PROJECTILE MOTION

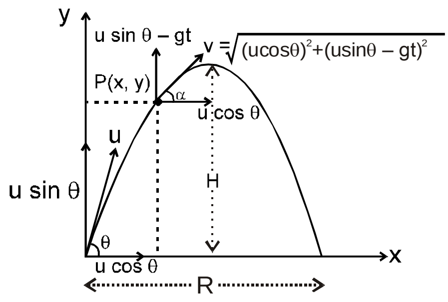
Contents

Syllabus

Kinematics in one and two dimensions (Cartesian coordinates only), Projectiles.

PROJECTILE MOTION

1. PROJECTILE THROWN AT AN ANGLE WITH HORIZONTAL



- Consider a projectile thrown with a velocity u making an angle θ with the horizontal.
- Initial velocity u is resolved in components in a coordinate system in which horizontal direction is taken as x-axis, vertical direction as y-axis and point of projection as origin.

  $u_x = u\cos\theta \qquad\qquad u_y = u\sin\theta$

- Again this projectile motion can be considered as the combination of horizontal and vertical motion. Therefore,

	Horizontal direction	**Vertical direction**
(a)	Initial velocity $u_x = u\cos\theta$	Initial velocity $u_y = u\sin\theta$
(b)	Acceleration $a_x = 0$	Acceleration $a_y = g$
(c)	Velocity after time t, $v_x = u\cos\theta$	Velocity after time t, $v_y = u\sin\theta - gt$

1.1 Time of flight :

The displacement along vertical direction is zero for the complete flight.
Hence, along vertical direction net displacement = 0

$$\Rightarrow \quad (u\sin\theta)\,T - \frac{1}{2}gT^2 = 0 \qquad \Rightarrow \qquad \mathbf{T = \frac{2u\sin\theta}{g}}$$

1.2 Horizontal range :

$$R = u_x\,.T \qquad \Rightarrow \qquad R = u\cos\theta.\ \frac{2u\sin\theta}{g}$$

$$\mathbf{R = \frac{u^2\sin 2\theta}{g}}$$

1.3 Maximum height :

At the highest point of its trajectory, particle moves horizontally, and hence vertical component of velocity is zero.
Using 3rd equation of motion i.e.

$v^2 = u^2 + 2as$

we have for vertical direction

$$0 = u^2\sin^2\theta - 2gH \qquad \Rightarrow \qquad \mathbf{H = \frac{u^2\sin^2\theta}{2g}}$$

1.4 Resultant velocity :

$\vec{v} = v_x\hat{i} + v_y\hat{j} \quad = u\cos\theta\ \hat{i} + (u\sin\theta - gt)\ \hat{j}$

Where, $|\ \vec{v}\ | = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$ and $\tan\alpha = v_y / v_x$.

Also, $v\cos\alpha = u\cos\theta \quad \Rightarrow v = \dfrac{u\cos\theta}{\cos\alpha}$

2. EQUATION OF TRAJECTORY

The path followed by a particle (here projectile) during its motion is called its **Trajectory**. Equation of trajectory is the relation between instantaneous coordinates (Here x & y coordinate) of the particle.

If we consider the horizontal direction,

$x = u_x.t$

$x = u\cos\theta.\ t \qquad ...(1)$

For vertical direction :

$y = u_y\ .\ t - 1/2\ gt^2$

$= u\sin\theta\ .\ t - 1/2\ gt^2 \qquad ...(2)$

Eliminating 't' from equation (1) & (2)

$$y = u\sin\theta\ .\ \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \quad \mathbf{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

This is an equation of parabola called as trajectory equation of projectile motion.

Other forms of trajectory equation :

- $\mathbf{y = x\tan\theta - \dfrac{gx^2(1+\tan^2\theta)}{2u^2}}$

- $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

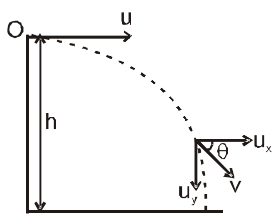
$\Rightarrow \quad y = x\tan\theta\left[1 - \dfrac{gx}{2u^2\cos^2\theta\tan\theta}\right]$

$\Rightarrow \quad y = x\tan\theta\left[1 - \dfrac{gx}{2u^2\sin\theta\cos\theta}\right]$

$\Rightarrow \quad \mathbf{y = x\tan\theta\left[1 - \dfrac{x}{R}\right]}$

3. PROJECTILE THROWN PARALLEL TO THE HORIZONTAL FROM SOME HEIGHT

Consider a projectile thrown from point O at some height h from the ground with a velocity u. Now we shall study the characteristics of projectile motion by resolving the motion along horizontal and vertical directions.



	Horizontal direction	**Vertical direction**
(i)	Initial velocity $u_x = u$	Initial velocity $u_y = 0$
(ii)	Acceleration $a_x = 0$	Acceleration $a_y = g$ (downward)

3.1 Time of flight :

This is equal to the time taken by the projectile to return to ground. From equation of motion

$S = ut + \frac{1}{2}at^2$, along vertical direction, we get

$$-h = u_y t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \quad h = \frac{1}{2}gt^2 \quad \Rightarrow \quad \mathbf{t = \sqrt{\frac{2h}{g}}}$$

3.2 Horizontal range :

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

$$R = u_x \, . \, t$$

$$R = u\sqrt{\frac{2h}{g}}$$

3.3 Velocity at a general point P(x, y) :

$$v = \sqrt{u_x^2 + u_y^2}$$

Here horizontal velocity of the projectile after time t

$v_x = u$

velocity of projectile in vertical direction after time t

$v_y = 0 + (-g)t = -gt = gt$ (downward)

$\therefore \quad v = \sqrt{u^2 + g^2t^2}$ and $\tan\theta = v_y/v_x$

3.4 Velocity with which the projectile hits the ground :

$V_x = u$

$V_y^2 = 0^2 - 2g(-h)$

$V_y = \sqrt{2gh}$

$V = \sqrt{V_x^2 + V_y^2} \quad \Rightarrow \mathbf{V = \sqrt{u^2 + 2gh}}$

3.5 Trajectory equation :

The path traced by projectile is called the trajectory.

After time t,

$x = ut$ ....(1)

$y = \frac{-1}{2}gt^2$ ....(2)

From equation (1)

$t = x/u$

Put the value of t in equation (2)

$$\mathbf{y = \frac{-1}{2}g \, . \, \frac{x^2}{u^2}}$$

This is trajectory equation of the particle projected horizontally from some height.

