BRAIN MAP Rotational Motion

Centre of mass : Centre of mass of a system is defined as that point where whole mass of the system is supposed to be concentrated for the translational motion of the system.

Two particle system : The position vector of CM of two particle system is,

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

n particle system : In case of n particle system, the position vector of CM is

$$\vec{R} = \frac{\sum_{i=1}^{i=n} m_i\vec{r}_i}{\sum_{i=1}^{i=n} m_i}$$

Velocity of centre of mass of a system of particle, $\vec{v}_{cm} = \frac{d\vec{R}}{dt}$

Acceleration of centre of mass of a system of particles, $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}$

Sum of all external forces acting on the system of particles, $\vec{F}_{ext} = M\vec{a}_{cm}$

Angular velocity : It is the time rate of change of angular displacement, $\vec{\omega} = \frac{d\vec{\theta}}{dt}$

$\vec{v} = \vec{\omega} \times \vec{r}$

Angular acceleration : It is the time rate of change of angular velocity,

$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$, $\vec{a} = \vec{\alpha} \times \vec{r}$

Equations of rotational motion :

- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- $\omega^2 - \omega_0^2 = 2\alpha\theta$

Law of conservation of angular momentum : If no external torque acts on a system, the total angular momentum of the system remains unchanged.

If $\vec{\tau}_{ext} = 0$; $\frac{d\vec{L}}{dt} = 0$

$\therefore$ $\vec{L} =$ constant

Also, $I_1\omega_1 = I_2\omega_2$

Moment of force or Torque : It is equal to the product of force and perpendicular distance

$\vec{\tau} = \vec{r} \times \vec{F}$; $|\vec{\tau}| = rF\sin\theta = rF_\perp$

Relation between torque and angular acceleration

$\vec{\tau} = I\vec{\alpha}$

Angular momentum : It is the product of linear momentum and perpendicular distance of line of action of linear momentum vector from the axis of rotation, in vector form

$\vec{L} = \vec{r} \times \vec{p}$

Angular momentum of a rigid body rotating about an axis

$\vec{L} = I\vec{\omega}$

Relation between torque and angular momentum : The net external torque on a body is equal to the rate of change of angular momentum

$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$

Moment of inertia : It is the quantity that measures the inertia of rotational motion of the body.

For a particle, $I = mr^2$

$I = \int r^2 dm$

Unit : kg m^2

Radius of gyration : It is equal to the root mean square distance of the constituent particles of the body from the given axis,

$$K = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}, \quad I = MK^2$$

Theorem of parallel axis

$I = I_{cm} + mh^2$

Theorem of perpendicular axis

$I_z = I_x + I_y$

Applicable to planar bodies only

Kinetic energy and M.I. : The energy possessed by the body on account of its rotation about a given axis is the kinetic energy of rotation

i.e., K.E. $= \frac{1}{2}I\omega^2$

Work done by torque τ in rotating a body by an angle

$= \int_0^\theta \tau d\theta$

Instantaneous power, $P = \tau\omega$

M.I. of few bodies of regular shape :

- M.I. of a rod about an axis through its CM and perpendicular to rod, $I = \frac{1}{12}Ml^2$
- M.I. of a circular ring about an axis through its centre and perpendicular to its plane, $I = MR^2$
- M.I. of a circular disc about an axis through its centre and perpendicular to its plane, $I = \frac{1}{2}MR^2$
- M.I. of a right circular solid cylinder about its symmetry axis, $I = \frac{1}{2}MR^2$
- M.I. of a right circular hollow cylinder about its axis $= MR^2$
- M.I. of a solid sphere about its diameter, $I = \frac{2}{5}MR^2$
- M.I. of a spherical shell about its diameter, $I = \frac{2}{3}MR^2$

Equilibrium of rigid body :

- Net external force acting on the body must be zero *i.e.,* $\Sigma\vec{F}_{ext} = 0$
- Net external torque on the body must be zero *i.e.,* $\vec{\tau}_{ext} = 0$

Centre of gravity : It is the point where the weight of the body act and total gravitational torque on the body is zero

i.e., $\sum_{i=1}^{i=n} m_i\vec{r}_1 = 0$

Principle of Moments : In rotational equilibrium the algebraic sum of the moments of all forces acting on the body, about a fixed point is zero.

Symbols used

m_1, m_2 = mass of particles

$\vec{R}$ = position vector of centre of mass

$\vec{r}_1$, $\vec{r}_2$ = position vector of particles

M = total mass

$\vec{\omega}$ = angular velocity

$d\vec{\theta}$ = angular displacement

$\vec{\tau}$ = moment of force or torque

$\vec{L}$ = angular momentum

$\vec{p}$ = linear momentum

$\vec{\omega}_0$ = initial angular velocity

$\vec{\alpha}$ = angular acceleration

K = radius of gyration

n = number of particles

I = moment of inertia

I_{cm} = moment of inertia about centre of mass

I_x, I_y, I_z = moment of inertia about x, y and z axes respectively

M.I. = moment of inertia

l = length of the rod