Day 27

Ray Optics

Day 27 Outlines ...

- Reflection of Light
- Refraction of Light
- Lens
- Total Internal Reflection
- Deviation of Light by a Prism
- Dispersion of Light by a Prism

Reflection of Light

According to the laws of reflection, (i) the incident ray, reflected ray and the normal drawn on the reflecting surface at the point of incidence lie in the same plane, and (ii) the angle of incidence $\angle i$ = angle of reflection $\angle r$.

Laws of reflection are true for reflection from a polished mirror or from an unpolished surface or for diffused reflection.

Whenever reflection takes place from a denser medium, the reflected rays undergo a phase change of π.

Reflection from a Plane Mirror

1. If a ray is incident on a plane mirror at an angle of incidence i, then it suffers a deviation of $(\pi - 2i)$.
2. While keeping an object fixed, a plane mirror is rotated in its plane by an angle θ, then the reflected ray rotates in the same direction by an angle 2θ.
3. Focal length as well as the radius of curvature of a plane mirror is infinity. Power of a plane mirror is zero.

4. If two plane mirrors are inclined to each other at an angle θ, the total number of images formed of an object kept between them, is $n = \frac{2\pi}{\theta}$ or, $\left(\frac{2\pi}{\theta} - 1\right)$, whichever is odd.

5. The minimum size of a plane mirror fixed on a wall of a room, so that a person at the centre of the room may see the full image of the wall behind him, should be $\frac{1}{3}$rd the size of the wall.

Reflection from a Spherical Mirror

1. A spherical mirror is a part of a hollow sphere whose one surface is polished so that it becomes reflecting. The other surface of the mirror is made opaque.
2. Images formed by a concave mirror may be real or virtual, may be inverted or erect, and may be smaller, larger or equal in size to that of the object. The image is virtual and erect when the object is placed between the pole and the principal focus of concave mirror. In all other cases, the image formed is real and inverted one.
3. Image formed by a convex mirror is virtual, erect and diminished in size irrespective of the position of the object. Moreover, image is formed in between the pole and the principal focus of the mirror.
4. The focal length of a spherical mirror is half of its radius of curvature, *i.e.*, $f = \frac{R}{2}$.

Mirror Formula

Let an object be placed at a distance u from the pole of a mirror and its image is formed at a distance v from the pole. Then, according to **mirror formula** $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

- The power of a mirror (in dioptre), is given as

$$P = \frac{1}{f(\text{in metre})}$$

- If a thin object of height h is placed perpendicular to the principal axis of a mirror and the height of its image be h', then the transverse or lateral magnification produced is given by

$$m = \frac{h'}{h} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

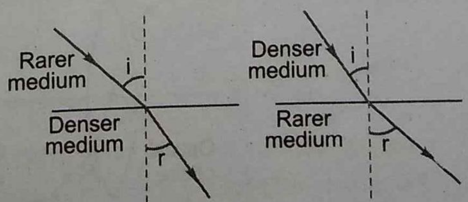
Negative sign of magnification means the inverted image and positive sign means an erect image.

- When a small sized object is placed along the principal axis, then its longitudinal (or axial) magnification is given by

$$\text{Axial magnification} = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Refraction of Light

When light passes from one medium, say air, to another medium, say glass, a part is reflected back into the first medium and the rest passes into the second medium. When it passes into the second medium, it either bends towards the normal or away from the normal. This phenomenon is known as refraction.



(a) (b)

Snell's Law

For a given pair of media, the ratio of the sine of angle of incidence (i) to the sine of angle of refraction (r) is a constant, which is called the refractive index of 2nd medium, w.r.t. 1st medium. Thus,

$$\frac{\sin i}{\sin r} = \text{constant} = n_{21} = \frac{n_2}{n_1}$$

Refractive Index

Refractive index is a unitless, dimensionless and a scalar quantity.

The refractive index of a medium w.r.t. vacuum (or free space) is known as its absolute refractive index. It is defined as the ratio of the speed of light in vacuum (c) to the speed of light in a given medium (v).

$$\therefore \quad n = \frac{c}{v}$$

Value of absolute refractive index of a medium can be 1 or more than 1 but never less than 1.

When light travels from one material medium to another, the ratio of the speed of light in the 1st medium to that in the 2nd medium is known as the relative refractive index of 2nd medium, w.r.t. the 1st medium. Thus,

$$n_{21} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{n_2}{n_1}$$

- *When light undergoing refraction through several media finally enters the Ist medium itself, then*

$$n_{21} \times n_{32} \times n_{13} = 1$$

or $$n_{32} = \frac{n_{31}}{n_{21}}$$

- *When the object is in denser medium and the observer is in rarer medium, then real and apparent depth have the relationship,* $\frac{\text{Real depth}}{\text{Apparent depth}} = n_{21}$

i.e., real depth > apparent depth

$$\text{Shift } y = h - h' = \left(1 - \frac{1}{n_{21}}\right)h$$

where h and h' are real and apparent depths.

Refraction from a Spherical Surface

Let an object be placed in a medium of refractive index n_1 at a distance u from the pole of a spherical surface of radius of curvature R and after refraction, its image is formed in a medium of refractive index n_2 at a distance v, then

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

The relation is true for all surfaces, whether the image formed is real or virtual.

Lens

A lens is part of a transparent refracting medium bound by two surfaces, with atleast one of the two surfaces being a curved one. The curved surface may be spherical or cylindrical.

Images formed by convex lens may be real or virtual, may be inverted or erect and may be smaller, larger or equal in size to that of the object. The image is virtual, erect and magnified when the object is placed between the optical centre and the principal focus of a convex lens. In all other cases, the image formed is real and inverted one.

Image formed by a concave lens is virtual, erect and diminished in size irrespective of the position of the object. Moreover, image is formed somewhere in between the optical centre and the principal focus of the lens.

The lens formula is given by $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For a thin object of height h placed perpendicular to the principal axis at a distance u, if the height of image formed is h', then **lateral** or **transverse magnification** m is given by

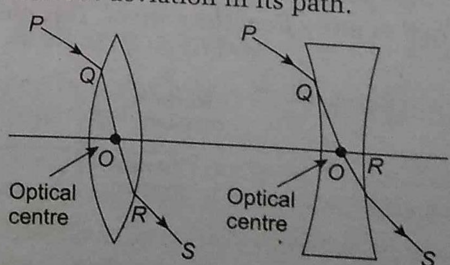
$$m = \frac{h'}{h} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

For a small sized object placed linearly along the principal axis, its **axial** or **longitudinal magnification** is given by

$$\text{Axial magnification} = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f+u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Some Definitions Relating Lenses

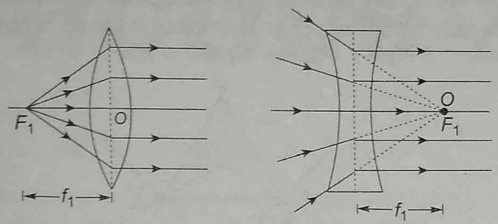
Optical centre The optical centre is a point within or outside the lens, at which incident rays refract without deviation in its path.



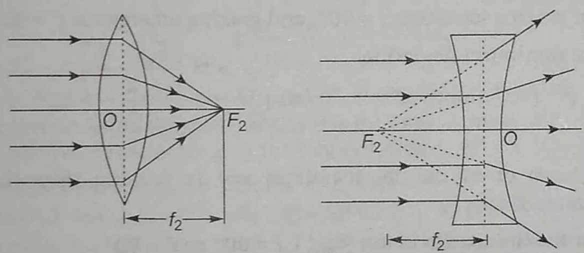
Principal axis The straight line passing through the optical centre of lens is called principal axis of lens.

Principal focus Lens has two principal foci.

(i) **First principal focus** It is a point on the principal axis of lens, the rays starting from which (convex lens) or appear to converge at which (concave lens) become parallel to principal axis after refraction.



(ii) **Second principal focus** It is the point on the principal axis at which the rays coming parallel to the principal axis converge (convex lens) or appear to diverge (concave lens) after refraction from the lens.



Both the foci of convex lens are real, while that of concave lens are virtual.

Focal length The distance between focus and optical centre of lens is called focal length of lens.

Newton's Formula

Let an object be situated at a distance x_1 from the first principal focus and its image is formed at a distance x_2 from the second principal focus, then $x_1 x_2 = f^2$

Power of Lens

The **power of a lens** is mathematically given by the reciprocal of its focal length,

i.e., power $\quad P = \dfrac{1}{f(\text{m})}$

SI unit of power is dioptre (D). Power of a converging lens is positive and that of a diverging lens is negative.

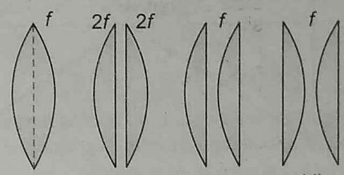
Lens Maker's Formula

For a lens having surfaces with radii of curvature R_1 and R_2 respectively, its focal length is given by $P = \dfrac{1}{f} = (n_{21} - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

where, $n_{21} = \dfrac{n_2}{n_1}$ = refractive index of the lens material w.r.t. the surroundings

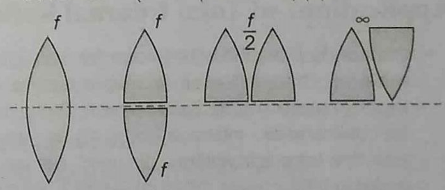
Cutting of a Lens

- If a symmetrical convex lens of focal length f is cut into two parts along its optic axis, then focal length of each part (a plano-convex lens) is $2f$. However, if the two parts are joined as shown in the figure, the focal length of the combination is again f.



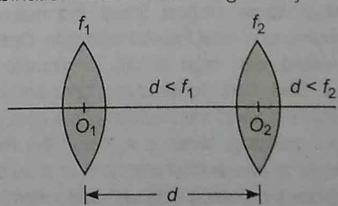
(a) (b) (c) (d)

- If a symmetrical convex lens of focal length f is cut into two parts along the principal axis, then the focal length of each part remains unchanged, as f(b). If these two parts are joined with the curved ends on one side, the focal length of the combination is $\dfrac{f}{2}$(c). But on joining the two parts in opposite sense, the net focal length becomes ∞ (or net power = 0) (d).



(a) (b) (c) (d)

The equivalent focal length of co-axial combination of two lenses is given by



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

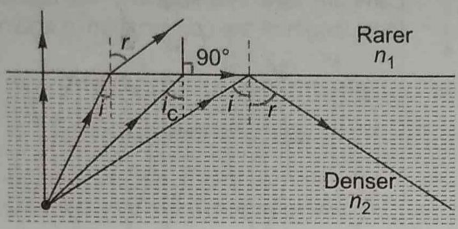
If a number of lenses are in contact, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots$$

- *If two thin lenses of focal lengths f_1 and f_2 are in contact, then their equivalent focal length,*
$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$
- *In terms of power, $P_{eq} = P_1 + P_2$*

Total Internal Reflection (TIR)

When a ray of light goes from a denser to a rarer medium, it bends away from the normal. For a certain angle of incidence i_C, the angle of refraction in rarer medium becomes 90°. The angle i_C is called the **critical angle.**



$$\sin i_C = \frac{n_1}{n_2} = \frac{1}{n_{21}}$$

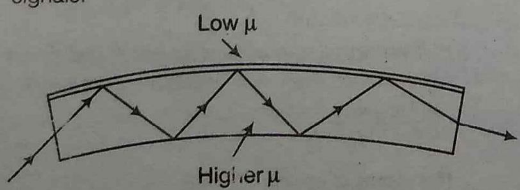
For the angle of incidence greater than the critical angle ($i > i_C$) in the denser medium, the light ray is totally internally reflected back into the denser medium itself.

Conditions for Total Internal Reflection

(i) The light ray should travel from the denser medium towards the rarer medium. (ii) The angle of incidence should be the greater than the critical angle.

Applications of Total Internal Reflection

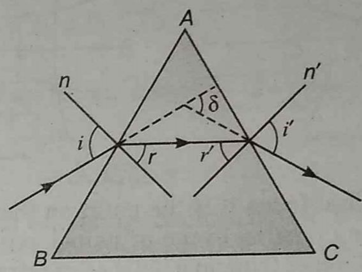
- **Diamonds** Diamonds are known for their spectacular brilliance. Their brilliance is mainly due to the total internal reflection of light inside them. The critical angle for diamond-air interface ($\cong 24.4°$) is very small, therefore once light enters a diamond, it is very likely to undergo total internal reflection inside it. By cutting the diamond suitably, multiple total internal reflections can be made which are responsible for the shine of diamond.
- **Optical fibres** Optical fibres too make use of the phenomena of total internal reflection. Optical fibres are fabricated with high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of cladding. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflection along the length of the fibre and finally comes out at the other end. Since, light undergoes total internal reflection at each stage there is no appreciable loss in intensity of light. Optical fibres are used for transmission of optical signals.



Deviation by a Prism

A prism is a homogeneous, transparent medium bounded by tw plane surfaces inclined at an angle A with each other. The surfaces are called as refracting surfaces and the angle betwe them is called angle of prism A. Deviation produced by a prism i

$$\delta = i + i' - A \Rightarrow r + r' = A$$



For grazing incidence $i = 90°$, and grazing emergence $i' = 90°$.

For minimum deviation

(i) $i = i'$ and $r = r'$ (ii) $\mu = \dfrac{\sin\left(\dfrac{\delta_m + A}{2}\right)}{\sin\dfrac{A}{2}}$

In case of minimum deviation, ray is passing through prisı symmetrically.

For maximum deviation (δ_{max}), $i = 90°$ or $i' = 90°$

For thin prism, $\delta = (\mu - 1) A$

Dispersion by a Prism

Dispersion of light is the phenomenon of splitting of white ligh into its constituent colours on passing light through a prism. Thi is because different colours have different wavelength, and henc different refractive indices

Angular dispersion $= \delta_v - \delta_r = (n_v - n_r) A$, where n_v and n represent refractive index for violet and red lights.

Dispersive power, $\omega = \dfrac{n_v - n_r}{n - 1}$, where $n = \dfrac{n_v + n_r}{2}$ is the mea refractive index.

> *By combining two prisms with angle A and A′ and RF n and n′ respectively we can create conditions of*
>
> *(a) Dispersion without deviation when,* $A' = -\dfrac{(n-1)A}{(n'-1)}$
>
> *(b) Deviation without dispersion when* $A' = -\left[\dfrac{n_v - n_r}{n'_v - n'_r}\right] A$

Refraction Through a Prism

A ray of light suffers two refractions at the two surfaces o passing through a prism.

Angle of deviation through a prism $\delta = i + e - A$

where i is the angle of incidence, e is the angle of emergence an A is the angle of prism.

Practice Zone

DAY 27

1. Light travels through a glass plate of thickness t and having refractive index n. If c be the speed of light in vacuum, the time taken by the light to travel through this thickness of glass is
(a) $\frac{t}{nc}$ (b) tnc
(c) $\frac{nt}{c}$ (d) $\frac{tc}{n}$

2. A ray of light is incident on the surface of separation of a medium at an angle 45° and is refracted in the medium at an angle 30°. What will be the speed of light in the medium?
(a) 1.96×10^8 ms^{-1} (b) 2.12×10^8 ms^{-1}
(c) 3.18×10^8 ms^{-1} (d) 3.33×10^8 ms^{-1}

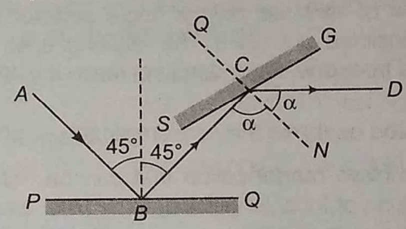
3. The optical path of a monochromatic light is same if it goes through 4.0 cm of glass or 4.5 cm of water. If the refractive index of glass is 1.53, the refractive index of the water is
(a) 1.30 (b) 1.36
(c) 1.42 (d) 1.46

4. A wire mesh consisting of very small squares is viewed at a distance of 8 cm through a magnifying lens of focal length 10 cm, kept close to the eye. The magnification produced by the lens is
(a) 5 (b) 8
(c) 10 (d) 20

5. A double convex lens made of glass (refractive index $n = 1.5$) has the radii of curvature of both the surfaces as 20 cm. Incident light rays parallel to the axis of the lens will converge at a distance L such that
(a) $L = 20$ cm (b) $L = 10$ cm
(c) $L = 40$ cm (d) $L = \frac{20}{3}$ cm

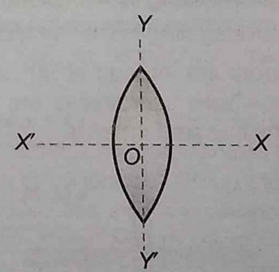
6. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image **[NCERT Exemplar]**
(a) moves away from the lens with a uniform speed 5 m/s
(b) moves away from the lens with a uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration

7. A light ray is incident on a horizontal plane mirror at an angle of 45°. At what angle should a second plane mirror be placed in order that the reflected ray finally be reflected horizontally from the second mirror as shown in figure is



(a) $\theta = 45°$
(b) $\theta = 60°$
(c) $\theta = 22.5°$
(d) $\theta = 15.3°$

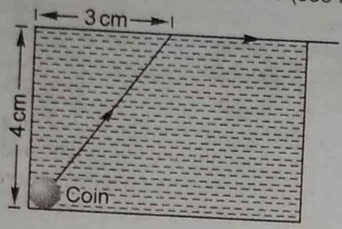
8. An equiconvex lens is cut into two halves along (i) XOX' and (ii) along YOY' as shown in figure. Let f, f' and f'' be the focal lengths of the complete lens, of each half in case (i), and of each half in case (ii) respectively.
Choose the correct statement from the following.



(a) $f' = 2f, f'' = f$ (b) $f' = f, f'' = f$
(c) $f' = 2f, f'' = 2f$ (d) $f' = f, f'' = 2f$

9. A concave lens and a convex lens have the same focal length of 20 cm and both are kept in contact. The combination is used to view an object 5 cm long kept at a distance of 20 cm from the lens combination. As compared to the object, the image will be
(a) magnified and inverted
(b) diminished and erect
(c) of the same size and erect
(d) of the same size and inverted

10. A small coin is resting on the bottom of a beaker filled with a liquid. A ray of light from the coin travels upto the surface of the liquid and moves along its surface (see figure).



How fast is the light travelling in the liquid?

(a) 1.8×10^8 ms^{-1} (b) 2.4×10^8 ms^{-1}
(c) 3.0×10^8 ms^{-1} (d) 1.2×10^8 ms^{-1}

11. A source of light lies on the angle bisector of two plane mirrors inclined at angle θ. The values of θ, so that the light reflected from one mirror does not reach the other mirror will be

(a) $\theta \geq 120°$ (b) $\theta \geq 90°$ (c) $\theta \leq 120°$ (d) $\theta < 30°$

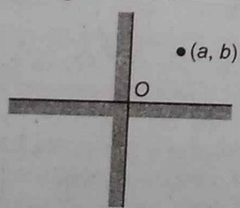
12. The maximum magnification that can be obtained with a convex lens of focal length 2.5 cm is (The least distance of distinct vision is 25 cm)

(a) 10 (b) 0.1 (c) 62.5 (d) 11

13. A beam of light composed of red and green rays is incident obliquely at a point on the face of a rectangular glass slab. When coming out of the opposite parallel face, the red and green rays emerge from

(a) two points propagating in two different non-parallel directions
(b) two points propagating in two different parallel directions
(c) one point propagating in two different directions
(d) one point propagating in the same direction

14. Two plane mirrors are inclined at 90°. An object is placed between them whose coordinates are (a, b). The position vectors of all the images formed is



(a) $-a\hat{i} - b\hat{j}, a\hat{i} - b\hat{j}, -a\hat{i} + b\hat{j}$
(b) $-a\hat{i} + b\hat{j}, -a\hat{j} - b\hat{j}, a\hat{i} - b\hat{j}$
(c) $a\hat{i} + b\hat{j}, -a\hat{i} - b\hat{j}, a\hat{i} - b\hat{j}$
(d) None of the above

15. When the distance between the object and the screen is more than 4F, we can obtain image of an object on the screen for the two positions of a lens. It is called displacement method. In one case, the image is magnified and in the other case it is diminished. Then, the ratio of the size of image to the diminished image is

(a) $\frac{(D+d)^2}{(D-d)^2}$ (b) $\frac{D}{d}$ (c) $\frac{D^2}{d^2}$ (d) $\frac{D+d}{D-d}$

16. The speed at which the image of the luminous point object is moving, if the luminous point object is moving at speed v_0 towards a spherical mirror, along its axis is
(Given, R = radius of curvature, u = object distance)

(a) $v_i = -v_0$ (b) $v_i = -v_0\left[\frac{R}{2u-R}\right]^2$
(c) $v_i = -v_0\left(\frac{2u-R}{R}\right)$ (d) $v_i = -v_0\left(\frac{R}{2u-R}\right)$

17. The refracting angle of prism is A and refractive index of material of prism is $\cot\frac{A}{2}$. The angle of minimum deviation is

(a) $180° - 3A$ (b) $180° + 2A$
(c) $90° - A$ (d) $180° - 2A$

18. To make an achromatic combination, a convex lens of focal length 42 cm having dispersive power of 0.14 is placed in contact with a concave lens of dispersive power 0.21. The focal length of the concave lens should be

(a) 63 cm (b) 21 cm
(c) 42 cm (d) 14 cm

19. The reflective surface is given by $y = 2\sin x$. The reflective surface is facing positive axis. What is the least value of coordinate of the point where a ray parallel to positive x-axis becomes parallel to positive y-axis after reflection?

(a) $\left(\frac{\pi}{3}, \sqrt{3}\right)$ (b) $\left(\frac{\pi}{2}, \sqrt{2}\right)$
(c) $\left(\frac{\pi}{3}, \sqrt{2}\right)$ (d) $\left(\frac{\pi}{4}, \sqrt{3}\right)$

Directions *(Q. Nos. 20 to 24) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

20. Statement I The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small as compared to their radii of curvature.

Statement II Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

21. **Statement I** Endoscopy involves use of optical fibres to study internal organs.

Statement II Optical fibres are based on the phenomenon of total internal reflection.

22. **Statement I** The refractive index of diamond is $\sqrt{6}$ and that of liquid is $\sqrt{3}$. If the light travels from diamond to the liquid, it will be totally reflected when the angle of incidence is 30°.

Statement II $n = \frac{1}{\sin C}$, where n is the refractive index of diamond with respect to liquid.

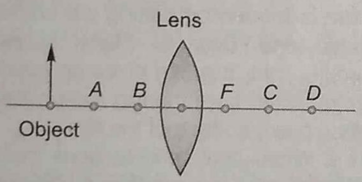
23. **Statement I** A double convex lens ($n = 1.5$) has a focal length 10 cm. When the lens is immersed in water ($n = 4/3$), its focal length becomes 40 cm.

Statement II $\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

24. **Statement I** The mirrors used in search lights are parabolic and not concave.

Statement II In a concave spherical mirror, the image formed is always virtual.

Directions *(Q. Nos. 25 to 28) The figure is a scaled diagram of an object and a converging lens surrounded by air. F is the focal point of lens as shown.*



25. At which of the labelled points can the image be formed?
(a) *A* (b) *B* (c) *C* (d) *D*

26. Which option describes the image most accurately?
(a) Real, erect (b) Real, inverted
(c) Virtual, erect (d) Virtual, inverted

27. If a parallel beam of blue light is focussed at *F*, then the parallel beam of red light is focussed at
(a) *F*
(b) *D*
(c) to the left of and close to *F*
(d) to the right of and close to *F*

28. The whole system is immersed in a liquid having the refractive index greater than the refractive index of the lens material. Then, mark the correct option for this new situation.
(a) The image will be real
(b) The image will be inverted
(c) The image will be formed on the same side of the lens as the object
(d) The image will be enlarged relative to the object

Directions *(Q. Nos. 29 to 31) The radius of curvature of the curved face of a thin plano-convex lens is 10 cm and it is made of glass of refractive index 1.5. A small object is approaching the lens with a speed of 1 cms^{-1} moving along the principal axis.*

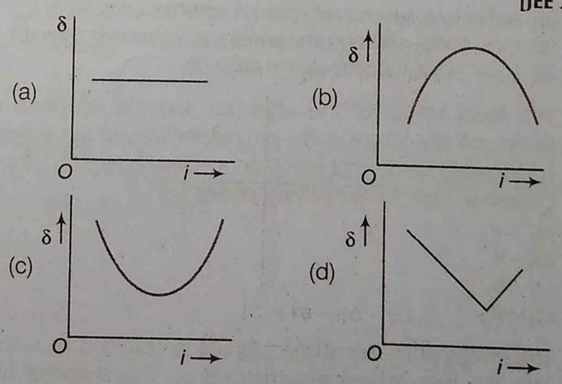
29. The focal length of the lens is
(a) 5 cm (b) 10 cm
(c) 15 cm (d) 20 cm

30. When the object is at a distance of 30 cm from the lens, the magnitude of the speed of its image is
(a) 1 cm s^{-1} (b) 2 cm s^{-1}
(c) 3 cm s^{-1} (d) 4 cm s^{-1}

31. When the object is at a distance of 30 cm from the lens, the magnitude of the rate of change of the lateral magnification is
(a) 0.1 per second
(b) 0.2 per second
(c) 0.3 per second
(d) 0.4 per second

32. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of lens **[JEE 2013]**
(a) 15 cm (b) 20 cm
(c) 30 cm (d) 10 cm

33. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **[JEE 2013]**



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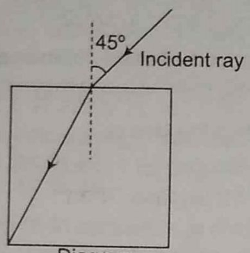
34. A light ray falls on a square glass slab as showin in the diagram. The index of refraction of the glass, if total internal reflection is to occur at the vertical face, is equal to **[JEE Main 2013]**



Diagram

(a) $\frac{(\sqrt{2}+1)}{2}$ (b) $\sqrt{\frac{5}{2}}$
(c) $\frac{3}{2}$ (d) $\sqrt{\frac{3}{2}}$

35. A printed page is pressed by a glass of water. The refractive index of the glass and water is 1.5 and 1.33, respectively. If the thickness of the bottom of glass is 1 cm and depth of water is 5 cm, how much the page will appear to be shifted if viewed from the top? **[JEE Main Online 2013]**

(a) 1.033 cm
(b) 3.581 cm
(c) 1.3533 cm
(d) 1.90 cm

36. Light is incident from a medium into air at two possible angles of incidence (a) 20° and (b) 40°. In the medium, light travels 3.0 cm in 0.2 ns. The ray will **[JEE Main Online 2013]**

(a) suffer total internal reflection in both cases (a) and (b)
(b) suffer total internal reflection in case (b) only
(c) have partial reflection and partial transmission in case (b)
(d) have 100% transmission in case (a)

37. The focal length of the objective and the eyepiece of a telescope are 50 cm and 5 cm respectively. If the telescope is focussed for distinct vision on a scale distant 2 m from its objective, then its magnifying power will be

(a) −4
(b) −8
(c) +8
(d) −2

38. The image of an illuminated square is obtained on a screen with the help of a converging lens. The distance of the square from the lens is 40 cm. The area of the image is 9 times that of the square. The focal length of the lens is **[JEE Main Online 2013]**

(a) 36 cm (b) 27 cm
(c) 60 cm (d) 30 cm

39. A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is **[AIEEE 2012]**

(a) 58.59 degree (b) 58.77 degree
(c) 58.65 degree (d) 59 degree

40. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is **[AIEEE 2011]**

(a) $\frac{1}{15}$ m/s (b) 10 m/s (c) 15 m/s (d) $\frac{1}{10}$ m/s

41. This question has a paragraph followed by two statements, statement 1 and statement 2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement 1 When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of II.

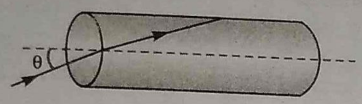
Statement 2 The centre of the interference pattern is dark. **[AIEEE 2011]**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1
(c) Statement 1 is false, Statement 2 is true
(d) Statement 1 is true, Statement 2 is false

42. In an optics experiments, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be **[AIEEE 2009]**

(a) $(2f, 2f)$ (b) $\left(\frac{f}{2}, \frac{f}{2}\right)$
(c) (f, f) (d) $(4f, 4f)$

43. A transparent solid cylinder rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure. **[AIEEE 2009]**



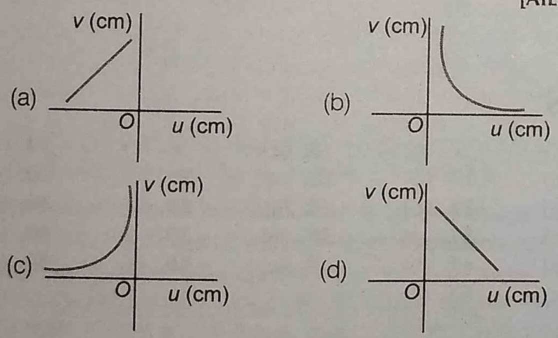
The incident angle θ for which the light ray grazes along the wall of the rod is

(a) $\sin^{-1}\left(\frac{1}{2}\right)$
(b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

44. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be **[AIEEE 2008]**

(a) 30° for both the colours
(b) greater for the violet colour
(c) greater for the red colour
(d) equal but not 30° for both the colours

45. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like **[AIEEE 2007]**



46. In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then, **[AIEEE 2007]**

(a) $x < f$
(b) $f < x < 2f$
(c) $x = 2f$
(d) $x > 2f$

47. Two lenses of power –15 D and +5 D are in contact with each other. The focal length of the combination is **[AIEEE 2007]**

(a) –20 cm
(b) –10 cm
(c) +20 cm
(d) +10 cm

48. A biconvex lens of focal length f forms a circular image of radius r of sun in the focal plane. Then, which option is correct? **[AIEEE 2006]**

(a) $\pi r^2 \propto f$
(b) $\pi r^2 \propto f^2$
(c) If lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$
(d) If f is doubled, intensity will increase

49. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be the angles of minimum deviation for the red and blue light respectively in a prism of this glass. Then, **[AIEEE 2006]**

(a) $D_1 < D_2$
(b) $D_1 = D_2$
(c) D_1 can be less than or greater than D_2 depending upon the angle of prism
(d) $D_1 > D_2$

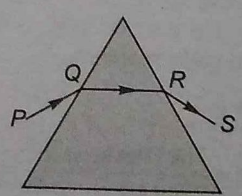
50. A convex lens is in contact with a concave lens. The magnitude of the ratio of their focal length is $\frac{3}{2}$. Their equivalent focal length is 30 cm. What are their individual focal lengths? **[AIEEE 2005]**

(a) –75, 50
(b) –10, 15
(c) 75, 50
(d) –15, 10

51. A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the water surface, the radius of this circle in cm, is **[AIEEE 2006]**

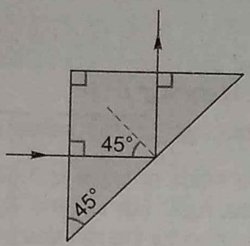
(a) $36\sqrt{7}$
(b) $\frac{36}{\sqrt{7}}$
(c) $36\sqrt{5}$
(d) $4\sqrt{5}$

52. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation, which of the following is true? **[AIEEE 2004]**



(a) PQ is horizontal
(b) QR is horizontal
(c) RS is horizontal
(d) Either PQ or RS is horizontal

53. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that for the refractive index n as **[AIEEE 2004]**



(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$

(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

54. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object? **[AIEEE 2004]**

(a) 20 cm (b) 30 cm

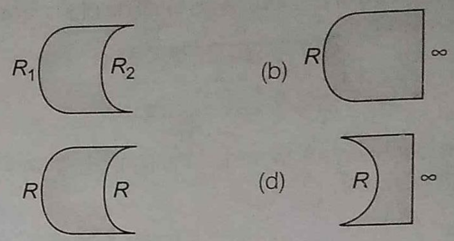
(c) 60 cm (d) 80 cm

55. To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**

(a) 60° (b) 90° (c) 120° (d) 30°

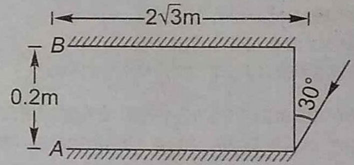
56. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams. **[AIEEE 2002]**

(a) (b) (c) (d)



57. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure.

The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is **[AIEEE 2002]**



(a) 28 (b) 30

(c) 32 (d) 34

Answers

1. (c)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (a)
11. (a)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (c)
21. (a)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (d)
31. (b)	**32.** (c)	**33.** (c)	**34.** (d)	**35.** (d)	**36.** (a)	**37.** (d)	**38.** (a)	**39.** (c)	**40.** (a)
41. (b)	**42.** (a)	**43.** (d)	**44.** (a)	**45.** (c)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (a)	**50.** (d)
51. (b)	**52.** (b)	**53.** (b)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (b)			

Hints & Solutions

1. Speed of light in given medium,

$$v = \frac{c}{n}$$

$\therefore$ Time taken $= \frac{\text{Thickness}}{\text{Speed}} = \frac{t}{c/n} = \frac{nt}{c}$

2. $$n = \frac{\sin i}{\sin r} = \frac{c}{v}$$

Hence, $$v = \frac{c \sin r}{\sin i}$$

$$= \frac{3 \times 10^8 \times \sin 30^\circ}{\sin 45^\circ}$$

$$= \frac{3 \times 10^8}{\sqrt{2}} = 2.12 \times 10^8 \text{ ms}^{-1}$$

3. As optical paths are equal, hence

$$n_g \cdot x_g = n_w \cdot x_w$$

$\Rightarrow$ $$n_w = n_g \cdot \frac{x_g}{x_w}$$

$$= 1.53 \times \frac{4.0}{4.5} = 1.36$$

4. $$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \quad \ldots(\text{i})$$

Given, $f=10$ cm (as lens is converging)

$u=-8$ cm (as object is placed on left side of the lens)

Substituting these values in Eq. (i), we get

$$\frac{1}{10}=\frac{1}{v}-\frac{1}{-8}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{10}-\frac{1}{8}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{8-10}{80}$$

$$\therefore \quad v=\frac{80}{-2}$$

$$=-40 \text{ cm}$$

Hence, magnification produced by the lens

$$m=\frac{v}{u}=\frac{-40}{-8}=5$$

5. Here, $n=1.5$, as per sign convention followed

$R_1=+20$ cm and $R_2=-20$ cm

$$\therefore \quad \frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1.5-1)\left[\frac{1}{(+20)}-\frac{1}{(-20)}\right]$$

$$=0.5\times\frac{2}{20}$$

$$=\frac{1}{20}$$

$$\Rightarrow \quad f=20 \text{ cm}$$

Incident rays travelling parallel to the axis of lens will converge at its second principal focus. Hence, $L=+20$ cm

6. When object would approach the lens then image would move away from the lens with non-uniform acceleration. This can be seen from the fact that when object is far away, image would be formed at focus P. When object approaches the lens then image would move away such that when object approaches to focus, then image would approach to infinity.

7. From the figure CD = emergent ray, and CD is parallel to PQ and BC is a line intersecting these parallel lines.

So, $\angle DCB + \angle CBQ = 180°$

$$\angle DCN + \angle NCB + \angle CBQ = 180°$$

$$\alpha+\alpha+45°=180°$$

$$\alpha=67.5°$$

But $\angle NCS = 90°$

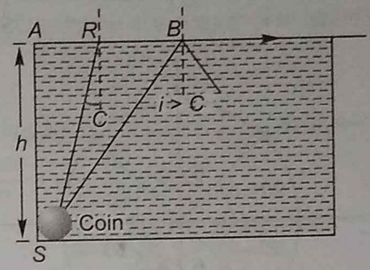
So, second mirror is at angle of 22.5° with horizontal.

8. When lens is cut along XOX', $f'=f$. However, when equiconvex lens is cut along YOY', each part behaves as a plano-convex lens of focal length $f''=2f$.

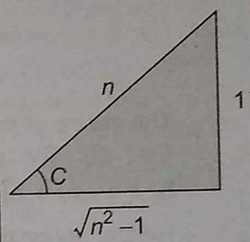
9. When a concave lens is joined in contact with a convex lens of same focal length, the combination behaves as a glass plate of infinite focal length or zero power.

Hence, the image of an object will be of the same size and erect.

10. As shown in figure, a light ray from the coin will not emerge out of liquid, if $i>C$.



Therefore, minimum radius R corresponds to $i=C$. In ΔSAB,



$$\frac{R}{h}=\tan C$$

or $$R=h\tan C$$

or $$R=\frac{h}{\sqrt{n^2-1}}$$

$$\left[\because \sin C=\frac{1}{n} \text{ and } \tan C=\frac{\sin C}{\sqrt{1-\sin^2 C}}\right]$$

Given, $R=3$ cm, $h=4$ cm

Hence, $$\frac{3}{4}=\frac{1}{\sqrt{n^2-1}}$$

or $$n^2=\frac{25}{9} \quad \text{or} \quad n=\frac{5}{3}$$

But $$n=\frac{c}{v}$$

or $$v=\frac{c}{n}=\frac{3\times10^8}{5/3}=1.8\times10^8 \text{ ms}^{-1}$$

11. $n=\frac{360}{\theta}$

Number of images, $N=n$, which is odd

$$=n-1$$

For the given condition, no successive reflection takes place. So, the number of images will be $N\le 2$

$$n-1\le 2$$

$$n\le 3$$

$$\frac{360}{\theta}\le 3$$

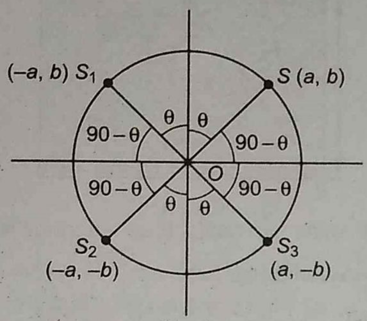
$$120\le\theta \quad \Rightarrow \quad \theta\ge 120°$$

12. Maximum magnification is obtained when image is formed at near point of eye. In that case,

$$m=1+\frac{D}{f}=1+\frac{25}{2.5}=11$$

13. In any medium other than air or vacuum, the velocities of different colours are different. Therefore, both red and green colours are refracted at different angles of refractions. Hence, after emerging from glass slab through opposite parallel face, they appear at two different points and move in two different parallel directions.

14. The images formed by combination of two plane mirrors are lying on a circle whose radius is equal to OS.



The centre of the circle is lying on meeting point of mirrors (*i.e.*, O). The position of images from diagram is for $S_1, r_1 = -a\hat{i} + b\hat{j}, r_2 = -a\hat{i} + b\hat{j}, r_3 = a\hat{i} + b\hat{j}$.

Hence, option (b) is true.

15. The ratio of the diminished image to the object is

$$m_2 = \frac{I_2}{O} = \frac{D-d}{D+d}$$

and the ratio of image to the object is $\frac{D+d}{D-d}$

Hence, $$\frac{m_1}{m_2} = \frac{I_1}{O} \times \frac{O}{I_2} = \frac{(D+d)^2}{(D-d)^2}$$

16. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating both sides

$$-\frac{1}{v^2}\frac{dv}{dt} = \frac{1}{u^2}\frac{du}{dt}$$

$$\frac{dv}{dt} = v_i = -\left(\frac{v}{u}\right)^2 \frac{du}{dt} = -\left(\frac{v}{u}\right)^2 v_0$$

Again $$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{2}{R} - \frac{1}{u} = \frac{2u-R}{Ru}$$

$$v = \frac{uR}{2u-R}$$

$$v_i = -\left(\frac{v}{u}\right)^2 v_0 = -v_0\left(\frac{R}{2u-R}\right)^2$$

17. $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} = \cot\frac{A}{2} = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\Rightarrow \sin\left(\frac{A+\delta_m}{2}\right) = \cos\frac{A}{2} = \sin\left(90 - \frac{A}{2}\right)$$

$$\Rightarrow \frac{A+\delta_m}{2} = 90° - \frac{A}{2}$$

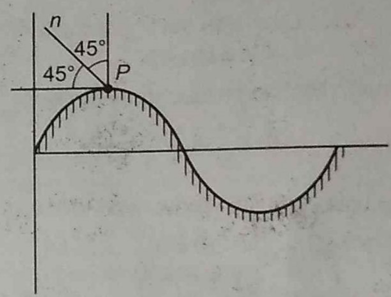
$$\Rightarrow \delta_m = (180° - 2A)$$

18. For an achromatic combination, the condition is

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

Here, $\omega_1 = 0.14, f_1 = 42$ cm thus we get $f_2 = -63$ cm

19. Let the incidence point is $P(x, y)$



$$m = \tan 45° = 1 \quad \text{(from law of reflection)}$$

$$y = 2\sin x$$

$$\therefore \quad m = \frac{dy}{dx} = 2\cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

The corresponding value of y is $2\sin\frac{\pi}{3} = \sqrt{3}$

20. Laws of reflection can be applied to any type of surface.

21. Optical fibre consists of a very long and thin fibre of quart glass. When a light ray is incident at one end of the fibr making a small angle of incidence, it suffers refraction fro air to quartz and strikes the fibre-layer interface at an angle incidence greater than the critical angle.

It therefore, suffers total internal reflection and strikes i opposite interface. At this interface also, the angle incidence is greater than the critical angle. So, it again suffe total internal reflection. Thus, optical fibre is based on tot internal reflection.

Endoscopy is a process for viewing internal organs of huma body. This process use a device endoscope which is base on total internal reflection.

22. Refraction index of diamond w.r.t. liquid

$${}_l n_d = \frac{1}{\sin C}$$

$$\therefore \quad \frac{\sqrt{6}}{\sqrt{3}} = \frac{1}{\sin C}$$

or $\sin C = \frac{1}{\sqrt{2}} = \sin 45° \therefore C = 45°$

For total internal reflection angle coincidence should be greater than critical angle.

But here angle of incidence is lower than critical angle, so total internal reflection does not occur in light.

23. In water,

$$\frac{1}{f_w} = \left(\frac{n_l - n_m}{n_m}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

$$\frac{1}{f_a} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

$$\frac{f_w}{f_a} = \frac{(n_l - 1)}{\left(\frac{n_l - n_m}{n_m}\right)} = \frac{(1.5 - 1)(4/3)}{(1.5 - 4/3)}$$

$$= \frac{0.5 \times 4/3}{\frac{3}{2} - \frac{4}{3}} = \frac{(2/3)6}{1}$$

$\therefore$ $f_w = 4f_a = 4 \times 10 = 40\,\text{cm}$

24. In search lights, an intense parallel beam of light is produced. In case of parabolic mirror, when source is at the focus beam of light produced over the entire cross-section is parallel beam. But in concave spherical mirror, due to it large aperture, marginal rays give a divergent beam.

25. As object is between infinity and $2F$, image will be between F and $2F$ and the point C is lying in this region.

26. Real, inverted, diminished.

27. Wavelength of blue light is smaller than wavelength of red light,

$$\lambda_B < \lambda_R \Rightarrow \eta_B > \eta_R$$

So, $\Rightarrow$ $f_B < f_R$ (by lens maker formulae)

(But f_B and f_R differ by very small value)

So, image for red light would be on the right side of and very close to F.

28. Now, lens behaves as a diverging lens, so lens will form virtual, erect and diminished image on the same side of lens as the object.

29. $\frac{1}{f} = (n-1) \times \frac{1}{R} = (1.5 - 1) \times \frac{1}{10}$ gives $f = 20$ cm

30. Lens formula is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{...(i)}$$

(Here, $u = -30$ cm and $f = +20$ cm)

or $\frac{1}{v} - \frac{1}{-30} = \frac{1}{20}$

which gives $v = 50$ cm

Differentiating Eq. (i) with respect to time t, we get

$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$$

or $\frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt}$

Speed of image $= \left(\frac{v^2}{u^2}\right) \times$ speed of object

$$= \left(\frac{60}{30}\right)^2 \times 1 = 4\ \text{cms}^{-1}$$

31. Linear magnification is given by

$$m = \frac{v}{u} \quad \text{...(i)}$$

Differentiating Eq. (i) with respect to time, we have

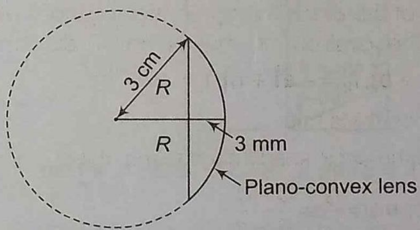
$$\frac{dm}{dt} = -\frac{v}{u^2}\frac{du}{dt} + \frac{1}{u}\frac{dv}{dt}$$

$$= \frac{1}{u^2}\left(-v\frac{du}{dt} + u\frac{dv}{dt}\right)$$

$$= \frac{1}{(30)^2}(-60 \times 1 - 30 \times 4) = -0.2 \text{ per second}$$

$\therefore$ Magnitude of $\frac{dm}{dt} = 0.2$ per second.

32.



By Pythagoras theorem

$$R^2 = 3^2 + (R - 3\text{mm})^2$$

$\Rightarrow$ $R^2 = 3^2 + R^2 - 2Rx(3\text{mm}) + (3\text{mm})^2$

$\Rightarrow$ $R \simeq 15\,\text{cm}$

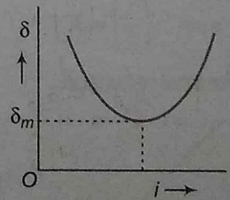
Also, $\mu = \frac{c}{v}$

$$\mu = \frac{3 \times 10^8}{2 \times 10^8} = \frac{3}{2}$$

As, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow$ $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{15} - 0\right] \Rightarrow f = 30\text{cm}$

33. Angle of deviation depends upon the angle of incidence. If we determine experimentally the angles of deviation corresponding to different angles of incidence, then the plot between i and δ that we will get is shown below



34. From figure, we get $r = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

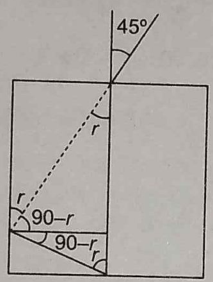
For critical angle, $\sin C = \frac{1}{\mu}$

Now, by Snell's law, we have

$$\frac{\mu}{1} = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 45°}{\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{3}}}$$

$$\mu = \sqrt{\frac{3}{2}}$$



35. Given, $\mu = 1.5, t_1 = 5$ cm, $\mu_2 = 1.33$ and $t_2 = 1$ cm

Change in path $= \Delta t_1 + \Delta t_2$

$$= \left(1 - \frac{1}{\mu_1}\right) \times t_1 + \left(1 - \frac{1}{\mu_2}\right) \times t_2$$

$$= \left(1 - \frac{1}{1.5}\right) \times 5 + \left(1 - \frac{1}{1.33}\right) \times 1 \simeq 1.9 \text{cm}$$

36. Speed of light in medium

$$= \frac{3 \times 10^{-2} \times 10}{0.2 \times 10^{-9}}$$

$$= \frac{3}{2} \times 10^8 \text{ ms}^{-1} = 1.5 \times 10^8 \text{ ms}^{-1}$$

As, $\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

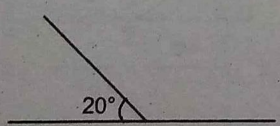
$$\frac{\mu}{1} = \frac{3 \times 10^8}{1.5 \times 10^8}$$

$\Rightarrow$ $\mu = 2$

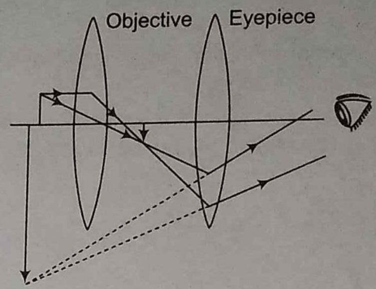
We have, $\sin C = \frac{1}{2}$

$\Rightarrow$ $C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

Hence, it is that internal reflection.



37.



For objective $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{(-200)} = \frac{1}{50}$$

$$\frac{1}{v} = \frac{1}{50} + \frac{1}{200} = \frac{1}{10}\left[\frac{1}{5} + \frac{1}{20}\right]$$

$$= \frac{1}{10} \times \frac{5}{20} = \frac{1}{40}$$

$\Rightarrow$ $v = 40$

$\therefore$ $m_o = \frac{40}{200} = \frac{1}{5}$

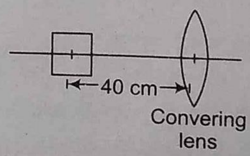
Now, for eyepiece $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

and similarly get the magnification, M_e

Now, total magnification $= m_o \times m_e$

will give the result as -2.

38. As magnification $= 9 = \frac{v}{u}$



$\therefore$ $v = 9u = 9 \times 40 = 360$

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{-40} = \frac{1}{f}$$

$$\frac{1}{360} + \frac{1}{40} = \frac{1}{f} \Rightarrow \left[\frac{1}{9} + 1\right]\frac{1}{40} = \frac{1}{f}$$

$\Rightarrow$ $\frac{10}{9} \times \frac{1}{40} = \frac{1}{f} \Rightarrow f = 36$ cm

39. 1 Vernier scale division $= \frac{29}{30}$ main scale division

$$1 \text{ VSD} = \frac{29}{30} \times 0.5° = \left(\frac{29}{60}\right)^°$$

Thus, least count = 1 MSD − 1VSD

$$= \left(\frac{1}{2}\right)^° - \left(\frac{29}{60}\right)^° = \left(\frac{1}{60}\right)^°$$

So, reading = main scale reading + vernier scale reading

$$= \text{MSR} + n \times \text{LC}$$

$$= 58.5° + 9 \times \left(\frac{1}{60}\right)^{\circ}$$

$$= 58.65°$$

40. $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{-1}{u^2}\frac{du}{dt} - \frac{1}{v^2}\frac{dv}{dt} = 0$$

$$\frac{dv}{dt} = \frac{-v^2}{u^2}\left(\frac{du}{dt}\right)$$

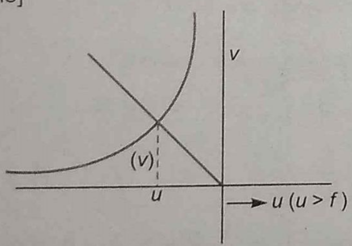
But $$\frac{v}{u} = \frac{f}{u-f}$$

$$\therefore \quad \frac{dv}{dt} = -\left(\frac{f}{u-f}\right)^2\left(\frac{du}{dt}\right)$$

$$= \left(\frac{0.2}{-2.8-0.2}\right)^2 \times 15 = \frac{1}{15}\,\text{ms}^{-1}$$

40. Both statements 1 and 2 are correct but statement 2 does not explain statement 1.

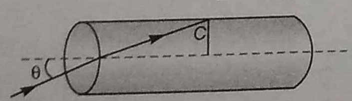
41. It is possible when object kept at centre of curvature because then only position of object and image would be same. *i.e.*, $u = v$ [which is the point of intersection between curve and straight line]



$$u = v$$

$$u = 2f, v = 2f$$

43. $$\sin C = \frac{\sqrt{3}}{2} \quad \dots\text{(i)}$$



$$\sin r = \sin(90° - C) = \cos C = \frac{1}{2}$$

$$\frac{\sin\theta}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\sin\theta = \frac{2}{\sqrt{3}} \times \frac{1}{2}$$

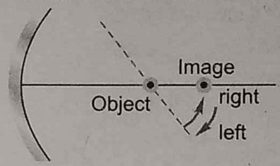
$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

44. At minimum deviation $(\delta = \delta_m)$

$$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30° \quad \text{(For both colours)}$$

45. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ = constant,

46. Since object and image move in opposite directions, the positioning should be as shown in the figure. Object lies between focus and centre of curvature $f < x < 2f$.

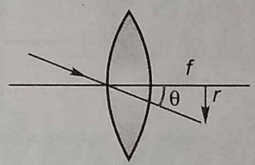


47. Power of combined lens is

$$P = P_1 + P_2 = -15 + 5 = -10\,\text{D}$$

$$\therefore \quad f = \frac{1}{P} = \frac{100}{-10}\,\text{cm or } f = -10\,\text{cm}$$

48.



$$r = f\tan\theta$$

or $$r \propto f$$

$$\therefore \quad \pi r^2 \propto f^2$$

49. $D = (n-1)A$

For blue light n is greater than that for red light, so $D_2 > D_1$.

50. Let focal length of convex lens is $+f$, then focal length of concave lens would be $-\frac{3}{2}f$.

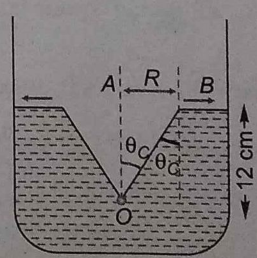
From the given condition,

$$\frac{1}{30} = \frac{1}{f} - \frac{2}{3f} = \frac{1}{3f}$$

$$\therefore \quad f = 10\,\text{cm}$$

Therefore, focal length of convex lens = + 10 cm and that of concave lens = – 15 cm.

51. The situation is shown in figure.



$$\sin\theta_C = \frac{1}{\mu}$$

$$\tan\theta_C = \frac{AB}{OA}$$

$$\therefore \quad AB = OA\tan\theta_C$$

$$\text{or} \quad AB = \frac{OA}{\sqrt{n^2-1}}$$

$$= \frac{12}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{36}{\sqrt{7}}$$

52. During minimum deviation the ray inside the prism is parallel to the base of the prism in case of an equilateral prism.

53. For total internal reflection from glass-air interface, critical angle C must be less than angle of incidence.

i.e., $C < i$

or $C < 45°$ $(\because \angle i = 45°)$

But $n = \dfrac{1}{\sin C}$

$\Rightarrow$ $C = \sin^{-1}\left(\dfrac{1}{n}\right)$

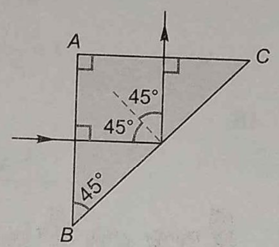
or $\sin^{-1}\left(\dfrac{1}{n}\right) < 45°$

or $\dfrac{1}{n} < \sin 45°$

or $n > \dfrac{1}{\sin 45°}$

or $n > \dfrac{1}{(1/\sqrt{2})}$

or $n > \sqrt{2}$



54. A plano-convex lens behaves as a concave mirror if its one surface (curved) is silvered. The rays refracted from plane surface are reflected from curved surface and again refract from plane surface. Therefore, in this lens two refractions and one reflection occur.

Let the focal length of silvered lens is F.

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m}$$

$$= \frac{2}{f} + \frac{1}{f_m}$$

where, f = focal length of lens before silvering,

f_m = focal length of spherical mirror.

$$\frac{1}{F} = \frac{2}{f} + \frac{2}{R} \quad \text{...(i)} \; (\because R = 2f_m)$$

Now, $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ ...(ii)

Here, $R_1 = \infty, R_2 = 30$ cm

$\therefore$ $\dfrac{1}{f} = (1.5-1)\left(\dfrac{1}{\infty} - \dfrac{1}{30}\right)$

or $\dfrac{1}{f} = \dfrac{0.5}{30} = -\dfrac{1}{60}$

or $f = -60$ cm

Hence, from Eq. (i)

$$\frac{1}{F} = \frac{2}{60} + \frac{2}{30} = \frac{6}{60}$$

$$F = 10 \text{ cm}$$

Again given that,

size of object = size of image

i.e., $O = I$

$\therefore$ $m = -\dfrac{v}{u} = \dfrac{I}{O}$

$\Rightarrow$ $\dfrac{v}{u} = -1$

or $v = -u$

Thus, from lens formula

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{-u} - \frac{1}{u}$$

$$\frac{1}{10} = -\frac{2}{u}$$

$\therefore$ $u = -20$ cm

Hence, to get a real image, object must be placed at a distance 20 cm on the left side of lens.

55. Number of images, $n = \dfrac{360°}{\theta} - 1$

where θ is angle between mirrors.

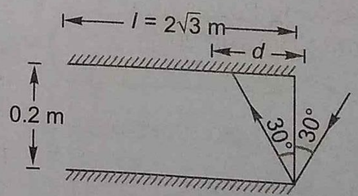
$\therefore$ $3 = \dfrac{360°}{\theta} - 1$ or $\theta = 90°$

56. $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

For no dispersion, $d\left\{\dfrac{1}{f}\right\} = 0$ or $R_1 = R_2 = R$.

So (c) is correct option.

57. $d = 0.2 \tan 30° = \dfrac{0.2}{\sqrt{3}}$



$$\frac{l}{d} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$$

Therefore, maximum number of reflections are 30.

Day 28

Optical Instruments

Day 28
Outlines ...

- Microscope
- Telescope
- Resolving Power of an Optical Instrument

Microscope

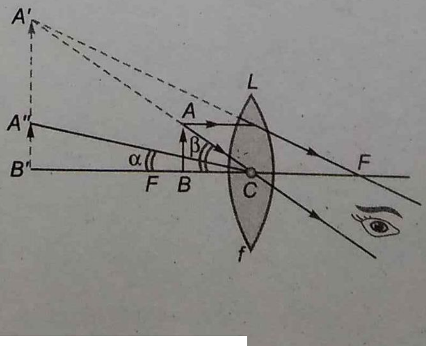
It is an optical instrument which forms a magnified image of a small nearby object and thus, increases the visual angle subtended by the image at the eye so that the object is seen to be bigger and distinct.

1. **Simple Microscope** (Magnifying Glass)

It consists of a single convex lens of small focal length and forms a magnified image of an object placed between the optical centre and the principal focus of the lens. If the image is formed at the near point of eye.

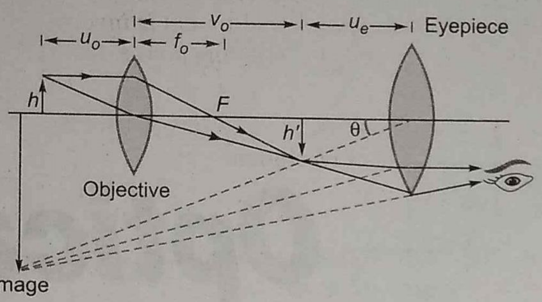
Then $m = \left(1 + \frac{D}{f}\right)$ but if the image is formed at infinity, then $m = \frac{D}{f}$ where,

D = normal distance (25 cm), f = focal length of magnifying lens.



2. Compound Microscope

It consists of two lenses of small focal length and small apertures. Also, the focal length and aperture of objective lens are smaller than that of eyepiece. The image formed by the objective lens is real, inverted and magnified. This image acts as the object for the eyepiece and the final image is highly magnified, virtual and inverted w.r.t. the original object.



If m_o and m_e be the magnifications produced by the objective and the eyepiece respectively, then total magnification of microscope $m = m_o \times m_e$

If final image is formed at the near point (D) of the eye, then

$$m = -\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right) \text{ or } m = -\frac{L}{f_o}\left(1+\frac{D}{f_e}\right) \text{(approx)}$$

If final image is formed at infinity, then

$$m = -\frac{v_o}{u_o}\cdot\frac{D}{f_e} = -\frac{L}{f_o}\cdot\frac{D}{f_e} \text{(approx)}$$

- Huygens eyepiece is free from chromatic and spherical aberration but it cannot be used for measurement purposes.
- Ramsden's eyepiece can be used for precise measurement as cross wires can be fixed in this eyepiece. It slightly suffers from spherical and chromatic aberrations.

Telescope

Telescope is an optical instrument which increases, the visual angle at the eye by forming the image of a distant object at the least distance of distinct vision, so that the object is seen distinct and bigger.

Refracting Telescope

It consists of an objective lens of large focal length f_o and large aperture.

The eyepiece consists of a convex lens of small aperture and small focal length f_e. Distance between the two lenses is set as

$$L = f_o + f_e$$

In normal adjustment, the final image is formed at infinity and magnifying power of the telescope is

$$m = -\frac{f_o}{f_e}$$

In practical adjustment, the final image is formed at the near point of the observer's eye. In this arrangement,

$$m = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

Reflecting Telescope

It consists of an objective which is a large paraboloid concave mirror of maximum possible focal length f_o and the eyepiece is a convex lens of small focal length and small aperture. Magnifying power, $m = -\frac{f_o}{f_e}$

Reflecting type telescope is considered superior as it is free from spherical and chromatic aberrations, is easy to install and maintain and can produce image of greater intensity.

