a sphere of radius R. A very small spherical ball slips on this wire. The time taken by this ball to slip from A to B is



(a) $\frac{2\sqrt{gR}}{g\cos\theta}$ (b) $2\sqrt{gR}\,\frac{\cos\theta}{g}$ (c) $2\sqrt{\frac{R}{g}}$ (d) $\frac{gR}{\sqrt{g\cos\theta}}$

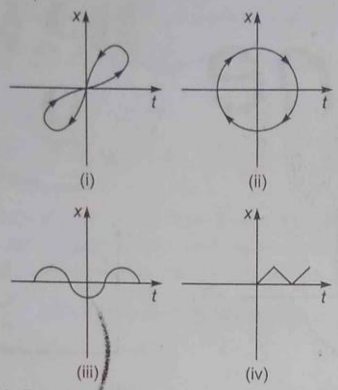
8. The velocity-time graph of a body in a straight line is as shown in figure.



The displacement of the body in five seconds is

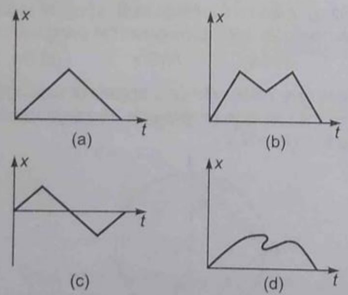
(a) 2 m (b) 3 m (c) 4 m (d) 5 m

9. Look at the graphs (i) to (iv) in figure carefully and choose, which of these can possibly represent one-dimensional motion of particle?

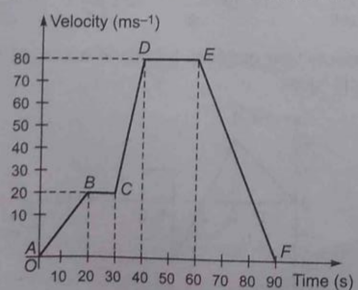


(a) Both (i) and (ii)
(b) Only (iv)
(c) Only (iii)
(d) Both (iii) and (iv)

10. Which of the following distance-time graphs is not possible?

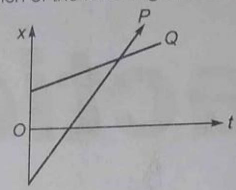


11. The velocity *versus* time curve of a moving point is shown in the figure below. The maximum acceleration is



(a) $1\,ms^{-2}$ (b) $4\,ms^{-2}$ (c) $2\,ms^{-2}$ (d) $1.5\,ms^{-2}$

12. Figure shows the time-displacement curve of the particles P and Q. Which of the following statement is correct?



(a) Both P and Q move with uniform equal speed
(b) P is accelerated and Q move with uniform speed but the speed of P is more than the speed of Q
(c) Both P and Q move with uniform speeds but the speed of P is more than the speed of Q
(d) Both P and Q move with uniform speeds but the speed of Q is more than the speed of P

13. A rod of length l leans by its upper end against a smooth vertical wall, while its other end leans against the floor. The end that leans against the wall moves uniformly downwards. Then,
(a) the other end also move uniformly
(b) the speed of other end goes on increasing
(c) the speed of other end goes on decreasing
(d) the speed of other end first decreases and then increases

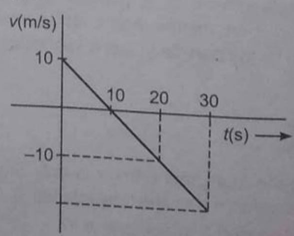
14. A vehicle travels half the distance L with speed v_1 and the other half with speed v_2, then its average speed is **[NCERT Exemplar]**

(a) $\frac{v_1 + v_2}{2}$ (b) $\frac{2v_1 + v_2}{v_1 + v_2}$ (c) $\frac{2v_1v_2}{v_1 + v_2}$ (d) $\frac{L(v_1 + v_2)}{v_1v_2}$

15. A car moving with a speed of $25\,ms^{-1}$ takes a U-turn in 5 s, without changing its speed. The average acceleration during these 5 s is
(a) $10\,ms^{-2}$ (b) $5\,ms^{-2}$ (c) $2\,ms^{-2}$ (d) $7\,ms^{-2}$

16. The correct statement from the following is
(a) A body having zero velocity will not necessarily have zero acceleration
(b) A body having zero velocity will necessarily have zero acceleration
(c) A body having uniform speed can have only uniform acceleration
(d) A body having non-uniform velocity will have zero acceleration

17. The velocity-time plot for a particle moving on a straight line is as shown , then



(a) the particle has a constant acceleration
(b) the particle has never turned around
(c) the average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s
(d) Both *a* and c are correct

18. A point initially at rest moves along the *x*-axis. Its acceleration varies with time as $a = (5t + 6)\,ms^{-2}$. If it starts from the origin, the distance covered by it in 2 s is
(a) 18.66 m (b) 14.33 m
(c) 12.18 m (d) 6.66 m

19. A body starts from the origin and moves along the axis such that the velocity at any instant is given by $v = 4t^3 - 2t$ where, t is in second and the velocity in ms^{-1}. Find the acceleration of the particle when it is at a distance of 2 m from the origin.
(a) 28 ms^{-2} (b) 22 ms^{-2} (c) 12 ms^{-2} (d) 10 ms^{-2}

20. The motor of an electric train can give it an acceleration of 1 ms^{-2} and brakes can give a negative acceleration of 3 ms^{-2}. The shortest time in which the train can make a trip between the two stations 1215 m apart is
(a) 113.6 s (b) 56.9 s
(c) 60 s (d) 55 s

21. A body is thrown vertically upwards in air, when air resistance is taken into account, the time of ascent is t_1 and time of descent is t_2, then which of the following is true?
(a) $t_1 = t_2$ (b) $t_1 < t_2$ (c) $t_1 > t_2$ (d) $t_1 \geq t_2$

22. A balloon is going upwards with velocity 12 ms^{-1}. It releases a packet when it is at a height of 65 m from the ground. How much time the packet will take to reach the ground if $g = 10\,ms^{-2}$?
(a) 5 s (b) 6 s (c) 7 s (d) 8 s

23. A body moving with an uniform acceleration describes 12 m in the 3rd second of its motion and 20 m in the 5th second. Find the velocity after the 10th second.
(a) 40 ms^{-1} (b) 42 ms^{-1}
(c) 52 ms^{-1} (d) 4 ms^{-1}

24. A ball is dropped from the top of a building. The ball takes 0.5 s to fall past the 3 m length of window some distance from the top of building. With what speed does the ball pass the top of window?
(a) 6 ms^{-1} (b) 12 ms^{-1}
(c) 7 ms^{-1} (d) 3.5 ms^{-1}

25. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, the distance travelled by the car is
(a) $\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$ (b) $\frac{1}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)t^2$
(c) $\frac{1}{2}\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)t^2$ (d) $\frac{1}{2}\left(\frac{\alpha^2-\beta^2}{\alpha\beta}\right)t^2$

