Day 4

Laws of Motion

Day 4

Outlines ...

- Concept of Forces
- Newton's Laws of Motion
- Connected Motion
- Friction

Concept of Forces

A push or a pull exerted on any object, is defined to be a force. It is a vector quantity. Thus, we denote it with an arrow over it, just as we do for velocity and acceleration. Some types of forces may be

(i) Contact forces, (ii) Non-contact forces

Contact forces involve physical contact between two objects. Early scientists, including Newton, were uneasy with the concept of forces that act between two disconnected objects. To overcome this conceptual difficulty, Michael Faraday introduced the concept of a field. The corresponding forces are called **non-contact or field forces**.

The known fundamental forces in nature are all field forces. These are, in order of decreasing strength, (1) strong nuclear forces between subatomic particles, (2) electromagnetic forces between electric charges, (3) weak nuclear forces, which arise in certain radioactive decay process and (4) gravitational force of attraction between objects.

The relative strengths of the gravitational force, the weak force, the electromagnetic force and the strong nuclear force are

$$F_g : F_w : F_e : F_s :: 1 : 10^{23} : 10^{36} : 10^{38}$$

Newton's Laws of Motion

Newton's laws of motion are of central importance in classical mechanics (physics). A large number of laws and results may be derived by Newton's laws. These laws describe the relationship between a body and the forces acting upon it and its motion in response to said forces. There are three laws of motion stated by Newton, these laws are based on experimental observations.

Newton's laws are not valid in the non-inertial frames, they have to be modified by introducing the concept of pseudo force.

First Law

If the net force ΣF exerted on an object is zero, the object continues in its original state of motion (or rest). That is, if $\Sigma F = 0$, an object at rest remains at rest and or object moving with constant velocity. This is **Newton's first law.**

Newton's first law of motion reveals **inertia** a fundamental property of matter. As a result, Newton's first law of motion is also known as the **law of inertia** Inertia is that property of the body by virtue of which the body preserves its state of rest or of uniform motion in a straight line.

Newton's first law is not true in all reference frames, but we can always find reference frames in which it is true. Such frames are called inertial reference frames or simply inertial frames. Thus, an inertial reference frame is one in which the Newton's laws hold.

Linear Momentum

The linear momentum of a body is defined as a product of mass and velocity of the body *i.e.*,

$$\mathbf{p} = m\mathbf{v}$$

Being the product of a scalar quantity (mass) and a vector quantity (velocity), the momentum is a vector quantity.

A body at rest cannot possess linear momentum and a moving body always possesses linear momentum.

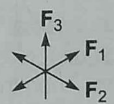
Second Law

The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass.

$$a = \frac{\Sigma F}{m}$$

This is the **Newton's second law.**

The equation in vector form, $\Sigma \mathbf{F} = m\mathbf{a}$



Concurrent Forces

where, $\mathbf{a}$ is the acceleration of the object, m is its mass and $\Sigma \mathbf{F}$ represents the vector sum of all the forces acting on the object.

It also states that the time rate of change of momentum of body is equal to the net external force exerted on that body

$$i.e.,\ \mathbf{F}_{net} = \frac{d\mathbf{P}}{dt}$$

If $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3 \ldots$ are the concurrent forces acting at the same point, then the point will be in equilibrium if

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \ldots = 0$$

Impulse

- When a force of large magnitude acts on an object for a small time interval, the force is called impulsive force. In such cases, we measure the total effect of force.
- Impulse of a force is a measure of the total effect of force. Mathematically, impulse $\mathbf{J} = \mathbf{F}_{av} dt = \int_0^{\Delta t} \mathbf{F}\, dt$
- For a force-time (F-t) graph, the area under the graph gives the value of the impulse. As, $\mathbf{F} = \frac{d\mathbf{p}}{dt}$

  Hence, impulse $\mathbf{J} = \int_0^{\Delta t} \mathbf{F}\, dt = \int_0^{\Delta t} \frac{d\mathbf{p}}{dt} dt = \int_{p_i}^{p_f} d\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$

  This relation is known as the **impulse-momentum theorem.**
- If for a given impulse, (*i.e.*, for a given change in momentum) time Δt of action of the force is small, then the force exerted, $\mathbf{F}_{av}$ is large. On the other hand, if time Δt is comparatively large, then the force $\mathbf{F}_{av}$ has a smaller magnitude.
- *If n bullets each of mass m and velocity v are fired from a gun, then the average force acting on the gun is mnv.*

Third Law

If two objects interact, the force F_{12} which the object 1 exerts on object 2 is equal in magnitude but opposite in direction to the force F_{21} which the object 2 exerts on object 1. This is **Newton's third law**. $F_{12} = -F_{21}$

Problem Solving through Newton's Laws of Motion

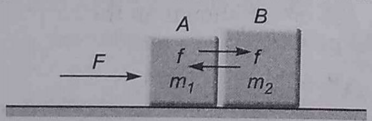
We make use of Newton's laws of motion to study the motion of a single object or a system of bodies under the action of a force or a set of forces. To solve such problems we proceed as follows

- Consider each individual object as a point object.
- Draw a free body diagram. A free body diagram consists of a diagrammatic representation of a single object as a sub-system of particles isolated from its surroundings and showing all the forces acting on it. The forces may be contact forces or field forces.
- Now, check whether under the effect of forces, the system can accelerate or not. If yes, then find the direction and magnitude of acceleration.
- In case of a composite problem of a number of objects connected together by strings/springs or pulleys etc find the relationship of accelerations of different objects of the system by specifying the constraint relationship, if any.
- If various external forces are in different directions then resolve these forces in two or three, mutually perpendicular directions as the case may be, and find the accelerations separately along these directions. Then the resultant acceleration will be equal to the vector sum of these accelerations.

Connected Motion

In Physics, motion is a change in position of an object with respect to time and its reference point. Motion is observed by attaching a frame of reference to a body and measuring its change in position relative to that frame. Hence, the connected motion describes the relative motion between two objects whether it is between two blocks and a plane or it is between a block and an inclined plane.

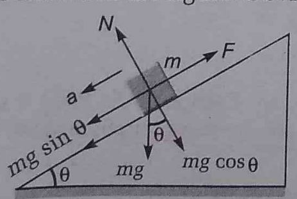
1. If two blocks of masses m_1 and m_2 are placed on a perfectly smooth surface and are in contact, then



Acceleration of the blocks, $a = \dfrac{F}{m_1 + m_2}$

and the contact force (acting normally) between the two blocks is $f = m_2 a = \dfrac{F m_2}{(m_1 + m_2)}$

2. For a block of mass m placed on a fixed, perfectly smooth inclined plane of angle θ, the forces acting on the block are shown in the figure. Obviously, here

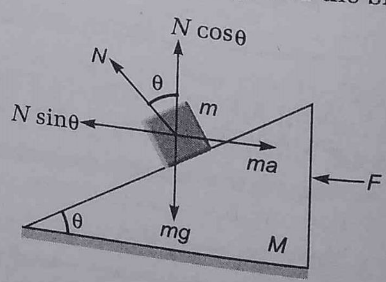


$a = g \sin\theta$

⇒

3. If a block of mass m is placed on a smooth movable wedge of mass M, which in turn is placed on smooth surface, then a force **F** is applied on the wedge, horizontally.

The acceleration of the wedge and the block is



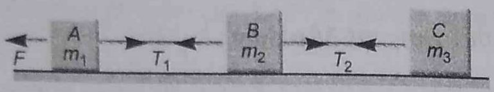
$$a = \frac{F}{(M + m)}$$

Force on the block

$$F = (M + m)a$$
$$= (M + m)g \tan\theta$$

4. For a block system shown in the figure, acceleration of the system

$$a = \frac{F}{m_1 + m_2 + m_3}$$



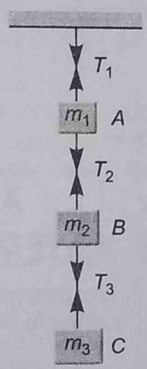
Tension in the string

$$T_1 = (m_2 + m_3)a = \frac{F(m_3 + m_2)}{(m_1 + m_2 + m_3)}$$

and tension

$$T_2 = m_3 a = \frac{F\, m_3}{m_1 + m_2 + m_3}$$

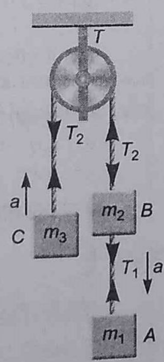
5. For a block system suspended freely from a rigid support as shown in the figure, the acceleration of the system $a = 0$.



String tension

$$T_1 = (m_1 + m_2 + m_3)g$$
$$T_2 = (m_2 + m_3)g$$
and $$T_3 = m_3 g$$

6. For a block system and a pulley shown in the figure, value of the acceleration of the system



$$a = \frac{(m_1 + m_2 - m_3)g}{(m_1 + m_2 + m_3)}$$

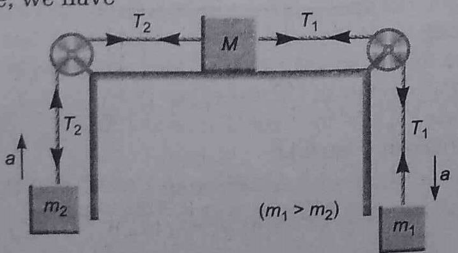
Tension $T_1 = \dfrac{2m_1 m_3 g}{(m_1 + m_2 + m_3)}$

Tension $T_2 = \dfrac{2m_3(m_1 + m_2)g}{(m_1 + m_2 + m_3)}$

and tension T

$$T = 2T_2 = \frac{4\, m_3(m_1 + m_2)g}{(m_1 + m_2 + m_3)}$$

7. For the pulley and block arrangement shown in the figure, we have



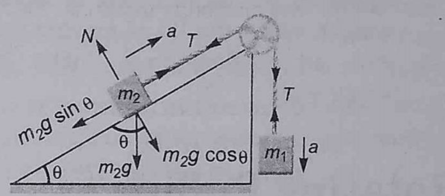
Net acceleration

$$a = \frac{\text{Net accelerating force}}{\text{Total mass}} = \frac{(m_1 - m_2)g}{(m_1 + m_2 + M)}$$

Tension $T_1 = m_1(g - a) = \dfrac{(M + 2m_2)m_1 g}{(M + m_1 + m_2)}$

and Tension $T_2 = m_2(g + a) = \dfrac{(M + 2m_1)m_2 g}{(M + m_1 + m_2)}$

8. For the system of block and pulley, with a smooth inclined plane shown in the figure, we have



Net acceleration

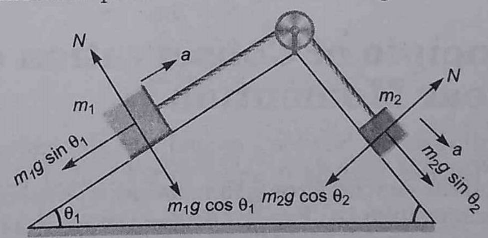
$$a = \frac{(m_1 - m_2 \sin\theta)g}{(m_1 + m_2)}, \text{ if } m_1 g > m_2 g \sin\theta$$

and $$a = \frac{(m_2 \sin\theta - m_1)g}{m_1 + m_2}, \text{ if } m_1 g < m_2 g \sin\theta$$

and tension in the string

$$T = m_1(g - a) = \frac{m_1 m_2(1 + \sin\theta)}{(m_1 + m_2)}$$

9. For a pulley and block system on a smooth double inclined plane as shown in the figure, we have



Net acceleration

$$a = \frac{(m_1 \sin\theta_1 - m_2 \sin\theta_2)g}{(m_1 + m_2)},$$

for $\theta_1 > \theta_2$, $m_1 > m_2$

and tension in the string

$$T = m_1(g \sin\theta_1 - a) = \frac{m_1 m_2(\sin\theta_1 + \sin\theta_2)g}{(m_1 + m_2)}$$

Apparent Weight of an Object in a Moving Lift

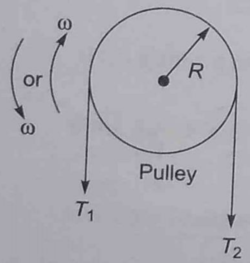
Earth attracts every body towards its centre. The force of attraction exerted by the earth on the body is called ***gravity force****. If m be the mass of the body then the gravity force on it is mg. Normally, the weight of a body is equal to the gravity force w = mg.*

But when the body is on an accelerating platform, the weight of the body appears to be changed. The changed weight is termed as apparent weight. Here, we consider the apparent weight of a man standing in a moving lift.

Let a person or an object be situated in a lift, which is in a state of motion. Then

1. Apparent weight of an object is the same as its actual weight on the ground, (i.e., mg) when either the lift is at rest or is in motion with a constant velocity either in the upward or in the downward direction.
2. Apparent weight of object is more than its actual weight, when the lift is moving vertically upwards with an acceleration a. Infact, apparent weight $= m(g + a)$.
3. The apparent weight is $m(g - a)$ *i.e.*, less than i actual weight, when the lift is moving downward with an acceleration a.
4. If the lift is moving downwards with an acceleratior $a = g$, the object is in a state of weightlessness. Thus, a freely falling body is always weightless.

Rotating Pullies



If pulley is not massless and it is rotating then tensions in either side in string are not same even the string is of the same material.

$$|T_1 - T_2| \cdot R = I\alpha$$

where, I = moment of inertia of the pulley. and α = angular acceleration of the pulley.

If a pulley is massless, net force on it is zero even if it is accelerated.

Principle of Conservation of Linear Momentum

According to the Newton's second law, the forces acting on an isolated particle is equal to the rate of change of linear momentum *i.e.*, $\mathbf{F} = \frac{d\mathbf{p}}{dt}$

If, $\mathbf{F} = 0, \frac{d\mathbf{p}}{dt} = 0$ or $\mathbf{p}$ = constant

i.e., if the force applied on a particle is zero, its linear momentum remains constant. This is the principle of **conservation of linear momentum** of a particle.

For a gun-bullet system, the force exerted by the gun is an internal force and so the total momentum of the system remains unchanged.

Applications of Principle of Conservation of Linear Momentum

The propulsion of rockets and jet planes is based on the principle of conservation of linear momentum.

- Upward thrust on the rocket, $F = \frac{u\,dm}{dt} - mg$

  and if effect of gravity is neglected, then $F = \frac{u\,dm}{dt}$
- Instantaneous upward velocity of the rocket

  $$v = u \ln\left(\frac{m_0}{m}\right) - gt$$

  and neglecting the effect of gravity

  $$v = u \ln\left(\frac{m_0}{m}\right) = 2.303u \log_{10}\left(\frac{m_0}{m}\right)$$

  where, m_0 = initial mass of the rocket including that of the fuel

  v_0 = initial velocity of the rocket at any time t

  m = mass of the rocket left

  v = velocity acquired by the rocket.
- Burnt out speed of the rocket is the speed attained by the rocket when the whole of fuel of the rocket has been burnt. The maximum velocity acquired by the rocket is when the whole the fuel of the rocket has been used up the residual mass (m_r) of the rocket is equal to mass of the empty container of the fuel.

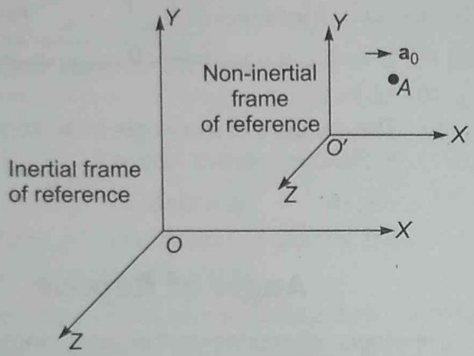
  $$v_b = u \log_e\left(\frac{m_0}{m_r}\right) = 2.303u \log_{10}\left(\frac{m_0}{m_r}\right)$$

Fictitious Force or Pseudo Force

Earth is rotating about its axis and revolving around the sun. Any rotating or revolving body is always accelerated, therefore, earth is infact a non- inertial frame.

A force appears to act on a body due to acceleration of the body in a non-inertial frame.

This apparent force is called the **fictitious force** or **pseudo force**.



Suppose point A is an accelerating point then its acceleration with respect to O' is given as

$$\mathbf{a}_{AO'} = \mathbf{a}_{AO} - \mathbf{a}_{O'O}$$

$\Rightarrow$ $$\mathbf{a} = \mathbf{a}_{AO} - \mathbf{a}_{O'O}$$

or $$m\mathbf{a} = m\mathbf{a}_{AO} - ma_O$$

Here, ma_O is the pseudo force.

The direction of this force is opposite to the acceleration of the non-inertial frame. where, m is the mass of the body.

Equilibrium of Concurrent Forces

If a number of forces act at the same point, they are called concurrent forces.

Consider that a body is under the action of a number of forces. Suppose that the body remains in equilibrium under the action of these forces *i.e.*, the body remains in its state of rest or of uniform motion along a straight line, when acted upon these forces.

The condition that body may be in equilibrium or the number of forces acting on the body may be in equilibrium is that these forces should produce a zero resultant force.

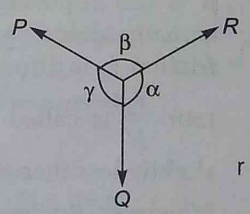
Therefore, the resultant of three concurrent forces will be zero and hence they will be in equilibrium , if they can be represented completely by the three sides of any triangle.

In case, a number of forces act at a point, then they will be in equilibrium, if they can be represented completely by the sides of a closed polygon taken in order.

Lami Theorem

For three concurrent forces in equilibrium posision.

If three forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other. Thus, if the forces are P, Q and R; α, β, γ be the angles between Q and R, R and P, P and Q espectively, also the forces are in equilibrium, we have



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Friction

Whenever an object actually slides or rolls over the surface of another body or tends to do so, a force opposing the relative motion starts acting between these two surfaces in contact. It is known as friction or the force due to friction. ***Force of friction acts in a tangential direction to the surfaces in contact.***

Types of Friction

The four types of friction are given below

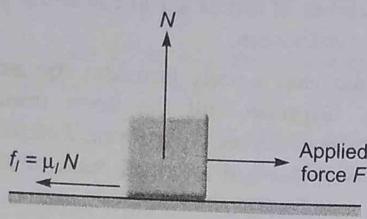
1. **Static Friction**

   It is the opposing force that comes into play when one body is at rest and a force acts to move it over the surface of another body.

   If the applied force F is such that the body remains at rest inspite of the action of the force on it, the frictional force $f_s = F$ is static friction. It is a self adjusting force and is always equal and opposite to the applied force.

2. Limiting Friction

It is the limiting (maximum) value of static friction when a body is just on the verge of starting its motion over the surface of another body.



The force of limiting friction f_l between the surfaces of two bodies is directly proportional to the normal reaction at the point of contact. Mathematically,

$$f_l \propto N$$

or $$f_l = \mu_l N$$

$\Rightarrow$ $$\mu_l = \frac{f_l}{N}$$

where, μ_l is the **coefficient of limiting friction** for the given surfaces in contact.

3. Kinetic Friction

It is the opposing force that comes into play when one body is actually slides over the surface of another body. Force of kinetic friction f_k is directly proportional to the normal reaction N and the ratio $\frac{f_k}{N}$ is called **coefficient of kinetic friction** μ_k, value of μ_k is slightly less than μ_e $(\mu_k < \mu_l)$.

Whenever limiting friction is converted into kinetic friction, body started motion with a lurch.

4. Rolling Friction

It is the opposing force that comes into play when a body of symmetric shape (wheel or cylinder or disc, etc.) rolls over the surface of another body. Force of rolling friction f_r is directly proportional to the normal reaction N and inversely proportional to the radius(r) of the wheel. Thus,

$$f_r \propto \frac{N}{r}$$

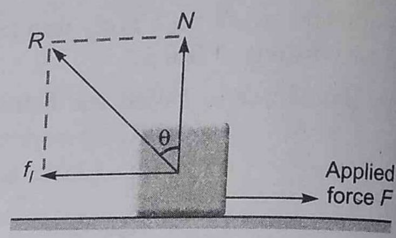
or $$f_r = \mu_r \frac{N}{r}$$

The constant μ_r is known as the **coefficient of rolling friction** μ_r has the unit and dimensions of length. Magnitudewise $\mu_r << \mu_k$ or μ_l

- *The value of rolling friction is much smaller than the value of sliding friction.*
- *Ball bearings are used to reduce the wear and tear and energy loss against friction.*

Angle of Friction

- Angle of friction is defined as the angle θ which the resultant R of the force of limiting friction f_l and normal reaction N, subtends with the normal reaction.

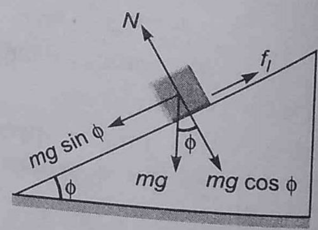


- The, tangent of the angle of friction is equal to the coefficient of friction.

i.e., $$\mu = \tan\theta$$

Angle of Repose

- Angle of repose is the least angle of the inclined plane (of given surface) with the horizontal such that the given body placed over the plane, just begins to slide down, without getting accelerated.



- The tangent of the angle of repose is equal to the coefficient of friction.

Hence, we conclude that angle of friction is (θ) equal to the angle of repose (φ).

- In limiting condition,

$$f_1 = mg\sin\phi$$

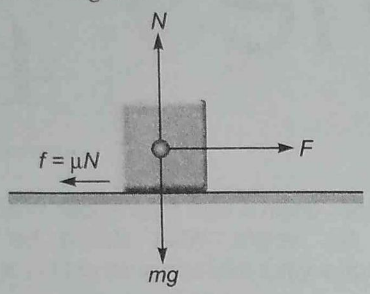
$$N = mg\cos\phi$$

$$\frac{f_1}{N} = \tan\phi$$

$\therefore$ $$\frac{f_1}{N} = \mu_s = \tan\phi$$

Acceleration of a Block on Applying a Force on a Rough Surface

1. Acceleration of a block on a horizontal surface is as shown in the figure.

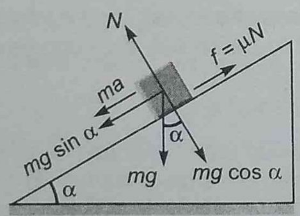


$$a = \frac{F - f}{m} = \frac{F - \mu mg}{m} = \frac{F}{m} - \mu g$$

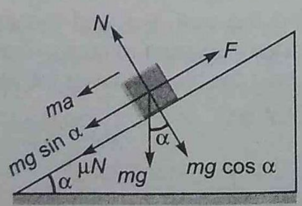
where, μ = coefficient of kinetic friction between the two surfaces in contact.

2. Acceleration of block sliding down a rough inclined plane as shown in the figure, is given by

$$a = g(\sin \alpha - \mu \cos \alpha)$$

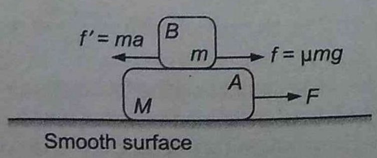


3. Retardation of a block sliding up a rough inclined plane, as shown in the figure is $a = g(\sin \alpha + \mu \cos \alpha)$



4. Motion of two bodies, one resting on the other

(i) Let a body A of mass M is placed on a smooth surface and a block B of mass m be placed on A. Let coefficient of friction between surfaces of A and B be μ . If a force F is applied on the lower body A as shown in figure, then



Common acceleration of the two bodies

$$a = \frac{F}{(M + m)}$$

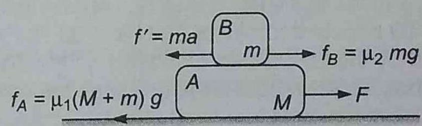
pseudo force acting on block B due to the accelerated motion $f' = ma$. The pseudo force tends to produce a relative motion between bodies A and B and consequently a frictional force, $f = \mu N = \mu\, mg$ is developed. For equilibrium,

$$ma \le \mu\, mg$$

or $$a \le \mu g$$

If under the influence of force F the acceleration produced exceeds μg, two bodies will not move together and the block B will slide backwards and after some time fall from body A.

(ii) Let friction is also present between the ground surface and body A. Let μ_1 = coefficient of friction between the given surface and body A and μ_2 = coefficient of friction between the surfaces of bodies A and B. If a force F is applied on the lower body A as shown in figure, then



Net accelerating force $= F - f_A$

$$= F - \mu_1 (M + m)\, g$$

$\therefore$ Net acceleration,

$$a = \frac{F - \mu_1 (M + m)\, g}{(M + m)}$$

$$= \frac{F}{(M + m)} - \mu_1 g$$

Pseudo force acting on the block B,

$$f' = ma$$

The pseudo force tends to produce a relative motion between the bodies A and B and consequently a frictional force $f_B = \mu_2 mg$ is developed. For equilibrium,

$$ma \le \mu_2 mg$$

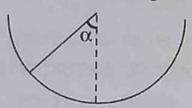
or $$a \le \mu_2 g$$

If acceleration produced under the effect of force F is more than $\mu_2 g$, then two bodies will not move together.

Practice Zone

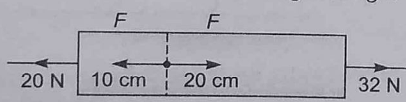
DAY 4

1. An insect crawls up a hemispherical surface very slowly. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(a) $\cot\alpha = 3$
(b) $\sec\alpha = 3$
(c) $\operatorname{cosec}\alpha = 3$
(d) None of these

2. The figure below shows a uniform rod of length 30 cm having a mass of 3.0 kg. The strings shown in the figure are pulled by constant forces of 20 N and 32 N. Find the force exerted by the 20 cm part of the rod on the 10 cm part. All the surfaces are smooth and the strings are light



(a) 36 N (b) 12 N (c) 64 N (d) 24 N

3. Two trains A and B are running in the same direction on parallel tracks such that A is faster than B. If packets of equal weight are exchanged between the two, then
(a) A will be retarded but B will be accelerated
(b) A will be accelerated but B will be retarded
(c) there will not be any change in the velocity of A but B will be accelerated
(d) there will not be any change in the velocity of B, but A will be accelerated

4. A given object takes n times more time to slide down a 45° rough inclined plane in comparison to slides down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

(a) $\frac{1}{1-n^2}$ (b) $1-\frac{1}{n^2}$ (c) $\sqrt{1-\frac{1}{n^2}}$ (d) $\sqrt{\frac{1}{1-n^2}}$

5. A light string passing over a smooth light pulley, connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $(g/8)$, then the ratio of masses is
(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3

6. A man slides down a light rope whose breaking strength is η times his weight. What should be his maximum acceleration so that the rope does not break?
(a) $g(1-\eta)$
(b) ηg
(c) $\frac{g}{1+\eta}$
(d) $\frac{g}{1-\eta}$

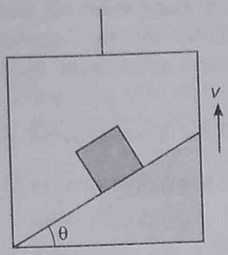
7. A lift is moving down with an acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively
(a) g, g (b) a, a (c) $(g-a), g$ (d) a, g

8. If M is the mass of a rocket, r is the rate of ejection of gases with respect to the rocket, then acceleration of the rocket, $\frac{dv}{dt}$ is equal to
(a) $\frac{ru}{(m-rt)}$
(b) $\frac{(m-rt)}{ru}$
(c) $\frac{ru}{(m+rt)}$
(d) $\frac{ru}{m}$

9. A block of mass M is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is μ and the acceleration due to gravity is g, then minimum force required to be applied by the finger to hold the block against the wall? **[NCERT Exemplar]**
(a) $\frac{Mg}{2\mu}$
(b) $\frac{Mg}{\mu}$
(c) $\frac{2M}{\mu g}$
(d) $\frac{2\,Mg}{\mu}$

10. Five forces inclined at an angle of 72° w.r.t. each other act on a particle of mass m placed at the origin of coordinates. Four forces are of magnitude F_1 and one has a magnitude F_2. Find the resultant acceleration of the particle.
(a) $\frac{F_2 - F_1}{m}$
(b) Zero
(c) $\frac{F_2 + F_1}{m}$
(d) $\frac{F_2 - 4F_1}{m}$

11. A block A is able to slide on the frictionless incline of angle θ and length l, kept inside an elevator going up with uniform velocity v. Time taken by the block to slide down the length of the incline, if released from rest is



(a) $\sqrt{\dfrac{2l}{(g+a)\sin\theta}}$ (b) $\sqrt{\dfrac{2l}{g}}$

(c) $\sqrt{\dfrac{2l}{g\sin\theta}}$ (d) $\sqrt{\dfrac{2l}{\sin\theta}}$

12. Two bodies of equal masses are connected by a light inextensible string passing over a smooth frictionless pulley. The amount of mass that should be transferred from one to another, so that both the masses move 50 m in 5 s is

(a) 30% (b) 40% (c) 70% (d) 50%

13. A wooden block of mass M resting on a rough horizontal surface, is pulled with a force F at an angle with the horizontal. If μ is the coefficient of kinetic friction between the block and the surface, then acceleration of the block is

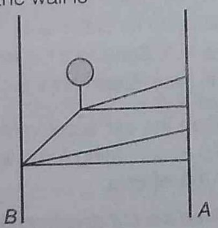
(a) $\dfrac{F}{M}(\cos\phi + \mu\sin\phi) - \mu g$ (b) $\dfrac{F}{M}\sin\phi$

(c) $\mu F\cos\phi$ (d) $\mu F\sin\phi$

14. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the string and the balance reads 49 N, when the lift is stationary. If the lift moves downwards with an acceleration of 5 ms^{-2}, the reading of the spring balance would be

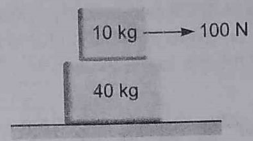
(a) 24 N (b) 74 N (c) 15 N (d) 49 N

15. A person 40 kg is managing to be at rest between two vertical walls by pressing one wall A by his hands and feet and B with his back. The coefficient of friction is 0.8 between his body and the wall. The force with which the person pushes the wall is



(a) 100 N (b) 50 N (c) 150 N (d) 200 N

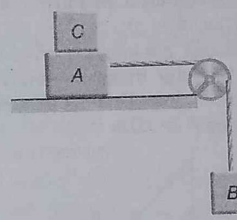
16. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on the top of the slab. The static coefficients of friction between the block and the slab is 0.60, while the kinetic coefficient is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8\ ms^{-2}$, the resultant acceleration of the slab will be



(a) 0.98 ms^{-2} (b) 1.47 ms^{-2}

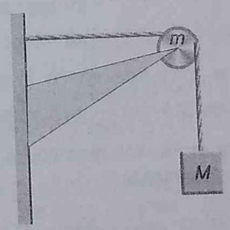
(c) 1.52 ms^{-2} (d) 6.1 ms^{-2}

17. Two masses A and B of 10 kg and 5 kg are connected with a string passing over a frictionless pulley, fixed at the corner of a table (as shown in figure). The coefficient of friction between the table and the block is 0.2. The minimum mass of C that may be placed on A to prevent it from moving is equal to



(a) 15 kg (b) 10 kg (c) 5 kg (d) zero

18. A string of negligible mass, going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by



(a) $\sqrt{2}Mg$ (b) $\sqrt{[(M+m)^2 + m^2]}g$

(c) $2Mg$ (d) $\sqrt{[(M+m) + m]^2}g$

19. A bullet is fired from a gun. The force on the bullet is given by $F = [600 - 2\times 10^5 t]$, where F is in newton and t is in second. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet?

(a) 9 N-s (b) Zero (c) 0.9 N-s (d) 1.8 N-s

20. A plane is inclined at an angle θ with the horizontal. A body of mass m rests on it. If the coefficient of friction is μ, then the minimum force that has to be applied parallel to the inclined plane so as to make the body to just move up the inclined plane, is

(a) $mg\sin\theta$ (b) $\mu mg\cos\theta$

(c) $\mu mg\cos\theta - mg\sin\theta$ (d) $\mu mg\cos\theta + mg\sin\theta$

21. A ball of mass m is thrown vertically upwards with a velocity v. If air exerts an average resisting force F, the velocity with which the ball returns to the thrower is

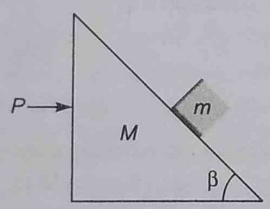
(a) $v\sqrt{\frac{mg}{mg+F}}$ (b) $v\sqrt{\frac{F}{mg+F}}$

(c) $v\sqrt{\frac{mg-F}{mg+F}}$ (d) $v\sqrt{\frac{mg+F}{mg}}$

22. A flexible uniform chain of mass m and length l suspended vertically so that its lower end just touches the surface of a table. When the upper end of the chain is released, it falls with each link coming to rest the instant it strikes the table. The force exerted by the chain on the table at the moment when y part of the chain has already rested on the table is

(a) $\frac{3myg}{l}$ (b) $\frac{3mg}{l}$ (c) $\frac{2}{3}\frac{mg}{l}$ (d) $\frac{1}{3}\frac{mg}{l}$

23. Two wooden blocks are moving on a smooth horizontal surface, such that the mass m remains stationary with respect to the block of mass M as shown in the figure. The magnitude of force P is



(a) $g\tan\beta$ (b) $mg\cos\beta$

(c) $(M+m)\operatorname{cosec}\beta$ (d) $(M+m)g\tan\beta$

24. A satellite in a force-free space sweeps out stationary interplanetary dust at a rate $\frac{dM}{dt}=\alpha v$, where M is the mass and v is the velocity of the satellites and α is a constant. The deceleration of the satellite is

(a) $\frac{2\alpha v^2}{M}$ (b) $-\frac{\alpha v^2}{M}$ (c) $-\frac{\alpha v^2}{2M}$ (d) $-\alpha v^2$

25. A motorcycle and a car are moving on a horizontal road with the same velocity. If they are bought to rest by application of brakes, which provides them equal retarding forces, then

(a) motorcycle will stop at a shorter distance
(b) car will stop at a shorter distance
(c) both will stop at the same distance
(d) Nothing can be predicted

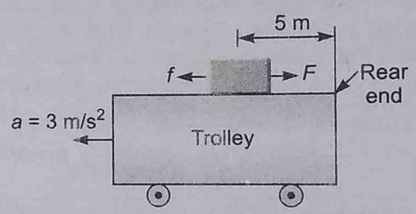
26. A block of mass 200 g is moving with a velocity of 5 m/sec along the positive x-direction. At time $t=0$, when the body is at $x=0$, a constant force 0.4 N is directed along the negative x-direction , is applied on the body for 10 s. What is the position x of the body at $t=2.5$ s?

(a) $x=1.75$ m (b) $x=1.25$ m
(c) $x=1.0$ m (d) $x=1.5$ m

27. A car of mass m starts from rest and acquires a velocity along east $v=v\hat{i}$ $(v>0)$ in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is

(a) $\frac{mv}{2}$ eastward and is exerted by the car engine
(b) $\frac{mv}{2}$ eastward and is due to the friction on the tyres exerted by the road
(c) more than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road
(d) $\frac{mv}{2}$ exerted by the engine

28. A block of mass 10 kg is placed at a distance of 5 m from the rear end of a long trolley as shown in the figure. The coefficient of friction between the block and the surface below is 0.2. Starting from rest, the trolley is given an uniform acceleration of 3 m/s^2. At what distance from the starting point will the block fall off the trolley? (Given $g=10$ m/s^2)



(a) 15 m (b) 25 m
(c) 20 m (d) 10 m

Directions *(Q. Nos. 29 to 33) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is ture, Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true, Statement II is not the correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

29. Statement I When the car accelerates horizontally along a straight road, the accelerating force is given by the push of the rear axle on the wheels.

Statement II When the car accelerates, the rear axle rotates with a greater frequency.

30. Statement I The work done in bringing a body down from the top to the base along a frictionless inclined plane is the same as the work done in bringing it down the vertical side.

Statement II The gravitational force on the body along the inclined plane is the same as that along the vertical side.

31. Statement I A bullet is fired from a rifle. If the rifle recoils freely, the kinetic energy of the rifle is less than that of the bullet.

Statement II In the case of a rifle-bullet system, the law of conservation of momentum is violated.

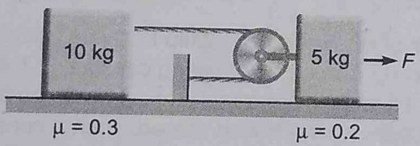
32. Statement I A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

Statement II For every action there is an equal and opposite reaction.

33. Statement I It is easier to pull a heavy object than to push it on a level ground.

Statement II The magnitude of frictional force depends on the nature of the two surfaces in contact.

Directions (Q. Nos. 34 to 36) *Two blocks of masses 10 kg and 5 kg are placed on a rough horizontal floor as shown in the figure. The strings and pulley are light and pulley is frictionless. The coefficient of friction between 10 kg block and surface is 0.3 while that between 5 kg block and surface is 0.2. A time varying horizontal force, $F = 5\,t$ Newton (t is in sec) is applied on the 5 kg block as known. [Take $g = 10\ ms^{-2}$]*



34. The motion of the block starts at $t = t_0$, then t_0 is

(a) 14 s (b) 8 s
(c) 9 s (d) 12 s

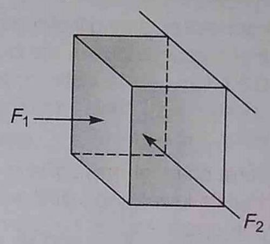
35. The frictional force between the 10 kg block and the surface at $t = \frac{t_0}{2}$, is in between

(a) zero and 10 N (b) 10 N and 35 N
(c) 12.5 N and 17.5 N (d) 12.5 N and 50 N

36. The acceleration of the 5 kg block at $t = 2t_0$ is

(a) $12\ ms^{-2}$ (b) $\frac{14}{5}\ ms^{-2}$
(c) $\frac{14}{9}\ ms^{-2}$ (d) $2\ ms^{-2}$

Directions (Q. Nos. 37 to 39) *A block of mass 4 kg is pressed against a rough wall by two perpendicular horizontal forces F_1 and F_2 as shown in the figure.*



Coefficient of static friction between the block and the floor is 0.6 and that of kinetic friction is 0.5. Take $g = 10\ ms^{-2}$.

37. For $F_1 = 300\,N$ and $F_2 = 100\,N$, find the direction and magnitude of the frictional force acting on the block.

(a) 180 N, vertically upwards
(b) 40 N, vertically upwards
(c) 107.7 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in the upward direction
(d) 91.6 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in the upward direction

38. For $F_1 = 150\,N$ and $F_2 = 100\,N$, find the direction and magnitude of the frictional force acting on the block.

(a) 90 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in the upward direction
(b) 75 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in the upward direction
(c) 107.7N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in the upward direction
(d) zero

39. The velocity of a particle moving in the x-y plane is given by

$$\frac{dx}{dt} = 8\pi \sin 2\pi t \text{ and } \frac{dy}{dt} = 5\pi \cos 2\pi t$$

where, $t = 0$, $x = 8$ and $y = 0$. The path of particle is

(a) a straight line
(b) an ellipse
(c) a circle
(d) a parabola

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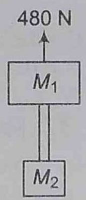
40. A body starts from rest on a long inclined plane of slope 45°. The coefficient of friction between the body and the plane varies as $\mu = 0.3\,x$, where x is distance travelled down the plane. The body will have maximum speed (for $g = 10\ \text{m/s}^2$) when x is equal to **[JEE Main Online 2013]**

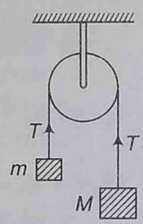
(a) 9.8 m (b) 27 m
(c) 12 m (d) 3.33 m

41. Two blocks of mass $M_1 = 20$ kg and $M_2 = 12$ kg are connected by a metal rod of mass 8 kg. The system is pulled vertically up by applying a force of 480 N as shown. The tension at the mid-point of the rod is **[JEE Main Online 2013]**



(a) 144 N (b) 96 N
(c) 240 N (d) 190 N

42. Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If $M = 2\,m$, then the stress produced in the wire is **[JEE Main Online 2013]**

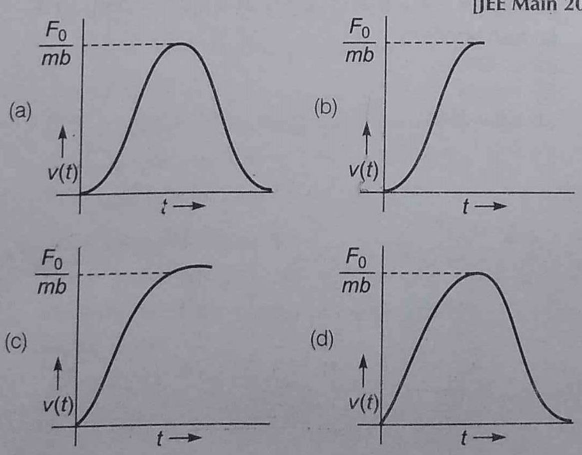


(a) $\frac{2\,mg}{3A}$ (b) $\frac{4\,gm}{3A}$ (c) $\frac{gm}{A}$ (d) $\frac{3\,mg}{4A}$

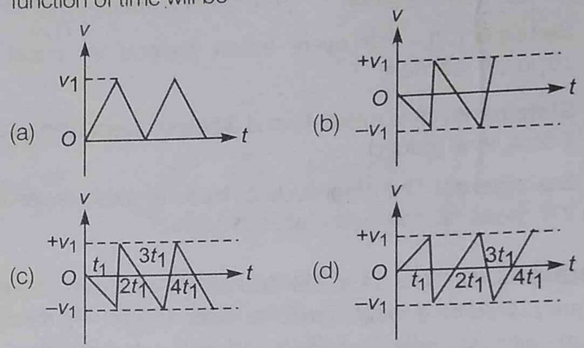
43. A boy of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be **[JEE Main Online 2013]**

(a) 15 m (b) 12.5 m (c) 15.5 m (d) 17 m

44. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves? **[JEE Main 2013]**



45. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time the height as function of time will be **[AIEEE 2009]**



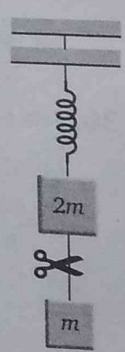
46. A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of $5.00\ \text{ms}^{-1}$. The magnitude of its momentum is recorded as **[AIEEE 2008]**

(a) $17.6\ \text{kg ms}^{-1}$ (b) $17.565\ \text{kg ms}^{-1}$
(c) $17.56\ \text{kg ms}^{-1}$ (d) $17.57\ \text{kg ms}^{-1}$

47. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m. **[AIEEE 2007]**

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{m}$
(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$

48. System shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of the masses $2\,m$ and m just after the string is cut, will be **[AIEEE 2006]**



(a) $\frac{g}{2}$ upwards, g downwards
(b) g upwards, $\frac{g}{2}$ downwards
(c) g upwards, $2g$ downwards
(d) $2g$ upwards, g downwards

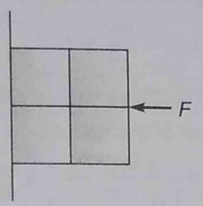
49. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves by 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10\ \text{ms}^{-2}$. **[AIEEE 2006]**

(a) 4 N (b) 16 N
(c) 20 N (d) 22 N

50. A player catches a cricket ball of mass 150 g, moving at a rate of 20 ms^{-1}. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to **[AIEEE 2006]**

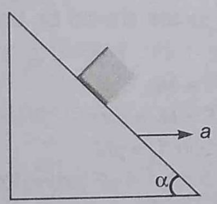
(a) 150 N (b) 3 N
(c) 30 N (d) 300 N

51. A block of mass m is at rest under the action of a force F, acting against a wall, as shown in the figure. Which of the following statement is incorrect? **[AIEEE 2005]**



(a) $f = mg$ (where, f is the frictional force)
(b) $F = N$ (where, N is the normal force)
(c) F will not produce torque
(d) N will not produce torque

52. A block is kept on a frictionless inclined surface with an angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then, a is equal to **[AIEEE 2005]**

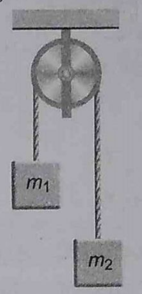


(a) $\frac{g}{\tan\alpha}$ (b) $g\,\text{cosec}\,\alpha$
(c) g (d) $g\tan\alpha$

53. The upper-half of an inclined plane with an inclination ϕ, is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by **[AIEEE 2005]**

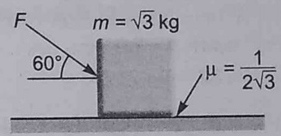
(a) $2\sin\phi$ (b) $2\cos\phi$
(c) $2\tan\phi$ (d) $\tan\phi$

54. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg, tied to a string, are hanging over a light frictionless pulley. What is the acceleration of the masses produced when lift is free to move? ($g = 9.8\ ms^{-2}$) **[AIEEE 2004]**



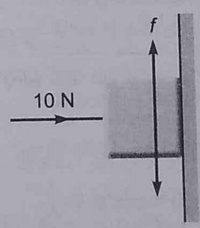
(a) $0.2\ ms^{-2}$ (b) $9.8\ ms^{-2}$
(c) $5\ ms^{-2}$ (d) $4.8\ ms^{-2}$

55. What is the maximum value of the force F such that the block shown in the arrangement, does not move? **[AIEEE 2003]**



(a) 20 N (b) 10 N
(c) 12 N (d) 15 N

56. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is **[AIEEE 2003]**



(a) 20 N (b) 50 N
(c) 100 N (d) 2 N

Answers

1. (a)	2. (d)	3. (a)	4. (b)	5. (b)	6. (a)	7. (c)	8. (a)	9. (b)	10. (a)
11. (c)	12. (b)	13. (a)	14. (a)	15. (d)	16. (a)	17. (a)	18. (b)	19. (c)	20. (d)
21. (c)	22. (a)	23. (d)	24. (b)	25. (c)	26. (b)	27. (b)	28. (a)	29. (a)	30. (c)
31. (c)	32. (b)	33. (b)	34. (a)	35. (c)	36. (c)	37. (c)	38. (b)	39. (b)	40. (d)
41. (d)	42. (b)	43. (d)	44. (c)	45. (c)	46. (a)	47. (c)	48. (a)	49. (d)	50. (c)
51. (d)	52. (d)	53. (c)	54. (a)	55. (a)	56. (d)				

Hints & Solutions

1. As it is clear from the figure

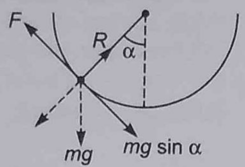
$$F = mg\sin\alpha$$

and $R = mg\cos\alpha$

$\Rightarrow \quad \dfrac{F}{R} = \tan\alpha$

i.e., $\quad \mu = \tan\alpha = \dfrac{1}{3}$

$\Rightarrow \quad \cot\alpha = 3$



2. Net force on the rod $f = 32 - 20 = 12\,\text{N}$

Acceleration of the rod $= \dfrac{f}{m} = \dfrac{12}{3} = 4\,\text{ms}^{-2}$

Equation of motion of the 10 cm part is

$$F - 20 = m \times a = 1 \times 4$$
$$F = 4 + 20 = 24\,\text{N}$$

Similarly, equation of motion of 20 cm part is

$$32 - F = m \times a = 2 \times 4$$
$$F = 32 - 8 = 24\,\text{N}$$

3. Initially, the momentum of the packet in train A is more than in train B. When packets are changed, the packet reaching train A being of lower momentum will retard the train A but packet reaching train B, being of higher momentum will accelerate B.

4. $s = ut + \dfrac{1}{2}at^2 = 0 + \dfrac{1}{2}at^2,\ t = \sqrt{\dfrac{2s}{a}}$

For the smooth plane $a = g\sin\theta$

For the rough plane $a' = g(\sin\theta - \mu\cos\theta)$

$$t' = \sqrt{\frac{2s}{g(\sin\theta - \mu\cos\theta)}} = nt = n\sqrt{\frac{2s}{g\sin\theta}}$$

$$n^2 g(\sin\theta - \mu\cos\theta) = g\sin\theta$$

When, $\theta = 45°,\ \sin\theta = \cos\theta = \dfrac{1}{\sqrt{2}}$

Solving it, we get $m = \left(1 - \dfrac{1}{n^2}\right)$

5. As, $\quad a = \dfrac{(m_1 - m_2)g}{m_1 + m_2} = \dfrac{g}{8}$

$$\frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{8}$$

or $\quad 8m_1 - 8m_2 = m_1 + m_2$

or $\quad 7m_1 = 9m_2$

$$\frac{m_1}{m_2} = \frac{9}{7}$$

6. As, $\quad mg - R = ma$

$$mg - \eta mg = ma$$
$$mg(1 - \eta) = ma$$
$$a = g(1 - \eta)$$

7. When dropped, acceleration of the ball is g as will be observed by a man standing stationary on the ground. The man inside the lift is having its own downward acceleration , a. Therefore, relative acceleration of the ball as observed by the man in the lift will must be $= (g - a)$.