- The large aperture of telescope objective helps in forming a brighter image.
- If diameter of pupil of human eye is d and that of telescope be D, then image formed by telescope will be $\left(\frac{D}{d}\right)^2$ times brighter than the image of the same object seen directly by the unaided eye.

Resolving Power of an Optical Instrument

***Resolving power** of an optical instrument is its ability to produce distinct images of two points of an object (or two nearby objects) very close together. Resolving power of an optical instrument is inverse of its limit of resolution. Smaller the limit of resolution of a device, higher is its resolving power. Limit of resolution of a normal human eye is 1′ .*

The minimum distance (or angular distance) between two points of an object whose images can be formed distinctly by the lens of an optical instrument, is called its **limit of resolution**.

Resolving Power of a Telescope

If the aperture (diameter) of the telescope objective be the D, then the minimum angular separation ($d\theta$) between two distant objects, whose images are just resolved by the telescope, is

$$d\theta = \frac{1.22\lambda}{D}$$

and resolving power of the telescope,

$$\text{RP} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

Resolving Power of a Microscope

The least distance (d) between two points, whose images are just seen distinctly by a microscope, is given by

$$d = \frac{1.22\lambda}{2n_m \sin\theta}$$

where, λ = wavelength of light used to illuminate the object,

n_m = refractive index of the medium between the object and the objective lens, and

θ = semi angle of the cone of light from the point object.

The term $n_m \sin\theta$ is generally called the **numerical aperture** of the microscope the objective.

$\therefore$ Resolving power of the microscope,

$$\text{RP} = \frac{1}{d} = \frac{2\,n_m \sin\theta}{1.22\lambda} = \frac{(\text{NA})}{0.61\lambda}$$

To see the objects which are too small for the naked eye, we usually use the optical instrument named as microscope. Investigating such as small objects is called microscopy. Microscopic means invisible to the eye unless aided by a microscope.

Practice Zone

DAY 28

1. From an aircraft flying at an altitude of 2000 m, photograph of the ground are taken from a camera whose size of the film is 18 cm × 18 cm and the focal length of camera is 50 cm. The area of the ground that can be photographed by the camera is
(a) 648900 cm^2 (b) 32831 cm^2
(c) 518400 m^2 (d) 18000 cm^2

2. A telescope consists of two lenses of focal length 10 cm and 1 cm. The length of the telescope when an object is kept at a distance of 60 cm from the objective and the final image is formed at least distant of distinct vision is
(a) 15.05 cm (b) 12.96 cm (c) 13.63 cm (d) 14.44 cm

3. A light source is placed at a distance b from a screen. The power of the lens required to obtain k-fold magnified image is
(a) $\frac{k+1}{kb}$ (b) $\frac{(k+1)^2}{kb}$ (c) $\frac{kb}{k+1}$ (d) $\frac{kb}{(k-1)^2}$

4. The focal length of plano-convex lens, the convex surface of which is silvered is 0.3 m. If μ of the lens is $\frac{7}{4}$, the radius of curvature of the convex surface is
(a) 0.45 m (b) 1.05 m
(c) 3 m (d) 0.9 m

5. A convergent doublet of separated lens, corrected for spherical aberration, are separated by 2 cm and has an equivalent focal length of 10 cm. The focal length of its component lenses are
(a) $f_1 = 18$ cm, $f_2 = 10$ cm
(b) $f_1 = 20$ cm, $f_2 = 28$ cm
(c) $f_1 = 20$ cm, $f_2 = 18$ cm
(d) $f_1 = 24$ cm, $f_2 = 18$ cm

6. To obtain a magnified image at the distance of distinct vision with simple microscope, where should the object be placed?
(a) Away from the focus
(b) At focus
(c) Between the focus and the optical centre
(d) None of the above

7. To obtain the maximum magnification with a simple microscope, where should the eye be placed?
(a) Close to the lens
(b) Half way between the focus and the optical centre
(c) Close to the focus
(d) None of the above

8. The final image formed by a microscope is
(a) real and magnified (b) real and diminished
(c) virtual and diminished (d) virtual and magnified

9. Between the primary and secondary rainbows, there is a dark band known as Alexandar's dark band. This is because
(a) there is no light scattered into this region
(b) light scattered into this region interfere constructively
(c) light is absorbed in this region.
(d) light scattered into this region interfere destructively.

10. In a compound microscope, the objective produces a magnification of 10, while the eyepiece produces a magnification of 5, then the over all magnification achieved by a compound microscope is
(a) 2 (b) 50 (c) 0.5 (d) 25.00

11. If f_o and f_e be the focal lengths of the objective and eyepiece of astronomical tube scope, the length of the tube is
(a) $f_o + f_e$ (b) $f_o - f_e$
(c) $\sqrt{f_o f_e}$ (d) $\frac{f_o f_e}{f_o + f_e}$

12. An astronomical telescope is set for normal adjustment and the distance between its objective and eyepiece is 1.05 cm. The magnifying power of the telescope is 20. What is the focal length of the objective?
(a) 2 m (b) 1 m
(c) 0.5 m (d) 0.25 m

13. The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and the eyepiece is found to be 20 cm. The focal length of lenses are
(a) 18 cm, 2 cm (b) 11 cm, 9 cm
(c) 10 cm, 10 cm (d) 15 cm, 5 cm

14. A simple telescope, consisting of an objective of focal length 60 cm and a single eye lens of focal length 5 cm is focussed on a distant object in such a way that parallel rays emerge from the eye lens. If the object subtends an angle of $2°$ at the objective, the angular width of the image is

(a) $10°$ (b) $24°$
(c) $50°$ (d) $(1/6)°$

15. A magnifying glass is used as the object to be viewed can be brought closer to the eye than the normal near point. This results in **[NCERT Exemplar]**

(a) the formation of virtual erect image and larger angle to be subtended by the object at the eye and hence viewed in greater detail
(b) increase in field of view
(c) infinite magnification at near point
(d) a diminished but clear image

16. In a laboratory four convex lenses L_1, L_2, L_3 and L_4 of focal lengths 2, 4, 6 and 8cm respectively are available. Two of these lenses form a telescope of length 10 cm and magnifying power 4. The objective and eye lenses are respectively

(a) L_2, L_3 (b) L_1, L_4
(c) L_1, L_2 (d) L_4, L_1

17. The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54cm. The focal length of eye lens and objective lens will be respectively

(a) 6 cm and 48 cm
(b) 48 cm and 6 cm
(c) 8 cm and 64 cm
(d) 6 cm and 60 cm

18. For compound microscope $f_o = 1$ cm, $f_e = 2.5$ cm. An object is placed at distance 1.2cm from objective lens. What should be the length of microscope for normal adjustment?

(a) 8.5 cm
(b) 8.3 cm
(c) 6.5 cm
(d) 6.3 cm

Directions *(Q. Nos. 19 to 21) Each of these questions contains two statements Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

19. **Statement I** The resolving power of a telescope is more if the diameter of the objective lens is more.

Statement II Objective lens of large diameter collects more light.

20. **Statement I** The focal length of the objective of telescope is larger than that of eyepiece.

Statement II The resolving power of telescope increases when the aperture of objective is small.

21. **Statement I** Resolving power of an optical instrument is reciprocal to its limit of resolution.

Statement II Smaller the distance between two point objects the instrument can resolve, higher is its resolving power.

Directions *(Q. Nos. 22 to 24) A telescope is an optical instrument that is used to examine distant objects. Two types of telescopes are in use-refracting and reflecting telescopes. A refracting astronomical telescope consists of two converging lenses called the objective and the eyepiece.*

The objective faces the distant object. The image of the object is formed at the focal plane of the objective. The position of the eyepiece is adjusted till this image is within the first focus of the eyepiece. A highly magnified final image is formed which is seen by the eye held close to the eyepiece. If both the object and the final image are at infinity, the telescope is said to be in normal adjustment.

22. In a refracting astronomical telescope, the final image is

(a) real, inverted and magnified
(b) real, erect and magnified
(c) virtual, erect and magnified
(d) virtual, inverted and magnified

23. The magnifying power of a telescope is high if

(a) both the objective and eyepiece have short focal lengths
(b) both the objective and the eyepiece have long focal lengths
(c) the objective has a short focal length and the eyepiece has a long focal length
(d) the objective has a long focal length and the eyepiece has a short focal length

24. The resolving power of a telescope is increased, if

(a) the objective of a bigger diameter is used
(b) the objective of a smaller diameter is used
(c) the objective of a higher focal length is used
(d) the eyepiece of a shorter focal length is used

AIEEE & JEE Main *Archive*

25. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement I Very large size telescopes are reflecting telescopes instead of refracting telescopes.

Statement II It is easier to provide mechanical support to large size mirrors than large size lenses. **[JEE Main Online 2013]**

(a) Statement I is true, Statement II is false
(b) Statement I is false, Statement II is true.
(c) Statement I is true, Statement II is true and Statement II is the correct explanation of Statement I
(d) Statement I is true Statement II is true but Statement II is not the correct explanation of Statement I

26. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take, wavelength of light = 500 nm] **[AIEEE 2005]**

(a) 5 m
(b) 1 m
(c) 6 m
(d) 3 m

27. An astronomical telescope has a large aperture to **[AIEEE 2002]**

(a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion

Answers

1. (c)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (b)
11. (a)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (c)
21. (a)	**22.** (d)	**23.** (d)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (b)			

Hints & Solutions

1. As object is very far from the camera lens.

So, $v = f = 0.50$ cm

Linear magnification $m = \frac{v}{u} = \frac{0.5}{2000}$

Arial magnification $= \frac{\text{area of image } (A_I)}{\text{area of object } (A_o)} = m^2$

$$\frac{A_I}{A_o} = m^2 = \left(\frac{v}{u}\right)^2$$

$$A_I = 18\text{ cm} \times 18\text{ cm} = 0.18 \times 0.18\text{ m}^2$$

$$A_o = \left(\frac{u}{v}\right)^2 \times A_I = \left(\frac{2000}{0.5}\right)^2 \times 0.18 \times 0.18 = 518400\text{ m}^2$$

2. Two lenses used are eyepiece and objective.

For eyepiece, $f_e = 1$ cm, $D = v_e = 25$ cm

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{25} - \frac{1}{u_e} = 1 \Rightarrow u_e = -\frac{25}{26}\text{cm}$$

For objective $u_o = -60$ cm, $f_o = 10$ cm

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} + \frac{1}{60} = \frac{1}{10}$$

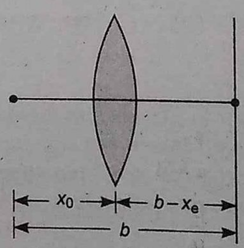
$$\Rightarrow \quad \frac{1}{v_o} = \frac{1}{10} - \frac{1}{60}$$

$$\Rightarrow \quad \frac{1}{v_o} = \frac{5}{60}$$

$$\Rightarrow \quad v_o = \frac{60}{5} = 12\text{ cm}$$

Length of telescope, $L = v_o + u_o = 12 + \frac{25}{26} = 12.96$ cm

3. Distance of light source from lens is x_0 and distance of screen from lens is $(b - x_0)$.



∴ Image is formed on screen, hence $(b - x_0)$ is also the image distance.

Image is formed on the screen so m will be negative.

$$m = -k = \frac{v}{u}$$

$$\therefore \quad v = -ku$$

Here, $u = -x_0$, $v = b - x_0$

$$\therefore \quad b - x_0 = kx_0$$

$$x_0 = \frac{b}{1+k}$$

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{b - x_0} - \frac{1}{-x_0} = p$$

$$\Rightarrow \quad p = \frac{b}{x_0(b - x_0)}$$

Putting the vlaue of x_0, we have $p = \frac{(k+1)^2}{kb}$

4. $P = 2P_L + P_m = \frac{2}{f_L} - \frac{1}{f_m} = \frac{2}{f_L} + \frac{2}{R}$

Here, $\frac{1}{f_L} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \left(\frac{7}{4} - 1\right)\left(\frac{1}{\infty} + \frac{1}{R}\right)$

$$\frac{1}{f_L} = \frac{3}{4R}$$

$$\therefore \quad P = -\frac{1}{F} = \frac{6}{4R} + \frac{2}{R} = \frac{6+8}{4R} = \frac{14}{4R}$$

$$\Rightarrow \quad -\frac{1}{0.3} = \frac{14}{4R}$$

$$\Rightarrow \quad R = \frac{-14 \times 0.3}{4} = -\frac{4.2}{4} = -1.05 \quad \text{m}$$

5. Since, the doublet is corrected for spherical aberration, it satisfies the following condition.

$$f_1 - f_2 = d = 2 \text{ cm}$$

$$\Rightarrow \quad f_1 - f_2 = 2 \text{ cm}$$

Let equivalent focal length = F

$$F = \frac{f_1 f_2}{f_1 + f_2 - d} = 10 \text{ cm}$$

On solving, $f_1 = 20$ cm, $f_2 = 18$ cm

10. $M = M_o \times M_e = 10 \times 5 = 50$

12. Here, $m = f_o / f_e$ and $L = f_o + f_e = 1.05$

or $\quad f_e = 1.05 - f_o$ and $\quad 20 = f_o/(1.05 - f_o)$

This gives $f_o = 1$ m and $f_e = 0.05$ m

13. $f_o + f_e = 20$ cm, $M = f_o/f_e = 9$. This gives $f_e = 2$ cm and $f_o = 18$ cm.

14. It is a case of normal adjustment.

Hence, $\quad M = f_o / f_e$

Also, $\quad \mu = \beta / \alpha$

Therefore, $\quad \frac{\beta}{\alpha} = \frac{f_o}{f_e}$

Here, $f_o = 60$ cm, $f_e = 5$ cm, $\alpha = 2°$, hence $\beta = 24°$

16. Length of tube = 10 cm

$$f_o + f_e = 10 \text{ cm} \qquad \ldots(i)$$

Magnification $\quad m = \frac{f_o}{f_e} = 4$

$$f_o = 4 f_e$$

Putting in Eq. (i), $5f_e = 10$ cm

or $\quad f_e = 2$ cm and $f_o = 8$ cm, $f_e = 2$ cm

Hence, L_4 and L_1 will be used.

Note in telescope, objective always have larger focal length than eyepiece.

17. As $L = f_o + f_e = 54$ cm and $|m| = \frac{f_o}{f_e} = 8$

On simplification, we get

$$f_e = +6 \text{ cm} \quad \text{and} \quad f_o = +48 \text{ cm}$$

18. When final image is formed at normal adjustment, then length of compound microscope,

$$L = v_o + u_e = \frac{u_o f_o}{(u_o + f_o)} + \frac{f_e D}{f_e + D}$$

$$= \frac{-1.2 \times 1}{-1.2 + 1} + \frac{2.5 \times 25}{2.5 + 25} = 6 + 2.27 = 8.27 \approx 8.3 \text{ cm}$$

19. Resolving power of telescope is $= \frac{a}{1.22\lambda}$

where a is the diameter of objective lens and λ is the wavelength of light used. It is obvious that on increasing a, more light is collected by objective lens and so, the image formed is more bright. Thus, resolving power of telescope increases.

20. The magnifying power of telescope in relaxed state is

$$m = \frac{f_o}{f_e}$$

So, for high magnification, the focal length of objective length should be larger than that of eyepiece.

Resolving power of a telescope $= \frac{d}{1.22\lambda}$

For high resolving power, diameter (d) of objective should be higher.

23. The magnifying power of a telescope (if the object is at infinity) is given by $\quad M = \frac{f_o}{f_e} \cdot \frac{D + f_e}{D}$

where, D = least distance of distinct vision, where the final image is formed.

24. The resolving power of a telescope is given by RP $= \frac{d}{1.22\lambda}$

The resolving power is independent of f_o or f_e.

25. As very large size telescope needs mechanical support to large size mirror than size of lense. So, in order to fulfil this mechanical support telescope is reflecting instead of refracting telescope.

26. We know, $\quad \frac{y}{D} \geq 1.22\frac{\lambda}{d} \Rightarrow D \leq \frac{yd}{1.22\lambda}$

$$\therefore \quad D \leq \frac{10^{-3} \times 3 \times 10^{-3}}{1.22 \times 5 \times 10^{-7}} = 5 \text{ m}$$

$$\Rightarrow \quad D_{max} = 5 \text{ m}$$

27. An astronomical telescopetic as a large aperture to have high resolution.

Day 29

Wave Optics

Day 29

Outlines ...

- Wave Nature of Light
- Interference of Light
- Diffraction of Light
- Polarisation of Light

Concept of Wave Optics

Wave optics describes the connection between waves and rays of light. According to wave theory of light, the light is a form of energy which travels through a medium in the form of transverse wave motion. The speed of light in a medium depends upon the nature of medium.

Wave Nature of Light

According to Huygen, light is a form of energy, which travels in the form of waves through a hypothetical medium 'ether'. The medium was supposed to be all pervading, transparent, extremely light, perfectly elastic and an ideal fluid.

Light waves transfer energy as well as momentum and travel in the free space with a constant speed of 3×10^8 ms^{-1}. However, in a material medium, their speed varies from medium to medium depending on the refractive index of the medium.

Wavefront

A wavefront is the locus of all those points (ether particles) which are vibrating in the same phase. The shape of the wavefront depends on the nature and dimension of the source of light.

(i) In an isotropic medium, for a point source of light, the wavefront is spherical in nature.
(ii) For a line (slit) source of light, the wavefront is cylindrical in shape.
(iii) For a parallel beam of light, the wavefront is a plane wavefront.

Huygens' Principle

Every point on a given wavefront acts as secondary source of light and emits secondary wavelets which travel in all directions with the speed of light in the medium. A surface touching all these secondary wavelets tangentially, in the forward direction, gives the new wavefront at that instant of time.

Interference of Light

Interference of light is the phenomenon of redistribution of light energy in space when two or more light waves of same frequency (or same wavelength) emitted by two coherent sources, travelling in a given direction, superimpose on each other.

If a_1 and a_2 be the amplitudes of two light waves of same frequency and ϕ be the phase difference between them, then the amplitude of resultant wave is given by $A_R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$

and in terms of intensity of light $I_R = I_1 + I_2 + 2\sqrt{I_1I_2} \cos\phi$

Condition for Constructive Interference

If at some point in space, the phase difference between two waves, $\phi = 0°$ or $2n\pi$ (or path difference between two waves, $\Delta = 0$ or $n\lambda$),

where n is an integer, then

$$A_R = a_1 + a_2$$

or

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2}$$

is maximum.

Such an interference is called **constructive interference.**

Condition for Destructive Interference

If at some point in space, the phase difference between two waves, $\phi = (2n-1)\pi$ [or path difference, $\Delta = (2n-1)\frac{\lambda}{2}$, then at such points $A_R = (a_1 - a_2)$ and $I_R = I_1 + I_2 - 2\sqrt{I_1I_2}$ is minimum leading to a **destructive interference.**

$$\frac{I_{max}}{I_{min}} = \frac{I_1 + I_2 + 2\sqrt{I_1I_2}}{I_1 + I_2 - 2\sqrt{I_1I_2}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2 = \left[\frac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\frac{r+1}{r-1}\right]^2$$

where, $r = \frac{a_1}{a_2}$ = amplitude ratio.

» *For sustained interference, coherent light beams are must. For good contrast we prefer intensities (or amplitudes) of superposing waves to be equal or nearly equal.*

» *Laser light of a particular wavelength, even from two independent laser sources, can produce interference fringes.*

Laws of Reflection

When the light incidence on a plane then its some part is reflected and some is refracted. The laws of reflection which relates the angle of incidence to the angle of reflection given as

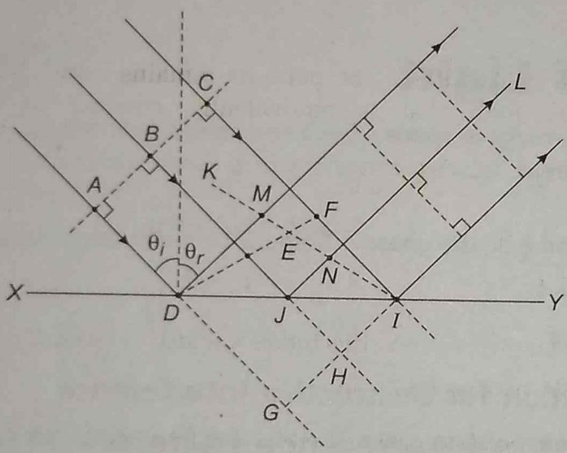
(i) Angle of incidence = Angle of reflection *i.e.*, $\theta_i = \theta_R$

This can be proved using the fact that ΔDIF and ΔDIM are congruent.

(ii) Similarly, it can be proved that the incident ray; reflected ray and normal to surface, all be in same plane.

Law of Reflection using Huygens' Principle

The figure illustrates Huygens' construction for a narrow, parallel beam of light to prove the laws of reflection. Huygens' principle must be modified to accommodate the case in which a wavefront, such as AC, encounters a plane interface, such as XY at an angle. Here, angle of incidence is θ_i. Since, points along the plane wavefront do not arrive at the interface simultaneously, allowance is made for these differences in constructing the wavelets that determine the reflected wavefront.



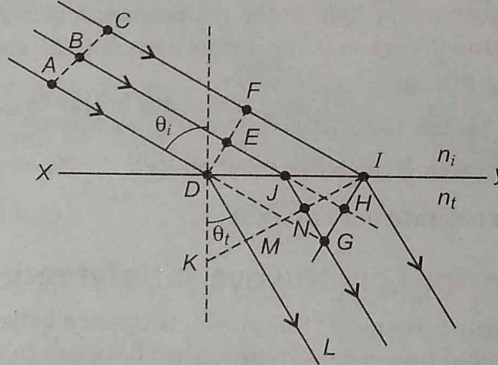
If the interface XY were not present, Huygens construction would produce the wavefront GI at the instance ray CF reached the interface at I. The intrusion of the reflecting surface however means that during the same time interval required for ray CF to progress from F to I ray BE has progressed from E to $J$and then a distance equivalent to JH after reflection.

Thus, a wavelet of radius JH centered at J is down above the reflecting surface. Similarly, a wavelet of radius DG is drawn centered at D to represent the propagation after reflection of the lower part of the beam. The new wavefront which must now be tangent to these wavelets at points M and N and include the point I is shown as KI in the figure. A representative reflective ray is DL, shown perpendicular to the reflected wavefront. The normal PD drawn for this ray is used to define angles of incidence and reflection for the beam.

Law of Refraction using Huygens' Principle

If the speed of light in vacuum is c then speed in up medium is $\frac{c}{n_i}$, where n_i is the refractive index. Simila light in the lower medium is $\frac{c}{n_i}$.

The points D, E and F on the incident wavefront arrive points D, J and I of the plane interface XY at differ times. In the absence of the refracting surface, wavefront GI is formed at the instant ray CF reaches During this progress of ray CF from F to I in time t, howe the ray AD has entered the lower medium where the spe is different.



Thus, a wavelet of radius $V_t t$ is constructed with centre D. The radius DM can also be expressed as

$$DM = V_t t \qquad \left[\because DG = V_i t \Rightarrow t = \frac{DG}{V_i}\right.$$

$$= V_t\left(\frac{DG}{V_i}\right) = \left(\frac{n_i}{n_t}\right) DG$$

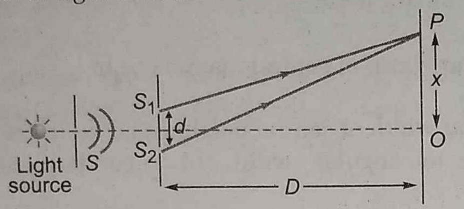
Similarly, a wavelet of radius $\left(\frac{n_i}{n_t}\right) JH$ is drawn centered J. The new wavefront KI includes point I on the interfa and is tangent to the two wavelets at points M and N. T geometric relationship between the angles θ_i and θ_t, forme by the representative incident ray AD and refracted ray L is Snell's law which may be expressed as

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Young's Double Slit Experiment

This experiment is a demonstration that matter and energy can display characteristics of both waves and particles and demonstrates the fundamentally probabilistic nature of quantum mechanical phenomena.

The arrangement is shown in figure. Monochromatic light of same wavelength is used.



For light waves reaching a point P, situated at a distance x from central point Δ, the path difference

$$\Delta = S_2P - S_1P = \frac{xd}{D}$$

(i) If $\frac{xd}{D} = n\lambda$, then we get the ***n*th bright fringe**. Hence, position of bright fringes on the screen are given by the relation $x = \frac{nD\lambda}{d}$.

(ii) If $\frac{xd}{D} = (2n-1)\frac{\lambda}{2}$, then we get the ***n*th dark fringe**. Hence, for nth dark fringe $x = \frac{(2n-1)D\lambda}{2d}$.

Fringe width β The separation between any two consecutive bright and dark fringes is called fringe width β and is given by, $\beta = \frac{D\lambda}{d}$

Angular fringe width of interference pattern

$$\alpha = \frac{\beta}{D} = \frac{\lambda}{d}$$

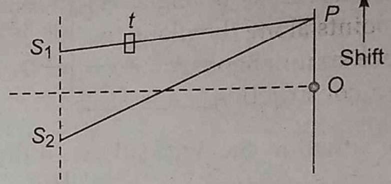
If the whole apparatus of Young's double slit experiment is immersed in a transparent medium of refractive index n_m, then fringe width in the medium

$$\beta_m = \frac{D\lambda}{n_m d} = \frac{\beta}{n_m}$$

For two interfering waves if ϕ_0 is the originating phase difference and Δ be the path difference between them then total phase difference them is $\phi = \phi_0 + \frac{2\pi}{\lambda}\Delta$.

Displacement of Fringe Pattern

When a thin transparent plate is introduced in the path of one of the interfering waves trains, it is found that the entire fringe pattern is shifted through a constant distance. This shift takes place towards the wave train, in the path of which the plate is introduced.



The fringe width of the patterns remains same. Effective optical path that is equivalently covered in air is $S_1P + t(\mu - 1)$.

Thus, the path difference

$$= S_2P - S_1P - t(\mu - 1)$$
$$= \frac{xd}{D} - t(\mu - 1)$$

[Optical path = refractive index × width of the material]

$\therefore$ Extra path $= \mu t - t = (\mu - 1)t$

Shift in Interference Pattern

Additional path difference due to sheet $= (n_m - 1)t$

Fringe shift $= \frac{D}{d}(n_m - 1)t = \frac{\beta}{\lambda}(n_m - 1)t$

If due to presence of thin film the fringe pattern shifts by n fringes, then

$$n = \frac{(n_m - 1)t}{\lambda}$$

or

$$t = \frac{n\lambda}{(n_m - 1)}$$

Interference in Thin Films

White light passes through a thin film, whose thickness is comparable to the wavelength of light, we get to see various colours due to interference of light waves reflected from the two surfaces of thin film.

For normal incidence, the interference maxima is given by the condition

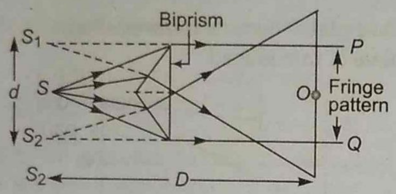
$$2n_m t = (2n-1)\frac{\lambda}{2}$$

where, t = thickness of film and
n_m = refractive index of film.

For dark band the condition is, $2n_m t = n\lambda$

Fresnel's Biprism

Fresnel's biprism is a device for obtaining **coherent sources**. The biprism is a glass prism of very small base angles.



Comparing the situation geometrically with the double-slit experiment fringe pattern width $G = w = \frac{D\lambda}{d}$

where, D = distance of screen from the line joining S_1S_2.

λ = wavelength of light used,

d = distance between S_1 and S_2.

Calculation of

Let a = distance of source S from biprism

deviation = $(\mu - 1)\alpha$ (for small angle prism)

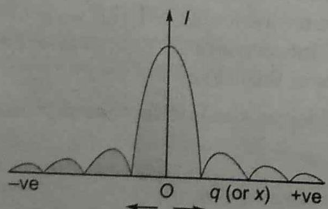
$\Rightarrow \quad d = 2a\delta = 2a(\mu - 1)\alpha$

Diffraction of Light

Diffraction of light is the phenomenon of bending of light around the edges of an aperture or obstacle and entry of light even in the region of geometrical shadow when size of aperture/obstacle is comparable to the wavelength of light used.

Fraunhoffer's Diffraction at a Single Slit

As a result of diffraction, due to single slit we obtain a broad, bright maxima symmetrical w.r.t. the centre point O (mid point of screen) and on either side of it we get secondary diffraction maxima of successively falling intensity and poor contrast as shown in figure.



Condition of diffraction minima is given by

$$a \sin\theta = n\lambda \text{ ; where } n = 1, 2, 3, 4, \ldots, k$$

But the condition of secondary diffraction maxima is

$$a \sin\theta = (2n - 1)\frac{\lambda}{2}; \text{ where } n = 1, 2, 3, 4, \ldots, k$$

Angular position of nth secondary minima is given by

$$\sin\theta = \theta = n\frac{\lambda}{a}$$

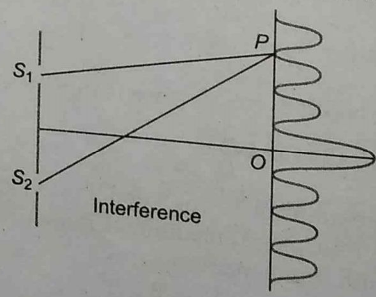
The angular width of central maxima is $2\theta = \frac{2\lambda}{a}$.

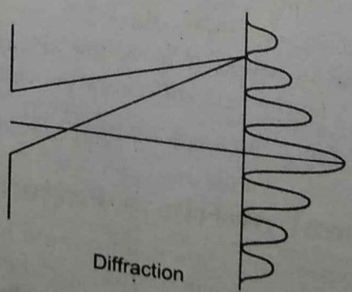
The angular width of central maxima is double the width as compared to angular width of secondary diffraction maxima.

If one of the two slits in Young's double slit experiment is closed, the interference will not take place and we shall be able to observe diffraction pattern due to a single slit.

Difference between Interference and Diffraction

- Interference is the superposition of waves from two different wavefronts, whereas diffraction is due to superposition of wavelets from different points of the same wavefront.
- Interference all maxima are equally bright and minima equally dark (perfectly black). In case of diffraction, maxima are of decreasing intensity, minima are not perfectly black.
- In case of interference, there are large number of maxima and minima, but in case of diffraction bands are few.



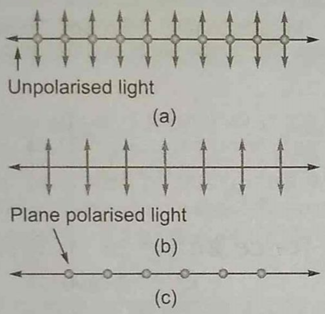


Polarisation of Light

Polarisation of light is the phenomenon of restricting the vibrations of light (electric vector) in a particular direction perpendicular to the direction of propagation of light wave. Such light is called ***plane polarised****.*

Ordinary light, emitted by sun or an electric lamp etc., propagating in a given direction, consists of many independent waves whose vibrations are randomly oriented in a plane which is perpendicular to the direction of propagation of light. Such light is said to be **unpolarised light**.

(i) Unpolarised light and plane polarised light are generally represented as shown in the figure



(ii) Only transverse waves can be polarised. Thus, it is proved that light waves are transverse waves.

(iii) Intensity of plane polarised light received from a tourmaline crystal or polaroid is half the intensity of the incident unpolarised light.

(iv) Nicol prism is another good device to produce plane polarised light. It is based on the property of double refraction.

- *A plane containing the vibrations of polarised light is called plane of vibration.*
- *A plane perpendicular to the plane of vibration is called plane of polarisation.*
- *Polarisation can take place only in transverse waves.*

Optical Rotation

Optical rotation is the property of rotating the plane polarised light by passing the same through an optically active substance. *Such substances are of two types*

1. Leavo Rotatory

These are substances which are left handed and rotate the plane of polarisation in an anti-clockwise direction.

2. Dextro Rotatory

These are substances which are right handed and rotate the plane of polarisation in clockwise direction. The angle of rotation depends on the length of the material density/concentration of the material, wavelength of light used and temperature which the experiment is carried out.

Polaroids

Polaroids polarise light. If the number of needle shaped crystals quinine and iodosulphate with their axes parallel to one another are packed between two sheets of plastic. This arrangement serves as the polaroids.

The important uses are

- These reduce excess glare and hence sun glasses are fitted with polaroid sheets.
- These are also used to reduce headlight glare of cars.
- They are used to improve colour contrast in old oil paintings.

Law of Malus

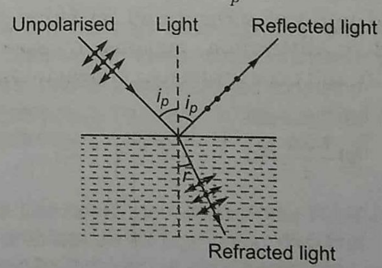
According to **Malus law**, the intensity of emergent light out of an analyser varies as $I \propto \cos^2\theta$ or $I = I_0 \cos^2\theta$, where θ is the angle between the planes of transmission of the polariser and the analyser.

Brewster's Law

When a beam of unpolarised light is reflected from the surface (unpolished) of a transparent medium of refractive index n at the polarising angle i_p, the reflected light is completely plane polarised.

According to Brewster's law,

$$\tan i_p = n$$

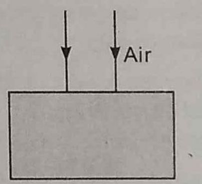


Moreover, in such an event, the reflected and transmitted rays are mutually perpendicular. Thus, angle of refraction $r = (90° - i_p)$.

Practice Zone

DAY 29

1. A parallel beam of light of intensity I_0 is incident on a glass plate, 25% of light is reflected by upper surface and 50% of light in reflected from lower surface. The ratio of maximum to minimum intensity in interference region of reflected ray is



(a) $\left(\dfrac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (b) $\left(\dfrac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$

(c) $\dfrac{5}{8}$ (d) $\dfrac{8}{5}$

2. For the same objective, find the ratio of the least separation between two points to be distinguished by a microscope for light of 5000 Å and electrons accelerated through 100 V used as the illuminating substance.

(a) 1×10^{-4}
(b) 0.2×10^{-3}
(c) 5×10^{-2}
(d) 7×10^{-4}

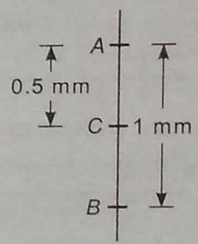
3. The path difference between two wavefronts emitted by coherent sources of wavelength 5460 Å is 2.1 μ. The phase difference between the wavefronts at that point is

(a) 7.692 rad (b) 7.692 π rad
(c) $\dfrac{7.692}{\pi}$ rad (d) $\dfrac{7.692}{3\pi}$ rad

4. The Young's double slit experiment is performed with blue and with green light of wavelengths 4360 Å and 5460 Å respectively. If x is the distance of the 4th maxima from the central one, then

(a) $x_{(blue)} = x_{(green)}$
(b) $x_{(blue)} > x_{(green)}$
(c) $x_{(blue)} < x_{(green)}$
(d) $x_{(blue)}/x_{(green)} = 5460/4360$

5. In Young's double slit experiment, the length of band is 1mm. The fringe width is 0.021 mm. The number of fringes is



(a) 45 (b) 46
(c) 49 (d) 48

6. In a Young's double slit experiment, two coherent sources are placed 0.90 mm apart and the fringes are observed one metre away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of monochromatic light used will be

(a) 60×10^{-4} cm (b) 10×10^{-4} cm
(c) 10×10^{-5} cm (d) 6×10^{-5} cm

7. A beam of ordinary unpolarised light passes through a tourmaline crystal C_1 and then it passes through another tourmaline crystal C_2 which is oriented such that its principal plane is parallel to that of C_2. The intensity of emergent light is I_0. Now, C_2 is rotated by 60° about the ray. The emergent ray will have an intensity

(a) $2\,I_0$ (b) $I_0/2$
(c) $I_0/4$ (d) $I_0/\sqrt{2}$

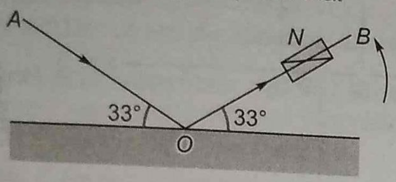
8. The first diffraction minimum due to single slit diffraction is θ, for a light of wavelength 5000 Å. If the width of slit is 1×10^{-4} cm. Then, the value of θ is

(a) 30° (b) 45°
(c) 60° (d) 15°

9. Angular width (θ) of central maximum of a diffraction pattern of a single slit does not depend upon

(a) distance between the slit and the source
(b) wavelength of light used
(c) width of the slit
(d) frequency of light used

10. A beam of light AO is incident on a glass slab ($\mu = 1.54$) in a direction as shown in the figure. The reflected ray OB is passed through a nicol prism. On viewing through a nicol prism, we find on rotating the prism that



(a) the intensity is reduced down to zero and remains zero
(b) the intensity reduces down somewhat and rises again
(c) there is no change in intensity
(d) the intensity gradually reduces to zero and then again increases.

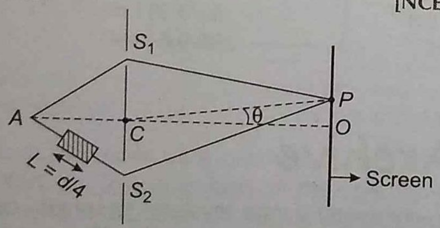
11. An unpolarised beam of light is incident on a group of four polarising sheets which are arranged in such a way that the characteristic direction of each polarising sheet makes an angle of 30° with that of the preceding sheet. The percentage of incident light transmitted by the first polariser will be

(a) 100% (b) 50% (c) 25% (d) 125%

12. A 5% solution of cane sugar placed in a tube of length 40 cm causes the optical rotation of 20°. How much length of 10% solution of the same substance will cause 35° rotation.

(a) 20 cm (b) 35 cm (c) 18 cm (d) 45 cm

13. A small transparent slab containing material of $\mu = 1.5$ is placed along AS_2 (figure). What will be the distance from O of the principal maxima and of the first minima on either side of the principal maxima obtained in the absence of the glass slab. **[NCERT Exemplar]**



$AC = CO = D, S_1C = S_2C = d << D$

(a) $\frac{5}{\sqrt{235}}$ below point O (b) $\frac{5}{\sqrt{231}}$ below point O

(c) $\frac{5}{\sqrt{220}}$ below point O (d) $\frac{5}{\sqrt{110}}$ below point O

14. Two light rays having the same wavelength λ in vacuum are in phase initially. Then the first ray travels a path L_1 through a medium of refractive index μ_1 while the second ray travels a path of length L_2 through a medium of refractive index μ_2. The two waves are then combined to observe interference. The phase difference between the two waves is

(a) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$ (b) $\frac{2\pi}{\lambda}(L_2 - L_1)$

(c) $\frac{2\pi}{\lambda}\left(\frac{L_1}{\mu_1} - \frac{L_2}{\mu_2}\right)$ (d) $\frac{2\pi}{\lambda}(\mu_2 L_1 - \mu_1 L_2)$

15. White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance d (>> b) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are

(a) $\lambda = \frac{2b^2}{3d}$ (b) $\lambda = \frac{b^2}{3d}$

(c) $\lambda = \frac{2b^2}{d}$ (d) $\lambda = \frac{3b^2}{d}$

16. A ray of light is incident on the surface of a glass plate of refractive index. 1.732 at the polarising angle. The angle of refraction of the ray is

(a) 45°
(b) 60°
(c) 15°
(d) 30°

17. Specific rotation of sugar solution is 0.01 rad-m^2/kg. If 200 kg/m^3 of impure sugar solution is taken in a polarimeter tube of length 0.25m and an optical rotation of 0.4 rad is observed, then percentage of purity of sugar in the sample is

(a) 11%
(b) 20%
(c) 80%
(d) 89%

Directions *(Q. Nos. 18 to 20) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

18. **Statement I** The thick film shows no interference pattern. Take thickness of the order of a few cms.

Statement II For interference pattern to be observed path difference between two waves is of the order of few wavelengths.

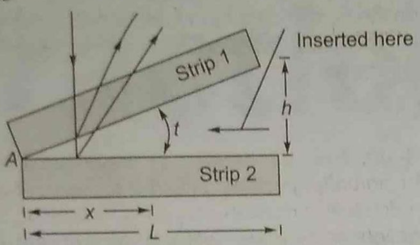
19. **Statement I** To observe diffraction of light, the size of obstacle/aperture should be of the order of 10^{-7} m.

Statement II 10^{-7}m is the order of wavelength of the visible light.

20. **Statement I** For a given medium, the polarising angle is 60°. The critical angle for this medium is 35°.

Statement II $\mu = \tan i_p$.

Directions (Q. Nos. 21 and 22) *Thin interference film is used to measure the thickness of slides, paper etc. The arrangement is as shown in figure.*



For sake of clarity, the two strips are shown thick. Consider the wedge formed in between strips 1 and 2, if the interference pattern because of the two waves reflected from wedge surface is observed, then from the observed data, we can compute thickness of paper, refractive index of the medium filled in wedge, number of bands formed etc. Consider the strips to be thick as compared to the wavelength of light and assume that the light is incident normally.

Neglect the effect due to reflection from top surface of strip 1 and bottom surface of strip 2. Take $L = 5$ cm, $\lambda_{air} = 400$ nm.

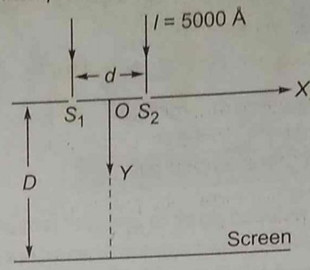
21. Consider an air wedge formed by two glass plates having refractive index 1.5, by placing a piece of paper of thickness 20 μm. Determine the number of dark bands formed.

(a) 1000 (b) 100 (c) 5000 (d) 500

22. For strip 1, refractive index = 1.34. For strip 2, refractive index = 1.6 and the wedge is filled with a medium having refractive index 1.5, then

(a) the band at the contact point would be dark
(b) the band at the contact point would be bright
(c) at contact point, no maxima or minima occurs
(d) at contact point, uniform illumination would be there

Directions (Q. Nos. 23 and 25) *In the figure shown alongside, light of wavelength $\lambda = 5000$ Å is incident on the slits (in a horizontally fixed place).*



Here, $d = 1$ mm, $D = 1$ m

Take origin at O and XY plane as shown in the figure. The screen is released from rest from the initial position as shown above. (Take $g = 10\ ms^{-2}$)

23. The velocity of central maxima at $t = 5$ s, is

(a) $50\ ms^{-1}$ along Y-axis
(b) $50\ ms^{-1}$ along negative Y-axis
(c) $25\ ms^{-1}$ along Y-axis
(d) $3 \times 10^8\ ms^{-1}$ along Y-axis

24. Velocity of the 2nd maxima w.r.t. central maxima at $t = 2$ s, is

(a) $8\ \hat{i}\ cms^{-1} + 20\ \hat{j}\ ms^{-1}$ (b) $8\ \hat{i}\ cms^{-1}$
(c) $2\ \hat{i}\ cms^{-1}$ (d) $86\ \hat{i}\ ms^{-1}$

25. Acceleration of the 3rd maxima w.r.t. the 3rd maxima on other side of central maxima at $t = 3$ s, is

(a) $0.02\ \hat{i}\ ms^{-2}$ (b) $0.03\ \hat{i}\ ms^{-2}$
(c) $10\ \hat{j}\ ms^{-2}$ (d) $0.6\ \hat{i}\ ms^{-2}$

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26. n identical waves each of intensity I_0 interfere with each other. The ratio of maximum intensities if the interference is (i) coherent and (ii) incoherent is **[JEE Main Online 2013]**

(a) n^2 (b) $\frac{1}{n}$
(c) $\frac{1}{n^2}$ (d) n

27. The source that illuminates the double-slit in 'double-slit interference experiment' emits two distinct monochromatic waves of wavelength 500 nm and 600 nm, each of them producing its own pattern on the screen. At the central point of the pattern when path difference is zero, maxima of both the patterns coincide and the resulting interference pattern is most distinct at the region of zero path difference. But as one moves out of this central region, the two fringe systems are gradually out of step such that maximum due to one wave length coincides with the minimum due to the other and the combined fringe system becomes completely indistinct. This may happen when path difference in nm is **[JEE Main Online 2013]**

(a) 2000
(b) 3000
(c) 1000
(d) 1500

28. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I In Young's double slit experiment, the number of fringes observed in the field of view is small with longer wavelength of light and is large with shorter wavelength of light.

Statement II In the double slit experiment the fringe width depends directly on the wavelength of light.

[JEE Main Online 2013]

(a) Statement I is true, Statement II is true and the Statement II is not the correct explanation of Statement I
(b) Statement I is false and the Statement II is true
(c) Statement I is true, Statement II is true and the Statement II is correct explanation of Statement I
(d) Statement I is true and the Statement II is false

29. Statement I Short wave transmission is achieved due to the total internal reflection of the electromagnetic wave from an appropriate height in the ionosphere.

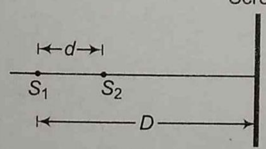
Statement II Refractive index of a plasma is independent of the frequency of electromagnetic waves. **[JEE Main Online 2013]**

(a) Statement I is true, Statement II is false
(b) Statement I is false, Statement II is true
(c) Statement I is true, Statement II is true but Statement II is not the correct explanation of Statement I
(d) Statement I is true, Statement II is true and Statement II is the correct explanation of Statement I

30. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is **[JEE Main 2013]**

(a) I_0 (b) $I_0/2$ (c) $I_0/4$ (d) $I_0/8$

31. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be **[JEE Main 2013]**



(a) points (b) straight lines
(c) semi-circle (d) concentric circles

32. In a Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ, is given by **[AIEEE 2012]**

(a) $\frac{I_m}{9}(4+5\cos\phi)$ (b) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$
(c) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

33. In Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case I_2, then the ratio $\frac{I_1}{I_2}$ is **[AIEEE 2011]**

(a) 4 (b) 2 (c) 1 (d) 0.5

34. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? **[AIEEE 2012]**

(a) 7.2 m (b) 2.4 m
(c) 3.2 m (d) 5.6 m

Directions *(Q. Nos. 35 to 37) are based on the following paragraph. An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I)=\mu_0+\mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.*

35. As the beam enters the medium, it will **[AIEEE 2010]**

(a) diverge
(b) converge
(c) diverge near the axis and converge near the periphery
(d) travel as a cylindrical beam

36. The initial shape of the wavefront of the beam is **[AIEEE 2010]**

(a) convex
(b) concave
(c) convex near the axis and concave near the periphery
(d) planar

37. The speed of light in the medium is **[AIEEE 2010]**

(a) minimum on the axis of the beam
(b) the same everywhere in the beam
(c) directly proportional to the intensity I
(d) maximum on the axis of the beam

38. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4^{th} bright fringe of the unknown light. From this data, the wavelength of the unknown light is **[AIEEE 2009]**

(a) 393.4 nm
(b) 885.0 nm
(c) 442.5 nm
(d) 776.8 nm

39. Three sound waves of equal amplitudes have frequencies $(\nu-1)$, ν, $(\nu+1)$. They superpose to give beat. The number of beats produced per second will be **[AIEEE 2009]**

(a) 4 (b) 3
(c) 2 (d) 1

40. In Young's double slit experiment, the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, I/I_0 is equal to **[AIEEE 2007]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

41. In Young's double slit experiment, the intensity at a point is (1/4) of the maximum intensity. Angular position of this point is **[AIEEE 2005]**

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

42. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is **[AIEEE 2005]**

(a) hyperbola (b) circle
(c) straight line (d) parabola

43. When an unpolarised light of intensity I_0 is incident on a polarising sheet, the intensity of the light which does not get transmitted is **[AIEEE 2005]**

(a) $\frac{1}{2}I_0$ (b) $\frac{1}{4}I_0$

(c) zero (d) I_0

44. In a YDSE, bi-chromatic light of wavelength 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is **[AIEEE 2004]**

(a) 4 mm (b) 5.6 mm (c) 14 mm (d) 28 mm

45. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double slit experiment, is **[AIEEE 2004]**

(a) infinite (b) five (c) three (d) zero

46. A mixture of light consisting of wavelength 590 nm and an unknown wavelength, illuminates the Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is **[AIEEE 2003]**

(a) 885.0 nm (b) 442.5 nm
(c) 776.8 nm (d) 393.4 nm

Answers

1. (a)	2. (b)	3. (a)	4. (c)	5. (c)	6. (d)	7. (c)	8. (c)	9. (a)	10. (d)
11. (b)	12. (b)	13. (b)	14. (a)	15. (b)	16. (d)	17. (c)	18. (a)	19. (a)	20. (b)
21. (b)	22. (b)	23. (a)	24. (c)	25. (b)	26. (d)	27. (d)	28. (c)	29. (a)	30. (c)
31. (d)	32. (d)	33. (b)	34. (d)	35. (b)	36. (d)	37. (a)	38. (c)	39. (c)	40. (d)
41. (c)	42. (d)	43. (d)	44. (c)	45. (b)	46. (b)				

Hints & Solutions

1. The intensity of light reflected from upper surface is

$$I_1 = I_0 \times 25\% = I_0 \times \frac{25}{100} = \frac{I_0}{4}$$

Intensity of transmitted light from upper surface is

$$I = I_0 - \frac{I_0}{4} = \frac{3I_0}{4}$$

$\therefore$ Intensity of reflected light from lower surface is

$$I_2 = \frac{3I_0}{4} \times \frac{50}{100} = \frac{3I_0}{8}$$

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{\frac{I_0}{4}} + \sqrt{\frac{3I_0}{8}}\right)^2}{\left(\sqrt{\frac{I_0}{4}} - \sqrt{\frac{3I_0}{8}}\right)^2} = \frac{\left(\frac{1}{2} + \sqrt{\frac{3}{8}}\right)^2}{\left(\frac{1}{2} - \sqrt{\frac{3}{8}}\right)^2}$$

2. $d_{min} = \frac{1.22\lambda}{2\sin\beta}$

where, λ is the wavelength of light and β is the angle subtended by the objective at the object.

For the light of wavelength 5500 Å

$$d_{min} = \frac{1.22 \times 5500 \times 10^{-10}}{2\sin\beta} \quad \ldots\text{(i)}$$

For electrons accelerated through 100 V, the de-Broglie wavelength

$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{100}} = 0.12 \times 10^{-9}\text{m}$$

$$d_{min} = \frac{1.22 \times 0.12 \times 10^{-9}}{2\sin\beta}$$

Ratio of the least separation

$$\therefore \quad \frac{d'_{min}}{d_{min}} = \frac{0.12 \times 10^{-9}}{5500 \times 10^{-10}} = 0.2 \times 10^{-3}$$

3. Phase difference $= \frac{2\pi}{\lambda} x = \frac{2\pi \times 2.1 \times 10^{-6}}{5460 \times 10^{-10}} = 7.692\,\pi$ rad

4. Distance of nth maxima, $x = n\lambda \frac{D}{d} \propto \lambda$

As, $\lambda_b < \lambda_g$

$\therefore$ $x_{\text{blue}} < x_{\text{green}}$

5. The number of fringes on either side of C of screen is

$$n_1 = \left[\frac{AC}{\beta}\right] = \left[\frac{0.5}{0.021}\right] = [23.8] \approx 24$$

Total number of fringes $= 2n_1 +$ fringe at centre

$$= 2n_1 + 1 = 2 \times 24 + 1$$
$$= 48 + 1 = 49$$

6. $$x = (2n-1)\frac{\lambda}{2}\frac{D}{d}$$

or $$\lambda = \frac{2xd}{(2n-1)D} = \frac{2 \times 10^{-3} \times 0.9 \times 10^{-3}}{(2 \times 2 - 1) \times 1}$$
$$= 6 \times 10^{-7}\ \text{m} = 6 \times 10^{-5}\ \text{cm}$$

7. Intensity of light from $C_2 = I_0$

On rotating through 60°,

$$I = I_0 \cos^2 60° = I_0 \left(\frac{1}{2}\right)^2 = I_0 / 4$$

8. The distance of first diffraction minimum from the central principal maximum is

$$x = \frac{D\lambda}{d}$$
$$\frac{x}{D} = \frac{\lambda}{d}$$
$$\Rightarrow \quad d = \frac{\lambda}{\sin\theta}$$
$$\Rightarrow \quad \sin\theta = \frac{\lambda}{d} = \frac{5000 \times 10^{-8}}{1 \times 10^{-4}} = 0.5 = \sin 30°$$
$$\Rightarrow \quad \theta = 30°$$

9. Angular width of central maximum

$$\theta = \frac{\beta}{D} = \frac{2\lambda D}{aD} = \frac{2\lambda}{a}$$

θ does not depend upon D, distance between slit and source.

10. As $i_p = \tan^{-1}(1.54) = 57°$

and in the figure given in question $i = 90° - 33° = 57° = i_p$

$\therefore$ Reflected light along OB is plane polarised. On rotating the nicol prism, intensity gradually reduces to zero and then increases again.

11. First polariser just polarises the unpolarised light. Therefore, intensity of polarised light transmitted from first polariser is

$$\frac{1}{2} I_0 = 50\%\ I_0$$

12. Specific rotation $S = \frac{\theta}{lc}$, where θ is in degrees, l is in decimeters, c is in g/cc.

$$S = \frac{\theta_1}{l_1 c_1} = \frac{\theta_2}{l_2 c_2}$$

Given, $\theta_1 = 20°$ for $c_1 = 5\%$ and $l_1 = 40$ cm

$\theta_2 = 35°$ for $c_2 = 10\%$

$$\frac{20}{40 \times 5} = \frac{35}{l_2 \times 10}$$

$$\Rightarrow \quad l_2 = \frac{35 \times 40 \times 5}{20 \times 10} = 35\ \text{cm}$$

13. In case of transparent glass slab of refractive index μ, the path difference $= 2d \sin\theta + (\mu - 1)L$

For the principal maxima, (path difference is zero)

i.e., $2d \sin\theta_0 + (\mu - 1)L = 0$

or $\sin\theta_0 = -\frac{L(\mu - 1)}{2d} = \frac{-L(0.5)}{2d}$ $\quad [\therefore L = d/4]$

or $\sin\theta_0 = \frac{-1}{16}$ or $OP = D\tan\theta_0 = D\sin\theta_0 = \frac{-D}{16}$

For the first minima, the path difference is $\pm\frac{\lambda}{2}$

$$\therefore \quad 2d\sin\theta_1 + 0.5L = \pm\frac{\lambda}{2}$$

or $$\sin\theta_1 = \frac{\pm\lambda/2 - 0.5L}{2d} = \frac{\pm\lambda/2 - d/8}{2d} = \frac{\pm\lambda/2 - \lambda/8}{2\lambda}$$
$$= \pm\frac{1}{4} - \frac{1}{16}$$

[$\because$ The diffraction occurs if the wavelength of waves in nearly equal to the slid width (d)]

On the positive side $\sin\theta'_1 = +\frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

On the negative side $\sin\theta''_1 = -\frac{1}{4} - \frac{1}{16} = -\frac{5}{16}$

The first principal maxima on the positive side is at distance

$$D\tan\theta'_1 = D\frac{\sin\theta'_1}{\sqrt{1 - \sin^2\theta'_1}}$$
$$= D\frac{3}{\sqrt{16^2 - 3^2}} = \frac{3D}{\sqrt{247}} \text{ above point } O$$

The first principal minima on the negative side is at distance

$$D\tan\theta''_1 = \frac{5}{\sqrt{16^2 - 5^2}} = \frac{5}{\sqrt{231}} \text{ below point } O.$$

14. Optical path for 1st ray $= \mu_1 L_1$

Optical path for 2nd ray $= \mu_2 L_2$

$\therefore$ Path difference $= (\mu_1 L_1 - \mu_2 L_2)$

Now, phase difference $= \frac{2\pi}{\lambda} \times$ (path difference)

$$= \frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$$

15. Path difference $= S_2P - S_1P$

From the figure,

$(S_2P)^2 - (S_1P)^2 = b^2$ or $(S_2P - S_1P)(S_2P + S_1P) = b^2$

or $\quad (S_2P - S_1P) = \frac{b^2}{2d}$

For dark fringes, $\frac{b^2}{2d} = (2n+1)\frac{\lambda}{2}$

For $n = 0$, $\frac{b^2}{2d} = \frac{\lambda}{2}$ or $\lambda = \frac{b^2}{d}$

For $n = 1$, $\frac{b^2}{2d} = \frac{3\lambda}{2}$ or $\lambda = \frac{b^2}{3d}$

16. From Brewster's law,

$$\mu = \tan\theta_p$$
$$1.732 = \tan\theta_p$$
$$\Rightarrow \quad \theta_p = \tan^{-1}(1.732) = 60°$$

Since the angle between i_p and r is 90 ° when the ray is incident at polarising angle, then

$$\theta_p + r = 90°$$
$$r = 90 - \theta_p = 90 - 60 = 30°$$

17. Thus, we have the specific rotation of the sugar solution, $s = 0.01\,\text{rad-m}^2/\text{kg}$

Concentration of impure sugar solution, $C_t = 200\ \text{kg/m}^3$

Length of the polarimeter, be $l = 0.25\,\text{m}$

Thus optical rotation, $\theta = 0.4\,\text{rad}$

Since, $\quad C = \frac{\theta}{sl} = \frac{0.4}{0.01 \times 0.25} = 160\ \text{kg/m}^3$

$\therefore$ % of purity of sugar in the solution

$$= \frac{C}{C_1} = \frac{160}{200} = 0.8 = 80\%$$

19. For diffraction to occur, the size of an obstacle/aperture is comparable to the wavelength of light wave. The order of wavelength of light wave is 10^{-7} m, so diffraction occurs.

20. From the relation,

$$\mu = \tan i_p = \tan 60° = \sqrt{3} \quad \text{...(i)}$$

and $\quad \sin C = \frac{1}{\mu} = \frac{1}{\sqrt{3}} \quad$ [from Eq. (i)]

$\therefore \quad C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = \sin^{-1}(0.577)$

$$= 35°$$

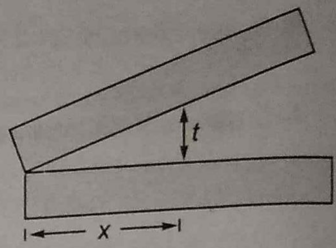
21. For constructive interference,

$$2t = (2n-1)\frac{\lambda_{\text{air}}}{2}$$

For destructive interference,

$$2t = n\lambda_{\text{air}}$$

As due to reflection at top surface of bottom layer an additional path difference of $\frac{\lambda}{2}$ occurs.



Contact point would be dark, let at distance x, n^{th} dark band forms.

$\Rightarrow \quad 2t = n\lambda_{\text{air}}$ and $\frac{t}{x} = \frac{h}{L}$

$\Rightarrow \quad \frac{2hx}{L} = n\lambda_{\text{air}}$

$\Rightarrow \quad x = n\left[\frac{\lambda_{\text{air}} \times L}{2h}\right]$

$\therefore \quad x = n\left[\frac{400 \times 10^{-9} \times 5 \times 10^{-2}}{2 \times 20 \times 10^{-6}}\right] = (5 \times 10^{-4} n)$

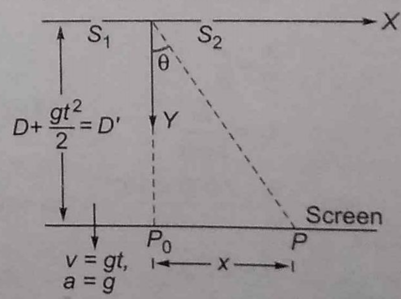
Maximum value of n would be for $x = L$

$\Rightarrow \quad n_{\text{max}} = \frac{5 \times 10^{-2}}{5 \times 10^{-4}} = 100$

22. For reflection from bottom layer of strip 1 as well as for top surface of strip 2, phase shift of π takes place. So, condition for maxima is $2t = n\lambda_{\text{medium}}$ and for minima is $2t = (2n-1)\frac{\lambda_{\text{medium}}}{2}$

So, contact point would be bright as here path difference is 0.

