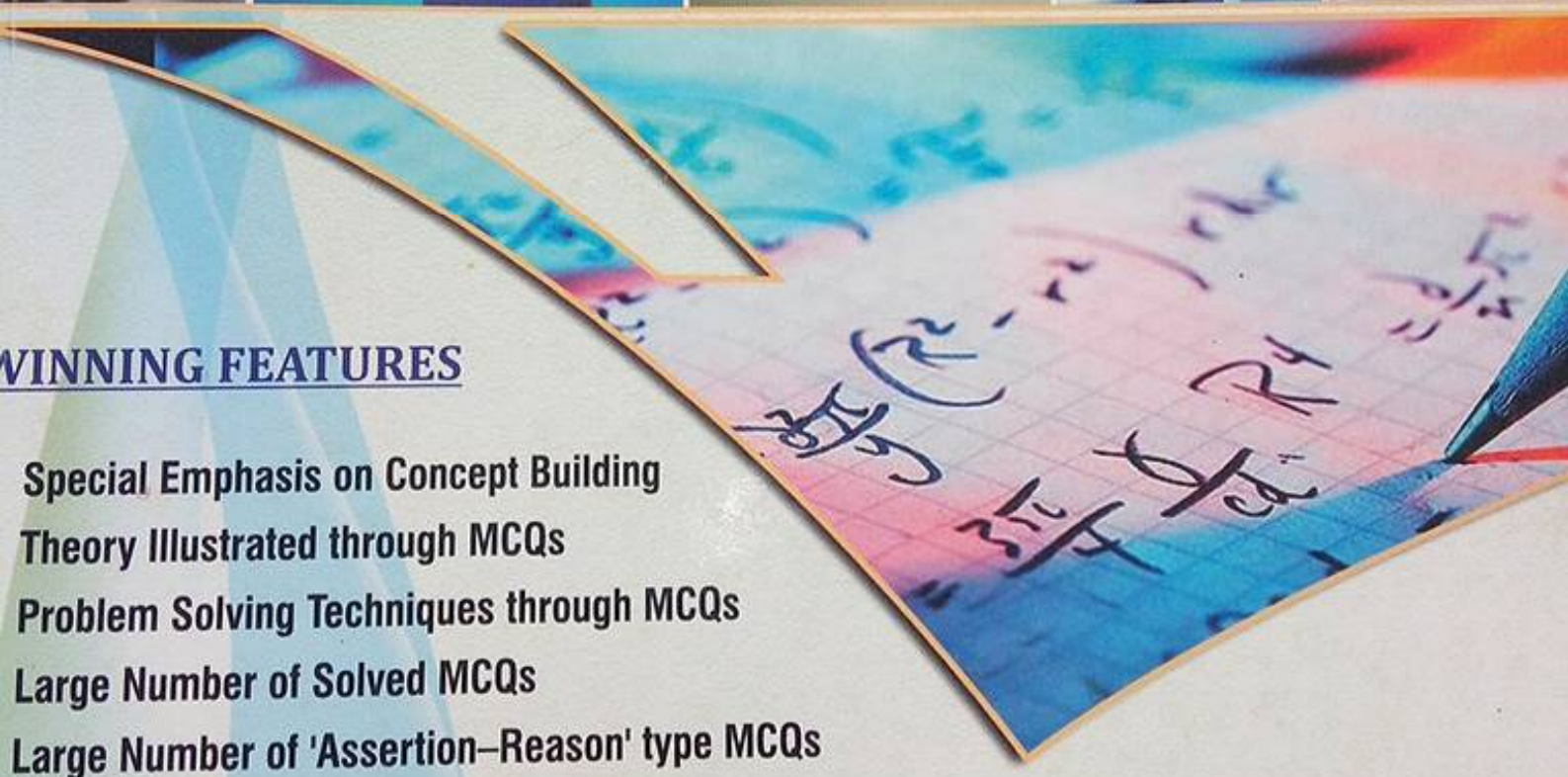
OBJECTIVE MATHEMATICS

Volume I

for JEE (Main & Advanced) & other Engineering Entrance Examinations

R.D. SHARMA





WINNING FEATURES

- Special Emphasis on Concept Building
- Theory Illustrated through MCQs
- Problem Solving Techniques through MCQs
- Large Number of Solved MCQs
- Large Number of 'Assertion–Reason' type MCQs
- Exercises containing MCQs
- Chapter–Test at the end of each Chapter

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OBJECTIVE MATHEMATICS

VOLUME - I

for

JEE (Main & Advanced) *& other*
Engineering Entrance Examinations

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ALGEBRAIC INEQUATIONS

1. INEQUATIONS

A statement involving variable (s) and the sign of inequality viz., $>$ or, $<$, or, $\geq$ or, $\leq$ is called an inequation.

An inequation may contain one or more variables. Also, it may be linear or quadratic or cubic etc.

If a is a non-zero real number and x, y are variables, then $ax+b<0$, $ax+b\leq 0$, $ay+b>0$ and $ay+b\geq 0$ are linear inequations in one variable.

If $a\neq 0, b\neq 0, c$ real numbers and x, y, z are variables, then $ax+by<c$, $ay+bz\leq c$, $ax+by>c$ and $ax+by\geq c$ are linear inequations in two variables.

Inequations $2x-3<0$, $3y\geq 5$, $\frac{x}{2}-7\leq 0$, $\frac{2x-3}{4}\geq -\frac{x}{2}+7$ and $-2+t\leq\frac{5}{3}$ are linear inequations in one variable. $2x-3y<6$, $x+y\geq -4$ are linear inequations in two variables. Inequations of the form $\frac{2x-3}{x-1}<4$, $x^2-5x+6\geq 0$ etc are not linear inequations.

SOLUTION *A solution of an inequation is the value (s) of the variable (s) that makes it a true statement.*

For example, $x=9$ is a solution of the inequation $\frac{3-2x}{5}<\frac{x}{3}-4$, because for $x=9$ it reduces to $-3<-1$ which is a true statement. But, $x=6$ is not its solution. For $x=6$, the inequation reduces to $-\frac{9}{5}<-2$ which is not true. For any real number x, we have $x^2+1>0$. So, the solution set of the inequation $x^2+1>0$ is the set R of all real numbers and the solution set of $x^2+1<0$ is the null set ϕ.

For solving an inequation, we use the following rules:

RULE 1 *Same number may be added to (or subtracted from) both sides of an inequation without changing the sign of inequality.*

RULE 2 *Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However, the sign of inequality is reversed when both sides of an inequation are multiplied or divided by a negative number.*

RULE 3 *Any term of an inequation may be taken to the other side with its sign changed without affecting the sign of inequality.*

ILLUSTRATION 1 *The solution set of the inequation* $\frac{1}{2}\left(\frac{3}{5}x+4\right)\geq\frac{1}{3}(x-6)$ *is*

(a) $[120,\infty)$ (b) $(-\infty, 120]$ (c) $[0, 120]$ (d) $[-120, 0]$

Ans. (b)

SOLUTION We have,

$$\frac{1}{2}\left(\frac{3}{5}x+4\right)\geq\frac{1}{3}(x-6)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3x+20}{5}\right)\geq\frac{1}{3}(x-6)$$

$$\Rightarrow \frac{3x+20}{10}\geq\frac{x-6}{3}$$

$$\Rightarrow 3(3x+20)\geq 10(x-6) \quad \left[\text{Multiplying both sides by 30 i.e. LCM of 10 and 3}\right]$$

$$\Rightarrow 9x+60\geq 10x-60$$

$$\Rightarrow 9x-10x\geq -60-60$$

$$\Rightarrow -x\geq -120$$

$$\Rightarrow x\leq 120$$

$$\Rightarrow x\in(-\infty, 120]$$

Hence, the solution set of the given inequation is $(-\infty, 120]$.

REMARK *The solution set of simultaneous inequations is the intersection of their solution sets.*

ILLUSTRATION 2 *If* $\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8}$ *and* $\frac{2x-1}{12}-\frac{x-1}{3}<\frac{3x+1}{4}$, *then x belongs to the interval*

(a) $(3,\infty)$ (b) $(0,\infty)$ (c) $(-\infty, 3)$ (d) $(-\infty, 0)$

Ans. (a)

SOLUTION We have,

$$\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8} \text{ and } \frac{2x-1}{12}-\frac{x-1}{3}<\frac{3x+1}{4}$$

$$\Rightarrow \frac{10x+3x}{8}>\frac{39}{8} \text{ and } \frac{2x-1-4x+4}{12}<\frac{3x+1}{4}$$

$$\Rightarrow \frac{13x}{8}>\frac{39}{8} \text{ and } \frac{-2x+3}{12}<\frac{3x+1}{4}$$

$$\Rightarrow 13x>39 \text{ and } -2x+3<9x+3$$

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Clearly, $x^{12}-x^9+x^4-x+1>0$ for all $x<0$.

Hence, $x^{12}-x^9+x^4-x+1>0$ for all $x\in(-\infty,\infty)$.

EXAMPLE 17 *The number of positive integral solutions of* $x^2+9<(x+3)^2<8x+25$, *is*

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (d)

SOLUTION We have,

$$x^2+9<(x+3)^2<8x+25$$

$$\Rightarrow\quad x^2+9<x^2+6x+9<8x+25$$

$$\Rightarrow\quad x^2+9<x^2+6x+9 \text{ and } x^2+6x+9<8x+25$$

$$\Rightarrow\quad 6x>0 \text{ and } x^2-2x-16<0$$

$$\Rightarrow\quad x>0 \text{ and } 1-\sqrt{17}<x<1+\sqrt{17}$$

$$\Rightarrow\quad 0<x<1+\sqrt{17}$$

$$\Rightarrow\quad x=1,2,3,4,5$$

EXAMPLE 18 *If* $x^2-ax+1-2a^2>0$ *for all* $x\in R$, *then*

(a) $a\in(-2/3,2/3)$ (b) $a\in[-2/3,2/3]$

(c) $a\in(-2/3,1)$ (d) $a\in(0,2/3)$

Ans. (a)

SOLUTION We have,

$$x^2-ax+(1-2a^2)>0 \text{ for all } x\in R$$

$$\Rightarrow\quad a^2-4(1-2a^2)<0 \Rightarrow 9a^2-4<0 \Rightarrow -\frac{2}{3}<a<\frac{2}{3}$$

EXAMPLE 19 *The least integral value of* k *for which*

$(k-2)x^2+8x+k+4\geq 0$ *for all* $x\in R$, *is*

(a) 5 (b) 4 (c) 3 (d) *none of these*

Ans. (b)

SOLUTION We have,

$$(k-2)x^2+8x+(k+4)\geq 0 \text{ for all } x\in R$$

$$\Rightarrow\quad k-2>0 \text{ and } 64-4(k-2)(k+4)\leq 0$$

$$\Rightarrow\quad k>2 \text{ and } k^2+2k-24\geq 0$$

$$\Rightarrow\quad k>2 \text{ and } (k+6)(k-4)\geq 0$$

$$\Rightarrow\quad k>2 \text{ and } k\leq -6 \text{ or } k\geq 4 \Rightarrow k\geq 4$$

Hence, the least integral value of k is 4.

EXAMPLE 20 *If* $9^{x+1}+(a^2-4a-2)3^x+1>0$ *for all* $x\in R$, *then*

(a) $a\in R$ (b) $a\in R^+$ (c) $a\in[1,\infty)$ (d) $a\in R-\{2\}$

Ans. (d)

SOLUTION We have,

$$9^{x+1}+(a^2-4a-2)3^x+1>0 \text{ for all } x\in R$$

$$\Rightarrow\quad 9y^2+(a^2-4a-2)y+1>0 \text{ for all } y>0, \text{ where } y=3^x$$

$$\Rightarrow\quad y\left(9y+\frac{1}{y}+a^2-4a-2\right)>0 \text{ for all } y>0$$

$$\Rightarrow\quad \left(3\sqrt{y}-\frac{1}{\sqrt{y}}\right)^2+(a-2)^2>0 \text{ for all } y>0$$

$$\Rightarrow\quad a\neq 2 \Rightarrow a\in R-\{2\}$$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The solution set of the inequation $\frac{x+4}{x-3}\leq 2$, is

(a) $(-\infty,3)\cup(10,\infty)$ (b) $(3,10]$

(c) $(-\infty,3)\cup[10,\infty)$ (d) none of these

2. If $\frac{3(x-2)}{5}\geq\frac{5(2-x)}{3}$, then x belongs to the interval

(a) $(2,\infty)$ (b) $[2,\infty)$

(c) $(-\infty,2]$ (d) none of these

3. The solution set of the inequation $\frac{2x+4}{x-1}\geq 5$, is

(a) $(1,3)$ (b) $(1,3]$

(c) $(-\infty,1)\cup[3,\infty)$ (d) none of these

4. The solution set of the inequation $\frac{4x+3}{2x-5}<6$, is

(a) $(5/2,33/8)$ (b) $(-\infty,5/2)\cup(33/8,\infty)$

(c) $(5/2,\infty)$ (d) $(33/8,\infty)$

5. The number of integral solutions of $2(x+2)>x^2+1$, is

(a) 2 (b) 3 (c) 4 (d) 5

6. If $|3x+2|<1$, then x belongs to the interval

(a) $(-1,-1/3)$ (b) $[-1,-1/3]$

(c) $(-\infty,-1)$ (d) $(-1/3,\infty)$

7. The solution set of the inequation $|2x-3|<|x+2|$, is

(a) $(-\infty,1/3)$ (b) $(1/3,5)$

(c) $(5,\infty)$ (d) $(-\infty,1/3)\cup(5,\infty)$

8. The solution set of the inequation $\left|\frac{3}{x}+1\right|>2$, is

(a) $(0,3]$ (b) $[-1,0)$

(c) $(-1,0)\cup(0,3)$ (d) none of these

9. The solution set of the inequation $0<|3x+1|<\frac{1}{3}$, is

(a) $(-4/9,-2/9)$ (b) $[-4/9,-2/9]$

(c) $(-4/9,-2/9)-\{-1/3\}$ (d) $[-4/9,-2/9]-\{-1/3\}$

10. The solution set of the inequation $\frac{x^2-3x+4}{x+1}>1, x\in R$, is

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Clearly, $x^{12}-x^9+x^4-x+1>0$ for all $x<0$.
Hence, $x^{12}-x^9+x^4-x+1>0$ for all $x\in(-\infty,\infty)$.

EXAMPLE 17 *The number of positive integral solutions of* $x^2+9<(x+3)^2<8x+25$, *is*

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (d)

SOLUTION We have,

$$x^2+9<(x+3)^2<8x+25$$
$$\Rightarrow\quad x^2+9<x^2+6x+9<8x+25$$
$$\Rightarrow\quad x^2+9<x^2+6x+9 \text{ and } x^2+6x+9<8x+25$$
$$\Rightarrow\quad 6x>0 \text{ and } x^2-2x-16<0$$
$$\Rightarrow\quad x>0 \text{ and } 1-\sqrt{17}<x<1+\sqrt{17}$$
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(c) $a\in(-2/3,1)$ (d) $a\in(0,2/3)$

Ans. (a)

SOLUTION We have,

$$x^2-ax+(1-2a^2)>0 \text{ for all } x\in R$$
$$\Rightarrow\quad a^2-4(1-2a^2)<0 \Rightarrow 9a^2-4<0 \Rightarrow -\frac{2}{3}<a<\frac{2}{3}$$

EXAMPLE 19 *The least integral value of* k *for which*

$(k-2)x^2+8x+k+4\geq 0$ *for all* $x\in R$, *is*

(a) 5 (b) 4 (c) 3 (d) *none of these*

Ans. (b)

SOLUTION We have,

$$(k-2)x^2+8x+(k+4)\geq 0 \text{ for all } x\in R$$
$$\Rightarrow\quad k-2>0 \text{ and } 64-4(k-2)(k+4)\leq 0$$
$$\Rightarrow\quad k>2 \text{ and } k^2+2k-24\geq 0$$
$$\Rightarrow\quad k>2 \text{ and } (k+6)(k-4)\geq 0$$
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(a) $a\in R$ (b) $a\in R^+$ (c) $a\in[1,\infty)$ (d) $a\in R-\{2\}$

Ans. (d)

SOLUTION We have,

$$9^{x+1}+(a^2-4a-2)3^x+1>0 \text{ for all } x\in R$$
$$\Rightarrow\quad 9y^2+(a^2-4a-2)y+1>0 \text{ for all } y>0, \text{ where } y=3^x$$
$$\Rightarrow\quad y\left(9y+\frac{1}{y}+a^2-4a-2\right)>0 \text{ for all } y>0$$
$$\Rightarrow\quad \left(3\sqrt{y}-\frac{1}{\sqrt{y}}\right)^2+(a-2)^2>0 \text{ for all } y>0$$
$$\Rightarrow\quad a\neq 2 \Rightarrow a\in R-\{2\}$$

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

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   (a) $(-\infty,3)\cup(10,\infty)$ (b) $(3,10]$
   (c) $(-\infty,3)\cup[10,\infty)$ (d) none of these

2. If $\frac{3(x-2)}{5}\geq\frac{5(2-x)}{3}$, then x belongs to the interval
   (a) $(2,\infty)$ (b) $[2,\infty)$
   (c) $(-\infty,2]$ (d) none of these

3. The solution set of the inequation $\frac{2x+4}{x-1}\geq 5$, is
   (a) $(1,3)$ (b) $(1,3]$
   (c) $(-\infty,1)\cup[3,\infty)$ (d) none of these

4. The solution set of the inequation $\frac{4x+3}{2x-5}<6$, is
   (a) $(5/2,33/8)$ (b) $(-\infty,5/2)\cup(33/8,\infty)$
   (c) $(5/2,\infty)$ (d) $(33/8,\infty)$

5. The number of integral solutions of $2(x+2)>x^2+1$, is
   (a) 2 (b) 3 (c) 4 (d) 5

6. If $|3x+2|<1$, then x belongs to the interval
   (a) $(-1,-1/3)$ (b) $[-1,-1/3]$
   (c) $(-\infty,-1)$ (d) $(-1/3,\infty)$

7. The solution set of the inequation $|2x-3|<|x+2|$, is
   (a) $(-\infty,1/3)$ (b) $(1/3,5)$
   (c) $(5,\infty)$ (d) $(-\infty,1/3)\cup(5,\infty)$

8. The solution set of the inequation $\left|\frac{3}{x}+1\right|>2$, is
   (a) $(0,3]$ (b) $[-1,0)$
   (c) $(-1,0)\cup(0,3)$ (d) none of these

9. The solution set of the inequation $0<|3x+1|<\frac{1}{3}$, is
   (a) $(-4/9,-2/9)$ (b) $[-4/9,-2/9]$
   (c) $(-4/9,-2/9)-\{-1/3\}$ (d) $[-4/9\ -2/9]-\{-1/3\}$

10. The solution set of the inequation $\frac{x^2-3x+4}{x+1}>1, x\in R$, is

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(a) $(3, \infty)$ (b) $(-1, 1) \cup (3, \infty)$
(c) $[-1, 1] \cup [3, \infty)$ (d) none of these

11. The solution set of the inequation $\frac{2}{|x-4|} > 1, x \neq 4$, is
(a) $(2, 6)$ (b) $(2, 4) \cup (4, 6)$
(c) $(-\infty, 2) \cup (6, \infty)$ (d) none of these

12. The solution set of the inequation $\frac{1}{|x|-3} < \frac{1}{2}$, is
(a) $(-\infty, -5) \cup (5, \infty)$
(b) $(-3, 3)$
(c) $(-\infty, -5) \cup (-3, 3) \cup (5, \infty)$
(d) none of these

13. The solution set of the inequation $\left|\frac{2x-1}{x-1}\right| > 2$, is
(a) $(3/4, 1) \cup (1, \infty)$ (b) $(3/4, \infty)$
(c) $(-\infty, 3/4)$ (d) none of these

14. The solution set of the inequation $\frac{|x-2|}{x-2} < 0$, is
(a) $(2, \infty)$ (b) $(-\infty, 2)$ (c) R (d) $(-2, 2)$

15. The solution set of the inequation $\left|x + \frac{1}{x}\right| > 2$, is
(a) $R - \{0\}$ (b) $R - \{-1, 0, 1\}$
(c) $R - \{1\}$ (d) $R - \{-1, 1\}$

16. The solution set of the inequation
$|x-1| + |x-2| + |x-3| \geq 6$, is
(a) $[0, 4]$ (b) $(-\infty, -2) \cup [4, \infty)$
(c) $(-\infty, 0] \cup [4, \infty)$ (d) none of these

17. The solution set of $x^2 + 2 \leq 3x \leq 2x^2 - 5$, is
(a) ϕ (b) $[1, 2]$
(c) $(-\infty, -1] \cup [5/2, \infty)$ (d) none of these

18. The set of values of x satisfying inequations $|x-1| \leq 3$ and $|x-1| \geq 1$, is
(a) $[2, 4]$ (b) $(-\infty, 2] \cup [4, \infty)$
(c) $[-2, 0] \cup [2, 4]$ (d) none of these

19. The solution set of the inequation
$x^2 + (a+b)x + ab < 0, a < b$, is
(a) (a, b) (b) $(-\infty, a) \cup (b, \infty)$
(c) $(-b, -a)$ (d) $(-\infty, -b) \cup (-a, \infty)$

20. The number of integral solutions of $x^2 - 3x - 4 < 0$, is
(a) 3 (b) 4
(c) 6 (d) none of these

21. The solution set of $|x^2 - 10| \leq 6$, is
(a) $(2, 4)$ (b) $(-4, -2)$
(c) $(-4, -2) \cup (2, 4)$ (d) $[-4, -2] \cup [2, 4]$

22. The solution set of the inequation $\left|x + \frac{1}{x}\right| < 4$, is
(a) $(2-\sqrt{3}, 2+\sqrt{3}) \cup (-2-\sqrt{3}, -2+\sqrt{3})$
(b) $R - (2-\sqrt{3}, 2+\sqrt{3})$
(c) $R - (-2-\sqrt{3}, -2+\sqrt{3})$
(d) none of these

23. The solution set of $x^2 + x + |x| + 1 < 0$, is
(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) R (d) ϕ

24. If $|x-1| + |x| + |x+1| \geq 6$, then x belongs to
(a) $(-\infty, 2]$ (b) $(-\infty, -2] \cup [2, \infty)$
(c) R (d) ϕ

25. If $\left|\frac{x^2+6}{5x}\right| \geq 1$, then x belongs to
(a) $(-\infty, -3)$
(b) $(-\infty, -3) \cup (3, \infty)$
(c) $(-\infty, -3] \cup [-2, 0) \cup (0, 2] \cup [3, \infty)$
(d) R

26. The greatest negative integer satisfying $x^2 - 4x - 77 < 0$ and $x^2 > 4$, is
(a) -4 (b) -6
(c) -7 (d) none of these

27. If $2 - 3x - 2x^2 \geq 0$, then
(a) $x \leq -2$ (b) $-2 \leq x \leq \frac{1}{2}$
(c) $x \geq -2$ (d) $x \leq \frac{1}{2}$

28. The solution of $6 + x - x^2 > 0$, is
(a) $-1 < x < 2$ (b) $-2 < x < 3$
(c) $-2 < x < -1$ (d) none of these
[CEE (Delhi) 2000]

29. If $4 \leq x \leq 9$, then
(a) $(x-4)(x-9) \leq 0$ (b) $(x-4)(x-9) \geq 0$
(c) $(x-4)(x-9) < 0$ (d) $(x-4)(x-9) > 0$

30. The set of real values of x for which
$\frac{10x^2 + 17x - 34}{x^2 + 2x - 3} < 8$, is
(a) $(-5/2, 2)$ (b) $(-3, -5/2) \cup (1, 2)$
(c) $(-3, 1)$ (d) none of these.

31. The solution set of the inequation $|x| - 1 < 1 - x$, is
(a) $(-1, 1)$ (b) $(0, \infty)$
(c) $(-1, \infty)$ (d) none of these

32. If S is the set of all real x such that
$\frac{2x}{2x^2 + 5x + 2} > \frac{1}{x+1}$, then S is equal to
(a) $(-2, -1)$ (b) $(-2/3, 0)$.
(c) $(-2/3, -1/2)$ (d) $(-2, -1) \cup (-2/3,$
[CEE (Delhi) 2

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33. The set of all real values of x for which
$$\frac{8x^2+16x-51}{(2x-3)(x+4)}<3, \text{ is}$$
(a) $(3/2, 5/2)$ (b) $(-4,-3)$
(c) $(-4,-3)\cup(3/2, 5/2)$ (d) none of these

34. The set of all integral values of x for which
$$5x-1<(x+1)^2<7x-3, \text{ is}$$
(a) ϕ (b) {1} (c) {2} (d) {3}

35. The number of integral solutions of $\frac{x+1}{x^2+2}>\frac{1}{4}$, is
(a) 1 (b) 2 (c) 5 (d) none of these
[JEE (Orissa) 2002]

36. The solution set of $x^2+2\le 3x\le 2x^2-5$, is
(a) ϕ (b) $[1, 2]$
(c) $(-\infty,-1]\cup[5/2,\infty)$ (d) none of these

37. If x is an integer satisfying $x^2-6x+5\le 0$ and $x^2-2x>0$, then the number of possible values of x, is
(a) 3 (b) 4 (c) 2 (d) infinite

38. If p, q, r, s, t are numbers such that $p+q<r+s$, $q+r<s+t$, $r+s<t+p$, $s+t<p+q$, then the largest and smallest numbers are
(a) p and q respectively (b) r and t respectively
(c) r and q respectively (d) q and p respectively

39. The number of integral solutions of $\frac{x+2}{x^2+1}>\frac{1}{2}$ is
(a) 4 (b) 5
(c) 3 (d) none of these

Answers

(c)	2.	(b)	3.	(b)	4.	(b)	5.	(b)	6.	(a)	7.	(b)	22.	(a)	23.	(d)	24.	(b)	25.	(c)	26.	(d)	27.	(b)	28.	(b)
(c)	9.	(c)	10.	(b)	11.	(b)	12.	(c)	13.	(a)	14.	(b)	29.	(a)	30.	(b)	31.	(d)	32.	(d)	33.	(c)	34.	(d)	35.	(c)
(b)	16.	(c)	17.	(a)	18.	(c)	19.	(c)	20.	(b)	21.	(d)	36.	(a)	37.	(a)	38.	(a)	39.	(c)						

utions of **Exercises** and **Chapter-tests** are available in a separate book on "**Solutions of Objective Mathematics**".

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2

LOGARITHMS

1. DEFINITION

If a is a positive real number, other than 1 and x is a rational number such that $a^x = N$, then we say that logarithm of N to base a is x or x is the logarithm of N to the base a, written as $\log_a N = x$.

Thus, $a^x = N \Leftrightarrow \log_a N = x$.

NOTE 1 *It should be noted that "log" is the abbreviation of the word "logarithm".*

It follows from the above definition that:

(i) $10^3 = 1000 \Leftrightarrow \log_{10} 1000 = 3$

(ii) $3^4 = 81 \Leftrightarrow \log_3 81 = 4$

(iii) $7^0 = 1 \Leftrightarrow \log_7 1 = 0$

(iv) $81^{1/4} = 3 \Leftrightarrow \log_{81} 3 = \frac{1}{4}$

NOTE 2 *For any positive real number a, we have $a^1 = a$.*

$\therefore \quad \log_a a = 1.$

It also follows from the above definition that:

(i) $\log_2 64 =$ (A real number x such that $2^x = 64$) $= 6$

(ii) $\log_5 625 =$ (A real number x such that $5^x = 625$) $= 4$

(iii) $\log_9 3 =$ (A real number x such that $9^x = 3$) $= \frac{1}{2}$

(iv) $\log_{10} 0.01 =$ (A real number x such that $10^x = 0.01$) $= -2$

Thus, the logarithm of a number to a given positive real number ($\neq 1$) as base is the index or the power to which the base must be raised in order to make it equal to the given number.

ILLUSTRATION 1 *If $\log_3 y = x$ and $\log_2 z = x$, then 72^x is equal to*

(a) yz^3 (b) y^2z^3 (c) y^3z^2 (d) y^3z^3

Ans. (b)

SOLUTION We have,

$$\log_3 y = x \text{ and } \log_2 z = x \Rightarrow y = 3^x \text{ and } z = 2^x$$

$$\therefore \quad 72^x = (2^3 \times 3^2)^x = (2^x)^3 (3^x)^2 = (z)^3 (y)^2 = y^2z^3$$

ILLUSTRATION 2 *If* $\frac{\log_{10} a}{2} = \frac{\log_{10} b}{3} = \frac{\log_{10} c}{5}$, *then* $bc =$

(a) a (b) a^2 (c) a^3 (d) a^4

Ans. (d)

SOLUTION We have,

$$\frac{\log_{10} a}{2} = \frac{\log_{10} b}{3} = \frac{\log_{10} c}{5} = \lambda \text{ (say)}$$

$$\Rightarrow \quad \log_{10} a = 2\lambda, \log_{10} b = 3\lambda, \log_{10} c = 5\lambda$$

$$\Rightarrow \quad a = 10^{2\lambda}, b = 10^{3\lambda}, c = 10^{5\lambda}$$

$$\Rightarrow \quad bc = 10^{8\lambda} = (10^{2\lambda})^4 = a^4$$

ILLUSTRATION 3 *If $\log_4 5 = x$ and $\log_5 6 = y$, then $\log_2 3$ is equal to*

(a) $2xy + 1$ (b) $2xy - 1$ (c) $2x + 1$ (d) $2y + 1$

Ans. (b)

SOLUTION We have,

$$\log_4 5 = x \text{ and } \log_5 6 = y$$

$$\Rightarrow \quad 5 = 4^x \text{ and } 6 = 5^y$$

$$\Rightarrow \quad 6 = (4^x)^y$$

$$\Rightarrow \quad 2 \times 3 = 4^{xy}$$

$$\Rightarrow \quad 2 \times 3 = (2^2)^{xy} \Rightarrow 3 = 2^{2xy-1} \Rightarrow \log_2 3 = 2xy - 1$$

2. LAWS OF LOGARITHMS

Following are the fundamental laws of logarithms to any base a, where $a > 0$ and $a \neq 1$.

FIRST LAW *If m, n are positive rational numbers, then*

$$\log_a (mn) = \log_a m + \log_a n$$

i.e. the log of the product of two numbers is equal to the sum of their logarithms.

GENERALISATION

(i) *If m, n, p are positive rational numbers, then*

$$\log_a (mnp) = \log_a m + \log_a n + \log_a p$$

(ii) *If $x_1, x_2, \ldots, x_n$ are positive rational numbers, then*

$$\log_a (x_1 x_2 \ldots x_n) = \log_a x_1 + \log_a x_2 + \ldots + \log_a x_n$$

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Ans. (c)

SOLUTION We know that $AM \geq GM$. So, statement-2 is false.

Now,

$$\log_2 x^3 - \log_x (0.125) = 3\log_2 x - \log_x\left(\frac{1}{8}\right)$$
$$= 3\left(\log_2 x + \log_x 2\right)$$
$$\geq 3 \times 2 \qquad \left[\because \log_2 x + \log_x 2 \geq 2\right]$$

EXAMPLE 8 *Statement–1: If* $x^{\log_x (3-x)^2} = 25$, *then* $x = -2$

Statement–2: $a^{\log_a x} = x$, *if* $a > 0, x > 0$ *and* $a \neq 1$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION $x^{\log_x (3-x)^2} = 25$

$\Rightarrow \quad (3-x)^2 = 25$ provided that $x > 0$ and $x \neq 1$

$\Rightarrow \quad 3 - x = \pm 5 \qquad [\because a^{\log_a x} = x]$

$\Rightarrow \quad x = -2, x = 8$

Hence, statement-2 is true but statement-1 is false.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

Note: In the following exercises $\log x$ *stands for* $\log_{10} x$ *unless stated otherwise.*

1. The value of $7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80}$, is
   (a) $\log 2$ (b) $\log 3$
   (c) $\log 5$ (d) none of these

2. The value of $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5}$ is
   (a) 5 (b) 2 (c) $\frac{5}{2}$ (d) $\frac{3}{2}$

3. The value of $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$ is
   (a) $\log 2$ (b) 1 (c) 0 (d) none of these

4. The value of $2^{\log_3 7} - 7^{\log_3 2}$ is
   (a) $\log 2$ (b) 1 (c) 0 (d) none of these

5. The value of $\frac{3 + \log 343}{2 + \frac{1}{2}\log\left(\frac{49}{4}\right) + \frac{1}{3}\log\left(\frac{1}{125}\right)}$ is
   (a) 3 (b) 2 (c) 1 (d) $\frac{3}{2}$

6. If $\log_{30} 3 = x$, $\log_{30} 5 = y$, then $\log_{30} 8 =$
   (a) $3(1-x-y)$ (b) $x - y + 1$
   (c) $1 - x - y$ (d) $2(x - y + 1)$

7. If $\log_a x$, $\log_b x$, $\log_c x$ are in A.P., where $x \neq 1$, then $c^2 =$
   (a) $(ab)^{\log_a b}$ (b) $(ac)^{\log_a b}$
   (c) $(ab)^{\log_b a}$ (d) $(ac)^{\log_b a}$

8. If $a^2 + 4b^2 = 12ab$, then $\log(a + 2b) =$
   (a) $\frac{1}{2}(\log a + \log b - \log 2)$ (b) $\log\frac{a}{2} + \log\frac{b}{2} + \log 2$
   (c) $\frac{1}{2}(\log a + \log b + 4\log 2)$ (d) $\frac{1}{2}(\log a - \log b + 4\log 2)$

9. If $9a^2 + 4b^2 = 18ab$, then $\log(3a + 2b) =$
   (a) $\log 5 + \log 3 + \log a + \log 5b$
   (b) $\log 5 + \log 3 + \log 3a + \log b$
   (c) $\log 5 + \log a + \log b$
   (d) none of these

10. The value of
    $$\log_5\left(1 + \frac{1}{5}\right) + \log_5\left(1 + \frac{1}{6}\right) + \log_5\left(1 + \frac{1}{7}\right) + \ldots + \log_5\left(1 + \frac{1}{624}\right)$$ is
    (a) 5 (b) 4 (c) 3 (d) 2

11. If $\log(x - y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$, then $\frac{x}{y} + \frac{y}{x} =$
    (a) 25 (b) 26 (c) 27 (d) 28

12. If $2^{\log_{10} 3\sqrt{3}} = 3^{k\log_{10} 2}$, then $k =$
    (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 3 (d) 2

13. If $\log_{10} 343 = 2.5353$, then the least positive integer n such that $7^n > 10^5$, is
    (a) 1 (b) 6 (c) 5 (d) 4

14. If $\log_{10} 2 = 0.3010$, then $\log_5 64 =$
    (a) $\frac{602}{233}$ (b) $\frac{233}{602}$ (c) $\frac{202}{633}$ (d) $\frac{633}{202}$

15. If $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$, then $x =$
    (a) 4 (b) 9 (c) 83 (d) 10

16. The value of $3^{\frac{4}{\log_4 9}} + 27^{\frac{1}{\log_{36} 9}} + 81^{\frac{1}{\log_5 3}}$, is
    (a) 890 (b) 860 (c) 857 (d) none of these

17. The value of $\log_{\sqrt{2}}\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$, is
    (a) $\frac{15}{16}$ (b) $\frac{7}{16}$ (c) $\frac{15}{8}$ (d) $\frac{31}{32}$

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18. The value of $a^{\frac{\log_b(\log_b x)}{\log_b a}}$, is
(a) $\log_a x$ (b) $\log_b x$ (c) $\log_x a$ (d) $\log_x b$

19. The value of $\frac{\log_a(\log_b a)}{\log_b(\log_a b)}$ is
(a) $\log_b a$ (b) $\log_a b$ (c) $-\log_a b$ (d) $-\log_b a$

20. If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$, then x equals to
(a) 8 (b) 4 (c) 2 (d) 16

21. If $\log_{10}\left(98 + \sqrt{x^2 - 12x + 36}\right) = 2$, then x =
(a) 4 (b) 8 (c) 12 (d) 4, 8

22. If $a^x = b^y = c^z = d^w$, then $\log_a(bcd)$ equals to
(a) $\frac{1}{x}\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$ (b) $x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$
(c) $\frac{y+z+w}{x}$ (d) none of these

[EAMCET 2007]

23. If $\log_5\left(\log_5(\log_2 x)\right) = 0$, then the values of x is
(a) 32 (b) 125 (c) 625 (d) 125

[JEE (WB) 2008]

24. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$, is
(a) 3 (b) 1 (c) 2 (d) 0

25. If $\log_2 a + \log_4 b + \log_4 c = 2$
$\log_9 a + \log_3 b + \log_9 c = 2$
$\log_{16} a + \log_{16} b + \log_4 c = 2$, then
(a) $a=\frac{2}{3}, b=\frac{27}{8}, c=\frac{32}{3}$ (b) $a=\frac{27}{8}, b=\frac{2}{3}, c=\frac{32}{3}$
(c) $a=\frac{32}{3}, b=\frac{27}{8}, c=\frac{2}{3}$ (d) $a=\frac{2}{3}, b=\frac{32}{3}, c=\frac{27}{8}$

26. If $y = 2^{1/\log_x 8}$, then x equal to
(a) y (b) y^2 (c) y^3 (d) none of these

[JEE (WB) 2007]

27. If $\log_y x = \log_z y = \log_x z$, then
(a) $x<y<z$ (b) $x>y\geq z$ (c) $x<y\leq z$ (d) $x=y=z$

28. If $3^{2x+1}.4^{x-1} = 36$, then x =
(a) $\log_{36} 48$ (b) $\log_{48} 36$ (c) $\log_{24} 12$ (d) $\log_{12} 24$

29. If $\log_x\left\{\log_4\left(\log_x(5x^2+4x^3)\right)\right\} = 0$, then
(a) 2 (b) 3 (c) 4 (d) 5

30. If $\frac{1}{\log_x 10} = \frac{2}{\log_a 10} - 2$, then x =
(a) $\frac{a}{2}$ (b) $\frac{a}{100}$ (c) $\frac{a^2}{10}$ (d) $\frac{a^2}{100}$

31. If $\log_{12} 27 = a$, then $\log_6 16$ =
(a) $\frac{3-a}{3+a}$ (b) $4\left(\frac{3-a}{3+a}\right)$
(c) $3\left(\frac{4-a}{4+a}\right)$ (d) $3\left(\frac{4+a}{4-a}\right)$

32. If $(4.2)^x = (0.42)^y = 100$, then $\frac{1}{x} - \frac{1}{y}$ =
(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) -1

33. If $\log_8 x = 2.5$ and $\log_2 y = 5$, then x =
(a) $y^{3/2}$ (b) $2y$ (c) y (d) $\frac{y}{2}$

34. If $\log_e 2 . \log_x 27 = \log_{10} 8 . \log_e 10$, then x =
(a) 1 (b) 3 (c) 2 (d) 4

35. If $3 + \log_5 x = 2\log_{25} y$, then x equals to
(a) $\frac{y}{125}$ (b) $\frac{y}{25}$ (c) $\frac{y^2}{625}$ (d) $3-\frac{y^2}{25}$

36. If $\frac{\log a}{3} = \frac{\log b}{4} = \frac{\log c}{5}$, then ca equals
(a) $2b$ (b) b^2 (c) $8b$ (d) $4b$

37. If $2^x . 3^{2x} = 100$, then x belongs to
(a) $(0,3)$ (b) $(1,3)$ (c) $(1,2)$ (d) $(0,2)$

38. If $x^{2\log_{10} x} = 1000x$, then x equals to
(a) $10, \sqrt{10}$ (b) $10^{-1}, 10\sqrt{10}$
(c) $10\sqrt{10}$ (d) $\sqrt{10}$

39. If $\log_{10} 5 = x$, then $\log_5 1250$ equals to
(a) $3-\frac{1}{x}$ (b) $2+\frac{1}{x}$ (c) $3+\frac{1}{x}$ (d) $2-\frac{1}{x}$

40. If $5^{3x^2\log_{10} 2} = 2^{\left(x+\frac{1}{2}\right)\log_{10} 25}$, then x equals to
(a) $1, -\frac{1}{3}$ (b) 1 (c) $1, -\frac{1}{2}$ (d) $-\frac{1}{3}, 1$

41. If $\log_x\left(4x^{\log_5 x}+5\right) = 2\log_5 x$, then x equals to
(a) 4, 5 (b) $-1, 5$ (c) $4, -1$ (d) $5, \frac{1}{5}$

42. The equation $\log_e x + \log_e(1+x) = 0$ can be written as
(a) $x^2+x-1=0$ (b) $x^2+x+1=0$
(c) $x^2+x-e=0$ (d) $x^2+x+e=0$

43. If $x = \log_3 5$, $y = \log_{17} 25$ which one of the following is correct?
(a) $x < y$ (b) $x = y$
(c) $x > y$ (d) none of these

44. If $x = \log_a(bc)$, $y = \log_b(ca)$ and $z = \log_c(ab)$, then which of the following is correct?
(a) $x+y+z = 1$ (b) $\frac{1}{1+x}+\frac{1}{1+y}+\frac{1}{1+z}=1$
(c) $xyz = 1$ (d) none of these

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45. The value of $\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$, is
(a) 2 (b) 3 (c) 5 (d) 7

46. If $\frac{\log x}{a-b} = \frac{\log y}{b-c} = \frac{\log z}{c-a}$, then xyz is equal to
(a) 0 (b) 1 (c) -1 (d) 2

47. If $\log_2 7 = x$, then x is:
(a) a rational number such that $0 < x < 2$
(b) an irrational number such that $2 < x < 3$
(c) a rational number such that $2 < x < 3$
(d) a prime number of the form $7x + 2$

48. If $2^{\frac{3}{\log_3 x}} = \frac{1}{64}$, then $x =$
(a) 3 (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{3}}$

49. If $a = 1 + \log_x yz$, $b = 1 + \log_y zx$, $c = 1 + \log_z xy$, then $ab + bc + ca =$
(a) 0 (b) $2abc$ (c) abc (d) $a^2 + b^2 + c^2$

50. If $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2}$, then $x^{a-b} \cdot y^{b-c} \cdot z^{c-a} =$
(a) 0 (b) -1 (c) 1 (d) 2

51. If $\log(2a - 3b) = \log a - \log b$, then $a =$
(a) $\frac{3b^2}{2b-1}$ (b) $\frac{3b}{2b-1}$ (c) $\frac{b^2}{2b+1}$ (d) $\frac{3b^2}{2b+1}$

52. If $\frac{\log 3}{x-y} = \frac{\log 5}{y-z} = \frac{\log 7}{z-x}$, then $3^{x+y}\ 5^{y+z}\ 7^{z+x} =$
(a) 0 (b) 2 (c) 1 (d) none of these

53. $\frac{\log_2 a}{3} = \frac{\log_2 b}{4} = \frac{\log_2 c}{5\lambda}$ and $a^{-3} b^{-4} c = 1$ then $\lambda =$
(a) 3 (b) 4 (c) 5 (d) -5

54. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then $a^a b^b c^c =$
(a) 0 (b) 1 (c) abc (d) none of these

55. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots \text{to } \infty\right)}$ is
(a) 0.16 (b) 1 (c) 0.4 (d) 4

56. If $a^2 + 4b^2 = 12ab$, then $\log(a + 2b) =$
(a) $\frac{1}{2}(\log a + \log b - 2)$ (b) $\log \frac{a}{2} + \log \frac{b}{2} + \log 2$
(c) $\frac{1}{2}(\log a + \log b + 4\log 2)$ (d) $\frac{1}{2}(\log a - \log b + 4\log 2)$

57. The value of $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$ is
(a) $\log 2$ (b) $\log 3$ (c) 1 (d) 0

58. If $\log(x + y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$, then
(a) $x + y = 0$ (b) $x - y = 0$
(c) $xy = 1$ (d) $x^2 + xy + y^2 = 0$

59. The number of real solutions of the equation $\log(-x) = 2\log(x + 1)$, is
(a) 0 (b) 1 (c) 2 (d) 4

60. The solution of the equation $\log_\pi\left(\log_2(\log_7 x)\right) = 0$, is
(a) 7^2 (b) π^2 (c) 2^2 (d) *none of these*

61. If $\log_4 2 + \log_4 4 + \log_4 16 + \log_4 x = 6$, then $x =$
(a) 4 (b) 64 (c) 32 (d) 8
[EAMCET 2006]

62. The number of real values of the parameter k for which the equation
$$(\log_{16} x)^2 - \log_{16} x + \log_{16} k = 0$$
with real coefficients will have exactly one solution, is
(a) 2 (b) 1 (c) 4 (d) none of these

63. If $a^x = b$, $b^y = c$, $c^z = a$, then the value of xyz is
(a) 0 (b) 1 (c) 2 (d) 3

64. The value of $\log_b a \times \log_c b \times \log_a c$, is
(a) 0 (b) 1 (c) $\log abc$ (d) 10

65. If $\log_a ab = x$, then the value of $\log_b ab$ is
(a) $\frac{x-1}{x}$ (b) $\frac{x}{x-1}$ (c) $\frac{x}{x+1}$ (d) $\frac{x+1}{x}$

66. If $x = -2$, then the value of $\log_4\left(\frac{x^2}{4}\right) - 2\log_4(4x^4)$, is
(a) 2 (b) -4 (c) -6 (d) 0

67. The value of $\sqrt{\log_{0.5}^2 4}$, is
(a) -2 (b) $\sqrt{-4}$ (c) 2 (d) none of these
[CEE (Delhi) 2002]

Answers

1. (a) 2. (c) 3. (c) 4. (c) 5. (a) 6. (a) 7. (b)
8. (c) 9. (d) 10. (c) 11. (c) 12. (b) 13. (b) 14. (a)
15. (d) 16. (c) 17. (c) 18. (b) 19. (c) 20. (a) 21. (d)
22. (b) 23. (a) 24. (b) 25. (a) 26. (c) 27. (d) 28. (a)
29. (d) 30. (d) 31. (b) 32. (c) 33. (a) 34. (b) 35. (a)
36. (b) 37. (c) 38. (b) 39. (c) 40. (a) 41. (a) 42. (a)
43. (c) 44. (b) 45. (c) 46. (b) 47. (b) 48. (c) 49. (c)
50. (c) 51. (a) 52. (c) 53. (c) 54. (b) 55. (d) 56. (c)
57. (a) 58. (b) 59. (b) 60. (a) 61. (c) 62. (b) 63. (b)
64. (b) 65. (b) 66. (c) 67. (a)

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CHAPTER TEST

Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct. Mark the correct alternative.

1. If $x^{\frac{3}{2}\left(\log_2 x - 3\right)} = \frac{1}{8}$, then x equals to
   (a) 2 (b) 3 (c) 5 (d) 6

2. If $\log_4 (3x^2 + 11x) > 1$, then x lies in the interval
   (a) $(-4, 1/3)$ (b) $(-4, 2)$
   (c) $[-4, 1/3]$ (d) $(-\infty, -4) \cup (1/3\ \infty)$

3. If $\log_6 (x+3) - \log_6 x = 2$, then x =
   (a) $\frac{1}{35}$ (b) $\frac{3}{35}$ (c) $\frac{2}{35}$ (d) $-\frac{3}{35}$

4. If $2^x \cdot 9^{2x+3} = 7^{x+5}$, then x =
   (a) $\frac{5\log 7 + 6\log 3}{\log 162 - \log 7}$ (b) $\frac{5\log 7 - 6\log 3}{\log 162 + \log 7}$
   (c) $\frac{5\log 7 - 6\log 3}{\log 162 - \log 7}$ (d) none of these

5. If $\log_7 \left\{\log_5 (\sqrt{x+5} + \sqrt{x})\right\} = 0$ then x =
   (a) 3 (b) 4 (c) 2 (d) none of these

6. If $\log_6 \left\{\log_4 (\sqrt{x+4} + \sqrt{x})\right\} = 0$, then x =
   (a) 1 (b) $\frac{5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{9}{4}$

7. If $x^{\log_x (x^2 - 4x + 5)} = (x-1)$, then x =
   (a) 1 (b) 2 (c) 4 (d) 5

8. If $\log_3 \left\{\log_6 \left(\frac{x^2 + x}{x - 1}\right)\right\} = 0$ then x =
   (a) -1 (b) 1 (c) 3 (d) 4

9. If $\log_8 \left\{\log_2 \left(\log_3 (x^2 - 4x + 85)\right)\right\} = \frac{1}{3}$, then x equals to
   (a) 5 (b) 4 (c) 3 (d) 2

10. If $x = \log_2 3$ and $y = \log_{1/2} 5$, then
    (a) $x > y$ (b) $x < y$
    (c) $x = y$ (d) none of these

11. If $\log_{x+2} \left(x^3 - 3x^2 - 6x + 8\right) = 3$, then x equals to
    (a) 1 (b) 2 (c) 3 (d) none of these

12. If $(2.3)^x = (0.23)^y = 1000$, then $\frac{1}{x} - \frac{1}{y}$ equals to
    (a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

13. If $10^{x-1} + 10^{-x-1} = \frac{1}{3}$, then x equals to
    (a) $\pm\log_{10} 3$ (b) $2\log_3 10$ (c) $\log_3 3$ (d) $\log_2 10$

14. The value of $\log_2 \left[\log_2 \left\{\log_3 \left(\log_3 27^3\right)\right\}\right]$ is
    (a) 1 (b) 0 (c) 3 (d) 2

15. If $2\log_8 a = x$, $\log_2 2a = y$ and $y - x = 4$, then x =
    (a) 10 (b) 16 (c) 4 (d) 6

16. If $\log_{10} x = y$, then $\log_{10^3} x^2$ equals
    (a) $\frac{1}{3}y$ (b) $\frac{2}{3}y$ (c) $\frac{3}{2}y$ (d) $3y$

17. If $\log_3 x \times \log_x 2x \times \log_{2x} y = \log_x x^2$, then y equals
    (a) 9 (b) 18 (c) 27 (d) 81

18. The number of solutions of $\log_2 (x-1) = 2\log_2 (x-3)$ is
    (a) 2 (b) 1 (c) 6 (d) 7

[JEE (WB) 2007]

19. If $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > x$, then the greatest integral value of x is
    (a) 2 (b) 3 (c) π (d) none of these

20. Let $x \in (1, \infty)$ and n be a positive integer greater than 1. If $f_n(x) = \frac{n}{\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \ldots + \frac{1}{\log_n x}}$, then $(n!)^{f_n(x)}$ equals to
    (a) n^x (b) x^n (c) n^n (d) n^{nx}

21. If $\log_2 \sin x - \log_2 \cos x - \log_2 (1 - \tan^2 x) = -1$, then x =
    (a) $\frac{n\pi}{2} + \frac{\pi}{8}, n \in Z$ (b) $n\pi - \frac{\pi}{8}, n \in Z$
    (c) $\frac{n\pi}{4} + \frac{\pi}{2}, n \in Z$ (d) none of these

Answers

1. (a) 2. (d) 3. (b) 4. (c) 5. (b) 6. (d) 7. (b)
8. (c) 9. (d) 10. (a) 11. (d) 12. (c) 13. (a) 14. (b)
15. (d) 16. (b) 17. (a) 18. (b) 19. (a) 20. (b) 21. (a)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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3

MISCELLANEOUS EQUATIONS AND INEQUATIONS

SOLVED (MCQs)

EXAMPLE 1 *The number of real solutions of the equation* $e^x = x$, *is*

(a) 1 (b) 2 (c) 0 (d) *none of these*

Ans. (c)

SOLUTION It is evident from the graphs of $y = e^x$ and $y = x$ that they do not intersect.

Hence, the equation $e^x = x$ has no solution.

ALITER We observe that

$e^x > x$ for all $x \in R$

$\therefore \quad e^x = x$ has no solution.

EXAMPLE 2 *The number of real solutions of the equation* $e^x + x = 0$, *is*

(a) 0 (b) 1 (c) 2 (d) *none of these*

Ans. (b)

SOLUTION It is evident from Fig. 1 that the curves $y = e^x$ and $y = -x$ intersect exactly at one point. So, the equation

$e^x = -x$ or, $e^x + x = 0$ has one real solution.

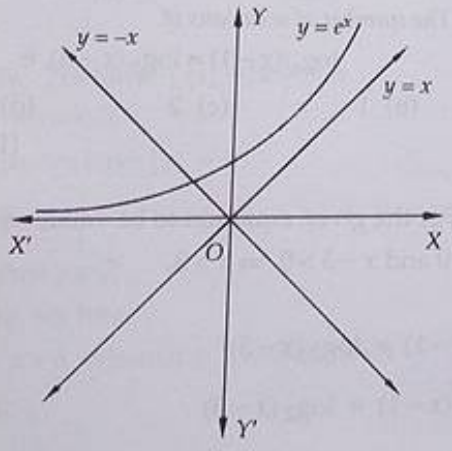


Fig. 1

EXAMPLE 3 *How many roots does the following equation possess?*

$$3^{|x|}\{2 - |x|\} = 1$$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION We have,

$$3^{|x|}\{2 - |x|\} = 1 \qquad \ldots(i)$$

$$\Rightarrow \quad 2 - |x| = 3^{-|x|}$$

In order the determine the number of roots of (i), it is sufficient to find the points of intersection of the curves $y = 2 - |x|$ and $y = 3^{-|x|}$. Graphs of these two curves are shown in Fig. 2.

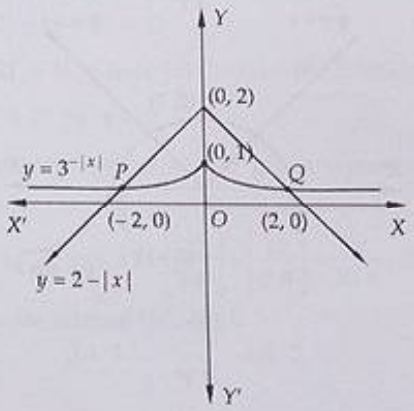


Fig. 2

We observe that the two curves intersect at two points P and Q. Hence, the equation (i) has two real solutions. Also, the solutions belong to the interval $(-2, 2)$.

EXAMPLE 4 *The number of real solutions of the equation* $\log_a x = |x|, 0 < a < 1$, *is*

(a) 0 (b) 1 (c) 2 (d) *none of these*

Ans. (b)

SOLUTION The number of real solutions of the equation $\log_a x = |x|$ is equal to the number of points of intersection of the curves

$$y = \log_a x \text{ and } y = |x|$$

The graphs of these two curves are shown in Fig. 3.

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SOLUTION Consider the curves $y = \cos^2 x - \cos x$ and $y = x$. The number of solutions of this equation is same as the number of points where these two curves intersect. We observe that $y = \cos^2 x - \cos x$ is a continuous curve in $[0, \pi/2]$ and remains below x-axis whereas $y = x$ remains above x-axis. These two curves cut at $(0, 0)$. Hence, the equation $\cos^2 x - \cos x = x$ has exactly one solution in $[0, \pi/2]$.

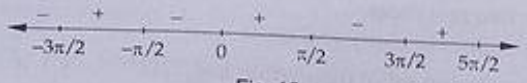


Fig. 16

EXAMPLE 86 *The number of points in* $(-\infty, \infty)$ *for which* $x^2 - x \sin x - \cos x = 0$, *is*

(a) 6 (b) 4 (c) 2 (d) 0

Ans. (c) **[JEE (Adv) 2013]**

SOLUTION We have to find the number of solution of the equation

$$x^2 - x \sin x - \cos x = 0 \text{ or, } x^2 = x \sin x + \cos x$$

Let $f(x) = x^2$ and $g(x) = x \sin x + \cos x$

Clearly, the number of solutions of the given equation is same as the number of points of intersection of the curves $y = f(x)$ and $y = g(x)$. The curve $y = f(x)$ is a parabola with vertex at $(0, 0)$ and y-axis as axis of symmetry.

Now, $g(x) = x \sin x + \cos x$

$\Rightarrow \quad g'(x) = x \cos x$

The changes in signs of $g'(x)$ are shown in the Figure 17.

Clealry, $x = 0, \frac{3\pi}{2}$ are points of relative minima and $x = \pm\frac{\pi}{2}$, $x = \frac{5\pi}{2}$ etc are points of relative maxima. Also, $g(x) = x \sin x + \cos x$ is an even function and $g(0) = 1$, $g(\pi/2) = \pi/2$ and $g(\pi) = -1$. So, a rough sketch of $y = g(x)$ is as shown in the following figure.

Clearly, $y = f(x)$ and $y = g(x)$ intersect at two points.

Hence, given equation has two solutions.

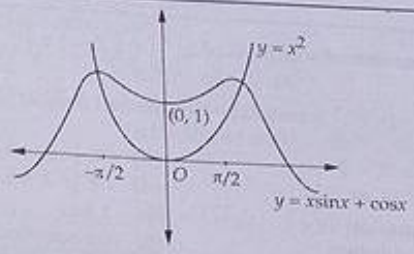


Fig. 17

EXAMPLE 87 *Let* $a \in R$ *and let* $f: R \to R$ *be given by* $f(x) = x^5 - 5x + a$, *then*

(a) $f(x)$ *has three real roots if* $a > 4$

(b) $f(x)$ *has only one real root if* $a > 4$

(c) $f(x)$ *has three real roots if* $a < -4$

(d) $f(x)$ *has three real roots if* $-4 < a < 4$

Ans. (b), (d) **[JEE (Adv) 2014]**

SOLUTION Let $g(x) = x^5 - 5x$ and $h(x) = -a$. Then, the number of real roots of $f(x) = 0$ is the number of points of intersection of the curves $y = g(x)$ and $y = h(x)$. Clearly, these two curves intersect at three points if $-4 < a < 4$.

So, $f(x)$ has three real roots if $-4 < a < 4$.

The two curves $y = g(x)$ and $y = h(x)$ intersect at exactly one point if $a > 4$ or $a < -4$.

So, $f(x)$ has only one real root if $a < -4$ or $a > 4$.

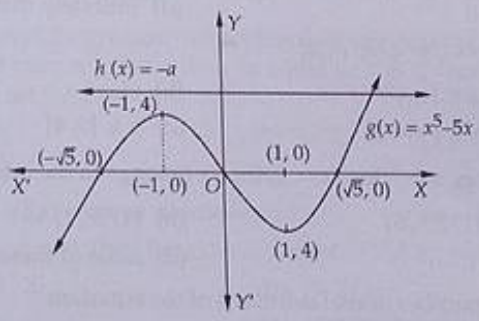


Fig. 18

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The number of real solutions of the equation $e^{-x} = x$, is
   (a) 0 (b) 1 (c) 2 (d) none of these
2. The number of real solutions of the equation
   $$e^{|x|} - |x| = 0, \text{ is}$$
   (a) 0 (b) 1 (c) 2 (d) none of these
3. The number of real solutions of the equation
   $$3^{-|x|} - 2^{|x|} = 0, \text{ is}$$
   (a) 0 (b) 1 (c) 2 (d) none of these
4. The number of real solutions of $\log_2 x + |x| = 0$, is
   (a) 0 (b) 1 (c) 3 (d) none of these
5. The number of real solutions of the equation
   $$\log_{0.5} x = |x|, \text{ is}$$
   (a) 1 (b) 0 (c) 2 (d) none of these
6. The number of solutions of the equation $\cos x + |x| = 0$ is
   (a) 0 (b) 1 (c) 2 (d) 3
7. The number of solutions of the equation
   $$2 \cos(e^x) = 3^x + 3^{-x}, \text{ is}$$
   (a) 0 (b) 1 (c) 2 (d) none of these

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8. The number of solutions of $3^{|x|} = |2 - |x||$ is
(a) 0 (b) 2 (c) 4 (d) infinite

9. If $[x]^2 = [x+6]$, where $[x]$ = the greatest integer less than or equal to x, then x must be such that
(a) $x = 3, -2$ (b) $x \in [-2, -1)$
(c) $x \in [3, 4)$ (d) $x \in [-2, -1) \cup [3, 4)$

10. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has
(a) no solution (b) one solution
(c) two solutions (d) more than two solutions

11. The equation $\sqrt{4x+9} - \sqrt{11x+1} = \sqrt{7x+4}$ has
(a) no solution (b) one solution
(c) two solutions (d) more than two solutions

12. The number of real roots of the equation
$$(\sin 2^x)(\cos 2^x) = \frac{2^x + 2^{-x}}{2},$$ is
(a) 1 (b) 2 (c) 3 (d) none of these

13. The number of irrational solutions of the equation
$$\sqrt{x^2 + \sqrt{x^2+11}} + \sqrt{x^2 - \sqrt{x^2+11}} = 4,$$ is
(a) 0 (b) 2 (c) 4 (d) 11

14. The total number of roots of the equation
$$|x - x^2 - 1| = |2x - 3 - x^2|$$ is
(a) 1 (b) 2
(c) 0 (d) infinitely many

15. If $3^{x/2} + 2^x > 25$, then
(a) $x \in [4, \infty)$ (b) $(4, \infty)$
(c) $x \in (-\infty, 4]$ (d) $x \in [0, 4]$

16. If $(\log_5 x)^2 + (\log_5 x) < 2$, then x belongs to the interval
(a) $(1/25, 5)$ (b) $(1/5, 1/\sqrt{5})$
(c) $(1, \infty)$ (d) none of these

17. The number of real solutions of the equation
$$27^{1/x} + 12^{1/x} = 2 \cdot 8^{1/x},$$ is
(a) 1 (b) 2 (c) 0 (d) infinite

18. The set of all real numbers satisfying the inequation
$$2^x + 2^{|x|} \geq 2\sqrt{2},$$ is
(a) $(1/2, \infty)$
(b) $\left(-\infty, \log_2(\sqrt{2}-1)\right)$
(c) $(-\infty, 1/2)$
(d) $[1/2, \infty) \cup \left(-\infty, \log_2(\sqrt{2}-1)\right)$

19. The set of all real numbers satisfying the inequation
$$x^{(\log_{10} x)^2 - 3(\log_{10} x) + 1} > 1000,$$ is
(a) $(0, 1000)$ (b) $(1000, \infty)$
(c) $(0, 100)$ (d) none of these

20. The solution set of the inequation
$$\log_{\sin 2\pi/3}(x^2 - 3x + 2) \geq 2,$$ is
(a) $[1/2, 1)$ (b) $(2, 5/2]$
(c) $[1/2, 1) \cup (2, 5/2]$ (d) $[1/2, 5/2]$

21. The equation $e^x = x(x+1)$, $x < 0$ has
(a) no real roots set
(b) exactly one real root
(c) two real roots
(d) infinitely many real roots

22. The solution set of inequation $\log_{1/3}(2^{x+2} - 4^x) \geq -2$, is
(a) $(-\infty, 2 - \sqrt{13})$ (b) $(-\infty, 2 + \sqrt{13})$
(c) $(-\infty, 2)$ (d) none of these

23. If $x, y \in R$, then $\frac{1}{2}\left(x + y + |x - y|\right) = x$ holds iff
(a) $x > y$ (b) $x < y$ (c) $x = y$ (d) $x \geq y$

24. The number of integral roots of the equation
$$e^{x-8} + 2x - 17 = 0,$$ is
(a) 1 (b) 2 (c) 4 (d) 8
[JEE (Orissa) 2002]

25. The solution set of the inequation
$$\log_{1/3}(x^2 + x + 1) + 1 > 0,$$ is
(a) $(-\infty, -2) \cup (1, \infty)$ (b) $[-1, 2]$
(c) $(-2, 1)$ (d) R

26. If $\log_3 x - \log_x 27 < 2$, then x belongs to the interval
(a) $(1/3, 27)$ (b) $(1/27, 3)$
(c) $(1/9, 9)$ (d) none of these

27. $\log_{16} x^3 + \left(\log_2 \sqrt{x}\right)^2 < 1$ iff x lies in
(a) $(2, 16)$ (b) $(0, 1/16)$
(c) $(1/16, 2)$ (d) none of these

28. The number of solutions of the equation
$$\log_{x-3}(x^3 - 3x^2 - 4x + 8) = 3,$$ is
(a) 1 (b) 2 (c) 3 (d) 4

29. If $0 < a < 1$, then the solution set of the inequation
$$\frac{1 + (\log_a x)^2}{1 + (\log_a x)} > 1,$$ is
(a) $(1, 1/a)$
(b) $(0, a)$
(c) $(1, 1/a) \cup (0, a)$
(d) none of these

30. The number of solution(s) of the inequation $\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} \leq 4 - 2x - x^2$, is
(a) 1 (b) 2
(c) 4 (d) infinitely many

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Answers

1. (b) 2. (a) 3. (b) 4. (b) 5. (a) 6. (a) 7. (a) 8. (b) 9. (d) 10. (a) 11. (b) 12. (d) 13. (b) 14. (a) 15. (b) 16. (a) 17. (c) 18. (d) 19. (d) 20. (c) 21. (b) 22. (c) 23. (d) 24. (a) 25. (c) 26. (d) 27. (c) 28. (a) 29. (c) 30. (a)

CHAPTER TEST

Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct. Mark the correct alternative in each case.

1. If $3^x + 2^{2x} \geq 5^x$, then the solution set for x is
(a) $(-\infty, 2]$ (b) $[2, \infty)$ (c) $[0, 2]$ (d) $[2]$
[JEE (Orissa) 2002]

2. The number of real solutions of the equation $1 - x = [\cos x]$, is
(a) 1 (b) 2
(c) 3 (d) none of these

3. The number of solutions of
$[\sin x + \cos x] = 3 + [-\sin x] + [-\cos x]$
in the internal $[0, 2\pi]$ is (where [.] denotes the the greatest integer function).
(a) 0 (b) 4 (c) infinite (d) 1

4. Let $x = \frac{a+2b}{a+b}$ and $y = \frac{a}{b}$, where a and b are positive integers. If $y^2 > 2$, then
(a) $x^2 \leq 2$ (b) $x^2 < 2$ (c) $x^2 > 2$ (d) $x^2 \geq 2$
[JEE (Orissa) 2003]

5. The solution set contained in R of the inequation $3^x + 3^{1-x} - 4 < 0$, is
(a) $(1, 3)$ (b) $(0, 1)$ (c) $(1, 2)$ (d) $(0, 2)$
[EAMCET 2003]

6. If $(\sin\alpha)^x + (\cos\alpha)^x \geq 1$, $0 < \alpha < \frac{\pi}{2}$, then
(a) $x \in [2, \infty)$ (b) $x \in (-\infty, 2]$
(c) $x \in [-1, 1]$ (d) none of these

7. The solution set of the inequation
$5^{(1/4)\left(\log_5 x\right)^2} \geq 5\, x^{(1/5)\left(\log_5 x\right)}$, is
(a) $(0, 5^{-2\sqrt{5}}]$ (b) $[5^{2\sqrt{5}}, \infty)$
(c) $(0, 5^{-2\sqrt{5}}] \cup [5^{2\sqrt{5}}, \infty)$ (d) $(0, \infty)$

8. The number of real roots of the equation $x^2 + x + 3 + 2\sin x = 0$ in the interval $[-\pi, \pi]$, is
(a) 2 (b) 4
(c) 6 (d) none of these

9. The number of real roots of the equation $1 + 3^{x/2} = 2^x$, is
(a) 0 (b) 1
(c) 2 (d) none of these

10. The number of roots of the equation $\sin \pi x = \left|\log |x|\right|$, is
(a) 2 (b) 4 (c) 5 (d) 6

11. The number of roots of the equation $[\sin^{-1} x] = x - [x]$, is
(a) 0 (b) 1
(c) 2 (d) none of these

12. The number of values of a for which the system of equations $2^{|x|} + |x| = y + x^2 + a$ and $x^2 + y^2 = 1$ has only one solution where a, x, y are real, is
(a) 1 (b) 2
(c) finitely many but more than 2 (d) infinitely many

13. The number of real solutions (x, y, z, t) of simultaneous equations
$2y = \frac{11}{x} + x, 2z = \frac{11}{y} + y, 2t = \frac{11}{z} + z, 2x = \frac{11}{t} + t$, is
(a) 0 (b) 1 (c) 2 (d) 4

14. If the sum of the greatest integer less than or equal to x and the least integer greater than or equal to x is 5, then the solution set for x is
(a) $(2, 3)$ (b) $(0, 5)$
(c) $[5, 6)$ (d) none of these

15. If x, y, z are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then the exhaustive set of values of x, is
(a) $[2/3, 2]$ (b) $[0, 2/3]$
(c) $[0, 2]$ (d) $[-1/3, 2/3]$

16. The product of real roots of the equation $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$, is
(a) 720 (b) 20 (c) 36 (d) none of these

17. $x^8 - x^5 - \frac{1}{x} + \frac{1}{x^4} > 0$, is satisfied for
(a) only positive values of x
(b) only negative values of x
(c) all real numbers except zero
(d) only for $x > 1$

18. The number of solutions of the equation
$\frac{(1 + e^{x^2})\sqrt{1 + x^2}}{\sqrt{1 + x^4} - x^2} = 1 + \cos x$, is
(a) 1 (b) 2 (c) 3 (d) 4

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19. The number of real roots of the equation

$1 + a_1 x + a_2 x^2 + \ldots + a_n x^n = 0$, where

$|x| < \frac{1}{3}$ and $|a_n| < 2$, is

(a) n if n is even (b) 1 if n is odd

(c) 0 for any $n \in N$ (d) none of these

20. Let a, b be integers and $f(x)$ be a polynomial with integer coefficients such that $f(b) - f(a) = 1$. Then, the value of $b - a$, is

(a) 1 (b) -1 (c) $1, -1$ (d) none of these

21. Let $P_n(x) = 1 + 2x + 3x^2 + \ldots + (n+1)x^n$ be a polynomial such that n is even. Then, the number of real roots of $P_n(x)$, is

(a) 0 (b) n (c) 1 (d) none of these

22. Let n be an odd positive integer. Then, the number of real roots of the polynomial

$P_n(x) = 1 + 2x + 3x^2 + \ldots + (n+1)x^n$, is

(a) 0 (b) n (c) 1 (d) none of these

23. Let n be an odd integer such that the polynomial $P_n(x) = 1 + 2x + 3x^2 + \ldots + (n+1)x^n$ has exactly one real root. This real root α satisfies

(a) $-1 < \alpha < 0$ (b) $0 < \alpha < 1$

(c) $0 \le \alpha \le 1$ (d) $-1 \le \alpha \le 0$

24. The number of positive real roots of $x^4 - 4x - 1 = 0$, is

(a) 3 (b) 2 (c) 1 (d) 0

25. The number of negative real roots of $x^4 - 4x - 1 = 0$, is

(a) 3 (b) 2 (c) 1 (d) 0

26. The number of complex roots of the equation $x^4 - 4x - 1 = 0$, is

(a) 3 (b) 2 (c) 1 (d) 0

27. Let $F(x)$ be a function defined by $F(x) = x - [x]$, $0 \ne x \in R$, where $[x]$ is the greatest integer less than or equal to x. Then, the number of solutions of $F(x) + F\left(\frac{1}{x}\right) = 1$,

(a) 0 (b) infinite (c) 1 (d) 2

28. The number of negative integral solutions of

$x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{|x-3|+4} + 2^{x-1}$,

(a) none (b) only one

(c) two (d) four

[CEE (Delhi) 19

29. The number of solutions of

$\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} = 4 - 2x - x^2$, is

(a) 1 (b) 2 (c) 3 (d) 4

30. The number of real solutions of $1 + |e^x - 1| = e^x(e^x - 2)$,

(a) 1 (b) 2 (c) 3 (4) 4

[IPU 20

Answers

1. (a) **2.** (b) **3.** (c) **4.** (b) **5.** (b) **6.** (b) **7.** (c) **8.** (d) **9.** (b) **10.** (d) **11.** (b) **12.** (a) **13.** (c) **14.** (a) **15.** (a) **16.** (b) **17.** (c) **18.** (a) **19.** (c) **20.** (c) **21.** (a) **22.** (c) **23.** (a) **24.** (c) **25.** (c) **26.** (b) **27.** (b) **28.** (a) **29.** (a) **30.** (a)

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INEQUALITIES

1. ELEMENTARY PROPERTIES OF INEQUALITIES

PROPERTY-1 *If a, b, c are real numbers such that $a > b$ and $b > c$, then $a > c$.*

More generally, if $a_1, a_2, a_3, \ldots, a_{n-1}, a_n$ are real numbers such that $a_1 > a_2 > a_3 > a_4 > \ldots > a_{n-1} > a_n$, then $a_1 > a_n$.

PROPERTY-2 *If a, b are real numbers, then*

$$a > b \Rightarrow a + c > b + c \quad \text{for all } c \in R$$

Also, $$a > b \Rightarrow a - c > b - c \quad \text{for all } c \in R.$$

PROPERTY-3 *If a, b are two real numbers, then*

$$a > b \Rightarrow ac > bc \quad \text{and} \quad \frac{a}{c} > \frac{b}{c} \text{ for all } c > 0, c \in R$$

$$\text{Also, } a > b \Rightarrow ac < bc \quad \text{and} \quad \frac{a}{c} < \frac{b}{c} \text{ for all } c < 0, c \in R.$$

PROPERTY-4 *If $a > b > 0$, then* $\frac{1}{a} < \frac{1}{b}$.

PROPERTY-5 *If $a_1 > b_1, a_2 > b_2, a_3 > b_3, \ldots, a_n > b_n$, then*

$$a_1 + a_2 + a_3 + \ldots + a_n > b_1 + b_2 + b_3 + \ldots + b_n$$

PROPERTY-6 *If $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ are positive real numbers such that*

$a_1 > b_1, a_2 > b_2, \ldots, a_n > b_n$, *then* $a_1 a_2 a_3 \ldots a_n > b_1 b_2 b_3 \ldots b_n$

PROPERTY-7 *If a, b are positive real numbers such that $a < b$ and if n is any positive rational number, then*

(i) $a^n < b^n$ (ii) $a^{-n} > b^{-n}$ (iii) $a^{1/n} < b^{1/n}$

PROPERTY-8 *If $0 < a < 1$ and n is any positive rational number, then*

(i) $0 < a^n < 1$ (ii) $a^{-n} > 1$

PROPERTY-9 *If $b > 1$ and n is any positive rational number, then*

(i) $b^n > 1$ (ii) $0 < b^{-n} < 1$

PROPERTY-10 *If $0 < a < 1$ and m, n are positive rational numbers, then*

(i) $m > n \Rightarrow a^m < a^n$ (ii) $m < n \Rightarrow a^m > a^n$

PROPERTY-11 *If $a > 1$ and m, n are positive rational numbers, then*

(i) $m > n \Rightarrow a^m > a^n$ (ii) $m < n \Rightarrow a^m < a^n$

PROPERTY-12 (i) *If $a > 1$ and $x > y > 0$, then* $\log_a x > \log_a y$

(ii) *If $0 < a < 1$ and $x > y > 0$, then* $\log_a x < \log_a y$.

PROPERTY-13 *For real number a, we have*

$$-|a| \le a \le |a|$$

PROPERTY-14 *If $r \ge 0$ and a is any real number, then*

$$|x - a| \le r \Leftrightarrow a - r \le x \le a + r$$

PROPERTY-15 *If a and b are positive real numbers, then*

$$a < b \Rightarrow \frac{a + c}{b + c} > \frac{a}{b} \quad \text{for all } c > 0$$

$$a > b \Rightarrow \frac{a + c}{b + c} < \frac{a}{b} \quad \text{for all } c > 0$$

PROPERTY-16 (i) *If a is a positive real number, then* $a + \frac{1}{a} \ge 2$.

(ii) *If a is a negative real number, then* $a + \frac{1}{a} \le -2$.

PROPERTY-17 $|a + b| \le |a| + |b|$

More generally, we have

$$|a_1 + a_2 + \ldots + a_n| \le |a_1| + |a_2| + \ldots + |a_n|$$

2. SOME INEQUALITIES

THEOREM 1 *(Arithmetic-geometric Mean Inequality): If $a_1, a_2, \ldots, a_n$ are n distinct positive real numbers, then*

$$\frac{a_1 + a_2 + \ldots + a_n}{n} > (a_1 a_2 \ldots a_n)^{1/n}$$

i.e. A.M. > G.M.

REMARK 1 *If $a_1 = a_2 = \ldots = a_n$, then A.M. = G.M.*

Particular cases: For $n = 2$ and 3 the above inequality becomes

$$\frac{a_1 + a_2}{2} > \sqrt{a_1 a_2}, \quad \frac{a_1 + a_2 + a_3}{3} > \sqrt[3]{a_1 a_2 a_3}$$

REMARK 2 *If $a_1, a_2, \ldots, a_n$ are n distinct positive real numbers, then*

$H = \dfrac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}}$ *is called the harmonic mean of $a_1, a_2, \ldots, a_n$*

and, we have

A.M. > G.M. > H.M.

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$$\frac{\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}}{n} > \left(\frac{1+2+\ldots+n}{n}\right)^{-1/2}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}} > n\sqrt{\frac{2}{n+1}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}} > \sqrt{n}\sqrt{\frac{2n}{n+1}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}} > \sqrt{n} \quad [\because\ 2n > n+1 \text{ for all } n \geq 2]$$

So, statement–1 is true.

We have,

$n(n+1) < (n+1)(n+1)$ for all $n \geq 2$

$\Rightarrow \quad \sqrt{n(n+1)} < n+1$ for all $n \geq 2$

So, statement–2 is true.

Clearly, statement–2 is not a correct explanation for statement–1.

EXAMPLE 8 *Statement–1: If a, b, c are positive numbers in AP such that $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca} = 1$, then the least value of b is $\sqrt{3}$.*

Statement–2: A.M. $\geq$ G.M.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION We have,

$$\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca} = 1$$

$$\Rightarrow \quad a+b+c = abc$$

$$\Rightarrow \quad 3b = abc \quad [\because\ a+c = 2b]$$

$$\Rightarrow \quad ac = 3$$

Now, A.M. $\geq$ G.M.

$$\Rightarrow \quad \frac{a+c}{2} \geq \sqrt{ac} \Rightarrow b \geq \sqrt{3}.$$

EXAMPLE 9 *If a, b, c are three unequal positive numbers, then*

Statement-1: The product of their sum and the sum of their reciprocals exceeds 9.

Statement-2: AM of n positive numbers exceeds their HM

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement-2 is true.

$$AM = \frac{a+b+c}{3} \text{ and } HM = \frac{1}{\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}} = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

Now,

$AM > HM$

$$\Rightarrow \quad \frac{a+b+c}{3} > \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

$$\Rightarrow \quad (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) > 9$$

So, statement-1 is also true and statement-2 is a correct explanation for statement-1.

EXERCISE

SECTION-I

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $0 < \theta < \pi$, then the minimum value of $\sin^5\theta + \text{cosec}^5\theta$ is
   (a) 0 (b) 1 (c) 2 (d) none of these

2. If $a > 1, b > 1, c > 1, d > 1$, then the minimum value of $\log_b a + \log_a b + \log_d c + \log_c d$ is
   (a) 1 (b) 2 (c) 3 (d) 4

3. The minimum value of $4^x + 4^{1-x}, x \in R$, is
   (a) 1 (b) 2 (c) 4 (d) none of these

4. If $x = \log_{2^2} 2 + \log_{2^3} 2^2 + \log_{2^4} 2^3 + \ldots + \log_{2^{n+1}} 2^n$, then
   (a) $x \geq \left(\frac{1}{n+1}\right)^{1/n}$ (b) $x \geq n\left(\frac{1}{n+1}\right)^{1/n}$ (c) $x \geq \left(\frac{n}{n+1}\right)^{1/n}$ (d) none of these

5. The least value of $5^{\sin x - 1} + 5^{-\sin x - 1}$ is
   (a) 10 (b) $\frac{5}{2}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

6. If a_1, a_2, a_3 be any positive real numbers, then which of the following statements is not true.
   (a) $3a_1, a_2, a_3 \leq a_1^3 + a_2^3 + a_3^3$
   (b) $\frac{a_1}{a_2}+\frac{a_2}{a_3}+\frac{a_3}{a_1} \geq 3$
   (c) $(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right) \geq 9$
   (d) $(a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^3 \leq 27$ [JEE (Orissa) 2002]

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7. For $\theta > \pi/3$, the value of $f(\theta) = \sec^2\theta + \cos^2\theta$ always lies in the interval
(a) $(0, 2)$ (b) $[0, 1]$ (c) $(1, 2)$ (d) $[2, \infty)$
[JEE (Orissa) 2002]

8. If a and b are two different positive real numbers then which of the following statement is true?
(a) $2\sqrt{ab} > a + b$ (b) $2\sqrt{ab} < a + b$
(c) $2\sqrt{ab} = a + b$ (d) none of these

9. If $0 < x < \frac{\pi}{2}$, then the minimum value of $\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x}$ is
(a) $\sqrt{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 1

10. If a, b, c are positive real numbers such that $a + b + c = 2$ then, which one of the following is true?
(a) $(2-a)(2-b)(2-c) \geq 8abc$ (b) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 2$
(c) $(2-a)(2-b)(2-c) < 8abc$ (d) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$

11. If a, b, c are positive real numbers such that $a + b + c = p$ then, which of the following is true?
(a) $(p-a)(p-b)(p-c) \geq \frac{8}{27}p^3$
(b) $(p-a)(p-b)(p-c) \geq 8abc$
(c) $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \geq p$
(d) none of these

12. The product of n positive numbers is n^n, then their sum is
(a) a positive integer (b) divisible by n
(c) equal to $n + \frac{1}{n}$ (d) never less than n^2

13. If roots of the equation $x^4 - 8x^3 + bx^2 + cx + 16 = 0$ are positive, then
(a) $b = 8 = c$ (b) $b = -24, c = -32$
(c) $b = 24, c = -32$ (d) $b = 24, c = 32$

14. If a, b are distinct positive real numebrs, then which one of the following is true?
(a) $a^4 + b^4 > a^3 b + ab^3$ (b) $a^4 + b^4 < a^3 b + ab^3$
(c) $a^3 + b^3 < a^2 b + ab^2$ (d) none of these

15. $2^{\sin^2 x} + 2^{\cos^2 x}$ is
(a) ≤ 2 (b) ≥ 2 (c) ≤ 1 (d) ≥ 1

16. If $xyz = abc$, then the least value of $bcx + cay + abz$ is
(a) $3abc$ (b) $6abc$ (c) abc (d) $4abc$

17. The number of ordered 4-tuples (x, y, z, w) where $x, y, z, w \in [0, 10]$ which satisfy the inequality
$2^{\sin^2 x} \times 3^{\cos^2 y} \times 4^{\sin^2 z} \times 5^{\cos^2 w} \geq 120$, is
(a) 81 (b) 144
(c) 0 (d) infinite

18. The range of ab if $|a| \leq 1$ and $a + b = 1$, $(a, b \in R)$, is
(a) $[0, 1/4]$ (b) $[-2, 1/4]$
(c) $[1/4, 2]$ (d) $[0, 2]$

19. If $x^2 + \frac{1}{x^2} = A$ and $x - \frac{1}{x} = B$, where $x \in R$ and $B > 0$, then the minimum value of $\frac{A}{B}$ is
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\sqrt{2} + 2$ (d) none of these

20. The least perimeter of a cyclic quadrilateral of given area A square units is
(a) $\sqrt{A}$ (b) $2\sqrt{A}$ (c) $3\sqrt{A}$ (d) $4\sqrt{A}$

21. A stick of length 20 units is to be divided into n parts so that the product of the lengths of the parts is greater than unity. The maximum possible value of n is
(a) 18 (b) 19 (c) 20 (d) 21

22. If x, y, z are positive real numbers such that $x^2 + y^2 + z^2 = 27$, then $x^3 + y^3 + z^3$ has
(a) minimum value 81 (b) maximum value 81
(c) minimum value 27 (d) maximum value 27

23. If $x_1, x_2, \ldots, x_n$ are real numbers, then the largest value of the expression
$\sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \ldots + \sin x_n \cos x_1$ is
(a) n (b) $\frac{n}{2}$ (c) $\frac{n}{4}$ (d) $\sqrt{n^2 - 1}$

24. The minimum value of the sum of the lengths of diagonals of a cyclic quadrilateral of area a^2 square units is
(a) $\sqrt{2}a$ (b) $2\sqrt{2}a$
(c) $2a$ (d) none of these

25. The minimum value of
$|\sin x + \cos x + \tan x + \sec x + \text{cosec}\, x + \cot x|$ is
(a) $2\sqrt{2} - 1$ (b) $2\sqrt{2} + 1$
(c) $\sqrt{2} - 1$ (d) $\sqrt{2} + 1$

26. The expression $(a+b+c)(b+c-a)(c+a-b)(a+b-c) \leq k b^2 c^2$, then k can take the value
(a) 4 (b) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{2}$

27. If n is even and $n \geq 4, x_1, x_2, \ldots, x_n \geq 0$ and $x_1 + x_2 + \ldots + x_n = 1$, then $P = x_1 x_2 + x_2 x_3 + \ldots + x_{n-} x_n$ cannot exceed
(a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$
(c) $\frac{1}{2n}$ (d) none of these

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SECTION-II

ASSERTION–REASON TYPE MCQs

Each of the following questions contains STATEMENT–1 (Assertion) and STATEMENT–2 (Reason). Each of these questions has following 4 choices (1), (2), (3), and (4) out of which only one is the correct answer.

1. *Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.*
2. *Statement–1 is True, Statement–2 is True; Statement–2 is not a correct explanation for Statement–1.*
3. *Statement–1 is True, Statement–2 is False.*
4. *Statement–1 is False, Statement – 2 is True.*

28. If $\log_4 (x+2y) + \log_4 (x-2y) = 1$, then

Statement–1: $\left(|x| - |y|\right)_{\min} = \sqrt{3}$

Statement–2: AM $\geq$ GM

(a) 1 (b) 2 (c) 3 (d) 4

29. Statement–1: If a, b, c be the length of the sides of ΔABC, then

$$\left|\frac{a-b}{a+b} + \frac{b-c}{b+c} + \frac{c-a}{c+a}\right| \leq \frac{1}{8}$$

Statement–2: $|x+y+z| \leq |x| + |y| + |z|$

(a) 1 (b) 2 (c) 3 (d) 4

Answers

1. (c) **2.** (d) **3.** (c) **4.** (b) **5.** (c) **6.** (c) **7.** (d) **8.** (b) **9.** (d) **10.** (a) **11.** (b) **12.** (d) **13.** (c) **14.** (a) **15.** (b) **16.** (a) **17.** (b) **18.** (b) **19.** (a) **20.** (d) **21.** (b) **22.** (a) **23.** (b) **24.** (b) **25.** (a) **27.** (d)

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SETS

1. SETS

A set is a well defined collection of objects. Here term 'well defined' means that there is some definite rule on the basis of which one can decide whether an object is in the collection or not. Sets are generally denoted by capital letters e.g. A, B, C, X, Y, Z etc. If an object 'a' is in set A, then we say that 'a' is an element of set A or 'a' belongs to set A and we write $a \in A$. If 'a' is not in set A, then we say that 'a' does not belong to set A and we write $a \notin A$.

The collection of cricketers in the world who have played at least five test matches is a set. However, the collection of good cricket players of India is not a set, because the term "good player" is vague and it is not well defined.

ILLUSTRATION 1 *Which of the following collection is a set?*

(a) *The collection of all girls in your class.*

(b) *The collection of intelligent girls in your class.*

(c) *The collection of beautiful girls in your class.*

(d) *The collection of tall girls in your class.*

Ans. (a)

SOLUTION Clearly, collections in (b), (c) and (d) are not well defined collections. So, they do not form a set.

A set is often described in one of the following two ways:

ROSTER METHOD *In this method, a set is described by listing elements, separated by commas, within braces { }.*

For example, the set of vowels of English Alphabet may be described as $\{a, e, i, o, u\}$.

The order in which the elements are written in a set makes no difference. Also, the repetition of an element has no effect.

SET BUILDER METHOD *In this method, a set is described by a characterising property, say $P(x)$, of its elements x. In such a case, the set is described by $\{x : P(x) \text{ holds}\}$ which is read as 'the set of all x such that $P(x)$ holds'.*

The set of all even integers can be written as

$$E = \{x : x = 2n, n \in Z\}$$

The set $\{1, 4, 9, 16, ...\}$ can be written as $\{x : x = n^2, n \in N\}$ or $\{x^2 : x \in N\}$.

ILLUSTRATION 2 *If B is the set whose elements are obtained by adding 1 to each of the even numbers, then the set builder notation of B is*

(a) $B = \{x : x \text{ is even}\}$ (b) $B = \{x : x \text{ is odd and } x > 1\}$

(c) $B = \{x : x \text{ is odd and } x \in Z\}$ (d) $B = \{x : x \text{ is an integer}\}$

[CEE (Delhi) 2002]

Ans. (c)

SOLUTION When we add 1 to even integers, we obtain odd integers

$\therefore \quad B = \{x : x \text{ is an odd integer}\} = \{x : x \text{ is odd and } x \in Z\}$

2. TYPES OF SETS

EMPTY SET *A set is said to be empty or null or void set, if it has no element and it is denoted by ϕ or $\{\}$.*

ILLUSTRATION 3 *Which of the following is the empty set ?*

(a) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$

(b) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$

(c) $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$

(d) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

Ans. (b)

SOLUTION We observe that there is no real number x such that $x^2 + 1 = 0$

$\therefore \quad \{x : x \text{ is a real number and } x^2 + 1 = 0\} = \phi$

SINGLETON SET *A set consisting of a single element is called a singleton set.*

FINITE SET *A set is called a finite set if it is either a void set or its elements can be listed (counted, labelled) by natural numbers 1, 2, 3, ... and the process of listing terminates at a certain natural number n (say).*

The number n in the above definition is called the cardinal number or order of a finite set A and is denoted by $n(A)$.

A set whose elements cannot be listed by natural numbers 1, 2, 3,....., n for any natural number n is called an infinite set.

ILLUSTRATION 4 *Which of the following sets is not finite?*

(a) $\{(x, y) : x^2 + y^2 \le 1 \le x + y,\ x, y \in R\}$

(b) $\{(x, y) : x^2 + y^2 \le 1 \le x + y, x, y \in Z\}$

(c) $\{(x, y) : x^2 \le y \le |x|, x, y \in Z\}$

(d) $\{(x, y) : x^2 + y^2 = 1, x, y \in Z\}$

Ans. (a)

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EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, then
   (a) $x = 39$ (b) $x = 63$
   (c) $39 \le x \le 63$ (d) none of these

2. In a battle 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs. The minimum value of x is
   (a) 10 (b) 12
   (c) 15 (d) none of these

3. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, ..., 18\}$ and N is the universal set, then $A' \cup ((A \cup B) \cap B')$ is
   (a) A (b) N
   (c) B (d) none of these

4. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, then $X \cup Y$ is equal to
   (a) X (b) Y
   (c) N (d) none of these

5. If sets A and B are defined as
   $A = \{(x, y) : y = e^x, x \in R\}$
   $B = \{(x, y) : y = x, x \in R\}$, then
   (a) $B \subset A$ (b) $A \subset B$
   (c) $A \cap B = \phi$ (d) $A \cup B = A$

6. Let $A = \{(x, y) : y = e^x, x \in R\}$,
   $B = \{(x, y) : y = e^{-x}, x \in R\}$. Then,
   (a) $A \cap B = \phi$ (b) $A \cap B \neq \phi$
   (c) $A \cup B = R^2$ (d) none of these

7. Suppose $A_1, A_2, ..., A_{30}$ are thirty sets each with five elements and $B_1, B_2, ..., B_n$ are n sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^{n} B_i = S$. If each element of S belongs to exactly ten of the A_i's and exactly 9 of the B_j's, then the value of n is
   (a) 15 (b) 135 (c) 45 (d) 90
   [CEE (Delhi) 2009]

8. If sets A and B are defined as
   $$A = \left\{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\right\},$$
   $$B = \{(x, y) : y = -x, x \in R\}, \text{ then}$$
   (a) $A \cap B = A$ (b) $A \cap B = B$
   (c) $A \cap B = \phi$ (d) none of these

9. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then
   (a) $d = bc$ (b) $c = bd$
   (c) $b = cd$ (d) none of these

10. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$, then
    (a) $d = bc$ (b) $c = bd$
    (c) $b = cd$ (d) none of these.

11. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then the number of families which buy A only is
    (a) 3100 (b) 3300 (c) 2900 (d) 1400

12. In the above question, the number of families which buy none of A, B and C is
    (a) 4000 (b) 3300 (c) 4200 (d) 5000

13. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of the set of all determinants with value –1. Then
    (a) C is empty
    (b) B has as many elements as C
    (c) $A = B \cup C$
    (d) B has twice as many elements as C.

14. If A and B are two sets, then $A \cap (A \cup B)$ equals
    (a) A (b) B
    (c) ϕ (d) none of these

15. If A is a finite set having n elements, then $P(A)$ has
    (a) $2n$ elements (b) 2^n elements
    (c) n elements (d) none of these

16. If $A = \{\phi, \{\phi\}\}$, then the power set of A is
    (a) A (b) $\{\phi, \{\phi\}, A\}$
    (c) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$ (d) none of these

17. Let A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?
    (a) 3 (b) 6 (c) 9 (d) 18

18. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are
    (a) $m = 7, n = 6$ (b) $m = 6, n = 3$
    (c) $m = 5, n = 1$ (d) $m = 8, n = 7$

19. If X and Y are two sets, then $X \cap (Y \cup X)'$ equals
    (a) X (b) Y (c) ϕ (d) none of these
    [CEE (Delhi) 2005

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20. Let $A = \{x : x$ is a multiple of 3$\}$ and $B = \{x : x$ is a multiple of 5$\}$. Then, $A \cap B$ is given by
(a) $\{3, 6, 9, ...\}$ (b) $\{5, 10, 15, 20, ...\}$
(c) $\{15, 30, 45, ...\}$ (d) none of these

21. Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$. Then, $A \cup (B \cap C)$ is
(a) $\{3\}$ (b) $\{1, 2, 3, 4\}$
(c) $\{1, 2, 5, 6\}$ (d) $\{1, 2, 3, 4, 5, 6\}$

22. Let A and B be two non-empty subsets of a set X such that A is not a subset of B. Then,
(a) A is a subset of complement of B
(b) B is a subset of A
(c) A and B are disjoint
(d) A and the complement of B are non-disjoint.

23. Let U be the universal set for sets A and B such that $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then, $n(A' \cap B')$ is equal to 300, provided that $n(U)$ is equal to
(a) 600 (b) 700 (c) 800 (d) 900

24. Out of 800 boys in a school 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is
(a) 160 (b) 240 (c) 216 (d) 128

25. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examinations is
(a) 11 (b) 12 (c) 13 (d) 14

26. Let Z be the set of all integers and
$A = \{(a, b) : a^2 + 3b^2 = 28, a, b \in Z\}$ and
$B = \{(a, b) : a > b, a, b \in Z\}$. Then, the number of elements in $A \cap B$, is
(a) 2 (b) 4 (c) 6 (d) 5

27. In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. If each student has taken either Mathematics or Economics or both, then the number of students who have taken Economics but not Mathematics is
(a) 7 (b) 25 (c) 18 (d) 32

28. If $A = \left\{(x, y) : y = \frac{4}{x}, x \neq 0\right\}$ and
$B = \{(x, y) : x^2 + y^2 = 8, x, y \in R\}$, then
(a) $A \cap B = \phi$
(b) $A \cap B$ contains one point only
(c) $A \cap B$ contains two points only
(d) $A \cap B$ contains 4 points only.

29. If $A = \{x : x$ is a multiple of 4$\}$ and,
$B = \{x : x$ is a multiple of 6$\}$, then $A \cap B$ consists of multiples of
(a) 16 (b) 12 (c) 8 (d) 4

30. If $A = \{(x, y) : x^2 + y^2 = 4; x, y \in R\}$ and
$B = \{(x, y) : x^2 + y^2 = 9; x, y \in R\}$, then
(a) $A - B = \phi$ (b) $B - A = B$
(c) $A \cap B \neq \phi$ (d) $A \cap B = A$

31. If $A = \{(x, y) : y^2 = x; x, y \in R\}$ and
$B = \{(x, y) : y = |x|; x, y \in R\}$, then
(a) $A \cap B = \phi$
(b) $A \cap B$ is a singleton set
(c) $A \cap B$ contains two elements only
(d) $A \cap B$ contains three elements only.

32. If $A = \left\{\theta : \cos\theta > -\frac{1}{2}, 0 \leq \theta \leq \pi\right\}$ and
$B = \left\{\theta : \sin\theta > \frac{1}{2}, \frac{\pi}{3} \leq \theta \leq \pi\right\}$, then
(a) $A \cap B = \{\theta : \pi/3 \leq \theta \leq 2\pi/3\}$
(b) $A \cap B = \{\theta : -\pi/3 \leq \theta \leq 2\pi/3\}$
(c) $A \cup B = \{\theta : -5\pi/6 \leq \theta \leq 5\pi/6\}$
(d) $A \cup B = \{\theta : 0 \leq \theta \leq \pi/6\}$

33. In rule method the null set is represented by
(a) $\{\}$ (b) Φ
(c) $\{x : x \neq x\}$ (d) $\{x : x = x\}$

34. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to
(a) A (b) B (c) Φ (d) $A \cap B^c$

35. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then, $n(A^c \cap B^c) =$
(a) 400 (b) 600 (c) 300 (d) 200

36. If $A = \{x : x$ is a multiple of 3$\}$ and,
$B = \{x : x$ is a multiple of 5$\}$, then $A - B$ is
(a) $\overline{A} \cap B$ (b) $A \cap \overline{B}$ (c) $\overline{A} \cap \overline{B}$ (d) $\overline{A \cap B}$

37. For any three sets A_1, A_2, A_3, let $B_1 = A_1$, $B_2 = A_2 - A_1$ and $B_3 = A_3 - (A_1 \cup A_2)$, then which one of the following statement is always true.
(a) $A_1 \cup A_2 \cup A_3 \supset B_1 \cup B_2 \cup B_3$
(b) $A_1 \cup A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$
(c) $A_1 \cup A_2 \cup A_3 \subset B_1 \cup B_2 \cup B_3$
(d) none of these [JEE (Orissa) 2002]

38. In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus is
(a) 80% (b) 40% (c) 60% (d) 70%

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39. An investigator interviewed 100 students to determine the performance of three drinks: milk, coffee and tea. The investigator reported that 10 students take all three drinks milk, coffee and tea; 20 students take milk and coffee; 25 students take milk and tea; 30 students take coffee and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of three drinks is
(a) 10 (b) 20 (c) 25 (d) 30
[JEE (Orissa) 2003]

40. Two finite sets have m and n elements. The number of elements in the power set of first set is 48 more than the total number of elements in power set of the second set. Then, the values of m and n are :
(a) 7, 6 (b) 6, 3 (c) 6, 4 (d) 7, 4

41. In a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone ?
(a) 35 (b) 48 (c) 60 (d) 22

42. If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$ and $C = \{3, 4, 6\}$, then $(A \cup B) \cap C$ is
(a) $\{3, 4, 6\}$ (b) $\{1, 2, 3\}$
(c) $\{1, 4, 3\}$ (d) none of these
[JEE (Orissa) 2003]

43. In a class of 45 students, 22 can speak Hindi and 12 can speak English only. The number of students, who can speak both Hindi and English, is
(a) 9 (b) 11 (c) 23 (d) 17

44. In a certain town 25% families own a cell phone, 15% families own a scooter and 65% families own neither a cell phone nor a scooter. If 500 families own both a cell phone and a scooter, then the total number of families in the town is
(a) 10000 (b) 20000 (c) 30000 (d) 40000

45. Let A be a set respresented by the squares of natural numbers and x, y are any two elements of A. Then,
(a) $x - y \in A$ (b) $xy \in A$ (c) $x + y \in A$ (d) $\frac{x}{y} \in A$
[JEE (Orissa) 2007]

Answers

1. (c) **2.** (a) **3.** (b) **4.** (b) **5.** (c) **6.** (b) **7.** (c)
8. (c) **9.** (a) **10.** (d) **11.** (b) **12.** (a) **13.** (b) **14.** (a)
15. (b) **16.** (c) **17.** (b) **18.** (b) **19.** (c) **20.** (c) **21.** (b)
22. (d) **23.** (b) **24.** (a) **25.** (d) **26.** (c) **27.** (c) **28.** (c)
29. (b) **30.** (b) **31.** (d) **32.** (a) **33.** (c) **34.** (d) **35.** (c)
36. (b) **37.** (a) **38.** (c) **39.** (b) **40.** (c) **41.** (c) **42.** (a)
43. (b) **44.** (c) **45.** (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d), out of which only one is correct. Mark the correct choice in each case.