4. PROJECTION FROM A TOWER

Case (i) : Horizontal projection

$u_x = u$; $u_y = 0$; $a_y = -g$

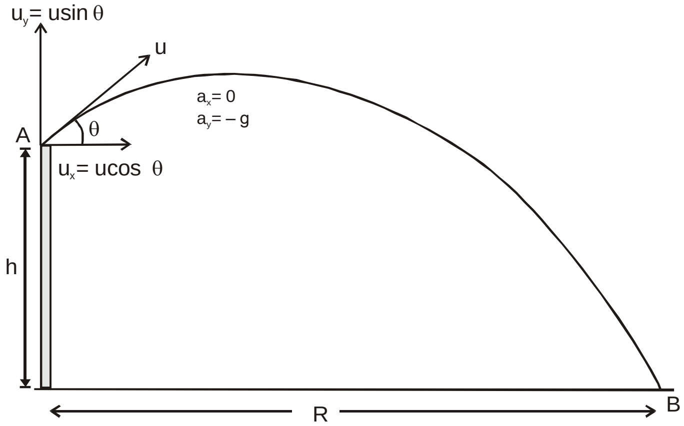
This is same as previous section (section 4)

Case (ii) : Projection at an angle θ above horizontal

$u_x = u\cos\theta$;

$u_y = u\sin\theta$;

$a_y = -g$



Equation of motion between A & B (in Y direction)

$S_y = -h$, $u_y = u\sin\theta$, $a_y = -g$, $t = T$

$$S_y = u_y t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad -h = u\sin\theta\, t - \frac{1}{2} g t^2$$

Solving this equation we will get time of flight, T.

And range, $R = u_x T = u\cos\theta\, T$

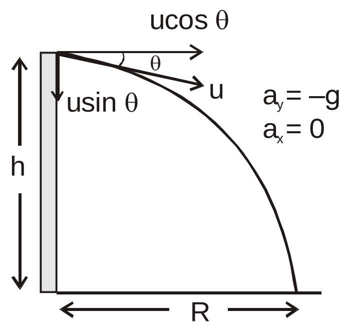
Also, $v_y^2 = u_y^2 + 2a_y S_y$

$= u^2 \sin^2\theta + 2gh$

$v_x = u\cos\theta$

$$v_B = \sqrt{v_y^2 + v_x^2} \quad \Rightarrow \quad \mathbf{v_B = \sqrt{u^2 + 2gh}}$$

Case (iii) : Projection at an angle θ below horizontal



$u_x = u\cos\theta$;

$u_y = -u\sin\theta$;

$a_y = -g$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$S_y = -h$, $u_y = -u\sin\theta$, $t = T$, $a_y = -g$

$$\Rightarrow -h = -u\sin\theta\, T - \frac{1}{2} g T^2 \quad \Rightarrow \quad h = u\sin\theta\, T + \frac{1}{2} g T^2 \;.$$

Solving this equation we will get time of flight, T.

And range, $R = u_x T = u\cos\theta\, T$

$v_x = u\cos\theta$

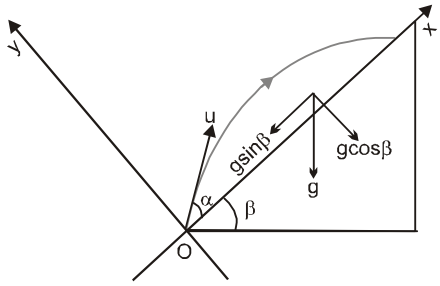
$v_y^2 = u_y^2 + 2a_y S_y = u^2\sin^2\theta + 2(-g)(-h)$

$v_y^2 = u^2\sin^2\theta + 2gh$

$$v_B = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$$

5. PROJECTION ON AN INCLINED PLANE

Case (i) : Particle is projected up the incline



Here α is angle of projection w.r.t. the inclined plane. x and y axis are taken along and perpendicular to the incline as shown in the diagram.

In this case: $a_x = -g\sin\beta$

$u_x = u\cos\alpha$

$a_y = -g\cos\beta$

$u_y = u\sin\alpha$

Time of flight (T) :

When the particle strikes the inclined plane y becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

Maximum height (H) :

When half of the time is elapsed y coordinate is equal to maximum distance from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2_\perp}{2g_\perp}$$

Range along the inclined plane (R):

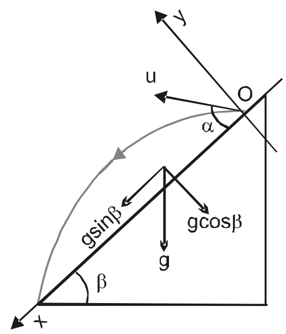
When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$$

Case (ii) : Particle is projected down the incline



In this case :

$a_x = g\sin\beta$

$u_x = u\cos\alpha$

$a_y = -g\cos\beta$

$u_y = u\sin\alpha$

Time of flight (T) :

When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2 \qquad \Rightarrow \qquad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \qquad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H) :

When half of the time is elapsed y coordinate is equal to maximum height of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \qquad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2{}_\perp}{2g_\perp}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \qquad R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \qquad R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$

Standard results for projectile motion on an inclined plane :

	Up the Incline	Down the Incline
Range	$\frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$	$\frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\alpha}{g\cos\beta}$	$\frac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\frac{\pi}{4}-\frac{\beta}{2}$	$\frac{\pi}{4}+\frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

Here α is the angle of projection with the incline and β is the angle of incline.