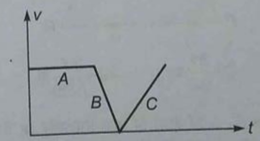
26. A stone falls freely from rest and the total distance covered by it in the last second of its motion equals the distance covered by it in the first three seconds of its motion. The stone remains in the air for
(a) 6 s (b) 5 s (c) 7 s (d) 4 s

27. A train accelerating uniformly from rest attains a maximum speed of 40 ms^{-1} in 20 s. It travels at this speed for 20 s and is brought to rest with an uniform retardation in the next 40 s. What is the average velocity during this period?
(a) $\frac{80}{3}$ ms^{-1} (b) 25 ms^{-1}
(c) 40 ms^{-1} (d) 30 ms^{-1}

28. A stone is dropped from a certain height and reaches the ground in 5 s. If the stone is stopped after 3 s of its fall and then allowed to fall again, then the time taken by the stone to reach the ground after covering the remaining distance is
(a) 2 s (b) 3 s
(c) 4 s (d) None of these

29. A boy sitting on the top most berth in the compartment of a train which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother sitting vertically below his hands at a distance of about 2 m. The apple will fall
(a) precisely in the hand of his brother
(b) slightly away from the hand of his brother in the direction of motion of the train
(c) slightly away from the hand of his brother in the direction opposite to the direction of motion of the train
(d) None of the above

30. The velocity-time graph of a body is shown in the figure. It implies that at point *B*



(a) the force is zero
(b) there is a force acting towards the motion
(c) there is a force which opposes the motion
(d) there is only a gravitational force

31. A point moves with a uniform acceleration and v_1, v_2, v_3 denote the average velocities in three successive intervals of time t_1, t_2, t_3. Which of the following relations is correct?
(a) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$
(b) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$
(c) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$
(d) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$

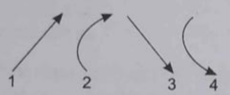
32. A bus starts moving with an acceleration of 1 ms^{-2}. A boy who is 48 m behind the bus starts running at 10 ms^{-1}. After what time will the boy be able to catch the bus?

(a) 6 s
(b) 8 s
(c) 10 s
(d) The boy cannot catch the bus

33. A train is moving along a straight path with a uniform acceleration. Its engine passes a pole with a velocity of 60 kmh^{-1} and the end (guard's van) passes across the same pole with a velocity of 80 kmh^{-1}. The middle point of the train will pass the same pole with a velocity

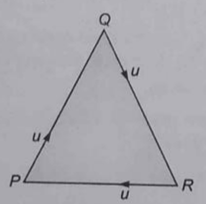
(a) 70 kmh^{-1} (b) 70.7 kmh^{-1}
(c) 65 kmh^{-1} (d) 75 kmh^{-1}

34. Particle P is moving in a straight line with a uniform velocity while the particle Q is moving in a straight line with an increasing velocity. Which of the following curves represents the apparent path of Q with respect to P?



(a) 1 (b) 2 (c) 3 (d) 4

35. Three persons P, Q and R of the same mass travel with the same speed u along an equilateral triangle of side d such that each one faces the other always. After how much time will they meet each other?



(a) $\frac{d}{u}$ s (b) $\frac{2d}{3u}$ s (c) $\frac{2d}{\sqrt{(3)}u}$ s (d) $\frac{d}{\sqrt{(3)}\,u}$ s

36. Four persons K, L, M and N are initially at the corners of a square of side of length d. If every person starts moving with the same speed v such that K is always headed towards L, L towards M, M is headed directly towards N and N towards K, then the four persons will meet after

(a) $\frac{d}{v}$ s (b) $\sqrt{(2)}\,\frac{d}{v}$ s
(c) $\frac{d}{\sqrt{(2)}v}$ s (d) $\frac{d}{2v}$ s

37. A juggler keeps 5 balls going with one hand, so that at any instant, 4 balls are in air and one ball in the hand. If each ball rises to a height of 20 m, then the time for which each ball stays in the hand is ($g = 10$ ms^{-2})

(a) 1 s (b) 1.5 s (c) 2 s (d) 4 s

38. From the top of a tower of height 50 m, a ball is thrown vertically upwards with a certain velocity. It hits the ground 10 s after it is thrown up. How much time does it take to cover a distance AB where A and B are two points 20 m and 40 m below the edge of the tower? ($g = 10$ ms^{-2})

(a) 2.0 s (b) 1 s
(c) 0.5 s (d) 0.4 s

39. A point moves in a straight line so that its displacement x metre at time t second is given by $x^2 = 1 + t^2$. Its acceleration in ms^{-2} at time (t) second is

(a) $\frac{1}{x^3}$ (b) $\frac{1}{x} - \frac{1}{x^2}$
(c) $-\frac{t}{x^2}$ (d) $\frac{1}{x} - \frac{t^2}{x^3}$

40. Car A is moving with a speed of 36 kmh^{-1} on a two lane road. Two cars B and C, each moving with a speed of 54 kmh^{-1} in opposite directions on the other lane are approaching car A. At certain instant of time, when the distance AB = distance, AC = 1 km, the driver of car B decides to overtake A before C does. What must be the minimum acceleration of car B so as to avoid an accident?

(a) 1 ms^{-2} (b) 4 ms^{-2}
(c) 2 ms^{-2} (d) 3 ms^{-2}

41. The displacement x of a particle varies with time, according to the relation $x = \frac{a}{b}(1 - e^{-bt})$. Then

(a) the particle can not reach a point at a distance x from its starting position, if $x' > a/b$
(b) at $t = 1/b$, the displacement of the particle is nearly $(2/3)(a/b)$
(c) the velocity and acceleration of the particle at $t = 0$ are a and $-ab$ respectively
(d) the particle will come back its starting point as $t \rightarrow \infty$

42. From the top of a tower, a stone is thrown up which reaches the ground in time t_1. A second stone thrown down, with the same speed, reaches the ground in time t_2. A third stone released from rest, from the same location, reaches the ground in a time t_3. Then,

(a) $\frac{1}{t_3} = \frac{1}{t_2} - \frac{1}{t_1}$ (b) $t_3^2 = t_1^2 - t_2^2$
(c) $t_3 = \frac{t_1 + t_2}{2}$ (d) $t_3 = \sqrt{t_1 t_2}$

43. The acceleration in ms^{-2} of a particle is given by, $a = 3t^2 + 2t + 2$, where, t is time. If the particle starts out with a velocity $v = 2$ ms^{-1} at $t = 0$, then the velocity at the end of 2 s is

(a) 36 ms^{-1} (b) 18 ms^{-1} (c) 12 ms^{-1} (d) 27 ms^{-1}

44. A bullet moving with a velocity of 100 ms^{-1} can just penetrate two plancks of equal thickness. The number of such plancks penetrated by the same bullet, when the velocity is doubled, will be

(a) 4 (b) 6 (c) 8 (d) 10

Directions (Q. Nos. 45 to 48) *Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true, Statement II is not the correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

45. Statement I A particle moving with a constant velocity, changes its direction uniformly.

Statement II In a uniform motion, the acceleration is zero.

46. Statement I Two objects moving with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ in the opposite directions, have their relative velocity along the direction of the one with a larger velocity.

