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Complete Physics—JEE Main

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Physics and Measurement

REVIEW OF BASIC CONCEPTS

1. The SI System of Units

The internationally accepted standard units of the fundamental physical quantities are given in Table 1.1.

Table 1.1 Fundamental SI Units

Physical Quantity	*Name of the Unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Angle in a plane	radian	rad
Solid angle	steradian	sr

2. Dimensions of Physical Quantities

The dimensions of a physical quantity are the powers to which the fundamental units of mass (M), length (L) and time (T) must be raised to represent the unit of that quantity. The dimensional formula of a physical quantity is an expression that tells us how and which of the fundamental quantities enter into the unit of that quantity.

In *mechanics*, the dimensional formula is written in terms of the dimensions of mass, length and time (M, L and T). In *heat and thermodynamics*, in addition to M, L and T, we need to mention the dimension of temperature in kelvin (K). In *electricity and magnetism*, in addition to M, L and T, we need to mention the dimension of current or charge per unit time (A or QT^{-1}).

Example 1 Find the dimensional formula of (a) velocity, (b) acceleration, (c) force, (d) work, (e) energy and (f) pressure.

Solution

(a) Velocity $(v) = \dfrac{\text{distance}}{\text{time}} = \dfrac{dx}{dt} = \dfrac{[\mathrm{L}]}{[\mathrm{T}]}$

$= [\mathrm{L\,T^{-1}}] = [\mathrm{M^0\,LT^{-1}}]$

(b) Acceleration $(a) = \dfrac{dv}{dt} = \dfrac{[\mathrm{M^0\,L\,T^{-1}}]}{[\mathrm{T}]} = [\mathrm{M^0\,L\,T^{-2}}]$

(c) Force $(F) = ma = [\mathrm{M}] \times [\mathrm{M^0\,L\,T^{-2}}] = [\mathrm{M\,L\,T^{-2}}]$

(d) Work $(W) = Fx = [\mathrm{M\,L\,T^{-2}}] \times [\mathrm{L}] = [\mathrm{M\,L^2\,T^{-2}}]$

(e) Energy = work $= [\mathrm{M\,L^2\,T^{-2}}]$

(f) Pressure $= \dfrac{\text{force}}{\text{area}} = \dfrac{[\mathrm{MLT^{-2}}]}{[\mathrm{L^2}]} = [\mathrm{ML^{-1}\,T^{-2}}]$

Table 1.2 gives the dimensional formulae of some important physical quantities.

3. Principle of Homogeneity of Dimensions

Consider a simple equation,

$$A + B = C.$$

If this is an equation of Physics, i.e. if A, B and C are physical quantities, then this equation says that one physical quantity A, when added to another physical quantity B, gives a third physical quantity C. This equation will have no meaning in Physics if the nature (i.e. the dimensions) of the quantities on the left-hand side of the equation is not the same as the nature of the quantity on the right-hand side. For example, if A is a length, B must also be a length and the result of addition of A and B must express a length. In other words, the dimensions of both sides of a physical equation must be identical. This is called the *principle of homogeneity of dimensions*.

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Table 1.2 Dimensional Formulae of some Physical Quantities

Physical Quantity	*Dimensional Formula*	*Physical Quantity*	*Dimensional Formula*
Area	$M^0L^2T^0$	Heat energy	ML^2T^{-2}
Volume	$M^0L^3T^0$	Entropy	$ML^2T^{-2}K^{-1}$
Density	$ML^{-3}T^0$	Specific heat	$M^0L^2T^{-2}K^{-1}$
Velocity	M^0LT^{-1}	Latent heat	$M^0L^2T^{-2}$
Acceleration	M^0LT^{-2}	Molar specific heat	$ML^2T^{-2}K^{-1}\ mol^{-1}$
Momentum	MLT^{-1}	Thermal conductivity	$MLT^{-3}K^{-1}$
Angular momentum	ML^2T^{-1}	Wien's constant	M^0LT^0K
Force	MLT^{-2}	Stefan's constant	$ML^0T^{-3}K^{-4}$
Energy, work	ML^2T^{-2}	Boltzmann's constant	$ML^2T^{-2}K^{-1}$
Power	ML^2T^{-3}	Molar gas constant	$ML^2T^{-2}K^{-1}\ mol^{-1}$
Torque, couple	ML^2T^{-2}	Electric charge	TA or Q
Impulse	MLT^{-1}	Electric current	A or QT^{-1}
Frequency	$M^0L^0T^{-1}$	Electric potential	$MLT^{-2}Q^{-1}$ or $ML^2T^{-3}A^{-1}$
Angular frequency	$M^0L^0T^{-1}$	Electric field	$MLT^{-2}Q^{-1}$ or $MLT^{-3}A^{-1}$
Angular acceleration	$M^0L^0T^{-2}$	Capacitance	$M^{-1}L^{-2}T^4A^2$ or $M^{-1}L^{-2}T^2Q^2$
Pressure	$ML^{-1}T^{-2}$	Inductance	$ML^2T^0Q^{-2}$ or $ML^2T^{-2}A^{-2}$
Elastic modulii	$ML^{-1}T^{-2}$	Resistance	$ML^2T^{-1}Q^{-2}$ or $ML^2T^{-3}A^{-2}$
Stress	$ML^{-1}T^{-2}$	Magnetic flux	$ML^2T^{-1}Q^{-1}$ or $ML^2T^{-2}A^{-1}$
Moment of inertia	ML^2T^0	Magnetic flux density or Magnetic induction field	$ML^0T^{-1}Q^{-1}$ or $ML^0T^{-2}A^{-1}$
Surface tension	ML^0T^{-2}	Permeability	MLQ^{-2} or $MLT^{-2}A^{-2}$
Viscosity	$ML^{-1}T^{-1}$	Permittivity	$M^{-1}L^{-3}T^2Q^2$ or $M^{-1}L^{-3}T^4A^2$
Gravitational constant	$M^{-1}L^2T^{-2}$	Planck's constant	ML^2T^{-1}

Example 2 The distance x traveled by a body varies with time t as

$x = at + bt^2$, where a and b are constants.

Find the dimensions of a and b.

Solution The dimensions of each term on the right hand of the given equation must be the same as those of the left hand side. Hence

Dimensions of at = dimensions of x

or $$[a] = \frac{[x]}{[t]} = \frac{[L]}{[T]} = [LT^{-1}] = [M^0LT^{-1}]$$

Dimensions of bt^2 = dimensions of x

or $$b = \frac{x}{t^2} = \frac{[L]}{[T^2]} = [LT^{-2}] = [M^0LT^{-2}]$$

Example 3 The pressure P, volume V and temperature T of a gas are related as

$$\left(P + \frac{a}{V^2}\right)(V - b) = cT$$

where a, b, and c are constants. Find the dimensions of $\frac{a}{b}$.

Solution Dimensions of $\frac{a}{V^2}$ = dimensions of P.

$\therefore$ Dimensions of a = dimensions of PV^2

Also dimensions of b = dimensions of V

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$$\therefore \text{ Dimensions of } \frac{a}{b} = \frac{[PV^2]}{[V]} = [PV] = [ML^{-1}T^{-2}] \times [L^3]$$
$$= [ML^2T^{-2}]$$

NOTE

1. Trigonometric function (sin, cos, tan, cot etc) are dimensionless. The arguments of these functions are also dimensionless
2. Exponential functions are dimensionless. Their exponents are also dimensionless

Example 4 When a plane wave travels in a medium, the displacement y of a particle located at x at time t is given by

$$y = a \sin(bt + cx)$$

where a, b and c are constant. Find the dimensions of $\frac{b}{c}$.

Solution Terms bt and cx must be dimensionless. Hence

$$[b] = \frac{1}{[t]} = [T^{-1}]$$

and
$$[c] = \frac{1}{[x]} = \frac{1}{[L]} = [L^{-1}]$$

$$\therefore \quad \left[\frac{b}{c}\right] = [LT^{-1}] = [M^0 LT^{-1}]$$

Note that the dimensions of a are the same as those of y.

Example 5 In the expression

$$P = \frac{a^2}{b} e^{-ax}$$

P is pressure, x is a distance and a and b are constants. Find the dimensional formula for b.

Solution $\left[\frac{a^2}{b}\right] = [P]$

Also ax is dimensionless. Hence $[a] = [L^{-1}]$.

$$\therefore \quad [b] = \frac{[a^2]}{[P]} = \frac{[L^{-2}]}{[ML^{-1}T^{-2}]} = [M^{-1}L^{-1}T^2]$$

The principle of homogeneity of dimensions can also be used to find the dependence of a physical quantity on other physical quantities.

Example 6 The time period (t) of a simple pendulum may depend upon m the mass of the bob, l the length of the string and g the acceleration due to gravity. Find the dependence of t on m, l and g.

Solution

Let $\quad t \propto m^a l^b g^c$

or $\quad t = k\, m^a l^b g^c$,

where k is a dimensionless constant.

Writing the dimensions of each quantity, we have

$$[T] = [M]^a [L]^b [LT^{-2}]^c$$

or
$$[M^0 L^0 T] = [M^a L^{b+c} T^{-2c}]$$

According to the principle of homogeneity of dimensions, the dimensions of all the terms on either side of this equation must be the same. Equating the powers of M, L and T, we have

$$a = 0,\ b + c = 0 \text{ and } -2c = 1,$$

which give $b = \frac{1}{2}$ and $c = -\frac{1}{2}$.

Hence
$$t = k\, m^0 l^{1/2} g^{(-1/2)}$$

$$\Rightarrow \quad t = k\sqrt{\frac{l}{g}}$$

Thus t is independent of the mass of the bob and is directly proportional to $\sqrt{l}$ and inversely proportional to $\sqrt{g}$.

4. Significant Figures

The number significant figure in any measurement indicates the degree of precision of that measurement. The degree of precision is determined by the least count of the measuring instrument. Suppose a length measured by a metre scale (of least count = 0.1 cm) is 1.5 cm, then it has two significant figures, namely 1 and 5. Measured with a vernier callipers (of least count = 0.01 cm) the same length is 1.53 cm and it then has three significant figures. Measured with a screw gauge (of least count = 0.001 cm) the same length may be 1.536 cm which has four significant figures.

It must be clearly understood that we cannot increase the accuracy of a measurement of changing the unit. For example, suppose a measurement of mass yields a value 39.4 kg. It is understood that the measuring instrument has a least count of 0.1 kg. In this measurement, three figures 3, 9 and 4 are significant. If we change 39.4 kg to 39400 g or 39400000 mg, we cannot change the accuracy of measurement. Hence 39400 g or 39400000 mg still have three significant figures; the zeros only serve to indicate only the magnitude of measurement.

Estimation of Appropriate Significant Figures in Calculations

The importance of significant figures lies in calculation to find the result of addition or multiplication of measured quantities having a different number of significant figures. The least accurate quantity determines the accuracy of the sum or product. The result must be rounded off to the appropriate digit.

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Rules for Rounding off

The following rules are used for dropping figures that are not significant

1. If the digit to be dropped is less than 5, the next (preceding) digit to be retained is left unchanged. For example, if a number 5.34 is to be rounded off to two significant figures, the digit to be dropped is 4 which is less than 5. Hence the next digit, namely 3, is not changed. The result of the indicated rounding-off is therefore, 5.3.
2. If the digit to be dropped is more than 5, the preceding digit to be retained is increased by 1. For examples 7.536 is rounded off as 7.54 to three significant figures.
3. If the digit to be dropped happens to be 5, then
   (a) the preceding digit to be retained is increased by 1 if it is odd, or
   (b) the preceding digit is retained unchanged if it is even.

   For example, 6.75 is rounded off to 6.8 to two significant figures and 4.95 is rounded off to 5.0 but 3.45 is rounded off to 3.4.

Rule for Finding Significant Figures

1. For addition and subtraction, we use the following rule.
   Find the sum or difference of the given measured quantities and then round off the final result such that it has the same number of digits after the decimal place as in the least accurate quantity (i.e., the quantity which has the least number of significant figures)

Example 7 Four objects have masses 2.5 kg, 1.54 kg, 3.668 kg and 5.1278 kg. Find the total mass up to appropriate significant figures.

Solution

$$M = 2.5 + 1.54 + 3.668 + 5.1278 = 12.8358 \text{ kg}$$

In this example, the least accurate quantity is 2.5 kg. This mass is accurate only up to the first decimal place in kg. Hence the final result much be rounded off to the first decimal place in kg. The correct result up to appropriate significant figures is $M = 12.8$ kg.

2. We use the following rule to determine the number of significant figures in the result of multiplication and division of various physical quantities.
   Do not worry about the number of digits after the decimal place. Round off the result so that it has the same number of significant figures as in the least accurate quantity.

Example 8 A man runs 100.2 m in 10.3 s. Find his average speed up to appropriate significant figure.

Solution Average speed $(v) = \dfrac{100.2\,\text{m}}{10.3\,\text{s}} = 9.728155 \text{ ms}^{-1}$

The distance 100.2 m has four significant figures but the time 10.3 s has only three. Hence the value of the result must be round off to three significant figures. The correct result is $v = 9.73 \text{ ms}^{-1}$

5. Least Counts of Some Measuring Instruments

1. Least count of metre scale = 1 mm = 0.1 cm
2. Vernier constant (or least count) of vernier callipers
   = value of 1 main scale division – value of 1 vernier scale division = 1 M.S.D. – 1 V.S.D

   Let the value of 1 M.S.D = a unit

   If n vernier scale divisions coincide with m main scale divisions, then value of

$$1.\text{V.S.D} = \frac{m}{n} \text{ of } 1 \text{ M.S.D} = \frac{ma}{n} \text{ unit}$$

$$\therefore \quad \text{Least count} = a - \frac{ma}{n} = \left(1 - \frac{m}{n}\right) a \text{ unit}$$

3. Least count of a micrometer screw is found by the formula

$$\text{Least count} = \frac{\text{Pitch of screw}}{\text{Total number of divisions on circular scale}}$$

   where pitch = lateral distance moved in one complete rotation of the screw.

Example 9 In a vernier callipers, 19 divisions of the main scale exactly coincide with 20 divisions of the vernier scale. If the smallest division of the main scale is 1.0 mm, find the vernier constant of the callipers.

Solution Value of 1 M.S.D. = 1.0 mm

20 divisions of the vernier scale = 19 M.S.D.

$$= 19 \times 1.0 \text{ mm} = 19 \text{ mm}$$

$$\therefore \quad \text{Value of 1 V.S.D.} = \frac{19}{20} \text{ mm}$$

Least count or vernier constant = 1 M.S.D. – 1 V.S.D

$$= 1.0 \text{ mm} - \frac{19}{20} \text{ mm}$$

$$= \frac{1}{20} \text{ mm} = 0.05 \text{ mm}$$

$$= 0.005 \text{ cm}$$

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6. Order of Accuracy: Proportionate Error

The order of accuracy of the result of measurements is determined by the least counts of the measuring instruments used to make those measurements. Suppose a length x is measured with a metre scale, then the error in x is $\pm \Delta x$, where Δx = least count of metre scale = 0.1 cm. If the same length is measured with vernier callipers of least count 0.01 cm, then $\Delta x = 0.01$ cm.

Fractional or proportionate error is defined as $\frac{\Delta x}{x}$.

Maximum percentage error = $\frac{\Delta x}{x} \times 100$.

1. **Error in Sum:** Suppose a quantity is given by

   $$a = x + y$$

   Then $\Delta a = \Delta x + \Delta y$ is the maximum error and

   $$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x + y)}$$

2. **Error in Difference:** If $a = x - y$, then the maximum error is

   $$\Delta a = \Delta x + \Delta y$$

   We take the worst case in which errors add up.

   $$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x - y)}$$

3. **Error in Product and Division:** Suppose we determine the value of a physical quantity u by measuring three quantities x, y and z whose true values are related to u by the equation

   $$u = x^{\alpha} y^{\beta} z^{-\gamma}$$

   Let the expected small errors in the measurement of quantities x, y and z be respectively $\pm \delta x$, $\pm \delta y$ and $\pm \delta z$ so that the error in u by using these observed quantities is $\pm \delta u$. The numerical values of δx, δy and δz are given by the least count of the instruments used to measure them.

   Taking logarithm of both sides we have

   $$\log u = \alpha \log x + \beta \log y - \gamma \log z$$

   Partial differentiation of the above equation gives

   $$\frac{\Delta u}{u} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} - \gamma \frac{\Delta z}{z}$$

   The proportional or relative error in u is $\Delta u/u$. The values of Δx, Δy and Δz may be positive or negative and in some cases the terms on the right hand side may counteract each other. This effect cannot be relied upon and it is necessary to consider the worst case which is the case when all errors *add* up giving an error Δu given by the equation:

   $$\left(\frac{\Delta u}{u}\right)_{\max} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} + \gamma \frac{\Delta z}{z}$$

   Thus to find the maximum proportional error in u, multiply the proportional errors in each factor (x, y and z) by the numerical value of the power to which each factor is raised and then add all the terms so obtained.

   The sum thus obtained will give the maximum proportional error in the result of u. When the proportional error of a quantity is multiplied by 100, we get the percentage error of that quantity.

Example 10 In an experiment for determining density (ρ) of a rectangular metal block, a student makes the following measurements.

Mass of block (m) = 39.3 g
Length of block (x) = 5.12 cm
Breadth of block (y) = 2.56 cm
Thickness of block (z) = 0.37 cm

The uncertainty in the measurement of m is $\pm$ 0.1 g and in the measurement of x, y and z is $\pm$ 0.01 cm. find the value of ρ (in g cm^{-3}) up to appropriate significant figures, stating the uncertainty in the value of ρ.

Solution $\rho = \frac{m}{x\,y\,z} = \frac{39.3}{5.12 \times 2.56 \times 0.37} = 8.1037\,\text{g cm}^{-3}$

$$\left(\frac{\Delta \rho}{\delta}\right)_{\max} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$= \frac{0.1}{39.3} + \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37}$$

$$= 0.0353$$

$\therefore$ $\Delta \rho = 0.0353 \times \rho = 0.0353 \times 8.1037$

$= 0.286$ gcm^{-3}

Round off error $\Delta \rho$ to the first significant figure as $\Delta \rho = 0.3$ gcm^{-3}.

Hence $\rho = 8.1037$ gm^{-3} is not accurate to the fourth decimal place. In fact, it is accurate only up to the first deciamal place. Hence the value of ρ much be rounded off as 8.1 and the result of the measurement is written as

$$\rho = (8.1 \pm 0.3)\ \text{g cm}^{-3}$$

NOTE

1. Errors always add; they never cancel other.
2. The quantity which is raised to the highest power contributes the maximum error and hence it must be measured to a high degree of accuracy.

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Applications

1. For a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$
2. For a sphere of radius r,

$$\text{Surface area } A = 4\pi r^2 \Rightarrow \frac{\Delta A}{A} = \frac{2\Delta r}{r}$$

$$\text{Volume } V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta V}{V} = \frac{3\Delta r}{r}$$

3. Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{2\Delta R}{R} + \frac{\Delta M}{M}$$

4. For resistances connected in series

$$R_s = R_1 + R_2 \Rightarrow \frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

5. For resistances connected in parallel

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad -\frac{\Delta R_p}{R_p^2} = -\frac{\Delta R_1}{R_1^2} - \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \frac{\Delta R_p}{R_P^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

6. Kinetic energy K and linear momentum p are related as

$$K = \frac{p^2}{2m} \quad \Rightarrow \quad \frac{\Delta K}{K} = \frac{2\Delta p}{p}$$

SECTION I

Multiple Choice Questions with One Correct Choice

1. Which one of the following is *not* a unit of length?
(a) angström (b) light year
(c) fermi (d) radian

2. The unit of impulse is the same as that of
(a) moment of force
(b) linear momentum
(c) rate of change of linear momentum
(d) force

3. Which pair of quantities has dimensions different from the other three pairs?
(a) Impulse and linear momentum
(b) Planck's constant and angular momentum
(c) Moment of inertia and moment of force
(d) Young's modulus and pressure

4. The dimensions of the coefficient of viscosity are
(a) ML^2T^{-2} (b) MLT^{-1}
(c) $ML^{-1}T^{-1}$ (d) $ML^{-1}T^{-2}$

5. The dimensions of surface tension are
(a) ML^0T^{-2} (b) MLT^{-2}
(c) $ML^{-1}T^{-2}$ (d) $ML^{-2}T^{-2}$

6. The SI unit of the universal gravitational constant G is
(a) $Nm\,kg^{-2}$ (b) $Nm^2\,kg^{-2}$
(c) $Nm^2\,kg^{-1}$ (d) $Nm\,kg^{-1}$

7. The dimensions of the coefficient of thermal conductivity are
(a) $MLT^{-3}K^{-1}$ (b) $MLT^{-2}K^{-1}$
(c) $MLT^{-1}K^{-1}$ (d) $MLT^{-2}K^{-2}$

8. The SI unit of Stefan's constant is
(a) $W\,s^{-1}\,m^{-2}\,K^{-4}$ (b) $J\,s\,m^{-2}\,K^{-4}$
(c) $J\,s^{-1}\,m^{-2}\,K^{-1}$ (d) $Wm^{-2}\,K^{-4}$

9. The SI unit of magnetic permeability is
(a) $A\,m^{-1}$
(b) A m
(c) $H\,m^{-1}$
(d) No unit; it is a dimensionless number

10. The quantities L/R and RC (where L, C and R stand for inductance, capacitance and resistance respectively) have the same dimensions as those of
(a) velocity (b) acceleration
(c) time (d) force

11. The dimensions of entropy are
(a) $M^0L^{-1}T^0K$ (b) $M^0L^{-2}T^0K^2$
(c) $MLT^{-2}K$ (d) $ML^2T^{-2}K^{-1}$

12. What is the physical quantity whose dimensions are ML^2T^{-2}?
(a) kinetic energy (b) pressure
(c) momentum (d) power

13. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(a) torque (b) surface tension
(c) viscosity (d) stress

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14. The dimensions of angular momentum are
(a) MLT^{-1} (b) ML^2T^{-1}
(c) $ML^{-1}T$ (d) ML^0T^{-2}

15. The gravitational force F between two masses m_1 and m_2 separated by a distance r is given by $F = \frac{G m_1 m_2}{r^2}$ where G is the universal gravitational constant. What are the dimensions of G?
(a) $M^{-1}L^3T^{-2}$ (b) ML^3T^{-2}
(c) ML^2T^{-3} (d) $M^{-1}L^2T^{-3}$

16. The equation of state of a real gas can be expressed as $\left(P + \frac{a}{V^2}\right)(V - b) = cT$ where P is the pressure, V the volume, T the absolute temperature and a, b and c are constants. What are the dimensions of a?
(a) $M^0L^3T^{-2}$ (b) ML^5T^{-2}
(c) $M^0L^3T^0$ (d) $ML^{-2}T^5$

17. The equation of state for n moles of an ideal gas is

$$PV = nRT$$

where R is the universal gas constant and P, V and T have the usual meanings. What are the dimensions of R?
(a) $M^0LT^{-2}K^{-1}mol^{-1}$
(b) $ML^2T^{-2}K^{-1}mol^{-1}$
(c) $M^0L^2T^{-2}K^{-1}mol^{-1}$
(d) $ML^{-2}T^{-2}K^{-1}mol^{-1}$

18. The SI unit of the universal gas constant R is
(a) erg $K^{-1}mol^{-1}$ (b) watt $K^{-1}mol^{-1}$
(c) newton $K^{-1}mol^{-1}$ (d) joule $K^{-1}mol^{-1}$

19. According to the quantum theory, the energy E of a photon of frequency ν is given by

$$E = h\nu$$

where h is Planck's constant. What is the dimensional formula for h?
(a) ML^2T^{-2} (b) ML^2T^{-1}
(c) ML^2T (d) ML^2T^2

20. What is the SI unit of Planck's constant?
(a) watt second (b) watt per second
(c) joule second (d) joule per second

21. The dimensions of Planck's constant are the same as those of
(a) energy (b) power
(c) angular frequency (d) angular momentum

22. Time period T of a simple pendulum may depend on m, the mass of the bob, l, the length of the string and g, the acceleration due to gravity, i.e.

$$T \propto m^a l^b g^c$$

What are the values of a, b and c?
(a) $0, \frac{1}{2}, -\frac{1}{2}$ (b) $0, -\frac{1}{2}, \frac{1}{2}$
(c) $\frac{1}{2}, 0, -\frac{1}{2}$, (d) $-\frac{1}{2}, 0, \frac{1}{2}$

23. The volume V of water passing any point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation

$$V \propto A^\alpha u^\beta t^\gamma$$

which one of the following will be true?
(a) $\alpha = \beta = \gamma$ (b) $\alpha \neq \beta = \gamma$
(c) $\alpha = \beta \neq \gamma$ (d) $\alpha \neq \beta \neq \gamma$

24. Which one of the following relations is dimensionally consistent where h is height to which a liquid of density ρ rises in a capillary tube of radius, r, T is the surface tension of the liquid, θ the angle of contact and g the acceleration due to gravity?
(a) $h = \frac{2T\cos\theta}{r\rho g}$ (b) $h = \frac{2Tr}{\rho g\cos\theta}$
(c) $h = \frac{2\rho g\cos\theta}{2Tr}$ (d) $h = \frac{2Tr\rho g}{\cos\theta}$

25. The frequency n of vibrations of uniform string of length l and stretched with a force F is given by

$$n = \frac{p}{2l}\sqrt{\frac{F}{m}}$$

where p is the number of segments of the vibrating string and m is a constant of the string. What are the dimensions of m?
(a) $ML^{-1}T^{-1}$ (b) $ML^{-3}T^0$
(c) $ML^{-2}T^0$ (d) $ML^{-1}T^0$

26. When a wave traverses a medium, the displacement of a particle located at x at time t is given by

$$y = a\sin(bt - cx)$$

where a, b and c are constants of the wave. The dimensions of b are the same as those of
(a) wave velocity (b) amplitude
(c) wavelength (d) wave frequency

27. In Q.26, the dimensions of $\frac{b}{c}$ are the same as those of
(a) wave velocity (b) wavelength
(c) wave amplitude (d) wave frequency

28. The van der Waal equation for n moles of a real gas is

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$$\left(P+\frac{a}{V^2}\right)(V-b)=nRT$$

where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are van der Waal constants. The dimensions of a are the same as those of

(a) PV (b) PV^2
(c) P^2V (d) P/V

29. In Q.28, the dimensions of b are the same as those of

(a) P (b) V
(c) PV (d) nRT

30. In Q.28, the dimensions of nRT are the same as those of

(a) energy (b) force
(c) pressure (d) specific heat

31. In Q. 28, the dimensional formula for ab is

(a) ML^2T^{-2} (b) ML^4T^{-2}
(c) ML^6T^{-2} (d) ML^8T^{-2}

32. If velocity (V), acceleration (A) and force (F) are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimensions of Young's modulus would be

(a) FA^2V^{-2} (b) FA^2V^{-3}
(c) FA^2V^{-4} (d) FA^2V^{-5}

33. The dimensions of permittivity (ε_0) of vacuum are

(a) $M^{-1}L^{-3}T^4A^2$ (b) $ML^{-3}T^2A^2$
(c) $M^{-1}L^3T^4A^2$ (d) $ML^3T^2A^2$

34. What are the dimensions of permeability (μ_0) of vacuum?

(a) $MLT^{-2}A^2$ (b) $MLT^{-2}A^{-2}$
(c) $ML^{-1}T^{-2}A^2$ (d) $ML^{-1}T^{-2}A^{-2}$

35. The dimensions of $1/\sqrt{\mu_0\varepsilon_0}$ are the same as those of

(a) velocity (b) acceleration
(c) force (d) energy

36. The dimensions of specific heat are

(a) $MLT^{-2}K^{-1}$ (b) $ML^2T^{-2}K^{-1}$
(c) $M^0L^2T^{-2}K^{-1}$ (d) $M^0LT^{-2}K^{-1}$

37. What are the dimensions of latent heat?

(a) ML^2T^{-2} (b) $ML^{-2}T^{-2}$
(c) M^0LT^{-2} (d) $M^0L^2T^{-2}$

38. What are the dimensions of Boltzmann's constant?

(a) $MLT^{-2}K^{-1}$ (b) $ML^2T^{-2}K^{-1}$
(c) $M^0LT^{-2}K^{-1}$ (d) $M^0L^2T^{-2}K^{-1}$

39. The dimensions of potential difference are

(a) $ML^2T^{-3}A^{-1}$ (b) $MLT^{-2}A^{-1}$
(c) $ML^2T^{-2}A$ (d) $MLT^{-2}A$

40. What are the dimensions of electrical resistance?

(a) $ML^2T^{-2}A^2$ (b) $ML^2T^{-3}A^{-2}$
(c) $ML^2T^{-3}A^2$ (d) $ML^2T^{-2}A^{-2}$

41. The dimensions of electric field are

(a) $MLT^{-3}A^{-1}$ (b) $MLT^{-2}A^{-1}$
(c) $MLT^{-1}A^{-1}$ (d) MLT^0A^{-1}

42. The dimensions of magnetic induction field are

(a) $ML^0T^{-1}A^{-1}$ (b) $M^0LT^{-1}A^{-1}$
(c) $MLT^{-2}A^{-1}$ (d) $ML^0T^{-2}A^{-1}$

43. What are the dimensions of magnetic flux?

(a) $ML^2T^{-2}A^{-1}$ (b) $ML^2T^{-2}A^{-2}$
(c) $ML^{-2}T^{-2}A^{-1}$ (d) $ML^{-2}T^{-2}A^{-2}$

44. The dimensions of self inductance are

(a) $ML^2T^{-2}A^{-1}$ (b) $ML^2T^{-2}A^{-2}$
(c) $ML^{-2}T^{-2}A^{-1}$ (d) $ML^{-2}T^{-2}A^{-2}$

45. The dimensions of capacitance are

(a) $M^{-1}L^{-2}TA^2$ (b) $M^{-1}L^{-2}T^2A^2$
(c) $M^{-1}L^{-2}T^3A^2$ (d) $M^{-1}L^{-2}T^4A^2$

46. If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be

(a) $V^{-2}F^0E^{-1}$ (b) V^0FE^2
(c) $VF^{-2}E^0$ (d) $V^{-2}F^0E$

47. Frequency (n) of a tuning fork depends upon length (l) of its prongs, density (ρ) and Young's modulus (Y) of its material. Then frequency and Young's modulus will be related as

(a) $n \propto \sqrt{Y}$ (b) $n \propto Y$
(c) $n \propto \frac{1}{\sqrt{Y}}$ (d) $n \propto \frac{1}{Y}$

48. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of free space and E = electric field) are

(a) MLT^{-1} (b) ML^2T^{-2}
(c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}

49. Of the following quantities, which one has dimensions different from the remaining three

(a) Energy per unit volume
(b) Force per unit area
(c) Product of voltage and charge per unit volume
(d) Angular momentum

50. If the time period t of a drop of liquid of density d, radius r, vibrating under surface tension s is given

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by the formula $t = \sqrt{d^a r^b s^c}$ and if $a = 1, c = -1$, then b is

(a) 1 (b) 2
(c) 3 (d) 4

51. A pair of physical quantities having the same dimensional formula is

(a) angular momentum and torque
(b) torque and energy
(c) entropy and power
(d) power and angular momentum

52. In the measurement of a physical quantity $X = \frac{A^2 B}{C^{1/3} D^3}$. The percentage errors introduced in the measurements of the quantities A, B, C and D are 2%, 2%, 4% and 5% respectively. Then the minimum amount of percentage of error in the measurement of X is contributed by:

(a) A (b) B
(c) C (d) D

53. Which of the following has the dimensions $ML^{-1}T^{-1}$?

(a) Surface tension
(b) Coefficient of viscosity
(c) Bulk modulus
(d) Angular momentum

54. Pressure gradient dp/dx is the rate of change of pressure with distance. What are the dimensions of dp/dx?

(a) $ML^{-1}T^{-1}$ (b) $ML^{-2}T^{-2}$
(c) $ML^{-1}T^{-2}$ (d) $ML^{-2}T^{-1}$

55. If E, M, J and G respectively denote energy, mass, angular momentum and gravitational constant, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of

(a) length (b) angle
(c) mass (d) time

56. If e, ε_0, h and c respectively represent electronic charge, permittivity of free space, Planck's constant and speed of light, then $\frac{e^2}{\varepsilon_0 hc}$ has the dimensions of

(a) current
(b) pressure
(c) angular momentum
(d) angle

57. If L, R, C and V respectively represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of

(a) current (b) $\frac{1}{\text{current}}$
(c) charge (d) $\frac{1}{\text{charge}}$

58. If E and B respectively represent electric field and magnetic induction field, then the ratio $\frac{E}{B}$ has the dimensions of

(a) displacement (b) velocity
(c) acceleration (d) angle

59. If C and V respectively represent the capacitance of a capacitor and the potential difference between its plates, then the dimensions of CV^2 are

(a) ML^2T^{-2} (b) $ML^3T^{-2}I^{-1}$
(c) $ML^2T^{-1}I^{-1}$ (d) $M^0L^0T^0$

60. If h and e respectively represent Planck's constant and electronic charge, then the dimensions of $\left(\frac{h}{e}\right)$ are the same as those of

(a) magnetic field (b) electric field
(c) magnetic flux (d) electric flux

61. If energy E, velocity V and time T are chosen as the fundamental units, the dimensional formula for surface tension will be

(a) EV^2T^{-2} (b) $EV^{-1}T^{-2}$
(c) $EV^{-2}T^{-2}$ (d) $E^2V^{-1}T^{-2}$

62. The number of particles crossing a unit area perpendicular to the x-axis in a unit time is given by

$$n = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$$

where n_1 and n_2 are the number of particles per unit volume at $x = x_1$ and $x = x_2$ respectively and D is the diffusion constant. The dimensions of D are

(a) M^0LT^{-2} (b) $M^0L^2T^{-4}$
(c) M^0LT^{-3} (d) $M^0L^2T^{-1}$

63. A gas bubble from an explosion under water oscillates with a period proportional to $P^a d^b E^c$ where P is the static pressure, d is the density of water and E is the energy of explosion. Then a, b and c respectively are

(a) $\frac{-5}{6}, \frac{1}{2}, \frac{1}{3}$ (b) $\frac{1}{2}, \frac{-5}{6}, \frac{1}{3}$
(c) $\frac{1}{3}, \frac{1}{2}, \frac{-5}{6}$ (d) 1, 1, 1

64. In a system of units in which the unit of mass is a kg, unit of length is b metre and the unit of time is c second, the magnitude of a calorie is

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(a) $\frac{4.2c}{ab^2}$ (b) $\frac{4.2c^2}{ab^2}$

(c) $\frac{abc}{4.2}$ (d) $\frac{4.2}{abc}$

65. The error in the measurement of the radius of a sphere is 1%. The error in the measurement of the volume is

(a) 1% (b) 3%
(c) 5% (d) 8%

66. If the error in the measurement of the volume of a sphere is 6%, then the error in the measurement of its surface area will be

(a) 2% (b) 3%
(c) 4% (d) 7.5%

67. A physical quantity X is represented by $X=(M^xL^yT^{-z})$. The maximum percentage errors in the measurement of M, L and T respectively are a%, b% and c%. The maximum percentage error in the measurement of X will be

(a) $(ax + by - cz)$ percent
(b) $(ax + by + cz)$ percent
(c) $(ax - by + cz)$ percent
(d) $(ax - by - cz)$ percent

68. The percentage errors in the measurements of the length of a simple pendulum and its time period are 2% and 3% respectively. The maximum error in the value of the acceleration due to gravity obtained from these measurements is

(a) 5% (b) 1%
(c) 8% (d) 10%

69. The moment of inertia of a body rotating about a given axis is 6.0 kg m^2 in the SI system. What is the value of the moment of inertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?

(a) 2.4×10^3 (b) 2.4×10^5
(c) 6.0×10^3 (d) 6.0×10^5

70. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

(a) resistance (b) charge
(c) voltage (d) current

71. The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula

$$\eta = \frac{\pi}{8}\frac{R^4}{l}\frac{P}{Q}$$

where R = radius of the capillary tube,
l = length of the tube,
P = pressure difference between its ends, and
Q = volume of liquid flowing per second.

Which quantity must be measured most accurately?

(a) R (b) l
(c) P (d) Q

72. The mass m of the heaviest stone that can be moved by the water flowing in a river depends on v, the speed of water, density (d) of water and the acceleration due to gravity (g). Then m is proportional to

(a) v^2 (b) v^4
(c) v^6 (d) v^8

73. The speed (v) of ripples depends upon their wavelenth (λ), density (ρ) and surface tension (σ) of water. Then v is proportional to

(a) $\sqrt{\lambda}$ (b) λ

(c) $\frac{1}{\lambda}$ (d) $\frac{1}{\sqrt{\lambda}}$

74. The period of revolution (T) of a planet moving round the sun in a circular orbit depends upon the radius (r) of the orbit, mass (M) of the sun and the gravitation constant (G). Then T is proportional to

(a) $r^{1/2}$ (b) r
(c) $r^{3/2}$ (d) r^2

75. If energy (E), momentum (p) and force (F) are chosen as fundamental units, the dimensions of mass in the new system will be

(a) $E^{-1}p^2F^0$ (b) $E^1p^{-2}F^0$
(c) $E^{-1}p^2F^{-2}$ (d) $E^{-2}p^1F^2$

76. If the velocity of light (c), gravitational constant (G) and planck's constant (h) are chosen as fundamental units, the dimensions of time in the new system will be

(a) $c^{-5/2}G^2h^{-1/2}$ (b) $c^{-3/2}G^{-2}h^2$
(c) $c^2G^{-2}h^{1/2}$ (d) $c^{-5/2}G^{1/2}h^{1/2}$

77. The amplitude of a damped oscillator of mass m varies with time t as

$$A = A_0e^{(-at/m)}$$

The dimensions of a are

(a) ML^0T^{-1} (b) M^0LT^{-1}
(c) MLT^{-1} (d) $ML^{-1}T$

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78. A student measures the value of g with the help of a simple pendulum using the formula

$$g = \frac{4\pi^2 L}{T^2}$$

The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?

(a) $\Delta L = 0.5$ cm, $\Delta T = 0.5$ s
(b) $\Delta L = 0.2$ cm, $\Delta T = 0.2$ s
(c) $\Delta L = 0.1$ cm, $\Delta T = 1.0$ s
(d) $\Delta L = 0.1$ cm, $\Delta T = 0.1$ s

79. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with a uncertainty of ± 0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained from the reading is

(a) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2
(b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2
(d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

80. In a vernier callipers, one main scale division is x cm and n divisions of the vernier scale coincide with $(n - 1)$ divisions of the main scale. The least count (in cm) of the callipers is

(a) $\left(\frac{n-1}{n}\right)x$ (b) $\frac{nx}{(n-1)}$

(c) $\frac{x}{n}$ (d) $\frac{x}{(n-1)}$

ANSWERS

1. (d) **2.** (b) **3.** (c)
4. (c) **5.** (a) **6.** (b)
7. (a) **8.** (d) **9.** (c)
10. (c) **11.** (d) **12.** (a)
13. (d) **14.** (b) **15.** (a)
16. (b) **17.** (b) **18.** (d)
19. (b) **20.** (c) **21.** (d)
22. (a) **23.** (b) **24.** (a)
25. (d) **26.** (d) **27.** (a)
28. (b) **29.** (b) **30.** (a)
31. (d) **32.** (c) **33.** (a)
34. (b) **35.** (a) **36.** (c)
37. (d) **38.** (b) **39.** (a)
40. (b) **41.** (a) **42.** (d)
43. (a) **44.** (b) **45.** (d)
46. (d) **47.** (a) **48.** (c)
49. (d) **50.** (c) **51.** (b)
52. (c) **53.** (b) **54.** (b)
55. (b) **56.** (d) **57.** (b)
58. (b) **59.** (a) **60.** (c)
61. (c) **62.** (d) **63.** (a)
64. (b) **65.** (b) **66.** (c)
67. (b) **68.** (c) **69.** (b)
70. (d) **71.** (a) **72.** (c)
73. (d) **74.** (c) **75.** (a)
76. (d) **77.** (a) **78.** (d)
79. (b) **80.** (c)

➔ SOLUTIONS

1. Choices (a), (b) and (c) are units of length. Choice (d) the radian is a unit of angle in a plane.

2. Impulse = force × time

$$\frac{d\mathbf{p}}{dt} \times dt$$

$= d\mathbf{p}$

= change in momentum

Hence the correct choice is (b).

3. The dimensions of moment of inertia are ML^2T^0 and of moment of force are $ML^2\,T^{-2}$. All other pairs in (a), (b) and (d) have identical dimensions.

4. The viscous force acting on a spherical body of radius r moving with a speed v in a fluid of coefficient of viscosity η is given by

$$F = 6\pi\,\eta\,r\,v \text{ or } \eta = \frac{F}{6\pi r v}$$

$\therefore$ Dimensions of η

$$= \frac{\text{dimension of } F}{\text{dimension of } r \times \text{dimension of } v}$$

$$= \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$$

Hence the correct choice is (c).

5. Surface tension = force/length = $MLT^{-2}/L = ML^0T^{-2}$. Hence the correct choice is (a).

6. From Newton's law of gravitation, the force of attraction F between two bodies of masses m_1 and m_2, separated by a distance r is given by

$$F = G\,\frac{m_1 m_2}{r^2};$$

G = gravitational constant

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or $$G = \frac{Fr^2}{m_1 m_2}$$

$$\therefore \quad \text{Unit of } G = \frac{\text{unit of } F \times \text{unit of } r^2}{\text{unit of } m_1 m_2}$$

$$= \frac{\text{newton} \times (\text{metre})^2}{(\text{kg})^2} = \text{N m}^2 \text{ kg}^{-2}$$

Hence the correct choice is (b).

7. The coefficient of thermal conductivity is defined as the rate of flow of heat energy per unit area of cross-section per unit temperature gradient. The dimensions of rate of flow of heat are $ML^2 T^{-2}/T = ML^2 T^{-3}$. The dimensions of area are L^2 and the dimensions of temperature gradient are = dimensions of temperature/dimension of length $= KL^{-1}$. Hence the dimensions of the coefficient of thermal conductivity are

$$\frac{ML^2 T^{-3}}{L^2 \times KL^{-1}} = MLT^{-3} K^{-1}$$

Thus the correct choice is (a).

8. From Stefan's law, the total energy emitted per second by a unit area of a black body is proportional to the fourth power of its absolute temperature, i.e.

$$E \propto T^4 \text{ or } \quad E = \sigma T^4$$

where σ is the stefan's constant. Thus

$$\sigma = \frac{E}{T^4}$$

$\therefore$ SI unit of σ

$$= \frac{\text{SI unit of energy per second}}{\text{SI unit of area} \times (\text{SI unit of temperature})^4}$$

$$= \frac{\text{watt}}{(\text{metre})^2 \times (\text{kelvin})^4} = \text{W m}^{-2} \text{ K}^{-4}$$

Hence the correct choice is (d).

9. The correct choice is (c).

10. L/R is the time constant of an L-R circuit and CR is the time constant of a C-R circuit. The dimension of the time constant is the same as that of time. Hence the correct choice is (c).

11. Entropy S is defined through the relation

$$dS = \frac{dQ}{T}$$

where dS is the change in entropy, dQ the change is heat energy and T is the absolute temperature.

$\therefore$ Dimensions of entropy

$$= \frac{\text{dimensions of heat energy}}{\text{dimensions of temperature}} = \frac{ML^2 T^{-2}}{K}$$

$$= ML^2 T^{-2} K^{-1}$$

Hence the correct choice is (d).

12. The dimensions of energy are $ML^2 T^{-2}$. The dimensions of pressure, momentum and power are $ML^{-1} T^{-2}$, MLT^{-1} and $ML^2 T^{-3}$ respectively. Thus the correct choice is (a).

13. $ML^{-1} T^{-2}$ are the dimensions of force per unit area. Out of the four choices, stress is the only quantity that is force per unit area. Hence the correct choice is (d).

14. The angular momentum **L** of a particle with respect to point whose position vector is **r** is given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **p** is the linear momentum of the moving particle.

$\therefore$ Dimensions of **L** = dimension of **r** $\times$ dimensions of **p** $= L \times MLT^{-1} = ML^2 T^{-1}$

Thus the correct choice is (b).

15. Since $G = \frac{Fr^2}{m_1 m_2}$, the dimensions of G are

$$(G) = \frac{\text{dimensions of } F \times \text{dimensions of } r^2}{\text{dimensions of } m_1 m_2}$$

$$= \frac{MLT^{-2} \times L^2}{M^2} = M^{-1} L^3 T^{-2}$$

Thus the correct choice is (a).

16. From the principle of homogeneity of dimensions, the dimensions of $\frac{a}{V^2}$ must be the same as those of P. Therefore dimensions of a = dimensions of $P \times$ dimensions of V^2

$= ML^{-1} T^{-2} \times (L^3)^2 = ML^{-1} T^{-2} \times L^6 = ML^5 T^{-2}$.

Thus the correct choice is (b).

17. Since $R = \frac{PV}{nT}$, the dimensions of R are given by

$$(R) = \frac{\text{dimensions of } P \times \text{dimensions of } V}{\text{dimensions of } n \times \text{dimensions of } T}$$

$$= \frac{ML^{-1} T^{-2} \times L^3}{\text{mol} \times K} = ML^2 T^{-2} \text{ mol}^{-1} K^{-1}$$

Thus the correct choice is (b).

18. Now $R = \frac{PV}{nT}$. Since the product PV has the same units as those of work, namely, joule; the unit of R is

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$$\frac{\text{unit of } PV}{\text{unit of } n \times \text{unit of } T} = \frac{\text{J}}{\text{mol} \times \text{K}} = \text{JK}^{-1}\,\text{mol}^{-1}.$$

Hence the correct choice is (d).

19. Dimensions of $h = \dfrac{\text{dimension of } E}{\text{dimension of } \nu}$

$$= \frac{\text{ML}^2\text{T}^{-2}}{\text{T}^{-1}} = \text{ML}^2\,\text{T}^{-1}$$

Thus the correct choice is (b).

20. Unit of $h = \dfrac{\text{unit of } E}{\text{unit of } \nu} = \dfrac{\text{joule}}{(\text{second})^{-1}} = \text{joule second.}$

Thus the correct choice is (c).

21. The correct choice is (d).

22. The dimensions of the two sides of proportionality are

$$\text{T} = \text{M}^a\,\text{L}^b\,(\text{LT}^{-2})^c$$

where LT^{-2} are the dimensions of g.

$$\therefore \qquad \text{T} = \text{M}^a\,\text{L}^b\,\text{L}^c\,\text{T}^{-2c} = \text{M}^a\,\text{L}^{b+c}\,\text{T}^{-2c}$$

Equating the powers of dimensions on both sides, we have $a = 0$, $b + c = 0$ and $-2c = 1$

which give $\quad c = -\dfrac{1}{2}$, $b = \dfrac{1}{2}$ and $a = 0$.

Thus the correct chioce is (a).

23. The dimensions of the two sides of proportionality are

$$\text{L}^3 = \text{L}^{2\alpha}(\text{LT}^{-1})^{\beta}\text{T}^{\gamma} = \text{L}^{2\alpha+\beta}\text{T}^{\gamma-\beta}$$

Equating the powers of dimensions on both sides, we have

$$2\alpha + \beta = 3$$

$$\gamma - \beta = 0$$

which give $\beta = \gamma$ and $\alpha = \dfrac{1}{2}(3 - \beta)$, i.e. $\alpha \neq \beta = \gamma$.

Thus the correct choice is (b).

24. Since $\cos\theta$ is dimensionless, using the dimensions of T, r, ρ and g, it is easy to see that choice (a) is only one that is dimensionally consistent. The dimensions of h are the same as those of $T/r\rho g$.

25. Squaring both sides of the given relation, we get

$$n^2 = \frac{p^2}{4\,l^2} \cdot \frac{F}{m} \quad \text{or} \quad m = \frac{p^2 F}{4\,l^2 n^2}$$

$\therefore$ dimensions of m

$$= \frac{\text{dimensions of } F}{\text{dimensions of } l^2 \times \text{dimensions of } n^2}$$

($\because$ p is a dimensionless number)

$$= \frac{\text{MLT}^{-2}}{\text{L}^2 \times (\text{T}^{-1})^2} = \text{ML}^{-1}\,\text{T}^0$$

Hence the correct choice is (d).

26. Since the argument of a sine function (or any trigonometric function) must be dimensionless, bt and cx are dimensionless. Since bt is dimensionless, the dimensions of b = dimensions of $1/t = \text{T}^{-1}$, which are the dimensions of frequency. Hence the correct choice is (d).

27. Dimensions of bt = dimensions of cx, as they are both dimensionless.

$\therefore$ Dimensions of $\dfrac{b}{c}$ = dimensions of $\dfrac{x}{t} = \dfrac{\text{L}}{\text{T}} = \text{LT}^{-1}$.

Hence the correct choice is (a).

28. From the principle of homogeneity, the dimensions of $\dfrac{a}{V^2}$ must be the same as those of P, i.e. dimensions of $\dfrac{a}{V^2}$ = dimensions of P

$\therefore$ dimensions of a = dimensions of PV^2. Hence the correct choice is (b).

29. The correct choice is (b).

30. The dimensions of nRT = dimensions of PV

$$= \text{ML}^{-1}\,\text{T}^{-2} \times \text{L}^3 = \text{ML}^2\,\text{T}^{-2}$$

which are the dimensions of energy. Hence the correct choice is (a)

31. The dimensions of $\dfrac{ab}{V^2}$ are the same as those of PV.

$\therefore$ Dimensions of ab = dimensions of $(PV) \times V^2 = \text{ML}^2\,\text{T}^{-2} \times \text{L}^6 = \text{ML}^8\,\text{T}^{-2}$

Hence the correct choice is (d).

32. Dimensions of Young's modulus Y are $\text{ML}^{-1}\,\text{T}^{-2}$. The dimension of V, A and F in terms of M, L and T are

$$(V) = \text{LT}^{-1},\ (A) = \text{LT}^{-2}$$

and $\qquad (F) = \text{MLT}^{-2}$

Let $\qquad (Y) = (V^a\,A^b\,F^c)$

Putting dimensions of Y, V, A and F. we have

$$(\text{ML}^{-1}\,\text{T}^{-2}) = (\text{LT}^{-1})^a \times (\text{LT}^{-2})^b \times (\text{MLT}^{-2})^c$$

or $\quad \text{M}^1\,\text{L}^{-1}\,\text{T}^{-2} = \text{M}^c\,\text{L}^{a+b+c}\,\text{T}^{-a-2b-2c}$

Equating powers of M, L and T we have

$$c = 1,\ a + b + c = -1$$

and $\quad -a - 2b - 2c = -2$

which give $\qquad a = -4$, $b = 2$ and $c = 1$.

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Thus $(Y) = (FA^2V^{-4})$
Thus the correct choice is (c).

33. According to Coulomb's law of electrostatics, force F between two charges q_1 and q_2 a distance r apart in vacuum, is given by

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$$

or $$\varepsilon_0 = \frac{1}{4\pi F}\cdot\frac{q_1q_2}{r^2}$$

$$\therefore \quad \text{Dimensions of } \varepsilon_0 = \frac{Q^2}{MLT^{-2}\times L^2}$$

$$= M^{-1}L^{-3}T^2Q^2 = M^{-1}L^{-3}T^4A^2 \quad \left(\because A = \frac{Q}{T}\right)$$

The correct choice is (a).

34. The force per unit length between two long wires carrying currents I_1 and I_2 a distance r apart in vacuum, is given by

$$f = \frac{\mu_0}{4\pi}\cdot\frac{I_1I_2}{r} \quad \text{or} \quad \mu_0 = \frac{4\pi rf}{I_1I_2}$$

$$\therefore \quad \text{Dimensions of } \mu_0 = \frac{L\times MLT^{-2}\times L^{-1}}{A^2}$$

$$= MLT^{-2}A^{-2}$$

Therefore, the correct choice is (b).

35. Dimensions of

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\left(MLT^{-2}\,A^{-2}\times M^{-1}L^{-3}T^4A^2\right)^{\frac{1}{2}}}$$

$$= \frac{1}{\left(L^{-2}T^2\right)^{\frac{1}{2}}} = LT^{-1}$$

which are the dimensions of velocity. Hence the correct choice is (a).

36. The heat energy content H of a body of mass m at temperature θ is given by $H = ms\theta$ where s is the specific heat. Therefore

$$s = \frac{H}{m\theta}$$

$\therefore$ Dimensions of s

$$= \frac{\text{dimensions of heat energy}}{\text{dimension of mass}\times\text{dimension of temperature}}$$

$$= \frac{ML^2T^{-2}}{M\times K} = M^0L^2T^{-2}K^{-1}$$

Thus the correct choice is (c).

37. Latent heat L is the amount of heat energy H required to change the state of a unit mass without producing any change in temperature. Thus

$$L = \frac{H}{m}$$

$$\therefore \quad \text{Dimensions of } L = \frac{ML^2T^{-2}}{M}$$

$$= L^2T^{-2} = M^0L^2T^{-2}$$

Thus the correct choice is (d).

38. According to the law of equipartition of energy, the energy per degree of freedom of a gas atom or molecule at a temperature θ kelvin is given by

$$E = \frac{1}{2}k\theta \quad \text{or} \quad k = \frac{2E}{\theta}$$

where k is the Boltzmann's constant.

$$\therefore \quad \text{Dimensions of } k = \frac{\text{dimensions of } E}{\text{dimension of } \theta}$$

$$= = \frac{ML^2T^{-2}}{K} = ML^2T^{\ 2}K^{-1}$$

39. The potential difference V between two points is the amount of work done in moving a unit charge from one point to the other.

Thus, $$V = \frac{\text{work done}}{\text{charge moved}} = \frac{W}{q}$$

$$\therefore \quad \text{Dimensions of } V = \frac{ML^2T^{-2}}{Q} = ML^2\,T^{-2}\,Q^{-1}$$

$$= ML^2\,T^{-3}\,A^{-1} \quad (\because Q = AT)$$

Hence the correct choice is (a).

40. From Ohm's law, resistance R is given by

$$R = \frac{\text{potential difference}}{\text{current}}$$

$$\therefore \text{ Dimensions of } R = \frac{ML^2T^{-3}A^{-1}}{A} = ML^2T^{-3}A^{-2}$$

Thus the correct choice is (b).

41. Force F experienced by a charge q in an electric field E is given by

$$F = qE \text{ or } E = \frac{F}{q}$$

$$\therefore \text{ Dimensions of } E = \frac{\text{dimensions of } F}{\text{dimensions of } Q} = \frac{MLT^{-2}}{AT}$$

$$= MLT^{-3}A^{-1}.$$

42. The force F, experienced by a charge q moving with speed v perpendicular to the direction of a uniform magnetic induction field B is given by

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$$F = qvB \text{ or } B = \frac{F}{qv}$$

$$\therefore \text{ Dimensions of } B = \frac{MLT^{-2}}{Q \times LT^{-1}} = ML^0T^{-1}Q^{-1}$$

$$= ML^0T^{-2}A^{-1} \qquad (\because Q = AT)$$

Hence the correct choice is (d).

43. The magnetic flux ϕ linked with a circuit of area A in a magnetic induction field B is given by

$$\phi = BA \cos \theta$$

where θ is the angle between the field and area vectors.

$\therefore$ Dimensions of ϕ = dimensions of BA

($\because \cos \theta$ is dimensionless)

$$= ML^0T^{-2}A^{-1} \times L^2 = ML^2T^{-2}A^{-1}$$

Thus the correct choice is (a).

44. The self inductance L of a coil in which the current varies at a rate $\frac{dI}{dt}$ is given by

$$e = -L \frac{dI}{dt}$$

where e is the e.m.f. induced in the coil. Now, the dimensions of e.m.f. are the same as those of potential difference, namely, $ML^2T^{-3}A^{-1}$.

Now, $$L = -\frac{e}{\frac{dI}{dt}}$$

$\therefore$ Dimensions of L

$$= \frac{\text{dimensions of } e}{\text{dimensions of } I / \text{dimensions of } t}$$

$$= \frac{ML^2T^{-3}A^{-1}}{A/T} = ML^2T^{-2}A^{-2}$$

Thus the correct choice is (b).

45. When a capacitor of capacitance C is charged to a potential difference V, the charge Q on the capacitor plates is given by

$$Q = CV \text{ or } C = \frac{Q}{V}$$

$$\therefore \text{ Dimensions of } C = \frac{\text{dimensions of } Q}{\text{dimensions of } V}$$

$$= \frac{AT}{ML^2T^{-3}A^{-1}} = M^{-1}L^{-2}T^4A^2$$

Hence the correct choice is (d).

46. Let $m \propto V^a F^b E^c$

$\therefore [M^1] = [L T^{-1}]^a \times [ML T^{-2}]^b \times [ML^2 T^{-2}]^c$

Equating the powers of M, L and T and solving, we get $a = -2$, $b = 0$ and $c = 1$. Hence the dimensional formula of $m = [V^{-2} F^0 E]$ which is choice (d).

47. Let $n \propto l^a \rho^b Y^c$

Putting dimensions of all the quantities, we have

$$(T^{-1}) \propto L^a (ML^{-3})^b (ML^{-1} T^{-2})^c$$

Equating powers of M, L and T on both sides, we get

$$b + c = 0,\ a - 3b - c = 0 \text{ and } -2c = -1$$

which give $a = -1$, $b = -\frac{1}{2}$ and $c = \frac{1}{2}$. Thus

$$n \propto l^{-1} \rho^{1/2} Y^{1/2}$$

Hence the correct choice is (a).

48. We know that

$$F = \frac{q_1 q_2}{(4\pi\varepsilon_0) r^2} \text{ and } E = \frac{F}{q}$$

$$\therefore \qquad \varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

$$\text{Hence } \frac{1}{2}\varepsilon_0 E^2 = \frac{q_1 q_2}{8\pi F r^2} \times \frac{F^2}{q^2} = \frac{q_1 q_2}{8\pi q^2} \times \frac{F}{r^2}$$

$$\therefore \text{ Dimensions of } \frac{1}{2}\varepsilon_0 E^2 = \text{dimensions of } \frac{F}{r^2}$$

$$= \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Hence the correct choice is (c).

49. Energy per unit volume, force per unit area and product of voltage and charge density all have dimensions of ML^2T^{-2}, but the dimensions of angular momentum are ML^2T^{-1}. Hence the correct choice is (d).

50. Given $t = d^{a/2} r^{b/2} s^{c/2}$. Substituting dimensions, we have

$$(T) = (ML^{-3})^{a/2} (L)^{b/2} (MT^{-2})^{c/2}$$

$$= M^{(a+c)/2} \cdot L^{(-3a/2 + b/2)} T^{-c}$$

Equating powers of L, we have, $-\frac{3}{2}a + \frac{b}{2} = 0$. Given $a = 1$.

$$\therefore \qquad -\frac{3}{2} + \frac{b}{2} = 0 \quad \text{or} \quad b = 3,$$

which is choice (c).

51. Both torque and energy have the dimensions of force × distance. Hence the correct choice is (b).

52. Given $X = \frac{A^2 B}{C^{1/3} D^3}$

Taking logarithm of both sides, we have

$$\log X = 2 \log A + \log B - \frac{1}{3} \log C - 3 \log D$$

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Partially differentiating, we have

$$\frac{\Delta x}{x} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{1}{3}\frac{\Delta C}{C} - 3\frac{\Delta D}{D}$$

$$\text{Percentage error in } A = 2\,\frac{\Delta A}{A} = 2 \times 2\% = 4\%$$

$$\text{Percentage error in } B = \frac{\Delta B}{B} = 2\%$$

$$\text{Percentage error in } C = \frac{1}{3}\,\frac{\Delta C}{C} = \frac{1}{3} \times 4\% = \frac{4}{3}\%$$

$$\text{Percentage error in } D = 3\,\frac{\Delta D}{D} = 3 \times 5\% = 15\%$$

We find that the minimum percentage error is contributed by C. Hence the correct choice is (c).

53. The correct choice is (b)

54. The correct choice is (b)

55. Dimensions of J and G are ML^2T^{-1} and $M^{-1}L^3T^{-2}$ respectively. The correct choice is (b).

56. Dimensions of ε_0 and h are $M^{-1}L^{-3}T^4A^2$ and ML^2T^{-1} respectively. The correct choice is (d).

57. RC has the dimensions of time (T). V has the dimensions of emf which has the dimensions of $L\,\frac{dI}{dt}$. The correct choice is (b).

58. The force **F** on a particle of charge q moving with a velocity **v** in **E** and **B** fields is given by

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Hence the dimensions of E are the same as those of vB. The correct choice is (b).

59. Energy stored in a capacitor of capacitance C having a potential difference V between its plates is given by

$$U = \frac{1}{2}CV^2$$

Hence, the dimensions of CV^2 = dimensions of energy. Hence the correct choice is (a).

60. Dimensions of $\left(\frac{h}{e}\right) = \frac{(ML^2T^{-1})}{AT} = ML^2T^{-2}A^{-1}$

Dimensions of $B = MT^{-2}A^{-1}$

Magnetic flux = $B \times$ area. The correct choice is (c).

61. Let surface tension $\sigma = E^a V^b T^c$. Using the dimensions of σ, E, V and T and equating powers of M, L and T, find the values of a, b and c. The correct choice is (c).

62. Dimensions of n (number of particles per unit area per unit time) = $L^{-2}T^{-1}$. Dimensions of n_1 or n_2 (number of particles per unit volume) = L^{-3}. Dimensions of x_1 or x_2 = L. The correct choise is (d).

63. The correct choice is (a).

64. Let n_1 be the magnitude (i.e. numerical value) of a physical quantity when the fundamental units are (M_1, L_1, T_1) and n_2 the magnitude of the same physical quantity when the fundamental units are (M_2, L_2, T_2), then, it is obvious that

$$n_1\,(M_1^x\,L_1^y\,T_1^z) = n_2\,(M_2^x\,L_2^y\,T_2^z) \qquad \text{(i)}$$

where x, y and z are the dimensions of the physical quantity in mass, length and time respectively. Now, we know that 1 calorie = 4.2 joule = 4.2 kg m^2s^{-2}. Therefore, in the first system of units $n_1 = 4.2$, $x = 1$, $y = 2$ and $z = -2$. Hence, in the second system of units in which $M_2 = a$ kg, $L_2 = b$ m and $T_2 = c$ s, we have from (i)

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^x\left(\frac{L_1}{L_2}\right)^y\left(\frac{T_1}{T_2}\right)^z$$

$$= n_1\left(\frac{1\,\text{kg}}{a\,\text{kg}}\right)^x\left(\frac{1\,\text{m}}{b\,\text{m}}\right)^y\left(\frac{1\,\text{s}}{c\,\text{s}}\right)^z$$

$$= 4.2\left(\frac{1}{a}\right)^1\left(\frac{1}{b}\right)^2\left(\frac{1}{c}\right)^{-2}$$

$$= \frac{4.2c^2}{ab^2}, \text{ which is choice (b).}$$

65. $V = \frac{4}{3}\pi r^3$. Taking logarithm of both sides, we have

$$\log V = \log 4 + \log \pi + 3 \log r - \log 3$$

Differentiating, we get

$$\frac{\delta V}{V} = 3\,\frac{\delta r}{r} = 3 \times 1\% = 3\%$$

66. $\frac{\delta V}{V} = 3\,\frac{\delta r}{r}$ or $6\% = 3\,\frac{\delta r}{r}$ or $\frac{\delta r}{r} = 2\%$.

Now surface area $s = 4\pi r^2$ or $\log s = \log 4\pi + 2 \log r$

$$\therefore \quad \frac{\delta s}{s} = 2\,\frac{\delta r}{r} = 2 \times 2\% = 4\%.$$

67. $X = M^x L^y T^{-z}$ or $\log X = x \log M + y \log L - z \log T$. Differentiating, we have

$$\frac{\Delta X}{X} = x\,\frac{\Delta M}{M} + y\,\frac{\Delta L}{L} - z\,\frac{\Delta T}{T}$$

The maximum error in X is (since all errors add up)

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$$\frac{\Delta X}{X} = x\,\frac{\Delta M}{M} + y\,\frac{\Delta L}{L} + z\,\frac{\Delta T}{T}$$

$= (xa + yb + zc)\%$.

The correct choice is (b).

68. $T = 2\pi\sqrt{\dfrac{l}{g}}$ or $g = 4\pi^2\,\dfrac{l}{T^2}$

or $\log g = \log(4\pi^2) + \log l - 2\log T$. The maximum error in g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T} = 2\% + 2 \times 3\% = 8\%$$

69. The dimensions of moment of inertia are (ML^2). We have

$$n_1(u_1) = n_2(u_2)$$

or $$n_1(M_1L_1^2) = n_2(M_2L_2^2)$$

$$\therefore \qquad n_2 = \frac{n_1(M_1L_1^2)}{(M_2L_2^2)} = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2$$

Given $n_1 = 6.0$, $M_1 = 1$ kg, $L_1 = 1$ m, $M_2 = 10$ g and $L_2 = 5$ cm. Therefore,

$$n_2 = 6.0 \times \left(\frac{1\text{ kg}}{10\text{ g}}\right) \times \left(\frac{1\text{ m}}{5\text{ cm}}\right)^2$$

$$= 6.0 \times \left(\frac{1000\text{ g}}{10\text{ g}}\right) \times \left(\frac{100\text{ cm}}{5\text{ cm}}\right)^2$$

$$= 6.0 \times 100 \times (20)^2 = 2.4 \times 10^5$$

70. The capacitance of a parallel plate capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$\therefore$ Dimension of $\varepsilon_0 L\,\dfrac{\Delta V}{\Delta t}$

$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because\ Q = CV)$$

Hence the correct choice is (d).

71. The correct choice is (a).

The maximum permissible error in η is given by the relation

$$\frac{\Delta\eta}{\eta} = 4\frac{\Delta R}{R} + \frac{\Delta l}{l} + \frac{\Delta P}{P} + \frac{\Delta Q}{Q}$$

It is clear that the error in the measurement of R is magnified four times on account of the occurrence of R^4 in the formula. Hence the radius (R) of the capillary tube must be measured most accurately. Thus the quantity which is raised to the highest power needs the most accurate measurement.

72. Take $m \propto v^a\, d^b\, g^c$ and show that $a = 6$.

73. Take $v \propto \lambda^a\, \rho^b\, \sigma^c$ and show that $a = -\dfrac{1}{2}$.

74. Take $T \propto r^a\ M^b\, G^c$ and show that $a = \dfrac{3}{2}$.

75. Take $M = E^a \rho^b F^c$ and show that $a = -1$, $b = 2$ and $c = 0$.

76. The correct choice is (d).

77. The exponent is a dimensionless number. Hence at/m is dimensionless. Therefore,

$$\text{Dimension of } a = \frac{\text{dimension of } m}{\text{dimension of } t} = \frac{\text{M}}{\text{T}} = \text{M L}^0 \text{T}^{-1}$$

78. The proportionate error in the measurement of g is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Hence Δg will be minimum if ΔL and ΔT are minimum. Thus the correct choice is (d).

79. $$Y = \frac{FL}{Al} = \frac{4MgL}{\pi d^2 l} \qquad (1)$$

where

$M = 1.0$ kg (exact), $g = 9.8$ ms^{-2} (exact)
$L = 2$ m (exact), $l = 0.8$ mm $= 0.8 \times 10^{-3}$ m
$\Delta l = \pm 0.05$ mm, $d = 0.4$ mm $= 0.4 \times 10^{-3}$ m
$\Delta d = \pm 0.01$ mm

Substituting the values of M, g, L, d and l in Eq. (1) we get

$$Y = 2.0 \times 10^{11}\ \text{Nm}^{-2}$$

From Eq. (1) the proportionate uncertainty in Y is given by

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

Since the values of M, g and L are exact, $\Delta M = 0$, $\Delta g = 0$ and $\Delta L = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l} = \frac{2 \times 0.01\text{ mm}}{0.4\text{ mm}} + \frac{0.05\text{ mm}}{0.8\text{ mm}}$$

$$= 0.05 + 0.0625 = 0.1125$$

$$\therefore \quad \Delta Y = 0.1125 \times Y = 0.1125 \times 2.0 \times 10^{11}$$

$$= 0.225 \times 10^{11}\ \text{Nm}^{-2}$$

Since the value of Y is correct only up to the first decimal place, the value of ΔY must be rounded off to the first decimal place. Thus $\Delta Y = 0.2 \times 10^{11}$ Nm^{-2}. Therefore, the result of the experiment is

$$Y + \Delta Y = (2.0 \pm 0.2) \times 10^{11}\ \text{Nm}^{-2}$$

Hence the correct choice is (b).

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80. Vernier constant = value of 1 M.S.D – value of 1 V.S.D.

Now n V.S.D = $(n-1)$ M.S.D = $(n-1)\,x$ cm

$\therefore \quad 1 \text{ V.S.D} = \left(\frac{n-1}{n}\right) x \text{ cm}$

$\therefore \quad \text{V.C.} = x \text{ cm} - \left(\frac{n-1}{n}\right) x \text{ cm} = \frac{x}{n} \text{ cm}$

Hence the correct choice is (c).

SECTION II

Multiple Choice Questions Based on Passage

Questions 1 and 2 are based on the following passage.

Passage-I

The dimensional method is a very convenient way of finding the dependence of a physical quantity on other physical quantities of a given system. This method has its own limitations. In a complicated situation, it is often not easy to guess the factors on which a physical quantity will depend. Secondly, this method gives no information about the dimensionless proportionality constant. Thirdly, this method is used only if a physical quantity depends on the product of other physical quantities. Fourthly, this method will not work if a physical quantity depends on another quantity as a trignometric or exponential function. Finally, this method does not give complete information in cases where a physical quantity depends on more than three quantities in problems in mechanics.

1. The dimensional method cannot be used to obtain denpendence of

(a) the height to which a liquid rises in a capillary tube on the angle of contact

(b) speed of sound in an elastic medium on the modulus of elactricity.

(c) height to which a body, projected upwards with a certain velocity, will rise on time t.

(d) the decrease in energy of a damped oscillator on time t.

2. In dimensional method, the dimensionless proportionality constant is to be determined

(a) experimentally

(b) by a detailed mathematical derivation

(c) by using the principle of dimensional homogeneity.

(d) by equating the powers of M, L and T.

Questions 3, 4 and 5 are based on the following passage.

Passage-II

In the study of physics, we often have to measure the physical quantities. The numerical value of a measured quantity can only be approximate as it depends upon the least count of the measuring instrument used. The number of significant figures in any measurement indicates the degree of precision of that measurement. The importance of significant figures lies in calculation. A mathematical calculation cannot increase the precision of a physical measurement. Therefore, the number of significant figures in the sum or product of a group of numbers cannot be greater than the number that has the least number of significant figures. A chain cannot be stronger than its weakest link.

3. A bee of mass 0.000087 kg sits on a flower of mass 0.0123 kg. What is the total mass supported by the stem of the flower upto appropriate significant figures?

(a) 0.012387 kg (b) 0.01239 kg

(c) 0.0124 kg (d) 0.012 kg

4. The radius of a uniform wire is $r = 0.021$ cm. The value π is given to be 3.142. What is the area of cross-section of the wire upto appropriate significant figures?

(a) 0.0014 cm^2 (b) 0.00139 cm^2

(c) 0.001386 cm^2 (d) 0.0013856 cm^2

5. A man runs 100.5 m in 10.3 s. His average speed upto appropriate significant figures is

(a) 9.76 ms^{-1} (b) 9.757 ms^{-1}

(c) 9.7573 ms^{-1} (d) 9.8 ms^{-1}

➔ ANSWERS

1. The correct choices are (a), (c) and (d). The height of a liquid in a capillary tube depends on cos θ, where θ is the angle of contact. The height S to which a body rises is given by $S = ut + \frac{1}{2} at^2$, which is a sum of two terms ut and $\frac{1}{2} at^2$. The energy of a damped oscillator decreases exponentially with time.

2. The correct choices are (a) and (b).

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3. Total mass = 0.000087 + 0.0123 = 0.012387 kg. The mass of the bee is accurate upto sixth decimal place in kg, whereas the mass of the flower is accurate only upto the fourth decimal place. Hence the sum must be rounded off to the fourth decimal place. Therefore the correct choice is (c).
4. $A = \pi r^2 = 3.142 \times (0.021)^2 = 0.00138562\ \text{cm}^2$. Now, there are only two significant figures in 0.021 cm. Hence the result must be rounded off to two significant figure as $A = 0.0014\ \text{cm}^2$, which is choice (a).
5. Average speed $= \dfrac{100.5\ \text{m}}{10.3\ \text{s}} = 9.7573\ \text{ms}^{-1}$

   The distance has four significant figures but the time has only three. Hence the result must be rounded off to three signigicant figure to $9.76\ \text{ms}^{-1}$. Thus the correct choice is (a).

SECTION III

Assertion-Reason Type Questions

In the following questions, **Statement-1(Assertion)** is followed by **Statement-2 (Reason)**. Each question has the following four options out of which only **one** choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

   The order of accuracy of measurement depends on the least count of the measuring instrument.

   Statement-2

   The smaller the least count the greater is the number of significant figures in the measured value.

2. **Statement-1**

   The dimensional method cannot be used to abtain the dependence of the work done by a force **F** on the angle θ between force **F** and displacement x.

   Statement-2

   All trignometric functions are dimensionless.

3. **Statement-1**

   The mass of an object is 13.2 kg. In this measurement there are 3 significant figures.

   Statement-2

   The same mass when expressed in grams as 13200 g has five significant figures.

➔ SOLUTIONS

1. The correct choice is (b).
2. Work done is $W = Fx \cos\theta$. Since $\cos\theta$ is dimensionless, the dependence of W on θ cannot be determined by the dimensional method. Hence the correct choice is (a)
3. The correct choice is (c). The degree of accuracy (and hence the number of significant figures) of a measurement cannot be increased by changing the unit.

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2

Kinematics

REVIEW OF BASIC CONCEPTS

1. Scalar and Vector Quantities

A scalar quantity has only magnitude but no direction, such as distance, speed, mass, area, volume, time, work, energy, power, temperature, specific heat, charge, potential, etc.

A vector quantity has both magnitude and direction, such as displacement, velocity, acceleration, force, momentum, torque, electric field, magnetic field, etc.

2. Parallelogram Law of Vector Addition

The procedure of finding the resultant of two vectors is known as the *parallelogram law of vector addition* and may be stated as follows.

If the two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

Figure 2.1 show two vectors **A** and **B** of magnitudes A and B inclined at an angle α. The magnitude R of the resultant vector **R** is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\alpha}$$

The angle β which the resultant vector **R** subtends with vector **A** is given by

$$\tan\beta = \frac{B\sin\alpha}{A + B\cos\alpha} \quad \text{or} \quad \sin\beta = \frac{B\sin\alpha}{R}$$

In vector notation the resultant vector is written as **R** = **A** + **B**.

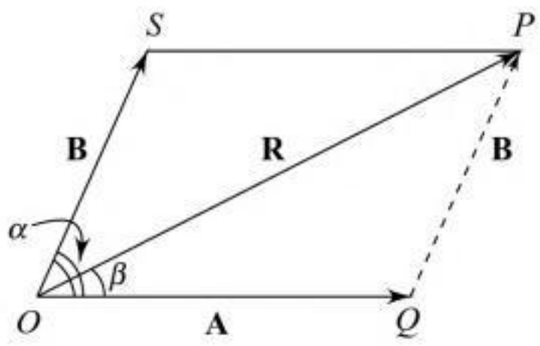

Fig. 2.1

Special Cases

(i) *When the two vectors are in the same direction*, i.e. $\alpha = 0°$, then $R = \sqrt{A^2 + B^2 + 2AB\cos 0°} = A + B$. Therefore, the magnitude of the resultant is equal to the sum of the magnitudes of the two vectors. Also $\tan\beta = 0$ or $\beta = 0$, i.e. the direction of the resultant is the same as that of either vector.

(ii) *When the two vectors are in opposite directions*, i.e., $\alpha = 180°$, then $R = \sqrt{A^2 + B^2 + 2AB\cos 180°} = A - B$. Also $\beta = 0$

(iii) When the two vectors are at right angles to each other, i.e. $\alpha = 90°$, then

$R = \sqrt{A^2 + B^2 + 2AB\cos 90°} = \sqrt{A^2 + B^2}$. Also $\tan\beta = \frac{B}{A}$.

Note: $R_{max} = A + B$ and $R_{min} = A - B$

Example 1 The following pairs of forces act on a particle at an angle θ which can have any value

(a) 2 N and 3 N (b) 3 N and 3 N
(c) 2 N and 6 N and (d) 3 N and 8 N

The resultant of which pair cannot have a magnitude of 4 N?

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Solution

For pair (a) : $R_{min} = A - B = 3 - 2 = 1$ N
and $R_{max} = A + B = 3 + 2 = 5$ N
For pair (b) : $R_{min} = 0$, $R_{max} = 6$ N
For pair (c) : $R_{min} = 4$ N, $R_{max} = 8$ N
For pair (d) : $R_{min} = 5$ N, $R_{max} = 11$ N

Hence the correct answer is (d).

Example 2 The magnitude of the resultant of two vectors of the same magnitude is equal to the magnitude of either vector. Find the angle between the two vectors.

Solution Given $A = B$ and $R = A$ or B.

$$R = \sqrt{A^2 + B^2 + 2\,AB\cos\theta}$$

$$\Rightarrow \quad A = \sqrt{A^2 + A^2 + 2\,A^2\cos\theta} = \sqrt{2A^2(1+\cos\theta)}$$

$$\Rightarrow \quad A^2 = 2A^2(1 + \cos\theta) \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$$

3. Triangle Law of Vector Addition

The parallelogram law of vector addition yields the triangle law of vector addition. In Fig. 2.1, vector QP = vector OS = **B**. In triangle OQP, vector OP = **R**. Hence, the triangle law of vector addition may be stated as follows:

If the two vectors are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented in magnitude and direction by the third side of the triangle taken in the opposite order.

4. Subtraction of Vectors

Suppose we wish to subtract a vector **B** from a vector **A**. Since

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

the subtraction of vector **B** from vector **A** is equivalent to the addition of vector – **B** to vector **A**. Hence the procedure to find (**A** – **B**) is as follows:

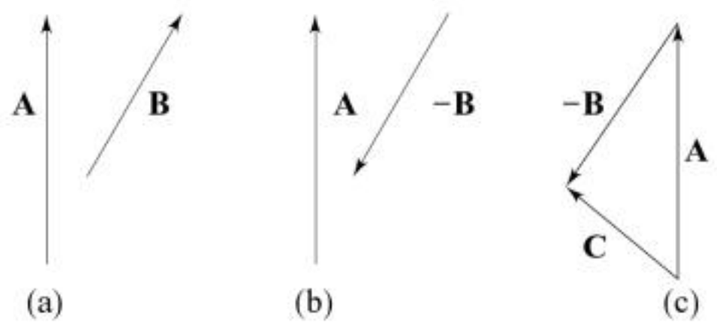

Fig. 2.2

Choose a convenient scale and draw the vectors **A** and **B** as shown in Fig. 2.2 (a). If **B** is to be subtracted from **A**, draw the vector negative of **B**, i.e. draw the vector – **B** [see Fig. 2.2 (b)]. Now shift the vector – **B** parallel to itself so that the tail of – **B** is at the head of **A**. Vector **C** is the sum of vectors **A** and – **B**, i.e. [see Fig. 2.2 (c)]

$$\mathbf{C} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$$

5. The Unit Vector

A vector whose magnitude is unity is called a unit vector. A unit vector is represented by $\hat{\mathbf{A}}$ where

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{A}$$

The unit vectors along x, y and z axes of a rectangular co-ordinate system are represented by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ respectively.

6. Resolution of a Vector into Rectangular Components

Consider a vector **A** in the x–y plane making an angle θ with the x-axis. The x and y components of **A** are $\mathbf{A}_x$ and $\mathbf{A}_y$. The magnitudes of $\mathbf{A}_x$ and $\mathbf{A}_y$ are (Fig. 2.3)

$A_x = A\cos\theta$, along x-direction

and $A_y = A\sin\theta$, along y-direction

Thus $\mathbf{A}_x = A_x\,\hat{\mathbf{i}} = (A\cos\theta)\,\hat{\mathbf{i}}$

and $\mathbf{A}_y = A_y\,\hat{\mathbf{j}} = (A\sin\theta)\,\hat{\mathbf{j}}$

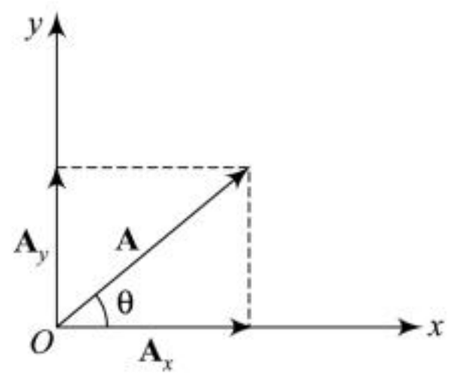

Fig. 2.3

From parallelogram law

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y = (\mathbf{A}\cos\theta)\,\hat{\mathbf{i}} + (\mathbf{A}\sin\theta)\,\hat{\mathbf{j}}$$

The magnitude of **A** in terms of the magnitudes of its rectangular components is

$$A = \sqrt{A_x^2 + A_y^2}$$

Also $$\tan\theta = \frac{A_y}{A_x}$$

Example 3 Resolve **A** into x and y components. The magnitude of **A** is 4 units.

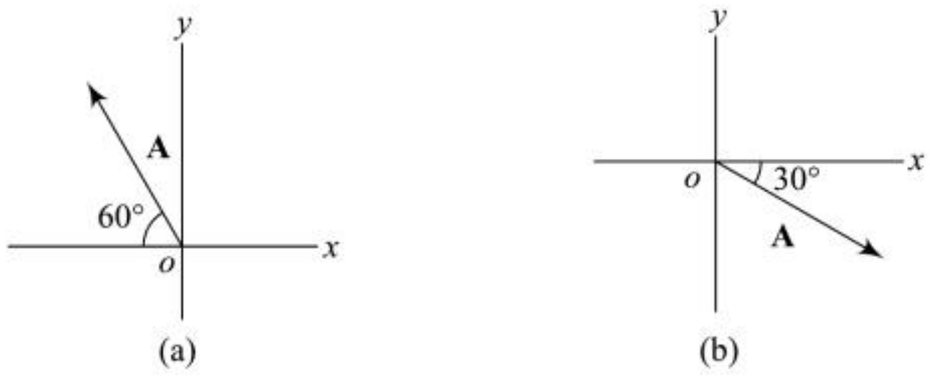

Fig. 2.4

Solution

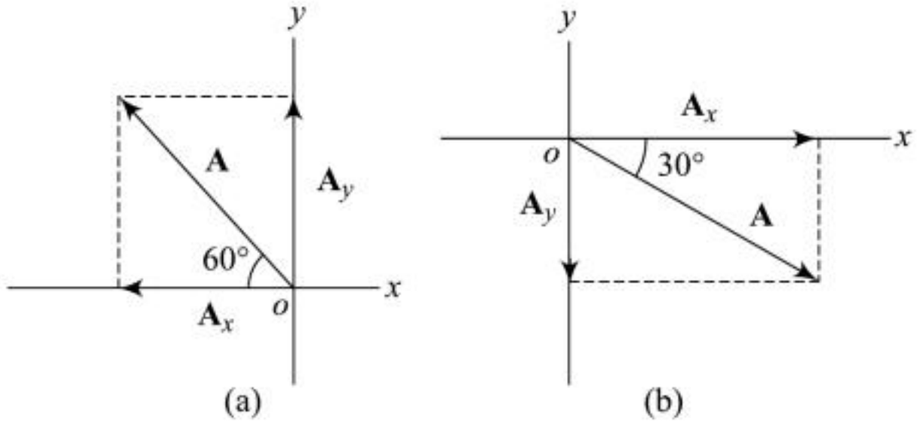

Fig. 2.5

In Fig. 2.5(a): $\mathbf{A}_x = -\,\mathrm{A}\cos 60° = -\,4 \times \frac{1}{2} = -2$ units

$$\mathbf{A}_y = +\,\mathbf{A}\sin 60° = +\,4 \times \frac{\sqrt{3}}{2}$$

$$= +\,2\sqrt{3} \text{ units}$$

In Fig. 2.5(b): $\mathbf{A}_x = +\,\mathbf{A}\cos 30° = +\,4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ units

$$\mathbf{A}_y = -\,\mathbf{A}\sin 30° = -\,4 \times \frac{1}{2} = -2 \text{ units}$$

NOTE

The magnitude of a component along $+x$ direction or $+y$ direction is taken to be positive while the magnitude of a component along $-x$ direction or $-y$ direction is taken to be negative.

7. Addition of Vectors Using Components

Two or more vectors are added by using components as follows:

(a) Resolve each vector into its rectangular components.

(b) Add the magnitudes of all the x-components taking into account their signs.

R_x = sum of x-components of all the vectors

(c) Similarly R_y = sum of y-components of all the vectors

(d) Magnitude of resultant is

$$R = \sqrt{R_x^2 + R_y^2}$$

(e) The angle θ which the resultant subtends with the x-axis is given by

$$\tan\theta = \frac{R_y}{R_x}$$

Example 4 Find the resultant of two vectors $\mathbf{A} = 4$ units and $\mathbf{B} = 3$ units shown in Fig. 2.6 by using

(a) parallelogram law and

(b) components method.

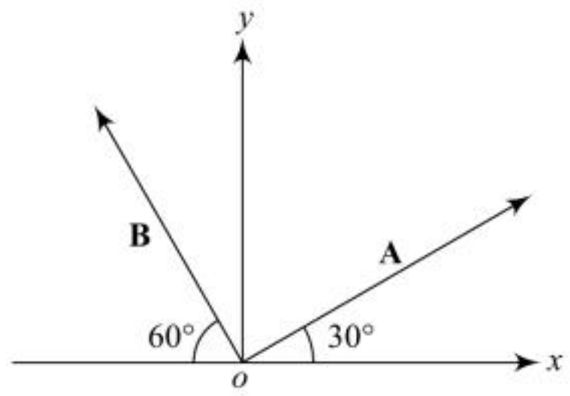

Fig. 2.6

Solution

(a) Angle between the two vectors is (Fig. 2.7)

$$\theta = 90°$$

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3\cos 90°}$$

$$= 5 \text{ units}$$

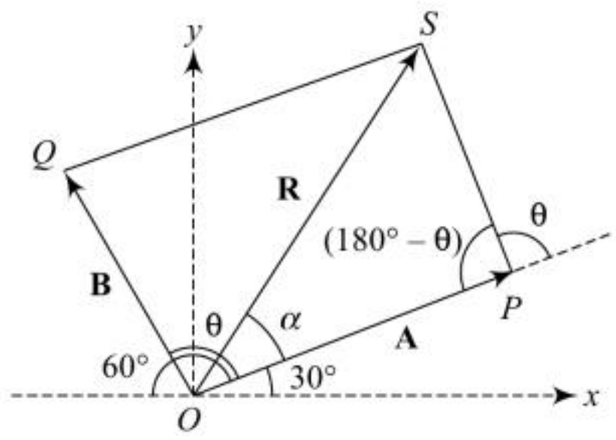

Fig. 2.7

$$\frac{R}{\sin(180° - \theta)} = \frac{B}{\sin\alpha}$$

$$\Rightarrow \quad \sin\alpha = \frac{B\sin\theta}{R} = \frac{3 \times \sin 90°}{5} = \frac{3}{5}$$

$$\therefore \quad \alpha = \sin^{-1}\left(\frac{3}{5}\right) \quad \Rightarrow \quad \alpha \simeq 37°$$

(b) The x and y components of **A** and **B** are [see Fig. 2.8]

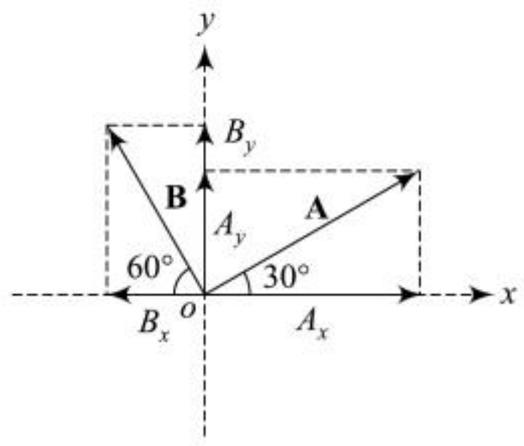

Fig. 2.8

$$A_x = A\cos 30° = 4\times\frac{\sqrt{3}}{2}$$
$$= 2\sqrt{3} \text{ units along } +x \text{ direction}$$
$$A_y = A\sin 30° = 4\times\frac{1}{2}$$
$$= 2 \text{ units along } +y \text{ direction}$$

and $$B_x = B\cos 60° = -\,3\times\frac{1}{2}$$
$$= -\,3/2 \text{ units along } -x \text{ direction}$$

and $$B_y = B\sin 60° = 3\times\frac{\sqrt{3}}{2}$$ units along $+y$ direction

$\therefore$ $R_x = A_x + B_x = \left(2\sqrt{3} - 3/2\right)$ along $+x$ direction

$$R_y = A_y + B_y = \left(2+\frac{3}{2}\sqrt{3}\right) \text{along } +y \text{ direction}$$

$$R_2 = R_x^2 + R_y^2 = \left(2\sqrt{3}-\frac{3}{2}\right)^2 + \left(2+\frac{3\sqrt{3}}{2}\right)^2$$
$$= 25$$

$\Rightarrow$ $R = 5$ units

The angle subtended by the resultant with the x-axis is given by

$$\tan\alpha = \frac{R_y}{R_x} = \frac{\left(2+\frac{3\sqrt{3}}{2}\right)}{\left(2\sqrt{3}-\frac{3}{2}\right)} = \frac{4.6}{1.96} = 2.347$$

giving $\alpha = 67°$ above the x-axis. Therefore, the angle which the resultant subtends with **A** is $67° - 30° = 37°$.

