

Trishna's

IIT JEE

Super Course in

physics

Mechanics I



Super Course in Physics

MECHANICS I

for the IIT-JEE

Volume 1

Trishna Knowledge Systems
A division of
Triumphant Institute of Management Education Pvt. Ltd

PEARSON

Chandigarh • Delhi • Chennai

**T.I.M.E.**

Triumphant Institute of
Management Education Pvt. Ltd.

T.I.M.E. is the No. 1 institute for the IIT-JEE courses. It is the largest, and the most successful, classroom-based entrance exam training institute in India. For more details, visit www.time4education.com

The aim of this publication is to supply information taken from sources believed to be valid and reliable. This is not an attempt to render any type of professional advice or analysis, nor is it to be treated as such. While much care has been taken to ensure the veracity and currency of the information presented within, neither the publisher nor its authors bear any responsibility for any damage arising from inadvertent omissions, negligence or inaccuracies (typographical or factual) that may have found their way into this book.

Copyright © Trishna Knowledge Systems, 2011

Licensees of Pearson Education in South Asia

No part of this eBook may be used or reproduced in any manner whatsoever without the publisher's prior written consent.

This eBook may or may not include all assets that were part of the print version. The publisher reserves the right to remove any material present in this eBook at any time.

ISBN 9788131759721

eISBN 9789332510227

Head Office: A-8(A), Sector 62, Knowledge Boulevard, 7th Floor, NOIDA 201 309, India

Registered Office: 11 Local Shopping Centre, Panchsheel Park, New Delhi 110 017, India

Contents

Chapter 1	Units, Dimensions and Error Analysis in Experiments	1.1—1.56
	STUDY MATERIAL	
	• <i>Physical Quantities and Units</i> • <i>Dimensions and Dimensional Analysis</i> • <i>Errors and Approximations—Calculation of Fractional and Percentage Error</i> • <i>Error Analysis in Experiments</i>	
Chapter 2	Motion in One Dimension	2.1—2.90
	STUDY MATERIAL	
	• <i>State of Rest and Motion</i> • <i>Classification of Motion into One, Two and Three Dimensions</i> • <i>Motion in One Dimension</i> • <i>Kinematic Equations of Motion (For Uniformly Accelerated or Uniformly Decelerated Motions Only)</i> • <i>Motion Under Gravity</i> • <i>Relative Motion of Particles in One Dimensional Motion</i> • <i>Graphical Representations of One Dimensional Motion</i>	
Chapter 3	Motion in Two Dimensions	3.1—3.86
	STUDY MATERIAL	
	• <i>Analysis of Two Dimensional Motion</i> • <i>Projectile Motion</i> • <i>For Additional Reading</i> • <i>Relative Motion in Two Dimensions</i> • <i>Circular Motion</i>	
Chapter 4	Laws of Motion	4.1—4.126
	STUDY MATERIAL	
	• <i>Introductory Concepts</i> • <i>Newton's Laws of Motion</i> • <i>Basic Forces in Nature</i> • <i>Common Forces in Mechanics</i> • <i>Application of Laws of Motion to Solve Problems</i> • <i>Problems Involving Constraints</i> • <i>Reference Frame: Inertial and Accelerated Frames</i> • <i>Friction</i> • <i>Circular Motion and Centripetal Force</i>	

Preface

The IIT-JEE, the most challenging amongst national level engineering entrance examinations, remains on the top of the priority list of several lakhs of students every year. The brand value of the IITs attracts more and more students every year, but the challenge posed by the IIT-JEE ensures that only the *best* of the aspirants get into the IITs. Students require thorough understanding of the fundamental concepts, reasoning skills, ability to comprehend the presented situation and exceptional problem-solving skills to come on top in this highly demanding entrance examination.

The pattern of the IIT-JEE has been changing over the years. Hence an aspiring student requires a step-by-step study plan to master the fundamentals and to get adequate practice in the various types of questions that have appeared in the IIT-JEE over the last several years. Irrespective of the branch of engineering study the student chooses later, it is important to have a sound conceptual grounding in Mathematics, Physics and Chemistry. A lack of proper understanding of these subjects limits the capacity of students to solve complex problems thereby lessening his/her chances of making it to the top-notch institutes which provide quality training.

This series of books serves as a source of learning that goes beyond the school curriculum of Class XI and Class XII and is intended to form the backbone of the preparation of an aspiring student. These books have been designed with the objective of guiding an aspirant to his/her goal in a clearly defined step-by-step approach.

- **Master the Concepts and Concept Strands!**

This series covers all the concepts in the latest IIT-JEE syllabus by segregating them into appropriate units. The theories are explained in detail and are illustrated using solved examples detailing the different applications of the concepts.

- **Let us First Solve the Examples—Concept Connectors!**

At the end of the theory content in each unit, a good number of “Solved Examples” are provided and they are designed to give the aspirant a comprehensive exposure to the application of the concepts at the problem-solving level.

- **Do Your Exercise—Daily!**

Over 200 unsolved problems are presented for practice at the end of every chapter. Hints and solutions for the same are also provided. These problems are designed to sharpen the aspirant’s problem-solving skills in a step-by-step manner.

- **Remember, Practice Makes You Perfect!**

We recommend you work out ALL the problems on your own – both solved and unsolved – to enhance the effectiveness of your preparation.

A distinct feature of this series is that unlike most other reference books in the market, this is not authored by an individual. It is put together by a team of highly qualified faculty members that includes IITians, PhDs etc from some of the best institutes in India and abroad. This team of academic experts has vast experience in teaching the fundamentals and their application and in developing high quality study material for IIT-JEE at T.I.M.E. (Triumphant Institute of Management Education Pvt. Ltd), the number 1 coaching institute in India. The essence of the combined knowledge of such an experienced team is what is presented in this self-preparatory series. While the contents of these books have been organized keeping in mind the specific requirements of IIT-JEE, we are sure that you will find these useful in your preparation for various other engineering entrance exams also.

We wish you the very best!

CHAPTER

1

UNITS, DIMENSIONS AND ERROR ANALYSIS IN EXPERIMENTS

■■ CHAPTER OUTLINE

Preview

STUDY MATERIAL

Physical Quantities and Units

Dimensions and Dimensional Analysis

- Concept Strands (1-4)

Errors and Approximations—Calculation of Fractional and Percentage Error

- Concept Strand 5

Error Analysis in Experiments

- Concept Strands (6-13)

CONCEPT CONNECTORS

- 10 Connectors

TOPIC GRIP

- Subjective Questions (5)
- Straight Objective Type Questions (5)
- Assertion–Reason Type Questions (3)
- Linked Comprehension Type Questions (3)
- Multiple Correct Objective Type Questions (3)
- Matrix-Match Type Question (1)

IIT ASSIGNMENT EXERCISE

- Straight Objective Type Questions (30)
- Assertion–Reason Type Questions (5)
- Linked Comprehension Type Questions (9)
- Multiple Correct Objective Type Questions (4)
- Matrix-Match Type Questions (2)

PHYSICAL QUANTITIES AND UNITS

Physics is the study of natural phenomena based on measurements. A physical quantity is one which has got a precise definition and which is used for explaining a natural phenomenon. Physical quantities should be amenable for measurement, either direct or indirect.

Unit is a standard quantity of the same kind with which a physical quantity is compared, for measuring it. For example: if length is to be measured we compare it with a standard metre scale.

Magnitude of a physical quantity 'X' = nu, where n = a real number and u = unit of measurement. For some physical quantities, there may be only a number and no unit. Example: refractive index, specific gravity. Such physical quantities are pure numbers.

It is possible to measure a physical quality in different units.

If n_1 is the numerical value in unit u_1 , n_2 in unit u_2 etc., then $x = n_1 u_1 = n_2 u_2 = \text{constant}$. Larger the unit, smaller the numerical value and vice versa.

Fundamental and derived units

The physical quantities mass, length and time are called fundamental physical quantities. The units of the fundamental physical quantities are called fundamental units.

Any physical quantity which can be derived from the fundamental physical quantities is called a derived physical quantity. The units of derived physical quantities are called derived units. Example, speed (m s^{-1}), acceleration (m s^{-2}) force (kg m s^{-2}) etc., are derived quantities.

The basic units in SI are units of **mass, length, time, temperature, electric current, luminous intensity and quantity of matter.**

The supplementary units in SI are units of **plane angle and solid angle.**

Unit systems

(a) CGS System

Length = centimetre (cm)
Mass = gram (g)
Time = second (s)

(b) FPS System

Length = foot (ft)
Mass = pound (lb)
Time = second (s)

(c) MKS System

Length = metre (m)
Mass = kilogram (kg)
Time = second (s)

(d) SI (*Le Systeme International d' unites*)

Length = metre (m)
Mass = kilogram (kg)
Time = second (s)
Temperature = kelvin (K)
Electric Current = ampere (A)
Luminous Intensity = candela (cd)
Quantity of matter = mole (mol)

The two supplementary units in SI are

Plane angle = radian (rad)
Solid angle = steradian (sr)

Definition of SI units

(a) Basic units

Metre (m) is the length equal to 1,650,763.3 wavelengths in vacuum of the radiations emitted due to transition between the levels $2p_{10}$ and $5d_5$ of the isotope of Krypton 86 atom.

It is the distance travelled by light in vacuum in $\frac{1}{299792458}$ second.

Kilogram (kg) is the mass of a platinum-iridium cylinder (90% Pt + 10% Ir) having its diameter equal to its height and each equal to 3.9 cm.

Second (s) is the time taken by 9,192,631,770 vibrations corresponding to the transition between two hyperfine levels of caesium 133 atom.

Kelvin (K) is the unit of temperature defined as $\frac{1}{273.16}$ fraction of the thermodynamic temperature at the triple point of water.

Candela (cd) is the luminous intensity in a perpendicular direction of a surface of $\frac{1}{6,00,000}$ m^2 of a black body

at a temperature of freezing platinum (2046.65 K) under a pressure of 1,01,325 newton per square metre. It is also the luminous intensity in a given direction due to a source which emits monochromatic radiation of frequency

540×10^{12} Hz and of which radiant intensity in that direction is $\frac{1}{683}$ watt per steradian.

Ampere (A) is that constant current which when passed through each of two straight parallel conductors of infinite length and of negligible cross-section held one metre apart in vacuum, exert a mutual force of 2×10^{-7} newton per metre.

Mole (mol) is the amount of substance which contains the same number of elementary units (atoms) as there are atoms in 0.012 kg of carbon 12. This number is called Avogadro constant and its value is $6.022045 \times 10^{23} \text{ mol}^{-1}$.

(b) Supplementary units

Radian (rad) is the plane angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

Steradian (sr) is the solid angle subtended at the centre of a sphere by its surface whose area is equal to the square of the radius of the sphere.

Rules for writing units and their symbols

The full name of unit in honour of great scientists should be written in small letters.

The symbols or abbreviations of unit, named after great scientists should be written with capital initial letter. Example, 'W' for watt, 'N' for newton, 'A' for ampere etc.,

The symbol of unit should be written either in full or in agreed abbreviation. Example, for second either use s or second and not sec.

Plurals should not be used for units. N s means newton second (impulse) and not newton (force).

Double prefixes shall not be used. Example, ' $\mu\mu\text{s}$ ' is not allowed; write ' μs '.

No hyphens (-) shall be used in units. Example, N-m not permitted.

Half space to be kept between different units in a derived unit. Example, N m.

No more than one slash shall be used in a unit. Example, m/s^2 and not m/s/s . It is better to express this as m s^{-2} .

No full stop to be used after units. Example, N and not N.

Order of magnitude

The order of magnitude of a numerical quantity (N) is the nearest power of 10 to which the value can be written.

The order of magnitude of numerical quantity N is $\times$ such that $0.5 \leq \frac{N}{10^x} \leq 5$

Large units of length

(a) Astronomical unit (AU)

One astronomical unit is the average distance between the Earth and Sun.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m} \sim 1.5 \times 10^{11} \text{ m}$$

(b) Light year (ly)

One light year is the total distance traveled by light at a speed of $3 \times 10^8 \text{ m s}^{-1}$ in vacuum during one year.

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

(c) Parallaxic second (parsec)

One parallaxic second is the distance at which an arc of length equal to one astronomical unit (1 AU) subtends an angle of one second at a point.

$$1 \text{ parsec} = 3.084 \times 10^{16} \text{ m} = 3.26 \text{ ly}$$

The largest unit of length is parsec.

Small units of length

(a) micron (μm)

$$1 \text{ micron} = 10^{-6} \text{ m}$$

(b) nanometer (nm)

$$1 \text{ nanometer} = 10^{-9} \text{ m}$$

(c) angstrom ($\text{\AA}$)

$$1 \text{ angstrom} = 10^{-10} \text{ m}$$

(d) fermi

$$1 \text{ fm} = 10^{-15} \text{ m}$$

It is the unit of length in which size of nucleus is expressed.

Smallest unit of length is fermi (fm).

Smallest unit of area is barn. ($1 \text{ barn} = 10^{-28} \text{ m}^2$). It is used for measuring the cross-sectional area of nuclei.

1.4 Units, Dimensions and Error Analysis in Experiments

Other units of mass

- (a) **Atomic mass unit (amu or u)**

$$1 \text{ u} = 1.67 \times 10^{-27} \text{ kg}$$

- (b) **1 quintal** = 100 kg

- (c) **1 metric ton** = 1000 kg

- (d) **Chandrasekhar Limit (CSL)**

$$1 \text{ CSL} = 1.4 \text{ times mass of Sun.}$$

This is the largest unit of mass.

Other units of time

- (a) **Solar day** is the time taken by Earth to complete one rotation about its own axis with respect to the Sun.

- (b) **Sidereal day** is the time taken by Earth to complete one rotation about its axis with respect to a fixed star.

- (c) **Solar year** is the time taken by the Earth to complete one revolution around the Sun.

$$\begin{aligned} \text{One solar year} &= 365.25 \text{ solar day} \\ &= 366.25 \text{ sidereal day} \end{aligned}$$

- (d) **Lunar month** is the time taken by Moon to complete one revolution around the Earth.

$$1 \text{ lunar month} = 27.3 \text{ day}$$

- (e) **Shake** 1 shake = 10^{-8} s

DIMENSIONS AND DIMENSIONAL ANALYSIS

The dimensions of a physical quantity are the powers to which the fundamental units have to be raised to obtain its units.

The expression representing a physical quantity in terms of the powers of the fundamental units is known as the dimensional formula of that physical quantity.

The equation obtained by equating the symbol of a physical quantity with its dimensional formula is called dimensional equation.

Principle of homogeneity

Principle of homogeneity states that the dimensions of the fundamental quantities on two sides of a physical relation must be same.

Example, If $M^a L^b T^c = M^x L^y T^z$, then according to principle of homogeneity, $a = x$, $b = y$ and $c = z$

This means only physical quantities of the same type can be added or subtracted.

Uses of dimensional analysis

(a) To convert one system of units into another

If a physical quantity $x = nu = n_1 u_1 = n_2 u_2$ etc., $x = M^a L^b T^c$ is its dimensional equation, M_1, L_1, T_1 are the units in system 1, M_2, L_2, T_2 are the units in system 2, then

$$\begin{aligned} n_1 u_1 &= n_2 u_2 \\ n_2 &= n_1 \left(\frac{u_1}{u_2} \right) = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c \end{aligned}$$

(b) Checking the correctness of a given physical relation

An equation will be dimensionally correct only if the dimensions of fundamental units on LHS = dimensions of fundamental units on RHS

(c) To derive relationships between various physical quantities

CONCEPT STRANDS

Concept Strand 1

Write the dimensional formula of velocity.

Solution

Velocity is expressed in m / s

$$\Rightarrow [v] = [m / s] = LT^{-1}$$

Hence the dimensional formula of velocity is LT^{-1} .

The above example shows a special case of the principle of homogeneity, which states that the quantities which are compared, equated, added or subtracted will have the same dimensions.

Concept Strand 2

Write the dimensions of force.

Solution

$$F = ma$$

$$[F] = [ma]$$

$$[a] = \left[\frac{s}{t^2} \right] = LT^{-2} \Rightarrow [F] = MLT^{-2}$$

Hence the dimensions of force are 1 in mass(M), 1 in length (L) and -2 in time (T).

Concept Strand 3

Find the relation for the force on a small sphere dropped in a viscous fluid given that it depends on its radius, speed and viscosity in the fluid.

Solution

$F = K \cdot r^a \eta^b v^c$ where K is the constant of proportionality

$$[r] = L$$

$$[\eta] = ML^{-1} T^{-1}$$

$$[v] = LT^{-1}$$

$$\Rightarrow [F] = MLT^{-2}$$

$$\Rightarrow MLT^{-2} = M^b L^{a-b+c} T^{-b-c}$$

Equating powers on either side,

$$b = 1$$

$$a - b + c = 1$$

$$a + c = 2$$

$$-b - c = -2$$

$$-c = -1$$

$$c = 1$$

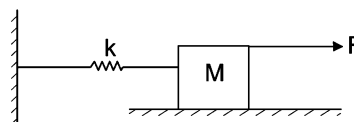
$$\Rightarrow a = 1$$

$$\Rightarrow F = K r \eta v$$

In order to apply the principles of dimensional analysis, we should not have any dimensional constant in the relation.

Concept Strand 4

The maximum velocity attained by a mass acted upon by a constant force depends on k, m, F. Find v.

**Solution**

$$\text{Let } v = F^a m^b k^c$$

$$[v] = LT^{-1}$$

$$[F] = MLT^{-2}$$

$$[m] = M$$

$$[k] = MT^{-2}$$

$$\Rightarrow LT^{-1} = (MLT^{-2})^a (M)^b (MT^{-2})^c$$

$$= M^{a+b+c} L^a T^{-2a-2c}$$

Equating powers on both sides, $a = 1$

$$-2a - 2c = -1$$

$$2c = 1 - 2a = -1 \Rightarrow$$

$$c = \frac{-1}{2}$$

$$a + b + c = 0$$

$$b = -a - c = -1 + \frac{1}{2}$$

$$= \frac{-1}{2} \Rightarrow v \propto \frac{F}{\sqrt{mk}}$$

Groups of quantities having same dimensional formula

- (a) Angular momentum (L), Planck's constant (h)
- (b) Frequency, angular velocity, velocity gradient, rate of nuclear disintegration, Hubble constant (H)
- (c) Linear momentum, impulse
- (d) Latent heat and gravitational potential
- (e) Surface tension, surface energy, spring constant, force constant

- (f) Work, energy, torque, couple
- (g) Stress, pressure, Young's modulus, Bulk modulus, energy density, Rigidity modulus
- (h) Velocity, speed, distance covered in the nth second
- (i) Acceleration, gravitational intensity

It must be noted that there can be physical quantities with units but not having any dimension.

However, no physical quantity can exist with dimension and without unit.

1.6 Units, Dimensions and Error Analysis in Experiments

The following quantities have units but do not have dimensions:

Table 1

Sl.No	Quantity	Unit
(i)	Plane angle	radian (rad)
(ii)	Solid angle	steradian (sr)
(iii)	Shear strain	radian (rad)
(iv)	Luminous efficiency	lumen/watt
(v)	Loudness of sound	bel (B)

Dimensions of some constants

- (a) Gravitational constant (G) = $M^{-1}L^3T^{-2}$
- (b) Planck's constant (h) = ML^2T^{-1}
- (c) Gas constant (R) = $ML^2T^{-2}\theta^{-1}mol^{-1}$
- (d) Boltzmann Constant (k) = $ML^2T^{-2}\theta^{-1}$
- (e) Stefan's Constant (σ) = $MT^{-3}\theta^{-4}$
- (f) Wien's Constant (b) = $L\theta$
- (g) Rydberg Constant (R_H) = L^{-1}
- (h) Avogadro Number (N_A) = mol^{-1}
- (i) Bohr Magnetron (μ_B) = L^2I
- (j) Hubble Constant (H) = T^{-1}

Dimensional formula of some physical quantities

Table 2

Quantity	Formula	Dimensional formula
Area	length $\times$ breadth	$M^0L^2T^0$
Volume	length $\times$ breadth $\times$ height	$M^0L^3T^0$
Density	$\frac{\text{mass}}{\text{volume}}$	$M^1L^{-3}T^0$

Quantity	Formula	Dimensional formula
Velocity	$\frac{\text{displacement}}{\text{time}}$	$M^0L^1T^{-1}$
Acceleration	$\frac{\text{velocity}}{\text{time}}$	$M^0L^1T^{-2}$
Force	mass $\times$ acceleration	$M^1L^1T^{-2}$
Work	force $\times$ displacement	$M^1L^2T^{-2}$
Power	$\frac{\text{work}}{\text{time}}$	$M^1L^2T^{-3}$
Momentum	mass $\times$ velocity	$M^1L^1T^{-1}$
Kinetic energy	$\frac{1}{2} \times \text{mass} \times \text{velocity}^2$	$M^1L^2T^{-2}$
Pressure	$\frac{\text{force}}{\text{area}}$	$M^1L^{-1}T^{-2}$
Potential energy }	mass $\times$ acceleration due to gravity $\times$ height	$M^1L^2T^{-2}$
Impulse	force $\times$ time	$M^1L^1T^{-1}$
Torque	force $\times$ arm of couple	$M^1L^2T^{-2}$

Table of prefixes

Table 3

Multiplier	Prefix	Symbol	Multiplier	Prefix	Symbol
10^{-1}	deci	d	10^1	deka	da
10^{-2}	centi	c	10^2	hector $\delta.r$	h
10^{-3}	milli	m	10^3	kilo	k
10^{-6}	micro	μ	10^6	mega	M
10^{-9}	nano	n	10^9	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E
10^{-21}	zepto	z	10^{21}	zeta	Z
10^{-24}	yocto	y	10^{24}	yotta	Y

ERRORS AND APPROXIMATIONS—CALCULATION OF FRACTIONAL AND PERCENTAGE ERROR

Accuracy and precision

Accuracy of a measurement indicates how close the measured values are to the actual value. It is the relative

freedom from errors. Smaller the least count, higher the accuracy of measurement.

Precision: The closeness of the various measured values to each other is called precision.

Error in the measurement of a physical quantity is the difference between its actual value and its measured value. Errors may be broadly classified into systematic errors and random errors.

Systematic errors

Errors arising from faulty calibration of the measuring instrument are known as systematic errors. As a result, the same amount of error occurs in all readings obtained with that instrument.

As an example, consider the zero error in a screw gauge. Once the error is detected, a suitable correction can be applied for the reading obtained.

Systematic errors may also occur due to the observer. For example, error due to parallax in measuring the deflection of the pointer in a compass box used in a tangent galvanometer. This error can be reduced by taking care to see that the pointer and its image seen in the plane mirror coincide, while taking readings.

Random errors

Even when systematic errors are taken into account, when the same measurement is repeated a number of times, different readings may be obtained. These errors are called random errors. These errors are compensated for by repeating the measurement a number of times and taking the mean value.

Estimation of errors

Absolute error

It is the difference between the true value of the physical quantity and the measured value. The arithmetic mean of a number of readings is taken as the true value.

$$\langle x \rangle = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

$\langle x \rangle$ represents the arithmetic mean and $x_1, x_2, \dots, x_n$ represent the readings obtained

Absolute errors are

$$\Delta x_1 = \langle x \rangle - x_1$$

$$\Delta x_2 = \langle x \rangle - x_2$$

$$\Delta x_3 = \langle x \rangle - x_3$$

.

.

.

$$\Delta x_n = \langle x \rangle - x_n$$

The final value of absolute errors is the mean of all the individual absolute errors.

$$\text{Thus, } \langle \Delta x \rangle = \frac{|\Delta x_1| + |\Delta x_2| + \dots + |\Delta x_n|}{n} = \frac{1}{n} \sum_{i=1}^n \Delta x_i$$

$\langle x \rangle$ may be positive or negative

The correct value of the physical quantity $\langle x \rangle \pm \langle \Delta x \rangle$

Fractional error and percentage error

If the error in the measurement of a physical quantity x is

Δx then the ratio $\frac{\Delta x}{x}$ is called the fractional error and

$\left[\frac{\Delta x}{x} \times 100 \right]$ is called the percentage error.

Estimation of error in the final result of an experiment

The final result obtained in an experiment generally depends on a number of measurements made with instruments of varying precision or least count. Measurement of each physical quantity that occurs in the final formula contributes its own error. All these errors have a cumulative effect on the final result.

- (i) Let z be a physical quantity depending on the variables x and y , given by $z = x + y$

If the error in x is Δx and the error in y is Δy , we have, for the error in z ,

$$\Delta z = \Delta x + \Delta y,$$

and z is represented along with its error as $z \pm \Delta z$.

This means the quantity $z \pm \Delta z$, lies somewhere between $z - \Delta z$ and $z + \Delta z$.

$$z = x + y$$

i.e., z lies between $z - \Delta x$ x $y - \Delta y$ and $z + \Delta x$ and $y + \Delta y$

i.e., $\Delta z = \Delta x + \Delta y$ is the uncertainty in z .

- (ii) If z is given by

$$z = xy$$

and if errors in x and y are Δx and Δy respectively, then z lies between

$$(x - \Delta x)(y - \Delta y) \text{ and } (x + \Delta x)(y + \Delta y).$$

i.e., between $xy - x\Delta y - y\Delta x + \Delta x\Delta y$

and

$$xy + x\Delta y + y\Delta x + \Delta x\Delta y.$$

For small Δx and Δy , $\Delta x\Delta y$ can be neglected so that z lies between

1.8 Units, Dimensions and Error Analysis in Experiments

$$x + y + x\Delta y + y\Delta x$$

and

$$x + y - x\Delta y - y\Delta x$$

$$\Rightarrow \Delta z = x\Delta y + y\Delta x$$

$$\text{or } \frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

(iii) If z is given by $z = x^m y^n$,

Applying binomial theorem,

$$\frac{\Delta z}{z} = |m| \frac{\Delta x}{x} + |n| \frac{\Delta y}{y}$$

Note that we take the modulus values for m and n as errors are additive.

CONCEPT STRAND

Concept Strand 5

Suppose the density ρ of a right circular cylinder of radius r and height h is to be measured. Let Δr be the error in the measurement of r and Δh be the error in the measurement of height h of the cylinder. If volume $V = \pi r^2 h$, calculate $\frac{\Delta V}{V}$ and $\frac{\Delta \rho}{\rho}$.

Solution

If ΔV is the error in the measurement of V , then,
$$\frac{\Delta V}{V} = 2 \frac{\Delta r}{r} + \frac{\Delta h}{h}$$

If m is the mass of the cylinder, $\rho = \frac{M}{V}$. If ΔM is the error in the measurement of mass and $\Delta \rho$ the resulting error in the calculation of the density ρ , then

$$\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{2\Delta r}{r} + \frac{\Delta h}{h}$$

In general, if a physical quantity L depends on a number of measured quantities $A, B, C, \dots$ etc., in accordance with the relation

$$L = A^x \cdot B^y \cdot C^z, \dots$$

and if $\Delta A, \Delta B, \Delta C$ are the errors in the measured values of $A, B, C, \dots$ respectively then the fractional error in the calculated value of L is given by,

$$\frac{\Delta L}{L} = x \frac{\Delta A}{A} + y \frac{\Delta B}{B} + z \frac{\Delta C}{C} + \dots$$

We notice that the quantity having the highest power and the lowest value has the greatest effect on the final result. Thus, any physical quantity of small value and large power has to be measured with the greatest possible accuracy.

Significant figures

Significant figures in a measurement are those digits that are measured correctly in an experimental observation plus one more digit that is uncertain.

Larger the number of significant digits after the decimal in a measurement, higher is the accuracy of measurement.

Significant figures do not change if we measure a physical quantity in different units.

Significant figures indicate the extent to which the readings taken in the measurement of a physical quantity are reliable. Every quantity can be written as a decimal number between 1 and 10 multiplied by appropriate power of 10.

Suppose the length of an object as measured with a scale of least count 0.1 cm is 18.9 cm. It has three

significant figures. The first two figures namely 1 and 8 are reliable, the third figure 9 is not reliable. The number of significant figures decides the accuracy of measurement. Greater the accuracy of our results, larger the number of significant figures we can use and vice versa. We determine the number of significant figures as follows:

1. The extreme non-zero digit at the left gives the most significant figure.
2. If the number is non-decimal the extreme right non-zero digit is the least significant figure.
3. If the number is decimal the last digit on the right is the least significant figure.
4. The number of digits from the most to the least significant figure gives the number of significant figures.

Again, 0.00841, 8.41, 841, 8410, 8.40, 0.0840 all have three significant figures.

Calculations become simpler without sacrificing accuracy if non significant figures are dropped. If the figure after the least significant figure is less than 5 it is dropped without altering the least significant figure.

For example: 2.543 becomes 2.54. If it is greater than five the least significant figure is increased by one.

For example: 2.546 becomes 2.55. If the figure after the least significant figure is 5 the least significant figure should be unaltered if it is even and should be increased by 1 if it is odd.

For example: 3.465 becomes 3.46, 3.455 becomes 3.46
Addition or subtraction is carried out as follows:

$$\begin{array}{rrr} 5.2245 & 6.3152 & 4.6218 \\ +1.23 & +1.20 & -1.824 \\ \hline = 6.4545 & = 7.5152 & = 2.7978 \\ = 6.45 & = 7.52 & = 2.798 \end{array}$$

Multiplication and division are carried out as follows:

$$\begin{aligned} 2.62 \times 10^{-4} \times 1.2 &= 3.144 \times 10^{-4} = 3.1 \times 10^{-4} \\ \frac{2.2 \times 10^{-3}}{1.41 \times 10^{-2}} &= 1.560 \times 10^{-1} = 1.6 \times 10^{-1} \end{aligned}$$

Rules for counting significant figures

- All non-zero digits are significant.
Example, 453.27 has five significant digits
- All zeros between two non-zero digits are significant.
Example, 4073.006 has seven significant digits
- In a number less than one, all zeros to the right of the decimal point and to the left of non-zero digit are not significant.
Example, 0.0007312 has only four significant digits
- All zeros to the right of the non-zero digit in a number are not significant
Example, 123000 has only three significant digits

- All zeros to the right of the last non-zero digit in a number become significant, when they come from a measurement.
Example, $S = 1010$ m has four significant digits

- In a number with decimal points, all zeros to the right of a non-zero digit are significant.
Example, 1.5300 has five significant digits

- The powers of ten are not counted as significant digits.
Example, 1.23×10^{23} has only three significant digits
 1.230×10^{23} has four significant digits only

Rules of rounding off

Rule 1

If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example, If 7.314 is to be rounded off to 2 decimal places then it becomes 7.31.

Rule 2

If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example, If 7.318 is to be rounded off to 2 decimal places, then it becomes 7.32.

Rule 3

If the digit to be dropped is 5, followed by digit other than zero, then the preceding digit is raised by one.

Example, If 34.6251 is to be rounded off to 2 decimal, it becomes 34.63.

Rule 4

If the digit to be dropped is 5, followed by zero, then the preceding digit is left unchanged if it is even.

Example, If 34.6250 is to be rounded off to 2 decimal it becomes 34.62.

Rule 5

If the digit to be dropped is 5, followed by zero, then the preceding digit is raised by one if it is odd.

Example, If 34.6750 is to be rounded off to 2 decimals it becomes 34.68.

SUMMARY

$$\bar{x} = x_{\text{mean}} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\Delta x_i = \bar{x} - x_i,$$

$$\bar{\Delta x} = \frac{|\Delta x_1| + |\Delta x_2| + \dots + |\Delta x_n|}{n}$$

$x_1, x_2, \dots, x_n$ are the various observed/measured values and $\bar{x}$ is the average $\langle x \rangle$

$\Delta x_i \rightarrow$ absolute error in the i th observation/measurement

$\bar{x} \rightarrow$ true value or mean value

$x_i \rightarrow$ observed/measured value for i th measurement

$\bar{\Delta x} \rightarrow$ Mean absolute error and is the same as $\langle \Delta x \rangle$

1.10 Units, Dimensions and Error Analysis in Experiments

$$\delta x = \frac{\Delta \bar{x}}{\bar{x}}$$

$$x = \bar{x} \pm \Delta \bar{x}$$
$$\Delta Z = \Delta A + \Delta B$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

$$\frac{\Delta Z}{Z} = n \frac{\Delta A}{A}$$

$$\frac{\Delta Z}{Z} = m \frac{\Delta A}{A} + n \frac{\Delta B}{B}$$

$$\frac{\Delta Z}{Z} = p \frac{\Delta A}{A} + q \frac{\Delta B}{B} + r \frac{\Delta C}{C}$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{\Delta A}{A+B} - \frac{\Delta B}{A+B}$$

$\delta x \rightarrow$ relative error

$\Delta \bar{x} \rightarrow$ Mean absolute error

$\bar{x} \rightarrow$ True value or mean value

$\delta x \times 100 =$ % relative error

$x \rightarrow$ Final result of measured physical quantity

If $Z = A \pm B$,

$\Delta Z \rightarrow$ Absolute error in sum or difference

If $Z = AB$ or $Z = A/B$

$\frac{\Delta Z}{Z} \rightarrow$ Relative error in product or quotient

If $Z = A^n$

If $Z = A^m B^n$

If $Z = \frac{A^p B^q}{C^r}$

If $Z = \frac{AB}{(A+B)}$

ERROR ANALYSIS IN EXPERIMENTS

Experiment 1: Vernier Calipers

Also

$$LC = 1 \text{ MSD} - 1 \text{ VSD}$$

Aim

To determine the length and radii of a hollow cylinder.

Apparatus

Vernier calipers and the hollow cylinder.

Principle

Slide calipers or vernier calipers consists of a main scale and an auxiliary scale called vernier scale for measuring lengths correct to any desired fraction of the smallest division on the main scale. In a forward reading vernier, a certain number of divisions on the main scale are divided into a larger number of divisions on the vernier scale. The difference between one main scale division (MSD) and one vernier scale division (VSD) is called the least count of the vernier. The least count of the vernier is the smallest distance that can be measured with the vernier. Least count (LC) is given by

$$LC = \frac{\text{Value of 1 MSD}}{\text{Number of divisions on the vernier scale}} = \frac{S}{N}$$

Description

The slide calipers consists of a rectangular metal strip AB (Fig.1.1). Along the edge of this strip two scales are marked. They form the main scales. A strip AP is fixed to the end A such that its edge is at right angles to AB. It is called the fixed jaw. Q is a movable jaw on which two verniers are etched one on either side. When Q is moved, the verniers slide along the main scales. By tightening the screw S the movable jaw can be fixed in any desired position. The projections M and N of the jaws can be used to measure the internal diameter of a hollow cylinder. When the two jaws touch each other the zero of the vernier scale coincides with the zero of the main scale.

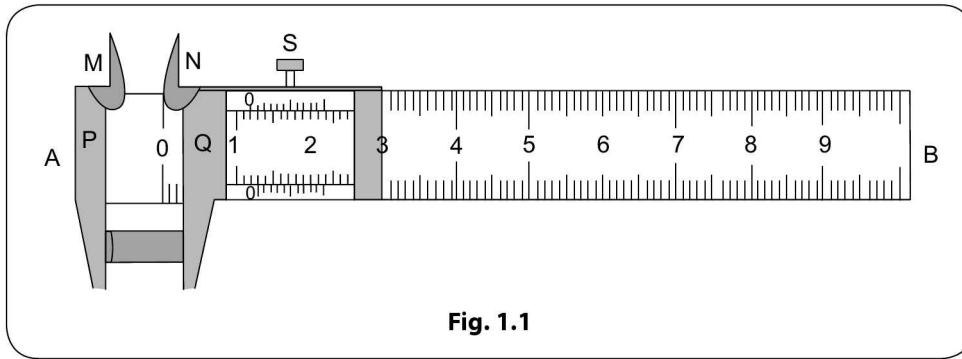


Fig. 1.1

Procedure

Determination of Least Count:

The value of each division on the main scale (S) and the number of division on the vernier scale (N) are noted. Then the least count (LC) is calculated using the relation

$$LC = \frac{S}{N}$$

Measurement of the length of a cylinder

The given cylinder is held lengthwise between the two jaws of the slide calipers. The screw S is tightened so that the cylinder is tightly gripped between the jaws. If the zero on the vernier coincides with a main scale division then the main scale reading (MSR) itself is the total reading. If the zero lies between two main scale divisions then the smaller one is taken as MSR. The division on the vernier which coincides with a main scale division is taken as coinciding vernier division (CVD). The total reading (TR) is calculated as $TR = MSR + (CVD \times LC)$.

This gives the length of the cylinder. The experiment is repeated changing the position of the cylinder. In each case, the reading on the main scale and vernier scale are noted and the length of the cylinder is calculated. The readings are tabulated and the mean length of the cylinder is calculated. The external diameter of the cylinder can be determined in a similar manner. The readings are tabulated as shown.

To determine the internal diameter of the given cylinder the projections M and N are inserted into the cylinder. The movable jaw N is adjusted such that the cylinder is firmly held. The readings are taken as explained above.

Observations

Length of 1 MSD = S = ... Total number of vernier scale divisions, N = ...

$$\text{Least count } LC = \frac{S}{N}$$

Object	Dimension	Trial No.	MSR cm	CVD	TR = MSR + (CVD × LC) cm	Mean cm
Cylinder	Length	1				
		2				
		3				
Cylinder	External Diameter	1				
		2				
		3				
Cylinder	Internal Diameter	1				
		2				
		3				

Result

- (1) Length of cylinder = ...cm = ... × 10⁻² m
- (2) External diameter of cylinder, D = ...cm = ... × 10⁻² m
∴ external radius, R = D/2 = ... × 10⁻² m
- (3) Internal diameter of cylinder, d = ...cm = ... × 10⁻² m
∴ internal radius, r = d/2 = ... × 10⁻² m

Error calculation

$$\text{Percentage error in length} = \frac{\Delta L}{L} \times 100$$

$$\begin{aligned} \text{Percentage error in radius} &= \frac{\Delta r}{r} \times 100 \\ &= \frac{LC}{r} \times 100 \end{aligned}$$

CONCEPT STRAND

Concept Strand 6

In determining the radius of a cylinder using slide calipers the following readings were obtained. Calculate the mean radius and error in its measurement.

Least count of vernier = 0.01 cm

Trial No.	MSR (cm)	CVD	TR = MSR + (CVD × LC) cm
1	1.4	7	1.47
2	1.4	8	1.48
3	1.4	6	1.46

Mean diameter,

$$d = \frac{1.47 + 1.48 + 1.46}{3} = 1.47 \text{ cm}$$

∴ radius,

$$r = \frac{d}{2} = \frac{1.47}{2} = 0.735 \text{ cm}$$

$$= 0.735 \times 10^{-2} \text{ m}$$

Calculations of error in the measurement of radius

$$\begin{aligned} \text{Percentage error} &= \frac{\Delta r}{r} \times 100 = \frac{LC}{r} \times 100 \\ &= \frac{0.01}{0.735} \times 100 = 1.36\% \end{aligned}$$

Zero error and zero correction

In the above discussion, we have considered that the zero of the main scale coincides with the zero of the vernier scale when the two jaws touch each other. If this is not so, then there is a **zero error** for the instrument and consequently, a **zero correction** has to be applied to the measured value. Zero error or zero correction has to be considered with sign, which can be either positive or negative.

$$\text{Zero correction} = - \text{zero error}$$

Zero correction is algebraically added to the measured length. If the zero of the vernier scale (VS) lies to the right side of the zero of the main scale (MS), when the two jaws of the vernier are in contact, the zero error is **positive**. If N is the main scale reading on the left side of zero of vernier scale and the xth division of the VS coincides with any main scale division, then

zero error = $+(N + x(LC))$, where LC = least count (or vernier constant) of the vernier.

$$\text{Zero correction} = -[N + x(LC)]$$

If the zero of the vernier scale lies to the left side of the zero of the main scale, when the two jaws of the vernier are in contact, the zero error is **negative**.

If N is the main scale reading on the left side of zero of vernier scale (obviously $N < \text{zero}$, so sign of N is nega-

tive) and the xth division of the VS coincides with any main scale division, then

$$\text{zero error} = -|N| + x(LC) \text{ and}$$

$$\text{zero correction} = |N| - x(LC)$$

Sources of error

- The vernier scale may be loose on the main scale.
- The jaws may not be at 90° to the main scale.
- The graduations on the scales may not be correct and clear.
- Parallax may occur in making observations.

Precautions to be taken

- The motion of vernier scale on main scale must be made smooth (by oiling, if required).
- Least count (or vernier constant) and zero error should be found out carefully and correctly recorded.
- The object (whose dimensions are to be measured) shall be gripped between the jaws firmly but without undue pressure on it from the jaws.
- Observations must be repeated a few times for different orientations.

Experiment 2: Screw Gauge

Aim

To determine the dimension of the given object using a screw gauge.

Principle

The construction of a screw gauge is based on the principle of the micrometer screw. When an accurately cut screw working in a fixed nut is turned, the distance through which the tip of the screw moves is directly proportional to the number of rotations given to the screw head. When it is given one complete rotation, the tip of the screw moves through a distance equal to the distance between two consecutive threads on the screw. This distance is called the pitch of the screw.

Apparatus

Screw gauge, glass plate, lead shot and wire.

Description

The screw gauge consists of a U shaped metallic frame (Fig. 1.2). To one arm of this frame a hollow cylinder C is attached. On this cylinder a scale graduated in mm is etched. This scale is called the pitch scale. A straight line is marked parallel to the axis of the cylinder and is called the pitch line. An accurately cut screw S works through the threads cut on the inner surface of the cylinder. It ends in a plane surface at B. To the screw head H a sleeve L is attached. The edge of the sleeve is divided into a number of equal parts (usually 50 or 100 parts). This scale is called the head scale. To the other arm of the frame opposite to the plane end B another plane stud A, known as the fixed plug is attached. When A and B are in contact the zero of the head scale generally

coincides with the pitch line. Otherwise the instrument is said to have a **zero error**. When the screw head is rotated through one division on the head scale, the tip of the screw moves through a distance equal to $1/n$ times the pitch (n = number of divisions on the head scale). This distance is called the **least count**. It represents the smallest dimension that can be accurately measured with the screw gauge.

Procedure

The pitch, least count and zero error of the screw gauge are determined as follows:

The tip of the screw is brought into contact with the fixed plug, by giving the screw head certain number of complete rotations. The distance uncovered on the pitch scale is noted. Pitch is calculated using the relation.

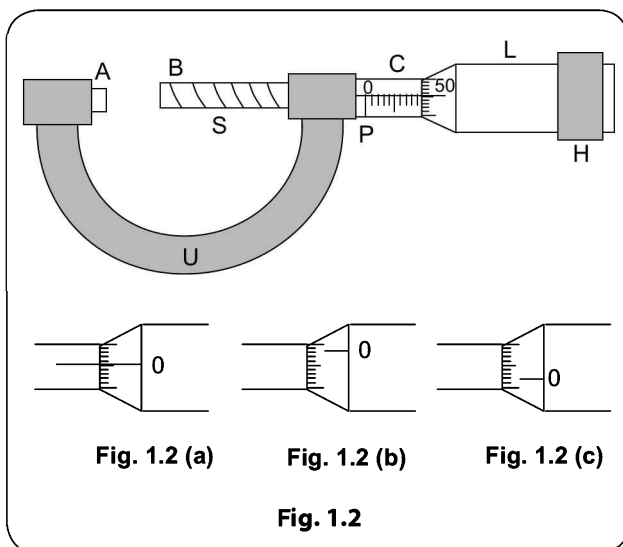
$$\text{Pitch} = \frac{\text{Distance uncovered on pitch scale}}{\text{Number of rotations given to screwhead}}$$

The least count of the screw gauge is calculated using the relation

$$\text{LC} = \frac{\text{Pitch}}{\text{Number of head scale divisions}}$$

To determine the zero error, the tip of the screw is made to touch A by turning the screw head. If the zero of the head scale coincides with the pitch line (Fig. 1.2(a)) then there is no **zero error**. If the zero of the head scale is above the pitch line **zero error is negative** (Fig. 1.2(b)). In the case of negative (–) zero error, the zero of the linear scale (pitch scale) is hidden from circular scale. If the zero of the head scale is below the pitch line **zero error is positive** (Fig. 1.2(c)). In the case of positive (+) zero error, the zero of the linear scale (pitch scale) is not hidden from circular scale. The number of divisions by which the zero of the head scale is above or below the pitch line gives the zero error (ZE).

To determine the diameter of the given wire it is held between A and B by turning the screw head. The pitch scale reading (PSR) and head scale reading (HSR) are noted. The diameter of the wire is calculated using the formula $TR = PSR + (HSR - ZE) LC$. The experiment is repeated several times slightly varying the position of the wire and its diameter is calculated. The readings are tabulated. The average of all the total readings gives the mean diameter. The diameter of the lead shot (sphere) and the thickness of the plate are determined in a similar manner.



1.14 Units, Dimensions and Error Analysis in Experiments

Object	Dimension	Trial No.	PSR mm	HSR	TR = PSR + (HSR - ZE) LCmm
Wire	Diameter	1			
		2			
		3			
Lead shot	Diameter	1			
		2			
		3			
Glass plate	Thickness	1			
		2			
		3			

Result

Diameter of wire = ... mm = ... $\times 10^{-3}$ m

Diameter of sphere = ... mm = ... $\times 10^{-3}$ m

Thickness of glass plate = ... mm = ... $\times 10^{-3}$ m

Error calculation

$$\text{Percentage error in diameter} = \frac{\Delta d}{d} \times 100 = \frac{LC}{d} \times 100$$

CONCEPT STRAND

Concept Strand 7

Calculate the volume of the given sphere using the following data obtained while determining the radius of a sphere using a screw gauge.

Distance covered on pitch scale = 4 mm

Number of rotations given to screw head = 4

Number of division on head scale = 100

Zero error = -3

$$\text{Pitch} = \frac{\text{Distance covered on pitch scale}}{\text{Number of rotations given}}$$

$$= \frac{4 \text{ mm}}{4} = 1 \text{ mm}$$

$$\therefore LC = \frac{\text{Pitch}}{\text{Number of head scale divisions}} = \frac{1}{100} = 0.01 \text{ mm}$$

Object	Dimension	Trial no.	PSR mm	HSR	TR = PSR + (HSR - ZE) LC (mm)
Sphere	Diameter	1	1	80	1.83
		2	1	81	1.84
		3	1	80	1.83

$$\text{Average diameter } d = \frac{1.83 + 1.84 + 1.83}{3} = 1.833 \text{ mm} = 1.833 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \text{Volume of sphere } V &= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(\frac{d}{2} \right)^3 \\ &= \frac{4}{3} \pi \times \left(\frac{1.833 \times 10^{-3}}{2} \right)^3 \\ &= 3.22 \times 10^{-9} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Percentage error in diameter} &= \frac{\Delta d}{d} \times 100 \\ &= \frac{LC}{d} \times 100 \\ &= \frac{0.01}{1.833} \times 100 \\ &= 0.55\% \end{aligned}$$

Percentage error in volume

$$\begin{aligned} &= \frac{3 \Delta d}{d} \times 100 \\ &= 3 \times \frac{\Delta d}{d} \times 100 \\ &= 3 \times 0.55 = 1.65\% \end{aligned}$$

Sources of error

- (i) The screw may have friction, preventing smooth rotation.
- (ii) The screw gauge may have backlash error.
- (iii) Circular scale divisions may not be of equal size.
- (iv) The experimental wire may not be of uniform cross-section.

Precautions to be taken

- (i) To avoid backlash error, the screw must be always turned only in one direction, while taking an observation.
- (ii) At each place, take readings in pairs. i.e., in two directions at right angles to each other.
- (iii) The wire must be straight and free from kinks.
- (iv) Always rotate the screw by the ratchet and stop as soon as it gives one tick sound.

Experiment 3: Simple Pendulum

Aim

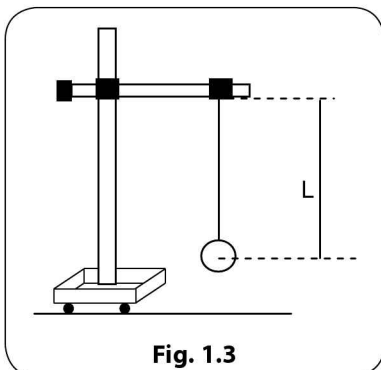
To determine the acceleration due to gravity (g) at the given place.

Principle

The period T of a simple pendulum of length ℓ is given by the equation $T = 2\pi\sqrt{\frac{\ell}{g}}$, provided the amplitude of oscillation is small. By setting up a pendulum of length ℓ and determining its period of oscillation T , the acceleration due to gravity is calculated using the equation $g = 4\pi^2(\ell/T^2)$.

Apparatus

Metallic bob with hook, stand, split cork, cotton thread, vernier calipers, stop clock and metre scale.



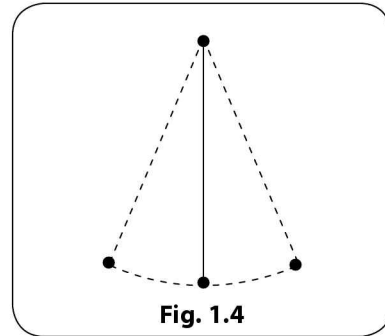
Procedure

The average diameter and hence radius r of the bob is measured using slide calipers. One end of the string is tied to the hook of the bob. The other end of the string is passed between the split pieces of a cork (Fig. 1.3). The length ℓ of the string is adjusted such that $L = \ell + r = 0.5$ m. The split cork is tightly fixed in the clamp of the stand. The pendulum is made to oscillate with a small amplitude in a vertical plane (Fig. 1.4). The time taken by the pendulum to execute 30 oscillations is found. The measurement is repeated three times. The mean time for 30 oscillations t is calculated and the period of oscillation $T = t/30$ s is also calculated.

The experiment is repeated increasing the length L of the pendulum to 0.6 m, 0.7 m, 0.8 m, 0.9 m and 1 m. In each case, the period of oscillation T is calculated. For each trial, the value of $\frac{L}{T^2}$ is calculated. The mean value of $\frac{L}{T^2}$ is found. g at the given place is calculated using the relation

$$g = 4\pi^2 \left(\frac{L}{T^2} \right)_{\text{average}}.$$

Observations



Determination of the radius of the bob

Value of 1 MSD = $S = \dots\dots$

Total number of VSD = $N = \dots\dots\dots$

$$LC = \frac{S}{N}$$

Trial No.	MSR ($\times 10^{-2}$ m)	CVD	TR = MSR + (CVD $\times$ LC) $\times 10^{-2}$ m
1			
2			
3			

Mean TR = $d = \dots\dots\dots \times 10^{-2}$ m

Radius $r = \frac{d}{2} = \dots\dots\dots \times 10^{-2}$ m

1.16 Units, Dimensions and Error Analysis in Experiments

Determination of period

Trial No.	$L \times 10^{-2} \text{ m}$	Time for 30 oscillation in s			Mean time for 30 Oscillations t in s	T in s $T = \frac{t}{30}$	$\left(\frac{L}{T^2}\right) \text{ms}^{-2}$
		Trial 1	Trial 2	Trial 3			
1	50						
2	60						
3	70						
4	80						
5	90						
6	100						

$$\left(\frac{L}{T^2}\right)_{\text{average}} =$$

$$g = 4\pi^2 \left(\frac{L}{T^2}\right)_{\text{average}} =$$

Result

Acceleration due to gravity at the given place is $g = \dots\dots\dots \text{m s}^{-2}$

CONCEPT STRAND

Concept Strand 8

Calculate acceleration due to gravity using the observation given below:

Radius of bob = 1 cm

Trial No.	$\ell \times 10^{-2} \text{ m}$	$L = (\ell + 1) \times 10^{-2} \text{ m}$	Time for 30 oscillations			Mean time, t (s)	Time for 1 oscillation (s)	$\frac{L}{T^2} \text{ m s}^{-2}$
			Trial 1	Trial 2	Trial 3			
1	49	50	43	42	43	42.67	1.42	0.248
2	59	60	47	46	46	46.33	1.54	0.253
3	69	70	51	50	50	50.33	1.68	0.248
4	79	80	54	53	54	53.67	1.79	0.250
5	89	90	57	57	57	57	1.9	0.249
6	99	100	60	60	61	60.33	2.01	0.248

Solution

Average value of $\left(\frac{L}{T^2}\right) = 0.249 \text{ m s}^{-2}$

$$\therefore g = 4\pi^2 \left(\frac{L}{T^2}\right)_{\text{average}} = 4 \times (3.14)^2 \times 0.249 = 9.82 \text{ m s}^{-2}$$

Sources of error

- (i) The string may not be weightless and inextensible.
- (ii) Point of suspension may not be rigid.
- (iii) The amplitude of oscillations may not be small.
- (iv) The bob may spin.
- (v) Air currents may disturb oscillations.
- (vi) Stop watch may be erroneous or inaccurate.
- (vii) There may be delay in starting and stopping the stop watch.
- (viii) There may be error in counting.

Precautions to be taken

- (i) There must be no air turbulence. Laboratory fans should be switched off. Mechanical vibrations in the experiment set up should be eliminated.
- (ii) String should be strong, light and inextensible.
- (iii) Point of suspension should be fixed in the rigid support.
- (iv) Lower faces of split cork should be in the same level.
- (v) Splitting should be perpendicular to the plane of vibration of the pendulum.
- (vi) Amplitude of oscillations must be small (so that $\sin\theta \approx \theta$).
- (vii) Bob should move along a straight line and it shall not spin during oscillations.
- (viii) Length of pendulum should include length of hook and radius of bob.
- (ix) Stop watch must be accurate.
- (x) Counting should be done properly, starting from zero.
- (xi) Large number of readings should be taken.

Experiment 4: Determination of Young's modulus of a wire using Searle's apparatus

Aim

To determine the Young's modulus of the material of a wire using Searle's apparatus.

Apparatus

Searle's apparatus, two long steel wires of same length and diameter, a metre scale, a screw gauge, eight half kg slotted weights and a 1 kg hanger.

Description of Searle's apparatus

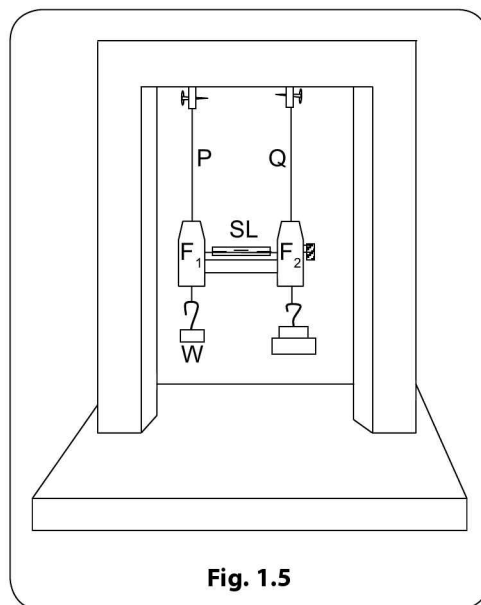


Fig. 1.5

Searle's apparatus consists of two rigid chucks F_1 and F_2 which are connected by a horizontal beam, so that only vertical relative motion between the chucks is possible. A spirit level (SL) is supported horizontally with one end resting on the top of a micrometer screw (spherometer), which is working through a nut along a vertical scale. In case of any relative motion between F_1 and F_2 , the SL no longer remains horizontal and screw has to be rotated to bring the bubble of SL back to original position. The distance through which the screw has to be moved, gives the relative motion. The entire apparatus is suspended vertically from two rigid supports with the help of the identical wires P and Q, one through F_1 and the other through F_2 . The wire P (through F_1) is the standard wire (reference wire) and a standard weight W (1 kg) is suspended from the bottom of F_1 . The wire Q (through F_2) is the experimental wire which we load by placing slotted weights (of weight 1 kg) attached to F_2 .

Procedure

- (i) Measure the length (L) of the experimental wire (Q) from the point where it leaves the fixed support (at top) to the point where it is fixed in the chuck F_2 .
- (ii) Measure the diameter (d) of the experimental wire using screw gauge. Area of cross section $A = \frac{\pi}{4} d^2$.

1.18 Units, Dimensions and Error Analysis in Experiments

- (iii) Initially, when there is no load in the hanger, the bubble in the spirit level (SL) has to be exactly at the centre.
- (iv) Load the experimental wire in steps of 0.5 kg by placing them on the hanger. Maximum permissible value is half of the breaking force. As a result of loading, the wire gets elongated by small amounts and SL gets disturbed from horizontal setting.
- (v) The increase in length is measured by turning the screw of spherometer upwards so as to restore the SL.

$\Delta L = n \times \text{pitch} + f \times \text{least count}$, where
 n = number of turns of spherometer
 f = difference in circular scale reading.

- (vi) The load on the hanger is reduced in the same steps and SL is restored to its horizontal position. The mean value of the two observations (during loading and unloading) gives the true increase in length of the wire for the corresponding load.
- (vii) Prepare the observation table as shown.

Load on the hanger	Spherometer positions		Mean $\ell = \frac{\ell_1 + \ell_2}{2}$	Extension $\Delta \ell = \ell$
	Loading (ℓ_1)	Unloading (ℓ_2)		
0.5 kg				
1.0 kg				

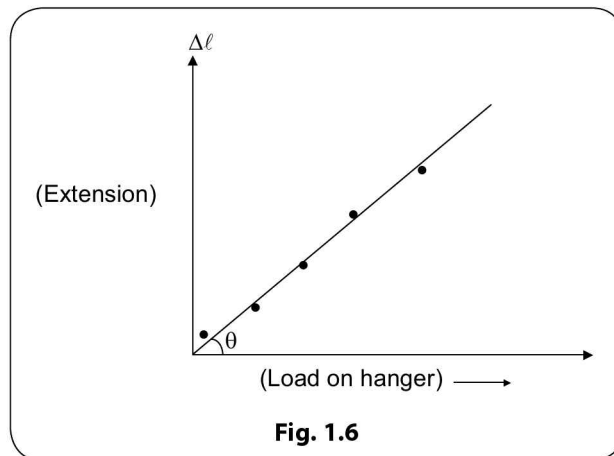
Calculations

Plot the extension ($\Delta \ell$) versus load on hanger graph.

$$\tan \theta = \frac{\Delta \ell}{\text{load}} = \frac{\Delta \ell}{mg}$$

$$\text{We have } Y = \frac{mgL}{A \Delta \ell} \Rightarrow$$

$$Y = \frac{L}{A} \times \frac{1}{\tan \theta}$$



L and $A \left(= \frac{\pi d^2}{4} \right)$ are determined earlier.

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + 2 \frac{\Delta d}{d} + \frac{\Delta(\Delta \ell)}{\Delta \ell}$$

Sources of error

- (i) Both wires may not be identical.
- (ii) Experimental wire may not be of uniform cross-section throughout.
- (iii) The slotted weights may not have standard weight.

Precautions to be taken

- (i) Both the wires should be identical.
- (ii) Both the wires should be supported from the same rigid support.
- (iii) Any kink present in the experimental wire shall be removed before starting the experiment.
- (iv) Diameter of the wire should be measured at different places and along two mutually perpendicular directions at every place.
- (v) Wire should be loaded and unloaded in the same steps.
- (vi) Slotted weights should be added and removed gently.
- (vii) After every loading and unloading, wait for at least two minutes so that the wire stabilizes and then make the measurements.

CONCEPT STRAND

Concept Strand 9

In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. With a weight of 50 N suspended from the wire, the extension measured during loading was 0.126 cm and during unloading was 0.124 cm respectively. If the least count of the micrometer used for measuring the extension is 0.001 cm, find the maximum error in the measurement of Young's modulus of the material of the wire from these data.

Solution

$$\Delta \ell = \frac{0.126 + 0.124}{2} = 0.125 \text{ cm}$$

$$= 1.25 \times 10^{-3} \text{ m}$$

$$L = 1.10 \text{ m}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (5 \times 10^{-4})^2 \text{ m}^2; F = 50 \text{ N}$$

$$Y = \frac{FL}{A \Delta \ell} = \frac{50 \times 1.1}{\frac{\pi}{4} \times (5 \times 10^{-4})^2 \times 1.25 \times 10^{-3}}$$

$$= 2.24 \times 10^{11} \text{ N m}^{-2}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + 2 \frac{\Delta d}{d} + \frac{\Delta(\Delta \ell)}{\Delta \ell}$$

$$= \frac{0.1}{110} + 2 \times \frac{0.001}{0.05} + \frac{0.001}{0.125}$$

$$= 0.0009 + 0.04 + 0.008 = 0.0489$$

$$\therefore \Delta Y = 0.0489 Y = 0.0489 \times 2.24 \times 10^{11}$$

$$= 1.09 \times 10^{10} \text{ N m}^{-2}$$

$$\therefore \text{Maximum error} = 1.09 \times 10^{10} \text{ N m}^{-2}$$

Experiment 5: Specific heat of a liquid using a calorimeter

Aim

To determine the specific heat of the given liquid by electrical method.

