DAY 3

SCALARS & VECTORS

OUTLINES

1. Scalar Quantity
2. Vector Quantity
3. Properties of Vectors
4. The Scalar Product
5. The Vector Product
6. Motion in a Plane
7. Projectile Motion

Scalar *and* Vector Quantities

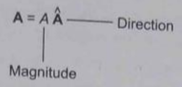
A scalar quantity is one whose specification is completed with its magnitude only. Two or more than two similar scalar quantities can be added according to the ordinary rules of algebra. e.g., mass, distance speed, energy etc.

A ***vector quantity*** *is a quantity that has magnitude as well as direction. Not all physical quantities have a direction. Temperature, energy, mass, and time for example, do not "point" in the spatial sense. We call such quantities scalars, and we deal with them by the rules of ordinary algebra.*

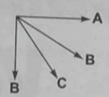
General Points Regarding Vectors

1. The vector having zero magnitude is called **zero vector.** It is written as 0. The initial and final points of a zero vector overlap, so its direction is arbitrary (not known to us).
2. A vector of unit magnitude is known as an **unit vector**. Unit vector for $\mathbf{A}$ is $\hat{\mathbf{A}}$ (read as A cap).

3. As shown in figure, vector has essentially two ingredients, magnitude and direction.

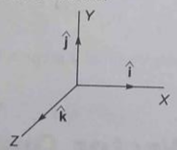


4. Vectors producing straight line linear effect are called **polar vectors** e.g., force, momentum, velocity, displacement.
5. The rotational effect of a polar vector gives rise to a new vector called **axial vector** (acting along the axis of rotation) *e.g.*, rotational effect of force (polar vector) is torque (an axial vector).
6. Vectors having common initial point are called **co-initial vectors**. The vectors **A**, **B**, **C** and **D** are called co-initial vectors.



Orthogonal Unit Vectors

The unit vectors along x-axis, y-axis and z-axis are denoted by $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. These are the orthogonal unit vectors.



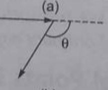
$$\hat{\mathbf{i}} = \frac{\mathbf{x}}{x}, \hat{\mathbf{j}} = \frac{\mathbf{y}}{y}, \hat{\mathbf{k}} = \frac{\mathbf{z}}{z}$$

(i) To find angle between two vectors, the two vectors from a point are drawn such that their arrow heads should be away from that point. The angle obtained in this way, is the **angle between the vectors**.

(ii) If heads coincide or tails coincide then internal angle is the angle between two vectors as in Fig. (a).

(a)

(iii) If head coincides with tail then external angle is the angle between the two vectors as in Fig. (b).



(b)

Equality of Two Vectors

Two vectors **A** and **B** are equal if they have the same magnitude and the same direction. This property allows us to translate a vector parallel to itself in a diagram without affecting the vector. In fact, for most purposes, any vector can be moved parallel to itself without being affected.

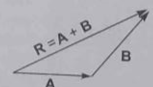
Laws of Vector Addition

When two or more vectors are added, they must all have the same units. It would be meaningless to add a velocity vector to a displacement vector because they are different physical quantities.

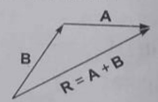
Here, we show a geometrical or graphical method for adding vectors. To add vector B to vector A geometrically, first draw A on a piece of graph paper to some scale. Draw A so that its direction is specified relative to a coordinate system.

Then draw vector **B** to the same scale and with the tail of **B** starting at the tip of **A**, as in **Fig.** (a). Vector **B** must be drawn along the direction that makes the proper angle relative to vector **A**. The resultant vector **R** = **A** + **B** is the vector drawn from the tail of **A** to the tip of **B**. This is known as the **triangle method of addition of vectors.**

When two vectors are added, the sum is independent of the order of the addition. That is, **A** + **B** = **B** + **A**. This can be seen from the geometric construction in **Fig. (b)**, and is called the commutative law of vector addition.

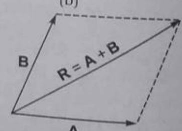


(a)



(b)

An alternative graphical procedure for adding two vectors, known as the **parallelogram method of addition of vectors** is shown in figure. In this construction, the tails of vectors **A** and **B** are joined together and the resultant vector **R** is the diagonal of the parallelogram formed with **A** and **B** as its sides.

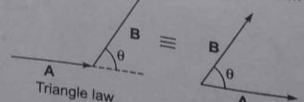


$$|\mathbf{R}| = |\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and angle α is given by

» *The negative of the vector A is defined as the vector that when added to A gives zero for the vector sum. This means that A and –A have the same magnitude but have opposite directions.*

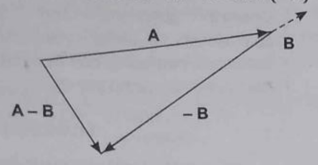
» *Triangle law and parallelogram law, both are equivalent. The equivalency is shown below.*



Triangle law Parallelogram law

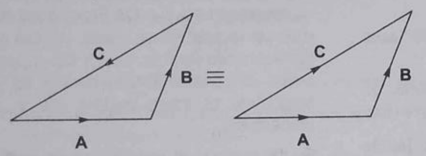
Subtraction of Vectors

Vector subtraction makes use of the definition of the negative of a vector. We define the operation **A** − **B** as vector − **B** added to vector **A**. **A** − **B** = **A** + (−**B**)



Thus, vector subtraction is really a special case of vector addition. The geometric construction for subtracting two vectors is shown in the above figure.

If the vectors form a closed n sided polygon with all the sides in the same order, then the resultant is 0.



Multiplication or Division of a Vector by a Scalar

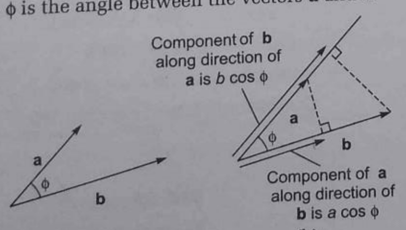
The multiplication or division of a vector by a scalar gives a vector. For example, if vector **A** is multiplied by the scalar number 3, the result, written as 3**A**, is a vector with a magnitude three times that of **A**, pointing in the same direction as **A**. If we multiply vector **A** by the scalar − 3, the result is − 3**A**, a vector with a magnitude three times that of **A**, pointing in the direction opposite to **A** (because of the negative sign).

The Scalar Product

The scalar product of two vectors **a** and **b** in Fig. (a) is written as $\mathbf{a}\cdot\mathbf{b}$ and is defined to be

$$\mathbf{a}\cdot\mathbf{b} = ab\cos\phi \qquad \text{...(i)}$$

where, ϕ is the angle between the vectors **a** and **b**.



(a) (b)

Because of the notation, $\mathbf{a}\cdot\mathbf{b}$ is also known as the **dot product** and is spelled as "**a** dot **b**."

(i) Dot product of the vectors with itself is equal to the square of the magnitude of the vector

$$\mathbf{a}\cdot\mathbf{a} = \mathbf{a}\cdot\mathbf{a}\cos 0° \Rightarrow \mathbf{a}\cdot\mathbf{a} = a^2 \qquad (\cos 0° = 1)$$

If $\theta = 180°$ *i.e.*, vectors are antiparallel,

Then, $\mathbf{a}\cdot\mathbf{b} = ab(-1)$ $\quad [\because \cos 180° = -1]$

$$\mathbf{a}\cdot\mathbf{b} = -ab$$

i.e., If two vectors are antiparallel then their dot product equals the negative product of the magnitudes of vectors.

If $\theta = 90°$ *i.e.*, vectors are perpendicular

$$\mathbf{a}\cdot\mathbf{b} = ab\cos 90° = ab\,(0) = 0$$

Vectors are perpendicular ⇔ Dot product = 0

(ii) If $\mathbf{a} = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}} + a_z\,\hat{\mathbf{k}}$ and $\mathbf{b} = b_x\,\hat{\mathbf{i}} + b_y\,\hat{\mathbf{j}} + b_z\,\hat{\mathbf{k}}$ and θ is the angle between **a** and **b**, then $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{ab}$ where,

$\mathbf{a}\cdot\mathbf{b} = ab\cos\theta$

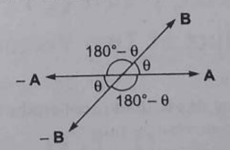
The component of **a** parallel to **b** in the vector form is

$$\mathbf{c} = \frac{(\mathbf{a}\cdot\mathbf{b})\,\mathbf{b}}{|\mathbf{b}|^2}$$

The component of **a** perpendicular to **b** in vector form is

$$\mathbf{d} = \mathbf{a} - \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b}$$

If angle between **A** *and* **B** *is* θ, *then*



(a) Angle between −**A** and **B** is $(180° - \theta)$

(b) Angle between **A** and −**B** is $(180° - \theta)$

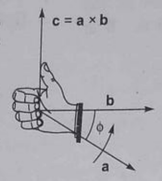
(c) Angle between −**A** and −**B** is θ

- *The dot product of force* **F** *and displacement* **s** *gives work (scalar quantity) i.e.,* $\mathbf{F}\cdot\mathbf{s} = W$.
- *The dot product of force* (**F**) *and velocity* (**v**) *is equal to power (scalar quantity) i.e.,* $\mathbf{F}\cdot\mathbf{v} = P$.
- *The dot product of magnetic induction* (**B**) *and area vector* (**A**) *is equal to the magnetic flux* (ϕ) *linked with the surface (scalar quantity)* $\mathbf{B}\cdot\mathbf{A} = \phi_B$.

The Vector Product

The vector product of **a** and **b**, written as $\mathbf{a} \times \mathbf{b}$, produces a third vector **c** whose magnitude is $c = ab \sin \phi$

where, ϕ is the smaller of the two angles between **a** and **b**.



Because of the notation, $\mathbf{a} \times \mathbf{b}$ is also known as the **cross product**, and it is spelled as "**a** cross **b**".

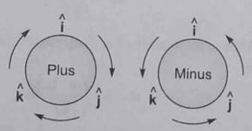
- *If a and b are parallel or antiparallel,* $\mathbf{a} \times \mathbf{b} = 0$.
- *The magnitude of* $\mathbf{a} \times \mathbf{b}$ *which can be written as* $|\mathbf{a} \times \mathbf{b}|$*, is maximum, when a and b are perpendicular to each other.*

$$\mathbf{a} \times \mathbf{a} = aa \sin 0° \,\hat{\mathbf{n}} \qquad (\sin 0° = 0)$$
$$\mathbf{a} \times \mathbf{a} = 0$$

i.e., cross product of vectors with itself is 0.