1. Which of the following cannot be the number of elements in the power set of any finite set?
(a) 26 (b) 32 (c) 8 (d) 16

2. If $A = \{1, 2, 3, 4\}$, then the number of subsets of set A containing element 3, is
(a) 24 (b) 28 (c) 8 (d) 16

3. If $A = \{1, 2, 3, 4\}$, then the number of subsets of A that contain the element 2 but not 3, is
(a) 16 (b) 4 (c) 8 (d) 24

4. If $n(u) = 100$, $n(A) = 50$, $n(B) = 20$ and $n(A \cap B) = 10$, then $n\{(A \cup B)^c\}$
(a) 60 (b) 30 (c) 40 (d) 20

5. Let Z denote the set of integers, then
$\{x \in Z : |x - 3| < 4\} \; n \; \{x \in Z : |x - 4| < 5\} =$
(a) $\{-1, 0, 1, 2, 3, 4\}$
(b) $\{-1, 0, 1, 2, 3, 4, 5\}$
(c) $\{0, 1, 2, 3, 4, 5, 6\}$
(d) $\{-1, 0, 1, 2, 3, 5, 6, 7, 8, 9\}$

6. If A_n is the set of first n prime numbers, then $\bigcup_{n=2}^{10} A_n =$
(a) $\{2, 3, 5, 7, 11, 13, 17, 19\}$
(b) $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
(c) $\{3, 5\}$
(d) $\{2, 3\}$

7. In Q. No. 6, $\bigcap_{n=3}^{10} A_n =$
(a) $\{3, 5, 7, 11, 13, 17, 19\}$ (b) $\{2, 3, 5\}$
(c) $\{2, 3, 5, 7, 11, 13, 17\}$ (d) $\{3, 5, 7\}$

8. Let $A_1, A_2, A_3, \ldots, A_{100}$ be 100 sets such that $n(A_i) = i + 1$ and $A_1 \subset A_2 \subset A_3 \subset \ldots \subset A_{100}$, then $\bigcup_{i=1}^{100} A_i$ contains..... elements
(a) 99 (b) 100 (c) 101 (d) 102

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9. If A and B are two sets such that $n(A)=7$, $n(B)=6$ and $(A\cap B)\neq\phi$. Then the greatest possible value of $n(A\,\Delta\,B)$, is
(a) 11 (b) 12 (c) 13 (d) 10

10. If A and B are two sets such that $n(A)=7$, $n(B)=6$ and $n(A\cap B)\neq\phi$. The least possible value of $n(A\,\Delta\,B)$, is
(a) 1 (b) 7 (c) 6 (d) 13

11. If $A_1, A_2, \ldots, A_{100}$ are sets such that $n(A_i)=i+2$, $A_1\subset A_2\subset A_3\ldots\subset A_{100}$ and $\overset{100}{\underset{i=3}{\cap}} A_i=A$, then $n(A)=$
(a) 3 (b) 4 (c) 5 (d) 6

12. If A, B and C are three non-empty sets such that A and B are disjoint and the number of elements contained in A is equal to those contained in the set of elements common to the sets A and C, then $n(A\cup B\cup C)$ is necessarily equal to
(a) $n(B\cup C)$ (b) $n(A\cup C)$
(c) both (a) and (b) (d) none of these

13. If
$$A=\left\{n:\frac{n^3+5n^2+2}{n}\text{ is an integer and itself is an integer}\right\},$$
then the number of elements in the set A, is
(a) 1 (b) 2 (c) 3 (d) 4

14. If
$$A=\left\{p\in N: p\text{ is a prime and }p=\frac{7n^2+3n+3}{n}\text{ for some }n\in N\right\},$$
then the number of elements in the set A is
(a) 1 (b) 2 (c) 3 (d) 4

15. A, B and C are three non-empty sets. If $A\subset B$ and $B\subset C$, then which of the following is true?
(a) $B-A=C-B$ (b) $A\cap B\cap C=B$
(c) $A\cup B=B\cap C$ (d) $A\cup B\cup C=A$

16. If $A=\{1,2,3,4,5,6\}$, then how many subsets of A contain the elements 2, 3 and 5?
(a) 4 (b) 8 (c) 16 (d) 32

17. If S is the set of squares and R is the set of rectangles, then $(S\cup R)-(S\cap S)$ is
(a) S
(b) R
(c) set of squares but not rectangles
(d) set of rectangles but not squares

18. If P is the set of all parallelograms, and T is the set of all trapeziums, then $P\cap T$ is
(a) P (b) T (c) ϕ (d) none of these

19. If $n(A\cap B)=10$, $n(B\cap C)=20$ and $n(A\cap C)=30$, then the greatest possible value of $n(A\cap B\cap C)$ is
(a) 15 (b) 20 (c) 10 (d) 4

20. If $n(A\cap B)=5$, $n(A\cap C)=7$ and $n(A\cap B\cap C)=3$, then the minimum possible value of $n(B\cap C)$ is
(a) 0 (b) 1 (c) 3 (d) 2

21. A and B are any two non-empty sets and A is proper subset of B. If $n(A)=5$, then find the minimum possible value of $n(A\,\Delta\,B)$
(a) is 1 (b) is 5
(c) cannot be determined (d) none of these

22. If A, B and C are three non-empty sets such that any two of them are disjoint, then $(A\cup B\cup C)\cap(A\cap B\cap C)=$
(a) A (b) B (c) C (d) ϕ

23. If $A=\left\{p: p=\dfrac{(n+2)(2n^5+3n^4+4n^3+5n^2+6)}{n^2+2n}, n,p\in Z^+\right\}$ then the number of elements in the set A, is
(a) 2 (b) 3 (c) 4 (d) 6

24. If A, B and C are three sets such that $A\supset B\supset C$, then $(A\cup B\cup C)-(A\cap B\cap C)=$
(a) $A-B$ (b) $B-C$
(c) $A-C$ (d) none of these

25. If $A_1\subset A_2\subset A_3\subset\ldots\subset A_{50}$ and $n(A_i)=i-1$, then $n\left(\overset{50}{\underset{i=11}{\cap}} A_i\right)=$
(a) 49 (b) 50 (c) 11 (d) 10

26. If $n(A_i)=i+1$ and $A_1\subset A_2\subset A_3\subset\ldots\subset A_{99}$, then $n\left(\overset{99}{\underset{i=1}{\cup}} A_i\right)=$
(a) 99 (b) 98 (c) 100 (d) 101

27. In a class, 70 students wrote two tests viz; test-I and test-II. 50% of the students failed in test-I and 40% of the students in test-II. How many students passed in both tests?
(a) 21 (b) 7 (c) 28 (d) 14

28. In an election, two contestants A and B contested x% of the total voters voted for A and $(x+20)$ % for B. If 20% of the voters did not vote, then $x=$
(a) 30 (b) 25 (c) 40 (d) 35

29. In a rehabilitation programme, a group of 50 families were assured new houses and compensation by the government. Number of families who got both is equal to the number of families who got neither of the two. The number of families who got new houses is 6 greater than the number of families who got compensation. How many families got houses?
(a) 22 (b) 28 (c) 23 (d) 25

30. In an office, every employee likes at least one of tea, coffee and milk. The number of employees who like only tea, only coffee, only milk and all the three are all equal. The numbe of employees who like only tea and coffee, only coffee an milk and only tea and milk are equal and each is equal t the number of employees who like all the three. Then possible value of the number of employees in the office
(a) 65 (b) 90 (c) 77 (d) 85

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Answer

1. (a) 2. (c) 3. (b) 4. (c) 5. (c) 6. (b) 7. (b) 8. (c) 9. (a) 10. (a) 11. (c) 12. (a) 13. (d) 14. (a) 15. (c) 16. (b) 17. (d) 18. (a) 19. (c) 20. (c) 21. (d) 22. (d) 23. (c) 24. (c) 25. (d) 26. (c) 27. (b) 28. (c) 29. (b) 30. (c)

Solutions of Exercises and Chapter-tests are available in a separate book on "Solutions of Objective Mathematics".

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CARTESIAN PRODUCT OF SETS AND RELATIONS

1. CARTESIAN PRODUCT OF SETS

If A and B are any two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the set A with set B and is denoted by $A \times B$.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

ILLUSTRATION 1 *If* $A = \{1, 2\}$ *and* $B = \{0, 1\}$, *then* $A \times B =$

(a) $\{(1, 0), (1, 1), (2, 0), (2, 1)\}$ (b) $(1,0), (2, 1)\}$

(c) $\{(1, 1), (1, 2), (0, 1), (0, 2)\}$ (d) *none of these*

Ans. (a)

SOLUTION We have,

$$A \times B = \{1, 2\} \times \{0, 1\} = \{(1, 0), (1, 1), (2, 0), (2, 1)\}$$

ILLUSTRATION 2 *If* $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, *then* $(A \cap B) \times A$ *is*

(a) $\{(1, 3), (2, 3), (3, 3)\}$ (b) $\{(3, 1), (3, 2), (3, 3)\}$

(c) $\{(1, 3), (3, 1), (3, 2)\}$ (d) *none of these*

Ans. (b)

SOLUTION We have,

$$A \cap B = \{3\} \text{ and } A = \{1, 2, 3\}$$

$$\therefore \quad (A \cap B) \times A = \{(3, 1), (3, 2), (3, 3)\}$$

ILLUSTRATION 3 *If* $A = \{x \in R : 0 < x < 1\}$ *and* $B = \{x \in R : -1 < x < 1\}$, *then* $A \times B$ *is the set of all points lying*

(a) *inside the rectangle having vertices at* $(1, 1), (0, 1), (0, -1)$ *and* $(1, -1)$

(b) *inside the rectangle having vertices at* $(1, 0), (1, 1), (0, 1)$ *and* $(0, 0)$

(c) *on the sides of the rectangle whose vertices are at* $(1, 1), (0, 1), (0, -1)$ *and* $(1, -1)$

(d) *inside or on the sides of the rectangle whose vertices are at* $(1, 1), (0, 1), (0, -1)$ *and* $(1, -1)$.

Ans. (a)

SOLUTION Clearly, A represents the line segment between $(0, 0)$ and $(1, 0)$ and B represents the line segment between $(0, -1)$ and $(0, 1)$. So, $A \times B$ represents the region in xy-plane lying inside the rectangle having vertices at $(1, 1), (0, 1), (0, -1)$ and $(1, -1)$.

REMARK *If* $A = \phi$ *or* $B = \phi$, *then* $A \times B$ *is defined as the null set.*

NOTE *In general* $A \times B \neq B \times A$ *and* $A \times B = B \times A \Leftrightarrow A = B$.

If A, B, C and D are any four sets, then the following are some useful results on cartesian products of sets. Proofs of these results are available in Author's class XI and XII books.

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

(iv) $A \times B = B \times A \Rightarrow A = B$

(v) If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$ and $A \times C \subseteq B \times C$ for any set C.

(vi) If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

(vii) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(viii) $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

(ix) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

(x) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

(xi) If A and B have n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.

2. RELATIONS

RELATION *Let* A *and* B *be two sets. Then, a relati from* A *to* B *is a subset of* $A \times B$.

If $(a, b) \in R$, then we write aRb which is read as a is related to b by the relation R. If $(a, b) \notin R$, then we write $a \not R b$ and we say that a is not related to b by the relation R.

ILLUSTRATION 4 *If* $A = \{a, b, c, d\}$ *and* $B = \{1, 2, 3\}$, *then which of the following is a relation from* A *to* B?

(a) $R_1 = \{(a, 1), (2, b), (c, 3)\}$

(b) $R_2 = \{(a, 1), (d, 3), (b, 2), (b, 3)\}$

(c) $R_3 = \quad , a), (2, b), (3, c)\}$

(d) $\quad (a, 1), (b, 2), (c, 3), (3, d)\}$

. (b)

SOLUTION We observe that $(a, 1), (c, 3) \in A \times B$ $(2, b) \notin A \times B$. So, R_1 is not a relation from A to B.

Clearly, $R_2 \subseteq A \times B$. So, it is a relation from A to B.

Since $(2, b) \in R_3$ but $(2, b) \notin A \times B$. So, $R_3 \not\subseteq A \times B$. Hence not a relation from A to B.

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are equivalence relations on set $A = \{a, b, c\}$. But, $R \cup S$ is not an equivalence relation on A, because

$(a, b) \in R \cup S$, $(b, c) \in R \cup S$ but $(a, c) \notin R \cup S$.

EXAMPLE 4 *Statement–1: If R is an equivalence relation on a set A, then R^{-1} is also an equivalence relation.*

Statement–2: $R = R^{-1}$ iff R is a symmetric relation.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION Both the statements are true (see section 3.1).

EXAMPLE 5 Let R be the set of real numbers.

Statement–1: $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement–2: $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c) **[AIEEE 2011]**

SOLUTION *Reflexivity:* For any $x \in R$, we have

$x - x = 0$, which is an integer

$\Rightarrow$ $(x, x) \in A$ for all $x \in R$

$\Rightarrow$ A is reflexive relation on R.

Symmetry: Let $(x, y) \in A$. Then,

$x - y$ is an integer

$\Rightarrow$ $y - x$ is an integer

$\Rightarrow$ $(y, x) \in A$

$\therefore$ A is symmetric relation on R.

Transitivity: Let $(x, y) \in A$ and $(y, z) \in A$. Then,

$x - y$ is an integer and $y - z$ is an integer

$\Rightarrow$ $x - z$ is an integer

$\Rightarrow$ $(x, z) \in A$

$\therefore$ A is a transitive relation on R.

Hence, A is an equivalence relation on R.

So, statement-1 is true.

We have,

$0 = 0 \times 2$, where 0 is a rational number

$\Rightarrow$ $(0, 2) \in B$

But, $2 \neq \alpha \times 0$ for any rational number α.

Thus, $(0, 2) \in B$ but $(2, 0) \notin B$.

So, B is not an equivalence relation on R.

Hence, statement-2 is not true.

EXAMPLE 6 *Consider the following relation R on the set of real square matrices of order 3.*

$R = \{(A, B) : A = P^{-1} BP$ *for some invertible matrix* $P\}$

Statement–1: R is an equivalence relation.

Statement–2: For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1} M^{-1}$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a) **[AIEEE 2011]**

SOLUTION *Reflexivity of R:* For any square matrix A of order 3, we have

$A = I^{-1} AI$, where I is invertible

$\Rightarrow$ $(A, A) \in R$

So, R is reflexive.

Symmetry of R: Let $(A, B) \in R$. Then,

$A = P^{-1} BP$ for some invertible matrix P

$\Rightarrow$ $B = PAP^{-1} = (P^{-1})^{-1} A P^{-1}$ for some invertible matrix P^{-1}

$\Rightarrow$ $(B, A) \in R$

So, R is symmetric.

Transitivity of R: Let $(A, B) \in R$ and $(B, C) \in R$. Then,

$A = P^{-1} BP$ and $B = Q^{-1} CQ$ for some invertible matrices P and Q

$\Rightarrow$ $A = P^{-1} (Q^{-1} CQ) P$

$\Rightarrow$ $A = (QP)^{-1} C (QP)$

$\Rightarrow$ $(A, C) \in R$

$\Rightarrow$ R is transitive.

Hence, R is an equivalence relation.

Consequently, statement-1 is correct.

Clearly, statement-2 is also true.

Also, we have used this statement for proving the correctness of statement-1.

Hence, statement-2 is a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out to which only one is correct.

1. Let R be a relation from a set A to a set B, then
   (a) $R = A \cup B$ (b) $R = A \cap B$
   (c) $R \subseteq A \times B$ (d) $R \subseteq B \times A$
2. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is
   (a) 2^{mn} (b) $2^{mn} - 1$ (c) $2mn$ (d) m^n
3. If R is a relation on a finite set having n elements, then the number of relations on A is
   (a) 2^n (b) 2^{n^2} (c) n^2 (d) n^n
4. Let R be a relation on a set A such that $R = R^{-1}$, then R is
   (a) reflexive (b) symmetric
   (c) transitive (d) none of these
5. Let R be a reflexive relation on a set A and I be the identity relation on A. Then,
   (a) $R \subset I$ (b) $I \subset R$ (c) $R = I$ (d) none of these
6. Let A be the non-void set of the children in a family. The relation 'x is a brother of y' on A is
   (a) reflexive (b) symmetric
   (c) transitive (d) none of these
7. Two points P and Q in a plane are related if $OP = OQ$, where O is a fixed point. This relation is
   (a) partial order relation (b) equivalence relation

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(c) reflexive but not symmetric
(d) reflexive but not transitive

8. The relation R defined in N as $a\ R\ b \Leftrightarrow b$ is divisible by a is
(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) symmetric and transitive
(d) none of these

9. Given the relation $R = \{(1, 2), (2, 3)\}$ on the set $A = \{1, 2, 3\}$, the minimum number of ordered pairs which when added to R make it an equivalence relation is
(a) 5 (b) 6 (c) 7 (d) 8

10. An integer m is said to be related to another integer n if m is a multiple of n. Then, the relation is
(a) reflexive and symmetric (b) reflexive and transitive
(c) symmetric and transitive (d) equivalence relation

11. Let $A = \{1, 2, 3, 4\}$, and let $R = \{(2, 2), (3, 3), (4, 4), (1, 2)\}$ be a relation on A. Then, R is
(a) reflexive (b) symmetric
(c) transitive (d) none of these

12. Let R_1 be a relation defined by
$R_1 = \{(a, b) \mid a \geq b, a, b \in R\}$. Then, R_1 is
(a) an equivalence relation on R
(b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive
(d) neither transitive nor reflexive but symmetric

13. Let R be a reflexive relation on a finite set A having n elements, and let there be m ordered pairs in R. Then,
(a) $m \geq n$ (b) $m \leq n$
(c) $m = n$ (d) none of these

14. The void relation on a set A is
(a) reflexive
(b) symmetric and transitive
(c) reflexive and symmetric
(d) reflexive and transitive

15. Let R be an equivalence relation on a finite set A having n elements. Then, the number of ordered pairs in R is
(a) less than n
(b) greater than or equal to n
(c) less than or equal to n
(d) none of these

16. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is
(a) reflexive but not symmetric
(b) reflexive but not transitive
(c) symmetric and transitive
(d) neither symmetric nor transitive

17. Let R and S be two equivalence relations on a set A. Then,
(a) $R \cup S$ is an equivalence relation on A
(b) $R \cap S$ is an equivalence relation on A
(c) $R - S$ is an equivalence relation on A
(d) none of these

18. The relation 'is subset of' on the power set $P(A)$ of a set A is
(a) symmetric (b) anti-symmetric
(c) equivalence relation (d) none of these

19. Let $P = \{(x, y) \mid x^2 + y^2 = 1, x, y \in R\}$. Then, P is
(a) reflexive (b) symmetric
(c) transitive (d) antisymmetric

20. Let $R = \{(a, a)\}$ be a relation on a set A. Then, R is
(a) symmetric
(b) antisymmetric
(c) symmetric and antisymmetric
(d) neither symmetric nor antisymmetric

21. Which one of the following relations on R is an equivalence relation?
(a) $a\ R_1\ b \Leftrightarrow |a| = |b|$ (b) $a\ R_2\ b \Leftrightarrow a \geq b$
(c) $a\ R_3\ b \Leftrightarrow a$ divides b (d) $a\ R_4\ b \Leftrightarrow a < b$

22. Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then, R is
(a) reflexive (b) symmetric
(c) antisymmetric (d) transitive

23. If R is an equivalence relation on a set A, then R^{-1} is
(a) reflexive only
(b) symmetric but not transitive
(c) equivalence
(d) none of these

24. Let R and S be two non-void relations on a set A. Which of the following statements is false?
(a) R and S are transitive $\Rightarrow R \cup S$ is transitive
(b) R and S are transitive $\Rightarrow R \cap S$ is transitive
(c) R and S are symmetric $\Rightarrow R \cup S$ is symmetric
(d) R and S are reflexive $\Rightarrow R \cap S$ is reflexive

[CEE (Delhi) 2009]

25. If R be a relation $<$ from $A = \{1, 2, 3, 4\}$ to $B = \{1, 3, 5\}$ i.e. $(a, b) \in R \Leftrightarrow a < b$, then $R \circ R^{-1}$ is
(a) $\{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$
(b) $\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$
(c) $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$
(d) $\{(3, 3), (3, 4), (4, 5)\}$

26. If R is a relation from a set A to a set B and S is a relation from B to a set C, then the relation SoR
(a) is from A to C (b) is from C to A
(c) does not exist (d) none of these

27. If $R \subset A \times B$ and $S \subset B \times C$ be two relations, then $(SoR)^{-1} =$
(a) $S^{-1} o R^{-1}$ (b) $R^{-1} o S^{-1}$ (c) SoR (d) RoS

28. In the set $A = \{1, 2, 3, 4, 5\}$, a relation R is defined by $R = \{(x, y) : x, y \in A \text{ and } x < y\}$. Then, R is
(a) reflexive (b) symmetric
(c) transitive (d) none of these

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29. Let $A = \{p, q, r\}$. Which of the following is not an equivalence relation on A ?
(a) $R_1 = \{(p, q), (q, r), (p, r), (p, p)\}$
(b) $R_2 = \{(r, q), (r, p), (r, r), (q, q)\}$
(c) $R_3 = \{(p, p), (q, q), (r, r), (p, q)\}$
(d) none of these

30. In order that a relation R defined on a non-empty set A is an equivalence relation, it is sufficient, if R
(a) is reflective
(b) is symmetric
(c) is transitive
(d) possesses all the above three properties

31. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (i.e. $n \mid m$). Then, R is
(a) reflexive and symmetric
(b) transitive and symmetric
(c) equivalence
(d) reflexive, transitive but not symmetric

32. Let R and S be two relations on a set A. Then, which one of the following is not true?
(a) R and S are transitive, then $R \cup S$ is also transitive
(b) R and S are transitive, then $R \cap S$ is also transitive
(c) R and S are reflexive, then $R \cap S$ is also reflexive
(d) R and S are symmetric, then $R \cup S$ is also symmetric

33. Let R be a relation on the set N be defined by $\{(x, y) \mid x, y \in N, 2x + y = 41\}$. Then, R is
(a) reflexive (b) symmetric
(c) transitive (d) none of these

34. Let L be the set of all straight lines in the Euclidean plane. Two lines l_1 and l_2 are said to be related by the relation R iff l_1 is parallel to l_2. Then, the relation R is not
(a) reflexive (b) symmetric
(c) transitive (d) none of these

35. For real numbers x and y, we write $x\,Ry \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then, the relation R is
(a) reflexive (b) symmetric
(c) transitive (d) none of these

36. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is/are not relations from X to Y ?
(a) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$
(b) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
(c) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$
(d) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

37. Let n be a fixed positive integer. Define a relation R on the set Z of integers by, $a\,R\,b \Leftrightarrow n \mid a - b$. Then, R is not
(a) reflexive (b) symmetric
(c) transitive (d) none of these

38. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha\,R\,\beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. Then R is
(a) reflexive (b) symmetric
(c) transitive (d) none of these

Answers

1. (c) 2. (a) 3. (b) 4. (b) 5. (b) 6. (c) 7. (b)
8. (a) 9. (c) 10. (b) 11. (c) 12. (b) 13. (a) 14. (b)
15. (b) 16. (a) 17. (b) 18. (b) 19. (b) 20. (c) 21. (a)
22. (b) 23. (c) 24. (a) 25. (c) 26. (a) 27. (b) 28. (c)
29. (d) 30. (d) 31. (d) 32. (a) 33. (d) 34. (d) 35. (a)
36. (d) 37. (d) 38. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. If $n(A \times B) = 45$, then $n(A)$ cannot be
(a) 15 (b) 17 (c) 5 (d) 9

2. Set builder form of the relation
$R = \{(-2, -7), (-1, -4), (0, -1), (1, 2), (2, 5)\}$ is
(a) $\{(a, b) : b = 2a - 3; a, b, \in Z\}$
(b) $((x, y) : y = 3x - 1; x, y \in Z\}$
(c) $\{(a, b) : b = 3a - 1; a, b \in N\}$
(d) $\{(u, v) : v = 3u - 1; -2 \le u < 3$ and $u \in Z\}$

3. If $A = \{a, b, c, l, m, n\}$, then the maximum number of elements in any relation on A is
(a) 12 (b) 16 (c) 32 (d) 36

4. If $A = \{1, 2, 3\}$, then the relation
$R = \{(1, 1), (2, 2), (3, 1), (1, 3)\}$ is
(a) reflexive (b) symmetric
(c) transitive (d) equivalence

5. If relation R is defined as:
aRb if "a is the father of b". Then, R is
(a) reflexive (b) symmetric
(c) transitive (d) none of these

6. If $R = \{(a, b) : |a + b| = a + b\}$ is a relation defined on a set $\{-1, 0, 1\}$, then R is
(a) reflexive (b) symmetric
(c) anti symmetric (d) transitive

7. A relation between two persons is defined as follows:
$aRb \Leftrightarrow a$ and b born in different months. Then, R is

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(a) reflexive (b) symmetric
(c) transitive (d) equivalence

8. Which of the following is an equivalence relation?
(a) is father of (b) is less than
(c) is congruent to (d) is an uncle of

9. Consider the following statements:
(i) Every reflexive relation is antisymmetric
(ii) Every symmetric relation is antisymmetric.
Which one among (i) and (ii) is true?
(a) (i) alone is true
(b) (ii) alone is true
(c) Both (i) and (ii) are true
(d) Neither (i) nor (ii) is true

10. Let A be a set of compartments in a train. Then the relation R defined on A as aRb iff "a and b have the link between them", then which of the following is true for R?
(a) reflexive (b) symmetric
(c) transitive (d) equivalence

11. If a set has 13 elements and R is a reflexive relation on A with n elements, then
(a) $13 \le n \le 26$ (b) $0 \le n \le 26$
(c) $13 \le n \le 169$ (d) $0 \le n \le 169$

12. The relation 'is not equal to' is defined on R, is
(a) reflexive only (b) symmetric only
(c) transitive only (d) equivalence

13. If A is a non-empty set, then which of the following is false?
p : Every reflexive relation is a symmetric relation
q : Every antisymmetric relation is reflexive
Which of the following is/are true?
(a) p alone (b) q alone
(c) Both p and q (d) Neither p nor q

14. Let X be the set of all engineering colleges in a state of Indian Republic and R be a relation on X defined as two colleges are related iff they are affiliated to the same university, then R is
(a) only reflexive (b) only symmetric
(c) only transitive (d) equivalence

15. If $R = \{(a, b) : a + b = 4\}$ is a relation on N, then R is
(a) reflexive (b) symmetric
(c) antisymmetric (d) transitive

16. If A is a non-empty set, then which of the following is false?
p : There is at least one reflexive relation on A
q : There is at least one symmetric relation on A
(a) p alone (b) q alone
(c) Both p and q (d) Neither p nor q

17. If $A = \{x, y, z\}$, then the relation
$R = \{(x, x), (y, y), (z, z), (z, x), (z, y)\}$ is
(a) symmetric (b) antisymmetric
(c) transitive (d) both (a) and (b)

18. Consider the following statements:
p : Every reflexive relation is symmetric relation
q : Every anti-symmetric relation is reflexive.
Which of the following is/are true?
(a) p alone (b) q alone
(c) Both p and q (d) Neither p nor q

19. The relation "is a factor of" on the set N of all natural numbers is not
(a) reflexive (b) symmetric
(c) antisymmetric (d) transitive

20. The relation $R = \{(1, 3), (3, 5)\}$ is defined on the set with minimum number of elements of natural numbers. The minimum number of elements to be included in R so that R is an equivalence relation, is
(a) 5 (b) 6 (c) 7 (d) 8

21. If a set A contains n elements, then which of the following cannot be the number of reflexive relations on the set A?
(a) 2^n (b) 2^{n-1} (c) 2^{n^2-1} (d) 2^{n+1}

22. In a set of teachers of a school, two teachers are said to be related if they "teach the same subject", then the relation is
(a) reflexive and symmetric
(b) symmetric and transitive
(c) reflexive and transitive
(d) equivalence

23. If $A = \{4, 6, 10, 12\}$ and R is a relation defined on A as "two elements are related iff they have exactly one common factor other than 1". Then the relaiton R is
(a) antisymmetric (b) only transitive
(c) only symmetric (d) equivalence

24. In a set of ants in a locality, two ants are said to be related iff they walk on a same straight line, then the relation is
(a) reflexive and symmetric
(b) symmetric and transitive
(c) reflexive and transtive
(d) equivalence

25. Let R be a relation defined on S, the set of squares on a chess board such that xRy iff x and y share a common side. Then, which of the following is false for R?
(a) Reflexive (b) symmetric
(c) Transitive (d) All the above

26. X is the set of all residents in a colony and R is a relation defined on X as follows:
"Two persons are related iff they speak the same language"
The relation R is
(a) only symmetric
(b) only reflexive
(c) both symmetric and reflexive but not transitive
(d) equivalence

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27. Let $A = \{ONGC, BHEL, SAIL, GAIL, IOCL\}$ and R be a relation defined as "two elements of A are related if they share exactly one letter". The relation R is
(a) anti-symmetric (b) only transitive
(c) only symmetric (d) equivalence

28. Let A be the set of all animals. A relation R is defined as "aRb iff a and b are in different zoological parks". Then R is
(a) only reflexive (b) only symmetric
(c) only transitive (d) equivalence

29. On the set of human beings a relation R is defined as follows:
"aRb iff a and b have the same brother". Then R is
(a) only reflexive (b) only symmetric
(c) only transitive (d) equivalence

30. Let A be the set of all students in a school. A relation R defined on A as follows:
"aRb iff a and b have the same teacher"
The relation R is
(a) reflexive (b) symmetric
(c) transitive (d) equivalence

31. If A and B are two sets such that $n(A \cap \bar{B}) = n(\bar{A} \cap B) = 10$ and $n(A \cup B) = 24$, then $n(A \times B) =$
(a) 105 (b) 210
(c) 70 (d) none of these

Answers

1. (b) 2. (d) 3. (d) 4. (b) 5. (d) 6. (b) 7. (b)
8. (c) 9. (d) 10. (b) 11. (c) 12. (b) 13. (d) 14. (d)
15. (b) 16. (d) 17. (d) 18. (d) 19. (b) 20. (a) 21. (d)
22. (d) 23. (c) 24. (d) 25. (c) 26. (d) 27. (c) 28. (
29. (d) 30. (d) 31. (b)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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1. FUNCTION AS A RELATION

Let A and B be two non-empty sets. A relation from A to B is said to be a function from A to B, if

(i) *for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$.*

(ii) $(a, b) \in f$ *and* $(a, c) \in f \Rightarrow b = c$.

Thus, a non-empty subset of $A \times B$ is a function from A to B if each element of A appears in some ordered pair in f and no two ordered pairs in f have the same first element.

We can also say that a subset f of $A \times B$ is a function from A to B if corresponding to each element in A there exists a unique ordered pair in $A \times B$.

ILLUSTRATION 1 *Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ be two sets. Which one of the following subsets of $A \times B$ defines a function from A to B?*

(a) $f_1 = \{(1, 2), (2, 3), (3, 4)\}$

(b) $f_2 = \{(1, 2), (1, 3), (2, 3), (3, 4)\}$

(c) $f_3 = \{(1, 3)(2, 4)\}$

(d) $f_4 = \{(1, 4), (2, 4), (3, 4), (2, 3)\}$

Ans. (a)

SOLUTION We observe that corresponding to each element in A there is unique ordered pair in f_1. So, f_1 is a function from A to B. f_2 is not a function, becasue there are two ordered pairs $(1, 2)$ and $(1, 3)$ in f_3 corresponding to $1 \in A$.

As there is no ordered pair in f_3 corresponding to $3 \in A$. So, it is not a function from A to B.

Similarly, f_4 is not a function from A to B as there are two ordered pairs $(2, 3)$ and $(2, 4)$ in f_4 corresponding to $2 \in A$.

ILLUSTRATION 2 *If $A = \{1, 2, 3, 4\}$, then which of the following are functions from A to itself?*

(a) $f_1 = \{(x, y) : y = x + 1\}$ (b) $f_2 = \{(x, y) : x + y > 4\}$

(c) $f_3 = \{(x, y) : y < x\}$ (d) $f_4 = \{(x, y) : x + y = 5\}$

Ans. (d)

SOLUTION We have, $f_1 = \{(1, 2), (2, 3), (3, 4)\}$,

$f_2 = \{(1, 4)\ (2, 3), (2, 4), (3, 3), (3, 4), (4, 4), (4, 1), (4, 2), (4, 3)\}$,

$f_3 = \{(2, 1), (3,1)\ (3, 2)\ ...\}$ and $f_4 = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$.

Obviously, f_4 is a function from A to itself and f_1, f_2, f_3 are not functions from A to itself.

2. FUNCTION AS A CORRESPONDENCE

DEFINITION *Let A and B be two non-empty sets. Then, a function 'f' from set A to set B is a rule or method or correspondence which associates elements of set A to elements of set B such that*

(i) *all elements of set A are associated to elements in set B.*

(ii) *an element of set A is associated to a unique element in set B.*

In other words, a function 'f' from a set A to a set B associates each element of set A to a unique element of set B.

Terms such as "map" (or "mapping"), "correspondence" are used as synonyms for "function". If f is a function from a set A to a set B, then we write $f : A \to B$ or $A \xrightarrow{f} B$, which is read as *f is a function from A to B* or *f maps A to B*.

If an element $a \in A$ is associated to an element $b \in B$, then b is called *'the f-image of a'* or *'image of a under f'* or *'the value of the function f at a'*. Also, a is called the pre-image of b under the function f. We write it as $b = f(a)$

If $f : A \to B$, then the set A is known as the domain of f and the set B is known as its co-domain. The set of all f-images of elements of A is known as the range of f or image set of A under f and is denoted by $f(A)$.

Thus, $f(A) = \{f(x) : x \in A\}$ = Range of f.

Clearly, $f(A) \subseteq B$.

If the domain A of a function $f : A \to B$ is a finite set, then f can be described by listing the values which it attains at different points in its domain. For example, if $A = \{1, 2, 3\}$ then a function $f : A \to R$ can be described as $f(0) = 1, f(1) = 0, f(2) = 3$ and $f(3) = 8$. The same function can also be described as $f(x) = x^2 - 1$. If $f : A \to B$ is such that A is an infinite set, then f cannot be described by listing the images of elements in its domain. In such a case, the function is generally described by a general rule. For example, $f : Z \to R$ given by $f(x) = x^2 + 3$ etc.

ILLUSTRATION 1 *If function $f = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is described by a linear relation, then $f(x)$ =*

(a) $2x - 1$ (b) $2x + 1$ (c) $x + 2$ (d) $x - 2$

Ans. (a)

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EXAMPLE 9 *Let $f:[-1,3]\to R$ be defined as $f(x)=4x^3-12x$*
Statement–1: The image of $[-1,3]$ under f is not the interval $[f(-1),f(3)]$.
Statement–2: f is not an injective map.
(a) 1 (b) 2 (c) 3 (d) 4
Ans. (a)
SOLUTION We have,

$$f'(x) = 12x^2-12 = 12(x-1)(x+1)$$

The signs of $f'(x)$ for different values of x are as shown below.



Fig. 12

So, $f(x)$ attains a local maximum at $x=-1$ and local minimum at $x=1$.
Clearly, $f(x)$ is decreasing in $[-1,1]$ and increasing in $[1,3]$.
So, f is not an injective map and the image of $[-1,3]$ is $[f(1),f(3)] = [-8,72]$.

EXAMPLE 10 *Let f be a function defined by $f(x)=(x-1)^2+1, (x\ge 1)$.*
Statement-1: The set $\{x: f(x)=f^{-1}(x)\} = \{1,2\}$
Statement-2: f is a bijection and $f^{-1}(x)=1+\sqrt{x-1}, x\ge 1$.
(a) 1 (b) 2 (c) 3 (d) 4
Ans. (a) **[AIEEE 2011]**
SOLUTION It is evident from the graph of $f(x)$ that $f(x)$ is a bijection for all $x\ge 1$ and hence invertible.
Now, $fof^{-1}(x) = x$ for all $x\ge 1$

$\Rightarrow f\left(f^{-1}(x)\right) = x$ for all $x\ge 1$

$\Rightarrow \left\{f^{-1}(x)-1\right\}^2+1 = x$ for all $x\ge 1$

$\Rightarrow f^{-1}(x) = 1+\sqrt{x-1}$ for all $x\ge 1$

So, statement-2 is correct.
Now, $\{x: f(x)=f^{-1}(x)\} = \{x: f(x)=x\} = \{x:(x-1)^2+1=x\} = \{1,2\}$
Hence, statement-1 is also correct and statement-2 is a correct explanation of statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $f(x)=(a-x^n)^{1/n}$, where $a>0$ and $n\in N$, then $fof(x)$ is equal to
(a) a (b) x (c) x^n (d) a^n

2. Let $f(x)$ be defined on $[-2,2]$ and is given by

$$f(x)=\begin{cases}-1, & -2\le x\le 0\\ x-1, & 0<x\le 2\end{cases}$$

and $g(x)=f(|x|)+|f(x)|$. Then, $g(x)$ is equal to
(a) $\begin{cases}-x, & -2\le x<0\\ 0, & 0\le x<1\\ x-1, & 1\le x\le 2\end{cases}$ (b) $\begin{cases}-x, & -2\le x<0\\ 0, & 0\le x<1\\ 2(x-1), & 1\le x\le 2\end{cases}$
(c) $\begin{cases}-x, & -2\le x<0\\ x-1, & 0\le x\le 2\end{cases}$ (d) none of these

3. Which of the following functions from Z to itself are bijections?
(a) $f(x)=x^3$ (b) $f(x)=x+2$
(c) $f(x)=2x+1$ (d) $f(x)=x^2+x$

4. Which of the following functions from $A=\{x:-1\le x\le 1\}$ to itself are bijections?
(a) $f(x)=\frac{x}{2}$ (b) $g(x)=\sin\left(\frac{\pi x}{2}\right)$
(c) $h(x)=|x|$ (d) $k(x)=x^2$

5. If $f:R\to R$ be a mapping defined by $f(x)=x^3+5$, then $f^{-1}(x)$ is equal to
(a) $(x+5)^{1/3}$ (b) $(x-5)^{1/3}$ (c) $(5-x)^{1/3}$ (d) $5-x$

6. Let $f:A\to B$ and $g:B\to C$ be bijections, then $(fog)^{-1}=$
(a) $f^{-1}og^{-1}$ (b) fog (c) $g^{-1}of^{-1}$ (d) gof

7. Let $f:R\to R$, $g:R\to R$ be two functions given by $f(x)=2x-3, g(x)=x^3+5$. Then, $(fog)^{-1}(x)$ is equal to
(a) $\left(\frac{x+7}{2}\right)^{1/3}$ (b) $\left(x-\frac{7}{2}\right)^{1/3}$ (c) $\left(\frac{x-2}{7}\right)^{1/3}$ (d) $\left(\frac{x-7}{2}\right)^{1}$

8. Let $f:R\to R$ be a function defined by $f(x)=\cos(5x+2)$. Then, f is
(a) injective (b) surjective (c) bijective (d) none of these

9. Let $f:N\to N$ be defined by $f(x)=x^2+x+1$, then f is
(a) one-one onto (b) many one onto
(c) one-one but not onto (d) none of these
[CEE (Delhi) 2009]

10. Let $A=\{x:-1\le x\le 1\}$ and $f:A\to A$ such that $f(x)=x|x|$, then f is
(a) a bijection (b) injective but not surjective
(c) surjective but not injective
(d) neither injective nor surjective

11. If $f(x)=\frac{3x+2}{5x-3}$, then
(a) $f^{-1}(x)=f(x)$ (b) $f^{-1}(x)=-f(x)$
(c) $(fof)(x)=-x$ (d) $f^{-1}(x)=-\frac{1}{19}f(x)$

12. If $f(x)=2^x$, then $f(0), f(1), f(2), \ldots$ are in
(a) AP (b) GP (c) HP (d) arbitrary

13. If the function $f:R\to A$ given by $f(x)=\frac{x^2}{x^2+1}$ is a surjection, then $A=$
(a) R (b) $[0,1]$ (c) $(0,1]$ (d) $[0,1)$

14. Which of the following functions is inverse of itself?
(a) $f(x)=\frac{1-x}{1+x}$ (b) $g(x)=5^{\log x}$
(c) $h(x)=2^{x(x-1)}$ (d) none of these

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15. If $f(x)=\frac{x-1}{x+1}$, then $f(2x)$ is
(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{3f(x)+1}{f(x)+3}$ (c) $\frac{f(x)+3}{f(x)+1}$ (d) $\frac{f(x)+3}{3f(x)+1}$

16. Given $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $fog(x)$ equals
(a) $-f(x)$ (b) $3f(x)$ (c) $[f(x)]^3$ (d) none of these

17. If $f(x)=a^x$, which of the following equalities do not hold?
(a) $f(x+2)-2f(x+1)+f(x)=(a-1)^2f(x)$
(b) $f(-x)f(x)-1=0$
(c) $f(x+y)=f(x)f(y)$
(d) $f(x+3)-2f(x+2)+f(x+1)=(a-2)^2f(x+1)$

18. The interval in which the function $y=\frac{x-1}{x^2-3x+3}$ transforms the real line is
(a) $(0,\infty)$ (b) $(-\infty,\infty)$ (c) $[0,1]$ (d) $[-1/3,1]$

19. Let $f(x)=|x-1|$. Then,
(a) $f(x^2)=[f(x)]^2$ (b) $f(|x|)=|f(x)|$
(c) $f(x+y)=f(x)+f(y)$ (d) none of these

20. If $f(x)=ax+b$ and $g(x)=cx+d$, then $f(g(x))=g(f(x))$ is equivalent to
(a) $f(a)=g(c)$ (b) $f(b)=g(b)$
(c) $f(d)=g(b)$ (d) $f(c)=g(a)$

21. Which of the following functions is (are) not an injective map(s)?
(a) $f(x)=|x+1|,\ x\in[-1,\infty)$
(b) $g(x)=x+\frac{1}{x},\ x\in(0,\infty)$
(c) $h(x)=x^2+4x-5,\ x\in(0,\infty)$
(d) $k(x)=e^{-x},\ x\in[0,\infty)$

22. If $f(x)$ is defined on $[0,1]$ by the rule
$$f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ 1-x, & \text{if } x \text{ is irrational.}\end{cases}$$
Then, for all $x\in[0,1]$, $f(f(x))$ is
(a) constant (b) $1+x$ (c) x (d) none of these

23. Let $f(x)=x$ and $g(x)=|x|$ for all $x\in R$. Then, the function $\phi(x)$ satisfying $|\phi(x)-f(x)|^2+|\phi(x)-g(x)|^2=0$, is
(a) $\phi(x)=x,\ x\in[0,\infty)$ (b) $\phi(x)=x,\ x\in R$
(c) $\phi(x)=-x,\ x\in(-\infty,0]$ (d) $\phi(x)=x+|x|,\ x\in R$

24. Let $f(x)=\frac{ax+b}{cx+d}$. Then, $fof(x)=x$ provided that
(a) $d=-a$ (b) $d=a$ (c) $a=b=c=d=1$ (d) $a=b=1$

25. If $f(x)=(ax^2+b)^3$, then the function g satisfying $f(g(x))=g(f(x))$ is given by
(a) $g(x)=\left(\frac{b-x^{1/3}}{a}\right)^{1/2}$ (b) $g(x)=\frac{1}{(ax^2+b)^3}$
(c) $g(x)=(ax^2+b)^{1/3}$ (d) $g(x)=\left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

26. If a function $f:[2,\infty)\to B$ defined by $f(x)=x^2-4x+5$ is a bijection, then $B=$
(a) R (b) $[1,\infty)$ (c) $[4,\infty)$ (d) $[5,\infty)$

27. The function $f:R\to R$ defined by $f(x)=(x-1)(x-2)(x-3)$, is
(a) one-one but not onto (b) onto but not one-one
(c) both one and onto (d) neither one-one nor onto

28. Let $A=\{x,y,z\}$, $B=\{u,v,w\}$ and $f:A\to B$ be defined by $f(x)=u, f(y)=v, f(z)=w$. Then, f is
(a) surjective but not injective
(b) injective but not surjective
(c) bijective
(d) none of these

29. If $f:R\to R$, defined by $f(x)=x^2+1$, then the values of $f^{-1}(17)$ and $f^{-1}(-3)$ respectively are
(a) $\phi,\{4,-4\}$ (b) $\{3,-3\},\phi$
(c) $\{4,-4\},\phi$ (d) $\{4,-4\},\{2,-2\}$

30. The function $f:N\to N$ (N is the set of natural numbers) defined by $f(n)=2n+3$, is
(a) surjective (b) injective
(c) bijective (d) none of these

31. The composite mapping fog of the maps $f:R\to R$, $f(x)=\sin x$ and $g:R\to R, g(x)=x^2$, is
(a) $x^2\sin x$ (b) $(\sin x)^2$ (c) $\sin x^2$ (d) $\frac{\sin x}{x^2}$

32. Let $f:R\to R$ be defined by $f(x)=3x-4$. Then, $f^{-1}(x)$ is
(a) $\frac{x+4}{3}$ (b) $\frac{x}{3}-4$ (c) $3x+4$ (d) none of these

33. $f:R\to R$ is a function defined by $f(x)=10x-7$. If $g=f^{-1}$, then $g(x)=$
(a) $\frac{1}{10x-7}$ (b) $\frac{1}{10x+7}$ (c) $\frac{x+7}{10}$ (d) $\frac{x-7}{10}$

34. Let $A=\{x\in R: x\le 1\}$ and $f:A\to A$ be defined as $f(x)=x(2-x)$. Then, $f^{-1}(x)$ is
(a) $1+\sqrt{1-x}$ (b) $1-\sqrt{1-x}$ (c) $\sqrt{1-x}$ (d) $1\pm\sqrt{1-x}$

35. If $f(x)=x^n$, $n\in N$ and $gof(x)=ng(x)$, then $g(x)$ can be
(a) $n|x|$ (b) $3x^{1/3}$ (c) e^x (d) $\log|x|$

36. If the function $f:R\to R$ be such that $f(x)=x-[x]$, where $[x]$ denotes the greatest integer less than or equal to x, then $f^{-1}(x)$, is
(a) $\frac{1}{x-[x]}$ (b) $[x]-x$ (c) not defined (d) none of these

37. $f:R\to R$ given by $f(x)=5-3\sin x$, is
(a) one-one (b) onto
(c) one-one and onto (d) none of these

38. Let $f:A\to B$ be a function defined by $f(x)=\sqrt{3}\sin x+\cos x+4$. If f is invertible, then
(a) $A=[-2\pi/3,\pi/3]$, $B=[2,6]$
(b) $A=[\pi/6,5\pi/6]$, $B=[-2,2]$
(c) $A=[-\pi/2,\pi/2]$, $B=[2,6]$
(d) $A=[-\pi/3,\pi/3]$, $B=[2,6]$

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39. Let $f: A \to B$ and $g: B \to A$ be two functions such that $gof = I_A$. Then,
(a) f is an injection and g is a surjection
(b) f is a surjection and g is an injection
(c) f and g both are injections
(d) f and g both are surjections

40. Let $f: A \to B$ and $g: B \to A$ be two functions such tat $fog = I_B$. Then,
(a) f and g both are injections
(b) f and g both are surjections
(c) f is an injection and g is a surjection
(d) f is a surjection and g is an injection.

41. Let $f: A \to B$ and $g: B \to C$ be two functions such that $gof: A \to C$ is onto. Then,
(a) f is onto (b) g is onto
(c) f and g both are onto (d) none of these

42. Let $f: A \to B$ and $g: B \to C$ be two functions such that $gof: A \to C$ is one-one. Then,
(a) f is one-one (b) g is one-one
(c) f and g both are one-one (d) none of these.

43. Let $f: A \to B$ and $g: B \to C$ be two functions such that $gof: A \to C$ is onto and g is one-one. Then,
(a) f is one-one (b) f is onto
(c) f is both one-one and onto (d) none of these

44. Let $f: A \to B$ and $g: B \to C$ be two functions such that $gof: A \to C$ is one-one and $f: A \to B$ is onto. Then, $g: B \to C$ is
(a) one-one (b) onto
(c) one-one and onto (d) none of these

45. Let $[x]$ denote the greatest integer less than or equal to x. If $f(x) = \sin^{-1} x, g(x) = [x^2]$ and $h(x) = 2x, \frac{1}{2} \le x \le \frac{1}{\sqrt{2}}$, then
(a) $fogoh(x) = \pi/2$ (b) $fogoh(x) = \pi$
(c) $hofog = hogof$ (d) $hofog \ne hogof$

46. If $f(x) = \sin^2 x, g(x) = \sqrt{x}$ and $h(x) = \cos^{-1} x, 0 \le x \le 1$, then
(a) $hogof = fogoh$ (b) $gofoh = fohog$
(c) $fohog = hogof$ (d) none of these

47. If $f(x) = (25 - x^4)^{1/4}$ for $0 < x < \sqrt{5}$, then $f\left(f\left(\frac{1}{2}\right)\right) =$
(a) 2^{-4} (b) 2^{-3} (c) 2^{-2} (d) 2^{-1}

48. If $X = \{1, 2, 3, 4\}$, then one-one onto mappings $f: X \to X$ such that $f(1) = 1, f(2) \ne 2, f(4) \ne 4$ are given by
(a) $f = \{(1, 1), (2, 3), (3, 4), (4, 2)\}$
(b) $f = \{(1, 2), (2, 4), (3, 3), (4, 2)\}$
(c) $f = \{(1, 2), (2, 4), (3, 2), (4, 3)\}$
(d) none of these

Answers

1. (b)	2. (b)	3. (b)	4. (b)	5. (b)	6. (c)	7. (d)	29. (c)	30. (b)	31. (c)	32. (a)	33. (c)	34. (b)	35. (d)
8. (d)	9. (c)	10. (a)	11. (a)	12. (b)	13. (d)	14. (a)	36. (c)	37. (d)	38. (a)	39. (a)	40. (d)	41. (b)	42. (a)
15. (b)	16. (b)	17. (d)	18. (d)	19. (d)	20. (c)	21. (b)	43. (b)	44. (a)	45. (c)	46. (d)	47. (d)	48. (a)	
22. (c)	23. (a)	24. (a)	25. (d)	26. (b)	27. (b)	28. (c)							

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct alternative in each case.

1. The number of bijective functions from set A to itself when A contains 106 elements is
(a) 106 (b) $(106)^2$ (c) 106 ! (d) 2^{106}

2. $f(x) = |\sin x|$ has an inverse if its domain is
(a) $[0, \pi]$ (b) $[0, \pi/2]$ (c) $[-\pi/4, \pi/4]$ (d) none of these

3. The function $f: [-1/2, 1/2] \to [-\pi/2, \pi/2]$ defined by $f(x) = \sin^{-1}(3x - 4x^3)$ is
(a) bijection
(b) injection but not a surjection
(c) surjection but not an injection
(d) neither an injection nor a surjection

4. Let $f: R \to R$ be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$. Then,
(a) f is a bijection (b) f is an injection only
(c) f is surjection on only
(d) f is neither an injection nor a surjection

5. Let $f: (e, \infty) \to R$ be defined by $f(x) = \log[\log(\log x)]$, then
(a) f is one-one but not onto
(b) f is onto but not one-one
(c) f is both one-one and onto
(d) f is neither one-one nor onto

6. Let $f: R - \{n\} \to R$ be a function defined by $f(x) = \frac{x - m}{x - n}$, where $m \ne n$. Then,
(a) f is one-one onto (b) f is one-one into
(c) f is many one onto (d) f is many one into

7. The inverse of the function $f: R \to (-1, 3)$ is given by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$
(a) $\log\left(\frac{x-1}{x+1}\right)^{-2}$ (b) $\log\left(\frac{x-2}{x-1}\right)^{1/2}$
(c) $\log\left(\frac{x}{2-x}\right)^{1/2}$ (d) $\log\left(\frac{x-1}{3-x}\right)^{1/2}$

8. The inverse of the function $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} + 1$ is given by
(a) $\frac{1}{2}\log_{10}\left(\frac{x}{2-x}\right)$ (b) $\log_{10}\left(\frac{x}{2-x}\right)$

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(c) $\frac{1}{2}\log_{10}\left(\frac{x}{1-x}\right)$ (d) none of these

9. Let $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$, $(x\neq 0)$ then $f(x)$ equals
(a) x^2-2 for all x (b) x^2-2 for all $|x|\geq 2$
(c) x^2-2 for all $|x|<2$ (d) none of these

10. Let $f:R\to R$, $g:R\to R$ be two functions given by $f(x)=2x-3, g(x)=x^3+5$. Then, $(fog)^{-1}x$ is equal to
(a) $\left(\frac{x-7}{2}\right)^{1/3}$ (b) $\left(\frac{x+7}{2}\right)^{1/3}$ (c) $\left(x-\frac{7}{2}\right)^{1/3}$ (d) $\left(\frac{x-2}{7}\right)^{1/3}$

11. If $g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$ then, $f(x)$ is equal to
(a) $1+2x^2$ (b) $2+x^2$ (c) $1+x$ (d) $2+x$

12. If $f(x)=\frac{1-x}{1+x}$, $x\neq 0,-1$ and $\alpha=f\left(f(x)\right)+f\left(f\left(\frac{1}{x}\right)\right)$, then
(a) $\alpha>2$ (b) $\alpha<-2$ (c) $|\alpha|>2$ (d) $\alpha=2$

13. Let $f:R\to R$ be a function defined by $f(x)=\frac{x^2-8}{x^2+2}$
Then, f is
(a) one-one but not onto (b) one-one and onto
(c) onto but not one-one (d) neither one-one nor onto

14. Let $f:(-\infty,2]\to(-\infty,4]$ be a function defined by $f(x)=4x-x^2$. Then, $f^{-1}(x)$ is
(a) $2-\sqrt{4-x}$ (b) $2+\sqrt{4-x}$ (c) $2\pm\sqrt{4-x}$ (d) not defined

15. The inverse of the function $f:R\to R$ given by $f(x)=\log_a(x+\sqrt{x^2+1})$ $(a>0, a\neq 1)$, is
(a) $\frac{1}{2}(a^x+a^{-x})$ (b) $\frac{1}{2}(a^x-a^{-x})$
(c) $\frac{1}{2}\left(\frac{a^x+a^{-x}}{a^x-a^{-x}}\right)$ (d) not defined

16. $f:R\to R$ is defined by $f(x)=\frac{e^{x^2}-e^{-x^2}}{e^{x^2}+e^{-x^2}}$, is
(a) one-one but not onto (b) many-one but onto
(c) one-one and onto (d) neither one-one nor onto

17. If $f(x)=\log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to
(a) $\{f(x)\}^2$ (b) $\{f(x)\}^3$ (c) $2f(x)$ (d) $3f(x)$

18. If $f(x)=\frac{2^x+2^{-x}}{2}$, then $f(x+y)f(x-y)$ is equals to
(a) $\frac{1}{2}[f(2x)+f(2y)]$ (b) $\frac{1}{2}[f(2x)-f(2y)]$
(c) $\frac{1}{4}[f(2x)+f(2y)]$ (d) $\frac{1}{4}[f(2x)-f(2y)]$

19. The function $f:R\to R$ given by $f(x)=x^2+x$, is
(a) one-one and onto (b) one-one and into
(c) many-one and onto (d) many one and into

20. Let $f:R\to R$ and $g:R\to R$ be given by $f(x)=3x^2+2$ and $g(x)=3x-1$ for all $x\in R$. Then,
(a) $fog(x)=27x^2-18x+5$ (b) $fog(x)=27x^2+18x-5$
(c) $gof(x)=9x^2-5$ (d) $gof(x)=9x^2+15$

21. The function $f:R\to R$, defined by $f(x)=[x]$, where $[x]$ denotes the greatest integer less than or equal to x, is
(a) one-one (b) onto (c) one-one and onto
(d) neither one-one nor onto

22. Let $f(x)=x, g(x)=1/x$ and $h(x)=f(x)g(x)$. Then, $h(x)=1$, if
(a) x is any rational number
(b) x is a non–zero real number
(c) x is a real number (d) x is a rational number

23. Let X and Y be subsets of R, the set of all real numbers. The function $f:X\to Y$ defined by $f(x)=x^2$ for $x\in X$ is one-one but not onto, if (Here, R^+ is the set of all positive real numbers)
(a) $X=Y=R^+$ (b) $X=R$, $Y=R^+$
(c) $X=R^+$, $Y=R$ (d) $X=Y=R$

24. If the functions f and g are defined by $f(x)=3x-4$, $g(x)=3x+2$ for $x\in R$, respectively then $g^{-1}\left(f^{-1}(5)\right)=$
(a) 1 (b) 1/2 (c) 1/3 (d) 1/4

25. If $f(x)=\frac{\sin^4 x+\cos^2 x}{\sin^2 x+\cos^4 x}$ for $x\in R$, then $f(2010)=$
(a) 1 (b) 2 (c) 3 (d) 4

26. If the function $f:R\to R$ is defined by $f(x)=\cos^2 x+\sin^4 x$, then $f(R)=$
(a) $[3/4, 1)$ (b) $(3/4, 1]$ (c) $[3/4, 1]$ (d) $(3/4, 1)$

27. Let $A=\{x\in R: x\neq 0, -4\leq x\leq 4\}$ and $f:A\in R$ be defined by $f(x)=\frac{|x|}{x}$ for $x\in A$. Then, the range of f is
(a) $\{1,-1\}$ (b) $\{x:0\leq x\leq 4\}$ (c) $\{1\}$ (d) $\{x:-4\leq x\leq 0\}$

28. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=2x+3$ and $g(x)=x^2+7$, then the values of x such that $g\left(f(x)\right)=8$ are
(a) 1, 2 (b) −1, 2 (c) −1, −2 (d) 1, −2

29. If $f:[-2,2]\to R$ is defined by
$$f(x)=\begin{cases}-1, \text{ for } -2\leq x\leq 0\\ x-1, \text{ for } 0\leq x\leq 2\end{cases}$$
then $\{x\in[-2,2]: x\leq 0 \text{ and } f(|x|)=x\}=$
(a) $\{-1\}$ (b) $\{0\}$ (c) $\{-1/2\}$ (d) ϕ

30. The function $f:R\to R$ defined by $f(x)=6^x+6^{|x|}$, is
(a) one-one and onto (b) many one and onto
(c) one-one and into (d) many one and into

Answers

1. (c) 2. (b) 3. (a) 4. (d) 5. (c) 6. (b) 7. (d)
8. (a) 9. (b) 10. (a) 11. (b) 12. (c) 13. (d) 14. (a)
15. (b) 16. (a) 17. (c) 18. (a) 19. (d) 20. (a) 21. (d)
22. (b) 23. (c) 24. (c) 25. (a) 26. (c) 27. (a) 28. (c)
29. (c) 30. (c)

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8

MATHEMATICAL REASONING

1. MATHEMATICAL LOGIC

ASSERTIVE SENTENCE *A sentence that makes an assertion is called an assertive sentence or a declarative sentence.*

For example, "Mars supports life" is an assertive or a declarative sentence. "Any two individuals are always related" is also a declarative sentence.

IMPERATIVE SENTENCE *A sentence that expresses a request or a command is called an imperative sentence.*

For example, "Please bring me a cup of tea" is an imperative sentence.

EXCLAMATORY SENTENCE *A sentence that expresses some strong feeling is called an exclamatory sentence.*

For example, "How big is the whale fish !" is an exclamatory sentence.

INTERROGATIVE SENTENCE *A sentence that asks some question is called an interrogative sentence.*

For example, "What is your age ?" is an interrogative sentence. In this chapter, we shall be discussing about a specific type of sentences which will called as statements or propositions.

2. STATEMENTS OR PROPOSITIONS

DEFINITION *A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.*

A statement is assumed to be either true or false. A true statement is also known as a valid statement. If a statement is false, we say that it is an invalid statement. A statement cannot be both true and false at the same time. This fact is known as the law of the excluded middle.

A sentence which is both true and false simultaneously is not a statement, rather it is a paradox.

OPEN STATEMENT *A declarative sentence containing variable (s) is an open statement if. It becomes a statement when the variable (s) is (are) replaced by some definite value (s).*

Consider the following sentences :

(i) x is an integer. (ii) $x^2 - 5x + 4 = 0$

We observe that the truth or falsity of these sentences depend on the values of variable x. For some values of x like $2, 3, -4, -9$ etc sentence (i) is true whereas it is false for other values of x like $\frac{3}{4}, \frac{9}{5}, \frac{12}{23}, \frac{-9}{4}$ etc. Similarly, sentence (ii) is true for $x = 1, 4$, but it is false for all other values of x.

So, these two sentences are open statements.

TRUTH SET *The set of all those values of the variable (s) in an open statement for which it becomes a true statement is called the turth set of the open statement.*

The truth set of the open statement $x^2 - 3x + 2 = 0$ is the set $\{1, 2\}$.

TRUTH VALUE The truth or falsity of a statement is called its truth value.

If a statement is true, then we say that its truth value is 'True' or 'T'. On the other hand the truth value of a false statement is 'False' or 'F'.

SIMPLE STATEMENT *Any statement or proposition whose truth value does not explicity depend on another statement is said to be a simple statement.*

In other words, a statement is said to be simple if it cannot be broken down into simpler statements, that is, if it is not composed of simpler statements.

Consider the following statements:

(i) $\sqrt{2}$ is an irrational number.

(ii) The set of real numbers is an infinite set.

(iii) $2 + 5 < 4$

All these statements are simple statements.

COMPOUND STATEMENTS *If a statement is combination of two or more simple statements, then it is said to be a compound statement or a compound proposition.*

(i) "Roses are red and violets are blue" is a compound statement as it is composed of the statements ; "Roses are red" and "Violets are blue".

(ii) "The school works or a holiday is declared" is a compound statement as it is a combination of the statements: "The school works" and " A holiday is declared".

(iii) "John is intelligent or studies every night" is also a compound statement as it is composed of the statements: "John is intelligent" and "John studies every night".

The simple statements which form a compound statement are known as its sub-statements or components or constituents.

If $p, q, r, \ldots$ are sub-statements of a compound statement S, then we write the compound statement as $S(p, q, r, \ldots)$.

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EXAMPLE 1 *Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".*

Statement–1: r is equivalent to either q or p.

Statement–2: r is equivalent to $\sim (p \Leftrightarrow \sim q)$

(a) 1 (b) 2 (c) 3 (d) 4 **[AIEEE 2008]**

Ans. (c)

SOLUTION We have,

$r = (\sim p \Leftrightarrow q)$

$\Rightarrow r = (p \vee q) \wedge (\sim q \vee \sim p)$ $[\because p \Leftrightarrow q = (\sim p \vee q) \wedge (\sim q \vee p)]$

$\Rightarrow r = [(p \vee q) \wedge \sim q] \vee [(p \vee q) \wedge \sim p]$

$\Rightarrow r = [(p \wedge \sim q) \vee (q \wedge \sim q)] \vee [(p \wedge \sim p) \vee (q \wedge \sim p)]$

$\Rightarrow r = [(p \wedge \sim q) \vee c] \vee [c \vee (q \wedge \sim p)]$

$\Rightarrow r = (p \wedge \sim q) \vee (q \wedge \sim p)$

$\Rightarrow r =$ either q or p

So, statement–1 is true.

Statement–2 is

$\sim (p \Leftrightarrow \sim q)$

$\sim [(\sim p \vee \sim q) \wedge (p \vee q)]$ $[\because p \Leftrightarrow q = (\sim p \vee q) \wedge (\sim q \vee p)]$

$= \sim [\sim (p \wedge q) \wedge (p \vee q)] = (p \wedge q) \vee (\sim p \wedge \sim q)$

Clearly, this is not equivalent to r.

EXAMPLE 2 *Statement–1:* $\sim (p \Leftrightarrow \sim q)$ *is equivalent to* $p \Leftrightarrow q$.

Statement–2: $\sim (p \Leftrightarrow \sim q)$ *is a tautology.*

(a) 1 (b) 2 (c) 3 (d) 4 **[AIEEE 2009]**

Ans. (c)

SOLUTION Let r_1 be the statement $\sim (p \Leftrightarrow \sim q)$. Then,

$r_1 = \sim (p \Leftrightarrow \sim q)$

$\Rightarrow r_1 = \sim [(\sim p \vee \sim q) \wedge (p \vee q)]$ $[\because p \Leftrightarrow q = (p \vee \sim q) \wedge (q \vee \sim p)]$

$\Rightarrow r_1 = \sim [\sim (p \wedge q) \wedge (p \vee q)]$

$\Rightarrow r_1 = (p \wedge q) \vee (\sim p \wedge \sim q)$

$\Rightarrow r_1 = [(p \wedge q) \vee \sim p] \wedge [(p \wedge q) \vee \sim q]$

$\Rightarrow r_1 = [(p \vee \sim p) \wedge (q \vee \sim p)] \wedge [(p \vee \sim q) \wedge (q \vee \sim q)]$

$\Rightarrow r_1 = [t \wedge (q \vee \sim p)] \wedge [(p \vee \sim q) \wedge t]$

$\Rightarrow r_1 = (q \vee \sim p) \wedge (p \vee \sim q)$

$\Rightarrow r_1 = (p \Leftrightarrow q)$

So, statement–1 is true.

Also, $p \Leftrightarrow q$ is not a tautology. So, statement–2 is false.

Clearly, this is not equivalent to r.

EXAMPLE 3 *Statement-1:* $(p \wedge \sim q) \wedge (\sim p \wedge q)$ *is a fallacy.*

Statement-2: $(p \to q) \Leftrightarrow (\sim q \to \sim p)$ *is a tautology.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION We know that $p \to q \Leftrightarrow \sim p \vee q$. Therefore,

$\sim q \to \sim p \Rightarrow \sim (\sim q) \vee \sim p \Leftrightarrow q \vee \sim p \Leftrightarrow \sim p \vee q$

Thus, $(p \to q) \Leftrightarrow (\sim q \to \sim p)$

So, statement-2 is true.

Now, $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$= (\sim q \wedge p) \wedge (\sim p \sim \wedge q)$

$= \sim q \wedge (p \wedge \sim p) \wedge q$

$= \sim q \wedge c \wedge q$

$= (\sim q \wedge q) \wedge c$

$= c \wedge c = c$

So, statement-1 is true.

Thus, both the statements are true and statement-2 is not a correct explanation for statement.

EXAMPLE 4 *Let p and q be any two propositions.*

Statement-1: $(p \to q) \leftrightarrow q \vee \sim p$ *is a tautology*

Statement-2: $\sim (\sim p \wedge q) \wedge (p \vee q) \leftrightarrow p$ *is a fallacy.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d) **[JEE (Main) 2014]**

SOLUTION Do yourself

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. Logical equivalent proposition to the proposition $\sim (p \vee q)$ is
   (a) $\sim p \wedge \sim q$ (b) $\sim p \vee \sim q$ (c) $\sim p \to \sim q$ (d) $\sim p \leftrightarrow \sim q$
2. Let p and q be two propositions. Then the inverse of the implication $p \to q$ is
   (a) $q \to p$ (b) $\sim p \to \sim q$ (c) $q \to p$ (d) $\sim q \to \sim p$
3. Let p and q be two propositions. Then the contrapositive of the implication $p \to q$ is
   (a) $\sim q \to \sim p$ (b) $\sim p \to \sim q$ (c) $q \to p$ (d) $p \leftrightarrow q$
4. If p and q are two simple propositions, then $p \to q$ is false when
   (a) p is true and q is true (b) p is false and q is true
   (c) p is true and q is false (d) both p and q are false
5. If p and q are two simple propositions, then $p \leftrightarrow \sim q$ is true when
   (a) p and q both are true (b) both p and q are false
   (c) p is false and q is true (d) none of these
6. For any three propositions p, q and r, the proposition $(p \wedge q) \wedge (q \wedge r)$ is true when
   (a) p, q, r are all false (b) p, q, r are all true
   (c) p, q are true and r is false
   (d) p is true and q and r are false
7. If p and q are two propositions, then $\sim (p \leftrightarrow q)$ is
   (a) $\sim p \wedge \sim q$ (b) $\sim p \vee \sim q$
   (c) $(p \wedge \sim q) \vee (\sim p \wedge q)$ (d) none of these
8. $p \wedge (q \wedge r)$ is logically equivalent to
   (a) $p \vee (q \wedge r)$ (b) $(p \wedge q) \wedge r$
   (c) $(p \vee q) \vee r$ (d) $p \to (q \wedge r)$
9. $\sim (\sim p) \leftrightarrow p$ is
   (a) a tautology (b) a contradiction

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(c) neither a contradiction nor a tautology
(d) none of these

10. Which of the following is a proposition ?
(a) I am a lion (b) A half open door is half closed
(c) A triangle is a circle and 10 is a prime number
(d) Logic is an interesting subject

11. Which of the following propositions is a tautology?
(a) $(\sim p \vee \sim q) \vee (p \vee \sim q)$ (b) $(\sim p \vee \sim q) \wedge (p \vee \sim q)$
(c) $\sim p \wedge (\sim p \vee \sim q)$ (d) $\sim q \wedge (\sim p \vee \sim q)$

12. Which of the following proposition is a tautology ?
(a) $\sim (p \to q) \vee (p \wedge \sim q)$ (b) $(p \to q) \to (p \wedge \sim q)$
(c) $(p \to q) \vee (p \wedge \sim q)$ (d) $(p \to q) \wedge (p \wedge \sim q)$

13. Which of the following propositions is a contradiction?
(a) $(\sim p \vee \sim q) \vee (p \vee \sim q)$ (b) $(p \to q) \vee (p \wedge \sim q)$
(c) $(\sim p \wedge q) \wedge (\sim q)$ (d) $(\sim p \wedge q) \vee (\sim q)$

14. $p \wedge q \to p$ is
(a) a tautology (b) a contradiction
(c) neither a tautology n or a contradiction
(d) none of these

15. Which of the following is false ?
(a) $p \vee \sim p$ is a tautology (b) $\sim (\sim p) \leftrightarrow p$ is a tautology
(c) $\big(p \wedge (p \to q)\big) \to p$ is a contradiction
(d) $p \wedge \sim p$ is a contradiction

16. Given that water freezes below zero degree celsius.
Consider the following statements :
p : Water froze this morning, q : This morning temperature was below 0°C
Which of the following is correct ?
(a) p and q are logically equivalent
(b) p is the inverse of q (c) p is the converse of q
(d) p is the contrapositive of q

17. If p, q, r have truth values T, F, T respectively, which of the following is true ?
(a) $(p \to q) \wedge r$ (b) $(p \to q) \wedge \sim r$
(c) $(p \wedge q) \wedge (p \vee r)$ (d) $q \to (p \wedge r)$

18. If $p \to (q \vee r)$ is false, then the truth values of p, q, r are respectively
(a) T, T, T (b) F, T, T (c) F, F, F (d) T, F, F

19. The negation of the proposition $q \vee \sim (p \wedge r)$ is
(a) $\sim q \vee (p \wedge r)$ (b) $\sim q \wedge (p \wedge r)$
(c) $\sim p \vee \sim q \vee \sim r$ (d) none of these

20. Which of the following is logically equivalent to $p \wedge q$?
(a) $p \to \sim q$ (b) $\sim p \vee \sim q$ (c) $\sim (p \to \sim q)$ (d) $\sim (\sim p \wedge \sim q)$

21. Which of the following is logically equivalent to $\sim (\sim p \to q)$?
(a) $p \wedge q$ (b) $p \wedge \sim q$ (c) $\sim p \wedge q$ (d) $\sim p \wedge \sim q$

22. Which of the following is a contradiction ?
(a) $(p \wedge q) \wedge \big(\sim (p \vee q)\big)$ (b) $p \vee (\sim p \wedge q)$
(c) $(p \to q) \to p$ (d) none of these

23. The negative of the proposition : "If a number is divisible by 15, then it is divisible by 5 or 3".
(a) If a number is divisible by 15, then it is not divisible by 5 and 3
(b) A number is divisible by 15 and it is not divisible by 5 and 3
(c) A number is divisible by 15 and it is not divisible by 5 or 3
(d) A number is not divisible by 15 or it is not divisible by 5 and 3

24. Consider the proposition : "If the pressure increases, the volume decreases". The negation of this propositions is
(a) If the pressure does not increase the volume does not decrease
(b) If the volume increases, the pressure decreases
(c) If the volume does not decrease, the pressure does not increase
(d) If the volume decreases, then the pressure increases.

25. Consider the proposition : "If we control population growth, we prosper". Negative of this proposition is
(a) If we do not control population growth, we prosper
(b) If we control population, we do not prosper
(c) We control population but we do not prosper
(d) We do not control population but we prosper

26. The negation of $q \vee \sim (p \wedge r)$ is
(a) $\sim q \vee \sim (p \wedge r)$ (b) $\sim q \vee (p \wedge r)$
(c) $\sim q \wedge (p \wedge r)$ (d) $\sim q \wedge \sim (p \wedge r)$

27. The negation of $p \wedge \sim (q \wedge r)$ is
(a) $\sim p \vee (q \wedge r)$ (b) $\sim p \vee (\sim q \vee \sim r)$
(c) $p \vee (q \wedge r)$ (d) $\sim p \wedge (q \vee r)$

28. The contrapositive of $(\sim p \wedge q) \to \sim r$ is
(a) $(p \wedge q) \to r$ (b) $(p \vee q) \to r$
(c) $r \to (p \vee \sim q)$ (d) none of these

29. $p \to q$ is logically equivalent to
(a) $p \wedge \sim q$ (b) $\sim p \to \sim q$
(c) $\sim q \to \sim p$ (d) none of these

30. Which of the following is logically equivalent to $\sim (p \leftrightarrow q)$?
(a) $(p \wedge \sim q) \wedge (q \wedge \sim p)$ (b) $p \vee q$
(c) $(p \wedge \sim q) \vee (q \wedge \sim p)$ (d) none of these

31. Which of the following is logically equivalent to $\sim (\sim p \to q)$?
(a) $p \wedge q$ (b) $p \wedge \sim q$ (c) $\sim p \wedge q$ (d) $\sim p \wedge \sim q$

32. Which of the following is logically equivalent to $(p \wedge q)$?
(a) $p \to q$ (b) $\sim p \wedge \sim q$ (c) $p \wedge \sim q$ (d) $\sim (p \to \sim q)$

33. The contrapositive of $2x + 3 = 9 \Rightarrow x \neq 4$ is
(a) $x = 4 \Rightarrow 2x + 3 \neq 9$ (b) $x = 4 \Rightarrow 2x + 3 = 9$
(c) $x \neq 4 \Rightarrow 2x + 3 \neq 9$ (d) $x \neq 4 \Rightarrow 2x + 3 = 9$

34. The proposition $(p \to \sim p) \wedge (\sim p \to p)$ is a
(a) tautology (b) contradiction
(c) neither a tautology nor a contradiction
(d) tautology and contradiction

35. Consider the following statements:
p : I shall pass, q : I study
The symbolic representation of the proposition "I sha pass iff I study" is
(a) $p \to q$ (b) $q \to p$ (c) $p \to \sim q$ (d) $p \leftrightarrow$

36. The proposition $p \to \sim (p \wedge \sim q)$ is
(a) a contradiction (b) a tautology
(c) either a tautology or a contradiction

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(d) neither a tautology nor a contradiction

37. $\sim (p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to
(a) $\sim p$ (b) p (c) q (d) $\sim q$

38. The negation of the compound proposition $p \vee (\sim p \vee q)$ is
(a) $(p \wedge \sim q) \wedge \sim p$ (b) $(p \wedge \sim q) \vee \sim p$
(c) $(p \wedge \sim q) \vee \sim p$ (d) none of these

39. Let p be the proposition : Mathematics is interesting and let q be the proposition that Mathematics is difficult, then the symbol $p \wedge q$ means
(a) Mathematics is interesting implies that Mathematics is difficult
(b) Mathematics is interesting implies and is implied by Mathematics is difficult
(c) Mathematics is interesting and Mathematics is difficult
(d) Mathematics is interesting or Mathematics is difficult

40. The inverse of the proposition $(p \wedge \sim q) \to r$ is
(a) $\sim r \to \sim p \vee q$ (b) $\sim p \vee q \to \sim r$
(c) $r \to p \wedge \sim q$ (d) none of these

Answers

1. (a) 2. (b) 3. (a) 4. (c) 5. (c) 6. (b) 7. (c)
8. (b) 9. (a) 10. (c) 11. (a) 12. (c) 13. (c) 14. (a)
15. (c) 16. (a) 17. (d) 18. (d) 19. (b) 20. (c) 21. (d)
22. (a) 23. (b) 24. (c) 25. (c) 26. (c) 27. (a) 28. (c)
29. (c) 30. (c) 31. (d) 32. (d) 33. (a) 34. (b) 35. (d)
36. (d) 37. (a) 38. (a) 39. (c) 40. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. Which of the following sentences is a statement?
(a) AArushi is a pretty girl
(b) What are you doing?
(c) Oh! It is amazing
(d) 2 is the smallest prime number

2. The property $\sim (p \wedge q) \equiv \sim p \vee \sim q$ is called
(a) associative law (b) De morgan's law
(c) commutative law (d) idempotent law

3. When does the inverse of the statement $\sim p \Rightarrow q$ results in T?
(a) p and q both are true (b) p is true and q is false
(c) p is false and q is false (d) both (b) and (c)

4. Which of the following is a contradiction?
(a) $p \vee q$ (b) $p \wedge q$ (c) $p \vee \sim p$ (d) $p \wedge \sim p$

5. In which of the following cases, $p \Rightarrow q$ is true?
(a) p is true, q is true (b) p is false, q is true
(c) p is true, q is false (d) none of these

6. Which of the following is equivalent to $p \Rightarrow q$?
(a) $p \Rightarrow q$ (b) $q \Rightarrow p$
(c) $(p \Rightarrow q) \wedge (q \Rightarrow p)$ (d) none of these

7. Which of the following pairs are logically equivalent?
(a) Conditional, Contrapositive
(b) Conditional, Inverse (c) Contrapositive, Converse
(d) Inverse, Contrapositive

8. Which of the following is contingency?
(a) $p \vee \sim p$ (b) $p \wedge q \Rightarrow p \vee q$ (c) $p \wedge \sim q$ (d) none of these

9. The statement $p \vee q$ is
(a) a tautology (b) a contradiction
(c) contingency (d) none of these

10. Which of the following is a tautology?
(a) $p \wedge q$ (b) $p \vee q$ (c) $p \vee \sim p$ (d) $p \wedge \sim p$

11. The statement $p \Rightarrow p \vee q$ is
(a) a tautology (b) a contradiction
(c) both a tautology and contradiction
(d) neither a tautology nor a contradiction

12. What are the truth values of $(\sim p \Rightarrow \sim q)$ and $\sim (\sim p \Rightarrow q)$ respectively, when p and q always speak true in any argument?
(a) T, T (b) F, F (c) T, F (d) F, T

13. If truth value of $p \vee q$ is true, then truth value of $\sim p \wedge q$ is
(a) false if p is true (b) true if p is true
(c) false if q is true (d) true if q is true

14. If p and q are two statements, then $p \vee \sim (p \Rightarrow \sim q)$ is equivalent to
(a) $p \wedge \sim q$ (b) p (c) q (d) $\sim p \wedge q$

15. The contropositive of the statement $\sim p \Rightarrow (p \wedge \sim q)$ is
(a) $p \Rightarrow (\sim p \vee q)$ (b) $p \Rightarrow (p \wedge q)$
(c) $p \Rightarrow (\sim p \wedge q)$ (d) $\sim p \vee q \Rightarrow p$

16. $\sim [\sim p \wedge (p \Leftrightarrow q)] \equiv$
(a) $p \vee q$ (b) $q \wedge p$ (c) T (d) F

17. If a compound statement r is contradiction, then the truth value of $(p \Rightarrow q) \wedge r \wedge p\ [p \Rightarrow \sim r]$ is
(a) TM (b) F (c) T or F (d) none of these

18. When does the value of the statement $p (\wedge r) \Leftrightarrow (r \wedge q)$ become false?
(a) p is T, q is F (b) p is, r is F
(c) p is F, q is F and r is F (d) none of these

19. If p always speaks against q, then $p \Rightarrow p \vee \sim q$ is
(a) a tautology (b) contradiction
(c) contingency (d) none of these

20. Which of the following connectives satisfy commutative law?
(a) $\wedge$ (b) $\vee$ (c) $\Leftrightarrow$ (d) all the above

Answer

1. (d) 2. (b) 3. (d) 4. (d) 5. (a) 6. (c) 7. (a)
8. (c) 9. (c) 10. (c) 11. (a) 12. (c) 13. (a) 14. (b)
15. (d) 16. (a) 17. (b) 18. (d) 19. (a) 20. (d)

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9

COMPLEX NUMBERS

1. INTEGRAL POWERS OF IOTA (i)

Euler was the first mathematician to introduce the symbol i (iota) for the square root of -1 with the property $i^2 = -1$. He also called this symbol as the imaginary unit.

Positive integral powers of i : We have,

$$i = \sqrt{-1}$$

$$\therefore \quad i^2 = -1,\ i^3 = i^2 \times i = -i,\ i^4 = (i^2)^2 = (-1)^2 = 1$$

The value of i^n for $n > 4$ is i^r, where r is the remainder when n is divided by 4.

Negative integral powers of i : By the law of indices, we have

$$i^{-1} = \frac{1}{i} = \frac{i^3}{i^4} = i^3 = -i,\ i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$$

$$i^{-3} = \frac{1}{i^3} = \frac{i}{i^4} = i,\ i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$$

If $n > 4$, then $i^{-n} = \frac{1}{i^r}$, where r is the remainder when n is divided by 4.

NOTE i^0 *is defined as* 1.

ILLUSTRATION 1 *If $i = \sqrt{-1}$ and $n \in N$, then*

$$i^n + i^{n+1} + i^{n+2} + i^{n+3} \text{ is equal to}$$

(a) 1 (b) i (c) i^n (d) 0

Ans. (d)

SOLUTION We have,

$$i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n (1 + i + i^2 + i^3) = i^n (1 + i - 1 - i) = 0$$

ILLUSTRATION 2 *If $i = \sqrt{-1}$, then $\{i^n + i^{-n} : n \in Z\}$ is equal to*

(a) $\{0, 2\}$ (b) $\{0, -2\}$ (c) $\{0, -2, 2\}$ (d) $\{0, -2i\}$

Ans. (c)

SOLUTION We have the following cases:

CASE I *When $n = 4m, m \in Z$*

In this case, we have

$$i^n = i^{4m} = (i^4)^m = 1 \text{ and } i^{-n} = \frac{1}{i^n} = \frac{1}{1} = 1$$

$$\therefore \quad i^n + i^{-n} = 2$$

CASE II *When $n = 4m + 1, m \in Z$*

In this case, we have

$$i^n = i^{4m+1} = (i^4)^m i^1 = i \text{ and, } i^{-n} = \frac{1}{i^n} = \frac{1}{i} = -i$$

$$\therefore \quad i^n + i^{-n} = i - i = 0$$

CASE III *When $n = 4m + 2, m \in Z$*

In this case, we have

$$i^n = i^{4m+2} = (i^4)^m i^2 = i^2 = -1$$

$$\therefore \quad i^n + i^{-n} = (-1) + (-1) = -2$$

CASE IV *When $n = 4m + 3, m \in Z$*

In this case, we have

$$i^n = i^{4m+3} = (i^4)^m i^3 = i^3 = -i \text{ and, } i^{-n} = \frac{1}{i^n} = \frac{1}{i^3} = i$$

$$\therefore \quad i^n + i^{-n} = (-i) + (i) = 0$$

Hence, $S = \{-2, 0, 2\}$

ILLUSTRATION 3 *The value of $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$ equals*

(a) i (b) $i - 1$ (c) $-i$ (d) 0

[IIT 1998]

Ans. (b)

SOLUTION We have,

$$\sum_{n=1}^{13} (i^n + i^{n+1})$$

$$= \sum_{n=1}^{13} i^n + \sum_{n=1}^{13} i^{n+1}$$

$$= i\left(\frac{1 - i^{13}}{1 - i}\right) + i^2\left(\frac{1 - i^{13}}{1 - i}\right) = i\,\frac{(1 - i)}{1 - i} - \left(\frac{1 - i}{1 - i}\right) = i - 1$$

ILLUSTRATION 4 *If n is an odd integer, then $(1 + i)^{6n} + (1 - i)^{6n}$ is equal to*

(a) 0 (b) 2 (c) -2 (d) *none of the*

Ans. (a)

SOLUTION We have,

$$(1 + i)^{6n} + (1 - i)^{6n} = \{(1 + i)^2\}^{3n} + \{(1 - i)^2\}^{3n}$$

$$= (2i)^{3n} + (-2i)^{3n} = 2^{3n} \{i^{3n} + (-i)^{3n}\}$$

$$= 2^{3n} \{i^{3n} - i^{3n}\} = 0 \qquad [\because n \text{ is o}$$

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$\Rightarrow \quad p^2 = 4z_1 z_2 \cos^2\frac{\alpha}{2} \Rightarrow p^2 = 4q\cos^2\frac{\alpha}{2}$

Hence, statement–1 is true.

EXAMPLE 12 *Statement–1: The locus of point z satisfying* $\left|\frac{3z+i}{2z+3+4i}\right| = \frac{3}{2}$ *is a straight line.*

Statement–2: The locus of a point equidistant from two fixed points is a straight line representing the perpendicular bisector of the segment joining the given points.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement–2 is true.

We have, $\left|\frac{3z+i}{2z+3+4i}\right| = \frac{3}{2}$

$\Rightarrow \quad \left|z+\frac{1}{3}i\right| = \left|z+\frac{3}{2}+2i\right|$

$\Rightarrow \quad \left|z-\left(0-\frac{1}{3}i\right)\right| = \left|z-\left(-\frac{3}{2}-2i\right)\right|$

$\Rightarrow$ z is equidistant from $\left(0,-\frac{1}{3}\right)$ and $\left(-\frac{3}{2},-2\right)$

$\Rightarrow$ Locus of z is a straight line. [Using statement–2]

EXAMPLE 13 *Statement–1: If a, b, c are distinct real numbers and* $w (\neq 1)$ *is a cube root of unity, then* $\left|\frac{a+bw+cw^2}{aw^2+b+cw}\right| = 1$

Statement–2: For any non-zero complex number z, $|z/\bar{z}| = 1$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION We have,

$$aw^2+b+cw = \frac{1}{w}(aw^3+bw+cw^2) = \frac{1}{w}(a+bw+cw^2)$$

$\Rightarrow \quad a+bw+cw^2 = w(aw^2+b+cw)$

$\Rightarrow \quad \left|\frac{a+bw+cw^2}{aw^2+b+cw}\right| = |w| = 1$

So, statement–1 is true.

For any non-zero complex number z, we have

$$\left|\frac{z}{\bar{z}}\right| = \frac{|z|}{|\bar{z}|} = 1 \qquad [\because |z| = |\bar{z}|]$$

So, statement–2 is true. But, it is not a correct explanation for statement–1.

EXAMPLE 14 *Let z be a unimodular complex number.*

Statement–1: $arg\left(z^2+\bar{z}\right) = \arg(z)$

Statement-2: $\bar{z} = \cos(\arg z) - i\sin(\arg z)$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION We have, $|z| = 1$.

$\therefore \quad z = \cos\theta + i\sin\theta$, where $\theta = \arg(z)$

$\Rightarrow \quad \bar{z} = \cos\theta - i\sin\theta = \cos(\arg(z)) - i\sin(\arg z)$

So, statement-2 is true.

Now, $\arg(z^2+\bar{z}) = \arg(\cos 2\theta + i\sin 2\theta + \cos\theta - i\sin\theta)$

$= \tan^{-1}\left(\frac{\sin 2\theta - \sin\theta}{\cos 2\theta + \cos\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\arg(z)$

So, statement-1 is false.

EXAMPLE 15 *Let z and w be complex numbers such that* $|z| = |w|$ *and arg z denote the principal argument of z.*

Statement-1: If $arg\, z + arg\, w = \pi$, *then* $z = -\bar{w}$

Stagement-2: $|z| = |w|$ *implies* $arg\, z - arg\, \bar{w} = \pi$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c) **[AIEEE 2012]**

SOLUTION Let $z = r(\cos\theta + i\sin\theta)$ and $w = r(\cos\alpha + i\sin\alpha)$. Then,

$$arg(z) + arg(w) = \pi \Rightarrow \theta + \alpha = \pi \Rightarrow \theta = \pi - \alpha$$

$\therefore \quad z = r(-\cos\alpha + i\sin\alpha) = -r(\cos\alpha - i\sin\alpha) = -|w|$

So, statement-1 is true.

Clearly, $|z| = |w|$ need not imply that $arg(z) - arg(\bar{w}) = \pi$, because if we take $z = i$ and $w = -i$, then $|z| = |w|$. But,

$$arg(z) - arg(\bar{w}) = arg(i) - arg(i) = 0 \neq \pi$$

Hence, statement-1 is true and statement-2 is false.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. Which of the following is correct?
   (a) $1 + i > 2 - i$ (b) $2 + i > 1 + i$
   (c) $2 - i > 1 + i$ (d) none of these
2. If $a = \sqrt{2}\,i$, then which of the following is correct?
   (a) $a = 1 + i$ (b) $a = 1 - i$
   (c) $a = -(\sqrt{2})\,i$ (d) none of these
3. Let z_1, z_2 be two complex numbers such that $z_1 + z_2$ and $z_1 z_2$ both are real, then
   (a) $z_1 = -z_2$ (b) $z_1 = \bar{z}_2$ (c) $z_1 = -\bar{z}_2$ (d) $z_1 = z_2$
4. If the complex numbers z_1, z_2, z_3 are in AP, then they lie on a
   (a) circle (b) parabola (c) line (d) ellipse
5. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is
   (a) 3 (b) 2 (c) 4 (d) none of these
   [JEE (WB) 2006]
6. The locus of the point z satisfying $\arg\left(\frac{z-1}{z+1}\right) = k$, where k is non-zero real, is
   (a) a circle with centre on y-axis
   (b) a circle with centre on x-axis
   (c) a straight line parallel to x-axis
   (d) a straight line making an angle of $60°$ with the x-axis.
7. If $\sqrt{x+iy} = \pm(a+ib)$, then $\sqrt{-x-iy}$ is equal to
   (a) $\pm(b+ia)$ (b) $\pm(a-ib)$

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(c) $\pm (b - ia)$ (d) none of these

8. The locus of the points z satisfying the condition $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$ is a
(a) parabola (b) circle
(c) pair of staight lines (d) none of these

9. If $(\sqrt{3}+i)^{10}=a+ib$, then a and b are respectively
(a) 128 & $128\sqrt{3}$ (b) 64 & $-64\sqrt{3}$
(c) 512 & $-512\sqrt{3}$ (d) none of these

10. If $\text{Re}\left(\frac{z-8i}{z+6}\right)=0$, then z lies on the curve
(a) $x^2+y^2+6x-8y=0$ (b) $4x-3y+24=0$
(c) $x^2+y^2-8=0$ (d) none of these

11. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then
(a) $\text{Re}(z)=0$ (b) $\text{Im}(z)=0$
(c) $\text{Re}(z)>0,\ \text{Im}(z)>0$ (d) $\text{Re}(z)>0,\ \text{Im}(z)<0$

12. If $z=x+iy$ and $w=\frac{1-iz}{z-i}$, then $|w|=1$ implies that in the complex plane
(a) z lies on imaginary axis (b) z lies on real axis
(c) z lies on unit circle (d) none of these

13. Let $3-i$ and $2+i$ be affixes of two points A and B in the argand plane and P represents the complex number $z=x+iy$. Then, the locus of P if $|z-3+i|=|z-2-i|$ is
(a) circle on AB as diameter
(b) the line AB
(c) the perpendicular bisector of AB
(d) none of these

14. POQ is a straight line through the origin O. P and Q represent the complex numbers $a+ib$ and $c+id$ respectively and $OP=OQ$. Then, which one of the following is not true?
(a) $|a+ib|=|c+id|$ (b) $a+b=c+d$
(c) $\arg(a+ib)=\arg(c+id)$ (d) none of these

15. If $z_1=a+ib$ and $z_2=c+id$ are complex numbers such that $|z_1|=|z_2|=1$ and $\text{Re}(z_1\bar{z}_2)=0$, then the pair of complex numbers $w_1=a+ic$ and $w_2=b+id$ satisfy
(a) $|w_1|=1$ (b) $|w_2|=1$
(c) $\text{Re}(w_1\bar{w}_2)=0$ (d) all of these

16. Let z_1 and z_2 be two complex numbers such that $z_1\neq z_2$ and $|z_1|=|z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$
(a) cannot be zero (b) is real and positive
(c) is real and negative (d) is purely imaginary

17. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$, is
(a) -1 (b) 0 (c) $-i$ (d) i

18. The equation $\bar{b}z+b\bar{z}=c$, where b is a non-zero complex constant and c is a real number, represents
(a) a circle (b) a straight line
(c) a pair of straight lines (d) none of these

19. If $|a_i|<1,\ \lambda_i\geq 0$ for $i=1,2,.....,n$ and $\lambda_1+\lambda_2+.....+\lambda_n=1$, then the value of $|\lambda_1 a_1+\lambda_2 a_2+....+\lambda_n a_n|$ is
(a) equal to 1 (b) less than 1
(c) greater than 1 (d) none of these

20. For any two complex numbers z_1, z_2 and any two real numbers a and b, $|az_1-bz_2|^2+|bz_1+az_2|^2=$
(a) $(a+b)(|z_1|^2+|z_2|^2)$
(b) $(a^2+b^2)(|z_1|^2+|z_2|^2)$
(c) $(a^2+b^2)(|z_1|+|z_2|)$
(d) none of these [JEE (WB) 2007]

21. Common roots of the equations $z^3+2z^2+2z+1=0$ and $z^{1985}+z^{100}+1=0$ are
(a) w, w^2 (b) $1, w, w^2$ (c) $-1, w, w^2$ (d) $-w, -w^2$

22. If z_1 and z_2 are two complex numbers such that $\left|\frac{z_1-z_2}{1-\bar{z}_1 z_2}\right|=1$, then which one of the following is not true?
(a) $|z_1|=1, |z_2|=1$ (b) $z_1=e^{i\theta}, \theta\in R$
(c) $z_2=e^{i\theta}, \theta\in R$ (d) all of these

23. The points representing cube roots of unity
(a) are collinear
(b) lie on a circle of radius $\sqrt{3}$
(c) from an equilateral triangle
(d) none of these

24. If z_1 and z_2 are two complex numbers such that $\left|\frac{z_1-z_2}{z_1+z_2}\right|=1$, then
(a) $z_1=kz_2,\ k\in R$ (b) $z_1=ikz_2,\ k\in R$
(c) $z_1=z_2$ (d) none of these

25. If z_1, z_2 are two complex numbers such that $\left|\frac{z_1-z_2}{z_1+z_2}\right|=1$ and $iz_1=kz_2$ where $k\in R$, then the angle between z_1-z_2 and z_1+z_2 is
(a) $\tan^{-1}\left(\frac{2k}{k^2+1}\right)$ (b) $\tan^{-1}\left(\frac{2k}{1-k^2}\right)$
(c) $-2\tan^{-1}k$ (d) $2\tan^{-1}k$

26. If n is a positive integer greater than unity and z is a complex number satisfying the equation $z^n=(z+1)^n$, then
(a) $\text{Re}(z)<0$ (b) $\text{Re}(z)>0$
(c) $\text{Re}(z)=0$ (d) none of these

27. If n is a positive integer greater than unity and z is a complex number satisfying the equation $z^n=(z+1)^n$, then
(a) $\text{Im}(z)<0$ (b) $\text{Im}(z)>0$

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(c) $\text{Im}(z)=0$ (d) none of these

28. If at least one value of the complex number $z=x+iy$ satisfy the condition $|z+\sqrt{2}| = \sqrt{a^2-3a+2}$ and the inequality $|z+i\sqrt{2}| < a$, then
(a) $a>2$ (b) $a=2$ (c) $a<2$ (d) none of these

29. Given z is a complex number with modulus 1. Then, the equation $\left(\frac{1+ia}{1-ia}\right)^4 = z$ has
(a) all roots, real and distinct
(b) two real and two imaginary
(c) three roots real and one imaginary
(d) one root real and three imaginary

30. The centre of a regular polygon of n sides is located at the point $z=0$, and one of its vertex z_1 is known. If z_2 be the vertex adjacent to z_1, then z_2 is equal to
(a) $z_1\left(\cos\frac{2\pi}{n} \pm i\sin\frac{2\pi}{n}\right)$ (b) $z_1\left(\cos\frac{\pi}{n} \pm i\sin\frac{\pi}{n}\right)$
(c) $z_1\left(\cos\frac{\pi}{2n} \pm i\sin\frac{\pi}{2n}\right)$ (d) none of these

31. If the points z_1, z_2, z_3 are the vertices of an equilateral triangle in the Argand plane, then which one of the following is not correct?
(a) $\frac{1}{z_1-z_2}+\frac{1}{z_2-z_3}+\frac{1}{z_3-z_1}=0$
(b) $z_1^2+z_2^2+z_3^2 = z_1z_2+z_2z_3+z_3z_1$
(c) $(z_1-z_2)^2+(z_2-z_3)^2+(z_3-z_1)^2=0$
(d) $z_1^3+z_2^3+z_3^3+3z_1z_2z_3=0$

32. For any complex number z, the minimum value of $|z|+|z-1|$ is
(a) 1 (b) 0 (c) 1/2 (d) 3/2

33. The inequality $|z-4| < |z-2|$ represents
(a) $\text{Re}(z) > 0$ (b) $\text{Re}(z) < 0$
(c) $\text{Re}(z) > 2$ (d) $\text{Re}(z) > 3$

[AIEEE 2002]

34. Number of non-zero integral solutions of the equation $(1-i)^n = 2^n$ is
(a) 1 (b) 2 (c) infinite (d) none of these

35. If $\text{Im}\left(\frac{2z+1}{iz+1}\right) = -2$, then locus of z is
(a) a circle (b) a parabola
(c) a straight line (d) none of these

36. If $z\,(\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z| =$
(a) 1 (b) 2 (c) 3 (d) 5

37. The value of $x^4+9x^3+35x^2-x+4$ for $x=-5+2\sqrt{-4}$ is
(a) 0 (b) -160 (c) 160 (d) -164

38. If z_1, z_2, z_3 are vertices of an equilateral triangle with z_0 its centroid, then $z_1^2+z_2^2+z_3^2 =$
(a) z_0^2 (b) $9z_0^2$ (c) $3z_0^2$ (d) $2z_0^2$

39. If z_1, z_2 are two complex numbers such that $\text{Im}(z_1+z_2)=0$, $\text{Im}(z_1z_2)=0$, then
(a) $z_1=-z_2$ (b) $z_1=z_2$ (c) $z_1=\bar{z}_2$ (d) none of these

40. If $z^2+z|z|+|z|^2=0$, then the locus of z is
(a) a circle (b) a straight line
(c) a pair of straight lines (d) none of these

41. If $\log_{\sqrt{3}}\left(\frac{|z|^2-|z|+1}{2+|z|}\right) > 2$, then the locus of z is
(a) $|z|=5$ (b) $|z|<5$ (c) $|z|>5$ (d) none of these

42. Let $z\,(\neq 2)$ be a complex number such that $\log_{1/2}|z-2| > \log_{1/2}|z|$, then
(a) $\text{Re}(z)>1$ (b) $\text{Im}(z)>1$ (c) $\text{Re}(z)=1$ (d) $\text{Im}(z)=1$.