Principle

Specific heat of a liquid is the quantity of heat required to raise the temperature of unit mass of the liquid through 1 K. The given liquid taken in a calorimeter is heated by passing a current through a coil immersed in the liquid. Equating the heat gained by the calorimeter and its contents to the electrical work done, the specific heat of the given liquid is calculated using,

$$S_l = \frac{1}{m_l} \left\{ \frac{VIt}{\theta_2 - \theta_1} - m_1 S_c \right\}$$

Apparatus

Joule's calorimeter, battery, key, rheostat, ammeter, voltmeter, resistance coil, thermometer and stop clock.

Joule's calorimeter consists of a calorimeter provided with an insulating lid. The ends of the heating coil are

connected to two terminals fixed to the lid. The lid also contains holes to insert the stirrer and a thermometer. The calorimeter is placed inside a wooden box. The space between the calorimeter and the box is filled with insulating material.

Circuit Diagram

Connections are made as shown in the circuit diagram Fig. 1.7. The resistance coil in the joule's calorimeter is connected in series with a battery, a plug key, a standard ammeter and a rheostat. A standard voltmeter is connected in parallel with the coil.

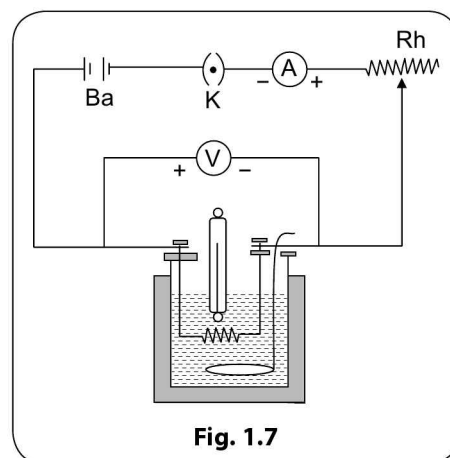


Fig. 1.7

1.20 Units, Dimensions and Error Analysis in Experiments

Procedure

1. The calorimeter is taken out. The mass m_1 of the empty calorimeter with stirrer is determined using a physical balance.
2. Enough liquid is poured into the calorimeter so that the coil will be completely immersed in the liquid (Fig. 1.7). The calorimeter with liquid and stirrer is weighed and its mass m_2 is determined.
3. The calorimeter is replaced in the wooden box. The key K is closed. The rheostat is adjusted so that a suitable current flows. The ammeter reading and the voltmeter reading are noted. The key is opened. The liquid is stirred well and the initial temperature θ_1 °C of the liquid and the calorimeter is noted.
4. The key K is closed. Simultaneously a stop clock is started. The readings of the ammeter I and that of the voltmeter V are noted. The liquid is continuously stirred. When the temperature rises by, say 8 °C (<10°C to avoid loss of heat) the key K is removed and the stop clock is stopped. After stirring the liquid, its maximum temperature θ_2 °C is noted. The time t for which the current is passed is recorded.
5. Radiation correction is applied by cooling the calorimeter and contents for t/2 second. The fall in temperature $\Delta\theta$ °C during this period is noted. The observations are recorded as shown below:

Mass of calorimeter + stirrer = m_1 = ...kg

Mass of calorimeter + stirrer + liquid = m_2 = ...kg

Mass of liquid = $m_2 - m_1 = m_l$ = -----kg

Initial temperature of liquid = θ_1 = ...°C

Final temperature of liquid = θ_2 = ...°C

Radiation correction = fall in temperature during t/2 seconds = $\Delta\theta$ = -----°C

Final corrected temperature = $\theta_2' = (\theta_2 + \Delta\theta) = \dots$ °C

Ammeter reading, I = ...A

Voltmeter reading, V = ...V

Time for which current is passed, t = ...second

Specific heat of calorimeter, S_c = ...J kg⁻¹ K⁻¹ (given)

Specific heat of liquid $S_l = \frac{1}{m_l} \left\{ \frac{VIt}{\theta_2' - \theta_1} - m_1 S_c \right\}$

Result

Specific heat of the given liquid J kg⁻¹ K⁻¹

Precautions

1. The electrical circuit should be checked thoroughly before replacing the calorimeter with contents.
2. Stopping the stop clock and opening the key have to be simultaneous.
3. The contents must be stirred well during the experiment.
4. The rise in temperature should not be greater than 10°C. This is to reduce the error due to loss of heat.

CONCEPT STRAND

Concept Strand 10

The following observations were recorded in an experiment to determine the specific heat of a liquid by electrical method. Calculate the specific heat of liquid.

Solution

Mass of calorimeter + stirrer = 42.3×10^{-3} kg

Mass of calorimeter + stirrer + liquid = 137.5×10^{-3} kg

Initial temperature of liquid = 27 °C

Final temperature of liquid = 34.2 °C

Radiation correction = 1 °C

Ammeter reading = 1 A

Voltmeter reading = 4.8 V

Time for which current flows = 12 minute

Specific heat of the material of the calorimeter = 375 J kg⁻¹ K⁻¹

Observations

Mass of calorimeter and stirrer $m_1 = 42.3 \times 10^{-3}$ kg

Mass of calorimeter, stirrer and liquid $m_2 = 137.5 \times 10^{-3}$ kg

Mass of liquid $m_l = (137.5 - 42.3) \times 10^{-3} = 95.2 \times 10^{-3}$ kg

Specific heat of calorimeter $S_c = 375$ J kg⁻¹ K⁻¹

Initial temperature of liquid $\theta_1 = 27$ °C

Final temperature of liquid $\theta_2 = 34.2$ °C

Radiation correction $\Delta\theta = 1$ °C

Final corrected temperature of liquid $\theta_2^1 = \theta_2 + \Delta\theta = 35.2^\circ\text{C}$

$$(\theta_2^1 - \theta) = (35.2 - 27) = 8.2^\circ\text{C}$$

Ammeter reading $I = 1\text{ A}$

Voltmeter reading $V = 4.8\text{ V}$

Time for which current is passed $t = 12\text{ minute} = 720\text{ second}$

$$\text{Specific heat of the liquid } S_\ell = \frac{1}{m_\ell} \left[\frac{VIt}{\theta_2^1 - \theta_1} - m_1 S_c \right]$$

$$S_\ell = \frac{1}{95.2 \times 10^{-3}} \left[\frac{4.8 \times 1 \times 720}{8.2} - 42.3 \times 10^{-3} \times 375 \right]$$

$$\begin{aligned} S_\ell &= \frac{10^3}{95.2} [421.5 - 15.94] \\ &= \frac{10^3 \times 405.6}{95.2} = 4.26 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} \end{aligned}$$

Specific heat of liquid $= 4.26 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$

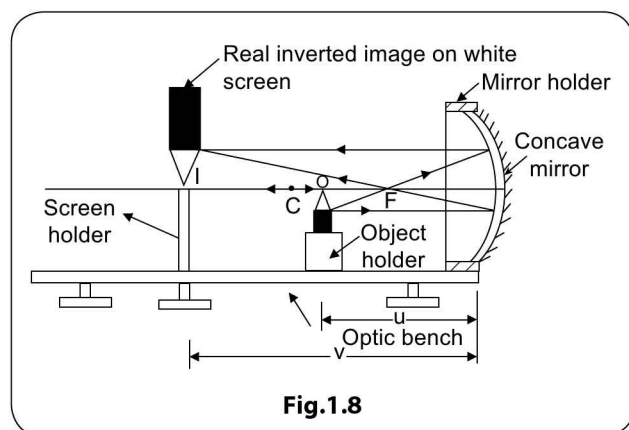
Experiment 6: Focal length of a concave mirror and a convex lens using u-v method

Aim

A. To find the focal length of a concave (converging) mirror.

Apparatus

An optic bench with three uprights, out of which two outer uprights are with lateral movement, a concave mirror, a mirror holder, two optic needles (one thin and one thick), a knitting needle and a metre (or half metre) scale. (If the optic bench has a metre scale etched or attached to it, no separate metre scale is needed).



Procedure

The concave (converging) mirror is held by a mirror holder on the optic bench and its position on the optic bench is locked. An object (optic needle) is mounted on a movable upright placed in front of the mirror. For a given position of the object, the distance of the object

from the pole of the mirror is measured as u . The white screen (mounted on a movable upright) is moved, till a sharp, inverted image of the object is obtained on it. The distance of screen from mirror (v) is measured. This is repeated for different positions of object and the corresponding positions of image (on the screen). Note down the readings and tabulate the readings as shown in the table below.

Object distance (u)	Image distance (v)	Focal length (f)

Calculation

The magnitude of the focal length (f) of mirror is given by the formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore f = \frac{uv}{(u + v)}$$

(Note: No sign convention applied to this formula)

After determining the focal lengths for various object distances, average them out. Conventionally, the focal length of concave mirror is taken with a negative ($-$) sign. The relative error in the focal length is obtained as

$$\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{u} - \frac{\Delta u}{(u + v)} - \frac{\Delta v}{(u + v)}$$

$$\Rightarrow \frac{\Delta f}{f} = f \left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right]$$

Sources of error

- The uprights may not be vertical.
- Removal of parallax may not be perfect.

1.22 Units, Dimensions and Error Analysis in Experiments

(iii) Index error (or bench error) may occur. In this experiment, the distances are to be measured from the pole of the mirror. The measured distances (or observed distances) may not be equal to actual distances and consequently an error is crept in measurement of the distances. This error is called index error or bench error.

Index error = observed distance – actual distance
It can be either positive or negative
 $\therefore$ Index correction = – (Index error)
Index correction is always added algebraically to the observed distance to get the corrected distance.

Precautions to be taken

- (i) The uprights must be vertical.
- (ii) Tip to tip parallax should be removed between the object needle O and the image of the needle I.
- (iii) To locate the position of the image, the eyes should be at least 30 cm away from the needle.

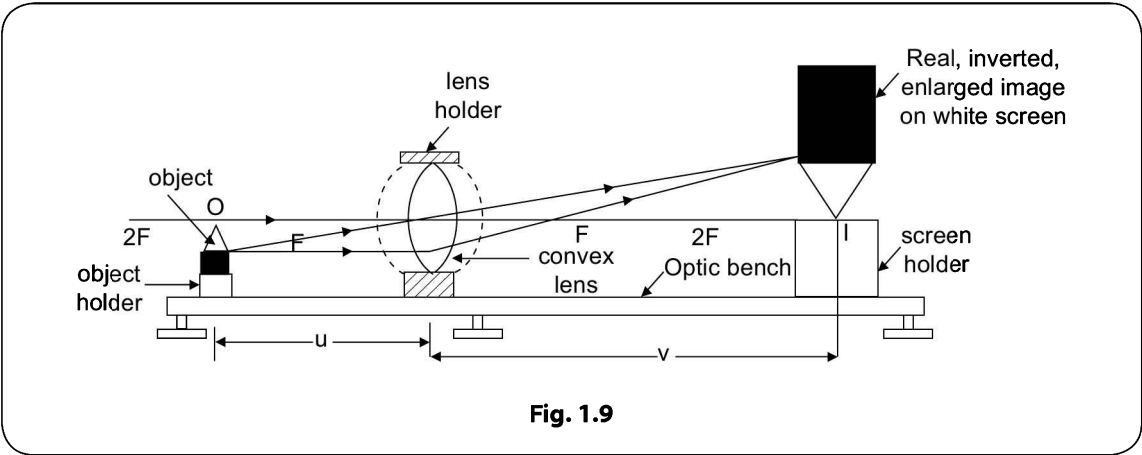
- (iv) The pole of the mirror, tips of the object and image needles should all lie at the same height above the optic bench.
- (v) Index correction for u and v should be applied.
- (vi) Principal axis of concave mirror must be horizontal and parallel to the scale (on the optic bench).

Aim

B. To find the focal length of a convex (converging) lens

Apparatus

An optic bench with three uprights (central upright fixed and two outer uprights with lateral movement), a convex lens with lens holder, two optic needles (one thin, other thick), a knitting needle and a metre scale (or a half metre scale). If the optic bench is etched or attached with a metre scale, no separate scale is necessary.



The convex lens (converging) is held by a lens holder on the optic bench and its position on the bench is fixed. A bright object (optic needle) is placed on an upright on one side of the lens at certain distance. A white screen on another upright at the other side of the lens is moved, till a sharp, inverted image of the object is obtained on the screen. The positions of the object and screen are locked and their distances from the lens (u and v respectively) are measured. Keeping the position of lens fixed, the position of object is varied and the corresponding positions of image on screen are noted. The readings are noted down in the observation table.

Object distance (u)	Image distance (v)	Focal length (f)

Calculation

The magnitude of the focal length (f) of the convex lens is given by the formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

$$\Rightarrow f = \frac{uv}{(u + v)}$$

(Note: No sign convention to be applied to this formula)

After determining the focal lengths for various object distances, average them out.

Conventionally, the focal length of a convex lens is taken as positive (+).

$$\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} - \frac{\Delta u}{(u + v)} - \frac{\Delta v}{(u + v)}$$

A very precise value of $\frac{\Delta f}{f}$ is obtained as

$$\frac{\Delta f}{f} = f \left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right]$$

Sources of error

- (i) The uprights may not be vertical.
- (ii) Removal of parallax may not be perfect.
- (iii) Index error (or bench error) may exist. In this experiment, the distances are to be measured from the optic centre of the lens. The measured distances (or observed distances) may not be equal to the actual distances and consequently an error is crept in measurements.

Precautions to be taken

- (i) The uprights must be vertical.
- (ii) Tip to tip parallax should be removed between the object needle O and the image of the needle I.
- (iii) To locate the position of the image, the eyes should be at least 30 cm away from the needle. The optic centre of the lens, tips of the object and image needles should all lie at the same height above the optic bench.
- (iv) Index correction for u and v should be applied.
- (v) Principal axis of convex lens must be horizontal and parallel to the scale (on the optic bench).

Experiment 7: Speed of sound using resonance column

Aim

To determine the speed of sound in air and hence to find the speed of sound at 0 °C.

Principle

An excited tuning fork of known frequency f is held over the mouth of an open pipe partially immersed in water. The lengths of the air column ℓ_1 and ℓ_2 that resonate in unison with the tuning fork are determined. The speed of air at laboratory temperature $t^\circ \text{C}$ is given by $v_t = 2f(\ell_2 - \ell_1)$. The speed of sound in air at 0 °C is given by $v_0 = \frac{v_t}{\sqrt{1 + t / 273}}$.

Apparatus

Two hollow cylindrical tubes of same diameter but different lengths, a set of tuning forks of known frequencies, rubber pad, stand, metre scale and thermometer.

Procedure

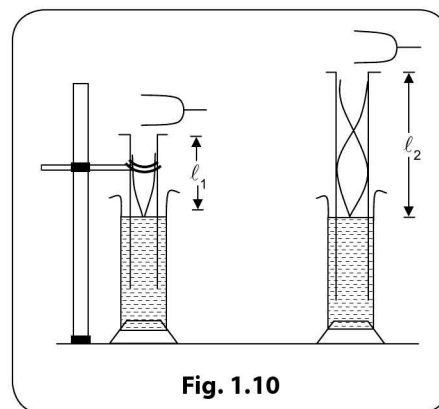


Fig. 1.10

Laboratory temperature $t^\circ \text{C}$ is measured using a thermometer. The glass jar is filled with water. The shorter tube is immersed partially in water. It is then clamped so that it is vertical. One of the given tuning forks is excited and held horizontally above the mouth of the tube. The height of the tube above water is varied until maximum sound is heard. The tube is clamped in this position. The length ℓ_1 of the tube above the surface of water, known as the first resonating length, is measured with a scale. The experiment is repeated with the longer tube using the same tuning fork. The second resonating length ℓ_2 is measured as before. The speed of sound at laboratory temperature $t^\circ \text{C}$ is calculated using the relation $v_t = 2f(\ell_2 - \ell_1)$. The experiment is repeated with two other tuning forks of different frequencies. In each case the resonating lengths ℓ_1 and ℓ_2 are measured and the value of v_t is calculated. The mean value v_t is calculated. The results are tabulated as shown.

1.24 Units, Dimensions and Error Analysis in Experiments

Trial No.	Frequency f Hz	Resonating lengths	$v_b = 2f(l_2 - l_1)$ m s ⁻¹
1		l_1 m l_2 m	
2			
3			
4			

Velocity of sound in air at 0 °C is calculated using the relation $v_0 = \frac{v_t}{\sqrt{1 + t / 273}}$ ms⁻¹

CONCEPT STRAND

Concept Strand 11

The following readings were recorded in an experiment to determine the velocity of sound in air using a resonance column. Calculate velocity of sound in air at the laboratory temperature and hence the velocity at 0 °C.

Trial No.	f (Hz)	Resonating lengths		$V_t = 2f(\ell_2 - \ell_1)$ m s ⁻¹
		$\ell_1 \times 10^{-2}$ m	$\ell_2 \times 10^{-2}$ m	
1	510	16.2	50.1	345.78
2	480	18.4	54.5	346.56
3	425	20.3	60.8	344.25

Result

Velocity of sound in air at t °C = $v_t = \dots\dots\dots$ m s⁻¹

Velocity of sound in air at 0 °C = $v_0 = \dots\dots\dots$ m s⁻¹

Sources of error and precautions

1. While finding ℓ_1 and ℓ_2 , the tube must be held vertically. The excited tuning fork must be directly above the tube in such a way that the waves travel along the axis of the tube.
2. Resonance must be detected carefully by trial and error method.
3. The ear should not be placed close to the fork or the open end of the tube.

Mean value of $V_t = 345.53$ m s⁻¹

Laboratory temperature, t = 26 °C

$$\therefore V_0 = \frac{V_t}{\sqrt{1 + t / 273}} = \frac{345.53}{\sqrt{1 + 26 / 273}} = 330.17 \text{ m s}^{-1}$$

Note:

$$\text{End correction } e = \frac{\ell_2 - 3\ell_1}{2}$$

Experiment 8: Verification of Ohm's law using voltmeter and ammeter

PART-A

Aim

To verify Ohm's law.

Principle

Ohm's law states that temperature and other physical conditions remaining the same, the current through a conductor is directly proportional to the potential difference across its ends.

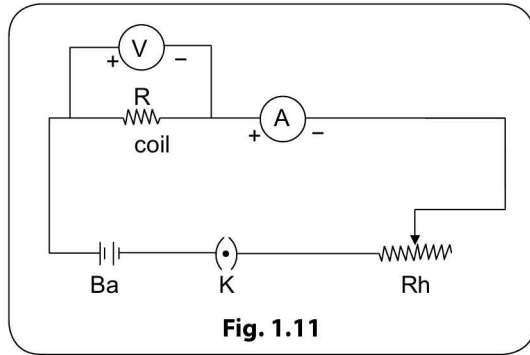
If I be the current through a conductor when the potential difference across its ends is V, then

$$\frac{V}{I} = \text{constant}$$

A known current I is passed through a coil. The potential difference V across its ends is measured. Then V/I is shown to be a constant thereby verifying Ohm's law.

Apparatus

Resistance coil, battery, rheostat, plug key, ammeter and voltmeter.

Circuit diagram

Connections are made as shown in the circuit diagram (Fig. 1.11). The coil of resistance R is connected in series with a battery, key, rheostat and an ammeter A . A voltmeter V is connected across the coil.

Procedure

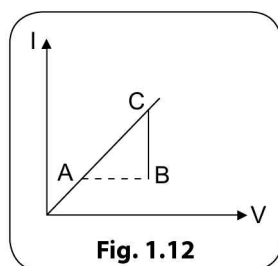
1. The circuit is completed by closing the key. The rheostat is adjusted so that a small current flows through the circuit. The reading of the ammeter (I ampere), and that of the voltmeter (V volt) are noted. The ratio V/I is calculated.
2. Using the rheostat the current in the circuit is varied. In each case the ratio V/I is calculated. The results are tabulated. The ratio V/I is found to be constant within the limits of experimental error. This verifies Ohm's law.

Trail No.	V(volt)	I (ampere)	$R = V/I$ (ohm)
1			
2			
3			

The mean value of $V/I = \text{Resistance of the coil} = \dots\dots\dots\Omega$

Result: Resistance of the coil $R = \dots\dots\dots\Omega$

$\frac{V}{I}$ is found to be a constant. Thus Ohm's law is verified.



A graph of voltage V versus current I is found to be a straight line passing through the origin. Thus $V \propto I$ and Ohm's Law is verified.

Resistance $R = \dots\dots\Omega$

PART B**Aim**

To verify the laws of combination of resistances using Ohm's law.

(1) Law of resistances in series**Principle**

According to the law of resistances in series, the effective resistance of two or more resistances in series is equal to the sum of the individual resistances, $R_s = R_1 + R_2$

$$(R_s)_{\text{theoretical}} = R_1 + R_2$$

The effective resistance of two standard resistance coils of resistances R_1 and R_2 connected in series is determined experimentally by measuring the p.d. across the combination and current through the combination.

$$(R_s)_{\text{expt}} = \frac{V}{I}$$

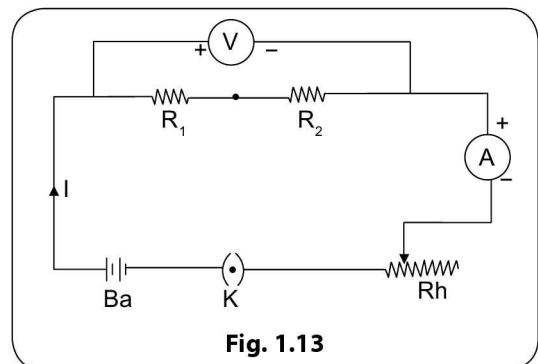
The experimental value is compared with the theoretical value and the law is verified.

Apparatus

Battery, plug key, rheostat, two standard resistance coils, ammeter and voltmeter.

Circuit diagram

Two standard resistance coils of known resistance R_1 and R_2 are connected in series (joined end to end). The



1.26 Units, Dimensions and Error Analysis in Experiments

combination is connected in series with a battery, a plug key, rheostat and ammeter. The voltmeter is connected across the combination.

Procedure

1. The circuit is completed by closing the plug key. The rheostat is adjusted for a suitable small current. The ammeter reading I and voltmeter reading V are noted. The experimental value of effective resistance of the series combination is calculated using,

$$(R_s)_{\text{expt}} = \frac{V}{I}$$
2. The experiment is repeated for different values of I by adjusting the rheostat. In each case, the corresponding value of V is noted and the effective resistance is calculated. The readings are tabulated and the mean value of effective resistance is calculated.
3. The theoretical value of effective resistance of the combination is calculated using $(R_s)_{\text{theoretical}} = R_1 + R_2$. The experimental value is compared with the theoretical value. It is found that they agree within the limits of experimental errors. This verifies the law of combination of resistances in series.

Trial No.	V(volt)	I (ampere)	$(R_s)_{\text{expt}} = V/I$ (ohm)
1.			
2.			
3.			

Mean $(R_s)_{\text{expt}} = \text{-----} \Omega$

$R_1 = \text{-----} \Omega$, $R_2 = \text{-----} \Omega$

$(R_s)_{\text{theoretical}} = R_1 + R_2 = \text{-----} \Omega$

It is found that $(R_s)_{\text{expt}} = (R_s)_{\text{theoretical}}$, within the limits of experimental errors.

Result

$(R_s)_{\text{expt}} = \text{.....} \Omega$ $(R_s)_{\text{theoretical}} = \text{.....} \Omega$

The law of combination of resistances in series is verified.

(2) Law of resistances in parallel

Principle

According to the law of resistances in parallel, the reciprocal of the effective resistance of two or more resistances in

parallel is equal to the sum of the reciprocals of individual resistances.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$(R_p)_{\text{theoretical}} = \frac{R_1 R_2}{R_1 + R_2}$$

The effective resistance of two standard resistance coils of resistances R_1 and R_2 connected in parallel is determined experimentally by measuring the p.d. across the combination and current through the combination.

$$(R_p)_{\text{expt}} = \frac{V}{I}$$

The experimental value is compared with the theoretical value and the law is verified.

Apparatus

Battery, plug key, rheostat, ammeter, voltmeter and two standard resistance coils.

Circuit diagram

Two standard resistance coils of known resistances R_1 and R_2 are connected in parallel. The combination is connected in series with a battery, a plug key, a rheostat and an ammeter. A voltmeter is connected across the combination.

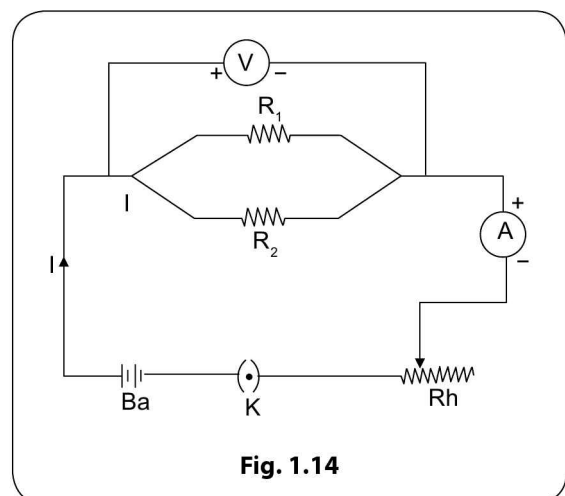


Fig. 1.14

Procedure

1. The experiment is performed as explained under part (1). The readings are tabulated as shown below and the mean value of effective resistance of the combination is calculated using,

$$(R_p)_{\text{expt}} = \frac{V}{I}$$

The theoretical value of effective resistance of the combination is calculated using

$$(R_p)_{\text{theoretical}} = \frac{R_1 R_2}{R_1 + R_2}$$

It is found that $(R_p)_{\text{expt}} = (R_p)_{\text{theoretical}}$, within the limits of experimental errors. This verifies the law of combination of resistances in parallel.

Result

$$(R_p)_{\text{expt}} = \dots \Omega$$

$$(R_p)_{\text{theoretical}} = \dots \Omega$$

The law of combination of resistances in parallel is verified.

Sources of error and precautions

1. Current (I) must be small to minimize the heating effect.
2. Zero errors, if any in ammeter and voltmeter must be taken into account.
3. Suitable standard resistance coils should be used.
4. The positive terminal of the battery must be connected to the positive terminal of the ammeter and voltmeter.

Estimation of error

$$R = \frac{V}{I}$$

The fractional error in the measured value of resistance in any trial is calculated using the formula.

$$\frac{dR}{R} = \frac{dV}{V} + \frac{dI}{I}$$

dV = the smallest value of voltage that can be measured with the given voltmeter

dI = the smallest value of current that can be measured with the given ammeter

The percentage error = $(dR/R) \times 100$

CONCEPT STRAND

Concept Strand 12

The following readings were obtained in an experiment to verify the laws of combination of resistances. Verify the laws. Given: $R_1 = 2 \Omega$ and $R_2 = 3 \Omega$. Calculate the percentage error in a trial if the smallest value of current read with the ammeter is 0.1 A and the smallest value of voltage read with the voltmeter is 0.1 V.

Solution

1. Resistances in series

Trial No.	V (volt)	I (ampere)
1	0.5	0.1
2	0.9	0.2
3	1.4	0.3

2. Resistances in parallel

Trial No.	V (volt)	I (ampere)
1	0.6	0.5
2	1.0	0.8
3	1.3	1.0

Observations

Resistance in series

$R_1 = 2 \Omega$ and $R_2 = 3 \Omega$ (given)

$$(R_s)_{\text{theoretical}} = R_1 + R_2 = 2 + 3 = 5 \Omega$$

Trial No.	V (Volt)	I (ampere)	$(R_s)_{\text{expt}} = \frac{V}{I} (\Omega)$
1	1.4	0.3	4.67
2	1.9	0.4	4.75
3	2.5	0.5	5.00

$$\text{Mean } (R_s)_{\text{expt}} = \frac{4.67 + 4.75 + 5.00}{3} = 4.81 \Omega$$

$(R_s)_{\text{expt}} = (R_s)_{\text{theoretical}}$, within the limits of experimental errors.

The law of resistances in series is verified.

1.28 Units, Dimensions and Error Analysis in Experiments

Resistances in parallel

$$R_1 = 2 \, \Omega \text{ and } R_2 = 3 \, \Omega \quad (\text{given})$$

$$(R_p)_{\text{theoretical}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2 \times 3}{5} = 1.2 \, \Omega$$

Trial No.	V (volt)	I (ampere)	$(R_p)_{\text{expt}} = \frac{V}{I} (\Omega)$
1	0.6	0.5	1.20
2	1.0	0.8	1.25
3	1.3	1.0	1.30

$$\text{Mean } (R_p)_{\text{expt}} = \frac{1.20 + 1.25 + 1.30}{3} = 1.25 \, \Omega$$

$(R_p)_{\text{expt}} = (R_p)_{\text{theoretical}}$, within the limits of experimental errors.

The law of resistances in parallel is verified

Calculation of percentage of error in the value of resistance in a trial

$$R = \frac{V}{I},$$

$$\frac{dR}{R} = \frac{dV}{V} + \frac{dI}{I}$$

Taking $V = 1.4$, $I = 0.3$

$dV = 0.1 \, \text{V}$ $dI = 0.1 \, \text{A}$ (given)

$$\frac{dR}{R} = \frac{0.1}{1.4} + \frac{0.1}{0.3} = 0.07 + 0.33 = 0.4$$

$$\text{Percentage of error} = \frac{dR}{R} \times 100 = 0.4 \times 100 = 40\%$$

Experiment 9: Specific resistance of the material of a wire using a metre bridge

Aim

To determine the resistivity (or specific resistance) of the material of a wire using a metre bridge.

Principle

The resistance X of the given wire is measured using a metre bridge. The mean diameter d of the wire is measured using a screw gauge. The length L of the wire is measured using a meter scale. The resistivity of the material of the wire is calculated using the formula.

$$\rho = \frac{(X)\pi d^2}{4L}$$

Apparatus

Metre bridge, Leclanche cell, plug key, sensitive galvanometer, slider, resistance box, metre scale, screw gauge, experimental wire, connecting wires. The metre bridge consists of a uniform wire AC of length $1 \, \text{m}$.

Circuit Diagram

A metre bridge works on the principle of Wheatstone's network. Electrical connections are made as shown in the circuit diagram (Fig. 1.14). The experimental wire X whose resistance is to be determined is connected in the gap on

the left of the bridge. A standard resistance box S is connected in the gap on the right. A Leclanche cell of emf E in series with a plug key is connected between the terminals A and C . One terminal of a sensitive galvanometer G is connected to B , the other terminal is connected to a slider using a long connection wire so that the slider can move easily from one end of the bridge wire to the other.

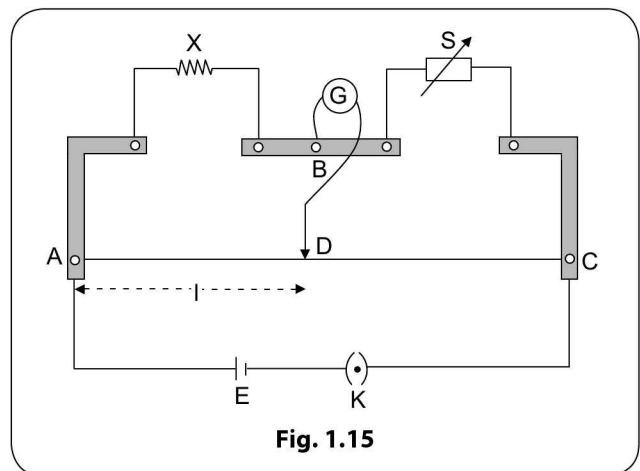


Fig. 1.15

Procedure

(i) Determination of resistance

1. A suitable resistance S is unplugged in the resistance box. The plug key is closed. The slider is first pressed on the bridge wire near the end A and then near the end B . The full scale deflections in the

galvanometer in opposite directions ensures proper electrical connections.

- The slider is gently moved along the bridge wire from end A until the deflection in the galvanometer becomes zero. The point D on the wire at which galvanometer shows zero deflection is called the balance point. The balancing length $AD = \ell$ is measured with the metre scale fixed below the bridge wire. The resistance S unplugged in the box is noted. The resistance x of the experimental wire is calculated using the formula.

$$X = \frac{S\ell}{1 - \ell}$$

- The experiment is repeated for different values of S. In each case, the corresponding balancing length is measured and the resistance of the wire is calculated. The readings are tabulated as shown below and the mean value of x is calculated.

Trial No.	S (Ω)	ℓ (m)	$X = \frac{S\ell}{1 - \ell}$ (Ω)
1			
2			
3			

Mean value of $X = \dots\Omega$

(ii) Determination of length

- The experimental wire connected in the left gap S is taken out. It is stretched without any kinks. The length L of the wire is measured using a metre scale.

(iii) Determination of diameter

- The pitch, least count and zero error of the given screw gauge are determined.

$$\text{Pitch} = \frac{\text{Distance moved on the pitch scale}}{\text{Number of rotations given to the head scale}} = \dots\text{mm}$$

Least count,

$$L.C = \frac{\text{Pitch}}{\text{No. of divisions on the head scale}} = \dots\text{mm}$$

Zero error, $Z = \dots\text{HSD}$

$$T.R = PSR + (HSR - Z) LC = \dots\text{mm}$$

- The diameter of the wire is determined at several places along the length of the wire. The readings are tabulated as shown and the mean diameter of the wire is calculated.

Trial No.	P.S.R (mm)	H.S.R	$T.R = PSR + (HSR - Z) \times LC$ (mm)
1.			
2.			
3.			

Mean diameter $d = \dots\text{mm} = \dots \times 10^{-3} \text{ m}$

- The electrical resistivity of the material of the given wire is calculated using the formula

$$\rho = \frac{(X)\pi d^2}{4L} = \dots\Omega \text{ m}$$

Result

The resistivity of the material of the given wire = $\dots\Omega \text{ m}$.

Source of error and precautions

- The value of S must be adjusted such that the balance point lies around middle portion of the bridge wire. This is to reduce the error in the measured value.
- The plugs in the resistance box must be tightened to ensure good electrical contact.
- The exact balance point must be found out by trial and error.
- Since the diameter d occurs as d^2 in the formula for resistivity it must be determined accurately. d being small, the error in the measured value will be large.

CONCEPT STRAND

Concept Strand 13

The following readings were obtained in an experiment to determine the resistivity of the material of a wire using a metre bridge. Calculate the resistivity of the material of the wire.

Solution

Determination of resistance

Trial No.	S in Ω	ℓ in m
1	2	0.545
2	3	0.448
3	4	0.374

1.30 Units, Dimensions and Error Analysis in Experiments

Determination of diameter

Least count of screw gauge = 0.01 mm

Zero error $Z = 2$ HSD

Trial No.	P.S.R mm	H.S.R
1	0	37
2	0	38
3	0	36

Length of the wire, $L = 60$ cm

Observations

Determination of resistance

Trial No	S (Ω)	ℓ (m)	$X = \frac{S\ell}{1-\ell} \Omega$
1	2	0.545	2.396
2	3	0.448	2.435
3	4	0.374	2.390

Mean $x = 2.407 \Omega$

$$X = \frac{S\ell}{1-\ell} = \frac{2 \times 0.545}{1-0.545} = \frac{2 \times 0.545}{0.455} = 2.396 \Omega$$

$$X = \frac{S\ell}{1-\ell} = \frac{3 \times 0.448}{1-0.448} = \frac{3 \times 0.448}{0.552} = 2.435 \Omega$$

$$X = \frac{S\ell}{1-\ell} = \frac{4 \times 0.374}{1-0.374} = \frac{4 \times 0.374}{0.626} = 2.390 \Omega$$

$$\text{Mean } X = \frac{2.396 + 2.435 + 2.390}{3} = 2.407 \Omega$$

Determination of the diameter of the wire

Least count of screw gauge LC = 0.01 mm

Zero error $Z = -2$ HSD

Trial No.	PSR (mm)	HSR	TR = PSR + (HSR - Z) LC diameter d (mm)
1	0	37	0.39
2	0	38	0.40
3	0	36	0.38

Mean diameter $d = 0.39$ mm $= 3.9 \times 10^{-4}$ m

$$\begin{aligned} \text{TR} &= \text{PSR} + (\text{HSR} - Z) \text{LC} = 0 + [37 - (-2)] 0.01 \\ &= 0 + (39 \times 0.01) = 0.39 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Mean diameter} &= \frac{0.39 + 0.40 + 0.38}{3} = 0.39 \text{ mm} \\ &= 3.9 \times 10^{-4} \text{ m} \end{aligned}$$

Calculation of resistivity

Length of the wire $L = 60$ cm $= 0.60$ m

Diameter of the wire $d = 3.9 \times 10^{-4}$ m

Resistance of the wire $x = 2.407 \Omega$

$$\text{Resistivity of the material of the wire } \rho = \frac{(X)\pi d^2}{4L}$$

$$\begin{aligned} \rho &= \frac{2.407 \times 3.14 \times (3.9 \times 10^{-4})^2}{4 \times 0.60} \\ &= \frac{2.407 \times 3.14 \times 3.9^2 \times 10^{-8}}{4 \times 0.6} = 47.9 \times 10^{-8} \end{aligned}$$

$$\rho = 0.479 \times 10^{-6} \Omega \text{ m} = 0.479 \mu\Omega \text{ m.}$$

The resistivity of the material of the wire $= 0.479 \mu\Omega \text{ m}$

Experiment 10: Measurement of unknown resistance using a post office box

Aim

To find the resistance (X) of a given wire using a Post Office Box and hence to determine the specific resistance of the material of the wire.

Apparatus

Post office box, a cell, a galvanometer, unknown resistance wire, a screw gauge, a metre rod, connecting wires, 2 rheostats and a piece of sand paper.

The post office box is a testing device based on the principle of Wheatstone's bridge. It has pegs and spring arms to close electrical circuits and measure properties of the circuit under test. These boxes were used in the United Kingdom by the engineers from the then General Post Office, who were responsible for UK telecommunications to trace electrical faults. Hence the name 'Post Office Box' for this instrument.

A typical Post Office Box is a wooden box with a hinged lid and a metal or bakelite panel showing circuit connections. Coils of wire are wound non-inductively, mounted in the body of the box and have a negligible temperature coefficient of resistance.

Pairs of ratio arms (P and Q) are each 10 Ω , 100 Ω and 1000 Ω . Hence the ratio of resistances of P (between A and B of Fig. 1.16) and Q (between B and C of Fig. 1.16) can be 10 : 10; 100 : 10; 1000 : 10; 100 : 10; 100 : 100; 100 : 1000, 1000 : 10, 1000 : 100 or 1000 : 1000 respectively.

Resistance arm R (between A and D of Fig. 1.16) contains a number of coils from 1 Ω to 5000 Ω with a plug for infinite resistance.

Connection Diagram

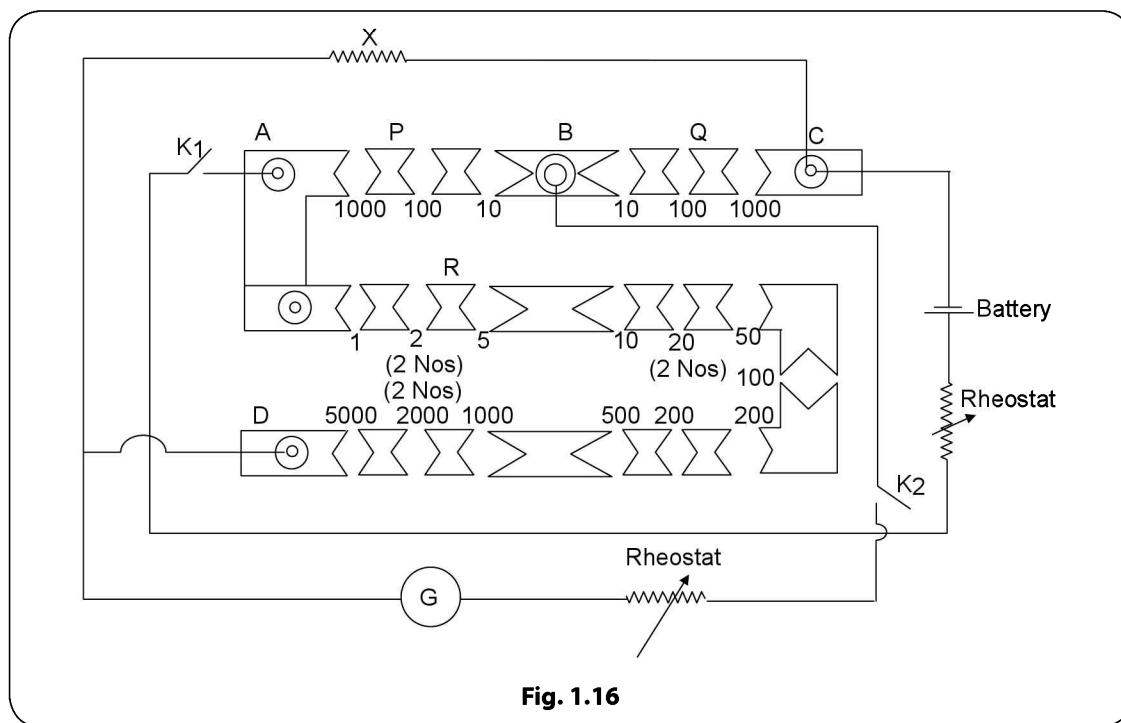


Fig. 1.16

Across AC, a battery, a rheostat and a key K_1 are connected while across BD, a galvanometer, a rheostat and a key K_2 are connected. The wire of unknown resistance X is connected between C and D. Note that the positive terminal of the battery is connected to A. The equivalent electric circuit is as follows:

When the Wheatstone's bridge is balanced, $\frac{P}{Q} = \frac{R}{X} \Rightarrow$

$$X = \left(\frac{Q}{P} \right) R$$

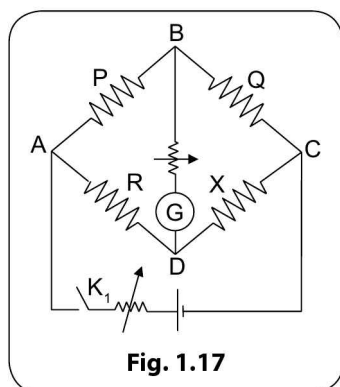


Fig. 1.17

Working procedure

The ratio arms (P and Q) are first adjusted so that they have resistances of 100 Ω each. If the value of resistance across R is so adjusted that on pressing key K_1 first and K_2 afterwards, the galvanometer shows no deflection, it means bridge is in balanced condition and $X = \frac{Q}{P} R$.

However, a more practical method is as follows. The ratio arms (P and Q) are first adjusted so that they have resistances of 100 Ω each. The resistance in the resistance arm is now adjusted so that the deflection in the galvanometer

1.32 Units, Dimensions and Error Analysis in Experiments

is in one direction (say for $R = R_0$) and in opposite direction, if $R = R_0 + 1 \Omega$. Now we can say that the resistance of resistance arm R lies between R_0 and $R_0 + 1 \Omega$. Hence $R =$

$(R_0 + 0.5) \Omega$. Hence $x = \frac{Q}{P} R = R \left(\because \frac{Q}{P} = 1 \right)$. Now repeat the experiment for $P = 100 \Omega$ and $Q = 1000 \Omega$ (i.e., $\frac{P}{Q} = \frac{1}{10}$), for $P = 10 \Omega$ and $Q = 1000 \Omega$ (i.e., $\frac{P}{Q} = \frac{1}{100}$),

for $P = 1000 \Omega$, $Q = 100 \Omega$ (i.e., $\frac{P}{Q} = 10$) etc., and compute

X. To reduce random errors, take the average value. The specific resistance of the material of the wire is given by $\rho = \frac{X\pi d^2}{4L}$, where d = diameter of wire, L = length of wire.

It must be noted that the accuracy of the post office box experiment depends upon the choice of the ratio $\frac{Q}{P}$.

If $\frac{Q}{P} = 1$, then the value of the unknown resistance x is obtained within $\pm 1 \Omega$. If $\frac{Q}{P} = \frac{1}{10}$, then the unknown resistance

$x = R \left(\frac{1}{10} \right)$ is accurately measured upto $\pm 0.1 \Omega$.

If $\frac{Q}{P} = \frac{1}{100}$, then the unknown resistance $x = R \left(\frac{1}{100} \right)$ is accurately measured upto $\pm 0.01 \Omega$.

Sources of error

- (i) The instrument screws may be loose so that the connecting wires do not make proper contact.
- (ii) The resistance plugs may be unclean.
- (iii) The resistances may change during the experiment due to joule heating.
- (iv) The experimental wire may not be of uniform thickness.
- (v) Insensitivity of the Wheatstone's bridge.

Precautions to be taken

- (i) The ends of the connecting wires as well as unknown resistance wire should be cleaned by sand paper.
- (ii) All connections should be neat, clean and tight.
- (iii) All plugs of the post office box should be clean and tight.
- (iv) The experimental wire should be straight and must not form any loop.
- (v) For taking an observation, the cell key (K_1) should be always pressed first and then the galvanometer key (K_2).
- (vi) The current should pass through the circuit for minimum time. This will avoid error due to change in resistance by Joule's heating.
- (vii) The length of the wire (L) should be measured from the points where the wire comes out of the terminals of the post office box.

CONCEPT CONNECTORS

Connector 1: Mass of a particle was measured as 1.6749×10^{-27} kg. The velocity of light is 2.99792458×10^8 m s⁻¹. Calculate the rest energy of the particle using the Einstein formula $E = mc^2$.

Solution:

$$m = 1.6749 \times 10^{-27} \text{ kg}$$

$$c = 2.99792458 \times 10^8 \text{ m s}^{-1}$$

$$E = mc^2$$

$$= 1.6749 \times 10^{-27} (2.99792458)^2 \times 10^{16} \text{ J}$$

$$= 15.05325049 \times 10^{-11} \text{ J}$$

Since m is given to five significant figures, the result should be expressed as

$$E = 15.053 \times 10^{-11} \text{ J}$$

Connector 2: A pendulum of length ℓ with a bob of negligible mass, oscillates with very small amplitude. Derive the expression for the time period by dimensional analysis.

Solution: Experimentally it is found that the time period does not depend on the mass of the bob and the oscillation angle, provided they have small values. The only parameters on which the time period will depend are the length of the pendulum and the acceleration due to gravity

$$\therefore T = K \ell^\alpha g^\beta \text{ where, } K, \alpha \text{ and } \beta \text{ are constants}$$

Dimensionally

$$[T] = [K] L^\alpha [L T^{-2}]^\beta$$

$$= L^{\alpha + \beta} T^{-2\beta}$$

Equating powers on both sides

$$\therefore \alpha + \beta = 0;$$

$$\beta = \frac{-1}{2}$$

$$\Rightarrow \alpha = \frac{1}{2}$$

$$\therefore T = K \ell^{\frac{1}{2}} g^{-\frac{1}{2}}$$

i.e., $T = K \sqrt{\frac{\ell}{g}}$; $K = 2\pi$ is determined experimentally and cannot be evaluated by dimensional analysis.

$$\therefore T = 2\pi \sqrt{\frac{\ell}{g}}$$

Connector 3: A progressive harmonic wave in the x -direction in a medium is represented by the equation $y = A \sin [\omega (x' - t + \alpha)]$ where, ω is the angular frequency and α a constant. Identify A , x' and α and their dimensions.

Solution: The general expression for a progressive harmonic wave is

$$y = A \sin(kx - \omega t + \phi)$$

where, y is the displacement at x , A the amplitude, k the wave number, ω the angular frequency, t the time and ϕ the phase constant.

1.34 Units, Dimensions and Error Analysis in Experiments

Comparing with the given expression,

(i) A is the amplitude with dimension L

$$\begin{aligned} \text{(ii) } \omega x' &= kx \Rightarrow x' = \frac{kx}{\omega} \\ &= \frac{2\pi}{\lambda} \cdot \frac{1}{\omega} \cdot x \quad (\lambda \text{ is the wavelength}) \\ &= \frac{x}{v} \left(\because \frac{2\pi}{\lambda\omega} = \frac{2\pi}{\lambda \cdot 2\pi\nu} = \frac{1}{\lambda\nu} = \frac{1}{v} \right) \\ &= \tau \end{aligned}$$

having the dimension T and representing the time taken by the wave to reach x from where it started at x = 0

$$\text{(iii) } \omega\alpha = \phi \Rightarrow \alpha = \frac{\phi}{\omega}$$

which is the phase constant defined not as a constant angle but as (angle $\times$ time).

$\therefore$ dimension of α is T

Aliter:

An easy answer is as follows:

The argument of sine function is dimensionless (angle). Since ω is outside the bracket, inside the bracket, All the three factors x' , t and α should have the same dimension T.

Connector 4: The electrostatic force between two point charges q_1 and q_2 separated by a distance r is given by the relation

$$F = K \frac{q_1 q_2}{r^2}$$

where K is a constant depending on the medium in which the charges are situated. Find the dimensional formula of the constant K.

Solution: F has dimensional formula MLT^{-2} .

$K \frac{q_1 q_2}{r^2}$ has dimensional formula

$$[K] \left[\frac{AT \cdot AT}{L^2} \right] = [K] \left[\frac{A^2 T^2}{L^2} \right]$$

Equating the dimensions,

$$[MLT^{-2}] = [K] \left[\frac{A^2 T^2}{L^2} \right]$$

$$\therefore K = ML^3 T^{-4} A^{-2}$$

Connector 5: If Planck's constant h, velocity of light c and energy E are chosen as the fundamental units, what is the dimension of length?

Solution: Energy has dimensional equation

$$\begin{aligned} [E] &= ML^2 T^{-2} \\ &= [ML^2 T^{-1}] [T^{-1}] \\ &= [ML^2 T^{-1}] [LT^{-1}] [L^{-1}] \end{aligned}$$

$\therefore$ Re arranging,

$$[L] = [ML^2 T^{-1}] [LT^{-1}] [E^{-1}]$$

But $ML^2 T^{-1} = MLT^{-1} L = \text{momentum} \times \text{length} = \text{angular momentum}$

$$[L] = [hcE^{-1}]$$

Aliter:

$$\text{SI unit of } h = \text{joule second} = [ML^2T^{-1}]$$

$$E = \text{joule} = [ML^2T^{-2}]$$

$$c = \text{metre per second} = [LT^{-1}]$$

$$\frac{h}{E} = \text{second} = [T]$$

$$\frac{h}{E} \times c = \text{second} \times \frac{\text{metre}}{\text{second}} = \text{metre} [L]$$

$$\therefore [L] = \frac{hc}{E} = hcE^{-1}$$

Connector 6: The following are the measured values of the diameter (d) of a steel ball in a bearing when a screw gauge of least count 0.001 cm was used.

Trial	1	2	3	4	5	6	7	8	9	10
d(cm)	0.254	0.256	0.248	0.248	0.247	0.253	0.246	0.246	0.252	0.250

What is the volume of the steel ball?

$$\text{Solution: } (d)_{\text{average}} = \frac{0.254 + 0.256 + 0.248 + 0.248 + 0.247 + 0.253 + 0.246 + 0.246 + 0.252 + 0.250}{10} = 0.250 \text{ cm}$$

The value is to be reported as 0.250 cm because the precision is equal to the least count of the screw gauge which is 0.001 cm.

$$\begin{aligned} \text{Volume } V &= \frac{4}{3} \pi \left(\frac{d}{2} \right)^3 = \frac{4}{3} \times 3.14159265 \times \left(\frac{1}{8} \right)^3 \text{ cm}^3 \\ &= 0.00818123 \text{ cm}^3 = 8.18 \times 10^{-3} \text{ cm}^3 \end{aligned}$$

Connector 7: Show that angular momentum can be expressed by the unit J s.

$$\text{Solution: } \frac{d\bar{L}}{dt} = \bar{J} \text{ where, } \bar{L} \text{ is angular momentum and } \bar{J} \text{ is torque.}$$

$$\therefore d\bar{L} = \bar{J} dt$$

$$= \text{Force} \times \text{moment arm} \times \text{time}$$

$$\text{Dimensionally, } [\bar{L}] = [MLT^{-2}] \times [L] \times [T]$$

$$= [ML^2T^{-2}] \times [T] = \text{Energy} \times \text{time} = \text{J s}$$

Connector 8: What is the dimensional formula of electrical resistance?

$$\text{Solution: } R = \frac{V}{A} = \frac{\text{Energy} / \text{Charge}}{A}$$

Equating dimensions

$$[R] = \left[\frac{ML^2T^{-2} / A.T}{A} \right] = [ML^2T^{-3}A^{-2}]$$

1.36 Units, Dimensions and Error Analysis in Experiments

Aliter:

Electric power = $I^2 R$

$$\Rightarrow [R] = \frac{\text{Power}}{I^2} = \frac{[ML^2T^{-3}]}{[A^2]}$$

$$\Rightarrow [R] = [ML^2T^{-3}A^{-2}]$$

Connector 9: The SI unit of viscosity is poiseuille (Pℓ). What is its dimensional formula?

Solution:

$$\text{viscosity} = \frac{\text{shear stress}}{\text{strain rate}}$$

$$\text{Strain rate} = \frac{d}{dt} \left(\frac{\Delta \ell}{\ell} \right) = \frac{1}{\ell} \frac{d(\Delta \ell)}{dt} = \frac{1}{\ell} \frac{d\ell}{dt} = \frac{v}{\ell}$$

$$\therefore \text{viscosity} = \frac{F/A}{v/\ell} = \text{pressure} \times \text{time} (N m^{-2} s)$$

$$\text{Dimensionally, } [\text{viscosity}] = [MLT^{-2}L^{-2}T] = ML^{-1}T^{-1}$$

Connector 10: A new system of fundamental units M' , L' and T' is defined such that force F' , velocity v' and acceleration a' in the new system are related to those in the old system by

$$F' = \beta^3 F$$

$$v' = \beta v$$

$$a' = \frac{\beta^2}{\alpha} a$$

where, α and β are constants. How are M' , L' and T' related to the old M , L and T ?

Solution:

Dimensionally

$$[F'] = M' L' (T')^{-2} = \beta^3 [F] = \beta^3 [MLT^{-2}] \quad \text{--- (1)}$$

$$[v'] = ()^{-1} = \beta [v] = \beta [LT^{-1}] \quad \text{--- (2)}$$

$$[a'] = L' (T')^{-2} = \frac{\beta^2}{\alpha} [a] = \frac{\beta^2}{\alpha} [LT^{-2}] \quad \text{--- (3)}$$

Squaring (2) and dividing by (3) algebraically,

$$\frac{(L')^2 (T')^{-2}}{(L') (T')^{-2}} = \frac{\beta^2 L^2 T^{-2}}{\beta^2 L T^{-2} / \alpha}$$

$$\therefore L' = \alpha L \quad \text{--- (4)}$$

Substituting in (2)

$$\alpha L (T')^{-1} = \beta L T^{-1}$$

$$T' = \frac{\alpha}{\beta} T \quad \text{--- (5)}$$

Substituting (4) and (5) in (1)

$$M' (\alpha L) \left(\frac{\alpha}{\beta} T \right)^{-2} = \beta^3 M L T^{-2}$$

$$\frac{M'}{\alpha} = \beta M$$

$$\therefore M' = \alpha \beta M \quad \text{--- (6)}$$

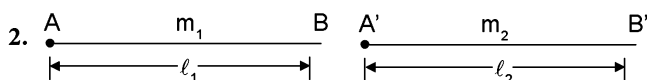
Equations (4), (5) and (6) are the required relations.

TOPIC GRIP



Subjective Questions

1. In a manufacturing line, a parallel combination of 12 ohm resistors give $3\ \Omega \pm 10\%$. If three resistors used in a combination are connected in series, what is the maximum voltage that can be applied across that series combination to keep the current within safe limit of 1A?



Two rods AB and A' B' of mass m_1 and m_2 and lengths ℓ_1 and ℓ_2 respectively, can rotate in a vertical plane about horizontal axes through A and A' respectively. They are released from horizontal position at the same instant and reach vertical position in time T_1 and T_2 respectively

Given that $\frac{m_1}{m_2} = 3$ and $\frac{\ell_1}{\ell_2} = 4$, determine $\frac{T_1}{T_2}$.

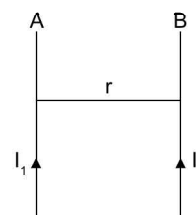
3. In an experiment to measure the internal resistance of a cell using a potentiometer, the error in measurement of length is ± 1 mm. The balancing length obtained with the cell alone is 20 cm. The cell is shunted with $10\ \text{ohm} \pm 2\%$ resistor and the balancing length obtained now is 19 cm. What is the percentage error in the calculation of internal resistance of the cell?
4. In a resonance column experiment using an organ pipe dipped in water and tuning fork of frequency $300\ \text{Hz} \pm 3\ \text{Hz}$, the first and second resonance column lengths are 26.1 cm and 81.1 cm and inside radius of the pipe is 1.90 cm. Determine the percentage error in the calculated value of velocity of sound in air from the above data.
5. A stretched string of mass per unit length m and length ℓ is kept under tension T and is put in vibration in one segment. Establish the formula for its frequency of vibration by dimensional analysis.



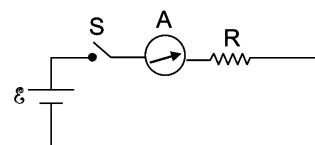
Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

6. The force per unit length of wire B, exerted by wire A is given by $F = \frac{k I_1 I_2}{r}$. Where r is the separation between the parallel wires carrying currents I_1 and I_2 . The dimensional formula of k is
 - (a) $ML^2T^{-2}I^2$
 - (b) $MLT^{-2}I^{-2}$
 - (c) ML^2T^2
 - (d) $M^2L^2T^2I^{-2}$



7. In the circuit shown, the energy delivered by battery is calculated using a stop watch with an error of $\pm 1\%$ in time measurement. The resistor used is having $\pm 5\%$ tolerance and the ammeter error is $\pm 10\%$. The switch is closed at $t = 0$ and opened at time t . The percentage error in the calculated amount of energy used is
 - (a) 16%
 - (b) 21%
 - (c) 26%
 - (d) 32%



1.38 Units, Dimensions and Error Analysis in Experiments

8. Which of the following quantities cannot be expressed as joule/m³?
(a) stress (b) power dissipated per unit volume
(c) energy density (d) pressure
9. The pairs of physical quantities that do not have the same dimensional representation are
(a) volt per meter and newton per coulomb (b) torque and angular momentum
(c) pressure and bulk modulus of elasticity (d) impulse of force and momentum
10. The impedance Z of a transverse wave on a stretched string is given by $Z = \sqrt{\delta T}$ where δ = mass per unit length and T = string tension. Then, which of the following is not correct?
(a) $\frac{Z}{\delta}$ may represent the velocity of the wave along the string
(b) the SI unit for Z is kg s^{-1}
(c) the SI unit for $\frac{Z}{\delta}$ is m s^{-2}
(d) the dimensional formula for T is MLT^{-2}



Assertion–Reason Type Questions

Directions: Each question contains Statement 1 and Statement 2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement 1 is True, Statement 2 is True; Statement 2 is a correct explanation for Statement 1
(b) Statement 1 is True, Statement 2 is True; Statement 2 is NOT a correct explanation for Statement 1
(c) Statement 1 is True, Statement 2 is False
(d) Statement 1 is False, Statement 2 is True

11. Statement 1

Distance moved in one second prior to the n th second by a body with initial velocity u and moving under uniform acceleration a is equal to: $\left[u + \frac{1}{2}a(2n - 1)\right]k$; where k has dimension of $[T]$ and value 1 in this case.
and

Statement 2

$u + \frac{1}{2}a(2n - 1)$ represents the average velocity for an interval of 1 s prior to n th second.

12. Statement 1

The values of diameter of a wire measured using vernier are 1.115 cm; 1.110 cm; 1.120 cm. From these three readings we can conclude that the precision of the measurement is 0.001 cm.
and

Statement 2

Since the third decimal is figuring precision has to be better than 0.01 cm.