(i) If two vectors are perpendicular to each other, we have $\theta = 90°$ and therefore, $\sin \theta = 1$. So that, $\mathbf{a} \times \mathbf{b} = ab\,\hat{\mathbf{n}}$.

(ii) The vectors **a**, **b** and $\mathbf{a} \times \mathbf{b}$ thus form a right handed system of mutually perpendicular vectors. It follows at once from the above that in case of the orthogonal triad of unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ (each perpendicular to each other).

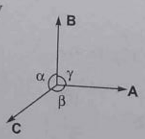


$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}},\ \hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}} \text{ and } \hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

Cross Product of Two Vectors in Determinant Form

For two vectors **a** and **b**, their cross product is given by

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix},$$



where, $\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$

and $\mathbf{b} = b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}}$.

- $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ *is called the scalar triple product. It is a scalar quantity.*

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$$

- *Properties of scalar triple product*
  - *(a)* $[\mathbf{ABC}] = [\mathbf{BCA}] = [\mathbf{CAB}]$
  - *(b)* $[\mathbf{ABC}] = -[\mathbf{BAC}]$
  - *(c) If vectors are coplanar,* $[\mathbf{ABC}] = 0$

Resolution of a Vector

The process of splitting a vector into various parts or components is called resolution of vector. These parts of a vector may act in different directions and are called components of vector. We can resolve a vector into number of components. *Generally, there are three components.*

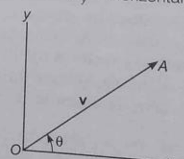
- Components along *x*-axis called *x*-component.
- Components along *y*-axis called *y*-component.
- Components along *z*-axis called *z*-component.

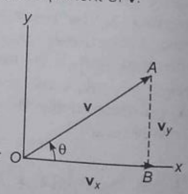
Consider a vector v acting at a point making an angle θ with positive *x*-axis. Vector **v** is represented by a line **OA** From point *A* drawn a perpendicular *AB* on *x*-axis. Let **OB** and **BA** represent two vectors. Vector **OB** is parallel to *x*-axis and vector **BA** is parallel to *y* axis. Magnitude of these vectors are $\mathbf{v}_x$ and $\mathbf{v}_y$ respectively.

By the method of head to tail we notice that the sum of these vectors is equal to vector **v**. Thus $\mathbf{v}_x$ and $\mathbf{v}_y$ are the rectangular components of vector **v**.

$\mathbf{v}_x$ = Horizontal component of **v**.

$\mathbf{v}_y$ = Horizontal component of **v**.





Consider right angled triangle ΔOAB

$$\cos \theta = \frac{\mathbf{OB}}{\mathbf{OA}}$$
$$\mathbf{OB} = \mathbf{OA} \cos \theta$$
$$\mathbf{v}_x = \mathbf{v} \cos \theta$$

Magnitude of vertical component.

Consider right angled triangle ΔOAB

$$\sin \theta = \frac{\mathbf{BA}}{\mathbf{OA}}$$
$$\mathbf{BA} = \mathbf{OA} \sin \theta$$
$$\mathbf{v}_y = \mathbf{v} \sin \theta$$

Motion in a Plane

If an object changes its position with respect to its surroundings with time, then it is called in motion. If an object does not change its position with respect to its surroundings with time then it is called at rest. Motion of a particle in a plane is called a two dimensional motion. For example, motion of (i) projectiles, (ii) charged particles in uniform electric and magnetic fields. In two dimensional motion both the x and y-coordinates of the position of a particle change with time.

Displacement, Velocity and Acceleration in Two Dimensions

1. The **displacement** of the object is defined as the change in two dimensional position vector.

$$\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$$

2. The **average velocity** of a particle during the time interval Δt is the ratio of the displacement covered to the time interval for this displacement.

$$\mathbf{v} = \frac{\Delta \mathbf{r}}{\Delta t}$$

3. The **instantaneous velocity** is defined as the average velocity $\frac{\Delta \mathbf{r}}{\Delta t}$ in the limit Δt approaching zero.

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

The direction of the instantaneous velocity vector is along a line that is tangent to the path of the particle and is in the direction of motion of the particle.

The **average acceleration** of an object when velocity changes by $\Delta \mathbf{v}$ in the time interval Δt is a vector defined as the ratio $\frac{\Delta \mathbf{v}}{\Delta t}$.

That is, $$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$$

The instantaneous acceleration is defined as the average acceleration in the $\lim_{\Delta t \to 0}$.

Projectile Motion

We considered objects moving along straight line paths, such as the x-axis. Now, let us look at some cases in which an object moves in a plane. By this we mean that the object has a motion in both the x and the y-directions simultaneously. Which is another way of saying, it moves in two dimensions. The motion of a particle thrown in a vertical plane, making an angle with the horizontal ($\neq 90°$), is an example of two dimensional motion. This is called ***projectile motion****.*

Time of flight

It is defined as the total time for which the projectile remains in air, $T = \frac{2u \sin \theta}{g}$

Maximum height

It is defined as the maximum vertical distance covered by projectile, $H = \frac{u^2 \sin^2 \theta}{2g}$

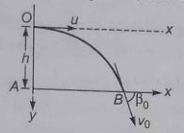
Horizontal range

It is defined as the maximum distance covered in horizontal distance, $R = \frac{u^2 \sin 2\theta}{g}$

- *Horizontal range is maximum when it is thrown at an angle of 45° from the horizontal* $R_{max} = \frac{u^2}{g}$
- *For angle of projection θ and $(90° - \theta)$ the horizontal range is same.*

A Projectile Fired Horizontally from a Certain Height

Let a particle be projected horizontally with a velocity u from a height h above the ground. Then taking the point of projection O as the origin, horizontal direction as x-axis and vertically upward direction as y-axis, we find that



(i) Equation of trajectory is $y = \frac{g}{2u^2} x^2$,

which represents the equation of a parabola.

(ii) Time taken by the projectile to reach the ground

$$T = \sqrt{\frac{2h}{g}}$$

The time is the same as that taken by an object falling freely from the same height.

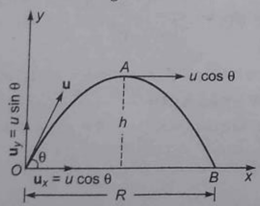
(iii) Horizontal range $AB = R = uT = u\sqrt{\frac{2h}{g}}$

(iv) Velocity of projectile at any instant of time t is

$\mathbf{v} = (u\hat{\mathbf{i}} + gt\,\hat{\mathbf{j}}) \Rightarrow v = \sqrt{u^2 + g^2t^2}$ and $\tan\beta = \frac{gt}{u}$

A Projectile Fired at an Angle of Elevation θ from the Ground

Let a particle be projected at an angle of elevation θ from the ground level at point O with an initial velocity **u**. The velocity may be resolved into (i) a horizontal component $\mathbf{u}_x = u\cos\theta$ and (ii) a vertical component $\mathbf{u}_y = u\sin\theta$. Moreover, here, $a_x = 0$ and $a_y = -g$. From these values of **u** and **a**, we get the following results.



(i) Equation of the trajectory, $y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$

which is the equation of a parabola symmetric about the y-axis (*i.e.*, vertical direction)

(ii) Vertical height covered, $h = \frac{u^2\sin^2\theta}{2g}$

(iii) Time of flight, $T = \frac{2u\sin\theta}{g}$

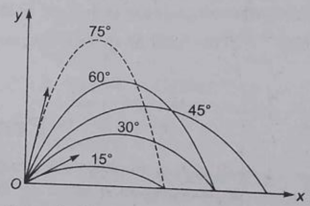
For complementary angles ϕ and $(90° - \phi)$, if T_ϕ and $T_{(90°-\phi)}$ are the time of flight and R is the range, then

$$T_\phi\, T_{90°-\phi} = \frac{2R_\phi}{g} = \frac{2R_{90°-\phi}}{g} = \frac{2R}{g} \quad \text{e.g.,}$$

$$T_{1°}T_{89°} = \frac{2R_{1°}}{g} = \frac{2R_{89°}}{g}$$

(iv) Horizontal range, $R = OB = u_x\,T = \frac{u^2\sin 2\theta}{g}$

For a given range R, there are two different angles of projection, one of them being the compliment of the other (15° or 75°, 30° or 60°) as in figure. However, the vertical height and the time of flight will be different for the two values of θ.



(v) Maximum horizontal range

$$R_{max} = \frac{u^2}{g}, \text{ when } \theta = 45°$$

For a given speed of projection, R depends on θ, the angle of projection. Range will be maximum for a given speed of projection, when $\sin 2\theta = 1$. *i.e.*, $2\theta = 90°$ or $\theta = 45°$. Hence, for maximum range, the angle of projection must be 45°.

(vi) **Velocity at any instant *t*** Let the instantaneous velocity be **v**, inclined at an angle β with the horizontal. Then

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

and $\tan\beta = \frac{u\sin\theta - gt}{u\cos\theta}$

If K is the kinetic energy at the point of launch, then kinetic energy at the highest point is

$$K' = \frac{1}{2}mv_x^2 = \frac{1}{2}mu^2\cos^2\theta \quad \text{or}$$

$$K' = K\cos^2\theta$$

(vii) **Relation between R and h** It is found that

$$\frac{R}{h} = 4\cot\theta$$

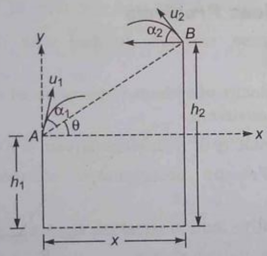
and if $\theta = 45°$, then

$$\frac{R_{max}}{h_{max}} = 4 \text{ or } R_{max} = 4\,h_{max}$$

(viii) **Two angles of projection for the same range** It is found that the range of a projectile is the same when it is projected either at an angle of elevation $\theta°$ or at an angle of $(90° - \theta)$.