23. At any time t, the situation is as shown in the figure below.



Central maxima is always lying on Y-axis at P_0. Its velocity at any time t is given by

$$v = gt$$

along positive Y-axis.

So, required velocity $= 50\ \text{ms}^{-1}$ along Y-axis

24. Path difference corresponding to point P,

$$\Delta x = d\sin\theta = d\tan\theta$$
$$\Delta x = \frac{d \times x}{D'}$$

For 2nd maxima, $\quad \Delta x = 2\lambda$

$\Rightarrow \quad dx = 2\lambda D'$

$\Rightarrow \quad x = \frac{2\lambda D'}{d}$

Location of central maxima is $\left(0, D + \frac{gt^2}{2}\right)$ or $(0, D')$

Location of 2nd maxima is $\left(\frac{2\lambda D'}{d}, D'\right)$

Velocity of 2nd maxima w.r.t. central maxima is

$$\frac{\mathbf{v}_{2nd}}{D} = \frac{2\lambda}{d}[0 + gt]\,\hat{\mathbf{i}}$$
$$= \frac{2\lambda}{d} \times gt\,\hat{\mathbf{i}} = 2\,\hat{\mathbf{i}}\ \text{cms}^{-1}$$

25. Location of 3rd maxima is $\left(\frac{3\lambda D'}{d}, D'\right)$

Location of 3rd maxima on other side is $\left(-\frac{3\lambda D'}{d}, D'\right)$

$$\frac{\mathbf{a}_{3rd}}{-3rd} = \left[\frac{3\lambda}{d}g - \left(-\frac{3\lambda g}{d}\right)\right]\hat{\mathbf{i}}$$
$$= \frac{6\lambda g}{d}\,\hat{\mathbf{i}} = 0.03\,\hat{\mathbf{i}}\ \text{ms}^{-2}$$

26. **When interference is coherent**

When two waves of intensities I_1 and I_2 having a phase difference ϕ interfere, the resultant intensity is given as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \ldots\text{(i)}$$

The intensity will maximum, then $\phi = 0$ or $\cos\phi = 1$ maximum intensity.

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_1})^2$$

In case, n identical waves each of intensities I_0 interfere, $I_{max} = (\sqrt{I_0} + \sqrt{I_0} + \sqrt{I_0} + \ldots + n \text{ times})^2$

$$= (n(\sqrt{I_0})^2 \quad \ldots\text{(i)}$$

$$\therefore \quad I_{max} = n^2 I_0 \quad \ldots\text{(ii)}$$

When interference is incoherent

Since, the average value of $\cos\phi$, over a complete cycle is zero

The Eq. (i), becomes,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \times 0 = I_1 + I_2 \quad \ldots\text{(iii)}$$

In case, n identical waves, each of intensities I_0 interfere,

Minimum intensity, $I_{min} = I_0 + I_0 + I_0 + \ldots n$ times

$$I_{min} = nI_0 \quad \ldots\text{(iv)}$$

$$\therefore \quad \text{Ratio } \frac{I_{max}}{I_{min}} = \frac{n^2 I_0}{nI_0} = n$$

27. $\because \quad n\lambda_1 = \left(n + \frac{1}{2}\right)\lambda_1$

$$\Rightarrow \quad n \times 500 \times 10^{-9} = \left(n + \frac{1}{2}\right) \times 600 \times 10^{-9} \Rightarrow n = \frac{3}{4}$$

Now from the formula, $\Delta x = n\lambda$ or $\left(n + \frac{1}{2}\right)\lambda$

we get, $\Delta x = 1500$ nm

28. The number of fringe is smaller increase of larger wavelength is used while incase of smaller wavelength is used the number of fringe is larger.

Also, fringe width is given by

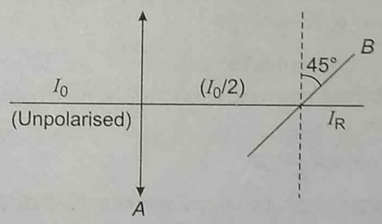
$$\beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \lambda$$

29. Short wave transmission is achieved due to the total internal reflection of the electromagnetic wave from an appropriate height in the ionosphere but refractive index of a plasma is dependent of the frequency of electromagnetic waves.

This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

30. This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate of decay modulated singal voltage for proper detection of modulated signal.

Relation between intensities is



$$I_R = \left(\frac{I_0}{2}\right)\cos^2(45°) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

31. It will be concentric circles.

32. Given, $a_1 = 2a_2 \Rightarrow I_1 = 4I_2 = 4I_0$

$$\therefore \quad I_m = (\sqrt{I_1} + \sqrt{I_2})^2 = (3\sqrt{I_2})^2 = 9I_2 = 9I_0$$
$$= I_o = \frac{I_m}{9}$$

Now, Resultant Intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= 4I_0 + I_0 + 2\sqrt{4I_0 I_0}\cos\phi$$
$$= 5I_0 + 4I_0\cos\phi = \frac{I_m}{9}(5 + 4\cos\phi)$$
$$= \frac{I_m}{9}[1 + 4(1 + \cos\phi)]$$
$$= \frac{I_m}{9}(1 + 8\cos^2\phi/2) \qquad \left[(1 + \cos\theta) = 2\cos^2\frac{\phi}{2}\right]$$

33. For coherent sources

$$I_1 = 4I_0\cos^2\phi/2 = 4I_0$$

For incoherent sources

$$I_2 = I_0 + I_0 = 2I_0$$

$$\therefore \quad \frac{I_1}{I_2} = 2$$

34. Shift in image position due to glass plate,

$$S = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{1}{1.5}\right) \times 1\,\text{cm} = \frac{1}{3}\text{cm}$$

For focal length of the lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{-240} = \frac{20+1}{240}$$

$$\Rightarrow \quad f = \frac{240}{21}\text{cm}$$

Now, to get back image on the film, lens has to form image at $\left(12 - \frac{1}{3}\right)\text{cm} = \frac{35}{3}\text{cm}$ such that the glass plate will shift the image on the film.

As $\quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{3}{35} - \frac{21}{240}$

$$\frac{1}{u} = \frac{48 \times 3 - 7 \times 21}{1680} = -\frac{1}{560}$$

$$\Rightarrow \quad u = -5.6\text{ m}$$

35. As intensity is maximum at axis,

$\therefore$ μ will be maximum and speed will be minimum on the axis of the beam.

$\therefore$ beam will converge.

37. For a parallel cylindrical beam, wavefront will be planar.

38. $3\lambda_1 = 4\lambda_2$

$$\Rightarrow \quad \lambda_2 = \frac{3}{4}\lambda_1 = \frac{3}{4} \times 590$$

$$= \frac{1770}{4} = 442.5\text{nm}$$

39. Maximum number of beats $= \nu + 1 - (\nu - 1) = 2$

40. Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

i.e., $\quad \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$

As, $\quad I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

or $\quad \frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right)$

or $\quad \frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}$

41. $\quad I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$\therefore \quad \frac{I_{max}}{4} = I_{max}\cos^2\frac{\phi}{2}$

$$\cos\frac{\phi}{2} = \frac{1}{2} \quad \text{or} \quad \frac{\phi}{2} = \frac{\pi}{3}$$

$$\therefore \quad \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right)\Delta x \qquad \text{...(i)}$$

where $\Delta x = d\sin\theta$ Substituting in Eq. (i), we get

$$\sin\theta = \frac{\lambda}{3d} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

42. Shape of interference fringes formed on a screen is parabolic in nature.

43. $\quad I = I_0\cos^2\theta$

Intensity of polarised light $= \frac{I_0}{2}$

$\therefore$ Intensity of untransmitted light $= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$

44. Let nth minima of 400 nm coincides with mth minima of 560 nm, then

$$(2n-1)\left(\frac{400}{2}\right) = (2m-1)\left(\frac{560}{2}\right)$$

or $\quad \frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = \dots$

i.e., 4th minima of 400 nm coincides with 3rd minima of 560 nm.

Location of this minima is,

$$Y_1 = \frac{(2 \times 4 - 1)(1000)(400 \times 10^{-6})}{2 \times 0.1} = 14\text{ mm}$$

Next 11th minima of 400 nm will coincide with 8th minima of 560 nm.

Location of this minima is,

$$Y_2 = \frac{(2 \times 11 - 1)(1000)(400 \times 10^{-6})}{2 \times 0.1} = 42\text{ mm}$$

$\therefore$ Required distance $= Y_2 - Y_1 = 28$ mm

45. For possible interference maxima of the screen, the condition is

$$d\sin\theta = n\lambda \qquad \text{...(i)}$$

Given, $\quad d =$ slit-width $= 2\lambda$

$\therefore \quad 2\lambda\sin\theta = n\lambda$

or $\quad 2\sin\theta = n$

The maximum value of $\sin\theta$ is 1, hence,

$$n = 2 \times 1 = 2$$

Thus, Eq. (i) must be satisfied by 5 integer values i.e., $-2, -1, 0, 1, 2$. Hence, the maximum number of possible interference maxima is 5.

46. We have $\frac{3D\lambda_k}{d} = \frac{4D\lambda_\mu}{d}$

where λ_k is the known wavelength and λ_μ is the unknown wavelength. Thus, we get

$$\lambda_\mu = \frac{3\lambda_k}{4}$$

$$= \frac{3}{4} \times 590 = 442.5\,\text{nm}$$

Unit Test 6 (Optics)

DAY 30

1. Minimum light intensity that can be perceived by normal human eye is about 10^{-10} W m^{-2}. What is the minimum number of photons of wavelength 660 nm that must enter the pupil in one second, for one to see the object? Area of cross-section of the pupil is 10^{-4} m^2.

(a) 3.3×10^2 (b) 3.3×10^3
(c) 3.3×10^4 (d) 3.3×10^5

2. If two + 5 D lenses are mounted at some distance apart, the equivalent power will always be negative, if the distance is

(a) greater than 40 cm (b) equal to 10 cm
(c) less than 10 cm (d) less than 20 cm

3. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is **[NCERT Exemplar]**

(a) 7.5° (b) 5°
(c) 15° (d) 2.5°

4. Two beams of light of intensities 4I and I interfere. The intensity on the screen, where the phase difference in $\pi/2$, will be

(a) 5 I (b) 6 I
(c) 8 I (d) 9 I

5. A thin glass prism $\mu = 1.5$ is immersed in water $\mu = 1.3$. If the angle of deviation in air for particular ray be D, then that in water will be

(a) 0.2 D (b) 0.3 D (c) 0.5 D (d) 0.6 D

6. The refractive index of the material of equilateral prism is $\sqrt{3}$. The angle of minimum deviation for the prism is

(a) 30° (b) 41°
(c) 49° (d) 60°

7. A thin convergent glass lens $\mu = 1.5$ has a power of + 5.0 D. When this lens is immersed in a liquid of refractive index μ_l it acts as a divergence lens of focal length 100 cm. The value of μ_l should be

(a) 3/2 (b) 4/3
(c) 5/3 (d) 2

8. A thin convex lens of crown glass having refractive index 1.5 has power 1 D. What will be the power of similar convex lens of refractive index 1.6?

(a) 0.6 D (b) 0.8 D
(c) 1.2 D (d) 1.6 D

9. A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is **[NCERT Exemplar]**

(a) blue (b) green
(c) violet (d) red

10. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image **[NCERT Exemplar]**

(a) moves away from the lens with uniform speed 5 m/s
(b) moves away from the lens with uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration

11. A passenger in an aeroplane shall **[NCERT Exemplar]**

(a) never see a rainbow
(b) may see a primary and a secondary rainbow as concentric circles
(c) may see a primary and a secondary rainbow as concentric arcs
(d) shall never see a secondary rainbow

12. Distance of distinct vision is 25 cm. The focal length of the convex lens is 5 cm. It can act as a magnifier of magnifying power

(a) less than 5
(b) 5
(c) 6
(d) more than 6

13. A myopic person having far point 80 cm uses spectacles of power –1.0 D. How far can he see clearly?

(a) 1 m
(b) 2 m
(c) 4 m
(d) More than 4 m

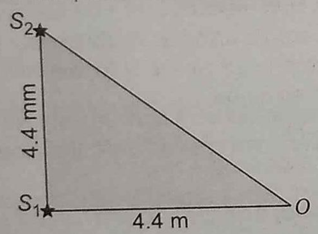
14. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will **[NCERT Exemplar]**

(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side on which the object lies
(d) act as a concave lens irrespective of side on which the object lies

15. Monochromatic light of wavelength 800 nm is used in double slit experiment. One of the slit is covered with a transparent slab of thickness 2.4×10^{-5} m. The refractive index of the material of slab is 1.4. What is the number of fringes that will shift due to introduction of the sheet?

(a) 14 (b) 12 (c) 16 (d) 10

16. Two coherent sources are 4.4 mm apart and 4.4 m from the screen as shown in the figure. If the sources emit light of wavelength 440 nm which produce an interference pattern on the screen. The pattern of the interference at point O is



(a) constructive only
(b) destructive only
(c) cannot be predicted
(d) may be constructive or destructive

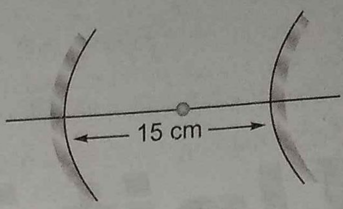
17. A lens behaves as a converging lens in air and a diverging lens in water. The refractive index of the material of the lens is

(a) equal to unity
(b) equal to 1.33
(c) between unity and 1.33
(d) greater than 1.33

18. Angular width of central maximum in the Fraunhoffer's diffraction pattern is measured. Slit is illuminated by the light of wavelength 6000 Å. If slit is illuminated by light of another wavelength, angular width decreases by 30%. Wavelength of light used is

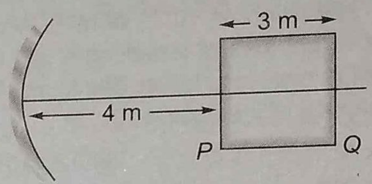
(a) 3500 Å (b) 4200 Å
(c) 4700 Å (d) 6000 Å

19. A convex mirror and a concave mirror of radius 10 m each are placed 15 m apart facing each other. An object is placed mid way between them. If the reflection first take place in the concave mirror and then in another mirror, the position of the final image is



(a) at infinity
(b) at the mid point of distance between two mirrors
(c) on the pole of concave mirror
(d) on the pole of convex mirror

20. A cube of side 3 m is placed in front of a concave mirror of focal length 2m with its face P at a distance 4 m and face Q at a distance 7 m from the mirror. What is distance between the images of face P and Q ?



(a) 1.2 m (b) 2.4 m
(c) 2.1 m (d) 2.2 m

21. A diminished image of an object is to be obtained on a screen 1 m from it. This can be achieved by approximately placing.

(a) a concave mirror of subitable focal length 0.25 m
(b) a convex mirror of suitable focal length
(c) a concave lens of suitable focal length
(d) a convex lens of focal length less than 0.25 m

22. A convex lens and a concave lens are placed in contact. The ratio of the magnitude of the power of the convex lens to that of the concave lens is 4 : 3. If the focal length of the convex lens is 12 cm, the focal length of the combination will be

(a) 16 cm (b) 24 cm
(c) 32 cm (d) 48 cm

23. The radius of curvature of a thin plano-convex lens is 10 cm and the refractive index of its glass is 1.5. If the plane surface is silvered, then it will behave like a

(a) concave mirror of focal length 10 cm
(b) concave mirror of focal length 20 cm
(c) convex mirror of focal length 10 cm
(d) convex mirror of focal length 20 cm

24. When the plane surface of a plano-convex lens of refractive index 1.5 is silvered, it behaves like a concave mirror of focal length 30 cm. When its convex surface is silvered, it will behave like a concave mirror of focal length

(a) 10 cm
(b) 20 cm
(c) 30 cm
(d) 45 cm

25. Two stars are situated at a distance of 8 light years from the earth. These are to be just resolved by a telescope of diameter 0.25 m. If the wavelength of light used is 5000 Å, then the distance between the stars must be

(a) 3×10^{10} m (b) 3.35×10^{11} m
(c) 1.95×10^{11} m (d) 4.32×10^{10} m

26. The refractive index of air is 1.0003. The thickness of air column which will have one more wavelength of yellow light ($\lambda = 6000$ Å), then in the same thickness in vacuum is

(a) 2 mm (b) 2 cm
(c) 2 m (d) 2 km

27. A thin symmetric convex lens of refractive index 1.5 and radius of curvature 0.3 m is immersed in water of refractive index 4/3. Its focal length in water is

(a) 0.15 m (b) 0.30 m (c) 0.60 m (d) 1.20 m

28. A parallel beam of sodium light of wavelength 6000Å is incident on a thin glass plate of $\mu = 1.5$, such that the angle of incidence in the plate is 60°. The smallest thickness of the plate which will make it appear dark by reflected light is

(a) 1260 Å (b) 2440 Å
(c) 3260 Å (d) 4000 Å

29. Two nicols are oriented with their principal planes making an angle of 60°. The percentage of incident unpolarised light which passes through the system is

(a) 50% (b) 100%
(c) 12.5% (d) 37.5%

30. In the visible region of the spectrum the rotation of the plane of polarization is given by $\theta = a + \frac{b}{\lambda^2}$

The optical rotation produced by a particular material is found to be 30° per mm at $\lambda = 5000$ Å and 50° per mm at $\lambda = 4000$Å. The value of constant a will be

(a) $+\frac{50°}{9}$ per mm (b) $-\frac{50°}{9}$ per mm
(c) $+\frac{9°}{50}$ per mm (d) $-\frac{9°}{50}$ per mm

31. The refracting angle of a prism is A and the refractive index of the prism is $\cot(A/2)$. The angle of minimum deviation is

(a) $180°-3A$ (b) $180° + 2A$
(c) $90°-A$ (d) $180°-2A$

32. A 20 cm length of a certain solution causes right handed rotation of 38°. A 30 cm length of another solution causes left-handed rotation of 24°. The optical rotation caused by 30 cm length of a mixture of the above solution in the volume ratio 1 : 2 is

(a) left handed rotation of 14°
(b) right handed rotation of 14°
(c) left handed rotation of 3°
(d) right handed rotation of 3°

33. Light passes successively through two polarimeters tubes each of length 0.29 m. The first tube contains dextro rotatory solution of concentration 60 kg m^{-3} and specific rotation 0.01 rad m^2 kg^{-1}. The second tube ntains laevorotatory solution of concentration 30 kgm^{-3} and specific rotation 0.02 rad m^2 kg^{-1}. The net rotation produced is

(a) 15° (b) 0°
(c) 20° (d) 10°

34. Polarising angle for water is 53°4'. If light is incident at this angle on the surface of water and reflected, the angle of refraction is

(a) 53° 4′ (b) 126° 56′
(c) 36° 56′ (d) 30° 4′

35. The distance between the first and the sixth minima in the diffraction pattern of a single slit is 0.5 mm. The screen is 0.5 m away from the slit. If the wavelength of light used is 5000 Å, then the slit width will be

(a) 5 mm (b) 2.5 mm
(c) 1.25 mm (d) 1.0 mm

Directions *(Q. Nos. 36 to 42) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

36. Statement I Angle of deviation depends on the angle of prism.

Statement II For thin prism $\delta = (\mu - 1) A$.
where, δ = angle of deviation,
μ = refractive index,
A = angle of prism.

37. Statement I Glass is transparent but its powder seems opaque. When water is poured over it, it becomes transparent.

Statement II Light gets refracted throught water.

38. Statement I If convex lens is kept in water its convergent power decreases.

Statement II Focal length of convex lens in water increases.

39. Statement I Danger signals are made of red colours.

Statement II Velocity of red light is maximum and thus, more visibility in dark.

40. Statement I The clouds in sky generally appear to be whitish.

Statement II Diffraction due to clouds is efficient in equal measure at all wavelengths.

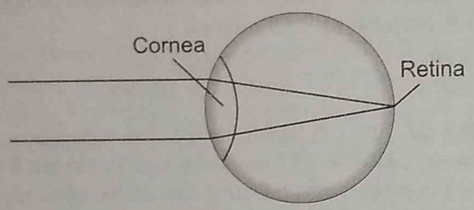
41. **Statement I** The resolving power of a telescope is more if the diameter of the objective lens is more.

Statement II Objective lens of large diameter collects more light.

42. **Statement I** The focal length of the objective of the telescope is larger than that of eyepiece.

Statement II The resolving power of telescope increases when the aperture of objective is small.

Directions (Q. Nos. 43 to 47) *Figure shows a simplified model of the eye that is based on the assumption that all of the refraction of entering light occurs at the cornea. The cornea is a converging lens located at the outer surface of the eye with fixed focal length approximately equal to 2 cm. Parallel light rays coming from a very distant object are refracted by the cornea to produce a focused image on the retina. The retina then transmits electrical impulse along the optic nerve to the brain.*



Two common defects of vision are myopia and hypermetropia. Myopia, sometimes referred to as near sightedness, occurs when the cornea focuses the image of a distant object in front of the retina. Hypermetropia, sometimes referred to as far sightedness, occurs when the cornea focuses the image of a nearby object behind the retina. Both of these problems can be corrected by introducing another lens in front of the eye so that the two lens system produces a focused image on the retina. If an object is so far away from the lens system that its distance may be taken as infinite, then the following relationship holds

$$\frac{1}{f_c} + \frac{1}{f_l - x} = \frac{1}{v}$$

where f_c is the focal length of the cornea, f_l, is the focal length of the correcting lens, x is the distance from the correcting lens to the cornea, and v is the image distance measured from the cornea.

*(**Note** The index of refraction is 1.0 for air and 1.5 for glass.)*

43. How far away should the retina be from the cornea for normal vision?

(a) 0.5 cm (b) 1.0 cm (c) 2.0 cm (d) 4.0 cm

44. For a distant object, the image produced by the cornea is

(a) real and inverted (b) real and upright
(c) virtual and inverted (d) virtual and upright

45. What kind of lens would be suitable to correct myopia and hypermetropia respectively?

(**Note** Assume that the correcting lens is at the focal point of the cornea so that $x = f_c$.)

(a) Converging, converging
(b) Converging, diverging
(c) Diverging, diverging
(d) Diverging, converging

46. The focal length of a woman's cornea is 1.8 cm, and she wears a correcting lens with a focal length of –16.5 cm at a distance $x = 1.5$ cm from her cornea. What is the image distance v measured from the cornea for a distant object?

(a) 1.0 cm (b) 1.5 cm
(c) 2.0 cm (d) 2.5 cm

47. In the case of contact lens, the cornea and the correcting lens are actually touching and act together as a single lens. If the focal length of both the cornea and the contact lens are doubled, then the image distance v for a distant object would be

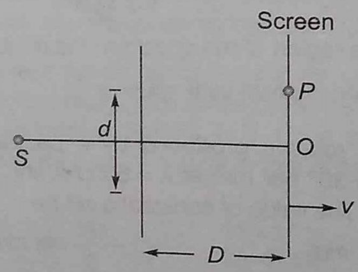
(a) $\frac{1}{4}$ the old value

(b) $\frac{1}{2}$ the old value

(c) the same as the old value

(d) twice the old value

Directions (Q. Nos. 48 and 49) *In YDSE apparatus shown in figure wavelength of light used is λ. The screen is moved away from the source with a constant speed v. Initial distance between screen and plane of slits was D.*



48. At a point P on the screen the order of fringe will

(a) increase
(b) decrease
(c) remain constant
(d) first increase then decrease

49. Suppose P is the point where 5th order maxima was lying at $t = 0$. Then, after how much time third order minima will lie at this point?

(a) $\frac{2D}{v}$ (b) $\frac{D}{v}$

(c) $\frac{3D}{2v}$ (d) $\frac{3D}{v}$

Answer *with* Solutions

1. (*c*) $I = 10^{-10}$ Wm^{-2} $= 10^{-10}$ Js^{-1}m^{2}. Let the number of photons required per second be n.

Then, $\frac{nhv}{10^{-4}} = 10^{-10}$

Hence, $n = 10^{-10} \times 10^{-4}/hv$

$= 10^{-14} \frac{\lambda}{hc}$

$= \frac{10^{-14} \times 660 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}}$

$= 3.3 \times 10^{4}$

2. (*a*) When two lenses are separated by some distance x, then equivalent power

$$P = P_1 + P_2 - x P_1 P_2$$

$\therefore$ $P = 5 + 5 - x \times 5 \times 5$

or $P = 10 - 25\,x$

Power will be negative if $10 - 25x$ will be negative

$$25x > 10$$

$$x > \frac{10}{25}\text{ m}$$

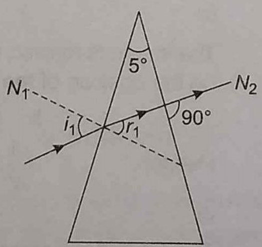
$$x > \frac{10}{25} \times 100$$

$$x > 40\text{ cm}$$

3. (*a*) Here, $A = 5°$, $i_1 = ?$

As the ray emerges from the other face of prism normally, $i_2 = 0°$

$\therefore$ $r_2 = 0°$, Fig, 6(EP).5.



As $r_1 + r_2 = A$

$r_1 + r_2 = A$

$r_1 = A - r_2 = 5 - 0 = 5°$

From $\mu = \frac{\sin i_1}{\sin r_1}$, $\sin i_1 = \mu \sin r_1$

$= 1.5 \times \sin 5°$

$= 1.5 \times 0.087 = 0.1305$

$i_1 = \sin^{-1}(0.1305) = 7.5°$.

4. (*a*) $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi = 4I + I + 2 \times 2I \cos\frac{\pi}{2} = 5I$

5. (*b*) $\delta \cong (\mu - 1)A$

For air $D = (1.5 - 1)\,A$

For water $\delta = ({}_g\mu_w - 1)A$

$= \left(\frac{1.5}{1.3} - 1\right)A$

Hence, $\delta = \frac{0.2}{1.3} \times \frac{D}{0.5} \cong 0.3D$

6. (*d*) $\mu = \frac{\sin i}{\sin r} = \frac{\sin\frac{A + \delta_m}{2}}{\sin\frac{A}{2}}$

$\Rightarrow$ $\sqrt{3} = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin 30°}$, because $A = 60°$

or $\sin\frac{A + \delta_m}{2} = \frac{\sqrt{3}}{2} = \sin 60°$

or $A + \delta_m = 2 \times 60 = 120$.

This gives $\delta_m = 60°$.

Where δ_m is minimum deviation.

7. (*c*) When the lens in air, we have

$$P_a = \frac{1}{f_a} = \frac{\mu_g - \mu_a}{\mu_a}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

When the lens is in liquid, we have

$$P_l = \frac{1}{f_l} = \frac{\mu_g - \mu_l}{\mu_l}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here, $P_a = 5, P_l = -1, \mu_a = 1, \mu_g = 1.5$

On solving, we get, $\mu_l = \frac{5}{3}$

8. (*c*) $P = \frac{1}{f}(\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Hence, $\frac{P_2}{P_1} = \left(\frac{\mu_2 - 1}{\mu_1 - 1}\right)$

i.e., $\frac{P_2}{1} = \frac{1.6 - 1}{1.5 - 1}$

Hence, $P_2 = 1.2$

9. (*d*) In air, all the colours of light travel with the same velocity, but in glass, velocities of different colours are different. Velocity of red colour is largest and velocity of violet colour is smallest. Therefore, after travelling through the glass slab, red colour will emerge first.

10. (*c*) When an object apprqaches a convergent lens from the left of the lens with a uniform speed of 5 m/s, the image moves away from the lens with a non uniform acceleration. For example, it $f = 20$ m and $v = -50$ m; -45 m, -40 m and -35 m; we get $v = 33.3$ m; 36 m; 40 m and 46.7 m. Clearly, image moves away from the lens with a non uniform acceleration. Choice (c) is correct.

11. (*b*) In an aeroplane, a passenger may observe a primary and a secondary rainbow as concentric circles.

12. (*c*) $M = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$

13. (*c*) Use $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Here, $v = -80$ cm, $f = -100$ cm

Hence, $\frac{1}{-80} - \frac{1}{u} = -\frac{1}{100}$ or $-\frac{1}{u} = -\frac{1}{100} + \frac{1}{80}$

$= \frac{-80 + 100}{80 \times 100}$

This gives $u = -400$ cm $= -4$ m

14. (*c*) Here, $\mu = 1.5$

If object lies on plane side; $R_1 = \infty, R_2 = -20$ cm

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{\infty} + \frac{1}{20}\right) = \frac{1}{40}$$

$f = +40$ cm. The lens behaves as convex.

If object lies on its curved side,

$R_1 = 20$ cm, $R_2 = \infty$,

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{\infty}\right) = \frac{1}{40}$$

$f' = 40$ cm. The lens behaves as convex. Choice c is correct

15. (*b*) The total fringe shift is $H = \frac{\beta}{\lambda}(\mu - 1)t$

The number of fringes that will shift

$$= \frac{\text{total fringe shift}}{\text{fringe width}}$$

or $$n = \frac{\frac{\beta}{\lambda}(\mu - 1)t}{\beta} = \frac{(\mu - 1)t}{\lambda}$$

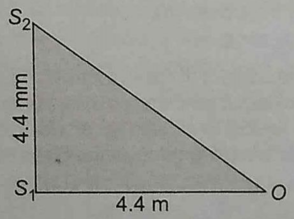
or $$n = \frac{(1.4 - 1) \times 2.4 \times 10^{-5}}{800 \times 10^{-9}}$$

or $$n = \frac{0.4 \times 2.4 \times 10^{-5}}{8 \times 10^{-7}}$$

or $$n = 12$$

16. (*a*) The path difference at point of observation is given by

$$\Delta = S_2O - S_1O$$



From the figure,

$$S_2O = [(4.4)^2 + (4.4 \times 10^{-3})^2]^{1/2}$$

or $$S_2O = 4.4[1 + (10^{-3})^2]^{1/2}$$

or $$S_2O = 4.4\left[1 + \frac{1}{2}(10^{-3})^2\right]$$

Therefore, $\Delta = 4.4\left[1 + \frac{1}{2}(10^{-3})^2 - 1\right] = \frac{4.4 \times 10^{-6}}{2} = 2.2 \times 10^{-6}$ m

Interference will be constructive if path difference is an intergral multiple of wavelength

$$n = \frac{\Delta}{\lambda} = \frac{2.2 \times 10^{-6}}{440 \times 10^{-9}} = 5$$

Hence, pattern of interference at point O is constructive.

17. (*c*) The focal length f of the lens in air is given by

$$\frac{1}{f} = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

where μ_g is the refractive index of the lens. If μ_w is the refractive index of water, the focal length in water (f') is given by

$$\frac{1}{f'} = \left(\frac{\mu_g - \mu_w}{\mu_w}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since the lens placed in air is convergent, f is positive. Therefore, $\mu_g > 1$. For the lens to be divergent when placed in water, f' must be negative, *i.e.*, $\mu_g < \mu_w$. Now, $\mu_w = 1.33$. Hence, μ_g must lies between 1 and 1.33.

18. (*b*) The condition for minima is given by

$$d\sin\theta = n\lambda$$

For $n = 1$, we have

$$d\sin\theta = \lambda$$

If angle is small, then $\sin\theta = \theta \Rightarrow \quad d\theta = \lambda$

Half angular width $\theta = \frac{\lambda}{d}$

Full angular width $2\theta = 2\frac{\lambda}{d}$

Also, $$\omega' = \frac{2\lambda'}{d}$$

$\therefore$ $$\frac{\lambda'}{\lambda} = \frac{\omega'}{\omega}$$

or $$\lambda' = \lambda\frac{\omega'}{\omega} = 6000 \times 0.7 = 4200 \text{ Å}$$

19. (*d*) For reflection from concave mirror

Applying, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, we get

$$\frac{1}{-5} = \frac{1}{v} - \frac{1}{7.5}$$

or $$\frac{1}{v} = -\frac{1}{5} + \frac{1}{7.5}$$

or $$v = -15 \text{ cm}$$

The image is formed on the pole of the convex mirror, which will be the position of the object for convex mirror. Therefore,

$$u = 0 \text{ and } f = 5 \text{ cm}$$

Hence, $$\frac{1}{v} = \frac{1}{5} - \frac{1}{0} = \frac{1}{5} - \infty = \infty$$

or $$v = 0$$

Therefore, final image is formed on the pole of convex mirror.

20. (*a*) For surface P, we have

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \text{ or } v_1 = 4 \text{ m}$$

For surface Q, we have

$$v_2 = \frac{1}{2} - \frac{1}{7} = \frac{5}{14}$$

or $$v_2 = \frac{14}{5} = 2.8 \text{ m}$$

Therefore, $v_1 - v_2 = (4 - 2.8) = 1.2$ m

21. (*d*) Convex mirror and concave lens do not form real image. For concave mirror $v > u$, so image will be enlarged, hence only convex lens can be used for the purpose

22. (*d*) Given, $f_1 = +12$ cm and $\frac{|P_1|}{|P_2|} = \frac{4}{3}$ or $\frac{|f_2|}{|f_1|} = \frac{4}{3}$

Since f_2 is negative, $\frac{f_2}{f_1} = -\frac{4}{3}$

Hence, $f_2 = -\frac{4}{3}f_1 = -\frac{4}{3} \times 12 = -16$ cm

The focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{12} + \frac{1}{-16} = \frac{1}{48}$$

which gives $F = 48$ cm

23. (*a*) When the plane surface of a plano-convex lens is silvered, it behaves like a concave mirror of focal length f given by

$$\frac{1}{f} = \frac{2(\mu - 1)}{R}$$

or $$f = \frac{R}{2(\mu - 1)} = \frac{10}{2(1.5 - 1)} = 10 \text{ cm}$$

24. (*a*) When the plane surface is silvered the focal length f_1 is given by

$$\frac{1}{f_1} = \frac{2(\mu - 1)}{R} \quad \text{... (i)}$$

But when the convex surface is silvered, the focal length f_2 is given by

$$\frac{1}{f_2} = \frac{2\mu}{R} \quad \text{... (ii)}$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{f_1}{f_2} = \frac{\mu}{\mu - 1} = \frac{1.5}{1.5 - 1} = 3$$

or $$f_2 = \frac{f_1}{3} = \frac{30}{3} = 10 \text{ cm}$$

25. (*c*) Limit of resolution of the telescope

$$\alpha = \frac{1.22\lambda}{a} = \frac{d}{x}$$

or $$d = \frac{1.22\lambda x}{a} = \frac{1.22 \times 5 \times 10^{-7} \times 8 \times 10^{16}}{0.25} = 1.95 \times 10^{11} \text{ m}$$

26. (*a*) Let d in cm be the thickness of air column = thickness of vacuum column (given). Number of waves of wavelength $\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm in a thickness d cm in vacuum is

$$n_v = \frac{d}{\lambda}$$

Since the refractive index of air $\mu = 1.0003$, the wavelength in air will be $$\lambda_a = \frac{\lambda}{\mu}$$

Therefore, number of waves of wavelength λ_a in dim of air is

$$n_a = \frac{d}{\lambda_a} = \frac{d\mu}{\lambda}$$

Given that $n_a + 1 = n_v$

Hence, $$\frac{d\mu}{\lambda} + 1 = \frac{d}{\lambda}$$

$$d = \frac{\lambda}{\mu - 1} = \frac{6 \times 10^5 \text{cm}}{1.0003 - 1}$$

$= 0.2$ cm $= 2$ mm

27. (*d*) $\frac{1}{f} = \frac{\mu_g - \mu_w}{\mu_w}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

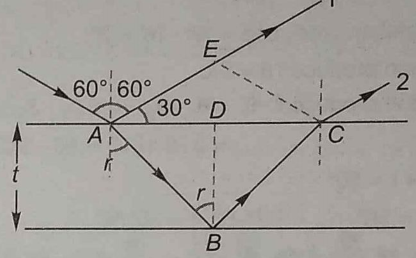
For a symmetric lens

$$R_2 = -R_1 = -0.3 \text{ m}$$

Therefore, $$\frac{1}{f} = \frac{(1.5 - 4/3)}{4/3}\left[\frac{1}{0.3} + \frac{1}{0.3}\right]$$

which gives $f = 1.2$ m

28. (*b*)



$$\sin r = \frac{\sin 60°}{\mu} = \frac{\sqrt{3}/2}{1.5}$$

$\therefore$ $r = 35.3°$

$AB = t \sec r = 1.23t$

$AC = 2(AD) = 2(t \tan r) = 1.42t$

$AE = AC \cos 30° = 1.23t$

Now, net path difference between 1 and 2,

$$\Delta X = \mu(2AB) - AE$$

$$= (1.5)(2)(1.23t) - 1.23t = 2.46t$$

For minimum intensity, $\Delta X = \lambda$

or $(2.46t) = 6000$

or $t = 2440$ Å

29. (*c*) Intensity of polarised light from first polarizer

$$= \frac{100}{2} = 50$$

From law of Malus intensity from second nicol

$$I = 50 \cos^2 60° = \frac{50}{4} = 12.5$$

30. (*b*) $$\theta = a + \frac{b}{\lambda^2}$$

$$30 = a + \frac{b}{(5000)^2}$$

and $$50 = a + \frac{b}{(4000)^2}$$

Solving for a, we get $a = -\frac{50°}{9}$ per mm

31. (*d*) Given, $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \cot\left(\frac{A}{2}\right) = \frac{\cos(A/2)}{\sin(A/2)}$$

or $$\sin\left(\frac{A + \delta_m}{2}\right) = \cos\left(\frac{A}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$\therefore$ $$\frac{A + \delta_m}{2} = 90° - \frac{A}{2}$$

or $$\delta_m = 180° - 2A$$

32. (*d*) As, $\theta \propto l$

Volume ratio 1 : 2 in a tube of length 30 cm means 10 cm length of first solution and 20 cm length of second solution.

Rotation produced by 10 cm length of first solution

$$\theta_1 = \frac{38°}{20} \times 10 = 19°$$

Rotation produced by 20 cm length of second solution

$$\theta_2 = -\frac{24°}{30} \times 20 = -16°$$

$\therefore$ Total rotation produced = 19°–16° = 3°

33. (*b*) Rotation produced $\theta = S/c$

Net rotation produced, $\theta_r = \theta_1 - \theta_2 = l(S_1c_1 - S_2c_2)$

$$= 0.29 \times [0.01 \times 60 - 0.02 \times 30] = 0$$

34. (*c*) $i_p + r = 90°$

or $r = 90° - i_p = 90° - 53°4' = 36°56'$

35. (*b*) For the first minimum, $a \sin\theta_1 = \lambda \approx a\theta_1 = \frac{ad_1}{D}$

For the sixth minimum, $a \sin\theta_6 = 6\lambda = a\theta_6 = \frac{ad_6}{D}$

$\therefore$ $$(6\lambda - \lambda) = \frac{a}{D}(d_6 - d_1)$$

or $$a = \frac{5D\lambda}{(d_6 - d_1)} = \frac{5 \times 0.5 \times 5 \times 10^{-7}}{0.5 \times 10^{-3}}$$

$$= 2.5 \times 10^{-3} \text{ m} = 2.5 \text{ mm}$$

36. (*a*) The relation between angle of deviation δ for a thin prism an angle of prism and refractive index of material of prism is given by $\delta = (\mu -) A$

37. (*a*) We know very well that glass is transparent. But when the glass is powdered, the irregular reflections occur from the surface of powdered glass and finally the light returns back into the same medium. Because of it the powdered glass looks opaque. When we pour water over the powdered glass, refraction of light takes place and it becomes transparent.

38. (*a*) The focal length f_w of convex lens in water of refractive index μ_w is given by

$$\frac{1}{f_w} = \left(\frac{\mu_g - \mu_w}{\mu_w}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

Here, μ_g is the refractive index of lens (glass).

The focal length of lens in air is given by

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we conclude the focal length of lens in water (f_w) is greater than focal length of lens in air (f_a). Therefore, the focal length of lens in water get increased consequently power decreases.

39. (*c*) Red colour consists of longest wavelength and it scatteres least. Therefore, signals of red colour are being seen from the long distances. Hence, the signals are made of red colour light.

40. (*c*) The clouds consist of dust particles and water droplets. The scattering of sun light by the big dust particles and water drops is not in accordance with the Rayleigh law. But they scatter the light of all the colours by the same amount. Hence, the clouds are seen generally white.

41. (*a*) Resolving power of telescope is $= \frac{a}{1.22\lambda}$

where a is the diameter of objective lens and λ is the wavelength of light used. It is obvious that on increasing a, more light is collected by objective lens and so, the image formed is more bright. Thus, resolving power of telescope increases.

42. (*c*) The magnifying power of telescope in relaxed state is $m = \frac{f_o}{f_e}$

So, for high magnification, the focal length of objective length should be larger than that of eyepiece.

Resolving power of a telescope $= \frac{d}{1.22\lambda}$

For high resolving power, diameter (d) of objective should be higher.

43. (*c*) In a normal eye parallel light rays are refracted by the cornea to produce a focused image on the retina. Therefore, the image distance equals the focal length. If the image is to be focused on the retina, then in the normal eye the retina must be at the focal length of the cornea, which is 2 cm.

44. (*a*) For a convex lens (cornea) if object lies beyond focus image is real and inverted.

45. (*d*) Myopia occurs when the cornea focuses the image of a distant object in fornt of the retina. This defect can, therefore corrected by a lens that would lead to an increased net focal length. Therefore, we need a diverging lens. In hypermetropia opposite in the case.

46. (*c*) Substituting the given values in the given equation,

$$\frac{1}{v} = \frac{1}{1.8} + \frac{1}{-16.5 - 1.5} \text{ or } v = 2 \text{ cm}$$

47. (*d*) Putting $x = 0$ in given equation, $\frac{1}{v} = \frac{1}{f_c} + \frac{1}{f_l}$

If f_c and f_l both are doubled, then $\frac{1}{v}$ will become $\frac{1}{2}$ times or v will become two times.

48. (*b*) $$\beta = \frac{n\lambda D}{d}$$

or $$n = \frac{\beta d}{\lambda D}$$

As D will increase order will decrease.

49. (*b*) $$\frac{5\lambda D}{d} = (2 \times 3 - 1)\frac{\lambda D'}{2d}$$

or $$5D = \frac{5}{2}D' = \frac{5}{2}(D + vt)$$

$\therefore$ $$t = \frac{D}{v}$$

Day 31

Dual Nature of Matter

Day 31

Outlines ...

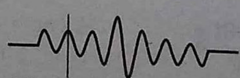
- Dual Nature of Radiation
- Photoelectric Effect
- Davisson-Germer Experiment

Dual Nature of Radiation

The concept of wave nature of matter arose from the dual character of the radiation which sometimes behaves as a wave and sometimes as a particle.

In the phenomena where light interacts with itself, such as dispersion, interference, diffraction and polarisation, the wave nature of light dominates. In the phenomena where the light interacts with matter, such as in Photoelectric effect, Compton effect, Raman effect, the particle nature of light dominates.

Matter waves travel as a wave packet with decreasing amplitude on either side of the present position.



Wave packet

(i) The de-Broglie wavelength of a particle of mass m and moving with velocity v is given by

$$\lambda = \frac{h}{mv}$$

(ii) If m_0 be the rest mass of the particle, then

$$\lambda = \frac{h[1 - v^2/c^2]^{1/2}}{m_0 v}$$

(iii) The de-Broglie wavelength of a particle of mass m and kinetic energy K is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

(iv) If a particle of mass m carrying charge q_0 is accelerated through a potential V, then its de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mq_0V}}$$

(v) The de-Broglie wavelength of a gas molecule of mass m at temperature T (in Kelvin) is given by

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

Here k is Boltzmann constant.

(vi) If an electron is accelerated through a potential V volts, then its de-Broglie wavelength is given by

$$\lambda \cong \left[\frac{150}{V}\right]^{1/2} \text{Å}$$

Matter Waves *and* Wave Nature of Particles

In case of light some phenomenon like diffraction and interference can be explained on the basis of its wave character. However, the certain other phenomenon such as black body radiation and photoelectric effect can be explained only on the basis of its particle nature. Thus light is said to have a dual character. Such studies on light wave were made by Einstein in 1905. Louis-de-Broglie, in 1942 extended the idea of photons to material particles such as electron and he proposed that matter also has a dual character as wave and as particle.

de-Broglie Wavelength Relation

According to de-Broglie, a wave is associated with energy moving particle. These waves are called de-Broglie waves or matter waves.

(i) In the nth orbit, we have

$$L = mvr = \frac{nh}{2\pi} \Rightarrow \lambda = \frac{h}{mv} = \frac{2\pi r}{n}$$

(ii) Number of waves in nth orbit is $N = \frac{2\pi r}{\lambda} = n$

Thus, the number of de-Broglie waves in nth orbit is n.

- *Expression for the wavelength associated with charged particles accelerated through a potential difference V.*
  - *(i) Electron* $\lambda_e = \frac{12.27}{\sqrt{V}}$ Å
  - *(ii) Proton* $\lambda_p = \frac{0.286}{\sqrt{V}}$ Å
  - *(iii) Deuteron* $\lambda_d = \frac{0.202}{\sqrt{V}}$ Å
  - *(iv) α-particle* $\lambda_\alpha = \frac{0.101}{\sqrt{V}}$ Å
- *Wavelength of atoms at temperature T*

$$\lambda = \frac{\eta}{\sqrt{2mkT}}$$

- *The de-Broglie wavelength associated with ordinary objects is of the order of* 10^{-24} *m.*

Emission of Electrons

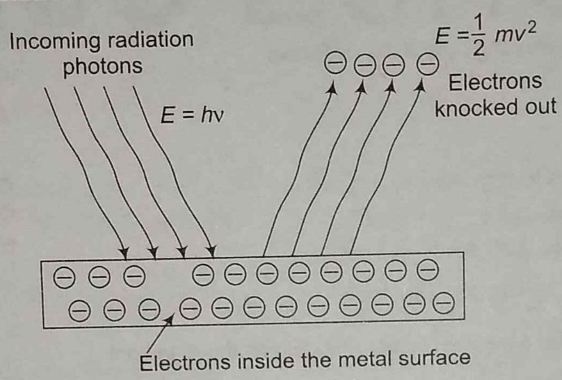
At room temperature, the free electrons move randomly within the conductor, but they don't leave the surface of the conductor due to attraction of positive charges. Some external energy is required to emit electrons from a metal surface. Minimum energy is required to emit the electrons which are just on the surface of the conductor. This minimum energy is called the **work function** (denoted by W) of the conductor. Work function is the property of the metallic surface. The energy required to liberate an electron from metal surface may arise from various sources such as heat, light, electric field etc. *Depending on the nature of source of energy, the following methods are possible*

- **Thermionic emission** The energy to the free electrons can be given by heating the metal. The electrons so emitted are known as **thermions.**
- **Field emission** When a conductor is put under strong electric field the free electrons on it experience an electric force in the opposite direction of field. Beyond a certain limit electrons start coming out of the metal surface. Emission of electrons from a metal surface by this method is called the field emission.
- **Secondary emission** Emission of electrons from a metal surface by the bombardment of high speed electrons or other particles is known as secondary emission.
- **Photoelectric emission** Emission of free electrons from a metal surface by falling light (or any other electromagnetic wave which has an energy greater than the work function of the metal) is called photoelectric emission. The electrons so emitted are called **photoelectrons.**

Photoelectric Effect

Photoelectric effect is the phenomenon of emission of electrons (known as photoelectrons) from the surface of metals when light radiation of suitable frequency is incident on them.

For a given metal, there exits a certain minimum frequency of light radiation below which no photoelectric emission takes place. This minimum frequency of radiation is known as the **threshold frequency** (ν_0).



(i) The minimum energy of incident radiation needed to eject the electrons from metal surface is known as the **work function** (ϕ_0) of that surface. Work function is related to threshold frequency as

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

where, λ_0 = threshold wavelength,

In electron-volt units, $$\phi(\text{eV}) = \frac{hc}{e\lambda_0} = \frac{12400}{\lambda(\text{Å})}$$

(ii) According to **Einstein's photoelectric equation** $h\nu = \phi_0 + K_{max}$,

where $K_{max} = \frac{1}{2}mv^2{}_{max}$ = Maximum kinetic energy of ejected photoelectron and ν is the frequency of incident light photon.

As $\phi_0 = h\nu_0$, hence, Einstein's equation may be written as $h(\nu - \nu_0) = K_{max} = \frac{1}{2}mv^2{}_{max}$

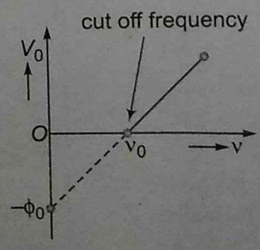
(iii) If in a photoelectric tube we apply a negative potential, then for a certain minimum negative potential the photoelectric current becomes zero. This negative potential is known as **stopping potential** (V_0).

It is, thus, a measure of maximum kinetic energy of photoelectrons, *i.e.*,

$$eV_0 = K_{max} = \frac{1}{2}mv^2{}_{max}$$

(iv) Photoelectric emission is an instantaneous phenomenon. Time lag between the incidence of light photon and emission of photoelectron is of the order of 10^{-9} s.

(v) Variation of stopping potential V_0 with frequency ν of incident radiation is shown in figure.



$$eV_0 = h(\nu - \nu_0) = h\nu - \phi_0$$

$$\Rightarrow \quad V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

Thus, V_0-ν graph is a straight line whose slope is $\frac{h}{e}$ and intercept is $-\phi_0$ eV. The graph meets the ν-axis at ν_0.

» For same wavelength, the ratio of accelerating potential is as follows

(i) $\frac{V_p}{V_\alpha} = \frac{m_\alpha q_\alpha}{m_p q_p} = \frac{4 \times 2}{1 \times 1} = 8$

(ii) $\frac{V_p}{V_d} = \frac{m_d q_d}{m_p q_p} = \frac{2 \times 1}{1 \times 1} = 2$

(iii) $\frac{V_d}{V_\alpha} = \frac{m_\alpha q_\alpha}{m_d q_d} = \frac{4 \times 2}{2 \times 1} = 4$

» The shortest X-rays wavelength emitted when the electrons incident on the target are accelerated through potential V will be

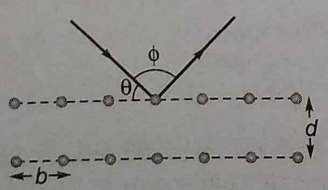
$$\lambda = \frac{hc}{eV} = \frac{1240}{V}\,\text{nm}$$

It is also called as Duane-Hunt law.

Davisson-Germer Experiment

This experiment verified the wave nature of electrons using Ni crystal.

Let b = distance between the lattice points and d = distance between the scattering planes. Also let θ = glancing angle for the incident beam of electrons and ϕ be the angle between the incident and diffracted beams. Then for a glancing incidence, we have $2d\sin\theta = n\lambda$ and for normal incidence, we have $b\sin\phi = n\lambda$.



Bragg's Equations

For normal incidence,

$$b\sin\phi = n\lambda$$

For glancing incidence,

$$2d\sin\theta = n\lambda$$

Also $\quad \phi + 2\theta = 180°$

Hertz and Lenard's observations on the basis of experiments on photoelectric effect are

- The rate of emission of photoelectrons from the surface of a metal varies directly as the intensity of the incident light.
- The maximum kinetic energy of the emitted photo electrons is independent of the intensity of the incident light.
- The maximum kinetic energy of the photoelectrons increases linearly with the increase in the frequency of the incident light.
- If the frequency of the incident light is below a certain lowest value then no photoelectrons are emitted from the metal.
- As soon as the light is incident on the surface of the metal, the photoelectrons are emitted instantly.
- The work function ϕ and threshold frequency ν_0 varies from metal to metal.

Practice Zone DAY 31

1. An electron of mass m and charge q is accelerated from rest in a uniform electric field of strength E. The velocity acquired by it as it travels a distance l is
(a) $\sqrt{2Eq\,l/m}$ (b) $\sqrt{2Eq\,/ml}$
(c) $\sqrt{2Em/ql}$ (d) $\sqrt{Eq\,/ml}$

2. The voltage applied to an electron microscope to produce electrons of wavelength 0.50 Å is
(a) 602 V (b) 50 V
(c) 138 V (d) 812 V

3. What will be the number of photons emitted per second by a 10 W sodium vapour lamp assuming that 90% of the consumed energy is converted into light? [Wavelength of sodium light is 590 nm, and $h = 6.63 \times 10^{-34}$ J-s]
(a) 0.267×10^{18} (b) 0.267×10^{19}
(c) 0.267×10^{20} (d) 0.267×10^{17}

4. A radiotransmitter operates at a frequency 1000 kHz and a power of 66 kW. Find the number of photons emitted per second.
(a) 10^{27} (b) 10^{28} (c) 10^{29} (d) 10^{30}

5. Two monochromatic beams A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what inference can you make about their frequencies? **[NCERT Exemplar]**
(a) $\gamma_B = 2\gamma_A$ (b) $\gamma_B = \gamma_A$
(c) $\gamma_A = 2\gamma_B$ (d) $\gamma_B > \gamma_A$

6. When a surface 1 cm thick is illuminated with light of wavelength λ, the stopping potential is V_0, but when the same surface is illuminated by light of wavelength 3λ, the stopping potential is $V_0/6$, the threshold wavelength for metallic surface is
(a) 4λ (b) 5λ (c) 3λ (d) 2λ

7. A photocell with a constant potential difference of V volt across it is illuminated by a point source from a distance of 25 cm. When the source is moved to a distance of 1m, the electrons emitted by the photocell
(a) carry 1/4th their previous energy
(b) are 1/16th as numerous as before
(c) are 1/4th as numerous as before
(d) carry 1/4th their previous momentum

8. The wavelength of the photoelectric threshold for silver is λ_0. The energy of the electron ejected from the surface of silver by an incident light of wavelength λ ($\lambda < \lambda_0$) will be
(a) $hc(\lambda_0 - \lambda)$ (b) $\dfrac{hc}{\lambda_0 - \lambda}$
(c) $\dfrac{h}{c} + \left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_0}\right)$ (d) $hc\left(\dfrac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)$

9. In a photoelectric effect measurement, the stopping potential for a given metal is found to be V_0 volt when radiation of wavelength λ_0 is used. If radiation of wavelength $2\lambda_0$ is used with the same metal then the stopping potential (in volt) will be
(a) $\dfrac{V_0}{2}$ (b) $2V_0$ (c) $V_0 + \dfrac{hc}{2e\lambda_0}$ (d) $V_0 - \dfrac{hc}{2e\lambda_0}$

10. An electron of mass m and charge e are initially of rest. It gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t is
(a) $\dfrac{-h}{eEt^2}$ (b) $\dfrac{-nh}{eEt^2}$
(c) $\dfrac{-h}{eE}$ (d) $\dfrac{-eht}{E}$

11. Specific heat of water is 4.2 J/g °C. If light of frequency 3×10^9 Hz is used to heat 400 g of water from 20°C to 40°C, the number of moles of photon's needed will be
(a) 1.69×10^{29} (b) 1.69×10^{28}
(c) 2.80×10^4 (d) 2.8×10^5

12. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV. **[NCERT Exemplar]**
(a) 2.83 eV (b) 2 eV
(c) 1.02 eV (d) 3.42 eV

13. The eye can detect 5×10^4 photons $m^{-2}s^{-1}$ of light of wavelength 500 nm. The ear can hear intensity upto 10^{-13} Wm^{-2}. As a power detector, which is more sensitive
(a) Sensitivity of eye is one-fifth of the ear
(b) Sensitivity of eye is five times that of the ear
(c) Both are equally sensitive
(d) Eye cannot be used as a power detector

14. A metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$ and when the same surface is illuminated with light of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is

(a) $4\lambda/3$ (b) 6λ (c) 8λ (d) 4λ

Directions *(Q. Nos. 15 to 17) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

15. Statement I As intensity of incident light (in photoelectric effect) increases, the number of photoelectrons emitted per unit time increases.

Statement II More intensity of light means more energy per unit area per unit time.

16. Statement I The relative velocity of two photons travelling in opposite directions is the velocity of light.

Statement II The rest mass of photon is zero.

17. Statement I Electron capture occurs more than positron emission in a heavy nucleus.

Statement II In a heavy nucleus, electrons are relatively close to nucleus.

Directions *(Q. Nos. 18 to 20) Light from the sun arrives at the earth, an average of 1.5×10^{11} m away, at the rate of 1.4×10^3 W/m^2 of area the perpendicular to the direction of light. Assume that sunlight is monochromatic with a frequency of 5×10^{14} Hz.*

18. The number of photons reaching per second on each square metre of the earth's surface directly facing the sun are

(a) 12.4×10^{29} (b) 4.2×10^{21}
(c) 1.2×10^{45} (d) 1.4×10^{13}

19. How many photons per second are per square meter emitted by the sun surface?

(a) 12.4×10^{29} (b) 4.2×10^{21}
(c) 1.2×10^{45} (d) 1.4×10^{13}

20. Near to surface of the earth, how many photons per cubic metre are there?

(a) 12.4×10^{29} (b) 4.2×10^{21}
(c) 1.2×10^{45} (d) 1.4×10^{13}

AIEEE & JEE Main *Archive*

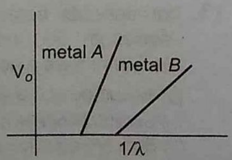
21. Protons of an electromagnetic radiation has an energy 11 keV each. To which region of electromagnetic spectrum does it belong? **[JEE Main Online 2013]**

(a) X-ray region (b) Ultra violet region
(c) Infrared region (d) Visible region

22. Orbits of a particle moving in a circle are such that the perimeter of the orbit equals an integer number of de-Broglie wavelengths of the particle. For a charged particle moving in a plane perpendicular to a magnetic field, the radius of the n^{th} orbital will therefore be proportional to **[JEE Main Online 2013]**

(a) n^2 (b) n
(c) $n^{1/2}$ (d) $n^{1/4}$

23. In an experiment on photoelectric effect, a student plots stopping potential V_0 against reciprocal of the wavelength λ of the incident light for two different metals A and B. These are shown in the figure.



Looking at the graphs, you can most appropriately say that **[JEE Main Online 2013]**

(a) Work function of metal B is greater than that of metal A
(b) Work function of metal A is greater than that of metal B
(c) Students data is not correct
(d) None of the above

24. Electrons are accelerated through a potential difference V_0 and protons are accelerated through a potential difference 4 V. The de-Broglie wavelength are λ_e and λ_p for electrons and protons respectively. The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by

(Given m_e is mass of electrons and m_p is mass of proton). **[JEE Main Online 2013]**

(a) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$ (b) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_e}{m_p}}$
(c) $\frac{\lambda_e}{\lambda_p} = \frac{1}{2}\sqrt{\frac{m_e}{m_p}}$ (d) $\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$

25. A copper ball of radius 1 cm and work function 4.47 eV is irradiated with ultraviolet radiation of wavelength 2500 Å. The effect of irradiation results in the emission of electrons from the ball. Further the ball will acquire charge and due to this there will be a finite value of the potential on the ball. The charge acquired by the ball is **[JEE Main Online 2013]**

(a) 5.5×10^{-13} C
(b) 7.5×10^{-13} C
(c) 4.5×10^{-12} C
(d) 2.5×10^{-11} C

26. This question has Statement I and Statement II. Of the four choices given the statements, choose the one that describes the two statements.

Statement I Davisson-Germer experiment established the wave nature of electrons.