13. Statement 1

If we consider electric charge $[Q]$ and energy $[J]$ as basic units, then the dimension of capacitance is $[Q^2J^{-1}]$
and

Statement 2

Energy stored in a capacitor = $\frac{1}{2}CV^2$ (symbols have usual meaning)



Linked Comprehension Type Questions

Directions: This section contains a paragraph. Based upon the paragraph, multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage 1

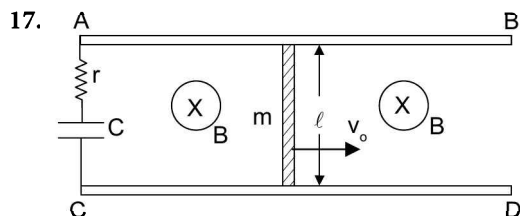
A satellite is launched into an orbit of radius r around the Earth, so that the expected period is T . Then:

14. GM/r has the dimensional formula (symbols have usual meaning):
 (a) ML^2T^2 (b) LT^{-3} (c) L^2T^{-2} (d) $M^{-2}L^{-2}T^{-2}$
15. A 2% error possible in orbital radius observed can result in an error in velocity calculated equal to
 (a) 1% (b) 2% (c) 3% (d) $\frac{1}{3}\%$
16. With the above observed error of 2% in r , the calculated error in time period is
 (a) 1% (b) 2% (c) 3% (d) 4%



Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers of which ONE OR MORE answers will be correct.



In the set up shown, a conducting rod of mass m and length ℓ is set in motion with an initial velocity v_0 and a magnetic field $\vec{B}$ exists as shown. r is the resistor and C is the capacitor connected as shown to the ends of the conducting rails AB and CD . Both rod and rails have no resistance and no friction anywhere. In this context, a formula can be derived as given below:

$$x = \frac{rC \cdot m}{B^2 \ell^2 C + m} \log f(r, B, \ell, C, m)$$

- (a) x has dimension $M^0 L^0 T$
 - (b) x represents acceleration of the rod
 - (c) $B^2 \ell^2 C$ has dimensions $ML^0 T^0$
 - (d) Dimensional integrity of the equation is not possible
18. A wire of length ℓ , is bent into a coil of radius b and is held in a magnetic field B with the plane of the coil normal to the field. A current i is sent through the coil. Which of the following are dimensionally correct?
- (a) The magnetic field $B = \frac{\mu_0 i}{b}$ where i is current
 - (b) The magnetic flux $\phi = \frac{\mu_0 i \ell}{b}$
 - (c) The magnetic energy $U = \mu_0 \ell i^2$
 - (d) the torque on the coil $\tau = \frac{\mu_0 i^2}{2b^2}$

1.40 Units, Dimensions and Error Analysis in Experiments

19. The phase velocity of a surface wave of wave length λ on a liquid of surface tension T and density ρ is given by

$$v_p = \sqrt{\left[\frac{x\lambda}{2\pi} + \frac{2\pi T}{\rho y} \right]}. \text{ Then}$$

- (a) the SI unit of x is m^2s^{-2} (b) the quantity y has the dimensions of $\text{M}^0\text{L}^1\text{T}^0$
(c) the operation $x\lambda + \frac{T}{\rho y}$ is not possible (d) the dimensional formula for xy is L^2T^{-2}



Matrix-Match Type Question

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

20. Match the columns, dimensionally.

symbols used $B \rightarrow$ magnetic field, $L \rightarrow$ inductance, $C \rightarrow$ capacitance, $R \rightarrow$ resistance, $V \rightarrow$ voltage, $i \rightarrow$ current, $\ell \rightarrow$ length of a rod

Column I

- (a) RC
(b) CV^2
(c) L/R
(d) $Bi\ell$

Column II

- (p) MLT^{-2}
(q) T
(r) One or more terms has $[I^*]$ in its dimension ($x \rightarrow$ non zero value)
(s) ML^2T^{-2}

IIT ASSIGNMENT EXERCISE



Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

21. Viscosity is measured in N s m^{-2} in SI units. The number of equivalent CGS unit (poise) for 1 SI unit is
 (a) 0.1 (b) 10 (c) 20 (d) 100
22. Specific heat of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$. It is equivalent to $x \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ where x is ($1 \text{ cal} = 4.2 \text{ J}$)
 (a) 1 (b) 2.2 (c) 4.2 (d) 4.8
23. Let one kilowatt hour be taken as one unit of energy denoted by u . The work done by a force of 5 N producing a displacement of 1 km is
 (a) $\frac{1}{720}u$ (b) $\frac{1}{120}u$ (c) $\frac{1}{36} \times 10^3 u$ (d) $5 \times 10^3 u$
24. The potential energy U of a particle varies with distance x from a fixed origin as $U = \frac{a\sqrt{x}}{b+x^3}$, where, a and b are constants. Then,
 (a) the dimensional formula for a is $\text{ML}^{\frac{9}{2}}\text{T}^{-2}$ (b) the dimensional formula for $\frac{a}{b}$ is $\text{ML}^{\frac{3}{2}}\text{T}^{-1}$
 (c) the operation $(a+b)$ is permissible. (d) the operation $x + b^{\frac{1}{3}}$ is not possible.
25. The thrust F due to an airplane propeller is dependent on the following parameters: angular velocity ω , speed of advance v_0 , propeller diameter D , air viscosity μ , air density ρ and velocity of sound c . Which of the following is not correct?
 (a) the group of parameters formed as $\frac{F}{\rho\omega^2 D^4}$ is dimensionless
 (b) $\frac{\rho\omega D^2}{\mu}$ is a dimensionless quantity
 (c) The parameters which do not have the dimension of mass are ω , v_0 , D , c and μ .
 (d) The parameters which have the dimension of length are F , v_0 , D , c , μ and ρ
26. The dimensional formula of $\frac{\mu_0}{B^2}$, where μ_0 = Permeability of free space and B = magnetic field, is
 (a) $\text{M}^{-1}\text{L}\text{T}^2$ (b) $\text{M}^{-1}\text{L}^{-1}\text{T}$ (c) $\text{M}^{-1}\text{L}^{-2}\text{T}^2$ (d) $\text{M}^{-1}\text{L}^{-1}\text{T}^2$
27. An equation, derived from physical laws, is expressed as $\frac{A}{B} + C = 0$ where A and B have the dimensions of capacitance and magnetic field respectively. What is the dimensional formula of C in mass, length, time, charge system?
 (a) $\text{M}^{-3}\text{L}^{-2}\text{T}^4\text{Q}$ (b) $\text{M}^{-2}\text{L}^{-2}\text{T}^3\text{Q}^4$ (c) $\text{M}^{-3}\text{L}^{-1}\text{T}^3\text{Q}^4$ (d) $\text{M}^{-3}\text{L}^{-2}\text{T}^4\text{Q}^4$
28. The dot product of three vectors $\vec{A}$, $\vec{B}$ and $d\vec{\ell}$ is written as $(\vec{B} \times d\vec{\ell}) \cdot \vec{A}$, where $\vec{B}$ is magnetic field, $d\vec{\ell}$ is the displacement. This dot product represents electromotive force (emf). The SI unit of the vector quantity $\vec{A}$ is
 (a) newton (b) newton per coulomb (c) metre second⁻¹ (d) metre second⁻²

1.42 Units, Dimensions and Error Analysis in Experiments

29. The Lagrange's equation of motion for a planet orbiting the Sun is written as the differential equation $p \frac{d^2 u}{d\theta^2} = \frac{GMm^2}{P} - \frac{P}{r}$, where, r = radius, $u = \frac{1}{r}$, θ = angle in radian, G = the universal gravitational constant, m = mass of the Sun. The quantity P has the dimensions of
 (a) Time period (b) Momentum (c) Angular momentum (d) Force
30. The unsteady state heat conduction in a solid medium is represented by the equation $\frac{\partial^2 T}{\partial x^2} + \frac{Q}{A} = \frac{BC}{A} \left(\frac{\partial T}{\partial t} \right)$ where, T is temperature, x is the X -coordinate, Q is the rate of energy generation per unit volume of the medium, C the specific heat capacity on mass basis and t is the time. The dimensional representation of A is
 (a) $MLT^{-2} \Theta^{-1}$ (b) $ML^3T^{-1} \Theta^{-1}$ (c) $MLT^{-3} \Theta^{-1}$ (d) $ML^3T^{-3} \Theta^{-1}$
31. In the above question the SI unit of the quantity B is
 (a) kg (b) m^3 (c) $kg\ m^{-3}$ (d) $J\ s^{-1}$
32. The parameter d in the equation $d = \sqrt{\frac{16I\rho}{\pi Bv}}$, where, I is current, ρ electrical resistivity, B magnetic field, v velocity, may represent
 (a) mass density (b) diameter (c) relative density (d) area
33. The following differential equation arises in the Rutherford's scattering experiment. $\frac{d^2 x}{dt^2} = \frac{aB}{J} (\cos \psi) \frac{d\psi}{dt}$ where $F = \frac{B}{r^2}$.
 F is force, r distance from the origin, x the X - coordinate, J the angular momentum, ψ the angle and t is time. The dimensional representation of the parameter a is
 (a) $M^0 L^0 T^{-1}$ (b) $M^0 LT^{-2}$ (c) $M^0 L^0 T^0$ (d) $ML^{-1} T^0$
34. The volume flow rate Q of a fluid is given by the equation $Q = \sqrt{p} \int_0^H Lx^{\frac{1}{2}} dx$. Where, L is channel width, x the X coordinate, H the height of the channel. The parameter p has the SI unit
 (a) $m\ s^{-1}$ (b) $m\ s^{-2}$ (c) $m^{\frac{1}{2}}\ s^{-1}$ (d) $m^{-\frac{1}{2}}\ s^{-1}$
35. The Clausius–Clapeyron equation is written as $\frac{dp}{dT} = \frac{L}{(v_1 - v_2)T}$ where, p is pressure T = Boiling point temperature, L = latent heat of vapourisation. The parameters v_1, v_2 must have the dimensions of
 (a) $ML^{-3}\Theta^0$ (b) $M^{-3}L\Theta^{-1}$ (c) $M^{-1}L^3\Theta^0$ (d) $M^0L\Theta^{-1}$
36. The dimensional formula for the quantity LC , where L is inductance and C is capacitance is
 (a) $M^0L^0T^{-2} I^0$ (b) $M^0L^0T^{-1} I^{-1}$ (c) $M^0L^0T^2 I^0$ (d) $M^1L^1T^{-2} I^0$
37. The quantity $\frac{\mu_0 i_1 i_2}{MR}$, where μ_0 is the permeability of free space, i_1, i_2 the currents, M mass and R distance of separation, can represent
 (a) angular acceleration (b) momentum (c) angular momentum (d) power
38. Bernoulli equation for fluid flow is written as $C = \frac{p}{\rho} + \frac{1}{2} \frac{v^2}{mK} + KgZ$, where, p is pressure, ρ density, v velocity, g acceleration due to gravity, Z height. The dimensionless parameters in the equation are
 (a) K alone (b) K and m (c) K and C (d) m and C

39. The pair of quantities having the same dimensional representation is
 (a) torque, angular momentum (b) stress, energy
 (c) pressure, Young's modulus (d) force, momentum
40. The SI unit of the quantity $\frac{1}{2}\rho L v^2$, where ρ is density, L length, v velocity, is
 (a) newton (b) newton metre (c) kg/s^2 (d) kg m^2
41. The $(n + 1)$ th division of a vernier scale coincides with the n th division of the main scale and one main scale division is equal to a unit. The least count of the vernier is
 (a) $\frac{n}{(n + 1)}\text{unit}$ (b) $\frac{an}{(n + 1)}\text{unit}$ (c) $\frac{a}{(n + 1)}\text{unit}$ (d) $\frac{(n + 1)}{an}\text{unit}$
42. A uniform wire of length $\ell = 5 \pm 0.05$ cm and radius $r = 0.5 \pm 0.005$ cm has mass $m = 0.4 + 0.004$ g. The maximum percentage error in its density is
 (a) 1 (b) 2 (c) 4 (d) 5.8
43. Students A, B and C perform an experiment to determine the acceleration due to gravity (g) at a place using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table
 Least count for time = 0.1 s
 Least count for length = 0.1 cm

Student	Length of pendulum (cm)	No of oscillations (n)	Total time for (n) oscillations	Time period(s)
A	64.0	8	128.0	16.0
B	64.0	4	64.0	16.0
C	20.0	4	36.0	9.0

If E_A , E_B and E_C are the percentage errors in g (i.e., $\frac{\Delta g}{g} \times 100$) for students A, B and C respectively.

- (a) E_B is maximum (b) E_A is minimum (c) E_C is minimum (d) $E_A = E_B$
44. The diameter of a steel ball is measured by three students A, B and C using the same screw gauge. A takes 10 readings, B takes 15 readings and C takes 18 readings. The measured value will be most accurate for
 (a) A (b) B (c) C (d) All the three
45. A certain length was measured by three students A, B and C using instruments of least count 0.1 cm, 0.01 cm and 0.001 cm respectively. Each one makes 20 measurements. The measurements will be more precise for
 (a) A (b) B (c) C (d) All of the three
46. The pitch of a screw gauge is 0.5 mm and there are 100 divisions on the circular scale. When the plane face of the screw is in contact with the plane face of the stud on the frame, the zero of the circular scale is below the pitch line by 6 divisions. The zero error of the screw gauge is
 (a) +0.030 mm (b) 0.06 mm (c) -0.030 mm (d) +0.060 mm
47. In the experiment to determine the speed of sound using a resonance column,
 (a) the prongs of the tuning fork are kept in a vertical plane.
 (b) the prongs of the tuning fork are kept in a horizontal plane.
 (c) the prongs of the tuning fork can be kept either in a vertical plane or in a horizontal plane.
 (d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air.

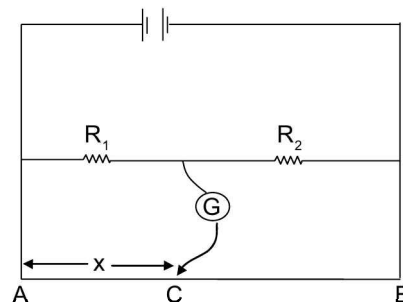
1.44 Units, Dimensions and Error Analysis in Experiments

48. In an experiment to determine the focal length (f) of a concave mirror by the $u-v$ method, a student places an object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/ her eye towards left, the image appears to the right of the object pin. Then

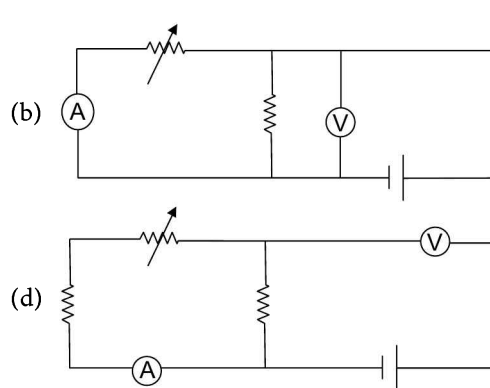
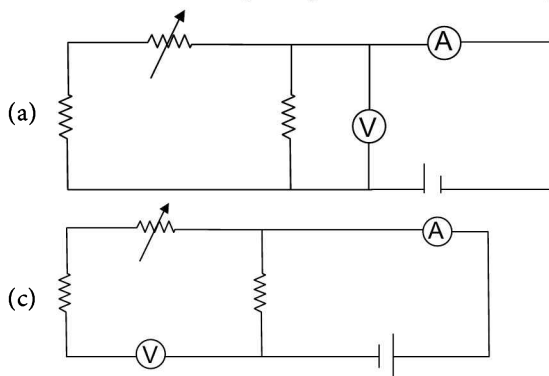
- (a) $x < f$ (b) $f < x < 2f$
(c) $x = 2f$ (d) $x > 2f$

49. The arrangement for metre bridge experiment is shown in figure. If $AC = x$, corresponds to null deflection of galvanometer, what would be its value, if the radius of wire AB is doubled?

- (a) x (b) $\frac{x}{4}$
(c) $4x$ (d) $2x$



50. Which of the following set ups can be used to verify Ohm's law?



Assertion-Reason Type Questions

Directions: Each question contains Statement 1 and Statement 2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement 1 is True, Statement 2 is True; Statement 2 is a correct explanation for Statement 1
(b) Statement 1 is True, Statement 2 is True; Statement 2 is NOT a correct explanation for Statement 1
(c) Statement 1 is True, Statement 2 is False
(d) Statement 1 is False, Statement 2 is True

51. Statement 1

All physical quantities having same dimension of basic SI units are same.

and

Statement 2

Basic unit of any physical quantity is truly represented by product of basic units of the system of units raised to appropriate power.

52. Statement 1

In the equation $KC = B - A$, where, K is a constant, K may have non-zero dimensions.

and

Statement 2

Dimensions of A and B are same.

53. Statement 1

In Ohm's law experiment, the potential drop across a resistance wire was measured as 10.0 V and the current was measured as 1.00 A. The maximum permissible error in resistance is $\pm 2\%$.
and

Statement 2

In this experiment, the least current of the ammeter is 0.01 A.

54. Statement 1

In the experiment to measure the specific heat of a liquid using a calorimeter, while performing the experiment, we keep the value of electric current constant by adjusting the rheostat.
and

Statement 2

Changing current will damage the heating coil.

55. Statement 1

In Post Office Box experiment, the battery key is pressed before the galvanometer key.
and

Statement 2

If the galvanometer key is pressed first and then we press the battery key, the galvanometer may get damaged.

**Linked Comprehension Type Questions**

Directions: This section contains a paragraph. Based upon the paragraph, multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage 1

If we have taken force (F), momentum (p) and energy (E) as fundamental units instead of M, L, T then also we can describe other physical quantities in terms of these units.

56. Dimensional formula of time in terms of the above units is

- (a) $F^2 p^2 E^1$ (b) $F^{-2} p^1 E^{-2}$ (c) $F^1 p^{-2} E^0$ (d) $F^{-1} p^1 E^0$

57. Dimensional formula of L in the above units is

- (a) $F^1 p^1 E^{-1}$ (b) $F^{-1} p^0 E^1$ (c) $F^1 p^0 E^1$ (d) $F^1 p^2 E^0$

58. Planck's constant (h) has the dimensional formula of

- (a) $F^{-1} p^{-1} E^{-1}$ (b) $F^{-1} p^1 E^1$ (c) $F^{-2} p^1 E^0$ (d) $F^1 p^1 E^0$

Passage 2

The density of a cube of mass 2500.5 gram was measured as follows. The cube was immersed completely in a jar filled with water. The overflowed quantity of water was poured in a measuring jar and it was measured to be 1000 cm^3 .

Density = $\frac{\text{Mass}}{\text{Volume}}$ is calculated.

59. The density of the material of the cube (in g cm^{-3}) is

- (a) 2.5005 (b) 2.500 (c) 3 (d) 2.501

60. The side of the cube (in cm) can be expressed as

- (a) 10 (b) 1×10^1 (c) 10.000 (d) 1.000×10^1

1.46 Units, Dimensions and Error Analysis in Experiments

61. The maximum percentage error in the measurement of the surface area of this cube by the above method is

- (a) $\frac{1}{60}\%$ (b) $\frac{1}{6}\%$ (c) $\frac{1}{600}\%$ (d) $\frac{1}{6000}\%$

Passage 3

In a resonance tube experiment to determine the speed of sound through air, using a tuning fork of frequency 400 Hz, the first resonance occurs at water level equal to 20.20 cm and the second resonance occurs at 62.50 cm. The speed of sound at 0°C is known to be 330 m s⁻¹.

62. The end correction is (in cm)

- (a) 0.75 (b) 0.85 (c) 0.95 (d) 1.05

63. The maximum possible error in the speed of sound is

- (a) 160 cm s⁻¹ (b) 16 cm s⁻¹ (c) 8 cm s⁻¹ (d) 64 cm s⁻¹

64. The laboratory temperature (at the time of experiment) is nearly

- (a) 14 °C (b) 22 °C (c) 28 °C (d) 35 °C



Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers of which ONE OR MORE answers will be correct.

65. Which of the following can be grouped together as having same dimensions?

- (a) Young's modulus (b) stress (c) pressure (d) thrust

66. The Lorentz force $\vec{F}$ acting on a particle of charge Q in an electric field $\vec{E}$ and magnetic field $\vec{B}$ is $\vec{F} = Q(\vec{E} + a\vec{B})$.

The motion of such a particle in X-Y plane is described by $\frac{d^2 v_x}{dt^2} = -\frac{B^2 Q^2}{y} v_x$ where, v_x = X-component of velocity

and t is time. The solution of this differential equation is $v_x = A \sin(\omega t + \phi)$ where, $\omega = \frac{BQ}{Z}$. Then,

- (a) the quantity 'A' has dimension of LT⁻¹.
(b) the dimensional formula for y is M²L⁰T⁰.
(c) the dimensional formula for Z is ML⁰T⁰.
(d) the SI unit of the quantity ϕ is radian.

67. The torque acting on a turbine wheel is $\tau = \rho Q (p_2 - p_1)$ where $Q = \int v dA$, v = velocity, dA = Area element and ρ = Density. Then,

- (a) The quantities p_1, p_2 represent angular momentum per unit mass.
(b) The SI unit of Q is m³s⁻¹.
(c) The quantity p_1, p_2 represent momentum per unit mass.
(d) The quantity τv represents power.

68. Identify the correct Statement/Statements from the following:

- (a) Zero error has to be subtracted from the observed reading to get actual reading.
(b) Zero error has to be added to the observed reading to get actual reading.
(c) Zero correction has to be subtracted from the observed reading to get actual reading
(d) Zero correction has to be added to the observed reading to get the actual reading.



Matrix-Match Type Questions

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

69. In the table, some physical quantities are given in Column I and possible SI units are given in Column II. Match the physical quantities in Column I with the units in Column II. The following abbreviations are used.

G – Universal gravitational constant

R_e – Radius of Earth

M_e – Mass of Earth

M_s – Mass of Sun

R – Universal gas constant

T – Absolute temperature

M – Molar mass

F – force

q – charge

B – Magnetic induction

Column I

(a) $\frac{GM_e}{R_e}$

(b) $\frac{3RT}{M}$

(c) $GM_e M_s$

(d) $\frac{F^2}{q^2 B^2}$

Column II

(p) (farad) (volt)² kg⁻¹

(q) kilogram (metre)³ (second)⁻²

(r) (volt) (coulomb) (metre)

(s) (metre)² (second)⁻²

70. Column I gives four physical quantities. Select the appropriate units from the choices given in Column II. Some of the physical quantities may have more than one choice correct.

Column I

(a) Magnetic Flux

(b) Magnetic Induction

(c) Inductance

(d) Magnetic permeability (μ)

Column II

(p) volt second (ampere)⁻¹

(q) henry (metre)⁻¹

(r) newton (ampere metre)⁻¹

(s) (joule) (ampere)⁻¹

(t) newton (ampere)⁻²

SOLUTIONS

ANSWER KEYS

Topic Grip

1. 43.2 V
2. 2
3. $\pm 22.5\%$
4. $\pm 1.36\%$
5. $f = \frac{1}{2\ell} \sqrt{\frac{T}{m}}$
6. (b)
7. (c)
8. (b)
9. (b)
10. (c)
11. (b)
12. (d)
13. (b)
14. (c)
15. (a)
16. (c)
17. (a), (c)
18. (a), (c)
19. (b), (d)
20. (a) $\rightarrow$ q, r
(b) $\rightarrow$ s, r

- (c) $\rightarrow$ q, r
- (d) $\rightarrow$ p, r

IIT Assignment Exercise

21. (b)
22. (a)
23. (a)
24. (a)
25. (c)
26. (a)
27. (d)
28. (c)
29. (c)
30. (c)
31. (c)
32. (b)
33. (c)
34. (b)
35. (c)
36. (c)
37. (a)
38. (b)
39. (c)
40. (c)
41. (c)
42. (c)
43. (b)
44. (c)
45. (c)
46. (a)
47. (a)
48. (b)
49. (a)
50. (a)
51. (d)
52. (b)
53. (b)

54. (c)
55. (a)
56. (d)
57. (b)
58. (b)
59. (b)
60. (d)
61. (a)
62. (c)
63. (b)
64. (a)
65. (a), (b), (c)
66. (a), (b), (c), (d)
67. (a), (b)
68. (a), (d)
69. (a) $\rightarrow$ (p), (s)
(b) $\rightarrow$ (p), (s)
(c) $\rightarrow$ (q), (r)
(d) $\rightarrow$ (p), (s)
70. (a) $\rightarrow$ (s)
(b) $\rightarrow$ (r)
(c) $\rightarrow$ (p)
(d) $\rightarrow$ (q), (t)

HINTS AND EXPLANATIONS

Topic Grip

1. Since the resistor values are equal

$$\frac{1}{R_e} = \frac{n}{R} \Rightarrow \frac{1}{3} = \frac{n}{12} \Rightarrow n = 4$$

taking differentials of the above equation

$$-R_e^{-2} \Delta R_e = -n R^{-2} \Delta R \quad (2)$$

$$\text{Divide } \frac{(2)}{(1)}$$

$$\Rightarrow -\Delta R_e R_e^{-2} R_e = -\frac{n R^{-2} \Delta R \cdot R}{n}$$

$$\frac{\Delta R_e}{R_e} = \frac{\Delta R}{R} \text{ the combination has the same percentage}$$

error as the individual resistors. Hence the resistors are $12 \Omega \pm 10\%$. When they are connected in series, error addition

$$R_e' = 48 \pm 4 \text{ (1.2)} = 48 \pm 4.8 \text{ ohm}$$

$$R_e' \text{ (min)} = 48 - 4.8 = 43.2 \text{ ohm. } I_{\max} = 1 \text{ A}$$

$$\therefore V_{\max} = 43.2 \text{ V}$$

2. $T = 2\pi \sqrt{\ell/g}$ is applicable only for small oscillation.

Use dimensional analysis: T is a function: $\ell^x m^y g^z$
 $\theta. \ell^x \rightarrow L^x; m^y \rightarrow M^y; g^z = (LT^{-2})^z = L^z T^{-2z}; \theta$ has no dimension: For dimensional equality

$$T = L^x M^y L^z T^{-2z} = L^{(x+z)} M^y T^{-2z}$$

$$\therefore y = 0; (x+z) = 0 \Rightarrow x = -z$$

$$-2z = 1 \Rightarrow z = -\frac{1}{2} \Rightarrow \therefore x = \frac{1}{2}$$

$$\therefore T = k \ell^{1/2} m^0 g^{-1/2} = k \sqrt{\frac{\ell}{g}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{\ell_1}{\ell_2}} = \sqrt{4} = 2$$

3. Internal resistance:

$$r = \frac{R(\ell_1 - \ell_2)}{\ell_1} = \frac{R \cdot \Delta \ell}{\ell_1} \text{ [R} \rightarrow \text{shunting resistance]}$$

$$\frac{dr}{r} = \frac{dR}{R} + \frac{d(\Delta \ell)}{\Delta \ell} + \frac{d\ell_1}{\ell_1}$$

$$\frac{dR}{R} = \pm 2\%; \Delta \ell = \ell_1 - \ell_2 = 20 - 19 = 1 \text{ cm}$$

errors add up: Hence $d(\Delta \ell) = (1 + 1) = 2 \text{ mm}$

$$\therefore \frac{d(\Delta \ell)}{\Delta \ell} = \frac{2}{10} \times 100 = 20\%$$

$$\frac{d\ell_1}{\ell_1} = \frac{1}{200} \times 100 = 0.5\%$$

$$\therefore \frac{dr}{r} \text{ (in percentage)} = 2 + 20 + 0.5 = 22.5\%$$

4. Let e be the end correction for first resonance

$$f = \frac{v}{4(\ell_1 + e)} \Rightarrow (\ell_1 + e) = \frac{v}{4f} \quad (1)$$

$$\text{for second resonance: } f = \frac{3v}{4(\ell_2 + e)}$$

$$\Rightarrow (\ell_2 + e) = \frac{3v}{4f} \quad (2)$$

$$(2) - (1) \Rightarrow (\ell_2 + e) - (\ell_1 + e) = \frac{3v}{4f} - \frac{v}{4f} = \frac{v}{2f}$$

$$(\ell_2 - \ell_1) = \frac{v}{2f} \Rightarrow v = 2f \Delta \ell \text{ (where } \Delta \ell = \ell_2 - \ell_1)$$

$$\therefore \frac{dv}{v} = \frac{df}{f} + \frac{d(\Delta \ell)}{\Delta \ell}$$

since $\Delta \ell = (\ell_2 - \ell_1)$ errors add up. From the data presented it can be concluded that accuracy of measurement is $\pm 0.1 \text{ mm}$. Higher accuracy will need the data to be presented as 81.10 cm etc. Hence $d(\Delta \ell) = 0.1 + 0.1 = 0.2 \text{ cm}$

$$\therefore \frac{d(\Delta \ell)}{\Delta \ell} = \frac{0.2}{(81.1 - 26.1)} = \frac{0.2}{55} \times 100\% = 0.36\%$$

$$\frac{df}{f} = \frac{3}{300} \times 100\% = 1\%$$

$$\therefore \frac{dv}{v} = \pm 1.36\%$$

5. The frequency f will depend on ℓ , T and m and f has dimension of T^{-1} .

$$\therefore [f] = [\ell^x T^y m^z] \Rightarrow T^{-1} = L^x (MLT^{-2})^y (ML^{-1})^z$$

$$M^0 L^0 T^{-1} = L^{x+y-z} M^{y+z} T^{-2y}$$

1.50 Units, Dimensions and Error Analysis in Experiments

$$\therefore \text{Equating the powers} \Rightarrow -2y = -1 \Rightarrow y = +\frac{1}{2}$$

$$y + z = 0 \Rightarrow z = -y = -\frac{1}{2}$$

$$x + y - z = 0 \Rightarrow x = -y + z = -\frac{1}{2} - \frac{1}{2} = -1$$

$$\therefore f = k \ell^{-1} T^{\frac{1}{2}} m^{-\frac{1}{2}} = k \frac{1}{\ell} \sqrt{\frac{T}{m}}$$

$$\text{By experiment } k = \frac{1}{2}$$

$$\therefore f = \frac{1}{2\ell} \sqrt{\frac{T}{m}}$$

6. Since the formula gives the force per unit length

$$[F] = \left[\frac{k I_1 I_2}{r} \ell \right], \text{ where } \ell \text{ is total length}$$

$$\therefore [k] \rightarrow [F] [I_1 I_2]^{-1} = MLT^{-2} I^{-2}$$

$$\begin{aligned} 7. E &= i^2 R t \Rightarrow \frac{dE}{E} = 2 \frac{di}{i} + \frac{dR}{R} + \frac{dt}{t} \\ &= 2 \times 10 + 5 + 1 = 26\% \end{aligned}$$

$$8. \text{Stress} = \frac{N}{m^2} = \frac{Nm}{m^3} = \frac{J}{m^3},$$

$$\text{Pressure} = \frac{N}{m^2} = \frac{J}{m^3}$$

$$\text{Energy density} = \frac{J}{m^3}$$

$$\text{Power / volume} = \frac{J}{s \cdot m^3}$$

9. Angular momentum = Torque $\times$ time

$$10. \frac{Z}{\delta} = \sqrt{\frac{T}{\delta}} \text{ has units } \sqrt{\frac{N}{kg/m}} = \frac{m}{s}$$

$$Z \text{ has units } \frac{kg \cdot m}{m \cdot s}$$

11. For a body with initial velocity u at $t = 0$: velocity at the end of n th second ($t = n$) = $u + an$.

Velocity at $t = (n - k) = u + a(n - k)$. Average velocity during the interval of k seconds prior to $t = n$

$$= \frac{u + an + u + an - ak}{2}$$

$$= u + \frac{1}{2}a(2n - k)$$

$$\therefore \text{distance moved in } k \text{ second}$$

$$= \left[u + \frac{1}{2}a(2n - k) \right] \cdot k$$

You can check that this is dimensionally correct.
Statement 2 does not fully support statement 1

12. Precision can be 0.005 cm. Each main division in 0.1 cm (1 mm). 19 main divisions are taken and divided into 20 vernier division. Each vernier division = $\frac{19}{20} \times 0.1$ cm.

Each main division = 0.1 cm

$$\text{Least count} = 0.1 - \frac{1.9}{20} = \frac{2 - 1.9}{20}$$

$$= \frac{0.1}{20} = 0.005 \text{ cm}$$

13. Both statements are correct but statement 2 does not substantiate fully statement 1.

$$E = \frac{Q^2}{2C} \Rightarrow C = \frac{Q^2}{2E} \Rightarrow \left[\frac{Q^2}{2E} \right] = [Q^2 J^{-1}]$$

14. The final total energy of the satellite is

$$\frac{-GMm}{2r}. \text{ Hence } \left[\frac{GMm}{2r} \right] \text{ has dimension } ML^2 T^{-2}$$

Removing m from the expression

$$\frac{GM}{r} \text{ has the dimension } L^2 T^{-2}$$

$$15. v^2 = \frac{GM}{r} \quad (1)$$

$\Rightarrow$ Taking differentials

$$2v dv = -\frac{GM dr}{r^2} \quad (2)$$

$$(2) \div (1) \Rightarrow \frac{2v dv}{v^2} = -\frac{GM dr}{r^2} \cdot \frac{r}{GM}$$

$$\Rightarrow 2 \frac{dv}{v} = -\frac{dr}{r} = 2\%$$

$$\text{neglecting sign: } \frac{dr}{r} = 2 \frac{dv}{v} \Rightarrow \frac{dv}{v} = 1\%$$

$$16. T^2 = \frac{4\pi^2}{GM} r^3 = kr^3$$

$T^{2/3} = kr$ - (1) Taking differentials of the equation

$$\Rightarrow \frac{2}{3} \cdot T^{-\frac{1}{3}} dT = k dr \quad (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{2}{3} \frac{dT}{T^{-\frac{1}{3}} T^{\frac{2}{3}}} = \frac{dr}{r} \Rightarrow \frac{2}{3} \frac{dT}{T} = \frac{dr}{r} = 2\%$$

$$\frac{dT}{T} = 3\%$$

17. $\log(x)$ has no dimension, hence neglect the term

$$[x] = \left[\frac{rCm}{B^2 \ell^2 C + m} \right] = \left[\frac{rC}{\frac{B^2 \ell^2 C}{m} + 1} \right]$$

rC has dimension of T (time constant)

($\because$ for dimensional homogeneity of denominator:

$\frac{B^2 \ell^2 C}{m}$ has no dimension; since 1 is without dimension)

$\therefore [B^2 \ell^2 C] = [M]$. Dimensional homogeneity is possible

Note: $B^2 \ell^2 C \rightarrow [MT^{-2}I^{-1}]^2 L^2 [M^{-1} L^{-2} T^4 I^2] \rightarrow M$

18. $\left[\frac{\mu_0 i}{b} \right] = \frac{MLT^{-2}I^{-2}}{L} I$

$$= MI^{-1}T^{-2} = [B] = [\text{magnetic field}]$$

$$\left[\frac{\mu_0 i \ell}{b} \right] = [B] \cdot [\ell] \neq [\text{magnetic flux}]$$

$\therefore$ (ii) is wrong

$$[\mu_0 \ell i^2] = [Bi\ell] [\ell] = [\text{energy}]$$

$$\frac{\mu_0 i^2}{2b^2} = \frac{[B] \cdot [i]}{[b]} \neq [\text{Force}]$$

19. $\frac{x\lambda}{2\pi}$ must have the dimension of v^2 i.e., $\left(\frac{m^2}{s^2} \right)$

$$x = \frac{m}{s^2} \Rightarrow (a) \text{ is wrong}$$

$$\frac{T}{\rho y} = \frac{m^2}{s^2}$$

$$y = \frac{N/m}{\frac{kg}{m^3} \left(\frac{m^2}{s^2} \right)} = m$$

$$xy = \frac{m}{s^2} \cdot m = \frac{m^2}{s^2} \Rightarrow [xy] = [L^2 T^{-2}]$$

20. (a) RC is Time constant; dimension only (T) $\rightarrow q, r$

- (b) $\frac{1}{2} CV^2$ is energy in a capacitor $\rightarrow (s) (r)$

- (c) L/R is time constant $\rightarrow (q) (r)$

- (d) $Bi\ell$ force on a rod carrying current i and moving perpendicular to a magnetic field $\rightarrow (p) (r)$

All the terms used, except ℓ has dimension (I^*) where x is a non zero value.

IIT Assignment Exercise

21. $Q = n_1 u_1 = n_2 u_2 \Rightarrow n_1 = 1, n_2 = x$

$$x = \frac{1 \times N s m^{-2}}{(\text{CGS units})} = \frac{MLT^{-2} TL^{-2}}{\text{CGS units}}$$

$$= \frac{M}{m} \left(\frac{L}{\ell} \right)^{-1}$$

(Time units in both are same)

$$= \left(\frac{kg}{g} \right) \left(\frac{m}{cm} \right)^{-1} = 10^3 \cdot (10^2)^{-1} = 10$$

22. $n_1 \left[\text{cal} \cdot g^{-1} (^\circ C)^{-1} \right] = 4200 \left[J \cdot kg^{-1} K^{-1} \right]$

$$n_1 = 4200 \left[\frac{1/4.2 \text{ cal}}{1 \text{ cal}} \right] \left[\frac{10^3 g}{g} \right]^{-1} \left[\frac{^\circ C}{^\circ C} \right]^{-1}$$

$$= \frac{4200}{4.2} \times \frac{1}{10^3} = 1 \text{ cal } g^{-1} ^\circ C^{-1}$$

23. $1u = 1 \text{ kilowatt hour} = 1000 \text{ watt hour}$
 $= 1000 \times 3600 \text{ watt second} (\because 1 \text{ hour} = 3600 \text{ second})$
 $= 3.6 \times 10^6 \frac{J}{s} \times s (\because W = J s^{-1})$
 $= 3.6 \times 10^6 J$

Work done, $W = FS$

$$= 5 N \times 1000 m$$

$$= 5000 N m$$

$$= 5000 J$$

$$= \frac{5000}{3.6 \times 10^6} = \frac{5}{3600} u$$

$$= \frac{1}{720} u$$

24. b must have the same dimension as x^3 that is L^3 .

$$a = \frac{N m m^3}{m^{\frac{1}{2}}} = \frac{kg m}{s^2} \times \frac{m^3}{m^{\frac{1}{2}}}$$

$$= kg m^{\frac{9}{2}} s^{-2}$$

$$\frac{a}{b} = \frac{kg m^{\frac{9}{2}} s^{-2}}{m^3} = kg m^{\frac{3}{2}} s^{-2} \text{ a and b have different dimensions.}$$

$\therefore a + b$ is not possible.

$$x + b^{\frac{1}{3}} = L + L, \text{ possible}$$

1.52 Units, Dimensions and Error Analysis in Experiments

$$25. \frac{F}{\rho \omega^2 D^4} = \frac{MLT^{-2}}{ML^{-3}(T^{-1})^2 L^4} = M^0 L^0 T^0$$

$$\frac{\rho \omega D^2}{\mu} = \frac{ML^{-3} \cdot T^{-1} L^2}{ML^{-1} T^{-1}} = M^0 L^0 T^0$$

$$\omega = T^{-1}, v_0 = LT^{-1} = c, \mu = ML^{-1} T^{-1}$$

$$26. \frac{B^2}{2\mu_0} = \text{energy density}$$

$$\frac{J}{m^3} = \frac{ML^2 T^{-2}}{L^3} = ML^{-1} T^{-2}$$

$$\frac{\mu_0}{B^2} = M^{-1} L T^2$$

$$27. A = \text{farad} = \frac{\text{coulomb}}{\text{watt}} = \frac{(\text{colomb})}{(\text{joule}/\text{colomb})}$$

$$= \frac{(\text{colomb})^2}{\text{joule}} = \frac{Q^2}{ML^2 T^{-2}}$$

$$B = \frac{\text{newton}}{\text{ampere metre}} = \frac{N}{IL} = \frac{MLT^{-2}}{IL} = M I^{-1} T^{-2}$$

$$= M (IT)^{-1} T^{-1} = MQ^{-1} T^{-1}$$

$$\therefore B^2 = M^2 Q^{-2} T^{-2}$$

$$\therefore \frac{A}{B^2} = \frac{Q^2}{ML^2 T^{-2}} \div M^2 Q^{-2} T^{-2}$$

$$= M^{-3} L^{-2} T^4 Q^4$$

C has same dimensions as $\frac{A}{B^2}$ by principle of homogeneity of dimensions

$$28. \bar{A} = \frac{\text{volt}}{\left(\frac{N}{Am}\right)(m)}; \text{volt} = J C^{-1} = \frac{Nm}{As}$$

$$\bar{A} = m s^{-1}$$

$$29. \frac{P}{r} \text{ and } \frac{GMm^2}{P} \text{ have the same dimensions.}$$

$$\therefore P \text{ has the dimension of } \sqrt{GMm^2 r}$$

$$\frac{GMm^2}{r^2} r^3 = N \cdot \text{kg} \cdot m^3$$

$$= \text{kg} \frac{m}{s^2} \cdot \text{kg} \cdot m^3 = \left(\text{kg} \frac{m^2}{s}\right)^2$$

$$P = \text{kg} \frac{m}{s} = \text{Angular momentum.}$$

30. Each term on LHS should have same dimensions.

$$\frac{\partial^2 T}{\partial x^2} \rightarrow \frac{K}{m^2}$$

$$\therefore A \rightarrow \frac{Q}{\left(\frac{K}{m^2}\right)} = \frac{\text{watt}}{m^3 \times \frac{K}{m^2}} = (ML^2 T^{-3}) L^{-1} \cdot \Theta^{-1}$$

$$= MLT^{-3} \Theta^{-1}$$

31. Since each term of the equation should have same dimension:

$$\left[\frac{Q}{A}\right] = \left[\frac{BC}{A} \frac{\partial T}{\partial t}\right]$$

$$\Rightarrow [B] = \frac{Q}{C \left(\frac{\partial T}{\partial t}\right)}$$

$$\Rightarrow B \rightarrow \frac{J s^{-1}}{m^3 \times \frac{J}{\text{kgK}} \times \frac{K}{s}} = \frac{\text{kg}}{m^3}$$

32. $\rho = \Omega \text{ m}$; $Bv\ell = \text{volt}$

$$Bv = \text{volt m}^{-1}$$

$$d = \sqrt{\frac{16I\rho}{\pi Bv}} = \sqrt{\frac{16 A \Omega m}{(\text{volt})}} \times m$$

$$A\Omega = \text{volt}$$

$$d = \sqrt{m^2} = m$$

33. $B = N m^2$, $J = (N m) s$

$$\frac{B}{J} \cos \psi \frac{d\psi}{dt} = \frac{Nm^2}{Nmss} = \frac{m}{s^2};$$

$$\frac{d^2 x}{dt^2} = \frac{m}{s^2}$$

$$a = \frac{m/s^2}{m/s^2}$$

$$34. \sqrt{P} = \frac{m^3/s}{m\left(\frac{1}{m^2}\right)(m)} = \frac{m^{\frac{1}{2}}}{s}$$

$$P = \frac{m}{s^2}$$

$$35. v_1 - v_2 = \frac{\text{energy/kg}}{\text{pressure}} = \frac{ML^2 T^{-2}}{M \cdot \frac{MLT^{-2}}{L^2}} = \frac{L^3}{M}$$

$$36. \frac{1}{\sqrt{LC}} = \text{frequency} = s^{-1}$$

$$LC = s^2 \Rightarrow M^0 L^0 T^2 I^0$$

$$37. \frac{\mu_0 i_1 i_2}{2\pi R} \text{ is force per unit length, when two wires carrying current are kept parallel.}$$

$$\therefore \mu_0 i_1 i_2 \text{ is force N} = \text{kg} \frac{\text{m}}{\text{s}^2}$$

$$\Rightarrow \therefore \frac{\mu_0 i_1 i_2}{MR} = \frac{1}{s^2}$$

$$38. C = \frac{N}{m^2 \times \frac{\text{kg}}{m^3}} = \frac{m^2}{s^2}$$

$$v^2 = \frac{m^2}{s^2} \Rightarrow mK \text{ is dimensionless}$$

$$KZ g = \frac{m^2}{s^2} \Rightarrow K(m) \frac{m}{s^2} = \frac{m^2}{s^2}$$

$$\therefore K \text{ is dimensionless}$$

$$\Rightarrow m \text{ is dimensionless}$$

$$39. \text{Pressure, Young's modulus } \frac{N}{m^2}$$

$$40. \frac{\text{kg}}{m^3} \times m \times \frac{m^2}{s^2} = \frac{\text{kg}}{s^2}$$

$$41. LC = \frac{\text{One main scale division}}{(\text{No. of vernier scale division that coincide with } n \text{ divisions on mainscale})}$$

$$= \frac{a}{(n+1)}$$

Aliter:

$$(n+1) \text{ VSD} = n \text{ MSD}$$

$$1 \text{ VSD} = \frac{n}{(n+1)} \text{ MSD}$$

$$LC = 1 \text{ MSD} - 1 \text{ VSD}$$

$$= 1 \text{ MSD} - \frac{n}{(n+1)} \text{ MSD}$$

$$= \frac{1 \text{ MSD}}{(n+1)} = \frac{a}{(n+1)}$$

$$42. \text{Density } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\pi r^2 \ell}$$

$$\Rightarrow \frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{2\Delta r}{r} + \frac{\Delta \ell}{\ell}$$

($\therefore$ error additive in nature)

$$\therefore \frac{\Delta \rho}{\rho} = \frac{0.004}{0.4} + \frac{2 \times 0.005}{0.5} + \frac{0.05}{5}$$

$$= \frac{1}{100} + \frac{2}{100} + \frac{1}{100} = \frac{4}{100}$$

$$\therefore \text{Percentage error in density}$$

$$= \frac{4}{100} \times 100 = 4\%$$

$$43. \text{We have } T = 2\pi \sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2 \ell}{T^2} = \frac{4\pi^2 \ell n^2}{t^2}$$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2 \frac{\Delta t}{t}$$

For student A

$$E_A = 100 \times \frac{\Delta g}{g} = 100 \left[\frac{0.1}{64.0} + \frac{2 \times 0.1}{128.0} \right] = 0.3\%$$

For student B

$$E_B = 100 \times \frac{\Delta g}{g} = 100 \left[\frac{0.1}{64.0} + \frac{2 \times 0.1}{64.0} \right] = 0.5\%$$

For student C

$$E_C = 100 \times \frac{\Delta g}{g} = 100 \left[\frac{0.1}{20} + \frac{2 \times 0.1}{36.0} \right] = 1\%$$

$$\therefore E_A \text{ is minimum}$$

44. Accuracy in measurement relates to how close the measured value is to the true value. The average value of the measured values is equated to the true value. When more number of measurements are taken, the average value becomes more accurate. Since C has taken most readings, with the same instrument, the measurements made by C will be more accurate.

45. Precision in measurement is related to what resolution or limit with which the quantity is measured, which in turn, depends upon the least count of the measuring instrument (i.e., number of significant digits). Smaller the least count, more precise will be the measurement. Hence the readings taken by C will be more precise

46. Least count (LC)

$$= \frac{\text{pitch}}{\text{No of divisions on circular scale}}$$

$$= \frac{0.5 \text{ mm}}{100} = 0.005 \text{ mm}$$

1.54 Units, Dimensions and Error Analysis in Experiments

$$\begin{aligned}\text{zero error (positive)} &= + 6 \text{ division} \times \text{LC} \\ &= 6 \times 0.005 \text{ mm} \\ &= + 0.030 \text{ mm}\end{aligned}$$

47. The prongs of the turning fork are to be kept in vertical plane to reduce the damping effect due to air friction.

48. If $x < f$, image will be virtual (erect and not inverted). Since image is inverted, x is not less than f . If $x = 2f$, object and image are at same location. Hence $x \neq 2f$. If $f < 2f$, the image is formed beyond $2f$. So on moving the eye to the left, the image will appear to the right (check by drawing the ray diagram).

If $x > 2f$, image is formed between $2f$ and f . So on moving the eye to the left, the image will also appear on the left side of object. (Check by drawing the ray diagram)

Hence $f < x < 2f$ is the correct answer.

49. The balancing length of the metre bridge is independent of the diameter of the wire, provided the material of wire is the same in both cases and $\frac{R_1}{R_2}$ remains unchanged.

50. In (c) and (d), voltmeter is connected in series which is not correct. In (b), the voltmeter measures the p.d across net resistance in circuit but ammeter measures current along only one path. In (a), the voltmeter measures p.d across net resistance and the ammeter measures the current through net resistance. Hence net resistance of circuit, $R_{eq} = \frac{V}{A}$ can be measured.

51. Many physical quantities have same dimensions of basic units.
Eg. Young's modulus, pressure and stress all have same dimensions.

Statement 2 is correct.

52. Statement 2 is correct, but does not explain statement

1. Dimension of $K = \frac{B}{C}$ or $\frac{A}{C}$. If C has same dimensions as A and B then K is dimensionless. Otherwise K is a constant with dimensions.

53. Both statements are true but statement 2 does not explain statement 1 completely

$$\Delta V = 0.1 \text{ V}, \Delta I = 0.01 \text{ A}, V = 10.0 \text{ V}, I = 1.00 \text{ A}$$

$$R = \frac{V}{I} \Rightarrow \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{0.1}{10} + \frac{0.01}{1} = 0.01 + 0.01 = 0.02$$

54. The electric energy given to the liquid is VIt , where $V = p.d$, $I =$ steady current and

$t =$ duration of time for which current is passed through the liquid. If I is varying, it will become difficult to calculate the electric energy given to the heating coil.

55. The back emf in the galvanometer coil can be very large (due to self induction of the galvanometer coil) if current tries to develop suddenly.

56. $T = F^x(p)^y E^z = (MLT^{-2})^x (MLT^{-1})^y (ML^2T^{-2})^z$

$$T = M^x L^x T^{-2x} M^y L^y T^{-y} M^z L^{2z} T^{-2z}$$

$$T = M^{x+y+z} L^{x+y+2z} T^{-2x-y-2z}$$

$$\therefore x + y + z = 0 \quad \text{--- (1) ;}$$

$$x + y + 2z = 0 \quad \text{--- (2)}$$

$$-2x - y - 2z = 1 \quad \text{--- (3)}$$

Solve : (2) - (1) $\Rightarrow z = 0$. Substitute in (1)

$$x = -y. \text{ Substitute in (3)}$$

$$\Rightarrow -2(-y) - y - 2(0) = 1$$

$$y = 1 ; x = -1 ; z = 0$$

$$T = F^{-1}(p)^1 E^0$$

Aliter:

$$F = M L T^{-2} \quad \text{---(i)}$$

$$p = M L T^{-1} \quad \text{---(ii)}$$

$$E = M L^2 T^{-2} \quad \text{---(iii)}$$

$$(ii) \div (i) \Rightarrow \frac{p}{F} = \frac{MLT^{-1}}{MLT^{-2}} = T$$

$$T = pF^{-1} \text{ or } T = F^{-1} p^1 E^0$$

57. $L = (MLT^{-2})^x (MLT^{-1})^y (MLT^{-2})^z$

Solving as above:

$$L = M^{x+y+z} L^{x+y+2z} T^{-2x-y-2z}$$

$$x + y + z = 0 \quad \text{--- (1);}$$

$$x + y + 2z = 1 \quad \text{--- (2)}$$

$$-2x - y - 2z = 0 \quad \text{--- (3)}$$

Solving as above (2) - (1) we get

$$z = 1; (3) + (1) \text{ we get } -x = z \Rightarrow x = -1$$

$$\Rightarrow \text{From (1) } y = (-x - z) = 0$$

$$L = F^{-1} p^0 E^1$$

Aliter:

$$(iii) \div (i)$$

$$\Rightarrow \frac{E}{F} = \frac{ML^2 T^{-2}}{MLT^{-2}} = L$$

$$L = F^{-1} E = F^{-1} p^0 E^1$$

$$58. E = h\nu \Rightarrow [h] = \left[\frac{E}{\nu} \right] = [ET]$$

$$= E^1 F^{-1} p^1$$

Aliter:

$$h = \text{joule second}$$

$$= \text{Energy} \times \text{time}$$

$$= E t$$

$$= E F^{-1} p^1 E^0 [t = F^{-1} p^1 E^0 \text{ from (44)}]$$

$$= F^{-1} p^1 E^1$$

$$59. \rho = \frac{m}{V} = \frac{2500.5}{1000} = 2.5005$$

Since V has 4 significant digits (because it comes from direct measurement) and m has 5 significant digits, ρ must be expressed in 4 significant digits (smaller of 4 and 5)

$\therefore$ The last digit 5 has to be dropped in ρ . Since 5 is not followed by any number and the digit preceding 5 is zero (even), ρ has to be expressed as 2.500 g cm^{-3} .

$$60. \text{Side } a = \sqrt[3]{V} = \sqrt[3]{1000} = 10 \text{ cm}$$

However, it has to be expressed in 4 significant digits. 10 has got only 1 significant digit (not coming from direct measurement, zero is insignificant)

1×10^1 has only one significant digit.

10.000 has five significant digits,

1.000×10^1 has 4 significant digits.

$$61. S = \text{surface area} = 6a^2 = 6 \times 10 \times 10 = 600 \text{ cm}^2$$

This has to be expressed in 4 significant digits.

$$\therefore S = 600.0 \text{ cm}^2$$

$$\Delta S = 0.1 \text{ cm}^2$$

$$\therefore \frac{\Delta S}{S} = \frac{0.1}{600} = \frac{1}{6000}$$

$$\frac{\Delta S}{S} \% = \frac{1}{6000} \times 100 = \frac{1}{60} \%$$

$$62. \ell_1 + e = \frac{\lambda}{4} \Rightarrow 4\ell_1 + 4e = \lambda - (1)$$

$$\ell_2 + e = \frac{3\lambda}{4}$$

$$\Rightarrow \ell_2 + e = 3(\ell_1 + e)$$

$$\ell_2 - 3\ell_1 = 2e$$

$$\Rightarrow e = \frac{(\ell_2 - 3\ell_1)}{2}$$

$$\therefore e = \frac{(62.50 - 3 \times 20.20)}{2}$$

$$= \frac{1.90}{2} = 0.95 \text{ cm}$$

$$63. v = 2f(\ell_2 - \ell_1)$$

$$= 2 \times 400 [62.50 - 20.20] = 800 \times 42.30$$

$$= 33840 \text{ cm s}^{-1} = 338.4 \text{ m s}^{-1}$$

$$\Delta v = 2f(\Delta \ell_2 + \Delta \ell_1)$$

$$= 2 \times 400 [0.01 + 0.01]$$

$$= 800 \times 0.02 = 16 \text{ cm s}^{-1}$$

$$64. v_0 = \frac{v_\theta}{\sqrt{1 + \frac{\theta}{273}}} \Rightarrow \sqrt{1 + \frac{\theta}{273}} = \frac{v_\theta}{v_0} = \frac{338.4}{330} = 1.025$$

$$\therefore 1 + \frac{\theta}{273} = (1.025)^2 = 1.05$$

$$\Rightarrow \frac{\theta}{273} = 1.05 - 1 = 0.05$$

$$\therefore \theta = 273 \times 0.05 = 13.65^\circ \text{C} \approx 14^\circ \text{C}$$

65. Young's modulus, stress, pressure have SI unit N/m^2
Thrust is force N.

66. $Q(\bar{v} \times \bar{B})$ represents magnetic force

$$\therefore a \text{ represents velocity} \Rightarrow [BQ] = \left[\frac{F}{v} \right]$$

$$\therefore \left[\frac{B^2 Q^2 v_x}{y} \right] = \left(\frac{\text{N}}{\text{m/s}} \right)^2 \times \frac{(\text{m/s})}{y}$$

$$= \frac{d^2 v_x}{dt^2} (\text{data given}) = \frac{\text{acceleration}}{t}$$

$$y = \left(\frac{\text{N}}{\text{acceleration}} \right)^2 = \text{mass}^2$$

$$[QB] = \left[\frac{F}{v} \right] \quad [\omega] = \left[\frac{BQ}{Z} \right] \quad (\text{Data given})$$

$$[Z] = \left[\frac{F}{v} \right] \frac{1}{[\omega]} = \frac{\text{MLT}^{-2} \cdot \text{T}}{\text{LT}^{-1}} = \text{M}; \phi \text{ is measured in radian}$$

Aliter:

$$v_x = A \sin(\omega t + \phi); (\sin \omega t + \phi) \text{ is dimensionless}$$

$\therefore A$ has same dimension as v_x (ie velocity)

$\therefore [A] = \text{M}^0 \text{L T}^{-1} \rightarrow (a) \text{ is correct}$

$$\omega t = \frac{\text{rad}}{\text{s}} \times \text{s} = \text{radian}$$

1.56 Units, Dimensions and Error Analysis in Experiments

ωt and ϕ have same dimensions

ϕ is measured in radian $\Rightarrow$ (d) is correct

$$\omega = \frac{BQ}{Z} \Rightarrow Z = \frac{BQ}{\omega}$$

$$[Z] = \frac{[B][Q]}{[\omega]} = \frac{\left[\frac{N}{Am}\right][As]}{\left[\frac{1}{s}\right]} = \left[\frac{Ns^2}{m}\right]$$

$$= \left[\frac{kg\,ms^{-2}s^2}{m}\right]$$

$\Rightarrow Z = M L^0 T^0 \Rightarrow$ (c) is correct

$$67. P_2 - P_1 = \frac{\tau}{\rho Q} = \frac{Nm}{\frac{kg}{m^3} \left(\frac{m^3}{s}\right)} = \frac{m^2}{s}$$

Angular momentum per unit mass

$$= (mvr) \, kg^{-1} = kg \, ms^{-1} \, m \cdot kg^{-1}$$

$$= m^2 s^{-1}$$

$\Rightarrow$ Hence they have same units.

$$Q = \frac{m}{s} (m^2) = \frac{m^3}{s}$$

$$\tau v = Nm \times \frac{m}{s} = \frac{J}{s} (m) = \text{watt (m)}$$

68. Conceptual. Zero error is subtracted while zero correction is added to the observed reading to get the actual reading.

Zero correction = - (zero error)

$$69. (a) \frac{GM_\epsilon}{R_\epsilon} = \text{gravitational potential} = \text{joule } kg^{-1}$$

$$= \text{farad (volt)}^2 kg^{-1}$$

$$= m^2 s^{-2}$$

(a) $\rightarrow p, s$

$$(b) \sqrt{\frac{3RT}{M}} = \text{rms velocity} = (\text{metre}) (\text{second})^{-1}$$

$$\therefore \frac{3RT}{M} = (\text{metre})^2 (\text{second})^{-2}$$

$$(\text{farad}) (\text{volt})^2 kg^{-1}$$

$$= \left(\frac{\text{coulomb}}{\text{volt}}\right) \times (\text{volt})^2 kg^{-1}$$

$$= (\text{coulomb} \times \text{volt}) kg^{-1}$$

$$= (\text{joule}) kg^{-1}$$

$$= (kg \, m^2 \, s^{-2}) kg^{-1}$$

$$= m^2 s^{-2} = \frac{3RT}{M}$$

$\therefore$ (b) $\rightarrow p, s$

$$(c) \quad GM_\epsilon M_s = \frac{GM_s}{R_s^2} \cdot M_\epsilon R_s^2 = (g_s \cdot M_\epsilon) R_s^2$$

$$= (F) R_s^2 = N \, m^2$$

$$= J \, m = kg \, m^3 \, s^{-2}$$

$$= (\text{volt}) (\text{coulomb}) (\text{metre})$$

$$(\because \text{joule} = \text{volt} \times \text{coulomb})$$

$\therefore$ (c) $\rightarrow (q, r)$

$$(d) F = qvB \Rightarrow \frac{F}{qB} = v$$

$$\therefore \frac{F^2}{q^2 B^2} = v^2 = (\text{metre})^2 (\text{second})^{-2}$$

$$= (\text{farad}) (\text{volt})^2 kg^{-1}$$

(d) $\rightarrow (p, s)$

$$70. (a) \rightarrow \text{Magnetic flux} = Wb = J \, A^{-1}$$

$$= (\text{joule}) (\text{ampere})^{-1}$$

$$(b) \rightarrow B = \frac{N}{Am} = \text{newton (ampere metre)}^{-1}$$

$$(c) \rightarrow L = \frac{Wb}{A} = \frac{Vs}{A} = \text{volt second (ampere)}^{-1}$$

$$(d) \rightarrow \mu = \text{henry metre}^{-1} = \text{newton (ampere)}^{-2}$$

CHAPTER

2

MOTION IN ONE DIMENSION

■■ CHAPTER OUTLINE

Preview

STUDY MATERIAL

State of Rest and Motion

Classification of Motion into One, Two and Three Dimensions

- Concept Strands (1-2)

Motion in One Dimension

- Concept Strands (3-6)

Kinematic Equations of Motion (For Uniformly Accelerated or Uniformly Decelerated Motions Only)

- Concept Strands (7-10)

Motion Under Gravity

- Concept Strands (11-14)

Relative Motion of Particles in One Dimensional Motion

- Concept Strand (15)

Graphical Representations of One Dimensional Motion

CONCEPT CONNECTORS

- 20 Connectors

TOPIC GRIP

- Subjective Questions (10)
- Straight Objective Type Questions (5)
- Assertion–Reason Type Questions (5)
- Linked Comprehension Type Questions (6)
- Multiple Correct Objective Type Questions (3)
- Matrix-Match Type Question (1)

IIT ASSIGNMENT EXERCISE

- Straight Objective Type Questions (80)
- Assertion–Reason Type Questions (3)
- Linked Comprehension Type Questions (3)
- Multiple Correct Objective Type Questions (3)
- Matrix-Match Type Questions (1)

ADDITIONAL PRACTICE EXERCISE

- Subjective Questions (40)
- Assertion–Reason Type Questions (10)
- Linked Comprehension Type Questions (9)
- Multiple Correct Objective Type Questions (8)
- Matrix-Match Type Questions (13)

STATE OF REST AND MOTION

All objects in this universe, small or large, are in constant motion. The stars, the Sun, the Earth and the planets are all constantly moving. Even those objects on the Earth that appear to be at rest are moving with the Earth in its orbit around the Sun. The description of motion of objects without considering the cause of the motion is known as **kinematics**. In kinematics, we do not consider the mass or size of the moving objects. Hence in kinematics all moving bodies, irrespective of their geometric sizes, are treated as **particles or point masses**.