8. Here, initial mass of the rocket $= M$

$$\frac{dm}{dt} = r$$

Relative velocity of gases wrt rocket $= v$

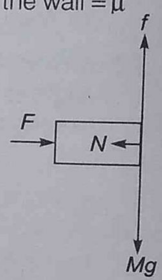
then, acceleration of the rocket

$$a = \frac{F}{m} = \frac{u(dm/dt)}{\left(M - \dfrac{dm}{dt} \times t\right)} = \frac{ur}{M - rt}$$

9. Given, mass of the block $= M$

Coefficient of friction between the block and the wall $= \mu$

Let, a force F be applied on the block to hold the block against the wall. The normal reaction of mass be N and force of friction acting upward be f. In equilibrium, vertical and horizontal forces should be balanced separately.



$$f = Mg \qquad \ldots\text{(i)}$$

and $\quad F = N \qquad \ldots$(ii)

But force of friction $(f) = \mu N$

$$= \mu F \text{ [using Eq. (ii)]} \qquad \ldots\text{(iii)}$$

From Eqs. (i) and (iii), we get

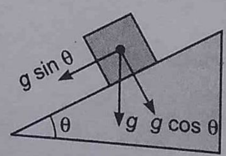
$$\mu F = Mg \quad \text{or} \quad F = \frac{Mg}{\mu}$$

10. According to polygon law, resultant of four forces, each of magnitude F_1 acting at an angle of 72°, is along the fifth side of the polygon taken an in opposite order. As F_2 is acting along this side of polygon, therefore the net force on the particle

$$= F_2 - F_1$$

$$\text{Acceleration} = \frac{F_2 - F_1}{m}$$

11.



From equation of motion,

$$\frac{1}{2}g\sin\theta\, t^2 = l$$

$\Rightarrow \quad t = \sqrt{\dfrac{2l}{g\sin\theta}}$

12. As, $s = ut + \frac{1}{2}at^2$

$$50 = 0 \times 5 + \frac{1}{2} \times a \times 5^2$$

$$a = \frac{100}{25} = 4\ \text{ms}^{-2}$$

Let, mass of one become m_1 and that of other m_2, where $m_1 > m_2$. As m_1 moves downwards with acceleration $a = 4\ \text{ms}^{-2}$

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

So, $$4 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)10$$

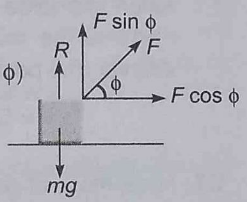
$$\left(\frac{m_1 - m_2}{m_1 + m_2}\right) = \frac{a}{g} = \frac{4}{10} = \frac{2}{5}$$

∴ Percentage of mass transferred

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times 100 = \frac{2}{5} \times 100 = 40\%$$

13. Here, $R = Mg - F\sin\phi$

$$f = \mu R = \mu(Mg - F\sin\phi)$$



$$F\cos\phi - f = Ma$$

$$a = \frac{1}{M}[F\cos\phi - f]$$

$$a = \frac{1}{M}[F\cos\phi - \mu(Mg - F\sin\phi)]$$

$$= \frac{F}{M}\cos\phi - \mu g + \frac{\mu F}{M}\sin\phi = \frac{F}{M}(\cos\phi + \mu\sin\phi) - \mu g$$

14. When the lift is stationary

$$R = mg$$

$$49 = m \times 9.8$$

$$m = \frac{49}{9.8}\ \text{kg} = 5\ \text{kg}$$

If, a is the downward acceleration of the lift.

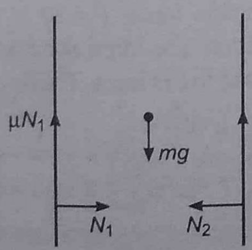
then, $R = m(g - a) = 5(9.8 - 5) = 24\text{N}$

15. Balanced horizontal force $N_1 = N_2$

Balanced vertical force $2\mu N_1 = mg$

$$\mu N_1 = \frac{mg}{2} = \frac{400}{2} = 200\ \text{N}$$

∴ Man pushes the wall with 200 N



16. Limiting force of friction of block on slab

$$\mu\, m_1 g = 0.6 \times 10 \times 9.8 = 58.8\text{N}$$

Since, the applied force = 100 N on block, which is greater than the force of limiting friction, the block will accelerate on the slab, due to which, the force acting on the slab will be that due to the kinetic friction ($\mu_k m_1 g$). Hence, acceleration of the slab $a = \frac{\mu_k m_1 g}{m_2} = \frac{0.4 \times 10 \times 9.8}{40} = 0.98\ \text{ms}^{-2}$

17. Let, T = tension in the string,

f = frictional force between block A and the table,

m = minimum mass of C

For just the motion of block A on the table.

$$T = f = \mu R = \mu(m + m')g$$

$$= 0.2\ (10 + m')g \quad \text{... (i)}$$

For just the motion of block B

$$T = 5g \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$5g = 0.2(10 + m')\ g$$

$$5 = 2 + 02\ m'$$

$$m' = (5 - 2)/\ 0.2 = 15\ \text{kg}$$

18. Force on the pulley, by the clamp

= Resultant of forces $(M + m)g$ acting along horizontally and mg acting vertically downwards

$$= \sqrt{(M + mg)^2 + (mg)^2} = \sqrt{[(M + m)^2 + m^2]}g$$

19. As, $F = 600 - 2 \times 10^5 t$

At, $t = 0, F = 600\text{N}$

$F = 0$, on leaving the barrel,

$$0 = 600 - 2 \times 10^5 t$$

$$t = \frac{600}{2 \times 10^5} = 3 \times 10^{-3}\text{s}$$

This is the time spent by the bullet in the barrel

Average force $= \frac{600 + 0}{2} = 300\text{N}$

Average impulse imparted $= F \times t$

$$= 300 \times 3 \times 10^{-3} = 0.9\ \text{N-s}$$

20. To move the body up the inclined plane, the force required,

$$= mg\sin\theta + \mu R$$

$$= mg\sin\theta + \mu mg\cos\theta$$

21. For an upward motion

Retarding force $= mg + F$

Retardation $(a) = \frac{mg + F}{m}$

Distance $$s = \frac{v^2}{2a} = \frac{v^2 m}{2(mg + F)} \quad \text{... (i)}$$

For the downward motion, net force $= mg - F$

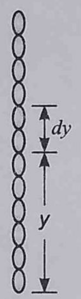
$\therefore$ Acceleration $(a') = \dfrac{mg - F}{m}$

Distance $s' = \dfrac{v'^2}{2a'} = \dfrac{v'^2 m}{2(mg - F)}$

As $s = s'$

$\therefore$ $v' = v\sqrt{\dfrac{mg - F}{mg + F}}$

22. F = force on the table due to the weight of the chain on the table + momentum of the chain transmitted on the table



$F = F_1 + F_2$

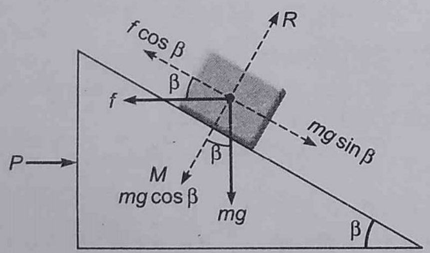
$F_1 = \dfrac{m}{l} yg, dp = dmV = \dfrac{m}{l} dy \sqrt{2gy}$

$\dfrac{dp}{dt} = F_2 = \dfrac{m}{l}\dfrac{dy}{dt}\sqrt{2gy}$

$= \dfrac{m}{l}(2gy)\left[\dfrac{dy}{dt} = \sqrt{2gy}\right]$

$\therefore F = \dfrac{m}{l} yg + \dfrac{2myg}{l} = \dfrac{3myg}{l}$

23. Different forces involved are shown in the figure



Acceleration of the system $a = \dfrac{P}{M + m}$

Force on block of mass $(m) = \dfrac{Pm}{M + m}$

If, f is the reaction of m on M, then

$f = \dfrac{Pm}{M + m}$

As it is clear from the figure

$f\cos\beta = mg\sin\beta$

$\dfrac{Pm}{(M + m)}\cos\beta = mg\sin\beta$

$P = g(M + m)\dfrac{\sin\beta}{\cos\beta}$

or, $P = (M + m)g\tan\beta$

24. It is known that the thrust $= -v\left(\dfrac{dM}{dt}\right) = -v(\alpha v)$

Hence, the retardation produced $= \dfrac{\text{Thrust}}{\text{Mass}} = -\dfrac{\alpha v^2}{M}$

25. We know that, $v^2 = u^2 + 2as$, so $s = \dfrac{v^2 - u^2}{2a}$

Thus, we see that s is independent of size and mass of the body and hence at both will stop the same distance, though here we did not account for the air resistance.

26. Given, $u = 5$ m/s, along positive x-direction

$F = -0.4$ N, along negative x-direction

$M = 200\text{ g} = 0.2\text{ kg}$

Thus, the acceleration $a = \dfrac{F}{M} = -\dfrac{0.4}{0.2} = -2\text{ m/s}^2$

The negative sign showing the retardation.

The position of the object at time t is given by

$x = x_0 + ut + \dfrac{1}{2}at^2$

At, $t = 0$, the body is at $x = 0$, therefore $x_0 = 0$.

Hence, $x = ut + \dfrac{1}{2}at^2$

Since, the force acts during the time interval from $t = 0$ to $t = 10$ s, the motion is accelerated only within this time interval. The position of the body at $t = 25$ s is given by,

$x = 5 \times 2.5 + \dfrac{1}{2} \times (-2) \times (2.5)^2 = 1.25\text{ m}$

27. Here, mass of car $= m$

As it starts from rest, $u = 0$

Final velocity along east, $v = v\hat{\mathbf{i}}$

$t = 2$ s.

From $v = u + at$

$u\hat{\mathbf{i}} = 0 + \mathbf{a} \times 2, \mathbf{a} = \dfrac{v}{2}\hat{\mathbf{i}}$

$\mathbf{F} = m\mathbf{a} = \dfrac{mv}{2}\hat{\mathbf{i}}$

i.e., force of car is $\dfrac{mv}{2}$ eastward.

This is due to friction on the tyres exerted by the road.

28. Since, the block is placed on the trolley, the acceleration of the block equal to acceleration of the trolley $= a = 3\text{ m/s}^2$. Therefore, the force acting on the block is

$F = ma = 10 \times 3 = 30$ N.

The weight mg of the block is balanced by the normal reaction R. As the trolley accelerates in the forward direction, it exerts a reaction force, $F = 30$ N on the block in the backward direction. The force of friction opposes this force and acts opposite to this force. The force of limiting friction is given by, $\mu = \dfrac{f}{R} = \dfrac{f}{Mg}$

$f = \mu mg = 0.2 \times 10 \times 10 = 20\text{N}$

Thus, the block is acted upon by two forces, a force $F = 30$ N towards right and a frictional force $F = 20$ N towards left. The net force on the block is towards right and is given by

$$F' = F - f = 30 - 20 = 10 \text{ N}$$

Due to this force, the block experiences an acceleration towards the rear end of the trolley and is given by

$$a' = \frac{F'}{m} = \frac{10}{10} = 1 \text{ m/s}^2.$$

Let, t be the time taken for the block to fall off from the rear end for the trolley. Then the block has to travel a distance $s' = 5$ m to fall off. Now, since the trolley starts from rest, so, $u = 0$ and using $s = ut + \frac{1}{2}at^2$, we can determine t as $\sqrt{10}$ s.

The distance covered by the trolley in this time,

$$t = \sqrt{10} \text{ s}, s' = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 3 \times 10 = 15 \text{m}$$

29. When a car accelerates, the engine rotates the rear axle which exerts a push on the wheels to move.

30. Work done in bringing the body against the gravitational force, depends only on the initial and final positions, not upon the path taken. But gravitational force on the body along the inclined plane is not the same as that along the vertical side.

31. If the bullet is fired from the rifle, the momentum of bullet-rifle system is conserved.

It means, $M_b v_b = M_r v_r$ ... (i)

and $$\frac{E_{k(b)}}{E_{k(r)}} = \frac{\frac{1}{2}M_b v_b^2}{\frac{1}{2}M_r v_r^2} = \frac{M_r}{M_b}$$

As, $M_r > M_b$ (mass of rifle is greater than the mass of bullet).

Hence, $E_{k(b)} > E_{k(r)}$

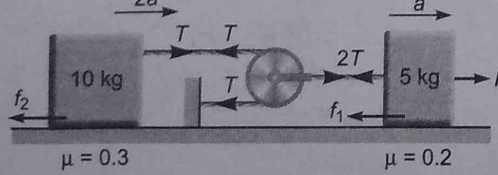
So, the kinetic energy of bullet is greater than the kinetic energy of rifle.

32. The cloth can be pulled out without dislodging the dishes from the table due to law of inertia, which is Newton's first law. While, Statement II is true, but it is Newton's third law.

33. Both Statements are correct. But Statement II does not explain correctly, Statement I.

Correct explanation is there is increase in normal reaction when the object is pushed and there is decrease in normal reaction when the object is pulled (but strictly, not horizontally).

34. From constraint theory, we can relate the acceleration of the 10 kg and 5 kg blocks.



Limiting frictional force between 5 kg blocks and surface is,

$$f_{L_1} = 0.2 \times 5 \times 10 = 10 \text{ N}$$

Limiting frictional force between 10 kg block and surface is,

$$f_{L_2} = 0.3 \times 10 \times 10 = 30 \text{ N}$$

For the 5 kg block, $P - 2T - f_1 = 0$ [In equilibrium, *i.e.*, when blocks are not moving]

For the 10 kg block, $T - f_2 = 0$

For no motion of the blocks, $f_1 \le f_{L_1}$ and $f_2 \le f_{L_2}$

So, $P - 2T \le f_{L_1}$ and $T \le f_{L_2}$

$\Rightarrow \quad P \le f_{L_1} + 2f_{L_2}$

So, for motion to take place, $P \ge f_{L_1} + 2f_{L_2}$

$\Rightarrow \quad 5\,t_0 = f_{L_1} + 2f_{L_2} = 10 + 2 \times 30$

$\Rightarrow \quad t_0 = 14$ s

35. At, $t = \frac{t_0}{2} = 7$ s, the equation for 5 kg and 10 kg blocks are

$$35 - 2T - f_1 = 0 \text{ and } T - f_2 = 0$$

$\Rightarrow \quad 35 = f_1 + 2f_2$

and we know that, $t = 7$ s, both the blocks are at rest. So, $f_1 \le f_{L1}$ and $f_2 \le f_{L_2}$.

Solving the above equation, we get $0 \le f_1 \le 10$ N

$$12.5 \text{ N} \le f_2 \le 17.5 \text{ N}$$

and $$12.5 \text{ N} \le T \le 17.5 \text{ N}$$

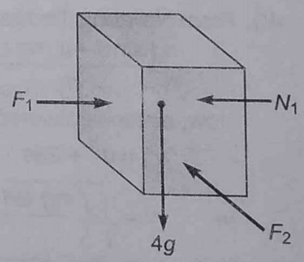
36. At, $t = 2t_0 = 28$ s, equation for the blocks are

$$140 - 2T - 10 = 5a$$

and $$T - 30 = 10 \times 2a$$

$$a = \frac{14}{9} \text{ ms}^{-2}$$

37. The forces acting on the block are F_1, F_2, mg, normal contact force and frictional force. Here frictional force won't act along the vertical direction as the component of resultant force acting along the surface on the body and is not along the vertical direction and direction of the frictional force is either opposite to the motion of block (direction of acceleration of block) if it is moving or not moving.



$$N_1 = 300 \text{ N}$$

So, $$f_L = \mu N_1 = 0.6 \times 300 = 180 \text{ N}$$

Resultant of $4g$ and F_2 is 107.7 N, making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal.

As the force applied along the surface is $> f_L$; so the block does not move the friction is static in nature.

$f = 107.7$ N making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction.

38. For, $F_1 = 150\,N$, $f_L = 0.6 \times 150 = 90\,N$

As component of resultant force along the surface is 107.7 N and is greater than f_L, so kinetic friction comes into existence *i.e.*, frictional force acquires the value

$$f = \mu_k N_1 = 0.5 \times 150 = 75\,N.$$

Its direction is opposite to component of resultant force along the surface.

39. *y-x* graph gives to the shape of path of particle

$$\frac{dx}{dt} = 8\pi \sin 2\pi t$$

$$\int_8^x dx = \int_0^t 8\pi \sin 2\pi t \, dt$$

$$\Rightarrow \quad x - 8 = -\frac{8\pi}{2\pi}[\cos 2\pi t]_0^t$$

$$\Rightarrow \quad x - 8 = 4[1 - \cos 2\pi t]$$

$$\Rightarrow \quad -\cos 2\pi t = \frac{x-12}{4} \quad \text{...(i)}$$

Also, $$\frac{dy}{dt} = 5\pi \cos 2\pi t$$

$$\int_0^y dy = 5\pi \int_0^t \cos 2\pi t \, dt$$

$$y = \frac{5}{2} \sin 2\pi t \quad \text{...(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$\left(\frac{x-12}{4}\right)^2 + \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

This is equation of ellipse.

40. From Newton's IInd law

$$\frac{mg \sin\theta - \mu\, mg \cos\theta}{m} = a$$

Now, distance covered by the particle

$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{mg \sin\theta - \mu\, mg \cos\theta}{m}\right)x}$$

$$= \sqrt{2\,gx \sin\theta - 0.6\,x^2\, g \cos\theta}$$

v should be maximum when $\frac{dv}{dx} = 0$

$$\Rightarrow \quad \frac{d\sqrt{2gx \sin\theta - 0.6\,x^2\, g\cos\theta)}}{dx} = 0$$

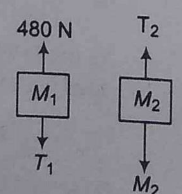
By differentiating, we get $x = 3.33$m

41. For block of mass M_1

$$\frac{480 - T_1 - 20\,g}{20} = a$$

Also, for block of mass M_2

$$\frac{T_2 - 12\,g}{12} = a$$



Since, a is common for the all the individuals of the system

$$\Rightarrow \quad \frac{480 - T_1 - 20\,g}{20} = \frac{T_2 - 12\,g}{12}$$

After taking $g = 10\text{ m/s}^2$ this gives

$$5T_2 + 3T_1 = 1440 \quad \text{...(i)}$$

Now, for the metal rod, tension at both of its end are dissimilar and $T_1 - T_2 = 80$ $(\because g = 10\text{ m/s}^2)$ ...(ii)

Now, from Eqs. (i) and (ii) we get,

$$T_1 = 230\,N$$

and $$T_2 = 150\,N$$

$\therefore$ Tension at mid-point $= T_1 - 4g = 190\,N$

42. Tension, $T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$

$$= \left(\frac{2m \times 2m}{m + 2m}\right)g \quad \text{where } m_1 = m \text{ and } m_2 = 2m$$

$$= \frac{4}{3} mg$$

$$\because \quad \text{Stress} = \frac{\text{Force (Tension)}}{\text{Area}}$$

$$= \frac{\frac{4}{3}mg}{A} = \frac{4\,mg}{3A}$$

43. As horizontal external force on the cart boy system is zero. So, the position of CM for the system will not change

Therefore, $20 \times 10 = 80 \times n$ (backward)

$$n = \frac{200}{80} = 2.5\,m$$

So, the displacement of boy w.r.t. ground

$$= 10 - 2.5 = 7.5\,m$$

$\therefore$ The distance of boy from the wall

$$= 25 - 7.5 = 17.5\,m$$

44. As the force is exponentially decreasing, so it's acceleration, *i.e.*, rate of increase of velocity will decrease with time. Thus, the graph of velocity will be an increasing curve with decreasing slope with time.

$$a = \frac{F}{m} = \frac{F_0}{m} e^{-bt} = \frac{dv}{dt}$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{F_0}{m} e^{-bt} = \frac{dv}{dt}$$

$$\Rightarrow \quad v = \frac{F_0}{m}\left(\frac{1}{-b}\right) e^{-bt}\Big|_0^t = \frac{F_0}{mb} e^{-bt}\Big|_t^0$$

$$= \frac{F_0}{mb}(e^0 - e^{-bt})$$

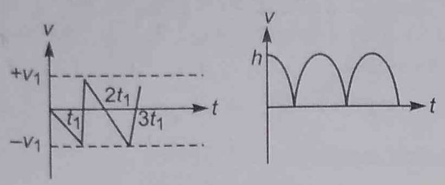
$$= \frac{F_0}{mb}(1 - e^{-bt})$$

with $$v_{max} = \frac{F_0}{mb}$$

45. $h = \frac{1}{2}gt^2$, (parabolic)

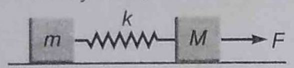
$v = -gt$ and after the collision, $v = gt$ (straight line)

Collision is perfectly elastic, then ball reaches to same height again and again with same velocity



46. As, $p = mv = 3.513 \times 5.00 \approx 17.6\,\text{kg-ms}^{-1}$

47. Acceleration of the system,



$$a = \frac{F}{m+M}$$

So, force acting on the mass is

$$F = ma = \frac{mF}{m+M}$$

48. Initially under the equilibrium of mass m

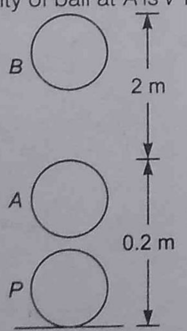
$$T = mg$$

Now, the string is cut. Therefore, $T = mg$ force is decreased on the mass m upwards and downwards on the mass $2m$.

$\therefore \quad a_m = \frac{mg}{m} = g$ (downwards)

and $\quad a_{2m} = \frac{mg}{2m} = \frac{g}{2}$ (upwards)

49. The situation is shown in the figure. At an initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B, let acceleration of ball during PA is $a\,\text{ms}^{-2}$ [assumed to be constant] in an upward direction and velocity of ball at A is $v\ \text{ms}^{-1}$.



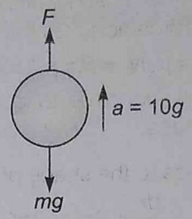
Then for PA, $\quad v^2 = 0^2 + 2a \times 0.2$

For AB, $\quad 0 = v^2 - 2 \times g \times 2$

$\Rightarrow \quad v^2 = 2g \times 2$

From above equations,

$$a = 10g = 100\ \text{ms}^{-2}$$



Then for PA, FBD of ball is

$$F - mg = ma$$

[F is the force exerted by hand on the ball]

$\Rightarrow \quad F = m(g + a)$

$= 0.2\,(11g)$

$= 22\,\text{N}$

50. This is the question based on impulse-momentum theorem.

$$|F\Delta t| = |\text{Change in momentum}|$$

$\Rightarrow \quad F \times 0.1 = |p_f - p_i|$

As the ball will stop after catching,

$$p_i = mv_i = 0.15 \times 20 = 3, p_f = 0$$

$\Rightarrow \quad F \times 0.1 = 3$

$\Rightarrow \quad F = 30\,\text{N}$

51. This is the equilibrium of coplanar forces.

Hence, $\quad \Sigma F_x = 0$

$\therefore \quad F = N$

$\therefore \quad \Sigma F_y = 0$

$f = mg$

$\Sigma \tau_c = 0$

$\therefore \quad \tau_N + \tau_f = 0$

Since, $\quad \tau_f \neq 0$

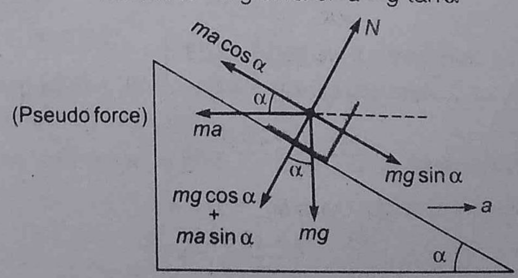
$\therefore \quad \tau_N \neq 0$

Thus, N will not produce torque.

52. In the frame attached to the wedge, the force diagram of block is shown in the figure. From free body diagram of the wedge,

For block to remain stationary,

$$ma\cos\alpha = mg\sin\alpha \text{ or } a = g\tan\alpha$$



53. According to the work-energy theorem,

$$\Sigma W = \Delta K = 0$$

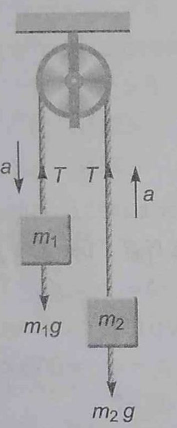
$\Rightarrow$ Work done by friction + Work done by gravity = 0

$$\Rightarrow -(\mu \, mg \cos \phi)\frac{l}{2} + mgl \sin \phi = 0$$

or $$\frac{\mu}{2} \cos \phi = \sin \phi$$

or $$\mu = 2 \tan \phi$$

54. On releasing, the motion of the system will be according to the figure



$$m_1 g - T = m_1 a \quad \dots \text{(i)}$$

and $$T - m_2 g = m_2 a \quad \dots \text{(ii)}$$

On solving,

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \quad \dots \text{(iii)}$$

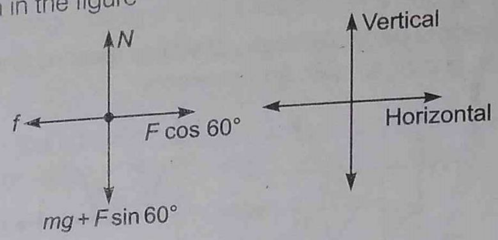
Here, $m_1 = 5 \text{ kg}, m_2 = 4.8 \text{ kg}$

$$g = 9.8 \text{ ms}^{-2}$$

$\therefore$ $$a = \left(\frac{5 - 4.8}{5 + 4.8}\right) \times 9.8$$

$$= \frac{0.2}{9.8} \times 9.8 = 0.2 \text{ ms}^{-2}$$

55. Free Body Diagram (FBD) of the block (shown by a dot) is shown in the figure



For vertical equilibrium of the block

$$N = mg + F \sin 60°$$

$$= \sqrt{3} g + \sqrt{3} \frac{F}{2} \quad \dots \text{(i)}$$

For no motion, force of friction

$$f \geq F \cos 60°$$

or $$\mu N \geq F \cos 60°$$

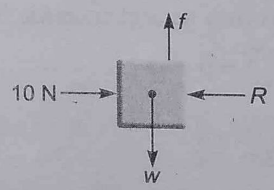
or $$\frac{1}{2\sqrt{3}}\left(\sqrt{3} g + \frac{\sqrt{3} F}{2}\right) \geq \frac{F}{2}$$

or $$g \geq \frac{F}{2}$$

or $$F \leq 2g \quad \text{or } 20 \text{ N}$$

Therefore, maximum value of F is 20 N.

56. Let, R be the normal contact force by wall on the block.



$$R = 10 \text{ N}$$

$$f_L = w$$

and $$f = \mu R$$

$\therefore$ $$\mu R = w$$

or $$w = 0.2 \times 10 = 2 \text{ N}$$

Day 5

Circular Motion

Day 5
Outlines ...

- Concept of Circular Motion
- Three Possible Types of Circular Motion
- Forces in Circular Motion

Concept of Circular Motion

The circular motion differs from the linear motion in one very important aspect that in a circular motion particles move along circular track such that direction of motion changes continuously unlike in a linear motion. Therefore, circular motion is described in terms of angular displacement i.e., angle turned by the rotating body in a unit time.

Terms Related to Circular Motion

The important terms used in circular motion are given as

1. Angular Displacement

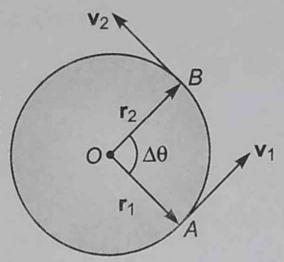
It is defined as the angle turned by the particle from some reference line. Angular displacement $\Delta\theta$ is usually measured in radians.

Finite angular displacement $\Delta\theta$ is a scalar but an infinitesimally small displacement is a vector.

2. Angular Velocity

It is defined as the rate of change of the angular displacement of the body.

$\therefore$ Angular velocity $\omega = \lim_{\Delta t \to 0} \left(\frac{\Delta\theta}{\Delta t}\right) = \frac{d\theta}{dt}$



It is an axial vector whose direction is given by the right hand rule. Its unit is rad/s.

3. Angular Acceleration

It is the rate of change of angular velocity.

Thus, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Its unit is rad/s^2.

4. Velocity

A particle in a circular motion has two types of velocities and corresponding two speeds.

(i) **Linear velocity** (v) or **speed** (v)

$$\mathbf{v} = \frac{d\mathbf{s}}{dt} \text{ and } v = |\mathbf{v}| = \left|\frac{d\mathbf{s}}{dt}\right|$$

(ii) **Angular velocity** (ω) or **speed** (ω)

$$\omega = \frac{d\theta}{dt} \text{ and } \omega = |\omega| = \left|\frac{d\theta}{dt}\right| = \frac{d\theta}{dt}$$

Relation between linear speed (v) and angular speed (ω) is

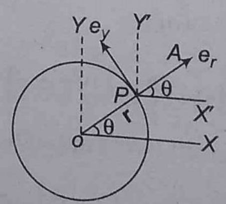
$$v = r\omega$$

In vector form, $\mathbf{v} = \omega \times \mathbf{r}$, which is a relation between linear velocity ($\mathbf{v}$) and angular velocity (ω).

Here, $\mathbf{r}$ is the position vector of particle with respect to the centre of the circle.

5. Acceleration

Acceleration of a particle in circular motion has two components



(i) **Tangential acceleration** (a_t), which is the component of **a** in the direction of velocity.

a_t = component of **a** along **v**

$$= \frac{dv}{dt} = \frac{d|\mathbf{v}|}{dt}$$

(ii) **Radial acceleration** (a_r), which is the component of **a** towards the centre of the circular motion. This is responsible for a change in the direction of velocity.

$$a_r = \frac{v^2}{r} = r\omega^2$$

The radial unit vector

$$\mathbf{e}_r = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$$

The tangential unit vector

$$\mathbf{e}_t = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$$

Velocity of the particle

$$\mathbf{v} = \omega r(-\mathbf{i}\sin\theta + \mathbf{j}\cos\theta)$$

Acceleration of the particle

$$\mathbf{a} = -\omega^2 r\,\mathbf{e}_r + \frac{dv}{dt}\mathbf{e}_t$$

And also, $\frac{rd\omega}{dt} = \frac{d(r\omega)}{dt} = \frac{dv}{dt}$

- *If the speed of the particle changes, the particle experiences a tangential force $F_t\left(= m\frac{dv}{dt}\right)$ along with the centripetal force (F_c), the net force then being given by $|\mathbf{F}| = \sqrt{F_c^2 + F_t^2}$*
- *Angle made by* **F** *with the tangent $\theta = \tan^{-1}\left(\frac{F_c}{F_t}\right)$*

Relation between Linear and Angular Variables

- If a reference line on a rigid body rotates by an angle θ, a point within the body at a position r from the rotation axis moves a distance s along a circular arc, where s is given by $s = \theta r$
  The angle θ must be measured in radian.
- The period of revolution (T) for the motion of each point and for the rigid body itself is given by $T = \frac{2\pi}{\omega}$.
- When a particle is moving along a curved path, then its tangential and angular velocities are related by $\mathbf{v} = \omega \times \mathbf{r}$, where $\mathbf{r}$ is the vector joining the location of particle and the point about which ω has been computed.
- In other way, we can write, $v = r\omega$, where v is the component of velocity perpendicular to $\mathbf{r}$ or we can say v_t is the tangential velocity.
- If we differentiate above equation, we get,

$$\frac{d\mathbf{v}}{dt} = \frac{d\omega}{dt} \times \mathbf{r}$$

i.e., $\mathbf{a}_t = \alpha \times \mathbf{r}$

where $\mathbf{a}_t$ is the tangential component of acceleration which is responsible for changing the magnitude of velocity.

Three Possible Types of Circular Motion

In circular motion, the direction of velocity definitely changes. Hence, a_r can never be zero. But speed may remain constant ($a_t = 0$), may be increasing (a_t = positive) or may be decreasing (a_t = negative).

Accordingly, we can classify circular motion in the following three types

1. Uniform Circular Motion

In which the speed of the particle remains constant. Therefore, $a_t = 0$ and $a_c = \frac{v^2}{r}$ or $r\omega^2$.

Thus, net acceleration is equal to a_r. Angle between **v** and **a** is 90°.

In circular motion, the centripetal force

$$F = \frac{mv^2}{r} \neq 0$$

so the body is not in equilibrium and the linear momentum of the body is not conserved, though the magnitude of linear momentum in a uniform circular motion is constant, but its direction changes continuously.

2. Circular Motion with an Increasing Speed

In which speed of the particle increases. Therefore, a_t is positive or in the direction of the velocity.

Thus, net acceleration in this case will be

$$a = \sqrt{a_t^2 + a_r^2}$$

where, $a_t = \frac{dv}{dt}$ or $\frac{d|\mathbf{v}|}{dt}$

3. Circular Motion with Decreasing Speed

In which speed of the particle decreases. Therefore, a_t is negative or in an opposite direction to that of velocity. Thus, net acceleration in this case is also $a = \sqrt{a_t^2 + a_r^2}$, but angle between **v** and **a** is obtuse.

where, $a_r = \frac{dv}{dt}$ or $\frac{d|\mathbf{v}|}{dt}$

and $a_r = \frac{v^2}{r}$ or $r\omega^2$

*The radial acceleration (a_r) is also sometimes called **normal acceleration** (a_n).*

Regarding circular motion, the following possibilities exist

(i) If $a_r = 0$ and $a_t = 0$, then $a = 0$ and the motion is uniform and translatory.

(ii) If $a_r = 0$ and $a_t \neq 0$, then $a = a_t$ and the motion is accelerated and translatory.

(iii) If $a_r \neq 0$ but $a_t = 0$, then $a = a_r$ and the motion is uniform and circular.

(iv) If $a_r \neq 0$ and $a_t \neq 0$, then $a = \sqrt{a_t^2 + a_r^2}$ and the motion is non-uniform and circular.

Forces in Circular Motion

In circular motion of an object two kinds of forces occur which are described below.

Centripetal Force

The centripetal force is the force required to move a body along a circular path with a constant speed. Centripetal force never acts by itself. It is to be provided by some agency in order to maintain the uniform circular motion of an object. The direction of the centripetal force is along the radius, acting towards the centre of the circle, on which the given body is moving.

Centripetal force, $F = \frac{mv^2}{r} = mr\omega^2 = mr\,4\pi^2 v^2 = mr\frac{4\pi^2}{T^2}$

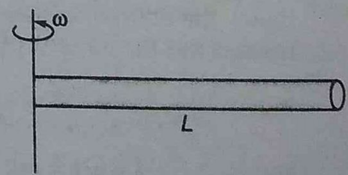
Centrifugal Force

Centrifugal force is a virtual force due to incorporated effects of inertia. The centrifugal force is a pseudo force experienced by a non-inertial observer moving in a circular path with a constant speed on account of its directional inertia. Mathematically,

$$\text{Centrifugal force} = \frac{mv^2}{r} = mr\omega^2$$

However, it is directed radially outwards, *i.e.*, in a direction opposite to that of the centripetal force.

e.g., If a tube, filled with an incompressible fluid of mass m and closed at both the ends is rotated with a constant angular velocity ω about an axis passing through one end, as shown in the figure,



then the force exerted by the liquid at the other end is $\frac{1}{2}mL\omega^2$.

Applications of Centripetal and Centrifugal Forces

Some of the most important applications of centripetal and centrifugal forces are given below.

1. Motion of a Vehicle on a Level Circular Road

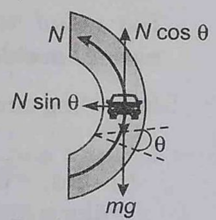
When a vehicle negotiates a circular path, it requires a centripetal force. In such cases the lateral force of friction may provide the requisite centripetal force. Thus, for maintaining its circular path required centripetal force

$$\left(\frac{mv^2}{r}\right) \le \text{frictional force } (\mu mg) \Rightarrow \quad v_{\max} = \sqrt{\mu rg}$$

where, μ = coefficient of friction between road and vehicle tyres and r = radius of circular path.

2. Banking of a Curved Road

For the safe journey of a vehicle on a curved (circular) road, without any risk of skidding, the road is slightly raised towards its outer end. Let the road be banked at an angle θ from the horizontal, as shown in the figure.



Banket track

$$v_{\max} = \sqrt{rg \cdot \tan\theta}$$

If b = width of the road and h = height of the outer edge of the road as compared to the inner edge, then $\tan\theta = \frac{v^2}{rg} = \frac{h}{b}$

In case of friction is present between road and tyre

then $v_{\max} = \sqrt{\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}}$

where, μ_s = coefficient of static friction.

3. Bending of a Cyclist

If a curved road is not banked, then a cyclist/scooterist bends away from the vertical, while negotiating a turn on a curved road. The angle θ at which the cyclist bends from the vertical, is given by $\tan\theta = \frac{v^2}{rg}$

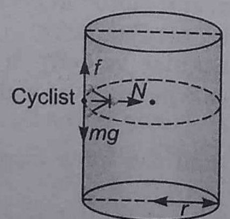
4. Motion of a Cyclist in a Death Well

For equilibrium of cyclist in a death well, as shown in the figure, the normal reaction N provides the centripetal force needed and the force of friction balances his weight mg.

Thus, $\quad N = \frac{mv^2}{r}$

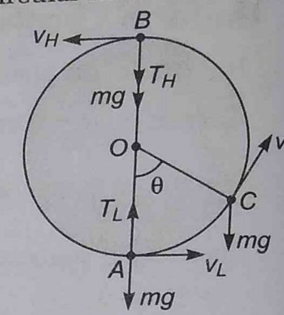
and $\quad f = \mu N = mg$

$\Rightarrow \quad v_{\max} = \sqrt{\frac{rg}{\mu}}$



5. Motion along a Vertical Circle

It is an example of non-uniform circular motion in which speed of object decreases due to effect of gravity as the object goes from its lowest position A to highest position B.



(i) At the lowest point A, the tension T_L and the weight mg are in mutually opposite directions and their resultant provides the necessary centripetal force,

$$\text{i.e.}, T_L - mg = \frac{mv_L^2}{r} \text{ or } T_L = mg + \frac{mv_L^2}{r}$$

(ii) At the highest point B, tension T_H and the weight mg are in the same direction and hence,

$$T_H + mg = \frac{mv_H^2}{r} \text{ or } T_H = \frac{mv_H^2}{r} - mg$$

Moreover, v_L and v_H are correlated as $v_H^2 = v_L^2 - 2gr$

(iii) In general, if the revolving particle, at any instant of time, is at position C, inclined at an angle θ from the vertical, then

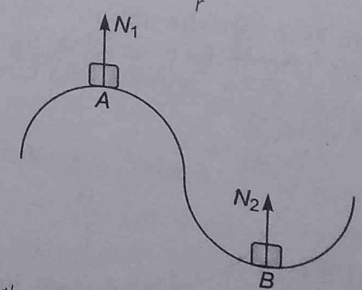
$$v^2 = v_L^2 - 2gr(1 - \cos\theta) \text{ and } T = mg\cos\theta + \frac{mv^2}{r}$$

(iv) In the critical condition of just looping the vertical loop, (*i.e.*, when the tension just becomes zero at the highest point B), we obtain the following results

$$T_H = 0, T_L = 6\,mg, v_L = \sqrt{5\,rg} \text{ and } v_H = \sqrt{rg}$$

In general, $\quad T_L - T_H = 6\,mg$

- When a vehicle is moving over a convex bridge, the maximum velocity $v = \sqrt{rg}$, where r is the radius of the road.
- When the vehicle is at the maximum height, the reaction of the road, is $N_1 = mg - \frac{mv^2}{r}$



- When the vehicle is moving in a dip B, then

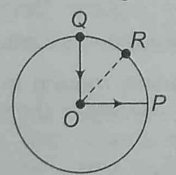
$$N_2 = mg + \frac{mv^2}{r}$$

Practice Zone DAY 5

1. A wheel of radius R is rolling on a straight line without slipping on a plane surface, the plane of the wheel is vertical. For the instant when the axis of the wheel is moving with a speed v relative to the surface, the instantaneous velocity of any point P on the rim of the wheel relative to the surface is
   (a) v (b) $v(1+\cos\theta)$
   (c) $v\sqrt{2(1+\cos\theta)}$ (d) $v(1-\cos\theta)$

2. A mass of 2 kg is whirled in a horizontal circle with the help of a string, at an initial speed of 5 rev/min. Keeping the radius constant, the tension in the string is doubled. The new speed is nearly
   (a) 14 rpm (b) 10 rpm (c) 2.25 rpm (d) 7 rpm

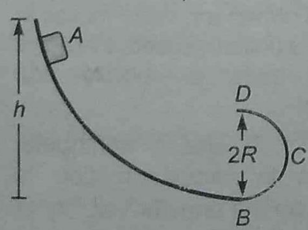
3. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path $OPRQO$ as shown in figure. If he maintains constant speed of 10 m/s, what is his accelearation at point R in magnitude and direction? **[NCERT Exemplar]**



   (a) $0.1\,m/s^2$ along RO (b) $0.01\,m/s^2$ along OR
   (c) $1\,m/s^2$ along RO (d) $0.1\,rad/s^2$ along RO

4. A weightless thread can bear a tension upto 3.7 kg-wt. A stone of mass 500 g is tied to it and revolved in a circular path of radius 4 m in a vertical plane. If $g = 10\,ms^{-2}$, then the maximum angular velocity of the stone will be
   (a) 4 rad s^{-1} (b) 16 rad s^{-1} (c) $\sqrt{21}$ rad s^{-1} (d) 2 rad s^{-1}

5. A frictionless track $ABCD$ ends in a semicircular loop of radius R. A body slides down the track from point A which is at a height $h = 5$ cm. Maximum value of R for the body to successfully complete the loop is



   (a) 5 cm (b) $\frac{15}{4}$ cm (c) $\frac{10}{3}$ cm (d) 2 cm

6. The angular speed of a flywheel making 120 rev/min is
   (a) $\pi\,rads^{-1}$ (b) $2\pi\,rads^{-1}$ (c) $4\pi\,rads^{-1}$ (d) $6\pi\,rads^{-1}$

7. A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved?
   (a) 14 ms^{-1} (b) 3 ms^{-1}
   (c) 3.92 ms^{-1} (d) 5 ms^{-1}

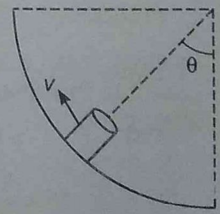
8. A stone of mass 1 kg tied to the end of a string of length 1m, is whirled in a horizontal circle with a uniform angular velocity of 2 rad s^{-1}. The tension of the string is (in N)
   (a) 2 (b) $\frac{1}{3}$ (c) 4 (d) $\frac{1}{4}$

9. A small body of mass m slides down from the top of a hemisphere of radius R. The surface of block and hemisphere are frictionless. The height at which the body lose contact with the surface of the sphere is
   (a) $\frac{3}{2}R$ (b) $\frac{2}{3}R$ (c) $\frac{1}{2}R$ (d) $\frac{1}{3}R$

10. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is
    (a) 1 : 1 (b) $m_1 : m_2$
    (c) $r_1 : r_2$ (d) $m_1m_2 : r_1r_2$

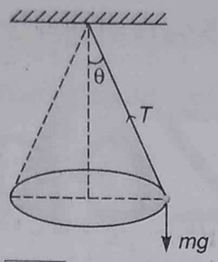
11. A wheel is rotating at 900 rpm about its axis. When the power is cut off, it comes to rest in 1 min. The angular retardation in $rads^{-2}$ is
    (a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/8$

12. The skate board negotiates the circular surface of radius 4.5 m. At $\theta = 45°$, its speed of centre of mass is 6 m/s. The combined mass of skate board and the person is 70 kg and his centre of mass is 0.75 m from the surface. The normal reaction between the surface and the skate board wheel is



    (a) 500 N
    (b) 2040 N
    (c) 1045 N
    (d) zero

13. A heavy sphere of mass m is suspended by a string of length l. The sphere is made to revolve about a vertical line passing through the point of suspension, in a horizontal circle such that the string always remains inclined to the vertical making an angle θ. What is the period of revolution?



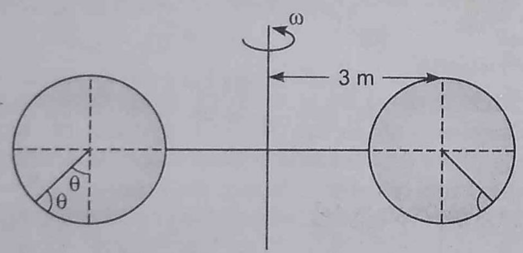
(a) $T = 2\pi\sqrt{\frac{l}{g}}$ (b) $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

(c) $T = 2\pi\sqrt{\frac{l\sin\theta}{g}}$ (d) $T = 2\pi\sqrt{\frac{l\tan\theta}{g}}$

14. A car is moving in a circular horizontal track of radius 10m with a constant speed of 10 m/s. A plump bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with the track is

(a) 60° (b) 30° (c) 45° (d) zero

15. Two small spherical balls are free to move on the inner surface of the rotating spherical chamber of radius $R = 0.2$ m. If the balls reach a steady state at angular position $\theta = 45°$, the angular speed ω of device is

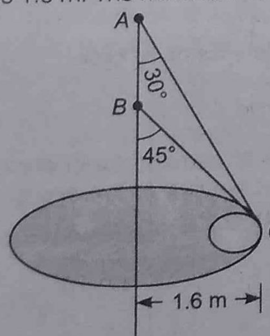


(a) 8 rad/s (b) 2 rad/s
(c) 3.64 rad/s (d) 9.34 rad/s

16. A particle is moving along a circular path of radius 5m, moving with a uniform speed of 5 m/s. What will be the average acceleration, when the particle completes half revolution?

(a) zero (b) $10/\pi$ m/s^2 (c) 10 m/s^2 (d) None

17. Two wires AC and BC are tied at C of small sphere of mass 5 kg, which revolves at a constant speed v in the horizontal circle of radius 1.6 m. The minimum value of v is



(a) 8.01 m/s (b) 1.6 m/s (c) 0 (d) 3.9 m/s

18. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.5 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn? **[NCERT Exemplar]**

(a) 0.9 m/s at 75° to the velocity
(b) 0.6 m/s at 54° to the velocity
(c) 0.3 m/s at 75° to the velocity
(d) 0.7 m/s at 68° to the velocity

Directions *(Q. Nos. 18 to 23) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below :*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

19. Statement I A car is moving in a horizontal circular plane with varying speed, then the frictional force is neither pointing towards the radial direction nor along the tangential direction.

Statement II Components of the frictional force are providing the necessary tangential and centripetal acceleration, in the above situation.

20. Statement I A particle moving in a vertical circle, has a maximum kinetic energy at the highest point of its motion.

Statement II The magnitude of the velocity remains constant for a particle moving in a horizontal plane.

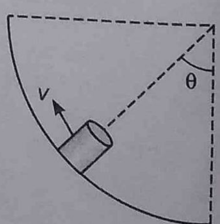
21. Statement I The centripetal forces and the centrifugal forces never cancel out.

Statement II They do not act at the same time.

22. Statement I Improper banking of roads causes wear and tear of tyres.

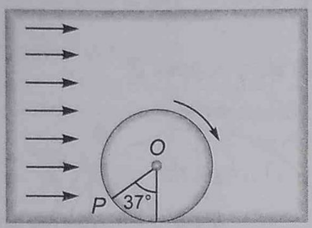
Statement II The necessary centripetal force in that event is provided by friction between the tyres and roads.

23. Statement I When a particle moves in a circle with a uniform speed, there is a change in both its velocity and acceleration.



Statement II The centripetal acceleration in circular motion is dependent on the angular velocity of the body.

Directions *(Q. Nos. 24 to 26) A cabin is falling freely and inside the cabin a disc of mass M and radius R undergoes uniform and a pure rolling motion with the help of some external agent. Inside the cabin, a wind is blowing in the horizontal direction which imparts an acceleration a to all the objects present in the cabin, in horizontal direction. [Disc still performs uniform and a pure rolling motion]. A very small particle gets separated from the disc from the point P and after some time it passes through the centre of the disc O.*



24. The angular velocity of the disc is

(a) $\frac{1}{3}\times\sqrt{\frac{7a}{6R}}$ (b) $\sqrt{\frac{8a}{15R}}$

(c) $\frac{4}{9}\times\sqrt{\frac{7a}{6R}}$ (d) $\frac{16}{9}\sqrt{\frac{8a}{15R}}$

25. The time taken by the particle to reach O from P is

(a) $\frac{4}{3}\sqrt{\frac{15R}{8a}}$ (b) $4\sqrt{\frac{6R}{7a}}$ (c) $3\sqrt{\frac{6R}{7a}}$ (d) $\frac{3}{4}\sqrt{\frac{15R}{8a}}$

26. The revolution made by the disc in the time interval computed in Q. No. 24 is

(a) 8 (b) $\frac{6}{5\pi}$

(c) $\frac{5\pi}{6}$ (d) $\frac{2}{3\pi}$

Passage *A thin circular loop of wire of radius R rotates about its vertical diameter with an angular frequency ω. The small bead on the wire remains at the lower most point. Assume that all surfaces are frictionless.* **[NCERT Exemplar]**

27. The value of ω is

(a) equal to $\frac{g}{R}$

(b) smaller than and equal to $\sqrt{\frac{g}{R}}$

(c) greater than and equal to $\sqrt{\frac{g}{R}}$

(d) greater than and equal to $\frac{g}{R}$

28. What is the angle made by the radius vector joining the centre to lead with the vertical downward direction when $\omega=\sqrt{\frac{2g}{R}}$?