NOTE

The components method is more useful if more than two vectors have to be added.

8. Scalar or Dot Product

The scalar (or dot) product of two vectors **A** and **B** is defined as the product of the magnitudes of **A** and **B** and the cosine of the angle between them, i.e.

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

The scalar product is represented by putting a dot between the two vectors. The scalar product of two vectors is a scalar quantity.

Properties of the Scalar Product

(i) The scalar product is commutative, i.e. $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$

(ii) $\mathbf{A}\cdot\mathbf{A} = A^2$ or $A = \sqrt{\mathbf{A}\cdot\mathbf{A}}$. This is true because in this case $\theta = 0°$.

(iii) If two vectors **A** and **B** are perpendicular to each other, then $\theta = 90°$ and $\mathbf{A}\cdot\mathbf{B} = AB\cos 90° = 0$. Note that $\mathbf{A}\cdot\mathbf{B}$ can be zero when neither **A** nor **B** is zero.

(iv) For unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ along the three rectangular axes x, y and z, we have

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

and $$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$

(v) $\mathbf{A}\cdot\mathbf{B} = (A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} + A_z\,\hat{\mathbf{k}})\cdot(B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}} + B_z\,\hat{\mathbf{k}})$
$= A_xB_x + A_yB_y + A_zB_z$

Some Examples of Scalar Product

(i) Work done is $W = \mathbf{F}\cdot\mathbf{s}$ where **F** is the force vector and **s** is the displacement vector.

(ii) Power consumed is $P = \mathbf{F}\cdot\mathbf{v}$ where **F** is the force vector and **v** is the velocity vector.

(iii) Electric current is $I = \mathbf{J}\cdot\mathbf{A}$ where **J** is the current density vector and **A** is the area vector.

(iv) Magnetic flux is $\Phi = \mathbf{B}\cdot\mathbf{A}$ where **B** is the magnetic induction vector and **A** is the area vector.

9. Vector or Cross Product

If the smaller angle between two vectors **A** and **B** is θ, then the vector or cross product of vectors **A** and **B** is defined as

$$\mathbf{A}\times\mathbf{B} = (AB\sin\theta)\,\hat{\mathbf{n}}$$

where A and B are the magnitudes of vectors **A** and **B** and $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane containing **A** and **B**. The vector product of two vectors **A** and **B** is equal to a vector **C**, i.e.

$$\mathbf{A}\times\mathbf{B} = \mathbf{C}$$

The magnitude of vector **C** is given by

$$C = AB\sin\theta$$

The direction of **C** is perpendicular to the plane formed by **A** and **B** and is given by the right hand screw rule.

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Properties of a Vector Product

(i) Vector product is anticommutative, i.e. $(\mathbf{A} \times \mathbf{B}) = -(\mathbf{B} \times \mathbf{A})$

(ii) $\mathbf{A} \times \mathbf{A} = 0$, i.e. the vector product of a vector by itself is zero. This is because, in this case, $\theta = 0$, and hence $\sin\theta = 0$.

Therefore $\mathbf{A} \times \mathbf{A} = AA \sin\theta = 0$

Hence, the condition for two vectors to be parallel ($\theta = 0°$) or antiparallel ($\theta = 180°$) is that their vector product should be zero.

If $\mathbf{A} \times \mathbf{B} = 0$, it means either (i) $\mathbf{A}$ is zero or, (ii) $\mathbf{B} = 0$ or (iii) the angle θ between them is 0° or 180°.

(iii) The distributive law holds for both scalar and vector products, i.e.

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

(iv) $(\lambda\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (\lambda\mathbf{B}) = \lambda(\mathbf{A} \times \mathbf{B})$; λ a real number.

(v) $|\mathbf{A} \times \mathbf{B}|^2 = |\mathbf{A}|^2 |\mathbf{B}|^2 - (\mathbf{A} \cdot \mathbf{B})^2$.

(vi) $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the three mutually perpendicular unit vectors at the origin O and along OX, OY and OZ respectively; the right-hand rule gives:

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}, \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}, \quad \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

(vii) $$(\mathbf{A} \times \mathbf{B}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{\mathbf{i}}\,(A_yB_z - A_zB_y) + \hat{\mathbf{j}}\,(A_zB_x - A_xB_z) + \hat{\mathbf{k}}\,(A_xB_y - A_yB_x)$$

Some Examples of Vector Product

(i) Torque = $\mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is the position vector and $\mathbf{F}$ is the force vector.

(ii) Linear velocity = $\omega \times \mathbf{r}$ where ω is the angular frequency vector and $\mathbf{r}$ is the position vector.

(iii) Angular momentum = $\mathbf{r} \times \mathbf{p}$ where $\mathbf{r}$ is the position vector and $\mathbf{p}$ is the linear momentum vector.

Example 5 Find the angle between vectors $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$.

Solution $\mathbf{A} \cdot \mathbf{B} = AB \cos\theta$

Magnitude of $\mathbf{A}$ is $A = \sqrt{1^2 + 1^2} = \sqrt{2}$

Magnitude of $\mathbf{B}$ is $B = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$$\therefore \quad (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = \sqrt{2} \times \sqrt{2} \cos\theta$$

$$\Rightarrow \quad \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} - \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 2\cos\theta \qquad (\because \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0)$$

$$\Rightarrow \quad 1 - 1 = 2\cos\theta \text{ gives } \cos\theta = 0$$

or $\theta = 90°$

Example 6 Two vectors $\mathbf{A}$ and $\mathbf{B}$ are inclined at an angle θ. Using definition of scalar product, show that the magnitude of the resultant of vectors $\mathbf{A}$ and $\mathbf{B}$ is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Solution $\mathbf{R} = \mathbf{A} + \mathbf{B}$. Therefore,

$$\mathbf{R} \cdot \mathbf{R} = (\mathbf{A} + \mathbf{B}) \cdot (\mathbf{A} + \mathbf{B}) = \mathbf{A} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{A} + \mathbf{B} \cdot \mathbf{B}$$

$$\Rightarrow \quad R^2 = A^2 + 2\,\mathbf{A}\,\mathbf{B} + B^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Example 7 Given $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ and $\mathbf{B} = 5\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. Find the magnitude and direction of $(\mathbf{A} \times \mathbf{B})$.

Solution $(\mathbf{A} \times \mathbf{B}) = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}) \times (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$$= 10\,\hat{\mathbf{i}} \times \hat{\mathbf{i}} + 8\,\hat{\mathbf{i}} \times \hat{\mathbf{j}} - 15\,\hat{\mathbf{j}} \times \hat{\mathbf{i}} - 12\,\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$(\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0 \text{ and } \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}})$$

$$= 0 + 8\hat{\mathbf{k}} + 15\hat{\mathbf{k}} - 0$$

$$= 23\,\hat{\mathbf{k}}$$

Thus the magnitude of $(\mathbf{A} \times \mathbf{B})$ is 23 units and its direction is along + z axis.

10. Position Vector and Displacement Vector

The position vector of a particle describes its instantaneous position with respect to the origin of the chosen frame of reference. It is a vector joining the origin to the particle and is denoted by vector $\mathbf{r}$.

For one-dimensional motion (say along x-axis), $\mathbf{r} = x\hat{\mathbf{i}}$, where x is the distance of the particle from origin O.

For two-dimension motion (say in the x–y plane), $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$, where (x, y) are the x and y coordinates of the particle.

For three-dimensional motion, $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

Displacement vector

If $\mathbf{r}_1$ is the position vector of a particle at time t_1, and $\mathbf{r}_2$ at time t_2, then the displacement vector is given by

$$\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1$$

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Vector **s** is the resultant of vectors $\mathbf{r}_2$ and $-\mathbf{r}_1$ (Fig. 2.9).

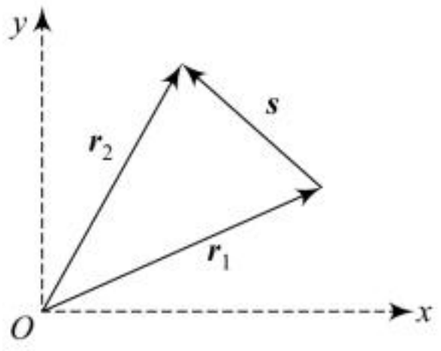

Fig. 2.9

The displacement vector is a vector joining the initial and the final positions of the particle after a given interval of time and its direction is from the initial to the final position.

Example 8 A particle moves from A to B along a circle of radius R. Find the path length and the magnitude of the displacement in terms of R and θ. [see Fig. 2.10]

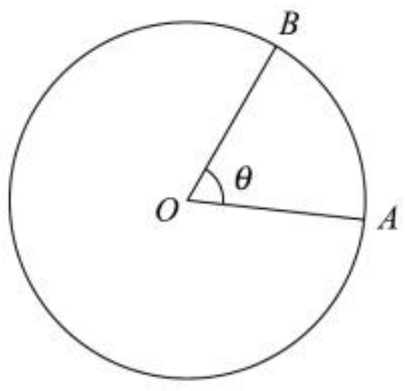

Fig. 2.10

Solution Path length = arc $AB = R\theta$

As shown in Fig. 2.11, the magnitude of displacement is

$$s = AB = AC + CB$$

$$= 2\,AC$$

$$\Rightarrow \quad s = 2R \sin(\theta/2)$$

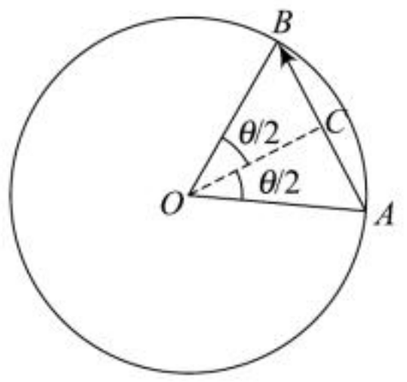

Fig. 2.11

11. Instantaneous Velocity and Average Velocity

The rate of change of displacement with time at a given instant is called instantaneous velocity and is given by

$$v = \frac{dx}{dt}$$

The average velocity in a given time interval is defined as

$$v_{av} = \frac{\text{total displacement}}{\text{time interval}}$$

Example 9 The position of a particle moving along x-axis is given by $x = 2t - 3t^2 + t^3$, where x is in metre and t in second.

(a) Find the velocity of the particle at $t = 2$ s.
(b) Find the average speed of the particle in the time interval from $t = 2$ s to $t = 4$ s.

Solution

(a) $x = 2t - 3t^2 + t^3$

$$v = \frac{dx}{dt} = 2 - 6t + 3t^2$$

$$\therefore \quad v \text{ at } (t = 2 \text{ s}) = 2 - 6 \times 2 + 3 \times (2)^2 = 2 \text{ ms}^{-1}$$

(b) Position at $t = 2$ s is

$$x_1 = 2 \times 2 - 3 \times (2)^2 + (2)^3 = 0$$

Position at $t = 4$ s is

$$x_2 = 2 \times 4 - 3 \times (4)^2 + (4)^3 = 24 \text{ m}$$

$\therefore$ Displacement is $x_2 - x_1 = 24 - 0 = 24$ m. Time interval $= 4 - 2 = 2$ s. Therefore,

$$\text{Average velocity} = \frac{24}{2} = 12 \text{ ms}^{-1}$$

12. Instantaneous Acceleration

The rate of change of velocity with time at a given instant is called instantaneous acceleration and is given by

$$a = \frac{dv}{dt}$$

13. Equations of One Dimensional Motion with Constant Acceleration

Let x_0 be the position of a particle at time $t = 0$ and let u be its velocity at $t = 0$. It is given a constant acceleration a for time t. As a result it moves in a straight line to a position x and acquires a velocity v.

The particle suffers a displacement $s = x - x_0$ in time t. The equations of motion of the particle are

$$v = u + at \tag{2.1}$$

$$x = x_0 + ut + \frac{1}{2}at^2$$

or

$$s = ut + \frac{1}{2}at^2 \tag{2.2}$$

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and $$v^2 - u^2 = 2\,a(x - x_0)$$

or $$v^2 - u^2 = 2a\,s \qquad (2.3)$$

1. While solving numerical problems of bodies moving in a straight horizontal direction, we will consider only the magnitudes of u, v, a and s and take care of their direction by assigning positive or negative sign to the quantity. For example, $+a$ will mean acceleration $-a$ will mean retardation (or deceleration).
2. In the case of a body falling vertically under gravity or projected vertically upwards, we use the following sign conventions.
   Quantities directed vertically upwards are taken to be positive and those directed vertically downwards are taken to be negative. Since the acceleration due to gravity is directed downward for a body moving vertically up or falling vertically down, we take $a = -g = -9.8\ \text{ms}^{-2}$ in Eqs. (2.1), (2.2) and (2.3).
3. Displacement in the nth second is given by
   s_n = displacement in n seconds
   − displacement in $(n-1)$ seconds

$$= u_n + \frac{1}{2}a(n)^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$\Rightarrow \qquad s_n = u + \frac{a}{2}(2n-1) \qquad (2.4)$$

Applications

(i) If a body moving with constant acceleration, starts from A with initial velocity u and reaches B with a velocity v, then the velocity midway between A and B is

$$v' = \sqrt{\frac{u^2 + v^2}{2}}$$

(ii) A body starting from rest has an acceleration a for a time t_1 and comes to rest under a retardation b for a time t_2. If s_1 and s_2 are the distances travelled in t_1 and t_2,

(a) $\dfrac{s_1}{s_2} = \dfrac{b}{a} = \dfrac{t_1}{t_2}$

(b) Total distance travelled $(s_1 + s_2)$

$$= \frac{1}{2}\left(\frac{ab}{a+b}\right)T^2, \text{ where } T = t_1 + t_2.$$

(c) Maximum velocity attained is

$$v_{\max} = \left(\frac{ab}{a+b}\right)T$$

(d) Average velocity over the whole trip is

$$v_{\text{av}} = \frac{v_{\max}}{2}.$$

(iii) At time $t = 0$ a body is thrown vertically upwards with a velocity u. at time $t = T$, another body is thrown vertically upwards with the same velocity u. The two bodies will meet at time

$$t = \frac{T}{2} + \frac{u}{g}$$

(iv) A body is dropped from rest and at the same time another body is thrown downward with a velocity u from the same point, then
   (a) the acceleration of each body is g,
   (b) their relative velocity is always u,
   (c) their separation will be x after a time $t = \dfrac{x}{u}$.

(v) From the top of a building, body A is thrown upwards with a certain speed, body B is thrown downwards with the same speed and body C is dropped from rest from the same point. If t_1, t_2 and t_3 are their respective times of reach the ground, then

$$t_3 = \sqrt{t_1\,t_2}$$

(vi) A body of mass m is dropped from a height h on a heap of sand. If it penetrates a depth x in the sand.
   (a) the average retardation in sand is given by

$$a = \frac{gh}{x}$$

   because loss in PE (mgh) = work done against the resistive force of sand (max).
   (b) total average force exerted by sand is

$$F = mg + ma = m(g + a)$$

(vii) A body is thrown vertically upward with a velocity u. If the resistive force due to air-friction produces a constant acceleration (or retardation) a $(< g)$
   (a) the net acceleration during upward motion $= g + a$,
   (b) the net acceleration during downward motion $= g - a$,
   (c) the maximum height attained is

$$h = \frac{u^2}{2(g+a)}$$

   (d) the time taken to reach the maximum height is

$$t_1 = \sqrt{\frac{2h}{(g+a)}} = \frac{u}{(g+a)}$$

   (e) the time of descent is

$$t_2 = \sqrt{\frac{2h}{(g-a)}} = \frac{u}{\left(g^2 - a^2\right)^{1/2}}$$

   (f) the speed with which the body hits the ground is

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$$v = \sqrt{2(g-a)h} = u\sqrt{\frac{(g-a)}{(g+a)}}$$

NOTE

Note that t_1 is less than t_2.

Some useful tips

(i) If a body, starting from rest, moves with a constant acceleration, the distances covered by it in 1s, 2s, 3s …… are in the ratio $1^2 : 2^2 : 3^2 \ldots = 1 : 4 : 9$ ……

(ii) If a body starts from rest and moves with a contant acceleration, the distance covered by it in the lst, 2nd, 3rd….seconds are in the ratio 1 : 3 : 5…

(iii) A body is projected upward with a certain speed. If air resistance is nelected, the speed with which it hits the ground = the speed of projection.

(iv) If a body is projected upwards with a velocity u, the maximum height attained is proportional to u^2 and the time of ascent is proportional to u.

(v) For a freely falling body,
  (a) velocity ∝ time
  (b) distance fallen ∝ (time)^2
  (c) velocity ∝ $\sqrt{\text{distance fallen}}$

14. Graphical Representation

1. *Displacement – time (x – t) graphs (Fig. 2.12)*
   Fig. 2.16(*a*) : Body at rest
   Fig. 2.16(*b*) : Body in uniform motion
   Fig. 2.16(*c*) : Body subjected to acceleration ($a > 0$)
   Fig. 2.16(*d*) : Body subjected to retardation ($a < 0$)
   Fig. 2.16(*e*) : Body accelerating and then decelerating

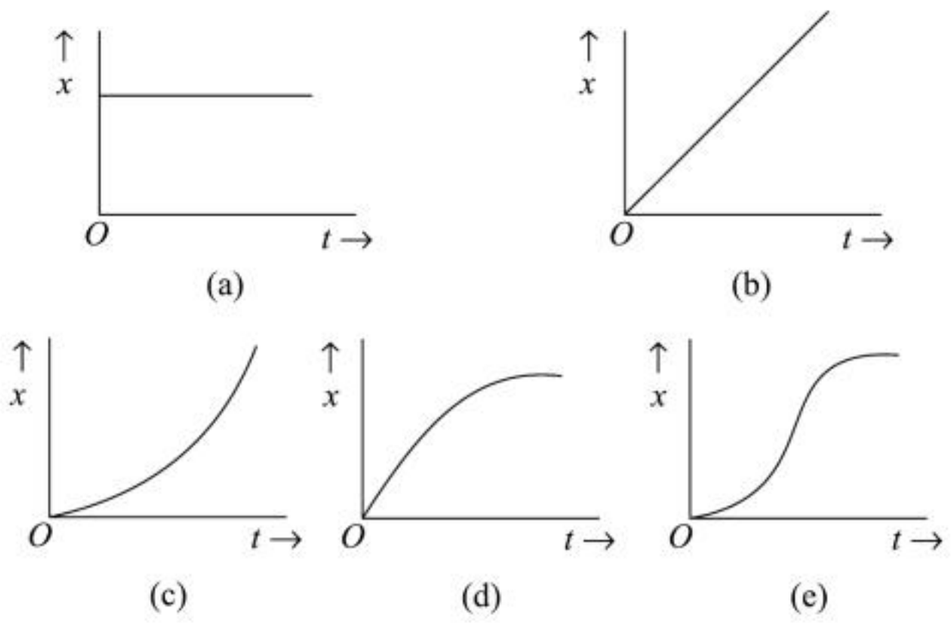

Fig. 2.12

NOTE

The slope of $x - t$ graph gives velocity for uniform motion [Fig. 2.16(*b*)]. For non-uniform motion [Fig. 2.16(*c*), (*d*) and (*e*)], the slope of the tangent to the curve at a point gives velocity at that instant.

2. Velocity-time ($v - t$) graphs for uniformly accelerated motion (Fig. 2.13)

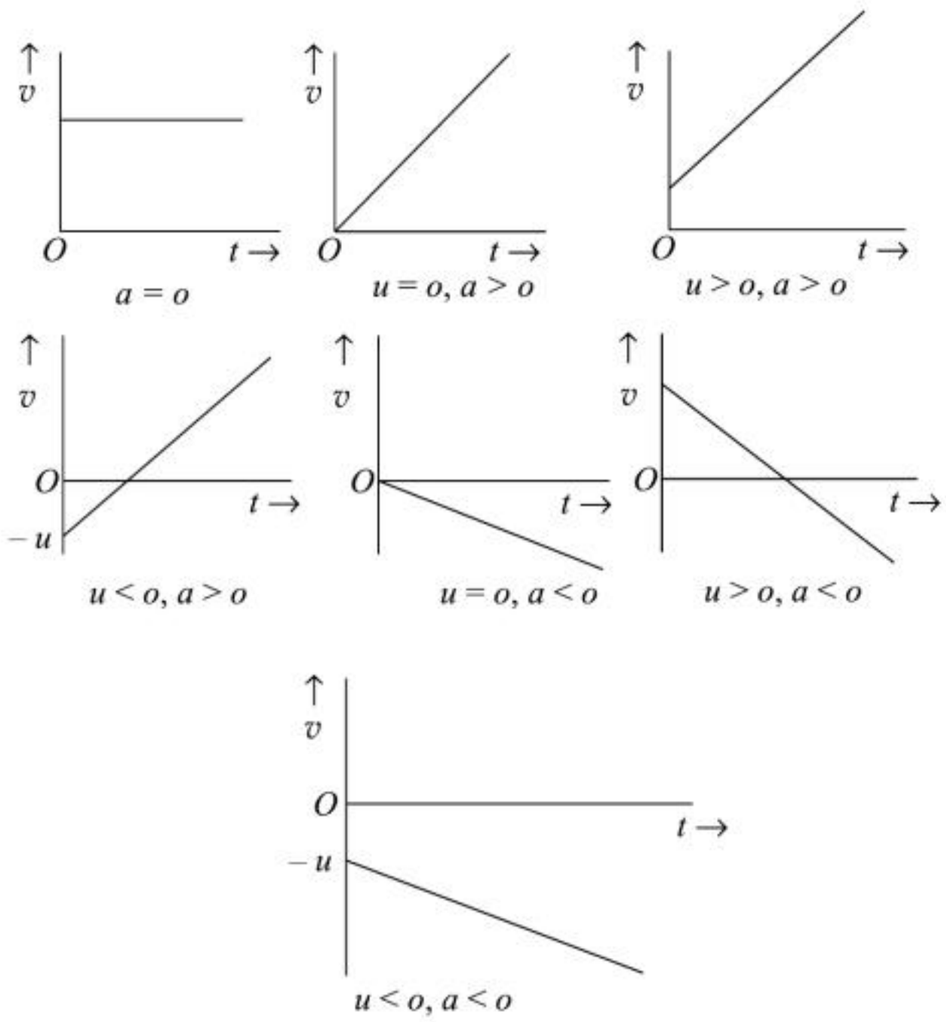

Fig. 2.13

NOTE

Acceleration = slope of ($v - t$) graph
Displacement = area under ($v - t$) graph

3. Acceleration – time ($a - t$) graph [Fig. 2.14]

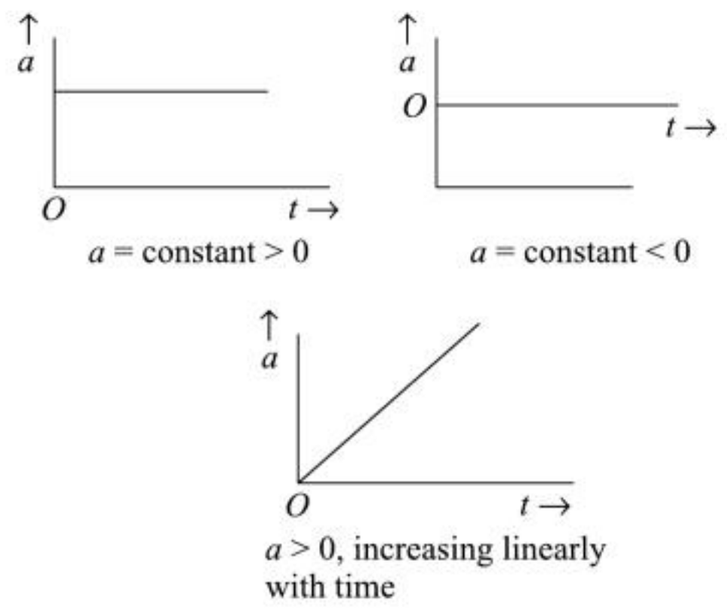

Fig. 2.14

4. $(a-t)$, $(v-t)$, $(x-t)$ graphs for free fall [Fig 2.15]

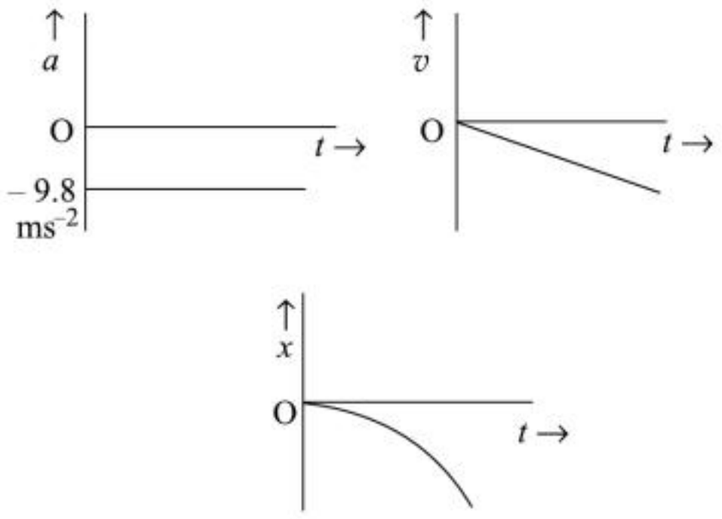

Fig. 2.15

Example 10 The displacement x (in metres) of a body varies with time t (in seconds) as

$$x = -\frac{2}{3}t^2 + 16t + 2$$

How long does the body take to come to rest?

Solution $v = \dfrac{dx}{dt} = -\dfrac{4}{3}t + 16$

$$0 = -\frac{4}{3}t + 16 \quad \Rightarrow \quad t = 12 \text{ s}$$

Example 11 From the top of a building 40 m tall, a ball a thrown vertically upwards with a velocity of 10 ms $^{-1}$. (a) After how long will the ball hit the ground? (b) After how long will the ball take to pass through the point from where it was projected? (c) With what velocity will it hit the ground? Take $g = 10$ ms^{-2}.

Solution

(a) $s = -40$ m, $u = +10$ ms $^{-1}$, $a = -10$ ms^{-2}

Now $\quad s = ut + \dfrac{1}{2}at^2$

$$\Rightarrow \quad -40 = 10t + \frac{1}{2} \times (-10)t^2$$

$$\Rightarrow \quad t^2 - 2t - 8 = 0$$

$$\Rightarrow \quad (t+2)(t-4) = 0$$

$\Rightarrow$ $t = -2$ s or 4 s. The negative value of t is not possible.

Hence the ball will hit the ground after 4 s.

(b) $s = 0$, $u = +10$ ms $^{-1}$ and $a = -10$ ms $^{-2}$.
Therefore $0 = 10t - 5t^2 \quad \Rightarrow \quad t = 2$ s

(c) $v = u + at = 10 - 10 \times 4 = 10 - 40 = -30$ ms $^{-1}$.
The negative sign indicates the velocity v is directed downwards.

Example 12 Figure 2.16 shows the velocity – time graph of a body moving in a straight line. Find (a) the distance travelled by the body in 20 s, (b) the displacement of the body in 20 s and (c) the average velocity in the time interval $t = 0$ to $t = 20$ s.

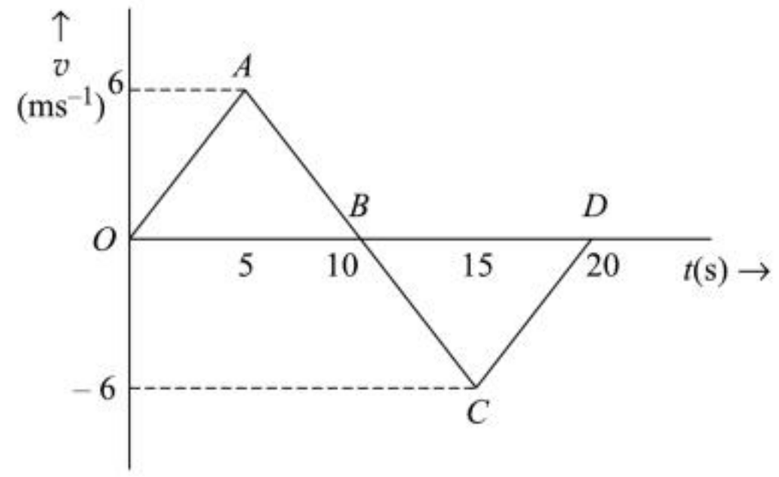

Fig. 2.16

Solution

(a) Distance = area under speed – time graph which is shown in Fig. 2.17.

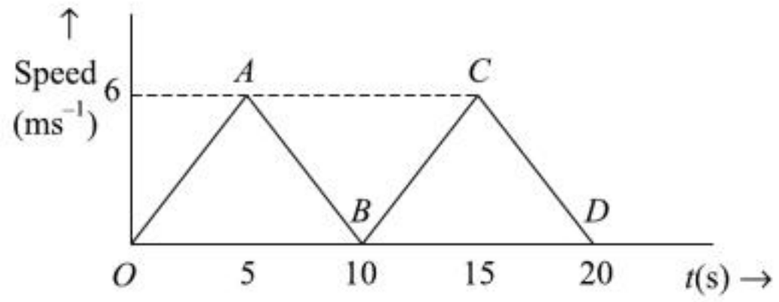

Fig. 2.17

$\therefore$ Distance travelled in 20 s = area of OAB + area of BCD in Fig 2.21

$$= \frac{1}{2} \times 6 \times 10 + \frac{1}{2} \times 6 \times 10 = 60 \text{ m}$$

(b) Displacement in 20 s = area of OAB – area of BCD in Fig. 2.20

$$= 30 - 30 = 0$$

(c) Average velocity $= \dfrac{\text{displacement}}{\text{time}} = 0$

Example 13 A body moving in a straight line covers a distance of 14 m in the 5th second and 20 m in the 8th second. How much distance will it cover in the 15th second?

Solution $S_n = u + \dfrac{a}{2}(2n-1)$

$$S_5 = u + \frac{9a}{2} \qquad (1)$$

$$S_8 = u + \frac{15a}{2} \qquad (2)$$

Solving (1) and (2), we get $a = 2$ ms $^{-2}$ and $u = 5$ ms $^{-1}$.

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$$\therefore \qquad S_{15} = u + \frac{29a}{2} = 5 + \frac{29 \times 2}{2} = 34 \text{ m}$$

15. Relative Velocity in One Dimension

If two bodies A and B are moving in a straight line with velocities v_A and v_B respectively, the relative velocity of A with respect to B is defined as

$$v_{AB} = v_A - v_B$$

The relative velocity of B with respect to A will be

$$v_{BA} = v_B - v_A$$

Example 14 A police van moving on a highway with a speed of 10 ms^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 30 ms^{-1}. If the muzzle speed of the bullet is 140 ms^{-1}, with what speed will the bullet hit the thief's car?

Solution Speed of police van is $v_V = 10 \text{ ms}^{-1}$, speed of thief's car is $v_C = 30 \text{ ms}^{-1}$. Relative velocity of bullet with respect to van is $v_{BV} = 140 \text{ ms}^{-1}$. Let v_B be the velocity of the bullet. $v_{BV} = v_B - v_V$. Hence $v_B = v_{BV} + v_V = 140 + 10 = 150 \text{ ms}^{-1}$. The bullet will hit the car with a speed

$$v_{BC} = v_B - v_C = 150 - 30 = 120 \text{ ms}^{-1}.$$

Example 15 From the top of a tower 60 m tall, a body is thrown vertically down with a velocity of 10 ms^{-1}. At the same time, another body is thrown vertically upward from the ground with a velocity of 20 ms^{-1}. (a) After how long will the two bodies meet? (b) At what height above the ground do they meet? Take $g = 10 \text{ ms}^{-2}$.

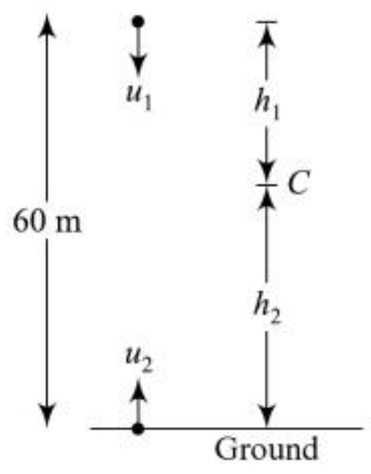

Fig. 2.18

Solution

(a) Suppose the bodies meet at C and let t be the time at which they meet.

For body 1: $s = -h_1, u_1 = -10 \text{ ms}^{-1}, a = -10 \text{ ms}^{-2}$

$$\therefore \qquad -h_1 = -10t + \frac{1}{2} \times (-10)t^2$$

Which gives $h_1 = 5t\,(t + 2)$ (1)

For body 2: $s = +h_2, u_2 = +20 \text{ ms}^{-1}$, $a = -10 \text{ ms}^{-2}$

$$\therefore \qquad h_2 = 20t - 5t^2 = 5t\,(-t + 4) \qquad (2)$$

Adding (1) and (2), $h_1 + h_2 = 30t$ or $60 = 30t$ $\Rightarrow$ $t = 2$ s.

(b) Using $t = 2$ s in Eq. (2), $h_2 = 5 \times 2\,(4 - 2) = 20$ m

Alternative method

Relative velocity of body 1 with respect to body 2 is

$$u_{12} = u_1 - u_2 = -10 - (20) = -30 \text{ ms}^{-1}$$

Relative displacement of body 1 with respect of body 2 is

$$s_{12} = -h_1 - (h_2) = -(h_1 + h_2) = -60 \text{ m}$$

Relative acceleration is $a_{12} = a_1 - a_2 = -g - (-g) = 0$. Using

$$s = ut + \frac{1}{2}at^2, \text{ we have } -60 = -30t \quad \Rightarrow \quad t = 2 \text{ s}.$$

$$h_2 = 20t - 5t^2 = 20 \times 2 - 5 \times (2)^2 = 20 \text{ m}$$

Example 16 The driver of train A moving at a speed of 30 ms^{-1} sights another train B moving on the same track at a speed of 10 ms^{-1} in the same direction. He immediately applies brakes and achieves a uniform retardation of 2 ms^{-2}. To avoid collision, what must be the minimum distance between trains A and B when the driver of A sights B?

Solution Relative initial velocity A w.r.t. B is,

$$u_{AB} = u_A - u_B = 30 - 10 = 20 \text{ ms}^{-1}$$

Relative retardation of A w.r.t B is

$$a_{AB} = a_A - a_B = -2 - 0 = -2 \text{ ms}^{-2}$$

To avoid collision, the relative final velocity of A w.r.t. B v_{AB} must be zero. Minimum relative displacement of A w.r.t. B is s_{AB} which is found by using the relation $v^2 - u^2 = 2as$ which gives

$$0 - (20)^2 = 2 \times (-2)\, s_{AB} \quad \Rightarrow \quad s_{AB} = 100 \text{ m}$$

16. Solving Problems Involving Non-uniform Acceleration

(a) *Finding velocity and displacement if the dependence of acceleration on time is given.*

Use $a(t) = \dfrac{dv}{dt}$ which gives $dv = a\,dt$. Substitute the given expression for a in terms of t and integrate both sides.

$$\int_u^v dv = \int_0^t a(t)\,dt$$

and obtain the expression for $v(t)$.

To find displacement x, use $v(t) = \dfrac{dx}{dt}$ which gives $dx = v(t)\,dt$.

Substitute the expression for v and integrate both sides

$$\int_{x_0}^{x} dx = \int_0^t v(t)\,dt$$

Example 17 A particle starts from rest at $x = 0$. Its acceleration at time $t = 0$ is 5 ms^{-2} which varies with time

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as shown in Fig. 2.19. Find (a) the maximum speed of the particle and (b) its displacement in time interval from $t = 0$ to $t = 2$ s.

Solution

(a) Slope of graph is $m = -\frac{5}{6}\,\text{m s}^{-2}$ per second. Intercept $c = 5\ \text{ms}^{-2}$. Using $y = mx + c$, the acceleration a varies with time as

$$a = -\frac{5}{6}t + 5$$

$$\Rightarrow \qquad \frac{dv}{dt} = -\frac{5}{6}t + 5$$

Integrating $$\int_0^v dv = \int_0^t \left(-\frac{5}{6}t + 5\right) dt$$

$$\Rightarrow \qquad v = -\frac{5}{12}t^2 + 5t \qquad (1)$$

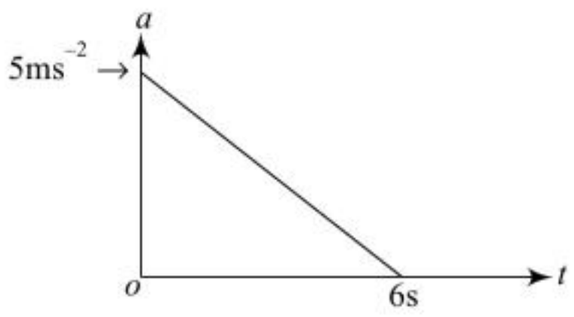

Fig. 2.19

It follows from graph that deceleration becomes zero at $t = 6$ s. Hence v will be maximum at $t = 6$ s. Using $t = 6$ s in Eq. (1) gives $v_{\max} = 15\ \text{ms}^{-1}$.

(b) From $v = \frac{dx}{dt}$, we have $dx = v\,dt$. Integrating

$$\int_0^x dx = \int_0^t v\,dt = \int_0^2 \left(-\frac{5}{12}t^2 + 5t\right) dt$$

$$\Rightarrow \qquad x = \left|-\frac{5}{12}\times\frac{t^3}{3} + 5\times\frac{t^2}{2}\right|_0^2$$

$$= -\frac{5}{12}\times\frac{2^3}{3} + 5\times\frac{2^2}{2} = \frac{80}{9}\,\text{m}$$

(c) *Finding velocity and displacement if the dependence of acceleration on displacement is given*

Use $$a(x) = \frac{dv}{dt} = \frac{dx}{dt}\times\frac{dv}{dx} = v\,\frac{dv}{dx}$$

$$\left(\because v = \frac{dx}{dt}\right)$$

or $$v\,dv = a(x)\,dx$$

Substitute the given expression of $a(x)$ in terms of x and integrate.

$$\int_v^v v\,dv = \int_{x_0}^x a(x)\,dx$$

Hence we get an expression for $v(x)$ in terms of x.

To find displacement, we use $v(x) = \frac{dx}{dt} \Rightarrow \frac{dx}{v(x)} = dt$ and integrate

$$\int_{x_0}^x \frac{dx}{v(x)} = \int_0^t dt$$

where v is the expression obtained above.

Example 18 A particle is moving along the x-axis with an acceleration $a = 2x$ where a is in ms^{-2} and x is in metre. If the particle starts from rest at $x = 1$ m, find its velocity when it reaches the position $x = 3$ m.

Solution $$a = v\,\frac{dv}{dx} \quad \Rightarrow \quad 2x = v\,\frac{dv}{dx}$$

$$\Rightarrow \quad v\,dv = 2x\,dx$$

$$\therefore \qquad \int_0^v v\,dv = 2\int_1^3 x\,dx$$

$$\Rightarrow \qquad \frac{v^2}{2} = 2\left|\frac{x^2}{2}\right|_1^3 = 3^2 - 1 = 8$$

which gives $v = 4\ \text{ms}^{-1}$.

(c) *Finding velocity and displacement if the dependence of acceleration on velocity is given*

$$a(v) = \frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{a(v)} = dt$$

Integrating

$$\int_u^v \frac{dv}{a(v)} = \int_0^t dt$$

Hence we obtain the expression for $v(t)$.

$$v(t) = \frac{dx}{dt} \quad \Rightarrow \quad dx = v(t)\,dt$$

Integrating

$$\int_{x_0}^x dx = \int_0^t v(t)\,dt$$

Example 19 A particle initially (i.e. at $t = 0$) moving with a velocity u is subjected to a retarding force which decelerates it at a rate $a = -k\sqrt{v}$ where v is the instantaneous velocity and k is a positive constant. (a) Find the time taken by the particle to come to rest. (b) Find the distance the particle travels during this time.

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Solution

(a) $a = \dfrac{dv}{dt} \Rightarrow -k\sqrt{v} = \dfrac{dv}{dt} \Rightarrow \dfrac{dv}{\sqrt{v}} = -k\,dt$

Integrating

$$\int_u^v \frac{dv}{\sqrt{v}} = -k\int_0^t dt$$

$$\Rightarrow 2\left(v^{1/2} - u^{1/2}\right) = -kt$$

Putting $v = 0$, we get $t = \dfrac{2\sqrt{u}}{k}$

(b) To find the stopping distance, we use

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow -k\sqrt{v} = v\frac{dv}{dx} \Rightarrow dx = -\frac{\sqrt{v}\,dv}{k}$$

Integrating

$$\int_0^x dx = -\frac{1}{k}\int_u^0 \sqrt{v}\,dv$$

$$\Rightarrow x = \frac{2u^{3/2}}{3k}$$

17. Motion in Two Dimensions: Projectile Motion

(i) *A body projected horizontally with a velocity u from a height h.* [Fig. 2.20] Horizontal and vertical distances covered in time t are

$$x = ut \tag{2.5}$$

$$y = \frac{1}{2}gt^2 \tag{2.6}$$

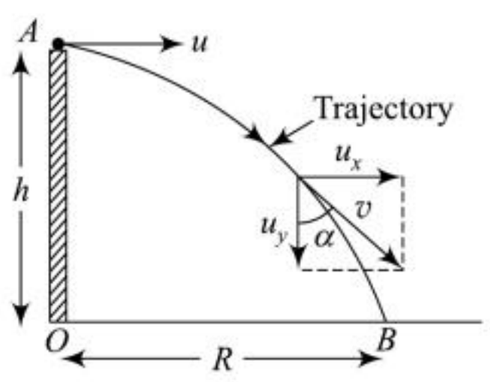

Fig. 2.20

Differentiating Eqs. (2.5) and (2.6) w.r.t time t, we get the horizontal and vertical velocities.

$$v_x = \frac{dx}{dt} = u \tag{2.6}$$

$$v_y = \frac{dy}{dt} = gt \tag{2.7}$$

Equation to trajectory

Eliminating t from Eqs. (2.5) and (2.6), we get

$$y = \left(\frac{g}{2u^2}\right)x^2$$

Since $y \propto x^2$, the trajectory of the body is parabolic.

Time of flight (t_f)

Putting $y = h$ and $t = t_f$ in Eq. (2.6), we get $t_f = \sqrt{\dfrac{2h}{g}}$

Horizontal range (R)

Putting $t = t_f$ and $x = R$ in Eq. (2.5), we get

$$R = u\,t_f = u\sqrt{\frac{2h}{g}}$$

Magnitude of resultant velocity at time t is

$$v = \left(u^2 + g^2 t^2\right)^{1/2}$$

The angle α which the resultant velocity vector subtends with the vertical is given by

$$\tan\alpha = \frac{u}{gt}$$

(ii) *A body projected from the ground with a velocity u at an angle θ with the horizontal.* (Fig. 2.21)

The horizontal and vertical distances covered in time t are

$$x = (u\cos\theta)t \tag{2.8}$$

and

$$y = (u\sin\theta)t - \frac{1}{2}gt^2 \tag{2.9}$$

Horizontal and vertical components of the velocity at time t are

$$v_x = u\cos\theta \tag{2.10}$$

and

$$u_y = u\sin\theta - gt \tag{2.11}$$

Magnitude of resultant velocity at time t is

$$v = \left(v_x^2 + v_y^2\right)^{1/2}$$

The angle α subtended by the resultant velocity vector with the horizontal is given by

$$\tan\alpha = \frac{v_y}{v_x}$$

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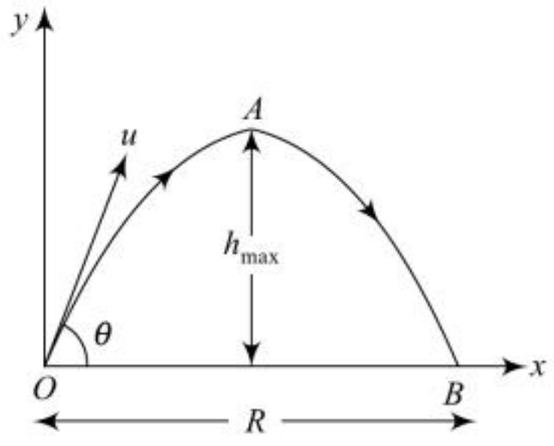

Fig. 2.21

Equation of trajectory

Eliminating t from Eqs. (2.8) and (2.9), we get

$$y = (\tan\theta)x - \frac{gx^2}{2u^2\cos^2\theta}$$

Time of flight (t_f)

Putting $y = 0$ and $t = t_f$ in Eq. (2.9), we get

$$t_f = \frac{2u\sin\theta}{g}$$

Maximum height attained (h_{max})

Put $t = t_f/2$ and $y = h_{max}$ in Eq. (2.9), we get

$$h_{max} = \frac{u^2\sin^2\theta}{2g}$$

Horizontal range (R)

Putting $t = t_f$ and $x = R$ in Eq. (2.8) we get

$$R = \frac{u^2\sin(2\theta)}{g}$$

(iii) *A body projected from a height h with a velocity u at an angle θ with the horizontal. (Fig. 2.22)*

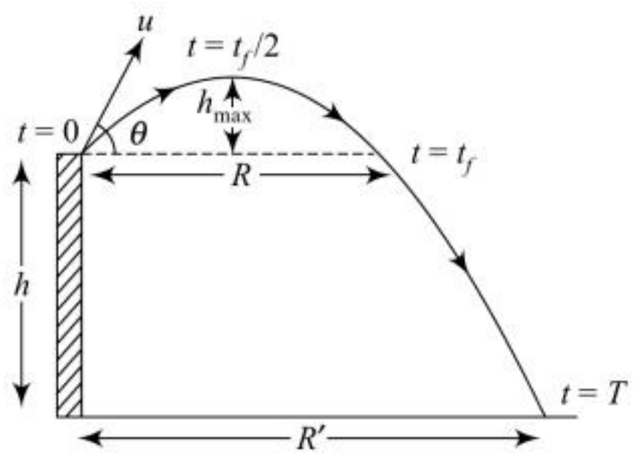

Fig. 2.22

If T is the total time of flight, then we have

$$h = u_y T - \frac{1}{2}gT^2$$

$$= (u\sin\theta)T - \frac{1}{2}gT^2$$

which gives $$T = \frac{u\sin\theta}{g} + \left(\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}\right)^{1/2}$$

The horizontal range is $R' = (u\cos\theta)T$

Applications

(i) The horizontal range is the same for angles θ and $(90° - \theta)$.

(ii) The horizontal range is maximum for $\theta = 45°$. $R_{max} = u^2/g$

(iii) When horizontal range is maximum, $h_{max} = \frac{R_{max}}{4}$

(iv) At the point of projection, KE $= \frac{1}{2}mu^2$, PE $= 0$. Total energy $E = \frac{1}{2}mu^2$.

(v) At the highest point, KE $= \frac{1}{2}mu^2\cos^2\theta$

and PE = total energy – KE

$$= \frac{1}{2}mu^2 - \frac{1}{2}mu^2\cos^2\theta = \frac{1}{2}mu^2\sin^2\theta.$$

(vi) To find R and h_{max} from the equation of trajectory

$y = ax - bx^2$

where a and b are constants, refer to Fig. 2.23.

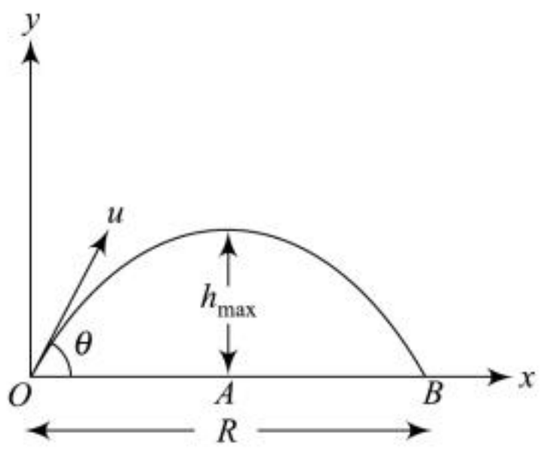

Fig. 2.23

(a) At O and B, $y = 0$. Putting $y = 0$ in the above equation, we have $0 = ax - bx^2 \Rightarrow x = 0$, $x = a/b$. Therefore $R = \frac{a}{b}$.

(b) At A, $y = h_{max}$ and $x = \frac{R}{2} = \frac{a}{2b}$. Using these values in $y = ax - bx^2$, we get $h_{max} = \frac{a^2}{4b}$.

(vii) If A and B are two points at the same horizontal level on a trajectory at a height h from the ground, (see Fig. 2.24), then

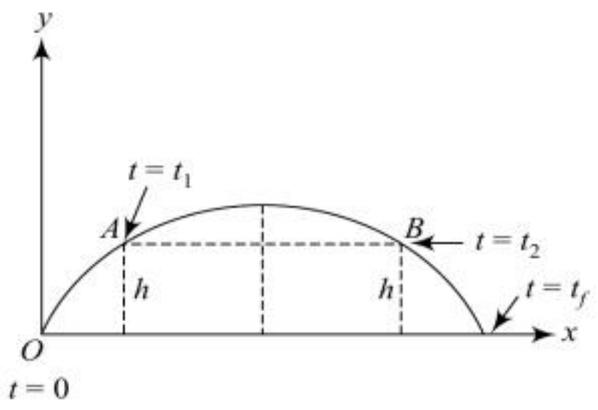

Fig. 2.24

(a) $t_f = \dfrac{2u\sin\theta}{g} = t_1 + t_2$

(b) $h = \dfrac{1}{2} g t_1 t_2$

(c) Average velocity during time interval $(t_2 - t_1)$ is $v_{av} = u\cos\theta$
($\because$ during this interval, the vertical displacement is zero)

(viii) *Velocity and Direction of Motion of Projectile at any Height.* Let P be a point on the trajectory of a projectile at a height h and let v be the velocity of the projectile at that height. If α is the angle which the velocity vector makes with the horizontal, then the horizontal and vertical components of the velocity are given by $v_x = u_x$ = constant

or $$v\cos\alpha = u\cos\theta \qquad \text{(i)}$$

and $$v_y^2 = u_y^2 - 2gh$$

or $$(v\sin\alpha)^2 = (u\sin\theta)^2 = -2gh \qquad \text{(ii)}$$

Squaring Eq. (i) and then adding to Eq. (ii), we get

$$v^2 = v_x^2 + v_y^2 = u_2 - 2gh \quad \text{or} \quad v = (u^2 - 2gh)^{1/2}$$

This gives the speed of the projectile at height h. The direction of the velocity vector (i.e., direction of motion) is obtained by taking the square root of Eq. (ii) and then dividing by Eq. (i). We get

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\left(u^2\sin^2\theta - 2gh\right)^{1/2}}{u\cos\theta}$$

(ix) *Time of Flight and Range of a Projectile on an Inclined Plane*
Consider an inclined plane OAB making an angle α with the horizontal (Fig. 2.25). Let a body be projected with a velocity u at an angle θ with the horizontal. Let us choose the x-axis along the plane OA and y-axis perpendicular to the plane OA. Let the body hit the inclined plane at point P so that $R = OP$ is the range on the inclined plane. The x and y components of the velocity of the projectile are

$$v_x = u\cos(\theta - \alpha) \quad \text{and} \quad v_y = u\sin(\theta - \alpha)$$

The x and y components of acceleration due to gravity are $-g\sin\alpha$ and $-g\cos\alpha$ respectively, as shown in Fig. 2.25. Let T_f be the time of flight on the inclined plane. Since the net vertical displacement in time T_f is zero (i.e., $h = 0$), we have

$$0 = v_y T_f - \frac{1}{2} g\cos\alpha\, T_f^2$$

or $$0 = v_y - \frac{1}{2} g\cos\alpha\, T_f$$

or $$0 = u\sin(\theta - \alpha) - \frac{1}{2} g\cos\alpha\, T_f$$

or $$T_f = \frac{2u\sin(\theta - \alpha)}{g\cos\alpha} \qquad \text{(iii)}$$

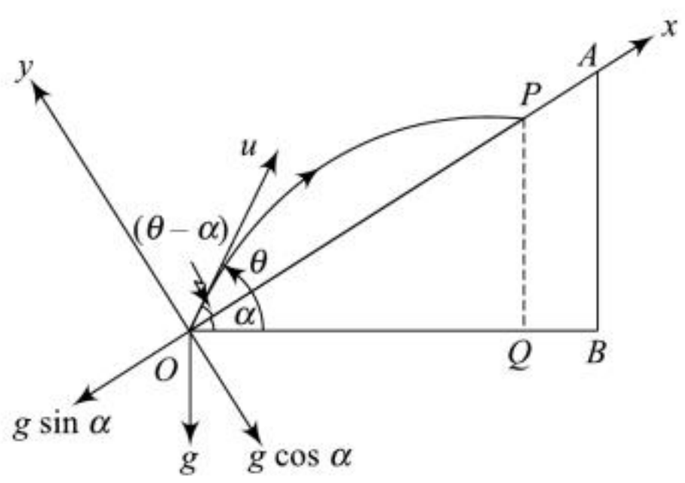

Fig. 2.25

During this time, the horizontal component of velocity $u\cos\theta$ remains constant. Hence, horizontal distance OQ is

$$OQ = (u\cos\theta)T_f$$

$\therefore$ Range of the projectile on the inclined plane is

$$R = OP = \frac{OQ}{\cos\alpha} = \frac{(u\cos\theta)\,T_f}{\cos\alpha} \qquad \text{(iv)}$$

Using Eq. (iii) in Eq. (iv), we get

$$R = \frac{2u^2\sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

18. Relative Velocity in Two Dimensions

The relative velocity of a body B with respect to body A is defined as

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

If vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ are inclined to each other at an angle θ as shown in Fig. 2.26, the relative velocity $\mathbf{v}_{BA}$ is found as follows.

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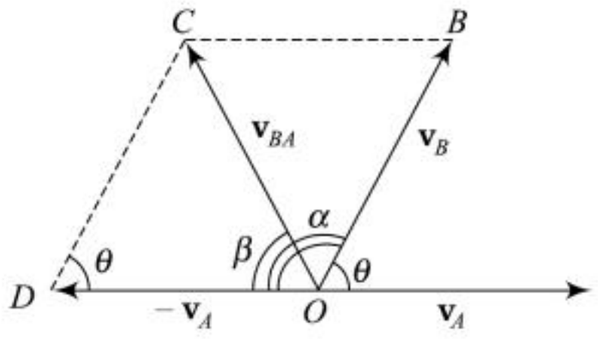

Fig. 2.26

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_B + (-\mathbf{v}_A)$$

Thus, the magnitude and direction of vector $\mathbf{v}_{BA}$ can be found by finding the resultant of vectors $\mathbf{v}_B$ and $\mathbf{v}_A$ which is vector **OC** as shown in Fig. 2.26.

Magnitude of vector $\mathbf{v}_{BA}$ is given by

$$v_{BA} = \left(v_A^2 + v_B^2 + 2\,v_A\,v_B\cos\alpha\right)^{1/2}$$

$$= \left(v_A^2 + v_B^2 - 2\,v_A\,v_B\cos\theta\right)^{1/2}$$

$$(\because \alpha = 180° - \theta)$$

The angle β which the resultant vector **OC** subtends with vector **OD** is given by

$$\frac{OC}{\sin\theta} = \frac{CD}{\sin\beta}$$

$$\Rightarrow \qquad \sin\beta = \frac{CD\sin\theta}{OC} = \frac{v_B\sin\theta}{v_{BA}}$$

Special Case

(i) If vector v_A and v_B are in the same direction, $\theta = 0°$, then $v_{BA} = \sqrt{v_A^2 + v_B^2 - 2\,v_A\,v_B} = v_B - v_A$.

(ii) If vector v_A and v_B are in opposite direction, $\theta = 180°$, then $v_{BA} = v_B + v_A$.

Applications

(i) *To cross the river of width d along the shortest path which is PQ*, the boat must move along *PR* making an angle $(90° + \theta)$ with the direction of the stream such that the direction of the resultant velocity **v** is along *PQ*. Angle θ is given by (see Fig. 2.27)

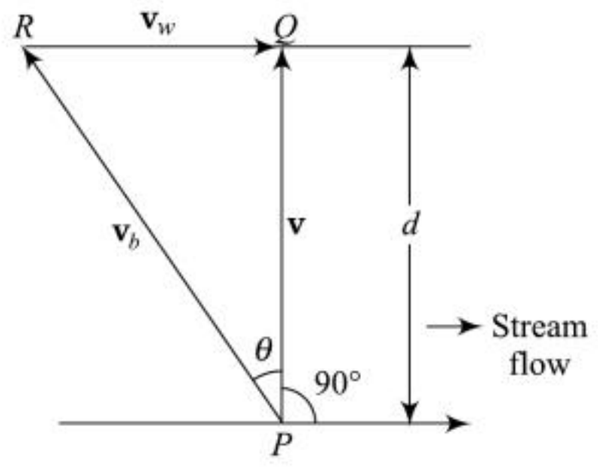

Fig. 2.27

$$\sin\theta = \frac{v_\omega}{v_b}$$

Also $$v = \sqrt{v_b^2 - v_\omega^2}$$

The time taken to cross the river along the shortest path is given by

$$t = \frac{d}{v} = \frac{d}{\sqrt{v_b^2 - v_\omega^2}}$$

(b) *To cross the river in the shortest time*, the boat should move along *PQ*. The shortest time is given by

$$t = \frac{d}{v_b}$$

At this time, the boat will reach the point *R* on the opposite bank of the river at a distance *x* from the point *Q* (Fig. 2.28). From the Figure, we have

$$x = d\tan\theta,$$

but $$\tan\theta = \frac{\mathbf{v}_w}{\mathbf{v}_b}$$ Therefore,

$$x = d\left(\frac{\mathbf{v}_w}{\mathbf{v}_b}\right) \qquad (2.12)$$

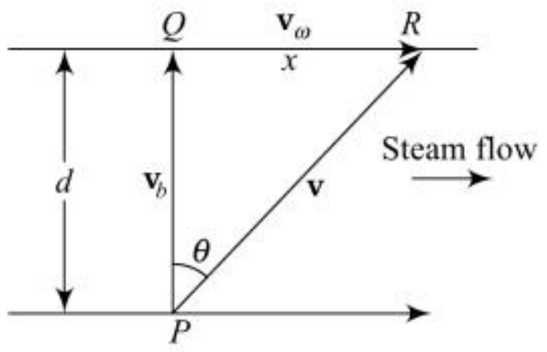

Fig. 2.28

(ii) *Holding an Umbrella to Project from Rain* Let $\mathbf{v}_r$ be the velocity of the rain falling vertically downward and $\mathbf{v}_m$ the velocity of a man walking from north to south direction (Fig. 2.29). In order to protect himself from rain, he must hold his umbrella in the direction of the resultant velocity **v**, which is given by

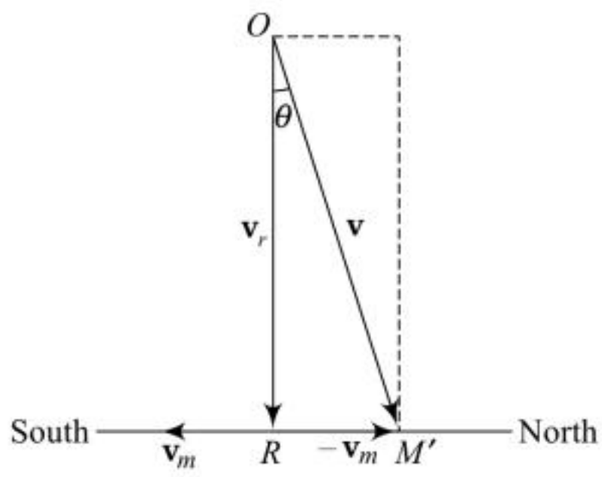

Fig. 2.29

$$v = \sqrt{v_r^2 + v_m^2}$$

This is the speed with which the rain strikes the umbrella. If θ is the angle subtended by the resultant velocity **v** with the vertical, then from triangle ORM', we have

$$\tan\theta = \frac{RM'}{OR} = \frac{v_m}{v_r}$$

or $$\theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Thus, the man must hold the umbrella at an angle θ with the vertical towards north.

19. Uniform Circular Motion

(i) *For a body moving in a horizontal circle*

The centripetal acceleration of a body of mass m moving in a circle of radius R with a constant speed v (or angular speed ω) is

$$a_c = \omega v = \omega^2 R = \frac{v^2}{R}$$

Centripetal force is

$$f_c = m\, a_c = m\, \omega^2\, R = \frac{mv^2}{R}$$

(ii) *For a body moving in a vertical circle*

The minimum speed to complete the circle when the body is at the top of the circle is $v = \sqrt{Rg}$.

The minimum speed to complete the circle when the body is at the bottom of the circle is $v = \sqrt{5Rg}$.

SECTION I

Multiple Choice Questions with One Correct Choice

1. The magnitude of the resultant of two vectors of magnitudes 3 units and 4 units in 5 units. What is the angle between two vectors?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) π

2. The magnitude of the resultant of two equal vectors is equal to the magnitude of either vector. What is the angle between the two vectors?

(a) 60° (b) 90°
(c) 120° (d) 150°

3. In Q.2, the angle between the resultant and either vector will be

(a) 60° (b) 90°
(c) 120° (d) 150°

4. Given $\mathbf{P} = \mathbf{A} + \mathbf{B}$ and $\mathbf{Q} = \mathbf{A} - \mathbf{B}$. If the magnitudes of vectors **P** and **Q** are equal, what is the angle between vectors **A** and **B**?

(a) zero (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

5. If $\mathbf{A}\cdot\mathbf{B} = \frac{1}{2}AB$, what is the angle between **A** and **B**?

(a) zero (b) 30°
(c) 60° (d) 90°

6. If $|\mathbf{A}\times\mathbf{B}| = \frac{1}{2}AB$, what is the angle between **A** and **B**?

(a) zero (b) 30°
(c) 60° (d) 90°

7. If $\mathbf{A}\cdot\mathbf{B}$ = magnitude of $\mathbf{A}\times\mathbf{B}$, then the angle between vectors **A** and **B** is

(a) 30° (b) 45°
(c) 60° (d) 75°

8. Given $\mathbf{A}\cdot\mathbf{B} = 0$ and $\mathbf{A}\times\mathbf{C} = 0$. What is the angle between **B** and **C**?

(a) 45° (b) 90°
(c) 135° (d) 180°

9. The resultant of two vectors **A** and **B** subtends an angle of 45° with either of them. The magnitude of the resultant is

(a) zero (b) $\sqrt{2}\,A$
(c) A (d) $2A$

10. Vector **C** is the sum of two vectors **A** and **B** and vector **D** is the cross product of vectors **A** and **B**. What is the angle between vectors **C** and **D**?

(a) zero (b) 60°
(c) 90° (d) 180°

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11. Given $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ and $\mathbf{D} = \mathbf{B} \times \mathbf{A}$. What is the angle between **C** and **D**?

(a) zero (b) 60°
(c) 90° (d) 180°

12. The resultant of two vectors of magnitudes 3 units and 4 units is 1 unit. What is the value of their dot product?

(a) – 12 units (b) – 7 units
(c) – 1 unit (d) zero

13. The resultant of two vectors of magnitudes 3 units and 4 units is 1 unit. What is the magnitude of their cross product?

(a) 12 units (b) 7 units
(c) 1 unit (d) zero

14. Three vectors **A**, **B** and **C** are related as **A** + **B** = **C**. If vector **C** is perpendicular to vector **A** and the magnitude of **C** is equal to the magnitude of **A**, what will be the angle between vectors **A** and **B**?

(a) 45° (b) 90°
(c) 135° (d) 180°

15. The magnitude of the resultant of (**A** + **B**) and (**A** – **B**) is

(a) $2A$ (b) $2B$
(c) $\sqrt{A^2 + B^2}$ (d) $\sqrt{A^2 - B^2}$

16. In Q.15, what is the angle between the resultant vector and vector **A**?

(a) zero (b) $\cos^{-1}\left(\frac{A}{B}\right)$
(c) $\cos^{-1}\left(\frac{B}{A}\right)$ (d) $\cos^{-1}\left(\frac{A-B}{A+B}\right)$

17. If $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along x-axis and y-axis respectively, the magnitude of vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ will be

(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

18. In Q.17, the angle subtended by vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ with the x-axis is

(a) 30° (b) 45°
(c) 0° (d) 75°

19. If $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along x, y and z-axes respectively, the angle θ between the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and vector $\hat{\mathbf{i}}$ is given by

(a) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(c) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

20. Given that $0.2\ \hat{\mathbf{i}} + 0.6\ \hat{\mathbf{j}} + a\,\hat{\mathbf{k}}$ is a unit vector. What is the value of a?

(a) $\sqrt{0.3}$ (b) $\sqrt{0.4}$
(c) $\sqrt{0.6}$ (d) $\sqrt{0.8}$

21. Given $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{k}}$. What is the value of the scalar product of **A** and **B**?

(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

22. The cross product of vectors **A** and **B** in Q. 21 is

(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$
(c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

23. Given $\mathbf{A} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$. The component of vector **A** along vector **B** is

(a) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (b) $\frac{3}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(c) $\frac{5}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (d) $\frac{7}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

24. In Q.23 above, what is the component of vector **A** perpendicular to vector **B** and in the same plane as **B**?

(a) $\frac{1}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$ (b) $\frac{3}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$
(c) $\frac{5}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$ (d) $\frac{7}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$

25. A stone falls from rest. The distance covered by the stone in the last second of its motion equals the distance covered by it during the first three seconds of its motion. How long does the stone take to reach the ground?

(a) 4 s (b) 5 s
(c) 6 s (d) 9 s

26. The acceleration a (in ms^{-2}) of a body, starting from rest varies with time t (in s) according to the relation

$$a = 3t + 4$$

The velocity of the body at time $t = 2$s will be

(a) 10 ms^{-1} (b) 12 ms^{-1}
(c) 14 ms^{-1} (d) 16 ms^{-1}

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27. Two balls of masses m_1 and m_2 are thrown vertically upward with the same speed u. If air resistance is neglected, they will pass through their point of projection in the downward direction with a speed

(a) $\frac{m_1 u}{m_2}$ (b) $\frac{m_2 u}{m_1}$

(c) $\frac{(m_1 + m_2)u}{(m_1 - m_2)}$ (d) u

28. The displacement y (in metres) of a body varies with time t (in seconds) as

$$y = -\frac{2}{3}t^2 + 16t + 2$$

How long does the body take to come to rest?

(a) 8 seconds (b) 10 seconds

(c) 12 seconds (d) 16 seconds

29. The velocity-time graph of a stone thrown vertically upward with an initial velocity of 30 ms^{-1} is shown in the Fig. 2.30. The velocity in the upward direction is taken as positive and that in the downward direction as negative. What is the maximum height to which the stone rises?

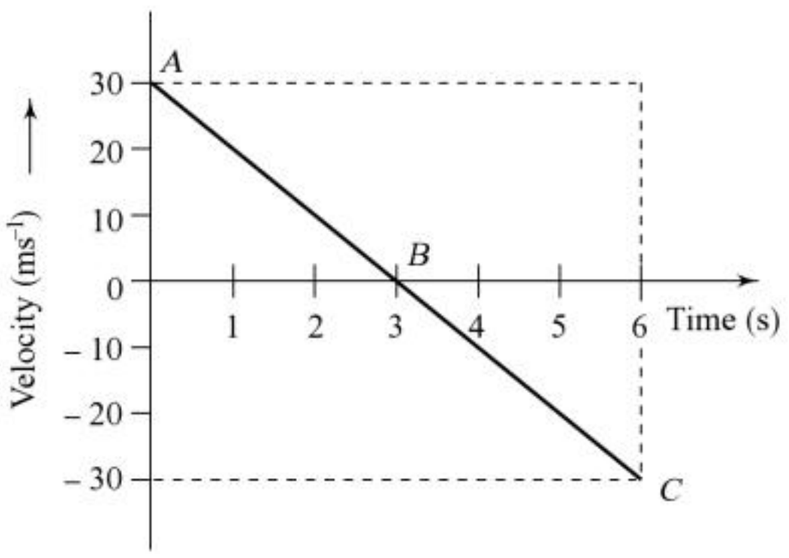

Fig. 2.30

(a) 30 m (b) 45 m

(c) 60 m (d) 90 m

30. At $t = 0$, an arrow is fired vertically upwards with a speed of 98 ms^{-1}. A second arrow is fired vertically upwards with the same speed at $t = 5$ s. Then

(a) the two arrows will be at the same height above the ground at $t = 10$ s

(b) the two arrows will reach back their starting points at $t = 20$ s and at $t = 30$ s

(c) the ratio of the speeds of the first and the second arrows at $t = 20$ s will be 2 : 1

(d) the maximum height attained by either arrow will be 980 m

31. The area under the velocity-time graph between any two instants $t = t_1$ and $t = t_2$ gives the distance covered in time $\delta t = t_2 - t_1$

(a) only if the particle moves with a uniform velocity

(b) only if the particle moves with a uniform acceleration

(c) only if the particle moves with an acceleration increasing at a uniform rate

(d) in all cases irrespective of whether the motion is one of uniform velocity, or of uniform acceleration or of variable acceleration.

32. The displacement x of a particle varies with time according to the relation $x = \frac{a}{b}\left(1 - e^{-bt}\right)$. Then

(a) At $t = 1/b$, the displacement of the particle is nearly (1/3) (a/b)

(b) The velocity and acceleration of the particle at $t = 0$ are a and ab respectively

(c) The particle cannot reach a point at a distance x' from its starting position if $x' > a/b$

(d) The particle will come back to its starting point as $t \to \infty$

33. The variation in the speed of a car during its two hour journey is shown in the graph of Fig. 2.31. The magnitude of the maximum acceleration of the car occurs during the interval

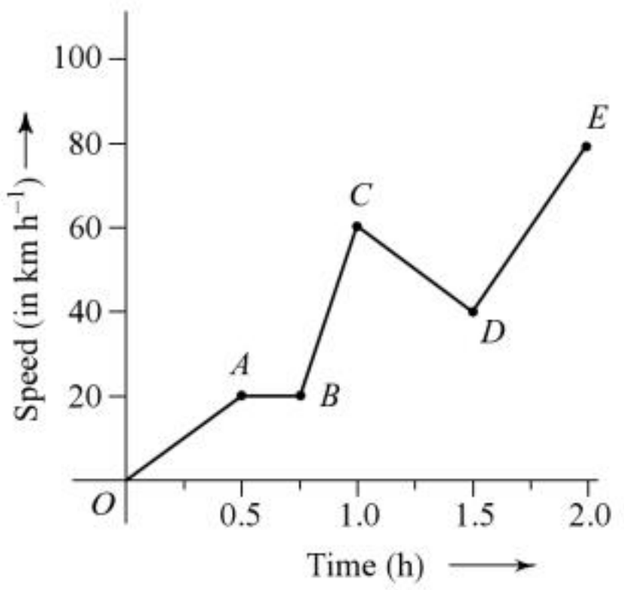

Fig. 2.31

(a) OA (b) BC

(c) CD (d) DE

34. A parachutist drops freely from an aeroplane for 10 s before the parachute opens out. Then he descends with a net retardation of 2.5 ms^{-2}. If he bails out of the plane at a height of 2495 m and $g = 10$ ms^{-2}, his velocity on reaching the ground will be

(a) 2.5 ms^{-1} (b) 7.5 ms^{-1}

(c) 5 ms^{-1} (d) 10 ms^{-1}

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35. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, the maximum velocity acquired by the car is given by

(a) $\frac{\alpha\beta}{\alpha+\beta}\,t$ (b) $\frac{\alpha+\beta}{\alpha\beta}\,t$

(c) $\frac{\alpha^2+\beta^2}{\beta\alpha}\,t$ (d) $\frac{\alpha^2-\beta^2}{\beta\alpha}\,t$

36. The distance travelled by the car in Q.35 above is given by

(a) $\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$ (b) $\frac{1}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)t^2$

(c) $\frac{1}{2}\left(\frac{\alpha^2+\beta^2}{\alpha}\right)t^2$ (d) $\frac{1}{2}\left(\frac{\alpha^2-\beta^2}{\beta}\right)t^2$

37. The acceleration of a particle, starting from rest, varies with time according to the relation $a = kt + c$, where k and c are constants of motion.

The velocity v is the particle after a time t will be

(a) $k\,t + ct$ (b) $(1/2)\,k\,t^2 + c\,t$

(c) $1/2\,(kt^2 + ct)$ (d) $kt^2 + (1/2)ct$

38. The acceleration of a particle, starting from rest, varies with time according to the relation $a = -s\,\omega^2 \cos \omega t$, where s and ω are constants. The displacement of this particle at a time t will be

(a) $-\frac{1}{2}\,(s\,\omega^2 \sin \omega t)t^2$ (b) $s\,\omega \sin \omega t$

(c) $s\,\omega \cos \omega t$ (d) $s \cos \omega t$

39. A freely falling body, falling from a tower of height h covers a distance $h/2$ in the last second of its motion. The height of the tower is (take $g = 10\ \text{ms}^{-2}$) nearly

(a) 58 m (b) 50 m

(c) 60 m (d) 55 m

40. A balloon is rising vertically upwards at a velocity of $10\ \text{ms}^{-1}$. When it is at a height of 45 m from the ground, a parachutist bails out from it. After 3s he opens his parachute and decelerates at a constant rate of $5\ \text{ms}^{-2}$. What was the height of the parachutist above the ground when he opened his parachute? Take $g = 10\ \text{ms}^{-2}$.

(a) 15 m (b) 30 m

(c) 45 m (d) 60 m

41. In Q.40, how far is the parachutist from the balloon at $t = 3$s?

(a) 15 m (b) 30 m

(c) 45 m (d) 60 m

42. In Q.40, with what velocity does the parachutist hit the ground?

(a) $10\ \text{ms}^{-1}$ (b) $20\ \text{ms}^{-1}$

(c) $30\ \text{ms}^{-1}$ (d) $40\ \text{ms}^{-1}$

43. In Q.40, after how long does the parachutist hit the ground after his exit from the balloon?

(a) 4 s (b) 5 s

(c) 6 s (d) 7 s

44. Car A is moving with a speed of 36 km h^{-1} on a two-lane road. Two cars B and C, each moving with a speed of 54 km h^{-1} in opposite directions on the other lane are approaching car A. At a certain instant when the distance AB = distance AC = 1 km, the driver of car B decides to overtake A before C does. What must be the minimum acceleration of car B so as to avoid an accident?

(a) $1\ \text{ms}^{-2}$ (b) $2\ \text{ms}^{-2}$

(c) $3\ \text{ms}^{-2}$ (d) $4\ \text{ms}^{-2}$

45. The driver of a train A moving at a speed of $30\ \text{ms}^{-1}$ sights another train B moving on the same track towards his train at a speed of $10\ \text{ms}^{-1}$. He immediately applies brakes and achieves a uniform retardation of $4\ \text{ms}^{-2}$. To avoid head-on collision, what must be the minimum distance between the trains?

(a) 100 m (b) 200 m

(c) 300 m (d) 400 m

46. A cyclist starts from the centre O of a circular track of radius r =1 km, reaches the edge P of the track and then cycles along the circumference and stops at point Q as shown in Fig. 2.32. If the total time taken is 10 min, what is the average velocity of the cyclist?

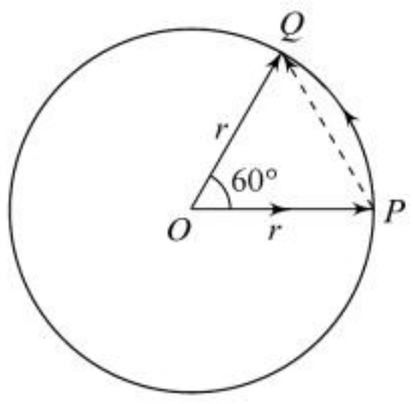

Fig. 2.32

(a) 3 km h^{-1} (b) 6 km h^{-1}

(c) 9 km h^{-1} (d) 12 km h^{-1}

47. In Q.46 above, the average speed of the cyclist is approximately equal to

(a) $12.3\ ms^{-1}$ (b) $13.4\ ms^{-1}$
(c) $14.4\ ms^{-1}$ (d) $15.4\ ms^{-1}$

48. Rain is falling vertically with a speed of $4\ ms^{-1}$. After some time, wind starts blowing with a speed of $3\ ms^{-1}$ in the north to south direction. In order to protect himself from rain, a man standing on the ground should hold his umbrella at an angle θ given by

(a) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards south

(b) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the vertical towards north

(c) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards south

(d) $\theta = \cot^{-1}\left(\frac{3}{4}\right)$ with the vertical towards north

49. In Q.48 above, with what speed does the rain strike the umbrella?

(a) $3\ ms^{-1}$ (b) $4\ ms^{-1}$
(c) $5\ ms^{-1}$ (d) $6\ ms^{-1}$

50. A swimmer can swim in still water with a speed of $5\ ms^{-1}$. While crossing a river his average speed is $3\ ms^{-1}$. If he crosses the river in the shortest possible time, what is the speed of flow of water?

(a) $2\ ms^{-1}$ (b) $4\ ms^{-1}$
(c) $6\ ms^{-1}$ (d) $8\ ms^{-1}$

51. A body, starting from rest, moves in a straight line with a constant acceleration a for a time interval t during which it travels a distance s_1. It continues to move with the same acceleration for the next time interval t during which it travels a distance s_2. The relation between s_1 and s_2 is

(a) $s_2 = s_1$ (b) $s_2 = 2s_1$
(c) $s_2 = 3s_1$ (d) $s_2 = 4s_1$

52. In Q.51, if v_1 is the velocity of the body at the end of first time interval and v_2 that at the end of the second time interval, the relation between v_1 and v_2 is

(a) $v_2 = v_1$ (b) $v_2 = 2v_1$
(c) $v_2 = 3v_1$ (d) $v_2 = 4v_1$

53. A body dropped from the top of a tower hits the ground after 4s. How much time does it take to cover the first half of the distance from the top of the tower?

(a) 1 s (b) 2 s
(c) $2\sqrt{2}$ s (d) $\sqrt{3}$ s

54. A body moving in a straight line with constant acceleration of $10\ ms^{-2}$ covers a distance of 40 m in the 4^{th} second. How much distance will it cover in the 6^{th} second?

(a) 50 m (b) 60 m
(c) 70 m (d) 80 m

55. A body, moving in a straight line with an initial velocity of $5\ ms^{-1}$ and a constant acceleration, covers a distance of 30 m in the 3^{rd} second. How much distance will it cover in the next 2 seconds?

(a) 70 m (b) 80 m
(c) 90 m (d) 100 m

56. A body, moving in a straight line, with an initial velocity u and a constant acceleration a, covers a distance of 40 m in the 4^{th} second and a distance of 60 m in the 6^{th} second. The values of u and a respectively are

(a) $10\ ms^{-1}, 5\ ms^{-2}$ (b) $10\ ms^{-1}, 10\ ms^{-2}$
(c) $5\ ms^{-1}, 5\ ms^{-2}$ (d) $5\ ms^{-1}, 10\ ms^{-2}$

57. A body, starting from rest and moving with a constant acceleration, covers a distance s_1 in the 4^{th} second and a distance s_2 in the 6^{th} second. The ratio s_1/s_2 is

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$
(c) $\frac{6}{11}$ (d) $\frac{7}{11}$

58. A car, starting from rest, has a constant acceleration a_1 for a time interval t_1 during which it covers a distance s_1. In the next time interval t_2, the car has a constant retardation a_2 and comes to rest after covering a distance s_2 in time t_2. Which of the following relations is correct?

(a) $\frac{a_1}{a_2} = \frac{s_1}{s_2} = \frac{t_1}{t_2}$ (b) $\frac{a_1}{a_2} = \frac{s_2}{s_1} = \frac{t_1}{t_2}$
(c) $\frac{a_1}{a_2} = \frac{s_1}{s_2} = \frac{t_2}{t_1}$ (d) $\frac{a_1}{a_2} = \frac{s_2}{s_1} = \frac{t_2}{t_1}$

59. In Q. 58, if the total distance covered by the car is s, the maximum speed attained by it will be

(a) $\left(2s.\frac{a_1a_2}{a_1+a_2}\right)^{\frac{1}{2}}$ (b) $\left(2s.\frac{a_1a_2}{a_1-a_2}\right)^{\frac{1}{2}}$
(c) $\left(\frac{s}{2}.\frac{a_1a_2}{a_1+a_2}\right)^{\frac{1}{2}}$ (d) $\left(\frac{s}{2}.\frac{a_1a_2}{a_1-a_2}\right)^{\frac{1}{2}}$

60. A car, starting from rest, is accelerated at a constant rate α until it attains a speed v. It is then retarded at a constant rate β until it comes to rest. The average speed of the car during its entire journey is

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(a) zero (b) $\frac{\alpha v}{2\beta}$

(c) $\frac{\beta v}{2\alpha}$ (d) $\frac{v}{2}$

61. A simple pendulum is hanging from the ceiling of a compartment of a train. It is observed that the string is inclined towards the rear of the train. If follows that the train is

(a) at rest (b) accelerated
(c) decelerated (d) in uniform motion.

62. The displacement of a body from a reference point, is given by

$$\sqrt{x} = 2t + 3$$

where x is in metres and t in seconds. This shows that the body is

(a) at rest (b) accelerated
(c) decelerated (d) in uniform motion

63. In Q.62, what is the initial velocity of the body?

(a) 2 ms^{-1} (b) 3 ms^{-1}
(c) 6 ms^{-1} (d) 12 ms^{-1}

64. In Q.62, what is the acceleration of the body?

(a) 2 ms^{-2} (b) 3 ms^{-2}
(c) 6 ms^{-2} (d) 8 ms^{-2}

65. A car moving at a speed v is stopped in a certain distance when the brakes produce a deceleration a. If the speed of the car was nv, what must be the deceleration of the car to stop it in the same distance and in the same time?

(a) $\sqrt{n}\,a$ (b) na
(c) n^2a (d) n^3a

66. A car is moving at a certain speed. The minimum distance over which it can be stopped is x. If the speed of the car is doubled, what will be the minimum distance over which the car can be stopped during the same time?

(a) $4x$ (b) $2x$
(c) $x/2$ (d) $x/4$

67. A bullet is fired vertically upwards with an initial velocity of 50 ms^{-1}. It covers a distance h_1 during the first second and a distance h_2 during the last 3 seconds of its upward motion. If $g = 10\ ms^{-2}$, h_1 and h_2 will be related as

(a) $h_1 = 3h_2$ (b) $h_1 = 2h_2$

(c) $h_1 = h_2$ (d) $h_1 = \frac{h_2}{3}$

68. A ball is thrown vertically downward with a velocity u from the top of a tower. It strikes the ground with a velocity $3u$. The time taken by the ball to reach the ground is given by

(a) $\frac{u}{g}$ (b) $\frac{2u}{g}$

(c) $\frac{3u}{g}$ (d) $\frac{4u}{g}$

69. In Q.68, the height of the tower is given by

(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$

(c) $\frac{3u^2}{g}$ (d) $\frac{4u^2}{g}$

70. The distance x covered by a body moving in a straight line in time t is given by the relation

$$2x^2 + 3x = t$$

If v is the velocity of the body at a certain instant of time, its acceleration will be

(a) $-v^3$ (b) $-2v^3$
(c) $-3v^3$ (d) $-4v^3$

71. The distance x covered by a body moving in a straight line in time t is given by

$$x^2 = t^2 + 2t + 3$$

The acceleration of the body will vary as

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$

(c) $\frac{1}{x^3}$ (d) $\frac{1}{x^4}$

72. The motion of a body is given by the equation

$$\frac{dV(t)}{dt} = 6.0 - 3V(t)$$

where $V(t)$ is the speed (in ms^{-1}) at time t (in second). If the body was at rest at $t = 0$, which of the following statements is wrong?

(a) the terminal speed is 2.0 ms^{-1}
(b) the magnitude of the initial acceleration is 6.0 ms^{-2}
(c) the speed varies with time as $V(t) = 2(1 - e^{-3t})$ ms^{-1}
(d) the speed is 2.0 ms^{-1} when the acceleration is half the initial value.

73. A body, moving in a straight line, covers half the distance with a speed V, the remaining part of the

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distance was covered with a speed V' for half the time and with a speed V'' for the other half of the time. What is the average speed of the body?

(a) $\frac{2V(V'+V'')}{(2V+V'+V'')}$ (b) $\frac{V(V'+V'')}{(2V+V'+V'')}$

(c) $\frac{2V'V''}{(V+V'+V'')}$ (d) $\frac{V'V''}{(V+V'+V'')}$

74. A car is moving on a straight horizontal road with a speed v. When brakes are applied to give a constant retardation a, the car is stopped in a shortest distance S. If the car were moving on the same road with a speed 3 v and the same retardation a is applied, the shortest distance in which the car is stopped will be

(a) 3 S (b) 6 S
(c) 9 S (d) 27 S

75. The maximum height attained by a projectile and its horizontal range are equal to each other if the projectile is projected at an angle θ given by

(a) $\theta = \tan^{-1}(2)$ (b) $\theta = \tan^{-1}(3)$
(c) $\theta = \tan^{-1}(4)$ (d) $\theta = \tan^{-1}(5)$

76. A person aims a gun at a target located at a horizontal distance of 100 m. If the gun imparts a horizontal speed of 500 ms^{-1} to the bullet, at what height above the target must he aim his gun in order to hit it? Take $g = 10\ ms^{-2}$.

(a) 10 cm (b) 20 cm
(c) 50 cm (d) 100 cm

77. When a projectile is at the highest point of its trajectory, the directions of its velocity and acceleration are

(a) parallel to each other
(b) anti-parallel to each other
(c) inclined to each other at an angle of 45°
(d) perpendicular to each other

78. A ball is projected horizontally with a velocity of 5 ms^{-1} from the top of a building 19.6 m high. How long will the ball take to hit the ground?

(a) $\sqrt{2}$ s (b) 2 s
(c) $\sqrt{3}$ s (d) 3 s

79. A bomb is dropped from an aeroplane when it is at a height h directly above a target. If the aeroplane is moving horizontally at a speed v, the distance by which the bomb will miss the target is given by

(a) $2v\sqrt{\frac{h}{g}}$ (b) $v\sqrt{\frac{h}{g}}$

(c) $v\sqrt{\frac{2h}{g}}$ (d) $v\sqrt{\frac{h}{2g}}$

80. An enemy plane is flying horizontally at an altitude of 2 km with a speed of 300 ms^{-1}. An armyman with an anti–aircraft gun on the ground sights the enemy plane when it is directly overhead and fires a shell with a muzzle speed of 600 ms^{-1}. At what angle with the vertical should the gun be fired so as to hit the plane?