If the position of a particle is not changing with respect to its surroundings with the passage of time, the particle is said to be in the state of rest.

If the position of a particle is changing with respect to its surroundings with the passage of time, the particle is said to be in the state of motion.

Since the description of motion of a particle is always with respect to its surroundings, to describe the motion of a particle, an observer is also needed. Since no particle is at permanent rest or in permanent motion, absolute rest or absolute motion is impossible.

The description of motion of a particle is done using six terms, which are time, distance, speed, displacement, velocity and acceleration. These terms will be explained later in this chapter. Out of these terms, the three terms distance, speed and time are scalars (i.e., they are physical quantities that require only a magnitude to describe them completely) while the other three terms displacement, velocity and acceleration are vectors (i.e., physical quantities that require both magnitude and direction to define them completely).

In any process involving change, we must be able to assign time to the various events that occur. To measure the motion of a particle, we have to measure its positions in space at different instants (of time).

For the purpose of time measurement, some instant of time must be chosen as the origin and be assigned the value of zero time. This is called origin for representation of time. The unit of time may be second, minute, hour, day, week, month or year as per convenience.

The events that occurred before the zero point of time will be given a negative number of units of time, while those that occur after zero point of time will be given a positive number of time unit. **The difference in time between two events is called the time interval.** It is important to note that if the zero of time measurements coincides with

the beginning of an event, then negative time will have no meaning for that event.

To fix the position of a body in space, we require a reference point (or the observer). Any arbitrary point in space can be taken as a reference point. We call this the **origin**. At the origin, we draw three arbitrary mutually perpendicular **axes**. The origin along with the three mutually perpendicular axes is called the **Cartesian coordinate system**, which serves as a **frame of reference** for the description of motion. The observer or the describer of the motion is located at the origin of the frame of reference.

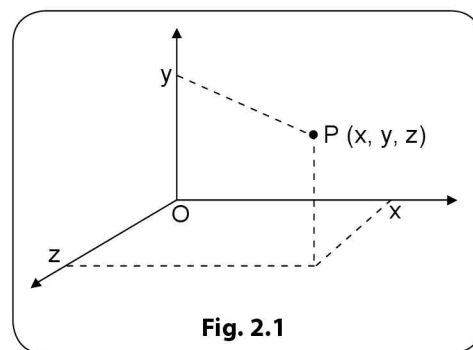


Fig. 2.1

A straight line drawn from the origin to the position of the particle at certain time is called the **position vector** of the particle at that time. Its magnitude is equal to the length of the line connecting the origin and the position of the particle and direction is from the origin to the particle. The **position coordinates** of the particle are expressed as x (measured along or parallel to X-axis), y (measured along or parallel to the Y-axis) and z (measured along or parallel to the Z-axis).

Fig. 2.1 shows the position of a point P (the particle), which can be reached from the origin O by moving x unit along the X-axis, y unit parallel to the Y-axis and z unit parallel to the Z-axis. These three values x , y and z are the three **position coordinates** of the point P written as (x, y, z) . These three coordinates define the distance of the particle in x , y , z directions from the origin O.

A particle is at rest, if its position vector remains the same at all times (i.e., its x , y and z co-ordinates remain the same at all times). A particle is moving, if its position vector keeps changing with time (i.e., one or more of its x , y and z co-ordinates keep changing at all times).

CLASSIFICATION OF MOTION INTO ONE, TWO AND THREE DIMENSIONS

Motion in one dimension

During the motion of a particle, if only one out of its three coordinates x , y and z undergoes a change, then the motion of the particle is one-dimensional motion.

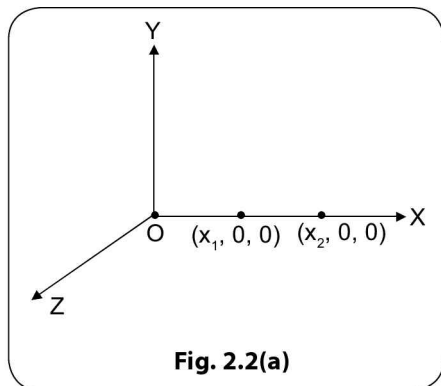
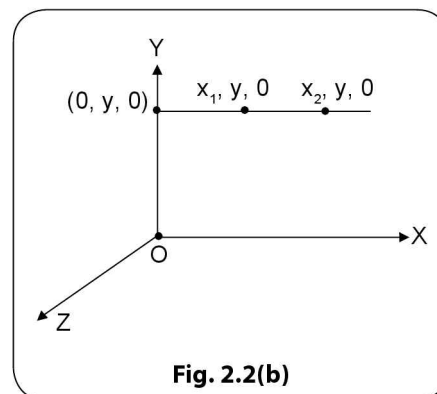


Figure 2.2(a) shows the motion of a particle in one dimension along the X-direction, from $(x_1, 0, 0)$ to $(x_2, 0, 0)$.



Only the X- co-ordinate of the particle is changing, while the other two coordinates (Y and Z) do not change. Note that the other two coordinates need not remain zero; they can have any constant nonzero value as shown in Fig. 2.2(b).

Thus ***all one-dimensional motions take place along a straight line.***

CONCEPT STRAND

Concept Strand 1

Consider the following cases:

- A particle travels along a path having position coordinates $(2 \text{ m}, 0, 4 \text{ m})$, $(4 \text{ m}, 0, 4 \text{ m})$, $(8 \text{ m}, 0, 4 \text{ m})$ and $(13 \text{ m}, 0, 4 \text{ m})$ at time $t = 1 \text{ s}$, 2 s , 3 s and 4 s respectively.
- A particle travels along a straight line and is at distances 2 m , 10 m , 25 m and 40 m from the origin at time $t = 1 \text{ s}$, 5 s , 7 s and 10 s respectively.
- A particle is at positions $(-5 \text{ m}, 0)$, $(-1 \text{ m}, 0)$, $(-3 \text{ m}, 0)$ and $(2 \text{ m}, 0)$ at time $t = 1 \text{ s}$, 5 s , 7 s and 12 s respectively.

- A particle passes through the points $(0, 1)$, $(0, 2)$, $(5, 0)$, $(5, 2)$ as time progresses.

Which of the above motions is/are one-dimensional motion/motions?

Solution

(a), (b) and (c) are one-dimensional motions as the motion of the particle is along a straight line. Motion represented by (d) is not one-dimensional as all the points representing the positions of the particle are not collinear.

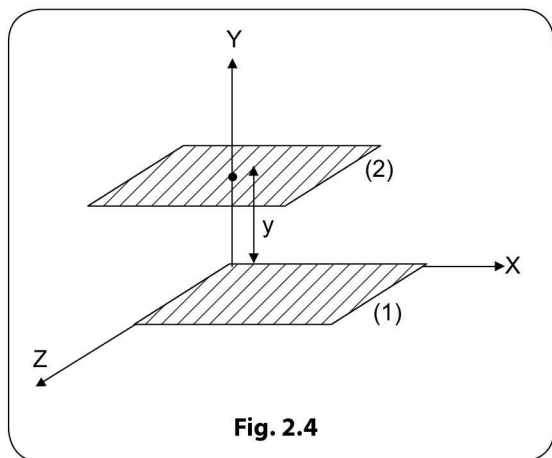
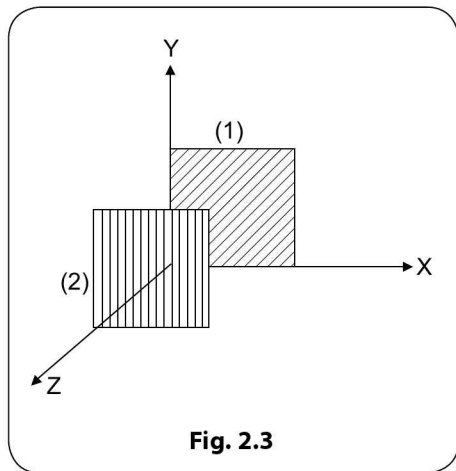
Motion in two dimensions

Motion of a particle, in which two of its position coordinates change with the passage of time while the third position co-ordinate remains constant at all time, is called two-dimensional motion.

Clearly, ***two-dimensional motion of a particle occurs on a plane surface.*** For example, if the motion of the particle takes place on the plane (1) (Fig. 2.3), the X and Y coordinates of the positions of the particle change continuously, while the Z coordinate remains constant. In this plane, the z coordinate has the value zero because it passes through the origin.

2.4 Motion in One Dimension

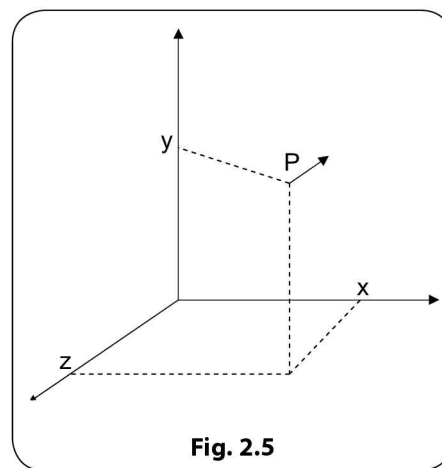
If the motion of the particle takes place on the plane (2), the X and Y position coordinates of the particle change but Z coordinate is constant and has a non zero-value.



Similarly, in Fig. 2.4, for the motion of a particle on the plane (1), its X and Z position coordinates change while the Y coordinate remains zero. But, if its motion is on the plane (2), its X and Z position coordinates change, while Y coordinate has a non-zero (constant) value.

Motion in three dimensions

If all the three position coordinates of a particle change with the passage of time, the motion of the particle is called three-dimensional motion.



The point P representing the particle moves in three dimensions in the direction shown by the arrow in Fig. 2.5. At any time during the motion, the point P is represented by three coordinates x, y and z.

CONCEPT STRAND

Concept Strand 2

Consider the following motions.

- The striker hitting the side of a carrom board at an angle.
- The striker hitting the side of a carrom board normally.
- The flight of a football in air on a windy day.

Which of the above is/are two-dimensional motion/ motions?

Solution

- Both X and Y coordinates change in this case with no change in the Z coordinates $\Rightarrow$ two-dimensional.
- Only one coordinate changes $\Rightarrow$ one-dimensional (lies along a straight line)
- All three coordinates change $\Rightarrow$ three-dimensional (does not lie on a plane surface)

MOTION IN ONE DIMENSION

In this section, we consider only the motion of a particle in one dimension or equivalently, motion along a straight line.

Distance

The length of the actual path along which the particle changes its position from its initial position to its final position in a given interval of time is called the distance covered by the particle. The following points are applicable to distance.

- Distance is a scalar quantity i.e., it does not take direction into consideration.
- Distance covered is always a positive quantity.
- For a moving particle, the distance covered can never be zero.
- The distance covered by a particle never decreases with time, it always increases.
- The unit of distance is the unit of length i.e., cm, m, km etc., and has dimensional formula L. **SI unit of distance is metre (m).**
- There can be infinite number of paths for a particle between its initial and final positions.

Displacement

The change in the position (or position vector) of a particle in a particular direction during a given time interval is called the displacement of the particle. The following points are applicable to displacement:

- Displacement is a vector quantity i.e., it has both magnitude and direction.
- For a moving particle, the displacement can be positive, negative or zero.
- Displacement of a particle does not give any idea about the nature of the path followed by the particle.
- The displacement of a particle is along the unique path (which is the shortest path between the initial and final positions of the particle), which may or may not be the actual path travelled by the particle. This path is a straight path.
- The shortest distance between the initial and final positions of the particle gives the magnitude of displacement.
- The displacement of a particle does not depend upon the choice of origin. That means the displacement remains the same, even if origin is shifted.

- The unit of displacement is the unit of length i.e., cm, m, km etc and has dimensional formula L. **SI unit of displacement is metre (m).**
- When the motion of the particle is along a straight line and in the same direction, magnitude of displacement = distance. Otherwise, magnitude of displacement is less than distance. **In general, the magnitude of displacement $\leq$ distance.**
- When a particle returns to its starting point, its displacement is zero but distance is not zero.

As a particle moves along a straight line, say for example the X-axis, as shown in Fig. 2.6, its position (distance from the origin $x = 0$) changes continuously. The motion of the particle is described by stating its position coordinate as time progresses. Let the body be at $-x_1$ at the instant of time t_1 .

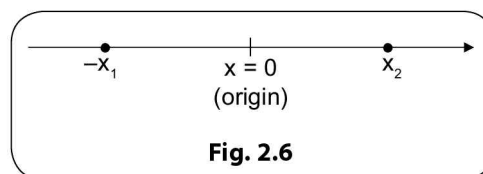


Fig. 2.6

The negative sign is written to denote that the position of the particle is to the left of the origin. Let the particle be at $+x_2$ at the instant of time t_2 . This time, the position of the particle is to the right of the origin and hence a positive sign.

By convention, the distance travelled in the positive direction of the X-axis is assigned a positive sign and that along the negative direction of the X-axis, a negative sign. The particle has travelled from the point $-x_1$ towards the right to the point x_2 , that is in the positive direction of the X-axis, a distance of magnitude given by the difference of the position coordinates of the end point and the beginning point, $x_2 - (-x_1) = x_2 + x_1$. The particle has undergone a **displacement** of magnitude $x_2 + x_1$. Displacement is the difference in the final and initial positions of the particle.

We may come across situations where the displacement is zero but the distance travelled is finite. For example, consider a particle that starts at the origin, goes to x_2 and back to the origin. The distance travelled is $2x_2$, whereas the displacement is zero. In the former case, we do not ask in which direction the particle moves as we are concerned only about the total distance it has travelled, $|x_2| + |x_2| = 2|x_2|$. On the other hand, in the latter case, the

2.6 Motion in One Dimension

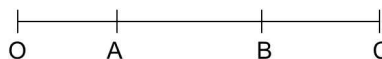
displacement is $x_2 + (-x_2) = 0$. Therefore, displacement is obtained by adding together all parts of the particle's

path, forward and backward, with the proper positive or negative sign.

CONCEPT STRAND

Concept Strand 3

A particle starts from the origin and moves along a straight line. Its path is given by O – B – A – C – B. If OA = 11 cm, OB = 16 cm and OC = 18 cm, what is the distance traveled by the particle?



Solution

$$\begin{aligned}\text{Distance} &= |\text{OB}| + |\text{BA}| + |\text{AC}| + |\text{CB}| \\ &= 16 + 5 + 7 + 2 = 30 \text{ cm}\end{aligned}$$

Speed

The rate at which distance is covered by a particle is called its speed.

The following points are applicable to speed.

- (a) Speed is a scalar quantity.
- (b) Speed is either positive or zero but never negative.
- (c) Speed is an indication of “how fast the particle is moving”. A fast moving particle has high speed and a slow moving particle has low speed. A particle at rest has zero speed.

The SI unit of speed is m s^{-1} and its dimensional formula is LT^{-1} . Also $1 \text{ km h}^{-1} = \frac{5}{18} \text{ m s}^{-1}$

Uniform speed

A particle is said to be moving with uniform speed, if it covers equal distances in equal intervals of time, however small these intervals may be. If v is the uniform speed of the particle, then the distance S covered by it in time interval t is given by

$$S = vt$$

If a train travels 1000 m in one minute, 500 m in 30 second, 250 m in 15 second and 125 m in 7.5 second and so on, then the train is moving with uniform speed.

Note:

A particle moving with uniform speed in the same direction is said to be in uniform motion.

Thus, a particle in uniform motion moves in a straight line in the same direction and for any time interval,

the magnitude of its displacement = distance covered by it.

Variable speed

A particle is said to move with variable speed, if it covers unequal distances in equal intervals of time.

Instantaneous speed

The speed of a moving particle at any instant of time is called its instantaneous speed. **Speedometer** of a vehicle measures its instantaneous speed.

Assume that a particle travels a distance ΔS in time Δt . Here, the Greek letter Δ in front of S denotes that the distance referred to is a small quantity. Mathematically instantaneous speed v is defined as

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t} = \frac{dS}{dt}$$

Average speed

When a particle is moving with variable speed, the average speed of the particle is the ratio of the total distance travelled by it to the total time taken. In fact average speed is the average of all instantaneous speeds.

Average speed = Total distance travelled/total time taken

If a particle is moving with uniform speed, then its average speed and instantaneous speed are both equal to the uniform speed.

Velocity

The time rate of change of position of a particle is called the velocity of the particle. That is velocity of a particle is its time rate of displacement.

Velocity = Displacement/Time i.e.,

$$\bar{v} = (\bar{x}_2 - \bar{x}_1) / (t_2 - t_1)$$

The following points are applicable to velocity.

- Velocity is a vector quantity. (A horizontal line above the letter denotes a vector)
- Velocity can be positive, negative or zero.
- The SI unit of velocity is m s^{-1} and its dimensional formula is LT^{-1} .
- Velocity of a particle is an indication of the rate at which a particle is changing position. If a person rapidly jumps one step forward and one step backward, it might result in a frenzy of activity but zero velocity, as there is no change in his position!
- Velocity of a particle changes when its magnitude changes, or its direction changes or both change. In this case, the particle is said to move with **Variable velocity**.
- The direction of velocity of a particle determines the direction of motion of the particle.

Uniform velocity

A particle is said to move with uniform velocity, if it covers equal displacements in equal intervals of time, however small these intervals may be. In this case, the particle is in uniform motion and hence it moves in a straight line, in the same direction.

Displacement = Uniform velocity $\times$ time i.e.,

$$\bar{S} = \bar{v}t$$

Note:

A particle moving with uniform velocity is said to be in uniform motion.

Instantaneous velocity

It is defined as the velocity of the particle at any instant of time or at any point of its path. Mathematically, the instantaneous velocity $\bar{v}$ is defined as

$$\bar{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \bar{x}}{\Delta t} = \frac{d\bar{x}}{dt}$$

Magnitude of the instantaneous velocity is equal to the instantaneous speed.

Average velocity

The ratio of the total displacement of a particle to the total time taken by it is defined as the average velocity of the particle.

Average velocity = Total Displacement/Total time.

When a particle returns to its starting point, its average velocity is zero but its average speed is not zero.

The average velocity of a particle for its motion during certain time interval is the uniform velocity with which the particle should move during that time interval to undergo the same displacement as in the actual motion of the particle during the same time interval.

CONCEPT STRAND

Concept Strand 4

'A' set out from her home along the straight road to school, which is 12 min walk at a distance of 1.2 km on the same road. After walking for 3 minutes she realized that she had forgotten her lunch box. She went back, picked up her lunch box and returned immediately. Assuming that A maintained the same speed of walking, calculate her average speed and average velocity.

Solution

Displacement is 1.2 km and the usual time for the walk is 12 min

$$\text{Average speed of A} = \left(\frac{1.2 + 0.3 + 0.3}{12 + 3 + 3} \right) = 0.1 \text{ km/min}$$

$$\text{Average velocity of A} = 1.2/(12+6) = 0.067 \text{ km/min}$$

2.8 Motion in One Dimension

Expressions for average velocity

- (i) In one dimensional motion we need not use the vector sign for the displacement, velocity, acceleration etc. Instead, we will use + or - sign wherever necessary. If a particle is at $x = 0$ (origin) at $t = 0$ and at x at time t , then the magnitude of its average velocity for

that time interval $v = v_{av} = \frac{x - 0}{t - 0} \Rightarrow$ The magnitude

of its displacement x after time interval t is given by $x = v_{av} t$

- (ii) If a particle moves in the same direction, with a uniform velocity of magnitude v_1 for t_1 second, v_2 for t_2 second etc.,

Magnitude of average velocity,

$$v_{av} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$v_{av} = \frac{v_1 t_1 + v_2 t_2 + \dots + v_n t_n}{(t_1 + t_2 + \dots + t_n)}$$

- (iii) In the above expression, if $t_1 = t_2 = \dots = t_n = t$, then

$$v_{av} = \frac{t(v_1 + v_2 + \dots + v_n)}{nt}$$

$$v_{av} = \frac{v_1 + v_2 + \dots + v_n}{n}$$

Hence if a particle travels in the same direction with velocities of different magnitudes in equal intervals of time, the magnitude of the average velocity of the motion is equal to the arithmetic mean of the magnitudes of the velocities.

- (iv) In the expression in (ii) above, if $v_1 t_1 = v_2 t_2 = \dots = v_n t_n = S$ (i.e., equal magnitudes of displacements in different intervals of time with velocities of different magnitudes in the same direction), then

$$t_1 = \frac{S}{v_1}, t_2 = \frac{S}{v_2}, \dots, t_n = \frac{S}{v_n} \text{ and}$$

$$v_{av} = \frac{S + S + \dots + S}{\left[\frac{S}{v_1} + \frac{S}{v_2} + \dots + \frac{S}{v_n} \right]} = \frac{n}{\left[\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n} \right]} \Rightarrow$$

$$\frac{n}{v_{av}} = \left[\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n} \right]$$

Special case:

If a particle travels in the same direction, a distance S_1 with velocity of magnitude v_1 and a distance S_2 with velocity of magnitude v_2 , then the magnitude of the average velocity of motion.

$$v_{av} = \frac{(S_1 + S_2)}{\left(\frac{S_1}{v_1} + \frac{S_2}{v_2} \right)} = \frac{(S_1 + S_2) v_1 v_2}{(S_1 v_2 + S_2 v_1)}$$

$$\text{If } S_1 = S_2, v_{av} = \frac{2v_1 v_2}{(v_1 + v_2)}$$

$$\Rightarrow \frac{2}{v_{av}} = \frac{1}{v_1} + \frac{1}{v_2}$$

Note:

Thus the average velocity for the total journey in this case is the harmonic mean of the two velocities.

CONCEPT STRANDS

Concept Strand 5

A particle traveling in the same direction, covers the first 10 m with a velocity of magnitude 2 m s^{-1} , the next 12 m with velocity of magnitude 3 m s^{-1} and the next 8 m with velocity of magnitude 4 m s^{-1} . Find the magnitude of the average velocity for the whole journey.

Solution

$$\begin{aligned} \text{Total time} &= \frac{10}{2} + \frac{12}{3} + \frac{8}{4} \\ &= 5 + 4 + 2 = 11 \text{ s} \end{aligned}$$

$$\text{Total distance} = 10 + 12 + 8 = 30 \text{ m}$$

$$\text{Average velocity} = \frac{30}{11} = 2.73 \text{ m s}^{-1}$$

Concept Strand 6

A, B and C together set out to the same school from the same home along a straight road. The school is at a distance of 1.2 km on the same road. After walking for 3 minutes, A realized that she had forgotten her lunch box. She went back, picked up her lunch box and returned immediately. B waited for 2 minutes, did not find A and resumed her

journey reaching the school in 9 minutes from the point where she waited. A returned and not finding B, continued walking, and reached the school. C, on the other hand, went straight and reached school in 12 minutes. Assuming that all had the same speed of walking, calculate the average speed of each and the distance at which B waited for A.

Solution

Displacement is 1.2 km and the speed of walking is the same for all three.

Average speed of C (same as magnitude of average velocity of C) = $1.2/12 = 0.1$ km/min

Average speed of B (same as magnitude of average velocity of B) = $1.2/(3+2+9) = 0.086$ km/min

Position where B waited = $3 \times 0.1 = 0.3$ km from home.

Average speed of A = $(1.2+0.6)/(3+6+9) = 0.1$ km/min

Magnitude of average velocity of A = $1.2/(3+6+9) = 0.067$ km/min

Acceleration

The time rate of change of velocity of a particle is called its acceleration. A particle moving with acceleration is in non-uniform motion. The following are applicable to acceleration.

- Acceleration is a vector quantity.
- A change in velocity means a change in its magnitude, or in its direction or both. Any such change will result in acceleration.
- For a particle moving in the same direction, if the magnitude of the final velocity is more than that of the initial velocity, the acceleration is positive and the body is speeding up.
- For a particle moving in the same direction, if the magnitude of the final velocity is less than that of the initial velocity, the acceleration is negative and the body is slowing down.
- If acceleration is zero, the body is either at rest or in uniform motion.
- A body under **free fall** near the surface of Earth experiences a constant downward acceleration equal to the acceleration due to gravity (9.81 m s^{-2}).
- The SI unit of acceleration is m s^{-2} and its dimensional formula is LT^{-2} .

- If a particle is moving with uniform speed, it may or may not have acceleration but a particle moving with uniform velocity has no acceleration.
- If a particle is moving with variable speed, it has acceleration.
- It is not necessary that a particle in one-dimensional motion has the same direction for its velocity and acceleration. If they are in the same direction, the particle speeds up and if they are in the opposite directions, the particle slows down.
- Negative acceleration is also called retardation or deceleration.

Uniform acceleration (Constant acceleration)

If the magnitude and direction of the acceleration of a particle remains constant at all time, the particle is said to be moving with constant acceleration. The change in magnitude of the velocity of a particle in uniform acceleration (either increase or decrease but not both) per unit time remains constant and this change is always in the same direction. If the magnitude of acceleration changes or the direction of acceleration changes or both magnitude and direction of acceleration of a particle change, then the particle is said to execute non-uniformly accelerated motion (or variable acceleration).

KINEMATIC EQUATIONS OF MOTION (FOR UNIFORMLY ACCELERATED OR UNIFORMLY DECELERATED MOTIONS ONLY)

Consider a particle in one-dimensional motion having an initial velocity of magnitude 'u' and a uniform acceleration of magnitude 'a'. Let v be the magnitude of its velocity after a time interval 't' and 'S' the magnitude of the displacement of the particle during that time interval.

In one-dimensional straight line motion, we need not use the vector signs for quantities like the displacement, velocity, acceleration, etc., instead we can represent them with $\pm$ signs, representing forward and backward directions.

2.10 Motion in One Dimension

Equation for final velocity v

The acceleration of the body is defined as

$$a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{v - u}{t}$$

$$u + at = v \quad \text{----- (1)}$$

Note:

If it is uniform retardation (instead of uniform acceleration), equation (1) will become

$$u - at = v \quad \text{----- (2)}$$

Equation for displacement S

The displacement of the particle in the time interval 't' is given by

$$S = (\text{average velocity}) \times \text{time} = \left(\frac{u + v}{2} \right) t$$

$$\text{Hence } S = \left(\frac{u + v}{2} \right) t \quad \text{----- (3)}$$

But $v = u + at$ from equation (1).

$$\therefore S = \left(\frac{u + u + at}{2} \right) t$$

$$= ut + \frac{1}{2}at^2 \Rightarrow$$

$$S = ut + \frac{1}{2}at^2 \quad \text{----- (4)}$$

For decelerated motion, the equation is

$$S = ut - \frac{1}{2}at^2 \quad \text{----- (5)}$$

CONCEPT STRAND

Concept Strand 7

A car is travelling on a straight road at 72 km h^{-1} when brakes are applied. If the deceleration is 2 m s^{-2} and if the brakes are applied for 7 s, find the distance travelled by the car during this time.

Solution

$$u = 72 \times \frac{5}{18} = 20 \text{ m s}^{-1}$$

$$t = 7 \text{ s}; a = -2 \text{ m s}^{-2}$$

$$S = ut + \frac{1}{2}at^2 = 20 \times 7 + \frac{1}{2} \times (-2) \times 7^2 = 140 - 49 = 91 \text{ m}$$

Relation between v , u , a and S

We know that

$$v - u = at \quad \text{--- (i)}$$

$$S = \left(\frac{u + v}{2} \right) t$$

$$\Rightarrow v + u = \frac{2S}{t} \quad \text{--- (ii)}$$

$$(i) \times (ii) \Rightarrow (v + u)(v - u) = at \times \frac{2S}{t} \Rightarrow$$

For accelerated motion, the relation is

$$v^2 - u^2 = 2aS \quad \text{--- (6)}$$

For decelerated motion, the relation is

$$v^2 - u^2 = -2aS \quad \text{--- (7)}$$

CONCEPT STRAND

Concept Strand 8

A train traveling on straight tracks has a speed of 90 km h^{-1} as its engine passes a pole and it has a speed of 72 km h^{-1}

when the guard van passes the same pole. If the length of the train is 225 m, find the magnitude of the deceleration of the train.

Solution

$$v^2 - u^2 = 2aS$$

$$v = 20 \text{ m s}^{-1};$$

$$u = 25 \text{ m s}^{-1}$$

$$\begin{aligned} a &= \frac{v^2 - u^2}{2S} \\ &= \frac{20^2 - 25^2}{2 \times 225} = \frac{-225}{2 \times 225} = -0.5 \text{ m s}^{-2} \\ \Rightarrow \text{Deceleration} &= 0.5 \text{ m s}^{-2} \end{aligned}$$

Equation for the distance travelled in the n th second

Distance travelled by a particle in the n th second = Distance travelled by the particle in n second – Distance travelled by the particle in $(n - 1)$ second.

i.e., $S_n = un + \frac{1}{2}an^2 - [u(n - 1) + \frac{1}{2}a(n - 1)^2]$, which on simplification gives

for accelerated motion,

$$S_n = u + a \left(n - \frac{1}{2} \right) \quad \text{--- (8)}$$

for decelerated motion,

$$S_n = u - a \left(n - \frac{1}{2} \right) \quad \text{--- (9)}$$

CONCEPT STRANDS**Concept Strand 9**

Identical blocks are slid across a rough horizontal floor at 10 m s^{-1} from one point to another, at the rate of one block per second. As the 8th block is released, the first one reaches its destination. What is the distance between the first and the second blocks at this instant? Consider that the floor produces a uniform retardation of 1 m s^{-2} on each block.

Solution

$$S_n = u + a \left(n - \frac{1}{2} \right) = 10 + (-1) \left(7 - \frac{1}{2} \right) = 10 - \frac{13}{2} = 3.5 \text{ m}$$

Concept Strand 10

A car travels on a straight road to a point 4 km away with a speed of 20 m s^{-1} . At the halfway point, the driver decides

that the average speed for the whole journey should be 25 m s^{-1} . What acceleration should be employed?

Solution

$$v_{av} = 25 \text{ m s}^{-1} = \frac{4000}{t}$$

$$\Rightarrow t = \frac{4000}{25} = 160 \text{ s}$$

$$\text{Halfway is reached at time} = \frac{2000}{20} = 100 \text{ s}$$

$$\Rightarrow 2000 = 20 \times 60 + \frac{1}{2}a \times 60^2 \Rightarrow \frac{1}{2} \times a \times 60^2 = 800$$

$$\therefore a = \frac{800 \times 2}{3600} = \frac{4}{9} \text{ m s}^{-2}$$

MOTION UNDER GRAVITY

It is found that all bodies near the surface of Earth, irrespective of their size and mass, experience the same constant magnitude of vertically downward acceleration. This acceleration is called acceleration due to gravity and its

magnitude is nearly 9.8 m s^{-2} . This is also called **free fall acceleration**. Consequently, when a body is released from a small height above the surface of Earth, it falls down vertically with uniform acceleration and its speed increases.

2.12 Motion in One Dimension

On the other hand, when a body is thrown up with certain velocity, this downward acceleration on it reduces the upward velocity of the body and it slows down as it gains height. At the highest point, its instantaneous velocity is zero (though its acceleration is not zero). Free fall near the surface of Earth can be analysed as a uniformly accelerated (or uniformly retarded) one-dimensional motion.

Kinematic equations of vertical motion near the surface of Earth

Equations of motion 1 to 9 can be adapted for bodies moving vertically up or down near the surface of the Earth. This is shown in Table 2.1. The height of fall (or rise) at time 't' is denoted as 'H'.

Table 2.1

Upward motion	Downward motion
$v = u - gt$	$v = u + gt$
$H = ut - \frac{1}{2}gt^2$	$H = ut + \frac{1}{2}gt^2$
$v^2 - u^2 = -2gH$	$v^2 - u^2 = 2gH$
$H_n = u - g\left(n - \frac{1}{2}\right)$	$H_n = u + g\left(n - \frac{1}{2}\right)$

We derive below some useful equations for a body thrown up or dropped from a height. Air resistance is neglected throughout.

- (i) The velocity of the body at its maximum height $H_{\max}$ is $v = 0$. Then from $v = u - gt$, $t = \frac{u}{g}$

$$\text{Time for going up } t = \frac{u}{g} \quad \text{--- (10)}$$

For a body coming down after reaching its maximum height,

$$\text{Time to hit the ground} = \frac{v}{g} = \frac{u}{g} \quad (\text{by symmetry}). \quad \text{--- (11)}$$

$$\text{Total time of flight } T = 2t = \frac{2u}{g} \quad \text{--- (12)}$$

Maximum height reached, $H_{\max}$ can be obtained from the equation $v^2 - u^2 = -2gH$
Since $v = 0$,

$$H_{\max} = \frac{u^2}{2g} \quad \text{--- (13)}$$

On substituting from (12)

$H_{\max}$ can also be written as

$$H_{\max} = \frac{1}{8}gT^2 \quad \text{--- (14)}$$

$$\text{Time of flight } T = \sqrt{\frac{8H_{\max}}{g}} \quad \text{--- (15)}$$

$$\text{Velocity on hitting the ground} = \text{initial velocity} = \sqrt{2gH_{\max}} \quad \text{--- (16)}$$

If a body is dropped from a height h , the time taken to reach ground is

$$t_{\text{fall}} = \sqrt{\frac{2h}{g}} \quad \text{--- (17)}$$

CONCEPT STRANDS

Concept Strand 11

A body is dropped from a small height H above the surface of the Earth. If the first half of the height $\frac{H}{2}$ is covered by it in time t_1 , what is the time taken by it to cover the second half of the height?

Solution

The total time to reach ground is t .

$$\text{Since } \frac{H}{2} = \frac{1}{2}gt_1^2$$

$$\text{and } H = \frac{1}{2}gt^2,$$

$$\frac{t_1^2}{t^2} = \frac{1}{2} \Rightarrow$$

$$t = \sqrt{2} t_1$$

$\therefore$ Time taken to cover the second half

$$t_2 = (t - t_1) = \sqrt{2} t_1 - t_1$$

$$= (\sqrt{2} - 1)t_1 = 0.414t_1$$

Concept Strand 12

A ball is thrown vertically up with a velocity of 30 m s^{-1} and simultaneously another ball was dropped from some height so that they hit each other just when the upward going ball was at its maximum height. What was the height from which the ball was dropped? ($g = 10 \text{ m s}^{-2}$)

Solution

Upward going ball: $u = 30 \text{ m s}^{-1}$, $v = 0$,

$$\therefore t = \frac{u}{g} = 3 \text{ s}$$

$$H = \frac{u^2}{2g} = 45 \text{ m}$$

Downward moving ball: $u = 0$, $t = 3 \text{ s}$

$$H = \frac{1}{2}gt^2 = 45 \text{ m}$$

The height from where the second ball was dropped = $45 + 45 = 90 \text{ m}$

Concept Strand 13

A particle is projected vertically from a hole of depth 10 m with an initial speed of 40 m s^{-1} . To what height above the ground will the particle rise? ($g = 10 \text{ m s}^{-2}$)

Solution

$$u = 40 \text{ m s}^{-1};$$

$$g = -10 \text{ m s}^{-2}$$

$$v^2 = u^2 + 2gh$$

$$0 = 40^2 - 2 \times 10 \times h$$

$$h = \frac{40^2}{20} = 80 \text{ m as measured from the hole.}$$

$\Rightarrow$ Height above the ground = $80 - 10 = 70 \text{ m}$

Concept Strand 14

A body at rest falls from a height h_1 above the ground on to a platform at a height h_2 above the ground, where its velocity is reversed with out any change in its magnitude. Subsequently, it falls to the ground. Find its total time of flight.

Solution

With the origin fixed on the ground, the free fall of the body from a height h_1 on to the platform at a height h_2 above the ground is given by

$$(h_2 - h_1) = \frac{1}{2}(-g)t_1^2 \Rightarrow t_1 = \sqrt{\frac{2(h_1 - h_2)}{g}}$$

The body bounces elastically on the platform reversing the velocity. Hence its motion is symmetric to the former. The time for upward motion is

$$t_2 = \sqrt{\frac{2(h_1 - h_2)}{g}}$$

Subsequent fall of the body from the height h_1 to the

ground takes time $t_3 = \sqrt{\frac{2h_1}{g}}$

$\therefore$ the total time taken is

$$\begin{aligned} T_{\text{fall}} &= \sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2(h_1 - h_2)}{g}} \\ &= \sqrt{\frac{2}{g}}(\sqrt{h_1} + 2\sqrt{h_1 - h_2}) \end{aligned}$$

RELATIVE MOTION OF PARTICLES IN ONE DIMENSIONAL MOTION

Suppose two particles are moving along the same line, either in the same direction or in opposite directions, with different velocities. Two trains moving along parallel tracks or two cars moving along a straight road are some common examples.

Relative displacement of A with respect to B = Position of A – Position of B and

Relative displacement of B with respect to A = Position of B – Position of A.

2.14 Motion in One Dimension

If particles A and B are on the same side of the origin, the magnitude of their relative displacement is equal to the difference of the magnitudes of their positions.

If particles A and B are on either side of the origin, the magnitude of their relative displacement is equal to the sum of the magnitudes of their positions.

If the velocities of two particles A and B moving in the same direction are v_A and v_B , respectively, we may define the relative velocity of A with respect to B as

$$\begin{array}{ccc} \xrightarrow{V_A} & & \\ \xrightarrow{V_B} & \Rightarrow & \xrightarrow{V_{AB} = V_A - V_B} \end{array}$$

Similarly, the relative velocity of B with respect to A is

$$V_{BA} = V_B - V_A \Rightarrow \xleftarrow{V_B - V_A}$$

If the bodies are moving in opposite directions, the magnitude of the relative velocity of A with respect to B is

$$\begin{array}{ccc} \xrightarrow{V_A} & & \\ \xleftarrow{V_B} & \Rightarrow & \xrightarrow{V_{AB} = V_A - (-V_B) = V_A + V_B} \end{array}$$

and its direction is in the direction of motion of A.

Also, the magnitude of the relative velocity of B with respect to A is

$$\begin{array}{c} V_{BA} = V_B - (-V_A) = V_B + V_A \\ \xleftarrow{V_B + V_A} \end{array}$$

and its direction is in the direction of motion of B.

Relative acceleration of A with respect to B = Acceleration of A – Acceleration of B and

Relative acceleration of B with respect to A = Acceleration of B – Acceleration of A.

If the accelerations of A and B are in the same direction, the magnitude of their relative acceleration is equal to the difference of the magnitudes of their accelerations.

If the accelerations of A and B are in opposite directions, the magnitude of their relative acceleration is equal to the sum of the magnitudes of their accelerations.

For two particles in free fall near the surface of the Earth, their relative acceleration is zero as each particle has an acceleration equal to 'g' (the acceleration due to gravity) in the vertically downward direction.

By using modified kinematic equations of motion, the problems involving relative motions of any two particles A and B can be solved.

In the kinematic equations of motion derived earlier, replace initial velocity 'u' with initial relative velocity ($u_A - u_B$), replace final velocity 'v' with final relative velocity ($v_A - v_B$), replace uniform acceleration 'a' with uniform relative acceleration ($a_A - a_B$) and replace displacement 'S' with relative displacement ($S_A - S_B$) respectively.

The kinematic equations of motion derived earlier are actually the kinematic equations of relative motion between two particles, out of which one particle is considered at rest and having zero acceleration.

CONCEPT STRAND

Concept Strand 15

A body is dropped from a point P at a height of 50 m above the ground. Two seconds later another body is thrown vertically up from ground from a point Q, which is vertically below P, with an initial speed of 20 m s^{-1} . At what time after the release of the second body will they meet?

Solution

In two seconds, the body that was dropped would attain a velocity

$$\begin{aligned} v &= at = 10 \times 2 \\ &= 20 \text{ m s}^{-1} \text{ in the downward direction.} \end{aligned}$$

During this time, it would have covered a distance

$$s = \frac{1}{2}at^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

Hence initial vertical separation between the two particles = $50 - 20 = 30 \text{ m}$

For the relative motion of the two particles, we have

$$S_R = u_R t + \frac{1}{2} a_R t^2,$$

where S_R = relative displacement, u_R = relative initial velocity and

a_R = relative acceleration of the moving particles.

Magnitude of relative displacement = $50 - 20 = 30 \text{ m}$

Magnitude of initial relative velocity = $20 + 20 = 40 \text{ m s}^{-1}$

Magnitude of relative acceleration = 0

$$\Rightarrow t = \frac{30}{40} = 0.75 \text{ s is the time after the release of the second body at which the two particles will collide.}$$

GRAPHICAL REPRESENTATIONS OF ONE DIMENSIONAL MOTION

A graphical representation of the motion of a particle is helpful in analysing the motion of the particle. The various graphs that are commonly used are

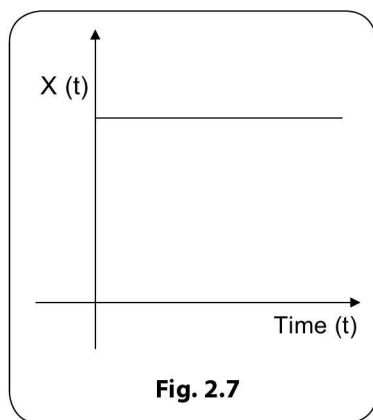
- (i) Position – Time (x–t) graph
- (ii) Velocity – Time (v–t) graph and
- (iii) Acceleration – Time (a–t) graph.

Position–time graph (x–t graph)

A graph showing the position of the particle at various instants of time is called a position time graph. This graph is useful in determining the instantaneous velocities, average velocities, instantaneous positions and displacements of the particle.

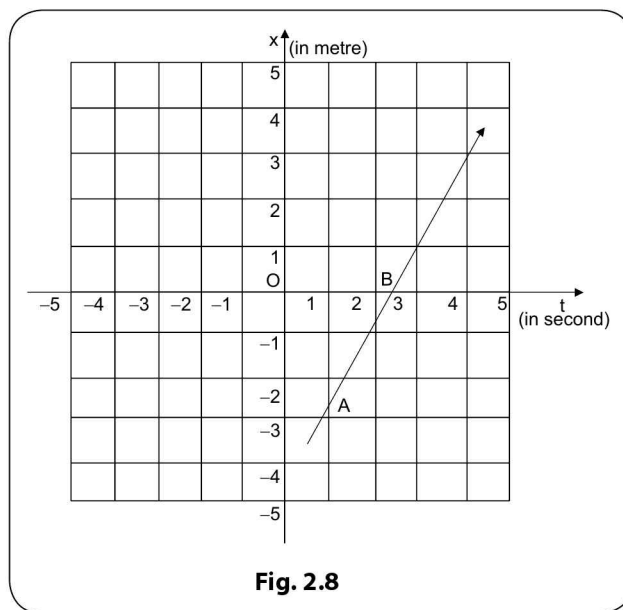
(a) Position–time graph of a particle at rest

For a particle at rest, since there is no change in its position with the passage of time, its position–time graph is a straight line parallel to the time axis as shown in Fig 2. 7.



(b) Position–time graph of a particle moving with uniform velocity

Consider the motion of a particle along a straight line represented by the Fig. 2.8. Here, the Abscissa represents time in second and the Ordinate represents the position of the particle in metre. The Ordinate (Y-axis) represents the straight path of the particle. The position–time graph is a straight line inclined to the time axis. The origin represents the observer, whose position is $x = 0$ at all times and this origin coincides with the origin for time measurements. The particle is at



–2.8 m at time 1 s, passes the origin (observer) at time 2.47 s and is at 2.9 m at 4 s. It can be seen that the displacements are equal in equal intervals of time and hence it represents a uniform motion (or motion with uniform velocity). In the interval from 1 s to 2.47 s, the particle has moved from –2.8 m to 0 m and in the interval 2.47 s to 4.0 s, it has moved from 0 m to 2.9 m. Therefore, by calculating the ratio of displacement to the time interval, we find that the average velocity is constant. i.e., the motion of the particle is with uniform velocity.

Considering the motion from A to B, average velocity,

$$v_{AV} = (0.0) - (-2.8) / (2.47 - 1.0)$$

$$= (2.9 - 0.0) / (4.0 - 2.47) = 1.90 \text{ m s}^{-1}$$

The position–time graph (x–t graph) drawn above, representing the motion of the particle, is a straight line with a positive slope. The slope with the time axis, of the tangent to the x–t graph at any point on the x–t graph, gives the instantaneous velocity of the particle at the corresponding time. The above graph shows that the particle is moving either away from the observer or towards the observer at a constant rate, because the observer's position is at the origin (0,0) all the time. In the region AB of the graph, the particle is moving towards the observer as the distance between particle and observer is decreasing with increasing time. Beyond B, the particle is moving away from the observer as the distance between the observer and the particle is increasing with time.

2.16 Motion in One Dimension

The following points may be noted:

- The magnitude of the average velocity of the particle upto a certain time is the slope of the line (with the time axis) joining the origin to the point on the x - t graph corresponding to that time. In the case of the position – time graph shown above, this slope is the same for every point on the position–time graph. So the average velocity of the particle is the same at all times. Hence the particle is in uniform motion.
- The slope of the tangent at any point on the position–time graph with the time axis, gives the magnitude of the instantaneous velocity of the particle. This rule is applicable for all types of one-dimensional motion (not necessarily uniform motion).
- The distance from the origin to the projection on the position axis, from any point on the position–time graph, gives the instantaneous position (i.e., the position of the particle at the corresponding time) of the particle.

Now consider a particle travelling in the negative X direction with uniform velocity as shown in Fig. 2.9. The particle is now moving in the negative direction towards the observer. The average velocity v_{AV} is now in the negative direction of position.

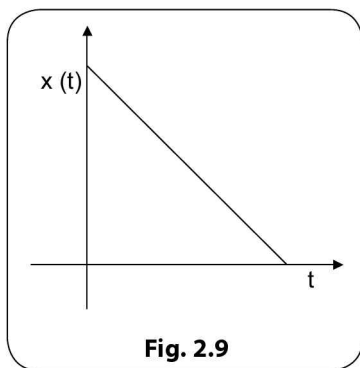


Fig. 2.9

(c) Position–time graph of a particle moving with uniform acceleration

With constant acceleration, the particle can either move away from the observer or move towards the observer. Correspondingly, the following position time graphs are possible.

The curvature of the graphs in Fig. 2.10 can be understood as follows; divide the horizontal time-axis into equal intervals of time. Measure the displacements in the successive intervals of time. Because of the acceleration, the successive displacements are found to have ever increasing

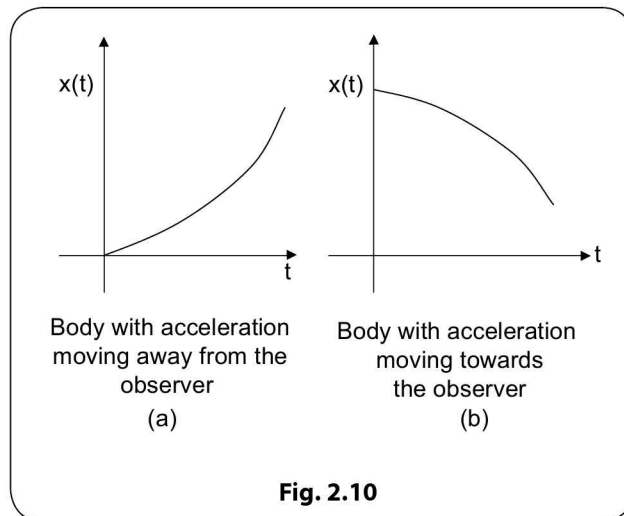


Fig. 2.10

values. For the particle moving away from the observer, the curve bends upwards and for the particle approaching the observer, the curve bends downwards. The slope with the time axis, of the tangents at the various points on the graph, is increasing with time, indicating that the instantaneous velocity of the particle is increasing with time. That is, the particle is accelerating.

(d) Position–time graph of a particle moving with uniform deceleration

As time advances, the displacement becomes smaller and smaller the curve bends as shown in Fig. 2.11. The slope with the time axis, of the tangents at the various points on the graph, is decreasing with time, indicating that the instantaneous velocity of the particle is decreasing with time. That is, the particle is decelerating.

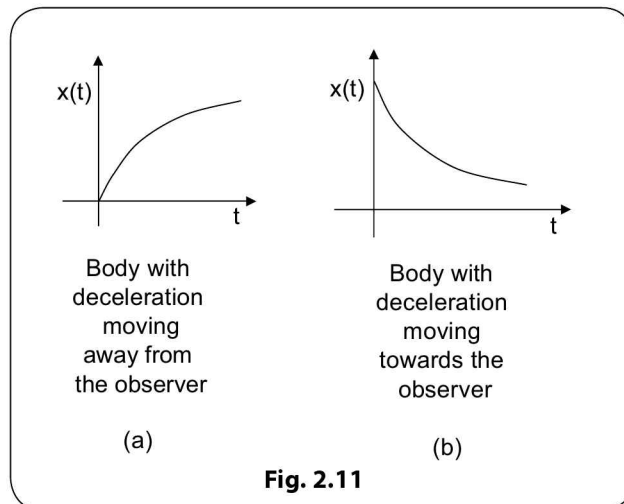


Fig. 2.11

Velocity–time graph (v–t graph)

The graph showing the relation between the instantaneous velocities of a particle at various instants of time is called the velocity–time graph. This graph is useful in determining the instantaneous velocities, instantaneous accelerations, average velocities, average accelerations and displacements of a particle and hence it is a very important graph.

If a particle is at rest, its velocity time graph is a straight line that coincides with the time axis.

(a) v–t graph of a particle moving with uniform velocity

The v–t graph shown in Fig. 2.12, represents the uniform motion of a particle. Since the velocity remains a constant and is independent of time during uniform motion, the velocity–time graph of uniform motion will be a straight line parallel to the time axis but not coinciding with the time axis.

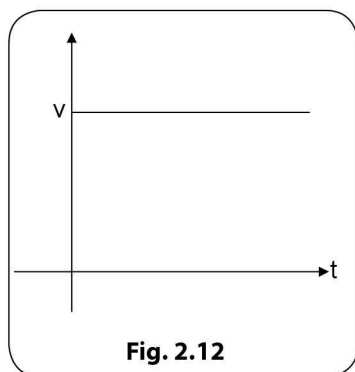


Fig. 2.12

The displacement of a particle in uniform motion can be found out from the velocity–time graph as shown in Fig. 2.13

The displacement of the particle in uniform motion during the time interval from t_1 to t_2 can be shown to be equal to the area under the velocity–time graph (the hatched area). The displacement during that time interval is equal to uniform velocity times the time interval.

Hence displacement $x_2 - x_1 = v_{AV} (t_2 - t_1)$.

The hatched area of the rectangle has sides v_{AV} along the vertical axis and $(t_2 - t_1)$ along the horizontal axis and its area is equal to $v_{AV} (t_2 - t_1)$ which is the same as the displacement during that time interval.

We know that if a body moves in the same direction, with a uniform velocity v_1 for t_1 second, v_2 for t_2 second etc

$$\text{Average velocity, } v_{av} = \frac{\text{Total displacement}}{\text{Total time}}$$

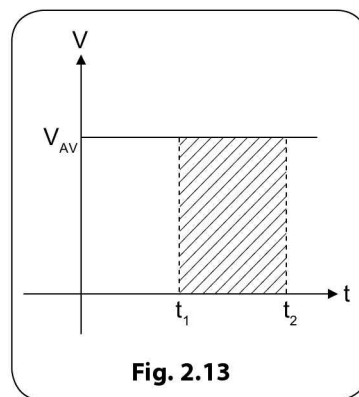


Fig. 2.13

$$v_{av} = \frac{v_1 t_1 + v_2 t_2 + \dots + v_n t_n}{(t_1 + t_2 + \dots + t_n)}$$

As before, the total displacement can be found out as the area under the velocity–time graph as shown in Fig. 2.14.

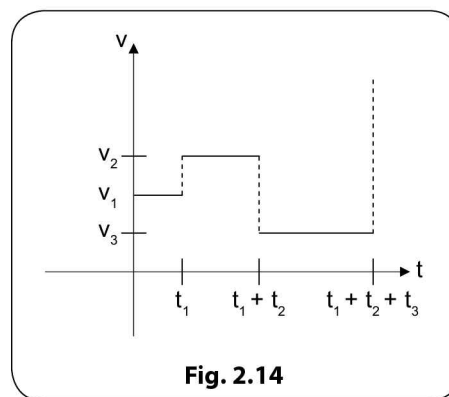


Fig. 2.14

(b) v–t graph of a particle moving with uniform acceleration

Suppose the velocity of a particle at the instant $t = 0$ is ' u ' and it becomes ' v ' in a time interval ' t ', under a uniform acceleration ' a '. Then, recalling kinematic equation (i)

$$v = u + at$$

The above equation with v and t as variables represents a straight line. Therefore, for uniformly accelerated motion, the velocity–time graph will be a straight line inclined to the time axis. The slope of the velocity–time graph with the time axis gives the magnitude of the acceleration of the particle as shown in Fig. 2.15. Again, the displacement can be calculated by measuring the area under the velocity–time graph. All areas above the time axis are treated as positive

2.18 Motion in One Dimension

and the areas below the time axis are treated as negative. The algebraic sum of these areas gives the magnitude of displacement.

Note:

The sum of the areas, without considering the signs of the areas, gives the distance traveled by the particle.

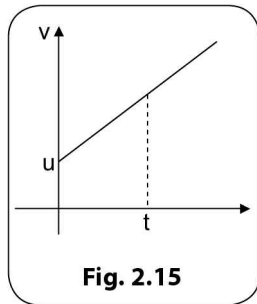


Fig. 2.15

(c) v - t graph of a particle moving with uniform deceleration

Suppose the velocity of a particle at the instant $t = 0$ is ' u ' and it becomes ' v ' in a time interval ' t ', under a uniform deceleration ' a '. Then, we have $v = u - at$.

The above equation with v and t as variables represents a straight line with negative slope. Therefore, for uniformly decelerated motion, the velocity-time graph will be a straight line inclined to the time axis with negative slope as shown in Fig. 2.16.

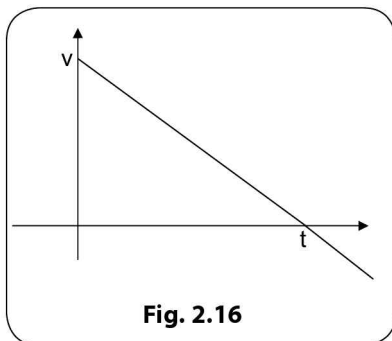


Fig. 2.16

Acceleration-time graph (a - t graph)

The graph showing the relation between the instantaneous accelerations of a particle at various instants of time is called the acceleration-time graph. This graph is useful in determining the instantaneous accelerations, average acceleration and change in velocity in a given time interval of a particle. The acceleration-time graph of a

particle in uniform motion (zero acceleration) is a straight line parallel to the time axis and coinciding with the time axis.

(a) a - t graph of a particle in uniformly accelerated motion

In the a - t graph, a horizontal line drawn parallel to the time axis and above it (Fig. 2.17) represents uniform acceleration of a particle. The increase in velocity of the particle during an interval of time can be found out from the graph as the area under the a - t graph.

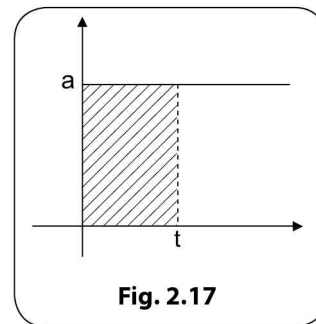


Fig. 2.17

Suppose the velocity of a particle at the instant $t = 0$ is ' u ' and it becomes ' v ' in a time interval ' t ', under uniform acceleration ' a ', then the change in its velocity during that time interval ' t ' is equal to $v - u = at$ (because $v = u + at$).

In Fig. 2.17, this is equal to the hatched area shown.

(b) a - t graph of a particle in uniformly decelerated motion

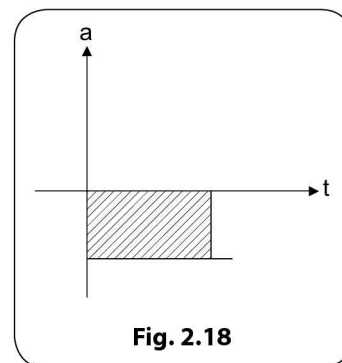


Fig. 2.18

In the a - t graph, a horizontal line drawn parallel to the time axis and below it (Fig. 2.18) represents uniform deceleration (or retardation) of a particle. The area under the a - t graph gives the reduction in the velocity of the particle during that time interval due to deceleration.

SUMMARY

$$v = u \pm at$$

$$S = \left(\frac{v + u}{2} \right) t$$

$$S = ut \pm \frac{1}{2} at^2$$

$$v^2 - u^2 = \pm 2aS$$

$$S_n = u \pm a \left(n - \frac{1}{2} \right)$$

$$\frac{n}{v_{AV}} = \frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n}$$

$$v = \frac{2v_1v_2}{v_1 + v_2}$$

$$v_{AV} = \frac{v_1 + v_2 + \dots + v_n}{n}$$

$$v_{AV} = \frac{v_1 + v_2}{2}$$

$$v_{AV} = \frac{(S_1 + S_2)v_1v_2}{(S_1v_2 + S_2v_1)}$$

$$v = u - gt$$

$$S = ut - \frac{1}{2}gt^2$$

$$v^2 - u^2 = -2gh$$

$$H_{\max} = \frac{u^2}{2g} = \frac{1}{2}gt_1^2 = \frac{1}{8}gT^2$$

$$t_1 = \frac{u}{g}, T = \frac{2u}{g}$$

$$t_1 = \sqrt{\frac{2H_{\max}}{g}}, T = 2\sqrt{\frac{2H_{\max}}{g}}$$

$$u = gt_1 = \sqrt{2gH_{\max}} = \frac{gT}{2}$$

$$S_n = u \pm g \left(n - \frac{1}{2} \right)$$

Kinematic equation of motion (applicable only for uniformly accelerated or uniformly decelerated motions).

$u \rightarrow$ initial velocity

$v \rightarrow$ velocity after time interval ' t '

$t \rightarrow$ time interval

$S \rightarrow$ displacement in time interval ' t '

$S_n \rightarrow$ distance travelled by the particle in the n th second

$+a \rightarrow$ for uniform acceleration

$-a \rightarrow$ for uniform retardation or deceleration

$v \rightarrow$ average speed of whole journey, when ' n ' equal distances are travelled with different speeds $v_1, v_2, \dots, v_n$. Average speed in such case is the harmonic mean of individual speeds.