43. If $g(x)$ and $h(x)$ are two polynomials such that the polynomial $P(x)=g(x^3)+xh(x^3)$ is divisible by x^2+x+1, then which one of the following is not true?
(a) $g(1)=h(1)=0$ (b) $g(1)=h(1)\neq 0$
(c) $g(1)=-h(1)$ (d) $g(1)+h(1)=0$

44. If $|z_1|=|z_2|=|z_3|$ and $z_1+z_2+z_3=0$, then z_1, z_2, z_3 are vertices of
(a) a right angled triangle (b) an equilateral triangle
(c) isosceles triangle (d) scalene triangle

45. If $x_n=\cos\frac{\pi}{3^n}+i\sin\frac{\pi}{3^n}$, then $x_1, x_2, x_3, \ldots x_\infty$ is equal to
(a) 1 (b) -1 (c) i (d) $-i$

46. If $(a_1+ib_1)(a_2+ib_2)\ldots(a_n+ib_n) = A+iB$, then $(a_1^2+b_1^2)(a_2^2+b_2^2)\ldots(a_n^2+b_n^2)$ is equal to
(a) 1 (b) A^2+B^2
(c) $A+B$ (d) $\frac{1}{A^2}+\frac{1}{B^2}$

47. If $(a_1+ib_1)(a_2+ib_2)\ldots(a_n+ib_n) = A+iB$, then $\sum_{i=1}^{n}\tan^{-1}\left(\frac{b_i}{a_i}\right)$ is equal to
(a) $\frac{B}{A}$ (b) $\tan\left(\frac{B}{A}\right)$ (c) $\tan^{-1}\left(\frac{B}{A}\right)$ (d) $\tan^{-1}\left(\frac{A}{B}\right)$

48. If $\cos\alpha+2\cos\beta+3\cos\gamma=\sin\alpha+2\sin\beta+3\sin\gamma=0$, then the value of $\sin 3\alpha+8\sin 3\beta+27\sin 3\gamma$ is
(a) $\sin(\alpha+\beta+\gamma)$ (b) $3\sin(\alpha+\beta+\gamma)$
(c) $18\sin(\alpha+\beta+\gamma)$ (d) $\sin(\alpha+2\beta+3\gamma)$

49. If α, β, γ are the cube roots of p, $p<0$, then for any x, y and z the values of $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ are
(a) w, w^2 (b) $-w, -w^2$ (c) $1, -1$ (d) none of these

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50. If the complex numbers $z_1 = a + i, z_2 = 1 + ib, z_3 = 0$ form the vertices of equilateral triangle (a, b are real numbers between 0 and 1), then

(a) $a = \sqrt{3} - 1,\ b = \frac{\sqrt{3}}{2}$ (b) $a = 2 - \sqrt{3},\ b = 2 - \sqrt{3}$

(c) $a = 1/2,\ b = 3/4$ (d) none of these

51. If w is a complex cube root of unity, then the value of $\frac{a + bw + cw^2}{c + aw + bw^2} + \frac{a + bw + cw^2}{b + cw + aw^2}$ is

(a) 1 (b) 0 (c) 2 (d) -1

52. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c = (1 - r)a + rb$ and $w = (1 - r)u + rv$, where r is a complex number, then the two triangles

(a) have the same area (b) are similar

(c) are congruent (d) none of these

53. If $z = re^{i\theta}$, then $|e^{iz}|$ is equal to

(a) $e^{-r\sin\theta}$ (b) $re^{-r\sin\theta}$ (c) $e^{-r\cos\theta}$ (d) $re^{-r\cos\theta}$

54. If a complex number z lies in the interior or on the bondary of a circle or radius 3 and centre at $(-4, 0)$, then the greatest and least values of $|z + 1|$ are

(a) 5, 0 (b) 6, 1 (c) 6, 0 (d) none of these

55. Let z_1 and z_2 be two complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$. Then,

(a) z_1, z_2 are collinear

(b) z_1, z_2 and the origin form a right angled triangle

(c) z_1, z_2 and the origin form an equilateral triangle

(d) none of these

56. If z_1, z_2, z_3 be vertices of an equilateral triangle occurring in the anticlockwise sense, then

(a) $z_1^2 + z_2^2 + z_3^2 = 2(z_1 z_2 + z_2 z_3 + z_3 z_1)$

(b) $\frac{1}{z_1 + z_2} + \frac{1}{z_2 + z_3} + \frac{1}{z_3 + z_1} = 0$

(c) $z_1 + w z_2 + w^2 z_3 = 0$ (d) none of these

57. Let z be a complex number satisfying $|z - 5i| \le 1$ such that amp z is minimum. Then, z is equal to

(a) $\frac{2\sqrt{6}}{5} + \frac{24i}{5}$ (b) $\frac{24}{5} + \frac{2\sqrt{6}i}{5}$

(c) $\frac{2\sqrt{6}}{5} - \frac{24i}{5}$ (d) none of these

58. If $|z - 25i| \le 15$, then

$|$maximum amp (z) $-$ minimum amp $(z)|$ =

(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\pi - 2\cos^{-1}\left(-\frac{3}{5}\right)$

(c) $\frac{\pi}{2} + \cos^{-1}\left(\frac{3}{5}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right)$

59. Let z be a complex number (not lying on x-axis) of minimum modulus such that $\left|z + \frac{1}{z}\right| = 1$. Then,

(a) Im $(z) = 0$ (b) Re $(z) = 0$

(c) amp $(z) = \pi$ (d) none of these

60. The maximum distance from the origin of coordinates to the point z satisfying the equation $\left|z + \frac{1}{z}\right| = a$ is

(a) $\frac{1}{2}\left(\sqrt{a^2 + 1} + a\right)$ (b) $\frac{1}{2}\left(\sqrt{a^2 + 2} + a\right)$

(c) $\frac{1}{2}\left(\sqrt{a^2 + 4} + a\right)$ (d) none of these

61. The expression $\tan\left\{i \log\left(\frac{a - ib}{a + ib}\right)\right\}$ reduces to

(a) $\frac{ab}{a^2 + b^2}$ (b) $\frac{2ab}{a^2 - b^2}$ (c) $\frac{ab}{a^2 - b^2}$ (d) $\frac{2ab}{a^2 + b^2}$

62. The complex numbers z_1, z_2, z_3, z_4 taken in that order on the Argand plane represent the vertices of a parallelogram iff

(a) $z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$

(c) $z_1 + z_2 = z_3 + z_4$ (d) none of these

63. The locus of the points representing the complex number z for which $|z| - 2 = |z - i| - |z + 5i| = 0$ is

(a) a circle with centre at the origin

(b) a straight line passing through the origin

(c) the single point $(0, -2)$ (d) none of these

64. For $n = 6k, k \in Z$, $\left(\frac{1 - i\sqrt{3}}{2}\right)^n + \left(\frac{-1 - i\sqrt{3}}{2}\right)^n$ has the value

(a) -1 (b) 0 (c) 1 (d) 2

65. The product of all values of $(\cos\alpha + i\sin\alpha)^{3/5}$ is

(a) 1 (b) $\cos\alpha + i\sin\alpha$

(c) $\cos 3\alpha + i\sin 3\alpha$ (d) $\cos 5\alpha + i\sin 5\alpha$

66. If $C^2 + S^2 = 1$, then $\frac{1 + C + iS}{1 + C - iS}$ is equal to

(a) $C + iS$ (b) $C - iS$ (c) $S + iC$ (d) $S - iC$

67. The centre of a square $ABCD$ is at $z = 0$. The affix of the vertex A is z_1. Then, the affix of the centroid of the triangle ABC is

(a) $z_1(\cos\pi \pm i\sin\pi)$ (b) $\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$

(c) $z_1\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$ (d) $\frac{z_1}{3}\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$

68. The number of solutions of the system of equations $\text{Re}(z^2) = 0;\ |z| = 2$ is

(a) 4 (b) 3 (c) 2 (d) 1

69. The vector $z = -4 + 5i$ is turned counterclockwise through an angle of 180° and stretched $1\frac{1}{2}$ times. The complex number corresponding to newly obtained vector is

(a) $6 - \frac{15}{2}i$ (b) $-6 + \frac{15}{2}i$ (c) $6 + \frac{15}{2}i$ (d) none of these

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70. The value of $[\sqrt{2}\{\cos(56^\circ 15') + i\sin(56^\circ 15')\}]^8$ is
(a) $4i$ (b) $8i$ (c) $16i$ (d) $-16i$

71. Find the complex number z satisfying the equations
$\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1$
(a) 6 (b) $6 \pm 8i$ (c) $6+8i,\ 6+17i$ (d) none of these

72. The vertices B and D of a parallelogram are $1-2i$ and $4+2i$. If the diagonals are at right angles and $AC = 2BD$, the complex number representing A is
(a) $\frac{5}{2}$ (b) $3i - \frac{3}{2}$ (c) $3i-4$ (d) $3i+4$

73. If for complex numbers z_1 and z_2, $\arg(z_1) - \arg(z_2) = 0$, then $|z_1 - z_2|$ is equal to
(a) $|z_1| + |z_2|$ (b) $|z_1| - |z_2|$
(c) $||z_1| - |z_2||$ (d) 0

74. The join of $z_1 = a + ib$ and $z_2 = \frac{1}{-a+ib}$ passes through
(a) origin (b) $z = 1 + i0$ (c) $z = 0 + i$ (d) $z = 1 + i$

75. If z_1, z_2, z_3, z_4 are the affixes of four points in the Argand plane, and z is the affix of a point such that
$|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$,
then z_1, z_2, z_3, z_4 are
(a) concyclic (b) vertices of a triangle
(c) vertices of a rhombus (d) in a straight line

76. The value of $\sum_{r=1}^{8}\left(\sin\frac{2r\pi}{9} + i\cos\frac{2r\pi}{9}\right)$ is
(a) -1 (b) 1 (c) i (d) $-i$

77. If $z_1, z_2, z_3, \ldots, z_n$ are n nth roots of unity, then for $k = 1, 2, \ldots, n$
(a) $|z_k| = k|z_{n+1}|$ (b) $|z_{k+1}| = k|z_k|$
(c) $|z_{k+1}| = |z_k| + |z_{k+1}|$ (d) $|z_k| = |z_{k+1}|$

78. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals
(a) 0 (b) $\pi/2$ (c) $3\pi/2$ (d) π

79. If $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = 0$, then
(a) $z_1 = z_2$ (b) $z_1 = \bar{z}_2$ (c) $z_1 z_2 = 1$ (d) none of these

80. If one vertex of a square whose diagonals intersect at the origin is $3(\cos\theta + i\sin\theta)$, then the two adjacent vertices are
(a) $\pm 3(\sin\theta - i\cos\theta)$ (b) $\pm(\sin\theta + i\cos\theta)$
(c) $\pm(\cos\theta - i\sin\theta)$ (d) none of these

81. The value of z satisfying the equation
$\log z + \log z^2 + \ldots + \log z^n = 0$, is
(a) $\cos\frac{4m\pi}{n(n+1)} + i\sin\frac{4m\pi}{n(n+1)},\ m = 1, 2, \ldots$
(b) $\cos\frac{4m\pi}{n(n+1)} - i\sin\frac{4m\pi}{n(n+1)},\ m = 1, 2, \ldots$
(c) $\sin\frac{4m\pi}{n} + i\cos\frac{4m\pi}{n},\ m = 1, 2, \ldots$
(d) 0

82. If $|z_1| = |z_2| = \ldots = |z_n| = 1$, then the value of $|z_1 + z_2 + \ldots + z_n|$, is
(a) n (b) $\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$ (c) 0 (d) none of these

83. If $w\ (\neq 1)$ be a cube root of unity and $(1+w)^7 = A + Bw$, then A and B are respectively the numbers:
(a) 0, 1 (b) 1, 1 (c) 1, 0 (d) $-1, 1$

84. If $w\ (\neq 1)$ is a cube root of unity, then
$\begin{vmatrix} 1 & 1+i+w^2 & w^2 \\ 1-i & -1 & w^2-1 \\ -i & -i+w-1 & -1 \end{vmatrix}$ equals
(a) 0 (b) 1 (c) i (d) w
[AIEEE 2002]

85. Let z and w be two non-zero complex numbers such that $|z| = |w|$ and $\arg(z) + \arg(w) = \pi$. Then, z equals
(a) w (b) $-w$ (c) $\bar{w}$ (d) $-\bar{w}$
[AIEEE 2002]

86. If $z \neq 0$ be a complex number and $\arg(z) = \pi/4$, then
(a) $\mathrm{Re}(z) = \mathrm{Im}(z)$ only (b) $\mathrm{Re}(z) = \mathrm{Im}(z) > 0$
(c) $\mathrm{Re}(z^2) = \mathrm{Im}(z^2)$ (d) none of these

87. $(1+i)^8 + (1-i)^8 =$
(a) 2^8 (b) 2^5 (c) $2^4\cos\frac{\pi}{4}$ (d) $2^8\cos\frac{\pi}{8}$

88. The smallest positive integer n for which $(1+i)^{2n} = (1-i)^{2n}$ is
(a) 4 (b) 8 (c) 2 (d) 12

89. If α and β are different complex numbers with $|\beta| = 1$, then $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$ is equal to
(a) 0 (b) 1/2 (c) 1 (d) 2

90. For any complex number z, the minimum value of $|z| + |z-1|$ is
(a) 1 (b) 0 (c) 1/2 (d) 3/2

91. If $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$, the modulus and argument form of $(1 + \cos 2\alpha) + i\sin 2\alpha$ is
(a) $-2\cos\alpha\{\cos(\pi+\alpha) + i\sin(\pi+\alpha)\}$
(b) $2\cos\alpha\{\cos\alpha + i\sin\alpha\}$
(c) $2\cos\alpha\{\cos(-\alpha) + i\sin(-\alpha)\}$
(d) $-2\cos\alpha\{\cos(\pi-\alpha) + i\sin(\pi-\alpha)\}$

92. If the roots of $(z-1)^n = i(z+1)^n$ are plotted in the Argand plane, they are
(a) on a parabola (b) concyclic
(c) collinear (d) The vertices of a triangle

93. Area of the triangle formed by 3 complex numbers $1+i, i-1, 2i$ in the Argand plane is
(a) 1/2 (b) 1 (c) $\sqrt{2}$ (d) 2

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94. If w is a complex cube root of unity, then $(1-w+w^2)^6+(1-w^2+w)^6=$
(a) 0 (b) 6 (c) 64 (d) 128
[JEE (WB) 2008]

95. The locus represented by the equation $|z-1|=|z-i|$ is
(a) a circle of radius 1
(b) an ellipse with foci at 1 and $-i$
(c) a line through the origin
(d) a circle on the line joining 1 and $-i$ as diameter

96. If $z=i\log(2-\sqrt{3})$, then $\cos z=$
(a) i (b) $2i$ (c) 1 (d) 2

97. If $a=\cos\alpha+i\sin\alpha$, $b=\cos\beta+i\sin\beta$, $c=\cos\gamma+i\sin\gamma$ and $\frac{b}{c}+\frac{c}{a}+\frac{a}{b}=1$, then
$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)$ is equal to
(a) 3/2 (b) $-3/2$ (c) 0 (d) 1

98. If z_1, z_2, z_3 are vertices of an equilateral triangle inscribed in the circle $|z|=2$ and if $z_1=1+i\sqrt{3}$, then
(a) $z_1=-2,\ z_3=1-i\sqrt{3}$ (b) $z_2=2,\ z_3=1-i\sqrt{3}$
(c) $z_2=-2,\ z_3=-1-i\sqrt{3}$ (d) $z_2=1-i\sqrt{3},\ z_3=-1-i\sqrt{3}$

99. The general value of θ which satisfies the equation $(\cos\theta+i\sin\theta)(\cos3\theta+i\sin3\theta)(\cos5\theta+i\sin5\theta)\ldots(\cos(2n-1)\theta+i\sin(2n-1)\theta)=1$ is
(a) $\frac{r\pi}{n^2}$ (b) $\frac{(r-1)\pi}{n^2}$ (c) $\frac{(2r+1)\pi}{n^3}$ (d) $\frac{2r\pi}{n^2}$

100. If z is a complex number such that $z\neq0$ and $\mathrm{Re}(z)=0$, then
(a) $\mathrm{Re}(z^2)=0$ (b) $\mathrm{Im}(z^2)=0$
(c) $\mathrm{Re}(z^2)=\mathrm{Im}(z^2)$ (d) none of these

101. If $z+z^{-1}=1$, then $z^{100}+z^{-100}$ is equal to
(a) i (b) $-i$ (c) 1 (d) -1

102. Let A, B and C represent the complex numbers z_1, z_2, z_3 respectively on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number
(a) $z_1+z_2-z_3$ (b) $z_2+z_3-z_1$
(c) $z_3+z_1-z_2$ (d) $z_1+z_2+z_3$

103. Number of solutions of the equation $z^2+|z|^2=0$, where $z\in C$ is
(a) 1 (b) 2 (c) 3 (d) infinity many

104. The number of solutions of the equation $z^2+\bar{z}=0$ is
(a) 2 (b) 4 (c) 6 (d) none of these

105. The centre of a square is at the origin and $1+i$ is one of its vertices. The extremities of its diagonals which does not pass through this vertex are
(a) $1-i,\ -1+i$ (b) $1-i,\ -1-i$
(c) $-1+i,\ -1-i$ (d) none of these

106. Let z and w be two complex numbers such that $|z|\le1$, $|w|\le1$ and $|z+iw|=|z-i\bar{w}|=2$. Then, z equals
(a) 1 or i (b) i or $-i$ (c) 1 or -1 (d) i or -1

107. The system of equations $|z+1-i|=\sqrt{2}$ and $|z|=3$ has
(a) no solution (b) one solution
(c) two solutions (d) none of these

108. The triangle with vertices at the points z_1, z_2 $(1-i)z_1+iz_2$ is
(a) right angled but not isosceles
(b) isosceles but not right angled
(c) right angled and isosceles
(d) equilateral

109. Let α and β be two fixed non-zero complex numbers and 'z' a variable complex number. If the lines $\alpha\bar{z}+\bar{\alpha}z+1=0$ and $\beta\bar{z}+\bar{\beta}z-1=0$ are mutually perpendicular, then
(a) $\alpha\beta+\bar{\alpha}\bar{\beta}=0$ (b) $\alpha\beta-\bar{\alpha}\bar{\beta}=0$
(c) $\bar{\alpha}\beta-\alpha\bar{\beta}=0$ (d) $\alpha\bar{\beta}+\bar{\alpha}\beta=0$

110. The centre of a square is at $z=0$. A is z_1, then the centroid of the triangle ABC is
(a) $z_1(\cos\pi\pm i\sin\pi)$ (b) $\frac{1}{3}z_1(\cos\pi\pm i\sin\pi)$
(c) $z_1\left(\cos\frac{\pi}{2}\pm i\sin\frac{\pi}{2}\right)$ (d) $\frac{1}{3}z_1\left(\cos\frac{\pi}{2}\pm i\sin\frac{\pi}{2}\right)$

111. If $z=x+iy$, then the equation $\left|\frac{2z-1}{z+1}\right|=m$ does not represent a circle when $m=$
(a) 1/2 (b) 1 (c) 2 (d) 3

112. If $x^2-2x\cos\theta+1=0$, then $x^{2n}-2x^n\cos n\theta+1$ is equal to
(a) $\cos2n\theta$ (b) $\sin2n\theta$ (c) 0 (d) none of these

113. If $p^2-p+1=0$, then the value of p^{3n} can be
(a) 1 (b) -1 (c) 0 (d) none of these

114. If $n\in Z$, then $\frac{2^n}{(1-i)^{2n}}+\frac{(1+i)^{2n}}{2^n}$ is equal to
(a) 0 (b) 2 (c) $[1+(-1)^n]i^n$ (d) none of these

115. If $\arg(z_1z_2)=0$ and $|z_1|=|z_2|=1$, then
(a) $z_1+z_2=0$ (b) $z_1\bar{z}_2=1$ (c) $z_1=\bar{z}_2$ (d) none of these

116. If ω is a complex cube root of unity, then $\frac{(1+i)^{2n}-(1-i)^{2n}}{(1+\omega^4-\omega^2)(1-\omega^4+\omega^2)}$ is equal to
(a) 0, if n is an even integer (b) 0 for all $n\in Z$
(c) $2^{n-1}i$ for all $n\in N$ (d) none of these

117. If z is a complex number satisfying $z+z^{-1}=1$, then z^n+z^{-n}, $n\in N$ has the value
(a) $2(-1)^n$, when n is a multiple of 3
(b) $(-1)^n$ when n is not a multiple of 3
(c) $(-1)^{n+1}$ when n is a multiple of 3
(d) 0 when n is not a multiple of 3

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118. The polynomial $x^{3m} + x^{3n+1} + x^{3k+2}$ is exactly divisible by $x^2 + x + 1$ if
(a) m, n, k are rational (b) m, n, k are integers
(c) m, n, k are positive integers (d) none of these

119. If z is a non-real 7^{th} root of -1, then $z^{86} + z^{175} + z^{289}$ is equal to
(a) 0 (b) -1 (c) 3 (d) 1

120. The locus of point z satisfying $\text{Re}(z^2) = 0$, is
(a) a pair of straight lines (b) a circle
(c) a rectangular hyperbola (d) none of these

121. The curve represented by $\text{Im}(z^2) = k$, where k is a non-zero real number, is
(a) a pair of straight lines (b) an ellipse
(c) a parabola (d) a hyperbola

122. If $\log_{\tan 30^\circ}\left(\frac{2|z|^2 + 2|z| - 3}{|z| + 1}\right) < -2$, then
(a) $|z| < 3/2$ (b) $|z| > 3/2$
(c) $|z| > 2$ (d) $|z| < 2$

123. The roots of the cubic equation $(z + \alpha\beta)^3 = \alpha^3, \alpha \neq 0$ represent the vertices of a triangle of sides of length
(a) $\frac{1}{\sqrt{3}}|\alpha\beta|$ (b) $\sqrt{3}|\alpha|$ (c) $\sqrt{3}|\beta|$ (d) $\frac{1}{\sqrt{3}}|\alpha|$

124. The roots of the cubic equation $(z + \alpha\beta)^3 = \alpha^3, \alpha \neq 0$
(a) represent sides of an equilateral triangle
(b) represent the sides of an isosceles triangle
(c) represent the sides of a triangle whose one side is of length $\sqrt{3}\,\alpha$
(d) none of these

125. If $\alpha, \beta\, \gamma$ *and* δ are the roots of the equation $x^4 - 1 = 0$, then the value of
$\frac{a\alpha + b\beta + c\gamma + d\delta}{a\gamma + b\delta + c\alpha + d\beta} + \frac{a\gamma + b\delta + c\alpha + d\beta}{a\alpha + b\beta + c\gamma + d\delta}$, is
(a) 3β (b) 0 (c) 2γ (d) none of these

126. If ω is a complex cube root of unity, then the equation $|z - \omega|^2 + |z - \omega^2|^2 = \lambda$ will represent a circle, if
(a) $\lambda \in (0, 3/2)$ (b) $\lambda \in [3/2, \infty)$
(c) $\lambda \in (0, 3)$ (d) $\lambda \in [3, \infty)$

127. Let ω be a complex cube root of unity. If the equation
$$|z - \omega|^2 + |z - \omega^2|^2 = \lambda$$
represents a circle with points representing ω and ω^2 as the end points of a diameter, then $\lambda =$
(a) 4 (b) 3 (c) 2 (d) $\sqrt{2}$

128. The equation $z\bar{z} + (4 - 3i)z + (4 + 3i)\bar{z} + 5 = 0$ represents a circle of radius
(a) 5 (b) $2\sqrt{5}$ (c) 5/2 (d) none of these

129. If z is a complex number in the Argand plane such that $\arg\left(\frac{z - 3\sqrt{3}}{z + 3\sqrt{3}}\right) = \frac{\pi}{3}$ then the locus of z is
(a) $|z - 3i| = 6$
(b) $|z - 3i| = 6, \text{Im}(z) > 0$
(c) $|z - 3i| = 6, \text{Im}(z) < 0$
(d) none of these

130. Let $z = 1 - t + i\sqrt{t^2 + t + 2}$, where t is a real parameter. The locus of z in the argand plane is
(a) a hyperbola (b) an ellipse
(c) a straight line (d) none of these

131. If $|z - 4 - 3i| \leq 1$ and m and n are the least and greatest value of $|z|$ and λ is the least value of $\frac{x^4 + x^2 + 4}{x}$ in the interval $(0, \infty)$, then λ is equal to
(a) m (b) n (c) $m + n$ (d) none of these

132. If $1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$ are the n, n^{th} roots of unity and z_1 *and* z_2 are any two complex numbers, then $\sum_{r=0}^{n-1} \left| z_1 + \alpha^r z_2 \right|^2 =$
(a) $n\left(|z_1|^2 + |z_2|^2\right)$ (b) $(n-1)\left(|z_1|^2 + |z_2|^2\right)$
(c) $(n+1)\left(|z_1|^2 + |z_2|^2\right)$ (d) none of these

133. If $z_r\, (r = 0, 1, 2, \ldots, 6)$ be the roots of the equation $(z + 1)^7 + z^7 = 0$, then $\sum_{r=0}^{6} \text{Re}(z_r) =$
(a) 0 (b) 3/2 (c) 7/2 (d) $-7/2$

134. The least positive integer n for which
$\left(\frac{1 + i}{1 - i}\right)^n = \frac{2}{\pi}\sin^{-1}\left(\frac{1 + x^2}{2x}\right)$, where $x > 0$, is
(a) 2 (b) 4 (c) 8 (d) 12

135. The area of the triangle formed by the points representing $-z$, iz and $z - iz$ in the Argand plane is
(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $\frac{1}{4}|z|^2$

136. If $z_0 = \frac{1 - i}{2}$, then the value of the product $(1 + z_0)(1 + z_0^2)(1 + z_0^{2^2})(1 + z_0^{2^3}) \ldots (1 + z_0^{2^n})$ must be
(a) $(1 - i)\left(1 + \frac{1}{2^{n-1}}\right)$, if $n \geq 1$ (b) $(1 - i)\left(1 - \frac{1}{2^{2^n}}\right)$, if $n > 1$
(c) $(1 - i)\left(1 - \frac{1}{2^{n-1}}\right)$, if $n \geq 1$ (d) $(1 - i)\left(1 + \frac{1}{2^{2^n}}\right)$, if $n >$

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Answers

1. (d)	2. (a)	3. (b)	4. (c)	5. (c)	6. (a)	7. (c)
8. (d)	9. (c)	10. (a)	11. (b)	12. (b)	13. (c)	14. (a)
15. (d)	16. (d)	17. (d)	18. (b)	19. (b)	20. (b)	21. (a)
22. (d)	23. (c)	24. (b)	25. (c)	26. (a)	27. (d)	28. (a)
29. (a)	30. (a)	31. (d)	32. (a)	33. (d)	34. (d)	35. (a)
36. (a)	37. (b)	38. (c)	39. (c)	40. (c)	41. (c)	42. (a)
43. (b)	44. (b)	45. (c)	46. (b)	47. (c)	48. (c)	49. (a)
50. (b)	51. (d)	52. (b)	53. (a)	54. (c)	55. (c)	56. (c)
57. (a)	58. (b)	59. (b)	60. (c)	61. (b)	62. (b)	63. (c)
64. (d)	65. (c)	66. (a)	67. (d)	68. (a)	69. (a)	70. (c)
71. (c)	72. (b)	73. (c)	74. (a)	75. (a)	76. (d)	77. (d)
78. (a)	79. (b)	80. (a)	81. (a)	82. (b)	83. (b)	84. (a)
85. (d)	86. (b)	87. (b)	88. (c)	89. (c)	90. (a)	91. (a)
92. (b)	93. (b)	94. (d)	95. (c)	96. (d)	97. (d)	98. (a)
99. (d)	100. (b)	101. (d)	102. (d)	103. (d)	104. (b)	105. (a)
106. (c)	107. (a)	108. (c)	109. (d)	110. (d)	111. (c)	112. (c)
113. (d)	114. (d)	115. (c)	116. (a)	117. (a)	118. (b)	119. (b)
120. (a)	121. (d)	122. (c)	123. (b)	124. (d)	125. (d)	126. (b)
127. (b)	128. (b)	129. (b)	130. (a)	131. (b)	132. (a)	133. (d)
134. (b)	135. (c)	136. (b)				

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The locus of the centre of the circle which touches the circles $|z-z_1| = a$ and $|z-z_2| = b$ externally (z, z_1 and z_2 are complex numbers) will be
   (a) an ellipse (b) a hyperbola
   (c) a circle (d) none of these

2. For positive integers n_1, n_2 the value of the expression $(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$, where $i = \sqrt{-1}$ is a real number if and only if
   (a) $n_1 = n_2 + 1$ (b) $n_1 = n_2 - 1$
   (c) $n_1 = n_2$ (d) $n_1 > 0, n_2 > 0$

3. The modulus of $\sqrt{2i} - \sqrt{-2i}$ is
   (a) 2 (b) $\sqrt{2}$ (c) 0 (d) $2\sqrt{2}$

4. The triangle formed by the points 1, $\frac{1+i}{\sqrt{2}}$ and i as vertices in the Argand diagram is
   (a) scalene (b) equilateral
   (c) isosceles (d) right-angled

5. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is
   (a) 2 (b) -2 (c) 1 (d) 0

6. If $\alpha + i\beta = \tan^{-1}(z)$, $z = x + iy$ and α is constant, the locus of 'z' is
   (a) $x^2 + y^2 + 2x \cot 2\alpha = 1$ (b) $\cot 2\alpha (x^2 + y^2) = 1 + x$
   (c) $x^2 + y^2 + 2y \tan 2\alpha = 1$ (d) $x^2 + y^2 + 2x \sin 2\alpha = 1$

7. If $\cos A + \cos B + \cos C = 0$, $\sin A + \sin B + \sin C = 0$ and $A + B + C = 180^\circ$, then the value of $\cos 3A + \cos 3B + \cos 3C$ is
   (a) 3 (b) -3 (c) $\sqrt{3}$ (d) 0

8. The value of the expression
   $2(1+w)(1+w^2) + 3(2+w)(2+w^2) + 4(3+w)(3+w^2) + \ldots + (n+1)(n+w)(n+w^2)$, where w is an imaginary cube root of unity is
   (a) $\left\{\frac{n(n+1)}{2}\right\}^2$ (b) $\left\{\frac{n(n+1)}{2}\right\}^2 - n$
   (c) $\left\{\frac{n(n+1)}{2}\right\}^2 + n$ (d) none of these

9. The value of the expression $\left(1+\frac{1}{w}\right)\left(1+\frac{1}{w^2}\right) + \left(2+\frac{1}{w}\right)\left(2+\frac{1}{w^2}\right) + \left(3+\frac{1}{w}\right)\left(3+\frac{1}{w^2}\right) + \ldots + \left(n+\frac{1}{w}\right)\left(n+\frac{1}{w^2}\right)$, where w is an imaginary cube root of unity is
   (a) $\frac{n(n^2+2)}{3}$ (b) $\frac{n(n^2-2)}{3}$
   (c) $\frac{n(n^2+1)}{3}$ (d) none of these

10. The condition that $x^{n+1} - x^n + 1$ shall be divisible by $x^2 - x + 1$ is that
    (a) $n = 6k+1$ (b) $n = 6k-1$
    (c) $n = 3k+1$ (d) $n = 3k-1$

11. The expression $(1+i)^{n_1} + (1+i^3)^{n_2}$ is real iff
    (a) $n_1 = -n_2$ (b) $n_1 = 4r + (-1)^r n_2$
    (c) $n_1 = 2r + (-1)^r n_2$ (d) none of these

12. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then
    (a) $x = 3, y = 1$ (b) $x = 1, y = 3$
    (c) $x = 0, y = 3$ (d) none of these

13. The centre of a square $ABCD$ is at $z = 0$. A is z_1, then the centroid of the triangle ABC is
    (a) $z_1(\cos\pi \pm i\sin\pi)$ (b) $\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$
    (c) $z_1(\cos\pi/2 \pm i\sin\pi/2)$ (d) $\frac{z_1}{3}(\cos\pi/2 \pm i\sin\pi/2)$

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14. If $\cos\alpha+2\cos\beta+3\cos\gamma=\sin\alpha+2\sin\beta+3\sin\gamma=0$ and $\alpha+\beta+\gamma=0$, then $\cos 3\alpha+8\cos 3\beta+27\cos 3\gamma=$
(a) 0 (b) 3 (c) 18 (d) -18

15. If $\cos\alpha+2\cos\beta+3\cos\gamma=\sin\alpha+2\sin\beta+3\sin\gamma=0$ and $\alpha+\beta+\gamma=n\pi$, then $\sin 3\alpha+8\sin 3\beta+27\sin 3\gamma=$
(a) 0 (b) 3 (c) 8 (d) -18

Sum of the series $\sum_{r=0}^{n}(-1)^r\,{}^nC_r\left|i^{5r}+i^{6r}+i^{7r}+i^{8r}\right|$, is
(a) 2^n (b) $2^{n/2+1}$
(c) $n^n+2^{n/2+1}$ (d) $2^n+2^{n/2+1}\cos\frac{n\pi}{4}$

17. If $az_1+bz_2+cz_3=0$ for complex numbers z_1, z_2, z_3 and real numbers a, b, c, then z_1, z_2, z_3 lie on a
(a) straight line
(b) circle
(c) depends on the choice of a, b, c
(d) none of these

18. If $2z_1-3z_2+z_3=0$, then z_1, z_2, z_3 are represented by
(a) three vertices of a triangle
(b) three collinear points
(c) three vertices of a rhombus
(d) none of these

19. If $\mathrm{Re}\left(\frac{z+4}{2z-i}\right)=\frac{1}{2}$, then z is represented by a point lying on
(a) a circle (b) an ellipse
(c) a straight line (d) none of these

20. If z_1, z_2, z_3, z_4 represent the vertices of a rhombus taken in the anticlockwise order, then
(a) $z_1+z_2+z_3+z_4=0$ (b) $z_1+z_2=z_3+z_4$
(c) $\mathrm{amp}\,\frac{z_2-z_4}{z_1-z_3}=\frac{\pi}{2}$ (d) $\mathrm{amp}\,\frac{z_1-z_2}{z_3-z_4}=\frac{\pi}{2}$

21. Let $a\in R$. If the origin and the non-real roots of $2z^2+2z+a=0$ form the three vertices of an equilateral triangle in the argand plane, then $a=$
(a) 1 (b) 2 (c) -1 (d) none of these

22. If $|z_1|=|z_2|=|z_3|=1$ and z_1, z_2, z_3 represent the vertices of an equilateral triangle, then
(a) $z_1+z_2+z_3=0$ and $z_1z_2z_3=1$
(b) $z_1+z_2+z_3=1$ and $z_1z_2z_3=1$
(c) $z_1z_2+z_2z_3+z_3z_1=0$ and $z_1+z_2+z_3=0$
(d) $z_1z_2+z_2z_3+z_3z_1=0$ and $z_1z_2z_3=1$

23. If P, P' represent the complex number z_1 and its additive inverse respectively, then the equation of the circle with PP' as a diameter is
(a) $\frac{z}{z_1}=\frac{\bar{z}_1}{\bar{z}}$ (b) $z\bar{z}+z_1\bar{z}_1=0$
(c) $z\bar{z}_1+\bar{z}z_1=0$ (d) none of these

24. Let $A(z_1), B(z_2), C(z_3)$ be the vertices of an equilateral triangle ABC in the Argand plane, then the number $\left(\frac{z_2-z_3}{2z_1-z_2-z_3}\right)$ is
(a) purely real
(b) purely imaginary
(c) a complex number with non-zero real and imaginary parts
(d) none of these

25. The area of the triangle whose vertices are i, ω and ω^2, where $i=\sqrt{-1}$ and ω, ω^2 are complex cube roots of unity, is
(a) $\frac{3\sqrt{3}}{2}$ sq. units (b) $\frac{3\sqrt{3}}{4}$ sq. units
(c) 0 (d) $\frac{\sqrt{3}}{4}$ sq. units

26. The complex number satisfying $|z+1|=|z-1|$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ is
(a) $(\sqrt{2}+1)+0i$ (b) $0+(\sqrt{2}+1)i$
(c) $0+(\sqrt{2}-1)i$ (d) $(-\sqrt{2}+1)+0i$

27. If A, B, C are three points in the Argand plane representing the complex numbers z_1, z_2, z_3 such that $z_1=\frac{\lambda z_2+z_3}{\lambda+1}$, where $\lambda\in R$, then the distance of A from the line BC is
(a) λ (b) $\frac{\lambda}{\lambda+1}$ (c) 1 (d) 0

28. If $z(\bar{z}+\bar{\alpha})+\bar{z}(z+\alpha)=0$ where α is a complex constant, then z is represented by a point on
(a) a circle (b) a straight line
(c) a parabola (d) none of these

29. Let A, B, C be three collinear points which are such that $AB\,.\,AC=1$ and the points are represented in the Argand plane by the complex numbers $0, z_1$ and z_2 respectively, Then,
(a) $z_1z_2=1$ (b) $z_1\bar{z}_2=1$
(c) $|z_1|\,|z_2|=1$ (d) none of these

30. If z_1, z_2, z_3, z_4 are the four complex numbers represented by the vertices of a quadrilateral taken in order such that $z_1-z_4=z_2-z_3$ and $\arg\left(\frac{z_4-z_1}{z_2-z_1}\right)=\pm\frac{\pi}{2}$, then the quadrilateral is
(a) a rhombus (b) a square
(c) a rectangle (d) not a cyclic quadrilateral

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31. If z be a complex number, then $|z-3-4i|^2+|z+4+2i|^2=k$ represents a circle, if k is equal to
(a) 30 (b) 40 (c) 55 (d) 35

32. In the argand plane, if O, P and Q represent respectively the origin O and the complex numbers z and $z+iz$ respectively, then $\angle OPQ$ is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

33. If $\frac{2z_1}{3z_2}$ is purely imaginary number, then $\left|\frac{z_1-z_2}{z_1+z_2}\right|^4$ is equal to
(a) 3/2 (b) 1 (c) 2/3 (d) 4/9

34. If ω is an imaginary cube root of unity, then the value of $\sin\left\{(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right\}$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{-1}{\sqrt{3}}$ (d) $-\frac{\sqrt{3}}{2}$

35. If the centre of a regular hexagon is at the origin and one of its vertices on argand diagram is $1+2i$, then its perimeter is
(a) $2\sqrt{5}$ (b) $6\sqrt{2}$ (c) $4\sqrt{5}$ (d) $6\sqrt{5}$

36. If $z^2+(p+iq)z+(r+is)=0$, where, p, q, r, s are non-zero has real roots, then
(a) $pqs=s^2+q^2r$ (b) $pqr=r^2+p^2s$
(c) $prs=q^2+r^2p$ (d) $qrs=p^2+s^2q$

37. Let z_1, z_2, z_3 be three vertices of an equilateral triangle circumscribing the circle $|z|=\frac{1}{2}$. If $z_1=\frac{1}{2}+i\frac{\sqrt{3}}{2}$ and z_1, z_2, z_3 were in anticlockwise sense, then z_2 is
(a) $1+i\sqrt{3}$ (b) $1-i\sqrt{3}$ (c) 1 (d) -1

38. If ω is the complex cube root of unity, then the value of $\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\ldots\right)}$
(a) -1 (b) 1 (c) $-i$ (d) i

39. The locus of $z=i+2\exp\left\{i\left(\theta+\frac{\pi}{4}\right)\right\}$, where θ is parameter, is
(a) a circle (b) an ellipse
(c) a parabola (d) hyperbola

40. If z lies on the circle $|z-1|=1$, then $\frac{z-2}{z}$ is
(a) purely real (b) purely imaginary
(c) positive real (d) none of these

41. If $a>0$ and the equation $|z-a^2|+|z-2a|=3$ represents an ellipse, then a belongs to the interval
(a) (1, 3) (b) $(\sqrt{2}, \sqrt{3})$ (c) (0, 3) (d) (1, 2)

42. For any complex number z, the minimum value of $|z|+|z-2i|$, is
(a) 0 (b) 1 (c) 2 (d) 4

43. The greatest and the least value of $|z_1+z_2|$, if $z_1=24+7i$ and $|z_2|=6$, are respectively
(a) 31, 19 (b) 25, 19 (c) 31, 25 (d) none of these

44. For all complex numbers z_1, z_2 satisfying $|z_1|=12$ and $|z_2-3-4i|=5$, the minimum value of $|z_1-z_2|$ is
(a) 0 (b) 2 (c) 7 (d) 17

45. If $k>1$, $|z_1|<k$ and $\left|\frac{k-z_1\bar{z}_2}{z_1-kz_2}\right|=1$, then
(a) $|z_2|<k$ (b) $|z_2|=k$ (c) $z_2=0$ (d) $|z_2|>k$

46. If $|z-i|=1$ and $\arg(z)=\theta$, where $0<\theta<\frac{\pi}{2}$, then $\cot\theta-\frac{2}{z}$ equals
(a) $2i$ (b) $-i$ (c) i (d) $1+i$

47. If $\mathrm{Re}(z)<0$, then the value of $\left|1+z+z^2+\ldots+z^n\right|$ cannot exceed
(a) n (b) $n|z|^n+1$
(c) $|z^n|-\frac{1}{|z|}$ (d) $|z|^n+\frac{1}{|z|}$

48. Let z_1, z_2, z_3 be three complex numbers satisfying $\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}=0$. Let $z_k=r_k(\cos\alpha_k+i\sin\alpha_k)$ and $w_k=\frac{\cos 2\alpha_k+i\sin 2\alpha_k}{z_k}$ for $k=1, 2, 3$. If w_1, w_2 and w_3 are the affixes of points A_1, A_2 and A_3 respectively in the Argand plane, then $\Delta A_1A_2A_3$ has its
(a) incentre at the origin
(b) centroid at the origin
(c) circumcentre at the origin
(d) orthocentre at the origin

Answers

1. (b) 2. (d) 3. (a) 4. (c) 5. (a) 6. (a) 7. (b)
8. (c) 9. (a) 10. (a) 11. (b) 12. (d) 13. (d) 14. (c)
15. (a) 16. (d) 17. (c) 18. (b) 19. (c) 20. (c) 21. (d)
22. (a) 23. (a) 24. (b) 25. (d) 26. (b) 27. (d) 28. (a)
29. (b) 30. (c) 31. (c) 32. (c) 33. (b) 34. (a) 35. (d)
36. (a) 37. (d) 38. (a) 39. (a) 40. (b) 41. (c) 42. (c)
43. (a) 44. (b) 45. (d) 46. (c) 47. (d) 48. (b)

Solution of Exercises and Chapter-tests are available in a separate book on "Solution of Objective Mathematics".

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10

SEQUENCES AND SERIES

1. ARITHMETIC PROGRESSION

DEFINITION *A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called an arithmetic progression (A.P.), if the difference of any term and the previous term is always same.*

i.e., $a_{n+1} - a_n$ = Constant $(= d)$ for all $n \in N$

or, $a_{n+1} - a_n$ is independent of n.

The constant difference 'd' is called the common difference.

GENERAL TERM *The n^{th} term a_n of an A.P. with first term 'a' and common difference 'd' is given by*

$$a_n = a + (n-1)\,d$$

If an A.P. consists of finite number of terms, say, m. Then, nth term from the end is $(m - n + 1)^{th}$ term from the beginning.

i.e., n^{th} term from the end

$= (m - n + 1)^{th}$ term from the beginning

$= a + (m - n + 1 - 1)\,d = a + (m - n)\,d$

We can also compute n^{th} term from the end by taking the last term as first term and common difference as '$-d$'.

Thus, if l is the last term. Then,

$$n^{th} \text{ term from the end} = l + (n-1)(-d) = l - (n-1)\,d$$

ILLUSTRATION 1 *Let T_r be the r^{th} term of an AP, for $r = 1, 2, \ldots$ If for some positive integers m, n we have $T_m = 1/n$ and $T_n = 1/m$, then T_{mn} equals*

(a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$ (c) 1 (d) 0

[IIT 1998, JEE (WB) 2008]

Ans. (c)

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_m = \frac{1}{n} \Rightarrow a = (m+1)\,d = \frac{1}{n} \quad \ldots(i)$$

and, $$T_n = \frac{1}{m} \Rightarrow a + (n-1)\,d = \frac{1}{m} \quad \ldots(ii)$$

Solving (i) and (ii), we get $d = \frac{1}{mn}$ and $a = \frac{1}{mn}$

$$\therefore \quad T_{mn} = a + (mn-1)\,d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = 1$$

ILLUSTRATION 2 *If $a_1, a_2, a_3, \ldots, a_{n+1}$ are in A.P., then*

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \ldots + \frac{1}{a_n a_{n+1}}, \text{ is}$$

(a) $\frac{n-1}{a_1 a_{n+1}}$ (b) $\frac{1}{a_1 a_{n+1}}$ (c) $\frac{n+1}{a_1 a_{n+1}}$ (d) $\frac{n}{a_1 a_{n+1}}$

Ans. (d)

SOLUTION Let d be the common difference of the given A.P. Then,

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \ldots + \frac{1}{a_n a_{n+1}}$$

$$= \frac{1}{d}\left\{\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 a_3} + \ldots + \frac{a_{n+1} - a_n}{a_n a_{n+1}}\right\}$$

$$= \frac{1}{d}\left\{\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \ldots + \frac{1}{a_n} - \frac{1}{a_{n+1}}\right\}$$

$$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{n+1}}\right) = \frac{a_{n+1} - a_1}{d\,a_1 a_{n+1}} = \frac{nd}{d\,a_1 a_{n+1}} = \frac{n}{a_1 a_{n+1}}$$

ILLUSTRATION 3 *If $a_1, a_2, a_3, \ldots, a_n$ are in A.P., where $a_i > 0$ for all i, then the value of*

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \text{ is}$$

(a) $\frac{1}{\sqrt{a_1} + \sqrt{a_n}}$ (b) $\frac{1}{\sqrt{a_1} - \sqrt{a_n}}$

(c) $\frac{n}{\sqrt{a_1} - \sqrt{a_n}}$ (d) $\frac{n-1}{\sqrt{a_1} - \sqrt{a_n}}$

Ans. (d)

SOLUTION Let d be the common difference of the A.P. Then,

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$$

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$$\Rightarrow S_n = \sum_{r=1}^{2m} r^2 + 2\sum_{r=1}^{m} (2r)^2$$

$$\Rightarrow S_n = \frac{2m(2m+1)(4m+1)}{6} + \frac{8m(m+1)(2m+1)}{6}$$

$$\Rightarrow S_n = \frac{2m(2m+1)}{6}\{(4m+1)+4(m+1)\}$$

$$\Rightarrow S_n = \frac{1}{6}(2m)(2m+1)\left(4(2m)+5\right) = \frac{n(n+1)(4n+5)}{6}$$

So, statement–2 is true.

If n is odd, then

$$S_n = 1^2 + 3\times 2^2 + 3^2 + 3\times 4^2 + \ldots + 3\times(n-1)^2 + n^2$$

$$\Rightarrow S_n = \frac{(n-1)(n-1+1)\left(4(n-1)+5\right)}{6} + n^2$$

Replacing n by $n-1$ in statement–2.

$$\Rightarrow S_n = \frac{n(n-1)(4n+1)}{6} + n^2$$

$$\Rightarrow S_n = \frac{n}{6}\{(n-1)(4n+1)+6n\}$$

$$\Rightarrow S_n = \frac{n}{6}(4n^2+3n-1) = \frac{n(n+1)(4n-1)}{6}$$

So, statement–1 is true and statement–2 is a correct explanation for statement–1.

EXAMPLE 12 *Statement-1:* $1.3.5\ldots(2n-1) \le n^n$ *for all* $n \in N$
Statement-2: $GM \le AM$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement-2 is true.

Using statement-2, we get

$$\{1.3.5\ldots(2n-1)\}^{1/n} \le \frac{1+3+5\ldots+(2n-1)}{n}$$

$$\Rightarrow \{1.3.5.\ldots(2n-1)\}^{1/n} \le \frac{n^2}{n}$$

$$\Rightarrow \{1.3.5\ldots(2n-1)\} \le n^n \text{ for all } n \in N$$

Hence, statement-1 is true and statement-2 is a correct explanation for statement-1.

EXAMPLE 13 *Let* $a_1, a_2, a_3, \ldots, a_n$ *be an A.P.*

Statement-1: $\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \ldots + \frac{1}{a_n a_1} = \frac{2}{a_1 + a_n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)$

Statement-2: $a_r + a_{n-r+1} = a_1 + a_n$ *for* $1 \le r \le n$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Let d be the common difference of the given A.P. Then,

$$a_r + a_{n-r+1} = \{a_1 + (r-1)d\} + \{a_1 + (n-r+1-1)d\}$$
$$= a_1 + a_1 + (n-1)d$$
$$= a_1 + a_n \text{ for } 1 \le r \le n$$

So, statement-2 is true.

Now, $\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \ldots + \frac{1}{a_n a_1}$

$$= \frac{1}{a_1 + a_n}\left\{\frac{a_1 + a_n}{a_1 a_n} + \frac{a_1 + a_n}{a_2 a_{n-1}} + \frac{a_1 + a_n}{a_3 a_{n-2}} + \ldots + \frac{a_1 + a_n}{a_n a_1}\right\}$$

$$= \frac{1}{a_1 + a_n}\left\{\frac{a_1 + a_n}{a_1 a_n} + \frac{a_2 + a_{n-1}}{a_2 a_{n-1}} + \frac{a_3 + a_{n-2}}{a_3 a_{n-2}} + \ldots + \frac{a_1 + a_n}{a_n a_1}\right\}$$

[Using statement-2]

$$= \frac{1}{a_1 + a_n}\left\{\left(\frac{1}{a_n} + \frac{1}{a_1}\right) + \left(\frac{1}{a_{n-1}} + \frac{1}{a_2}\right) + \left(\frac{1}{a_{n-2}} + \frac{1}{a_3}\right) + \ldots + \left(\frac{1}{a_1} + \frac{1}{a_n}\right)\right\}$$

$$= \frac{2}{a_1 + a_n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)$$

So, statement-1 is also true and statement-2 is a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If pth, qth and rth terms of a G.P. are x, y, z respectively, then $x^{q-r} y^{r-p} z^{p-q}$ is equal to
   (a) 0 (b) 1 (c) –1 (d) none of these

2. a, b, c are in A.P., then $\frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ are in
   (a) A.P. (b) G.P. (c) H.P. (d) none of these

3. If x, y, z are in G.P. and $x+3$, $y+3$, $z+3$ are in H.P., then
   (a) $y = 2$ (b) $y = 3$ (c) $y = 1$ (d) $y = 0$

4. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P., then
   (a) a, b, c, are in A.P. (b) a^2, b^2, c^2 are in A.P.
   (c) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. (d) none of these

5. If a, b, c are in A.P. as well as in G.P. then
   (a) $a = b \ne c$ (b) $a \ne b = c$
   (c) $a \ne b \ne c$ (d) $a = b = c$

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6. The value of $2.\overline{357}$ is
(a) $\frac{2355}{1001}$ (b) $\frac{2355}{999}$ (c) $\frac{2355}{1111}$ (d) none of these

7. If $\frac{3+5+7+\ldots\ldots+n \text{ terms}}{5+8+11+\ldots\ldots+10 \text{ terms}} = 7$, then the value of n is
(a) 35 (b) 36 (c) 37 (d) 40

8. If $x, 1, z$ are in A.P. and $x, 2, z$ are in G.P., then $x, 4, z$ are in
(a) AP (b) G.P. (c) H.P. (d) none of these

9. Sum of three numbers in G.P. be 14. If one is added to first and second and 1 is subtracted from the third, the new numbers are in A.P. The smallest of them is
(a) 2 (b) 4 (c) 6 (d) 8

10. If first and $(2n-1)^{th}$ terms of an A.P., G.P. and H.P. are equal and their nth terms are a, b, c respectively, then
(a) $a = b = c$ (b) $a + c = b$
(c) $a > b > c$ and $ac - b^2 = 0$ (d) none of these

11. The sum of first two terms of an infinite G.P. is 1 and every term is twice the sum of the successive terms. Its first term is
(a) 1 / 3 (b) 2 / 3 (c) 3 / 4 (d) 1 / 4

12. If x, y, z are in G.P. and $a^x = b^y = c^z$, then
(a) $\log_b a = \log_a c$ (b) $\log_c b = \log_a c$
(c) $\log_b a = \log_c b$ (d) none of these

13. If the sum of an infinite G.P. be 3 and the sum of the squares of its term is also 3, then its first term and common ratio are
(a) 3/2 , 1/2 (b) 1/2 , 3/2
(c) 1 , 1/2 (d) none of these

14. If a, b, c, d are in G.P. and $a^x = b^y = c^z = d^u$, then x, y, z, u are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

15. If a, b, c are in H.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

16. The sum of first n terms of the series
$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots.$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is
(a) $\frac{n(n+1)}{2}$ (b) $\frac{n^2(n+1)}{2}$
(c) $\frac{n(n+1)^2}{2}$ (d) $\left\{\frac{n(n+1)}{2}\right\}^2$
[AIEEE 2004]

17. If x, y, z are pth, qth and rth terms respectively, of an A.P. and also of G.P., then $x^{y-z} y^{z-x} z^{x-y}$, is equal to
(a) xyz (b) 0 (c) 1 (d) none of these

18. If $x = 1 + a + a^2 + \ldots\ldots \infty$ and $y = 1 + b + b^2 + \ldots\ldots \infty$ where a and b are proper fractions, then $1 + ab + a^2 b^2 + \ldots\ldots \infty$ equals
(a) $\frac{xy}{y + x - 1}$ (b) $\frac{x + y}{x - y}$
(c) $\frac{x^2 + y^2}{x - y}$ (d) none of these
[JEE (WB) 2006]

19. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $d/a, e/b, f/c$ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these
[CEE (Delhi) 2006]

20. If a, b, c are in A.P., p, q, r are in H.P. and ap, bq, cr are in G.P., then $\frac{p}{r} + \frac{r}{p}$ is equal to
(a) $\frac{a}{c} - \frac{c}{a}$ (b) $\frac{a}{c} + \frac{c}{a}$ (c) $\frac{b}{q} + \frac{q}{b}$ (d) $\frac{b}{q} - \frac{q}{b}$

21. The sum of integers from 1 to 100 which are divisible by 2 or 5, is
(a) 3000 (b) 3010 (c) 3150 (d) 3050
[AIEEE 2002]

22. The sum of first 10 terms of the series
$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \ldots\ldots$ is
(a) $\left(\frac{x^{20} - 1}{x^2 - 1}\right)\left(\frac{x^{22} + 1}{x^{20}}\right) + 20$ (b) $\left(\frac{x^{18} - 1}{x^2 - 1}\right)\left(\frac{x^{11} + 1}{x^9}\right) + 2$
(c) $\left(\frac{x^{18} - 1}{x^2 - 1}\right)\left(\frac{x^{11} - 1}{x^9}\right) + 20$ (d) none of these

23. The geometric mean between -9 and -16 is
(a) 12 (b) -12 (c) -13 (d) none of these

24. The sum of n terms of an A.P. is $3n^2 + 5$. The number of term which equals 159 is
(a) 13 (b) 21 (c) 27 (d) none of these

25. If p, q, r are in A.P., then pth, qth and rth terms of any G.P. are in
(a) A.P. (b) G.P.
(c) reciprocals of these terms are in A.P.
(d) none of these

26. If $\log 2$, $\log(2^x - 1)$ and $\log(2^x + 3)$ are in A.P., then $2, 2^x - 1, 2^x + 3$ are in
(a) A.P. (b) H.P. (c) G.P. (d) none of these

27. If S denotes the sum to infinity and S_n the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots.$, such that $S - S_n < \frac{1}{1000}$, then the least value of n is
(a) 8 (b) 9 (c) 10 (d) 11

28. If x, y, z are positive integers, then $(x + y)(y + z)(z + x)$ is
(a) $= 8xyz$ (b) $> 8xyz$ (c) $< 8xyz$ (d) none of these

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29. The sides a, b, c of ΔABC are in G.P., where $\log a - \log 2b$, $\log 2b - \log 3c$, $\log 3c - \log a$ are in A.P., then the ΔABC is
(a) acute angled (b) obtuse angled
(c) right angled (d) none of these

30. If a, b, c are in A.P., $b-a, c-b$ and a are in G.P., then $a:b:c$ is
(a) 1 : 2 : 3 (b) 1 : 3 : 5 (c) 2 : 3 : 4 (d) 1 : 2 : 4

31. If $x^a = x^{b/2} z^{b/2} = z^c$, then a, b, c are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

32. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying odd places, the common ratio will be equal to
(a) 2 (b) 3 (c) 4 (d) 5

33. If the interior angles of a polygon are in A.P. with common difference 5° and smallest angle is 120°, then the number of sides of the polygon is
(a) 9 or 16 (b) 9 (c) 16 (d) 13

34. For what value of b, will the roots of the equation $\cos x = b, -1 \le b \le 1$ when arranged in ascending order of their magnitudes, form an A.P.?
(a) -1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1/2

35. If the first and $(2n-1)$ st terms of an A.P., G.P. and H.P. are equal and their nth terms are a, b and c respectively, then
(a) $a = b = c$ (b) $a \ge b \ge c$
(c) $a + c = b$ (d) none of these

36. The sum to infinity of the series
$$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots\ldots, \text{ is}$$
(a) $\frac{16}{35}$ (b) $\frac{11}{8}$ (c) $\frac{35}{16}$ (d) $\frac{8}{6}$

37. The sum of all two digit numbers which when divided by 4, yield unity as remainder is,
(a) 1012 (b) 1201 (c) 1212 (d) 1210

38. The determinant
$$\Delta = \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$$
is equal to zero, if
(a) a, b, c are in A.P. (b) a, b, c are in G.P.
(c) a, b, c are in H.P. (d) α is a root of $ax^2 + bx + c = 0$

39. The sum of the series
$(1+2) + (1+2+2^2) + (1+2+2^2+2^3) + \ldots$ up to n terms is
(a) $2^{n+2} - n - 4$ (b) $2(2^n - 1) - n$
(c) $2^{n+1} - n$ (d) $2^{n+1} - 1$

[JEE (WB) 2008]

40. If a, b, c are in H.P., then the value of $\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right)$ is
(a) $\frac{2}{bc} - \frac{1}{b^2}$ (b) $\frac{1}{4}\left(\frac{3}{c^2} + \frac{2}{ca} - \frac{1}{a^2}\right)$
(c) $\frac{3}{b^2} - \frac{2}{ab}$ (d) all of these

41. The 5th term of the sequence $\frac{10}{9}, \frac{1}{3}\sqrt{\frac{20}{3}}, \frac{2}{3}, \ldots\ldots$ is
(a) $\frac{1}{3}$ (b) 1 (c) $\frac{2}{5}$ (d) $\sqrt{\frac{2}{3}}$

[JEE (WB) 2008]

42. If $x^{18} = y^{21} = z^{28}$, then $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

43. If d, e, f are in G.P. and the two quadratic equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then
(a) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in H.P. (b) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.
(c) $dbf = aef + cde$ (d) $b^2 df = ace^2$

44. The sum of n terms of the following series
$1 + (1+x) + (1+x+x^2) + \ldots\ldots$ is
(a) $\frac{1-x^n}{1-x}$ (b) $\frac{x(1-x^n)}{1-x}$
(c) $\frac{n(1-x) - x(1-x^n)}{(1-x)^2}$ (d) none of these

[JEE (WB) 2007]

45. For a sequence $< a_n >$, $a_1 = 2$ and $\frac{a_{n+1}}{a_n} = \frac{1}{3}$. Then, $\sum_{r=1}^{20} a_r$ is
(a) $\frac{20}{2}[4 + 19 \times 3]$ (b) $3\left(1 - \frac{1}{3^{20}}\right)$
(c) $2(1 - 3^{20})$ (d) none of these

46. If $< a_n >$ is an arithmetic sequence, then
$$\Delta = \begin{vmatrix} a_m & a_n & a_p \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix} \text{ equals}$$
(a) 1 (b) -1 (c) 0 (d) none of these

47. $(\underbrace{666\ldots.6}_{n\text{-digits}})^2 + (\underbrace{888\ldots.8}_{n\text{-digits}})$ is equal to
(a) $\frac{4}{9}(10^n - 1)$ (b) $\frac{4}{9}(10^{2n} - 1)$
(c) $\frac{4}{9}(10^n - 1)^2$ (d) none of these

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48. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)....(x-n)$ is

(a) $\frac{1}{24} n(n+1)(n-1)(3n+2)$

(b) $\frac{1}{24} n(n^2-1)(3n+2)$

(c) $\frac{n(n+1)(2n+1)}{6}$

(d) none of these

49. The sum of the series $1^2+1+2^2+2+3^2+3+\ldots+n^2+n$ is

(a) $\frac{n(n+1)}{2}$ (b) $\left\{\frac{n(n+1)}{2}\right\}^2$

(c) $\frac{n(n+1)(n+2)}{3}$ (d) $\frac{n(n+1)(n+2)(n+3)}{4}$

[JEE (WB) 2007]

50. If $H_1, H_2, ..., H_n$ be n harmonic means between a and b, then $\frac{H_1+a}{H_1-a}+\frac{H_n+b}{H_n-b}$ is equal to

(a) 0 (b) n (c) $2n$ (d) 1

1. If a, b, c be respectively the pth, qth and rth terms of a H.P., then

$$\Delta = \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$ equals

(a) 1 (b) 0 (c) -1 (d) none of these

52. If a, b, c are in G.P. and $a-b, c-a$ and $b-c$ are in H.P., then $a+4b+c$ is equal to

(a) -3 (b) 0 (c) 3 (d) none of these

53. The cubes of the natural numbers are grouped as $1^3, (2^3, 3^3), (4^3, 5^3, 6^3), ...$, then the sum of the numbers in the nth group is

(a) $\frac{1}{8} n^3(n^2+1)(n^2+3)$ (b) $\frac{1}{16} n^3(n^2+16)(n^2+12)$

(c) $\frac{n^3}{12}(n^2+2)(n^2+4)$ (d) none of these

54. Let a and b be roots of $x^2-3x+p=0$ and let c and d be the roots of $x^2-12x+q=0$, where a, b, c, d form an increasing GP Then the ratio of $q+p : q-p$ is equal to

(a) 8 : 7 (b) 11 : 10 (c) 17 : 15 (d) none of these

55. If S_1, S_2, S_3 be the sum of n, $2n$, $3n$ terms respectively of an A.P., then

(a) $S_3 = S_1 + S_2$ (b) $S_3 = 2(S_1 + S_2)$

(c) $S_3 = 3(S_2 - S_1)$ (d) none of these

56. If a, b, c, d, e, f are A.M.'s between 2 and 12, then $a+b+c+d+e+f$ is equal to

(a) 14 (b) 42 (c) 84 (d) none of these

57. If a, b, c are in G.P., then $\log_a x, \log_b x, \log_c x$ are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

58. If x, y, z are in H.P., then $\log(x+z)+\log(x-2y+z)$ is equal to

(a) $\log(x-z)$ (b) $2\log(x-z)$

(c) $3\log(x-z)$ (d) $4\log(x-z)$

59. If a, b, c, d are in HP, then $ab+bc+cd$ is equal to

(a) $3ad$ (b) $(a+b)(c+d)$

(c) $3ac$ (d) none of these

60. The sum of $i-2-3i+4$ ..... upto 100 terms, where $i=\sqrt{-1}$ is

(a) $50(1-i)$ (b) $25i$

(c) $25(1+i)$ (d) $100(1-i)$

61. If a, b, c are in H.P., then the value of $\frac{b+a}{b-a}+\frac{b+c}{b-c}$ is

(a) 1 (b) 2 (c) 3 (d) none of these

62. If a, b, c are in H.P., then

(a) $\frac{a-b}{b-c}=\frac{a}{c}$ (b) $\frac{b-c}{c-a}=\frac{b}{a}$

(c) $\frac{c-a}{a-b}=\frac{c}{b}$ (d) none of these

63. If a, b, c, are in A.P. ; b, c, d are in G.P. and c, d, e, are in H.P., then a, c, e, are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

64. If $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P., then $a, \frac{1}{b}, c$ are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

65. The sum of n terms of an A.P. is $a\,n(n-1)$. The sum of the squares of these terms is

(a) $a^2\,n^2\,(n-1)^2$

(b) $\frac{a^2}{6} n(n-1)(2n-1)$

(c) $\frac{2a^2}{3} n(n-1)(2n-1)$

(d) $\frac{2a^2}{3} n(n+1)(2n+1)$

66. If the sum of first p terms, first q terms and first r terms of an A.P. be x, y and z respectively. Then,

$$\frac{x}{p}(q-r)+\frac{y}{q}(r-p)+\frac{z}{r}(p-q) \text{ is}$$

(a) 0 (b) 2 (c) pqr (d) $\frac{8xyz}{pqr}$

67. Let $S_n=\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\ldots+\frac{1+2+\ldots+n}{1^3+2^3+\ldots+n^3}$; $n=1,2,3,...$ Then, S_n is not greater than

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

68. If a, b, c are in A.P.; a, x, b are in G.P. and b, y, c are in G.P., then x^2, b^2, y^2 are in

(a) H.P. (b) G.P. (c) A.P. (d) none of these

69. If $\log(x+z)+\log(x-2y+z)=2\log(x-z)$, then x, y, z are in

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(a) H.P. (b) G.P. (c) A.P. (d) none of these

70. If $\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0$ and $b \neq a+c$, then a, b, c are in
(a) G.P. (b) H.P. (c) A.P. (d) none of these

71. If arithmetic mean of two positive numbers is A, their geometric mean is G and harmonic mean H, then H is equal to
(a) $\frac{G^2}{A}$ (b) $\frac{A^2}{G^2}$ (c) $\frac{A}{G^2}$ (d) $\frac{G}{A^2}$

[JEE (WB) 2006]

72. If $(1-p)\,(1+2x+4x^2+8x^3+16x^4+32x^5) = 1-p^6,\ p \neq 1$ then a value of p/x is
(a) 1/2 (b) 2 (c) 1/4 (d) 4

73. If a, b, c are in G.P., then $\log_a \lambda$, $\log_b \lambda$, $\log_c \lambda$ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

74. If the sum of the series $1+\frac{3}{x}+\frac{9}{x^2}+\frac{27}{x^3}+\ldots$ to ∞ is a finite number, then
(a) $x < 3$ (b) $x > \frac{1}{3}$ (c) $x < \frac{1}{3}$ (d) $x > 3$

75. If H be the H.M. between a and b, then the value of $\frac{H}{a}+\frac{H}{b}$, is
(a) 2 (b) $\frac{ab}{a+b}$ (c) $\frac{a+b}{ab}$ (d) none of these

[JEE (WB) 2006]

76. The sum of n terms of two arithmetic progressions are in the ratio $2n+3: 6n+5$, then the ratio of their 13th terms is
(a) 53 : 155 (b) 27 : 87 (c) 29 : 83 (d) 31 : 89

[JEE (WB) 2006]

77. If $x=\sum_{n=0}^{\infty} a^n$, $y=\sum_{n=0}^{\infty} b^n$, $z=\sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. such that $|a|<1$, $|b|<1$, and $|c|<1$, then x, y, z are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

78. $x^{1/2} \cdot x^{1/4} \cdot x^{1/8} \cdot x^{1/16} \ldots$ to ∞ is equal to
(a) 0 (b) 1 (c) x (d) ∞

79. If a, b, c be in arithmetic progression, then the value of $(a+2b-c)\,(2b+c-a)\,(a+2b+c)$ is
(a) $16\,abc$ (b) $4\,abc$ (c) $8\,abc$ (d) $3\,abc$

[JEE (WB) 2008]

80. If a, b, c are in G.P. and $\log_c a$, $\log_b c$, $\log_a b$ are in A.P., then the common difference of the A.P. is
(a) 3 (b) 3/2 (c) 1/2 (d) 2/3

81. If $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences given by $a_n = (x)^{\frac{1}{2^n}} + (y)^{\frac{1}{2^n}}$ and $b_n = (x)^{\frac{1}{2^n}} - (y)^{\frac{1}{2^n}}$ for all $n \in N$. Then, $a_1\, a_2\, a_3 \ldots a_n$ is equal to
(a) $x-y$ (b) $\frac{x+y}{b_n}$ (c) $\frac{x-y}{b_n}$ (d) $\frac{xy}{b_n}$

82. The sum of the squares of three distinct real numbers which are in G.P. is S^2. If their sum is αS, then
(a) $1 < \alpha^2 < 3$ (b) $\frac{1}{3} < \alpha^2 < 3$
(c) $1 < \alpha < 3$ (d) $\frac{1}{3} < \alpha < 1$

83. If there be n quantities in G.P., whose common ratio is r and S_m denotes the sum of the first m terms, then the sum of their products, taken two by two is
(a) $S_m\, S_{m-1}$ (b) $\frac{r}{r+1} S_m\, S_{m-1}$
(c) $\frac{r}{r-1} S_m\, S_{m-1}$ (d) $\frac{r+1}{r} S_m\, S_{m-1}$

84. The value of $\sum_{r=1}^{n} \log\left(\frac{a^r}{b^r-1}\right)$ is
(a) $\frac{n}{2}\log\left(\frac{a^n}{b^n}\right)$ (b) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^n}\right)$
(c) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^n-1}\right)$ (d) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^n+1}\right)$

85. After inserting n A.Ms between 2 and 38, the sum of the resulting progression is 200. The value of n is
(a) 10 (b) 8 (c) 9 (d) none of these.

86. An A.P., a G.P. and a H.P. have the same first and last terms and the same odd number of terms. The middle terms of the three series are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

87. If a, b, c be in G.P. and $a+x, b+x, c+x$ in H.P., then the value of x is (a, b, c are distinct numbers)
(a) c (b) b (c) a (d) none of these

88. The maximum sum of the series $20+19\frac{1}{3}+18\frac{2}{3}+\ldots$ is
(a) 310 (b) 300
(c) 320 (d) none of these

89. If $2(y-a)$ is the H.M. between $y-x$ and $y-z$, then $x-a,\ y-a, z-a$ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

90. If the roots of the equation $x^3-12x^2+39x-28=0$ are in A.P., then their common difference will be
(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

91. If the sum of first n natural numbers is 1/5 times the sum of their squares, then the value of n is
(a) 5 (b) 6 (c) 7 (d) 8

92. $\log_3 2, \log_6 2 \log_{12} 2$ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

93. The value of $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots\ldots \infty$, is
(a) 9 (b) 1 (c) 3 (d) none of these

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94. The following consecutive terms $\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ of a series are in
(a) H.P. (b) G.P. (c) A.P. (d) A.P., G.P

95. The sum of all 2 digit odd numbers is
(a) 2475 (b) 2530 (c) 4905 (d) 5049

96. If the sum of the series 2, 5, 8, 11, ... is 60100, then n, the number of terms, is
(a) 100 (b) 200 (c) 150 (d) 250

97. Given two numbers a and b. Let A denote the single A.M. and S denote the sum of n A.M.'s between a and b, then S/A depends on
(a) n, a, b (b) n, b (c) n, a (d) n

98. Let $\sum_{r=1}^{n} r^4 = f(n)$, then $\sum_{r=1}^{n} (2r-1)^4$ is equal to
(a) $f(2n) - 16 f(n)$ (b) $f(2n) - 7f(n)$
(c) $f(2n-1) - 8f(n)$ (d) none of these

99. The value of $0.4\dot{2}\dot{3}$, is
(a) $\frac{419}{999}$ (b) $\frac{419}{990}$ (c) $\frac{423}{1000}$ (d) none of these

100. If a, b, c are in A.P. and a^2, b^2, c^2 are in H.P., then
(a) $a = b = c$ (b) $2b = 3a + c$
(c) $b^2 = \sqrt{(ac/8)}$ (d) none of these

101. The H.M. of two numbers is 4 and the arithmetic mean A and geometric mean G satisfy the relation $2A + G^2 = 27$, the numbers are
(a) 6, 3 (b) 5, 4 (c) 5, – 2.5 (d) – 3, 1

102. The sixth term of an A.P. is equal to 2 the value of the common difference of the A.P. which makes the product $a_1 a_4 a_5$ least is given by
(a) $x = 8/5$ (b) $x = 5/4$
(c) $x = 2/3$ (d) none of these

103. If $\frac{x+y}{1-xy}, y, \frac{y+z}{1-yz}$ be in A.P., then $x, \frac{1}{y}, z$ will be in
(a) A.P. (c) G.P. (c) H.P. (d) none of these

104. If a, b, c be in A.P., b, c, d are in G.P., and c, d, e are in H.P., then a, c, e will be in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

105. Three non-zero real numbers form an A.P. and the squares of these numbers taken in the same order form a G.P. Then, the number of all possible values of common ratios of the G.P. is
(a) 1 (b) 2 (d) 3 (d) none of these

106. If pth, qth, rth and sth terms of an A.P. are in G.P., then $p-q, q-r, r-s$ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

107. The nth term of the sequence 4, 14, 30, 52, 80, 114, ..., is
(a) $n^2 + n + 2$ (b) $3n^2 + n$
(c) $3n^2 - 5n + 2$ (d) $(n+1)^2$

108. If $|x| < 1$ and $|y| < 1$, the sum to infinity of the sequence $x + y, (x^2 + xy + y^2), (x^3 + x^2y + xy^2 + y^3), \ldots$, is
(a) $\frac{x+y-xy}{1-x-y+xy}$ (b) $\frac{x+y+xy}{1-x-y+xy}$
(c) $\frac{x}{1-x} + \frac{y}{1-y}$ (d) $\frac{(x-y)(x+y-xy)}{1-x-y+xy}$

109. If S_1, S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms respectively of an A.P., then
$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2) =$
(a) 0 (b) 1 (c) $S_1 S_2 S_3$ (d) $n_1 n_2 n_3$

110. If $|a| < 1$ and $|b| < 1$, then the sum of the series
$a(a+b) + a^2(a^2+b^2) + a^3(a^3+b^3) + \ldots$ upto ∞, is
(a) $\frac{a}{1-a} + \frac{ab}{1-ab}$ (b) $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$
(c) $\frac{b}{1-b} + \frac{a}{1-a}$ (d) $\frac{b^2}{1-b^2} + \frac{ab}{1-ab}$

111. If $\log_x a, a^{x/2}$ and $\log_b x$ are in G.P., then x is equal to
(a) $\log_a(\log_b a)$ (b) $\log_a(\log_e a) + \log_a(\log_e b)$
(c) $-\log_a(\log_a b)$ (d) $\log_a(\log_e b) - \log_a(\log_e a)$

112. If a, b, c, d are in G.P., then
$(a^3+b^3)^{-1}, (b^3+c^3)^{-1}, (c^3+d^3)^{-1}$ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

113. If, for $0 < x < \pi/2$,
$y = \exp[(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty) \log_e 2]$
is a zero of the quadratic equation $x^2 - 9x + 8 = 0$, then the value of $\frac{\sin x + \cos x}{\sin x - \cos x}$, is
(a) 0 (b) $2+\sqrt{3}$ (c) $2-\sqrt{3}$ (d) none of these

114. The value of $0.2^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\right)}$, is
(a) 4 (b) log 4 (c) log 2 (d) none of these

115. If the sum of an infinitely decreasing G.P. is 3, and the sum of the squares of its terms is 9/2, the sum of the cubes of the terms is
(a) $\frac{105}{13}$ (b) $\frac{108}{13}$ (c) $\frac{729}{8}$ (d) none of these

116. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ to $\infty = \frac{\pi^2}{6}$, then
$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$ equals
(a) $\pi^2/8$ (b) $\pi^2/12$ (c) $\pi^2/3$ (d) $\pi^2/2$

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117. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty\right)}$, is
(a) 2 (b) 3 (c) 4 (d) none of these

118. If the sum of the first n terms of a series be $5n^2+2n$, then its second term is
(a) $\frac{56}{15}$ (b) $\frac{27}{14}$ (c) 17 (d) 16

119. If $x, |x+1|, |x-1|$ are first three terms of an A.P., then the sum of its first 20 terms is
(a) 360, 180 (b) 350, 180
(c) 150, 100 (d) none of these

120. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. and $a_i > 0$ for each $i = 1, 2, 3, \ldots, n$, then
$\sum_{r=1}^{n-1} \frac{1}{a_{r+1}^{2/3}+a_{r+1}^{1/3}\,a_r^{1/3}+a_r^{2/3}}$ is equal to
(a) $\frac{n+1}{a_{n-1}^{2/3}+a_{n-1}^{1/3}\,a_1^{1/3}+a_1^{2/3}}$ (b) $\frac{n-1}{a_n^{2/3}+a_n^{1/3}\,a_1^{1/3}+a_1^{2/3}}$
(c) $\frac{n-1}{a_n^{2/3}+a_n^{1/3}\,a_1^{1/3}+a_1^{2/3}}$ (d) $\frac{n+1}{a_{n+1}^{2/3}+a_{n+1}^{1/3}\,a_1^{1/3}+a_1^{2/3}}$

121. If $\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0$ and $b \neq a+c$, then a, b, c are in
(a) G.P. (b) H.P.
(c) A.P. (d) none of these

122. If a, b, c are in H.P., then the value of
$\frac{(ac+ab-bc)(ab+bc-ac)}{(abc)^2}$, is
(a) $\frac{(a+c)(3a-c)}{4a^2c^2}$ (b) $\frac{2}{bc}+\frac{1}{b^2}$
(c) $\frac{2}{bc}-\frac{1}{a^2}$ (d) $\frac{(a-c)(3a+c)}{4a^2c^2}$

123. The set of all possible values of x for which 13 is the A.M. of 5^{1+x} and 5^{1-x}, is
(a) $5, \frac{1}{5}$ (b) $\{-1, 1\}$
(c) $\{0, 1\}$ (d) none of these

124. If a, b, c are in A.P., then $a+\frac{1}{bc}, b+\frac{1}{ca}, c+\frac{1}{ab}$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

125. The coefficient of x^{49} in the product
$(x-1)(x-3)(x-5)\ldots(x-99)$, is
(a) -99^2 (b) 1
(c) -2500 (d) none of these

126. The coefficient of x^{15} in the product
$(1-x)(1-2x)(1-2^2x)(1-2^3x)\ldots(1-2^{15}x)$ is equal to
(a) $2^{105}-2^{121}$ (b) $2^{121}-2^{105}$
(c) $2^{120}-2^{104}$ (d) none of these

127. If $S_n = \sum_{r=1}^{n} a_r = \frac{1}{6}n(2n^2+9n+13)$, then $\sum_{r=1}^{n}\sqrt{a_r}$ equals
(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+2)}{2}$
(c) $\frac{n(n+3)}{2}$ (d) $\frac{n(n+5)}{2}$

128. If $\sum_{r=1}^{n} a_r = \frac{1}{6}n(n+1)(n+2)$ for all $n \geq 1$, then
$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{a_r}$ is
(a) 2 (b) 3 (c) 3/2 (d) 6

129. Sum to n terms of the series
$\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}+\ldots\ldots$, is
(a) $\frac{n^3}{3(n+1)(n+2)(n+3)}$ (b) $\frac{n^3+6n^2-3n}{6(n+2)(n+3)(n+4)}$
(c) $\frac{15n^2+7n}{4n(n+1)(n+5)}$ (d) $\frac{n^3+6n^2+11n}{18(n+1)(n+2)(n+3)}$

Answers

1. (b) 2. (d) 3. (b) 4. (b) 5. (d) 6. (b) 7. (a)
8. (c) 9. (a) 10. (c) 11. (c) 12. (c) 13. (a) 14. (c)
15. (c) 16. (b) 17. (c) 18. (a) 19. (a) 20. (b) 21. (d)
22. (a) 23. (b) 24. (c) 25. (b) 26. (c) 27. (d) 28. (b)
29. (b) 30. (a) 31. (c) 32. (c) 33. (b) 34. (a) 35. (b)
36. (c) 37. (d) 38. (b) 39. (a) 40. (d) 41. (c) 42. (a)
43. (a) 44. (c) 45. (b) 46. (c) 47. (b) 48. (b) 49. (c)
50. (c) 51. (b) 52. (b) 53. (c) 54. (c) 55. (c) 56. (b)
57. (c) 58. (b) 59. (a) 60. (a) 61. (b) 62. (a) 63. (b)
64. (c) 65. (c) 66. (a) 67. (c) 68. (c) 69. (a) 70. (b)
71. (a) 72. (b) 73. (c) 74. (d) 75. (a) 76. (a) 77. (c)
78. (c) 79. (a) 80. (b) 81. (c) 82. (b) 83. (b) 84. (c)
85. (b) 86. (b) 87. (b) 88. (a) 89. (b) 90. (c) 91. (c)
92. (c) 93. (c) 94. (c) 95. (a) 96. (b) 97. (d) 98. (a)
99. (b) 100. (a) 101. (a) 102. (c) 103. (c) 104. (b) 105. (c)
106. (b) 107. (b) 108. (a) 109. (a) 110. (b) 111. (a) 112. (b)
113. (b) 114. (a) 115. (b) 116. (a) 117. (c) 118. (c) 119. (b)
120. (c) 121. (b) 122. (a) 123. (b) 124. (a) 125. (c) 126. (a)
127. (c) 128. (a) 129. (d)

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CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. Let $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$, then the sum to n terms of the series
$$\frac{1^2}{1^3} + \frac{1^2+2^2}{1^3+2^3} + \frac{1^2+2^2+3^2}{1^3+2^3+3^3} + \ldots, \text{ is}$$
(a) $\frac{4}{3}H_n - 1$ (b) $\frac{4}{3}H_n + \frac{1}{n}$
(c) $\frac{4}{3}H_n$ (d) $\frac{4}{3}H_n - \frac{2}{3}\frac{n}{n+1}$

2. The sum of n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$, is
(a) $2^n - n - 1$ (b) $1 - 2^{-n}$
(c) $n + 2^{-n} - 1$ (d) $2^n - 1$

3. If A_1, A_2 are two A.M's, G_1, G_2 are two G.M's and H_1, H_2 are two H.M's between two numbers, then $\frac{A_1 + A_2}{H_1 + H_2}$ is equal to
(a) $\frac{H_1 H_2}{G_1 G_2}$ (b) $\frac{G_1 G_2}{H_1 H_2}$ (c) $\frac{H_1 H_2}{A_1 A_2}$ (d) $\frac{G_1 G_2}{A_1 A_2}$

4. If $(m+1)^{\text{th}}$, $(n+1)^{\text{th}}$ and $(r+1)^{\text{th}}$ terms of an AP are in G.P; and m, n, r are in HP, then the ratio of the first term and common difference of this AP is
(a) $n/2$ (b) $-n/2$ (c) $n/3$ (d) $-n/3$

5. Given that n arithmetic means are inserted between two sets of numbers $a, 2b$, and $2a, b$, where $a, b \in R$. Suppose further that mth mean between these two sets of numbers is same, then the ratio $a : b$ equals
(a) $n - m + 1 : m$ (b) $n - m + 1 : n$
(c) $m : n - m + 1$ (d) $n : n - m + 1$

6. If a, b, c are in G.P., then $a + b, 2b, b + c$ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

7. If in a progression $a_1, a_2, a_3, \ldots$ etc., $(a_r - a_{r+1})$ bears a constant ratio with $a_r . a_{r+1}$, then the terms of the progression are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

8. If in an A.P. $a_1 = \log_{10} a$, $a_{n+1} = \log_{10} b$ and $a_{2n+1} = \log_{10} c$, then a, b, c are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

9. The sum of the series
$1^2 - 2^3 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots - 2008^2 + 2009^2$ is
(a) 2019045 (b) 1005004
(c) 2000506 (d) none of these

10. If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a, b, c are non-zero numbers, then a, b, c are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

11. If S_n denotes the sum of n terms of an A.P. with common difference d, then
(a) $d = S_n - S_{n-1} + S_{n-2}$ (b) $d = S_n - 2S_{n-1} + S_{n-2}$
(c) $d = S_n - 2S_{n-1} + S_{n-2}$ (d) none of these

12. If the lengths of sides of a right angled triangle are in A.P. then their ratio is
(a) 2 : 3 : 4 (b) 3 : 4 : 5
(c) 4 : 5 : 6 (d) none of these

13. The sum of 11 terms of an A.P. whose middle term is 30,
(a) 320 (b) 330 (c) 340 (d) 350

14. The maximum sum of the series
$20 + 19\frac{1}{3} + 18\frac{2}{3} + 18 + \ldots$, is
(a) 310 (b) 290 (c) 320 (d) none of these

15. If three numbers are in G.P., then the numbers obtained by adding the middle number to each of these numbers are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

16. If $p, q, r, s \in N$ and they are four consecutive terms of an A.P., then pth, qth, rth and sth terms of a G.P. are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

17. If x, y, z be three positive prime numbers. The progression in which $\sqrt{x}, \sqrt{y}, \sqrt{z}$ can be three terms (not necessarily consecutive) is
(a) A.P. (b) G.P. (c) H.P. (d) none of the

18. If a, b, c are in H.P., then $\frac{1}{b-a} + \frac{1}{b-c} =$
(a) $\frac{1}{a} + \frac{1}{b}$ (b) $\frac{1}{a} + \frac{1}{c}$ (c) $\frac{1}{b} + \frac{1}{c}$ (d) none the these

19. If three numbers are in H.P., then the numbers obtained by subtracting half of the middle number from each of them are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

20. a, b, c, d, e are five numbers in which the first three are in A.P. and the last three are in H.P. If the three numbers in the middle are in G.P., then the numbers in the odd place are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

21. In a G.P. with alternatively positive and negative terms any term is the A.M. of the next two terms. Then, the common ratio of the G.P. is
(a) -1 (b) -3 (c) -2 (d) $-1/2$

22. If a, b, c, d are in H.P., then $ab + bc + cd =$
(a) ad (b) $2ad$ (c) $3ad$ (d) none of these

23. If $\sum_{r=1}^{\infty} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}$, then $\sum_{r=1}^{\infty} \frac{1}{r^2}$ is equal to

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(a) $\frac{\pi^2}{24}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) none of these

24. If it is given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots$ to $\infty=\frac{\pi^4}{90}$, then the value of $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots$ to ∞ is equal to

(a) $\frac{\pi^4}{96}$ (b) $\frac{\pi^4}{45}$ (c) $\frac{89\pi^4}{90}$ (d) none of these

25. The minimum number of terms from the beginning of the series

$$20+22\frac{2}{3}+25\frac{1}{3}+\ldots.$$

so that the sum may exceed 1568, is

(a) 25 (b) 27 (c) 28 (d) 29

26. The sum of the series

$1-3+5-7+9-11+\ldots.$ to n terms is

(a) $-n$, when n is even (b) $2n$, when n is even

(c) $-n$, when n is odd (d) $2n$, when n is odd

27. If a, b, c are three unequal positive quantities in H.P., then

(a) $a^{3/2}+c^{3/2}>2b^{1/2}$ (b) $a^5+c^5>2b^5$

(c) $a^2+c^2>2b^3$ (d) none of these

28. Fifth term of a G.P. is 2, then the product of its 9 terms is

(a) 256 (b) 512 (c) 1024 (d) none of these

29. $1^3-2^3+3^3-4^3+\ldots.+9^3=$

(a) 425 (b) -425 (c) 475 (d) -475

30. Sum of infinite number of terms of a G.P. is 20 and sum of their squares is 100. The common ratio of the G.P. is

(a) 5 (b) 3/5 (c) 8/5 (d) 1/5

31. If $1, \log_9(3^{1-x}+2), \log_3(4.3^x-1)$ are in A.P., then x equals

(a) $\log_3 4$ (b) $1-\log_3 4$

(c) $1-\log_4 3$ (d) $\log_4 3$

32. Two sequences $<a_n>$ and $<b_n>$ are defined by

$$a_n=\log\left(\frac{5^{n+1}}{3^{n-1}}\right),\ b_n=\left\{\log\left(\frac{5}{3}\right)\right\}^n, \text{ then}$$

(a) $<a_n>$ is an A.P. and $<b_n>$ is a G.P.

(b) $<a_n>$ and $<b_n>$ both are G.P.

(c) $<a_n>$ and $<b_n>$ both are A.P.

(d) $<b_n>$ is a G.P. and $<b_n>$ is neither an A.P nor a G.P.

33. The sum of the series

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots.+\frac{1}{\sqrt{n^2-1}+\sqrt{n^2}} \text{ equals}$$

(a) $\frac{2n+1}{\sqrt{n}}$ (b) $\frac{\sqrt{n}+1}{\sqrt{n}+\sqrt{n-1}}$

(c) $\frac{n+\sqrt{n^2-1}}{2\sqrt{n}}$ (d) $n-1$

34. If a and b are two different positive real numbers, then which of the following statements is true?

(a) $2\sqrt{ab}>a+b$ (b) $2\sqrt{ab}<a+b$

(c) $2\sqrt{ab}=a+b$ (d) none of these

35. In the sequence $\{1\}, \{2, 3\}, \{4, 5, 6\}, \{7, 8, 9, 10\}, \ldots..$ of sets, the sum of the elements in 50th set is

(a) 62525 (b) 65255 (c) 56255 (d) 55625

36. If the first term of an A.P. is 2 and common difference is 4, then the sum of its 40 terms is

(a) 3200 (b) 1600 (c) 200 (d) 2800

37. If $1+\frac{1+2}{2}+\frac{1+2+3}{3}+\ldots.$ to n terms is S, then S is equal to

(a) $\frac{n(n+3)}{4}$ (b) $\frac{n(n+2)}{4}$

(c) $\frac{n(n+1)(n+2)}{6}$ (d) n^2

38. The sum of 10 terms of the series $\sqrt{2}+\sqrt{6}+\sqrt{18}+\ldots.$ is

(a) $121(\sqrt{6}+\sqrt{2})$ (b) $243(\sqrt{3}+1)$

(c) $\frac{121}{\sqrt{3}-1}$ (d) $242(\sqrt{3}-1)$

39. In a G.P. if the $(m+n)^{th}$ term is p and $(m-n)^{th}$ term is q, then its m^{th} term is

(a) 0 (b) pq (c) $\sqrt{pq}$ (d) $\frac{1}{2}(p+q)$

40. The fourth, seventh and tenth terms of a G.P. are p, q and r respectively, then

(a) $p^2=q^2+r^2$ (b) $p^2=qr$

(c) $q^2=pr$ (d) $r^2=p^2+q^2$

41. The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is

(a) 2489 (b) 4735 (c) 2632 (d) 2317

42. Let the harmonic mean and the geometric mean of two numbers be in the ratio 4 : 5. The two numbers are in the ratio

(a) 1 : 1 (b) 2 : 1 (c) 3 : 1 (d) 4 : 1

43. The sum of the series

$1+2.2+3.2^2+4.2^3+5.2^4+\ldots.+100.2^{99}$, is

(a) 99×2^{100} (b) $99\times2^{100}+1$

(c) 100×2^{100} (d) none of these

44. If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P., then

(a) a, b, c are in A.P. (b) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(c) a, b, c are in H.P. (d) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P.

45. If the m^{th}, n^{th} and p^{th} terms of an A.P. and G.P. be equal and be respectively x, y, z, then

(a) $x^y y^z z^x = x^z y^x z^y$

(b) $(x-y)^x (y-z)^y = (z-x)^z$

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(c) $(x-y)^z (y-z)^x = (z-x)^y$
(d) none of these

46. If the 7^{th} term of an H.P. is $1/10$ and 12^{th} term is $1/25$, then 20^{th} term is
(a) $\frac{1}{37}$ (b) $\frac{1}{41}$ (c) $\frac{1}{45}$ (d) $\frac{1}{49}$

47. The length of a side of a square is "a" metre. A second square is formed by joining the middle points of this square. Then a third square is formed by joining the middle points of the sides of the second square and so on. Then, the sum of the areas of squares which carried upto infinity is
(a) a^2 (b) $2a^2$ (c) $3a^2$ (d) $4a^2$

48. The harmonic mean of the roots of the equation
$(5+\sqrt{2})x^2 - (4+\sqrt{5})x + (8+2\sqrt{5}) = 0$, is
(a) 2 (b) 4 (c) 6 (d) 8

49. If three positive real numbers a, b, c $(c > a)$ are in H.P., then $\log(a+c) + \log(a-2b+c)$ is equal to
(a) $2\log(c-b)$ (b) $2\log(a+c)$
(c) $2\log(c-a)$ (d) $\log a + \log b + \log c$

50. In a H.P. p^{th} term is q and q^{th} term is p. Then, $(pq)^{th}$ term is
(a) $\frac{p+q}{pq}$ (b) 0 (c) $\frac{pq}{p+q}$ (d) 1

51. The sum of the series $\frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81} + \ldots$ to n terms is
(a) $n - \frac{1}{2}(3^{-n} - 1)$ (b) $n - \frac{1}{2}(1 - 3^{-n})$
(c) $n + \frac{1}{2}(3^n - 1)$ (d) $n - \frac{1}{2}(3^n - 1)$

52. If three real numbers a, b, c are in H.P., then which one of the following is true?
(a) $\frac{1}{a}, b, \frac{1}{c}$ are in A.P. (b) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in H.P.
(c) ab, bc, ca are in H.P. (d) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in H.P.

53. If the sums of n terms of the series
$704 + \frac{1}{2}(704) + \frac{1}{4}(704) + \ldots$ and, $1984 - \frac{1}{2}(1984) + \frac{1}{4}(1984) \ldots$
are equal, then n =
(a) 5 (b) 3 (c) 4 (d) 10

54. If $2\times2^2 + 3\times2^3 + 4\times2^4 + \ldots + n\times2^n = 2^{n+10}$, then n =
(a) 510 (b) 512 (c) 513 (d) 508

55. If $1^2 + 2^2 + 3^2 + \ldots + 2003^2 = (2003)(4007)(334)$
and,
$1\times2003 + 2\times2002 + 3\times2001 + \ldots + 2003\times1 = (2003)(334)x$
then x =
(a) 2005 (b) 2004 (c) 2003 (d) 2001

56. The sum to n terms of the series
$(n^2-1^2) + 2(n^2-2^2) + 3(n^2-3^2) + \ldots$, is
(a) $\frac{n^2}{4}(n^2-1)$ (b) $\frac{n}{4}(n+1)^2$ (c) 0 (d) $2n(n^2-1)$

57. The sum upto $(2n+1)$ terms of the series
$a^2 - (a+d)^2 + (a+2d)^2 - (a+3d)^2 + \ldots$, is
(a) $a^2 + 3nd^2$ (b) $a^2 + 2nad + n(n-1)d^2$
(c) $a^2 + 3nad + n(n-1)d^2$ (d) $a^2 + 2nad + n(2n+1)d^2$

58. If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$, then the value of $S_n = 1 + \frac{3}{2} + \frac{5}{3} + \ldots + \frac{2n-1}{n}$ is
(a) $H_n + n$ (b) $2n - H_n$
(c) $(n-1) + H_n$ (d) $H_n + 2n$

59. The sum to n terms of the series
$1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$, is
(a) $2(n-1) + \frac{1}{2n-1}$ (b) $2n - \frac{1}{2^n}$
(c) $2 + \frac{1}{2^n}$ (d) $2n - 1 + \frac{1}{2^n}$

60. If $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots + \frac{1}{2^n - 1}$, then
(a) $a_{100} < 100$ (b) $a_{100} > 100$
(c) $a_{200} < 100$ (d) none of these

Answers

1. (d)	2. (c)	3. (b)	4. (b)	5. (c)	6. (c)	7. (c)	8. (b)	9. (a)	10. (c)
11. (c)	12. (b)	13. (b)	14. (a)	15. (c)	16. (b)	17. (d)	18. (b)	19. (b)	20. (b)
21. (c)	22. (c)	23. (c)	24. (a)	25. (d)	26. (a)	27. (b)	28. (b)	29. (a)	30. (b)
31. (b)	32. (a)	33. (d)	34. (b)	35. (a)	36. (a)	37. (d)	38. (b)	39. (a)	40. (b)
41. (a)	42. (a)	43. (c)	44. (b)	45. (a)	46. (c)	47. (c)	48. (d)	49. (b)	50. (a)
51. (a)	52. (b)	53. (a)	54. (c)	55. (a)	56. (a)	57. (d)	58. (b)	59. (a)	60. (a)

Solution of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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11

QUADRATIC EXPRESSIONS AND EQUATIONS

1. SOME DEFINITIONS

REAL POLYNOMIAL *Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and x be a real variable. Then,*

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

is called a real polynomical of real variable x with real coefficients.

For example, $2x^3 - 6x^2 + 11x - 6$ and $x^2 - 4x + 3$ are real polynomials.

COMPLEX POLYNOMIAL *If $a_0, a_1, a_2, \ldots, a_n$ are complex numbers and x is a varying complex number, then*

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_{n-1} x^{n-1} + a_n x^n$$

is called a complex polynomial or, a polynomial of complex variable with complex coefficients.

For example, $2x^2 - (3 + 7i)\, x + (9i - 3)$ is a complex polynomial.

DEGREE OF A POLYNOMIAL *A polynomial $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n$, real or complex, is a polynomial of degree n, if $a_n \neq 0$.*

For example, the polyonomials $2x^3 - 7x^2 + x + 5$ and $(3 - 2i)\, x^2 - ix + 5$ are of degree 3 and 2 respectively.

A polynomial of second degree is generally called a quadratic polynomial. Polynomials of degree 3 and 4 are known as cubic and biquadratic polynomials.

POLYNOMIAL EQUATION *If $f(x)$ is a polynomial, real or complex, then $f(x) = 0$ is called a polynomial equation.*

If $f(x)$ is a polynomial of second degree, then $f(x) = 0$ is called a quadratic equation. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where $a, b, c \in C$, such that $a \neq 0$.

ROOTS OF AN EQUATION *The values of the variable satisfying the given equation are called its roots.*

In other words, $x = \alpha$ is a root of the equation $f(x) = 0$, if $f(\alpha) = 0$.

The real roots of an equation $f(x) = 0$ are the x-coordinates of the points where the curve $y = f(x)$ crosses x-axis.

2. POSITION OF ROOTS OF A POLYNOMIAL EQUATION

If $f(x) = 0$ is a polynomial equation with real coefficients and a, b are two real numbers such that $f(a)\, f(b) < 0$, then the equation $f(x) = 0$ has at least one real root or an odd number of real roots between a and b. In case $f(a)$ and $f(b)$ are of the same sign, then either no real root or an even number of real roots of $f(x) = 0$ lie between a and b.

ILLUSTRATION 1 *If $a, b, c, d \in R$ such that $a < b < c < d$, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$*

(a) *are imaginary*
(b) *are equal*
(c) *are real and distinct lying between a and b*
(d) *real and distinct lying between a and d.*

Ans. (d)

SOLUTION Let $f(x) = (x - a)(x - c) + 2(x - b)(x - d)$. Then,

$f(a) = 2(a - b)(a - d) > 0$ $[\because a - b < 0$ and $a - d < 0]$

$f(b) = (b - a)(b - c) < 0$ $[\because b - a > 0$ and $b - c < 0]$

and, $f(d) = (d - a)(d - b) > 0$ $[\because d - a > 0$ and $d - b > 0$

Thus, one root of $f(x) = 0$ lies between a and b and another roo lies between b and d. Hence, the roots of the given equation ar real and distinct and between a and d.

DEDUCTIONS

(i) *Every equation of an odd degree has at least one real root, who sign is opposite to that of its last term, provided that the coefficient first term is positive.*

(ii) *Every equation of an even degree whose last term is negative the coefficient of first term positive, has at least two real roots, positive and one negative.*

(iii) *If an equation has only one change of sign, it has one +ve and no more.*

(iv) *If all the terms of an equation are positive and the equ involves no odd powers of x, then all its roots are complex.*

3. DESCARTE'S RULE OF SIGNS

The maximum number of positive real roots of a polyn equation $f(x) = 0$ is the number of changes of signs from po to negative and negative to positive in $f(x)$.

Consider the equation $x^3 + 6x^2 + 11x - 6 = 0$. The signs various terms are :

$$+ \quad + \quad + \quad -$$

Clearly, there is only one change of sign in the exp $x^3 + 6x^2 + 11x - 6$. So, the given equation has at mo positive real root.

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As x is real.

$\therefore \quad 4(y+1)^2 - 16(y-1)^2 \geq 0$

$\Rightarrow \quad (3y-1)(y-3) \leq 0 \Rightarrow \frac{1}{3} \leq y \leq 3$

So, statement–2 is true.

Let $\alpha = 3^{x+1}$. Then,

$$\frac{9\times 3^{2x}+6\times 3^x+4}{9\times 3^{2x}-6\times 3^x+4} = \frac{(3^{x+1})^2+2\times 3^{x+1}+4}{(3^{x+1})^2-2\times 3^{x+1}+4}$$

$$\Rightarrow \quad \frac{9\times 3^{2x}+6\times 3^x+4}{9\times 3^{2x}-6\times 3^x+4} = \frac{\alpha^2+2\alpha+4}{\alpha^2-2\alpha+4}$$

From statement–2, we have

$$\frac{1}{3} \leq \frac{\alpha^2-2\alpha+4}{\alpha^2+2\alpha+4} \leq 3 \text{ for all } \alpha \in R$$

$$\Rightarrow \quad \frac{1}{3} \leq \frac{\alpha^2+2\alpha+4}{\alpha^2-2\alpha+4} \leq 3 \text{ for all } \alpha \in R$$

$$\Rightarrow \quad \frac{1}{3} \leq \frac{9\times 3^{2x}+6\times 3^x+4}{9\times 3^{2x}-6\times 3^x+4} \leq 3 \text{ for all } x \in R$$

So, statement–1 is true and statement–2 is a correct explanation for statement–1.

EXAMPLE 19 *Let a, b, c be real numbers such that $ax^2+bx+c = 0$ and $x^2+x+1 = 0$ have a common root.*

Statement-1: $a = b = c$

Statement-2: Two quadratic equations with real coefficients cannot have only one imaginary root common.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION The equation $x^2+x+1 = 0$ has imaginary roots w and w^2. If $ax^2+bx+c=0$ and $x^2+x+1=0$ has one imaginary root common, then other imaginary root must also be common as imaginary roots occur in pairs. So, statement-2 is true.

Given equations have both roots common.

$$\therefore \quad \frac{a}{1} = \frac{b}{1} = \frac{c}{1} \Rightarrow a = b = c$$

So, statement-1 is also true and statement-2 is a correct explanation for statement-1.

EXAMPLE 20 *Statement-1: The cubic equation $4x^3-15x^2+14x-5 = 0$ has a root in the internal (2, 3).*

Statement-2: If $f(x)$ is a polynomial equation which has two real roots α, β $(\alpha<\beta)$, then $f'(x)=0$ will have a root γ such that $\alpha<\gamma<\beta$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Let $f(x) = x^4-5x^3+7x^2-5x+6$.

Clearly, $f(2) = 0$ and $f(3) = 0$.

So, $x = 2$ and $x = 3$ are real roots of $f(x) = 0$.

Hence, by Rolle's theorem, $f'(x) = 0$ has a root in (2, 3) i.e. $4x^3-15x^2+14x-5 = 0$ has a root in (2, 3).

So, both the statements are true and statement-2 is a correct explanation for statement-1.

EXAMPLE 21 *Statement-1: The equation*

$$\frac{\pi^e}{x-e}+\frac{e^\pi}{x-\pi}+\frac{\pi^\pi+e^e}{x-\pi-e} = 0 \text{ has real roots.}$$

Statement-2: If $f(x)$ is a polynomial and a, b are two real numbers such that $f(a)f(b)<0$, then $f(x)=0$ has an odd number of real roots between a and b.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Statement-2, being a standard result, is true.

Let $f(x) = \frac{\pi^e}{x-e}+\frac{e^\pi}{x-\pi}+\frac{\pi^\pi+e^e}{x-\pi-e}$. Then,

$$f(x) = \pi^e(x-\pi)(x-\pi-e)+e^\pi(x-e)(x-\pi-e) + (\pi^\pi+e^e)(x-e)(x-\pi)$$

Now, $f(e) = \pi^{e+1}(\pi-e) > 0, f(\pi) = -e^{\pi+1}(\pi-e) < 0$

and, $f(\pi+e) = (\pi^\pi+e^e)\pi e > 0$

Clearly, $e<\pi<\pi^\pi+e^e$ such that $f(e)>0, f(\pi)<0$ and $f(\pi+e)>0$

So, $f(x)$ has real roots in (e, π) and $(\pi, \pi+e)$.

EXAMPLE 22 *Consider a quadratic equation $ax^2+bx+c = 0$, where $2a+3b+6c=0$ and let $g(x) = a\frac{x^3}{3}+b\frac{x^2}{2}+cx$.*

Statement-1 The quadratic equation has at least one root in the internal (0, 1).

Statement-2 The Rolle's Theorem is applicable to function $g(x)$ on the interval [0, 1].

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a) **[AIEEE 2012]**

SOLUTION Clearly, $g(x) = \frac{a}{3}x^3+\frac{b}{2}x^2+cx$ is differentiable on (0, 1) and continous on [0, 1]. Also, $g(0)=0$ and $g(1) = \frac{a}{3}+\frac{b}{2}+c = \frac{2a+3b+6c}{6} = 0$. Therefore, $g(0) = g(1) = 0$.