(a) 30° (b) 60°
(c) 75° (d) 45°

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29. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is **[AIEEE 2012]**

(a) $m_1r_1 : m_2r_2$ (b) $m_1 : m_2$ (c) $r_1 : r_2$ (d) 1 : 1

30. A particle moves in a circular path with decreasing speed. Choose the correct statement. **[AIEEE 2005]**

(a) Angular momentum remains constant
(b) Acceleration **a** is acting towards the centre
(c) Particle moves in a spiral path with decreasing radius
(d) The direction of angular momentum remains constant

31. Which of the following statements is false for a particle moving in a circle with a constant angular speed? **[AIEEE 2004]**

(a) The velocity vector is tangent to the circle
(b) The acceleration vector is tangent to the circle
(c) The acceleration vector points towards the centre of the circle
(d) The velocity and acceleration vectors are perpendicular to each other

32. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that **[AIEEE 2004]**

(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line

33. A particle undergoes a uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle, remain conserved? **[AIEEE 2003]**

(a) Centre of the circle
(b) On the circumference of the circle
(c) Inside the circle
(d) Outside the circle

34. The minimum velocity (in ms^{-1}) with which a car driver must traverse a curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is **[AIEEE 2002]**

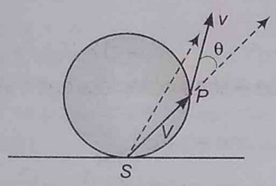
(a) 60 (b) 30
(c) 15 (d) 25

Answers

1. (c)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (a)
11. (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (d)
21. (c)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (c)
31. (b)	**32.** (c)	**33.** (a)	**34.** (b)						

Hints & Solutions

1. $v' = \sqrt{v^2 + v^2 + 2v^2 \cos\theta}$



$\Rightarrow \quad v' = v\sqrt{2(1+\cos\theta)}$

2. As, $T = F_c = m r \omega^2 = mr\, 4\pi^2 n^2$

When T is doubled, n becomes $\sqrt{2}$ times, which is nearly 7 rpm.

3. Acceleration of the cyclist at point R

= centripetal acceleration (a_c)

$$a_c = \frac{v^2}{r} = \frac{(10)^2}{1000} = \frac{100}{1000}$$

$= 0.1\ \text{m/s}^2$, along RO

4. As, $T_{max} = mr\omega^2 + mg$

$$3.7 \times 10 = 0.5 \times 4\omega^2 + 0.5 \times 10$$

or $\quad \omega^2 = \frac{32}{2} = 16$

or $\quad \omega = 4\ \text{rads}^{-1}$

5. Velocity at the bottom is $\sqrt{2gh}$

For completing the loop,

$$\sqrt{2gh} = \sqrt{5gR}$$

Hence, $\quad R = 2h/5 = (2 \times 5)/5 = 2\ \text{cm}$

6. As, $\omega = \frac{2\pi n}{t} = \frac{2\pi \times 120\ \text{rad}}{60\ \text{s}} = 4\pi\ \text{rads}^{-1}$

7. As, $T = mv^2/r$

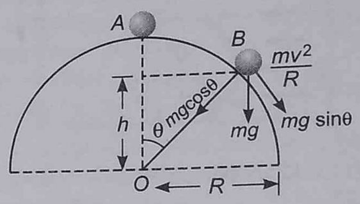
Hence, $\quad v = \sqrt{Tr/m}$

$= \sqrt{25 \times 1.96 / 0.25}$

$= 14\ \text{ms}^{-1}$

8. As, $T = mr\omega^2 = 1 \times 1 \times (2)^2 = 4\ \text{N}$

9. Suppose body slips at point B



$$mg\cos\theta = \frac{mv^2}{R} \qquad [v = \sqrt{2g(R-h)}]$$

$$g\cos\theta = \frac{2g(R-h)}{R}$$

$$\cos\theta = \frac{2(R-h)}{R}$$

$$\frac{h}{R} = \frac{2(R-h)}{R} \qquad \left[\cos\theta = \frac{h}{R}\right]$$

$\therefore \quad h = \frac{2}{3}R$

10. $\frac{\omega_1}{\omega_2} = \frac{2\pi/T_1}{2\pi/T_2}$. Here $T_1 = T_2 = t$

$\Rightarrow \quad \omega_1 : \omega_2 = 1:1$

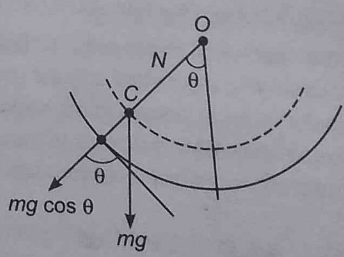
11. Use $\qquad \omega = \omega_0 + \alpha t \qquad \ldots$(i)

Here, $\omega_0 = 900\ \text{rpm} = (2\pi \times 900)/60\ \text{rad s}^{-1}$

$\omega = 0$ and $t = 60$

Then, Eq. (i) gives $\alpha = -\frac{\pi}{2}\ \text{rad s}^{-2}$

12.



From the diagram,

$$N - mg\cos\theta = \frac{mv^2}{r}$$

where, $\quad r = CO = 4.5 - 0.75 = 3.75\ \text{m}$

$\therefore \quad N = \dfrac{mv^2}{r} + mg\cos\theta$

From the figure, $\theta = 45°$, $m = 70$ kg, $g = 9.8$ m/s^2,

$v' = 6$ m/s, $r = 4.5$ m

Putting these values in above equation, we get

$$N = 1045\,\text{N}$$

13. Here, $\dfrac{mv^2}{r} = T\sin\theta$ and $mg = T\cos\theta$

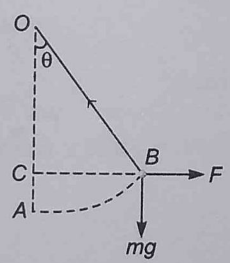
Dividing these two, we get

$$\tan\theta = \frac{v^2}{rg} = \frac{r\omega^2}{g} = \frac{4\pi^2}{gT^2}\,l\sin\theta$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

14. Centrifugal force on the rod, $F = \dfrac{mv^2}{r}$ along BF.

Let θ be the angle, which the rod makes with the vertical forces, acting on the rod, are shown in figure.



Resolving mg and F into two rectangular components, we have

Forces parallel to the rod

$$mg\cos\theta + \frac{mv^2}{r}\sin\theta = T$$

Force perpendicular to the rod

$$mg\sin\theta - \frac{mv^2}{r}\cos\theta$$

The rod would be balanced if

$$mg\sin\theta - \frac{mv^2}{r}\cos\theta = 0$$

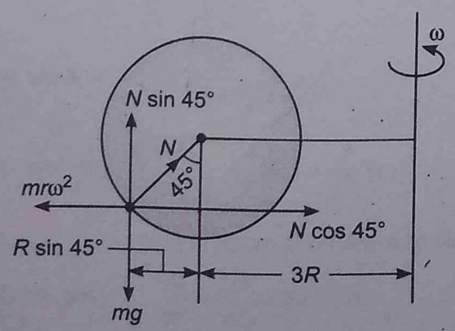
$$mg\sin\theta = \frac{mv^2}{r}\cos\theta$$

This gives $\quad \tan\theta = \dfrac{v^2}{rg} = \dfrac{(10)^2}{10\times 10} = 1 = \tan 45°$

Here $\quad \theta = 45°$

15. Given, $\quad R = 0.2$ m

From the figure $\quad r = 3R + R\sin 45°$



In the frame of rotating spherical chamber

$$N\cos 45° = mr\omega^2$$

$$N\sin 45° = mg$$

$$\Rightarrow \quad \tan 45° = \frac{mg}{mr\omega^2} = \frac{g}{r\omega^2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{g}{3R + \frac{R}{\sqrt{2}}}}$$

$$= 3.64\ \text{rad/s}$$

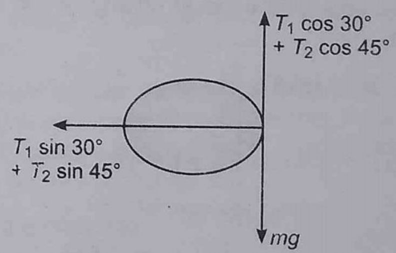
16. The change in velocity, when the particle completes half the revolution is given by

$$\Delta v = 5\text{m/s} - (-5\text{m/s}) = 10\text{m/s}$$

Now, the time taken to complete half the revolution is given by $t = \dfrac{\pi r}{v} = \pi$

So, the average acceleration $= \dfrac{\Delta v}{t} = \dfrac{10}{\pi}$ m/s^2

17.



From the figure,

$$T_1\cos 30° + T_2\cos 45° = mg$$

$$T_1\sin 30° + T_2\sin 45° = \frac{mv^2}{r}$$

$$\Rightarrow \quad T_1 = \frac{mg - \frac{mv^2}{r}}{\frac{(\sqrt{3}-1)}{2}}$$

But $T_1 \geq 0$

$$\frac{mg - \frac{mv^2}{r}}{\frac{\sqrt{3}-1}{2}} \geq 0$$

$$\Rightarrow \quad mg \geq \frac{mv^2}{r}$$

$$\Rightarrow \quad v \leq \sqrt{rg}$$

$$\therefore \quad v_{max} = \sqrt{rg} = \sqrt{1.6\times 9.8} = 3.96\ \text{m/s}$$

18. Speed of the cyclist $(v) = 27$ km/h

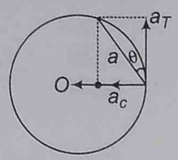
$$= 27 \times \frac{5}{18} \qquad \left(\because 1\,\text{km/h} = \frac{5}{18}\,\text{m/s}\right)$$

$$= \frac{15}{2}\ \text{m/s}$$

Radius of the circular turn $(r) = 80\,m$

$\therefore$ Centripetal acceleration acting on the cyclist

$$a_c = \frac{v^2}{r} = \frac{(15/2)^2}{80} = \frac{225}{4 \times 80}\ m/s^2 = 0.70\,m/s^2$$



Tangential acceleration applied by brakes

$$a_T = 0.5\,m/s^2$$

Centripetal acceleration and tangential acceleration act perpendicular to each other.

$\therefore$ Resultant acceleration $a = \sqrt{a_c^2 + a_T^2}$

$$= \sqrt{(0.7)^2 + (0.5)^2} = \sqrt{0.49 + 0.25} = \sqrt{0.74} = 0.86\,m/s^2$$

If resultant acceleration makes an angle θ with the direction of velocity, then

$$\tan\theta = \frac{a_c}{a_T} = \frac{0.7}{0.5} = 1.4 = \tan 54°28'$$

$$\theta = 54°28'$$

19. In the present case, the tangential component of frictional force is responsible for changing the speed of car while component along the radial direction is providing necessary centripetal force, hence net friction force is neither towards radial or along tangential direction.

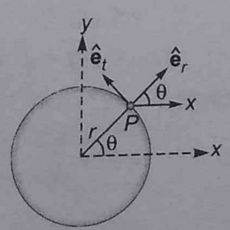
20. As the kinetic energy at the highest point is zero.

21. We know that centripetal and centrifugal forces act at the same time on two different bodies. Thus, they never cancel out.

22. If the roads are not properly banked, the force of friction between tyres and road provides the necessary centripetal force, which causes the wear and tear of tyres.

23. A particle in a circular motion has the shown feature. The velocity of particle in circular motion

$$\mathbf{v} = r\omega\,\hat{\mathbf{e}}_t \quad \dots (i)$$



Thus, we see that velocity of the particle is $r\omega$ along $\hat{\mathbf{e}}_t$ or in tangent direction. So, it changes as the particle rotates the circle. Acceleration of the particle

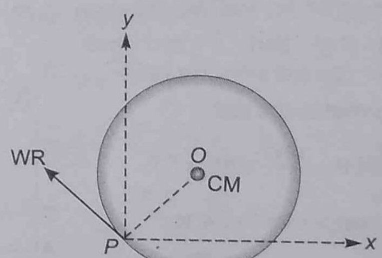
$$\mathbf{a} = -\omega^2 r\,\hat{\mathbf{e}}_r + \frac{dv}{dt}\hat{\mathbf{e}}_t \quad \dots (ii)$$

Thus, acceleration of a particle moving in a circle has two components one along $\hat{\mathbf{e}}_t$ (along tangent) and the other along $\hat{\mathbf{e}}_r$ (or towards centre). Of these the first one is called the tangential $\mathbf{a}_x$ and other is called centripetal $\mathbf{a}_r$.

From Eq. (ii), it is obvious that acceleration depends on angular velocity (ω) of the body.

24-26 Let us solve this situation w.r.t. cabin frame of reference. Acceleration of detached particle w.r.t. cabin in horizontal direction is a (towards right) and is zero in vertical direction. Let angular velocity of disc be ω and velocity of its centre of mass be v, then from pure rolling motion condition, $v - R\omega = 0$ *i.e.*, $v = R\omega$

Initial velocity of the particle is,



$$\mathbf{u}_P = (v - R\omega\cos 37°)\,\hat{\mathbf{i}} + (R\omega\sin 37°)\,\hat{\mathbf{j}} = \frac{R\omega}{5}\hat{\mathbf{i}} + \frac{3R\omega}{5}\hat{\mathbf{j}}$$

For the required situation, position vector of the final location of particle is, $\mathbf{r} = (R\sin 37° + vt)\hat{\mathbf{i}} + (R\cos 37°)\hat{\mathbf{j}}$, where t is the time taken by particle to reach from P to O.

$$R\cos 37° = \frac{3R\omega}{5}t$$

$$\Rightarrow \quad t = \frac{4}{3\omega},$$

and $\quad R\sin 37° + vt = \left(\frac{R\omega}{5}\right)t + \frac{1}{2}at^2$ where, $v = R\omega$

$$\Rightarrow \quad \omega = \sqrt{\frac{8a}{15R}}, \quad \text{(answer question 24)}$$

$$t = \frac{4}{3} \times \sqrt{\frac{15R}{8a}} \quad \text{(answer question 25)}$$

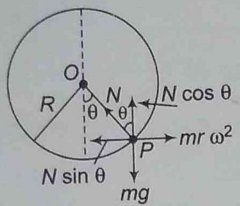
Number of revolutions made in time t is,

$$n = \frac{\omega t}{2\pi} = \frac{2}{3\pi} \quad \text{(answer question 26)}$$

27-28. Different forces acting at point P.

Weight of the bead mg, acting vertically downwards.

Normal reaction of the loop, acting towards the centre. It can be revolved into horizontal component $N \cos\theta$ and vertical component $N \sin\theta$.



In equilibrium, $$mg = N\cos\theta \quad \dots(i)$$

and $$mr\omega^2 = N\sin\theta \quad \dots(ii)$$

But radius of the horizontal circle $r = R\sin\theta$

$$m(R\sin\theta)\omega^2 = N\sin\theta$$

or $$mR\omega^2 = N \quad \dots(iii)$$

Substituting value of N from Eq. (iii) in Eq. (i), we get

$$mg = mR\omega^2\cos\theta$$

or $$\cos\theta = \frac{g}{R\omega^2} \quad \dots(iv)$$

But $\cos\theta \le 1$, therefore the bead will remain at its lower most point for

$$\frac{g}{R\omega^2} \le 1$$

or $$\omega \le \sqrt{\frac{g}{R}}$$ (answer question 27)

When $$\omega = \sqrt{\frac{2g}{R}}$$

From Eq.(iv) $$\cos\theta = \frac{g}{R \times 2g/R} = \frac{1}{2}$$

or $$\theta = 60°$$ (answer question 28)

29. As their period of revolution is same, so, their angular speed of cetripetal acceleration is circular path, $a = \omega^2 r$.

Thus, $$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

30. When a particle moves in a spiral path with decreasing radius.

31. For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and the acceleration vector always points towards the centre of circle or is always along radius of the circle. Since, tangential vector is perpendicular to radial vector, therefore, velocity vector will be perpendicular to the acceleration vector. But in no case acceleration vector is tangent to the circle.

32. When a force of constant magnitude acts on the velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

33. In uniform circular motion, the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remains conserved.

34. Using the relations

$$\frac{mv^2}{r} = \mu R, R = mg$$

$$\Rightarrow \quad \frac{mv^2}{r} = \mu mg \text{ or } v^2 = \mu rg$$

$$\therefore \quad v^2 = 0.6 \times 150 \times 10$$

or $$v = 30\,\text{ms}^{-1}$$

Day 6

Work, Energy and Power

Day 6

Outlines ...

- Work
- Energy
- Work Energy Theorem
- Power
- Collision

Work

Work is said to be done when a force applied on a body displaces the body through a certain distance, in the direction of force.

The work done by the force **F** in displacing the body through a distance **s** is

$$W = (F\cos\theta)s = Fs\cos\theta = \mathbf{F}\cdot\mathbf{s}$$

where, $F\cos\theta$ is the component of the force, acting along the direction of the displacement produced.

SI unit of work is joule (J).

$$1\,\text{J} = 1\,\text{N-m}$$

Work is a scalar quantity. *Work can be of three types*

(i) Positive work (ii) Negative work and (iii) Zero work.

(i) **Positive work** means that force or the component of force is parallel to the displacement. Mathematically, work is said to be positive, if value of the angle θ between the directions of **F** and **s** is either zero or an acute angle.

(ii) **Negative work** means that force or the component of force is opposite (or anti-parallel) to the displacement. Mathematically, work is said to be negative, if value of angle θ between the directions of **F** and **s** is either 180° or an obtuse angle.

(iii) As work done $W = \mathbf{F} \cdot \mathbf{s} = Fs\cos\theta$, hence **work done can be zero**, if
  (a) No force is being applied on the body, *i.e.*, $F = 0$.
  (b) Although the force is being applied on a body but it is unable to cause any displacement in the body, *i.e.*, $F \neq 0$ but $s = 0$.
  (c) Both F and s are finite but the angle θ between the directions of force and displacement is 90°. In such a case

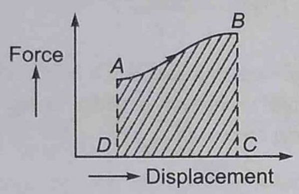
$$W = \mathbf{F} \cdot \mathbf{s} = Fs\cos\theta = Fs\cos 90° = 0$$

Work Done in Different Conditions

- Work done by a variable force is given by

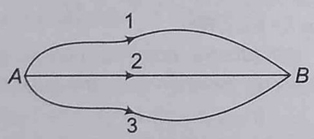
$$W = \int \mathbf{F} \cdot d\mathbf{s}$$

It is equal to the area under the force-displacement graph, along with proper sign.



Work done = Area *ABCDA*

- Work done by constant force and conservative force is path independent *i.e.*, it depends only on initial and final position.



$$W_1 = W_2 = W_3$$

Suppose we consider the work due to force of gravity.

- Work done in displacing any body under the action of a number of forces is equal to the work done by the resultant force.
- In equilibrium (static or dynamic), the resultant force is zero, therefore resultant work done is zero.
- The force (or field) is said to be **conservative**, if the **work done** by the force, also referred to as the line integral of the force, *i.e.*, $\int \mathbf{F} \cdot d\mathbf{l}$ is independent of the path followed between any two points. In such cases, the work done simply depends upon the initial and the final positions. Moreover, work done by a conservative force for a closed path is always zero. Gravitational force, force of gravity, electrostatic force, elastic force are some examples of conservative forces (fields).
- The force is said to be **non-conservative**, if the **work done** by the force depends on the actual path followed by the body for a displacement between any two points. Friction, viscous force, damping force, etc., are some examples of non-conservative forces.
- Work done by the force of gravity on a particle of mass m is given by $W = mgh$, where h is height through particle one displaced. Work done by the couple for an angular displacement θ is given by $W = \tau \cdot \theta$, where τ is the torque of the couple.

Energy

Energy is defined as the capacity or ability of a body to do work. Energy is scalar and its units and dimensions are the same as that of work. Thus, SI unit of energy is J. There are so many types of energy e.g., kinetic, potential, electrostatic, magnetic, geothermal, elastic, solar etc. Some of them are described below.

Some other commonly used units of energy are

$$1 \text{ erg} = 10^{-7} \text{ J},$$
$$1 \text{ cal} = 4.186 \text{ J} \cong 4.2 \text{ J},$$
$$1 \text{ kcal} = 4186 \text{ J}, 1 \text{ kWh} = 3.6 \times 10^6 \text{ J},$$

and 1 electron volt $= 1 \text{ eV} = 1.60 \times 10^{-19}$ J

1. Kinetic Energy

Kinetic Energy (KE) is the capacity of a body to do work by virtue of its motion. Motion may be either translational or rotational.

A body of mass m, moving with a velocity v, has a kinetic energy

$$K = \frac{1}{2}mv^2$$

Thus, $K \propto m$

$K \propto v^2$

Kinetic energy of a body is always positive irrespective of the sign of velocity v. Negative kinetic energy is impossible.

Kinetic energy is correlated with momentum as

$$K = \frac{p^2}{2m} \text{ or } p = \sqrt{2mK}$$

Kinetic energy for a system of particle will be

$$K = \frac{1}{2}\sum_i m_i v_i^2$$

Kinetic energy depends on the frame of reference. Kinetic energy of a passenger sitting in a running train is zero in the frame of reference of the train but is finite in the frame of reference of the earth.

2. Potential Energy

Potential energy is the energy stored in a body or a system by virtue of its position in a field of force or due to its configuration. Potential energy is also called **mutual energy** or **energy of the configuration**.

It is measured by the amount of work that a body can do in passing from its present position or configuration to some standard position or configuration, called **zero position** or **the zero configuration**.

Change in potential energy of a body between any two points is equal to the negative of work done by the conservative force in displacing the body between these two points, without there being any change in kinetic energy. Thus,

$$dU = -dW = -\mathbf{F}\cdot d\mathbf{r}$$

and

$$U_2 - U_1 = -W = -\int_{r_1}^{r_2} \mathbf{F}\cdot d\mathbf{r}$$

Value of the potential energy in a given position can be defined only by assigning some arbitrary value to the reference point. Generally, reference point is taken at infinity and potential energy at infinity is taken as zero. In that case,

$$U = -W = -\int_{\infty}^{r} \mathbf{F}\cdot d\mathbf{r}$$

Potential energy is a scalar quantity but has a sign. It may be positive as well as negative.

Like kinetic energy, the potential energy also depends on the frame of reference.

- *The potential energy of a body, subjected to a conservative force is uncertain, upto a certain limit. This is because the point of zero potential energy is a matter of choice.*
- *Every mechanical force is not associated with a potential energy. The work done by the force of friction over a closed path is not zero because no potential energy cannot be associated with friction.*

Generally, potential energy is of two types

(i) Gravitational Potential Energy

It is the energy associated with the state of separation between two bodies which interact *via* the gravitational force. The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is $U = \frac{-Gm_1m_2}{r}$.

Generally, one of the two bodies is our earth of mass M and radius R. If m is the mass of the other body, situated at a distance $r (r \geq R)$ from the centre of earth, the potential energy of the body

$$U(r) = -\frac{GMm}{r}$$

- *If a body of mass m is raised to a height h from the surface of earth, the change in potential energy of the system (earth+body) comes out to be*

$$\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

or $\Delta U \approx mgh$ if $h << R$

Thus, the potential energy of a body at height h, i.e., mgh is really the change in potential energy of the system for h << R

- *For the gravitational potential energy, the zero of the potential energy is chosen to be the ground.*

(ii) Elastic Potential Energy

Whenever an elastic body (say a spring) is either stretched or compressed, work is being done against the elastic spring force.

The work done is $W = \frac{1}{2}kx^2$

where k is spring constant and x is the displacement. And elastic potential energy

$$U = \frac{1}{2}kx^2$$

Elastic potential energy is always positive.

(iii) Electric Potential Energy

The electric potential energy of two point charges q_1 and q_2 separated by a distance r in vaccum is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

where, $\frac{1}{4\pi\varepsilon_0} = 9.1 \times 10^9 \frac{\text{N}-\text{m}^2}{\text{C}^2}$ = Constant

Work-Energy Theorem

- According to this theorem, work done by all the forces (conservative or non-conservative, external or internal) acting on a particle or an object is equal to the change in kinectic energy of it.

i.e., $W_{net} = W_c + W_{nc} + W_{ext} = \Delta K \cdot E = K_f - K_i$

- If work is being done by the force on the body, then its kinetic energy increases, i.e., $\Delta K = +ve$. On the other hand, if work is being done by the body on the force, then its kinetic energy decreases i.e., $\Delta K = -ve$. The theorem is true for all type of forces.
  (i) The work-energy theorem is not independent of the Newton's second law. It may be viewed as scalar form of second law.
  (ii) When a body moves along a circular path, with uniform speed, there is no change in its kinetic energy. By the work-energy theorem, the work done by the centripetal force is zero.

Law of Conservation of Energy

When both conservative as well as non-conservative forces act on a system, then the mechanical energy of the system, may be transformed into other forms of energy.

In such a scenario, we define a general law of conservation of energy, according to which "Energy can neither be created nor destroyed and total energy of an isolated system should remain conserved." Of course energy may be transformed from one form to another form, such that energy lost in one form exactly reappears in the other form. The law is an universal law and is true always.

We define the change in potential energy of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^f \mathbf{F} \cdot d\mathbf{r}$$

where, W is work done by internal force on the system as system passes from the position i to f.

$$U_f - U_i = -W = -(K_f - K_i)$$

$$\Rightarrow \quad U_i + K_i = U_f + K_f$$

Law of Conservation of Energy (Statement)

Total mechanical energy of a system remains constant if the internal forces are conservative and external forces do not work.

If non-conservative internal forces operate within the system or external forces do work on the system, then apply work-energy theorem.

$$\Rightarrow \quad W_c + W_{nc} + W_{ext} = K_f - K_i \qquad [\because W_c = -(U_f - U_i)]$$

$$\Rightarrow \quad W_{nc} + W_{ext} = E_f - E_i$$

Power

Power is defined as the rate of doing work. If an agent does work W in time t, then its average power is given by

$$P_{av} = \frac{W}{t}$$

The shorter is the time taken by a person or a machine in performing a particular task, the larger is the power of that person or machine.

Power is a scalar quantity and its SI unit is watt, where

$$1 W = 1 J/s$$

Instantaneous power,

$$\mathbf{P}_{inst} = \frac{dW}{dt} = \frac{\mathbf{F} \cdot d\mathbf{s}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

Some other commonly used units of power are

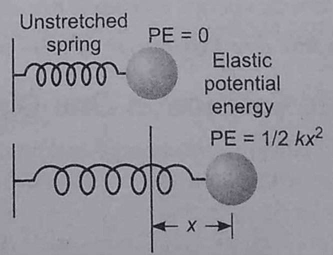
$$1 kW = 10^3 W, 1 MW = 10^6 W,$$

$$1 HP = 746 W$$

The power of a system is defined as the rate of change of kinetic energy per unit time. Mathematically, Power $P = \frac{d}{dt}(KE) = \frac{dK}{dt}$

Potential Energy of a Spring

Elastic potential energy is potential energy stored as a result of deformation of an elastic object such as stretching of a spring. It is equal to work done to stretch the spring, which depends upon the spring constant k as well as the distance stretched. From Hooke's law, the force required to stretch the spring will be directly proportional to the amount of stretch.



$$F = -kx \qquad \text{(Force by spring)}$$

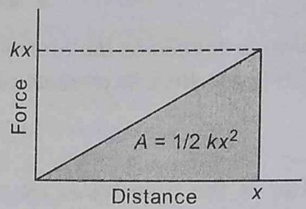
Then, the work done to stretch the spring by a distance x is

$$W = PE = \frac{1}{2}kx^2 \qquad \text{(work done by stretcher)}$$

Since, the change in potential energy of an object between two positions is equal to the work that must be done to move the object from one point to the other, the calculation of potential energy is equivalent to calculating work. Since, the force required to stretch a spring changes with distance, the calculation of the work involves an integral.

$$W = \int_0^x kx\,dx = k\,\frac{x^2}{2}$$

Work can also be visualized as the area under the force curve. *Here object is displaced slowly.*

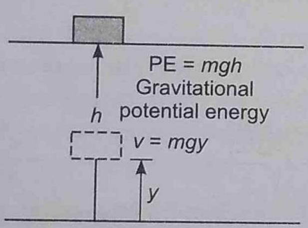


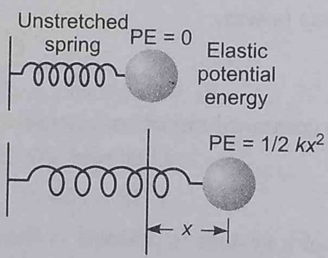
If the potential energy function U is known, the force at any point can be obtained by taking the derivative of the potential.

$$F_y = -\frac{dU}{dy} = \frac{-d}{dy}(mgy)$$

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right)$$

$$F_y = -mg,\ F_x = -kx$$



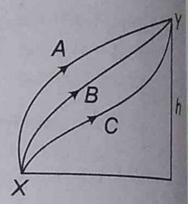


Conservative Force & Non-conservative Force

- **Conservative force** may be defined as one for which work done in moving between two points X and Y is independent of the path taken between the two points

  Now, consider the given example in which we are going from X to Y through different paths A, B, C then work against gravity, will be always mgh.

  All paths between A and B require the same amount of work, if the force is a conservative force.



- **Non-conservative force** can only arise in classical physics due to neglected degrees of freedom. Examples are friction and non-elastic material stress. *e.g.*,
- If a body (say an automobile) is moving on a surface, then a force of kinetic friction ($f_k = \mu_k N$) starts acting on the body.
- To maintain the motion, a force $|\mathbf{F}| = f_k = \mu_k N$ is to be continuously applied on the body.

  Hence, Work done $W = Fs = \mu_k Ns$

  where, s is the displacement in the direction of the force applied against friction.

Collision

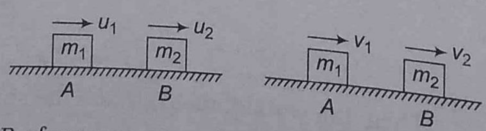
It is defined as an isolated event in which two or more colliding bodies exert relatively strong forces on each other for a relatively short time, as a result of which the kinetic energy and momentum of the colliding particles, change.

Total linear momentum and total energy of the colliding bodies remain conserved in all sorts of collisions. However, the total kinetic energy of the colliding bodies may or may not remain conserved before and after the collision. From the view point of kinetic energy, collisions are classified as (a) elastic collisions, and (b) inelastic collisions.

The collision is in fact a redistribution of total momentum of the particles.

1. Perfectly Elastic Collision in One Dimension

In a perfectly elastic collision, total energy and total linear momentum of colliding particles remain conserved. Moreover, the forces involved in interaction are conservative in nature and the total kinetic energy before and after the collision, remains unchanged.



In above figure two bodies A and B of masses m_1 and m_2 and having initial velocities u_1 and u_2 in one dimension, collide elastically and after collision move with velocities v_1 and v_2, then we find that

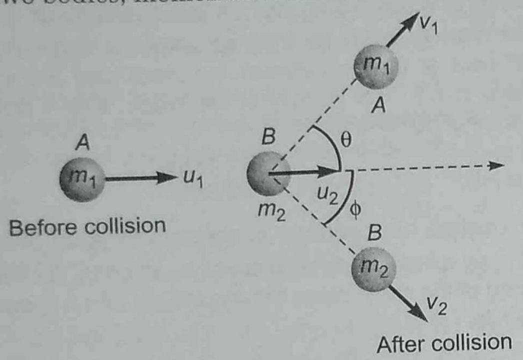
(i) Relative velocity of approach = Relative velocity of separation, *i.e.*, $u_1 - u_2 = v_2 - v_1$

(ii) $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

2. Perfectly Elastic Collision in a Plane

In a two dimensional (or oblique) collision between two bodies, momentum remains conserved,



∴ Along the x-axis

$$m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi \quad \text{... (i)}$$

and along the y-axis

$$0 = m_1v_1\sin\theta - m_2v_2\sin\phi \quad \text{... (ii)}$$

As the total kinetic energy remains unchanged,

Hence, $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

We can solve these equations provided that either the value of θ or ϕ is known to us.

3. Inelastic Collision in One Dimension

In an inelastic collision, the total linear momentum as well as total energy remain conserved but total kinetic energy after collision is not equal to kinetic energy before collision.

Rebounding of a Ball on Collision with the Floor

1. Speed of the ball after the nth rebound
$$v_n = e^n v_0 = e^n\sqrt{2gh_0}$$
2. Height covered by the ball after the nth rebound
$$h_n = e^{2n}h_0$$
3. Total distance (vertical) covered by the ball before it stops bouncing
$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + \ldots = h_0\left(\frac{1+e^2}{1-e^2}\right)$$
4. Total time taken by the ball before it stops bouncing
$$T = t_0 + t_1 + t_2 + t_3 + \ldots$$
$$= \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \ldots = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$$

Coefficient of Restitution (e)

For a collision, it is defined as the ratio of relative velocity of separation to the relative velocity of approach.

Thus, Coefficient of restitution $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

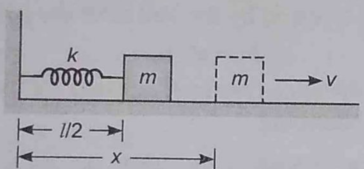
For a perfectly elastic collision $e = 1$.

If $0 < e < 1$, the collision is said to be partially elastic. But if, $e = 0$, the collision is said to be perfectly inelastic.

- In a perfectly inelastic collision, $e = 0$ which means that $v_2 - v_1 = 0$ or $v_2 = v_1$
- It can be shown that for an inelastic collision the final velocities of the colliding bodies are given by
$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{(1+e)m_2}{(m_1 + m_2)}u_2$$
and $$v_2 = \frac{(1+e)m_1}{(m_1 + m_2)}u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$
- If a particle of mass m, moving with velocity u, hits an identical stationary target inelastically, then final velocities of projectile and target are correlated as
i.e., $m_1 = m_2 = m$ and $u_2 = 0$; $\dfrac{v_1}{v_2} = \dfrac{1-e}{1+e}$

Practice Zone DAY 6

1. A block of mass m is pushed against a spring of spring constant k, fixed to one end of the wall. The block can slide on a frictionless table. The natural length of the spring is l and it is compressed to half its natural length when the block is released. The velocity of the block as a function of its distance x from the wall is



(a) $\sqrt{\frac{k}{m}\left(\frac{l^2}{4}-(l-x)^2\right)}$ (b) $\sqrt{\frac{k}{m}}(l-x)^2$

(c) $\sqrt{\frac{k}{m}\left(\frac{l^2}{4}+(l+2x)^2\right)}$ (d) $\sqrt{\frac{m}{k}}(l+x)^2$

2. A cyclist rides up a hill with a constant velocity. Determine the power developed by the cyclist if the length of the connecting rod of the pedal is $r = 25$cm, the time of revolution of the rod is $t = 2$ s and the mean force exerted by his foot on the pedal is $F = 15$ kgf.

(a) 115.6 W (b) 215.6 W

(c) 15.6 W (d) 11.56 W

3. An open water tight railway wagon of mass 5×10^3 kg coasts with an initial velocity of 1.2 ms^{-1} on a railway track without friction. Rain falls vertically downwards on the wagon.

What change occurs in the kinetic energy of the wagon, after it has collected 10^3 kg of water?

(a) 900 J (b) 300 J

(c) 600 J (d) 1200 J

4. A spring gun having a spring of spring constant k is placed at a height h. A ball of mass m is placed in its barrel and compressed by a distance x. Where shall we place a box on the ground so that the ball lands in the box?

(a) $\frac{kh}{mg}x$ (b) $\sqrt{\frac{2kh}{mg}}x$

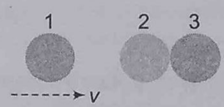
(c) $\sqrt{\frac{kh}{2mg}}x$ (d) $\frac{kh}{2mg}$

5. A block of mass 0.5 kg has an initial velocity of 10 ms^{-1} while moving down an inclined plane of angle 30°, the coefficient of friction between the block and the inclined surface is 0.2. The velocity of the block, after it covers a distance of 10 m, is

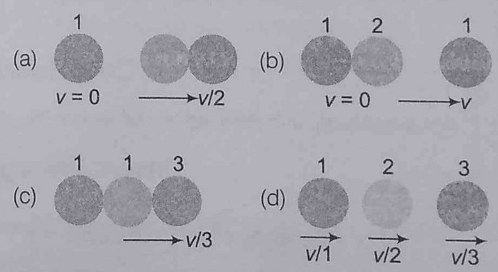
(a) 17 ms^{-1} (b) 13 ms^{-1}

(c) 24 ms^{-1} (d) 8 ms^{-1}

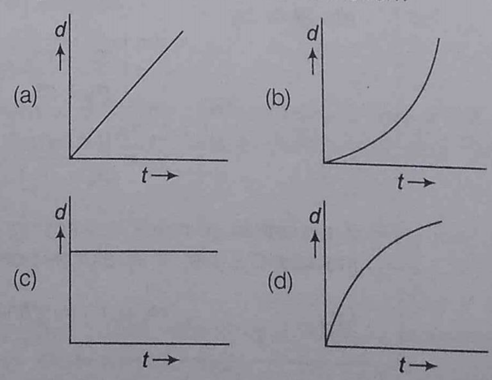
6. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in



If the collision is elastic, which of the following is a possible result after collision?



7. A body is moving unidirectionallly under the influence of a source of constant power supplying energy. Which of the diagrams shown in figure correctly shows the displacement-time curve for its motion?



8. A 1.5 kg block is initially at rest on a horizontal frictionless surface, when a horizontal force in the positive direction of x-axis, is applied to the block. The force is given by $\mathbf{F} = (4 - x^2)\,\hat{\mathbf{i}}$ N, where x is in metre and the initial position of the block is at $x = 0$. The maximum kinetic energy of the block between $x = 0$ and $x = 2.0$ m is

(a) 6.67 J (b) 5.33 J (c) 8.67 J (d) 2.44 J

9. A particle is moving in a region where the potential U is given by $U = k(x^2 + y^2 + z^2)$. The force acting on the particle is

(a) $k(\hat{i}x + \hat{j}y + \hat{k}z)$ (b) $-k(\hat{i}x + \hat{j}y + \hat{k}z)$
(c) $-2k(\hat{i}x + \hat{j}y + \hat{k}z)$ (d) zero

10. The particle is released from a height h. At a certain height, its kinetic energy is two times its potential energy. Height and speed of the particle at that instant are

(a) $\frac{h}{3}, \sqrt{\frac{2gh}{3}}$ (b) $\frac{h}{3}, 2\sqrt{\frac{gh}{3}}$
(c) $\frac{2h}{3}, \sqrt{\frac{2gh}{3}}$ (d) $\frac{h}{3}, \sqrt{2gh}$

11. A uniform chain of length l and weight W is hanging from its ends A and B which are close together. At a given instant end B is released. The tension at A when B has fallen a distance $x < \frac{l}{2}$ is

(a) $\frac{W}{2}\left[\frac{3x}{l} - 2\right]$ (b) $\frac{W}{2}\left[3 - \frac{3x}{4}\right]$
(c) $\frac{W}{2} \cdot \left[1 + \frac{3x}{l}\right]$ (d) $\frac{W}{2}\left[\frac{3x}{l} + 4\right]$

12. Two balls of masses m_1 and m_2 are separated from each other and a charge is placed between them. The whole system is at rest on the ground. Suddenly, the charge explodes and the masses are pushed apart. The mass m_1 travels a distance S_1 and then it stops. If the coefficient of friction between the balls and the ground are the same, mass m_2 stops after covering the distance

(a) $S_2 = \frac{m_1}{m_2} S_1$ (b) $S_2 = \frac{m_2}{m_1} S_1$
(c) $S_2 = \frac{m_1^2}{m_2^2} S_1$ (d) $S_2 = \frac{m_2^2}{m_1^2} S_1$

13. A shell is fired from a cannon with a velocity $v\,\text{ms}^{-1}$ at an angle θ with the horizontal direction. At the highest point in its path, it explodes into 2 pieces of equal masses. One of the pieces retraces its path to the cannon. The speed in ms^{-1} of the other piece, immediately after the explosion is

(a) $3v\cos\theta$ (b) $2v\cos\theta$ (c) $v\cos\theta$ (d) $\sqrt{\frac{3}{2}}\,v\cos\theta$

14. A ball of mass m and density ρ is immersed in a liquid of density 3ρ at a depth h and is released. To what height will the ball jump up above the surface of liquid? (Neglect the resistance of water and air)

(a) h (b) $\frac{h}{2}$ (c) $3h$ (d) $2h$

15. Power supplied to a particle of mass 2 kg varies with time as $P = 3t^2/2$ watt, where t is in second. If velocity of the particle at $t = 0$ is $v = 0$, the velocity of the particle at $t = 2$ s, will be

(a) $1\,\text{ms}^{-1}$ (b) $4\,\text{ms}^{-1}$ (c) $2\,\text{ms}^{-1}$ (d) $2\sqrt{2}\,\text{ms}^{-1}$

16. A pendulum consists of a wooden bob of mass (m) and length (l). A bullet of mass (m_1) is fired towards the pendulum with a speed v_1. The bullet emerges out of the bob with a speed $v_1/3$, and the bob just completes the vertical circle. The value of v_1 is

(a) $\left(\frac{m}{m_1}\right)\sqrt{5gl}$ (b) $\frac{m}{m_1}\sqrt{\frac{g}{l}}$
(c) $\frac{2}{3}\left(\frac{m_1}{m}\right)\sqrt{5gl}$ (d) $\frac{3}{2}\left(\frac{m}{m_1}\right)\sqrt{5gl}$

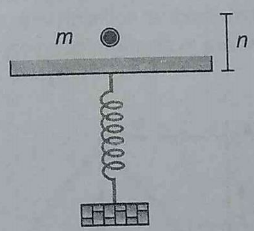
17. A force $\mathbf{F} = -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and is then taken parallel to the y-axis to the point (a, a). The total work done by the force is

(a) $-2ka^2$ (b) $2ka^2$ (c) $-ka^2$ (d) ka^2

18. A body of mass 500 g is taken up an inclined plane of length 10 m and height 5m and then released to slide down to the bottom. The coefficient of friction between the body and the plane is 0.1. What is the amount of work done in the round trip?

(a) 5 J (b) 15 J
(c) $5\sqrt{3}$ J (d) $\frac{5}{\sqrt{3}}$ J

19. A ball of mass m is dropped from a height h on a massless platform fixed at the top of a vertical spring as shown below. The platform is depressed by a distance x. What will be the value of the spring constant?

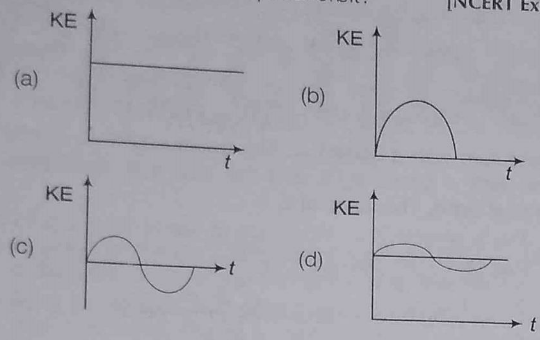


(a) $2mg/x$ (b) $2mgh/x$
(c) $2mg(h + x)/x^2$ (d) $2mg(h) + 2mghx/x^2$

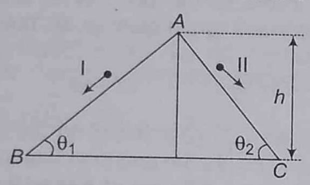
20. A loaded trolley and an unloaded trolley are both moving with the same kinetic energy. The mass of the loaded trolley is three times that of the unloaded trolley. Brakes are applied to both of them so as to exert an equal retarding force. If t_1 and t_2 be the time taken by the unloaded and loaded trolleys respectively, before coming to a stop, then

(a) $t_1 = \sqrt{3}t_2$ (b) $t_1 = 3t_2$
(c) $t_2 = 3t_1$ (d) $t_1 = \frac{2}{3}t_2$

21. Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit? **[NCERT Exemplar]**



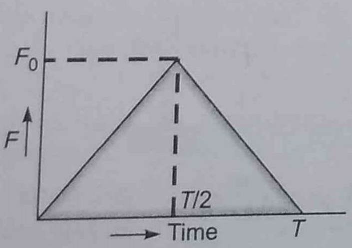
22. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.



Which of the following statement is correct ?

(a) Both the stones reach the bottom at the same time but not with the same speed.
(b) Both the stones reach the bottom with the same spped and stone I reaches the bottom earlier than stone II.
(c) Both the stones reach the bottom with the same speed and and stone II reaches the bottom earlier than stone I.
(d) Both the stones reach the bottom at different times and with different speeds.

23. A particle of mass m moving with a velocity u makes an elastic one dimensional collision with a stationary particle of mass m. They are in contact for a very short interval of time T.



The force of interaction increases from zero to F_0 linearly in a time interval $\frac{T}{2}$ and decreases linearly to zero in further time interval $\frac{T}{2}$. The magnitude of F_0 is

(a) $\frac{mu}{T}$
(b) $\frac{2mu}{T}$
(c) $\frac{mu}{2T}$
(d) None of these

Directions *(Q. Nos. 24 to 31) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

24. Statement I Two bodies of different masses have the same momentum, and their kinetic energies are in the inverse ratio of their masses.

Statement II Kinetic energy of body is given by the relation.

$$KE = \frac{1}{2}mv^2$$

25. Statement I When a machine-gun fires n bullets per second with kinetic energy K, then the power of the machine-gun is $P = nK$.

Statement II Power = $\frac{\text{Work}}{\text{Time}} = \frac{nK}{1}$

26. Statement I A body cannot have energy without having momentum but it can have momentum without having energy.

Statement II Momentum and energy have different dimensions.

27. Statement I Kinetic energy is conserved in an elastic collision.

Statement II Kinetic energy is conserved in an inelastic collision.

28. Statement I An object is displaced from point $A(2\,\text{m}, 3\,\text{m}, 4\,\text{m})$ to a point B $(1\,\text{m}, 2\,\text{m}, 3\,\text{m})$ under a constant force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$N. The work done by the force in this process is –9 J.

Statement II Work done by a force, an object can be given by the relation,

$$W = \int_{r_1}^{r_2} \mathbf{F} \cdot \mathbf{dr} \quad \text{or} \quad W = \mathbf{F} \cdot \mathbf{s}$$

29. Statement I A quick collision between two bodies is more violent than a slow collision; even when the initial and the final velocities are identical.

Statement II The momentum is greater in the first case.

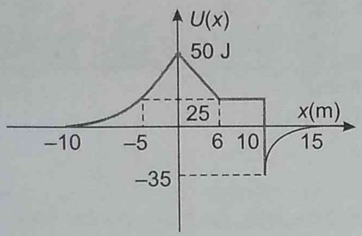
30. Statement I A man carrying a load on his head and walking in a street, does not do any work against the force of gravity.

Statement II When a body moves with an uniform velocity, no work is done.

31. **Statement I** According to the law of conservation of mechanical energy, change in potential energy is equal and opposite to the change in kinetic energy.

Statement II Mechanical energy is not conserved.

Directions (Q. Nos. 32 to 34) *The figure shows the variation of potential energy of a particle as a function of x, the x-coordinate of the region. It has been assumed that potential energy depends only on x. For all other values of x, U is zero, i.e., for $x < -10$ and $x > 15$, $U = 0$.*



32. If the total mechanical energy of the particle is 25 J, then it can be found in the region

(a) $-10 < x < -5$ and $6 < x < 15$
(b) $-10 < x < 0$ and $6 < x < 10$
(c) $-5 < x < 6$
(d) $-10 < x < 10$

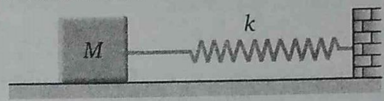
33. If the total mechanical energy of the particle is – 40 J, then it can be found in the region

(a) $x < -10$ and $x > 15$
(b) $-10 < x < -5$ and $6 < x < 15$
(c) $10 < x < 15$
(d) It is not possible.