Statement II The relative velocity between two bodies moving with velocity $\mathbf{v}_1$ and $\mathbf{v}_2$ is given by $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$

47. Statement I The average speed of the particle in a uniform motion is the same as its average velocity, if the particle moves in a straight line.

Statement II The average velocity is the rate of change of position of a particle moving along a straight path, per unit time.

48. Statement I The area of a velocity-time graph gives the distance travelled.

Statement II The slope of the velocity-time graph gives the acceleration of the body.

Directions (Q. Nos. 49 and 50) *A particle initially (i.e., at time $t=0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates as, $a = -k\sqrt{v}$ where v is the instantaneous velocity and k is a positive constant.*

49. The particle comes to rest in a time

(a) $\frac{2\sqrt{u}}{k}$ (b) $\frac{\sqrt{u}}{k}$
(c) $2k\sqrt{u}$ (d) $k\sqrt{u}$

50. The distance covered by the particle before coming to rest is

(a) $\frac{u^{3/2}}{k}$ (b) $\frac{2u^{3/2}}{k}$
(c) $\frac{3u^{3/2}}{2k}$ (d) $\frac{2u^{3/2}}{3k}$

Directions (Q. Nos. 51 to 54) *A particle is moving along x-axis. Its initial velocity is 40 ms^{-1} along positive x-axis and an acceleration of 10 ms^{-2} along negative x-axis. Particle starts from $x = 10$ m.*

51. Velocity of particle is zero at ..... second.

(a) 6 (b) 4
(c) 8 (d) 2

52. Maximum x-coordinate of particle (in positive direction) is .... m.

(a) 90 (b) 60
(c) 120 (d) 30

53. Velocity of particle at origin is ..... ms^{-1}.

(a) $30\sqrt{2}$ (b) $20\sqrt{2}$
(c) $-20\sqrt{2}$ (d) $-30\sqrt{2}$

54. Particle is at time .... second at origin.

(a) $4+3\sqrt{2}$ (b) $4+\sqrt{2}$
(c) $2+3\sqrt{2}$ (d) $3+2\sqrt{2}$

Directions (Q. Nos. 55 and 56) *For two particles A and B (moving with constant velocities) to collide, relative velocity of A with respect to B should be along AB. At time $t=0$, particle A is at (1m, 2m) and B is at (5m, 5m). Velocity of B along x-direction is 2 ms^{-1} and along y- direction is 4 ms^{-1}. Velocity of particle A is $\sqrt{2}$ v at 45° with the x-axis. A collides with B.*

55. Value of v is .... ms^{-1}.

(a) 5 (b) 15
(c) 25 (d) 10

56. Time when A will collide with B is ....

(a) 0.5 s (b) 1.5 s
(c) 4 s (d) 3 s

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57. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is **[JEE Main Online 2013]**

(a) 4.4 m
(b) 2.4 m
(c) 3.6 m
(d) 1.6 m

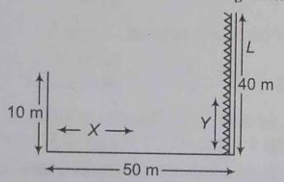
58. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40$ m. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity $v = 20$ m/s, on a horizontal surface? ($g = 10\ m/s^2$) **[JEE Main Online 2013]**

(a) 30° (b) 60°
(c) 75° (d) 45°

59. A projectile of mass M is fired so that the horizontal range is 4 km. At the highest point the projectile explodes in two parts of masses $M/4$ and $3M/4$ respectively and the heavier part starts falling down vertically with zero initial speed. The horizontal range (distance from point of firing) of the lighter part is **[JEE Main Online 2013]**

(a) 16 km (b) 1 km
(c) 10 km (d) 2 km

60. A person lives in a high-rise building on the bank of a river 50 m wide. Across the river is a well light tower of height 40 m. When the person, who is at a height of 10 m, looks through a polarizer at an appropriate angle at light of the tower reflecting from the river surface, he notes that intensity of light coming from distance X from his building is the least and this corresponds to the light coming from light bulbs at height 'Y' on the tower. The values of X and Y are respectively close to refractive index of water $\simeq \frac{4}{3}$. **[JEE Main Online 2013]**

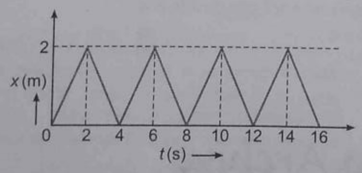


(a) 25 m, 10 m (b) 13 m, 27 m
(c) 22 m, 13 m (d) 17 m, 20 m

61. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where, v is the instantaneous speed. The time taken by the object, to come to rest, would be **[AIEEE 2011]**

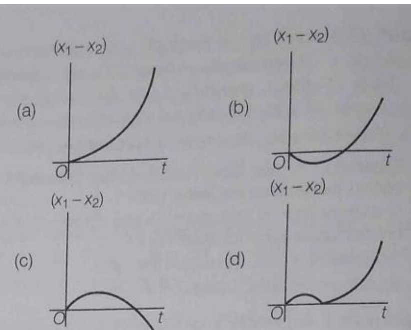
(a) 2 s (b) 4 s (c) 8 s (d) 1 s

62. The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[AIEEE 2010]**



(a) 0.4 N-s (b) 0.8 N-s (c) 1.6 N-s (d) 0.2 N-s

63. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t? **[AIEEE 2008]**



64. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is **[AIEEE 2007]**

(a) $v_0 + 2g + 3f$ (b) $v_0 + \frac{g}{2} + \frac{f}{3}$
(c) $v_0 + g + f$ (d) $v_0 + \frac{g}{2} + f$

65. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[AIEEE 2006]**

(a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

66. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance travelled is 15 s, then **[AIEEE 2005]**

(a) $s = ft$ (b) $s = \frac{1}{6}ft^2$
(c) $s = \frac{1}{72}ft^2$ (d) $s = \frac{1}{4}ft^2$

67. A small block slides without friction down an inclined plane starting from rest. Let s_n be the distance travelled from $t = n - 1$ to $t = n$. Then, $\frac{s_n}{s_{n+1}}$ is **[AIEEE 2004]**

(a) $\frac{2n-1}{2n}$ (b) $\frac{2n+1}{2n-1}$ (c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n}{2n+1}$

68. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $\frac{T}{3}$ s? **[AIEEE 2004]**

(a) $\frac{h}{9}$ m from the ground (b) $\frac{7h}{9}$ m from the ground
(c) $\frac{8h}{9}$ m from the ground (d) $\frac{17h}{18}$ m from the ground

69. Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is **[AIEEE 2002]**

(a) 1 : 1 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

Answers

1. (a)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (b)
21. (b)	**22.** (a)	**23.** (b)	**24.** (d)	**25.** (a)	**26.** (b)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (c)
31. (b)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (a)	**37.** (a)	**38.** (d)	**39.** (d)	**40.** (a)
41. (b)	**42.** (d)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (d)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (d)
51. (b)	**52.** (a)	**53.** (d)	**54.** (a)	**55.** (d)	**56.** (a)	**57.** (c)	**58.** (b)	**59.** (c)	**60.** (b)
61. (a)	**62.** (b)	**63.** (b)	**64.** (b)	**65.** (a)	**66.** (c)	**67.** (c)	**68.** (c)	**69.** (d)	