Thus, Rolle's theorem is applicable to the function $g(x)$ on [0, 1]. Consequently, there exists $\alpha \in (0, 1)$ such that $g'(\alpha) = 0$ i.e. $a\alpha^2+b\alpha+c=0$. Hence, $g(x) = ax^2+bx+c$ has a root $\alpha \in (0, 1)$. So, both the statements are true and statement-2 is a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $A = \{x : f(x) = 0\}$ and $B = \{x : g(x) = 0\}$, then $A \cap B$ will be the set of roots of the equation

(a) $|f(x)|^2+|g(x)|^2 = 0$ (b) $\frac{f(x)}{g(x)}$ (c) $\frac{g(x)}{f(x)}$ (d) none of these

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2. Let x_1, x_2 be the roots of the equation $x^2 - 3x + p = 0$ and let x_3, x_4 be the roots of the equation $x^2 - 12x + q = 0$. If the numbers x_1, x_2, x_3, x_4 (in order) form an increasing G.P., then
(a) $p = 2,\ q = 16$ (b) $p = 2,\ q = 32$
(c) $p = 4,\ q = 16$ (d) $p = 4,\ q = 32$

3. The roots of the equation $|x^2 - x - 6| = x + 2$ are
(a) $-2, 1, 4$ (b) $0, 2, 4$ (c) $0, 1, 4$ (d) $-2, 2, 4$

4. If the equation $x^3 - 3x + a = 0$ has distinct roots between 0 and 1, then the value of a is
(a) 2 (b) 1/2 (c) 3 (d) none of these

5. If $f(x) = ax^2 + bx + c,\ g(x) = -ax^2 + bx + c$ where $ac \neq 0$, then $f(x)\, g(x) = 0$ has
(a) at least three real roots
(b) no real roots
(c) at least two real roots
(d) two real roots and two imaginary roots

6. The equation $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2},\ 0 \le x \le \frac{\pi}{2}$ has
(a) no real solution (b) one real solution
(c) more than one real solution (d) none of these

7. The number of real roots of the equation
$(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ is
(a) 1 (b) 2 (c) 3 (d) none of these

8. The roots of the equation $\log_2(x^2 - 4x + 5) = (x - 2)$ are
(a) 4, 5 (b) 2, −3 (c) 2, 3 (d) 3, 5

9. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then, the equation $ax^2 + bx + c = 0$ has
(a) real roots (b) complex roots
(c) exactly one root (d) none of these

10. The value of k for which the equation
$3x^2 + 2x(k^2 + 1) + k^2 - 3k + 2 = 0$
has roots of opposite signs, lies in the interval
(a) $(-\infty, 0)$ (b) $(-\infty, -1)$
(c) $(1, 2)$ (d) $(3/2, 2)$

11. If p and q are roots of the quadratic equation
$x^2 + mx + m^2 + a = 0$, then the value of $p^2 + q^2 + pq$ is
(a) 0 (b) a (c) $-a$ (d) $\pm m^2$

12. The condition that one root of the equation $ax^2 + bx + c = 0$ may be double of the other, is
(a) $b^2 = 9ac$ (b) $2b^2 = 9ac$
(c) $2b^2 = ac$ (d) $b^2 = ac$

13. If $e^{\cos x} - e^{-\cos x} = 4$, then the value of $\cos x$ is
(a) $\log_e(2 + \sqrt{5})$ (b) $-\log_e(2 + \sqrt{5})$
(c) $\log_e(-2 + \sqrt{5})$ (d) none of these

14. If one root of the equation $5x^2 + 13x + k = 0$ is reciprocal of other, then the value of k is
(a) 0 (b) 5 (c) 1/6 (d) 6

15. If the equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ have both roots common, then the value of $(2r - p)$ is
(a) 0 (b) 1/2
(c) 1 (d) none of these

16. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is
(a) 3 (b) 2
(c) 1 (d) none of these

17. The number of quadratic equations which are unchanged by squaring their roots is
(a) 2 (b) 4
(c) 6 (d) none of these

18. If the equation $x^2 - 3kx + 2e^{2\log k} - 1 = 0$ has real roots such that the product of roots is 7, then the value of k is
(a) ± 1 (b) 2 (c) ± 3 (d) none of these

19. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is
(a) 49/4 (b) 4/49 (c) 4 (d) none of these
[AIEEE 2004]

20. If the roots of the equation $x^2 - px + q = 0$ differ by unity, then
(a) $p^2 = 4q$ (b) $p^2 = 4q + 1$
(c) $p^2 = 4q - 1$ (d) none of these

21. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is
(a) $ab\,x^2 - (a + b)cx + (a + b)^2 = 0$
(b) $ac\,x^2 - (a + c)bx + (a + c)^2 = 0$
(c) $ac\,x^2 + (a + c)bx - (a + c)bx - (a + c)^2 = 0$
(d) none of these

22. The number of roots of the equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ is
(a) 1 (b) 2 (c) 0 (d) infinitely many

23. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is
(a) 4 (b) 3 (c) 2 (d) 1
[AIEEE 2003]

24. If the equation $\frac{a}{x-a} + \frac{b}{x-b} = 1$ has roots equal in magnitude but opposite in sign, then the value of $a + b$ is
(a) −1 (b) 0 (c) 1 (d) none of these

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25. If one root of the equation $ax^2 + bx + c = 0$ is reciprocal of the one root of the equation $a_1x^2 + b_1x + c_1 = 0$, then
(a) $(aa_1 - cc_1)^2 = (bc_1 - b_1a)(b_1c - a_1b)$
(b) $(ab_1 - a_1b)^2 = (bc_1 - b_1c)(ca_1 - c_1a)$
(c) $(bc_1 - b_1c)^2 = (ca_1 - a_1c)(ab_1 - a_1b)$
(d) none of these

26. If $\sin\alpha$ and $\cos\alpha$ are roots of the equation $px^2 + qx + r = 0$, then
(a) $p^2 - q^2 + 2pr = 0$ (b) $(p + r)^2 = q^2 - r^2$
(c) $p^2 + q^2 - 2pr = 0$ (d) $(p - r)^2 = q^2 + r^2$

27. If $x = c$ is a root of order 2 of a polynomial $f(x)$, then $x = c$ is also a root of the polynomial
(a) $f'(x)$ (b) $f''(x)$ (c) $f'''(x)$ (d) none of these

28. If $x - c$ is a factor of order m of the polynomial $f(x)$ of degree n $(1 < m < n)$, then $x = c$ is a root of the polynomial
(a) $f^m(x)$ (b) $f^{m-1}(x)$
(c) $f^n(x)$ (d) none of these

29. If a and b are two distinct real roots of the polynomial $f(x)$ such that $a < b$, then there exists a real number c lying between a and b, such that
(a) $f(c) = 0$ (b) $f'(c) = 0$
(c) $f''(c) = 0$ (d) none of these

30. If $ax^3 + bx - c$ is divisible by $x^2 + bx + c$, then 'a' is a root of the equation
(a) $cx^2 - bx - 1 = 0$ (b) $ax^2 - bx - 1 = 0$
(c) $bx^2 - ax - 1 = 0$ (d) none of these

31. If α, β are the roots of $x^2 + px + q = 0$ and also of $x^{2n} + p^n x^n + q^n = 0$ and if $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $x^n + 1 + (x + 1)^n = 0$, then n is
(a) an odd integer (b) an even integer
(c) any integer (d) none of these

32. Root(s) of the equation $9x^2 - 18|x| + 5 = 0$ belonging to the domain of definition of the function $f(x) = \log(x^2 - x - 2)$ is (are)
(a) $\frac{-5}{3}, \frac{-1}{3}$ (b) $\frac{5}{3}, \frac{1}{3}$ (c) $\frac{-5}{3}$ (d) $\frac{-1}{3}$

33. If $x = 1 + i$ is a root of the equation $x^3 - ix + 1 - i = 0$, then the other real root is
(a) 1 (b) -1
(c) 0 (d) none of these

34. Let a, b, c be real numbers, $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies
(a) $\gamma = \frac{\alpha + \beta}{2}$ (b) $\gamma = \alpha + \frac{\beta}{2}$
(c) $\gamma = \alpha/2 + \beta$ (d) $\alpha < \gamma < \beta$

35. If α, β are the roots of the equation $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always (p, q, r, s are real numbers)
(a) two real roots
(b) two negative roots
(c) two positive roots
(d) one positive and one negative roots

36. The equation $(\cos p - 1)x^2 + \cos p\, x + \sin p = 0$, in variable x, has real roots. Then, p belongs to the interval
(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

37. The number of solutions of the equation $2\sin(e^x) = 5^x + 5^{-x}$, is
(a) 0 (b) 1
(c) 2 (d) Infinitely many

38. Let $f(x)$ be a quadratic expression which is positive for all real x and $g(x) = f(x) + f'(x) + f''(x)$, then for any real x,
(a) $g(x) < 0$ (b) $g(x) > 0$
(c) $g(x) = 0$ (d) $g(x) \geq 0$

39. If c and d are roots of the equation $(x - a)(x - b) - k = 0$, then a, b are roots of the equation
(a) $(x - c)(x - d) - k = 0$ (b) $(x - c)(x - d) + k = 0$
(c) $(x - a)(x - c) + k = 0$ (d) $(x - b)(x - d) + k = 0$

40. If A is the A.M. of the roots of the equation $x^2 - 2ax + b = 0$ and G is the G.M. of the roots of the equation $x^2 - 2bx + a^2 = 0$, then
(a) $A > G$ (b) $A \neq G$
(c) $A = G$ (d) none of these

41. If α, β are roots of $x^2 + px - q = 0$ and γ, δ are roots of $x^2 + px + r = 0$, then the value of $(\alpha - \gamma)(\alpha - \delta)$ is
(a) $p + q$ (b) $q - r$ (c) $r - q$ (d) $q + r$

42. If the roots of the equation $\frac{1}{x + a} + \frac{1}{x + b} = \frac{1}{c}$ are equal in magnitude but opposite in sign, then their product is
(a) $\frac{1}{2}(a^2 + b^2)$ (b) $-\frac{1}{2}(a^2 + b^2)$
(c) $\frac{1}{2}ab$ (d) $-\frac{1}{2}ab$

43. If the ratio of the roots of the equation $x^2 + px + q = 0$ be equal to the ratio of the roots of $x^2 + lx + m = 0$, then
(a) $p^2m = q^2l$ (b) $pm^2 = q^2l$
(c) $p^2l = q^2m$ (d) $p^2m = l^2q$

44. If $x + 1$ is a factor of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$, then the value of p is
(a) -4 (b) 0 (c) 4 (d) 2

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45. If x^2-3x+2 is a factor of x^4-px^2+q, then the values of p and q are
(a) 5, −4 (b) 5, 4 (c) −5, 4 (d) −5, −4.

46. If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, then it is equal to
(a) $\frac{p-p'}{q-q'}$ (b) $\frac{p+p'}{q+q'}$ (c) $\frac{q'-q}{p-p'}$ (d) $\frac{q+q'}{p+p'}$

47. If the expressions $x^2-11x+a$ and $x^2-14x+2a$ have a common factor, then the values of 'a' are
(a) 0, 24 (b) 0, −24 (c) 1, −1 (d) −2, 1

48. If a, b, c are in G.P., then the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

49. If the equations $x^2+ax+b=0$ and $x^2+bx+a=0$ have a common root, then the numerical value of $a+b$ is
(a) 1 (b) 0 (c) −1 (d) none of these
[EAMCET 2002, JEE (Orissa) 2002]

50. If the roots of the equation $x^2+a^2=8x+6a$ are real, then a belongs to the interval
(a) [2, 8] (b) [−2, 8]
(c) [−8, 2] (d) none of these

51. If the sum of the roots of the equation $(a+1)x^2+(2a+3)x+(3a+4)=0$ is −1, then the product of the roots is
(a) 0 (b) 1 (c) 2 (d) 3

52. If one root of the equation $8x^2-6x-a-3=0$ is the square of the other, then the values of a are
(a) 4, −24 (b) 4, 24 (c) −4, −24 (d) −4, 24

53. If $b_1 b_2 = 2(c_1+c_2)$, then at least one of the equations $x^2+b_1x+c_1=0$ and $x^2+b_2x+c_2=0$ has
(a) real roots
(b) purely imaginary roots
(c) imaginary roots
(d) none of these

54. The values of x satisfying $|x^2+4x+3|+(2x+5)=0$ are
(a) $-4, -1-\sqrt{3}$ (b) $4, 1+\sqrt{3}$
(c) $-4, 1-\sqrt{3}$ (d) $-4, 1+\sqrt{3}$

55. If $a \le 0$, then the real values of x satisfying $x^2-2a|x-a|-3a^2=0$ are
(a) $a(1-\sqrt{2}), a(-1+\sqrt{6})$ (b) $a(1+\sqrt{2}), a(1-\sqrt{6})$
(c) $a(1-\sqrt{2}), a(1-\sqrt{6})$ (d) none of these

56. If $2+i\sqrt{3}$ is a root of $x^2+px+q=0$ where $p, q \in R$, then
(a) $p=-4, q=7$ (b) $p=4, q=7$
(c) $p=4, q=-7$ (d) $p=-4, q=-7$.

57. If $\tan\alpha$ and $\tan\beta$ are roots of the equation $x^2+px+q=0$ with $p \neq 0$, then
(a) $\sin^2(\alpha+\beta)+p\sin(\alpha+\beta)\cos(\alpha+\beta)+q\cos^2(\alpha+\beta)=q$
(b) $\tan(\alpha+\beta)=\frac{p}{q+1}$
(c) $\cos(\alpha+\beta)=-p$ (d) $\sin(\alpha+\beta)=1-q$

58. If α is a root of the quadratic equation $x^2+6x-2=0$, then another root β is
(a) $\alpha^2+5\alpha-8$ (b) $\frac{\alpha}{3\alpha-1}$
(c) $\frac{2\alpha^2+12\alpha-6}{\alpha}$ (d) all of these

59. If the sum of the roots of the equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then
(a) c^2b, a^2c, b^2a are in A.P. (b) c^2b, a^2c, b^2a are in G.P.
(c) $\frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in G.P. (d) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in G.P.

60. For real values of x, the expression $\frac{(x-b)(x-c)}{(x-a)}$ will assume all real values provided
(a) $a \le c \le b$ (b) $b \ge a \ge c$
(c) $b \le c \le a$ (d) $a \ge b \ge c$

61. If $a(p+q)^2+2bpq+c=0$ and $a(p+r)^2+2bpr+c=0$, then $qr=$
(a) $p^2+\frac{c}{a}$ (b) $p^2+\frac{a}{c}$ (c) $p^2+\frac{a}{b}$ (d) $p^2+\frac{b}{a}$

62. The roots of $ax^2+2bx+c=0$ and $bx^2-2\sqrt{ac}\,x+b=0$ are simultaneously real, then
(a) $a=b, c=0$ (b) $ac=b^2$
(c) $4b^2=ac$ (d) none of these

63. If a, b, c are real and $x^3-3b^2x+2c^3$ is divisible by $x-a$ and $x-b$, then
(a) $a=-b=-c$ (b) $a=2b=2c$
(c) $a=b=c$ or $a=-2b=-2c$ (d) none of these

64. If the equation $x^2+px+q=0$ has roots u and v where p, q are non-zero constants. Then,
(a) $qx^2+px+1=0$ has roots $\frac{1}{u}$ and $\frac{1}{v}$
(b) $(x-p)(x+q)=0$ has roots $u+v$ and uv
(c) $x^2+p^2x+q^2=0$ has roots u^2 and v^2
(d) $x^2+qx+p=0$ has roots $\frac{u}{v}$ and $\frac{v}{u}$

65. For the equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2(x)-\frac{5}{4}}=\sqrt{2}$ which one of the following is not true?
(a) has at least one real solution
(b) has exactly three real solutions
(c) has exactly one irrational solution
(d) all of these

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66. If $\cos\alpha$ is a root of $25x^2+5x-12=0$, $-1<x<0$. Then, the value of $\sin 2\alpha$ is
(a) $\frac{12}{25}$ (b) $\frac{-12}{25}$ (c) $\frac{-24}{25}$ (d) $\frac{20}{25}$

67. If $a, b, c \in R$ and $a+b+c=0$, then the quadratic equation $4ax^2+3bx+2c=0$ has
(a) one positive and one negative root
(b) imaginary roots
(c) real roots
(d) none of these

68. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has
(a) both roots in $[a, b]$
(b) both roots in $(-\infty, a)$
(c) roots in $(-\infty, a)$ and other in (b, ∞)
(d) both roots in (b, ∞)

69. The roots of the quadratic equation $(a+b-2c)x^2-(2a-b-c)x+(a-2b+c)=0$ are
(a) $a+b+c$ and $a-b+c$ (b) $\frac{1}{2}$ and $a-2b+c$
(c) $a-2b+c$ and $\frac{1}{a+b-c}$ (d) none of these

70. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2+b|x|+c=0$ is
(a) 2 (b) 4 (c) 0 (d) none of these

71. Real roots of the equation $x^2+5|x|+4=0$ are
(a) $-1, -4$ (b) $1, 4$
(c) $-4, 4$ (d) none of these

72. If a and b are the non-zero distinct roots of $x^2+ax+b=0$, then the least value of x^2+ax+b is
(a) $\frac{2}{3}$ (b) $\frac{9}{4}$ (c) $-\frac{9}{4}$ (d) 1

73. If x is real then $\frac{x^2-2x+4}{x^2+2x+4}$ takes values in the interval
(a) $[1/3, 3]$ (b) $(1/3, 3)$
(c) $(3, 3)$ (d) $(-1/3, 3)$

74. If $a<c<b$, then the roots of the equation $(a-b)^2x^2+2(a+b-2c)x+1=0$ are
(a) imaginary (b) real
(c) one real and one imaginary (d) equal and imaginary

75. If α, β are the roots of $x^2+bx-c=0$, then the equation whose roots are b and c is
(a) $x^2+\alpha x-\beta=0$
(b) $x^2-x(\alpha+\beta+\alpha\beta)-\alpha\beta(\alpha+\beta)=0$
(c) $x^2+(\alpha+\beta-\alpha\beta)x-\alpha\beta(\alpha+\beta)=0$
(d) $x^2+x(\alpha+\beta+\alpha\beta)+\alpha\beta(\alpha+\beta)=0$.

76. If α, β are roots of $ax^2+bx+c=0$, then the equation $ax^2-bx(x-1)+c(x-1)^2=0$ has roots
(a) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ (b) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$
(c) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ (d) $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$

77. If $\alpha\neq\beta$ and $\alpha^2=5\alpha-3$, $\beta^2=5\beta-3$, then the equation having α/β and β/α as its roots is
(a) $3x^2+19x+3=0$ (b) $3x^2-19x+3=0$
(c) $3x^2-19x-3=0$ (d) $x^2-16x+1=0$
[AIEEE 2002]

78. The expression $y=ax^2+bx+c$ has always the same sign as, c if
(a) $4ac<b^2$ (b) $4ac>b^2$ (c) $ac<b^2$ (d) $ac>b^2$

79. If α, β and γ are the roots of $x^3+8=0$, then the equation whose roots are α^2, β^2 and γ^2 is
(a) $x^3-8=0$ (b) $x^3-16=0$
(c) $x^3+64=0$ (d) $x^3-64=0$

80. Given that $ax^2+bx+c=0$ has no real roots and $a+b+c<0$, then
(a) $c=0$ (b) $c>0$ (c) $c<0$ (d) $c=0$

81. If $x \in R$, then the expression 9^x-3^x+1 assumes
(a) all real values
(b) all real values greater than 0
(c) all real values greater than 3/4
(d) all real values greater than 1/4

82. The values of 'a' for which the roots of the equation $x^2+x+a=0$ are real and exceed 'a' are
(a) $0<a<1/4$ (b) $a<1/4$
(c) $a<-2$ (d) $-2<a<0$

83. Let α, β be the roots of $x^2+bx+1=0$. Then, the equation whose roots are $-\left(\alpha+\frac{1}{\beta}\right)$ and $-\left(\beta+\frac{1}{\alpha}\right)$, is
(a) $x^2=0$ (b) $x^2+2bx+4=0$
(c) $x^2-2bx+4=0$ (d) $x^2-bx+1=0$

84. The roots α, β and γ of an equation $x^3-3ax^2+3bx-c=0$ are in H.P. Then,
(a) $\beta=\frac{1}{a}$ (b) $\beta=b$ (c) $\beta=\frac{b}{c}$ (d) $\beta=\frac{c}{b}$

85. If b and c are odd integers, then the equation $x^2+bx+c=0$ has
(a) two odd roots
(b) two integer roots, one odd and one even
(c) no integer roots
(d) none of these

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86. If the equations $ax^2+bx+c=0$ and $x^3+3x^2+3x+2=0$ have two common roots, then
(a) $a=b\neq c$ (b) $a=-b=c$
(c) $a=b=c$ (d) none of these

87. If both the roots of the equation $ax^2+bx+c=0$ are zero, then
(a) $b=c=0$ (b) $b=0,\ c\neq 0$
(c) $b\neq 0,\ c=0$ (d) $b\neq 0,\ c\neq 0$

88. Let $\alpha, \beta, \gamma, \delta$ be the roots of $x^4+x^2+1=0$. Then, the equation whose roots are $\alpha^2, \beta^2, \gamma^2, \delta^2$ is
(a) $(x^2-x+1)^2=0$ (b) $(x^2+x+1)^2=0$
(c) $x^4-x^2+1=0$ (d) $x^2+x+1=0$

89. The number of real roots of $\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$ is
(a) 0 (b) 2 (c) 4 (d) 6

90. The roots of the equation $(3-x)^4+(2-x)^4=(5-2x)^4$ are
(a) all real (b) all imaginary
(c) two real and two imaginary (d) none of these

91. The real roots of $|x|^3-3x^2+3|x|-2=0$ are
(a) 0, 2 (b) ± 1 (c) ± 2 (d) 1, 2

92. The number of positive integral roots of $x^4+x^3-4x^2+x+1=0$ is
(a) 0 (b) 1 (c) 2 (d) 4

93. If x, y, z are real and distinct, then $x^2+4y^2+9z^2-6yz-3zx-2xy$ is always
(a) non-negative (b) non-positive
(c) zero (d) none of these

94. The value of 'a' for which the equations $x^3+ax+1=0$ and $x^4+ax^2+1=0$ have a common root is
(a) 2 (b) -2 (c) 0 (d) none of these

95. If $\frac{x^2-bx}{ax-c}=\frac{\lambda-1}{\lambda+1}$ has roots equal in magnitude and opposite in sign then the value of λ is
(a) $\frac{a-b}{a+b}$ (b) $\frac{a+b}{a-b}$ (c) c (d) $\frac{1}{c}$

96. The values of 'a' for which $(a^2-1)x^2+2(a-1)x+2$ is positive for any x are
(a) $a\geq 1$ (b) $a\leq 1$ (c) $a>-3$ (d) $a\leq -3$ or $a\geq 1$

97. If α, β are the roots of the equation $x^2+\sqrt{\alpha}x+\beta=0$, then the values of α and β are
(a) $\alpha=1,\ \beta=-1$ (b) $\alpha=1,\ \beta=-2$
(c) $\alpha=2,\ \beta=1$ (d) $\alpha=2,\ \beta=-2$

98. If a, b, c are in A.P. and if $(b-c)x^2+(c-a)x+a-b=0$ and $2(c+a)x^2+(b+c)x=0$ have a common root then
(a) a^2, b^2, c^2 are in A.P. (b) a^2, c^2, b^2 are in A.P.
(c) a^2, c^2, b^2 are in G.P. (d) none of these

99. If the expression $mx-1+\frac{1}{x}$ is always non-negative, then the minimum value of m must be
(a) $-\frac{1}{2}$ (b) 0 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

100. The set of values of p for which the roots of the equation $3x^2+2x+p(p-1)=0$ are of opposite signs is
(a) $(-\infty, 0)$ (b) $(0, 1)$ (c) $(1, \infty)$ (d) $(0, \infty)$

101. Let α and β be the roots of the equation $x^2+x+1=0$. The equation whose roots are α^{19}, β^7 is
(a) $x^2-x-1=0$ (b) $x^2-x+1=0$
(c) $x^2+x-1=0$ (d) $x^2+x+1=0$

102. If p and q are the roots of the equation $x^2+px+q=0$, then
(a) $p=1$ (b) $p=1$ or 0
(c) $p=-2$ (d) $p=-2$ or 0 [AIEEE 2002]

103. If p, q, r are positive and are in A.P., the roots of the quadratic equation $px^2+qx+r=0$ are real for
(a) $\left|\frac{r}{p}-7\right|\geq 4\sqrt{3}$ (b) $\left|\frac{p}{r}-7\right|<4\sqrt{3}$
(c) all p and r (d) no p and r

104. If two equations $a_1x^2+b_1x+c_1=0$ and, $a_2x^2+b_2x+c_2=0$ have a common root, then the value of $(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)$, is
(a) $-(a_1c_2-a_2c_1)^2$ (b) $(a_1a_2-c_1c_2)^2$
(c) $(a_1c_1-a_2c_2)^2$ (d) $(a_1c_2-c_1a_2)^2$

105. The value of p for which the difference between the roots of the equation $x^2+px+8=0$ is 2 are
(a) ± 2 (b) ± 4 (c) ± 6 (d) ± 8

106. If $f(x)=2x^3+mx^2-13x+n$ and 2, 3 are roots of the equation $f(x)=0$, then the values of m and n are
(a) $-5, -30$ (b) $-5, 30$ (c) 5, 30 (d) none of these

107. If the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are equal, then a, b, c are in
(a) H.P. (b) G.P. (c) A.P. (d) none of these

108. If $7^{\log_7(x^2-4x+5)}=x-1$, x may have values
(a) 2, 3 (b) 7 (c) $-2, -3$ (d) 2, -3

109. If α, β are the roots of $ax^2+bx+c=0$ the equation whose roots are $2+\alpha, 2+\beta$ is
(a) $ax^2+x(4a-b)+4a-2b+c=0$
(b) $ax^2+x(4a-b)+4a+2b+c=0$
(c) $ax^2+x(b-4a)+4a+2b+c=0$
(d) $ax^2+x(b-4a)+4a-2b+c=0$

110. For the equation $|x|^2+|x|-6=0$
(a) there is only one root
(b) there are only two distinct roots
(c) there are only three distinct roots
(d) there are four distinct roots

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111. Two students while solving a quadratic equation in x, one copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The correct roots are
(a) $3, -2$ (b) $-3, 2$ (c) $-6, -1$ (d) $6, -1$

112. If 8, 2 are the roots of $x^2 + ax + \beta = 0$, and 3, 3 are the roots of $x^2 + \alpha x + b = 0$, then the roots of $x^2 + ax + b = 0$ are
(a) $8, -1$ (b) $-9, 2$ (c) $-8, -2$ (d) $9, 1$

113. If one root of $x^2 - x - k = 0$ is square of the other, then $k =$
(a) $2 \pm \sqrt{3}$ (b) $3 \pm \sqrt{2}$ (c) $2 \pm \sqrt{5}$ (d) $5 \pm \sqrt{2}$

114. If a, b are odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0, a \neq 0$ are
(a) rational (b) irrational
(c) non-real (d) none of these

115. The value of p for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2, lies in
(a) $(4/5, 2)$ (b) $(-1, -4/5)$
(c) $(2, \infty)$ (d) $(-\infty, -1)$

116. The value of 'c' for which $|\alpha^2 - \beta^2| = 7/4$, where α and β are the roots of $2x^2 + 7x + c = 0$, is
(a) 4 (b) 0 (c) 6 (d) 2

117. The value of 'k' for which one of the roots of $x^2 - x + 3k = 0$, is double of one of the roots of $x^2 - x + k = 0$ is
(a) 1 (b) -2 (c) 2 (d) none of these

118. The equations $ax^2 + bx + a = 0$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots in common. Then, $a + b$ must be equal to
(a) 1 (b) -1 (c) 0 (d) none of these

119. The graph of the function $y = 16x^2 + 8(a + 5)x - 7a - 5$ is strictly above the x-axis, then 'a' must satisfy the inequality
(a) $-15 < a < -2$ (b) $-2 < a < -1$
(c) $5 < a < 7$ (d) none of these

120. The number of real solutions of the equation
$(5 + 2\sqrt{6})^{x^2 - 3} + (5 - 2\sqrt{6})^{x^2 - 3} = 10$, is
(a) 2 (b) 4 (c) 6 (d) none of these

121. The number of real roots of the equation
$2x^4 + 5x^2 + 3 = 0$, is
(a) 4 (b) 1 (c) 0 (d) 3

122. If x, a, b, c are real and $(x - a + b)^2 + (x - b + c)^2 = 0$, then a, b, c are in
(a) H.P. (b) G.P. (c) A.P. (d) none of these

123. If the roots of the equation
$(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$
are equal then a, b, c are in
(a) G.P. (b) A.P. (c) H.P. (d) none of these

124. If a, b, c are all positive and in H.P., then the roots of $ax^2 + 2bx + c = 0$ are
(a) real (b) imaginary
(c) rational (d) equal

125. If the equation $ax^2 + 2bx - 3c = 0$ has no real roots and $\frac{3c}{4} < a + b$, then
(a) $c < 0$ (b) $c > 0$ (c) $c \geq 0$ (d) $c = 0$

126. If the roots of the equation $x^2 + 2ax + b = 0$, are real and distinct and they differ by at most $2m$, then b lies in the interval
(a) $(a^2 - m^2, a^2)$ (b) $[a^2 - m^2, a^2)$
(c) $(a^2, a^2 + m^2)$ (d) none of these

127. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of the determinant
$$\begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos\alpha \\ \cos(\alpha - \beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}, \text{ is}$$
(a) $\sin(\alpha + \beta)$ (b) $\sin\alpha \sin\beta$
(c) $1 + \cos(\alpha + \beta)$ (d) none of these

128. Let α, β be the roots of the equation $ax^2 + bx + c = 0$, and let $\alpha^n + \beta^n = S_n$ for $n \geq 1$. Then, the value of the deteminant
$$\begin{vmatrix} 3 & 1 + S_1 & 1 + S_2 \\ 1 + S_1 & 1 + S_2 & 1 + S_3 \\ 1 + S_2 & 1 + S_3 & 1 + S_4 \end{vmatrix} \text{ is}$$
(a) $\frac{b^2 - 4ac}{a^4}$ (b) $\frac{(a + b + c)(b^2 + 4ac)}{a^4}$
(c) $\frac{(a + b + c)(b^2 - 4ac)}{a^4}$ (d) $\frac{(a + b + c)^2 (b^2 - 4ac)}{a^4}$

129. Let $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$, $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$. Then, the equation whose roots are α, β is
(a) $x^2 - x + 2 = 0$ (b) $x^2 + x - 2 = 0$
(c) $x^2 - x - 2 = 0$ (d) $x^2 + x + 2 = 0$

130. If $m \in Z$ and the equation $mx^2 + (2m - 1)x + (m - 2) = 0$ has rational roots, then m is of the form
(a) $n(n + 2), n \in Z$ (b) $n(n + 1), n \in Z$
(c) $n(n - 2), n \in Z$ (d) none of these

131. If $(1 + k)\tan^2 x - 4\tan x - 1 + k = 0$ has real roots $\tan x_1$ and $\tan x_2$, then
(a) $k^2 \leq 5$ (b) $k^2 \geq 6$
(c) $k = 3$ (d) none of these

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132. If the sum of the squares of the roots of the equation $x^2-(\sin\alpha-2)x-(1+\sin\alpha)=0$ is least, then $\alpha=$
(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/2$ (d) $\pi/6$

133. p, q, r and s are integers. If the A.M. of the roots of $x^2-px+q^2=0$ and GM of the roots of $x^2-rx+s^2=0$ are equal, then
(a) q is an odd integer (b) r is an even integer
(c) p is an even integer (d) s is an odd integer

134. If α, β, γ be the roots of $x^3+a^3=0\ (a\in R)$, then the number of equation(s) whose roots are $\left(\frac{\alpha}{\beta}\right)^2$ and $\left(\frac{\alpha}{\gamma}\right)^2$, is
(a) 1 (b) 2 (c) 3 (d) 6

135. If α, β are the roots of $ax^2+c=bx$, then the equation $(a+cy)^2=b^2y$ in y has the roots
(a) α^{-1}, β^{-1} (b) α^2, β^2
(c) $\alpha\beta^{-1}, \alpha^{-1}\beta$ (d) $\sqrt{\alpha}, \sqrt{\beta}$

136. If the equation $2x^2-7x+1=0$ and $ax^2+bx+2=0$ have a common root, then
(a) $a=2, b=-7$ (b) $a=-\frac{7}{2}, b=1$
(c) $a=4, b=-14$ (d) none of these

137. The common roots of the equations $x^3+2x^2+2x+1=0$ and $1+x^{2002}+x^{2003}=0$ are (where ω is a complex cube root of unity)
(a) ω, ω^2 (b) $1, \omega^2$
(c) $-1, -\omega$ (d) $\omega, -\omega^2$

138. If $f(x)=\sum_{k=2}^{n}\left(x-\frac{1}{k-1}\right)\left(x-\frac{1}{k}\right)$, then the product of roots of $f(x)=0$ as $n\to\infty$, is
(a) -1 (b) 0
(c) 1 (d) none of these

Answers

1. (a) 2. (b) 3. (d) 4. (d) 5. (c) 6. (a) 7. (d)
8. (c) 9. (a) 10. (c) 11. (c) 12. (b) 13. (d) 14. (b)
15. (a) 16. (b) 17. (b) 18. (b) 19. (a) 20. (b) 21. (d)
22. (c) 23. (a) 24. (b) 25. (a) 26. (a) 27. (a) 28. (b)
29. (b) 30. (a) 31. (b) 32. (c) 33. (b) 34. (d) 35. (a)
36. (d) 37. (a) 38. (b) 39. (b) 40. (c) 41. (d) 42. (b)
43. (d) 44. (c) 45. (b) 46. (c) 47. (a) 48. (a) 49. (c)
50. (b) 51. (c) 52. (d) 53. (a) 54. (a) 55. (a) 56. (a)
57. (a) 58. (e) 59. (a) 60. (b) 61. (a) 62. (b) 63. (c)
64. (a) 65. (d) 66. (c) 67. (c) 68. (d) 69. (d) 70. (c)
71. (d) 72. (c) 73. (a) 74. (a) 75. (d) 76. (c) 77. (b)
78. (b) 79. (d) 80. (c) 81. (c) 82. (c) 83. (c) 84. (d)
85. (c) 86. (c) 87. (a) 88. (b) 89. (a) 90. (c) 91. (c)
92. (c) 93. (a) 94. (b) 95. (a) 96. (d) 97. (b) 98. (b)
99. (c) 100. (b) 101. (d) 102. (b) 103. (a) 104. (d) 105. (c)
106. (b) 107. (a) 108. (a) 109. (d) 110. (b) 111. (d) 112. (d)
113. (c) 114. (a) 115. (d) 116. (c) 117. (b) 118. (c) 119. (a)
120. (b) 121. (c) 122. (c) 123. (a) 124. (b) 125. (a) 126. (b)
127. (d) 128. (d) 129. (d) 130. (b) 131. (a) 132. (c) 133. (c)
134. (a) 135. (b) 136. (c) 137. (a) 138. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The set of values of 'a' for which
$x^2+ax+\sin^{-1}(x^2-4x+5)+\cos^{-1}(x^2-4x+5)=0$
has at least one real root is given by
(a) $(-\infty, -\sqrt{2\pi}]\cup[\sqrt{2\pi}, \infty)$ (b) $(-\infty, -\sqrt{2\pi})\cup(\sqrt{2\pi}, \infty)$
(c) R (d) none of these

2. The set of possible values of a for which
$x^2-(a^2-5a+5)x+(2a^2-3a-4)=0$
has roots whose sum and product are both less than 1 is
(a) $(-1, 5/2)$ (b) $(1, 4)$ (c) $[1, 5/2]$ (d) $(1, 5/2)$

3. The equation $(a+2)x^2+(a-3)x=2a-1, a\neq -2$ has rational roots for
(a) all rational values of a except $a=-2$
(b) all real values of a except $a=-2$
(c) rational values of $a>1/2$
(d) none of these

4. If $\sin\theta, \sin\alpha\cos\theta$ are in G.P., then the roots of $x^2+2x\cot\alpha+1=0$ are always
(a) equal (b) real
(c) imaginary (d) greater than 1

5. If α, β are the roots of $x^2-3x+a=0, a\in R$ and $\alpha<1<\beta$, then
(a) $a\in(-\infty, 2)$ (b) $a\in(-\infty, 9/4]$
(c) $a\in(2, 9/4]$ (d) none of these

6. If the equations $ax^2+bx+c=0$ and $cx^2+bx+a=0, a\neq c$ have a negative common root, then $a-b+c=$
(a) 0 (b) 2 (c) 1 (d) none of these

7. If the roots of $x^3-12x^2+39x-28=0$ are in A.P., then their common difference is
(a) ±1 (b) ±2 (c) ±3 (d) ±4

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8. If the roots of $a_1x^2+b_1x+c_1=0$ are α_1, β_1 and those of $a_2x^2+b_2x+c_2=0$ are $\alpha_2\ \beta_2$ such that $\alpha_1\alpha_2=\beta_1\beta_2=1$, then
(a) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (b) $\frac{a_1}{c_2}=\frac{b_1}{b_2}=\frac{c_1}{a_2}$
(c) $a_1a_2=b_1b_2=c_1c_2$ (d) none of these

9. If the roots of the equation $ax^2-4x+a^2=0$ are imaginary and the sum of the roots is equal to their product, then $a=$
(a) -2 (b) 4 (c) 2 (d) none of these

10. If a, b, c are real numbers in G.P. such that a and c are positive, then the roots of the equation $ax^2+bx+c=0$
(a) are real and are in ratio $b:ac$
(b) are real
(c) are imaginary and are in ratio $1:\omega$, where ω is a complex cube root of unity
(d) are imaginary and are in ratio $-1:\omega$

11. If the absolute value of the difference of the roots of the equation $x^2+ax+1=0$ exceeds $\sqrt{3a}$, then
(a) $a\in(-\infty,-1)\cup(4,\infty)$ (b) $a\in(4,\infty)$
(c) $a\in(-1,4)$ (d) $a\in[0,4)$

12. If α, β are roots of the equation $375x^2-25x-2=0$ and $S_n=\alpha^n+\beta^n$, then $\lim_{n\to\infty}\sum_{r=1}^{n}S_r$ is equal to
(a) 7/116 (b) 1/12 (c) 29/348 (d) none of these

13. The quadratic equations $x^2+(a^2-2)x-2a^2=0$ and $x^2-3x+2=0$ have
(a) no common root for all $a\in R$
(b) exactly one common root for all $a\in R$
(c) two common roots for some $a\in R$
(d) none of these

14. If roots of $ax^2+bx+c=0, a, b, c\in R, a\neq 0$ are imaginary; then
(a) $ac>0$ (b) $ab>0$ (c) $bc>0$
(d) exactly two of ab, bc and ca are positive

15. The value of m for which the equation $x^3-mx^2+3x-2=0$ has two roots equal in magnitude but opposite in sign, is
(a) 4/5 (b) 3/4 (c) 2/3 (d) 1/2

16. The equation formed by decreasing each root of $ax^2+bx+c=0$ by 1 is $2x^2+8x+2=0$, then
(a) $a=-b$ (b) $b=-c$ (c) $c=-a$ (d) $b=a+c$

17. If the roots of $ax^2-bx-c=0$ change by the same quantity, then the expression in a, b, c that does not change is
(a) $\frac{b^2-4ac}{a^2}$ (b) $\frac{b-4c}{a}$ (c) $\frac{b^2+4ac}{a^2}$ (d) $\frac{b^2-4ac}{a}$

18. If $x^2-2r\,a_r\,x+r=0\,;\ r=1,2,3$ are three quadratic equations of which each pair has exactly one root common, then the number of solutions of the triplet (a_1, a_2, a_3) is
(a) 1 (b) 2 (c) 9 (d) 27

19. If x^2+px+1 is a factor of the cubic polynomial ax^3+bx+c, then
(a) $a^2+c^2=-ab$ (b) $a^2-c^2=-ab$
(c) $a^2-c^2=ab$ (d) none of these

20. If $(x-1)^3$ is a factor of $x^4+ax^3+bx^2+cx-1$, then the other factor is
(a) $x-3$ (b) $x+1$ (c) $x+2$ (d) $x-1$

21. If α is a root of the equation $2x(2x+1)=1$, then the other root is
(a) $3\alpha^3-4\alpha$ (b) $-2\alpha(\alpha+1)$
(c) $4\alpha^3-3\alpha$ (d) none of these

22. If one root of the equation $(a-b)x^2+ax+1=0$ be double the other and if $a\in R$, then the greatest value of b is
(a) 9/8 (b) 7/8 (c) 8/9 (d) 8/7

23. If the equations $ax^2+bx+c=0$ and $2x^2+3x+4=0$ have a common root, then $a:b:c$
(a) 2:3:4 (b) 1:2:3
(c) 4:3:2 (d) none of these

24. If the equation $x^3+ax^2+b=0, b\neq 0$ has a root of order 2, then
(a) $a^2+2b=0$ (b) $a^2-2b=0$
(c) $4a^3+27b+1=0$ (d) $4a^3+27b=0$

25. If the roots of the equation $x^2-bx+c=0$ are two consecutive integers, then b^2-4c is equal to
(a) 1 (b) 0 (c) 2 (d) none of these

26. If $a, b, c\in R$ and the equations $ax^2+bx+c=0$ and $x^3+3x^2+3x+2=0$ have two roots in common, then
(a) $a=b\neq c$ (b) $a=b=-c$
(c) $a=b=c$ (d) none of these

27. Let S denote the set of all real values of a for which the roots of the equation $x^2-2ax+a^2-1=0$ lie between 5 and 10, then S equals
(a) $(-1,2)$ (b) $(2,9)$ (c) $(4,9)$ (d) $(6,9)$

28. The sum of the real roots of the equation $|x-2|^2+|x-2|-2=0$, is
(a) 4 (b) 3 (c) 2 (d) 10

29. The product of the real roots of the equation $\left|2x+3\right|^2-3\left|2x+3\right|+2=0$, is
(a) 5/4 (b) 5/2 (c) 5 (d) 2

30. If $a+b+c=0$ and $a\neq c$ then the roots of the equation $(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$, are
(a) real and unequal (b) real and equal
(c) imaginary (d) none of these

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31. If $\sec\alpha$ and $\tan\alpha$ are the roots of $ax^2+bx+c=0$, then
(a) $a^2-b^2+2ac=0$ (b) $a^3+b^3+c^3-2abc=0$
(c) $a^4+4ab^2c=b^4$ (d) none of these

32. If the roots of the equation $x^3+bx^2+3x-1=0$ form a non-decreasing H.P., then
(a) $b\in(-3,\infty)$ (b) $b=-3$
(c) $b\in(-\infty,-3)$ (d) none of these

33. Let $[x]$ denote the greatest integer less than or equal to x. Then, in $[0,3]$ the number of solutions of the equation $x^2-3x+[x]=0$, is
(a) 6 (b) 4 (c) 2 (d) 0

34. The number of non-zero solutions of the equation
$x^2-5x-(Sgn\,(x))\,6=0$, is
(a) 1 (b) 2 (c) 3 (d) 4

35. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as other, is
(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

36. If α,β,γ are the roots of the equation $x^3+ax^2+bx+c=0$, then $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=$
(a) $\frac{a}{c}$ (b) $-\frac{b}{c}$ (c) $\frac{b}{a}$ (d) $\frac{c}{a}$

37. If α,β and γ are the roots of $x^3+qx+r=0$, then $\sum\frac{\alpha}{\beta+\gamma}$, is
(a) 3 (b) $q+r$ (c) q/r (d) -3

38. If α and β are the roots of the equation $ax^2+bx+c=0$, then $(1+\alpha+\alpha^2)(1+\beta+\beta^2)$, is
(a) 0 (b) positive
(c) negative (d) none of these

39. If α and β are the roots of $x^2+px+1=0$ and γ and δ are the roots of $x^2+qx+1=0$, then the value of $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$, is
(a) p^2-q^2 (b) q^2-p^2 (c) p^2 (d) q^2
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40. The maximum number of real roots of the equation $x^{2n}-1=0$, is
(a) 2 (b) 3 (c) n (d) $2n$

41. The value of k for which the equation $(k-2)x^2+8x+k+4=0$ has both roots real, distinct and negative, is
(a) 0 (b) 2 (c) 3 (d) -4

42. If $x^{2/3}-7x^{1/3}+10=0$, then the set of values of x, is
(a) $\{12,5\}$ (b) $\{8\}$ (c) ϕ (d) $\{8,125\}$

43. If $x^2+2ax+10-3a>0$ for all $x\in R$, then
(a) $-5<a<2$ (b) $a<-5$
(c) $a>5$ (d) $2<a<5$

44. If the difference between the roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a\neq b$, then
(a) $a+b+4=0$ (b) $a+b-4=0$
(c) $a-b-4=0$ (d) $a-b+4=0$

45. Product of the real roots of the equation $t^2x^2+|x|+9=0,(t\neq0)$
(a) is always positive (b) is always negative
(c) does not exist (d) none of these

46. If the sum of the squares of the roots of the equation $x^2-(a-2)x-(a+1)=0$ is least, then the value of a is
(a) 0 (b) 2 (c) -1 (d) 1

47. If $x^2+ax+10=0$ and $x^2+bx-10=0$ have a common root, then a^2-b^2 is equal to
(a) 10 (b) 20 (c) 30 (d) 40

48. If $x^2+px+q=0$ is the quadratic equation whose roots are $a-2$ and $b-2$ where a,b are the roots of $x^2-3x+1=0$, then
(a) $p=1,q=5$ (b) $p=1,q=-5$
(c) $p=-1,q=1$ (d) $p=1,q=-1$

49. If α and β are the roots of the equation $x^2-ax+b=0$ and $A_n=\alpha^n+\beta^n$, then which one of the following is true?
(a) $A_{n+1}=aA_n+bA_{n-1}$ (b) $A_{n+1}=bA_n+aA_{n-1}$
(c) $A_{n+1}=aA_n-bA_{n-1}$ (d) $A_{n+1}=bA_n-aA_{n-1}$

50. Let a,b,c be real. If $ax^2+bx+c=0$ has two real roots α and β where $\alpha<-2$ and $\beta>2$, then
(a) $4-\frac{2b}{a}+\frac{c}{a}<0$ (b) $4+\frac{2b}{a}-\frac{c}{a}<0$
(c) $4-\frac{2b}{a}+\frac{c}{a}=0$ (d) $4+\frac{2b}{a}+\frac{c}{a}=0$

Answers

1. (a) 2. (d) 3. (a) 4. (b) 5. (a) 6. (a) 7. (c)
8. (b) 9. (c) 10. (c) 11. (a) 12. (c) 13. (b) 14. (a)
15. (c) 16. (b) 17. (c) 18. (b) 19. (c) 20. (b) 21. (c)
22. (a) 23. (a) 24. (d) 25. (a) 26. (c) 27. (d) 28. (a)
29. (b) 30. (a) 31. (c) 32. (b) 33. (c) 34. (a) 35. (b)
36. (b) 37. (d) 38. (b) 39. (b) 40. (a) 41. (c) 42. (d)
43. (a) 44. (a) 45. (d) 46. (d) 47. (d) 48. (d) 49. (c)
50. (a)

Solutions of Exercises and Chapter-tests are available in a separate book on "Solutions of Objective Mathematics".

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12

MATHEMATICAL INDUCTION

1. PRINCIPLES OF MATHEMATICAL INDUCTION

STATEMENTS *A sentence or description which can be judged to be true or false is called a statement.*

Following are some examples of statements:

(i) 2 divides 6,
(ii) Jaipur is the capital of Rajasthan
(iii) There are 5 days in a week
(iv) $(x+1)$ is a factor of x^2-3x+2
(v) $A \cup B = B \cup A$.

Clearly, statements (i), (ii) and (v) are true statements whereas (iii) and (iv) are false statements.

MATHEMATICAL STATEMENTS *Statements involving mathematical relations are known as the mathematical statements.*

Clearly, statements (i), (iv) and (v) are mathematical statements.

If $P(n)$ is the statement "$n(n+1)(n+2)$ is divisible by 12", then statements $P(3)$ and $P(4)$ are true, but that $P(5)$ is not true.

FIRST PRINCIPLE OF MATHEMATICAL INDUCTION

Let $P(n)$ be a statement involving the natural number n such that
(I) $P(1)$ is true i.e. $P(n)$ is true for $n=1$.
and, (II) $P(m+1)$ is true, whenever $P(m)$ is true
i.e. $P(m)$ is true $\Rightarrow P(m+1)$ is true.
Then, $P(n)$ is true for all natural numbers n.

SECOND PRINCIPLE OF MATHEMATICAL INDUCTION

Let $P(n)$ be a statement involving the natural number n such that
(I) $P(1)$ is true i.e. $P(n)$ is true for $n=1$.
and, (II) $P(m+1)$ is true, whenever $P(n)$ is true for all n, where $1 \le n \le m$.
Then, $P(n)$ is true for all natural numbers.

2. SOME USEFUL RESULTS

Following results will be very helpful in answering objective type problems on mathematical induction.

(i) $1+2+\ldots+n = \Sigma n = \dfrac{n(n+1)}{2}$

(ii) $1^2+2^2+\ldots+n^2 = \Sigma n^2 = \dfrac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^3+\ldots+n^3 = \Sigma n^3 = \left\{\dfrac{n(n+1)}{2}\right\}^2$

(iv) $1^4+2^4+\ldots+n^4 = \Sigma n^4 = \dfrac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

(v) The sum of first n even natural numbers $= \Sigma 2n = n(n+1)$

(vi) The sum of first n odd natural numbers $= \Sigma(2n-1) = n^2$

(vii) $\Sigma n(n+1) = \dfrac{n(n+1)(n+2)}{3}$

(viii) $\Sigma n(n+1)(n+2) = \dfrac{n(n+1)(n+2)(n+3)}{4}$

(ix) $x^n - y^n = (x-y)(x^{n-1}+x^{n-2}y+x^{n-3}y^2+\ldots+xy^{n-2}+y^{n-1})$
for all $n \in N$

(x) $x^n + y^n = (x+y)(x^{n-1}-x^{n-2}y+x^{n-3}y^2+\ldots-xy^{n-2}+y^{n-1})$
when n is an odd natural number.

SECTION - I

SOLVED MCQs

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

EXAMPLE 1 *The sum of the terms in the n^{th} bracket of the series* $(1)+(2+3+4)+(5+6+7+8+9)+\ldots$, *is*

(a) $(n-1)^3+n^3$ (b) $(n+1)^3+8n^2$

(c) $\dfrac{(n+1)(n+2)}{6n}$ (d) $(n+1)^3+n^3$

Ans. (a)

SOLUTION For $n=1$, we have

Sum of the terms in first bracket $=1$

and, $(n-1)^3+n^3 = (1-1)^3+1^3 = 1$

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(b) $S(k) \Rightarrow S(k+1)$

(c) $S(k) \not\Rightarrow S(k+1)$

(d) $S(1)$ *is correct*

[AIEEE 2004]

Ans. (b)

SOLUTION Let $S(k)$ be true. Then,

$$1+3+5+\ldots+(2k-1) = 3+k^2 \quad \ldots(i)$$

Now,

$$S(k+1) = 1+3+5+\ldots+(2k-1)+(2k+1)$$

$$= (3+k^2)+2k+1 = 3+(k+1)^2 \quad \text{[Uing (i)]}$$

$\Rightarrow$ $S(k+1)$ is true.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

[AIEEE 2011]

SOLUTION It can be checked by using PMI that statement-2 is true for all $n \in N$.

So, statement-2 is correct.

Now,

$$(n+1)^7 - n^7 - 1 = \{(n+1)^7 - (n+1)\} - \{n^7 - n\}$$

Using statement-2, $\{(n+1)^7 - (n+1)\}$ and $\{n^7 - n\}$ both are divisible by 7. Therefore, $(n+1)^7 - n^7 - 1$ is also divisible by 7.

So, statement-1 is correct and statement-2 is a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. For all $n \in N$, $49^n + 16n - 1$ is divisible by

(a) 64 (b) 8 (c) 16 (d) 4

2. If $n \in N$, then $n(n^2-1)$ is divisible by

(a) 6 (b) 16 (c) 36 (d) 24

3. For all $n \in N$, $7^{2n} - 48n - 1$ is divisible by

(a) 25 (b) 26 (c) 1234 (d) 2304

4. For all $n \in N$, $5^{2n} - 1$ is divisible by

(a) 6 (b) 11 (c) 24 (d) 26

5. For all $n \in N$, $n^3 + 2n$ is divisible by

(a) 3 (b) 8 (c) 9 (d) 11

6. For all $n \in N$, $4^n - 3n - 1$ is divisible by

(a) 3 (b) 8 (c) 9 (d) 11

7. For all $n \in N$, $3^{3n} - 26^n - 1$ is divisible by

(a) 24 (b) 64 (c) 17 (d) 676

8. If $n \in N$, then $3^{2n} + 7$ is divisible by

(a) 3 (b) 8 (c) 9 (d) 11

9. For all $n \in N$, $3n^5 + 5n^3 + 7n$ is divisible by

(a) 3 (b) 5 (c) 10 (d) 15

10. The nth terms of the series $3+7+13+21+\ldots$. is

(a) $4n-1$ (b) n^2+2n

(c) n^2+n+1 (d) n^2+2

11. The nth term of the series $4+14+30+52+80+114+\ldots$.is

(a) $5n-1$ (b) $2n^2+2n$

(c) $3n^2+n$ (d) $2n^2+2$

12. $3+13+29+51+79+\ldots$. to n terms =

(a) $2n^2+7n^3$ (b) n^2+5n^3

(c) n^3+2n^2 (d) none of these

13. The sum to n terms of the series $1^3+3^3+5^3+\ldots$ is

(a) $n^2(n^2-1)$ (b) $n^2(2n^2-1)$

(c) $n^2(2n^2+1)$ (d) $n^2(n^2+1)$

[EAMCET 2001]

14. If $10^n + 3.4^{n+2} + \lambda$ is exactly divisible by 9 for all $n \in N$, then the least positive integral value of λ is

(a) 5 (b) 3 (c) 7 (d) 1

15. If $x^n - 1$ is divisible by $x-k$, then the least positive integral value of k is

(a) 1 (b) 2 (c) 3 (d) 4

16. If a, b are distinct rational numbers, then for all $n \in N$ the number $a^n - b^n$ is divisible by

(a) $a-b$ (b) $a+b$

(c) $2a-b$ (d) $a-2b$

17. If n is an odd positive integer, then $a^n + b^n$ is divisible by

(a) $a+b$ (b) $a-b$

(c) a^2+b^2 (d) none of these

18. If n is an even positive integer, then $a^n + b^n$ is divisible by

(a) $a+b$ (b) $a-b$

(c) a^2-b^2 (d) none of these

19. For all $n \in N$, $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7}{15n}$ is

(a) an integer (b) a natural number

(c) a positive fraction (d) none of these

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20. The sum of n terms of the series $1+(1+a)+(1+a+a^2)+(1+a+a^2+a^3)+\ldots$, is

(a) $\frac{n}{1-a}-\frac{a(1-a^n)}{(1-a)^2}$ (b) $\frac{n}{1-a}+\frac{a(1-a^n)}{(1-a)^2}$

(c) $\frac{n}{1-a}+\frac{a(1+a^n)}{(1-a)^2}$ (d) $-\frac{n}{1-a}+\frac{a(1-a^n)}{(1-a)^2}$

21. If $3+5+9+17+33+\ldots$ to n terms $=2^{n+1}+n-2$, then nth term of LHS is

(a) 3^n-1 (b) $2n+1$ (c) 2^n+1 (d) $3n-1$

22. $7^{2n}+3^{n-1}.2^{3n-3}$ is divisible by

(a) 24 (b) 25 (c) 9 (d) 13

23. For all $n \in N$, $10^n+3.4^{n+2}+5$ is divisible by

(a) 23 (b) 3 (c) 9 (d) 207

24. If $n \in N$, then $x^{2n-1}+y^{2n-1}$ is divisible by

(a) $x+y$ (b) $x-y$

(c) x^2+y^2 (d) none of these

25. The sum of the cubes of three consecutive natural numbers is divisible by

(a) 7 (b) 9 (c) 25 (d) 26

26. $\frac{(n+2)!}{(n-1)!}$ is divisible by

(a) 6 (b) 11 (c) 24 (d) 26

27. For all $n \in N$, n^4 is less than

(a) 10^n (b) 4^n (c) 5^n (d) 10^{10}

28. For all $n \in N$, $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots.+\frac{1}{\sqrt{n}}$ is

(a) equal to $\sqrt{n}$

(b) less than or equal to $\sqrt{n}$

(c) greater than or equal to $\sqrt{n}$

(d) none of these

29. For all $n \in N$, Σn

(a) $< \frac{(2n+1)^2}{8}$ (b) $> \frac{(2n+1)^2}{8}$

(c) $= \frac{(2n+1)^2}{8}$ (d) none of these

30. For all $n \in N$, $\cos\theta \cos 2\theta \cos 4\theta \ldots.. \cos 2^{n-1}\theta$ equals to

(a) $\frac{\sin 2^n\theta}{2^n \sin\theta}$ (b) $\frac{\sin 2^n\theta}{\sin\theta}$

(c) $\frac{\cos 2^n\theta}{2^n \cos 2\theta}$ (d) $\frac{\cos 2^n\theta}{2^n \sin\theta}$

Answers

1. (a) 2. (a) 3. (d) 4. (c) 5. (a) 6. (c) 7. (d) 8. (b) 9. (d) 10. (c) 11. (c) 12. (c) 13. (b) 14. (a) 15. (a) 16. (a) 17. (a) 18. (d) 19. (b) 20. (a) 21. (c) 22. (b) 23. (c) 24. (a) 25. (b) 26. (a) 27. (a) 28. (b) 29. (a) 30. (a)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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13

PERMUTATIONS AND COMBINATIONS

1. FACTORIAL

FACTORIAL *The continued product of first n natural numbers is called the "n factorial" and is denoted by* $\lfloor n$ *or* $n!$ *i.e.*

$$n! = 1\times2\times3\times4\times\ldots\times(n-1)\times n$$

For example,

$$2! = 1\times2$$
$$3! = 1\times2\times3$$
$$4! = 1\times2\times3\times4$$
$$5! = 1\times2\times3\times4\times5 \text{ and so on.}$$

Clearly, $n! = (n-1)!\,n$ for all $n \in N$.

ILLUSTRATION 1 *If* $a_n = n\,(n!)$, *then* $\sum_{r=1}^{100} a_r$ *is equal to*

(a) $101!$ (b) $100!-1$ (c) $101!-1$ (d) $101!+1$

Ans. (c)

SOLUTION We have,

$$\sum_{r=1}^{100} a_r = \sum_{r=1}^{100} r\,(r!) = \sum_{r=1}^{100} \{(r+1)-1\}\,r!$$

$$\Rightarrow \sum_{r=1}^{100} a_r = \sum_{r=1}^{100} \{(r+1)!-r!\}$$

$$\Rightarrow \sum_{r=1}^{100} a_r = (2!-1!)+(3!-2!)+(4!-3!)+\ldots.+(101!-100!)$$

$$\Rightarrow \sum_{r=1}^{100} a_r = 101!-1$$

ILLUSTRATION 2 *If* $a_n = \dfrac{n}{(n+1)!}$, *then* $\sum_{n=1}^{50} a_n$ *is equal to*

(a) $\dfrac{50!-1}{50!}$ (b) $\dfrac{51!-1}{51!}$ (c) $\dfrac{1}{2\,(n-1)!}$ (d) *none of these*

Ans. (b)

SOLUTION We have,

$$a_n = \frac{n}{(n+1)!}$$

$$\Rightarrow a_n = \frac{(n+1)-1}{(n+1)!}$$

$$\Rightarrow a_n = \frac{1}{n!}-\frac{1}{(n+1)!}$$

$$\therefore \sum_{n=1}^{50} a_n = \sum_{n=1}^{50}\left\{\frac{1}{n!}-\frac{1}{(n+1)!}\right\}$$

$$\Rightarrow \sum_{n=1}^{50} a_n = \left(\frac{1}{1!}-\frac{1}{2!}\right)+\left(\frac{1}{2!}-\frac{1}{3!}\right)+\left(\frac{1}{3!}-\frac{1}{4!}\right)+\ldots+\left(\frac{1}{50!}-\frac{1}{51!}\right)$$

$$\Rightarrow \sum_{n=1}^{50} a_n = 1-\frac{1}{51!} = \frac{51!-1}{51!}$$

ILLUSTRATION 3 *If* $n!, 3\times n!$ *and* $(n+1)!$ *are in G.P., then* $n!, 5\times n!$ *and* $(n+1)!$ *are in*

(a) *A.P.* (b) *G.P.* (c) *H.P.* (d) *none of these*

Ans. (a)

SOLUTION It is given that $n!, 3\times n!$ and $(n+1)!$ are in G.P.

$$\therefore (3\times n!)^2 = n!\times(n+1)!$$

$$\Rightarrow 9\times(n!)^2 = n!\times(n+1)\,n!$$

$$\Rightarrow 9 = n+1$$

$$\Rightarrow n = 8$$

$$\therefore n! = 8!,\ 5\times n! = 5\times8! \text{ and } (n+1)! = 9!$$

We observe that

$$2\,(5\times n!) = 10\times8! = (9+1)\times8! = 9\times8!+8!$$

$$\Rightarrow 2\,(5\times n!) = 9!+8! = n!+(n+1)!$$

Hence, $n!, 5\times n!$ and $(n+1)!$ are in A.P.

ILLUSTRATION 4 *The value of* $1.1!+2.2!+3.3!+\ldots+n.n!$, *is*

(a) $(n+1)!$ (b) $(n+1)!+1$

(c) $(n+1)!-1$ (d) *none of these*

[CEE (Delhi) 2005]

Ans (c)

SOLUTION We have,

$$1.1!+2.2!+3.3!+\ldots+n.n!$$

$$= \sum_{r=1}^{n} r\,(r!)$$

$$= \sum_{r=1}^{n} [(r+1)\,r!-r!]$$

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Ans. (d)

SOLUTION Let $S = \sum_{r=0}^{n} {}^{2n+1}C_r$. Then,

$$S = \sum_{r=0}^{n} {}^{2n+1}C_{2n+1-r} \qquad [\because {}^{n}C_r = {}^{n}C_{n-r}]$$

$$\therefore \quad 2S = \sum_{r=0}^{2n+1} {}^{2n+1}C_r = 2^{2n+1} \Rightarrow S = 2^{2n}.$$

So, statement–2 is true.

The number of ways of selecting at least one and at most n books from $(2n+1)$ books is

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 2^{2n} - 1$$

$$\therefore \quad 2^{2n} - 1 = 255 \Rightarrow 2^{2n} = 2^8 \Rightarrow n = 4$$

So, statement–1 is false.

EXAMPLE 9 *Statement–1: The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{9}C_3$. Statement–2: The number of ways of choosing any 3 places from 9 different places is ${}^{9}C_3$.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b) **[AIEEE 2011]**

SOLUTION The number of ways of distributing n items among r persons such that each person receives at least one item is ${}^{n-1}C_{r-1}$.

Therefore, the number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{10-1}C_{4-1} = {}^{9}C_3$. So, statement – 1 is true.

Clearly, statement-2 is also true but it is not a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The value of $\sum_{r=1}^{n} \frac{{}^{n}P_r}{r!}$ is

(a) 2^n (b) $2^n - 1$ (c) 2^{n-1} (d) $2^n + 1$

2. If ${}^{12}P_r = {}^{11}P_6 + 6 \cdot {}^{11}P_5$ then r is equal to

(a) 6 (b) 5
(c) 7 (d) none of these

3. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$, then

(a) $n \geq 4$ (b) $n > 5$
(c) $n > 7$ (d) none of these

4. ${}^{n}C_r + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2}$ is equal to

(a) ${}^{n+1}C_r$ (b) ${}^{n}C_{r+1}$
(c) ${}^{n-1}C_{r+1}$ (d) none of these

5. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$, then n is equal to

(a) 4 (b) 6 (c) 3 (d) 8

6. If ${}^{35}C_{n+7} = {}^{35}C_{4n-2}$, then all the values of n are given by

(a) 28 (b) 3, 6 (c) 3 (d) 6

7. If ${}^{n}C_{r-1} = 36$, ${}^{n}C_r = 84$ and ${}^{n}C_{r+1} = 126$, then

(a) $n = 8, r = 4$ (b) $n = 9, r = 3$
(c) $n = 7, r = 5$ (d) none of these

8. If ${}^{n-1}C_6 + {}^{n-1}C_7 > {}^{n}C_6$, then

(a) $n > 4$ (b) $n > 12$ (c) $n \geq 13$ (d) $n > 13$

9. Which of the following is incorrect?

(a) ${}^{n}C_r = {}^{n}C_{n-r}$ (b) ${}^{n}C_r = {}^{n-1}C_r + {}^{n}C_{n-r}$
(c) ${}^{n}C_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$ (d) $r!\,{}^{n}C_r = {}^{n}P_r$

10. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then the value of r is

(a) 40 (b) 41 (c) 42 (d) none of these

11. $\sum_{r=0}^{m} {}^{n+r}C_n$ is equal to

(a) ${}^{n+m+1}C_{n+1}$ (b) ${}^{n+m+2}C_n$
(c) ${}^{n+m+3}C_{n-1}$ (d) none of these

[JEE (WB) 2006]

12. Every body in a room shakes hands with every body else. The total number of hand shakes is 66. The total number of persons in the room is

(a) 11 (b) 12 (c) 13 (d) 14

13. On the occasion of Diwali festival each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is

(a) ${}^{20}C_2$ (b) $2 \cdot {}^{20}C_2$
(c) $2 \times {}^{20}P_2$ (d) none of these

14. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57:16$, then the value of n is

(a) 20 (b) 19 (c) 18 (d) 17

15. The exponent of 3 in 100 !, is

(a) 33 (b) 44 (c) 48 (d) 52

16. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then, the number of words which have at least one letter repeated is

(a) 69760 (b) 30240
(c) 99748 (d) none of these

17. If 7 points out of 12 are in the same straight line, then the number of triangles formed is

(a) 19 (b) 158 (c) 185 (d) 201

18. A polygon has 44 diagonals, the number of its sides is

(a) 9 (b) 10 (c) 11 (d) 12

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19. A polygon has 170 diagonals. How many sides will it have?
(a) 12 (b) 17 (c) 20 (d) 25

20. The number of all possible selections of one or more questions from 10 given questions, each question having an alternative is
(a) 3^{10} (b) $2^{10}-1$ (c) $3^{10}-1$ (d) 2^{10}

21. The number of ways of painting the faces of a cube with six different colours is
(a) 1 (b) 6
(c) 6 ! (d) none of these

22. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw?
(a) 129 (b) 84
(c) 64 (d) none of these

23. The number of different permutations of the word BANANA, is
(a) 720 (b) 60 (c) 120 (d) 360

24. A person wishes to make up as many different parties as he can out of his 20 friends such that each party consists of the same number of persons. The number of friends he should invite at a time, is
(a) 5 (b) 10
(c) 8 (d) none of these

25. The number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated is
(a) $4!\cdot 4!$ (b) $\frac{8!}{4!}$
(c) 288 (d) none of these

26. The total number of words which can be formed out of the letters a, b, c, d, e, f taken 3 together, such that each word contains at least one vowel, is
(a) 72 (b) 48
(c) 96 (d) none of these

27. The smallest value of x satisfying the inequality ${}^{10}C_{x-1} > 2\cdot {}^{10}C_x$ is
(a) 7 (b) 10 (c) 9 (d) 8

28. The number of arrangements which can be made using all the letters of the word *LAUGH*, if the vowels are adjacent, is
(a) 10 (b) 24 (c) 48 (d) 120

29. The number of ways of choosing a committee of 4 women and 5 men from 10 women and 9 men, if Mr. *A* refuses to serve on the committee when Ms. *B* is a member of the committee, is
(a) 20580 (b) 21000
(c) 21580 (d) all the above

30. A set contains $(2n+1)$ elements. The number of subsets of this set containing more than n elements is equal to
(a) 2^{n-1} (b) 2^n (c) 2^{n+1} (d) 2^{2n}

31. The number of ways in which any four letters can be selected from the word 'CORGOO' is
(a) 15 (b) 11
(c) 7 (d) none of these

32. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is
(a) 216 (b) 240 (c) 600 (d) 3125

[AIEEE 2002]

33. If 7 points out of 12 are in the same straight line, then the number of triangles formed is
(a) 19 (b) 158 (c) 185 (d) 201

34. If a denotes the number of permutations of $x+2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x-11$ things taken all at a time such that $a = 182\,b\,c$, then the value of x is
(a) 15 (b) 12 (c) 10 (d) 18

35. There are n number of seats and m number of people have to be seated, then how many ways are possible to do this $(m < n)$?
(a) ${}^nP_m$ (b) ${}^nC_m$
(c) ${}^nC_n \times (m-1)!$ (d) ${}^{n-1}P_{m-1}$

36. There are 5 letters and 5 directed envelopes. The number of ways in which all the letters can be put in wrong envelope is
(a) 119 (b) 44 (c) 59 (d) 40

37. The number of ways of selecting 10 balls from unlimited number of red, black, white and green balls is
(a) 286 (b) 84
(c) 715 (d) none of these

38. The total number of different combinations of letters which can be made from the letters of the word MISSISSIPPI is
(a) 150 (b) 148
(c) 149 (d) none of these

39. The total number of natural numbers of six digits that can be made with digits 1, 2, 3, 4, if all digits are to appear in the same number at least once, is
(a) 1560 (b) 840 (c) 1080 (d) 480

40. The total number of seven-digit numbers the sum of whose digits is even is
(a) 9000000 (b) 4500000
(c) 8100000 (d) none of these.

41. All possible four-digit numbers are formed using the digits 0, 1, 2, 3 so that no number has repeated digits. The number of even numbers among them is
(a) 9 (b) 18
(c) 10 (d) none of these

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42. If the permutations of a, b, c, d, e taken all together be written down in alphabetical order as in dictionary and numbered, then the rank of the permutation *debac* is
(a) 90 (b) 91 (c) 92 (d) 93

43. The number of natural numbers smaller than 10^4, in the decimal notation of which all the digits are different is
(a) 5274 (b) 5265
(c) 4676 (d) none of these

44. If eight persons are to address a meeting, then the number of ways in which a specified speaker is to speak before another specified speaker is
(a) 2520 (b) 20160
(c) 40320 (d) none of these

45. The total number of proper divisors of 38808 is
(a) 72 (b) 70 (c) 69 (d) 71

46. All possible two factors products are formed from the numbers 1, 2, 3, 4, ..., 200. The number of factors out of the total obtained which are multiples of 5 is
(a) 5040 (b) 7180
(c) 8150 (d) none of these

47. In an examination there are three multiple choice questions and each question has four choices. Number of sequences in which a student can fail to get all answers correct is
(a) 11 (b) 15 (c) 80 (d) 63

48. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary, then the rank of the word RANDOM is
(a) 614 (b) 615 (c) 613 (d) 616

49. The sum of all the numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time is
(a) 93324 (b) 66666
(c) 84844 (d) none of these

50. The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is
(a) 18 (b) 108
(c) 432 (d) none of these

51. If the letters of the word *MOTHER* are written in all possible orders and these words are written out as in a dictionary, then the rank of the word MOTHER is
(a) 240 (b) 261 (c) 308 (d) 309

52. The total number of numbers greater than 1000, but not greater than 4000, that can be formed with the digits 0, 1, 2, 3, 4 when the repetition of digits allowed is
(a) 375 (b) 374
(c) 376 (d) none of these
[AIEEE 2002]

53. The number of ways in which 5 picturers can be hung from 7 picture nails on the wall is
(a) 7^5 (b) 5^7
(c) 2520 (d) none of these

54. The number of all four digit numbers which are divisible by 4 that can be formed from the digits 1, 2, 3, 4 and 5 is
(a) 125 (b) 30
(c) 95 (d) none of these

55. The number of all five digit numbers which are divisible by 4 that can be formed from the digits 0, 1, 2, 3, 4 (without repetition) is
(a) 36 (b) 30
(c) 34 (d) none of these

56. The number of ways in which $m + n$ $(n \le m+1)$ different things can be arranged in a row such that no two of the n things may be together is
(a) $\frac{(m+n)!}{m!\,n!}$ (b) $\frac{m!\,(m+1)!}{(m+n)!}$
(c) $\frac{m!\,(m+1)!}{(m-n+1)!}$ (d) none of these

57. All possible two-factor products are formed from the numbers 1, 2, ..., 100. The number of factors out of the total obtained which are multiple of 3 is
(a) 2211 (b) 4950
(c) 2739 (d) none of these

58. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then the number of ways in which they can be seated is
(a) $\frac{m!\,n!}{(m+n)!}$ (b) $\frac{m!\,(m+1)!}{(m-n+1)!}$
(c) $\frac{m!\,n!}{(m-n+1)!}$ (d) none of these

59. The total number of ways in which six '+' and four '−' signs can be arranged in a line such that no two '−' signs occur together is
(a) 35 (b) 15
(c) 30 (d) none of these

60. If in a chess tournament each contestant plays once against each of the others and in all 45 games are played, then the number of participants is
(a) 9 (b) 10
(c) 15 (d) none of these

61. Total number of four digit odd numbers that can be formed by using 0, 1, 2, 3, 5, 7 is
(a) 216 (b) 375 (c) 400 (d) 720
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62. A committee of 5 is to be formed from 9 ladies and 8 men. If the committee commands a lady majority, then the number of ways this can be done is
(a) 2352 (b) 1008 (c) 3360 (d) 3456

63. The number of ordered triplets of positive integers which are solutions of the equation $x + y + z = 100$ is
(a) 6005 (b) 4851
(c) 5081 (d) none of these

64. The number of straight lines can be formed out of 10 points of which 7 are collinear

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(a) 26 (b) 21
(c) 25 (d) none of these

65. The number of ways in which ten candidates $A_1, A_2, \ldots A_{10}$ can be ranked such that A_1 is always above A_{10} is
(a) 5 ! (b) 2 (5 !) (c) 10 ! (d) $\frac{1}{2}$ (10 !)

66. In Q. 65, the number of ways in which A_1 and A_2 are next to each other is
(a) 9 ! (b) 2 (9 !)
(c) $\frac{1}{2}$ (9 !) (d) none of these

67. The total number of all proper factors of 75600 is
(a) 120 (b) 119
(c) 118 (d) none of these

68. The total number of ways in which 11 identical apples can be distributed among 6 children is
(a) 252 (b) 462
(c) 42 (d) none of these

69. The number of ways in which a pack of 52 cards be divided equally amongst four players in order is
(a) $^{52}C_{13}$ (b) $^{52}C_4$
(c) $\frac{52!}{(13!)^4}$ (d) $\frac{52!}{(13!)^4 4!}$

70. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card
(a) $\frac{52!}{(17!)^3}$ (b) $\frac{52!}{(17!)^3 3!}$
(c) $\frac{51!}{(17!)^3}$ (d) $\frac{51!}{(17!)^3 3!}$

71. The total number of ways of dividing 15 things into groups of 8, 4 and 3 respectively is
(a) $\frac{15!}{8!4!(3!)^2}$ (b) $\frac{15!}{8!4!3!}$
(c) $\frac{15!}{8!4!}$ (d) none of these

72. There are 3 copies each of 4 different books. The number of ways in which they can be arranged in a shelf is
(a) $\frac{12!}{(3!)^4}$ (b) $\frac{12!}{(4!)^3}$
(c) $\frac{12!}{(3!)^4 4!}$ (d) $\frac{12!}{(4!)^3 3!}$

73. The number of ways in which 12 books can be put in 3 shelves, 4 on each, is
(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(3!)(4!)^3}$
(c) $\frac{12!}{(3!)^3 4!}$ (d) none of these

74. The total number of ways in which 12 persons can be divided into three groups of 4 persons each is
(a) $\frac{12!}{(3!)^3 4!}$ (b) $\frac{12!}{(4!)^3}$ (c) $\frac{12!}{(4!)^3 3!}$ (d) $\frac{12!}{(3!)^4}$

75. The number of ways in which 12 balls can be divided between two friends, one receiving 8 and the other 4, is
(a) $\frac{12!}{8!4!}$ (b) $\frac{12!2!}{8!4!}$
(c) $\frac{12!}{8!4!2!}$ (d) none of these

76. The total number of ways in which 2 n persons can be divided into n couples is
(a) $\frac{2n!}{n!n!}$ (b) $\frac{2n!}{(2!)^n}$
(c) $\frac{2n!}{n!(2!)^n}$ (d) none of these

77. The total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty paise coins and 7 twenty five paise coins is
(a) 28 (b) 56
(c) $^{37}C_6$ (d) none of these

78. The number of ways in which thirty five apples can be distributed among 3 boys so that each can have any number of apples, is
(a) 1332 (b) 666
(c) 333 (d) none of these

79. A person goes in for an examination in which there are four papers with a maximum of m marks from each paper. The number of ways in which one can get 2 m marks is
(a) $^{2m+3}C_3$ (b) $\frac{1}{3}(m+1)(2m^2+4m+1$
(c) $\frac{1}{3}(m+1)(2m^2+4m+3)$ (d) none of these

80. m parallel lines in a plane are intersected by a family of n parallel lines. The total number of parallelograms so formed is
(a) $\frac{(m-1)(n-1)}{4}$ (b) $\frac{mn}{4}$
(c) $\frac{m(m-1)n(n-1)}{2}$ (d) $\frac{mn(m-1)(n-1)}{4}$

81. 1, 2, 3, 4 are four numbers. How many numbers can be made using all four numbers?
(a) 1 (b) 3 (c) 2 (d) 4

82. There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played between themselves proved to exceed by 66 the number of games that the men played with the women. The number of participants is
(a) 6 (b) 11
(c) 13 (d) none of these

83. A parallelogram is cut by two sets of m lines parallel to its sides. The number of parallelograms thus formed is

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(a) $({}^{m}C_2)^2$ (b) $\left({}^{m+1}C_2\right)^2$

(c) $\left({}^{m+2}C_2\right)^2$ (d) none of these

84. There are n straight lines in a plane, no two of which are parallel, and no three pass through the same point. Their points of intersection are joined. Then, the number of fresh lines thus obtained is

(a) $\frac{n(n-1)(n-2)}{8}$ (b) $\frac{n(n-1)(n-2)(n-3)}{6}$

(c) $\frac{n(n-1)(n-2)(n-3)}{8}$ (d) none of these

85. There are 18 points in a plane such that no three of them are in the same line except five points which are collinear. The number of triangles formed by these points is

(a) 805 (b) 806

(c) 816 (d) none of these

86. There are five different green dyes, four differnt blue dyes and three different red dyes. The total number of combinations of dyes that can be chosen taking at least one green and one blue dye is

(a) 3255 (b) 2^{12}

(c) 3720 (d) none of these

87. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4; and then the men select the chairs from amongst the remaining. The number of possible arrangements is

(a) ${}^{4}C_3 \cdot {}^{4}C_2$ (b) ${}^{4}C_2 \cdot {}^{4}P_3$

(c) ${}^{4}P_2 \cdot {}^{4}P_3$ (d) none of these

88. A student is allowed to select at most n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select a book is 63, then the value of n is

(a) 6 (b) 3

(c) 4 (d) none of these

89. In a steamer there are stalls for 12 animals and there are cows, horses and calves (not less than 12 of each) ready to be shipped; the total number of ways in which the shipload can be made is

(a) 3^{12} (b) 12^3 (c) ${}^{12}P_3$ (d) ${}^{12}C_3$

90. The number of numbers that can be formed by using digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places

(a) 3! 4! (b) 34 (c) 18 (d) 12

91. Number of ways in which Rs 18 can be distributed amongst four persons such that no body receives less than Rs 4 is

(a) 4^2 (b) 2^4

(c) 4! (d) none of these

92. A box contains two white balls, three black balls and four red balls. The number of ways in which three balls can be drawn from the box, so that at least one of the ball is black is

(a) 74 (b) 64 (c) 8 4 (d) 20

93. The number of ways in which four letters can be selected from the word *degree* is

(a) 7 (b) 6

(c) $\frac{6!}{3!}$ (d) none of these

94. The total number of arrangements which can be made out of the letters of the word 'Algebra', without altering the relative position of vowels and consonants is

(a) $\frac{7!}{2!}$ (b) $\frac{7!}{2!5!}$ (c) 4!3! (d) $\frac{4!3!}{2}$

95. The number of ways in which seven persons can be arranged at a round table if two particular persons may not sit together is

(a) 480 (b) 120

(c) 80 (d) none of these

96. The total number of ways in which 4 boys and 4 girls can form a line, with boys and girls alternating, is

(a) $(4!)^2$ (b) 8!

(c) $2(4!)^2$ (d) $4! \cdot {}^{5}P_4$

97. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(a) $8 \times {}^{6}C_4 \times {}^{7}C_4$ (b) $6 \times 7 \times {}^{8}C_4$

(c) $6 \times 8 \times {}^{7}C_4$ (d) $7 \times {}^{6}C_4 \times {}^{8}C_4$

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98. A code word consists of three letters of the English alphabet followed by two digits of the decimal system. If neither letter nor digit is repeated in any code word, then the total number of code words is

(a) 1404000 (b) 16848000

(c) 2808000 (d) none of these

99. The number of permutations of all the letters of the word 'EXERCISES' is

(a) 60480 (b) 30240

(c) 10080 (d) none of these

100. A father with 8 children takes 3 at a time to the Zoological Gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is

(a) 336 (b) 112

(c) 56 (d) none of these

101. A father with 8 children takes them 3 at a time to the Zoological Gardens, as often as he can without taking the same 3 children together more than once. The number of times each child will go to the garden is

(a) 56 (b) 21

(c) 112 (d) none of these

102. If the letters of the word *LATE* be permuted and the words so formed be arranged as in a dictionary. Then, the rank of *LATE* is

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(a) 12 (b) 13 (c) 14 (d) 15

103. The total number of ways of arranging the letters *AAAA BBB CC D E F* in a row such that letters C are separated from one another is
(a) 2772000 (b) 1386000
(c) 4158000 (d) none of these

104. The sides *AB*, *BC*, *CA* of a triangle *ABC* have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices is
(a) 220 (b) 204 (c) 205 (d) 195

105. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is
(a) 9 (b) 24
(c) 12 (d) none of these

106. The number of all the possible selections which a student can make for answering one or more questions out of eight given questions in a paper, when each question has an alternative is
(a) 256 (b) 6560
(c) 6561 (d) none of these

107. The greatest possible number of points of intersection of 8 straight lines and 4 circles is
(a) 32 (b) 64 (c) 76 (d) 104

108. A lady gives a dinner party to 5 guests to be selected from nine friends. The number of ways of forming the party of 5, given that two of the friends will not attend the party together is
(a) 56 (b) 126
(c) 91 (d) none of these

109. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is
(a) 10^2 (b) 1023 (c) 2^{10} (d) 10 !

110. The number of ways in which four persons be seated at a round table, so that all shall not have the same neighbours in any two arrangements is
(a) 24 (b) 6 (c) 3 (d) 4

111. At an election there are five candidates and three members to be elected, and an elector may vote for any number of candidates not greater than the number to be elected. Then the number of ways in which an elector may vote is
(a) 25 (b) 30
(c) 32 (d) none of these

112. There are n different books and p copies of each. The number of ways in which a selection can be made from them is
(a) n^p (b) p^n
(c) $(p+1)^n - 1$ (d) $(n+1)^p - 1$

113. A library has a copies of one book, b copies of each of two books, c copies of each of three books, and single copies of d books. The total number of ways in which these books can be distributed is
(a) $\frac{(a+b+c+d)!}{a!\,b!\,c!}$ (b) $\frac{(a+2b+3c+d)!}{a!\,(b!)^2\,(c!)^3}$
(c) $\frac{(a+2b+3c+d)!}{a!\,b!\,c!}$ (d) none of these

114. The total number of arrangements of the letters in the expression $a^3 b^2 c^4$ when written at full length, is
(a) 1260 (b) 2520
(c) 610 (d) none of these

115. The total number of selections of fruit which can be made from 3 bananas, 4 apples and 2 oranges, is
(a) 39 (b) 315
(c) 512 (d) none of these

116. The total number of ways of dividing mn things into n equal groups, is
(a) $\frac{(mn)!}{m!\,n!}$ (b) $\frac{(mn)!}{(n)^m\,m!}$
(c) $\frac{(mn)!}{(m!)^n\,n!}$ (d) none of these

117. The total number of permutations of 4 letters that can be made out of the letters of the word *EXAMINATION*, is
(a) 2454 (b) 2436
(c) 2545 (d) none of these

118. The value of
$$^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{s=1}^{5} {}^{47-s}C_{40-s}, \text{ is}$$
(a) $^{46}C_7$ (b) $^{46}C_8$ (c) $^{47}C_7$ (d) $^{47}C_8$

119. If $^nC_{n-r} + 3\,.\,{}^nC_{n-r+1} + 3\,.\,{}^nC_{n-r+2} + {}^nC_{n-r+3} = {}^xC_r$, then $x =$
(a) $n+1$ (b) $n+2$ (c) $n+3$ (d) $n+4$

120. If n is even and
$^nC_0 < {}^nC_1 < {}^nC_2 < \ldots < {}^nC_r > {}^nC_{r+1} > {}^nC_{r+2} > \ldots > {}^nC_n$,
then, $r =$
(a) $\frac{n}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n-2}{2}$ (d) $\frac{n+2}{2}$

121. The number of ways in which one can post 5 letters in 7 letter boxes is
(a) 35 (b) $^7P_5$ (c) 7^5 (d) 5^7

122. The number of 2×2 matrices having elements 0 and 1, is
(a) 8 (b) 16 (c) 4 (d) none of these

123. Three dice are rolled. The number of possible outcomes in which at least one die shows 5 is
(a) 215 (b) 36 (c) 125 (d) 91

124. The number of numbers of four different digits that can be formed from the digits of the number 12356 such that the numbers are divisible by 4, is
(a) 36 (b) 48 (c) 12 (d) 24

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125. If m and n are positive integers more than or equal to 2, $m > n$, then $(mn)!$ is divisible by
(a) $(m!)^n$, $(n!)^m$ and $(m+n)!$ but not by $(m-n)!$
(b) $(m+n)!$, $(m-n)!$, $(m!)^n$ but not by $(n!)^m$
(c) $(m!)^n$, $(n!)^m$, $(m+n)!$ and $(m-n)!$
(d) $(m!)^n$ and $(n!)^m$ but not by $(m+n)!$ and $(m-n)!$

126. The number of integers which lie between 1 and 10^6 and which have the sum of the digits equal to 12, is
(a) 8550 (b) 5382 (c) 6062 (d) 8055

127. A man invites a party to $(m+n)$ friends to dinner and places m at one round table and n at another. The number of ways of arranging the guests is
(a) $\frac{(m+n)!}{m!\,n!}$ (b) $\frac{(m+n)!}{(m-1)!\,(n-1)!}$
(c) $(m-1)!\,(n-1)!$ (d) none of these

128. The number of ways of arranging m positive and $n < m+1$ negative signs in a row so that no two negative signs are together is
(a) ${}^{m+1}P_n$ (b) ${}^{n+1}P_m$ (c) ${}^{m+1}C_n$ (d) ${}^{n+1}C_m$

129. Out of 10 consonants four vowels, the number of words that can be formed using six consonants and three vowels is
(a) ${}^{10}P_6 \times {}^6P_3$ (b) ${}^{10}C_6 \times {}^6C_3$
(c) ${}^{10}C_6 \times {}^4C_3 \times 9!$ (d) ${}^{10}P_6 \times {}^4P_3$

130. Four couples (husband and wife) decide to form a committee of four members. The number of different committees that can be formed in which no couple finds a place is
(a) 10 (b) 12 (c) 14 (d) 16

131. In a test there are n questions. In the test 2^{n-i} students gave wrong answers to at least i questions, where $i = 1, 2, \ldots, n$. If the total number of wrong answers given is 2047, then, $n =$
(a) 12 (b) 11 (c) 10 (d) none of these

132. The number of numbers of 9 distinct digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than that in the middle is
(a) 48 (b) 576 (c) 8! (d) none of these

133. The number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour, is
(a) $9! \times 10!$ (b) $5 \times (9!)^2$
(c) $(9!)^2$ (d) none of these

134. There are n seats round a table numbered $1, 2, 3, \ldots, n$. The number of ways in which m ($\leq n$) persons can take seat is
(a) ${}^nC_m$ (b) ${}^nC_m \times m!$
(c) $(m-1)!$ (d) $(m-1)! \times (n-1)!$

135. A shopkeeper sells three varieties of perfumes and he has a large number of bottles of the same size of each variety in his stock. There are 5 places in a row in his showcase. The number of different ways of displaying the three varieties of perfumes in the show case is
(a) 6 (b) 50
(c) 150 (d) none of these

136. If $r > p > q$, the number of different selections of $p+q$ things taking r at a time, where p things are identical and q other things are identical, is
(a) $p+q-r$ (b) $p+q-r+1$
(c) $r-p-q+1$ (d) none of these

Answers

1. (b) 2. (a) 3. (c) 4. (d) 5. (a) 6. (b) 7. (b)
8. (d) 9. (b) 10. (b) 11. (a) 12. (b) 13. (b) 14. (b)
15. (c) 16. (a) 17. (c) 18. (c) 19. (c) 20. (c) 21. (a)
22. (c) 23. (b) 24. (b) 25. (a) 26. (c) 27. (d) 28. (c)
29. (a) 30. (d) 31. (c) 32. (a) 33. (c) 34. (b) 35. (a)
36. (b) 37. (a) 38. (c) 39. (a) 40. (b) 41. (c) 42. (d)
43. (a) 44. (b) 45. (b) 46. (b) 47. (d) 48. (a) 49. (a)
50. (b) 51. (d) 52. (a) 53. (c) 54. (a) 55. (b) 56. (c)
57. (c) 58. (b) 59. (a) 60. (b) 61. (d) 62. (d) 63. (b)
64. (c) 65. (d) 66. (b) 67. (c) 68. (d) 69. (c) 70. (b)
71. (b) 72. (a) 73. (a) 74. (c) 75. (b) 76. (c) 77. (a)
78. (b) 79. (c) 80. (d) 81. (d) 82. (c) 83. (c) 84. (c)
85. (b) 86. (c) 87. (d) 88. (b) 89. (a) 90. (c) 91. (d)
92. (b) 93. (a) 94. (d) 95. (a) 96. (c) 97. (d) 98. (a)
99. (b) 100. (c) 101. (b) 102. (c) 103. (b) 104. (c) 105. (a)
106. (b) 107. (d) 108. (c) 109. (b) 110. (c) 111. (a) 112. (c)
113. (b) 114. (a) 115. (d) 116. (c) 117. (a) 118. (d) 119. (c)
120. (a) 121. (c) 122. (b) 123. (d) 124. (a) 125. (c) 126. (c)
127. (d) 128. (c) 129. (c) 130. (d) 131. (b) 132. (b) 133. (b)
134. (b) 135. (c) 136. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. Seven women and seven men are to sit round a circular table such that there is a man on either side of every women; the number of seating arrangements is
(a) $(7!)^2$ (b) $(6!)^2$ (c) $6! \times 7!$ (d) $7!$

2. There are $(n+1)$ white and $(n+1)$ black balls each set numbered 1 to $n+1$. The number of ways in which the balls can be arranged in a row so that the adjacent balls are of different colours is

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(a) $(2n+2)!$ (b) $(2n+2)!\times 2$
(c) $(n+1)!\times 2$ (d) $2\{(n+1)!\}^2$

3. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is
(a) $9(10!)$ (b) $2(10!)$
(c) $45(8!)$ (d) $10!$

4. The number of committees of 5 persons consisting of at least one female member, that can be formed from 6 males and 4 females, is
(a) 246 (b) 252 (c) 6 (d) none of these

5. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is
(a) ${}^{16}C_{11}$ (b) ${}^{16}C_5$ (c) ${}^{16}C_9$ (d) ${}^{20}C_9$

6. In a football championship, 153 matches were played. Every team played one match with each other. The number of teams participating in the championship is
(a) 17 (b) 18 (c) 9 (d) none of these

7. How many numbers between 5000 and 10,000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, each digit appearing not more than once in each number ?
(a) $5\times {}^8P_3$ (b) $5\times {}^8C_8$
(c) $5!\times {}^8P_3$ (d) $5!\times {}^8C_3$

8. If x, y and r are positive integers, then ${}^xC_r+{}^xC_{r-1}\,{}^yC_1+{}^xC_{r-2}\,{}^yC_2+\ldots+{}^yC_r=$
(a) $\frac{x!\,y!}{r!}$ (b) $\frac{(x+y)!}{r!}$ (c) ${}^{x+y}C_r$ (d) ${}^{xy}C_r$

9. In how many ways can 5 red and 4 white balls be drawn from a bag containing 10 red and 8 white balls
(a) ${}^8C_5\times {}^{10}C_4$ (b) ${}^{10}C_5\times {}^8C_4$
(c) ${}^{18}C_9$ (d) none of these

10. All the letters of the word 'EAMCET' are arranged in all possible ways. The number of such arrangements in which no two vowels are adjacent to each other is
(a) 360 (b) 144 (c) 72 (d) 54

11. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is
(a) 10^2 (b) 1023 (c) 2^{10} (d) $10!$

12. How many 10 digits numbers can be written by using the digits 1 and 2
(a) ${}^{10}C_1+{}^9C_2$ (b) 2^{10} (c) ${}^{10}C_2$ (d) $10!$

13. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total number of m points are taken on I_1; n points on I_2, k points on I_3. The maximum number of triangles formed with vertices at these points are
(a) ${}^{m+n+k}C_3$ (b) ${}^{m+n+k}C_3-{}^mC_3-{}^nC_3-{}^kC_3$
(c) ${}^mC_3+{}^nC_3+{}^kC_3$ (d) none of these

14. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is
(a) 6 (b) 18 (c) 12 (d) 9

15. The number of diagonals that can be drawn by joining the vertices of an octagon is
(a) 28 (b) 48
(c) 20 (d) none of these

16. The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is
(a) 432 (b) 108 (c) 36 (d) 18

17. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is
(a) 11 (b) 12 (c) 27 (d) 63

18. There are 10 points in a plane, out of these 6 are collinear. The number of triangles formed by joining these points is
(a) 100 (b) 120
(c) 150 (d) none of these

19. Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner ?
(a) 61 (b) 62 (c) 63 (d) 64

20. Let P_m stand for ${}^mP_m$. Then,
$1+P_1+2P_2+3P_3+\ldots+n\cdot P_n$ is equal to
(a) $(n-1)!$ (b) $n!$
(c) $(n+1)!-1$ (d) none of these

21. The number of ways four boys can be seated around a round table in four chairs of different colours is
(a) 24 (b) 12 (c) 23 (d) 64

22. We are required to form different words with the help of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then m_1/m_2 is equal to
(a) 42 (b) 30 (c) 6 (d) 1/30

23. In a college examination, a candidate is required to answer 6 out of 10 questions which are divided into two sections each containing 5 questions. Further the candidate is not permitted to attempt more than 4 questions from either of the section. The number of ways in which he can make up a choice of 6 questions, is
(a) 200 (b) 150 (c) 100 (d) 50

24. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by
(a) $7!\times 5!$ (b) $6!\times 5!$ (c) 30 (d) $5!\times 4!$

25. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

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(a) 346 (b) 140 (c) 196 (d) 280

26. The number of all possible words that can be formed using the letters of the word "MATHEMATICS" is

(a) $\frac{11!}{2!2!2!}$ (b) 11! (c) ${}^{11}C_1$ (d) none of these

27. In a group of boys, two boys are brothers and in this group, 6 more boys are there. In how many ways, they can sit if the brothers are not to sit alongwith each other :

(a) 4820 (b) 1410 (c) 2830 (d) none of these

28. Sita has 5 coins each of the different denomination. The number of different sums of money she can form is

(a) 32 (b) 25 (c) 31 (d) none of these

29. The number of ways in which 5 boys and 3 girls can be seated in a row so that each girl is between two boys is

(a) 2880 (b) 1880 (c) 3800 (d) 2800

30. How many nine digit numbers can be formed by using the digits 2, 2, 3, 3, 5, 5, 8, 8, 8 so that the odd digits occupy even positions?

(a) 7560 (b) 180 (c) 16 (d) 60

31. The number of 4-digit even numbers that can be formed using 0, 1, 2, 3, 4, 5, 6 without repetition is

(a) 120 (b) 300 (c) 420 (d) 20

32. The number of ways in which n ties can be selected from a rack displaying 3 n different ties is

(a) $\frac{3n!}{2n!}$ (b) $3\times n!$ (c) $(3n)!$ (d) $\frac{3n!}{n!\,2n!}$

33. There are n-points $(n>2)$ in each of two parallel lines. Every point on one line is joined to every point on the other line by a line segment drawn within the lines. The number of points (between the lines) in which these segments intersect is

(a) ${}^{2n}C_2-2\cdot{}^{n}C_1+2$ (b) ${}^{2n}C_2-2\times{}^{n}C_2$

(c) ${}^{n}C_2\times{}^{n}C_2$ (d) none of these

34. The maximum number of points of intersection of 8 circles is

(a) 16 (b) 24 (c) 28 (d) 56

35. The maximum number of points into which 4 circles and 4 straight lines intersect is

(a) 26 (b) 50 (c) 56 (d) 72

36. In a circket championship there are 36 matches. The number of teams if each plays one match with other is

(a) 8 (b) 9 (c) 10 (d) none of these

37. The expression

${}^{n}C_r+4\cdot{}^{n}C_{r-1}+6\cdot{}^{n}C_{r-2}+4\cdot{}^{n}C_{r-3}+{}^{n}C_{r-4}$ equals

(a) ${}^{n+4}C_r$ (b) $2\cdot{}^{n+4}C_{r-1}$

(c) $4\cdot{}^{n}C_r$ (d) $11\cdot{}^{n}C_r$

38. The total number of numbers greater than 100 and divisible by 5, that can be formed from the digits 3, 4, 5, 6 if no digit is repeated is

(a) 24 (b) 48 (c) 30 (d) 12

39. The number of words which can be made out of the letters of the word "MOBILE" when consonants always occupy odd places, is

(a) 20 (b) 36 (c) 30 (d) 720

40. How many numbers greater than 24000 can be formed by using the digits 1, 2, 3, 4, 5 when no digit is repeated, is

(a) 36 (b) 60 (c) 84 (d) 120

41. The number of words that can be formed out of the letters of the word 'COMMITTEE' is

(a) $\frac{9!}{(2!)^3}$ (b) $\frac{9!}{(2!)^2}$ (c) $\frac{9!}{2!}$ (d) 9!

42. How many different committees of 5 can be formed from 6 men and 4 women on which exact 3 men and 2 women serve ?

(a) 6 (b) 20 (c) 60 (d) 120

43. The number of ways to arrange the letters of the word CHEESE are

(a) 120 (b) 240 (c) 720 (d) 6

44. If ${}^{k+5}P_{k+1}=\frac{11(k-1)}{2}\cdot{}^{k+3}P_k$, then the values of k are

(a) 7 and 11 (b) 6 and 7 (c) 2 and 11 (d) 2 and 6

45. The number of proper divisors of 38808 is

(a) 70 (b) 72 (c) 71 (d) none of these

46. There are 15 persons in a party and each person shakes hand with another. The total number of hand shakes is

(a) ${}^{15}P_2$ (b) ${}^{15}C_2$ (c) 15! (d) $2\times15!$

47. In how many ways 5 different beads can be arranged to form a necklace?

(a) 12 (b) 120 (c) 60 (d) 24

48. The number of ways that 8 beads of different colours be strung as a necklace is

(a) 2520 (b) 2880 (c) 5040 (d) 4320

49. There are n-points in a plane of which p points are collinear. How many lines can be formed from these points?

(a) ${}^{n}C_2-{}^{p}C_2+1$ (b) ${}^{n}C_2-{}^{p}C_2$

(c) $n-{}^{p}C_2$ (d) ${}^{n}C_2-{}^{p}C_2-1$

50. The number of four-digit even numbers that can be formed using 0, 1, 2, 3, 4, 5, 6 without repetition is

(a) 120 (b) 300 (c) 420 (d) 20

51. The number of ways in which 21 objects can be grouped into three groups of 8, 7, and 6 objects is

(a) $\frac{20!}{8!+7!+6!}$ (b) $\frac{21!}{8!\,7!}$

(c) $\frac{21!}{8!7!6!}$ (d) $\frac{21!}{8!+7!+6!}$

52. Let l_1 and l_2 be the two lines intersecting at P. If A_1, B_1, C_1 are points on l_1 ; A_2, B_2, C_2, D_2, E_2 are points on l_2 and if none of these coincides with P, then the number of triangles formed by these eight points is

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(a) 56 (b) 55 (c) 46 (d) 45

53. Consider the fourteen lines in the plane given by $y = x + r$, $y = -x + r$, where $r \in \{0, 1, 2, 3, 4, 5, 6\}$. The number of squares formed by these lines whose diagonals are of length 2 is
(a) 9 (b) 16 (c) 25 (d) 36

54. The number of words that can be formed out of the letters of the word "ARTICLE" so that vowels occupy even places is
(a) 574 (b) 36 (c) 754 (d) 144

55. The number of ways in which 9 persons can be divided into three equal groups is
(a) 1680 (b) 840 (c) 560 (d) 280

56. A dictionary is printed consisting of 7 lettered words only that can be made with letters of the word "CRICKET". If the words are printed in the alphabetical order, as in the ordinary dictionary, then the number of words before the word CRICKET is
(a) 530 (b) 480 (c) 531 (d) 481

57. The number of times the digit 5 will be written when listing the integers from 1 to 1000, is
(a) 271 (b) 272 (c) 300 (d) none of these

58. How many number lying between 999 and 10000 can be formed with the help of digits 0, 2, 3, 6, 7, 8 when the digits are not repeated?
(a) 100 (b) 200 (c) 300 (d) 400

59. The number of different words that can be formed from the letters of the word 'PENCIL' so that no two vowels are together, is
(a) 120 (b) 260 (c) 240 (d) 480

60. How many four digit numbers can be formed using digits 1, 2, 3, 4, 5 such that at least one of the number is repeated?
(a) $4^4 - 5!$ (b) $4^5 - 4!$ (c) $5^4 - 4!$ (d) $5^4 - 5!$

Answer

1. (c) 2. (d) 3. (a) 4. (a) 5. (c) 6. (b) 7. (a)
8. (c) 9. (b) 10. (c) 11. (b) 12. (b) 13. (b) 14. (b)
15. (c) 16. (b) 17. (d) 18. (a) 19. (c) 20. (c) 21. (a)
22. (b) 23. (a) 24. (b) 25. (c) 26. (a) 27. (d) 28. (c)
29. (a) 30. (d) 31. (c) 32. (d) 33. (c) 34. (d) 35. (b)
36. (b) 37. (a) 38. (d) 39. (b) 40. (c) 41. (a) 42. (d)
43. (a) 44. (b) 45. (a) 46. (b) 47. (a) 48. (a) 49. (a)
50. (c) 51. (c) 52. (d) 53. (c) 54. (d) 55. (d) 56. (a)
57. (c) 58. (c) 59. (d) 60. (d)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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14

BINOMIAL THEOREM AND ITS APPLICATIONS

1. BINOMIAL THEOREM

THEOREM *If x and a are real numbers, then for all $n \in N$,*

$$(x+a)^n = {}^nC_0\, x^n a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + \ldots\ldots\ldots + {}^nC_r\, x^{n-r} a^r + \ldots + {}^nC_{n-1}\, x^1 a^{n-1} + {}^nC_n\, x^0 a^n$$

i.e. $$(x+a)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} a^r$$

REMARK *The above expansion is also valid when x and a are complex numbers.*

1.1 PROPERTIES OF BINOMIAL EXPANSION

PROPERTY-1 *We have,*

$$(x+a)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} a^r$$

Since r can have values from 0 to n, the total number of terms in the expansion is $(n+1)$.