EXERCISE # 1

PART - I : OBJECTIVE QUESTIONS

** Marked Questions are having more than one correct option.*

Section (A) : Definition, Projectile on a horizontal plane

A-1. A particle move along the parabolic path $x = y^2 + 2y + 2$ in such a way that the y-component of velocity vector remains 5m/s during the motion. The magnitude of the acceleration of the particle is :

(A) 50 m/s^2 (B) 100 m/s^2 (C) $10\sqrt{2}$ m/s^2 (D) 0.1 m/s^2

A-2. If 4 seconds be the time in which a projectile reaches a point P of its path and 5 seconds the time from P till it reaches the horizontal plane through the point of projection . The height of P above the horizontal plane will be - [g = 9.8 m/sec^2]

(A) 98 meters (B) 49 meters (C) 196 meters (D) 147 meters

A-3. During projectile motion, acceleration of a particle at the highest point of its trajectory is :

(A) g (B) zero

(C) less than g (D) dependent upon projection velocity

A-4. In case of an oblique projectile, the velocity is perpendicular to acceleration :

(A) Once only (B) Twice (C) Never (D) Four times

A-5. The velocity of projection of a projectile is $(6\hat{i} + 8\hat{j})$ ms^{-1}. The horizontal range of the projectile is :

(A) 4.9 m (B) 9.6 m (C) 19.6 m (D) 14 m

A-6. A projectile is fired with velocity u making angle θ with the horizontal. What is the change in velocity (from initial) when it is at the highest point :

(A) u $\cos\theta$ (B) u (C) u $\sin\theta$ (D) (u $\cos\theta$ – u)

A-7. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flight in two cases, then the product of two times of flight is :

(A) $t_1t_2 \propto R^2$ (B) $t_1t_2 \propto R$ (C) $t_1t_2 \propto \frac{1}{R}$ (D) $t_1t_2 \propto \frac{1}{R^2}$

A-8.* In case of projectile motion if two projectiles A and B are projected with same speed at angles 15° and 75° respectively to the horizontal then :

(A) $H_A > H_B$ (B) $H_A < H_B$ (C) $T_A > T_B$ (D) $T_A < T_B$

A-9. Four bodies P, Q, R and S are projected with equal velocities having angles of projection 15°, 30°, 45° and 60° with the horizontal respectively. The body having shortest range is :

(A) P (B) Q (C) R (D) S

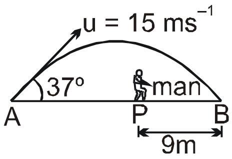
A-10. If for a given angle of projection, the horizontal range is doubled, the time of flight becomes :

(A) 4 times (B) 2 times (C) $\sqrt{2}$ times (D) $1/\sqrt{2}$ times

A-11. A projectile is fired with a speed u at an angle θ with the horizontal. Its speed when its direction of motion makes an angle 'α' with the horizontal is :

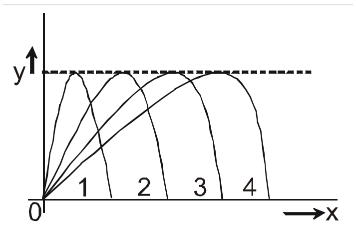
(A) u $\sec\theta \cos\alpha$ (B) u $\sec\theta \sin\alpha$ (C) u $\cos\theta \sec\alpha$ (D) u $\sin\theta \sec\alpha$

A-12. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.



(A) 3 ms^{-1} (B) 5 ms^{-1} (C) 9 ms^{-1} (D) 12 ms^{-1}

A-13. Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first :



(A) 1, 2, 3, 4 (B) 2, 3, 4, 1 (C) 3, 4, 1, 2 (D) 4, 3, 2, 1

A-14. The velocity at the maximum height of a projectile is half of its initial velocity u. Its range on the horizontal plane is:

(A) $\frac{2u^2}{3g}$ (B) $\frac{\sqrt{3}u^2}{2g}$ (C) $\frac{u^2}{3g}$ (D) $\frac{u^2}{2g}$

A-15. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target in the same horizontal plane. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target.

(A) $\frac{5\pi}{36}$ rad (B) $\frac{11\pi}{36}$ rad (C) $\frac{7\pi}{36}$ rad (D) $\frac{13\pi}{36}$ rad.

A-16. A particle moves along the parabolic path $y = ax^2$ in such a way that the x component of the velocity remains constant, say c. The acceleration of the particle is :

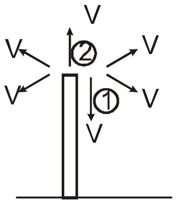
(A) $ac\hat{k}$ (B) $2ac^2\hat{j}$ (C) $ac^2\hat{j}$ (D) $a^2c\hat{j}$

Section (B) : Projectile from a tower

B-1.* One stone is projected horizontally from a 20 m high cliff with an initial speed of 10 ms^{-1}. A second stone is simultaneously dropped from that cliff. Which of the following is true?

(A) Both strike the ground with the same speed.
(B) The ball with initial speed 10 ms^{-1} reaches the ground first.
(C) Both the balls hit the ground at the same time.
(D) Both strike the ground with different speed

B-2.* Particles are projected from the top of a tower with same speed at different angles as shown. Which of the following are True ?



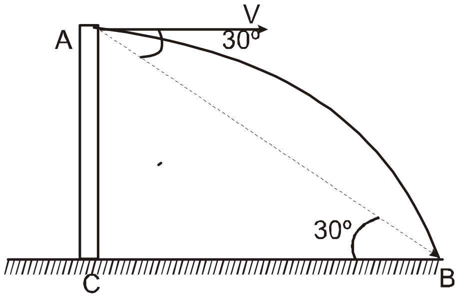
(A) All the particles would strike the ground with (same) speed.

(B) All the particles would strike the ground with (same) speed simultaneously.

(C) Particle 1 will be the first to strike the ground.

(D) Particle 1 strikes the ground with maximum speed.

B-3. An object is thrown horizontally from a point 'A' from a tower and hits the ground 3s later at B. The line from 'A' to 'B' makes an angle of 30° with the horizontal. The initial velocity of the object is : (take g = 10 m/s^2)



(A) $15\sqrt{3}$ m/s (B) 15 m/s (C) $10\sqrt{3}$ m/s (D) $25/\sqrt{3}$ m/s

B-4. A body of mass m is projected horizontally with a velocity v from the top of a tower of height h and it reaches the ground at a distance x from the foot of the tower. If a second body of mass 2m is projected horizontally from the top of a tower of height 2h, it reaches the ground at a distance 2x from the foot of the tower. The horizontal velocity of the second body is :

(A) v (B) 2v (C) $\sqrt{2}\,v$ (D) $\frac{v}{2}$

B-5.* A heavy stone is thrown from a cliff of height h in a given direction. The speed with which it hits the ground:

(A) must depend on the speed of projection

(B) must be larger than the speed of projection

(C) must be independent of the speed of projection

(D) may by smaller than the speed of projection

Section (C) : Equation of trajectory

C-1. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/sec. The cartesian equation of its path is : (g = 10 m/s^2)

(A) $y = 2x - 5x^2$ (B) $y = x - 5x^2$ (C) $4y = 2x - 5x^2$ (D) $y = 2x - 25x^2$

C-2. The equation of projectile is $y = 16x - \frac{5x^2}{4}$. The horizontal range is :

(A) 16 m (B) 8 m (C) 3.2 m (D) 12.8 m

C-3. A ball is projected so as to pass a wall at a distance a from the point of projection at an angle of 45° and falls at a distance b on the other side of the wall. If h is height of wall then :

(A) $h = a\sqrt{2}$ (B) $h = b\sqrt{2}$ (C) $h = \frac{\sqrt{2}\ ab}{a+b}$ (D) $h = \frac{ab}{a+b}$

C-4.* A particle moves in the xy plane with a constant acceleration 'g' in the negative y-direction. Its equation of motion is $y = ax - bx^2$, where a and b are constants. Which of the following are correct?