Hints & Solutions

1. Impulse = area under $(F-t)$ graph $= 18.51\,\text{Ns}$

$$\therefore \quad \Delta v = \frac{\text{Impulse}}{m} = \frac{18.51}{2} \qquad (\because J = \Delta P)$$

$$= 9.26\ \text{ms}^{-1}$$

$$v_f = v_i + \Delta v = 5 + 9.25 = 14.26\,\text{ms}^{-1}$$

2. Case I, Relative velocity is $v_1 + v_2 = 8$

Case II, Relative velocity is $v_1 - v_2 = 2$

On solving, $v_1 = 5\,\text{ms}^{-1}$, $v_2 = 3\,\text{ms}^{-1}$

3. Given, $x = 400$ m, $y = 300$ m, $z = 1200$ m

$$\therefore \quad r = \sqrt{x^2 + y^2 + z^2} = \text{magnitude of displacement}$$

$$= \sqrt{(400)^2 + (300)^2 + (1200)^2}$$

$$= 10^2\sqrt{16 + 9 + 144}$$

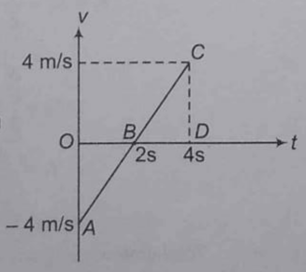
$$= 1300\ \text{m}$$

4. Here, $x = (t-2)^2$, Velocity $v = \frac{dx}{dt} = 2(t-2)$ m/s

Acceleration $a = \frac{dv}{dt} = 2\,\text{ms}^{-2}$ (*i.e.*, uniform)

When, $t = 0, v = -4$ m/s,
$t = 2\,s, v = 0$
$t = 4\,s, v = 4$ m/s

Velocity (v) – time (t) graph of this motion is as shown in figure.



Distance travelled

= area AOB + area BCD

$$= \frac{4\times 2}{2} + \frac{4\times 2}{2} = 8\,\text{m}$$

5. In graph (*b*), for one value of displacement, there are two timings. As a result of it, for one time, the average velocity is positive and for other time is equivalent negative. Due to it, the average velocity for the two timings (equal to time period) can vanish.

6. As, $u = 0$ and a is a constant

$$\frac{l}{4} = \frac{1}{2}a(2)^2 \qquad \text{...(i)}$$

$$l = \frac{1}{2}at^2 \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{l}{l/4} = \frac{t^2}{(2)^2}, \Rightarrow t^2 = 16$$

$$t = 4\,\text{s}$$

7. Acceleration of the body down the plane $= g\cos\theta$

Distance travelled by ball in time t second

$$= AB = \frac{1}{2}(g\cos\theta)t^2$$

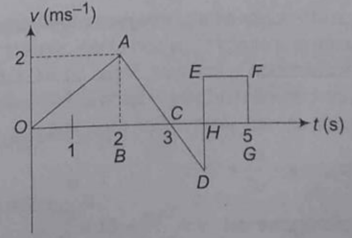
From $\Delta\, ABC$, $AB = 2R\cos\theta$

$$2R\cos\theta = \frac{1}{2}g\cos\theta\, t^2$$

$$t^2 = \frac{4R}{g} \text{ or } t = 2\sqrt{\frac{R}{g}}$$

8. Displacement is the algebraic sum of area under velocity-time graph.

As, displacement = area of triangles + area of rectangle



$\Delta\, OAB + \Delta\, ABC + \Delta CDH +\ HEFG$

$$= \frac{1}{2}\times 2\times 2 + \frac{1}{2}\times 1\times 2 + \frac{1}{2}\times 1\times 2 + 1\times 1$$

$$= 2 + 1 + 1 + 1 = 5$$

9. In one dimensional motion, there is a single value of distance at one particular time.

10. The distance travelled can never be negative in one dimensional motion.

11. Maximum acceleration is represented by the maximum slope of the velocity-time graph. Thus, it is the portion *CD* of the graph, which has a slope $= \frac{60-20}{40-30} = 4\,\text{ms}^{-2}$

12. As *x-t* graph is a straight line in either case, velocity of both is uniform. As the slope of x - t graph for P is greater, therefore, velocity of P is greater than that of Q.

13. If $(x, 0)$ and $(y, 0)$ are the coordinates of the end points of the rod at a given location, then, $x^2 + y^2 = l^2$

Differentiating it wrt time t, we get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -y\frac{dy/dt}{x} \text{ and } v_x = -\frac{y}{x}v_y$$

As, y decreases, x increases, so v_x decreases.

v_x becomes zero when y is zero.

14. Time taken to travel first half distance, $t_1 = \frac{L/2}{v_1} = \frac{L}{2v_1}$

Time taken to travel second half distance, $t_2 = \frac{L/2}{v_2} = \frac{L}{2v_2}$

Total time $= t_1 + t_2 = \frac{L}{2v_1} + \frac{L}{2v_2} = \frac{L}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]$

Average speed $= \frac{\text{Total distance}}{\text{Total time}} = \frac{L}{\frac{L}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]} = \frac{2v_1v_2}{v_1 + v_2}$.

15. Average acceleration

$$= \frac{\text{Change in velocity}}{\text{Time}} = \frac{25-(-25)}{5} = 10\,\text{ms}^{-2}$$

16. When a body is projected vertically upwards, at the highest point of its motion, the velocity of the body becomes zero but acceleration is not zero.

17. The slope of velocity-time graph gives acceleration. Since, the given graph is a straight line and slope of graph is constant. Hence acceleration is constant. Thus, (a) is correct. The area of *v-t* graph between 0 to 10 s is same as between 10 s to 20 s. Hence (c) is correct. Also option (d) is true.

18. Acceleration, $a = \frac{dv}{dt} = 5t + 6$

On integrating, we get $v = \frac{5}{2}t^2 + 6t = \frac{dx}{dt}$

Integrating again, $x = \frac{5}{6}t^3 + \frac{6}{2}t^2$

At, $t = 2\text{s}, x = \frac{5}{6} \times 8 + 3 \times 4 = 18.66\,\text{m}$

19.
$$v = 4t^3 - 2t \quad \text{...(i)}$$
$$\frac{dx}{dt} = 4t^3 - 2t$$

On integration, we get

$$x = 2 = t^4 - t^2 \quad \text{...(ii) (if } t^2 = \alpha)$$
$$= \alpha^2 - \alpha$$
$$\alpha^2 - \alpha - 2 = 0$$
$$(\alpha - 2)(\alpha + 1) = 0$$
$$\therefore \alpha = 2$$