Condition for Collision of Two Projectiles from Different Heights

Consider two particles are projected from A and B with initial speeds u_1 and u_2, respectively as shown in the figure.



$$\mathbf{a}_{12} = -g\,\hat{\mathbf{j}} - (-g\,\hat{\mathbf{j}}) = 0$$

As relative acceleration of 1 w.r.t. 2 is zero, so particles will collide only when velocity of 1 w.r.t. 2 is along line AB.

v_{12x} = component of velocity of 1 w.r.t. 2 along the x-axis
$= u_1 \cos\alpha_1 + u_2 \cos\alpha_2$

u_{12y} = component of velocity of 1 w.r.t. 2 along the y-axis
$= u_1 \sin\alpha_1 - u_2 \sin\alpha_2$

Let the velocity of 1 w.r.t. 2 be making an angle β with the positive x-axis, then

$$\tan\beta = \frac{u_{12y}}{u_{12x}} = \frac{u_1\sin\alpha_1 - u_2\sin\alpha_2}{u_1\cos\alpha_1 + u_2\cos\alpha_2}$$

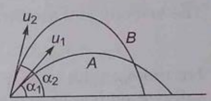
For collision to take place, $\theta = \beta$

$$\Rightarrow \quad \tan\theta = \tan\beta$$

$$\Rightarrow \quad \frac{h_2 - h_1}{x} = \frac{u_1\sin\alpha_1 - u_2\sin\alpha_2}{u_1\cos\alpha_1 + u_2\cos\alpha_2}$$

Path of the Projectile as Seen from Another Projectile

Consider two particles A and B, projected from the same point with initial speeds u_1 and u_2, respectively as shown in the figure.



$$\mathbf{a}_{AB} = 0$$

$$u_{ABx} = v_{ABx} = u_1\cos\alpha_1 - u_2\cos\alpha_2$$

$$u_{ABy} = v_{ABy} = u_1\sin\alpha_1 - u_2\sin\alpha_2$$

$$x_{AB} = v_{ABx} \times t \text{ and } y_{AB} = v_{ABy} \times t$$

$$\Rightarrow \quad \frac{x_{AB}}{y_{AB}} = \text{constant}$$

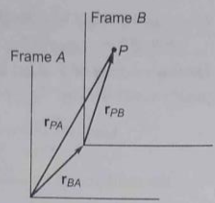
So, motion of one projectile as seen from other projectile will be a straight line.

Important Points & Formulae of Projectile Motion

- At highest point, the linear momentum is $mu\cos\theta$ and the kinetic energy is $\frac{1}{2}m(u\cos\theta)^2$.
- The horizontal displacement of the projectile after t seconds $x = (u\cos\theta)t$
- The vertical displacement of the projectile after t seconds
  $$y = (u\sin\theta)t - \frac{1}{2}gt^2$$
- Equation of the path of projectile
  $$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$
- The path of a projectile is parabolic.
- Kinetic energy at lowest point $= \frac{1}{2}mu^2$
- Linear momentum at lowest point $= mu$
- Acceleration of projectile is constant throughout the motion and it acts vertically downwards being equal to g.
- Angular momentum of projectile $= mu\cos\theta \times h$, where h denotes the height.
- In case of angular projection, the angle between velocity and acceleration varies from $0° < \theta < 180°$.
- The maximum height occurs when the projectile covers a horizontal distance equal to half of the horizontal range, *i.e.*, $R/2$.
- When the maximum range of projectile is R, then its maximum height is $R/4$.

Relative Motion in Two Dimensions

The description of the motion of an object in two or three dimensions depends on the choice of the coordinate system. Figure shows two reference frames in two dimensions. The vectors $\mathbf{r}_{PA}$ and $\mathbf{r}_{PB}$ are the position vectors of the object P in reference frame A and in reference frame B, respectively. Vector $\mathbf{r}_{BA}$ is the position of observer B (located at the origin of reference frame B) w.r.t. frame A. The position vector of the object P in reference frame B can be obtained from the position vector in reference frame A.

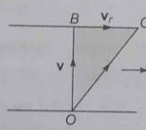


$$\mathbf{r}_{PA} = \mathbf{r}_{BA} + \mathbf{r}_{PB}$$

The velocity and acceleration of the object P are therefore,

$$\mathbf{v}_{PA} = \mathbf{v}_{BA} + \mathbf{v}_{PB} \text{ and } \mathbf{a}_{PA} = \mathbf{a}_{BA} + \mathbf{a}_{PB}$$

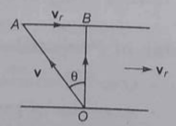
For crossing the river in the shortest time, the boat should sail perpendicular to the flow. If the width of the river is d and v is the velocity of the boat in still water, then



$$t = \frac{d}{v} \quad \text{and} \quad OC = \sqrt{d^2 + \left(v_r \times \frac{d}{v}\right)^2}$$

For crossing the river by the shortest distance, the boat moves such that the horizontal component of the velocity balances the speed of flow. Time of crossing,

where v_r = velocity of river flow



$$t = \frac{d}{\sqrt{v^2 - v_r^2}}$$

Applications of the Concept of Relative Motion

1. Rain Problems

In this case, the three velocities with which we have to deal, are

$\mathbf{v}_{RG} \Rightarrow$ Velocity of rain w.r.t. ground

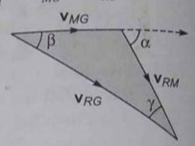
$\mathbf{v}_{MG} \Rightarrow$ Velocity of man w.r.t. ground

$\mathbf{v}_{RM} \Rightarrow$ Velocity of rain w.r.t. man

From the equation of relative motion,

$$\mathbf{v}_{RM} = \mathbf{v}_{RG} - \mathbf{v}_{MG} \quad \text{or} \quad \mathbf{v}_{RM} + \mathbf{v}_{MG} = \mathbf{v}_{RG}$$

From this equation we can construct a vector diagram, which would be very helpful in solving the questions.



The vector diagram for a particular situation is shown in figures.

From sine law, $\dfrac{|\mathbf{v}_{MG}|}{\sin\gamma} = \dfrac{|\mathbf{v}_{RM}|}{\sin\beta} = \dfrac{|\mathbf{v}_{RG}|}{\sin(\pi - \alpha)}$

Remember $|\mathbf{v}_{MG}|, |\mathbf{v}_{RM}|$ or $|\mathbf{v}_{RG}|$ are not representing the length of triangles.

2. River Boat Problems

In this case, we have to deal with the following velocities

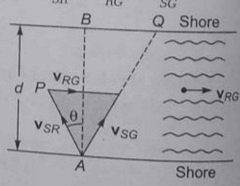
$\mathbf{v}_{SR} \Rightarrow$ velocity of swimmer/steamer/boat w.r.t. river/water/stream.

$\mathbf{v}_{RG} \Rightarrow$ Velocity of river/water/stream w.r.t. ground.

$\mathbf{v}_{SG} \Rightarrow$ Velocity of swimmer steamer/boat w.r.t. ground.

From relative motion equation $\mathbf{v}_{SR} + \mathbf{v}_{RG} = \mathbf{v}_{SG}$

For the situation shown in the figure, a boat is crossing the river and it is heading (moving) along AP as shown, then the time taken by the boat to cross the river is,



$$t = \frac{d}{|\mathbf{v}_{SR} \cos\theta|}$$

and it will reach the point Q due to river flow. The distance $BQ = (|\mathbf{v}_{RG}| - |\mathbf{v}_{SR}| \sin\theta)\, t$ is termed as **drift**.

3. Aeroplane Problems

Here the following velocities are to be analyzed

$\mathbf{v}_{PA} \Rightarrow$ Velocity of plane w.r.t. air.

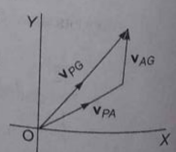
$\mathbf{v}_{AG} \Rightarrow$ Velocity of air w.r.t. ground.

$\mathbf{v}_{PG} \Rightarrow$ Velocity of plane w.r.t. ground.

The relative motion equation would be

$$\mathbf{v}_{PG} = \mathbf{v}_{PA} + \mathbf{v}_{AG}$$

as shown in figure below



1. Unit vector perpendicular to vector $\mathbf{A} = 3\hat{i} + \hat{j}$ and $\mathbf{B} = 2\hat{i} - \hat{j} - 5\hat{k}$ both, is

(a) $\pm \frac{3\hat{j} - 2\hat{k}}{\sqrt{11}}$ (b) $\pm \frac{(\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{11}}$

(c) $\pm \frac{-\hat{j} + 2\hat{k}}{\sqrt{13}}$ (d) $\pm \frac{\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{13}}$

2. Given, $\mathbf{A} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{B} = -\hat{i} - \hat{j} - \hat{k}$. $(\mathbf{A} - \mathbf{B})$ will make an angle with **A** as

(a) 0° (b) 180° (c) 90° (d) 60°

3. Component of the vector $\mathbf{A} = 2\hat{i} + 3\hat{j}$ along the vector $\mathbf{B} = (\hat{i} + \hat{j})$ is

(a) $\frac{5}{\sqrt{2}}$ (b) $4\sqrt{2}$

(c) $\frac{\sqrt{2}}{3}$ (d) None of these

4. Which of the following is a unit vector?

(a) $\hat{i} + \hat{j}$ (b) $\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

(c) $\sin\theta\,\hat{i} + 2\cos\theta\,\hat{j}$ (d) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j})$

5. The component of a vector along any other direction is

(a) always less than its magnitude
(b) always greater than its magnitude
(c) always equal to its magnitude
(d) None of the above

6. At what angle should the two forces $2P$ and $\sqrt{2}P$ act so that the resultant force is $P\sqrt{10}$?

(a) 45° (b) 60°
(c) 90° (d) 120°

7. If three vectors along coordinate axes represent the adjacent sides of a cube of length b, then the unit vector along its diagonal passing through the origin will be

(a) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$ (b) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}b}$

(c) $\hat{i} + \hat{j} + \hat{k}$ (d) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

8. If **A** and **B** are two non-zero vectors having equal magnitude, the angle between the vectors **A** and **A** − **B** is

(a) 0°
(b) 90°
(c) 180°
(d) dependent on the orientation of **A** and **B**

9. Unit vector parallel to the resultant of vectors $4\hat{i} - 3\hat{j}$ and $8\hat{i} + 8\hat{j}$ will be

(a) $\frac{(24\hat{i} + 5\hat{j})}{13}$ (b) $\frac{(12\hat{i} + 5\hat{j})}{13}$

(c) $\frac{(6\hat{i} + 5\hat{j})}{13}$ (d) None of these

10. A force of 5 N acts on a particle along a direction making an angle of 60° with vertical. Its vertical component will be

(a) 10 N (b) 3 N (c) 4 N (d) 2.5 N

11. A vector is represented by $3\hat{i} + \hat{j} + 2\hat{k}$. Its length in x-y plane is

(a) 2 (b) $\sqrt{14}$ (c) $\sqrt{10}$ (d) $\sqrt{5}$

12. If $\frac{|\mathbf{a} + \mathbf{b}|}{|\mathbf{a} - \mathbf{b}|} = 1$, then angle between **a** and **b** is

(a) 0° (b) 45° (c) 90° (d) 60°

13. Which one of the following expression is correct?

(a) $|\mathbf{a} - \mathbf{b}| = |\mathbf{a}| - |\mathbf{b}|$ (b) $|\mathbf{a} - \mathbf{b}| \le |\mathbf{a}| - |\mathbf{b}|$
(c) $|\mathbf{a} - \mathbf{b}| \ge |\mathbf{a}| - |\mathbf{b}|$ (d) $|\mathbf{a} - \mathbf{b}| > |\mathbf{a}| - |\mathbf{b}|$