(A) The x-component of its velocity is constant.

(B) At the origin, the y-component of its velocity is $a\sqrt{\frac{g}{2b}}$.

(C) At the origin, its velocity makes an angle $\tan^{-1}(A)$ with the x-axis.

(D) The particle moves exactly like a projectile.

C-5. The path of one projectile as seen by an observer on another projectile is a/an :

(A) straight line (B) parabola (C) ellipse (D) circle

Section (D) : Projectile on an inclined plane

D-1. Find time of flight of projectile thrown horizontally with speed 10 ms^{-1} from a long inclined plane which makes an angle of $\theta = 45°$ from horizontal :

(A) $\sqrt{2}$ sec (B) $2\sqrt{2}$ sec (C) 2 sec (D) None of these

D-2. On an inclined plane of inclination 30º, a ball is thrown at an angle of 60º with the horizontal from the foot of the incline with a velocity of $10\sqrt{3}$ ms^{-1}. If $g = 10$ ms^{-2}, then the time in which ball will hit the inclined plane is :

(A) 1 sec. (B) 6 sec. (C) 2 sec. (D) 4 sec.

PART - II : MISCELLANEOUS OBJECTIVE QUESTIONS

COMPREHENSION

Comprehensions # 1 :

The trajectory of a projectile in a vertical plane is $y = \sqrt{3}\, x - 2x^2$. [$g = 10$ m/s^2]

1. Angle of projection θ is :

(A) 30° (B) 60° (C) 45° (D) $\sqrt{3}$ rad

2. Maximum height is H :

(A) $\frac{8}{3}$ (B) $\frac{3}{8}$ (C) $\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$

3. Range OA is :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\sqrt{3}$ (D) $\frac{3}{8}$

4. Time of flight of the projectile is :

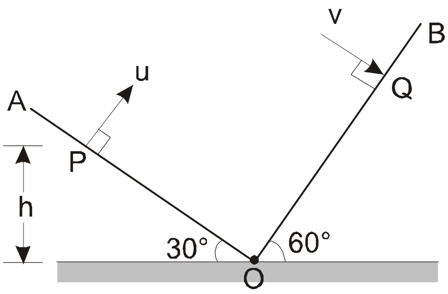
(A) $\sqrt{\frac{3}{10}}$ sec (B) $\sqrt{\frac{10}{3}}$ sec (C) 1 sec (D) 2 sec

5. Radius of curvature of the path of the porjectile at the topmost point P is :

(A) $\frac{1}{2}$ m (B) 1 m (C) 4 m (D) $\frac{1}{4}$ m

Comprehensions # 2 :

Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity u = $10\sqrt{3}\ \frac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q (Take g = 10 m/s^2). Then :



6. The time of flight is from P to Q is :
(A) 5 Sec. (B) 2 sec (C) 1 sec (D) None of these

7. The speed with which the particle strikes the plane OB is :
(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

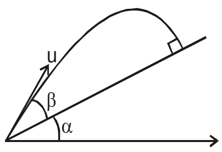
8. The height h of point P from the ground is :
(A) $10\sqrt{3}$ m (B) 10 m (C) 5 m (D) 20 m

9. The distance PQ is :
(A) 20 m (B) $10\sqrt{3}$ m (C) 10 m (D) 5 m

MATCH THE COLUMN

10. Match the following.

The projectile collides perpendicularly with the inclined plane. (Refer the figure)

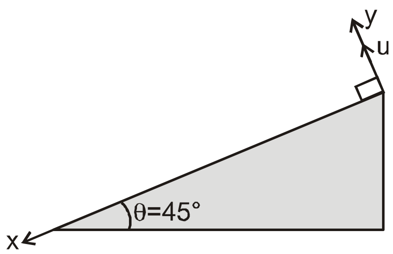


Column I	**Column II**
(A) Maximum height attained by the projectile from the ground	(p) zero
(B) Maximum height attained by the projectile from Inclined plane	(q) g
(C) Acceleration of the projectile before striking the inclined plane	(r) $\frac{u^2 \sin^2 \beta}{2\ g\cos\alpha}$
(D) Horizontal component of acceleration of the projectile.	(s) $\frac{u^2 \sin^2(\alpha+\beta)}{2\ g}$

11. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in column-I with the values given in column-II. (g in column II is acceleration due to gravity) :

Column I	Column II
(A) Horizontal component of velocity	(p) $\frac{a}{b}$
(B) Time of flight	(q) $\frac{a^2}{4b}$
(C) Maximum height	(r) $\sqrt{\frac{g}{2b}}$
(D) Horizontal range	(s) $\sqrt{\frac{2a^2}{bg}}$

12. An inclined plane makes an angle $\theta = 45°$ with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at t = 0 sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in column I with the results in column II. (g in column II is acceleration due to gravity.)



Column I	Column II
(A) The instant of time at which velocity of stone is parallel to x-axis	(p) $\frac{2\sqrt{2}u}{g}$
(B) The instant of time at which velocity of stone makes an angle $\theta = 45°$ with positive x-axis.	(q) $\frac{2u}{g}$
(C) The instant of time till which (starting from t = 0) component of displacement along x-axis become half the range on inclined plane is	(r) $\frac{\sqrt{2}u}{g}$
(D) Time of flight on inclined plane is	(s) $\frac{u}{\sqrt{2}g}$

EXERCISE # 2

PART - I : MIXED OBJECTIVE

*** Marked Questions are having more than one correct option.***

Single Correct Answer Type :

1. A large number of bullets are fired in all directions with same speed *u*. What is the maximum area on the ground on which these bullets will spread :

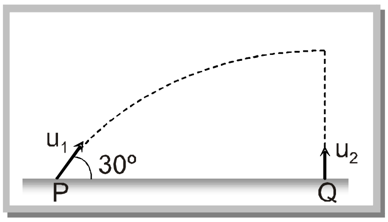
(A) $\pi \frac{u^2}{g}$ (B) $\pi \frac{u^4}{g^2}$ (C) $\pi^2 \frac{u^4}{g^2}$ (D) $\pi^2 \frac{u^2}{g^2}$

2. Two projectiles *A* and *B* are projected with same speed at angles 30° and 60° to horizontal, then choose the wrong statement? (symbols have their usual meaning).