$v_{AV} \rightarrow$ average speed of whole journey, when 2 equal distances are covered with different speeds v_1 and v_2 .

$v_{AV} \rightarrow$ average speed of whole journey, when distances are covered in n equal intervals of time with different speeds $v_1, v_2, \dots, v_n$. Average speed in such case is the arithmetic mean of the individual speeds.

$v_{AV} \rightarrow$ average speed of whole journey, when distances are covered in two equal intervals of time with speeds ' v_1 ' and ' v_2 ' in each time interval.

$v_{AV} \rightarrow$ average speed for the whole journey, where ' v_1 ' is the speed for (distance S_1) and ' v_2 ' is the speed for (distance S_2)

For vertical upward motion under gravity ($g = \text{constant}$),

$u =$ initial velocity (upwards)

$g =$ acceleration due to gravity

$h =$ height above ground at time interval ' t '

$t_1 =$ time taken to reach maximum height

$H_{\max} =$ maximum height above ground

$T =$ total time of flight

$v =$ velocity after time interval ' t ' (if ' v ' is positive, particle is still travelling up, if v is zero particle is at maximum height and if ' v ' is negative, particle is travelling downwards).

Physical quantities in the upward direction are taken as positive. So all physical quantities in downward direction will be negative. Time is always positive.

2.20 Motion in One Dimension

$$v = u + gt$$

$$h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh$$

For free fall ($u = 0$)

$$h_{\max} = \frac{1}{2}gt_1^2 = \frac{v_1^2}{2g}$$

$$v_1 = \sqrt{2gh_{\max}} = gt_1$$

$$t_1 = \sqrt{\frac{2h_{\max}}{g}} = \frac{v_1}{g}$$

$$(a) \quad v_1 = a, v_2 = 2a, v_3 = 3a, \dots$$

$$v_1 : v_2 : v_3 : \dots : v_n \\ :: 1 : 2 : 3 : \dots : n$$

$$(b) \quad S(1) = \frac{1}{2}a, S(2)$$

$$= 2a, \dots, S(n) = \frac{1}{2}an^2$$

$$S(1) : S(2) : \dots : S(n)$$

$$:: 1^2 : 2^2 : \dots : n^2 = 1 : 4 : \dots : n^2$$

$$(c) \quad S_1 = \frac{a}{2},$$

$$S_2 = \frac{3a}{2}, S_3 = \frac{5a}{2}, \dots$$

$$S_n = (2n-1) \frac{a}{2}$$

$$S_1 : S_2 : S_n :: 1 : 3 : \dots : (2n-1)$$

$$(d) \quad S_{1g} = \frac{g}{2}, S_{2g} = \frac{3g}{2},$$

$$S_{3g} = \frac{5g}{2}, \dots$$

$$t_2 = (\sqrt{2} - 1)t_1 = 0.414t_1$$

$$t = t_1 + t_2 = 1.414t_1 = \sqrt{2}t_1$$

$v_{AB} = v_A - v_B$ if A and B travel in the same direction.

$v_{AB} = v_A + v_B$, if A and B travel in the opposite directions.

$v_{BA} = v_B - v_A$, if A and B travel in the same direction.

For vertical free fall or thrown downwards,

u = initial downward velocity

g = acceleration due to gravity

v = velocity after 't' second

h = depth of fall (from the point of release)

$H_{\max}$ = maximum depth of free-fall from the point of release ($u = 0$)

t_1 = time taken to reach ground in free fall

v_1 = velocity with which particle hits ground in free fall.

For particle starting from rest and moving with uniform acceleration (Example, free fall near earth),

a = uniform acceleration v_1, v_2, v_3, v_n velocity after 1, 2, 3 ... 'n' seconds from start

$S(1), S(2), \dots, S(n)$ are the total displacements at the end of 1, 2, ... n seconds from start,

$S_1, S_2, \dots, S_n$ are the distance travelled in the 1st, 2nd, ... nth second.

$S_{1g}, S_{2g}, \dots, S_{ng}$ are the distances travelled in the 1st, 2nd ... nth second in free fall near surface of Earth.

For particle falling freely from rest, ' t_1 ' is the time taken to fall through first half of the height, ' t_2 ' is the time taken to fall through the second half of the height and ' t ' is the time taken to fall through the total height.

v_{AB} = velocity of A relative to B

v_{BA} = velocity of B relative to A

v_A = velocity of A

v_B = velocity of B

CONCEPT CONNECTORS

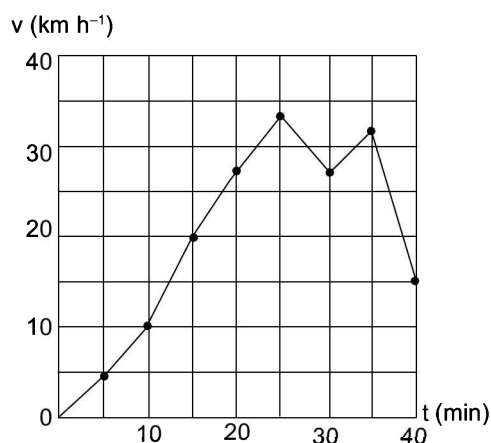
Connector 1: The speedometer of a motor car had the following readings (km h^{-1}) at intervals of 5 minute

Time (min)	0	5	10	15	20	25	30	35	40
Reading (km h^{-1})	0	4	10	20	26	34	26	32	16

- Plot the speed vs time graph
- Calculate the average speed from 0 to 40 min.
- Calculate the instantaneous speed at 18 min.

Solution:

(i)



$$(ii) \quad v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{\text{Area enclosed by speed vs time graph}}{\text{Time}}$$

$$= \frac{1}{8} \sum_{i=0}^7 \left(\frac{v_i + v_{i+1}}{2} \right) = \frac{160}{8} = 20 \text{ km h}^{-1}$$

(iii) At 18 min,

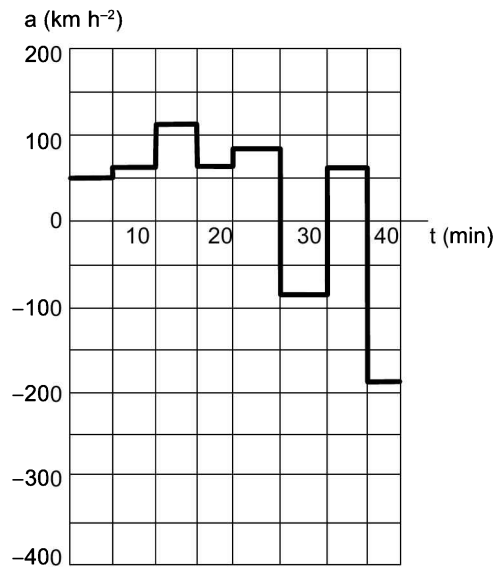
$$\begin{aligned} v_{inst} &= v_{t=15} + \left(\frac{v_{t=20} - v_{t=15}}{5} \right) \times 3 \\ &= 20 + \left(\frac{26 - 20}{5} \right) \times 3 = 23.6 \text{ km h}^{-1} \end{aligned}$$

Connector 2: In the above problem, considering that the particle was travelling in the same direction throughout the journey.

- plot the average acceleration vs time graph.
- calculate the average acceleration for the whole journey.
- calculate the average acceleration for time interval 15 minute to 20 minute

2.22 Motion in One Dimension

Solution: (i) Average acceleration (km h^{-2}) = $\frac{\text{average velocity (km h}^{-1}\text{)}}{\text{time interval (min.)}}$



(ii) From the graph

$$a_{av} = \frac{\text{change in velocity}}{\text{Time}} = \frac{(16 \text{ km h}^{-1} - 0)}{40 \text{ min}} = \frac{16 \times 60}{40} \text{ km h}^{-2} = 24 \text{ km h}^{-2}$$

(iii) a_{inst} = acceleration during the time interval 15 min to 20 min

$$= \frac{(v_{20} - v_{15})}{5 \text{ min}} = \frac{60 \times (26 - 20)}{5} \text{ km h}^{-2} = 72 \text{ km h}^{-2}$$

Connector 3: The total distance travelled (in m) by a particle is given by $S = 6t^2$ where t is the time (in s). Find the

- average speed of the particle in the time interval from 0 to 5 s and
- instantaneous speed at $t = 0.5$ s.

Solution: (i) The distance travelled during $t = 0$ to $t = 5$ s, $S = 6 \times 5^2 = 150$ m.

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total time}} = \frac{150}{5} = 30 \text{ ms}^{-1}$$

(ii) The instantaneous speed is $v_t = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{6t_2^2 - 6t_1^2}{t_2 - t_1}$

$$= \frac{6(t_1 + \Delta t)^2 - 6t_1^2}{(t_1 + \Delta t) - t_1} = \frac{12t_1 \Delta t}{\Delta t} = 12t_1$$

(neglecting $(\Delta t)^2$ in the numerator which is extremely small)

Aliter: Instantaneous speed $= \frac{ds}{dt} = \frac{d}{dt} (6t^2) = 12t$

At $t = 0.58$, $v = 12 \times 0.5 = 6 \text{ m s}^{-1}$

The instantaneous speed at $t = 0.5$ s is

$v = 12 \times 0.5 = 6 \text{ m s}^{-1}$.

Connector 4: A particle travelling in a straight path in the same direction covers the first half of the distance with a velocity v_1 and the second half of the distance with two different velocities v_2 and v_3 in equal intervals of time. Find the average velocity of the particle for the entire journey.

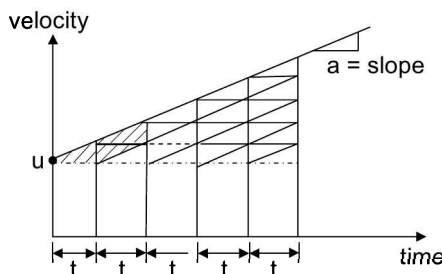
Solution: Let the total displacement of the body be $2S$ and let t_1 be the time taken to cover the first half. During the second half let the time of travel be $2t$, so that it travels with v_2 and v_3 velocities in time t each.

$$\text{Then the average velocity} = \frac{2S}{t_1 + 2t}$$

$$\text{Here } t_1 = \frac{S}{v_1} \text{ and } (v_2 + v_3)t = S, \text{ so that } t = \frac{S}{(v_2 + v_3)}$$

$$\begin{aligned} \therefore v_{av} &= \frac{2S}{\frac{S}{v_1} + 2\left(\frac{S}{v_2 + v_3}\right)} \\ &= \frac{2}{\frac{1}{v_1} + \frac{2}{v_2 + v_3}} = \frac{2v_1(v_2 + v_3)}{v_2 + v_3 + 2v_1} \end{aligned}$$

Connector 5: Prove that for a body under constant acceleration, the distances covered in successive equal time intervals are in arithmetic progression.



Solution: Let a = acceleration,
 t = interval of time,
 u = initial velocity.

Graphical solution:

We can see that the additional distance covered in the second interval is equal to twice the area of triangle hatched in the first interval $= 2 \times \frac{1}{2} at^2 = at^2$

The additional distance covered in third interval is equal to the additional distance covered in second interval $= at^2$ and so on

$\therefore$ The distances covered in successive intervals form an A.P with common difference of $2 \cdot \frac{1}{2} at^2 = at^2$
 If $u = 0$ (as a special case such as free falling body)

The distances covered form an AP (first term say $S = \frac{1}{2} at^2$), $S, 3S, 5S, \dots$, so on

Analytical proof:

$$\text{Total distance covered in time } nt = unt + \frac{1}{2}an^2t^2 \quad \text{--- (1)}$$

$$\text{Total distance covered in time } (n-1)t = u(n-1)t + \frac{1}{2}a(n-1)^2t^2 \quad \text{--- (2)}$$

$$\text{Distance covered during the } n\text{th interval} = S_n = (1) - (2)$$

$$S_n = ut - \frac{1}{2}at^2 + ant^2 \quad \text{--- (3)}$$

2.24 Motion in One Dimension

Distance covered in the $(n - 1)$ th interval

$$S_{n-1} = ut - \frac{1}{2}at^2 + ant^2 - at^2 \quad \text{--- (4)}$$

$$S_n - S_{n-1} = at^2 \text{ (independent of } n\text{)}$$

$\therefore S_1, S_2, S_3 \dots$ etc form an AP with common difference at^2 .

Notes:

1. If we assume $u = 0$, the AP will be $\frac{1}{2}at^2, \frac{1}{2}at^2 + at^2, \frac{1}{2}at^2 + 2at^2$]

$$\Rightarrow \frac{1}{2}at^2, \frac{3}{2}at^2, \frac{5}{2}at^2 \dots\dots$$

$$\Rightarrow S, 3S, 5S \text{ (where } S = \frac{1}{2}at^2\text{)}$$

2. The total distance covered in n intervals will be

$$\frac{n}{2} \left[\dots + \left(- \right) \right] = \quad \text{--- (5)}$$

Connector 6: A vehicle can accelerate or decelerate at a maximum value of 1 m s^{-2} and can attain a maximum speed of 20 m s^{-1} . If it starts from rest, what is the shortest time it can travel 1000 m , in a straight path, if it is to come to rest at the end of 1000 m ?

Solution: $t_1 = \frac{20 - 0}{1} = 20 \text{ s}, S_1 = \left(\frac{0 + 20}{2} \right) \cdot 20 = 200 \text{ m}$

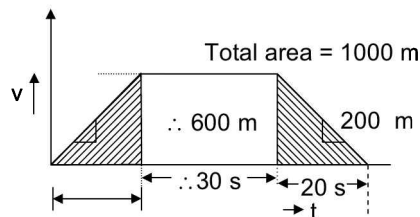
$$t_2 = \frac{0 - 20}{-1} = 20 \text{ s}, S_2 = \left(\frac{20 + 0}{2} \right) \cdot 20 = 200 \text{ m}$$

$\therefore$ Distance travelled at uniform velocity,
 $S_3 = [1000 - (200 + 200)] = 600 \text{ m at } 20 \text{ m s}^{-1}$

$$\Rightarrow t_3 = 30 \text{ s}$$

$$\Rightarrow t_1 + t_2 + t_3 = 70 \text{ s.}$$

Graphical:



Connector 7: A particle moving on a straight line, starts from rest, moves with an acceleration a , reaches a velocity v , moves with a constant velocity v for some time, then with a retardation b and comes to rest. If total distance covered is ℓ , then show that the total time taken is $\frac{\ell}{v} + \frac{v}{2} \left(\frac{1}{a} + \frac{1}{b} \right)$

Solution: There are three parts of journey.

Part 1: with acceleration a

$$t_1 = \frac{v}{a}; \ell_1 = \frac{1}{2} \cdot a \cdot \frac{v^2}{a^2} = \frac{1}{2} \cdot \frac{v^2}{a}$$

Part 2: with uniform velocity v

$$t_2 = \frac{\ell_2}{v}$$

Part 3: with retardation b

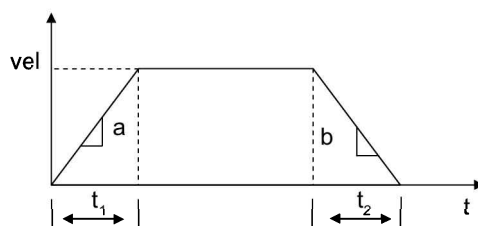
$$t_3 = \frac{v}{b}; \quad \ell_3 = \frac{1}{2} \cdot b \cdot \frac{v^2}{b^2} = \frac{1}{2} \frac{v^2}{b}$$

$$\ell_2 = \ell - (\ell_1 + \ell_3) = \ell - \frac{1}{2} v^2 \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$t = t_1 + t_2 + t_3 = \frac{v}{a} + \frac{\ell}{v} - \frac{v}{2} \left(\frac{1}{a} + \frac{1}{b} \right) + \frac{v}{b} = \frac{\ell}{v} + \frac{v}{2} \left(\frac{1}{a} + \frac{1}{b} \right)$$

Aliter:

(Graphical solution)



If another particle covers the same distance by travelling with uniform velocity v , then its journey time will be ℓ/v and it will be less than the first particle by $\left(\frac{t_1}{2} + \frac{t_2}{2} \right)$ (as shown).

$$\therefore \text{Journey time of first particle is } \frac{\ell}{v} + \frac{t_1}{2} + \frac{t_2}{2} = \frac{\ell}{v} + \frac{v}{2} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$(\because t_1 = \frac{v}{a} \text{ and } t_2 = \frac{v}{b})$$

Connector 8: A particle starts at point A, moves on a straight line with acceleration 'a' for some time, and then moves with retardation 'd' till it stops. If distance travelled is S and travel time is t , show that $\frac{1}{a} + \frac{1}{d} = \frac{t^2}{2S}$

Solution: If v is the maximum speed reached and t' is the time taken to do so then,

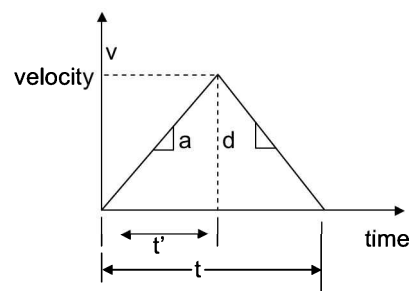
$$a = \frac{v}{t'}, \quad d = \frac{v}{t - t'}$$

$$\frac{1}{a} + \frac{1}{d} = \frac{t}{v} \quad (1)$$

$$S = \text{area under } v - t \text{ graph} = \frac{1}{2} vt' + \frac{1}{2} v(t - t')$$

$$S = \frac{1}{2} vt \Rightarrow v = \frac{2S}{t} \text{ Substituting in (1)}$$

$$\frac{1}{a} + \frac{1}{d} = \frac{t^2}{2S}$$



2.26 Motion in One Dimension

Connector 9: If a particle moving along a straight line with constant acceleration covers three successive equal distances in time intervals t_1, t_2, t_3 respectively, prove that $\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$

Solution: Let the equal distance covered in each interval be S . Then distance covered from t_A to $t_D = 3S \Rightarrow 3S =$

$$\frac{v_A + v_D}{2} \times \text{total time}$$

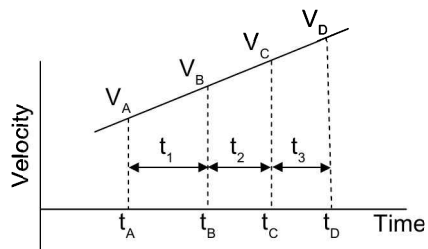
$$3S = \left(\frac{v_A}{2} + \frac{v_D}{2} \right) (t_1 + t_2 + t_3)$$

$$\Rightarrow \frac{3S}{t_1 + t_2 + t_3} = \frac{v_A}{2} + \frac{v_D}{2}$$

$$= \frac{v_A}{2} + \frac{v_B}{2} - \frac{v_B}{2} - \frac{v_C}{2} + \frac{v_C}{2} + \frac{v_D}{2}$$

$$= \left(\frac{v_A}{2} + \frac{v_B}{2} \right) - \left(\frac{v_B}{2} + \frac{v_C}{2} \right) + \left(\frac{v_C}{2} + \frac{v_D}{2} \right) = \frac{S}{t_1} - \frac{S}{t_2} + \frac{S}{t_3}$$

$$\Rightarrow \frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$



Connector 10: A balloon going upwards with a velocity of 12 m s^{-1} is at a height of 65 m from the ground when a packet is dropped from it. How much time will the packet take to reach the ground? ($g = 10 \text{ m s}^{-2}$)

Solution: Let h be the height of the balloon from the ground and t be the time taken by the packet to reach the ground. Taking upward direction positive

$$s = ut + \frac{1}{2}at^2$$

$$u = 12 \text{ m s}^{-1}; S = -65 \text{ m}; a = -10 \text{ m s}^{-2}$$

$$\Rightarrow -65 = 12t - \frac{1}{2} \times 10t^2$$

$$5t^2 - 12t - 65 = 0 \Rightarrow t = 5 \text{ s}$$

Thus, the packet will reach the ground in 5 second.

Connector 11: A cricket ball thrown vertically upwards from the top of a tower with a speed 20 m s^{-1} reaches the ground in 6 s . Find the height of the tower. ($g = 10 \text{ m s}^{-2}$)

Solution: Let the height of the tower be h . Since the displacement is in the downward direction, taking upward direction as positive, with $u = 20 \text{ m s}^{-1}$, $t = 6 \text{ s}$ and $a = -10 \text{ m s}^{-2}$, we have,

$$S = ut + \frac{1}{2}at^2 \Rightarrow -h = 20 \times 6 - \frac{1}{2} \times 10 \times 6^2 \Rightarrow h = 60 \text{ m}$$

Connector 12: A parachutist jumps out from an aircraft flying horizontally and opens the parachute after dropping through a distance of 45 m , decelerating at 2 m s^{-2} . If he reaches the ground with a speed of 2 m s^{-1} , how long was he in the air? At what height did he jump out of the aircraft? ($g = 10 \text{ m s}^{-2}$)

Solution: Let the time taken for the free fall be t_1 , and taking downward direction as positive, we have

$$S = ut + \frac{1}{2}at^2$$

$$u = 0; S = 45 \text{ m}; a = 10 \text{ m s}^{-2}$$

$$\Rightarrow 45 = 0 + \frac{1}{2} \times 10 \times t_1^2$$

$$t_1 = \sqrt{\frac{45 \times 2}{10}} = 3 \text{ s}$$

Let v_s be the velocity of the parachutist just before the parachute opens.

$$v_s = u + at = 0 + 10 \times 3 = 30 \text{ m s}^{-1}$$

Let the time taken to reach the ground after the parachute opens be t_2

$$v = u + at$$

$$0 = 30 - 2 \times t_2 \Rightarrow t_2 = \frac{30}{2} = 15 \text{ s}$$

$$2 = 30 - 2 \times t_2 \Rightarrow t_2 = \frac{28}{2} = 14 \text{ s}$$

The parachutist was in the air for $(t_1 + t_2) = 3 + 14 = 17 \text{ s}$

Let the height through which the parachutist falls after opening the parachute be h .

$$v^2 = u^2 + 2aS$$

$$0 = 30^2 - 2 \times h \Rightarrow h = \frac{30^2}{2} = 450 \text{ m}$$

$$2^2 - 30^2 = 2 \times (-2) \times h$$

$$h = 224 \text{ m}$$

The height at which the parachutist jumps out from the aircraft = $224 + 45 = 269 \text{ m}$.

Connector 13: A cricket bowler tosses a ball vertically upwards with an initial speed of 12 m s^{-1} .

(a) How long does the ball take to reach its maximum height?

(b) How long does the ball take to reach 7 m above its release point? ($g = 10 \text{ m s}^{-2}$)

Solution:

(a) Take the upward direction as positive. Once the ball leaves the bowler, till it returns, its acceleration is that of free fall acceleration, $a = -10 \text{ m s}^{-2}$

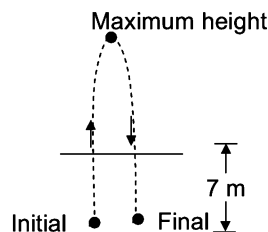
Again, velocity at maximum height is zero

$$\text{Now, } v = u + at$$

$$v = 0, u = 12 \text{ m s}^{-1}, a = -10 \text{ m s}^{-2}$$

$$\therefore 0 = 12 - 10t \Rightarrow -12 = -10t$$

$$\Rightarrow t = \frac{12}{10} = 1.2 \text{ s}$$



$$(b) \text{ Now, } S = ut + \frac{1}{2}at^2$$

$$7 = 12t + \frac{1}{2}(-10) \times t^2 \Rightarrow 5t^2 - 12t + 7 = 0$$

$$(5t - 7)(t - 1) = 0$$

$$t = 1.4 \text{ s or } 1 \text{ s}$$

We take the smaller value, $t = 1 \text{ s}$

2.28 Motion in One Dimension

Connector 14: An object is dropped from top of a building. During the last second of its fall it covers $\frac{16}{25}$ th of the height of building. Find the height of the building ($g = 10 \text{ m s}^{-2}$).

Solution: $\downarrow + \quad h = \frac{1}{2}gt^2 \quad \text{--- (1)}$

$$h - \frac{16}{25}h = \frac{1}{2}g(t-1)^2$$

$$\Rightarrow \frac{9}{25}h = \frac{1}{2}g(t-1)^2 \quad \text{--- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{5}{3} = \frac{t}{t-1} \Rightarrow t = 2.5 \text{ s}$$

$$\therefore (1) \Rightarrow h = 31.25 \text{ m.}$$

Connector 15: A body thrown vertically up from top of a building takes time t_1 to reach ground; If thrown vertically down with same velocity it takes time t_2 to reach ground; if dropped it takes time t_3 to reach ground. Show that $t_3 = \sqrt{t_1 t_2}$.

Solution: Treating upward direction as positive:
Let u be the initial velocity of throw,

$$\text{Then } -h = ut_1 - \frac{1}{2}gt_1^2 \quad \text{--- (1)}$$

$$-h = -ut_2 - \frac{1}{2}gt_2^2 \quad \text{--- (2)}$$

$$-h = -\frac{1}{2}gt_3^2 \quad \text{--- (3)}$$

(1) $\times t_2 +$ (2) $\times t_1$ gives

$$h(t_1 + t_2) = \frac{1}{2}gt_1 t_2 (t_1 + t_2) \Rightarrow h = \frac{1}{2}gt_1 t_2$$

Substituting (3)

$$\frac{1}{2}gt_3^2 = \frac{1}{2}gt_1 t_2 \Rightarrow t_3 = \sqrt{t_1 t_2}$$

Connector 16: A jet plane travelling at a speed of 500 kmph ejects its products of combustion at a speed of 1500 kmph relative to the plane. What is the speed of the exhaust of the jet with respect to an observer on the ground?

Solution: Speed of the plane $= v_p = 500 \text{ kmph}$. Relative speed of the ejected products with respect to the plane $v_r = 1500 \text{ kmph}$. Let v_p be the speed of the ejected products with respect to the observer on the ground.

$$v_r = v_p - (-v_p)$$

$$1500 = v_p + 500$$

$$v_p = 1500 - 500 = 1000 \text{ kmph}$$

Connector 17: Two balls A and B are thrown simultaneously. B is thrown vertically upwards with a speed of 10 m s^{-1} from the ground, while A is thrown vertically downwards from a height of 20 m with the same speed along the same vertical line. At what height do the balls collide? ($g = 10 \text{ m s}^{-2}$)

Solution: Since the relative acceleration is zero, their relative motion is uniform.

$$\text{The time for collision} = \frac{\text{Relative Distance}}{\text{Relative Velocity}} = \frac{20}{20} = 1 \text{ s.}$$

$$\text{Time of flight of B} = \frac{2u}{g} = \frac{2 \times 10}{10} = 2 \text{ s} > 1 \text{ s}$$

Let the height above the ground where they collide be h . Then, for the body B,

$$S = ut + \frac{1}{2}at^2 \Rightarrow h = 10 \times 1 + \frac{1}{2} \times (-10) \times 1^2 = 5 \text{ m, above the ground}$$

Aliter:

Time of flight of B = 2 × time until collision

∴ collision takes place when B is at its maximum height

$$\therefore h = \frac{u^2}{2g} = \frac{(10)^2}{2 \times 10} = 5 \text{ m, above the ground}$$

Connector 18: A body at rest is dropped from a height of 250 m. Simultaneously another body is dropped from rest from a point 180 m above the ground. What is the difference in their heights after they have fallen for (i) 2 s and (ii) 3 s? How does the difference vary with time? ($g = 10 \text{ m s}^{-2}$)

Solution: Consider relative motion

$$\text{Relative acceleration} = g - g = 0$$

$$\text{Initial relative velocity} = 0$$

$$\Rightarrow \text{Relative distance} = 250 - 180 = 70 \text{ m}$$

$$\text{Time to fall} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 180}{10}} = 6 \text{ s}$$

The difference is constant at 70 m for $t < 6 \text{ s}$ and decreases thereafter.

Connector 19: Two cars on straight parallel roads are ℓ distance apart and approach with speeds v_1 and v_2 respectively at an instant. If their accelerations are a_1 and a_2 , along the directions of their motion respectively.

(a) at what time will they meet?

(b) plot the relative velocity vs time graph.

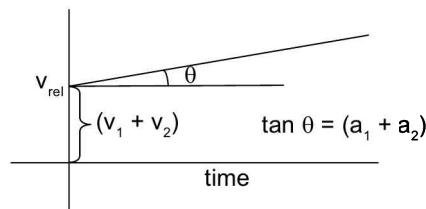
Solution:

$$(a) v_{\text{rel}} = v_1 + v_2$$

$$a_{\text{rel}} = a_1 + a_2$$

$$\ell = (v_1 + v_2)t + \frac{1}{2}(a_1 + a_2)t^2$$

$$t = \frac{-(v_1 + v_2) + \sqrt{(v_1 + v_2)^2 + 2(a_1 + a_2)\ell}}{a_1 + a_2}$$



$$(b) a_{\text{rel}} = a_1 + a_2 = \text{constant.} \Rightarrow v_{\text{rel}} = (v_1 + v_2) + (a_1 + a_2)t$$

Connector 20: An elevator ascends with an acceleration of 0.2 m s^{-2} . At the instant its velocity is 1 m s^{-1} , a loose bolt in the ceiling of the lift drops. The ceiling is 5 m from the floor of the elevator. Determine the time until the bolt hits the floor of the lift and the distance it has fallen. (use $g = 9.8 \text{ m s}^{-2}$)

2.30 Motion in One Dimension

Solution: Treat $\downarrow$ + and treat $S = 0$ at bottom floor of lift at $t = 0$. Let bolt hit floor at $t = t_1$.

Then for bolt: $u = +1 \text{ m s}^{-1}$

$$a = -g = -9.8 \text{ m s}^{-2}$$

(note $a = -g$ and not $-g + 0.2$)

$$\therefore S_1 = S_0 + ut + \frac{1}{2}at^2$$

$$\Rightarrow S_1 = +5 + t_1 + \frac{1}{2}(-9.8)t_1^2 = 5 + t_1 - 4.9t_1^2$$

For floor: $u = +1 \text{ m s}^{-1}$

$$a = +0.2 \text{ m s}^{-2}$$

$$S_2 = t_1 + \frac{1}{2}(0.2)t_1^2 = t_1 + 0.1t_1^2.$$

When they meet $S_1 = S_2 \Rightarrow 5t_1^2 = 5 \Rightarrow t_1 = 1 \text{ s}$

$$\therefore \text{Distance travelled by bolt} = t_1 - 4.9t_1^2 = -3.9 \text{ m}$$

Aliter:

Let us do same problem using relative motion concept:

Bolt: b, floor: f

$$\uparrow + \text{So, } S_{b/f} = +5 \text{ m}$$

$$u_{b/f} = 0$$

$$a_{b/f} = -g - (+0.2) = -10 \text{ ms}^{-2}$$

$$S'_{b/f} = 0$$

$$\therefore 0 = (+5 + 0) \times t_1 + \frac{1}{2}(-10)t_1^2 \Rightarrow t_1 = 1 \text{ s}.$$

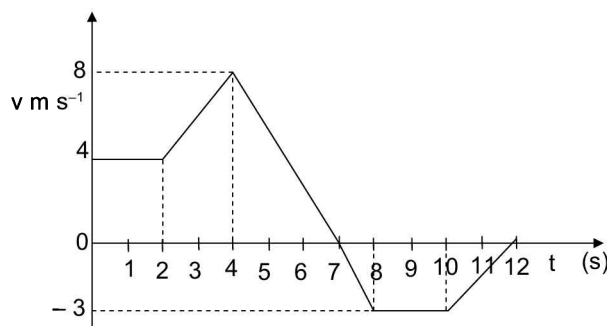
$$\therefore \text{Distance travelled by bolt} = ut + \frac{1}{2}gt^2 = -1 \times 1 + \frac{1}{2}(9.8) \cdot 1^2 = 3.9 \text{ m} \downarrow$$

TOPIC GRIP



Subjective Questions

- A particle travelling along a straight line travels the first 50 m with a uniform velocity of 5 m s^{-1} , travels at a uniform velocity of 8 m s^{-1} for the next 5 s and travels at a uniform velocity of 12 m s^{-1} for the next 10 s.
 - What is the average velocity of the journey?
 - If the last part of the journey (during last 10 s), was performed at a constant acceleration of 2 m s^{-2} from an initial velocity of 8 m s^{-1} , what is the average velocity of the journey?
- A stone falls from a balloon at a height of 100 m above ground.
 - How much time is required for the stone to reach the ground, if
 - the balloon is ascending uniformly at 5 m s^{-1} ?
 - the balloon is descending uniformly at 5 m s^{-1} ?
 - the balloon is stationary?
 - If t_1 , t_2 and t are the answers to (i), (ii) and (iii) respectively, then $t_1 t_2 = t^2$. Does it hold for any initial height of the balloon and any initial velocity of the balloon?
($g = 10 \text{ m s}^{-2}$)
- The velocity–time graph of a particle in 1 D motion is given below.
 - Draw the acceleration–time graph from $t = 0$ to $t = 12 \text{ s}$.
 - Find displacement in the interval $t = 3 \text{ s}$ to $t = 6 \text{ s}$.
 - Find displacement from $t = 0$ till $t = 12 \text{ s}$.
 - Find distance covered from $t = 0$ till $t = 12 \text{ s}$.



- A car accelerates from rest for a distance αs , after which it decelerates for a distance βs and comes to rest in total time t . Find the maximum velocity and the initial acceleration.
- A particle starts from rest and travels horizontally with a constant acceleration. At some point it hits a wall where its velocity and acceleration are reversed without any change in their magnitudes. If the particle returns to the initial point, find the ratio of the time taken for the forward and reverse journey.
- A train running on a straight track starts from station A, runs with constant acceleration of 0.5 m s^{-2} for 30 s, then maintains uniform speed for some time, applies brake (giving constant retardation) 150 m ahead of station B and stops at B. If the total running time was 170 s, calculate the distance between the stations.
- From top of a tall building, object A was dropped at $t = 0$ and object B was thrown down at $t = 2 \text{ s}$ with a velocity of 40 m s^{-1} . From a spot on the ground vertically down, object C was thrown up at $t = 1 \text{ s}$ with a velocity of 30 m s^{-1} . All three objects collide. What was the height of the building? ($g = 10 \text{ m s}^{-2}$)

2.32 Motion in One Dimension

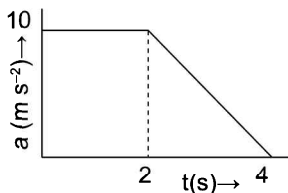
8. A platform starts from rest on the ground and moves up with a constant acceleration of 10 m s^{-2} . If two particles are released from the platform at time t_1 and t_2 respectively, find the ratio of the time T_1 and T_2 taken by the two particles to hit the ground ($g = 10 \text{ m s}^{-2}$).
9. Two particles are projected vertically upwards with speed 20 m s^{-1} and 100 m s^{-1} respectively from the top of a tower. If their times of flight are in the ratio 1:3, find the height of the tower ($g = 10 \text{ m s}^{-2}$).
10. A boat moves from P to Q (downstream) in 2.5 h and from Q to P (upstream) in 5.0 h respectively, when the engine is producing maximum speed on the boat in each case. If due to increased water flow, the speed of flow of the river gets doubled, how long will it take the boat (moving with maximum possible speed) to move from
 - (i) P to Q (downstream)?
 - (ii) Q to P (upstream)?



Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

11. A ball thrown vertically upwards from level ground is observed twice, at a height H above the ground, within a time interval t . The initial velocity of the ball was
 - (a) $\sqrt{8gH + g^2 t^2}$
 - (b) $\sqrt{8gH + \left(\frac{gt}{2}\right)^2}$
 - (c) $\frac{1}{2}\sqrt{8gH + g^2 t^2}$
 - (d) $\sqrt{8gH + 4g^2 t^2}$
12. Acceleration–time graph of a particle starting from rest and moving along a straight line is as shown. The velocity of the particle after 4 s is (in m/s).



- (a) 10
 - (b) 20
 - (c) 15
 - (d) 30
13. A particle, having an initial velocity of 5 m s^{-1} moves along a straight line with a constant acceleration of 2 m s^{-2} for 10 s. Its displacement in the last second is
 - (a) 25 m
 - (b) 30 m
 - (c) 24 m
 - (d) 27 m
14. A body is thrown up from a tower. If the ratio of distance to displacement is n , the ratio of time for upward and downward motion is
 - (a) $\frac{1}{n}$
 - (b) $\frac{1}{\sqrt{n}}$
 - (c) $\sqrt{\frac{n-1}{n+1}}$
 - (d) $\sqrt{1 - \frac{1}{n}}$
15. Two passenger trains pass (at the same instant) through points A and B, which are 200 km apart. They run on parallel tracks towards each other at speeds of 60 km/h and 45 km/h respectively. They will meet in
 - (a) $2\frac{1}{2}$ hour
 - (b) $1\frac{3}{4}$ hour
 - (c) $1\frac{19}{21}$ hour
 - (d) $1\frac{16}{21}$ hour



Assertion–Reason Type Questions

Directions: Each question contains Statement–1 and Statement–2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1
- (b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1
- (c) Statement–1 is True, Statement–2 is False
- (d) Statement–1 is False, Statement–2 is True

16. Statement 1

For a particle in 1D uniformly accelerated motion, if displacement from origin is zero at time $t = t_1$ and at $t = t_2$, ($t_1, t_2 \neq 0$) then $\vec{r}_0 = \vec{a}$ where $\vec{r}_0$ and $\vec{a}$ are initial position vector and acceleration vector respectively.

and

Statement 2

For 1D motion with constant acceleration, displacement is quadratic in t , $ax^2 + bx + c = 0$ cannot have two positive roots if a and c have opposite signs

17. Statement 1

If two particles in their vertical motions under gravity, cross each other once, it is quite possible they may cross each other again.

and

Statement 2

$s = ut + \frac{1}{2}at^2$ can have two roots for a given value of s .

18. Statement 1

If velocity of a particle in 1D motion increases with time parabolically, then during any given interval of time, it would cover less distance than it would if it had travelled with constant acceleration equal to the average acceleration as per parabolic law.

and

Statement 2

Area under the referred parabola is less than area under the chord.

19. STATEMETNT – 1

If two cars travel along same straight line in same direction, each with constant acceleration and if the distance between them decreases with time, then they cannot be travelling with same acceleration.

and

Statement 2

Area under relative velocity vs time graph gives relative displacement and slope of the graph gives relative acceleration.

20. Statement 1

For a particle travelling along a straight line such that velocity increases with time, its average acceleration in any given time interval cannot be more than both its initial and final instantaneous accelerations

and

Statement 2

Average acceleration is ratio of change in velocity to time.



Linked Comprehension Type Questions

Directions: This section contains 2 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

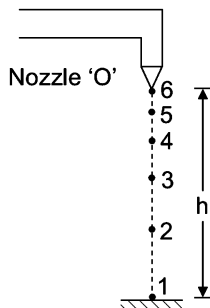
Passage I

At $t = 0$, particle A starts from a point O from rest and moves with constant acceleration of 5 m s^{-2} along a straight line. At $t = 3 \text{ s}$, particle B starts from the same point O and moves along the same line with constant velocity of 40 m s^{-1} . A third particle C starts from O at $t = 6 \text{ s}$ with initial velocity 'u' and moves with uniform acceleration of 10 m s^{-2} to overtake A at the same instant when A overtakes B. Then,

21. B overtakes A at
 (a) 16 s (b) 12 s (c) 8 s (d) 4 s
22. A overtakes B at
 (a) 10 s (b) 8 s (c) 12 s (d) 16 s
23. The initial velocity of C is
 (a) 20 m s^{-1} (b) 30 m s^{-1} (c) 35 m s^{-1} (d) 40 m s^{-1}

Passage II

Drops of water are falling down at regular intervals through a nozzle O which is at a height of h above the ground. The 6th drop comes out of the nozzle by the time the first one hits the ground. Then at that instant



24. The height of the second drop above the ground is
 (a) $\frac{9h}{25}$ (b) $\frac{16h}{25}$
 (c) $\frac{4h}{25}$ (d) $\frac{12h}{25}$
25. The distance between the fourth and second drop is
 (a) $\frac{h}{25}$ (b) $\frac{12h}{25}$
 (c) $\frac{7h}{25}$ (d) $\frac{16h}{25}$
26. The time interval between the release of two successive drops is
 (a) $\frac{2}{5} \sqrt{\frac{2h}{g}}$ (b) $\frac{3}{5} \sqrt{\frac{2h}{g}}$ (c) $\frac{4}{5} \sqrt{\frac{2h}{g}}$ (d) $\frac{1}{5} \sqrt{\frac{2h}{g}}$



Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

27. A body starting from rest, uniformly accelerates for first 10 s and then uniformly decelerates for next 20 s and comes to a halt. It covers 300 m during the whole journey. Then
- maximum velocity is 20 m s^{-1} .
 - velocity at 5 seconds after start is 10 m s^{-1} .
 - velocity at 5 seconds before stop is 5 m s^{-1} .
 - distance covered in accelerated motion is 200 m.
28. A body moving with uniform retardation in +X direction has a velocity $\sqrt{k\ell}$ at origin and $\sqrt{\ell}$ at a distance ℓ from origin.
- The time to travel ℓ distance from origin is $t = \frac{2\sqrt{\ell}}{(\sqrt{k} + 1)}$.
 - Acceleration $a = \left(\frac{k-1}{2} \right)$.
 - Its velocity at $\frac{\ell}{2}$ is $\sqrt{\frac{\ell(k+1)}{2}}$.
 - Its velocity at $\frac{\ell}{2}$ is $\sqrt{\frac{\ell}{2}(k-1)}$.
29. A ball A is dropped from a 180 m tall tower. After 1 s another ball B is projected from right below the tower with initial velocity of 25 m s^{-1}
- B reaches ground earlier than A.
 - Both reach ground together.
 - When they meet, their relative velocity is 25 m s^{-1} .
 - When they meet, $v_A > v_B$ and their velocities will be in same direction.



Matrix-Match Type Question

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

30. A, B, and C are non-zero constants, all greater than zero. The position of a particle in one dimensional motion, at time 't' from start, is represented by (a), (b), (c) and (d) in column I.

Column I	Column II
(a) $A + Bt + Ct^2$	(p) an accelerating body.
(b) $Bt - Ct^2$	(q) it can be a body moving vertically up.
(c) $A - Ct^2$	(r) a body which starts from the origin of the coordinate system.
(d) Ct^2	(s) A body which starts from a point other than the origin.

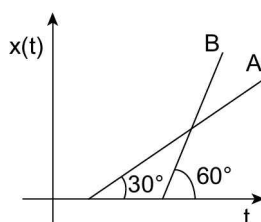
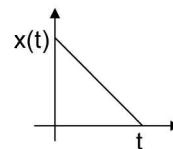
IIT ASSIGNMENT EXERCISE



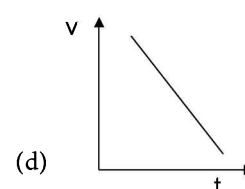
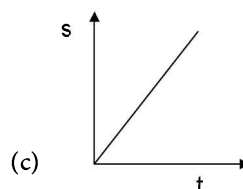
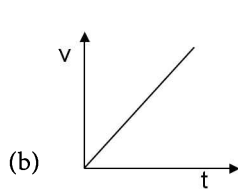
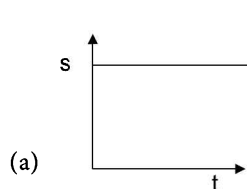
Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

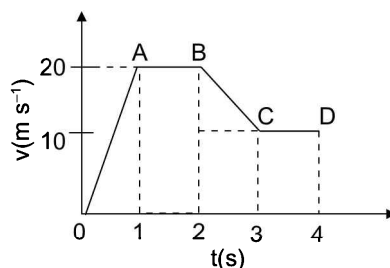
31. When a bus takes a hairpin curve on a hill, the motion will be
 (a) two dimensional (b) three dimensional
 (c) one or two dimensional (d) two or three dimensional
32. A person takes 3 steps forward and 1 step backward, again 3 steps forward and one step backward and so on. Each step is 1 foot long and takes 1 second. The time in which the person falls in a pit 7 feet away from the starting point will be (assume the person cannot stand at the edge of the pit).
 (a) 12 s (b) 11 s (c) 14 s (d) 13 s
33. A car takes t second to travel from A to B. During the first half of the time it travels with a speed v_1 and for the second half it travels with a speed v_2 . The average speed is
 (a) $\frac{2v_1v_2}{v_1 + v_2}$ (b) $\frac{v_1v_2}{v_1 + v_2}$ (c) $\frac{v_1 + v_2}{2}$ (d) $\frac{v_1 + v_2}{v_1v_2}$
34. The position–time graph of a body in uniform motion is as shown. Then,
 (a) the body is moving away from the observer.
 (b) the body is moving towards the observer.
 (c) such a position–time graph is not possible in the case of uniform motion.
 (d) the body is moving with decreasing velocity.
35. The position time graphs of two bodies moving in the same direction are as shown. The ratio of their speeds $v_A : v_B$ will be



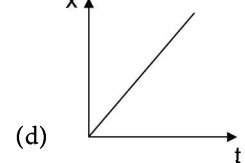
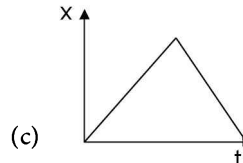
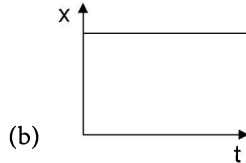
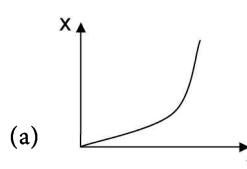
- (a) 3 : 1 (b) 1 : 3 (c) $\sqrt{3} : 1$ (d) $1 : \sqrt{3}$
36. A particle moves along a straight line with uniform acceleration and v_1 , v_2 and v_3 denote the average velocities in the three successive intervals of time $(0, t_1)$, (t_1, t_2) and (t_2, t_3) respectively. Which of the following relations is correct?
 (a) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$
 (b) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$
 (c) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_1 - t_3)$
 (d) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$
37. Which of the following graphs shown in figure represents motion with uniform speed along a straight line?
 $s \rightarrow$ displacement
 $v \rightarrow$ velocity



38. The variation of velocity of a particle moving along a straight line is illustrated in the figure. The distance travelled by the particle in 4 s is



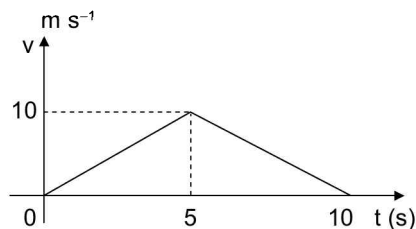
- (a) 60 m (b) 55 m (c) 30 m (d) 25 m
39. A train 200 m long passes over a bridge 600 m long. The time taken by the train to cross the bridge when the train moves with a uniform speed of 10 m s^{-1} is
- (a) 100 s (b) 40 s (c) 80 s (d) 60 s
40. A body moves from A to B with a constant speed of 20 m s^{-1} and returns from B to A with a constant speed of 40 m s^{-1} . The average speed of the body for the whole journey is
- (a) $(80/3) \text{ m s}^{-1}$ (b) 30 m s^{-1} (c) 60 m s^{-1} (d) 40 m s^{-1}
41. Which of the following displacement time graphs represents one-dimensional uniform motion?



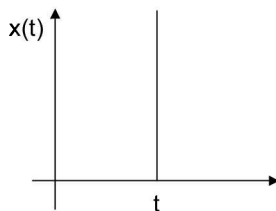
42. If a body, starting from rest and moving with constant acceleration, covers 10 m in the first second, the distance travelled by it in the next second will be
- (a) 10 m (b) 20 m (c) 30 m (d) 40 m
43. A car starting from rest and moving on a straight road with constant acceleration, attains a speed of 40 m s^{-1} in 20 s. Its average acceleration is
- (a) 0.5 m s^{-2} (b) 2 m s^{-2} (c) 4 m s^{-2} (d) 8 m s^{-2}
44. A body starting from rest and moving along a straight line with uniform acceleration attains a speed of 10 m s^{-1} in 2 s. The distance travelled by this body in the first 4 s of its motion will be
- (a) 10 m (b) 20 m (c) 30 m (d) 40 m
45. A bullet hitting a wooden block loses half its velocity when it penetrates 60 cm. If the retardation is uniform, it will come to rest after penetrating a further distance of
- (a) 10 cm (b) 20 cm (c) 30 cm (d) 40 cm

2.38 Motion in One Dimension

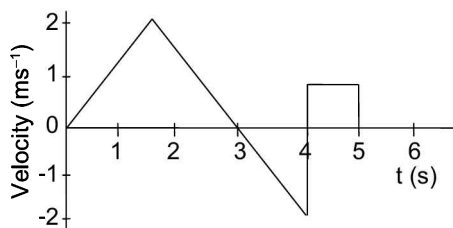
46. The velocity–time graph of a body in one dimensional motion is as shown. The distance covered by the body in 10 second is



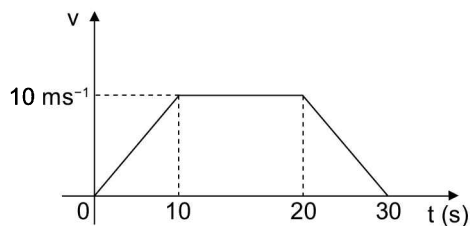
- (a) 25 m (b) 50 m (c) 100 m (d) 150 m
47. In the above question, the distance travelled by the body in the fifth second is
 (a) 4 m (b) 3 m (c) 9 m (d) 7 m
48. In Q. No. 46 the distance travelled by the body in the 10th second is
 (a) 1 m (b) 3 m (c) 5 m (d) 7 m
49. The position time graph of a body is as shown. Then the,



- (a) body moves with a varying velocity. (b) body moves with a varying acceleration.
 (c) motion is impossible. (d) body is at rest.
50. From the given velocity–time graph of a body in 1 D motion, the displacement of the body in 5 s is

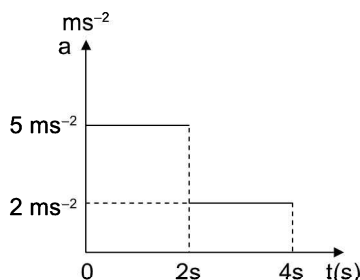


- (a) 2 m (b) 3 m (c) 4 m (d) 5 m
51. From the given graph the total distance travelled by the body in 30 s,

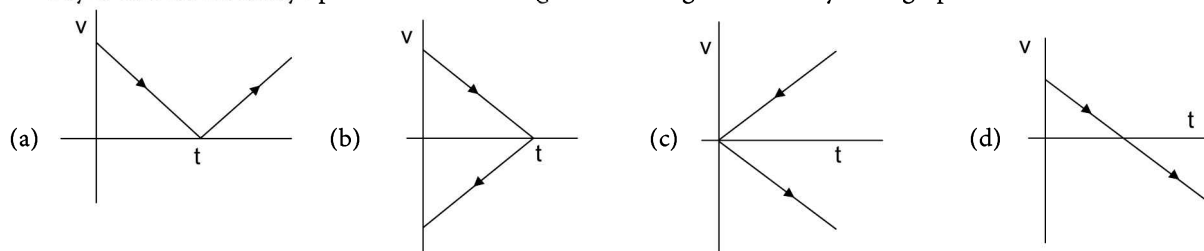


- (a) 200 m (b) 250 m (c) 300 m (d) 400 m

52. In the above question the distance travelled by the body in the 12th second will be
 (a) 10 m (b) 20 m (c) 100 m (d) 200 m
53. Figure shows a $a - t$ graph of a body starting from rest. Then the velocity of the body after 2 s is



- (a) 5 m s^{-1} (b) 10 m s^{-1} (c) 20 m s^{-1} (d) 15 m s^{-1}
54. In the above case distance travelled by the body in the first 2 s is
 (a) 5 m (b) 10 m (c) 20 m (d) 15 m
55. In Q.No. 53, velocity of the body at the end of 4 s is
 (a) 10 m s^{-1} (b) 7 m s^{-1} (c) 14 m s^{-1} (d) 20 m s^{-1}
56. A body is thrown vertically upward. Which among the following is its velocity time graph?