(a) 30° (b) 45°
(c) 60° (d) 75°

81. In Q.80, at what minimum altitude should the enemy plane fly to avoid being hit? Take $g = 10\ ms^{-2}$.

(a) 12.5 km (b) 13.5 km
(c) 14.5 km (d) 15.5 km

82. The ceiling of a tunnel is 5 m high. What is the maximum horizontal distance that a ball thrown with a speed of 20 ms^{-1}, can go without hitting the ceiling of the tunnel? Take $g = 10\ ms^{-2}$.

(a) $10\sqrt{3}$ m (b) $20\sqrt{3}$ m
(c) $30\sqrt{3}$ m (d) $40\sqrt{3}$ m

83. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m/s at an angle of elevation of 30°. The ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (Take $g = 10\ m/s^2$)

(a) 2 : 1 (b) 3 : 1
(c) 3 : 2 (d) 1.5 : 1

84. In Q.83, the horizontal displacement (from the foot of the tower) of the ball is approximately equal to

(a) 50 m (b) 60 m
(c) 70 m (d) 80 m

85. A cannon on a level plain is aimed at an angle α above the horizontal and a shell is fired with a muzzle velocity v_0 towards a vertical cliff a distance R away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

(a) $R\sin\alpha - \frac{1}{2}\frac{gR^2}{v_0^2\sin^2\alpha}$

(b) $R\cos\alpha - \frac{1}{2}\frac{gR^2}{v_0^2\cos^2\alpha}$

(c) $R\tan\alpha - \frac{1}{2}\frac{gR^2}{v_0^2\cos^2\alpha}$

(d) $R\tan\alpha - \frac{1}{2}\frac{gR^2}{v_0^2\sin^2\alpha}$

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86. It is possible to project a particle with a given velocity in two possible ways so as to make it pass through a point P at a distant r from the point of projection. The product of the times taken to reach this point in the two possible ways is then proportional to

(a) $1/r$ (b) r

(c) r^3 (d) $\frac{1}{r^2}$

87. A gun kept on a straight horizontal road is used to hit a car travelling on the same road away from the gun at a uniform speed of $10\sqrt{2}$ ms^{-1}. The car is at a distance of 150 m from the gun when it is fired at an angle of 45° to the horizontal. With what speed should the shell be projected so that it hits the car? Take $g = 10\ ms^{-2}$.

(a) $20\ ms^{-1}$ (b) $30\ ms^{-1}$

(c) $40\ ms^{-1}$ (d) $50\ ms^{-1}$

88. In Q.87, what is the distance of the car from the gun when the shell hits it?

(a) 250 m (b) 500 m

(c) 750 m (d) 1000 m

89. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 ms^{-1} at an angle of 30° to the horizontal. If $g = 10\ ms^{-2}$, after how long will the ball hit the ground?

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

90. A projectile has a maximum range of 200 m. What is the maximum height attained by it?

(a) 25 m (b) 50 m

(c) 75 m (d) 100 m

91. A body thrown along a frictionless inclined plane of angle of inclination 30° covers a distance of 40 m along the plane. If the body is projected with the same speed at angle of 30° with the ground, it will have a range of (take $g = 10\ ms^{-2}$)

(a) 20 m (b) $20\sqrt{2}$ m

(c) $20\sqrt{3}$ m (d) 40 m

92. Which of the following remains constant during the motion of a projectile fired from a planet?

(a) kinetic energy
(b) momentum
(c) vertical component of velocity
(d) horizontal component of velocity

93. A body is projected with kinetic energy K at an angle of 60° with the horizontal. Its kinetic energy at the highest point of its trajectory will be

(a) $2K$ (b) K

(c) $K/2$ (d) $K/4$

94. A body, projected with a certain kinetic energy, has a horizontal range R. The kinetic energy will be minimum at a position of the projectile when its horizontal range is

(a) R (b) $3R/4$

(c) $R/2$ (d) $R/4$

95. At what angle (θ) with the horizontal should a body be projected so that its horizontal range equals the maximum height it attains?

(a) $\theta = \tan^{-1}\left(\sqrt{2}\right)$ (b) $\theta = \tan^{-1}\left(2\sqrt{2}\right)$

(c) $\theta = \tan^{-1}\left(2\sqrt{3}\right)$ (d) $\theta = \tan^{-1}(4)$

96. The horizontal distance x and the vertical height y of a projectile at a time t are given by

$$x = at \text{ and } y = bt^2 + ct$$

where a, b and c are constants. What is the magnitude of the velocity of the projectile 1 second after it is fired?

(a) $[a^2 + (2b + c)^2]^{1/2}$ (b) $[2a^2 + (b + c)^2]^{1/2}$

(c) $[2a^2 + (2b + c)^2]^{1/2}$ (d) $[a^2 + (b + 2c)^2]^{1/2}$

97. In Q.96, the angle (θ) with the horizontal at which the projectile is projected is given by

(a) $\theta = \tan^{-1}\left(\frac{a}{b}\right)$ (b) $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

(c) $\theta = \tan^{-1}\left(\frac{a}{c}\right)$ (d) $\theta = \tan^{-1}\left(\frac{c}{a}\right)$

98. In Q.96, the acceleration due to gravity is given by

(a) $-2a$ (b) $-2b$

(c) $-2c$ (d) $-ac + b$

99. In Q. 96, the magnitude of the initial velocity of the projectile is given by

(a) $(a^2 + b^2)^{1/2}$ (b) $(b^2 + c^2)^{1/2}$

(c) $(a^2 + c^2)^{1/2}$ (d) $(b^2 - 4ac)^{1/2}$

100. Four projectiles are projected with the same speed at angles 20°, 35°, 60° and 75° with the horizontal. The range will be the longest for the projectile whose angle of projection is

(a) 20° (b) 35°

(c) 60° (d) 75°

101. A projectile thrown at an angle of 30° with the horizontal has a range R_1 and attains a maximum height h_1. Another projectile thrown, with the same velocity, at an angle 30° with the vertical, has a range

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R_2 and attains a maximum height h_2. The relation between R_1 and R_2 is

(a) $R_1 = \frac{R_2}{2}$ (b) $R_1 = R_2$

(c) $R_1 = 2R_2$ (d) $R_1 = 4R_2$

102. In Q.101 what is the relation between h_1 and h_2?

(a) $h_1 = \frac{h_2}{4}$ (b) $h_1 = \frac{h_2}{3}$

(c) $h_1 = \frac{h_2}{2}$ (d) $h_1 = h_2$

103. The maximum height attained by a projectile is increased by 10% by increasing its speed of projection, without changing the angle of projection. The percentage increase in the horizontal range will be

(a) 20% (b) 15%
(c) 10% (d) 5%

104. In Q.103, what is the percentage increase in the time of flight of the projectile?

(a) 20% (b) 15%
(c) 10% (d) 5%

105. The maximum height attained by projectile is increased by 10% by changing the angle of projection, without changing the speed of projection. The percentage increase in the time of flight will be

(a) 20% (b) 15%
(c) 10% (d) 5%

106. The speed of projection of projectile is increased by 5%, without changing the angle of projection. The percentage increase in the range will be

(a) 2.5% (b) 5%
(c) 7.5% (d) 10%

107. In Q. 106, what is the percentage increase in the maximum height attained by the projectile?

(a) 2.5% (b) 5%
(c) 7.5% (d) 10%

108. In Q. 106, what is the percentage increase in the time of flight of the projectile?

(a) 2.5% (b) 5%
(c) 7.5% (d) 10%

109. A projectile attains a certain maximum height when projected from earth. If it is projected at the same angle and with the same initial speed from the moon, where the acceleration due to gravity is one–sixth that on earth, by what factor will the maximum height of the projectile increase?

(a) $\sqrt{3}$ (b) 3
(c) $\sqrt{6}$ (d) 6

110. A projectile has a range R and time of flight T. If the range is doubled (by increasing the speed of projection, without changing the angle of projection), the time of flight will become

(a) $\frac{T}{\sqrt{2}}$ (b) $\sqrt{2}\,T$
(c) $\frac{T}{2}$ (d) $2\,T$

111. A projectile has the same range R when the maximum height attained by it is either h_1 or h_2. Then R, h_1 and h_2 will be related as

(a) $R = \sqrt{h_1 h_2}$ (b) $R = 2\sqrt{h_1 h_2}$
(c) $R = 3\sqrt{h_1 h_2}$ (d) $R = 4\sqrt{h_1 h_2}$

112. A body is projected horizontally from a point above the ground. The motion of the body is described by the equations

$$x = 2t$$

and

$$y = 5t^2$$

where x and y are the horizontal and vertical displacements (in m) respectively at time t. The trajectory of the body is

(a) a straight line (b) a circle
(c) an ellipse (d) a parabola

113. What is the velocity with which the body in Q. 112 is projected?

(a) 2 ms^{-1} (b) 2.5 ms^{-1}
(c) 3.5 ms^{-1} (d) 5 ms^{-1}

114. What is the magnitude of the velocity of the body in Q. 112, 0.2 second after it is projected?

(a) $\sqrt{2}$ ms^{-1} (b) $2\sqrt{2}$ ms^{-1}
(c) $3\sqrt{2}$ ms^{-1} (d) $4\sqrt{2}$ ms^{-1}

115. The velocity of the body in Q. 112 at time $t = 0.2$s is inclined with the vertical at an angle of

(a) 22.5° (b) 30°
(c) 45° (d) 60°

116. A body is projected at time $t = 0$ from a certain point on a planet's surface with a certain velocity at a certain angle with the planet's surface (assumed horizontal). The horizontal and vertical displacements x and y (in metres) respectively vary with time t (in seconds) as

$$x = 10\sqrt{3}\,t$$

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$$y = 10\,t - t^2$$

What is the magnitude and direction of the velocity with which the body is projected?

(a) 20 ms^{-1} at an angle of 30° with the horizontal
(b) 20 ms^{-1} at an angle of 60° with the horizontal
(c) 10 ms^{-1} at an angle of 30° with the horizontal
(d) 10 ms^{-1} at an angle of 60° with the horizontal

117. What is the value of acceleration due to gravity on the planet's surface in Q. 116?

(a) 1.0 ms^{-2} (b) 2 ms^{-2}
(c) 4 ms^{-2} (d) 9.8 ms^{-2}

118. What is the maximum height which the body in Q.116 will attain?

(a) 25 m (b) 50 m
(c) 75 m (d) 100 m

119. What is the total time of flight of the body in Q.116?

(a) 2 s (b) 5 s
(c) 10 s (d) 20 s

120. What is the distance between the point from where the body in Q. 116 is projected and the point where it strikes the planet's surface?

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m
(c) $75\sqrt{3}$ m (d) $100\sqrt{3}$ m

121. Which one of the following statements is NOT true about the motion of a projectile?

(a) The time of flight of a projectile is proportional to the speed with which it is projected.
(b) the horizontal range of a projectile is proportional to the square–root of the speed with which it is projected.
(c) For a given speed of projection, the angle of projection for maximum range is 45°.
(d) At maximum height, the acceleration due to gravity is perpendicular to the velocity of the projectile.

122. A plumb line is hanging from the ceiling of a train. If the train moves along a horizontal track with a uniform acceleration a, the plumb line gets inclined to the vertical at a angle

(a) $\tan^{-1}\left(\frac{a}{g}\right)$ (b) $\tan^{-1}\left(\frac{g}{a}\right)$
(c) $\sin^{-1}\left(\frac{a}{g}\right)$ (d) $\cos^{-1}\left(\frac{g}{a}\right)$

123. A block is placed on the top of a smooth inclined plane of inclination θ kept on the floor of a lift. When the lift is descending with a retardation a, the block is released. The acceleration of the block relative to the incline is

(a) $g \sin\theta$ (b) $a \sin\theta$
(c) $(g - a)\sin\theta$ (d) $(g + a)\sin\theta$

124. A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at angle of 60° with the vertical with the same initial speed. At the highest point, the ratio of their potential energies will be

(a) 4 : 1 (b) 3 : 2
(c) 2 : 3 (d) 2 : 1

125. Displacement (x) of a particle is related to time (t) as

$$x = at + bt^2 - ct^3$$

where a, b and c are constants of motion. The velocity of the particle when its acceleration is zero is given by

(a) $a + \frac{b^2}{c}$ (b) $a + \frac{b^2}{2c}$
(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{4c}$

126. The angle between vectors $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{B} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ is

(a) zero (b) 45°
(c) 90° (d) 180°

127. A vector **A** is along the positive z-axis and its vector product with another vector **B** is zero, then vector **B** could be

(a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ (b) $4\hat{\mathbf{i}}$
(c) $\hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $-7\hat{\mathbf{k}}$

128. **A** is a vector which when added to the resultant of vectors $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ and $(\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ yields a unit vector along the y–axix. Then vector **A** is ?

(a) $-3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ (b) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$
(c) $3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ (d) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

129. The angle between two vectors **A** and **B** is θ. Vector **R** is the resultant of the two vectors. If **R** makes an angle $\frac{\theta}{2}$ with **A**, then

(a) $A = 2B$ (b) $A = \frac{B}{2}$
(c) $A = B$ (d) $AB = 1$

130. A river is flowing from west to east at a speed of 5 m/minute. In what direction should a man on the south bank of the river capable of swimming at 10 m/minute in still water swim to cross the river in the shortest time?

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(a) East (b) West
(c) South (d) North

131. In Q. 130 above, in what direction should the man swim so that he crosses the river by swimming the shortest distance?

(a) North (b) 60° west of north
(c) 30° west of north (d) 30° west of south

132. A body is thrown vertically up with a velocity u. It passes three points A, B and C in its upward journey with velocities $\frac{u}{2}$, $\frac{u}{3}$ and $\frac{u}{4}$ respectively. The ratio $\frac{AB}{BC}$ is

(a) $\frac{20}{7}$ (b) 2
(c) $\frac{10}{7}$ (d) 1

133. A body is thrown vertically up with a velocity u. It passes a point at a height h above the ground at time t_1 while going up and at time t_2 while falling down. Then the relation between u, t_1 and t_2 is

(a) $t_1 + t_2 = \frac{2u}{g}$ (b) $t_2 - t_1 = \frac{2u}{g}$
(c) $t_1 + t_2 = \frac{u}{g}$ (d) $t_2 - t_1 = \frac{u}{g}$

134. In Q. 133 above, the relation between t_1, t_2 and h is

(a) $t_1 t_2 = \frac{2h}{g}$ (b) $t_1 t_2 = \frac{h}{g}$
(c) $(t_1 + t_2)^2 = \frac{2h}{g}$ (d) $(t_1 + t_2)^2 = \frac{h}{g}$

135. A body dropped from a height H above the ground strikes an inclined plane at a height h above the ground. As a result of the impact, the velocity of the body becomes horizontal. The body will take the maximum time to reach the ground if

(a) $h = \frac{H}{4}$ (b) $h = \frac{H}{2\sqrt{2}}$
(c) $h = \frac{H}{2}$ (d) $h = \frac{H}{\sqrt{2}}$

136. A body is dropped from the roof of a multi-storeyed building. It passes the ceiling of the 15th storey at a speed of 20 ms^{-1}. If the height of each storey is 4 m, the number of storeys in the building is (take $g = 10\ ms^{-2}$ and neglect air resistance)

(a) 20 (b) 25
(c) 30 (d) 35

137. Two balls A and B are projected from the same location simultaneously. Ball A is projected vertically upwards and ball B at 30° to the vertical. They reach the ground simultaneously. The velocities of projection of A and B are in the ratio

(a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$
(c) $\sqrt{3} : 2$ (d) $2 : \sqrt{3}$

138. A body is projected with a velocity $\mathbf{v} = (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$ ms^{-1}. The maximum height attained by the body is (take $g = 10\ ms^{-2}$)

(a) 0.8 m (b) 8 m
(c) 80 m (d) 800 m

139. In Q. 138 above, the time of flight of the body is

(a) 0.8 s (b) 1.0 s
(c) 4.0 s (d) 8.0 s

140. A body is projected with a velocity $\mathbf{u}$ at an angle θ with the horizontal. The velocity of the body will become perpendicular to the velocity of projection after a time t given by

(a) $\frac{2u \sin\theta}{g}$ (b) $\frac{u \sin\theta}{g}$
(c) $\frac{2u}{g \sin\theta}$ (d) $\frac{u}{g \sin\theta}$

141. A body is projected at an angle θ with the horozontal. When it is at the highest point, the ratio of the potential and kinetic energies of the body is

(a) $\tan\theta$ (b) $\tan^2\theta$
(c) $\cot\theta$ (d) $\cot^2\theta$

142. The equation of motion of a projectile is $y = ax - bx^2$ where a and b are constants of motion. The horizontal range of the projectile is

(a) $\frac{a}{b}$ (b) $\frac{\sqrt{a}}{2b}$
(c) $\frac{a^2}{2b}$ (d) $\frac{a^2}{4b}$

143. In Q. 142 above, the greatest height attained by the projectile is

(a) $\frac{a}{b}$ (b) $\frac{\sqrt{a}}{2b}$
(c) $\frac{a^2}{2b}$ (d) $\frac{a^2}{4b}$

144. In Q. 142 above, the time of flight of the projectile is

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(a) $a\sqrt{\frac{2}{bg}}$ (b) $\frac{a}{\sqrt{bg}}$

(c) $\sqrt{\frac{2a}{bg}}$ (d) $\sqrt{\frac{a}{2bg}}$

145. A particle moving in a straight line covers half the distance with a speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speeds of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is

(a) 4.0 m/s (b) 5.0 m/s

(c) 5.5 m/s (d) 4.8 m/s

146. A car is travelling at a velocity of 10 km/h on a straight road. The driver of the car throws a parcel with a velocity of $10\sqrt{2}$ km/h when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with the direction of the car,

(a) 135° (b) 45°

(c) $\tan^{-1}\left(\sqrt{2}\right)$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

147. The initial velocity of a particle is $\mathbf{u} = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ ms^{-1}. It is moving with uniform acceleration $\mathbf{a} = (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})$ ms^{-2}. The magnitude of its velocity after 10 s is

(a) 3 ms^{-1} (b) 4 ms^{-1}

(c) 5 ms^{-1} (d) 10 ms^{-1}

148. The horizontal and vertical displacements of a projectile at time 't' are $x = 36t$ and $y = 48t - 4.9t^2$ respectively where x and y are in metre and t is in second. Initial velocity of the projectile in m/s is:

(a) 15 (b) 30

(c) 45 (d) 60

149. The displacement x of a particle moving in one dimension is related to time t by the equation

$$t = \sqrt{x} + 3$$

where x is in metres and t in seconds. The displacement of the particle when its velocity is zero is

(a) zero (b) 4 m

(c) 1 m (d) 0.5 m

150. A particle initially (i.e, at $t = 0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates at a rate

$$a = -k\sqrt{v}$$

where v is the instantaneous velocity and k is a positive constant. The time T taken by the particle to come to rest is given by

(a) $T = \frac{2\sqrt{u}}{k}$ (b) $T = \frac{2u}{k}v$

(c) $T = \frac{2u^{3/2}}{k}$ (d) $T = \frac{2u^2}{k}$

151. A particle starts from rest. Its acceleration at time $t = 0$ is 5 ms^{-2} which varies with time as shown in Fig. 2.33. The maximum speed of the particle will be

(a) 7.5 ms^{-1} (b) 15 ms^{-1}

(c) 30 ms^{-1} (d) 7.5 ms^{-1}

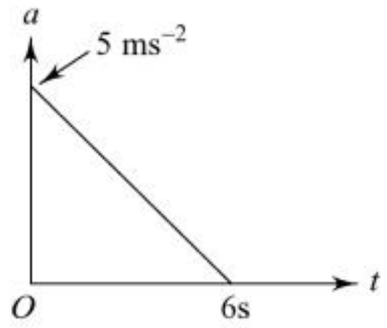

Fig. 2.33

152. Figure 2.34 shows the variation of velocity (v) of a body with position (x) from the origin O. Which of the graphs shown in Fig. 2.35 correctly represents the variation of the acceleration (a) with position (x)?

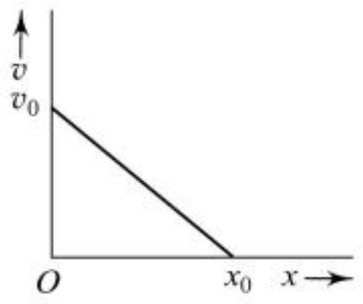

Fig. 2.34

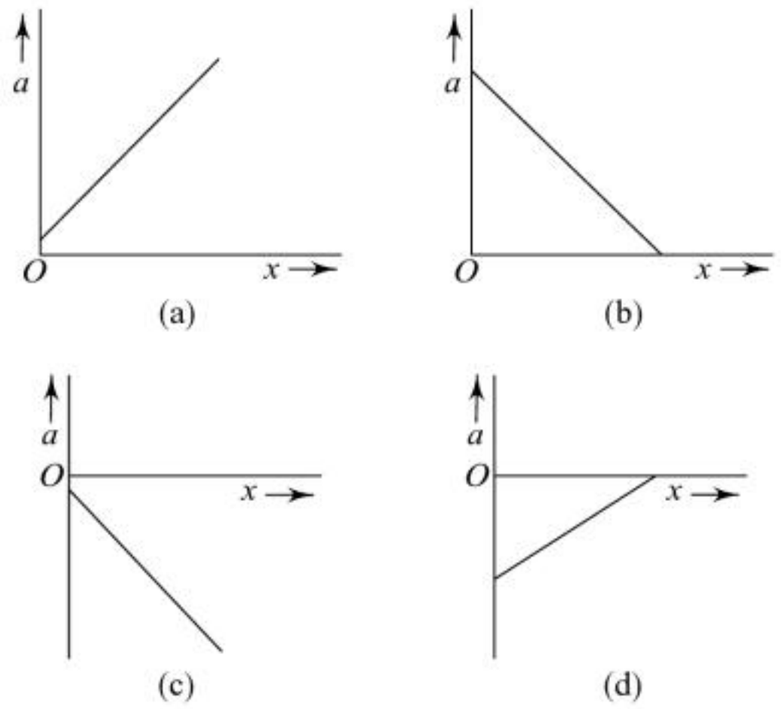

Fig. 2.35

153. The velocity (v) of a body moving along the postive x-direction varies with displacement (x) from the origin as $v = k\sqrt{x}$, where k is a constant. Which of the graphas shown in Fig. 2.36 correctly represents the displacement-time ($x - t$) graph of the motion?

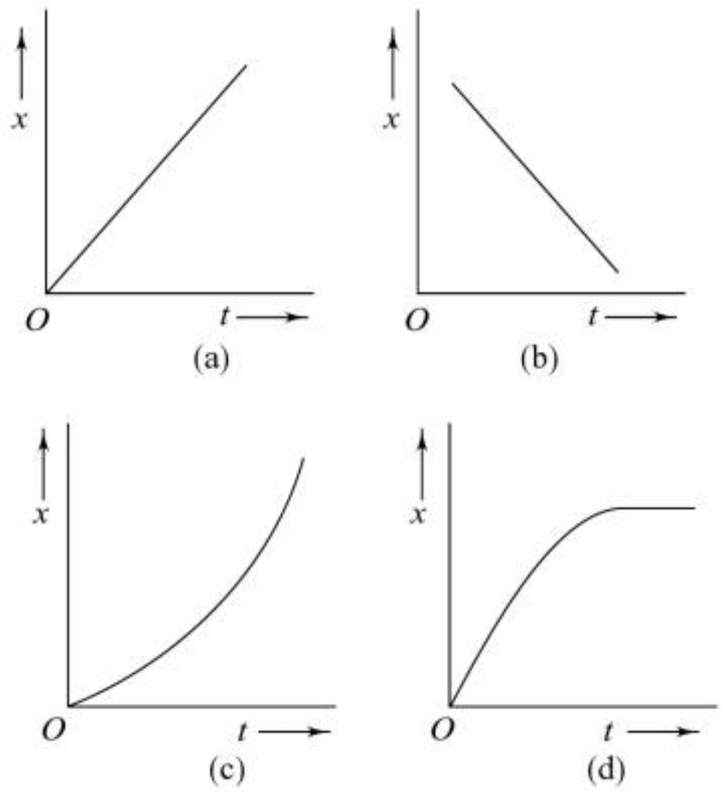

Fig. 2.36

ANSWERS

1. (b) **2.** (c) **3.** (a)
4. (c) **5.** (c) **6.** (b)
7. (b) **8.** (b) **9.** (b)
10. (c) **11.** (d) **12.** (a)
13. (d) **14.** (a) **15.** (a)
16. (a) **17.** (b) **18.** (b)
19. (a) **20.** (c) **21.** (a)
22. (d) **23.** (c) **24.** (a)
25. (b) **26.** (c) **27.** (d)
28. (c) **29.** (b) **30.** (c)
31. (d) **32.** (c) **33.** (b)
34. (c) **35.** (a) **36.** (a)
37. (b) **38.** (d) **39.** (a)
40. (b) **41.** (c) **42.** (a)
43. (b) **44.** (a) **45.** (b)
46. (b) **47.** (a) **48.** (b)
49. (c) **50.** (b) **51.** (c)
52. (b) **53.** (c) **54.** (b)
55. (c) **56.** (d) **57.** (d)
58. (d) **59.** (a) **60.** (d)
61. (b) **62.** (b) **63.** (d)
64. (d) **65.** (c) **66.** (a)
67. (c) **68.** (b) **69.** (d)
70. (d) **71.** (c) **72.** (d)
73. (a) **74.** (c) **75.** (c)
76. (b) **77.** (d) **78.** (b)
79. (c) **80.** (a) **81.** (b)
82. (b) **83.** (a) **84.** (c)
85. (c) **86.** (b) **87.** (d)
88. (a) **89.** (d) **90.** (b)
91. (c) **92.** (d) **93.** (d)
94. (c) **95.** (d) **96.** (a)
97. (d) **98.** (b) **99.** (c)
100. (b) **101.** (b) **102.** (b)
103. (c) **104.** (d) **105.** (d)
106. (d) **107.** (d) **108.** (b)
109. (d) **110.** (b) **111.** (d)
112. (d) **113.** (a) **114.** (b)
115. (c) **116.** (a) **117.** (b)
118. (a) **119.** (c) **120.** (d)
121. (b) **122.** (a) **123.** (d)
124. (a) **125.** (c) **126.** (a)
127. (d) **128.** (a) **129.** (c)
130. (d) **131.** (c) **132.** (a)
133. (a) **134.** (a) **135.** (c)
136. (a) **137.** (c) **138.** (a)
139. (a) **140.** (d) **141.** (b)
142. (a) **143.** (d) **144.** (a)
145. (d) **146.** (a) **147.** (d)
148. (d) **149.** (a) **150.** (a)
151. (b) **152.** (d) **153.** (c)

➔ SOLUTIONS

1. The magnitude R of the resultant vector **R** of two vectors **A** and **B** inclined at an angle θ is given by $R^2 = A^2 + B^2 + 2\,AB\cos\theta$

$$\cos\theta = \frac{R^2 - A^2 - B^2}{2AB} = \frac{(5)^2 - (3)^2 - (4)^2}{2 \times 3 \times 4} = 0$$

$\therefore \quad \theta = \frac{\pi}{2}$. Hence the correct choice is (b).

2. $R^2 = A^2 + B^2 + 2AB\cos\theta$. It is given that $R = A = B$. Putting these values we have

$$A^2 = A^2 + A^2 + 2A^2\cos\theta$$

or $\cos\theta = -\frac{1}{2}$ which gives $\theta = 120°$

Hence the correct choice is (c).

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3. From the triangle law of vector addition, it is clear that the vectors **R**, **A** and **B** in Q.2 must be represented by the three sides of an equilateral triangle because the magnitudes of the three vectors are equal. Hence the angle α between vectors **R** and **A** is 60°. Alternatively, the value of α is given by

$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta} = \frac{\sin\theta}{1+\cos\theta} \quad (\because A = B)$$

$$= \frac{\sin 120°}{1+\cos 120°} = 1.732$$

which gives $\alpha = 60°$. Hence the correct choice is (a).

4. Let θ be the angle between **A** and **B**. Then magnitudes of vectors **P** and **Q** are given by

$$P^2 = A^2 + B^2 + 2AB\cos\theta$$

and $$Q^2 = A^2 + B^2 - 2AB\cos\theta$$

Since $P = Q$, it follows that $\cos\theta = 0$ or $\theta = \pi/2$. Hence correct choice is (c).

5. Given $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = \frac{1}{2}AB$ or $\cos\theta = \frac{1}{2}$. Therefore $\theta = 60°$. Hence correct choice is (c).

6. Given $|\mathbf{A}\times\mathbf{B}| = AB\sin\theta = \frac{1}{2}AB$ or $\sin\theta = \frac{1}{2}$. Therefore $\theta = 30°$. Hence the correct choice is (b).

7. Given $AB\cos\theta = AB\sin\theta$ or $\tan\theta = 1$ which gives $\theta = 45°$. Hence the correct choice is (b).

8. Since $\mathbf{A}\cdot\mathbf{B} = 0$, it follows that **A** is perpendicular to **B**. Also $\mathbf{A}\times\mathbf{C} = 0$. Therefore **A** is perpendicular to **C**. Hence **B** is perpendicular to **C**. Therefore, the correct choice is (b).

9. As shown in Fig. 2.37, the angle θ between vectors **A** and **B** is 90°. Also $A = B$. Therefore, the magnitude of the resultant is given by

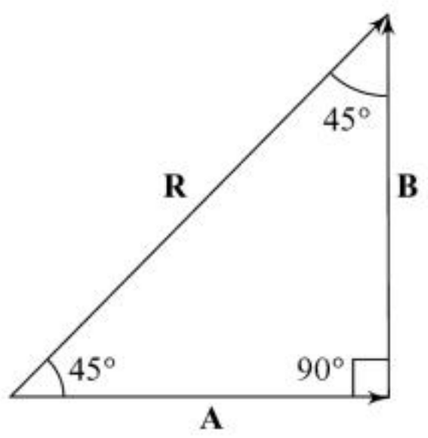

Fig. 2.37

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$
$$= A^2 + A^2 + 2A^2\cos 90°$$
$$= 2A^2$$

or $R = \sqrt{2}A$. Hence the correct choice is (b).

10. Vector **C** lies in the plane containing vectors **A** and **B**, and vector **D** is perpendicular to both **A** and **B**. Hence **D** must be perpendicular to **C**. Hence the correct choice is (c).

11. Since $\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$, $\mathbf{C} = -\mathbf{D}$, *i.e.* vectors **C** and **D** are in opposite directions. Hence the correct choice is (d).

12. Let θ be the angle between the two vectors. The resultant is given by

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

Putting the values of R, A and B we get

$$(1)^2 = (3)^2 + (4)^2 + 2\times 3\times 4\times\cos\theta$$

or $$\cos\theta = -1 \text{ or } \theta = 180°$$

Now $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = 3\times 4\times\cos 180° = -12$
Hence the correct choice is (a).

13. The magnitude of $\mathbf{A}\times\mathbf{B} = AB\sin\theta = 3\times 4\times\sin 180° = 0$. Hence the correct choice is (d).

14. Since $\mathbf{A}+\mathbf{B} = \mathbf{C}$, vector **C** is the resultant of vectors **A** and **B**. Using the triangle law of vector addition (see Fig. 2.38), we have $\theta = 45°$ ($\because A = C$)
Thus, the correct choice is (a).

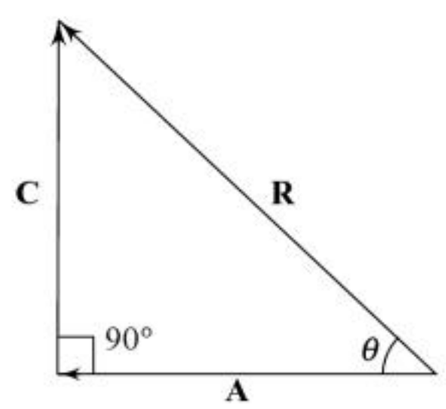

Fig. 2.38

15. The resultant **R** of vectors $(\mathbf{A}+\mathbf{B})$ and $(\mathbf{A}-\mathbf{B})$ is
$\mathbf{R} = (\mathbf{A}+\mathbf{B}) + (\mathbf{A}-\mathbf{B}) = 2\mathbf{A}$
$\therefore$ The magnitude of the resultant $= 2A$. Hence the correct choice is (a)

16. Since $\mathbf{R} = 2\mathbf{A}$, **R** is parallel to **A**. Hence the correct choice is (a).

17. Let $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ represent the magnitudes of vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ respectively. Since $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors, $\hat{\mathbf{i}} = 1$ and $\hat{\mathbf{j}} = 1$. Therefore, the magnitude of vector

$$\hat{\mathbf{i}} + \hat{\mathbf{j}} = \sqrt{\hat{\mathbf{i}}^2 + \hat{\mathbf{j}}^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}.$$

Thus, the correct choice is (b).

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18. The angle subtended by vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ with the x–axis is given by

$$\tan\theta = \frac{\hat{\mathbf{i}}}{\hat{\mathbf{j}}} = \frac{1}{1} = 1$$

or $\theta = 45°$ which is choice (b).

19. $$\cos\theta = \frac{\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right).\hat{\mathbf{i}}}{\left(\hat{\mathbf{i}}^2+\hat{\mathbf{j}}^2+\hat{\mathbf{k}}^2\right)^{1/2}\times\hat{\mathbf{i}}} = \frac{\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}+\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}+\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}}{(1+1+1)^{1/2}\times 1}$$

$$= \frac{1+0+0}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$(\because\ \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0 \text{ and } \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = 1)$$

Hence the correct choice is (a).

20. Here $(0.2)^2 + (0.6)^2 + a^2 = 1$ or $a^2 = 1 - 0.04 - 0.36$, $= 0.6$ or $a = \sqrt{0.6}$. So the correct choice is (c).

21. $\mathbf{A}\cdot\mathbf{B} = \left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}}+\hat{\mathbf{k}}\right) = \hat{\mathbf{i}}\cdot\hat{\mathbf{i}}+\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}+\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}+\hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 1 + 0 + 0 + 0 = 1$

Thus, the correct choice is (a).

22. $\mathbf{A}\times\mathbf{B} = \left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)\times\left(\hat{\mathbf{i}}+\hat{\mathbf{k}}\right) = \hat{\mathbf{i}}\times\hat{\mathbf{i}}+\hat{\mathbf{i}}\times\hat{\mathbf{k}}+\hat{\mathbf{j}}\times\hat{\mathbf{i}}+\hat{\mathbf{j}}\times\hat{\mathbf{k}}$

$= 0 - \hat{\mathbf{j}} - \hat{\mathbf{k}} + \hat{\mathbf{i}}$ which is choice (d).

23. The component of vector $\mathbf{A}$ along vector $\mathbf{B} = (\mathbf{A}\cdot\mathbf{B})\hat{\mathbf{B}}$ where $\hat{\mathbf{B}} = \frac{\mathbf{B}}{B}$ where B is the magnitude of vector $\mathbf{B}$. Now

$$(\mathbf{A}\cdot\mathbf{B}) = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$$

$$= 2\,\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\,\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} + 3\,\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}$$

$$= 2 + 0 + 0 + 3 = 5$$

Also $$\hat{\mathbf{B}} = \frac{\mathbf{B}}{B} = \frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$$

Thus, the answer is $\frac{5}{\sqrt{2}}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$ which is choice (c).

24. Since $\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right) = 0$, vector $\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$ is perpendicular to vector $\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$. Let $\hat{\mathbf{i}}-\hat{\mathbf{j}} = \mathbf{C}$. Now

$$(\mathbf{A}\cdot\mathbf{C}) = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{i}}\right)\cdot\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$$

The required component is

$$(\mathbf{A}\cdot\mathbf{C})\ \frac{\mathbf{C}}{C} = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)\cdot\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{|\hat{\mathbf{i}}-\hat{\mathbf{j}}|}$$

$$\left(\because \text{ Magnitude of } \hat{\mathbf{i}}-\hat{\mathbf{j}} = \sqrt{1+1} = \sqrt{2}\right)$$

$$= -\frac{1}{\sqrt{2}}\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right) = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{j}}-\hat{\mathbf{i}}\right)$$

Thus, the correct choice is (a).

25. Let n seconds be the time taken by the stone to reach the ground. The distance covered in the last second = distance covered in n seconds – distance covered in $(n-1)$ seconds $= \frac{1}{2}gn^2 - \frac{1}{2}g(n-1)^2$. The distance covered in the first 3 seconds $= \frac{1}{2}g\times(3)^2$. It is given that

$$\frac{1}{2}gn^2 - \frac{1}{2}g(n-1)^2 = \frac{1}{2}g\times(3)^2$$

or $$n^2 - (n-1)^2 = 9$$

which gives $n = 5$s. Hence the correct choice is (b).

26. Since $a = \frac{dv}{dt}$, we have $v = \int_0^t a\,dt = \int_0^t (3t+4)\,dt$

$$= \frac{3t^2}{2} + 4t. \text{ At } t = 2\text{s},$$

$$v = \frac{3}{2}\times(2)^2 + 4\times 2 = 14\ \text{ms}^{-1}.$$

Hence the correct choice is (c).

27. For a given initial speed, the maximum height attained and the downward speed at the point of projection are independent of the masses of balls. Hence the correct choice is (d).

28. Given $y = -\frac{2}{3}t^2 + 16t + 2$. Comparing it with $s = ut + \frac{1}{2}at^2$, we have $u = 16\ \text{ms}^{-1}$ and $\frac{a}{2} = -\frac{2}{3}$ or $a = -\frac{4}{3}\ \text{ms}^{-2}$. For a body to come to rest, the final velocity $v = 0$. Using these values of u, v and a in relation

$$v = u + at$$

we have $$0 = 16 - \frac{4}{3}t$$

or $$t = 12\ \text{s}$$

Hence the correct choice is (c).

29. Given $u = 30\ \text{ms}^{-1}$. From Fig. 2.34, the acceleration a = slope of line $AB = \frac{30\ \text{ms}^{-1}}{-3s} = -10\,\text{ms}^{-2}$. The maximum height is reached when final velocity $v = 0$. Using the values of u, v and a in relation $v^2 - u^2 = 2as$, we have

$$0 - 30\times 30 = 2\times -10\times s$$

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or $$s = \frac{30 \times 30}{20} = 45 \text{ m}$$

Hence the correct choice is (b).

30. The two arrows will be at the same height at $t = n$ if $98 - 4.9n^2 = 98\,(n-5) - 4.9\,(n-5)^2$. This gives $n = 12.5$ s. The time to reach the highest point is given by $0 = 98 - 9.8\,t$ i.e., $t = 10$ s. therefore, time of flight is 2×10 s or 20 s. The speed of the first arrow at $t =$ 20 s is 98 ms^{-1} again while that of the second arrow is $(98 - 9.8 \times 0.5)$ ms^{-1} *i.e.* 49 ms^{-1}. The maximum heights attained are 490 m each.

Hence the correct choice is (c)

31. The area under the velocity–time graph always gives the distance covered as it is given by the integral $\int_{t_1}^{t_2} v\,dt$ which is true irrespective of whether the body has a uniform velocity or a uniform or non-uniform acceleration. Hence the correct choice is (d).

32. Velocity of the particle is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{\frac{a}{b}\left(1 - e^{-bt}\right)\right\} = ae^{-bt}$$

Acceleration of the particle is given by

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}\left(ae^{-bt}\right) = -abe^{-bt}$$

At $t = 1/b$, the displacement of the particle is

$$x = \frac{a}{b}\left(1 - e^{-1}\right) \simeq \frac{a}{b}\left(1 - \frac{1}{3}\right)$$

$$\simeq \frac{2}{3}\frac{a}{b} \qquad \left(\because\ e^{-1} \simeq \frac{1}{3}\right)$$

Hence choice (a) is wrong. At $t = 0$, the values v and α respectively are $v = ae^{-0} = a$ and $\alpha = -abe^{-0}$ $= -ab$. Hence choice (b) is also wrong. The displacement x is maximum when $t \to \infty$, *i.e.*, $x_{\max} = \frac{a}{b}\left(1 - e^{-\infty}\right) = \frac{a}{b}$. Hence choice (c) is correct.

33. The acceleration is given by the slope of the speed—time graph. The slope of the graph is maximum during the interval *BC*. Hence the correct choice is (b).

34. The velocity of the parachutist at the end of 10 seconds = 10 $g = 10 \times 10 = 100$ ms^{-1} and the distance fallen in 10 seconds = $v^2/2g = 100 \times 100/2 \times 10 = 500$ m. The distance travelled after he bails out is $s = 2495 - 500 = 1995$ m. For this distance $u = 100$ ms^{-1} and $a = -2.5$ ms^{-2}. Therefore, the final velocity v is given by

$$v^2 - u^2 = 2as$$

or $$v^2 = u^2 + 2as$$

$$= (100)^2 - 2 \times 2.5 \times 1995$$

which gives $v = 5$ ms^{-1}. Hence, the correct choice is (c).

35. Let t_1 be the time during which the car accelerates at a rate α. The velocity at the end of time t_1 will be

$$v \text{ at } t_1 = u + \alpha t_1 = 0 + \alpha t_1 = \alpha t_1 \qquad (\because\ u = 0)$$

The time during which the car decelerates is $t_2 = t - t_1$. For this time t_2, the initial velocity is αt_1 and the final velocity is zero and the acceleration is $-\beta$. Therefore

$$0 = \alpha t_1 - \beta\,(t - t_1)$$

which gives $t_1 = \frac{\beta t}{\alpha + \beta}$. Therefore, maximum velocity = v at $t_1 = \alpha t_1 = \frac{\alpha\beta\, t}{\alpha + \beta}$.

Hence the correct choice is (a).

36. The distance travelled in time t_1 is

$$s_1 = ut_1 + \frac{1}{2}\,\alpha t_1^2 = \frac{1}{2}\,\alpha t_1^2 \qquad (\because\ u = 0)$$

$$= \frac{1}{2}\alpha\left(\frac{\beta}{\alpha + \beta}\right)^2 t^2 \qquad \left(\because\ t_1 = \frac{\beta t}{\alpha + \beta}\right)$$

Also, the distance travelled in time t_2 is given by

$$-2\beta s_2 = 0^2 - \left(\frac{\alpha\beta}{\alpha + \beta}\right)^2 t^2$$

or $$s_2 = \frac{1}{2}\beta\left(\frac{\alpha}{\alpha + \beta}\right)^2 t^2$$

Therefore, $$s_1 + s_2 = \frac{1}{2}\alpha\left(\frac{\beta}{\alpha + \beta}\right)^2 t^2$$

$$+\frac{1}{2}\beta\left(\frac{\alpha}{\alpha + \beta}\right)^2 t^2$$

$$= \frac{1}{2}\frac{\alpha\beta}{\alpha + \beta}t^2$$

Hence the correct choice is (a).

37. Since $a = \frac{dv}{dt}$, we have $v = \int a\,dt = \int (kt + c)\,dt$ $= \frac{1}{2}\,kt^2 + ct$. Hence the correct choice is (b).

38. The velocity of the particle at time t is given by

$$v = \int a\,dt$$

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$$= \int \left(-s\omega^2 \cos \omega t\right) dt$$

$$= -\, s\omega \sin \omega t$$

Now, the displacement is given by

$$x = \int v\, dt = -\, s \int \omega \sin \omega t\, dt$$

$$= s \cos \omega t$$

Thus, the correct choice is (d).

39. We have $\frac{h}{2} = \frac{1}{2}\, g\,(2n-1)$ so that $n = \frac{1}{2}\left(\frac{h}{g}+1\right)$.

Hence h is given by

$$h = \frac{1}{2}\, gn^2 = \frac{1}{2}\, g \left\{\frac{1}{4}\left(\frac{h}{g}+1\right)^2\right\}$$

$$= \frac{1}{8}\, g \left(\frac{h^2}{g^2}+\frac{2h}{g}+1\right)$$

$$= \frac{1}{8}\left(\frac{h^2}{g}+2h+g\right)$$

Simplifying and putting $g = 10\ \text{ms}^{-2}$, we get

$$h^2 - 60h + 100 = 0$$

The positive root of this quadratic gives $h \simeq 58$ m. Hence the correct choice is (a).

40. When the parachutist bails out, he shares the velocity of the balloon and has an upward velocity of $10\ \text{ms}^{-1}$, *i.e.* $u = +10\ \text{ms}^{-1}$. Also $g = -10\ \text{ms}^{-2}$ (acting downwards). The displacement in $t = 3$s is given by

$$s = ut + \frac{1}{2}\, gt^2$$

$$= 10 \times 3 + \frac{1}{2} \times (-10) \times (3)^2$$

$$= -15\ \text{m}$$

Since the displacement is negative, it is directed downwards. So the height from the ground when he opened his parachute $= 45 - 15 = 30$ m. Thus, the correct choice is (b).

41. In time 3s, the balloon has risen through 30m (as the velocity of the balloon is $10\ \text{ms}^{-1}$ upwards). Hence the parachutist is now $30 + 15 = 45$ m away from the balloon. Thus, the correct choice is (c).

42. The velocity of the parachutist 3s after he bails out is

$$v = u + gt$$

$$= 10 + (-10) \times 3 = -20\ \text{ms}^{-1}$$

(directed downwards)

At $t = 3$s, his initial velocity is $u = -20\ \text{ms}^{-1}$ and to hit the ground, his displacement $s = -30$ m (see solution of Q. 40). Now $a = +5\ \text{ms}^{-2}$ (directed upwards). The time taken to hit the ground is given by

$$s = ut + \frac{1}{2}\, gt^2$$

or $$-30 = -20\, t + \frac{5}{2}\, t^2$$

or $$t^2 - 8t + 12 = 0$$

or $$(t-6)\,(t-2) = 0$$

which gives $t = 6$ s or 2 s. If $t = 6$ s, then the velocity with which he hits the ground is $v = u + at = -20 + 5 \times 6 = 10\ \text{ms}^{-1}$. This is positive, *i.e.* v is directed upwards, which is not possible. Thus the correct answer is $t = 2$ s, in which case, the velocity with which he hits the ground is

$$v = -20 + 5 \times 2 = -10\ \text{ms}^{-1}$$

which is negative as it should be. Thus, the correct choice is (a).

43. The total time the parachutist takes (after his exit from the balloon) to hit the ground is $= 3\text{ s} + 2\text{ s} = 5$ s which is choice (b).

44. Let us suppose that cars A and B are moving in the positive x–direction. Then car C is moving in the negative x–direction. Therefore, $v_A = +36\ \text{km h}^{-1} = +10\ \text{ms}^{-1}$, $v_B = +54\ \text{km h}^{-1} = +15\ \text{ms}^{-1}$ and $v_C = -54\ \text{km h}^{-1} = -15\ \text{ms}^{-1}$. The relative velocity B with respect to A is $v_{BA} = v_B - v_A = 15 - 10 = 5\ \text{ms}^{-1}$. The relative velocity of C with respect to A is $v_{CA} = v_C - v_A = -15 - 10 = -25\ \text{ms}^{-1}$. At time $t = 0$, the distance between A and B = distance between A and $C = 1$ km $= 1000$ m. The car C will cover a distance $AC = 1000$ m and just reach car A at a time t given by

$$t = \frac{AC}{|v_{CA}|} = \frac{1000\ \text{m}}{25\ \text{ms}^{-1}} = 40\ s$$

Car B will overtake car A just before car C does and avoid an accident, if it acquires a minimum acceleration a such that it covers a distance $s = AB = 1000$ m in time $t = 40$ s, travelling at a relative speed $u = v_{BA} = 5\ \text{ms}^{-1}$. Putting these values in relation

$$s = ut + \frac{1}{2}\, at^2$$

We get $$1000 = 5 \times 40 + \frac{1}{2} \times a \times (40)^2$$

which gives $a = 1\ \text{ms}^{-2}$ which in choice (a).

45. The relative speed of train A with respect to train $B = 30 + 10 = 40\ \text{ms}^{-1}$. The minimum distance now is given by

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$$0 - (40)^2 = 2 \times 4 \times s$$

which gives $s = 200$ m which is choice (b).

46. Average velocity $= \dfrac{\text{net displacement}}{\text{total time}}$

The displacement of the cyclist in moving from P to Q along the circumference = shortest distance between P and Q = straight line distance PQ (shown by broken lines in the figure) in the direction from P to Q. The net displacement in moving from O to P and then from P to Q = resultant of vector displacement OP and PQ. From the triangle law of vector addition, the magnitude of the net displacement = $OQ = 1$ km. Now, the time taken = 10 min = $\dfrac{1}{6}$ h. Therefore, average velocity is 1 km/ $\dfrac{1}{6}$ h = 6 km h^{-1}, which is choice (b).

47. Average speed $= \dfrac{\text{total distance}}{\text{total time}}$.

Total distance = O to P (along radius) + P to Q (along the arc)

$$= r + \frac{r\pi}{3} \qquad \left(\because 60° = \frac{\pi}{3} \text{ rad}\right)$$

$$= 1 \text{ km} + 1 \text{ km} \times \frac{3.14}{3} = 2.05 \text{ km}$$

$\therefore$ Average speed $= \dfrac{2.05 \text{ km}}{\frac{1}{6} \text{h}} \simeq 12.3$ km h^{-1} which is choice (a).

48. Velocity of rain $(v_r) = 4$ ms^{-1} vertically downwards. Velocity of wind $(v_w) = 3$ ms^{-1} from north to south direction. A rain drop is acted upon by two velocities v_r and v_w as shown in Fig. 2.43. From the triangle law, the resultant velocity of the rain drop is $v = OW$. In order to protect himself from rain, he must hold his umbrella at an angle θ with the vertical (towards north) given by

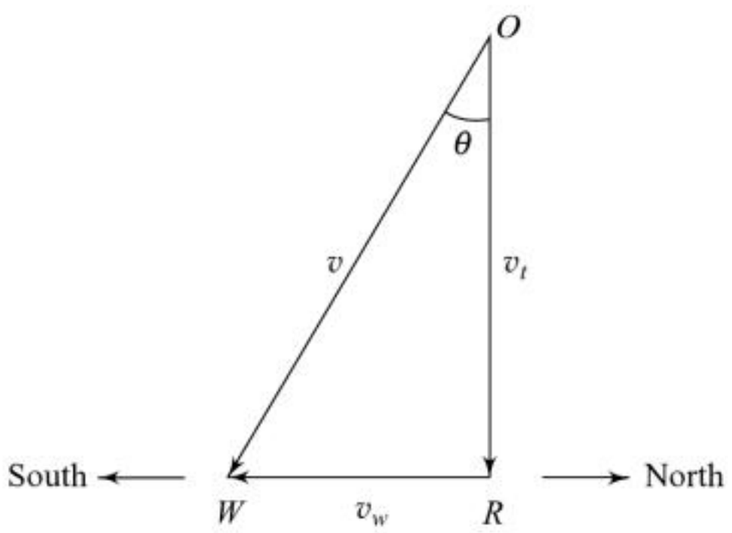

Fig. 2.39

$$\tan \theta = \frac{RW}{OR} = \frac{v_w}{v_r} = \frac{3}{4}$$

Thus, the correct choice is (b).

49. The magnitude v of the resultant velocity gives the speed with which the rain strikes the umbrella, which is given by

$$v = [v_r^2 + v_w^2]^{1/2} = [16 + 9]^{1/2} = 5 \text{ ms}^{-1}$$

Hence the correct choice is (c).

50. In order to cross the river in the shortest time, the resultant velocity v of the swimmer must be perpendicular to the velocity v_w of water, as shown in Fig. 2.40. It follows from the figure that $v_s^2 = v^2 + v_w^2$ or $v_w^2 = v_s^2 - v^2 = 25 - 9 = 16$ or $v_w = 4$ ms^{-1} which is choice (b).

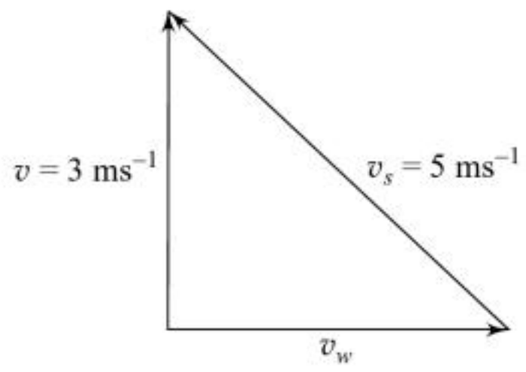

Fig. 2.40

51. Since the initial velocity is zero, the distance travelled in the first time interval t is

$$s_1 = 0 + \frac{1}{2} at^2 = \frac{1}{2} at^2$$

The velocity of the body at the end of this time interval is $v = 0 + at = at$. This is the initial velocity for the next time interval t during which the body travels a distance.

$$s_2 = ut + \frac{1}{2} at^2 = at^2 + \frac{1}{2} at^2 = \frac{3}{2} at^2 \quad (\because u = at)$$

$\therefore s_2 = 3 s_1$. Thus the correct choice is (c).

52. Here $v_1 = 0 + at = at$ and $v_2 = v_1 + at = at + at = 2at$. Therefore, $v_2 = 2v_1$. Hence the correct choice is (b).

53. Let h be the height of the tower. Then $h = \dfrac{1}{2} gt^2 = \dfrac{1}{2} g(4)^2 = 8g$.

The time t taken to fall through $\dfrac{h}{2} = 4g$ is given by

$$\frac{h}{2} = \frac{1}{2} gt^2 \text{ or } 4g = \frac{1}{2} gt^2 \text{ or } t^2 = 8 \text{ or } t = 2\sqrt{2} \text{ s.}$$

Thus, the correct choice is (c).

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54. The distance covered in the nth second is given by

$$s_n = u + a\left(n - \frac{1}{2}\right)$$

$$\therefore \quad s_4 = u + a\left(4 - \frac{1}{2}\right) = u + \frac{7}{2}a$$

or $$40 = u + \frac{7}{2} \times 10 \text{ or } u = 40 - 35$$
$$= 5 \text{ ms}^{-1}$$

$$\therefore \quad s_6 = 5 + 10 \times \left(6 - \frac{1}{2}\right)$$

which gives $s_6 = 60$ m.

Thus, the correct choice is (b).

55. Now, $s_n = u + a\left(n - \frac{1}{2}\right)$. Therefore,

$$30 = 5 + a\left(3 - \frac{1}{2}\right) \text{ which gives } a = 10 \text{ ms}^{-2}.$$

$$\therefore \quad s_4 = 5 + 10\left(4 - \frac{1}{2}\right) = 40 \text{ m}$$

and $$s_5 = 5 + 10\left(5 - \frac{1}{2}\right) = 50 \text{ m}$$

$\therefore s = s_4 + s_5 = 40 + 50 = 90$ m. Thus the correct choice is (c).

56. Given $$40 = u + \frac{7a}{2}$$

and $$60 = u + \frac{11a}{2}$$

These equations give $u = 5$ ms^{-1} and $a = 10$ ms^{-2}. Thus, the correct choice is (d).

57. Since $u = 0$, we have

$$s_1 = 0 + \frac{7a}{2} = \frac{7a}{2}$$

and $$s_2 = 0 + \frac{11a}{2} = \frac{11a}{2}$$

$$\therefore \quad s_1/s_2 = 7/11, \text{ which is choice (d).}$$

58. Since the initial velocity of the car is zero, its velocity at the end of the first time interval t_1 is $v = 0 + a_1t_1 = a_1t_1$. This is the initial velocity for the next time interval t_2. Since the final velocity is zero, we have, from $v = u + at$,

$$0 = a_1 t_1 - a_2 t_2 \qquad (\because u = a_1 t_1)$$

Now, the distance covered in the first time interval t_1 is given by

$$2a_1 s_1 = v^2 - u^2 = a_1^2 t_1^2$$

$$(\because v = a_1 t_1 \text{ and } u = 0)$$

or $$s_1 = \frac{1}{2} a_1 t_1^2 \qquad \text{(i)}$$

The distance covered in the next time interval t_2 is given by

$$-2a_2 s_2 = 0 - a_1^2 t_1^2$$

$$(\because v = 0 \text{ and } u = a_1 t_1 \text{ now})$$

or $$s_2 = \frac{1}{2} \frac{a_1^2}{a_2} t_1^2 = \frac{1}{2} \frac{a_2^2 t_2^2}{a_2}$$

$$(\because a_1 t_1 = a_2 t_2)$$

or $$s_2 = \frac{1}{2} a_2 t_2^2 \qquad \text{(ii)}$$

From (i) and (ii) we get

$$\frac{s_1}{s_2} = \frac{a_1 t_1^2}{a_2 t_2^2} = \frac{a_2}{a_1} \frac{a_1^2 t_1^2}{a_2^2 t_2^2} = \frac{a_2}{a_1} \qquad (\because a_1 t_1 = a_2 t_2)$$

Thus we have $\frac{s_2}{s_1} = \frac{a_1}{a_2} = \frac{t_2}{t_1}$ which is choice (d).

59. The maximum speed v attained by the car = speed it attains at the end of time interval t_1 during which it is accelerated. As shown above, this speed is $v = a_1t_1 = a_2t_2$.

Now $$s_1 = \frac{1}{2} a_1 t_1^2 = \frac{v^2}{2a_1} \qquad (\because v = a_1 t_1)$$

and $$s_2 = \frac{1}{2} a_2 t_2^2 = \frac{v^2}{2a_2} \qquad (\because v = a_2 t_2)$$

$$\therefore \quad s = s_1 + s_2 = \frac{v^2}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$$

or $$v = \left[2s. \frac{a_1 a_2}{a_1 + a_2}\right]^{1/2}$$

Hence the correct choice is (a).

60. The distance s_1 covered by the car during the time it is accelerated is given by $2\alpha s_1 = v^2$, which gives $s_1 = v^2/2\alpha$. The distance s_2 covered during the time the car is decelerated is, similarly given by $s_2 = v^2/2\beta$. Therefore, the total distance covered is

$$s = s_1 + s_2 = \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad \text{(i)}$$

If t_1 is the time of acceleration and t_2 that of deceleration, then $v = \alpha t_1 = \beta t_2$ or $t_1 = v/\alpha$ and $t_2 = v/\beta$. Therefore, the total time taken is

$$t = t_1 + t_2 = v\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad \text{(ii)}$$

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From (i) and (ii), the average speed of the car is given by

$$\frac{\text{total distance}}{\text{total time}} = \frac{s}{t} = \frac{v}{2}$$

Hence the correct choice is (d).

61. If the train is at rest or in uniform motion, its acceleration is zero. The only acceleration acting on the bob is the acceleration due to gravity. Hence the string will be vertical. Hence choice (a) and (d) are not possible. The string is inclined along the direction of the resultant of the acceleration **a** of train acting horizontally and the acceleration due to gravity **g** acting vertically. In Fig. 2.41, vector $\mathbf{a}_r$ is the resultant of vectors **a** and **g** (use triangle law). Hence, the string is inclined opposite to the direction of the acceleration of the train. Thus the correct choice is (b).

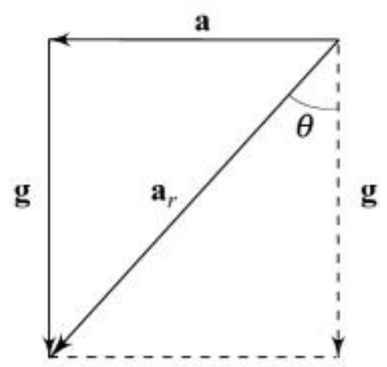

Fig. 2.41

62. Squaring both sides, we have

$$x = 12t + 4t^2 + 9$$

Since displacement x changes with time t, the body cannot be at rest. The velocity of the body is given by

$$v = \frac{dx}{dt} = 12 + 8t$$

Since the velocity v changes with time t, the body is not in uniform motion; it is accelerated because v increases with t. Hence the correct choice is (b).

63. We have seen that $v = 12 + 8t$. Comparing it with $v = u + at$ we find that $u = 12\ \text{ms}^{-1}$. Hence the correct choice is (d).

64. Now $v = 12 + 8t$. Comparing it with $v = u + at$, we find that $a = 8\ \text{ms}^{-2}$. Alternatively, acceleration a is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(12 + 8t) = 8$$

or $a = 8\ \text{ms}^{-2}$. Hence the correct choice is (d).

65. The distance over which the car can be stopped is given by $2as = v^2$ or $a = v^2/2s$. If v becomes nv, the value a^* of a to stop the car in the same distance is $a^* = (nv)^2/2s = n^2v^2/2s$. Thus $a^* = n^2a$. Hence the correct choice is (c).

66. Let a be the maximum retardation the brakes of the car can produce. Then $2ax = v^2$. If v is doubled to $2v$, then the minimum distance x^* is given by $2ax^* = (2v)^2 = 4v^2$. Thus $x^* = 4x$. Hence the correct choice is (a).

67. The total time taken by the bullet to reach the highest point (where its velocity becomes zero) is given by $0 = u - gt$ or $t = u/g = 50/10 = 5$ s. The distance it covers in the first 1 second of its upward motion is $h_1 = ut - \frac{1}{2}gt^2 = 50 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 50 - 5 = 45$ m.

Now, the velocity of the bullet at the end of first 2 seconds is $v = u - gt = 50 - 10 \times 2 = 30\ \text{ms}^{-1}$. This is the initial velocity for the last 3 seconds of upward motion. The distance covered in the last 3 seconds is $h_2 = 30 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = 90 - 45 = 45$ m. Thus $h_1 = h_2$, which is choice (c).

68. From $v = u + gt$, we have $3u = u + gt$ or $2u = gt$ or $t = 2u/g$. Hence the correct choice is (b).

69. From $2gh = v^2 - u^2$, we have $2gh = (3u)^2 - u^2 = 8u^2$ or $h = 4u^2/g$. Hence the correct choice is (d).

70. Differentiating $2x^2 + 3x = t$ with respect to t we have

$$4x\frac{dx}{dt} + \frac{3dx}{dt} = 1 \qquad \text{(i)}$$

Now $\frac{dx}{dt} = v$. Therefore, $4xv + 3v = 1$ or $4x + 3 = 1/v$. Differentiating Eq. (i) with respect to time t, we have

$$4\left(\frac{dx}{dt}\right)^2 + 4x\frac{d^2x}{dt^2} + 3\frac{d^2x}{dt^2} = 0$$

or $$4v^2 + 4xa + 3a = 0$$

or $$a = -\frac{4v^2}{4x + 3} \qquad \text{(ii)}$$

where $a = \frac{d^2x}{dt^2}$ is the acceleration. But $4x + 3 = 1/v$. Using this in Eq. (ii) we get $a = -4v^3$. Hence the correct choice is (d).

71. Given, $x^2 = t^2 + 2t + 3$. Differentiating, we have

$$2x\frac{dx}{dt} = 2t + 2 \quad \text{or} \quad x\frac{dx}{dt} = t + 1 \qquad \text{(i)}$$

Differentiating again, we have

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$$\left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2} = 1.$$

or $$x^2\left(\frac{dx}{dt}\right)^2 + x^3 a = x^2 \qquad \text{(ii)}$$

Where $a = \dfrac{d^2x}{dt^2}$ is the acceleration. Using (i) in (ii) we have

$$(t+1)^2 + x^3 a = x^2$$

or $$t^2 + 2t + 1 + x^3 a = x^2 \qquad \text{(iii)}$$

But it is given that $x^2 = t^2 + 2t + 3$. Using this in Eq. (iii) we get $a = 2/x^3$. Hence the correct choice is (c).

72. Given $\dfrac{dV(t)}{dt} = 6.0 - 3V(t)$ (i)

Since at $t = 0$, $V(0) = 0$ (given), the acceleration at $t = 0$ is

$$a(0) = \left.\frac{dV(t)}{dt}\right|_{t=0} = 6.0 - 0 = 6.0\ \text{ms}^{-2}$$

Thus, choice (b) is correct. To check whether choice (c) is correct, differentiating $V(t)$, given in choice (c), with respect to t, we get

$$\frac{dV(t)}{dt} = \frac{d}{dt}\left[2\left(1-e^{-3t}\right)\right]$$

$$= 2\frac{d}{dt}\left(1 - e^{-3t}\right) = 6e^{-3t}$$

$$= 6 - 6\left(1 - e^{-3t}\right) = 6 - 3V(t)$$

$$(\because\ V(t) = 2\left(1 - e^{-3t}\right)]$$

which is Eq. (i). Hence choice (c) is also correct. Now let us check if choice (a) is correct or not. Terminal speed is the speed at time $t \to \infty$. Putting $t \to \infty$ in

$$V(t) = 2\left(1 - e^{-3t}\right) \qquad \text{(ii)}$$

We have

$$V(\infty) = 2\left(1 - e^{-\infty}\right) = 2(1 - 0) = 2\ \text{ms}^{-1}$$

Hence, choice (a) is correct. Finally, let us check choice (d). The acceleration will be half the initial value, *i.e.*, it will be equal to $6.0/2 = 3.0\ \text{ms}^{-1}$ at time t^* given by [use Eq. (i)]

$$3.0 = 6.0 - 3V(t^*)$$

or $$V(t^*) = 1$$

or $$2(1 - e^{-3t^*}) = 1$$

or $$e^{-3t^*} = 0.5$$

or $$-3t^* \ln e^{-1} = \ln(0.5)$$

or $$-3t^* = -0.693$$

or $$t^* = 0.231\ \text{s}$$

Putting this value of t in Eq. (ii) we have

$$V(\text{at } t = 0.231\ \text{s}) = 2\left(1 - e^{-3\times 0.231}\right)$$
$$= 2(1 - e^{-0.693}) = 2(1 - 0.5)$$
$$= 1.0\ \text{ms}^{-1}$$

so choice (d) is wrong.

73. Let the total distance be $2s$ and let t_1 and t_2 be the times taken to traverse the first half and the second half of the distance respectively. Then $t_1 = s/V$. For the second half of the distance, the distance s_1 is covered in time $\dfrac{t_2}{2}$ with speed V' is $s_1 = V'\dfrac{t_2}{2}$ and the distance s_2 is covered in time $\dfrac{t_2}{2}$ with speed V' is $s_2 = V''\dfrac{t_2}{2}$, so that $s = s_1 + s_2 = V'\dfrac{t_2}{2} + V''\dfrac{t_2}{2} = (V' + V'')\ \dfrac{t_2}{2}$, which gives

$$t_2 = \frac{2s}{(V' + V'')}$$

$\therefore$ Total time taken $= t_1 + t_2$

$$= \frac{s}{V} + \frac{2s}{V' + V''} = \frac{s(2V + V' + V'')}{V(V' + V'')}$$

Hence average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$$= \frac{2s \times V(V' + V'')}{s(2V + V' + V'')} = \frac{2V(V' + V'')}{(2V + V' + V'')}$$

Thus, the correct choice is (a).

74. The shortest stopping distance S is given by

$$0 - v^2 = -2aS$$

or $S = \dfrac{v^2}{2a}$. Thus, for a given value of a, $S \propto v^2$. If v is increased by a factor of 3, S will increase by a factor $(3)^2 = 9$. Hence the correct choice is (c).

75. The maximum height ($u^2 \sin^2\theta/2g$) and the horizontal range ($= u^2 \sin 2\theta/g$) are equal if $\sin^2\theta/2 = \sin 2\theta = 2\sin\theta\cos\theta$ or $\tan\theta = 4$ or $\theta = \tan^{-1}(4)$. Hence the correct choice is (c).

76. The bullet covers a distance of 100 m in time $t = 100/500 = 0.2$ s. The vertical downward distance moved in time $t = 0.2$ s is $\dfrac{1}{2}gt^2 = \dfrac{1}{2} \times 10 \times (0.2)^2$ $= 0.2\ \text{m} = 20\ \text{cm}$. Hence the correct choice is (b).

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77. At the highest point of the trajectory, the velocity of the projectile is parallel to the ground but its acceleration is directed vertically downwards. Hence the correct choice is (d).

78. The time taken to reach the ground is independent of the horizontal velocity of projection. Since the initial downward velocity is zero (the initial velocity has only a horizontal component), the time of flight is given by

$$h = \frac{1}{2} gt^2$$

which gives $$t = \sqrt{\frac{2h}{g}} = \left(\frac{2 \times 19.6}{9.8}\right)^{1/2} = 2 \text{ s}$$

Hence the correct choice is (b).

79. The time taken by the bomb to hit the ground is given by (as shown above)

$$t = \sqrt{\frac{2h}{g}}$$

Now, the horizontal speed of the aeroplane (and also of the bomb) is v.

$\therefore$ Horizontal range = horizontal velocity $\times$ t

$$= vt = v\sqrt{\frac{2h}{g}}$$

Hence the correct choice is (c).

80. Let G be the position of the gun and E that of the enemy plane flying horizontally with speed $u = 300$ ms^{-1}, when the shell is fired with a speed $v_0 = 600$ ms^{-1} in a direction θ with the horizontal (Fig. 2.42). The horizontal component of v_0 is

$$v_x = v_0 \cos \theta$$

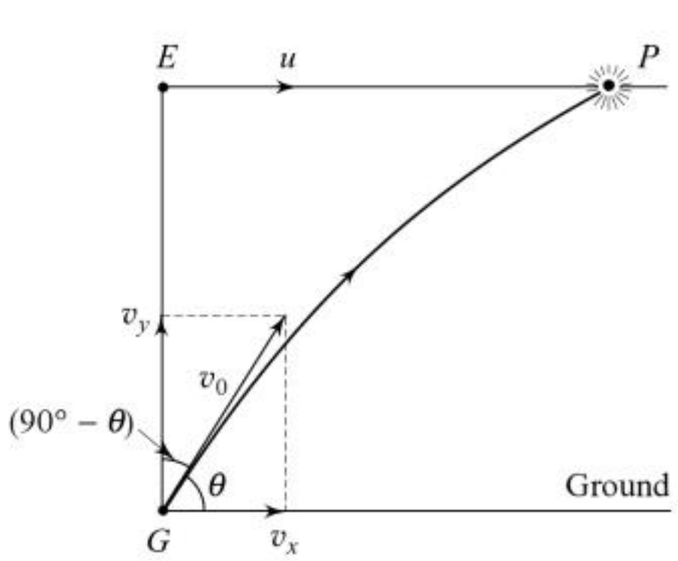

Fig. 2.42

Let the shell hit the plane at point P and let t be the time taken for the shell to hit the plane. It is clear that the shell will hit the plane, if the horizontal distance EP travelled by the plane in time t = the distance travelled by the shell in the horizontal direction in the same time, *i.e.*, $u \times t = v_x \times t$ or $u = v_x = v_0 \cos \theta$ or $\cos \theta = \frac{u}{v_0} = \frac{300}{600} = 0.5$ or $\theta = 60°$.

Therefore angle with the vertical = $90° - \theta = 30°$. Hence the correct choice is (a).

81. To avoid being hit, the plane should have a minimum altitude = maximum height attained by the shell which is

$$h_{\max} = \frac{v_0^2 \sin^2 \theta}{2g} = \frac{(600)^2 \times \sin^2 (60°)}{2 \times 10}$$

$$= 13.5 \times 10^3 \text{ m} = 13.5 \text{ km}.$$

which is choice (b).

82. Given $h_{\max} = 5$ m, $u = 20$ ms^{-1} and $g = 10$ ms^{-2}. We know that

$$h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

or $$\sin \theta = \frac{1}{u} \times \sqrt{2g h_{\max}} = \frac{1}{20} \times \sqrt{2 \times 10 \times 5} = 0.5$$

which gives $\theta = 30°$. Therefore

$$R_{\max} = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 60°}{10} = 20\sqrt{3} \text{ m}$$

Hence the correct choice is (b).

83. The time of flight $t_f = \frac{2u \sin \theta}{g} = \frac{2 \times 20 \times 0.5}{10} =$ 2 s. The initial downward velocity = 20 sin 30° = 10 ms^{-1}. The time taken to fall through a height of 40 m with velocity 10 ms^{-1} is given by

$$40 = 10 \times t + \frac{1}{2} \times 10 \times t^2$$

or $$t^2 + 2t - 8 = 0$$

which gives $t = 2$s. Hence the total time to hit the ground = $t_f + t = 2 + 2 = 4$ s. Thus, the correct choice is (a).

84. The horizontal distance covered in 4s = $20 \times \cos 30° \times 4 \simeq 70$ m. Hence the correct choice is (c).

85. The time taken to move a horizontal distance R is $t = R/(v_0 \cos \alpha)$. Therefore, the vertical distance moved in this time is given by

$$h = u_0 t - \frac{1}{2} gt^2$$

$$= v_0 \sin \alpha \times \frac{R}{v_0 \cos \alpha} - \frac{1}{2} g \left(\frac{R}{v_0 \cos \alpha}\right)^2$$

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$$= R \tan \alpha - \frac{gR^2}{2v_0^2 \cos^2 \alpha}$$

which is choice (c).

86. The range of a projectile is $r = 2v_0^2 \cos\theta \sin\theta/g$, the two possible angles of projection are θ and $(90° - \theta)$. The time of flight corresponding to these two angles are

$$t_1 = \frac{2v_0 \sin\theta}{g} \text{ and}$$

$$t_2 = \frac{2v_0 \sin(90° - \theta)}{g} = \frac{2v_0 \cos\theta}{g}$$

so that $t_1 t_2 = \dfrac{4v_0^2 \sin\theta \cos\theta}{g^2} = \dfrac{2r}{g}$. Thus $t_1 t_2 \propto r$.

Hence the correct choice is (b).

87. Let at $t = 0$, O and A are the positions of the gun and the car (Fig. 2.43). Let us say that at time $t = t_0$, the shell and the car reach B simultaneously so that the shell hits the car when it is at a distance OB from the gun. Let u be the speed of projection of the shell. Then, initial horizontal component of velocity of the shell $= u \cos 45° = \dfrac{u}{\sqrt{2}}$, and initial vertical component $= u \sin 45° = u/\sqrt{2}$. Therefore, the time of flight $= \dfrac{2u}{\sqrt{2}\, g} = \sqrt{2}\, u/g$. The car takes this time to cover the distance AB while the shell covers the distance OB in this time. Now $OB = OA + AB = 150 \text{ m} + AB$. Distance AB is given by

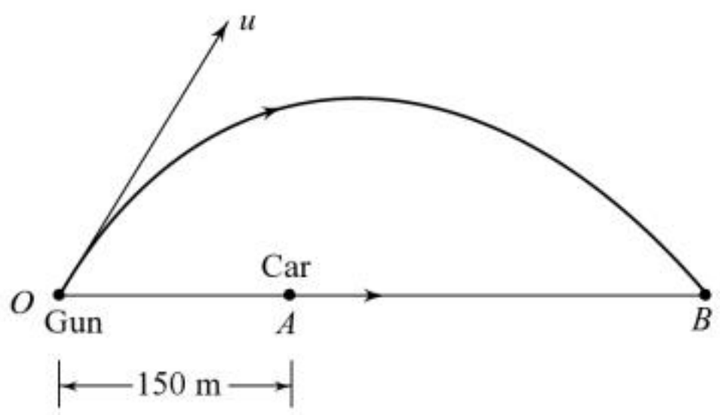

Fig. 2.43

$$AB = 10\sqrt{2} \times \sqrt{2} \times \frac{u}{g} = 20\, u/g$$

and $$OB = \frac{u}{\sqrt{2}} \times \frac{\sqrt{2}\, u}{g} = u^2/g$$

$\therefore$ $$\frac{u^2}{g} = 150 + \frac{20u}{g}$$

or $\quad u^2 - 20u - 1500 = 0 \qquad (\because g = 10 \text{ ms}^{-2})$

The positive root of this quadratic equation is $u = 50 \text{ ms}^{-1}$. Hence the correct choice is (d).

88. The distance of the car from the gun when the shell hits is

$$OB = \frac{u^2}{g} = \frac{50 \times 50}{10} = 250 \text{ m, which is choice (a).}$$

89. The horizontal range $R = AC$ is

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{20 \times 20 \times \sin 60°}{10}$$

$$= 20\sqrt{3} \text{ m}$$

At C, the velocity of the ball is again 20 ms^{-1} directed down as shown in Fig. 2.44.

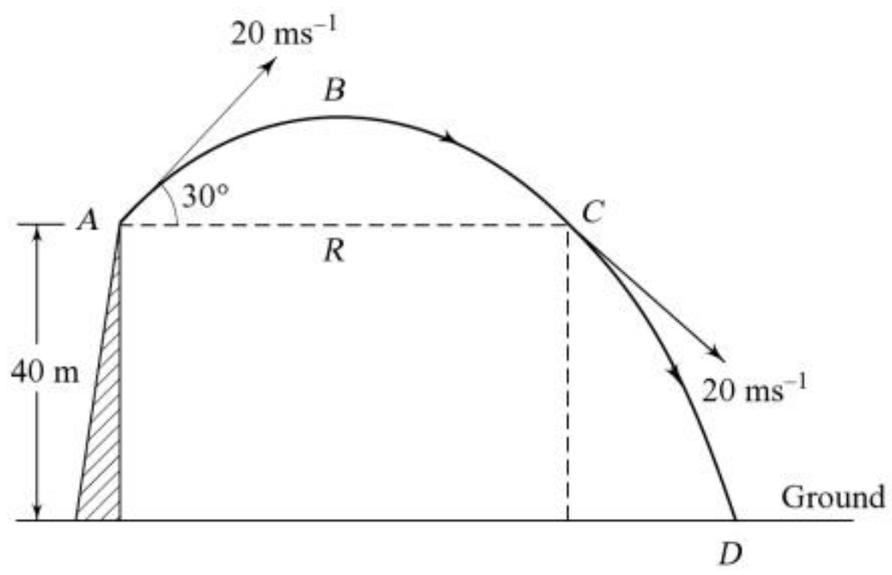

Fig. 2.44

The downward vertical component of this velocity is $20 \sin 30° = 10 \text{ ms}^{-1}$. The ball will hit the ground at D after travelling a vertical distance $h = 40$ m. If t_1 is the time taken for this, then

$$h = ut_1 + \frac{1}{2} gt_1^2$$

or $\quad 40 = 10\, t_1 + 5\, t_1^2$

or $\quad t_1^2 + 2\, t_1 - 8 = 0$

The positive root of this quadratic equation is $t_1 = 2$s. Now, the time of flight from A to C via B is

$$t_2 = \frac{2u \sin\theta}{g} = \frac{2 \times (20) \times \sin 30°}{10}$$

$$= \frac{2 \times 20 \times \sin 30°}{10} = 2 \text{ s}$$

$\therefore$ Total time taken $= t_1 + t_2 = 2\text{s} + 2\text{s} = 4\text{s}$. Hence the correct choice is (d).

90. For maximum range $\theta = 45°$. Hence $R_{max} = u^2/g$ and $h_{max} = u^2/4g$. Thus $h_{max} = R_{max}/4 = 200/4 = 50$ m. Hence the correct choice is (b).

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91. Let u be the initial speed with which the body is thrown along the inclined plane. As shown in Fig. 2.45, the effective deceleration is given by

$$a = g \sin\theta$$

$$= g \sin 30° = \frac{g}{2} = 5 \text{ ms}^{-2}$$

The body stops after covering a distance, say, s along the plane, which is given by $-2as = 0 - u^2$ or $u = \sqrt{2as} = \sqrt{2 \times 5 \times 40} = 20 \text{ ms}^{-1}$. A projectile projected at angle $\theta = 30°$ with this speed will have a range of

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{20 \times 20 \times \sin 60°}{10} = 20\sqrt{3} \text{ m}$$

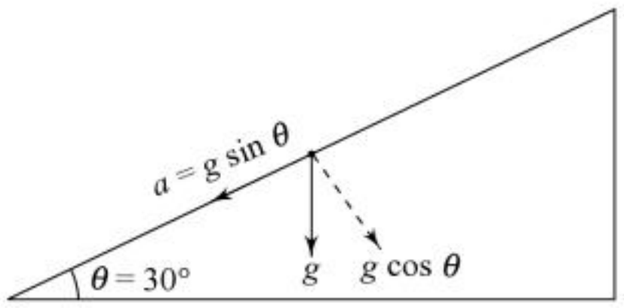

Fig. 2.45

Hence the correct choice is (c).

92. Since the velocity of the projectile changes continuously, both kinetic energy and momentum undergo a change with time. Only the vertical component of velocity changes due to gravity; the horizontal component always remains constant. Hence the correct choice is (d).

93. At the highest point, the velocity has only the horizontal component $v_x = v \cos\theta = v \cos 60° = v/2$. Now kinetic energy $\frac{1}{2} mv^2$ is proportional to v^2. Since the velocity is reduced to half, the kinetic energy becomes one–fourth, *i.e.*, $K/4$. Hence the correct choice is (d).

94. Kinetic energy is minimum when the projectile is at the highest point of its trajectory. At the highest point, its range = half the horizontal range. Hence the correct choice is (c).

95. Now $R = \frac{u^2 \sin 2\theta}{g}$ and $h_{\max} = \frac{u^2 \sin^2\theta}{2g}$. Given $R = h_{\max}$. Therefore $\sin 2\theta = \frac{\sin^2\theta}{2}$ or $2 \sin\theta \cos\theta = \frac{\sin^2\theta}{2}$ or $\tan\theta = 4$. Hence the correct choices is (d).

96. The horizontal component of velocity is

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(at) = a \quad \text{(i)}$$

The vertical component of velocity is

$$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(bt^2 + ct\right) = 2bt + c \quad \text{(ii)}$$

The value of v_y at $t = 1$ s is $(2b + c)$. Therefore, the magnitude of velocity at $t = 1$ s is

$$v = (v_x^2 + v_y^2)^{1/2} = [a^2 + (2b + c)^2]^{1/2}$$

Thus, the correct choice is (a).

97. If a projectile is projected with an initial velocity u at an angle θ with the horizontal, the horizontal and vertical components of its velocity at time t are given by

$$v_x = u \cos\theta \quad \text{(iii)}$$

and $$v_y = u \sin\theta - gt \quad \text{(iv)}$$

Comparing (iii) and (iv) with (i) and (ii) above we have $u \cos\theta = a$ and $u \sin\theta = c$. Dividing, we get, $\tan\theta = c/a$. Hence the correct choice is (d).

98. Comparing (iv) with (ii) we have $g = -2b$. Hence the correct choice is (b).

99. We have seen above that $u \cos\theta = a$ and $u \sin\theta = c$. Squaring and adding we get: $u^2 = a^2 + c^2$ or $u = (a^2 + c^2)^{1/2}$. Hence the correct choice is (c).

100. Range $R = u^2 \sin 2\theta/g$. For the same u, $R \propto \sin 2\theta$. Since $\sin 2\theta$ is the largest for $\theta = 35°$, the correct choice is (b).

101. The range is the same for θ and $(90° - \theta)$. Hence $R_1 = R_2$ for $\theta = 30°$ or $60°$. Thus the correct choice is (b).

102. Since $h_{\max} = \frac{u^2 \sin^2\theta}{2g}$ and u is the same for both projectiles, we have

$$\frac{h_1}{h_2} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{1}{3} \text{ or } h_1 = \frac{h_2}{3}.$$

Thus the correct choice is (b).

103. We know that $h = \frac{u^2 \sin^2\theta}{2g}$. The increase δh in h when u changes by δu can be obtained by partially differenting this expression. Thus

$$\delta h = \frac{2u\,\delta u \sin^2\theta}{2g}$$

$$\therefore \quad \frac{\delta h}{h} = \frac{2\delta u}{u}$$

Since h is increased by 10 %, $\frac{\delta h}{h} = 0.1$. Therefore,

$$\frac{2\delta u}{u} = 0.1.$$

Now, range $R = \frac{u^2 \sin 2\theta}{g}$. Therefore,

$$\delta R = \frac{2u\,\delta u \sin 2\theta}{g}$$

$$\therefore \quad \frac{\delta R}{R} = \frac{2\delta u}{u} = \frac{\delta h}{h} = 0.1$$

Thus $\delta R = 0.1\ R$. Hence R also increases by 10%. Thus, the correct choice is (c).

104. The time of flight is $T = \frac{2u \sin\theta}{g}$. Therefore

$$\delta T = \frac{2\delta u \sin\theta}{g}$$

which give $\frac{\delta T}{T} = \frac{\delta u}{u}$

But $\frac{\delta u}{u} = 0.05$. Therefore $\frac{\delta T}{T} = 0.05$

or $\delta T = 0.05\ T$. Hence T increases by 5%. Thus, the correct choice is (d).

105. Given $h = \frac{u^2 \sin^2\theta}{2g}$. Differentiating partially we get ($\because v_0$ = constant) $\delta h = \frac{u^2}{2g}\ 2 \sin\theta \cos\theta\ \delta\theta$.

Thus $\frac{\delta h}{h} = \frac{2\cos\theta\,\delta\theta}{\sin\theta} = 0.1$ (given).

Therefore, $\frac{\cos\theta\,\delta\theta}{\sin\theta} = 0.05$. We also have $T = \frac{2u\sin\theta}{g}$ which gives $\delta T = \frac{2u\cos\theta\,\delta\theta}{g}$. Thus $\frac{\delta T}{T} = \frac{\cos\theta\,\delta\theta}{\sin\theta}$. But $\frac{\cos\theta\,\delta\theta}{\sin\theta} = 0.05$. Therefore, $\frac{\delta T}{T} = 0.05$ or $\delta T = 0.05\ T$. Hence T increases by 5%. Thus the correct choice is (d).

106. We have seen above that $\frac{\delta R}{R} = \frac{2\delta u}{u} = 2 \times 0.05 = 0.1$ $\left(\because \frac{\delta u}{u} = 0.05\right)$ or $\delta R = 0.1\ R$. Hence, R will increase by 10%. Thus, the correct choice is (d).

107. We have seen that $\frac{\delta h}{h} = \frac{2\delta u}{u} = 2 \times 0.05 = 0.1$ or $\delta h = 0.1\ h$. Hence h increases by 10%. Thus, the correct choice is (d).

108. Now $\frac{\delta T}{T} = \frac{\delta u}{u} = 0.05$ or $\delta T = 0.05\ T$. Thus, T increases by 5%. Hence, the correct choice is (b).

109. Let h_e and h_m be the maximum heights attained by the projectile when projected from earth and moon respectively. Now

$$h_e = \frac{u^2 \sin^2\theta}{2g_e} \text{ and } h_m = \frac{u^2 \sin^2\theta}{2g_m} \text{ which give}$$

$\frac{h_m}{h_e} = \frac{g_e}{g_m} = 6$. Hence the correct choice is (d).

110. Now $R = \frac{2u^2 \sin\theta\cos\theta}{g^2}$ and $T^2 = \frac{4u^2 \sin^2\theta}{g^2}$.

From these two equations we have $T^2 = 2R \tan\theta$ or $T \propto \sqrt{R}$. Hence the correct choice is (b).

111. The range of a projectile is the same for two angles of projection θ and $90° - \theta$. For these two angles of projection, the maximum heights are

$$h_1 = \frac{u^2 \sin^2\theta}{2g} \text{ and}$$

$$h_2 = \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$$

$$\therefore \quad h_1 h_2 = \frac{u^4 \sin^2\theta \cos^2\theta}{4g^2}.$$

$$\text{Also } R^2 = \frac{4u^4 \sin^2\theta \cos^2\theta}{g^2}.$$

Which give $R^2 = 16\ h_1 h_2$ or $R = 4\sqrt{h_1 h_2}$. Hence the correct choice is (d).

112. Given $x = 2t$ and $y = 5t^2$. Eliminating t we get $y = \frac{5x^2}{4}$, i.e. $y \propto x^2$. Hence, the trajectory of the body is parabolic. Thus, the correct choice is (d).

113. Let u be the velocity with which the body is projected and g be the acceleration due to gravity. Then, we have

$$x = u\,t \text{ and } y = -\frac{1}{2}\ gt^2$$

Thus, the magnitude of the velocity of projection is the coefficient of t in the expression for x. Now, given that $x = 2t$. Hence $u = 2\ \text{ms}^{-1}$ which is choice (a).

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114. The x and y components of velocity are

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(2t) = 2 \text{ ms}^{-1}$$

and $$v_y = \frac{dy}{dt} = \frac{d}{dt}(5t^2) = 10t$$

At $t = 0.2$ s, $v_y = 10 \times 0.2 = 2$ ms^{-1}. The magnitude of velocity at $t = 0.2$ s is

$$v = (v_x^2 + v_y^2)^{1/2} = \sqrt{4+4} = 2\sqrt{2} \text{ ms}^{-1}.$$

Hence the correct choice is (b).

115. The angle of the velocity vector with the vertical at time t is given by

$$\tan \alpha = \frac{u}{|g|t}$$

We have seen that $u = 2$ ms^{-1}. Comparing

$y = -\frac{1}{2} gt^2$ with $y = 5t^2$, we get, $-\frac{1}{2} g = 5$ or $g = -10$ ms^{-2}. Therefore, at $t = 0.2$ s,

$$\tan \alpha = \frac{2}{|-10| \times 0.2} = 1$$

which gives $\alpha = 45°$. Hence the correct choice is (c).