(A) $R_A = R_B$ (B) $H_B = 3H_A$ (C) $\sqrt{3}\, T_B = T_A$ (D) All of these

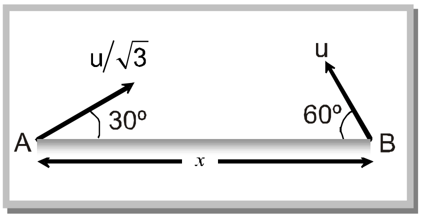
3. Two projectiles are thrown simultaneously in the same plane from the same point, their velocities are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal.
If $v_1 \cos \theta_1 = v_2 \cos\theta_2$, then choose the incorrect statement :
(A) one particle will remain exactly below or above the other particle
(B) the trajectory of one with respect to other will be a vertical straight line
(C) both will have the same range
(D) none of these

4. A particle P is projected with velocity u_1 at an angle of 30º with the horizontal. Another particle Q is thrown vertically upwards with velocity u_2 from a point vertically below the highest point of path of P. The necessary condition for the two particles to collide at the highest point is :



(A) $u_1 = u_2$ (B) $u_1 = 2u_2$ (C) $u_1 = \frac{u_2}{2}$ (D) $u_1 = 4u_2$

5. Two particles are separated at a horizontal distance *x* as shown in figure. They are projected at the same time as shown in figure with different initial speed. The time after which the horizontal distance between the particles become zero is :



(A) u/2x (B) x/u (C) 2u/x (D) u/x

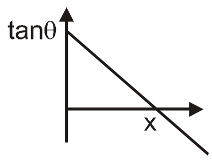
6. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a, b are constants, x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. The maximum height attained is :

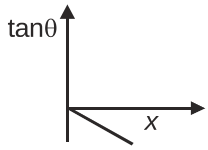
(A) $\frac{a^2}{4b}$ (B) $\frac{a^2}{8b}$ (C) $\frac{a^2}{8b}$ (D) $\frac{a^2}{2gb}$

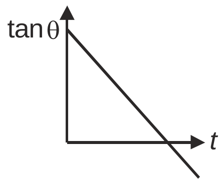
7. Particle is dropped from the height of 20m from horizontal ground. A constant force acts on the particle in horizontal direction due to which horizontal acceleration of the particle becomes 6 ms^{-2}. Find the horizontal displacement of the particle till it reaches ground.

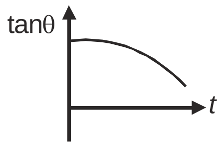
(A) 6 m (B) 10 m (C) 12 m (D) 24 m

8. A particle is dropped from a tower in a uniform gravitational field at $t = 0$. The particle is blown over by a horizontal wind with constant velocity. Slope of trajectory of particle ($\tan\theta$) with horizontal varies according to:

(A) 

(B) 

(C) 

(D) 

9. A projectile is fired with a velocity at right angle to the slope which is inclined at an angle q with the horizontal. The expression for the range R along the incline is :

(A) $\frac{2v^2}{g}\sec\theta$ (B) $\frac{2v^2}{g}\tan\theta$ (C) $\frac{2v^2}{g}\tan\theta\sec\theta$ (D) $\frac{v^2}{g}\tan^2\theta$

Multiple Correct Answer(s) Type :

10.* Two particles are projected from the same point with the same speed, at different angles θ_1 and θ_2 to the horizontal. They have the same horizontal range. Their time of flight are t_1 and t_2 respectively. Then :

(A) $\theta_1 + \theta_2 = 90°$ (B) $\frac{t_1}{t_2} = \tan\theta_1$ (C) $\frac{t_1}{t_2} = \tan\theta_2$ (D) $\frac{t_1}{\sin\theta_1} = \frac{t_2}{\sin\theta_2}$

11.* A projectile is projected at an angle of elevation α. After t seconds, it appears to have an angle of elevation β as seen from the point of projection. Which of the following are correct :

(A) the y-coordinate of the position of the projectile at time t is $(v\sin\alpha)\ t - \frac{1}{2}gt^2$, where v is the velocity of projection

(B) the x-coordinate of the position of projectile at time t is $(v\cos\alpha)\ t$, where v is the velocity of projection

(C) $\tan\beta = \tan\alpha - \frac{gt}{2v\cos\alpha}$

(D) velocity of projection is $\frac{gt\cos\beta}{2\sin(\alpha-\beta)}$

12.* A cannon ball has a range R on a horizontal plane. h and h' are the greatest heights in the two paths for which this is possible. Now, which of the following are correct :

(A) there are two possible angles of projection θ and $(90° - \theta)$ for which the horizontal range is R

(B) if $h = \frac{v^2\sin^2\theta}{2g}$ then $h' = \frac{v^2\cos^2\theta}{2g}$

(C) $R = \frac{v^2\cos 2\theta}{g}$

(D) $R = 4\sqrt{hh'}$

13.* Choose the correct alternative (s) :

(A) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is 2h.

(B) The angle of projection for a projectile motion whose range R is n times the maximum height is $\tan^{-1}(4/n)$

(C) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R\tan q$ where q is the angle of projection.

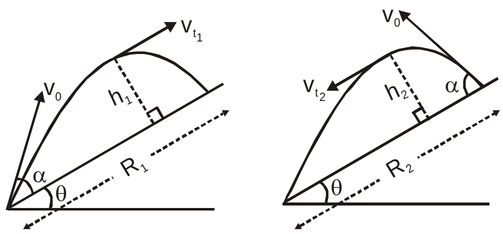
(D) A ball is thrown vertically up. Another ball is thrown at an angle q with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two balls 1 : 1.