14. A man walks 20 m at an angle 60°, towards north of east. How far towards east, has he travelled?

(a) 10 m (b) 20 m
(c) $20\sqrt{3}$ m (d) $\frac{10}{\sqrt{3}}$ m

15. If two vectors $2\hat{i} + 3\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are parallel to each other, then value of λ is

(a) zero (b) 2 (c) 3 (d) 4

16. If a unit vector is represented by $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$, then the value of c is

(a) 1 (b) $\sqrt{0.11}$ (c) $\sqrt{0.01}$ (d) $\sqrt{0.39}$

17. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is **[NCERT Exemplar]**

(a) $v_0 + 2g + 3f$
(b) $v_0 + \frac{g}{2} + \frac{f}{3}$
(c) $v_0 + g + f$
(d) $v_0 + \frac{g}{2} + f$

18. Given that $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$, out of the three vectors, two are equal in magnitude and the magnitude of the third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then, angle between the vectors are

(a) 30°, 60°, 90°
(b) 45°, 45°, 90°
(c) 90°, 135°, 45°
(d) 90°, 135°, 135°

19. A vector having magnitude of 30 unit, makes equal angles with each of the X, Y and Z axes. The components of the vector along each of X, Y and Z axes are

(a) $10\sqrt{3}$ unit (b) $\frac{10}{\sqrt{3}}$ unit (c) $15\sqrt{3}$ unit (d) 10 unit

20. If $\mathbf{F}_1$ and $\mathbf{F}_2$ are two vectors of equal magnitude F such that $|\mathbf{F}_1 \cdot \mathbf{F}_2| = |\mathbf{F}_1 \times \mathbf{F}_2|$, then $|\mathbf{F}_1 + \mathbf{F}_2|$ is equal to

(a) $\sqrt{(2+\sqrt{2})}F$
(b) $2F$
(c) $F\sqrt{2}$
(d) None of these

21. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is

(a) $-2abv^2$ (b) $2bv^3$ (c) $-2av^3$ (d) $2av^2$

22. Pick up the correct statement.

(a) Path of a projectile as seen by another projectile is a parabola
(b) A body, whatever be its motion, is always at rest in a frame of reference fixed to the body itself
(c) Area under a-t graph, gives velocity
(d) Area under a-t graph gives a change in velocity

23. P, Q and R are three coplanar forces acting at a point and are in equilibrium. Given, $P = 1.9318$ kg-wt, $\sin\theta_1 = 0.9659$, the value of R in kg-wt is

(a) 0.965 (b) 2 (c) 1 (d) 1/2

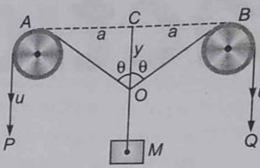
24. $\int \frac{dv}{\sqrt{2av - v^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right]$, on the basis of dimensional analysis, the value of n is

(a) 0 (b) -2 (c) 3 (d) 1

25. A boat takes 2 h to travel 8 km and back in a still water lake. If the velocity of water is 4 kmh^{-1}, the time taken for going upstream of 8 km and coming back is

(a) 2 h
(b) 2 h 40 min
(c) 1 h and 20 min
(d) Cannot be estimated from the given information

26. In the arrangement shown in the figure, the ends P and Q of an unstretchable string move downwards with an uniform speed u. Pulleys A and B are fixed. Mass M moves upward with a speed



(a) $2u\cos\theta$
(b) $\frac{u}{\cos\theta}$
(c) $\frac{2u}{\cos\theta}$
(d) $u\cos\theta$

27. A river is flowing from west to east with a speed of 5 m/min. A man on the south bank of the river, is capable of swimming at 10 m/min in still water, he wants to swim across the river in the shortest time. He should swim in a direction

(a) due north
(b) 30° east of north
(c) 30° west of north
(d) 60° east of north

28. A ball rolls off the top of a stair way with a horizontal velocity of u ms^{-1}. If the steps are h metre high and b metre wide, the ball will hit the edge of the nth step, where n is

(a) $\frac{2hu}{gb^2}$
(b) $\frac{2hu^2}{gb^2}$
(c) $\frac{2hu^2}{gb}$
(d) $\frac{hu^2}{gb^2}$

29. The range of a projectile fired at an angle of 15° is 50 m, If it is fired with the same speed at an angle of 45°, its range will be

(a) 25 m
(b) 50 m
(c) 100 m
(d) 77.6 m

30. A projectile is fired at an angle of 30° with the horizontal such that the vertical component of its initial velocity is 80 m/s^{-1}. Its time of flight is T. Its velocity at $t = \frac{T}{4}$ has a magnitude of nearly ($g = 10\ ms^{-2}$)

(a) 180 ms^{-1}
(b) 155 ms^{-1}
(c) 145 ms^{-1}
(d) 140 ms^{-1}

31. The maximum height attained by a projectile is found to be equal to 0.433 times its horizontal range. The angle of projection of this projectile is

(a) 30°
(b) 45°
(c) 60°
(d) 75°

32. A ball A is thrown up vertically with a speed u and at the same instant another ball B is released from a height h. At time t, the speed of A relative to B is **[NCERT Examplar]**

(a) u
(b) $2u$
(c) $u - gt$
(d) $\sqrt{u^2 - gt}$

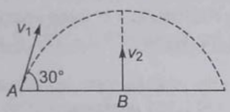
33. A ball is thrown from the ground with a velocity of $20\sqrt{3}$ ms^{-1} making an angle of 60° with the horizontal. The ball will be at a height of 40 m from the ground after a time t equal to ($g = 10$ ms^{-2})

(a) $\sqrt{2}$ s (b) $\sqrt{3}$ s (c) 2 s (d) 3 s

34. A particle of mass m is projected with a velocity v at an angle of 60° with the horizontal. When the particle is at its maximum height, the magnitude of its angular momentum about the point of projection is

(a) zero (b) $\frac{3mv^3}{16g}$ (c) $\frac{\sqrt{3}\,mv^3}{16g}$ (d) $\frac{3\,mv^3}{8g}$

35. A body is projected with a velocity v_1 from the point A as shown in the figure. At the same time, another body is projected vertically upwards from B with velocity v_2. The point B lies vertically below the highest point. For both the bodies to collide, $\frac{v_2}{v_1}$ should be



(a) 2 (b) 0.5
(c) $\sqrt{\frac{3}{2}}$ (d) 1

36. A person aims a gun at a bird from a point, at a horizontal distance of 100 m. If the gun can induce a speed of 500 ms^{-1} to the bullet, at what height above the bird must he aim his gun in order to hit it? ($g = 10$ ms^{-2})

(a) 10 cm (b) 20 cm (c) 50 cm (d) 100 cm

37. A car is travelling with a velocity of 10 kmh^{-1} on a straight road. The driver of the car throws a parcel with a velocity of $10\sqrt{2}$ kmh^{-1} when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with the direction of the car.

(a) 135° (b) 45°
(c) $\tan^{-1}(\sqrt{2})$ (d) $\tan\left(\frac{1}{2}\right)$

38. A cannon ball has the same range R on a horizontal plane for two angles of projection. If h_1 and h_2 are the greatest heights in the two paths for which this is possible, then

(a) $R = (h_1h_2)^{1/4}$ (b) $R = 3\sqrt{h_1h_2}$
(c) $R = 4\sqrt{h_1h_2}$ (d) $R = \sqrt{h_1h_2}$

39. Two paper screens A and B are separated by a distance of 200 m. A bullet pierces A and then B. The hole in B is 40 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is

(a) 200 ms^{-1} (b) 400 ms^{-1}
(c) 600 ms^{-1} (d) 700 ms^{-1}

40. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection is **[NCERT Exemplar]**

(a) 60° (b) $\tan^{-1}(\sqrt{3}/2)$
(c) $\tan^{-1}(1/2)$ (d) 45°

41. A swimmer crosses a flowing stream of width d to and fro in time t_1. The time taken to cover the same distance up and down the stream is t_2. Then the time the swimmer would take to swim across a distance $2d$ in still water is

(a) $\frac{t_1^2}{t_2}$ (b) $\frac{t_2^2}{t_1}$
(c) $\sqrt{t_1t_2}$ (d) $(t_1 + t_2)$

42. Ratio of minimum kinetic energies of two projectiles with the same mass is 4 : 1. The ratio of the maximum height attained by them is also 4 : 1. The ratio of their range would be

(a) 2 : 1 (b) 4 : 1
(c) 8 : 1 (d) 16 : 1

43. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1}. Then the time after which its inclination with the horizontal is 45°, is

(a) 15 s (b) 10.98 s
(c) 5.49 s (d) 2.745 s

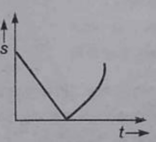
44. A projectile projected with a velocity u at an angle θ passes through a given height h two times at t_1 and t_2. Then,

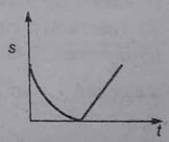
(a) $t_1 + t_2 = T$ (time of flight) (b) $t_1 + t_2 = \frac{T}{2}$
(c) $t_1 + t_2 = 2T$ (d) $\sqrt{t_1t_2} = T$

45. A ship A is moving Westwards with a speed of 10 kmh^{-1} and a ship B, 100 km South of A is moving Northwards with a speed of 10 kmh^{-1}. The time after which the distance between them is shortest and the the shortest distance between them are

(a) 0 h, 100 km (b) 5h, $50\sqrt{2}$ km
(c) $5\sqrt{2}$ h, 50 km (d) $10\sqrt{2}$ h, $50\sqrt{2}$ km

46. A ball is thrown vertically upwards. Which of the following plots represent the speed-time graph of the ball during its flight, if the air resistance is not ignored?