$\alpha = -1$, which is not possible

$t^2 = \alpha = 2$ or $t = \sqrt{2}$,

Differentiating Eq. (i) w.r.t. t,

$$\frac{dv}{dt} = 12t^2 - 2$$
$$a = 12 \times 2 - 2 = 22\,\text{ms}^{-2}$$

20. Let s_1 be the distance travelled by the train moving with acceleration $1\,\text{ms}^{-2}$ for time t_1 and s_2 be the distance travelled by the train moving with retardation $3\,\text{ms}^{-2}$ for time t_2. If v is the velocity of the train after time t_1, then

$$v = 1 \times t_1 \quad \text{...(i)}$$
$$s_1 = \frac{1}{2} \times 1 \times t_1^2 = \frac{t_1^2}{2} \quad \text{...(ii)}$$

Also, $v = 3t_2$ ...(iii)

and $s_2 = vt_2 - \frac{1}{2} \times 3 \times t_2^2$

$$= t_1t_2 - \frac{3}{2}t_2^2$$

From Eqs. (i) and (iii), we get

$$t_1 = 3t_2$$

or $t_2 = \frac{t_1}{3}$

$$s_1 + s_2 = \frac{t_1^2}{2} + t_1 \times \frac{t_1}{3} - \frac{3}{2} \times \frac{t_1^2}{9}$$
$$= \frac{2}{3}t_1^2$$
$$1215 = \frac{2}{3}t_1^2$$
$$t_1 = \sqrt{\frac{3 \times 1215}{2}}$$
$$= 42.69\,\text{s}$$

Total time $= t_1 + t_2 = t_1 + \frac{t_1}{3}$

$$= 56.92\,\text{s}$$

21. When a body is thrown up, its velocity goes on decreasing as air resistance is small. When a body falls down, its velocity goes on increasing as air resistance is large, t_2 increases.

22. $a =$

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22. $a = +g = 10\ ms^{-2}$, $s = 65\ m$, $t = ?$

As, $$s = ut + \frac{1}{2}at^2$$

$\Rightarrow$ $$65 = -12t + 5t^2$$

$$5t^2 - 12t - 65 = 0$$

This gives, $$t = \frac{12 \pm \sqrt{144 + 1300}}{10}$$

$$= \frac{12 \pm 38}{10} = 5\ s$$

23. Using, $$D_n = u + \frac{a}{2}(2n - 1)$$

$$12 = u + \frac{a}{2}(2 \times 3 - 1) \quad \ldots(i)$$

$$20 = u + \frac{a}{2}(2 \times 5 - 1) \quad \ldots(ii)$$

Subtract Eq. (i) from Eq. (ii),

$$8 = \frac{a}{2}(10 - 6) = 2a$$

$$a = 4\ ms^{-2}$$

From Eq. (i), $$12 = u + \frac{4}{2} \times 5$$

$$u = 2\ ms^{-1}$$

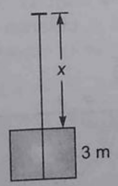
From, $$v = u + at = 2 + 4 \times 10$$

$$= 42\ ms^{-1}$$

24. From $s = ut + \frac{1}{2}at^2$,

$$x = 0 + \frac{1}{2} \times 10t^2 = 5t^2 \quad \ldots(i)$$

Also, $$x + 3 = 0 + \frac{1}{2} \times 10(t + 0.5)^2 = 5\left(t^2 + \frac{1}{4} + t\right) \quad \ldots(ii)$$



Subtract Eq. (i) from Eq. (ii)

$$3 = 5\left(\frac{1}{4} + t\right) = \frac{5}{4} + 5t$$

$$3 - \frac{5}{4} = 5t$$

$$\frac{7}{4} = 5t \text{ or } t = \frac{7}{20}\ s$$

From $v = u + at$,

$$v = 0 + 10 \times \frac{7}{20} = 3.5\ ms^{-1}$$

25. From principle of homogeneity, we check the dimensions on both sides

$$\frac{1}{2}\left(\frac{\alpha\beta}{\alpha + \beta}\right)t^2 = \frac{(acc)^2}{acc}t^2 = (acc)t^2$$

$$= [LT^{-2}] \times [T^2] = [L] = \text{distance}$$

26. As, $$D_n = u + \frac{g}{2}(2n - 1)$$

$$= 0 + \frac{g}{2}(2n - 1)$$

Distance travelled in the first three second

From $$s = ut + \frac{1}{2}at^2$$

$$s_3 = 0 \times 3 + \frac{1}{2} \times g \times 3^2 = \frac{9}{2}g$$

As, $$D_n = s_3$$

$$\frac{g}{2}(2n - 1) = \frac{9}{2}g$$

$$2n - 1 = 9$$

$$n = 5s$$

27. As, $$v = u + at_1 \quad \ldots(i)$$

$$40 = 0 + a \times 20$$

$$a = 2\ ms^{-2}$$

Now, $v^2 - u^2 = 2as$

$$40^2 - 0 = 2 \times 2 \times s_1$$

$$s_1 = 400\ m$$

$$s_2 = v \times t_2 \quad \ldots(ii)$$

$$= 40 \times 20 = 800\ m$$

and $$v = u + at \quad \ldots(iii)$$

$$0 = 40 + a \times 40,$$

$$a = -1\ ms^{-2}$$

Also, $$v^2 - u^2 = 2as$$

$$0^2 - 40^2 = 2(-1)\ s_3$$

$$s_3 = 800\ m$$

$\therefore$ Total distance travelled $= s_1 + s_2 + s_3$

$$= 400 + 800 + 800 = 2000\ m$$

and total time taken $= 20 + 20 + 40 = 80\ s$

$\therefore$ Average velocity $= \frac{2000}{80} = 25\ ms^{-1}$

28. From $s = ut + \frac{1}{2}at^2$,

$$s = 0 + \frac{1}{2} \times 10 \times 5^2 = 125\ m$$

Distance covered in 3 s,

$$= 0 + \frac{1}{2} \times 10 \times 3^2 = 45\ m$$

Distance to be covered = 125 − 45 = 80 m

From $s = ut + \frac{1}{2}at^2$, $80 = 0 + \frac{1}{2} \times 10t^2$

$\Rightarrow$ $t^2 = \frac{80}{5} = 16$

$\therefore$ $t = 4$ s

29. To stop the train, force is being applied on the train in the backward direction. Due to inertia of motion, the apple falls slightly away from the hand in the direction of motion of the train.

30. At B, slope of v-t curve is negative *i.e.*, velocity decreases with time. Hence, there is a force (and hence retardation) which opposes the motion.