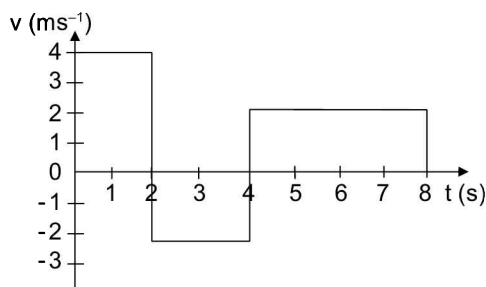


57. The displacement of a body moving along a straight line is given by $x = pt^2 + qt + r$ where p, q, r are constants and ' t ' represents time. The acceleration of the body will
 (a) remain a constant (b) decrease
 (c) increase (d) first increase and then decrease
58. The velocity of a body depends on time according to the equation $v = 20 + 0.1 t^2$. The body is undergoing
 (a) uniform acceleration (b) non-uniform acceleration
 (c) uniform deceleration (d) zero acceleration
59. A particle accelerates from rest at a constant rate for some time moving along a straight line and attains a velocity of 8 m s^{-1} . Immediately it decelerates at a constant rate and comes to rest. If the total time taken is 4 s, the distance travelled is
 (a) 32 m (b) 16 m (c) 4 m (d) 2 m
60. A body moves along a straight line with constant acceleration for 20 s after it starts from rest. It covers a distance S_1 in the first 10 seconds and S_2 in the next 10 seconds of its motion. Relation between S_1 and S_2 is
 (a) $S_2 = 3S_1$ (b) $S_2 = 4S_1$ (c) $S_2 = 2S_1$ (d) $S_2 = S_1$
61. A particle moves along X-axis in such a way that its x-coordinate varies with time t according to the relation $x = 2 - 5t + 6t^2$. The initial velocity (at $t = 0$) of the particle is
 (a) -5 unit (b) 3 unit (c) 6 unit (d) -6 unit

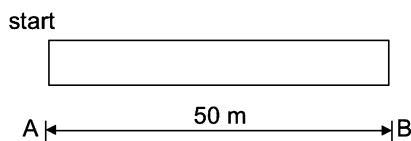
2.40 Motion in One Dimension

62. The motors of an electric train can give it a uniform acceleration of 1 m s^{-2} and the brakes can give it a uniform negative acceleration of 3 m s^{-2} . The shortest time in which the train can make a trip, starting from a station and stopping at another station that is separated from the starting station by a distance of 1350 m on straight track, is
 (a) 15 s (b) 30 s (c) 60 s (d) 120 s
63. An electron starting from rest and moving along a straight line has a velocity that increases linearly with time, i.e., $v = kt$ where $k = 2 \text{ m s}^{-2}$. The distance covered by the electron in the first 3 second will be
 (a) 9 m (b) 16 m (c) 27 m (d) 36 m
64. The displacement of a particle moving along a straight line at time 't' is given by $x = 1 + 2t + 3t^2$. The instantaneous acceleration is
 (a) Zero (b) 6 unit (c) $(2 + 6t)$ unit (d) $6t$ unit
65. From the top of a tower of height 20 m, a boy drops one stone; one second later he throws another stone vertically downwards so that both hit the ground together. The initial velocity of the second stone is ($g = 10 \text{ m s}^{-2}$)
 (a) 10 m s^{-1} (b) 15 m s^{-1} (c) 20 m s^{-1} (d) 25 m s^{-1}
66. A body released from the top of a tower of height 'h' takes 't' second to reach the ground. At time $\frac{1}{2}t$ the height of the body from the ground is
 (a) $\frac{h}{4}$ (b) $\frac{h}{3}$ (c) $\frac{h}{2}$ (d) $\frac{3h}{4}$
67. A body dropped from a tower travels half of the total distance in the last second of its motion. The total time of fall will be ($g = 10 \text{ m s}^{-2}$)
 (a) $\sqrt{2} \text{ s}$ (b) 2 s (c) $2 + \sqrt{2} \text{ s}$ (d) $2\sqrt{2} \text{ s}$
68. Two bodies are thrown vertically upwards, with their initial velocities in the ratio 2 : 3. The ratio of maximum heights attained will be
 (a) $\sqrt{2} : \sqrt{3}$ (b) 4 : 9 (c) 2 : 3 (d) Data is insufficient
69. A balloon is at a height of 12.8 m above the ground and is ascending with a velocity of 12 m s^{-1} . A 2 kg body is dropped from it. It will reach the ground in ($g = 10 \text{ m s}^{-2}$)
 (a) 1.5 s (b) 2 s (c) 3.2 s (d) 4.5 s
70. A body starts from rest and falls through a height of 19.6 m. The time it takes to fall through the last 4.9 m of its fall is
 (a) 0.4 s (b) 0.268 s (c) 0.05 s (d) 0.1 s
71. Two bodies of masses m_1 and m_2 are dropped from rest from heights h_1 and h_2 . The ratio of their respective time to reach the ground is
 (a) $\sqrt{m_1} h_1 : \sqrt{m_2} h_2$ (b) $h_1 : h_2$ (c) $m_1 \sqrt{h_1} : m_2 \sqrt{h_2}$ (d) $\sqrt{h_1} : \sqrt{h_2}$
72. A body is released from a certain height. Another one is released from the same height a second later. The separation between the bodies 2 s after the release of the second body is
 (a) 24.5 m (b) 4.9 m (c) 9.8 m (d) 14.7 m
73. A stone thrown vertically upwards with a speed of 5 m s^{-1} attains a height of h_1 . Another stone thrown vertically upwards from the same point with a velocity 10 m s^{-1} attains a height of h_2 . The relation between h_1 and h_2 is
 (a) $h_1 = h_2/2$ (b) $h_2 = 2h_1$ (c) $h_1 = 3h_2$ (d) $h_2 = 4h_1$
74. The distance travelled by a body falling freely from rest in first, second and third second of its fall are in the ratio
 (a) 1 : 2 : 3 (b) 1 : 3 : 5 (c) 1 : 4 : 9 (d) 1 : 5 : 11

75. When a body is thrown up, at the highest point, its velocity is zero. Its acceleration is (upward positive)
 (a) Zero (b) g (c) $g/2$ (d) $-g$
76. A man stands on the edge of a cliff at a height above the ground. He throws a ball A straight up with an initial speed u and another ball B straight down with the same initial speed. When the balls hit the ground,
 (a) speed of ball A = speed of ball B (b) speed of ball A > speed of ball B
 (c) speed of ball B > speed of ball A (d) speed of ball A $\geq$ speed of ball B
77. A train of length 200 m travelling at 30 m s^{-1} overtakes another train of length 300 m travelling at 20 m s^{-1} in the same direction on a parallel track. The time taken by the first train to cross the second is
 (a) 30 s (b) 50 s (c) 10 s (d) 20 s
78. A wooden block is dropped from the top of a building 100 m high above ground. Simultaneously a bullet is fired from the foot of the building upwards with a velocity of 100 m s^{-1} . The height above the ground at which they meet is
 (a) 90.2 m (b) 4.9 m (c) 95.1 m (d) 9.8 m
79. A stone is projected vertically upwards with a speed $\sqrt{2gh}$. At the same time another stone is dropped downwards from a point in the same vertical line located at a height h above the ground. The height of the point above the ground where the two bodies collide is
 (a) $\frac{2h}{3}$ (b) $\frac{h}{2}$ (c) $\frac{3h}{4}$ (d) $\frac{h}{4}$
80. Time taken by a boat to travel a distance $2d$ in still water is t_{still} . The same boat travels in flowing river; going downstream for a distance of d and returning to the starting point, taking a total time of t_{flow} . The correct relationship is
 (a) $t_{\text{still}} = t_{\text{flow}}$ (b) $t_{\text{flow}} < t_{\text{still}}$ (c) $t_{\text{still}} < t_{\text{flow}}$ (d) $t_{\text{still}} = 2 t_{\text{flow}}$
81. The velocity–time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 8 s are



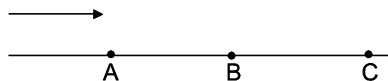
- (a) 12 m, 20 m (b) 14 m, 12 m (c) 16 m, 20 m (d) 12 m, 16 m
82. An swimmer completes one round (A to B and back) of a pool of length 50 m in 80 second. The displacement of the swimmer at the end of 5 minute and 12 second is



- (a) 10 m (b) 50 m (c) 40 m (d) 30 m

2.42 Motion in One Dimension

83. The greatest acceleration or deceleration that a train may have is a . The minimum time in which the train can start from one station to stop at the next (at a straight distance of s) is
- (a) $\sqrt{\frac{s}{a}}$ (b) $\sqrt{\frac{2s}{a}}$ (c) $\frac{1}{2}\sqrt{\frac{s}{a}}$ (d) $2\sqrt{\frac{s}{a}}$
84. A bullet fired into a large block of wood of uniform resistance, hits the block with a velocity of 48 m s^{-1} . It penetrates a distance of 60 cm and the velocity is then reduced to 24 m s^{-1} . It then penetrates a further distance of (in cm)
- (a) 50 (b) 16 (c) 18 (d) 20
85. A uniformly accelerating train passes a person on the ground. If the speeds of the points 1, 2 and 3 on the train as they pass the person are u_1 , u_2 and u_3 respectively and $\frac{u_1}{u_2} = \frac{u_2}{u_3}$, then $\frac{s_1}{s_2}$ is (where, s_1 is the distance between 1 and 2; s_2 between 2 and 3 respectively),
- (a) $\frac{s_1}{s_2} = \frac{u_2}{u_3}$ (b) $\frac{s_1}{s_2} = \left(\frac{u_1}{u_2}\right)^2$ (c) $\frac{s_1}{s_2} = \frac{u_3}{u_1}$ (d) $\frac{s_1}{s_2} = \frac{u_3 - u_2}{u_2 - u_1}$
86. A car travels in a straight line with varying velocity given by $v = |t - t_0| \text{ m s}^{-1}$ for $0 \leq t \leq 2t_0$ where, t = time and t_0 = time at which velocity is zero. The distance travelled by the car in this interval is
- (a) t_0^2 (b) $2 t_0^2$ (c) $\frac{3}{2} t_0^2$ (d) 0
87. Two bodies move with uniform acceleration along a straight line. The ratio of the initial speeds of the bodies is 2:1 and the ratio of their accelerations is 1:2 respectively. Then the ratio of the instantaneous speeds of the bodies at any time t is
- (a) 1:1 (b) 1:4 (c) 2:1 (d) Ratio varies with time
88. A long, straight rope lying on a floor is pulled along its length with uniform acceleration along its length. If the velocities of the front and back ends of the rope when it passes through a particular point on the floor are 2 m s^{-1} and 4 m s^{-1} respectively, the velocity of the midpoint of the rope when it passes through the same point is (in m s^{-1})
- (a) $2\sqrt{2}$ (b) $\sqrt{10}$ (c) 3 (d) 2.67
89. A point object starts from rest from the origin and moves with a uniform acceleration (in m s^{-2}) of $\alpha \hat{i} + \beta \hat{j} + \gamma \hat{k}$. The distance of the object from the origin after 1 second is
- (a) $\frac{\sqrt{\alpha^2 + \beta^2 + \gamma^2}}{2}$ (b) $\frac{\sqrt{\alpha\beta + \beta\gamma + \gamma\alpha}}{2}$ (c) $(\alpha + \beta + \gamma)$ (d) $(\alpha + \beta + \gamma)^2$
90. A particle moving along a straight line with uniform acceleration covers distances $AB = 8 \text{ m}$ and $BC = 18 \text{ m}$ in one second each. The speed of the particle at point B is (in m s^{-1})

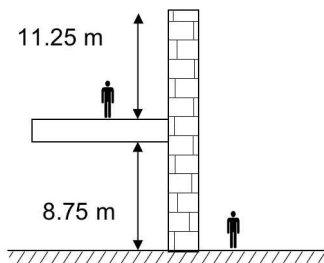


- (a) 13 (b) 8 (c) 5 (d) 10
91. Three points A, B and C are on a straight line such that $AB = BC$. A particle moving with uniform acceleration passes A and C with speeds of 6 m s^{-1} and 8 m s^{-1} respectively. Another particle moving with uniform acceleration passes A and B with speeds of 12 m s^{-1} and 14 m s^{-1} respectively. If t_1 and t_2 are the times taken by the particles to cross AB, $\frac{t_1}{t_2}$ is nearly
- (a) 2.0 (b) 1.3 (c) $\sqrt{2}$ (d) 0.5

92. A stone falls into a well. If it hits the water in 3 s from its release, the depth to water surface from the mouth of the well is ($g = 10 \text{ m s}^{-2}$)
 (a) 100 m (b) 50 m (c) 30 m (d) 45 m
93. A balloon with a basket, with a man inside, starts from rest and rises vertically with a uniform acceleration of 4.9 m s^{-2} . The man releases a ball 2 second after the balloon leaves the ground. The maximum height above the ground attained by the ball is
 (a) 9.8 m (b) 14.7 m (c) 19.6 m (d) 24.5 m
94. If an object falling vertically from rest takes Δt second less and acquires a velocity of $u \text{ m s}^{-1}$ more in falling through the same distances on 2 planets having accelerations due to gravity of $2g$ and $8g$ respectively, then
 (a) $u = 4g\Delta t$ (b) $u = 2g\Delta t$ (c) $u = 10g\Delta t$ (d) $u = 2\sqrt{2}g\Delta t$
95. If air resistance is to be considered while analysing the motion of a particle thrown vertically upwards, which of the following relationships between time of ascent (t_{asc}) and time of descent (t_{des}) is correct?
 (a) $t_{\text{asc}} = t_{\text{des}}$ (b) $t_{\text{asc}} > t_{\text{des}}$
 (c) $t_{\text{asc}} < t_{\text{des}}$ (d) Either b or c depending on the mass of the projectile.
96. A particle is thrown vertically upwards with a speed of 50 m s^{-1} . The ratio of the distance travelled in the 5th second to that in the 6th second is ($g = 10 \text{ m s}^{-2}$)
 (a) 2:1 (b) 5:3 (c) 4:5 (d) 1:1
97. A boy P stands on top of a tower. Another boy Q throws a ball vertically up from the ground so that it just reaches P and the time taken is t_0 . The speed with which P should throw the ball vertically downwards so that it reaches the ground in time $\frac{t_0}{2}$ is
 (a) gt_0 (b) $2gt_0$ (c) $\frac{3}{4}gt_0$ (d) $\frac{gt_0}{4}$
98. A man throws balls into the air one after another. He always throws a ball when the previous one thrown has just reached the highest point. The height to which each ball rises, if he throws 5 balls per second is ($g = 10 \text{ m s}^{-2}$)
 (a) 0.31 m (b) 0.20 m (c) 0.42 m (d) 0.53 m
99. Two stones are dropped from the top of a tower at half a second apart. The time after dropping the first stone at which the distance between the two stones is 20 m is ($g = 10 \text{ m s}^{-2}$)
 (a) 4 s (b) 3.75 s (c) 4.25 s (d) 2 s
100. In a circus, a juggler throws balls vertically upwards at intervals of 2 s. He throws a ball when the previous one is at the highest point. Then the maximum height to which each ball rises is ($g = 10 \text{ m s}^{-2}$)
 (a) 20 m (b) 45 m (c) 25 m (d) 15 m
101. The speed with which a stone dropped from a cliff 160 m high, hits the sea below is nearly ($g = 10 \text{ m s}^{-2}$)
 (a) 150 m s^{-1} (b) 56 m s^{-1} (c) 60 m s^{-1} (d) 59 m s^{-1}
102. Two bodies are thrown vertically upwards from walls of heights 2.2 m and 1.05 m respectively, with an initial velocity of 10 m s^{-1} . The ratio of their times of flight is ($g = 10 \text{ m s}^{-2}$)
 (a) $\frac{22}{21}$ (b) $\frac{12}{11}$ (c) $\frac{44}{23}$ (d) $\frac{13}{11}$
103. A body is thrown vertically up with a speed of 10 m s^{-1} from the top of a wall of height 3.45 m. The height from which another body should be dropped simultaneously such that the two bodies reach the ground at the same time is ($g = 10 \text{ m s}^{-2}$)
 (a) 40 m (b) 24.5 m (c) 6.9 m (d) 20 m

2.44 Motion in One Dimension

104. Three construction workers can throw bricks vertically up with speeds of 4 m s^{-1} , 6 m s^{-1} and 12 m s^{-1} respectively. Working together, they can stack them on a platform of maximum height (in m)
 (a) 5 (b) 10 (c) 15 (d) 20
105. A body is thrown vertically up with a speed of 10 m s^{-1} from a tower of height 15 m. The magnitude of its average velocity for the journey is (in m s^{-1})
 (a) 5 (b) 10 (c) 15 (d) 20
106. A man is standing on a platform behind a wall of height 11.25 m as shown. The platform he is standing on is at a height of 8.75 m. The minimum time (in s) in which a person on the ground on the other side of the wall can throw him a tool is ($g = 10 \text{ m s}^{-2}$)



- (d) 2 (b) 3.5 (c) 4.5 (d) 5
107. An elevator car whose floor to ceiling distance is equal to 2.7 m, starts from rest and ascends with a constant acceleration of 1.2 m s^{-2} . Two seconds later, a bolt drops from the car's ceiling. The time taken by the bolt to hit the floor of the elevator is
 (a) 1 s (b) 0.8 s (c) 1.2 s (d) 0.7 s
108. The positions of three particles are as follows:
 (i) $x = 3t + 2.5t^2$
 (ii) $x = \frac{5}{2}t(2+t)$
 (iii) $x = 6 + 3t - 2.5t^2$
 Relative motion is uniform between
 (a) (i) and (ii) (b) (ii) and (iii)
 (c) (i) and (iii) (d) both (i) and (ii) and (i) and (iii)
109. Inside a lift, which is moving vertically upwards with a uniform acceleration 'a', a body is projected vertically upwards with a speed 'u' with respect to the lift. Its time of flight inside the lift is
 (a) $\frac{2u}{a+g}$ (b) $\frac{2u}{a-g}$ (c) $\frac{2u}{g}$ (d) $\frac{2u}{a}$
110. An elevator is going down with a constant acceleration. A coin dropped from a point 1.8 m above the elevator floor takes one second to reach the floor. The magnitude of the acceleration of the elevator (in m s^{-2}) is ($g = 10 \text{ m s}^{-2}$)
 (a) 3.6 (b) 5 (c) 7.2 (d) 6.4



Assertion–Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
 (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
 (c) Statement-1 is True, Statement-2 is False
 (d) Statement-1 is False, Statement-2 is True

111. Statement 1

A bus starting from rest, moves on a straight road and away from a person at some distance behind the bus, with an acceleration along the straight road given by the expression $a = kt^3$ (where k = a constant and t = time). When the person travels on the straight road with the minimum constant speed required for catching the bus, the speeds of the bus and man are the same at the instant he catches the bus.

and

Statement 2

For the minimum possible speed, the displacement time plot of the person is tangent to that of the bus.

112. Statement 1

For a body thrown vertically up from the ground, the distance covered in the first second is equal to the distance covered in the last second only if the initial velocity is greater than 10 m s^{-1} ($g = 10 \text{ m s}^{-2}$).

and

Statement 2

For the upward and downward journeys to take more than one second, the initial speed has to be greater than 10 m s^{-1} .

113. Statement 1

Two particles A and B are projected vertically upwards from the top of two identical towers, one on Earth and the other on Moon, with the same initial speed. The magnitudes of the displacement of A and B in the first second of projection can be the same.

and

Statement 2

The acceleration due to gravity on the surface of Earth is 6 times that on the surface of Moon.

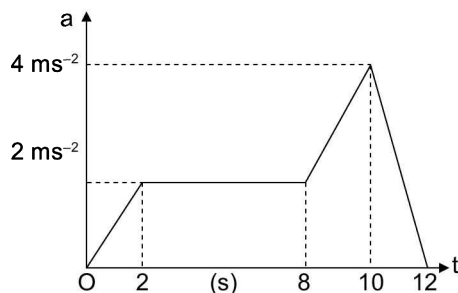


Linked Comprehension Type Questions

Directions: This section contains a paragraph. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage I

The acceleration–time curve of a particle moving along a straight line with initial velocity of 1 ms^{-1} , is as shown



114. The velocity of the particle at time $t = 4 \text{ s}$ is (in m s^{-1})

(a) 4

(b) 8

(c) 6

(d) 7

2.46 Motion in One Dimension

115. The displacement of the particle during the interval $t = 2$ s to $t = 8$ s is
 (a) 54 m (b) 12 m (c) 24 m (d) 48 m
116. The maximum velocity attained by the particle is
 (a) 24 m s^{-1} (b) 25 m s^{-1} (c) 26 m s^{-1} (d) 30 m s^{-1}



Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

117. A bullet plunges vertically into a retarding medium with an initial velocity u . Its velocity is reduced by 20% in a distance of 0.9 m in the medium. If a is the uniform retardation in the medium, then:
 (a) The total distance travelled by the bullet in the medium is 2.5 m.
 (b) The total distance travelled by the bullet in the medium is 1.6 m.
 (c) Its average velocity in the medium is numerically equal to $\frac{\sqrt{5a}}{2}$.
 (d) The time of travel in the medium is numerically equal to $\sqrt{\frac{5}{a}}$.
118. A body A is given an initial velocity v in the X-direction at $t = 0$; its velocity is increased by v in the same direction at the end of every second. Another body B starting from zero velocity is given a uniform acceleration ' a ' in X-direction. Both travel the same distance for n second.
 (a) Acceleration $a = v \left(1 + \frac{1}{n} \right)$.
 (b) The distance travelled by the first body in 20 s is 210 v metre.
 (c) If n is large, in 5th second both travel the same distance.
 (d) If n is large, for even values of n , in the $\left(\frac{n}{2} \right)$ second both travel equal distance; if n is odd, at $\frac{n+1}{2}$ second they travel equal distance.
119. A body projected vertically upward with a velocity v at $t = 0$ is found at a height h after 1 second and after a further 6 second, it is found at the same height ($g = 10 \text{ m s}^{-2}$). Then
 (a) h is 30 m (b) u is 40 m s^{-1}
 (c) maximum height is 80 m. (d) distance moved in 5th second is 5 m



Matrix-Match Type Question

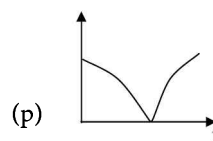
Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

120.

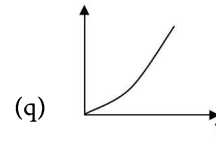
Column I

- (a) The position time graph of a body in 1D motion with constant acceleration (positive, negative or zero).

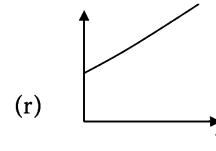
Column II



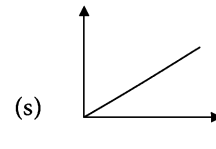
- (b) The position–time graph of a body in 1D motion with constant velocity.



- (c) The position time graph of a body dropped from a height H rebounding to the same height.



- (d) The velocity time graph of a body in 1D motion with constant acceleration.



ADDITIONAL PRACTICE EXERCISE

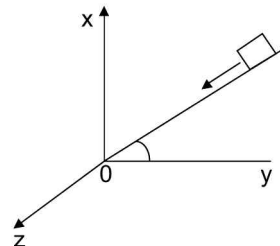


Subjective Questions

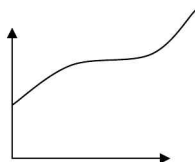
Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

121. A block released from rest on an inclined plane slides down as shown. The motion of the block with respect to the given system is

(a) one dimensional. (b) two dimensional.
(c) three dimensional. (d) one and two dimensional.



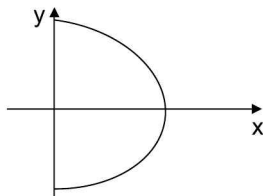
122.



The above graph can represent

(a) distance vs t (b) displacement vs t (c) (a) or (b) (d) neither (a) nor (b)

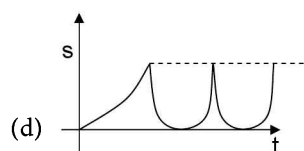
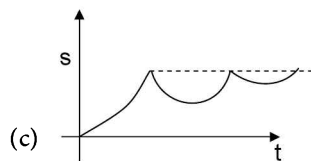
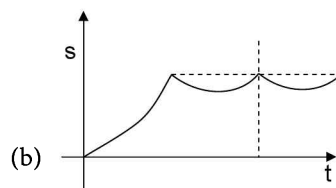
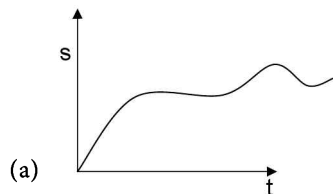
123.



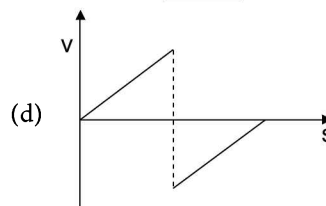
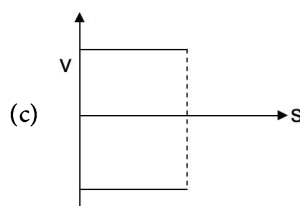
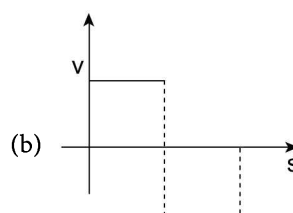
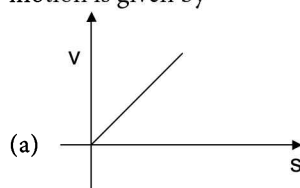
The graph shown was obtained for a body undergoing uniformly accelerated motion along a straight line. The plot could be (y and x -axes respectively)

(a) time vs displacement (b) velocity vs displacement
(c) time vs speed (d) (a) or (b)

124. A ball is released from a height above a floor and comes to rest after a few bounces. The displacement time graph of the motion is given by

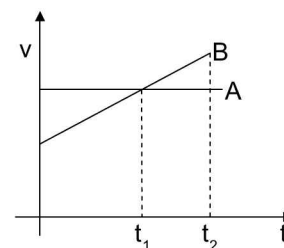


125. A ball is bouncing back and forth with constant speed between two walls. The velocity vs displacement graph of the motion is given by



126. Two bodies A and B, initially at the origin and moving along the same straight line have velocities as shown. If the total distance covered by A and B in time t_2 is the same, then

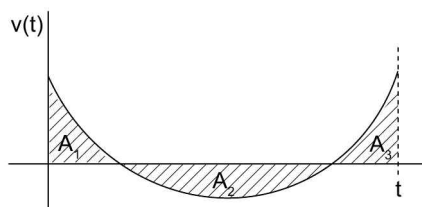
- (a) $t_1 = \frac{t_2}{2}$
 (b) B overtakes A at $t = t_1$
 (c) average velocity of A > average velocity of B
 (d) both (a) and (b) are true



127. A particle starting from rest and moving along a straight line has acceleration a for time t_1 and acceleration $-a$ for time t_2 . If the total displacement of the body is zero, then $\frac{t_2}{t_1}$ is

- (a) 1 (b) 2 (c) $\sqrt{2}$ (d) $\sqrt{2} + 1$

128. The velocity time graph of a particle starting from the origin at time $t = 0$ and moving along a straight line is as shown. If A_1 , A_2 and A_3 are the shaded areas and $A_2 > 2A_1$ and $A_3 < A_1$, the number of times the particle crosses the origin is



- (a) 0 (b) 1 (c) 2 (d) 3

129. A sports car is moving on a straight road at a speed of 288 kmph. The air brakes are applied till the speed reaches 144 kmph, producing a uniform deceleration of 10 m s^{-2} . At 144 kmph speed, the regular brakes are applied till the car stopped, producing a uniform deceleration of 5 m s^{-2} . The distance travelled by the car, from the instant of applying the air brakes till it stopped, is (in m)

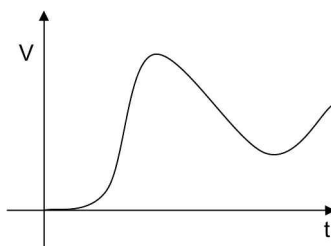
- (a) 320 (b) 400 (c) 512 (d) 640

130. A particle starts from rest and travels a total distance of 18 m along a straight path. The first half of the distance was travelled with a uniform acceleration of 2 m s^{-2} and the rest with uniform velocity. The average velocity for the whole journey is (in m s^{-1})

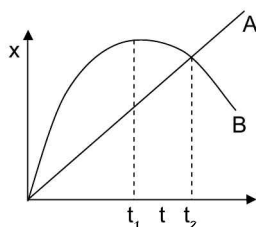
- (a) 3 (b) 4 (c) 6 (d) 9

2.50 Motion in One Dimension

131. A particle starts from rest and travels a straight distance of 18 m. The first half of the distance is covered with a uniform acceleration of 2 m s^{-2} while the second half is covered with a uniform acceleration of 2.5 m s^{-2} . The total time taken for the journey is
- (a) 4.2 s (b) 3.6 s (c) 5 s (d) 3 s
132. Train A can move with a uniform acceleration of 3 m s^{-2} while speeding up and with a uniform deceleration of 5 m s^{-2} on applying the brakes. Train B can move with a uniform acceleration of 4 m s^{-2} while speeding up and with a uniform deceleration of 4 m s^{-2} on applying the brakes. If t_A and t_B are the minimum times taken by train A and train B respectively to start from one station and stop at the next station while travelling on straight tracks, then
- (a) $t_A = t_B$ (b) $t_A > t_B$
 (c) $t_A < t_B$ (d) $\frac{t_A}{t_B}$ depends on the distance between stations.
133. The velocity time graph of a body in one dimensional motion is as shown. The number of times the average acceleration is equal to the instantaneous acceleration during the motion is

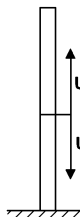
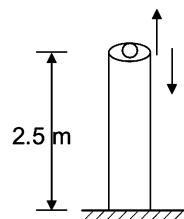


- (a) 0 (b) 1 (c) 2 (d) 3
134. The displacement time graph of two bodies A and B in 1 D motion in the same direction is as shown



- (a) their relative velocity is zero at a time $t < t_1$
 (b) their relative velocity is zero at a time $t_1 < t < t_2$
 (c) the distance between the bodies is maximum at $t = t_1$
 (d) the relative acceleration is zero at some point
135. A body starts with a certain initial velocity and moves with a constant deceleration along a straight line. If the distances travelled in the 5th and 7th second are in the ratio $\frac{31}{27}$, the body will come to a stop in time
- (a) 40 s (b) 30 s (c) 20 s (d) 10 s
136. A body starts with a certain initial velocity and moves with constant deceleration along a straight line. If distances travelled in the 5th and 7th second are in the ratio $\frac{31}{27}$, the distance travelled by the body before it stops is
- (a) 50 m (b) 100 m (c) 150 m (d) data insufficient

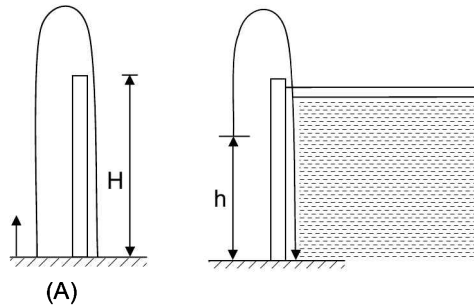
137. A body is dropped from a tower of height 45 m. Another body is thrown up at the same instant with speed u from a height of 30 m. If they reach the ground simultaneously, u is ($g = 10 \text{ m s}^{-2}$)
 (a) 5 m s^{-1} (b) 10 m s^{-1} (c) 15 m s^{-1} (d) 20 m s^{-1}
138. A body dropped from a tower hits an obstacle 20 m below so that its speed is halved. If the total time of flight is 4 s, the height of the tower (in m) above the ground is ($g = 10 \text{ m s}^{-2}$)
 (a) 20 (b) 40 (c) 60 (d) 80
139. A paratrooper jumps from a height H . The parachute can provide a uniform deceleration of 2 m s^{-2} . The height above the ground at which the parachute should be opened so that he touches ground with zero speed is ($g = 10 \text{ m s}^{-2}$)
 (a) $\frac{H}{6}$ (b) $\frac{4H}{5}$ (c) $\frac{5H}{6}$ (d) $\frac{6H}{7}$
140. A body is projected vertically upwards with a speed of 12 m s^{-1} on the surface of a planet, where the acceleration due to its gravity is 8 m s^{-2} . The time (in s) for which it stays above a height of 5 m is
 (a) 1 (b) 2 (c) 3 (d) 4
141. A body is thrown vertically up from a wall of height 2.2 m with a speed of 10 m s^{-1} . The distance travelled in the last second of its motion is ($g = 10 \text{ m s}^{-2}$)
 (a) 5 m (b) 6 m (c) 7 m (d) 8 m
142. A body dropped from a height of 20 m takes 1 s to cover a vertical pole on ground. If the body is dropped from a height of 40 m, the time taken to cover the same pole is ($g = 10 \text{ m s}^{-2}$)
 (a) 0.5 s (b) $\frac{1}{\sqrt{2}} \text{ s}$ (c) $(2\sqrt{2} - \sqrt{5}) \text{ s}$ (d) $\frac{1}{\sqrt{3}} \text{ s}$
143. A body A is dropped inside a pillar that is in the form of a tube and its inside has a constant downward acceleration of 5 m s^{-2} . The velocity with which another body B should be simultaneously thrown up from the top of the pillar, so that it falls outside the tube and both bodies A and B reach the ground at the same time, is ($g = 10 \text{ m s}^{-2}$)
 (a) 1.25 m s^{-1} (b) 2.5 m s^{-1}
 (c) 5 m s^{-1} (d) 7.5 m s^{-1}
144. Two bodies are projected vertically, from the mid point of a tower with equal speeds, so that one just reaches the top of the tower. The ratio of times of flight of the two bodies is



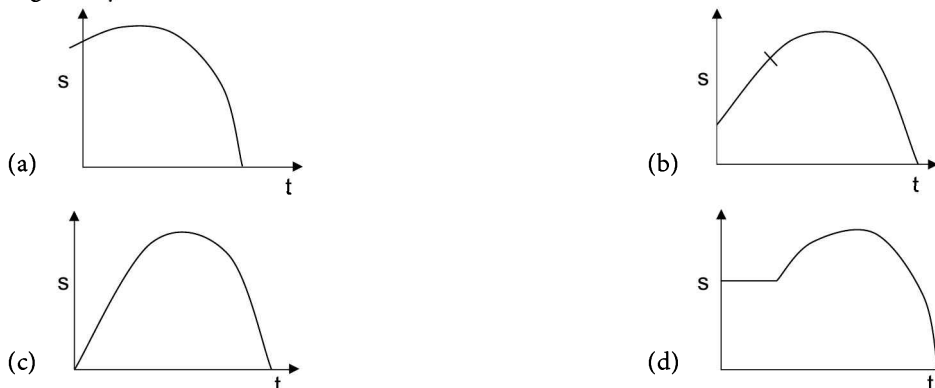
- (a) 3 (b) 2 (c) $\sqrt{3}$ (d) $3 + 2\sqrt{2}$
145. A juggler throws balls up with equal speed. He releases a ball when the previous one has reached half the maximum height. The maximum number of balls moving in the upward direction at any one instant is
 (a) 1 (b) 2 (c) 3 (d) 4
146. In the above problem, if the time in air of each ball is 2 s, to have maximum number of balls in air, the reaction time should be about (ms)
 (a) 50 (b) 100 (c) 240 (d) 400

2.52 Motion in One Dimension

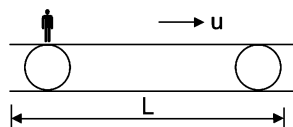
147. A body is thrown from ground vertically upwards into an empty tank of height H in minimum time. The tank is now filled with water so that the acceleration inside is reduced to $\frac{g}{2}$. The height h from which the body should be thrown so that the time is unchanged is



- (a) $0.5 H$ (b) $0.66 H$ (c) $0.8 H$ (d) $0.9 H$
148. Three bodies are thrown vertically up simultaneously as shown in the figure, so that all three collide. Then
- (a) $u_2 = \frac{u_1 + u_3}{2}$ (b) $u_2 = \sqrt{u_1 u_3}$
- (c) $u_2 = \sqrt{\frac{u_1^2 + u_3^2}{2}}$ (d) $\frac{2}{u_2} = \frac{1}{u_1} + \frac{1}{u_3}$
-
149. Two bodies connected by a uniform flexible string have equation of motion along a straight line given by $x_1 = 3t$ and $x_2 = 6t + 2t^2 + 10$ respectively, all 'x' in metre and 't' in second. The time taken for the initial velocity of the mid-point of the string to double in magnitude is
- (a) 2.25 s (b) 1.25 s (c) 2.5 s (d) 3.75 s
150. In a straight portion of a river, from a point A on the bank of the river as you move to a point P at a distance of 'x' m from A in the downstream direction, the speed of flow of river is equal to $2\sqrt{9 + 0.1x}$ m s⁻¹. A boat plies between A and another point B downstream. If the engine of the boat can produce a maximum speed of 36 kmph on the boat in still water, the maximum range AB is (in m)
- (a) 100 (b) 160 (c) 320 (d) 500
151. A body is thrown up, and another body is thrown down, from a tower of height h . The relative displacement vs time is given by



152. An aircraft carrier is designed to land aircraft of minimum speed 144 kmph. If the maximum speed of the carrier is 36 kmph, the minimum percentage increase in the length of the runway to just land aircraft of minimum speed 188 kmph is (consider that the aircraft moves horizontally and in the same direction as the carrier, at the time of landing. The initial length of runway is designed for landing when the carrier is at rest).
- (a) 5 (b) 10 (c) 18 (d) 20
153. A ball is dropped from a height 'h' above the ground. It loses 50% of its velocity on every impact with the ground before bouncing up. What is the total distance travelled by the ball when it finally comes to rest on the ground?
- (a) 1.67 h (b) 2 h (c) 1.33 h (d) 2.5 h
154. The uniform acceleration with which a man should start running from rest, behind a bus that goes past him with a constant speed of 5 m s^{-1} on a straight road, to catch it in 20 m distance from start, is
- (a) 1 m s^{-2} (b) 2.5 m s^{-2} (c) 5 m s^{-2} (d) 10 m s^{-2}
155. Two cars with the same speed v have constant decelerations of a_1 and a_2 ($a_1 < a_2$) respectively at a point on the road. If they are travelling in the same direction on parallel roads, the difference in times in which they reach their destinations (where they stop moving), is
- (a) $v \left(\frac{1}{a_1} - \frac{1}{a_2} \right)$ (b) $v \left(\frac{1}{a_1} + \frac{1}{a_2} \right)$
- (c) $\frac{v}{2} \left(\frac{1}{a_1} - \frac{1}{a_2} \right)$ (d) $\frac{1}{2} \left(\frac{v}{a_1 - a_2} \right)$
156. Two bodies are projected vertically upwards from a point with speeds of 20 m s^{-1} and 15 m s^{-1} respectively. The second body rebounds from the ground with the same speed, and they meet in mid air. The relative velocity (in m s^{-1}) when they meet is ($g = 10 \text{ m s}^{-2}$)
- (a) 15 (b) 25 (c) 35 (d) 45
157. A stone is dropped into a well. From the open mouth of the well, the water level is at a depth of 25 m. The sound of splash is heard after 2.31 s from its release. The speed of stone in air inside the well is (in m s^{-1}) (given $g = 10 \text{ m s}^{-2}$)
- (a) 400 (b) 300 (c) 357 (d) 260
158. A man standing on a horizontal conveyor belt moving with a constant speed u , starts running with a constant acceleration a . The speed with which the man exits the conveyor belt after covering a length 'L' on it is



- (a) $\sqrt{u^2 + 2aL}$ (b) $\sqrt{u^2 + aL}$ (c) $\sqrt{2aL}$ (d) u
159. Two bodies are projected vertically upwards with speeds 20 m s^{-1} and 15 m s^{-1} respectively from the same point. After the second body bounces once, they collide. If t_1 and t_2 are the time of flight of the second particle before bouncing and time interval from bouncing till collision with first particle, then $\frac{t_1}{t_2}$ is
- (a) 2 (b) 3 (c) 4 (d) 5
160. Two bodies separated by a distance of 's' start moving towards each other with speeds of v and $2v$ respectively. The uniform acceleration with which the first body should move so that they meet at the middle is
- (a) $\frac{v^2}{s}$ (b) $\frac{v^2}{2s}$ (c) $\frac{8v^2}{s}$ (d) $\frac{4v^2}{s}$



Assertion-Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- (c) Statement-1 is True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True

161. Statement 1

If a body starting with velocity v , gets its velocity doubled at every equal intervals of distance, then its average velocity for n equal intervals of distance is $\frac{nv}{2}$, if n is very large,

and

Statement 2

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

162. Statement 1

For a body moving under constant acceleration, the total distance moved during a time duration, t has no bearing on the average velocity during that duration.

and

Statement 2

$$\text{Displacement } \bar{s} = \bar{v}_{\text{avg}} \times t$$

163. Statement 1

Particle A is in uniform motion while particle B starts from rest and moves with uniform acceleration along a straight line. At an instant, the displacements of A (S_A) and B (S_B) are related as $S_A : S_B = 2 : 1$, and the magnitude of the velocities of A (V_A) and B (V_B) are equal at that instant.

and

Statement 2

$v = at$ for uniformly accelerated motion of a body, starting from rest.

164. Statement 1

If the average velocity and average acceleration of a particle in 1-D motion is zero, during certain time interval, it need not be at rest in that duration.

and

Statement 2

The average values do not impose restrictions on the instantaneous values at every point.

165. Statement 1

A body thrown up will be having initial retardation and then acceleration under the influence of constant $\bar{g}$.

and

Statement 2

Effective acceleration vector can be same for a body undergoing retardation or acceleration

166. Statement 1

Whatever may be the initial velocity of a vertical projectile, if its time of flight T is an odd integer, then its displacement in $\left(\frac{T+1}{2}\right)^{\text{th}}$ second is zero.

and

Statement 2

For a body under uniform acceleration, displacement in n^{th} second is $\bar{u} + \frac{1}{2}a(2n-1)$.

167. Statement 1

If the time of flight of a vertical projectile is an even integer T , then its displacement in $\left(\frac{T}{2}\right)^{\text{th}}$ second is numerically equal to $\frac{g}{2}$.

and

Statement 2

The displacement in the n^{th} second for a vertical projectile is $\bar{s} = \bar{u} - \frac{1}{2}g(2n-1)$.

168. Statement 1

When drops of water are falling down from a tap at regular intervals, at the instant when a drop just leaves the tap, the separations between adjacent drops are in arithmetic progression.

and

Statement 2

The distance travelled by the drop in the n^{th} second is $\frac{1}{2}g(2n-1)$.

169. Statement 1

When two bodies are projected vertically upward from a point on ground at different instants and with different velocities, if either of them is observed from the reference frame of the other body while both of them are in motion, then the observed body will be in linear motion with uniform velocity.

and

Statement 2

Their relative velocity is constant while they are in motion.

170. Statement 1

When two particles travelling in a straight line in the same direction, have accelerations which are proportional to their velocity (same proportionality constant), there is only one speed for each particle at which the relative acceleration is unchanged.

and

Statement 2

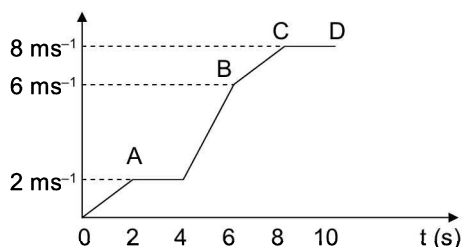
In the above case, if the relative velocity is zero, relative acceleration is also zero.



Linked Comprehension Type Questions

Directions: This section contains 3 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage I



Using the velocity time graph answer the following questions

171. During the motion of the body, the acceleration maximum at
 (a) $t = 0$ to $t = 2$ s (b) $t = 2$ s to $t = 4$ s (c) $t = 4$ s to $t = 6$ s (d) $t = 8$ s to $t = 10$ s
172. The average acceleration till time $t = 10$ s is
 (a) 2 m s^{-2} (b) 0.8 m s^{-2} (c) 1 m s^{-2} (d) 0.5 m s^{-2}
173. The displacement of the body at time $t = 6$ s is
 (a) 6 m (b) 12 m (c) 18 m (d) 14 m

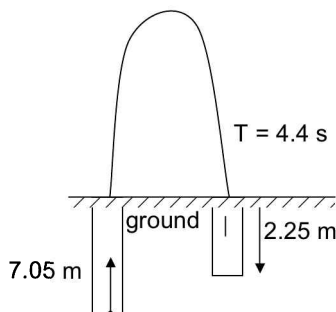
Passage II

A particle is moving along X-axis and its velocity 'v' varies with time 't' (in s) as $v = v_0 \left[1 - \frac{t}{T} \right]$, where, $v_0 = 10 \text{ cm s}^{-1}$ and $T = 5$ s respectively. At the instant $t = 0$, the particle leaves the origin in the +X direction. Then

174. The magnitude of the displacement of the particle at time $t = 6$ s is
 (a) 12 cm (b) 36 cm (c) 24 cm (d) 6 cm
175. The magnitude of the velocity of the particle at time $t = 5$ s is
 (a) zero (b) 10 cm s^{-1} (c) 5 cm s^{-1} (d) 2.5 cm s^{-1}
176. The acceleration of the particle at time $t = 5$ s is
 (a) 2 cm s^{-2} along negative X-axis. (b) 2 cm s^{-2} along positive X-axis.
 (c) 1 cm s^{-2} along negative X-axis. (d) 1 cm s^{-2} along positive X-axis.

Passage III

A body is projected vertically up from inside of hole of depth 7.05 m and falls into another nearby hole of depth 2.25 m. The time the particle spends outside the holes is 4.4 s ($g = 10 \text{ m s}^{-2}$)



177. The maximum height (in m) above the ground the particle rises is
 (a) 48.4 (b) 24.2 (c) 96.8 (d) 12.1
178. The time taken to get out of the first hole is (s)
 (a) 0.2 (b) 0.3 (c) 0.4 (d) 0.5
179. The time it moves in the second hole is (s)
 (a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4



Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

180. The uniform accelerations of a particle in the regions A, B and C are 5 m s^{-2} , 5 m s^{-2} and 9 m s^{-2} respectively and the thicknesses of these regions are 1.3 m, 1.1 m and 0.5 m respectively. A particle of speed 7 m s^{-1} enters from the left, in a direction normal to the regions A, B and C
- | | | |
|---|---|---|
| A | B | C |
|---|---|---|
- (a) The speeds at the end of regions A, B and C are in the ratio $6 : 5 : 4$.
 (b) The speeds at the end of regions A, B and C are in the ratio $\sqrt{3} : \sqrt{2} : 1$.
 (c) The time spent in the regions A, B and C are in the ratio $\frac{1}{5} : \frac{1}{2.2} : 1$.
 (d) The time spent in regions A, B and C are in the ratio $\frac{1}{5} : \frac{1}{5} : \frac{1}{9}$.
181. A body, which can move with constant acceleration of 2 m s^{-2} and constant deceleration of 4 m s^{-2} starts from rest at A along a straight line and stops at B and covers that total distance with maximum average velocity possible. If the distance moved during deceleration is 2 m, then
 (a) maximum velocity = 4 m s^{-1} (b) average velocity = 2 m s^{-1}
 (c) total distance AB = 6 m (d) total time of travel = 4 s
182. A vehicle starting from rest and moving under uniform acceleration along a straight line covers the distance between two points separated by 126 m in 6 s and the velocity at the 2nd point is 30 m s^{-1} . Then
 (a) its velocity at first point is 12 m s^{-1} (b) its acceleration is 2 m s^{-2}
 (c) the first point is 22 m from the starting point. (d) it reaches the first point in 4 s after start.
183. Displacement of a particle, at the origin at $t = 0$, is given by $s = -9t^2 + 20t$ where, 't' is in second and 's' in metre. Then,
 (a) it will come back to origin only once.
 (b) it will come back to origin with a velocity of -20 m s^{-1}
 (c) it will come back to origin with a velocity of $+20 \text{ m s}^{-1}$
 (d) it will come back to origin with a velocity of -10 m s^{-1}
184. A body projected vertically up with an initial velocity v , travels 45 m in the first second of its flight ($g = 10 \text{ m s}^{-2}$)
 (a) Its initial velocity is 50 m s^{-1} . (b) The maximum height it reaches is 125 m.
 (c) Its time of flight is 9 s (d) Its time of flight is 10 s.
185. A ball thrown up vertically covers 25 m in its first one second of flight. Then
 (a) Initial velocity $u = 30 \text{ m s}^{-1}$
 (b) Time of flight = 3 s

2.58 Motion in One Dimension

- (c) Maximum height $H = 45 \text{ m}$
- (d) Ratio of distance covered in the last second to the total distance of the flight is $\frac{5}{18}$
186. A ball thrown vertically up from the top of a tower, with a velocity of 30 m s^{-1} reaches ground in 8 s. Then
- maximum height reached from ground is 45 m.
 - velocity of the ball when it reaches ground is 50 m s^{-1} .
 - height of the tower is 80 m.
 - height of the tower is 90 m.
187. A ball is dropped from a height of 60 m. From a point vertically below on ground, a ball is projected vertically upwards with a speed of 30 m s^{-1} at the same instant. Then
- their initial relative velocity is 30 m s^{-1} .
 - Relative velocity at the time of collision is 40 m s^{-1} .
 - Time for collision is 2 s.
 - They collide at a height of 20 m from ground.



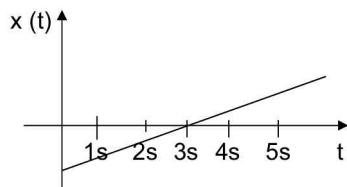
Matrix-Match Type Questions

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

188. Given 'u' and 'a' in column I are constants and not equal to zero. Time is represented by 't' and position by 'x'.

Column I

- The displacement of a particle in 1D motion is given by $x = u(t - 3) + a(t - 3)^2$
- The displacement of a particle in 1D motion is given by $x = ut + at^2$
- The position time graph of a body in 1D motion is given below



Column II

- Acceleration of the particle is $2a$.
 - The motion is uniformly accelerated.
 - At time $t = 3 \text{ s}$, the particle is at the origin.
 - The particle does uniform motion.
- $v^2 = u^2 + 4as$ is the expression for the velocity of a particle in 1D motion where s is the displacement

189. A man starts from rest at $t = 0$, on a bike to cross a point 100 m away on a straight road in minimum time. In the last 15 m of the stretch, the speed is constant at 10 m s^{-1} . The constant acceleration or deceleration of the bike is 10 m s^{-2} . Match the columns.

Column I

- At $t = 1 \text{ s}$
- At $t = 3 \text{ s}$
- At $t = 5 \text{ s}$
- At $t = 6 \text{ s}$

Column II

- stops deceleration
- $v = 10 \text{ m s}^{-1}$
- $v = 30 \text{ m s}^{-1}$
- $v = v_{\text{max}}$

190. A body is projected vertically upwards with a velocity of 40 m s^{-1} at $t = 0$. Match the modulus value or sign of the quantity referred in column I to the values given in column II. Take upward values as +ve.

Column I

Column II

(a) Maximum distance moved during any one second	(p) $\frac{g}{2}$
(b) Displacement in 4th second	(q) g
(c) Distance moved in 5th second	(r) $\frac{7g}{2}$
(d) velocity just after 3 second	(s) +ve

191. A particle moving along a straight line starts from rest at $t = 0$, moves with a constant acceleration of 5 m s^{-2} for 9 s, then with zero acceleration for 6 s, then with a constant acceleration of 3 m s^{-2} for 5 s, then with a constant retardation of 10 m s^{-2} till coming to rest.
- Determine the time at which the instantaneous acceleration is equal to the average acceleration till that time and plot the velocity time graph.
 - Determine the time at which the instantaneous velocity is equal to the average velocity till that time and plot the displacement–time graph.
192. Body A begins to move with an initial velocity of 2 m s^{-1} and a constant acceleration a . 10 s later, body B starts from the same point with an initial velocity of 12 m s^{-1} in the same direction as 'A' and with the same constant acceleration a .
- What is the maximum value of 'a' for which body B will overtake A?
 - If overtaking takes place at $t = 25 \text{ s}$, what is a ?
193. Three cars A, B and C proceed in the same direction along a straight road as detailed below, all starting from the origin. Car A was at rest at $t = 0$, then moving with uniform velocity of 20 m s^{-1} . Car B started from rest at $t = 10 \text{ s}$, with a constant acceleration of 8 m s^{-2} . Car C was started at $t = 8 \text{ s}$, with an initial velocity of 10 m s^{-1} , moved with a uniform acceleration of 5 m s^{-2} till $t = 16 \text{ s}$ and then was uniformly decelerating when a collision occurred, involving all three cars.
- At what time and at what distance from the origin did the collision occur?
 - What was the deceleration of car C?
 - Which car hit which other car from behind?
194. A ball is dropped from a height h on a floor where the rebound velocity is e times the impact velocity ($0 < e < 1$). Find the time taken by the ball to come to rest.
195. Two particles are thrown from a height h . One is thrown vertically up while the other is thrown vertically down with the same speed. Analyse and plot the graph between the distance of separation and time while both bodies are moving under gravity.
196. A particle is thrown up vertically from the ground with speed u and another particle is thrown down with the same speed at the same time from a height h . Find the minimum value of u so that they collide in air.
197. A vessel contains a fluid to a height h_f such that it exerts an additional constant retardation of a ($a < g$) (in addition to the gravity effects) on a ball moving in it. Find the height above the fluid surface from where the ball should be dropped so that it emerges from the fluid after collision with bottom (assume ball velocity reverses on collision with the bottom surface).
198. (i) A particle thrown vertically up so as to reach a maximum height H in time T . It is at height h in time t . Show that
- $$\frac{t}{T} = 1 \pm \sqrt{1 - \frac{h}{H}}.$$
- (ii) Particle A is thrown vertically up and after some time particle B is thrown vertically up from the same point with the same velocity. They collide 7 seconds after particle A was thrown and at a height $7/16$ of maximum height reached by A. What was the time interval between their projections?

2.60 Motion in One Dimension

199. Two bodies s distance apart move towards each other with a constant velocity of v . A particle travels with a velocity u from one body to the other and back and so on. Find the distance between the two bodies when the particle completes its n th trip (Motion from any body to the other is defined as a trip). Find the number of trips made by the particle by the time the two bodies collide. Find the distance travelled by the particle by the time the two bodies collide.
200. Two cars A and B are travelling along a straight road with uniform velocities v_A and v_B . A is behind B by 'd' at some instant when A applies brakes giving retardation a .
- (i) Prove that the condition for no collision between the cars is $(v_A - v_B) < \sqrt{2ad}$.
 - (ii) Determine if a vehicle which is capable of a maximum retardation of 20 m s^{-2} and travelling at 162 km/h , will be able to avoid collision with a stationary vehicle 50 m in front.

SOLUTIONS

ANSWERS KEYS

Topic Grip

1. (i) 8.4 m s^{-1}
(ii) 10.8 m s^{-1}
2. (a) (i) 5 s
(ii) 4 s
(iii) $\sqrt{20} \text{ s}$
(b) holds for any h and u
3. (i) $-$
(ii) $17\frac{2}{3} \text{ m}$
(iii) 21.5 m
(iv) 42.5 m
4. $v_{\max} = 2s \frac{(\alpha + \beta)}{t}$
 $a_i = \frac{2s(\alpha + \beta)^2}{\alpha t^2}$
5. $\sqrt{2} + 1$
6. 2175 m
7. 85 m
8. t_1/t_2
9. 105 m
10. (i) 2 h
(ii) 10 h
11. (c) 12. (d) 13. (c)
14. (c) 15. (c) 16. (a)
17. (d) 18. (a) 19. (d)
20. (d) 21. (d) 22. (c)
23. (b) 24. (a) 25. (b)
26. (d)
27. (a), (b), (c)
28. (a), (c)
29. (b), (d)
30. (a) $\rightarrow (p), (s)$
(b) $\rightarrow (p), (q), (r)$
(c) $\rightarrow (p), (s)$
(d) $\rightarrow (p), (r)$

IIT Assignment Exercise

31. (b) 32. (b) 33. (c)
34. (b) 35. (b) 36. (b)
37. (c) 38. (b) 39. (c)
40. (a) 41. (d) 42. (c)
43. (b) 44. (d) 45. (b)
46. (b) 47. (c) 48. (a)
49. (c) 50. (b) 51. (a)
52. (a) 53. (b) 54. (b)
55. (c) 56. (d) 57. (a)
58. (b) 59. (b) 60. (a)
61. (a) 62. (c) 63. (a)
64. (b) 65. (b) 66. (d)
67. (c) 68. (b) 69. (c)
70. (b) 71. (d) 72. (a)
73. (d) 74. (b) 75. (d)
76. (a) 77. (b) 78. (c)
79. (c) 80. (c) 81. (a)
82. (a) 83. (d) 84. (d)
85. (b) 86. (a) 87. (d)
88. (b) 89. (a) 90. (a)
91. (a) 92. (d) 93. (b)
94. (a) 95. (c) 96. (d)
97. (c) 98. (b) 99. (c)
100. (a) 101. (b) 102. (a)
103. (b) 104. (b) 105. (a)
106. (b) 107. (d) 108. (a)
109. (a) 110. (d) 111. (a)
112. (d) 113. (b) 114. (d)
115. (a)
116. (b)
117. (a), (c), (d)
118. (a), (b), (d)
119. (b), (c), (d)
120. (a) $\rightarrow (q), (r), (s)$
(b) $\rightarrow (r), (s)$
(c) $\rightarrow (p)$
(d) $\rightarrow (r), (s)$

Additional Practice Exercise

121. (b) 122. (c) 123. (b)
124. (c) 125. (c) 126. (a)
127. (d) 128. (b) 129. (b)
130. (b) 131. (a) 132. (b)
133. (c) 134. (a) 135. (c)
136. (d) 137. (a) 138. (c)
139. (c) 140. (b) 141. (c)
142. (c) 143. (b) 144. (d)
145. (d) 146. (c) 147. (b)
148. (a) 149. (a) 150. (b)
151. (c) 152. (b) 153. (a)
154. (b) 155. (a) 156. (b)
157. (c) 158. (a) 159. (d)
160. (c) 161. (a) 162. (b)
163. (b) 164. (a) 165. (b)
166. (a) 167. (c) 168. (b)
169. (a) 170. (d) 171. (c)
172. (b) 173. (d) 174. (c)
175. (a) 176. (a) 177. (b)
178. (b)
179. (a)
180. (a), (d)
181. (a), (b), (c)
182. (a), (d)
183. (a), (b)
184. (a), (b), (d)
185. (a), (c), (d)
186. (b), (c)
187. (a), (c)
188. (a) $(\rightarrow p, q, r)$
(b) $(\rightarrow p, q)$
(c) $(\rightarrow r, s)$
(d) $(\rightarrow p, q)$
189. (a) $(\rightarrow q)$
(b) $(\rightarrow r, s)$
(c) $(\rightarrow p, q)$
(d) $(\rightarrow q)$

2.62 Motion in One Dimension

190. (a) ($\rightarrow$ r, s)

(b) ($\rightarrow$ p, s)

(c) ($\rightarrow$ p, s)

(d) ($\rightarrow$ q, s)

191. (i) $t = 15 \text{ s}$ to 20 s

(ii) $t = \sqrt{493} \text{ s}$

192. (i) 1 m s^{-2}

(ii) 0.65 m s^{-2}

193. (i) 20 s , 400 m

(ii) 5 m s^{-2}

(iii) C hit A and B hit C

194. $\sqrt{\frac{2h}{g}} \left[\frac{1+e}{1-e} \right]$

195. $s = 2ut$ (linear plot)

196. $u = \frac{\sqrt{gh}}{2}$

197. $h = \frac{2a}{g} h_f$

198. $6s$

199. $s_n = \frac{s(u-v)^n}{(u+v)^n}; n = \infty; d = \frac{us}{2v}$

200. (ii) collision cannot be avoided.

HINTS AND EXPLANATIONS

Topic Grip

1. (i) Let the total distance travelled = x m
 Then $x_1 = 50 \text{ m} = 5 \text{ (m s}^{-1}) \times t_1 \text{ (s)} \Rightarrow t_1 = 10 \text{ s}$
 $x_2 = 8 \text{ (m s}^{-1}) \times 5 \text{ (s)} = 40 \text{ m}$
 $x_3 = 12 \text{ (m s}^{-1}) \times 10 \text{ (s)} = 120 \text{ m}$
 $x = x_1 + x_2 + x_3 = 210 \text{ m}$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{210}{10 + 5 + 10} = 8.4 \text{ m s}^{-1}$$

- (ii) $x_3 = 8 \times 10 + \frac{1}{2} \times 2 \times 10^2$ (using $s = ut + \frac{1}{2}at^2$)
 $= 180 \text{ m}$

$$\therefore \text{Average velocity} = \frac{50 + 40 + 180}{10 + 5 + 10}$$

$$= \frac{270}{25} = 10.8 \text{ m s}^{-1}$$

2. Use vertical upwards as positive.

(a) (i) $-100 = 5t_1 - 5t_1^2 \Rightarrow t_1 = \frac{1 \pm \sqrt{81}}{2}; t_1 = 5 \text{ s}$

(ii) $-100 = -5t_2 - 5t_2^2$
 $\Rightarrow t_2 = \frac{-1 \pm \sqrt{81}}{2}; t_2 = 4 \text{ s}$

(iii) $-100 = -5t^2 \Rightarrow t = \sqrt{20} \text{ s}$

- (b) For height 'h' and initial speed of balloon u.

case (i): $-h = ut_1 - \frac{1}{2}gt_1^2$ — (1)

case (ii): $-h = -ut_2 - \frac{1}{2}gt_2^2$ — (2)

case (iii): $-h = -\frac{1}{2}gt^2$ — (3)

(1) $\times t_2 + (2) \times t_1 \Rightarrow$

$$-h(t_1 + t_2) = -\frac{1}{2}gt_1t_2(t_1 + t_2)$$

$$\Rightarrow h = \frac{1}{2}gt_1t_2$$

Using (3) $\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \Rightarrow t_1t_2 = t^2$

$\therefore$ holds good for any h and u, as long as 'g' remains constant.

3. (i) Refer v - t graph

$$t = 0 \text{ to } t = 2 \text{ s}$$

Uniform velocity $\Rightarrow$ Acceleration = 0

$$t = 2 \text{ s to } t = 4 \text{ s: Slope} = \frac{4}{2}$$

$$\text{Acceleration} = +2 \text{ m s}^{-2}$$

$$t = 4 \text{ s to } t = 7 \text{ s: Slope} = \frac{-8}{3}$$

$$\text{Acceleration} = \frac{-8}{3} \text{ m s}^{-2}$$

$$t = 7 \text{ s to } t = 8 \text{ s: Slope} = -3$$

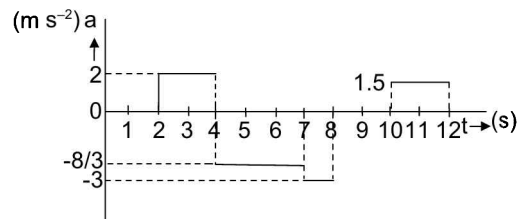
$$\text{Acceleration} = -3 \text{ m s}^{-2}$$

$$t = 8 \text{ s to } t = 10 \text{ s}$$

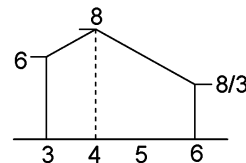
Uniform velocity $\Rightarrow$ Acceleration = 0

$$t = 10 \text{ s to } t = 12 \text{ s, Slope} = \frac{+3}{2}$$

$$\text{Acceleration} = 1.5 \text{ m s}^{-2}$$



- (ii) Displacement during a time interval is same as Area under v-t graph for the time interval:



Displacement

(for $t = 3 \text{ s to } t = 6 \text{ s}$)

$$= \left(\frac{6+8}{2} \right) 1 + \left(\frac{8+\frac{8}{3}}{2} \right) \times 2 = 17\frac{2}{3} \text{ m}$$

2.64 Motion in One Dimension

(iii) Displacement ($t = 0$ to $t = 12$ s)

$$= \text{Area } (t = 0 \text{ to } t = 7 \text{ s}) - \text{Area}$$

$$(t = 7 \text{ s to } t = 12 \text{ s})$$

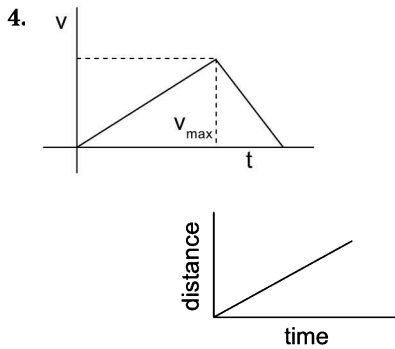
$$= 4 \times 2 + \left(\frac{4 + 8}{2} \right) \times 2 + \frac{1}{2} \cdot 8 \cdot 3 -$$

$$\left(\frac{1}{2} \times 3 \times 1 + 2 \times 3 + \frac{1}{2} \times 3 \times 2 \right) = 21.5 \text{ m}$$

(Area below the X-axis shows negative displacement.)