Statement II If electrons have wave nature, they can interfere and show diffraction. **[AIEEE 2012]**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

27. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I A metallic surface is irradiated by a monochromatic light of frequency $\nu > \nu_0$ (*the threshold frequency*). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement II The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light. **[AIEEE 2011]**

(a) Statement I is true, Statement II is true, Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is false

28. Statement I When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement II Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light. **[AIEEE 2010]**

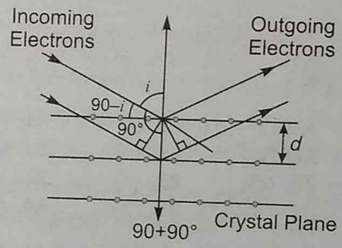
(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

29. If a source of power 4 kW produces 10^{20} photons/second, the radiation belong to a part of the spectrum called **[AIEEE 2010]**

(a) X-rays
(b) ultraviolet rays
(c) microwaves
(d) γ-rays

Directions *(Q. Nos. 30 to 32) are based on the following paragraph.*

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure). **[AIEEE 2008]**



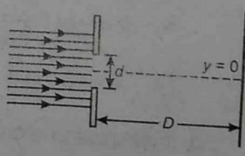
30. Electrons accelerated by potential V are diffracted from a crystal. If $d = 1$ Å and $i = 30°$, V should be about ($h = 6.6\times 10^{-34}$ J-s, $m_e = 9.1\times 10^{-31}$ kg, $e = 1.6\times 10^{-19}$C) **[AIEEE 2008]**

(a) 2000 V
(b) 50 V
(c) 500 V
(d) 1000 V

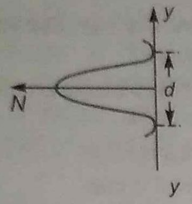
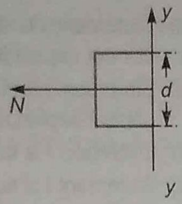
31. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de-Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer) **[AIEEE 2008]**

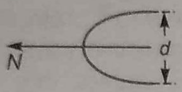
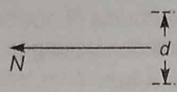
(a) $d \sin i = n\lambda_{dB}$
(b) $2d \cos i = n\lambda_{dB}$
(c) $2d \sin i = n\lambda_{dB}$
(d) $d \cos i = n\lambda_{dB}$

32. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de-Broglie wavelength. They are detected on a screen at a distance D from the slit (see figure).



Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y = 0$ corresponds to the middle of the slit) ? **[AIEEE 2008]**

(a) 
(b) 

(c) 
(d) 

33. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is **[AIEEE 2007]**

(a) $\frac{\nu}{c}$ (b) $h\nu c$

(c) $\frac{h\nu}{c^2}$ (d) $\frac{h\nu}{c}$

34. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5 V. The incident radiation lies in **[AIEEE 2006]**

(a) ultraviolet region (b) infrared region

(c) visible region (d) X-ray region

35. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is? (use hc = 1240 eV-nm)

(a) 1.41 eV (b) 1.51 eV

(c) 1.68 eV (d) 3.09 eV

36. Two identical, photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then **[AIEEE 2003]**

(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$ (b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$

(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$ (d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

Answers

1. (a)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (a)
11. (c)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (d)
21. (b	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (c)	**28.** (d)	**29.** (a)	**30.** (b)
31. (b)	**32.** (d)	**33.** (d)	**34.** (a)	**35.** (a)	**36.** (a)				

Hints & Solutions

1. As $v^2 = u^2 + 2as$;

So, $v^2 = 0 + 2\frac{qEl}{m}$ or $v = \sqrt{\frac{2qEl}{m}}$

2. de-Broglie wavelength is $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

But $E = eV$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$V = \frac{h^2}{2me\lambda^2}$$

$$V = \frac{(6.62 \times 10^{-34})^2}{(0.5 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}$$

$\Rightarrow V = 601.98\,V \approx 602\,V$

3. Energy of photon, $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{590 \times 10^{-9}}$

$$= \frac{6.63 \times 3}{59} \times 10^{-18}$$

Light energy produced per second $= \frac{90}{100} \times 10$

$= 9$ W

Number of photons emitted per sec

$$= \frac{9 \times 59}{6.63 \times 3 \times 10^{-18}} = 2.67 \times 10^{19} = 0.267 \times 10^{20}$$

4. Number of photons emitted per second $= \frac{p}{h\nu}$

$$= \frac{66}{6.6 \times 10^{-34} \times (1000 \times 10^3)} = 10^{29}$$

5. $\text{Intensity}_A = \text{Intensity}_B$

The number of photons of beam $A = n_A$

The number of photons of beam $B = n_B$

According to question, $n_A = 2n_B$

Let ν_A be the frequency of beam A and ν_B be the frequency of beam B.

∴ Intensity ∝ Energy of photons

$I \propto (h\nu) \times$ Number of photons

∴ $\frac{I_A}{I_B} = \frac{n_A \nu_A}{n_B \nu_B}$

According to question, $I_A = I_B$

∴ $n_A \nu_A = n_B \nu_B \Rightarrow \frac{\nu_A}{\nu_B} = \frac{n_B}{n_A} = \frac{1}{2}$

So, $\nu_B = 2\,\nu_A$

6. From Einstein's photoelectric equation, we have

$$eV_0 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \quad \ldots(i)$$

$$\frac{eV_0}{6} = hc\left[\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right] \quad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$6 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right)}$$

$$\frac{6}{3\lambda} - \frac{6}{\lambda_0} = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{5}{\lambda_0}$$

$$\Rightarrow \quad \frac{\lambda_0}{\lambda} = 5$$

$$\Rightarrow \quad \lambda_0 = 5\lambda$$

7. Photoelectric current I is directly proportional to intensity of light and intensity $\propto \frac{1}{(\text{distance})^2}$

$$I \propto \frac{1}{r^2}$$

$$I_{25} \propto \frac{1}{(25)^2} \quad \ldots(i)$$

$$I_{100} \propto \frac{1}{(100)^2} \quad [1 \text{ m} = 100 \text{ cm}] \quad \ldots(ii)$$

$$\therefore \quad \frac{I_{25}}{I_{100}} = \frac{(100)^2}{(25)^2} = 16$$

$$\Rightarrow \quad I_{100} = \frac{I_{25}}{16}$$

8. $E_k = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc\left[\frac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right]$

9.

$$eV_0 = \frac{hc}{\lambda_0} - W_0$$

and

$$eV' = \frac{hc}{2\lambda_0} - W_0$$

Subtracting them, we have

$$e(V_0 - V') = \frac{hc}{\lambda_0}\left[1 - \frac{1}{2}\right] = \frac{hc}{2\lambda_0}$$

or

$$V' = V_0 - \frac{hc}{2e\lambda_0}$$

10. Here the initial velocity is $u = 0$

Since $a = \frac{eE}{m}$, $v = ?$ at $t = t$

So we get, using $v = u + at = 0 + \frac{eE}{m}t$

This gives $\lambda = \frac{h}{mv} = \frac{h}{m(eEt/m)} = \frac{h}{eEt}$

The rate of change of de-Broglie wavelength is

$$\frac{d\lambda}{dt} = \frac{h}{eE} \times \left(\frac{-1}{t^2}\right) = \frac{-h}{eEt^2}$$

11.

$$E = nhr$$

$$ms\,\Delta T = nhr$$

$$\therefore \quad n = \frac{ms\,\Delta T}{hr}$$

$$= \frac{400 \times 4.2 \times (40 - 20)}{6.626 \times 10^{-34} \times 3 \times 10^9}$$

$$= 1.69 \times 10^{28} \text{ photons}$$

$$= \frac{1.69 \times 10^{28}}{6.023 \times 10^{23}}$$

$$= 2.8 \times 10^4 \text{ mole photons}$$

12. Given, wavelength $\lambda_1 = 600 \text{ nm} = 600 \times 10^{-9}$ m

Energy correspond to $\lambda_1 = E_1$

Again, wavelength $\lambda_2 = 400 \text{ nm} = 400 \times 10^{-9}$ m

Energy correspond to $\lambda_2 = E_2$

Let the work function of metal is ϕ.

According to question $2E_1 = E_2$

i.e.,

$$2\left(\frac{hc}{\lambda_1} - \phi_0\right) = \frac{hc}{\lambda_2} - \phi_0$$

or

$$\frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} = 2\phi_0 - \phi_0$$

or

$$hc\left[\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right] = \phi_0$$

$$\phi_0 = 6.63 \times 10^{-34} \times 3 \times 10^8\left[\frac{2}{600 \times 10^{-9}} - \frac{1}{400 \times 10^{-9}}\right]$$

or

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-7}}\left[\frac{1}{3} - \frac{1}{4}\right]$$

or

$$= 6.63 \times 3 \times 10^{-9}\left[\frac{4-3}{12}\right] \text{J}$$

$$= \frac{6.63 \times 3 \times 10^{-19}}{1.6 \times 10^{-19} \times 12} = 1.02 \text{ eV}$$

13. Power of eye $= \frac{5 \times 10^4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-19}}$

$$= 0.2 \times 10^{-13} \text{ Wm}^{-2}$$

The power of ear $= 1 \times 10^{-13}$ Wm^{-2}

Thus the sensitivity of eye is five times more than that of the ear.

14. According to the Einstein's photo electric effect

$$h\nu - h\nu_0 = \frac{1}{2}mv^2 = eV$$

$$\Rightarrow \quad \left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right) = eV$$

where, λ_0 = threshold wavelength.

Now for the first case

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = e(3V_0) \quad \ldots(i)$$

For the second case

$$\frac{hc}{2\lambda} - \frac{hc}{\lambda_0} = e(V_0) \quad \ldots(ii)$$

Dividing Eq. (i) by Eq.(ii), we get

$$\frac{hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)} = \frac{3eV_0}{eV_0}$$

$$\Rightarrow \quad \frac{\frac{\lambda_0 - \lambda}{\lambda\lambda_0}}{\frac{\lambda_0 - 2\lambda}{2\lambda \cdot \lambda_0}} = \frac{3eV_0}{eV_0}$$

$$\lambda_0 = 4\lambda$$

15. From quantum theory of light, as intensity of light increases means number of photons/area/time increases and hence more photons take part in ejecting the photoelectron thus, increasing the number of photoelectrons.

16. Velocity of first photon $= u = c$

Velocity of second photon $= v = -c$

Now relative velocity of first photon with respect to second photon

$$= \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{c - (-c)}{1 - \frac{c \times (-c)}{c^2}}$$

$$= \frac{2c}{1 + \frac{c^2}{c^2}}$$

$$= \frac{2c}{1+1} = \frac{2c}{2} = c$$

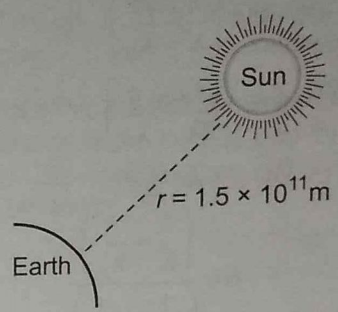
Also, the rest of mass of photon is zero.

17. In a heavy nucleus, the electrons are relatively close to nucleus, which promotes their interaction with the nucleus. Thus, electron capture is more often than positron emission in decay of heavy nucleus.

Since nearly all unstable nuclei found in nature are having high Z, positron emission was not discovered until several decades after electron emission had been established.

18. Intensity of sun light as received by the earth's surface is

$$I = 1.4 \times 10^3 \, Wm^{-2}$$



So power of the light at the sun's surface is,

$$P = 4\pi r^2 \times I$$

$$= 4\pi \times (1.5 \times 10^{11})^2 \times 1.4 \times 10^3 \, W$$

$$q = 3.96 \times 10^{26} \, W$$

Let n_0 photon/s are emitted by sun's surface, then

$$n_0 h\nu = P$$

$$\Rightarrow \quad n_0 \times 6.626 \times 10^{-34} \times 5 \times 10^{14} = 3.96 \times 10^{26}$$

$$\Rightarrow \quad n_0 = 1.2 \times 10^{45}$$

19. Let out of these n_0 photon/s the photons reaching earth are n/area/time, then $I = nh\nu$

$$\Rightarrow \quad n = \frac{1.4 \times 10^3}{6.626 \times 10^{-34} \times 5 \times 10^{14}}$$

$$= 4.2 \times 10^{21} \text{ photon/m}^2\text{-s.}$$

20. Number of photons reaching earth's surface on each square metre are $4.2 \times 10^{21} \, s^{-1}$.

In time Δt, the photons reaching earth's surface per square metre are $4.2 \times 10^{21} \, \Delta t$, and in the same time Δt, the photons travel a distance of $c\Delta t$ where c is the speed of light. So, the number of photons in each m^3 near the surface of the earth, is $\frac{4.2 \times 10^{21} \Delta t}{c\Delta t} = 1.4 \times 10^{13}$

21. As, $E = \frac{hc}{\lambda}$ and $\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{11 \times 1.6 \times 10^{-19}}$

$$= 1.125 \times 10^{-7} \, m$$

Hence, UV region.

22. As, $2\pi r = n\lambda \Rightarrow r = \frac{n\lambda}{2\pi}$

Now, de-Broglie equation $\lambda = \frac{h}{P}$

$$\Rightarrow \quad mV_n = \frac{h}{\lambda} = \frac{h}{\frac{2\pi r_n}{n}} = \frac{nh}{2\pi r_n}$$

Also, for charged particle moving in a magnetic field

$$r_n = \frac{mV_n}{qB} = \frac{nh}{(2\pi r_n) q B} \Rightarrow r_n^2 = \frac{nh}{2\pi q B}$$

$$\therefore \quad r_n \propto n^{1/2}$$

23. We have, $eV_0 = \frac{hc}{\lambda} - \phi \Rightarrow V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$

$$V_0 = mx + c$$

$\because$ Data is not sufficient.

24. We have, $E = qV$, we know that $E = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2E}{m}} \quad \text{as,} \quad \lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2E}{m}}}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mqV}} \quad \ldots\text{(i)}$$

For electron, $\lambda_e = \frac{h}{\sqrt{2m_e qV}}$ ...(ii)

For protron $\lambda_p = \frac{h}{\sqrt{2m_p qV}}$

$\therefore$ $\lambda_p = \frac{h}{\sqrt{2m_p q \cdot 4V}}$ $(\because V = 4V)$ ...(iii)

Dividing Eq. (ii) by (iii), we get

$$\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$$

25. As, $\frac{1}{4\pi\varepsilon_0}\frac{Q}{1 \times 10^{-2}} = \frac{\frac{hc}{\lambda} - \phi}{Q} \Rightarrow Q = 5.5 \times 10^{-13}$ C

26. Davisson and Germer experimentally established wave nature of electron by observing diffraction pattern while bombarding electrons on Ni crytal.

27. Maximum kinetic energy $(KE)_{max}$ is given by

$$(KE)_{max} = h\nu - h\nu_0.$$

When frequency is increased $(KE)_{max}$ will increase stopping potential is that negative voltage given to the anode at which photocurrent stops, hence doubling frequency will not effect it, also

If $\nu' = 2\nu$

$\therefore$ $K'_{max} = eV'_0 = h(2\nu_1 - \nu_0)$

$K'_{max} = 2k_{max} + h\nu_0$

$\therefore$ $K'_{max} > 2K_{max}$ and $\Rightarrow V'_0 > 2V_0$

Hence, $(KE)_{max}$ and stopping potential are linearly dependent on the frequency of incident light.

28. Since the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays

$$KE_{photoelectron} = h\nu - \phi$$

Stopping potential is to stop the fastest photoelectron

$$V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

So, KE_{max} and V_0 both increases.

But KE ranges from zero to KE_{max} because of loss of energy due to subsequent collisions before getting ejected and not due to range of frequencies in the incident light.

29. $4 \times 10^3 = 10^{20} \times hf$

$$f = \frac{4 \times 10^3}{10^{20} \times 6.023 \times 10^{-34}}$$

$$f = 6.64 \times 10^{16} \text{ Hz}$$

The obtained frequency lies in the band of X-rays.

30. For constructive interference,

$$2d\cos i = n\lambda = \frac{h}{\sqrt{2meV}}$$

On substituting values get, $V = 50$ volt

31. Expression is given by $2d\cos i = n\lambda_{dB}$

32. As diffraction pattern has to be wider than slit width, So, (d) is the correct option.

33. The momentum of the photon

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

34. From Einstein's photoelectric equation

$$h\nu = h\nu_0 + eV_0 = 6.2 + 5 = 11.2 \text{ eV}$$

$$\Rightarrow \quad \frac{hc}{\lambda} = 11.2 \text{ eV or } \lambda = \frac{hc}{11.2} = 1108.9 \text{ Å}$$

which belongs to ultraviolet region.

35. $E_\lambda = \frac{1240}{400}$ eV $= 3.1$ eV

$$K.E = \frac{1}{2}mv^2 = eV_0 = 1.68 \text{ eV}$$

$$\Rightarrow \quad 3.1 \text{ eV} = W_0 + 1.68 \text{ eV}$$

$$W_0 = (3.1 - 1.68) \text{ eV}$$

$$W_0 = 1.42 \text{ eV}$$

36. $hf = hf_0 + \frac{1}{2}mv^2$

$$\Rightarrow \quad v_1^2 = \frac{2hf_1}{m} - \frac{2hf_0}{m}$$

$$v_2^2 = \frac{2hf_2}{m} - \frac{2hf_0}{m}$$

$$\therefore \quad v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$$

Day 32

Atoms

Day 32

Outlines ...

- Scattering of α-particles
- Rutherford's Model of the Atom
- Bohr's Model
- Hydrogen Spectrum

Theory of Atoms

Atom is the smallest particle of an element which contain all properties of element. Molecule is a single atom or a group of atoms joined by chemical bonds. It is the smallest unit of a chemical compound that can have an independent existance. Nuclei refers to a nucleus of an atom, having a given number of nucleons. It is a general term referring to all known isotopes– both stable and unstable of the chemical elements. Thus 0^{16} and 0^{17} are different nuclides.

Scattering of α-particles

In 1911, Rutherford successfully explained the scattering of α-particles on the basis of nuclear model of the atom.

Number of α-particles scattered through angle θ is given by

$$N(\theta) \propto \frac{Z^2}{\sin^4(\theta/2)\,K^2}$$

where K is the kinetic energy of the α-particle and Z is the atomic number of the metal.

Rutherford's Model of the Atom

On the basis of scattering of α-particles, Rutherford postulated the following model of the atom

(i) Atom is a sphere of diameter about 10^{-10} m. Whole of its positive charge and most of its mass is concentrated in the central part called the nucleus.

(ii) The diameter of the nucleus is of the order of 10^{-14} m.

(iii) The space around the nucleus is virtually empty with electrons revolving around the nucleus in the same way as the planets revolve around the sun.

(iv) The electrostatic attraction of the nucleus provides centripetal force to the orbiting electrons.

(v) Total positive charge in the nucleus is equal to the total negative charge of the orbiting electrons.

(vi) Rutherford's model suffers from the following drawbacks

(a) stability of the atomic model

(b) nature of energy spectrum

Bohr's Model

Bohr added the following postulates to the Rutherford's model of the atom

(i) The electrons revolve around the nucleus only in certain permitted orbits, in which the angular momentum of the electron is an integral multiple of $h/2\pi$, where h is the Planck's constant.

(ii) The electrons do not radiate energy while revolving in the permitted orbits.

That is, the permitted orbits are stationary, non-radiating orbits.

(iii) The energy is radiated only when the electron jumps from an outer permitted orbit to some inner permitted orbit. (Absorption of energy makes the electron jump from inner orbit to outer orbit).

(iv) If energy of the electron in nth and mth orbits be E_n and E_m respectively, then while the electron jumps from nth to mth orbit the radiation frequency ν is emitted, such that $E_n - E_m = h\nu$

This is called the **Bohr's frequency equation**.

- *Radius of the orbit of electron in a hydrogen atom in its stable state, corresponding to $n = 1$, is called Bohr's radius. Value of Bohr's radius is $r_0 = 0.529$ Å ≈ 0.53 Å.*
- *The time period of an electron in orbital motion in the Bohr's orbit is*

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.53\text{Å}}{\frac{c}{137}} = 1.52 \times 10^{-6} \text{ s}$$

and the frequency of revolution is $f = \frac{1}{T} = 6.5757 \times 10^{15}$ cps

Some Characteristics of an Atom

(i) The **orbital radius** of the electron is

$$r_n = 4\pi\varepsilon_0 \frac{n^2h^2}{4\pi^2 me^2}$$

(ii) The **orbital velocity** of electron is

$$v_n = \frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{nh}$$

(iii) **Orbital frequency** is given by

$$f = \frac{1}{T} = \frac{v}{2\pi r}$$

$$= \frac{me^4}{4\varepsilon_0^2 n^3 h^3}$$

(iv) The **total energy** of the orbital electron is

$$E = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \left[\frac{2\pi me^4}{h^2}\right]\left[\frac{1}{n^2}\right]$$

$$\text{KE} = \frac{me^4}{8n^2h^2\varepsilon_0^2}$$

$$\text{PE} = -\frac{me^4}{4n^2h^2\varepsilon_0^2}$$

(v) The **velocity of the orbital electron** may be written as

$$v_n = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\pi e^2}{h}\right]\left(\frac{1}{n}\right)$$

$$= \frac{c}{n}\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi e^2}{ch}\right] = \frac{c}{n}\alpha$$

Here $\frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{ch} = \alpha$ is called fine structure constant. It is a dimensionless quantity.

The value of α comes out to be $\frac{1}{137} = 0.0073$

(vi) The kinetic, potential and total energies of the electron with r as the radius of the orbit are as follows $KE = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right]$ $PE = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$

and $E = -\frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right]$

Therefore, they are related to each other as follows $KE = -E$ and $PE = 2E$

(vii) For a hydrogen atom

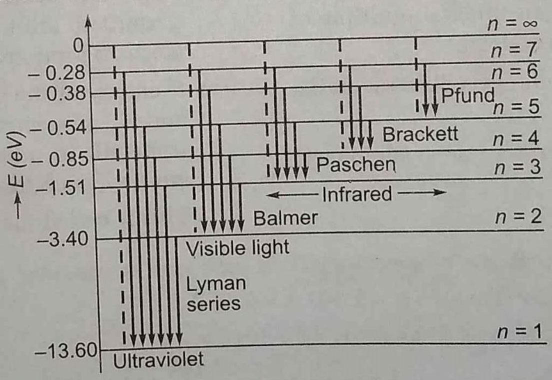
$$r_n \propto n^2, v_n \propto \frac{1}{n} \text{ and } |E| \propto \frac{1}{n^2}$$

The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is $\Delta L = (n+1)\frac{h}{2\pi} - \frac{nh}{2\pi} = \frac{h}{2\pi}$

Hydrogen Spectrum

Hydrogen spectrum consists of spectral lines classified as five spectral series of hydrogen atom. Out of these five, Lyman series lies in the ultraviolet region of spectrum,

Balmer series lies in the visible region and the remaining three series, lie in the infrared region of spectrum.



Total number of emission spectral lines from some excited state n, to another energy $n_2 (< n_1)$ is given by $\frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$

e.g., total number of lines from $n_1 = n$ *to* $n_2 = 1$ are $\frac{n(n-1)}{2}$

The five spectral series of hydrogen atom are given below

1. Lyman Series

Spectral lines of Lyman series correspond to the transition of electron from higher energy levels (orbits) $n_i = 2, 3, 4, \ldots$ to ground energy level (1st orbit) $n_f = 1$.

For Lyman series, $\frac{1}{\lambda} = \bar{v} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right]$ where $n = 2, 3, 4, \ldots$

It is found that a term $Rch = 13.6$ eV $= 2.17 \times 10^{-18}$ J. The term Rch is known as Rydberg's energy.

2. Balmer Series

Electronic transitions from $n_i = 3, 4, 5, \ldots$ to $n_f = 2$, give rise to spectral lines of Balmer series.

Thus, for a Balmer series line

$$\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(2)^2} - \frac{1}{n^2}\right],$$

where $n = 3, 4, 5, \ldots$

3. **Paschen Series**

Lines of this series lie in the infrared region and correspond to electronic transition from $n_i = 4,5,6,...$ to $n_f = 3$.

Thus, $\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(3)^2} - \frac{1}{n^2}\right]$, where $n = 4, 5, 6...$

4. **Brackett Series**

It too lies in the infrared region and corresponds to transition from $n_i = 5, 6, 7, ...$ to $n_f = 4$.

Thus, for Brackett series

$$\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(4)^2} - \frac{1}{n^2}\right], \text{ where } n = 5, 6, 7, ...$$

5. **Pfund Series**

It lies in the far infrared region of spectrum and corresponds to electronic transitions from higher orbits $n_i = 6,7,8,...$ to orbit having $n_f = 5$. Thus, we have

$$\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(5)^2} - \frac{1}{n^2}\right] \text{ where } n = 6, 7, 8, ...$$

- *Radius of the nth orbit in hydrogen atom is the $r_n = 0.53n^2$ Å*
- *If elements with $n > 4$ were not allowed to exist, then total number of elements would be*

$$2 \times [1^2 + 2^2 + 3^2 + 4^2] = 60$$

Ionisation Energy and Potential

Ionisation energy of an atom is defined as the energy required to ionise it *i.e.*, to make the electron jump from its present orbit to infinity.

Thus, ionisation energy of hydrogen atom in the ground state $= E_\infty - E_1 = 0 - (-13.6 \text{ eV}) = +13.6 \text{ eV}$

The potential through which an electron is to be accelerated so that it acquires energy equal to the ionisation energy is called the **ionisation potential**.

Therefore, ionisation potential of hydrogen atom in its ground state is 13.6V.

Excitation Energy and Potential

Excitation energy is the energy required to excite an electron from a lower energy level to a higher energy level. The potential through which an electron is accelerated so as to gain requisite ionisation energy is called the **ionisation potential**.

Thus, first excitation energy of hydrogen atom

$$= E_2 - E_1 = -3.4 - (-13.6) \text{ eV}$$
$$= +10.2 \text{ eV}$$

Similarly second excitation energy of hydrogen

$$= E_3 - E_1$$
$$= -1.51 - (-13.6) = 12.09 \text{ eV}$$

- *Total energy of a closed system is always negative and its magnitude is the binding energy of the system.*
- *Kinetic energy of a particle can't be negative, while the potential energy can be zero, positive or negative. It is basically depends on the reference point where we have taken it as zero.*

Practice Zone

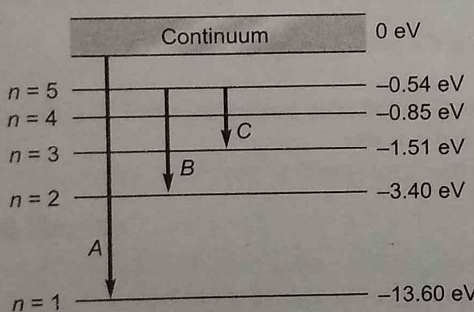
DAY 32

1. The ground state energy of H-atom is 13.6 eV. The energy needed to ionize H-atom from its second excited state
(a) 1.51 eV
(b) 3.4 eV
(c) 13.6 eV
(d) 12.1 eV

2. The potential energy between a proton and an electron is $PE = \frac{e^2}{4\pi\varepsilon_0(3R^3)}$, then the radius of the Bohr's orbit is
(a) $\frac{4\pi^2 e^2 m}{4\pi\varepsilon_0 n^2 h^2}$
(b) $\frac{6\pi^2 e^2 m}{4\pi\varepsilon_0 n^3 h^3}$
(c) $\frac{e^2 m}{4\pi\varepsilon_0 h^3}$
(d) $\frac{2\pi e^2 m}{4\pi\varepsilon_0 nh^3}$

3. An electron jumps from the 4th orbit to the 2nd orbit of hydrogen atom. Given the Rydberg's constant $R = 10^5 \text{cm}^{-1}$, the frequency in hertz of the emitted radiation will be
(a) $\frac{3}{16} \times 10^5$
(b) $\frac{3}{16} \times 10^{15}$
(c) $\frac{9}{16} \times 10^{15}$
(d) $\frac{3}{4} \times 10^{15}$

4. In figure, the energy levels of the hydrogen atom have been shown along with some transitions marked *A*, *B*, *C*. The transitions *A*, *B* and *C* respectively represents



(a) the first member of the Lyman series, third member of Balmer series and second member of Paschen series
(b) the ionisation potential of H, second member of Balmer series and third member of Paschen series
(c) the series limit of Lyman series, second member of Balmer series and second member of Paschen series
(d) the series limit of Lyman series, third member of Balmer series and second member of Paschen series

5. In a hypothetical Bohr hydrogen atom, the mass of the electron is doubled. The energy E and radius r_0 of the first orbit will be (a_0 is the Bohr radius)
(a) $E_0 = -27.2$ eV ; $r_0 = a_0/2$
(b) $E_0 = -27.2$ eV ; $r_0 = a_0$
(c) $E_0 = -13.6$ eV ; $r_0 = a_0/2$
(d) $E_0 = -13.6$ eV ; $r_0 = a_0$

6. The ratio of the kinetic energy to the energy of an electron in a Bohr orbit is
(a) –1
(b) 2
(c) 1 : 2
(d) None of these

7. Taking the Bohr's radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about **[NCERT Exemplar]**
(a) 53 pm
(b) 27 pm
(c) 18 pm
(d) 13 pm

8. To explain theory of hydrogen atom, Bohr considered
(a) quantisation of linear momentum
(b) quantisation of angular momentum
(c) quantisation of angular frequency
(d) quantisation of energy

9. An hydrogen atom moves with a velocity u and makes a head on inelastic collision with another stationary H-atom. Both atoms are in ground state before collision. The minimum value of u if one of them is to be given a minimum excitation energy is
(a) $2.64 \times 10^4 \text{ ms}^{-1}$
(b) $6.24 \times 10^4 \text{ ms}^{-1}$
(c) $2.02 \times 10^6 \text{ ms}^{-1}$
(d) $6.24 \times 10^8 \text{ ms}^{-1}$

10. A hydrogen like ion having wavelength difference between first Balmer and Lyman series equal 593Å has Z equal to
(a) 2
(b) 3
(c) 4
(d) 1

11. Energy E of a hydrogen atom with principal quantum number n is given by $E = -\frac{13.6}{n^2}$ eV. The energy of a photon ejected when the electron jumps from $n = 3$ state to $n = 2$ state of hydrogen, is approximately
(a) 1.5 eV
(b) 0.85 eV
(c) 3.4 eV
(d) 1.9 eV

12. In the Bohr model of the hydrogen atom, let R, V and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number n?
(a) E/v (b) R/E (c) vR (d) RE

13. In Rutherford's experiment, the number of alpha particles scattered through an angle of 90° is 28 per minute. Then the number of particles scattered through an angle of 60° per minute by the same nucleus is
(a) 28 per minute (b) 112 per minute
(c) 12.5 per minute (d) 7 per minute

14. A small particle of mass m moves such that potential energy $PE = \frac{1}{2} mr^2\omega^2$. Assuming Bohr's model of quantisation of angular momentum and circular orbit, radius of nth orbit is proportional to
(a) $\sqrt{n}$ (b) $\sqrt{n^3}$ (c) $\frac{1}{\sqrt{n}}$ (d) $\frac{1}{\sqrt{n^3}}$

Directions *(Q. Nos. 15 to 18) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

15. **Statement I** Balmer series lies in the visible region of electromagnetic spectrum.
Statement II $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, where $n = 3, 4, 5$.

16. **Statement I** The ionisation potential of hydrogen is found to be 13.6 eV, the ionisation potential of doubly ionised lithium is 122.4 eV.
Statement II Energy in the nth state of hydrogen atom is $E_n = -\frac{13.6}{n^2}$.

17. **Statement I** Bohr had to postulate that the electrons in stationary orbits around the nucleus do not radiate.
Statement II According to classical physics all moving electrons radiate.

18. **Statement I** The different lines of emission spectra (like Lyman, Balmer etc) of atomic hydrogen gas are produced by diferent atoms.
Statement II The sample of atomic hydrogen gas consists of millions of atoms.

Directions *(Q. Nos. 19 to 20) In a mixture of H - He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.*

19. The quantum number n of the state finally populated in He^+ ions is
(a) 2 (b) 3 (c) 4 (d) 5

20. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is
(a) 6.5×10^{-7}m (b) 5.6×10^{-7}m
(c) 4.8×10^{-7}m (d) 4.0×10^{-7}m

21. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon. **[NCERT Exemplar]**
(a) 9.7×10^{-8} m and 3.1×10^{15} Hz
(b) 7.6×10^{-6} m and 2.6×10^{14} Hz
(c) 2.9×10^{-10} m and 4.9×10^{12} Hz
(d) 8.6×10^{-9} m and 3.1×10^{14} Hz

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22. From the following, the quantity (constructed from the basic constants of nature), that has the dimensions, as well as correct order of magnitude, vis-a-vis typical atomic size, is
(a) $\frac{h^2}{4\pi\varepsilon_0 me^2}$ (b) $\frac{4\pi\varepsilon_0 h^2}{me^2}$ (c) $\frac{me^1}{4\pi\varepsilon_0 h^2}$ (d) $4\pi\varepsilon_0 e^2$

23. In the Bohr's model of hydrogen-like atom the force between the nucleus and the electron is modified as $F = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$, where β is a constant. For this atom, the radius of the nth orbit in terms of the Bohr radius $\left(a_0 = \frac{\varepsilon_0 h^2}{m\pi e^2}\right)$ is
(a) $r_n = a_0 n - \beta$ (b) $r_n = a_0 n^2 + \beta$
(c) $r_n = a_0 n^2 - \beta$ (d) $r_n = a_0 n + \beta$

24. The half-life of a radioactive element A is the same as the mean-life of another radioactive element B. Initially both substances have the same number of atoms, then
(a) A and B both decay at the same rate always
(b) A and B will decay at the same rate initially
(c) A will decay at a faster rate than B initially
(d) B will decay at a faster rate than A initially

25. In the Bohrs model an electron moves in a circular orbit around the proton. Considering the orbiting electron to be a circular current loop, the magnetic moment of the hydrogen atom, when the electron is in nth excited state, is

(a) $\left(\frac{e}{2m}\right)\frac{n^2h}{\pi}$ (b) $\left(\frac{e}{m}\right)\frac{nh}{2\pi}$ (c) $\left(\frac{e}{2m}\right)\frac{nh}{2\pi}$ (d) $\left(\frac{e}{m}\right)\frac{n^2h}{2\pi}$

26. In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to **[JEE Main 2013]**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^3/2}$ (d) $\frac{1}{n^3}$

27. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then, the number of spectral lines in the emission spectra will be **[AIEEE 2012]**

(a) 2 (b) 3 (c) 5 (d) 6

28. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer) **[AIEEE 2012]**

(a) $\frac{(m_1+m_2)^2 n^2h^2}{2m_1^2m_2^2r^2}$ (b) $\frac{n^2h^2}{2(m_1+m_2)r^2}$

(c) $\frac{2n^2h^2}{(m_1+m_2)r^2}$ (d) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$

29. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radition will be obtained in the transition from **[AIEEE 2009]**

(a) $2 \to 1$ (b) $3 \to 2$ (c) $4 \to 2$ (d) $5 \to 3$

30. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$, where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the nth orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then which of the following is true? **[AIEEE 2008]**

(a) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$ (b) T_n independent of n, $r_n \propto n$

(c) $T_n \propto \frac{1}{n}, r_n \propto n$ (d) $T_n \propto \frac{1}{n}, r_n \propto n^2$

31. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **[AIEEE 2007]**

(a) 802 nm (b) 823 nm (c) 1882 nm (d) 1648 nm

32. Which of the following transitions in hydrogen atoms emit photons of highest frequency? **[AIEEE 2007]**

(a) $n = 2$ to $n = 6$ (b) $n = 6$ to $n = 2$

(c) $n = 2$ to $n = 1$ (d) $n = 1$ to $n = 2$

33. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of **[AIEEE 2004]**

(a) 1Å (b) 10^{-10} cm

(c) 10^{-12} cm (d) 10^{-15} cm

34. The manifestation of band structure in solids is due to **[AIEEE 2004]**

(a) Heisenberg's uncertainty principle
(b) Pauli's exclusion principle
(c) Bohr's correspondence principle
(d) Boltzmann's law

35. The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number. **[AIEEE 2003]**

(a) $r_n \propto n$ (b) $r_n \propto \frac{1}{n}$ (c) $r_n \propto n^2$ (d) $r_n \propto \frac{1}{n^2}$

36. If the atom $_{100}Fm^{257}$ follows the Bohr model and the radius of last orbit of $_{100}Fm^{257}$ is n times the Bohr radius, then find n **[AIEEE 2003]**

(a) 100 (b) 200 (c) 4 (d) 1/4

37. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is **[AIEEE 2002]**

(a) 10.2 eV (b) zero (c) 3.4 eV (d) 6.8eV

38. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition **[AIEEE 2002]**

(a) $2 \to 1$ (b) $3 \to 2$

(c) $4 \to 2$ (d) $5 \to 4$

Answers

1. (a)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (b)
11. (d)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (c)
21. (a)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (d)	**28.** (d)	**29.** (d)	**30.** (b)
31. (b)	**32.** (b)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (d)	**37.** (c)	**38.** (d)		

Hints & Solutions

1. Second excited state corresponds to $n = 3$

$\therefore \quad E = \frac{13.6}{(3)^2} \text{ eV} = 1.51 \text{ eV}$

2. $F = \frac{-dPE}{dR} = -\frac{e^2}{4\pi\varepsilon_0 R^4} = \frac{mv^2}{R} = \frac{e^2}{4\pi\varepsilon_0 R^4}$. Also $mvR = \frac{nh}{2\pi}$

$\therefore \quad \frac{m}{R}\left(\frac{nh}{2\pi mR}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 R^4} \Rightarrow R = \frac{4\pi^2 (e^2 m)}{4\pi\varepsilon_0 n^2 h^2}$

3. $v = \frac{c}{\lambda} = c.R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 3 \times 10^8 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$= \frac{9}{16} \times 10^{15} \text{ Hz}$

4. *A* represents series limit of Lyman series, *B* represents third member of Balmer series and *C* represents second member of Paschen series.

5. As $\quad r \propto \frac{1}{m}$

$\therefore \quad r_0 = \frac{1}{2} a_0$

As $\quad E \propto m$

$\therefore \quad E_0 = 2(-13.6) = -27.2 \text{ eV}$

6. $K = \frac{me^4}{8\varepsilon_0^2 n^2 h^2}$; $E = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$

$\Rightarrow \quad E = -K \Rightarrow \frac{K}{E} = -1$

7. On the basis of Bohr's model, $r = \frac{n^2 \lambda^2}{4\pi^2 mKZe^2} = \frac{a_0 n^2}{Z}$

Let Li^{++} ion, $Z = 3, n = 1$ for ground state. Given, $a_0 = 53$ pm

$r = \frac{53 \times (1)^2}{3} = 18 \text{ pm}$

8. While proposing his theory of hydrogen atom Bohr considered quantisation of angular momentum as the essential condition for the stationary orbits.

9. $\quad mu = 2mv \Rightarrow v = \frac{u}{2}$

$\Delta E = \frac{1}{2} mu^2 - \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{mu^2}{4}$

$\frac{1}{4} mu^2 = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\frac{1}{4}(1.0078)(1.66 \times 10^{-27})u^2 = 10.2 \times 1.6 \times 10^{-19}$

$\Rightarrow \quad u = 6.24 \times 10^4 \text{ ms}^{-1}$

10. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$, $\Delta\lambda = \frac{1}{RZ^2\left(\frac{1}{4} - \frac{1}{9}\right)} - \frac{1}{RZ^2\left(\frac{1}{1} - \frac{1}{4}\right)}$

$593 = \frac{36}{5RZ^2} - \frac{4}{3RZ^2}$

$Z^2 = \frac{88}{15R\,\Delta\lambda} = \frac{88}{15(1.097 \times 10^7)(593 \times 10^{-10})} = 9$

$\Rightarrow \quad Z = 3$

11. Given, $\quad E_n = -\frac{13.6}{n^2}$ eV.

$E_3 = -\frac{13.6}{(3)^2} \text{ eV} = -\frac{13.6}{9} \text{ eV}$ and $E_2 = -\frac{13.6}{(2)^2} \text{ eV} = -\frac{13.6}{4} \text{ eV}$

So, $\Delta E = E_3 - E_2 = -\frac{13.6}{9} - \left(-\frac{13.6}{4}\right)$

$= 1.9 \text{ eV}$ (approximately)

12. According to the Bohr's postulate.

$mvr = \frac{nh}{2\pi} \Rightarrow vR = n\left(\frac{h}{2\pi m}\right) \Rightarrow vR \propto n$

Thus, *vR* is directly proportional to the principal quantum number.

13. According to Rutherford's scattering formula, if the α-particle scattered at angles θ is directly proportional to $\frac{1}{\sin^4(\theta/2)}$, then

$N_\theta = \frac{K}{\sin^4(\theta/2)}$

when $\theta = 90°, N_\theta = 28 \text{ min}^{-1}$

$\Rightarrow \quad 28 = \frac{K}{\sin(45°)} = 4K \Rightarrow K = 7$

Thus $\quad N_\theta = \frac{7}{\sin^4(\theta/2)}$

Hence, the number of α-particles scattered at an angle of 60° per minute is $N'_\theta = \frac{7}{\sin^4 30°} = \frac{7}{(1/2)^4} = 7 \times 16 = 112$

14. $F = \frac{-du}{dr} = -m\omega^2 r$. Since, $mvr = \frac{nh}{2\pi}$ or $mr^2\omega = \frac{nh}{2\pi}$

$v = r\omega \Rightarrow r^2 = \frac{nh}{2\pi m\omega} \Rightarrow r = \sqrt{\frac{nh}{2\pi m\omega}} \Rightarrow r \propto \sqrt{n}$

15. The wavelength in Balmer series is given by

$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3, 4, 5..$

$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{36}{5R}$

$\lambda_{max} = \frac{36 \times 1}{5 \times 1.097 \times 10^7} = 6563 \text{ Å}$

and $\quad \frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$\lambda_{min} = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 3646 \text{ Å}$

The wavelength, 6563 Å and 3646 Å lie in visible region. Therefore, Balmer series lies in visible region.

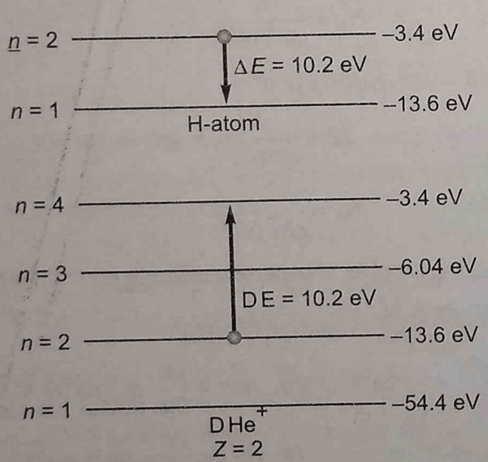
16. From Bohr's theory the energy of hydrogen atom in the nth state is given by $E_n = -\frac{13.6}{n^2}$ eV. For an atom of atomic number Z, with one electron in the outer orbit (singly ionised He or doubly lithium) we use. $E_n = -\frac{13.6Z^2}{n^2}$ eV, where Z is the atomic number. Hence, ground state energy of doubly ionised lithium is $\frac{-13.6 \times 9}{(1)^2} = -122.4$eV

Ionisation potential (potential to be applied to electron to overcome this energy) is 122.4V.

17. Bohr postulated that, electron instead of revolving in any orbit around the nucleus, revolve only in some specific orbits. These orbits are called the non-radiating orbits or the stationary orbits. The electrons revolving in these orbits do not radiate any energy. They radiate only when they go from one orbit to the next lower orbit.

18. A single atom can have only the transition at time, we are observing different lines due to large number of transitions taking place simultaneously that occurred in different atoms of the sample.

19.



Energy given by H-atom in transition from $n = 2$ to $n = 1$ is equal to energy taken by He^+ atom in transition from $n = 2$ to $n = 4$.

20. Visible light lies in the range, $\lambda_1 = 4000$Å to $\lambda_2 = 7000$Å. Energy of photons corresponding to these wavelengths (in eV) would be

$$E_1 = \frac{12375}{4000} = 3.09\text{eV} \text{ and } E_2 = \frac{12375}{7000} = 1.77 \text{ eV}$$

From energy level diagram of He^+ atom we can see that in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04) = 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{2.64} \text{Å} = 4687.5\text{Å} = 4.8 \times 10^{-7}\text{m}$$

21. For ground state $n_1 = 1$ to $n_2 = 4$

Energy absorbed by photon, $E = E_2 - E_1$

$$= 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \times 1.6 \times 10^{-19} \text{J}$$

$$= 13.6\left(\frac{1}{1} - \frac{1}{4^2}\right) \times 1.6 \times 10^{-18}$$

$$= 13.6 \times 1.6 \times 10^{-19}\left(\frac{15}{16}\right) = 20.4 \times 10^{-19}$$

or $E = h\nu = 20.4 \times 10^{-19}$

$$\text{Frequency } \nu = \frac{20.4 \times 10^{-19}}{h} = \frac{20.4 \times 10^{-19}}{6.63 \times 10^{-34}} = 3.076 \times 10^{15}$$

$$= 3.1 \times 10^{15} \text{ Hz}$$

Wavelength of photon $\lambda = \frac{c}{\nu}$

$$= \frac{3 \times 10^8}{3.076 \times 10^{15}} = 9.74 \times 10^{-8} \text{m}$$

Thus, the wavelength is 9.7×10^{-8} m and frequency is 3.1×10^{15} Hz.

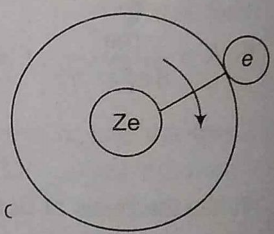
22. The typical atomic size given by $\frac{4\pi\varepsilon_0 h^2}{me^2}$.

23. As, force between nucleus and electron is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

where, r = atomic radus

and e = electronic charge



From question,

$$F_e = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right), \text{ where } \beta = c$$

We know that, $r_n = \frac{\varepsilon_0 h^2 n^2}{Zm\pi e^2}$

$$r_0 = \frac{\varepsilon_0 h^2}{Zm\pi e^2}$$

$\therefore \quad r_n = r_0 n^2$

As, we consider the nth electron which attracted by nucleus while itself is repelled by the electron of inner shell to the consider shell.

24. As we know that $nt_{1/2} = t_{av}$

$$\Rightarrow \quad \frac{t_{av}}{t_{1/2}} = n$$

Hence, B will decay at a faster rate than A initially.

25. As, $i = \frac{e}{T}$ and magnetic moment $M = iA$ $\quad (\because A = \pi r^2)$

$$\therefore \quad M = \frac{e}{T} \cdot \pi r^2 \quad \ldots\text{(i)}$$

Now, $T = \frac{2\pi r}{v}$

It becomes, $M = \frac{\frac{e}{2\pi r}\cdot \pi r^2}{v} = \frac{evr}{2}$ ...(ii)

Also, $mvr = \frac{nh}{2\pi},\ vr = \frac{nh}{2\pi m}$

Putting this value in Eq. (ii), we get $M = \frac{e\cdot nh}{2\cdot 2\pi m} = \left(\frac{e}{2m}\right)\frac{nh}{2\pi}$

26. $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = K\left[\frac{1}{(n-1)^3} - \frac{1}{n^2}\right] = \frac{K2n}{n^2(n-1)^2} = \frac{2K}{n^3} \propto \frac{1}{n^3}$$

27. In emission spectrum, number of bright lines is given by

$$\frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

28. Rotational kinetic energy of the two body system rotating about their centre of mass is

$$\text{RKE} = \frac{1}{2}\mu\omega^2 r^2,$$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass

and angular momentum, $L = \mu\omega r^2 = \frac{nh}{2\pi}$

$$\therefore\ \text{RKE} = \frac{1}{2}\mu\omega^2 r^2 = \frac{1}{2}\mu\cdot\left(\frac{nh}{2\pi\mu r^2}\right)^2 r^2$$

$$= \frac{n^2h^2}{8\pi^2\mu r^2} = \frac{n^2h^2}{2\mu r^2}$$

$$= \frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2} \qquad \left[\text{Here, } h^2 = \frac{\lambda}{4\pi}\right]$$

29. Infrared radiation corresponds to least value of $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ *i.e.*, from Paschen, Brackett and Pfund series. Thus, the transition corresponds to $5 \to 3$.

30. $\frac{mv^2}{r_n} = \frac{k}{r_n}$ given $mvr_n = \frac{nh}{2\pi}$ from Bohr's theory.

Solving, $r_n \propto n$ and T_n is independent of n.

31. The series in U-V region is Lyman series. Longest wavelength corresponds to minimum energy which occurs in transition from $n = 2$ to $n = 1$.

$$\therefore\quad 122 = \frac{\frac{1}{R}}{\frac{1}{(1)^2} - \frac{1}{(2)^2}} \quad \text{...(i)}$$

The smallest wavelength in the infrared region corresponds to maximum energy of Paschen series.

$$\therefore\quad \lambda = \frac{\frac{1}{R}}{\frac{1}{(3)^2} - \frac{1}{\infty}} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get $\lambda = 823.5$ nm

32. Emission spectrum would rises when electron makes a jump from higher energy level to lower energy level.

Frequency of emitted photon is proportional to change in energy of two energy levels, *i.e.*, $\nu = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

33. Here, $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$$\therefore\quad 5\,\text{MeV} = \frac{9\times10^9\times(2e)\times(92e)}{r} \quad \left(\because\ \frac{1}{2}mv^2 = 5\,\text{MeV}\right)$$

$$\Rightarrow\quad r = \frac{9\times10^9\times2\times92\times(1.6\times10^{-19})^2}{5\times10^6\times1.6\times10^{-19}}$$

or $r = 5.3\times10^{-14}\,\text{m} \approx 10^{-12}$ cm

34. According to Pauli's exclusion principle, the electronic configuration of number of subshells existing in a shell and number of electrons entering each subshell is found. Hence, on the basis of Pauli's exclusion of band structure in solids can be explained.

35. $U = eV = eV_0 \ln\left(\frac{r}{r_0}\right)$ and $|F| = \left|-\frac{dU}{dr}\right| = \frac{eV_0}{r}$

This force will provide the necessary centripetal force. Hence,

$$\frac{mv^2}{r} = \frac{eV_0}{r}$$

or $v = \sqrt{\frac{eV_0}{m}}$ ...(i)

Moreover, $mvr = \frac{nh}{2\pi}$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we have

$$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}} \text{ or } r_n \propto n$$

36. $r_m = \left(\frac{m^2}{Z}\right)(0.53\,\text{Å}) = (n\times0.53)\,\text{Å}$

$$\therefore\quad \frac{m^2}{Z} = n$$

$m = 5$ (for ${}_{100}\text{Fm}^{257}$ the outermost shell)

and $Z = 100$

$$\therefore\quad n = \frac{(5)^2}{100} = \frac{1}{4}$$

37. Energy required to remove an electron from nth orbit is,

$$E_n = -\frac{13.6}{n^2}$$

Here, $n = 2$

Therefore, $E_2 = -\frac{13.6}{(2)^2} = -3.4$ V

38. Energy of infrared radiation is less than the energy of ultraviolet radiation. In options (a), (b) and (c), energy released will be more, while in option (d) only, energy released will be less.

Day 33

Nuclei

Day 33
Outlines ...

- Concept of Nucleus
- Composition and Size of Nucleus
- Radioactivity
- Law of Radioactive Decay
- Mass Defect and Binding Energy
- Nuclear Fission
- Nuclear Fusion

Concept of Nucleus

In every atom the positive charge and mass is densely concentrated at the centre of the atom forming its nucleus. Nuclear radius is of the order of 10^{-15} m. In nucleus the number of protons is equal to the atomic number of that element and the remaining particles to fulfil the mass number are the neutrons i.e., number of protons = atomic number z (say) and number of neutrons.

Composition *and* Size of Nucleus

As mass of an atom is very small hence, we define a new unit of mass, called 1 atomic mass unit (1u), which is $\frac{1}{12}$ th the mass of one atom of carbon-12.

$$1\ \text{u} = 1.660539 \times 10^{-27}\ \text{kg} \simeq 1.66 \times 10^{-27}\ \text{kg} = 931.5\ \text{MeV}$$

A nucleus has a structure of its own. It consists of protons and neutrons. Electr[ons] cannot exist inside the nucleus. A proton is a positively charged particle hav[ing] mass (m_p) of 1.007276 u and charge $(+e) = +1.602 \times 10^{-19}$ C.

Number of protons Z inside the nucleus of an atom is exactly equal to the num[ber] of electrons revolving around the nucleus of that atom. This number is called atomic number. A neutron is a neutral particle having mass $m_n = 1.008665$ u. T[he] number of neutrons in the nucleus of an atom is called the **neutron number**. The sum of the number of protons and neutrons is called the mass number. Thus, $A = N + Z$. The radius of a nucleus depends only on its mass numbe[r] according to the relation $r = r_0 A^{1/3}$, where r_0 is a constant having a value of 1.2 f[m]

Isotopes, Isobars *and* Isotones

Isotopes

Isotopes of an element are nuclides having same atomic number Z but different mass number A (or different neutron number N). Isotopes of an element have identical electronic configuration and hence, identical chemical properties. ${}_1^1H$, ${}_1^2H$, ${}_1^3H$ and ${}_6^{11}C$, ${}_6^{12}C$, ${}_6^{14}C$, etc, are isotopes.

Isobars

Nuclides having same mass number A but different atomic number Z are called isobars. Isobars represent different chemical properties. In isobars number of protons Z as well as number of neutrons N, differ but total nucleon (or mass) number $A = N + Z$ is the same. ${}_1^3H$, ${}_2^3He$ and ${}_6^{14}C$, ${}_7^{14}N$ are isobars.

Isotones

Nuclides with different atomic number Z and different mass number A but same neutron number are called isotones.

Thus, for isotones $N = (A - Z)$ is constant. ${}_1^3H$, ${}_2^4He$ and ${}_{80}^{198}Hg$, ${}_{79}^{197}Au$ are examples of isotones.

Properties of Nucleus

The nuclear properties are described below

- **Nuclear size**
  - (a) Size of the nucleus is of the order of fermi (1 fermi $= 10^{15}$ m)
  - (b) The radius of the nucleus is given by $R = R_0 A^{1/3}$, where $R_0 = 1.3$ fermi and A is the mass number.
  - (c) The size of the atom is of the order of 10^{-19} m.
- **Volume** The volume of nucleus is $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3$
- **Density**
  - (a) Density $= \frac{\text{Mass of nucleus}}{\text{Volume of the nucleus}} = \frac{Am_p}{\frac{4}{3}\pi (R_0 A^{1/3})^3} = \frac{m_p}{\frac{4}{3}\pi R_0^3}$

    where, $m_p = 1.6 \times 10^{-27}$ kg = mass of proton and $R_0 = 1.3$ fermi.
  - (b) Density of nuclear matter is of the order of 10^{17} kg/m^3
  - (c) Density of nuclear matter is independent of the mass number.

Radioactivity

Radioactivity is the phenomenon of spontaneous emission of radiations by heavier nucleus. Some naturally occurring radioactive substances are uranium, thorium, polonium, radium, neptunium etc. In fact, all elements having atomic number $Z > 82$ are radioactive in nature.

Radiations emitted by radioactive substances are of 3 types, namely (i) α-particles, (ii) β- particles, and (iii) γ- rays.

α-particles are positively charged particles with charge $q_\alpha = +2e$ and mass $m_\alpha = 4m_p$. Thus, α-particles may be considered as helium nuclei (or doubly charged helium ions). Ionising power of α-particles is maximum but, their penetrating power is minimum.

β-particles are negatively charged particles with rest mass as well as charge same as that of electrons. But origin of β-particles is from the nucleus. Their ionising power is lesser than that of α-particles, but speed as well as penetrating power is much greater than that of α-particles. Generally, β- decay means β^--decay.

γ-rays are electromagnetic radiations of extremely short wavelengths. Thus, γ-rays travel with the speed of light. Their ionising power is least but penetrating power is extremely high. These are not deflected either in an electric or a magnetic field.

Proton was discovered through artificial disintegration of nitrogen by α-particles as follows

$$_2He^4 + {}_7N^{14} \longrightarrow {}_8O^{17} + {}_1H^1$$

Law of Radioactive Decay

According to **Rutherford-Soddy's law** *for radioactive decay, "the rate of decay of a radioactive material at any instant is proportional to the quantity of that material actually present at that time." Mathematically,*

$$-\frac{dN}{dt} \propto \lambda \quad or \quad \frac{dN}{dt} = -\lambda N$$

Here, λ is a proportionality constant, known as the **decay constant** (or disintegration constant). Unit of λ is s^{-1} or day^{-1} or $year^{-1}$ etc. It can be shown that number of nuclei present after time t is given by $N = N_0 e^{-\lambda t}$

where N_0 = number of nuclei present at time $t = 0$.

Again number of nuclei decayed in time t will be

$N - N_0 = N_0 [1 - e^{-\lambda t}]$

= number of **daughter nuclei** produced at time t

Activity

The activity of a radioactive substance is defined as the rate of disintegration (or the count rate) of that substance. Mathematically, activity is defined as

$$R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

where $R_0 = \lambda N_0$ = initial value of activity.

Different Units of Activity

Units of activity are

- 1 becquerel = 1 Bq = 1 disintegration per second (SI unit)
- 1 curie = 1 Ci = 3.7×10^{10} Bq
- 1 rutherford = 1 Rd = 10^6 Bq

Half-Life Period ($T_{1/2}$)

Half-life period of a radioactive sample is the time in which half of the particles of the sample initially present, gets disintegrated. Alternately, it is the time in which, activity of the sample falls to one-half of its initial value.

Thus, for $t = \frac{T}{2}$, $\quad N = \frac{N_0}{2}$ and $R = \frac{R_0}{2}$

The half-life period is related to decay constant λ as

$$T_{1/2} = \frac{0.693}{\lambda}$$

After n half-lives the quantity of a radioactive substance left intact (undecayed) is given by

$$N = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

Mean Life Period (τ)

1. Mean life of a radioactive sample is the time at which both N and R have been reduced to $\frac{1}{e}$ or e^{-1} (or 36.8%) of their initial values. It is found that $\tau = \frac{1}{\lambda}$
2. Half-life $T_{1/2}$ and mean life τ of a radioactive sample are correlated as $T_{1/2} = 0.693\,\tau$ or $\tau = 1.44 T_{1/2}$

Nuclear Fission

Nuclear fission is the process of splitting of a heavy nucleus ($^{235}_{92}U$ or $^{239}_{94}Pu$) into two lighter nuclei of comparable masses along with the release of a large amount of energy ($\simeq$ 200 MeV) after bombardment by slow neutrons.

A characteristic nuclear fission reaction equation for $^{235}_{92}U$ is

$$^{1}_{0}n(\text{slow}) + {}^{235}_{92}U \longrightarrow {}^{236}_{92}U \longrightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n + Q$$

In the fission of uranium, the percentage of mass converted into energy is about 0.1%.

Controlled Chain Reaction and Nuclear Reactor

In the fission of one nucleus of $^{235}_{92}U$, on an average, $2\frac{1}{2}$ neutrons are released. These released neutrons may further trigger more fissions causing more neutrons being formed, which in turn may cause more fission. Thus, a self sustained nuclear chain reaction is formed. To maintain the nuclear chain reaction at a steady (sustained) level the extra neutrons produced, are absorbed by suitable neutron absorbents like cadmium or boron.

Neutrons formed as a result of fission have a energy of about 2 MeV, whereas for causing further fission, we need slow thermal neutrons having an energy of about 0.3 eV. For this purpose suitable material called a **moderator** is used, which slow down the neutrons. Water, heavy water and graphite are commonly used as moderators.

A **nuclear reactor** is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction and can be employed for producing electrical power, for producing different isotopes and for various other uses.

Reproduction Factor H_2

Reproduction factor (k) of a nuclear chain reaction is defined as

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss / Absorption of neutrons}}$$

(i) If $k = 1$, then the chain reaction will be steady and the reactor is said to be **critical**.

(ii) If $k > 1$, then the chain reaction is accelerated and it may cause explosion in the reactor. Such a reactor is called **super-critical**.