31. Suppose velocity at O = zero

As average velocity in interval t_1 is v_1,

$\therefore$ Velocity at $A = v_1$

As average velocity in interval t_2 is v_2,

$\therefore$ Velocity at $B = (v_2 - v_1)$

As average velocity in interval t_3 is v_3,

Velocity at $C = (v_3 - v_2 + v_1)$

Using $v = u + at$

$v_1 = 0 + at_1$ ...(i)

$(v_2 - v_1) = 0 + a(t_1 + t_2)$ ...(ii)

$(v_3 - v_2 + v_1) = 0 + a(t_1 + t_2 + t_3)$ ...(iii)

Subtract Eq. (i) from Eq. (iii), we get

$(v_3 - v_2) = a(t_2 + t_3)$ ...(iv)

Divide Eq. (ii) by Eq. (iv), we get

$$\frac{(v_2 - v_1)}{(v_3 - v_2)} = \frac{a(t_1 + t_2)}{a(t_2 + t_3)}$$

$$\frac{(v_1 - v_2)}{(v_2 - v_3)} = \frac{t_1 + t_2}{t_2 + t_3}$$

32. The bus moves a distance, $x = \frac{1}{2}at^2$

The man moves a distance, $x + 48 = v \times t$

$$\frac{1}{2} \times 1 \times t^2 + 48 = 10t$$

$$t^2 - 20t + 96 = 0$$

$$(t - 12)(t - 8) = 0$$

$$t = 12 \text{ s}, t = 8 \text{ s}$$

Man will cross the bus after 8 s. But again after 12 s bus will cross him due to accelerated motion.

33. From, $v^2 - u^2 = 2as$

$$\frac{80^2 - 60^2}{2a} = s$$

$$s = \frac{6400 - 3600}{2a} = \frac{1400}{a}$$

The middle point of the train is to cover a distance

$$\frac{s}{2} = \frac{700}{a}$$

From, $v^2 - u^2 = 2as$

$$v^2 - 60^2 = 2a \times \frac{700}{a} = 1400$$

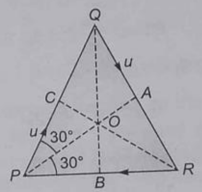
$$v^2 = 1400 + 3600$$

$$v = \sqrt{5000} = 70.7 \text{ km h}^{-1}$$

34. Velocity of Q w.r.t. P is increasing, therefore, slope of displacement-time graph is increasing as in case of curve (2).

35. The person at P will travel a distance PO, with velocity along $PO = u \cos 30°$

Here,



$$PO = PB \sec 30°$$

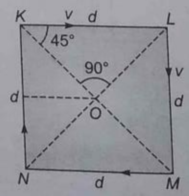
$$= \frac{d}{2} \times \frac{2}{\sqrt{3}} = \frac{d}{\sqrt{3}}$$

$\therefore$ Time of meeting,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{d/\sqrt{3}}{u \cos 30°}$$

$$= \frac{d/\sqrt{3}}{u\sqrt{3}/2} = \frac{2d}{3u} \text{ second}$$

36. The four persons, K, L, M and N will meet at O. *i.e.*, the centre of the diagonal of the square $KLMN$. The person K will travel a distance KO, with velocity along $KO = v \cos 45° = \frac{v}{\sqrt{2}}$



Here, $KO = d \cos 45° = \frac{d}{\sqrt{2}}$

Time of meeting,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{d/\sqrt{2}}{v/\sqrt{2}} = \frac{d}{v} \text{ s}$$

37. Time for one ball in hand,

$$= \frac{T}{n-1} = \frac{2\sqrt{\frac{2x}{g}}}{(n-1)} = \frac{2}{(n-1)}\sqrt{\frac{2x}{g}}$$

$$= \frac{2}{(5-1)}\sqrt{\frac{2 \times 20}{10}} = 1\text{ s}$$

38. Given, $v = -u$, $a = g = 10\text{ ms}^{-2}$, $s = 50\text{ m}$, $t = 10\text{ s}$

As, $$s = ut + \frac{1}{2}at^2,$$

$$\Rightarrow \quad 50 = -u \times 10 + \frac{1}{2} \times 10 \times 10^2$$

On solving, $u = 45\text{ ms}^{-1}$

If, t_1 and t_2 are the timings taken by the ball to reach the points A and B respectively, then

$$20 = -45\, t_1 + \frac{1}{2} \times 10 \times t_1^2$$

$$40 = -45\, t_2 + \frac{1}{2} \times 10 \times t_2^2$$

On solving, we get $t_1 = 9.4\text{ s}$ and $t_2 = 9.8\text{ s}$

Time taken to cover the distance AB,

$$= (t_2 - t_1) = 9.8 - 9.4 = 0.4\text{ s}$$

39. As, $x^2 = (1 + t^2)$ or $x = (1 + t^2)^{1/2}$

Velocity, $$\frac{dx}{dt} = \frac{1}{2}(1 + t^2)^{-1/2} \times 2t = t\,(1 + t^2)^{-1/2}$$

Acceleration, $$\frac{d^2x}{dt^2} = t\left(-\frac{1}{2}\right) \times (1 + t^2)^{-3/2} \times 2t + (1 + t^2)^{-1/2}$$

$$= \frac{1}{x} - \frac{t^2}{x^3}$$

40. Let us suppose that the cars A and B are moving in the positive x-direction. Then, car C is moving in the negative x-direction.

Therefore, $v_A = 36\text{ kmh}^{-1} = 10\text{ ms}^{-1}$

$v_B = 54\text{ kmh}^{-1} = 15\text{ ms}^{-1}$

and $v_C = -54\text{ kmh}^{-1} = -15\text{ ms}^{-1}$

Thus, the relative speed of B with respect to A is,

$$v_{BA} = v_B - v_A = 15 - 10 = 5\text{ ms}^{-1}$$

and the relative speed of C with respect to A is,

$$v_{CA} = v_C - v_A = -15 - 10 = -25\text{ ms}^{-1}$$

At time $t = 0$, the distance between A and B = distance between A and $C = 1\text{ km} = 1000\text{ m}$.

The car C covers a distance $AC = 1000\text{ m}$ and reaches car A at a time t given by

$$t = \frac{AC}{|v_{CA}|} = \frac{1000\text{ m}}{25\text{ ms}^{-1}} = 40\text{ s}$$

Car B will overtake car A just before car C does and the accident can be avoided if it acquires a minimum acceleration a such that it covers a distance, $s = AB = 1000\text{ m}$ in time $t = 40\text{ s}$ travelling with a relative speed of $u = v_{BA} = 5\text{ms}^{-1}$.

This gives, from $s = ut + \frac{1}{2}at^2$, $a = 1\text{ms}^{-1}$

41. Velocity of the particle is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{\frac{a}{b}(1 - e^{-bt})\right\} = ae^{-bt}$$

Acceleration of the particle is given by

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}(ae^{-bt}) = -abe^{-bt}$$

At $t = 1/b$, the displacement of the particle is

$$x = \frac{a}{b}(1 - e^{-1}) \approx \frac{a}{b}\left(1 - \frac{1}{3}\right) = \frac{2}{3}\frac{a}{b} \qquad \left(\because e^{-1} \approx \frac{1}{3}\right)$$

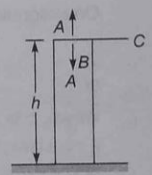
Thus, choice (b) is correct. At $t = 0$, the value v and α are $v = ae^{-0} = a$ and $\alpha = -ab\,e^{-0} = ab$