PROPERTY-2 *The sum of the indices of x and a in each term is n.*

PROPERTY-3 *We have,*

$${}^nC_r = {}^nC_{n-r}, \quad r = 0, 1, 2, \ldots, n$$

$$\Rightarrow \quad {}^nC_0 = {}^nC_n,\ {}^nC_1 = {}^nC_{n-1},\ {}^nC_2 = {}^nC_{n-2} = \ldots$$

So, the coefficients of terms equidistant from the beginning and the end are equal. These coefficients are known as the binomial coefficients.

PROPERTY-4 *Replacing a by $-a$, in the expansionof $(x+a)^n$, we get*

$$(x-a)^n = {}^nC_0\, x^n a^0 - {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 - {}^nC_3\, x^{n-3} a^3 \ldots\ldots + \ldots + (-1)^r\, {}^nC_r\, x^{n-r} a^r + \ldots + (-1)^n\, {}^nC_n\, x^0 a^n.$$

i.e., $$(x-a)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^{n-r} a^r$$

Thus, the terms in the expansion of $(x-a)^n$ are alternatively positive and negative, the last term is positive or negative according as n is even or odd.

PROPERTY-5 Putting $x = 1$ and $a = x$ in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \ldots + {}^nC_r\, x^r + \ldots + {}^nC_n\, x^n$$

i.e., $$(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r$$

This is the expansion of $(1+x)^n$ in ascending powers of x.

PROPERTY-6 Putting $a = 1$ in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} + {}^nC_2\, x^{n-2} + \ldots\ldots\ldots + {}^nC_r\, x^{n-r} + \ldots + {}^nC_{n-1}\, x + {}^nC_n$$

i.e., $$(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r}$$

This is the expansion of $(1+x)^n$ in descending powers of x.

PROPERTY-7 Putting $x = 1$ and $a = -x$ in the expansion of $(x+a)^n$, we get

$$(1-x)^n = {}^nC_0 - {}^nC_1\, x + {}^nC_2\, x^2 - {}^nC_3\, x^3 + \ldots + (-1)^r\, {}^nC_r\, x^r + \ldots + (-1)^n\, {}^nC_n\, x^n.$$

i.e., $$(1-x)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^r$$

ILLUSTRATION 1 *If in the expansion of $(1+x)^m (1-x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, then m is*

(a) 6 (b) 9 (c) 12 (d) 24

[IIT 1999, JEE (WB) 2007]

Ans. (c)

SOLUTION We have,

$$(1+x)^m (1-x)^n = \left({}^mC_0 + {}^mC_1\, x + {}^mC_2\, x^2 + \ldots + {}^mC_m\, x^m\right) \times \left({}^nC_0 - {}^nC_1\, x + {}^nC_2\, x^2 \ldots\ldots + (-1)^n\, {}^nC_n\, x^n\right)$$

$$= {}^mC_0 \cdot {}^nC_0 - \left({}^mC_0\, {}^nC_1 - {}^nC_0\, {}^mC_1\right) x + \left({}^mC_0\, {}^nC_2 + {}^nC_0\, {}^mC_2 - {}^mC_1 \cdot {}^nC_1\right) x^2 + \ldots\ldots$$

It is given that the coefficients of x and x^2 in the expansion of $(1+x)^m (1-x)^n$ are 3 and -6 respectively.

$$\therefore \quad -({}^mC_0\, {}^nC_1 - {}^nC_0\, {}^mC_1) = 3$$

and, $$\quad {}^mC_0\, {}^nC_2 + {}^nC_0\, {}^mC_2 - {}^mC_1\, {}^nC_1 = -6$$

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EXAMPLE 12 *Let* $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$; $S_2 = \sum_{j=1}^{10} j\cdot{}^{10}C_j$ *and* $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement–1 $S_3 = 50\times 2^9$.

Statement–2 $S_1 = 90\times 2^8$ *and* $S_2 = 10\times 2^8$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION We have,

$$S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j = \sum_{j=1}^{10} j\times\frac{10}{j}\,{}^{9}C_{j-1} = 10\sum_{j=1}^{10}{}^{9}C_{j-1} = 10\times 2^9$$

$$S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j = \sum_{j=1}^{10} j(j-1)\times\frac{10}{j}\times\frac{9}{j-1}\times{}^{8}C_{j-2}$$

$$\Rightarrow\quad S_2 = 10\times 9\sum_{j=2}^{10}{}^{8}C_{j-2} = 90\times 2^8$$

and, $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j = \sum_{j=1}^{10}\{j(j-1)+j\}\,{}^{10}C_j$

$$\Rightarrow\quad S_3 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j + \sum_{j=1}^{10} j\,{}^{10}C_j = 10\times 9\times 2^8 + 10\times 2^9$$

$$\Rightarrow\quad S_3 = 11\times 10\times 2^8$$

So, statement–2 is true and statement–1 is false.

EXAMPLE 13 *Let* $(1+x)^{36} = a_0 + a_1x + a_2x^2 + \ldots + a_{36}x^{36}$. *Then,*

Statement-1: $a_0 + a_3 + a_6 + \ldots + a_{36} = \frac{2}{3}(2^{35}+1)$

Statement-2: $a_0 + a_2 + a_4 + \ldots + a_{36} = 2^{35}$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION We have,

$$a_0 + a_1 + a_2x^2 + a_3x^3 + \ldots + a_{36}x^{36} = (1+x)^{36} \quad \ldots(i)$$

Putting $x = 1, \omega, \omega^2$ successively, we get

$$(a_0 + a_3 + a_6 + \ldots + a_{36}) + (a_1 + a_4 + a_7 + \ldots + a_{34}) + (a_2 + a_5 + \ldots + a_{35}) = 2^{36} \quad \ldots(ii)$$

$$(a_0 + a_3 + a_6 + \ldots + a_{36}) + \omega(a_1 + a_4 + a_7 + a_{34}) + \omega^2(a_2 + a_5 + \ldots + a_{35}) = 1 \quad \ldots(iii)$$

$$(a_0 + a_3 + a_6 + \ldots + a_{36}) + \omega^2(a_1 + a_4 + a_7 + \ldots + a_{34}) + \omega(a_2 + a_5 + \ldots + a_{35}) = 1 \quad \ldots(iv)$$

Adding these three, we get

$$3(a_0 + a_3 + a_6 + \ldots + a_{36}) = 2(2^{35}+1)$$

$$\Rightarrow\quad a_0 + a_3 + a_6 + \ldots + a_{36} = \frac{2}{3}(2^{35}+1)$$

So, statement-1 is true.

Putting $x = -1$ in (i) and adding it to (ii), we get

$$a_0 + a_2 + a_4 + \ldots + a_{36} = 2^{35}$$

So, statement-2 is true. But, statement-2 is not a correct explanation for statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If A and B are coefficients of x^r and x^{n-r} respectively in the expansion of $(1+x)^n$, then
   (a) $A = B$ (b) $A + B = 0$ (c) $A = rB$ (d) $A = nB$

2. Coefficient of x^{-4} in $\left(\frac{3}{2} - \frac{3}{x^2}\right)^{10}$ is
   (a) $\frac{405}{226}$ (b) $\frac{504}{289}$ (c) $\frac{450}{263}$ (d) none of these
   [JEE (WB) 2006]

3. The number of terms in the expansion of $(1 + 2x + x^2)^{20}$, when expanded in descending powers of x, is
   (a) 20 (b) 21 (c) 40 (d) 41

4. The greatest coefficient in the expansion of $(1+x)^{2n}$, is
   (a) ${}^{2n}C_n$ (b) ${}^{2n}C_{n-1}$
   (c) ${}^{2n}C_{n+1}$ (d) ${}^{2n}C_{2n-1}$
   [JEE (WB) 2006]

5. The number of terms in the expansion of $(2x + 3y - 4z)^n$, is
   (a) $n+1$ (b) $n+3$
   (c) $\frac{(n+1)(n+2)}{2}$ (d) none of these

6. Given positive integers $r > 1$, $n > 2$ and the coefficients of $(3r)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal. Then,
   (a) $n = 2r$ (b) $n = 3r$ (c) $n = 2r + 1$ (d) none of these
   [AIEEE 2002]

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7. The number of terms in the expansion of $\left(1+5\sqrt{2}\,x\right)^9+\left(1-5\sqrt{2}\,x\right)^9$, is
(a) 5 (b) 7 (c) 9 (d) 10

8. The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$, is
(a) 2^{20} (b) 2^{19}
(c) $2^{19}+\frac{1}{2}\,{}^{20}C_{10}$ (d) $2^{19}-\frac{1}{2}\,{}^{20}C_{10}$

9. The coefficient of x^{-10} in $\left(x^2-\frac{1}{x^3}\right)^{10}$, is
(a) -252 (b) 210 (c) $-5!$ (d) -120
[JEE (WB) 2008]

10. If the coefficient of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in A.P., then the value of r, is
(a) 5, 9 (b) 6, 9 (c) 7, 9 (d) none of these

11. If C_r stands for ${}^nC_r$, then the sum of first $(n+1)$ terms of the series $a\,C_0-(a+d)\,C_1+(a+2d)\,C_2-(a+3d)\,C_3+\ldots$, is
(a) $\frac{a}{2^n}$ (b) $n\,a$ (c) 0 (d) none of these

12. If $(1+x+x^2)^n=C_0+C_1x+C_2x^2+\ldots$, then the value of $C_0C_1-C_1C_2+C_2C_3-\ldots$, is
(a) 3^n (b) $(-1)^n$ (c) 2^n (d) none of these

13. If the coefficients of second, third and fourth terms in the expansion of $(1+x)^n$ are in A.P., then the value of n, is
(a) 2 (b) 7 (c) 6 (d) 8

14. If the binomial coefficients of 2nd, 3rd and 4th terms in the expansion of
$$\left\{\sqrt{2^{\log_{10}(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_{10}3}}\right\}^m$$
are in A.P. and the 6th term is 21, then the value(s) of x, is (are)
(a) 1, 3 (b) 0, 2 (c) 4 (d) -1

15. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}}+x^2\log_{10}x\right)^8$ is 5600, then x equals
(a) 1 (b) $\log_e 10$ (c) 10 (d) x does not exist

16. If the $(r+1)^{th}$ term in the expansion of $\left\{\sqrt[3]{\frac{a}{\sqrt{b}}}+\sqrt{\frac{b}{\sqrt[3]{a}}}\right\}^{21}$ contains a and b to one and the same power, then the value of r, is
(a) 9 (b) 10 (c) 8 (d) 6

17. If the coefficients of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in A.P., then
(a) $2n^2+9n+7=0$ (b) $2n^2-9n+7=0$
(c) $2n^2-9n-7=0$ (d) none of these

18. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^6$ is
(a) 900 (b) 909 (c) 990 (d) 999

19. If $(1+x-2x^2)^6=1+C_1x+C_2x^2+C_3x^3+\ldots+C_{12}x^{12}$, then the value of $C_2+C_4+C_6+\ldots+C_{12}$, is
(a) 30 (b) 32 (c) 31 (d) none of these

20. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is p and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are q and r, then
(a) $p+q=r$ (b) $p+r=q$ (c) $p=q+r$ (d) $p+q+r=0$

21. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$ equal to
(a) $\frac{a_2}{a_2+a_3}$ (b) $\frac{1}{2}\frac{a_2}{a_2+a_3}$
(c) $\frac{2a_2}{a_2+a_3}$ (d) $\frac{2a_3}{a_2+a_3}$

22. The coefficient of x^r $(0\le r\le(n-1))$ in the expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots+(x+2)^{n-1}$, is
(a) ${}^nC_r\,(3^r-2^n)$ (b) ${}^nC_r\,(3^{n-r}-2^{n-r})$
(c) ${}^nC_r\,(3^r+2^{n-r})$ (d) none of these

23. If $(1-x+x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$, then $a_0+a_2+a_4+\ldots+a_{2n}$ is equal to
(a) $\frac{3^n+1}{2}$ (b) $\frac{3^n-1}{2}$
(c) $\frac{3^{n-1}+1}{2}$ (d) $\frac{3^{n-1}-1}{2}$

24. The coefficient of x^m in
$(1+x)^p+(1+x)^{p+1}+\ldots+(1+x)^n$, $p\le m\le n$ is
(a) ${}^{n+1}C_{m+1}$ (b) ${}^{n-1}C_{m-1}$
(c) ${}^nC_m$ (d) ${}^nC_{m+1}$

25. If the coefficient of x^7 in the expansion of $(ax^2+b^{-1}x^{-1})^{11}$ is equal to the coefficient of x^{-7} in $(ax-b^{-1}x^{-2})^{11}$, then $ab=$
(a) 1 (b) 2 (c) 3 (d) 4
[AIEEE 2005, JEE (WB) 2008]

26. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$, is
(a) 30 (b) 60 (c) 40 (d) none of these
[IP (Delhi) 2003]

27. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$, is

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(a) $\frac{25840}{9}$ (b) $\frac{24840}{9}$

(c) $\frac{26840}{9}$ (d) none of these

28. If $T_0, T_1, T_2, \ldots, T_n$ represent the terms in the expansion of $(x+a)^n$, then the value of $(T_0 - T_2 + T_4 - T_6 \ldots)^2 + (T_1 - T_3 + T_5 + \ldots)^2$, is
(a) $(x^2 - a^2)^n$ (b) $(x^2 + a^2)^n$
(c) $(a^2 - x^2)^n$ (d) none of these

29. The total number of terms in the expansion of $(x+y)^{100} + (x-y)^{100}$ after simplification, is
(a) 50 (b) 51
(c) 202 (d) none of these

30. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio $1:7:42$, then the value of n, is
(a) 60 (b) 70
(c) 55 (d) none of these

31. If the second, third and fourth term in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, then the value of n is
(a) 15 (b) 20 (b) 10 (d) 5

32. The value of $\frac{18^3 + 7^3 + 3 \cdot 18 \cdot 7 \cdot 25}{3^6 + 6 \cdot 243 \cdot 2 + 15 \cdot 181 \cdot 4 + 20 \cdot 27 \cdot 8 + 15 \cdot 9 \cdot 16 + 6 \cdot 3 \cdot 32 + 64}$, is
(a) 10 (b) 1 (c) 2 (d) 20

33. If the coefficients of $(2r+4)^{\text{th}}$ and $(r-2)^{\text{th}}$ terms in the expansion of $(1+x)^{18}$ are equal, then the value of r, is
(a) 5 (b) 6 (c) 7 (d) 9

34. The middle term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$, is
(a) ${}^{10}C_1 \frac{1}{x}$ (b) ${}^{10}C_5$ (c) ${}^{10}C_6$ (d) ${}^{10}C_7 x$

35. The 14th term from the end in the expansion of $\left(\sqrt{x} - \sqrt{y}\right)^{17}$ is
(a) ${}^{17}C_5 x^6 (-\sqrt{y})^5$ (b) ${}^{17}C_6 (\sqrt{x})^{11} y^3$
(c) ${}^{17}C_4 x^{13/2} y^2$ (d) none of these

36. If $[x]$ denotes the greatest integer less than or equal to x and $F = R - [R]$ where $R = (5\sqrt{5} + 11)^{2n+1}$, then RF is equal to
(a) 4^{2n+1} (b) 4^{2n}
(c) 4^{2n-1} (d) none of these

37. If $[x]$ denotes the greatest integer less than or equal to x, then $[(6\sqrt{6} + 14)^{2n+1}]$
(a) is an even integer (b) is an odd integer
(c) depends on n (d) none of these

38. If $n \in N$ such that $(7 + 4\sqrt{3})^n = I + F$, where $I \in N$ and $0 < F < 1$. Then, the value of $(I+F)(1-F)$ is
(a) 0 (b) 1 (c) 7^{2n} (d) 2^{2n}

39. If the ratio of the 7th term from the beginning to the seventh term from the end in the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^x$ is $\frac{1}{6}$ then x, is
(a) 9, (b) 6, 15
(c) 12, 9 (d) none of these

40. The sum of the coefficients of the polynomial $(1 + x - 3x^2)^{2143}$ is
(a) -1 (b) 1
(c) 0 (d) none of these

41. If the sum of the coefficients in the expansion of $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$ vanishes, then the value of α, is
(a) 2 (b) -1 (c) 1 (d) -2

42. If $n \in N$, then the sum of the coefficients in the expansion of the binomial $(5x - 4y)^n$, is
(a) 1 (b) -1 (c) n (d) 0

43. If the sum of the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ is a and if the sum of the coefficients in the expansion of $(1 + x^2)^n$ is b, then
(a) $a = 3b$ (b) $a = b^3$
(c) $b = a^3$ (d) none of these

44. If the coefficient of $(r+1)^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ be equal to that of $(r+3)^{\text{th}}$ term, then
(a) $n - r + 1 = 0$ (b) $n - r - 1 = 0$
(c) $n + r + 1 = 0$ (d) none of these

45. The coefficient of the middle term in the expansion of $(1+x)^{2n}$, is
(a) $\frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{n!} 2^n$ (b) $\frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{(n!)^2} 2^n$
(c) $\frac{(2n)!}{(n!)^2} 2^{2n}$ (d) none of these

46. The coefficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$, is
(a) 30 (b) 60
(c) 40 (d) none of these
[JEE (WB) 2007]

47. If the coefficients of r^{th} and $(r+1)^{\text{th}}$ terms in the expansion of $(3 + 7x)^{29}$ are equal, then r =
(a) 15 (b) 21 (c) 14 (d) none of these
[JEE (WB) 2007]

48. If there is a term containing x^{2r} in $\left(x + \frac{1}{x^2}\right)^{n-3}$, then

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(a) $n-2r$ is a positive integral multiple of 3
(b) $n-2r$ is even
(c) $n-2r$ is odd
(d) none of these

49. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also, is

(a) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (b) $\frac{n+1}{n} < x < \frac{n}{n+1}$

(c) $\frac{n}{n+4} < x < \frac{n+4}{4}$ (d) none of these

50. If the fourth term in the expansion of $\left|\sqrt{x^{\left(\frac{1}{\log x+1}\right)}} + x^{1/12}\right|^6$ is equal to 200 and $x > 1$, then x is equal to

(a) $10^{\sqrt{2}}$ (b) 10 (c) 10^4 (d) none of these

51. The interval in which x must lie so that the numerically greatest term in the expansion of $(1-x)^{21}$ has the numerically greatest coefficient, is

(a) $\left[\frac{5}{6}, \frac{6}{5}\right]$ (b) $\left(\frac{5}{6}, \frac{6}{5}\right)$ (c) $\left(\frac{4}{5}, \frac{5}{4}\right)$ (d) $\left[\frac{4}{5}, \frac{5}{4}\right]$

52. If the r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ coefficients of $(1+x)^n$ are in A.P., then n is a root of the equation

(a) $x^2 - x(4r+1) + 4r^2 - 2 = 0$
(b) $x^2 + x(4r+1) + 4r^2 - 2 = 0$
(c) $x^2 + x(4r+1) + 4r^2 + 2 = 0$
(d) none of these **[AIEEE 2005]**

53. The remainder when 5^{99} is divided by 13, is

(a) 6 (b) 8 (c) 9 (d) 10

54. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then the value of $\sum_{r=0}^{n} (r+1) C_r$, is

(a) $n\,2^n$ (b) $(n+1)\,2^{n-1}$
(c) $(n+2)\,2^{n-1}$ (d) $(n+2)\,2^{n-2}$

55. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficient in the expansion of $(1+x)^n$, then the value of $a C_0 + (a+b) C_1 + (a+2b) C_2 + \ldots + (a+nb) C_n$, is

(a) $(a+nb)\,2^n$ (b) $(a+nb)\,2^{n-1}$
(c) $(2a+nb)\,2^{n-1}$ (d) $(2a+nb)\,2^n$

56. Let $(1+x)^n = \sum_{r=0}^{n} C_r\, x^r$ and,

$$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n\frac{C_n}{C_{n-1}} = \frac{1}{k} n(n+1),$$

then the value of k, is

(a) 1/2 (b) 2 (c) 1/3 (d) 3

57. The value of the sum of the series

$3 \cdot {}^nC_0 - 8\,{}^nC_1 + 13\,{}^nC_2 - 18\,{}^nC_3 + \ldots$ upto $(n+1)$ terms, is

(a) 0 (b) 3^n (c) 5^n (d) none of these

58. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, then for n odd, ${C_1}^2 + {C_3}^2 + {C_5}^2 + \ldots + {C_n}^2$ is equal to

(a) 2^{2n-2} (b) 2^n
(c) $\frac{(2n)!}{2\,(n!)^2}$ (d) $\frac{(2n)!}{(n!)^2}$

59. If $C_0, C_1, C_2, \ldots, C_n$ are coefficients in the binomial expansion of $(1+x)^n$, then $C_0 C_2 + C_1 C_3 + C_2 C_4 + \ldots + C_{n-2} C_n$ is equal to

(a) $\frac{(2n)!}{(n-2)!\,(n+2)!}$ (b) $\frac{(2n)!}{((n-2)!)^2}$
(c) $\frac{(2n)!}{((n+2)!)^2}$ (d) none of these

60. The value of $2C_0 + \frac{2^2}{2} C_1 + \frac{2^3}{3} C_2 + \frac{2^4}{4} C_3 + \ldots + \frac{2^{11}}{11} C_{10}$, is

(a) $\frac{3^{11}-1}{11}$ (b) $\frac{2^{11}-1}{11}$
(c) $\frac{11^3-1}{11}$ (d) $\frac{11^2-1}{11}$

61. If m, n, r are positive integers such that $r < m, n$, then ${}^mC_r + {}^mC_{r-1}\,{}^nC_1 + {}^mC_{r-2}\,{}^nC_2 + \ldots + {}^mC_1\,{}^nC_{r-1} + {}^nC_r$ equals

(a) $({}^nC_r)^2$ (b) ${}^{m+n}C_r$
(c) ${}^{m+n}C_r + {}^mC_r + {}^nC_r$ (d) none of these

62. The value of

$$\frac{1}{81^n} - \frac{10}{81^n}\,{}^{2n}C_1 + \frac{10^2}{81^n}\,{}^{2n}C_2 - \frac{10^3}{81^n}\,{}^{2n}C_3 + \ldots + \frac{10^{2n}}{81^n}, \text{ is}$$

(a) 2 (b) 0 (c) 1/2 (d) 1

63. The term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^4 \left(x + \frac{1}{x}\right)^3$, is

(a) -3 (b) 0 (c) 1 (d) 3

64. The positive value of a so that the coefficients of x^5 and x^{15} are equal in the expansion of $\left(x^2 + \frac{a}{x^3}\right)^{10}$

(a) $\frac{1}{2\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $2\sqrt{3}$

65. If n is a positive integer and $C_k = {}^nC_k$, then $\sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2$ equals

(a) $\frac{n(n+1)(n+2)}{12}$ (b) $\frac{n(n+1)^2(n+2)}{12}$
(c) $\frac{n(n+1)(n+2)^2}{12}$ (d) none of these

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66. The coefficient of x^{50} in the expression
$(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \ldots + 1001\,x^{1000}$ is
(a) ${}^{1000}C_{50}$ (b) ${}^{1001}C_{50}$ (c) ${}^{1002}C_{50}$ (d) ${}^{1000}C_{51}$

67. If C_r stands for ${}^nC_r$, then the sum of the series
$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 + \ldots + (-1)^n (n+1) C_n^2]$,
where n is an even positive integer, is equal to
(a) 0 (b) $(-1)^{n/2}(n+1)$
(c) $(-1)^{n/2}(n+2)$ (d) $(-1)^n n$

68. The coefficient of x^n in the expansion of $\frac{1}{(1-x)(3-x)}$, is
(a) $\frac{3^{n+1}-1}{2\cdot 3^{n+1}}$ (b) $\frac{3^{n+1}-1}{3^{n+1}}$
(c) $2\left(\frac{3^{n+1}-1}{3^{n+1}}\right)$ (d) none of these

69. The coefficient of x^n in the expansion of $\frac{(1+x)^2}{(1-x)^3}$, is
(a) n^2+2n+1 (b) $2n^2+n+1$
(c) $2n^2+2n+1$ (d) n^2+2n+2

70. The coefficient of x^n in the expansion of $(1-2x+3x^2-4x^3+\ldots)^{-n}$, is
(a) $\frac{(2n)!}{n!}$ (b) $\frac{(2n)!}{(n!)^2}$
(c) $\frac{1}{2}\frac{(2n)!}{(n!)^2}$ (d) none of these

71. If $(r+1)^{\text{th}}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r, is
(a) 5 (b) 6 (c) 4 (d) 7

72. The coefficient of x^7 in the expansion of $(x-2x^2)^{-3}$, is
(a) 67485 (b) 67548 (c) 67584 (d) 67845

73. The coefficient of x^n in the expansion of $(1+x+x^2+\ldots)^{-n}$, is
(a) 1 (b) $(-1)^n$ (c) n (d) $n+1$

74. If the binomial expansion of $(a+bx)^{-2}$ is $\frac{1}{4} - 3x + \ldots$, then $(a, b) =$
(a) (2, 12) (b) (2, 8)
(c) (−2, −12) (d) none of these

75. If $C_r = {}^nC_r$ and $(C_0+C_1)(C_1+C_2)\ldots(C_{n-1}+C_n) = k\frac{(n+1)^n}{n!}$, then the value of k, is
(a) $C_0 C_1 C_2 \ldots C_n$ (b) $C_1^2 C_2^2 \ldots C_n^2$
(c) $C_1 + C_2 + \ldots + C_n$ (d) none of these

76. If the third term in the binomial expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m, is
(a) 2 (b) 1/2 (c) 3 (d) 4

77. If p is nearly equal to q and $n > 1$, such that $\frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q} = \left(\frac{p}{q}\right)^k$, then the value of k, is
(a) n (b) $\frac{1}{n}$ (c) $n+1$ (d) $\frac{1}{n+1}$

78. If $y = 3x + 6x^2 + 10x^3 + \ldots$, then $x =$
(a) $\frac{4}{3} - \frac{1\cdot 4}{3^2\cdot 2}y^2 + \frac{1\cdot 4\cdot 7}{3^2\cdot 3}y^3 \ldots$
(b) $-\frac{4}{3} + \frac{1\cdot 4}{3^2\cdot 2}y^2 - \frac{1\cdot 4\cdot 7}{3^2\cdot 3}y^3 + \ldots$
(c) $\frac{4}{3} + \frac{1\cdot 4}{3^2\cdot 2}y^2 + \frac{1\cdot 4\cdot 7}{3^2\cdot 3}y^3 + \ldots$
(d) none of these

79. If $y = \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \ldots$, then the value of $y^2 + 2y$ is
(a) 2 (b) −2
(c) 0 (d) none of these

80. If $(1+2x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r =$
(a) $({}^nC_r)^2$ (b) ${}^nC_r \cdot {}^nC_{r+1}$
(c) ${}^{2n}C_r$ (d) ${}^{2n}C_{r+1}$

81. The coefficient of x^3 in $\left(\sqrt{x^5} + \frac{3}{\sqrt{x^3}}\right)^6$, is
(a) 0 (b) 120 (c) 420 (d) 540

82. The number of non-zero terms in the expansion of $(1+3\sqrt{2}x)^9 + (1-3\sqrt{2}x)^9$, is
(a) 9 (b) 0 (c) 5 (d) 10

83. The coefficient of y in the expansion of $(y^2 + c/y)^5$, is
(a) $29c$ (b) $10c$ (c) $10c^3$ (d) $20c^2$

84. The greatest coefficient in the expansion of $(1+x)^{10}$, is
(a) $\frac{10!}{5!6!}$ (b) $\frac{10!}{(5!)^2}$
(c) $\frac{10!}{5!7!}$ (d) none of these

85. The approximate value of $(7.995)^{1/3}$ correct to four decimal places, is
(a) 1.9995 (b) 1.9996 (c) 1.9990 (d) 1.9991

86. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$, is
(a) ${}^nC_4$ (b) ${}^nC_4 + {}^nC_2$
(c) ${}^nC_4 + {}^nC_1 + {}^nC_4 \times {}^nC_2$ (d) ${}^nC_4 + {}^nC_2 + {}^nC_1 \times {}^nC_2$

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87. The remainder when $32^{(32)^{(32)}}$ is divided by 7, is
(a) 1 (b) 2 (c) 3 (d) 4

88. If x^m occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{2n}$, the coefficient of x^m, is
(a) $\frac{(2n)!}{m!\,(2n-m)!}$ (b) $\frac{(2n)!\,3!\,3!}{(2n-m)!}$
(c) $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ (d) none of these

89. If $n>1$, then $(1+x)^n - nx - 1$ is divisible by
(a) $2x$ (b) x^2 (c) x^3 (d) x^4

90. The number of terms with integral coefficients in the expansion of $(7^{1/3}+5^{1/2}x)^{600}$, is
(a) 100 (b) 50 (c) 101 (d) none of these

91. The term independent of x in the expansion of $(1-x)^2\left(x+\frac{1}{x}\right)^{10}$, is
(a) ${}^{11}C_5$ (b) ${}^{10}C_5$ (c) ${}^{10}C_4$ (d) none of these

92. The range of values of the term independent of x in the expansion of $\left(x\sin^{-1}\alpha+\frac{\cos^{-1}\alpha}{x}\right)^{10}$, $\alpha\in[-1,1]$, is
(a) $\left[-\frac{{}^{10}C_5\,\pi^{10}}{2^5}, \frac{{}^{10}C_5\,\pi^{10}}{2^{20}}\right]$ (b) $\left[\frac{{}^{10}C_5\,\pi^2}{2^{20}}, \frac{{}^{10}C_5\,\pi^2}{2^5}\right]$
(c) $[1,2]$ (d) $(1,2)$

93. If the sum of the coefficients in the expansion of $(\alpha x^2 - 2x + 1)^{35}$ is equal to the sum of the coefficient in the expansion of $(x-\alpha y)^{35}$, then $\alpha =$
(a) 0 (b) 1
(c) any real number (d) none of these

94. If the coefficients of r^{th} and $(r+1)^{th}$ terms in the expansion of $(3+7x)^{29}$ are equal, then r equals
(a) 15 (b) 21 (c) 14 (d) none of these

95. The sum of the magnitudes of the coefficients in the expansion of $(1-x+x^2-x^3)^n$, is
(a) 0 (b) 2^n (c) 3^n (d) 4^n

96. If the ninth term in the expansion of
$$\left\{3^{\log_3\sqrt{25^{x-1}+7}} + 3^{-1/8\log_3(5^{x-1}+1)}\right\}^{10}$$
is equal to 180 and $x>1$, then x equals
(a) $\log_{10}15$ (b) $\log_5 15$
(c) $\log_e 15$ (d) none of these

97. If $n>3$, then
$xyz\,C_0 - (x-1)(y-1)(z-1)\,C_1 + (x-2)(y-2)(z-2)\,C_2 - (x-3)(y-3)(z-3)\,C_3 + \ldots + (-1)^n (x-n)(y-n)(z-n)\,C_n$ equals
(a) xyz (b) $nxyz$ (c) $-xyz$ (d) 0

98. The coefficient of x^5 in the expansion of $\frac{1+x^2}{1+x}$, $|x|<1$, is
(a) -1 (b) 2 (c) 0 (d) -2

99. The digit at unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$, is
(a) 0 (b) 1 (c) 2 (d) 3

100. The expression
$$\frac{1}{\sqrt{4x+1}}\left\{\left(1+\frac{\sqrt{4x+1}}{2}\right)^7 - \left(1-\frac{\sqrt{4x+1}}{2}\right)^7\right\}$$
is a polynomial in x of degree
(a) 7 (b) 5 (c) 4 (d) 3

101. Let $(1+x)^n = \sum_{r=0}^{n} a_r x^r$. Then,
$\left(1+\frac{a_1}{a_0}\right)\left(1+\frac{a_2}{a_1}\right)\ldots\left(1+\frac{a_n}{a_{n-1}}\right)$ is equal to
(a) $\frac{(n+1)^{n+1}}{n!}$ (b) $\frac{(n+1)^n}{n!}$
(c) $\frac{n^{n-1}}{(n-1)!}$ (d) $\frac{(n+1)^{n-1}}{(n-1)!}$

102. If n is an odd natural number and ${}^nC_0 < {}^nC_1 < {}^nC_2 < \ldots < {}^nC_r > {}^nC_{r+1} > {}^nC_{r+2} > \ldots > {}^nC_n$, then $r =$
(a) $\frac{n}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n-2}{2}$ (d) does not exist

103. The coefficient of x^5 in the expansion of $(2-x+3x^2)^6$, is
(a) -4692 (b) 4692 (c) 2346 (d) -5052

104. If $(1+2x+3x^2)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{20} x^{20}$, then a_1 equals
(a) 10 (b) 20 (c) 210 (d) none of these

105. The coefficient of $x^8 y^6 z^4$ in the expansion of $(x+y+z)^{18}$, is not equal to
(a) ${}^{18}C_{14}\times{}^{14}C_8$ (b) ${}^{18}C_{10}\times{}^{10}C_6$
(c) ${}^{18}C_6\times{}^{12}C_8$ (d) ${}^{18}C_6\times{}^{14}C_6$

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106. If $\sum_{r=0}^{2n} a_r (x-100)^r = \sum_{r=0}^{2n} b_r (x-101)^r$ and $a_k = \frac{2^k}{{}^kC_n}$ for all $k \geq n$, then b_n equals

(a) $2^n (2^{n+1} - 1)$

(b) $2^n (2^n + 1)$

(c) $2^n (2^n - 1)$

(d) $2^{n+1} (2^n - 1)$

107. The value of
$1 \times 2 \times 3 \times 4 + 2 \times 3 \times 4 \times 5 + 3 \times 4 \times 5 \times 6 + \ldots + n(n+1)(n+2)(n+3)$, is

(a) $\frac{1}{5}(n+1)(n+2)(n+3)(n+4)(n+5)$

(b) $\frac{1}{5} n (n+1)(n+2)(n+3)(n+4)$

(c) $\frac{1}{5} n (n+1)(n+2)(n+3)(n+4)$

(d) ${}^{n+4}C_5$

Answers

1. (a) 2. (d) 3. (b) 4. (c) 5. (c) 6. (a) 7. (a)
8. (c) 9. (b) 10. (a) 11. (c) 12. (d) 13. (b) 14. (b)
15. (c) 16. (a) 17. (b) 18. (c) 19. (c) 20. (c) 21. (c)
22. (b) 23. (a) 24. (a) 25. (a) 26. (b) 27. (a) 28. (b)
29. (b) 30. (c) 31. (d) 32. (b) 33. (b) 34. (b) 35. (c)
36. (a) 37. (a) 38. (b) 39. (a) 40. (a) 41. (c) 42. (a)
43. (b) 44. (b) 45. (a) 46. (b) 47. (b) 48. (a) 49. (a)
50. (b) 51. (b) 52. (a) 53. (b) 54. (c) 55. (c) 56. (b)
57. (a) 58. (c) 59. (a) 60. (a) 61. (b) 62. (d) 63. (b)
64. (a) 65. (b) 66. (c) 67. (c) 68. (a) 69. (c) 70. (b)
71. (a) 72. (c) 73. (b) 74. (a) 75. (a) 76. (b) 77. (b)
78. (d) 79. (a) 80. (c) 81. (d) 82. (c) 83. (c) 84. (b)
85. (b) 86. (d) 87. (d) 88. (c) 89. (b) 90. (c) 91. (a)
92. (a) 93. (b) 94. (b) 95. (d) 96. (b) 97. (d) 98. (d)
99. (b) 100. (d) 101. (b) 102. (b) 103. (d) 104. (b) 105. (d)
106. (a) 107. (b)

CHAPTER TEST

Each question has four choices (a), (b), (c) and (d), out of which only one is correct. Mark the correct choice.

1. The term independent of x in the expansion of $(1+x)^n (1+1/x)^n$, is

(a) $C_0^2 + 2C_1^2 + 3 \cdot C_2^2 + \ldots + (n+1) C_n^2$

(b) $(C_0 + C_1 + \ldots + C_n)^2$

(c) $C_0^2 + C_1^2 + \ldots + C_n^2$

(d) none of these

2. The expression $[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$ is a polynomial of degree

(a) 5 (b) 6 (c) 7 (d) 8

3. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$, is

(a) ${}^{100}C_{47}$ (b) ${}^{100}C_{53}$

(c) $-{}^{100}C_{53}$ (d) $-{}^{100}C_{100}$

4. The value of $C_0 + 3C_1 + 5C_2 + 7C_3 \ldots + (2n+1) C_n$ is equal to

(a) 2^n (b) $2^n + n \cdot 2^{n-1}$

(c) $2^n \cdot (n+1)$ (d) none of these

5. The largest term in the expansion of $(3 + 2x)^{50}$ where $x = 1/5$ is

(a) 5th, 6th (b) 51st (c) 6th, 7th (d) 7th, 8th

6. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is

(a) 0 (b) 2^{49} (c) 2^{50} (d) 2^{51}

7. The term independent of x in $\left\{\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right\}^{10}$, is

(a) $\frac{9}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{7}{4}$

8. If the coefficients of x^7 and x^8 in $(2 + x/3)^n$ are equal, then n is equal to

(a) 56 (b) 55 (c) 45 (d) 15

9. If the rth term in the expansion of $(x/3 - 2/x^2)^{10}$ contains x^4, then r is equal to

(a) 2 (b) 3 (c) 4 (d) 5

10. If the third term in the expansion of $[x + x^{\log_{10} x}]^5$ is 10^6, then x (> 1) may be

(a) 1 (b) 10 (c) $10^{-5/2}$ (d) 10^2

11. The value of $(x > 1)$, for which the 6th term in the expansion of

$$\left\{2^{\log_2 \sqrt{(9^{x-1}+7)}} + \frac{1}{2^{(1/5)\log_2 (3^{x-1}+1)}}\right\}^7$$

is 84 is equal to

(a) 4 (b) 3 (c) 2 (d) 1

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12. In the expansion of $(1+x)^{2n}$ $(n \in N)$, the coefficients of $(p+1)^{th}$ and $(p+3)^{th}$ terms are equal, then
(a) $p = n-2$ (b) $p = n-1$
(c) $p = n+1$ (d) $p = 2n-2$

13. In the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$, the coefficient of x^{39}, is
(a) 1365 (b) -1365
(c) 455 (d) -455

14. The value of $C_0^2 + 3 \cdot C_1^2 + 5 \cdot C_2^2 + \ldots$ to $(n+1)$ terms, is
(a) ${}^{2n-1}C_{n-1}$ (b) $(2n+1)\,{}^{2n-1}C_n$
(c) $2(n+1) \cdot {}^{2n-1}C_{n-1}$ (d) ${}^{2n-1}C_n + (2n+1)\,{}^{2n-1}C_{n-1}$

15. The value of $\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + \ldots$, is
(a) $\frac{2^{n-2}}{(n-1)!}$ (b) $\frac{2^{n-1}}{n!}$
(c) $\frac{2^n}{n!}$ (d) $\frac{2^n}{(n-1)!}$

16. The coefficient of $x^n y^n$ in the expansion of $(1+x)(1+y)(x+y)^n$, is
(a) $\sum_{r=0}^{n} C_r^2$ (b) $\sum_{r=0}^{n} C_{r+2}^2$ (c) $\sum_{r=0}^{n} C_{r+3}^2$ (d) $\sum_{r=0}^{n} C_r^3$

17. If $(1+x-2x^2)^6 = 1 + a_1 x + a_2 x^2 + \ldots + a_{12} x^{12}$, then $a_2 + a_4 + a_6 + \ldots + a_{12} =$
(a) 30 (b) 65 (c) 31 (d) 63

18. The ratio of the coefficient of x^{15} to the term independent of x in $\left(x^2 + \frac{2}{x}\right)^{15}$, is
(a) 1/4 (b) 1/16 (c) 1/32 (d) 1/64

19. The number of terms in the expansion of $(x+y+z)^{10}$, is
(a) 11 (b) 33 (c) 66 (d) 1000

20. In the expansion of $(1+x)^{30}$, the sum of the coefficients of odd powers of x, is
(a) 2^{30} (b) 2^{31} (c) 0 (d) 29

21. The number of terms in the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n$, $n \in N$, is
(a) $2n$ (b) $3n$ (c) $2n+1$ (d) $3n+1$

22. The constant term in the expansion of $(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}$ is
(a) ${}^{22}C_{10}$ (b) 0 (c) ${}^{22}C_{11}$ (d) none of these

23. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$, the constant term, is
(a) ${}^{15}C_6$ (b) 0 (c) $-{}^{15}C_6$ (d) 1

24. The middle term in the expansion of $\left(1 - \frac{1}{x}\right)^n (1-x)^n$, is
(a) ${}^{2n}C_n$ (b) $-{}^{2n}C_n$
(c) $-{}^{2n}C_{n-1}$ (d) none of these

25. The number of terms whose values depend on x in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$, is
(a) $2n+1$ (b) $2n$
(c) n (d) none of these

26. The number of rational terms in the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$, is
(a) 3 (b) 18 (c) 4 (d) 16

27. The coefficient of x^6 in $\{(1+x)^6 + (1+x)^7 + \ldots + (1+x)^{15}\}$ is
(a) ${}^{16}C_9$ (b) ${}^{16}C_5 - {}^6C_5$
(c) ${}^{16}C_6 - 1$ (d) none of these

28. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}$, $n \in N$, $x > 0$, is
(a) n (b) $n+1$ (c) $n-1$ (d) $2n$

29. In the expansion of $\left(x + \sqrt{x^2-1}\right)^6 + \left(x - \sqrt{x^2-1}\right)^6$, the number of terms, is
(a) 7 (b) 14 (c) 6 (d) 4

30. The last term in the binomial expansion of $\left(\sqrt[3]{2} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3 \cdot \sqrt[3]{9}}\right)^{\log_3 8}$. Then, the 5th term from the beginning is
(a) ${}^{10}C_6$ (b) $2 \times {}^{10}C_4$
(c) $\frac{1}{2} \times {}^{10}C_4$ (d) none of these

31. The coefficient of $x^6 a^{-2}$ in the expansion of $\left(\frac{x^2}{a} - \frac{a}{x}\right)^{12}$, is
(a) ${}^{12}C_6$ (b) $-{}^{12}C_5$
(c) 0 (d) none of these

32. If in the expansion of $(1+ax)^n$, $n \in N$, the coefficients of x and x^2 are 8 and 24 respectively, then
(a) $a = 2, n = 4$ (b) $a = 4, n = 2$
(c) $a = 2, n = 6$ (d) $a = -2, n = 4$

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33. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n \in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then n =
(a) 25 (b) 20
(c) 15 (d) none of these

34. The coefficient of x^{20} in the expansion of $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$, is
(a) ${}^{30}C_{10}$ (b) ${}^{30}C_{25}$
(c) 1 (d) none of these

35. If the sum of the numerical coefficients in the binomial expansion of $\left(\frac{1}{x}+2x\right)^n$ is equal to 6561, then the term independent of x, is
(a) ${}^8C_4$ (b) ${}^8C_4 \times 2^4$
(c) ${}^6C_4 \times 2^4$ (d) none of these

36. The sum ${}^{40}C_0 + {}^{40}C_1 + {}^{40}C_2 + \ldots + {}^{40}C_{20}$ is equal to
(a) $2^{40}+\frac{40!}{(20!)^2}$ (b) $2^{39}-\frac{1}{2}\times\frac{40!}{(20!)^2}$
(c) $2^{39}+{}^{40}C_{20}$ (d) none of these

37. If $(1+x)^{2n} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$, then $(a_0 - a_2 + a_4 - a_6 + \ldots - a_{2n})^2 + (a_1 - a_3 + a_5 - a_7 + \ldots + a_{2n-1})^2$ is equal to
(a) 2^n (b) 4^n
(c) 0 (d) none of these

38. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is
(a) 6th term (b) 7th term
(c) 5th term (d) 8th term

39. The number of integral terms in the expansion of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$, is
(a) 35 (b) 32 (c) 33 (d) 34

40. The coefficient of term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{x^2}\right)^{10}$, is
(a) 5/3 (b) 4/5 (c) 6 (d) 1/2

41. If the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a is
(a) $\frac{9}{7}$ (b) $\frac{7}{9}$ (c) $-\frac{9}{7}$ (d) $-\frac{7}{9}$

42. r and n are positive integers such that $r > 1$, $n > 2$ and coefficients of $(r+2)^{th}$ term and $(3r)^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals
(a) $3r$ (b) $3r+1$ (c) $2r$ (d) $2r+1$

43. If in the expansion of $(1+x)^{20}$, the coefficients of r^{th} and $(r+4)^{th}$ terms are equal, then the value of r, is
(a) 7 (b) 8 (c) 9 (d) 10

44. The coefficient of x^5 in the expansion of $(x+3)^6$ is
(a) 18 (b) 6 (c) 12 (d) 10

45. Coefficient of x^n in the expansion of $\frac{(1+x)^n}{1-x}$
(a) $4n$ (b) 2^n (c) n^2 (d) $\frac{n(n+1)}{2}$

46. The sum of the rational terms in the expansion of $(2^{1/5}+\sqrt{3})^{20}$, is
(a) 71 (b) 85 (c) 97 (d) none of these

47. The expression
$$\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$$
is a polynomial of degree
(a) 6 (b) 8 (c) 10 (d) 12

48. If the sum of the coefficients of the first, second and third terms in the expansion of $\left(x^2+\frac{1}{x}\right)^m$ is 46, then the coefficient of the term independent of x, is
(a) 84 (b) 92 (c) 98 (d) 106

49. The coefficient of x^4 in the expansion of $(1+x+x^3+x^4)^{10}$, is
(a) ${}^{40}C_4$ (b) ${}^{10}C_4$ (c) 210 (d) 310

50. The coefficient of x^5 in $(1+x^2)^5(1+x)^4$ is
(a) 20 (b) 30 (c) 60 (d) 55

51. If in the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n \in N$, sum of the coefficients of x^5 and x^{10} is zero, then n =
(a) 5 (b) 10 (c) 15 (d) 20

52 $\sum_{k=1}^{\infty} k\left(1+\frac{1}{n}\right)^{k-1}$ =
(a) $n(n-1)$ (b) $n(n+1)$
(c) n^2 (d) $(n+1)^2$

53. The coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$, is
(a) 476 (b) 496 (c) 506 (d) 528

54. The expansion of $(8-3x)^{3/2}$ in terms of powers of x is valid only if
(a) $x > \frac{8}{3}$ (b) $|x| < \frac{8}{3}$ (c) $x < \frac{3}{8}$ (d) $x < \frac{8}{3}$

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55. If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a, is
(a) $-\frac{7}{9}$ (b) $-\frac{9}{7}$ (c) $\frac{7}{9}$ (d) $\frac{9}{7}$

56. If $x=\frac{1}{3}$, then the greatest term in the expansion of $(1+4x)^8$ is the
(a) 3rd term (b) 6th term (c) 5th term (d) 4th term

57. $\frac{1}{n!}+\frac{1}{2!\,(n-2)!}+\frac{1}{4!\,(n-4)!}+\ldots$ is equal to
(a) $\frac{2^{n-1}}{n!}$ (b) $\frac{2^n}{(n+1)!}$ (c) $\frac{2^n}{n!}$ (d) $\frac{2^{n-2}}{(n-1)!}$

58. The coefficient of x^n in the binomial expansion of $(1-x)^{-2}$, is
(a) $\frac{2^n}{2!}$ (b) $n+1$ (c) n (d) 2^n

59. The coefficient of x^6 in the expansion of $(1+x+x^2)^{-3}$ is
(a) 6 (b) 5 (c) 4 (d) 3

60. $\sum\limits_{0\le i<j\le 10}\sum {}^{10}C_j\ {}^{j}C_i$ is equal to
(a) $2^{10}-1$ (b) 2^{10} (c) $3^{10}-1$ (d) 3^{10}

Answers

1. (c) 2. (c) 3. (c) 4. (c) 5. (c) 6. (b) 7. (c)
8. (b) 9. (b) 10. (b) 11. (c) 12. (b) 13. (d) 14. (c)
15. (b) 16. (d) 17. (c) 18. (c) 19. (c) 20. (d) 21. (c)
22. (a) 23. (c) 24. (a) 25. (b) 26. (a) 27. (a) 28. (a)
29. (d) 30. (a) 31. (c) 32. (a) 33. (c) 34. (b) 35. (b)
36. (d) 37. (b) 38. (d) 39. (c) 40. (a) 41. (a) 42. (c)
43. (c) 44. (a) 45. (b) 46. (d) 47. (a) 48. (a) 49. (d)
50. (c) 51. (c) 52. (c) 53. (a) 54. (d) 55. (d) 56. (b)
57. (a) 58. (b) 59. (d) 60. (c)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on "**Solutions of Objective Mathematics**".

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15

EXPONENTIAL AND LOGARITHMIC SERIES

1. THE NUMBER e

Leonhard Euler, the great Swiss mathematician introduced and named the number e in his calculus text in 1748. It is defined as follows.

DEFINITION *The sum of the infinite infinite series*

$$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots\infty$$

is denoted by the number e.

THEOREM 1 *Prove that* $e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

PROOF Let n be a natural number greater than 1, then $\frac{1}{n}<1$.

Using binomial theorem, we have

$$\left(1+\frac{1}{n}\right)^n=1+n\left(\frac{1}{n}\right)+\frac{n(n-1)}{2!}\left(\frac{1}{n}\right)^2+\frac{n(n-1)(n-2)}{3!}\left(\frac{1}{n}\right)^3+\ldots$$

$$\Rightarrow\quad \left(1+\frac{1}{n}\right)^n=1+1+\frac{1}{2!}\left(1-\frac{1}{n}\right)+\frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)+\ldots$$

This is an identity and it is true for all values of n, however large to be.

If n is infinitely large i.e. $n\to\infty$, then

$\frac{1}{n},\frac{2}{n},\frac{3}{n}$ etc. will approach to zero.

$$\therefore\quad \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots$$

$$\Rightarrow\quad \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots=e$$

Hence,

$$e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

THEOREM 2 *Prove that* $2<e<3$.

PROOF We have,

$$e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots\infty$$

$$\Rightarrow\quad e=2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots\infty$$

$$\Rightarrow\quad e-2=\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots\infty$$

$$\Rightarrow\quad e-2=\text{a positive number}$$

$$\Rightarrow\quad e>2 \qquad \ldots(i)$$

We know that

$$n!>2^{n-1}\text{ for all }n>2$$

$$\therefore\quad \frac{1}{n!}<\frac{1}{2^{n-1}}\text{ for all }n>2$$

$$\Rightarrow\quad \frac{1}{3!}<\frac{1}{2^2},\ \frac{1}{4!}<\frac{1}{2^3},\ \frac{1}{5!}<\frac{1}{2^4},\ldots$$

$$\Rightarrow\quad \frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\ldots<\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots$$

$$\Rightarrow\quad 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}\ldots<1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots$$

$$\Rightarrow\quad 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots<1+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots$$

$$\Rightarrow\quad e<1+\frac{1}{\left(1-\frac{1}{2}\right)}\qquad\left[\because 1+\frac{1}{2}+\frac{1}{2^2}+\ldots=\frac{1}{\left(1-\frac{1}{2}\right)}=2\right]$$

$$\Rightarrow\quad e<1+2=3 \qquad \ldots(ii)$$

From (i) and (ii), we get $2<e<3$.

Hence, e lies between 2 and 3.

THEOREM 3 *Prove that* $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\infty$

or,
$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

PROOF Let n be a natural number greater than 1. Then, for any x, we have

$$\left(1+\frac{1}{n}\right)^{nx}=1+nx\cdot\frac{1}{n}+\frac{(nx)(nx-1)}{2!}\frac{1}{n^2}+\frac{(nx)(nx-1)(nx-2)}{3!}\frac{1}{n^3}+\ldots\infty$$

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EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. A matrix $A = [a_{ij}]$ is an upper triangular matrix, if
   (a) it is a square matrix and $a_{ij} = 0, i < j$
   (b) it is a square matrix and $a_{ij} = 0, i > j$
   (c) it is not a square matrix and $a_{ij} = 0, i > j$
   (d) it is not a square matrix and $a_{ij} = 0, i < j$

2. If A is any $m \times n$ matrix such that AB and BA are both defined, then B is an
   (a) $m \times n$ matrix (b) $n \times m$ matrix
   (c) $n \times n$ matrix (d) $m \times m$ matrix

3. If $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $E(\alpha)\,E(\beta)$ is equal to
   (a) $E(0^\circ)$ (b) $E(\alpha\beta)$ (c) $E(\alpha+\beta)$ (d) $E(\alpha-\beta)$

4. If $E(\theta) = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and θ and ϕ differ by an odd multiple of $\pi/2$, then $E(\theta)\,E(\phi)$ is a
   (a) null matrix (b) unit matrix
   (c) diagonal matrix (d) none of these

5. If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$ and, $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$ are two matrices such that the product AB is the null matrix, then $(\alpha - \beta)$ is
   (a) 0 (b) multiple of π
   (c) an odd multiple of $\pi/2$ (d) none of these

6. The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is
   (a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ (d) none of these

7. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B equals
   (a) $I\cos\theta + J\sin\theta$ (b) $I\sin\theta + J\cos\theta$
   (c) $I\cos\theta - J\sin\theta$ (d) $-I\cos\theta + J\sin\theta$

8. If A is a square matrix such that $AA^T = I = A^TA$, then A is
   (a) a symmetric matrix (b) a skew-symmetric matrix
   (c) a diagonal matrix (d) an orthogonal matrix.

9. If A is an orthogonal matrix, then A^{-1} equals
   (a) A (b) A^T (c) A^2 (d) none of these

10. If $D = \text{diag}\,(d_1, d_2, d_3, \ldots, d_n)$, where $d_i \neq 0$ for all $i = 1, 2, \ldots, n$, then D^{-1} is equal to
    (a) D (b) diag $(d_1^{-1} d_2^{-1}, \ldots, d_n^{-1})$
    (c) I_n (d) none of these

11. If $A = \begin{bmatrix} b & b^2 \\ -a^2 & -ab \end{bmatrix}$, then A is
    (a) Iidempotent (b) involutory (c) nilpotent (d) scalar

12. If A is a 3×4 matrix and B is a matrix such that A^TB and BA^T are both defined, then order of B is
    (a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3

13. Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, then the values of x and y are
    (a) $x = -\frac{1}{11}, y = \frac{2}{11}$ (b) $x = -\frac{1}{11}, y = -\frac{2}{11}$
    (c) $x = \frac{1}{11}, y = \frac{2}{11}$ (d) $x = \frac{1}{11}, y = -\frac{2}{11}$

14. If A, B are two square matrices such that $AB = A$ and $BA = B$, then
    (a) A, B are idempotent (b) only A is idempotent
    (c) only B is idempotent (d) none of these

15. The inverse of a symmetric matrix is
    (a) symmetric (b) skew-symmetric
    (c) diagonal matrix (d) none of these

16. The inverse of a diagonal matrix is
    (a) a symmetric matrix (b) a skew-symmetric matrix
    (c) a diagonal matrix (d) none of these

17. If A is a symmetric matrix and $n \in N$, then A^n is
    (a) symmetric (b) skew-symmetric
    (c) a diagonal matrix (d) none of these

18. If A is a skew-symmetric matrix and n is a positive integer, then A^n is
    (a) a symmetric matrix (b) skew-symmetric matrix
    (c) diagonal matrix (d) none of these

19. If A is a skew-symmetric matrix and n is odd positive integer, then A^n is
    (a) a symmetric matrix (b) a skew-symmetric matrix
    (c) a diagonal matrix (d) none of these

20. If A is a skew-symmetric matrix and n is an even positive integer, then A^n is
    (a) a symmetric matrix (b) a skew-symmetric matrix
    (c) a diagonal matrix (d) none of these

21. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n, then $a_{ii} =$
    (a) 0 for some i (b) 0 for all $i = 1, 2, \ldots, n$
    (c) 1 for some i (d) 1 for all $i = 1, 2, \ldots, n$.

22. If A, B are symmetric matrices of the same order then $AB - BA$ is
    (a) symmetric matrix (b) skew-symmetric matrix
    (c) null matrix (d) unit matrix

23. If A and B are matrices of the same order, then $(A+B)^2 = A^2 + 2AB + B^2$ is possible, iff
    (a) $AB = I$ (b) $BA = I$ (c) $AB = BA$ (d) none of these

24. If $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$, then trace of matrix A is
    (a) 17 (b) 25 (c) 3 (d) 12

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25. If A is a skew-symmetric matrix, then trace of A is
(a) 1 (b) -1 (c) 0 (d) none of these

26. If $A=\begin{bmatrix}1 & x\\ x^2 & 4y\end{bmatrix}a$, $B=\begin{bmatrix}-3 & 1\\ 1 & 0\end{bmatrix}$ and adj $A+B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then the values of x and y are respectively
(a) $(1, 1)$ (b) $(-1, 1)$ (c) $(1, 0)$ (d) none of these

27. If A is a square matrix of order $n\times n$ and k is a scalar, then adj (kA) is equal to
(a) k adj A (b) k^n adj A (c) k^{n-1} adj A (d) k^{n+1} adj A

28. If A is a singular matrix, then adj A is
(a) singular (b) non-singular
(c) symmetric (d) not defined

29. If A is a square matrix or order $n\times n$, then adj (adj A) is equal to
(a) $|A|^n A$ (b) $|A|^{n-1} A$ (c) $|A|^{n-2} A$ (d) $|A|^{n-3} A$

30. If A is a singular matrix, then A adj A is
(a) identity matrix (b) null matrix
(c) scalar matrix (d) none of these

31. If $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$ and A (adj A) $=k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then the value of k is
(a) $\sin x \cos x$ (b) 1 (c) 2 (d) 3

32. If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ and $n\in N$, then A^n is equal to
(a) $2^n A$ (b) $2^{n-1} A$ (c) nA (d) none of these

33. If $A=\begin{bmatrix}a & b\\ b & a\end{bmatrix}$ and $A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}$, then
(a) $\alpha=a^2+b^2,\ \beta=ab$ (b) $\alpha=a^2+b^2,\ \beta=2ab$
(c) $\alpha=a^2+b^2,\ \beta=a^2-b^2$ (d) $\alpha=2ab,\ \beta=a^2+b^2$.
[AIEEE 2003]

34. If A is a square matrix, then adj A^T- (adj $A)^T$ is equal to
(a) $2|A|$ (b) $2|A|I$
(c) null matrix (d) unit matrix

35. If $A=\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$ and $A^2-kA-5I_2=O$, then the value of k is
(a) 3 (b) 5 (c) 7 (d) -7

36. If $A=[a_{ij}]$ is a scalar matrix, then trace of A is
(a) $\sum_i \sum_j a_{ij}$ (b) $\sum_i a_{ij}$ (c) $\sum_j a_{ij}$ (d) $\sum_i a_{ii}$

37. If $A=[a_{ij}]$ is a scalar matrix of order $n\times n$ such that $a_{ii}=k$ for all i, then trace of A is equal to
(a) nk (b) $n+k$ (c) n/k (d) none of these

38. If $A=[a_{ij}]$ is a scalar matrix of order $n\times n$ such that $a_{ii}=k$ for all i, then $|A|=$
(a) nk (b) $n+k$ (c) n^k (d) k^n

39. If $A=[a_{ij}]$ is a square matrix of order $n\times n$ and k is a scalar, then $|kA|=$
(a) $k^n|A|$ (b) $k|A|$ (c) $k^{n-1}|A|$ (d) none of these

40. Let $F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, where $\alpha\in R$. Then, $\left(F(\alpha)\right)^{-1}$ is equal to
(a) $F(-\alpha)$ (b) $F(\alpha^{-1})$ (c) $F(2\alpha)$ (d) none of these

(a) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$

41. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ and $G(y)=\begin{bmatrix}\cos x & 0 & \sin x\\ 0 & 1 & 0\\ -\sin x & 0 & \cos x\end{bmatrix}$, then $[F(x)\,G(y)]^{-1}$ is equal to
(a) $F(-x)\,G(-y)$ (b) $F(x^{-1})\,G(y^{-1})$
(c) $G(-y)\,F(-x)$ (d) $G(y^{-1})\,F(x^{-1})$

42. If $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then the matrix A equals
(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}-i & 0\\ 0 & -i\end{bmatrix}$ (c) $\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

43. If $\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & \tan\theta\\ -\tan\theta & 1\end{bmatrix}^{-1}=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$, then
(a) $a=1, b=1$ (b) $a=\cos 2\theta, b=\sin 2\theta$
(c) $a=\sin 2\theta, b=\cos 2\theta$ (d) none of these

44. If A and B are matrices such that AB and $A+B$ both are defined, then
(a) A and B can be any two matrices
(b) A and B are square matrices not necessarily of the same order
(c) A, B are square matrices of the same order
(d) number of columns of A is same as the number of rows of B

45. If a matrix A is such that $3A^3+2A^2+5A+I=0$, then A^{-1} is equal to
(a) $-(3A^2+2A+5)$ (b) $3A^2+2A+5$
(c) $3A^2-2A-5$ (d) none of these

46. A and B be 3×3 matrices. Then, $AB=O$ implies
(a) $A=O$ and $B=O$
(b) $|A|=O$ and $|B|=O$
(c) either $|A|=O$ or $|B|=O$
(d) $A=O$ or $B=O$

47. Which of the following is incorrect?
(a) $A^2-B^2=(A+B)(A-B)$
(b) $(A^T)^T=A$
(c) $(AB)^n=A^nB^n$ where A, B commute
(d) $(A-I)(I+A)=0\Leftrightarrow A^2=I$.

48. If A is an invertible matrix, then which of the following is correct
(a) A^{-1} is multivalued (b) A^{-1} is singular
(c) $(A^{-1})^T\neq(A^T)^{-1}$ (d) $|A|\neq 0$

49. Which of the following is/are incorrect?
(i) adjoint of a symmetric matrix is symmetric
(ii) adjoint of a unit matrix is a unit matrix
(iii) A (adj A) = (adj A) $A=|A|I$
(iv) adjoint of a diagonal matrix is a diagonal matrix
(a) (i) (b) (ii) (c) (iii) and (iv) (d) none of these

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50. If $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is to be the square root of two-rowed unit matrix, then α, β and γ should satisfy the relation
(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2-\beta\gamma=0$
(c) $1-\alpha^2+\beta\gamma=0$ (d) $\alpha^2+\beta\gamma-1=0$

51. If for a matrix A, $A^2+I=O$, where I is the identity matrix, then A equals
(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

52. If $A=[a_{ij}]_{m\times n}$ is a matrix of rank r, then
(a) $r=\min(m,n)$ (b) $r<\min(m,n)$
(c) $r\le\min(m,n)$ (d) none of these

53. If I_n is the identity matrix of order n, then rank of I_n is
(a) 1 (b) n (c) 0 (d) none of these

54. If $A=[a_{ij}]_{m\times n}$ is a matrix of rank r and B is a square submatrix of order $r+1$, then
(a) B is invertible (b) B is not invertible
(c) B may or may not be invertible (d) none of these

55. The rank of a null matrix is
(a) 0 (b) 1 (c) does not exist (d) none of these

56. If $A=[a_{ij}]_{m\times n}$ is a matrix and B is a non-singular square submatrix of order r, then
(a) rank of A is r (b) rank of A is greater than r
(c) rank of A is less than r (d) none of these

57. Which of the following is correct?
(a) Determinant is a square matrix
(b) Determinant is a number associated to a matrix
(c) Determinant is a number associated to a square matrix
(d) none of these

58. If a square matrix A is orthogonal as well as symmetric, then
(a) A is involutory matrix (b) A is idempotent matrix
(c) A is a diagonal matrix (d) none of these

59. Let A be a skew-symmetric matrix of odd order, then $|A|$ is equal to
(a) 0 (b) 1 (c) -1 (d) none of these

60. Let A be a skew-symmetric matrix of even order, then $|A|$
(a) is a square (b) is not a square
(c) is always zero (d) none of these

61. If A is an orthogonal matrix, then
(a) $|A|=0$ (b) $|A|=\pm1$
(c) $|A|=\pm2$ (d) none of these

62. Let A be a non-singular square matrix. Then, $|\text{adj } A|$ is equal to
(a) $|A|^n$ (b) $|A|^{n-1}$ (c) $|A|^{n-2}$ (d) none of these

63. Let $A=[a_{ij}]_{n\times n}$ be a square matrix, and let c_{ij} be cofactor of a_{ij} in A. If $C=[c_{ij}]$, then
(a) $|C|=|A|$ (b) $|C|=|A|^{n-1}$
(c) $|C|=|A|^{n-2}$ (d) none of these

64. If A is a non-singular square matrix of order n, then the rank of A is
(a) equal to n (b) less than n
(c) greater than n (d) none of these

65. If A is a matrix such that there exists a square submatrix of order r which is non-singular and every square submatrix of order $r+1$ or more is singular, then
(a) rank $(A)=r+1$ (b) rank $(A)=r$
(c) rank $(A)>r$ (d) rank $(A)\ge r+1$

66. Let A be a matrix of rank r. Then,
(a) rank $(A^T)=r$ (b) rank $(A^T)<r$
(c) rank $(A^T)>r$ (d) none of these

67. Let $A=[a_{ij}]_{m\times n}$ be a matrix such that $a_{ij}=1$ for all i, j. Then
(a) rank $(A)>1$ (b) rank $(A)=1$
(c) rank $(A)=m$ (d) rank $(A)=n$

68. If A is a non-zero column matrix of order $m\times1$ and B is a non-zero row matrix of order $1\times n$, then rank of AB equals
(a) m (b) n (c) 1 (d) none of these

69. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ is
(a) 1 (b) 2 (c) 3 (d) 4

70. If A is an invertible matrix, then det (A^{-1}) is equal to
(a) det (A) (b) $\frac{1}{\det(A)}$ (c) 1 (d) none of these

71. If A and B are two matrices such that rank of $A=m$ and rank of $B=n$, then
(a) rank $(AB)=mn$ (b) rank $(AB)\ge$ rank (A)
(c) rank $(AB)\ge$ rank (B)
(d) rank $(AB)\le\min($rank A, rank $B)$

72. If $A=\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$, then $(A+B)^{-1}=$
(a) is a skew-symmetric matrix (b) $A^{-1}+B^{-1}$
(c) does not exist (d) none of these

73. Let $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to
(a) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$ (b) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$
(c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$ (d) $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$

74. If $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $\lim_{n\to\infty}\frac{1}{n}A^n$ is
(a) a null matrix (b) an identity matrix
(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) none of these

75. If $A=\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB=I_3$, then $x+y$ equals
(a) 0 (b) -1 (c) 2 (d) none of these

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76. If $A=\begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}$, then $\lim_{n\to\infty}\frac{1}{n}A^n$ is
(a) $\begin{bmatrix}0 & a\\ 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ (d) none of these

77. If the matrix $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is commutative with the matrix $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, then
(a) $a=0, b=c$ (b) $b=0, c=d$
(c) $c=0, d=a$ (d) $d=0, a=b$

[JEE (WB) 2008]

Answers

1. (b) 2. (b) 3. (c) 4. (a) 5. (c) 6. (c) 7. (a)
8. (d) 9. (b) 10. (b) 11. (c) 12. (a) 13. (b) 14. (a)
15. (a) 16. (c) 17. (a) 18. (d) 19. (b) 20. (a) 21. (b)
22. (b) 23. (c) 24. (a) 25. (c) 26. (a) 27. (c) 28. (a)
29. (c) 30. (b) 31. (b) 32. (b) 33. (b) 34. (c) 35. (b)
36. (d) 37. (a) 38. (d) 39. (a) 40. (a) 41. (c) 42. (a)
43. (b) 44. (b) 45. (d) 46. (c) 47. (a) 48. (d) 49. (d)
50. (d) 51. (b) 52. (c) 53. (b) 54. (b) 55. (c) 56. (d)
57. (c) 58. (a) 59. (a) 60. (a) 61. (b) 62. (b) 63. (b)
64. (a) 65. (b) 66. (a) 67. (b) 68. (c) 69. (c) 70. (b)
71. (d) 72. (d) 73. (c) 74. (a) 75. (a) 76. (a) 77. (c)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. If A is an invertible matrix and B is a matrix, then
(a) rank (AB) = rank (A) (b) rank (AB) = rank (B)
(c) rank (AB) > rank (A) (d) rank (AB) > rank (B)

2. The order of $[x\ y\ z]\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ is
(a) 3×1 (b) 1×1 (c) 1×3 (d) 3×3

3. If $A=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$, then A^{-1} is
(a) $\begin{bmatrix}1/a & 0 & 0\\ 0 & 1/b & 0\\ 0 & 0 & 1/c\end{bmatrix}$ (b) $\begin{bmatrix}-1/a & 0 & 0\\ 0 & -1/b & 0\\ 0 & 0 & -1/c\end{bmatrix}$
(c) $\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & 1/c\end{bmatrix}$ (d) none of these

4. The inverse of the matrix $\begin{bmatrix}1 & 3\\ 3 & 10\end{bmatrix}$ is equal to
(a) $\begin{bmatrix}10 & 3\\ 3 & 1\end{bmatrix}$ (b) $\begin{bmatrix}10 & -3\\ -3 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 3\\ 3 & 10\end{bmatrix}$ (d) $\begin{bmatrix}-1 & -3\\ -3 & -10\end{bmatrix}$

5. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then $A^4=$
(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

6. If $X=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, the value of X^n is equal to
(a) $\begin{bmatrix}3n & -4n\\ n & -n\end{bmatrix}$ (b) $\begin{bmatrix}2+n & 5-n\\ n & -n\end{bmatrix}$
(c) $\begin{bmatrix}3^n & (-4)^n\\ 1^n & (-1)^n\end{bmatrix}$ (d) none of these

7. If $A=\begin{bmatrix}5 & 2\\ 3 & 1\end{bmatrix}$, then $A^{-1}=$
(a) $\begin{bmatrix}1 & -2\\ -3 & 5\end{bmatrix}$ (b) $\begin{bmatrix}-1 & 2\\ 3 & -5\end{bmatrix}$ (c) $\begin{bmatrix}-1 & -2\\ -3 & -5\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}$

8. For the system of equations:
$x+2y+3z=1$
$2x+y+3z=2$
$5x+5y+9z=4$
(a) there is only one solution
(b) there exists infinitely many solution
(c) there is no solution
(d) none of these

9. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then $A^2=$
(a) $\begin{bmatrix}8 & -5\\ -5 & 3\end{bmatrix}$ (b) $\begin{bmatrix}8 & -5\\ 5 & 3\end{bmatrix}$
(c) $\begin{bmatrix}8 & -5\\ -5 & -3\end{bmatrix}$ (d) $\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$

10. If $A=\begin{bmatrix}4 & x+2\\ 2x-3 & x+1\end{bmatrix}$ is symmetric, then $x=$
(a) 3 (b) 5 (c) 2 (d) 4

11. If $A+B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $A-2B=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$, then $A=$
(a) $\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}$ (b) $\begin{bmatrix}2/3 & 1/3\\ 1/3 & 2/3\end{bmatrix}$
(c) $\begin{bmatrix}1/3 & 1/3\\ 2/3 & 1/3\end{bmatrix}$ (d) none of these

12. $\begin{bmatrix}-6 & 5\\ -7 & 6\end{bmatrix}^{-1}=$
(a) $\begin{bmatrix}-6 & 5\\ -7 & 6\end{bmatrix}$ (b) $\begin{bmatrix}6 & -5\\ -7 & 6\end{bmatrix}$ (c) $\begin{bmatrix}6 & 5\\ 7 & 6\end{bmatrix}$ (d) $\begin{bmatrix}6 & -5\\ 7 & -6\end{bmatrix}$

13. From the matrix equation $AB=AC$ we can conclude $B=C$ provided that
(a) A is singular (b) A is non-singular

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(c) A is symmetric (d) A is square.

14. If I_3 is the identity matrix of order 3, then $(I_3)^{-1} =$
(a) 0 (b) $3I_3$ (c) I_3 (d) not necessarily exists.

15. Let a, b, c be positive real numbers. The following system of equations in x, y and z
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,\ \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has
(a) no solution (b) unique solution
(c) infinitely many solutions (d) finitely many solutions

16. If A and B are two matrices such that $A+B$ and AB are both defined, then
(a) A & B are two matrices not necessarily of same order
(b) A and B are square matrices of same order
(c) number of columns of A = number of rows of B
(d) none of these

17. A and B are two square matrices of same order and A' denotes the transpose of A, then
(a) $(AB)=B'A'$ (b) $(AB)'=A'B'$
(c) $AB=0 \Rightarrow |A|=0$ or $|B|=O$
(d) $AB=0 \Rightarrow A=0$ or $B=0$

18. Consider the system of equations
$a_1x+b_1y+c_1z=0$
$a_2x+b_2y+c_2z=0$
$a_3x+b_3y+c_3z=0$
if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$, then the system has
(a) more than two solutions
(b) one trivial and one non-trivial solutions
(c) no solution
(d) only trivial solution (0, 0, 0)

19. The system of linear equations
$x+y+z=2$
$2x+y-z=3$
$3x+2y+kz=4$ has a unique solution if
(a) $k\neq 0$ (b) $-1<k<1$ (c) $-2<k<2$ (d) $k=0$

20. If A and B are square matrices of order 3 such that $|A|=-1, |B|=3$, then $|3AB|$ equals
(a) -9 (b) -81 (c) -27 (d) 81

21. If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ will always be less than
(a) 3 (b) 2 (c) 1 (d) none of these

22. Let $A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of A, then α is
(a) 5 (b) -1 (c) 2 (d) -2

23. Let $A=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is
(a) $A^2=I$ (b) $A=-I$, where I is a unit matrix
(c) A^{-1} does not exist (d) A is a zero matrix

24. If $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $\text{adj}\,A=\begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & -1 \end{bmatrix}$, then $x+y=$
(a) 6 (b) -1 (c) 3 (d) 1

25. If A is a square matrix such that $A(\text{adj}\,A)=\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then $|\text{adj}\,A|=$
(a) 4 (b) 16 (c) 64 (d) 256

26. If n is a natural number. Then $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^n$, is
(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is even (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is odd
(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is a natural number (d) none of these

27. If $x^2+y^2+z^2\neq 0$, $x=cy+bz$, $y=az+cx$ and $z=bx+ay$, then $a^2+b^2+c^2+2abc=$
(a) 2 (b) $a+b+c$ (c) 1 (d) $ab+bc+ca$

28. If A is a singular matrix, then $A(\text{adj}\,A)$ is a
(a) scalar matrix (b) zero matrix
(c) identity matrix (d) orthogonal matrix

29. If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, I is the unit matrix of order 2 and a, b are arbitrary constants, then $(aI+bA)^2$ is equal to
(a) a^2I-abA (b) $a^2I+2abA$
(c) a^2I+b^2A (d) none of these

30. If $A=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then which one of the following is not correct?
(a) A is orthogonal matrix (b) A' is orthogonal matrix
(c) $|A|=1$ (d) A is not invertible

Answers

1. (b) 2. (b) 3. (a) 4. (b) 5. (a) 6. (d) 7. (b) 8. (a) 9. (d) 10. (b) 11. (c) 12. (a) 13. (b) 14. (c) 15. (b) 16. (b) 17. (a) 18. (a) 19. (a) 20. (a) 21. (a) 22. (a) 23. (c) 24. (a) 25. (b) 26. (a) 27. (c) 28. (b) 29. (b) 30. (d)

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17

DETERMINANTS

1. DETERMINANT

DEFINITION *Every square matrix can be associated to an expression or a number which is known as its determinant. If $A = [a_{ij}]$ is a square matrix of order n, then the determinant of A is denoted by det A or, $|A|$.*

If $A = [a_{11}]$ is a square matrix of order 1, then the determinant of A is defined as

$$|A| = a_{11} \quad or, \quad |a_{11}| = a_{11}$$

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a square matrix of order 2, then the expression $a_{11}a_{22} - a_{12}a_{21}$ is defined as the determinant of A

i.e.,
$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \qquad ...(i)$$

ILLUSTRATION 1 $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$

(a) 7 (b) 10 (c) 13 (d) 17

Ans. (b)

SOLUTION We have,

$$\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$$

$$= \begin{vmatrix} \log_3 2^9 & \log_{2^2} 3 \\ \log_3 2^3 & \log_{2^2} 3^2 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_{2^3} 3 \\ \log_3 2^2 & \log_3 2^2 \end{vmatrix}$$

$$= \begin{vmatrix} 9\log_3 2 & \frac{1}{2}\log_2 3 \\ 3\log_3 2 & \frac{2}{2}\log_2 3 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \frac{1}{3}\log_2 3 \\ 2\log_3 2 & 2\log_3 2 \end{vmatrix}$$

$$= \left(9 - \frac{3}{2}\right) \times \left(2 - \frac{2}{3}\right) = \frac{15}{2} \times \frac{4}{3} = 10 \quad \left[\because \log_2 3 \times \log_3 2 = 1\right]$$

Thus, the determinant of a square matrix of order 2 is equal to the product of the diagonal elements minus the product of off-diagonal elements.

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order 3, then the expression

$$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{32}a_{21} - a_{11}a_{23}a_{32} - a_{22}a_{13}a_{31} - a_{12}a_{21}a_{33}$$

is defined as the determinant of A

i.e.,
$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

or,
$$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{32}a_{21} - a_{11}a_{23}a_{32} - a_{22}a_{31}a_{13} - a_{33}a_{12}a_{21} \qquad ...(ii)$$

or,
$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{33}a_{21} - a_{23}a_{31}) + a_{13}(a_{32}a_{21} - a_{22}a_{31})$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

[Using(i)]

$$= (-1)^{1+1}a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Thus, the determinant of a square matrix of order 3 is the sum of the product of elements a_{1j} in first row with $(-1)^{1+j}$ times the determinant of a 2×2 sub-matrix obtained by leaving the first row and column passing through the element.

The above expansion of $|A|$ is known as the expansion along first row.

There are three rows and three columns in a square matrix of order 3. The expression (ii) for the determinant of a square matrix of order 3 can be arranged in various forms to obtain the expansion of $|A|$ along any of its rows or columns. Infact, to

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Statement–1: $\Delta_2 = \Delta_1^2$

Statement–2: If A is a square matrix of order n and B is the matrix of cofactors of elements of A, then $|B| = |A|^{n-1}$.

(a) 1 (b) 2 (c) 3 (d) 4

An a)

ON Clearly, statement–2 is true. (see Theorem 5 on 8)

$|\text{adj } A| = |A|^{n-1}$

$|B| = |A|^{n-1}$ $[\because |B| = |\text{adj } A|]$

$\Delta_2 = \Delta_1^2$ $\left[\because \Delta_2 \text{ is the determinant of the matrix of cofactors}\right]$

Hence, both the statement are true and statement–2 is a correct explanation for statement–1.

EXAMPLE 5 *Let a, b, c be distinct real numbers and D be the determinant given by*

$$D = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$$

Statement–1: If D > 0, then abc > – 8

Statement–2: A.M. > G.M.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION We have,

$$D = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = abc - (a+b+c) + 2$$

$\therefore \quad D > 0$

$\Rightarrow \quad abc - (a+b+c) + 2 > 0$

$\Rightarrow \quad abc + 2 > a + b + c$ ...(i)

But, A.M. > G.M.

$\Rightarrow \quad \dfrac{a+b+c}{3} > (abc)^{1/3}$

$\Rightarrow \quad a+b+c > 3\,(abc)^{1/3}$

From (i) and (ii), we get

$abc + 2 > 3\,(abc)^{1/3}$

$\Rightarrow \quad x^3 + 2 > 3x$, where $x = (abc)^{1/3}$

$\Rightarrow \quad x^3 - 3x + 2 > 0 \Rightarrow (x-1)^2 (x+2) > 0$

$\Rightarrow \quad x + 2 > 0 \Rightarrow (abc)^{1/3} + 2 > 0 \Rightarrow (abc)^{1/3} > -2 \Rightarrow abc > -8$

EXAMPLE 6 *Statement-1: Determinant of a skew-symmetric matrix of order 3 is zero.*

Statement-2: For any matrix A, Det (A) = Det (A^T) *and Det* $(-A) = -$ *Det (A)*

where Det (B) denotes the determinant of matrix B. Then,

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c) [AIEEE 2011]

SOLUTION Let A be a skew-symmetric matrix of order 3. Then

$A^T = -A$

$\Rightarrow \quad Det\,(A^T) = Det\,(-A)$

$\Rightarrow \quad Det\,(A) = (-1)^3\,Det\,(A)$

$\Rightarrow \quad Det\,(A) = -\,Det\,(A)$

$\Rightarrow \quad 2\,Det\,(A) = 0$

$\Rightarrow \quad Det\,(A) = 0$

So, statement-1 is true.

For any square matrix of order n, we have

$Det\,(A^T) = Det\,(A)$ and $Det\,(-A) = (-1)^n\,Det\,(A)$

So, statement-2 is not true.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If a, b, c, are in A.P., then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$, is

(a) 3 (b) – 3 (c) 0 (d) none of these

2. If $p+q+r = 0 = a+b+c$, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$, is

(a) 0 (b) $pa + qb + rc$ (c) 1 (d) none of these

3. If $a > 0, b > 0, c > 0$ are respectively the p^{th}, q^{th}, r^{th} terms of a GP, then the value of the determinant $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$, is

(a) 1 (b) 0 (c) – 1 (d) none of these **[AIEEE 2002]**

4. If A is an invertible matrix, then det (A^{-1}) is equal to

(a) det (A) (b) $\dfrac{1}{\det(A)}$ (c) 1 (d) none of these

5. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix}$ is equal to

(a) 1 (b) – 1 (c) 0 (d) none of these

6. If A, B, C are the angles of a triangle, then the determinant $\Delta = \begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix}$ is equal to

(a) 1 (b) – 1

(c) $\sin A + \sin B + \sin C$ (d) none of these

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7. The determinant $\Delta = \begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by

(a) x^5 (b) x^4 (c) x^4+1 (d) x^4-1

8. Let $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$. Then, the value of $\sum_{r=1}^{n} D_r$ is

(a) $\alpha\beta\gamma$ (b) $2^n\alpha + 2^n\beta + 4^n\gamma$
(c) $2\alpha+3\beta+4\gamma$ (d) none of these

9. The determinant $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is equal to zero, if

(a) a, b, c, are in A.P. (b) a, b, c, are in G.P.
(c) a, b, c, are in H.P. (d) α is a root of $ax^2+bx+c=0$

10. If $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$, then $\sum_{a=1}^{n} \Delta_a$ is equal to

(a) 0 (b) 1
(c) $\left\{\frac{n(n+1)}{2}\right\}\left\{\frac{a(a+1)}{2}\right\}$ (d) none of these

11. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in GP, then the determinant $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

(a) 0 (b) 1 (c) 2 (d) none of these
[AIEEE 2004, 2005]

12. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then the value of xyz is

(a) -2 (b) -1 (c) -3 (d) none of these

13. If $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, then the value of k, is

(a) 1 (b) 2 (c) 3 (d) 4

14. If A is a square matrix of order n such that its elements are polynomial in x and its r-rows become identical for $x = k$, then

(a) $(x-k)^r$ is a factor of $|A|$
(b) $(x-k)^{r-1}$ is a factor of $|A|$
(c) $(x-k)^{r+1}$ is a factor $|A|$
(d) $(x-k)^r$ is a factor of A

15. If $a^2+b^2+c^2=0$ and $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = k\,a^2b^2c^2$, then the value of k is

(a) 2 (b) 1 (c) 4 (d) 3

16. If $a^{-1}+b^{-1}+c^{-1}=0$ such that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$, then the value of λ is

(a) 0 (b) abc (c) $-abc$ (d) none of these

17. The value of the determinant $\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$ is

(a) $4\cos\alpha\cos\beta\cos\gamma$ (b) $2\cos\alpha\cos\beta\cos\gamma$
(c) $4\sin\alpha\sin\beta\sin\gamma$ (d) none of these

18. If A, B, C are the angles of a triangle, then the value of $\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is

(a) $\cos A \cos B \cos C$ (b) $\sin A \sin B \sin C$
(c) 0 (d) none of these

19. If $\begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix} = f(x) - xf'(x)$ then $f(x)$ is equal to

(a) $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$
(b) $(x+\alpha)(x+\beta)(x+\gamma)(x+\delta)$
(c) $2(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$
(d) none of these

20. If x, y, z are in AP, then the value of the det A is, where $A = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$

(a) 0 (b) 1 (c) 2 (d) none of these

21. If $a \neq b \neq c$, then the value of x satisfying the equation $\begin{vmatrix} 0 & x-a & a-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$ is

(a) a (b) b (c) c (d) 0

22. If a, b, c are different, then the value of x satisfying $\begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix} = 0$ is

(a) a (b) b (c) c (d) 0

23. The system of equations

$$kx+y+z=1$$
$$x+ky+z=k$$
$$x+y+kz=k^2$$

have no solution, if k equals

(a) 0 (b) 1 (c) -1 (d) -2

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24. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = k a^2 b^2 c^2$, then k is equal to

(a) 3 (b) 2
(c) 4 (d) none of these

25. The value of the determinant

$$\Delta = \begin{vmatrix} 2a_1 b_1 & a_1 b_2 + a_2 b_1 & a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 & 2a_2 b_2 & a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 & a_3 b_2 + a_2 b_3 & 2a_3 b_3 \end{vmatrix}$$ is

(a) 1 (b) -1
(c) 0 (d) $a_1 a_2 a_3 b_1 b_2 b_3$.

26. If B is a non-singular matrix and A is a square matrix, then Det $(B^{-1}AB)$ is equal to

(a) Det (B) (b) Det (A)
(c) Det (B^{-1}) (d) Det (A^{-1})

27. If $0 < \theta < \pi$ and the system of equations

$$(\sin\theta)\, x + y + z = 0$$
$$x + (\cos\theta)\, y + z = 0$$
$$(\sin\theta)\, x + (\cos\theta)\, y + z = 0$$

has a non-trivial solution, then θ =

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

28. If the determinant $\begin{vmatrix} b-c & c-a & a-b \\ b'-c' & c'-a' & a'-b' \\ b''-c'' & c''-a'' & a''-b'' \end{vmatrix}$ is expressible as $m \begin{vmatrix} a & b & c \\ a' & b' & c' \\ a'' & b'' & c'' \end{vmatrix}$, then the value of m is

(a) -1 (b) 0 (c) 1 (d) 2

29. If $a \neq b$, then the system of equation

$$ax + by + bz = 0$$
$$bx + ay + bz = 0$$
$$bx + by + az = 0$$

will have a non-trivial solution, if

(a) $a + b = 0$ (b) $a + 2b = 0$
(c) $2a + b = 0$ (d) $a + 4b = 0$

30. If $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$,

where a, b, c are all different, then the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix}$$

vanishes when

(a) $a + b + c = 0$ (b) $x = \frac{1}{3}(a+b+c)$
(c) $x = \frac{1}{2}(a+b+c)$ (d) $x = a + b + c$

31. Suppose $p, q, r \neq 0$ and the system of equations

$$(p+a)\, x + by + cz = 0$$
$$ax + (q+b)\, y + cz = 0$$
$$ax + by + (r+c)\, z = 0$$

has a non-trivial solution, then the value of $\frac{p}{a} + \frac{q}{b} + \frac{r}{c}$ is

(a) -1 (b) 0 (c) 1 (d) 2

32. One factor of $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & cb \\ ca & cb & c^2+x \end{vmatrix}$ is

(a) x^2 (b) $(a^2+x)(b^2+x)(c^2+x)$
(c) $\frac{1}{x}$ (d) none of these

33. The equation $\begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0$, where a, b, c are different, is satisfied by

(a) $x = 0$ (b) $x = a$
(c) $x = \frac{1}{3}(a+b+c)$ (d) $x = a + b + c$

34. Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta < 2\pi$. Then, which of the following is not correct?

(a) Det $(A) = 0$ (b) Det $(A) \in (-\infty, 0)$
(c) Det $(A) \in [2, 4]$ (d) Det $(A) \in [-2, \infty)$

35. If a, b, c are non-zero real numbers, then

$\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix}$ vanishes, when

(a) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$ (b) $\frac{1}{a} - \frac{1}{b} - \frac{1}{c} = 0$
(c) $\frac{1}{b} + \frac{1}{c} - \frac{1}{a} = 0$ (d) $\frac{1}{b} - \frac{1}{c} - \frac{1}{a} = 0$

36. The value of the determinant $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$ is

(a) $k(a+b)(b+c)(c+a)$
(b) $kabc(a^2+b^2+c^2)$
(c) $k(a-b)(b-c)(c-a)$
(d) $k(a+b-c)(b+c-a)(c+a-b)$

37. The system of simultaneous equations

$$kx + 2y - z = 1$$
$$(k-1)\, y - 2z = 2$$
$$(k+2)\, z = 3$$

have a unique solution if k equals

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(a) -2 (b) -1 (c) 0 (d) 1

[I.P. (Delhi) 2003]

38. The value of the determinant $\begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix}$, where ω is an imaginary cube root of unity, is

(a) $(1-\omega)^2$ (b) 3

(c) -3 (d) none of these.

39. If a, b, c are non-zero real numbers such that $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$, then

(a) $\frac{1}{a}+\frac{1}{b\omega}+\frac{1}{c\omega^2}=0$ (b) $\frac{1}{a}+\frac{1}{b\omega^2}+\frac{1}{c\omega}=0$

(c) $\frac{1}{a\omega}+\frac{1}{b\omega^2}+\frac{1}{c}=0$ (d) all the above

40. If the system of equations

$$x + a y + a z = 0$$
$$b x + y + b z = 0$$
$$c x + c y + z = 0$$

where a, b and c are non-zero non-unity, has a non-trivial solution, then the value of $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}$ is

(a) 0 (b) 1 (c) -1 (d) $\frac{abc}{a^2+b^2+c^2}$

41. The determinant $\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\sin\alpha & \sin\alpha & \cos\beta \end{vmatrix}$ is independent of

(a) α (b) β (c) α and β (d) neither α nor β

42. If ω is a cube root of unity, then for polynomial is

$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

(a) 1 (b) ω (c) ω^2 (d) 0

43. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$, then

(a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

(c) $\frac{d}{dx}(\Delta_1) = 3\Delta_2^2$ (d) $\Delta_1 = 3(\Delta_2)^{3/2}$

44. If $y = \sin mx$, then the value of the determinant

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$$, where $y_n = \frac{d^n y}{dx^n}$, is

(a) m^9 (b) m^2

(c) m^3 (d) none of these

45. If $a \neq p$, $b \neq q$, $c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$ is

(a) 0 (b) 1 (c) -1 (d) 2

46. $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} =$

(a) $(x+p)(x+q)(x-p-q)$ (b) $(x-p)(x-q)(x+p+q)$

(c) $(x-p)(x-q)(x-p-q)$ (d) $(x+p)(x+q)(x+p+q)$

47. The factors of $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$ are

(a) $x-a, x-b$, and $x+a+b$

(b) $x+a, x+b$ and $x+a+b$

(c) $x+a, x+b$ and $x-a-b$

(d) $x-a, x-b$ and $x-a-b$

48. Let $\omega = -\frac{1}{2}+i\frac{\sqrt{3}}{2}$, then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$, is

(a) 3ω (b) $3\omega(\omega-1)$

(c) $3\omega^2$ (d) $3\omega(1-\omega)$

49. If $a+b+c=0$ one root of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

(a) $x=1$ (b) $x=2$

(c) $x=a^2+b^2+c^2$ (d) $x=0$

[IP (Delhi) 2003]

50. Suppose $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and

$$D' = \begin{vmatrix} a_1+p b_1 & b_1+q c_1 & c_1+r a_1 \\ a_2+p b_2 & b_2+q c_2 & c_2+r a_2 \\ a_3+p b_3 & b_3+q c_3 & c_3+r a_3 \end{vmatrix}$$. Then,

(a) $D' = D$ (b) $D' = D(1-pqr)$

(c) $D' = D(1+p+q+r)$ (d) $D' = D(1+pqr)$

51. A and B are two non-zero square matrices such that $AB = O$. Then,

(a) both A and B are singular

(b) either of them is singular

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(c) neither matrix is singular
(d) none of these

52. The roots of the equation $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$, are

(a) 1, 2 (b) −1, 2 (c) 1, −2 (d) −1, −2

53. From the matrix equation $AB = AC$ we can conclude $B = C$ provided that

(a) A is singular (b) A is non-singular
(c) A is symmetric (d) A is square

54. If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$, then

(a) $\Delta_1 + \Delta_2 = 0$ (b) $\Delta_1 + 2\Delta_2 = 0$
(c) $\Delta_1 = \Delta_2$ (d) $\Delta_1 = 2\Delta_2$

55. The value of $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix}$, is

(a) 1 (b) 0 (c) −1 (d) 67

56. The value of $\begin{vmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{vmatrix}$, is

(a) 4 (b) $x+y+z$ (c) xyz (d) 0

57. A root of the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$, is

(a) a (b) b (c) 0 (d) 1

58. Let a, b, c be positive real numbers. The following system of equations in x, y and z

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, $-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

(a) no solution
(b) unique solution
(c) infinitely many solutions
(d) finitely many solutions

59. If A and B are two matrices such that $A + B$ and AB are both defined, then

(a) A and B are two matrices not necessarily of same order
(b) A and B are square matrices of same order
(c) number of columns of A = number of rows of B
(d) none of these

60. If ω is an imaginary cube root of unity, then the value of $\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$, is

(a) $a^3 + b^3 + c^3$ (b) $a^2 b - b^2 c$
(c) 0 (d) $a^3 + b^3 + c^3 - 3abc$

[JEE (WB) 2006]

61. If α, β are non-real numbers satisfying $x^3 - 1 = 0$, then the value of $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$ is equal to

(a) 0 (b) λ^3 (c) $\lambda^3 + 1$ (d) $\lambda^3 - 1$

62. The value of the determinant $\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ is equal to

(a) −4 (b) 0 (c) 1 (d) 4

63. In a third order determinant, each element of the first column consists of sum of two terms, each element of the second column consists of sum of three terms and each element of the third column consists of sum of four terms. Then it can be decomposed into n determinants, where n has the value

(a) 1 (b) 9 (c) 16 (d) 24

64. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$, is

(a) 6 (b) 3
(c) 0 (d) none of these

65. For positive numbers x, y, z (other than unity) the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 3 & \log_y z \\ \log_z x & \log_z y & 5 \end{vmatrix}$, is

(a) 0 (b) $\log x \log y \log z$
(c) 1 (d) 8

66. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$ is equal to

(a) 0 (b) −1 (c) 1 (d) 10

67. If $\Delta = \begin{vmatrix} 3 & 4 & 5 & x \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix}$, then Δ equals

(a) $(y - 2z + 3x)^2$ (b) $(x - 2y + z)^2$
(c) $(x + y + z)^2$ (d) $x^2 + y^2 + z^2 - xy - yz - zx$

68. If A, B and C are the angles of a triangle and $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$, then the triangle ABC is

(a) isosceles (b) equilateral
(c) right angled isosceles (d) none of these

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69. If a, b, c are the sides of a ΔABC and A, B, C are respectively the angles opposite to them, then

$$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos(B-C) \\ c\sin A & \cos(B-C) & 1 \end{vmatrix}$$ equals

(a) $\sin A - \sin B \sin C$ (b) abc (c) 1 (d) 0

70. If $0 \le [x] < 2, -1 \le [y] < 1$ and $1 \le [z] < 3$, $[\cdot]$ denotes the greatest integer function, then the maximum value of the determinant

$$\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$$, is

(a) 2 (b) 6 (c) 4 (d) none of these

71. Coefficient of x in

$$f(x) = \begin{vmatrix} x & (1+\sin x)^3 & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$$, is

(a) 0 (b) 1
(c) -2 (d) cannot be determined

72. The determinant $\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$ has the value, where A, B, C are angles of a triangle.