34. If the particle is isolated and its total mechanical energy is 60 J, then

(a) the particle can be found anywhere from $-\infty < x < \infty$
(b) the particle's maximum kinetic energy is 95 J
(c) the particle's kinetic energy is not zero anywhere on the x-axis.
(d) All of the above

Directions (Q. Nos. 35 to 37) *The block shown in the figure is acted upon by a spring having a spring constant k and a weak frictional force of constant magnitude f. The block is pulled a distance x_0 from the equilibrium and is released. It then oscillates many times and finally comes to rest. Friction here, causes the damping of the oscillations.*



35. Mark the correct statement.

(a) The decrease in amplitude is same for each cycle of oscillation and is equal to $\frac{3f}{k}$
(b) The decrease in amplitude is same for each cycle of oscillation and is equal to $\frac{4f}{k}$
(c) The decrease in amplitude is different for each cycle of oscillation
(d) The decrease in amplitude is different for each cycle of oscillation and for every subsequent cycle it decreases by a factor of 2

36. The number of cycles (n), the mass oscillates, before coming to rest is given by

(a) $\frac{kx_0}{f} = 1 + 2^n$
(b) $kx_0 = f(1 + e^{-2n})$
(c) $\frac{kx_0}{f} = 4n + 1$
(d) $\frac{kx_0}{f} - 1 = n$

37. The total distance travelled by the block from the instant of time of its release to the instant of time when it finally comes to rest, is

(a) $\left(\frac{kx_0}{f} - 1\right)\left(x_0 - \frac{f}{2k}\right) - \frac{2f}{k}\left[\frac{1}{4}\left(\frac{kx_0}{f} - 1\right)^2 - 1\right]$

(b) $\frac{1}{4}\left(\frac{kx_0}{f} - 1\right)\left(x_0 - \frac{f}{2k}\right) - \frac{2f}{k}\left[\frac{1}{64}\left(\frac{kx_0}{f} - 1\right)^2 - 1\right]$

(c) $\frac{kx_0}{f}\left(x_0 - \frac{f}{2k}\right) - \frac{2f}{k}\left[\frac{1}{4}\left(\frac{kx_0}{f}\right)^2 - 1\right]$

(d) $\frac{1}{4}\frac{kx_0}{f}\left(x_0 - \frac{f}{2k}\right) - \frac{2f}{k}\left[\frac{1}{64}\left(\frac{kx_0}{f}\right)^2 - 1\right]$

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38. A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself.

The centre of gravity rises by 0.5 m before he leaps. After the leap the centre of gravity rises by another 1 m. The maximum power delivered by the muscles is (Take $g = 10\ \text{ms}^{-2}$). **[JEE Main Online 2013]**

(a) 6.26×10^3 W at the start
(b) 6.26×10^3 W at take off
(c) 6.26×10^4 W at the start
(d) 6.26×10^4 W at take off

39. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement I If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement II $k_1 < k_2$ **[AIEEE 2012]**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement I
(d) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I

40. At time $t = 0$ s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to **[AIEEE 2011]**
(a) $\sqrt{t}$
(b) constant
(c) t
(d) $\frac{1}{\sqrt{t}}$

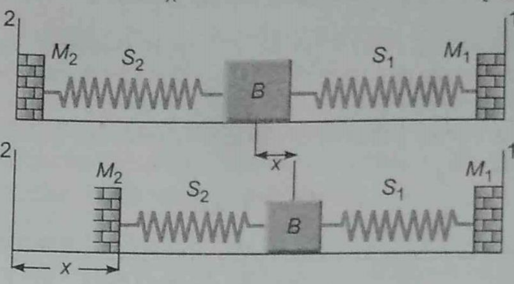
41. Statement I Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
Statement II Principle of conservation of momentum holds true for all kinds of collisions. **[AIEEE 2010]**
(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is false

42. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is **[AIEEE 2010]**
(a) $\frac{b^2}{2a}$
(b) $\frac{b^2}{12a}$
(c) $\frac{b^2}{4a}$
(d) $\frac{b^2}{6a}$

43. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[AIEEE 2008]**
(a) 200 J – 500 J
(b) 2×10^5 J – 3×10^5 J
(c) 20000 J – 50000 J
(d) 2000 J – 5000 J

44. A block of mass 0.50 kg is moving with a speed of 2.00 m/s on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[AIEEE 2008]**
(a) 0.16 J
(b) 1.00 J
(c) 0.67 J
(d) 0.34 J

45. A block B is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively. The other ends are attached to two supports M_1 and M_2, which are not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards the wall 1 by a small distance x and is then released. The block returns and covers a maximum distance y, towards the wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is **[AIEEE 2008]**



(a) 4
(b) 2
(c) $\frac{1}{2}$
(d) $\frac{1}{4}$

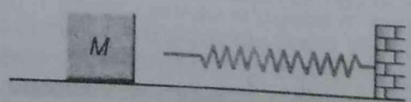
46. A 2 kg block slides on a horizontal floor with a speed of 4ms^{-1}. It strikes an uncompressed spring, and compresses it till the block comes to rest. The kinetic frictional force is 15 N and spring constant is 10000 Nm^{-1}. The spring gets compressed by **[AIEEE 2007]**
(a) 5.5 cm
(b) 2.5 cm
(c) 11.0 cm
(d) 8.5 cm

47. The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$ J
The total mechanical energy of the particle is 2 J. Then, the maximum speed (in ms^{-1}) is **[AIEEE 2006]**
(a) $\frac{3}{\sqrt{2}}$
(b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$
(d) 2

48. A bullet fired into a fixed target losses half of its velocity after penetrating distance of 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to its motion? **[AIEEE 2005]**
(a) 3.0 cm
(b) 2.0 cm
(c) 1.5 cm
(d) 1.0 cm

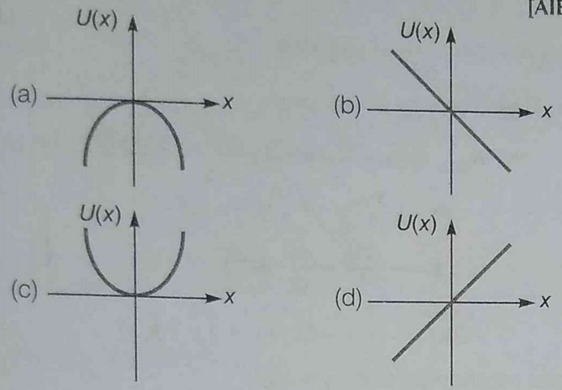
49. A body of mass m is accelerated uniformly from rest to a speed v in a time interval T. The instantaneous power delivered to the body as a function of time, is given by **[AIEEE 2005, 04]**
(a) $\frac{mv^2}{T^2}t$
(b) $\frac{mv^2}{T^2}t^2$
(c) $\frac{1}{2}\frac{mv^2}{T^2}t$
(d) $\frac{1}{2}\frac{mv^2}{T^2}t^2$

50. A block of mass M moving on a frictionless horizontal surface, collides with a spring of spring constant k and compresses it by length L. The maximum momentum of the block, after collision is **[AIEEE 2005]**



(a) $L\sqrt{Mk}$
(b) $\frac{kL^2}{2M}$
(c) zero
(d) $\frac{ML^2}{k}$

51. A particle is placed at the origin and a force $F = kx$ acts on it (where k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ *versus* x will be (where U is the potential energy function) **[AIEEE 2004]**



52. A particle moves in a straight line with retardation proportional to its displacement. The loss in kinetic energy of the particle, for any displacement x, is proportional to **[AIEEE 2004]**

(a) x^2 (b) e^x
(c) x (d) $\log_e x$

53. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? **[AIEEE 2004]**

(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J

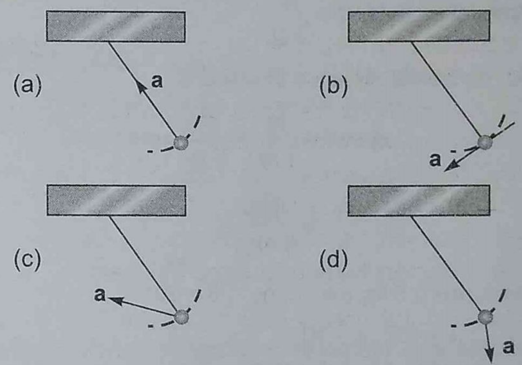
54. A force $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ N is applied over a particle which displaces it from its origin to the points $\mathbf{r} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}})$ m. The work done on the particle in Joules is **[AIEEE 2004]**

(a) −7 (b) +7
(c) +10 (d) +13

55. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to **[AIEEE 2003]**

(a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$

56. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector **a** is correctly shown in **[AIEEE 2002]**



Answers

1. (a)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (b)
11. (c)	**12.** (c)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (a)
21. (d)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (a)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (a)	**30.** (b)
31. (c)	**32.** (a)	**33.** (d)	**34.** (d)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (d)
41. (a)	**42.** (c)	**43.** (d)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (d)	**49.** (a)	**50.** (a)
51. (a)	**52.** (a)	**53.** (b)	**54.** (b)	**55.** (b)	**56.** (c)				

Hints & Solutions

1. Let at any instant block is at a place x, then applying the law of conservation of energy

$$\frac{1}{2}k\left(\frac{l}{2}\right)^2 = \frac{1}{2}k(l-x)^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k}{m}\left(\frac{l^2}{4} - (l-x)^2\right)}$$

2. $v = r\omega = r\dfrac{2\pi}{t} = \dfrac{1}{4} \times \dfrac{2\pi}{2} = \dfrac{\pi}{4}\,\text{m s}^{-1}$

$$P = F \times v$$

$$= (15 \times 9.8) \times \frac{\pi}{4}$$

$$= 115.6\ \text{W}$$

3. If v' is the final velocity of the wagon, then applying the principle of conservation of linear momentum, we get

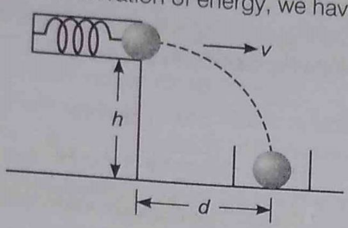
$$5 \times 10^3 \times 1.2 = (5 \times 10^3 + 10^3) \times v'$$

$$v' = 1\,\text{ms}^{-1}$$

$$\text{Change in KE} = \frac{1}{2}(6 \times 10^3) \times 1^2 - \frac{1}{2}(5 \times 10^3)(1.2)^2$$

$= -\,600$ J (minus sign for the loss in kinetic energy)

4. From law of conservation of energy, we have



$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k}{m}}\, x$$

Time taken to fall, $t = \sqrt{\frac{2h}{g}}$

So, horizontal distance travelled is

$$d = vt = \sqrt{\frac{k}{m}}\, x \sqrt{\frac{2h}{g}}\, x$$

$$= \sqrt{\frac{2kh}{mg}}\, x$$

5. Here, m = 0.5 kg, u = 10 ms^{-1}, $\theta = 30°$

$\mu = 0.2$, s = 10 m

Acceleration down the plane,

$$a = g(\sin\theta - \mu\cos\theta)$$

$$= 10\,(\sin 30° - 0.2\cos 30°) = 3.268\,\text{ms}^{-2}$$

From, $v^2 = u^2 + 2as$

$$= 10^2 + 2\,(3.268) \times 10 = 165.36$$

$$v = \sqrt{165.36} \approx 13\ \text{ms}^{-1}$$

6. As the ball bearings are identical, their masses are equal. In elastic collision, their velocities are interchanged. In collision between 1 and 2; velocity of 1 becomes zero and velocity of 2 becomes v. In collision between 2 and 3, velocity of 2 becomes zero and velocity of 3 becomes v.

7. Here, $P = [\text{ML}^2\text{T}^{-3}] = \text{constant}$

As mass M of body is fixed

$$[\text{L}^2\text{T}^{-3}] = \text{constant}$$

$$\Rightarrow \quad \frac{[\text{L}^2]}{[\text{T}^3]} = \text{constant} \Rightarrow [\text{L}] \propto [\text{T}^{3/2}]$$

$\Rightarrow$ displacement $\propto t^{3/2}$

8. From the work-energy theorem, kinetic energy of the block at a distance x is

$$K = \int_0^x F\,dx = \int_0^x (4 - x^2)\,dx = 4x - \frac{x^3}{3}$$

For kinetic energy to be maximum, $\frac{dK}{dx} = 0$

$$\frac{d}{dx}\left(4x - \frac{x^3}{3}\right) = 0$$

$$4 - x^2 = 0 \ \text{ or } \ x = \pm 2\text{m}$$

At $x = +2m$, $\frac{d^2K}{dx^2}$ = negative

i.e., kinetic energy is maximum.

$$K_{max} = 4x - \frac{x^3}{3} = 4\,(2) - \frac{2^3}{3} = 5.33\ \text{J}$$

9.
$$U = k\,(x^2 + y^2 + z^2) = kr^2$$

$$F = -\frac{dU}{dr} = \frac{-d}{dr}(kr^2)$$

$$-2k\mathbf{r} = -kr = -2k\,(\hat{\mathbf{i}}\,x + \hat{\mathbf{j}}\,y + \hat{\mathbf{k}}\,z)$$

10. Total mechanical energy = mgh

As, $\frac{\text{KE}}{\text{PE}} = \frac{2}{1}$

$$\text{KE} = \frac{2}{3}mgh \ \text{ and } \ \text{PE} = \frac{1}{3}mgh$$

Height from the ground at this instant, $h' = \frac{h}{3}$, and speed of the particle at this instant,

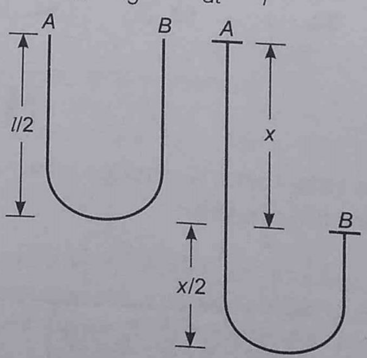
$$v = \sqrt{2g(h - h')} = \sqrt{2g\left(\frac{2h}{3}\right)} = 2\sqrt{\frac{gh}{3}}$$

11. Mass per unit length $\lambda = \frac{m}{l} = \frac{W}{lg}$

Velocity $v^2 = 2g\left(\frac{x}{2}\right)$ or $v = \sqrt{gx}$

Change in momentum when an element dx falls is $\frac{W}{lg}\sqrt{gx}\,dx$

$$F = \frac{dp}{dt} = \frac{W}{lg}\sqrt{gx}\,\frac{dx}{dt} = \frac{W}{l}x \qquad \left(\because \frac{dx}{dt} = \sqrt{gx}\right)$$



Tension at A = Weight of half the chain + Weight of $\frac{x}{2}$ length + F

$$= \frac{W}{2} + \frac{Wx}{2l} + \frac{Wx}{l}$$

$$= \frac{W}{2}\left[1 + \frac{3x}{l}\right]$$

12. From the conservation of momentum, we get

$$m_1v_1 = m_2v_2 \quad \text{...(i)}$$

also, $\frac{1}{2}m_1v_1^2 = f_1S_1 = \mu m_1 g S_1$ ...(ii)

and $\frac{1}{2}m_2v_2^2 = f_2S_2 = \mu_2 g S_2$ ...(iii)

where, μ = coefficient of friction

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{m_1v_1^2}{m_2v_2^2} = \frac{m_1S_1}{m_2S_2} \Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1} \quad \text{[from Eq. (i)]}$$

Thus, we get $\frac{v_1}{v_2} = \frac{m_1S_1}{m_2S_2}$ or $\frac{m_2}{m_1} = \frac{m_1S_1}{m_2S_2} \Rightarrow S_2 = \frac{m_1^2}{m_2^2}S_1$

13. Velocity at the highest point $= v\cos\theta$

Applying the principle of conservation of linear momentum, we get $2m(v\cos\theta) = m(-v\cos\theta) + mv'$

$$v' = 3v\cos\theta$$

14. Volume of ball $= m/\rho$,

Acceleration of the ball inside the liquid,

$$a = \frac{F_{net}}{m} = \frac{\text{upthrust} - \text{weight}}{m}$$

$$a = \frac{\left(\frac{m}{\rho}\right)(3\rho)g - mg}{m} = 2g, \text{ upwards}$$

Velocity of the ball on reaching the surface,

$$v = \sqrt{2ah} = \sqrt{4gh}$$

The ball will jump to a height, $H = \frac{v^2}{2g} = \frac{4gh}{2g} = 2h$

15. From the work-energy theorem, $\Delta KE = W_{net}$

$$K_f - K_i = \int P\,dt$$

$$\frac{1}{2}mv^2 - 0 = \int_0^2 \left(\frac{3}{2}t^2\right)dt$$

$$\frac{1}{2}(2)v^2 = \frac{3}{2}\left[\frac{t^3}{3}\right]_0^2 = 4$$

$$v = 2\text{ ms}^{-1}$$

16. $v = \sqrt{5gl}$ ...(i)

Using the principle of conservation of linear momentum,

$$mv = m_1(v_1 - v_1/3) = m_1\left(\frac{2}{3}v_1\right)$$

$$v = \frac{2}{3}\frac{m_1}{m}v_1 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{2}{3}\frac{m_1}{m}v_1 = \sqrt{5gl} \Rightarrow v_1 = \frac{3}{2}\frac{m}{m_1}\sqrt{5gl}$$

17. $W_1 = \int_0^a \mathbf{F}\,d\mathbf{x} = \int_0^a -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\hat{\mathbf{i}}dx = \int_0^a -k(0\hat{\mathbf{i}} + x\hat{\mathbf{j}})\hat{\mathbf{i}}dx = \text{zero}$

$$W_2 = \int_0^a \mathbf{F}\cdot d\mathbf{y} = \int_0^a -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\hat{\mathbf{j}}dy$$

$$= \int_0^a -k(a\hat{\mathbf{i}} + a\hat{\mathbf{j}})\hat{\mathbf{j}}dy = -ka\int_0^a dy = -ka^2$$

Total work done, $W = W_1 + W_2 = 0 - ka^2 = -ka^2$

18. Work done in the round trip = total work done against friction while moving up and down the plane $= 2(\mu\, mg\cos\theta) \times s$

$$= 2\left(0.1 \times 0.5 \times 10 \times \frac{\sqrt{3}}{2} \times 10\right) = 5\sqrt{3}\text{ J}$$

19. Here, $mg(h + x) = \frac{1}{2}kx^2 \Rightarrow kx^2 = 2mg(h + x)$

$$\Rightarrow \quad k = \frac{2mg(h + x)}{x^2}$$

20. Let mass of the unloaded trolley $= M$

Then, $\frac{\text{Time taken by the unloaded trolley } (t_1)}{\text{Time taken by the loaded trolley } (t_2)} = \sqrt{\frac{\text{mass of loaded trolley}}{\text{mass of unloaded trolley}}}$

$$\Rightarrow \quad \frac{t_1}{t_2} = \sqrt{\frac{3M}{M}} = \sqrt{3} \Rightarrow t_1 = \sqrt{3}t_2$$

21. As the earth moves ones around the sun in its elliptical orbit, its KE, is maxium when it is closest to the sun and minimum when it is farthest from the sun. As KE of earth is never zero during its motion choice (d) is correct.

22. As both surfaces I and II are frictionless and two stones slide from rest from the same height, therefore, both the stones reach the bottom with same speed $\left(\frac{1}{2}mv^2 = mgh\right)$. As accelaration down plane II is larger ($a_2 = g\sin\theta_2$ is greater than $a_1 = g\sin\theta_1$), therefore, stone II reaches the bottom earlier than stone I.

23. The collision will cause an exchange of velocities. The change in momentum of any particle $= mu$, which is equal to the impulse = area under the force-time graph

$$mu = \frac{1}{2}F_0 \times T \Rightarrow F_0 = 2\frac{mu}{T}$$

24. According to the principle of conservation of momentum

$$m_1v_1 = m_2v_2 \Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1}$$

Again, $\frac{KE_1}{KE_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{m_1}{m_2} \times \left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$

$$\therefore \quad KE \propto \frac{1}{m}$$

25. $\text{Power} = \frac{\text{work}}{\text{time}} = \frac{n \times K}{1} = nK$

26. We know that a body may not have momentum but may have potential energy by virtue of its position as in case of a stretched or a compressed spring. But when the body does not contain energy then its kinetic energy is zero hence, its momentum is also zero.

Dimensions of momentum $(mv) = [MLT^{-1}]$

Dimensions of energy $\left(\frac{1}{2}mv^2\right) = [ML^2T^{-2}]$

They are not the same.

27. In an elastic collision, the kinetic energy and momentum are conserved. In an inelastic collision, the momentum is conserved but the kinetic energy is not conserved.

28. $W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r}$

$$\int_{(2m, 3m, 4m)}^{(1m, 2m, 3m)} (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}})$$

$$= [2x + 3y + 4z]_{(2m, 3m, 4m)}^{(1m, 2m, 3m)} = -9\text{ J}$$

Alternate

Since, $\mathbf{F}$ = constant, we can also use

$W = \mathbf{F} \cdot \mathbf{s}$

Here, $\mathbf{s} = \mathbf{r}_f - \mathbf{r}_i = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$= (-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

$\therefore \quad W = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})(-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) = -2 - 3 - 4 = -9\text{ J}$

29. Momentum $p = mv$ or $p \propto v$

i.e., momentum is directly proportional to the velocity, so the momentum is greater in a quicker collision between two bodies than in a slower one. Hence, due to greater momentum, quicker collision between two bodies will be more violent even if the initial and the final velocities are identical.

30. If a man walking on a road carries a load on his head, then no work is done against gravity.

When a body moves in an uniform velocity, then its acceleration will be zero. Hence, no work will be done.

31. According to the conservation of mechanical energy, as for the conservative forces, the sum of kinetic energy and potential energy remains constant and throughout the motion, it is independent of time. This is the law of conservation of mechanical energy.

i.e., KE + PE = TE = constant.

32. Kinetic energy can never be negative. Total mechanical energy $= K + U$

Here, if $E = 25$ J, then $K = 25 - U$

For K to be non-negative, $U < 25$ J, which is the case for $-\infty < x < -5$ and $6 < x < \infty$.

33. Concept used is the same as above.

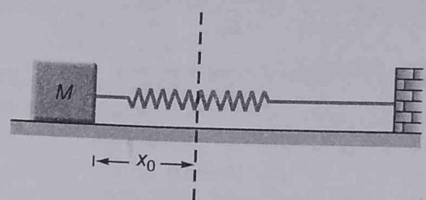
If $E = -40$ J, $K = -40 - U$

It means $U < -40$ for K to be positive but from given variations. It is clear that the minimum value of U is -35 J, so this is not possible.

34. Here, $E = 60$ J and as the system is isolated, the total mechanical energy remains conserved. It is clear from the given variation that U varies from -35 J to 50 J.

So, $K = E - U$ will be from 10 J to 95 J.

35. Let x_1, x_2 represent the right extreme and the left extreme positions of the block after the completion of the Ist half and the Ist complete oscillations, respectively. These distances are measured from the natural positions of the spring.



Similarly, x_3, x_4 represent the same for the IInd half and the IInd complete oscillations and after completion of the n^{th} cycle of oscillation, the position is described as x_{2n}.

From the work-energy theorem, $\frac{k\,x_0^2}{2} - \frac{k\,x_1^2}{2} = f(x_0 + x_1)$

[For the left and the right extreme positions of the Ist half and of the Ist cycle]

$$\Rightarrow \quad x_0 - x_1 = \frac{2f}{k}$$

Now, applying the work-energy theorem for the right and the left extreme positions of the IInd half of the Ist cycle, we get

$$\frac{k\,x_1^2}{2} - \frac{k\,x_2^2}{2} = f(x_1 + x_2) \Rightarrow x_1 - x_2 = 2f/k$$

Similarly, for other subsequent cycles, we get

$$x_2 - x_3 = 2f/k$$
$$x_3 - x_4 = 2f/k$$
$$\vdots$$
$$x_{2n-1} - x_{2n} = 2f/k$$

where, x_{2n} is the elongation in the spring from its natural length, after n cycles.

Decrease in amplitude after the Ist cycle is,

$$\Delta A_2 = x_0 - x_2 = 4f/k$$

Decrease in amplitude after the IInd cycle is,

$$\Delta A_2 = x_2 - x_4 = 4f/k$$

and this continues, so the decrease in amplitude is same after each cycle and is equal to $4f/k$.

36. At static equilibrium, $k\,x = f \Rightarrow \quad x = f/k$

Let the block stops after n cycles, then $x_{2n} = x$

Now, from the equations above, $x_0 - x_{2n} = 2n \times \frac{2f}{k}$

$$\Rightarrow \quad x_{2n} = x_0 - \frac{4fn}{k} = \frac{f}{k} \Rightarrow n = \frac{1}{4}\left[\frac{kx_0}{f} - 1\right]$$

37. Distance travelled before coming to rest,

$$s=(x_0+2x_1+x_2)+(x_2+2x_3+x_4)+\dots+(x_{2n-2}+2x_{2n-1}+x_{2n})$$

$$=x_0+2x_1+2x_2+2x_3+\dots+2x_{2n-2}+2x_{2n-1}+x_{2n}$$

$$=x_0+2[x_1+x_2+x_3+\dots+x_{2n-1}]+x_{2n}$$

$$=x_0+2[x_0\times(2n-1)]-\frac{2\times 2f}{k}(4n^2-1)+x_0-\frac{4nf}{k}$$

$$=\left[\frac{kx_0}{f}-1\right]\left[x_0-\frac{f}{2k}\right]-\frac{2f}{k}\left[\frac{1}{4}\left(\frac{kx_0}{f}-1\right)^2-1\right]$$

38. As, $P=F\cdot v=\frac{dP}{dt}=F\cdot\frac{dv}{dt}$

To deliver the maximum power $\frac{dP}{dt}=0$, which gives

$P_{max}=6.26\times 10^3$ W

39. As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about statement I. But as in statement II, $k_1<k_2$

Then, for same force

$$W=Fx=F\frac{F}{k}=\frac{F^2}{k}$$

$$\Rightarrow \quad W\propto\frac{1}{k}\quad i.e., W_1>W_2$$

But for same displacement.

$$W=F\,x=\frac{1}{2}kxx=\frac{1}{2}kx^2$$

$$\Rightarrow \quad W\propto k, i.e., W_1<W_2$$

Thus, in the light of statement II, statement I is false.

40. Given $\frac{dk}{dt}=$ constant

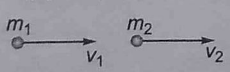
$$\Rightarrow \quad k\propto t \quad \Rightarrow \quad v\propto\sqrt{t}$$

Also, $p=Fv=\frac{dk}{dt}=$ constant

$$\Rightarrow \quad F\propto\frac{1}{v} \quad \Rightarrow \quad F\propto\frac{1}{\sqrt{t}}$$

41. If it is a completely inelastic collision then

$$m_1v_1+m_2v_2=m_1v+m_2v$$



$$v=\frac{m_1v_1+m_2v_2}{m_1+m_2}$$

Kinetic energy $=\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}$

As $\mathbf{p_1}$ and $\mathbf{p_2}$ both simultaneously cannot be zero therefore, total kinetic energy cannot be lost.

42. $U(x)=\frac{a}{x^{12}}-\frac{b}{x^6}$

$$U(x=\infty)=0$$

As, $\quad F=-\frac{dU}{dx}=-\left[\frac{12a}{x^{13}}+\frac{6b}{x^7}\right]$

At equilibrium, $F=0$

$$\therefore \quad x^6=\frac{2a}{b}$$

$$\therefore \quad U_{\text{at equilibrium}}=\frac{a}{\left(\frac{2a}{b}\right)^2}-\frac{b}{\left(\frac{2a}{b}\right)}=\frac{-b^2}{4a}$$

$$\therefore \quad D=[U(x=\infty)-U_{\text{at equilibrium}}]=\frac{b^2}{4a}$$

43. Approximate = 60 kg

Approximate velocity = 10 ms^{-1}

Approximate kinetic energy $=\frac{1}{2}\times 60\times 100=3000$ J

kinetic energy range = 2000 to 5000 J

44. $m_1u_1+m_2u_2=(m_1+m_2)v \quad \Rightarrow \quad v=2/3\ ms^{-1}$

Energy loss $=\frac{1}{2}(0.5)\times(2)^2-\frac{1}{2}(1.5)\times\left(\frac{2}{3}\right)^2=0.67$ J

45. From energy conservation,

$$\frac{1}{2}kx^2=\frac{1}{2}(4k)y^2$$

$$\therefore \quad \frac{y}{x}=\frac{1}{2}$$

46. $a=\frac{F_k}{m}=\frac{15}{2}=7.5ms^{-2}$

Now, $\quad ma=\frac{1}{2}kx^2$

$$\Rightarrow \quad 2\times 7.5=\frac{1}{2}\times 10000\times x^2$$

or $\quad x^2=3\times 10^{-3}$

or $\quad x=0.055$ m $=5.5$ cm

47. $V(x)=\left(\frac{x^4}{4}-\frac{x^2}{2}\right)$

For minimum value of V, $\frac{dV}{dx}=0\Rightarrow\frac{4x^3}{4}-\frac{2x}{2}=0$

$$\Rightarrow \quad x=0, x=\pm 1$$

So, $\quad V_{min}(x=\pm 1)=\frac{1}{4}-\frac{1}{2}=\frac{-1}{4}$ J

Now, $K_{max}+V_{min}=$ total mechanical energy

$$\Rightarrow \quad K_{max}=\left(\frac{1}{4}\right)+2 \text{ or } K_{max}=\frac{9}{4}$$

or $\quad \frac{mv^2}{2}=\frac{9}{4}$ or $v=\frac{3}{\sqrt{2}}ms^{-1}$

48. According to the work-energy theorem,

$$W = \Delta K$$

Case I $-F \times 3 = \frac{1}{2} m \left(\frac{v_0}{2}\right)^2 - \frac{1}{2} m v_0^2$

where F is the resistive force and v_0 is the initial speed.

Case II Let, the further distance travelled by the bullet before coming to rest is s.

$\therefore \quad -F(3+s) = K_f - K_i = -\frac{1}{2} m v_0^2$

$\Rightarrow \quad -\frac{1}{8} m v_0^2 (3+s) = -\frac{1}{2} m v_0^2$

or $\quad \frac{1}{4}(3+s) = 1$ or $\frac{3}{4} + \frac{s}{4} = 1$

or $\quad s = 1\,\text{cm}$

49. $F = ma = \frac{mv}{T} \qquad \left(\because a = \frac{v-0}{T}\right)$

Instantaneous power $= Fv = mav = \frac{mv}{T} at = \frac{mv}{T}\frac{v}{T} t = \frac{mv^2}{T^2} t$

50. According to the conservation of energy,

$$\frac{1}{2} kL^2 = \frac{1}{2} Mv^2 \Rightarrow kL^2 = \frac{(Mv)^2}{M}$$

or $\quad MkL^2 = p^2 \qquad (\because p = Mv)$

$\Rightarrow \quad p = L\sqrt{Mk}$

51. From $F = -\frac{dU}{dx}$

$$\int_0^{U(x)} dU = -\int_0^x F\,dx = -\int_0^x (kx)\,dx$$

$\therefore \quad U(x) = -\frac{kx^2}{2}$ as $U(0) = 0$

52. $a = -kx$ or $\frac{vdv}{dx} = -kx \Rightarrow v\,dv = -kx\,dx$

Let, the velocity change from v_0 to v.

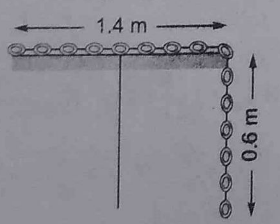
$\Rightarrow \quad \int_{v_0}^{v} dv = -\int_0^x k\,x\,dx$

$\Rightarrow \quad \frac{v^2 - v_0^2}{2} = -\frac{k\,x^2}{2}$

$\Rightarrow \quad m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mk\,x^2}{2}$

$\Rightarrow \quad \Delta K \propto x^2 \qquad [\Delta K$ is loss in kinetic energy$]$

53. Mass per unit length $= \frac{M}{L} = \frac{4}{2} = 2\,\text{kg m}^{-1}$



The mass of 0.6 m of chain $= 0.6 \times 2 = 1.2$ kg

The height of the centre of mass of the hanging part

$$h = \frac{0.6 + 0}{2} = 0.3\ \text{m}$$

Hence, work done in pulling the chain on the table = work done against gravity the force of gravity

i.e., $\quad W = mgh = 12 \times 10 \times 0.3 = 3.6$ J

54. Work done in displacing the particle

$$w = \mathbf{F} \cdot \mathbf{r} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (2\hat{2} - \hat{\mathbf{j}})$$

$$= 5 \times 2 + 3(-1) + 2 \times 0 = 10 - 3 = 7\ \text{J}$$

55. $\quad P =$ constant

$\Rightarrow \quad Fv = P \qquad [\because P = \text{force} \times \text{velocity}]$

$\Rightarrow \quad Ma \times v = P \Rightarrow va = \frac{P}{M}$

$\Rightarrow \quad v \times \left[\frac{v\,dv}{ds}\right] = \frac{P}{M} \qquad \left[\because a = \frac{v\,dv}{ds}\right]$

$\Rightarrow \quad \int_0^v v^2 dv = \int_0^s \frac{P}{M} ds$

[Assuming at $t = 0$ it starts from rest, *i.e.*, from $s = 0$]

or $\quad \frac{v^3}{3} = \frac{P}{M} s \Rightarrow v = \left(\frac{3P}{M}\right)^{1/3} s^{1/3}$

$\Rightarrow \quad \frac{ds}{dt} = ks^{1/3} \qquad \left[k = \left(\frac{3P}{M}\right)^{1/3}\right]$

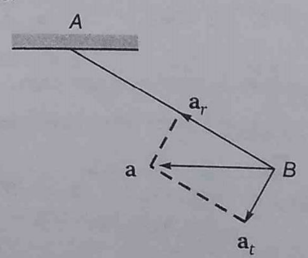
$\Rightarrow \quad \int_0^s \frac{ds}{s^{1/3}} = \int_0^t k\,dt$

or $\quad \frac{s^{2/3}}{2/3} = kt \Rightarrow s^{2/3} = \frac{2}{3} kt$

or $\quad s = \left(\frac{2}{3} k\right)^{3/2} \times t^{3/2}$

or $\quad s \propto t^{3/2}$

56. Net acceleration **a** of the bob in position B has two components.



(a) $\mathbf{a}_r$ = radial acceleration (towards BA)

(b) $\mathbf{a}_t$ = tangential acceleration (perpendicular to BA)

Therefore, direction of **a** is correctly shown in option (c).

Day 7

Rotational Motion

Day 7

Outlines ...

- Centre of Mass
- Rigid Bodies
- Concepts of Rotational Motion
- Torque
- Angular Momentum

Centre of Mass

*The **centre of mass** of a body is a point where the whole mass of the body is supposed to be concentrated for describing its translatory motion. In a uniform gravitational field, the centre of mass and the centre of gravity of a system are coincident.*

If all the external forces acting on the body /system of bodies, were to be applied at the centre of mass is zero, then state of rest/motion of the body/system of bodies, shall remain unaffected. Centre of mass of a two particles system consisting of two particles of masses m_1, m_2 and respective position vectors $\mathbf{r}_1, \mathbf{r}_2$ is given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

If $m_1 = m_2 = m$ (say), then $\mathbf{r}_{CM} = \dfrac{\mathbf{r}_1 + \mathbf{r}_2}{2}$

Thus, the centre of mass of two equal masses lies exactly at the centre of the line joining the two masses.

(x_1, y_1) and (x_2, y_2) are the coordinates of the location of the two particles, the coordinates of their centre of mass are given by $x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$ and $y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$

Centre of mass of n-particles system which consists n-particles of masses $m_1, m_2, \ldots, m_n$ with $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ as their position vectors at a given instant of time. The $\mathbf{r}_{CM}$ of the system at that instant is given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \ldots + m_n\mathbf{r}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_i\mathbf{r}_i}{M}$$

Further
$$\left.\begin{aligned} x_{CM} &= \frac{\sum_{i=1}^{n} m_ix_i}{M} \\ y_{CM} &= \frac{\sum_{i=1}^{n} m_iy_i}{M} \\ z_{CM} &= \frac{\sum_{i=1}^{n} m_iz_i}{M} \end{aligned}\right\} \quad \ldots\text{(i)}$$

In case of a body with a continuous mass distribution, we can replace the summations in above Eq. (i) by the following integrals

$$\Sigma\, mx \rightarrow \int x\,dm; \quad \Sigma\, my \rightarrow \int y\,dm; \quad \Sigma\, mz \rightarrow \int z\,dm$$

Then, the coordinates of the centre of mass of a body of mass M will be

$$x_{CM} = \frac{1}{M}\int x\,dm$$
$$y_{CM} = \frac{1}{M}\int y\,dm$$
$$z_{CM} = \frac{1}{M}\int z\,dm$$

The equivalent vector representation for the centre of mass will be
$$\mathbf{r}_{CM} = \frac{1}{M}\int \mathbf{r}\,dm$$

- *Velocity of centre of mass of n-particles system is given by*
$$\mathbf{v}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{v}_i}{M}$$
- *Similarly, acceleration of centre of mass is given by*

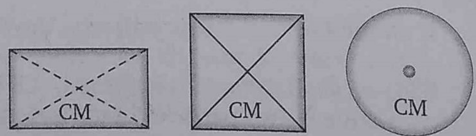
or $$\mathbf{a}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{a}_i}{M}$$
- *In accordance with Newton's second law of motion*
$$\mathbf{F}_{CM} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n$$
Or $$\mathbf{F}_{CM} = \sum_{i=1}^{n} \mathbf{F}_i$$
- *This is called as equation of motion of centre of mass.*
- *In pure translation, every portcle of the body moves with the same velocity at any instant of time.*

Rigid Bodies

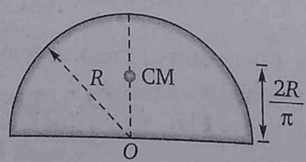
A rigid body is defined as that body which does not undergo any change in shape (or) volume when external forces are applied on it. When a force is applied on a rigid body, the distance between any two particles of the body will remain unchanged, however, larger the forces may be.

Centre of Mass of Some Rigid Bodies

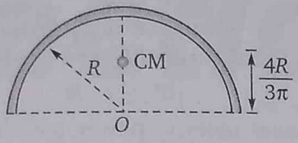
1. Centre of mass of a uniform rectangular, square or circular plate lies at its centre.



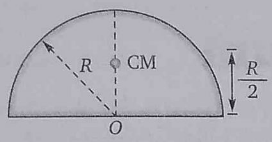
2. Centre of mass of a uniform semicircular ring lies at a distance of $h = \frac{2R}{\pi}$ from its centre, on the axis of symmetry where R is the radius of the ring.



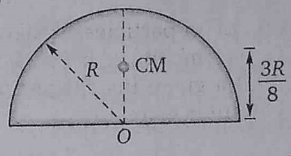
3. Centre of mass of a uniform semicircular disc of radius R lies at a distance of $h = \frac{4R}{3\pi}$ from the centre on the axis of symmetry as shown in figure.



4. Centre of mass of a hemispherical shell of radius R lies at a distance of $h = \frac{R}{2}$ from its centre on the axis of symmetry as shown in figure.



5. Centre of mass of a solid hemisphere of radius R lies at a distance of $h = \frac{3R}{8}$ from its centre on the axis of symmetry.



If some mass or area is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formula

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 - m_2\mathbf{r}_2}{m_1 - m_2} \text{ or } \mathbf{r}_{CM} = \frac{A_1\mathbf{r}_1 - A_2\mathbf{r}_2}{A_1 - A_2}$$

In rotation about a fixed axis, every particle of the rigid body moves in a circle which is in a plane perpendicular to the axis and has its centre on the axis.

Concept of Rotational Motion

In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.

(i) Rotational motion is characterised by angular displacement $\mathbf{d\theta}$ and angular velocity $\omega = \frac{\mathbf{d\theta}}{dt}$.

(ii) If angular velocity is not uniform, then rate of change of angular velocity is called the angular acceleration.

Thus, angular acceleration $\alpha = \frac{\mathbf{d\omega}}{dt}$.

SI unit of angular acceleration is rad/s^2.

(iii) Angular acceleration α and linear tangential acceleration $\mathbf{a}_t$ are correlated as $\mathbf{a}_t = \alpha \times \mathbf{r}$

If angular acceleration α is uniform, then equations of rotational motion may be written as

(i) $\omega = \omega_0 + \alpha t$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 - \omega_0^2 = 2\alpha\theta$

(iv) $\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n - 1)$

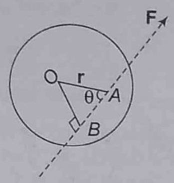
Torque

Torque (or moment of a force) is the turning effect of a force applied at a point on a rigid body about the axis of rotation.

Mathematically, torque $\tau = \mathbf{r} \times \mathbf{F} = |\mathbf{r} \times \mathbf{F}|\hat{\mathbf{n}} = rF\sin\theta\,\hat{\mathbf{n}}$

where $\hat{\mathbf{n}}$ is a unit vector along the axis of rotation. Torque is an axial vector and its SI unit is newton-metre (N-m).

(i) The torque about axis of rotation is independent of choice of origin O so long as it is chosen on the axis of rotation AB.



(ii) Only normal component of force contributes towards the torque. Radial component of force does not contribute towards the torque.

(iii) A torque produces angular acceleration in a rotating body.

Torque $\tau = I\alpha$

(iv) Moment of a couple (or torque) is given by product of position vector $\mathbf{r}$ between the two forces and either force $\mathbf{F}$. Thus,

$$\tau = \mathbf{r} \times \mathbf{F}$$

(v) If under the influence of an external torque τ the given body rotates by $\mathbf{d}\theta$, then work done

$$dW = \tau \cdot d\theta$$

(vi) In rotational motion, power may be defined as the scalar product of torque and angular velocity, *i.e.*, power $P = \tau \cdot \omega$

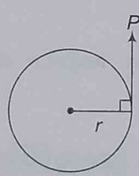
Angular Momentum

The moment of linear momentum of a given body about an axis of rotation is called as its angular momentum. If $\mathbf{p}$ *be the linear momentum of a particle and* $\mathbf{r}$ *is its position vector from the point of rotation, then*

Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p} = r\,p \sin\theta\,\hat{\mathbf{n}}$

where, $\hat{\mathbf{n}}$ is a unit vector in the direction of rotation. Angular momentum is an axial vector and its SI unit is kg-m^2s^{-1} or J-s.

(i) For rotational motion of a rigid body angular momentum is equal to the product of angular velocity and moment of inertia of the body about the axis of rotation. Mathematically, $\mathbf{L} = I\omega$



(ii) According to the second law of rotational motion, the rate of change of angular momentum of a body is equal to the external torque applied on it and takes place in the direction of torque. Thus,

$$\tau = \frac{d\mathbf{L}}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\alpha$$

(iii) Total effect of a torque applied on a rotating body in a given time is called angular impulse. Angular impulse is equal to total change in angular momentum of the system in given time. Thus, angular impulse

$$\mathbf{J} = \int_0^{\Delta t} \tau\, dt = \Delta\mathbf{L} = \mathbf{L}_f - \mathbf{L}_i$$

(iv) The angular momentum of a system of particles about the origin is

$$L = \sum^{n} r_i \times p_i$$

Law of Conservation of Angular Momentum

According to the law of conservation of angular momentum, if no external torque is acting on a system, then total vector sum of angular momentum of different particles of the system remains constant.

We know that $\frac{d\mathbf{L}}{dt} = \tau_{ext}$

Hence, if $\tau_{ext} = 0$, then $\frac{d\mathbf{L}}{dt} = 0 \Rightarrow L = \text{constant}$

therefore, in the absence of any external torque, total angular momentum of a system must remain conserved.

Conservation of angular momentum in earth-moon system results in the transfer of angular momentum from earth to moon. This in turn results in slowing down the rotation rate of earth and in gradual increase of the radius of moon's orbit.

Practice Zone DAY 7

1. Angular momentum is
(a) moment of momentum
(b) product of mass and angular velocity
(c) product of moment of inertia and velocity
(d) moment in angular motion

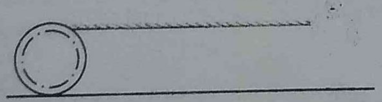
2. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed
(a) zero (b) v_0 (c) $\sqrt{2}v_0$ (d) $2v_0$

3. Friction coefficient is $\left(\frac{1}{7}\right) g \sin\theta$ and the sphere is released from rest on the incline
(a) it will stay at rest
(b) it will make pure translation motion
(c) it will translate and rotate about the centre
(d) the angular momentum of the sphere about its centre will remain constant

4. The motor of an engine is rotating about its axis with an angular velocity of 100 rev/m. It comes to rest in 15 s, after being switched off. Assuming constant angular deceleration. What are the numbers of revolutions made by it before coming to rest?
(a) 12.5 (b) 40 (c) 32.6 (d) 15.6

5. If the radius of earth contracts $\frac{1}{n}$ of its present day value length of the day will be approximately
(a) $\frac{24}{n}$h (b) $\frac{24}{n^2}$h (c) $24\,n$h (d) $24\,n^2$h

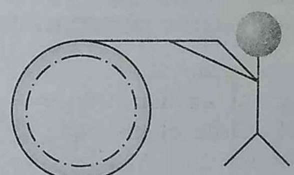
6. A string of negligible thickness is wrapped several times around a cylinder kept on a rough horizontal surface. A man standing at a distance λ_m from the cylinder holds one end of the string and pulls the cylinder towards him. There is no slipping anywhere. The length in (m) of the string passed through the hand of the man while the cylinder reached his hands is



(a) 1 (b) 2 (c) 3 (d) 4

7. A particle of mass 5 g is moving with a uniform speed of $3\sqrt{2}$ cm s^{-1} is in the *xy*-plane along the line $y = 2\sqrt{5}$ cm. The magnitude of its angular momentum about the origin in g-cm^2 s^{-1} is
(a) zero (b) 30 (c) $30\sqrt{2}$ (d) $30\sqrt{10}$

8. A string of negligible thickness is wrapped several times around a cylinder kept on a rough horizontal surface. A boy standing at a distance l from the cylinder holds one end of the string and pulls the cylinder toward him. Assuming no slipping the length of the thread passed through the hands of the man is



(a) $\frac{l}{2}$ (b) l (c) $2l$ (d) $\frac{3l}{2}$

9. A uniform disc of radius a and mass m, is rotating freely with angular speed ω in a horizontal plane, about a smooth fixed vertical axis through its centre. A particle, also of mass m, is suddenly attached to the rim of the disc and rotates with it. The new angular speed is
(a) $\frac{\omega}{6}$ (b) $\frac{\omega}{3}$ (c) $\frac{\omega}{2}$ (d) $\frac{\omega}{5}$

10. The door of an almirah is 6ft high, 1.5 ft wide and weigh 8 kg. The door is supported by two hinges situated at a distance of 1ft from the ends. Assuming forces exerted on the hinges are equal then the magnitude of force is
(a) 15 N (b) 10 N (c) 28 N (d) 43 N

11. When a body is projected at an angle with the horizontal in the uniform gravitational field of the earth, the angular momentum of the body about the point of projection, as it proceeds along the path
(a) remains constant
(b) increases
(c) decreases
(d) initially decreases and after its highest point increases

12. Two boys of masses 10 kg and 8 kg are moving along a vertical rope, the former climbing up with acceleration of 2 m/s^2 while later coming down with uniform velocity of 2 m/s. Then, tension in rope at fixed support will be ($g = 10\ m/s^2$)

(a) 200 N (b) 120 N
(c) 180 N (d) 160 N

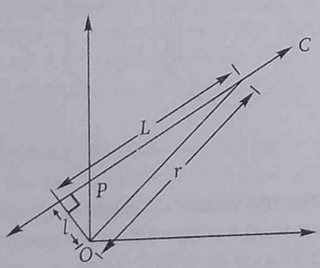
13. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad/s. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis? **[NCERT Exemplar]**

(a) 3200 J, 62.5 J-s (b) 3125 J, 62.5 J-s
(c) 3500 J, 68 J-s (d) 3400 J, 63.5 J-s

14. A uniform rod *AB* of mass *m* and length *l* at rest on a smooth horizontal surface. An impulse *P* is applied to the end *B*. The time taken by the rod to turn through at right angle is

(a) $2\pi\frac{ml}{P}$ (b) $2\frac{\pi P}{ml}$
(c) $\frac{\pi}{12}\frac{ml}{P}$ (d) $\frac{\pi P}{ml}$

15. A particle of mass *m* moves along line *PC* with velocity *v* as shown. What is the angular momentum of the particle about *O*?



(a) *mvL* (b) *mvl* (c) *mvr* (d) zero

16. A ring and a disc having the same mass, roll without slipping with the same linear velocity. If the kinetic energy of the ring is 8 J, that of the disc must be

(a) 2 J (b) 4 J (c) 6 J (d) 16 J

Directions *(Q. Nos. 17 to 20) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given here*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

17. Statement I Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Statement II By the principle of conservation of energy, the total kinetic energies of both the cylinders are equal when they reach the bottom of the incline.

18. When a disc rotates with uniform angular velocity, which of the following is not true? **[NCERT Exemplar]**

(a) The sense of rotation remains same.
(b) The orientation of the axis of rotation remains same.
(c) The speed of rotation is non-zero and remains same.
(d) The angular acceleration is non-zero and remains same.

19. Assertion The centre of mass of a body will change with the change in shape and size of the body.

Reason $r = \dfrac{\sum_{i=1}^{i=n} m_i i_i}{\sum_{i=1}^{i=n} m_i}$

20. Statement I Total torque on a system is independent of the origin if the total external force is zero.

Statement II Torque due to couples is independent of the origin.