116. We know that the position coordinates x and y are given by

$$x = (u \cos \theta)t \qquad \text{(i)}$$

and $$y = (u \sin \theta)\, t - \frac{1}{2} gt^2 \qquad \text{(ii)}$$

Comparing Eq. (i) with $x = 10\sqrt{3}\ t$, we have, $u \cos \theta = 10\sqrt{3}$ ms^{-1}. Also, comparing Eq. (ii) with $y = 10t - t^2$, we have, $u \sin \theta = 10$ ms^{-1}. These equations give $u^2 = 10^2 + 10^2 \times 3 = 400$ or $u = 20$ ms^{-1} and $\tan \theta = \frac{1}{\sqrt{3}}$ which gives $\theta = 30°$. Hence the correct choice is (a).

117. Comparing $y = 10t - t^2$ with Eq. (ii) above, we get, $-1 = -\frac{g}{2}$ or $g = 2$ ms^{-2}. Hence the correct choice is (b).

118. We have seen above that $u = 20$ ms^{-1}, $\theta = 30°$ and $g = 2$ ms^{-2}. Now, the maximum height attained is

$$h = \frac{u^2 \sin^2 \theta}{2g} = \frac{(20)^2 \times \sin^2 30°}{2 \times 2} = 25 \text{ m}$$

Hence the correct choice is (a).

119. The time of flight is

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 20 \times \sin 30°}{2} = 10 \text{ s}$$

Thus, the correct choice is (c).

120. The horizontal range is

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 60°}{2} = 100\sqrt{3} \text{ m}$$

Hence, the correct choice is (d).

121. Since $R \propto u^2$, choice (b) is incorrect.

122. When the train is at rest or moving with a uniform velocity, the plumb line hangs vertically along OB (Fig. 2.46). If the train moves with an acceleration a, the plumb line gets inclined along OC, the direction of the resultant of accelerations a and g. It is clear from the figure that $\tan \theta = a/g$. Hence the correct choice is (a).

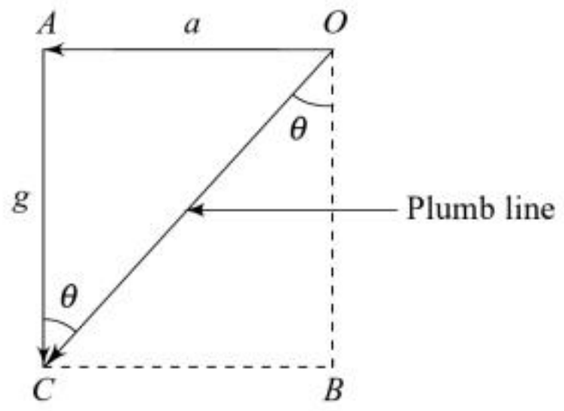

Fig. 2.46

123. When the lift is descending with a retardation (negative acceleration) a, the effective value of g is $g_{\text{eff}} = g + a$. The component of this acceleration along the inclined plane is $g_{\text{eff}} \sin \theta = (g + a) \sin \theta$. Hence the correct choice is (d).

124. The maximum height attained by the first ball is

$$h_1 = \frac{u^2}{2g}$$

where u is the initial speed of projection. The maximum height attained by the second ball is ($\because \theta = 90° - 60° = 30°$)

$$h_2 = \frac{u^2 \sin^2 (30°)}{2g} = \frac{u^2}{8g}$$

Now, PE of ball 1 at height $h_1 = mgh_1$ and that of ball 2 at height $h_2 = mgh_2$. Therefore, the ratio of potential energies $= \frac{h_1}{h_2} = \frac{u^2}{2g} \times \frac{8g}{u^2} = 4$. Hence the correct choice is (a).

125. Velocity is $v = \frac{dx}{dt} = a + 2bt - 3ct^2$ (i)

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Acceleration is $a = \dfrac{d^2x}{dt^2} = 2b - 6ct$

Acceleration is zero at time t given by $0 = 2b - 6ct$ or $t = \dfrac{b}{3c}$. Putting this value of t in Eq. (i).

We have $$v = a + 2b \cdot \frac{b}{3c} - 3c \times \frac{b^2}{9c^2}$$
$$= a + \frac{b^2}{3c}$$

Thus, the correct choice is (c).

126. We know that $\mathbf{A} \cdot \mathbf{B} = AB \cos\theta$ (i)

$$\mathbf{A} \cdot \mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}})$$
$$= 3 \times 6 + 4 \times 8 + 5 \times 10$$
$$= 100 \quad \text{(ii)}$$

The magnitudes of **A** and **B** are

$$A = \sqrt{(3)^2 + (4)^2 + (5)^2} = \sqrt{50} \quad \text{(iii)}$$

and $$B = \sqrt{(6)^2 + (8)^2 + (10)^2} = \sqrt{200} \quad \text{(iv)}$$

Using (ii), (iii) and (iv) in (i), we have

$$100 = \sqrt{50} \times \sqrt{200} \cos\theta = 100 \cos\theta$$

or $\cos\theta = 1$ or $\theta =$ zero, which is choice (a).

127. The vector product of two non-zero vectors is zero if they are in the same direction. Hence, vector **B** must be parallel to vector **A**, i.e. along $\pm z$-axis. Thus the correct choice is (d).

128. Given $\mathbf{A} + (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1\hat{\mathbf{j}}$

or $\mathbf{A} = -3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6\hat{\mathbf{k}}$, which is choice (a).

129. The angle α which the resultant **R** makes with **A** is given by

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

given $\alpha = \dfrac{\theta}{2}$. Hence

$$\tan\left(\frac{\theta}{2}\right) = \frac{B\sin\theta}{A + B\cos\theta}$$

or $$\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{2B\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{A + B\cos\theta}$$

which gives $A + B\cos\theta = 2B\cos^2\left(\dfrac{\theta}{2}\right)$

or $A + B\left[2\cos^2\left(\dfrac{\theta}{2}\right) - 1\right] = 2B\cos^2\left(\dfrac{\theta}{2}\right)$

which gives $A = B$, which is choice (c).

130. Refer to Fig. 2.47, where **u** is velocity of water current. Let **v** be the velocity of the man and let $AB = w$ be the width of the river. The river is crossed with a velocity equal to the component of **v** along AB, which is $v\cos\theta$, where θ is the angle between the north direction and the direction of velocity **v**.

$\therefore$ Time taken to cross the river $= \dfrac{w}{v\cos\theta}$.

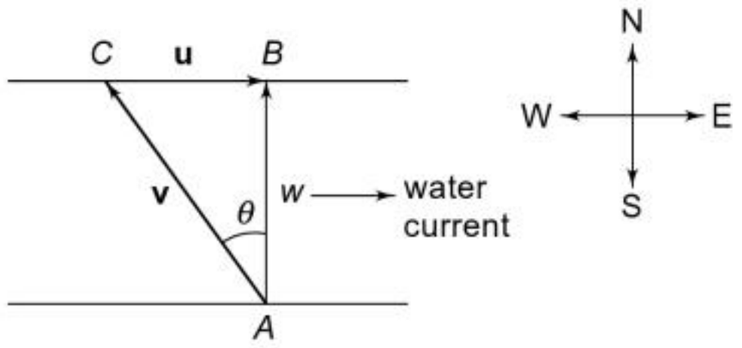

Fig. 2.47

This time is the shortest if $\theta = 0^\circ$. Hence the man should swim in the north direction, which is choice (d).

131. The distance will be the shortest if the resultant of velocities **u** and **v** is along AB. Thus,

$$\sin\theta = \frac{u}{v} = \frac{5\text{m/min}}{10\text{m/min}} = \frac{1}{2}$$

or $\theta = 30^\circ$,

i.e., He should swim 30° west of north, which is choice (c).

132. $$-2g(AB) = \left(\frac{u}{3}\right)^2 - \left(\frac{u}{2}\right)^2$$
$$= \frac{u^2}{9} - \frac{u^2}{4} = -\frac{5u^2}{36} \quad \text{(i)}$$

and $$-2g(BC) = \left(\frac{u}{4}\right)^2 - \left(\frac{u}{3}\right)^2$$
$$= \frac{u^2}{16} - \frac{u^2}{9} = -\frac{7u^2}{144} \quad \text{(ii)}$$

Dividing (i) by (ii), we get $\dfrac{AB}{BC} = \dfrac{20}{7}$ which is choice (a).

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133. We know that $h = ut - \frac{1}{2} gt^2$

or $t^2 - \frac{2u}{g} t + \frac{2h}{g} = 0$

The roots of this quadratic equation are t_1 and t_2. The sum of the roots is $t_1 + t_2 = \frac{2u}{g}$. Hence the correct choice is (a).

134. Product of roots is $t_1 t_2 = \frac{2h}{g}$ which is choice (a).

135. The time t_1 taken by the body to strike the inclined plane is given by

$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

The time t_2 taken by the body to reach the ground after striking the plane is

$$t_2 = \sqrt{\frac{2h}{g}}$$

Total time $t = t_1 + t_2 = \sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}$

Time t will be maximum if $\frac{dt}{dh} = 0$, *i.e.,* if

$$\frac{d}{dh}\left[\sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}\right] = 0$$

or $$\sqrt{\frac{2}{g}}\left[-\frac{1}{2}(H-h)^{-1/2} + \frac{1}{2}h^{-1/2}\right] = 0$$

or $\frac{1}{\sqrt{H-h}} = \frac{1}{\sqrt{h}}$ or $H - h = h$ or $h = \frac{H}{2}$.

Hence the correct choice is (c).

136. Let n be the number of storeys above the 15th storey. Then height fallen is $h = 4n$ metres.

Using $v^2 - u^2 = 2gh$, we have

$$(20)^2 - 0 = 2 \times 10 \times 4n$$

which gives $n = \frac{400}{80} = 5$. Therefore, the total number of storeys = 15 + 5 = 20, which is choice (a).

137. $t_A = \frac{2u_A}{g}$ and $t_B = \frac{2u_B \sin\theta}{g} = \frac{2u_B \sin 60°}{g}$

$$= \frac{\sqrt{3}u_B}{g}$$

Given $t_A = t_B$. Therefore

$$\frac{2u_A}{g} = \frac{\sqrt{3}u_B}{g} \quad \text{or} \quad \frac{u_A}{u_B} = \frac{\sqrt{3}}{2}$$

Hence the correct choice is (c).

138. $\mathbf{v} = (3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$ ms^{-1}. The magnitude of velocity is

$v = \sqrt{(3)^2 + (4)^2} = 5$ ms^{-1}. The angle subtended by the velocity vector with the horizontal (x-axis) is given by

$\tan\theta = \frac{4}{3}$ which gives $\sin\theta = \frac{4}{5}$.

Now, $h_{\max} = \frac{v^2 \sin^2\theta}{2g} = \frac{(5)^2 \times \left(\frac{4}{5}\right)^2}{2 \times 10} = 0.8$ m

Hence the correct choice is (a).

139. $T_f = \frac{2v\sin\theta}{g} = \frac{2 \times 5 \times 4}{5 \times 10} = 0.8$ s. Hence the correct choice is (a).

140. Velocity of projection is $\mathbf{v}_0 = (u\cos\theta)\hat{\mathbf{i}} + (u\sin\theta)\hat{\mathbf{j}}$. At time t, the velocity of the body is

$$\mathbf{v} = (u\cos\theta)\hat{\mathbf{i}} + (u\sin\theta - gt)\hat{\mathbf{j}}$$

The dot product of $\mathbf{v}_0$ and $\mathbf{v}$ is

$$\mathbf{v}_0 \cdot \mathbf{v} = u^2\cos^2\theta + u\sin\theta(u\sin\theta - gt)$$

or $$\mathbf{v}_0 \cdot \mathbf{v} = u^2 - (u\sin\theta)\,gt \qquad \text{(i)}$$

Since v is perpendicular to $\mathbf{v}_0$, $\mathbf{v}_0 \cdot \mathbf{v} = 0$. Using this in (i), we have

$$0 = u^2 - (u\sin\theta)gt \quad \text{or} \quad t = \frac{u}{g\sin\theta}$$

Hence the correct choice is (d).

141. $\text{PE} = mg\,h_{\max} = mg\left(\frac{u^2\sin^2\theta}{2g}\right) = \frac{mu^2\sin^2\theta}{2}$

$\text{KE} = \frac{1}{2} m(u\cos\theta)^2$. Hence

$$\frac{\text{PE}}{\text{KE}} = \tan^2\theta$$

Thus, the correct choice is (b).

142. Given $y = ax - bx^2$. Comparing this equation with

$$y = (\tan\theta)x - \frac{gx^2}{2u^2\cos^2\theta}$$

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We have $a = \tan\theta$ and $b = \dfrac{g}{2u^2\cos^2\theta}$

Horizontal range $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2u^2\sin\theta\cos\theta}{g}$

$$= \frac{2u^2\cos^2\theta}{g}(\tan\theta) = \frac{a}{b}$$

Hence the correct choice is (a).

143. $h_{max} = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u^2\cos^2\theta}{2g}(\tan^2\theta)$

$$= \frac{2u^2\cos^2\theta}{4g}(\tan^2\theta) = \frac{a^2}{4b}$$

Hence the correct choice is (d).

144. $t_f = \dfrac{2u\sin\theta}{g} = \dfrac{2u\cos\theta}{g}(\tan\theta)$ (i)

Now $b = \dfrac{g}{2u^2\cos^2\theta}$ or $u\cos\theta = \sqrt{\dfrac{g}{2b}}$ (ii)

Using (ii) in (i), we get

$t_f = a\sqrt{\dfrac{2}{bg}}$, which is choice (a).

145. Let the total distance be S. The time taken to travel the first half, *i.e.*, $\dfrac{S}{2}$ is

$$t_1 = \frac{S/2}{3} = \frac{S}{6}$$

Let t_2 be the time taken to cover a distance S_1 with speed 4.5 m/s and t_3 that to cover distance S_2 with speed 7.5 m/s.

Then $S_1 = 4.5\,t_2$ and $S_2 = 7.5\,t_3$

Now $S_1 + S_2 = \dfrac{S}{2}$ and $t_2 = t_3$ (given).

Therefore $\dfrac{S}{2} = S_1 + S_2$

$$= (4.5 + 7.5)t_2 \quad \text{or} \quad t_2 = \frac{S}{24}.$$

$\therefore$ Total time taken $t_1 = t_1 + t_2 = \dfrac{S}{6} + \dfrac{S}{24} = \dfrac{5S}{24}$

$\therefore$ Average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{S}{5S/24}$

$$= \frac{24}{5} = 4.8 \text{ m/s},$$

which is choice (d).

146. In the figure v_c represents the velocity of the car and v_r that of the parcel. M is the position of the man. From parallelogram law, the direction of the resultant velocity v_r must be along the direction along which the man is standing. It follows from Fig. 2.48 that angle θ is given by

$$\sin\theta = \frac{v_c}{v_p} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ or } \theta = 45^\circ$$

Hence the correct choice is (a).

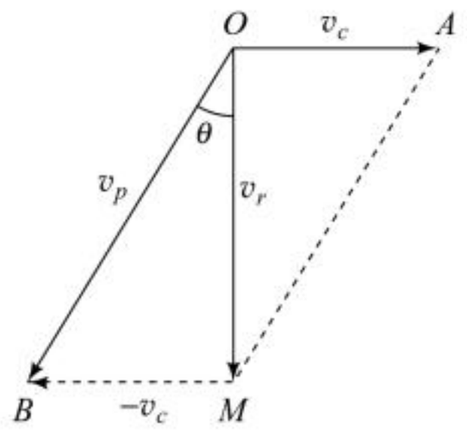

Fig. 2.48

147. $\mathbf{v} = \boldsymbol{u} + \boldsymbol{at} = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) + (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}}) \times 10$

$$= (8\hat{\mathbf{i}} + 6\hat{\mathbf{j}}) \text{ ms}^{-1}$$

The x and y components of v and $v_x = 8$ ms^{-1} and $v_y = 6$ ms^{-1}. The magnitude of v is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(8)^2 + (6)^2} = 10 \text{ ms}^{-1}$$

Hence the correct choice is (d).

148. Given: $x = 36t$ (i)

and $y = 48t - 4.9t^2$ (ii)

For a body projected with a velocity u at an angle θ with the horizontal, the x and y displacements at time t are given by

$$x = (u\cos\theta)t \quad \text{(iii)}$$

and $y = (u\sin\theta)t - \dfrac{1}{2}gt^2$ (iv)

Comparing (i) with (iii) and (ii) with (iv), we have

$u\cos\theta = 36$

and $u\sin\theta = 48$

Squaring and adding, we have

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$$u^2 (\cos^2 \theta + \sin^2 \theta) = (36)^2 + (48)^2$$

or $u^2 = 3600$ or $u = 60 \text{ ms}^{-1}$, which is choice (d).

149. Given $t = \sqrt{x} + 3$. Squaring, we have

$$x = t^2 - 6t + 9 \qquad \text{(i)}$$

$$\text{velocity} \quad v = \frac{dx}{dt} = \frac{d}{dt}(t^2 - 6t + 9) = 2t - 6 \qquad \text{(ii)}$$

Find t from Eq. (ii) when $v = 0$. Use this value of t is Eq. (i). The correct choice is (a).

150. Given $a = -k\sqrt{v}$ or $\frac{dv}{dt} = -k\sqrt{v}$. Thus $v^{-1/2}\, dv = -k\, dt$

Integrating, we get $2v^{1/2} = -kt + c$. Using the given initial condition ($v = u$ at $t = 0$), we get $c = 2\sqrt{u}$. Thus, we have

$$2(v^{1/2} - u^{1/2}) = -kt$$

Now, use $t = T$ and $v = 0$. The correct choice is (a).

151. The slope of the line is $m = -\frac{5}{6} \text{ ms}^{-2}$ per second and its intercept is $c = 5 \text{ ms}^{-2}$. Using $y = mx + c$, the acceleration a (in ms^{-2}) as a function of time t is given by

$$a = -\frac{5}{6} t + 5$$

or $$\frac{dv}{dt} = -\frac{5}{6} t + 5$$

or $$v = \int_0^t \left(\frac{5}{6} t + 5 \right) dt$$

or $$v = -\frac{5}{12} t^2 + 5t + k \qquad \text{(i)}$$

where k is the constant of integration. Since the particle starts from rest, $v = 0$ at $t = 0$. Using this in (i) we ger $k = 0$. Hence

$$v = -\frac{5}{6} t^2 + 5t \qquad \text{(ii)}$$

It follows from the graph that the deceleration becomes zero at $t = 6$ s. Hence, the speed of the particle will be maximum at $t = 6$ s. Putting $t = 6$ s in Eq. (ii), we have

$$v_{\max} = -\frac{5}{12} \times (6)^2 + 5 \times 6$$

$$= -15 + 30 = 15 \text{ ms}^{-1}$$

Hence the correct choice is (b).

152. The slope of the given v versus x graph is $m = -\frac{v_0}{x_0}$ and intercept is $c = +v_0$. Hence v varies with x as

$$v = -\left(\frac{v_0}{x_0}\right) x + v_0 \qquad \text{(i)}$$

where v_0 and x_0 are constants of motion. Differentiating with respest to time t, we have

$$\frac{dv}{dt} = -\left(\frac{v_0}{x_0}\right) \frac{dx}{dt}$$

or $$a = -\left(\frac{v_0}{x_0}\right) v \qquad \text{(ii)}$$

Using Eq. (i) in Eq. (ii), we get

$$a = -\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0} x + v_0\right)$$

or $$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus, the graph of a versus x is a straight line having a positive slope $= \left(\frac{v_0}{x_0}\right)^2$ and negative intercept $= -\frac{v_0^2}{x_0}$. Hence the correct choice is (d).

153. Given $v = k\sqrt{x} \Rightarrow v^2 = k^2 x$. Differentiating, we have

$$2v\frac{dv}{dt} = k^2 \frac{dx}{dt} = k^2 v \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{k^2}{2}$$

$$\Rightarrow \quad \int dv = \frac{k^2}{2} \int dt$$

$$\Rightarrow \quad v = \frac{k^2 t}{2}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{k^2 t}{2}$$

$$\Rightarrow \quad \int dx = \frac{k^2}{2} \int t\, dt$$

$$\Rightarrow \quad x = \frac{k^2}{4} t^2$$

Thus $x \propto t^2$. Hence the correct choice is (c)

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SECTION II

Multiple Choice Questions Based on Passage

Questions 1 to 4 are based on the following passage.

Passage I

It must be clearly understood that distance is not the same as displacement. Distance is a scalar quantity and is given by the total length of the path travelled by the body in a certain interval of time. Displacement is a vector quantity and is given by the shortest distance (in a specified direction) between the initial and the final positions of the body. The direction of the displacement vector is from the initial position (starting point) to the final position (end point) of the motion. Speed is a scalar quantity. The average speed is defined as

$$v = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Velocity is a vector quantity. The average velocity is defind as

$$v = \frac{\text{net displacement}}{\text{time taken}}$$

The direction of the velocity vector is the same as that of the displacement vector. Acceleration is defined as the rate of change of velocity and it is a vector quantity.

1. A cyclist starts from centre O of a circular track of radius $r = 1$ km, reaches edge P of the track and then cycles along the circumference and stops at point Q as shown in Fig. 2.49. the displacement of the cyclist is

(a) $r\left(1+\frac{\pi}{6}\right)$ (b) r

(c) $r\left(1+\frac{\pi}{3}\right)$ (d) $\frac{\pi r}{3}$

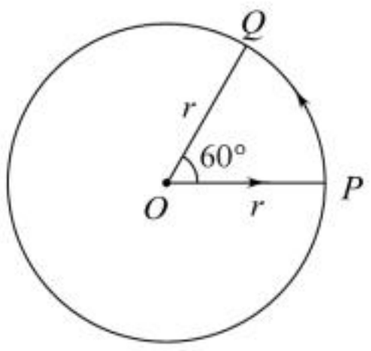

Fig. 2.49

2. In Q.1 above the magnitude of the velocity in km h^{-1} of the cyclist is

(a) 12.3 (b) 12.0
(c) 3.0 (d) 6.0

3. In Q.1 the distance travelled by the cyclist is approximately

(a) 2 km (b) 2.01 km
(c) 2.05 km (d) 1 km

4. In Q.1, the average speed (in km h^{-1}) of the cyclist is

(a) 12.3 (b) 6.15
(c) 6.0 (d) 12.0

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1. Net displacement is OQ = shortest distance between the starting point O and end point Q = 1 km = r. Therefore, the correct choice (b).

2. Average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{1\,\text{km}}{\frac{1}{6}\text{h}} = 6\ \text{km}^{-1}$

Hence the correct choice is (d).

3. Distance travelled = $OP + PQ$ (along the arc)

$$= r + \frac{2\pi r}{6} = r\left(1+\frac{\pi}{3}\right)$$

$$= 1\ \text{km}\left(1+\frac{3.14}{3}\right) = 2.05\ \text{km}$$

Hence the correct choice is (c).

4. Average speed $= \frac{\text{distance}}{\text{time}} = \frac{2.05\,\text{km}}{\frac{1}{6}\text{h}}$

$$= 12.3\ \text{km h}^{-1}$$

Thus the correct choice is (a).

Questions 5 to 7 are based on the following passage.

Passage II

Ball A is rolled along the positive x-direction with a speed of 5 ms^{-1} towards a bigger ball B 20 m away. After collision with ball B, ball A retraces the path and reaches its starting point with a speed of 4 ms^{-1}.

5. The average velocity of ball A during the time interval from 0 to 4 s is

(a) 5 ms^{-1} along positive x-direction.
(b) 5 ms^{-1} along negative x-direction.

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(c) $\frac{40}{9}$ ms^{-1} along negative x-direction.

(d) $\frac{14}{3}$ ms^{-1} along positive x-direction.

6. The average valocity of ball A during the time interval 0 to 9 s is

(a) 4.5 ms^{-1} along positive x-direction.
(b) 4.5 ms^{-1} along negative x-direction.
(c) $\frac{40}{9}$ ms^{-1} along negative x-direction.
(d) zero

7. The average velocity of ball A during the time interval 0 to 6 s is

(a) 2 ms^{-1} along positive x-direction.
(b) 2 ms^{-1} along negative x-direction.
(c) $\frac{14}{3}$ ms^{-1} along positive x-direction.
(d) $\frac{14}{3}$ ms^{-1} along negative x-direction.

➔ SOLUTIONS

5. Time taken by ball A to reach ball B is

$$t_1 = \frac{20}{5} = 4 \text{ s}$$

During the time interval 0 to 4 s, the net displacement of ball A = 20 m along positive x-direction. Therefore, the velocity of ball A during 0 to 4s = 5 ms^{-1} along positive x-direction, which is choice (a).

6. Time taken by ball A to retrace its path and reach the starting point after collision with ball B is

$$t_2 = \frac{20}{4} = 5 \text{ s}$$

Therefore, the net displacement in time interval 0 to 9 s = 0. Hence average velocity = 0, which is choice (d).

7. During the time interval 0 to 6 s, ball A covers a distance of 20 m along positive x-direction up to ball B (which takes 4 s) and in the next 2 s, it covers a distance of 4 $ms^{-1} \times 2$ s = 8 m along the negative x-direction. Therefore,

Net displacement of A from 0 to 6 s = 20 – 8 = 12 m along positive x-direction. Hence, average velocity of A during 0 and 6 s is

$$\frac{12}{6\text{s}} = 2 \text{ ms}^{-1} \text{ along positive } x\text{-direction}$$

Thus the correct choice is (a).

Questions 8 to 10 are based on the following passage.

Passage III

The speed-time graph of the motion of a body is shown in Fig. 2.50.

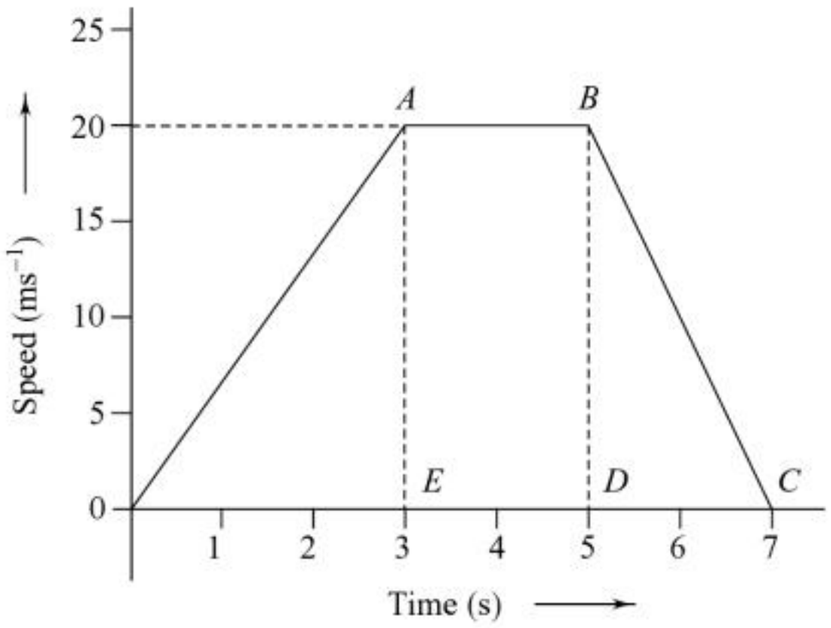

Fig. 2.50

8. The accelerations of the body during the last 2 seconds is

(a) $\frac{20}{3}$ ms^{-2} (b) $-\frac{20}{7}$ ms^{-2}
(c) –10 ms^{-2} (d) zero

9. The ratio of distance travelled by the body during the last 2 seconds to the total distance travelled by it is

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$
(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

10. The average speed of the car during the whole journey is

(a) 10 ms^{-1} (b) 20 ms^{-1}
(c) $\frac{90}{7}$ ms^{-1} (d) $\frac{40}{7}$ ms^{-1}

➔ SOLUTIONS

8. Acceleration during last 2 seconds = slope of line BC

$$= \frac{0-20}{7-5} = -10 \text{ ms}^{-2}$$

Thus the correct choice is (c).

9. Distance travelled in last two seconds

= area of triangle BCD

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$$= \frac{1}{2} \times BD \times DC$$

$$= \frac{1}{2} \times 20 \times 2 = 20 \text{ m}$$

Total distance travelled = area of trapesium $OABC$

$$= \frac{1}{2}(AB + OC) \times AE$$

$$= \frac{1}{2}(2 + 7) \times 20 = 90 \text{ m}$$

Hence the correct choice is (b).

10. Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$$= \frac{90\,\text{m}}{7\,\text{s}} = \frac{90}{7}\,\text{ms}^{-1}$$

Thus the correct choice is (c).

Questions 11 and 12 are based on the following passage.

Passage IV

A particle initially (i.e. at time $t = 0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates at a rate

$$a = -k\sqrt{v}$$

where v is the instantaneous velocity and k is a positive constant.

11. The particle comes to rest in a time

(a) $\dfrac{2\sqrt{u}}{k}$ (b) $\dfrac{\sqrt{u}}{k}$

(c) $2k\sqrt{u}$ (d) $k\sqrt{u}$

12. The distance covered by the particle before coming to rest is

(a) $\dfrac{u^{3/2}}{k}$ (b) $\dfrac{2u^{3/2}}{k}$

(c) $\dfrac{3u^{3/2}}{2k}$ (d) $\dfrac{2u^{3/2}}{3k}$

➔ SOLUTIONS

11. Given $a = -kv^{1/2}$ or $\dfrac{dv}{dt} = -kv^{1/2}$

Thus $v^{-1/2}dv = -k\,dt$

Integrating, we have

$$\int v^{-1/2}dv = -k\int dt$$

or $$2v^{1/2} = -kt + c \qquad \text{(i)}$$

where c is the constant of integration. Given that at $t = 0$, $v = u$. Using this in (i) we get $2u^{1/2} = c$. Using this value of c in (i),

we have $$2(v^{1/2} - u^{1/2}) = -kt \qquad \text{(ii)}$$

Let τ be the time taken by the particle to come to rest. Then, $v = 0$ at $t = \tau$. Using this in (ii), we get

$$2(0 - u^{1/2}) = -k\tau \quad \text{or} \quad \tau = \frac{2u^{1/2}}{k} \qquad \text{(iii)}$$

Hence the correct choice is (a).

12. To find the distance s covered in this time, we use Eq. (i) to get

$$v^{1/2} = u^{1/2} - \frac{kt}{2}$$

Squaring, we have

$$v = u - ktu^{1/2} + \frac{k^2t^2}{4}$$

But $$v = \frac{ds}{dt}$$

Therefore, $$\frac{ds}{dt} = u - kt\,u^{1/2} + \frac{k^2t^2}{4}$$

Integrating from $t = 0$ to $t = \tau$, we have

$$s = \left| ut - \frac{ku^{1/2}t^2}{2} + \frac{k^2t^3}{12} \right|_0^{\tau}$$

or $$s = u\tau - \frac{1}{2}ku^{1/2}\tau^2 + \frac{1}{12}k^2\tau^3 \qquad \text{(iv)}$$

Substituting the value of τ from (iii) in (iv), we get

$$s = \frac{2u^{3/2}}{k} - \frac{4u^{3/2}}{2k} + \frac{8u^{3/2}}{12k} \quad \text{or} \quad s = \frac{2u^{3/2}}{3k},$$

which is choice (d).

Questions 13 to 17 are based on the following passage.

Passage V

Relative Velocity

If two objects A and B are moving in a straight line with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ respectively, the relative velocity of object A with respect to object B is given by

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

It follows that the relative velocity of object B with respect to object A will be

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

If objects A and B are moving along different directions in a plane, the relative velocity of one object with respect to

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the other is found by using the parallelogram law of vector addition. To find $\mathbf{v}_{AB}$, we find the resultant of vectors $\mathbf{v}_A$ and $-\mathbf{v}_B$. To find $\mathbf{v}_{BA}$, we find the resultant of vectors $\mathbf{v}_B$ and $-\mathbf{v}_A$.

13. Two persons P and Q are standing 54 m apart on a long horizontal belt moving with a speed of 4 ms^{-1} in the direction from P to Q. Person P rolls a round stone towards person Q with a speed of 9 ms^{-1} with respect to the belt. The velocity of stone with respect to an observer on a stationary platform is

(a) 13 ms^{-1} in the direction to motion of the belt
(b) 13 ms^{-1} opposite to the direction of motion of the belt
(c) 5 ms^{-1} in the direction of motion of the belt.
(d) 5 ms^{-1} opposite to the direction of motion of the belt.

14. What will be the answer to Q. 13 above if person Q rolls the stone towards person P with a speed of 9 ms^{-1} with respect to the belt?

(a) 13 ms^{-1} in the direction of motion of the belt
(b) 13 ms^{-1} opposite to the direction of motion of the belt
(c) 5 ms^{-1} in the direction of motion of the belt.
(d) 5 ms^{-1} opposite to the direction of motion of the belt.

15. In Q. 13 above, what is the time taken by the stone to travel from P to Q?

(a) $\frac{54}{13}$ s (b) $\frac{54}{5}$ s

(c) 6s (d) $\frac{27}{2}$ s

16. Rain is falling vertically with a speed of 4 ms^{-1}. A man riding a bicycle is travelling at a speed of 3 ms^{-1} in the north to south direction. In order to protect himself from rain, the man should hold his umbrella at an angle θ given by

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ in the north-west direction

(b) $\tan^{-1}\left(\frac{3}{4}\right)$ in the south-west direction

(c) $\tan^{-1}\left(\frac{4}{3}\right)$ in the north-east direction

(d) $\tan^{-1}\left(\frac{4}{3}\right)$ in the south-east direction

17. In Q. 16 above, with what speed does the rain strike his umbrella?

(a) 1 ms^{-1} (b) 7 ms^{-1}
(c) 5 ms^{-1} (d) 3.5 ms^{-1}

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13. Let us choose the positive direction to be the direction from P to Q, *i.e.,* the direction of motion of the belt. Then, the velocity of the belt is

$$v_B = +4 \text{ ms}^{-1}$$

Speed of the stone with respect to the belt is

$$v_{SB} = +9 \text{ ms}^{-1}$$

If v_S is the speed of the stone with respect to a stationary observer, we have

$$v_{SB} = v_S - v_B$$

or $$v_S = v_{SB} + v_B = +9 + 4 = +13 \text{ ms}^{-1}$$

The positive sign shows that the direction of velocity of stone is from P to Q, i.e. in the direction of motion of the belt. Hence the correct choice is (a).

14. In the case $v_B = +4 \text{ ms}^{-1}$ but $v_{SB} = -9 \text{ ms}^{-1}$. Hence $v_S = v_{SB} + v_B = -9 + 4 = -5 \text{ ms}^{-1}$

The negative sign indicates that the direction of velocity of stone is opposite to the direction of motion of the belt. Hence the correct choice is (d).

15. Since persons P, Q and the stone are located on the belt, the speed of the stone relative to P or Q will be 9 ms^{-1}.

$\therefore$ Time taken $= \frac{54 \text{ m}}{9 \text{ ms}^{-1}} = 6$ s, which is choice (c).

16. Refer to Fig. 2.51.

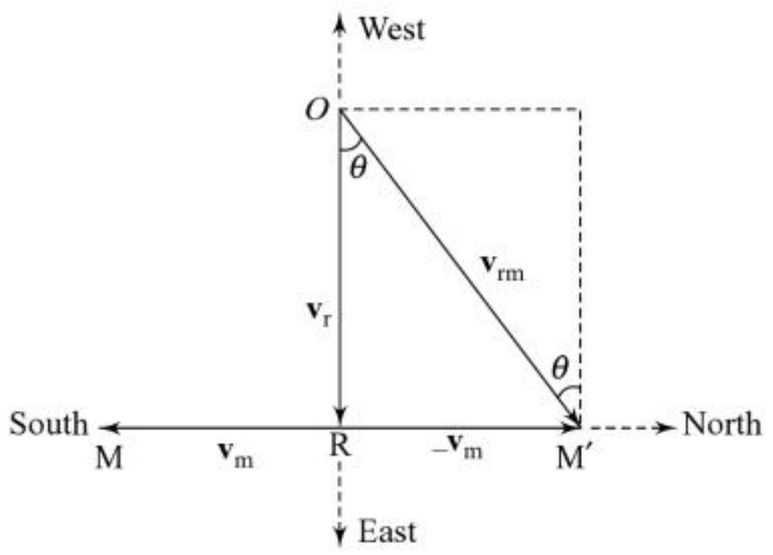

Fig. 2.51

Let $\mathbf{OR} = \mathbf{v}_r$ and $\mathbf{RM} = \mathbf{v}_m$ respectively represent the velocities of the rain and the man. Given $\mathbf{v}_r = 4 \text{ ms}^{-1}$ vertically downward from west to east direction.

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$\mathbf{v}_m = 3\ \text{ms}^{-1}$ from north to south direction.
In order to protect himself from rain, the man must hold his umbrella in the direction of the relative velocity $\mathbf{v}_{rm}$, which is given by

$$\mathbf{v}_{rm} = \mathbf{v}_r - \mathbf{v}_m = \mathbf{v}_r + (-\mathbf{v}_m)$$

Thus, we vector $\mathbf{v}_{rm}$ is the resultant of vectors $\mathbf{v}_r$ and $-\mathbf{v}_m$. In Fig. 2.55, vector $\mathbf{RM'} = -\mathbf{v}_m$ from the parallelogram law of vector addition, the resultant velocity is given by vector $\mathbf{OM'}$. If θ is the angle subtended by the resultant velocity $\mathbf{v}_{rm}$ with the vertical, then from triangle $\mathbf{ORM'}$, we have

$$\tan\theta = \frac{\text{RM}'}{\text{OR}} = \frac{v_m}{v_r} = \frac{3}{4}$$

Hence the correct choice is (b).

17. Since $\mathbf{v}_r$ and $\mathbf{v}_m$ are perpendicular to each other,

$$v_{rm} = \left(v_r^2 + v_m^2\right)^{1/2} = \left[(4)^2 + (3)^2\right]^{1/2} = 5\ \text{ms}^{-1}$$

Hence the correct choice is (c).

Questions 18 to 21 are based on the following passage.

Passage VI

Two objects are projected from the same point with the same speed u at angles of projection α and β with the horizontal respectively. They strike the ground at the same point at a distance R from the point of projection. The respective maximum heights attained by the objects are h_1 and h_2 and t_1 and t_2 are the respective times of flight.

18. R, h_1 and h_2 are related as

(a) $R = \sqrt{h_1 h_2}$ (b) $R = \sqrt{2h_1 h_2}$
(c) $R = 2\sqrt{2h_1 h_2}$ (d) $R = 4\sqrt{2h_1 h_2}$

19. The ratio $\frac{t_1}{t_2}$ is equal to

(a) $\sin\alpha$ (b) $\cos\alpha$
(c) $\tan\alpha$ (d) $\cot\alpha$

20. The ratio h_1/h_2 is equal to

(a) $\sin^2\alpha$ (b) $\cos^2\alpha$
(c) $\tan^2\alpha$ (d) $\cot^2\alpha$

21. The sum $(h_1 + h_2)$ is equal to

(a) $\frac{u^2}{g}\sin^2\alpha$ (b) $\frac{u^2}{g}\cos^2\alpha$
(c) $\frac{u^2}{g}$ (d) $\frac{u^2}{2g}$

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Since the horizontal range in the same $\alpha + \beta = 90°$. Therefore $\beta = 90° - \alpha$ and we have

$$h_1 = \frac{u^2}{2g}\sin^2\alpha \quad \text{(i)}$$

$$h_2 = \frac{u^2}{2g}\sin^2\beta = \frac{u^2}{2g}\cos^2\alpha \quad \text{(ii)}$$

$$R = \frac{u^2}{2g} \times 2\sin\alpha\cos\alpha \quad \text{(iii)}$$

$$t_1 = \frac{2u\sin\alpha}{g} \quad \text{(iv)}$$

$$t_2 = \frac{2u\sin\beta}{g} = \frac{2u}{g}\cos\alpha \quad \text{(v)}$$

18. From Eqs. (i), (ii) and (iii), we have

$$R = \frac{u^2}{g} \times 2 \times \sqrt{\frac{2gh_1}{u^2} \times \frac{2gh_2}{u^2}} = 4\sqrt{h_1 h_2}$$

Hence the correct choice is (d)

19. From Eqs. (iv) and (v) it follows that the correct choice is (c).

20. From Eqs. (i), (ii), (iv) and (v), we have

$$\frac{h_1}{h_2} = \tan^2\alpha$$

Hence the correct choice is (c).

21. From Eqs. (i) and (ii), we find that

$h_1 + h_2 = \frac{u^2}{2g}$, which is choice (d).

Questions 22 to 25 are based on the following passage.

Passage VII

The vertical displacement y of a projectile varies with the horizontal displacement x as

$$y = ax - bx^2$$

where a and b are constants.

22. The trajectory of the projectile is a

(a) straight line (b) circle
(c) parabola (d) hyperbola

23. The horizontal range of the projectile is

(a) a (b) b
(c) $\sqrt{\frac{a}{b}}$ (d) $\frac{a}{b}$

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24. The maximum height attained by the projectile is

(a) $\frac{a}{b}$ (b) $\frac{2a}{b}$

(c) $\frac{a^2}{2b}$ (d) $\frac{a^2}{4b}$

25. The time of flight of the projectile is

(a) $\frac{a}{\sqrt{bg}}$ (b) $a\sqrt{\frac{2}{bg}}$

(c) $\frac{2a}{\sqrt{bg}}$ (d) $2a\sqrt{\frac{2}{bg}}$

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22. The given equation

$$y = ax - bx^2 \qquad \text{(i)}$$

is the equation of a parabola. Hence the correct choice is (c).

23. The value of y is zero at $x = 0$ and $x = R$ (horizontal range). Putting $y = 0$ and $x = R$ in Eq. (i), we get $R = a/b$, which is choice (d).

24. Differentiating Eq. (1) with respect to time t, we have

$$\frac{dy}{dt} = a\frac{dx}{dt} - 2bx\frac{dx}{dt}$$

$$\Rightarrow \qquad v_y = va v_x - 2vxv_x = (a - 2bx)v_x \qquad \text{(ii)}$$

At the maximum height $v_y = 0$. Using this in Eq. (ii), we get $(a - 2bx) = 0$ or $x = a/2b$. Putting this value of x in Eq. (i), we have (since $y = h_{max}$ at this value of x)

$$h_{max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$$

Hence the correct choice is (d).

25. The time t to reach the maximum height is given by

$$h_{max} = \frac{1}{2}gt^2$$

$$\Rightarrow \qquad t = \sqrt{\frac{2h_{max}}{g}} = \sqrt{\frac{2a^2}{4bg}} = \frac{a}{\sqrt{2bg}}$$

Therefore, the time of flight is

$t_f = 2t = a\sqrt{\frac{2}{bg}}$, which is choice (b)

Questions 26 to 32 are based on the following passage.

Passage VIII

The position vector **r** with respect to the origin of a particle varies with time t as

$$\mathbf{r} = (at)\,\hat{\mathbf{i}} + (bt - ct^2)\,\hat{\mathbf{j}}$$

where a, b and c are constants.

26. The trajectory of the particle is a

(a) straight line (b) circle
(c) parabola (d) none of these

27. The magnitude of the initial velocity of the particle is

(a) $\sqrt{a^2 + c^2}$ (b) $\sqrt{b^2 + c^2}$
(c) $\sqrt{a^2 + b^2}$ (d) $(a + b - 2c)$

28. The angle θ with the horizontal along which the particle is projected is given by

(a) $\sin\theta = \frac{b}{c}$ (b) $\cos\theta = \frac{a}{c}$

(c) $\tan\theta = \frac{b}{a}$ (d) $\tan\theta = \frac{2c}{\sqrt{a^2 + b^2}}$

29. The time of flight of the particle is

(a) $\frac{b}{c}$ (b) $\frac{ab}{c}$

(c) $\frac{c}{a}$ (d) $\frac{b}{ac}$

30. The acceleration due to gravity at that place is

(a) $2a$ (b) $2b$
(c) $2c$ (d) none of these

31. The maximum height to which the particle rises is

(a) $\frac{2b^2}{c}$ (b) $\frac{b^2}{2c}$

(c) $\frac{b^2}{4c}$ (d) $\frac{4b^2}{c}$

32. The horizontal range of the particle is

(a) $\frac{ab}{c}$ (b) $\frac{ac}{b}$

(c) $\frac{bc}{a}$ (d) abc

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26. Comparing Eq. $\mathbf{r} = (at)\,\hat{\mathbf{i}} + (bt - ct^2)\,\hat{\mathbf{j}}$ with Eq. $\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$, we get

$$x = at \qquad \text{(i)}$$

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and $y = bt - ct^2$ (ii)

Eliminating t from (i) and (ii) we get

$y = \left(\frac{b}{a}\right)x - \left(\frac{c}{a^2}\right)x^2$ which is the equation of a parabola. Hence the correct choice is (c).

27. From Eqs. (i) and (ii), we have

$$v_x = \frac{dx}{dt} = a \quad \text{(iii)}$$

$$v_y = \frac{dy}{dt} = b - 2ct \quad \text{(iv)}$$

Putting $t = 0$ in Eq. (iii), the initial values of v_x and v_y are a and b respectively. The initial speed of the particle is

$u = \sqrt{a^2 + b^2}$, which is choice (c).

28. If the particle is projected with an initial velocity u at an angle θ with the horizontal, then the horizontal displacement x and vertical displacement y at time t are

$$x = (u \cos \theta)t \quad \text{(v)}$$

and $$y = (u \sin \theta)t - \frac{1}{2} gt^2 \quad \text{(vi)}$$

Comparing Eqs. (v) and (vi) with Eqs. (iii) and (iv) we have $u \cos \theta = a$ and $u \sin \theta = b$ which give $\tan \theta = b/a$, which is choice (c).

29. When $t = t_f$, $y = 0$. Putting $y = 0$ and $t = t_f$ in Eq. (ii) we get $0 = t_f(b - ct_f)$ gives $t_f = 0$ and $t_f = b/c$. But $t_f = 0$ is not possible. Hence the correct choice is (a).

30. Comparing Eq. (vi) with Eq. (ii), we get $g = 2c$, which is choice is (c).

31. Now $y = h_{max}$ when $t = \frac{1}{2} t_f = \frac{b}{2c}$. Putting $y = h_{max}$ and $t = b/2c$ in Eq. (ii) we get

$$h_{max} = b \times \frac{b}{2c} - c\left(\frac{b}{2c}\right)^2 = \frac{b^2}{4c}$$

32. $$R = (u \cos \theta)t_f \quad \text{(vii)}$$

Now $u = \sqrt{a^2 + b^2}$, $\tan \theta = \frac{b}{a}$ which gives

$\cos \theta = \frac{a}{\sqrt{a^2 + b^2}}$ and $t_f = \frac{b}{c}$. Putting these values in Eq. (vii), we get $R = \frac{ab}{c}$. Hence the correct choice is (a).

SECTION III

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four options out of which only ONE choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true but Statement-2 is NOT the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1

A body moving in a straight line may have non-zero acceleration at an instant when its speed is zero.

Statement-2

If a body is thrown vertically upwards, its speed at the instant when it reaches the highest point is zero but its acceleration is 9.8 ms^{-2}.

2. Statement-1

A body moving in a straight line with a constant speed must have a zero acceleration.

Statement-2

A body moving along a curve with a constant speed may have a zero acceleration.

3. Statement-1

A wooden ball and a steel ball of the same mass, released from the same height in air, do not reach the ground at the same time.

Statement-2

The apparent weight of a body in a meduim depends on the density of the body relative to that of the meduim.

4. Statement-1

If the displacement-time graph of the motion of a body is a straight line parallel to the time axis, then it follows that the body is at rest.

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Statement-2

Velocity is equal to the rate of change of displacement.

5. **Statement-1**

If the voelocity-time graph of the motion of a body is a curve, then the body is either uniformly accelerated or uniformly retarded.

Statement-2

The slope of the velocity-time graph gives the acceleration.

6. **Statement-1**

A body is projected horizontally with a velocity u from the top of a building of height h. It hits the ground after a time $t = \sqrt{2h/g}$.

Statement-2

The vertical and horizontal motions can be treated independently.

7. **Statement-1**

A body is projected from the ground with kinetic energy K at an angle of 60° with the horizontal. If air resistance is neglected, its kinetic energy when it is at the highest point of its trajectory will be $K/4$.

Statement-2

At the highest point of the trajectory, the directions of the velocity and acceleration of the body are perpendular to each other.

8. **Statement-1**

One end of a string of length R is tied to stone of mass m and the other end to a small pivot on a frictionless vertical board. The stone is whirled in a vertical circle with the pivot as the centre. The minimum speed the stone must have, when it is at the topmost point on the circle, so that the string does not slack is $\sqrt{gR}$.

Statement-2

At the topmost point on the circle, the centripetal force is provided partly by tension in the string and partly by the weight of the stone.

9. **Statement-1**

The maximum range on an inclined plane when a body is projected upwards from the base of the plane is less than that when it is projected downwards from the top of the same plane with the same speed.

Statement-2

The maximum range along an inclined plane is independent of the angle of inclination of the plane.

10. **Statement-1**

In projectile motion, the velocity of the body at a point on it trajectory is equal to the slope at that point.

Statement-2

The velocity vector at a point is always along the tangent to the trajectory at that point.

11. **Statement-1**

In a uniform circular motion, the centripetal force is always perpendicular to the velocity vector.

Statement-2

Then the force does no work on the body and its kinetic energy remains constant.

12. **Statement-1**

In a non-uniform circular motion, the particle has two accelerations-one along the tangent to the circle and the other towards the centre of the circle.

Statement-2

In a non-uniform circular motion, the magnitude and the direction of the velocity vector both change with time.

13. **Statement-1**

In a non-uniform circular motion, the acceleration of the particle is equal to sum of the tangential acceleration and the centripetal acceleration.

Statement-2

The two accelerations are perpendicular to each other.

14. **Statement-1**

In a uniform circular motion, the kinetic energy of the body remains constant.

Statement-2

The momentum of the body does not change with time.

15. **Statement-1**

In a uniform circular motion, the acceleration is always directed towards the centre of the circle.

Statement-2

Otherwise the speed of the body moving along the circle will change with time.

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1. The correct choice is (b).
2. The correct choice is (c). The velocity of a body moving along a curve continuously changes

because its direction of motion is changing. Hence a body moving along a curve with a constant speed has acceleration called centripetal acceleration.

3. The correct choice is (a). The effective acceleration due to gravity in a meduim is given by

$$g_{\text{eff}} = g\left(1 - \frac{\rho}{\sigma}\right)$$

where ρ = density of the medium and σ = density of the body.

4. The correct choice is (a). If the displacement-time graph is parallel to the time axis, then, rate of change of displacement is zero.

5. The correct choice is (d). If the velocity-time graph is a curve, the slope of the graph is not constant.

6. The correct choice is (a). The time taken by the body to hit the ground is the same as if it was dropped from that height and fell freely under gravity.

7. The correct choice is (b). If m is the mass of the body and u its velocity of projection, the initial kinetic energy is

$$K = \frac{1}{2}mu^2$$

At the highest point, the horizontal velocity is (u cos 60°) and vertical velocity is zero. Hence the kinetic energy at the highest point is

$$K' = \frac{1}{2}m\,(u\cos 60°)^2 = \frac{1}{4} \times \frac{1}{2}mu^2 = \frac{K}{4}$$

At the highest point of the trajectory, the velocity of the body is horizontal (parallel to the ground) but its acceleration is g directed vertically downwards.

8. The correct choice is (a). When the stone is at the topmost point A on the circle, the centripetal force is provided by ($mg + T$) as shown in Fig. 2.52.

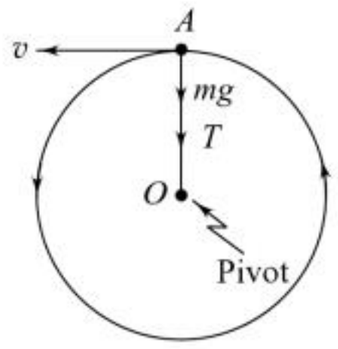

Fig. 2.52

Thus $$\frac{mv^2}{R} = mg + T$$

When the stone is at A, the string will not slack if tension $T = 0$, which gives $\frac{mv^2}{R} = mg \quad \Rightarrow v = \sqrt{Rg}$.

9. The range along the inclined plane when a body is projected with velocity u at an angle θ with the horizontal is given by

$$R = \frac{2u^2 \sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

$$= \frac{u^2}{g\cos^2\alpha}[\sin(2\theta - \alpha) - \sin\alpha]$$

where α is the angle of inclination of the plane. Range R will be maximum of $\sin(2\theta - \alpha) = 1$ or $2\theta - \alpha = 90°$ in which case

$$R_{\max} = \frac{u^2}{g\cos^2\alpha}[1 - \sin\alpha] = \frac{u^2}{g(1+\sin\alpha)}$$

If the body is projected downwards from the top of the same inclined plane, the maximum range will be

$$R'_{\max} = \frac{u^2}{g(1-\sin\alpha)}$$

Thus $R'_{\max} > R_{\max}$. Since the range R depends on angle α, Statement 2 is false. Hence the correct choice is (c).

10. The correct choice is (d). At the highest point on the trajectory, the slope is zero but velocity is u cos θ.

11. The correct choice is (a).

12. The correct choice is (a).

13. The correct choice is (d). The acceleration of the particle is given by

$$a = \sqrt{a_c^2 + a_t^2}$$

where a_c = centripetal acceleration and a_t = tangential acceleration.

14. The correct choice is (c). The speed of the body remains constant but the momentum changes with time because the direction of the velocity vector changes with time.

15. The correct choices is (a). If the acceleration vector is not directed towards the centre of the circle, it will have a component along the tangent, as a result the speed of the body will change and the motion no longer remains uniform.

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3

Laws of Motion

REVIEW OF BASIC CONCEPTS

1. Newton's First Law of Motion

Newton's first law of motion states that *every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by an external unbalanced force.*

2. Newton's Second Law of Motion

Newton's second law of motion states that *the rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction in which the force acts.*

Linear Momentum Newton defined linear momentum as the product of the mass and the velocity of a body.

$$\mathbf{p} = m\mathbf{v}$$

where $\mathbf{v}$ is the velocity at a certain instant of time. Differentiating this equation with respect to time, we get

$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} \qquad (\because m \text{ is constant})$$

$$F = m\mathbf{a}$$

where $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$ is the acceleration produced.

NOTE

(1) Force = slope of momentum–time ($p - t$) graph.
(2) Change in momentum = area under the force–time ($F - t$) graph.

Example 1 A constant force acts for 0.5s on a body of mass 1.5 kg initially at rest. When the force ceases to act, the body is found to travel a distance of 5.0 m in 2.0 s in the direction of the force. Find the magnitude of the force applied.

Solution According to Newton's first law, when the force ceases to act, the body will move with a uniform velocity given by

$$v = \frac{5}{2} = 2.5 \text{ ms}^{-1}$$

Using $u = 0$, $v = 2.5$ ms^{-2} and $t = 0.5$ s in $v = u + at$, we get $a = 5$ ms^{-2}. From Newton's second law,

$$\text{Force } F = ma = 1.5 \times 5 = 7.5 \text{ N}.$$

Example 2 Two forces each of magnitude 10 N act on a body of mass 5 kg at an angle of 120°. Find the magnitude of acceleration produced.

Solution $F_1 = F_2 = 10$ N and $\theta = 120°$. Resultant force is

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{10^2 + 10^2 + 2 \times 10 \times 10 \times \left(-\frac{1}{2}\right)} = 10 \text{ N}$$

$$\therefore \qquad a = \frac{F}{m} = \frac{10}{5} = 2 \text{ ms}^{-2}$$

Example 3 The velocity of a body of mass 2 kg changes from $\mathbf{v}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ ms^{-1} to $\mathbf{v}_2 = (-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ ms^{-1} in 3 s. Find (a) the magnitude of the change in momentum of the body and (b) the magnitude of the force applied.

Solution

(a) Change im momentum = final momentum – initial momentum

or
$$\Delta\mathbf{p} = \mathbf{m}\mathbf{v}_2 - m\mathbf{v}_1$$

$$= m[(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})]$$

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$$= m(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$$\therefore \quad |\Delta \mathbf{p}| = \sqrt{(-5)^2 + (-1)^2 + (4)^2}$$

$$= 2 \times \sqrt{42} = 12.96 \text{ kg ms}^{-1}$$

(b) $|\mathbf{F}| = \dfrac{|\Delta \mathbf{p}|}{\Delta t} = \dfrac{12.96}{3} = 4.32 \simeq 4.3 \text{ N}$

Example 4 A particle of mass 1 g is moving along the positive x-axis under the influence of a force.

$$F = -\frac{k}{x^2}$$

where $k = 10^{-3}$ Nm^2. When the particle is at $x = 1.0$ m, its velocity $v = 0$. Find (a) the magnitude of its velocity when, it reaches $x = 0.5$ m and (b) its position when its speed is 1 ms^{-1}.

Solution

$$F = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt} = mv\frac{dv}{dx}$$

Given, $F = -\dfrac{k}{x^2}$. Therefore

$$-\frac{k}{x^2} = mv\frac{dv}{dx} \Rightarrow v\,dv = -\frac{k}{mx^2}dx$$

Integrating, we have

$$\int v\,dv = -\frac{k}{m}\int x^{-2}\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{k}{mx} + c \qquad \text{(i)}$$

where c is the constant of integration. Given $v = 0$ when $x = 1.0$ m. Using this in eq. (i), we get $c = -\dfrac{k}{m}$. Equation (i) becomes

$$\frac{v^2}{2} = \frac{k}{mx} - \frac{k}{m}$$

$$\Rightarrow \quad v = \left[\frac{2k}{m}\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

$$v = \left[\frac{2 \times 10^{-3}}{10^{-3}}\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

$[\because k = 10^{-3} \text{ Nm}^2 \text{ and } m = 10^{-3} \text{ kg}]$

$$\Rightarrow \quad v = \left[2\left(\frac{1}{x} - 1\right)\right]^{1/2}$$

(a) when $x = 0.5$ m, $v = \left[2\left(\dfrac{1}{0.5} - 1\right)\right]^{1/2} = \sqrt{2}$ ms^{-1}

(b) when $v = 1$ ms^{-1}, $1 = \left[2\left(\dfrac{1}{x} - 1\right)\right]^{1/2} \Rightarrow x = 0.67$ m

3. Newton's Third Law of Motion

Newton's third law of motion states that *whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first, or, to every action there is an equal and opposite reaction*. The action and reaction forces act on different body.

4. Law of Conservation of Linear Momentum

The *law of conservation of linear momentum* may be stated as '*when no net external force acts on a system consisting of several particles, the total linear momentum of the system is conserved, the total linear momentum being the vector sum of the linear momentum of each particle in the system*'.

Recoil of a Gun

The gun and the bullet constitute a two-body system. Before the gun is fired, both the gun and the bullet are at rest. Therefore, the total momentum of the gun-bullet system is zero. After the gun is fired, the bullet moves forward and the gun recoils backwards. Let m_b and m_g be the masses of the bullet and the gun. If $\mathbf{v}_b$ and $\mathbf{v}_g$ are their respective velocities after firing, the total momentum of the gun-bullet system after firing is ($m_b \mathbf{v}_b + m_g \mathbf{v}_g$). From the law of conservation of momentum, the total momentum after and before the gun is fired must be the same, i.e.

$$m_b \mathbf{v}_b + m_g \mathbf{v}_g = 0$$

or $$\mathbf{v}_g = -\frac{m_b \mathbf{v}_b}{m_g}$$

The negative sign indicates that the gun recoils in a direction opposite to that of the bullet. In terms of magnitudes, we have

$$v_g = \frac{m_b v_b}{m_g}$$

5. Impulse

Consider a collision between two bodies A and B moving in the same straight line. Let Δt be the duration of the collision, i.e. the time for which the bodies were in contact during which time the transfer of momentum took place. We assume that the bodies continue moving in the same straight line after the collision with velocities different from their initial velocities.

Impulse of a force is the product of the average force and the time for which the force acts and it is equal to the change in momentum of the body during that time. Impulse is a vector and is measured in kg m s^{-1} or N s.

$$\mathbf{I} = \mathbf{F}_{av}\,\Delta t = \Delta \mathbf{p}$$

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Example 5 A ball of mass m is moving with a velocity $\mathbf{v}$ towards a rigid vertical wall. After striking the wall, the ball deflects through an angle θ without change in its speed. Obtain the expression for the impulse imparted to the ball.

Solution Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the initial and final velocities of the ball [Fig. 3.1(a)]

$$\text{Impulse} = \text{change in momentum}$$
$$= m\mathbf{v}_2 - m\mathbf{v}_1 = m(\mathbf{v}_2 - \mathbf{v}_1) = m[\mathbf{v}_2 + (-\mathbf{v}_1)]$$

or
$$\text{Impulse} = m\Delta\mathbf{v}$$

where $\Delta\mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$ is the resultant of $\mathbf{v}_2$ and $-\mathbf{v}_1$ [Fig. 3.1(b)].

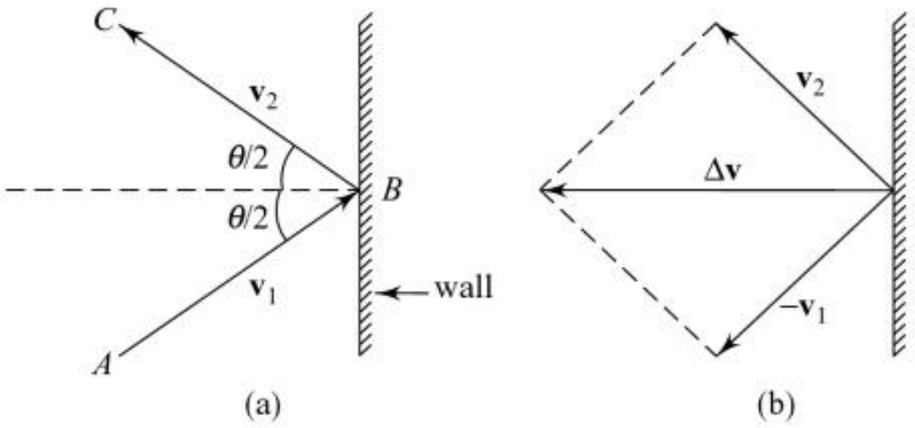

Fig. 3.1

Magnitude of $\Delta\mathbf{v}$ is
[$\because$ magnitude of $\mathbf{v}_1$ = magnitude of $\mathbf{v}_2 = v$]

$$\Delta v = \sqrt{v_1^2 + v_1^2 + 2v_1v_2\cos\theta}$$
$$= \sqrt{v^2 + v^2 + 2v^2\cos\theta} = \sqrt{2v^2(1+\cos\theta)}$$
$$= 2v\cos\left(\frac{\theta}{2}\right)$$

The direction of impulse is perpendicular to the wall and away from it.

6. Contact Forces

(1) *Normal Reaction*

The force exerted by one body when placed on the surface of another body is known as contact force. If the two surfaces in contact are perfectly smooth (i.e., frictionless), then the contact force acts only perpendicular (normal) to their surface of contact and is known as *normal reaction* (R).

If a block of mass m is placed on a horizontal frictionless surface [Fig. 3.2 (a)], the normal reaction $R = mg$. If the block is placed on an inclined plane of inclination α [Fig. 3.2 (b)], the normal reaction $R = mg\cos\alpha$

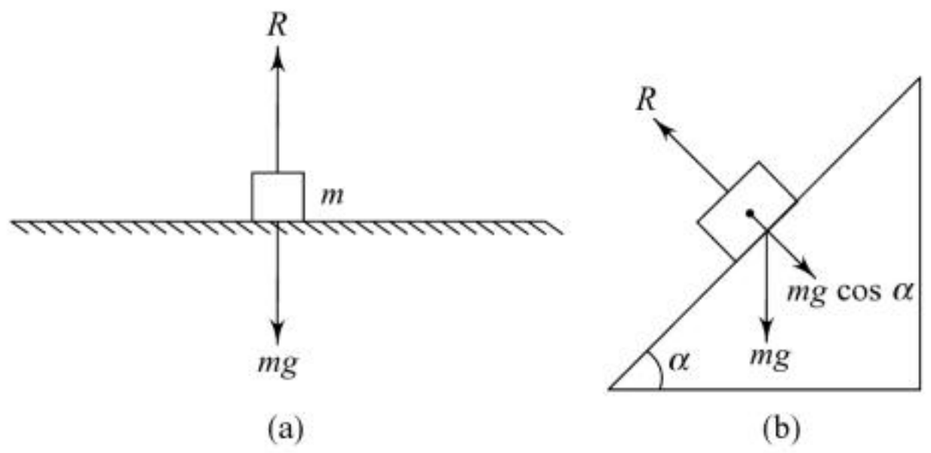

Fig. 3.2

If there is friction between the surface of contact, then the component of the contact force perpendicular to their surface gives the normal reaction and the other component which acts along the tangent to the surface of contact gives the force of friction. The normal reaction, tension and friction are examples of contact force.

(2) Tension

The force in a string is called tension (T). If the string is massless, the tension has the same magnitude at all points of the string. Tension in the string always acts away from the body to which it is attached. If the string passes over a frictionless pulley and its ends are attached to two bodies, the tension in the entire string has the same magnitude and its direction is towards its point of contact with the pulley.

6. Friction

Friction is the force which comes into play when one body slides or rolls over the surface of another body and acts in a direction tangential to the surfaces in contact and opposite to the direction of motion of the body.

The maximum (or limiting) force of friction when a body just begins to slide over the surface of another body is called the *limiting friction*. The force of friction just before one body begins to slide over another is called the *limiting* or *static friction* (f_s). The coefficient of limiting or static friction (μ_s) is defined as

$$\mu_s = \frac{f_s}{R}$$

where R is the normal reaction, i.e. the normal force pressing the two surfaces together.

The force necessary to maintain a body in uniform motion over the surface of another body, after motion has started, is called the *kinetic* or *sliding friction* (f_k). The *coefficient of kinetic friction* (μ_k) is defined as

$$\mu_k = \frac{f_k}{R}$$

Note that μ_k is always less than μ_s.

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Angle of Friction Angle of friction is the angle between the resultant of the force of limiting friction (f) and the normal reaction (R). In Fig. 3.3, θ is the angle of friction, which is given by

$$\tan\theta = \frac{f}{R} = \mu$$

$$\theta = \tan^{-1}(\mu)$$

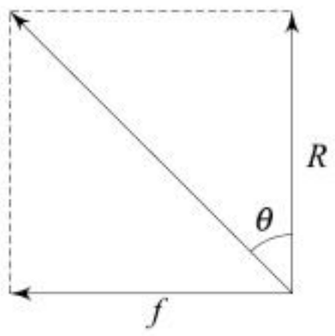

Fig. 3.3

Angle of Repose Suppose a body is placed on an inclined plane. The angle of inclination is gradually increased until the body just begins to slide along the plane. When this happens the angle of inclination α of the inclined surface with the horizontal is called the angle of repose (see Fig. 3.4). It follows from the figure that

Force of limiting friction $(f) = mg \sin\alpha$

Force of normal reaction $(R) = mg \cos\alpha$

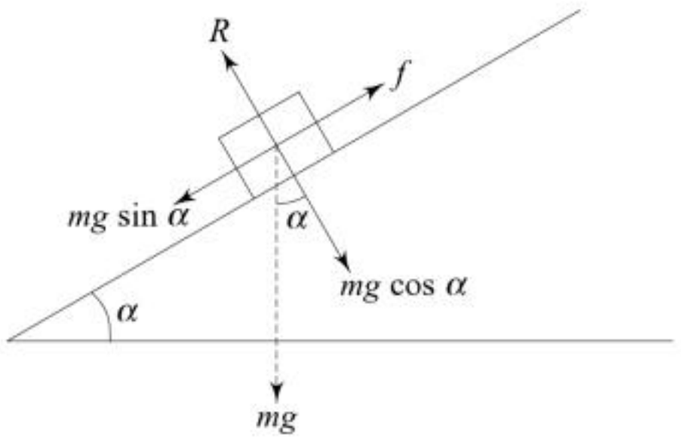

Fig. 3.4

Therefore, $\tan\alpha = \frac{f}{R} = \mu = \tan\theta$

or $\alpha = \theta = \tan^{-1}(\mu)$

7. Solving Problems in Mechanics by Free Body Diagram Method

In mechanics, we often have to handle problems which involve a group of bodies connected to one another by strings, pulleys, springs, etc. They exert forces on one another. Furthermore, there are frictional forces and the force of gravity acting on each body in the group. To solve such complicated problems, it is always convenient to choose one body in the group, find the magnitude and the direction of the forces acting on this body by all the remaining bodies in the group. Then we find the resultant of all the forces acting on the body to obtain the net force exerted on it. We then use the laws of motion to determine the dynamics of the body. We apply this procedure to all other bodies in the group one by one. It is useful to draw a separate diagram for each body, showing the directions of the different forces acting on it. Such a diagram is called the *free body diagram* (F.B.D.) of the body.

(1) Two masses tied to a string going over a frictionless pulley Consider two bodies of masses m_1 and m_2 ($m_1 > m_2$) connected by a string which passes over a pulley, as shown in Fig. 3.5(a). When the bodies are released, the heavier one moves downwards and the lighter one moves up.

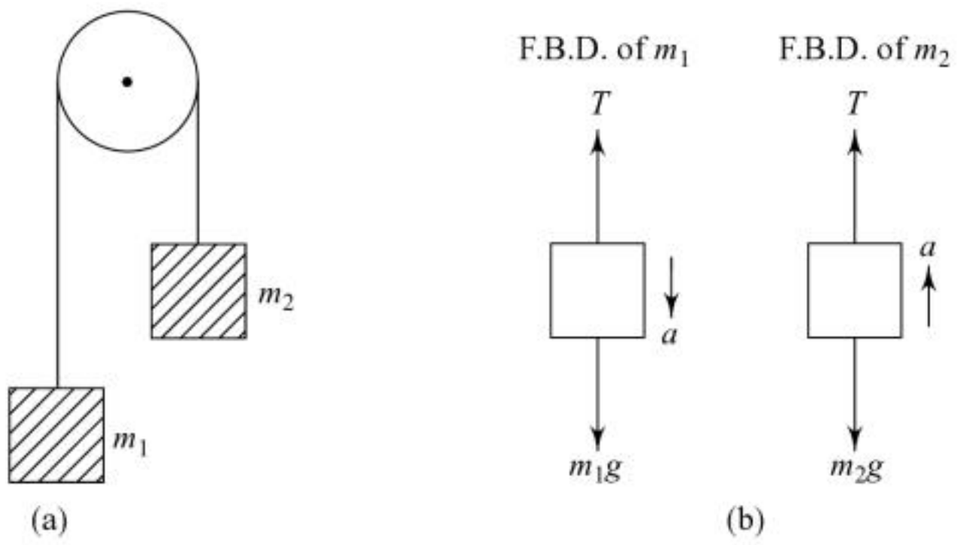

Fig. 3.5

Net force in the direction of motion of m_1 is $m_1g - T$. Therefore, the equation of motion of m_1 is

$$m_1g - T = m_1a \qquad \text{(i)}$$

Net force in the direction of motion of m_2 is $(T - m_2g)$. Therefore, the equation of motion of m_2 is

$$T - m_2g = m_2a \qquad \text{(ii)}$$

From (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

and $$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

(2) Two masses in contact Figure 3.6(a) shows two blocks of masses m_1 and m_2 placed in contact on a horizontal frictionless surface. A force F is applied to mass m_1. As a result, the masses move with a common acceleration a. To find a and the contact force on m_2, we draw the free body diagrams as shown in Figs. 3.6(b) and (c).

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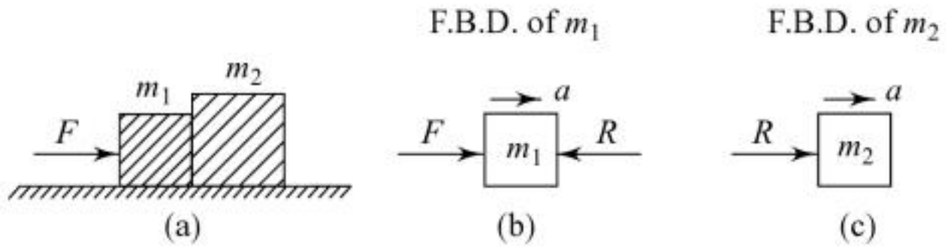


Fig. 3.6

R = normal reaction force between the blocks. From Figs. 3.6(b)and (c), we get

$$F - R = m_1 a \qquad \text{(i)}$$

and
$$R = m_2 a \qquad \text{(ii)}$$

Adding (i) and (ii) we get

$$F = (m_1 + m_2)a$$

$$\Rightarrow \qquad a = \frac{F}{(m_1 + m_2)}$$

Contact force on m_2 is

$$F_2 = m_2\, a = \frac{m_2 F}{(m_1 + m_2)}$$

(3) Three masses in contact Figure 3.7(a) shows three blocks of masses m_1, m_2, and m_3 placed in contact on a horizontal frictionless surface. A force F is applied to m_1. As a result, the three masses move with a common acceleration a. To find a and the contact forces on m_2 and m_3, we draw the free body diagrams as shown in Figs. 3.7(b) (c) and (d).

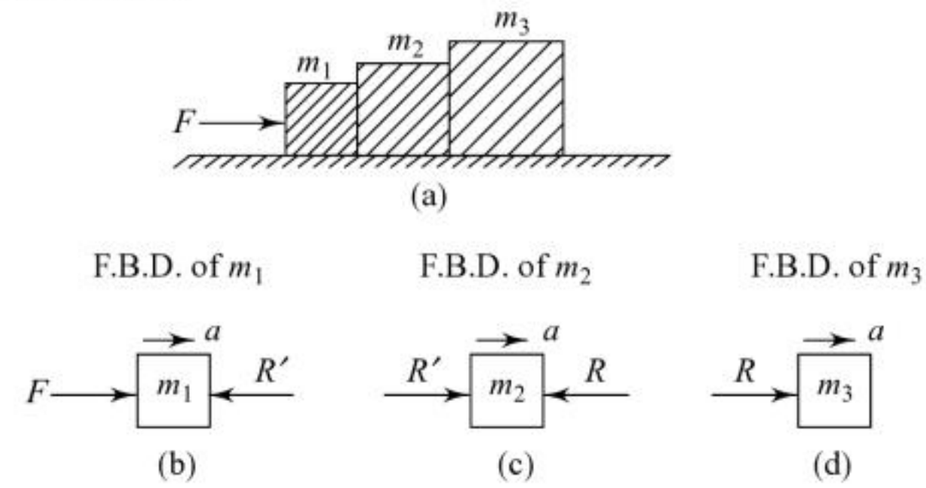


Fig. 3.7

R' = contact force on m_2 = reaction force between m_1 and m_2

R = contact force on m_3 = reaction force between m_2 and m_3

It follows from Figs. 3.7(b), (c) and (d) that

$$F - R' = m_1 a \qquad \text{(i)}$$

$$R' - R = m_2 a \qquad \text{(ii)}$$

and
$$R = m_3 a \qquad \text{(iii)}$$

Adding (i), (ii) and (iii) we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

Contact force on m_2 is $F_2 = R'$ which from (ii) is given by

$$F_2 = R' = R + m_2 a$$

Using (iii) we have

$$F_2 = m_3 a + m_2 a$$

$$= (m_2 + m_3)a$$

$$\Rightarrow \qquad F_2 = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

Contact force on m_3 is

$$F_3 = R = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(4) Two masses connnected with a string Figure 3.8(a) shows two blocks of masses m_1 and m_2 connected with a string and lying on a horizontal frictionless surface. A force F is applied to m_2. As a result, the masses move with a common acceleration a. To find a and force exerted on m_1, we draw the free body diagrams as shown in Figs. 3.8(b) and (c). T is the tension in the string.

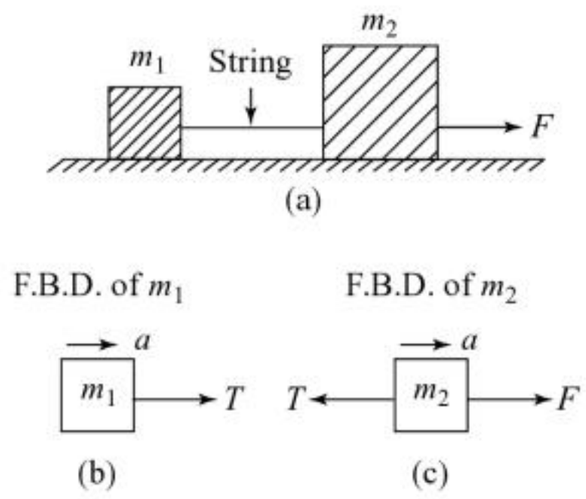


Fig. 3.8

It follows from Figs. 3.8(b) and (c) that

$$T = m_1 a \qquad \text{(i)}$$

and
$$F - T = m_2 a \qquad \text{(ii)}$$

Adding (i) and (ii) we get

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = m_1 a = \frac{m_1 F}{m_1 + m_2}$$

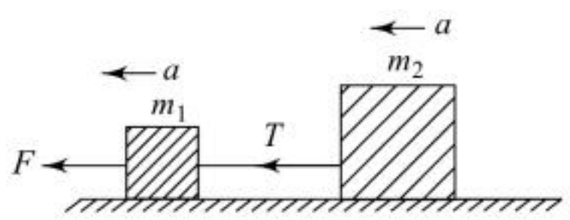


Fig. 3.9

If force F is applied on mass m_1 as shown in Fig. 3.9, then

$$a = \frac{F}{m_1 + m_2}$$

Tension in the string is

$$T = \frac{m_2 F}{m_1 + m_2}$$

(5) Three masses connected by strings Figure 3.10 (a) shows three blocks of masses m_1, m_2 and m_3 connected by two strings and placed on a horizontal frictionless surface. A force F is applied to m_1. As a result, the blocks move with a common acceleration a. To find a and the forces acting on m_2 and m_3, we draw free body diagrams as shown in Fig. 3.10(b) and (c) and (d). T is the tension in the string between m_1 and m_2 and T' is the tension in the string between m_2 and m_3.

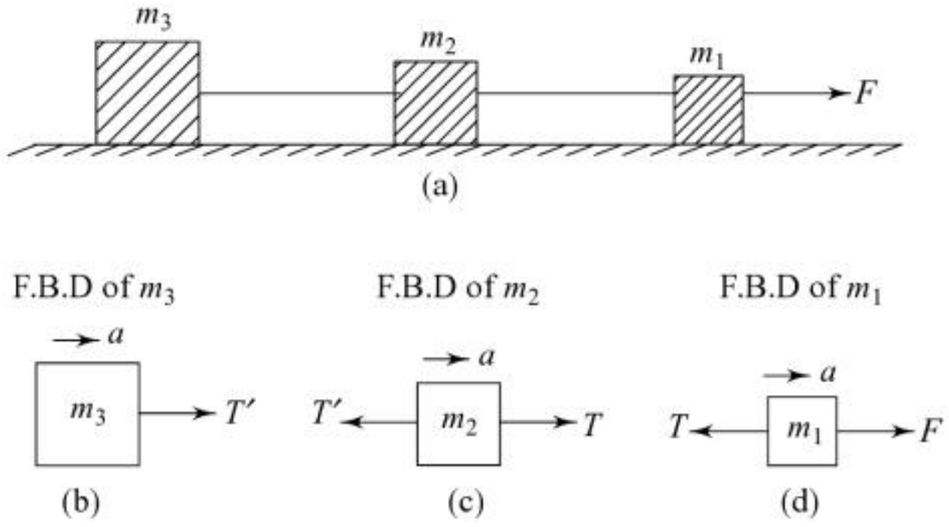


Fig. 3.10

It follows from Figs. 3.10(b), (c) and (d) that

$$T' = m_3 a \quad \text{(i)}$$

$$T - T' = m_2 a \quad \text{(ii)}$$

and $$F - T = m_1 a \quad \text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_1 and m_2 is T, which is obtained by adding (i) and (ii).

$$T = (m_2 + m_3)a = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

The tension in the string between m_2 and m_3 is T', which from (i) is given by

$$T' = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

(6) Two masses connected by a string and suspended from a support Two blocks of masses m_1 and m_2 are connected by two strings and suspended from a support as shown in Fig. 3.11(a). Mass m_2 is pulled down by a force F. The tension T is the string between m_1 m_2 and tension T' in the string between m_1 and the support can be found from the free body diagrams as shown in Fig. 3.11(b) and (c).

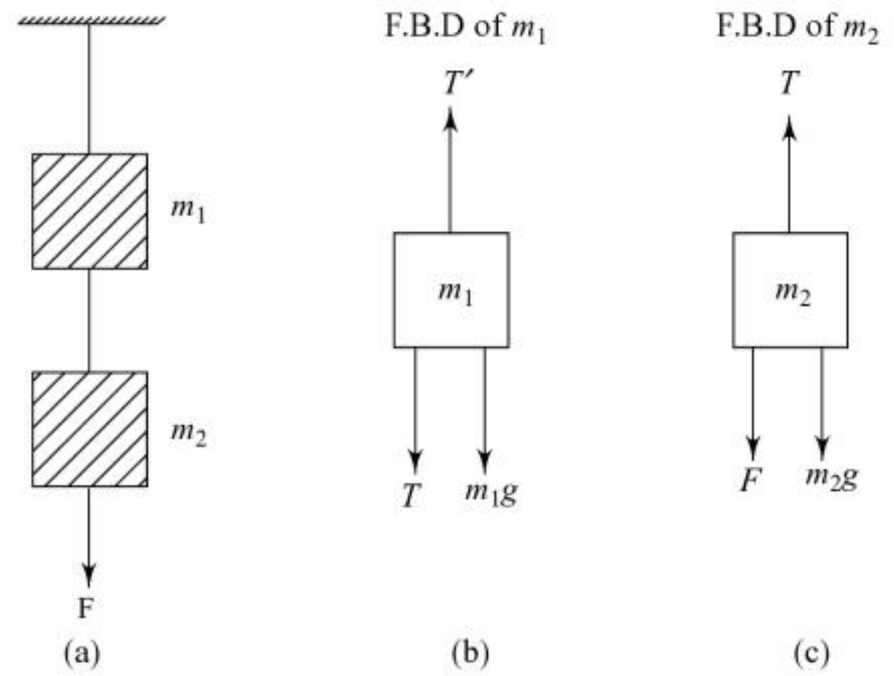


Fig. 3.11

$$T' = T + m_1 g \quad \text{(i)}$$

$$T = F + m_2 g \quad \text{(ii)}$$

Using (ii) and (i), we get

$$T' = F + (m_1 + m_2)g$$

(7) Two blocks connected by a string passing over a frictionless pulley fixed at the edge of a horizontal table Consider a block of mass m_1 lying on a frictionless table connected through a pulley to another block of mass m_2 hanging vertically (Fig. 3.12). When the system is released, let acceleration of the blocks be a.

From free body diagrams, the equations of motion of m_1 and m_2 are

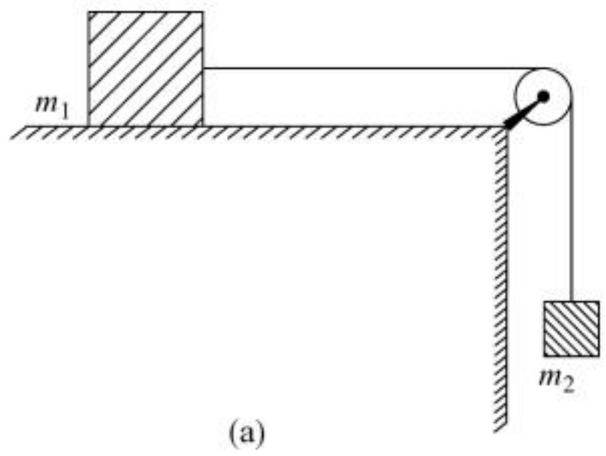
$$T = m_1 a \quad \text{(i)}$$

and $$m_2 g - T = m_2 a \quad \text{(ii)}$$

Adding (i) and (ii), we get

$$a = \frac{m_2 g}{(m_1 + m_2)}$$

Also $$T = m_1 a = \frac{m_1 m_2 g}{(m_1 + m_2)}$$



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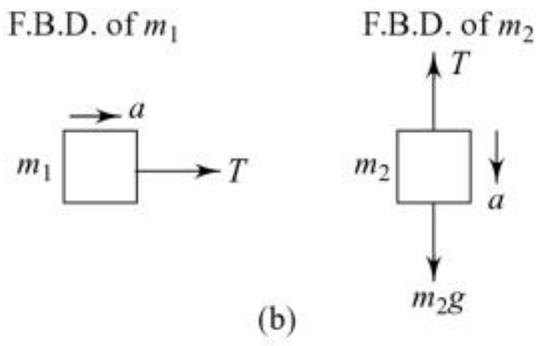


Fig. 3.12

If the table top is frictionless, the blocks will move even if $m_2 < m_1$.

If μ is the coefficient of friction between block m_1 and the table, the force of friction is

$$f = \mu R = \mu m_1 g$$

From F.B.D. of m_1 (Fig. 3.13), Eq. (i) becomes

$$T - f = m_1 a \quad \Rightarrow \quad T - \mu m_1 g = m_1 a$$

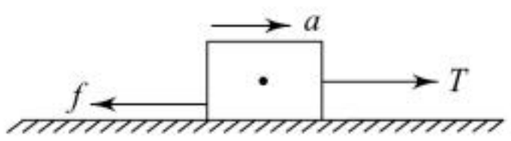
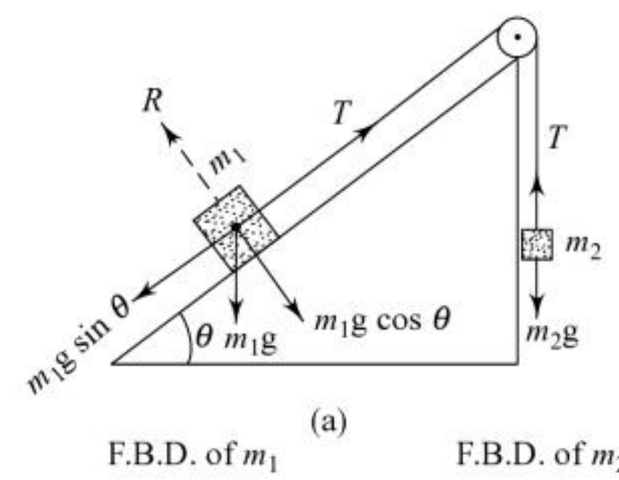


Fig. 3.13

(8) Two blocks connected by a string passing over a frictionless pulley fixed at the top of an inclined plane Let T be the tension in the string. Since the pulley is frictionless, the tension is the same throughout the string (Fig. 3.14). There are the following two cases:

(a) Mass m_1 moving up along the incline with acceleration a [Fig. 3.14]



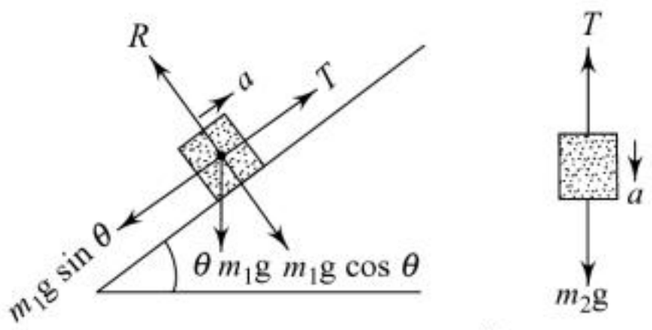


Fig. 3.14

The equations of motion of m_1 and m_2 [see Fig. 3.14(b)]

$$T - m_1 g \sin\theta = m_1 a \qquad \text{(i)}$$

$$m_2 g - T = m_2 a \qquad \text{(ii)}$$

which give

$$a = \frac{(m_2 - m_1 \sin\theta) g}{(m_1 + m_2)}$$

and

$$T = m_2 (g - a)$$

If μ is the coefficient of friction between m_1 and the inclined plane, the frictional force $f = \mu R = \mu m_1 g \cos\theta$ will act down the plane because the block m_1 is moving up the plane. In this case, Eq. (i) is replaced by

$$T - m_1 g \sin\theta - f = m_1 a$$

$$\Rightarrow \quad T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a$$

(b) Mass m_1 moves down the incline with acceleration a

In this case, we get $m_1 g \sin\theta - T = m_1 a$ and $T - m_2 g = m_2 a$ which give

$$a = \frac{(m_1 \sin\theta - m_2) g}{(m_1 + m_2)}$$

and $\quad T = m_2 (g + a)$

(9) Two blocks connected by a string passing over a frictionless pulley fixed at the top of a double inclined plane Let the block of mass m_1 move up along the inclined plane of angle of inclination θ_1, and the block of mass m_2 move down the inclined plane of angle of inclination θ_2 (Fig. 3.15). Let T be the tension in the string. Then, for m_1 and m_2, we have

$$T - m_1 g \sin\theta_1 = m_1 a$$

and

$$m_2 g \sin\theta_2 - T = m_2 a$$

Eliminating T, we get

$$a = \frac{(m_2 \sin\theta_2 - m_1 \sin\theta_1) g}{(m_1 + m_2)}$$

Also

$$T = m_1 (a + g \sin\theta_1)$$

$$= m_2 (g \sin\theta_2 - a)$$

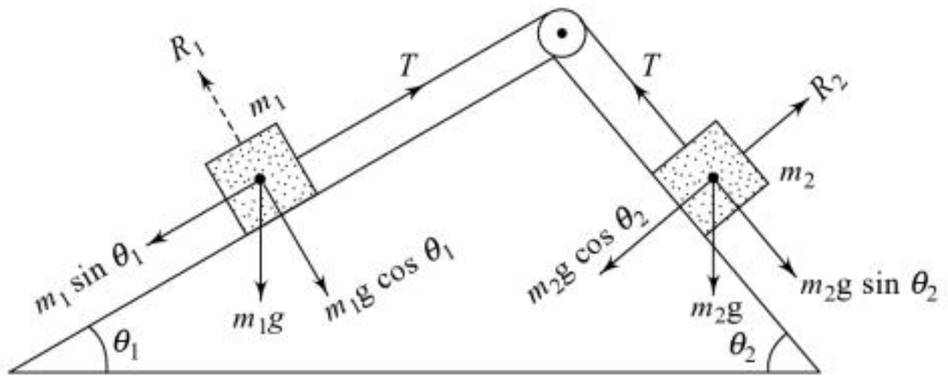


Fig. 3.15

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(a) the surface is frictionless and
(b) the coefficient of kinetic friction between the surface and the block is $\mu = 0.2$. Take $g = 10\ \text{ms}^{-2}$.