(iii) If $k < 1$, then chain reaction gradually slows down and comes to a halt. Such a reactor is called **sub-critical**.

The reactors giving fresh nuclear fuel which often exceeds the nuclear fuel used in known as breeder reactor.

Nuclear Fusion

Nuclear fusion is the process in which two or more light nuclei combine to form a single large nucleus.

The mass of the single nucleus so formed is less than the sum of the masses of parent nuclei and this difference in mass, results in the release of tremendously large amount of energy.

The fusion reaction going on in the central core of sun is a multistep process but the net reaction is

$$4\,{}_1^1\text{H} + 2\varepsilon^- \longrightarrow {}_2^4\text{He} + 2\nu + 6\gamma + 26.7 \text{ MeV}$$

When two positively charged particles (protons or deuterons) combine to form a larger nucleus, the process is hindered by the Coulombian repulsion between them.

To overcome the Coulombian repulsion, the charged particles are to be given an energy of atleast 400 keV. For this, proton/deuterons must be heated to a temperature of about 3×10^9 K. Nuclear fusion reaction is therefore, known as **thermo nuclear fusion** reaction.

Mass Defect *and* Binding Energy

- The difference in mass of a nucleus and its constituent nucleons is called the mass defect of that nucleus. Thus,
  Mass defect $\Delta M = Zm_p + (A - Z)m_n - M$
  where M is the mass of a given nucleus.
- **Packing fraction** of an atom is the difference between mass of nucleus and its mass number per nucleon. Thus,
  Packing fraction $= \dfrac{M - A}{A}$
- The energy equivalent of the mass defect of a nucleus is called its **binding energy.**
  Thus, binding energy $\Delta E_b = \Delta M c^2 = [ZM_p + (A - Z)M_n - M]c^2$
  If masses are expressed in atomic mass units,
  then $\Delta E_b = \Delta M \times 931.5$ MeV
  $= [ZM_p + (A - Z)M_n - M] \times 931.5$ MeV
- Binding energy per nucleon (ΔE_{bn}) is the average energy needed to separate a nucleus into its individual nucleons.
  Thus, $\Delta E_{\text{bn}} = \dfrac{\Delta E_b}{A}$
- As the rest mass energy of each of electron and positron is $E_0 = m_0 c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$ J $= 0.51$ MeV
- Therefore, an energy of atleast 1.02 MeV is needed for pair production.

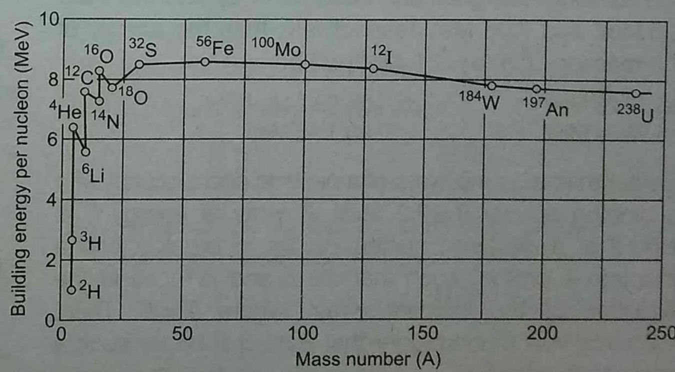
Mass Excess

Let A be the mass number of a nucleus. Let m_d (atomic mass units) be the mass of the neutral atom and au is the mass of the nuclide in amu then excess mass = ($mu - au$)

$$= (m - a)\frac{931.5}{c^2} \text{ MeV} \times c^2 = (m - a)\, 931.5 \text{ MeV}$$

Packing fraction $\rho = (m - A)/A$

The figures shows binding energy per nucleon *versus* mass number. The nuclides showing binding energy per nucleon greater than 7.5 MeV/nucleon are stable.



- *Nucleons attract each other when they are separated by a distance of 10^{-14} m.*
- *The density of nucleus is of the order of 10^{17} kgm^{-3}.*

Practice Zone DAY 33

1. A free state ^{191}Ir nucleus with excitation energy $E = 129$ keV comes to ground state emitting a γ-ray. The fractional change in the γ-ray energy due to recoil of nucleus is
(a) 4.18×10^{-8} (b) 3.63×10^{-7}
(c) 2.63×10^{-4} (d) 9.3×10^{-10}

2. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of
(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

3. A particle moving with a velocity of $\frac{1}{100}$th of that of light, will cross a nucleus in about
(a) 10^{-8} s (b) 10^{-12} s (c) 10^{-17}s (d) 10^{-20} s

4. The half-life of a radioactive sample is 10h. The total number of disintegration in 10th hour measured from a time when the activity was one Ci is
(a) 0.53×10^{-3} (b) 6.91×10^{13}
(c) 2.63×10^{-3} (d) 9.91×10^{13}

5. The binding energies per nucleon of Li^7 and He^4 are 5.6 MeV and 7.06 MeV respectively, then the energy of the reaction $Li^7 + p = 2\,[{}_2He^4]$ will be
(a) 17.28 MeV (b) 39.2 MeV
(c) 28.24 MeV (d) 1.46 MeV

6. Deuteron is a bound state of a neutron and a proton with a binding energy $B = 2.2$ MeV. A γ-ray of energy E is aimed at a deuteron nucleus to try to break it into a (neutron + proton) such that the n and p move in the direction of the incident γ-ray. Where $E \neq B$. Then, calculate how much bigger that B must E be for such a process to happen? **[NCERT Exemplar]**
(a) $\frac{B^2}{2mc^2}$ (b) $\frac{B}{2mc^2}$
(c) $\frac{B^2}{4mc^2}$ (d) $\frac{3B}{4mc^2}$

7. Two nucleons are at a separation of 1 fm. The net force between them is F_1 if both are neutrons, F_2 if both are protons, and F_3 if one is a proton and the other is a neutron.
(a) $F_1 > F_2 > F_3$ (b) $F_2 > F_1 > F_3$
(c) $F_1 = F_3 > F_2$ (d) $F_1 = F_2 > F_3$

8. Are the nucleons fundamental particles or do they consist of still smaller parts? One way to find out is to probe a nucleon just the Rutherford probed an atom. What should be the kinetic energy of an electron for it to be able to probe a nucleon? Assume the diameter of a nucleon to be approximately 10^{-15} m. **[NCERT Exemplar]**
(a) 10^9 eV
(b) 10^{19} eV
(c) 10^{29} eV
(d) 10^{11} eV

9. A radioactive sample decays by two different processes. Half-life for the first process is t_1 and for the second process is t_2. The effective half-life is
(a) $t_1 + t_2$ (b) $t_1 - t_2$
(c) $(t_1 + t_2)/2$ (d) $\frac{t_1 t_2}{t_1 + t_2}$

10. Consider $x \xrightarrow{-\alpha} y \xrightarrow{-\alpha} z$, where half-lives of x and y are Z year and one. month. The ratio of atoms of x and y when transient equilibrium $[T_{1/2}(x) > T_{1/2}(y)]$ has been established is
(a) 1 : 22 (b) 1 : 26
(c) 26 : 1 (d) 23 : 1

11. A radioactive nucleus undergoes a series of decays according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, these numbers of A_4 are
(a) 172, 69 (b) 177, 69
(c) 171, 69 (d) 172, 68

12. A piece of wood from the ruins of an ancient building was found to have a ^{14}C activity of 12 disintegrations per minute per gram of its carbon content. The ^{14}C activity of the living wood is 16 disintegrations per minute per gram. How long ago did the tree, from which the wooden sample came, die? Given half-life of ^{14}C is 5760 yr? **[NCERT Exemplar]**

(a) 2391 yr (b) 2300 yr
(c) 2250 yr (d) 2261 yr

13. A sample of a radioactive element has a mass of 10 g at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives is

(a) 1.35 g (b) 2.50 g
(c) 3.70 g (d) 6.30 g

14. If $N_{t_1} = N_0 e^{-\lambda t_1}$, then the number of atoms delayed during the time interval from t_1 and t_2 $(t_1 > t_2)$, will be

(a) $N_{t_1} - N_{t_2} = N_0 [e^{-\lambda t_1} - e^{-\lambda t_2}]$
(b) $N_{t_2} - N_{t_1} = N_0 [e^{-\lambda t_2} - e^{-\lambda t_1}]$
(c) $N_{t_2} - N_{t_1} = N_0 [e^{-\lambda t_2} - e^{\lambda t_1}]$
(d) None of the above

15. On fission of one nucleus of U^{235}, the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 kW. Number of nuclei fissioned per second in the reactor is

(a) 3.125×10^{16} (b) 6.25×10^{10}
(c) 3.125×10^{32} (d) 6.25×10^{20}

16. The sequence of decay of a radioactive nucleus is $N_0 \xrightarrow{\alpha} N_1 \xrightarrow{\beta} N_2 \xrightarrow{\alpha} N_3 \xrightarrow{\alpha} N_4$. If Nucleon number and atomic number of N_2 are 176 and 71 respectively, then what are their values for N and N_4 ?

(a) 168, 67 and 180, 71
(b) 67, 168 and 180, 72
(c) 180, 67 and 72, 180
(d) None of these

17. In a nuclear reactor, U^{235} undergoes fission liberating 200 MeV of energy per fission. The reactor has 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, the total mass of uranium required is (Avogadro's number $= 6.02 \times 10^{26}$/K-mol, 1 eV= 1.6×10^{-19} J)

(a) 3.84×10^4 kg (b) 9.28×10^6 kg
(c) 3.84×10^8 kg (d) 9.28×10^4 kg

Directions *(Q. Nos. 18 to 22) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

18. Statement I A certain radioactive substance has a half-life period of 30 days. Its disintegration constant is 0.0231 day^{-1}.

Statement II Decay constant varies inversely as half-life.

19. Statement I Half-life of a certain radioactive element is 100 days. After 200 days, fraction left undecayed will be 50%.

Statement II $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, where symbols have usual meaning.

20. Statement I In a decay, daughter nucleus shifts two places to the left from the parent nucleus.

Statement II An alpha particle carries four units of mass.

21. Statement I Energy is released in nuclear fission.

Statement II Total binding energy of the fission fragments is larger than the total binding energy of the parent nucleus.

22. Statement I If half-life period and the mean-life of a radioactive element are denoted by T and T_m respectively then $T < T_m$.

Statement II Mean-life $= \frac{1}{\text{decay constant}}$

Directions *(Q. Nos. 23 to 25) We have two radioactive nuclei A and B. Nucleus A converts into C after emitting two α-particles and three β-particles. Nucleus B converts into C after emitting one α-particle and five β-particles. At time t = 0, nuclei of A are $4N_0$ and that of B are N_0. Half-life of A (into the conversion of C) is 1 min and that of B is 2 min. Initially the number of nuclei of C are zero.*

23. If atomic numbers and mass numbers of A and B are Z_1, Z_2, A_1 and A_2 respectively. Then,

(a) $Z_1 - Z_2 = 6$ (b) $A_1 - A_2 = 4$
(c) Both (a) and (b) are correct (d) Both (a) and (b) are wrong

24. What are the number of nuclei of C, when the number of nuclei of A and B become equal?

(a) $2N_0$ (b) $3N_0$
(c) $\frac{9N_0}{2}$ (d) $\frac{5N_0}{2}$

25. At what time rate, are the disintegrations of A and B equal?

(a) 4 min (b) 6 min
(c) 8 min (d) 2 min

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26. When uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as $_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3X + Q$ (energy) where three particles names X are produced and energy Q is released. What is the name of the particle X? **[JEE Main Online 2013]**

(a) electron (b) α-particle
(c) neutron (d) neutrino

27. Assume that a neutron breaks into a proton and an electron. The energy released during this process is (mass of neutron $= 1.6725 \times 10^{-27}$ kg, mass of proton $= 1.6725 \times 10^{-27}$ kg, mass of electron $= 9 \times 10^{-31}$ kg) **[AIEEE 2012]**

(a) 0.9 MeV (b) 7.10 MeV (c) 6.30 MeV (d) 5.4 MeV

28. The half-life of a radioactive substance is 20 min. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is **[AIEEE 2011]**

(a) 14 min (b) 20 min (c) 28 min (d) 7 min

29. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then **[AIEEE 2010]**

(a) $E_2 = 2E_1$ (b) $E_1 > E_2$ (c) $E_2 > E_1$ (d) $E_1 = 2E_2$

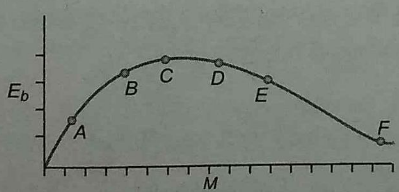
30. The speed of daughter nuclei is **[AIEEE 2010]**

(a) $c\frac{\Delta m}{M + \Delta m}$ (b) $c\sqrt{\frac{2\Delta m}{M}}$ (c) $c\sqrt{\frac{\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M + \Delta m}}$

31. A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **[AIEEE 2010]**

(a) $\frac{A - Z - 8}{Z - 4}$ (b) $\frac{A - Z - 4}{Z - 8}$
(c) $\frac{A - Z - 12}{Z - 4}$ (d) $\frac{A - Z - 4}{Z - 2}$

32. The below is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei.



Consider four reactions **[AIEEE 2009]**

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$

where ε is the energy released? In which reactions ε is positive?

(a) (i) and (iv) (b) (i) and (iii)
(c) (ii) and (iv) (d) (ii) and (iii)

Directions *Question number 30 is Assertion-Reason ty questions. This question contains two statements : Statement (Assertion) and Statement II (Reason). This question also has fo alternative choices, only one of which is the correct answer. Yo have to select the correct choice.* **[AIEEE 200**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false; Statement II is th correct explation for Statement I
(c) Statement I is true, Statement II is true; Statement II is not th correct explanation for Statement I
(d) Statement I is true, Statement II is false

33. Statement I Energy is released when heavy nuclei underg fission or light nuclei undergo fusion.

Statement II For heavy nuclei, binding energy for pe nucleon increases with increasing Z while for light nuclei. decreases with increasing Z.

34. A radioactive sample S_1 having an activity of 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 can be **[AIEEE 2008]**

(a) 20 yr and 5 yr, respectively
(b) 20 yr and 10 yr, respectively
(c) 10 yr each
(d) 5 yr each

35. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially, they have the same number of atoms. Then, **[AIEEE 2007]**

(a) X will decay faster than Y
(b) Y will decay faster than X
(c) Y and X have same decay rate initially
(d) X and Y decay at same rate always

36. If M_o is the mass of an oxygen isotope $_8O^{17}$, M_p and M_n are the masses of a proton and a neutron, respectively, the nuclear binding energy of the isotope is **[AIEEE 2007]**

(a) $(M_o - 8M_p)c^2$ (b) $(M_o - 8M_p - 9M_n)c^2$
(c) M_oc^2 (d) $(M_o - 17M_n)c^2$

37. Half-life of a radioactive substance A is 4 days. The probability that a nucleus will decay in two half-lives is **[AIEEE 2006]**

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 1

Answers

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (d)
11. (a)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (b)
21. (a)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (b)
31. (b)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (b)	**36.** (b)	**37.** (b)			

Hints & Solutions

1. Momentum $p = \frac{E}{c} = \frac{0.129}{c}$

KE (recoil of nucleus) $= \frac{p^2}{2m} = \frac{(0.129)^2}{2 \times c^2 m}$

$$= \frac{(0.129)^2}{2 \times 931 \times 191} = 4.679 \times 10^{-8} \text{ MeV}$$

It is the first order approximation.

$$\frac{\delta E_x}{E_\gamma} = \frac{4.679 \times 10^{-5} \text{ keV}}{129 \text{ keV}} = 3.627 \times 10^{-7}$$

2. The distance of closest approach, $r_0 = \frac{(Ze)(2e)}{4\pi\varepsilon_0(E)}$

$$= \frac{2 \times 92 (1.6 \times 10^{-19})^2 \times 9 \times 10^9}{5 \times 1.6 \times 10^{-13}} = 10^{-12} \text{ cm}$$

3. The required time, $t = \frac{\text{nuclear distance}}{\text{velocity}}$

$$= \frac{10^{-14}}{3 \times 10^8 / 100} \approx 10^{-20} \text{ s}$$

4. As, $-\frac{dN}{dt} = \lambda N$, $N = \frac{3.7 \times 10^{10} \times 3.6 \times 10^4}{0.693}$

Also, $N = N_0 e^{-\lambda t}$, $\Delta N = N_1 - N_2$

$$\Delta N = -(N_0 e^{-\lambda \times 10 \times 3600} - N_0 e^{-\lambda \times 9 \times 3600})$$

$$= \frac{3.7 \times 10^{14} \times 3.6}{0.693} [0.535 - 0.5] = 6.91 \times 10^{13}$$

5. The reaction is ${}_3\text{Li}^7 + {}_1p^1 \longrightarrow 2\,({}_2\text{He}^4)$

$\therefore$ $E_p = 2E\,({}_2\text{He}^4) - E\,(\text{Li}) = 2\,(4 \times 7.06) - 7 \times 5.6$

$= 56.48 - 39.2 = 17.28$ MeV

6. Binding energy $B = 2.2$ MeV

From the energy conservation law,

$$E - B = K_n + K_p = \frac{p_n^2}{2m} + \frac{p_p^2}{2m} \quad \ldots\text{(i)}$$

From conservation of momentum

$$p_n + p_p = \frac{E}{c} \quad \ldots\text{(ii)}$$

As $E = B$, Eq. (i) $\quad p_n^2 + p_p^2 = 0$

It only happen if $\quad p_n = p_p = 0$

So, the Eq. (ii) cannot satisfy and the process cannot take place.

Let $E = B + X$, where $X << B$ for the process to take place

Put value of p_n from Eq. (ii) in Eq. (i), we get

$$X = \frac{\left(\frac{E}{C} - p_p\right)^2}{2m} + \frac{p_p^2}{2m}$$

or $$2p_p^2 - \frac{2Ep_p}{c} + \frac{E^2}{c^2} - 2mX = 0$$

Using the formula of quadratic equation, we get

$$p_p = \frac{\frac{2E}{c} + \sqrt{\frac{4E^2}{c^2} - 8\left(\frac{E^2}{c^2} - 2mX\right)}}{4}$$

For the real value p_p, the discriminant is positive

$$\frac{4E^2}{c^2} = 8\left(\frac{E_2}{c_2} - 2mX\right)$$

$$16m\,X = \frac{4E^2}{c^2} = \frac{E^2}{4mc^2} \simeq \frac{B^2}{4mc^2}$$

7. Nuclear force of attraction between any two nucleons (n-n, p-p, p-n) is same. The difference comes up only due to electrostatic force of repulsion between two protons.

$\therefore$ $\quad F_1 = F_3 \neq F_2$

As, $\quad F_2 < F_3$ or F_1

$\therefore$ $\quad F_1 = F_3 > F_2$

8. Each particle (neutron and proton) present inside the nucleus is called a **nucleon**.

Let λ be the wavelength $\lambda = 10^{-15}$ m.

To detect separate parts inside a nucleon, the electron must have wavelength less than 10^{-15} m.

We know that $\lambda = \frac{h}{p}$ and KE = PE $\quad \ldots$(i)

$$\text{Energy} = \frac{hc}{\lambda} \quad \ldots\text{(ii)}$$

From Eq. (i) and Eq. (ii), we get

Kinetic energy of electron,

$$\text{KE} = \text{PE} - \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{10^{-15} \times 1.6 \times 10^{-19}} \text{ eV} = 10^9 \text{ eV}$$

9. As $\lambda = \lambda_1 + \lambda_2 \Rightarrow \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} = \frac{t_2 + t_1}{t_1 t_2}$

or $t = \frac{t_1 t_2}{t_1 + t_2}$

10. $N_2 = \frac{\lambda_1 N_1^0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$

When $(T_{1/2})_1 > (T_{1/2})_2$ at transient equilibrium

$$\lambda_1 < \lambda_2$$

$$e^{-\lambda_2 t} << e^{-\lambda_1 t}$$

$$\therefore \quad N_2 = \frac{\lambda_1 N_1^0 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} = \frac{\lambda_1 N_1}{\lambda_2 - \lambda_1}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\frac{0.693}{1} - \frac{0.693}{2 \times 12}}{\frac{0.693}{2 \times 12}} = \frac{23}{1}$$

11. Decrease in mass number due to two α -particles $= 2 \times 4 = 8$

Decrease in charge number due to loss of two α-particles = 4
Increase in charge number due to loss of one β -particle = 1
Net decrease in charge number $= 4 - 1 = 3$

12. Given, $R = 12$ dis/min per g, $R_0 = 16$ dis/min per g

$T_{1/2} = 5760$ yr

Let t be the time span of the tree.

According to radioactive decay law, $R = R_0 e^{-\lambda t}$

or $\frac{R}{R_0} = e^{-\lambda t}$ or $e^{\lambda t} = \frac{R_0}{R}$

Taking log on both the sides $\lambda t \log_e e = \log_e \frac{R_0}{R}$

$$\lambda t = \left(\log_{10} \frac{16}{12}\right) \times 2.303$$

$$t = \frac{2.303 (\log 4 - \log 3)}{\lambda}$$

$$= \frac{2.303\,(0.6020 - 4771) \times 5760}{0.6931} \qquad \left(\because \lambda = \frac{0.6931}{T_{1/2}}\right)$$

$= 2391.20$ yr

13. The relation of mean life and decay constant is,

$$t = 2\tau = \frac{2}{\lambda}, \text{ where } \tau = \frac{1}{\lambda}$$

Then we get from the equation, $m = m_0 e^{-\lambda t}$

$$\Rightarrow \quad m = 10 \times e^{-\lambda \times 2/\lambda} = 10 \times e^{-2} = 10 \times 0.135 = 1.35 \text{ g}$$

14. Since, $N_{t_1} = N_0 e^{-\lambda t_1}$ and $N_{t_2} = N_0 e^{-\lambda t_2}$

Then the number of atoms decayed during the time interval t_1 to t_2 is $= N_{t_1} - N_{t_2} = N_0 [e^{-\lambda t_1} - e^{-\lambda t_2}]$

15. Power received from the reactor is

$$P = 1000 \text{ kW} = 1000 \times 1000 = 10^6 \text{Js}^{-1}$$

Also, $1 \text{MeV} = 1.6 \times 10^{-13}$ J

Number of nuclei fissioned per second $= \frac{10^6}{200 \times 1.6 \times 10^{-13}}$
$= 3.125 \times 10^{16} \text{s}^{-1}$

16. As mass number of each α-particle is 4 units and its charge is 2 units, therefore for N_4

$A = 176 - 8 = 168$ and $Z = 71 - 4 = 67$

Now the charge of β is –1 and its mass number is zero

so, $A = 176 + 0 + 4 = 180$ and $Z = 71 - 2 + 2 = 71$

17. The reactor generates 1000 MW power, that is, it generates energy at the rate of $1000 \times 10^6 \text{ W} = 10^9 \text{ Js}^{-1}$

Total energy generated in 10 yr is

$$E = (10^9 \text{ Js}^{-1}) \times 10 \times 365 \times 24 \times 60 \times 60)$$

$$= 3.15 \times 10^{17} \text{ J} = \frac{3.15 \times 10^{17}}{1.6 \times 10^{-13}} = 1.97 \times 10^{30} \text{ MeV}$$

In the reactor 200 MeV energy is liberated in the fission of nucleus of U^{235} atom.

$\therefore$ Total number of U^{235} atoms required is

$$\frac{1.97 \times 10^{30}}{200} = 0.985 \times 10^{28}$$

1 kmol that is 235 kg of U^{235} has 6.02×10^{26} atoms

Therefore, total mass of U^{235} having 0.985×10^{28} atoms is

$$\frac{235}{6.02 \times 10^{26}} \times (0.985 \times 10^{28}) = 3.84 \times 10^3 \text{ kg}$$

Since, efficiency of reactor is 10%, actual mass of U^{235} required is $(3.84 \times 10^3) \times \frac{100}{10} = 3.84 \times 10^4$ kg

18. From the relation, $\lambda = \frac{0.6931}{T} = \frac{0.6931}{30} = 0.0231 \text{ day}^{-1}$

19. Number of half-lives $n = \frac{t}{T} = \frac{200}{100} = 2$

The fraction left undecayed is given by

$$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 25\%$$

20. We know that an α-particle carries 2 units of positive charge and four units of mass. On α-decay, charge number of parent nucleus decreases by 2 units. As classification or grouping of elements is based on charge number, hence daughter nucleus shifts two places to the left from the parent nucleus.

21. In a nuclear fission, when a bigger nucleus is fissioned into two light weight nuclei, then due to mass defect some energy is released. According to concept of binding energy, fission can occur because the total mass energy will decrease; that is ΔE_{bn} (binding energy) will increase.

We see that for high mass nucleide ($A = 240$), the binding energy per nucleon is about 7.6 MeV/nucleon. For the middle weight nucleides ($A = 120$), it is about 8.5 MeV/nucleon. Thus, binding energy of fission fragments is larger than the total binding energy of the parent nucleus.

22. We know that half-life period T and decay constant λ are related by the equation.

$$T = \frac{0.6931}{\lambda} \quad \ldots(i)$$

While mean-life T_m is related with λ by the equation

$$T_m = \frac{1}{\lambda} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get $T = 0.6931 T_m$ or $T < T_m$

23. $Z_1 - 2 \times 2 + 3 \times 1 = Z_2 - 2 \times 1 + 5 \times 1 = Z_c$

$\therefore$ $Z_1 - Z_2 = 4$ and $A_1 - 4 \times 2 = A_2 - 1 \times 4 = A_c$

$\therefore$ $A_1 - A_2 = 4$

24. $A : 4N_0 \xrightarrow{1 \text{ min}} 2N_0 \xrightarrow{1 \text{ min}}$

$N_0 \xrightarrow{1 \text{ min}} \frac{N_0}{2} \xrightarrow{1 \text{ min}} \frac{N_0}{4}$

$B : N_0 \xrightarrow{2 \text{ min}} \frac{N_0}{4} \xrightarrow{2 \text{ min}} \frac{N_0}{4}$

After 4 min $N_A = N_B = \frac{N_0}{4}$

$\therefore$ $N_c = (4N_0 + N_0) - \left(\frac{N_0}{4} + \frac{N_0}{4}\right) = \frac{9N_0}{2}$

25. Given, $R_A = R_B$

$\therefore$ $\lambda_A N_A = \lambda_B N_B$

or $\left(\frac{\ln 2}{T_A}\right)(4N_0\, e - \lambda_a t) = (N_0)\left(\frac{\ln 2}{T_B}\right) e - \lambda_b t$

or $e^{(\lambda a - \lambda b)\, t} = 8$

$\therefore$ $(\lambda_A - \lambda_B)t = \ln 8 = 3(\ln 2)$

$\therefore$ $\left(\frac{\ln 2}{1} - \frac{\ln 2}{2}\right)t = 3 \ln(2)$ or $t = 6$ min

26. The fission of ${}_{92}U^{235}$ is represented by

${}_{92}U^{235} + {}_0 n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q$

The name of the particle X is neutron $({}_0n^1)$.

27. According to given data, mass of neutron and proton are equal which do not permit the breaking up of neutron and proton. But if we take standard mass of neutron as 1.6750×10^{-27} kg, then Energy released = mass defect $\times c^2$

$= (m_n - m_p - m_e) \times c^2$

$= \frac{(1.6750 \times 10^{-27} - 1.6725 \times 10^{-27} - 9 \times 10^{-31})}{1.66 \times 10^{-27}} \times 931.5$ MeV

$= \frac{16 \times 10^{-31}}{1.66 \times 10^{-27}} \times 931.5 \text{ MeV} = \frac{1.6 \times 0.9315}{1.66}$ MeV ≈ 0.9 MeV

28. $N_1 = N_0 - \frac{1}{3}N_0 = \frac{2}{3}N_0,\ N_2 = N_0 - \frac{2}{3}N_0 = \frac{1}{3}N_0$

$$\frac{N_1}{N_2} = \left(\frac{1}{2}\right)^n \Rightarrow n = 1$$

$\therefore t_2 - t_1 =$ one half-life $= 20$ min

29. After decay, the daughter nuclei will be more stable hence, binding energy per nucleon will be more than that of their parent nucleus.

30. Conserving the momentum $\Rightarrow 0 = \frac{M}{2}v_1 - \frac{M}{2}v_2$

$$v_1 = v_2 \quad \ldots(i)$$

$$\Delta mc^2 = \frac{1}{2}\frac{M}{2}v_1^2 + \frac{1}{2}\frac{M}{2}v_2^2 \quad \ldots(ii)$$

$$\Delta mc^2 = \frac{M}{2}v_1^2 \Rightarrow \frac{2\Delta mc^2}{M} = v_1^2$$

$$v_1 = c\sqrt{\frac{2\Delta m}{M}}$$

31. In positive β-decay a proton is transformed into a neutron and a positron is emitted.

$$p^+ \longrightarrow n^0 + e^+$$

Number of neutrons initially was $A - Z$

Number of neutrons after decay is $(A-Z) - 3 \times 2$ (due to α particles) $+ 2 \times 1$ (due to positive β-decay)

The number of protons will reduce by 8

[as 3×2 (due to α-particles) + 2 (due to positive β - decay)]

Hence, atomic number reduces by 8.

So, the ratio of number of neutrons to that of protons

$$= \frac{A - Z - 4}{Z - 8}$$

32. 1st reaction is fusion and 4th reaction is fission.

33. Here, statement I is correct and statement II is wrong can be directly concluded from binding energy nucleon curve.

34. Activity of $S_1 = \frac{1}{2}$ (activity of S_2)

or $\lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$ or $\frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$

or $\frac{T_1}{T_2} = \frac{2N_1}{N_2}$ $\quad \left(T = \text{half-life} = \frac{\ln 2}{\lambda}\right)$

Given, $N_1 = 2N_2$

$\therefore$ $\frac{T_1}{T_2} = 4$

35. According to question $T_{1/2}(X) = \tau(\lambda) \Rightarrow \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$

or $\lambda_Y = \frac{\lambda_X}{0.693} \Rightarrow \lambda_Y > \lambda_X$

So, Y will decay faster than X.

36. Binding energy BE $= (M_{\text{nucleus}} - M_{\text{nucleons}})c^2$

$$= (M_o - 8M_p - 9M_n)c^2$$

37. After two half-lives 1/4th fraction of nuclei will remain undecayed. Or, 3/4th fraction will decay. Hence, the probability that a nucleus decays in two half-lives is 3/4.

Day 34

Electronic Devices

Day 34

Outlines ...

- Semiconductor Physics
- Semiconductor Diode
- Diode as a Rectifier
- Special Purpose-Diodes
- Transistor

Semiconductor Physics

Certain substances like silicon, germanium, etc., offer resistances to the flow of charge which lies in between that of offered by conductors and insulators. They are called ***semiconductors.***

Resistivity alone does not determine the semiconductor nature of the substances. *Some of the typical properties of the semiconductors are given below*

(i) They have negative temperature coefficient of resistance.

(ii) Their conductivity changes appreciably when a suitable trivalent or pentavalent impurity is added to them.

A pure semiconductor in which no impurity of any sort has been mixed, is called **intrinsic semiconductor**. Germanium ($E_g = 0.72$ eV) and silicon ($E_g = 1.1$ eV) are intrinsic semiconductors.

At 0K these behave as 100% insulators. But at any other temperature they have thermally generated charge carriers and thus behave as semiconductor.

In an **intrinsic semiconductor** the number of free electrons in conduction band n_e is exactly equal to the number of holes n_h in valence band. Thus, $n_e = n_h = n_i$ where n_i is called the number density of intrinsic carriers.

Conductivity of an intrinsic semiconductor is $\sigma = e(n_e \cdot \mu_e + n_h \cdot \mu_h)$

where n_e is free electron density, n_h the hole density and μ_e and μ_h their respective mobilities.

Electrical conductivity of pure semiconductor is very small. To increase the conductivity of a pure semiconducting material it is doped with a controlled quantity (1 in 10^5 or 10^6) of suitable impurity. Such a doped semiconductor is called an **extrinsic semiconductor.**

n-type Semiconductor

To prepare an n-type semiconductor a pentavalent impurity, e.g., P, As, Sb is used as a dopant with Si or Ge.

Such an impurity is called **donor impurity** because each dopant atom provides one free electron.

In n-type semiconductor $n_e >> n_h$, *i.e.*, electrons are majority charge carriers and the holes are minority charge carriers such that $n_e \cdot n_h = n_i^2$.

A n-type semiconductor is electrically neutral and is not negatively charged.

p-type Semiconductor

To prepare a p-type semiconductor a trivalent impurity, e.g., B, Al, In, Ga, etc., is used as a dopant with Si or Ge. Such an impurity is called **acceptor impurity** as each impurity atom wants to accept an electron from the crystal lattice. Thus, effectively each dopant atom provides a **hole**.

In p-type semiconductor $n_h >> n_e$, *i.e.*, holes are majority charge carriers and electrons minority charge carriers such that $n_h \cdot n_e = n_i^2$ A p-type semiconductor is electrically neutral and is not positively charged.

The number of free electrons in a semiconductor varies with temperature as $T^{3/2}$.

Semiconductor Diode

*A **p-n junction** is obtained by joining a small p-type crystal with a small n-type crystal without employing any other binding material in between them.*

Whenever a p-n junction is formed, electrons from n-region diffuse through the junction into p-region and the holes from p-region diffuse into n-region.

As a result of which neutrality of both n and p-regions is disturbed and a thin layer of immobile negative charged ions appear near the junction in the p-crystal and a layer of positive ions appear near the junction in n-crystal. This layer containing immobile ions is called **depletion layer**. The thickness of depletion layer is approximately of the order of 10^{-6} m.

The potential difference developed across the p-n junction due to diffusion of electrons and holes is called the **potential barrier** V_b (or emf of fictitious battery). For germanium diode barrier potential is 0.3 V but for Si diode its value is 0.7 V. The barrier electric field developed due to it is of the order of 10^5 Vm^{-1}.

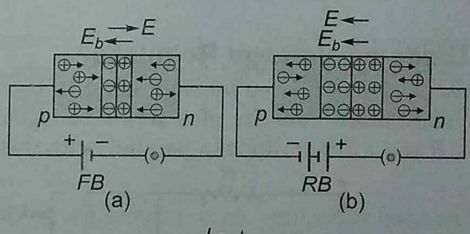
Mobility of Charge Carriers

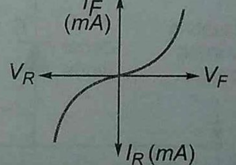
The mobility of a charge carrier is defined as the velocity gained by it per unit electric field. That is $\mu = V_d/E$

I-V Characteristics in Forward and Reverse Bias

When we join an external potential source such that p-side of p-n junction is joined to positve of voltage source and n-side to negative of source, the junction is said to be **forward biased** and applied electric field E opposes the barrier electric field E_b. As a result width of depletion layer is reduced and on applying a voltage $V > V_b$, a forward current begins to flow. Resistance offered by p-n junction in forward bias is small (about 10-50 Ω).

If connections of potential source are reversed [Fig (b)] the junction is said to be reverse biased and in this case E and E_b, being in same direction, are added up. So, the depletion layer broadens and potential barrier is fortified. Consequently, an extremely small leakage current flows across the junction due to minority charge carriers and junction resistance is extremely high ($\simeq 10^5$ Ω).





(c)

For a sufficiently high reverse bias voltage (25 V or even more) the reverse current suddenly increases. This voltage is called **Zener voltage** or **breakdown voltage** or **avalanche voltage**.

- *A p-n junction behaves as a voltage controlled switch. In forward bias, it acts like ON switch and in reverse bias as OFF switch.*
- *The p-n junction can be presumed as a capacitor, in which the depletion layer acts as dielectric.*

Diode as a Rectifier

Junction diode allows current to pass only when it is forward biased. So, if an alternating voltage is applied across a diode, the current flows only in that part of the cycle, when the diode is forward biased. This property is used to rectify alternating voltages and the circuit used for this purpose is called a rectifier, and the process is known as rectification.

In **half-wave rectifier** only one diode is used. In it no current flow takes place and no output signal is obtained.

Even during one half cycle the output obtained is a mixture of DC and AC.

$$\text{The ripple factor} = \frac{\text{Effective AC component of voltage}}{\text{Effective DC component of voltage}}$$

$$= 1.21 \text{ or } 121\%$$

The **average output** in one cycle is

$$V_{DC} = \frac{V_0}{\pi} \text{ and } I_{DC} = \frac{I_0}{\pi}$$

The **ripple frequency** for half-wave rectifier is same as that of **AC input signal**. In **full-wave rectifier** two p-n junction diodes have been joined in complimentary modes. In this rectifier, we obtain a continuous unidirectional current through the load resistor R_L.

Ripple factor in full-wave rectifier $\frac{V_{AC}}{V_{DC}} = 0.48 = 48\%$

The **average output** in one cycle is

$$V_{DC} = \frac{2}{\pi} V_0 \Rightarrow I_{DC} = \frac{2}{\pi} I_0$$

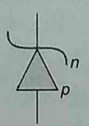
- *The ripple frequency for full-wave rectifier is twice that of AC input signal.*
- *The peak inverse voltage for half wave rectifier is E_0 and that for full wave rectifier is $2E_0$, where E_0 is the peak voltage of input AC.*

Special Purpose Diodes

The special purpose diode that perform many different functions. e.g., diodes are used to regulate voltage (zener diodes), to produce light, Light Emitting Diode (LED) are given below.

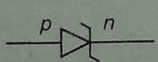
Zener Diode

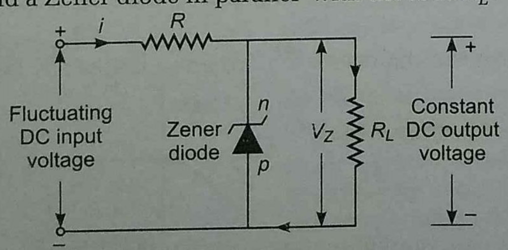
It is a highly doped p-n junction diode which is not damaged by high reverse current. It is always used in reverse bias in break down voltage region and is chiefly used as a voltage regulator.



Zener Diode as Voltage Regulator

The following circuit is used for stabilizing voltage across a load R_L. The circuit consists of a series voltage-dropping resistance R and a Zener diode in parallel with the load R_L.





The Zener diode is selected with Zener voltage V_z equal to the voltage desired across the load. The fluctuating DC input voltage may be the DC output of a rectifier. Whenever the input voltage increases, the excess voltage appears as increased voltage across the resistance R. This causes an increase in the input current i. This increase is taken away by the Zener diode while the current through the load and hence the voltage across it remains constant at V_z. Likewise a decrease in the input voltage causes a decrease in the input current i. The current through the diode decreases correspondingly, again maintaining the load current.

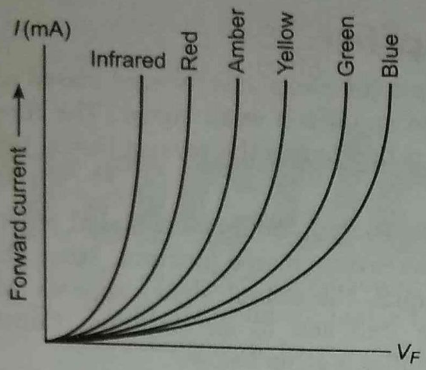
Light Emitting Diode (LED)

It is a specially designed diode made of GaAsP, GaP, etc. When used in forward biased, it emits characteristic, almost monochromatic light.

In reverse biased it works like a normal diode.

I-V Characteristics of LED

LEDs are current dependent devices with its forward voltage drop (V_F) depending on the forward biased LED current. Light emitting diode I-V characteristics as given below



Before LED can emit any form of light it needs a current to flow through it, as it is a current dependent device with their light output intensity being directly proportional to the forward current flowing through the LED.

Photo Diode

It is a special diode used in reverse bias which conducts only when light of suitable wavelengths is incident on the junction of diode. The energy of incident light photon must be greater than the band gap of semiconductor. Materials used are Cds, Se, Zns

Solar Cell

It is a special *p-n* junction in which one of the semiconductors is made extremely thin so that solar radiation falling on it reaches junction of diode without any absorption. A solar cell directly converts solar energy into electrical energy. Popularly used solar cells, Ni-cd, PbS cell

Transistor

A transistor is a combination of two p-n junctions joined in series. A junction transistor is known as ***Bipolar Junction Transistor*** *(BJT). It is a three terminal device.*

Transistors are of two types— (i) n-p-n transistor, (ii) p-n-p transistor

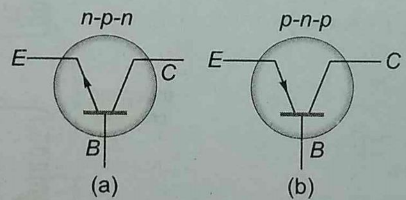
A transistor has three regions

(i) An **emitter** (*E*), which is most heavily doped and is of moderate size. It supplies large number of charge carriers, which are free electrons in a *n-p-n* transistor and holes in a *p-n-p* transistor.

(ii) A **base** (*B*), which is very lightly doped and is very thin (thickness $\simeq 10^{-5}$ m).

(iii) A **collector** (*C*), which is moderately doped and is thickest.

A transistor is symbolically represented as shown in figures.



For proper functioning of a transistor, the **emitter-base junction is forward biased** but the **collector-base junction is reverse biased.**

In a *n-p-n* transistor electrons flow from emitter towards the base and constitute a current I_E. Due to larger reverse bias at base-collector junction, most of these electrons further pass into the collector, constituting a collector current I_C. But a small percentage of electrons (less than 5%) may combine with holes present in base. These electrons constitute a base current I_B. It is self evident that

$$I_E = I_C + I_B$$

Action of *p-n-p* transistor is also same but with one difference that holes are moving from emitter to base and then to collector.

A transistor can be connected in either of the following three configurations

(i) Common Emitter (CE) configuration

(ii) Common Base (CB) configuration

(iii) Common Collector (CC) configuration. Generally, we prefer common emitter configuration because power gain is maximum in this configuration

Characteristics of a Transistor

In common emitter configuration variation of current on the input side with input voltage (I_E *versus* V_{BE}) is known as the **input characteristics** and the variation in the output current with output voltage (I_C *versus* V_{CE}) is known as **output characteristics**. *From these characteristics, we obtain the values of following parameters.*

- Input resistance

$$r_i = \left|\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE} = \text{constant}}$$

- Output resistance

$$r_o = \left|\frac{\Delta V_{CE}}{\Delta I_C}\right|_{I_B = \text{constant}}$$

- AC current gain

$$\beta = \left|\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE} = \text{constant}}$$

The current gain for common-emitter configuration β *ranges from 20 to 200.*

- Transconductance

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}} = \frac{\beta}{r_i}$$

- *A transistor can be used as an amplifier. The voltage gain of an amplifier will be given by*

$$A_V = \frac{V_o}{V_i} = \beta \cdot \frac{R_C}{R_B}$$

where R_C and R_B are net resistances in collector and base circuits respectively.

- *In common base configuration, AC current gain is defined as*

$$\alpha = \left|\frac{\Delta I_C}{\Delta I_E}\right|_{V_{CE} = \text{constant}}$$

- *Value of* α *is slightly less than 1. In fact* $0.95 \le \alpha \le 1$.
- *Current gains* α *and* β *are correlated as*

$$\beta = \frac{\alpha}{1-\alpha}$$

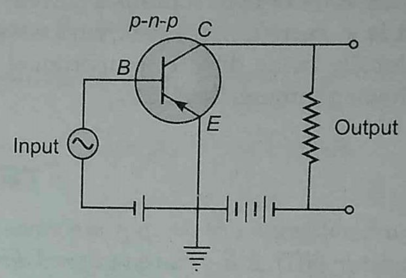
or

$$\alpha = \frac{\beta}{1+\beta}$$

Transistor as an Amplifier

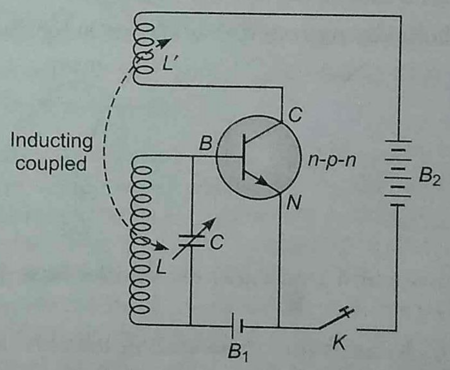
A transistor consisting of two *p-n* junctions, one forward biased and the other reverse biased can be used to amplify a weak signal. The forward biased junction has a low resistance path whereas the reverse biased junction has a high resistance path.

The weak input signal is applied across the forward biased junction and the output signal is taken across the reverse biased junction. Since, the input and output currents are almost equal, the output signal appears with a much higher voltage. The transistor thus acts as an amplifier. Common-emitter configuration of transistor amplifier is given below



Transistor as an Oscillator

An electronic oscillator is a device that generates electrical oscillations of constant amplitude and of a desired frequency, without any external input. The circuit providing such oscillation is known as a tank scillator is using positive feed back.



Some of the properties o fthe oscillator are

1. Oscillator is using positive feedback.
2. To work as an oscillator

   $|A\beta| = 1; \beta \rightarrow$ feedback factor
3. f = frequency of oscillation $= \frac{1}{2\pi} \times \frac{1}{\sqrt{LC}}$

Practice Zone

DAY 34

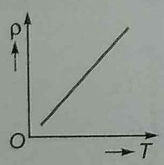
1. If the resistivity of copper is 1.7×10^{-6} Ω–m, then the mobility of electrons in copper, if each atom of copper contributes one free electron for conduction is [the atomic weight of copper is 63.54 and density is 8.96 g/cc

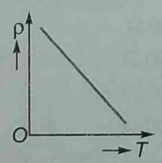
(a) 23.36 cm^2/ Vs (b) 503.03 cm^2/ Vs
(c) 43.25 cm^2/ Vs (d) 88 cm^2/ Vs

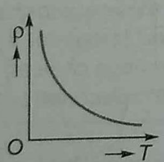
2. The conductivity of a semiconductor increases with increase in temperature because **[NCERT Exemplar]**

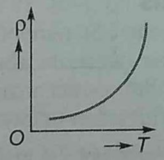
(a) number density of free current carriers increases
(b) relaxation time increases
(c) both number density of carriers and relaxation time increase
(d) number density of current carriers increases, relaxation time decreases but effect of decrease in relaxation time is much less than increase in number density

3. The temperature (T) dependence of resistivity (ρ) of a semiconductor is represented by

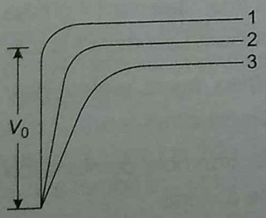
(a) 

(b) 

(c) 

(d) 

4. In figure, V_0 is the potential barrier across a p-n junction, when no battery is connected across the junction **[NCERT Exemplar]**



(a) 1 and 3 both correspond to forward bias of junction
(b) 3 corresponds to forward bias of junction and 1 corresponds to reverse bias of junction
(c) 1 corresponds to forward bias and 3 corresponds to reverse bias of junction
(d) 3 and 1 both correspond to reverse bias of junction

5. In BJT, maximum current flows in which of the following?

(a) Emitter region
(b) Base region
(c) Collector region
(d) Equal in all the regions

6. A tungsten emitter works at 2500 K. To increase the emission current density by 20%, how much change in the work function is required (Given, log 2 = 0.3, log 3 = 0.477)

(a) 0.016 eV (b) 0.039 eV
(c) 2.54 eV (d) 0.254 eV

7. In n-type silicon, which of the following statement is true **[NCERT Exemplar]**

(a) Electrons are majority carriers and trivalent atoms are the dopants
(b Electrons are minority carriers and pentavalent atoms are the dopants
(c) Holes are minority carriers and pentavalent atoms are the dopants
(d) Holes are majority carriers and trivalent atoms are the dopants

8. Application of a forward bias to a p-n junction

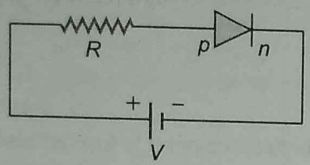
(a) increases the number of donors on the n-side
(b) increases the electric field in the depletion zone
(c) increases the potential difference across the depletion zone
(d) widens the depletion zone

9. In an unbiased p-n junction, holes diffuse from the p-region to n-region because **[NCERT Exemplar]**

(a) free electrons in the n-region attract them
(b) they move across the junction by the potential difference
(c) hole concentration in p-region is more as compared to n-region
(d) All of the above

10. Zener breakdown in a semiconductor diode occurs, when
(a) forward current exceeds certain value
(b) reverse bias exceeds certain value
(c) forward bias exceeds certain value
(d) potential barrier is reduced to zero

11. For the given circuit of *p-n* junction diode, which of the following statements is correct?



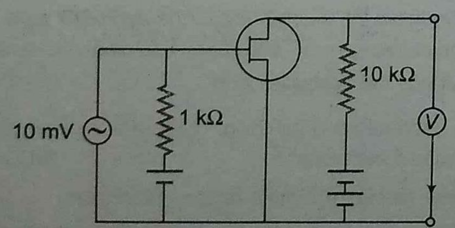
(a) In forward biasing the voltage across R is V
(b) In forward biasing the voltage across R is $2V$
(c) In reverse biasing the voltage across R is V
(d) In reverse biasing the voltage across R is $2V$

12. When forward bias is applied to a *p-n* junction, what happens to the potential barrier V_B and the width of charge depleted region x?
(a) V_B increases, x decreases (b) V_B decreases, x increases
(c) V_B increases, x increases (d) V_B decreases, x decreases

13. Carbon, silicon and germanium atoms have four valence electrons each. Their valence and conduction bands are separated by energy band gaps represented by $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$ respectively. Which one of the following relationship is true in their case?
(a) $(E_g)_C > (E_g)_{Si}$ (b) $(E_g)_C = (E_g)_{Si}$
(c) $(E_g)_C < (E_g)_{Ge}$ (d) $(E_g)_C < (E_g)_{Si}$

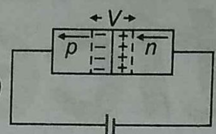
14. What is the plate current in a diode valve under the space charge limited operation, when the plate potential is 60 V ? In a diode valve, the plate current is 320 mA, when the plate potential is 240 V.
(a) 30 mA (b) 20 mA
(c) 40 mA (d) 10 mA

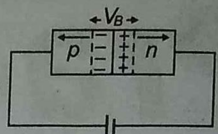
15. In the following a common emitter configuration an *n-p-n* transistor with current gain $\beta = 100$ is used. The output voltage of the amplifier will be

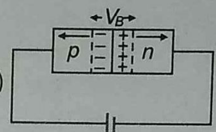


(a) 10 mV (b) 0.1 V
(c) 1.0 V (d) 10 V

16. In the case of forward biasing of *p-n* junction, which one of the following figures correctly depicts the direction of the flow of charge carriers?

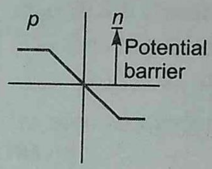
(a)


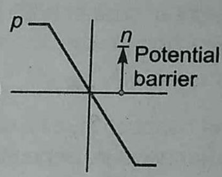
(b)


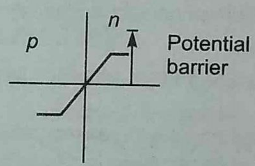
(c)


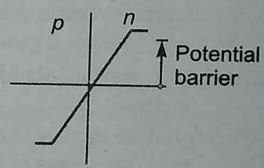
(d) None of these

17. In a forward biased *p-n* junction diode, the potential barrier in the depletion region will be of the form

(a)


(b)


(c)


(d)


18. The input resistance of a common emitter transistor amplifier, if the output resistance is 500 k Ω, the current gain $\alpha = 0.98$ and the power gain is 6.0625×10^6 is
(a) 198 Ω
(b) 300 Ω
(c) 100 Ω
(d) 400 Ω

Directions *(Q. Nos. 19 to 23) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

19. Statement I When base region has larger width, the collector current increases.

Statement II Electron-hole combination in base results in increases of base current.

20. **Statement I** If forward current changes by 1.5 mA when forward voltage in semiconductor diode is changed from 0.5 to 2 V, the forward resistance of diode will be 1Ω.

Statement II The forward resistance is given by $R_f = \frac{\Delta V_f}{\Delta I_f}$

21. **Statement I** A Zener diode is used to get constant voltage at variable current under reverse bias.

Statement II The most popular use of Zener diode is as voltage regulator.

22. **Statement I** In a common emitter transistor amplifier the input current is much less than output current.

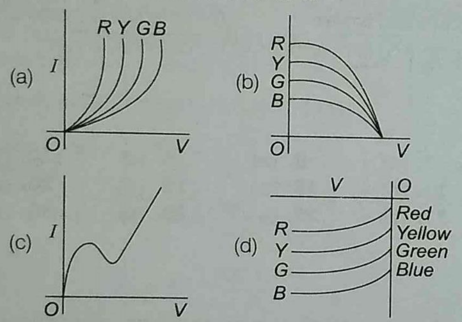
Statement II The common emitter transistor amplifier has very high input impedance.

23. **Statement I** Light emitting diode (LED) emits spontaneous radiation.

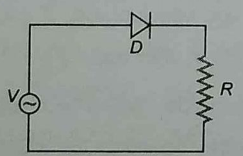
Statement II LED are forward biased *p-n* junctions.

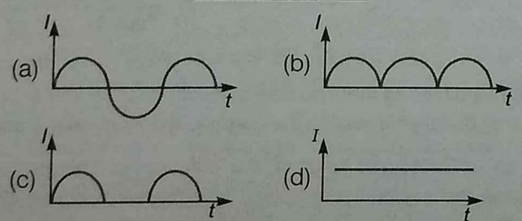
AIEEE & JEE Main Archive

24. The *I-V* characteristic of an LED is **[AIEEE-2012]**



25. A *p-n* junction (*D*) shown in the figure can act as a rectifier. An alternating current source (*V*) is connected in the circuit. **[AIEEE 2009]**





26. A working transistor with its three legs marked *P*, *Q* and *R* is tested using a multimeter. No conduction is found between *P* and *Q*. By conneting the common (negative) terminal of the multimeter to *R* and the other (positive) terminal to *P* or *Q*. Some resistance is seen on the multimeter. Which of the following is the true for the transistor? **[AIEEE 2008]**

(a) It is an *n-p-n* transistor with *R* as base
(b) It is a *p-n-p* transistor with *R* as collector
(c) It is a *p-n-p* transistor with *R* as emitter
(d) It is an *n-p-n* transistor with *R* as collector

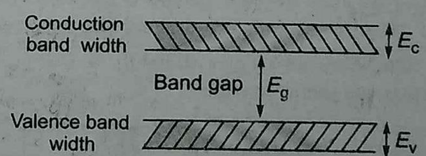
27. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate? **[AIEEE 2007]**

(a) The number of free conduction electrons is significant in C but small in Si and Ge
(b) The number of free conduction electrons is negligibly small in all the three
(c) The number of free electrons for conduction is significant in all the three
(d) The number of free electrons for conduction is significant only in Si and Ge but small in C

28. In a common-base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be **[AIEEE 2006]**

(a) 49 (b) 50 (c) 51 (d) 48

29. If the lattice constant of this semiconductor is decreased, then which of the following is correct? **[AIEEE 2006]**

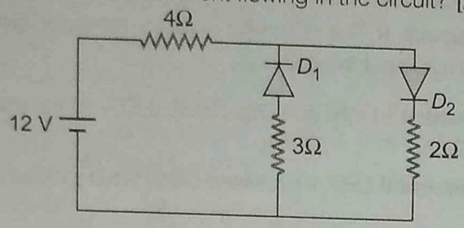


(a) All E_c, E_g, E_v increase
(b) E_c and E_v increase, but E_g decreases
(c) E_c and E_v decrease, but E_g increases
(d) All E_c, E_g, E_v decrease

30. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities? **[AIEEE 2006]**

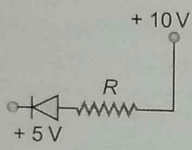
(a) 5/8 (b) 4/5 (c) 5/4 (d) 4/7

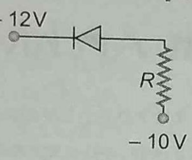
31. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit? **[AIEEE 2006]**

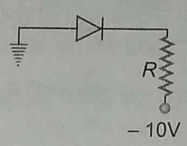


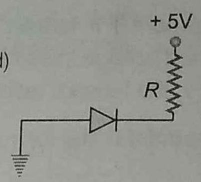
(a) 1.71 A (b) 2.00 A (c) 2.31 A (d) 1.33 A

32. In the following, which one of the diodes is reverse biased? **[AIEEE 2006]**

(a)


(b)


(c)


(d)


33. In a full wave rectifier circuit operating from, 50 Hz mains frequency, the fundamental frequency in the ripple would be **[AIEEE 2005]**

(a) 50 Hz (b) 25 Hz (c) 100 Hz (d) 70.7 Hz

34. For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ ($h_{fe} = 50$ and $h_{oe} = 25\mu AV^{-1}$), the current gain is **[AIEEE 2004]**

(a) – 5.2 (b) – 15.7 (c) – 24.8 (d) – 48.78

35. In the middle of the depletion layer of reverse biased *p-n* junction, the **[AIEEE 2003]**

(a) electric field is zero (b) potential is maximum
(c) electric field is maximum (d) potential is zero

Answers

1. (c)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (a)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (d)	**20.** (d)
21. (a)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (c)
31. (b)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (a)					

Hints & Solutions

1. Mobility of electron $(\mu) = \frac{\sigma}{ne}$ ...(i)

Resistivity $\rho = \frac{1}{\sigma}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\mu = \frac{1}{ne\rho} \quad \text{...(iii)}$$

n = number of free electrons per unit volume

$$n = \frac{N_0 \times d}{\text{atomic weight}} = \frac{6.023 \times 10^{23} \times 8.96}{63.54} = 8.5 \times 10^{22} \quad \text{...(iv)}$$

From Eqs. (iii) and (iv), we get

$$\mu = \frac{1}{8.5 \times 10^{22} \times 1.6 \times 10^{-19} \times 1.7 \times 10^{-6}} = 43.25 \text{cm}^2/\text{Vs}$$

2. Based on the theory discussed we can conclude that when temperature increases, number density of current carriers increases in the semiconductor, relaxation time decreases but effect of relaxation time will be ignored.

3. The resistivity of a semiconductor decreases with increase in temperature exponentially. Hence, option (c) is correct.

4. When *p-n* junction is forward biased, it opposes the potential barrier across junction. When is reverse biased it supports the same.

5. Maximum current flows in emitter region.

6. $$JI = AT^2 e^{-W/kT}$$

Current density and work function, $J' = AT^2 e^{-W'/kT}$

$\frac{J'}{J} = e^{-W'/kT} + W/kT$ and $1.2 = e^{-(W'-W)/kT}$

Taking log on both side $W - W' = kT \log_e 1.20 = 0.039$ eV

7. *n*-type is obtained by doping the Ge or Si with pentavalent atoms. In *n*-type semiconductor, electrons are majority carriers and holes are minority carriers, hence option (c) is correct.

8. In forward biasing more number of electrons enter in *n*-side from battery thereby increasing the number of donors on *n*-side.

9. In an unbiased *p-n* junction, the diffusion of charge carriers across the junction takes place from higher concentration to lower concentration. Thus option (c) is correct.

10. Zener breakdown in a semiconductor diode occurs when reverse bias exceeds certain value, which is known as breakdown or Zener or Avalanche voltage.

11. In forward biasing for an ideal diode resistance of diode is zero and whole resistance in the circuit is *R*. Hence, voltage across *R* is *V*.

12. In a *p-n* junction in forward bias potential barrier V_B as well as the width of charge depleted region *x* decrease.