(a) 0 (b) 1
(c) $\sin A \sin B$ (d) $\cos A \cos B \cos C$

73. Using the factor theorem it is found that $a+b$, $b+c$ and $c+a$ are three factors of the determinant

$$\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$$

The other factor in the value of the determinant is

(a) 4 (b) 2
(c) $a+b+c$ (d) none of these

74. The value of $\begin{vmatrix} a & a^2-bc & 1 \\ b & b^2-ca & 1 \\ c & c^2-ab & 1 \end{vmatrix}$, is

(a) 1 (b) -1 (c) 0 (d) $-abc$

75. If $\alpha+\beta+\gamma=\pi$, then the value of the determinant

$$\begin{vmatrix} e^{2i\alpha} & e^{-i\gamma} & e^{-i\beta} \\ e^{-i\gamma} & e^{2i\beta} & e^{-i\alpha} \\ e^{-i\beta} & e^{-i\alpha} & e^{2i\gamma} \end{vmatrix}$$, is

(a) 4 (b) -4
(c) 0 (d) none of these

76. If $a \ne b \ne c$, are value of x which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$

is given by

(a) $x=0$ (b) $x=c$
(c) $x=b$ (d) $x=a$

77. The repeated factor of the determinant

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$, is

(a) $z-x$ (b) $x-y$
(c) $y-z$ (d) none of these

78. The value of the determinant

$$\Delta = \begin{vmatrix} \frac{1-a_1^3b_1^3}{1-a_1b_1} & \frac{1-a_1^3b_2^3}{1-a_1b_2} & \frac{1-a_1^3b_3^3}{1-a_1b_3} \\ \frac{1-a_2^3b_1^3}{1-a_2b_1} & \frac{1-a_2^3b_2^3}{1-a_2b_2} & \frac{1-a_2^3b_3^3}{1-a_2b_3} \\ \frac{1-a_3^3b_1^3}{1-a_3b_1} & \frac{1-a_3^3b_2^3}{1-a_3b_2} & \frac{1-a_3^3b_3^3}{1-a_3b_3} \end{vmatrix}$$, is

(a) 0
(b) dependent only on a_1, a_2, a_3
(c) dependent only on b_1, b_2, b_3
(d) dependent on $a_1, a_2, a_3, b_1, b_2, b_3$

79. The determinant

$$\Delta = \begin{vmatrix} b & c & b\alpha+c \\ c & d & c\alpha+d \\ b\alpha+c & c\alpha+d & a\alpha^3-c\alpha \end{vmatrix}$$

is equal to zero, if

(a) b, c, d are in A.P.
(b) b, c, d are in G.P.
(c) b, c, d are in H.P.
(d) α is a root of $ax^3+bx^2-cx-d=0$

80. $\Delta = \begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix} =$

(a) 0 (b) abc
(c) $\frac{1}{abc}$ (d) none of these

81. If $\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix} = A_0+A_1x+A_2x^2+A_3x^3$,

then A_1 is equal to

(a) abc (b) 0 (c) 1 (d) none of these

82. If $a \ne 0, b \ne 0, c \ne 0$, then

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$ is equal to

(a) abc (b) $abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$
(c) 0 (d) $1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

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83. If $1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$, then

$$\Delta=\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}$$ *is equal to*

(a) 0 (b) abc

(c) $-abc$ (d) none of these

84. If a, b and c are all different from zero and

$$\Delta=\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=0$$, then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ *is*

(a) abc (b) $\frac{1}{abc}$ (c) $-a-b-c$ (d) -1

85. In a ΔABC, a, b, c are sides and A, B, C are angles opposite to them, then the value of the determinant

$$\begin{vmatrix}a^2 & b\sin A & c\sin A\\ b\sin A & 1 & \cos A\\ c\sin A & \cos A & 1\end{vmatrix}$$, *is*

(a) 0 (b) 1 (c) 2 (d) 3

86. If $\begin{vmatrix}-12 & 0 & \lambda\\ 0 & 2 & -1\\ 2 & 1 & 15\end{vmatrix}=-360$, then the value of λ is

(a) -1 (b) -2 (c) -3 (d) 4

Answers

1. (c) 2. (a) 3. (b) 4. (b) 5. (a) 6. (d) 7. (b)
8. (d) 9. (b) 10. (a) 11. (a) 12. (b) 13. (b) 14. (a)
15. (c) 16. (b) 17. (d) 18. (c) 19. (a) 20. (a) 21. (d)
22. (d) 23. (d) 24. (c) 25. (c) 26. (b) 27. (d) 28. (b)
29. (b) 30. (b) 31. (a) 32. (a) 33. (c) 34. (c) 35. (a)
36. (c) 37. (b) 38. (b) 39. (d) 40. (c) 41. (a) 42. (d)
43. (b) 44. (d) 45. (d) 46. (b) 47. (a) 48. (d) 49. (d)
50. (a) 51. (b) 52. (b) 53. (b) 54. (a) 55. (b) 56. (b)
57. (c) 58. (b) 59. (b) 60. (c) 61. (b) 62. (d) 63. (d)
64. (c) 65. (d) 66. (c) 67. (b) 68. (a) 69. (d) 70. (c)
71. (c) 72. (a) 73. (a) 74. (c) 75. (b) 76. (a) 77. (a)
78. (d) 79. (b) 80. (a) 81. (b) 82. (b) 83. (a) 84. (d)
85. (a) 86. (c)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. Consider the system of equations

$$a_1x+b_1y+c_1z=0$$
$$a_2x+b_2y+c_2z=0$$
$$a_3x+b_3y+c_3z=0$$

If $\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$, then the system has

(a) more than two solutions
(b) one trivial and one non-trivial solutions
(c) no solution
(d) only trivial solution (0, 0, 0)

2. If $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$ for $x\neq 0, y\neq 0$, then D is divisible by

(a) x but not y (b) y but not x
(c) neither x nor y (d) both x and y

3. If $\begin{vmatrix}x+a & b & c\\ a & x+b & c\\ a & b & x+c\end{vmatrix}=0$, then x equals

(a) $a+b+c$ (b) $-(a+b+c)$
(c) $0, a+b+c$ (d) $0, -(a+b+c)$

4. $\begin{vmatrix}\sin^2 x & \cos^2 x & 1\\ \cos^2 x & \sin^2 x & 1\\ -10 & 12 & 2\end{vmatrix}=$

(a) 0 (b) $12\cos^2 x-10\sin^2 x$
(c) $12\sin^2 x-10\cos^2 x-2$ (d) $10\sin 2x$

5. The system of linear equations

$$x+y+z=2$$
$$2x+y-z=3$$
$$3x+2y+kz=4$$

has a unique solution, if

(a) $k\neq 0$ (b) $-1<k<1$
(c) $-2<k<2$ (d) $k=0$

6. The roots of the equation

$$\begin{vmatrix}3x^2 & x^2+x\cos\theta+\cos^2\theta & x^2+x\sin\theta+\sin^2\theta\\ x^2+x\cos\theta+\cos^2\theta & 3\cos^2\theta & 1+\frac{\sin 2\theta}{2}\\ x^2+x\sin\theta+\sin^2\theta & 1+\frac{\sin 2\theta}{2} & 3\sin^2\theta\end{vmatrix}$$

(a) $\sin\theta, \cos\theta$ (b) $\sin^2\theta, \cos^2\theta$
(c) $\sin\theta, \cos^2\theta$ (d) $\sin^2\theta, \cos\theta$

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(c) $-a-b-c$ (d) -1

A, B, C are angles opposite

eterminant

c) 2 (d) 3

he value of λ, is

-3 (d) 4

(a) 55. (b) 56. (b)
b) 62. (d) 63. (d)
a) 69. (d) 70. (c)
) 76. (a) 77. (a)
) 83. (a) 84. (d)

oice.

$-(a+b+c)$

$\sin^2 x - 10\sin^2 x$

$\cos 2x$

< 1

$\begin{vmatrix} \sin\theta + \sin^2\theta & \cdots \\ \frac{\sin 2\theta}{2} & \cdots \\ \sin^2\theta & \cdots \end{vmatrix} = 0$

$\cos^2\theta$

$\cos\theta$

$\begin{vmatrix} bc & bc'+b'c & b'c' \\ ca & ca'+c'a & c'a' \\ ab & ab'+a'b & a'b' \end{vmatrix}$ is equal to

(a) $(ab-a'b')(bc-b'c')(ca-c'a')$
(b) $(ab+a'b')(bc+b'c')(ca+c'a')$
(c) $(ab'-a'b)(bc'-b'c)(ca'-c'a)$
(d) $(ab'+a'b)(bc'+b'c)(ca'+c'a)$

9. If α, β, γ are the cube roots of 8, then $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$

(a) 0 (b) 1 (c) 8 (d) 2

10. One root of the equation

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0, \text{ is}$$

(a) 8/3 (b) 2/3 (c) 1/3 (d) 16/3

11. If a, b and c are non-zero real numbers, then

$$\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} \text{ is equal to}$$

(a) abc (b) $a^2b^2c^2$ (c) $ab+bc+ca$ (d) 0

12. If x, y, z are in A.P., then the value of the det (A) is, where

$$A = \begin{bmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{bmatrix}$$

(a) 0 (b) 1
(c) 2 (d) none of these

13. The value of $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$, is

(a) $6abc$ (b) $a+b+c$ (c) $4abc$ (d) abc

14. If a, b, c are non-zero real numbers and if the equations $(a-1)x = y+z, (b-1)y = z+x, (c-1)z = x+y$ has non-trivial solution, then $ab+bc+ca$ is equal to

(a) $a+b+c$ (b) abc
(c) 1 (d) none of these

15. If $a \neq b, b, c$ satisfy $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$, then $abc =$

(a) $a+b+c$ (b) 0 (c) b^3 (d) $ab+bc$

16. The value of $\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$, is

(a) 8 (b) -8 (c) 400 (d) 1

17. The value of $\Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ is equal to

(a) $9a^2(a+b)$ (b) $9b^2(a+b)$
(c) $a^2(a+b)$ (d) $b^2(a+b)$

18. If all the elements in a square matrix A of order 3 are equal to 1 or -1, then $|A|$, is

(a) an odd number (b) an even number
(c) an imaginary number (d) a real number

19. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are

(a) $-1, -2$ (b) $-1, 2$ (c) $1, -2$ (d) $1, 2$

20. If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$, then $\lim_{x \to 0} \frac{f(x)}{x^2}$, is

(a) 3 (b) -1 (c) 0 (d) 1

21. If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & \sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$$

then the triangle must be

(a) equilateral (b) isosceles
(c) any triangle (d) right angled

22. If $\begin{vmatrix} x & 2 & 3 \\ 2 & 3 & x \\ 3 & x & 2 \end{vmatrix} = \begin{vmatrix} 1 & x & 4 \\ x & 4 & 1 \\ 4 & 1 & x \end{vmatrix} = \begin{vmatrix} 0 & 5 & x \\ 5 & x & 0 \\ x & 0 & 5 \end{vmatrix} = 0$, then the value of x equals $(x \in R)$:

(a) 0 (b) 5 (c) -5 (d) none of these

23. If $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, then x is equal to

(a) $0, 2a$ (b) $a, 2a$
(c) $0, 3a$ (d) none of these

24. If $\Delta_1 = \begin{vmatrix} 7 & x & 2 \\ -5 & x+1 & 3 \\ 4 & x & 7 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & 2 & 7 \\ x+1 & 3 & -5 \\ x & 7 & 4 \end{vmatrix}$, then the value of x for which $\Delta_1 + \Delta_2 = 0$, is

(a) 2 (b) 0
(c) any real number (d) none of these

25. If $\Delta_1 = \begin{vmatrix} 10 & 4 & 3 \\ 17 & 7 & 4 \\ 4 & -5 & 7 \end{vmatrix}, \Delta_2 = \begin{vmatrix} 4 & x+5 & 3 \\ 7 & x+12 & 4 \\ -5 & x-1 & 7 \end{vmatrix}$ such that $\Delta_1 + \Delta_2 = 0$, then

(a) $x = 5$ (b) $x = 0$
(c) x has no real value (d) none of these

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25. If $\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix} = 0$, then

(a) $d = 0$ (b) $a+d = 0$

(c) $d = 0$ or $a+d = 0$ (d) none of these

26. If $\Delta_k = \begin{vmatrix} k & 1 & 5 \\ k^2 & 2n+1 & 2n+1 \\ k^3 & 3n^2 & 3n+1 \end{vmatrix}$, then $\sum_{k=1}^{n} \Delta_k$ is equal to

(a) $2\sum_{k=1}^{n} k$ (b) $2\sum_{k=1}^{n} k^2$ (c) $\frac{1}{2}\sum_{k=1}^{n} k^2$ (d) 0

27. If the system of equations

$bx+ay = c, cx+az = b, cy+bz = a$

has a unique solution, then

(a) $abc = 1$ (b) $abc = -2$

(c) $abc = 0$ (d) none of these

28. If a, b, c are non-zero real numbers, then the system of equations

$(\alpha+a)x+\alpha y+\alpha z = 0$

$\alpha x+(\alpha+b)y+\alpha z = 0$

$\alpha x+\alpha y+(\alpha+c)z = 0$

has a non-trivial solution, if

(a) $\alpha^{-1} = -(a^{-1}+b^{-1}+c^{-1})$ (b) $\alpha^{-1} = a+b+c$

(c) $\alpha+a+b+c = 1$ (d) none of these

29. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b, c are the p^{th}, q^{th} and r^{th} terms of a H.P., is

(a) $p+q+r$ (b) $(a+b+c)$

(c) 1 (d) none of these

30. If $A = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$ and $B = \begin{vmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{vmatrix}$, then

(a) $A = 2B$ (b) $A = B$

(c) $A = -B$ (d) none of these

Answer

1. (a) 2. (d) 3. (d) 4. (a) 5. (a) 6. (a) 7. (c) 8. (a) 9. (b) 10. (d) 11. (a) 12. (c) 13. (b) 14. (c) 15. (b) 16. (b) 17. (b) 18. (b) 19. (d) 20. (b) 21. (c) 22. (c) 23. (d) 24. (a) 25. (c) 26. (b) 27. (c) 28. (a) 29. (d) 30. (c)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on "**Solutions of Objective Mathematics**".

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PROBABILITY

1. SOME DEFINITIONS

ELEMENTARY EVENT *If a random experiment is performed, then each of its outcomes is known as an elementary event.*

SAMPLE SPACE *The set of all possible outcomes of a random experiment is called the sample space assoicated with it and it is generally denoted by S.*

In other words, the set of all elementary events assoicated to a random experiment is called its sample space.

Consider the experiment of tossing two coins together or a coin twice. In this experiment the possible outcomes are :

Head on first and Head on second,
Head on first and Tail on second,
Tail on first and Head on second,
Tail on first and Tail on second.

If we define

HH = Getting head on both coins,
HT = Getting head on first and tail on second,
TH = Getting tail on first and head on second,
TT = Getting tail on both coins.

Then,

HH, HT, TH and TT are elementary events associated to the random experiment of tossing of two coins. The sample space associated to this experiment is given by

$$S = \{HH, HT, TH, TT\}$$

Similarly, the sample space associated to the random experiment of tossing three coins simultaneously or tossing a coin three times is given by

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, THT, TTT\}$$

Consider the random experiment in which two dice are tossed together or a die is tossed twice. If we define

E_{ij} = Getting number i on the upper face of first die and number j on the upper face of second die.

where $i = 1, 2, ..., 6$ and $j = 1, 2, ..., 6$.

Then, E_{ij} are elementary events assoicated to this experiment and are generally denoted by (i, j).

Thus, $(1, 1)$, $(1, 2)$, ... , $(1, 6)$, $(2, 1)$ ... , $(2, 6)$, $(3, 1)$, ..., $(3, 6)$, $(4, 1)$..., $(4, 6)$, $(5, 1)$, ..., $(5, 6)$ and $(6, 1)$, ... , $(6, 6)$ are 36 elementary events assoicated to the random experiment of tossing two dice and the sample space assoicated to it is given by

$$S = \{(1, 1), (1, 2) ....., (6, 6)\}$$

Let there be a bag containing 3 white and 2 black balls. Let the white balls be denoted by $W_1, W_2 W_3$ and black balls be denoted by B_1, B_2. If we draw two balls from the bag, then there are ${}^5C_2$ elementary events associated to this experiment. These elementary events are :

$B_1 W_1, B_1 W_2, B_1 W_3, B_2 W_1, B_2 W_2, B_2 W_3, W_1 W_2, W_1 W_3, W_2 W_3$, and $B_1 B_2$. The set of all these elementary events is the sample space associated to the experiment.

A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 red and 4 black balls ; if it shows a tail, we throw a die. If we denote three red balls as R_1, R_2 and R_3 and four black balls as B_1, B_2, B_3 and B_4. Then the elementary events associated to this experiment are:

$HR_1, HR_2, HR_3, HB_1, HB_2, HB_3, HB_4, T1, T2, T3, T4, T5$ and $T6$.

The set of these elementary events is the sample space associated to the given random experiment.

REMARK 1 *Elementary events assoicated to a random experiment are also known as indecomposable events.*

COMPOUND EVENT *A subset of the sample space associated to a random experiment is said to define a compound event if it is disjoint union of single element subsets of the sample space.*

As we know that single element subsets of sample space associated to a random experiment are elementary events associated to the random experiments. So, an event associated to a random experiment is a compound event if it is obtained by combining two or more elementary events.

Compound events are also known as decomposable events.

Consider the random experiment of throwing a die. The sample space associated to this experiment is $S = \{1, 2, 3, 4, 5, 6\}$. If we define an event E as 'Getting an even number'. In set theoretical notations this event is $E = \{2, 4, 6\}$. Clearly, it is obtained by combining three elementary events viz. 'getting 2, getting 4' and 'getting 6'. So, E is a compound event.

REMARK 2 *If there are n elementary events associated to a random experiment, then the sample space associated to it has n elements and so there are 2^n subsets of it. Out of these 2^n subsets there are n single*

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Number of A.P's with common difference 2 is = 14

(1, 3, 5, 7 ; 2, 4, 6, 8; ...; 14, 16, 18, 20)

Number of A.P's with common difference 3 is = 11

(1, 4, 7, 10; 2, 5, 8, 11; ... ; 11, 14, 17, 20)

and so on.

The maximum common difference can be 6 and the number of such A.P.'s is = 2

(1, 7, 13, 19; 2, 8, 14, 20)

$\therefore$ Total number of ways in which chosen numbers are in A.P.

$= 17 + 14 + 11 + \ldots + 2 = 57$

$\therefore$ Probability that the chosen numbers are in AP $= \frac{57}{{}^{20}C_4} = \frac{1}{85}$

Clearly, the set of all possible values of common difference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6\}$

So, statement–2 is not true.

EXAMPLE 14 *Statement-1: The probabilities of solving a new problem by 3 students are* $\frac{1}{2}, \frac{1}{3}$ *and* $\frac{1}{4}$ *respectively. The probability that the problem will be solved by them is* $\frac{1}{4}$.

Statement-2: If A, B and C are three independent events, then the probability that at least one of them happens is $1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement-2 is true.

Required probability $= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{3}{4}$

So, statement-1 is also true and statement-2 is a correct explanation for statement-1.

EXAMPLE 15 *Let A and B be two independent events.*

Statement-1: If $P(A) = 0.3$ *and* $P(A \cup \overline{B}) = 0.8$, *then* $P(B) = \frac{2}{7}$

Statement-2: For any event E, $P(\overline{E}) = 1 - P(E)$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Clearly, statement-2 is true.

If A and B are independent events such that

$P(A \cup \overline{B}) = 0.8$

$\Rightarrow 1 - P(\overline{A})\,P(\overline{B}) = 0.8$

$\Rightarrow 1 - \left(1 - P(A)\right)P(B) = 0.8$

$\Rightarrow 1 - 0.7\,P(B) = 0.8 \Rightarrow P(B) = \frac{2}{7}$

So, statement-1 is true and statement-2 is a correct explanation for statement-1.

EXERCISE

SECTION - I

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If A and B are two independent events, then the probability that only one of A and B occur is
   (a) $P(A) + P(B) - 2P(A \cap B)$
   (b) $P(A) + P(B) - P(A \cap B)$
   (c) $P(A) + P(B)$
   (d) none of these

2. If A and B are two events, then P (neither A nor B) equals
   (a) $1 - P(A \cup B)$ (b) $P(\overline{A}) + P(\overline{B})$
   (c) $1 - P(A) - P(B)$ (d) none of these

3. A drawer contains 5 brown socks and 4 blue socks well mixed. A man reaches the drawer and pulls out 2 socks at random. What is the probability that they match?
   (a) 4/9 (b) 5/8 (c) 5/9 (d) 7/12

4. If A and B are any two events, the probability that exactly one of them occurs is
   (a) $P(A) + P(B) + 2P(A \cap B)$
   (b) $P(A) + P(B) - P(A \cap B)$
   (c) $P(\overline{A}) + P(\overline{B}) + 2P(\overline{A} \cap \overline{B})$
   (d) $P(A \cap \overline{B}) + P(\overline{A} \cap B)$

5. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$, is
   (a) 0.4 (b) 0.8 (c) 1.2 (d) 1.4

6. Three identical dice are rolled. The probability that the same number will appear on each of them, is
   (a) 1/6 (b) 1/36 (c) 1/18 (d) 3/28

7. If A and B are two independent events, the probability that both A and B occur is 1/8 and the probability that neither of them occurs is 3/8. The probability of the occurrence of A, is
   (a) $\frac{1}{2}, \frac{1}{4}$ (b) $\frac{1}{3}, \frac{1}{4}$ (c) $\frac{1}{4}, \frac{1}{6}$ (d) $\frac{1}{5}, \frac{1}{2}$

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8. Let E and F be two independent events. The probability that both E and F happen is $1/12$ and the probability that neither E nor F occurs is $1/2$. Then,
(a) $P(E)=\frac{1}{3}, P(F)=\frac{1}{4}$ (b) $P(E)=\frac{1}{2}, P(F)=\frac{1}{6}$
(c) $P(E)=\frac{1}{6}, P(F)=\frac{1}{2}$ (d) $P(E)=\frac{1}{4}, P(F)=\frac{2}{3}$

9. If m rupee coins and n ten paise coins are placed in a line, then the probability that the extreme coins are ten paise coins, is
(a) ${}^{m+n}C_m$ (b) $\frac{n(n-1)}{(m+n)(m+n-1)}$
(c) ${}^{m+n}P_m$ (d) ${}^{m+n}P_n$

10. If A and B are two events such that $P(A)=3/4$ and $P(B)=5/8$, then
(a) $P(A\cup B)\geq\frac{3}{4}$ (b) $P(A'\cap B)\leq\frac{1}{4}$
(c) $\frac{3}{8}\leq P(A\cap B)\leq\frac{5}{8}$ (d) all of these

11. If there are 6 girls and 5 boys who sit in a row, then the probability that no two boys sit together is
(a) $\frac{6!6!}{2!11!}$ (b) $\frac{7!5!}{2!11!}$ (c) $\frac{6!7!}{2!11!}$ (d) none of these

12. A mapping is selected at random from the set of all the mappings of the set $A=\{1, 2, ..., n\}$ into itself. The probability that the mapping selected is an injection is
(a) $\frac{1}{n^n}$ (b) $\frac{1}{n!}$ (c) $\frac{(n-1)!}{n^{n-1}}$ (d) $\frac{n!}{n^{n-1}}$

13. In Q. 12 the probability that the mapping is a bijection, is
(a) $\frac{1}{n^n}$ (b) $\frac{1}{n!}$ (c) $\frac{(n-1)!}{n^{n-1}}$ (d) $\frac{n!}{n^{n-1}}$

14. Let A and B be two finite sets having m and n elements respectively such that $m\leq n$. A mapping is selected at random from the set of all mappings from A to B. The probability that the mapping selected is an injection, is
(a) $\frac{n!}{(n-m)!\,m^n}$ (b) $\frac{n!}{(n-m)!\,n^m}$
(c) $\frac{m!}{(n-m)!\,n^m}$ (d) $\frac{m!}{(n-m)!\,m^n}$

15. In Q. 14 if $m>n$ then the probability that the mapping selected is an injective map is
(a) $\frac{n!}{(n-m)!\,m^n}$ (b) $\frac{n!}{(n-m)!\,n^m}$
(c) $\frac{{}^nC_m}{n^m}$ (d) none of these

16. Two dice are rolled one after the other. The probability that the number on the first is smaller than the number on the second is
(a) 1/2 (b) 7/18 (c) 3/4 (d) 5/12

17. A natural number is chosen at random from the first 100 natural numbers. The probability that $x+100/x>50$, is
(a) 1/10 (b) 11/50 (c) 11/20 (d) none of these

18. A binary operation is chosen at random from the set of all binary operations on a set A containing n elements. The probability that the binary operation is commutative, is
(a) $\frac{n^n}{n^{n^2}}$ (b) $\frac{n^{n/2}}{n^{n^2}}$ (c) $\frac{n^{n/2}}{n^{n^2/2}}$ (d) none of these

19. In the above question the probability that the binary operation is non-commutative is
(a) $\frac{n^{n^2}-n^{n(n+1)}}{n^{n^2}}$ (b) $\frac{n^{n^2}-n^{\frac{n(n+1)}{2}}}{n^{n^2}}$
(c) $\frac{n^{n^2/2}-n^{\frac{n(n+1)}{2}}}{n^{n^2/2}}$ (d) none of these

20. A four figure number is formed of the figures 1, 2, 3, 5 with no repetitions. The probability that the number is divisible by 5 is
(a) 3/4 (b) 1/4 (c) 1/8 (d) none of these

21. In the above question the probability that the number is odd is
(a) 3/4 (b) 1/4 (c) 1/8 (d) none of these

22. Suppose $n\,(\geq 3)$ persons are sitting in a row. Two of them are selected at random. The probability that they are not together is
(a) $1-\frac{2}{n}$ (b) $\frac{2}{n-1}$ (c) $1-\frac{1}{n}$ (d) none of these

23. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is
(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250

24. In shuffling a pack of playing cards, four are accidently dropped. The probability that the missing cards should be one from each suit is
(a) $\frac{1}{256}$ (b) $\frac{1}{270725}$ (c) $\frac{2197}{20825}$ (d) none of these.

25. If A and B are independent events such that $P(A)>0$, $P(B)>0$, then
(a) A and B are mutually exclusive
(b) A and $\bar{B}$ are dependent
(c) $\bar{A}$ and B are dependent
(d) $P(A/B)+P(\bar{A}/B)=1$

26. If A and B are independent events such that $P(A)>0$, $P(B)>0$, then
(a) A and B are mutually exclusive
(b) A and $\bar{B}$ are independent
(c) $P(A\cup B)=P(\bar{A})P(\bar{B})$
(d) $P(A/B)=P(\bar{A}/B)$

27. Twelve balls are distributed among three boxes. The probability that the first box contains 3 balls is

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(a) $\frac{110}{9}\left(\frac{2}{3}\right)^{10}$ (b) $\frac{9}{110}\left(\frac{2}{3}\right)^{10}$ (c) $\frac{^{12}C_3}{12^3}\times 2^9$ (d) $\frac{^{12}C_3}{3^{12}}$

28. A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is
(a) 1/2 (b) 5/9 (c) 4/9 (d) 2/3

29. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur is 0.14. Then the probability that neither A nor B occurs is
(a) 0.39 (b) 0.25 (c) 0.11 (d) none of these.

30. For any two events A and B in a sample space which one of the ofllowing is incorrect?
(a) $P(A/B)\geq \frac{P(A)+P(B)-1}{P(B)}$, $P(B)\neq 0$, is always true
(b) $P(A\cap \bar{B}) = P(A) - P(A\cap B)$ does not hold
(c) $P(A\cup B) = 1 - P(\bar{A})\,P(\bar{B})$, if A and B are independent
(d) $P(A\cup B) = 1 - P(\bar{A})\,P(\bar{B})$, if A and B are disjoint.

31. The odds against a certain event are 5 : 2 and the odds in favour of another independent event are 6 : 5. The probability that at least one of the events will happen is
(a) 25/77 (b) 52/77 (c) 12/77 (d) 65/77

32. A bag contains four tickets marked with numbers 112, 121, 211, 222. One ticket is drawn at random from the bag. Let E_i $(i = 1, 2, 3)$ denote the event that ith digit on the ticket is 2. Then, which one of the following is incorrect?
(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_3 and E_1 are independent
(d) E_1, E_2, E_3 are independent

33. A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept aside the first. This process is repeated till all the balls are drawn from the box. The probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red is
(a) $\frac{1}{1260}$ (b) $\frac{1}{7560}$ (c) $\frac{1}{126}$ (d) none of these

34. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, III are p, q and 1/2 respectively. If the probability that the student is successful is 1/2, then
(a) $p = q = 1$ (b) $p = q = \frac{1}{2}$
(c) $p = 0, q = 1$ (d) there are infinite values of p and q

35. A cricket club has 15 members, of whom only 5 can bowl. If the names of 15 members are put into a box and 11 are drawn at random, then the probability of obtaining an eleven containing at least 3 bowlers is
(a) 7/13 (b) 6/13 (c) 11/15 (d) 12/13.

36. If $\frac{1+4p}{p}, \frac{1-p}{4}, \frac{1-2p}{2}$ are probabilities of three mutually exclusive events, then
(a) $\frac{1}{3}\leq p\leq \frac{1}{2}$ (b) $\frac{1}{2}\leq p\leq \frac{2}{3}$
(c) $\frac{1}{6}\leq p\leq \frac{1}{2}$ (d) none of these

37. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is,
(a) 16/81 (b) 1/81 (c) 80/81 (d) 65/81

38. Three numbers are chosen from 1 to 30. The probability that they are not consecutive is
(a) $\frac{144}{145}$ (b) $\frac{143}{145}$ (c) $\frac{142}{145}$ (d) none of these

39. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of the experiment are performed. The probability that the event A happens at least once is
(a) 0.936 (b) 0.784 (c) 0.904 (d) none of these

40. If A and B are two events such that $P(A) > 0$ and $P(B)\neq 1$, then $P(\bar{A}/\bar{B})$ is equal to
(a) $1 - P(A/\bar{B})$ (b) $1 - P(\bar{A}/B)$
(c) $\frac{1-P(A\cap B)}{P(\bar{B})}$ (d) $\frac{P(\bar{A})}{P(\bar{B})}$

41. A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing single event is
(a) 0.56 (b) 0.54 (c) 0.38 (d) 0.94

42. Let S be a set containing n elements. Two subsets A and B of S are chosen at random. The probability that $A\cup B = S$ is
(a) $\frac{^{2n}C_n}{2^{2n}}$ (b) $\left(\frac{3}{4}\right)^n$ (c) $\frac{1}{^{2n}C_n}$ (d) none of these

43. Fifteen coupons are numbered 1, 2, 3, ..., 15. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on selected coupon is 9, is
(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$ (c) $\left(\frac{3}{5}\right)^7$ (d) none of these

44. If $P(A\cap B) = \frac{1}{2}$, $P(\bar{A}\cap\bar{B}) = \frac{1}{3}$, $P(A) = p$, $P(B) = 2p$, then the value of p is given by
(a) 1/3 (b) 7/18 (c) 4/9 (d) 1/2

45. If $P(A\cup B) = 3/4$ and $P(\bar{A}) = 2/3$, then $P(\bar{A}\cap B)$ is equal to
(a) 1/12 (b) 7/12 (c) 5/12 (d) 1/2

46. The probability that in a random arrangement of the letters of the word 'UNIVERSITY', the two I's do not come together is
(a) 4/5 (b) 1/5 (c) 1/10 (d) 9/10

47. A letter is taken out at random from 'ASSISTANT' and another is taken out from 'STATISTICS'. The probability that they are the same letters, is

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(a) 1/45 (b) 13/90 (c) 19/90 (d) none of these

48. Out of 40 consecutive integers, two are chosen at random, the probability that their sum is odd is
(a) 14/29 (b) 20/39 (c) 1/2 (d) none of these

49. Three integers are chosen at random from the first 20 integers. The probability that their product is even, is
(a) 2/19 (b) 3/29 (c) 17/19 (d) 4/29

50. Three dice are thrown. The probability that the same number will appear on each of them, is
(a) 1/6 (b) 1/18 (c) 1/36 (d) none of these

51. If A and B are independent events and $P(C) = 0$, then
(a) A and C are independent
(b) B and C are independent
(c) A, B and C are independent
(d) all of these.

52. If two squares are chosen at random on a chess board, the probability that they have a side in common is
(a) 1/9 (b) 1/18 (c) 2/7 (d) none of these

53. An ordinary cube has four blank faces, one face marked 2 and another marked 3. Then the probability of obtaining 9 in 5 throws, is
(a) $\frac{31}{7776}$ (b) $\frac{5}{2592}$ (c) $\frac{5}{1944}$ (d) $\frac{5}{1296}$

54. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are
(a) $\frac{1}{9}, \frac{1}{3}$ (b) $\frac{1}{16}, \frac{1}{4}$ (c) $\frac{1}{4}, \frac{1}{2}$ (d) none of these

55. The probability that the 13th day of a randomly chosen month is a Friday, is
(a) $\frac{1}{12}$ (b) $\frac{1}{7}$ (c) $\frac{1}{84}$ (d) none of these

56. There are 20 cards. 10 of these cards have the letter '*I*' printed on them and the other 10 have the letter '*T*' printed on them. If three cards are picked up at random and kept in the same order, the probability of making word *IIT* is
(a) $\frac{4}{27}$ (b) $\frac{5}{38}$ (c) $\frac{1}{8}$ (d) $\frac{9}{80}$

57. Two coins and a die are tossed. The probability that both coins fall heads and the die shows a 3 or 6, is
(a) 1/8 (b) 1/12 (c) 1/16 (d) none of these

58. A die is rolled three times, the probability of getting a larger number than the previous number each time is
(a) $\frac{15}{216}$ (b) $\frac{5}{54}$ (c) $\frac{13}{216}$ (d) $\frac{1}{18}$

59. One hundred cards are numbered from 1 to 100. The probability that a randomly chosen card has a digit 5 is
(a) $\frac{1}{100}$ (b) $\frac{9}{100}$ (c) $\frac{19}{100}$ (d) none of these

60. Three six faced dice are tossed together, then the probability that exactly two of the three numbers are equal is
(a) 165/216 (b) 177/216 (c) 51/216 (d) 90/216

61. If a party of n persons sit at a round table, then the odds against two specified individuals sitting next to each other are
(a) $2 : n-3$ (b) $n-3 : 2$ (c) $n-2 : 2$ (d) $2 : n-2$

62. A and B stand in a ring with 10 other persons. If the arrangement of the persons is at random, then the probability that there are exactly 3 persons between A and B is
(a) 2/11 (b) 9/11 (c) 1/11 (d) none of these

63. Three letters, to each of which corresponds an envelope, are placed in the envelopes at random. The probability that all the letters are not placed in the right envelopes is
(a) 1/6 (b) 5/6 (c) 1/3 (d) 2/3

64. The sum of two positive quantities is equal to $2n$. The probability that their product is not less than 3/4 times their greatest product is
(a) 3/4 (b) 1/2 (c) 1/4 (d) none of these.

65. If p is the probability that a man aged x will die in a year, then the probability that out of n men $A_1, A_2, \ldots, A_n$ each aged x, A_1 will die in a year and be first to die, is
(a) $1-(1-p)^n$ (b) $(1-p)^n$
(c) $\frac{1}{n}[1-(1-p)^n]$ (d) $\frac{1}{n}(1-p)^n$

66. If the letters of the word 'MISSISSIPPI' are written down at random in a row, the probability that four *S*'s come consecutively is
(a) $\frac{8}{165}$ (b) $\frac{4}{165}$ (c) $\frac{161}{165}$ (d) none of these

67. If the letters of the word "MISSISSIPPI" are written down at random, in a row the probability that no two '*S*' occur together is
(a) $\frac{5}{33}$ (b) $\frac{7}{33}$ (c) $\frac{6}{31}$ (d) none of these

68. If the letters of the word 'REGULATIONS' be arranged at random, the probability that there will be exactly 4 letters between *R* and *E* is
(a) $\frac{6}{55}$ (b) $\frac{3}{55}$ (c) $\frac{49}{55}$ (d) none of these

69. If the letters of the word 'REGULATION' be arranged at random, the probability that there will be exactly 4 letters between *R* and *E* is
(a) 1/10 (b) 1/9 (c) 1/5 (d) 1/2

70. A bag contains a white and b black balls. Two players A and B alternately draw a ball from the bag, replacing the ball each time after draw. A begins the game. If the probability of A winning (that is drawing a white ball) is twice the probability of B winning, then the ratio $a : b$ is equal to
(a) 1:2 (b) 2:1 (c) 1:1 (d) none of these

71. Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is
(a) $\frac{9}{87}$ (b) $\frac{12}{87}$ (c) $\frac{15}{87}$ (d) $\frac{47}{87}$

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72. Cards are drawn one by one without replacement from a pack of 52 cards. The probability that 10 cards will precede the first ace is
(a) $\frac{241}{1456}$ (b) $\frac{164}{4165}$ (c) $\frac{451}{884}$ (d) none of thes

73. If A and B are two independent events, then A and $\bar{B}$ are
(a) not independent (b) also independent
(c) mutually exclusive (d) none of these.

74. A bag contains 4 tickets numbered 1, 2, 3, 4 and another bag contains 6 tickets numbered 2, 4, 6, 7, 8, 9. One bag is chosen and a ticket is drawn. The probability that the ticket bears the number 4 is
(a) 1/48 (b) 1/8 (c) 5/24 (d) none of these

75. A six faced die is a biased one. It is thrice more likely to show an odd number than show an even number. It is thrown twice. The probability that the sum of the numbers in the two throws is even, is
(a) 5/9 (b) 5/8 (c) 1/2 (d) none of these

76. A bag contains an assortment of blue and red balls. If two balls are drawn at random, the probability of drawing two red balls is five times the probability of drawing two blue balls. Furthermore, the probability of drawing one ball of each colour is six times the probability of drawing two blue balls. The number of red and blue balls in the bag is
(a) 6, 3 (b) 3, 6 (c) 2, 3 (d) none of these

77. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is
(a) 3/11 (b) 4/11 (c) 2/11 (d) none of these

78. Three letters are written to different persons and addresses to three envelopes are also written. Without looking at the addresses, the probability that the letters go into right envelopes is
(a) 1/27 (b) 1/6 (c) 1/9 (d) none of these

79. A coin is tossed three times. The probability of getting head and tail alternately is
(a) 1/8 (b) 1/2 (c) 1/4 (d) none of these.

80. A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_3 = 30$, is
(a) $\frac{{}^{20}C_2}{{}^{50}C_5}$ (b) $\frac{{}^{29}C_2}{{}^{50}C_5}$ (c) $\frac{{}^{20}C_2 \times {}^{29}C_2}{{}^{50}C_5}$ (d) none of these

81. One ticket is selected at random from 100 tickets numbered 00, 01, 02, ..., 99. Suppose A and B are the sum and product of the digit found on the ticket. Then $P(A = 7/B = 0)$ is given by
(a) 2/13 (b) 2/19 (c) 1/50 (d) none of these

82. If the probability that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is
(a) $p + q$ (b) $p + q - 2pq$
(c) $p + q - pq$ (d) $p + q + pq$

83. Four positive integers are taken at random and are multiplied together. Then the probability that the product ends in an odd digit other than 5, is
(a) 609/625 (b) 16/625 (c) 2/5 (d) 3/5

84. Two cards are drawn from a well shuffled deck of 52 cards. The probability that one is red card and the other is a queen is
(a) 4/51 (b) 16/221 (c) 50/663 (d) none of these

85. The probability that the roots of the equation $x^2 + nx + \frac{1}{2} + \frac{n}{2} = 0$ are real where $n \in N$ such that $n \le 5$, is
(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

86. A matrix is chosen at random from the set of all 2×2 matrices with elements 0 and 1 only. The probability that the value of the determinant of the matrix chosen is positive, is
(a) 1/2 (b) 3/16 (c) 11/16 (d) 13/16

87. If p is chosen at random in the interval $0 \le p \le 5$, the probability that the roots of the equation $x^2 + px + \frac{p}{4} + \frac{1}{2} = 0$ are real, is
(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

88. Dialing a telephone number an old man forgets the last two digits remembering only that these are different dialled at random. The probability that the number is dialled correctly, is
(a) 1/45 (b) 190 (c) 1/100 (d) none of these

89. Three squares of a chess board are chosen at random, the probability that two are of one colour and one of another is
(a) 16/21 (b) 8/21 (c) 32/12 (d) none of these

90. If n integers taken at random are multiplied together, then the probability that the last digit of the product is 1, 3, 7 or 9, is
(a) $\frac{2^n}{5^n}$ (b) $\frac{4^n - 2^n}{5^n}$ (c) $\frac{4^n}{5^n}$ (d) none of these

91. In Q. 90, the probability that the last digit is 2, 4, 6 or 8, is
(a) $\frac{8^n}{5^n}$ (b) $\frac{8^n - 2^n}{5^n}$ (c) $\frac{4^n - 2^n}{5^n}$ (d) none of these

92. The probability that a teacher will give an unannounced test during any class meeting is 1/5. If a student is absent twice, then the probability that the student will miss at least one test, is
(a) 4/5 (b) 2/5 (c) 7/5 (d) 9/25

93. Fifteen coupons are numbered 1, 2, ..., 15 respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is
(a) $\left(\frac{9}{10}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$ (c) $\left(\frac{3}{5}\right)^7$ (d) none of these

94. Five coins whose faces are marked 2, 3 are thrown. The probability of obtaining a total of 12, is
(a) 5/32 (b) 11/16 (c) 5/16 (d) 10/16

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SECTION - II

ASSERATION REASON TYPE MCQS

95. *Statement–1*: If $\frac{1}{5}(1+5p), \frac{1}{3}(1+2p), \frac{1}{3}(1-p)$ and $\frac{1}{5}(1-3p)$ are probabilities of four mutually exclusive events, then p can take infinite number of values.
Statement–2: If A, B, C and D are four mutually exclusive events, then
$P(A), P(B), P(C), P(D) \geq 0$
and $P(A)+P(B)+P(C)+P(D) \leq 1$
(a) 1 (b) 2 (c) 3 (d) 4

96. *Statement–1*: A natural number is chosen at random. The probability that the sum of the digits of its square is 93, is 0.
Statement–2: A number is divisible by 31 iff sum of its digits is divisible by 31.
(a) 1 (b) 2 (c) 3 (d) 4

97. Let $m \in N$ and suppose three numbers are chosen at random from the numbers 1, 2, 3, ..., m.
Statement–1: If $m = 2n$ for some $n \in N$, then the chosen numbers are in A.P. with probability $\frac{3}{2(2n-1)}$.
Statement–2: If $m = 2n+1$, then the chosen numbers are in A.P. with probability $\frac{3n}{4n^2-1}$
(a) 1 (b) 2 (c) 3 (d) 4

98. A set P contains n elements. Two subsets A and B of P are chosen independently.
Statement–1: Probability that $A \cap B = A$ is $(3/4)^n$.
Statement–2: Probability that $A \cup B = P$ is $(1/2)^n$.
(a) 1 (b) 2 (c) 3 (d) 4

99. Let A and B be two events such that $P(A) > 0$.
Statement–1: If $P(A)+P(B) > 1$, then
$P(B/A) \geq 1 - P(\bar{B})/P(A)$
Statement–2: If $P(A/\bar{B}) \geq P(A)$, then $P(A) \geq P(A/B)$.
(a) 1 (b) 2 (c) 3 (d) 4

100. Let A, B and C be three events associated to a random experiment.
Statement–1: If $A \cap B \subseteq C$, then $P(C) \geq P(A)+P(B)-1$.
Statement–2: If $P\{(A \cap B) \cup (B \cap C) \cup (C \cap A)\}$
$\leq \min\{P(A \cup B), P(B \cup C), P(C \cup A)\}$
(a) 1 (b) 2 (c) 3 (d) 4

101. From an urn containing a white and b black balls k (< , b) balls are drawn and laid aside, their colour is un noticed. Then another ball, that is $(k+1)^{th}$ ball, is drawn
Statement–1: Probability that $(k+1)^{th}$ ball drawn is white is $\frac{a}{a+b}$.
Statement–2: Probability that $(k+1)^{th}$ ball drawn is black is $\frac{a}{a+b}$.
(a) 1 (b) 2 (c) 3 (d) 4

Answers

1. (a) 2. (a) 3. (a) 4. (d) 5. (c) 6. (b) 7. (a)
8. (a) 9. (b) 10. (d) 11. (c) 12. (c) 13. (c) 14. (b)
15. (d) 16. (d) 17. (c) 18. (c) 19. (b) 20. (b) 21. (a)
22. (a) 23. (b) 24. (c) 25. (d) 26. (b) 27. (a) 28. (c)
29. (a) 30. (d) 31. (b) 32. (d) 33. (a) 34. (d) 35. (d)
36. (d) 37. (a) 38. (a) 39. (b) 40. (a) 41. (b) 42. (b)
43. (d) 44. (b) 45. (c) 46. (a) 47. (c) 48. (b) 49. (c)
50. (c) 51. (d) 52. (b) 53. (d) 54. (a) 55. (c) 56. (b)
57. (b) 58. (b) 59. (c) 60. (d) 61. (b) 62. (a) 63. (b)
64. (b) 65. (c) 66. (b) 67. (b) 68. (a) 69. (b) 70. (c)
71. (d) 72. (b) 73. (b) 74. (c) 75. (b) 76. (a) 77. (b)
78. (b) 79. (c) 80. (c) 81. (b) 82. (b) 83. (b) 84. (c)
85. (d) 86. (b) 87. (c) 88. (b) 89. (a) 90. (a) 91. (c)
92. (d) 93. (c) 94. (c) 95. (a) 96. (c) 97. (b) 98. (c)
99. (c) 100. (b) 101. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. Four tickets marked 00, 01, 10, 11, respectively are placed in a bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on tickets thus drawn is 23, is
(a) 25/256 (b) 100/256 (c) 231/256 (d) none of these

2. Two persons each makes a single throw with a pair of dice. The probability that the throws are unequal is given by
(a) $1/6^3$ (b) $73/6^3$ (c) $51/6^3$ (d) none of these

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3. A and B toss a fair coin each simultaneously 50 times. The probability that both of them will not get tail at the same toss is

(a) $\left(\frac{3}{4}\right)^{50}$ (b) $\left(\frac{2}{7}\right)^{50}$ (c) $\left(\frac{1}{8}\right)^{50}$ (d) $\left(\frac{7}{8}\right)^{50}$

4. A bag contains $(2n+1)$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $n+1$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

5. Seven digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a random order. The probability that this seven digit number is divisible by 9 is

(a) 2/9 (b) 1/5 (c) 1/3 (d) 1/9

6. Three dice are thrown. The probability that the sum of the numbers appearing is 15, is

(a) 1/216 (b) 1/72 (c) 5/108 (d) 1/18

7. Probability that a student will succeed in I.I.T. entrance test is 0.2 and that he will succeed in Roorkee entrance test is 0.5. If the probability that he will be successful at both the places is 0.3, then the probability that he does not succeed at both the places is

(a) 0.4 (b) 0.3 (c) 0.2 (d) 0.6

8. A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. The probability that actually there was a six, is

(a) 4/9 (b) 5/9 (c) 3/10 (d) none of these

9. In an entrance test there are multiple choice questions. There are four possible answers to each question, of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is

(a) 37/40 (b) 1/37 (c) 36/37 (d) 1/9

10. A letter is known to have come either from London or Clifton; on the postmark only the two consecutive letters ON are legible. The probability that it came from London is

(a) 5/17 (b) 12/17 (c) 17/30 (d) 3/5

11. An urn contains 6 white and 4 black balls. A fair die is rolled and that number of balls are chosen from the urn. The probability that the balls selected are white is

(a) 1/5 (b) 1/6 (c) 1/7 (d) 1/8

12. An urn contains five balls. Two balls are drawn and are found to be white. The probability that all the balls are white is

(a) 3/4 (b) 3/5 (c) 3/10 (d) 1/2

13. A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, then the probability that the missing cards is black, is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{{}^{25}C_{13}}{{}^{51}C_{13}}$

14. A biased die is tossed and the respective probabilities for various faces to turn up are

Face	:	1	2	3	4	5	6
Probability	:	0.1	0.24	0.19	0.18	0.15	0.14

If an even face has turned up, then the probability that it is face 2 or face 4, is

(a) 0.25 (b) 0.42 (c) 0.75 (d) 0.5

15. In a bag there are three tickets numbered 1, 2, 3. A ticket is drawn at random and put back, and this is done four times. The probability that the sum of the numbers is even, is

(a) 41/81 (b) 39/81 (c) 40/81 (d) none of these

16. Two friends A and B have equal number of daughters. There are three cinema tickets which are to be distributed among the daughters of A and B. The probability that all the tickets go to daughters of A is 1/20. The number of daughters each of them have is

(a) 4 (b) 5 (c) 6 (d) 3

17. A bag contains n white and n black balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each pair consists of one white and one black ball is

(a) $\frac{1}{{}^{2n}C_n}$ (b) $\frac{2^n}{{}^{2n}C_n}$ (c) $\frac{2^n}{n!}$ (d) $\frac{2^n}{(2n)!}$

18. A bag contains 10 white and 3 black balls. Balls are drawn one-by-one without replacement till all the black balls are drawn. The probability that the procedure of drawing balls will come to an end at the seventh draw is

(a) $\frac{105}{286}$ (b) $\frac{15}{286}$ (c) $\frac{181}{286}$ (d) none of these

19. If $A_1, A_2, \ldots, A_n$ are n independent events such that $P(A_i) = \frac{1}{i+1}, i = 1, 2, \ldots, n$. The probability that none of the n events occurs, is

(a) $\frac{n}{n+1}$ (b) $\frac{1}{n+1}$ (c) $\frac{n}{(n+1)(n+2)}$ (d) none of these

20. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral equals

(a) 1/2 (b) 1/5 (c) 1/10 (d) 1/20

21. Let $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$. Then,

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A^c \cup B^c) = P(A^c) + P(B^c)$

(c) $P((A \cup B)^c) = P(A^c)\,P(B^c)$

(d) $P(A/B) = P(B)$

22. The probability that a number selected at random from the set of numbers 1, 2, 3, 4, ..., 100 is a cube, is

(a) $\frac{1}{25}$ (b) $\frac{2}{25}$ (c) $\frac{3}{25}$ (d) $\frac{4}{25}$

23. For any two independent events E_1 and E_2, $P\{(E_1 \cup E_2) \cap (\bar{E}_1) \cap (\bar{E}_2)\}$ is

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(a) $\leq 1/4$ (b) $> 1/4$ (c) $\geq 1/2$ (d) none of these

24. If A and B are two events than the value of the determinant choosen at random from all the determinants of order 2 with entries 0 or 1 only is positive or negative respectively. Then,
(a) $P(A) > P(B)$ (b) $P(A) < P(B)$
(c) $P(A) = P(B) = 1/2$ (d) $P(A) = P(B)$

25. The probability that a man will live 10 more years is 1/4 and the probability that his wife will live 10 more years is 1/3. Then the probability that neither will be alive in 10 years, is
(a) 5/12 (b) 1/2 (c) 7/12 (d) 11/12.

26. The probability that at least one of the events A and B occurs is 0.7 and they occur simultaneously with probability 0.2. Then, $P(\overline{A}) + P(\overline{B}) =$
(a) 1.8 (b) 0.6 (c) 1.1 (d) 1.4

27. A man alternately tosses a coin and throws a die beginning with the coin. The probability that he gets a head in the coin before he gets a 5 or 6 in the dice is
(a) 3/4 (b) 1/2 (c) 1/3 (d) none of these

28. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Then,
(a) $P(A) = 1/2, P(B) = 1/3$ (b) $P(A) = 1/2, P(B) = 1/6$
(c) $P(A) = 1/3, P(B) = 1/6$ (d) none of these

29. A, B, C are any three events. If $P(S)$ denotes the probability of S happening, then $P(A \cap (B \cup C)) =$
(a) $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$
(b) $P(A) + P(B) + P(C) - P(B)\,P(C)$
(c) $P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$
(d) $P(A) + P(B) + P(C)$

30. In a class of 125 students 70 passed in Mathematics, 55 in Statistics and 30 in both. The probability that a student selected at random from the class, has passed in only one subject is
(a) 13/25 (b) 3/25 (c) 17/25 (d) 8/25

31. A box contains 10 mangoes out of which 4 are rotten. 2 mangoes are taken out together. If one of them is found to be good, the probability that the other is also good is
(a) 1/3 (b) 8/15 (c) 5/13 (d) 2/3

32. A lot consists of 12 good pencils, 6 with minor defects and 2 with major defects. A pencil is choosen at random. The probability that this pencil is not defective is
(a) 3/5 (b) 3/10 (c) 4/5 (d) 1/2

33. 3 mangoes and 3 apples are in a box. If 2 fruits are chosen at random, the probability that one is a mango and the other is an apple is
(a) 3/5 (b) 5/6 (c) 1/36 (d) none of these

34. There are 3 bags which are known to contain 2 white and 3 black, 4 white and 1 black and 3 white and 7 black balls respectively. A ball is drawn at random from one of the bags and found to be a black ball. Then the probability that it was drawn from the bag containing the most black balls is
(a) 7/15 (b) 5/19 (c) 3/4 (d) none of these

35. Among the workers in a factory only 30% receive bonus and among those receiving bonus only 20% are skilled. The probability that a randomly selected worker is skilled and is receiving bonus is
(a) 0.03 (b) 0.02 (c) 0.06 (d) 0.015

36. If two events A and B are such that $P(A^c) = 0.3, P(B) = 0.4, P(A \cap B^c) = 0.5$, then $P(B/A \cup B^c) =$
(a) 0.20 (b) 0.25 (c) 0.30 (d) 0.35

37. An almirah stores 5 black and 4 white socks well mixed. A boy pulls out 2 socks at random. The probability that 2 are of the same colour is
(a) 4/9 (b) 5/8 (c) 5/9 (d) 7/12

38. There are 4 white and 4 black balls in a bag and 3 balls are drawn at random. If balls of same colour are identical, the probability that none of them is black, is
(a) 1/4 (b) 1/14 (c) 1/2 (d) none of these

39. Cards are drawn one-by-one at random from a well shuffled pack of 52 playing cards untill 2 aces are obtained from the first time. The probability that 18 draws are required for this, is
(a) 3/34 (b) 17/455
(c) 561/15925 (d) none of these

40. Five different objects A_1, A_2, A_3, A_4, A_5 are distributed randomly in 5 places marked 1, 2, 3, 4, 5. One arrangement is picked at random. The probability that in the selected arrangement, none of the object occupies the place corresponding to its number, is
(a) 119/120 (b) 1/15
(c) 11/30 (d) none of these

41. A father has 3 children with at least one boy. The probability that he has 2 boys and one girl is
(a) 1/4 (b) 1/3 (c) 2/3 (d) none of these

42. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for the job. The probability that at least one of the selected persons will be a woman is
(a) 25/39 (b) 14/39 (c) 5/13 (d) 10/13

43. A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are choosen at random. It is given that the maximum number on the two choosen tickets is not more than 10. The probability that the minimum number is 5 is
(a) 13/15 (b) 1/330 (c) 1/3 (d) 1/9

44. A wire of length l is cut into three pieces. What is the probability that the three pieces form a triangle ?
(a) 1/2 (b) 1/4 (c) 2/3 (d) none of these

45. If, $x \in [0, 5]$, then what is the probability that $x^2 - 3x + 2 \geq 0$?
(a) 4/5 (b) 1/5 (c) 2/5 (d) none of these

46. If $a \in [-20, 0]$, then the probability that the graph of the function $y = 16x^2 + 8(a+5)x - 7a - 5$ is strictly above the x-axis is
(a) 1/2 (b) 1/17 (c) 17/20 (d) none of these

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47. The probability that A can solve a problem is 2/3 and B can solve it is 3/4. If both attempt the problem, what is the probability that the problem gets solved ?
(a) 11/12 (b) 7/12 (c) 5/12 (d) 9/12

48. A bag contains $(2n+1)$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $(n+1)$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then $n =$
(a) 10 (b) 11 (c) 12 (d) 13

49. Five boys and three girls are seated at random in a row. The probability that no boy sits between two girls is
(a) 1/56 (b) 1/8 (c) 3/28 (d) none of these

50. A and B are two independent events such that $P(A) = \frac{1}{5}$, $P(A \cup B) = \frac{7}{10}$. Then, $P(\bar{B}) =$
(a) 3/8 (b) 2/7 (c) 7/9 (d) none of these

51. A and B are two independent events such that their probabilities are $\frac{3}{10}$ and $\frac{2}{5}$ respectively. The probability of exactly one of the events happening is
(a) 23/50 (b) 1/2 (c) 31/50 (d) none of these

52. There are 7 seats in a row. Three persons take seats at random. The probability that the middle seat is always occupied and no two persons are consecutive is
(a) 9/70 (b) 9/35 (c) 4/35 (d) none of these

53. 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the some boy does not receive both the pens is
(a) 5/11 (b) 7/11 (c) 2/3 (d) 6/11

54. 4 five-rupee coins, 3 two-rupee coins and 2 one-rupee coins are stacked together in a column at random. The probability that the coins of the same denominator are consecutive is
(a) 13/9! (b) 1/210 (c) 1/35 (d) none of these

55. Two cards are drawn at random from a pack of 52 cards. The probability of getting at least a spade and an ace is
(a) 1/34 (b) 8/221 (c) 1/26 (d) 2/51

56. A and B appeared for an interview for two posts. The probability of A's selection is 1/3 and that of B's selection is 2/5. The probability that only one of them is selected, is
(a) 7/15 (b) 8/15 (c) 2/15 (d) 4/15

57. Let S be the sample space associated with the random experiment of throwing simultaneously two unbiased dice with six faced (numbered 1 to 6) and let $E_k = \{(a, b) \in S : ab = k\}$ for $k \geq 1$. If $p_k = P(E_k)$ for $k \geq 1$, then correct among the following is
(a) $p_1 < p_{30} < p_4 < p_6$ (b) $p_{36} < p_6 < p_2 < p_4$
(c) $p_1 < p_{11} < p_4 < p_6$ (d) $p_{36} < p_{11} < p_6 < p_4$

58. For $k = 1, 2, 3$ the box B_k contains k red balls and $k+1$ white balls. Let $P(B_1) = \frac{1}{2}$, $P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$. A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from B_2, is
(a) $\frac{35}{78}$ (b) $\frac{14}{39}$ (c) $\frac{10}{13}$ (d) $\frac{12}{13}$

59. Four numbers are chosen at random from $\{1, 2, 3, ..., 40\}$. The probability that they are not consecutive is
(a) $\frac{1}{2470}$ (b) $\frac{4}{7969}$ (c) $\frac{2469}{2470}$ (d) $\frac{7965}{7969}$

60. A number n is chosen at random from $\{1, 2, 3, ..., 1000\}$. The probability that n is a number which leaves a remainder 1 when divided by 7, is
(a) $\frac{71}{580}$ (b) $\frac{143}{1000}$ (c) $\frac{72}{500}$ (d) $\frac{71}{1000}$

Answers

(a) 2. (d) 3. (a) 4. (a) 5. (d) 6. (c) 7. (d)
a) 9. (b) 10. (b) 11. (a) 12. (d) 13. (b) 14. (c)
c) 16. (d) 17. (b) 18. (b) 19. (b) 20. (c) 21. (c)
a) 23. (a) 24. (d) 25. (b) 26. (c) 27. (a) 28. (a)
) 30. (a) 31. (c) 32. (a) 33. (a) 34. (a) 35. (c)
36. (b) 37. (a) 38. (a) 39. (c) 40. (c) 41. (b) 42. (a)
43. (d) 44. (b) 45. (a) 46. (d) 47. (a) 48. (a) 49. (c)
50. (a) 51. (a) 52. (c) 53. (a) 54. (b) 55. (c) 56. (a)
57. (a) 58. (b) 59. (c) 60. (b)

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19

DISCRETE PROBABILITY DISTRIBUTIONS

1. RANDOM VARIABLE AND ITS PROBABILITY DISTRIBUTION

DEFINITION *Let S be the sample space associated with a given random experiment. Then, a real valued function X which assigns to each event $w \in S$ to a unique real number $X(w)$ is called a random variable.*

In other words, a random variable is a real valued function having domain as the sample space associated with a random experiment.

Thus, a random variable associated with a given random experiment associates every event to a unique real number as discussed below.

Consider a random experiment of tossing three coins. The sample space of eight possible outcomes of this experiment is given by

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let X be a real valued function on S, defined by

$X(w)$ = Number of heads in $w \in S$.

Then, X is a random variable such that :

$X(HHH) = 3$, $X(HHT) = 2$, $X(HTH) = 2$, $X(THH) = 2$

$X(HTT) = 1$, $X(THT) = 1$, $X(TTH) = 1$ and $X(TTT) = 0$.

Also, if w denotes the event "Getting two heads", then

$$w = \{HTH, THH, HHT\}$$

and, $X(w) = 2$

Similarly, X associates every other compound event to a unique real number. For the random variable X, we have range $(X) = \{0, 1, 2, 3\}$ and we say that X is a random variable such that it assumes values 0, 1, 2, 3. This random variable can also be described as the number of heads in a single throw of three coins.

Now, consider the random experiment of throwing an unbiased die. Let Y be a real valued function defined on the sample space $S = \{1, 2, 3, 4, 5, 6\}$ associated with the random experiment, defined by

$$Y(w) = \begin{cases} 1, \text{ if the outcome is an even number} \\ -1, \text{ if the outcome is an odd number} \end{cases}$$

Clearly, Y is a random variable such that :

$$Y(1) = -1, Y(2) = 1, Y(3) = -1, Y(4) = 1, Y(5) = -1$$

and, $Y(6) = 1$.

Here, range $(Y) = \{-1, 1\}$. Therefore, we say that Y is a random variable such that it assumes values -1 and 1.

Finally, consider a random experiment of tossing three coins. Let X be a real valued function defined on the sample space $S = \{HHH, HHT, HTH, THH, HTT, TTH, THT, TTT\}$ such that

$X(w)$ = Number of tails in $w \in S$.

Then, X is a random variable such that

$$X(HHH) = 0, X(HHT) = 1, X(HTH) = 1, X(THH) = 1,$$
$$X(HTT) = 2, X(THT) = 2, X(TTH) = 2 \text{ and } X(TTT) = 3.$$

Clearly, range of X is $\{0, 1, 2, 3\}$

1.1 PROBABILITY DISTRIBUTION

Consider a random experiment in which three coins are tossed simultaneously (or a coin is tossed three times). Let X be a random variable defined on the sample space $S = \{HHH, HTH, THH, HHT, THT, TTH, HTT, TTT\}$ such that

$X(w)$ = Number of heads in $w \in S$.

Clearly, X assumes value 0, 1, 2, 3.

Now,

$P(X = 0)$ = Probability of getting no head = $P(TTT) = \frac{1}{8}$

$P(X = 1)$ = Probability of getting one head

$= P(HTT \text{ or } THT \text{ or } TTH) = \frac{3}{8}$

$P(X = 2)$ = Probability of getting two heads

$= P(HHT \text{ or } THH \text{ or } HTH) = \frac{3}{8}$

and,

$P(X = 3)$ = Probability of getting 3 heads

$= P(HHH) = \frac{1}{8}$

These values of X and the corresponding probabilities can be exhibited as under :

X :	0	1	2	3
$P(X)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Thus tabular representation of the values of a random variable X and the corresponding probabilities is known as its probability distribution.

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EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. A random variable has the following probability distribution.

x :	0	1	2	3	4	5	6	7
$p(x)$:	0	$2p$	$2p$	$3p$	p^2	$2p^2$	$7p^2$	$2p$

The value of p, is
(a) 1/10 (b) -1
(c) $-1/10$ (d) none of these

2. If X is a random-variable with distribution given below:

X :	0	1	2	3
$P(X=x)$:	k	$3k$	$3k$	k

The value of k and its variance are
(a) 1/8, 22/27 (b) 1/8, 23/27
(c) 1/8, 24/27 (d) 1/8, 3/4

3. If in a distribution each x is replaced by corresponding value of $f(x)$, then the probability of getting $f(x_i)$ when the probability of getting x_i is p_i, is
(a) p_i (b) $f(p_i)$
(c) $f\left(\frac{1}{p_i}\right)$ (d) none of these

4. A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is
(a) ${}^{11}C_6\,(0.24)^5$ (b) ${}^{11}C_6\,(0.4)^6\,(0.6)^5$
(c) ${}^{11}C_6\,(0.6)^6\,(0.4)^5$ (d) none of these

5. The probability that a man can hit a target is 3/4. He tries 5 times. The probability that he will hit the target at least three times is
(a) 291/364 (b) 371/464
(c) 471/502 (d) 459/512

6. Six ordinary dice are rolled. The probability that at least half of them will show at least 3 is
(a) $41\times\frac{2^4}{3^6}$ (b) $\frac{2^4}{3^6}$
(c) $20\times\frac{2^4}{3^6}$ (d) none of these

7. Two persons each make a single throw with a pair of dice. The probability that the throws are unequal is given by:
(a) $\frac{1}{6^3}$ (b) $\frac{73}{6^3}$
(c) $\frac{51}{6^3}$ (d) none of these

8. If the range of a random variable X is $0, 1, 2, 3, 4, \ldots$ $P(X=k)=\left(\frac{k+1}{3^k}\right)a$ for $k\geq 0$, then $a=$
(a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{8}{27}$ (d) none of these

9. In an experiment the success is twice that of failure. If the experiment is repeated 6 times, the probability that there are 4 times favourable is
(a) $\frac{64}{729}$ (b) $\frac{192}{729}$ (c) $\frac{240}{729}$ (d) none of these

10. The probability that a candidate secures a seat in Engineering through "EAMCET" is 1/10. 7 candidates are selected at random from a centre. The probability that exactly two will get seats is
(a) $15\,(0.1)^2\,(0.9)^5$ (b) $20\,(0.1)^2\,(0.9)^5$
(c) $21\,(0.1)^2\,(0.9)^5$ (d) $23\,(0.1)^2\,(0.9)^5$

11. Six coins are tossed simultaneously. The probability of getting at least 4 heads is
(a) 11/64 (b) 11/32 (c) 15/44 (d) none of these

12. Fifteen coupons are numbered 1, 2, 3, ..., 15 respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is
(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$
(c) $\left(\frac{3}{5}\right)^5$ (d) none of these

13. Two players toss 4 coins each. The probability that they both obtain the same number of heads is
(a) 5/256 (b) 1/16
(c) 35/128 (d) none of these

14. A box contains 24 identical balls of which 12 are white and 12 black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on 7th draw is
(a) 5/64 (b) 27/32 (c) 5/32 (d) none of these

15. Two dice are tossed 6 times. Then the probability that they show an exactly four of the tosses is
(a) $\frac{225}{18442}$ (b) $\frac{116}{20003}$
(c) $\frac{125}{15552}$ (d) none of these

16. If X follows a binomial distribution with parameters n and p. If $4\,(P(X=4))=P(X=2)$, then $p=$
(a) 1/2 (b) 1/4 (c) 1/6 (d) none of these

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17. The number of times a die must be tossed to obtain a 6 at least once with probability exceeding 0.9 is at least
(a) 13 (b) 19
(c) 25 (d) none of these

18. Seven chits are numbered 1 to 7. Four chits are drawn one by one with replacement. The probability that the least number appearing on any selected chit is 5, is
(a) $\left(\frac{3}{7}\right)^4$ (b) $\left(\frac{6}{7}\right)^3$ (c) $\frac{5\times4\times3}{7^3}$ (d) $\left(\frac{3}{4}\right)^4$

19. If the mean of a binomial distribution is 25, then its standard deviation lies in the interval given below:
(a) [0, 5) (b) (0, 5] (c) [0, 25) (d) (0, 25]

20. The value of C for which $P(X=k)=Ck^2$ can serve as the probability function of a random variable X that takes value 0, 1, 2, 3, 4 is
(a) $\frac{1}{30}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{1}{15}$

21. In order to get at least once a head with probability ≥ 0.9, the number of times a coin needs to be tossed is
(a) 3 (b) 4
(c) 5 (d) none of these

22. The probability that a man will hit a target in shooting practice is 0.3. If he shoots 10 times, the probability that he hits the target is
(a) 1 (b) $1-(0.7)^{10}$
(c) $(0.7)^{10}$ (d) $(0.3)^{10}$

23. If A and B each toss three coins. The probability that both get the same number of heads, is
(a) 1/9 (b) 3/16 (c) 5/16 (d) 3/8

24. A box contains 100 bulbs out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective, is
(a) $\left(\frac{1}{10}\right)^5$ (b) $\left(\frac{1}{2}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\frac{9}{10}$

25. A box contains tickets numbered 1 to 20. 3 tickets are drawn from the box with replacement. The probability that the largest number on the tickets is 7 is
(a) $\frac{2}{19}$ (b) $\frac{7}{20}$
(c) $1-\left(\frac{7}{20}\right)^3$ (d) none of these

26. A fair coin is tossed a fixed number of times. If the probability of getting 4 heads equals the probability of getting 7 heads, then the probability of getting 2 heads is
(a) $\frac{55}{2048}$ (b) $\frac{3}{4096}$
(c) $\frac{1}{1024}$ (d) none of these

27. A coin is tossed n times. The probability that head will turn up an odd number of times, is
(a) $\frac{1}{2}$ (b) $\frac{n+1}{2n}$ (c) $\frac{n-1}{2n}$ (d) $\frac{2^{n-1}-1}{2^n}$

28. Two coins are tossed five times. The probability that an odd number of heads are obtained, is
(a) $\left(\frac{1}{2}\right)^5$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) none of these

29. A six faced die is so biased that it is twice as likely to show an even number as an odd number when thrown. If it is thrown twice the probability that the sum of two numbers thrown is even, is
(a) 4/9 (b) 5/9 (c) 1/9 (d) none of these

30. An unbiased coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then n =
(a) 7 (b) 14 (c) 16 (d) 19

31. An unbiased coin is tossed n times. Let X denote the number of times head occurs. If $P(X=4)$, $P(X=5)$ and $P(X=6)$ are in AP, then the value of n can be
(a) 7, 14 (b) 10, 14
(c) 12, 7 (d) none of these

32. An unbiased coin is tossed a fixed number of times. If the probability of getting 4 heads equals the probability of getting 7 heads, then the probability of getting 2 heads is
(a) 55/2048 (b) 1/1024
(c) 3/4096 (d) none of these

33. A carton contains 20 bulbs, 5 of which are defective. The probability that, if a sample of 3 bulbs is chosen at random from the carton, 2 will be defective, is
(a) 1/16 (b) 3/64 (c) 9/64 (d) 2/3

34. In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. The minimum number of bombs which should be dropped to give a 99% chance or better of completely destroying the target is
(a) 10 (b) 11
(c) 12 (d) none of these

35. The probability of a man hitting a target is 1/4. If he fires n times such that the probability of his hitting the target at least once is greater than 2/3, then the value of n is
(a) 3 (b) 4
(c) 5 (d) none of these

36. If the mean and S.D. of a binomial distribution are 20 and 4 respectively, then the number of trials is
(a) 50 (b) 25 (c) 100 (d) 80

37. The probability of India winning a test match against West Indies is 1/2. Assuming independence from match to match the probability that in a 5 match series India's second win occurs at the third test is

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(a) 1/8 (b) 1/4 (c) 1/2 (d) 2/3

38. The mean and standard deviation of a binomial variate X are 4 and $\sqrt{3}$ respectively. Then, $P(X \geq 1) =$

(a) $1-\left(\frac{1}{4}\right)^{16}$ (b) $1-\left(\frac{3}{4}\right)^{16}$

(c) $1-\left(\frac{2}{3}\right)^{16}$ (d) $1-\left(\frac{1}{3}\right)^{16}$

39. The probability distribution of a random variable X is given by

$X = x$:	0	1	2	3	4
$P(X=x)$:	0.4	0.3	0.1	0.1	0.1

The variance of X is

(a) 1.76 (b) 2.45 (c) 3.2 (d) 4.8

[EAMCET 2007]

40. For a binomial variate X with $n = 6$, if $P(X = 4)$, then its variance is

(a) 8/9 (b) 1/4 (c) 9/8 (d) 4

Answers

1. (a) **2.** (d) **3.** (a) **4.** (a) **5.** (d) **6.** (a) **7.** (c)
8. (b) **9.** (d) **10.** (c) **11.** (b) **12.** (d) **13.** (c) **14.** (a)
15. (c) **16.** (d) **17.** (a) **18.** (a) **19.** (a) **20.** (a) **21.** (b)
22. (b) **23.** (c) **24.** (c) **25.** (d) **26.** (a) **27.** (a) **28.** (d)
29. (b) **30.** (b) **31.** (a) **32.** (a) **33.** (c) **34.** (a) **35.** (b)
36. (b) **37.** (b) **38.** (b) **39.** (a) **40.** (c)

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CARTESIAN CO-ORDINATE SYSTEM

1. DISTANCE BETWEEN TWO POINTS

The distance between any two points in the plane is the length of the line segment joining them.

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e., $PQ = \sqrt{(\text{Difference of abscissae})^2 + (\text{Difference of ordinates})^2}$

NOTE *If O is the origin and $P(x, y)$ is any point, then from the above formula, we have*

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

SOME USEFUL POINTS

(I) *In order to prove that a given figure is a*

(i) *square, prove that the four sides are equal and the diagonals are also equal.*

(ii) *rhombus, prove that the four sides are equal.*

(iii) *rectangle, prove that opposite sides are equal and the diagonals are also equal.*

(iv) *a parallelogram, prove that the opposite sides are equal.*

(v) *parallelogram but not a rectangle, prove that its opposite sides are equal but the diagonals are not equal.*

(vi) *a rhombus but not a square, prove that its all sides are equal but the diagonals are not equal.*

(II) *For three points to be collinear, prove that the sum of the distances between two pairs of points is equal to the third pair of points.*

ILLUSTRATION 1 *If the point (x, y) is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, then*

(a) $ax = by$ (b) $bx = ay$ (c) $ax + by = 0$ (d) $bx + ay = 0$

Ans. (b)

SOLUTION Let $P(x, y)$, $Q(a+b, b-a)$ and $R(a-b, a+b)$ be the given points. Then,

$$PQ = PR \qquad \text{[Given]}$$

$$\Rightarrow \sqrt{[x-(a+b)]^2 + [y-(b-a)]^2} = \sqrt{[x-(a-b)]^2 + [y-(a+b)]^2}$$

$$\Rightarrow [x-(a+b)]^2 + [y-(b-a)]^2 = [x-(a-b)]^2 + [y-(a+b)]^2$$

$$\Rightarrow bx = ay$$

ILLUSTRATION 2 *If P and Q are two points whose coordinates are $(at^2, 2at)$ and $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$ respectively and S is the point $(a, 0)$. Then, harmonic mean of SP and SQ is*

(a) a (b) $4a$ (c) $2a$ (d) $\frac{2}{a}$

Ans. (c)

SOLUTION We have,

$$SP = \sqrt{(at^2 - a)^2 + (2at - 0)^2} = a(t^2 + 1)$$

Replacing t by $-\frac{1}{t}$ in SP, we get $SQ = \frac{a}{t^2}(1 + t^2)$

$$\therefore \quad \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(t^2+1)} + \frac{t^2}{a(t^2+1)}$$

$$\Rightarrow \quad \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$$

$$\Rightarrow \quad \frac{2SPSQ}{SP + SQ} = 2a \Rightarrow HM \text{ of } SP \text{ and } SQ \text{ is } 2a$$

ILLUSTRATION 3 *A triangle with vertices $(4, 0)$, $(-1, -1)$, $(3, 5)$, is*

(a) *isosceles and right angled*

(b) *isosceles but not right angled*

(c) *right angled but not isosceles*

(d) *neither right angled nor isosceles*

Ans. (a) **[AIEEE 2002]**

SOLUTION Let ABC be the triangle with vertices $A(4, 0)$, $B(-1, -1)$ and $C(3, 5)$. Then,

$$AB = AC = \sqrt{26} \text{ and } BC = \sqrt{52}$$

Clearly, $AB^2 + AC^2 = BC^2$

Hence, the triangle is right-angled isosceles triangle.

ILLUSTRATION 4 *If the coordinates of the mid-points of side AB and AC of ΔABC are $D(3, 5)$ and $E(-3, -3)$ respectively, the BC =*

(a) 10 (b) 15 (c) 20 (d) 30

Ans. (c)

SOLUTION We have,

$$BC = 2DE = 2\sqrt{(3+3)^2 + (5+3)^2} = 20$$

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$\Rightarrow$ C is an obtuse angle

$\Rightarrow$ ΔABC is an obtuse angled isosceles triangle.

So, statement–1 is true.

Statement–2 is not true as the triangle with angles 120°, 40°, 20° is obtuse angled but not isosceles.

EXAMPLE 6 *Statement–1: The quadrilaterial whose vertices (in order) are $A(1, 0)$, $B(0, 3)$, $C(-2, 0)$ and $D(0, 2)$ cannot be convex.*

Statement–2: A quadrilateral $ABCD$ (in order) is convex iff when any diagonal is taken then the remaining vertices must be on the opposite sides of it.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Statement–2 is correct because if one angle, say D of the quadrilateral $ABCD$ which is not convex is more than 180°, then B and D lie on the same side of the diagonal AC.

Statement–2 is ture. Also, statement–2 is a correct explanation for statement–1.

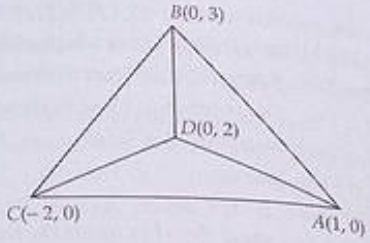


Fig. 12

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out ot which only one is correct.

1. If the vertices of a triangle are at $O(0, 0)$, $A(a, 0)$ and $B(0, a)$. Then, the distance between its circumcentre and orthocentre is

(a) $\frac{a}{2}$ (b) $\frac{a}{\sqrt{2}}$ (c) $\sqrt{2}a$ (d) $\frac{a}{4}$

2. The angles A, B and C of a ΔABC are in A.P. If $AB = 6$, $BC = 7$, then AC =

(a) 5 (b) 7
(c) 8 (d) none of these

3. If the distance between the points $P(a\cos 48°, 0)$ and $Q(0, a\cos 12°)$ is d, then $d^2 - a^2$ =

(a) $\frac{a^2}{4}(\sqrt{5}-1)$ (b) $\frac{a^2}{4}(\sqrt{5}+1)$

(c) $\frac{a}{8}(\sqrt{5}-1)$ (d) $\frac{a^2}{8}(\sqrt{5}+1)$

4. If the centroid of the triangle formed by the points (a, b), (b, c) and (c, a) is at the origin, then $a^3 + b^3 + c^3$ =

(a) 0 (b) abc
(c) $3abc$ (d) $-3abc$

5. If $O'(4, 8/3)$ is the orthocentre of the triangle ABC the coordinates of whose vertices are $O(0, 0)$, $A(8, 0)$ and $B(4, 6)$, then the coordinates of the orthocentre of $\Delta O'AB$ are

(a) (0, 0) (b) (8, 0)
(c) (4, 6) (d) none of these

6. If the coordinates of two vertices of an equilateral triangle ae (2, 4) and (2, 6), then the coordiantes of its third vertex are

(a) $(\sqrt{3}, 5)$ (b) $(2\sqrt{3}, 5)$
(c) $(2+\sqrt{3}, 5)$ (d) (2, 5)

7. If O is the origin and $P(2, 3)$ and $Q(4, 5)$ are two points, then $OP \cdot OQ \cos \angle POQ$ =

(a) 8 (b) 15 (c) 22 (d) 23

8. If O is the origin and $P(x_1, y_1)$, $Q(x_2, y_2)$ are two points, then $OP \cdot OQ \sin \angle POQ$ =

(a) $x_1x_2 + y_1y_2$ (b) $x_1y_2 + x_2y_1$
(c) $|x_1y_2 - x_2y_1|$ (d) none of these

9. If $O(0, 0)$, $A(4, 0)$ and $B(0, 3)$ are the vertices of a triangle OAB, then the coordinates of the excentre opposite to the vertex $O(0, 0)$ are

(a) (12, 12) (b) (6, 6)
(c) (3, 3) (d) none of these

10. If $P(3, 7)$ is a point on the line joining $A(1, 1)$ and $B(6, 16)$, then the harmonic conjugate Q of point P has the coordinates

(a) (9, 29) (b) (−9, 29)
(c) (9, −29) (d) (−9, −29)

11. The coordinates of the centroid of a triangle having its circumcentre and orthocentre at (7/2 , 5/2) and (2, 1) respectively, are

(a) (3, 2) (b) (13/6 , 3/2)
(c) (5/2 , 3/2) (d) (3/2 , 5/2)

12. The mid-points of the sides of a ΔABC are $D(6, 1)$, $E(3, 5)$ and $F(-1, -2)$, then the coordinates of the vertex opposite to D are

(a) (−4, 2) (b) (−4, 5)
(c) (2, 5) (d) (10, 8)

13. If the coordinates of orthocentre O' and centroid G of a ΔABC are (0, 1) and (2, 3) respectively, then the coordinates of the circumcentre are

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(a) (3, 2) (b) (1, 0) (c) (4, 3) (d) (3, 4)

14. The ratio in which the x-axis divides the line segment joining (3, 6) and (4, − 3) is
(a) 2 : 1 (b) 1 : 2
(c) 3 : 4 (d) none of these

15. If C and D are the points of internal and external division of line segment AB in the same ratio, then AC, AB, AD are in
(a) AP (b) GP (c) HP (d) AGP

16. If $G(1, 4)$ is the centroid of triangle ABC having its two vertices A and B at (4, − 3) and (− 9, 7) respectively, then area of the triangle ABC in square units, is
(a) $\frac{138}{2}$ (b) $\frac{319}{2}$ (c) $\frac{183}{2}$ (d) $\frac{381}{2}$

17. A triangle with vertices (4, 0), (− 1, − 1), (3, 5) is
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles **[AIEEE 2002]**

18. In order to remove xy-term from the equation $5x^2 + 4\sqrt{3}xy + 9y^2 - 8 = 0$ the coordinate axes must be rotated through an angle
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

19. In order to remove first degree terms from the equation $2x^2 + 7y^2 + 8x - 14y + 4 = 0$, the origin is shifted at the point
(a) (− 2, 1) (b) (1, 2) (c) (2, 1) (d) (1, − 2)

20. If the equation $2x^2 + y^2 - 4x - 4y = 0$ is transformed to the equation $2X^2 + Y^2 - 8X - 8Y + 18 = 0$ by shifting the origin at a point P without rotating the coordinates axes, then the coordinates of P are
(a) (1, 2) (b) (1, − 2) (c) (− 1, 2) (d) (− 1, − 2)

21. If by shifting the origin at (1, 1) the coordinates of a point P become $(\cos\theta, \cos\phi)$, then the original coordiantes of P were
(a) $(2\cos^2\theta/2, 2\cos^2\phi/2)$ (b) $(2\sin^2\theta/2, 2\sin^2\phi/2)$
(c) $(2\cos\theta/2, 2\cos\phi/2)$ (d) $(2\sin\theta/2, 2\sin\phi/2)$

22. By rotating the coordinates axes through 30° in anticlockwise sense the equation $x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2$ changes to
(a) $X^2 - Y^2 = 3a^2$ (b) $X^2 - Y^2 = a^2$
(c) $X^2 - Y^2 = 2a^2$ (d) none of these

23. The sides of a ΔABC are $BC = 5$, $CA = 4$ and $AB = 3$. If A is at the origin and the bisector of the internal angle A meets BC in D (12/7, 12/7), then the coordinates of the incentre, are
(a) (2, 2) (b) (2, 3) (c) (3, 2) (d) (1, 1)

24. The harmonic conjugate of (4, − 2) with respect to (2, − 4) and (7, 1) is
(a) (− 8, − 14) (b) (2, 3)
(c) (− 2, − 3) (d) (13, − 5)

25. If the coordinates of the centroid and a vertex of an equilateral triangle are (1, 1) and (1, 2) respectively, then the coordinates of another vertex, are
(a) $\left(\frac{2-\sqrt{3}}{2}, -\frac{1}{2}\right)$ (b) $\left(\frac{2+3\sqrt{3}}{2}, -\frac{1}{2}\right)$
(c) $\left(\frac{2+\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) none of these

26. The transformed equation of $3x^2 + 3y^2 + 2xy - 2 = 0$ when the coordinate axes are rotated through an angle of 45°, is
(a) $X^2 + 2Y^2 = 1$ (b) $2X^2 + Y^2 = 1$
(c) $X^2 + Y^2 = 1$ (d) $X^2 + 3Y^2 = 1$ **[EAMCET 2008]**

27. The transformed equation of $x^2 + 6xy + 8y^2 = 10$ when the axes are rotated through an angle $\pi/4$ is
(a) $15x^2 - 14xy + 3y^2 = 20$ (b) $15x^2 + 14xy - 3y^2 = 20$
(c) $15x^2 + 14xy + 3y^2 = 20$ (d) $15x^2 - 14xy - 3y^2 = 20$ **[EAMCET 2006]**

28. Let $0 \le \theta \le \frac{\pi}{2}$ and $x = X\cos\theta + Y\sin\theta$, $y = X\sin\theta - Y\cos\theta$ such that $x^2 + 4xy + y^2 = aX^2 + bY^2$, where a, b are constants, then
(a) $a = -1, b = 3, \theta = \frac{\pi}{4}$ (b) $a = 1, b = -3, 2 = \frac{\pi}{3}$
(c) $a = 3, b = -1, \theta = \frac{\pi}{4}$ (d) $a = 3, b = -1, \theta = \frac{\pi}{3}$ **[CEE (Delhi) 2002]**

29. If $x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$ and $x^2 + 4xy + y^2 = AX^2 + BY^2, 0 \le \theta \le \frac{\pi}{2}$, then
(a) $\theta = \frac{\pi}{6}$ (b) $\theta = \frac{\pi}{4}$ (c) $A = -6$ (d) $B = 1$ **[CEE (Delhi) 2002]**

Answers

1. (b) 2. (d) 3. (d) 4. (c) 5. (a) 6. (c) 7. (d) 8. (c) 9. (b) 10. (d) 11. (a) 12. (a) 13. (d) 14. (a) 15. (c) 16. (c) 17. (a) 18. (c) 19. (a) 20. (d) 21. (a) 22. (b) 23. (d) 24. (a) 25. (c) 26. (b) 27. (c) 28. (c) 29. (b)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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21

STRAIGHT LINES

1. STRAIGHT LINE

A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

Every first degree equation in x, y represents a straight line. So, $ax + by + c = 0$ is the general equation of a line.

It should be noted that there are only two unknowns in the equation of a straight line because equation of every straight line can be put in the form $ax + by + 1 = 0$ where a, b are two unknowns.

1.1 SLOPE (GRADIENT) OF A LINE

The trigonometrical tangent of the angle that a line makes with the positive direction of the x-axis in anticlockwise sense is called the slope or gradient of the line.

The slope of a line is generally denoted by m.

REMARK The angle of inclination of a line with the positive direction of x-axis in anticlockwise sense always lies between 0° and 180°.

If (x_1, y_1) and (x_2, y_2) are coordinates of any two points on a line, then its slope m is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Difference of ordinates}}{\text{Difference of abscissae}}$$

ILLUSTRATION 1 *The inclination of the straight line passing through the point A (−3, 6) and the mid-point of the line joining the point B (4, −5) and C (−2, 9), with x-axis is*

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{4}$

Ans. (d)

SOLUTION The coordinates of the mid-point D of line segment BC are $(1, 2)$.

$\therefore$ Slope of $AD = \frac{6-2}{-3-1} = -1$

$\Rightarrow$ $\tan\theta = -1$, where θ is the angle made by AD with x-axis.

$\Rightarrow$ $\theta = \frac{3\pi}{4}$

So, AD makes angle $\frac{3\pi}{4}$ with X-axis.

ILLUSTRATION 2 *The points $P(a, b)$, $Q(c, d)$, $R(a, d)$ and $S(c, b)$, where a, b, c, d are distinct real numbers, are*

(a) *collinear* (b) *vertices of a square*

(c) *vertices of a rhombus* (d) *concyclic*

Ans. (b)

SOLUTION We have,

Slope of PS = 0 = Slope of QR

and,

Slope of $PR = \infty$ = Slope of QS

$\Rightarrow$ PS and QR are parallel to x-axis and PR and QS are parallel to y-axis.

$\Rightarrow$ $PQRS$ is a rectangle.

Also, $PQ = RS$.

Hence, $PQRS$ is a square.

ILLUSTRATION 3 *If the line passing through (3, 4) and (x, 5) makes 135° angle with the positive direction of x-axis, then x =*

(a) −2 (b) 2 (c) −1 (d) 1

Ans. (b)

SOLUTION Since the line passing through (3, 4) and (x, 5) makes an angle of 135° with x-axis. Therefore, its slope is $\tan 135° = -1$. But, the slope of the line is also equal to

$$\frac{5-4}{x-3} \qquad \left[\text{Using}: m = \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$$\therefore \quad -1 = \frac{5-4}{x-3} \Rightarrow -x + 3 = 1 \Rightarrow x = 2.$$

ILLUSTRATION 4 *If the points A (−2, −5), B (2, −2) and C (8, a) are collinear, then a =*

(a) −5/2 (b) 5/2 (c) 3/2 (d) 1/2

Ans. (b)

SOLUTION Since A, B, C are collinear points.

$\therefore$ Slope of AB = Slope of $AC \Rightarrow \frac{3}{4} = \frac{a+5}{10} \Rightarrow a = \frac{5}{2}$

ILLUSTRATION 5 *If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in GP with same common ratio, then the points $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$*

(a) *lie on a straight line* (b) *lie on an ellipse*

(d) *lie on a circle* (d) *are vertices of a triangle*

[IIT 1999, AIEEE 2003]

Ans. (a)

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Ans. (c) **[IIT 2007]**

SOLUTION Statement–2 is not true (See plane geometry).

In ΔOPQ, OR is the bisector of $\angle POQ$.

$$\therefore \quad \frac{OP}{OQ} = \frac{PR}{RQ} \qquad \left[\because \text{Bisector of an angle divides the opposite side in the ratio of the sides containing the angle}\right]$$

$$\Rightarrow \quad \frac{PR}{RQ} = \frac{2\sqrt{2}}{5}$$

So, statement–1 is true.

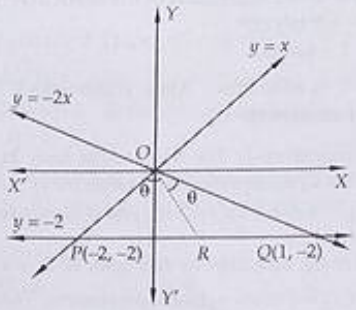


Fig. 46

EXAMPLE 14 *The lines* $L_1 : y - x = 0$ *and* $L_2 : 2x + y = 0$ *intersect the line* $L_3 : y + 2 = 0$ *at* P *and* Q *respectively. The bisectors of the acute angle between* L_1 *and* L_2 *intersect* L_3 *at* R.

Statement 1: The ratio $PR : RQ$ *equals* $2\sqrt{2} : \sqrt{5}$

Statement-2 In any triangle, bisector of an angle divides the triangle into two similar trianges.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION The coordinates of P and Q are $(-2, -2)$ and $(1, -2)$ respectively.

$\therefore \quad OP = 2\sqrt{2}$ and $OQ = \sqrt{5}$

Clearly, statment-2 is true. Also, from statement-2, we get

$$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$$

Hence, both the statements are correct and statement-2 is a correct explanation for statement-1.

EXAMPLE 15 *Let* L *be the line* $y = 2x$, *in the two dimensional plane.*

Statement 1: The image of the point $(0, 1)$ *in* L *is the point* $\left(\frac{4}{5}, \frac{3}{5}\right)$

Statement-2 The points $(0, 1)$ *and* $\left(\frac{4}{5}, \frac{3}{5}\right)$ *lie on opposite sides of the line* L *and are at equal distance from it.*

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a) **[AIEEE 2012]**

SOLUTION Given points $A\,(0, 1)$ and $B\left(\frac{4}{5}, \frac{3}{5}\right)$ are such that the mid-point of AB lies on $y = 2x$ and AB is perpendicular to line $y = 2x$. So, B is the image of A in the line $y = 2x$.

The given line is $L \equiv 2x - y = 0$.

For $x = 0$, $y = 1$, we have

$$L' = 2 \times 0 - 1 = -1 < 0$$

For $x = \frac{4}{5}, y = \frac{3}{5}$, we have

$$L'' 2 \times \frac{4}{5} - \frac{3}{5} = 1 > 0$$

Clearly, $L'\,L'' < 0$. So, points $A\,(1, 0)$ and $B\left(\frac{4}{5}, \frac{3}{5}\right)$ are on opposite sides of the line $y = 2x$.

We observe that $(0, 1)$ and $(-2, -5)$ are equidistant points on the opposite sides of $y = 2x$. But, they are not mirror images of each other. So, statement-2 is not a correct explanation of statement-1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If $P\,(1, 0)$, $Q\,(-1, 0)$ and $R\,(2, 0)$ are three given points, then the locus of point S satisfying the relation $SQ^2 + SR^2 = 2\,SP^2$ is
   (a) a straight line parallel to x-axis
   (b) circle through origin
   (c) circle with centre at the origin
   (d) a straight line parallel to y-axis

2. The locus of the mid-point of the portion intercepted between the axes by the line $x \cos\alpha + y \sin\alpha = p$, where p is a constant is
   (a) $x^2 + y^2 = 4p^2$ (b) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$
   (c) $x^2 + y^2 = \frac{4}{p^2}$ (d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$
   [AIEEE 2002]

3. The locus of the point of intersection of lines $x \cos\alpha + y \sin\alpha = a$ and $x \sin\alpha - y \cos\alpha = b$ is (α is a variable)
   (a) $2(x^2 + y^2) = a^2 + b^2$ (b) $x^2 - y^2 = a^2 - b^2$
   (c) $x^2 + y^2 = a^2 + b^2$ (d) none of these

4. The position of a moving point in the xy plane at time t is given by $\left(u \cos\alpha \cdot t,\ u \sin\alpha \cdot t - \frac{1}{2} g t^2\right)$, where u, α, g are constants. The locus of the moving point is

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(a) a circle (b) a parabola
(c) an ellipse (d) none of these

5. If $A(\cos\alpha, \sin\alpha)$, $B(\sin\alpha, -\cos\alpha)$, $C(1, 2)$ are the vertices of a ΔABC, then as α varies the locus of its centroid is
(a) $x^2+y^2-2x-4y+1=0$
(b) $3(x^2+y^2)-2x-4y+1=0$
(c) $x^2+y^2-2x-4y+3=0$
(d) none of these.

6. If A and B are two fixed points, then the locus of a point which moves in such a way that the angle APB is a right angle is
(a) a circle (b) an ellipse
(c) a parabola (d) none of these

7. If a variable line passes through the point of intersection of the lines $x+2y-1=0$ and $2x-y-1=0$ and meets the coordinate axes in A and B, then the locus of the mid point of AB is
(a) $x+3y=0$ (b) $x+3y=10$
(c) $x+3y=10xy$ (d) none of these

8. If a variable line drawn through the point of intersection of straight lines $\frac{x}{\alpha}+\frac{y}{\beta}=1$ and $\frac{x}{\beta}+\frac{y}{\alpha}=1$ meets the coordinate axes in A and B, then the locus of the mid point of AB is
(a) $\alpha\beta(x+y)=xy(\alpha+\beta)$ (b) $\alpha\beta(x+y)=2xy(\alpha+\beta)$
(c) $(\alpha+\beta)(x+y)=2\alpha\beta xy$ (d) none of these

9. The nearest point on the line $3x-4y=25$ from the origin is
(a) $(-4, 5)$ (b) $(3, -4)$ (c) $(3, 4)$ (d) $(3, 5)$

10. The distance between the lines given by $(x+7y)^2+4\sqrt{2}(x+7y)-42=0$, is
(a) 4/5 (b) $4\sqrt{2}$ (c) 2 (d) $10\sqrt{2}$

11. The image of the point $(-1, 3)$ by the line $x-y=0$ is
(a) $(3, -1)$ (b) $(1, -3)$ (c) $(-1, -1)$ (d) $(3, 3)$

12. If $A(1, 1)$, $B(\sqrt{3}+1, 2)$ and $C(\sqrt{3}, \sqrt{3}+2)$ be three vertices of a square, then the diagonal through B is
(a) $y=(\sqrt{3}-2)x+(3-\sqrt{3})$ (b) $y=0$
(c) $y=x$ (d) none of these

13. If $(-4, 0)$ and $(1, -1)$ are two vertices of a triangle of area 4 square units, then its third vertex lies on
(a) $y=x$ (b) $5x+y+12=0$
(c) $x+5y-4=0$ (d) none of these

14. The equation of a straight line passing through the point of intersection of $x-y+1=0$ and $3x+y-5=0$ and perpendicular to one of them, is
(a) $x+y+3=0$ (b) $x-y-3=0$
(c) $x-3y-5=0$ (d) $x-3y+5=0$

15. The distance between the lines $5x-12y+65=0$ and $5x-12y-39=0$, is
(a) 4 (b) 16 (c) 2 (d) 8
[JEE (WB) 2008]

16. Locus of the centroid of triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$, where t is a parameter, is
(a) $(3x-1)^2+(3y)^2=a^2-b^2$ (b) $(3x-1)^2+(3y)^2=a^2+b^2$
(c) $(3x+1)^2+(3y)^2=a^2+b^2$ (d) $(3x+1)^2+(3y)^2=a^2-b^2$
[JEE (WB) 2007]

17. The equation of the line with gradient $-3/2$ which is concurrent with the lines $4x+3y-7=0$ and $8x+5y-1=0$ is
(a) $3x+2y-2=0$ (b) $3x+2y-63=0$
(c) $2y-3x-2=0$ (d) none of these

18. The point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ lies on the line
(a) $x-y=0$ (b) $(x+y)(a+b)=2ab$
(c) $(lx+my)(a+b)=(l+m)ab$ (d) all of these

19. The equation of the bisector of the acute angle between the lines $3x-4y+7=0$ and $12x+5y-2=0$, is
(a) $99x-27y-81=0$ (b) $11x-3y+9=0$
(c) $21x+77y-101=0$ (d) $21x+77y+101=0$
[JEE (WB) 2007]

20. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1-a_2)x+(b_1-b_2)y+c=0$, then the value of c, is
(a) $a_1^2-a_2^2+b_1^2-b_2^2$ (b) $\sqrt{a_1^2+b_1^2-a_2^2-b_2^2}$
(c) $\frac{1}{2}(a_1^2+a_2^2+b_1^2+b_2^2)$ (d) $\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$
[AIEEE 2003]

21. The equations of perpendicular bisectors of the sides AB and AC of a triangle ABC are $x-y+5=0$ and $x+2y=0$, respectively. If the point A is $(1, -2)$, the equation of the line BC is
(a) $23x+14y-40=0$ (b) $23x+14y+40=0$
(c) $14x+23y-40=0$ (d) $14x+23y+40=0$

22. If each of the points $(x_1, 4)$, $(-2, y_1)$ lies on the line joining the points $(2, -1)$, $(5, -3)$, then the point $P(x_1, y_1)$ lies on the line
(a) $6(x+y)-25=0$ (b) $2x+6y+1=0$
(c) $2x+3y-6=0$ (d) $6(x+y)+25=0$

23. The equation of a line which passes through $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta+y\,\mathrm{cosec}\,\theta=a$ is
(a) $x\cos\theta+y\sin\theta=2a\cos 2\theta$
(b) $x\sin\theta-y\cos\theta=2a\sin 2\theta$
(c) $x\sin\theta+y\cos\theta=2a\cos 2\theta$
(d) none of these

24. The ends of the base of an isosceles triangle are at $(2a, 0)$ and $(0, a)$. The equation of one side is $x=2a$. The equation of the other side is
(a) $x+2y-a=0$ (b) $x+2y=2a$
(c) $3x+4y-4a=0$ (d) $3x-4y+4a=0$

25. The straight lines $x+y=0$, $3x+y-4=0$ and $x+3y-4=0$ form a triangle which is
(a) right angled (b) equilateral
(c) isosceles (d) none of these
[JEE (WB) 2006]

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26. Three straight lines $ax+by=c$, $bx+cy=a$ and $cx+ay=b$ are concurrent, if

(a) $a+b=c$ (b) $b+c=a$

(c) $c+a=b$ (d) $a+b+c=0$

[JEE (WB) 2006]

27. The equation of one side of a rectangle is $3x-4y-10=0$ and the coordinates of two of its vertices are $(-2, 1)$ and $(2, 4)$. Then, the area of the rectangle is

(a) 20 sq. units (b) 40 sq. units

(c) 10 sq. units (d) 30 sq. units

[IP (Delhi) 2003]

28. Given four lines whose equations are $x+2y-3=0$, $2x+3y-4=0$, $3x+4y-7=0$ and $4x+5y-6=0$, then the lines are

(a) concurrent (b) sides of a square

(c) sides of a rhombus (d) none of these

29. All points lying inside the triangle formed by the points $(1, 3)$, $(5, 0)$ and $(-1, 2)$ satisfy

(a) $3x+2y\geq 0$ (b) $2x+y-13\geq 0$

(c) $2x-3y+12\leq 0$ (d) $-2x+y\geq 0$

30. The distance of the point $(3, 5)$ from the line $2x+3y-14=0$ measured parallel to the line $x-2y=1$ is

(a) $7/\sqrt{5}$ (b) $7/\sqrt{13}$ (c) $\sqrt{5}$ (d) $\sqrt{13}$

31. Let the base of a triangle lie along the line $x=a$ and be of length $2a$. The area of this triangle is a^2 if the vertex lies on the lines

(a) $x=-a, x=2a$ (b) $x=0, x=a$

(c) $x=a/2, x=-a$ (d) none of these

32. The equation of a straight line passing through $(1, 2)$ and having intercept of length 3 between the straight lines $3x+4y=24$ and $3x\ 4y=12$ is

(a) $7x+24y-55=0$ (b) $24x+7y-38=0$

(c) $24x-7y-10=0$ (d) $7x-24y+41=0$

33. The point $(4, 1)$ undergoes the following three transformations successively:

(I) Reflection about the line $y=x$.

(II) Translation through a distance 2 units along the positive direction of x-axis.