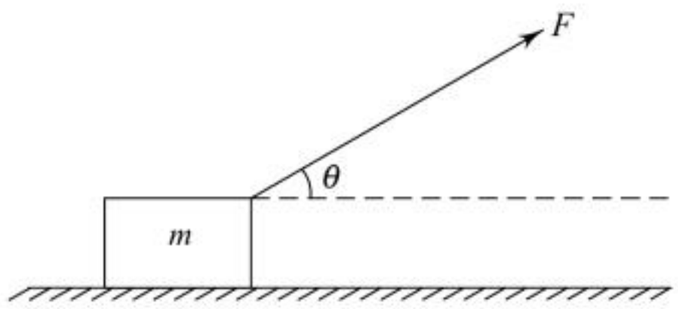

Fig. 3.18

Solution The free body diagrams of the block in the two cases are shown in Fig. 3.19.

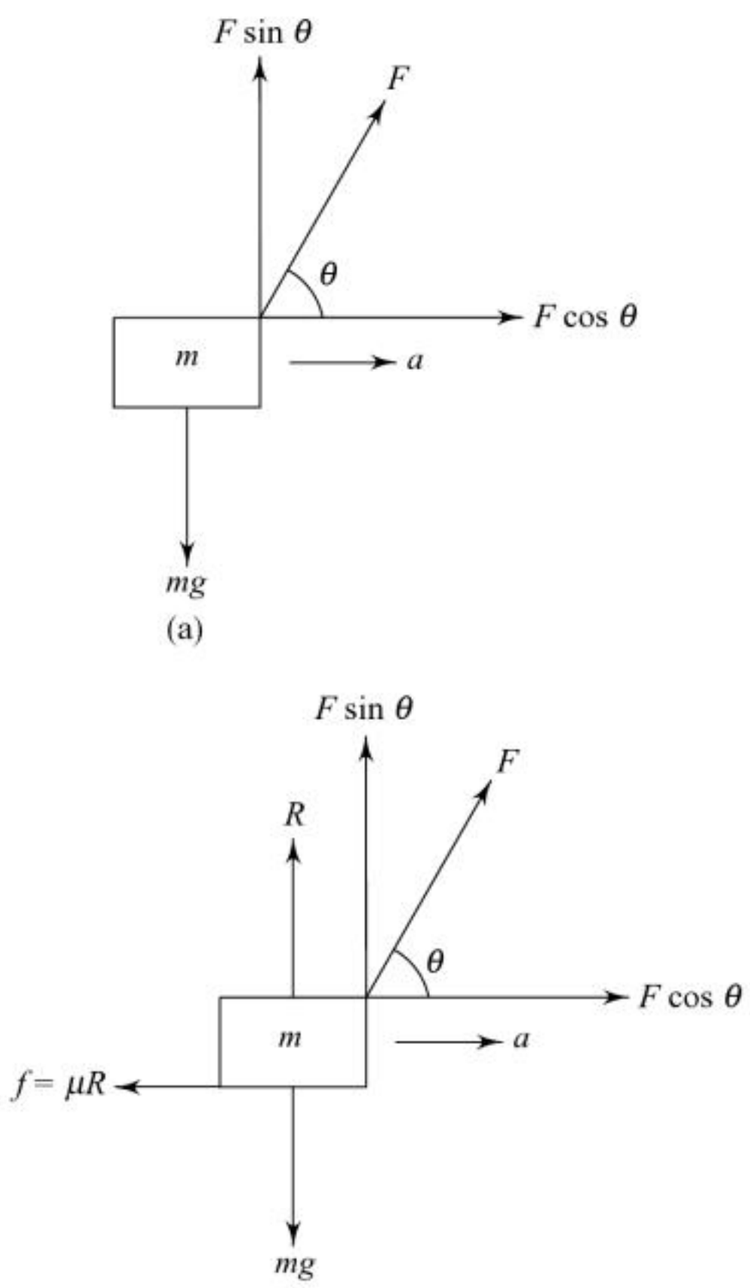

Fig. 3.19

(a) From Fig. 3.19(a)

$$F\cos\theta = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta}{m} = \frac{10\times\cos 60°}{1} = 5\ \text{ms}^{-2}$$

(b) From Fig. 3.19(b)

$$F\cos\theta - f = ma$$

$$\Rightarrow \quad F\cos\theta - \mu mg = ma$$

$$\Rightarrow \quad a = \frac{F\cos\theta - \mu mg}{m}$$

$$= \frac{10\cos 60° - 0.2\times 1\times 10}{1}$$

$$= 3\ \text{ms}^{-2}$$

From Fig. 3.19(b) we also have $F\sin\theta + R = mg$ or $F\sin\theta = mg - R$. Since $F\sin\theta < mg$, the block does not move upwards.

Example 7 Two blocks of masses $m_1 = 2$ kg and $m_2 = 3$ kg are suspended from a rigid support by means of strings AB and CD as shown in Fig. 3.20. String AB has negligible mass and string CD has mass 0.5 kg/m. Each string has length 50 cm. Find the tension (a) at mid-point P of string AB and (b) at point Q of string CD where $CQ = 20$ cm. Take $g = 10\ \text{ms}^{-2}$.

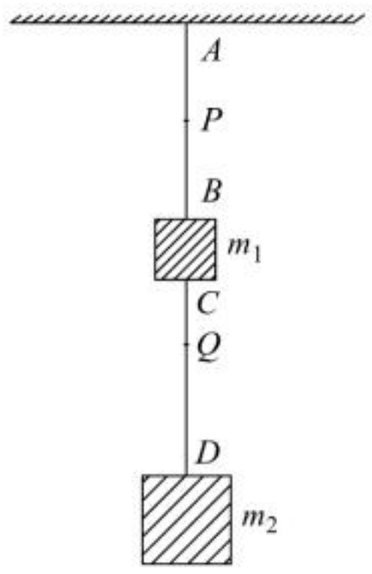

Fig. 3.20

Solution Mass of string CD is $m = 0.5 \times 0.5 = 0.25$ kg
Since string AB is massless, the tension in AB is the same at every point.

(a) Total mass below point $P = m_1 + m + m_2$

$$= 2 + 0.25 + 3 = 5.25\ \text{kg}$$

$$\therefore \quad \text{Tension at } P = 5.25 \times 10 = 52.5\ \text{N}$$

(b) Total mass below point Q = mass of length $QD + m_2$

$$= 0.5 \times 0.3 + 3 = 3.15\ \text{kg}$$

$$\therefore \quad \text{Tension at } Q = 3.15 \times 10 = 31.5\ \text{N}$$

Example 8 A block of mass $m = 100$ g is placed on an inclined plane of inclination $\theta = 30°$ as shown in Fig. 3.21. There is no friction between the block and the inclined plane. What minimum acceleration a should be given to the system to the left so that the block does not slide down the plane?

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$$\Rightarrow \qquad \mu > \frac{(2\pi\nu)^2 r}{g}$$

$\nu = 30$ rev/min $= \dfrac{30}{60} = 0.5$ Hz,

$r = 0.3$ m and $g = 9.8$ ms^{-2}.

$$\therefore \qquad \mu_{\min} = \frac{4\pi^2 v^2 r}{g}$$

$$= \frac{4 \times (3.14)^2 \times (0.5)^2 \times 0.3}{9.8}$$

$$= 0.3$$

Example 25 A small sphere of mass $m = 500$ g moving on the inner surface of a large hemispherical bowl of radius $R = 5$ m describes a horizontal circle at a distance $OC = 2.5$ m below the centre O of the bowl as shown in Fig. 3.47. Find the force exerted by the sphere on the bowl and the time period of revolution of the sphree around the circle. Take $g = 10$ ms^{-2}.

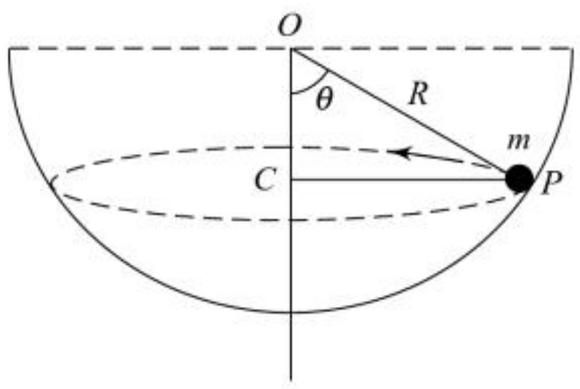

Fig. 3.47

Solution Given $OP = 5$ m and $OC = 2.5$ m. Therefore $\cos\theta = \dfrac{OC}{OP} = \dfrac{1}{2} \Rightarrow \theta = 60°$.

Radius of circle is $r = CP = OP \sin\theta = 5 \sin 60° = \dfrac{5\times\sqrt{3}}{2}$ m

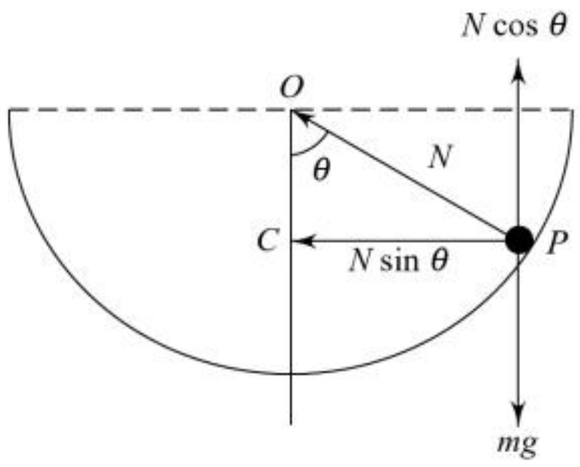

Fig. 3.48

Figure 3.48 shows the forces acting on the sphere, N is the normal reaction.

Net force towards centre C of the circle $= N \sin\theta$

$$\Rightarrow \qquad mr\omega^2 = N\sin\theta \qquad \text{(i)}$$

Also $\qquad mg = N\cos\theta \qquad$ (ii)

From (ii), $\qquad N = \dfrac{mg}{\cos\theta} = \dfrac{0.5\times 10}{\cos 60°} = 10$ N

From (i), $\qquad m\omega^2 R\sin\theta = N\sin\theta$

$$\Rightarrow \qquad \omega = \sqrt{\frac{N}{mR}} = \sqrt{\frac{10}{0.5\times 5}} = 2 \text{ rad s}^{-1}$$

$\therefore$ Time period $\qquad T = \dfrac{2\pi}{\omega} = \pi$ second $= 3.14$ s

Example 26 A steel metre rod of mass $m = 1.5$ kg rests with its upper end against a smooth vertical wall and its lower end on rough horizontal floor. What should be the minimum coefficient of friction between the ground and the rod so that it can be inclined at an angle of 30° with the floor without slipping?

Solution Refer to Fig. 3.49. Length of rod $AB = 1$ m

N_1 = normal reaction of the wall

N_2 = normal reaction of the floor

The frictional force (f) between the rod and the floor acts along AD. The weight mg of the rod acts at its midpoint (centre of mass) C so that $AC = AB/2$.

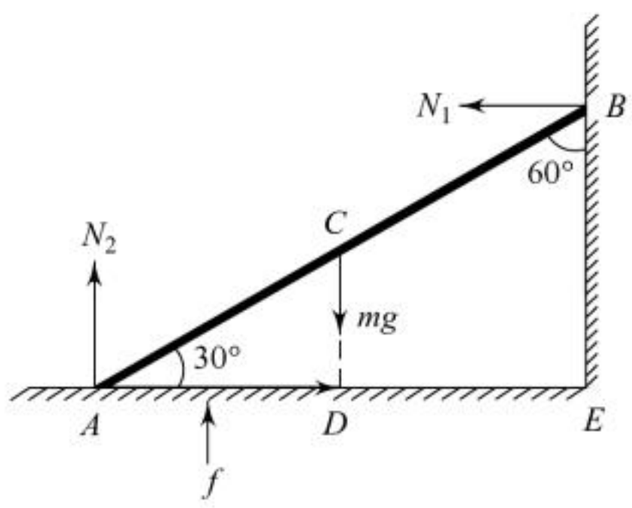

Fig. 3.49

For translational equilibrium,

$$N_2 = mg \quad \text{and} \quad N_1 = f$$

Taking moments of forces about A, we have for rotational equilibrium

$$N_2 \times 0 - mg \times AD + N_1 \times BE = 0$$

$$\Rightarrow \quad 0 - N_2 \times AD + f \times BE = 0 \ (\because mg = N_2, N_1 = f)$$

$$\Rightarrow \qquad \frac{f}{N_2} = \frac{AD}{BE} = \frac{AC\cos 30°}{AB\sin 30°}$$

$$\Rightarrow \qquad \mu = \frac{1}{2} \times \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} = 0.87$$

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31. A train starts from rest with a constant acceleration $a = 2\ \text{ms}^{-2}$. After 5 s, a stone is dropped from the window of the train. If the window is at a height of 2 m from the ground, what is the magnitude of the velocity of the stone 0.2 s after it was dropped? Take $g = 10\ \text{ms}^{-2}$.

(a) $4\sqrt{6}\ \text{ms}^{-1}$ (b) $10\ \text{ms}^{-1}$
(c) $2\sqrt{26}\ \text{ms}^{-1}$ (d) $12\ \text{ms}^{-1}$

32. In Q.31, the angle, which the resultant velocity vector makes with the horizontal is given by

(a) $\theta = \tan^{-1}(0.1)$ (b) $\theta = \tan^{-1}(0.2)$
(c) $\theta = \tan^{-1}(0.3)$ (d) $\theta = \tan^{-1}(0.4)$

33. In Q.31, the acceleration of the stone after it is dropped is given by

(a) $g + a$ (b) $\sqrt{g^2 + a^2}$
(c) a (d) g

34. A boy of mass 50 kg is standing on a weighing machine placed on the floor of a lift. The machine reads his weight in newton. What is the reading of the machine when the lift is stationary?

(a) zero (b) 50 N
(c) 490 N (d) 500 N

35. In Q.34, what is the reading of the machine if the lift is moving upwards with a uniform speed of $10\ \text{ms}^{-1}$?

(a) 470 N (b) 480 N
(c) 490 N (d) 500 N

36. In Q.34, what is the reading of the machine if the lift is moving downwards with a uniform acceleration of $5\ \text{ms}^{-2}$?

(a) 240 N (b) 490 N
(c) 720 N (d) none of these

37. In Q.34, what is the reading of the machine if the lift is moving upwards with a uniform acceleration of $5\ \text{ms}^{-2}$?

(a) 240 N (b) 490 N
(c) 740 N (d) none of these

38. In Q.34, what would be the reading of the machine if the connecting rope of the lift suddenly breaks and the lift begins to fall freely under gravity?

(a) zero (b) 240 N
(c) 490 N (d) 720 N

39. A rope which can withstand a maximum tension of 400 N is hanging from a tree. If a monkey of mass 30 kg climbs on the rope, in which of the following cases will the rope break? Take $g = 10\ \text{ms}^{-2}$ and neglect the mass of the rope.

(a) The monkey climbs up with a uniform speed of $5\ \text{ms}^{-1}$.
(b) The monkey climbs up with a uniform acceleration of $2\ \text{ms}^{-2}$.
(c) The monkey climbs up with a uniform acceleration of $5\ \text{ms}^{-2}$.
(d) The monkey climbs down with a uniform acceleration of $5\ \text{ms}^{-2}$.

40. A stream of a liquid of density ρ flowing horizontally with a speed v gushes out of a tube of radius r and hits at a vertically wall nearly normally. Assuming that the liquid does not rebound from the wall, the force exerted on the wall by the impact of liquid is given by

(a) $\pi r \rho v$ (b) $\pi r \rho v^2$
(c) $\pi r^2 \rho v$ (d) $\pi r^2 \rho v^2$

41. A shell explodes into three fragments of equal masses. Two fragments fly off at right angles to each other with speeds of $9\ \text{ms}^{-1}$ and $12\ \text{ms}^{-1}$. What is the speed of the third fragment?

(a) $9\ \text{ms}^{-1}$ (b) $12\ \text{ms}^{-1}$
(c) $15\ \text{ms}^{-1}$ (d) $18\ \text{ms}^{-1}$

42. A cricket ball of mass 150 g is moving with a velocity of $12\ \text{ms}^{-1}$ and is hit by a bat so that the ball is turned back with a velocity of $20\ \text{ms}^{-1}$. The force of the blow acts for 0.1 s. What is the average force exerted on the ball by the bat?

(a) 18 N (b) 30 N
(c) 48 N (d) 60 N

43. Two billiard balls each of mass 50 g, moving in opposite directions each with a speed $6\ \text{ms}^{-1}$, collide and rebound with the same speed. The impulse imparted to each ball due to the other is

(a) 0.3 Ns (b) 0.6 Ns
(c) 0.9 Ns (d) 1.2 Ns

44. A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is given by

(a) $mv\cos(\theta)$ (b) $mv\sin(\theta)$
(c) $2mv\cos\left(\frac{\theta}{2}\right)$ (d) $2mv\sin\left(\frac{\theta}{2}\right)$

45. Figure 3.54 shows the position–time (x–t) graph of one-dimensional motion of a body of mass 0.4 kg. What is the time interval between consecutive impulses received by the body?

(a) 2 s (b) 4 s
(c) 8 s (d) 16 s

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79. In Q.77, the contact force acting on mass m_3 is

(a) 12 N (b) 10 N
(c) 8 N (d) 6 N

80. Three blocks of masses $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are connected by massless strings and placed on a horizontal frictionless surface as shown in Fig. 3.62. A force $F = 12$ N is applied to mass m_1 as shown. The acceleration of the system is

(a) 12 ms^{-2} (b) 6 ms^{-2}
(c) 4 ms^{-2} (d) 2 ms^{-2}

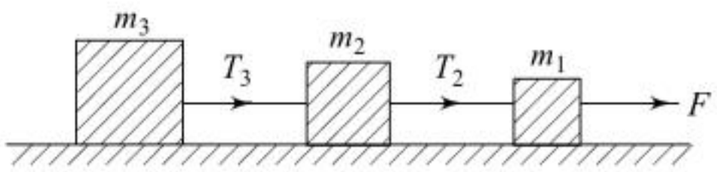

Fig. 3.62

81. In Q.80, what is tension T_2 between masses m_1 and m_2?

(a) 12 N (b) 10 N
(c) 8 N (d) 6 N

82. In Q.81, what is tension T_3 between masses m_2 and m_3?

(a) 12 N (b) 10 N
(c) 8 N (d) 6 N

83. A ball of mass m is connected to a ball of mass M by means of a massless spring. The balls are pressed so that the spring is compressed. When released, ball of mass m moves with acceleration a. The magnitude the acceleration of mass M will be

(a) $\frac{ma}{(M+m)}$ (b) $\frac{Ma}{(M+m)}$
(c) $\frac{ma}{M}$ (d) $\frac{Ma}{m}$

84. A boy, while catching a ball, experiences an impulse of 6 Ns. If the mass of the ball is 200 g, what was the speed of the ball before it was caught?

(a) 10 ms^{-1} (b) 20 ms^{-1}
(c) 30 ms^{-1} (d) 40 ms^{-1}

85. A car moving at a speed v is stopped by a retarding force F in a distance s. If the speed of the car were 3 v, the force needed to stop it within the same distance s will be

(a) 3 F (b) 6 F
(c) 9 F (d) 12 F

86. A car moving at a speed v is stopped by a retarding force F in a distance s. If the retarding force were $3F$, the car will be stopped in a distance

(a) $\frac{s}{3}$ (b) $\frac{s}{6}$
(c) $\frac{s}{9}$ (d) $\frac{s}{12}$

87. Two blocks of masses m and M are placed on a horizontal frictionless table connected by a spring as shown in Fig. 3.63. Mass M is pulled to the right with a force F. If the acceleration of mass m is a, the acceleration of mass M will be

(a) $\frac{(F-ma)}{M}$ (b) $\frac{(F+ma)}{M}$
(c) $\frac{F}{M}$ (d) $\frac{am}{M}$

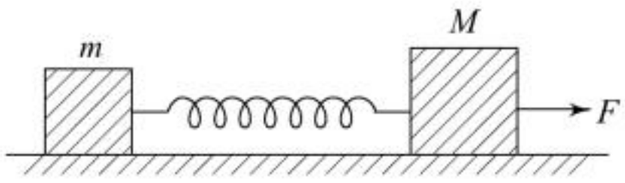

Fig. 3.63

88. A boy wants to climb down a rope. The rope can withstand a maximum tension equal to two–thirds the weight of the boy. If g is the acceleration due to gravity, the minimum acceleration with which the boy should climb down the rope should be

(a) $\frac{g}{3}$ (b) $\frac{2g}{3}$
(c) g (d) zero

89. A mass m is suspended from a rigid support P by means of a massless string as shown in Fig. 3.64. A horizontal force F is applied at point O of the string. The system is in equilibrium when the string makes an angle θ with the vertical. Then the relation between the tension T, force F and angle θ is

(a) $F = T \sin\theta$ (b) $F = T \cos\theta$
(c) $F = \frac{T}{\sin\theta}$ (d) $F = \frac{T}{\cos\theta}$

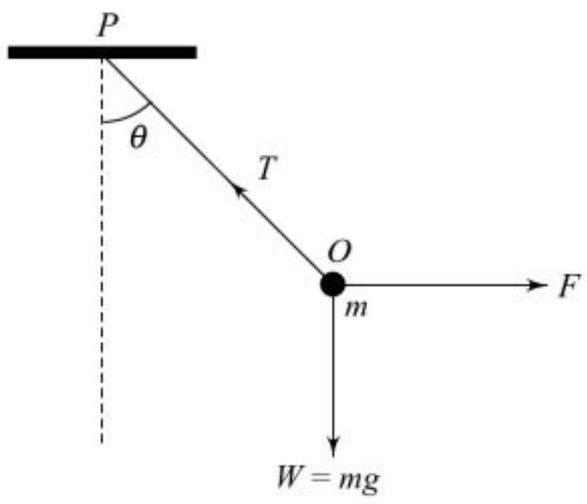

Fig. 3.64

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(a) 7.5 cm (b) 10 cm
(c) 12.5 cm (d) 15 cm

160. A train rounds an unbanked circular bend of radius 50 m at a speed of 54 km h^{-1}. If $g = 10\ ms^{-2}$, the angle of banking required to prevent wearing out of rails is given by

(a) $\theta = \tan^{-1}(0.15)$ (b) $\theta = \tan^{-1}(0.25)$
(c) $\theta = \tan^{-1}(0.35)$ (d) $\theta = \tan^{-1}(0.45)$

161. One end of a string of length R is tied to a stone of mass m and the other end to a small pivot on a frictionless vertical board. The stone is whirled in a vertical circle with the pivot as the centre. The minimum speed the stone must have, when it is at the topmost point on the circle, so that the string does not slack is given by

(a) $\sqrt{gR}$ (b) $\sqrt{mgR}$
(c) $\sqrt{2gR}$ (d) $\sqrt{2mgR}$

162. In Q.161, the minimum speed the stone must have, when it is at the lowermost point on the circle, so that the stone can complete the circle is given by

(a) $\sqrt{4gR}$ (b) $\sqrt{4mgR}$
(c) $\sqrt{5gR}$ (d) $\sqrt{5mgR}$

163. The pilot of an aircraft, who is not tied to his seat, can loop a vertical circle in air without falling out at the top of the loop. What is the minimum speed required so that he can successfully negotiate a loop or radius 4 km? Take $g = 10\ ms^{-2}$.

(a) 100 ms^{-1} (b) 200 ms^{-1}
(c) 300 ms^{-1} (d) 400 ms^{-1}

164. A disc of radius $r = 20$ cm is rotating about its axis with an angular speed of 20 rad s^{-1}. It is gently placed on a horizontal surface which is perfectly frictionless (Fig. 3.78). What is the linear speed of point A on the disc?

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

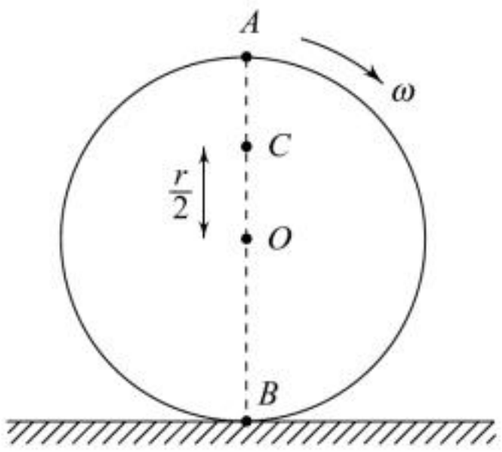

Fig. 3.78

165. In Q.164, the linear speed of point B on the disc will be

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

166. In Q.164, what is the linear speed of point C on the disc?

(a) 1 ms^{-1} (b) 2 ms^{-1}
(c) 3 ms^{-1} (d) 4 ms^{-1}

167. The combined mass of a rider and a motorcycle is 200 kg. What is the necessary frictional force if the rider is to negotiate a curve of 80 m radius at a speed of 72 km h^{-1}? Take $g = 10\ ms^{-2}$.

(a) 500 N (b) 750 N
(c) 1000 N (d) 1250 N

168. In Q.167, the angle at which rider must lean to avoid falling is given by

(a) $\theta = \tan^{-1}(0.2)$ (b) $\theta = \tan^{-1}(0.3)$
(c) $\theta = \tan^{-1}(0.4)$ (d) $\theta = \tan^{-1}(0.5)$

169. A car, moving at a speed of 54 km h^{-1}, is to go round a curved road of radius 30 m. If the curved road is not banked, what must be the coefficient of friction between the tyres and the road for the car to negotiate the curve? Take $g = 10\ ms^{-2}$.

(a) zero (b) 0.25
(c) 0.50 (d) 0.75

170. A string of length $L = 1$ m is fixed at one end and carries a mass of 100 g at the other end. The string makes $\sqrt{5}/\pi$ revolutions per second about a vertical axis passing through its second end. What is the angle of inclination of the string with the vertical? Take $g = 10\ ms^{-2}$.

(a) 30° (b) 45°
(c) 60° (d) 75°

171. In Q.170, the tension in the string is

(a) 2 N (b) 3 N
(c) 4 N (d) 5 N

172. In Q.170, what is the linear speed of the mass?

(a) $\sqrt{10}$ ms^{-1} (b) $\sqrt{15}$ ms^{-1}
(c) $2\sqrt{5}$ ms^{-1} (d) 5 ms^{-1}

173. A simple pendulum of length $r = 1$ m and bob mass 100 g is swinging with an angular amplitude of 60°. What is the tension in the string when the bob passes through the equilibrium position? Take $g = 10\ ms^{-2}$.

(a) 1 N (b) 2 N
(c) 3 N (d) 4 N

174. A spring which obey's Hooke's law extends by 1 cm when a mass is hung on it. It extends by a

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12. The contact force exerted by A on B is $F_2 = \dfrac{m_2 F}{(m_1 + m_2)}$, which is choice (d).

13. Let m_A and m_B be the masses of skaters A and B and a_A and a_B their respective accelerations, when they pull at each other. From Newton's third law, action and reaction forces are equal in magnitude, i.e.

$$m_A a_A = m_B a_B$$

or $$m_A \frac{v_A}{t} = m_B \frac{v_B}{t}$$

or $$m_A v_A = m_B v_B$$

or $$m_A^2 v_A^2 = m_B^2 v_B^2 \qquad \text{(i)}$$

where v_A and v_B are their respective speeds and t is the time taken for them to meet. Let s_A and s_B be the distances travelled by them when they meet, we have,

$$2a_A s_A = v_A^2 \quad \text{and} \quad 2a_B s_B = v_B^2$$

Using these equations in Eq. (i), noting that $m_A\ a_A = m_B\ a_B$, we get $\dfrac{s_A}{s_B} = \dfrac{m_B}{m_A} = \dfrac{70}{50} = \dfrac{7}{5}$. Since $s_A + s_B = 6$ m; $s_A = 3.5$ and $s_B = 2.5$ m. Hence, the correct choice is (d).

14. If a body has a constant velocity, both its speed and its direction of motion are constant. Hence, choice (a) is incorrect. A body having a constant speed can have an acceleration, called centripetal acceleration ($= v^2/R$) which is not variable. Hence, choices (b) and (c) are also incorrect. A body having a constant speed can have a varying velocity, e.g. a body moving in a circle with a constant speed; its direction of motion and hence its velocity is continuously changing with time. Hence, the only correct choice is (d).

15. As long as the coin is in the hand of the person, it shares the acceleration of the train; it has the inertia of motion. When he tosses the coin, it falls behind him opposite to the direction of accelerated motion but now it no longer shares the acceleration of the train. Hence the correct choice is (a).

16. The reaction force offered by the wall to the bullets = the force exerted by bullets on the wall (third law of motion) = the rate of change of momentum of bullets (second law of motion). Now, total mass of n bullets $= Nm$. Momentum of n bullets $= Nmv$. If n bullets are fired per second, the change of momentum per second $= nNmv$. Hence, the correct choice is (a).

17. The effective acceleration is $g_{\text{eff}} = g + a$. Therefore, the effective weight of the man is

$$Mg_{\text{eff}} = M(g + a) = Mg\left(1 + \frac{a}{g}\right) = W\left(1 + \frac{a}{g}\right)$$

Hence the correct choice is (a).

18. Since $v = \sqrt{2gh}$, the correct choice is (c).

19. The correct choice is (c).

20. Since the shell is at rest, the initial momentum is zero. After it explodes, the total momentum of the equal fragments A and B must be zero, which is possible only if they fly off in opposite directions with equal speeds. Hence the correct choice is (c).

21. Since the bomb is at rest, its momentum is zero. From the principle of conservation of momentum, it follows that, after explosion, the total momentum of all the fragments must be zero. Hence the correct choice is (a).

22. The component of weight Mg of the block along the inclined plane $= Mg \sin \theta$. The minimum frictional force to be overcome is also $Mg \sin \theta$. To make the block just move up the plane the minimum force applied must overcome the component $Mg \sin \theta$ of gravitational force as well as the frictional force $Mg \sin \theta = 2\ Mg \sin \theta$. Hence the correct choice is (d).

23. Frictional force on the block = coefficient of friction × normal reaction $= \mu mg \cos (90° - \theta)$ ($\because$ θ is the angle with the vertical. $\therefore$ $(90° - \theta)$ is the angle with the horizontal) $= 0.7 \times 2 \times 9.8 \times \cos 60° = 0.7 \times 9.8$ N

Hence the correct choice is (d).

24. Now, $u = 20\ \text{ms}^{-1}$, $t = 5$s and $v = 0$ (since the car is brought to rest). The acceleration of the car is

$$a = \frac{v - u}{t} = \frac{0 - 20}{5} = -4\ \text{ms}^{-2}$$

The negative sign indicates retardation and its magnitude is $4\ \text{ms}^{-2}$.

$\therefore$ Retardation force = mass × acceleration

$$= (940 + 60) \times 4 = 4000\ \text{N}$$

Hence the correct choice is (d).

25. Now $2as = v^2 - u^2$. Therefore

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (20)^2}{2 \times (-4)} = 50\ \text{m}$$

Hence the correct choice is (a).

26. Since the rocket moves vertically upwards against gravity, the blast must produce a total acceleration (for lift–off) $= g + a$. Therefore, thrust $= m\ (g + a)$. Thus the correct choice is (c).

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$a = -g \sin\theta - g \sin\theta = -2g \sin\theta$

Let s be the distance moved up the plane before the block comes to rest. Then, from $v^2 - u^2 = 2as$, we have

$$0 - u^2 = 2 \times (-2g \sin\theta) \times s$$

or $$s = \frac{u^2}{4g \sin\theta}$$

Hence, the correct choice is (c).

49. The weight $W = mg$ of the block can be resolved into two rectangular components: one along the plane ($W \sin\theta$) and the other perpendicular to it ($W \cos\theta$). Let R be the magnitude of the normal reaction and f_s be the force of sliding friction (see Fig. 3.83).

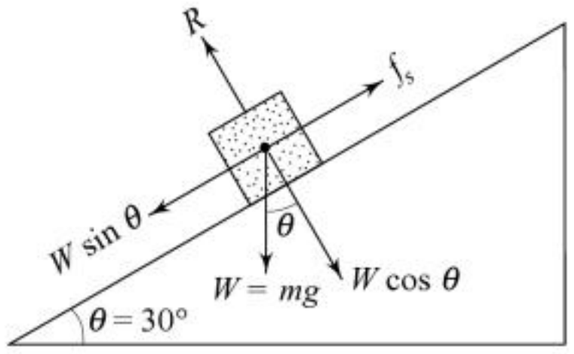

Fig. 3.83

When these forces are in equilibrium, the block just begins to slide, i.e.

$$f_s = W \sin\theta$$

Also $$R = W \cos\theta$$

$$\therefore \text{ Coefficient of sliding friction is } \mu_s = \frac{f_s}{R} = \frac{W \sin\theta}{W \cos\theta}$$

$$= \tan\theta = \tan 30°.$$

Hence, the correct choice is (c).

50. The minimum force F, applied parallel to the plane, that would just make the block move up the plane, must not only overcome the force of friction f_s but also the component $mg \sin\theta$ of the weight along the plane. Therefore,

$$F = f_s + mg \sin\theta$$

But $f_s = mg \sin\theta$. Therefore, $F = 2\ mg \sin\theta = 2 \times 5 \times 10 \times \sin 30° = 50$ N.

Hence the correct choice is (b).

51. Since the block is placed on the trolley, the acceleration of the block = acceleration of the trolley $a = 3\ \text{ms}^{-2}$. Therefore, the force acting on the block is

$$F = ma = 10 \times 3 = 30 \text{ N}$$

The weight mg of the block is balanced by the normal reaction R. As the trolley accelerates in the forward direction, it exerts a reaction force $F = 30$ N on the block in the backward direction, as shown in the figure. The force of friction will oppose this force and will act in a direction opposite to that of F. The force of limiting friction f is given by

$$\mu = \frac{f}{R} = \frac{f}{mg}$$

or $$f = \mu mg = 0.2 \times 10 \times 10 = 20 \text{ N}$$

Thus, the block is acted upon by two forces-force $F = 30$ N towards the right and frictional force $f = 20$ N towards the left (see Fig. 3.55). The net force on the block towards the right, i.e. towards the rear end of the trolley is

$$F' = F - f = 30 - 20 = 10 \text{ N}$$

Due to this force, the block experiences an acceleration towards the rear end which is given by

$$a' = \frac{F'}{m} = \frac{10}{10} = 1\ \text{ms}^{-2}$$

Let t be the time taken for the block to fall from the rear end of the trolley. Clearly, the block has to travel a distance $S' = 5$ m to fall off the trolley. Since the trolley starts from rest, initial velocity $u = 0$. Now t can be obtained from the relation

$$s = ut + \frac{1}{2}\ at^2$$

Putting $s = 5$ m, $u = 0$ and $a = a' = 1\ \text{ms}^{-2}$, we get $t = \sqrt{10}$ s.

The distance covered by the trolley in time $t = \sqrt{10}$ s is ($\because u = 0$)

$$s' = ut + \frac{1}{2}\ at^2 = 0 + \frac{1}{2} \times 3 \times 10 = 15 \text{ m}$$

Hence, the correct choice is (a).

52. The acceleration of each object is

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{(5-3) \times 10}{(5+3)}$$

$$= 2.5\ \text{ms}^{-2}$$

Hence, the correct choice is (d).

53. The tension in the string is

$$T = \frac{2m_1 m_2}{(m_1 + m_2)} \cdot g = \frac{2 \times 5 \times 3 \times 10}{(5+3)} = 37.5 \text{ N}$$

Thus, the correct choice is (b).

54. Since the masses start from rest, $u = 0$. The distance moved by the 5 kg mass in 0.2 s is

$$s = ut + \frac{1}{2}\ at^2 = 0 + \frac{1}{2} \times 2.5 \times (0.2)^2 = 0.05 \text{ m} = 5 \text{ cm}$$

Hence, the correct choice is (a).

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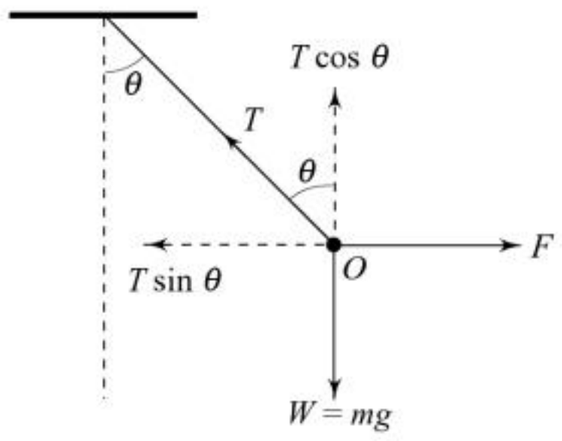

Fig. 3.88

91. Squaring Eqs. (i) and (ii) and adding we get $T^2 = W^2 + F^2$. Hence the correct choice is (d).

92. Dividing (i) by (ii) we get $F = W \tan \theta$, which is choice (a).

93. The common acceleration of the masses is

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{(6-4)\times 10}{(6+4)} = 2 \text{ ms}^{-2}$$

Hence the correct choice is (a).

94. The tension in the string is

$$T = \frac{2m_1 m_2 g}{(m_1 + m_2)} = \frac{2\times 6\times 4\times 10}{(6+4)} = 48 \text{ N}$$

Hence the correct choice is (b).

95. If the lift rises upwards with an acceleration a, the effective value of g is $g_{\text{eff}} = g + a$. Hence

$$T = \frac{2m_1 m_2}{(m_1 + m_2)} \times (g + a) = \frac{2\times 6\times 4}{10} \times (10+5)$$
$$= 72 \text{ N}$$

Thus, the correct choice is (d).

96. In this case, $g_{\text{eff}} = g - a$. Hence

$$T = \frac{2m_1 m_2}{(m_1 + m_2)} \times (g - a) = 24 \text{ N}$$

Thus, the correct choice is (a).

97. Since the acceleration of the train is perpendicular to the acceleration due to gravity, the acceleration vector of the train has no component along the vertical direction. Hence, in this case, $g_{\text{eff}} = g$. Thus, the correct choice is (b).

98. Let h be the height of each inclined plane. Then, the distances along the plane are $s_1 = \dfrac{h}{\sin\theta_1}$ and $s_2 = \dfrac{h}{\sin\theta_2}$ respectively. The accelerations of the block are $a_1 = g \sin\theta_1$ and $a_2 = g \sin\theta_2$ respectively. Now, since the block is released from rest, the velocity of the block when it reaches the bottom of the planes is $v_1^2 = 2a_1 s_1$ and $v_2^2 = 2a_2 s_2$ respectively. But $v_1 = a_1 t_1$ and $v_2 = a_2 t_2$ or $a_1^2 t_1^2 = 2a_1 s_1$ and $a_2^2 t_2^2 = 2a_2 s_2$. These equations give

$$\frac{t_2^2}{t_1^2} = \frac{a_1}{a_2}\cdot\frac{s_2}{s_1} = \frac{g\sin\theta_1}{g\sin\theta_2}\cdot\frac{h}{\sin\theta_2}\cdot\frac{\sin\theta_1}{h}$$
$$= \frac{\sin^2\theta_1}{\sin^2\theta_2}$$

$$\Rightarrow \quad \frac{t_2}{t_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

Hence, the correct choice is (c).

99. Let m be the mass of the rope and l its length. The tension T at a distance x from the support = weight of length $(l - x)$ of the rope $= \dfrac{mg}{l} \times (l - x)$ or

$$T = \frac{6\times 10}{3} \times (3-1) = 40 \text{ N}$$

Hence the correct choice is (c).

100. The acceleration of the block sliding down the smooth inclined plane is $a_1 = g \sin\theta$ and down the rough inclined plane is $a_2 = g\sin\theta - \mu g\cos\theta$. Given $t_1 = t$ and $t_2 = 2t$. If the length of the inclined plane is s, we have

$$s = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$$

or $$a_1 t_1^2 = a_2 t_2^2$$

or $$g\sin\theta \times t^2 = (g\sin\theta - \mu g\cos\theta)\times (2t)^2$$

or $$\sin\theta = 4(\sin\theta - \mu\cos\theta)$$

which gives $$\mu = \frac{3}{4}\tan\theta = \frac{3}{4} \qquad (\because \theta = 45°)$$

Hence the correct choice is (d).

101. We use the relation $v^2 - u^2 = 2as$. Since $u = 0$, we have $v^2 = 2as$.
Now $v_1^2 = 2a_1 s$ or $v^2 = 2g\sin\theta \times s$

and $v_2^2 = 2\,a_2 s$ or $\dfrac{v^2}{n^2} = 2(g\sin\theta - \mu g\cos\theta)\times s$

Dividing, we get or $n^2(\sin\theta - \mu\cos\theta) = \sin\theta$

which gives $\mu = \left(1 - \dfrac{1}{n^2}\right)\tan\theta$, which is choice (a).

102. The force of friction between block m and block $M = \mu_1 mg$, where μ_1 is the coefficient of friction between the two blocks. Now, the force of friction between block M (with block m on top of it) and the horizontal surface $= \mu_2 (M + m)g$, where μ_2 is

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Substituting the values of l_0 and k, and solving we get

$$l_3 = 3l_2 - 2l_1$$

Hence the correct choice is (c).

128. Refer to Fig. 3.91. The velocity of the shell at the highest point is $v = u \sin\theta = 100 \times \sin 30° = 50\ \text{ms}^{-1}$ parallel to the positive x-direction. Let m be the mass of the shell. Then the mass of the lighter fragment is $m_1 = \frac{m}{3}$ and its momentum is $p_1 = m_1 v_1$; where $v_1 = 200\ \text{ms}^{-1}$. The direction of p_1 is vertically upwards. The mass of the heavier fragment is $m_2 = \frac{2m}{3}$ and its momentum is $p_2 = m_2 v_2$, where v_2 is the speed of the heavier fragment at the time of explosion. Let momentum vector p_2 substend an angle α with the x-direction as shown. From the law of conservation of momentum, the component $p_2 \sin\alpha$ of p_2 along y-direction must balance with p_1 and the component $p_2 \cos\alpha$ must balance with p, i.e.

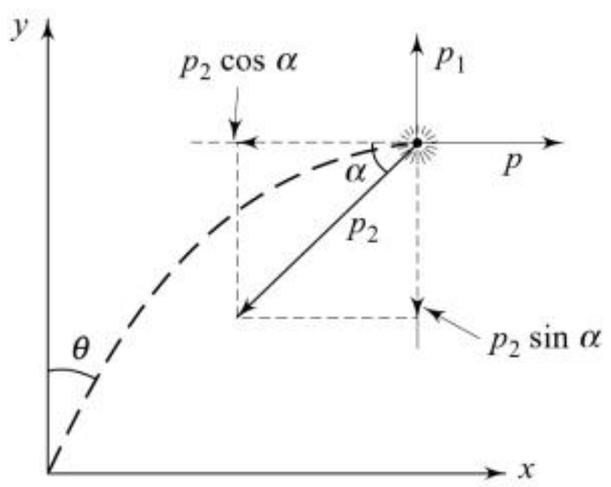

Fig. 3.91

$$p_2 \sin\alpha = p_1 \text{ or } m_2 v_2 \sin\alpha = m_1 v_1$$

or $$\frac{2mv_2}{3} \sin\alpha = \frac{mv_1}{3}$$

or $$2v_2 \sin\alpha = v_1 \quad \text{(i)}$$

and $p_2 \cos\alpha = p$ or $m_2 v_2 \cos\alpha = mv$

or $$\frac{2mv_2}{3} \cos\alpha = mv \text{ or}$$

$$2v_2 \cos\alpha = 3v \quad \text{(ii)}$$

Squaring and adding (i) and (ii) we have

$$4v_2^2 = v_1^2 + 9v^2$$

or $$2v_2 = (v_1^2 + 9v^2)^{1/2}$$

Now $v = 50\ \text{ms}^{-1}$ and $v_1 = 200\ \text{ms}^{-1}$ (given). Using these values, we get $v_2 = 125\ \text{ms}^{-1}$. Hence the correct choice is (a).

129. The forces acting on the balloon are its weight acting downwards and upthrust F acting upwards. Thus

$$F - Mg = Ma \quad \text{(i)}$$

When mass m is removed, we have

$$F - (M - m)\, g = (M - m)\, a' \quad \text{(ii)}$$

where a' is the new acceleration. Eliminating F from (i) and (ii) and simplifying we get

$$a' = \frac{Ma + mg}{M - m}$$

which is choice (a).

130. Let m be the mass per unit length of the rope. Let x be the part of the rope on the floor at time t. The weight of this part is

$$F_1 = mgx$$

Now, if a small part dx falls on the floor in time dt, the force exerted by it is

$$F_2 = \text{rate of change of momentum}$$

$$= \frac{(m\,dx)v}{dt}$$

Now $\frac{dx}{dt} = v$, where v is the velocity of that part of the rope at that instant. But $v^2 = 2gx$. Hence $F_2 = mv^2 = m \times (2gx) = 2mgx$. Total force $F = F_1 + F_2 = mgx + 2mgx = 3mgx = 3F_1$

Hence the correct choice is (c).

131. Given $\frac{d^2x}{dt^2} = 2t - t^2$ or $\frac{dv}{dt} = 2t - t^2$. Integrating we get

$$v = t^2 - \frac{t^3}{3} = t^2\left(1 - \frac{t}{3}\right)$$

The body will reverse its direction of motion at a time t when $v = 0$, i.e. at $t = 3$ s. Now, since $v = \frac{dx}{dt}$, we have $\frac{dx}{dt} = t^2 - \frac{t^3}{3}$

Integrating, we have

$$x = \frac{t^3}{3} - \frac{t^4}{12} = \frac{t^3}{3}\left(1 - \frac{t}{4}\right)$$

$$\therefore \quad x\ (\text{at } t = 3\,\text{s}) = \frac{(3)^3}{3}\left(1 - \frac{3}{4}\right) = \frac{(3)^2}{4}\ \text{m}$$

Hence the correct choice is (b).

132. The radius of the circular motion of the bead is $r = L$. The linear acceleration of the bead is $a = \alpha r = \alpha L$. If m is the mass of the bead, then

Force acting on the bead $= ma = m\,\alpha\,L$

$\therefore$ Reaction force acting on the bead is $R = m\,\alpha\,L$

The bead starts slipping when frictional force between the bead and the rod becomes equal to centrifugal force acting on the bead, i.e.

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$= Ak\,[-\,\hat{\mathbf{i}}\,\sin(kt) - \hat{\mathbf{j}}\,\cos(kt)]$

Now $\vec{F}\cdot\vec{p} = Ak[-\,\hat{\mathbf{i}}\,\sin(kt) - \hat{\mathbf{j}}\,\cos kt]$

$\cdot A[\,\hat{\mathbf{i}}\,\cos(kt) - \hat{\mathbf{j}}\,\sin(kt)]$

$= A^2k[-\sin(kt)\cos(kt) + \cos(kt)\sin kt]$

$= 0$

($\because\ \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = 1$ and $\hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = 0$). Hence angle between $\vec{F}$ and $\vec{P}$ is 90°, which is choice (d).

SECTION II

Multiple Choice Questions Based on Passage

Question 1 to 3 are based on the following passage.

Passage I

Reference Frames

We define the position of a body in terms of a frame of reference. Rest and motion are relative; there is nothing like absolute rest or absolute motion. The position or state of motion of a body may appear different from different frames of reference. For example, the passengers and everything else in a moving train are at rest in a reference frame situated in the train but they are in motion in a reference frame situated on the platform. Similarly, a stone dropped by a passenger from the window of a railway carriage in uniform motion appears to him to fall vertically downwards but to a person outside the carriage, it appears to follow a parabolic path.

Newton's first law of motion does not hold in all frames of reference; it holds only for inertial frames of reference. An inertial reference frame is a frame which moves with a constant velocity.

1. A reference frame attached to the earth
   (a) is an inertial frame by definition
   (b) cannot be an inertial frame because the earth is revolving round the sun
   (c) is an inertial frame because Newton's laws of motion are applicable in this frame
   (d) cannot be an inertial frame because the earth is rotating about its own axis.

2. Which of the following observers are inertial?
   (a) A child revolving in a merry-go-round
   (b) A driver in a car moving with a constant velocity
   (c) A pilot in an aircraft which is taking off
   (d) A passenger in a train which is slowing down to a stop.

3. Choose the correct statements from the following.
   (a) An inertial frame is non-accelerating.
   (b) An inertial frame is non-rotating.
   (c) A reference frame moving at a constant velocity with respect to an inertial frame is also an inertial frame.
   (d) Newton's laws of motion hold for both inertial and non-inertia frames.

➔ ANSWERS

1. The velocity of the earth changes with time (due to a change in its direction) as it revolves round the sun. Therefore, a frame attached to the earth is accelerated. Accelerated frames and rotating frames of reference are not inertial frames. Hence the correct choices are (b) and (d).

2. The observers in (a), (c) amd (d) are all accelerating. Hence, they are non-inertial. Only the driver in (b) is inertial since his motion is not accelerated.

3. The correct choices are (a), (b) and (c).

Questions 4 to 6 are based on the following passage.

Passage II

Two bodies A and B of masses m and $2\,m$ respectively are moving with equal linear momenta. They are subjected to the same retarding force.

4. If x_1 and x_2 are the respective distances moved by them before stopping, then x_1/x_2 is

   (a) $\frac{1}{2}$ (b) 1

   (c) $\sqrt{2}$ (d) 2

5. If t_1 and t_2 are the respective times taken by them to stop, then t_1/t_2 is

   (a) 1 (b) 2

   (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

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(a) $\frac{3mg}{2}$ (b) $3\,mg$

(c) $\frac{4mg}{3}$ (d) $\frac{2mg}{2}$

➔ SOLUTIONS

24. Since the inclined plane is smooth and $m_2 > m_1$, block m_1 will up the plane and block m_2 will move vertically with a common acceleration a. If T is the tension in the string, the free-body diagrams of masses m_1 and m_2 are as shown in Fig. 3.109

The equations of motion of the blocks are

$$T - m_1 g \sin\theta = m_1 a \qquad \text{(i)}$$

and

$$m_2 g - T = m_2 a \qquad \text{(ii)}$$

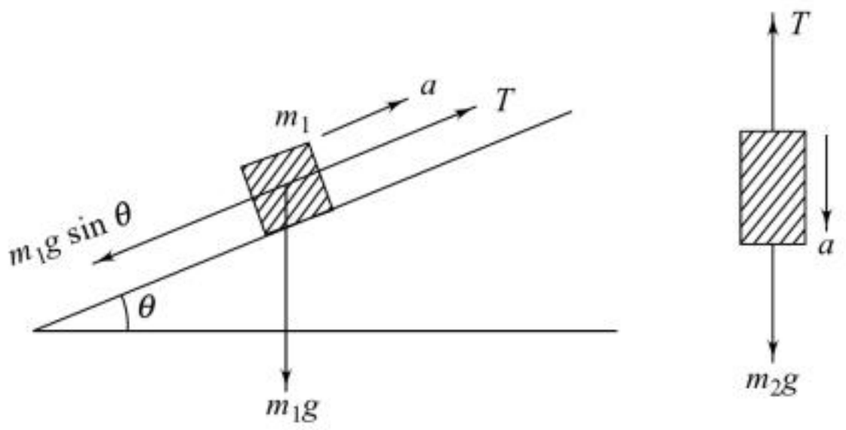

Fig. 3.109

Equations (i) and (ii) give

$$a = \frac{(m_2 - m_1 \sin\theta)g}{(m_1 + m_2)}$$

$$= \frac{(2m - m \times \sin 30°)g}{(m + 2m)} = \frac{g}{2}$$

Hence the correct choice is (c).

25. From Eqs. (i) and (ii), we get

$$T = m_2(g - a) = 2m \times \left(g - \frac{g}{2}\right) = mg$$

Hence the correct choice is (a).

26. Since the blocks remain at rest, the equations of motions of blocks m_1 and m_2 are (here f is the frictional force on m_1)

$$T - m_1 g \sin\theta - f = 0$$

and

$$T = m_2 g$$

These equations give

$$f = m_2 g - m_1 g \sin\theta$$

$$= 2\,m \times g - m \times g \times \sin 30°$$

$$= 2\,mg - \frac{mg}{2} = \frac{3mg}{2},$$

which is choice (b).

Questions 27 to 29 are based on the following passage.

Passage IX

Two blocks of masses $m_1 = 3\,m$ and $m_2 = 2\,m$ are suspended from a rigid support by two inextensible uniform wires A and B. Wire A has negligible mass and wire B has a mass $m_3 = m$, as shown in Fig. 3.110. The whole system of blocks, wires and the support have an upward acceleration a.

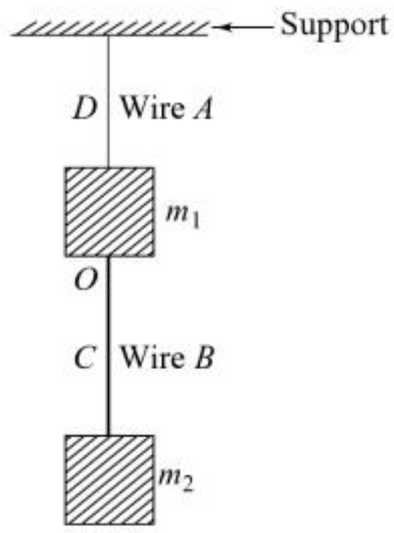

Fig. 3.110

27. The tension at the mid-point C of wire B is

(a) $\frac{1}{2}m(g + a)$ (b) $\frac{3}{2}m(g - a)$

(c) $\frac{3}{2}m(g + a)$ (d) $\frac{5}{2}m(g + a)$

28. The tension at point O of wire B is

(a) $3m(g + a)$ (b) $3m(g - a)$

(c) $2m(g + a)$ (d) $2m(g - a)$

29. The tension at the mid-point D of wire A is

(a) $2m(g + a)$ (b) $4m(g - a)$

(c) $6m(g + a)$ (d) $8m(g - a)$

➔ SOLUTIONS

27. Refer to Fig. 3.111. Let T be the tension at the mid-point C of wire B. Then

$$T - \left(m_2 + \frac{m_3}{2}\right)g = \left(m_2 + \frac{m_2}{2}\right)a$$

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4

Work, Energy and Power

REVIEW OF BASIC CONCEPTS

1. Work Done by a Force

(a) Work done by a constant force

When a constant force **F** acting on a body produces a displacement **S**, then the work done by the force is given by

$$W = \mathbf{F} \cdot \mathbf{S} = FS\cos\theta$$

where θ is the angle between the force vector **F** and the displacement vector **S** [see Fig. 4.1]. F and S are the magnitudes of **F** and **S** respectively.

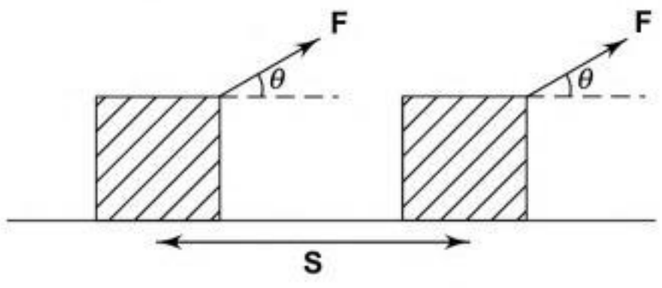

Fig. 4.1

(i) If θ is acute, $\cos\theta$ is positive. Hence work is positive for acute θ. In this case the force increases the speed of the body.

(ii) If $\theta = 90°$, $W = 0$, i.e. if the force is perpendicular to displacement work done by the force is zero.

(iii) If θ is obtuse, W is negative. In this case the force decreases the speed of the body.

(iv) If $\theta = 0$, i.e. force **F** is in the same direction as displacement **S**, then $W = FS$

(v) If $\theta = 180°$, force **F** is opposite to **S** (example frictional force), $W = -FS$. Work done by frictional and viscous force is always negative.

(b) Work done by a variable force

Suppose a force **F** is not constant but depends on the positive vector **r** of the body, then the work done by the force **F** in moving the body from a position r_1 to a position r_2 is given by

$$W = \int_{r_1}^{r_2} \mathbf{F} \cdot \mathbf{dr}$$

Example 1 A box is dragged on a horizontal floor by a rope which makes an angle of 60° with the horizontal. Find the work done if a force of 150 N is applied to drag the box through a distance of 10 m.

Solution

$$W = FS\cos\theta = 150 \times 10 \times \cos 60°$$
$$= 750 \text{ J}$$

Example 2 A horizontal force F pulls a 20 kg box at a constant velocity along a horizontal floor. If the coefficient of friction between the box and the floor is 0.25, find the work done by force F in moving the box through a distance of 2 m.

Solution Since the box is moved at a constant velocity, the applied force F just overcomes the frictional force f, i.e.

$$F = f = \mu mg$$

$\therefore$ Work done $W = FS\cos\theta = \mu mgS\cos 0°$

$$= 0.25 \times 20 \times 9.8 \times 2 = 98 \text{ J}$$

Example 3 A block of mass $m = 5$ kg slides down from the top of an inclined plane of inclination $\theta = 30°$ with the horizontal. The coefficient of sliding friction between

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(d) the kinetic energy of the block when it reaches the bottom of the plane. What conclusion will you draw from your answers to (b), (c) and (d)? Take $g = 10\ \text{ms}^{-2}$.

Solution Refer to fig. 4.5.

$$m = 0.5\ \text{kg},\ \mu = 0.2,\ l\ (= AC) = 2.5\ \text{m}$$

and $$h\ (= AB) = 1.5\ \text{m}$$

$$\sin\theta = \frac{AB}{AC} = \frac{1.5}{2.5} = 0.6$$

$$\therefore \quad \cos\theta = \sqrt{1 - \sin^2\theta} = 0.8$$

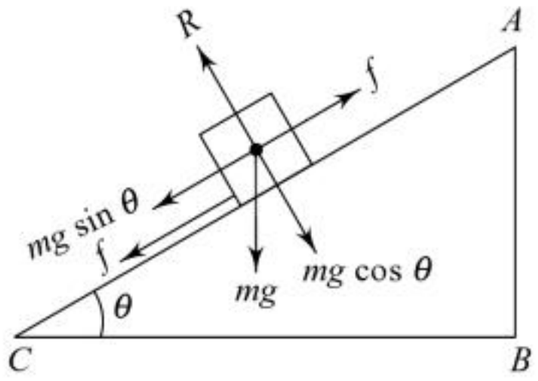

Fig. 4.5

(a) Work done by gravitational force to take the block from C to A is

$$W_1 = (-mg\sin\theta)l$$

The negative sign indicates that the gravitational force $mg\sin\theta$ is opposite to the displacement which is from C to A. Work done by the gravitational force to move the block from A to C is (because now the gravitational force is in the direction of the displacement)

$$W_2 = (+mg\sin\theta)l$$

$\therefore$ Total work done by the gravitational force over the round trip is

$$W_G = W_1 + W_2 = 0$$

This shows that gravitational force is conservative.

(b) Frictional force $f = \mu R = \mu mg\cos\theta$. The applied force to move the block from C to A is

$$F = mg\sin\theta + f = mg\sin\theta + \mu mg\cos\theta$$
$$= mg\ (\sin\theta + \mu\cos\theta)$$

Work done by applied force to move the block from C to A is

$$W_a = F \times AC = mg\ (\sin\theta + \mu\cos\theta) \times l$$
$$= 0.5 \times 10\ (0.6 + 0.2 \times 0.8) \times 2.5$$
$$= 9.5\ \text{J}$$

(c) Friction always opposes motion. When the block is moved from C to A, frictional force f is opposite to direction. Hence work done by frictional force in the upward journey is

$$W_3 = -fl$$

When the block slides from A to C, f acts upwards along the plane and is opposite to the displacement. Hence work done by frictional force in the downward journey is

$$W_4 = -fl$$

$\therefore$ Total work done by frictional force over the round trip is

$$Wf = W_3 + W_4 = -2fl$$
$$= -2\ \mu mg\cos\theta \times l$$
$$= -2 \times 0.2 \times 0.5 \times 10 \times 0.8 \times 2.5$$
$$= -4\ \text{J}$$

(d) When the block is A, its initial velocity $u = 0$. Let v be the velocity when it reaches C. Since f acts upwards, the net force on the block when it slides down is

$$F' = mg\sin\theta - f = mg\ (\sin\theta - \mu\cos\theta)$$

$\therefore$ Acceleration

$$a = \frac{F'}{m} = g\ (\sin\theta - \mu\cos\theta)$$
$$= 10\ (0.6 - 0.2 \times 0.8)$$
$$= 4.4\ \text{ms}^{-2}$$

From

$v^2 - u^2 = 2as$, we have

$$v^2 - 0 = 2 \times 4.4 \times 2.5 \quad \Rightarrow \quad v^2 = 22\ \text{m}^2\text{s}^{-2}$$

$\therefore$ Kinetic energy at $C = \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 0.5 \times 22$$
$$= 5.5\ \text{J}$$

Conclusion

Initial knietic energy at $A = 0$. Therefore, change in K.E.$= 5.5 - 0 = 5.5$ J. Now total work done is

$$W = W_G + W_a + W_f$$
$$= 0 + 9.5 - 4 = 5.5\ \text{J}$$

Thus, work done = change in kinetic energy. This is the work-energy principle.

Example 9 A block of mass $m = 500$ g is placed at the top of an inclined plane of inclination $\theta = 60°$. The length of the plane is 2 m. The block is released from rest. Find its speed when it reaches the bottom of the plane if

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Example 22 Two steel balls of the same mass m moving in opposite directions with the same speed u collide head-on. If the collision is perfectly elastic, predict the result of the collision.

Solution $m_1 = m_2 = m$, $u_1 = u$ and $u_2 = -u$. Let v_1 and v_2 be their velocities after collision.

Total momentum before collision $= m_1u_1 + m_2u_2 = m(u - u) = 0$

Total momentum after collision $= mv_1 + mv_2 = m(v_1 + v_2)$

From conservation of momentum,

$$0 = m(v_1 + v_2) \Rightarrow v_2 = -v_1$$

since $e = 1$, we have

$$v_2 - v_1 = u_1 - u_2 = u - (-u) = 2u$$

Putting $v_1 = -v_2$, we get $v_2 = u$. Also $v_1 = -u$. Thus, after the collision, the two balls move in opposite directions with equal speeds, each equal to u but their directions are reversed.

Example 23 A ball mass 2 kg moving with a velocity of 8 ms^{-1} collides head-on with another ball of mass of 4 kg moving with a velocity of 2 ms^{-1} moving in the same direction. The collision is elastic and the coefficient restitution is $e = 0.5$.

(a) Find the velocities of the balls after the collision.
(b) Calculate the loss of kinetic energy due to collision.

Solution Refer to Fig 4.10 again.

(a) Given $m_1 = 2$ kg, $m_2 = 4$ kg, $u_1 = 8$ ms^{-1}, $u_2 = 2$ ms^{-1} and $e = 0.5$

From conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad 2 \times 8 + 4 \times 2 = 2v_1 + 4v_2$$

$$\Rightarrow \quad 12 = v_1 + 2v_2 \qquad \text{(i)}$$

Since $e = 0.5$, we have

$$v_2 - v_1 = e(u_1 - u_2) = 0.5 \times (8 - 2) = 3 \qquad \text{(ii)}$$

Eliminating v_2 from (i) and (ii) we get $v_1 = 2$ ms^{-1}. Using this in (i) or (ii), we get $v_2 = 5$ ms^{-1}

(b) Kinetic energy before collision is

$$K_i = \frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2$$

$$= \frac{1}{2} \times 2 \times (8)^2 + \frac{1}{2} \times 4 \times (2)^2 = 72 \text{ J}$$

Kinetic energy after collision is

$$K_f = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$$

$$= \frac{1}{2} \times 2 \times (2)^2 + \frac{1}{2} \times 4 \times (5)^2 = 54 \text{ J}$$

$\therefore$ Loss of K.E. $= K_i - K_f = 72 - 54 = 18$ J

Example 24 In Example 23, what is the loss of kinetic energy if the ball of mass 4 kg is moving towards the mass of mass 2 kg, their speeds being the same?

Solution In this case $u_2 = -2$ ms^{-1}. Equations (i) and (ii) become

$$4 = v_1 + 2v_2 \qquad \text{(iii)}$$

and $$v_2 - v_1 = 5 \qquad \text{(iv)}$$

Equations (iii) and (iv) give $v_1 = -2$ ms^{-1} and $v_2 = 3$ ms^{-1}

$$K_i = \frac{1}{2} \times 2 (8)^2 + \frac{1}{2} \times 4 \times (-2)^2 = 72 \text{ J}$$

$$K_f = \frac{1}{2} \times 2 \times (-2)^2 \times \frac{1}{2} \times 4 \times (3)^2 = 22 \text{ J}$$

$\therefore$ Loss of K.E $= 72 - 22 = 50$ J

Example 25 Two blocks B and C of masses 1 kg and 2 kg respectively are connected by a massless elastic spring of spring constant 150 Nm^{-1} and placed on a horizontal frictionless surface as shown in Fig. 4.15. A third block A of mass 1 kg moves with a velocity of 3 ms^{-1} along the line joining B and C and collides with B. If the collision is perfectly elastic and the natural length of the spring is 80 cm, find the minimum separation between blocks B and C.

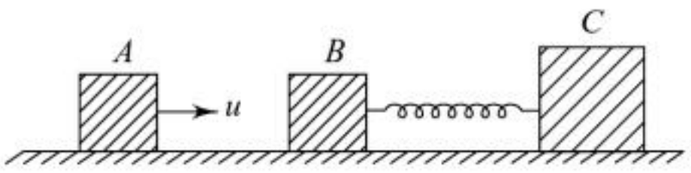

Fig. 4.15

Solution Given $m_A = m_B = 1$ kg, $m_C = 2$ kg and $u = 3$ ms^{-1}. Block A will collide with block B. Since they have equal masses and the collision is perfectly elastic, A will come to rest and B will move to the right with a velocity u. Block B will compress the spring. Hence block C will accelarate and block B will retard until both B and C move with the same velocity. Let this common velocity be v. Since no external force acts, the momentum of B and C is conserved, i.e.

$$m_B u = (m_B + m_C) v$$

$$\Rightarrow \quad 1 \times 3 = (1 + 2) v \quad \Rightarrow \quad v = 1 \text{ ms}^{-1}$$

If x is the maximum compression, then from the principle of conservation of energy,

$$\frac{1}{2} m_Au^2 = \frac{1}{2} (m_B + m_C) v^2 + \frac{1}{2} kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times 1 \times (3)^2 = \frac{1}{2} \times (1 + 2) \times (1)^2 + \frac{1}{2} \times 150 \times x^2$$

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which gives $x = 0.2 \text{ m} = 20 \text{ cm}$

$\therefore$ Minimum separation between B and $C = 80 \text{ cm} - 20 \text{ cm} = 60 \text{ cm}$

Example 26 A block of $m_1 = m$ is moving on a frictionless horizontal surface with velocity $u_1 = 2u$ towards another block of mass $m_2 = 3m$ moving on the same surface with velocity $u_2 = u$ in the same direction. A massless spring of force constant k is attached to m_2 as shown in Fig. 4.16. When block m_1 collides with the spring, show that the maximum compression of the spring is given by $x = \frac{u}{2}\sqrt{\frac{3m}{k}}$.

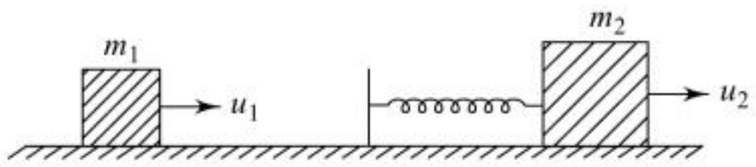

Fig. 4.16

Solution When block m_1 collides with spring, it begins to get compressed. As a result m_2 gains speed. The compression of the spring is maximum at the instant when the relative velocity of m_1 with respect to m_2 is zero, i.e. when both m_1 and m_2 have equal velocities. Let v be the common velocity of the blocks. From conservation of momentum,

$$m_1u_1 + m_2 u_2 = (m_1 + m_2)v$$

$$\Rightarrow \quad 2mu + 3mu = (m + 3m)v$$

$$\Rightarrow \quad v = \frac{5u}{4}$$

From the law of conservation of energy

Loss in K.E. = gain in P.E of spring

If x is the maximum compression, then

$$\left(\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times m \times (2u)^2 + \frac{1}{2} \times 3m \times u^2 - \frac{1}{2}(m + 3m) \times \left(\frac{5u}{4}\right)^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{3}{4}mu^2 = kx^2 \Rightarrow x = \frac{u}{2}\sqrt{\frac{3m}{k}}$$

7. Useful Formulae and Tips

1. A body of mass m is dropped from a height h. Due to the friction of air, it will hit the ground with a speed less than $\sqrt{2gh}$. If v is the speed with which it hits the ground, the work done by friction is

$$W_f = \frac{1}{2}mv^2 - mgh = \frac{1}{2}m(v^2 - 2gh)$$

If friction is absent, $W_f = 0$, then $v = \sqrt{2gh}$.

2. Two block A and B of masses m_1 and m_2 are released from the same height at the same time. Block A slides along an inclined plane of inclination θ and block B falls vertically downwards (Fig. 4.17)

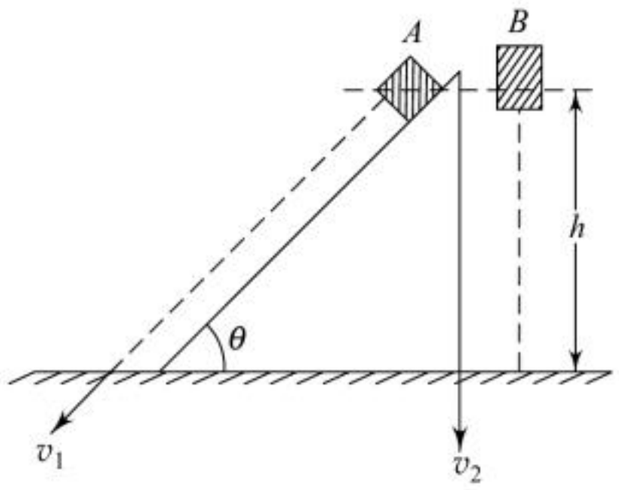

Fig. 4.17

If the inclined plane is frictionless, gain in KE = loss in PE, i.e.

$$\frac{1}{2}m_1v_1^2 = m_1gh \quad \Rightarrow \quad v_1 = \sqrt{2gh}$$

If the air friction is neglected.

$$\frac{1}{2}m_2v_2^2 = m_2gh \quad \Rightarrow \quad v_2 = \sqrt{2gh}$$

Thus both block will hit the ground with the same speed independent of the mass. But the times taken to reach the ground will be different.

For block A, $t_1 = \sqrt{\frac{2h}{g\sin^2\theta}}$

For block B, $t_2 = \sqrt{\frac{2h}{g}}$

3. If a block of mass m in contact with a spring compressed by a distance x is released, the block will leave the spring with a velocity v determined from

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

which gives $v = \sqrt{\frac{k}{m}}x$, where k is the spring constant.

4. If a block of mass m moving with speed u comes in contract with a relaxed spring of spring constant k, its velocity v when the spring is compressed by an amount x is obtained from.

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$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

which gives $v = \left(\frac{k\,x^2}{m} - u^2\right)^{1/2}$

5. If two springs of spring constants k_1 and k_2 are stretched by the same force F, then $F = k_1x_1 = k_2\,x_2$. Potential energy stored is

$$U_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}Fx_1$$

and $$U_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}Fx_2$$

which give $$\frac{U_1}{U_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

6. If the two springs are stretched by the same amount x, then $F_1 = k_1x$ and $F_2 = k_2x$.

$U_1 = \frac{1}{2}k_1x^2$ and $U_2 = \frac{1}{2}k_2x^2$.

$$\frac{U_1}{U_2} = \frac{k_1}{k_2} = \frac{F_1}{F_2}$$

7. A chain has a length L and mass M. A part L/n is hanging at the edge of the table. The length of the chain lying on the table is $(L - L/n)$. Then work done against gravity to pull the the hanging part on the table $= \frac{MgL}{2n^2}$

8. If a body of mass m moving with velocity v is stopeed in a distance x by a retarding force F, then

$$\frac{1}{2}mv^2 = Fx$$

(a) If two bodies of masses m_1 and m_2 moving with the same velocity are subjected to the same retarding force, the ratio of the stopping distance is

$$\frac{x_1}{x_2} = \frac{m_1}{m_2}$$

(b) If the two bodies are moving with equal kinetic energy and are stooped by the same retarding force, then

$$x_1 = x_2$$

(c) If the two bodies are moving with equal liner momentum and are stopped by the same force, then.

$$\frac{p^2}{2m} = Fx$$

and $$\frac{x_1}{x_2} = \frac{m_2}{m_1}$$

SECTION I

Multiple Choice Questions with One Correct Choice

1. A raindrop of radius r falls from a certain height h above the ground. The work done by the gravitational force is proportional to

(a) r (b) r^2
(c) r^3 (d) r^4

2. A block of mass m moves up an inclined plane of angle of inclination θ. If μ is the coefficient of friction between the block and the inclined plane, the work done in moving the block through a distance x along the plane is given by

(a) $mgx\,(\sin\theta + \mu\cos\theta)$
(b) $mgx\sin\theta$
(c) $mgx\,(\sin\theta - \mu\cos\theta)$
(d) $\mu\,mgx\cos\theta$

3. A pendulum has a length l. Its bob is pulled aside from its equilibrium position through an angle α and then released. The speed of the bob when it passes through the equilibrium position is given by

(a) $\sqrt{2gl}$ (b) $\sqrt{2gl\cos\alpha}$
(c) $\sqrt{2gl(1-\cos\alpha)}$ (d) $\sqrt{2gl(1-\sin\alpha)}$

4. A small ball is pushed from a height h along a smooth hemispherical bowl. With what speed should the ball be pushed so that it just reaches the top of the opposite end of the bowl? The height of the top of the bowl is R.

(a) $\sqrt{2gh}$ (b) $\sqrt{2gR}$
(c) $\sqrt{2g\,(R+h)}$ (d) $\sqrt{2g\,(R-h)}$

5. Two particles of masses m and $4m$ have linear momenta in the ratio of 2 : 1. What is the ratio of their kinetic energies?

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(a) $\sqrt{2}$ (b) 2
(c) 4 (d) 16

6. Two particles of masses m and $4m$ have kinetic energies in the ratio of 2:1. What is the ratio of their linear momenta?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

7. A moving bullet hits a solid target resting on a frictionless surface and gets embedded in it. What is conserved in this process?

(a) momentum and kinetic energy
(b) kinetic energy alone
(c) momentum alone
(d) neither momentum nor kinetic energy

8. A bullet is fired from a rifle which recoils after firing. The ratio of the kinetic energy of the rifle to that of the bullet is

(a) zero (b) one
(c) less than one (d) more than one

9. A bullet is fired at a plank of wood with a speed of 200 ms^{-1}. After passing through the plank, its speed reduces to 180 ms^{-1}. Another bullet, of the same mass and size but moving with a speed of 100 ms^{-1} is fired at the same plank. What would be the speed of this bullet after passing through the plank? Assume that the resistance offered by the plank is the same for both the bullets?

(a) 48 ms^{-1} (b) 49 ms^{-1}
(c) 50 ms^{-1} (d) 51 ms^{-1}

10. A particle of mass m has half the kinetic energy of another particle of mass $m/2$. If the speed of the heavier particle is increased by 2 ms^{-1}, its new kinetic energy equals the original kinetic energy of the lighter particle. The ratio of the original speeds of the lighter and heavier particles is

(a) 1:1 (b) 1:2
(c) 1:3 (d) 1:4

11. In Q. 10, what is the original speed of the heavier particle?

(a) $2(1+\sqrt{2})$ ms^{-1} (b) $2(1-\sqrt{2})$ ms^{-1}
(c) $(2\sqrt{2}+1)$ ms^{-1} (d) $(2\sqrt{2}-1)$ ms^{-1}

12. Two identical cylindrical vessels, with their bases at the same level, each contains a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the vessels are interconnected?

(a) $A\rho g\,(h_1-h_2)^2$ (b) $A\rho g\,(h_1+h_2)^2$
(c) $A\rho g\left(\frac{h_1-h_2}{2}\right)^2$ (d) $A\rho g\left(\frac{h_1+h_2}{2}\right)^2$

13. The bob of a pendulum is released from a horizontal position A as shown in Fig. 4.18. The length of the pendulum is 2 m. If 10% of the initial energy of the bob is dissipated as heat due to the friction of air, what would be the speed of the bob when it reaches the lowermost point B? Take $g = 10\ ms^{-2}$.

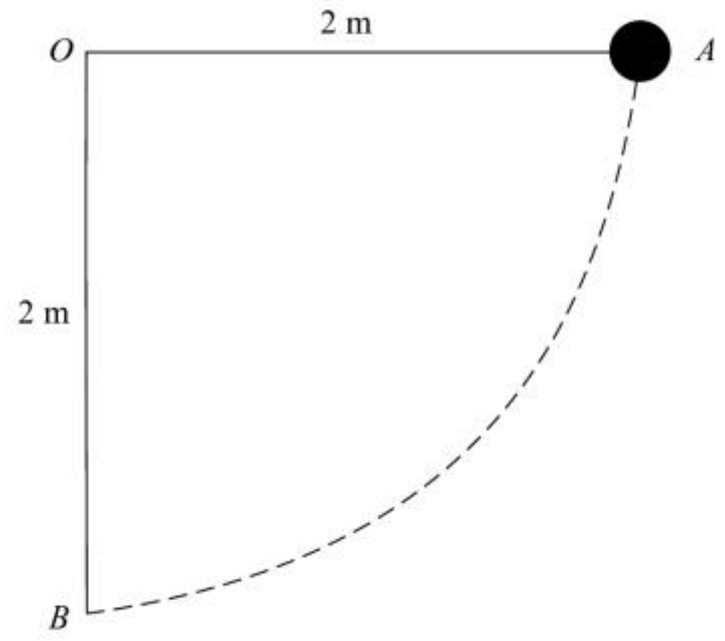

Fig. 4.18

(a) 3 ms^{-1} (b) 4 ms^{-1}
(c) 5 ms^{-1} (d) 6 ms^{-1}

14. A wooden block of mass 0.9 kg is suspended from the ceiling of a room by thin wires. A bullet of mass 0.1 kg moving horizontally with a speed of 10 ms^{-1} strikes the block and sticks to it. What is the height to which the block rises? Take $g = 10\ ms^{-2}$.

(a) 2.5 m (b) 5.0 m
(c) 7.5 m (d) 10.0 m

15. In Q. 14, what is the loss in kinetic energy of the system due to impact?

(a) 450 J (b) 400 J
(c) 350 J (d) 300 J

16. Two identical balls marked 2 and 3, in contact with each other and at rest on a horizontal frictionless table, are hit head-on by another identical ball marked 1 moving initially with a speed v as shown in Fig. 4.19. What is observed, if the collision is elastic?

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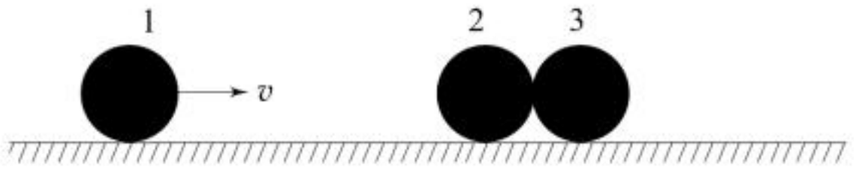

Fig. 4.19

(a) Ball 1 comes to rest and balls 2 and 3 roll out with speed $\frac{v}{2}$ each.

(b) Balls 1 and 2 come to rest and ball 3 rolls out with speed v.

(c) Balls 1, 2 and 3 roll out with speed $\frac{v}{3}$ each.

(d) Balls 1, 2 and 3 come to rest.

17. The bob A of a pendulum released from a height h hits head–on another bob B of the same mass of an identical pendulum initially at rest. What is the result of this collision? Assume the collision to be elastic (see Fig. 4.20).

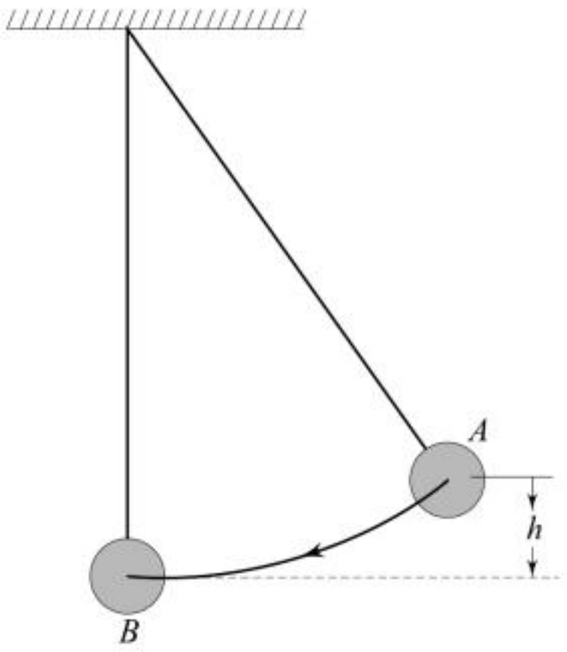

Fig. 4.20

(a) Bob A comes to rest at B and bob B moves to the left attaining a maximum height h.

(b) Bobs A and B both move to the left, each attaining a maximum height $\frac{h}{2}$.

(c) Bob B moves to the left and bob A moves to the right, each attaining a maximum height $\frac{h}{2}$.

(d) Both bobs come to rest.

18. A steel ball falls from a height h on a floor for which the coefficient of restitution is e. The height attained by the ball after two rebounds is

(a) eh (b) e^2h

(c) e^3h (d) e^4h

19. Two balls marked 1 and 2 of the same mass m and a third ball marked 3 of mass M are arranged over a smooth horizontal surface as shown in Fig. 4.21. Ball 1 moves with a velocity v_1 towards balls 2 and 3. All collisions are assumed to be elastic. If $M < m$, the number of collisions between the balls will be

(a) one (b) two

(c) three (d) four

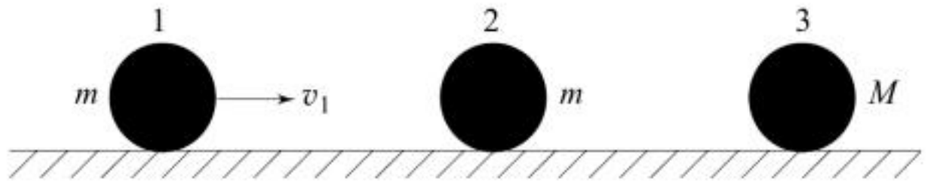

Fig. 4.21

20. In Q. 19, if $M > m$, the number of collisions between the balls will be

(a) one (b) two

(c) three (d) four

21. The distance x moved by a body of mass 0.5 kg by a force varies with time t as

$$x = 3t^2 + 4t + 5$$

where x is expressed in metre and t in second. What is the work done by the force in the first 2 seconds?

(a) 25 J (b) 50 J

(c) 75 J (d) 100 J

22. A body of mass m is dropped from a height h above the ground. The velocity v of the body when it has lost half its initial potential energy is given by

(a) $v = \sqrt{gh}$ (b) $v = \sqrt{2gh}$

(c) $v = \sqrt{\frac{gh}{2}}$ (d) $v = 2\sqrt{gh}$

23. A body of mass m is thrown vertically upwards with a velocity v. The height h at which the kinetic energy of the body is half its initial value is given by

(a) $h = \frac{v^2}{g}$ (b) $h = \frac{v^2}{2g}$

(c) $h = \frac{v^2}{3g}$ (d) $h = \frac{v^2}{4g}$

24. A car of mass m moving at a speed v is stopped in a distance x by the friction between the tyres and the road. If the kinetic energy of the car is doubled, its stopping distance will be

(a) $8x$ (b) $4x$

(c) $2x$ (d) x

25. A body of mass m is dropped from a certain height. It has a velocity v when it is at a height h above the

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ground. Which of the following will remain constant during the free fall?

(a) $v^2 + 2gh$ (b) $v^2 - 2gh$

(c) $v + \sqrt{\frac{gh}{2}}$ (d) $v - 2\sqrt{gh}$

26. A body of mass m moving with a speed v suffers a perfectly inelastic collision with another body of M at rest. The speed of the composite body will be

(a) $\left(\frac{m+M}{m}\right)v$ (b) $\left(\frac{m}{m+M}\right)v$

(c) $\left(\frac{M}{m+M}\right)v$ (d) $\left(\frac{m+M}{M}\right)v$

27. In Q. 26, the ratio of the final kinetic energy of the system to the initial kinetic energy is

(a) $\frac{m}{m+M}$ (b) $\frac{M}{m+M}$

(c) $\frac{m+M}{m}$ (d) $\frac{m+M}{M}$

28. A ball of mass m moving horizontally at a speed v collides with the bob of a simple pendulum at rest. The mass of the bob is also m. If the collision is perfectly inelastic, the height to which the two balls rise after the collision will be given by

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$

(c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

29. In Q. 28, the ratio of the kinetic energy of the system immediately after the collision to that before the collision will be

(a) 1 : 1 (b) 1 : 2

(c) 1 : 3 (d) 1 : 4

30. In Q.28, if the collision is perfectly elastic, the bob of the pendulum will rise to a height of

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$

(c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

31. A body of mass m_1 moving at a constant speed undergoes an elastic collision with a body of mass m_2 initially at rest. The ratio of the kinetic energy of mass m_1 after the collision to that before the collision is

(a) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ (b) $\left(\frac{m_1 + m_2}{m_1 - m_2}\right)^2$

(c) $\left(\frac{2m_1}{m_1 + m_2}\right)^2$ (d) $\left(\frac{2m_2}{m_1 + m_2}\right)^2$

32. In Q. 31, the ratio of the kinetic energies of masses m_2 and m_1 after the collision is

(a) $\frac{m_1 m_2}{(m_1 - m_2)^2}$ (b) $\frac{\sqrt{2} m_1 m_2}{(m_1 - m_2)^2}$

(c) $\frac{2m_1 m_2}{(m_1 - m_2)^2}$ (d) $\frac{4m_1 m_2}{(m_1 - m_2)^2}$

33. A ball P of mass 2 kg undergoes an elastic collision with another ball Q at rest. After collision, ball P continues to move in its original direction with a speed one-fourth of its original speed. What is the mass of ball Q?