(a)


(b)


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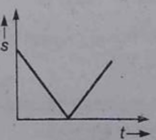
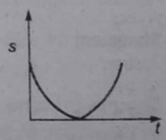
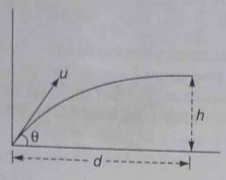


(d)


47. If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u if stone is launched at an angle θ? **[NCERT Exemplar]**



(a) $\frac{g}{\cos\theta}\sqrt{\frac{d}{2(d\tan\theta - h}}$ (b) $\frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

(c) $\sqrt{\frac{gd^2}{h\cos^2\theta}}$ (d) $\sqrt{\frac{gd^2}{(d-h)}}$

48. A ball whose kinetic energy is E, is projected at an angle of 45°, to the horizontal. The kinetic energy of the ball at the highest point in its flight will be

(a) $E/\sqrt{2}$ (b) E
(c) 0 (d) $E/2$

49. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flight in the two cases, then the product of the two time of flight is directly proportional to

(a) R (b) $\frac{1}{R}$

(c) $\frac{1}{R^2}$ (d) R^2

Directions *(Q. Nos. 50 to 59) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below :*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

50. **Statement I** Rain is falling vertically downwards with a velocity of 3 kmh^{-1}. A man walks with a velocity of 4 kmh^{-1}. Relative velocity of rain w.r.t. man is 5 kmh^{-1}.

Statement II Relative velocity of rain w.r.t. man is given by

$$\mathbf{v}_{rm} = \mathbf{v}_r - \mathbf{v}_m$$

51. **Statement I** For the projection angle $\tan^{-1}(4)$, the horizontal and maximum height of a projectile are equal.

Statement II The maximum range of a projectile is directly proportional to the square of velocity and inversely proportional to the acceleration due to gravity.

52. **Statement I** In order to hit a target, a man should point his rifle in the same direction as the target.

Statement II The horizontal range of bullet is dependent on the angle of projection with the horizontal.

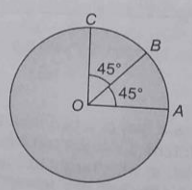
53. **Statement I** The trajectory of projectile is quadratic in x and linear in y.

Statement II y-component of the trajectory is independent of the x-component.

54. **Statement I** In a javelin throw, the athlete throws the projectile at an angle slightly more than 45°.

Statement II The maximum range does not depend upon the angle of projection.

55. **Statement I** The resultant of three vectors **OA**, **OB** and **OC** as shown in the figure is $R(1+\sqrt{2})$. R is the radius of the circle.



Statement II **OA** + **OC** is acting along **OB** and (**OA** + **OC**) + **OB** is acting along **OB**.

56. **Statement I** Two forces acting at a point, will have the resultant force directed outwards from the point at which the forces are acting.

Statement II The resultant of two forces, acts along the diagonal formed by the parallelograph with the sides being the given forces.

57. **Statement I** If **A** is parallel to **B**, then **A** × **B** is a null vector.

Statement II The cross product of two vectors is given by $\mathbf{A}\times\mathbf{B} = AB\sin\theta$

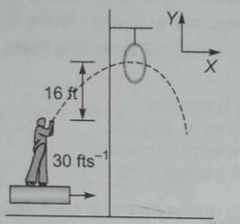
58. **Statement I** Scalars can be added algebraically.

Statement II Vectors cannot be added algebraically.

59. **Statement I** Angle between $\hat{i}+\hat{j}$ and $\hat{i}$ is 45°.

Statement II $\hat{i}+\hat{j}$ is equally include to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is 90°.

Directions *(Q. Nos. 60 to 63) A man is riding on a flat car travelling with a constant velocity of 30 ft s^{-1}. He wishes to throw a ball through a stationary hoop (something like basket ball game) 16 ft above the height of his hands in such a manner that ball is moving horizontally when it passes through the hoop. To do this he throws the ball with a speed of 40 fts^{-1} w.r.t. himself (Take g = 32 fts^{-2}). [Coordinate axes are as shown in the figure]*



60. The initial velocity of the ball w.r.t. the trolley frame of reference is (fts^{-1})

(a) $32\hat{i} + 24\hat{j}$
(b) $40\hat{i} + 0\hat{j}$
(c) $24\hat{i} + 32\hat{j}$
(d) $30\hat{i} + 26.45\hat{j}$

61. The initial velocity of the ball w.r.t. the ground frame of reference is (fts^{-1})

(a) $62\hat{i} + 24\hat{j}$
(b) $70\hat{i}$
(c) $24\hat{i} + 62\hat{j}$
(d) $54\hat{i} + 32\hat{j}$

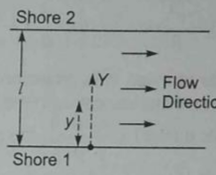
62. How many second after he releases the ball, it pass through the loop?

(a) 1 s (b) 2 s (c) 0.5 s (d) 3 s

63. The horizontal distance between hoop and the ball when the person releases the ball is

(a) 24 ft
(b) 54 ft
(c) 12 ft
(d) 72 ft

Directions *(Q. Nos. 64 to 66) The velocity of flow of stream between the two parallel shores, shore 1 and shore 2 is varying uniformly from 0 to v over the width of the river. A boat starts rowing with constant speed u (relative to stream) from shore 1 to 2 reach shore 2.*



64. If the boat is moving in such a way that w.r.t. observer on shore it is always moving along a perpendicular line to shores, then angle made by the boat's bow with the perpendicular line as a function of y is given by [Take $u > v$]

(a) $\cos^{-1}\left(\frac{uy}{vt}\right)$
(b) $\cos^{-1}\left(\frac{vy}{ut}\right)$
(c) $\tan^{-1}\left(\frac{vy}{ut}\right)$
(d) $\sin^{-1}\left(\frac{vy}{ut}\right)$

65. For the situation mentioned above the time taken by the boat to cross the river is

(a) $\frac{1}{u}\sin^{-1}\left(\frac{v}{u}\right)$
(b) $\frac{1}{v}\sin^{-1}\left(\frac{u}{v}\right)$
(c) $\frac{1}{v}\sin^{-1}\left(\frac{v}{u}\right)$
(d) $\frac{1}{u}\sin^{-1}\left(\frac{u}{v}\right)$

66. If $u = \frac{3v}{4}$, then upto what value of y the observer will see that boat to be moving along a line perpendicular to the shores?

(a) $\frac{3t}{4}$
(b) $\frac{t}{2}$
(c) $\frac{4t}{3}$
(d) $\frac{2t}{3}$

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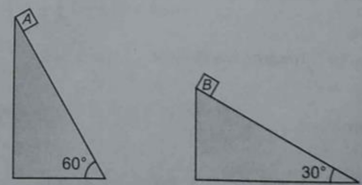
67. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is **[AIEEE 2011]**

(a) $\pi\frac{v^4}{g^2}$
(b) $\frac{\pi}{2}\frac{v^4}{g^2}$
(c) $\pi\frac{v^2}{g^2}$
(d) $\pi\frac{v^2}{g}$

68. Two fixed frictionless inclined plane making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? **[AIEEE 2010]**



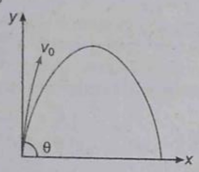
(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

69. For a particle in uniform circular motion the acceleration **a** at a point $P(R, \theta)$ on the circle of radius R is (here θ is measured from the x-axis) **[AIEEE 2010]**

(a) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ (b) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

(c) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ (d) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

70. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



[AIEEE 2010]

(a) $-mgv_0t^2\cos\theta\,\hat{j}$ (b) $mgv_0t\cos\theta\,\hat{k}$

(c) $-\frac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$ (d) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$

71. A particle has an initial velocity $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\,\hat{i} + 0.3\,\hat{j}$. Its speed after 10 s is **[AIEEE 2009]**

(a) 10 units (b) $7\sqrt{2}$ units

(c) 7 units (d) 8.5 units

72. A particle is projected at angle of 60° with the horizontal having a kinetic energy K. The kinetic energy at the highest point is **[AIEEE 2007]**

(a) K (b) zero

(c) $K/4$ (d) $K/2$

73. A particle is moving Eastwards with a velocity of 5 ms^{-1}. In 10 s, the velocity changes to 5 ms^{-1} Northwards The average acceleration in this time is **[AIEEE 2005]**

(a) $\frac{1}{\sqrt{2}}ms^{-2}$ towards North-East

(b) $\frac{1}{2}ms^{-2}$ Towards North

(c) zero

(d) $\frac{1}{\sqrt{2}}ms^{-2}$ Towards North-West

74. A force $\mathbf{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\mathbf{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle in joule is **[AIEEE 2004]**

(a) -7 (b) $+7$

(c) $+10$ (d) $+13$

75. A particle is acted upon by a force of constant magnitude which is always acting perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that **[AIEEE 2004]**

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a straight line

76. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 ms^{-1} at an angle of 30° with the horizontal. How far from the throwing point, will the ball be at the height of 10 m from the ground?

$[g = 10\ ms^{-2}, \sin 30° = \frac{1}{2}, \cos 30° = \frac{\sqrt{3}}{2}]$ **[AIEEE 2003]**

(a) 5.20 m (b) 4.33 m

(c) 2.60 m (d) 8.66 m

77. The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by

(a) $t^2\sqrt{\alpha^2 + \beta^2}$ (b) $\sqrt{\alpha^2 + \beta^2}$

(c) $3t\sqrt{\alpha^2 + \beta^2}$ (d) $3t^2\sqrt{\alpha^2 + \beta^2}$

78. A ball whose kinetic energy is E, is projected at an angle of 45° with respect to the horizontal. The kinetic energy of the ball at the highest point of its flight will be **[AIEEE 2002]**

(a) E (b) $\frac{E}{\sqrt{2}}$

(c) $\frac{E}{2}$ (d) zero

79. If $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$, then the angle between **A** and **B** is **[AIEEE 2004]**

(a) π (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Answers

1. (b)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (d)
11. (c)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (a)
21. (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (c)
31. (c)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (d)	**40.** (c)
41. (a)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (b)	**48.** (d)	**49.** (a)	**50.** (a)
51. (b)	**52.** (d)	**53.** (c)	**54.** (d)	**55.** (a)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (a)	**60.** (c)
61. (d)	**62.** (a)	**63.** (b)	**64.** (d)	**65.** (c)	**66.** (a)	**67.** (a)	**68.** (d)	**69.** (c)	**70.** (c)
71. (b)	**72.** (c)	**73.** (d)	**74.** (b)	**75.** (c)	**76.** (d)	**77.** (d)	**78.** (c)	**79.** (a)	

Hints & Solutions

1. The unit vector in the normal direction is

$$\hat{\mathbf{n}} = \pm \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A}|\,|\mathbf{B}|\,\sin\theta}$$

Here, $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & 0 \\ 2 & -1 & -5 \end{vmatrix}$

$$= -5\hat{\mathbf{i}} + 15\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

$$|\mathbf{A}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$|\mathbf{B}| = \sqrt{(2)^2 + (-1)^2 + (-5)^2} = \sqrt{30}$$

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|} = \frac{1}{2\sqrt{3}}$$

$\therefore$ $$\sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{11}}{2\sqrt{3}}$$

$\therefore$ $$\hat{\mathbf{n}} = \pm \frac{-5\hat{\mathbf{i}} + 15\hat{\mathbf{j}} - 5\hat{\mathbf{k}}}{\sqrt{10}\cdot\sqrt{30}\cdot\dfrac{\sqrt{11}}{2\sqrt{3}}} = \pm\frac{(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{11}}$$

2. $\mathbf{A} - \mathbf{B} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}} = 2\mathbf{A}$

i.e., $\mathbf{A} - \mathbf{B}$ and $\mathbf{A}$ are parallel.