AIEEE & JEE Main Archive

21. A ring of mass *M* and radius *R* is rotating about its axis with angular velocity ω. Two identical bodies each of mass *m* are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be **[JEE Main Online 2013]**

(a) $\frac{m(M+2m)}{M}\omega^2R^2$ (b) $\frac{Mm}{(M+2m)}\omega^2R^2$
(c) $\frac{Mm}{(M-2m)}\omega^2R^2$ (d) $\frac{(M+m)M}{(M+2m)}\omega^2R^2$

22. A particle of mass 2 kg is moving such that at time *t*, its position, in metre, is given by $\mathbf{r}(t) = 5\hat{\mathbf{i}} - 2t^2\hat{\mathbf{j}}$. The angular momentum of the particle at $t = 2$ s about the origin in kg $m^{-2}\ s^{-1}$ is **[JEE Main Online 2013]**

(a) $-80\hat{\mathbf{k}}$
(b) $(10\hat{\mathbf{i}} - 16\hat{\mathbf{j}})$
(c) $-40\hat{\mathbf{k}}$
(d) $40\hat{\mathbf{k}}$

23. A hoop of radius r and mass m rotaing with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE Main 2013]**

(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$
(c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

24. A wheel has angular acceleration of 3.0 rad/s^2 and an initial angular speed of 2 rad/s^2. In a time of 2 s, it has rotated through an angle (in radian) of **[AIEEE 2008]**

(a) 6 (b) 10
(c) 12 (d) 4

25. Angular momentum of the particle rotating with a central force is constant due to **[AIEEE 2007]**

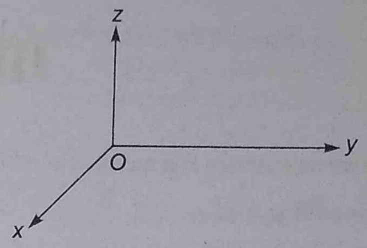
(a) constant force
(b) constant linear momentum
(c) zero torque
(d) constant torque

26. Consider a two particles system with particles having masses m_1 and m_2.

If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position? **[AIEEE 2006]**

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

27. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, – 1) is **[AIEEE 2006]**

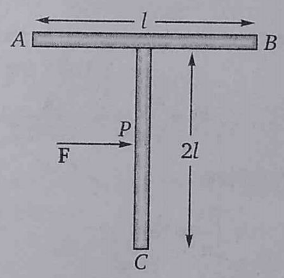


(a) $F(\hat{i}-\hat{j})$ (b) $-F(\hat{i}+\hat{j})$ (c) $F(\hat{i}+\hat{j})$ (d) $-F(\hat{i}-\hat{j})$

28. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$ **[AIEEE 2006]**

(a) $\frac{\omega(m+2M)}{m}$ (b) $\frac{\omega(m-2M)}{(m+2M)}$ (c) $\frac{\omega m}{(m+M)}$ (d) $\frac{\omega m}{(m+2M)}$

29. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[AIEEE 2005]**



(a) $\frac{2}{3}l$ (b) $\frac{3}{2}l$ (c) $\frac{4}{3}l$ (d) l

Answers

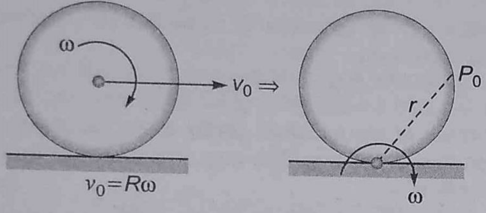
1. (a)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (d)
11. (b)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (d)	**19.** (a)	**20.** (a)
21. (b)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (a)	**27.** (c)	**28.** (d)	**29.** (c)	

Hints & Solutions

1. $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

i.e., angular momentum is a moment of momentum.

2. $v_p = r\omega = (\sqrt{2}R)\omega = \sqrt{2}\, v_0$



3.
$$\mu < \mu_{min}$$
But $\mu \neq 0$

Therefore, sphere will roll with forward slipping.

4.
$$0 = \omega_0 - \alpha t$$
$$\alpha = \frac{\omega_0}{t} = \frac{(100 \times 2\pi)/60}{15} = 0.7 \text{ rads}^{-2}$$

Now, angle rotated before coming to rest,
$$\theta = \frac{\omega_0^2}{2\alpha} = \frac{\left(\frac{100 \times 2\pi}{60}\right)^2}{2 \times 0.7} = 78.33 \text{ rad}$$

Number of rotations
$$n = \frac{\theta}{2\pi} = 12.5$$

5.
$$I\omega = \text{constant}$$
or $\frac{R^2}{T} = \text{constant}$ $\left(\text{As } I \propto R^2 \text{ and } \omega \propto \frac{1}{T}\right)$

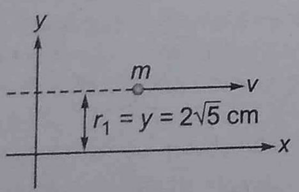
$\therefore$ $T \propto R^2$

As $R' = \frac{1}{n} R$

$\therefore$ $T' = \frac{T}{n^2} = \frac{24}{n^2}\text{h}$

6. In case of pure rolling velocity of topmost point is 2 times the velocity of centre of mass.

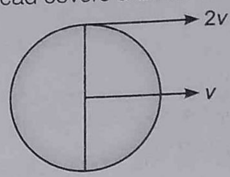
7.
$$L = mvr_\perp$$



$= (5)(3\sqrt{2})(2\sqrt{5})$

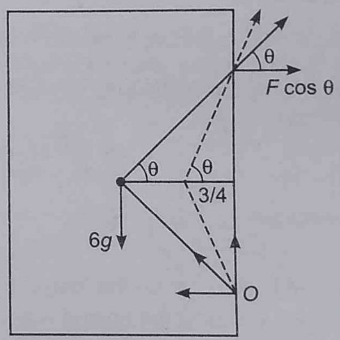
$= 30\sqrt{10}$ g-cm^{2}s^{-1}

8. If velocity of centre of mass is v, then velocity of contact is 0 and that of the top is $2v$, hence when centre of mass covers a distance l, thread covers a distance $2l$.



9.
$$I_1\omega_1 = I_2\omega_2$$
$$\therefore \quad \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{\left(\frac{ma^2}{2}\right)}{\left(\frac{ma^2}{2}\right) + ma^2}\,\omega = \frac{\omega}{3}$$

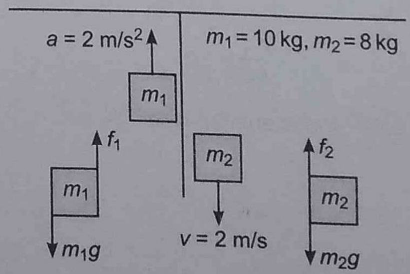
10. $\tan\theta = \frac{\frac{2}{3}}{4} = \frac{8}{3}$, $\cos\theta = \frac{7}{\sqrt{73}}$



Taking torque about O,
$$8g\left(\frac{3}{4}\right) = 4F\cos\theta \Rightarrow 6g = 4F\left(\frac{3}{\sqrt{73}}\right)$$
or $F = 5\sqrt{73} = 43$ N

11.
$$\tau_0 = mg \times r_\perp$$
As $r_\perp$ is continuously increasing or torque is continuously increasing on the particle. Hence, angular momentum is continuously increasing.

12. Since, m_2 moves with constant velocity



$\therefore \quad f_2 = m_2 g$

$f_2 = 8 \times 10 = 80 \text{ N}$

Since, mass of boy m_1 moves with $a = 2 \text{ m/s}^2$ in upward direction

$$f_1 - m_1 g = m_1 a$$

$$\therefore \quad f_1 = m_1 g + m_1 a$$

$$= 10 \times 10 + 10 \times 2 = 120 \text{ N}$$

For equilibrium of rope,

$$T = f_1 + f_2 = 120 + 80 = 200 \text{ N}$$

13. Moment of inertia of the solid cylinder about its axis of symmetry,

$$I = \frac{1}{2} MR^2 = \frac{1}{2} \times 20 \times (0.25)^2$$

$$= 10 \times 0.0625 = 0.625 \text{ kg-m}^2$$

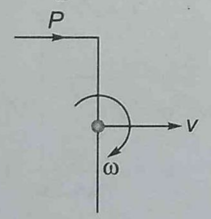
Kinetic energy associated with the rotation of the cylinder is given by

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.625 \times (100)^2$$

$$= 0.3125 \times 10000 = 3125 \text{ J}$$

Angular momentum, $L = I\omega = 0.625 \times 100 = 62.5 \text{ J-s}$

14. Angular impulse $= P \times \frac{l}{2}$ = change in angular moment



$$\therefore \quad \frac{Pl}{2} = I\omega = \left(\frac{ml^2}{12}\right)\omega$$

$$\therefore \quad \omega = \frac{6P}{ml}$$

Now, $t = \frac{\theta}{\omega} = \frac{\pi/2}{6P/ml} = \frac{\pi\, ml}{12\, P}$

15. Angular momentum of particle about O,

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

$$|\mathbf{L}| = mr\, v \sin\theta = mv(r \sin\theta) = mvl$$

16. For ring, $\frac{K_R}{K_T} = 1$

For disc, $\frac{K_R}{K_T} = \frac{1}{2}$

For ring, total kinetic energy $= 2(K_T) = 8 \text{ J}$ (given)

Hence, $K_T = 4 \text{ J}$

For disc, total kinetic energy $= \frac{3}{2} K_T$

$$= \frac{3}{2} \times 4 = 6 \text{ J}$$

17. In case of pure rolling on inclined plane,

$$a = \frac{g \sin\theta}{1 + I/mR^2}$$

$$I_{\text{Solid}} < I_{\text{Hollow}}$$

$$\therefore \quad a_{\text{Solid}} > a_{\text{Hollow}}$$

$\therefore$ Solid cylinder will reach the bottom first. Further, in case pure rolling on stationary ground, work done by friction is zero. Therefore, mechanical energy of both the cylinders will remain constant.

$\therefore \quad (\text{KE})_{\text{Hollow}} = (\text{KE})_{\text{Solid}}$ = decrease in PE $= mgh$

18. When a disc rotates with uniform angular velocity, angular acceleration of the disc is zero.

19. Position vector of centre of mass depends on masses of particles and their location. Therefore change, in shape size of body do change the centre of mass.

20. If net force on the system is zero, it can be resolved into two equal and opposite forces which can be considered to form a couple.

21. By conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$ ...(i)

where, $I_1 = mR^2$ ...(ii)

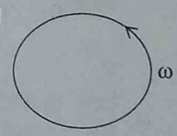
and $I_2 = 2mR^2 + MR^2$ ...(iii)

Now, change in

$$\text{KE} = \frac{1}{2} L_1 I_1^2 - \frac{1}{2} L_2 I_2^2 \quad \text{...(iv)}$$

Substituting the values from Eqs. (i), (ii) and (iii) into Eq. (iv), we get,

Change in KE $= \left(\frac{Mm}{M + 2m}\right)\omega^2 R^2$



22. Given, $m = 2$ kg, $\mathbf{r}(t) = 5\hat{\mathbf{i}} - 2t^2\hat{\mathbf{j}}$

Angular momentum $(L) = \mathbf{r} \times \mathbf{p}$

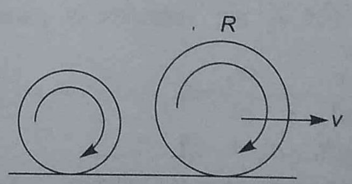
$\therefore$ Velocity, $v = \frac{dr}{dt} = \frac{d}{dt}(5\hat{\mathbf{i}} - 2t^2\hat{\mathbf{j}}) = -8\hat{\mathbf{j}}$ (at $t = 2$s)

$$\therefore \quad p = mv = 2 \times (-8\hat{\mathbf{j}}) = -16\hat{\mathbf{j}}$$

Therefore, $L = \mathbf{r} \times \mathbf{p} = (5\hat{\mathbf{i}} - 2t^2\hat{\mathbf{j}}) \times -16\hat{\mathbf{j}}$ (at $t = 2$ s)

$$= -80\hat{\mathbf{k}}$$

23.



From conservation of angular momentum

$$mr^2\omega_0 = mvr + mr^2 \times \frac{v}{r}$$

$$\Rightarrow \quad v = \frac{\omega_0 r}{2}$$

24. Angular acceleration is time derivative of angular speed and angular speed is time derivative of angular displacement.

By definition $\alpha = \frac{d\omega}{dt}$

i.e., $d\omega = \alpha dt$

So, if in time t the angular speed of a body changes from ω_0 to ω

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha dt$$

If α is constant

$$\omega - \omega_0 = \alpha t \quad \text{...(i)}$$

Now, as by definition

$$\omega = \frac{d\theta}{dt} \Rightarrow \theta = \frac{d\omega}{dt}$$

Eq. (i) becomes

$$\frac{d\theta}{dt} = \omega_0 + \alpha t$$

i.e., $d\theta = (\omega_0 + at)dt$

So, if in time t angular displacement is θ

$$\int_0^{\theta} d\theta = \int_0^t (\omega_0 + \alpha t)dt$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Given, $\alpha = 3.0 \text{ rads}^{-2}, \omega_0 = 2.0 \text{ rads}^{-1}, t = 2s$

Hence, $\theta = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2$

or $\theta = 4 + 6 = 10$ rad

Eqs. (i) and (ii) are similar to first and second equations of linear motion.

25. Torque due to central force is zero

$$\tau = \frac{d}{dt}(L) = 0$$

$\therefore$ $L =$ constant

26. To keep the centre of mass at the same position, velocity of centre of mass is zero, so

$$\frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = 0$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are velocities of particles 1 and 2 respectively.

$$\Rightarrow \quad m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} = 0 \quad \left[\because \mathbf{v}_1 = \frac{d\mathbf{r}_1}{dt} \text{ and } \mathbf{v}_2 = \frac{d\mathbf{r}_2}{dt}\right]$$

$$\Rightarrow \quad m_1 d\mathbf{r}_1 + m_2 d\mathbf{r}_2 = 0$$

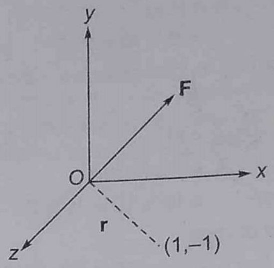
$d\mathbf{r}_1$ and $d\mathbf{r}_2$ represent the change in displacement of particles.

Let 2nd particle has been displaced by distance x.

$$\Rightarrow \quad m_1(d) + m_2(x) = 0$$

or $$x = -\frac{m_1 d}{m_2}$$

27. $\tau = \mathbf{r} \times \mathbf{F}$

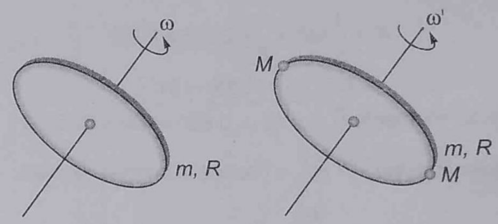


$$\tau = (\hat{i} - \hat{j}) \times (-F\hat{k})$$

$$= F[(-\hat{i} \times \hat{k}) + (\hat{j} \times \hat{k})]$$

$$= F[\hat{j} + \hat{i}] = F[\hat{i} + \hat{j}]$$

28. As no external torque is acting on the system, angular momentum of system remains conserved.



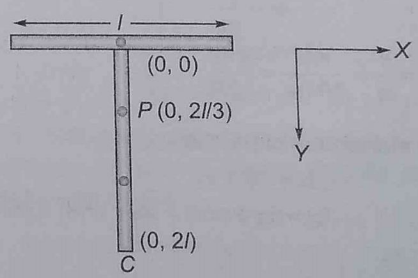
i.e., $I_1\omega = I_2\omega'$

$$\Rightarrow \quad mR^2\omega = (mR^2 + 2MR^2)\omega'$$

$$\Rightarrow \quad \omega' = \left(\frac{m}{m + 2M}\right)\omega$$

29. For pure translatory motion, net torque about centre of mass should be zero.

Thus, F is applied at centre of mass of system.



$$OP = \frac{m_1 \times 0 + m_2 \times l}{m_1 + m_2}$$

where m_1 and m_2 are masses of horizontal and vertical section of the object. Assuming object is uniform,

$$m_2 = 2m_1 \Rightarrow OP = \frac{2l}{3}$$

$$\therefore \quad PC = \left(l - \frac{2l}{3} + l\right) = \left(2l - \frac{2l}{3}\right) = \frac{4l}{3}$$

Day 8

Rigid Body

Day 8

Outlines ...

- Moment of Inertia
- Radius of Gyration
- Theorems for Moment of Inertia
- Moment of Inertia for Simple Geometrical Objects
- Rigid Body Rotation

Moment of Inertia

Moment of inertia of a rotating body is its property to oppose any change in its state of uniform rotation.

If in a given rotational system particles of masses $m_1, m_2, m_3, \ldots$ be situated at normal distances $r_1, r_2, r_3, \ldots$ from the axis of rotation, then moment of inertia of the system about the axis of rotation is given by

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots = \Sigma\, mr^2$$

For a rigid body having continuous mass distribution

$$I = \int dm\, r^2$$

SI unit of moment of inertia is kg-m^2. It is neither a scalar nor a vector *i.e.*, it is a tensor.

- *Moment of inertia of a body about a given axis is equal to twice the K.E of rotation of the body rotating with unit angular velocity about the given axis.*

  i.e., $I = 2 \times$ KE of rotation
- *Moment of inertia of a part of a rigid body (symmetrically cut from the whole mass) is the same as that of the whole body.*

Radius of Gyration

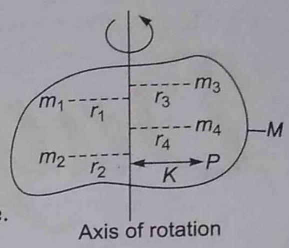
Radius of gyration of a given body about a given axis of rotation is the normal distance of a point from the axis, where if whole mass of the body is placed, then its moment of inertia will be exactly same as it has with its actual distribution of mass.

Thus, radius of gyration

$$K = \sqrt{\frac{I}{M}}$$

or $$K = \left[\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}\right]^{1/2}$$

SI unit of radius of gyration is metre.



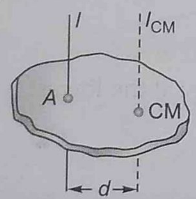
Radius of gyration depends upon shape and size of the body, position and configuration of the axis of rotation and also on distribution of mass of body w.r.t. axis of rotation.

Theorems on Moment of Inertia

There are two theorems in moment of inertia i.e., theorem of parallel axis and theorem of perpendicular axis. Both the theorems are described below.

1. Theorem of Parallel Axes

Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about a parallel axis passing through its centre of mass I_{CM} and the product of mass of body (M) and square of normal distance d between the two axes.

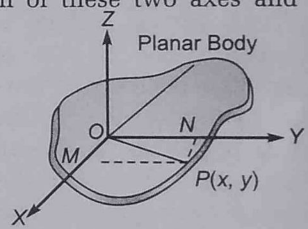


Theorem of parallel axes is applicable for any type of rigid body whether it is a two-dimensional or three-dimensional while the theorem of perpendicular axes is applicable for laminar type or two-dimensional bodies only.

Mathematically, $I = I_{CM} + Md^2$

2. Theorem of Perpendicular Axes

The sum of moment of inertia of a plane laminar body about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis passing through the point of intersection of these two axes and perpendicular to the plane of laminar body. If I_x and I_y be moment of inertia of the body about two perpendicular axes in its own plane and I_z be the moment of inertia about an axis passing through point O and perpendicular to the plane of lamina, then $I_z = I_x + I_y$



In theorem of perpendicular axes, the point of intersection of the three axes (x, y and z) may be any point on the plane of body (it may even lie outside the body). This point may or may not be the centre of mass of the body.

Moment of Inertia for Simple Geometrical Objects

- **Uniform Ring of Mass M and Radius R** About an axis passing through the centre and perpendicular to plane of ring $I = MR^2$. About a diameter $I = \frac{1}{2}MR^2$
- **Uniform Circular Disc of Mass M and Radius R** About an axis passing through the centre and perpendicular to plane of disc $I = \frac{1}{2}MR^2$. About a diameter $I = \frac{1}{4}MR^2$
- **Thin Uniform Rod of Mass M and Length l** About an axis passing through its centre and perpendicular to the rod, $I = \frac{1}{12}Ml^2$
- **Uniform Solid Cylinder of Mass M, Length l and Radius R** About its own axis, $I = \frac{1}{2}MR^2$ About an axis passing through its centre and perpendicular to its length $I = M\left[\frac{l^2}{12} + \frac{R^2}{4}\right]$
- **Uniform Solid Sphere** About its diameter $I = \frac{2}{5}MR^2$. About its tangent $I = \frac{7}{5}MR^2$

(i) Expression for moment of inertia of a lamina about an axis passing through origin making an angle θ with x-axis is $I = I_x \cos^2\theta + I_y \sin^2\theta - 2F\sin\theta\cos\theta$

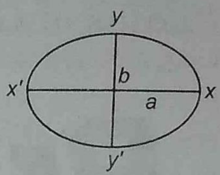
where, $F = \Sigma mxy$ = product of inertia.

(ii) Calculation for moment of inertia by digits

Moment of inertia about an axis of symmetry =

$$\frac{\text{Mass} \times \text{the sum of squares of perpendicular semi-axis}}{3 \text{ or } (4 \text{ or } 5)}$$

where denominator to be 3 or 4 or 5 according as the body is rectangular, elliptical (including circular) on ellipsoidal (including spherical) *e.g.*, for ellipse, $I_z = \frac{M}{4}(a^2 + b^2)$



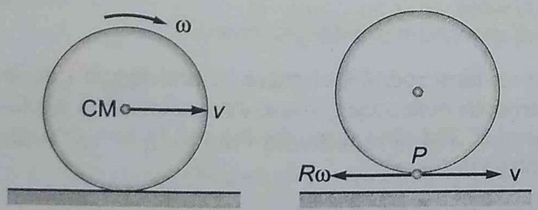
Rigid Body Rotation

If a rigid body of moment of inertia I about a given axis is rotating with an angular speed ω, then its rotational kinetic energy is given by, $K_R = \frac{1}{2}I\omega^2$ *Rotational kinetic energy is a scalar having SI unit joule (J).*

Rotational kinetic energy is related to angular momentum as per relation, $K_R = \frac{L^2}{2I}$ or $L = \sqrt{2IK_R}$

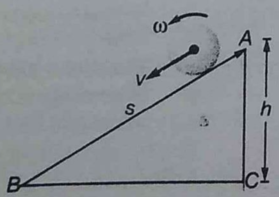
Pure Rolling Motion

Let a rigid body, having symmetric surface about its centre of mass, is being spined at a certain angular speed and placed on a surface so that plane of rotation is perpendicular to the surface. If now the body is simultaneously given a translational motion too, then the net motion is called **rolling motion.** If the given body rolls over a surface such that there is no relative motion between the body and the surface at the point of contact, then the motion is called **rolling without slipping**.



(a) (b)

1. In pure rolling motion without slipping, there is no relative motion between the rolling body and the surface over which rolling motion takes place at the point of contact. As shown in previous figure, it is possible when $v = R\omega$, where v = translational velocity of the centre of mass of rolling body.
2. Friction is responsible for rolling motion without slipping.
3. Rolling motion of a body may be considered as a pure rotation about an axis through point of contact with a constant angular velocity ω.



4. Let a symmetric body starts rolling from the top point A of an inclined plane $(u = 0)$ of length s and height h, then

(i) Acceleration in motion. $a = \frac{g\sin\theta}{\left(1 + \frac{k^2}{R^2}\right)}$

(ii) Final velocity at the lowest point B, $v = \sqrt{\frac{2hg}{\left(1 + \frac{k^2}{R^2}\right)}}$

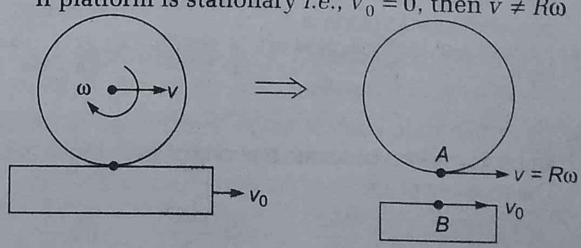
(iii) Time of descent

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1 + \frac{k^2}{R^2}\right)} = \sqrt{\frac{2s}{g\sin\theta}\left(1 + \frac{k^2}{R^2}\right)}$$

Impure Rolling Motion

In impure rolling motion, the point of contact of the body with the platform is not relatively at rest w.r.t. platform on which it is performing rolling motion, as a result sliding occurs at point of contact.

1. For impure rolling motion, $v_{AB} \neq 0$ *i.e.*, $v - R\omega \neq v_0$. If platform is stationary *i.e.*, $v_0 = 0$, then $v \neq R\omega$



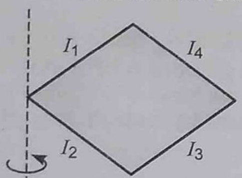
2. As in impure rolling motion, velocity of point of contact is not zero relative to the platform. Kinetic friction comes into the existence *i.e.*, $\mu_K N$.

Practice Zone DAY 8

1. A wheel has mass of the rim 1 kg, having 50 spokes each of mass 5g. The radius of the wheel is 40 cm. The moment of inertia

(a) 0.273 kg-m^2 (b) 1.73 kg-m^2
(c) 0.173 kg-m^2 (d) 2.73 kg-m^2

2. The moment of inertia of a system of four rods each of length l and mass m about the axis shown is



(a) $\frac{2}{3}ml^2$ (b) $2\ ml^2$ (c) $3\ ml^2$ (d) $\frac{8}{3}ml^2$

3. The surface density of a circular disc of radius a depends on the distance as $\rho(r) = A + Br$. The moment of inertia about the line perpendicular to the plane of the disc is

(a) $\pi a^4\left(\frac{A}{2}+\frac{2a}{5}B\right)$ (b) $\pi a^4\left(\frac{A}{2}+\frac{2B}{5}\right)$
(c) $2\pi a^3\left(\frac{A}{2}+\frac{Ba}{5}\right)$ (d) None of these

4. A hoop of radius 2 m, weight 100 kg. It rolls along horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it? **[NCERT Exemplar]**

(a) 4.8 J (b) 4.0 J (c) 6.2 J (d) 7.2 J

5. Mass of bigger disc having radius $2R$ is M. A disc of radius R is cut from bigger disc as shown. Moment of inertia of disc about an axis passing through periphery and perpendicular to plane is

(a) $\frac{27\ MR^2}{8}$ (b) $\frac{29\ MR^2}{8}$ (c) $3.5\ MR$ (d) $2MR^2$

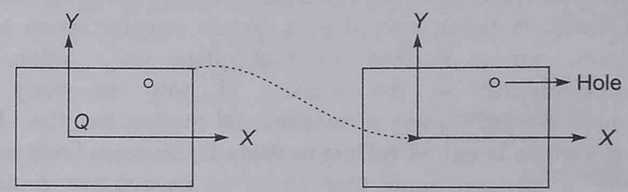
6. Three thin rods each of length L and mass M are placed along x, y and z-axis such that one of each rod is at origin. The moment of inertia of this system about z-axis is

(a) $\frac{2}{3}ML^2$ (b) $\frac{4ML^2}{3}$ (c) $\frac{5\ ML^2}{3}$ (d) $\frac{ML^2}{3}$

7. The ratio of the radii of gyration of a circular disc and a circular ring of the same radii about a tangential axis perpendicular to plane of disc or ring is

(a) 1 : 2 (b) $\sqrt{5} : \sqrt{6}$
(c) 2 : 3 (d) $\frac{\sqrt{3}}{2}$

8. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. The moment of inertia about the z-axis, then **(NCERT Exemplar)**



(a) increased
(b) decreased
(c) same
(d) changed in unpredicted manner

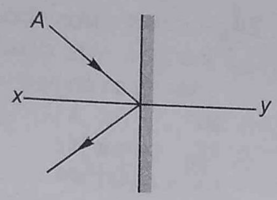
9. A uniform rod AB of mass m and length l at rest on a smooth horizontal surface. An impulse P is applied to the end B. The time taken by the rod to turn through at right angle is

(a) $2\pi\frac{ml}{P}$ (b) $2\frac{\pi P}{ml}$
(c) $\frac{\pi}{12}\frac{ml}{P}$ (d) $\frac{\pi P}{ml}$

Directions *(Q. Nos. 10 and 11) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given here*

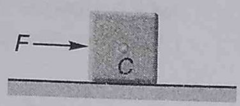
(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

10. Statement I A disc A moves on a smooth horizontal plane and rebounds elastically from a smooth vertical wall (Top view is shown in figure) in this case about any point on line xy. The angular momentum of the disc remains conserved.



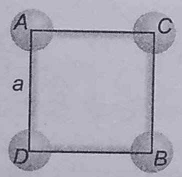
Statement II About any point in the plane, the torque experienced by disc is zero as gravity force and normal contact force balance each other.

11. Statement I A block is kept on a rough horizontal surface, under the action of a force F as shown in the figure. The torque of normal contact force about centre of mass is having zero value.



Statement II The point of application of normal contact force may pass through centre of mass.

Directions *(Q. Nos. 12 and 13) Four solid spheres each of mass m and radius r are located with their centres on four corners of a squares ABCD of side a as shown in the figure.*



12. The moment of inertia of the system of four spheres about diagonal AB is

(a) $\frac{m}{5}(8r^2 + 5a^2)$ (b) $\frac{m}{5}(7r^2 + 4a^2)$

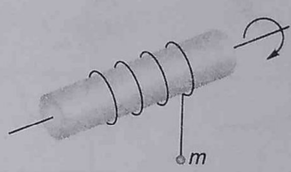
(c) $\frac{m}{5}(5r^2 + 8a^2)$ (d) $\frac{m}{5}(3r^2 + 5a^2)$

13. Four spheres of diameter $2a$ and mass M are placed with their centres on the four corners of a square of side b. Then the moment of Inertia of the system about an axis along one of the sides the square is

(a) $\frac{4}{5}Ma^2 + 2Mb^2$ (b) $\frac{8}{5}Ma^2 + 2Mb^2$

(c) $\frac{8}{5}Ma^2$ (d) $\frac{4}{5}Ma^2 + 4Mb^2$

Directions *(Q. Nos. 14 to 16) A solid cylinder of mass M and radius R is mounted on a frictionless horizontal axis so that it can freely rotate about this axis. A string of negligible mass is wrapped round the cylinder and a body of mass m is hung from the string as shown in figure. The mass is released from rest.*



14. The acceleration with which the mass falls is

(a) g (b) $\frac{mg}{M}$

(c) $\frac{mg}{(M+m)}$ (d) $\frac{2mg}{(M+2m)}$

15. The tension in the string is

(a) mg (b) $\frac{Mmg}{(M+m)}$

(c) $\frac{2Mmg}{(M+2m)}$ (d) $\frac{Mmg}{(M+2m)}$

16. The angular speed of the cylinder is proportional to h^n, where h is the height through which the mass falls. The value of n is

(a) zero (b) 1

(c) 1/2 (d) 2

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17. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc **[AIEEE 2011]**

(a) continuously decreases

(b) continuously increases

(c) first increases and then decreases

(d) remains unchanged

18. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ N (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2 the number of rotations made by the pulley before its direction of motion if reserved, is **[AIEEE 2011]**

(a) more than 3 but less than 6

(b) more than 6 but less than 9

(c) more than 9

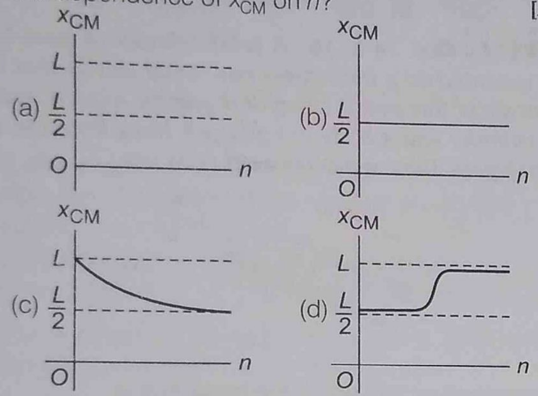
(d) less than 3

19. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[AIEEE 2009]**

(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$

(c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$

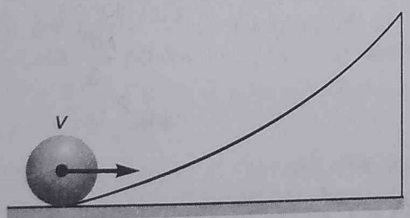
20. A thin rod of length L is lying along the x-axis with ends at $x = 0$ and $x = L$. Its linear density $\left(\frac{\text{mass}}{\text{length}}\right)$ varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximate the dependence of x_{CM} on n? **[AIEEE 2008]**



21. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[AIEEE 2008]**

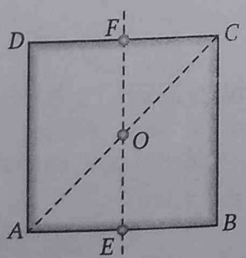
(a) $\frac{5}{6}ma^2$ (b) $\frac{1}{12}ma^2$ (c) $\frac{7}{12}ma^2$ (d) $\frac{2}{3}ma^2$

22. A small object of uniform density rolls up a curved surface with an initial velocity v. It reached up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is **[AIEEE 2007]**



(a) ring (b) solid sphere
(c) hollow sphere (d) disc

23. For the given uniform square lamina $ABCD$, whose centre is O. **[AIEEE 2007]**

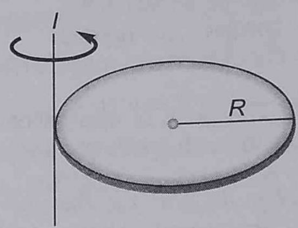


(a) $\sqrt{2}I_{AC} = I_{EF}$ (b) $I_{AD} = 3I_{EF}$
(c) $I_{AC} = I_{EF}$ (d) $I_{AC} = \sqrt{2}I_{EF}$

24. A round uniform body of radius R, mass M and moment of inertia I, rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then, its acceleration is **[AIEEE 2007]**

(a) $\frac{g\sin\theta}{1 + I/MR^2}$ (b) $\frac{g\sin\theta}{1 + MR^2/I}$
(c) $\frac{g\sin\theta}{1 - I/MR^2}$ (d) $\frac{g\sin\theta}{1 - MR^2/I}$

25. A solid sphere of radius R has moment of inertia I about its geometrical axis. It is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to **[AIEEE 2006]**



(a) $\frac{2}{\sqrt{15}}R$ (b) $\frac{2}{\sqrt{5}}R$
(c) $\frac{3}{\sqrt{15}}R$ (d) $\frac{\sqrt{3}}{\sqrt{15}}R$

26. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is **[AIEEE 2006]**

(a) $2ml^2$
(b) $\sqrt{3}ml^2$
(c) $3ml^2$
(d) ml^2

27. A particle moves in a circular path with decreasing speed. Choose the correct statement. **[AIEEE 2005]**

(a) Angular momentum remains constant
(b) Acceleration (a) is towards the centre
(c) Particle moves in a spiral path with decreasing radius
(d) The direction of angular momentum remains constant

28. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and, a body C of mass $\frac{2}{3}m$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards **[AIEEE 2005]**

(a) depends on height of breaking
(b) does not shift
(c) body C
(d) body B

Answers

1. (c)	2. (d)	3. (a)	4. (b)	5. (b)	6. (a)	7. (d)	8. (b)	9. (c)	10. (c)
11. (d)	12. (a)	13. (b)	14. (d)	15. (d)	16. (c)	17. (c)	18. (a)	19. (d)	20. (a)
21. (d)	22. (d)	23. (c)	24. (a)	25. (a)	26. (c)	27. (d)	28. (b)		

Hints & Solutions

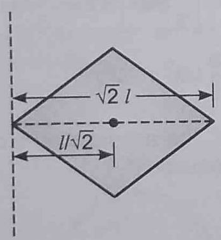
1. $I = mr^2 + 50\dfrac{ml^2}{3}$

$$= I \times (0.4)^2 + \frac{50\,(5 \times 10^{-3})\,(0.4)^2}{3}$$

$$= 0.16\,(1.083) = 0.173\ \text{kg-m}^2$$

2. Consider a square lamina, then $4m\left(\dfrac{l^2}{12} + \dfrac{l^2}{12}\right)$ about

$$\text{Centre of Mass} = \frac{2ml^2}{3}$$



Apply perpendicular axis theorem

$$= \frac{2ml^2}{3} + 4m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{8}{3}ml^2$$

3. $dm = 2\pi r\,dr\,(\rho) = (A + Br)\,(2\pi r\,dr)$

$$I = \int_0^a dmr^2 = \frac{\pi Aa^4}{2} + \frac{2\pi Ba^5}{5} = \pi a^4\left(\frac{A}{2} + \frac{2a}{5}B\right)$$

4. Given, $R = 2\ \text{m}$

$M = 100\ \text{kg}$

Speed of centre of mass $(v) = 20\ \text{cm/s} = 0.20\ \text{m/s}$

Work done to stop the hoop = Total kinetic energy of the hoop

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

But moment of inertia $I = Mr^2$ and angular velocity $\omega = \dfrac{v}{R}$

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}(Mr^2) \times \left(\frac{v^2}{R^2}\right)$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

$$= 100 \times (0.20)^2$$

$$= (100 \times 0.04)\ \text{J} = 4.0\ \text{J}$$

5. Surface density of motional disc is

$$\sigma = \frac{M}{\pi(2R)^2} = \frac{M}{4\pi R^2}$$

Mass of cutting portion is

$$m_1 = \sigma \times \pi R^2 = \frac{M}{4}$$

$$I = I_1 - I_2$$

where,

I_1 = Moment of inertia of disc about given axis without cutting portion and

I_2 = Moment of inertia due to cutting portion.

$$I = \frac{M(2R)^2}{2} + M(2R)^2 - \left[\frac{m_1R^2}{2} + m_1(3R)^2\right]$$

$$= 6MR^2 - \frac{19MR^2}{8}$$

$$= \frac{29MR^2}{8}$$

6. Moment of inertia of the rod lying along z-axis will be zero. Of the rods along x and y axis will be $\dfrac{ML^2}{3}$ each.

Hence, total moment of inertia is $\dfrac{2}{3}ML^2$.

7. Radius of gyration

$$K = \sqrt{\frac{I}{m}}$$

$$K_{disc} = \sqrt{\frac{\frac{1}{2}mR^2 + mR^2}{m}} = \sqrt{\frac{3}{2}}R$$

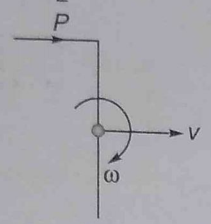
$$K_{ring} = \sqrt{\frac{mR^2 + mR^2}{m}} = \sqrt{2}R$$

$$\frac{K_{disc}}{K_{ring}} = \frac{\sqrt{\frac{3}{2}}}{\sqrt{2}} = \frac{\sqrt{3}}{2}$$

8. According to the theorem of perpendicular axes, $I_z = I_x + I_y$ with the hole, I_x and I_y, both decreases gluing the removed piece at the centre of square plate does not affect I_z.

Hence, I_z decreases overall.

9. Angular impulse $= P \times \frac{l}{2}$ = change in angular moment



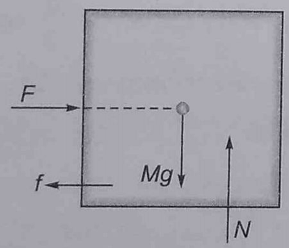
$\therefore \qquad \frac{Pl}{2} = I\omega = \left(\frac{ml^2}{12}\right)\omega$

$\therefore \qquad \omega = \frac{6P}{ml}$

Now, $\qquad t = \frac{\theta}{\omega} = \frac{\pi/2}{6P/ml} = \frac{\pi\, ml}{12\,P}$

10. The forces experienced by disc are gravity and normal contact force. In addition to these, impact force (during collision) will act on the disc along line xy. Gravity and normal contact force balance each other (in terms of force and torque both), but impact force causes non-zero torque acting on disc about all points except the points lying on its line of action *i.e.*, xy. So, angular momentum remains conserved about any point on xy.

11. Here, as the block is kept in equilibrium, the net torque experienced by the body about any point has to be zero. Here, due to F and Mg about C, the torque is zero but friction is providing non-zero torque in clockwise direction, now other force is N only which can produce non-zero force in anti-clockwise direction to make net torque zero and it is possible only when N does not pass through C.



12. The moment of inertia of the system of four spheres about diagonal AB is

I_{AB} = MI of A about AB + MI of B about AB + MI of C about AB + MI of D about AB

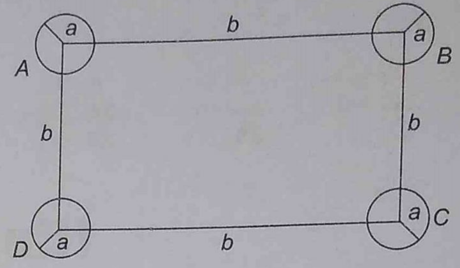
$$= \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{2}{5}mr^2 + \frac{1}{2}ma^2 + \frac{2}{5}mr^2 + \frac{1}{2}ma^2$$

$$= \frac{8}{5}mr^2 + ma^2$$

$$= m\left(\frac{8r^2}{5} + a^2\right)$$

$$= \frac{m}{5}(8r^2 + 5a^2)$$

13. We calculate moment of inertia of the system about



Moment of inertia of each of the spheres A and D about

$$AD = \frac{2}{5}Ma^2$$

Moment of inertia of each of the spheres B and C about AD

$$= \left(\frac{2}{5}Ma^2 + Mb^2\right)$$

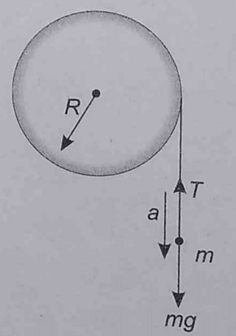
Using theorem of parallel axis, we get

Total moment of inertia,

$$= I\left[\frac{2}{5}Ma^2\right]\times 2 + \left[\frac{2}{5}Ma^2 + Mb^2\right]\times 2$$

$$I = \frac{8}{5}Ma^2 + 2Mb^2$$

14. Refering to figure, the acceleration a of the falling body is governed by the equation



$$ma = mg - T \qquad \text{...(i)}$$

where T is the tension in the string.

Torque on cylinder is

$$TR = I\alpha$$

$$\Rightarrow \qquad T = \frac{I\alpha}{R} = \left(\frac{1}{2}MR^2\right)\times\frac{a}{R^2} = \frac{1}{2}Ma \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$a = \frac{2\,mg}{(M+2m)} \qquad \text{...(iii)}$$

15. From Eqs. (ii) and (iii), we get

$$T = \frac{m\,Mg}{(M+2m)}$$

16. From conservation of energy, we have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mR^2\omega^2 + \frac{1}{2}\times\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{1}{4}(2m + M)R^2\omega^2$$

$$\omega = \left[\frac{4mgh}{(M + 2m)R^2}\right]^{1/2}$$

Thus, $\omega \propto h^{1/2}$.

Hence, value of n is $\frac{1}{2}$.

17. Moment of inertia $= \frac{1}{2}MR^2 + mx^2$

where, m = mass of insect

and x = distance of insect from centre

Clearly, as the insect moves along the diameter of the disc, MI first decreases, then increases.

By conservation of angular momentum, angular speed first increases, then decreases.

18. $F = 20t - 5t^2$

$$\alpha = \frac{FR}{I} = 4t - t^2$$

$$\Rightarrow \quad \frac{d\omega}{dt} = 4t - t^2$$

$$\Rightarrow \quad \int_0^\omega d\omega = \int_0^t (4t - t^2)\,dt$$

$$\Rightarrow \quad \omega = 2t^2 - \frac{t^3}{3}$$

When direction is reversed $\omega = 0$, *i.e.*, $t = 0$, 6s

Now, $d\theta = \omega\, dt$

$$\int_0^\theta d\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right) dt$$

$$\Rightarrow \quad \theta = \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6 = 144 - 108 = 36 \text{ rad}$$

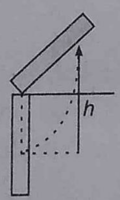
$\therefore$ Number of rotations $n = \frac{\theta}{2\pi} = \frac{36}{2\pi} < 6$

19. If centre of mass rises to a maximum height, then Loss in KE = Gain in PE, we get

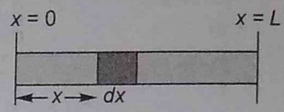
$$\frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}\times\frac{1}{3}ml^2\omega^2 = mgh$$

$$\Rightarrow \quad h = \frac{1}{6}\frac{l^2\omega^2}{g}$$



20. $$x_{CM} = \frac{\int x\,dm}{\int dm}$$



If $n = 0$

Then, $x_{CM} = \frac{L}{2}$

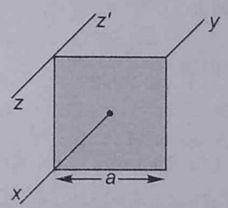
As n increases, the centre of mass shift away from $x = \frac{L}{2}$ which only option (a) is satisfying.

Alternately, you can use basic concept.

$$X_{CM} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n \times x dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= L\left[\frac{n+1}{n+2}\right]$$

21. Moment of inertia of square plate about xy is $\frac{ma^2}{6}$. Moment of inertia about zz' can be computed using parallel axes theorem



$$I_{zz'} = I_{xy} + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

22. $\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$

$$\therefore \quad I = \frac{1}{2}mR^2$$

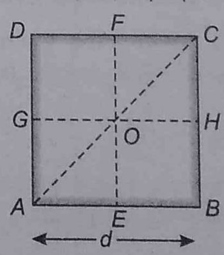
Therefore, the body is a disc.

23. Let the each side of square lamina is d.

So, $I_{EF} = I_{GH}$ (due to symmetry)

and $I_{AC} = I_{BD}$ (due to symmetry)

Now, according to theorem of perpendicular axes,



$$I_{AC} + I_{BD} = I_0$$

$$\Rightarrow \quad 2I_{AC} = I_0 \qquad \text{...(i)}$$

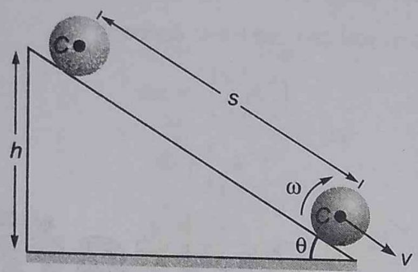
and $I_{EF} + I_{GH} = I_0$

$\Rightarrow$ $2 I_{EF} = I_0$ ...(ii)

From Eqs. (i) and (ii), we get

$$I_{AC} = I_{EF}$$

24. Assuming that no energy is used up against friction, the loss in potential energy is equal to the total gain in the kinetic energy.



i.e., $Mgh = \frac{1}{2} I \left(\frac{v^2}{R^2}\right) + \frac{1}{2} Mv^2$

or $\frac{1}{2} v^2 \left(M + \frac{I}{R^2}\right) = Mgh$

or $v^2 = \frac{2Mgh}{M + I/R^2}$

$= \frac{2gh}{1 + I/MR^2}$

If s be the distance covered along the plane, then

$$h = s \sin\theta$$

$\therefore$ $v^2 = \frac{2gs \sin\theta}{1 + I/MR^2}$

Now, $v^2 = 2as$

$\therefore$ $2as = \frac{2gs \sin\theta}{1 + I/MR^2}$

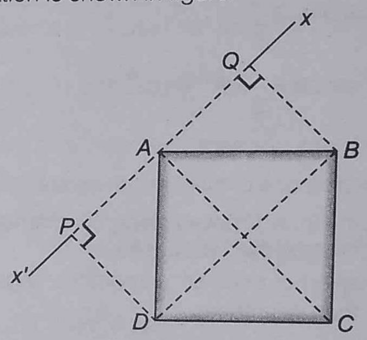
or $a = \frac{g \sin\theta}{1 + I/MR^2}$

25. $\frac{2}{5} MR^2 = \frac{1}{2} Mr^2 + Mr^2$

or $\frac{2}{5} MR^2 = \frac{3}{2} Mr^2$

$\therefore$ $r = \frac{2}{\sqrt{15}} R$

26. The situation is shown in figure.



$I_{xx'} = m \times DP^2 + m \times BQ^2 + m \times CA^2$

$= m \times 2 \times \left(\frac{\sqrt{2} l}{2}\right)^2 + m \times (\sqrt{2} l)^2$

$= 3ml^2$

27. $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

Direction of $(\mathbf{r} \times \mathbf{v})$, hence the direction of angular momentum remains the same.

28. The position of centre of mass remains unaffected because breaking of mass into two parts is due to internal forces.

Day 9

Gravitation

Day 9

Outlines ...

- Concept of Gravitation
- Universal Law of Gravitation
- Kepler's Laws of Planetary Motion
- Gravitational Field
- Gravitational Potential
- Escape Velocity
- Artificial Satellite
- Geostationary Satellite

Concept of Gravitation

Gravitation is the name given to the force of attraction between any two massive bodies of the universe. It was discovered by Newton in the year 1665, when he saw an apple falling down the tree. Gravitational force is the weakest force among the four fundamental forces of nature, but is the most important force because it played an important role in initiating the birth of stars, and controlling the entire structure of universe. The gravitational force has significant applications in the present advancement of physics.