14.* If T is the total time of flight, h is the maximum height & R is the range for horizontal motion, the x & y co-ordinates of projectile motion and time t are related as :

(A) $y = 4h\left(\frac{t}{T}\right)\left(1-\frac{t}{T}\right)$ (B) $y = 4h\left(\frac{X}{R}\right)\left(1-\frac{X}{R}\right)$

(C) $y = 4h\left(\frac{T}{t}\right)\left(1-\frac{T}{t}\right)$ (D) $y = 4h\left(\frac{R}{X}\right)\left(1-\frac{R}{X}\right)$

15.* Two balls are thrown from an inclined plane at angle of projection α with the plane, one up the inclined and other down the inclined as shown in figure. Then :



(A) $h_1 = h_2 = \frac{v_0^2 \sin^2\alpha}{2g\cos\theta}$

(B) $T_1 = T_2 \frac{2v_0 \sin\alpha}{g\cos\theta}$

(C) $R_2 - R_1 = g\sin\theta \,.\, T^2$ (where T represents time of flight)

(D) $vt_2 = vt_1$

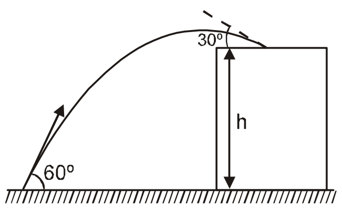
16.* A projectile is projected at an angle α (> 45°) with an initial velocity u. The time t at which its horizontal component will equal the vertical component in magnitude:

(A) $t = \frac{u}{g}(\cos\alpha - \sin\alpha)$ (B) $t = \frac{u}{g}(\cos\alpha + \sin\alpha)$

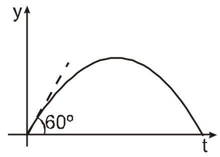
(C) $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$ (D) $t = \frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

PART - II : SUBJECTIVE QUESTIONS

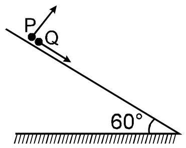
1. A stone projected at an angle of 60° from the ground level strikes at an angle of 30° on the roof of a building of height 'h= 30m '. Find the speed of projection(in m/s) of the stone.



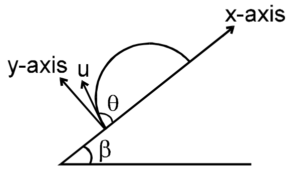
2. A particle is projected from the horizontal x-z plane, in vertical x-y plane where x-axis is horizontal and positive y-axis vertically upwards. The graph of 'y' coordinate of the particle v/s time is as shown. The range of the particle is $\sqrt{3}$ m . Then the speed of the projected particle is $\sqrt{20+x}$ in m/s . find x :



3. A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide after t = 4 second. The speed of projection of P is 5+ x (in m/s). find x .

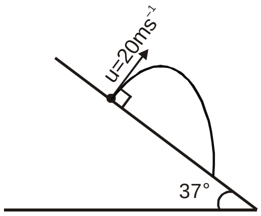


4. A particle is projected at an angle θ with an inclined plane making an angle β with the horizontal as shown in figure, speed of the particle is u, after time t find :

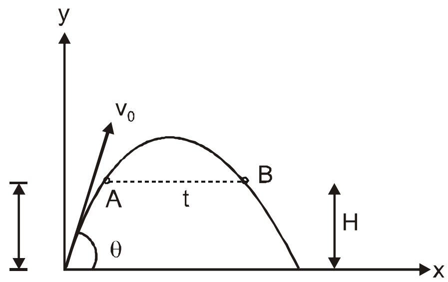


(a) x component of acceleration ?
(b) y component of acceleration ?
(c) x component of velocity ?
(d) y component of velocity ?
(e) x component of displacement ?
(f) y component of displacement ?
(g) y component of velocity when particle is at maximum distance from the incline plane ?

5. Find range of projectile on the inclined plane which is projected perpendicular to the incline plane with velocity 20 m/s as shown in figure.



6. The figure shows two position A and B at the same height h above the ground. If the maximum height of the projectile is H, Then determine the time t elapsed between the positions A and B in terms of H.



7. A particle is projected with velocity 20 m/s at an angle of 45° with horizontal. Find out the -

(a) horizontal Range [take $g = 10\ m/s^2$]

(b) Maximum Height

(c) Time of flight

(d) For which angle is T maximum, $T_{max} = ?$

(e) For which angle is R maximum, $R_{max.} = ?$

8. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high should above the ground can the cricketer throw the same ball.

9. The direction of motion of a projectile at a certain instant is inclined at an angle α to the horizontal. After t seconds it is inclined an angle β . Find the horizontal component of velocity of projection in terms of g, t , α and β.

10. A gun kept on a straight horizontal road is used to hit a car, traveling along the same road away from the gun with a uniform speed of $72 \times \sqrt{2}$ km/hour. The car is at a distance of 50 metre from the gun, when the gun is fired at an angle of 45° with the horizontal.

Find (i) the distance of the car from the gun when the shell hits it, (ii) the speed of projection of the shell from the gun. [$g = 10\ m/s^2$]

11. The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$, find the angle of projection. Also find the speed of projection. Where at t = 0, x = 0 and y = 0 also $\frac{d^2x}{dt^2} = 0$ & $\frac{d^2y}{dt^2} = -g$.

12. If at an instant the velocity of a projectile be u and its inclination to the horizontal be θ, at what time interval after that instant will the particle be moving at right angles to its former direction.

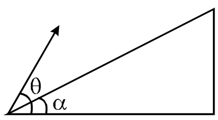
13. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m/sec at an angle of 30° to the horizontal. When and at what distance from the foot of the tower does the ball hit the ground ? What is the speed of the ball at this instant ?

14. The radius vector of a point A relative to the origin varies with time t as $\vec{r} = at\,\hat{i} - bt^2\,\hat{j}$, where a and b are positive constants and $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find:

(i) The equation of the point's trajectory y (x) ; plot this function

(ii)The time dependence of the velocity **v** and acceleration **a** vectors as well as of the moduli of these quantities.

15. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane inclination α as shown in figure.