(iv) Distance covered ($t = 0$ to $t = 12$ s)

$$= \text{area } (t = 0 \text{ to } t = 7 \text{ s}) + \text{area } (t = 7 \text{ s to } t = 12 \text{ s}) = 32 + 10.5 = 42.5 \text{ m}$$



$$\frac{1}{2} \cdot v_{\max} \cdot t = (\alpha + \beta)s$$

$$\Rightarrow v_{\max} = 2 \frac{(\alpha + \beta)s}{t}$$

$$v_{\max}^2 = 2 \cdot a_1 \cdot \alpha s \quad (\text{using } v^2 - u^2 = 2as)$$

$$\Rightarrow 2a_1 \alpha s =$$

$$\Rightarrow a_1 = \frac{2s(\alpha + \beta)^2}{\alpha t^2}$$

5. Let t_1 be time for forward motion and t_2 be time for reverse journey.

$$s_1 = \frac{1}{2}at_1^2; s_2 = -(at_1)t_2 - \frac{1}{2}at_2^2$$

$s_1 + s_2 = 0$ (Net displacement is zero. Because starting and final point are the same)

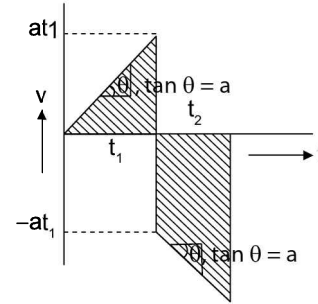
$$\Rightarrow t_1^2 - 2t_1t_2 - t_2^2 = 0$$

$$\Rightarrow t_1 = (\sqrt{2} + 1)t_2$$

$$\frac{t_1}{t_2} = \sqrt{2} + 1$$

Aliter:

Graphical method; equate the areas



$$\frac{1}{2}t_1 \cdot at_1 = at_1 \cdot t_2 + \frac{1}{2}t_2 \cdot a \cdot t_2$$

$$\Rightarrow \frac{t_1}{t_2} = 1 + \sqrt{2}$$

6. Method 1:

Part: 1 (accelerated motion)

$$v_1 = v_0 + a_1 t_1 = 0 + (0.5) 30 = 15 \text{ m s}^{-1}$$

$$S_1 = \left(\frac{v_0 + v_1}{2} \right) t_1 = (7.5)30 = 225 \text{ m}$$

Part 2: (Uniform motion)

$$v_2 = v_1 = 15 \text{ m s}^{-1}$$

$$S_2 = v_2 \cdot t_2 = 15t_2 \quad (t_2 \text{ unknown}) \quad \text{--- (1)}$$

Part 3: (Retarded motion)

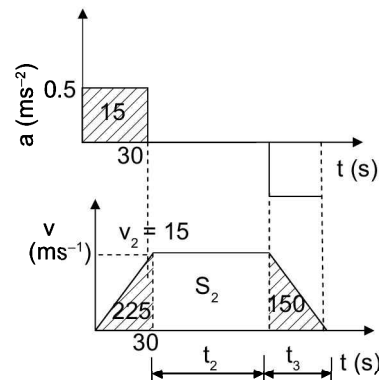
$$S_3 = 150 = \left(\frac{v_2 + v_3}{2} \right) t_3 = \left(\frac{15 + 0}{2} \right) t_3 \Rightarrow t_3 = 20 \text{ s}$$

$$t = 170 = t_1 + t_2 + t_3 = 30 + t_2 + 20 \Rightarrow t_2 = 120 \text{ s}$$

$$(1) \Rightarrow S_2 = 15t_2 = 15 \cdot 120 = 1800 \text{ m}$$

$$S = S_1 + S_2 + S_3 = 225 + 1800 + 150 = 2175 \text{ m}$$

Method 2: (Graphical)



$$v_2 = \text{area under } a - t \text{ (} t = 0 \text{ to } t = 30\text{)} \\ = 15 \text{ m s}^{-1}$$

$$S_1 = \text{area under } v - t \text{ (} t = 0 \text{ to } t = 30\text{)} \\ = 225 \text{ m}$$

$$S_3 = \text{area under } v - t \text{ during } t_3 = 150 \text{ m (given)}$$

$$\therefore t_3 = 20 \text{ s}$$

$$\therefore t_2 = 170 - 30 - 20 = 120 \text{ s}$$

$$\therefore S_2 = \text{area under } v - t \text{ during } t_2 \\ = 15 \times 120 = 1800 \text{ m}$$

$$\text{Hence } S = 225 + 1800 + 150 = 2175 \text{ m}$$

$$7. s_A = \frac{1}{2}gt^2 = 5t^2$$

where, t = time of meeting

$$s_B = 40(t - 2) + \frac{1}{2}g(t - 2)^2 \\ = 40t - 80 + 5t^2 + 20 - 20t$$

$$s_A = s_B \Rightarrow 20t = 60 \Rightarrow t = 3 \text{ s}$$

$$\therefore s_A = 5 \times 3^2 = 45 \text{ m}$$

$$s_C = 30(t - 1) - \frac{1}{2}g(t - 1)^2 \\ = 30 \times 2 - 5 \times 2^2 = 40 \text{ m}$$

$$\therefore \text{Height of building} = s_A + s_C = 85 \text{ m}$$

8. For the first particle, let T_1 be time to reach the surface after release. Then (upward direction is positive)

Speed of projection, $u = 10t_1$

Height from which body was projected, $h = \frac{1}{2}10t_1^2$

$$\frac{1}{2}10t_1^2 + 10t_1T_1 - \frac{1}{2}gT_1^2 = 0$$

$$\Rightarrow T_1 = (\sqrt{2} + 1)t_1, \text{ by taking } g = 10 \text{ m s}^{-2}$$

$$\text{Similarly, } T_2 = (\sqrt{2} + 1)t_2 \therefore \frac{T_1}{T_2} = \frac{t_1}{t_2}$$

$$9. T = \frac{u}{g} + \sqrt{\frac{2\left(h + \frac{u^2}{2g}\right)}{g}} \\ = \frac{u}{g} + \frac{1}{g}\sqrt{2gh + u^2}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{1}{3} = \frac{u_1 + \sqrt{u_1^2 + 2gh}}{u_2 + \sqrt{u_2^2 + 2gh}}$$

$$\frac{20 + \sqrt{2gh + 20^2}}{100 + \sqrt{2gh + 100^2}} = \frac{1}{3}$$

$$60 + 3\sqrt{2gh + 20^2} = 100 + \sqrt{2gh + 100^2}$$

$$3\sqrt{2gh + 20^2} = 40 + \sqrt{2gh + 100^2}$$

$$9(2gh + 20^2) = 40^2 + 2gh + 100^2 + 2 \times 40 \times \sqrt{2gh + 100^2}$$

$$8 \times 2gh + 9 \times 20^2 - 40^2 - 100^2 = 80\sqrt{2gh + 100^2}$$

$$2gh - 1000 = 10\sqrt{2gh + 100^2}$$

$$\Rightarrow h = 105 \text{ m, by taking } g = 10 \text{ m s}^{-2}$$

10. Let speed of river = $u \text{ km h}^{-1}$

Speed of boat = $v \text{ km h}^{-1}$

Distance between spots = $s \text{ km}$

$$\text{Then } \frac{s}{v + u} = 2.5$$

$$\Rightarrow \frac{v}{s} + \frac{u}{s} = \frac{1}{2.5} = 0.4 \quad \text{--- (1)}$$

$$\text{Similarly, } \frac{v}{s} - \frac{u}{s} = \frac{1}{5} = 0.2 \quad \text{--- (2)}$$

Adding (1) and (2)

$$\frac{2v}{s} = 0.6 \Rightarrow v = 0.3 \text{ s}$$

Substituting in (1), $u = 0.1 \text{ s}$

$$\therefore \frac{s}{v + 2u} = \frac{s}{(0.3 + 0.2)s} = \frac{1}{0.5} = 2h$$

$$\frac{s}{v - 2u} = \frac{s}{(0.3 - 0.2)s} = \frac{1}{0.1} = 10h$$

11. Let the ball be at a height h at time T and $T + t$

$$H = uT - \frac{1}{2}gT^2 \quad \text{--- (1)}$$

$$H = u(T + t) - \frac{1}{2}g(T + t)^2 \quad \text{--- (2)}$$

Equating (1) and (2) we get

$$T = \frac{2u - gt}{2g} \quad \text{--- (3)}$$

Substituting (3) in (1) we get

$$H = \frac{4u^2 - g^2t^2}{8g} \text{ or } u = \frac{1}{2}\sqrt{8gH + g^2t^2}$$

2.66 Motion in One Dimension

Aliter:

$$h = \frac{1}{2} g \left(\frac{t}{2} \right)^2 = \frac{1}{8} g t^2$$

$$\begin{aligned} H_{\max} &= H + h \\ &= H + \frac{1}{8} g t^2 \\ &= \frac{8H + g t^2}{8} \end{aligned}$$

$$\text{But } H_{\max} = \frac{u^2}{2g}$$

$$\Rightarrow \frac{8H + g t^2}{8} = \frac{u^2}{2g}$$

$$\Rightarrow u = \frac{1}{2} \sqrt{8gH + g^2 t^2}$$

12. Area under a-t graph represents change in velocity.

Initial velocity = 0

Hence the area represents final velocity.

Velocity after 4s = Area

$$= 10 \times 2 + \frac{1}{2} \times 10 \times 2 = 30 \text{ m s}^{-1}$$

13. Let u be the initial velocity of the particle moving with a constant acceleration a for a time t . The displacement in the n th second

$$\begin{aligned} S_n &= u + \frac{1}{2} a (2n-1) \\ &= \frac{1}{2} a (2n-1) \end{aligned}$$

14. $s_1 + s_2 = \text{distance}$

$s_2 - s_1 = \text{displacement}$

$$n = \frac{s_2 + s_1}{s_2 - s_1}$$

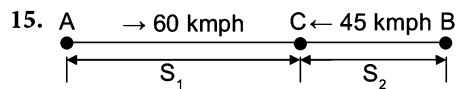
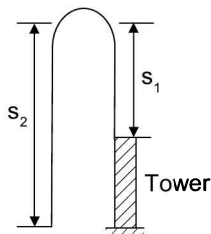
$$\Rightarrow \frac{(n+1)}{(n-1)} = \frac{s_2}{s_1}; s_1 = \frac{1}{2} g t_1^2$$

For downward journey,

$$s_2 = \frac{1}{2} g t_2^2$$

$$\Rightarrow \frac{t_2^2}{t_1^2} = \frac{s_2}{s_1} = \frac{(n+1)}{(n-1)}$$

$$\Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{n-1}{n+1}}$$



$$S_{\text{rel}} = S_1 + S_2 = 200 \text{ km}$$

$$v_{\text{rel}} = 60 + 45 = 105 \text{ km h}^{-1}$$

$$t = \frac{S_{\text{rel}}}{v_{\text{rel}}} = \left(\frac{200}{105} \right) \text{ h} = 1 \frac{19}{21} \text{ h}$$

16. $s = s_0 + ut + \frac{1}{2} at^2$. For $s = 0$ to have two positive values

for t , treating u positive, both s and a should be negative

(particle starts behind origin and has retardation)

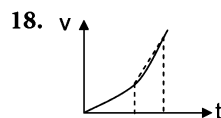
17. Relative displacement is not quadratic but linear

$$s_1 = u_1 t - \frac{1}{2} g t^2$$

$$s_2 = u_2 (t + a) - \frac{1}{2} g (t + a)^2 \quad [\text{assuming a general case of time lag}]$$

$$s_{1/2} = t [u_1 - u_2 \pm ag] - u_2 a + \frac{a^2}{2} g$$

$$s_{1/2} = 0 \text{ gives only one value for } t$$



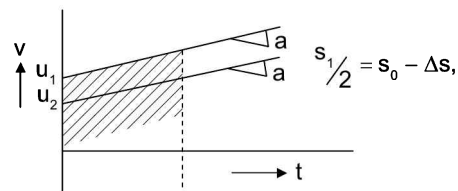
19. Statement-2 is perfectly right, but in this case there is an 'initial relative displacement so'

$$\therefore s_1 = u_1 t + \frac{1}{2} a_1 t^2$$

$$s_2 = s_0 + u_2 t + \frac{1}{2} a_2 t^2$$

$$s_{2/1} = s_0 + (u_2 - u_1) t + \frac{1}{2} (a_2 - a_1) t^2$$

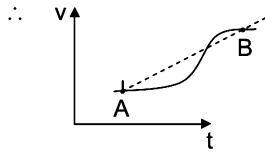
$$a_2 \text{ can be } = a_1 \text{ if } u_2 < u_1$$



decreases with t

$\therefore \Delta s$ increases with t .

20. Given slope of $v-t$ is +ve everywhere



Passage I

The distance traveled by A and B are the same when B overtakes A.

$$\therefore S = \frac{1}{2} \cdot 5 \cdot t^2 = 40(t - 3)$$

$$\therefore t^2 - 16t + 48 = 0$$

$$\therefore t = 4 \text{ s or } 12 \text{ s}$$

$\therefore$ At $t = 4$ s, B overtakes A

At $t = 12$ s, A overtakes B

The distance covered by C in 6 s is the same as the distance traveled by A in 12 s or the distance travelled by B in $(12 - 3) = 9$ s.

$$\therefore 6u + \frac{1}{2}at^2 = \left(\frac{1}{2} \right) \cdot 5 \cdot 9^2$$

$$\therefore u = 30 \text{ m s}^{-1}$$

key

21. $\rightarrow$ (d);

22. $\rightarrow$ c,

23. $\rightarrow$ b

Passage II

24. Since distances covered in consecutive seconds from start in free fall are in the ratio

$$1 : 3 : 5 : 7 : 9, \quad x + 3x + 5x + 7x + 9x = h$$

$$\text{i.e., } 25x = h$$

$$\text{or } x = \frac{h}{25}$$

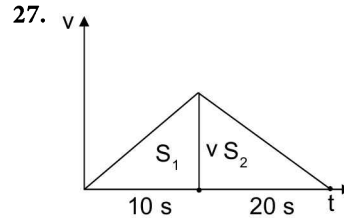
$$\text{Second drop is at a height of } 9x = \frac{9h}{25}$$

25. Distance between fourth and second drop is $12x = \frac{12h}{25}$

26. Time taken by the first drop is $t = \sqrt{\frac{2h}{g}}$ for 6 drops there are five intervals. Given the drops are released in equal intervals.

So time interval between any two successive drops

$$\text{is } \frac{1}{5} \sqrt{\frac{2h}{g}}$$



Velocity-time graph is as shown. Area under $v-t$ graph is displacement

$$\frac{1}{2} v \cdot 30 = 300$$

$$v = 20 \text{ m s}^{-1}$$

$$v = a_1 t_1 \Rightarrow 20 = a_1 \times 10 \Rightarrow a_1 = 2 \text{ m s}^{-2}$$

$$20 = a_2 \times 20 \Rightarrow a_2 = 1 \text{ m s}^{-2}$$

$$v_1 \text{ at 5 second after start} = a_1 t_1 = 2 \times 5 = 10 \text{ m s}^{-1}$$

$$v_2 \text{ at 5 second before stop} = a_2 t_2 = 1 \times 5 = 5 \text{ m s}^{-1}$$

$$S_1 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} \times 2 \times 10^2 = 100 \text{ m}$$

Ans: (a), (b), (c)

$$28. s = \left(\frac{u+v}{2} \right) \cdot t$$

$$t \left(\frac{\sqrt{k\ell} + \sqrt{\ell}}{2} \right) = \ell$$

$$t = \frac{2\ell}{\sqrt{\ell}(\sqrt{k} + 1)} = \frac{2\sqrt{\ell}}{(\sqrt{k} + 1)} \Rightarrow$$

acceleration a is obtained from $v^2 - u^2 = 2as$

$$\Rightarrow \ell - k\ell = 2a\ell$$

$$a = \frac{1-k}{2}$$

$$\text{velocity at } \frac{\ell}{2} \text{ is obtained from } u^2 - k\ell = 2 \cdot \left(\frac{1-k}{2} \right) \cdot \frac{\ell}{2}$$

$$\Rightarrow u^2 = k\ell - \frac{k\ell}{2} + \frac{\ell}{2} = \frac{k\ell}{2} + \frac{\ell}{2}$$

$$u = \sqrt{\frac{\ell}{2}(k+1)}$$

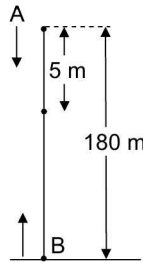
2.68 Motion in One Dimension

29. When B is projected, A is at $180 - \frac{1}{2}gt^2 = 175$ m above ground.

At that time $v_A = gt = 10 \text{ m s}^{-1}$

$$v_B = 25 \text{ m s}^{-1}$$

$\therefore v_r = 25 + 10 = 35 \text{ m s}^{-1}$, which will remain same through out



$$\text{Time when they meet: } t = \frac{d}{v_r} = \frac{175}{35} = 5 \text{ s}$$

$$\text{But at 5 s, For A: } S = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (5+1)^2$$

(since A is dropped 1s earlier)

$$= 180 \text{ m (i.e. at ground level)}$$

$$\text{for B: } T = \frac{2u}{g} = \frac{2 \times 25}{10} = 5 \text{ s i.e., both are at ground}$$

level $\Rightarrow$ They meet at ground level.

Their velocities will be in downward direction when they meet

$$v_A = gt = 10 \times 6 = 60 \text{ m s}^{-1}$$

$$v_B = 25 \text{ m s}^{-1} \Rightarrow v_A > v_B$$

30. (a)

$$x = A + Bt + Ct^2$$

$$v = B + 2Ct$$

$$a = 2C$$

$\Rightarrow$ The body is accelerating and moving up if $2C = -g \Rightarrow$ impossible as $C > 0$

$$x(0) = A \neq 0$$

$\Rightarrow$ (p, s)

(b)

$$x = Bt - Ct^2$$

$$v = B - 2Ct$$

$$a = -2C \rightarrow a \neq 0 \rightarrow \text{body is accelerating (p)}$$

$$\Rightarrow a < 0 \Rightarrow \text{deceleration}$$

$$(p, q, r)$$

(c)

$$x = A - Ct^2$$

$$v = -2Ct$$

$$a = -2C \rightarrow a \neq 0 \rightarrow \text{body is accelerating (p)}$$

$$a < 0, \text{ so deceleration}$$

$$v < 0 \Rightarrow \text{cannot be moving up}$$

$$x(0) = A \neq 0$$

$\Rightarrow$ (p), (s)

(d)

$$x = Ct^2$$

$$v = 2Ct$$

$$a = 2C \rightarrow a \neq 0, \rightarrow \text{body accelerating}$$

$$a > 0 \Rightarrow p$$

$$x(0) = 0 \Rightarrow r \Rightarrow (p, r)$$

IIT Assignment Exercise

31. The motion is three dimensional.

32. Distance covered in the first 4 steps = 2 feet

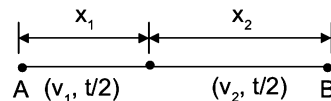
Distance covered in the first 8 steps = 4 feet

Distance covered in the first 11 steps = 7 feet

Since each step requires $t = 1$ s

total time = 11 s

33.



$$\text{Average speed} = \frac{x_1 + x_2}{t}$$

$$= \frac{v_1 \frac{t}{2} + v_2 \frac{t}{2}}{t} = \frac{v_1 + v_2}{2}$$

34. The observer is located at the origin (i.e., $x = 0$).

$\therefore$ Body is moving towards the observer.

35. For uniform motion

$$x = x_0 + vt$$

For straight line

$$x = x_0 + \tan \theta \cdot t$$

$$\Rightarrow v = \tan \theta$$

$$\frac{v_A}{v_B} = \frac{\tan \theta_1}{\tan \theta_2} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

$$v_A : v_B = 1:3.$$

36. Let u be the initial velocity

Let u_1 , u_2 , and u_3 be the velocities at the end of the three intervals of time

$$u_1 = u + at_1, u_2 = u + a(t_1 + t_2)$$

$$u_3 = u + a(t_1 + t_2 + t_3)$$

Since v_1 , v_2 and v_3 are the average velocities, and the motion is uniformly accelerated, we have

$$v_1 = \frac{u + u_1}{2} = u + \frac{1}{2}at_1$$

$$v_2 = \frac{u_1 + u_2}{2} = u + at_1 + \frac{1}{2}at_2$$

$$v_3 = \frac{u_2 + u_3}{2} = u + at_1 + at_2 + \frac{1}{2}at_3$$

$$\frac{v_1 - v_2}{v_2 - v_3} = \frac{-\frac{1}{2}at_1 - \frac{1}{2}at_2}{-\frac{1}{2}at_2 - \frac{1}{2}at_3} = \frac{t_1 + t_2}{t_2 + t_3}$$

$$\Rightarrow (v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$$

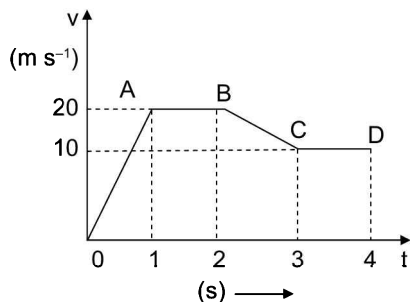
37. Speed = $\frac{\text{distance}}{\text{time}}$. If speed is constant,

$$\frac{\text{distance}}{\text{time}} = \text{constant } t \Rightarrow \text{distance} \propto \text{time}.$$

$$\text{Speed} = \frac{ds}{dt}$$

For uniform speed $\frac{ds}{dt}$ is constant. Graph (c) is the correct answer.

38.



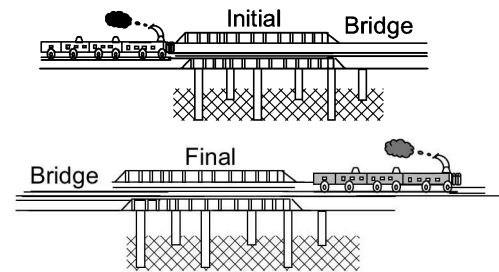
Distance travelled by the particle in 4s = area enclosed by OABCD and time axis

$$= \frac{1}{2} \times 1 \times 20 + 20 \times 1 + \frac{1}{2} \times 10 \times 1 + 10 \times 1 + 10 \times 1 = 55 \text{ m}$$

39. Total distance = 600 + 200 = 800 m

$$v = 10 \text{ m s}^{-1}$$

$$t = \frac{x}{v} = 80 \text{ s}$$



40. Let x be the distance between A and B.

$$\therefore t_1 = \frac{x}{v_1} = \frac{x}{20} \text{ s}$$

$$\therefore t_2 = \frac{x}{v_2} = \frac{x}{40} \text{ s}$$

$$\text{Average speed} = \frac{2x}{t_1 + t_2}$$

$$= \frac{2x}{\frac{x}{20} + \frac{x}{40}} = \frac{80}{3} \text{ m s}^{-1}$$

Aliter:

$$v_{AV} = \frac{2v_1v_2}{(v_1 + v_2)} = \frac{2 \times 20 \times 40}{(20 + 40)}$$

$$= \frac{80}{3} \text{ m s}^{-1}$$

41. For uniform motion, $x - t$ graph is a straight line with a positive slope; an $\theta = v$.

42. $S = ut + \frac{1}{2}at^2$

$$u = 0, S = 10 \text{ m}, t = 1 \text{ s}$$

$$\therefore a = \frac{2S}{t^2} = \frac{20}{1^2} = 20 \text{ m s}^{-2}$$

2.70 Motion in One Dimension

Distance travelled in the n th second

$$S_n = u_0 + \frac{a}{2}(2n - 1)$$

$$S_2 = 0 + \frac{20}{2}(2 \times 2 - 1) = 30 \text{ m}$$

Distances covered in consecutive time intervals from start are in the ratio 1 : 3 : 5 : 7

43. Average acceleration

$$= \frac{\text{Total change in velocity}}{\text{Total time}}$$

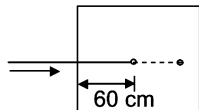
$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{40 - 0}{20} = 2 \text{ m s}^{-2}$$

44. $v = u + at$; $u = 0$; $v = 10 \text{ m s}^{-1}$; $t = 2 \text{ s}$

$$\therefore a = \frac{10}{2} = 5 \text{ m s}^{-2}$$

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{5}{2} \times 4^2 = 40 \text{ m}$$

45.



Let the initial velocity be v . When the bullet penetrates

$$60 \text{ cm}, v' = \frac{v}{2}$$

$$\therefore \left(\frac{v}{2}\right)^2 = v^2 + 2aS$$

$$\Rightarrow a = \frac{\frac{v^2}{4} - v^2}{2S} = \frac{-3v^2}{4} \times \frac{1}{2S} = \frac{-3v^2}{8 \times 60} \text{ m s}^{-2}$$

Further distance in which the bullet comes to rest is given by

$$0^2 - \left(\frac{v}{2}\right)^2 = 2aS'$$

$$S' = -\frac{v^2}{4} \times \frac{1}{2 \times (-3)v^2} \times 8 \times 60 = \frac{60}{3} = 20 \text{ cm}$$

Aliter:

$$v \rightarrow \frac{v}{2} \rightarrow 0$$

$\Rightarrow$ Equal time intervals ($\because a = \text{constant}$)

$$\Rightarrow 3x \rightarrow x \rightarrow 0; 3x = 60 \text{ cm} \Rightarrow x = 20 \text{ cm}$$

$\therefore$ Particle travels 20 cm more before stopping

46. Distance covered by the body in 10 s = area under speed time graph.

$$= \frac{1}{2} \times 10 \times 10 = 50 \text{ m}$$

47. Distance travelled by the body in the n th s is

$$x_n = u_0 + \frac{a}{2}(2n - 1)$$

$$u_0 = 0; a = \frac{v}{t} = \frac{10}{5} = 2 \text{ m s}^{-2}$$

$$\therefore x_n = 0 + \frac{2}{2}(2 \times 5 - 1) = 9 \text{ m}.$$

Aliter:

$$\text{Area between 4 s and 5 s} = \frac{(8 + 10)}{2} \times 1 = 9 \text{ m}$$

48. Distance travelled by the body in 10th second is

$$x_{10} = 10 + \frac{-2}{2}(2 \times 5 - 1) = 10 - 9 = 1 \text{ m}.$$

Aliter:

$$\text{Area between 9 s and 10 s} = \frac{1}{2} \times 2 \times 1 = 1 \text{ m}$$

49. Slope of the curve is $v = \frac{dx}{dt} = \infty$.

$\therefore$ body moves with infinite velocity.

$\Rightarrow$ Impossible

50. Displacement of the body in 5 second is

$$\frac{1}{2} \times 3 \times 2 - \frac{1}{2} \times 2 \times 1 + 1 \times 1 = 3 - 1 + 1 = 3 \text{ m}$$

51. Total distance travelled by the body = Area of trapezium

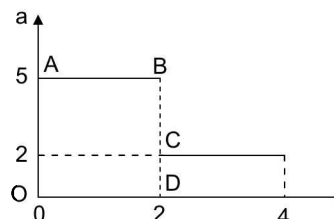
$$= \frac{1}{2}(10 + 30) \times 10 = 200 \text{ m}$$

52. The body is traveling with a uniform velocity of 10 m s^{-1}

from $t = 11$ to $t = 12 \text{ s}$

$$\therefore x_{12} = u.t = 10 \times 1 = 10 \text{ m}$$

53.



Velocity after 2 second

= area under graph OABD

$$= 5 \times 2 = 10 \text{ m s}^{-1}.$$

54. Distance travelled $S = ut + \frac{1}{2}at^2$

($\because$ a is constant from $t = 0$ to $t = 2$ s)

$$= \frac{1}{2} \times 5 \times 4 = 10 \text{ m}$$

55. $v = u + at = 10 + 2 \times 2$
 $= 14 \text{ m s}^{-1}$

56. When a body is thrown vertically upward, which is taken as the positive direction, velocity gradually decreases. After it reaches maximum height h , where the velocity is zero, it starts moving in the downward direction so that its velocity increases in the negative direction. Hence (d); or $a = -g = \text{constant}$
 $\Rightarrow$ curve (d).

57. $x = pt^2 + q + t + r$
 compare term by term with

$$s = s_0 + ut + \frac{1}{2} \cdot t^2$$

$$\Rightarrow \frac{a}{2} = p$$

$$\Rightarrow a = 2p$$

$\Rightarrow$ Acceleration is constant

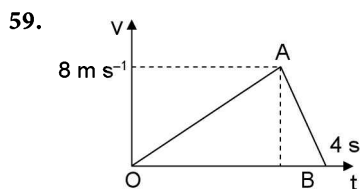
58. For uniform acceleration, $v = a_1 + a_2 t$, where a_1, a_2 are constants

$\Rightarrow$ non-uniform acceleration

Aliter:

$$a = \frac{dv}{dt} = 0.2 t \Rightarrow a \text{ is different at different times}$$

$\Rightarrow$ non-uniform acceleration



Distance covered by the particle

= area under graph

= area of $\triangle AOB$

$$= \frac{1}{2} \times t \times v = \frac{1}{2} \times 4 \times 8 = 16 \text{ m}$$

60. $S_1 = \frac{1}{2} at^2 = \frac{1}{2} a \times 10^2 = 50a$

$$S = \frac{1}{2} a \times (20^2) = 200a$$

$$S = S_1 + S_2 \Rightarrow S_2 = 200a - 50a = 150a$$

$$\Rightarrow S_2 = 3 S_1$$

61. $x = 2 - 5t + 6t^2$

comparing with $x_\theta = x_0 + ut + \frac{1}{2}at^2$

Initial velocity

$$v_0 = -5 \text{ unit}$$

Aliter:

$$v = \frac{dx}{dt} = -5 + 12t; \text{ At } t = 0, v = -5 \text{ unit}$$

62. Distance between stations = 1350 m.

Let v be the maximum velocity attained by the train. The train accelerates for a distance S_1 , and decelerates for a distance S_2 .

$$v^2 = 2\alpha x_1 = 2x_1 (\because \alpha = 1 \text{ m s}^{-2})$$

$$v^2 = 2\beta x_2 = 2 \times 3 \times x_2 = 6x_2 (\because \beta = 3 \text{ m s}^{-2})$$

$$x_1 + x_2 = 1350$$

$$= \frac{v^2}{2} + \frac{v^2}{6} = \frac{2v^2}{3}$$

$$\Rightarrow v = \sqrt{\frac{1350 \times 3}{2}} = 45 \text{ m s}^{-1}.$$

The train accelerates for a time t_1 and retards for time t_2 .

$$\text{Then } v = \alpha t_1, v = \beta t_2,$$

$$t = t_1 + t_2 = \frac{45}{1} + \frac{45}{3} = 60 \text{ s}$$

Aliter:

(i)

Let the time for acceleration be t_1 and there for deceleration be t_2

$$\Rightarrow v = 1 \cdot t_1 = 3 t_2 \Rightarrow \frac{t_1}{t_2} = 3$$

$$t = t_1 + t_2 = \frac{4}{3} t_1 \Rightarrow t_1 = \frac{3}{4} t$$

$$\text{Distance travelled} = \frac{1}{2} v \cdot t$$

$$= \frac{1}{2} \cdot 1 \cdot \frac{3}{4} t \cdot t = 1350 \text{ m}$$

$$t^2 = 450 \times 2 \times 4$$

$$\Rightarrow t = 60 \text{ s}$$

2.72 Motion in One Dimension

$$(ii) S = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$$

$$\Rightarrow t^2 = \frac{2S(\alpha + \beta)}{\alpha\beta} = 3600$$

$$\Rightarrow t = 60 \text{ s}$$

63. $k = 2 \text{ ms}^{-2}$ represents acceleration

$$\text{Hence } x = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times 3^2 = 9 \text{ m}$$

64. Comparing with

$$x = x_0 + ut + \frac{1}{2}at^2$$

$$\Rightarrow \frac{a}{2} = 3$$

$$\Rightarrow a = 6 \text{ unit}$$

65. Distance covered by the first stone $S = \frac{1}{2}gt^2 = h$

$$\therefore t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2 \text{ s}$$

Distance covered by the second stone

$$S = ut + \frac{1}{2}gt^2$$

Here $t = 2 - 1 = 1 \text{ s}$.

$$\therefore u = 20 - \frac{10}{2} = 20 - 5 = 15 \text{ m s}^{-1}$$

66. Time taken to fall through a height h , $t = \sqrt{\frac{2h}{g}}$.

$$\frac{t}{2} = \sqrt{\frac{2h'}{g}} \Rightarrow \frac{1}{2}\sqrt{\frac{2h}{g}} = \sqrt{\frac{2h'}{g}} \Rightarrow h' = \frac{h}{4}$$

$\therefore$ Height of the body from the ground

$$= h - \frac{h}{4} = \frac{3h}{4}$$

Aliter:

For $1^{\text{st}} \frac{t}{2} \text{ s}$, if distance covered is x , for next $\frac{t}{2} \text{ s}$ the distance covered is $3x$

$$\Rightarrow x + 3x = h$$

$$\Rightarrow x = \frac{h}{4}$$

$$\Rightarrow h' = 3x = \frac{3h}{4}$$

67. Distance travelled in the n th second

$$S_n = u + \frac{a}{2}(2n - 1)$$

$$S_n = \frac{S}{2} = 0 + \frac{10}{2}(2n - 1)$$

$$S = 10(2n - 1) \quad \text{--- (1)}$$

$$\text{Also } S = 0 + \frac{1}{2} \times 10 \times n^2 \quad \text{--- (2)}$$

Equating (1) and (2),

$$10(2n - 1) = 5n^2$$

$$n^2 - 4n + 2 = 0$$

$$n = \frac{+4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm \sqrt{8}}{2} = 2 + \sqrt{2} \text{ s (since } n > 1)$$

Aliter:

Let total time be T and height H .

$$\Rightarrow T = \sqrt{\frac{2H}{g}}$$

$$\text{Time to cover first } \frac{H}{2} = \sqrt{\frac{2\left(\frac{H}{2}\right)}{g}} = \frac{1}{\sqrt{2}}T$$

$$\therefore T\left(1 - \frac{1}{\sqrt{2}}\right) = 1 \Rightarrow T = \frac{\sqrt{2}}{(\sqrt{2} - 1)} = \frac{\sqrt{2}(\sqrt{2} + 1)}{(2 - 1)}$$

$$\Rightarrow T = (2 + \sqrt{2}) \text{ s}$$

68. $v_1^2 - u_1^2 = 2gh_1$ i.e., $0 - 4u^2 = 2gh_1$

$$0 - 9u^2 = 2gh_2$$

$$\frac{h_1}{h_2} = \frac{4}{9}$$

Aliter:

$$\frac{h_1}{h_2} = \frac{u_1^2}{u_2^2} = \frac{4}{9}$$

69. $S = 12.8 \text{ m}$, $u = -12 \text{ m s}^{-1}$

$$12.8 = -12t + \frac{1}{2} \times 10 \times t^2$$

$$5t^2 - 12t - 12.8 = 0$$

$$t = \frac{+12 \pm \sqrt{144 + 4 \times 5 \times 12.8}}{2 \times 5} = 3.2 \text{ s}$$

Aliter:

$$\text{Final velocity} = \sqrt{u^2 + 2gh}$$

$$= \sqrt{12^2 + 2 \times 10 \times 12.8} = 20 \text{ m s}^{-1}$$

$$\therefore \text{Time} = \frac{u + \sqrt{u^2 + 2gh}}{g} = \frac{12 - (-20)}{10} = 3.2 \text{ s}$$

$$70. S = ut + \frac{1}{2} at^2$$

$$19.6 = \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow t = \sqrt{\frac{2 \times 19.6}{9.8}} = 2 \text{ s}$$

Let t' be the time for distance $(19.6 - 4.9) \text{ m}$

$$19.6 - 4.9 = \frac{1}{2} \times 9.8 \times t'^2$$

$$\Rightarrow t' = \sqrt{\frac{14.7 \times 2}{9.8}} = \sqrt{3} \text{ s}$$

$\therefore$ Time for last 4.9 m fall is $2 - \sqrt{3} = 0.268 \text{ s}$.

$$71. t_1 = \sqrt{\frac{2h_1}{g}}; t_2 = \sqrt{\frac{2h_2}{g}}.$$

$$t_1 : t_2 = \sqrt{h_1} : \sqrt{h_2}$$

$$72. \text{ The distance travelled by the first body in 3 s, } x_1 = \frac{1}{2} \times 9.8 \times 3^2 = 44.1 \text{ m.}$$

$$\text{Distance travelled by the second body in 2 s is } x_2 = \frac{1}{2} \times 9.8 \times 2^2 = 19.6 \text{ m}$$

Hence their separation is $x_1 - x_2 = 24.5 \text{ m}$

$$73. \text{ For a body thrown vertically upwards, final velocity, } v = 0$$

$$v^2 - u^2 = 2ax$$

$$u^2 = 2gh$$

$$\frac{h_1}{h_2} = \frac{u_1^2}{u_2^2} = \frac{5^2}{10^2} = \frac{1}{4} \Rightarrow 4h_1 = h_2$$

$$74. S_n = \frac{a}{2}(2n - 1)$$

$$\therefore S_1 : S_2 : S_3 = 1 : 3 : 5$$

$$75. \text{ The body is uniformly accelerated.}$$

$$\Rightarrow \text{Acceleration} = -g$$

$$76. \text{ The body thrown up returns to the initial point with the same speed, but downward. Now the motions of both bodies are identical, and they reach the ground with the same speed.}$$

$$77. \text{ Relative velocity of overtaking} = 30 - 20 = 10 \text{ m s}^{-1}.$$

Total distance covered with this relative velocity during overtaking

$$= 200 + 300 = 500 \text{ m}$$

$$\text{Time taken} = 500/10 = 50 \text{ s.}$$

$$78. \text{ Relative acceleration} = 0 \Rightarrow \text{Relative motion is uniform}$$

$$\Rightarrow \frac{1}{\text{time}} = \frac{\text{relative velocity}}{\text{relative distance}} = \frac{100}{100}$$

$$\Rightarrow t = 1 \text{ s}$$

For the body

$$S = \frac{1}{2} \times 9.8 \times 1^2 = 4.9 \text{ m.}$$

$$\therefore \text{Distance from the ground} = 100 - 4.9 = 95.1 \text{ m.}$$

$$79. \text{ Relative velocity of 1 with respect to other remains constant. } u_{\text{rel}} \times t = h$$

$$\Rightarrow t = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}}$$

Height through which the dropped body falls =

$$\frac{1}{2}gt^2 = \frac{h}{4}$$

Height above ground at which collision takes place

$$= \frac{3h}{4}$$

$$80. \text{ Let the speed of the boat be } v \text{ and that of the river be } u. \text{ For the boat to return } v > u$$

$$t_{\text{still}} = \frac{2d}{v};$$

$$t_{\text{flow}} = t_{\text{down}} + t_{\text{up}}$$

$$= \frac{d}{v+u} + \frac{d}{v-u} = \frac{d \cdot 2v}{v^2 - u^2}$$

$$= \frac{2d}{v - \frac{u^2}{v}} > t_{\text{still}}$$

$$81. \text{ Displacement} = \text{area under the velocity-time graph with proper sign}$$

$$\text{So displacement} = 4 \times 2 - 2 \times 2 + 2 \times 4 = 12 \text{ m}$$

Distance = Total area under the speed-time graph

$$\text{So, distance} = 4 \times 2 + 2 \times 2 + 2 \times 4 = 20 \text{ m}$$

So 12 m, 20 m

2.74 Motion in One Dimension

82. Total time in second = $5 \times 60 + 12 = 312$ s

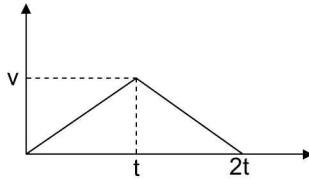
$$\frac{312}{80} = 3 + \frac{72}{80} \Rightarrow \text{Three full rounds completed}$$

He needs 8s more to complete the fourth round. i.e.,

$$\frac{8}{80} \times 100 = 10 \text{ m away.}$$

$$\therefore \text{Displacement} = 10 \text{ m}$$

83.



Let $2t$ be the time taken

$$\text{By symmetry, } s_1 = \frac{1}{2}at^2; s_2 = \frac{1}{2}at^2;$$

$$s = s_1 + s_2 = at^2 \Rightarrow t = \sqrt{\frac{s}{a}}$$

$$\therefore \text{Total time} = 2t = 2\sqrt{\frac{s}{a}}$$

Aliter:

$$v_{\max} = at$$

Area under the velocity time graph is the displacement

$$\Rightarrow s = \frac{1}{2} \cdot 2t \cdot at = at^2$$

$$t = \sqrt{\frac{s}{a}}$$

$$\Rightarrow \text{total time} = 2\sqrt{\frac{s}{a}}$$

84. Since the resistance is uniform, the acceleration is the same throughout. Let the acceleration be a . If s is the total penetration, $48^2 = 2as$. If s_1 is the total penetration after reaching speed of 24 m s^{-1} ,

$$24^2 = 2as_1$$

$$\Rightarrow \frac{s}{s_1} = 4$$

$$\text{But } s - s_1 = 60 \text{ cm}$$

$$\Rightarrow 4s_1 - s_1 = 60 \Rightarrow s_1 = 20 \text{ cm}$$

Aliter:

$$v \rightarrow \frac{v}{2} \rightarrow 0$$

$$3x \rightarrow x \rightarrow 0$$

$$3x = 60 \text{ cm}$$

$$x = \frac{60}{3} = 20 \text{ cm}$$

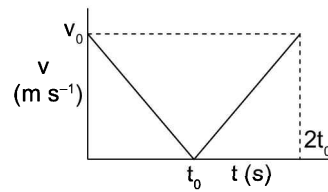
$$85. u_2^2 - u_1^2 = 2as_1$$

$$u_3^2 - u_2^2 = 2as_2$$

$$\frac{s_1}{s_2} = \frac{u_1^2(k^2 - 1)}{u_2^2(k^2 - 1)} = \left(\frac{u_1}{u_2}\right)^2$$

$$\text{where, } k = \frac{u_2}{u_1} = \frac{u_3}{u_2}$$

86.



The v - t graph is plotted in as shown:

$$v_0 = |0 - t_0| = t_0$$

Distance travelled = Area under the curve

$$= \left(\frac{1}{2} \times t_0 \times t_0\right) \times 2 = t_0^2$$

$$87. \frac{v_1}{v_2} = \frac{u_1 + a_1 t}{u_2 + a_2 t} = \frac{2k_1 + k_2 t}{k_1 + 2k_2 t} \text{ (Varies with time)}$$

Aliter:

$$v_{12} = (u_1 - u_2) + (a_1 - a_2)t \rightarrow \text{varies with time}$$

88. Let the front pass through point with speed u and the length of rope be L .

Speed of back v as it passes the point is given by $v^2 = u^2 + 2aL$

Speed of mid-point, v_1 is given by

$$v_1^2 = u^2 + 2a\left(\frac{L}{2}\right) = u^2 + \frac{v^2 - u^2}{2} = \frac{v^2 + u^2}{2}$$

$$v_1 = \sqrt{\frac{v^2 + u^2}{2}} = \sqrt{\frac{4 + 16}{2}} = \sqrt{10} \text{ m s}^{-1}$$

$$89. \bar{s} = \bar{u}t + \frac{1}{2}\bar{a}t^2 = \frac{1}{2}\bar{a}t^2; t = 1 \text{ s}$$

$$|\bar{s}| = \frac{1}{2}|\bar{a}|t^2 = \frac{\sqrt{\alpha^2 + \beta^2 + \gamma^2}}{2}$$

90. Let uniform acceleration be 'a' m s⁻²

Let speed at A be 'u' m s⁻¹

Speed at B = (u + a) m s⁻¹

Speed at C = (u + 2a) m s⁻¹

$$u + \frac{1}{2}a \times 1^2 = 8 \rightarrow u + \frac{1}{2}a = 8 \quad \text{--- (1)}$$

$$(u + a) + \frac{1}{2}a \times 1^2 = 18 \rightarrow u + \frac{3}{2}a = 18 \quad \text{--- (2)}$$

From (1) and (2), a = 10 m s⁻²

$$\therefore u = 8 - \frac{1}{2}a = 8 - 5 = 3 \text{ m s}^{-1}$$

$$v_B = u + a = 3 + 10 = 13 \text{ m s}^{-1}$$

$$91. v_B = \sqrt{\frac{6^2 + 8^2}{2}} = 5\sqrt{2} \text{ m s}^{-1}$$

$$v_{av1} = \frac{6 + 5\sqrt{2}}{2} \text{ m s}^{-1}$$

$$v_{av2} = \frac{12 + 14}{2} = 13 \text{ m s}^{-1}$$

$$t_1 = \frac{AB}{v_{AV1}}; t_2 = \frac{AB}{v_{AV2}} \Rightarrow \frac{t_1}{t_2} = \frac{v_{AV2}}{v_{AV1}}$$

$$\therefore \frac{t_1}{t_2} = \frac{v_{av2}}{v_{av1}} = \frac{13 \times 2}{(5\sqrt{2} + 6)} = \frac{26 \times (5\sqrt{2} - 6)}{14}$$

$$= 1.989 \approx 2.0$$

$$92. s = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 3^2 = 45 \text{ m}$$

93. The height of the balloon at time t = 2 s be h₁

$$h_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 4.9 \times (2)^2 = 9.8 \text{ m}$$

When ball is released, it has initial velocity upwards

$$v' = u + at = 0 + 4.9 \times 2 = 9.8 \text{ m s}^{-1}.$$

$\therefore$ Height attained by ball after it is out of balloon is given by

$$v^2 = u^2 - 2gh_2$$

(where, v = 0 and u = v').

$$\therefore 0 = (v')^2 - 2gh_2$$

$$\text{or } h_2 = \frac{v'^2}{2g} = \frac{(9.8)^2}{2 \times 9.8} = 4.9 \text{ m}$$

$$\therefore H = h_1 + h_2 = 9.8 + 4.9 = 14.7 \text{ m}$$

$$94. \frac{u}{\Delta t} = \frac{u_1 - u_2}{\Delta t_2 - \Delta t_1} = \frac{\sqrt{2a_1s} - \sqrt{2a_2s}}{\sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}}}$$

$$\Rightarrow u = (\sqrt{a_1 a_2}) \Delta t$$

$$\Rightarrow u = \sqrt{(2g)(8g)} \Delta t$$

$$\Rightarrow u = 4g \Delta t$$

95. During upward as well as downward motion, the air resistance opposes motion. Hence during upward motion the magnitude of acceleration > 9.8 m s⁻²; while during downward motion, it is less than 9.8 m s⁻². Hence t_{asc} < t_{des}.

96. Time of flight = $\frac{2u}{g} = 10 \text{ s}$; Distances travelled in the 5th s and 6th s are symmetric about the highest point reached.

Hence 1:1.

$$97. u \left(\frac{t_0}{2} \right) + \frac{1}{2}g \left(\frac{t_0}{2} \right)^2 = \frac{1}{2}gt_0^2$$

$$u \frac{t_0}{2} = \frac{3}{8}gt_0^2;$$

$$u = \frac{3}{4}gt_0$$

98. 5 balls thrown per second.

The time between two successive throws, $t = \frac{1}{5} = 0.2 \text{ s}$

By symmetry $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.2)^2 = 0.2 \text{ m}$

$$99. s_1 = \frac{1}{2}gt^2; s_2 = \frac{1}{2}g \left(t - \frac{1}{2} \right)^2$$

$$\frac{1}{2}gt^2 - \frac{1}{2}g \left(t - \frac{1}{2} \right)^2 = 20 \text{ m}$$

$$\Rightarrow t^2 - \left(t - \frac{1}{2} \right)^2 = \frac{40}{g} = 4$$

$$\Rightarrow t^2 - \left(t^2 + \frac{1}{4} - 2 \cdot \frac{1}{2}t \right) = 4$$

$$\Rightarrow t^2 - t^2 - \frac{1}{4} + t = 4$$

$$\Rightarrow t = 4.25 \text{ s}$$

2.76 Motion in One Dimension

Aliter:

$$\text{At } t = 0.5 \text{ s, relative distance} = \frac{1}{2}gt^2$$

$$= \frac{1}{2} \times 10 \times (0.5)^2$$

$$= 1.25 \text{ m}$$

$$\text{If } t = 0.5 \text{ s, relative velocity} = gt = 10 \times 0.5$$

$$= 5 \text{ m s}^{-1}$$

After both the bodies are airborne, the relative motion is uniform

$$\Rightarrow 20 = 1.25 + 5 \Delta t$$

$$\Delta t = 3.75 \text{ s}$$

$$\therefore \text{Time} = 3.75 + 0.5 = 4.25 \text{ s}$$

100. Time of ascent = Time of descent = 2 s

The distance travelled by a body, starting from rest and falling freely under gravity is

$$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 4 = 20 \text{ m}$$

101. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 160} = 40\sqrt{2} \approx 56 \text{ m s}^{-1}$

$$102. T = \frac{u}{g} + \sqrt{\frac{2\left(\frac{u^2}{2g} + h\right)}{g}}$$

$$= \frac{u + \sqrt{u^2 + 2gh}}{g}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{10 + \sqrt{10^2 + 2 \times 10 \times 2.2}}{10 + \sqrt{10^2 + 2 \times 10 \times 1.05}}$$

$$= \frac{10 + 12}{10 + 11} = \frac{22}{21}$$

$$103. t = \frac{u}{g} + \frac{\sqrt{u^2 + 2gh}}{g}$$

$$= \frac{10 + \sqrt{10^2 + 2 \times 10 \times 3.45}}{10}$$

$$= \frac{10 + 13}{10} = 2.3 \text{ s}$$

$$s = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 2.3^2 = 26.45 \text{ m}$$

(Nearest answer is 24.5 m)

$$104. H = \frac{u_1^2}{2g} + \frac{u_2^2}{2g} + \frac{u_3^2}{2g}$$

$$= \frac{4^2 + 6^2 + 12^2}{2 \times 9.8} = \frac{196}{2 \times 9.8} = 10 \text{ m}$$

$$105. v^2 = u^2 + 2as$$

$$= 10^2 + 2 \cdot 10 \cdot 15$$

$$= 400$$

$$v = 20 \text{ m s}^{-1}$$

$$v_{av} = \left(\frac{-20 + 10}{2} \right) = -5 \text{ m s}^{-1}$$

$$\Rightarrow |v_{av}| = 5 \text{ m s}^{-1}$$

Aliter:

$$T = \frac{u}{g} + \frac{\sqrt{u^2 + 2gh}}{g}$$

$$= \frac{10}{10} + \frac{\sqrt{10^2 + 2 \times 10 \times 15}}{10} = 3 \text{ s}$$

$$|s| = 15 \text{ m}$$

$$\therefore v_{AV} = \frac{|s|}{T} = \frac{15}{3} = 5 \text{ m s}^{-1}$$

$$106. T = \sqrt{\frac{2h_1}{g}} + \sqrt{\frac{2h_2}{g}}$$

$$= \sqrt{\frac{2 \times (11.25 + 8.75)}{10}} + \sqrt{\frac{2 \times 11.25}{10}}$$

$$= 2 + 1.5 = 3.5 \text{ s}$$

107. Let the elevator be the frame of reference with the observer at the bottom. In the problem, separation between the floor and the ceiling is 2.7 m and acceleration of the elevator is

$$a = 1.2 \text{ m s}^{-2} \text{ upwards}$$

$$a_{bc} = a_{bg} - a_{eg} = -g - a = -(g + a)$$

$$\therefore h = \frac{1}{2}[-(g + a)]t^2$$

$$\text{or } t = \sqrt{\frac{2h}{g + a}} = \sqrt{\frac{2 \times 2.7}{9.8 + 1.2}} = 0.7 \text{ s}$$

108. Relative uniform motion if Δx is linear in t . The relative acceleration should be zero. The accelerations are 2.5, 2.5 and -2.5 respectively

$$\Rightarrow \text{(i) and (ii) only}$$

109. Relative acceleration $a_{\text{rel}} = a + g$

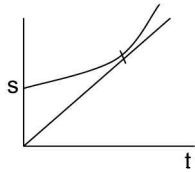
$$\Rightarrow \text{time of flight} = \frac{2u}{a + g}$$

110. The relative acceleration is $g - a$, where, a is the acceleration of the elevator $\frac{1}{2}(g - a)t^2 = 1.8 \text{ m}$, $t = 1 \text{ s}$

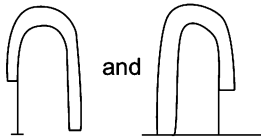
$$\Rightarrow \sqrt{2a_1s} = \sqrt{2 \times 2 \times 9}$$

$$10 - a = 3.6 \Rightarrow a = 6.4 \text{ m s}^{-2}$$

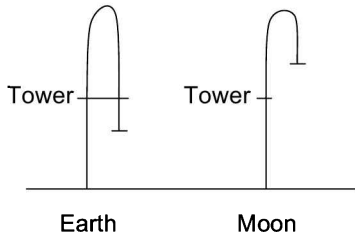
- 111.



112. The distance are same even for times $< 2 \text{ s}$
Example,



- 113.



Aliter:

$$\text{Let } u > 10 \text{ m s}^{-1}$$

$$\text{Then } S_e = u - \frac{g}{2}$$

$$S_m = u - \frac{g}{12}$$

$$\Rightarrow S_m > S_e$$

$$\text{Let } u = 0$$

$$S_e = \frac{g}{2}$$

$$S_m = \frac{g}{12}$$

$$S_e > S_m$$

$$\Rightarrow \text{for same } u \text{ in } (0, 10) S_m = S_e$$

$$u = 2.83 \text{ ms}^{-1} \rightarrow S_e = S_m$$

114. Area of acceleration time curve gives the change in velocity

$$\Delta v = \frac{1}{2} \times 2 \times 2 + 2 \times 2 = 6 \text{ m s}^{-1}$$

$$\therefore v = u + \Delta v = 1 + 6 = 7 \text{ m s}^{-1}$$

115. $u = u_0 + \frac{1}{2} \times 2 \times 2 = 1 + 2 \text{ m s}^{-1} = 3 \text{ m s}^{-1}$

$$S = ut + \frac{1}{2} at^2 = 3 \times 6 + \frac{1}{2} \times 2 \times 6^2 = 54 \text{ m}$$

116. $\Delta v = \frac{1}{2} \times 2 \times 2 + 6 \times 2 + \frac{2(4+2)}{2} + \frac{1}{2} \times 4 \times 2$
 $= 24 \text{ m s}^{-1}$

$$\therefore v_{\text{max}} = 1 + \Delta v = 1 + 24 = 25 \text{ m s}^{-1}$$

117. For the first 0.9 m distance:

$$(0.8u)^2 = u^2 - 2as \text{ [where } s = 0.9 \text{ m]} \Rightarrow 2as = u^2 - (0.8u)^2$$

If the bullet travels a further distance of 'd' metre, and comes to halt

$$0 = (0.8u)^2 - 2ad \Rightarrow 2ad = (0.8u)^2$$

$$\frac{2ad}{2as} = \frac{(0.8u)^2}{u^2 - (0.8u)^2} = \frac{0.64}{1 - 0.64} = \frac{64}{36}$$

$$\Rightarrow d = 0.9 \times \frac{64}{36} = 1.6 \text{ m}$$

$$\therefore \text{Total distance} = 1.6 + 0.9 = 2.5 \text{ m}$$

For the total distance travelled in the medium

$$\Rightarrow s = \frac{1}{2} at^2$$

$$2.5 = \frac{1}{2} at^2 \Rightarrow t = \sqrt{\frac{5}{a}}$$

$$v_{\text{avg}} \times t = s \Rightarrow v_{\text{avg}} \times \sqrt{\frac{5}{a}} = 2.5$$

$$\Rightarrow v_{\text{avg}} = 2.5 \sqrt{\frac{a}{5}} = \frac{\sqrt{5a}}{2}$$

Ans: (a), (c), (d)

118. $\begin{array}{ccccccc} v & & 2v & & 3v & & \dots & nv \\ \hline 1 \text{ s} & \bullet & 2 \text{ s} & \bullet & 3 \text{ s} & \bullet & \dots & n \text{ s} \end{array}$

For the body A, distance travelled in n second

$$= v \times 1 + 2v \times 1 + 3v \times 1 + \dots + nv \times 1$$

$$S_A = v(1 + 2 + \dots + n) = v \frac{n \times (n+1)}{2}$$

2.78 Motion in One Dimension

For second body B, distance moved in n second

$$S_B = \frac{1}{2} a n^2 \Rightarrow \frac{1}{2} a n^2 = v \frac{(n)(n+1)}{2}$$

$$a = v \left(1 + \frac{1}{n} \right) \Rightarrow \text{when } n \text{ is large: } a = v$$

$$\begin{aligned} \text{For A; distance travelled in } 20 \text{ s} &= v \frac{20 \times 21}{2} \\ &= 210 v \end{aligned}$$

For body B, the distance travelled in x th second

$$\frac{1}{2} a \times (2x - 1) = \frac{1}{2} v \left(1 + \frac{1}{n} \right) (2x - 1) = vx \text{ (for A)}$$

$$\frac{n+1}{n} (2x - 1) = 2x \Rightarrow n2x - n + 2x - 1 = n2x$$

$x = \text{Ans (a), (b), (d)}$

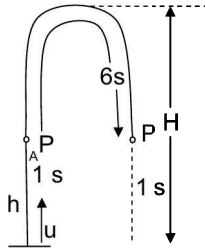
119. going up = 1 s;

Return to P = 6 s

Obviously from P to ground is 1 s again.

Hence time of flight

$$= 1 + 6 + 1 = 8 \text{ s}$$



For the fall from peak position,

$$\text{time } t = \frac{T}{2} = 4 \text{ s}$$

$$H = \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times 16 = 80 \text{ m}$$

$$u^2 = 2gH$$

$$\Rightarrow u = \sqrt{2 \times 10 \times 80} = 40 \text{ m s}^{-1}$$

$$h = ut - \frac{1}{2} g t^2$$

$$= 40 - \frac{1}{2} \times 10 = 35 \text{ m}$$

Since the body reaches peak position in 4 second and at that time its velocity is zero; in 5th second its fall is

$$h' = \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times 1 = 5 \text{ m}$$

120. (a)

for case (p), velocity jump from negative to positive $\Rightarrow$ infinite acceleration

(q) could be a parabola. (uniformly accelerated)

(r) and (s) one uniform motion with (r) with initial displacement.

(b)

It is uniform motion, $\Rightarrow$ (r), (s) by above.

(c)

Velocity increases (see (a) $\Rightarrow$ (p))

(d) The graph is linear $\Rightarrow$ (r), (s)

Additional Practice Exercise

121. Motion along a plane $\Rightarrow$ two dimensional

122. Displacement $\rightarrow$ single valued

Distance $\rightarrow$ slope positive, single valued

$$123. v^2 = u^2 + 2as$$

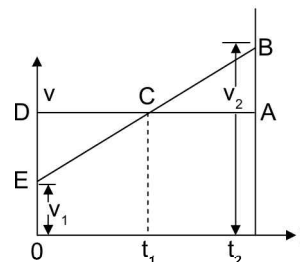
$$s = \frac{1}{2} a t^2 + ut + s_0$$

$$124. t = 2 \sqrt{\frac{2h}{g}}$$

as the heights fall, the time also falls.