(a) 0.9 kg (b) 1.2 kg

(c) 1.5 kg (d) 1.8 kg

34. A ball of mass m moving with a velocity v undergoes an oblique elastic collision with another ball of the same mass m but at rest. After the collision, if the two balls move with the same speeds, the angle between their directions of motion will be

(a) 30° (b) 60°

(c) 90° (d) 120°

35. In Q. 34, if the two balls move with different speeds after the collision, the angle between their directions of motion will be

(a) less than 90° (b) more than 90°

(c) exactly 90° (d) exactly 180°

36. A box is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$

(c) $t^{3/2}$ (d) t^2

37. A bullet, incident normally on a wooden plank, loses one-tenth of its speed in passing through the plank. The least number of such planks required to stop the bullet is

(a) 5 (b) 6

(c) 7 (d) 8

38. A bullet is fired normally on an immovable wooden plank. It loses 25% of its momentum in penetrating a thickness of 3.5 cm. The total thickness penetrated by the bullet is

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(a) 8 cm (b) 10 cm
(c) 12 cm (d) 14 cm

39. A bullet is fired normally on an immovable wooden plank. It loses 25% of its kinetic energy in penetrating a thickness x of the plank. What is the total thickness penetrated by the bullet?

(a) $2x$ (b) $4x$
(c) $6x$ (d) $8x$

40. A uniform rod of mass m and length l is made to stand vertically on one end. The potential energy of the rod in this position is

(a) $\frac{mgl}{4}$ (b) $\frac{mgl}{3}$
(c) $\frac{mgl}{2}$ (d) mgl

41. If the rod in Q. 40 is held inclined at an angle of 60° with the vertical, what will be the potential energy of the rod in this position?

(a) $\frac{mgl}{4}$ (b) $\frac{mgl}{3}$
(c) $\frac{mgl}{2}$ (d) mgl

42. A body, having kinetic energy k, moving on a rough horizontal surface, is stopped in a distance x. The force of friction exerted on the body is

(a) $\frac{k}{x}$ (b) $\frac{\sqrt{k}}{x}$
(c) $\frac{k}{\sqrt{x}}$ (d) kx

43. A body of mass m, having momentum p, is moving on a rough horizontal surface. If it is stopped in a distance x, the coefficient of friction between the body and the surface is given by

(a) $\mu = \frac{p^2}{2gm^2x}$ (b) $\mu = \frac{p^2}{2mgx}$
(c) $\mu = \frac{p}{2mgx}$ (d) $\mu = \frac{p}{2gm^2x}$

44. A uniform chain of mass M and length L is held on a horizontal frictionless table with $\frac{1}{n}$ th of its length hanging over the edge of the table. The work done is pulling the chain up on the table is

(a) $\frac{Mgl}{n}$ (b) $\frac{Mgl}{2n}$
(c) $\frac{Mgl}{n^2}$ (d) $\frac{Mgl}{2n^2}$

45. A body of mass $m = 1$ kg is dropped from a height $h = 40$ cm on a horizontal platform fixed to one end of an elastic spring, the other being fixed to a base, as shown in Fig. 4.22. As a result the spring is compressed by an amount $x = 10$ cm. What is the force constant of the spring. Take $g = 10\ \text{ms}^{-2}$.

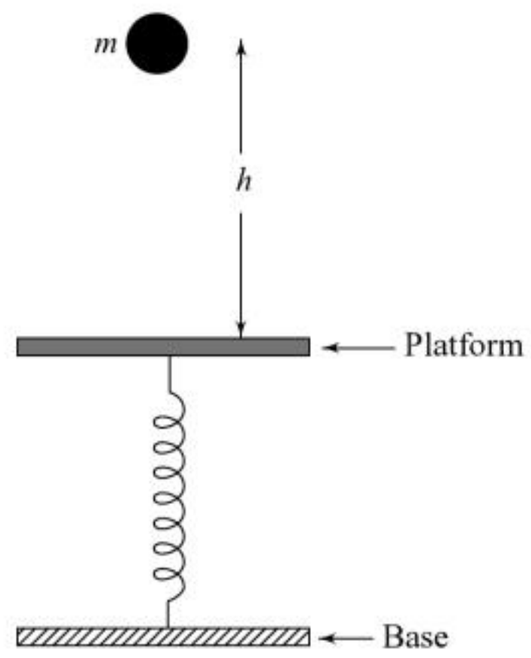

Fig. 4.22

(a) $600\ \text{Nm}^{-1}$ (b) $800\ \text{Nm}^{-1}$
(c) $1000\ \text{Nm}^{-1}$ (d) $1200\ \text{Nm}^{-1}$

46. A block of wood of mass M is suspended by means of a thread. A bullet of mass m is fired horizontally into the block with a velocity v. As a result of the impact, the bullet is embedded in the block. The block will rise to vertical height given by

(a) $\frac{1}{2g}\left(\frac{mv}{M+m}\right)^2$ (b) $\frac{1}{2g}\left(\frac{mv}{M-m}\right)^2$
(c) $\frac{1}{2g}\frac{mv^2}{(M+m)}$ (d) $\frac{1}{2g}\frac{mv^2}{(M-m)}$

47. A moving particle of mass m makes a head-on collision with a particle of mass $2m$ initially at rest. If the collision is perfectly elastic, the percentage loss of energy of the colliding particle is

(a) 50% (b) 66.7%
(c) 88.9% (d) 100%

48. A body of mass m moving with a velocity v in the x-direction collides with a body of mass M moving with a velocity V in the y-direction. They stick together during collision. Then

(a) the magnitude of the momentum of the composite body is $\sqrt{(mv^2) + (MV)^2}$

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(b) the composite body moves in a direction making a angle $\theta = \tan^{-1}\left(\frac{MV}{mv}\right)$ with the x-axis.

(c) the loss of kinetic energy as a result of collision is $\frac{1}{2}\frac{Mm}{(M+m)}(V^2+v^2)$

(d) all the above choices are correct.

49. A body of mass 2 kg moving with a velocity $(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ ms^{-1} collides with another body of mass 3 kg moving with a velocity $(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ in ms^{-1}. If they stick together, the velocity in ms^{-1} of the composite body is

(a) $\frac{1}{5}(8\hat{\mathbf{i}}+7\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ (b) $\frac{1}{5}(-4\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})$

(c) $\frac{1}{5}(8\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})$ (d) $\frac{1}{5}(-4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-3\hat{\mathbf{k}})$

50. A body of mass 2 kg has an initial velocity $\mathbf{v}_i = (\hat{\mathbf{i}}+\hat{\mathbf{j}})\,\text{ms}^{-1}$. After collision with another body its velocity becomes $\mathbf{v}_f = (5\hat{\mathbf{i}}+6\hat{\mathbf{j}}+\hat{\mathbf{k}})\,\text{ms}^{-1}$. If the impact time is 0.02 s, the average force of impact on the body (in newton) is

(a) $50\,(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}})$ (b) $50\,(4\hat{\mathbf{i}}-5\hat{\mathbf{j}}-\hat{\mathbf{k}})$

(c) $100\,(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}-\hat{\mathbf{k}})$ (d) $100\,(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}})$

51. A body falls from a height h on a horizontal surface and rebounds. Then it falls again, and again rebounds and so on. If the restitution coefficient is $\frac{1}{3}$, the total distance covered by the body before it comes to rest is

(a) $\frac{h}{4}$ (b) $\frac{5h}{4}$

(c) $2h$ (d) $3h$

52. In Q. 51 above, the total time taken by the body to come to rest is

(a) $\sqrt{\frac{2h}{g}}$ (b) $2\sqrt{\frac{2h}{g}}$

(c) $3\sqrt{\frac{2h}{g}}$ (d) $4\sqrt{\frac{2h}{g}}$

53. A body P strikes another body Q of mass that is p times that of body P and moving with a velocity that is $\frac{1}{q}$ of the velocity of body P. If body P comes to rest, the coefficient of restitution is

(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{q(p-1)}$

(c) $\frac{p-q}{q(p-1)}$ (d) $\frac{p+q}{q(p-1)}$

54. Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. Sphere A is projected along the groove and at the end of time T impinges on sphere B. If e is the coefficient of restitution, the second impact will occur after a time equal to

(a) T (b) eT

(c) $\frac{2T}{e}$ (d) $2\,eT$

55. Two masses M and m (with $M > m$) are connected by means of a pulley as shown in Fig. 4.23. The system is released. At the instant when mass M has fallen through a distance h, the velocity of mass m will be

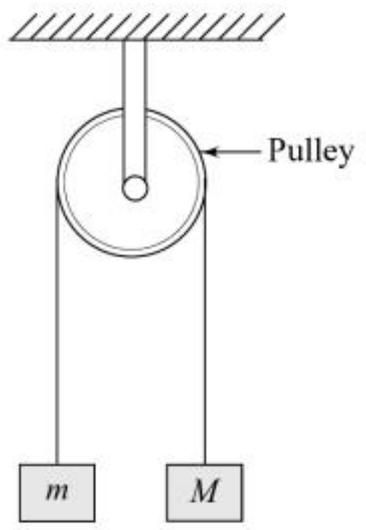


Fig. 4.23

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{2gh\,M}{m}}$

(c) $\sqrt{\frac{2gh\,(M-m)}{(M+m)}}$ (d) $\sqrt{\frac{2gh\,(M+m)}{(M-m)}}$

56. A mass m, lying on a horizontal frictionless surface is connected to mass M as shown in Fig. 4.24. The system is now released. The velocity of mass m when mass M as descended a distance h is

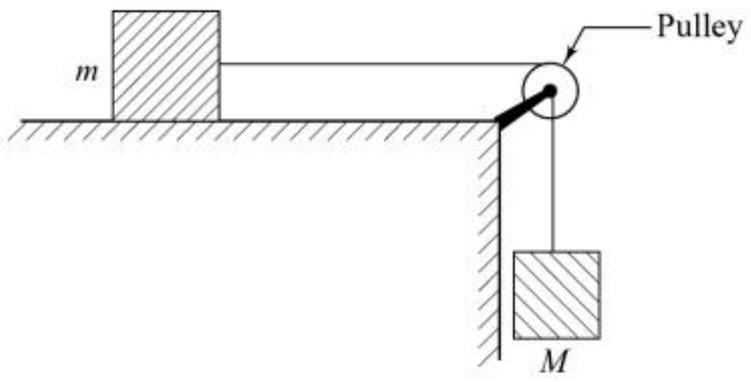


Fig. 4.24

(a) $\sqrt{\frac{2\,Mgh}{m}}$ (b) $\sqrt{\frac{2\,mgh}{M}}$

(c) $\sqrt{\frac{2\,Mgh}{(M+m)}}$ (d) $\sqrt{2gh}$

57. An escalator is moving downwards with a uniform speed u. A man of mass m is running upwards on it at a uniform speed v. If the height of the escalator is h, the work done by the man in going up the escalator is

(a) zero (b) mgh

(c) $\frac{mghu}{(v-u)}$ (d) $\frac{mgh\,v}{(v-u)}$

58. A uniform rope of length L is lying in a lump on a frictionless table. A small part of the rope is held hanging through a hole in the table just below the lump. The system is then released. The speed of the end of the rope as it leaves the hole is

(a) $\sqrt{2gL}$ (b) $2\sqrt{\frac{gL}{3}}$

(c) $\sqrt{\frac{2\,gL}{3}}$ (d) zero

59. The potential energy (in joule) of a body of mass 2 kg moving in the $x - y$ plane is given by

$$U = 6x + 8y$$

where the position coordinates x and y are measured in metre. If the body is at rest at point (6 m, 4 m) at time $t = 0$, it will cross the y-axis at time t equal to

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

60. In Q. 59 above, the speed of the body when it crosses the y-axis is

(a) zero (b) $5\ \text{ms}^{-1}$

(c) $10\ \text{ms}^{-1}$ (d) $20\ \text{ms}^{-1}$

61. A bullet of mass m is fired horizontally with a velocity v on a wooden block of mass M suspended from a support and gets embedded in it. The kinetic energy of the bullet + block system is

(a) $\frac{1}{2}\,mv^2$ (b) $\frac{1}{2}\,(M+m)v^2$

(c) $\frac{Mmv^2}{2(M+m)}$ (d) $\frac{m^2v^2}{2(M+m)}$

62. A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing it rises to 1.8 m. The ball loses it velocity on bouncing by a factor of

(a) $\frac{16}{25}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{9}{2}$

63. An isolated particle of mass m is moving in a horizontal plane $(x - y)$, along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at

(a) $y = -5$ cm (b) $y = +20$ cm

(c) $y = +5$ cm (d) $y = -20$ cm

64. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in Fig. 4.25) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

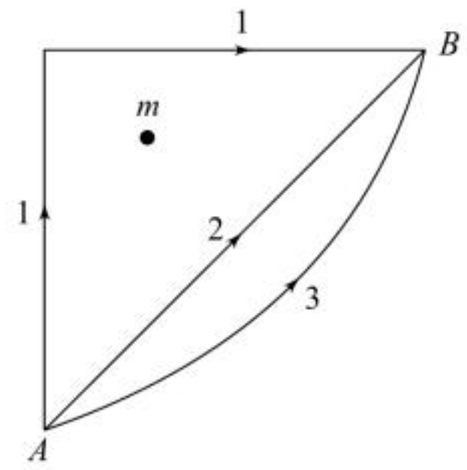


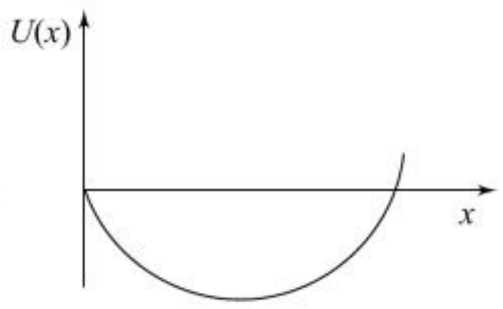
Fig. 4.25

(a) $W_1 > W_3 > W_2$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_3 < W_2$ (d) $W_1 < W_2 < W_3$

65. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is (see Fig. 4.26)

(a)



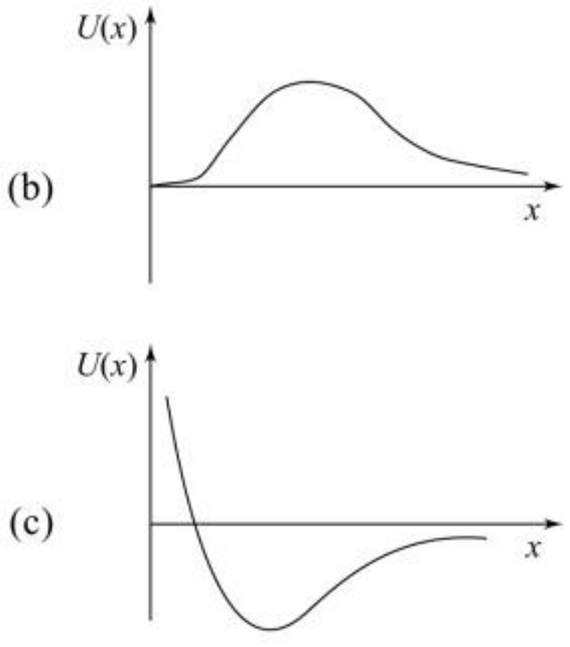


Fig. 4.26

66. A body of mass 6 kg is acted upon by a force which causes a displacement in it given by $x = \frac{t^2}{4}$ metre where t is the time in second. The work done by the force is 2 seconds is

(a) 12 J (b) 9 J
(c) 6 J (d) 3 J

67. A ladder 2.5 m long and of weight 150 N has its centre of mass 1 m from its bottom. A weight of 40 N is attached to the top end. The work required to raise the ladder from the horizontal position to the vertical position is

(a) 190 J (b) 250 J
(c) 285 J (d) 475 J

68. A body of mass 5 kg rests on a rough horizontal surface of coefficient of friction 0.2. The body is pulled through a distance of 10 m by a horizontal force of 25 N. The kinetic energy acquired by it is (take $g = 10\ \text{ms}^{-2}$)

(a) 200 J (b) 150 J
(c) 100 J (d) 50 J

69. A body is moving up an inclined plane of angle θ with an initial kinetic energy E. The coefficient of friction between the plane and the body is μ. The work done against friction before the body comes to rest is:

(a) $\frac{\mu\cos\theta}{E\cos\theta+\sin\theta}$ (b) $\mu E\cos\theta$

(c) $\frac{\mu E\cos\theta}{\mu\cos\theta-\sin\theta}$ (d) $\frac{\mu E\cos\theta}{\mu\cos\theta+\sin\theta}$

70. A particle falls from a height h on a fixed horizontal plate and rebounds. If e is the coefficient of restitution, the total distance travelled by the particle before it stops rebounding is

(a) $\frac{h(1+e^2)}{(1-e^2)}$ (b) $\frac{h(1-e^2)}{(1+e^2)}$

(c) $\frac{h(1-e^2)}{2(1+e^2)}$ (d) $\frac{h(1+e^2)}{2(1-e^2)}$

ANSWERS

1. (c)	**2.** (a)	**3.** (c)
4. (d)	**5.** (d)	**6.** (a)
7. (c)	**8.** (c)	**9.** (b)
10. (b)	**11.** (a)	**12.** (c)
13. (d)	**14.** (b)	**15.** (a)
16. (b)	**17.** (a)	**18.** (d)
19. (b)	**20.** (c)	**21.** (c)
22. (a)	**23.** (d)	**24.** (c)
25. (a)	**26.** (b)	**27.** (a)
28. (d)	**29.** (b)	**30.** (b)
31. (a)	**32.** (d)	**33.** (b)
34. (c)	**35.** (c)	**36.** (c)
37. (b)	**38.** (a)	**39.** (b)
40. (c)	**41.** (a)	**42.** (a)
43. (a)	**44.** (d)	**45.** (c)
46. (a)	**47.** (c)	**48.** (d)
49. (a)	**50.** (d)	**51.** (b)
52. (b)	**53.** (d)	**54.** (c)
55. (c)	**56.** (a)	**57.** (d)
58. (b)	**59.** (b)	**60.** (c)
61. (d)	**62.** (b)	**63.** (a)
64. (b)	**65.** (d)	**66.** (d)
67. (b)	**68.** (b)	**69.** (d)
70. (a)		

➔ SOLUTIONS

1. Mass of the drop m = volume × density of water = $\frac{4\pi}{3}r^3\rho$, where ρ is the density of water. Work done by gravitational force is

$$W = mgh = \frac{4\pi}{3}r^3\rho gh$$

Thus $W \propto r^3$. Hence the correct choice is (c).

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2. Since the block is moved up the incline, the force applied on it has to overcome the component $mg \sin \theta$ of its weight mg acting down the plane and the force of friction $\mu mg \cos \theta$ also acting down the plane. Thus

$$F = mg \sin \theta + \mu mg \cos \theta$$

$\therefore$ Work done $W = Fx = mgx (\sin \theta + \mu \cos \theta)$. Hence the correct choice is (a).

3. As shown in Fig. 4.27, the height attained by the bob when the string subtends an angle α with the vertical is

$$h = l - l \cos \alpha = l (1 - \cos \alpha)$$

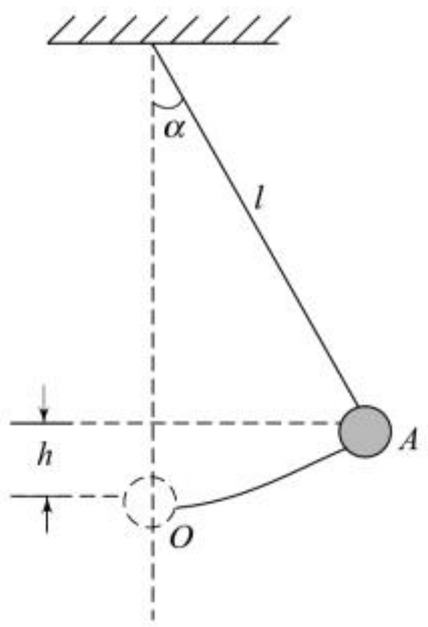

Fig. 4.27

Its potential energy at the highest point $A = mgh$, where m is the mass of the bob. Let v be the speed of the bob when it passes through O. Its kinetic energy at $O = \frac{1}{2} mv^2$. From the principle of conservation of energy, we have

$$\frac{1}{2} mv^2 = mgh$$

or $$v = \sqrt{2gh} = \sqrt{2gl(1 - \cos \alpha)}$$

Hence the correct choice is (c).

4. From the principle of conservation of energy, we have

$$\frac{1}{2} mv^2 + mgh = 0 + mgR$$

which gives $v = \sqrt{2g(R - H)}$. Hence the correct choice is (d).

5. Given $p_1 = m_1 v_1 = 2p$ and $p_2 = m_2 v_2 = p$, so that

$$\frac{m_1 v_1}{m_1 v_2} = 2$$

The ratio of their kinetic energies is

$$\frac{(\text{KE})_1}{(\text{KE})_2} = \frac{\frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_2 v_2^2} = \frac{m_1^2 v_1^2}{m_2^2 v_2^2} \cdot \frac{m_2}{m_1}$$

But $m_2 = 4\, m_1$ and $\frac{m_1 v_1}{m_2 v_2} = 2$. Therefore,

$$\frac{(\text{KE})_1}{(\text{KE})_2} = (2)^2 \times 4 = 16$$

Hence the correct choice is (d).

6. Given $(\text{KE})_1 = \frac{1}{2} m_1 v_1^2 = 2K$

and $(\text{KE})_2 = \frac{1}{2} m_2 v_2^2 = K$, so that

$$\frac{m_1 v_1^2}{m_2 v_2^2} = 2$$

or $$\frac{m_1^2 v_1^2}{m_2^2 v_2^2} = \frac{2m_1}{m_2}$$

or $$\frac{p_1}{p_2} = \sqrt{\frac{2m_1}{m_2}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}} \qquad (\because m_2 = 4\, m_1)$$

Hence the correct choice is (a)

7. The correct choice is (c).

8. Let M be the mass of the rifle and m that of the bullet and let V and v be their respective speeds. From the principle of conservation of momentum, we have $MV = mv$. The ratio of their kinetic energies is

$$\frac{K_r}{K_b} = \frac{\frac{1}{2} MV^2}{\frac{1}{2} mv^2} = \frac{MV^2}{mv^2} = \frac{(MV)^2}{(mv)^2} \cdot \frac{m}{M} = \frac{m}{M}$$

Since $m < M$, $K_r < K_b$. Hence the correct choice is (c).

9. Let m be the mass of each bullet. Since the resistance offered by the plank is the same for the two bullets, the amount of work done by the plank is the same for the two bullets. From work–energy principle, the decrease in the kinetic energy is the same for the two bullets.

Decrease in KE of first bullet $= \frac{1}{2} mu_1^2 - \frac{1}{2} mv_1^2$

$$= \frac{1}{2} m(200)^2 - \frac{1}{2} m(180)^2 \qquad \text{(i)}$$

If v_2 is the speed of the second bullet after passing through the plank, then

Decrease in KE of second bullet $= \frac{1}{2} mu_2^2 - \frac{1}{2} mv_2^2$

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15. Initial KE $= \frac{1}{2}\, mu^2$

Final KE $= \frac{1}{2}\,(m + M)v^2$

$\therefore$ Loss in KE $= \frac{1}{2}\, mu^2 - \frac{1}{2}\,(m + M)\,v^2$

$= \frac{1}{2} \times 0.1 \times (100)^2 - \frac{1}{2}\,(0.1 + 0.9) \times (10)^2$

$= 450$ J

Hence the correct choice is (a).

16. The system consists of three identical balls marked 1, 2 and 3. Let m be the mass of each ball. Before the collision,

KE of the system = KE of 1 + KE of 2 + KE of 3

$= \frac{1}{2}\, mv^2 + 0 + 0 = \frac{1}{2}\, mv^2$

Case (a) After the collision,

KE of the system = KE of 1 + KE of 2 + KE of 3

$= 0 + \frac{1}{2} m \left(\frac{v}{2}\right)^2 + \frac{1}{2} m \left(\frac{v}{2}\right)^2$

$= \frac{1}{4}\, mv^2$

Case (b)

KE of the system = KE of 1 + KE of 2 + KE of 3

$= 0 + 0 + \frac{1}{2} mv^2 = \frac{1}{2} mv^2$

Case (c)

KE of the system = KE of 1 + KE of 2 + KE of 3

$= \frac{1}{2} m\left(\frac{v}{3}\right)^2 + \frac{1}{2} m\left(\frac{v}{3}\right)^2 + \frac{1}{2} m\left(\frac{v}{3}\right)^2$

$= \frac{1}{6}\, mv^2$

Case (d)

KE of the system = 0

Now, in an elastic collision, the kinetic energy of the system remains unchanged. Hence choice (b) is the only possible result of the collision.

17. Suppose the bob A acquires a velocity v on reaching the bob B. In a head-on elastic collision between two bodies of the same mass when one of them is at rest, the velocities are exchanged after the collision. Hence the bob A will come to rest at the lowermost position (occupied by B before collision) and the bob B will move to the left attaining a maximum height h. Hence the correct choice is (a).

18. The velocity attained after a fall through a height h is given by

$$v^2 = 2gh$$

Thus $h \propto v^2$. The velocity after first rebound is ev. Therefore, the height attained after first rebound $= e^2h$. Velocity after second rebound is e^2v. Hence the height attained after second rebounds is e^4h. Thus the correct choice is (d).

19. The first collision will be between balls 1 and 2. Since both have the same mass, after the collision ball 1 will come to rest and ball 2 will move with speed v_1. This ball will collide with the stationary ball 3. After this second collision, let v_2 and v_3 be the speeds of balls 2 and 3 respectively. Since the collisions are elastic, v_2 and v_3 are given by

$$v_2 = \left(\frac{m - M}{m + M}\right) v_1 \qquad \text{(i)}$$

and $$v_3 = \left(\frac{2m}{m + M}\right) v_1 \qquad \text{(ii)}$$

If $M < m$, it follows from (i) and (ii) that $v_2 < v_3$ and both have the same direction. Therefore, ball 2 cannot collide with ball 3 again. Hence there are only two collisions. Thus, the correct choice is (b).

20. If $M > m$, we have from Eq. (i)

$$v_2 = -\left(\frac{M - m}{M + m}\right) v_1$$

The negative sign indicates that, after the second collision, ball 2 will move in opposite direction towards the ball 1 which is at rest after the first collision. Therefore, ball 2 will make another collision with ball 1.

Hence, in this case, there are three collisions in all between the balls. Thus the correct choice is (c).

21. Velocity $(v) = \frac{dx}{dt} = \frac{d}{dt}\,(3t^2 + 4t + 5) = 6t + 4.$

Acceleration is $a = \frac{dv}{dt} = \frac{d}{dt}\,(6t + 4) = 6\ \text{ms}^{-2}$. Therefore, applied force is $F = ma = 0.5 \times 6 = 3$ N. Now $t = 2$s, the distance moved is $x = 3 \times (2)^2 + 4 \times 2 + 5 = 25$ m $\quad (\because\ u = 4\text{ms}^{-1}$ and $x = 5$ m at $t = 0)$

$\therefore$ Work done $W = Fx = 3 \times 25 = 75$ J. Hence the correct choice is (c).

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22. Initial PE = mgh. Now, gain in KE = loss in PE. Thus

$$\frac{1}{2}\ mv^2 = \frac{1}{2}\ mgh$$

or $$v = \sqrt{gh}$$

Hence the correct choice is (a).

23. Initial KE = $\frac{1}{2}\ mv^2$. Now, gain in PE = loss in KE. Thus

$$mgh = \frac{1}{4}\ mv^2$$

or $$h = \frac{v^2}{4g}$$

Hence the correct choice is (d).

24. If a is the deceleration due to the force of friction f, then

$$2ax = v^2$$

or $$\frac{1}{2}\ mv^2 = max$$

or $$\text{KE} = fx \qquad (\because f = ma)$$

Thus if KE is doubled, x is also doubled. Hence the correct choice is (c).

25. The total energy = KE + PE remains constant during the free fall, i.e.

$$mgh + \frac{1}{2}\ mv^2 = \text{constant}$$

or $$gh + \frac{v^2}{2} = \text{constant}$$

Hence the correct choice is (a).

26. Initial momentum of the system = mv, since body of mass M is at rest. After the inelastic collision, the bodies stick together and the mass of the composite body is $(m + M)$. If V is the speed of the composite body, its momentum will be $(m + M)V$. From the principle of conservation of momentum, we have

$$mv = (m + M)V$$

or $$V = \left(\frac{m}{m + M}\right)v$$

Hence the correct choice is (b).

27. Initial KE = $\frac{1}{2}\ mv^2$. Final KE = $\frac{1}{2}\ (m + M)V^2$. Therefore,

$$\frac{\text{Final KE}}{\text{Initial KE}} = \left(\frac{m + M}{m}\right)\frac{V^2}{v^2} = \frac{m}{m + M}$$

Hence the correct choice is (a).

28. In a perfectly inelastic collision, two bodies stick together. After the collision, the speed of the ball and the bob (sticking together) is $v' = v/2$. The height to which they will rise is given by

$$v' = \sqrt{2gh'}$$

or $$h' = \frac{v'^2}{2g} = \frac{v^2}{8g}$$

Hence the correct choice is (d).

29. Mass of the ball and the bob sticking together is $m' = 2m$. KE after collision = $\frac{1}{2}\ m'\ v'^2 = \frac{1}{2} \times 2m \times \left(\frac{v}{2}\right)^2$ $= \frac{1}{4}\ mv^2$. KE before collision = $\frac{1}{2}\ mv^2$. Therefore, their ratio is 1:2. Hence the correct choice is (b).

30. In an elastic collision between two bodies of the same mass with one of them initially at rest, the moving body is brought to rest and the other moves with the same speed. Thus the ball will come to rest and the bob of the pendulum acquires a speed v. At this speed, it will rise to height h given by $h = v^2/2g$. Hence the correct choice is (b).

31. Let u_1 be the speed of mass m_1 before the collision. Here $u_2 = 0$. Therefore, the speeds of masses m_1 and m_2 after the collision respectively are

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

$\therefore$ KE of m_1 after collision = $\frac{1}{2}\ m_1 v_1^2 =$ $\frac{1}{2}\ m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$. KE of m_1 before collision = $\frac{1}{2}\ m_1 u_1^2$. The ratio of the two is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$.

Hence the correct choice is (a).

32. KE of m_2 after collision = $\frac{1}{2}\ m_2 v_2^2 =$ $\frac{1}{2}\ m_2\left(\frac{2m_1}{m_1 + m_2}\right)^2 u_1^2$.

KE of m_1 after collision = $\frac{1}{2}\ m_1 v_1^2 =$ $\frac{1}{2}\ m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$. Dividing the two, we find that the correct choice is (d).

33. Since the collision is elastic, both momentum and kinetic energy are conserved. If m is mass of ball Q

and v' its speed after the collision, the law of conservation of momentum gives

$$2v = 2 \times \frac{v}{4} + mv'$$

where v is the original speed of ball P. Thus

$$mv' = \frac{3v}{2} \text{ or } v' = \frac{3v}{2m} \qquad \text{(i)}$$

The law of conservation of energy gives

$$\frac{1}{2} \times 2 \times v^2 = \frac{1}{2} \times 2 \times \left(\frac{v}{4}\right)^2 + \frac{1}{2} mv'^2$$

$$\text{or} \qquad mv'^2 = \frac{15v^2}{8} \qquad \text{(ii)}$$

Using (i) in (ii) we get $m = 1.2$ kg. Hence the correct choice is (b).

34. Refer to Fig. 4.28.

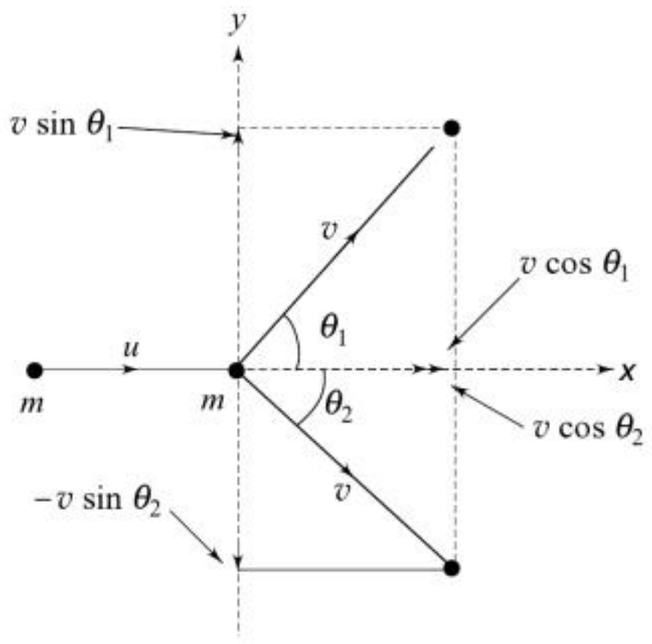

Fig. 4.28

Taking the components of the velocities along x and y–axes and using the law of conservation of momentum for x and y components we have

$$mu = mv \cos \theta_1 + mv \cos \theta_2 \qquad \text{(i)}$$

$$\text{and} \qquad 0 = mv \sin \theta_1 - mv \sin \theta_2 \qquad \text{(ii)}$$

From (ii) we get $\sin \theta_1 = \sin \theta_2$ or $\theta_1 = \theta_2$. Using $\theta_1 = \theta_2 = \theta$ in (i) we have

$$mu = 2\, mv \cos \theta$$

$$\text{or} \qquad \cos \theta = \frac{u}{2v} \qquad \text{(iii)}$$

Since the collision is elastic, kinetic energy is also conserved, i.e.

$$\frac{1}{2}\, mu^2 = \frac{1}{2}\, mv^2 + \frac{1}{2}\, mv^2$$

$$\text{or} \qquad u^2 = 2v^2 \text{ or } u = \sqrt{2}\, v \qquad \text{(iv)}$$

Using (iv) and (iii) we have $\cos \theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$.

Thus $\theta_1 + \theta_2 = 2\theta = 90°$. Hence the correct choice is (c).

35. Refer to Fig. 4.28 again. Let $\mathbf{v}_1$ be the velocity of the ball moving along direction θ_1 and $\mathbf{v}_2$ be the velocity of the ball moving along θ_2.

From momentum conservation,

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2$$

$$\Rightarrow \qquad \mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2$$

The magnitude of $\mathbf{u}$ is

$$u = \left[v_1^2 + v_2^2 + 2v_1 v_2 \cos(\theta_1 + \theta_2)\right]^{1/2} \qquad \text{(i)}$$

Since kinetic energy is also conversed,

$$\frac{1}{2} mu^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$$

$$\Rightarrow \qquad u^2 = v_1^2 + v_2^2 \qquad \text{(ii)}$$

Using (ii) in (i) we have

$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos(\theta_1 + \theta_2)$$

$$\Rightarrow \quad \cos(\theta_1 + \theta_2) = 0 \quad \Rightarrow \quad \theta_1 + \theta_2 = 90°$$

36. Power $P = Fv = mav = m \frac{dv}{dt} \cdot v$ or $v\, dv = \frac{P}{m} dt$.

Integrating, we have

$$\int v dv = \frac{P}{m} \int dt \qquad (\because P = \text{constant})$$

$$\text{or} \qquad \frac{v^2}{2} = \frac{Pt}{m}$$

$$\text{or} \qquad v = \sqrt{\frac{2P}{m}}\, t^{1/2}$$

$$\text{or} \qquad \frac{dx}{dt} = \sqrt{\frac{2P}{m}}\, t^{1/2}$$

$$\text{or} \qquad dx = \sqrt{\frac{2P}{m}}\, t^{1/2}\, dt$$

Integrating again, we have

$$\int d\, x = \sqrt{\frac{2P}{m}} \int t^{1/2} dt$$

$$\text{or} \qquad x = \frac{2}{3} \sqrt{\frac{2P}{m}}\, t^{3/2}$$

i.e. $x \propto t^{3/2}$. Hence the correct choice is (c).

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37. Let v be the speed of the bullet incident on the first plank. Its speed after it passes the plank $= \frac{9v}{10}$. If x is the thickness of the plank, the deceleration a due to the resistance of the plank is given by

$$2ax = v^2 - \left(\frac{9v}{10}\right)^2 = \frac{19v^2}{100} \qquad \text{(i)}$$

Suppose the bullet is stopped after passing through n such planks. Then the distance covered by the bullet is $s = nx$. Thus, we have

$$v^2 - 0 = 2as = 2anx$$

or $$n = \frac{v^2}{2ax} = \frac{v^2 \times 100}{19v^2} \text{ [use Eq. (i) above]}$$

$$= \frac{100}{19} = 5.26$$

Thus the minimum number of planks required is 6. Hence the correct choice is (b).

38. Let u cms^{-1} be the speed of the bullet. Since the mass of the bullet remains unchanged, its speed becomes $v = \frac{3u}{4}$ cms^{-1} after it penetrates a distance $x = 3.5$ cm. The retardation a due to the resistance of the wooden is given by

$$u^2 - v^2 = 2ax$$

or $$u^2 - \left(\frac{3u}{4}\right)^2 = 2a \times 3.5$$

which gives $a = \frac{u^2}{16}$ cms^{-2}. The bullet will come to rest when its velocity $v' = 0$. If x' is the thickness penetrated by the bullet, then

$$u^2 - v'^2 = 2ax'$$

or $$x' = \frac{u^2}{2a}. \text{ But } a = \frac{u^2}{16} \text{ cms}^{-2}. \text{ Therefore}$$

$$x' = \frac{u^2 \times 16}{2u^2} = 8 \text{ cm}$$

Hence the correct choice is (a).

39. Since the wood offers a constant deceleration and hence a constant retardation force, the bullet will lose the remaining 75% of its kinetic energy after penetrating a further distance of $3x$. Therefore, the total distance penetrated by the bullet before it comes to rest $= x + 3x = 4x$. Hence the correct choice is (b).

40. The potential energy in the vertical position = work done in raising it from horizontal position to vertical position. In doing so, the mid–point of the rod is raised through a height $h = l/2$. Since the entire mass of the rod can be assumed to be concentrated at the mid–point (centre of mass), the work done $= mgh = mgl/2$. Hence the correct choice is (c).

41. Refer to Fig. 4.29. $AD = AB = l$. In the inclined position, let the centre of mass C of the rod be at a height h above the ground, so that $AC = l/2$. In triangle ACE, we have

$$h = AC \sin 30° = \frac{l}{2} \sin 30° = \frac{l}{4}$$

$\therefore PE = mgh = \frac{mgl}{4}$, which is choice (a).

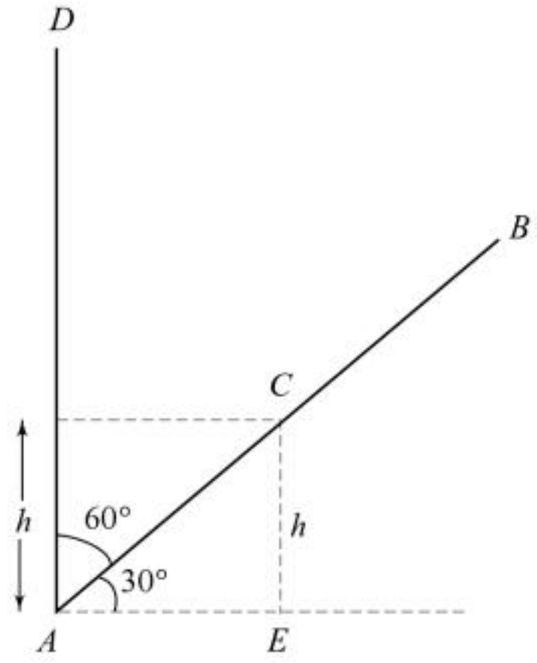

Fig. 4.29

42. Let f be the force of friction and m be the mass of the body. The retardation $a = f/m$. If v is the initial speed of the body, then

$$2ax = v^2$$

or $$max = \frac{1}{2}mv^2 = k$$

But $ma = f$. Therefore $fx = k$ or $f = k/x$. Hence the correct choice is (a).

43. Force of friction $= \mu mg$. Therefore, retardation $a = \mu mg/m = \mu g$. Also $2ax = v^2$ or $2am^2x = m^2v^2$. But $p = mv$. Therefore,

$$2\,am^2x = p^2$$

But $a = \mu g$. Therefore, $2\,\mu g\, m^2x = p^2$ or $\mu = \frac{p^2}{2gm^2x}$.

Hence the correct choice is (a).

44. The mass per unit length of the chain $m = \frac{M}{L}$.

The mass of the hanging portion of the chain is $m' = \frac{mL}{n}$. This mass can be assumed to be concen-

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trated at the centre of the hanging portion of the chain which is a distance of $x = \frac{L}{2n}$ from the edge of the table. Therefore, the work done in pulling the hanging portion of the chain on to the table top is

$$W = m'gx = \frac{mL}{n} \times g \times \frac{L}{2n} = \frac{mgL^2}{2n^2} = \frac{MgL}{2n^2}$$

Hence the correct choice is (d).

45. Since the platform is depressed by an amount x, the total work done on the spring is $mg\ (h + x)$. This work is stored in the spring in the form of potential energy $\frac{1}{2}\ kx^2$. Equating the two, we have

$$\frac{1}{2}\ kx^2 = mg\ (h + x)$$

or $$k = \frac{2m\ g(h + x)}{x^2}$$

Given, $h = 0.4$ m, $x = 0.1$ m, $m = 1$ kg and $g = 10\ \text{ms}^{-2}$. Substituting these values, we get $k = 1000\ \text{Nm}^{-1}$. Hence the correct choice is (c).

46. Let V be the velocity of the block with the bullet embedded in it at the time of impact. Then from the principle of conservation of momentum, we have

$$mv = (M + m)\ V$$

or $$V = \frac{mv}{(M + m)} \qquad \text{(i)}$$

If the block, with the bullet embedded in it, rises to a vertical height h, then from the principle of conservation of energy, we have

$$\frac{1}{2}\ (M + m)\ V^2 = (M + m)\ gh$$

or $$V = \sqrt{2gh} \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$\sqrt{2gh} = \frac{mv}{(M + m)}$$

Squaring this equation, we find that h is given correctly by choice (a).

47. Percentage loss of energy $= \frac{4mM}{(M + m)^2} \times 100$

$$= \frac{4m \times 2m}{(2m + m)^2} \times 100$$

$$= \frac{800}{9} = 88.9\%$$

Hence the correct choice is (c).

48. Refer to Fig. 4.30. Here $\mathbf{p} = m\mathbf{v}$ and $\mathbf{P} = M\mathbf{V}$. The resultant of $\mathbf{p}$ and $\mathbf{P}$ is

$$p_r = \sqrt{p^2 + P^2} = \sqrt{(mv)^2 + (MV)^2}$$

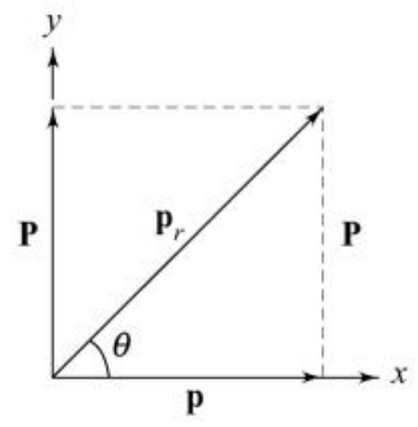

Fig. 4.30

which is choice (a). The angle which the resultant momentum p_r subtends with the x-axis is given by

$$\tan\theta = \frac{P}{p} = \frac{MV}{mv}, \text{ which is choice (b).}$$

$$\text{Loss of KE} = \left(\frac{1}{2}mv^2 + \frac{1}{2}MV^2\right) - \frac{1}{2}\left[\frac{m^2v^2 + M^2V^2}{(M + m)}\right]$$

$$= \frac{1}{2}\frac{Mm}{(M + m)}\ (V^2 + v^2), \text{ which is choice (c).}$$

Hence the correct choice is (d).

49. $\mathbf{p}_1 = 2\text{ kg}\ (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\ \text{ms}^{-1} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}})\ \text{kg ms}^{-1}$

$\mathbf{p}_2 = 3\text{ kg}\ (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\ \text{ms}^{-1} = (6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\ \text{kg ms}^{-1}$

Resultant momentum is

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$$

$$= (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}) + (6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$= (8\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\ \text{kg ms}^{-1}$$

Total mass $(m) = 2 + 3 = 5$ kg. Therefore, the velocity of composite body is

$$\mathbf{v} = \frac{\mathbf{P}}{m} = \frac{1}{5}(8\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\ \text{ms}^{-1}$$

Hence the correct choice is (a).

50. Change in momentum $= m\ (\mathbf{v}_f - \mathbf{v}_i) = 2\text{ kg}\ (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \hat{\mathbf{k}})\ \text{ms}^{-1} - 2\text{ kg}\ (\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-1} = 2\ (4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}})\ \text{kg ms}^{-1}$.

$$\text{Average force} = \frac{\text{change in momentum}}{\text{time of impact}}$$

$$= \frac{2(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}})}{0.02\text{ s}} = 100(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

Hence the correct choice is (d).

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51. Total distance $= h + 2e^2 h + 2e^4 h + ...$

$$= h + 2e^2 h (1 + e^2 + ...)$$

$$= h + \frac{2e^2 h}{1 - e^2} = h\left(\frac{1 + e^2}{1 - e^2}\right)$$

$$= h\left[\frac{1 + \left(\frac{1}{3}\right)^3}{1 - \left(\frac{1}{3}\right)^2}\right] = \frac{3h}{4}$$

Hence the correct choice is (b).

52. Total time $= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + \cdots$

$$= \sqrt{\frac{2h}{g}} + \sqrt{\frac{2h}{g}}\left(e + e^2 + \cdots\right)$$

$$= \sqrt{\frac{2h}{g}} + \sqrt{\frac{2h}{g}}\,\frac{e}{(1-e)} = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

$$= \sqrt{\frac{2h}{g}}\left(\frac{1 + \frac{1}{3}}{1 - \frac{1}{3}}\right) = 2\sqrt{\frac{2h}{g}}$$

Hence the correct choice is (b).

53. Given $m_Q = p\, m_P$ and $v_Q = v_P/q$. From the principle of conservation of momentum, we have (since body P comes to rest after collision)

$$m_P v_P + m_Q v_Q = m_Q v$$

where v is the velocity of body Q after collision. Thus

$$m_P v_P + p\, m_P \frac{v_P}{q} = p\, m_P v.$$

which gives $$\frac{v}{v_P} = \frac{p+q}{pq} \qquad \text{(i)}$$

Now, the coefficient of restitution is given by

$$e = \frac{v}{v_p - v_Q} = \frac{v}{v_p - \frac{v_p}{q}}$$

which gives $$\frac{v}{v_P} = \frac{e}{q}(q-1) \qquad \text{(ii)}$$

Equating (i) and (ii), we get $e = \dfrac{p+q}{p(q-1)}$ which is choice (d).

54. Refer to Fig. 4.31. If sphere A is projected with velocity v, the time taken by it to strike B is equal to $\frac{\pi r}{v} = T$ or $\pi r = Tv$. Now, the coefficient of restitution is given by

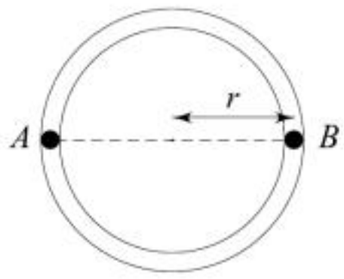

Fig. 4.31

$$e = \frac{v_B - v_A}{v}$$

where v_A and v_B are the velocities of A and B after the collision. Thus, $v_B - v_A = ev$. The spheres travel with this relative velocity. It is clear that one will overtake the other after travelling a distance $= 2\pi r$.

$\therefore$ Time taken $= \dfrac{2\pi r}{v_B - v_A} = \dfrac{2\pi r}{ev} = \dfrac{2Tv}{ev} = \dfrac{2T}{e}$

(since $\pi r = Tv$).

Hence the correct choice is (c).

55. If mass m falss through a distance h, mass m rises up through the same distance h. Let v be the common velocity of the masses when this happens. Now, loss in PE = gain in KE, i.e.

$$Mgh - mgh = \frac{1}{2}(M+m)v^2$$

which gives $v = \sqrt{\dfrac{2gh(M-m)}{(M+m)}}$, which is choice (c).

56. When M has descended a distance h, loss of PE $= Mgh$. If v is the common velocity of the masses, gain in KE $= \dfrac{1}{2}(M+m)v^2$. Hence

$$\frac{1}{2}(M+m)v^2 = Mgh$$

or $$v = \sqrt{\frac{2Mgh}{(M+m)}}$$, which is choice (c).

57. Relative speed of man with respect to escalator $= (v - u)$.

$\therefore$ Actual displacement of man per second $= (v - u)$. Hence, the actual displacement of man in going up the escalator of height h is $\dfrac{v\,h}{(v-u)}$. Therefore,

Work done $= mg \times \dfrac{v\,h}{(v-u)}$, which is choice (d)

58. Let the mass of the rope be M. If a small part of length x hangs through the hole, it weight

$$= \frac{Mx}{L}(g)$$

and its acceleration is $a = \dfrac{Mx}{L}\dfrac{g}{M} = \dfrac{xg}{L}$

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∴ Force on the remaining part of length $(L - x)$ = mass of part of length $(L - x) \times$ acceleration (a)

$$= \frac{M}{L}(L-x) \times \frac{xg}{L} = \frac{Mg}{L^2}(Lx - x^2)$$

If the rope falls through a distance dx, the work done by gravity is

$$dW = \frac{Mg}{L^2}(Lx - x^2)\,dx$$

∴ Total work done is

$$W = \int_0^L dW = \frac{Mg}{L^2}\int_0^L (Lx - x^2)\,dx$$

$$= \frac{Mg}{L^2}\left|\frac{Lx^2}{2} - \frac{x^3}{3}\right|_0^L = \frac{MgL}{6}$$

Since the centre of mass of the rope falls through a distance $\frac{L}{2}$, decrease in PE $= \frac{MgL}{2}$. From the principle of conservation of energy, we have
Decrease in PE – Work done (W) = Increase in KE

or $$\frac{MgL}{2} - \frac{MgL}{6} = \frac{1}{2}Mv^2$$

which gives $v = \sqrt{\frac{4gL}{3}}$, which is choice (b).

59. Given $U = 6x + 8y$ joule and mass $m = 2$ kg. Force along x-axis is

$$F_x = \frac{dU}{dx} = \frac{d}{dy}(6x + 8y) = 6 \text{ newton}$$

Force along y-axis is

$$F_y = \frac{dU}{dx} = \frac{d}{dy}(6x + 8y) = 8 \text{ newton}$$

Therefore, the x and y components of acceleration are

$$a_x = \frac{F_x}{m} = \frac{6}{2} = 3 \text{ ms}^{-2}$$

and $$a_y = \frac{F_y}{m} = \frac{8}{2} = 4 \text{ ms}^{-2}$$

∴ Resultant acceleration

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(3)^2 + (4)^2} = 5 \text{ ms}^{-2}$$

The x and y coordinates of the body at time t are

$$x = x_0 - \frac{1}{2}a_x t^2 = 6 - \frac{1}{2} \times 3 \times t^2$$

$$= \left(6 - \frac{3}{2}t^2\right) \text{ metre}$$

and $$y = y_0 - \frac{1}{2}a_y t^2 = 4 - \frac{1}{2} \times 4 \times t^2$$

$$= (4 - 2t^2) \text{ metre}$$

The body will cross the y-axis when $x = 0$, i.e. at time t given by $\left(6 - \frac{3}{2}t^2\right) = 0$ or $t = 2$ s. Hence the correct choice is (b).

60. $v_x = a_x t = 3 \text{ ms}^{-2} \times 2 \text{ s} = 6 \text{ ms}^{-1}$
$v_y = a_y t = 4 \text{ ms}^{-2} \times 2 \text{ s} = 8 \text{ ms}^{-1}$

∴ $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6)^2 + (8)^2} = 10 \text{ ms}^{-1}$$

Hence the correct choice is (c).

61. Initial momentum (p) = momentum of bullet + momentum of block = $mv + 0 = mv$. From the principle of conservation of momentum, final momentum of bullet + block system of mass $(M + m)$ = Initial momentum p. Now

$$\text{KE} = \frac{p^2}{2 \times (M \times m)} = \frac{m^2v^2}{2(M + m)}$$

Hence the correct choice is (d).

62. A ball dropped from a height h_1 on reaching the planet's surface will have a velocity given by

$$v_1 = \sqrt{2gh_1}$$

Let v_2 be the velocity with which the ball bounces. It will attain a height h_2 given by

$$v_2 = \sqrt{2gh_2}$$

∴ $$\frac{v_2}{v_1} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{1.8}{5}} = 0.6$$

or $1 - \frac{v_2}{v_1} = 1 - 0.6$ or $\frac{v_1 - v_2}{v_1} = 0.4 = \frac{2}{5}$

Hence the correct choice is (b).

63. Let m' and M be the masses of the lighter and heavier fragments respectively. Since the particle is moving along the x-axis, the y-component of momentum will be zero immediately after and before explosion, i.e.

$$m'v_y + MV_y = 0$$

where v_y and V_y are the velocities of the lighter and heavier fragments respectively immediately after explosion. Thus

$$V_y = -\left(\frac{m'}{M}\right)v_y = -\left(\frac{m/4}{3m/4}\right)v_y = -\frac{1}{3}v_y$$

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20. Tensions T_1, T_2 and T_3 are related as

(a) $T_1 = T_2 = T_3$ (b) $T_1 < T_2 < T_3$

(c) $T_1 > T_2 > T_3$ (d) $T_1 = T_3 < T_2$

21. Tension T_4 is given by

(a) $T_4 = \dfrac{mv^2}{R} + mg\cos\theta$

(b) $T_4 = \dfrac{mv^2}{R} - mg\cos\theta$

(c) $T_4 = \dfrac{mv^2}{R} + mg\sin\theta$

(d) $T_4 = \dfrac{mv^2}{R} - mg\sin\theta$

22. The minimum speed the object must have at the highest point A to complete the circle is

(a) $\sqrt{\dfrac{Rg}{2}}$ (b) $\sqrt{Rg}$

(c) $\sqrt{2Rg}$ (d) $2\sqrt{Rg}$

Here g is the acceleration due to gravity.

23. The minimum speed the object must have at the lowest point C to complete the circle is

(a) $\sqrt{Rg}$ (b) $\sqrt{2Rg}$

(c) $2\sqrt{2Rg}$ (d) $\sqrt{5Rg}$

24. At the minimum speed of Q.22, the tension in the string is

(a) 4 mg (b) 5 mg

(c) 6 mg (d) mg

➔ SOLUTIONS

20. Refer to Fig. 4.36.

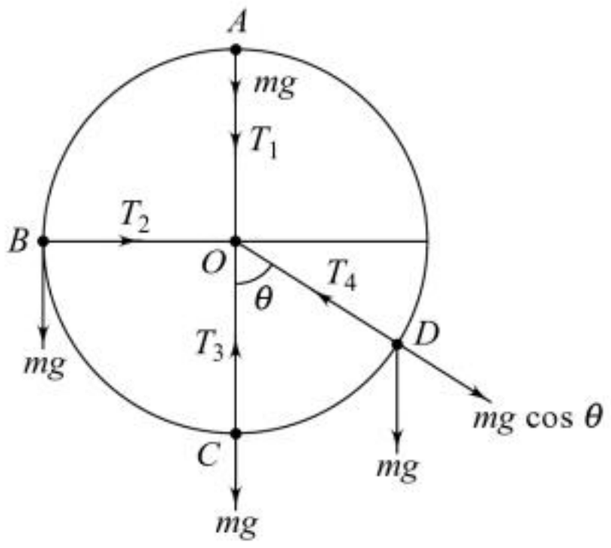

Fig. 4.36

When the object is at A, since the weight mg acts vertically downwards, the force towards centre O is.

$$F_1 = T_1 + mg$$

$$\Rightarrow \quad \frac{mv^2}{R} = T_1 + mg$$

$$\Rightarrow \quad T_1 = \frac{mv^2}{R} - mg$$

At point B, weight mg has no component along BO. Hence the force towards O is

$$F_2 = T_2 \quad \Rightarrow \quad \frac{mv^2}{R} = T_2$$

At point C, the weight mg acts in opposite direction to tension T_3. Thus at C, the force towards centre O is

$$F_3 = T_3 - mg$$

$$\Rightarrow \quad \frac{mv^2}{R} = T_3 - mg$$

$$\Rightarrow \quad T_3 = \frac{mv^2}{R} + mg$$

Hence the correct choice is (b).

21. At point D, the force towards the centre is

$$F_4 = T_4 - mg\cos\theta$$

$$\Rightarrow \quad \frac{mv^2}{R} = T_4 - mg\cos\theta$$

$$\Rightarrow \quad T_4 = \frac{mv^2}{R} + mg\cos\theta, \text{ which is choice (a).}$$

22. In order to keep a body of mass m in a circular path, the centripetal force, at the highest point A, must at least be equal to the weight of the body. Thus

$$\frac{mv_A^2}{R} = mg \quad \text{or} \quad v_A = \sqrt{Rg}$$

gives the minimum speed the body must have at the highest point so that it can complete the circle. Hence the correct choice is (b).

23. The minimum speed v_C of the body must have at the lowest point C is given by $v^2{}_C = v^2{}_A + 2 \times 2\,Rg$ where we have used $v^2 = u^2 + 2gh$, with $h = 2\,R$. Thus

$$v_C^2 = Rg + 4\,Rg = 5\,Rg$$

or $v_C = \sqrt{5Rg}$, which is choice (d).

24. The tension at this point is given by

$$T_3 = m\left(\frac{v_C^2}{R} + g\right) = m(5\,g + g) = 6\,mg$$

Hence the correct choice is (c).

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Questions 25 to 28 are based on the following passage.

Passage VIII

The kinetic energy of a particle moving along a circle of radius R depends on distance (s) as $K = as^2$ where a is a constant.

25. The centripetal force is given by

(a) $\frac{as^2}{2R}$ (b) $\frac{as^2}{R}$

(c) $\frac{2as^2}{R}$ (d) $\frac{4as^2}{R}$

26. The speed of the particle around the circle is

(a) $2s\left(\frac{a}{m}\right)^{1/2}$ (b) $s\left(\frac{a}{m}\right)^{1/2}$

(c) $s\left(\frac{2a}{m}\right)^{1/2}$ (d) $s\left(\frac{a}{2m}\right)^{1/2}$

27. The tangential force acting on the particle is

(a) mas (b) $2mas$
(c) as (d) $2as$

28. The net force acting on the particle is

(a) $2as\left(1+\frac{s}{R}\right)$ (b) $as\left(1+\frac{s^2}{R^2}\right)^{1/2}$

(c) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (d) zero

➔ SOLUTIONS

25. Given KE $(K) = \frac{1}{2}mv^2 = as^2$. Therefore, the centripetal force is

$$f_c = \frac{mv^2}{R} = \frac{2\times\left(\frac{1}{2}mv^2\right)}{R} = \frac{2as^2}{R},$$

which is choice (c).

26. The speed v of the particle around the circle is given by

$$\frac{1}{2}mv^2 = as^2 \quad \text{or} \quad v = s\left(\frac{2a}{m}\right)^{1/2}$$

Hence the correct choice is (c).

27. The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left[s\left(\frac{2a}{m}\right)^{1/2}\right] = \left(\frac{2a}{m}\right)^{1/2}\frac{ds}{dt}$$

But $\frac{ds}{dt} = v = s\left(\frac{2a}{m}\right)^{1/2}$. Therefore,

$$a_t = \left(\frac{2a}{m}\right)^{1/2} \times s\left(\frac{2a}{m}\right)^{1/2} = \frac{2as}{m}$$

$\therefore$ Tangential force is $f_t = ma_t$

$$= m \times \frac{2as}{m} = 2as, \text{ which is choice (d).}$$

28. Net force acting on the particle is

$$f = (f_c^2 + f_t^2)^{1/2}$$

$$= \left[\left(\frac{2as^2}{R}\right)^2 + (2as)^2\right]^{1/2}$$

$$= 2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$$

Thus the correct choice is (c).

Questions 29 to 31 are based on the following passage.

Passage IX

A conical pendulum consists of a string of length L fixed at one end carrying a body of mass m at the other end. The mass is revolved in a circle in the horizontal plane about a vertical axis passing through the fixed end of the string. The angular frequency of revolution of the body is ω. The string makes an angle θ with the vertical axis.

29. The tension in the string is

(a) $\frac{m\omega^2}{L}$ (b) $\frac{L\omega^2}{m}$

(c) $m\omega^2 L$ (d) $m\omega L^2$

30. The angle of inclination of the string with the vertical is given by

(a) $\cos\theta = \frac{g}{\omega^2 L}$ (b) $\sin\theta = \frac{g}{\omega^2 L}$

(c) $\cos\theta = \frac{\omega^2 L}{g}$ (d) $\sin\theta = \frac{\omega^2 L}{g}$

31. The linear speed of the body is

(a) ωL (b) $\omega L \sin\theta$
(c) $\omega L \cos\theta$ (d) $\omega L \tan\theta$

➔ SOLUTIONS

29. Let T be tension in the string. Figure 4.37 shows the forces acting on the system. Tension T can be resolved into two mutually perpendicular components.

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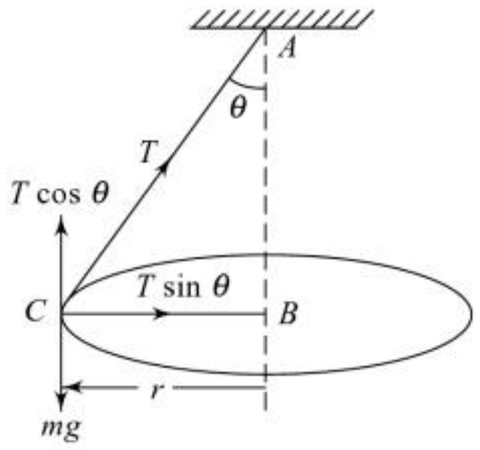

Fig. 4.37

The horizontal component $T \sin \theta$ provides the centripetal force for circular motion and the vertical component $T \cos \theta$ balances the weight mg.

Thus

$$T \cos \theta = mg \quad (1)$$

and $$T \sin \theta = \frac{mv^2}{r} = m\omega^2 r$$

But $r = L \sin \theta$. Therefore,

$$T \sin \theta = m\omega^2 L \sin \theta \text{ or } T = m\omega^2 L \quad (2)$$

Hence the correct choice is (c).

30. From (1), we have $\cos \theta = \frac{mg}{T}$ (3)

Using (2) in (3), we get

$$\cos \theta = \frac{mg}{m\omega^2 L} = \frac{g}{\omega^2 L}$$, which is choice (a).

31. Linear velocity is $v = \omega r = \omega L \sin \theta$, which is choice (b).

SECTION III

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each questions has the following four choices out of which only one choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1
A simple pendulum of length l is displaced from its mean position O to position A so that the string makes an angle θ_1 with the vertical and then released. If air resistances is neglected, the speed of the bob when the string makes an angle θ_2 with the vertical is $v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$.

Statement-2
The total momentum of a system is conserved if no external force acts on it.

2. Statement-1
A uniform rod of mass m and length l is held at an angle θ with the vertical. The potential energy of the rod in this position is $\frac{1}{2} mg\, l \cos \theta$.

Statement-2
The entire mass of the rod can be assumed to be concentrated at its centre of mass.

3. Statement-1
A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Statement-2
The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

4. Statement-1
A man carrying a bucket of water and walking on a rough level road with a uniform velocity does no work while carrying the bucket.

Statement-2
The work done on a body by a force **F** in giving it a displacement **S** is defined as

$$W = \mathbf{F} \cdot \mathbf{S} = FS \cos \theta$$

where θ is the angle between vectors **F** and **S**.

5. Statement-1
A crane P lifts a car up to a certain height in 1 min. Another crane Q lifts the same car up to the same height in 2 min. Then crane P consumes two times more fuel than crane Q.

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Statement-2
Crane P supplies two times more power than crane Q.

6. **Statement-1**
Two inclined frictionless tracks of different inclinations θ_1 and θ_2 meet at A from where two blocks P and Q of different masses m_1 and m_2 are allowed to slide down from rest, one on each track as shown in Fig. 4.38. Then blocks P and Q will reach the bottom with the same speed.

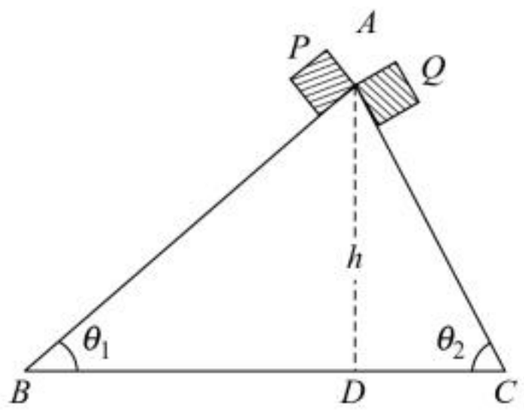

Fig. 4.38

Statement-2
Blocks P and Q have equal accelerations down their respective tracks.

7. **Statement-1**
In Q.6 above, block P will take a longer time to reach the bottom than block Q.

Statement-2
Block Q has a greater acceleration down the track than block P.

8. **Statement-1**
Comets move around the sun in highly elliptical orbits. The work done by the gravitational force of the sun on a comet over a complete orbit is zero.

Statement-2
The gravitational force is conservative.

9. **Statement-1**
The total energy of a system is always conserved irrespective of whether external forces act on the system.

Statement-2
If external forces act on a system, the total momentum and energy will increase.

10. **Statement-1**
The rate of change of the total linear momentum of a system consisting of many particles is proportional to the vector sum of all the internal forces due to inter-particle interactions.

Statement-2
The internal forces can change the kinetic energy of the system of particles but not the linear momentum of the system.

11. **Statement-1**
An elastic spring of force constant k is stretched by a small length x. The work done in extending the spring by a further length x is $2\,kx^2$.

Statement-2
The work done in extending an elastic spring by a length x is proportional to x^2.

12. **Statement-1**
Two identical balls B and C lie on a horizontal smooth straight groove so that they are touching. A third identical ball A moves at a speed v along the groove and collides with B (see Fig. 4.39). If the collisions are perfectly elastic, then after the collision, balls A and B will come to rest and ball C moves with velocity v to the right.

Fig. 4.39

Statement-2
In an elastic collision, linear momentum and kinetic energy are both conserved.

13. **Statement-1**
Two bodies A and B of masses m and $2\,m$ respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with a velocity u and collides elastically with A as shown in Fig. 4.40. At a certain instant t_0 after the collision, it is found that the velocities of A and B are the same $= u/3$.

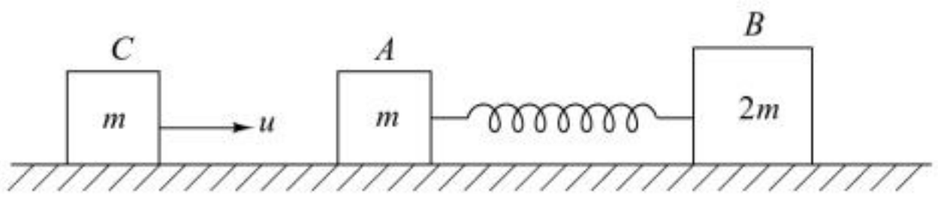

Fig. 4.40

Statement-2
In an elastic collision, the kinetic energy of the system is conserved.

14. **Statement-1**
In an inelastic collision between two bodies, the total energy does not change after the collision but the kinetic energy of the system decreases.

Statement-2
The loss of kinetic energy appears as heat in the system.

15. **Statement-1**
In a collision between two bodies, each body exerts an equal and opposite force on the other at each instant of time during the collision.

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Hence crane P supplies two times more power than crane Q. Thus the correct choice is (d).

6. The acceleration of blocks P and Q respectively are

$$a_1 = \frac{m_1 g \sin\theta_1}{m_1} = g \sin\theta_1$$

and $$a_2 = \frac{m_2 g \sin\theta_2}{m_2} = g \sin\theta_2$$

Since $\theta_2 > \theta_1$; $a_2 > a_1$. The potential energy of block P at $A = m_1gh$. When it reaches the bottom B, its kinetic energy is $\frac{1}{2} m_1 v_1^2$ where v_1 is its speed when it reaches B. Now P.E at A = K.E. at B. Hence

$$m_1 gh = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gh}\,.$$

Similarly $$m_2 gh = \frac{1}{2} m_2 v_2^2 \Rightarrow v_2 = \sqrt{2gh} = v_1.$$

Hence the correct choice is (c).

7. The correct choice is (a). If t_1 and t_2 are the times taken by P and Q to reach the bottom, then

$$v_1 = u_1 + a_1 t_1 = a_1 t_1 \qquad (\because u_1 = 0)$$

and $$v_2 = u_2 + a_2 t_2 = a_2 t_2 \qquad (\because u_2 = 0)$$

Now $v_1 = v_2$. Hence $a_1 t_1 = a_2 t_2$. Thus

$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

Since $a_2 > a_1$; $t_1 > t_2$.

8. The correct choice is (a). For a conservation force, the work done in moving a body from one point to another does not depend on the nature of the path and the work done over a closed path is zero, irrespective of the nature of the path.

9. Statement-1 is false; the total energy of an isolated system is conserved. Statement-2 is true. Hence the correct choice is (d).

10. Statement-1 is false and Statement-2 is true. The rate of change of momentum is proportional to the net external force acting on the system. Hence the correct choice is (d).

11. The correct choice is (d). Potential energy stored in the spring when it is extended by x is $U_1 = \frac{1}{2} kx^2$

Potential energy stored in the spring when it is further extended by x is

$$U_2 = \frac{1}{2} k(x + x)^2 = 2kx^2$$

$\therefore$ Work done = gain in potential energy = $U_2 - U_1$

$= 2\,kx^2 - \frac{1}{2} kx^2 = \frac{1}{2}\,kx^2$

12. The correct choice is (a). Linear momentum will be conserved if A comes to rest and B and C move to the right with a velocity $v/2$ each or A, B and C all move to the right with velocity $v/3$ each. It is easy to see that in these two cases, the kinetic energy is not conserved. Hence the only result of the collision is the one given in Statement-1.

13. The correct choice is (b). Let C collide with A at $t = 0$. Since the collision in elastic and A and C have equal masses, C will come to rest and A will move to the right with velocity u and at this instant the spring is uncompressed and B is at rest. Hence at $t = 0$, the momentum of the system = mu. When A moves to the right, it compresses the spring and as a result B beings to move to the right. Let v be the common velocity of A and B at time t_0. From the principle of conservation of linear momentum, we have

Momentum of C before collision = momentum of A after collision + momentum of B after collision or

$$mu = mv + (2m)\,v \Rightarrow v = \frac{u}{3}.$$

14. The correct choice is (a). The total energy (which includes all forms of energy) is conserved in any process.

15. The correct choice is (b). Statement-1 follows from Newton's third law of motion.

16. The correct choice is (a).

17. The correct choice is (d). The two colliding body need not get stuck after an inelastic collision.

18. The correct choice is (d). Since the velocities of the two bodies change due to collision, the linear momentum of each body will change but the total linear momentum of the system of two bodies is conserved.

19. The correct choice is (d). Due to change in velocity, the energy of each body changes on collision but the total energy of the system of two bodies is conserved.

20. The correct choice is (d). If an external force acts on a system, it is accelerated which will increase the total energy.

21. The correct choice is (a). If v and V are the velocities of Q and P after the collision, then from conservation of momentum and kinetic energy, we have

$$Mu = mv + MV \Rightarrow M(u - V) = mv \quad (1)$$

$$\frac{1}{2} Mu^2 = \frac{1}{2} mv^2 + \frac{1}{2} MV^2$$

$$\Rightarrow \quad M(u - V)(u + V) = mv^2 \quad (2)$$

From Equations (1), (2), we get

$$v = \frac{2Mu}{(M + m)} = \frac{2u}{\left(1 + \frac{m}{M}\right)}$$

If $M \gg m$, then v is maximum equal to $2u$ (since $\frac{m}{M}$ $\rightarrow$ zero).

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5

Rotational Motion

REVIEW OF BASIC CONCEPTS

1. Centre of Mass of Discrete Particles

For a system of particles, the centre of mass is defined as that point where the entire mass of the system is imagined to be concentrated, for considerations of its translational motion.

If $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ldots \mathbf{r}_n$ are the position vectors of masses $m_1, m_2, m_3, \ldots m_n$ respectively, the centre of mass of the system is

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + \cdots + m_n\mathbf{r}_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

$$= \frac{\sum_{N=1}^{n} m_n\mathbf{r}_n}{\sum_{N=1}^{n} m_n} = \frac{\sum_{N=1}^{n} m_n\mathbf{r}_n}{M}$$

where M is the total mass of the system of particles.

$\mathbf{r}_{CM}$ is the weighted average of all the position vectors of the particles of the system, the contribution of each particle being proportional to its mass.

For a system consisting of two particles, the centre of mass is

$$\mathbf{r}_{cm} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

If the masses are equal, i.e. $m_1 = m_2$, then

$$\mathbf{r}_{CM} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$$

which means that the centre of mass lies exactly in the middle of the line joining the two masses.

Example 1 Three particles of masses $m_1 = m$, and $m_2 = 2m$ and $m_3 = 3m$ are placed at the corners of an equilateral triangle of side a as shown in Fig. 5.1. Locate the centre of mass of the system.

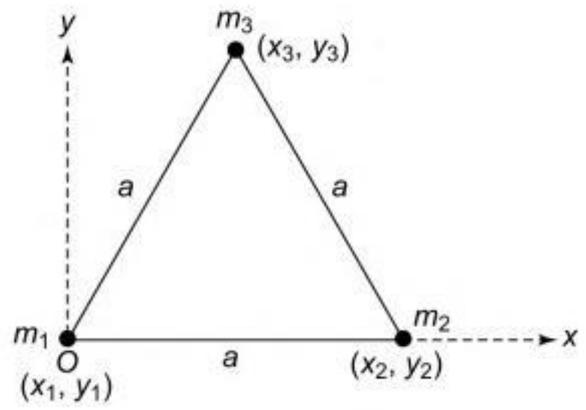

Fig. 5.1

Solution Take any one particle to be located at the origin O. The x and y coordinates of m_1, m_2, and m_3 respectively are $x_1 = 0$ and $y_1 = 0$, $x_2 = a$ and $y_2 = 0$ and $x_3 = \frac{a}{2}$ and $y_3 = \frac{\sqrt{3}a}{2}$

The x and y coordinates of the centre of mass are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + 2m \times a + 3m \times \frac{a}{2}}{m + 2m + 3m} = \frac{7a}{12}$$

and

$$y_{CM} = \frac{m_1\ y_1 + m_2\ y_2 + m_3\ y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + 2m \times 0 + 3m \times \frac{\sqrt{3}a}{2}}{m + 2m + 3m}$$

$$= \frac{\sqrt{3}a}{2}$$

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Example 2 Four particles of masses 1 kg, 1 kg, 2 kg, and 2 kg, are placed at the corners of a square of side 12 cm as shown in Fig. 5.2. Find the position vector of the centre of mass of the system.

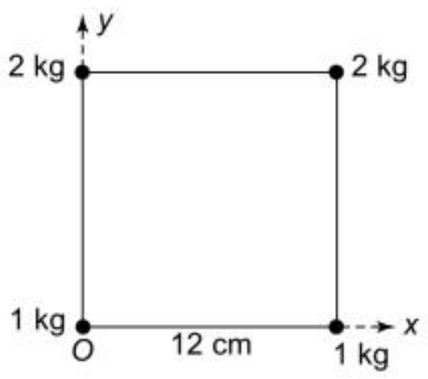

Fig. 5.2

Solution

$$x_{CM} = \frac{1\times 0+1\times 12+2\times 12+2\times 0}{1+1+2+2}$$

$$= 6 \text{ cm}$$

$$y_{CM} = \frac{1\times 0+1\times 0+2\times 12+2\times 12}{1+1+2+2}$$

$$= 8 \text{ cm}$$

The position vector of the centre of mass is

$$\mathbf{r}_{CM} = (6\hat{\mathbf{i}}+8\hat{\mathbf{j}}) \text{ cm}$$

2. Centre of Mass of a Body having Continuous Distribution of Mass

If a body has continuous distribution of mass, the position of its centre of mass is determined by dividing the body into a very large number of extremely small elements. If dm is the mass of the element and it is at a distance x and y from the origin of chosen coordinate system, then x and y coordinates of the centre of mass are given by

$$x_{CM} = \frac{\int x\, dm}{\int dm}$$

and

$$y_{CM} = \frac{\int y\, dm}{\int dm}$$

Example 3 Locate the centre of mass of a uniform rod of mass M and length L.

Solution We assume that the rod lies along the x-axis with one end at origin O.

Mass per unit length of the rod $= \frac{M}{L}$

Mass of element of length dx is [Fig 5.3]

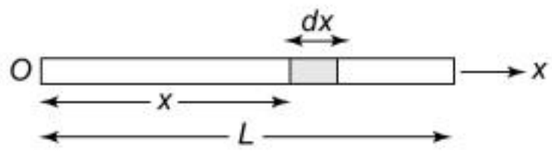

Fig. 5.3

$$dm = \frac{M}{L}\, dx$$

The x-coordinate of the centre of mass is

$$x_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} = \frac{1}{M}\int_0^L \frac{M}{L}\, x\, dx = \frac{L}{2}$$

The centre of mass is at the centre of the rod.

Example 4 A non-uniform rod of length L is lying along the x-axis with one end at origin O as shown in Fig. 5.4. The linear mass density (i.e. mass per unit length) λ varies with x as $\lambda = a + bx$, where a and b are constants. Find the distance of the centre of mass from origin O.

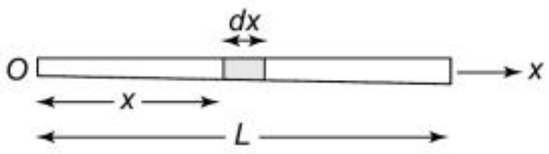

Fig. 5.4

Solution Mass of element is $dm = \lambda\, dx = (a + bx)\, dx$

$$x_{CM} = \frac{\int_0^L x\, dm}{\int_0^L dm} \qquad \text{(i)}$$

$$\int_0^L x\, dm = \int_0^L (a+bx)\, x\, dx$$

$$= a\left|\frac{x^2}{2}\right|_0^L + b\left|\frac{x^3}{3}\right|_0^L$$

$$= \frac{aL^2}{2} + \frac{bL^3}{3} \qquad \text{(ii)}$$

$$\Rightarrow \quad \int_0^L x\, dm = \frac{L^2}{6}(3a+2bL)$$

And

$$\int_0^L dm = \int_0^L (a+bx)\, dx = aL + \frac{bL^2}{2}$$

$$= \frac{L}{2}(2a+bL) \qquad \text{(iii)}$$

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Using (ii) and (iii) in (i), we get

$$x_{CM} = \frac{L(3a+2bL)}{3(2a+bL)}$$

Example 5 Locate the centre of mass of a uniform semi-circular ring of radius R and linear mass density λ.

Solution Let us take the centre of the ring at origin O. Consider a small element of arc length dl of the ring. Let θ be the angle which the radius vector of the element makes with the x-axis as shown in Fig. 5.5.

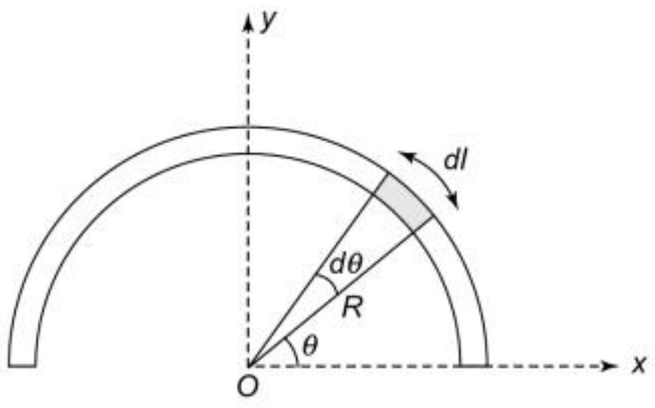

Fig. 5.5

Let $d\theta$ be the angle subtended by the element at the centre. Then $dl = R\,d\theta$. Mass of the element is

$$dm = \lambda dl = \lambda R d\theta$$

The x and y components of radius vector R are $x = R\cos\theta$ and $y = R\sin\theta$. Then

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\lambda R^2 \int_0^{\pi} (\cos\theta)\,d\theta}{\lambda R \int_0^{\pi} d\theta}$$

$$= \frac{\lambda R^2}{\lambda R\pi} |\sin\theta|_0^{\pi}$$

$$= \frac{R}{\pi}(\sin\pi - \sin 0) = 0$$

$$y_{CM} = \frac{\int y dm}{\int dm} = \frac{\lambda R^2 \int_0^{\pi} (\sin\theta)\,d\theta}{\lambda R \int_0^{\pi} d\theta}$$

$$= \frac{\lambda R^2}{\lambda R\pi} |-\cos\theta|_0^{\pi}$$

$$= -\frac{R}{\pi}(\cos\pi - \cos 0) = \frac{2R}{\pi}$$

Thus, the centre of mass is at a distance of $\frac{2R}{\pi}$ from origin O on the y-axis.

3. Finding Centre of Mass of a System when a Part of its Mass is Removed

Consider of system of mass M. If a mass m is removed, the remaining mass = $M - m$ which may written as $M + (-m)$. Then the x and y coordinates of the centre of mass of the remaining portion are given by

$$x_{CM} = \frac{Mx - mx'}{M - m}$$

and

$$y_{CM} = \frac{My - my'}{M - m}$$

where x and y are the coordinates of the centre of mass of the complete part and x' and y' are the coordinates of the centre of mass of the removed part.

Example 6 Four particles, each of mass 1 kg, are placed at the corners of a square of side 12 cm. If mass m_3 is removed, find the shift in the centre of mass of the system (see Fig. 5.6).

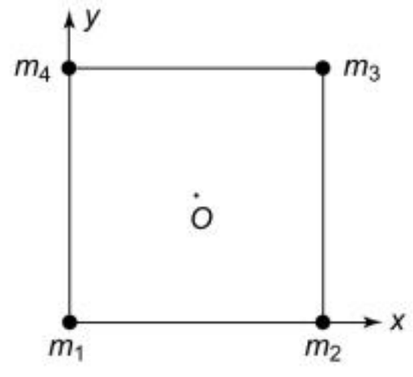

Fig. 5.6

Solution As shown in Example 2, the x and y coordinates of the original system are $x_{CM} = 6$ cm and $y_{CM} = 6$ cm

If mass m_3 is removed, the x and y coordinates of the centre of mass of the remaining system are

$$x'_{CM} = \frac{1\times 0 + 1\times 12\times 1\times 0}{1+1+1} = 4 \text{ cm}$$

$$y'_{CM} = \frac{1\times 0 + 1\times 0 + 1\times 12}{1+1+1} = 4 \text{ cm}$$

Thus $\mathbf{r} = 6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

and $\mathbf{r}' = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$\therefore$ shift $\Delta\mathbf{r} = \mathbf{r} - \mathbf{r}' = (6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}) - (4\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$= 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

Example 7 From a uniform thin disc of radius R and mass M, a circular portion of radius $r = R/2$ is removed as shown in Fig. 5.7. Find the centre of mass of the remaining part of the disc.

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and $$y_{CM} = \frac{My - my'}{M - m} = \frac{M \times 0 - \frac{M}{4} \times -\frac{a}{4}}{M - \frac{M}{4}} = +\frac{a}{12}$$

4. Velocity and Acceleration of Centre of Mass of a System of Particles

If $\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_n$ are the position vectors of masses $m_1, m_2, \ldots m_n$, respectively, the position vector of the centre of mass of the system of particles is given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \cdots + m_n\mathbf{r}_n}{m_1 + m_2 + \cdots + m_n}$$

The velocity of the centre of mass is given by

$$\mathbf{v}_{CM} = \frac{d\mathbf{r}_{CM}}{dt} = \frac{m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} + \cdots + m_n\frac{d\mathbf{r}_n}{dt}}{m_1 + m_2 + \cdots + m_n}$$

$$\Rightarrow \quad \mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots + m_n\mathbf{v}_n}{m_1 + m_2 + \cdots + m_n}$$

The acceleration of the centre of mass is given by

$$\mathbf{a}_{CM} = \frac{d\mathbf{v}_{CM}}{dt} = \frac{m_1\frac{d\mathbf{v}_1}{dt} + m_2\frac{d\mathbf{v}_2}{dt} + \cdots + m_n\frac{d\mathbf{v}_n}{dt}}{m_1 + m_2 + \cdots + m_n}$$

$$\Rightarrow \quad \mathbf{a}_{CM} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \cdots + m_n\mathbf{a}_n}{m_1 + m_2 + \cdots + m_n}$$

or $$\mathbf{a}_{CM} = \frac{\mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum \mathbf{F}_{ext}}{M}$$

If $\sum \mathbf{F}_{ext} = 0$, then $\mathbf{a}_{CM} = 0$, i.e $\mathbf{v}_{CM}$ = constant. Hence if *no net external force acts on a system, its centre of mass will remain at rest or will move with a constant velocity.*

Example 9 A boy of mass $m = 50$ kg stands at the end A of a flat plank AB of wood of mass $M = 100$ kg and length $l = 10$ m floating in the still water in a lake. The end B of the plank is at a distance of 30 m from the shore of the lake as shown in Fig. 5.10. The boy walks a distance of 6 m on the plank towards the shore. How far is the boy from the shore now? Neglect viscosity of water.

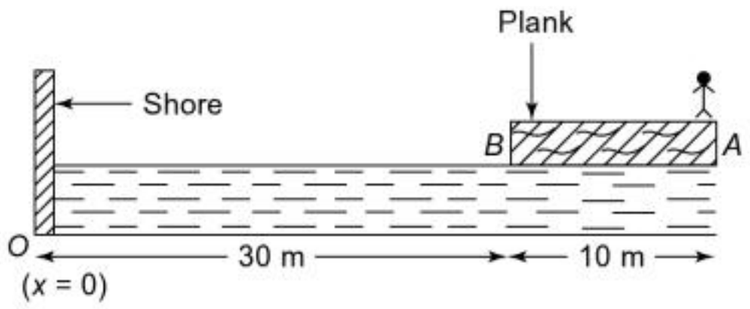

Fig. 5.10

Solution Initially the centre of mass of the system (plank + boy) is at rest. To walk, the boy exerts a force in the backward direction. The plank in turns exerts a reaction force on the boy in the forward direction. These forces are internal to the system. since no external force acts, the centre of mass of the system remains at rest even when the walks on the plank.

Let the shore be at the origin O ($x = 0$). Initially let x be the distance of the centre of mass of the plank from O. Then the distance of the centre of mass of the system (plank + boy) from O will be

$$x_{CM} = \frac{M \times x + m \times (30+10)}{M + m} = \frac{100x + 50 \times 40}{100 + 50}$$

$$= \frac{100x + 2000}{150}$$

Since the boy moves towards the shore and the centre of mass of the system has to remain at rest, the plank will move away from the shore. If x' is the distance moved by the plank, the distance of the centre of mass from the shore when the boy walks 6 m on the plank is given by

$$x'_{CM} = \frac{100(x + x') + 50(40 - 6 + x')}{100 + 50}$$

$$= \frac{100x + 150x' + 1700}{150}$$

Since $x_{CM} = x'_{CM}$,

$$\frac{100x + 2000}{150} = \frac{100x + 150x' + 1700}{150}$$

$$\Rightarrow \quad x' = \frac{300}{150} = 2 \text{ m}$$

$\therefore$ Distance of the boy from the shore = 40 – 6 + 2 = 36 m.

Example 10 A car of mass 1000 kg is moving with a velocity of 10 ms^{-1} towards another car of mass 1500 kg moving with a velocity of 15 ms^{-1} in the same direction. Find the velocity of the centre of mass of the two cars.