EXERCISE # 3

PART-I IIT-JEE (PREVIOUS YEARS PROBLEMS)

*** Marked Questions are having more than one correct option.**

1. A building 4.8 m high 2b meters wide has a flat roof. A ball is projected from a point on the horizontal ground 14.4 m away from the building along its width. If projected with velocity 16 m/s at an angle of 45° with the ground, the ball hits the roof in the middle, find the width 2b. Also find the angle of projection so that the ball just crosses the roof if projected with velocity 10 $\sqrt{3}$ m/s.(g=10m/s^2) **[REE '95, 6]**

2. A vertical pole has a red mark at some height. A stone is projected from a fixed point on the ground. When projected at an angle of 45° it hits the pole orthogonally 1 m above the mark. When projected with a different velocity at an angle of $\tan^{-1}(3/4)$, it hits the pole orthogonally 1.5 m below the mark. Find the velocity and angle of projection so that it hits the mark orthogonally to the pole. [g = 10 m/sec^2] **[REE '96, 6]**

3.* The coordinates of a particle moving in a plane are given by x (t) = a cos (p t) and y (t) = b sin (pt), where a, b (< a) and p are positive constants of appropriate dimensions then -
(A) the path of the particle is an ellipse
(B) the velocity and acceleration of the particle are normal to each other at t = π/2p
(C) the acceleration of the particle is always directed towards a focus
(D) the distance travelled by the particle in time interval t = 0 to = π/2p is a. **[JEE 1999, 3/200]**

4. Shots fired simultaneously from the top and foot of a vertical cliff at elevations of 30^0 and 60^0 respectively, strike an object simultaneously which is at a height of 100 meters from the ground and at a horizontal distance of 200 $\sqrt{3}$ meters from the cliff. Find the height of the cliff, the velocities of projection of the shots and the time taken by the shots to hit the object. (g = 10 m/sec^2.) **[REE-2000, 5/100]**

PART-II AIEEE (PREVIOUS YEARS PROBLEMS)

1. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be : **[AIEEE-2002, 4/300]**
(1) E (2) E/$\sqrt{2}$ (3) E/2 (4) zero

2. A body playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?
[g = 10 m/s^2, sin 30° = 1/2, cos 30° = $\frac{\sqrt{3}}{2}$] **[AIEEE-2002, 4/300]**
(1) 5.20 m (2) 4.33 m (3) 2.60 m (4) 8.66 m

3. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flights in the two cases, then the product of the two times of flights is directly proportional to : **[AIEEE-2004, 05, 4/300]**
(1) 1/R^2 (2) 1/R (3) R (4) R^2

4. A ball is thrown from a point with a speed v_0 at angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball? If yes, what should be the angle of projection? **[AIEEE-2004, 4/300]**
(1) Yes, 60° (2) Yes, 30° (3) No (4) Yes, 45°

5. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE-2007, 3/120]**
(1) K (2) Zero (3) K/4 (4) K/2

6. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is :
(1) 7$\sqrt{2}$ units (2) 7 units (3) 8.5 units (4) 10 units
[AIEEE-2009, 4/144]

EXERCISE # 4

BOARD PATTERN QUESTIONS

1. State, for each of the following physical quantities, if it is a scalar or a vector :
volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

2. Pick out the two scalar quantities in the following list :
force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, reaction as per Newton's third law, relative velocity.

3. Pick out the only vector quantity in the following list :
Temperature, pressure, impulse, time, power , total path lengthy, energy, gravitational potential, coefficient of friction, charge.

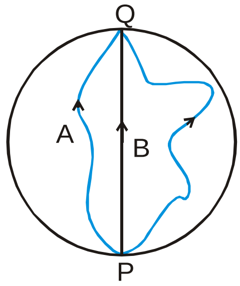
4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :
(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions,
(c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

5. Read each statement below carefully and state with reasons, if it is true or false :
(a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle. (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) Three vectors not lying in a plane can never add up to give a null vector.

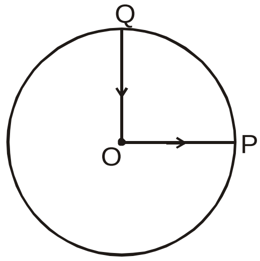
6. Establish the following vector inequalities geometrically or otherwise :
(a) $|a + b| \leq |a| + |b|$ (b) $|a + b| \geq |a| - |b|$
(c) $|a - b| \leq |a| + |b|$ (d) $|a - b| \geq |a| - |b|$
When does the equality sign above apply ?

7. Given a + b + c + d = 0, which of the following statements are correct :
(a) a, b, c and d must each be a null vector,
(b) The magnitude of (a + c) equals the magnitude of (b + d),
(c) The magnitude of a can never be greater than the sum of the magnitudes of b, c and d,
(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear ?

8. The girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure 4.23. What is the magnitude of the displacement vector for each ? For which girls is this equal to the actual length of path skate ?



9. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along OQ as shown in figure 4.24. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist ?



10. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eight turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity ? Are the two equal ?

12. Rain is falling vertically with a sped of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella ?

13. A man can swim with a sped of 4 km/h in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km/h and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank ?

14. In a harbour, wind is blowing at he speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a sped of 51 km/h to the north, what is the direction of the flag on the mast of the boat ?

15. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall ?

16. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

17. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?

18. An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900km/h. Compare its centripetal acceleration with the acceleration due to gravity.

19. Read each statement below carefully and state, with reasons, if it is rue or false :
(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre
(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point
(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

20. The position of a particle is given by

$$\vec{r} = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}\,m$$

where t is in seconds and the coefficients have the proper units for r to be in metres.
(a) Find the v and a of the particle ? (b) What is the magnitude and direction of velocity of the particle at t = 2 s ?

21. A particle starts from the origin at t = 0 s with a velocity of 10.0 $\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})$ m s^{-2}. (a) At what time is the x-coordinate of the particle 16 m ? What is the y-coordinate of the particle at that time ? (b) What is the speed of the particle at the time ?

22. $\hat{i}$ and $\hat{j}$ are unit vector along x- and y-axis respectively. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$, and $\hat{i}-\hat{j}$? What are the components of a vector A = $2\hat{i}+3\hat{j}$ along the directions of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

23. For any arbitrary motion in space, which of the following relations are true :
(a) $V_{average} = (1/2)\,(v\,(t_1) + v(t_2))$
(b) $V_{average} = [\,r\,(t_2) - r\,(t_1)]\,/\,(t_2 - t_1)$
(c) V (t) = v (0) at
(d) $r(t) = r(0) + v(0)\,t + (1/2)\,a\,t^2$
(e) $a_{average} = [v\,(t_2) - v\,(t_1)]\,/\,(t_2 - t_1)$
(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

24. Which of the following quantities are independent of the choice of orientation of the coordinate axes ?
a + b, $3a_x + 2b_y$, |a + b – c|, angle between b and c, a where is a scalar ?

25. Read each statement below carefully and state, with reasons and examples, if it is true or false :
A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
(c) must be dimensionless
(d) does not vary from one point to another in space
(e) has the same value for observers with different orientations of axes.

26. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft ?