Universal Law of Gravitation

Every body in the universe attracts every other body. Force of gravitational attraction between two bodies of mass m_1 and m_2 is given as

$$F = G\frac{m_1 m_2}{r^2}$$

The proportionality constant G is called universal gravitational constant. In SI system, value of gravitational constant G is $6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$. Dimensional formula of G is $[\text{M}^{-1}\text{L}^3\text{T}^{-2}]$.

Acceleration due to Gravity

The acceleration produced in the motion of a body under the effect of gravity (earth's gravitation) is called acceleration due to gravity (g).

If we consider a body of mass m placed at the surface of the earth, then force of gravity acting on it is,

$$F = \frac{GMm}{R^2}$$

and acceleration due to gravity

$$g = \frac{F}{m} = \frac{GM}{R^2} = \frac{4}{3}\pi G\rho R$$

where, ρ = mean density of the earth.

For the earth, mean value of g on the earth's surface is 9.8 ms^{-2}.

Variation with Altitude *and* Depth

- Value of acceleration due to gravity at a height h from the surface of the earth is given by

$$g' = \frac{gR^2}{(R+h)^2}$$

If $h << R$, then $g' = g\left[1 - \frac{2h}{R}\right]$

or $g - g' = \Delta g = \left(\frac{2h}{R}\right)g$

- Value of acceleration due to gravity g at a depth d from the surface of the earth is given by

$$g' = g\left(1 - \frac{d}{R}\right) \text{ or } g - g' = \Delta g = \left(\frac{d}{R}\right)g$$

At the centre of the earth $d = R$ and hence, $g' = 0$.

As earth rotates about its own axis with a period of 24 h (or angular velocity $\omega = 7.275 \times 10^{-5}$ rad s^{-1}), all bodies situated on the earth's surface move along the circular path and need centripetal force for that. A part of weight of body is used to provide centripetal force and consequently effective value of g changes to $g' = g - R\omega^2 \cos^2 \lambda$

where, λ = latitude angle of given place

At poles; $\lambda = 90°$ and hence $g_{pole} = g$

At equator; $\lambda = 0°$ and hence

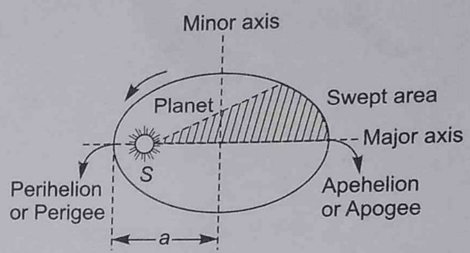
$$g_{equator} = g - R\omega^2$$

$\Rightarrow$ $g_{pole} - g_{equator} = R\omega^2$

The value of g also varies due to the non sphericity and non homogeneity of the earth.

Kepler's Laws of Planetary Motion

Based on trial and error, observation and using already compiled data by earlier physicists, Kepler discovered three empirical laws which accurately describe the motion of planets. These laws are



1. **Law of Orbits**

   Every planet moves in an elliptical orbit around the sun, with sun situated at one of the foci of the ellipse.

2. **Law of Areas**

   The line joining a planet to the sun sweeps equal areas in equal intervals of time, howsoever small these time intervals may be. Mathematically,

$$\frac{dA}{dt} = \frac{1}{2} r v = \text{constant} = \frac{L}{2m}$$

   where, L = angular momentum of planet and m = mass of planet.

3. **Law of Periods**

   The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet. Thus,

$$T^2 \propto r^3 \text{ or } \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

Gravitational Field

Gravitational field of a given body is the space around it, within which gravitational effect due to that body may be experienced.

Gravitational Field Intensity

Gravitational field intensity at any point in the field of given body is defined as the force experienced by a unit mass placed at that point. Mathematically,

Gravitational field intensity $\mathbf{I} = \lim_{m_0 \to 0} \frac{\mathbf{F}}{m_0}$

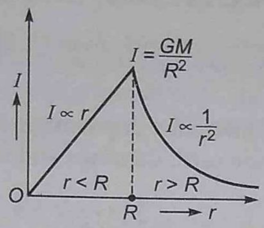
where, m_0 is a small test mass. The SI unit of gravitational intensity is N kg^{-1}.

Gravitational intensity at a point situated at a distance r from a point mass M is given by

$$I = \frac{GM}{r^2}$$

Gravitational intensity at a point due to the combined effect of different point masses is given by the vector sum of individual intensities.

Thus, $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \ldots$



Gravitational field intensity due to a solid sphere (e.g., earth) of mass M and radius R at a point distant r from its centre $(r > R)$ is $I = \frac{GM}{r^2}$

and at the surface of solid sphere $I = \frac{GM}{R^2}$

However, for a point $r < R$, we find that

$$I = \frac{GMr}{R^3}$$

Due to a body in the form of uniform shell gravitational field intensity at a point outside the shell $(r > R)$ is given by

$$I = \frac{GM}{r^2}$$

But at any point inside the shell, gravitational intensity is zero.

Gravitational Potential

Gravitational potential at any point in a gravitational field is defined as the work done in bringing a unit mass from infinity to that point. Thus, gravitational potential $V = \lim_{m_0 \to 0} \frac{W}{m_0}$ *Hence, gravitational potential is always negative.*

Gravitational potential is a scalar term and its SI unit is J kg^{-1}.

Gravitational potential due to a point mass is $V = -\frac{GM}{r}$

For solid sphere

1. At a point outside the solid sphere, (*e.g.*, earth), *i.e.*,
$$r > R,\ V = -\frac{GM}{r}$$
2. At a point on the surface of spherical, $V = -\frac{GM}{R}$
3. At a point inside the sphere, $(r < R)$
$$V = -\frac{GM}{2R^3}(3R^2 - r^2)$$
$$= -\frac{GM}{2R}\left[3 - \frac{r^2}{R^2}\right]$$
4. At the centre of solid sphere, $V = -\frac{3GM}{2R} = \frac{3}{2}V_{\text{surface}}$

For spherical shell

1. At a point outside the shell, $V = -\frac{GM}{r}$ where, $r > R$.
2. At a point on the surface of spherical shell,
$$V = -\frac{GM}{R}$$
3. At any point inside the surface of spherical shell
$$V = -\frac{GM}{R} = V_{\text{surface}}$$

The energy possessed by a body due to interaction forces acting inside the body is the self energy. For particles self energy is not defined. It is defined as the negative of work done by gravitational force in assembling the body from some reference position to make the desired shape of the body.

- *For hollow sphere, self energy* $U_s = -\frac{GM^2}{2R}$,
- *For solid sphere, self energy* $U_s = -\frac{3}{5} \times \frac{GM^2}{R}$

Relation between Gravitational Field *and* Gravitational Potential

If $\mathbf{r}_1$ and $\mathbf{r}_2$ are position of two points in the gravitation field $(\mathbf{I})$ then change in gravitational potential

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{r_1}^{r_2} \mathbf{I}.d\mathbf{r} \Rightarrow dV = -\mathbf{I}.d\mathbf{r} = I_x\hat{\mathbf{i}} + I_y\hat{\mathbf{j}} + I_z\hat{\mathbf{k}} \text{ then,}$$

and $$dr = dx\,\hat{\mathbf{i}} + dx\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}} = I_x\,\hat{\mathbf{i}}\,I_y\hat{\mathbf{j}} + I_z\hat{\mathbf{k}},\ dV = -\mathbf{I}.d\mathbf{r} = -I_x dx - I_y dy - I_z dz$$

Thus, $$\mathbf{I} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$$

Remember that partial differentiation indicates that variation of gravitational potential in counter along the variation of x coordinate then other coordinates (*i.e.*, y and z) are assumed to be constant.

Gravitational Potential Energy

Gravitational potential energy of a body or system is negative of work done by the conservative gravitational forces, F in bringing it from infinity to the present position.

Mathematically, gravitational potential energy

$$U = -W = -\int_{\infty}^{\mathbf{r}} \mathbf{F} \cdot \mathbf{dr}$$

The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by

$$U = -\frac{Gm_1m_2}{r}$$

The gravitational potential energy of mass m at the surface of the earth is

$$U = -\frac{GMm}{R}$$

Difference in potential energy of mass m at a height h from the earth's surface and at the earth's surface is

$$U_{(R+h)} - U_R = \frac{mgh}{1+\frac{h}{R}} \simeq mgh \text{ if } h << R$$

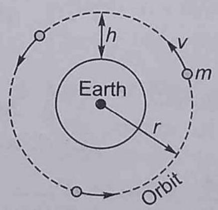
- *For three particles system,*

$$U = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right]$$

- *For n-particles system, $\frac{n(n-1)}{2}$ pairs form and total potential energy of the system is sum of potential energies of all such pairs*

Motion of a Satellite of Mass (m) around the Earth (M)

Consider a satellite of mass m revolving in a circle around the earth. If the satellite is at a height h above the earth's surface, the radius of its orbit is $r = R + h$, where R is the radius of the earth. The gravitational force between m and M provides the centripetal force necessary for a circular motion.



Escape Velocity

Escape velocity is the minimum velocity with which a body should be projected from a given surface so as to enable it to just overcome the gravitational pull of that surface (e.g.,earth). The value of escape velocity from the surface of a planet of mass M, radius R and acceleration due to gravity g is $v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

Escape velocity does not depend upon the mass or shape or size of the body as well as the direction of projection of the body. For earth value of escape velocity is 11.2 kms^{-1}.

Some Important Escape Velocities

Heavenly Body	Escape Velocity
Moon	2.3 kms^{-1}
Mercury	4.28 kms^{-1}
Earth	11.2 kms^{-1}
Jupiter	60 kms^{-1}
Sun	618 kms^{-1}
Neutron star	2×10^5 kms^{-1}

A planet will have atmosphere if the root mean square velocity of its atmospheric molecules is less than the escape velocity for the given planet. That is, why moon has no atmosphere ($v_e = 2.3 \text{ kms}^{-1}$) while jupiter has a thick atmosphere ($v_e = 60 \text{ kms}^{-1}$). Even the lightest hydrogen cannot escape from its surface.

Artificial Satellites

Artificial satellites are man made satellites launched from the earth. The path of these satellites are elliptical with the centre of earth at a foci of the ellipse. However, as a first approximation we may consider the orbit of satellite as circular.

Orbital Velocity of Satellite

When a satellite rotates around the earth, the gravitational force due to earth provides the necessary centripetal force for its orbital motion. The orbital velocity of satellite

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{gR^2}{(R+h)}}$$

If $h << R$ or $r \simeq R$, then,

$$v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.9 \text{ kms}^{-1}$$

Period of Revolution

It is the time taken by a satellite to complete one revolution around the earth.

Revolution period, $T = \frac{2\pi r}{v_o} = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}}$

If $r \simeq R$, then $T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$

Height of Satellite in Terms of Period

The height of the satellite (from the earth planet) can be determined by its time period and *vice-versa*.

As the height of the satellite in terms of time period, $h = r - R = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3} - R$

Energy of Satellite

Kinetic energy of satellite,

$$K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$$

Potential energy of satellite, $U = -\frac{GMm}{r}$

and total energy of satellite

$$E = K + U = -\frac{GMm}{2r} = -K$$

Binding Energy of Satellite

It is the energy required to remove the satellite from its orbit and take it to infinity.

Binding energy $= -E = +\frac{GMm}{2r}$

Angular Momentum of Satellite

Angular momentum of a satellite,

$$L = mv_0 r = \sqrt{m^2GMr}$$

During the motion of planet and satellite, the angular momentum will remain conserved.

Geostationary Satellite

A geosynchronous or geostationary satellite is a satellite in geosynchronous orbit with an orbital period the same as the earth's rotation period. Such a satellite returns to the same position in the sky after each sidereal day, and over the course of a day traces out a path in the sky. A special case of geosynchronous satellite is the geostationary satellite, which has a geostationary orbit (a circular geosynchronous orbit directly above the earth's equator).

Practice Zone

DAY 9

1. At the surface of a certain planet acceleration due to gravity is one-quarter of that on the earth. If a brass ball is transported on this planet, then which one of the following statements is not correct?

(a) The brass ball has the some mass on the other planet as on the earth
(b) The mass of the brass ball on this planet is a quarter of its mass as measured on the earth
(c) The weight of the brass ball on this planet is a quarter of the weight as measured on the earth
(d) The brass ball has the same volume on the other planet as on the earth

2. If both the mass and radius of the earth, each decreases by 50%, the acceleration due to gravity would

(a) remain same
(b) decrease by 50%
(c) decrease by 100%
(d) increase by 100%

3. A body is suspended on a spring balance in a ship sailing along the equator with a speed v'. If ω is the angular speed of the earth and ω_0 is the scale reading when the ship is at rest, the scale reading when the ship is sailing is

(a) ω_0
(b) zero
(c) $\omega_0\left(1 \pm \frac{2\omega v'}{g}\right)$
(d) $\omega_0\left(1 - \frac{g}{2\omega}\right)$

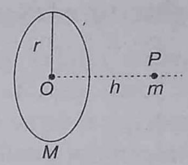
4. The maximum vertical distance through which a full dressed astronaut can jump on the earth is 0.5 m. Estimate the maximum vertical distance through which he can jump on the moon, which has a mean density 2/3rd that of the earth and radius one quarter that of the earth.

(a) 1.5 m
(b) 3 m
(c) 6 m
(d) 7.5 m

5. A uniform ring of mass M and radius R is placed directly above a uniform sphere of mass $8M$ and of same radius R. The centre of the ring is at a distance of $d = \sqrt{3}R$ from the centre of the sphere. The gravitational attraction between the sphere and the ring is

(a) $\frac{GM^2}{R^2}$
(b) $\frac{3GM^2}{2R^3}$
(c) $\frac{2GM^2}{\sqrt{2}R^2}$
(d) $\frac{\sqrt{3}GM^2}{R^2}$

6. A mass m is placed at P at a distance h along the normal through the centre O of a thin circular ring of mass M and radius r as shown in figure.



If the mass is removed further away such that OP becomes $2h$, by what factor, the force of gravitation will decrease, if $h = r$?

(a) $\frac{3\sqrt{2}}{4\sqrt{3}}$
(b) $\frac{5\sqrt{2}}{\sqrt{3}}$
(c) $\frac{4\sqrt{3}}{5}$
(d) $\frac{4\sqrt{2}}{5\sqrt{2}}$

7. The gravitational field in a region is given by $\mathbf{I} = (4\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{Nkg}^{-1}$. Work done by this field is zero when a particle is moved along the line

(a) $x + y = 6$
(b) $x + 4y = 6$
(c) $y + 4x = 6$
(d) $x - y = 6$

8. A satellite of mass m_s revolving in a circular orbit of radius r_s around the earth of mass M, has a total energy E. Then its angular momentum will be

(a) $(2Em_s r_s)^{1/2}$
(b) $(2Em_s r_s)$
(c) $(2Em_s r_s^2)^{1/2}$
(d) $(2Em_s r_s^2)$

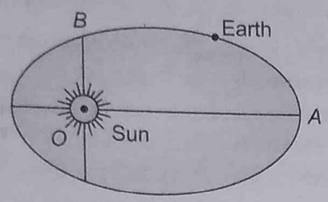
9. A mass M is split into two parts m and $(M - m)$, which are separated by a certain distance. The ratio m/M which maximizes the gravitational force between the parts is

(a) 1 : 4
(b) 1 : 3
(c) 1 : 2
(d) 1 : 1

10. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force, it exerts on a point mass m at $x = 0$, if the mass per unit length is $A + Bx^2$, is

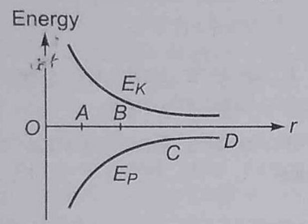
(a) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$
(b) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$
(c) $Gm\left[A\left(\frac{1}{(a+L)} - \frac{1}{a}\right) - BL\right]$
(d) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

11. The earth moves around the sun in an elliptical orbit as shown in the figure. The ratio $\frac{OA}{OB} = x$. The ratio of the speeds of the earth at B and at A is



(a) $\sqrt{x}$ (b) x
(c) x^2 (d) $x\sqrt{x}$

12. The curves for potential energy (E_P) and kinetic energy (E_K) of two particles system are shown in the figure. At what points the system will be bound energy?



(a) Only at point A (b) Only at point D
(c) At points A and D (d) At points A, B and C

13. A projectile is fired vertically upwards from the surface of the earth with a velocity kv_e, where v_e is the escape velocity and $k < 1$. If R is the radius of the earth, the maximum height to which it will rise measured from the centre of earth will be

(a) $\frac{1-k^2}{R}$ (b) $\frac{R}{1-k^2}$
(c) $R(1-k^2)$ (d) $\frac{R}{1+k^2}$

14. Two metallic spheres each of mass M are suspended by two strings each of length L. The distance between the upper ends of strings is L. The angle which the strings will make with the vertical due to mutual attraction of the spheres is

(a) $\tan^{-1}\left[\frac{GM}{gL}\right]$ (b) $\tan^{-1}\left[\frac{GM}{2gL}\right]$
(c) $\tan^{-1}\left[\frac{GM}{gL^2}\right]$ (d) $\tan^{-1}\left[\frac{2GM}{gL^2}\right]$

15. A body is released from a point distance r from the centre of earth. If R is the radius of the earth and $r > R$, then the velocity of the body at the time of striking the earth will be

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$
(c) $\sqrt{\frac{2gRr}{r-R}}$ (d) $\sqrt{\frac{2gR(r-R)}{r}}$

16. A research satellite of mass 200 kg circles the earth in an orbit of average radius $\frac{3R}{2}$, where R is the radius of earth. Assuming the gravitational pull on a mass of 1 kg on the earth's surface to be 10 N, the pull on the satellite will be

(a) 880 N (b) 889 N
(c) 885 N (d) 892 N

17. If earth of radius R, while rotating with angular velocity ω becomes stand still, what will be the effect on the weight of a body of mass m at a lattitude of 45°?

(a) Remains unchanged
(b) Decreases by $R\omega^2$
(c) Increases by $R\omega^2$
(d) Increases by $R\omega^2/2$

18. If one moves from the surface of the earth to the moon, what will be the effect on its weight?

(a) Weight of a person decreases continuously with height from the surface of the earth
(b) Weight of a person increases with height from the surface of the earth
(c) Weight of a person first decreases with height and then increases with height from the surface of the earth
(d) Weight of a person first increases with height and then decreases with height from the surface of the earth

19. A satellite goes along an elliptical path around earth. The rate of change of area swept by the line joining earth and the satellite is proportional to

(a) $r^{1/2}$ (b) r
(c) $r^{3/2}$ (d) r^2

20. A solid sphere is of density ρ and radius R. The gravitational field at a distance r from the centre of the sphere, when $r < R$, is

(a) $\frac{\rho\pi G R^3}{r}$ (b) $\frac{4\pi G\rho r^2}{3}$
(c) $\frac{4\pi G\rho R^3}{3r^2}$ (d) $\frac{4\pi G\rho r}{3}$

21. The co-lattitude (angle at which direct reception is possible) angle of a geostationary satellite as observed from the ground is

(a) $\tan^{-1}(0.15)$ (b) $\sin^{-1}(0.15)$
(c) $\cos^{-1}(0.15)$ (d) $\cot^{-1}(0.05)$

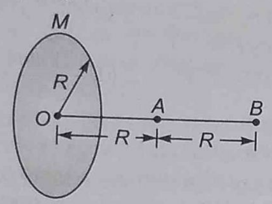
22. Two small satellites move in circular orbits around the earth, at distances r and $r + \Delta r$ from the centre of the earth. This time periods of rotation are T and $T + \Delta T$, ($\Delta r << r, \Delta T << T$). Then, ΔT is equal to

(a) $\frac{3}{2}T\frac{\Delta r}{r}$ (b) $\frac{2}{3}T\frac{\Delta r}{r}$
(c) $\frac{-3}{2}T\frac{\Delta r}{r}$ (d) $T\frac{\Delta r}{r}$

23. There is a crater of depth $\frac{R}{100}$ on the surface of the moon of radius R. A projectile is fired vertically upwards from the crater with a velocity equal to the escape velocity v from the surface of the moon. Maximum height attained by projectile is

(a) $99.5\,R$ (b) R
(c) ∞ (d) $66.2\,R$

24. A ring having non-uniform distribution of mass having mass M and radius R is being considered. A point mass m_0 is taken slowly from A to B along the axis of the ring. In doing so, work done by the external force against the gravitational force exerted by ring is



(a) $\frac{GMm_0}{\sqrt{2}R}$ (b) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right]$
(c) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{2}}\right]$
(d) It is not possible to find the required work as the nature of distribution of mass is not known

25. The gravitational force exerted by the sun on the moon is about twice as great as the gravitational force exerted by the earth on the moon, but still the moon is not escaping from gravitational influence of the earth. Mark the option which correctly explains the above system.

(a) At some point of time the moon will escape from earth *i.e.*, above statement is false
(b) Separation between the moon and sun is larger than the separation between the earth and moon
(c) The moon-earth system is bounded one and a minimum amount of energy is required to escape the moon from the earth
(d) None of the above

26. An artificial satellite of the earth is launched in circular orbit in equatorial plane of the earth and satellite is moving from west to east. With respect to a person on the equator, the satellite is completing one round trip in 24 h. Mass of the earth is, $M = 6 \times 10^{24}$ kg. For this situation, orbital radius of the satellite is

(a) 2.66×10^4 km (b) 6400 km
(c) 36,000 km (d) 29,600 km

27. What is the direction of areal velocity of the earth around the sun? **(NCERT Exemplar)**

(a) Perpendicular to positon of the earth w.r.t. the sun at the focus
(b) Perpendicular to velocity of the earth revolving in the elliptical path.
(c) Parallel to angular displacement.
(d) Both (a) and (b)

Directions *(Q. Nos. 28 to 34) Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

28. Statement I An astronaut in an orbiting space station above the earth experiences weightlessness.

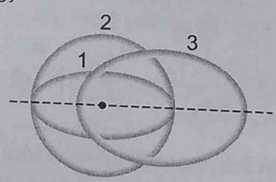
Statement II An object moving around the earth under the influence of the earth's gravitational force is in a state of 'free-fall'.

29. Statement I Kepler's laws for planetary motion are consequence of Newton's laws.

Statement II Kepler's laws can be derived by using Newton's laws.

30. Statement I Three orbits are marked as 1, 2 and 3. These three orbits have same semi-major axis although their shapes (eccentricities) are different.

The three identical satellites are orbiting in these three orbits, respectively. These three satellites have the same binding energy.



Statement II Total energy of a satellite depends on the semi-major axis of orbit according to the expression,

$$E = \frac{-GMm}{2r}$$

31. Statement I Two satellites are following one another in the same circular orbit. If one satellite tries to catch another (leading one) satellite, then it can be done by increasing its speed without changing the orbit.

Statement II The energy of the earth satellites system in circular orbits is given by $E = \frac{-GMm}{2r}$, where r is the radius of the circular orbit.

32. Statement I If the earth were a hollow sphere, gravitational field intensity at any point inside the earth would be zero.

Statement II Net force on a body inside the sphere is zero.

33. **Statement I** The length of the day is slowly increasing.

Statement II The dominant effect causing a slowdown in the rotation of the earth is the gravitational pull of other planets in the solar system.

34. **Statement I** The weight of a space craft varies as it goes from the earth to the moon.

Statement II Space craft is designed in such a way that it shows the variation in its weight when it goes from earth to sun.

Directions *(Q. Nos. 35 to 37) The satellites when launched from the earth are not given the orbital velocity initially, in practice, a multi-stage rocket propeller carries the spacecraft up to its orbit and during each stage rocket has been fired to increase the velocity to acquire the desired velocity for a particular orbit. The last stage of the rocket brings the satellite in circular/elliptical (desired) orbit. Consider a satellite of mass 150 kg in low circular orbit, in this orbit, we cannot neglect the effect of air drag. This air drag opposes the motion of satellite and hence total mechanical energy of earth-satellite system decreases means total energy becomes more negative and hence orbital radius decreases which causes the increase in kinetic energy. When the satellite comes in enough low orbit, the excessive thermal energy generation due to air friction may cause the satellite to burn up.*

35. It has been mentioned in passage that as r decreases, E decreases but K increases. The increase in K is [E = Total mechanical energy, r = Orbital radius, K = Kinetic energy] is

(a) due to increase in gravitational PE
(b) due to decrease in gravitational PE
(c) due to work done by air friction force
(d) Both (b) and (c)

36. If due to air drag, the orbital radius of the earth decreases from R to $R - \Delta R, \Delta R << R$, then the expression for increase in orbital velocity Δv is

(a) $\frac{\Delta R}{2}\sqrt{\frac{GM}{R^3}}$ (b) $-\frac{\Delta R}{2}\sqrt{\frac{GM}{R^3}}$ (c) $\Delta R\sqrt{\frac{GM}{R^3}}$ (d) $-\Delta R\sqrt{\frac{GM}{R^3}}$

37. For information given in question no. 36, the change in kinetic energy, ΔK is

(a) $-\frac{GMm}{R^2} \times \Delta R$ (b) $\frac{GMm}{R^2} \times \Delta R$
(c) $\frac{GMm}{2R^2} \times \Delta R$ (d) $-\frac{GMm}{2R^2} \times \Delta R$

Directions *(Q. Nos. 38 and 39) A satellite of mass m is revolving in a circular orbit of radius r around the earth of mass M. The speed of the satellite in its orbit is one-fourth the escape velocity from the surface of the earth.*

38. The height of the satellite above the surface of the earth is (R = radius of earth)

(a) $2R$ (b) $3R$ (c) $5R$ (d) $7R$

39. The magnitude of angular momentum of the satellite is

(a) $m\sqrt{GMR}$ (b) $\frac{m}{2}\sqrt{GMR}$
(c) $\frac{m}{2\sqrt{2}}\sqrt{GMR}$ (d) $2m\sqrt{GMR}$

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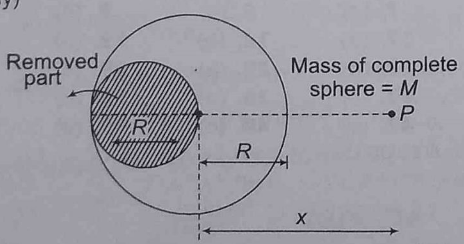
40. The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown has been 'removed out') at a very far off point, P located as shown, would be (nearly)



(a) $\frac{5}{6}\frac{GM}{R^2}$ (b) $\frac{8}{9}\frac{GM}{R^2}$ (c) $\frac{7}{8}\frac{GM}{R^2}$ (d) $\frac{6}{7}\frac{GM}{R^2}$

41. The gravitational field in a region is given by

$$\mathbf{E} = (5\text{N/kg})\,\hat{\mathbf{i}} + (12\text{ N/kg})\,\hat{\mathbf{j}}$$

If the potential at the origin is taken to be zero, then the ratio of the potential at the points (12 m, 0) and (0, 5m) is

(a) zero (b) 1 (c) $\frac{144}{25}$ (d) $\frac{25}{144}$

42. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? **[JEE Main 2013]**

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$
(c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{3R}$

43. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be **[AIEEE 2012]**

(a) 6.4×10^{11} J (b) 6.4×10^{8} J
(c) 6.4×10^{9} J (d) 6.4×10^{10} J

44. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is **[AIEEE 2011]**

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$ (c) $-\frac{9Gm}{r}$ (d) zero

45. The height at which the acceleration due to gravity becomes g (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is **[AIEEE 2009]**

(a) $2R$ (b) $\frac{R}{\sqrt{3}}$

(c) $\frac{R}{2}$ (d) $\sqrt{2}\,R$

46. This question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2008]**

Statement I For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.

Statement II If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(a) Statement I is true, Statement II is true Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement II is true; Statement II is false
(d) Statement I is false; Statement II is true

47. Average density of the earth **[AIEEE 2006]**

(a) does not depend on g
(b) is a complex function of g
(c) is directly proportional to g
(d) is inversely proportional to g

48. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of the earth. When both d and h are much smaller than the radius of the earth, then which one of the following is correct? **[AIEEE 2005]**

(a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$

(c) $d = 2h$ (d) $d = h$

49. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to **[AIEEE 2004]**

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$

(c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

50. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[AIEEE 2004]**

(a) gx (b) $\frac{gR}{R-x}$

(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

51. The time period of a satellite of earth is 5 h. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become **[AIEEE 2003]**

(a) 10 h (b) 80 h
(c) 40 h (d) 20 h

52. The escape velocity of body depends upon mass as **[AIEEE 2002]**

(a) m^0 (b) m^1 (c) m^2 (d) m^3

Answers

1. (b)	2. (d)	3. (c)	4. (b)	5. (d)	6. (d)	7. (c)	8. (c)	9. (c)	10. (b)
11. (b)	12. (d)	13. (b)	14. (c)	15. (d)	16. (b)	17. (d)	18. (b)	19. (a)	20. (d)
21. (b)	22. (a)	23. (a)	24. (b)	25. (c)	26. (a)	27. (d)	28. (a)	29. (d)	30. (a)
31. (d)	32. (a)	33. (d)	34. (c)	35. (b)	36. (a)	37. (c)	38. (d)	39. (c)	40. (c)
41. (b)	42. (a)	43. (d)	44. (c)	45. (a)	46. (c)	47. (c)	48. (c)	49. (a)	50. (d)
51. (c)	52. (a)								

Hints & Solutions

1. The mass of a body is always constant and does not change with position.

2. Here, $g = GM/R^2$ and

$$g' = \frac{G(M/2)}{(R/2)^2} = \frac{2GM}{R^2} = 2g$$

$\therefore$ % increase in $g = \left(\frac{g' - g}{g}\right) \times 100$

$$= \left(\frac{2g - g}{g}\right) \times 100 = 100\%$$

3. At the equator, $g_1 = g\left[1 - \frac{R\omega^2}{g}\right] = g - R\omega^2$

$$\omega_0 = mg_1 = m(g - R\omega^2) = m\left[g - \frac{v^2}{R}\right]$$

Speed of ship relative to the velocity of centre of the earth will be $v \pm v'$. The weight recorded by the spring balance in the sailing ship will be

$$\omega = m\left[g - \frac{(v \pm v')^2}{R}\right]$$

$$\frac{\omega}{\omega_0} = \left[1 - \frac{(v \pm v')^2}{Rg}\right]\left[1 - \frac{v^2}{Rg}\right]^{-1}$$

$$= \left[1 - \frac{(v \pm v')^2}{Rg}\right]\left[1 + \frac{v^2}{Rg}\right]$$

$$= 1 - \frac{(v \pm v')^2}{Rg} + \frac{v^2}{Rg} + \dots$$

$$= 1 \pm \frac{2vv'}{Rg} = 1 \pm \frac{2\omega v'}{g} \qquad (\because v = R\omega)$$

$$\omega = \omega_0\left[1 \pm \frac{2\omega v'}{g}\right]$$

4. On the moon, $g_m = \frac{4}{3}\pi G\,(R/4)(2\rho/3) = \frac{1}{6}\left(\frac{4}{3}\pi GR\rho\right) = \frac{1}{6}g$

Work done in jumping $= m \times g \times 0.5 = m \times (g/6)\,h_1$

$$h_1 = 0.5 \times 6 = 3.0 \text{ m}$$

5. Gravitational intensity due to the ring at a distance $d = \sqrt{3}R$ on its axis is

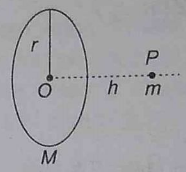
$$I = \frac{GMd}{(d^2 + R^2)^{3/2}} = \frac{GM \times \sqrt{3}R}{(3R^2 + R^2)^{3/2}}$$

$$= \frac{\sqrt{3}GM}{8R^2}$$

Force on sphere $= (8M)I = (8M) \times \frac{\sqrt{3}GM}{8R^2}$

$$= \frac{\sqrt{3}GM^2}{R^2}$$

6. Gravitational force acting on an object of mass m, placed at point P at a distance h along the normal through the centre of a circular ring of mass M and radius r is given by



$$F = \frac{GMmh}{(r^2 + h^2)^{3/2}}$$

When mass is displaced upto distance $2h$, then

$$F' = \frac{GMm \times 2h}{(r^2 + (2h)^2)^{3/2}}$$

$$= \frac{2GMmh}{(r^2 + 4h^2)^{3/2}}$$

When $h = r$, then

$$F = \frac{GMm \times r}{(r^2 \times r^2)^{3/2}} = \frac{GMm}{2\sqrt{2}r^2}$$

and
$$F' = \frac{2GMmr}{(r^2 + 4r^2)^{3/2}} = \frac{2GMm}{5\sqrt{5}\,r^2}$$

$\therefore$
$$\frac{F'}{F} = \frac{4\sqrt{2}}{5\sqrt{5}}$$

or
$$F' = \frac{4\sqrt{2}}{5\sqrt{5}}F$$

7. Work done by the gravitational field is zero, when displacement is perpendicular to gravitational field. Here, gravitational field,

$$I = 4\hat{i} + \hat{j}$$

If θ_1 is the angle which the line $y + 4x = 6$ makes with positive x-axis, then

$$\tan\theta_1 = \frac{1}{4}$$

$$\theta_1 = \tan^{-1}\left(\frac{1}{4}\right) = 14°6'$$

If θ_2 is the angle which the line $y + 4x = 6$ makes with positive x-axis, then

$$\theta_2 = \tan^{-1}(-4) = 75°56'$$

$$\theta_1 + \theta_2 = 90°$$

The line $y + 4x = 6$ is perpendicular to I.

8. Total energy of satellite, $E = -\frac{GMm_s}{2r_s}$ ...(i)

Orbital velocity of satellite, $v_s = \sqrt{\frac{GM}{r_s}}$

Angular momentum of satellite is given by

$$L = m_s v_s r_s = m_s \left(\frac{GM}{r_s}\right)^{1/2} r_s$$

$$= (GM\, m_s^2 r_s)^{1/2}$$

$$= (2\, E m_s r_s^2)^{1/2} \quad \text{[from Eq. (i)]}$$

9. As, $F = \frac{Gm(M-m)}{x^2}$

For maximum,

$$\frac{dF}{dm} = \frac{G}{x^2}(M-2m) = 0$$

or $$\frac{m}{M} = \frac{1}{2}$$

10. Mass of the element of length dx at a distance x from the origin $= (A + Bx^2)\, dx$

$$dF = \frac{Gm(A + Bx^2)dx}{x^2}$$

On integrating, $F = Gm \int_a^{a+L} \frac{(A+Bx^2)dx}{x^2}$

$$= Gm \int_a^{a+L} \left(\frac{A}{x^2} + B\right) dx$$

$$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

11. According to law of conservation of angular momentum;

$$mv_A \times OA = mv_B \times OB$$

$$\frac{v_B}{v_A} = \frac{OA}{OB} = x$$

12. The system will be bound at all these points where the total energy $= (E_P + E_K)$ is negative. In the given curve, at points A, B and C, the $E_P > E_K$.

13. From law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{mgh}{1 + \frac{h}{R}}$$

Here, $v = kv_e = k\sqrt{2gR}$

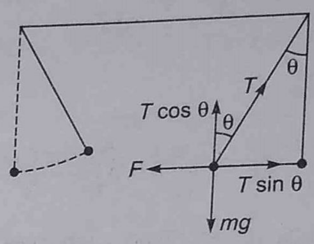
$$\frac{1}{2}m k^2 \cdot 2gR = \frac{mg\,(r-R)}{1 + \frac{r-R}{R}}$$

$$k^2 R\left[1 + \frac{r-R}{R}\right] = r - R$$

$$k^2 r = r - R$$

$$\Rightarrow \quad r = \frac{R}{1-k^2}$$

14. The metallic spheres will be at positions as shown in the figure.



From the figure

$$T\sin\theta = F = \frac{GM \times M}{L^2} = \frac{GM^2}{L^2}$$

and $$T\cos\theta = Mg$$

$$\Rightarrow \quad \tan\theta = \frac{GM}{gL^2}$$

or $$\theta = \tan^{-1}\left(\frac{GM}{gL^2}\right)$$

15. Using law of conservation of energy,

$$-\frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{r}$$

$$= GM\left(\frac{r-R}{rR}\right)$$

$$= gR\left(\frac{r-R}{r}\right) \quad \left(\because \frac{GM}{R^2} = g\right)$$

$$\therefore \quad v = \sqrt{2\,gR\,(r-R)/r}$$

16. Here, $mg = 10$ or $1 \times g = 10 \Rightarrow g = 10\,\text{ms}^{-2}$

Now, $$g' = g\frac{R^2}{r^2} = 10 \times \frac{R^2}{(3R/2)^2} = \frac{40}{9}$$

Pull on satellite $= m'g' = 200 \times \frac{40}{9} = 889\,\text{N}$

17. We have $g' = g - R\omega^2\cos^2\lambda$;

When $\lambda = 45°$,

$$g' = g - R\omega^2(1/\sqrt{2})^2 = g - R\omega^2/2$$

When earth stops rotating, $\omega = 0$, so $g' = g$

Increase in weight of body

$$= g - (g - gR\omega^2/2) = R\omega^2/2$$

18. The gravitational attraction on a body due to the earth decreases with height and increases due to the moon at a certain height. It becomes zero and with further increase in height the gravitational attraction of the moon becomes more than that of the earth.

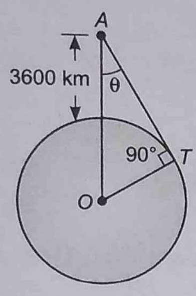
19. Areal velocity $= \frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m} = \frac{vr}{2}$

$$= \frac{r}{2}\sqrt{\frac{GM}{r}} = \frac{1}{2}\sqrt{GMr}$$

So, $\frac{dA}{dt} \propto \sqrt{r}$

20. Intensity, $I = \frac{GM}{R^3} r = \frac{Gr}{R^3}\left(\frac{4}{3}\pi R^3 \rho\right) = \frac{4\pi G\rho r}{3}$

21. Co-lattitute angle is the one up to which AT is tangent



$$\sin\theta = \frac{R}{36000 + R}$$

$$= \frac{6400}{36000 + 6400}$$

$\Rightarrow \quad \theta = \sin^{-1}(0.15)$

22. According to Kepler's law $T^2 \propto r^3$

$$T^2 = kr^3 \quad \dots \text{(i)}$$

Differentiating it, we have

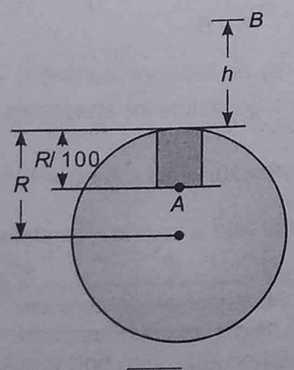
$$2T\,\Delta T = 3kr^2 \Delta r$$

Dividing it by Eq. (i), we get

$$\frac{2T\,\Delta T}{T^2} = \frac{3kr^2\Delta r}{kr^3}$$

$$\Delta T = \frac{3}{2} T \frac{\Delta r}{r}$$

23. Let particle be projected from A which reaches B at a height h.



It is projected with $v_e = \sqrt{\frac{2GM}{R}}$

KE + PE (at A) = PE (at B)

$$\frac{1}{2}m\left(\frac{2GM}{R}\right) - \frac{GmM}{2R^3}\left[3R^2 - \left(\frac{99}{100}\right)^2 R^2\right] = \frac{GMm}{R+h}$$

$$\Rightarrow \quad R + h = \frac{2R}{0.0199} \Rightarrow h = 99.5\,R$$

24. Even though the distribution of mass is unknown we can find the potential due to ring on any axial point because from any axial point the entire mass is at same distance (whatever would be the nature of distribution).

Potential at A due to ring is, $V_A = -\frac{GM}{\sqrt{2}R}$

Potential at B due to ring is, $V_B = -\frac{GM}{\sqrt{5}R}$

$$dU = U_f - U_i = U_B - U_A = m_0(V_B - V_A)$$

$$= \frac{GMm_0}{R}\left[-\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{2}}\right]$$

$$W_{gr} = -W_{ext}$$

$$\Rightarrow \quad W_{gr} = -dU = -W_{ext}$$

$$\therefore \quad W_{ext} = dU = \frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]$$

25. Option (c) is correct, a minimum amount of energy equal to |TE| of the moon-earth system has to be given to break (unbound) the system, the sun is exerting force on the moon but not providing any energy.

26. Here time period of satellite w.r.t. observer on equator is 24 h and the satellite is moving from west to east, so angular velocity of satellite w.r.t. the earth's axis of rotation (considered as fixed) is, $\omega = \frac{2\pi}{T_s} + \frac{2\pi}{T_e}$, where T_s and T_e are time periods of satellite and the earth, respectively

$$\omega = \frac{\pi}{6}(\text{h}^{-1}) = 1.45 \times 10^{-4} \text{rads}^{-1}$$

From, $\quad v = \sqrt{\frac{GM}{r}}$

$$\Rightarrow \quad r\omega = \sqrt{\frac{GM}{r}} \Rightarrow r^{3/2} = \frac{\sqrt{GM}}{\omega}$$

$$\Rightarrow \quad r = 2.66 \times 10^7 \text{m} = 2.66 \times 10^4 \text{ km}$$

27. Areal velocity of the earth around the sun is given by

$$\frac{d\mathbf{A}}{dt} = \frac{\mathbf{L}}{2m}$$

where, $\mathbf{L}$ is the angular momentum and m is the mass of the earth. But angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$

$$\therefore \quad \text{Areal velocity} \left(\frac{d\mathbf{A}}{dt}\right) = \frac{1}{2m}(\mathbf{r} \times m\mathbf{v}) = \frac{1}{2}(\mathbf{r} \times \mathbf{v})$$

Therefore, the direction of areal velocity $\left(\frac{d\mathbf{A}}{dt}\right)$ is the direction of $(\mathbf{r} \times \mathbf{v})$, *i.e.*, perpendicular to the plane containing $\mathbf{r}$ and $\mathbf{v}$ and directed as given by right hand rule.

28. Force acting on astronaut is utilised in providing necessary centripetal force, thus he feels weightlessness, as he is in a state of free fall.

29. Kepler's laws are based on observations, hit and trial method and already recorded data but later on Newton proved their correctness using his laws.

30. Total energy of earth (planet)-satellite system is independent of eccentricity of orbit and it depends on semi-major axis and masses of the planet and satellite.

31. Here, Statement I is wrong because as speed of one satellite increases, its kinetic energy and hence total energy increases *i.e.*, total energy becomes less negative and hence r increases *i.e.*, orbit changes.

32. It is clear that the net force on the body inside the hollow sphere is zero hence, then net gravitational field intensity $\left(\mathbf{E} = \frac{\mathbf{F}}{m}\right)$ at any point inside the earth must also be zero.

33. Length of the day depends upon the angular speed of earth about it's own axis, the earth is not slowing down due to conservation of angular momentum.

34. As the spacecraft moves away from the surface of the earth towards the moon, then there will be no change in the mass of spacecraft. However, its weight will keep on changing as described below

(a) It's weight will decrease in the beginning

(b) It will become zero at some point, where the force of attraction on the spacecraft due to the earth and that due to the moon becomes just equal and opposite.

(c) It will again start increasing as the spacecraft further moves towards the moon

35. From, work-energy theorem, $dK = -dU + W_{\text{air friction}}$

$W_{\text{air friction}}$ is negative, so $dK = -dU +$ (a negative quantity)

As K increases, it means U decreases by an amount greater than magnitude of $W_{\text{air friction}}$.

36. From, $v = \sqrt{\frac{GM}{R}}$

Take log on both sides and then differentiate

$$\frac{dv}{v} = \frac{-1 \times dR}{2R} \Rightarrow \frac{\Delta v}{v} = \frac{\Delta R}{2R} \qquad [\text{as } \Delta R = -dR]$$

$$\Delta v = \frac{\Delta R}{2}\sqrt{\frac{GM}{R^3}}$$

37. As, $K = \frac{mv^2}{2}$

$$\frac{dK}{K} = \frac{2\,dv}{v} \Rightarrow \frac{dK}{K} = \frac{2\Delta v}{v} \quad [\because dv = \Delta v \text{ as } v \text{ is increasing}]$$

$$= \frac{2\Delta R}{2R} = \frac{\Delta R}{R}$$

$$\Rightarrow \quad dK = K \times \frac{\Delta R}{R} = \frac{GMm}{2R} \times \frac{\Delta R}{R}$$

$$= \frac{GMm}{2R^2} \times \Delta R \qquad [\because \text{KE is increasing}]$$

38. As, $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$ ...(i)

$$v = \frac{v_e}{4} = \frac{1}{4}\sqrt{\frac{2GM}{R}} \quad \text{...(ii)}$$

Eqs. (i) and (ii) give $h = 7R$

39. Angular momentum,

$$L = mvR = m\sqrt{\frac{GM}{8R}} \times R = \frac{m}{2\sqrt{2}}\sqrt{GMR} \qquad (\because h = 7R)$$

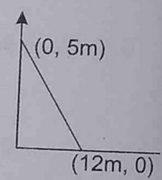
40. We have, $M' = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$

$$\therefore \quad = \frac{M}{8}$$

$\therefore$ Gravitational field at $P = \frac{GM}{R^2} - \frac{GM}{8R^2}$

$$= G \times \frac{M}{R^2}\left(1 - \frac{1}{8}\right) = \frac{7}{8}\frac{GM}{R^2}$$

41. As, $E = E_x\mathbf{i} + E_y\mathbf{j}$



Now, $V_x = E_x \cdot d_x = 5 \times 12 = 60$ V

and $V_y = E_y \times d_y = 12 \times 5 = 60$ V

So, $\therefore$ Ratio of potential at $= \frac{60}{60} = 1$

42. From conservation of planets

Total energy at the planet = Total energy at the altitude

$$\frac{-GMm}{R} + (\text{KE})_{\text{surface}} = \frac{-GMm}{3R} + \frac{1}{2}mv_A^2 \quad \text{...(i)}$$

In its orbit, the necessary centripetal force is provided by gravitational force

$$\therefore \quad \frac{mv_A^2}{(R+2R)} = \frac{GMm}{(R+2R)^2}$$

$$\Rightarrow \quad v_A^2 = \frac{Gm}{3R} \quad \text{...(ii)}$$

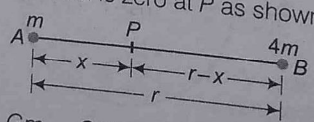
From Eqs. (i) and (ii), we get

$$(\text{KE})_{\text{surface}} = \frac{5}{6}\frac{GMm}{R}$$

43. Potential energy on the earth surface is $-mgR$ while in free space it is zero. So, to free the spaceship, minimum required energy is

$$K = mgR = 10^3 \times 10 \times 6400 \times 10^3 \text{ J} = 6.4 \times 10^{10} \text{ J}$$

44. Let gravitational field is zero at P as shown in figure.



$$\therefore \quad \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$\Rightarrow \quad 4x^2 = (r - x)^2$

$\Rightarrow \quad 2x = r - x$

$\Rightarrow \quad x = \frac{r}{3}$

$\therefore \quad V_P = -\frac{Gm}{x} - \frac{G(4m)}{r - x}$

$= -\frac{9Gm}{r} \qquad (\because x = r/3)$

45. As, $g' = \frac{GM}{(R+h)^2}$, acceleration due to gravity at height h

$\Rightarrow \quad \frac{g}{9} = \frac{GM}{R^2} \cdot \frac{R^2}{(R+h)^2} = g\left(\frac{R}{R+h}\right)^2$

$\Rightarrow \quad \frac{1}{9} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{3}$

$\Rightarrow \quad 3R = R + h \quad \Rightarrow \quad 2R = h$

46. Correct option is (c), you can make an analogy with Gauss law in electrostatics.

47. As, $g = \frac{GM}{R^2}; M = \left(\frac{4}{3}\pi R^3\right)\rho$

$\therefore \quad g = \frac{4G}{3}\frac{\pi R^3}{R^2}\rho$

or $\quad g = \left(\frac{4G\pi R}{3}\right)\rho \qquad (\rho = \text{average density})$

48. $g_h = g\left(1 - \frac{2h}{R}\right) \qquad \text{...(i)}$

and $\quad g_d = g\left(1 - \frac{d}{R}\right) \qquad \text{...(ii)}$

As per statement of the problem, $g_h = g_d$

i.e., $\quad g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$

$\Rightarrow \quad 2h = d$

49. The necessary centripetal force required for a planet to move around the sun = gravitational force exerted on it

i.e., $\quad \frac{mv^2}{R} = \frac{GM_e m}{R^n}$ or $v = \left(\frac{GM_e}{R^{n-1}}\right)^{1/2}$

Now, $\quad T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM_e}\right)^{1/2}$

$= 2\pi\left(\frac{R^2 \times R^{n-1}}{Gm_e}\right)^{1/2}$

$= 2\pi\left(\frac{R^{(n+1)/2}}{(Gm_e)^{1/2}}\right)$

or $\quad T \propto R^{(n+1)/2}$

50. The gravitational force exerted on satellite at a height x is

$$F_G = \frac{GM_e m}{(R+x)^2}$$

where, $\quad M_e$ = mass of the earth.