The displacement x is maximum, when

$$t \to \infty,$$

i.e., $$t_{max} = \frac{a}{b}(1 - e^{-\infty}) = \frac{a}{b}$$

42. We know that, $h = ut + \frac{1}{2}gt^2$



$$\Rightarrow \quad h = -ut + \frac{1}{2}gt^2$$

and $$t = \frac{2u}{g} = t_1 - t_2 \qquad \ldots\text{(i)}$$

Case I $$h = -ut_1 + \frac{1}{2}gt_1^2 \qquad \ldots\text{(ii)}$$

Case II $$h = +ut_2 + \frac{1}{2}gt_2^2 \qquad \ldots\text{(iii)}$$

Case III $$h = \frac{1}{2}gt_3^2 \qquad \ldots\text{(iv)}$$

This gives, $$\frac{2h}{g} = \frac{2u}{g}t_2 + t_2^2 \qquad \ldots\text{(v)}$$

Solving these, give us, $t_3^2 = (t_1 - t_2)t_2 + t_2^2$

$$\Rightarrow \quad t_3 = \sqrt{t_1 t_2}$$

43. Given, $$a = \frac{dv}{dt} = 3t^2 + 2t + 2$$

$$\Rightarrow \quad dv = (3t^2 + 2t + 2)\,dt$$

On integrating, this gives $\int_u^v dv = \int_0^t (3t^2 + 2t + 2)\,dt$

$$\Rightarrow \quad v - u = \left[\frac{3t^3}{3} + \frac{2t^2}{2} + 2t\right]_0^t$$

$$\Rightarrow \quad v = u + [t^3 + t^2 + 2t]_0^t = 18\text{ ms}^{-1}$$

44. Given that the initial velocity of the bullet in the first case is $u_1 = 10$ cm/s.

Initial number of plancks, $n_1 = 2$

Initial stopping distance $= s_1 = n_1 x = 2x$, with x as the thickness of one planck.

Similarly, Initial velocity of the bullet in second case,

$$u_2 = 20 \text{ cm/s}$$

We know that the relation for the stopping distance s is

$$v^2 = u^2 + 2as$$

Since, $v = 0$,

So, $2as = -u^2$

As, $s \propto u^2$

Hence, $$\frac{s_1}{s_2} = \left(\frac{u_1}{u_2}\right)^2 = \left(\frac{100}{200}\right)^2 = \frac{1}{4}$$

Thus, $s_2 = 4s_1 = 8x$

Hence, the number of plancks $= n_2 = \frac{s_2}{x} = 8$

49. Given, $a = -kv^{1/2}$ or $\frac{dv}{dt} = -kv^{1/2}$

Thus, $v^{-1/2}dv = -k\,dt$

On integrating, we have

$$\int v^{-1/2}dv = -k\int dt$$

or $$2v^{1/2} = -kt + c \quad \text{...(i)}$$

Where, c is the constant of integration. Given that at $t = 0$, $v = u$. Using this in Eq. (i), we get $2u^{1/2} = c$. Using this value of c in Eq. (i), we have

$$2(v^{1/2} - u^{1/2}) = -kt \quad \text{...(ii)}$$

Let τ be the time taken by the particle to come to rest. Then, $v = 0$ at $t = \tau$.

Using this in Eq. (ii), we get

$$2(0 - u^{1/2}) = -k\tau$$

or $$\tau = \frac{2u^{1/2}}{k} = \frac{2\sqrt{u}}{k} \quad \text{...(iii)}$$

50. To find the distance s covered in this time, we use Eq. (i) to get

$$v^{1/2} = u^{1/2} - \frac{kt}{2}$$

Squaring, we have

$$v = u - ktu^{1/2} + \frac{k^2t^2}{4}$$

But $$v = \frac{ds}{dt}$$

Therefore, $$\frac{ds}{dt} = u - kt\,u^{1/2} + \frac{k^2t^2}{4}$$

Integrating from $t = 0$ to $t = \tau$, we have

$$s = \left| ut - \frac{ku^{1/2}t^2}{2} + \frac{k^2t^3}{12} \right|_0^\tau$$

or $$s = u\tau - \frac{1}{2}ku^{1/2}\tau^2 + \frac{1}{12}k^2\tau^3 \quad \text{...(iv)}$$

Substituting the value of t from Eq. (iii) in Eq. (iv), we get

$$s = \frac{2u^{3/2}}{k} - \frac{4u^{3/2}}{2k} + \frac{8u^{3/2}}{12k} = \frac{2u^{3/2}}{3k}$$

51-54 Given, $u = 40 \text{ ms}^{-1}$, $a = -10 \text{ ms}^{-2}$

From, $v = u + at$

$0 = 40 - 10t$ or $t = 4$ s (answer question 51)

$$x_{max} = x_i + \left|\frac{u^2}{2a}\right|$$

$$= 10 + \frac{40^2}{2 \times 10} = 90 \text{ m} \quad \text{(answer question 52)}$$

At origin, $s = -10$ m, substituting in $v^2 = u^2 + 2as$,

We have, $v^2 = (40)^2 + (2)(-10)(-10) = 1800$

$v = -\sqrt{1800} \text{ ms}^{-1}$

$= -30\sqrt{2} \text{ ms}^{-1}$ (answer question 53)

From, $v = u + at$

$$t = \frac{v - u}{a} = \frac{-30\sqrt{2} - 40}{-10}$$

$= (4 + 3\sqrt{2})$ s (answer question 54)

55. According to question,

$(v_{AB})_x = (v - 2)$ m/s

and $(v_{AB})_y = (v - 4)$ m/s

The displace of particle along x, $(AB)_x = 4$ m and along y $(AB)_y = 3$ m $\mathbf{v}_{AB} \uparrow\uparrow \mathbf{AB}$ if

$$\frac{v-2}{4} = \frac{v-4}{3} \text{ or } v = 10 \text{ ms}^{-1}$$

56. $|\mathbf{v}_{AB}| = 10 \text{ ms}^{-1}$, $|\mathbf{AB}| = 5 \text{ ms}^{-1}$

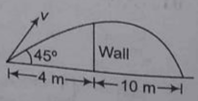
$\therefore$ $$t = \frac{|\mathbf{AB}|}{|\mathbf{v}_{AB}|} = 0.5 \text{ s}$$

57. As range $= 10 = \frac{u^2 \sin 2\theta}{g} \Rightarrow u^2 = 10\,g$

$\therefore$ $u = 10$ m/s (as $g = 10 \text{ m/s}^2$)

$$Y = x\tan\theta - \frac{1}{2}\frac{gx^2}{2v_0^2\cos^2\theta}$$

$$= 4\tan 45° - \frac{1}{2}\frac{g \times 16}{2\,v_0^2\cos^2 45°}$$



$$= 4 \times 1 - \frac{1}{2}\frac{10 \times 16}{2 \times 10 \times 10 \times \frac{1}{2}} = 4 - 0.8 = 2 \approx 3.6 \text{ m}$$

58. According to question $\frac{u^2}{g} = 40$

$\therefore \quad u = 20$ m/s and $T = \frac{2u \sin 45°}{g}$

$$= 2 \times 20 \times \frac{1}{\sqrt{2}} \times \frac{1}{10} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

When car is moving with speed, $v = 20$ m/s then

$$(v \cos\theta + 20) \times t = 40$$

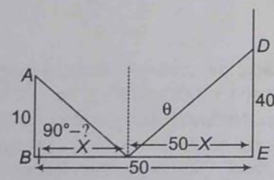
$$(20 \cos\theta + 2v) \times 2\sqrt{2} = 40$$

$\Rightarrow$
$$\cos^2 \frac{\theta}{2} = \frac{1}{2\sqrt{2}} \approx 60°$$

59. As, we consider the n th electron which attracted by nucleus while itself is repelled by the electron of inner shell to the consider shell.