Solution $$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{1000 \times 10 + 1500 \times 15}{1000 + 1500}$$

$$= 13 \text{ ms}^{-1}$$

5. Momentum Conservation and Centre of Mass Motion

We have seen that

$$\sum \mathbf{F}_{ext} = M\mathbf{a}_{CM} = M\frac{d\mathbf{v}_{CM}}{dt} = \frac{d}{dt}(M\mathbf{v}_{CM})$$

$$\Rightarrow \quad \sum \mathbf{F}_{ext} = \frac{d\mathbf{p}}{dt}$$

where $\mathbf{P} = M\mathbf{v}_{CM}$ is the *total linear momentum of the system of particles which is equal to the product of the total mass of the system and the velocity of the centre of mass.*

If $\sum \mathbf{F}_{\text{ext}} = 0, \quad \dfrac{d\mathbf{P}}{dt} = 0 \quad \Rightarrow \quad \mathbf{P} = \text{constant.}$

Thus, if no net external force acts on a system, the total linear momentum of the system remains constant; the total linear momentum being the vector sum of linear momentum of individual particles, i.e.

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 + \cdots + \mathbf{P}_n$$

Example 11 A boy of mass $m = 40$ kg is standing on a stationary long plank of mass $M = 260$ kg floating on still water in a lake. He starts running with a velocity $v = 6\ \text{ms}^{-1}$ relative to the plank. Find the velocity of the boy relative to a stationary observer on the bank of the lake.

Solution Since the system (boy + plank) is initially at rest, its momentum is zero. Since no external force acts on the system, the momentum of the system will remain zero. Let us assume that the boy runs in the positive x-direction.

Velocity of boy relative to the plank is

$$\mathbf{v}_{bp} = v\hat{\mathbf{i}}$$

Now $\mathbf{v}_{bp} = \mathbf{v}_b - \mathbf{v}_p$

where $\mathbf{v}_b$ = velocity of boy relative to ground

and $\mathbf{v}_p$ = velocity of plank relative to ground

Hence $\mathbf{v}_b = \mathbf{v}_{bp} + \mathbf{v}_p$

$$= v\hat{\mathbf{i}} + v_p\hat{\mathbf{i}}$$

$$= (v + v_p)\hat{\mathbf{i}}$$

Total momentum of plank + boy = 0

$$\Rightarrow \quad M\mathbf{v}_p + m\mathbf{v}_b = 0$$

$$\Rightarrow \quad Mv_p\hat{\mathbf{i}} + mv_b\hat{\mathbf{i}} = 0$$

$$\Rightarrow Mv_p\hat{\mathbf{i}} + m(v + v_p)\hat{\mathbf{i}} = 0$$

which gives $v_p = -\dfrac{mv}{M+m}$

$$\therefore \text{ Velocity of boy is } \mathbf{v}_b = \left(v - \frac{mv}{M+m}\right)\hat{\mathbf{i}} = \frac{Mv}{M+m}\hat{\mathbf{i}}$$

$$= \left(\frac{260 \times 6}{260 + 40}\right)\hat{\mathbf{i}}$$

$$= 5.2\,\hat{\mathbf{i}}\ \text{ms}^{-1}$$

Hence the velocity of boy relative to a stationary observer is $5.2\ \text{ms}^{-1}$ in the direction along which the boy is running.

6. Torque

If a force $\mathbf{F}$ acts on a particle P whose position vector with respect to the origin of an inertial reference frame is $\mathbf{r}$, the torque $\vec{\tau}$ acting on the particle with respect to the origin is defined as [Fig. 5.11]

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

In terms of magnitudes,

$$\tau = rF\sin\theta = F(r\sin\theta) = Fr_\perp$$

where θ is the angle between vectors $\mathbf{r}$ and $\mathbf{F}$.

Torque $\boldsymbol{\tau}$ is a vector quantity. Its magnitude is given by $\tau = rF\sin\theta$; its direction is normal to the plane containing vectors $\mathbf{r}$ and $\mathbf{F}$ and can be determined by the *right–hand screw rule.*

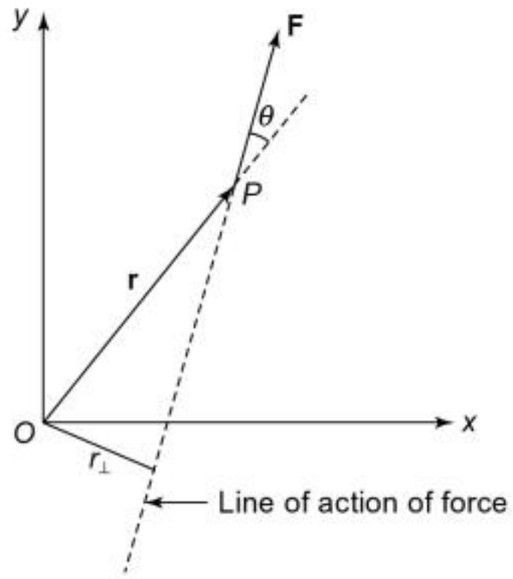

Fig. 5.11

Unit of Torque Torque has the same dimensions as those of work (both being force times distance) viz. ML^2T^{-2}. The two are, however, very different quantities. Work is a scalar, torque is a vector. To distinguish between the two we express work in joules and torque in newton–metre (N m).

Couple

Two equal antiparallel forces having different lines of action constitute a couple.

The moment of couple or torque = $Fr_\perp$ (Fig. 5.12)

= magnitude of either force × perpendicular distance between the two antiparallel forces

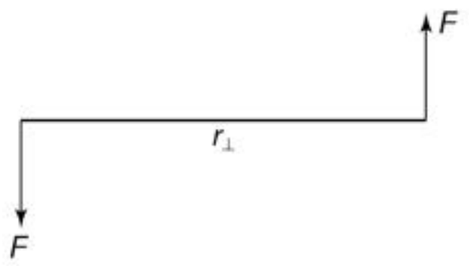

Fig. 5.12

Work done by torque

If a force F acts on a rigid body at perpendicular $r_\perp$ from the axis of rotation, the work done by the force in rotating the body through an angle $\Delta\theta$ is given by

$$\Delta W = Fr_\perp \Delta\theta = \tau \Delta\theta$$

= magnitude of torque × angular displacement

$$\text{Power} = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

Where ω is the angular velocity.

Example 12 A force $\mathbf{F} = \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{i}}\right)$ newton acts on a particle whose position vector with respect to origin O is $\vec{\mathbf{r}} = \left(4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}\right)$ metre. Find the magnitude and direction of the torque.

Solution

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$= \left(4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}\right) \times \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)$$

$$= 4\hat{\mathbf{i}} \times 2\hat{\mathbf{i}} + 12\hat{\mathbf{i}} \times \hat{\mathbf{j}} - 10\hat{\mathbf{j}} \times \hat{\mathbf{i}} - 15\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$= 0 + 12\hat{\mathbf{k}} + 10\hat{\mathbf{k}} - 0$$

$$= 22\ \hat{\mathbf{k}} \text{ newton metre}$$

The magnitude of torque is 22 Nm and its direction is along the positive z-axis.

Example 13 A rectangular plate $OPQR$ of dimensions 2 m × 3 m lies in the x-y plane as shown in Fig 5.13. A force $\mathbf{F} = \left(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)$ newton is applied at point Q. Find the torque of $\mathbf{F}$ (a) about origin O, (b) about point P and (c) about x-axis, y-axis and z-axis.

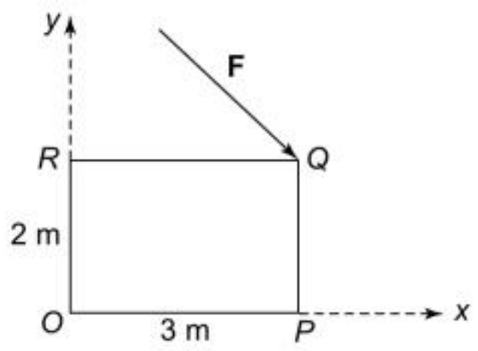

Fig. 5.13

Solution

(a) $\mathbf{r} = \overrightarrow{OQ} = \left(3\hat{\mathbf{i}} + 2\mathbf{j}\right)$ metre

Torque about O is

$$\tau_O = \mathbf{r} \times \mathbf{F}$$

$$= \left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right) \times \left(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)$$

$$= 9\hat{\mathbf{i}} \times \hat{\mathbf{i}} + 15\hat{\mathbf{i}} \times \hat{\mathbf{j}} + 6\hat{\mathbf{j}} \times \hat{\mathbf{i}} + 10\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$

$$= 0 + 15\hat{\mathbf{k}} - 6\hat{\mathbf{k}} + 0 = 9\hat{\mathbf{k}} \text{ Nm}$$

(b) Torque about P is

$$\boldsymbol{\tau}_P = \overrightarrow{PQ} \times \mathbf{F}$$

$$= 2\hat{\mathbf{j}} \times \mathbf{F}$$

$$= 2\hat{\mathbf{j}} \times \left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right) = -6\,\hat{\mathbf{k}} \text{ Nm}$$

(c) Torque about x-axis is

$$\boldsymbol{\tau}_x = \boldsymbol{\tau}_O \cdot \hat{\mathbf{i}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$$

Torque about y-axis is

$$\boldsymbol{\tau}_y = \boldsymbol{\tau}_O \cdot \hat{\mathbf{j}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = 0$$

Torque about z-axis is

$$\boldsymbol{\tau}_z = z_O\ \hat{\mathbf{k}} = 9\hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 9 \text{ Nm}$$

7. Angular Momentum

The angular momentum $\mathbf{L}$ of a particle P with respect to the origin of an inertial reference frame is defined as [Fig. 5.14]

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where $\mathbf{r}$ is the position vector of the particle and $\mathbf{p}$ its linear momentum. In terms of magnitudes,

$$L = rp \sin\theta = r_1 \times p$$

where θ is the angle between vectors $\mathbf{r}$ and $\mathbf{p}$. The dimensions of angular momentum are $(ML^2\,T^{-1})$ and its SI unit is kg m^2s^{-1}.

The direction of $\mathbf{L}$ is perpendicular to the plane containing the vectors $\mathbf{r}$ and $\mathbf{p}$ and its sense is given by the *right–hand rule*.

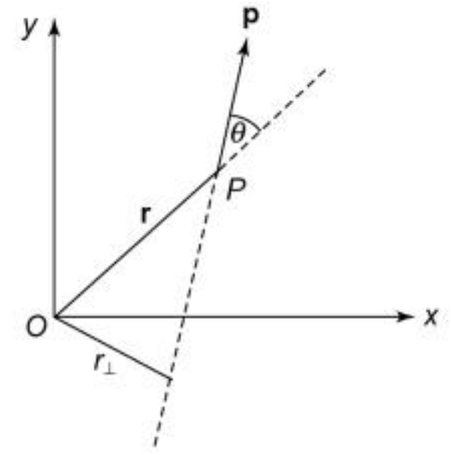

Fig. 5.14

Example 14 A body of mass m = 200 g is moving parallel to the x-axis with a velocity $\mathbf{v}$ = 30 cms^{-1} in the x-y plane as shown in Fig. 5.15. Calculate the magnitude of its angular momentum about origin O at any time t.

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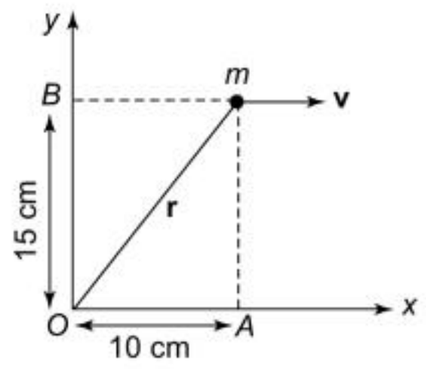

Fig. 5.15

Solution Magnitude of angular momentum $= r_\perp p$

$$= r_\perp mv$$

$$= OB \times m \times v$$

$$= 0.15 \times 0.2 \times 0.3$$

$$= 9 \times 10^{-3} \text{ kg m}^2\text{s}^{-1}$$

Example 15 A body of mass $m = 200$ g is projected with a velocity $u = 5$ ms^{-1} at an angle $\theta = 30°$ with the horizontal. Calculate the magnitude of the angular momentum of the body about the point of projection when it is at the highest point of its trajectory. Take $g = 10$ ms^{-2}.

Solution Refer to Fig. 5.16. At the highest point A, the body has only horizontal velocity

$$v = u \cos \theta$$

$$h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

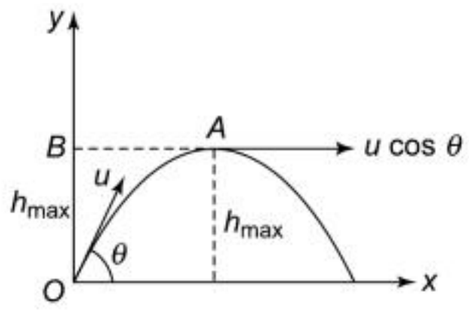

Fig. 5.16

Magnitude of angular momentum of the body about O when it is at point A is

$$L = mv \times OB$$

$$= mv \times h_{\max}$$

$$= m\,u\cos\theta \times \frac{u^2 \sin^2 \theta}{2g}$$

$$= \frac{mu^3 \sin^2 \theta \cos\theta}{2g}$$

$$= \frac{0.2 \times (5)^3 \times \sin^2 (30°)\cos(30°)}{2 \times 10}$$

$$= \frac{0.2 \times 125 \times \frac{1}{4} \times \frac{\sqrt{3}}{2}}{20}$$

$$= 0.27 \text{ kg m}^2\text{s}^{-1}$$

8. Relation between Torque and Angular Momentum

In linear motion, the relation between force **F** and linear momentum **p** is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

In rotational motion, the relation between torque τ and angular momentum **L** is

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$$

which states that *the torque acting on a particle is equal to the rate of change of angular momentum.*

9. Angular Impulse

In linear motion, impulse **I** is defined as

$$\mathbf{I} = \int \mathbf{F}\, dt$$

$$= \int \frac{d\mathbf{p}}{dt} dt$$

$$= d\mathbf{p} = \text{change in linear momentum}$$

In rotational motion, angular impulse **J** is defined as

$$\mathbf{J} = \int \tau\, dt$$

$$= \int \frac{d\mathbf{L}}{dt} dt$$

$$= d\mathbf{L} = \text{change in angular momentum}$$

10. Law of Conservation of Angular Momentum

If no external torque acts, the total angular momentum of a body or a system of particles is conserved.

We have seen that the rate of change of angular momentum of a particle is equal to the torque produced by the total force. If, in a certain situation, the torque itself vanishes, then it follows that the angular momentum of the particle will remain constant. This is the law of conservation of the angular momentum of a particle. One trivial situation is when the force vanishes. Then the torque vanishes too. The particle then moves freely in a straight line in accordance with Newton's first law in which case both linear and angular momenta are conserved.

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A general situation is when the torque vanishes without the force itself vanishing. The torque $\boldsymbol{\tau}$ will vanish if the component $\mathbf{F}_{\perp}$ (the angular component) of $\mathbf{F}$ vanishes but the radial component $\mathbf{F}_{\text{II}}$ does not. The radial component $\mathbf{F}_{\text{II}}$ is the component of $\mathbf{F}$ along the radius (or position) vector $\mathbf{r}$. Hence, *if the force acting on the particle is purely radial (i.e. if it is directed along or against its position vector) then the torque acting on the particle vanishes and its angular momentum is conserved and so is its areal velocity.*

11. Moment of Inertia

The moment of inertia of a rigid body about a particular axis may be defined as the sum of the products of the masses of all the particles constituting the body and the squares of their respective distances from the axis of rotation, i.e.

$$I = m_1 r_1^2 + m_2 r^2{}_2 + m_3 r_3^2 + \cdots + m_n r_n^2$$

$$= \sum_{\text{N=1}}^{n} m_n r_n^2$$

Its value depends upon the particular axis about which the body rotates and the way the mass is distributed in the body with respect to the axis of rotation.

In the case of a body which does not consist of separate, discrete particles but has a continuous and homogeneous distribution of matter in it, the summation is replaced by integration, so that

$$I = \int r^2 d\,m$$

where dm is the mass of an infinitesimally small element of the body at a distance r from the axis of rotation.

Moment of inertia is a scalar quantity. Its SI unit is kg m^2 and its dimensions are (ML^2).

12. Radius of Gyration

The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the entire mass of the body were concentrated, its moment of inertia about the given axis would be the same as with its actual distribution of mass. It is usually denoted by the letter K. If M is the mass of the body, its moment of inertia I in terms of its radius of gyration K can be written

$$I = MK^2$$

13. Moment of Inertia and Rotational Kinetic Energy

Kinetic energy of a rotating body is related to moment of inertia as

$$\text{KE} = \frac{1}{2} I\omega^2$$

where ω is the angular velocity (or frequency) of the body.

14. Moment of Inertia and Torque

The magnitude of torque is given by

$$\tau = I\alpha$$

where α is the angular acceleration of the body.

15. Moment of Inertia and Angular Momentum

The magnitude of angular momentum of a rotating body is given by

$$L = I\omega$$

16. Parallel Axes Theorem

If M is the total mass of a body and h the distance between two parallel axes, then according to parallel axes theorem

$$I = I_{\text{CM}} + Mh^2$$

17. Perpendicular Axes Theorem

The theorem of perpendicular axes for a body of plane lamina states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its own plane and intersecting each other at the point where the perpendicular axis passes through it.

If I_x and I_y are the moments of inertia of a plane lamina about the perpendicular axes x and y respectively which lie in the plane of the lamina and intersect each other at O, then the moment of inertia I of the lamina about an axis passing through O and perpendicular to its plane is given by

$$I = I_x + I_y$$

18. Expressions for moment of inertia of bodies of regular shapes about particular axes of rotation:

	Shape of body	*Axis of Rotation*	*Expression for Moment of Inertia*
1.	Circular ring of mass M and radius R	(i) through centre, perpendicular to plane of ring	MR^2
		(ii) any diameter	$(1/2)\,MR^2$

Contd...

Shape of body	*Axis of Rotation*	*Expression for Moment of Inertia*
	(iii) any tangent in the plane of ring	$(3/2)\,MR^2$
	(iv) any tangent perpendicular to plane of ring	$2\,MR^2$
2. Circular disc of mass M and radius R	(i) through centre, perpendicular to plane of disc	$(1/2)\,MR^2$
	(ii) any diameter	$(1/4)\,MR^2$
	(iii) tangent in the plane of the disc	$(5/4)\,MR^2$
	(iv) tangent perpendicular to plane of disc	$(3/2)\,MR^2$
3. Sphere of mass M and radius R	(i) any diameter	$(2/5)\,MR^2$
	(ii) any tangent plane	$(7/5)\,MR^2$
4. Hollow sphere of mass M and radius R	(i) any diameter	$\frac{2}{3}MR^2$
	(ii) any tangent plane	$\frac{5}{3}MR^2$
5. Cylinder of mass M, radius R and length L	(i) own axis	$(1/2)\,MR^2$
	(ii) through centre perpendicular to length	$M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$
	(iii) through end faces and $\perp$ to length	$M\left(\frac{R^2}{4}+\frac{L^2}{3}\right)$
6. One dimensional rod of mass M and length L	(i) centre of rod and $\perp$ to length	$ML^2/12$
	(ii) one end and $\perp$ to length	$ML^2/3$
7. Rectangular lamina of mass M, length L and breadth B	(i) length of lamina and in its plane	$MB^2/3$
	(ii) breadth of lamina and in its plane	$ML^2/3$
	(iii) centre of lamina and parallel or to length or breadth in its plane	$\frac{MB^2}{12}$ or $\frac{ML^2}{12}$
	(iv) centre of lamina and $\perp$ to its plane	$M\left(\frac{L^2+B^2}{12}\right)$
	(v) centre of length and $\perp$ to its plane	$M\left(\frac{L^2}{12}+\frac{B^2}{3}\right)$
	(vi) centre of breadth and $\perp$ to its plane	$M\left(\frac{L^2}{3}+\frac{B^2}{12}\right)$

19. Kinematics of Rotational Motion with Constant Angular Acceleration

Consider a body rotating with an initial angular velocity ω_0. It is given a constant angular acceleration α (by applying a constant torque) for a time t. As a result it acquires a final angular velocity ω and suffers an angular displacement θ in time t. The equations of rotational motion are

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

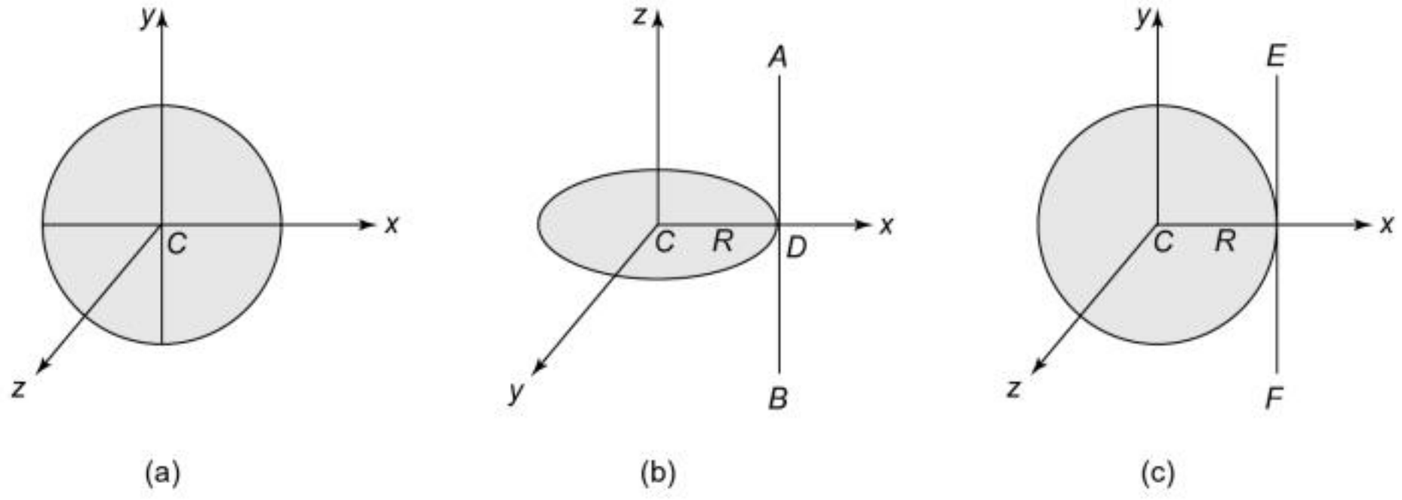
and $$2\alpha\theta = \omega^2 - \omega_0^2$$

Example 16 The moment of inertia of a uniform circular disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane is $\frac{1}{2}MR^2$. Find the moment of inertia of the disc

(a) about any diameter
(b) about an axis passing through a point on the edge of the disc and perpendicular to the disc
(c) about a tangent in the plane of the disc.

Solution The plane of the disc is the x-y plane.

(a) Using perpendicular axes theorem [Fig. 5.17(a)]



(a) (b) (c)

Fig. 5.17

$$I_x + I_y = I_z$$

Now $I_z = I_C = \frac{1}{2}MR^2$ (given).

From symmetry $I_y = I_x$.

$$\therefore \quad 2I_x = \frac{1}{2}MR^2 \quad \Rightarrow \quad I_x = I_y = \frac{1}{4}MR^2$$

(b) Using parallel axes theorem [Fig. 5.17(b)]

$$I_{AB} = I_z + M(CD)^2$$
$$= I_C + MR^2$$
$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

(c) Using parallel axes theorem [Fig. 5.17(c)]

$$I_{EF} = I_y + MR^2$$
$$= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

Example 17 A thin uniform disc of mass M and radius R has concentric hole of radius r. Find the moment of inertia of the disc about an axis passing through its centre and perpendicular to its plane.

Solution Mass per unit area of the disc is [Fig. 5.18]

$$m = \frac{M}{\pi\left(R^2 - r^2\right)}$$

Mass of the disc if it was complete (i.e. without hole) is

$$M_1 = m \times \pi R^2 = \frac{M}{\pi\left(R^2 - r^2\right)} \times \pi R^2$$
$$= \frac{M\,R^2}{R^2 - r^2}$$

Mass of the removed portion is

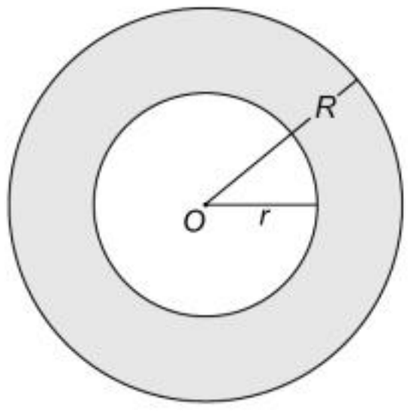


Fig. 5.18

$$M_2 = m \times \pi r^2 = \frac{Mr^2}{R^2 - r^2}$$

Since the two portions are concentric, the moment of inertia of the given disc about the given axis is

$$I = \frac{1}{2}M_1R^2 - \frac{1}{2}M_2\,r^2$$
$$= \frac{1}{2}\left[\frac{MR^4}{R^2 - r^2} - \frac{Mr^4}{R^2 - r^2}\right]$$
$$= \frac{M}{2}\left[\frac{R^4 - r^4}{R^2 - r^2}\right] = \frac{1}{2}M\left(R^2 + r^2\right)$$

Example 18 Find the moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its centre and making an angle θ with the rod.

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Example 24 A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely in a vertical plane.

(a) What is the angular acceleration of the rod when it is at an angle θ with the vertical?

(b) With what linear speed will the end B hit the floor?

Solution

(a) The entire mass of the rod acts at its centre of mass C. $AC = L/2$. The magnitude of the torque due to weight Mg is

$$\tau = Mg \times r_{\perp}$$

$$= Mg \times AD = Mg \times \frac{L}{2}\sin\theta$$

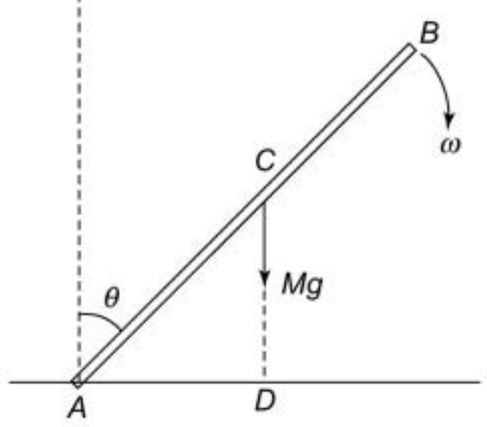

Fig. 5.24

Moment of inertia of the rod about A is $I = \dfrac{ML^2}{3}$

$$\therefore \quad \text{Angular acceleration } \alpha = \frac{\tau}{I} = \frac{MgL\sin\theta \times 3}{2 \times ML^2}$$

$$= \frac{3g\sin\theta}{L}$$

(b) When the end B hits the floor, the vertical distance through which C falls is $L/2$. From the law of conservation of energy,

Loss in P.E. = gain in K.E

$$Mg \times \frac{L}{2} = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{3g}{L}}$$

Linear speed of end $B = L\omega = \sqrt{3gL}$

Example 25 A particle of mass m is released from rest at point P located at a distance x_0 from origin O on the x-axis as shown in Fig. 5.25. It falls vertically along the negative y-axis.

(a) Find the magnitude and direction of the torque acting on the particle at time t when it reaches point Q whose position vector with respect to O is **r**.

(b) Find the magnitude of the angular momentum of the particle about O at this time t.

(c) Show that, in this example, $\tau = \dfrac{dL}{dt}$

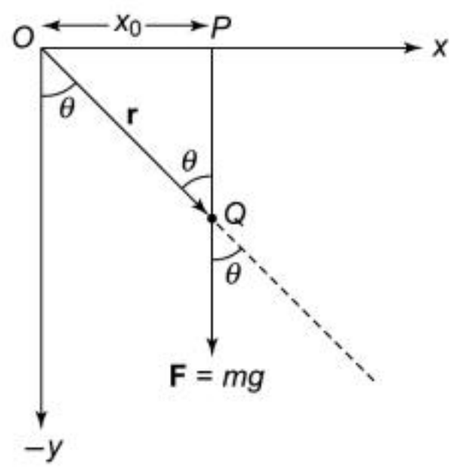

Fig. 5.25

Solution

(a) The torque is due to force of gravity $\mathbf{F} = m\mathbf{g}$. The magnitude of the torque of **F** about O is

$$\tau = rF\sin\theta = r\,mg\sin\theta$$

$$= rmg\,\frac{x_0}{r} \qquad \left(\because \sin\theta = \frac{x_0}{r}\right)$$

$$\Rightarrow \qquad \tau = mg\,x_0$$

From right hand rule, the direction of the torque is into the page .

(b) The magnitude of angular momentum about O is

$$L = rp\sin\theta = rmv\sin\theta$$

From $v = u + at$ we have $v = 0 + gt = gt$. Therefore

$$L = rm \times gt \times \frac{x_0}{r} = mg\,x_0 t$$

(c) $\dfrac{dL}{dt} = \dfrac{d}{dt}(mgx_0 t) = mgx_0 = \tau$

20. Rolling Motion without Slipping

(1) *Total Kinetic Energy*

The total kinetic energy of a body which is moving as well as rotating is equal to the sum of translational K.E. and rotational K.E., i.e.

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where m = mass of the body, v = linear velocity of its centre of mass, I = moment of inertia of the body about an axis passing through its centre of mass and ω = angular velocity of rotation.

(2) *Instantaneous Velocity of a Point on a Rolling Body*

Consider a wheel of radius $R(= AC = BC)$ rolling without slipping on a horizontal rough surface (Fig. 5.26).

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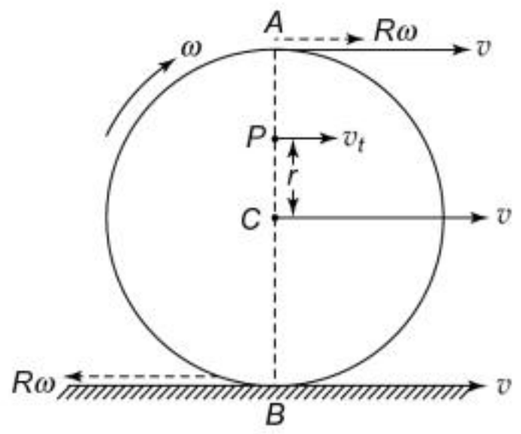

Fig. 5.26

Every point on the wheel has instantaneous velocity. For a point P at a distance r from the centre of mass C, the instantaneous velocity is the vector sum of velocity $\mathbf{v}$ of the centre of mass and tangential velocity $\mathbf{v}_t$ of point P relative to the centre of mass, i.e.

$$\mathbf{v}_P = \mathbf{v} + \mathbf{v}_t$$

where $v_t = r\omega$. Vector $\mathbf{v}_t$ is directed along the tangent to the circle of radius r about C.

For point A, $v_t = R\omega$ and $v = r\omega$. Since these velocities are in the same direction, the instantaneous velocity of A is

$$v_A = v + v_t = R\omega + R\omega = 2\,R\omega$$

For point B in contact with the horizontal surface,

$$v_B = v + v_t = R\omega - R\omega = 0$$

If a body rolls on a surface without slipping, the instantaneous velocity of the point of contact with the surface is zero.

(3) *A body Rolling without Slipping on a Rough Horizontal Surface*

Horizontal force F is applied at the centre of mass of a body (disc, ring, cylinder or sphere) of mass M and radius R (Fig. 5.27) on a rough horizontal surface. If f is the frictional force, and the body rolls without slipping $a_{CM} = \alpha R$.

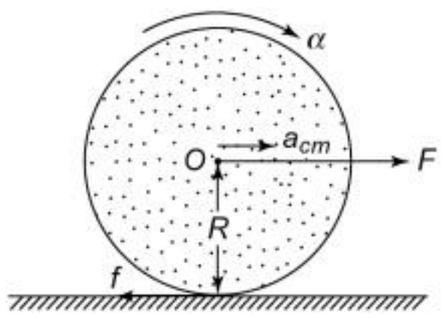

Fig. 5.27

$$F - f = Ma_{CM} \qquad \text{(i)}$$

$$\tau = fR = I\alpha \Rightarrow f = \frac{I\alpha}{R} = \frac{Ia_{CM}}{R^2} \qquad \text{(ii)}$$

From (i) and (ii), acceleration of centre of mass is

$$a_{CM} = \frac{F}{M\left(1 + \dfrac{I}{MR^2}\right)}$$

and

$$f = \frac{F}{\left(1 + \dfrac{MR^2}{I}\right)}$$

For a disc $I = \dfrac{1}{2}MR^2 \Rightarrow a_{CM} = \dfrac{2F}{3M}$

For a ring $I = MR^2 \Rightarrow a_{CM} = \dfrac{F}{2M}$

For a solid cylinder $I = \dfrac{1}{2}MR^2 \Rightarrow a_{CM} = \dfrac{2F}{3M}$

For a hollow cylinder $I = MR^2 \Rightarrow a_{CM} = \dfrac{F}{2M}$

For a solid sphere $I = \dfrac{2}{5}MR^2 \Rightarrow a_{CM} = \dfrac{5F}{7M}$

For a hollow sphere $I = \dfrac{2}{3}MR^2 \Rightarrow a_{CM} = \dfrac{3F}{5M}$

If the force is applied tangentially to the body as shown in Fig. 5.28, then

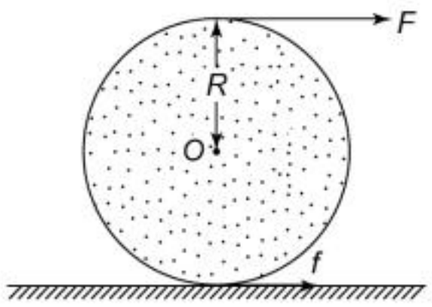

Fig. 5.28

$$a_{CM} = \frac{2F}{M\left(1 + \dfrac{I}{MR^2}\right)}$$

and

$$f = \frac{F\left(1 - \dfrac{I}{MR^2}\right)}{1 + \dfrac{I}{MR^2}}$$

(4) *A body Rolling without slipping on a Rough Inclined Plane*

A body (ring, disc, cylinder or sphere) of mass M and radius R is rolling (without slipping) down a rough inclined plane of inclination θ (Fig. 5.29).

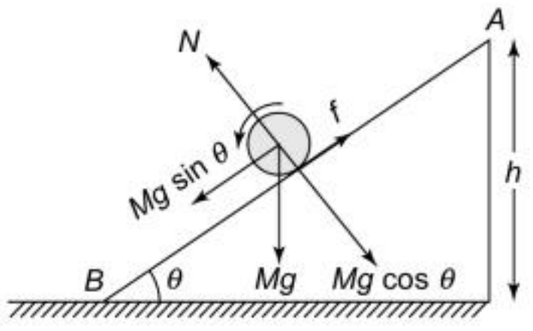

Fig. 5.29

For linear motion parallel to the plane

$$Mg \sin \theta - f = Ma \qquad \text{(i)}$$

where a = linear acceleration of the centre of mass.
For rotational motion about the axis through the centre of mass

$$\tau = I\,\alpha$$

$$\Rightarrow \qquad Rf = I\alpha \Rightarrow$$

$$f = \frac{I\alpha}{R} = \frac{I\alpha}{R^2} \qquad (\because\ a = R\alpha)\ \text{(ii)}$$

I is the moment of inertia about the centre of mass.
Using (i) and (ii), we get

$$a = \frac{g \sin \theta}{\left(1 + \dfrac{I}{MR^2}\right)}$$

For a ring $I = MR^2 \quad \Rightarrow \quad a = \dfrac{g \sin \theta}{2}$

For a disc $I = \dfrac{1}{2} MR^2 \quad \Rightarrow \quad a = \dfrac{2g \sin \theta}{3}$

For a solid cylinder $I = \dfrac{1}{2} MR^2 \quad \Rightarrow \quad a = \dfrac{2g \sin \theta}{3}$

For a hollow cylinder $I = MR^2 \quad \Rightarrow \quad a = \dfrac{g \sin \theta}{2}$

For a hollow sphere $I = \dfrac{2}{3} MR^2 \quad \Rightarrow \quad a = \dfrac{3g \sin \theta}{5}$

For a solid sphere $I = \dfrac{2}{5} MR^2 \quad \Rightarrow \quad a = \dfrac{5g \sin \theta}{7}$

Frictional force is $f = \dfrac{Ia}{R^2} = \dfrac{Mg \sin \theta}{\left(1 + \dfrac{MR^2}{I}\right)}$

Condition for rolling without slipping

To prevent slipping, $f \le \mu N$, where μ is the coefficient of static friction between the body and the plane and $N = Mg \cos \theta$ is the normal reaction.
Hence to avoid slipping,

$$\frac{Mg \sin \theta}{1 + \dfrac{MR^2}{I}} \le \mu\, Mg \cos \theta$$

$$\Rightarrow \qquad \mu \ge \frac{\tan \theta}{\left(1 + \dfrac{MR^2}{I}\right)}$$

Example 26 A solid cylinder of mass M and radius R is released from rest from top A of an inclined plane of height h and inclination θ as shown in Fig. 5.30. The cylinder rolls without slipping. Find (a) the speed at which it reaches bottom B of the plane and (b) the time it takes to reach B.

Solution The acceleration of the cylinder is

$$a = \frac{g \sin \theta}{1 + \dfrac{I}{MR^2}} = \frac{2g \sin \theta}{3} \qquad \left(\because I = \frac{1}{2} MR^2\right)$$

Distance travelled is $s = AB = \dfrac{h}{\sin \theta}$

(a) Using $v^2 - u^2 = 2as$, we have

$$v^2 - 0 = 2 \times \frac{2\ g \sin \theta}{3} \times \frac{h}{\sin \theta}$$

$$\Rightarrow \qquad v = \sqrt{\frac{4\ gh}{3}}$$

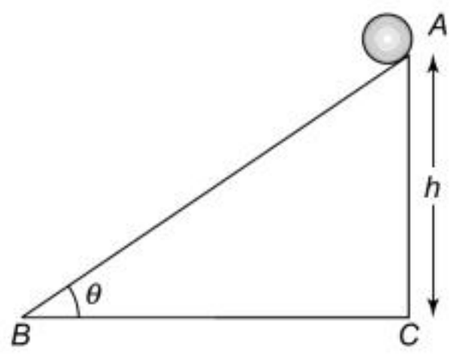

Fig. 5.30

Speed v can also be found from the law of conservation of energy. As the cylinder moves from A to B, it loses P.E. and gains K.E.

Loss in P.E. = gain in K.E.

or $$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

Now $I = \dfrac{1}{2} MR^2$ and $\omega = \dfrac{v}{R}$. Therefore

$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} \times \frac{1}{2} MR^2 \times \left(\frac{v}{R}\right)^2$$

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$$= \frac{3}{4} Mv^2$$

$$\Rightarrow \qquad v = \sqrt{\frac{4\,gh}{3}}$$

(b) From $v = u + at$, we have

$$\sqrt{\frac{4\,gh}{3}} = 0 + \frac{2\,g\sin\theta}{3}\,t$$

$$\Rightarrow \qquad t = \sqrt{\frac{3h}{g}} \cdot \frac{1}{\sin\theta}$$

Example 27 A billiard ball has mass $M = 250$ g and radius $R = 2.5$ cm and is initially at rest. A rod held horizontal at a height h above centre C hits the ball. The ball begins to roll without slipping. Find the value of h [see Fig. 5.31].

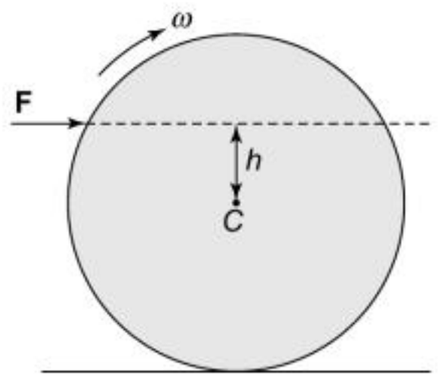

Fig. 5.31

Solution The horizontal force **F** imparts a linear impulse

$$I = \int F dt = \text{change in linear momentum}$$

$$\Rightarrow \qquad I = Mv - 0 = Mv = MR\omega \qquad \text{(i)}$$

where v is the velocity of the centre of mass of the ball. Since it rolls without slipping, $v = R\omega$, where ω is the angular velocity.

The torque due to **F** imparts an angular impulse

$$J = I\,h = \text{change in angular momentum}$$

$$= I\omega - 0 = I\omega$$

or $$Ih = \left(\frac{2}{5} M R^2\right) \omega \qquad \left(\because I = \frac{2}{5} M R^2\right) \text{(ii)}$$

Dividing (ii) by (i)

$$h = \frac{\frac{2}{5} M R^2 \omega}{M R\omega} = \frac{2R}{5} = \frac{2 \times 2.5\,\text{cm}}{5} = 1.0\ \text{cm}$$

Example 28 A turntable of radius $R = 10$ m is rotating making 98 revolutions in 10 s with a boy of mass $m = 60$ kg standing at its centre. He starts running along a radius. Find the frequency of the turntable when the boy is 4 m from the centre. The moment of inertia of the turntable about is axis 1000 kg m^2.

Solution Initial moment of inertia of the system is

M_1 = M.I. of turntable + M.I. of boy at the centre

$= 1000 + 0 = 1000$ kg m^2

Initial frequency $\nu_1 = 9.8$ rev/sec

Final moment of the system is

M_2 = M.I. of turntable + M.I. of boy at a distance 4 m from the centre of turn table

$= 1000 + 60 \times (4)^2 = 1960$ kg m^2

Since no external torque acts, the angular momentum of the system is conserved, i.e.

$$I_2\omega_2 = I_1\omega_1 \Rightarrow I_2\nu_2 = I_1\nu_1$$

$$\Rightarrow \qquad \nu_2 = \frac{I_1\nu_1}{I_2} = \frac{1000 \times 9.8}{1960} = 5\ \text{rev/s}$$

$$= 5\ \text{Hz}$$

Example 29 A uniform rod AB of length $L = 1$ m is sliding along two mutually perpendicular surfaces OP and OQ as shown in Fig. 5.32. When the rod subtends an angle $\theta = 30°$ with OQ, the end B has a velocity $\sqrt{3}$ ms^{-1}. Find the velocity of end A at that time.

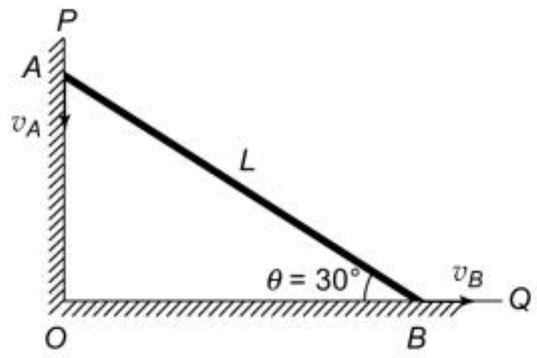

Fig. 5.32

Solution $OB = x$, $OA = y$ and $x^2 + y^2 = L^2$ and $x = L\cos\theta$

Differentiating $x^2 + y^2 = L^2$ with respect to t we have

$$2x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt} = 0$$

$$\Rightarrow \qquad 2xv_B + 2yv_A = 0$$

$$\Rightarrow \qquad v_A = -\frac{x}{y}\,v_B$$

$$\therefore \qquad |v_A| = \frac{x}{y}\,v_B = v_B \cot\theta$$

$$= \sqrt{3} \times \cot 30°$$

$$= \sqrt{3} \times \frac{1}{\sqrt{3}} = 1\ \text{ms}^{-1}$$

Example 30 A rope is wound around a hollow cylinder of mass $M = 3$ kg and radius $R = 40$ cm. If the rope is pulled with a force $F = 30$ N, find (a) the angular acceleration of the cylinder and (b) the linear acceleration of the rope.

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2. The centre of mass of a system of two particles of masses m_1 and m_2 is at a distance a_1 from mass m_1 and at a distance a_2 from mass m_2 such that

(a) $\frac{a_1}{a_2} = \frac{m_2}{m_1}$ (b) $\frac{a_1}{a_2} = \frac{m_1}{m_2}$

(c) $\frac{a_1}{a_2} = \frac{m_1}{(m_1 + m_2)}$ (d) $\frac{a_1}{a_2} = \frac{m_2}{(m_1 + m_2)}$

3. Three particles of the same mass lie in the x–y plane. The (x, y) coordinates of their positions are (1, 1), (2, 2) and (3, 3) respectively. The (x, y) coordinates of the centre of mass are

(a) (1, 2) (b) (2, 2)
(c) (4, 2) (d) (6, 6)

4. A child is standing at one end of a long trolley moving with a speed v on a smooth horizontal track. If the child starts running towards the other end of the trolley with a speed u, the centre of mass of the system (trolley + child) will move with a speed

(a) zero (b) $(v + u)$
(c) $(v - u)$ (d) v

5. Choose the only incorrect statement from the following:

(a) The position of the centre of mass of a system of particles does not depend upon the internal forces between particles.
(b) The centre of mass of a solid may lie outside the body of the solid.
(c) A body tied to a string is whirled in a circle with a uniform speed. If the string is suddenly cut, the angular momentum of the body will change from its initial value.
(d) The angular momentum of a comet revolving around a massive star, remains constant over the entire orbit.

6. A loaded spring gun of mass M fires a 'shot' of mass m with a velocity v at an angle of elevation θ. The gun is initially at rest on a horizontal frictionless surface. After firing, the centre of mass of the gun–shot system

(a) moves with a velocity $v\ m/M$
(b) moves with velocity $\frac{vm}{M} \cos\theta$ in the horizontal direction
(c) remains at rest
(d) moves with a velocity $\frac{v(M - m)}{(M + m)}$ in the horizontal direction.

7. If a man of mass M jumps to the ground from a height h and his centre of mass moves a distance x in the time taken by him to 'hit' the ground, the average force acting on him (assuming his retardation to be constant during his impact with the ground) is

(a) $M g\ h/x$ (b) $M g\ x/h$
(c) $M g\ (h/x)^2$ (d) $M g\ (x/h)^2$

8. Four particles of masses m, m, $2m$ and $2m$ are placed at the four corners of a square of side a as shown in Fig. 5.35. The (x, y) co-ordinates of the centre of mass are

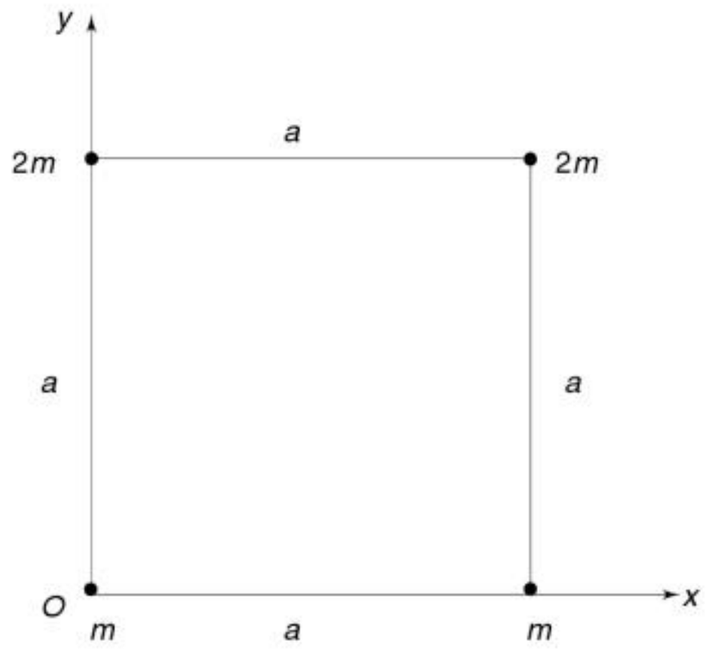

Fig. 5.35

(a) $\left(\frac{a}{2}, 2a\right)$ (b) $\left(\frac{a}{2}, a\right)$
(c) $\left(\frac{a}{2}, \frac{2a}{3}\right)$ (d) $\left(a, \frac{a}{3}\right)$

9. A carpet of mass M, made of an inextensible material, is rolled along its length in the form of a cylinder of radius R and kept on a rough floor. If the carpet is unrolled, without sliding, to a radius $R/2$, the decrease in potential energy is

(a) $\frac{1}{2} MgR$ (b) $\frac{5}{8} MgR$
(c) $\frac{3}{4} MgR$ (d) $\frac{7}{8} MgR$

10. Four tiny masses are connected by a rod of negligible mass as shown in Fig. 5.36.

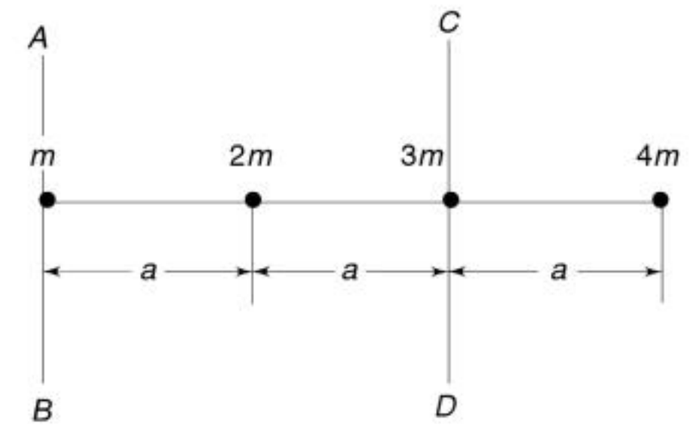

Fig. 5.36

The moment of inertia of the system about axis AB is

(a) $30\ ma^2$ (b) $40\ ma^2$
(c) $50\ ma^2$ (d) $60\ ma^2$

11. In Q. 10, the radius of gyration of the system about axis AB is

(a) $\sqrt{2}\ a$ (b) $\sqrt{3}\ a$
(c) $2\ a$ (d) $\sqrt{5}\ a$

12. In Q. 10, the moment of inertia of the system about axis CD is

(a) $5\ ma^2$ (b) $10\ ma^2$
(c) $30\ ma^2$ (d) $40\ ma^2$

13. In Q. 10, the radius of gyration of the system about axis CD is

(a) $\sqrt{2}\ a$ (b) a
(c) $2\sqrt{2}\ a$ (d) $2a$

14. Three point–masses m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of side a. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1?

(a) $(m_1 + m_2)\dfrac{a^2}{4}$ (b) $(m_2 + m_3)\dfrac{a^2}{4}$
(c) $(m_1 + m_3)\dfrac{a^2}{4}$ (d) $(m_1 + m_2 + m_3)\dfrac{a^2}{4}$

15. A molecule consists of two atoms, each of mass m, separated by a distance a. The moment of inertia of the molecule about its centre of mass is

(a) $2\ ma^2$ (b) ma^2
(c) $\dfrac{1}{2}\ ma^2$ (d) $\dfrac{1}{4}\ ma^2$

16. In Q. 15, if k is the average rotational kinetic energy of the molecule at room temperature, its frequency of rotation is

(a) $\dfrac{1}{\pi a}\sqrt{\dfrac{k}{m}}$ (b) $\dfrac{1}{2\pi a}\sqrt{\dfrac{k}{m}}$
(c) $\dfrac{1}{\pi a}\sqrt{\dfrac{2k}{m}}$ (d) $\dfrac{1}{2\pi a}\sqrt{\dfrac{2k}{m}}$

17. The moment of inertia of a solid sphere of mass M and radius R, about an axis through its centre, is $\dfrac{2}{5}\ MR^2$. The moment of inertia about an axis tangential to the surface of the sphere will be

(a) $\dfrac{4}{5}\ MR^2$ (b) MR^2
(c) $\dfrac{6}{5}\ MR^2$ (d) $\dfrac{7}{5}\ MR^2$

18. The moment of inertia of a uniform circular disc of mass M and radius R about any of its diameters is $\dfrac{1}{4}\ MR^2$. What is the moment of inertia of the disc about an axis passing through its centre and normal to the disc?

(a) MR^2 (b) $\dfrac{1}{2}\ MR^2$
(c) $\dfrac{3}{2}\ MR^2$ (d) $2\ MR^2$

19. In Q.18, what is the moment of inertia of the disc about an axis passing through a point on its edge and normal to the disc?

(a) MR^2 (b) $\dfrac{1}{2}\ MR^2$
(c) $\dfrac{3}{2}\ MR^2$ (d) $2\ MR^2$

20. When W joule of work is done on a flywheel, its frequency of rotation increases from ν_1 Hz to ν_2 Hz. The moment of inertia of the flywheel about its axis of rotation is given by

(a) $\dfrac{W}{2\pi^2\left(\nu_2^2 - \nu_1^2\right)}$ (b) $\dfrac{W}{2\pi^2\left(\nu_2^2 + \nu_1^2\right)}$
(c) $\dfrac{W}{4\pi^2\left(\nu_2^2 - \nu_1^2\right)}$ (d) $\dfrac{W}{4\pi^2\left(\nu_2^2 + \nu_1^2\right)}$

21. Two discs of moments of inertia I_1 and I_2 about their respective axes, rotating with angular frequencies ω_1 and ω_2 respectively, are brought into contact face to face with their axes of rotation coincident. The angular frequency of the composite disc will be

(a) $\dfrac{I_1\omega_1 - I_2\omega_2}{I_1 - I_2}$ (b) $\dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$
(c) $\dfrac{I_2\omega_1 + I_1\omega_2}{I_1 + I_2}$ (d) $\dfrac{I_2\omega_1 - I_1\omega_2}{I_1 - I_2}$

22. A solid cylinder of mass M and radius R rolls down an inclined plane of height h. The angular velocity of the cylinder when it reaches the bottom of the plane will be

(a) $\dfrac{2}{R}\sqrt{gh}$ (b) $\dfrac{2}{R}\sqrt{\dfrac{gh}{2}}$
(c) $\dfrac{2}{R}\sqrt{\dfrac{gh}{3}}$ (d) $\dfrac{1}{2R}\sqrt{gh}$

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23. In Q.22, the rotational kinetic energy of the cylinder when it reaches the bottom of the plane will be

(a) Mgh (b) $\frac{Mgh}{2}$

(c) $\frac{Mgh}{3}$ (d) $\frac{Mgh}{4}$

24. A light-weight boy holds two heavy dumb-bells of equal mass with outstretched arms while standing on a turn-table which is rotating at an angular frequency ω_1 when the dumb-bells are at a distance r_1 from the axis of rotation. The boy suddenly pulls the dumb-bells towards his chest until they are at a distance r_2 from the axis of rotation. The new angular frequency of rotation ω_2 of the turn-table will be equal to

(a) $\omega_1 \frac{r_2}{r_1}$ (b) $\omega_1 \frac{r_1^2}{r_2^2}$

(c) $\omega_1 \frac{r_1}{r_2}$ (d) $\omega_1 \frac{r_2^2}{r_1^2}$

25. A cord is wound around the circumference of a bicycle wheel (without tyre) of diameter 1 m. A mass of 2 kg is tied to the end of the cord and it is allowed to fall from rest. The weight falls 2 m in 4 s. The axle of the wheel is horizontal and the wheel rotates with its plane vertical. The angular acceleration produced is (Take $g = 10\ \text{ms}^{-2}$)

(a) 0.5 rad s^{-2} (b) 1.0 rad s^{-2}
(c) 2.0 rad s^{-2} (d) 4.0 rad s^{-2}

26. In Q. 25, the moment of inertia of the wheel about the horizontal axis is

(a) 10 kg m^2 (b) 20 kg m^2
(c) 30 kg m^2 (d) 40 kg m^2

27. If the earth were to suddenly contract to half its present size, without any change in its mass, the duration of the new day will be

(a) 6 hours (b) 12 hours
(c) 18 hours (d) 30 hours

28. Two circular discs have masses in the ratio of 1 : 2 and radii in the ratio of 2 : 1. The ratio of their moments of inertia about the diameter is

(a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) 8 : 1

29. A circular ring of mass M and radius R is rotating about its axis at an angular frequency ω. Two blocks, each of mass m, are gently placed on the opposite ends of a diameter of the ring. The angular frequency of the ring becomes ω'. The ratio ω'/ω is

(a) $\frac{M}{(M+2m)}$ (b) $\frac{2M}{(M+2m)}$

(c) $\frac{2m}{M}$ (d) $\frac{M}{2m}$

30. A solid sphere rolls down from the top of an inclined plane. Its velocity on reaching the bottom of the plane is v. When the same sphere slides down from the top of the plane, its velocity on reaching the bottom is v'. The ratio v'/v is

(a) $\sqrt{\frac{3}{5}}$ (b) 1

(c) $\sqrt{\frac{7}{5}}$ (d) $\frac{3}{\sqrt{5}}$

31. A circular disc rolls down an inclined plane without slipping. What fraction of its total energy is translational?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

32. A sphere rolls down an inclined plane without slipping. What fraction of its total energy is rotational?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

33. A circular disc is rolling down an inclined plane without slipping. If the angle of inclination is 30°, the acceleration of the disc down the inclined plane is

(a) g (b) $\frac{g}{2}$

(c) $\frac{g}{3}$ (d) $\frac{\sqrt{2}}{3} g$

34. A block of mass M is released from the top of an inclined plane. Its velocity on reaching the bottom of the plane is v. A circular disc of the same mass M rolls down the incline plane from the top. Its velocity on reaching the bottom is v'. The ratio v'/v will be

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{\frac{2}{3}}$

(c) 1 (d) $\frac{2\sqrt{2}}{3}$

35. Two circular loops A and B of radii R and $2R$ respectively are made of the same wire. Their moments

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of inertia about the axis passing through the centre and perpendicular to their plane are I_A and I_B respectively. The ratio I_A/I_B is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

36. Two circular loops A and B are made of the same wire and their radii are in the ratio $1:n$. Their moments of inertia about the axis passing through the centre and perpendicular to their plane are in the ratio $1:m$. The relation between m and n is

(a) $m = n$ (b) $m = n^2$

(c) $m = n^3$ (d) $m = n^4$

37. A small coin is placed at a distance r from the centre of a gramophone record. The rotational speed of the record is gradually increased. If the coefficient of friction between the coin and the record is μ, the minimum angular frequency of the record for which the coin will fly off is given by

(a) $\sqrt{\frac{2\mu g}{r}}$ (b) $\sqrt{\frac{\mu g}{2r}}$

(c) $\sqrt{\frac{\mu g}{r}}$ (d) $2\sqrt{\frac{\mu g}{r}}$

38. In Q. 37, what would be the minimum angular frequency at which two identical coins, placed one on top of the other, at the same location on the record, will fly off?

(a) $\sqrt{\frac{\mu g}{r}}$ (b) $\sqrt{\frac{2\mu g}{r}}$

(c) $2\sqrt{\frac{\mu g}{r}}$ (d) $2\sqrt{\frac{2\mu g}{r}}$

39. Three particles, each of mass m, are placed at the corners of a right angled triangle as shown in Fig. 5.37. If $OA = a$ and $OB = b$, the position vector of the centre of mass is (here **i** and **j** are unit vectors along x and y axes respectively).

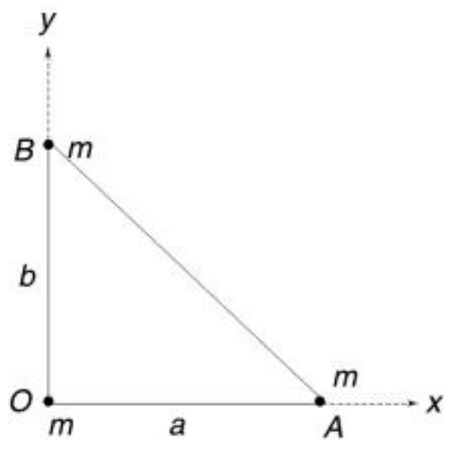

Fig. 5.37

(a) $\frac{1}{3}(a\mathbf{i} + b\mathbf{j})$ (b) $\frac{1}{3}(a\mathbf{i} - b\mathbf{j})$

(c) $\frac{2}{3}(a\mathbf{i} + b\mathbf{j})$ (d) $\frac{2}{3}(a\mathbf{i} - b\mathbf{j})$

40. Three particles each of mass m, are placed at the corners of an equilateral triangle of side a, as shown in Fig. 5.38. The position vector of the centre of mass is

(a) $\frac{a}{2}(\mathbf{i} + \mathbf{j}/\sqrt{3})$ (b) $\frac{a}{2}(3\,\mathbf{i} + \mathbf{j})$

(c) $\frac{a}{2}(3\,\mathbf{i} + \sqrt{3}\,\mathbf{j})$ (d) $\frac{a}{2}(3\,\mathbf{i} + \mathbf{j}/\sqrt{3})$

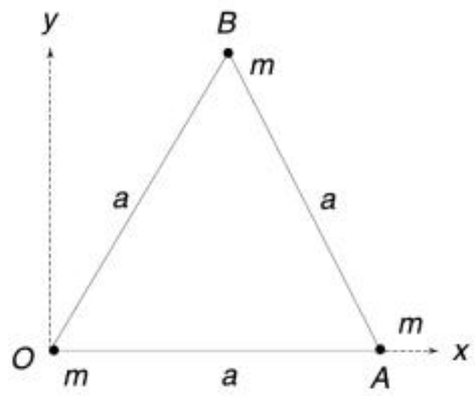

Fig. 5.38

41. A cylinder of mass m and radius r is rotating about its axis with a constant speed v. Its kinetic energy is

(a) $2\,mv^2$ (b) mv^2

(c) $\frac{1}{2}mv^2$ (d) $\frac{1}{4}mv^2$

42. A circular disc of mass m and radius r is rolling on a horizontal surface with a constant speed v. Its kinetic energy is

(a) $\frac{1}{4}mv^2$ (b) $\frac{1}{2}mv^2$

(c) $\frac{3}{4}mv^2$ (d) mv^2

43. Two solid spheres A and B, each of radius R, are made of materials of densities ρ_A and ρ_B respectively. Their moments of inertia about a diameter are I_A and I_B respectively. The ratio I_A/I_B is

(a) $\sqrt{\frac{\rho_A}{\rho_B}}$ (b) $\sqrt{\frac{\rho_B}{\rho_A}}$

(c) $\frac{\rho_A}{\rho_B}$ (d) $\frac{\rho_B}{\rho_A}$

44. A cylinder, released from the top of an inclined plane, rolls without sliding and reaches the bottom with

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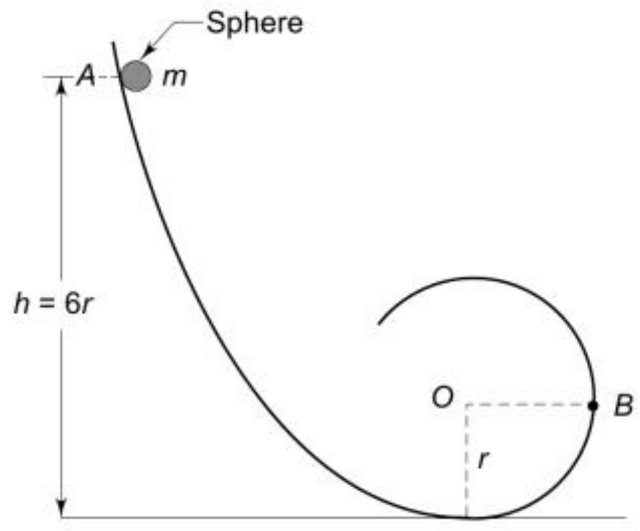

Fig. 5.42

51. In Q. 50 above, the horizontal force acting on the sphere when it reaches B is

(a) $10\,mg$ (b) mg

(c) $\frac{50}{7}\,mg$ (d) $\frac{22}{7}\,mg$

52. A solid sphere is rotating about a diameter at an angular velocity ω. If it cools so that its radius reduces to $\frac{1}{n}$ of its original value, its angular velocity becomes

(a) $\frac{\omega}{n}$ (b) $\frac{\omega}{n^2}$

(c) $n\omega$ (d) $n^2\omega$

53. In Q. 52 above, if the original rotational kinetic energy of the sphere is K, its new value will be

(a) $\frac{K}{n^2}$ (b) $\frac{K}{n^4}$

(c) n^2K (d) n^4K

54. A solid sphere is rotating about its diameter. Due to increase in room temperature, its volume increases by 0.5%. If no external torque acts, the angular speed of the sphere will

(a) increase by nearly $\frac{1}{3}\%$

(b) decrease by nearly $\frac{1}{3}\%$

(c) increase by nearly $\frac{1}{2}\%$

(d) decrease by nearly $\frac{2}{3}\%$

55. A cylinder of mass M has a length L that is $\sqrt{3}$ times its radius R. What is the ratio of its moment of inertia about its own axis and that about an axis passing through its centre and perpendicular to its axis?

(a) 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

56. A uniform rod of length L is suspended from one end such that it is free to rotate about an axis passing through that end and perpendicular to the length. What minimum speed must be imparted to the lower end so that the rod completes one full revolution?

(a) $\sqrt{2gL}$ (b) $2\sqrt{gL}$

(c) $\sqrt{6gL}$ (d) $2\sqrt{2gL}$

57. The height of a solid cylinder is four times its radius. It is kept vertically at time $t = 0$ on a belt which is moving in the horizontal direction with a velocity $v = 2.45\ t^2$ where v is in ms^{-1} and t is in second. If the cylinder does not slip, it will topple over at time t equal to

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

58. A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely on the floor in the vertical plane. The angular velocity of the rod when its end B strikes the floor is

(a) $\sqrt{\frac{g}{L}}$ (b) $\sqrt{\frac{2g}{L}}$

(c) $\sqrt{\frac{3g}{L}}$ (d) $2\sqrt{\frac{g}{L}}$

59. The moment of inertia of a thin rod of mass M and length L about an axis passing through the point at a distance $L/4$ from one of its ends and perpendicular to the rod is

(a) $\frac{7ML^2}{48}$ (b) $\frac{ML^2}{12}$

(c) $\frac{ML^2}{9}$ (d) $\frac{ML^2}{3}$

60. A circular disc of radius R is free to oscillate about an axis passing through a point on its rim and perpendicular to its plane. The disc is turned through an angle of 60° and released. Its angular velocity when it reaches the equilibrium position will be

(a) $\sqrt{\frac{g}{3R}}$ (b) $\sqrt{\frac{2g}{3R}}$

(c) $\sqrt{\frac{2g}{R}}$ (d) $2\sqrt{\frac{2g}{R}}$

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61. A massless and inextensible cord is wound round the circumference of a circular ring of mass M and radius R. The ring is free to rotate about an axis passing through its centre and perpendicular to its plane. A mass m is attached at the free end of the cord and is at rest. The angular speed of the ring when mass m has fallen through at height h is

(a) $\sqrt{\frac{2gh}{R^2}}$ (b) $\sqrt{\frac{2mgh}{MR^2}}$

(c) $\sqrt{\frac{2mgh}{(M+m)R^2}}$ (d) $\sqrt{\frac{2mgh}{(M+2m)R^2}}$

62. A circular portion of diameter R is cut out from a uniform circular disc of mass M and radius R as shown in Fig. 5.43. The moment of inertia of the remaining (shaded) portion of the disc about an axis passing through the centre O of the disc and perpendicular to its plane is

(a) $\frac{15}{32}MR^2$ (b) $\frac{7}{16}MR^2$

(c) $\frac{13}{32}MR^2$ (d) $\frac{3}{8}MR^2$

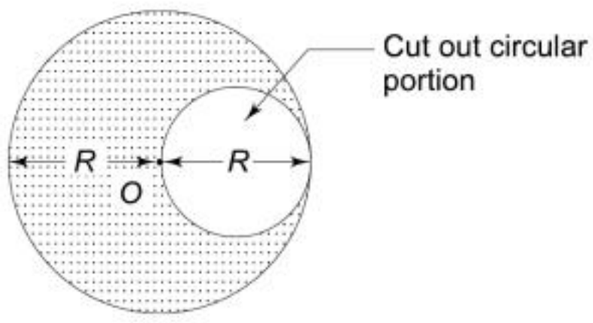

Fig. 5.43

63. The moment of inertia of a hollow sphere of mass M and internal and external radii R and $2R$ about an axis passing through its centre and perpendicular to its plane is

(a) $\frac{3}{2}MR^2$ (b) $\frac{13}{32}MR^2$

(c) $\frac{31}{35}MR^2$ (d) $\frac{62}{35}MR^2$

64. A man, standing on a turn-table, is rotating at a certain angular frequency with his arms outstretched. He suddenly folds his arms. If his moment of inertia with folded arms is 75% of that with outstretched arms, his rotational kinetic energy will

(a) increase by 33.3% (b) decrease by 33.3%
(c) increase by 25% (d) decrease by 25%

65. Two blocks of masses 1 kg and 2 kg are suspended at the end of a light string passing over a friction-less pulley of mass 4 kg and radius 10 cm. When the masses are released, the acceleration of the system is

(a) $\frac{g}{9}$ (b) $\frac{g}{7}$

(c) $\frac{g}{5}$ (d) $\frac{g}{3}$

66. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in second) applied tangentially. The force is then withdrawn. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion is reversed, is very nearly equal to

(a) $5\frac{1}{2}$ (b) $8\frac{1}{2}$

(c) $11\frac{1}{2}$ (d) $14\frac{1}{2}$

67. Four forces are applied to a wheel of radius 20 cm as shown in Fig. 5.44. The net torque produced by the forces is

(a) 5.4 Nm anticlockwise
(b) 1.8 Nm clockwise
(c) 2.0 Nm clockwise
(d) 5.4 Nm clockwise

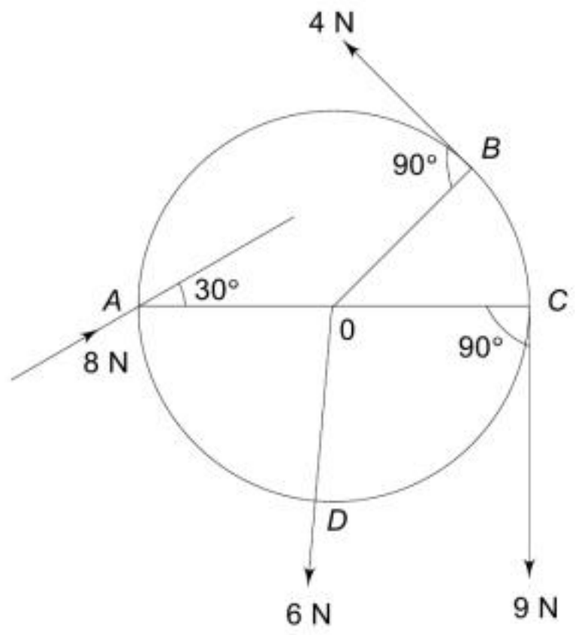

Fig. 5.44

68. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with angular velocity ω_0 about its axis perpendicular to the rod and passing through its mid point (see Fig. 5.45). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is

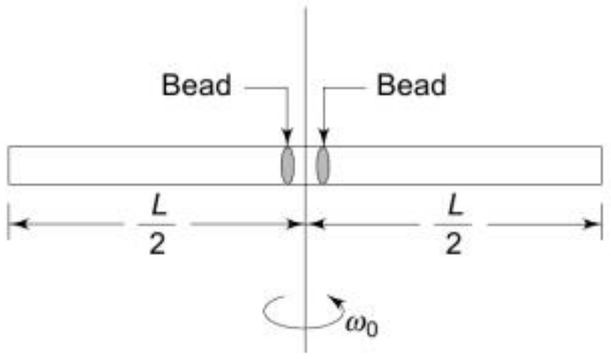

Fig. 5.45

(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+6m}$

(c) $\frac{(M+6m)\omega_0}{M}$ (d) ω_0

69. I_1, I_2, I_3 and I_4 are respectively the moments of inertia of a thin square plate $ABCD$ of uniform thickness about axes 1, 2, 3 and 4 which are in the plane of the plate. The moment of inertia of the plate about an axis passing through the centre O and perpendicular to the plane of the plate is (Fig. 5.46)

(a) $2(I_1 + I_2)$ (b) $2(I_3 + I_4)$

(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

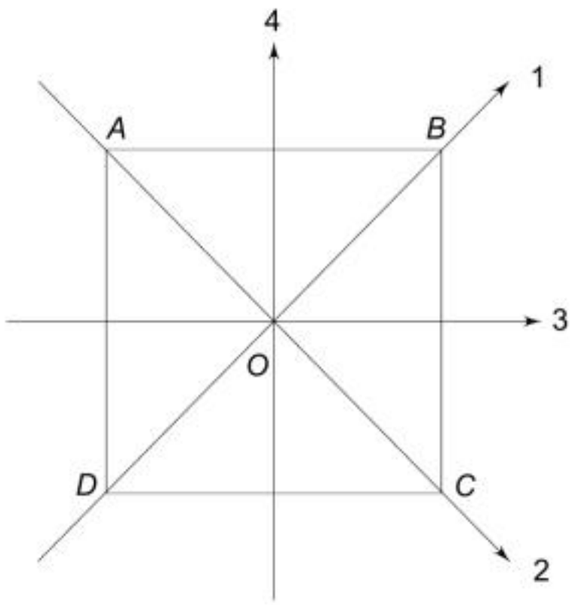

Fig. 5.46

70. A mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin

(a) is zero (b) remains constant

(c) goes on increasing (d) goes on decreasing

71. A smooth sphere A is moving on a frictionless horizontal surface with angular speed ω and centre of mass velocity v. It collides elastically and head-on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$

(c) $\omega_A = \omega$ (d) $\omega_B = \omega$

72. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Fig. 5.47. The magnitude of angular momentum of the disc about the origin O is

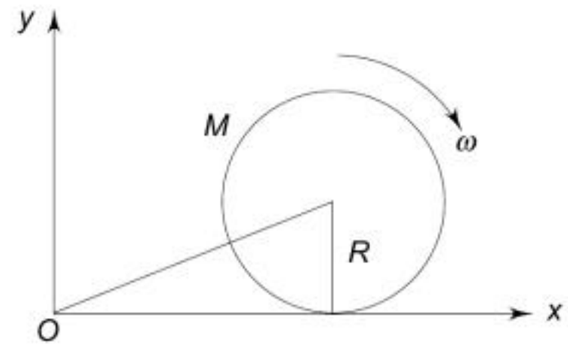

Fig. 5.47

(a) $\frac{1}{2} MR^2\omega$ (b) $MR^2\omega$

(c) $\frac{3}{2} MR^2\omega$ (d) $2MR^2\omega$

73. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown in Fig. 5.48. The moment of inertia of the loop about the axis XX' is

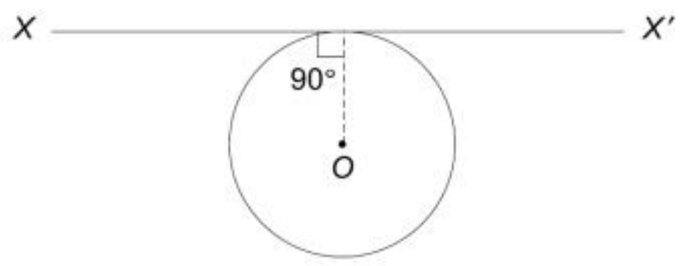

Fig. 5.48

(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

74. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\frac{M\omega^2 L}{2}$ (b) $M\omega^2 L$

(c) $\frac{M\omega^2 L}{4}$ (d) $\frac{M\omega^2 L^2}{2}$

75. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown in Fig. 5.49. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is

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88. A thin uniform disc has mass M and radius R. A circular hole of radius $R/3$ is made in the disc as shown in Fig. 5.50. The moment of inertia of the remaining portion of disc about an axis passing through O and perpendicular to the plane of the disc is

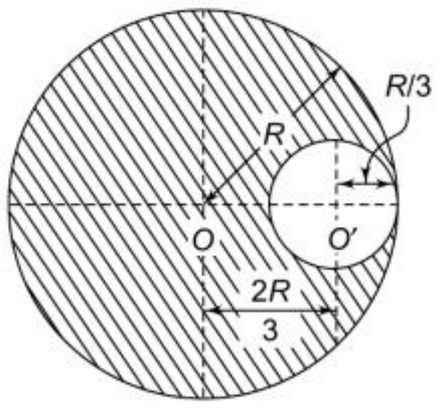

Fig. 5.50

(a) $\frac{1}{9}MR^2$ (b) $\frac{2}{9}MR^2$

(c) $\frac{1}{3}MR^2$ (d) $\frac{4}{9}MR^2$

89. A solid metallic sphere of radius R having moment of inertia equal to I about its diameter is melted and recast into a solid disc of radius r of a uniform thickness. The moment of inertia of the disc about an axis passing through its edge and perpendicular to its plane is also equal to I. The ratio r/R is

(a) $\frac{2}{\sqrt{15}}$ (b) $\frac{2}{\sqrt{10}}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{2}}$

90. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum length $h = \frac{3v^2}{4g}$, with respect to the initial position. The object is (see Fig. 5.51)

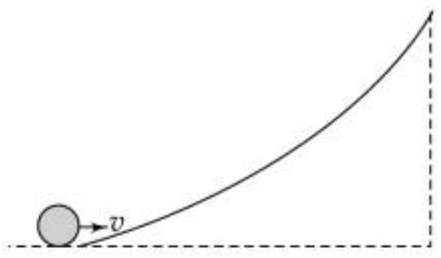

Fig. 5.51

(a) ring (b) solid sphere
(c) hollow sphere (d) disc

91. The mass per unit length of a non-uniform rod OP of length L varies as

$$m = k\frac{x}{L}$$

where k is a constant and x is the distance of any point on the rod from end O. The distance of the centre of mass of the rod from end O is

(a) $\frac{L}{3}$ (b) $\frac{2L}{3}$

(c) $\frac{L}{2}$ (d) $\frac{2L}{\sqrt{3}}$

92. A uniform thin rod of mass M and length L is hinged by a frictionless pivot at its end O, as shown Fig. 5.52. A bullet of mass m moving horizontally with a velocity v strikes the free end of the rod and gets embedded in it. The angular velocity of the system about O just after the collision is

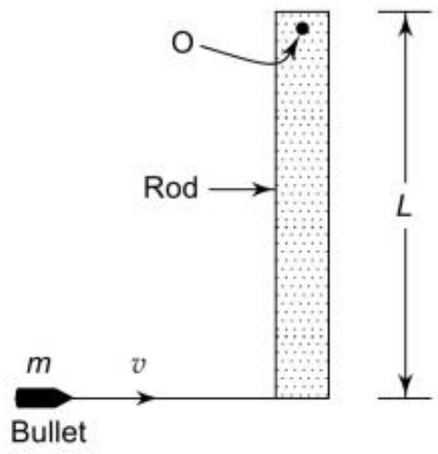

Fig. 5.52

(a) $\frac{mv}{L(M+m)}$ (b) $\frac{2mv}{L(M+2m)}$

(c) $\frac{3mv}{L(M+3m)}$ (d) $\frac{mv}{LM}$

93. A gramophone record of mass M and radius R is rotating at an angular velocity ω. A coin of mass m is gently placed on the record at a distance $r = R/2$ from its centre. The new angular velocity of the system is

(a) $\frac{2\omega M}{(2M+m)}$ (b) $\frac{2\omega M}{(M+2m)}$

(c) ω (d) $\frac{\omega M}{M}$

ANSWERS

1. (c)	**2.** (a)	**3.** (b)
4. (d)	**5.** (c)	**6.** (c)
7. (a)	**8.** (c)	**9.** (d)
10. (c)	**11.** (d)	**12.** (b)
13. (b)	**14.** (b)	**15.** (c)
16. (a)	**17.** (d)	**18.** (b)
19. (c)	**20.** (a)	**21.** (b)

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24. Let the mass of each dumb-bell be m. Then,
Total initial angular momentum $= mr_1^2\omega_1 + mr_1^2\omega_1$
$= 2mr_1^2\omega_1$

When the boy pulls the dumb-bells towards his chest, let the new value of period be T_2. It is given that

$$r_2 = 10 \text{ cm}$$

$$\omega_2 = \frac{2\pi}{T_2}$$

Final angular momentum $= 2mr_2^2\omega_2$
From the principle of conservation of angular momentum,
Initial angular momentum = Final angular momentum

or $\quad 2mr_1^2\omega_1 = 2mr_2^2\omega_2$

$$\frac{\omega_1}{\omega_2} = \frac{r_2^2}{r_1^2}$$

Hence the correct choice is (b).

25. We first find the linear acceleration a by using the relation

$$s = ut + \frac{1}{2}\,at^2$$

or $\quad 2 = 0 + \frac{1}{2} \times a \times (4)^2$

which gives $a = \frac{1}{4}$ ms^{-2}. Now $R = 0.5$ m. The angular acceleration α is

$$\alpha = \frac{a}{R} = \frac{1}{4 \times 0.5} = \frac{1}{2} = 0.5 \text{ rad s}^{-2}$$

Hence the correct choice is (a).

26. The torque produced by the force of the falling weight is

$$\tau = mg \times \text{moment arm} = mgR$$

$$= 2 \times 10 \times 0.5 = 10 \text{ Nm}$$

Now $\quad I = \frac{\tau}{\alpha} = \frac{10}{0.5} = 20 \text{ kg m}^2$

Hence the correct choice is (b).

27. Let M be the mass and R the initial radius of the earth. If ω is the angular velocity of the rotation of the earth, the duration T of the day is

$$T = \frac{2\pi}{\omega}$$

Let R' be the radius of the earth after contraction and ω' its angular velocity. From the conservation of angular momentum, we have

$$I\omega = I'\omega'$$

where $I\left(=\frac{2}{5}MR^2\right)$ and $I'\left(=\frac{2}{5}MR'^2\right)$ are the moments of inertia of the earth before and after contraction, respectively.

$$\therefore \quad \frac{2}{5}\,MR^2\omega = \frac{2}{5}\,MR'^2\omega'$$

or $\quad \omega' = \frac{R^2\omega}{R'^2} = 4\omega \qquad (\because R' = R/2)$

The duration T' of the new day will be (since $T = 2\pi/\omega$)

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{4\omega} = \frac{T}{4}$$

$$T' = \frac{24 \text{ hours}}{4} = 6 \text{ hours}$$

28. $I_1 = \frac{1}{4}\,M_1R_1^2$ and $I_2 = \frac{1}{4}\,M_2R_2^2$. Therefore,

$$\frac{I_1}{I_2} = \frac{M_1}{M_2}\cdot\frac{R_1^2}{R_2^2} = \frac{1}{2} \times (2)^2 = 2$$

Hence the correct choice is (b).

29. From the law of conservation of angular momentum, we have

$$I\omega = I'\omega'$$

Here $I = MR^2$ and $I' = (M + 2m)\,R^2$. Therefore

$$\frac{\omega'}{\omega} = \frac{I}{I'} = \frac{M}{(M+2m)}$$

Hence the correct choice is (a).

30. For rolling :

$$Mgh = \frac{1}{2}\,Mv^2 + \frac{1}{2}\,I\omega^2$$

$$= \frac{1}{2}\,Mv^2 + \frac{1}{2} \times \left(\frac{2}{5}MR^2\right) \times \frac{v^2}{R^2}$$

$$= \frac{1}{2}\,Mv^2 + \frac{1}{5}\,Mv^2 = \frac{7}{10}\,Mv^2$$

$$\left(\because I = \frac{2}{5}MR^2 \text{ and } \omega = \frac{v}{R}\right)$$

For sliding :

$Mgh = \frac{1}{2}\,Mv'^2$. Therefore $\frac{1}{2}\,Mv'^2 = \frac{7}{10}\,Mv^2$

or $\quad \frac{v'}{v} = \sqrt{\frac{7}{5}}$, which is choice (c).

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31. Rotational kinetic energy is

$$(\text{KE})_r = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{1}{4} MR^2\omega^2 \qquad (\because I = \frac{1}{2} MR^2)$$

where M is the mass of the disc and R its radius.
Translational kinetic energy is

$$(\text{KE})_t = \frac{1}{2} Mv^2 = \frac{1}{2} M(R\omega)^2 = \frac{1}{2} MR^2\omega^2$$

$$(\because v = R\omega)$$

Total energy, $\text{KE} = (\text{KE})_r + (\text{KE})_t = \frac{3}{4} MR^2\omega^2$

$$\therefore \quad \frac{(\text{KE})_t}{\text{KE}} = \frac{2}{3}$$

Hence the correct choice is (d).

32. $(\text{KE})_r = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5} MR^2\omega^2$

$$(\because I = \frac{2}{5} MR^2)$$

$$(\text{KE})_t = \frac{1}{2} Mv^2 = \frac{1}{2} MR^2\omega^2$$

$$\text{Total KE} = \frac{7}{10} MR^2\omega^2$$

$$\therefore \quad \frac{(\text{KE})_r}{\text{KE}} = \frac{2}{7}$$

Hence the correct choice is (a).

33. Downward force $F = Mg \sin\theta$. The effective mass of the rolling disc is $M_{\text{eff}} = M + \frac{I_{\text{CM}}}{R^2}$, where I_{CM} is the moment of inertia about its centre of mass, which is

$$I_{\text{CM}} = \frac{1}{2} MR^2$$

$$\therefore \quad M_{\text{eff}} = M + \frac{1}{2}\frac{MR^2}{R^2} = \frac{3M}{2}$$

$$\therefore \quad \text{Acceleration } a = \frac{F}{M_{\text{eff}}} = \frac{Mg\sin\theta}{3M/2}$$

$$= \frac{2}{3} g \sin 30° = \frac{g}{3}$$

Hence the correct choice is (c).

34. The acceleration of the block sliding down the plane is

$$a = g \sin\theta$$

where θ is the angle of inclination. If l is the length of the inclined plane, the velocity of the block on reaching the bottom is given by

$$v^2 = 2\,al = 2g \sin\theta \times l$$

or $$v = \sqrt{2gl\sin\theta}$$

The acceleration of the disc rolling down the plane is (as shown above)

$$a' = \frac{2}{3} g \sin\theta$$

Therefore, the velocity of the disc on reaching the bottom is given by

$$v'^2 = 2a'l = \frac{4}{3} gl\sin\theta \text{ or } v' = 2\sqrt{\frac{gl\sin\theta}{3}}$$

$$\therefore \quad \frac{v'}{v} = \sqrt{\frac{2}{3}}$$

Hence the correct choice is (b).

35. Let μ be the mass per unit length of the wire. The mass of loop A is $M_A = 2\pi R\mu$ and mass of loop B is $M_B = 4\pi R\mu$. Their moments of inertia respectively are

$$I_A = M_A R_A^2 = 2\pi R\mu \times R^2 = 2\pi\mu R^3$$

and $$I_B = M_B R_B^2 = 4\pi R\mu \times (2R)^2 = 16\pi\mu R^3$$

$$\therefore \quad \frac{I_A}{I_B} = \frac{1}{8}$$

Hence the correct choice is (d).

36. Here $M_A = 2\pi\mu R$ and $M_B = 2\pi n\mu R$. Their moments of inertia are

$$I_A = M_A R_A^2 = 2\pi\mu R \times R^2 = 2\pi\mu R^3$$

and $$I_B = M_B R_B^2 = 2\pi n\mu R \times (nR)^2 = 2\pi n^3\mu R^3$$

$$\therefore \quad \frac{I_B}{I_A} = n^3; \text{ but } \frac{I_B}{I_A} = m \text{ (given)}$$

Thus, $m = n^3$. Hence the correct choice is (c).

37. Let m be the mass of the coin. It will fly off when the centripetal force $mr\omega^2$ just exceeds the force of friction μmg. The minimum ω is given by

$$mr\omega^2 = \mu mg$$

or $$\omega = \sqrt{\frac{\mu g}{r}}$$

Hence the correct choice is (c).

38. The minimum angular frequency is independent of the mass. Hence the correct answer is still $\sqrt{\mu g / r}$ which is choice (a).

39. The (x, y) co-ordinates of the masses at O, A and B respectively are (refer to Fig. 5.36)

$(x_1 = 0, y_1 = 0)$, $(x_2 = a, y_2 = 0)$ and $(x_3 = 0, y_3 = b)$

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The (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + m \times a + m \times 0}{m + m + m} = \frac{a}{3}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m \times 0 + m \times 0 + m \times b}{m + m + m} = \frac{b}{3}$$

The position vector of the centre of mass is $x_{CM}\,\mathbf{i} + y_{CM}\,\mathbf{j}$

$$= \frac{a}{3}\,\mathbf{i} + \frac{b}{3}\,\mathbf{j} = \frac{1}{3}(a\mathbf{i} + b\mathbf{j}),$$

which is choice (a).