3. Component of $\mathbf{A}$ along $\mathbf{B} = A\cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{B} = \dfrac{2+3}{\sqrt{1+1}} = \dfrac{5}{\sqrt{2}}$

4. Let $\mathbf{A} = \cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}}$

$\therefore$ $|\mathbf{A}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

Therefore, it is a unit vector.

6. $P\sqrt{10} = \sqrt{4P^2 + 2P^2 + 4\sqrt{2}P^2\cos\theta}$

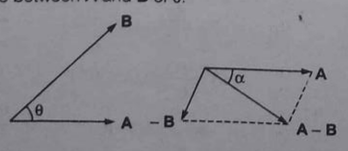
$\therefore$ $\theta = 45°$

7. Diagonal vector $\mathbf{A} = b\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}} + b\hat{\mathbf{k}}$

or $A = \sqrt{b^2 + b^2 + b^2} = \sqrt{3}\,b$

$\therefore$ $\hat{\mathbf{A}} = \dfrac{\mathbf{A}}{A} = \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

8. Suppose angle between two vectors $\mathbf{A}$ and $\mathbf{B}$ of equal magnitude is θ. Then, angle between $\mathbf{A}$ and $\mathbf{A} - \mathbf{B}$ will be $\dfrac{180° - \theta}{2}$ or $90° - \dfrac{\theta}{2}$. Hence, this angle will depend on the angle between $\mathbf{A}$ and $\mathbf{B}$ or θ.



9. Resultant of two given vectors is $12\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}$. Magnitude of this resultant will be $\sqrt{144 + 25} = 13$. Hence, a unit vector parallel to this resultant would be $\dfrac{12\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}}{13}$.

10. $F_v = 5\cos 60° = 2.5\,\text{N}$

11. In *x-y* plane vector is $3\,\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

$\therefore$ Length in *x-y* plane $= \sqrt{9 + 1} = \sqrt{10}$

12. $$\frac{|\mathbf{a} + \mathbf{b}|}{|\mathbf{a} - \mathbf{b}|} = 1$$

i.e., $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$

or $a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

$\therefore$ $\cos\theta = 0°$ or $\theta = 90°$

13. $\mathbf{a} - \mathbf{b}$ is nothing but addition of $\mathbf{a}$ and $-\mathbf{b}$

So, the magnitude of $\mathbf{a} - \mathbf{b}$ will lie between $|\mathbf{a}| + |\mathbf{b}|$ and $|\mathbf{a}| - |\mathbf{b}|$

14. $20\cos 60° = 10\,\text{m}$

15. The coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ should bear a constant ratio.

or $\dfrac{2}{-4} = \dfrac{3}{-6} = \dfrac{1}{-\lambda}$

or $\lambda = 2$

16. $\sqrt{(0.5)^2 + (0.8)^2 + (c)^2} = 1$

$\therefore$ $c^2 = 0.11$

or $c = \sqrt{0.11}$

17. $v = v_o + gt + ft^2$

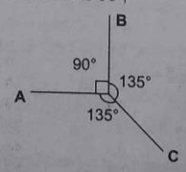
or $\dfrac{dx}{dt} = v_o + gt + ft^2$

$dx = (v_o + gt + ft^2)dt$

So, $\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)dt$

$x = v_o + \dfrac{g}{2} + \dfrac{f}{3}$

18. Angle between $\mathbf{A}$ and $\mathbf{B}$ is 90°,



between $\mathbf{B}$ and $\mathbf{C}$ is 135° and that between $\mathbf{A}$ and $\mathbf{C}$ is 135°.

19. $A_x = A_y = A_z$

Now, $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{3}\, A_x$

$\therefore \quad A_x = \frac{A}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

20. $FF\cos\theta = FF\sin\theta$

or $\tan\theta = 1$ or $\theta = 45°$

$\therefore \quad |\mathbf{F}_1 + \mathbf{F}_2| = \sqrt{F^2 + F^2 + 2FF\cos 45°}$

$= (\sqrt{2+\sqrt{2}})\,F$

21. $t = ax^2 + bx$

$\Rightarrow \quad 1 = 2ax\frac{dx}{dt} + \frac{bdx}{dt}$

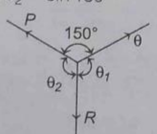
$\Rightarrow \quad v = \frac{1}{2ax + b}$

Thus, $\quad a = \frac{dv}{dt} = \frac{-2av}{(2ax+b)^2} = -2av^3$

22. $a = \frac{dv}{dt} \Rightarrow \int_{v_1}^{v_2} dv = \int_0^t a\,dt$

$\Rightarrow \Delta V$ = area under a-t graph and Δv represents a change in velocity. The path of projectile is a straight line as the relative velocity between the particles remains constant.

23. Here, $\frac{P}{\sin\theta_1} = \frac{Q}{\sin\theta_2} = \frac{R}{\sin 150°}$



$\Rightarrow \quad R = \frac{P\sin 150°}{\sin\theta_1} = \frac{1.9318 \times (1/2)}{0.9653} = 1$

24. [LHS] = [RHS]

$\left[\frac{dv}{\sqrt{2av - v^2}}\right] = \left[a^n \sin^{-1}\left(\frac{x}{a} - 1\right)\right]$

$\frac{[dv]}{[\sqrt{v^2}]} = [a^n]$

$\frac{[dv]}{[v]} = [a^n]$

$\Rightarrow \quad [M^0L^0T^0] = [a^n]$

$\Rightarrow \quad n = 0$

25. Total distance travelled by the boat in 2 h

$= 8 + 8 = 16\,\text{km}$

Therefore, speed of boat in still water,

$v_b = \frac{16}{2} = 8\,\text{km h}^{-1}$

Effective velocity when boat moves upstream

$= v_b - v_w = 8 - 4 = 4\,\text{km h}^{-1}$,

Therefore, time taken to travel $= \frac{8}{4} = 2\,\text{h}$

Effective velocity when boat moves down stream

$= v_b + v_w = 8 + 4 = 12\,\text{km h}^{-1}$

The time taken to travel 8 km distance

$= \frac{8}{12} = \frac{2}{3}\,\text{h} = 40\,\text{min}$

Total time taken = 2 h + 40 min = 2 h 40 min.

26. As P and Q move down, the length $l(= OA = OB)$ decreases at the rate of $u\ \text{ms}^{-1}$ (say).

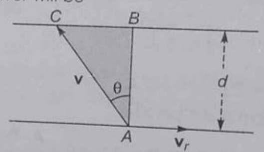
From given figure, $l^2 = a^2 + y^2$

Differentiating w.r.t. time

$2l\frac{dl}{dt} = 0 + 2y\frac{dy}{dt}$

$\therefore \quad \frac{dy}{dt} = \frac{l}{y}\frac{dl}{dt} = \frac{1}{\cos\theta}u = \frac{u}{\cos\theta}$

27. Let the swimmer starts swimming with velocity v along AC in a direction making an angle θ with AB as shown in the figure. If d is the width of the river, time taken by the swimmer to cross the river will be



$t = d/v\cos\theta$

As component of AB will be $v\cos\theta$, This time will be minimum, when $\cos\theta = \max = 1$, *i.e.*, $\theta = 0°$.

28. Let the ball strike the, nth step after t second. Vertical distance travelled by the ball $= nh = \frac{1}{2}gt^2$

Horizontal distance travelled by the ball $= nb = ut$

or $\quad t = \frac{nb}{u}$

$\Rightarrow \quad nh = \frac{1}{2}g\left(\frac{nb}{u}\right)^2$

or $\quad n = \frac{2u^2}{g}\frac{h}{b^2}$

29. $\frac{u^2}{g}\sin(2 \times 15°) = 50$

or $\quad \frac{u^2}{g} = \frac{50}{\sin 30°}$

$= \frac{50}{1/2} = 100$

$\text{HR} = \frac{u^2}{g}\sin(2 \times 45°)$

$= \frac{u^2}{g} = 100\,\text{m}$

30. Vertical component of the initial velocity,

$$u_y = u\sin 30° \text{ or } u = \frac{u_y}{\sin 30°} = \frac{80}{1/2} = 160\ \text{ms}^{-1}$$

$$t = \frac{T}{4} = \frac{2u\sin 30°}{4\times g} = \frac{2\times 80}{4\times 10} = 4\ \text{s}$$

$$v_x = u\cos 30° = 160\times\frac{\sqrt{3}}{2} = 80\sqrt{3}\ \text{ms}^{-1}$$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{u_x^2 + (u_y - gt)^2}$$

$$= \sqrt{(80\sqrt{3})^2 + (80 - 10\times 4)^2}$$

$$= 144.3\ \text{ms}^{-1} = 145\ \text{ms}^{-1}.$$

31. $\frac{u^2\sin^2\theta}{2g} = 0.433\frac{u^2}{g}\sin 2\theta$

or $\sin^2\theta = 0.433\times 4\sin\theta\cos\theta$

or $\tan\theta = 1.732 = \sqrt{3}$; or $\theta = 60°$

32. Refer figure at time t,

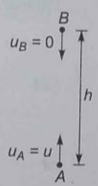
Velocity of A, $v_A = u - gt$ (upwards)

Velocity of B, $v_B = gt$ (downwards)

$= -gt$ (upwards)

Relative velocity of A w.r.t. B is

$v_{AB} = v_A - v_B = (u - gt) - (-gt) = u$



33. As, $s = u\sin\theta t - \frac{1}{2}gt^2$

So, $40 = 20\sqrt{3}\times\frac{\sqrt{3}t}{2} - \frac{1}{2}\times 10\times t^2$

or $5t^2 - 30t + 40 = 0$

or $t^2 - 6t + 8 = 0$ or $t = 2$ or 4

The minimum time $t = 2$ s

34. Maximum height,

$$H = \frac{v^2\sin^2 60°}{2g} = \frac{v^2}{2g}\times\frac{3}{4} = \frac{3v^2}{8g}$$

Momentum of the particle at the highest point

$$p = mv\cos 60° = \frac{mv}{2}$$

Angular momentum $= pH$

$$= \frac{mv}{2}\times\frac{3v^2}{8g}$$

$$= \frac{3mv^3}{16g}$$

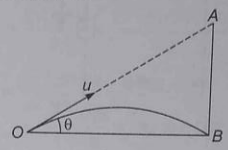
35. The two bodies will collide at the highest point if both cover the same vertical height in the same time. So,

$$\frac{v_1^2\sin^2 30°}{2g} = \frac{v_2^2}{2g}$$

or $$\frac{v_2}{v_1} = \sin 30°$$

$$= \frac{1}{2} = 0.5$$

36. Let the gun be fired with a velocity u from the point O on the bird at B, making an angle θ with the horizontal direction. Therefore, the height of the aim of the person be at height $BA\,(= h)$ above the bird.