60. In ΔABC, $\tan(90° - \theta) = \frac{10}{x}$

or
$$\cot\theta = \frac{10}{x} \quad \text{...(i)}$$



and from ΔCDE,

$$\tan\theta = \frac{10}{50 - x} \quad \text{...(ii)}$$

$\therefore$ Multiplying Eqs. (i) and (ii), we get

$$\cot\theta \times \tan\theta = \frac{10}{x} \times \frac{10}{50 - x}$$

$$1 = \frac{100}{(50 - x)x} \Rightarrow 50x - x^2 - 100 = 0$$

or $\quad x^2 - 50x + 100 = 0$

$\therefore$
$$x = \frac{50 \pm \sqrt{2500 - 400}}{2}$$

By solving this, we have, $x = 27$ m

Hence, the other distance $= 13$ m

61. Given, $\quad \frac{dv}{dt} = -2.5\sqrt{v}$

$\Rightarrow \quad \frac{dv}{\sqrt{v}} = -2.5dt$

$\Rightarrow \quad \int_{6.25}^{0} v^{-1/2}\, dv = -2.5 \int_0^t dt$

$\Rightarrow \quad -2.5\,[t]_0^t = [2\, v^{1/2}]_{6.25}^{0}$

$\Rightarrow \quad t = 2$ s

62. From the graph, it is a straight line so, uniform motion. Because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity $v_1 = \frac{2}{2} = 1\text{ms}^{-1}$

Final velocity $v_2 = \frac{2}{2} = -1\text{ms}^{-1}$

$$\mathbf{p}_i = mv_1 = 0.4 \text{ N-s}$$

$$\mathbf{p}_f = mv_2 = -0.4 \text{ N-s}$$

$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_i = -0.4 - 0.4 = -0.8$ N-s(where, $\mathbf{J}$ = impulse)

$\therefore \quad |\mathbf{J}| = 0.8$ N-s

63. As, $x_1(t) = \frac{1}{2}at^2$ and $x_2(t) = vt$

$\therefore \quad x_1 - x_2 = \frac{1}{2}at^2 - vt$ (parabola)

Clearly, graph (*b*) represents it correctly.

64. As, $v = v_0 + gt + ft^2$ or $\frac{dx}{dt} = v_0 + gt + ft^2$

$\Rightarrow \quad dx = (v_0 + gt + ft^2)\,dt$

So, $\quad \int_0^x dx = \int_0^1 (v_0 + gt + ft^2)\,dt$

$\Rightarrow \quad x = v_0 + \frac{g}{2} + \frac{f}{3}$

65. Given, $\quad v = \alpha\sqrt{x}$

or $\quad \frac{dx}{dt} = \alpha\sqrt{x} \qquad \left(\because v = \frac{dx}{dt}\right)$

or $\quad \frac{dx}{\sqrt{x}} = \alpha\, dt$

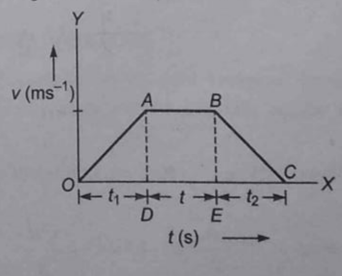
On integration $\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$

[$\because$ at $t = 0$, $x = 0$ and let at any time t, particle is at x]

$\Rightarrow \quad \left.\frac{x^{1/2}}{1/2}\right|_0^x = \alpha t$ or $x^{1/2} = \frac{\alpha}{2}t$

or $\quad x = \frac{\alpha^2}{4} \times t^2$ or $x \propto t^2$

66. The velocity-time graph for the given situation can be drawn as below. Magnitudes of slope of $OA = f$



and slope of $BC = \frac{f}{2}$

$$v = ft_1 = \frac{f}{2}t_2$$

$$t_2 = 2t_1$$

In graph area of ΔOAD gives distance,

$$s = \frac{1}{2}ft_1^2 \quad \ldots(i)$$

Area of rectangle $ABED$ gives distance travelled in time t

$$s_2 = (ft_1)t$$

Distance travelled in time t_2,

$$s_3 = \frac{1}{2}\frac{f}{2}(2t_1)^2$$

Thus, $s_1 + s_2 + s_3 = 15\ s$

$\Rightarrow \quad s + (ft_1)t + ft_1^2 = 15\ s$

or $\quad s + (ft_1)t + 2s = 15\ s \left(s = \frac{1}{2}ft_1^2\right)$

or $\quad (ft_1) = 12s \quad \ldots(ii)$

From Eqs. (i) and (ii), we have

$$\frac{12s}{s} = \frac{(ft_1)t}{\frac{1}{2}(ft_1)t_1} \text{ or } t_1 = \frac{t}{6}$$

From Eq. (i), we get $s = \frac{1}{2}f(t_1)^2$

$$= \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{1}{72}ft^2$$

67. Distance travelled in t^{th} second is,

$$s_t = u + at - \frac{1}{2}a$$

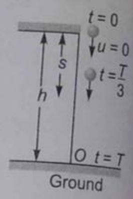
Given, $u = 0$

$$\therefore \quad \frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$$

68. From law of motion gives,

$$s = ut + \frac{1}{2}gT^2$$

or $\quad h = 0 + \frac{1}{2}gT^2 \quad (\because u = 0)$



$\Rightarrow \quad T = \sqrt{\left(\frac{2h}{g}\right)}$

At, $\quad t = \frac{T}{3}$ s,

$$s = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2$$

or $\quad s = \frac{1}{2}g \cdot \frac{T^2}{9}$

$\Rightarrow \quad s = \frac{g}{18} \times \frac{2h}{g} \quad \left(\because T = \sqrt{\frac{2h}{g}}\right)$

or $\quad s = \frac{h}{9}$ m

Hence, the position of ball from the ground

$$= h - \frac{h}{9} = \frac{8h}{9} \text{ m}$$

69. In this question, the cars are identical means coefficient of friction between the tyre and the ground is same for both the cars, as a result retardation is same for both the cars equal to μg. Let first car travel distance s_1, before stopping while second car travel distance s_2, then from

$$v^2 = u^2 - 2as$$

$\Rightarrow \quad 0 = u^2 - 2\mu g \times s_1$

$\Rightarrow \quad s_1 = \frac{u^2}{2\mu g}$

and $\quad 0 = (4u)^2 - 2\mu g \times s_2$

$\Rightarrow \quad s_2 = \frac{16u^2}{2\mu g} = 16s_1$

$\Rightarrow \quad \frac{s_1}{s_2} = \frac{1}{16}$