Since, gravitational force provides the necessary centripetal force, so $\frac{GM_e m}{(R+x)^2} = \frac{mv_o^2}{(R+x)}$

(where v_o is orbital speed of satellite)

$\Rightarrow \quad \frac{GM_e m}{(R+x)} = mv_o^2$

or $\quad \frac{gR^2 m}{(R+x)} = mv_o^2 \qquad \left(\because g = \frac{GM_e}{R^2}\right)$

or $\quad v_o = \sqrt{\left[\frac{gR^2}{(R+x)}\right]} = \left[\frac{gR^2}{(R+x)}\right]^{1/2}$.

51. According to Kepler's law, $T^2 \propto r^3$

$\Rightarrow \quad 5^2 \propto r^3 \qquad \text{...(i)}$

and $\quad (T')^2 \propto (4)^3 \qquad \text{...(ii)}$

From Eqs. (i) and (ii), we have

$$\frac{25}{(T')^2} = \frac{r^3}{64r^3}$$

or $\quad T' = \sqrt{1600}$

or $\quad T' = 40\,\text{h}$

52. Escape velocity does not depend on mass and angle of projection.

Unit Test 1 (Mechanics)

DAY 10

1. Taking into account the significant figures, what should be the value of $9.99 + 0.0099$?
(a) 9.9999 (b) 10.00
(c) 10.0 (d) 10

2. The maximum error in the measurement of mass and length of the cube are 3% and 2% respectively. The maximum error in the measurement of density will be
(a) 5% (b) 6%
(c) 7% (d) 9%

3. If C and L denote the capacitance and inductance, then the dimensional formula for $C\text{-}L$ is same as that for
(a) frequency (b) time period
(c) $(\text{frequency})^2$ (d) $(\text{time period})^2$

4. The dimensions of $(\text{velocity})^2 \div$ radius are the same as that of
(a) Planck's constant (b) Gravitational constant
(c) Dielectric constant (d) None of these

5. A soap bubble oscillates with time period T, which in turn depends on the pressure (p), density (ρ) and surface tension (σ). Which of the following correctly represents the expression for T^2?
(a) $\frac{\rho\sigma^2}{p^3}$ (b) $\frac{\rho p^3}{\sigma}$
(c) $\frac{p^3\sigma}{\rho}$ (d) $\frac{\rho}{p^3\sigma}$

6. An automobile travels on a straight road for 40 km at 30 km h^{-1}. It then continues in the same direction for another 40 km at 60 km h^{-1}. What is the average velocity of the car during its 80 km trip?
(a) 30 km h^{-1} (b) 50 km h^{-1}
(c) 40 km h^{-1} (d) 60 km h^{-1}

7. A particle had a speed of 18 ms^{-1} at a certain time 2.4 s later its speed was 30 ms^{-1} in the opposite direction. What were the magnitude and direction of the average acceleration of the particle during this 2.4 s interval?
(a) 10 ms^{-2} (b) 15 ms^{-2} (c) 20 ms^{-2} (d) 25 ms^{-2}

8. An electron has a constant acceleration of 3.2 ms^{-2}. At a certain instant its velocity is 9.6 ms^{-1}. What is its velocity 2.5 s earlier?
(a) 1.4 ms^{-1} (b) 1.6 ms^{-1}
(c) 2.4 ms^{-1} (d) 3.2 ms^{-1}

9. A rock is dropped from a 100 m high cliff. How long does it take to fall first 50 m and the second 50 m?
(a) 2 s, 3 s (b) 1.5 s, 2.5 s
(c) 1.2 s, 3.2 s (d) 3.2 s, 1.3 s

10. Two bodies of masses M_1 and M_2 are dropped from heights H_1 and H_2 respectively. They reach the ground after time T_1 and T_2 respectively. Which of the following relation is correct?
(a) $\frac{T_1}{T_2} = \left[\frac{H_1}{H_2}\right]^{1/2}$ (b) $\frac{T_1}{T_2} = \frac{H_1}{H_2}$ (c) $\frac{T_1}{T_2} = \left[\frac{M_1}{M_2}\right]^{1/2}$ (d) $\frac{T_1}{T_2} = \frac{M_1}{M_2}$

11. As a rocket is accelerating vertically upwards at 9.8 m s^{-2} near the earth's surface, it releases a projectile with zero speed relative to rocket. Immediately after release, the acceleration (in ms^{-2}) of the projectile is [Take, $g = 9.8\ ms^{-2}$]
(a) Zero (b) 9.8 ms^{-2}, up
(c) 9.8 ms^{-2}, down (d) 19.6 ms^{-2}, down

12. A bullet is fired in a horizontal direction with a muzzle velocity of 300 ms^{-1}. In the absence of air resistance, how far will it drop in travelling a horizontal distance of 150 m? [Take, $g = 10\ ms^{-2}$]
(a) 1.25 cm (b) 12.5 cm (c) 1.25 m (d) 1.25 mm

13. A fixed mortar fires a bomb at an angle of 53° above the horizontal with a muzzle velocity of 80 ms^{-1}. A tank is advancing directly towards the mortar on level ground at a constant speed of 5 ms^{-1}. The initial separation (at the instant mortar is fired) between the mortar and tank, so that the tank would be hit is [Take, $g = 10\ ms^{-2}$]
(a) 678.4 m (b) 614.4 m
(c) 64 m (d) None of these

14. The sum of the magnitudes of the two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitude of the forces?

(a) 12, 5 (b) 14, 4 (c) 5, 13 (d) 10, 8

15. A vector **a** is turned without a change in its length through a small angle $d\theta$. The value of $|\Delta \mathbf{a}|$ and Δa are respectively

(a) $0, a\,d\theta$ (b) $a\,d\theta, 0$
(c) 0, 0 (d) None of these

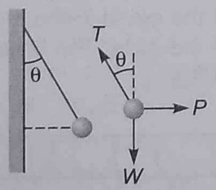
16. A particle moves towards east with a velocity of $5\ \text{ms}^{-1}$. After 10 s, its direction changes towards north with the same velocity. The average acceleration of the particle is

(a) zero (b) $\frac{1}{\sqrt{2}}\ \text{ms}^{-2}$, N-W
(c) $\frac{1}{\sqrt{2}}\ \text{ms}^{-2}$, N-E (d) $\frac{1}{\sqrt{2}}\ \text{ms}^{-2}$, S-W

17. The vectors from origin to the points A and B are $\mathbf{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\mathbf{B} = 2\hat{i} + \hat{j} - 2\hat{k}$ respectively. The area of the triangle OAB be

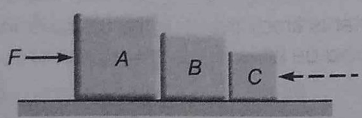
(a) $\frac{5}{2}\sqrt{17}$ sq units (b) $\frac{2}{5}\sqrt{17}$ sq units
(c) $\frac{3}{5}\sqrt{17}$ sq units (d) $\frac{5}{3}\sqrt{17}$ sq units

18. A metal sphere is hung by a string fixed to a wall. The sphere is pushed away from the wall by a stick. The forces acting on the sphere are shown in the second diagram. Which of the following statements is wrong?



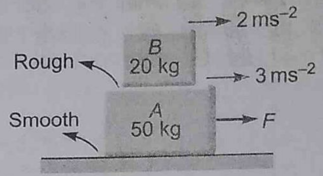
(a) $P = W \tan\theta$ (b) $\mathbf{T} + \mathbf{P} + \mathbf{W} = 0$
(c) $T^2 = P^2 + W^2$ (d) $T = P + W$

19. Three blocks A, B and C of masses 5 kg, 3 kg and 2 kg respectively are placed on a horizontal surface. The coefficient of friction between C and surface is 0.2 while between A and surface is zero and between B and surface is zero. If a force $F = 10\ \text{N}$ is first applied on A as shown and then in 2nd case on C (shown dotted), then the ratio of normal contact force between B and C in first with respect to the second case is [Take, $g = 10\ \text{ms}^{-2}$]



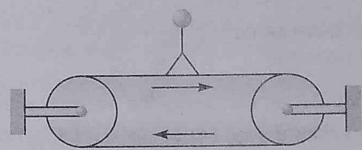
(a) $\frac{13}{12}$ (b) $\frac{12}{13}$ (c) $\frac{1}{2}$ (d) $\frac{2}{1}$

20. A 20 kg block is placed on top of a 50 kg block as shown. A horizontal force F acting on A causes an acceleration of $3\ \text{ms}^{-2}$ to A and $2\ \text{ms}^{-2}$ to B as shown. For this situation mark out the correct statement.



(a) The friction force between A and B is 40 N
(b) The net force acting on A is 150 N
(c) The value of F is 190 N
(d) All of the above

21. The figure below shows a man standing stationary w.r.t. a horizontal conveyor belt which is accelerating at $1\ \text{ms}^{-2}$. What is the net force on the man in this situation?



Take mass of the person to be as 70 kg. If the maximum acceleration of the belt, for which the man remains stationary w.r.t. the belt, is $3\ \text{ms}^{-2}$ then the coefficient of static friction between the man's shoes and the belt would be

(a) 70 N, 0.2 (b) 70 N, 0.3 (c) 700 N, 0.1 (d) 700 N, 0.3

22. A parachutist is in free fall before opening her parachute. The net force on her has a magnitude F and is directed downwards. This net force is somewhat less than, her weight w because of air resistance. Then, she opens her parachute. At the instant after her parachute fully inflates, the net force on her would be

(a) greater than F and still directed downwards
(b) less than F and still directed downwards
(c) zero
(d) directed upwards, but could be more or less than F

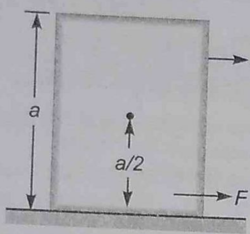
23. The drive shaft of an automobile rotates at 3600 rpm and transmits 80 HP up from the engine to the rear wheels. The torque developed by the engine is

(a) 16.58 N-m (b) 0.022 N-m
(c) 158.31 N-m (d) 141.6 N-m

24. A disk starts rotating from rest about its axis with an angular acceleration equal to $\alpha = 10\ \text{rads}^{-2}$, where t is time in seconds. At $t = 0$, the disk is at rest. The time taken by the disk to make its, first complete revolution is

(a) $\left[\frac{6\pi}{5}\right]^{1/3}$ (b) $\left[\frac{3\pi}{10}\right]^{1/3}$ (c) $\left[\frac{2\pi}{5}\right]^{1/2}$ (d) $\left[\frac{6\pi}{13}\right]^{1/3}$

25. A rectangular block of mass M and height a is resting on a smooth level surface. A force F is applied to one corner as shown in the figure. At what point should a parallel force $3F$ be applied in order that the block undergoes pure translational motion? Assume, the normal contact force between the block and surface, passes through the centre of gravity of the block.



(a) $\frac{a}{3}$, vertically above centre of gravity
(b) $\frac{a}{6}$, vertically above centre of gravity
(c) No such point exists
(d) It is not possible

26. A helicopter takes off along the vertical with 3 ms^{-2} with zero initial velocity. In a certain time t, then pilot switches off the engine. The sound dies away at the point of take off in 30 s. When engine is switched off, velocity of the helicopter is

(a) 80 ms^{-1} (b) 30 ms^{-1}
(c) 25 ms^{-1} (d) 100 ms^{-1}

27. To maintain a rotor at an uniform angular speed of 200 $rads^{-1}$, an engine needs to transmit a torque of 180 N-m. What is the power required by the engine? (Assume efficiency of the engine to be 80%)

(a) 36 kW
(b) 18 kW
(c) 45 kW
(d) 54 kW

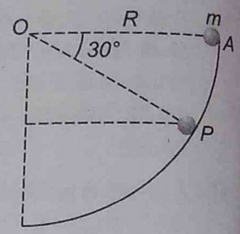
28. When a ball is whirled in a circle and the string supporting the ball is released, the ball flies off tangentially. This is due to

(a) the action of centrifugal force
(b) inertia for linear motion
(c) centripetal force
(d) some unknown cause

29. When a particle is moving in a vertical circle,

(a) its radial and tangential acceleration both are constant
(b) its radial and tangential acceleration both are varying
(c) its radial acceleration is constant but tangential acceleration is varying
(d) its radial acceleration is varying but tangential acceleration is constant

30. A particle of mass m slides on a quarter part of a smooth sphere of radius R as shown in the figure. It is released from rest at A, the normal contact force exerted by surface on the particle, when it reaches P is



(a) $\frac{mg}{2}$ (b) $\frac{3mg}{2}$ (c) $mg \times \frac{\sqrt{3}}{2} + mg$ (d) $\frac{mg\sqrt{3}}{2}$

31. If an object weighs 270 N at the earth's surface, what will be the weight of the object at an altitude equal to twice the radius of the earth?

(a) 270 N (b) 90 N (c) 30 N (d) 60 N

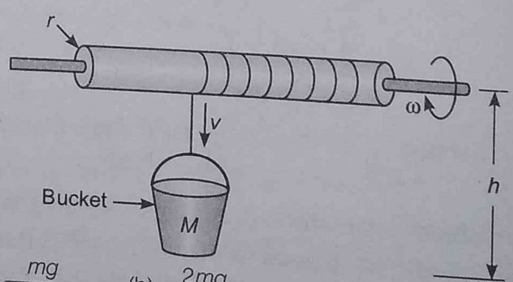
32. At its aphelion, the planet mercury is 6.99×10^{10} m from the sun, and at its perihelion it is 4.6×10^{10} m from the sun. If its orbital speed at aphelion is 3.88×10^4 ms^{-1}, then its perihelion orbital speed would be

(a) 3.88×10^4 ms^{-1} (b) 5.90×10^4 ms^{-1}
(c) 5.00×10^4 ms^{-1} (d) 5.5×10^4 ms^{-1}

33. If R is the radius of the orbit of a geosynchronous satellite and another satellite is orbiting around the earth in a circular orbit of radius $\frac{R}{2}$, then its time period would be

(a) $6\sqrt{2}$ h (b) 6 h
(c) 12 h (d) Cannot be determined

34. A cylinder of mass M and radius r is mounted on a frictionless axle over a well. A rope of negligible mass is wrapped around the cylinder and a bucket of mass m is suspended from the rope. The linear acceleration of the bucket will be

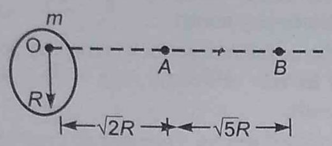


(a) $\frac{mg}{M+2m}$ (b) $\frac{2mg}{m+2M}$ (c) $\frac{mg}{2M+m}$ (d) $\frac{2mg}{M+2m}$

35. There is a crater of depth $\frac{R}{100}$ on the surface of the moon, where R is the radius of the moon. A projectile is fired vertically upwards from the crater with velocity equal to the escape velocity v from the surface of the moon. The maximum height attained by the projectile is

(a) 100 R (b) R (c) 85 R (d) 99.5 R

36. A circular ring having an uniformly distributed mass m and radius R is as shown in the figure. If a point



mass m is taken slowly from A to B, then work done by the external agent will be

(a) $-\frac{GMm}{R}\left[\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right]$ (b) $\frac{GMm}{R}\times\frac{1}{\sqrt{5}}$

(c) $\frac{GMm}{R}\left[\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right]$ (d) $\frac{GMm}{R}\left[\frac{1}{\sqrt{10}}\right]$

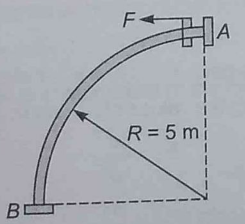
37. A rocket of initial mass (including fuel) 15000 kg ejects mass at a constant rate of 25 kgs^{-1} with a constant relative speed of 15 kms^{-1}. The acceleration of the rocket, 5 min after the blast is [Neglect gravity effect]

(a) 40 ms^{-2} (b) 50 ms^{-2} (c) 60 ms^{-2} (d) 45 ms^{-2}

38. An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 ms^{-1}. Frictional force of 2000 N is opposing its motion. The minimum power delivered by the motor to the elevator is [Take $g = 10\,ms^{-2}$]

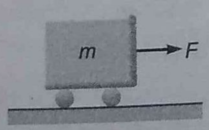
(a) 36 kW (b) 4 kW (c) 40 kW (d) -40 kW

39. A bead of mass $\frac{1}{2}$ kg starts from rest from a point A to B move in a vertical plane along a smooth fixed quarter ring of radius 5 m, under the action of a constant horizontal force $F = 5$N as shown. The speed of the bead as it reaches the point B is [Take $g = 10\,ms^{-2}$]



(a) 14.14 ms^{-1} (b) 7.07 ms^{-1} (c) 5 ms^{-1} (d) 25 ms^{-1}

40. A car (treat it as particle) of mass m is accelerating on a level smooth road under the action of a single force F. The power delivered to the car is constant and equal to P. If the velocity of the car at an instant is v, then after travelling how much distance, it becomes double?



(a) $\frac{7mv^3}{3P}$ (b) $\frac{4mv^3}{3P}$ (c) $\frac{mv^3}{P}$ (d) $\frac{18mv^3}{7P}$

Directions *(Q. Nos. 41 to 46) Each of these questions contains two statements Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

41. By considering the earth to be non spherical

Statement I As, one moves from equator to the pole of the earth, the value of accelaration due to gravity increases.

Statement II If the earth stops rotating about its own axis, the value of accelaration due to gravity will be same at pole and at equator.

42. Statement I Total torque on a system is independent of the origin if the total external force is zero.

Statement II Torque due to a couple is independent of the origin.

43. Statement I For a continuously moving particle, the average speed of the particle can never decrease as the time elapsed increases.

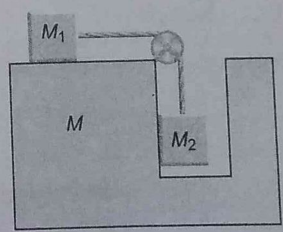
Statement II Average speed

$= \frac{\text{Total distance travelled}}{\text{Time elapsed}}$ and total distance travelled by a continuously moving particle can never decrease as time passes.

44. Statement I Acceleration of a moving particle can change its direction without any change in the direction of velocity.

Statement II If the change in velocity vector changes direction, the direction of acceleration vector also changes.

45. Statement I In the diagram shown, all the surfaces are frictionless. If the system is released from rest, the block of mass M moves towards left.

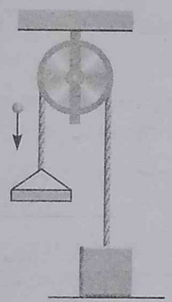


Statement II For the described situation normal force between M_2 and M (acting on M in left direction) is greater than the horizontal component of reaction force exerted by the hinge on M (towards right).

46. Statement I A person moves his hand on the top surface of a perfectly rigid block, which is rigidly fixed to the ground. In this case, work done by person's hand on the block is zero.

Statement II Work done by a force **F** in any time interval is the dot product of **F** with the displacement of point of application of **F** in same time interval in frame *S*.

Directions (Q. Nos. 47 to 50) *A pan of mass $m = 1.5$ kg and a block of mass $M = 3$ kg are connected to each other by a light inextensible string, passing over a light pulley as shown in the figure. Initially the block is resting on a horizontal floor. A ball of mass $m_0 = 0.5$ kg collides with the pan at a speed $v_0 = 20\ ms^{-1}$. Consider this instant of collision as $t = 0$. Assume collision to be perfectly inelastic. [Take, $g = 10\ ms^{-2}$]*



47. Mark the correct statement(s) for this situation.
(a) After the collision, the (pan + ball) system moves downward with decreasing speed
(b) After the collision, the block is moving upwards with same speed as with which the (ball + pan) system is moving downwards
(c) The block will jerk for a number of times during its motion
(d) All of the above

48. Find the time t at which the block strikes the floor for the first time.
(a) 1 s (b) 2 s (c) 5 s (d) 1.5 s

49. Find the velocity of (pan + ball) at $t = 2.6$ s? Assume that the block comes to rest instantaneously after striking the floor.
(a) 4 ms^{-1}, downward (b) 4 ms^{-1}, upward
(c) 0.6 ms^{-1}, upward (d) 0.4 ms^{-1}, downward

50. Find the maximum height reached by the block after the 2nd jerk.
(a) 0.2 m (b) 0.64 m
(c) 0.16 m (d) No jerk for the 2nd time

Answer *with* Solutions

1. (*d*) $9.99 + 0.0099 = 9.9999 = 10$

2. (*d*) Here, $\frac{\Delta M}{M} = 3\% = \pm 0.03$ and $\frac{\Delta l}{l} = 2\% = \pm 0.02$

Hence, $\frac{\Delta V}{V} = \frac{3\Delta l}{l} = \pm 0.06$

Now $\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V}$ $\left(\because \text{Density} = \frac{\text{Mass}}{\text{Volume}}\right)$

$= \pm 0.09 = 9\%$

3. (*d*) Time period of *C*-*L* oscillations is given by $2\pi\sqrt{LC}$.

Hence, $[LC] = [\text{time period}]^2$

4. (*d*) Dimensional formula of $(\text{velocity})^2 \div$ radius

$$= \frac{[M^0LT^{-1}]^2}{[M^0LT^0]} = [M^0LT^{-2}] = [\text{acceleration}]$$

Note that in circular motion, centripetal acceleration is $\frac{V^2}{R}$

5. (*a*) Here, $T^2 = p^a \rho^b \sigma^c$ ... (i)

Putting the dimensions of the quantities RHS, we get

$= [ML^{-1}T^{-2}]^a [ML^{-3}]^b [MT^{-2}]^c$

$= [M^{a+b+c} L^{-a-3b} T^{-2a-2c}]$

Hence, $a + b + c = 0$

$-a - 3b = 0$

and $-2a - 2c = 2$

On solving, we get $a = -3$ *b*, $b = 1$ and $c = 2$

So, after putting the values of *a*, *b* and *c* in Eq. (i), we get

$$T^2 = \frac{\rho\sigma^2}{p^3}$$

6. (*c*) Average velocity $(v_{av}) = \frac{\Delta x}{\Delta t}$

where, Δx is the displacement in a given time interval. For first part of the journey, car's time interval is

$$\Delta t_1 = \frac{40}{30} = 1.33 \text{ h}$$

For second part of the journey car's time interval is

$$\Delta t_2 = \frac{40}{60} = 0.67 \text{ h}$$

Hence, the total displacement

$$\Delta x = \Delta x_1 + \Delta x_2 = 40 + 40 = 80 \text{ km}$$

and the total time interval $\Delta t = \Delta t_1 + \Delta t_2 = 1.33 + 0.67 = 2$ h

The average velocity is

$$v_{av} = \frac{80}{2} = 40 \text{ kmh}^{-1}$$

7. (*c*) The average acceleration in the given interval is

$$a_{av} = \frac{(v_2 - v_1)}{(t_2 - t_1)}$$

Take $v_1 = 18\,\text{ms}^{-1}, v_2 = -30\,\text{ms}^{-1}, t_1 = 0$

and $t_2 = 2.4\,\text{s}$

$$a_{av} = \frac{-30 - 18}{2.4}$$

$$= -20\,\text{ms}^{-1}$$

The – ve sign indicates that the acceleration is opposite to the original direction of travel.

8. (*b*) Here, we calculate the velocity for a time interval before $t = 0$, the value we use for t is negative, $t = -2.5$ s.

Thus,

$$v = (9.6) + (3.2)(-2.5) \qquad [\because v = v_0 + \alpha t]$$

$$v = 1.6\,\text{ms}^{-1}$$

9. (*d*) Take the *y*-axis to be upward, at $v_0 = 0$

and solve $y = v_0 t - \frac{1}{2} g t^2$ for t

So, $t = \sqrt{-2y/g}$

Here, for $y = -50$ m

So, $t = \sqrt{-\frac{2(-50)}{9.8}} = 3.2$ s

Here, $y = -100$ m

$t = \sqrt{-\frac{2\,(-100)}{9.8}} = 4.5$ s

Hence, the difference is the time taken to fall the second 50 m.

$= 4.5 - 3.2 = 1.3$ s

10. (*a*) Distance of fall is independent of the mass of the bodies

$$H_1 = \frac{1}{2} g T_1^2$$

and $H_2 = \frac{1}{2} g T_2^2$

Hence, $\frac{T_1}{T_2} = \left[\frac{H_1}{H_2}\right]^{1/2}$

11. (*d*) As nothing has been mentioned that w.r.t. which frame of reference is to be found, it means we have to compute w.r.t. frame of reference of earth. As the object is released, its acceleration w.r.t. ground is only due to the influence of gravity of the earth and hence, is equal to 9.8 ms^{-2} in the downward direction.

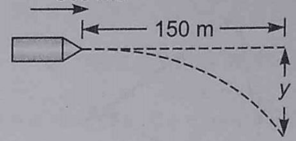
Acceleration of projectile w.r.t. rocket is

$$\mathbf{a} = \mathbf{a}_{PG} - \mathbf{a}_{HG}$$

$$= 9.8 - (-9.8)$$

$$= 19.6\,\text{ms}^{-2}, \text{downward}$$

12. (*c*) Let the bullet, dropped by *y* metre while covering a horizontal distance of 150 m.



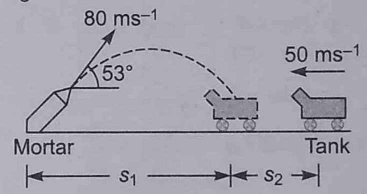
Let t be the time taken by the bullet to cover a horizontal distance of 150 m, then

$$150 = 300t$$

$$\Rightarrow \quad t = \frac{1}{2}\,\text{s}$$

$$y = \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times \frac{1}{4} = 1.25\,\text{m}$$

13. (*a*) The situation is shown clearly in figure. Time of flight of bomb is $T = \frac{2u\sin\theta}{g}$



$$= \frac{2 \times 80 \times 4}{10 \times 5} = 12.8\,\text{s}$$

Distance travelled by the tank in *T* seconds is

$$s_1 = 5T = 5 \times 12.8 = 64\,\text{m}$$

The horizontal distance travelled by bomb in *T* seconds is

$$s_2 = \frac{u^2 \sin 2\theta}{g}$$

$$s_2 = \frac{80^2 \times 2 \times \frac{3}{5} \times \frac{4}{5}}{10}$$

$$= 614.4\,\text{m} \qquad (\because \sin 2\theta = 2\sin\theta.\cos\theta)$$

So, required separation

$$s = s_1 + s_2 = 678.4\,\text{m}$$

14. (*c*) Let *P* be the smaller force and *Q* be the greater force, then according to the problem

$$P + Q = 18 \qquad \ldots \text{(i)}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12 \qquad \ldots \text{(ii)}$$

$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90° = \infty$$

$$\therefore \quad P + Q\cos\theta = 0 \qquad \ldots \text{(iii)}$$

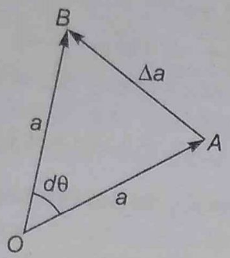
By solving Eqs. (i), (ii) and (iii), we get,

$$P = 5$$

and $$Q = 13$$

15. (*b*) From the figure $|\mathbf{OA}| = a$ and $|\mathbf{OB}| = a$

Also from the triangle rule



$$\mathbf{OB} - \mathbf{OA} = \mathbf{AB} = \Delta\mathbf{a}$$

$\Rightarrow$ $$|\Delta\mathbf{a}| = AB$$

Using, $$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$\Rightarrow$ $$AB = a.d\theta$$

So, $$|\Delta\mathbf{a}| = a.d\theta$$

Δa means a change in magnitude of the vector, *i.e.*, $|\mathbf{OB}| - |\mathbf{OA}| \Rightarrow a - a = 0$

So, $$\Delta a = 0$$

16. (*b*) $\Delta u = 2u\sin\left(\frac{\theta}{2}\right) = 2\times 5\times \sin 45° = \frac{10}{\sqrt{2}}$

$\therefore$ $$a = \frac{\Delta u}{\Delta t} = \frac{10/\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\text{ms}^{-2} \text{ in } N\text{-}W \text{ direction}$$

17. (*a*) Given $\mathbf{OA} = \mathbf{a} = 3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{OB} = \mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

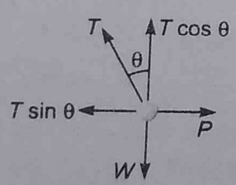
$$\therefore (\mathbf{a}\times\mathbf{b}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix} = (12-2)\hat{\mathbf{i}} + (4+6)\hat{\mathbf{j}} + (3+12)\hat{\mathbf{k}}$$

$$= 10\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 15\hat{\mathbf{k}}$$

$$\Rightarrow |\mathbf{a}\times\mathbf{b}| = \sqrt{10^2+10^2+15^2} = \sqrt{425} = 5\sqrt{17}$$

$$\text{Area of } \Delta OAB = \frac{1}{2}|\mathbf{a}\times\mathbf{b}| = \frac{5\sqrt{17}}{2} \text{ sq. units.}$$

18. (*c*)



As the metal sphere is in equilibrium under the effect of the three forces therefore, $\mathbf{T} + \mathbf{P} + \mathbf{W} = 0$

From the figure, $$T\cos\theta = W \quad \text{...(i)}$$
$$T\sin\theta = P \quad \text{... (ii)}$$

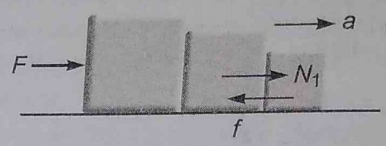
From Eqs. (i) and (ii), we get, $P = W\tan\theta$

and $$T^2 = P^2 + W^2 \quad (\text{as, } \sin^2\theta + \cos^2\theta = 1)$$

19. (*a*) Friction force between C and surface is
$$f = \mu\times 2g = 0.2\times 2\times 10 = 4\text{N}$$

Case I $(5+3+2)a = F - f$

$\Rightarrow$ $$a = \frac{10-4}{10} = \frac{6}{10}$$

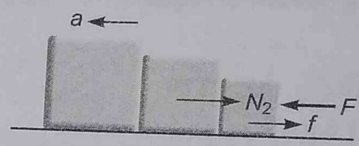


For C, $$N_1 - f = 2a$$

$\Rightarrow$ $$N_1 = 2\times\frac{6}{10} + 4 = \frac{26}{5}\text{N}$$

Case II $F - f = (5+3+2)a$

$\Rightarrow$ $$a = \frac{10-4}{10} = \frac{6}{10}$$

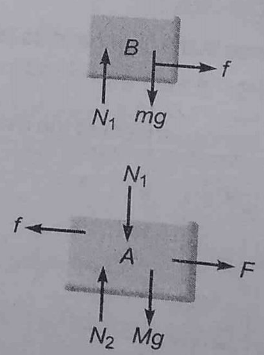


For C, $F - N_2 - f = 2a$

$\Rightarrow$ $$N_2 = 10 - 4 - 2\times\frac{6}{10}$$
$$= \frac{24}{5}\text{N}$$

Required ratio is, $$\frac{N_1}{N_2} = \frac{26}{24} = \frac{13}{12}$$

20. (*d*) As the acceleration of A and B are different, it means there is a relative motion between A and B. The free body diagram of A and B can be drawn as



For A, $$F - f = Ma_A = 50\times 3$$

For B, $$f = ma_B = 20\times 2$$

21. (*b*) As person remains stationary w.r.t. belt, so acceleration of the person w.r.t. ground is the same as that of belt w.r.t. ground. So, net force acting on the person is, $f = ma = 70\times 1 = 70\text{N}$. Let coefficient of static friction between the man's shoes and belt is μ_S, then

$\Rightarrow$ $$a_{max} = \mu_s g$$
$$\mu_s = 0.3$$

As, $$a_{max} = 3\,\text{ms}^{-2}$$

22. (*d*) As the parachute inflates fully, the force of air friction increases by a large amount and the parachutist starts decelerating, *i.e.*, net force acting on her is in upward direction but the magnitude of the net force cannot be determined from given information.

23. (*c*) From $P = \tau\omega$

$$\tau = \frac{P}{\omega}$$

It is given, $P = 80 \text{ HP} = 80 \times 746 \text{ W} = 59680 \text{ N-ms}^{-1}$

$$\omega = 3600 \text{ rpm} = \frac{3600}{60} \times 2\pi \text{ rads}^{-1}$$

$$= 120\pi \text{ rads}^{-1}$$

So, $\tau = 158.31 \text{ N-m}$

24. (*c*) As, $\omega = \omega_0 \pm \alpha t$

$\Rightarrow \quad \omega_0 = 10t$

or, $\frac{d\theta}{dt} = 10t$

On integrating, (for one complete revolution)

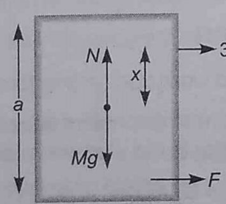
$$\int_0^{2\pi} d\theta = \int_0^T 10t\,dt$$

$$\Rightarrow \quad 2\pi = \frac{10T^2}{2}$$

$$\text{or} \quad \sqrt{\frac{2}{5}\pi} = T$$

$$\Rightarrow \quad T = \left(\frac{2\pi}{5}\right)^{\frac{1}{2}}$$

25. (*b*) The free body diagram of the block can be drawn as shown. As body has to move in a pure translational motion, the torque about the centre of gravity must be zero.



$$3F \times x = F \times \frac{a}{2} \Rightarrow x = \frac{a}{6}$$

26. (*a*) The altitude of the helicopter when engine is switched off $h = \frac{at_1^2}{2}$. Sound is not heard after $t_2 = t_1 + \frac{at_1^2}{2c}$, where c = speed of sound.

$$at_1^2 + 2ct_1 - 2ct_2 = 0$$

$$t_1 = \frac{-2c \pm \sqrt{4c^2 + 8cat^2}}{2a}$$

$$t_1 = -\frac{c}{a} + \sqrt{\frac{c^2}{a^2} + \frac{2c}{a}t_2}$$

$$\therefore \quad v = at_1 = -320 + \sqrt{(320)^2 + 2 \times 320 \times 3 \times 30}$$

$$= \sqrt{1600 \times (10)^2} - 320 = 400 - 320 = 80 \text{ ms}^{-1}$$

27. (*c*) Power required for rotor, $P = \tau \cdot \omega = 36$ kW

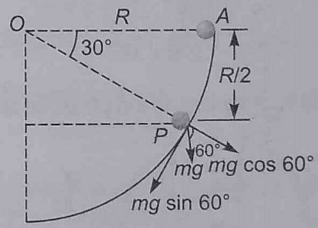
Power of engine, $P_0 = \frac{P}{0.8} = 45$ kW (as efficiency is 80%)

28. (*b*) This is due to moment of Intertia for linear motion of string.

29. (*b*) In a vertical circle, both radial and tangential components of the acceleration change direction at every instant.

30. (*b*) Apply Work-Energy theorem at *A* and *P*,

$$\frac{mv^2}{2} - 0 = mg \times \frac{R}{2}$$



Use dynamical equations at *P*,

$$N - mg\cos 60° = \frac{mv^2}{R}$$

$$\Rightarrow \quad N = mg + \frac{mg}{2} = \frac{3\,mg}{2}$$

31. (*c*) Let, *m* be the mass of the object and *g* is the acceleration due to gravity at the earth's surface then, $mg = 270$ N.

The acceleration due to gravity at an altitude of $2R_e$ is,

$$g' = \frac{GM}{(R_e + 2R_e)^2} = \frac{g}{9}$$

So, required weight $= mg' = \frac{mg}{9} = 30$ N

32. (*b*) From the conservation of angular momentum,

$$mv_A r_A = mv_P r_P$$

$$\Rightarrow \quad v_P = \frac{3.88 \times 10^4 \times 6.99 \times 10^{10}}{4.6 \times 10^{10}} = 5.90 \times 10^4 \text{ ms}^{-1}$$

33. (*a*) From the Kelper's law, $T^2 \propto r^3$

$$\Rightarrow \quad \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow \quad \frac{24}{T_2} = \left(\frac{R}{R/2}\right)^{3/2} = 2^{3/2}$$

$$\Rightarrow \quad T_2 = \frac{24}{2^{3/2}} = \frac{24}{2\sqrt{2}} = 6\sqrt{2}\text{ h}$$

34. (*d*) Weight of bucket acts downwards while tension T in opposite direction

$$mg - T = ma$$

Also, $\tau = I\alpha = rT$

$\Rightarrow$ $\frac{1}{2}Mr^2\alpha = rT$

or, $T = \frac{Ma}{2}$

$\therefore$ $a = \frac{mg - T}{m}$

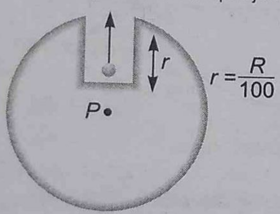
$\Rightarrow$ $a = \frac{mg - \frac{Ma}{2}}{m}$

$= \frac{2mg - Ma}{2m}$

$\Rightarrow$ $2ma + Ma = 2mg$

$\Rightarrow$ $a = \frac{2mg}{2m + M}$

35. (*d*) Let, v be the velocity with which projectile is projected, then



$$v = \sqrt{\frac{2GM}{R}}$$

where, M is the mass of the moon.

From energy conservation.

$$\frac{m \times v^2}{2} - \frac{GMm\left[3R^2 - 2\left(R - \frac{R}{100}\right)^2\right]}{2R^3} = -\frac{GMm}{R + h}$$

where, m is the mass of the projectile and h is the maximum height to which the projectile reaches.

Solving above equations, we get $h = 99.5R$

36. (*c*) Potential energy of the system when point mass is at A, is

$$U_i = -\frac{GMm}{\sqrt{2}R}$$

Potential energy of system when point mass is at B, is

$$U_f = -\frac{GMm}{\sqrt{5}R}$$

Work done by the force of gravity on the point mass, as it moves from A to B, is

$$W_{\text{gravitational}} = -dU = -(U_f - U_i)$$

$$= \frac{GMm}{R}\left[\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}}\right]$$

From work-energy theorem,

$$dK = 0 = W_{\text{grav}} + W_{\text{ext}}$$

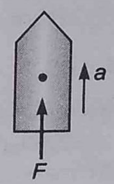
$$W_{\text{ext}} = -W_{\text{grav}}$$

$$= \frac{GMm}{R}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]$$

37. (*b*) Thrust force acting on the rocket is,

$$F = v_{\text{rel}}\frac{dm}{dt}$$

$$F = 15 \times 1000 \times 25\text{N}$$



Mass of rocket at $t = 5$ min after the blasting starts, is

$$m = 15000 - 25 \times 5 \times 60$$

$$= 7500$$

So, $F = ma$

$\Rightarrow$ $a = \frac{F}{m} = \frac{15 \times 25000}{7500}$

$= 50\ \text{ms}^{-2}$

Note : If gravity is not neglected, then $F - mg = ma$ check whether acceleration is constant here.

38. (*c*) The net downward force on the elevator is,

$$F_1 = mg + f$$

$$= 18000 + 2000$$

$$= 20000\ \text{N}$$

So, the motor has to work against this force.

To move the elevator with a constant speed, the minimum power delivered by the motor to the elevator must be,

$$P = \mathbf{F} \cdot \mathbf{v} = 20000 \times 2 = 40\ \text{kW}$$

39. (*a*) Applying the work-energy theorem,

$$\frac{1}{2} \times mv^2 - 0 = F \times R + mg \times R$$

$\Rightarrow$ $\frac{1}{2} \times \frac{1}{2} \times v^2 = 5 \times 5 + \frac{1}{2} \times 10 \times 5 = 50$

$$v = \sqrt{200} = 14.14\ \text{ms}^{-1}$$

40. (*a*) As, $P = Fv = mv\frac{dv}{ds} \times v$

$\Rightarrow$ $\int_v^{2v} mv^2 dv = \int_0^s P ds$

$\Rightarrow$ $s = \frac{7mv^3}{3P}$

41. (*c*) As one go from equator to pole of the earth the value of g increase due to decrease in latitude (λ). Also the earth is non spherical, this implies the value of g, at the poles and equitorial point on the earth's surface are unequal due to its different distances from earth's centre.

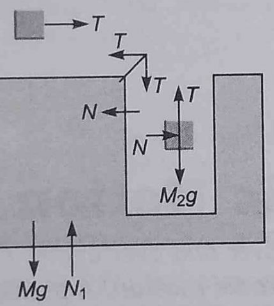
42. (*a*) If net force on the system is zero, it can be resolved into two equal and opposite forces which can be considered to form a couple.

43. (*b*) Here, both the statements are correct but II is not the correct explanation of *I*. As a particle is continuously moving, the total distance travelled is increasing with time but the particle can move slowly in some later time span as compared to earlier time span, resulting in decrease in average speed.

44. (*a*) $$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

i.e., direction of acceleration is same as that of the change in velocity vector so if direction of $\Delta\mathbf{v}$, changes, direction of **a** changes and hence, Statement I is explained by Statement II.

45. (*c*) If we draw the free body diagram of all the three blocks, they would be as shown in the figure.



Now, the forces responsible for the motion of *M* towards left are *N* and *T* (acting on *M* due to hinge).

So, it is clear that Statement I is correct and Statement II is wrong.

46. (*b*) Here, statement II is the standard definition of work done by a force. But work is done by a force on body w.r.t. some observer only when the particles of the body at which force acts get displaced from its position w.r.t. observer, this is because work is nothing but a way to transform or to transport energy and without the displacement of material particles, energy cannot be transported. But in Statement I the block is rigid. So there is no movement of particles even at microscopic level so work done by hand on the block is zero.

47. (*d*) **48.** (*b*) **49.** (*d*) **50.** (*c*)

Due to collision, an impulsive tension occurs in the string.

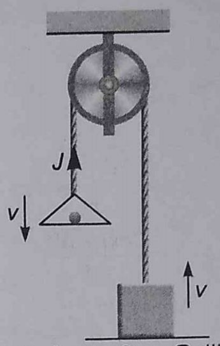
For (ball + pan) system Just after collision

$$-J = (m + m_0)v - m_0 v_0$$

For block, $J = Mv \Rightarrow v = 2\ \text{ms}^{-1}$

As the string is taut and the mass of block is greater than mass of (ball + pan), so the blocks and (ball + pan) are under deceleration.

Deceleration, $a = \dfrac{3g - 2g}{5} = 2\ \text{ms}^{-2}$



System Just after Collision

So, block will again come to ground after time t, given by

$$0 = 2t - \frac{1}{2} \times 2t^2 \Rightarrow t = 2\text{s}$$

At this instant, the (pan + ball) system is moving up with $v = 2\ \text{ms}^{-1}$ and the string becomes slack, so the system moves under gravity.

To determine the speed of the system at $t = 2.6$s, first find the time after which string is again jerked. That will happen when system of pan + ball crosses the time $t = 2$s instant during its downward journey if *i.e.*, after its time of flight of motion under gravity.

$$T = \frac{2 \times 2}{10} = 0.4\ \text{s}$$

i.e., string will again jerk at $t = 2.4$ s

For $t > 2.4$s, the system is again starts retarding with a retardation of $2\ \text{ms}^{-2}$.

Before this, we have to find the velocity of various components of the system just after jerk,

As, $-J' = 2v' - 2 \times 2$ and $J = 3v'$

$\Rightarrow$ $5v = 4$

$\Rightarrow$ $v = \dfrac{4}{5} = 0.8\ \text{ms}^{-1}$

With this velocity, the (ball + pan) moves down for 0.2 s under a retardation of $2\ \text{ms}^{-2}$.

So, the required velocity,

$$v = 0.8 - 2 \times 0.2$$
$$= 0.4\ \text{ms}^{-1}\ \text{downwards}$$

The maximum height attained by the block after 2nd jerk,

$$0 = (0.8)^2 - 2 \times 2H$$

$\Rightarrow$ $H = 0.16$ m

Day 11

Oscillations

Day 11

Outlines ...

Periodic Motion

A motion which repeats itself over and over again after a regular interval of time is called a ***periodic motion.*** *A periodic motion in which a body moves back and forth repeatedly about a fixed point (called mean position) is called* ***oscillatory*** *or* ***vibratory motion.***

Period The regular interval of time after which periodic motion repeats itself is called period of the motion.

Frequency The number of times of motion repeated in one second is called frequency of the periodic motion. Every oscillatory motion is periodic but every periodic motion is not an oscillatory motion.

Displacement as a Function of Time

In a periodic motion each displacement value is repeated after a regular interval of time, displacement can be represented as a function of time $y = f(t)$.

Periodic Function

A function which repeats its value after a fix interval of time is called a **periodic function.**

$$y(t) = y(t + T)$$

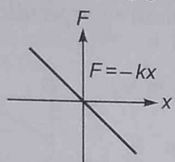
where T is the period of the function.

Trignometric functions $\sin\theta$ and $\cos\theta$ are simplest periodic functions having period of 2π.

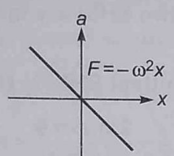
Simple Harmonic Motion

Simple Harmonic Motion (SHM) is that type of oscillatory motion in which the particle moves to and fro or back and forth about a fixed point under a restoring force whose magnitude is directly proportional to its displacement i.e., $F \propto x$ or $F = -kx$ where, k is a positive constant called the ***force constant*** *or* ***spring factor*** *and x is displacement.*

In SHM, $F = -kx$ or $a = -\omega^2 x$, *i.e.*, *F-x* graph or *a-x* graph is a straight line passing through the origin with a negative slope. The corresponding graphs are shown below



(a)
Slope = – k



(b)
Slope = – ω2

A simple harmonic motion may be mathematically expressed by a single sinusoidal (sine or cosine) function of time. One oscillation (or vibration) is said to be complete if the particle executing SHM moves from its mean position to one extreme, then to other extreme and finally back to its mean position. Time taken by the particle in completing one oscillation (or vibration) is called time period (T). Time period $T = \frac{2\pi}{\omega}$, here ω is referred as the angular frequency of SHM.

- *Differential equations of SHM For linear SHM,* $\frac{d^2y}{dt^2} + \omega^2 y = 0$, *For angular SHM,* $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$
- *All sine and cosine functions of t are simple harmonic in nature i.e., for the function*
- $y = A\sin(\omega t \pm \phi)$ *or* $y = A\cos(\omega t \pm \phi)$
- $\frac{d^2y}{dt^2}$ *is directly proportional to* $-y$. *Hence, they are simple harmonic in nature.*

Terms Related to SHM

The few important terms related to simple harmonic motion are given as

Displacement

The displacement of a particle executing SHM is, in general, expressed as $y = A\sin(\omega t + \phi)$

where A is the amplitude of SHM, ω the angular frequency $\left(\text{where } \omega = \frac{2\pi}{T} = 2\pi\nu\right)$ and ϕ the initial phase of SHM.

However, displacement may also be expressed as

$$x = A\cos(\omega t + \phi)$$

Velocity

The velocity of a particle executing SHM at an instant is defined as the time rate of change of its displacement at that instant. Velocity, $v = \omega\sqrt{A^2 - y^2}$

At the mean position ($y = 0$), during its motion $v = A\omega = v_{max}$ and at the extreme positions ($y = \pm A$) $v = 0$.

Velocity amplitude $= v_{max} = A\omega$

Acceleration

The acceleration of a particle executing SHM at an instant is defined as the time rate of change of velocity at that instant.

Acceleration, $a = -\omega^2 y$

The acceleration is also a variable. At the mean position ($y = 0$), acceleration $a = 0$ and at the extreme position ($y = \pm A$), the acceleration is $a_{max} = -A\omega^2$.

$\therefore$ Acceleration amplitude $a_{max} = A\omega^2$

Phase and Phase Relationship

Phase is that physical quantity which tells about the position and direction of motion of any particle at any moment. It is denoted by ϕ. In SHM, the velocity is ahead of the displacement by a phase $\frac{\pi}{2}$ and the acceleration is further ahead of the velocity by a phase of $\frac{\pi}{2}$.

Time Period

The time taken by a particle to complete one oscillation is called **time period**. It is denoted by T.

Acceleration in SHM, $a = -\omega^2 y$, hence,

$$|a| = \omega^2 |y| \Rightarrow \omega = \sqrt{\frac{|a|}{|y|}}$$

$\therefore$ Time period of SHM,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{|y|}{|a|}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

Composition of Two SHMs

If a particle is acted upon two separate forces each of which can produce a simple harmonic motion. The resultant motion of the particle would be a combination of two SHMS.

For which $\mathbf{F}_1 + \mathbf{F}_2 = m\dfrac{d^2\mathbf{r}}{dt}$

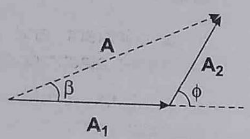
and $\mathbf{r}_1 + \mathbf{r}_2 = \mathbf{r}$ = resultant position of the particle

where m = mass of the particle.

$\mathbf{r}_1, \mathbf{r}_2$ = positions of the particle under two forces.

There are two cases

(i) When two SHM are in same direction the resultant is given by



$$x = x_1 + x_2$$
$$= A\sin(\omega t + \beta)$$

where, $x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin(\omega t + \phi)$

$$A = \sqrt{A_1^2 + 2A_1A_2\cos\delta + A_2} \text{ and } \tan\beta = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$$

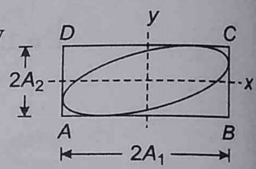
For any value of ϕ other than 0 and π resultant amplitude is between $|A_1 - A_2|$ and $A_1 + A_2$.