13. Carbon, silicon and germanium are semiconductors, $(E_g)_C = 5.2$ eV, $(E_g)_{Si} = 1.21$ eV and $(E_g)_{Ge} = 0.75$ eV

Thus, $(E_g)_C > (E_g)_{Ci}$ and $(E_g)_C > (E_g)_{Ge}$

14. Using Child Langmuir Law, $I \propto V^{3/2}$

$320 \propto (240)^{3/2}$ ...(i) $\quad I \propto (60)^{3/2}$ ...(ii)

Dividing Eq. (i) by Eq. (ii),

$$\frac{320}{I} = (4)^{3/2} \Rightarrow I = \frac{320}{8} \Rightarrow I = 40 \text{ mA}$$

15. Output voltage $= \beta \times$ input voltage $= 100 \times 10^{-2} = 1$V

16. Forward bias is obtained when the negative terminal of the battery is connected to the n-side and the positive terminal to the p-side of the semiconductor. Then, the negative terminal will repel free electrons in the n-section towards the junction and the positive terminal on the p-side will push the holes towards the junction.

17. The diode is forward biased, hence the potential barrier decreases and becomes less. Though in both the options (c) and (d). The diode is forward biased but in (d) the barrier width is less.

18. Power gain = Current gain × Voltage gain

Voltage gain $A_V = \beta \dfrac{R_2}{R_1}$

$$\text{Current gain} = \beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$$

$$A_V = 49\left(\frac{500 \times 10^3}{R_1}\right)$$

$$\text{Power gain } R_1 = 6.0625 \times 10^6 = 49 \times \left(\frac{500 \times 10^3}{R_1}\right) \times 49 = 198\ \Omega$$

19. When base region has larger width, electron-hole combination increases the base current. The output collector current decreases by the relation, $I_E = I_B + I_C$

20. $R_f = \dfrac{\Delta V_f}{\Delta I_f} = \dfrac{(2-0.5)}{1.5 \times 10^{-3}} = 10^3\ \Omega = 1\text{k}\Omega$

21. Zener diodes are specially designed junction diodes, which can operate in the reverse break down voltage region continuously without being damaged. The Zener diode is used as a voltage regulator as constant voltage at variable current under reverse bias is obtained from it.

22. The common emitter transistor amplifier has input resistance equal to 1kΩ (approx.) and output resistance equal to 10kΩ (approx.). The output current in CE amplifier is much larger than the input current.

23. When a junction diode is forward biased energy is released at the junction due to recombination of electrons and holes. In the junction diode made of gallium arsenide or indium phosphide, the energy is released in visible region. Such a junction diode is called Light Emitting Diode or LED. The radiated energy emitted by LED is equal or less than band gap of semiconductor.

24. For same value of current higher value of voltage is required for higher frequency.

25. Given figure is half wave rectifier.

26. Since, no conduction is found when multimeter is connected across P and Q, it means either both P and Q are n-type or p-type. So, it means R is base, when R is connected to common terminal and conduction is seen when other terminal is connected to P or Q. So, it means transistor is n-p-n with R as base.

27. The number of free electrons for conduction is significant only in Si and Ge but small in C, as C is an impurity.

28. $\beta = \dfrac{I_C}{I_B}$ and $I_E = I_C + I_B \therefore \beta = \dfrac{I_C}{I_E - I_C} = \dfrac{5.488}{5.60 - 5.488} = 49$

29. If lattice constant of semiconductor is decreased, then E_c and E_v decrease but E_g increases.

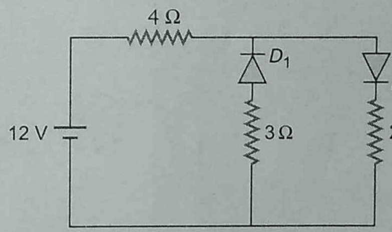
30. $I = n_e A v_d$

$$\therefore \quad \frac{I_e}{I_h} = \frac{n_e \times (v_d)_e}{n_h \times (v_d)_h}$$

Here, $\dfrac{n_e}{n_h} = \dfrac{7}{5}, \dfrac{I_e}{I_h} = \dfrac{7}{4}$

$$\therefore \quad \frac{7}{4} = \frac{7}{5} \times \frac{(v_d)_e}{(v_d)_h} \text{ or } \frac{(v_d)_e}{(v_d)_h} = \frac{5}{7} \times \frac{7}{4} = \frac{5}{4}$$

31. In the given circuit diode D_1 is reverse biased while D_2 is forward biased, so the circuit can be redrawn as



Apply KVL to get current flowing through the circuit

$$-12 + 4i + 2i = 0 \quad \text{or} \quad i = \frac{12}{6} = 2\text{A}$$

32. For reverse biasing of an ideal diode, the potential of n-side should be higher than potential of p-side. Only option (d) is satisfying the criterion for reverse biasing.

33. Given, $f = 50$ Hz and $T = \dfrac{1}{50}$

For full wave rectifier, $T_1 = \dfrac{T}{2} = \dfrac{1}{100}$ and $f_1 = 100$ Hz

34. For a transistor amplifier in common emitter configuration, current gain $A_i = \dfrac{h_{fe}}{1 + h_{oe} R_L}$

Where, h_{fe} and h_{oe} are hybrid parameters of a transistor.

$$\therefore \quad A_i = \frac{50}{1 + 25 \times 10^{-6} \times 1 \times 10^3} = -48.78$$

35. Due to the reverse biasing the width of depletion region increases and current flowing through the diode is almost zero. In this case electric field is almost zero at the middle of the depletion region.

Day 35

Gate Circuit

Day 35

Outlines ...

- Analog and Digital Electronics
- Logic Gates
- Three Basic Logic Gates
- Combination of Logic Gates
- Transistor as a Switch

Analog & Digital Electronics

In analog circuits, we use sinusoidal voltage waveforms, i.e., a continuous range of values of voltages are possible. Such signals are called ***analog signals****.*

The electronic circuits like amplifier, oscillator etc., are called ***analog circuits****. A pulse waveform type voltage signal has only two discrete values of voltage.*

The high level is termed as 1 and the low level is termed as 0. It is like binary digit system. Using the two levels of a signal we can develop a new form of electronics, known as ***digital electronics****.*

Devices based on digital electronics are vastly superior than electronic devices based on analog system.

Logic Gates

A logic gate is a digital electronic circuit which follows a logical relationship between its input and output. A logic gate may have one or more inputs but has only one output. Logic gates follow Boolean algebra, which consists of three basic operations, namely AND $(A \cdot B = Y)$, OR $(A + B = Y)$ and NOT $(\overline{A} = Y)$.

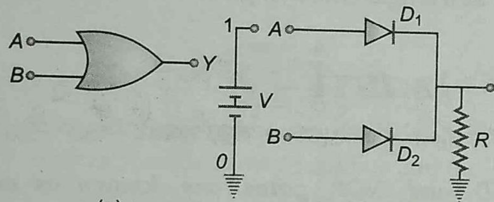
Three Basic Logic Gates

On the basis of types of operations *i.e.*, OR, AND, NOT etc, there are three basic logic gates. These gates with their detailed descriptions are given below.

1. OR Gate

It has two inputs, one output.

Logic symbol of OR gate is shown in **Fig.**(*a*). Its Boolean expression is $Y = A + B$ (being read as Y equals A or B).



(a) (b)

An OR gate can be realised by using two *p-n* junction diodes as shown in **Fig.** (*b*).

Truth table of OR gate is

0	0	0
1	0	1
0	1	1
1	1	1

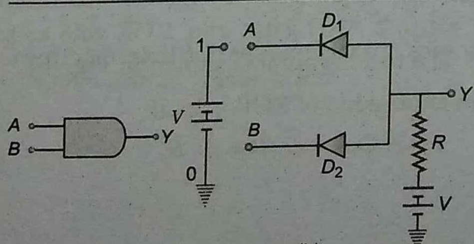
2. AND Gate

It also has two inputs, one output.

Boolean expression of AND gate is $Y = A \cdot B$ (being read as Y equals A and B).

Truth table of AND gate is

0	0	0
1	0	0
0	1	0
1	1	1



(a) (b)

Logic symbol of AND gate is shown in **Fig.** (a) and the actual circuit arrangement used to realise AND gate in practice has been shown in **Fig.** (b).

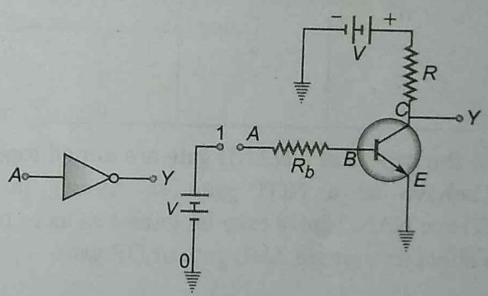
3. NOT Gate

It has an one input and one output.

Boolean expression of NOT gate $\overline{A} = Y$ (being read as A equals Y).

The truth table of the NOT gate is,

0	1
1	0



(a) (b)

The logic symbol and actual circuit for realisation of NOT gate have been shown in above figure.

(*i*) $A + 0 = A$	(*ii*) $A + A = A$
(*iii*) $A + 1 = 1$	(*iv*) $A + \overline{A} = 1$
(*v*) $A \cdot 0 = 0$	(*vi*) $A \cdot A = A$
(*vii*) $A \cdot 1 = A$	(*viii*) $A \cdot \overline{A} = 0$
(*ix*) $\overline{A + B} = \overline{A} \cdot \overline{B}$	(*x*) $\overline{A \cdot B} = \overline{A} + \overline{B}$

In the above expressions A and B are to be assigned the values 0 or 1.

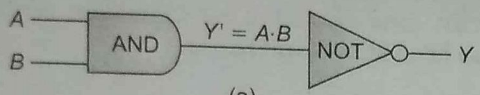
Here, if 0 implies low, then 1 implies high. Similarly, if 0 implies off, then 1 implies on. Also, if 0 stands for down then 1 stand for up vice-versa may also be true. It may be kept in mind than in digital electronics, 0 and 1 represent two states and not two values.

Combination of Logic Gates

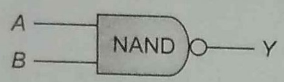
Some common combinations obtained from three fundamental gates OR, AND and NOT are as follows

1. NAND Gate

If the output of AND gate is joined to input of a NOT gate, the combination is a NAND gate.



(a)



(b)

Boolean expression of NAND gate is $\overline{A \cdot B} = Y$. Its logic symbol is shown in above figure.

The truth table of NAND gate is,

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

If both the inputs of a NAND gate are joined together, it will behave as a NOT gate as shown in figure. Two/Three NAND gates may be joined so as to have the same effect as a single AND gate or OR gate.

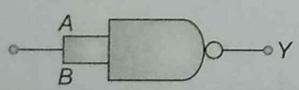
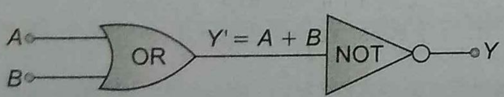


Fig. NAND as NOT gate

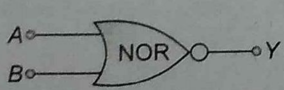
2. NOR Gate

If the output of OR gate is joined to input of a NOT gate, the combination is a NOR gate.

Boolean expression of NOR gate is $\overline{A + B} = Y$. Its logic symbol is shown in figure.



(a)

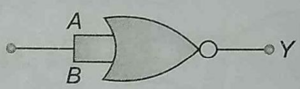


(b)

The truth table of NOR gate is

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

If both the inputs of a NOR gate are joined together, it will behave as a NOT gate as shown in figure. Two/Three NOR gates may be combined so as have the same effect as a single OR/AND gate.

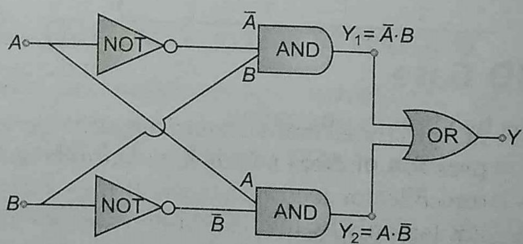


NOR as NOT gate

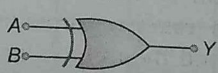
NAND and NOR gates are known as universal gate.

3. XOR Gate

The logic gate which gives high output (1) when either input A or input B, but not both of them, are high is called exclusive OR gate the XOR gate. If both inputs A and B are either high (1) or low (0), then the output is zero.



(a)



(b)

XOR gate can be obtained by having a combination of 2 NOT gates, 2 AND gates and 1 OR gate as shown in **Fig.** (a). The logic symbol is shown in **Fig.** (b).

Boolean expression of XOR gate, is

$$Y = A \oplus B = \bar{A} \cdot B + A \cdot \bar{B}$$

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

The Boolean expressions obey the commutative law, associative law as well as distributive law.

- Commutative law

  (i) $A + B = B + A$ (ii) $A \cdot B = B \cdot A$

- Associative law

  (iii) $A + (B + C) = (A + B) + C$ (iv) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

- Distributive law

  (v) $A \cdot (B + C) = A \cdot B + A \cdot C$ (vi) $A + \overline{A} \cdot B = A + B$

  (vii) $A + A \cdot B = A$ (viii) $A \cdot (A + B) = A$

  (ix) $A \cdot (\overline{A} + B) = A \cdot B$ (x) $\overline{A \cdot B} = \overline{A} + \overline{B}$

Transistor as a Switch

Transistors are commonly used as electronic switches, both for high power applications such as switched mode power supplies and for low power applications such as logic gates.

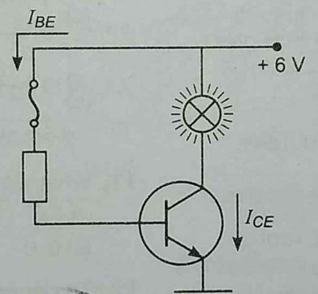


Fig. BJT used as an electronic switch, in grounded emitter configuration

In a grounded emitter transistor circuit, such as the light switch circuit shown, as the base voltage rises the emitter and collector currents rise exponentially. The collector voltage drops because of the collector load resistance. If the collector voltage were zero, the collector current would be limited only by the light bulb resistance and supply voltage.

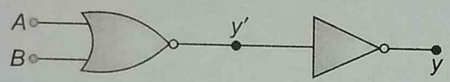
The transistor is then said to be saturated. It will have a very small voltage from collector to emitter. The transistor provides current gain, allowing a relatively large current in the collector to be switched by a much smaller current into the base terminal. The ratio of these currents varies depending on the type of transistor and even for a particular type, varies depending on the collector current.

In any switching circuit, values of input voltage would be chosen such that the output is either completely off or completely on. The transistor is acting as a switch and this type of operation is common in digital circuits.

Practice Zone

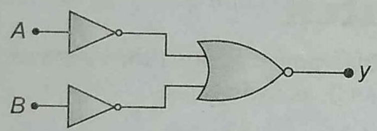
DAY 35

1. The circuit shown in figure below will act as **[NCERT Exemplar]**



(a) OR gate (b) AND gate
(c) XOR gate (d) None of these

2. The circuit shown below will acts as **[NCERT Exemplar]**



(a) AND gate (b) OR gate (c) NAND gate (d) XOR gate

3. In Boolean algebra $A + B = Y$ implies that

(a) sum of A and B is Y
(b) Y exists when A exists or B exists or both A and B exist
(c) Y exists only when A and B both exist
(d) Y exists when A or B exist but not when both A and B exist

4. In the Boolean algebra, which of the following is wrong?

(a) $1 + 0 = 1$ (b) $0 + 1 = 1$
(c) $1 + 1 = 1$ (d) None of these

5. The output of a 2-input OR gate is fed to a NOT gate, the new gate obtained is

(a) OR gate (b) NOT gate (c) NOR gate (d) XOR gate

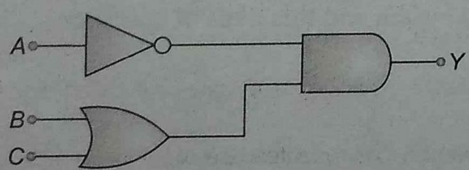
6. Digital circuit can be made by the repetitive use of

(a) OR gates (b) AND gates
(c) NOT gates (d) NAND gates

7. If $A = B = 1$, then in terms of Boolean algebra what is not equal to the value of $A \cdot B + A$?

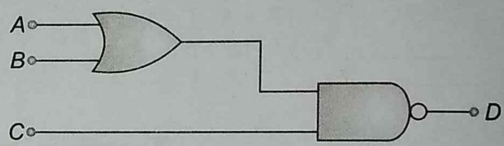
(a) $B \cdot A + B$ (b) $B + A$
(c) B (d) None of these

8. The Boolean equation for the circuit given in figure is



(a) $Y = \overline{A} \cdot B + C$ (b) $Y = \overline{A} \cdot (\overline{B} + \overline{C})$
(c) $Y = \overline{A} \cdot (B + \overline{C})$ (d) $Y = \overline{A} \cdot (B + C)$

9. For the given combination of gates, if the logic states of inputs A, B and C are as follows. $A = B = C = 0$ and $A = B = 1$, $C = 0$, then the logic states of output D are

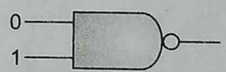


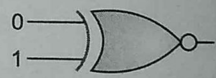
(a) 0, 0 (b) 0, 1 (c) 1, 0 (d) 1, 1

10. Which of the following gates will have an output of 1 ?

(a)


(b)

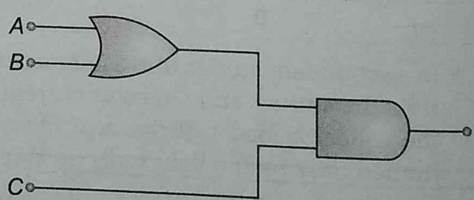

(c)


(d)


11. What will be the input of A and B for the Boolean expression $(\overline{A + B}) \cdot (\overline{A \cdot B}) = 1$?

(a) 0, 0 (b) 0, 1 (c) 1, 0 (d) 1, 1

12. To get an output 1 from the circuit shown in the figure, the input must be



(a) $A = 0, B = 1, C = 0$ (b) $A = 1, B = 0, C = 0$
(c) $A = 1, B = 0, C = 1$ (d) $A = 1, B = 1, C = 0$

13. The output of an OR gate is connected to both the inputs of a NAND gate , the truth table is

(a)

A	B	Y
1	0	0
1	1	1
0	1	0

(b)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(c)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

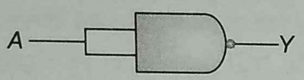
(d) None of these

14. The following truth table is for

A	B	Y
1	1	0
1	0	1
0	1	1
0	0	1

(a) NAND (b) AND (c) XOR (d) NOT

15. Which of the following is the truth table for the circuit below? **[NCERT Exemplar]**



(a)

A	Y
1	0
0	1

(b)

A	Y
0	0
1	1

(c)

A	Y
1	1
0	1

(d)

A	Y
0	1
0	0

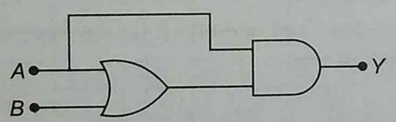
16. Which of the following is NOT equal to 1 in Boolean algebra?

(a) $A + 1$ (b) $A \cdot \overline{A}$ (c) $A + \overline{A}$ (d) $\overline{\overline{A} \cdot A}$

17. Which of the following is the truth table for NOT gate?

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

18. The truth table of the following combination of gates is



(a)

Inputs			
A	B	A · B	Y
0	0	0	1
0	1	1	0
0	0	0	1
1	1	1	0

(b)

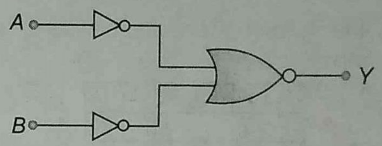
Inputs			
A	B	A · B	Y
1	1	0	1
0	1	0	1
1	0	0	0
0	1	1	1

(c)

Inputs			
A	B	A + B	Y = A.(A + B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

(d) None of the above

19. Which logic gate is represented by the following combination of logic gates?

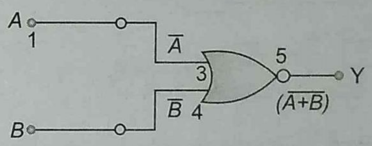


(a) OR (b) NAND
(c) AND (d) NOR

20. The output of OR gate is

(a) if either or both the inputs are 1
(b) only if both inputs are 1
(c) if either input is zero
(d) if both inputs are zero

21. Following diagram performs the logic function of



(a) AND gate (b) NAND gate
(c) OR gate (d) XOR gate

22. The truth table given below is for

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(a) AND (b) XOR (c) XNOR (d) OR

Directions *(Q. Nos. 23 to 25) Each of these questions contains two statements: Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below.*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

23. **Statement I**The logic gate NOT cannot be built using diode.
Statement II The output voltage and the input voltage of the diode have 180° phase difference.

24. **Statement I** NOT gate is also called invertor.
Statement II NOT gate inverts the input signal.

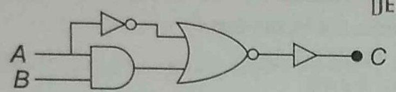
25. **Statement I** NAND or NOR gates are called digital building blocks.
Statement II The repeated use of NAND or NOR gates can produce all the basic or completed gates.

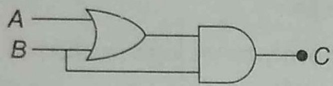
AIEEE & JEE Main Archive

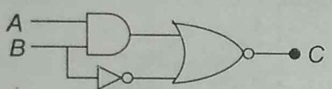
26. Which of the following circuits correctly represents the following truth table?

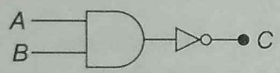
A	B	C
0	0	0
0	1	0
1	0	1
1	1	0

[JEE Main Online 2013]

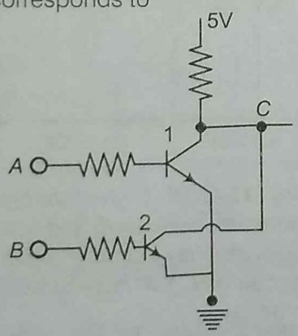
(a) 

(b) 

(c) 

(d) 

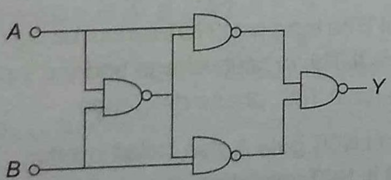
27. Consider two *n-p-n* transistors as shown in figure. If 0 V corresponds to false and 5 V corresponds to true, then the output at C corresponds to



[JEE Main Online 2013]

(a) *A* NAND *B* (b) *A* OR *B*
(c) *A* AND *B* (d) *A* NOR *B*

28. Truth table for system of four NAND gates as shown in figure is **[AIEEE 2012]**



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

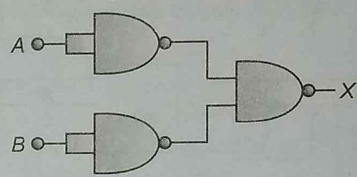
(d)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

29. The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a **[AIEEE 2011]**

(a) OR gate (b) NOT gate (c) NOR gate (d) AND gate

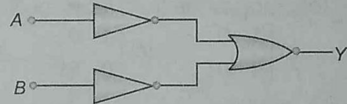
30. The combination of gates shown below yields

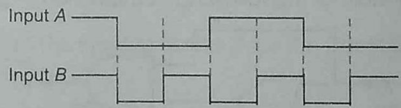


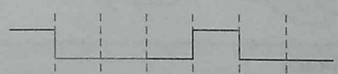
[AIEEE 2010]

(a) OR gate (b) NOT gate (c) XOR gate (d) NAND gate

31. The logic circuit shown below has the input waveforms *A* and *B* as shown. Pick out the correct output wavefrom. **[AIEEE 2009]**

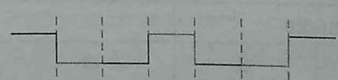




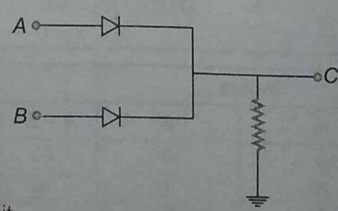
(a) 

(b) 

(c) 

(d) 

32. In the adjacent circuit, *A* and *B* represent two inputs and *C* represents the output, **[AIEEE 2008]**



The circuit represents

(a) NOR gate (b) AND gate
(c) NAND gate (d) OR gate

Answers

1. (a)	2. (a)	3. (b)	4. (d)	5. (c)	6. (d)	7. (d)	8. (d)	9. (d)	10. (c)
11. (a)	12. (c)	13. (a)	14. (a)	15. (a)	16. (b)	17. (c)	18. (c)	19. (c)	20. (a)
21. (a)	22. (c)	23. (b)	24. (a)	25. (a)	26. (a)	27. (a)	28. (a)	29. (c)	30. (a)
31. (a)	32. (d)								

Hints & Solutions

1. The output of NOR gate is made input for NOT gate.

$Y = \overline{\overline{A+B}} = A+B$

2. The output of two NOT gate is input for NOR gate

Hence, $Y = \overline{\overline{A}+\overline{B}} = \overline{\overline{A}}\cdot\overline{\overline{B}} = A\cdot B$

4. It is OR operation. Here, $1+0=0+1=1+1=1$

5. The combination of OR and NOT gates is NOR gate.

6. The repetitive use of NAND and NOR gate gives digital circuits.

7. $AB + A = 1\cdot 1 + 1 = 1+1 = 1$. Same is true for $B\cdot A + B$ as well as $B + A$.

8. $Y = \overline{A}\cdot(B+C)$

9. The output D for the given combination

$$D = \overline{(A+B)\cdot C} = \overline{(A+B)} + \overline{C}$$

If $A=B=C=0$, then $D = \overline{(0+0)} + \overline{0} = \overline{0}+\overline{0} = 1+1 = 1$

If $A=B=1, C=0$, then $D = \overline{(1+1)} + \overline{0} = \overline{1}+\overline{0} = 0+1 = 1$

10. For option (c), it is a NAND gate, its output $= \overline{0.1} = \overline{0} = 1$

11. The given Boolean expression can be written as

$$Y = \overline{(A+B)}\cdot\overline{(AB)} = (\overline{A}\cdot\overline{B})\cdot(\overline{A}+\overline{B})$$

$$= (\overline{A}\cdot\overline{A})\cdot\overline{B} + \overline{A}(\overline{B}\cdot\overline{B}) = \overline{A}\cdot\overline{B} + \overline{A}\cdot\overline{B} = \overline{A}\cdot\overline{B}$$

So, the truth table is

A	*B*	*Y*
0	0	1
1	0	0
0	1	0
1	1	0

12. The Boolean expression for the given combination is

$$Y = (A+B).\,C$$

The truth table is

A	*B*	*C*	*Y* = (*A* + *B*) *C*
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Hence, $A=1, B=0$ and $C=1$

13. When two inputs of a NAND gate are joined together, it works as a NOT gate. The OR gate connected to this NOT gate results is a NOR gate.

14. For NAND gate, $Y = \overline{AB}$

15. The output of the NAND gate is

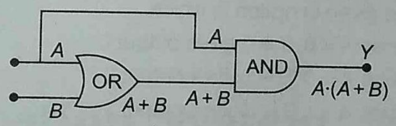
$Y = \overline{A\cdot A} = \overline{A} + \overline{A} = \overline{A}$

16. Here, $A\cdot\overline{A} = 0$ both when $A = 0$ or 1

17. For NOT gate, $Y = \overline{A}$

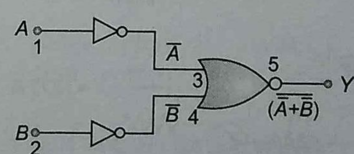
18. Let us draw the given combination pointing out that the first gate is OR gate second gate is AND gate. The inputs of the OR gate, are A and B, and its output is $A+B$ that is A OR B.

The inputs of the AND gate are A and $A+B$ and its output is $A\cdot(A+B)$ that is A AND (A or B). The truth table for the output is $Y = A.(A+B)$ is as follows



Inputs			
A	*B*	*A* + *B*	*Y* = *A* · (*A* + *B*)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

19. Lets mark the figure again, we have



At point (1), we have input A and at point (2), we have input B

At point (3) we have input $\overline{A}$ and at point (4), we have input $\overline{B}$

At point 5, we have the output of the NOR gate as

$$\overline{(\overline{A}+\overline{B})} = \overline{\overline{A}}\cdot\overline{\overline{B}} = A\cdot B$$

So, we get the result to be an AND gate.

20. The truth table for the OR gate is given as

Inputs		Output
A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

21. The operation of first gate is NAND gate and the second is NOT gate, so the resultant operation is that of an AND gate.

22. The truth table corresponds to the boolean operation $Y = A\cdot\overline{B} + \overline{A}\cdot B$. This is for XNOR and XOR gate.

23. NOT gate inverts the signal applied to it. But in diode, the input and output are in same phase. Thus, NOT gate cannot be built using diode.

24. NOT gate inverts the input signal *i.e.*, if input is 1 then output will be zero or *vice-versa*. Therefore, it is called as invertor. NOT gate inverts the input order means that for low input, it gives high output or for high input, it gives low output.

25. NAND or NOR gates are called universal (digital) building blocks because using repeared order of these two types of gates we can produce all the basic gates namely OR, AND or complex gates.

26. Observing the given gate we observe that gate would be same as given in option in which.

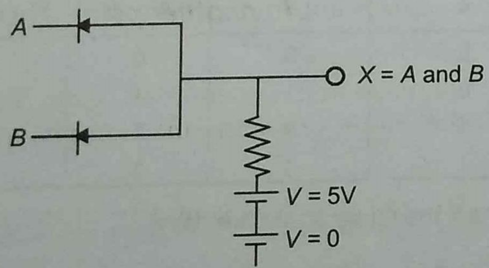
The values $A = 0, B = 0$ gives output 0

The values $A = 0, B = 1$ gives output 0

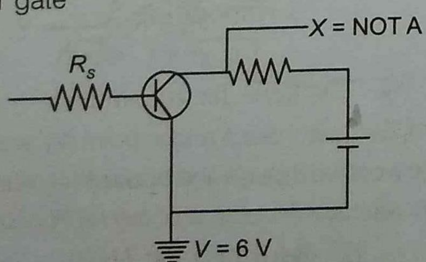
The values $A = 1, B = 0$ gives output 1

The value $A = 1$, and $B = 1$ gives output 0

27. From the figure of AND gate

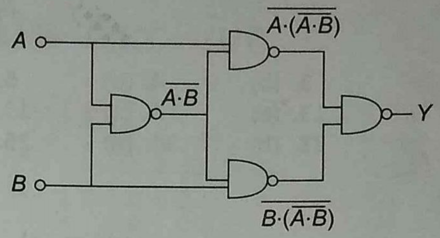


and NOT gate



Clearly, the function X = NOT (A AND B) of the logical variables A AND B is called NAND gate.

28.



$$Y = \overline{(\overline{(A\cdot\overline{(A\cdot B)})})\cdot(\overline{B\cdot\overline{(A\cdot B)}})}$$

$$= \overline{(\overline{A} + A\cdot B)\cdot(\overline{B} + A\cdot B)}$$

$$= \overline{(\overline{A} + A\cdot B)} + \overline{(\overline{B} + (A\cdot B))}$$

$$= A\cdot(\overline{A\cdot B}) + B\cdot(\overline{A\cdot B})$$

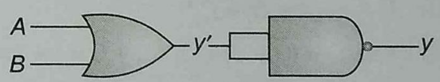
$$= A\cdot(\overline{A} + \overline{B}) + B\cdot(\overline{A} + \overline{B})$$

$$= A\cdot\overline{B} + B\cdot\overline{A}$$

A	B	$\overline{A}$	$\overline{B}$	$A\cdot\overline{B}$	$B\cdot\overline{A}$	Y
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

It is the truth table of XOR gate.

29.



$y' = A + B$ and $y = \overline{y'} = \overline{A+B}$

i.e., output of a NOR gate.

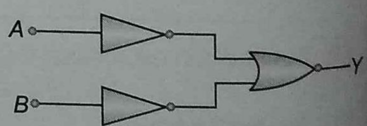
30. Truth table for given combination is

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

This comes out to be truth table of OR gate.

31. Truth table

A	B	Y
1	1	1
1	0	0
0	2	0
0	0	0



32. If we give the following inputs to A and B, then corresponding output is shows in table

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

The above table is similar to OR gate.

Day 36

Communication Systems

Day 36

Outlines ...

- Communication System
- Modulation
- Demodulation or Detection
- Propagation of Electromagnetic Waves

Communication System

The word communication refers to the faithful transmission of information or message signal from one point to another point where it reaches in an intelligible form.

Irrespective of its nature, every communication system has three essential elements. These are (i) transmitter, (ii) medium or communication channel and (iii) receiver.

In modern communication systems the information is generally in the form of electrical voltage/current signals.

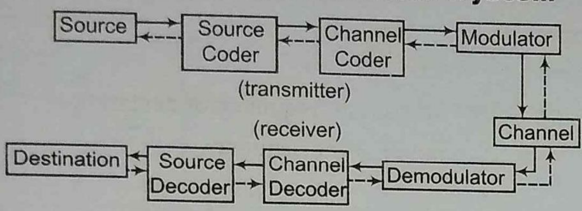
An electrical transducer converts some physical variables (pressure, displacement, force, temperature etc.) into corresponding variations in the electrical signal as its output.

Information converted in electrical form and suitable for transmission is called a signal, which can be either **analog** or **digital.**

Bandwidth refers to the frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

For speech signals (as in telephony) a band width of 2800 Hz is adequate. For transmission of musical programme a band width of about 20 kHz is required. For TV transmission generally a band width of 6 MHz is allocated to a channel.

Block Diagram of Communication System



Line Communication

For point to point communication we need line communication. Line communication may be in the form of electrical signal or optical signal.

Optical Communication

Optical communication uses light waves in the frequency range 10^{12} to 10^{16} Hz as the guided wave medium for propagation of audio frequency signal.

Main advantage of optical communication system lies in the fact that here very high band widths of MHz and even GHz are possible. Consequently a large number of messages can be transmitted through a single cable without any risk of their intermixing.

Moreover due to very little line loss the quality of reception is vastly superior.

An optical communication system consists of mainly three parts which are

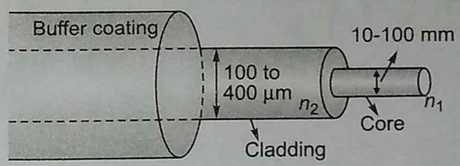
(i) optical source and modulator

(ii) optical fibre cable,

and (iii) optical signal detector.

Optical Fibre

Optical fibre make use of the principle of total internal reflection of light. The refractive index n_1 of central core is higher than refractive index n_2 of cladding.



Total internal reflection will take place at core-cladding interface if angle of incidence there is

$$i = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

For above condition to be fulfilled the light ray must enter the optical fibre at a maximum acceptance angle θ_0 from the axis of fibre such that

$$(\theta_0)_{max} = \sin^{-1}\left[\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right]$$

where, n_0 is the refractive index of the outer medium. For air, $n_0 = 1$ and then

$$(\theta_0)_{max} = \sin^{-1}(\sqrt{n_1^2 - n_2^2})$$

- *Numerical Aperture (NA)* $= n_0 \sin(\theta_0)_{max} = \sqrt{n_1^2 - n_2^2}$
- *All the information in optical fibre is carried out by the principal of total internal reflection and all the information is carried in core of the optical fibre.*
- *If angle of incidence is greater than* $(\theta_0)_{max}$, *then total internal reflection will not take place and some information will be lossed.*

Modulation

It is the process in which one of the characteristic parameters (amplitude, frequency or phase) of the carrier signal will be varied linearly in accordance with message signal amplitude variations.

Modulation is essential so that (i) height of transmitting antenna is not too high, (ii) there is no overlapping of signals from different sources, and (iii) radiating power of transmitting antenna is high enough.

For a carrier wave, which is a continuous sinusoidal wave, modulation is of three types namely Amplitude Modulation (AM), Frequency Modulation (FM) and Phase Modulation (PM).

1. Amplitude Modulation (AM)

(i) Amplitude Modulation (AM) is the process of changing the amplitude (A_c) of a carrier wave linearly in accordance with the amplitude of message signal (A_m).

(ii) The ratio $\mu = \frac{A_m}{A_c}$ is called the modulation index and in practice we maintain $\mu \le 1$, so as to avoid distortion.

(iii) In AM modulated wave signal we have carrier wave of frequency ω_c and two side bands of frequencies $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$ respectively. Thus, total band width of AM signals is $2\omega_m$.

- Upper Side Band Frequency (USB) $= \nu_c + \nu_m$
  where, ν_c = carrier frequency
  ν_m = signal frequency.
- Lower Side Band Frequency (LSB) $= \nu_c - \nu_m$
  where, ν_c = carrier frequency
  ν_m = signal frequency.
- Bandwidth $= \nu_{USB} - \nu_{LSB}$
  $= (\nu_c + \nu_m) - (\nu_c - \nu_m) = 2\nu_m$

(iv) AM technique is simpler and cost effective. However, it suffers from noisy reception, low efficiency, small operating range and poor audio quality.

(v) **Power dissipated** in AM wave, $P = P_c\left[1 + \frac{\mu^2}{2}\right]$ where $P_c = \frac{E_c^2}{2R}$ is power dissipated by unmodulated carrier wave where, μ = modulation index.

2. Frequency Modulation (FM)

(i) Frequency modulation is the process of changing the frequency of a carrier wave in accordance with the amplitude of message signal.

(ii) In FM modulated wave the amplitude of wave and hence total transmitted power remains constant.

(iii) Frequency of modulated signal consists of central band of frequency ω_c and side bands of frequencies $(\omega_c \pm \omega_m), (\omega_c \pm 2\omega_m), (\omega_c \pm 3\omega_m)$ .....
The number of side bands depends on the modulation index.

(iv) Total **bandwidth of** modulated signal $= 2n \cdot \omega_m$, where n = number of significant side band pairs.

(v) FM technique is more complex and costly. However, efficiency is more and audio quality is vastly improved. Noise level is negligible.

In the FM wave, **modulation index**

$$m_f = \frac{\delta}{\nu_m}$$

where, δ = maximum frequency of deviation
ν_m = modulation frequency.

Demodulation or Detection

(i) Demodulation is the process of extracting the audio frequency information/message signal from the modulated wave.

(ii) For AM modulated wave a *p-n* junction diode or a vacuum tube diode is used as a demodulator. A diode basically acts as a rectifier and, thus, reduces the modulated carrier wave into positive envelope only. This positive envelope is sent through a *R-C* circuit. Carrier wave passes through the capacitor and AF signal is regenerated across *R*.

Modem

Modems are used to interface two digital sources/receivers. **A modem contains a modulator and a demodulator arrangement** in a single compact unit. Modems are placed at both the ends of communication circuit. At the transmitting stage, modem changes the digital output from the computer/business machine into analog signal so that it can be easily sent *via* a communication channel (telephone line). The modem used at the receiver stage does the reverse process.

Fax

The electronic reproduction of a document as a whole at some distant place is called **FAX (Facsimile transmission)**. At the transmitting stage the document is converted into transmittable electronic code. At the receiving stage these codes are converted back into a copy of the original document.

Propagation of Electromagnetic Waves

It is that category of communication in which no line or cable is used as a communication channel and the modulated signal is propagated through free space. It is of following types

1. Ground Wave Propagation

(i) In ground wave (also known as surface wave) propagation, the radiowaves travel along the surface of earth following the curvature of earth.

(ii) In these waves continuously attenuation of signal takes place as the wave propagates. It is due to (i) induced currents in the ground, and (ii) diffraction of waves. The attenuation increases rapidly with increase in frequency of the waves.

The method does not suit at all for radiowaves of frequency more than 2 MHz.

- *Ground waves are vertically polarised.*
- *AM is an example of ground wave propagation.*

2. Sky Wave Propagation

(i) These waves travel upward and are reflected back from the ionosphere, which is a atmospheric layer around the earth at a distance of 80 to 400 km from the earth's surface.

(ii) In sky wave propagation attenuation of signal is very less and hence the transmission range is large.

(iii) Sky wave method can be used for radiowaves of frequencies 2 MHz to 20 MHz. However, by using special techniques upper range of this limit may be suitably increased.

(iv) There is a certain maximum frequency of radiowaves (called as the critical frequency) which can be reflected back by the ionosphere. So, the sky wave method fails for waves of higher frequencies.

- *Reflection occurs from ionisation layers due to bending of waves when it travels through different layers.*
- *The critical frequency ν_c of a sky wave for reflection from a layer of atmosphere is ... $\nu_c = 9(N_{max})^{1/2}$ where N_{max} is the number density of electrons per m^3 of the layer of atmosphere.*
- *Skip distance* $D_{skip} = 2h\sqrt{\left(\frac{\nu_{max}}{\nu_c}\right)^2 - 1}$

where, h = height of reflecting layer of atmosphere,

ν_{max} = maximum frequency of electromagnetic wave,

ν_c = critical frequency for the layer of atmosphere.

- *Refractive index μ, for a layer in sky wave propagation is*

$$\mu = \sqrt{1 - \frac{81.45N}{\nu^2}}$$

where N is the number density of electron per m^3

3. Space Wave Propagation

(i) In this method waves are transmitte[d] directly from the transmitting antenna [to] receiving antenna. Due to this reason it [is] also called the **line of sight** propagation. [It] occurs between 30 MHz to 300 MHz.

(ii) Range is limited due to curvature of earth. [If] h be the height of the transmitting antenn[a] then signal can be received upto [a] maximum distance $d = \sqrt{2Rh}$ where R [is] radius of earth.

(iii) In such a case area covered $A = \pi d^2 = 2\pi R$[h] and if population density is n the[n] population covered

$$= nA = 2\pi Rhn$$

(iv) If heights of transmitting and receivin[g] antennas be h_T and h_R respectively, the[n] effective range will be

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

(v) Relative permittivity of the layer [of] atmosphere is

$$\varepsilon_r = 1 - \frac{81.45N}{\nu^2}$$

4. Satellite Communication

(i) It is mainly done with the help of geostationary satellite orbiting the earth i[n] the equatorial plane from west to east at height of about 36000 km above the surfac[e] of earth so that its revolution time is 24 h.

(ii) The transmitted signal from the eart[h] station is uplinked to satellite. The satellit[e] receives it, demodulate it, amplify it an[d] remodulate it and transmit it back. Now it i[s] downlinked to earth station. A radi[o] transponder does all these jobs in a satellite.

(iii) Uplink frequency and downlink frequenc[y] are kept widely different so as to avoi[d] their interference in free space.

(iv) For world wide coverage three geostationar[y] satellites are required at 120° apart fro[m] each other.

Practice Zone DAY 36

1. Three waves A, B and C of frequencies 1600 kHz, 5 MHz and 60 MHz, respectively are to be transmitted from one place to another. Which of the following is the most appropriate mode of communication? **[NCERT Exemplar]**
   (a) A is transmitted *via* space wave while B and C are transmitted *via* sky wave.
   (b) A is transmitted *via* ground wave, B *via* sky wave and C *via* space wave.
   (c) B and C are transmitted *via* ground wave while A is transmitted *via* sky wave.
   (d) B is transmitted *via* ground wave while A and C are transmitted *via* space wave.

2. In optical communication system operating at 1200 nm, only 2% of the source frequency is available for TV transmission having a bandwidth of 5 MHz. The number of TV channels that can be transmitted is
   (a) 2 million (b) 10 million
   (c) 0.1 million (d) 1 million

3. For 100% modulation, the power carried by the side bands (P_{SB}) is given by
   (a) $P_{SB} = P$ (b) $P_{SB} = 3P$
   (c) $P_{SB} = \frac{1}{3}P$ (d) $P_{SB} = 0$

4. What is the modulation index if an audio signal of amplitude one half of the carrier amplitude is used in AM?
   (a) 1 (b) 0
   (c) 0.5 (d) > 1

5. For what value of m_a will the total power per cycle be maximum in the modulated wave?
   (a) 0 (b) 1 (c) 1/2 (d) > 1

6. How the sound waves can be sent from one place to another in space communication?
   (a) Through wires
   (b) Through space
   (c) By superimposing it on undamped electromagnetic waves
   (d) By superimposing it on damped electromagnetic waves

7. An EM wave of maximum frequency 300 kHz and critical frequency 100 kHz is to be transmitted to a height equal to 150 km. Calculate the skip distance.
   (a) 624 km (b) 849 km (c) 636 km (d) 942 km

8. A diode AM detector with the output circuit consisting of $R = 1\,k\Omega$ and $C = 1\,\mu F$ would be more suitable for detecting a carrier signal of
   (a) 10 kHz (b) 1 kHz
   (c) 0.75 kHz (d) 0.5 kHz

9. The channel width in FM broadcast in VHF is
   (a) 800 (b) 600
   (c) 400 (d) 200

10. The electron density of E, F_1, F_2 layers of ionosphere is 2×10^{11}, 5×10^{11} and $8 \times 10^{11}\,m^{-3}$ respectively. What is the ratio of critical frequency for reflection of radiowaves?
    (a) 2 : 4 : 3 (b) 4 : 3 : 2
    (c) 2 : 3 : 4 (d) 3 : 2 : 4

11. On a particular day, the maximum frequency reflected from the ionosphere is 9 MHz. On another day, it was found to increase by 1 MHz. What is the ratio of the maximum electron densities of the ionosphere on the two days?
    (a) 1.23 (b) 1.0 (c) 1.43 (d) 0.75

12. If sky wave with a frequency of 50 MHz is incident on D region at an angle of 30°, then angle of refraction is
    (a) 15° (b) 30°
    (c) 60° (d) 45°

13. A message signal of frequency ω_m is superposed on a carrier wave of frequency ω_c to get an Amplitude Modulated wave (AM) The frequency of the AM wave will be **[NCERT Exemplar]**
    (a) ω_m (b) ω_c
    (c) $\frac{\omega_c + \omega_m}{2}$ (d) $\frac{\omega_c - \omega_m}{2}$

14. To cover a population of 20 lakh, a transmitter tower should have a height of (Given radius of earth = 6400 km, population per square km = 1000) is
    (a) 25 m (b) 50 m
    (c) 75 m (d) 100 m

15. The TV transmission tower in Delhi has a height of 240 m. The distance up to which the broadcast can be received (Taking the radius of earth to be 6.4×10^6 m) is
    (a) 100 km (b) 60 km
    (c) 55 km (d) 50 km

16. In an amplitude modulated wave for audio-frequency of 500 cycle/s, the appropriate carrier frequency will be

(a) 50 cycle/s (b) 100 cycle/s
(c) 500 cycle/s (d) 50,000 cycle/s

17. The characteristic impedance of a coaxial cable is of the order of

(a) 50 Ω (b) 200 Ω
(c) 270 Ω (d) None of these

18. If μ_1 and μ_2 are the refractive indices of the materials of core and cladding of an optical fibre, then the loss of light due to its leakage can be minimized by having

(a) $\mu_1 > \mu_2$ (b) $\mu_1 < \mu_2$
(c) $\mu_1 = \mu_2$ (d) None of these

19. The minimum length of antenna required to transmit a radio signal of frequency 20 MHz is

(a) 5 m (b) 7.5 m (c) 2 m (d) 3.75 m

20. Repeaters are required for transmitting microwave terrestrial communication system over a 40-50 km distance because

(a) microwave power decreases rapidly with distance
(b) the curvature of the earth limits the distance over which the line of sight can be established
(c) signal to noise ratio decreases rapidly with distance
(d) signal distortion creeps in rapidly with distance

21. A speech signal of 3 kHz is used to modulate a carrier signal of frequency 1 MHz, using amplitude modulation. The frequencies of the side bands will be **[NCERT Exemplar]**

(a) 1·003 MHz and 0·997 MHz
(b) 3001 kHz and 2997 kHz
(c) 1003 kHz and 1000 kHz
(d) 1 MHz and 0.997 MHz

22. Maximum Usable Frequency (MUF) is F-region layers is x, when the critical frequency is 60 MHz and the angle of incidence is 70°, then x is

(a) 150 MHz (b) 170 MHz
(c) 175 MHz (d) 190 MHz

23. Frequencies higher than 10 MHz were found not being reflected by the ionosphere on a particular day at a place. The maximum electron density of the ionosphere on the day was near to

(a) $1.5 \times 10^{10}\ m^{-3}$ (b) $1.24 \times 10^{12}\ m^{-3}$
(c) $3 \times 10^{12}\ m^{-3}$ (d) None of these

24. Consider telecommunication through optical fibres. Which of the following statement is not true?

(a) Optical fibres can be of graded refractive index
(b) Optical fibres are subjected to electromagnetic interference from outside
(c) Optical fibres have extremely low transmission loss
(d) Optical fibres may have homogeneous core with a suitable cladding

Directions *(Q. Nos. 25 to 30) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

25. Statement I FM broadcast is preferred over AM broadcast.

Statement II Process of combining the message signals with carrier wave is called demodulation.

26. Statement I Higher the modulation index, the reception will be strong and clear.

Statement II The degree, to which the carrier wave is modulated is called modulation index.

27. Statement I Transducer in communication system converts electrical signal into a physical quantity.

Statement II For information signal is to be transmitted directly to long distances, modulation is necessary.

28. Statement I Modem is a demodulator.

Statement II It works only in a transmitting and receiving mode.

29. Statement I Optical fibre communication has immunity to cross-talk.

Statement II Optical interference between fibres is zero.

30. Statement I Television signals are received through sky-wave propagation.

Statement II The ionosphere reflects electromagnetic waves frequencies greater than a certain critical frequency.

31. Which of the following four alternatives is not correct? We need modulation

(a) to increase the selectivity
(b) to reduce the time lag between transmission and reception of the information signal
(c) to reduce the size of antenna
(d) to reduce the fractional bandwidth, that is the ratio of the signal bandwidth to the centre frequency.

AIEEE & JEE Main Archieve

32. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement II The state of ionosphere varies from hour to hour, day to day and season to season. **[AIEEE 2011]**

(a) Statement I is true, Statement II is true, Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I.

(c) Statement I is false, Statement II is true

(d) Statement I is true, Statement II is false

33. Consider telecommunication through optical fibres. Which of the following statements is not true? **[AIEEE 2003]**

(a) Optical fibres can be graded refractive index.

(b) Optical fibres are subjected to electromagnetic interference from outside.

(c) Optical fibres have extremely low transmission loss.

(d) Optical fibres may have homogeneous core with a suitable cladding.

Answers

1. (b)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (c)
11. (a)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (b)
21. (a)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (d)
31. (b)	**32.** (b)	**33.** (a)							

Hints & Solutions

1. For ground wave propagation, the frequency range is 530 kHz to 1710 kHz. For sky wave propagation, the frequency range is 1710 kHz to 40 MHz. For space wave propagation, the frequency range is 54 MHz to 4.2 GHz. Thus, option (b) is correct.

2. The frequency optical communication

$$\nu = \frac{c}{\lambda} \Rightarrow V = \frac{3\times10^8}{1200\times10^{-9}} = 25\times10^{13}\text{ Hz}$$

But only 2% of the source frequency is available for TV transmission $\nu = 2.5\times10^{14}\times 2\%$

$$\nu = 2.5\times10^{14}\times\frac{2}{100}$$

$$\nu = 5\times10^{12}\text{ Hz}$$

$$\text{Number of channels} = \frac{\nu'}{\text{bandwidth}}$$

$$\text{Number of channels} = \frac{5\times10^{12}}{5\times10^6} = 10^6 = 1\text{ million}$$

3. $$P_{SB} = \frac{1}{R}\left(\frac{m_aE_c}{2\sqrt{2}}\right)^2 + \frac{1}{\lambda}\left(\frac{m_aE_c}{2\sqrt{2}}\right)^2 \quad \left[\because P_c = \frac{E_c^2}{2R}\right]$$

$$P_{SB} = \frac{m_a^2E_c^2}{4R} = \frac{m_a^2P_c}{2} = \frac{P_c}{2} \quad [\therefore m_a = 1]$$

Also, $$P = P_c\left[1+\frac{m_a^2}{2}\right]$$

Here, $m_a = 1$

$$\Rightarrow \quad P = P_c\left[1+\frac{1}{2}\right] = \frac{3}{2}P_c$$

Hence, $$\frac{P_{SB}}{P} = \frac{P_c/2}{\frac{3}{2}P_c} = \frac{1}{3}$$

or $$P_{SB} = \frac{1}{3}P$$

4. Here, $E_m = \frac{1}{2}E_c$

Therefore, $E_{max} = E_c + E_m = E_c + \frac{1}{2}E_c = 1.5\,E_c$

$$E_{min} = E_c - E_m = E_c - \frac{1}{2}E_c = 0.5\,E_c$$

Also, $$m_a = \frac{E_{min} - E_{min}}{E_{min} + E_{min}} = \frac{1.5\,E_c - 0.5\,E_c}{1.5E_c + 0.5\,E_c}$$

$$m_a = \frac{E_c}{2.0E_c} = 0.5$$

5. Since, $P = P_c\left[1+\frac{m_a^2}{2}\right]$

Power will be maximum, if $m_a = 1$

Therefore, $P_{max} = P_c\left[1+\frac{1}{2}\right]$

$$= \frac{3}{2}P_c = 1.5\,P_c$$

7. $D_{Skip} = 2h\sqrt{\left(\frac{\nu}{\nu_c}\right)^2 - 1}$

$= 2 \times 150\sqrt{\left(\frac{300}{100}\right)^{-2} - 1}$

$= 2 \times 150 \times 2\sqrt{2} = 300 \times 2 \times 1.414$

$= 2.828 \times 300 = 848.4 \simeq 849\,\text{km}$

8. Given, $R = 1\,\text{k}\Omega$

$R = 1 \times 10^3\,\Omega, C = 1\,\mu\text{F} = 1 \times 10^{-6}\,\text{F}$

In this condition frequency of carrier signal.

$$\frac{1}{RC} << f_c.$$

$$\frac{1}{1 \times 10^3 \times 10^{-6}} << f_c$$

$\Rightarrow$ $f_c >> 1\,\text{kHz}$

Because frequency is greater than 1

$f_c = 10\,\text{kHz}$

10. Critical frequency for reflection of radio waves is given by

$\nu_c \propto N^{1/2}$

$\nu_{CE} : \nu_{CF_1} : \nu_{CF_2}$

$= (2 \times 10^{11})^{1/2} : (5 \times 10^{11})^{1/2} : (8 \times 10^{11})^{1/2} = 2 : 3 : 4$

11. $\frac{N'_{max}}{N_{max}} = \left(\frac{V'_c}{V_c}\right)^2$

$= \left(\frac{9+1}{9}\right)^2 = \left(\frac{10}{9}\right)^2 = 1.23$

12. For D-region, $N = 10^9\,\text{m}^{-3}$

$\mu = \sqrt{1 - \frac{81.45\,N}{\nu^2}}$

$= \sqrt{1 - \frac{81.45 \times 10^9}{(50 \times 10^6)^2}} \approx 1$

$\mu = \frac{\sin i}{\sin r} = 1$

or $\sin h = \sin i$

or $r = i = 30°$

13. In amplitude modulation the frequency of modulated wave is equal to the frequency of carrier wave. Thus, option (b) is correct.

14. Area of region covered $= \pi(2\pi R)$

In $1\,\text{km}^2 = 1000$ people

$\frac{1}{1000} \times 20 \times 10^6 = 20 \times 10^6 = A$

$20 \times 10^3 = \pi\,(2 \times h \times 6400)$

$\Rightarrow$ $h = \frac{20 \times 10^3}{\pi \times 2 \times 6400} = 50\text{m}$

15. $d = \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 240}\ \text{m} = 55\ \text{km}$

16. Carrier frequency is always greater than modulating frequency (i.e., audio frequency), so option (d) is appropriate carrier frequency.

17. The characteristic impedance of a coaxial cable is of the order of 270 Ω.

18. Refractive index of core is always greater than refractive index of cladding, to minimize the lost of light.

19. $\nu = 20\ \text{MHz} = 20 \times 10^6\ \text{Hz}$

Wavelength of antenna is, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{20 \times 10^6} = 15\ \text{m}$

For efficient radiation and reception, the height of transmitting and receiving antennas should be comparable to a quarter wavelength of the frequency used.

The minimum length of antenna $= \frac{\lambda}{4} = \frac{15}{4} = 3.75\ \text{m}$

20. To increases the range of transmission of microwaves, a number of antennas are erected in between the transmitting and receiving antennas. Such antennas in between the transmitting and receiving antennas are known as repeaters.

21. Here, $\nu = 3\,\text{kHz} = 0.003\,\text{MHz}$

Using amplitude modulation, the frequencies of the side band $= (\nu + \Delta\nu)$ and $(\nu - \Delta\nu)$

$= (1 + 0.003)$ and $(1 - 0.003) = 1.003\,\text{MHz}$ and $0.997\,\text{MHz}$

Thus, option (a) is correct.

22. $\text{MUF} = v_C\, s\, i = 60 \times 10^6 \times s\,70°$

$= 60 \times 10^6 \times \frac{1}{0.342} = 175.43 \times 10^6$

$= 175.43\ \text{MHz}$

23. $f^2 = \frac{80.6N}{\cos^2 i}$

where i is angle of incidence and N is electron density. For the wave not reflected from ionosphere, $i = 0$.

$\therefore$ $f^2 = \frac{80.6N}{\cos^2 0°} = 80.6N$

or $N = \frac{f^2}{80.6} = \frac{(10 \times 10^6)^2}{80.6}$

$= \frac{100}{80.6} \times 10^{12} = 1.24 \times 10^{12}\ \text{m}^{-3}$

24. *Some of the characteristics of an optical fibre are as follows*

(i) If works on the principle of total internal reflection.

(ii) It consists of core made up of glass/silica/plastic with refractive index n_1, which is surrounded by a glass of plastic cladding with refractive index $n_2 (n_2 > n_1)$. The refractive index of cladding can be either changing abruptly or gradually changing (graded index fibre).

(iii) There is a very little transmission loss through optical fibres.

(iv) There is no interference from stray electric and magnetic fields to the signals through optical fibres.

25. In AM modulation, the amplitude of the carrier signal varies in accordance with the information signal. AM signals are noisy because electrical noise signals significantly affect this. In FM modulation, amplitude of carrier wave is fixed while its frequency is changing. FM gives better quality transmission. It is preferred for transmission of music.

Demodulation is the process in which the original modulating voltage is recovered from the modulated wave.

26. The modulation index determines the strength and quality of the transmitted signal.

If the modulation index is small the amount of variation in the carrier amplitude will be small consequently the audio signal being transmitted will not be strong.

Hence, for high modulation index or greater degree of modulation, the audio signal reception will be clear and strong.

27. In any communication system information (a physical quantity) is first converted into an electrical signal by a device called transducer.

Most of the speech or information signal cannot be directly transmitted to long distances. For this an intermediate step of modulation is necessary in which the information signal is loaded or superimposed on a high frequency wave which acts as a carrier wave.

28. Modem is a modulating and demodulating device. It acts as a modulator in transmitting mode and as demodulator in receiving mode.

29. Optical communication is a system by which we transfer the informations on any distance from one location to other through optical range of frequency using optical fibre. The optical interference between fibres is zero. Hence optical fibre communication has immunity to cross talk.

30. In sky wave propagation the radiowaves which have frequency between 2 MHz to 30 MHz, are reflected back to the ground by the ionosphere. But radio waves having frequency greater than 30 MHz cannot be reflected by the ionosphere because at this frequency they penetrates the ionosphere. It makes the sky wave propagation less reliable for propagation of TV signal having frequency greater than 30 MHz.

Critical frequency is defined as the highest frequency that is returned to the earth by the ionosphere. Thus, about this frequency a wave whether it is electromagnetic will penetrate the ionosphere and is not reflected by it. Hence, choice (d) is correct.

31. Modulation does not change time lag between transmission and reception.

33. Some of the characteristics of an optical fibre are as follows

(i) It works on the principle of total internal reflection.

(ii) It consists of core made up of glass/silica/plastic with refractive index n_1, which is surrounded by a glass or plastic cladding with refractive index $n_2 (n_2 > n_1)$. The refractive index of cladding can be either changing abruptly or gradually changing (graded index fibre).

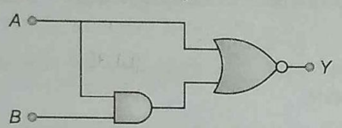
(iii) There is a very little transmission loss through optical fibres.