Here, horizontal range $= \frac{u^2\sin 2\theta}{g} = 100$

or $\frac{(500)^2\sin 2\theta}{10} = 100$

or $\sin 2\theta = \frac{100\times 10}{(500)^2} = \frac{1}{250} = \sin 14'$

or $2\theta = 14'$ or $\theta = 7' = \frac{7}{60}\times\frac{\pi}{180}$ radian

As, Angle $= \frac{\text{arc}}{\text{radius}}$

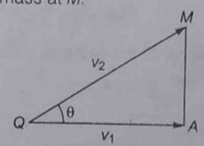
$\therefore$ $\theta = \frac{AB}{OB}$

or $AB = \theta\times OB$

$= \frac{7}{60}\times\frac{\pi}{180}\times(100\times 100)$ cm

$= 20$ cm

37. Let v_1 be the velocity of the car and v_2 be the velocity of the parcel. The parcel is thrown at an angle θ from Q, it reaches the mass at M.



$\therefore$ $\cos\theta = \frac{v_1}{v_2} = \frac{10}{10\sqrt{2}}$

$= \frac{1}{\sqrt{2}} = \cos 45°$

So, $\theta = 45°$

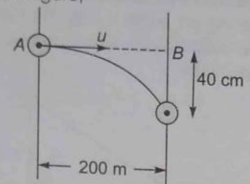
38. The cannon ball will have the same horizontal range for the angle of projection θ and $(90° - \theta)$. So,

$$h_1 = \frac{u^2\sin^2\theta}{2g} \text{ and } h_2 = \frac{u^2\cos^2\theta}{2g}$$

$$h_1h_2 = \frac{1}{4}\left(\frac{u^2\sin\theta\cos\theta}{g}\right)^2$$

$$= \frac{1}{4}\times\frac{R^2}{4} \quad\text{or}\quad R = 4\sqrt{h_1h_2}$$

39. Refering to the figure,



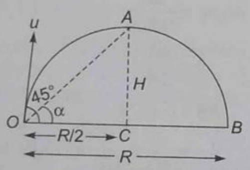
$$200 = ut \text{ or } t = \frac{200}{u}$$

Also, $$\frac{40}{100} = \frac{1}{2} \times 9.8 \left(\frac{200}{u}\right)^2$$

On solving, $u = 700 \text{ ms}^{-1}$.

40. Maximum height, $H = \dfrac{u^2 \sin^2 45°}{2g} = \dfrac{u^2}{4g} = AC$

Horizontal range, $R = \dfrac{u^2 \sin 2 \times 45°}{g} = \dfrac{u^2}{g}$



$\therefore$ $$OC = R/2 = u^2/(2g)$$

$$\tan \alpha = \frac{AC}{OC} = \frac{u^2/4g}{u^2/2g} = \frac{1}{2}$$

$\therefore$ $$\alpha = \tan^{-1}(1/2)$$

41. Let u be the velocity of the swimmer and v be the velocity of the river flow. Then

$$t_1 = \frac{2d}{\sqrt{u^2 - v^2}}$$

and $$t_2 = \frac{d}{u - v} + \frac{d}{u + v} = \frac{2ud}{u^2 - v^2}$$

If t is the time taken by the swimmer to swim a distance $2d$ in still water, then $t = \dfrac{2d}{u}$

$$t_2 \times t = \frac{2ud}{u^2 - v^2} \times \frac{2d}{u} = \frac{4d^2}{u^2 - v^2}$$

From above, $t_2 \times t = t_1^2$ or $t = \dfrac{t_1^2}{t_2}$

42. Given, $$\frac{\frac{1}{2} m(u_1 \cos \theta_1)^2}{\frac{1}{2} m(u_2 \cos \theta_2)^2} = \frac{4}{1}$$

or $$\frac{u_1 \cos \theta_1}{u_2 \cos \theta_2} = 2 \qquad \ldots(i)$$

and $$\left(\frac{u_1^2 \sin^2 \frac{\theta_1}{2g}}{u_2^2 \sin^2 \frac{\theta_2}{2g}}\right) = \frac{4}{1}$$

or $$\frac{u_1 \sin \theta_1}{u_2 \sin \theta_2} = 2 \qquad \ldots(ii$$

$\therefore$ $$\frac{R_1}{R_2} = \frac{u_1^2\, 2 \sin \theta_1 \cos \theta_1 / g}{u_2^2\, 2 \sin \theta_2 \cos \theta_2 / g} = \frac{u_1 \sin \theta_1}{u_2 \sin \theta_2} \times \frac{u_1 \cos \theta_1}{u_2 \cos \theta_2}$$

$$= 2 \times 2 = 4$$

43. Horizontal component of the velocity at angle 60°
= Horizontal component of the velocity at angle 45°.

i.e., $$u \cos 60° = v \cos 45°$$

$$147 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}}$$

or $$v = \frac{147}{\sqrt{2}} \text{ ms}^{-1}$$

Vertical component of $u = u \sin 60° = \dfrac{147\sqrt{3}}{2}$ m

Vertical component of $v = v \sin 45°$

$$= \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{147}{2} \text{ m}$$

But $$v_y = u_y + at$$

$\therefore$ $$\frac{147}{2} = \frac{147\sqrt{3}}{2} - 9.8\, t$$

or $$9.8\, t = \frac{147}{2}(\sqrt{3} - 1)$$

$\therefore$ $$t = 5.49 \text{ s}$$

44. For a projectile fired with a velocity u inclined at an angle θ with the horizontal

$$h = u \sin \theta\, t - \frac{1}{2} g t^2$$

$\Rightarrow$ $$g t^2 - 2u \sin \theta t + 2h = 0$$

$\therefore$ $$t = \frac{2u \sin \theta \pm \sqrt{4u^2 \sin^2 \theta - 8gh}}{2g}$$

$\Rightarrow$ $$t_1 = \frac{2u \sin \theta + \sqrt{4u^2 \sin^2 \theta - 8gh}}{2g}$$

and $$t_2 = \frac{2u \sin \theta - \sqrt{4u^2 \sin^2 \theta - 8gh}}{2g}$$

$\therefore$ $$t_1 + t_2 = \frac{2u \sin \theta}{g} = T$$

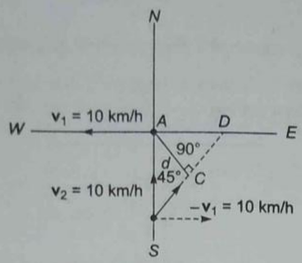
45. Let the ships A and B be at positions shown in figure when the distance between them is shortest.

Relative velocity of B w.r.t. A is

$$v_r = \sqrt{v_1^2 + v_2^2} = \sqrt{10^2 + 10^2}$$

$$= 10\sqrt{2} \text{ km/h along } BC$$

The shortest distance between A and C is d given by, $d = AC = AB \sin 45°$



$$= 100 \times \frac{1}{\sqrt{2}} = 50\sqrt{2} \text{ km}$$

Shortest time $t = \dfrac{d}{v_r} = \dfrac{50\sqrt{2}}{10\sqrt{2}} = 5\,\text{h}$

46. In going upwards, the velocity decreases because $v = u - gt$.

Again while going downwards, the velocity increases because $v = u + gt$

Thus, the correct result should have been (c), but since, the air resistance is not ignored, so the curve is not flat and hence, the result is (d).

47. $h = (u\sin\theta)t - \dfrac{1}{2}gt^2$ ...(i)

$d = u\cos\theta \times t$ or $t = \dfrac{d}{u\cos\theta}$

From Eq. (i), $h = u\sin\theta \times \dfrac{d}{u\cos\theta} - \dfrac{1}{2}g\dfrac{d^2}{u^2\cos^2\theta}$

or $= \dfrac{d}{\cos\theta}\sqrt{\dfrac{g}{2(d\tan\theta - h)}}$.

48. The maximum height attained by the particle is

$$h_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g}$$

Low potential energy at height

$$h_{max} = mgh_{max} = mg \times \frac{u^2}{4g} = \frac{mu^2}{4}$$

and maximum kinetic energy $= \dfrac{1}{2}mu^2$

Thus, KE at h_{max} = max KE − PE

$$= \frac{1}{2}mu^2 - \frac{1}{4}mu^2$$

$$= \frac{1}{4}mu^2 = \frac{E}{2}$$

49. We know that the range is $T = \dfrac{2u\sin\theta}{g}$

According to the question, the range of the projectile is the same for complementary angles,

So, $T_1 = \dfrac{2u\sin\theta}{g}, T_2 = \dfrac{2u\sin(90° - \theta)}{g}$

Again, the range of the projectile is

$$R = \frac{u^2\sin 2\theta}{g}$$

$\therefore \quad T_1 T_2 = \dfrac{2u\sin\theta}{g} \times \dfrac{2u\sin(90° - \theta)}{g}$

$$= \frac{2u^2\sin 2\theta}{g^2} = \frac{2R}{g}$$

$\therefore \quad T_1T_2 \propto R$

50. Velocity of rain

$\mathbf{v}_r = -3\hat{\mathbf{j}}$ (vertically downward)

Velocity of man

$\mathbf{v}_m = 4\hat{\mathbf{i}}$

$\therefore$ Relative velocity of rain w.r.t. man

$\mathbf{v}_{rm} = \mathbf{v}_r - \mathbf{v}_m = -3$ for $\hat{\mathbf{j}} - 4$ for $\hat{\mathbf{i}}$

$\therefore \quad |\mathbf{v}_{rm}| = \sqrt{(-3)^2 + (-4)^2} = 5\,\text{kmh}^{-1}$

51. Horizontal range of the projectile

$$R = \frac{u^2\sin 2\theta}{g}$$

$$R_{max} = \frac{u^2}{g} \quad (\theta = 45°)$$

The maximum height attained by the projectile

$$H = \frac{u^2\sin^2\theta}{2g}$$

If, $H = R$, then

$$\frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin 2\theta}{g}$$

$$\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$

So, $\tan\theta = 4$

$\therefore \quad \theta = \tan^{-1}(4)$

52. To hit a target, the man should aim his rifle at a point higher than the target as the bullet suffers a vertical deflection $(y = \frac{1}{2}gt^2)$ due to acceleration due to gravity.