(III) Rotation of coordinate axes through an angle $\pi/4$ about the origin in the clockwise direction

The final position of the point is given by the coordinates

(a) $\left(1/\sqrt{2}, 7/\sqrt{2}\right)$ (b) $(-2, 7\sqrt{2})$

(c) $\left(-1/\sqrt{2}, 7/\sqrt{2}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$

34. The y-intercept of the line which passes through $(2, 2)$ and is perpendicular to the line $3x+y=3$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

35. Points on the line $y=x$ whose perpendicular distance from the line $3x+4y=12$ are 4 have the coordinates

(a) $\left(-\frac{8}{7}, -\frac{8}{7}\right), \left(-\frac{32}{7}, -\frac{32}{7}\right)$ (b) $\left(\frac{8}{7}, \frac{8}{7}\right), \left(\frac{32}{7}, \frac{32}{7}\right)$

(c) $\left(-\frac{8}{7}, -\frac{8}{7}\right), \left(\frac{32}{7}, \frac{32}{7}\right)$ (d) none of these

36. The point $P(1, 1)$ is translated parallel to $2x=y$ in the first quadrant through a unit distance. The coordinates of the new position of P are

(a) $\left(1\pm\frac{2}{\sqrt{5}}, 1\pm\frac{1}{\sqrt{5}}\right)$ (b) $\left(1\pm\frac{1}{\sqrt{5}}, 1\pm\frac{2}{\sqrt{5}}\right)$

(c) $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ (d) $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

37. If a line joining two points $A(2, 0)$ and $B(3,1)$ is rotated about A in anti-clockwise direction through an angle 15°, then the equation of the line in the new position is

(a) $\sqrt{3}\,x-y=2\sqrt{3}$ (b) $\sqrt{3}\,x+y=2\sqrt{3}$

(c) $x+\sqrt{3}\,y=2\sqrt{3}$ (d) none of these

38. The limiting position of the point of intersection of the lines $3x+4y=1$ and $(1+c)x+3c^2y=2$ as c tends to 1, is

(a) $(-5, 4)$ (b) $(5, -4)$

(c) $(4, -5)$ (d) none of these

39. Let the perpendiculars from any point on the line $2x+11y=5$ upon the lines $24x+7y-20=0$ and $4x-3y-2=0$ have the lengths p_1 and p_2 respectively. Then,

(a) $2p_1=p_2$ (b) $p_1=p_2$

(c) $p_1=2p_2$ (d) none of these

40. $P(2, 1)$, $Q(4, -1)$, $R(3,2)$ are the vertices of a triangle and if through P and R lines parallel to opposite sides are drawn to intersect in S, then the area of $PQRS$ is

(a) 6 (b) 4 (c) 8 (d) 12

41. If a line passes through the point $(2, 2)$ and encloses a triangle of area A square units with the coordinate axes, then the intercepts made by the line on the coordinate axes are the roots of the equations

(a) $x^2\pm Ax\mp 2A=0$ (b) $x^2\pm Ax\pm 2A=0$

(c) $x^2\pm 2Ax\pm A=0$ (d) $x^2\pm 2Ax\mp A=0$

42. Points on the line $x+y=4$ which are equidistant from the lines $|x|=|y|$, are

(a) $(4, 0), (0, 4)$ (b) $(-4, 0), (0, -4)$

(c) $(4, 0), (-4, 0)$ (d) none of these

43. Two points A and B move on the coordinate axes such that the distance between them remains same. The locus of the mid-point of AB is

(a) a straight line (b) a pair of straight lines

(c) a circle (d) none of these

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44. If a straight line L is perpendicular to the line $5x - y = 1$ such that the area of the Δ formed by the line L and the coordinate axes is 5, then the equation of the line L is
(a) $x + 5y + 5 = 0$ (b) $x + 5y \pm \sqrt{2} = 0$
(c) $x + 5y \pm \sqrt{5} = 0$ (d) $x + 5y \pm 5\sqrt{2} = 0$

45. Let O be the origin, and let $A(1, 0)$, $B(0, 1)$ be two points. If $P(x, y)$ is a point such that $xy > 0$ and $x + y < 1$, then
(a) P lies either in side ΔOAB or in third quadrant
(b) P cannot be inside ΔOAB
(c) P lies inside the ΔOAB
(d) none of these

46. Two sides of an isosceles triangle are given by the equation $7x - y + 3 = 0$ and $x + y - 3 = 0$. If its third side passes through the point $(1, -10)$, then its equations are
(a) $x - 3y - 7 = 0$ or, $3x + y - 31 = 0$
(b) $x - 3y - 31 = 0$ or, $3x + y - 7 = 0$
(c) $x - 3y - 31 = 0$ or, $3x + y + 7 = 0$
(d) none of these

47. A line is drawn from $P(x_1, y_1)$ in the direction α with the x-axis, to meet $Ax + By + C = 0$ at Q. Then, the length PQ is equal to
(a) $\left|\dfrac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$ (b) $-\dfrac{Ax_1 + By_1 + C}{A\cos\alpha + B\sin\alpha}$
(c) $\dfrac{Ax_1 + By_1 + C}{A\cos\alpha\, B\sin\alpha}$ (d) $-\dfrac{Ax_1 + By_1 + C}{A\sin\alpha + B\cos\alpha}$

48. The value of k for which the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent, is
(a) 20 (b) -7 (c) 7 (d) -20
[EAMCET 2008]

49. The equations of the lines on which the perpendiculars from the origin make 30° angle with x-axis and which form a triangle of area $\dfrac{50}{\sqrt{3}}$ with axes, are
(a) $x + \sqrt{3}y \pm 10 = 0$ (b) $\sqrt{3}x + y \pm 10 = 0$
(c) $x \pm \sqrt{3}y - 10 = 0$ (d) none of these

50. The area of the triangle formed by y-axis, the straight line L passing through $(1, 1)$ and $(2, 0)$ and the straight line perpendicular to the line L and passing through $(1/2, 0)$, is
(a) $\dfrac{25}{8}$ sq. units (b) $\dfrac{25}{4}$ sq. units
(c) $\dfrac{25}{16}$ sq. units (d) $\dfrac{25}{2}$ sq. units

51. Points on the line $x + y = 4$ that lie at a unit distance from the line $4x + 3y - 10 = 0$, are
(a) $(3, 1)$ and $(-7, 11)$ (b) $(-3, 7)$ and $(2, 2)$
(c) $(-3, 7)$ and $(-7, 11)$ (d) none of these

52. The equation of the line passing through the intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and at distance $\sqrt{5}$ from the origin, is
(a) $2x - y = 5$ (b) $x + 2y = 5$
(c) $2x + y = 5$ (d) $x + 2y = 1$

53. The number of lines that are parallel to $2x + 6y + 7 = 0$ and have an intercept of length 10 between the coordinate axes, is
(a) 1 (b) 2 (c) 4 (d) infinitely many

54. The line $x + y = 4$ divides the line joining $(-1, 1)$ and $(5, 7)$ in the ratio $\lambda : 1$, then the value of λ is
(a) 2 (b) 1/2 (c) 3 (d) none of these

55. The image of the point $(1, 3)$ in the line $x + y - 6 = 0$, is
(a) $(3, 5)$ (b) $(5, 3)$ (c) $(1, -3)$ (d) $(-1, 3)$

56. A triangle ABC, right angled at A, has points A and B as $(2, 3)$ and $(0, -1)$ respectively. If $BC = 5$ units, then the point C, is
(a) $(-4, 2)$ (b) $(4, 2)$ (c) $(3, -3)$ (d) $(0, -4)$

57. The equation of the line passing through the intersection of $x - \sqrt{3}y + \sqrt{3} - 1 = 0$ and $x + y - 2 = 0$ and making an angle of 15° with the first line is
(a) $x - y = 0$ (b) $x - y + 1 = 0$
(c) $y = 1$ (d) $\sqrt{3}x - y + 1 - \sqrt{3} = 0$

58. In a rhombus $ABCD$ the diagonals AC and BD intersect at the point $(3, 4)$. If the point A is $(1, 2)$ the diagonal BD has the equation
(a) $x - y - 1 = 0$ (b) $x + y - 1 = 0$
(c) $x - y + 1 = 0$ (d) $x + y - 7 = 0$

59. The distance between the pair of lines given by $x^2 + y^2 + 2xy - 8ax - 8ay - 9a^2 = 0$ is
(a) $2\sqrt{5}a$ (b) $10\sqrt{a}$ (c) 10a (d) $5\sqrt{2}a$

60. The ratio in which the line $3x - 2y + 5 = 0$ divides the join of $(6, -7)$ and $(-2, 3)$, is
(a) 1 : 1 (b) 7 : 37 (c) 37 : 7 (d) none of these

61. One vertex of the equilateral triangle with centroid at the origin and one side as $x + y - 2 = 0$ is
(a) $(-1, -1)$ (b) $(2, 2)$ (c) $(-2, -2)$ (d) none of these

62. The distance of the line $x + y - 8 = 0$ from $(4, 1)$ measured along the direction whose slope is -2 is
(a) $3\sqrt{5}$ (b) $6\sqrt{5}$ (c) $2\sqrt{5}$ (d) none of these

63. The area enclosed within the curve $|x| + |y| = 1$, is
(a) 1 sq. units (b) 2 sq. units
(c) 3 sq. units (d) 4 sq. units
[JEE (WB) 2006]

64. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

65. If each of the points $(x_1, 4)$ and $(-2, y_1)$ lies on the line joining the points $(2, -1)$ and $(5, -3)$, then the point $P(x_1, y_1)$ lies on the line
(a) $x = 3y$ (b) $x = -3y$
(c) $y = 2x + 1$ (d) $2x + 6y + 1 = 0$.

66. The area bounded by the straight lines $y = 1$ and $\pm 2x + y = 2$ is
(a) 1/2 sq. unit (b) 1 sq. unit
(c) 3/2 sq. units (d) 2 sq. units

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67. The locus of a point P which divides the line joining $(1, 0)$ and $(2\cos\theta, 2\sin\theta)$ internally in the ratio $2:3$ for all θ, is a
(a) straight line (b) circle
(c) pair of straight lines (d) parabola

68. $A(-5, 0)$ and $B(3, 0)$ are two of the vertices of a triangle ABC. Its area is 20 square cms. The vertex C lies on the line $x - y = 2$. The coordinates of C are
(a) $(-7, -5)$ or $(3, 5)$ (b) $(-3, -5)$ or $(-5, 7)$
(c) $(7, 5)$ or $(3, 5)$ (d) $(-3, -5)$ or $(7, 5)$

69. The area of the parallelogram formed by the lines $x\cos\alpha + y\sin\alpha = p$, $x\cos\alpha + y\sin\alpha = q$, $x\cos\beta + y\sin\beta = r$ and $x\cos\beta + y\sin\beta = s$ for given values of p, q, r and s is least, if $(\alpha - \beta)$ =
(a) $\pm\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

70. The ratio in which the line $3x + 4y + 2 = 0$ divides the distance between $3x + 4y + 5 = 0$, and $3x + 4y - 5 = 0$, is
(a) $7:3$ (b) $3:7$ (c) $2:3$ (d) none of these

71. If the extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is $x = 2a$, then the area of the triangle is
(a) $5a^2$ sq. units (b) $\frac{5}{2}a^2$ sq. units
(c) $\frac{25a^2}{2}$ sq. units (d) none of these

72. The vertices of a ΔOBC are $(0, 0)$, $B(-3, -1)$ and $C(-1, -3)$. The equation of a line parallel to BC and intersecting sides OB and OC whose distance from the origin is $1/2$, is
(a) $x + y + \frac{1}{2} = 0$ (b) $x + y - \frac{1}{2} = 0$
(c) $x + y - \frac{1}{\sqrt{2}} = 0$ (d) $x + y + \frac{1}{\sqrt{2}} = 0$

73. The area (in square units) of the quadrilateral formed by two pairs of lines $l^2x^2 - m^2y^2 - n(lx + my) = 0$ and $l^2x^2 - m^2y^2 + n(lx - my) = 0$, is
(a) $\frac{n^2}{2|lm|}$ (b) $\frac{n^2}{|lm|}$ (c) $\frac{n}{2|lm|}$ (d) $\frac{n^2}{4|lm|}$
[EAMCET 2003]

74. The equation(s) of the bisector(s) of that angle between the lines $x + 2y - 11 = 0$, $3x - 6y - 5 = 0$ which contains the point $(1, -3)$ is
(a) $3x = 19$ (b) $3y = 7$
(c) $3x = 19$ and $3y = 7$ (d) none of these

75. The line $3x + 2y = 24$ meets y-axis at A and x-axis at B. The perpendicular bisector of AB meets the line through $(0, -1)$ parallel to x-axis at C. The area of the triangle ABC is
(a) 182 sq. units (b) 91 sq. units
(c) 48 sq. units (d) none of these

76. A ray of light coming from the point $(1, 2)$ is reflected at a point A on the x-axis and then passes through the point $(5, 3)$. The coordinates of the point A are
(a) $(13/5, 0)$ (b) $(5/13, 0)$
(c) $(-7, 0)$ (d) none of these
[JEE (Orissa) 2003]

77. If PM is the perpendicular from $P(2, 3)$ on the line $x + y = 3$, then the coordinates of M are
(a) $(2, 1)$ (b) $(-1, 4)$ (c) $(1, 2)$ (d) $(4, -1)$
[EAMCET 2005]

78. The incentre of the triangle formed by the lines $x = 0$, $y = 0$ and $3x\ 4y = 12$ is at
(a) $(1/2, 1/2)$ (b) $(1, 1)$ (c) $(1, 1/2)$ (d) $(1/2, 1)$

79. If one vertex of an equilateral triangle is at $(2, -1)$ and the base is $x + y - 2 = 0$, then the length of each side is
(a) $\sqrt{3/2}$ (b) $\sqrt{2/3}$ (c) $2/3$ (d) $3/2$

80. The area of the parallelogram formed by the lines $3x - 4y + 1 = 0$, $3x - 4y + 3 = 0$, $4x - 3y - 1 = 0$ and $4x - 3y - 2 = 0$, is
(a) $\frac{1}{6}$ sq. units (b) $\frac{2}{7}$ sq. units
(c) $\frac{3}{8}$ sq. units (d) none of these

81. Points $A(1, 3)$ and $C(5, 1)$ are opposite vertices of a rectangle $ABCD$. If the slope of BD is 2, then its equation is
(a) $2x - y = 4$ (b) $2x + y = 4$
(c) $2x + y - 7 = 0$ (d) $2x + y + 7 = 0$

82. The line $x + 2y = 4$ is translated parallel to itself by 3 units in the sense of increasing x and then rotated by $30°$ in the clockwise direction about the point where the shifted line cuts the x-axis. The equation of the line in the new position is
(a) $y = \tan(\theta - 30°)(x - 4 - 3\sqrt{5})$
(b) $y = \tan(30° - \theta)(x - 4 - 3\sqrt{5})$
(c) $y = \tan(\theta + 30°)(x + 4 + 3\sqrt{5})$
(d) $y = \tan(\theta - 30°)(x + 4 + 3\sqrt{5})$

83. The line PQ whose equation is $x - y = 2$ cuts the x-axis at P and Q is $(4, 2)$. The line PQ is rotated about P through $45°$ in the anticlockwise direction. The equation of the line PQ in the new position is
(a) $y = -\sqrt{2}$ (b) $y = 2$ (c) $x = 2$ (d) $x = -2$.

84. If the points $(1, 3)$ and $(5, 1)$ are two opposite vertices of a rectangle and the other two vertices lie on the line $y = 2x + c$, then the value of c is
(a) 4 (b) -4 (c) 2 (d) none of these.

85. In the above question the coordinates of the other two vertices are
(a) $(2, 0), (4, 4)$ (b) $(2, 4), (4, 0)$
(c) $(-2, 0), (4, -4)$ (d) $(2, 0), (-4, 4)$

86. If the vertices of a diagonal of a square are $(-2, 4)$ and $(-2, -2)$, then its other two vertices are at
(a) $(1, -1), (5, 1)$ (b) $(1, 1), (5, -1)$
(c) $(1, 1), (-5, 1)$ (d) none of these

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87. The equations of two sides of a square whose area is 25 square units are $3x-4y=0$ and $4x+3y=0$. The equations of the other two sides of the square are
(a) $3x-4y\pm 25=0,\ 4x+3y\pm 25=0$
(b) $3x-4y\pm 5=0,\ 4x+3y\pm 5=0$
(c) $3x-4y\pm 5=0,\ 4x+3y\pm 25=0$
(d) none of these

88. The centroid of the triangle formed by the pair of straight lines $12x^2-20xy+7y^2=0$ and the line $2x-3y+4=0$ is
(a) $\left(-\frac{7}{3},\frac{7}{3}\right)$ (b) $\left(-\frac{8}{3},\frac{8}{3}\right)$ (c) $\left(\frac{8}{3},\frac{8}{3}\right)$ (d) $\left(\frac{4}{3},\frac{4}{3}\right)$
[EAMCET 2006]

89. If the lines $ax+2y+1=0,\ bx+3y+1=0,\ cx+4y+1=0$ are concurrent, then a, b, c are in
(a) AP (b) GP (c) HP (d) none of these.

90. Two vertices of a triangle are (5, –1) and (–2, 3). If the orthocentre of the triangle is the origin, then coordinates of the third vertex are
(a) (4, 7) (b) (–4, –7)
(c) (–4, 7) (d) none of these

91. If the foot of the perpendicular from the origin to a straight line is at the point (3, – 4). Then, the equation of the line is
(a) $3x-4y=25$ (b) $3x-4y+25=0$
(c) $4x+3y-25=0$ (d) $4x-3y+25=0$

92. A rectangle has two opposite vertices at the points (1, 2) and (5, 5). If the other vertices lie on the line $x=3$, then their coordinates are
(a) (3, 1), (3, 3) (b) (3, 1), (3, 6)
(c) (3, 1), (3, 4) (d) none of these

93. The orthocentre of the triangle formed by the lines $xy=0$ and $x+y=1$ is
(a) (1/2, 1/2) (b) (1/3, 1/3)
(c) (0, 0) (d) (1/4, 1/4)
[JEE (Orissa) 2004]

94. The y-intercept of the line passing through (2, 2) and perpendicular to the line $3x+y=3$ is
(a) 1/3 (b) 2/3 (c) 1 (d) 4/3

95. A line passes through (2, 2) and is perpendicular to the line $3x+y=3$. Its y-intercept is
(a) 1/3 (b) 2/3 (c) 1 (d) 4/3

96. Three straight lines $2x+11y-5=0$, $24x+7y-20=0$ and $4x-3y-2=0$
(a) form a triangle (b) are only concurrent
(c) are concurrent with one line bisecting the angle between the other two
(d) none of these

97. A line passes through the point of intersection of the lines $3x+y+1=0$ and $2x-y+3=0$ and makes equal intercepts with the axes. The equation of the line, is
(a) $5x+5y-3=0$ (b) $x+5y-3=0$
(c) $5x-y-3=0$ (d) $5x+5y+3=0$

98. A straight line through the point (2, 2) intersects the lines $\sqrt{3}x+y=0$ and $\sqrt{3}x-y=0$ at the points A and B. The equation to the line AB so that the traiangle OAB is equilateral is
(a) $x-2=0$ (b) $y-2=0$
(c) $x+y-4=0$ (d) none of these

99. The image of the point (3, 8) in the line $x+3y=7$, is
(a) (1, 4) (b) (4, 1)
(c) (–1, –4) (d) (–4, –1)

100. If $a\neq b\neq c$, then the equations
$$(b-c)x+(c-a)y+(a-b)=0$$
and, $(b^3-c^3)x+(c^3-a^3)y+(a^3-b^3)=0$
will represent the same line, if
(a) $a+b=-c$ (b) $c+a=-b$
(c) $b+c=-a$ (d) $a+b+c=0$

101. The range of values of θ in the interval $(0,\pi)$ such that the points (3, 5) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x+y-1=0$, is
(a) $(0,\pi/2)$ (b) $(0,\pi/4)$
(c) $(\pi/4,\pi/2)$ (d) none of these

102. If $P(\sin\theta, 1/\sqrt{2})$ and $Q(1/\sqrt{2}, \cos\theta)$, $-\pi\le\theta\le\pi$ are two points on the same side of the line $x-y=0$, then θ belongs to the interval
(a) $(-\pi/4,\pi/4)\cup(\pi/4,3\pi/4)$ (b) $(-\pi/4,\pi/4)$
(c) $(\pi/4,3\pi/4)$ (d) none of these

103. If the point $(1,\alpha)$ always remains in the interior of the triangle formed by the lines $y=x$, $y=0$ and $x+y=4$, then α lies in the interval
(a) (0, 1) (b) [0, 1] (c) [0, 4] (d) none of these

104. Let ABC be an isosceles triangle with $AB=BC$. If base BC is parallel to x-axis and m_1 and m_2 are the slopes of medians drawn through the angular points B and C, then
(a) $m_1 m_2=-1$ (b) $m_1+m_2=0$
(c) $m_1 m_2=2$ (d) $m_1+2m_2=0$

105. If a, c, b are in G.P., then the line $ax+by+c=0$
(a) has a fixed direction
(b) always passes through a fixed point
(c) forms a triangle with the axes whose area is constant
(d) always cuts intercepts on the axes such that their sum is zero

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106. If the intercept made on the line $y = mx$ by the lines $y = 2$ and $y = 6$ is less than 5, then the range of values of m is
(a) $(-\infty, -4/3) \cup (4/3, \infty)$ (b) $(-4/3, 4/3)$
(c) $(-3/4, 3/4)$ (d) none of these

107. The straight line $4x + 3y = 12$ intersects the x-axis and y-axis at A and B respectively. Then the distance BI where I is the centre of the in-circle of ΔOAB, where B is the origin, is equal to
(a) $\sqrt{10}$ (b) $2\sqrt{5}$ (c) 3 (d) 2

108. The four sides of a quadrilateral are given by the equation $xy(x-2)(y-3) = 0$. The equation of the line parallel to $x - 4y = 0$ that divides the quadrilateral in two equal areas is
(a) $x - 4y + 5 = 0$ (b) $x - 4y - 5 = 0$
(c) $4y = x + 1$ (d) $4y + 1 = x$

109 The coordinates of the foot of the perpendicular from the point (2, 4) on the line $x + y = 1$ are
(a) $(1/2, 3/2)$ (b) $(-1/2, 3/2)$
(c) $(4/3, 1/2)$ (d) $(3/4, -1/2)$

110. The coordinates of three vertices of a quadrilateral in order are (6, 1), (7, 2) and (−1, 0). If the area of the quadrilateral is 4 square units, then the locus of the fourth vertex is
(a) $x - 7y = 1$ (b) $x - 7y + 15 = 0$
(c) $(x-7y)^2 + 14(x-7y) - 15 = 0$ (d) none of these

111. The range of values of α for which the points $(\alpha, 2+\alpha)$ and $\left(\frac{3\alpha}{2}, \alpha^2\right)$ lie on opposite sides of the line $2x + 3y = 6$, is
(a) $(-2, 1)$ (b) $(-\infty, -2) \cup (0, 1)$
(c) $(-2, 0) \cup (1, \infty)$ (d) $(-1, 0) \cup (2, \infty)$

112. A point moves such that the area of the triangled formed by it with the points (1, 5) and (3, −7) is 21 sq. units. The locus of the point is
(a) $6x + y - 32 = 0$ (b) $6x - y + 32 = 0$
(c) $x + 6y - 32 = 0$ (d) $6x - y - 32 = 0$

113. The orthocentre of the triangle whose vertices are (5, −2), (−1, 2) and (1, 4), is
(a) (1/5, 14/5) (b) (14/5, 1/5)
(c) (1/5, 1/5) (d) (14/5, 14/5)

114. If the pair of straight lines $xy - x - y + 1 = 0$ and the line $ax + 2y - 3 = 0$ are concurrent, then $a =$
(a) −1 (b) 0 (c) 3 (d) 1
[EAMCET 2002]

115. If (−2, 6) is the image of the point (4, 2) with respect to the line $L = 0$, then $L =$
(a) $3x - 2y + 5$ (b) $3x - 2y + 10$
(c) $2x + 3y - 5$ (d) $6x - 4y - 7$
[EAMCET 2002]

116. If the points (1, 2) and (3, 4) were to be on the same side of the line $3x - 5y + a = 0$, then
(a) $7 < a < 11$ (b) $a = 7$
(c) $a = 1$ (d) $a < 7$ or $a > 11$
[EAMCET 2000]

117. The coordinates of the image of the origin O with respect to the line $x + y + 1 = 0$ are
(a) $(-1/2, -1/2)$ (b) $(-2, -2)$
(c) (1, 1) (d) $(-1, -1)$
[EAMCET 2000]

118. A straight rod of length 9 units slides with its ends A, B always on the X and Y axis respectively. Then, the locus of the centroid of ΔOAB is
(a) $x^2 + y^2 = 3$ (b) $x^2 + y^2 = 9$
(c) $x^2 + y^2 = 1$ (d) $x^2 + y^2 = 81$
[EAMCET 2000]

119. The area of the triangle formed by the axes and the line $(\cos h\,\alpha - \sin h\,\alpha)x + (\cos h\,\alpha + \sin h\,\alpha)y = 2$ in square units, is
(a) 4 (b) 3 (c) 2 (d) 1
[EAMCET 2000]

Answers

1. (d) 2. (b) 3. (c) 4. (b) 5. (b) 6. (a) 7. (c) 8. (b) 9. (b) 10. (a) 11. (a) 12. (d) 13. (c) 14. (d) 15. (d) 16. (b) 17. (a) 18. (d) 19. (b) 20. (d) 21. (c) 22. (b) 23. (d) 24. (d) 25. (c) 26. (d) 27. (a) 28. (d) 29. (a) 30. (c) 31. (b) 32. (d) 33. (c) 34. (d) 35. (c) 36. (b) 37. (a) 38. (a) 39. (b) 40. (b) 41. (a) 42. (a) 43. (c) 44. (d) 45. (a) 46. (c) 47. (b) 48. (b) 49. (b) 50. (c) 51. (a) 52. (c) 53. (b) 54. (b) 55. (a) 56. (b) 57. (a) 58. (d) 59. (d) 60. (c) 61. (c) 62. (a) 63. (b) 64. (a) 65. (d) 66. (a) 67. (b) 68. (d) 69. (b) 70. (b) 71. (b) 72. (d) 73. (a) 74. (a) 75. (b) 76. (a) 77. (c) 78. (b) 79. (b) 80. (b) 81. (a) 82. (a) 83. (c) 84. (b) 85. (a) 86. (c) 87. (a) 88. (c) 89. (a) 90. (b) 91. (a) 92. (b) 93. (c) 94. (d) 95. (d) 96. (c) 97. (a) 98. (b) 99. (c) 100. (d) 101. (a) 102. (a) 103. (b) 104. (b) 105. (c) 106. (a) 107. (a) 108. (a) 109. (b) 110. (c) 111. (b) 112. (a) 113. (a) 114. (d) 115. (a) 116. (d) 117. (d) 118. (b) 119. (c)

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CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The orthocentre of the triangle with vertices $\left(2, \frac{\sqrt{3}-1}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(2, -\frac{1}{2}\right)$ is
   (a) $\left(\frac{3}{2}, \frac{\sqrt{3}-3}{6}\right)$ (b) $\left(2, -\frac{1}{2}\right)$
   (c) $\left(\frac{5}{4}, \frac{(\sqrt{3}-2)}{4}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{2}\right)$

2. The equations to a pair of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. The equations to its diagonals are
   (a) $x + 4y = 13$ and $y = 4x - 7$
   (b) $4x + y = 13$ and $4y = x - 7$
   (c) $4x + y = 13$ and $y = 4x - 7$
   (d) $y - 4x = 13$ and $y + 4x = 7$

3. The distance between the parallel lines $y = 2x + 4$ and $6x = 3y + 5$ is
   (a) $17/\sqrt{3}$ (b) 1 (c) $3/\sqrt{5}$ (d) $17\sqrt{5}/15$

4. P is a point on either of the two lines $y - \sqrt{3}\,|x| = 2$ at a distance of 5 units from their point of intersection. The coordinates of the foot of the perpendicular from P on the bisector of the angle between them are
   (a) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$ or, $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ depending on which the point P is taken.
   (b) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$ (c) $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ (d) $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

5. If one of the diagonals of a square is along the line $x = 2y$ and one of its vertices is (3, 0), then its sides through this vertex are given by the equations
   (a) $y - 3x + 9 = 0, 3y + x - 3 = 0$
   (b) $y + 3x + 9 = 0, 3y + x - 3 = 0$
   (c) $y - 3x + 9 = 0, 3y - x + 3 = 0$
   (d) $y - 3x + 3 = 0, 3y + x + 9 = 0$

6. The line which is parallel to x-axis and crosses the curve $y = \sqrt{x}$ at an angle of 45° is
   (a) $x = 1/4$ (b) $y = 1/4$ (c) $y = 1/2$ (d) $y = 1$

7. $P(3, 1)$, $Q(6, 5)$ and $R(x, y)$ are three points such that the angle PRQ is a right angle and the area of $\Delta RQP = 7$, then the number of such points R is
   (a) 0 (b) 1 (c) 2 (d) 4

8. The equation of the straight line which passes through the point (1, – 2) and cuts off equal intercepts from the axes will be.
   (a) $x + y = 1$ (b) $x - y = 1$
   (c) $x + y + 1 = 0$ (d) $x - y - 2 = 0$

9. The equation of the straight line which is perpendicular to $y = x$ and passes through (3, 2) will be given by
   (a) $x - y = 5$ (b) $x + y = 5$
   (c) $x + y = 1$ (d) $x - y = 1$

10. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is
    (a) 3/2 (b) 3/10 (c) 6 (d) none of these

11. The equation of the straight line passing through (1, 2) and perpendicular to $x + y + 1 = 0$ is
    (a) $y - x + 1 = 0$ (b) $y - x - 1 = 0$
    (c) $y - x + 2 = 0$ (d) $y - x - 2 = 0$

12. The straight lines $x + y = 0, 3x + y - 4 = 0, x + 3y - 4 = 0$ form a triangle which is
    (a) isosceles (b) equilateral
    (d) right angled (d) none of these

13. Triangle formed by $x^2 - 3y^2 = 0$ and $x = 4$ is
    (a) isosceles (b) equilateral
    (c) right angled (d) none of these

14. Orthocentre of the triangle whose sides are given by $4x - 7y + 10 = 0, x + y - 5 = 0$ and $7x + 4y - 15 = 0$ is
    (a) (– 1, – 2) (b) (1, – 2) (c) (– 1, 2) (d) (1, 2)

15. The line $(p + 2q)x + (p - 3q)y = p - q$ for different values of p and q passes through the fixed point
    (a) (3/2, 5/2) (b) (2/5, 2/5)
    (c) (3/5, 3/5) (d) (2/5, 3/5)

16. The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$, is
    (a) 7/2 (b) 4 (c) 7/10 (d) none of these

17. If the diagonals of a parallelogram $ABCD$ are along the lines $x + 5y = 7$ and $10x - 2y = 9$, then $ABCD$ must be a
    (a) rectangle (b) square
    (c) cyclic quadrilateral (d) rhombus

18. The straight lines $x + y - 4 = 0, 3x + y - 4 = 0$, $x + 3y - 4 = 0$ form a triangle which is
    (a) isosceles (b) right angled
    (c) equilateral (d) none of these

19. The orthocentre of the triangle formed by (0, 0), (8, 0), (4, 6) is
    (a) (4, 8/3) (b) (3, 4) (c) (4, 3) (d) (– 3, 4)

20. A point equidistant from the lines $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$ is
    (a) (1, – 1) (b) (1, 1) (c) (0, 0) (d) (0, 1)

21. The lines $2x + y - 1 = 0, ax + 3y - 3 = 0$ and $3x + 2y - 2 = 0$ are concurrent for
    (a) all a (b) $a = 4$ only
    (c) $-1 \le a \le 3$ (d) $a > 0$ only

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22. The diagonals of the parallelogram whose sides are $lx+my+n=0$, $lx+my+n'=0$, $mx+ly+n=0$, $mx+ly+n'=0$ include an angle
(a) $\pi/3$ (b) $\pi/2$
(c) $\tan^{-1}\left(\frac{l^2-m^2}{l^2+m^2}\right)$ (d) $\tan^{-1}\left(\frac{2lm}{l^2+m^2}\right)$

23. The equations to the sides of a triangle are $x-3y=0$, $4x+3y=5$ and $3x+y=0$. The line $3x-4y=0$ passes through
(a) the incentre
(b) the centroid
(c) the circumcentre
(d) the orthocentre, of the triangle

24. A straight line through $P(1,2)$ is such that its intercept between the axes is bisected at P. Its equation is
(a) $x+2y=5$ (b) $x-y+1=0$
(c) $x+y-3=0$ (d) $2x+y-4=0$

25. Two points $(a,0)$ and $(0,b)$ are joined by a straight line. Another point on this line, is
(a) $(3a,-2b)$ (b) (a^2,ab) (c) $(-3a,2b)$ (d) (a,b)

26. If the line $y=mx$ meets the lines $x+2y-1=0$ and $2x-y+3=0$ at the same point, then m is equal to
(a) 1 (b) -1 (c) 2 (d) -2

27. The equations $ax+by+c=0$ and $dx+ey+f=0$ represent the same straight line if and only if
(a) $\frac{a}{d}=\frac{b}{c}$ (b) $c=f$
(c) $\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$ (d) $a=d, b=e, c=f$

28. If the line segment joining $(2,3)$ and $(-1,2)$ is divided internally in the ratio $3:4$ by the line $x+2y=\lambda$, then λ =
(a) $\frac{41}{7}$ (b) $\frac{5}{7}$ (c) $\frac{36}{7}$ (d) $\frac{31}{7}$

29. A point moves in the xy-plane such that the sum of its distance from two mutually perpendicular lines is always equal to 3. The area of the locus of the point is
(a) 18 sq.units (b) 9/2 sq. units
(c) 9 sq. units (d) none of these

30. The vertices of a triangle are $(0,3)$, $(-3,0)$ and $(3,0)$. The coordinates of its orthocentre are
(a) $(0,2)$ (b) $(0,-3)$ (c) $(0,3)$ (d) $(0,-2)$

31. The lines $x\cos\alpha+y\sin\alpha=p_1$ and $x\cos\beta+y\sin\beta=p_2$ will be perpendicular, if
(a) $\alpha\pm\beta=\frac{\pi}{2}$ (b) $\alpha=\frac{\pi}{2}$
(c) $|\alpha-\beta|=\frac{\pi}{2}$ (d) $\alpha=\beta$

32. Family of lines $x\sec^2\theta+y\tan^2\theta-2=0$ for different real θ, is
(a) not concurrent (b) concurrent at $(1,1)$
(c) concurrent at $(2,-2)$ (d) concurrent at $(-2,2)$

33. The members of the family of lines $(\lambda+\mu)x+(2\lambda+-\mu)y=\lambda+2\mu$, where $\lambda\neq0, \mu\neq0$, pass through the point
(a) $(3,-1)$ (b) $-3,1$
(c) $(1,1)$ (d) none of these

34. The area of a pentagon whose vertices are $(4,1)$, $(3,6)$, $(-5,1)$, $(-3,-3)$ and $(-3,0)$ is
(a) 30 sq. units (b) 60 sq. units
(c) 9 sq. units (d) none of these

35. The foot of the perpendicular on the line $3x+y=\lambda$ drawn from the origin is C. If The line cuts the x-axis and y-axis at A and B respectively, then $BC:CA$, is
(a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

Answers

1. (b) 2. (c) 3 (d) 4. (b) 5. (a) 6. (c) 7. (c)
8. (c) 9. (b) 10. (b) 11. (b) 12. (a) 13. (b) 14. (d)
15. (d) 16. (c) 17. (d) 18. (a) 19. (a) 20. (c) 21. (a)
22. (b) 23. (d) 24. (d) 25. (a) 26. (b) 27. (c) 28. (a)
29. (a) 30. (c) 31. (c) 32. (c) 33. (a) 34. (a) 35. (d)

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PAIR OF STRAIGHT LINES

1. JOINT EQUATION OF A PAIR OF STRAIGHT LINES

If $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are equations of two straight lines, then their joint equation is

$$(a_1x+b_1y+c_1)(a_2x+b_2y+c_2)=0$$

This equation is of the form

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

which is known as the general equation of a straight line.

The joint equation of the straight lines $a_1x+b_1y=0$ and $a_2x+b_2y=0$ is of the form $ax^2+2hxy+by^2=0$ which is known as the homogeneous equation of second degree. Thus, the combined equation of a pair of straight lines passing through the origin is a homogeneous equation of second degree.

Consider the equation $ax^2+2hxy+by^2=0$. This equation can be re-written as

$$b\left(\frac{y}{x}\right)^2+2h\left(\frac{y}{x}\right)+a=0 \qquad \text{[Dividing throughout by } x^2\text{]}$$

$$\Rightarrow \quad \frac{y}{x}=\frac{-h\pm\sqrt{h^2-ab}}{b}$$

$$\Rightarrow \quad y=\left(\frac{-h\pm\sqrt{h^2-ab}}{b}\right)x$$

$$\Rightarrow \quad y=m_1x,\ y=m_2x, \text{ where}$$

$$m_1=\frac{-h+\sqrt{h^2-ab}}{b} \text{ and } m_2=\frac{-h-\sqrt{h^2-ab}}{b}$$

The values of m_1 and m_2 are (i) real and distinct if $h^2>ab$ (ii) real and equal if $h^2=ab$ and (iii) imaginary if $h^2<ab$.

Also, $m_1+m_2=-\frac{2h}{b}$ and $m_1m_2=\frac{a}{b}$

Thus, $ax^2+2hxy+by^2=0$ represents a pair of straight lines $y=m_1x$ and $y=m_2x$ which are real and distinct if $h^2>ab$, coincident if $h^2=ab$ and imaginary if $h^2<ab$. Also,

$$m_1+m_2=-\frac{2h}{b}=-\frac{\text{Coefficient of } xy}{\text{Coefficient of } y^2}$$

$$\text{and, } m_1m_2=\frac{a}{b}=\frac{\text{Coefficient of } x^2}{\text{Coefficient of } y^2}$$

NOTE *If $y=m_1x$ and $y=m_2x$ are the lines represented by $ax^2+2hxy+by^2=0$, then $y-m_1x$ and $y-m_2x$ are factors of $ax^2+2hxy+by^2$ and $y=m_1x, y=m_2x$ satisfy the equation $ax^2+2hxy+by^2=0$.*

ILLUSTRATION 1 *If the equation $2x^2+\lambda xy+2y^2=0$ represents a pair of real and distinct lines, then*

(a) $\lambda\in(-4,4)$ (b) $\lambda\in R$

(c) $\lambda\in(-\infty,-4)\cup(4,\infty)$ (d) $\lambda=4,-4$

Ans. (c)

SOLUTION The equation $2x^2+\lambda xy+2y^2=0$ will represent a pair of real and distinct lines, if

$$\left(\frac{\lambda}{2}\right)^2-2\times2>0 \qquad \text{[Using: } h^2>ab\text{]}$$

$$\Rightarrow \quad \lambda^2-16>0 \Rightarrow \lambda\in(-\infty,-4)\cup(4,\infty)$$

ILLUSTRATION 2 *If the pair of lines represented by $ax^2+2hxy+by^2=0, b\neq0$, are such that the sum of the slopes of the lines is three times the product of their slopes, then*

(a) $3b+2h=0$ (b) $3a+2h=0$

(c) $2a+3h=0$ (d) *none of these*

Ans. (b)

SOLUTION Let $y=m_1x$ and $y=m_2x$ be the lines represented by $ax^2+2hxy+by^2=0$. Then,

$$m_1+m_2=-\frac{2h}{b} \text{ and } m_1m_2=\frac{a}{b}$$

It is given that

$$m_1+m_2=3m_1m_2 \Rightarrow -\frac{2h}{b}=\frac{3a}{b} \Rightarrow 2h+3a=0$$

ILLUSTRATION 3 *The two straight lines given by*

$$x^2(\tan^2\theta+\cos^2\theta)-2xy\tan\theta+y^2\sin^2\theta=0$$

make with the axis of x angles such that the difference of their tangents is

(a) 4 (b) 3 (c) 2 (d) *none of these*

Ans. (c)

SOLUTION Let m_1 and m_2 be the slopes of the lines given by $x^2(\tan^2\theta+\cos^2\theta)-2xy\tan\theta+y^2\sin^2\theta=0$. Then,

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$\therefore$ By statement–2, the equation

$(a-b)x^2+(b-c)xy+(c-a)y^2=0$

respresents a pair of real and distinct lines.

EXAMPLE 2 *Statement–1: All chords of the curve $3x^2-y^2-2x+4y=0$ which subtend a right angle at the origin pass through a fixed point.*

Statement–2: The equation $ax+by+c=0$ represents a family of straight lines passing through a fixed point iff there is a linear relation between a, b and c.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Statement–2 is true (see Chapter on straight line).

Let $ax+by=1$ be a chord of the curve $3x^2-y^2-2x+4y=0$. The joint equation of the pair of lines joining the origin to the points of intersection of the curve and the chord $ax+by=1$ is

$3x^2-y^2-2x(ax+by)+4y(ax+by)=0$

$\Rightarrow \quad x^2(3-2a)+2xy(2a-b)+y^2(4b-1)=0$

Lines represented by this equation will be at right angle, if

$3-2a+4b-1=0$

$\Rightarrow \quad a-2b-1=0$

$\Rightarrow \quad ax+by=1$ passes through a fixed point (By statement–2)

EXAMPLE 3 *Statement–1: If a, b are non-zero real numbers such that $a+b=2$, then $ax^2+2xy+by^2+2ax+2by=0$ represents a pair of straight lines.*

Statement–2: If a, b are of opposite signs, then $ax^2+2xy+by^2=0$ represents a pair of distinct lines passing through the origin.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION The equation $ax^2+2xy+by^2+2ax+2by=0$ will represent a pair of straight lines, if

$a\times b\times 0+2\times b\times a\times 1-a\times(b)^2-b\times(a)^2-0\times 1^2=0$

[Using: $abc+2fgh-af^2-bg^2-ch^2=0$]

$\Rightarrow \quad 2ab-ab(a+b)=0$

$\Rightarrow \quad a+b=2 \qquad [\because a\neq 0, b\neq 0]$

So, statement–1 is true.

Lines represented by $ax^2+2xy+by^2=0$ will be real and distinct if $1-ab>0$, which is true as $ab<0$. So, statement–2 is also true.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. For what value of k is $4x^2+8xy+ky^2=9$ the equation of a pair of straight lines?
   (a) 0 (b) 4 (c) 9 (d) -9
2. The angle between the lines represented by $x^2-y^2=0$ is
   (a) 0° (b) 45° (c) 90° (d) 180°
3. If the angle between the two lines represented by $2x^2+5xy+3y^2+6x+7y+4=0$ is $\tan^{-1}(m)$, then m is equal to
   (a) 1/5 (b) -1
   (c) $-2/3$ (d) none of these
4. If one of the lines of $ax^2+2hxy+by^2=0$ bisects the angle between the axes, in the first quadrant, then
   (a) $h^2-ab=0$ (b) $h^2+ab=0$
   (c) $(a+b)^2=h^2$ (d) $(a+b)^2=4h^2$

   [EAMCET 2001]
5. The point of intersection of the two lines given by $2x^2-5xy+2y^2-3x+3y+1=0$ is
   (a) (1/2, 1/3) (b) $(-1/7,-1/7)$
   (c) $(-1/3, 1/3)$ (d) none of these
6. The value of 'a' for which the lines represented by $ax^2+5xy+2y^2=0$ are mutually perpendicular is
   (a) 2 (b) -2
   (c) $\frac{25}{8}$ (d) none of these
7. Joint equation of the diagonals of the square formed by the pairs of lines $xy+4x-3y-12=0$ and $xy-3x+4y-12=0$, is
   (a) $x^2-y^2+x-y=0$ (b) $x^2-y^2+x+y=0$
   (c) $x^2+2xy+y^2+x+y=0$ (d) $x^2-2xy+y^2+x-y=0$
8. Which of the following pair of straight lines intersect at right angle?
   (a) $2x^2=y(x+2y)$ (b) $(x+y)^2=x(y+3x)$
   (c) $2y(x+y)=xy$ (d) $y=\pm 2x$
9. The equation of the lines parallel to the line common to the pair of lines given by $6x^2-xy-12y^2=0$ and $15x^2+14xy-8y^2=0$ and the sum of whose intercepts on the axes is 7, is
   (a) $2x-3y=42$ (b) $3x+4y=12$
   (c) $5x-2y=10$ (d) none of these
10. The equation $x^2+k_1y^2+k_2xy=0$ represents a pair of perpendicular lines if
    (a) $k_1=-1$ (b) $k_1=2k_2$
    (c) $2k_1=k_2$ (d) none of these
11. If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel lines, then
    (a) $\frac{a}{h}=\frac{b}{h}=\frac{f}{g}$ (b) $\frac{a}{h}=\frac{h}{b}=\frac{f}{g}$
    (c) $\frac{a}{h}=\frac{h}{b}=\frac{g}{f}$ (d) none of these

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12. If the slope of one of the lines given by $ax^2 - 6xy + y^2 = 0$ is twice the other, then a =
(a) 1 (b) 2 (c) 4 (d) 8

13. If the equation $12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$ represents a pair of perpendicular straight lines, then
(a) $p = 12, q = 1$ (b) $p = 1, q = 12$
(c) $p = -1, q = 12$ (d) $p = 1, q = -12$

14. If θ is the acute angle between the lines given by $6x^2 + 5xy - 4y^2 + 7x + 13y - 3 = 0$, then the equation of the line passing through the point of intersection of these lines and making angle θ with the positive x-axis is
(a) $2x + 11y + 13 = 0$ (b) $11x - 2y + 13 = 0$
(c) $2x - 11y + 2 = 0$ (d) $11x + 2y - 11 = 0$

15. If the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is rotated about the origin through 90°, then their equations in the new position are given by
(a) $ax^2 - 2hxy + by^2 = 0$ (b) $ax^2 - 2hxy - by^2 = 0$
(c) $bx^2 - 2hxy + ay^2 = 0$ (d) $bx^2 + 2hxy + ay^2 = 0$

16. If the lines represented by $x^2 - 2pxy - y^2 = 0$ are rotated about the origin through an angle θ, one in clockwise direction and other in anti-clockwise direction, then the equation of the bisectors of the angle between the lines in the new position is
(a) $px^2 + 2xy - py^2 = 0$ (b) $px^2 + 2xy + py^2 = 0$
(c) $x^2 - 2pxy + y^2 = 0$ (d) none of these

17. The difference of the tangents of the angles which the lines $x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make with the x-axis is
(a) $2\tan\theta$ (b) 2 (c) $2\cot\theta$ (d) $\sin 2\theta$

18. If the two pairs of lines $x^2 - 2mxy - y^2 = 0$ and $x^2 - 2nxy - y^2 = 0$ are such that one of them represents the bisector of the angles between the other, then
(a) $mn + 1 = 0$ (b) $mn - 1 = 0$
(c) $\frac{1}{m} + \frac{1}{n} = 0$ (d) $\frac{1}{m} - \frac{1}{n} = 0$

19. If $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$, represents a pair of straight lines, then the value of λ is
(a) 4 (b) 3 (c) 2 (d) 1

20. The equation $y^2 - x^2 + 2x - 1 = 0$, represents
(a) a pair of st. lines (b) a circle
(c) a parabola (d) an ellipse

21. The angle between the straight lines $x^2 - y^2 - 2x - 1 = 0$, is
(a) 90° (b) 60° (c) 75° (d) 36°

[CEE (Delhi) 1997]

22. If the angle between two lines represented by $2x^2 + 5xy + 3y^2 + 7y + 4 = 0$ is $\tan^{-1} m$, then m =
(a) 1/5 (b) 1 (c) 7/5 (d) 7

23. A diagonal of the rectangle formed by the lines $x^2 - 7x + 6 = 0$ and $y^2 - 14y + 40 = 0$ is
(a) $5x + 6y = 0$ (b) $5x - 6y = 0$
(c) $6x - 5y + 14 = 0$ (d) $6x - 5y - 14 = 0$

24. The angle between the pair of lines $2x^2 + 5xy + 2y^2 + 3x + 3y + 1 = 0$, is
(a) $\cos^{-1}(4/5)$ (b) $\tan^{-1}(4/5)$
(c) 0 (d) $\pi/2$

25. The circumcentre of the triangle formed by the lines $xy + 2x + 2y + 4 = 0$ and $x + y + 2 = 0$, is
(a) (0, 0) (b) (−2, −2)
(c) (−1, −1) (d) (−1, −2)

26. The distance between the parallel lines $9x^2 - 6xy + y^2 + 18x - 6y + 8 = 0$, is
(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$ (c) $\frac{4}{\sqrt{10}}$ (d) $\sqrt{10}$

27. The joint equation of the straight lines $x + y = 1$ and $x - y = 4$, is
(a) $x^2 - y^2 = -4$ (b) $x^2 - y^2 = 4$
(c) $(x + y - 1)(x - y - 4) = 0$ (d) $(x + y + 1)(x - y + 4) = 0$

28. If the slope of one of the lines given by $ax^2 - 6xy + y^2 = 0$ is square of the other, then a =
(a) 8, −27 (b) −8, 27 (c) 1, 8 (d) −8, −27

29. The value of k such that $3x^2 - 11xy + 10y^2 - 7x + 13y + k = 0$ may represent a pair of straight lines, is
(a) 3 (b) 4 (c) 6 (d) 8

30. If $x^2 - kxy + y^2 + 2y + 2 = 0$ denotes a pair of straight lines then k =
(a) 2 (b) $1/\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$

31. The equations of the lines which are parallel to the line common to the pair of the lines given by $6x^2 - xy - 12y^2 = 0$ and $15x^2 + 14xy - 8y^2 = 0$ and at a distance of 7 units from it are
(a) $3x + 4y = \pm 35$ (b) $5x - 2y = \pm 7$
(c) $2x - 3y = \pm 7$ (d) none of these

32. If the slope of one of the lines given by $36x^2 + 2hxy + 72y^2 = 0$ is four times the other, then h^2 =
(a) 5040 (b) 4050
(c) 8100 (d) none of these

33. The combined equation of the pair of straight lines through the point (1, 0) and parallel to the lines represented by $2x^2 - xy - y^2 = 0$ is
(a) $2x^2 - xy - 2y^2 + 4x - y = 6$
(b) $2x^2 - xy - y^2 - 4x - y + 2 = 0$
(c) $2x^2 - xy - y^2 - 4x + y + 2 = 0$
(d) none of these

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Answers

1. (b) 2. (c) 3. (a) 4. (d) 5. (d) 6. (b) 7. (a)
8. (a) 9. (b) 10. (a) 11. (c) 12. (d) 13. (a) 14. (b)
15. (c) 16. (a) 17. (b) 18. (a) 19. (c) 20. (a) 21. (a)
22. (a) 23. (c) 24. (a) 25. (c) 26. (b) 27. (c) 28. (a)
29. (b) 30. (d) 31. (a) 32. (b) 33. (c)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The equation $x^3 - 6x^2y + 11xy^2 - 6y^3 = 0$ represents three straight lines passing through the origin, the slopes of which form an
   (a) A.P. (b) G.P. (c) H.P. (d) none of these

2. If the lines given by $ax^2 + 2hxy + by^2 = 0$ are equally inclined to the lines given by $ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$, then
   (a) λ is any real number (b) $\lambda = 2$
   (c) $\lambda = 1$ (d) none of these

3. The equations to the straight lines passing through the origin and making an angle α with the straight line $y + x = 0$ are given by
   (a) $x^2 + 2xy \sec 2\alpha + y^2 = 0$ (b) $x^2 - 2xy \sec 2\alpha + y^2 = 0$
   (c) $x^2 + 2xy \cos 2\alpha + y^2 = 0$ (d) none of these

4. If the lines joning the origin to the points of intersection of $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy - c = 0$ are at right angles, then
   (a) $g^2 + f^2 = c$ (b) $g^2 - f^2 = c$
   (c) $g^2 - f^2 = 2c$ (d) $g^2 + f^2 = c^2$

5. If the area of the triangle formed by the pair of lines given by $8x^2 - 6xy + y^2 = 0$ and the line $2x + 3y = a$ is 7, then a =
   (a) 14 (b) $14\sqrt{2}$
   (c) 28 (d) none of these

6. The lines bisecting the angle between the bisectors of the angles between the lines $ax^2 + 2hxy + by^2 = 0$ are given by
   (a) $(a - b)(x^2 - y^2) - 4hxy = 0$
   (b) $(a - b)(x^2 + y^2) + 4hxy = 0$
   (c) $(a - b)(x^2 - y^2) + 4hxy = 0$
   (d) none of these

7. The pair of lines $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ are rotated about the origin by $\pi/6$ in anticlockwise sense. The equation of the pair in the new position is
   (a) $\sqrt{3}x^2 - xy = 0$ (b) $x^2 - \sqrt{3}xy = 0$
   (c) $xy - \sqrt{3}y^2 = 0$ (d) none of these

8. If two of the lines given by the equation $ax^3 + bx^2y + cxy^2 + dy^3 = 0\ (a \neq 0)$ make complementary angles with x-axis in anticlockwise sense, then
   (a) $a(a - c) = d(b - d)$ (b) $d(a - c) = a(d - b)$
   (c) $a(a - c) = d(d - b)$ (d) none of these

9. If two of the lines given by the equation $ay^4 + bxy^3 + cx^2y^2 + dx^3y + ex^4 = 0$ are at right angle, then
   (a) $(c + a + e)(e - a)^2 + (b + d)(ad + be) = 0$
   (b) $(c + a + e)(e - a)^2 - (b + d)(ad + be) = 0$
   (c) $(c + a + e)(e + a)^2 + (b + d)(ad + be) = 0$
   (d) none of these

10. The pairs of straight lines $ax^2 + 2hxy - ay^2 = 0$ and $hx^2 - 2axy - hy^2 = 0$ are such that
    (a) one pair bisects the angle between the other pair
    (b) the lines of one pair are equally inclined to the lines of the other pair
    (c) the lines of each pair are perpendicular to other pair
    (d) all of these

11. The equations $a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$ and $ax^2 + 2hxy + by^2 = 0$ represent
    (a) two pairs of perpendicular straight lines
    (b) two pairs of parallel straight lines
    (c) two pairs of straight lines which are equally inclined to each other
    (d) none of these

12. If $\frac{x^2}{a} + \frac{y^2}{b} + \frac{2xy}{h} = 0$ respresents a pair of straight lines such that slope of one line is twice the other, then $ab : h^2$ is
    (a) 9 : 8 (b) 8 : 9 (c) 1 : 2 (d) 2 : 1

13. If the lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are equidistant from the origin, then
    (a) $f^4 - g^4 = c(bf^2 - ag^2)$ (b) $f^4 - g^4 = c(af^2 - bg^2)$
    (c) $f^4 - g^4 = c(ag^2 - bf^2)$ (d) none of these

14. The distance between the two lines represented by the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ is
    (a) 8/5 (b) 6/5
    (c) 11/5 (d) none of these

15. The equation of the image of the lines $y = |x|$ in the line mirror $x = 2$ is
    (a) $y = |x - 4|$ (b) $|y| = x + 4$
    (c) $|y| + 4 = x$ (d) none of these

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16. If the equation $3x^2 + xy - y^2 - 3x + 6y + k = 0$ represents a pair of straight lines, then the value of k is
(a) 9 (b) 1 (c) -9 (d) 0

17. The equation $x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$ represents a pair of lines which are parallel to each other. The distance between them is
(a) 4 (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) 2

18. The values of λ for which the equation $x^2 - y^2 - x + \lambda y - 2 = 0$ represents a pair of straight lines are:
(a) $-3, 1$ (b) $3, -3$ (c) $-1, 1$ (d) $3, 1$

19. Distance between the pair of lines represented by the equation $x^2 - 6xy + 9y^2 + 3x - 9y - 4 = 0$, is
(a) $\frac{15}{\sqrt{10}}$ (b) $\frac{1}{2}$ (c) $\sqrt{\frac{5}{2}}$ (d) $\frac{1}{\sqrt{10}}$

20. If the angle between the pair of straight lines represented by the equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$, is $\tan^{-1} 3$, where 'λ' is a non-negative real number. Then, λ =
(a) 2 (b) 0 (c) 3 (d) 1

Answers

1. (c) 2. (a) 3. (a) 4. (c) 5. (c) 6. (c) 7. (a) 8. (c) 9. (a) 10. (d) 11. (c) 12. (a) 13. (a) 14. (a) 15. (a) 16. (c) 17. (d) 18. (b) 19. (c) 20. (b)

Solutions of **Exercises** and **Chapter-tests** are available in a separate book on **"Solutions of Objective Mathematics"**.

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1. THE CIRCLE

A circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane is always constant.

The fixed point is called the centre of the *circle* and the constant distance is called the *radius* of the circle.

In Fig. 1, P is the moving point, C is the fixed point and CP is equal to the radius.

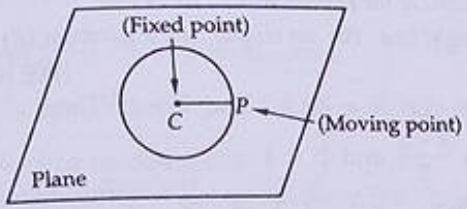


Fig. 1

By the equation of a circle is meant the equation of the circumference; it is a relation between the coordinates x, y of the moving point P, involving some constants depending upon the position of the centre and the length of the radius.

2. STANDARD EQUATION OF A CIRCLE

THEOREM *The equation of a circle whose centre is at (h, k) and radius a is given.*

$$(x-h)^2 + (y-k)^2 = a^2$$

PROOF Let C be the centre of the circle and its coordinates be (h, k). Let the radius of the circle be a and let $P(x, y)$ be any point on the circumference. Then,

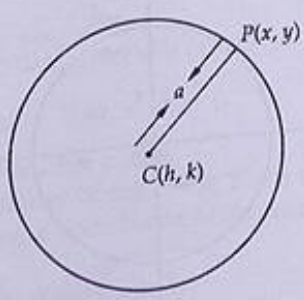


Fig. 2

$$CP = a$$

$$\Rightarrow \quad CP^2 = a^2$$

$$\Rightarrow \quad (x-h)^2 + (y-k)^2 = a^2$$

This is the relation between the coordinates of any point on the circumference and hence it is the required equation of the circle having centre at (h, k) and radius equal to a.

NOTE 1 *The above equation is known as the central form of the equation of a circle.*

NOTE 2 *If the centre of the circle is at the origin and radius is a, then from the above form the equation of the circle is*

$$x^2 + y^2 = a^2$$

ILLUSTRATION 1 *The equation of the circle whose centre is $(2, -3)$ and radius 5, is*

(a) $x^2 + y^2 + 4x - 6y - 12 = 0$ (b) $x^2 + y^2 - 4x + 6y - 12 = 0$

(c) $x^2 + y^2 - 6x + 4y - 12 = 0$ (d) *none of these*

Ans. (b)

SOLUTION The equation of the required circle is

$$(x-2)^2 + (y+3)^2 = 5^2 \Rightarrow x^2 + y^2 - 4x + 6y - 12 = 0$$

ILLUSTRATION 2 *The radius of the circle passing through the point $(6, 2)$, two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is*

(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

Ans. (b)

SOLUTION The two diameters intersect at $(8, -2)$ which is the centre of the circle. The circle passes through $(6, 2)$.

$$\therefore \quad \text{Radius} = \sqrt{(8-6)^2 + (-2-2)^2} = \sqrt{20}$$

ILLUSTRATION 3 *The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length $3a$ is*

(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$

(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

[AIEEE 2002, CEE (Delhi) 2003]

Ans. (c)

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So, $x^2+y^2-xy+x+y+1=0$ represents a pair of straight lines.

Hence, statement–1 is not true. However, statement–2 is true.

EXAMPLE 11 *Statement–1: The common tangent to the circles* $x^2+y^2+2x=0, x^2+y^2-6x=0$ *form an equilateral triangle.*
Statement–2: The given circles touch each other externallly.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION Centres of two circles are $C_1(-1, 0)$ and $C_2(3, 0)$ respectively and their radii are 1 and 3 respectively.
Clearly, C_1C_2 = Sum of the radii.
So, two circles touch each other externally.

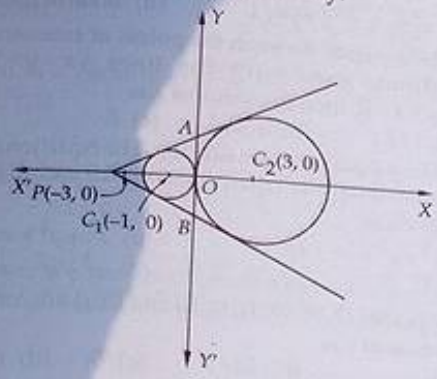


Fig. 94

Let P be the point of intersection of transverse common tangents. Then, P divides C_1C_2 externally in the ratio 1 : 3. So, coordinates of P are $(-3, 0)$.

The equation of any line passing through $P(-3, 0)$ is
$y = m(x+3)$
If it touches the circle $x^2+y^2+2x=0$, then

$$\left|\frac{2m}{\sqrt{m^2+1}}\right| = 1 \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

$\therefore \quad \angle APB = 60°$
Since ΔPAB is isosceles. Therefore, $\angle A = \angle B = 60°$
Hence, ΔPAB is equilateral.

EXAMPLE 12 *Statement-1: The length of the sides of quadrilateral can be in A.P.*
Statement-2: The length of tangents from an external point to a circle are equal.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION It is evident from Fig.23.95 that
$AB + DC = AD + BC$
$\Rightarrow$ AB, BC, AD, DC are in AP.

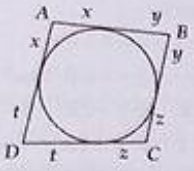


Fig. 95

So, lengths of sides of quadrilateral can be in AP.
Hence, statement-1 is true.
Statement-2 is a standard result in geometry. So, it is true. Also, Statment-2 is a correct explanation for statement-1.

SECTION - II

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The centre of the circle passing through $(0, 0)$ $(a, 0)$ and $(0, b)$ is
(a) (a, b) (b) $(a/2, b/2)$
(c) $(-a/2, -b/2)$ (d) $(-a, -b)$

2. The circle $x^2+y^2+4x-7y+12=0$ cuts an intercept on y-axis of length
(a) 3 (b) 4 (c) 7 (d) 1

3. A square is inscribed in the circle $x^2+y^2-2x+4y+3=0$ and its sides are parallel to the coordinate axes, then one vertex of the square is
(a) $(1+\sqrt{2}, -2)$ (b) $(1-\sqrt{2}, -2)$
(c) $(1, -2+\sqrt{2})$ (d) none of these

4. The straight line $y=mx+c$ cuts the circle $x^2+y^2=a^2$ in real points if
(a) $\sqrt{a^2(1+m^2)} < c$ (b) $\sqrt{a^2(1-m^2)} < c$
(c) $\sqrt{a^2(1+m^2)} > c$ (d) $\sqrt{a^2(1-m^2)} > c$

5. The area of the circle centred at (1, 2) and passing through (4, 6), is
(a) 5π (b) 10π (c) 25π (d) none of these

6. The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a circle, the condition will be
(a) $a=b$ and $c=0$ (b) $f=g$ and $h=0$
(c) $a=b$ and $h=0$ (d) $f=g$ and $c=0$

7. The equation of the circle passing through (4, 5) having the centre at (2, 2) is
(a) $x^2+y^2+4x+4y-5=0$ (b) $x^2+y^2-4x-4y-5=0$
(c) $x^2+y^2-4x-13=0$ (d) $x^2+y^2-4x-4y+5=0$

8. The locus of the centre of a circle of radius 2 which rolls on the outside of the circle $x^2+y^2+3x-6y-9=0$ is
(a) $x^2+y^2+3x-6y+5=0$
(b) $x^2+y^2+3x-6y-31=0$

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(c) $x^2+y^2+3x-6y+\frac{29}{4}=0$

(d) none of these

9. Four distinct points $(2k, 3k)$, $(1, 0)$, $(0, 1)$ and $(0, 0)$ lie on a circle for
(a) all integral values of k (b) $0<k<1$
(c) $k<0$ (d) for two values of k

10. A square is inscribed in the circle $x^2+y^2-2x+4y-93=0$ with its sides parallel to the coordinate axes. The coordinates of its vertices are
(a) $(-6,-9),(-6,5),(8,-9),(8,5)$
(b) $(-6,9),(-6,-5),(8,-9),(8,5)$
(c) $(-6,-9),(-6,5),(8,9),(8,5)$
(d) $(-6,-9),(-6,5),(8,-9),(8,-5)$

11. A line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2+y^2=r^2$ at A and B. Then, $PA \cdot PB$ is equal to
(a) $(\alpha+\beta)^2-r^2$ (b) $\alpha^2+\beta^2-r^2$
(c) $(\alpha-\beta)^2+r^2$ (d) none of these

12. The equation of a circle passing through $(3,-6)$ and touching both the axes is
(a) $x^2+y^2-6x+6y+9=0$
(b) $x^2+y^2+6x-6y+9=0$
(c) $x^2+y^2+30x-30y+225=0$
(d) $x^2+y^2+30x+30y+225=0$

13. The centre of a circle passing through the points $(0, 0)$, $(1, 0)$ and touching the circle $x^2+y^2=9$ is
(a) $(3/2, 1/2)$ (b) $(1/2, 3/2)$
(c) $(1/2, 1/2)$ (d) $(1/2, -\sqrt{2})$

[AIEEE 2002]

14. If $2x-4y=9$ and $6x-12y+7=0$ are parallel tangents to circle, then radius of the circle is
(a) $\frac{\sqrt{3}}{5}$ (b) $\frac{17}{6\sqrt{5}}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{17}{3\sqrt{5}}$

15. One of the diameters of the circle
$x^2+y^2-12x+4y+6=0$ is given by
(a) $x+y=0$ (b) $x+3y=0$ (c) $x=y$ (d) $3x+2y=0$

16. The length of the chord cut off by $y=2x+1$ from the circle $x^2+y^2=2$, is
(a) $5/6$ (b) $6/5$ (c) $6/\sqrt{5}$ (d) $\sqrt{5}/6$

17. Area of the circle in which a chord of length $\sqrt{2}$ makes an angle $\pi/2$ at the centre is
(a) $\pi/2$ (b) 2π (c) π (d) $\pi/4$

18. The coordinates of the middle point of the chord cut off by $2x-5y+18=0$ on the cirlce $x^2+y^2-6x+2y-54=0$ are
(a) $(1, 4)$ (b) $(2, 4)$ (c) $(4, 1)$ (d) $(1, 1)$

19. Equation of the circle which touches $3x+4y=7$ and passes through $(1, -2)$ and $(4, -3)$ is
(a) $x^2+y^2-94x+18y+55=0$
(b) $15x^2+15y^2-94x+18y+55=0$
(c) $15x^2+15y^2+94x+18y+55=0$
(d) $x^2+y^2-94x-18y+55=0$

20. If the lines $3x-4y-7=0$ and $2x-3y-5=0$ are diameters of a circle of area 49π square units, the equation of the circle is
(a) $x^2+y^2+2x-2y-62=0$ (b) $x^2+y^2-2x+2y-62=0$
(c) $x^2+y^2-2x+2y-47=0$ (d) $x^2+y^2+2x-2y-47=0$

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21. Equation of the circle with centre on the y-axis and passing through the origin and $(2, 3)$ is
(a) $x^2+y^2+13y=0$ (b) $3x^2+3y^2-13y=0$
(c) $x^2+y^2+13x+3=0$ (d) $6x^2+6y^2-13x=0$

22. If the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ cut the coordinate axes in concyclic points, then
(a) $|a_1a_2| = |b_1b_2|$ (b) $|a_1b_1| = |a_2b_2|$
(c) $|a_1b_2| = |a_2b_1|$ (d) none of these

23. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x-y+1=0$ and $x-2y+3=0$, then the value of λ is
(a) 2, 1/3 (b) 1/3, 1 (c) 6 (d) 3

24. $ABCD$ is a square whose side is a. The equation of the circle circumscribing the square, taking AB and AD as axes of reference, is
(a) $x^2+y^2+ax+ay=0$ (b) $x^2+y^2+ax-ay=0$
(c) $x^2+y^2-ax-ay=0$ (d) $x^2+y^2-ax+ay=0$

25. If the points $(2, 0)$, $(0, 1)$, $(4, 5)$ and $(0, c)$ are concyclic, then the value of c is
(a) 1 (b) 14/3 (c) 5 (d) none of these

26. A, B, C and D are the points of intersection with the coordinate axes of the lines $ax+by=ab$ and $bx+ay=ab$, then
(a) A, B, C, D are concyclic
(b) A, B, C, D form a parallelogram
(c) A, B, C, D form a rhombus
(d) none of these

27. Two perpendicular tangents to the circle $x^2+y^2=a^2$ meet at P. Then the locus of P has the equation
(a) $x^2+y^2=2a^2$ (b) $x^2+y^2=3a^2$
(c) $x^2+y^2=4a^2$ (d) none of these

28. The equations of the tangents drawn from the origin to the circle $x^2+y^2-2rx-2hy+h^2=0$, are
(a) $x=0, y=0$
(b) $y=0, (h^2-r^2)x-2rhy=0$
(c) $x=0, (h^2-r^2)x-2rhy=0$
(d) $x=0, (h^2-r^2)x+2rhy=0$

29. If from any point P on the circle $x^2+y^2+2gx+2fy+c=0$, tangents are drawn to the circle
$x^2+y^2+2gx+2fy+c\sin^2\alpha+(g^2+f^2)\cos^2\alpha=0$,
then the angle between the tangents is
(a) α (b) 2α (c) $\alpha/2$ (d) none of these

30. If the equation of a given circle is $x^2+y^2=36$, then the length of the chord which lies along the line $3x+4y-15=0$ is
(a) $3\sqrt{6}$ (b) $2\sqrt{3}$ (c) $6\sqrt{3}$ (d) none of these

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31. The common chord of $x^2+y^2-4x-4y=0$ and $x^2+y^2=16$ subtends at the origin an angle equal to
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

32. The equation of a circle passing through (1, 1) and points of intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ is
(a) $4x^2+4y^2-30x-10y-25=0$
(b) $4x^2+4y^2+30x-13y-25=0$
(c) $4x^2+4y^2-17x-10y+25=0$
(d) none of these

33. The tangents to $x^2+y^2=a^2$ having inclinations α and β intersect at P. If $\cot\alpha+\cot\beta=0$, then the locus of P is
(a) $x+y=0$ (b) $x-y=0$
(c) $xy=0$ (d) none of these

34. Equation of a circle with centre (4, 3) touching the circle $x^2+y^2=1$, is
(a) $x^2+y^2-8x-6y-9=0$
(b) $x^2+y^2-8x-6y+11=0$
(c) $x^2+y^2-8x-6y-11=0, x^2+y^2-8x-6y+9=0$
(d) none of these

35. The number of common tangents that can be drawn to the circles $x^2+y^2-4x-6y-3=0$ and $x^2+y^2+2x+2y+1=0$ is
(a) 1 (b) 2 (c) 3 (d) 4

36. If $3x+y=0$ is a tangent to the circle with centre at the point $(2,-1)$, then the equation of the other tangent to the circle from the origin is
(a) $x-3y=0$ (b) $x+3y=0$ (c) $3x-y=0$ (d) $2x+y=0$

37. The circles whose equations are $x^2+y^2+c^2=2ax$ and $x^2+y^2+c^2-2by=0$ will touch one another externally if
(a) $\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a^2}$ (b) $\frac{1}{c^2}+\frac{1}{a^2}=\frac{1}{b^2}$
(c) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ (d) none of these

38. The distance between the chords of contact of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ from the origin and the point (g, f), is
(a) g^2+f^2 (b) $\frac{1}{2}(g^2+f^2+c)$
(c) $\frac{g^2+f^2+c}{2\sqrt{g^2+f^2}}$ (d) $\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$

39. The condition that the chord $x\cos\alpha+y\sin\alpha-p=0$ of $x^2+y^2-a^2=0$ may subtend a right angle at the centre of the circle is
(a) $a^2=2p^2$ (b) $p^2=2a^2$ (c) $a=2p$ (d) $p=2a$

40. The locus of the centre of the circles which touch both the circles $x^2+y^2=a^2$ and $x^2+y^2=4ax$ externally has the equation
(a) $12(x-a)^2-4y^2=3a^2$ (b) $9(x-a)^2-5y^2=2a^2$
(c) $8x^2-3(y-a)^2=9a^2$ (d) none of these

41. Let P be a point on the circle $x^2+y^2=9$, Q a point on the line $7x+y+3=0$, and the perpendicular bisector of PQ be the line $x-y+1=0$. Then, the coordinates of P are
(a) (3, 0) (b) (0, 3)
(c) (72/25, −21/25) (d) (−72/25, −21/25)

42. The two lines through (2, 3) from which the circle $x^2+y^2=25$ intercepts chords of length 8 units have equations
(a) $2x+3y=13, x+5y=17$
(b) $y=3, 12x+5y=39$
(c) $x=2, 9x-11y=51$
(d) none of these

43. A line meets the coordinate axes in A and B. A circle is circumscribed about the triangle OAB. The distances from the end points A, B of the side AB to the tangent at O are equal to m and n respectively. Then, the diameter of the circle is
(a) $m(m+n)$ (b) $n(m+n)$
(c) $m-n$ (d) none of these

44. The coordinates of the point on the circle $x^2+y^2-12x-4y+30=0$, which is farthest from the origin are
(a) (9, 3) (b) (8, 5) (c) (12, 4) (d) none of these

45. The circles $x^2+y^2+x+y=0$ and $x^2+y^2+x-y=0$ intersect at an angle of
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

46. The equation of the circle having radius 5 and touching the circle $x^2+y^2-2x-4y-20=0$ at (5, 5) is
(a) $(x^2+y^2)+18x+16y+120=0$
(b) $(x^2+y^2)+18x-16y+120=0$
(c) $(x^2+y^2)-18x+16y+120=0$
(d) $(x^2+y^2)-18x-16y+120=0$

47. AB is a diameter of a circle and C is any point on the circumference of the circle. Then,
(a) the area of ΔABC is maximum when it is isosceles
(b) the area of ΔABC is minimum when it is isosceles
(c) the perimeter of ΔABC is maximum when it is isosceles
(d) none of these

48. The locus of the mid points of a chord of the circle $x^2+y^2=4$ which subtends a right angle at the origin is
(a) $x+y=2$ (b) $x^2+y^2=1$
(c) $x^2+y^2=2$ (d) $x+y=1$

49. The pole of the straight line $9x+y-28=0$ with respect to the circle $2x^2+2y^2-3x+5y-7=0$ is
(a) (3, 1) (b) (1, 3) (c) (3, −1) (d) (−3, 1)

50. If the tangents are drawn to the circle $x^2+y^2=12$ at the point where it meets the circle $x^2+y^2-5x+3y-2=0$, then the point of intersection of these tangents is
(a) (6, −6) (b) (6, 18/5)
(c) (6, −18/5) (d) none of these

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51. If the straight line $x-2y+1=0$ intersects the circle $x^2+y^2=25$ in points P and Q, then the coordinates of the point of intersection of tangents drawn at P and Q to the circle $x^2+y^2=25$ are
(a) (25, 50) (b) (–25, –50)
(c) (–25, 50) (d) (25, –50)

52. If the chord of contact of tangents drawn from the point (h, k) to the circle $x^2+y^2=a^2$ subtends a right angle at the centre, then
(a) $h^2+k^2=a^2$ (b) $2(h^2+k^2)=a^2$
(c) $h^2-k^2=a^2$ (d) $h^2+k^2=2a^2$

53. The equation of a circle which cuts the three circles
$x^2+y^2-3x-6y+14=0$
$x^2+y^2-x-4y+8=0$
$x^2+y^2+2x-6y+9=0$
orthogonally, is
(a) $x^2+y^2-2x-4y+1=0$ (b) $x^2+y^2+2x+4y+1=0$
(c) $x^2+y^2-2x+4y+1=0$ (d) $x^2+y^2-2x-4y-1=0$

54. The equation of a circle which passes through $(2a, 0)$ and whose radical axis in relation to the circle $x^2+y^2=a^2$ is $x=a/2$, is
(a) $x^2+y^2-ax=0$ (b) $x^2+y^2+2ax=0$
(c) $x^2+y^2-2ax=0$ (d) $x^2+y^2+ax=0$

55. If the circle $x^2+y^2+2gx+2fy+c=0$ bisects the circumference of the circle $x^2+y^2+2g'x+2f'y+c'=0$, then
(a) $2g(g-g')+2f(f-f')=c-c'$
(b) $2g'(g-g')+2f'(f-f')=c'-c$
(c) $2g'(g-g')+2f'(f-f')=c-c'$
(d) $2g(g-g')+2f(f-f')=c'-c$

56. The pole of a straight line with respect to the circle $x^2+y^2=a^2$ lies on the circle $x^2+y^2=9a^2$. If the straight line touches the circle $x^2+y^2=r^2$, then
(a) $9a^2=r^2$ (b) $9r^2=a^2$ (c) $r^2=a^2$ (d) none of these

57. The equation to the chord of the circle $x^2+y^2=9$ whose middle point is $(1, -2)$ is
(a) $x-2y=9$ (b) $x-2y-4=0$
(c) $x-2y-5=0$ (d) $x-2y+5=0$

58. The locus of the middle point of chords of the circle $x^2+y^2=a^2$ which pass through the fixed point (h, k) is
(a) $x^2+y^2-hx-ky=0$ (b) $x^2+y^2+hx+ky=0$
(c) $x^2+y^2-2hx-2ky=0$ (d) $x^2+y^2+2hx+2ky=0$

59. If the circles $(x-a)^2+(y-b)^2=c^2$ and $(x-b)^2+(y-a)^2=c^2$ touch each other, then
(a) $a=b\pm 2c$ (b) $a=b\pm\sqrt{2}c$
(c) $a=b\pm c$ (d) none of these

60. The equation of the image of the circle $x^2+y^2+16x-24y+183=0$ *by the line mirror* $4x+7y+13=0$ *is*
(a) $x^2+y^2+32x-4y+235=0$
(b) $x^2+y^2+32x+4y-235=0$
(c) $x^2+y^2+32x-4y-235=0$
(d) $x^2+y^2+32x+4y+235=0$

61. The number of the tangents that can be drawn from $(1, 2)$ to $x^2+y^2=5$, is
(a) 1 (b) 2 (c) 3 (d) 0

62. Equation of the circle through the origin and making intercepts of 3 and 4 on the positive sides of the axes is
(a) $x^2+y^2+3x+4y=0$ (b) $x^2+y^2-3x-4y=0$
(c) $x^2+y^2+3x-4y=0$ (d) $x^2+y^2-3x+4y=0$

63. Equation of the chord of the circle $x^2+y^2-4x=0$ whose mid point is $(1, 0)$ is
(a) $y=2$ (b) $y=1$ (c) $x=2$ (d) $x=1$

64. The tangent to $x^2+y^2=9$ which is parallel to y-axis and does not lie in the third quadrant touches the circle at the point
(a) (3, 0) (b) (–3, 0) (c) (0, 3) (d) (0, –3)

65. The two circles $x^2+y^2-5=0$ and $x^2+y^2-2x-4y-15=0$
(a) touch each other externally
(b) touch each other internally
(c) cut each other orthogonally
(d) do not intersect

66. If the circle $x^2+y^2+2x+3y+1=0$ cuts $x^2+y^2+4x+3y+2=0$ in A and B, then the equation of the circle on AB as diameter is
(a) $x^2+y^2+x+3y+3=0$
(b) $2x^2+2y^2+2x+6y+1=0$
(c) $x^2+y^2+x+6y+1=0$
(d) none of these

67. The circle $x^2+y^2=4$ cuts the circle $x^2+y^2-2x-4=0$ at the points A and B. If the circle $x^2+y^2-4x-k=0$ passes through A and B then the value of k is
(a) –4 (b) 0 (c) –8 (d) 4

68. If the circle $x^2+y^2+2gx+2fy+c=0$ is touched by $y=x$ at P such that $OP=6\sqrt{2}$, then the value of c is
(a) 36 (b) 144 (c) 72 (d) none of these

69. The number of common tangents of the circles $x^2+y^2-2x-1=0$ and $x^2+y^2-2y-7=0$ is
(a) 1 (b) 2 (c) 3 (d) 4

70. The length of the common chord of the circles $x^2+y^2+4x+1=0$ and $x^2+y^2+4y-1=0$ is
(a) $\sqrt{15/2}$ (b) $\sqrt{15}$ (c) $2\sqrt{15}$ (d) none of these

71. If a circle passes through (1, 2) and cuts the circle $x^2+y^2=4$ orthogonally then the equation of the locus of its centre is
(a) $2x+4y-9=0$ (b) $2x+4y+9=0$
(c) $2x-4y+9=0$ (d) none of these

72. The lines $3x-4y+4=0$ and $6x-8y-7=0$ are tangents to the same circle. Then, its radius is
(a) 1/4 (b) 1/2 (c) 3/4 (d) none of these

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74. The coordinates of the centre of the circle which intersects circles $x^2+y^2+4x+7=0$, $2x^2+2y^2+3x+5y+9=0$ and $x^2+y^2+y=0$ orthogonally are
(a) $(-2, 1)$ (b) $(-2, -1)$ (c) $(2, -1)$ (d) $(2, 1)$

75. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y=x+7$. If A and B are the points $(-3, 4)$ and $(5, 4)$ respectively, then the area of the rectangle is
(a) 16 sq. units (b) 24 sq. units
(c) 32 sq. units (d) none of these

76. The points of contact of tangents to the circle $x^2+y^2=25$ which are inclined at an angle of 30° to the x-axis are
(a) $(\pm 5/2, \pm 1/2)$ (b) $(\pm 1/2, \pm 5/2)$
(c) $(\mp 5/2, \mp 1/2)$ (d) none of these

77. If $(m_i, 1/m_i)$, $i=1, 2, 3, 4$ are concyclic points, then the value of $m_1 m_2 m_3 m_4$, is
(a) 1 (b) −1 (c) 0 (d) none of these

78. The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2+y^2=9$ and the line joining their points of contact, is
(a) $\frac{25}{192}$ sq. units (b) $\frac{192}{25}$ sq. units
(c) $\frac{384}{25}$ sq. units (d) none of these

78. The tangent at P, any point on the circle $x^2+y^2=4$, meets the coordinate axes in A and B, then
(a) length of AB is constant
(b) PA and PB are always equal
(c) the locus of the mid-point of AB is $x^2+y^2=x^2y^2$
(d) none of these

79. The equation of the circle which touches the axes of the coordinates and the line $\frac{x}{3}+\frac{y}{4}=1$ and whose centre lies in the first quadrant is $x^2+y^2-2cx-2cy+c^2=0$, where c is
(a) 1, 6 (b) 2, 1 (c) 3, 6 (d) 6, 4

80. If the chords of contact of the tangents from a point on the circle $x^2+y^2=a^2$ to the circle $x^2+y^2=b^2$ touch the circle $x^2+y^2=c^2$, then the roots of the equation $ax^2+2bx+c=0$, are
(a) imaginary (b) real and equal
(c) real and unequal (d) rational

81. If from the origin a chord is drawn to the circle $x^2+y^2-2x=0$, then the locus of the mid point of the chord has equation
(a) $x^2+y^2+x+y=0$ (b) $x^2+y^2+2x+y=0$
(c) $x^2+y^2-x=0$ (d) $x^2+y^2-2x+y=0$

82. The locus of a point represented by
$$x=\frac{a}{2}\left(\frac{t+1}{t}\right), y=\frac{a}{2}\left(\frac{t-1}{t}\right) \text{ is}$$
(a) an ellipse (b) a circle
(c) a pair of lines (d) none of these

83. If the circle $C_1: x^2+y^2=16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, the coordinates of the centre of C_2 are
(a) $(-9/5, 12/5)$, $(9/5, -12/5)$
(b) $(-9/5, -12/5)$, $(9/5, 12/5)$
(c) $(12/5, -9/5)$, $(-12/5, 9/5)$
(d) none of these

84. The locus of the mid-point of the chord of the circle $x^2+y^2-2x-2y-2=0$, which makes an angle of 120° at the centre, is
(a) $x^2+y^2-2x-2y+1=0$ (b) $x^2+y^2+x+y-1=0$
(c) $x^2+y^2-2x-2y-1=0$ (d) none of these

85. The two circles $x^2+y^2-2x-3=0$ and $x^2+y^2-4x-6y-8=0$ are such that
(a) they touch each other
(b) they intersect each other
(c) one lies inside the other
(d) each lies outside the other

86. The equation of the circle having its centre on the line $x+2y-3=0$ and passing through the point of intersection of the circles
$x^2+y^2-2x-4y+1=0$ and $x^2+y^2-4x-2y+4=0$ is
(a) $x^2+y^2-6x+7=0$ (b) $x^2+y^2-3x+4=0$
(c) $x^2+y^2-2x-2y+1=0$ (d) $x^2+y^2+2x-4y+4=0$

87. The equation of the circumcircle of the triangle formed by the lines $y+\sqrt{3}x=6$, $y-\sqrt{3}x=6$ and $y=0$ is
(a) $x^2+y^2-4y=0$ (b) $x^2+y^2+4x=0$
(c) $x^2+y^2-4y-12=0$ (d) $x^2+y^2+4x=12$

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88. The equation $x^2+y^2+4x+6y+13=0$ represents
(a) a circle
(b) a pair of two straight lines
(c) a pair of coincident straight lines
(d) a point

89. To which of the following circles, the line $y-x+3=0$ is normal at the point $\left(3+\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$?
(a) $\left(x-3-\frac{3}{\sqrt{2}}\right)^2+\left(y-\frac{3}{\sqrt{2}}\right)^2=9$
(b) $\left(x-\frac{3}{\sqrt{2}}\right)^2+\left(y-\frac{3}{\sqrt{2}}\right)^2=9$
(c) $x^2+(y-3)^2=9$ (d) $(x-3)^2+y^2=9$

90. Circles are drawn through the point (2, 0) to cut intercepts of length 5 units on the x-axis. If their centres lie in the first quadrant, then their equation is
(a) $x^2+y^2-9x+2ky+14=0$
(b) $3x^2+3y^2+27x-2ky+42=0$
(c) $x^2+y^2-9x-2ky+14=0$
(d) $x^2+y^2-2kx-9y+14=0$

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91. The equation of the circle which touches both the axes and the straight line $4x + 3y = 6$ in the first quadrant and lies below it is
(a) $4x^2 + 4y^2 - 4x - 4y + 1 = 0$
(b) $x^2 + y^2 - 6x - 6y + 9 = 0$
(c) $x^2 + y^2 - 6x - y + 9 = 0$
(d) $4(x^2 + y^2 - x - 6y) + 1 = 0$

92. The slope of the tangent at the point (h, h) of the circle $x^2 + y^2 = a^2$, is
(a) 0 (b) 1 (c) -1 (d) depends on h

93. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if
(a) $r < 2$ (b) $r > 8$ (c) $2 < r < 8$ (d) $2 \le r \le 8$

94. The locus of the centre of a circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis is given by the equation
(a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$
(c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$.

95. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is
(a) $2ax + 2by - (a^2 + b^2 + p^2) = 0$
(b) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ **[AIEEE 2005]**

96. The locus of the mid-point of the chords of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is
(a) $x + y = 2$ (b) $x^2 + y^2 = 1$
(c) $x^2 + y^2 = 2$ (d) $x + y = 1$

97. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their point of intersection and the point $(1, 1)$ is
(a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$
(c) $x^2 + y^2 - 4y + 2 = 0$ (d) none of these

98. The equation of the circle described on the common chord of the circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 + 2y = 0$ as diameter is
(a) $x^2 + y^2 + x - y = 0$ (b) $x^2 + y^2 - x - y = 0$
(c) $x^2 + y^2 - x + y = 0$ (d) $x^2 + y^2 + x + y = 0$

99. Origin is a limiting point of a coaxial system of which $x^2 + y^2 - 6x - 8y + 1 = 0$ is a member. The other limiting point is
(a) $(-2, -4)$ (b) $(3/25, 4/25)$
(c) $(-3/25, -4/25)$ (d) $(4/25, 3/25)$

100. A circle passes through the origin and has its centre on $y = x$. If it cuts $x^2 + y^2 - 4x - 6y + 10 = 0$ orthogonally, then the equation of the circle is
(a) $x^2 + y^2 - x - y = 0$ (b) $x^2 + y^2 - 6x - 4y = 0$
(c) $x^2 + y^2 - 2x - 2y = 0$ (d) $x^2 + y^2 + 2x + 2y = 0$

101. The number of common tangents to the circles $x^2 + y^2 - x = 0, x^2 + y^2 + x = 0$ is
(a) 2 (b) 1 (c) 4 (d) 3

102. Consider the circles $x^2 + (y-1)^2 = 9$, $(x-1)^2 + y^2 = 25$. They are such that
(a) these circles touch each other
(b) one of these circles lies entirely inside the other
(c) each of these circles lies outside the other
(d) they intersect in two point

103. A circle touches the x-axis and also touches the circle with centre (0, 3) and radius 2. The locus of the centre of the circle is
(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

104. The circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 4x + 6y + 4 = 0$
(a) touch externally (b) touch internally
(c) intersect at two points (d) do not intersect

105. The equation of the unit circle concentric with $x^2 + y^2 . 8x + 4y - 8 = 0$ is
(a) $x^2 + y^2 - 8x + 4y - 8 = 0$
(b) $x^2 + y^2 - 8x + 4y + 8 = 0$
(c) $x^2 + y^2 - 8x + 4y - 28 = 0$
(d) $x^2 + y^2 - 8x + 4y + 19 = 0$

106. The point $(\sin\theta, \cos\theta)$, θ being any real number, lie inside the circle $x^2 + y^2 - 2x - 2y + \lambda = 0$, if
(a) $\lambda < 1 + 2\sqrt{2}$ (b) $\lambda > 2\sqrt{2} - 1$
(c) $\lambda < -1 - 2\sqrt{2}$ (d) $\lambda > 1 + 2\sqrt{2}$

107. The range of values of $\theta \in [0, 2\pi]$ for which $(1 + \cos\theta, \sin\theta)$ is on interior point of the circle $x^2 + y^2 = 1$, is
(a) $(\pi/6, 5\pi/6)$ (b) $(2\pi/3, 5\pi/3)$
(c) $(\pi/6, 7\pi/6)$ (d) $(2\pi/3, 4\pi/3)$

108. The range of values of a for which the point $(a, 4)$ is outside the circles $x^2 + y^2 + 10x = 0$ and $x^2 + y^2 - 12x + 20 = 0$, is
(a) $(-\infty, -8) \cup (-2, 6) \cup (6, \infty)$ (b) $(-8, -2)$
(c) $(-\infty, -8) \cup (-2, \infty)$ (d) none of these

109. If (α, β) is a point on the chord PQ of the circle $x^2 + y^2 = 25$, where the coordinates of P and Q are $(3, -4)$ and $(4, 3)$ respectively, then
(a) $3 \le \alpha \le 4$ and $-4 \le \beta \le 3$
(b) $-4 \le \alpha \le 3$ and $3 \le \beta \le 4$
(c) $\alpha = 3$ and $-4 \le \beta \le 4$
(d) none of these

110. If the point $(\lambda, \lambda + 1)$ lies in the interior of the region bounded by $y = \sqrt{25 - x^2}$ and x-axis, then λ lies in the interval
(a) $(-4, 3)$ (b) $(-\infty, -1) \cup (3, \infty)$
(c) $(-1, 3)$ (d) none of these

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111. If the point $\left(-5+\frac{\lambda}{\sqrt{2}}, -3+\frac{\lambda}{\sqrt{2}}\right)$ is an interior point of the larger segment of the circle $x^2+y^2=16$ cut off by the line $x+y=2$, then
(a) $\lambda \in (-\infty, 5\sqrt{2})$ (b) $\lambda \in (4\sqrt{2}-\sqrt{14}, 5\sqrt{2})$
(c) $\lambda \in (4\sqrt{2}-\sqrt{14}, 4\sqrt{2}+\sqrt{14})$ (d) none of these

112. The abscissae of two points A and B are the roots of the equation $x^2+2ax-b^2=0$, and their ordinates are the roots of the equation $x^2+2px-q^2=0$. The radius of the circle with AB as diameter is
(a) $\sqrt{a^2+p^2}$ (b) $\sqrt{b^2+q^2}$
(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2+b^2+p^2+q^2}$

113. Three sides of a triangle have the equations $L_r \equiv y-m_r x-c_r=0$, $r=1,2,3$. If λ, μ, ν are non-zero real numbers such that $\lambda L_2 L_3+\mu L_3 L_1+\nu L_1 L_2=0$ represents the circumcircle of the triangle, then
(a) $\lambda(m_2+m_3)+\mu(m_3+m_1)+\nu(m_1+m_2)=0$
(b) $\lambda(m_2 m_3-1)+\mu(m_3 m_1-1)+\nu(m_1 m_2-1)=0$
(c) both (a) and (b) hold together
(d) none of these

114. The equation to the circle having $y=mx$ as a diameter where $y=mx$ is a chord of the circle, through the origin, of radius a and having the x-axis as diameter is
(a) $(1+m^2)(x^2+y^2)-2a(x+my)=0$
(b) $(1-m^2)(x^2+y^2)-2a(x+my)=0$
(c) $(1+m^2)(x^2+y^2)+2a(x+my)=0$
(d) none of these

115. The straight lines joining the origin to the points of intersection of the line $4x+3y=24$ with the curve $(x-3)^2+(y-4)^2=25$
(a) are coincident (b) are perpendicular
(c) make equal angles with x-axis (d) none of these

116. The locus of the point of intersection of the tangents to the circle $x=r\cos\theta, y=r\sin\theta$ at points whose parametric angles differ by a right angle is
(a) $x^2+y^2=\frac{r^2}{2}$ (b) $x^2+y^2=2r^2$
(c) $x^2+y^2=4r^2$ (d) none of these

117. If the chord of contact of tangents from a point $P(x_1, y_1)$ to the circle $x^2+y^2=a^2$ touches the circle $(x-a)^2+y^2=a^2$, then the locus of (x_1, y_1), is
(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

118. The circle S_1 with centre $C_1(a_1, b_1)$ and radius r_1 touches externally the circle S_2 with centre $C_2(a_2, b_2)$ and radius r_2. If the tangent at their common point passes through the origin, then
(a) $(a_1^2+a_2^2)+(b_1^2+b_2^2)=r_1^2+r_2^2$
(b) $(a_1^2-a_2^2)+(b_1^2-b_2^2)=r_1^2-r_2^2$
(c) $(a_1^2-b_2^2)+(a_2^2+b_2^2)=r_1^2+r_2^2$
(d) $(a_1^2-b_1^2)+(a_2^2+b_2^2)=r_1^2+r_2^2$

119. Two vertices of an equilateral triangle are $(-1, 0)$ and $(1, 0)$ and its third vertex lies above the x-axis. The equation of its circumcircle, is
(a) $x^2+y^2-\frac{1}{\sqrt{3}}y-1=0$ (b) $x^2+y^2+\frac{2}{\sqrt{3}}y-1=0$
(c) $x^2+y^2-\frac{2}{\sqrt{3}}y-1=0$ (d) none of these

120. The sum of the coefficients in the expansion of $(\alpha^2x^2-2\alpha x+1)^{51}$, as a polynomial in x, vanishes. Position of the point $(\alpha, 2\alpha^2)$ with respect to the circle $x^2+y^2=4$, is
(a) outside (b) inside (c) on side (d) cannot be decided

121. Tangents PT_1 and PT_2 are drawn from a point P to the circle $x^2+y^2=a^2$. If the point P lies on the line $px+qy-r=0$, then the locus of the circumcircle of the triangle PT_1T_2 is
(a) $px+qy=r/2$ (b) $2px+2py+r=0$
(c) $px+qy=r$ (d) $(x-p)^2+(y-q)^2=r^2$

122. The value of θ in $[0, 2\pi]$ so that circle $x^2+y^2+2(\sin\alpha)x+2(\cos\alpha)y+\sin^2\theta=0$ always lies inside the square of unit side length, is/are
(a) $(\pi/3, 2\pi/3)$ (b) $[4\pi/3, 5\pi/3]$
(c) $(\pi/4, 2\pi/3)$ (d) none of these

123. The value of α in $[0, 2\pi]$ so that $x^2+y^2+2\sqrt{\sin\alpha}\,x+(\cos\alpha-1)=0$ having intercept on x-axis always greater than 2 is/are
(a) $(\pi/4, 3\pi/2]$ (b) $(\pi/4, \pi]$
(c) $(\pi/4, 5\pi/4)$ (d) $[0, \pi]$

124. If in a ΔABC (whose circumcentre is at the origin), $a \le \sin A$, then for any point (x, y) inside the circumcircle of ΔABC, we have
(a) $|xy| < 1/8$ (b) $|xy| > 1/8$
(c) $1/8 < xy < 1/2$ (d) none of these

125. If P is a point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2+y^2+2x-4y-20=0$ and $x^2+y^2-4x+2y-44=0$ is 2:3, then the locus of P is a circle with centre
(a) $(7, -8)$ (b) $(-7, 8)$ (c) $(7, 8)$ (d) $(-7, -8)$
[EAMCET 2003]

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Answers

1. (b) 2. (d) 3. (d) 4. (c) 5. (c) 6. (c) 7. (b)
8. (b) 9. (d) 10. (a) 11. (b) 12. (a) 13. (d) 14. (b)
15. (b) 16. (c) 17. (c) 18. (a) 19. (b) 20. (c) 21. (b)
22. (a) 23. (a) 24. (c) 25. (b) 26. (a) 27. (a) 28. (c)
29. (b) 30. (c) 31. (d) 32. (b) 33. (c) 34. (c) 35. (c)
36. (a) 37. (c) 38. (d) 39. (a) 40. (a) 41. (a) 42. (b)
43. (d) 44. (a) 45. (d) 46. (d) 47. (a) 48. (c) 49. (c)
50. (b) 51. (c) 52. (d) 53. (a) 54. (c) 55. (c) 56. (b)
57. (c) 58. (a) 59. (b) 60. (d) 61. (a) 62. (b) 63. (d)
64. (a) 65. (b) 66. (b) 67. (d) 68. (c) 69. (a) 70. (a)
71. (a) 72. (c) 73. (b) 74. (c) 75. (d) 76. (a) 77. (b)
78. (c) 79. (a) 80. (b) 81. (c) 82. (d) 83. (a) 84. (a)
85. (b) 86. (a) 87. (c) 88. (d) 89. (d) 90. (c) 91. (a)
92. (c) 93. (c) 94. (d) 95. (a) 96. (c) 97. (b) 98. (d)
99. (b) 100. (c) 101. (d) 102. (b) 103. (b) 104. (c) 105. (d)
106. (c) 107. (d) 108. (a) 109. (a) 110. (c) 111. (b) 112. (d)
113. (c) 114. (a) 115. (b) 116. (b) 117. (b) 118. (b) 119. (c)
120. (a) 121. (a) 122. (d) 123. (b) 124. (a) 125. (b)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The locus of the centre of the circle which cuts the circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ orthogonally, is
   (a) an ellipse (b) the radical axis of the given circles
   (c) a conic (d) another circle

2. Given that the circles $x^2+y^2-2x+6y+6=0$ and $x^2+y^2-5x+6y+15=0$ touch, the equation to their common tangent is
   (a) $x=3$ (b) $y=6$
   (c) $7x-12y-21=0$ (d) $7x+12y+21=0$

3. The two circles $x^2+y^2-2x-2y-7=0$ and $3(x^2+y^2)-8x+29y=0$
   (a) touch externally (b) touch internally
   (c) cut each other orthogonally (d) do not cut each other

4. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$, is
   (a) (3/2, 1/2) (b) (1/2, 3/2)
   (c) (1/2, 1/2) (d) $(1/2, \pm\sqrt{2})$

5. The circle $x^2+y^2=4$ cuts the circle $x^2+y^2+2x+3y-5=0$ in A and B. Then the equation of the circle on AB as diameter is
   (a) $13(x^2+y^2)-4x-6y-50=0$
   (b) $9(x^2+y^2)+8x-4y+25=0$
   (c) $x^2+y^2-5x+2y+72=0$
   (d) none of these

6. One of the limit point of the coaxial system of circles containing $x^2+y^2-6x-6y+4=0$, $x^2+y^2-2x-4y+3=0$, is
   (a) (−1, 1) (b) (−1, 2) (c) (−2, 1) (d) (−2, 2)

7. A circle touches y-axis at (0, 2) and has an intercept of 4 units on the positive side of x-axis. The equation of the circle is
   (a) $x^2+y^2-4(\sqrt{2}x+y)+4=0$
   (b) $x^2+y^2-4(x+\sqrt{2}y)+4=0$
   (c) $x^2+y^2-2(\sqrt{2}x+y)+4=0$
   (d) none of these

8. The equation of the circle whose one diameter is PQ, where the ordinates of P, Q are the roots of the equation $x^2+2x-3=0$ and the abscissae are the roots of the equation $y^2+4y-12=0$, is
   (a) $x^2+y^2+2x+4y-15=0$ (b) $x^2+y^2-4x-2y-15=0$
   (c) $x^2+y^2+4x+2y-15=0$ (d) none of these

9. The length of the chord of the circle $x^2+y^2+4x-7y+2=0$ along the y-axis, is
   (a) 1 (b) 2 (c) 1/2 (d) none of these

10. The equation of any tangent to the circle $x^2+y^2-2x+4y-4=0$, is
    (a) $y=m(x-1)^2+3\sqrt{1+m^2}-2$ (b) $y=mx+3\sqrt{1+m^2}$
    (c) $y=mx+3\sqrt{1+m^2}-2$ (d) none of these

11. The angle between the pair of tangents drawn from the point (1, 1/2) to the circle $x^2+y^2+4x+2y-4=0$, is
    (a) $\cos^{-1}\frac{4}{5}$ (b) $\sin^{-1}\frac{4}{5}$ (c) $\sin^{-1}\frac{3}{5}$ (d) none of these

12. The intercept on the line $y=x$ by the circle $x^2+y^2-2x=0$ is AB. The equation of the circle with AB as diameter is
    (a) $x^2+y^2+x+y=0$ (b) $x^2+y^2=x+y$
    (c) $x^2+y^2-3x+y=0$ (d) none of these

13. If $3x+y=0$ is tangent to the circle having its centre at (2, −1), then the equation of other tangent to the circle from the origin is
    (a) $x-3y=0$ (b) $x+3y=0$
    (c) $3x-y=0$ (d) $2x+y=0$

14. The locus of the middle points of chords of length 4 of the circle $x^2+y^2=16$ is
    (a) a straight line (b) a circle of radius 2
    (c) a circle of radius $2\sqrt{3}$ (d) an ellipse

15. Two tangents to the circle $x^2+y^2=4$ at the points A and B meet at $P(-4, 0)$. The area of the quadrilateral $PAOB$, where O is the origin, is

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(a) 4 (b) $6\sqrt{2}$ (c) $4\sqrt{3}$ (d) none of these

16. A tangent is drawn to the circle $2(x^2+y^2)-3x+4y=0$ and it touches the circle at point A. If the tangent passes through the point $P(2, 1)$, then PA =
(a) 4 (b) 2 (c) $2\sqrt{2}$ (d) none of these

17. The length of the chord of the circle $x^2+y^2=25$ passing through $(5, 0)$ and perpendicular to the line $x+y=0$ is
(a) $5\sqrt{2}$ (b) $5/\sqrt{2}$ (c) $2\sqrt{5}$ (d) none of these

18. If the points $A(2, 5)$ and B are symmetrical about the tangent to the circle $x^2+y^2-4x+4y=0$ at the origin, then the coordinates of B are
(a) $(5, -2)$ (b) $(1, 5)$ (c) $(5, 2)$ (d) none of these

19. The equation of the circle of radius $2\sqrt{2}$ whose centre lies on the line $x-y=0$ and which touches the line $x+y=4$, and whose centre is coordinate satisfy $x+y>4$, is
(a) $x^2+y^2-8x-8y+24=0$ (b) $x^2+y^2=8$
(c) $x^2+y^2-8x+8y-24=0$ (d) none of these

20. The maximum number of points with rational coordinates on a circle whose centre is $(\sqrt{3}, 0)$ is
(a) one (b) two (c) four (d) infinite

21. The equation of a circle C is $x^2+y^2-6x-8y-11=0$. The number of real points at which the circle drawn with points $(1, 8)$ and $(0, 0)$ as the ends of a diameter cuts the circle, C is
(a) 0 (b) 1 (c) 2 (d) none of these

22. Two circles, each of radius 5, have a common tangent at $(1, 1)$ whose equation is $3x+4y-7=0$. Then their centres are
(a) $(4, -5), (-2, 3)$ (b) $(4, -3), (-2, 5)$
(c) $(4, 5), (-2, -3)$ (d) none of these

23. The number of points on the circle $2(x^2+y^2)=3x$ which are at a distance 2 from the point $(-2, 1)$ is
(a) 2 (b) 0 (c) 1 (d) none of these

24. A ray of light incident at the point $(-2, -1)$ gets reflected from the tangent at $(0, -1)$ to the circle $x^2+y^2=1$. The reflected ray touches the circle. The equation of the line along which the incident ray moved is
(a) $4x-3y+11=0$ (b) $4x+3y+11=0$
(c) $3x+4y+11=0$ (d) none of these

25. The point on the straight line $y=2x+11$ which is nearest to the circle $16(x^2+y^2)+32x-8y-50=0$, is
(a) $(9/2, 2)$ (b) $(-9/2, 2)$
(c) $(9/2, -2)$ (d) none of these

26. Extremities of a diagonal of a rectangle are $(0, 0)$ and $(4, 3)$. The equations of the tangents to the circumcircle of the rectangle which are parallel to the diagonal are
(a) $16x+8y\pm25=0$ (b) $6x-8y\pm25=0$
(c) $8x+6y\pm25=0$ (d) none of these

27. The equation of the circle which has a tangent $2x-y-1=0$ at $(3, 5)$ on it and with the centre on $x+y=5$, is
(a) $x^2+y^2+6x-16y+28=0$
(b) $x^2+y^2-6x+16y-28=0$
(c) $x^2+y^2+6x+6y-28=0$
(d) $x^2+y^2-6x-6y-28=0$

28. The angle of intersection of the circles $x^2+y^2=4$ and $x^2+y^2=2x+2y$, is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/4$

29. The normal at the point $(3, 4)$ on a circle at the point $(-1, -2)$. The equation of the circle, is
(a) $x^2+y^2+2x-2y-13=0$
(b) $x^2+y^2-2x-2y-11=0$
(c) $x^2+y^2-2x+2y+12=0$
(d) $x^2+y^2-2x-2y+14=0$

30. The inverse point of $(1, -1)$ with respect to $x^2+y^2=4$ is
(a) $(-1, 1)$ (b) $(-2, 2)$ (c) $(1, -1)$ (d) $(2, -2)$

31. A variable circle passes through the fixed point $(2, 0)$ and touches the y-axis. Then, the locus of its centre, is
(a) a parabola (b) a circle
(c) an ellipse (d) a hyperbola

32. The radius of the circle $r^2-2\sqrt{2}r(\cos\theta+\sin\theta)-5=0$, is
(a) 9 (b) 5 (c) 3 (d) 2

33. A straight rod of length 9 units with its ends A, B always on x and y axes respectively. Then, the locus of the centroid of ΔOAB, is
(a) $x^2+y^2=3$ (b) $x^2+y^2=9$
(c) $x^2+y^2=1$ (d) $x^2+y^2=81$

34. The radius of the larger circle lying in the first quadrant and touching the line $4x+3y-12=0$ and the coordinate axes, is
(a) 5 (b) 6 (c) 7 (d) 8

35. A line is at a constant distance c from the origin and meets the coordinate axes in A and B. The locus of the centre of the circle passing through O, A, B is
(a) $x^{-2}+y^{-2}=c^{-2}$ (b) $x^{-2}+y^{-2}=2c^{-2}$
(c) $x^{-2}+y^{-2}=3c^{-2}$ (d) $x^{-2}+y^{-2}=4c^{-2}$

36. The number of circles that touch all the straight lines $x+y-4=0$, $x-y+2=0$ and $y=2$, is
(a) 1 (b) 2 (c) 3 (d) 4

37. The number of integral values of 'a' for which the radius of the circle $x^2+y^2+ax+(1-a)y+5=0$ cannot exceed 5, is
(a) 14 (b) 18 (c) 16 (d) none of these

38. If the four points of the intersection of the lines $2x-y+11=0$ and $x-2y+3=0$ with the axes lie on a circle, then the coordinates of the centre of the circle are
(a) $(7/5, 5/2)$ (b) $(7/4, 5/4)$
(c) $(-7/4, 5/4)$ (d) $(7/4, -5/4)$

39. If $2x+3y-6=0$ and $9x+6y-18=0$ cuts the axes in concyclic points, then the centre of the circle, is
(a) $(2, 3)$ (b) $(3, 2)$ (c) $(5, 5)$ (d) $(5/2, 5/2)$

40. If the line $lx+my+n=0$ intersects the curve $ax^2+2hxy+by^2=1$ at P and Q such that the circle with

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PQ as a diameter passes through the origin, then $l^2 + m^2 =$.