40. Refer to Fig. 5.37. The (x, y) co-ordinates of the masses at O, A and B respectively are

$$(x_1 = 0, y_1 = 0), (x_2 = a, y_2 = 0), \left(x_3 = \frac{a}{2}, y_3 = \frac{a\sqrt{3}}{2}\right)$$

Therefore, the (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m \times 0 + m \times a + m \times a/2}{m + m + m} = \frac{a}{2}$$

$$y_{CM} = \frac{m \times 0 + m \times a + m \times a\sqrt{3}/2}{m + m + m}$$

$$= \frac{a}{2\sqrt{3}}$$

$\therefore$ Position vector of centre of mass is $\frac{a}{2}\left(\mathbf{i} + \frac{\mathbf{j}}{\sqrt{3}}\right)$.
Hence the correct choice is (a).

41. The kinetic energy (which is rotational) is $\frac{1}{2} I\omega^2$.

Now, The moment of inertia $I = \frac{1}{2} mr^2$ and $\omega = \frac{v}{r}$.

Therefore, KE $= \frac{1}{2} \times \frac{1}{2} mr^2 \times \left(\frac{v}{r}\right)^2 = \frac{1}{4} mv^2$,

which is choice (d).

42. The kinetic energy of a rolling disc consists of two parts: translational energy $= \frac{1}{2} mv^2$ and rotational energy $= \frac{1}{2} I\omega^2$.

$\therefore$ KE $= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$$= \frac{1}{2} mv^2 + \frac{1}{2} \times \left(\frac{1}{2} mr^2\right) \times \left(\frac{v}{r}\right)^2$$

$$(\because I = \frac{1}{2} mr^2)$$

$$= \frac{1}{2} mv^2 + \frac{1}{4} mv^2 = \frac{3}{4} mv^2$$

Hence the correct choice is (c).

43. Mass of sphere A, $M_A = \frac{4}{3} \pi R^3 \rho_A$, mass of sphere B, $M_B = \frac{4}{3} \pi R^3 \rho_B$. Now, $I_A = \frac{2}{5} M_A R^2$ and $I_B = \frac{2}{5} M_B R^2$. Therefore,

$$\frac{I_A}{I_B} = \frac{M_A}{M_B} = \frac{\rho_A}{\rho_B}$$

Hence the correct choice is (c).

44. When the cylinder rolls without sliding, the acceleration down the plane is (see solutions to Q. 33)

$$a_r = \frac{2}{3} g \sin\theta$$

When the cylinder slides without rolling, the acceleration is

$$a_s = g \sin\theta$$

where θ is the inclination of the plane.
If h is the height of the inclined plane, their speeds on reaching the bottom are given by

$$v_r = \sqrt{2a_r h} \text{ and } v_s = \sqrt{2a_s h}$$

Since $a_s > a_r$, it follows that $v_s > v_r$, which is choice (b).

45. When the sphere rolls down the plane, its acceleration is given by

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}}$$

Now, the moment of inertia of the sphere about its diameter is

$$I = \frac{2}{5} MR^2$$

Therefore, $a = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{5}{7} g \sin\theta$ (i)

For rolling without sliding, the frictional force f provides the necessary torque τ which is given by

$$\tau = \text{force} \times \text{moment arm} = fR$$

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But $\tau = I\alpha$, where α is the angular acceleration of the sphere. Thus, $I\alpha = fR$. Also, linear acceleration $a = \alpha R$. Therefore,

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} = \frac{2}{5}\,Ma \left(\because I = \frac{2}{5}MR^2\right)$$

Now, force of friction = $\mu \times$ normal reaction = $\mu Mg \cos\theta$. Thus $\mu Mg \cos\theta = \frac{2}{5}\,Ma$

or $$a = \frac{5}{2}\,\mu g \cos\theta \qquad \text{(ii)}$$

Equating (i) and (ii) we have

$$\frac{5}{7}\,g \sin\theta = \frac{5}{2}\,\mu g \cos\theta$$

or $$\mu = \frac{2}{7}\tan\theta$$

Hence the correct choice is (d).

46. Given $PQ = QR = RP = L$. The centre of mass is located at centroid C which cuts lines PS, QT and UR in the ratio 2 : 1. Let $h = CS = CT = UC$. In $\Delta\, PQS$, we have (see Fig. 5.56)

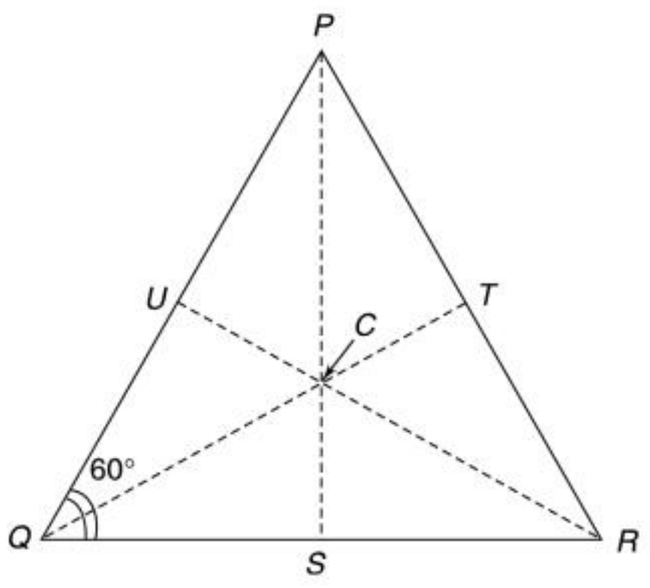

Fig. 5.56

$$PS = PQ \sin 60° = L \sin 60° = \frac{\sqrt{3}}{2}\,L.$$

$$\therefore \qquad h = \frac{PS}{3} = \frac{1}{3} \times \frac{\sqrt{3}}{2}\,L = \frac{L}{2\sqrt{3}}$$

Since the structure consists of three identical rods, its moment of inertia about an axis passing through its centre of mass C and perpendicular to its plane is, from parallel axes theorem,

$$I_c = 3\,(I + Mh^2)$$

where I is the moment of inertia of each rod about the axis passing through its centre and perpendicular to its length, which is given by

$$I = \frac{M\,L^2}{12}$$

Also $$Mh^2 = M\left(\frac{L}{2\sqrt{3}}\right)^2 = \frac{M\,L^2}{12}$$

$$\therefore \qquad I_c = 3\left(\frac{M\,L^2}{12} + \frac{M\,L^2}{12}\right)$$

$$= 3 \times \frac{M\,L^2}{6} = \frac{M\,L^2}{2}$$

Hence the correct choice is (a).

47. Refer to Fig. 5.38. Moment of inertia is a scalar quantity. So the moment of inertia of the structure is the sum of the moments of inertia of the four rods about the specified axis of rotation, i.e.,

$$I = I_1 + I_2 + I_3 + I_4$$

where I_1 = moment of inertia of rod 1 about an axis passing through its centre E and perpendicular to its plane = $\frac{M\,L^2}{12}$,

I_2 = moment of inertia of rod 2 about an axis passing through its centre F and perpendicular to its plane = $\frac{M\,L^2}{12}$,

I_3 = moment of inertia of rod 3 about a parallel axis at a distance $\frac{L}{2}$ from it = $M\left(\frac{L}{2}\right)^2 = \frac{M\,L^2}{4}$, and

I_4 = moment of inertia of rod 4 about a parallel axis at a distance $\frac{L}{2}$ from it = $\frac{M\,L^2}{4}$.

$$\therefore \qquad I = \frac{M\,L^2}{12} + \frac{M\,L^2}{12} + \frac{M\,L^2}{4} + \frac{M\,L^2}{4}$$

$$= \frac{2}{3}\,ML^2, \text{ which is choice (d).}$$

48. Refer to Fig. 5.57. It is clear from the figure that the moment of inertia of triangular sheet $ABC = \frac{1}{2} \times$ moment of inertia of a square sheet $ABCD$ about its diagonal AC or $I_t = \frac{1}{2}\,I_s$. Now, mass of square sheet $= M + M = 2M$. Therefore,

$$I_s = (2M)\,\frac{L^2}{12} = \frac{M\,L^2}{6}$$

$$\therefore \qquad I_t = \frac{I_s}{2} = \frac{M\,L^2}{12}$$

Hence the correct choice is (a).

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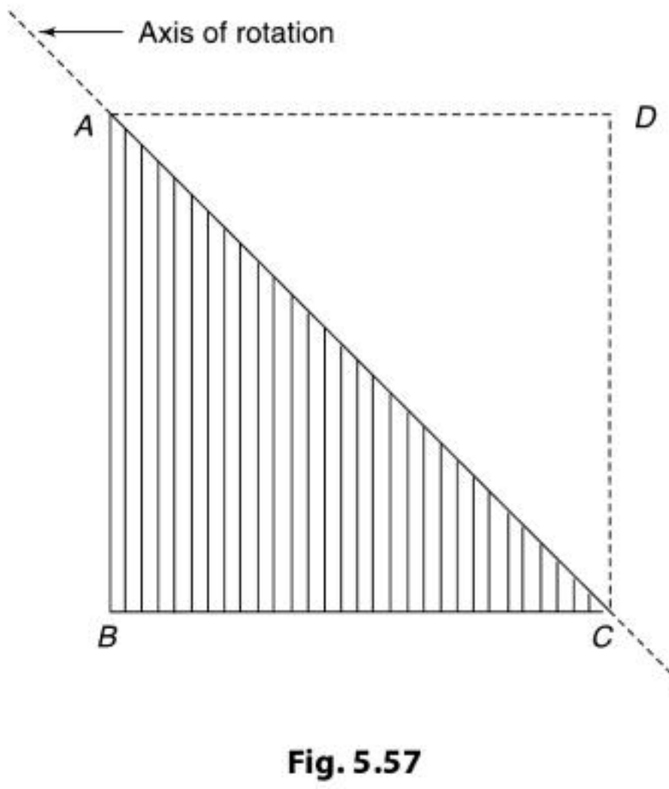

Fig. 5.57

49. $I_{BC} = \dfrac{m(AB)^2}{3} = \dfrac{ma^2}{3}$

$$I_{AB} = \frac{m(BC)^2}{3} = \frac{4}{3}ma^2$$

$$I_{HF} = \frac{m(AB)^2}{12} = \frac{ma^2}{12}$$

$$I_{EG} = \frac{m(BC)^2}{12} = \frac{ma^2}{3}$$

Thus, the moment of inertia about HF is the minimum, which is choice (c).

50. Let v be the speed of the sphere when it reaches B. Then, loss in PE = gain in translation KE + gain in rotational KE, i.e.

$$mg\,(6r - r) = \frac{1}{2}\,mv^2 + \frac{1}{2}\,I\omega^2$$

or $$5\,mgr = \frac{1}{2}\,mv^2 + \frac{1}{2}\times\frac{2}{5}\,mr^2 \times \frac{v^2}{r^2}$$

$$= \frac{1}{2}\,mv^2 + \frac{1}{5}\,mv^2 = \frac{7}{10}\,mv^2$$

or $v = \sqrt{\dfrac{50gr}{7}}$, which is choice (b).

51. Horizontal force $= \dfrac{mv^2}{r} = \dfrac{m}{r}\times\dfrac{50gr}{7} = \dfrac{50mg}{7}$

Hence the correct choice is (c).

52. $I\omega = I_1\omega_1$. $I = \dfrac{2}{5}\,mr^2$, $I_1 = \dfrac{2}{5}\,m\left(\dfrac{r}{n}\right)^2 = \dfrac{I}{n^2}$. Hence

$$I\omega = \frac{I_1}{n^2}\,\omega_1$$

or $\omega_1 = n^2\omega$, which is choice (d).

53. $K = \dfrac{1}{2}\,I\omega^2$, $K_1 = \dfrac{1}{2}\,I_1\,\omega_1^2 = \dfrac{1}{2}\times\dfrac{I}{n^2}\,(n^2\omega)^2$

$$= \left(\frac{1}{2}\,I\omega^2\right)\,n^2 = n^2K.$$

Hence the correct choice is (c).

54. $V = \dfrac{4}{3}\,\pi r^3$ or $\log V = \log\left(\dfrac{4\pi}{3}\right) + 3\log r$.

Differentiating, we have

$$\frac{\delta V}{V} = 3\frac{\delta r}{r} \text{ or } \frac{\delta r}{r} = \frac{1}{3}\frac{\delta V}{V} = \frac{1}{3}\times 0.5\% = \frac{1}{6}\%$$

Since no external torque acts, $I\omega$ = constant or $\dfrac{2}{5}\,mr^2\omega$ = constant or $r^2\omega$ = constant (c)

or $2\log r + \log\omega = \log c$. Differentiating, we have

$$\frac{2\delta r}{r} + \frac{\delta\omega}{\omega} = 0$$

or $$\frac{\delta\omega}{\omega} = -2\,\frac{\delta r}{r} = -2\times\frac{1}{6}\% = -\frac{1}{3}\%$$

The negative sign indicates that ω decreases. Hence the correct choice is (b).

55. $I_1 = \dfrac{1}{2}\,MR^2$, $I_2 = M\left(\dfrac{R^2}{4} + \dfrac{L^2}{12}\right)$

$$= M\left(\frac{R^2}{4} + \frac{\left(\sqrt{3}R\right)^2}{12}\right) = \frac{1}{2}\,MR^2$$

$\therefore \dfrac{I_1}{I_2} = 1$, which is choice (a).

56. In one full revolution the increase in PE = MgL, where M is the mass of the rod. Therefore,

$$MgL = \frac{1}{2}\,I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

or $$\omega = \sqrt{\frac{6g}{L}}\,.\ \text{Now } v = L\omega = L\sqrt{\frac{6g}{L}} = \sqrt{6gL}$$

Hence the correct choice is (c).

57. The cylinder will topple when the torque mgr equals the torque $ma\,\dfrac{h}{2}$ (see Fig. 5.58)

or $$a = \frac{2gr}{h} = \frac{g}{2} \qquad (\because h = 4r) \qquad \text{(i)}$$

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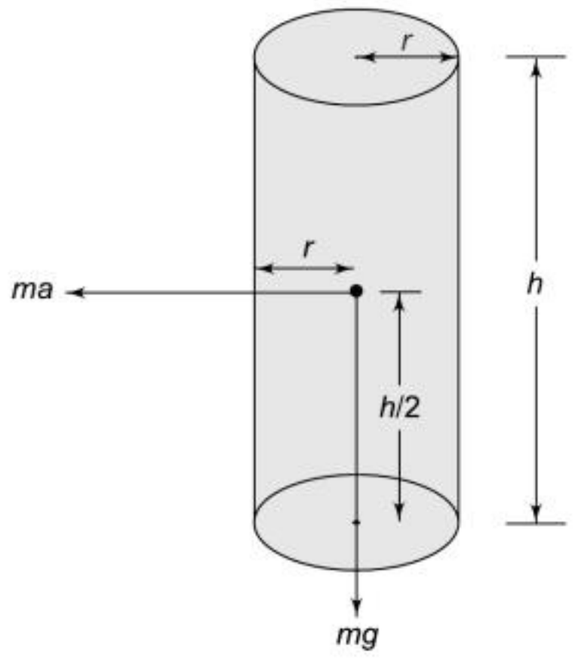

Fig. 5.58

Now $v = 2.45\,t^2$

$$\therefore \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(2.45t^2\right) = 4.9t \qquad \text{(ii)}$$

Equating (i) and (ii), we get $t = \dfrac{g}{2 \times 4.9} = \dfrac{9.8}{9.8} = 1$ s.

Hence the correct choice is (a).

58. Loss in PE = gain in rotational KE. As the centre of mass of the rod falls through a distance $L/2$, the loss in PE $= \dfrac{MgL}{2}$. Gain in KE $= \dfrac{1}{2}\, I\omega^2 = \dfrac{1}{2}\left(\dfrac{ML^2}{3}\right)\omega^2$

Equating the two, we have

$$\frac{MgL}{2} = \frac{ML^2\omega^2}{6}$$

or $\omega = \sqrt{\dfrac{3g}{L}}$, which is choice (c).

59. Using the parallel axes theorem, $I = I_{CM} + Mx^2$ where x is the distance of the axis of rotation from the C.M. (centre of mass) of the rod, which is $x = \dfrac{L}{2} - \dfrac{L}{4} = \dfrac{L}{4}$.

Also $I_{C.M.} = \dfrac{ML^2}{12}$. Hence

$I = \dfrac{ML^2}{12} + \dfrac{ML^2}{16} = \dfrac{7ML^2}{48}$, which is choice (a).

60. PE at $\theta = 60°$ is $Mgh\,(1 - \cos\theta)$ where h is the distance between the axis of rotation and the centre of mass of the disc. Gain in KE when the disc reaches the equilibrium position $= \dfrac{1}{2}\, I\omega^2$ where $I = I_{C.M.} + Mx^2 = \dfrac{1}{2}\, MR^2 + MR^2 = \dfrac{3}{2}\, MR^2$. Here x is the distance between the centre of mass and the axis of rotation, i.e. $x = R$.

Now PE = KE gives

$$MgR\,(1 - \cos 60°) = \frac{1}{2}\, I\omega^2 = \frac{3}{4}\, MR^2\,\omega^2 \qquad (\because\ h = R)$$

which gives $\omega = \sqrt{\dfrac{2g}{3R}}$, which is choice (b).

61. Loss in PE = gain in rotational KE. Thus

$$mgh = \frac{1}{2}\,(I + mR^2)\,\omega^2 = \frac{1}{2}\,(MR^2 + mR^2)\,\omega^2$$

$$= \frac{1}{2}\, R^2\,(M + m)\,\omega^2$$

or $\omega = \sqrt{\dfrac{2mgh}{(M+m)\,R^2}}$.

Hence the correct choice is (c).

62. Moment of inertia of complete disc about O is $I = \dfrac{1}{2}\, MR^2$. Mass of the cut-out part is $m = \left(\dfrac{M}{4}\right)$.

The moment of inertia of the cut-out portion about its own centre $I_0 = \dfrac{1}{2}\, mr^2 = \dfrac{1}{2}\left(\dfrac{M}{4}\right)\left(\dfrac{R}{2}\right)^2 = \dfrac{1}{32}$ MR^2 because $r = R/2$. From the parallel axes theorem, the moment of inertia of the cut out portion about O is

$$I_c = I_0 + mr^2 = \frac{1}{32}\, MR^2 + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$= \frac{3}{32}\, MR^2$$

$\therefore$ Moment of inertia of the shaded portion about O is $I_s = I - I_c = \dfrac{1}{2}\, MR^2 - \dfrac{3}{32}\, MR^2 = \dfrac{13}{32}\, MR^2$, which is choice (c).

63. We obtain the given hollow sphere as if a solid sphere of radius R has been removed from a solid sphere of radius $2R$. The mass of the given hollow sphere is (here ρ is the density of the material of the sphere)

$$M = M_1 - M_2$$

where $M_1 = \dfrac{4}{3}\,\pi\,(2R)^3\rho$ and $M_2 = \dfrac{4}{3}\,\pi R^3\rho$ are the masses of spheres of radii $2R$ and R respectively.

$$\therefore \quad M = \frac{28}{3}\,\pi R^3\rho \qquad \text{(i)}$$

The moment of inertia of the given hollow sphere is

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$$I = \frac{2}{5} M_1 (2R)^2 - \frac{2}{5} M_2 R^2$$

$$= \frac{2}{5} \times \frac{4}{3} \pi (2R)^3 \rho (2R)^2 - \frac{2}{5} \times \frac{4}{3} (\pi R^3 \rho) R^2$$

$$= \frac{2}{5} (32 - 1) \frac{4}{3} \pi R^5 \rho \qquad \text{(ii)}$$

Using (i) in (ii), we get $I = \frac{62}{35} MR^2$, which is choice (d).

64. Let I_1 and ω_1 be the moment of inertia and angular frequency when his arms are outstretched and I_2 and ω_2 those when his arms are folded. Then

$$I_1 \omega_1 = I_2 \omega_2$$

Given $I_2 = \frac{3}{4} I_1$. Hence $I_1 \omega_1 = \frac{3}{4} I_1 \omega_2$ or $\omega_2 = \frac{4}{3} \omega_1$.

Initial KE is $K_1 = \frac{1}{2} I_1 \omega_1^2$ and final KE is

$$K_2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} \times \frac{3I_1}{4} \times \left(\frac{4\omega_1}{3}\right)^2 = \frac{4}{3}\left(\frac{1}{2} I_1 \omega_1^2\right)$$

$$= \frac{4}{3} K_1$$

$\therefore$ Percentage increase in KE

$$= \frac{K_2 - K_1}{K_1} \times 100 = \frac{\frac{4}{3} K_1 - K_1}{K_1} \times 100$$

$$= \frac{100}{3} = 33.3\%$$

Hence the correct choice is (a).

65. Refer to Fig. 5.59. When the masses are released, the torque is

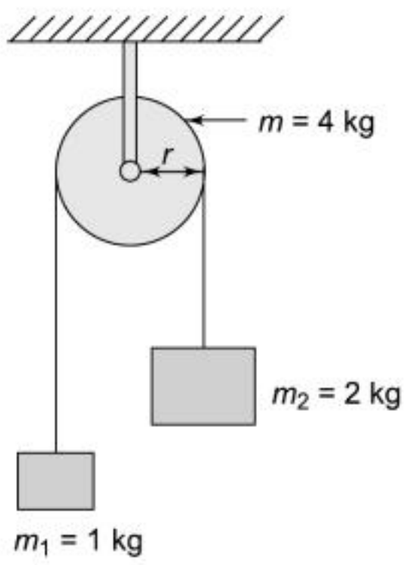

Fig. 5.59

$$\tau = (m_2 g - m_1 g) \times r$$

$$= (m_2 - m_1) gr \qquad \text{(i)}$$

But $\tau = I\alpha = I \frac{a}{r}$ $\qquad (\because a = r\alpha)$

where I = M.I. due to m_1 + M.I. due to m_2 + M.I. of pulley

$$= m_1 r^2 + m_2 r^2 + \frac{1}{2} mr^2$$

$$= \left(m_1 + m_2 + \frac{m}{2}\right) r^2$$

$\therefore$ $\tau = \left(m_1 + m_2 + \frac{m}{2}\right) ar$ (ii)

Equating (i) and (ii), we get

$$a = \frac{(m_2 - m_1) g}{\left(m_1 + m_2 + \frac{m}{2}\right)}$$

Putting $m_1 = 1$ kg, $m_2 = 2$ kg and $m = 4$ kg, we get $a = \frac{g}{5}$. Hence the correct choice is (c).

66. Torque $I\alpha = F \times r$ or $10\alpha = (20t - 5t^2) \times 2$ or $\alpha = (4t - t^2)$ rad s^{-2}. But $\alpha = \frac{d\omega}{dt}$. Therefore

$$\frac{d\omega}{dt} = 4t - t^2$$

Integrating, we have

$$\omega = \left(2t^2 - \frac{t^3}{3}\right) = t^2 \left(2 - \frac{t}{3}\right) \text{ rad s}^{-1}$$

The pulley reverses its direction when $\omega = 0$ momentarily, i.e. when $\left(2 - \frac{t}{3}\right) = 0$ or $t = 6$ s.

Now $\omega = \frac{d\theta}{dt}$. Thus

$$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

or $$\theta = \frac{2t^3}{3} - \frac{t^4}{12} = \frac{t^3}{3}\left(2 - \frac{t}{4}\right) \text{ rad}$$

For $t = 6$ s, $\theta = \frac{(6)^3}{3}\left(2 - \frac{6}{4}\right) = 36$ rad

One full rotation corresponds to $\theta = 2\pi$ rad. Therefore, number of rotations

$$= \frac{36}{2\pi} = \frac{36}{2 \times 3.14} = 5.7$$

Thus the closest choice is (a).

67. Magnitude of torque = $Fr \sin\theta$ where θ is the angle between the force vector and the radius vector. The torques are (see Fig. 5.43)

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$\tau_1 = 8\ \text{N} \times 0.2\ \text{m} \times \sin 30° = 0.8\ \text{Nm}$ (clockwise)

$\tau_2 = 4\ \text{N} \times 0.2\ \text{m} \times \sin 90° = 0.8\ \text{Nm}$ (anticlockwise)

$\tau_3 = 9\ \text{N} \times 0.2\ \text{m} \times \sin 90° = 1.8\ \text{Nm}$ (clockwise)

$\tau_4 = 6\ \text{N} \times 0.2\ \text{m} \times \sin 0° = 0$

$\therefore$ Net torque = + 0.8 Nm – 0.8 Nm + 1.8 Nm + 0 = 1.8 Nm clockwise. Hence the correct choice is (b).

68. From the principle of conservation of angular momentum, $I_0\omega_0 = I\omega$, where I_0 and ω_0 are the moment of inertia and angular velocity when the beads are at the centre of the rod and I and ω those when the beads are at the ends of the rod.

$$I_0 = \frac{ML^2}{12} \text{ and } I = \frac{ML^2}{12} + \frac{mL^2}{4} + \frac{mL^2}{4} = \frac{L^2}{12}(M+6m)$$

$$\therefore \quad \frac{ML^2}{12}\omega_0 = \frac{(M+6m)\omega L^2}{12}$$

or $$\omega = \frac{M\omega_0}{(M+6m)}$$

Hence the correct choice is (b).

69. Because of symmetry about axes 1 and 2, $I_1 = I_2$. Similarly, $I_3 = I_4$. From perpendicular axes theorem, it follows that the moment of inertia of the plate about an axis passing through the centre and perpendicular to the plane of the plate is

$$I = I_1 + I_2 = I_3 + I_4 = 2I_1 = 2I_3 \qquad (\because I_1 = I_2,\ I_3 = I_4)$$

or $$I_1 = I_3.$$

Thus $$I = I_1 + I_2 = I_3 + I_4 = I_1 + I_3.$$

Hence the correct choice is (c).

70. Refer to Fig. 5.60. The angular momentum of the mass at point $P(x, y)$ about origin O is defined as

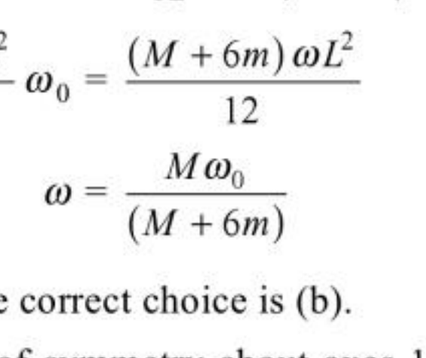

Fig. 5.60

$$\mathbf{L} = m\mathbf{r} \times \mathbf{v} = m\left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}}\right) \times \left(v\hat{\mathbf{i}}\right)$$

$$= myv\left(-\hat{\mathbf{k}}\right) \qquad \left(\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0 \text{ and } \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}\right)$$

Now m and v are constants. Also y remains constant as the mass moves parallel to the x-axis. Hence L remains constant. Thus the correct choice is (b).

71. Since there is no friction between the sphere and the horizontal surface and also between the spheres themselves, there will be no transfer of angular momentum from sphere A to sphere B due to the collision. Since the collision is elastic and the spheres have the same mass, the sphere A only transfers its linear velocity v to sphere B. Sphere A will continue to rotate with the same angular speed ω at a fixed location. Hence the correct choice is (c).

72. Refer to Fig. 5.61. Let $OC = R_C$ and let $\mathbf{v}_c$ be the velocity of the centre of mass of the disc. The linear momentum of the centre of mass is $\mathbf{p}_c = M\mathbf{v}_c$

If $\mathbf{L}_c$ is angular momentum of the disc about C, then the angular momentum about origin O is $\mathbf{L}_0 = \mathbf{L}_c + \mathbf{R}_c \times \mathbf{p}_c$

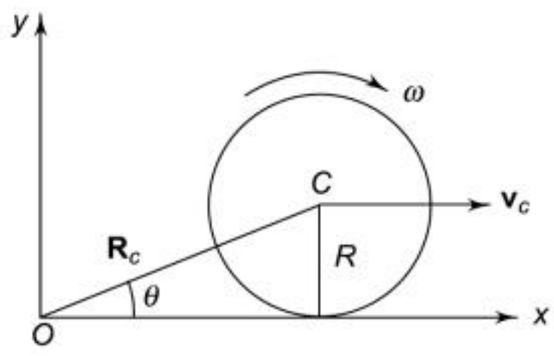

Fig. 5.61

$\therefore$ Magnitude $L_o = I_c\omega + R_c \times Mv_c \sin\theta$

$$= \frac{1}{2}MR^2\omega + MR_c v_c \sin \qquad \left(\because I_c = \frac{1}{2}MR^2\right)$$

$$= \frac{1}{2}MR^2\omega + MR \times R\omega \qquad (\because R_c \sin\theta = R \text{ and } v_c = R\omega)$$

$$= \frac{3}{2}MR^2\omega$$

Hence the correct choice is (c).

73. Let m be the mass of the loop and r its radius. The moment of inertia of the loop about an axis passing through the centre O is (Fig. 5.62)

$$I_O = \frac{1}{2}mr^2$$

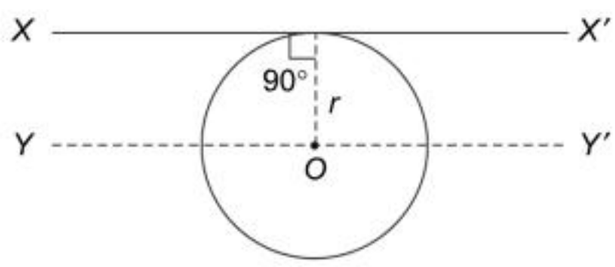

Fig. 5.62

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From the parallel axes theorem, the moment of inertia about XX' is

$$I = I_O + mr^2 = \frac{1}{2}\, mr^2 + mr^2 = \frac{3}{2}\, mr^2$$

The mass of the loop, $m = \rho L$ and radius $r = L/2\pi$. Hence

$$I = \frac{3}{2} \times \rho L \times \left(\frac{L}{2\pi}\right)^2 = \frac{3\rho L^3}{8\pi^2}$$

Thus the correct choice is (d).

74. The entire mass of the liquid can be regarded to be concentrated at the centre of mass of the tube which is at a distance of $r = \frac{L}{2}$ from the axis of revolution. The force exerted by the liquid at the other end of the tube is the centripetal force of a mass M revolving in a circle of radius $r = \frac{L}{2}$. Thus

$$F_c = \frac{Mv^2}{r} = \frac{M(r\omega)^2}{r} = Mr\omega^2 = \frac{ML\omega^2}{2}$$

Hence the correct choice is (a).

75. Torque due to F about A is $\tau_1 = FL$
Since the weight mg acts through the centre of mass of the block (which is at a distance of $L/2$ from the side of the block) the torque due to weight mg about A is

$$\tau_2 = mg\left(\frac{L}{2}\right)$$

The minimum force required to topple the block is obtained when τ_1 is slightly greater than τ_2, i.e. in the limit.

$$(\tau_1)_{\min} = \tau_2 \text{ or } F_{\min} L = mg\left(\frac{L}{2}\right) \text{ or } F_{\min} = \frac{mg}{2}$$

Hence the correct choice is (c).

76. Since no external force acts on the system, the centre of mass will remain at rest. Hence the correct choice is (a).

77. The velocity v_{CM} of the centre of mass can be obtained by using the principle of conservation of linear momentum,

$$MV = (M + m)\, v_{CM}$$

or
$$v_{CM} = \frac{MV}{(M+m)} = \frac{10\text{ kg} \times 14\text{ ms}^{-1}}{(10+4)\text{ kg}}$$
$$= 10\text{ ms}^{-1}$$

Hence the correct choice is (c).

78. When a cylinder rolls up or down an inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plane when the cylinder rolls up or down the plane. Thus, the correct choice is (b).

79. The correct choice is (d).

80. The magnitude of angular momentum of a rotating body is given by $L = I\omega$. If no torque acts, the angular momentum is conserved, i.e. $I\omega$ = constant. Hence $I_1\,\omega_1 = I_2\,\omega_2$. If K_1 and K_2 are the corresponding radii of gyration, then $I_1 = MK_1^2$ and $I_2 = MK_2^2$. Hence

$$MK_1^2\,\omega_1 = MK_2^2\,\omega_2$$

or
$$\frac{K_1}{K_2} = \frac{\sqrt{\omega_2}}{\sqrt{\omega_1}}, \text{ which is choice (c).}$$

81. Given: $l = 6R$. From parallel axes theorem, the moment of inertia about the given axis is given by

$$I = M\left(\frac{R^2}{4} + \frac{l^2}{3}\right)$$
$$= M\left[\frac{R^2}{4} + \frac{(6R)^2}{3}\right]$$
$$= M\left(\frac{R^2}{4} + \frac{36R^2}{3}\right) = \frac{49MR^2}{4}$$

Hence the correct choice is (d).

82. Areal velocity $A = \dfrac{\text{area swept by radius vector}}{\text{time taken}}$

Assuming that the orbit of the planet is a circle of radius R, then

$$A = \frac{\pi R^2}{T}$$

Now, time period $T = \dfrac{2\pi}{\omega}$. Hence

$$A = \frac{\pi R^2}{2\pi/\omega} = \frac{R^2\omega}{2}$$

or
$$\omega = \frac{2A}{R^2}$$

Angular momentum $L = I\omega = (MR^2) \times \dfrac{2A}{R^2} = 2\,MA$

Hence the correct choice is (b).

83. The mass of the rod can be considered to be concentrated at its centre ($x = L/2$) where $x = 0$ is the origin. Hence

$$R_{CM} = \frac{M_1 \times L/2 + 0}{M_1 + M_2} = \frac{LM_1}{2(M_1 + M_2)}$$

Hence the correct choice is (b).

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Equations (1) and (2) give $a = \dfrac{6F}{5M}$, which is choice (c)

5. From Eqs. (1) and (2) we get $f = \dfrac{Ma}{6} = \dfrac{F}{5}$. Hence the correct choice is (d).

Questions 6 to 11 are based on the following passage.

Passege III

Four solid spheres each of mass m and radius r are located with their centres on four corners of a square $ABCD$ of side a as shown in Fig. 5.67.

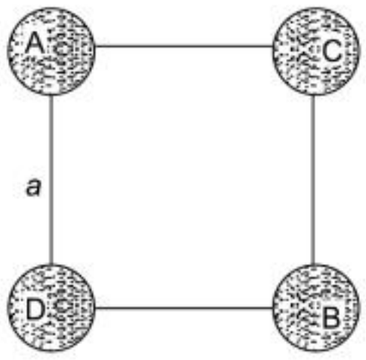

Fig. 5.67

6. The moment of inertia of sphere A about diagonal AB is

(a) $\dfrac{2}{3}mr^2$ (b) $\dfrac{2}{5}mr^2$

(c) $m\left(r^2 + \dfrac{a^2}{4}\right)$ (d) $m\left(\dfrac{a^2}{4} - r^2\right)$

7. The moment of inertia of sphere C about diagonal AB is

(a) $\dfrac{2}{5}mr^2$ (b) $\dfrac{2}{5}(2r^2 + 3a^2)$

(c) $\dfrac{m}{5}(5r^2 + 3a^2)$ (d) $\dfrac{m}{10}(4r^2 + 5a^2)$

8. The moment of inertia of sphere B about side AD is

(a) $\dfrac{2}{5}mr^2$ (b) $\dfrac{m}{5}(5a^2 + 2r^2)$

(c) $\dfrac{m}{5}(2r^2 + a^2)$ (d) $\dfrac{m}{5}(3a^2 + 5r^2)$

9. The moment of inertia of sphere D about side AD is

(a) $\dfrac{2}{5}mr^2$ (b) $\dfrac{2}{3}m(r^2 + a^2)$

(c) $\dfrac{m}{5}(3a^2 + 4r^2)$ (d) $\dfrac{m}{5}(5a^2 + 2r^2)$

10. The moment of inertia of the system of four spheres about diagonal AB is

(a) $\dfrac{m}{5}(8r^2 + 5a^2)$ (b) $\dfrac{m}{5}(7r^2 + 4a^2)$

(c) $\dfrac{m}{5}(5r^2 + 8a^2)$ (d) $\dfrac{m}{5}(3r^2 + 5a^2)$

11. The moment of inertia of the system of four spheres about side AD is

(a) $\dfrac{2m}{5}(2r^2 + 5a^2)$ (b) $\dfrac{m}{5}(7r^2 + 5a^2)$

(c) $\dfrac{2m}{5}(4r^2 + 5a^2)$ (d) $\dfrac{m}{5}(3r^2 + 5a^2)$

➔ SOLUTIONS

Refer to Fig. 5.68.

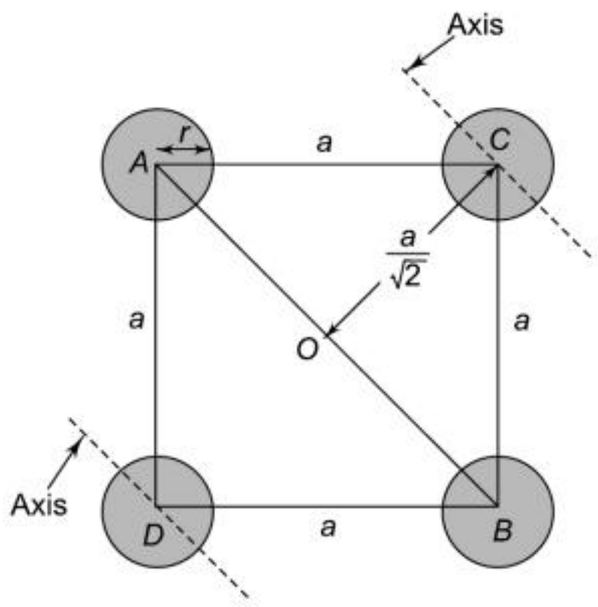

Fig. 5.68

The moment of inertia of spheres A and B about their common diameter $AB = \dfrac{2}{5}\ mr^2$ each. Also the moment of inertia of spheres C and D about an axis passing through their centre and parallel to $AB = \dfrac{2}{5}\ mr^2$ each. The distance of this axis (shown by broken lines) from the diagonal $AB = a/\sqrt{2}$. From the parallel axes theorem, the moment of inertia of spheres C and D about diagonal AB is

$$\frac{2}{5}\ mr^2 + m(CO)^2 = \frac{2}{5}\ mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2$$

$$= \frac{2}{5}\ mr^2 + \frac{ma^2}{2}$$

6. The correct choice is (b).
7. The correct choice is (d).
8. The correct choice is (b).
9. The correct choice is (a).

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21. Tension T_2 is

(a) $\frac{9mg}{5}$ (b) $\frac{7mg}{5}$

(c) mg (d) $\frac{3mg}{5}$

22. The magnitude of torque on the pulley is

(a) $2\,mgR$ (b) $\frac{2mgR}{3}$

(c) $\frac{2mgR}{5}$ (d) $3\,mgR$

➔ SOLUTIONS

Refer to Fig. 5.71. Since the pulley has a finite mass, the tensions T_1 and T_2 will not be equal. If a is the acceleration of the system, the equations of motion of masses

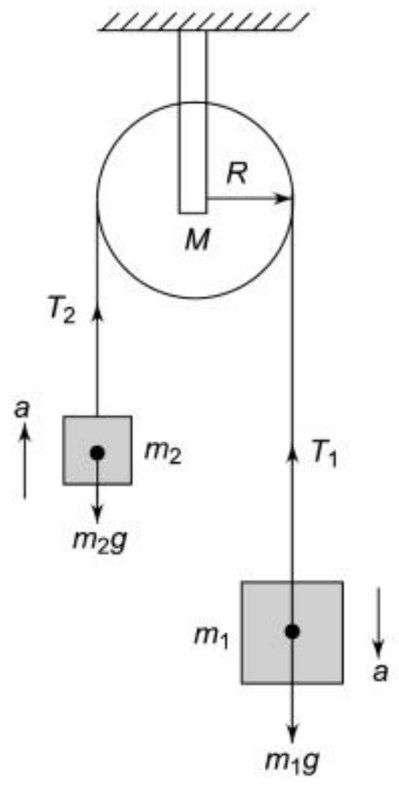

Fig. 5.71

$$m_1g - T_1 = m_1a \quad \Rightarrow \quad 3mg - T_1 = 3ma \qquad (1)$$

and $$T_2 - m_2g = m_2a \quad \Rightarrow \quad T_2 - mg = ma \qquad (2)$$

The resultant tension $(T_1 - T_2)$ exerts a torque on the pulley which is given by

$$\tau = (T_1 - T_2)R \qquad (3)$$

Also, $\tau = I\alpha$ where $I = \frac{1}{2}MR^2$ and α is the angular acceleration which is given by

$$a = \frac{a}{R}$$

$$\therefore \qquad \tau = \frac{1}{2}MR^2 \times \frac{a}{R} = \frac{1}{2}MRa = mRa \qquad (4)$$

$(\because\ M = 2m)$

From Eqs. (3) and (4) we get

$$T_1 - T_2 = ma \qquad (5)$$

19. Using Eqs. (1) and (2) and (5) and solving for a we get $a = \frac{2g}{5}$. Hence the correct choices is (c).

20. From Eqs. (1) and (2) we get $T_1 = 3m(g - a) = 3m\left(g - \frac{2g}{5}\right) = \frac{9mg}{5}$; which is choice (d).

21. Eqs. (1) and (2) give $T_2 = m(g + a) = m\left(g + \frac{2g}{5}\right) = \frac{7mg}{5}$ so choice (b) is correct.

22. From Eq. (4) $\tau = mRa = mR \times \frac{2g}{5} = \frac{2mgR}{5}$, which is choice (c).

Questions 23 to 26 are based on the following passage.

Passage VII

A small solid sphere of mass m rolls without slipping on the track shown in Fig. 5.72. The radius of the circular part of the track is R. The sphere starts from rest from point P at a height $H = 4.5\,R$ above the bottom.

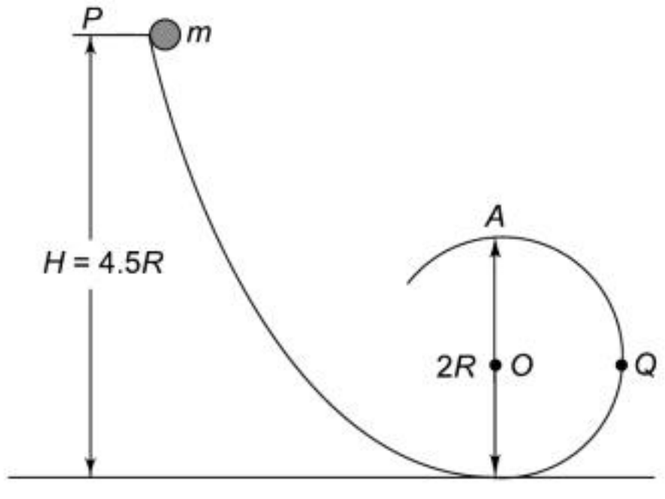

Fig. 5.72

23. The speed of the sphere when it reaches point Q on the track is

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$

(c) $\sqrt{5gR}$ (d) $\sqrt{7gR}$

24. The horizontal force acting on the sphere when it is at point Q is

(a) mg (b) $2\,mg$

(c) $3\,mg$ (d) $5\,mg$

25. The magnitude of the force acting on the sphere when it is at point Q is

(a) $\sqrt{4.5}\,mg$ (b) $\sqrt{5}\,mg$

(c) $\sqrt{26}\,mg$ (d) $3\sqrt{3}\,mg$

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26. What is the minimum value of height H so that the sphere can reach the top A of the circle?

(a) 2.4 R (b) 2.5 R
(c) 2.6 R (d) 2.7 R

➔ SOLUTIONS

23. Loss of P.E. at Q = Gain in K.E.

$$mg(H-R) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mg(4.5R-R) = \frac{1}{2}mv^2 + \frac{1}{2}\times\left(\frac{2}{5}mR^2\right)\times\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

which gives $v = \sqrt{5gR}$. Hence the correct choice is (c).

24. At point Q the velocity is directed tangentially. Thus, the horizontal force acting on the sphere at point Q is the centripetal force directed towards the centre O of the circular part of the track and is given by

$$F_h = \frac{mv^2}{R} = \frac{m}{R}\times(5\ gR) = 5\ mg,$$

which is choice (d).

25. The vertical force acting on the sphere is

$$F_v = \text{weight of the sphere} = mg$$

$\therefore$ Net force $F = \sqrt{F_h^2 + F_v^2} = \sqrt{26}\ mg$

Thus the correct choice is (c).

26. The sphere will rise to point A if it has a minimum speed at A which satisfies

$$\frac{mv_A^2}{R} = mg \quad\Rightarrow\quad v_A = \sqrt{Rg}$$

The minimum value of H is given by

$$mg(H_{\min} - 2R) = \frac{7}{10}mv_A^2 = \frac{7}{10}mRg$$

$\Rightarrow$ $H = 2R + 0.7R = 2.7R$, which is choice (d).

Questions 27 to 30 are based on the following passage.

Passage VIII

A uniform disc of mass M and radius R rolls without slipping down a plane inclined at an angle θ with the horizontal.

27. The acceleration of the centre of mass of the disc is

(a) $g\sin\theta$ (b) $\frac{2g\sin\theta}{3}$
(c) $\frac{g\sin\theta}{3}$ (d) $\frac{2g\cos\theta}{3}$

28. The frictional force on the disc is

(a) $\frac{Mg\sin\theta}{3}$ (b) $\frac{2Mg\sin\theta}{3}$
(c) $Mg\sin\theta$ (d) none of these

29. The magnitude of torque acting on the disc is

(a) MgR (b) $MgR\sin\theta$
(c) $\frac{2MgR\sin\theta}{3}$ (d) $\frac{MgR\sin\theta}{3}$

30. If the disc is replaced by a ring of the same mass M and the same radius R, the ratio of the frictional force on the ring to that on the disc will be

(a) $\frac{3}{2}$ (b) 2
(c) $\sqrt{2}$ (d) 1

➔ SOLUTIONS

The various forces acting on the disc are shown in the figure. As the disc rolls down the plane, the frictional force f acts upwards along the plane. This force f produces a torque which rotates the disc. If a is the linear acceleration of the centre of mass of the disc, the equation of motion is (see Fig. 5.73)

$$Mg\sin\theta - f = Ma \qquad (1)$$

If α is the angular acceleration and I is the moment of inertia of the disc about the axis of rotation, the torque acting on it is

$$\tau = I\alpha$$

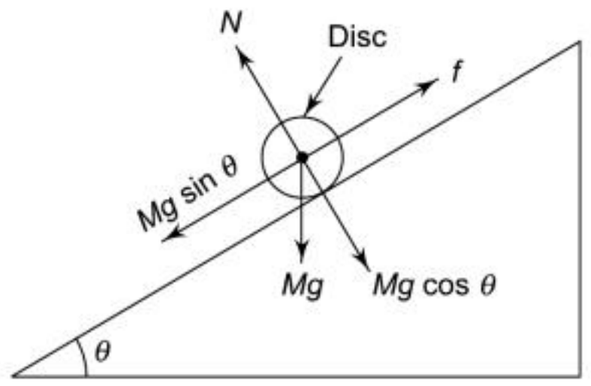

Fig. 5.73

Now $I = \frac{1}{2}MR^2$, $\alpha = \frac{a}{R}$ and $\tau = fR$. Hence

$$fR = \frac{1}{2}MR^2 \times \frac{a}{R} = \frac{1}{2}MRa$$

or $$f = \frac{1}{2}Ma \qquad (2)$$

27. From Eqs. (1) and (2), we get $a = \dfrac{2g\sin\theta}{3}$, which is choice (b).

28. $f = \dfrac{1}{2}Ma = \dfrac{Mg\sin\theta}{3}$, which is choice (a).

29. $\tau = fR = \dfrac{MgR\sin\theta}{3}$, which is choice (d).

30. For a ring, $I = MR^2$. The correct choice is (a).

Questions 31 to 33 are based on the following passage.

Passage IX

A solid cylinder of mass M and radius R is mounted on a frictionless horizontal axle so that it can freely rotate about this axis. A string of negligible mass is wrapped round the cylinder and a body of mass m is hung from the string as shown in Fig. 5.74. The mass is released from rest.

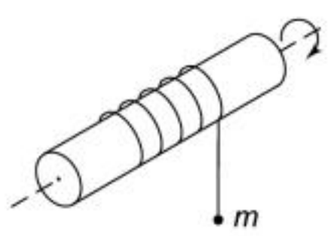

Fig. 5.74

31. The acceleration with which the mass falls is

(a) g (b) $\dfrac{mg}{M}$

(c) $\dfrac{mg}{(M+m)}$ (d) $\dfrac{2mg}{(M+2m)}$

32. The tension in the string is

(a) mg (b) $\dfrac{Mmg}{(M+m)}$

(c) $\dfrac{2Mmg}{(M+2m)}$ (d) $\dfrac{Mmg}{(M+2m)}$

33. The angular speed of the cylinder is proportional to h^n, where h is the height through which the mass falls. The value of n is

(a) zero (b) 1

(c) $\dfrac{1}{2}$ (d) 2

➔ SOLUTIONS

Referring to Fig. 5.75, the acceleration a of the falling body is governed by the equation

$$ma = mg - T \qquad (1)$$

where T is the tension in the string.
Torque on cylinder is

$$\tau = TR = I\alpha$$

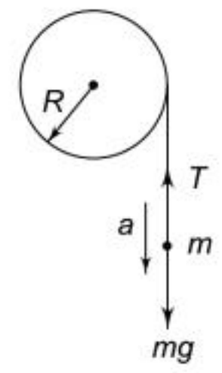

Fig. 5.75

$$\Rightarrow \quad T = \frac{I\alpha}{R}$$

$$= \left(\frac{1}{2}MR^2\right) \times \frac{a}{R^2}$$

$$= \frac{1}{2}Ma \qquad (2)$$

31. From Eqs. (1) and (2) we get

$$a = \frac{2mg}{(M+2m)}, \text{ which is choice (d).}$$

32. Equations (1) and (2) give $T = \dfrac{mMg}{(M+2m)}$, which is choice (d).

33. From conservation of energy, we have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mR^2\omega^2 + \frac{1}{2} \times \left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{1}{4}(2m + M)\,R^2\omega^2$$

$$\omega = \left[\frac{4mgh}{(M+2m)R^2}\right]^{1/2}$$

Thus $\omega \propto h^{1/2}$. Hence the correct choice is (c).

SECTION III

Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

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(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

1. Statement-1

A solid cylinder of mass M and radius R rolls down an inclined plane of height h. The rotational kinetic energy of the cylinder when it reaches the bottom of the plane is $Mgh/3$.

Statement-2

The total energy of the cylinder remains constant throughout its motion.

2. Statement-1

Two bodies A and B initially at rest, of masses $2m$ and m respectively move towards each other under mutual gravitational force of attraction. At the instant when the speed of A is v and that of B is $2v$, the speed of the centre of mass of system is $4v/3$.

Statement-2

The speed of the centre of mass of a system changes if an external force acts on the system.

3. Statement-1

The angular momentum of a particle moving in a circular orbit with a constant speed remains conserved about any point on the circumference of the circle.

Statement-2

If no net torque acts, the angular momentum of a system is conserved.

4. Statement-1

Two blocks of masses M and m (with $M > m$) are connected by a spring of negligible mass and placed on a horizontal frictionless surface. An impulse gives a velocity V to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

$$v_{CM} = \frac{MV}{(M+m)}$$

Statement-2

The principle of conservation of linear momentum is applicable to the centre of mass motion.

5. Statement-1

A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is $ML\,\omega^2$.

Statement-2

The entire mass of the liquid can be regarded as being concentrated at the centre of mass of the tube.

6. Statement-1

A thin wire of length L and mass m is bent into a circular loop of radius r as shown in Fig. 5.76. The moment of inertia of the loop about the XX' is $3mL^2/8\pi^2$.

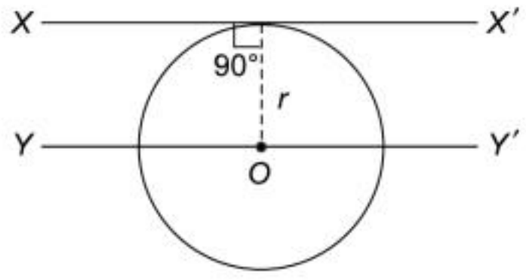

Fig. 5.76

Statement-2

According to the parallel axes theorem, the moment of inertia of the loop about XX' = moment of inertia about $YY' + mr^2$.

7. Statement-1

A thin uniform rod AB of mass M and length L is hinged at one end A to the horizontal floor. Initially it stands vertically. It is allowed to fall freely on the floor in the vertical plane. The angular velocity of the rod when its ends B strikes the floor is $\sqrt{3g/L}$.

Statement-2

The angular momentum of the rod about the hinge remains constant throughout its fall to the floor.

8. Statement-1

For a particle moving in a circle with a constant speed, the direction of the centripetal force depends on whether the particle is moving clockwise or anticlockwise along the circle.

Statement-2

The centripetal force is directed radially towards the centre of the circle.

9. Statement-1

A body tied to a string is moved in a circle with a uniform speed. If the string suddenly breaks, the angular momentum of the body becomes zero.

Statement-2

The torque on the body equals the rate of change of angular momentum.

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10. Statement-1

A solid cylinder and a solid sphere have the same mass M and the same radius R. If torques of equal magnitude are applied to them for the same time, the sphere will acquire greater angular speed.

Statement-2

For a given torque, the angular acceleration is inversely proportional to the moment of inertia.

11. Statement-1

If a disc rotating about its axis with a certain angular speed is gently placed on a horizontal frictionless surface, it will roll along the surface with the same angular speed.

Statement-2

No torque acts on the disc if the friction is absent.

12. Statement-1

Two solid spheres of the same radius, one made of steel and the other of aluminium are released from rest from the top of an inclined plane at the same time. The two spheres will reach the bottom at the same time.

Statement-2

The linear acceleration down the plane is independent of the mass of the sphere.

13. Statement-1

A sphere and a cylinder slide without rolling from rest from the top of an inclined plane. They will reach the bottom with the same speed.

Statement-2

Bodies of all shapes, masses and sizes slide down a plane with the same acceleration.

14. Statement-1

If a cylinder rolling with angular speed ω suddenly breaks up into two equal halves of the same radius, the angular speed of each piece becomes 2ω.

Statement-2

If no external torque acts, the angular momentum of a system is conserved.

15. Statement-1

Friction is necessary for a body to roll on a surface.

Statement-2

Friction provides the necessary tangential force and torque.

16. Statement-1

A sphere is rolling on a rough surface in the direction of the arrow as shown in Fig. 5.77. The force of friction at the point of contact will be in the direction of the arrow.

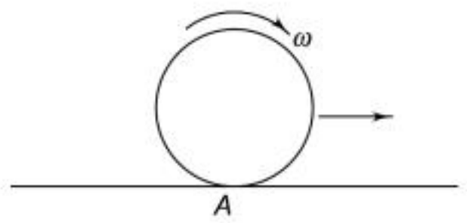

Fig. 5.77

Statement-2

Friction opposes motion.

17. Statement-1

During perfect rolling, the force of friction becomes zero.

Statement-2

The speed at the point of contact becomes zero.

18. Statement-1

During rolling, the acceleration of the point of contact is non zero.

Statement-2

The direction of the velocity changes with time.

➔ SOLUTIONS

1. The correct choice is (a).
From the law of conservation of energy, we have
Potential energy = Translational kinetic energy + Rotational kinetic energy

$$\text{or} \qquad Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$\text{or} \qquad Mgh = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{3}{4}MR^2\,\omega^2$$

$$\text{or} \qquad \omega^2 = \frac{4gh}{3R^2}$$

Now the rotational kinetic energy = $\frac{1}{2}I\omega^2$.
Substituting for ω^2 and I, we have

$$\text{Rotational kinetic energy} = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{4gh}{3R^2}$$

$$= \frac{Mgh}{3}$$

2. The correct choice is (d). Mutual gravitational force is internal to the system. Since no external force acts on the system, the centre of mass (which is initially at rest) will remain at rest.

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6

Gravitation

REVIEW OF BASIC CONCEPTS

1. Newton's Law of Gravitation

Newton's law of universal gravitation states as follows:

'Any two particles of matter anywhere in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, the direction of the force being along the line joining the particles, i.e. (Fig. 6.1)

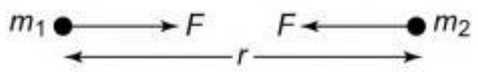

Fig. 6.1

$$F \propto \frac{m_1 m_2}{r^2}$$

where F is the magnitude of the force of attraction between two particles of masses m_1 and m_2 separated by a distance r.

In the form of an equation the law is written as

$$F = \frac{G m_1 m_2}{r^2}$$

where G is a constant called the universal gravitation constant. The value of this constant is to be determined experimentally and is found to be

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}.$$

2. Gravitational Force due to Multiple Masses

If a system consists of more than two masses, the gravitational force experienced by a given mass due to all other masses is obtained from the principle of superposition which states that '*the gravitational force experienced by one mass is equal to the vector sum of the gravitational forces exerted on it by all other masses taken one at a time.*'

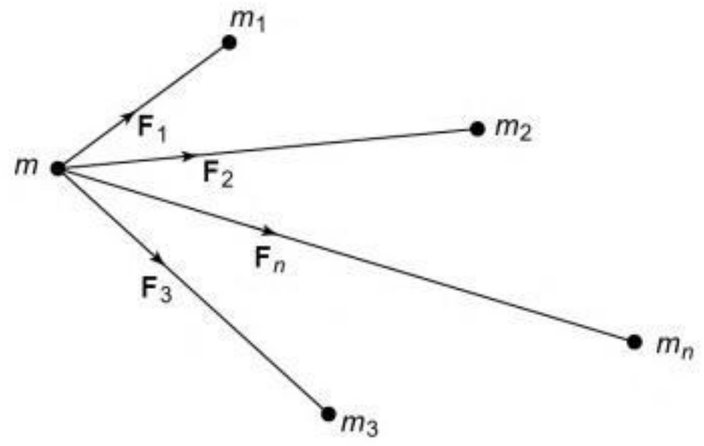

Fig. 6.2

The gravitational force on mass m due to masses m_1, m_2, m_3, ... m_n is given by (Fig. 6.2)

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_n$$

NOTE

(1) Gravitational force is alway attractive.
(2) Gravitational force between two masses does not depend on the medium between them.
(3) Gravitational force acts along the straight line joining the centres of the two bodies.

Example 1 Two bodies A and B of masses $m_1 = 1$ kg and $m_2 = 16$ kg respectively are placed 1.0 m apart. A third body C of mass $m = 3$ kg is placed on the line joining A and B. At what distance from A should C be placed so that it experiences no gravitational force?

Solution Let x metre be the distance between A and C (Fig. 6.3)

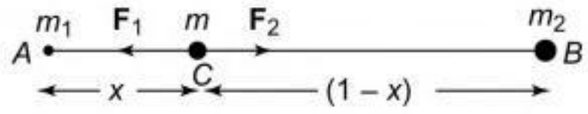

Fig. 6.3

Force exerted by A on C is

$$F_1 = \frac{G\,m_1 m}{x^2} \text{ directed towards } A$$

Force exerted by B on C is

$$F_2 = \frac{G\,m_2 m}{(1-x)^2} \text{ directed towards } B$$

C will experience no force if $F_1 = F_2$, i.e.

$$\frac{G\,m_1 m}{x^2} = \frac{G\,m_2 m}{(1-x)^2}$$

$$\frac{m_2}{m_1} = \frac{(1-x)^2}{x^2}$$

$$16 = \frac{(1-x)^2}{x^2}$$

$$4 = \frac{1-x}{x}$$

which gives $x = 0.2$ m. Note that if body C is placed to the left of body A or to the right of body B, it will experience a finite gravitational force.

Example 2 Three bodies, each of mass m, are placed at the vertices of an equilateral triangle of side a as shown in Fig. 6.4. Find the magnitude and direction of the force experienced by the body at vertex A.

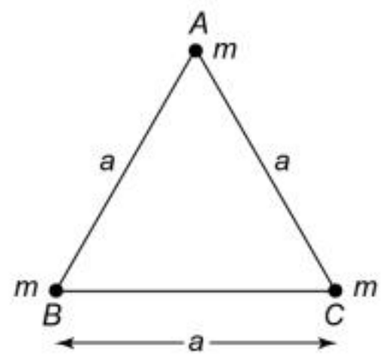

Fig. 6.4

Solution The forces exerted on the body at A by bodies at B and C are shown in Fig. 6.5.

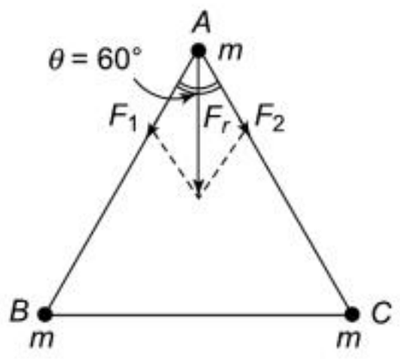

Fig. 6.5

$$F_1 = F_2 = \frac{Gm^2}{a^2} = F \text{ (say)}$$

The magnitude of the resultant force is

$$F_r = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$= \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ}$$

$$= \sqrt{3}\,F = \sqrt{3}\,\frac{Gm^2}{a^2},$$

directed vertically downwards.

3. Acceleration due to Gravity

Considering the earth as an isolated mass, a force is experienced by a body near it. This force is directed towards the centre of the earth and has a magnitude mg, where g is the acceleration due to gravity.

$$F = mg = \frac{GmM}{R^2}$$

where M is the mass of the earth and R its radius (nearly constant for a body in the vicinity of the earth)

$$\therefore \qquad g = \frac{GM}{R^2}$$

All bodies near the surface of the earth fall with the same acceleration which is directed towards the centre of the earth.

4. Variation of g

1. Variation with altitude The acceleration due to gravity of a body at a height h above the surface of the earth is given by

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

where g is the acceleration due to gravity on the surface of the earth. If h is very small compared to R, we can use binomial expansion and retain terms of order h/R. We then get

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Thus, the acceleration due to gravity decreases as the altitude (h) is increased.

2. Variation with depth The acceleration due to gravity at a depth d below the surface of the earth is given by

$$g_d = g\left(1 - \frac{d}{R}\right)$$

This equation shows that the acceleration due to gravity decreases with depth. At the centre of the earth where $d = R$, $g_d = 0$. Thus the acceleration due to gravity is maximum at the surface of the earth, decreases with increase in depth and becomes zero at the centre of the earth.

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3. Variation with Latitude Due to the rotation of earth about its axis, the value of g varies with latitude, i.e. from one place to another on the earth's surface. At poles, the effect of rotation on g is negligible. At the equator, the effects of rotation on g is the maximum. In general, the value of acceleration due to gravity at a place decreases with the decrease in the latitude of the place.

The accelration due to gravity at a place on earth where the latitude is ϕ is given by

$$g\phi = g - \omega^2 R \cos^2 \phi;$$

ω = angular velocity of rotation of earth.

At equator, $\phi = 0 \quad \Rightarrow \quad g_e = g - R\omega^2$ (minimum)

At poles, $\phi = 90° \quad \Rightarrow \quad g_p = g$ (maximum)

$$g\phi = g - 0.0337 \cos^2 \phi$$

Thus the value of g varies slightly from place to place on earth. Variation of g with altitude and depth is shown in Fig. 6.6.

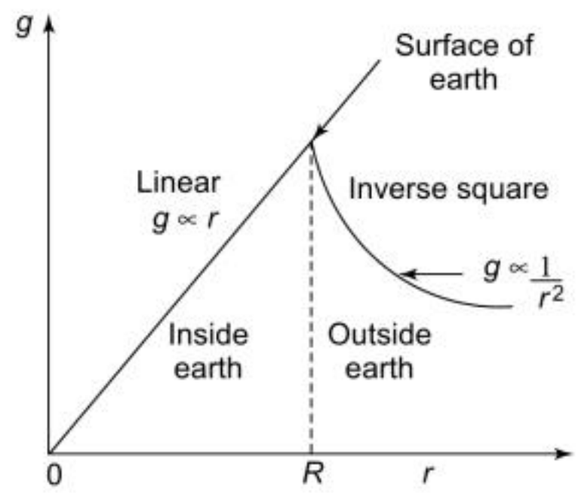

Fig. 6.6 Variation of g (gravitational acceleration)

Example 3 The acceleration due to gravity on the surface of the moon is 1.67 ms^{-2}. The mass of the earth is about 80 times that of the moon. Estimate the ratio of the radius of the earth to that of the moon.

Solution $g_e = \dfrac{G M_e}{R_e^2}$ and $g_m = \dfrac{G M_m}{R_m^2}$

$$\therefore \quad \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_m}\right)^{1/2}$$

$$= \left(80 \times \frac{1.67}{9.8}\right)^{1/2} \simeq 3.7$$

Example 4 Assuming that the earth is a sphere of radius R, at what altitude will the value of acceleration due to gravity be half its value at the surface of the earth?

Solution $\dfrac{g'}{g} = \left(\dfrac{R}{R+h}\right)^2 \Rightarrow \dfrac{1}{2} = \left(\dfrac{R}{R+h}\right)^2$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{R}{R+h}$$

which gives $h = \left(\sqrt{2} - 1\right) R.$

Example 5 A body weighs 63 N on the surface of the earth. How much will it weigh at a height equal to half the radius of the earth?

Solution $W = mg = 63$ N

$$W' = mg' = mg\left(\frac{R}{R+h}\right)^2$$

$$\therefore \quad W' = W\left(\frac{R}{R+h}\right)^2$$

$$= 63 \times \left(\frac{R}{R+R/2}\right)^2 = 28 \text{ N}$$

Example 6 Assuming the earth to be a sphere of uniform mass density, find the percentage decrease in the weight of a body when it is taken to the end of a tunnel 32 km below the surface of the earth. Radius of earth = 6400 km.

Solution $g' = g\left(1 - \dfrac{d}{R}\right)$

$$= g\left(1 - \frac{32}{6400}\right) = \frac{199\, g}{200}$$

Decrease in weight $= mg - mg'$

$$= mg\left(1 - \frac{199}{200}\right) = \frac{mg}{200}$$

Percentage decrease $= \dfrac{mg/200}{mg} \times 100 = 0.5\%$

Example 7 At what depth below the surface of the earth will the value of acceleration due to gravity become 90% of its value at the surface? $R = 6.4 \times 10^6$ m.

Solution $g' = 0.9\, g.$ $\qquad g' = g\left(1 - \dfrac{d}{R}\right)$

$$\therefore \quad 0.9 = 1 - \frac{d}{R}$$

$$\Rightarrow \quad d = 0.1\, R = 6.4 \times 10^5 \text{ m}$$

5. Gravitational Field Intensity

Just as the region around a magnet has magnetic field and the region around a charge has electric field, the region around a mass has gravitational field.

The gravitational field intensity (or simply gravitational field) at a point is defined as the gravitational force experienced by a unit mass placed at that point.

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$$W_2 = \int_{\infty}^{r} \frac{GMm}{r^2} dr$$

$$= GMm \int_{\infty}^{r} r^{-2} dr = -\frac{GMm}{r}$$

Total work $W = W_1 + W_2 = -\frac{GMm}{r}$

∴ Gravitational potential energy of the system is

$$U = W = -\frac{GMm}{r}$$

The zero of potential energy is assumed to at $r = \infty$. The negative sign indicates the potential energy is negative for any finite separation between the masses and increases to zero at infinite separation.

Expression for Increase in Gravitational Potential Energy

If the body m is moved away from M, the potential energy of the system increases.

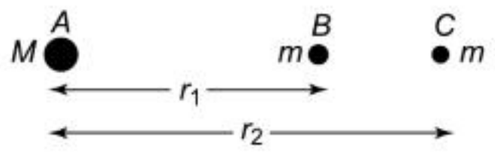

Fig. 6.10

$$\text{P.E. at } B = -\frac{GMm}{r_1}$$

$$\text{P.E. at } C = -\frac{GMm}{r_2}$$

$$\therefore \text{ Increase in P.E.} = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$

$$= GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If the body of mass M is the earth, then the increase in gravitational P.E. when a body of mass m is taken from the surface of the earth to a height h above the surface is given by (see Fig. 6.11), R = radius of the earth.

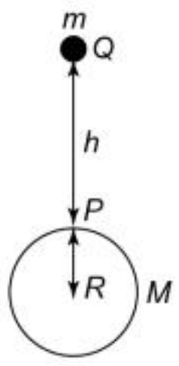

Fig. 6.11

$$\Delta U = \text{P.E. at } Q - \text{P.E at } P$$

$$= -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GmMh}{R(R+h)}$$

If $h << R$; $\quad \Delta U = \frac{GmMh}{R^2} = mgh \quad \left(\because \frac{GM}{R^2} = g\right)$

7. Gravitational Potential

Gravitational potential at a point P in the gravitational field of a body of M is defined as the amount of work done to bring a unit mass from infinity to that point along any path and without any acceleration, i.e., (Fig. 6.12)

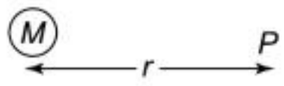

Fig. 6.12

$$V = \frac{W}{m} = -\frac{GMm}{r \times m}$$

$$\Rightarrow \quad V = -\frac{GM}{r}$$

Potential V is a scalar quantity. Hence the gravitational potential at a point P due to a number of masses $m_1, m_2, \ldots m_n$ at distances $r_1, r_2, \ldots r_n$ respectively from P is given by

$$V = V_1 + V_2 + \cdots + V_n$$

$$= -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \cdots + \frac{m_n}{r_n}\right)$$

The SI unit of V is J kg^{-1}.

Relation between Gravitational Field Intensity (I) and Gravitational Potential (V)

Gravitational field intensity and gravitational potential at a point are related as

$$I = -\frac{dV}{dr}$$

Gravitational Potential due to a Spherical Shell

(i) At a point outside the shell, $V = -\frac{GM}{r}$ (for $r > R$)
where M is the mass and R is the radius of the shell.

(ii) At a point on the surface of the shell, $V = -\frac{GM}{R}$

(iii) At a point inside the shell, $V = -\frac{GM}{R}$ (for $r < R$)

Figure 6.13 shows the variation V with r for a spherical shell.

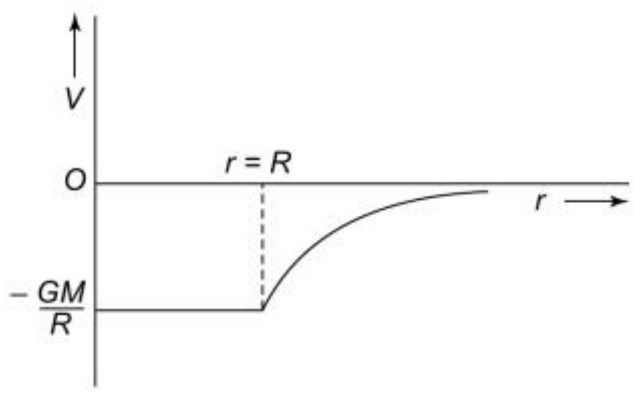

Fig. 6.13

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Gravitational Potential due to a Solid sphere of mass M and radius R

(i) For points outside the sphere ($r > R$), $V = -\dfrac{GM}{r}$

(ii) For points inside the sphere ($r < R$),

$$V = -\frac{3GM}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{6}\right)$$

(ii) At the centre of the sphere ($r = 0$)

$$V = -\frac{3GM}{2R}$$

(iv) On the surface of the sphere ($r = R$)

$$V = -\frac{GM}{R}$$

Example 8 Two masses $m_1 = 100$ kg and $m_2 = 8100$ kg are held 1 m apart.

(a) At what point on the line joining them is the gravitational field equal to zero? Find the gravitational potential at that point.

(b) Find the gravitational potential energy of the system. Given $G = 6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}$.

Solution Gravitational field at P due to m_1 is [Fig. 6.14]

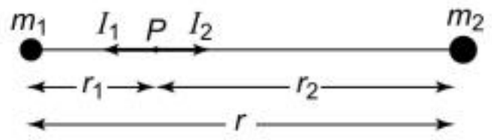

Fig. 6.14

$$I_1 = \frac{Gm_1}{r_1^2} \text{ directed towards } m_1$$

Gravitational field at P due to m_2 is

$$I_2 = \frac{Gm_2}{r_2^2} \text{ directed towards } m_2$$

Gravitational field at P will be zero if

$$I_1 = I_2 \quad \Rightarrow \quad \frac{Gm_1}{r_1^2} = \frac{Gm_2}{r_2^2}$$

or $$\frac{m_2}{m_1} = \left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \qquad 81 = \left(\frac{1 - r_1}{r_1}\right)^2 \quad (\because r_2 = r - r_1 \text{ and } r = 1 \text{ m})$$

$$\therefore \qquad 9 = \frac{1 - r_1}{r_1} \quad \Rightarrow \quad r_1 = 0.1 \text{ m}$$

Gravitational potential at P is

$$V = V_1 + V_2 = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$= -6.67 \times 10^{-11} \times \left(\frac{100}{0.1} \times \frac{8100}{0.9}\right)$$

$$= -6.67 \times 10^{-7}\ \text{J kg}^{-1}$$

(b) Gravitational potential energy of the system is

$$\text{G.P.E.} = -\frac{G\,m_1\,m_2}{r}$$

$$= -\frac{6.67 \times 10^{-11} \times 100 \times 8100}{1}$$

$$= -5.4 \times 10^{-5}\ \text{J}$$

Example 9 Three equal masses, each equal to m, are kept at the vertices of an equilateral triangle of side a. Find the gravitational field and gravitational potential at the centroid of the triangle.

Solution Refer to Fig. 6.15.

The centroid O divides the lines AD, BE and CF in the ratio 2:1. Also $AO = BO = CO = r$ (say). Now $AO = \dfrac{2}{3}$ and

$$AD = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}\,a}{2}$$

$$\therefore \qquad AO = \frac{2}{3} \times \frac{\sqrt{3}\,a}{2} = \frac{a}{\sqrt{3}}, \text{ i.e. } r = \frac{a}{\sqrt{3}}$$

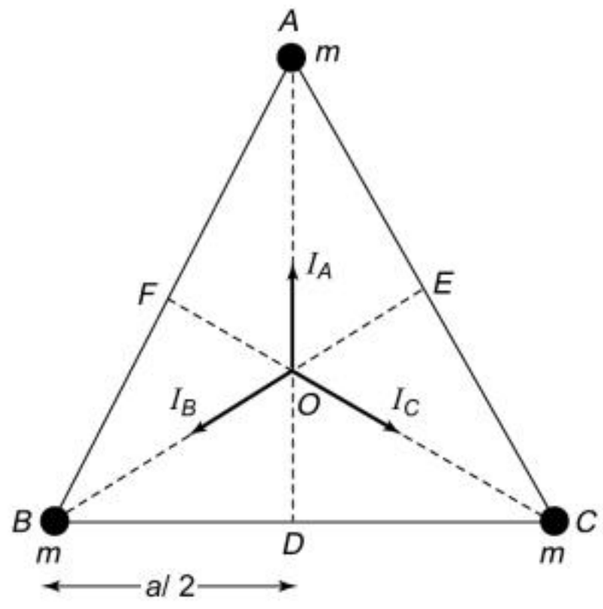

Fig. 6.15

The gravitational fields at O due masses m at A, B and C are $I_A = I_B = I_C = \dfrac{GM}{r_2} = I$. Their directions are shown in Fig. 6.14. The angle between any two of them is $\theta = 120°$.

The resultant of I_B and I_C is

$$I' = \sqrt{I^2 + I^2 + 2I^2 \cos 120°} = I$$

directed vertically down.

I' will cancel with I_A. Hence the net gravitational field at O is zero.

The gravitational potential at O is

$$V = V_1 + V_2 + V_3$$

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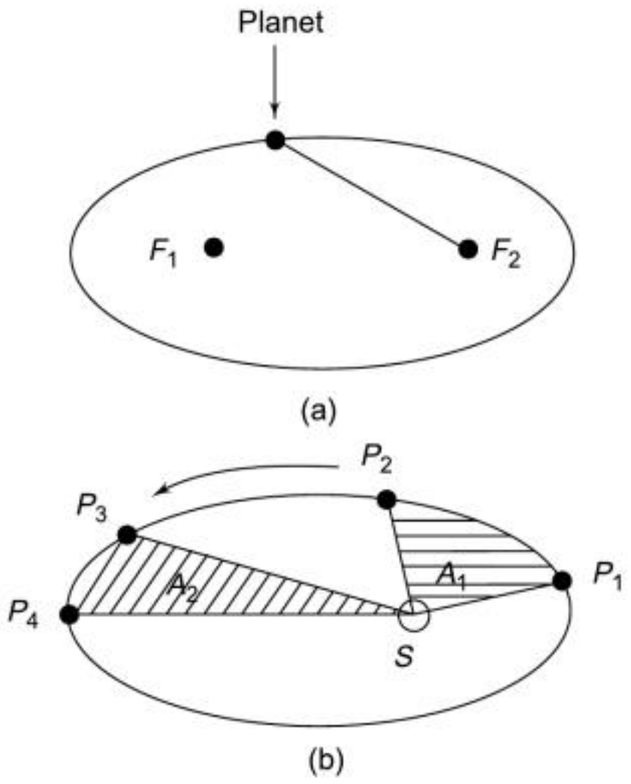

Fig. 6.16

If T_2 represents the period of a second planet about the sun, and r_2 its mean distance, then for this planet

$$T_2^2 \propto r_2^3$$

These two relations can be combined since the factor of proportionality is the same for both. Thus

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Example 12 The mass of Jupiter is 318 times that of the earth and its radius is 11.2 times that of the earth. Calculate the escape velocity from Jupiter's surface. Given the escape velocity from earth'surface = 11.2 km s^{-1}.

Solution For Jupiter : $v_J = \sqrt{\dfrac{2M_J G}{R_J}}$

For Earth : $v_J = \sqrt{\dfrac{2M_E G}{R_E}}$

$$\therefore \quad \frac{v_J}{v_E} = \sqrt{\frac{M_J}{M_E} \times \frac{R_E}{R_J}} = \sqrt{318 \times \frac{1}{11.2}} = 5.33$$

$$\therefore \quad v_J = 5.33\, v_E = 5.33 \times 11.2 = 59.7 \text{ km s}^{-1}$$

Example 13 Calculate the escape velocity of a body at a height 1600 km above the surface of the earth. Radius of earth = 6400 km.

Solution Work required to move a body of mass m from $r = R + h$ to $r = \infty$ is

$$W = \int_{R+h}^{\infty} F dr = GmM \int_{R+h}^{\infty} \frac{dr}{r^2}$$

$$= \frac{GmM}{R+h}$$

If v_e is the escape velocity, then

$$\frac{1}{2} m v_e^2 = \frac{GmM}{R+h}$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2gR^2}{R+h}} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Given $R = 6.4 \times 10^6$ m, and $h = 1.6 \times 10^6$ m and $g = 9.8$ ms^{-2}

$$\therefore \quad v_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{(6.4 + 1.6) 10^6}}$$

$$= 10 \times 10 \text{ ms}^{-1} = 10 \text{ km s}^{-1}$$

Example 14 A rocket is launched vertically from the surface of the earth with an initial velocity equal to half the escape velocity. Find the maximum height attained by it in terms of R where R is the radius of the earth. Ignore atmospheric resistance.

Solution On the surface of the earth, the total energy of the rocket is

$$E_i = \text{K.E.} + \text{P.E.} = \frac{1}{2} mv^2 - \frac{GmM}{R}$$

At the highest point, $v = 0$. If h is the maximum height attained, the energy of the rocket at height h is

$$E_f = -\frac{GmM}{(R+h)}$$

From conservation of energy, $E_i = E_f$, i.e.,

$$\frac{1}{2} mv^2 - \frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R} \times \frac{h}{R+h}} = v_e \sqrt{\frac{h}{R+h}}$$

Given $v = \dfrac{v_e}{2}$. Hence $\dfrac{v_e}{2} = v_e \sqrt{\dfrac{R}{R+h}} \quad \Rightarrow \quad h = \dfrac{R}{3}$

Example 15 A body is dropped from a height h equal to $\dfrac{R}{2}$, where R is the radius of the earth. Show that it will hit the surface of the earth with a speed $v = v_e/\sqrt{3}$, where v_e is the escape velocity from the surface of the earth.

Solution Total energy of the body at height h is

$$E_i = -\frac{GmM}{(R+h)} \qquad \text{(i)}$$

Total energy when it hits the surface of the earth is

$$E_f = \frac{1}{2} mv^2 - \frac{GmM}{R} \qquad \text{(ii)}$$

From conservation of energy, $E_i = E_f$, i.e.

$$-\frac{GmM}{(R+h)} = \frac{1}{2} mv^2 - \frac{GmM}{R}$$

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(a) the burning of fuel in its engine
(b) the ejection of hot gases from its exhaust
(c) the gravitational attraction of the sun
(d) the gravitational attraction of the earth.

17. An instrument package is released from an orbiting earth satellite by simply detaching it from the outer wall of the satellite. The package will
(a) go away from the earth and get lost in outer space
(b) fall to the surface of the earth
(c) continue moving along with the satellite in the same orbit and with the same velocity
(d) fall through a certain distance and then move in an orbit around the earth.

18. Two satellites A and B are orbiting around the earth in circular orbits of the same radius. The mass of A is 16 times that of B. The ratio of the period of revolution of B to that of A is
(a) 1 : 16 (b) 1 : 4
(c) 1 : 2 (d) 1 : 1

19. A satellite is moving around the earth in a stable circular orbit. Which one of the following statements will be wrong for such a satellite?
(a) It is moving at a constant speed.
(b) Its angular momentum remains constant.
(c) It is acted upon by a force directed away from the centre of the earth which counter- balances the gravitational pull of the earth.
(d) It behaves as if it were a freely falling body.

20. Astronauts in a stable orbit around the earth are said to be in a weightless condition. The reason for this is that
(a) the capsule and its contents are falling freely at the same rate
(b) there is no gravitational force acting on them
(c) the gravitational force of the earth balances that of the sun
(d) there is no atmosphere at the height at which they are orbiting.

21. The escape velocity from the earth is v_e. What is the escape velocity from a planet whose radius is twice that of the earth and mean density is the same as that of the earth?
(a) $v_e/2$ (b) v_e
(c) $2\,v_e$ (d) $4\,v_e$

22. Choose the wrong statement. The escape velocity of a body from a planet depends upon
(a) the mass of the body
(b) the mass of the planet
(c) the average radius of the planet
(d) the average density of the planet

23. Choose the wrong statement. The orbital velocity of a body in a stable orbit around a planet depends upon
(a) the average radius of the planet
(b) the height of the body above the planet
(c) the acceleration due to gravity
(d) the mass of the orbiting body

24. Choose the correct statement. In planetary motion
(a) the speed along the orbit remains constant
(b) the angular speed remains constant
(c) the total angular momentum remains constant.
(d) the radius of the orbit remains constant.

25. An object weighs W newton on earth. It is suspended from the lower end of a spring balance whose upper end is fixed to the ceiling of a space capsule in a stable orbit around the earth. The reading of the spring balance will be
(a) W (b) less than W
(c) more than W (d) zero.

26. If M is the mass of the earth, R its radius (assumed spherical) and G the universal gravitational constant, then the amount of work that must be done on a body of mass m so that it completely escapes from the gravity of the earth, is given by
(a) $\dfrac{GmM}{R}$ (b) $\dfrac{GmM}{2R}$
(c) $\dfrac{3GmM}{2R}$ (d) $\dfrac{3GmM}{4R}$

27. A rocket is fired from the earth to the moon. The distance between the earth and the moon is r and the mass of the earth is 81 times the mass of the moon. The gravitational force on the rocket will be zero, when its distance from the moon is
(a) $\dfrac{r}{20}$ (b) $\dfrac{r}{15}$
(c) $\dfrac{r}{10}$ (d) $\dfrac{r}{5}$

28. Assuming that the earth is a sphere of radius R, at what altitude will the value of the acceleration due to gravity be half its value at the surface of the earth?
(a) $h = \dfrac{R}{2}$ (b) $h = \dfrac{R}{\sqrt{2}}$
(c) $h = \left(\sqrt{2}+1\right)R$ (d) $h = \left(\sqrt{2}-1\right)R$

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ANSWERS

1. (c)	**2.** (b)	**3.** (c)
4. (c)	**5.** (d)	**6.** (d)
7. (a)	**8.** (b)	**9.** (c)
10. (d)	**11.** (b)	**12.** (b)
13. (b)	**14.** (d)	**15.** (a)
16. (d)	**17.** (d)	**18.** (c)
19. (b)	**20.** (d)	**21.** (b)
22. (b)	**23.** (d)	**24.** (d)
25. (b)	**26.** (a)	**27.** (a)
28. (c)	**29.** (b)	**30.** (a)
31. (c)	**32.** (c)	**33.** (b)
34. (b)	**35.** (c)	**36.** (c)
37. (a)	**38.** (a)	**39.** (c)
40. (c)		

➔ HINTS/SOLUTIONS

1. Value of 1 main scale division = 0.5°
30 vernier scale divisions
= 29 × main scale divisions
= 29 × 0.5°

$\therefore$ Value of 1 vernier scale division = $\frac{29}{30}\times 0.5°$

Least count = value of 1 m.s.d – value of 1 v.s.d.

$$= 0.5° - \frac{29}{30}\times 0.5° = \frac{0.5°}{30} = \frac{0.5}{30}\times 60 \text{ min}$$
$$= 1 \text{ min}$$

2. RC has dimension of time and V has the dimensions of $L\ \frac{dI}{dt}$. Hence

$$\left[\frac{L}{RCV}\right] = \left[\frac{1}{T}\times\frac{T}{A}\right] = \frac{1}{\text{current}}$$

3. Given $v = k\sqrt{x} \Rightarrow v^2 = k^2x$. Differentiating, we have

$$2v\frac{dv}{dt} = k^2\frac{dx}{dt} = k^2v \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{k^2}{2}$$

$$\Rightarrow \quad \int dv = \frac{k^2}{2}\int dt$$

$$\Rightarrow \quad v = \frac{k^2t}{2}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{k^2t}{2}$$

$$\Rightarrow \quad \int dx = \frac{k^2}{2}\int t\ dt$$

$$\Rightarrow \quad x = \frac{k^2}{4}t^2$$

Thus $x \propto t^2$.

4. From energy conservation [see Fig. P-81]

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL$$

$$\Rightarrow \quad v = \sqrt{u^2 - 2gL}$$

$$\Delta v = \sqrt{v^2 + (-u)^2} = \sqrt{2\left(u^2 - gL\right)}$$

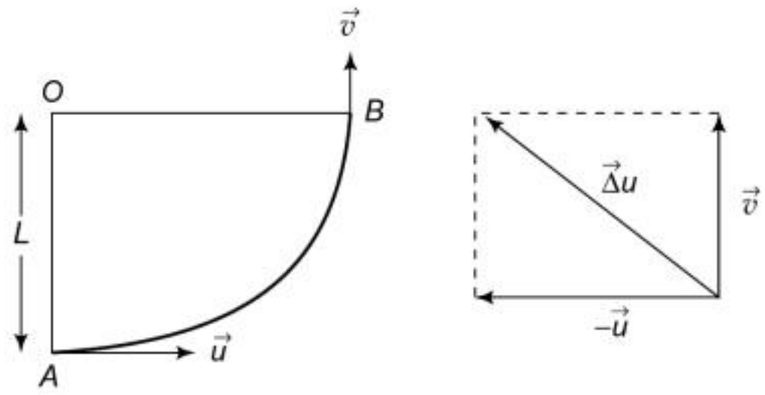

Fig. P-81

5. Given $v = \alpha\sqrt{x}$. Since $v = \frac{dx}{dt}$, we have

$$\frac{dx}{dt} = \alpha\sqrt{x}$$

or $$\frac{dx}{\sqrt{x}} = \alpha dt$$

Integrating, we have

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt$$

which gives $-2\sqrt{x} = \alpha\ t$ or $x = \frac{\alpha^2t^2}{4}$. Hence x is proportional to t^2.

6. Given $\vec{v} = K\left(y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}\right)$. Hence

$v_x = Ky$ and $v_y = Kx$. Therefore,

$$\frac{dx}{dt} = Ky \quad \text{and} \quad \frac{dy}{dt} = Kx.$$

Now $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{Kx}{Ky} = \frac{x}{y}$$

$$\Rightarrow \quad y\,dy = x\,dx$$

Integrating we get $\frac{y^2}{2} = \frac{x^2}{2} + c$

where c = constant of integration.
Hence $y^2 = x^2 + 2c$

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