(ii) When two SHM are mutually perpendicular to each other.

The resultant SHM is given by

$$\frac{x}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy\cos\phi}{A_1A_2} = \sin^2\phi$$

(ellipse)



where, $x = A_1 \sin \omega t$

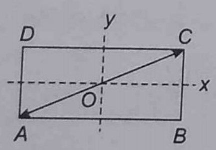
and $y = A_2 \sin(\omega t + \phi)$

Here x is always between $-A_1$ to $+A_1$ and y is always between $-A_2$ to $+A_2$.

Special Cases in Composition of Two SHMs

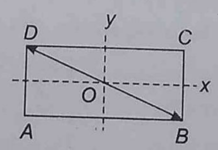
1. When $\phi = 0$,

$$y = \frac{A_2}{A_1}x$$



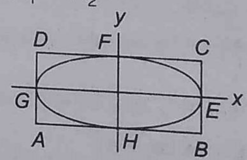
2. When $\phi = \pi$

$$y = -\frac{A_2}{A_1}x$$



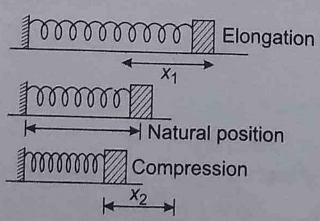
3. When $\phi = \pi/2$ If $A_1 = A_2 = A$ then

$$\frac{x}{A_1^2} + \frac{y}{A_2^2} = 1 \qquad x^2 + y^2 = A^2 \text{ (circle)}$$



Oscillations of a Spring

A spring pendulum consists of a point (small sized) mass m either suspended from a massless (or light) spring or placed on a smooth horizontal plane attached with a spring.



Here, $x_1 = x_2 = l$

If the mass is once pulled so as to stretch the spring and is then released, then a restoring force acts on it which continuously tries to restore its mean position, restoring force $F = -kl$, where k is force constant and l is the change in length of the spring under the restoring force the spring pendulum oscillates simple harmonically having time period and frequency given by

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ and } \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where k is the force constant of the spring and it is numerically equal to the force required to increase the length of the spring by unity.

If the spring is not light but has a mass m_s, then

$$T = 2\pi\sqrt{\frac{m + 1/3\,m_s}{k}}$$

If two masses m_1 and m_2, connected by a spring, are made to oscillate on a horizontal surface, then its period will be

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

where, $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ = reduced mass of the system.

Series Combination of Springs

If two springs of spring constants k_1 and k_2 are joined in series (horizontally and vertically), then their equivalent spring constant k_s is given by

(a)

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

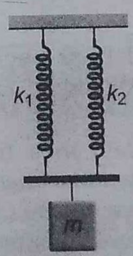
$$\Rightarrow \quad k_s = \frac{k_1 k_2}{k_1 + k_2}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k_s}}$$

$$= 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Parallel Combination of Springs

If the two springs of spring constants k_1 and k_2 are joined in parallel as shown, then their equivalent spring constant $k_p = k_1 + k_2$ hence,



(b)

$$T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

Force and Energy in SHM

We know that, the acceleration of body in SHM is $a = -\omega^2 x$.

Applying the equation of motion $\mathbf{F} = m\mathbf{a}$,

We have, $F = -m\omega^2 x = -kx$

where, $\omega = \sqrt{\dfrac{k}{m}}$

and $k = m\omega^2$

is a constant and sometimes it is called the elastic constant.

In SHM, the force is proportional and opposite to the displacement.

If a particle of mass m is executing SHM, then at a displacement x from the mean position, the particle possesses potential and kinetic energy.

At any displacement x,

Potential energy,

$$U = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} k x^2$$

Kinetic energy,

$$K = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$= \frac{1}{2} k (A^2 - x^2)$$

Total energy,

$$E = U + K$$

$$= \frac{1}{2} m\omega^2 A^2$$

$$= 2\pi^2 m\nu^2 A^2$$

If there is no friction, the total mechanical energy, $E = K + U$, of the system always remains constant even though K and U change.

Simple Pendulum

A simple pendulum, in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string. The motion of a simple pendulum is simple harmonic for very small angular displacement (α) whose time period and frequency are given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

and

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

where l is the effective length of the string and g is acceleration due to gravity.

(i) If a pendulum of length l at temperature $\theta°$C has a time period T, then on increasing the temperature by $\Delta\theta°$C its time period changes to $T \times \Delta T$,

where, $$\frac{\Delta T}{T} = \frac{1}{2}\alpha\,\Delta\theta$$

where α is the temperature coefficient of expansion of the string.

(ii) A second's pendulum is a pendulum whose time period is 2 s. At a place where $g = 9.8\ \text{ms}^{-2}$, the length of a second's pendulum is 0.9929 m (or 1 m approx).

(iii) If the bob of a pendulum (having density ρ) is made to oscillate in a non-viscous fluid of density σ, then it can be shown that the new period is

$$T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{\sigma}{\rho}\right)}}$$

(iv) If a pendulum is in a lift or in some other carriage moving vertically with an acceleration a, then the effective value of the acceleration due to gravity becomes $(g \pm a)$ and hence,

$$T = 2\pi\sqrt{\frac{l}{(g \pm a)}}$$

Here, positive sign is taken for an upward accelerated motion and negative sign for a downward accelerated motion.

(v) If a pendulum is made to oscillate in a freely falling lift or an orbiting satellite then the effective value of g is zero and hence, the time period of the pendulum will be infinity and therefore pendulum will not oscillate at all.

(vi) If the pendulum bob of mass m has a charge q and is oscillating in an electrical field E acting vertically downwards, then

$$T = 2\pi\sqrt{\frac{l}{\left(g \pm \frac{qE}{m}\right)}}$$

(vii) The positive sign is to be used if the electrical force is acting vertically downwards and negative sign if the electrical force is acting vertically upwards.

(viii) If pendulum of charge q is oscillating in an electric field E acting horizontally, then

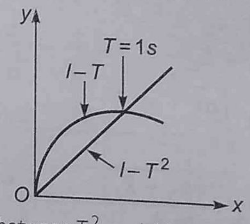
$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{q^2E^2}{m^2}}}}$$

(ix) If the length of a simple pendulum is increased to such an extent that $l \to \infty$, then its time period is

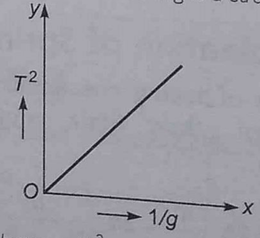
$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$$

where, R = radius of the earth.

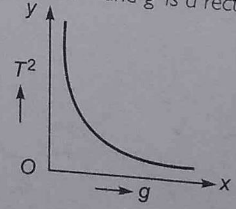
» *The graphs l-T and l-T^2 intersect at T = l s.*



» *The graph between T^2 and $1/g$ is a straight line.*



» *The graph between T^2 and g is a rectangular hyperbola.*



Free, Damped, Forced *and* Resonant Vibrations

As we know, a periodic motion in which a body moves black and forth repeatedly about a mean position is called oscillatory motion. The term vibration is sometimes used more narrowly to mean a mechanical oscillation but it is sometimes used as a synonym of oscillation. Some of the vibrations are described below.

Free Vibrations

If a body, capable of oscillating, is slightly displaced from its position of equilibrium and then released, it starts oscillating with a frequency of its own. Such oscillations are called free vibrations.The frequency with which a body oscillates is called the **natural frequency** and is given by

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here, a body continues to oscillate with a constant amplitude and a fixed frequency.

Damped Vibrations

The oscillations in which the amplitude decreases gradually with the passage of time are called **damped vibrations**.

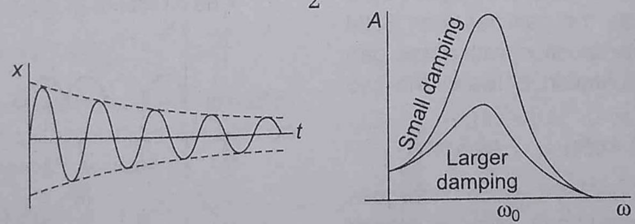
Damping force, $F_d = -bv$

where v is the velocity of the oscillator and b is a damping constant. The displacement of the oscillator is given by

$$x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)$$

where, ω' = the angular frequency $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

The mechanical energy E of the oscillator is given by $E(t) = \frac{1}{2}kA^2e^{-bt/m}$



(a) (b)

Forced Vibrations

The vibrations in which a body oscillates under the effect of an external periodic force, whose frequency is different from the natural frequency of the oscillating body, are called forced vibrations. In forced vibrations the oscillating body vibrates with the frequency of the external force and amplitude of oscillations is generally small.

Resonant Vibrations

It is a special case of forced vibrations in which the frequency of external force is exactly same as the natural frequency of the oscillator. As a result, the oscillating body begins to vibrate with a large amplitude leading to the phenomenon of resonance to occur. Resonant vibrations play a very important role in music and in tuning of station/channel in a radio/TV etc.

Practice Zone DAY 11

1. The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is **[NCERT Exemplar]**
(a) simple harmonic with period $2\pi/\omega$
(b) simple harmonic with period π/ω
(c) periodic but not simple harmonic
(d) non-periodic

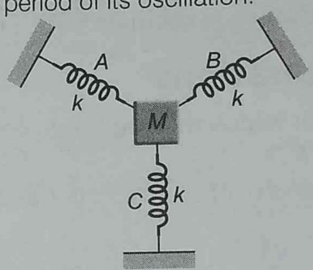
2. A body is executing SHM when its displacement from the mean position are 4 cm and 5 cm and it has velocity 10 cms^{-1} and 8 cms^{-1} respectively. Its periodic time t
(a) $\frac{2\pi}{2}$ s (b) π s
(c) $\frac{3\pi}{2}$ s (d) 2π s

3. Two particles execute simple harmonic motion on same straight line with same mean position, same time period 6 s and same amplitude 5 cm. Both the particles start SHM from their mean position (in same direction) with a time gap of 1 s. Find the maximum separation between the two particles during their motion
(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

4. A coin is placed on a horizontal platform, which undergoes horizontal SHM about a mean position O. The coin placed on the platform does not slip, when angular frequency of the SHM is ω. The coefficient of frequency between the coin and platform is μ. The amplitude of oscillation is gradually increased. The coin will be begin to slip on the platform for the first time
(a) at the mean position
(b) at the extreme position of the oscillation
(c) for an amplitude of $\mu g/\omega^2$
(d) for an amplitude of $g/\mu\omega^2$

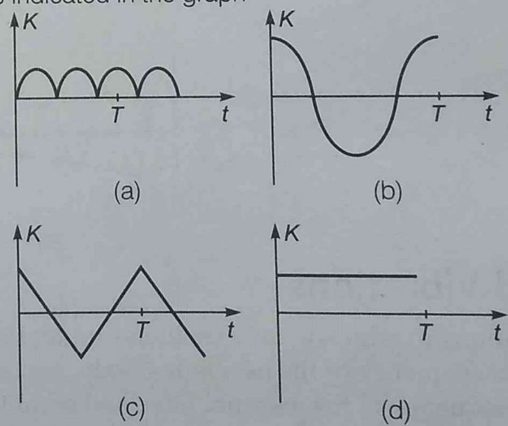
5. A block rests on a horizontal table, which is executing SHM in the horizontal direction with an amplitude a. If the coefficient of friction is μ, then the block just starts to slip when the frequency of oscillation is
(a) $\frac{1}{2\pi}\sqrt{\frac{\mu g}{a}}$ (b) $2\pi\sqrt{\frac{a}{\mu g}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{a}{\mu g}}$ (d) $\sqrt{\frac{a}{\mu g}}$

6. A particle of mass M is attached to three springs A, B and C having equal force constant k. If the particle is pushed a little towards any one of the springs and then left on its own, find the time period of its oscillation.



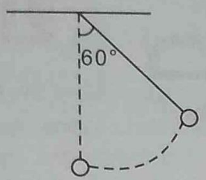
(a) $2\pi\sqrt{(M/k)}$ (b) $2\pi\sqrt{(2M/k)}$
(c) $2\pi\sqrt{(M/2k)}$ (d) $2\pi\sqrt{(M/3k)}$

7. A body performs SHM. Its kinetic energy K varies with time T as indicated in the graph



8. Two particles A and B are oscillating about a point O along a common line such that equation of A is given as $x_1 = a\cos\omega t$ and equation of B is given as $x_2 = b\sin\left(\omega t + \frac{3\pi}{2}\right)$. Then, the motion of A w.r.t. B is
(a) a simple harmonic motion with amplitude $(a - b)$
(b) a simple harmonic motion with amplitude $(a + b)$
(c) a simple harmonic motion with amplitude $\sqrt{a^2 + b^2}$
(d) not a simple harmonic motion but oscillatory motion

9. A pendulum of length $l = 1\,m$ is released from $\theta_0 = 60°$. The rate of change of speed of the bob at $\theta = 30°$ is ($g = 10\,m/s^2$).



(a) $5\sqrt{3}\,m/s^2$ (b) $5\,m/s^2$
(c) $10\,m/s^2$ (d) $2.5\,m/s^2$

10. The value of g decrease by 0.1% on a mountain as compared to sea level. If a simple pendulum is used to record the time, then the length must be
(a) increased by 0.1% (b) decreased by 0.1%
(c) increased by 0.2% (d) decreased by 0.2%

11. Two pendulums have time periods T and $\frac{5T}{4}$. They start SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completes one oscillation?
(a) 45° (b) 90°
(c) 60° (d) 30°

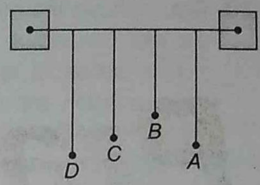
12. A piece of wood has dimension $a \times b \times c$. It is floating in a liquid of density ρ such that side a is vertical. It is now pushed down gently and released. The time period is
(a) $2\pi\sqrt{\rho a/g}$ (b) $2\pi\sqrt{abc/g}$
(c) $2\pi\sqrt{g/\rho a}$ (d) $2\pi\sqrt{bc/\rho g}$

13. A simple pendulum of length l is suspended from the roof of a train which is moving in a horizontal direction with an acceleration a. Then, the time period T is given by
(a) $2\pi\sqrt{l/g}$ (b) $2\pi\sqrt{l/(a^2+g^2)}$
(c) $2\pi\sqrt{l/(a+g)}$ (d) $2\pi\sqrt{l/(g-a)}$

14. The length of a spring is α when a force of 4 N is applied on it. The length of a spring is β when a force of 5 N is applied on it. Then find the length of the spring when a force of 9 N is applied on the spring.
(a) $5\beta - 4\alpha$ (b) $\beta - \alpha$ (c) $5\alpha - 4\beta$ (d) $9(\beta - \alpha)$

15. A simple pendulum of length l has a bob of mass m with a charge q on it. A vertical sheet of charge having surface, charge density σ passes through the point of suspension. At equilibrium, the spring makes an angle θ with the vertical. If the tension in the string, then,
(a) $\tan\theta = \frac{\sigma q}{2\varepsilon_0 mg}$ (b) $\tan\theta = \frac{\sigma q}{\varepsilon_0 mg}$
(c) $T > 2\pi\sqrt{\frac{l}{g}}$ (d) $T = 2\pi\sqrt{\frac{l}{g}}$

16. Four pendulums A, B, C and D are hung from the same elastic support as shown alongside. A and C are of the same length while B is smaller than A and D is larger than A. A is given a displacement then in steady state

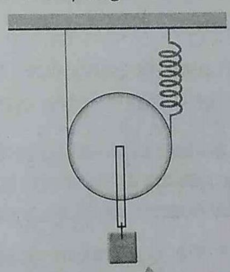


(a) D will vibrate with maximum amplitude
(b) C will vibrate with maximum amplitude
(c) B will vibrate with maximum amplitude
(d) All the four will oscillate with equal amplitude

17. A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a, and time period T. The speed of the pendulum at $x = a/2$ will be
(a) $\frac{\pi a\sqrt{3}}{T}$ (b) $\frac{\pi a\sqrt{3}}{2T}$
(c) $\frac{3\pi^2 a}{T}$ (d) $\frac{\pi a}{T}$

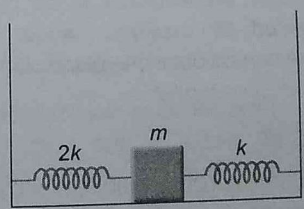
18. A mass m is suspended from a massless pulley which itself is suspended with the help of a massless extensible spring as shown below.

What will be the time period of oscillation of the mass? The force constant of the spring is k.



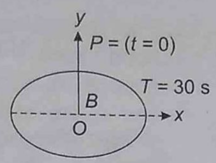
(a) $\pi\sqrt{m/k}$ (b) $2\pi\sqrt{m/k}$
(c) $4\pi\sqrt{m/k}$ (d) $2\pi\sqrt{m/2k}$

19. Two springs of force constant k and $2k$ are connected to a mass as shown below. The frequency of oscillation of the mass is



(a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$
(c) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

20. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is



[NCERT Exemplar]

(a) $x(t) = B\sin\left(\frac{2\pi r}{30}\right)$ (b) $x(t) = B\cos\left(\frac{\pi t}{15}\right)$

(c) $x(t) = B\sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$ (d) $x(t) = B\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

Directions (Q. Nos. 21 to 24) *Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

21. Statement I A particle performing SHM at certain instant is having velocity v. It again acquires a velocity v for the first time after a time interval of T second, then the time period of oscillation is T second.

Statement II A particle performing SHM can have the same velocity at two instants in one cycle.

22. Statement I A particle performing SHM while crossing the mean position is having a minimum potential energy, this minimum potential energy could be non-zero.

Statement II In the equilibrium position, the net force experienced by the particle is zero, hence potential energy would be zero at the mean position.

23. Statement I A circular metal hoop is suspended on the edge by a hook. The hoop can oscillate from one side to the other in the plane of the hoop, or it can oscillate back and forth in a direction perpendicular to the plane of the hoop. The time period of oscillation would be more when oscillations are carried out in the plane of the hoop.

Statement II Time period of physical pendulum is more if the moment of inertia of the rigid body about the corresponding axis, passing through the pivoted point is more.

24. Statement I The time period of a pendulum, in a satellite orbiting around the earth, is infinity.

Statement II Time period of a pendulum is inversely proportional to the square root of acceleration due to gravity.

Directions (Q. Nos. 25 to 27) *A platform is executing SHM in a vertical direction, with an amplitude of 5 cm and a frequency of $\frac{10}{\pi}$ vib/s. A block is placed on the platform at the lowest point of its path. [Take $g = 10\ ms^{-2}$]*

25. At what point will the block leave the platform?
(a) 2.5 cm from the mean position when acceleration is acting in a downward direction and velocity is in an upward direction
(b) 2.5 cm from the mean position when the platform is moving in an upward direction
(c) 2.5 cm above mean position when the platform is moving in a downward direction
(d) 2.5 cm below the mean position

26. Mark the correct statement(s).
(a) Normal contact force between the platform and the block is constant
(b) As platform approaches the mean position from bottom, the normal contact force between the block and the platform increases
(c) As the platform moves up and away from the mean position, the normal contact force between the block and the platform, decreases
(d) Both (b) and (c) are correct

27. At what point, the block returns to the platform?
(a) 1.3 cm above the equilibrium position
(b) 1.3 cm below the equilibrium position
(c) 4.3 cm above the equilibrium position
(d) 4.3 cm below the equilibrium position

Directions (Q. Nos. 28 to 30) *A particle is executing SHM on a straight line. A and B are two points at which its velocity is zero. The particle is crossing a certain point $X(AX < XB)$ at successive intervals of 1.2 s and 3.6 s with a speed of $4\ ms^{-1}$.*

28. Determine the amplitude of oscillation.
(a) 9.6 m (b) $\frac{9.6\sqrt{2}}{\pi}$ m
(c) $\frac{\pi \times 4\sqrt{2}}{3}$ m (d) 4 m

29. The maximum speed of the particle is
(a) $4\sqrt{2}\ ms^{-1}$ (b) $4\pi\ ms^{-1}$
(c) $\frac{8\sqrt{2}\pi^2}{14.4} ms^{-1}$ (d) $\frac{\pi}{0.6} ms^{-1}$

30. The ratio $\frac{AX}{XB}$ is
(a) $\frac{\sqrt{2}}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$
(c) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (d) Information is insufficient

AIEEE & JEE Main Archive

31. Bob of a simple pendulum of length l is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T, then **[JEE Main Online 2013]**

(a) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone

(b) $T = 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone

(c) $T > 2\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone

(d) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone

32. If the time period t of the oscillating of a drop of liquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{r^{2b} . s^c . d^{a/2}}$. It is observed that the time period is directly proportional to $\sqrt{\frac{d}{s}}$. The value of b should therefore be **[JEE Main Online 2013]**

(a) $\frac{3}{4}$ (b) $\sqrt{3}$ $\frac{3}{2}$ (d) $\frac{2}{3}$

33. Two simple pendulums of length 1 m and 4 m respectively are both given small displacement in the same direction at shorter pendulum has completed number of oscillations equal to **[JEE Main Online 2013]**

(a) 2 (b) 7 (c) 5 (d) 3

34. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10 s it will decreases to α times its original magnitude, where α equals **[JEE Main 2013]**

(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

35. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds **[AIEEE 2012]**

(a) $\frac{0.693}{b}$ (b) b (c) $\frac{1}{b}$ (d) $\frac{2}{b}$

36. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement I If stretched by the same amount, work done on S_1, will be more than that on S_2. **[AIEEE 2012]**

Statement II $k_1 < k_2$

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement I
(d) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I

37. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is **[AIEEE 2011]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

38. If a spring of stiffness k is cut into two parts A and B of length $l_A : l_B = 2:3$, then the stiffness of spring A is given by **[AIEEE 2011]**

(a) $\frac{5}{2}k$ (b) $\frac{3k}{5}$
(c) $\frac{2k}{5}$ (d) k

39. A wooden cube (density of wood d) of side l floats in a liquid of density ρ with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period T. Then, T is equal to **[AIEEE 2011]**

(a) $2\pi\sqrt{\frac{l\rho}{(\rho - d)g}}$

(b) $2\pi\sqrt{\frac{ld}{\rho g}}$

(c) $2\pi\sqrt{\frac{l\rho}{dg}}$

(d) $2\pi\sqrt{\frac{ld}{(\rho - d)g}}$

40. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then which of the following does not change with time? **[AIEEE 2009]**

(a) $a^2T^2 + 4\pi^2v^2$ (b) $\frac{aT}{x}$

(c) $aT + 2\pi v$ (d) $\frac{aT}{v}$

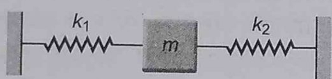
41. A wave travelling along the x-axis is described by the equation $y(x,t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are **[AIEEE 2008]**

(a) $\alpha = 25.00\pi, \beta = \pi$
(b) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$
(c) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$
(d) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

42. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then **[AIEEE 2007]**

(a) $A = x_0, \delta = -\frac{\pi}{4}$
(b) $A = x_0\omega^2, \delta = \frac{\pi}{4}$
(c) $A = x_0\omega^2, \delta = -\frac{\pi}{4}$
(d) $A = x_0\omega^2, \delta = \frac{3\pi}{4}$

43. Two springs of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[AIEEE 2007]**



(a) $\frac{f}{2}$
(b) $\frac{f}{4}$
(c) $4f$
(d) $2f$

44. The maximum velocity of a particle executing simple harmonic motion with an amplitude 7 mm, is 4.4 ms^{-1}. The period of oscillation is **[AIEEE 2006]**

(a) 0.01 s
(b) 10 s
(c) 0.1 s
(d) 100 s

45. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2, (k = 1 ms^{-2})$, where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is (Take $g = 10 \text{ ms}^{-2}$) **[AIEEE 2005]**

(a) $\frac{6}{5}$
(b) $\frac{5}{6}$
(c) 1
(d) $\frac{4}{5}$

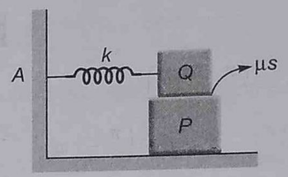
46. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos \pi t$. The phase difference of the velocity of particle 1, with respect to the velocity of particle 2 is **[AIEEE 2005]**

(a) $\frac{-\pi}{6}$
(b) $\frac{\pi}{3}$
(c) $\frac{-\pi}{3}$
(d) $\frac{\pi}{6}$

47. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till the water is coming out, the time period of oscillation would **[AIEEE 2005]**

(a) first increase and then decrease to the original value
(b) first decrease and then increase to the original value
(c) remain unchanged
(d) increase towards a saturation value

48. A block P of mass m is placed on a horizontal frictionless plane. A second block of the same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by a distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks? **[AIEEE 2004]**



(a) $kA/2$
(b) kA
(c) $\mu - mg$
(d) Zero

49. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, Then, **[AIEEE 2004]**

(a) $T = t_1 + t_2$
(b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$
(d) $T^{-2} = t_1^{-2} + t_2^{-2}$

50. The total energy of a particle, executing simple harmonic motion is **[AIEEE 2004]**

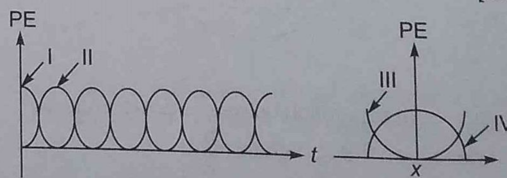
(a) $\propto x$
(b) $\propto x^2$
(c) independent of x
(d) $\propto x^{1/2}$

where x is the displacement from the mean position.

51. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then **[AIEEE 2004]**

(a) $\omega_1 = \omega_2$
(b) $\omega_1 > \omega_2$
(c) $\omega_1 < \omega_2$, when damping is small
$\omega_1 > \omega_2$, when damping is large
(d) $\omega_1 < \omega_2$

52. For a particle executing SHM, the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x. **[AIEEE 2003]**



(a) I and III
(b) II and IV
(c) II and III
(d) I and IV

Answers

1. (b)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (b)
11. (b)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (a)
21. (d)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (b)
31. (d)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (d)	**36.** (a)	**37.** (a)	**38.** (a)	**39.** (b)	**40.** (b)
41. (a)	**42.** (d)	**43.** (d)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (a)	**48.** (a)	**49.** (b)	**50.** (c)
51. (a)	**52.** (a)								

Hints & Solutions

1. Given, $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ ...(i)

Velocity, $v = \frac{dy}{dt} = 3 \times 2\,\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)$

Acceleration, $A = \frac{dv}{dt} = -4\omega^2 \times 3\cos\left(\frac{\pi}{4} - 2\,\omega t\right) = -4\,\omega^2 y$

As $A \propto y$ and negative sign shows that it is directed towards equilibrium (or mean position), hence particle will execute SHM. Comparing Eq. (i) with equation

$$y = r\cos(\phi - \omega' t)$$

We have, $\omega' = 2\omega$ or $\frac{2\pi}{T'} = 2\omega$

or $T' = \frac{\pi}{\omega}$

2. Using $v^2 = \omega^2(a^2 - y^2)$, we have

$$10^2 = \omega^2(a^2 - 4^2)$$
$$8^2 = \omega^2(a^2 - 5^2)$$

So, $10^2 - 8^2 = \omega^2(5^2 - 4^2) = (3\omega)^2$

$\Rightarrow$ $6 = 3\omega$

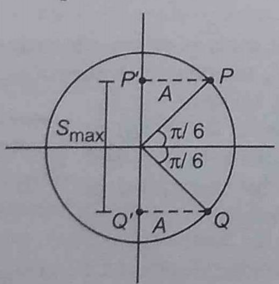
or $\omega = 2$

$\therefore$ $T = 2\pi/\omega$

$= 2\pi/2 = \pi$ second

3. Phase difference

$$\phi = \omega t = \frac{2\pi}{6} \times 1 = \frac{\pi}{3}\text{ rad}$$



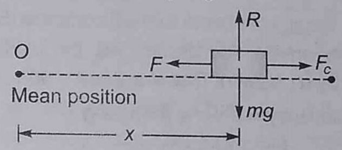
The maximum separation between the two particles is

$$S_{max} = 2A\sin\frac{\pi}{6}$$

or $S_{max} = 2 \times 5 \times \frac{1}{2} = 5\text{ cm}$

4. Let O be the mean position and x be the distance of the coin from O. The coin will slip if centrifugal force on the coin just becomes equal to the force of friction *i.e.*,

$$m x\omega^2 = \mu mg$$



From the diagram,

$$mA\omega^2 = \mu mg \text{ or } A = \mu g/\omega^2$$

5. Force of friction $= \mu mg = m\omega^2 a = m\,(2\pi\nu)^2 a$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{\mu g}{a}}$$

6. When the mass m is pushed in a downward direction through a distance x, the effective restoring force, in magnitude is

$$F = kx + kx\cos 60° + kx\cos 60° = 2kx$$

$\therefore$ Spring factor, $k' = 2k$

Inertia factor $= M$

As time period, $T = 2\pi\sqrt{\frac{M}{2k}}$

7. The frequency of kinetic energy is twice that of a particle executive SHM.

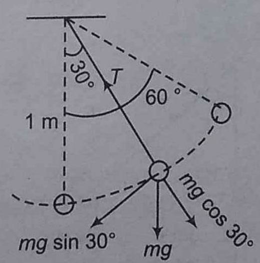
8. The displacement of A relative to B is $x = x_1 - x_2$

$$x = a\cos\omega t - b\sin\left(\omega t + \frac{3\pi}{2}\right)$$

$$= a\cos\omega t + b\cos\omega t = (a + b)\cos\omega t$$

Which is an simple harmonic motion with amplitude $(a + b)$.

9.



Rate of change of speed

$$\frac{dv}{dt} = \text{tangential acceleration}$$

$$= \frac{\text{tangential force}}{\text{mass}} = \frac{mg \sin 30°}{m}$$

$$= g \sin 30° = 10\left(\frac{1}{2}\right) \text{m/s}^2 = 5 \text{ m/s}^2$$

10. As, $T = 2\pi\sqrt{l/g}$

Taking log and differentiating the expression, keeping T constant we have

$$\frac{dl}{l} = \frac{dg}{g} = -\frac{0.1}{100}$$

$$\therefore \quad (dl/l) \times 100 = -0.1/100 \times 100$$

$$= -0.1\%$$

11. When bigger pendulum of time period (5T/4) completes one oscillation, the smaller pendulum will complete (5/4) oscillation. It means, the smaller pendulum will be leading the bigger pendulum by a phase of $T/4 \text{ s} = \pi/2 \text{ rad} = 90°$.

12. Force of buoyancy $= b \times c \times x \times \rho_w \times g$

$$= bcxg$$

$(\because \rho_w = 1)$

and mass of piece of wood $= abc\rho$

So, acceleration $= -bcxg/abc\rho = -(g/a\rho)x$

Hence, time period, $T = 2\pi\sqrt{\frac{\rho a}{g}}$

13. Effective acceleration $= \sqrt{a^2 + g^2}$

$\therefore$ Time period $T = 2\pi\sqrt{\frac{l}{(a^2 + g^2)}}$

14.

$$4 = k(\alpha - l)$$

$$5 = k(\beta - \alpha)$$

$$9 = k(\gamma - l)$$

$$\Rightarrow \quad \frac{4}{5} = \frac{\alpha - l}{\beta - l} \quad \text{or} \quad 4\beta - 4l = 5\alpha - 5l$$

$$l = 5\alpha - 4\beta$$

Now, $9\alpha - 9l = 4\gamma - 4l$

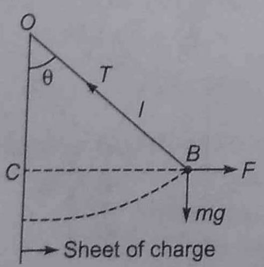
$$4\gamma = 9\alpha - 5l = 9\alpha - 5(5\alpha - 4\beta)$$

$$= 9\alpha - 25\alpha + 20\beta$$

$$= 20\beta - 16\alpha$$

$$\gamma = 5\beta - 4\alpha$$

15. In the figure we represent the electric intensity at B due to the sheet of charge,



$$E = \frac{1}{2}\frac{\sigma}{\varepsilon_0}$$

Force on bob due to the sheet of charge $F = qE = \frac{1}{2}\frac{\sigma q}{\varepsilon_0}$

As the bob is in equilibrium, so $\frac{mg}{OC} = \frac{F}{CB} = \frac{T}{BO}$

or $\quad \frac{CB}{OC} = \frac{F}{mg} = \frac{1/2\,\sigma q/\varepsilon_0}{mg} = \frac{\sigma q}{2\varepsilon_0 mg} = \tan\theta$

16. Hence, pendulum A and C will be in resonance, hence, C will vibrate with the maximum amplitude.

17. Since $v = \omega\sqrt{a^2 - y^2}$, also here $y = a/2$

$$\Rightarrow \quad v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\sqrt{\frac{3a^2}{4}} = \frac{2\pi}{T} \times \frac{\sqrt{3}a}{2} = \frac{\pi\sqrt{3}a}{T}$$

18. If mass m moves down a distance y, then the spring is pulled by $2y$ and the force with which the spring is pulled will be $F = R = mg/2$. Hence, $mg/2 = k(2y)$

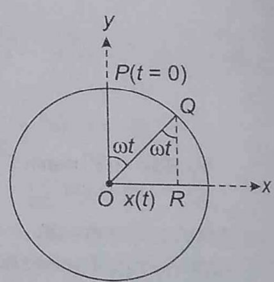
$$\Rightarrow \quad y/9 = m/4k$$

$$\Rightarrow \quad T = 2\pi\sqrt{y/9} = 2\pi\sqrt{m/4k} = \pi\sqrt{m/k}$$

19. The effective spring constant is $K = k + 2k = 3k$. The time period of oscillation is given by $T = 2\pi\sqrt{\frac{m}{k}}$

and $\nu = \frac{1}{T}$, so we get $\nu = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

20. Given, $T = 30$ s, $OQ = B$. The projection of the radius vector in the diameter of the circle when a particle is moving with uniform angular velocity (ω) on a circle of reference is SHM. Let the particle go from P to Q in time t.

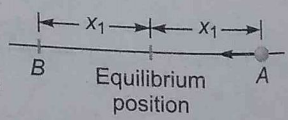


Then, $\angle POQ = \omega t = \angle OQP$. The projection of radius OQ on x-axis will be $OR = x(t)$ say.

In ΔOQR, $\sin \omega t = \frac{x(t)}{B}$

or $x(t) = B \sin \omega t = B \sin\frac{2\pi}{T}t = B \sin\frac{2\pi}{30}t$

21. Consider the situation as shown in the adjoint figure. Let us say at any instant t_1, the particle crosses A as shown, the particle again acquires the same velocity, when it crosses B let us say at instant t_2. According to statement I, $(t_2 - t_1)$ is the time period of SHM which is wrong.



22. At the mean position, $F = 0 = -\frac{dU}{dx} = 0$

$\Rightarrow U =$ constant which can be zero or non-zero.

23. When the hoop oscillates in its plane, moment of inertia is

$$I_1 = mR^2 + mR^2$$

i.e., $\quad I_1 = 2mR^2$

While when the hoop oscillates in a direction perpendicular to the plane of the hoop, moment of inertia is

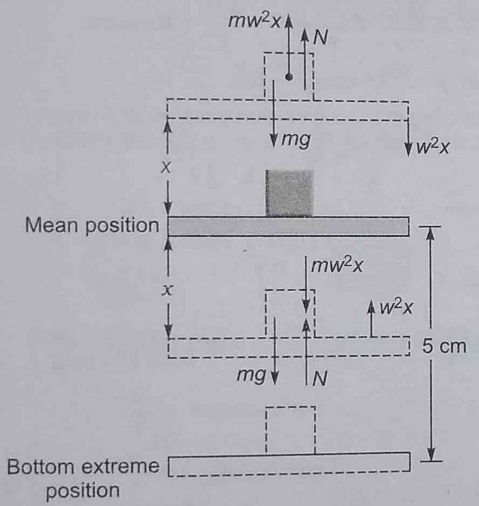
$$I_2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$$

Time period of physical pendulum is, $T = 2\pi\sqrt{\frac{I}{mgd}}$; d is same in both the cases.

24. From the relation of the time period, $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}}$

When the satellite is orbiting around the earth, the value of g inside it is zero. Hence, the time period of pendulum in a satellite will be infinity and it is also clear that time period of pendulum is inversely proportional to square root of acceleration due to gravity g.

25-27. Let the mass of block be m, the situation is as shown in the figure below



The free body diagram of the block (including inertial forces due to non-inertial nature of platform) when it is at a displacement x from the mean position on other side is shown.

When block is on the down side of the mean position, $mg + m\omega^2 x = N$ i.e., $N > mg$ always.

As the platform moves from the bottom extreme to the mean position, the value of x decreases and hence N also decreases.

When the block is on the upside of the mean position,

$$mg = N + m\omega^2 x \quad i.e., \quad N = mg - m\omega^2 x$$

As the platform moves away form the mean position in a vertically upward direction, the value of N decreases and may become zero at some position. When N becomes zero, the block leaves the contact of the platform and moves under gravity. Let $N = 0$ at a distance x from the mean position when the platform is moving away from the mean position in an upward direction,

$$0 = mg - m\omega^2 x$$

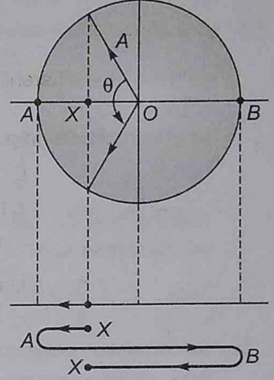
$$\Rightarrow \quad x = \frac{g}{\omega^2} = \frac{10}{(2\pi f)^2} = \frac{10}{(20)^2} = 0.025\text{ m} = 2.5\text{ cm}$$

At this instant the velocity of the block is the same as that of the platform, and is given by

$$v = \omega\sqrt{A^2 - x^2} = 20\sqrt{(0.05)^2 - (0.025)^2} = 0.866\text{ m/s}$$

The point at which be block hits the platform again, can be found out by carrying out the calculation or solving graphically.

The circular motion representation of the given SHM is shown in the figure. Below, we have shown the motion of particle a along straight line. Let ω be the angular frequency of SHM, then $\omega \times 1.2 = \theta$



From the given information,

$T = 1.2 + 3.6 = 4.8\text{ s}$

So, $\quad \theta = \frac{2\pi}{4.8} \times 1.2 = \frac{\pi}{2}$

Let $\quad OX = x_0$,

Then, $\cos\frac{\theta}{2} = \frac{x_0}{A}$

$\Rightarrow \quad x_0 = \frac{A}{\sqrt{2}}$ (as $\theta = 90°$)

where, A is the amplitude of oscillation.

From $\quad v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \quad 4^2 = \omega^2 A^2 - \omega^2 x_0^2 = \frac{\omega^2 A^2}{2}$

$\Rightarrow \quad v = \omega A = 4\sqrt{2}$ m/s, the maximum speed

$$A = \frac{4\sqrt{2}}{\omega} = \frac{4\sqrt{2}}{2\pi} \times 4.8 = \frac{9.6\sqrt{2}}{\pi}\text{ m}$$

$$\frac{AX}{XB} = \frac{A - x_0}{A + x_0} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$

31. $T < 2\pi\sqrt{\frac{l}{g}}$

As, current passed through in the coil which attracts the molecules of air to closer it, thus density of air increases which posses larger, damping that of air at above because due to gravity density at lower position is more than that of upper position.

32. As, $t = \sqrt{r^{2b} s^c d^{a/2}}$ ...(i)

and given $\quad t \propto \sqrt{\frac{d}{s}}$ i.e., $t \propto d^{1/2}$ & $s^{-1/2}$ ...(ii)

From Eq. (i), we get $t = r^b s^{\frac{c-1}{2}} s^{1/2} d^{\frac{a-2}{4}} d^{1/2}$

$= r^b s^{\frac{c-1}{2}} . d^{\frac{a-2}{4}} s^{1/2} d^{1/2}$ ...(iii)

Now from Eqs. (ii) and (iii), we get $b = 3/2$

33. Let T_1 and T_2 be the time period of shorter length and larger length pendulums respectively. According to question,

$$nT_1 = (n-1)T_2$$

So, $\quad n 2\pi\sqrt{\frac{1}{8}} = (n-1)2\pi\sqrt{\frac{4}{8}}$

or $\quad n = (n-1)2 = 2n - 2 \Rightarrow n = 2$

34. Amplitude of damped oscillator, $A = A_0 e^{-\frac{bt}{2m}}$

After 5 s, $\quad 0.9A_0 = A_0 e^{-\frac{b(5)}{2m}}$

$\Rightarrow \quad 0.9 = e^{-\frac{b(5)}{2m}}$ ...(i)

After 10 more second, $A = A_0 e^{\frac{-b(15)}{2m}} = A_0 \left(e^{-\frac{5b}{2m}}\right)^3$ ...(ii)

From Eqs. (i) and (ii), we get $A = 0.729 A_0$

Hence, $\alpha = 0.729$

35. For damped harmonic motion, $ma = -kx - mbv$

or $ma + mbv + kx = 0$

Solution to above equation is

$$x = A_0 e^{-\frac{bt}{2}} \sin \omega t; \text{ with } \omega^2 = \frac{k}{m} - \frac{b^2}{4m}$$

where amplitude drops exponentially with time.

$$A_\tau = A_0 e^{-\frac{b\tau}{2}}$$

Average time τ is that duration when amplitude drops by 63%, *i.e.*, becomes A_0/e

Thus, $A_\tau = \frac{A_0}{e} A_0 e^{-\frac{b\tau}{2}}$ or $\frac{b\tau}{2} = 1$ or $\tau = \frac{2}{b}$

36. As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about Statement I. But as in Statement II, $k_1 < k_2$

Then, for same force $W = F.x = F.\frac{F}{K} = \frac{F^2}{K}$

$\Rightarrow$ $W \propto \frac{1}{k}$ *i.e.*, $W_1 > W_2$

But for same displacement, $W = F.x = \frac{1}{2}kx.x = \frac{1}{2}kx^2$

$\Rightarrow$ $W \propto k$, *i.e.*, $W_1 < W_2$

Thus, in the light of Statement II, Statement I is false.

37. Let $x_1 = A\sin(\omega t + \phi_1)$ and $x_2 = A\sin(\omega t + \phi_1)$

$$x_2 - x_1 = A[\sin(\omega t + \phi_2) - \sin(\omega t + \phi_1)]$$

$$= 2A\cos\left(\frac{2\omega t + \phi_1 + \phi_2}{2}\right)\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$$

The resultant motion can be treated as a simple harmonic motion with amplitude $2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

Given, maximum distance between the particles $= X_0 + A$

$\therefore$ Amplitude of resultant SHM $= X_0 + A - X_0 = A$

$\therefore$ $2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A \Rightarrow \phi_2 - \phi_1 = \pi/3$

38. For spring $k \propto \frac{1}{l}$

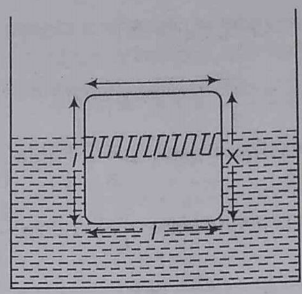
$\therefore$ $\frac{k_A}{k_B} = \frac{l_B}{l_A} \Rightarrow k_A = \frac{l_A + l_B}{l_A} k_A = \frac{5}{2}k$

39. Let at any instant, cube is at a depth x from the equilibrium position, then net force acting on the cube = upthrust on the portion of length x

$\therefore$ $F = -\rho l^2 xg = -\rho l^2 g\, x$ ...(i)

Negative sign shows that, force is opposite to x.

Hence, equation of SHM



$F = -kx$ ...(ii)

Comparing Eqs.(i) and (ii),

$$k = \rho l^2 g$$

$\therefore$ $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l^3 d}{\rho l^2 g}} = 2\pi\sqrt{\frac{ld}{\rho g}}$

40. As, $\frac{aT}{x} = \frac{\omega^2 xT}{x} = \frac{4\pi^2}{T^2} \times T = \frac{4\pi^2}{T}$ = constant.

41. Given, $y = 0.005\cos(\alpha x - \beta t)$

Comparing the equation with the standard form,

$$y = A\cos\left(\frac{x}{\lambda} - \frac{t}{T}\right)2\pi$$

we have, $2\pi/\lambda = \alpha$ and $2\pi/T = \beta$

$\Rightarrow$ $\alpha = 2\pi/0.08 = 25.00\,\pi$ and $\beta = \pi$

42. Given, $x = x_0\cos\left(\omega t - \frac{\pi}{4}\right)$

Acceleration, $a = \frac{d^2x}{dt^2} = -\omega^2 x_0 \cos\left(\omega t - \frac{3\pi}{4}\right)$

$$= \omega^2 x_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$$

So, $A = \omega^2 x_0$ and $\delta = \frac{3\pi}{4}$

43. We know that, $f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

When k_1 and k_2 are made four times their original value. Then,

and $f' = \frac{1}{2\pi}.2\sqrt{\frac{k_1 + k_2}{m}} = 2f$

44. Maximum velocity $v = A\omega$, (where A is the amplitude and ω is the angular frequency of oscillation).

$\therefore$ $4.4 = (7 \times 10^{-3}) \times 2\pi/T$

or $T = \frac{7 \times 10^{-3}}{4.4} \times \frac{2 \times 22}{7} = 0.01\text{ s}$

45. Given, $y = kt^2 \Rightarrow \frac{d^2y}{dt^2} = 2k$

or $a_y = 2\text{ m/s}^2$ (as $k = 1\text{ m/s}^2$)

$\therefore$ $T_1 = 2\pi\sqrt{\frac{l}{g}}$ and $T_2 = 2\pi\sqrt{\frac{l}{g + a_y}}$

$\therefore$ $\frac{T_1^2}{T_2^2} = \frac{g + a_y}{g} = \frac{10 + 2}{10} = \frac{6}{5}$

46. Given, $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$

$$\Rightarrow \frac{dy_1}{dt} = v_1 = 0.1 \times 100\, p \cos\left(100\,\pi\, t + \frac{\pi}{3}\right)$$

or $$v_1 = 10\,\pi \sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

or $$v_1 = 10\pi \sin\left(100\pi t + \frac{5\pi}{6}\right) \text{ and } y_2 = 0.1 \cos \pi t$$

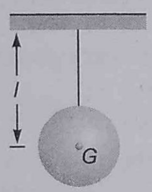
$$\Rightarrow \frac{dy_2}{dt} = v_2 = -0.1 \sin \pi t$$

or $$v_2 = 0.1 \sin(\pi t + \pi)$$

Hence, the phase difference

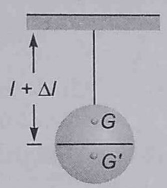
$$\Delta\phi = \phi_1 - \phi_2$$
$$= \left(100\pi t + \frac{5\pi}{6}\right) - (\pi t + \pi)$$
$$= \frac{5\pi}{6} - \pi = -\frac{\pi}{6} \qquad (\text{at } t = 0)$$

47.



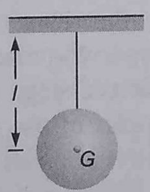
Spherical hollow ball filled with water

$$T = 2\pi\sqrt{\frac{l}{g}}$$



Spherical hollow ball half filled with water

$$T_1 = 2\pi\sqrt{\frac{l + \Delta l}{g}}$$



Spherical hollow ball

$$T_2 = 2\pi\sqrt{\frac{l}{g}} \text{ and } T_1 > T_2$$

Hence, time period first increases and then decreases to the original value.

48. Angular frequency of the system, $\omega = \sqrt{\frac{k}{m + m}} = \sqrt{\frac{k}{2m}}$

Maximum acceleration of the system will be, $\omega^2 A$ or $\frac{kA}{2m}$. This acceleration of the lower block, is provided by friction.

Hence, $$f_{max} = ma_{max} = m\omega^2 A$$
$$= m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$$

49. Time period of the spring,

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

Here, k be the force constant of spring.
For the first spring,

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \qquad \text{...(i)}$$

For the second spring

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \qquad \text{...(ii)}$$

The effective force constant in the series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

Time period of combination

$$T = 2\pi\sqrt{\left[\frac{m(k_1 + k_2)}{k_1 k_2}\right]}$$

$$\Rightarrow T^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2} \qquad \text{...(iii)}$$

From Eqs. (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

or $$t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

or $$t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$$

$$\Rightarrow t_1^2 + t_2^2 = T^2 \qquad \text{[from Eq. (iii)]}$$

50. In a simple harmonic motion, when a particle is displaced to a position from its mean position, its kinetic energy is converted into potential energy. Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on the displacement x.

51. For amplitude of oscillation and energy to be maximum, frequency of the force must be equal to the initial frequency and this is only possible in case of resonance. In resonance state, $\omega_1 = \omega_2$.

52. Potential energy is minimum (in this case zero) at the mean position ($x = 0$) and maximum at the extreme positions ($x = \pm A$). At time $t = 0, x = A$, the potential energy should be maximum. Therefore, graph I is correct. Further in graph III, potential energy is minimum at $x = 0$. Hence, this is also correct.