(a) $n^2(a+b)$ (b) $n^2(a+b)^2$ (c) $n^2(a^2-b^2)$ (d) $n^2(a^2+b^2)$

41. Two circles, each of radius 5, have a common tangent at $(1, 1)$ whose equation is $3x + 4y - 7 = 0$. Then, their centres are
(a) $(4, -5), (-2, 3)$ (b) $(4, -3), (-2, 5)$
(c) $(4, 5), (-2, -3)$ (d) none of these

42. PQ is a chord of the circle $x^2 + y^2 - 2x - 8 = 0$ whose mid-point is $(2, 2)$. The circle passing through P, Q and $(1, 2)$ is
(a) $x^2 + y^2 - 7x + 10y + 28 = 0$ (b) $x^2 + y^2 - 7x - 10y + 22 = 0$
(c) $x^2 + y^2 - 7x - 10y + 22 = 0$ (d) $x^2 + y^2 + 7x + 10y - 22 = 0$

43. The number of circles belonging to the system of circles $2(x^2 + y^2) + \lambda x - (1 + \lambda^2) y - 10 = 0$ and orthogonal to $x^2 + y^2 + 4x + 6y + 3 = 0$, is
(a) 2 (b) 1 (c) 0 (d) none of these

44. The equation of the circle passing through $(0, 0)$ and belonging to the system of circles of which $(3, 1)$ and $(-1, 5)$ are limiting points, is
(a) $x^2 + y^2 - x + 3y = 0$ (b) $x^2 + y^2 - 11x + 3y = 0$
(c) $x^2 + y^2 = 1$ (d) none of these

45. If $P(-1, -3)$ is a centre of similitude for the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x - 6y + 6 = 0$, then the length of the common tangent through P to the circles is
(a) 2 (b) 3 (c) 4 (d) 5

46. If $P(1, 1/2)$ is a centre of similitude for the circles $x^2 + y^2 + 4x + 2y - 4 = 0$ and $x^2 + y^2 - 4x - 2y + 4 = 0$, then the length of the common tangent through P to the circles is
(a) 4 (b) 3 (c) 2 (d) 1

47. The equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents a system of circles, λ being a parameter, passing through two fixed points P and Q. The circle on PQ as a diameter, is
(a) $x^2 + y^2 - 2y = 0$ (b) $x^2 + y^2 - 2x - 8 = 0$
(c) $x^2 + y^2 - 2y = 8$ (d) $x^2 + y^2 - 2x - 2y = 8$

48. $x = 1$ is the radical axis of the two orthogonally intersecting circles. If $x^2 + y^2 = 4$ is one of the circles, then the other circle is
(a) $x^2 + y^2 - 4x + 4 = 0$ (b) $x^2 + y^2 - 8x + 4 = 0$
(c) $x^2 + y^2 + 8x - 4 = 0$ (d) none of these

49. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure $45°$ at the major segment of the circle, then the value of m is
(a) 2 (b) -2 (c) -1 (d) none of these

50. The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$
(a) cut orthogonally (b) touch each other internally
(c) intersect in two points
(d) touch each other externally

51. The equation of the pair of straight lines parallel to x-axis and touching the circle $x^2 + y^2 - 6x - 4y - 12 = 0$, is
(a) $y^2 - 4y - 21 = 0$ (b) $y^2 + 4y - 21 = 0$
(c) $y^2 - 4y + 21 = 0$ (d) $y^2 + 4y + 21 = 0$

52. The equation of the circumcircle of the triangle formed by the lines $x = 0$, $y = 0$, $2x + 3y = 5$, is
(a) $6(x^2 + y^2) + 5(3x - 2y) = 0$
(b) $x^2 + y^2 - 2x - 3y + 5 = 0$
(c) $x^2 + y^2 + 2x - 3y - 5 = 0$
(d) $6(x^2 + y^2) - 5(3x + 2y) = 0$

53. The value of λ for which the circle $x^2 + y^2 + 2\lambda x + 6y + 1 = 0$ intersects the circle $x^2 + y^2 + 4x + 2y = 0$ orthogonally, is
(a) $\frac{11}{8}$ (b) -1 (c) $\frac{-5}{4}$ (d) $\frac{5}{2}$

54. The equation of the circle concentric to the circle $2x^2 + 2y^2 - 3x + 6y + 2 = 0$ and having double the area of this circle, is
(a) $8x^2 + 8y^2 - 24x + 48y - 13 = 0$
(b) $16x^2 + 16y^2 + 24x - 48y - 13 = 0$
(c) $16x^2 + 16y^2 - 24x + 48y - 13 = 0$
(d) $8x^2 + 8y^2 + 24x - 48y - 13 = 0$

55. The circles $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ intersect at an angle
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

56. The equation of the image of the circle $(x-3)^2 + (y-2) = 1$ in the mirror $x + y = 19$, is
(a) $(x-14)^2 + (y-13)^2 = 1$ (b) $(x-15)^2 + (y-14)^2 = 1$
(c) $(x-16)^2 + (y-15)^2 = 1$ (d) $(x-17)^2 + (y-16^2) = 1$

Answers

1. (b) 2. (a) 3. (c) 4. (d) 5. (a) 6. (a) 7. (a)
8. (a) 9. (a) 10. (a) 11. (b) 12. (b) 13. (a) 14. (c)
15. (c) 16. (b) 17. (a) 18. (c) 19. (a) 20. (b) 21. (c)
22. (c) 23. (b) 24. (b) 25. (b) 26. (b) 27. (a) 28. (d)
29. (b) 30. (d) 31. (a) 32. (c) 33. (b) 34. (b) 35. (d)
36. (d) 37. (c) 38. (c) 39. (d) 40. (a) 41. (c) 42. (b)
43. (a) 44. (b) 45. (b) 46. (c) 47. (b) 48. (b) 49. (c)
50. (d) 51. (a) 52. (d) 53. (c) 54. (c) 55. (d) 56. (d)

Solutions of Exercises and Chapter-tests are available in a separate book on "Solutions of Objective Mathematics".

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24

PARABOLA

1. CONIC SECTIONS

A conic section, as the name implies, is a section cut off from a circular (not necessary a right circular) cone by a plane in various ways. The shape of the section depends upon the position of the cutting plane.

Consider a double right circular cone of semi-vertical angle α and let it be cut by a plane inclined at an angle θ to the axis of the cone. We will get different sections (curves) as follows:

CASE I *If the plane passes through the vertex O*

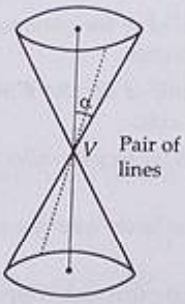


Fig. 1

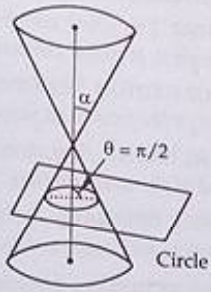


Fig. 2

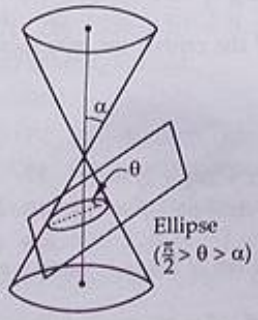


Fig. 3

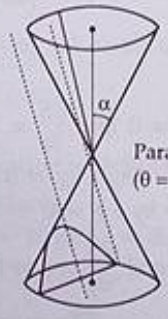


Fig. 4

The curve of intersection is a pair of straight lines passing through the vertex which are

(i) real and distinct for $\theta < \alpha$
(ii) coincident for $\theta = \alpha$ i.e. the plane touches the cone
(iii) imaginary for $\theta > \alpha$

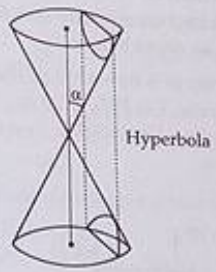


Fig. 5

CASE II *If the plane does not pass through the vertex O:*

The curve of intersection is called

(i) a circle if $\theta = \pi/2$
(ii) a parabola for $\theta = \alpha$ i.e., if the plane is parallel to the generator PQ
(iii) an ellipse for $\theta > \alpha$ $(\theta \neq \pi/2)$ i.e., if the plane cuts both the generating lines PQ and RS.
(iv) a hyperbola for $\theta < \alpha$ i.e., if the plane cuts both the cones.

1.1 ANALYTICAL DEFINITION

DEFINITION *A conic section or a conic is the locus of a point which moves in such a way that its distance from a fixed point always bears a constant ratio to its distance from a fixed line, all being in the same plane.*

Thus, if S is a fixed point in the plane of the paper and ZZ' is a fixed line in the same plane, then the locus of a point P which moves in the same plane in such a way that

$$\frac{SP}{PM} = \text{Constant} = e \text{ (say)}$$

is called a conic section or simply a conic.

The fixed point is called the focus of the conic section and the fixed line is known as its directrix. The constant ratio 'e' is called the eccentricity of the conic section

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Statement–2: The locus of the point of intersection of perpendicular tangents to a parabola is its directrix.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Statement–2 is true (see Theorem2 on page 24.17). Statement–1 is also true, because given tangents are drawn from the point $(-a, -b)$ which is a point on the directrix. Also, statement–2 is a correct explanation for statement–1.

EXAMPLE 7 *Given : A circle* $2x^2 + 2y^2 = 5$ *and a parabola* $y^2 = 4\sqrt{5}\,x$

Statement–1: An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement–2: If the line $y = mx + \frac{\sqrt{5}}{m}$, $m \neq 0$, *is their common tangent, then m satisfies* $m^4 - 3m^2 + 2 = 0$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b) **[JEE (Main) 2013]**

SOLUTION The equation of a tangent to $y^2 = 4\sqrt{5}x$ is

$y = mx + \frac{\sqrt{5}}{m}$, where m is the slope of the tangent.

If it touches the circle $2x^2 + 2y^2 = 5$, then

$$\left|\frac{\sqrt{5}/m}{\sqrt{1+m^2}}\right| = \sqrt{\frac{5}{2}}$$

$$\Rightarrow \quad m\sqrt{1+m^2} = \sqrt{2}$$

$$\Rightarrow \quad m^4 + m^2 - 2 = 0$$

$$\Rightarrow \quad (m^2+2)(m^2-1) = 0$$

$$\Rightarrow \quad m = \pm 1$$

Substituting these values in $y = mx + \frac{\sqrt{5}}{m}$, the equations of common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$. Also, values of m satisfy $m^4 - 3m^2 + 2 = 0$. Hence, both the statements are true. But, statement II is not a correct explanation of statement-I.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. If the focus and vertex of a parabola are the points (0, 2) and (0, 4) respectively, then its equation is
   (a) $y^2 = 8x + 32$ (b) $y^2 = -8x + 32$
   (c) $x^2 + 8y = 32$ (d) $x^2 - 8y = 32$
2. The equation of the directrix of the parabola $x^2 - 4x - 3y + 10 = 0$, is
   (a) $y = -\frac{5}{4}$ (b) $y = \frac{5}{4}$ (c) $y = -\frac{3}{4}$ (d) $x = \frac{5}{4}$
3. If the vertex of a parabola is the point $(-3, 0)$ and the directrix is the line $x + 5 = 0$, then its equation is
   (a) $y^2 = 8(x+3)$ (b) $x^2 = 8(y+3)$
   (c) $y^2 = -8(x+3)$ (d) $y^2 = 8(x+5)$
4. The angle made by a double ordinate of length $8a$ at the vertex of the parabola $y^2 = 4ax$, is
   (a) $\pi/3$ (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/6$
5. The coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4, are
   (a) $(1/2, \pm 2)$ (b) $(1, \pm 2\sqrt{2})$
   (c) $(2, \pm 4)$ (d) none of these
6. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ whose one vertex is at the vertex of the parabola. The length of its side is
   (a) $4a\sqrt{3}$ (b) $2a\sqrt{3}$ (c) $16a\sqrt{3}$ (d) $8a\sqrt{3}$
7. The coordinates of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are
   (a) (0, 2) (b) (2, 1) (c) (1, 2) (d) (–2, –1)
8. If y_1, y_2 and y_3 are the ordinates of the vertices of a triangle inscribed in the parabola $y^2 = 4ax$, then its area is
   (a) $\frac{1}{2a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$
   (b) $\frac{1}{4a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$
   (c) $\frac{1}{8a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$
   (d) none of these
9. The area of the triangle inscribed in the parabola $y^2 = 4x$ the ordinates of whose vertices are 1, 2 and 4 is
   (a) 7/2 sq.units (b) 5/2 sq. units
   (c) 3/2 sq. units (d) 3/4 sq. units
10. The length of the latusrectum of the parabola whose focus is (3, 3) and directrix is $3x - 4y - 2 = 0$, is
    (a) 2 (b) 1 (c) 4 (d) none of these
11. The length of the latusrectum of the parabola whose focus is $\left(\frac{u^2}{2g}\sin 2\alpha, -\frac{u^2}{2g}\cos 2\alpha\right)$ and directrix is $y = \frac{u^2}{2g}$ is
    (a) $\frac{u^2}{g}\cos^2\alpha$ (b) $\frac{u^2}{g}\cos 2\alpha$
    (c) $\frac{2u^2}{g}\cos 2\alpha$ (d) $\frac{2u^2}{g}\cos^2\alpha$
12. The locus of the points of trisection of the double ordinates of the parabola $y^2 = 4ax$ is
    (a) $y^2 = ax$ (b) $9y^2 = 4ax$ (c) $9y^2 = ax$ (d) $y^2 = 9ax$
13. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then
    (a) $4al + n = 0$ (b) $4al + 4am + n = 0$
    (c) $4am + n = 0$ (d) $al + n = 0$
14. The length of the chord of the parabola $y^2 = 4ax$ passing through the vertex and making an angle θ with the axis is
    (a) $4a\,\mathrm{cosec}^2\theta$ (b) $4a\cos\theta\,\mathrm{cosec}^2\theta$

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(c) $4a\cot\theta\,\text{cosec}^2\theta$ (d) $2a\,\text{cosec}^2\theta$

15. The parametric representation $(2+t^2, 2t+1)$ represents
(a) a parabola with focus at (2, 1)
(b) a parabola with vertex at (2, 1)
(c) an ellipse with centre at (2, 1)
(d) none of these

16. The ratio in which the line segment joining the points (4, – 6) and (3, 1) is divided by the parabola $y^2 = 4x$ is
(a) $\frac{-20\pm\sqrt{155}}{11}:1$ (b) $\frac{-2\pm2\sqrt{155}}{11}:1$
(c) $-20\pm2\sqrt{155}:11$ (d) $-20\pm\sqrt{155}:11$

17. If (a, b) is the mid point of a chord passing through the vertex of the parabola $y^2 = 4x$, then
(a) $a = 2b$ (b) $2a = b$ (c) $a^2 = 2b$ (d) $2a = b^2$

18. If the vertex and focus of a parabola are (3, 3) and (–3, 3) respectively, then its equation is
(a) $x^2 + 6x - 24y + 63 = 0$ (b) $x^2 - 6x + 24y - 63 = 0$
(c) $y^2 - 6y + 24x - 63 = 0$ (d) $y^2 + 6y - 24x + 63 = 0$

19. The locus of the middle points of the focal chord of the parabola $y^2 = 4ax$ is
(a) $y^2 = a(x-a)$ (b) $y^2 = 2a(x-a)$
(c) $y^2 = 4a(x-a)$ (d) none of these

20. If y_1, y_2 are the ordinates of two points P and Q on the parabola and y_3 is the ordinate of the point of intersection of tangents at P and Q, then
(a) y_1, y_2, y_3 are in AP (b) y_1, y_3, y_2 are in AP
(c) y_1, y_2, y_3 are in GP (d) y_1, y_3, y_2 are in GP

21. If the line $x + y = 1$ touches the parabola $y^2 - y + x = 0$, then the coordinates of the point of contact are
(a) (1, 1) (b) (1/2, 1/2) (c) (0, 1) (d) (1, 0)

22. The locus of the foot of the perpendicular from the focus upon a tangent to the parabola $y^2 = 4ax$ is
(a) the directrix (b) tangent at the vertex
(c) $x = a$ (d) none of these

23. The equation to the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is
(a) $x + 2y + 4 = 0$ (b) $2x + y - 4 = 0$
(c) $x - 2y - 4 = 0$ (d) $x - 2y + 4 = 0$

24. If t is the parameter for one end of a focal chord of the parabola $y^2 = 4ax$, then its length is
(a) $a\left(t+\frac{1}{t}\right)^2$ (b) $a\left(t-\frac{1}{t}\right)^2$ (c) $a\left(t+\frac{1}{t}\right)$ (d) $a\left(t-\frac{1}{t}\right)$
[JEE (WB) 2008]

25. The slope of the normal at the point $(at^2, 2at)$ of the parabola, $y^2 = 4ax$, is
(a) $1/t$ (b) t (c) $-t$ (d) $-1/t$

26. The normal at $(ap^2, 2ap)$ on $y^2 = 4ax$, meets the curve again at $(aq^2, 2aq)$ then
(a) $p^2 + pq + 2 = 0$ (b) $p^2 - pq + 2 = 0$
(c) $q^2 + pq + 2 = 0$ (d) $p^2 + pq + 1 = 0$

27. The length of the subnormal to the parabola $y^2 = 4ax$ at any point is equal to
(a) $a\sqrt{2}$ (b) $2\sqrt{2}a$ (c) $a/\sqrt{2}$ (d) 2a

28. The two parabolas $y^2 = 4x$ and $x^2 = 4y$ intersect at a point P, whose abscissae is not zero, such that
(a) they both touch each other at P
(b) they cut at right angles at P
(c) the tangents to each curve at P make complementary angles with the x-axis
(d) none of these

29. A set of parallel chords of the parabola $y^2 = 4ax$ have their mid-points on
(a) any straight line through the vertex
(b) any straight line through the focus
(c) a straight line parallel to the axis
(d) another parabola

30. The point on the curve $y^2 = ax$ the tangent at which makes an angle of 45° with x-axis will be given by
(a) $(a/2, a/4)$ (b) $(-a/2, a/4)$
(c) $(a/4, a/2)$ (d) $(-a/4, a/2)$

31. If $2x + y + k = 0$ is a normal to the parabola $y^2 = -8x$, then the value of k is
(a) –16 (b) – 8 (c) –24 (d) 24

32. If the parabolas $y^2 = 4x$ and $x^2 = 32y$ intersect at (16, 8) at an angle θ, then θ is equal to
(a) $\tan^{-1}(3/5)$ (b) $\tan^{-1}(4/5)$ (c) π (d) $\pi/2$

33. The normal at $(a, 2a)$ on $y^2 = 4ax$, meets the curve again at $(at^2, 2at)$, then the value of t is
(a) 1 (b) 3 (c) –1 (d) – 3

34. If a chord which is normal to the parabola $y^2 = 4ax$ at one end subtends a right angle at the vertex, then its slope is
(a) 1 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) 2

35. The equations of the normals at the ends of the latus rectum of the parabola $y^2 = 4ax$ are given by
(a) $x^2 - y^2 - 6ax + 9a^2 = 0$
(b) $x^2 - y^2 - 6ax - 6ay + 9a^2 = 0$
(c) $x^2 - y^2 - 6ay + 9a^2 = 0$
(d) none of these

36. The normal at the point $P(ap^2, 2ap)$ meets the parabola $y^2 = 4ax$ again at $Q(aq^2, 2aq)$ such that the lines joining the origin to P and Q are at right angle. Then,
(a) $p^2 = 2$ (b) $q^2 = 2$ (c) $p = 2q$ (d) $q = 2p$

37. If the normals at points 't_1' and 't_2' to the parabola $y^2 = 4ax$ meet on the parabola, then
(a) $t_1 t_2 = -1$ (b) $t_2 = -t_1 - \frac{2}{t_1}$
(c) $t_1 t_2 = 2$ (d) none of these
[AIEEE 2003]

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38. If the normals at two points P and Q of a parabola $y^2 = 4ax$ intersect at a third point R on the curve, then the product of ordinates of P and Q, is
(a) $4a^2$ (b) $2a^2$ (c) $-4a^2$ (d) $8a^2$

39. The angle between the tangents drawn from the origin to the parabola $y^2 = 4a(x-a)$, is
(a) 90° (b) 30° (c) $\tan^{-1}(1/2)$ (d) 45°

40. The angle between the tangents drawn from a point $(-a, 2a)$ to $y^2 = 4ax$, is
(a) $\pi/4$ (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/6$

41. The angle between tangents to the parabola $y^2 = 4ax$ at the points where it intersects with the line $x - y - a = 0$, is
(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/2$

42. If $P(-3, 2)$ is one end of the focal chord PQ of the parabola $y^2 + 4x + 4y = 0$, then the slope of the normal at Q is
(a) $-1/2$ (b) 2 (c) 1/2 (d) –2

43. If $x = my + c$ is a normal to the parabola $x^2 = 4ay$, then the value of c is
(a) $-2am - am^3$ (b) $2am + am^3$
(c) $-\frac{2a}{m} - \frac{a}{m^3}$ (d) $\frac{2a}{m} + \frac{a}{m^3}$

44. The equation of the normal at the point of contact of a tangent $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$, is
(a) $y = mx - 2am - am^3$ (b) $m^3y = m^2x - 2am^2 - a$
(c) $m^3y = 2am^2 - m^2x + a$ (d) none of these

45. If $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are two variable points on the curve $y^2 = 4ax$ and PQ subtends a right angle at the vertex, then $t_1 t_2$ is equal to
(a) –1 (b) –2 (c) –3 (d) –4

46. If the vertex of the parabola $y = x^2 - 8x + c$ lies on x-axis, then the value of c is
(a) –16 (b) –4 (c) 4 (d) 16

47. If the chord $y = mx + c$ subtends a right angle at the vertex of the parabola $y^2 = 4ax$, then the value of c is
(a) –4am (b) 4am (c) –2am (d) 2am

48. The equation of the tangent at the vertex of the parabola $x^2 + 4x + 2y = 0$ is
(a) $x = -2$ (b) $x = 2$ (c) $y = 2$ (d) $y = -2$

49. The locus of the point of intersection of perpendicular tangents to the parabola $x^2 = 4ay$ is
(a) $y = a$ (b) $y = -a$ (c) $x = a$ (d) $x = -a$

50. If $y = 2x + 3$ is a tangent to the parabola, $y^2 = 24x$, then its distance from the parallel normal is
(a) $5\sqrt{5}$ (b) $10\sqrt{5}$ (c) $15\sqrt{5}$ (d) $3\sqrt{5}$

51. If the normal at (1, 2) on the parabola $y^2 = 4x$ meets the parabola again at the point $(t^2, 2t)$, then the value of t is
(a) 1 (b) 3 (c) –3 (d) 1

52. The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h, k). The centroid of triangle PQR lies on
(a) $x = 0$ (b) $y = 0$ (c) $x = -a$ (d) $y = a$
[PET (MP) 1999]

53. If the point $P(4, -2)$ is the one end of the focal chord PQ of the parabola $y^2 = x$, then the slope of the tangent at Q is
(a) $-1/4$ (b) 1/4 (c) 4 (d) –4

54. If PSQ is the focal chord of the parabola $y^2 = 8x$ such that $SP = 6$. Then, the length SQ is
(a) 6 (b) 4 (c) 3 (d) none of these

55. The angle between the normals to the parabola $y^2 = 24x$ at points (6, 12) and (6, –12), is
(a) 30° (b) 45° (c) 60° (d) 90°

56. The common tangent to the parabola $y^2 = 4ax$ and $x^2 = 4ay$, is
(a) $x + y + a = 0$ (b) $x + y - a = 0$
(c) $x - y + a = 0$ (d) $x - y - a = 0$

57. The equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and parabola $y^2 = 4x$ above x-axis is
(a) $\sqrt{3}y = 3x + 1$ (b) $\sqrt{3}y = -x - 3$
(c) $\sqrt{3}y = x + 3$ (d) $\sqrt{3}y = -3x - 1$
[JEE (WB) 2007]

58. The length of the subtangent to the parabola $y^2 = 16x$ at the point whose abscissa is 4, is
(a) 2 (b) 4 (c) 8 (d) none of these

59. If P is a point on the parabola $y^2 = 4ax$ such that the subtangent and subnormal at P are equal, then the coordinates of P are
(a) $(a, 2a)$ or $(a, -2a)$ (b) $(2a, 2\sqrt{2}a)$ or $(2a, -2\sqrt{2}a)$
(c) $(4a, -4a)$ or $(4a, 4a)$ (d) none of these

60. The normal to the parabola $y^2 = 8x$ at the point (2, 4) meets the parabola again at the point
(a) (– 18, – 12) (b) (– 18, 12) (c) (18, 12) (d) (18, – 12)
[JEE (Orissa) 2003]

61. The graph represented by the equations $x = \sin^2 t$, $y = 2\cos t$, is
(a) a portion of a parabola (b) a parabola
(c) a part of a sine graph (d) a part of a hyperbola

62. The subtangent, ordinate and subnormal to the parabola $y^2 = 4ax$ at a point (different from the origin) are in
(a) AP (b) GP (c) HP (d) none of these

63. The tangents at the points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ are at right angles if
(a) $t_1t_2 = -1$ (b) $t_1t_2 = 1$ (c) $t_1t_2 = 2$ (d) $t_1t_2 = -2$

64. The equation of the parabola whose vertex is at (2, – 1) and focus at (2, – 3) is
(a) $x^2 + 4x - 8y - 12 = 0$ (b) $x^2 - 4x + 8y + 12 = 0$
(c) $x^2 + 8y = 12$ (d) $x^2 - 4x + 12 = 0$

65. If $P(1, 3)$ and $Q(1, 1)$ are two points on the parabola $y^2 = 4x$ such that a point dividing PQ internally in the ratio $1 : \lambda$ is an interior point of the parabola, then λ lies in the interval

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(a) (0, 1) (b) (−3/5, 1) (c) (1/2, 3/5) (d) none of these

66. The vertex of the parabola $y^2 + 6x - 2y + 13 = 0$, is
(a) (1, −1) (b) (−2, 1) (c) (3/2, 1) (d) (−7/2, 1)

67. The cartesian equation of the directrix of the parabola whose parametric equations are $x = 2t + 1, y = t^2 + 2$, is
(a) $y = 2$ (b) $y = 1$ (c) $y = -1$ (d) $y = -2$

68. If the vertex of a parabola is (0, 2) and the extremities of latusrectum are (−6, 4) and (6, 4), then, its equation is
(a) $x^2 - 4y + 8 = 0$ (b) $x^2 + 4y - 8 = 0$
(c) $x^2 - 8y + 16 = 0$ (d) $x^2 + 8y - 16 = 0$

69. A line L passing through the focus of the parabola $(y-2)^2 = 4(x+1)$ intersects the parabola in two distinct points. If m be the slope of the line L, then
(a) $m \in (-1, 1)$ (b) $m \in (-\infty, -1) \cup (1, \infty)$
(c) $m \in (-\infty, 0) \cup (0, \infty)$ (d) none of these

70. Let $y = f(x)$ be a parabola, having its axis parallel to y-axis, which is touched by the line $y = x$ at $x = 1$, then
(a) $f'(0) = f'(1)$ (b) $2f(0) = 1 - f'(0)$
(c) $f'(1) = 1$ (d) $f(0) + f'(0) + f''(0) = 1$

71. If two tangents drawn from a point P to the parabola $y^2 = 4x$ be such that the slope of one tangent is double of the other, then P lies on the curve.
(a) $9y = 2x^2$ (b) $9x = 2y^2$
(c) $2x = 9y^2$ (d) none of these

72. The angle between the tangents drawn from the point (3, 4) to the parabola $y^2 - 2y + 4x = 0$ is
(a) $\tan^{-1}(8\sqrt{5}/7)$ (b) $\tan^{-1}(12/\sqrt{5})$
(c) $\tan^{-1}(\sqrt{5}/7)$ (d) none of these

73. Set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 4$ touches the parabola $y^2 = 4x$, is
(a) $\left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$
(b) $(-\infty, -1) \cup (1, \infty)$ (c) $(-1, 1)$ (d) R

74. The mid-point of the line joining the common points of the line $2x - 3y + 8 = 0$ and $y^2 = 8x$, is
(a) (3, 2) (b) (5, 6) (c) (4, −1) (d) (2, −3)

Answers

1. (c) 2. (b) 3. (a) 4. (b) 5. (c) 6. (d) 7. (b)
8. (c) 9. (d) 10. (a) 11. (d) 12. (b) 13. (a) 14. (b)
15. (b) 16. (c) 17. (d) 18. (c) 19. (b) 20. (b) 21. (c)
22. (b) 23. (d) 24. (a) 25. (c) 26. (a) 27. (d) 28. (c)
29. (c) 30. (c) 31. (d) 32. (a) 33. (d) 34. (c) 35. (a)
36. (a) 37. (c) 38. (d) 39. (a) 40 (b) 41. (d) 42. (a)
43. (a) 44. (c) 45. (d) 46. (d) 47. (a) 48 (c) 49. (b)
50. (c) 51. (c) 52. (b) 53. (c) 54. (c) 55. (d) 56. (a)
57. (c) 58. (c) 59. (a) 60. (d) 61. (b) 62. (b) 63. (a)
64. (b) 65. (a) 66. (b) 67. (b) 68. (c) 69. (c) 70. (b)
71. (b) 72. (a) 73. (a) 74. (b) 75. (d)

CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. If $y = 2x + k$ is a tangent to the curve $x^2 = 4y$, then k is equal to
(a) 4 (b) 1/2 (c) −4 (d) −1/2

2. The normal drawn at a point $(at_1^2, 2at_1)$ of the parabola $y^2 = 4ax$ meets it again on the point $(at_2^2, 2at_2)$, then
(a) $t_1 = 2t_2$ (b) $t_1^2 = 2t_2$
(c) $t_1 t_2 = -1$ (d) none of these

3. The mid-point of the chord $2x + y - 4 = 0$ of the parabola $y^2 = 4x$ is
(a) (5/2, −1) (b) (−1, 5/2)
(c) (3/2, −1) (d) none of these

4. The two ends of latusrectum of a parabola are the points (3, 6) and (−5, 6). The focus is
(a) (1, 6) (b) (−1, 6) (c) (1, −6) (d) (−1, −6)

5. Locus of the middle points of all chords of the parabola $y^2 = 4x$ which are drawn through the vertex is
(a) $y^2 = 8x$ (b) $y^2 = 2x$
(c) $x^2 + 4y^2 = 16$ (d) $x^2 = 2y$

6. The vertex of the parabola $x^2 + 2y = 8x - 7$ is
(a) (4, 7/2) (b) (4, 9/2) (c) (9/2, 4) (d) (1, 0)

7. The point of contact of the line $x - 2y - 1 = 0$ with the parabola $y^2 = 2(x - 3)$, is
(a) (5, 2) (b) (5, −2) (c) (2, 5) (d) (5, 3)

8. The number of distinct normals that can be drawn from (11/4, 1/4) to the parabola $y^2 = 4x$, is
(a) 3 (b) 2 (c) 1 (d) 4

9. At what point on the parabola $y^2 = 4x$ the normal makes equal angles with the axes ?
(a) (4, 4) (b) (9, 6) (c) (4, −4) (d) (1, −2)

10. Three normals to the parabola $y^2 = x$ are drawn through a point (c, 0), then
(a) $c = 1/4$ (b) $c = 1/2$ (c) $c > 1/2$ (d) none of these

11. The normal chord of a parabola $y^2 = 4ax$ at (x_1, x_1) subtends a right angle at the

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(a) focus (b) vertex
(c) end of the latusrectum (d) none of these

12. AB, AC are tangents to a parabola $y^2 = 4ax$, p_1, p_2, p_3 are the lengths of the perpendiculars from A, B, C on any tangent to the curve, then p_2, p_1, p_3 are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

13. The circles on focal radii of a parabola as diameter touch
(a) the tangent at the vertex (b) the axis
(c) the directrix (d) none of these

14. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the three conormal points are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

15. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then, a point of intersection of the circle and the parabola is
(a) $(p/2, \pm p)$ (b) $(p, p/2)$
(c) $(-p/2, p)$ (d) $(-p/2, -p)$

16. The equation of the tangent to the parabola $y^2 = 8x$ which is perpendicular to the line $x - 3y + 8 = 0$ is
(a) $9x + 3y + 2 = 0$ (b) $3x + y + 2 = 0$
(c) $3x - y - 1 = 0$ (d) $9x - 3y + 2 = 0$

17. The tangent drawn at any point P to the parabola $y^2 = 4ax$ meets the directrix at the point K. The angle which KP subtends at the focus is
(a) 30° (b) 45° (c) 60° (d) 90°

18. The tangent and normal at the point $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ meet the x-axis in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through P, T, G is
(a) $\tan^{-1}(t^2)$ (b) $\cot^{-1}(t^2)$
(c) $\tan^{-1}(t)$ (d) $\cot^{-1}(t)$

19. The parabola $y^2 = 4ax$ passes through the point $(2, -6)$, then the length of its latusrectum is
(a) 18 (b) 9 (c) 6 (d) 16

20. A variable circle passes through the fixed point (2, 0) and touches y-axis. Then, the locus of its centre is
(a) a parabola (b) a circle
(c) an ellipse (d) a hyperbola

21. The locus of the poles of the focal chords of a parabola is ..... of the parabola
(a) the axis (b) a focal chord
(c) the directrix (d) the tangent at the vertex

22. If the slope of the focal chord of $y^2 = 16x$ is 2, then the length of the chord is
(a) 22 (b) 24 (c) 20 (d) 18

23. If $x - 2y - a = 0$ is a chord of the parabola $y^2 = 4ax$, then its length is
(a) $4a\sqrt{5}$ (b) $40a$ (c) $20a$ (d) $15a$

24. Equation of normal to the parabola $y^2 = 4x$ which passes through (3, 0) is
(a) $x + y = 3$ (b) $x + y + 3 = 0$
(c) $x - 2y = 3$ (d) none of these

25. The length of the normal of the parabola $y^2 = 4x$ which subtends a right angle at the vertex is
(a) $6\sqrt{3}$ (b) $3\sqrt{3}$ (c) 2 (d) 1

26. At what point on the parabola $y^2 = 4x$, the normal makes equal angles with the axis ?
(a) (4, 4) (b) (9, 6) (c) (4, −4) (d) (1, −2)

27. The circle on focal radii of a parabola as diameter touches the
(a) axis (b) directrix
(c) tangent at the vertex (d) none of these

28. Tangents are drawn at the ends of any focal chord of the parabola $y^2 = 16x$. Then which of the following statements about the point of intersection of tangents is true.
(a) its abscissae is independent of the extremities of the focal chord
(b) its ordinate is independent of the extremities of the focal chord
(c) it is at a distance of 8 units from the vertex of the parabola
(d) It is at a distance of 16 units from the focus of the parabola

29. The angle between the pair of tangents drawn from (1, 3) to the parabola $y^2 = 8x$, is
(a) $\tan^{-1} 2$ (b) $\tan^{-1} \frac{1}{2}$
(c) $\tan^{-1} \frac{1}{3}$ (d) $\tan^{-1} 3$

30. A variable tangent to the parabola $y^2 = 4ax$ meets the parabola $y^2 = -4ax$ at P and Q. The locus of the mid-point of PQ is
(a) $y^2 = -2ax$ (b) $y^2 = -ax$
(c) $y^2 = -\frac{4}{3}ax$ (d) $y^2 = -4ax$

Answers

1. (c) 2. (d) 3. (a) 4. (b) 5. (b) 6. (b) 7. (a)
8. (a) 9. (d) 10. (c) 11. (a) 12. (b) 13. (a) 14. (b)
15. (a) 16. (a) 17. (d) 18. (c) 19. (a) 20. (a) 21. (c)
22. (c) 23. (c) 24. (a) 25. (a) 26. (d) 27. (c) 28. (a)
29. (c) 30. (c)

Solutions of Exercises and Chapter-tests are available in a separate book on "Solutions of Objective Mathematics".

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ELLIPSE

1. DEFINITION

An ellipse is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distance from a fixed straight line (called directrix) is always constant which is always less than unity.

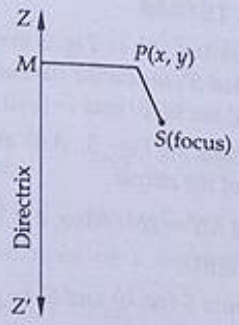


Fig. 1

The constant ratio is generally denoted by e and is known as the *eccentricity* of the ellipse.

If S is the focus, ZZ' is the directrix and P is any point on the ellipse, then by definition

$$\frac{SP}{PM} = e \quad \Rightarrow \quad SP = e\,PM$$

ILLUSTRATION 1 *The equation of the ellipse whose focus is (1, 0) and the directrix is $x + y + 1 = 0$ and eccentricity is equal to $1/\sqrt{2}$, is*

(a) $(x-1)^2 + y^2 = (x+y+1)^2$

(b) $2\{(x-1)^2 + y^2\} = (x+y+1)^2$

(c) $4\{(x-1)^2 + y^2\} = (x+y+1)^2$

(d) *none of these*

Ans. (c)

SOLUTION Let $S(1, 0)$ be the focus and ZZ' be the directrix. Let $P(x, y)$ be any point on the ellipse and PM be perpendicular from P on the directrix. Then, by definition, we have

$$SP = e\,PM, \quad \text{where } e = 1/\sqrt{2}$$

$$\Rightarrow \quad SP^2 = e^2\,PM^2$$

$$\Rightarrow \quad (x-1)^2 + (y-0)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 \left\{\frac{x+y+1}{\sqrt{1+1}}\right\}^2$$

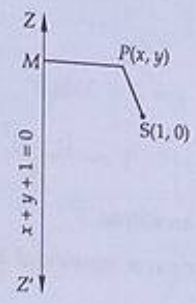


Fig. 2

$$\Rightarrow \quad 4\{(x-1)^2 + y^2\} = (x+y+1)^2$$

2. STANDARD FORM OF THE ELLIPSE

The equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2(1 - e^2)$ represents an ellipse having two foci at $S(ae, 0)$ and $S'(-ae, 0)$ and corresponding directrices as $x = \frac{a}{e}$ and $x = -\frac{a}{e}$ respectively as shown in Fig. 3

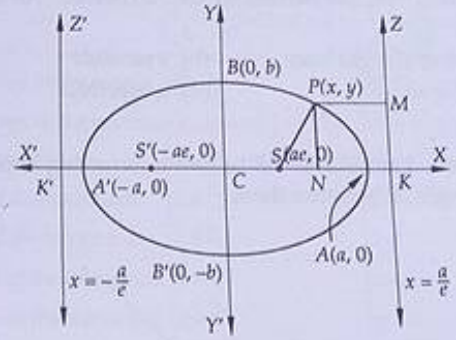


Fig. 3

REMARK *If the centre of an ellipse is at the point (x_1, y_1) and the directions of the axes are parallel to the coordinate axes, then its equation is*

$$\frac{(x-x_1)^2}{a^2} + \frac{(y-y_1)^2}{b^2} = 1$$

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EXAMPLE 6 *The tangents at a point P on the ellipse* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, *which is not an extremity of major axis meets a directrix at T.*
Statement–1: The circle on PT as diameter passes through the focus of the ellipse corresponding to the directrix on which T lies.
Statement–2: PT subtends a right angle at the focus of the ellipse corresponding to the directrix on which T lies.
(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION The equation of tangent at a point $P(x_1, y_1)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$.

This cuts the directrix $x=\frac{a}{e}$ at $T\left(\frac{a}{e}, \frac{(ae-x_1)b^2}{aey_1}\right)$.

The coordinates of the focus S are $(ae, 0)$.

$\therefore \quad m_1 = \text{Slope of } SP = \frac{y_1}{x_1-ae}$

and, $m_2 = \text{Slope of } ST = \frac{(ae-x_1)b^2}{aey_1(a/e-ae)} = \frac{ae-x_1}{y_1}$

Clearly, $m_1 m_2 = -1$.

So, PT subtends a right angle at the focus. Consequently, circle described on PT as a diameter passes through the focus of the ellipse.

Hence, both the statements are true and statement–2 is a correct explanation for statement–1.

EXAMPLE 7 *Let C be the locus of a point the sum of whose distances from the points* $S(\sqrt{3}, 0)$ *and* $S'(-\sqrt{3}, 0)$ *is 4.*
Statement–1: The curve C cuts off intercept $2\sqrt{3}$ *from the line* $2y-1=0$.
Statement–2: The equation of the curve C is $x^2+8y^2=5$.
(a) 1 (b) 2 (c) 3 (d) 4

Ans. (c)

SOLUTION The curve C represents an ellipse having its foci at $S(\sqrt{3}, 0)$ and $S'(-\sqrt{3}, 0)$ and major axis of length 4.

$\therefore \quad 2a=4$ and $2ae=2\sqrt{3} \Rightarrow a=2$ and $ae=\sqrt{3} \Rightarrow a=2, e=\frac{\sqrt{3}}{2}$

$\therefore \quad b^2=a^2(1-e^2) \Rightarrow b^2=4\left(1-\frac{3}{4}\right)=1$

Hence, the equation of C is $\frac{x^2}{4}+y^2=1$.

So, statement-2 is not true.

Solving $2y-1=0$ and $\frac{x^2}{4}+\frac{y^2}{1}=1$, we get $P(\sqrt{3}, 1/2)$ and $Q(-\sqrt{3}, 1/2)$ as points of intersection.
Clearly, $PQ = 2\sqrt{3}$.
So, statement-1 is true.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents an ellipse if
(a) $\Delta=0, h^2<ab$ (b) $\Delta\neq 0, h^2<ab$
(c) $\Delta\neq 0, h^2>ab$ (d) $\Delta\neq 0, h^2=ab$

2. The equation of the ellipse whose focus is $S(1,-1)$, directrix the line $x-y-3=0$ and eccentricity $1/2$, is
(a) $7x^2+2xy+7y^2-10x+10y+7=0$
(b) $7x^2+2xy+7y^2+7=0$
(c) $7x^2+2xy+7y^2+10x-10y-7=0$
(d) none of these

3. The equation to the ellipse (referred to its axes as the axes of x and y respectively) whose foci are $(\pm 2, 0)$ and eccentricity $1/2$, is
(a) $\frac{x^2}{12}+\frac{y^2}{16}=1$ (b) $\frac{x^2}{16}+\frac{y^2}{12}=1$
(c) $\frac{x^2}{16}+\frac{y^2}{8}=1$ (d) none of these

4. The equation of the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{2/5}$, is
(a) $3x^2+6y^2=33$ (b) $5x^2+3y^2=48$
(c) $3x^2+5y^2=32$ (d) none of these

5. The eccentricity of the ellipse $9x^2+5y^2-30y=0$ is
(a) $1/3$ (b) $2/3$ (c) $3/4$ (d) none of these

6. If the tangent at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the tangetns at the ends B and B' of minor axis at L and L' respectively, then $BL \cdot B'L' =$
(a) a^2 (b) b^2 (c) a^2+b^2 (d) a^2-b^2

7. If A and B are two fixed points and P is a variable point such that $PA+PB=4$, the locus of P is
(a) a parabola (b) an ellipse
(c) a hyperbola (d) none of these

8. The length of the latusrectum of the ellipse $3x^2+y^2=12$ is
(a) 4 (b) 3 (c) 8 (d) $4/\sqrt{3}$

9. The eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose latusrectum is half of its major axis is
(a) $1\sqrt{2}$ (b) $\sqrt{2}/3$ (c) $\sqrt{3}/2$ (d) none of these

10. The eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose latusrectum is half of its minor axis is
(a) $1/\sqrt{2}$ (b) $\sqrt{2}/3$ (c) $\sqrt{3}/2$ (d) none of these

11. If the focal distance of an end of the minor axis of an ellipse (referred to its axes as the axes of x and y respectively) is k and the distance between its foci is $2h$, then its equation is

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(a) $\frac{x^2}{k^2}+\frac{y^2}{h^2}=1$ (b) $\frac{x^2}{k^2}+\frac{y^2}{k^2-h^2}=1$

(c) $\frac{x^2}{k^2}+\frac{y^2}{h^2-k^2}=1$ (d) $\frac{x^2}{k^2}+\frac{y^2}{k^2+h^2}=1$

12. If $2y=x$ and $3y+4x=0$ are the equations of a pair of conjugate diameters of an ellipse, then the eccentricity of the ellipse is
(a) $\sqrt{2/3}$ (b) $\sqrt{2/5}$ (c) $\sqrt{1/3}$ (d) $\sqrt{1/2}$
[JEE (WB) 2008]

13. If θ is a parameter, then $x=a(\sin\theta+\cos\theta)$, $y=b(\sin\theta-\cos\theta)$ represents
(a) an ellipse (b) a circle
(c) a pair of straight lines (d) a hyperbola
[JEE (WB) 2008]

14. The distances from the foci of point $P(x_1, y_1)$ on the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ are
(a) $4\pm\frac{5}{4}y_1$ (b) $5\pm\frac{4}{5}x_1$ (c) $5\pm\frac{4}{5}y_1$ (d) none of these

15. The equation of the ellipse having vertices at $(\pm 5, 0)$ and foci $(\pm 4, 0)$ is
(a) $\frac{x^2}{25}+\frac{y^2}{16}=1$ (b) $9x^2+25y^2=225$
(c) $\frac{x^2}{9}+\frac{y^2}{25}=1$ (d) $4x^2+5y^2=20$ **[JEE (WB) 2008]**

16. The eccentricity of the conic $x^2-4x+4y^2=12$ is
(a) $\sqrt{3}/2$ (b) $2/\sqrt{3}$ (c) $\sqrt{3}$ (d) none of these

17. The parametric representation of a point on the ellipse whose foci are $(-1, 0)$ and $(7, 0)$ and eccentricity $1/2$, is
(a) $(3+8\cos\theta, 4\sqrt{3}\sin\theta)$ (b) $(8\cos\theta, 4\sqrt{3}\sin\theta)$
(c) $(3+4\sqrt{3}\cos\theta, 8\sin\theta)$ (d) none of these

18. If S and S' are two foci of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a<b)$ and $P(x_1, y_1)$ a point on it, then $SP+S'P$ is equal to
(a) $2a$ (b) $2b$ (c) $a+ex_1$ (d) $b+ey_1$

19. The eccentricity of the ellipse represented by the equation $25x^2+16y^2-150x-175=0$ is
(a) $2/5$ (b) $3/5$ (c) $4/5$ (d) none of these

20. The length of the latusrectum of the ellipse $5x^2+9y^2=45$ is
(a) $5/3$ (b) $10/3$ (c) $2\sqrt{5}/5$ (d) $\sqrt{5}/3$

21. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre $(0, 3)$, is
(a) 4 (b) 3 (c) $\sqrt{12}$ (d) 7/2
[JEE (WB) 2007]

22. The eccentricity of the conic $4x^2+16y^2-24x-32y=1$ is
(a) 1/2 (b) $\sqrt{3}$ (c) $\sqrt{3}/2$ (d) $\sqrt{3}/4$
[JEE (WB) 2006]

23. A set of points is such that each point is three times as far away from the y-axis as it is from the point $(4, 0)$. Then, the locus of the points is
(a) hyperbola (b) parabola (c) ellipse (d) circle

24. The foci of an ellipse are $(0, \pm 6)$ and the equations of the directrices are $y=\pm 9$. The equation of the ellipse is
(a) $5x^2+9y^2=4$ (b) $2x^2-6y^2=28$
(c) $6x^2+3y^2=45$ (d) $9x^2+5y^2=180$

25. All ellipse has its centre at $(1, -1)$ and semi-major axis=8 and it passes through the point $(1, 3)$. The equation of the ellipse is
(a) $\frac{(x+1)^2}{64}+\frac{(y+1)^2}{16}=1$ (b) $\frac{(x-1)^2}{64}+\frac{y+1)^2}{16}=1$
(c) $\frac{(x-1)^2}{16}+\frac{y+1)^2}{64}=1$ (d) $\frac{(x+1)^2}{64}+\frac{(y-1)^2}{16}=1$

26. Let LL' be the latusrectum and S be a focus of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If $\Delta SLL'$ is equilaterial, then the eccentricity of the ellipse is
(a) $1/\sqrt{5}$ (b) $1/\sqrt{3}$ (c) $1/\sqrt{2}$ (d) $\sqrt{2}/3$

27. The equation of the axes of the ellipse $3x^2+4y^2+6x-8y-5=0$, are
(a) $x+3, y=5$ (b) $x+3=0, y-5=0$
(c) $x-1=0, y=0$ (d) $x+1=0, y-1=0$

28. The equations to the directrices of the ellipse $4(x-3)^2+9(y+2)^2=144$ are
(a) $5x-15\pm 18\sqrt{5}=0$ (b) $5x+15\pm 2\sqrt{5}=0$
(c) $15x+5\pm 2\sqrt{5}=0$ (d) $15x-5\pm 18\sqrt{5}=0$

29. If the vertices of an ellipse are $(-12, 4)$ and $(14, 4)$ and eccentricity $12/13$, then the equation of the ellipse is
(a) $\frac{(x-4)^2}{25}+\frac{(y-1)^2}{169}=1$ (b) $\frac{(x-4)^2}{169}+\frac{(y-1)^2}{25}=1$
(c) $\frac{(x-1)^2}{169}+\frac{(y-4)^2}{25}=1$ (d) $\frac{(x+1)^2}{169}+\frac{(y+4)^2}{25}=1$

30. If the coordinates of the vertices of an ellipse are $(-6, 1)$ and $(4, 1)$ and the equation of a focal chord passing through the focus on the right side of the centre is $2x-y-5=0$. The equation of the ellipse is
(a) $\frac{(x+1)^2}{25}+\frac{(y+1)^2}{16}=1$ (b) $\frac{(x+1)^2}{25}+\frac{(y-1)^2}{16}=1$
(c) $\frac{(x-1)^2}{25}+\frac{(y+1)^2}{16}=1$ (d) none of these

31. If the tangent at the point $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ to the ellipse $16x^2+11y^2=256$ is also a tangent to the circle $x^2+y^2-2x=15$, then the value of ϕ is
(a) $\pm\pi/2$ (b) $\pm\pi/4$ (c) $\pm\pi/3$ (d) $\pm\pi/6$

32. A man running round a race course notes that the sum of the distances of two flag-posts from him is always 10 metres and the distance between the flag-posts is 8 metres. The area of the path he encloses in square metres is.
(a) 15π (b) 12π (c) 18π (d) 8π

33. The angle between the pair of tangents drawn from the point $(1, 2)$ to the ellipse $3x^2+2y^2=5$ is

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(a) $\tan^{-1}\left(\frac{12}{5}\right)$ (b) $\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)$

(c) $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$ (d) $\tan^{-1}(12\sqrt{5})$

34. The foci of the ellipse, $25(x+1)^2+9(y+2)^2=225$ are at,
(a) $(-1, 2)$ and $(-1, -6)$ (b) $(-2, 1)$ and $(-2, 6)$
(c) $(-1, -2)$ and $(-2, -1)$ (d) $(-1, -2)$ and $(-1, -6)$

35. If the coordinates of the centre, a focus and adjacent vertex are $(2, -3)$, $(3, -3)$ and $(4, -3)$ respectively, then the equation of the ellipse is
(a) $\frac{(x-2)^2}{4}+\frac{(y+3)^2}{3}=1$ (b) $\frac{(x-3)^2}{4}+\frac{(y-2)^2}{3}=1$
(c) $\frac{(x-2)^2}{8}+\frac{(y+3)^2}{6}=1$ (d) $\frac{(x+2)^2}{4}+\frac{(y-3)^2}{3}=1$

36. The eccentric angle of the point of contact of the line $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$ with the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is
(a) 0 (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/2$

37. If a tangent, having slope $-\frac{4}{3}$, to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes in points A and B respectively, then the area of ΔOAB is equal to
(a) 12 sq. units (b) 48 sq. units
(c) 64 sq. units (d) 24 sq. units

38. The equation of the chord of the ellipse $2x^2+5y^2=20$ which is bisected at the point $(2, 1)$ is
(a) $4x+5y+13=0$ (b) $4x+5y=13$
(c) $5x+4y+13=0$ (d) none of these

39. AB is a diameter of $x^2+9y^2=25$. The eccentric angle of A is $\pi/6$. Then, the eccentric angle of B is
(a) $5\pi/6$ (b) $-5\pi/6$ (c) $-2\pi/3$ (d) none of these

40. If one end of a diameter of the ellipse $4x^2+y^2=16$ is $(\sqrt{3}, 2)$, then the other end is
(a) $(-\sqrt{3}, 2)$ (b) $(\sqrt{3}, -2)$ (c) $(-\sqrt{3}, -\sqrt{2})$ (d) $(0, 0)$

41. The equation of a diameter conjugate to a diameter $y=\frac{b}{a}x$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
(a) $y=-\frac{b}{a}x$ (b) $y=-\frac{a}{b}x$ (c) $y=\frac{a}{b}x$ (d) none of these

42. If θ and ϕ are eccentric angles of the ends of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $(\theta-\phi)$ is equal to
(a) $\pm\pi/2$ (b) $\pm\pi$ (c) 0 (d) none of these

43. If A, A' are the vertices, S, S' are the foci and Z, Z' are the feet of the directrices of an ellipse with centre C, then CS, CA, CZ are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

44. The eccentricity of an ellipse whose pair of a conjugate diameter are $y=x$ and $3y=-2x$ is
(a) 2/3 (b) 1/3 (c) $1/\sqrt{3}$ (d) none of these

45. The locus of the point of intersection of tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which meet at right angle is
(a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

46. The number of normals that can be drawn from a piont to a given ellipse is
(a) 2 (b) 3 (c) 4 (d) 1 **[JEE (WB) 2006]**

47. The sum of the squares of the perpendiculars on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from two points on the minor axis each at a distance ae from the centre is
(a) $2a^2$ (b) $2b^2$ (c) a^2+b^2 (d) a^2-b^2

48. If the polar with respect to $y^2=4ax$ touches the ellipse $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$, the locus of its pole is
(a) $\frac{x^2}{\alpha^2}-\frac{y^2}{(4a^2\alpha^2/\beta^2)}=1$ (b) $\frac{x^2}{\alpha^2}+\frac{\beta^2y^2}{4a^2}=1$
(c) $\alpha^2x^2+\beta^2y^2=1$ (d) none of these

49. If p and q are the segments of a focal chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then
(a) $a^2(p+q)=2bpq$ (b) $b^2(p+q)=2apq$
(c) $a(p+q)=2b^2pq$ (d) $b(p+q)=2a^2pq$

50. If $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then the eccentric angle of the point of contact is equal to
(a) 0° (b) 90° (c) 45° (d) 60°

51. Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e. If A, A' are the vertices and S, S' are the foci of the ellipse, then Area $\Delta PSS'$: Area $\Delta APA'$ =
(a) $e^3:1$ (b) $e^2:1$ (c) $e:1$ (d) $1/e:1$

52. If $P(\theta)$ and $Q(\pi/2+\theta)$ are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Locus of the mid-point of PQ is
(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$
(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ (d) none of these

53. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having its centre $(0, 3)$ is
(a) 4 (b) 3 (c) $\sqrt{12}$ (d) 7/2

54. The centre of the ellipse $\frac{(x+y-2)^2}{9}+\frac{(x-y)^2}{16}=1$ is
(a) $(0, 0)$ (b) $(1, 1)$ (c) $(1, 0)$ (d) $(0, 1)$

55. In an ellipse the distance between its foci is 6 and its minor axis is 8. Then, its eccentricity is
(a) 4/5 (b) $1/\sqrt{52}$ (c) 3/5 (d) 1/2

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56. S and T are the foci of an ellipse and B is an end of the minor axis. If STB is an equilateral triangle, the eccentricity of the ellipse is
(a) 1/4 (b) 1/3 (c) 1/2 (d) 2/3

57. The length of the latusrectum of an ellipse is one third of its major axis. Its eccentricity would be
(a) 2/3 (b) $\sqrt{2/3}$ (c) $1/\sqrt{3}$ (d) $1/\sqrt{2}$

58. If the length of the major axis of an ellipse is three times the length of its minor axis, its eccentricity, is
(a) 1/3 (b) $1/\sqrt{3}$ (c) $1/\sqrt{2}$ (d) $2\sqrt{2}/3$

59. The distance between the foci of the ellipse $5x^2 + 9y^2 = 45$, is
(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 2

60. The length of the latusrectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{49} = 1$ is
(a) 98/6 (b) 72/7 (c) 72/14 (d) 98/12

61. The equation of the ellipse whose one focus is at (4, 0) and whose eccentricity is 4/5 is
(a) $\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$ (b) $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ (c) $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$ (d) $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$

62. The equation of the ellipse passing through (2, 1) having $e = 1/2$ is
(a) $3x^2 + 4y^2 = 16$ (b) $3x^2 + 5y^2 = 17$
(c) $5x^2 + 3y^2 = 23$ (d) none of these

63. If C is the centre of the ellipse $9x^2 + 16y^2 = 144$ and S is one focus. The ratio of CS to major axis, is
(a) $\sqrt{7} : 16$ (b) $\sqrt{7} : 4$ (c) $\sqrt{5} : \sqrt{7}$ (d) none of these

64. In an ellipse the distance between the foci is 8 and the distance between the directrices is 25. The length of major axis is
(a) $10\sqrt{2}$ (b) $20\sqrt{2}$ (c) $30\sqrt{2}$ (d) none of these

65. The centre of the ellipse $4x^2 + 9y^2 + 16x - 18y - 11 = 0$ is
(a) (−2, −1) (b) (−2, 1) (c) (2, −1) (d) none of these

66. If P is any point on the ellipse $9x^2 + 36y^2 = 324$ whose foci are S and S'. Then, $SP + S'P$ equals
(a) 3 (b) 12 (c) 36 (d) 324

67. An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 cm and 4 cm, the necessary length of the string and the distance between the pins respectively in cms. are
(a) $6, 2\sqrt{5}$ (b) $6, \sqrt{5}$ (c) $4, 2\sqrt{5}$ (d) none of these

68. Two perpendicular tangents drawn to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ intersect on the curve
(a) $x = a/e$ (b) $x^2 + y^2 = 41$ (c) $x^2 + y^2 = 9$ (d) $x^2 - y^2 = 41$

69. The distance of the point 'θ' on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from a focus is
(a) $a(e + \cos\theta)$ (b) $a(e - \cos\theta)$
(c) $a(1 + e\cos\theta)$ (d) $a(1 + 2e\cos\theta)$

70. If $y = mx + c$ is a tangent to the ellipse $x^2 + 2y^2 = 6$, then $c^2 =$
(a) $36/m^2$ (b) $6m^2 - 3$ (c) $3m^2 + 6$ (d) $6m^2 + 3$

71. Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with foci at S and S'. If A be the area of triangle PSS', then the maximum value of A is
(a) 24 sq. units (b) 12 sq. units
(c) 36 sq. units (d) none of these

72. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $y = mx + c$ intersect in real points only if
(a) $a^2m^2 < c^2 - b^2$ (b) $a^2m^2 > c^2 - b^2$
(c) $a^2m^2 \geq c^2 - b^2$ (d) $c \geq b$

73. Let E be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the points (1, 2) and (2, 1) respectively. Then
(a) Q lies inside C but outside E
(b) Q lies outside both C and E
(c) P lies inside both C and E
(d) P lies inside C but outside E

74. Equation of the ellipse with eccentricity 1/2 and foci at $(\pm 1, 0)$ is
(a) $\frac{x^2}{3} + \frac{y^2}{4} = 1$ (b) $\frac{x^2}{4} + \frac{y^2}{3} = 1$
(c) $\frac{x^2}{4} + \frac{y^2}{3} = \frac{4}{3}$ (d) none of these

75. If B and B' are the ends of minor axis and S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then area of the rhombus $SBS'B'$ will be
(a) 12. sq. units (b) 48 sq. units
(c) 24 sq. units (d) 36 sq. units

76. The length of the axes of the conic $9x^2 + 4y^2 - 6x + 4y + 1 = 0$, are
(a) 1/2, 9 (b) 3, 2/5 (c) 1, 2/3 (d) 3, 2

[JEE (Orissa) 2002]

Answers

1. (b)	2. (a)	3. (b)	4. (c)	5. (b)	6. (a)	7. (b)	8. (d)	9. (a)	10. (c)
11. (b)	12. (c)	13. (a)	14. (c)	15. (b)	16. (a)	17. (a)	18. (b)	19. (b)	20. (b)
21. (a)	22. (c)	23. (c)	24. (d)	25. (b)	26. (b)	27. (d)	28. (a)	29. (c)	30. (b)
31. (c)	32. (a)	33. (c)	34. (a)	35. (a)	36. (c)	37. (d)	38. (b)	39. (b)	40. (c)
41. (a)	42. (a)	43. (b)	44. (c)	45. (a)	46. (c)	47. (a)	48. (a)	49. (b)	50. (c)
51. (c)	52. (a)	53. (a)	54. (b)	55. (c)	56. (c)	57. (b)	58. (d)	59. (b)	60. (b)
61. (b)	62. (a)	63. (d)	64. (a)	65. (b)	66. (b)	67. (d)	68. (b)	69. (c)	70. (d)
71. (b)	72. (c)	73. (d)	74. (b)	75. (c)	76. (c)				

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CHAPTER TEST

Each of the following questions has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct choice.

1. The maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with vertex at one end of major axis is
(a) $\sqrt{3}\,ab$ (b) $\frac{3\sqrt{3}}{4}ab$ (c) $\frac{5\sqrt{3}}{4}ab$ (d) none of these

2. A tangent to the ellipse $x^2+4y^2=4$ meets the ellipse $x^2+2y^2=6$ at P and Q. The angle between the tangent at P and Q of the ellipse $x^2+2y^2=6$ is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

3. If the distance of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ from the centre is 2, then the eccentric angle of the point is
(a) $\frac{\pi}{4},\frac{3\pi}{4}$ (b) $\frac{\pi}{3},\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) none of these

4. If the minor axis of an ellipse subtends an angle of 60° at each focus of the ellipse, then its eccentricity is
(a) $\sqrt{3}/2$ (b) $1/\sqrt{2}$ (c) $2/\sqrt{3}$ (d) none of these

5. Let S and S' be two focii of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If the circle described on SS' as diameter touches the ellipse in real points, then the eccentricity of the ellipse is
(a) $2/\sqrt{3}$ (b) $\sqrt{3}/2$ (c) $1/\sqrt{2}$ (d) $1/\sqrt{3}$

6. The equation of the normal at the point $P(2, 3)$ on the ellipse $9x^2+16y^2=180$ is
(a) $3y=8x-10$ (b) $3y-8x+7=0$
(c) $8y+3x+7=0$ (d) $3x+2y+7=0$

7. For the ellipse $3x^2+4y^2+6x-8y-5=0$ the eccentricity is
(a) 1/3 (b) 1/2 (c) 1/4 (d) 1/5

8. Let S, S' be the focii and BB' be the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If $\angle BSS'=\theta$, then the eccentricity e of the ellipse is equal to
(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

9. If the length of the latusrectum of the ellipse $x^2\tan^2\theta+y^2\sec^2\theta=1$ is 1/2, then θ =
(a) $\pi/12, 5\pi/12$ (b) $\pi/6, 5\pi/6$
(c) $7\pi/12$ (d) none of these

10. The equation of the ellipse whose vertices are $(-4, 1)$, $(6, 1)$ and one of the focal chord is $x-2y-2=0$, is
(a) $\frac{(x-1)^2}{25}+\frac{(y-1)^2}{9}=1$ (b) $\frac{(x+1)^2}{25}+\frac{(y+1)^2}{9}=1$
(c) $\frac{(x-1)^2}{16}+\frac{(y-1)^2}{25}=1$ (d) $\frac{(x+1)^2}{16}+\frac{(y+1)^2}{25}=1$

11. If $(-4, 3)$ and $(8, 3)$ are the vertices of an ellipse whose eccentricity is 5/6 then the equation of the ellipse is
(a) $\frac{(x-2)^2}{11}+\frac{(y-3)^2}{36}=1$ (b) $\frac{(x-2)^2}{36}+\frac{(y-3)^2}{11}=1$
(c) $\frac{(x-3)^2}{36}+\frac{(y-2)^2}{11}=1$ (d) none of these

12. The area of the triangle formed by three points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are α, β and γ is
(a) $2ab\sin\frac{\alpha-\beta}{2}\cos\frac{\beta-\gamma}{2}\cos\frac{\gamma-\alpha}{2}$
(b) $2ab\sin\frac{\alpha-\beta}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\gamma-\alpha}{2}$
(c) $2ab\sin\frac{\alpha-\beta}{2}\sin\frac{\beta-\gamma}{2}\sin\frac{\gamma-\alpha}{2}$
(d) $2ab\cos\frac{\alpha-\beta}{2}\cos\frac{\beta-\gamma}{2}\cos\frac{\gamma-\alpha}{2}$

13. If the chord joining points $P(\alpha)$ and $Q(\beta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ subtends a right angle at the vertex $A(a, 0)$, then $\tan\alpha/2\tan\beta/2$ =
(a) a^2/b^2 (b) $-a^2/b^2$ (c) b^2/a^2 (d) $-b^2/a^2$

14. If $P(\alpha, \beta)$ is a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci S and S' and eccentricity e, then area of $\Delta SPS'$ is
(a) $ae\sqrt{a^2-\alpha^2}$ (b) $be\sqrt{b^2-\alpha^2}$
(c) $ae\sqrt{b^2-\alpha^2}$ (d) $be\sqrt{a^2-\alpha^2}$

15. If the tangent at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the tangents at the vertices A and A' in L and L' respectively, then $AL\cdot A'L'$ =
(a) $a+b$ (b) a^2+b^2 (c) a^2 (d) b^2

16. P is a point on the circle $x^2+y^2=c^2$. The locus of the mid-points of chords of contact of P with respect to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is
(a) $c^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=x^2+y^2$ (b) $c^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=x^2+y^2$
(c) $c^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=(x^2+y^2)^2$ (d) none of these

17. The locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is
(a) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=a^2+b^2$ (b) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=a^2-b^2$
(c) $\frac{a^6}{x^2}+\frac{b^6}{y^2}=(a^2-b^2)^2$ (d) $\frac{a^4}{x^2}+\frac{b^4}{y^2}=(a^2-b^2)^2$

18. The locus of the mid-points of focal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is
(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{a^2}$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a^2}$

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(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x^2}{a^4}$ (d) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a}$

19. The locus of points whose polars with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are at a distance d from the centre of the ellipse, is

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{d^2}$ (b) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{d^2}$

(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{d^4}$ (d) none of these

20. If the chord of contact of tangents drawn from a point P to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ subtends a right-angle at its centre, then P lies on

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2}+\frac{1}{b^2}$ (b) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\left(\frac{1}{a}+\frac{1}{b}\right)^2$

(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^4}+\frac{1}{b^4}$ (d) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$

21. The locus of the poles of tangents to the auxiliary circle with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2}$ (b) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{b^2}$

(c) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}$ (d) none of these

22. The locus of the poles of tangents to the director circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2+b^2}$ (b) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2+b^2}$

(c) $\frac{x^2}{a^6}+\frac{y^2}{b^6}=\frac{1}{a^2+b^2}$ (d) none of these

23. The locus of the mid-points of chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ that touch the circle $x^2+y^2=b^2$, is

(a) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{x^2}{a^4}+\frac{y^2}{b^4}$ (b) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=b^2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}\right)$

(c) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=a^2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}\right)$ (d) none of these

24. If any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes intercepts p and q on the coordinate axes, then $\frac{a^2}{p^2}+\frac{b^2}{q^2}=$

(a) 1 (b) 2 (c) 3 (d) 4

25. If the tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ make angles α and β with the major axis such that $\tan\alpha+\tan\beta=\lambda$, then the locus of their point of intersection is

(a) $x^2+y^2=a^2$ (b) $x^2+y^2=b^2$

(c) $x^2-a^2=2\lambda xy$ (d) $\lambda(x^2-a^2)=2xy$

26. If C is the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the normal at an end of a latusrectum cuts the major axis in G, then $CG=$

(a) ae (b) a^2e^2 (c) ae^3 (d) a^2e^3

27. If the normals at $P(\theta)$ and $Q(\pi/2+\theta)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meet the major axis in G and g respectively, then $PG^2+Qg^2=$

(a) $b^2(1-e^2)(2-e^2)$ (b) $a^2(1-e^2)(2-e^2)$

(c) $a^2(1+e^2)(2+e^2)$ (d) $b^2(1+e^2)(2+e^2)$

28. Let C be the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If the tangent at any point on the ellipse cuts the coordinate axes in P and Q respectively, then $\frac{a^2}{CP^2}+\frac{b^2}{CQ^2}=$

(a) 1 (b) 2 (c) 3 (d) 4

29. The tangent at point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the minor axis in Q and PR is drawn perpendicular to the minor axis. If C is the centre of the ellipse, then $CQ\cdot CR=$

(a) b^2 (b) $2b^2$ (c) a^2 (d) $2a^2$

30. If the lengths of major and semi-minor axes of an ellipse are 4 and $\sqrt{3}$ and their corresponding equations are $y-5=0$ and $x+3=0$, then the equation of the ellipse is

(a) $3x^2+4y^2+18x-40y+115=0$

(b) $4x^2-3y^2-24x+30y+99=0$

(c) $3x^2-4y^2-18x+40y+115=0$

(d) $4x^2+3y^2+24x-30y+99=0$

Answers

1. (b) **2.** (a) **3.** (a) **4.** (a) **5.** (c) **6.** (b) **7.** (b) **8.** (b) **9.** (a) **10.** (a) **11.** (b) **12.** (c) **13.** (d) **14.** (d) **15.** (d) **16.** (a) **17.** (c) **18.** (b) **19.** (b) **20.** (d) **21.** (c) **22.** (b) **23.** (b) **24.** (a) **25.** (d) **26.** (c) **27.** (b) **28.** (a) **29.** (a) **30.** (a)

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26

HYPERBOLA

1. HYPERBOLA

A hyperbola is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distance from a fixed line (called directrix) is always constant which is always greater than unity.

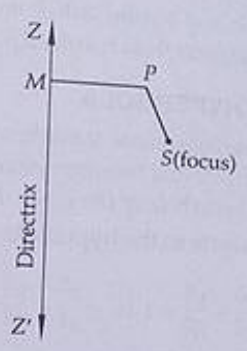


Fig. 1

The fixed point is called the focus, the fixed line is called the directrix, the constant ratio is generally denoted by e and is know as the eccentricity of the hyperbola.

If S is the focus, ZZ' is the directrix and P is any point on the hyperbola as shown in Fig. 1, then by definition, we have

$$\frac{SP}{PM} = e \Rightarrow SP = e \, . \, PM.$$

ILLUSTRATION *The equation of the hyperbola whose focus is (1, 2), directrix is the line $x + y + 1 = 0$ and eccentricity 3/2, is*

(a) $x^2 + y^2 + 18xy + 34x + 50y - 31 = 0$

(b) $x^2 + y^2 - 18xy - 2x - 14y + 31 = 0$

(c) $x^2 + y^2 + 18xy + 34x + 50y + 49 = 0$

(d) $x^2 + y^2 - 18xy + 34x + 50y - 31 = 0$

Ans. (a)

SOLUTION Let $S\,(1, 2)$ be the focus and let $P\,(x, y)$ be a point on the hyperbola. Draw perpendicular PM from P on the directrix $x + y + 1 = 0$. By definition, we have

$$SP = e\,PM$$

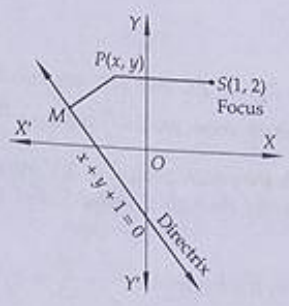


Fig. 2

$$\Rightarrow \quad \sqrt{(x-1)^2 + (y-2)^2} = \frac{3}{2}\left|\frac{x+y+1}{\sqrt{1^2+1^2}}\right|$$

$$\Rightarrow \quad (x-1)^2 + (y-2)^2 = \frac{9}{4}\left\{\frac{(x+y+1)^2}{2}\right\}$$

$$\Rightarrow \quad 8\,[(x-1)^2 + (y-2)^2] = 9\,[(x+y+1)^2]$$

$$\Rightarrow \quad x^2 + y^2 + 18\,xy + 34\,x + 50\,y - 31 = 0.$$

2. STANDARD EQUATION OF THE HYPERBOLA

The equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = a^2\,(e^2 - 1)$ represents hyperbola having its foci at $S\,(ae, 0)$ and $S'\,(-ae, 0)$ and corresponding directrices $x = \frac{a}{e}$ and $x = -\frac{a}{e}$ respectively as shown in Fig. 3.

VERTICES *In Fig. 3, the points A and A', where the curve meets the line joining the foci S and S', are called the vertices of the hyperbola. The coordinates of A and A' are (a, 0) and (– a, 0) respectively.*

TRANSVERSE AND CONJUGATE AXES *In Fig. 3, the straight line joining the vertices A and A' is called the transverse axis of the hyperbola. Its length AA' is generally taken to be 2 a.*

The straight line through the centre which is perpendicular to the transverse axis does not meet the hyperbola in real points. But if B, B' be the points on this line such that $CB = CB' = b$, the line BB' is called the conjugate axis such that $BB' = 2b$.

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SECTION - II

ASSERTION–REASON TYPE MCQs

In this section each question contains STATEMENT–1 (Assertion) and STATEMENT–2 (Reason). Each of these questions has following four choices (1), (2), (3) and (4), only one of which is the correct answer.

1. *Statement–1 is True, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.*
2. *Statement–1 is True, Statement–2 is True; Statement–2 is not a correct explanation for Statement–1.*
3. *Statement–1 is True, Statement–2 is False.*
4. *Statement–1 is False, Statement – 2 is True*

EXAMPLE 1 *Statement–1: If the foci of a hyperbola are at (4, 1) and (– 6, 1) and eccentricity is $\frac{5}{4}$, then the length of its transverse axis is 4.*

Statement–2: Distance between the foci of a hyperbola is equal to the product of its eccentricity and length of the transverse axis.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (d)

SOLUTION Distance between two foci of the hyperbola $= 2ae$

$\therefore \quad 4+6 = 2ae \Rightarrow 10 = 2\times\frac{5}{4}\times a \Rightarrow a = 4.$

Hence, length of transverse axis = 8.

So, statement–1 is not true.

Distance between foci $= 2ae$ = Transverse axis × Eccentricity.

So, statement–2 is true.

EXAMPLE 2 *A vertex of a brach of the hyperbola $x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$, B is one of the end points of its latusrectum and C is the focus of the hyperbola nearest to the point A.*

Statement–1: The area of ΔABC is $\left(\frac{\sqrt{3}}{2}-1\right)$ sq. units.

Statement–2: Eccentricity of the hyperbola is $\frac{\sqrt{3}}{2}$ and length of the conjugate axis is $2\sqrt{2}$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

SOLUTION See Example 28 on page 26.23.

EXAMPLE 3 *Statement–1: Tangents drawn from the point (2, – 1) to the hyperbola $x^2-4y^2=4$ are at right angle.*

Statement–2: The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the circle $x^2+y^2=a^2-b^2$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION Statement–2 is true (See Section 8 on page 26.8).

Since (2, – 1) is a point on the director circle $x^2+y^2=3$ of the hyperbola $x^2-4y^2=4$. So, statement–1 is also true. Also, statement–2 is a correct explation for statement–1.

EXAMPLE 4 *Statement–1: If 5/3 is the eccentricity of a hyperbola, then the eccentricity of its conjugate hyperbola is 5/4.*

Statement–2: If e and e′ are the eccentricities of hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ respectively, then $\frac{1}{e^2}+\frac{1}{e'^2}=1$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (a)

SOLUTION We have, $e^2 = 1+\frac{b^2}{a^2}$ and $e'^2 = 1+\frac{a^2}{b^2}$

$\therefore \quad \frac{1}{e^2}+\frac{1}{e'^2} = \frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2} = 1.$

So, statement–2 is true.

Let $e = \frac{5}{3}$ and $e' = \frac{5}{4}$. Then, $\frac{1}{e^2}+\frac{1}{e'^2} = 1$

So, statement–1 is also true.

Clearly, statement–2 is a correct explanation for statement–1.

EXERCISE

This exercise contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which only one is correct.

1. The equation of the hyperbola whose directrix $x+2y=1$, focus (2, 1) and eccentricity 2 is
   (a) $x^2+16xy-11y^2-12x+6y+21=0$
   (b) $x^2-16xy-11y^2-12x+6y+21=0$
   (c) $x^2-4xy-y^2-12x+6y+21=0$
   (d) none of these

2. The eccentricity of the hyperbola whose latusrectum is 8 and conjugate axis is equal to half the distance between the foci, is
   (a) 4/3 (b) $4/\sqrt{3}$
   (c) $2/\sqrt{3}$ (d) none of these

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3. The eccentricity of the hyperbola can never be equal to
(a) $\sqrt{\frac{9}{5}}$ (b) $2\sqrt{\frac{1}{9}}$ (c) $3\sqrt{\frac{1}{8}}$ (d) 2
[CEE (Delhi) 2008]

4. The eccentricity of a rectangular hyperbola is
(a) 2 (b) $\sqrt{2}$ (c) 0 (d) none of these

5. If t is a parameter, then $x = a\left(t + \frac{1}{t}\right), y = b\left(t - \frac{1}{t}\right)$ represents
(a) an ellipse (b) a circle
(c) a pair of straight lines (d) a hyperbola
[JEE (WB) 2008]

6. If $5x^2 + \lambda y^2 = 20$ represents a rectangular hyperbola, then λ equals
(a) 5 (b) 4 (c) –5 (d) none of these

7. The eccentricity of the hyperbola $3x^2 - 4y^2 = -12$ is
(a) $\sqrt{\frac{7}{3}}$ (b) $\frac{\sqrt{7}}{2}$ (c) $-\sqrt{\frac{7}{3}}$ (d) $-\frac{\sqrt{7}}{2}$

8. The locus of the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = \lambda$ and $\frac{x}{a} - \frac{y}{b} = \frac{1}{\lambda}$ (λ is a variable) is
(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

9. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a hyperbola, if
(a) $\Delta \neq 0, h^2 < ab$ (b) $\Delta \neq 0, h^2 > ab$
(c) $\Delta \neq 0, h^2 = ab$ (d) $\Delta \neq 0, a + b = 0$

10. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a rectangular hyperbola if
(a) $\Delta \neq 0, h^2 > ab, a + b = 0$ (b) $\Delta \neq 0, h^2 < ab, a + b = 0$
(c) $\Delta \neq 0, h^2 = ab, a + b = 0$ (d) none of these

11. The locus of middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is
(a) $3x - 4y = 4$ (b) $3y - 4x + 4 = 0$
(c) $4x - 3y = 3$ (d) $3x - 4y = 2$
[JEE (WB) 2007]

12. The value of m for which the line $y = mx + \frac{25}{\sqrt{3}}$ is a normal to the conic $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is
(a) $\pm\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $-\frac{\sqrt{3}}{2}$ (d) none of these
[JEE (WB) 2006]

13. The value of m for which the line $y = mx + 2$ becomes a tangent to the hyperbola $4x^2 - 9y^2 = 36$ is
(a) $\pm\frac{2}{3}$ (b) $\pm\frac{2\sqrt{2}}{3}$ (c) $\pm\frac{8}{9}$ (d) $\pm\frac{4\sqrt{2}}{3}$
[JEE (WB) 2006]

14. The equation $\frac{x^2}{12-k} + \frac{y^2}{8-k} = 1$ represents
(a) a hyperbola if $k < 8$ (b) an ellipse if $k > 8$
(c) a hyperbola if $8 < k < 12$ (d) none of these

15. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) and $x^2 - y^2 = c^2$ cut at right angles, then
(a) $a^2 + b^2 = 2c^2$ (b) $b^2 - a^2 = 2c^2$
(c) $a^2 - b^2 = 2c^2$ (d) $a^2 b^2 = 2c^2$

16. A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$ is
(a) $y = \frac{3}{\sqrt{7}}x + \frac{15}{\sqrt{7}}$ (b) $y = 3\sqrt{\frac{2}{7}}x + \frac{15}{\sqrt{7}}$
(c) $y = 2\sqrt{\frac{3}{7}}x + 15\sqrt{7}$ (d) none of these

17. The equation of a tangent parallel to $y = x$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is
(a) $x - y + 1 = 0$ (b) $x - y + 2 = 0$
(c) $x + y - 1 = 0$ (d) $x - y + 2 = 0$

18. The diameter of $16x^2 - 9y^2 = 144$ which is conjugate to $x = 2y$ is
(a) $y = \frac{16x}{9}$ (b) $y = \frac{32x}{9}$ (c) $x = \frac{16y}{9}$ (d) $x = \frac{32y}{9}$

19. The tangents from a point $(2\sqrt{2}, 1)$ to the hyperbola $16x^2 - 25y^2 = 400$ include an angle equal to
(a) $\pi/2$ (b) $\pi/4$ (c) π (d) $\pi/3$
[JEE (Orissa) 2005]

20. If the line $y = 3x + \lambda$ touches the hyperbola $9x^2 - 5y^2 = 45$, then the value of λ is
(a) 36 (b) 45 (c) 6 (d) 15

21. The equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ are
(a) $y = \pm x \pm \sqrt{b^2 - a^2}$ (b) $y = \pm x \pm \sqrt{a^2 - b^2}$
(c) $y = \pm x \pm (a^2 - b^2)$ (d) $y = \pm x \pm \sqrt{a^2 + b^2}$

22. A point moves in a plane so that its distances PA and PB from two fixed points A and B in the plane satisfy the relation $PA - PB = k$ ($k \neq 0$), then the locus of P is
(a) a parabola (b) an ellipse
(c) a hyperbola (d) a branch of a hyperbola

23. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is
(a) 1 (b) 5 (c) 7 (d) 9
[AIEEE 2003]

24. The curve represented by $x = a(\cosh\theta + \sinh\theta)$, $y = b(\cosh\theta - \sinh\theta)$, is
(a) a hyperbola (b) an ellipse (c) a parabola (d) a circle

25. The equation of the conic with focus at $(1, -1)$, directrix along $x - y + 1 = 0$ and with eccentricity $\sqrt{2}$ is
(a) $x^2 - y^2 = 1$ (b) $xy = 1$
(c) $2xy - 4x + 4y + 1 = 0$ (d) $2xy + 4x - 4y - 1 = 0$

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26. If the eccentricity of a hyperbola is $\sqrt{3}$, the eccentricity of its conjugate hyperbola is
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{\frac{3}{2}}$ (d) $2\sqrt{3}$

27. The area of the triangle formed by the lines $x+y=0, x-y=0$ and any tangent to the hyperbola $x^2-y^2=a^2$ is
(a) $4a^2$ sq. units (b) $3a^2$ sq. units
(c) $2a^2$ sq. units (d) a^2 sq. units

28. The angle between the asymptotes of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is equal to
(a) $2\tan^{-1}\left(\frac{b}{a}\right)$ (b) $\tan^{-1}\left(\frac{a}{b}\right)$
(c) $2\tan^{-1}\left(\frac{a}{b}\right)$ (d) $\tan^{-1}\left(\frac{b}{a}\right)$

29. If a normal of slope m to the parabola $y^2=4ax$ touches the hyperbola $x^2-y^2=a^2$, then
(a) $m^6-4m^4-3m^2+1=0$ (b) $m^6-4m^4+3m^2-1=0$
(c) $m^6+4m^4-3m^2+1=0$ (d) $m^6+4m^4+3m^2+1=0$

30. The equation of the common tangent to the hyperbola $3x^2-y^2=3$ and to the parabola $y^2=8x$ is
(a) $2x-y-1=0$ (b) $x-2y+1=0$
(c) $2x+y-1=0$ (d) $2x+y+1=0$

31. The circle drawn on the line segment joining the foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ as diameter cuts the asymptotes at
(a) (a,a) (b) (b,a) (c) $(\pm b,\pm a)$ (d) $(\pm a,\pm b)$

32. The angle between the asymptotes of the hyperbola $2x^2-2y^2=9$ is
(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/2$

33. The difference of the focal distances of any point on the hyperbola is equal to its
(a) latusrectum (b) eccentricity
(c) transverse axis (d) conjugate axis

34. If $P(x_1,y_1)$, $Q(x_2,y_2)$, $R(x_3,y_3)$ and $S(x_4,y_4)$ are four concyclic points on the rectangular hyperbola $xy=c^2$, the coordinates of orthocentre of the ΔPQR are
(a) $(x_4,-y_4)$ (b) (x_4,y_4) (c) $(-x_4,-y_4)$ (d) $(-x_4,y_4)$

35. The points of intersection of the curves whose parametric equations are $x=t^2+1$, $y=2t$ and $x=2s$, $y=2/s$ is given by
(a) $(1,-3)$ (b) $(2,2)$ (c) $(-2,4)$ (d) $(1,2)$

36. The equation of the hyperbola whose foci are $(6,5)$, $(-4,5)$ and eccentricity $5/4$, is
(a) $\frac{(x-1)^2}{16}-\frac{(y-5)^2}{9}=1$ (b) $\frac{x^2}{16}-\frac{y^2}{9}=1$
(c) $\frac{(x-1)^2}{16}-\frac{(y-5)^2}{9}=-1$ (d) none of these

37. The line $5x+12y=9$ touches the hyperbola $x^2-9y^2=9$ at the point
(a) $(-5,4/3)$ (b) $(5,-4/3)$
(c) $(3,-1/2)$ (d) none of these

38. The length of the latusrectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$, is
(a) $\frac{2a^2}{b}$ (b) $\frac{2b^2}{a}$ (c) $\frac{b^2}{a}$ (d) $\frac{a^2}{b}$

39. P is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then $OT\cdot ON$ is equal to
(a) e^2 (b) a^2 (c) b^2 (d) b^2/a^2

40. If the tangent at the point $(2\sec\theta, 3\tan\theta)$ to the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=1$ is parallel to $3x-y+4=0$, then the value of θ, is
(a) $45°$ (b) $60°$ (c) $30°$ (d) $75°$

41. The eccentricity of the hyperbola $\frac{\sqrt{1999}}{3}(x^2-y^2)=1$, is
(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) $\sqrt{3}$

42. The tangents to the hyperbola $x^2-y^2=3$ are parallel to the straight line $2x+y+8=0$ at the following points:
(a) $(2,1),(1,2)$ (b) $(2,-1),(-2,1)$
(c) $(-2,-1),(1,2)$ (d) $(-2,-1),(-1,-2)$

43. If the tangent at the point $(a\sec\alpha, b\tan\alpha)$ to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the transverse axis at T, then the distance of T from a focus of the hyperbola is
(a) $b(e-\cos\alpha)$ (b) $b(e+\cos\alpha)$
(c) $a(e+\cos\alpha)$ (d) $\sqrt{a^2e^2+b^2\cot^2\alpha}$

44. The equation of the hyperbola of given transverse axis 2a with its vertex mid-way between the centre and the corresponding focus is
(a) $3x^2-y^2=a^2$ (b) $3x^2-y^2=3a^2$
(c) $x^2-3y^2=a^2$ (d) $x^2-3y^2=a^2$

45. The locus of the mid-points of the chords of the circle $x^2+y^2=16$ which are tangents to the hyperbola $9x^2-16y^2=144$ is
(a) $(x^2+y^2)^2=16x^2-9y^2$ (b) $(x^2+y^2)^2=9x^2-16y^2$
(c) $(x^2-y^2)^2=16x^2-9y^2$ (d) none of these

46. If the latusrectum subtends a right angle at the centre of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then its eccentricity is
(a) $\frac{\sqrt{13}}{2}$ (b) $\frac{\sqrt{5}-1}{2}$ (c) $\frac{\sqrt{5}+1}{2}$ (d) $\frac{\sqrt{3}+1}{2}$

47. If the latusrectum of a hyperbola through one focus subtends $60°$ angle at the other focus, then its eccentricity e is
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{6}$

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48. If the latusrectum of a hyperbola forms an equilateral triangle with the vertex at the centre of the hyperbola, then the eccentricity of the hyperbola is
(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{11}+1}{2}$ (c) $\frac{\sqrt{13}+1}{2\sqrt{3}}$ (d) $\frac{\sqrt{13}-1}{2\sqrt{3}}$

49. Let LL' be the latusrectum through the focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and A' be the farther vertex. If $\Delta A'LL'$ is equilateral, then the eccentricity of the hyperbola is
(a) $\sqrt{3}$ (b) $\sqrt{3}+1$ (c) $\frac{\sqrt{3}+1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}+1}{\sqrt{3}}$

50. The angle between the asymptotes of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, is
(a) $\pi-2\tan^{-1}\frac{3}{4}$ (b) $\pi-2\tan^{-1}\frac{3}{2}$
(c) $\tan^{-1}\frac{3}{4}$ (d) $\pi-2\tan^{-1}\frac{4}{3}$

Answers

1. (b) 2. (c) 3. (b) 4. (b) 5. (d) 6. (c) 7. (a)
8. (d) 9. (b) 10. (c) 11. (a) 12. (a) 13. (b) 14. (c)
15. (c) 16. (b) 17. (a) 18. (b) 19. (a) 20. (c) 21. (b)
22. (c) 23. (c) 24. (a) 25. (c) 26. (c) 27. (d) 28. (a)
29. (d) 30. (d) 31. (d) 32. (d) 33. (c) 34. (d) 35. (b)
36. (a) 37. (b) 38. (a) 39. (b) 40. (c) 41. (a) 42. (b)
43. (c) 44. (b) 45. (a) 46. (c) 47. (b) 48. (c) 49. (d)
50. (a)

CHAPTER TEST

Each question has four choices (a), (b), (c) and (d) out of which only one is correct. Mark the correct alternative.

1. The equation of the tangent to the conic $x^2-y^2-8x+2y+11=0$ at $(2,1)$, is
(a) $x+2=0$ (b) $2x+1=0$
(c) $x-2=0$ (d) $x+y+1=0$

2. The value of m for which $y=mx+6$ is a tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{49}=1$, is
(a) $\sqrt{\frac{17}{20}}$ (b) $\sqrt{\frac{20}{17}}$ (c) $\sqrt{\frac{3}{20}}$ (d) $\sqrt{\frac{20}{3}}$

3. The equation of the tangent to the hyperbola $4y^2=x^2-1$ at the point $(1,0)$, is
(a) $x=1$ (b) $y=1$ (c) $y=4$ (d) $x=4$

4. The number of normals to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from an external point is
(a) 2 (b) 4 (c) 6 (d) 5

5. If e and e_1, are the eccentricities of the hyperbolas $xy=c^2$ and $x^2-y^2=c^2$, then $e^2+e_1^2$ is equal to
(a) 1 (b) 4 (c) 6 (d) 8

6. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then, $CP^2+CQ^2+CR^2+CS^2=$
(a) r^2 (b) $2r^2$ (c) $3r^2$ (d) $4r^2$

7. The equation of the pair of asymptotes of the hyperbola $xy-4x+3y=0$ is
(a) $xy-4x+3y-1=0$ (b) $xy-4x+3y-10=0$
(c) $xy-4x+3y-12=0$ (d) none of these

8. If the latusrectum of the hyperbola $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ is $\frac{9}{2}$, then its eccentricity is
(a) 4/5 (b) 5/4 (c) 3/4 (d) 4/3

9. If the chords of the rectangular hyperbola $x^2-y^2=a^2$ touch the parabola $y^2=4ax$, then the locus of their mid-points is
(a) $x^2(y-a)=y^3$ (b) $y^2(x-a)=x^3$
(c) $x(y^2-a)=y$ (d) $y(x^2-a)=x$

10. Tangents drawn from the point (c,d) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ make angles α and β with the x-axis. If $\tan\alpha\tan\beta=1$, then $c^2-d^2=$
(a) a^2-b^2 (b) b^2-a^2 (c) a^2+b^2 (d) none of these

11. If the tangent at (α,β) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cuts the auxiliary circle at points whose ordinates are y_1 and y_2, then $\frac{1}{y_1}+\frac{1}{y_2}=$
(a) $\frac{1}{\alpha}$ (b) $\frac{2}{\alpha}$ (c) $\frac{1}{\beta}$ (d) $\frac{2}{\beta}$

12. If the chords of contact of tangents drawn from P to the hyperbola $x^2-y^2=a^2$ and its auxiliary circle are at right angle, then P lies on
(a) $x^2-y^2=3a^2$ (b) $x^2-y^2=2a^2$
(c) $x^2-y^2=0$ (d) $x^2-y^2=1$

13. If the tangent at a point P on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cuts one of its directrices at Q, then the angle made by PQ at the corresponding focus, is

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(a) 45° (b) 30° (c) 60° (d) 90°

14. The mid-point of the chord intercepted by the hyperbola $9x^2 - 16y^2 = 144$ on the line $9x - 8y - 10 = 0$, is
(a) (1, 2) (b) (−1, 2) (c) (−2, 1) (d) (2, 1)

15. If the chord of contact of tangents drawn from P to the parabola $y^2 = 4ax$ touches the rectangular hyperbola $x^2 - y^2 = a^2$, then P lies on
(a) $4x^2 - y^2 = a^2$ (b) $y^2 - 4x^2 = 4a^2$
(c) $4x^2 + y^2 = 4a^2$ (d) $4y^2 - x^2 = 4a^2$

16. If the tangent at any point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the lines $bx - ay = 0$ and $bx + ay = 0$ in the points Q and R, then $CQ \cdot CR =$
(a) $a^2 b^2$ (b) $a^2 - b^2$
(c) $a^2 + b^2$ (d) none of these

17. If $lx + my + n = 0$ is a tangent to the rectangular hyperbola $xy = c^2$, then
(a) $l < m < 0$ (b) $l > 0, m < 0$
(c) $l < 0, m > 0$ (d) none of these

18. A tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cuts the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in P and Q. The locus of the mid-point of PQ is
(a) $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ (b) $\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
(c) $\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^2 = \frac{2x^2 y^2}{a^2 b^2}$ (d) none of these

19. The product of lengths of perpendicular from any point on the hyperbola $x^2 - y^2 = 8$ to its asymptotes is
(a) 8 (b) 6 (c) 2 (d) 4

20. The angle between the asymptotes of the hyperbola $3x^2 - y^2 = 3$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{5}$ (c) $\frac{2\pi}{3}$ (d) $\frac{2\pi}{5}$

21. The area of the triangle formed by any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with its asymptotes is
(a) $4a^2 b^2$ (b) $a^2 b^2$ (c) $4ab$ (d) ab

22. If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, then the relation between e_1 and e_2 is
(a) $3e_1^2 + e_2^2 = 2$ (b) $e_1^2 + 2e_2^2 = 3$
(c) $2e_1^2 + e_2^2 = 3$ (d) $e_1^2 + 3e_2^2 = 2$

23. The distance between the directrices of the hyperbola $x = 8 \sec\theta, y = 8 \tan\theta$, is
(a) $8\sqrt{2}$ (b) $16\sqrt{2}$ (c) $4\sqrt{2}$ (d) $6\sqrt{2}$

24. The straight line $x + y = \sqrt{2}\,p$ will touch the hyperbola $4x^2 - 9y^2 = 36$, if
(a) $p^2 = 2$ (b) $p^2 = 5$
(c) $5p^2 = 2$ (d) $2p^2 = 5$

25. The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{3} - \frac{y^2}{1} = 1$, is
(a) $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 3$
(c) $x^2 - y^2 = 3$ (d) $x^2 + y^2 = 4$

26. The product of perpendicular distances from any point on the hyperbola $9x^2 - 16y^2 = 144$ to its asymptotes is
(a) $\frac{25}{12}$ (b) $\frac{144}{25}$ (c) $\frac{144}{7}$ (d) $\frac{25}{144}$

27. The angle between the asymptotes of the hyperbola $27x^2 - 9y^2 = 24$ is
(a) 30° (b) 120° (c) 45° (d) 240°

28. The equation of the chord of contact of tangents from (1, 2) to the hyperbola $3x^2 - 4y^2 = 3$ is
(a) $3x - 16y = 3$ (b) $3x - 8y - 3 = 0$
(c) $\frac{x}{3} - \frac{y}{4} = 1$ (d) $\frac{x}{4} - \frac{y}{3} = 1$

29. Equation of the hyperbola whose vertices are $(\pm 3, 0)$ and foci at $(\pm 5, 0)$, is
(a) $16x^2 - 9y^2 = 144$ (b) $9x^2 - 16y^2 = 144$
(c) $25x^2 - 9y^2 = 225$ (d) $9x^2 - 25y^2 = 81$

30. The length of the semi-transverse axis of the rectangular hyperbola $xy = 32$ is
(a) 32 (b) 16 (c) 64 (d) 8

Answers

1. (c) 2. (a) 3. (a) 4. (b) 5. (b) 6. (d) 7. (c)
8. (b) 9. (b) 10. (c) 11. (d) 12. (c) 13. (d) 14. (d)
15. (c) 16. (c) 17. (a) 18. (a) 19. (d) 20. (c) 21. (d)
22. (c) 23. (a) 24. (d) 25. (a) 26. (b) 27. (b) 28. (b)
29. (a) 30. (d)

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