53. The equation of the trajectory of the projectile is

$$y = x\tan\theta - \frac{1}{2}\frac{g}{u^2\cos^2\theta}x^2$$

Thus, the statement (I) is true. Also it is clear that y-component depends on the x-component.

54. We know that when a body is projected from a plane above the surface of earth, then for the maximum range, the angle of projection must be slightly less than 45° as the range depends on the angle of projection.

55.

$$OA = OC$$

OA + **OC** is along **OB** (bisector) and its magnitude is

$$2R\cos 45° = R\sqrt{2}$$

(**OA** + **OC**) + **OB** is along **OB** and its magnitude is

$$R\sqrt{2} + R = R(1+\sqrt{2})$$

57. Since, **A** is parallel to **B** *i.e.*, **A** || **B**, so $\theta = 0°$

$\therefore$ $\mathbf{A}\times\mathbf{B} = AB\sin 0° = \mathbf{0}$. It means $\mathbf{A}\times\mathbf{B}$ is a null vector, where, null vector has a magnitude zero but has the direction.

58. Scalars quantities contain only magnitude not the direction. Hence, scalars can be added algebraically. On the other hand, vector quantities have both magnitude and direction and these cannot be added algebraically.

59. $\cos\theta = \dfrac{(\hat{i}+\hat{j})\cdot\hat{i}}{|(\hat{i}+\hat{j})\,\hat{i}\,|} = \dfrac{1}{\sqrt{2}} = \cos 45°$

So, $\theta = 45°$

60-63.

Let the initial velocity of the ball w.r.t. person be $\mathbf{v} = (\mathbf{v}_x\hat{i} + \mathbf{v}_y \hat{j})\,\text{fts}^{-1}$

where x and y are along horizontal and vertical directions, respectively.

Initial velocity of ball w.r.t. ground frame of reference is,

$$\mathbf{v}_{BG} = \mathbf{v} + 30\hat{i} = [(u_x + 30)\hat{i} + v_y \hat{j}]\,\text{fts}^{-1}$$

As the ball is passing through hoop when its vertical component of velocity is zero, it means it is at the topmost point of its flight, so the maximum height of the projectile is 16 ft.

From given information, $v_x^2 + v_y^2 = 40^2$

From projectile theory, maximum height

$$= \frac{(\text{vertical component of velocity})^2}{2g}$$

$$\Rightarrow \quad 16 = \frac{v_y^2}{2\times 32} \quad \Rightarrow \quad v_y = 32\,\text{fts}^{-1}$$

From $\quad v_x^2 + v_y^2 = 40^2,\ v_x = \sqrt{40^2 - 32^2} = 24\,\text{fts}^{-1}$

So, initial velocity of ball w.r.t. frame of reference attached with the trolley, is $\mathbf{v} = (24\hat{i} + 32\hat{j})\,\text{fts}^{-1}$

Initial velocity of the ball w.r.t. ground is,

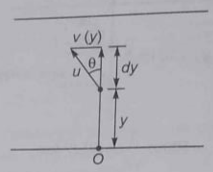
$$\mathbf{v}_{BG} = (54\hat{i} + 32\hat{j})\,\text{fts}^{-1}$$

The time taken by the ball to pass through the hoop measured from the instant of throw is equal to half of the time of flight, *i.e.*, required time, $t = \dfrac{T}{2} = \dfrac{v_y}{g} = \dfrac{32}{32} = 1$ s. Initial horizontal separation between the ball and hoop, to serve the purpose, is half of the range *i.e.*, $x = \dfrac{R}{2} = 54\times 1 = 54$ ft.

64-66.

Velocity of stream at a distance y from 0 is, $v(y) = \dfrac{vy}{t}$

Vector sum of velocity of boat w.r.t. river and of river w.r.t. ground gives the velocity of boat w.r.t. ground (observer on shore).



For a small change in y from y to $y + dy$, speed of stream can be taken as constant.

So, for required situation, $\sin\theta = \dfrac{v(y)}{u}$

$$\theta = \sin^{-1}\left(\frac{vy}{ut}\right)$$

Velocity of boat w.r.t. ground as a function of y is given by,

$$\frac{dy}{dt} = \sqrt{u^2 - [v(y)]^2} = \sqrt{u^2 - \left(\frac{vy}{t}\right)^2}$$

$$\Rightarrow \int_0^t \frac{dy}{\sqrt{u^2 - \left(\frac{uy}{t}\right)^2}} = \int_0^t dt$$

$$\Rightarrow \quad t = \frac{1}{v}\sin^{-1}\left(\frac{v}{u}\right)$$

From $\quad \sin\theta = \dfrac{vy}{ut}$

as value of $\sin\theta$ would be always less than or equal to 1, $\sin\theta \le 1$

$$\Rightarrow \quad \frac{vy}{ut} \le 1 \Rightarrow y \le \frac{ut}{v} = \frac{3t}{4}$$

68. $\quad mg\sin\theta = ma$

$\therefore \quad a = g\sin\theta$

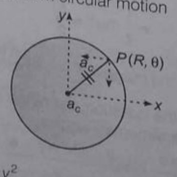
where a is along the inclined plane

$\therefore$ Vertical component of acceleration is $g\sin^2\theta$.

$\therefore$ Relative vertical acceleration of A with respect to B is

$g(\sin^2 60° - \sin^2 30°) = \dfrac{g}{2} = 4.9\,\text{ms}^{-2}$ (in vertical direction)

69. For a particle in uniform circular motion



$\mathbf{a} = \dfrac{v^2}{R}$ towards centre of circle

$\therefore \quad \mathbf{a} = \frac{v^2}{R}(-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}})$

or $\quad \mathbf{a} = -\frac{v^2}{R}\cos\theta\,\hat{\mathbf{i}} - \frac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$

70. $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

$$\mathbf{L} = m\left[v_0\cos\theta t\,\hat{\mathbf{i}} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{\mathbf{j}}\right] \times [v_0\cos\theta\,\hat{\mathbf{i}} + (v_0\sin\theta - gt)\,\hat{\mathbf{j}}]$$

$$= mv_0\cos\theta t\left[-\frac{1}{2}gt\right]\hat{\mathbf{k}}$$

$$= -\frac{1}{2}mgv_0t^2\cos\theta\,\hat{\mathbf{k}}$$

71. $\mathbf{u} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}};\ \mathbf{a} = 0.4\,\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}}$

$$\mathbf{u} = \mathbf{u} + \mathbf{a}t = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})\,10$$

$$= 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} = 7\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$$

Speed $= \sqrt{7^2 + 7^2} = 7\sqrt{2}$ unit

72. Kinetic energy at highest point,

$$(\text{KE})_H = \frac{1}{2}mv^2\cos^2\theta$$

$$= K\cos^2\theta$$

$$= K(\cos 60°)^2 = \frac{K}{4}$$

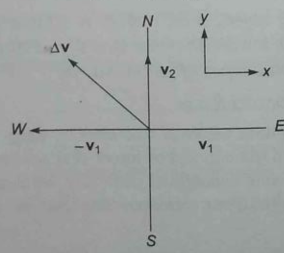
73. $\mathbf{v}_1 = +5\hat{\mathbf{i}}$

$\mathbf{v}_2 = +5\hat{\mathbf{j}}$

$$\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = 5\hat{\mathbf{j}} - 5\hat{\mathbf{i}}$$

$$|\Delta\mathbf{v}| = 5\sqrt{2}$$

$$\therefore \quad a = \frac{|\Delta\mathbf{v}|}{t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\,\text{ms}^{-2}$$



For direction,

$$\tan\alpha = -\frac{5}{5} = -1$$

Average acceleration is $\frac{1}{\sqrt{2}}\,\text{ms}^{-2}$ towards North-West.

74. Work done in displacing the particle

$$W = \mathbf{F}\cdot\mathbf{r} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

$$= 5 \times 2 + 3 \times (-1) + 2 \times 0$$

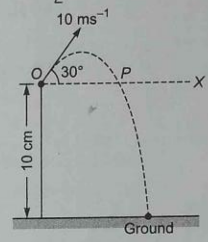
$$= 10 - 3 = 7\,\text{J}$$

75. When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

76. $$OP = R = \frac{u^2\sin 2\theta}{g}$$

$$= \frac{10^2 \times \sin(2 \times 30°)}{10}$$

$$= \frac{10\sqrt{3}}{2} = 5\sqrt{3} = 8.66\,\text{m}$$



77. Since, $X = \alpha t^3$ and $Y = \beta t^3$

$$\therefore \quad \mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \alpha t^3\hat{\mathbf{i}} + \beta t^3\hat{\mathbf{j}}$$

Now, $\quad \mathbf{v} = \frac{d\mathbf{r}}{dt} = \alpha t^2 \times 3\hat{\mathbf{i}} + \beta t^2 \times 3\hat{\mathbf{j}}$

Thus, $|\mathbf{v}| = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2}$

$$= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

78. At the highest point of its flight, vertical component of velocity is zero and only horizontal component is left which is

$$u_x = u\cos\theta$$

Given, $\quad \theta = 45°$

$$\therefore \quad u_x = u\cos 45° = \frac{u}{\sqrt{2}}$$

Hence, at the highest point kinetic energy

$$E' = \frac{1}{2}mu_x^2$$

$$= \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2}m\left(\frac{u^2}{2}\right)$$

$$= \frac{E}{2} \qquad \left(\because \frac{1}{2}mv^2 = E\right)$$

79. $(\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{A})$ (given)

$\Rightarrow \quad (\mathbf{A} \times \mathbf{B}) - (\mathbf{B} \times \mathbf{A}) = \mathbf{0}$

or $\quad (\mathbf{A} \times \mathbf{B}) + (\mathbf{B} \times \mathbf{A}) = \mathbf{0} \qquad [\because (\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{B})]$

or $\quad 2(\mathbf{A} \times \mathbf{B}) = \mathbf{0}$

$\Rightarrow \quad 2AB\sin\theta = 0$

or $\quad \sin\theta = 0 \qquad [\because |\mathbf{A}| = A \neq 0, |\mathbf{B}| = B \neq 0]$

$\Rightarrow \quad \theta = 0$ or π