(iv) There is no interference from stray electric and magnetic fields to the signals through optical fibres.

Unit Test 7 (Modern Physics)

DAY 37

1. If $A = 0 = B$ and $A = 1 = B$, then the output of the logic circuit shown below are respectively.



(a) 0, 0 (b) 0, 1 (c) 1, 0 (d) 1, 1

2. Ultraviolet light of wavelength 350 nm and intensity 1.00 W m^{-2} is incident on a potassium surface. If 0.5% of the photons participate in ejecting the photoelectrons, how many photo electrons, are emitted per second, if the potassium surface has an area of 1cm^2 ?

(a) 1.76×10^{18} photoelectrons/s

(b) 1.76×10^{14} photoelectrons/s

(c) 8.8×10^{11} photoelectrons/s

(d) The value of work function is required to complete the value of emitted photoelectrons/s.

3. A broken ligament is being welded back in place using 20 ms pulses from a 0.5W laser operating at a wavelength of 632 nm. The number of photons in 5 pulses of laser are

(a) 1.59×10^{-18} (b) 3.18×10^{-17}

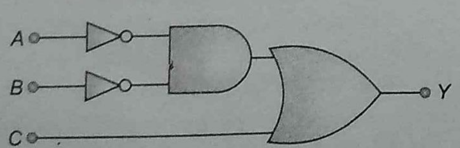
(c) 1.59×10^{23} (d) 3.18×10^{16}

4. Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about **[NCERT Exemplar]**

(a) 53 pm (b) 27 pm

(c) 18 pm (d) 13 pm

5. The Boolean expression for the logic circuit shown below is



(a) $Y = (A + B) + C$ (b) $Y = A \cdot B + C$

(c) $Y = (\bar{A} + \bar{B}) \cdot C$ (d) $Y = \bar{A} \cdot \bar{B} + C$

6. An electron collides with a hydrogen atom in its ground state and excites it to $n = 3$. The energy given to hydrogen atom in this inelastic collision is [Neglect the recoiling of hydrogen atom]

(a) 10.2 eV (b) 12.1 eV

(c) 12.5 eV (d) None of these

7. An X-ray tube operates at 50 kV. Consider that at each collision, an electron converts 50% of its energy into photons and 10% energy would be dissipated as thermal energy due to the collision then the wavelength of emitted by photons during 2^{nd} collision is [Take $hc = 1242$ eV-nm]

(a) 1.242 nm

(b) 1.242 Å

(c) 4.968 nm

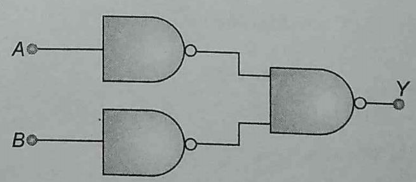
(d) 4.968 Å

8. For the nuclear reaction, ${}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4}$ the radium nucleus is initially at rest and the alpha particle carries the energy 5.3 MeV. The energy released in the reaction is

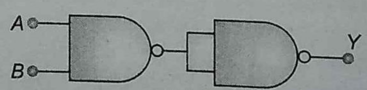
(a) 5.4 MeV (b) 5.0 MeV

(c) 300 MeV (d) 286 MeV

9. The combinations (M) and (N) of the NAND gates are shown below.



(M)



(N)

The output (Y) of (M) and (N) are equivalent to the output of

(a) OR gate and AND gate respectively

(b) AND gate and NOT gate respectively

(c) AND gate and OR gate respectively

(d) OR gate and NOT gate respectively

10. The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is $B = -\frac{me^4}{4n^2\varepsilon_0^2h^2}$ (m = electron mass).

If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$B = \frac{Me^4}{8n^2\varepsilon_0^2h^2}$ = (M = proton mass)

This last expression is not correct because **[NCERT Exemplar]**

(a) n would not be integral
(b) Bohr-quantisation applies only to electron
(c) the frame in which the electron is at rest is not inertial
(d) the motion of the proton would not be in circular orbits, even approximately

11. Light strikes a sodium surface, causing photoelectric emission. The stopping potential for the ejected electrons is 5.0 V, and the work function of sodium is 2.2 eV. What is the wavelength of the incident light?

(a) 100 nm (b) 170 nm (c) 150 nm (d) 200 nm

12. Light of wavelength 200 nm shines on an aluminium, 4.20 eV is required to eject an electron. What is the kinetic energy of the fastest ejected electrons?

(a) 0.5 eV (b) 1.00 eV (c) 2.00 eV (d) 4.00 eV

13. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because **[NCERT Exemplar]**

(a) of the electrons not being subject to a central force
(b) of the electrons colliding with each other
(c) of screening effects
(d) the force between the nucleus and an electron will no longer be given by Coulomb's law

14. An electron is trapped in a one dimensional infinite well of width 250 pm and is in its ground state. What is the longest wavelengths of light that can excite the electron from the ground state *via* a single photon absorption?

(a) $\lambda = \frac{4mL^2c}{h(n_i^2 - n_f^2)}$ (b) $\lambda = \frac{2mLc}{h(n_f^2 - n_i^2)}$

(c) $\lambda = \frac{8mL^2c}{h(n_f^2 - n_i^2)}$ (d) $\lambda = \frac{8mL}{h(n_f^2 - n_i^2)}$

15. What is the ratio of the shortest wavelength of the Balmer series to the shortest wavelength of the Lyman series?

(a) 1 : 4 (b) 2 : 4
(c) 4 : 1 (d) 1 : 1

16. In the ground state of the hydrogen atom, the electron has a total energy of −13.6 eV, its kinetic energy is

(a) 12.5 eV (b) 13.6 eV
(c) 14.9 eV (d) −27.2 eV

17. Two H atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is **[NCERT Exemplar]**

(a) 10.20 eV (b) 20.40 eV
(c) 13.6 eV (d) 27.2 eV

18. X-rays are produced in an X-ray tube by electrons accelerated through an electric potential difference of 50.0 kV. An electron makes three collisions in the target before coming to rest and loses half its remaining kinetic energy in each of the first two collisions. Determine the wavelengths of the resulting photon after the first two collisions. (Neglect the recoil of the heavy target atoms).

(a) 49.6 pm, 99.2 pm (b) 50.0 pm, 100 pm
(c) 30.6 pm, 60.2 pm (d) 77.7 pm, 114.4 pm

19. The X-rays are produced when 35.0 keV electrons strike a molybdenum ($Z = 42$) target. What value of λ_{min} result?

(a) 44.5 pm (b) 35.4 pm
(c) 50.0 pm (d) 47.7 pm

20. What is the binding energy per nucleon for ^{239}Pu. The atomic masses you will need are 239.052 16 u (^{239}Pu), 1.00783u (^{1}H) and 1.00867u (neutron)

(a) 8.35 MeV (b) 9.50 MeV
(c) 7.56 MeV (d) 5.96 MeV

21. Nuclear radii may be measured by scattering high energy electrons from nuclei. What is the de-Broglie wavelength for 200 MeV electrons?

(a) 8.28 fm (b) 7.98 fm (c) 6.45 fm (d) 6.20 fm

22. Consider an initially pure 3.4 g sample of ^{67}Ga, an isotope that has a half-life of 78 h. What is its initial decay rate?

(a) $8.00\times 10^{16}s^{-1}$ (b) $6.27\times 10^{16}s^{-1}$
(c) $7.53\times 10^{16}s^{-1}$ (d) $8.53\times 10^{15}s^{-1}$

23. Generally, more massive nuclides tend to be more unstable to alpha decay. For example, the most stable isotope of uranium, ^{238}U, has an alpha decay half-life of 4.5×10^9 yr. The most stable isotope of plutonium is ^{244}Pu with an 8.2×10^7 yr half-life. When half of an original sample of ^{238}U has decayed, what fractions of the original sample of isotopes of plutonium is left?

(a) 2.24×10^{-17} (b) 3.31×10^{-17}
(c) 4.43×10^{-17} (d) 5.54×10^{-17}

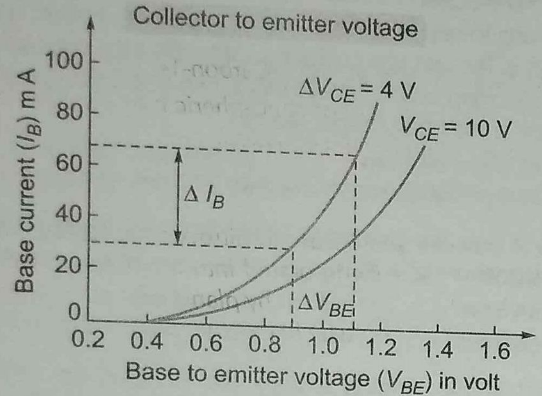
24. The cesium isotope ^{137}Cs is present in the fallout from above ground detonations of nuclear bombs. Because it decays with a slow (30.2 yr) half-life into ^{137}Ba, releasing considerable energy in the process, it is of environmental concern. The atomic masses of the Cs and Ba are 136.9071 and 136.9058 u, respectively; calculate the total energy released in such a decay.

(a) 1.21 MeV (b) 2.21 MeV (c) 3.33 MeV (d) 2.56 MeV

25. A photodiode is fabricated from a semiconductor with band gap of 2.8 eV. Can it detect a wavelength of 6000 nm?

(a) possible (b) not possible
(c) either possible (d) data insufficient

26. Calculate the input resistance of the transistor operating at $V_{CE} = 4\text{V}$ in CE configuration having its input characteristics as shown in figure



(a) 5756.6 Ω (b) 5656 Ω
(c) 5882.2 Ω (d) 6000 Ω

27. A semiconductor has the electron concentration of $6\times 10^{12}\ \text{cm}^{-3}$ and hole concentration of $9\times 10^{13}\ \text{cm}^{-3}$. What is the conductivity of this semiconductor? Given : Electron mobility $= 2.6\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ and hole mobility $= 0.02\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$.

(a) $1.754\ \text{Sm}^{-1}$ (b) $3.333\ \text{Sm}^{-1}$
(c) $5.000\ \text{Sm}^{-1}$ (d) $2.784\ \text{Sm}^{-1}$

28. On a particular day, the maximum frequency reflected from the ionosphere is 10 MHz. On another day, it was found to decrease to 8 MHz. What is the ratio of the maximum electron densities of the ionosphere on the two days?

(a) 20 : 10 (b) 30 : 15 (c) 25 : 16 (d) 24 : 11

29. A transmitting antenna at the top of a tower has a height 32 m and that of the receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in line of sight mode? Given radius of earth 6.4×10^6m.

(a) 50.0 km (b) 45.5 km
(c) 35.5 km (d) 30.2 km

30. A TV tower has a height of 75 m. What is the maximum area upto which this TV transmission can be received? Take radius of the earth as 6.4×10^6 m

(a) $3.02\times 10^3\text{km}^2$ (b) $2.02\times 10^3\text{km}^2$
(c) $3.02\times 10^4\text{km}^2$ (d) $4.02\times 10^3\text{km}^2$

31. The maximum peak to peak voltage of an AM wave is 26 mV and ninimum peak to peak voltage is 4 mV. Find modulation index.

(a) 0.62 (b) 0.73 (c) 0.83 (d) 0.93

32. Frequencies higher than 10 MHz are found not to be reflected by the ionosphere on a particular day at a place. What is the maximum electron density of the ionosphere?

(a) $2.025\times 10^{11}\text{e}^-\text{m}^{-3}$ (b) $2.235\times 10^{12}\text{e}^-\text{m}^{-3}$
(c) $1.235\times 10^{12}\text{e}^-\text{m}^{-3}$ (d) $3.235\times 10^{11}\text{e}^-\text{m}^{-3}$

Directions *(Q. Nos. 33 to 42) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

33. Statement I As intensity of incident light (in photoelectric effect) increases, the number of photoelectrons emitted per unit time increases.

Statement II More intensity of light means more energy per unit area per unit time.

34. Statement I Targets in X-ray tubes are made from high melting point metals.

Statement II Most of the energy of striking electrons is lost into collisions which simply appears as thermal energy.

35. Statement I The different lines of emission spectra (like Lyman, Balmer etc) of atomic hydrogen gas are produced by different atoms.

Statement II The sample of atomic hydrogen gas consists of millions of atoms.

36. Statement I When a heavy nucleus is decayed to stabilise, it emits α-particle rather than individual protons of $_2\text{He}^3$ nucleus.

Statement II Alpha particle is having high binding energy.

37. Statement I In α-decay, KE of emitted alpha particle is never equal to disintegration energy, Q of the reaction.

Statement II In α-decay, the daughter nucleus recoils as alpha particle is emitted to conserve the momentum.

38. Statement I Electron capture occurs more than positron emission in a heavy nucleus.

Statement II In a heavy nucleus, electrons are relatively close to nucleus.

39. Statement I A beam of charged particles is employed in the treatment of cancer.

Statement II Charged particles on passing through a material medium lose their energy by causing ionisation of the atoms along their path.

40. Statement I NAND or NOR gates are called digital building blocks.

Statement II The repeated use of NAND or NOR gates can produce all the basic or completed gates.

41. Statement I Microwave communication is preferred over optical communication.

Statement II Microwaves provide large number of channels and band width compared to optical signals.

42. Statement I Modem is a demodulator

Statement II It works only in a transmitting mode.

Directions (Q. Nos. 43 to 45) *A beam of light has three wavelengths 440 nm, 495 nm and 660 nm with a total intensity of 3.24×10^{-3} Wm^{-2} equally distributed amongst the three wavelengths. The beam falls normally on an area of 1.0 cm^2 of a clean metallic surface of work function 2.2 eV. Assume that there is no loss of light by reflection and each energetically capable photon ejects one electron and take $h = 6.6 \times 10^{-34}$ J-s.*

43. Photoelectric emission is caused by
(a) light of wavelength 440 nm alone
(b) light of wavelength 660 nm alone
(c) lights of wavelengths 440 nm and 495 nm
(d) lights of wavelengths 495 nm and 660 nm

44. The incident energy (in Js^{-1}) of each wavelength is
(a) 3.24×10^{-7}
(b) 1.62×10^{-7}
(c) 1.08×10^{-7}
(d) 0.81×10^{-7}

45. The total number of photoelectrons liberated per second is
(a) 4.9×10^{-11} (b) 5.1×10^{-11}
(c) 5.3×10^{-11} (d) 5.5×10^{-11}

Directions (Q. Nos. 46 to 47) *In hydrogen atom electron is orbiting in that orbit in which energy of Li^{+2} is equal to ground state energy of hydrogen. It now jumps to some higher energy state where its angular momentum becomes twice.*

46. What is the initial state of electron in hydrogen atom?
(a) 2 (b) 3
(c) 4 (d) 6

47. In final state
(a) radius of orbit becomes 4 times
(b) speed of electron will becomes half
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

Directions (Q. Nos. 48 to 50) *Carbon-14 (symbol $^{14}_{6}C$) is produced by the bombardment of atmospheric nitrogen with high energy neutrons according to the equation.*

$$^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}H$$

Radiocarbon is unstable and decays to nitrogen with a half-life of 5600 yr. The carbon-14 is incorporated into atmospheric carbon dioxide molecules which are taken in by plants when they breathe in carbon dioxide. Animals which eat the plants also take in carbon-14.

By measuring the ratio of the concentration of ^{14}C to ^{12}C in any ancient organism, say a tree, one can determine the date when the organism died.

48. A capsule contains 8 g of $^{14}_{6}C$ whose half-life is 5600 yr. After 16800 yr, the amount of $^{14}_{6}C$ left in the capsule will be
(a) 4 g (b) 2 g
(c) $\frac{8}{3}$ g (d) 1 g

49. Radiocarbon is produced in the atmosphere as a result of
(a) collisions between fast neutrons and nitrogen nuclei
(b) the action of cosmic rays on atmospheric oxygen
(c) the action of X-rays on carbon
(d) lighting discharge in atmosphere

50. Choose the only incorrect statement. In radioactive decay of an element
(a) α-particles may be emitted
(b) β-particles may be emitted
(c) γ-rays may be emitted
(d) the nucleus does not undergo any change

Hints & Solutions

1. (c) When $A = B = 0$, $Y = 1$. When $A = B = 1, Y = 0$

2. (c) Energy of photon,

$$E = \frac{hc}{\lambda} = \frac{1242}{350} \text{ eV} = 3.55 \text{ eV} = 5.68 \times 10^{-19} \text{ J}$$

Let n photons, per unit area per unit time are reaching the potassium surface, then

$$n = \frac{1.00}{5.68 \times 10^{-19}} = 1.76 \times 10^{18}$$

So, number of photons received by potassium surface per unit time is,

$n \times$ Area of potassium surface

$$= 1.76 \times 10^{18} \times 1 \times 10^{-4}$$
$$= 1.76 \times 10^{14}$$

Required number of photoelectrons emitted per unit time

$$= 1.76 \times 10^{14} \times \frac{0.5}{100}$$
$$= 8.8 \times 10^{11}$$

3. (c) The power of laser is 0.5 W, let n photons/s are incident by laser pulse on the broken ligament, then

$$n \times h\nu = 0.5 \text{ W}$$

$$\Rightarrow \quad \frac{nhc}{\lambda} = 0.5$$

$$\Rightarrow \quad n = \frac{0.5 \times \lambda}{hc}$$

$$= \frac{0.5 \times 632 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= 1.59 \times 10^{18} \text{ photons/s}$$

So, number of photons contained in 5 pulse are,

$$n \times 5 \times (20 \times 10^{3}) = 1.59 \times 10^{23}$$

4. (c) On the basis of Bohr's model,

$$r = \frac{n^2 h^2}{4\pi^2 mKZe^2} = a_0 \frac{n^2}{Z}$$

For Li^{++} ion, $Z = 3$; $n = 1$ for ground state

Given $a_0 = 53$ pm

$$\therefore \quad r = \frac{53 \times 1^2}{3} = 18 \text{ pm}$$

5. (d) The two inputs for the OR gate are $\bar{A} \cdot \bar{B}$ and C.

$$\therefore \quad Y = \bar{A} \cdot \bar{B} + C$$

6. (b) The energy taken by hydrogen atom corresponds to its transition from $n = 1$ to $n = 3$ state.

ΔE (given to hydrogen atom)

$$= 13.6\left(1 - \frac{1}{9}\right)$$

$$= 13.6 \times \frac{8}{9} = 12.1 \text{ eV}$$

7. (b) During 1st collision,

Initial energy of electron = 50 keV

Energy appearing as photon = 50% of 50 keV
= 25 keV

Energy lost in collision = 10% of 50 keV
= 5 keV

Energy left for 2nd collision
= (50 − 25 − 5) keV = 20 keV

For 2nd collision,

Initial energy = 20 keV

Energy of the emitted photon = 50% of 20 keV
= 10000 eV

So, required wavelength,

$$\lambda = \frac{1242}{10000} \text{nm} = 1.242 \text{ Å}$$

8. (a) As parent nucleus is at rest and emitted particle (α) carries some energy, daughter nucleus (Rn) recoils to conserve the momentum.

The energy released in the reaction appears in the form of kinetic energy of α-particle and the daughter nucleus.

$$Q = K_\alpha + K_D$$

From momentum conservation, $p_\alpha = p_D$

Solving above equation, we have

$$K_\alpha = \frac{M_D}{M_D + M_\alpha} \times Q$$

$$\Rightarrow \quad Q = \frac{M_D + M_\alpha}{M_D} \times K_\alpha = \frac{222 + 4}{222} \times 5.3 = 5.4 \text{ MeV}$$

9. (a) It follows from the logic symbol (A) that

$$X = \overline{\bar{A}\bar{B}}$$

for which the truth table is as follows

A	B	$\bar{A}$	$\bar{B}$	$\bar{A} \cdot \bar{B}$	$X = \overline{\bar{A}\bar{B}}$
0	0	1	1	1	0
1	0	0	1	0	1
0	1	1	0	0	1
1	1	0	0	0	1

This truth table satisfies the Boolean expression $X = A + B$, which is the OR gate. Hence, the logic symbol (A) is equivalent to an OR gate. It follows from logic symbol (B) that

$$X = \overline{\bar{A} \cdot \bar{B}} = A \cdot B$$

which is the Boolean expression for AND gate.

10. (c) In a hydrogen atom, electron revolving around a fixed proton nucleus has some centripetal acceleration. Therefore, its frame of reference is non inertial. In the frame of reference, where the electron is at rest, the given expression cannot be true for binding energy as the frame in which electron is at rest would not be inertial.

11. (*b*) The energy of an incident photon $E = hf = hc/\lambda$, kinetic energy of the most energetic electron emitted $K_m = E - \phi = (hc/\lambda) - \phi, V_0$ is related to kinetic energy by $eV_0 = K_m$,

so $eV_0 = (hc/\lambda) - \phi$ and

$$\lambda = \frac{hc}{eV_0 + \phi} = \frac{1240}{5.0 + 2.2} = 170 \text{ nm}$$

12. (*c*) $K_m = hf - \phi = (hc/\lambda) - \phi$

$$f = c/\lambda$$
$$K_m = \frac{1240}{200} - 4.20$$
$$= 2.00 \text{ eV}$$

13. (*a*) The simple Bohr model cannot be directly applied to calculate energy levels of an atom with many electrons. This is because all the electrons in the atom are not being subjected to one single central force.

14. (*c*) Energy levels are $E_n = n^2h^2 8/mL^2$,

$f = \Delta E/h = (h/8mL^2)(n_f^3 - n_i^2)$ and the wavelength of the light is

$$\lambda = \frac{c}{f} = \frac{8mL^2c}{h(n_f^2 - n_i^2)}$$

15. (*c*) The energy E of the photon emitted

$$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Frequency f of the electromagnetic wave $f = E/h$ and the wavelength $\lambda = c/f$.

Thus,

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E}{hc} = \frac{13.6}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For which $n_2 = \infty$. For the Balmer series, $n_1 = 2$ and the shortest wavelength is $\lambda_B = 4hc/13.6$. For the Lyman series, $n_1 = 1$ and the shortest wavelength is $\lambda_L = hc/13.6$. The ratio is $\lambda_B/\lambda_L = 4$

16. (*d*) If $a (= 5.292 \times 10^{-11} \text{m})$ is the Bohr radius, then the potential energy is $U = -\frac{e^2}{4\pi\varepsilon_0 a}$

$$= -\frac{(8.99 \times 10^9)(1.602 \times 10^{-19})^2}{5.292 \times 10^{-11}}$$
$$= -4.36 \times 10^{-18} = -27.2 \text{ eV}$$

17. (*a*) Initial KE of each of two hydrogen atoms in ground state = 13.6 eV.

∴ KE of both H atoms before collision = 2 × 13.6 eV = 27.2 eV.

As the collision is inelastic, linear momentum is conserved, but some KE is lost.

If one H atom goes over to first excited state and other remains in ground state, then their combined

$$\text{KE after collison} = \left(\frac{13.6}{2^2}\right) + \left(\frac{13.6}{1^2}\right) = 17.0 \text{ eV}$$

∴ Reduction in combined KE = 27.2 − 17.0 eV = 10.2 eV.

18. (*a*) Initial kinetic energy of the electron is 50.0 keV. After the first collision, the kinetic energy is 25 keV; after the second, it is 12.5 keV; and after the third it is zero. Energy of the photon produced in the first collision is 50.0 keV − 25.0 keV. The wavelength associated with this photon is

$$\lambda = \frac{1240}{25.0 \times 10^3}$$
$$= 4.96 \times 10^{-2} = 49.6 \text{ pm}$$

The energy of the photons produced in the second collision 12.5 keV and the wavelength associated with it is

$$\lambda = \frac{1240}{12.5 \times 10^3}$$
$$= 9.92 \times 10^{-2} \text{nm}$$
$$= 99.2 \text{ pm}$$

19. (*b*) The cut-off wavelength λ_{min} is characteristic of the incident electrons, not of the target material. This wavelength is the wavelength of a photon with energy equal to the kinetic energy of incident electron

$$\lambda_{min} = \frac{1240}{35 \times 10^3}$$
$$= 3.54 \times 10^{-2} \text{nm} = 35.4 \text{ pm}$$

20. (*c*) The binding energy is given by

$$\Delta E_{BE} = [Zm_H + (A - Z)m_n - M_{Pu}]c^2,$$

∴ $\Delta m = (94)(1.00783 \text{ u}) + (239 - 94)$
$(1.00867 \text{ u}) - (239.05216 \text{ u}) = 1.94101 \text{ u}$

$\Delta E_{BE} = (1.94101\text{u})(931.5 \text{ MeV/u}) = 1808$ MeV. Since, there are 239 nucleons, the binding energy per nucleon is

$$\Delta E_{BE} = \frac{E}{A} = \frac{1808}{239} = 7.56 \text{ MeV}$$

21. (*d*) The de-Broglie wavelength

$$\lambda = \frac{hc}{E} = \frac{1240}{200 \times 10^6}$$
$$= 6.20 \times 10^{-6} \text{nm} = 6.20 \text{ fm}$$

22. (*c*) Decay rate $R = \lambda N$,

$$R = R_0 = \lambda N_0,$$
$$\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(78 \text{ h})$$
$$= 8.89 \times 10^{-3} \text{h}^{-1} \text{ then } N_0 = M/m$$

Now, $m = (67\text{u})(1.661 \times 10^{-24} \text{g/u}) = 1.113 \times 10^{-22}$g

and $N_0 = (3.4 \text{ g})/(1.113 \times 10^{-22} \text{g}) = 3.05 \times 10^{22}$

Thus,

$$R_0 = (8.89 \times 10^{-3} \text{h}^{-1})(3.05 \times 10^{22})$$
$$= 2.71 \times 10^{20} \text{h}^{-1} = 7.53 \times 10^{16} \text{s}^{-1}$$

23. (*b*) $\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$,

$$\frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5 \times 10^9)}{8.2 \times 10^7} = 38.0$$

and $\frac{N}{N_0} = e^{-38.0} = 3.31 \times 10^{-17}$

24. (*a*) M_{Cs} be the mass of one atom of $^{137}Cs_{55}$ and M_{Ba} be the mass of one atom of $^{137}Ba_{56}$. To obtain the nuclear masses we must subtract the mass of 55 electrons from M_{Cs} and the mass of 56 electrons from M_{Ba}. The energy released is

$$Q = [(M_{Cs} - 55m) - (M_{Ba} - 56m) - m]c^2$$

where m is the mass of an electron. Once cancellations have been made $= (M_{Cs} - M_{Ba})c^2$ is obtained.

Thus,

$$Q = [136.9071\text{ u} - 136.9058\text{ u}]\,c^2$$
$$= (0.0013\text{u})c^2$$
$$= (0.0013\text{ u})(932\text{ MeV/u}) = 1.21\text{ MeV}$$

25. (*b*) $E = \frac{hc}{\lambda} = \frac{6.626\times 10^{-34}\times 3\times 10^8}{6000\times 10^{-9}}$

$= 3.313\times 10^{-20}$ J

$= \frac{3.313\times 10^{-20}}{1.6\times 10^{-19}}\text{eV} = 2.0706\times 10^{-1}\text{eV} = 0.21\text{ eV}$

Since $E < E_g$,

$\therefore$ Detection is not possible.

26. (*c*) From the figure and considering the dotted lines shown therein, we get

$\Delta V_{BE} \approx (1.1 - 0.9) \approx 0.2$ V

$\Delta I_B \approx (68 - 34) = 34\,\mu$A

$$r_i \approx \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}} \approx 5882.4\,\Omega$$

27. (*d*) Hole concentration > Electron concentration.

So, semiconductor is *p*-type.

Now, $e = 1.6\times 10^{-19}\text{C}, n_e = 6\times 10^{12}\times 10^6\text{m}^{-3}$

$= 6\times 10^{18}\text{ m}^{-3}$

$n_h = 9\times 10^{13}\times 10^6\text{m}^{-3} = 9\times 10^{19}\text{m}^{-3}$

$\mu_e = 2.6\text{ m}^2\text{V}^{-1}\text{s}^{-1}$ and $\mu_h = 0.02\text{ m}^2\text{V}^{-1}\text{s}^{-1}$

Again, $\sigma = e[n_e\mu_e + n_h\mu_h]$

or $\sigma = 1.6\times 10^{-19}[6\times 10^{18}\times 2.6 + 9\times 10^{19}\times 0.02]\text{ S m}^{-1}$

$= 1.6\times 10^{-19}\times 10^{18}[15.6 + 1.8]\text{ S m}^{-1}$

$= 1.6\times 17.4\times 10^{-1}\text{ S m}^{-1} = 2.784\text{ S m}^{-1}$

28. (*c*) $f_c = 10\text{ MHz}, f'_c = 8\text{ MHz}$

$$\frac{N_{max}}{N'_{max}} = \left(\frac{f_c}{f'_c}\right)^2 = \left(\frac{10}{8}\right)^2 = 25:16$$

29. (*b*) $d_m = \sqrt{2\times 64\times 10^5\times 32} + \sqrt{2\times 64\times 10^5\times 50}$ m

$= 64\times 10^2\times\sqrt{10} + 8\times 10^3\times\sqrt{10}$m

$= 144\times 10^2\sqrt{10}\text{m} = 45.5$ km

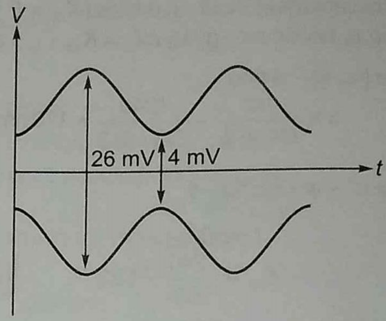
30. (*a*) $d = \sqrt{2Rh}$

$= \sqrt{2\times 6.4\times 10^6\times 75}$

$= 3.1\times 10^4\text{ m} = 31$ km

Area covered $= \pi d^2 = 3.02\times 10^3\text{ km}^2$

31. (*b*) Maximum voltage of AM wave,



$$V_{max} = \frac{26}{2}\text{mV} = 13\text{ mV}$$

Minimum voltage of AM wave,

$$V_{min} = \frac{4}{2}\text{mV} = 2\text{ mV}$$

$\therefore$ Modulation index, $m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$

$= \frac{13-2}{13+2} = \frac{11}{15} = 0.73$

32. (*c*) Critical frequency, $f_c = 10\text{ MHz} = 10^7$ Hz

$f_c = 9(N_{max})^{1/2}$

or $(N_{max})^{1/2} = \frac{f_c}{9}$

or $N_{max} = \left(\frac{f_c}{9}\right)^2$

or $N_{max} = \left(\frac{10^7}{9}\right)^2 e^-\text{m}^{-3}$

$= \frac{10^{14}}{81} e^-\text{m}^{-3} = 1.235\times 10^{12}\ e^-\text{m}^{-3}$

33. (*b*) Here both the statements are correct, but II is not the correct explanation for I. From quantum theory of light, as intensity of light increases means number of photons/ area × time increases and hence more photons take part in ejecting the photoelectrons thus increasing the number of photoelectrons.

34. (*a*) When highly energetic electrons strike the metal target, they (electron) suffer numerous collisions that heats up the metal. If melting point of metal target is low, then it may melt due to thermal energy which appears due to loss in kinetic energy of the moving electrons.

35. (*d*) A single atom can have only one transition at a time, we are observing different lines due to large number of transitions taking place simultaneously that occurred in different atoms of the sample.

36. (*a*) Alpha particles are emitted during decaying of heavy nucleus rather than individual protons or $^3_2$He nucleus because alpha particles have high binding energies. To escape from a nucleus, a particle must have high kinetic energy and only an alpha particle mass is sufficiently smaller than that of its constituent nucleons for such high energy to be available.

37. (*a*) In α-decay, the daughter nucleus recoils to conserve the momentum and

$$Q = K_\alpha + K_D$$

So, $K_\alpha < Q$

38. (*a*) In a heavy nucleus, the electrons are relatively close to nucleus, which promotes their interaction with the nucleus. Thus electron capture is more often than positron emission in decay of heavy nucleus.

39. (*b*) A radiation consists of a beam of charged particles. When radiation is used for cancer treatment, then on falling upon the cancerous tissues, it destroys the cancer cells. Statement II is also true.

40. (*a*) NAND or NOR gate are called universal (digital) building blocks because using repeated order of these two types of gates we can produce all the basis gates namely OR, AND and NOT gate or complex gates.

41. (*a*) Microwave communication is preferred over optical communication because microwaves provide large number of channels and wider band width compared to optical signals as information carrying capacity is directly proportional to band width. So, wider the band width, greater the information carrying capacity.

42. (*d*) Modem is a modulating and demodulating device. It acts as a modulator in transmitting mode as demodulator in receiving mode.

43. (*c*) The threshold wavelength is

$$\lambda_0 = \frac{hc}{W_0} = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{2.2\times1.6\times10^{-19}} = 6\times10^{-7}\text{ m} = 600\text{ nm}$$

Out of the three given wavelength, two wavelengths $\lambda_1 = 440$ nm and $\lambda_2 = 495$ nm will cause photoelectric emission as these wavelengths are less than λ_0.

44. (*c*) Intensity of each wavelength is

$$I = \frac{1}{3}\times 3.24\times10^{-3} = 1.08\times10^{-3}\text{ Wm}^{-2}.$$

Area of metal surface is $A = 1\text{ cm}^2 = 1\times10^{-4}\text{ m}^2$. Therefore, energy of each wavelength is $E = I\times A = 1.08\times10^{-7}\text{Js}^{-1}$

45. (*b*) Let n_1 be the number of photons of wavelength λ_1 incident per second. The energy of one photon $= \frac{hc}{\lambda_1}$

Hence, $E = \frac{n_1hc}{\lambda_1}$

or $n_1 = \frac{E\lambda_1}{hc} = \frac{(1.08\times10^{-7})\times(440\times10^{-9})}{(6.6\times10^{-34})\times(3\times10^{-8})}$

$= 2.4\times10^{11}$

Similarly, $n_2 = 2.7\times10^{11}$

∴ Total number $n = n_1 + n_2 = 5.1\times10^{11}$.

46. (*b*) Let E_0 be the energy of hydrogen atom in ground state.

Then, $E_0 = E_0\left(\frac{Z^2}{n^2}\right)$ $\left(\text{as } E\propto\frac{Z^2}{n^2}\right)$

or $n = Z$

or $n = 3$ (as $Z = 3$ for Li^{+2})

47. (*c*) $L_n = n\frac{h}{2\pi}$ or $L_n \propto n$

$r_n \propto \frac{n^2}{Z}$ while $v_n \propto \frac{Z}{n}$

L_n has doubled *i.e.*, n has become 2 times.

Hence, radius will become 4 times and speed will remain half.

48. (*d*) Number of half-lives $n = \frac{16800}{5600} = 3$

Therefore, the amount of C - 14 left

$$= \frac{8\text{ g}}{(2)^n} = \frac{8\text{ g}}{(2)^3} = 1\text{ g}$$

49. (*a*) Our atmosphere contains a large number of stable isotopes. When cosmic rays strike these isotopes, a number of radioisotopes are produced. One of these radioisotopes is carbon-14 which is produced by the bombardment of atmospheric nitrogen with high energy neutrons.

50. (*d*) In radioactive decay of an element α, β and γ-rays are emitted and the nucleus undergo changes, so option (d) is incorrect.

Day 38

Mock TEST 1

(Based on Complete Syllabus)

Instructions

1. *The test consists of 30 questions.*
2. *Candidates will be awarded marks for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
3. *There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response.*

1. A hoop of radius 2 m weight 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it? **[NCERT Exemplar]**

(a) 6.0 J (b) 2.5 J
(c) 4.0 J (d) 6.5 J

2. The length of the string of a simple pendulum is measured with a meter scale, is found to be 92.0 cm, the radius of the bob plus the hook is measured with the help of vernier calliper to be 2.17 cm. Mark out the correct statement.

(a) Least count of meter scale is 0.1 cm
(b) Least count of vernier callipers is 0.01 cm
(c) Effective length of simple pendulum is 94.2 cm
(d) All of the above

3. Two bodies *A* and *B* of equal mass are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitudes of *A* to that of *B* is

(a) $\frac{k_1}{k_2}$ (b) $\sqrt{\frac{k_1}{k_2}}$
(c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

4. What is the effect of increasing the intensity of the light that falls on the emitter in a photoelectric effect apparatus?

(a) Cut-off frequency decreases
(b) Stopping potential decreases
(c) Time delay for emission of photoelectron decreases
(d) Saturation photocurrent increases

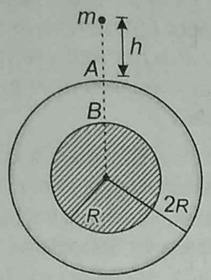
5. As the object moves from infinity to focus, then which is true, about the image formed by a single a concave mirror?

(a) Always real and speed of image continuously increases
(b) Always real and speed is initially smaller and finally larger than object speed
(c) Initially real and moving with speed smaller than object speed but later on image becomes virtual and moving with speed of object
(d) Always virtual and speed is less than object speed

6. Two bodies of different masses has been released from the top of tower. One is thrown in the horizontal direction while other is dropped, then which will reach the ground first?

(a) The body which has been thrown horizontally
(b) The body which has been dropped
(c) Both will reach the ground simultaneously
(d) Depends on the velocity with which the first body has been projected horizontally

Directions (Q. Nos. 7 to 9) *A solid sphere of mass M and radius R is surrounded by a spherical shell of the same mass M and radius 2R as shown. A small particle of mass m is released from rest from a height h (<< R) above the shell. There is a hole in the shell.*



7. In what time will it enter the hole at A?

(a) $2\sqrt{\frac{hR^2}{GM}}$ (b) $\sqrt{\frac{2hR^2}{GM}}$

(c) $\sqrt{\frac{hR^2}{GM}}$ (d) None

8. What time will it take to move from A to B?

(a) $= \frac{R^2}{\sqrt{GMh}}$ (b) $> \frac{R^2}{\sqrt{GMh}}$ (c) $< \frac{R^2}{\sqrt{GMh}}$ (d) None

9. With what approximate speed will it collide at B?

(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{GM}{2R}}$

(c) $\sqrt{\frac{3GM}{2R}}$ (d) $\sqrt{\frac{GM}{R}}$

10. An AC source producing emf

$$e = e_0 \,[\cos(100\,\pi\,\text{s}^{-1})t + \cos(500\,\pi\,\text{s}^{-1})t]$$

is connected in series with a capacitor and resistor. The steady state current in the circuit is found to be

$$I = I_1 \cos[(100\,\pi \text{s}^{-1})t + \phi_1] + I_2 \cos[(500\,\pi\,\text{s}^{-1})t + \phi_2]$$

(a) $I_1 > I_2$
(b) $I_1 = I_2$
(c) $I_1 < I_2$
(d) The information is insufficient to find the relation between I_1 and I_2

11. A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km/h. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 m/s. **[NCERT Exemplar]**

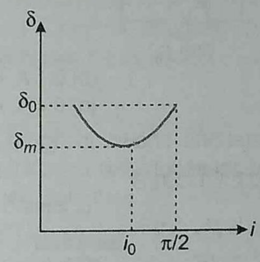
(a) 40.00 kHz (b) 50.53 kHz
(c) 45.93 kHz (d) 55.63 kHz

12. If electric potential due to some charge distribution is given by $V = 3/r^2$, where r is radial distance, then find electric field at (1, 1, 1)

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2\,(\hat{i}+\hat{j}+\hat{k})}{3}$

(c) $\frac{2}{8\,(\hat{i}+\hat{j}+\hat{k})}$ (d) $\frac{3}{2\,(\hat{i}+\hat{j}+\hat{k})}$

13. In the diagram, a plot between δ (deviation) *versus* i (angle of incidence) for a triangular prism is given. From the observed plot, some conclusions can be drawn. Mark out the correct conclusions.

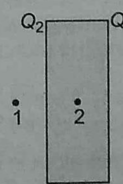


(a) The range of deviation for which two angles of incidence are possible for same deviation is $\delta_0 - \delta_m$
(b) The curve is unsymmetrical about i_0
(c) For a given δ, i is unique
(d) Both (a) and (b) are correct

14. Mark the correct option.

(a) When the branch of the circuit containing battery is open, then potential difference across the terminals of battery is same as emf
(b) When two terminals of a battery are shorted, then potential difference across the terminals of battery is zero
(c) If external resistance of the circuit changes, then potential difference across the terminals of the battery changes but emf remains same
(d) All of the above

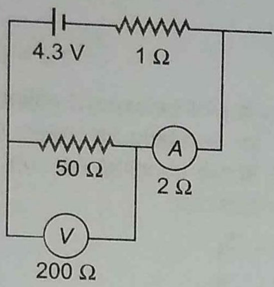
15. A non-conducting plate (infinite plane plate) is given a charge in such a way that Q_1 appears on one side and Q_2 on other side. The face area of plate is A. Find the electric field at points 1 and 2.



(a) $\frac{Q_1+Q_2}{2\varepsilon_0 A}, \frac{Q_2-Q_1}{2\varepsilon_0 A}$ (b) $\frac{Q_1-Q_2}{2\varepsilon_0 A}, \frac{Q_1+Q_2}{2\varepsilon_0 A}$

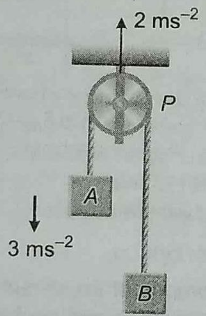
(c) $\frac{Q_1+Q_2}{\varepsilon_0 A}, \frac{Q_2-Q_1}{\varepsilon_0 A}$ (d) $\frac{Q_1-Q_2}{\varepsilon_0 A}, \frac{Q_1+Q_2}{\varepsilon_0 A}$

16. The emf and internal resistance of the battery shown in figure are 4.3 V and 1Ω respectively. The external resistance R is 50 Ω. The resistance of the voltmeter and ammeter are 200 Ω and 2 Ω respectively. Find the readings of the two meters.



(a) 0.1 A, 2 V (b) 0.1 A, 4 V
(c) 0. 4 A, 1 V (d) 0.4 A, 4 V

17. All the accelerations shown in figure are with respect to ground, find acceleration of B.



(a) 3 ms^{-2}, upward (b) 5 ms^{-2}, upward
(c) 3 ms^{-2}, downward (d) None of these

18. Three dielectric slabs of thickness $d/4$, $d/7$ and $d/2$ having dielectric constants 2, 8/7 and 4 respectively are inserted between the plates of a parallel plate capacitor having plate separation d and plate area A. Find the capacitance of the system.

(a) $\frac{118\varepsilon_0 A}{75d}$ (b) $\frac{88\varepsilon_0 A}{63d}$ (c) $\frac{226\varepsilon_0 A}{135d}$ (d) $\frac{284\varepsilon_0 A}{75d}$

19. The ratio of the acceleration due to gravity at the bottom of a deep mine and that on the surface of the earth is 978/980. Find the depth of the mine if the density of the earth is uniform throughout and the radius of the earth is 6300 km.

(a) 12.86 km (b) 13.0 km
(c) 25.38 km (d) 90.9 km

20. An electron of hydrogen atom is considered to be revolving around the proton in the circular orbit of radius $\frac{h^2}{4\pi^2 me^2}$ with velocity $\frac{2\pi e^2}{h}$. The equivalent current due to circulating charge is

(a) $\frac{4\pi^2 me^4}{h^3}$ (b) $\frac{4\pi^2 me^5}{h^3}$ (c) $\frac{4\pi^2 m^2 e^4}{h^3}$ (d) None

21. A sky wave with a frequency 55 MHz is incident on D-region of earth's atmosphere at 45°. The angle of refraction is (electron density for D-regions is 400 electron/cm^3)

(a) 60° (b) 45° (c) 30° (d) 15°

Directions (Q. Nos. 22 to 23) *Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

22. Statement I The rocket works on the principle of conservation of linear momentum.

Statement II Whenever there is the change in momentum of one body, the same change occurs in the momentum of the second body of the same system (having two bodies only) but in opposite direction.

23. Statement I If the half-life of a radioactive substance is 40 days, then 75% substance decays in 20 days.

Statement II $N = N_0 \left(\frac{1}{2}\right)^n$

where, $n = \frac{\text{Time elapsed}}{\text{Half-life period}}$

24. The voltage gain of an amplifier without feedback is 100. If a negative feedback is introduced, with a feedback fraction $\beta = 0.1$, then the gain of the feedback amplifier is

(a) 9.09 (b) 10 (c) 100.1 (d) 90.0

25. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. Find the effective thermal conductivity of the system.

(a) $K_1 + K_2$ (b) $\frac{K_1 K_2}{(K_1 + K_2)}$
(c) $\frac{(3K_1 + K_2)}{4}$ (d) $\frac{(K_1 + 3K_2)}{4}$

26. A 20 g bullet pierces through plate of mass $m_1 = 1$ kg and then comes to rest inside a second plate of mass $m_2 = 2.98$ kg. It is found that the two plates, initially at rest, now move with equal velocities. The percentage loss in the initial velocity of bullet when it is between m_1 and m_2 (neglect any loss of material of the bodies, due to action of bullet) will be

(a) 20% (b) 25% (c) 30% (d) 45%

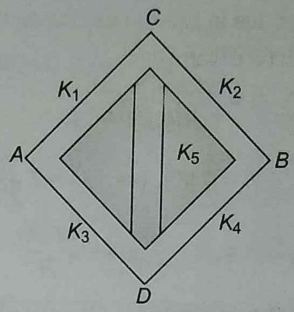
27. Two clocks are being tested against a standard clock located in a national laboratory. At 12 : 00 : 00 noon by the standard clock, the readings of the two clocks are

	Clock I	Clock II
Monday	12 : 00 : 05	10 : 15 : 06
Tuesday	12 : 01 : 15	10 : 14 : 59
Wednesday	11 : 59 : 08	10 : 15 : 18
Thursday	12 : 01 : 50	10 : 15 : 07
Friday	11 : 59 : 15	10 : 14 : 53
Saturday	12 : 01 : 30	10 : 15 : 24
Sunday	12 : 01 : 19	10 : 15 : 11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer?

(a) Clock I
(b) Clock II
(c) Both have same precision time interval measurements
(d) None of the above

28. Five rods of same dimensions are arranged as shown in the figure. They have thermal conductivities K_1, K_2, K_3, K_4 and K_5. When the points A and B are maintained at different temperatures, no heat flows through the central rod if



(a) $K_1 K_4 = K_2 K_3$ (b) $K_1 = K_4$ and $K_2 = K_3$
(c) $\frac{K_1}{K_4} = \frac{K_2}{K_3}$ (d) $K_1K_2 = K_3K_4$

29. A block of wood has a mass of 25 g. When a 5 g metal piece with a volume of 2 cm^3 is attached to the bottom of the block, the wood barely floats in water. What is the volume V of the wood?

(a) 28 cc (b) 35 cc (c) 48 cc (d) 12 cc

30. A body dropped from a height H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate the work done by air-friction.

(a) 2.8 mgH (b) $-1.3\ mgH$
(c) 1.3 mgH (d) $-40\ mgH$

Answer *with* Solutions

1. (*c*) Given, $R = 2$ m

$M = 100$ kg

Speed of centre of mass $(v) = 20$ cm/s $= 0.20$ m/s

Work done to stop the hoop = Total kinetic energy of the hoop

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

But moment of intertia $I = Mr^2$ and angular velocity $\omega = \frac{v}{R}$

$$\therefore \quad W = \frac{1}{2}Mv^2 + \frac{1}{2}(Mr^2)\times\left(\frac{v^2}{R^2}\right)$$

$$\therefore \quad = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

$$= 100 \times (0.20)^2 = (100 \times 0.04)\text{ J} = 4.0\text{ J}$$

2. (*c*) Effective length of the pendulum is $(92.0 + 2.17)$ cm $= 94.2$ cm after rounding off to 3 significant digits.

3. (*d*) Maximum velocity $= a\omega = a\sqrt{\frac{k}{m}}$

Given that, $a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}}$

$$\Rightarrow \quad \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

4. (*d*) Intensity cannot change cut-off frequency or stopping potential. High intensity means more number of photons, so chances of hitting to electron by them increases and hence more photoelectrons are emitted *i.e.,* photocurrent increases.

5. (*b*) As the object moves from infinity to centre of curvature, the image formed by a concave mirror would be real and is moving from focus to centre of curvature, but as the object crosses centre of curvature and moves towards focus the image is still real but moves from centre of curvature towards infinity and when the object is at focus the real image would be formed at infinity. So, image speed is smaller in beginning when the object is moving from infinity to centre of curvature and increases thereafter.

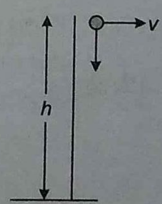
6. (*c*) Since, vertical displacement is same, as well as initial velocity in vertical downward direction is zero for both the bodies.

So, $h = \frac{gt_1^2}{2}$

(for horizontal throwing)

$h = \frac{gt_2^2}{2}$ (for dropping)

$\therefore \quad t_1 = t_2$



7. (*a*) Acceleration due to gravity near the surface of shell can be assumed to be uniform ($h << R$).

$$g = \frac{G(2M)}{(2R)^2} = \frac{GM}{2R^2}$$

From $$h = \frac{1}{2}gt^2,\ t = \sqrt{\frac{2h}{g}} = 2\sqrt{\frac{hR^2}{GM}}$$

8. (*c*) $$u_A = \sqrt{2gh} = \sqrt{2 \times \frac{GM}{2R^2}h} = \frac{\sqrt{GMh}}{R}$$

From A to B, field due to shell is zero, but field due to sphere is non-zero.

Hence, $$t_{AB} < \frac{R}{u_A} < \frac{R^2}{\sqrt{GMh}}$$

9. (*d*) $K_A \approx 0$: Potential between A and B due to shell is constant. From energy conservation we can write,

$$K_A + U_A = K_B + U_B \quad \text{or} \quad K_B = U_A - U_B = m(V_A - V_B)$$

or $$\frac{1}{2}mv_B^2 = m(V_A - V_B)$$

$\therefore$ $$v_B = \sqrt{2(V_A - V_B)} = \sqrt{2\left[-\frac{GM}{2R} + \frac{GM}{R}\right]} = \sqrt{\frac{GM}{R}}$$

10. (*c*) $$I_1 = \frac{e_0}{\sqrt{R^2 + \left(\frac{1}{\omega_1 C}\right)^2}} = \frac{e_0}{Z_1}, \quad \text{where } \omega_1 = 100\pi$$

$$I_2 = \frac{e_0}{\sqrt{R^2 + \left(\frac{1}{\omega_2 C}\right)^2}} = \frac{e_0}{Z_2}, \text{ where } \omega_2 = 500\pi$$

So, $Z_1 > Z_2$, therefore $I_1 < I_2$.

11. (*a*) SONAR frequency, $V_s = 40\text{ kHz} = 40 \times 10^3$ Hz

Speed of enemy submarine

$$v_e = 360\text{ km/h} = 360 \times \frac{5}{18}\text{ m/s} = 100\text{ m/s} \quad \left(\because 1\text{ km/h} = \frac{5}{18}\text{m/s}\right)$$

Speed of sound in water = 1450 m/s

Apparent frequency received by the submarine

$$V' = \left(\frac{v + v_0}{v}\right)V = \left(\frac{1450 + 100}{1450}\right) \times 40 \times 10^3 = 42.76 \times 10^3 \text{ Hz}$$

Now, the reflected waves have a different frequency

$$V'' = \left(\frac{v}{v + v_s}\right)V$$

Here, $v_s = 100$ m/s is velocity of enemy submarine

$$V'' = \left(\frac{1450}{1450 - 100}\right) \times 42.76 = 45.93 \times 10^3 \text{ Hz} = 45.93 \text{ kHz}$$

12. (*b*) As, $$V = \frac{3}{r^2}$$

$\therefore$ $$E = -\left(\frac{dV}{dr}\right)\hat{r} = -\frac{\partial}{\partial r}\left[\frac{3}{r^2}\right]\hat{r} = \frac{6}{r^4}\hat{r}$$

$\Rightarrow$ $$E = 6\frac{(\hat{i} + \hat{j} + \hat{k})}{(\sqrt{3})^4} = \left(\frac{2}{3}\right)(\hat{i} + \hat{j} + \hat{k})$$

13. (*d*)

14. (*d*) In the option (a), the circuit is open, as a result no current flows through it, so the potential difference across internal resistor is zero and hence, terminal potential difference is same as emf of battery.

For option (b), the terminal potential difference gets zero as battery is shorted although emf remains same.

In option (c), discharging of battery takes place, current has been withdrawn from battery, whose value is depending on the value of resistance, which inturn changes the value of terminal potential difference but emf remains same.

15. (*a*) At 1; $$E_1 = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_1}{2\varepsilon_0} = \frac{Q_1 + Q_2}{2A\varepsilon_0} \text{ towards left}$$

At 2; $$E_2 = \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_1}{2\varepsilon_0} = \frac{Q_1 - Q_2}{2A\varepsilon_0} \text{ towards right}$$

$$\sigma_2 = \frac{Q_2}{A} \quad \text{and} \quad \sigma_1 = \frac{Q_1}{A}$$

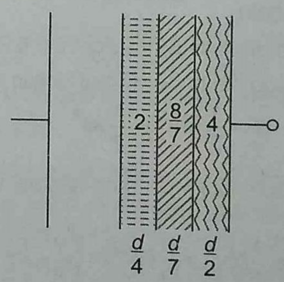
16. (*b*) First of all draw the equivalent circuit diagram, current flowing through circuit $$= \frac{4.3}{(50 \,||\, 200 + 2 + 1)} = 0.1 \text{ A}$$

Voltmeter reading = $4.3 - 0.1 \times 3 = 4$ V

17. (*d*) Consider downward direction as positive

$$\mathbf{a}_{AP} = -\mathbf{a}_{BP}$$
$$\mathbf{a}_{AG} = \mathbf{a}_{AP} + \mathbf{a}_{PG}$$
$$3 = \mathbf{a}_{AP} - 2$$
$\Rightarrow$ $$\mathbf{a}_{AP} = 5\text{ ms}^{-2}$$
$\therefore$ $$\mathbf{a}_{BG} = \mathbf{a}_{BP} + \mathbf{a}_{PG} = -5 - 2 = -7\text{ ms}^{-2}$$

18. (*d*) Here, three slabs are in series



$$C_1 = \frac{\varepsilon_0 A}{\left(\frac{d/4}{2} + \frac{d/7}{8/7} + \frac{d/2}{4}\right)} = \frac{\varepsilon_0 A}{\left(\frac{d}{8} + \frac{d}{8} + \frac{d}{8}\right)} = \frac{8\varepsilon_0 A}{3d}$$

$$C_2 = \frac{\varepsilon_0 A}{\left[d - \left(\frac{d}{4} + \frac{d}{7} + \frac{d}{2}\right)\right]} = \frac{28\varepsilon_0 A}{25d}$$

Now, $$C_{eq} = C_1 + C_2 = \frac{284\varepsilon_0 A}{75d}$$

19. (*a*) $$\frac{g'}{g} = \frac{978}{980} = 1 - \frac{d}{R} \quad \text{or} \quad \frac{d}{R} = 1 - \frac{978}{980} = \frac{2}{980}$$

or $$d = \frac{2R}{980} = \frac{2 \times 6300}{980} = 12.86 \text{ km}$$

20. (*b*) As, $T = \frac{2\pi r}{v} = \frac{2\pi \times h^2}{4\pi^2 me^2} \times \frac{h}{2\pi e^2} = \frac{h^3}{4\pi^2 me^4}$

$\therefore$ Current, $I = \frac{e}{T} = \frac{4\pi^2 me^5}{h^3}$

21. (*b*) $n_{\text{eff}} = n_0 \sqrt{1 - \left(\frac{80.5\,N}{v^2}\right)}$

$$= 1\sqrt{1 - \frac{80.5 \times (400 \times 10^6)}{(55 \times 10^6)^2}} = 1$$

Also $n_{\text{eff}} = \frac{\sin i}{\sin r}$

$\Rightarrow$ $\sin i = \sin r \Rightarrow r = i = 45°$

22. (*a*) Since, in the rocket fuel is undergoing combustion, the gases produced in this process leave the body of the rocket with large velocity and produce upthrust to the rocket. Let us assume that the fuel is undergoing combustion at the constant rate, then rate of change of momentum of the rocket will be constant. Since, more and more fuel will be burnt the mass of rocket will go on decreasing, so it will lead to increase the velocity of the rocket more and more rapidly.

23. (*d*) Here, $N = N_0 \left(\frac{1}{2}\right)^{t/T}$ or $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$ ...(i)

where T is the half-life period and $\frac{N}{N_0}$ is fraction of atoms left after time t. Here, $T = 40$ days and $\frac{N}{N_0} = \frac{25}{100} = \frac{1}{4} = 0.25$

Putting the values of T and $\frac{N}{N_0}$ in Eq. (i), we get

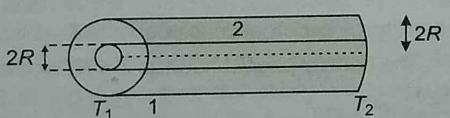
$$\frac{1}{4} = \left(\frac{1}{2}\right)^{t/40}$$

or $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/40}$ or $\frac{t}{40} = 2$

or $t = 80$ days

24. (*a*) $A_f = \frac{A}{1 + \beta A} = \frac{100}{1 + 0.1 \times 100} = \frac{100}{11} = 9.09$

25. (*d*) This can be considered as a parallel combination of two, one the inner cylinder and the other surrounding cylinder.



$A_1 = \pi R^2$,

$A_2 = \pi (4R^2 - R^2) = 3\pi R^2$

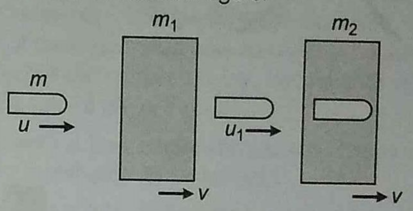
$L_1 = L_2 = L$

Heat is flowing only along the length of tube

$$H_{eq} = H_1 + H_2$$
$$= \frac{K_1 A_1 (T_1 - T_2)}{L} + \frac{K_2 A_2 (T_1 - T_2)}{L}$$

$\Rightarrow$ $K_{eq} \times 4 = K_1 + 3K_2 \Rightarrow K_{eq} = \frac{(K_1 + 3K_2)}{4}$

26. (*b*) The situation is shown in figure.



First, take 1st sheet and bullet as the system, $mu = mu_1 + m_1 v$
Now, take 2nd sheet and bullet as the system $mu_1 = (m_1 + m_2)v$
Solving this equation, we get

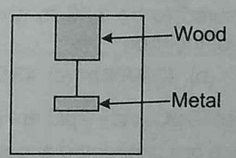
Percentage loss in $u = \frac{(u - u_1)}{u} \times 100\% = 25\%$

27. (*b*) A simple look at the data shows that average reading of clock I is much closer to the standard time that average reading of clock II. Therefore, zero error in clock II is much larger than the zero error in clock I. But zero error can always be corrected by applying necessary correction. Now, over the seven days, range of variation of clock I is 12 : 01 : 50 – 11 : 59 : 08 = 162 s and range of variation of clock II is 10 : 15 : 24 – 10 : 14 : 53 = 31 s. Therefore, precision of clock II is much better than the precision of clock I. We will prefer clock II.

28. (*a*) The arrangement of rods is analogous, the arrangement of resistances in a Wheatstone bridge balanced condition. Thus, no heat flows through the rod conductivity K_5, then

$$\frac{K_1}{K_3} = \frac{K_2}{K_4} \Rightarrow K_1 K_4 = K_2 K_3$$

29. (*a*) Let volume of wood is V cm^3, then total volume of displaced water is $(V + 2)$ cm^3, then for translational equilibrium,



$$(V + 2)\rho g = (25g + 5g)$$

where all the quantities are in CGS unit and ρ is the density of water.