27. A vector has magnitude and direction. Does it have a location in space ? Can it vary with time ? Will two equal vectors a and b at different locations in space necessarily have identical physical effects ? Give examples in support of your answer.

28. A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector ?

29. Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere ? Explain.

30. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away ? Assume the muzzle speed to the fixed, and neglect air resistance.

31. Amachine gun is mounted on the top of a tower 100 m high. At what angle should the gun be inclined to cover a maximum range of firing on the ground below ? the muzzle speed of the bullet is 150 ms^{-1} (take g = 10 m s^{-2})

32. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane. At what minimum altitude should the pilot fly the plane to avoid being hit ? (Take g = 10 m s^{-2}).

33. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.5 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?

34. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta\ (t)\ \tan^{-1}\ 2\hat{i}-\hat{j}+4.0\hat{k}\,m\left(\frac{v_{0y}}{v_{ax}}\right)$$

(b) Shows that the projection angle θ_0 for a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

ANSWERS

Exercise # 1

PART-I

A-1.	(A)	**A-2.**	(A)	**A-3.**	(A)	**A-4.**	(A)	**A-5.**	(B)	**A-6.**	(C)	**A-7.**	(B)
A-8.*	(B)	**A-9.**	(A)	**A-10.**	(C)	**A-11.**	(C)	**A-12.**	(B)	**A-13.**	(D)	**A-14.**	(B)
A-15.	(D)	**A-16.**	(B)	**B-1.***	(CD)	**B-2.***	(AC)	**B-3.**	(A)	**B-4.**	(C)	**B-5.***	(AB)
C-1.	(A)	**C-2.**	(D)	**C-3.**	(D)	**C-4.***	(ABCD)	**C-5.**	(A)	**D-1.**	(C)	**D-2.**	(C)

PART-II

1.	(B)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(D)	**6.**	(B)	**7.**	(A)

8. (C) **9.** (A) **10.** (A)–s ; (B)–r ; (C)–q ; (D)–p **11.** (A)–r ; (B)–s ; (C)–q ; (D)–p

12. (A) – r ; (B) – s ; (C) – q ; (D) – p

Exercise # 2

PART-I

1.	(B)	**2.**	(C)	**3.**	(C)	**4.**	(B)	**5.**	(B)	**6.**	(A)	**7.**	(C)
8.	(B)	**9.**	(C)	**10.***	(ABD)	**11.***	(ABCD)	**12.***	(ABD)	**13.***	(ABCD)	**14.***	(AC)
15.*	(ABC)	**16.***	(BC)										

PART-II

1. 30 **2.** 8 **3.** 5

4. (a) $-g\sin\beta$, (b) $-g\cos\beta$, (c) $u\cos\theta - g\sin\beta \times t$, (d) $u\sin\theta - g\cos\beta \times t$,

(e) $u\cos\theta \times t - \frac{1}{2}g\sin\beta \times t^2$, (f) $u\sin\theta \times t - \frac{1}{2}g\cos\beta \times t^2$, (g) zero.

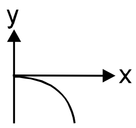
5. 75 m **6.** $t = \sqrt{\frac{8}{g}(H-h)}$

7. (a) 40 m (b) 10 m (c) $2\sqrt{2}$ sec. (d) $\theta = 90°$, 4 sec. (e) $\theta = 45°$, 40 m

8. 50 m **9.** $\frac{gt}{\tan\alpha - \tan\beta}$ **10.** (i) 1250 m (ii) 50 m/sec.

11. $\theta = 60°$, 2 m/s **12.** $(u/g)\ \text{cosec}\,\theta$. **13.** 4 s, $40\sqrt{3}$ m, $20\sqrt{3}$ m/s

14. (i) $y = -\frac{bx^2}{a^2}$



(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$, acceleration $= -2b\hat{j}$,

$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, |acceleration | = 2 b

15. $\cot(\theta - \alpha) = 2\tan\alpha$

Exercise # 3

PART-I

1. width of the roof is 9.6 m; $\theta = \tan^{-1}\frac{3}{2}$ or $\theta = 45°$

2. $\frac{\sqrt{3620}}{3}$ m/s , $\tan^{-1}\left(\frac{9}{10}\right)$

3.* (AB)

4. 400 m, $V_T = 40$ m/s , $V_F = 40\sqrt{3}$ m/s, T = 10 s.

PART-II

1. (3) **2.** (4) **3.** (3) **4.** (1) **5.** (3) **6.** (1)

Exercise # 4

1. Volume, mass, speed, density, number of moles, angular frequency are scalars; the rest are vectors.

2. Work, current

3. Impulse

4. Only (c) and (d) are permissible

5. (a) T, (b) F, (c) F, (d) T, (e) T

6. Hint : The sum (difference) of any two sides of a triangle is never less (greater) than the third side. Equality holds for collinear vectors.

7. All statements except (a) are correct

8. 400 m for each; B

9. (a) O; (b) O; (c) 21.4 km h^{-1}

10. Displacement of magnitude 1 km and direction 60° with the initial direction; total path length = 1.5 km (third turn); null displacement vector; path length = 3 km sixth turn); 866 m, 30°, 4 km (eight turn)

11. (a) 49.3 km h^{-1} ; (b) 21.4 km h^{-1}. No, the average speed equals average velocity magnitude only for a straight path.

12. About 18° with the vertical, towards the south.

13. 15 min, 750 m

14. East (approximately)

15. 150.5 m

16. 50 m

17. 9.9 m s^{-2}, along the radius at every point towards the centre.

18. 6.4 g

19. (a) False (true only for uniform circular motion) (b) True, (c) True

20. (a) $v(t) = (3.0\,\hat{i} - 4.0\,\hat{j})$ $\hat{a}(t) = -4.0\,\hat{j}$ (b) 8.54 m s^{-1}, 70° with x-axis.

21. (a) 2s, 24 m, 21.26 m s^{-1}

22. $\sqrt{2}$, 45° with the x-axis; $\sqrt{2}$, – 45° with the x-axis, $(5/\sqrt{2}, -1/\sqrt{2})$.

23. (b) and (e)

24. All except $3a_x + 2b_y$

25. Only (e) is true

26. 182.2 m s^{-1}

28. No. Rotations in general cannot be associated with vectors

29. A vector can be associated with a plane area

30. No

31. 43.8°

32. At an angle if $\sin^{-1}(1/3) = 19.5°$ with the vertical; 16 km

33. 0.86 m s^{-2}, 54.5° with the direction of velocity.