$\Rightarrow$ $(V + 2) \times 1 = 30$

$\therefore$ $V = 28$ cm^3

30. (*d*) The forces acting on the body are
force of gravity and air-friction
According to work-energy theorem,
total work done on the body = Gain in Kinetic energy

$$W = \frac{1}{2}mv^2 = \frac{1}{2}m(12\sqrt{gH})^2 = 0.72\,mgH$$

As work done by gravity, $W_1 = mgH$

$\therefore$ Work done by friction,

$$W_2 = W - W_1$$
$$= 0.72\,mgH - mgH = -0.28\,mgH$$

Day 39

Mock TEST 2

(Based on Complete Syllabus)

Instructions

1. *The test consists of 30 questions.*
2. *Candidates will be awarded marks for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
3. *There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response.*

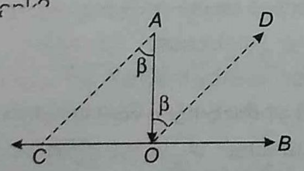
1. A carpet is to be installed in a room whose length is measured to be 12.71 m (four significant figures) and whose width is measured to be 3.46 m (three significant figures). Find the area of the room.

(a) $43.97\ m^2$ (b) $43.9766\ m^2$ (c) $43.98\ m^2$ (d) $44.0\ m^2$

2. If the charge of $10\mu C$ and $-2\mu C$ are given to two plates of a capacitor which are connected across a battery of 12 V, find the capacitance of the capactitor.

(a) 0.33 µF (b) 0.5 µF (c) 0.41 µF (d) 0.66 µF

3. A man can swim with a speed of 4 km/h in still water. How long does he take to cross a river 1 km wide, if the river flows steadily 3 km/h and he makes his strokes normal to the river current. How far down the river does he go when he reaches the other bank? **[NCERT Exemplar]**



(a) 600 km (b) 750 km (c) 800 km (d) 850 km

4. A body when projected vertically up, covers a total distance s, during its time of flight. If we neglect gravity then how much distance the particle will travel during the same time. Will it fall back?

(a) s, Yes (b) s, No (c) $2s$, Yes (d) $2s$, No

Directions *(Q. Nos. 5 to 7) Atomic number (Z) of an element is the number of protons present in the nucleus of an atom of the element. Mass number (A) is total number of protons and neutrons present in the nucleus of the atom of the element. The size of nucleus is given by $R = R_0 A^{1/3}$, where R_0 = a constant* $= 1.2 \times 10^{-15}$ m. Nucleus density $\rho = \frac{3m}{4\pi R_0^3}$ = constant = 2.20 kg / m_3.

5. Number of neutrons in a gold nucleus with $A = 197$ and $Z = 79$ is

(a) 79 (b) 197
(c) 118 (d) None of these

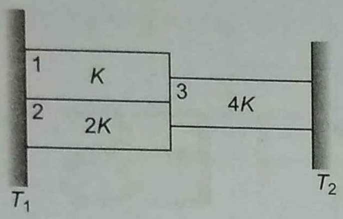
6. The density of hydrogen nucleus with $Z = 1$ is 2.29×10^{17} kgm^{-3}. The density of gold nucleus $Z = 79$ would be

(a) $\frac{2.29}{79} \times 10^{17}\ kg/m^3$
(b) $2.29 \times 79 \times 10^{17}\ kg/m^3$
(c) $2.29 \times 10^{17}\ kg/m^3$
(d) $\frac{2.29}{\sqrt{79}} \times 10^{17}\ kg/m^3$

7. If mass of a proton is 1.007825 amu and mass of a neutron is 1.008665 amu, then mass of $_3Li^7$ nucleus approximately be

(a) 7.058075 amu (b) 7.000000 amu
(c) 7.023475 amu (d) 7.034600 amu

8. Find the equivalent thermal resistance of the combination of rods as shown in the figure. Every rod has the same length l and cross-sectional area A. Thermal conductivities are mentioned in figure.



(a) $\frac{l}{4KA}$ (b) $\frac{7l}{4KA}$

(c) $\frac{7l}{12KA}$ (d) $\frac{l}{12KA}$

9. A particle is kept at rest at a distance R (earth's radius) above the earth's surface. The minimum speed with which it should be projected so that it does not return is

(a) $\sqrt{\frac{GM}{4R}}$ (b) $\sqrt{\frac{GM}{2R}}$

(c) $\sqrt{\frac{GM}{R}}$ (d) $\sqrt{\frac{2MG}{R}}$

10. In J J Thomson's experiment, a potential difference of 320 V is accelerating the electron. The electron beam is entering a region having uniform magnetic field 6×10^{-5} T acting perpendicular to it. Find the value of electric field in this region so that the electron does not experience any deflection. ($m_e = 9.1 \times 10^{-31}$ kg)

(a) 640 Vm^{-1} (b) 642 Vm^{-1}
(c) 637 Vm^{-1} (d) 644 Vm^{-1}

11. A wire of length 100 cm is connected to a cell of emf 2V and negligible internal resistance. The resistance of the wire is 3Ω. The additional resistance required to produce a potential difference of 1 mV/cm is

(a) 47 Ω (b) 57 Ω (c) 60 Ω (d) 55 Ω

12. A particular piano string is supposed to vibrate at a frequency of 440 Hz. In order to check its frequency, a tuning fork known to vibrate at a frequency of 440 Hz is sounded at the same time the piano key is struck, and a beat frequency of 4 beats/s is heard. Find the possible frequencies at which the string could be vibrating.

(a) 444 Hz, 436 Hz (b) 440 Hz, 436 Hz
(c) 444 Hz, 440 Hz (d) 449 Hz, 440 Hz

13. A uniform rectangular marble slab is 3.4m long and 2.0 m wide. It has a mass of 180 kg. If it is originally lying on the flat ground, how much work is needed to stand it on one end?

(a) 2.0 kJ (b) 3.0 J
(c) 3.0 kJ (d) 3000 kJ

Directions *(Q. Nos. 14 and 15) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

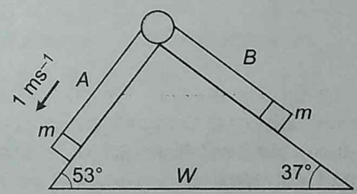
14. Statement I When the range of projectile is maximum, the time of flight is the largest.

Statement II Range is maximum when angle of projection is 45°.

15. Statement I 1 amu is equivalent to 931 Mev.

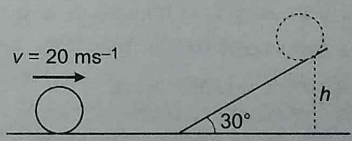
Statement II Energy equivalent (E) or mass (m) is $E = mc^2$

16. The two blocks shown in figure have equal masses and $\mu_s = \mu_k = 0.3$ for both blocks. Wedge W is fixed and block A is given initial speed of 1 ms^{-1} down the plane. How far will it move before coming to rest if inclines and strings are quite long ? [take $g = 10$ ms^{-2}]



(a) 0.45 m (b) 0.9 m
(c) + 4.8 m (d) Question is irrelevant

17. As shown in figure, a uniform solid sphere rolls on a horizontal surface at 20 ms^{-1}. It then rolls up the incline shown. What will be the value of h where the ball stops?



(a) 28.6 m (b) 8.6 m
(c) 6 m (d) 18.6 m

18. A 1.6 kg block on a horizontal surface is attached to a spring with a spring constant of 1.0×10^3 Nm^{-1}. The spring is compressed to a distance of 2.0 cm, and the block is released from rest. Calculate the speed of the block as it passes through the equilibrium position, $x = 0$, if the surface is frictionless.

(a) 0.75 ms^{-1} (b) 0.50 ms^{-1}
(c) 0.25 ms^{-1} (d) 2.25 ms^{-1}

19. A small conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is

(a) clockwise
(b) anti-clockwise
(c) zero
(d) clockwise or anti-clockwise depending on whether the current is increased or decreased

20. An inductor coil joined to a 6 V battery draws a steady current of 12 A. This coil is connected to a capacitor and an AC source of rms voltage 6 V in series. If the current in the circuit is in phase with the emf, the rms current will be

(a) 16.9 A (b) 12 A
(c) 8 A (d) 9.87 A

21. Two wires, each having a weight per unit length of 1.0×10^{-4} Nm^{-1}, are strung parallel to one another above earth's surface, one directly above the other. The wires are aligned in a north-south direction so that earth's magnetic field will not affect them. When their distance of separation is 0.10 m, what must be the current in each in order for the lower wire to levitate the upper wire? Assume that the wires carry the same currents, travelling in opposite directions.

(a) 2.7 A
(b) 0.1 A
(c) 3.5 A
(d) 7.1 A

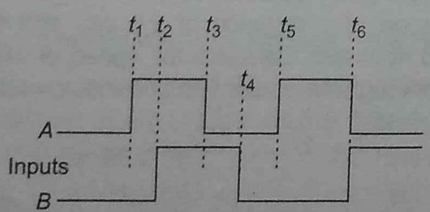
22. Calculate the minimum thickness of a soap-bubble film ($n = 1.33$) that will result in constructive interference in the reflected light if the film is illuminated by light with a wavelength in free space of 602 nm.

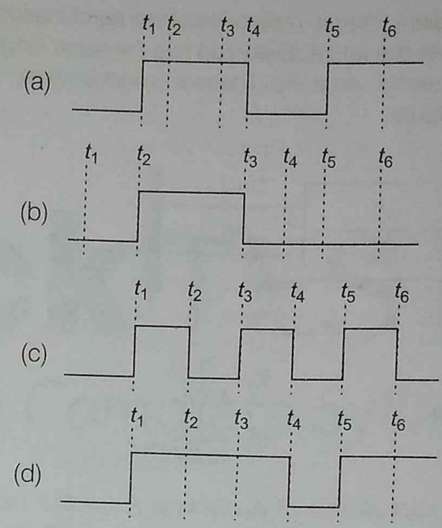
(a) 98 nm (b) 113 nm
(c) 125 nm (d) 25 nm

23. A ground receiver station is receiving a signal at 100 MHz, transmitted from a ground transmitter at a height of 300m located at a distance of 100km. Then [$N_{max} = 10^{12}$ per m^3]

(a) signal is coming *via* space wave
(b) signal is coming *via* sky wave
(c) signal is coming *via* satellite transponder
(d) None of the above

24. The output waveform (Y) of AND gate for the following inputs A and B given below is





25. A sled and its rider together weigh 800 N. They move down on a frictionless hill through a vertical distance of 10.0 m. Use conservation of mechanical energy to find the speed of the sled at the bottom of the hill, assuming the rider pushes off with an initial speed of 5.00 ms^{-1}. Neglect air resistance

(a) 21.5 ms^{-1} (b) 14.9 ms^{-1} (c) 4.9 ms^{-1} (d) 20.3 ms^{-1}

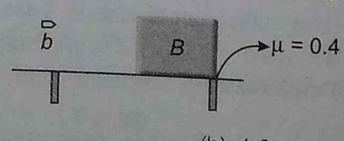
26. Satellite dishes do not have to change directions in order to stay focussed on a signal from a satellite. This means that the satellite always has to be found at the same location with respect to the surface of earth. For this to occur, the satellite must be at a height such that its revolution period is the same as that of earth, 24 h. At what height must the satellite be so to achieve this?

(a) $\left(\frac{T^2}{4\pi^2} GM_e\right)^{1/3}$ (b) $\left(\frac{T^2}{4\pi^2} GM_e\right)$
(c) $\left(\frac{T^2}{4\pi^2} GM_e\right)^{1/2}$ (d) $\left(\frac{T}{4\pi^2} GM_e\right)^{1/3}$

27. Water with a mass of 2.0 kg is held at constant volume in a container while 10.0 kJ of energy is slowly added by a flame. The container is not well insulated, and as a result 2.0 kJ of energy leaks out to the surroundings. What is the temperature increase of water?

(a) 0.28° C (b) 27° C (c) 0.96° C (d) 1.27° C

28. A 20 g bullet is fired horizontally with a speed of 600 ms^{-1} into a 7 kg block on a table top. The bullet b lodges in the block B. If the coefficient of kinetic friction between the block and the table top is 0.4, what is the distance the block will slide?

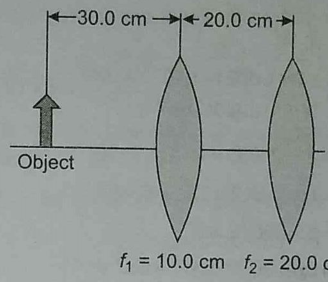


(a) 0.5 m (b) 1.2 m
(c) 0.37 m (d) 0.85 m

29. A uniform rope, of mass m per unit length, hangs vertically from a support so that the lower end just touches the table top. If it is released, then at the time a length y of the rope has fallen, the force on the table is equivalent to the weight of the length ky of the rope. Find the value of k.

(a) 1 (b) 2
(c) 3 (d) 3.5

30. Two converging lenses are placed 20.0 cm apart, as shown in figure. If the first lens has a focal length of 10.0 cm and the second has a focal length of 20.0 cm, locate the final image formed of an object 30.0 cm in front of the first lens.



(a) 6.67 cm left (b) 6.67 cm right
(c) 15.0 cm left (d) 15.0 cm right

Answer *with* Solutions

1. (*d*) If we multiply 12.71 m by 3.46 m, we will get an answer of 43.9766 m^2. In this example, we have only three significant figures in our least accurate measurement, so we should express our final answer as 44.0 m^2. Note that in the answer given, we used a general rule for rounding off numbers, which states that the last digit retained is to be increased by 1 if the first digit dropped was equal to 5 or greater.

2. (*b*) Charge of capacitor is the charge on facing surfaces of the plates of capacitor

$$Q = \left(\frac{q_1 - q_2}{2}\right) = \frac{[10-(-2)]}{2} = 6\,\mu C$$

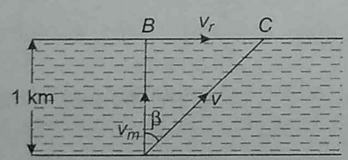
Potential difference across the capacitor = 12 V

So, $$C = \frac{Q}{V} = \left[\frac{(6\times 10^{-6})}{12}\right] F = 0.5\,\mu F$$

3. (*b*) Given, speed of man (v_m) = 4 km/h

Speed of river (v_t) = 3 km/h

Width of the river (d) = 1 km



Time taken by the man to cross the river (fig.)

$$t = \frac{\text{Width of the river}}{\text{Speed of the man}} = \frac{1\,\text{km}}{4\,\text{km/h}} = \frac{1}{4}\,\text{h} = \frac{1}{4}\times 60 = 15\,\text{min}$$

Distance travelled along the river $= v_r \times t$

$$= 3\times\frac{1}{4} = \frac{3}{4}\,\text{km} = \frac{3000}{4} = 750\,\text{m}$$

4. (*d*) Let particle is projected with speed u, so total time of flight

$$T = \left(\frac{2u}{g}\right)$$

and $$s = 2\times \text{maximum height} = 2\times\frac{u^2}{2g} = \frac{u^2}{g}$$

If there is no gravity than $s' = u\times T = \dfrac{2u^2}{g} = 2s$

If gravity is not there, it will never fall back.

5. (*c*) $N = A - Z = 197 - 79 = 118$

6. (*c*) Density of every nucleus is same $= 2.29\times 10^{17}\ \text{kgm}^{-3}$

7. (*a*) In $_3Li^7$, $Z = 3$; $N = A - Z = 7 - 3 = 4$

$\therefore$ Mass of nucleus $= Z_{mp} + (A-Z)m_n$
$= 3\times 1.007825 + 4\times 1.008665 = 7.058075$ amu

The actual mass of nucleus is slightly less than this calculated value.

8. (*c*) $R_1 = \dfrac{l}{KA}$, $R_2 = \dfrac{l}{2KA}$, $R_3 = \dfrac{l}{4KA}$

$$R_{eq} = \left(\frac{R_1R_2}{R_1+R_2} + R_3\right)$$

$$R_{eq} = \left[\frac{\frac{l}{KA}\cdot\frac{l}{2KA}}{\frac{l}{KA}+\frac{l}{2KA}} + \frac{l}{4KA}\right] = \left[\frac{\frac{l^2}{2K^2A^2}}{\frac{2l+l}{2KA}} + \frac{l}{4KA}\right]$$

$$= \frac{l}{3KA} + \frac{l}{4KA} = \frac{4l+3l}{12KA} = \frac{7l}{12KA}$$

[as rods 1 and 2 are in parallel and equivalent is in series with 3].

9. (*c*) Applying energy conservation law,

$$\frac{mv^2}{2} - \frac{GMm}{R+R} = 0$$

$$\Rightarrow \quad \frac{mv^2}{2} = \frac{GMm}{2R} \Rightarrow v = \sqrt{\frac{GM}{R}}$$

10. (c) $E=\sqrt{\frac{2qv}{M}}\times B$

$=\sqrt{\frac{2\times1.60218\times10^{-19}\times320}{9.1\times10^{-31}}}\times6\times10^{-5}$

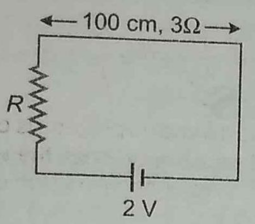
$=\sqrt{112.680\times10^{12}}\times6\times10^{-5}$

$=10.61512\times10^{6}\times6\times10^{-5}$

$=63.69072\times10^{1}$

$=636.9072\ \text{Vm}^{-1}\approx637\ \text{Vm}^{-1}$

11. (b) Let a resistance R is connected in series with the battery and wire.



Voltage drop across wire $=1\times10^{-3}\times100=0.1$ V

Let current in circuit is I.

$\therefore \quad 0.1=I\times3=3\times\frac{2}{R+3}$

$\Rightarrow \quad R=57\ \Omega$

12. (a) The number of beats per second is equal to the difference in frequency between the two sound sources. In this case, because one of the source frequencies is 440 Hz,
4 beat/s would be heard if the frequency of the string (the second source) were either 444 Hz or 436 Hz.

13. (c) The work done by gravity is the work done, as if all the mass were concentrated at the centre of mass. The work necessary to lift the object can be thought of as the work done against gravity and is just $W=mgh$, where h is the height through which the centre of mass is raised.

$$W=180\ (9.8)\ (1.7)=3.0\ \text{kJ}$$

14. (d) The horizontal range

$$R=\frac{u^2\sin2\theta}{g}$$

Time of flight $\quad T=\frac{2u\sin\theta}{g}$

Range is maximum when $\theta=45°$

So that, $\quad \sin2\theta=\sin90°=1$

$$R_{\text{max}}=\frac{u^2}{g}$$

Time of flight is maximum when $\theta=90°$

So that, $\quad \sin\theta=\sin90°=1$

$$T_{\text{max}}=\frac{2u}{g}$$

15. (b) Substituting $m=1$ amu $=1.67\times10^{-27}$ kg and $c=3\times10^{8}$ m/s, in the Energy-mass equivalence relation
$E=mc^2$

$=1.67\times10^{-27}\times(3\times10^{8})^2$

$=1.67\times10^{-27}\times9\times10^{16}$ J

$=\frac{1.67\times10^{-27}\times9\times10^{16}}{1.6\times10^{-13}}$ $\quad [\because 1\ \text{Mev}=1.6\times10^{-13}\ \text{V}]$

$=931$ MeV.

16. (a) Find acceleration

$2ma=mg\sin53°-\mu\,mg\cos53°$
$-\,mg\sin37°-\mu\,mg\cos37°$

Then, use $v^2=u^2+2as,\quad v=0,u=1\ \text{ms}^{-1}$

On solving, we get $s=0.45$ m

17. (a) The rotational and translational kinetic energy of the ball at the bottom will be changed to gravitational potential energy, when the sphere stops. We therefore write

$$\left(\frac{Mv^2}{2}+\frac{I\omega^2}{2}\right)_{\text{start}}=(Mgh)_{\text{end}}$$

For a solid sphere,

$$I=\frac{2\ Mr^2}{5}$$

Also, $\omega=\frac{v}{r}$. Then, above equation becomes

$$\frac{1}{2}Mv^2+\frac{1}{2}\left(\frac{2}{5}Mr^2\right)\left(\frac{v}{r}\right)^2=Mgh$$

or $\quad \frac{1}{2}v^2+\frac{1}{5}v^2=(9.8)\ h$

Using $v=20\ \text{ms}^{-1}$, gives $h=28.6$ m

18. (b) The initial elastic potential energy of the compressed spring is

$$PE_s=\frac{1}{2}kx_i^2$$

Because the block is always at the same height above earth's surface, the gravitational potential energy of the system remains constant. Hence, the initial potential energy stored in the spring is converted to kinetic energy at $x=0$. That is,

$$\frac{1}{2}kx_i^2=\frac{1}{2}mv_f^2$$

Solving for v_f gives

$$v_f=\sqrt{\frac{k}{m}}\,x_i=\sqrt{\frac{1.0\times10^3}{1.6}}\ (2.0\times10^{-2})=0.50\ \text{ms}^{-1}$$

19. (c) The angle between magnetic field and area vector is 90°, so the flux associated with coil is zero. Although magnetic field is changing but flux is remaining constant equal to zero, so emf induced and hence, current in the loop is equal to zero.

20. (b) Resistance of coil $=\frac{V_{DC}}{I_{DC}}=\frac{6}{12}=0.5\ \Omega$

In an AC circuit, the current is in phase with emf. This means that the net reactance of the circuit is zero. The impedance is equal to the resistance, *i.e.*, $Z=0.5\ \Omega$

$$\text{Rms current}=\frac{\text{rms voltage}}{Z}=\frac{6}{0.5}=12\ \text{A}$$

21. (*d*) If the upper wire is to float, it must be in equilibrium under the action of two forces : the force of gravity and magnetic repulsion. The weight per unit length here 1.0×10^{-4} Nm^{-1} must be equal and opposite the magnetic force per unit length. Because the currents are the same, we have

$$\frac{F_1}{l} = \frac{mg}{l} = \frac{\mu_0 I^2}{2\pi d}$$

$$1.0 \times 10^{-4} = \frac{(4\pi \times 10^{-7})(I^2)}{(2\pi)(0.10)}$$

We solve for the current to find

$$I = 7.1 \text{ A}$$

22. (*b*) Because $2nt = \frac{\lambda}{2}$, we have

$$t = \frac{\lambda}{4n} = \frac{602}{(4)(1.33)} = 113 \text{ nm}$$

23. (*c*) Maximum distance covered by space wave communication

$$= \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 300} = 62 \text{ km}$$

Since, receiver-transmitter distance is 100 km, this is ruled out for signal frequency.

Further f_c for ionospheric propagation is

$$f_c = 9(N_{max})^{1/2} = 9 \times (10^{12})^{1/2} = 9 \text{ MHz}$$

So, the signal of 100 MHz ($7f_c$) comes *via* the satellite mode.

24. (*b*) For $t \le t_1$; $A = 0, B = 0$; Hence $Y = 0$

For t_1 to t_2; $A = 1, B = 0$; Hence $Y = 0$

For t_2 to t_3; $A = 1, B = 1$; Hence $Y = 1$

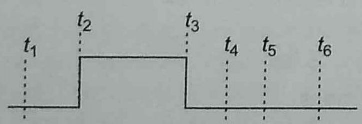
For t_3 to t_4; $A = 0, B = 1$; Hence $Y = 0$

For t_4 to t_5; $A = 0, B = 0$; Hence $Y = 0$

For t_5 to t_6; $A = 1, B = 0$; Hence $Y = 0$

For $t > t_6$; $A = 0, B = 1$; Hence $Y = 0$

Based on the above, the output waveform for AND gate can be drawn as given below



25. (*b*) The initial energy of the sled-rider-earth system includes kinetic energy because of the initial speed

$$\frac{1}{2}mv_i^2 + mg\,y_i = \frac{1}{2}mv_f^2 + mg\,y_f$$

or $$\frac{1}{2}v_i^2 + g\,y_i = \frac{1}{2}v_f^2 + g\,y_f$$

If we set the origin of our coordinates at the bottom of the incline, the initial and final y coordinates of the sled are $y_i = 10.0$ m and $y_f = 0$. Thus, we get

$$\frac{1}{2}v_i^2 + g\,y_i = \frac{1}{2}v_f^2 + 0$$

$$v_f^2 = v_i^2 + 2g\,y_i = (5.00)^2 + 2\,(9.80)(10.0)$$

$$v_f = 14.9 \text{ ms}^{-1}$$

26. (*a*) The force that produces the centripetal acceleration of the satellite is the gravitational force, so

$$\therefore \quad G\frac{M_e m}{r^2} = \frac{mv^2}{r} \quad \text{...(i)}$$

where M_e is earth's mass and r is the satellite's distance from the centre of earth.

Also, we find the speed of the satellite to be

$$v = \frac{d}{T} = \frac{2\pi r}{T} \quad \text{...(ii)}$$

where T is the orbital period of the satellite.

Solving Eqs. (i) and (ii) simultaneously for r yields

$$r = \left(\frac{T^2}{4\pi^2}GM_e\right)^{1/3}$$

27. (*c*) Recall that an isovolumetric process is one that takes place at constant volume. In such a process the work done is equal to zero because there is no change in volume. Thus, the first law of thermodynamics gives

$$\Delta U = Q$$

This indicates that the net energy Q added to the water goes into increasing the internal energy of the water. The net energy added to the water is

$$Q = 10.0 - 2.0 = 8.0 \text{ kJ}$$

Because $Q = mc\,\Delta T$, the temperature increase of the water is

$$\Delta T = \frac{Q}{mc} = \frac{8.0 \times 10^3}{(2.0)(4.186 \times 10^3)} = 0.96°\text{C}$$

28. (*c*) By conservation of momentum, the momentum of the block bullet system just after the interaction is $p = mv$. where m is the mass of bullet and v is its velocity before striking the block.

Hence, the kinetic energy of the system just after the lodging of bullet into the block is

$$K = \frac{p^2}{2(M+m)} = \frac{(mv)^2}{2(M+m)} \quad \text{...(i)}$$

The friction force does work

$$W_f = -f_s = -\mu_k(m+M)gs \quad \text{...(ii)}$$

in stopping the block where s is the distance traversed by block-bullet system on the table top.

From work-energy theorem.

$$\Delta K = W_f$$

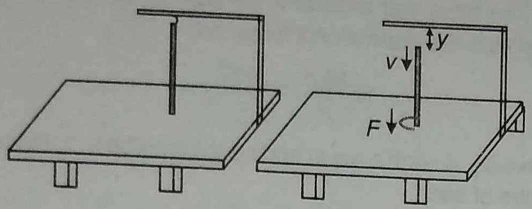
$$0 - K = -\mu_k(m+M)gs$$

Substituting values in Eqs.(i) and (ii), we get

$$\therefore \quad s = 0.37 \text{ m}$$

Also the rest mass of photon is zero.

29. (*c*) The descending part of the rope is in free fall. It has speed $v = \sqrt{2gy}$ at the instant all its points have descended a distance y. The length of the rope which lands on the table during an interval dt following this instant is $v\,dt$. The increment of momentum imparted to the table by this length in coming to rest is $m(v\,dt)v$. Thus, the rate at which momentum is transferred to the table is



$$\frac{dp}{dt} = mv^2 = (2\,my)\,g$$

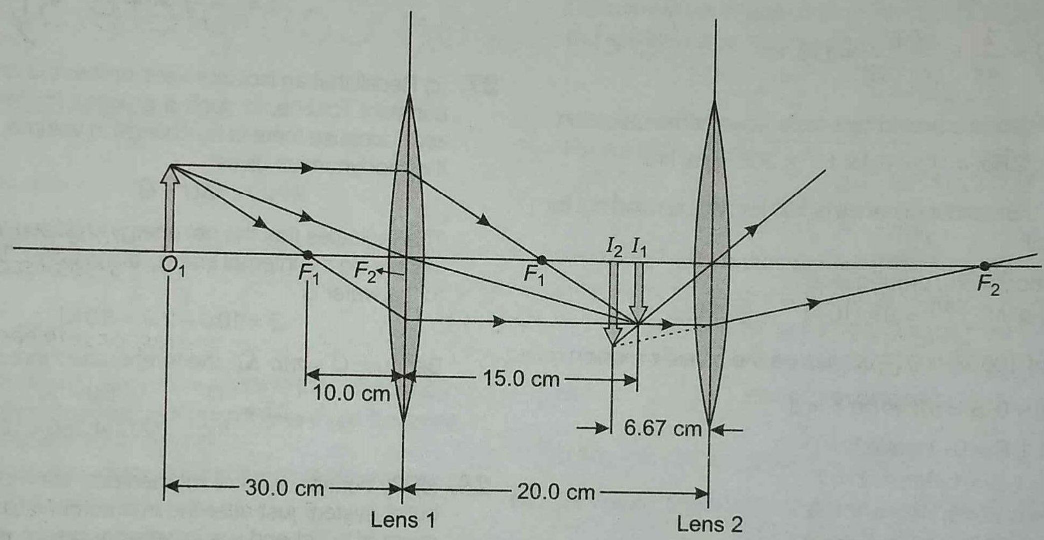
and this is the force arising from stopping the downward fall of the rope. Since, a length of rope y of the weight $(my)g$, already lies on the tabletop, the total force on the tabletop is $(2my)g + (my)g = (3my)\,g$, or the weight of a length $3y$ of rope.

So, $k = 3$

30. (*a*) First we make ray diagrams roughly to scale to see where the image from the first lens falls and how it acts as the object for the second lens. The location of the image formed by the first lens is found *via* the thinlens equation

$$\frac{1}{30.0} + \frac{1}{v_1} = \frac{1}{10.0}$$

$$v_1 = +\ 15.0\ \text{cm}$$



The image formed by this lens becomes the object for the second lens. Thus, the object distance for the second lens is 20.0 cm − 15.0 cm = 5.00 cm. We again apply the thinlens equation to find the location of the final image.

$$\frac{1}{5.00} + \frac{1}{v_2} = \frac{1}{20.0}$$

$$v_2 = -\ 6.67\ \text{cm}$$

Thus, the final image is 6.67 cm to the left of the second lens.

Day 40

Mock TEST 3

(Based on Complete Syllabus)

Instructions

1. *The test consists of 30 questions.*
2. *Candidates will be awarded marks for correct response of each question. 1/4 (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
3. *There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response.*

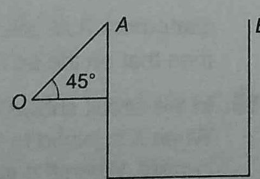
1. An organ pipe of length l, open at both ends is bound to vibrate in its first harmonic when sounded with a tuning fork of 480 Hz. What should be the length of a pipe closed at one end, so that it also vibrates in its first harmonic with the same tuning fork? **[NCERT Exemplar]**

(a) $L_c = 2L_0$ (b) $L_c = \frac{L_0}{3}$ (c) $L_c = \frac{L_0}{2}$ (d) $L_c = \frac{2L_0}{3}$

2. A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800 m from the foot of hill and can be moved on the ground at a speed of 2 m/s, so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g = 10\ m/s^2$. **[NCERT Exemplar]**

(a) 31 s (b) 27 s (c) 37 s (d) 45 s

3. A motor cyclist starts from the bottom of a slope of angle 45° and travels along the slope to jump clear of the valley AB shown in figure. The width of the valley is 160 m and the length of the slope is $160\sqrt{2}$m.



The minimum velocity with which he should leave the bottom O, so that he can clear the valley, is (nearest to in ms^{-1})

(a) 50 (b) 56 (c) 60 (d) 70

4. Choose the correct alternative.

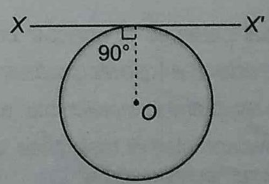
(a) Gravitational potential at curvature centre of a thin hemispherical shell of radius R and mass M is equal to GM/R

(b) Gravitational field strength at a point lying on the axis of a thin uniform circular ring of radius R and mass M is equal to $\frac{GMx}{(R^2 + x^2)^{3/2}}$, where x is distance of that point from centre of the ring

(c) Newton's law of gravitation for gravitational force between two bodies is applicable only when bodies have spherically symmetric distribution of mass

(d) None of the above

5. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

6. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R, break into N small droplets each of radius r. Estimate the drop in temperature.

(a) $\frac{S}{\rho_s}\left[\frac{1}{R}\right]$ (b) $\frac{2S}{\rho_s}\left[\frac{1}{r}-\frac{1}{R}\right]$ (c) $\frac{3S}{\rho_s}\left[\frac{1}{R}-\frac{1}{r}\right]$ (d) $\frac{2S}{\rho_s}\left[\frac{1}{R}-\frac{1}{r}\right]$

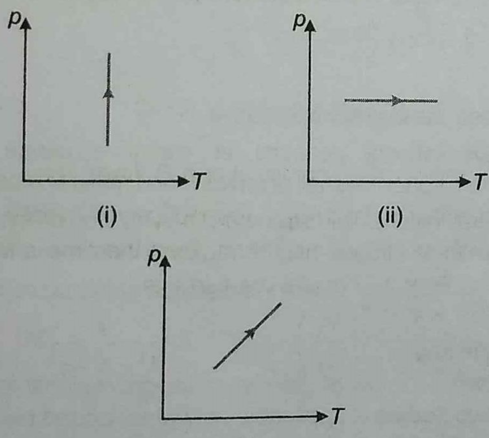
7. A man beats a drum at a certain distance from a mountain. He slowly increase the rate of beating and finds that the echo is not heard distinctly when the drum beating is at the rate of 40 per min. He moves by 80 m towards the mountain and finds that the echo is again not heard distinctly when the rate of beating of the drum is 1 per sec. What is the original distance of the man from the mountain?

(a) 120 m (b) 240 m (c) 270 m (d) 340 m

8. A coil, a capacitor and an AC source of rms voltage 24 V are connected in series. By varying the frequency of the source, a maximum rms current of 6 A is observed. If this coil and capacitor is connected to a battery of emf 12 V and internal resistance 4 Ω, the maximum current through it will be

(a) 2.4 A (b) 1.8 A (c) 1.5 A (d) 1.2 A

9. Pressure *versus* temperature graphs of an ideal gas are as shown in figure. Choose the wrong statement.



(i) (ii) (iii)

(a) Density of gas is increasing in graph (i)
(b) Density of gas is decreasing in graph (ii)
(c) Density of gas is constant in graph (iii)
(d) None of the above

10. Two non-ideal batteries of unequal emf 's are connected in parallel. Consider the following statements.

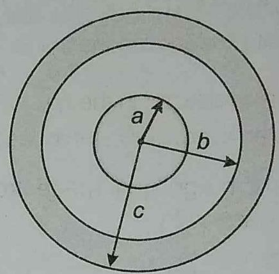
(A) The equivalent emf is smaller than either of the two emf 's.

(B) The equivalent internal resistance is smaller than either of the two internal resistances.

(a) Both A and B are correct (b) A is correct but B is wrong
(c) B is correct but A is wrong (d) Both A and B are wrong

11. A solid conducting sphere of radius a has a net positive charge $2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and has a net charge $-Q$.

The surface charge density on the inner and outer surfaces of the spherical shell will be



(a) $-\frac{2Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$ (b) $-\frac{Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$

(c) $0, \frac{Q}{4\pi c^2}$ (d) None of these

Directions *(Q. Nos. 12 to 14) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

12. Statement I Time period of oscillation of two magnets when like poles are in same direction (in a vibration magnetometer) is smaller, than the period of vibration when like poles are in opposite direction.

Statement II Moment of inertia increases in same position.

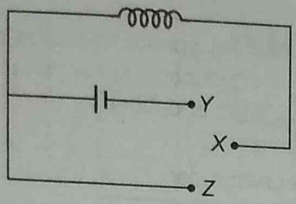
13. Statement I It is impossible for a ship to use the internal energy of sea water to operate its engine.

Statement II A heat engine is different from a refrigerator.

14. Statement I A tennis ball bounces higher on hills than in plains.

Statement II Acceleration due to gravity on the hill is greater than that on the surface of the earth.

15. In the circuit shown, the coil has inductance and resistance. When X is joined to Y, the time constant is τ during growth of current. When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z



(a) the total heat produced in the coil is $P\tau$

(b) the total heat produced in the coil is $\frac{1}{2}P\tau$

(c) the total heat produced in the coil is $2P\tau$

(d) the data given is not sufficient to reach a conclusion

16. The permanent magnet is made from which one of the following substances?

(a) Diamagnetic

(b) Paramagnetic

(c) Ferromagnetic

(d) Electromagnetic

17. Match **List I** (Phenomenon) with **List II** (Principle) and select the correct answer using the codes given below the lists.

	List I (Phenomenon)		List II (Principle)
I.	Blue colour of a sky	A.	Total internal reflection
II.	Glittering of diamond	B.	Dispersion of light
III.	Formation of rainbow	C.	Scattering of light
IV.	In the evening when the sun goes down below the horizon, it continues to remain visible for some time	D.	Refraction of light

Codes

(a) I-C, II-A, III-B, IV-D

(b) I-C, II-A, III-D, IV-B

(c) I-A, II-C, III-B, IV-D

(d) I-A, II-C, III-D, IV-B

18. A metal wire of linear mass density of 9.8 gm^{-1} is stretched with a tension of 10 kg-wt between two rigid supports which are 1m apart. The wire passes through the middle points between the poles of a permanent magnet and it vibrates in resonance when carrying on alternating current of frequency n. The frequency n of the alternating current is

(a) 25 Hz (b) 50 Hz

(c) 200 Hz (d) 100 Hz

19. In a given process of an ideal gas, $dW = 0$ and $dQ < 0$. Then, for the gas

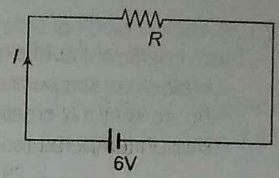
(a) the temperature will decrease

(b) the volume will increase

(c) the pressure will remain constant

(d) the temperature will increase

Directions (Q. Nos. 20 and 21) *The circuit shown in figure contains a resistance of $R = 6\,\Omega$ connected with a battery of emf 6 V.*



Given n = number of electrons per volume $= 10^{29}/m^3$, length of circuit = 10 cm, cross-section $A = 1\,mm^2$

20. The energy absorbed by electrons from initial state of no current (ignore thermal motion) to the state of drift velocity is **[NCERT Exemplar]**

(a) 1.5×10^{-18} J (b) 3.5×10^{-19} J

(c) 2×10^{-17} J (d) 3×10^{-15} J

21. Electrons give up energy at the rate of RI^2 per second to the thermal energy. What time scale would one associate with energy in question (20)?

(a) 3.3×10^{-18} s (b) 4.1×10^{-19} s (c) 2.4×10^{-17} s (d) 2.9×10^{-19} s

22. The potential energy of a particle of mass m is given by

$$U(x) = \begin{cases} E_0 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$, respectively. If the total energy of particle is $2E_0$, the ratio $\frac{\lambda_1}{\lambda_2}$ will be

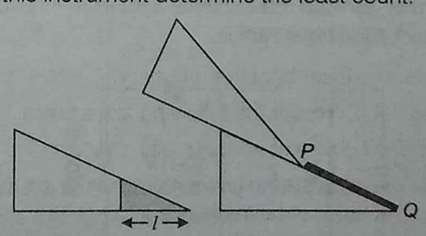
(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

23. Two radioactive nuclei A and B have their disintegration constant λ_A and λ_B, respectively. Initially, N_A and N_B number of nuclei are taken, then the time after which their undisintegrated nuclei are same is

(a) $\frac{\lambda_A \lambda_B}{(\lambda_A - \lambda_B)} \ln\left(\frac{N_B}{N_A}\right)$ (b) $\frac{1}{(\lambda_A + \lambda_B)} \ln\left(\frac{N_B}{N_A}\right)$

(c) $\frac{1}{(\lambda_B - \lambda_A)} \ln\left(\frac{N_B}{N_A}\right)$ (d) $\frac{1}{(\lambda_A - \lambda_B)} \ln\left(\frac{N_B}{N_A}\right)$

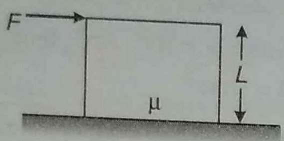
24. A student constructed a vernier callipers as shown. He used two identical inclines and tried to measure the length of line PQ. For this instrument determine the least count.



(a) $\frac{l(1 - \cos\theta)}{\cos\theta}$ units (b) $\frac{l}{\cos\theta}$ units

(c) $l(1 - \cos\theta)$ units (d) $\frac{1 - \cos\theta}{l}$ units

25. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



(a) infinitesimal
(b) $\frac{mg}{4}$
(c) $\frac{mg}{2}$
(d) $mg(1-\mu)$

26. A double star consists of two stars having masses M and $2M$. The distance between their centres is equal to r. They revolve under their mutual gravitational interaction. Then, which of the following statement(s) is/are correct?

(a) Heavier star revolves in orbit of radius $2r/3$
(b) Both of the stars revolve with the same period which is equal to $\frac{2\pi}{\sqrt{2\,GM/3}}\,r^{3/2}$
(c) Kinetic energy of heavier star is twice that of the other star
(d) None of the above

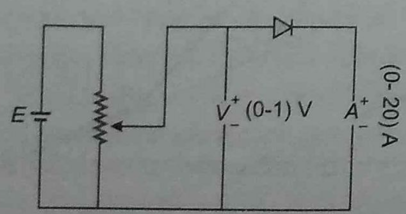
27. The fundamental frequency of a sonometer wire of length l is f_0. A bridge is now introduced at a distance of Δl from the centre of the wire ($\Delta l << l$). The number of beats heard if both sides of the bridges are set into vibration in their fundamental modes, are

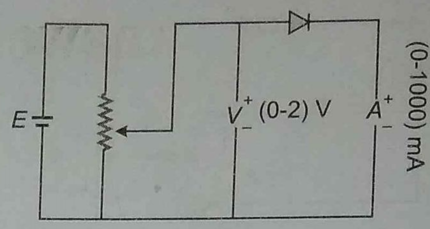
(a) $\frac{8f_0\,\Delta l}{l}$
(b) $\frac{f_0\,\Delta l}{l}$
(c) $\frac{2f_0\,\Delta l}{l}$
(d) $\frac{4f_0\,\Delta l}{l}$

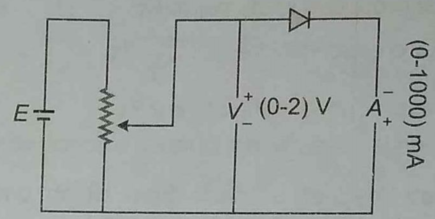
28. A mason is supplied with bricks by his assistant who is 3 m below him, the assistant tossing the brick vertically up. The speed of the brick when it reaches the mason is 2 ms^{-1}. What percentage of energy used up by the servant serves no useful purpose ?

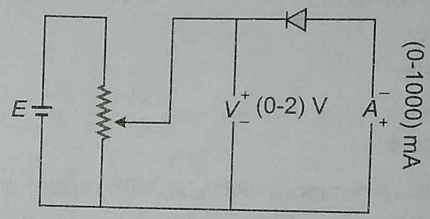
(a) 9.8%
(b) 4.9%
(c) 5.6%
(d) 10%

29. To plot forward characteristic of p-n junction diode, the correct circuit diagram is

(a)


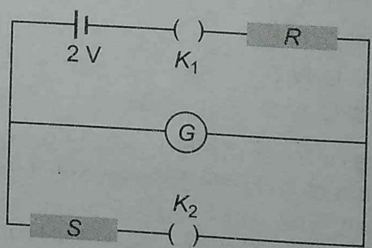
(b)


(c)


(d)


30. In determining resistance of galvanometer by using half deflection method, the following readings are noted down.

S.No.	Resistance (R) (ohm)	Deflection (div) θ	Shunt for $\frac{\theta}{2}$
1.	200	80	40 Ω
2.	280	60	42 Ω
3.	300	50	43 Ω
4.	450	40	45 Ω
5.	620	30	46.3 Ω

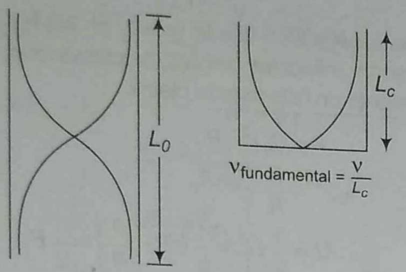


The circuit diagram is also shown for reference. Galvanometer has total 100 divisions and it can measure current upto 10 mA. The least count of galvanometer is

(a) 10 mA
(b) 1 mA
(c) 0.1 mA
(d) Information is insufficient

Answer *with* Solutions

1. (c)



The fundamental frequency, $\nu_0 = \frac{v}{2L_0}$

and frequency of open pipe, ν_0 = frequency of closed pipe ν_c

or $\frac{v}{2L_0} = \frac{v}{4L_c} \Rightarrow L_c = \frac{L_0}{2}$

2. (d) Given, height of the hill $(h) = 500$ m

$$u = 125 \text{ m/s}$$

To cross the hill, the vertical component of the velocity should be sufficient to cross such height.

$\therefore \quad u_y \geq \sqrt{2gh} \geq \sqrt{2 \times 10 \times 500} \geq 100$ m/s

But $\quad u^2 = u_x^2 + u_y^2$

$\therefore$ Horizontal component of initial velocity

$$u_x = \sqrt{u^2 - u_y^2} = \sqrt{(125)^2 - (100)^2} = 75 \text{ m/s}$$

Time taken to reach the top of the hill

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500}{10}} = 10 \text{ s}$$

Time taken to reach the ground from the top of the hill

$t' = t = 10$ s

Horizontal distance travelled in 10 s

$$x = u_x \times t = 75 \times 10 = 750 \text{ m}$$

$\therefore$ Distance through which canon has to be removed

$$= 800 - 750 = 50 \text{ m}$$

Speed with which canon can move = 2 m/s

$\therefore$ Time taken by canon $= \frac{50}{2}$

$$t'' = 25 \text{ s}$$

$\therefore$ Total time taken by a packet to reach on the ground

$$= t'' + t + t'$$
$$= 25 + 10 + 10 = 45 \text{ s}$$

3. (d) Velocity of take-off from A to clear the valley is given by

$$R = \frac{u^2}{g} \sin 2\alpha$$

$$\alpha = 45°, \quad u = \sqrt{gR} = 40 \text{ ms}^{-1}$$

Velocity to start from lowest point (due to retardation on inclined plane, $g \sin \alpha$), $v_0^2 = u^2 + 2g \sin \alpha \times s$

$$\therefore \quad v_0 = \sqrt{(40)^2 + 2 \times 10 \times \frac{1}{\sqrt{2}} \times 160 \times \sqrt{2}}$$
$$= \sqrt{4800} \simeq 70 \text{ ms}^{-1}$$

4. (c) Because every element of hemispherical shell is at a distance R from curvature centre, therefore gravitational potential at its centre $= -\frac{GM}{R}$, *i.e.*, option (a) is wrong.

Gravitational field strength at a point, lying on the axis of a thin uniform circular ring of radius R is $\frac{GMx}{(R^2 + x^2)^{3/2}}$

So, option (b) is wrong.

Newton's law of gravitation is applicable to only those bodies which have spherically symmetric distribution of mass. So, option (c) is correct.

5. (d) Mass of the ring, $M = \rho L$

Let R be the radius of the ring.

Then, $\quad L = 2\pi R \quad$ or $\quad R = \frac{L}{2\pi}$

Moment of inertia about XX' (from parallel axis theorem) will be given by $I_{XX'} = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2$

Putting values of M and R, $I_{XX'} = \frac{3}{2}(\rho L)\left(\frac{L^2}{4\pi^2}\right) = \frac{3}{8}\frac{\rho L^3}{\pi^2}$

6. (c) When a big drop of radius R, break into N droplets each of radius r, the volume remains constant.

$\therefore \quad$ Volume of big drop = $N \times$ Volume of small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3 \quad \text{or} \quad R^3 = Nr^3$$

or $\quad N = \frac{R^3}{r^3}$

Now, change in surface area $= 4\pi R^2 - N4\pi r^2 = 4\pi(R^2 - Nr^2)$

Energy released $\quad = S \times \Delta A \quad$ (where, S = surface tension)

$\quad = S \times 4\pi(R^2 - Nr^2)$

Due to releasing of this energy, the temperature is lowered. If ρ is the density and s is specific heat of liquid and its temperature is lowered by $\Delta\theta$ then,

Energy released = $ms\Delta\theta$

$$S \times 4\pi(R^2 - Nr^2) = \left(\frac{4}{3} \times R^3 \times \rho\right) s\Delta\theta$$

$$\Delta\theta = \frac{S \times 4\pi(R^2 - Nr^2)}{\frac{4}{3}\pi R^3 \rho \times s} = \frac{3S}{\rho s}\left[\frac{R^2}{R^3} - \frac{Nr^2}{R^3}\right]$$

$$= \frac{3S}{\rho s}\left[\frac{1}{R} - \frac{(R^3/r^3) \times r^2}{R^3}\right] = \frac{3S}{\rho s}\left[\frac{1}{R} - \frac{1}{r}\right]$$

7. (b) The echo is not heard distinctly when the echo and the next beat fall on the ear simultaneously, *i.e.*, time per beat = time taken by the reflected beat to reach the man.

Hence, $\quad \frac{2d}{v} = \frac{60}{40} = \frac{3}{2} \quad$ and $\quad \frac{2(d - 80)}{v} = 1$

This gives, $\quad d = 240$ m

8. (c) Let R be the resistance of coil + capacitor.

Then, $R = \frac{24}{6} = 4\,\Omega$

In the second case, $I_{max} = \frac{12}{R+4} = \frac{12}{8} = 1.5$ A

9. (a) As, $\rho = \frac{pM}{RT}$

Density ρ remains constant when p/T or volume remains constant. In graph (i) volume is decreasing, hence density is increasing; while in graphs (ii) and (iii) volume is increasing, hence, density is decreasing. Note that volume would have been constant in case the straight line in graph (iii) had passed through origin.

10. (a) In parallel,

$$E_{eq} = \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2} = E_1\left(\frac{r_2 + E_2/E_1 r_1}{r_1 + r_2}\right) = E_2\left(\frac{r_1 + E_1/E_2 \cdot r_2}{r_1 + r_2}\right)$$

Now, if $E_1 = E_2$. Then, $E_{eq} = E_1$

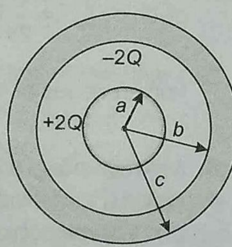
if $E_2 > E_1$. Then, $E_{eq} > E_1$ and $E_{eq} < E_2$

Similarly, if $E_1 > E_2$. Then, $E_{eq} > E_2$ but $E_{eq} < E_1$

11. (a) Due to induction inner surface of spherical shell has charge $-2Q$,

So, surface charge density on inner side $\sigma_{inner} = \frac{-2Q}{4\pi b^2}$

and surface charge density on outer side $\sigma_{outer} = \frac{Q}{4\pi c^2}$



12. (c) ***Case I*** When the like poles of two magnets are placed in same direction, then the time period of vibration is expressed as

$$T' = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B}} \quad \text{...(i)}$$

Case II When the like poles of two magnets are placed in opposite direction, then period of vibration is expressed as

$$T'' = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B}} \quad \text{...(ii)}$$

It is clear from Eqs. (i) and (ii) that $T' < T''$.

13. (b) For using the internal energy of sea water to operate the engine of a ship, the internal energy of the sea water has to be converted into mechanical energy. Since, whole of the internal energy cannot be converted into mechanical energy, a part has to be rejected to a colder body (sink). Since, no such body is available, the internal energy of the sea water cannot be used to operate the engine of the ship.

Note that a refrigerator is a heat engine working in the reverse direction.

14. (c) Suppose that the tennis ball bounces with a velocity u. It will go up, till its velocity becomes zero. If h is the height upto which it rises on the hill, then

$$(0)^2 - u^2 = 2(-g')h$$

where g' is acceleration due to gravity on the hill

$$\therefore \quad h = \frac{u^2}{2g'}$$

Since, the acceleration due to gravity on the hill (g') is less than that on earth (effect of height), it follows that tennis ball will bounce higher on hills than in plains.

15. (b) As, $P = (I_0)^2 \cdot R$

i.e., $(I_0)^2 = \frac{P}{R}$

$$\therefore \quad U = \frac{1}{2}LI_0^2 = \frac{1}{2}(\tau R)\left(\frac{P}{R}\right) = \frac{1}{2}P\tau$$

Hence, τ = time constant.

16. (c) If a magnet retains its attracting power for a long time it is said to be permanent, otherwise temporary. Permanent magnets are made of ferromagnetic substances.

17. (a) Blue colour of a sky → Scattering of Light

Glittering of diamond → Total Internel Reflection

Formation of rainbow → Dispersion of Light

It continues to remain visible for sometime → Reflection of Light

18. (b) Since, the tension $T = 10$ kg-wt $= 10 \times 9.8 = 98$ N and $m = 9.8 \times 10^{-3}$ kg-m^{-1}, $L = 1$m, so we get

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2\times 1} \times \sqrt{\frac{98}{9.8\times 10^{-3}}} = 50 \text{ Hz}$$

19. (a) From first law of thermodynamics

$$dQ = dU + dW$$

$$dQ = dU \quad (\text{if } dW = 0)$$

Since, $dQ < 0$

Therefore, $dU < 0$

or $U_{final} < U_{initial}$

or temperature will decrease.

20. (c) Given, $V = 6$ V, $R = 6\,\Omega$, $A = 1\times 10^{-6}$ m^2 and $l = 10$ cm $= 0.1$ m

The current in the circuit, $I = \frac{V}{R} = \frac{6}{6} = 1$A

Use the relation $I = ne\,Av_d$

Drift velocity of electrons,

$$v_d = \frac{I}{neA} = \frac{1}{10^{29}\times 1.6\times 10^{-19}\times 1\times 10^{-6}}$$

$$= \frac{1}{1.6}\times 10^{-4} \text{ m/s}$$

The energy of electrons, (KE) $= \frac{1}{2}mv^2$

$$= \frac{1}{2}\times m_e \times v_d^2 \times \text{Volume} \times \text{Number of electrons per volume}$$

$$= \frac{1}{2}\times 9.1\times 10^{-31}\times\left(\frac{10^{-4}}{1.6}\right)^2 \times A \times l \times n$$

[$\because$ Mass of electron $m_e = 9.1\times 10^{-31}$]

$$= \frac{9.1\times 10^{-39}}{2\times 1.6\times 1.6}\times 10^{-6}\times 0.1\times 10^{29}$$

$$= 2\times 10^{-17} \text{ J}$$

21. (a) Energy loss in the circuit $= I^2R = 1^2 \times 6 = 6$ J/s

All of the KE of electrons lost

$$= \frac{\text{Total KE}}{\text{Energy loss per second}} = \frac{2 \times 10^{-17}}{6} = 3.33 \times 10^{-18} \text{s}$$

22. (c) $KE = 2E_0 - E_0 = E_0$ (For $0 \le x \le 1$)

So, $\lambda_1 = \dfrac{h}{\sqrt{2mE_0}}$ ...(i)

Again $KE = 2E_0$ (For $x > 1$)

$\therefore$ $\lambda_2 = \dfrac{h}{\sqrt{2m2E_0}}$ ...(ii)

From Eqs. (i) and (ii), we get $\dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$

23. (c) After disintegration, $N_A e^{-\lambda_A t} = N_B e^{-\lambda_B t}$ (For $0 \le x \le 1$)

or $e^{(\lambda_B - \lambda_A)t} = \dfrac{N_B}{N_A}$ ...(i)

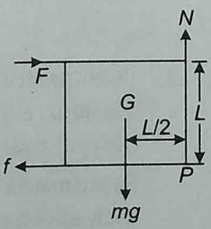
$\therefore$ $(\lambda_B - \lambda_A)t = \ln\left(\dfrac{N_B}{N_A}\right) \Rightarrow t = \dfrac{1}{\lambda_B - \lambda_A}\ln\left(\dfrac{N_B}{N_A}\right)$

24. (a) Let θ be the angle of incline. Here the incline kept horizontally is working as main scale while the other incline kept on horizontally placed incline is treated as vernier scale.

From the figure, it is clear that, 1 MSD $= \dfrac{l}{\cos\theta}$ unit and 1 VSD $= l$ unit, so LC of instrument is,

$$LC = 1 MSD - 1 VSD = \left(\frac{l}{\cos\theta} - l\right) = \frac{l(1 - \cos\theta)}{\cos\theta} \text{ units}$$

25. (c) At the critical condition, normal reaction N will pass through point P.



The block will topple when

$\tau_F > \tau_{mg}$

or $FL > (mg)\dfrac{L}{2}$ $\therefore F > \dfrac{mg}{2}$

Therefore, the minimum force required to topple the block is $F = \dfrac{mg}{2}$

26. (b) The centre of mass of the double star system remains stationary and both the stars revolve round in circular orbits which are concentric with the centre of mass.

The distance of centre of mass from the heavier star

$$= \frac{Mr + 2M \times 0}{M + 2M} = \frac{r}{3}$$

Hence, the heavier star revolves in a circle of radius $\dfrac{r}{3}$ while the lighter star in a circle of radius

So, option (a) is not correct.

Reduced mass of the system $= \dfrac{M \cdot 2M}{M + 2M} = \dfrac{2M}{3}$

Period of revolution of the double star system $= \dfrac{2\pi}{\sqrt{\dfrac{2GM}{3}}} r^{3/2}$

where r is the distance between two stars.

So, option (b) is correct.

KE of a star $= \dfrac{1}{2}mv^2$

KE of heavier star $E_1 = \dfrac{1}{2} \times 2M \times \left(\dfrac{r}{3}\omega\right)^2$

and that of lighter star $E_2 = \dfrac{1}{2}M\left(\dfrac{2r}{3}\omega\right)^2$

So, kinetic energy of lighter star is two times that of heavier star. So, option (c) is wrong.

27. (a) $f_0 = \dfrac{v}{2l}$

Beat frequency $= f_1 - f_2 = \dfrac{v}{2\left(\dfrac{l}{2} - \Delta l\right)} - \dfrac{v}{2\left(\dfrac{l}{2} + \Delta l\right)}$

$$= (f_0 l)\left[\frac{2}{l - 2\Delta l} - \frac{2}{l + 2\Delta l}\right] = 2f_0 l\left[\frac{4\Delta l}{l^2}\right] \approx \frac{8f_0 \Delta l}{l}$$

28. (c) Once the bricks leave the assistant's hands the only force that acts on them is gravitational force. Since this produces a constant acceleration $a = -g = -32$ fts^{-2}, the kinematic equation, $v^2 = v_0^2 - 2a(x - x_0)$

can be used to describe the motion. The initial velocity v_0 is found by putting known values in the above equation,

$$v_0^2 = 36 + 2 \times 32 \times 10 = 676 \Rightarrow v_0 = 26 \text{ fts}^{-1}$$

The kinetic energy given to each brick and supplied by the assistant is

$$E_1 = \frac{1}{2}mv_0^2 = \frac{1}{2} \times m \times 676 = 338m \text{ ft}^2\text{s}^{-2}$$

If the brick layer's assistant supplied only just enough energy to reach the required level and no more, the initial velocity being u, they would have zero velocity at the Mason's hand.

$\therefore$ $u^2 = 0 + 2g(x - x_0) = 2 \times 32 \times 10 = 640 \Rightarrow u = 8\sqrt{10}$ fts^{-1}

KE supplied in this case, $E_2 = \dfrac{1}{2}mu^2 = 320m$ ft^{2}s^{-2}

$\therefore$ Wasted energy $= E_1 - E_2$

$$\% \text{ waste} = \frac{E_1 - E_2}{E_1} \times 100 = \frac{338 - 320}{320} \times 100 = 5.6\%$$

29. (b) For forward bias mode the p-side of diode has to be at higher potential then n-side. The meters used are DC, so we have to be careful while connecting them w.r.t. polarity.

Last point is to decide the range of meters, the range of meters has to be in such a way that we can have the readings which leads to plot on realistic scale. If we take 0-20 A ammeter, then reading we read from this is tending to 0 to 5 divisions which is not fruitful.

30. (c) Least count of galvanometer is the current it can measure when the galvanometer needle deflects by 1 division.

$$LC = \frac{10}{100} = 0.1 \text{ mA}$$