125. Velocity is $+v$ or $-v$, and there are two values for every s .

126.



key (a)

Distance equal

$$\Rightarrow \frac{(v_1 + v_2)}{2} t = vt$$

$$v = \frac{v_1 + v_2}{2}$$

—(i)

Area of ΔEDC = Area of ΔABC

$$\Rightarrow \frac{v - v_1}{t_1} = \frac{v_2 - v}{(t_2 - t_1)} \quad \text{---(ii)}$$

Substitute value of v from (i) in (ii)

$$\Rightarrow \frac{t_1}{t_2 - t_1} = 1 \Rightarrow 2t_1 = t_2$$

$$\Rightarrow t_1 = \frac{t_2}{2}$$

$$127. S = \frac{1}{2} at_1^2 \quad \text{---(i)}$$

$$-S = at_1 \cdot t_2 - \frac{1}{2} at_2^2 \quad \text{---(ii) From (i) and (ii), we get}$$

$$\frac{t_2}{t_1} = \sqrt{2} + 1$$

$$128. A_1 + A_3 < 2A_1 < A_2$$

$$129. S = \frac{80^2 - 40^2}{2 \times 10} + \frac{40^2}{2 \times 5}$$

$$= 40^2 \left(\frac{3}{20} + \frac{1}{10} \right) = 400 \text{ m}$$

$$130. t_1 = \sqrt{\frac{2(S/2)}{a}} = \sqrt{\frac{2 \times (18/2)}{2}} = 3 \text{ s}$$

$$v = 3 \times 2 = 6 \text{ m s}^{-1}$$

$$t_2 = \frac{9}{6} = 1.5 \text{ s}$$

$$\Rightarrow T = t_1 + t_2 = 3 + 1.5 = 4.5 \text{ s}$$

$$v_{av} = \frac{S}{T} = \frac{18}{4.5} = 4 \text{ m s}^{-1}$$

$$131. t_1 = \sqrt{\frac{2(S/2)}{a_1}} = \sqrt{\frac{2 \times (18/2)}{2}} = 3 \text{ s}$$

$$v_1 = \sqrt{2a_1 \frac{S}{2}} = \sqrt{2 \times 2 \times \frac{18}{2}} = 6 \text{ m s}^{-1}$$

$$S_2 = v_1 t_2 + \frac{1}{2} a_2 t_2^2$$

$$\Rightarrow 9 = 6t_2 + \frac{1}{2} \cdot \frac{5}{2} \cdot t_2^2$$

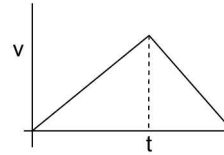
$$\Rightarrow \frac{5}{4} t_2^2 + 6t_2 - 9 = 0$$

$$\Rightarrow t_2 = \frac{-6 \pm \sqrt{36 - 4 \cdot \frac{5}{4} \cdot (-9)}}{2 \cdot \frac{5}{4}}$$

$$= \frac{-6 \pm 9}{\frac{5}{2}} = 1.2 \text{ s}$$

$$\therefore \text{Total time} = t_1 + t_2 = 3 + 1.2 = 4.2 \text{ s}$$

132.



$$v = a_1 t_1$$

$$v = a_2 t_2$$

$$T = (t_1 + t_2) = v \left(\frac{1}{a_1} + \frac{1}{a_2} \right)$$

$$\text{Distance} = \frac{1}{2} \cdot v \cdot T = \frac{1}{2} \cdot \frac{T^2}{\left(\frac{1}{a_1} + \frac{1}{a_2} \right)}$$

$$\Rightarrow \left(\frac{t_A}{t_S} \right)^2 = \left(\frac{a_{mB}}{a_{mA}} \right)$$

where, a_m represents the harmonic mean

$$\Rightarrow \frac{t_A}{t_B} > 1$$

$$t_A > t_B$$

133. key (c)

The tangent at the point should pass through the origin

134. At $t = 0$

$$v_B > v_A = a \text{ constant}$$

$$\text{At } t = t_1, v_B = 0$$

$$\text{At } t > t_1, v_B < 0$$

$$135. \frac{u + a \left(5 - \frac{1}{2} \right)}{u + a \left(7 - \frac{1}{2} \right)} = \frac{31}{27}$$

$$\Rightarrow 27u + 121.5a = 31u + 201.5a$$

2.80 Motion in One Dimension

$$\Rightarrow \frac{u}{a} = -20; 0 - 4 = at$$

$$t = \frac{-u}{a} = 20 \text{ s}$$

136. $v^2 = u^2 + 2as$

$$s = \frac{-u^2}{2a} = -\left(\frac{u}{a}\right) \frac{u}{2}$$

We can find $\frac{u}{a}$, but not u or a

$\Rightarrow$ data insufficient

137. Time = $\frac{(h_1 - h_2)}{u}$ (i) (From relative motion of bodies).

Also, from motion of 1st body,

$$\text{Time} = \sqrt{\frac{2h_1}{g}}$$

$$= \sqrt{\frac{2 \times 45}{10}} = 3 \text{ s. Substituting in (i)}$$

$$\Rightarrow \frac{45 - 30}{u} = 3$$

$$u = 5 \text{ m s}^{-1}$$

138. $t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2 \text{ s}$

(t = time taken to hit obstacle)

$$v = \sqrt{2gd} = \sqrt{2 \times 20 \times 10} = 20 \text{ m s}^{-1}$$

(v = velocity with which obstacle is hit)

$$v' = \frac{v}{2} = 10 \text{ m s}^{-1}$$

$$t' = 4 - 2 = 2 \text{ s}$$

$$h' = v't' + \frac{1}{2}g(t')^2$$

$$= 10 \times 2 + \frac{1}{2} \cdot 10 \cdot 2^2$$

$$= 20 + 20 = 40 \text{ m}$$

$$\therefore H = d + h' = 20 + 40 = 60 \text{ m}$$

139. (S_1 = height of free fall, before parachute opens;
 S_2 = height of fall with parachute open)

$$v_1^2 - 0 = 2gS_1$$

$$0 - v_1^2 = 2gS_2$$

$$\Rightarrow 0 = 2gS_1 - 2gS_2$$

$$2gS_1 - 2gS_2 = 0$$

$$\frac{S_1}{S_2} = \frac{a}{g} = \frac{2}{10} = \frac{1}{5}$$

$$\frac{H - S_2}{S_2} = \frac{1}{5}$$

$$\frac{S_2}{H} = \frac{5}{6}$$

$$S_2 = \frac{5}{6}H$$

140. $v^2 = u^2 + 2aS$

$$v^2 = 12^2 + 2 \times (-8) \times 5$$

$$v = 8 \text{ m s}^{-1}$$

$$\therefore \text{time} = \frac{2v}{g} = \frac{2 \times 8}{8} = 2 \text{ s}$$

141. v = velocity on hitting second

$$v^2 = u^2 + 2aS$$

$$v^2 = 10^2 + 2(-10)(-2.2)$$

$$= 144$$

$$v = \sqrt{144} = 12 \text{ m s}^{-1}$$

$$\therefore v' = \text{velocity at the beginning of last second}$$

$$\therefore v' = v - gt = 12 - 10 \cdot 1 = 2 \text{ m s}^{-1}$$

$$(t = 1 \text{ s for last second})$$

$$\therefore \text{Distance travelled in last second} =$$

$$\text{Average velocity} \times \text{time} = \frac{(v + v')}{2} \cdot t$$

$$= \frac{(12 + 2)}{2} \times 1 = 7 \text{ m}$$

142. Total time = $\sqrt{\frac{2 \times 20}{10}}$

(1st case)

$$= 2 \text{ s}$$

$$\therefore \text{Time till tower} = 1 \text{ s}$$

$$\therefore \text{Distance till pole} = \frac{1}{2} \cdot 10 \cdot 1^2 = 5 \text{ m}$$

$$\Rightarrow \text{Height of pole} = 20 - 5 = 15 \text{ m}$$

$$\text{Total time} = \sqrt{\frac{2 \times 40}{10}} = 2\sqrt{2} \text{ s}$$

(2nd case)

$$\text{Time till tower} = \sqrt{\frac{2 \times 25}{10}} = \sqrt{5} \text{ s}$$

$$\therefore \text{time} = (2\sqrt{2} - \sqrt{5})\text{s}$$

$$143. \frac{1}{2}at^2 = h \Rightarrow \frac{1}{2} \cdot 5 \cdot t^2 = 2.5$$

$t = 1 \text{ s}$; for particle falling outside,

$$-h = ut - \frac{1}{2}gt^2, t = 1 \text{ s}$$

$$-2.5 = u \cdot 1 - \frac{1}{2} \times 10 \times 1^2$$

$$u = 2.5 \text{ m s}^{-1}$$

$$144. \text{Ratio} = \frac{T + \frac{T}{\sqrt{2}}}{T - \frac{T}{\sqrt{2}}}$$

Where T is the time to go from top of the tower to ground.

$$\Rightarrow = \frac{\sqrt{2} + 1}{\sqrt{2} - 1} = 3 + 2\sqrt{2}$$

145. Let T be the time required to reach the highest point.

$\therefore$ Time required to reach half the height

$$= T - \frac{T}{\sqrt{2}} = (1 - 0.707)T = 0.293 T$$

$$\therefore \# \text{ balls} = \left[\frac{1}{0.293} \right] + 1 = 3 + 1 = 4$$

$$146. 2T = 2 \text{ s} \rightarrow T = 1 \text{ s}$$

max balls = 7, unit of which 6 are in air with time interval of 0.293 s

$$\begin{aligned} \text{Reaction time} &= 2 \text{ s} - 6 \times 0.293 \\ &= 0.242 \text{ s} = 242 \text{ ms} \end{aligned}$$

$$147. T = 2\sqrt{\frac{2H}{g}}, \text{ where } T = \text{time of flight in case - 1}$$

$$T_1 = \sqrt{\frac{2H}{g}} = \sqrt{2} \sqrt{\frac{2H}{g}}, \text{ where, } T_1 = \text{time of flight inside tank in case - 2}$$

$$\therefore T_2 = T - T_1 = (2 - \sqrt{2})\sqrt{\frac{2H}{g}}, \text{ where, } T_2 = \text{time in air, outside the tank.}$$

$$\Rightarrow (H - h) = \frac{1}{2}gT_2^2 = \frac{1}{2}g(2 - \sqrt{2})^2 \frac{2H}{g}$$

$$h = \left(1 - (2 - \sqrt{2})^2\right)H$$

$$\left(\sqrt{2} - 1\right) \approx$$

148. Considering relative motion of particles,

$$\frac{h}{u_2 - u_1} = \frac{h}{u_3 - u_2} = t, \text{ where } t = \text{time of travel}$$

$$\text{before collision } \therefore u_2 = \frac{u_1 + u_3}{2}$$

$$149. x = \frac{(x_1 + x_2)}{2}, \text{ where, } x = \text{location of midpoint of string}$$

$$= \frac{9}{2}t + t^2 + 5 \quad (\text{compare with } s = s_0 + ut + \frac{1}{2}at^2)$$

$$\Rightarrow u = \frac{9}{2} \text{ ms}^{-1}, a = 2 \text{ m s}^{-2}$$

$$\Rightarrow v = \frac{dS}{dt} = \frac{9}{2} + 2t; \text{ At } t = 0, v_0 = \frac{9}{2} \text{ m s}^{-1}, \text{ when}$$

$$v_0 \Rightarrow 2v_0 = 9 \text{ m s}^{-1}, \text{ we have}$$

$$2t = 9 - \frac{9}{2} = \frac{9}{2}$$

$$t = \frac{9}{4} = 2.25 \text{ s}$$

150. The speed of the boat should at least be equal to that of the river at B, in order to be able to make the return trip.

$$\Rightarrow 36 \text{ kmph} = 10 \text{ m s}^{-1} \text{ should be the speed of the river at the point}$$

$$\Rightarrow 10 = 2\sqrt{9 + \frac{x}{10}}$$

$$25 - 9 = \frac{x}{10}$$

$$x = 160 \text{ m}$$

151. Ans : (c) The initial and final relative displacements are zero. Only (c) meets this condition.

152. Min speed of aircrafts $v_1 = 144 \text{ kmph}$

$$= 144 \times \frac{5}{18} = 40 \text{ m s}^{-1}$$

Min speed of aircrafts $v_2 = 188 \text{ kmph}$

$$= 188 \times \frac{5}{18} \approx 52 \text{ m s}^{-1}$$

2.82 Motion in One Dimension

Speed of carrier $v_c = 36 \text{ kmph}$

$$= 36 \times \frac{5}{18} = 10 \text{ ms}^{-1}$$

We will consider that the length of runway (S_1) is designed for landing when carrier is at rest.

$$\therefore v_1^2 = 2a S_1$$

Now minimum length is S_2 and relative velocity

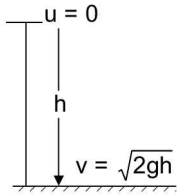
$$v = v_2 - v_c = S_2 - 10 = 42 \text{ m s}^{-1}$$

$$\therefore v^2 = 2a S_2$$

$$\therefore \frac{S_2}{S_1} = \left(\frac{v}{v_1} \right)^2 = \left(\frac{42}{40} \right)^2 = 1.10$$

$$\therefore \% \text{ increases in length} = 10$$

153.



$$v = \sqrt{2gh}; u_1 = \frac{1}{2}v = ev \left(\text{taking } \frac{1}{2} = e \right)$$

$$= e\sqrt{2gh}$$

$$h_1 = \frac{u_1^2}{2g} = \frac{e^2 \cdot 2gh}{2g} = e^2 h$$

Similarly, $u_2 = \frac{1}{2} u_1 = eu_1 = e^2 v = e^2 \sqrt{2gh}$

$$h_2 = \frac{u_2^2}{2g} = \frac{e^4 2gh}{2g} = e^4 h = (e^2)^2 h$$

Similarly, $h_3 = (e^2)^3 h$

Total distance =

$$h + 2h_1 + 2h_2 + \dots + \infty$$

$$= h + 2[h_1 + h_2 + \dots + \infty]$$

$$= h + 2[e^2 h + (e^2)^2 h + \dots + \infty]$$

$$= h + 2e^2 h [1 + e^2 + e^4 + \dots + \infty]$$

$$= h + 2e^2 h \times \frac{1}{(1 - e^2)} = h \frac{[1 - e^2 + 2e^2]}{(1 - e^2)}$$

$$= \frac{h[1 + e^2]}{[1 - e^2]} = \frac{h \left[1 + \left(\frac{1}{2} \right)^2 \right]}{\left[1 - \left(\frac{1}{2} \right)^2 \right]} = \frac{h \times 1.25}{0.75} = 1.67 h$$

$$154. ut = \frac{1}{2} \cdot at^2 = S;$$

$$u = \text{speed of bus} = 5 \text{ m s}^{-1}$$

$$a = \text{acceleration of mass}$$

$$S = \text{displacement} = 20 \text{ m}$$

$$T = \text{time taken}$$

$$t = \frac{S}{u}$$

$$\Rightarrow \frac{1}{2} \cdot a \left(\frac{S}{u} \right)^2 = S$$

$$\Rightarrow a = \frac{2u^2}{S} = \frac{2 \times 25}{20} = 2.5 \text{ m s}^{-2}$$

$$155. T_1 = \frac{v}{a_1} \text{ and } T_2 = \frac{v}{a_2}; \Delta T = T_1 - T_2$$

$$= v \left[\frac{1}{a_1} - \frac{1}{a_2} \right]$$

$$156. \text{ For the second particle, time of flight} = \frac{2 \times 15}{10} = 3 \text{ s}$$

$$\text{velocity of the first particle} = 20 - 3 \times 10$$

$$= -10 \text{ m s}^{-1}$$

$$\therefore \text{Relative velocity} = 15 - (-10) = 25 \text{ m s}^{-1}$$

157. Time taken to hit water,

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 25}{10}} = \sqrt{5} = 2.24 \text{ s}$$

Time taken by sound to reach open mouth of well

$$t_2 = T - t_1 = 2.31 - 2.24 = 0.07 \text{ s}$$

Speed of sound v

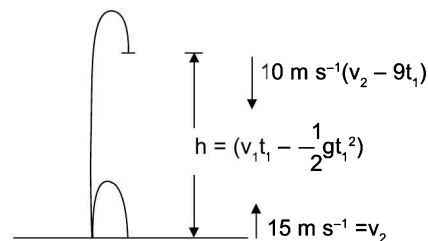
$$= \frac{\text{distance}}{\text{time}} = \frac{h}{t_2} = \frac{25}{0.07} \approx 357 \text{ m s}^{-1}$$

158. With respect to ground,

$$v^2 = u^2 + 2a.L$$

$$v = \sqrt{u^2 + 2aL}$$

159.



t_1 = time of flight of 2 body before bouncing

$$= 2 \frac{u_2}{g} = 2 \times \frac{15}{10} = 3 \text{ s}$$

h = position of particle 1 in time t_1

$$= (v_1 t_1 - \left(v_1 t_1 - \frac{1}{2} g t_1^2 \right)) = \left[v_1 - \frac{1}{2} g t_1 \right] t_1$$

$$= \left[20 - \frac{1}{2} \times 10 \times 3 \right] \times t_1 = (20 - 15) t_1 = 5 t_1$$

$$= v_{\text{rel}} (\text{after bouncing}) = +15 - (v - g t_1)$$

$$= (15 + g t_1 - v_1) = 15 + 10 \times 3 - 20 = 25 \text{ m s}^{-1}$$

$$h = v_{\text{rel}} \times t_2 = 25 \times t_2$$

$$\therefore 5 t_1 = 25 t_2$$

$$\frac{t_1}{t_2} = \frac{25}{5} = 5$$

$$160. \quad \frac{s}{2} = vt + \frac{1}{2} at^2 = 2vt$$

$$t = \frac{2v}{a} = \frac{s}{2} \cdot \frac{1}{2v}$$

$$a = \frac{8v^2}{s}$$

$$161. \quad \begin{array}{ccccccc} v_0 & x & 2v_0 & x & 4v_0 & x & 8v_0 \\ \hline \left(t_1 = \frac{x}{v_0} \right) & & \left(t_2 = \frac{x}{2v_0} \right) & & & & \end{array}$$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{total time}}$$

$$\begin{aligned} v_{\text{average}} &= \frac{nx}{\frac{x}{v_0} + \frac{x}{2v_0} + \frac{x}{4v_0} + \dots + \frac{x}{nv_0}} \\ &= \frac{nx}{\frac{x}{v_0} \left[1 + \frac{1}{2} + \frac{1}{4} + \dots \right]} = \frac{nx}{\frac{x}{v_0} \left[\frac{1}{1 - \frac{1}{2}} \right]} \end{aligned}$$

$$[\text{sum of geometric series} = \frac{a}{1-r} \text{ for } r \ll 1, n \text{ very}$$

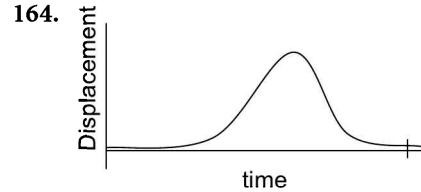
$$\text{large}] = \frac{nv_0}{2}$$

162. Distance is a scalar quantity, while velocity is a vector quantity. Only (average speed $\times$ time) gives the distance moved. Typical example is a body thrown up

$\Rightarrow$ average velocity for time of flight is zero, but average speed is $\frac{u}{2}$. Distance moved = $\frac{u}{2} T$ ($u \rightarrow$ initial velocity, $T \rightarrow$ time of flight)

$$163. \quad v_A t = 2 \left(\frac{1}{2} at^2 \right)$$

$$\Rightarrow v_A = at = v_B \rightarrow \text{final velocity of B}$$



165. For a body thrown up, effective acceleration vector is $\vec{g}$ acting down. When $\vec{v}$ and $\vec{g}$ are in opposite direction initially, we have retarded motion. When they are same direction we have accelerated motion. If $v = 0$ also it is accelerated, because $|\vec{v}|$ increases.

$$166. \quad T = \frac{2u}{g} \Rightarrow u = \frac{Tg}{2}$$

$$\Rightarrow \bar{s} = \bar{u} + \frac{1}{2} \bar{g} [2n - 1]$$

$$\Rightarrow \text{Take upwards as +ve}$$

$$\begin{aligned} \Rightarrow s &= \frac{Tg}{2} + \frac{1}{2} (-g) \left[2 \left(\frac{T+1}{2} \right) - 1 \right] \\ &= \frac{Tg}{2} - \frac{Tg}{2} = 0 \end{aligned}$$

$$167. \quad \bar{s} = \bar{u} + \frac{1}{2} \bar{a} (2n - 1) \Rightarrow$$

$\bar{u} + \frac{1}{2} \bar{g} (2n - 1)$: formula given in statement 2 is wrong

$$T = \frac{2u}{g} \Rightarrow u = \frac{gT}{2};$$

Take upward as +ve $\Rightarrow \bar{u} = \hat{j}u$

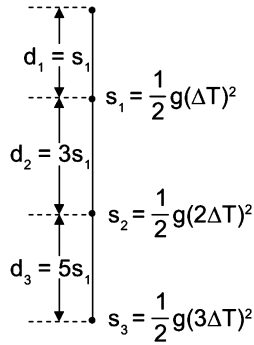
$$\bar{a} = \bar{g} = -\hat{j}g$$

$$\bar{S} = \hat{j} \left[u - \frac{1}{2} g (2n - 1) \right]$$

$$S = u - \frac{1}{2} g (2n - 1) = \frac{gT}{2} - \frac{g}{2} \left[\left(2 \cdot \frac{T}{2} - 1 \right) \right] = \frac{g}{2}$$

2.84 Motion in One Dimension

168.



The intervals need not be 1s, the formulae in statement 2 is only for a duration of 1s. As shown in the sketch, the separations are $\rightarrow s, 3s, 5s$. Hence are in arithmetic progression.

169. (Theoretical) $\Rightarrow v_1 = u_1 - gt_1, v_2 = u_2 - gt_2$
second body is projected in time Δt after the first body.

$$v_2 = u_2 - g(t_1 + \Delta t) \Rightarrow v_r = v_1 - v_2$$

$$= (u_1 - gt_1) - [u_2 - g(t_1 + \Delta t)]$$

$v_r = (u_1 - u_2) + g\Delta t = \text{constant}$. Hence its motion relative to the other is equivalent to motion in uniform velocity.

170. $a_1 = -kv_1$

$$a_2 = -kv_2$$

$$v_2 - v_1 = 0$$

$$\Rightarrow a_1 - a_2 = -k(v_1 - v_2) = 0$$

There are an infinite number of speeds

171. During $t = 0, t = 2$ s

$$t = 6 \text{ s}, t = 8 \text{ s}$$

Acceleration is same and it is 1 m s^{-2}

But during $t = 4 \text{ s}$ to $t = 6 \text{ s}$

Acceleration is 2 m s^{-2}

172. Slope of line OD is 0.8 m s^{-2}

so average acceleration is 0.8 m s^{-2} during 0 to 10 s

$$173. \text{Area} = \frac{1}{2} \times 2 \times 2 + 2 \times 2 + 2 \times \frac{(6+2)}{2} = 14 \text{ m}$$

174. Given velocity of the particle

$$v = v_0 \left[1 - \frac{t}{T} \right], T = 5 \text{ s}, v_0 = 10 \text{ cm s}^{-1}$$

$$v = v_0 \left[1 - \frac{t}{T} \right] \Rightarrow v = 10 - 2t,$$

compare with $v = u + at$

$$\Rightarrow a = -2 \text{ cm s}^{-2}$$

$$\Rightarrow u = 10 \text{ cm s}^{-1}$$

$$s = ut + \frac{1}{2}at^2 = 10 \times 6 + \frac{1}{2}(-2)6^2$$

$$= 60 - 36 = 24 \text{ cm}$$

175. at $t = 5$

$$v = 10 \left[1 - \frac{5}{5} \right] = 0 \text{ cm s}^{-1}$$

176. $a = -2 \text{ cm s}^{-2}$

$$177. H = \frac{1}{2} \cdot gt^2 = \frac{1}{8}gT^2 \quad (T = 2t)$$

$$= \frac{1}{8} \times 10 \times 4.4^2 = 24.2 \text{ m}$$

178. $H' = H + d_1$

$$= 24.2 + 7.05 = 31.25 \text{ m}$$

$$T' = \sqrt{\frac{2 \times H'}{g}} = \sqrt{\frac{2 \times 31.25}{10}} = 2.5 \text{ s}$$

$$\therefore \text{time} = 2.5 - 2.2 = 0.3 \text{ s}$$

179. $H'' = H + d_2 = 24.2 + 2.25 = 26.45 \text{ m}$

$$T'' = \sqrt{\frac{2H''}{g}} = \sqrt{\frac{2 \times 26.45}{10}} = 2.3 \text{ s}$$

$$\therefore \text{Time } 't_2' = T'' - T = 2.3 - 2.2 = 0.1 \text{ s}$$

180. $v_A^2 = v^2 - 2aS$

$$= 7^2 - 2 \times 5 \times 1.3 = 36$$

$$v_A = 6 \text{ m s}^{-1}$$

$$v_B^2 = v_A^2 - 2aS$$

$$= 36 - 2 \times 5 \times 1.1 = 25$$

$$v_B = 5 \text{ m s}^{-1}$$

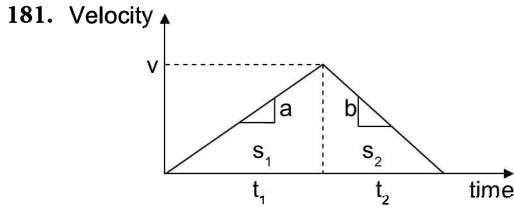
$$v_C^2 = v_B^2 - 2as = 25 - 2 \times 9 \times 0.5 = 16$$

$$v_C = 4 \text{ m s}^{-1} \rightarrow v_A : v_B : v_C = 6 : 5 : 4$$

Δv is same for all.

$$\text{Time spent} = \frac{\Delta v}{a}$$

$$\therefore \frac{1}{a_1} : \frac{1}{a_2} : \frac{1}{a_3} = \frac{1}{5} : \frac{1}{5} : \frac{1}{9}$$



Since maximum average velocity should correspond to minimum time possible, the velocity time graph should be as shown

$$v^2 = 2bs_2 = 2 \times 4 \times 2 = 16$$

$$v = 4 \text{ m s}^{-1} \Rightarrow \text{maximum speed}$$

$$v^2 = 2as_1 \Rightarrow s_1 = \frac{16}{2 \times 2} = 4 \text{ m}$$

$$\therefore \text{Total distance} = s_1 + s_2 = 6 \text{ m}$$

$$v = at_1 \Rightarrow t_1 = \frac{v}{a} = \frac{4}{2} = 2 \text{ s}$$

$$v = bt_2 \Rightarrow t_2 = \frac{v}{b} = \frac{4}{4} = 1 \text{ s}$$

$$\text{Total time} = 2 + 1 = 3 \text{ s}$$

$$\text{Average velocity} = \frac{\text{total distance}}{\text{total time}} = \frac{6}{3} = 2 \text{ m s}^{-1}$$

Key: (a), (b), (c)

182. Let a be the acceleration and v_A be the velocity at the first point A

$$\text{Then : } 30 = v_A + a \cdot 6 \quad \text{--- (1)}$$

$$\Rightarrow \text{use } s = ut + \frac{1}{2}at^2$$

$$126 = 6v_A + \frac{1}{2}a \times (6)^2 \Rightarrow 126 = 6v_A + 18a \quad \text{--- (2)}$$

$$\text{solve equations (1) and (2)} \Rightarrow v_A = 12 \text{ m s}^{-1}, a = 3 \text{ m s}^{-2}$$

$$\text{Distance to the first point} \Rightarrow v_A^2 = 2as_A \Rightarrow s_A$$

$$= \frac{(12)^2}{2 \times 3} = 24 \text{ m}$$

$$\text{Time to reach A} \Rightarrow s_A$$

$$= \frac{1}{2}at^2 \Rightarrow 24 = \frac{1}{2} \times 3 \times t^2 \Rightarrow t = 4 \text{ s}$$

Key: a, d

Aliter:

$$\left(\frac{v_A + v_B}{2} \right) t = S_{AB}; \text{ Given } t = 6 \text{ s,}$$

$$S_{AB} = 126 \text{ m, } v_B = 30 \text{ m s}^{-1}$$

$$\rightarrow v_A = 12 \text{ m s}^{-1}$$

$$a = \frac{(v_B - v_A)}{t} = 3 \text{ m s}^{-2}$$

$$t_A = \frac{v_A}{a} = \frac{12}{3} = 4 \text{ s}$$

$$183. S = -9t^2 + 20t$$

$$\text{compare with } s = ut + \frac{1}{2}at^2 \Rightarrow u = 20 \text{ m s}^{-1}$$

similar to body thrown up. It will return once, with $v = -20 \text{ m s}^{-1}$

184. Let the total time of flight be $2n$. Distance travelled in last second of its fall = distance travelled in first second = distance travelled in n th second of downward motion = $u + \frac{g}{2}[2n - 1]$

$$\Rightarrow 0 + \frac{1}{2} \cdot 10(2n - 1) = 45 \text{ m}$$

$$2n = 10 \text{ s}$$

$$\therefore \text{Total time of flight} = 10 \text{ s}$$

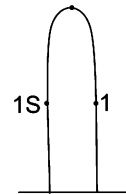
$$H = \frac{1}{2}g\left(\frac{T}{2}\right)^2 = \frac{1}{2} \times g \times (5)^2 = \frac{1}{2} \times 10 \times 25 = 125 \text{ m}$$

$$u^2 = 2gH = 2 \times 10 \times 125$$

$$\Rightarrow u = \sqrt{2500} = 50 \text{ m s}^{-1}$$

185. Consider a body dropped from height H . If it takes t s to reach ground distance covered in the last second.

$$h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = \frac{g}{2}(2t-1)$$



In this case : distance moved in the first second = distance moved in the last second.

$$\frac{g}{2}(2t-1) = 25 \Rightarrow 2t-1 = 5 \Rightarrow t = 3 \text{ s}$$

2.86 Motion in One Dimension

∴ time of flight $T = 2t = 6 \text{ s}$

$$v_{\text{final}} = gt = 30 \text{ m s}^{-1} = \text{Initial velocity}$$

$$H = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 9 = 45 \text{ m}$$

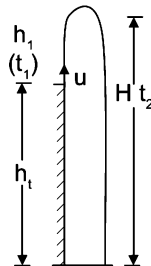
Ratio of distance covered in the last second to total

$$\text{distance} = \frac{25}{2H} = \frac{25}{90} = \frac{5}{18}$$

Key: a, c, d

186. The maximum height reached from the tower, h_1 is given by

$$\begin{aligned} u^2 &= 2gh_1 \Rightarrow h_1 \\ &= \frac{u^2}{2g} = \frac{900}{20} = 45 \text{ m} \end{aligned}$$



Time to reach maximum height t , is given by

$$\frac{1}{2}gt_1^2 = 45 \Rightarrow t_1^2 = 9$$

$$\Rightarrow t_1 = 3 \text{ s}$$

Hence time t_2 (from top position to ground)

$$= 8 - 3 = 5 \text{ s}$$

Maximum height H is given by

$$H = \frac{1}{2}gt_2^2 = \frac{1}{2} \times 10 \times 5^2 = 125 \text{ m}$$

∴ Height of the tower $= 125 - 45 = 80 \text{ m}$

$$v = 0 + gt_2 = 0 + 10 \times 5 = 50 \text{ m s}^{-1}$$

187. Relative velocity $= \text{constant} = u_1 + u_2 = 30 \text{ m s}^{-1}$

$$\text{Time of collision} = \frac{60}{30} = 2 \text{ s}$$

$$\text{Height of fall} = \frac{1}{2}gt^2 = 5 \times 4 = 20 \text{ m}$$

$$\text{Height of collision} = 60 - 20 = 40 \text{ m}$$

Key: (a), (c)

188. (a)

$$x = u(t - 3) + a(t - 3)^2$$

$$v = u + 2a(t - 3)$$

$$\text{acceleration} = 2a$$

$$x(3) = 0$$

(b)

$$x = ut + at^2$$

$$v = u + 2at$$

$$\text{acceleration} = 2a$$

$$x(3) = 3u + 9a \neq 0$$

(c)

$$\frac{dx}{dt} = \text{slope} = a \text{ constant } t$$

⇒ uniform motion

$$x(3) = 0$$

(d)

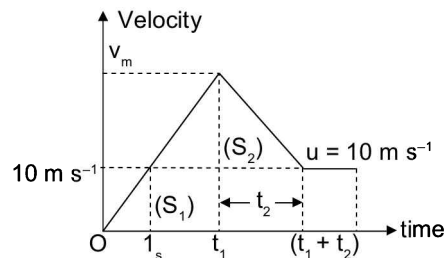
$$v^2 = u^2 + 4as = u^2 + 2(2a)s$$

$$\text{Acceleration} = 2a$$

$$s = ut + \frac{1}{2} \cdot 2at^2 = ut + at^2$$

$$s(3) \neq 0$$

189.



For minimum time, the velocity time graph will be as shown. Let s_1 represent the distance moved in acceleration and s_2 represent the distance moved in deceleration. Distance in uniform velocity $s_3 = 15 \text{ m}$ (given data)

$$v_{\text{max}} = v_m, u = 10 \text{ m s}^{-1}$$

$$v_m^2 = 2as_1 \quad \text{--- (1)}$$

$$v_m^2 - u^2 = 2as_2 \quad \text{--- (2)}$$

$$s_1 + s_2 = (100 - 15) = 85 \text{ m}$$

$$s_2 = (85 - s_1) \quad \text{--- (3) from equations (1), (2) \& (3)}$$

$$2as_1 - u^2 = 2a(85 - s_1)$$

$$(a = 10 \text{ m s}^{-2}, u = 10 \text{ m s}^{-1})$$

$$\Rightarrow 20s_1 - 100 = 20(85 - s_1)$$

Solving $s_1 = 45 \text{ m} \Rightarrow v_m^2 = 2as_1 = 2 \times 10 \times 45 \Rightarrow v_m = 30 \text{ m s}^{-1}$

$$at_1 = 30 \Rightarrow t_1 = \frac{30}{10} = 3 \text{ s}$$

(reaches a maximum velocity = 30 m s^{-1})

$$v_m^2 - u^2 = 2as_2 \Rightarrow 900 - 100 = 20s_2 \Rightarrow s_2 = 40 \text{ m}$$

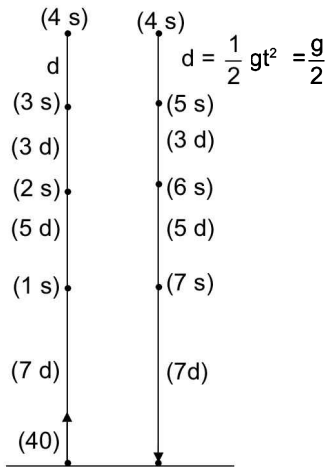
$$u = v_m - at_2 \Rightarrow 10 = 30 - 10t_2 \Rightarrow t_2 = 2 \text{ s}$$

$\therefore t_1 + t_2 = 5 \text{ s}$ (when deceleration ends and $v = 10 \text{ m s}^{-1}$)

Obviously at $t = 1 \text{ s}$, $v = 10 \text{ m s}^{-1}$ [$\because v = at$]

And at $t = 6 \text{ s}$, $v = 10 \text{ m s}^{-1}$ [in speed limit zone]

190.



$$\text{Time of flight } T = \frac{2u}{g} = \frac{2 \times 40}{10} = 8 \text{ s}$$

$\Rightarrow 4 \text{ s}$, at peak position Velocity at 3 s

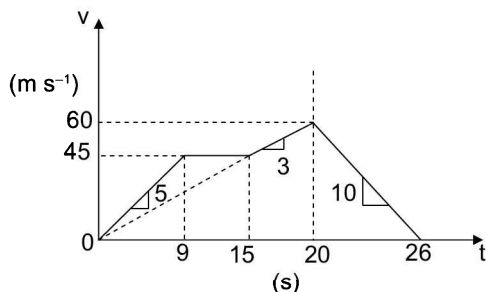
$$= |\text{velocity at } 5 \text{ s}| = g \times 1 = g$$

If we calculate the distance moved in 4th and

$$5\text{th second, both are equal to } d = \frac{1}{2}gt^2 = \frac{g}{2}$$

The distance moved in other unit intervals of time are in the ratio $1 : 3 : 5 : 7$, and are marked in the figure

191. (i)

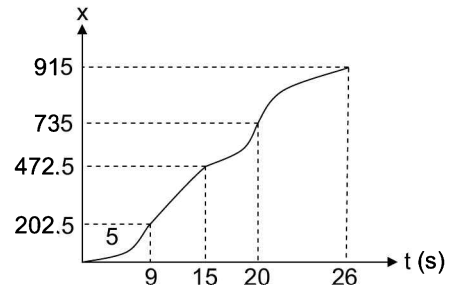


The instantaneous acceleration at t will equal average acceleration upto t if tangent of v - t graph passes through origin, as the body starts from rest

$$\left(a_{av} = \frac{v-0}{t-0} \right). \text{ Using the above, we can see that}$$

throughout the interval $t = 15$ to $t = 20$, average acceleration = instantaneous acceleration = 3 ms^{-2} and throughout the interval $t = 0$ to $t = 9$, the acceleration $a = 5 \text{ m s}^{-2}$.

(ii)



The instantaneous velocity at t will equal average velocity up to t , if tangent of s - t graph passes through origin. The body has acceleration ≥ 0 in the interval $t = 0$ to $t = 20 \text{ s}$. Thus average velocity < instantaneous velocity similarly by examining the s - t graph we can conclude that t will lie between $t = 20 \text{ s}$ to $t = 26 \text{ s}$ when avg. velocity = instantaneous velocity (This can also be verified by checking s/t values at beginning and end of each of time intervals and checking against velocity value; during the same intervals; as shown below:

Displacement during 0 to 20 s

$$= \frac{1}{2} \times 9 \times 45 + (15 - 9) \times 45 + \frac{(60 + 45)}{2} \times (20 - 15) = 735 \text{ m}$$

During interval $t = 20 \text{ s}$ to $t = 26 \text{ s}$ average velocity

$$= \frac{s}{t} = [735 + 60(t - 20) - \frac{1}{2} \times 10 \times (t - 20)^2] \frac{1}{t}$$

Instantaneous velocity = $60 - 10(t - 20)$

Equating both and solving, $t = \sqrt{493} \text{ s}$

192.

$$(i) \quad s_A = 2t + \frac{1}{2}at^2$$

$$s_B = 12(t - 10) + \frac{1}{2}a(t - 10)^2$$

$$= 12t - 120 + \frac{1}{2}at^2 + 50a - 10at$$

2.88 Motion in One Dimension

Condition for B to overtake A

$$s_B - s_A = 0 \text{ at same point} \Rightarrow 10t(1-a) = 120 - 50a$$

$$\Rightarrow t(1-a) = 12 - 5a$$

$$\Rightarrow t = \frac{12-5a}{(1-a)} = \frac{5(1-a)+7}{(1-a)} = 5 + \frac{7}{1-a}$$

$$t > 10 \Rightarrow \in \left(\frac{-2}{5}, 1 \right) \Rightarrow a_{\max} = 1 \text{ m s}^{-2}$$

$$(ii) \quad t = 25 \text{ s} \Rightarrow 25 = 5 + \frac{7}{1-a} \Rightarrow \frac{7}{1-a} = 20$$

$$\Rightarrow a = 0.65 \text{ m s}^{-2}$$

193. Let t_c be the time of collision and S be its distance from origin.

$$\text{Car A: } S = 20t_c$$

$$\text{Car B: } S = \frac{1}{2} \cdot 8(t_c - 10)^2 = 4t_c^2 + 400 - 80t_c$$

$$\therefore 20t_c = 4t_c^2 + 400 - 80t_c \Rightarrow t_c^2 - 25t_c + 100 = 0$$

$$t_c = 20 \text{ s or } 5 \text{ s}$$

$$t_c = 5 \text{ s to be ignored because B started only at } t = 10 \text{ s.}$$

$$\therefore t_c = 20 \text{ s and } S = 400 \text{ m}$$

Car C:

$$S = 10(16-8) + \frac{1}{2} \times 5(16-8)^2$$

$$+ [10 + 5(16-8)]$$

$$[20-16] - \frac{1}{2} \times a[20-16]^2$$

$$\Rightarrow S = 80 + 160 + 200 - 8a$$

$$\text{Putting } S = 400 \text{ m}$$

$$400 = 440 - 8a \Rightarrow a = 5 \text{ m s}^{-2} \text{ (deceleration)}$$

Since A was travelling with uniform velocity, it is obvious no other car has overtaken it till collision.

Since B started later than C, we should check whether B has overtaken C till $t = 16$ s.

$$S_c = 10t + \frac{5}{2}t^2$$

(Keeping $t = 0$ for car C)

$$S_B = \frac{1}{2} \cdot 8(t-2)^2 = 4(t-2)^2$$

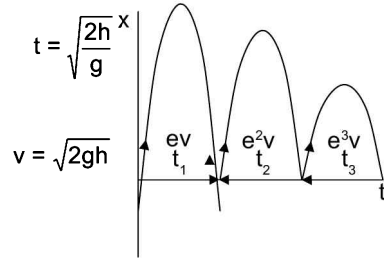
$$\text{Equating } 10t + \frac{5}{2}t^2 = 4t^2 + 16 - 16t$$

$$\Rightarrow t = \frac{26 \pm \sqrt{580}}{3} \text{ s}$$

None of the two values lie between $t = 2$ s and $t = 8$ s.

$\therefore$ B has not overtaken C. Hence C hit A and B hit C.

194.



$$t = \text{Time to fall from height } h = \sqrt{\frac{2h}{g}}$$

$$\text{Velocity, when it hits the ground, } v = \sqrt{2gh}$$

Velocity at first rebound = ev

$\therefore$ Time of flight t_1 after rebound is obtained by:

$$evt_1 - \frac{1}{2}gt_1^2 = 0 \Rightarrow t_1 = \frac{2ev}{g}$$

$$= \frac{2e}{g} \sqrt{2gh} = 2\sqrt{2e} \sqrt{\frac{h}{g}} = 2e \sqrt{\frac{2h}{g}}$$

$$\text{Similarly, } t_2 = 2e^2 \sqrt{\frac{2h}{g}}; t_3 = 2e^3 \sqrt{\frac{2h}{g}},$$

$$\therefore \text{ total time} = t + t_1 + t_2 + t_3 + \dots$$

$$= \sqrt{\frac{2h}{g}} [1 + 2e + 2e^2 + 2e^3 + \dots]$$

$$= \sqrt{\frac{2h}{g}} \left[1 + \frac{2e}{1-e} \right] = \sqrt{\frac{2h}{g}} \left[\frac{1+e}{1-e} \right]$$

195. $v_1 = -u + gt$ (Taking downward direction as positive)

$$v_2 = u + gt$$

$$v_{\text{rel}} = \Delta v = 2u = \text{constant}$$

$$s = v_{\text{rel}} \cdot t = 2ut \text{ (linear plot)}$$

$$196. \quad s_1 = ut - \frac{1}{2}gt^2;$$

$$s_2 = -ut - \frac{1}{2}gt^2 + h$$

$$s_1 = s_2; 2ut = h; t = \frac{h}{2u}$$

$$s_1 = ut - \frac{1}{2}gt^2$$

$$= u \cdot \frac{h}{2u} - \frac{1}{2}g \cdot \frac{h^2}{4u^2}; s_1 > 0$$

$$\text{i.e., } \frac{h}{2} - \frac{gh^2}{8u^2} > 0 \Rightarrow \frac{gh}{4u^2} < 1 \Rightarrow u > \frac{\sqrt{gh}}{2}$$

Aliter:

Relative velocity = $2u$

Relative distance = h

Relative acceleration = 0

$$t = \frac{h}{2u} < \text{time of flight of first ball} = \frac{2u}{g}$$

$$\Rightarrow u > \frac{\sqrt{gh}}{2}$$

197. Let v_{reb} be speed just after rebound and v_{impact} be speed just before hitting the liquid from air. For just coming out of fluid, the speed with which the ball comes out of the liquid is zero. While the ball is moving upwards in the fluid, it experiences a net deceleration of $(a + g)$. While the ball is moving downwards in the fluid, it experiences a net acceleration of $(g - a)$. Using the equation of motion for uniformly accelerated motion. $v^2 = u^2 + 2aS$ twice

$$v_{\text{reb}}^2 = 2(g + a)h_f$$

$$v_{\text{reb}}^2 = v_{\text{impact}}^2 + 2(g - a)h_f$$

$$v_{\text{impact}}^2 = v_{\text{reb}}^2 - 2(g - a)h_f$$

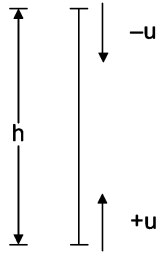
$$v_{\text{impact}}^2 = 2gh = 2(g + a)h_f - 2(g - a)h_f$$

$$2gh = 4ah_f \Rightarrow h = \frac{2a}{g} \cdot h_f$$

$$198. (i) T = \sqrt{\frac{2H}{g}} \quad \text{--- (1)}$$

$$u = \sqrt{2gH} \quad \text{--- (2)}$$

$$h = ut - \frac{1}{2}gt^2 \Rightarrow t^2 - \frac{2}{g}ut + \frac{2h}{g} = 0$$



$$\Rightarrow (\text{Substituting (2)}) \quad t^2 - \frac{2}{g}\sqrt{2gH}t + \frac{2h}{g} = 0$$

$$\Rightarrow t^2 - 2\sqrt{\frac{2H}{g}}t + \frac{2h}{g} \cdot \frac{H}{H} = 0$$

$$\Rightarrow (\text{using (1)}) \quad t^2 - 2Tt + T^2 \cdot \frac{h}{H} = 0$$

$$\Rightarrow t = \frac{2T \pm \sqrt{4T^2 - 4T^2 \frac{h}{H}}}{2} = T \left[1 \pm \sqrt{1 - \frac{h}{H}} \right]$$

$$\Rightarrow \frac{t}{T} = 1 \pm \sqrt{1 - \frac{h}{H}}$$

(minus applies when going up, plus applies when coming down)

[cross check: spl: cases]

$$h = H \Rightarrow t = T \text{ (checks OK)}$$

$$h = 0 \Rightarrow t = 0 \text{ or } 2T$$

- (ii) Let particle A be thrown at $t = 0$ and particle B at $t = \Delta t$. If H is max. height reached by A and at time $t = T$ and if h = height at which the particles collide at time t , then as per (a) above,

$$\frac{t}{T} = 1 + \sqrt{1 - \frac{h}{H}} \quad \text{--- (1)}$$

$$\frac{t - \Delta t}{T} = 1 - \sqrt{1 - \frac{h}{H}} \quad \text{--- (2)}$$

Dividing (2) by (1)

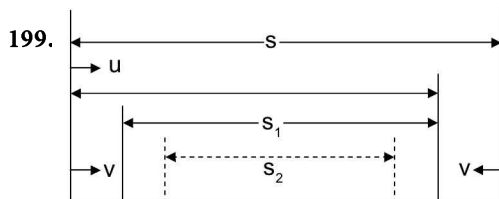
$$\frac{t - \Delta t}{t} = \frac{1 - \sqrt{1 - \frac{h}{H}}}{1 + \sqrt{1 - \frac{h}{H}}};$$

$$\text{Given } t = 7 \text{ s and } \frac{h}{H} = \frac{7}{16}$$

$$\Rightarrow \frac{7 - \Delta t}{7} = \frac{1 - \sqrt{1 - \frac{7}{16}}}{1 + \sqrt{1 - \frac{7}{16}}}$$

$$\Rightarrow 1 - \frac{\Delta t}{7} = \frac{1 - 3/4}{1 + 3/4} = \frac{1/4}{7/4} = \frac{1}{7} \Rightarrow \Delta t = 6 \text{ s}$$

2.90 Motion in One Dimension



Let v = velocity of the two bodies

u = velocity of the particle

Original distance = s ,

$$\text{time of flight} = \frac{s}{(u + v)}$$

$$s_1 = s - 2v \times t = s - \frac{s \cdot 2v}{v + u} = \frac{s(u - v)}{(v + u)}$$

$$\begin{aligned} \text{2nd time of flight } t_1 &= \frac{s(u - v)}{(v + u)} \cdot \frac{1}{(u + v)} \\ &= \frac{s(u - v)}{(u + v)^2} \end{aligned}$$

$$\begin{aligned} s_2 &= s_1 - 2vt_1 = \frac{s(u - v)}{(v + u)} - \frac{s(u - v)}{(v + u)^2} 2v \\ &= \frac{s(u - v)^2}{(u + v)^2} \end{aligned}$$

$$\text{Similarly, } s_3 = \frac{s(u - v)^3}{(u + v)^3}; s_n = \frac{s(u - v)^n}{(u + v)^n}$$

However small be the value of s_n , it is always possible for the particle to make a trip reducing the distance between the bodies to $1/3$ rd of s_n . Hence for s_n to become equal to zero, n should be infinite. The particle should make infinite trips.

$$\text{Time taken for collision } T_{\text{col}} = \frac{s}{2v}$$

Distance travelled by the particle by the time collision takes place = speed of particle $\times$ time till collision =

$$u \cdot \frac{s}{2v} = \frac{us}{2v}$$

200.

(i) To avoid collision; $v_{\text{rel}}^2 < 2ad$

$$v_{\text{rel}} = v_A - v_B$$

$$v_{\text{rel}}^2 < 2ad \quad v_A - v_B < \sqrt{2ad}$$

(ii) $v_B = 0$; $a = 20 \text{ m s}^{-2}$. $d = 50 \text{ m}$;

$$v_A = 162 \times \frac{5}{18} = 45 \text{ m s}^{-1}; \sqrt{2ad} = \sqrt{2 \times 20 \times 50}$$

$$= 44.7 \text{ m s}^{-1}$$

$\therefore v_A - v_B$ is greater than $\sqrt{2ad}$. Hence collision cannot be avoided.

CHAPTER

3

MOTION IN TWO DIMENSIONS

■■ CHAPTER OUTLINE

Preview

STUDY MATERIAL

Analysis of Two Dimensional Motion

Projectile Motion

- Concept Strands (1-4)

For Additional Reading

- Concept Strands (5-7)

Relative Motion in Two Dimensions

- Concept Strands (8-10)

Circular Motion

- Concept Strand (11)

CONCEPT CONNECTORS

- 20 Connectors

TOPIC GRIP

- Subjective Questions (10)
- Straight Objective Type Questions (5)
- Assertion–Reason Type Questions (5)

- Linked Comprehension Type Questions (6)
- Multiple Correct Objective Type Questions (3)
- Matrix-Match Type Question (1)

IIT ASSIGNMENT EXERCISE

- Straight Objective Type Questions (80)
- Assertion–Reason Type Questions (3)
- Linked Comprehension Type Questions (3)
- Multiple Correct Objective Type Questions (3)
- Matrix-Match Type Question (1)

ADDITIONAL PRACTICE EXERCISE

- Straight Objective Type Questions (40)
- Assertion–Reason Type Questions (10)
- Linked Comprehension Type Questions (9)
- Multiple Correct Objective Type Questions (8)
- Matrix-Match Type Questions (1)
- Subjective Questions

ANALYSIS OF TWO DIMENSIONAL MOTION

A particle moving on a plane surface is said to be in two-dimensional motion. The motion of a particle in two dimensions can be treated as the vector sum of two distinct and independent one-dimensional motions in two mutually perpendicular directions on the plane. Consequently, all vector physical quantities of the motion in two-dimensions can be represented as the vector sum of the corresponding physical quantities along two mutually perpendicular directions on the plane. For example, if a particle is executing a two dimensional motion on the XY plane, then its position, velocity and acceleration at time 't' on the XY plane can be represented as

- (i) $\vec{r}(t) = x\hat{i} + y\hat{j}$, where $\vec{r}(t)$ is the position vector of the particle in the XY plane, x is the position co-ordinate of the particle in the X-direction and y is the position co-ordinate of the particle in the Y-direction ($\hat{i}$ and $\hat{j}$ are respectively the unit vectors along X and Y directions that are mutually perpendicular). This gives us the magnitude of the position vector $r = \sqrt{x^2 + y^2}$.

- (ii) $\vec{v}(t) = v_x\hat{i} + v_y\hat{j}$, where $\vec{v}(t)$ is the velocity vector of the particle in the XY plane, v_x is the component of the velocity of the particle in the X-direction and v_y is the component of the velocity of the particle in the Y-direction. This gives us the magnitude of the velocity vector $v = \sqrt{v_x^2 + v_y^2}$.

- (iii) $\vec{a}(t) = a_x\hat{i} + a_y\hat{j}$, where $\vec{a}(t)$ is the acceleration vector of the particle in the XY plane, a_x is the component of the acceleration of the particle in the X-direction and a_y is the component of the acceleration of the particle in the Y-direction. This gives us the magnitude of the acceleration vector $a = \sqrt{a_x^2 + a_y^2}$.

All one-dimensional motions are special cases of two-dimensional motion. Typical cases in two-dimensional motion are the motion of projectiles, river crossing problems, rainfall problems etc.

PROJECTILE MOTION

A projectile is a particle that is thrown in any direction near the surface of the earth with certain small velocity which subsequently falls freely under the influence of gravity of the earth alone. The projectile's motion has two components. One component is along the horizontal direction and the other component is along the vertical direction. In the analysis of projectile motion, the following assumptions are made:

- The vertical motion of the projectile is subjected to uniform acceleration in the downward direction and this acceleration is equal to 'g' (acceleration due to gravity). Air resistance and other forces on the particle are negligible. There is no horizontal acceleration on the projectile. i.e., horizontal motion is with uniform velocity.
- Variation of acceleration due to gravity with height above the surface of the earth is negligible for projectile motion.
- The surface of the earth is considered to be flat and the curvature of the earth's surface is negligible.
- The effects of rotation of the earth about its own axis and the revolution of the earth around the sun have negligible effects on projectile motion.

Thus a projectile motion is the two-dimensional motion of a particle in the vertical plane, in which the horizontal motion of the particle is with uniform velocity and the vertical motion of the particle is with uniform downward acceleration.

There are three types of projectiles:

- Projectiles launched from level ground at an angle with the horizontal.
- Projectiles launched horizontally from certain height above the level ground. These are called as horizontal projectiles.
- Projectiles launched on inclined planes (either upwards or downwards).

We will discuss these three cases separately.

Projectiles launched from level ground at an angle with the horizontal

Consider a particle thrown upwards from level ground with an initial velocity of magnitude 'u', making an angle θ with the horizontal.

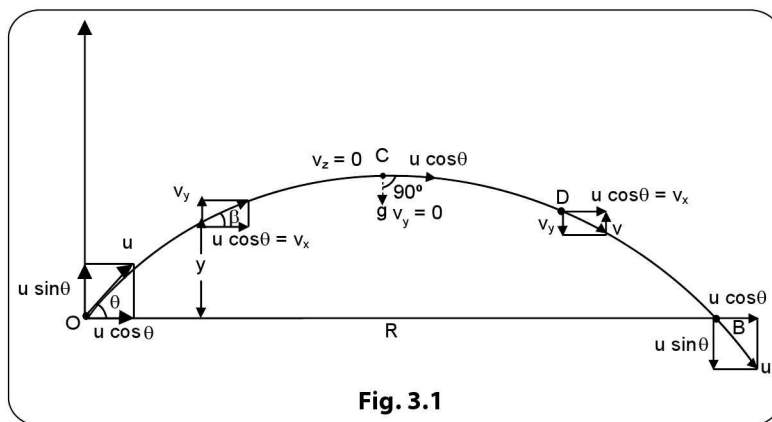


Fig. 3.1

The initial velocity of the projectile $\bar{u}$ can be resolved into two components, u_x along the horizontal (X direction) and u_y along the vertical (Y direction). We have $u_x = u \cos \theta$ and $u_y = u \sin \theta$, where θ is the angle of projection with the horizontal (Fig. 3.1). Hence the initial velocity of the projectile $\bar{u}$ is $\bar{u} = u \cos \theta \hat{i} + u \sin \theta \hat{j}$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along the X (horizontal) and Y (vertical) directions respectively. The horizontal component of initial velocity $u \cos \theta$ will remain constant throughout the motion. Hence the motion in the horizontal direction (X-direction) can be treated as uniform motion.

The vertical component of velocity u_y that is in the positive Y direction (vertically upwards) is subject to the force of gravity in the negative Y direction (vertically downwards). So, as time passes, the vertical component of velocity v_y will continuously decrease according to the equation

$$v_y = u \sin \theta - gt$$

At the position of maximum height (point C in figure), the vertical component of velocity becomes zero (i.e., $v_y = 0$ at point C). From the point C to B, the vertical component of velocity will increase in the downward direction. As it just reaches the point B, the vertical component of

velocity will become equal to the initial vertical component of velocity $u \sin \theta$ but in the downward direction.

The time interval from the instant of projection (or launch) to the instant of hitting the ground (or return) is called **the time of flight** (T) of the projectile.

The path along which the projectile moves (curve OCB in Fig. 3.1) is called **the trajectory** of the projectile.

The maximum horizontal distance travelled by the projectile (OB in Fig. 3.1) is called its **horizontal range** or simply range.

The maximum height reached by the projectile above the level ground (at point C in Fig. 3.1), is called its **maximum height**.

(i) Maximum height attained (H)

When the projectile attains the maximum height (or maximum displacement in the vertical direction), the vertical component of the velocity v_y becomes zero. So the maximum height attained can be calculated using the equation

$$v_y^2 - (u \sin \theta)^2 = -2gH$$

$$\text{or } 0 - u^2 \sin^2 \theta = -2gH$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

CONCEPT STRAND

Concept Strand 1

The ceiling of an indoor football stadium is 45 m high. A footballer kicked a ball with a speed u_1 so that it sailed through a maximum height of 20 m but did not reach his teammate. If he had kicked the ball at a higher speed of u_2 at the same angle, the ball would have grazed the ceiling and reached his teammate. How large should be u_2 in comparison to u_1 ?

Solution

Considering the vertical motion of the ball, we have

$$u_1^2 \sin^2 \theta = 2g \times 20 \quad \text{--- (1)}$$

$$u_2^2 \sin^2 \theta = 2g \times 45 \quad \text{--- (2)}$$

$$\frac{u_2^2}{u_1^2} = \frac{45}{20} \Rightarrow \frac{u_2}{u_1} = \frac{3}{2} \Rightarrow u_2 = \frac{3}{2}u_1$$

3.4 Motion in Two Dimensions

(ii) Time of flight (T)

Time taken by the projectile from the instant of projection, to the instant when it lands on the ground is called its time of flight. If t is the time taken to travel from the point of projection to the maximum height, then the time of flight $T = 2t$.

Remember, the time taken to travel from the point of projection to the maximum height t is the time taken for the vertical component of velocity to become zero. Therefore,

$$\begin{aligned} v_y &= 0 = u \sin \theta - gt \\ \Rightarrow t &= \frac{u \sin \theta}{g} \\ \text{Time of flight } T &= 2t = \frac{2u \sin \theta}{g} \end{aligned}$$

It is interesting to note that the time of flight of a projectile depends only on the vertical component of its initial velocity and the acceleration due to gravity (free fall acceleration).

(iii) Range of the projectile (R)

Range is the horizontal displacement from the point of projection to the landing point on the ground. Remember that the horizontal motion is uniform.

$$\therefore \text{Displacement} = R =$$

$$\text{Uniform horizontal velocity} \times \text{Time of flight}$$

$$= u \cos \theta \times \frac{2u \sin \theta}{g}$$

Since $\sin 2\theta = 2 \sin \theta \cos \theta$, we get

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\text{Now } \sin 2\theta = \sin (180 - 2\theta) = \sin 2(90 - \theta)$$

Therefore, range R can also be written as

$$R = \frac{u^2}{g} \sin 2(90 - \theta)$$

Thus, if you throw two projectiles from a point on the ground, with the same initial velocity u , but at different angles, such that one angle is complementary to the other angle, ($\theta_2 = 90^\circ - \theta_1$), both will fall at the same point on ground. i.e., Ranges will be equal as shown in Fig. 3.2.

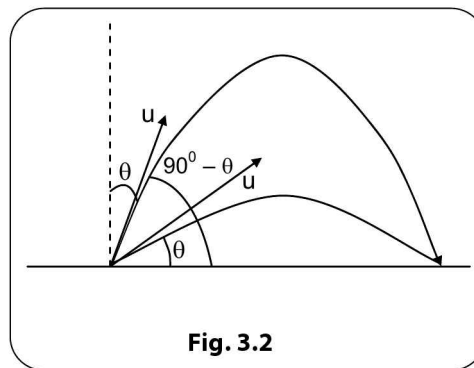


Fig. 3.2

However, the maximum height and time of flight will be more for higher angle of projection. R will be maximum when $\frac{u^2 \sin 2\theta}{g}$ is maximum. For a given u , R will be maximum when $\theta = 45^\circ$ (or $\theta = \frac{\pi}{4}$ radian) and maximum range = $\frac{u^2}{g}$

The situation is illustrated in Fig. 3.3.

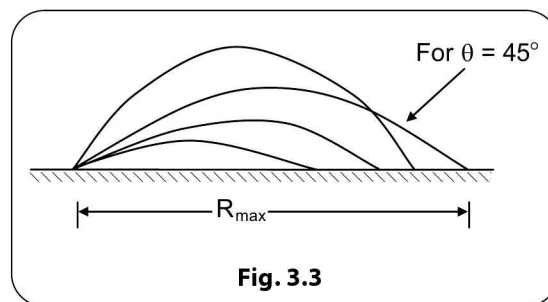


Fig. 3.3

CONCEPT STRANDS

Concept Strand 2

A fielder throws a cricket ball at 144 kmph from a point at a distance of 80 m from the wicket and hits it. At what angle to the horizontal did he throw the ball?

Solution

$$u = 144 \text{ kmph} = 144 \times \frac{5}{18} = 40 \text{ m s}^{-1}$$

$$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow 80 = \frac{40 \times 40 \sin 2\theta}{10}$$

$$\Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6} \text{ rad}$$

$$\text{or } \frac{5\pi}{6} \text{ rad}$$

$$\Rightarrow \theta = 15^\circ$$

$$\text{or } 75^\circ$$

The fielder obviously chooses the smaller angle, to minimize the time of flight.

Concept Strand 3

A stone thrown from a point on the ground at 30° to the horizontal, to hit another point on the ground 145 m away, fell short by 5 m. A second trial at a different angle (but the same speed) again fell short by the same distance. What were the differences in their flight times?

Solution

The range being the same, the second throw must have been at $90^\circ - 30^\circ = 60^\circ$. Therefore,

$$T = \frac{2u \sin \theta}{g}$$

$$\Rightarrow \frac{T_2}{T_1} = \sqrt{3} \quad \text{--- (1)}$$

$$T_1 T_2 = \frac{4u^2}{g^2} \times \sin 30^\circ \sin 60^\circ = \frac{2R}{g}$$

$$T_1 T_2 = \frac{2R}{g} = \frac{2 \times 140}{10} = 28 \quad \text{--- (2)}$$

Substituting from (1) in (2),

$$\sqrt{3} T_1^2 = 28 \Rightarrow T_1 = 4.02 \text{ s}$$

$$\therefore T_2 = 7 \text{ s}$$

$$\Rightarrow \text{Difference} = 7 - 4 = 3 \text{ s}$$

(iv) Velocity of the projectile t second after projection (v)

Consider a projectile with an initial velocity u making an angle θ with the horizontal. After t seconds of its projection, let its velocity be v . Let v make an angle β with the horizontal (See Fig. 3.1)

We can calculate v if we know the horizontal component (v_x) and vertical component (v_y)

$$\text{i.e., } v = \sqrt{v_x^2 + v_y^2}$$

Now in vector form $\vec{u} = u \cos \theta \hat{i} + u \sin \theta \hat{j}$. After a time t ,

$$\vec{v} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$$

$$\text{We know that } v_x = u_x = u \cos \theta$$

$$v_y = (u \sin \theta - gt)$$

$$\therefore v = \sqrt{(u \cos \theta)^2 + (u \sin \theta - gt)^2}$$

$$\Rightarrow v = \sqrt{u^2 + g^2 t^2 - 2ug t \sin \theta}$$

$$\text{Here } \tan \beta = \frac{v_y}{v_x}$$

$$= \frac{(u \sin \theta - gt)}{u \cos \theta}$$

$$\therefore \beta = \tan^{-1} \left(\frac{u \sin \theta - gt}{u \cos \theta} \right)$$

(v) Equation of the trajectory (path) of the projectile

We have seen that

$$x = u \cos \theta \cdot t \quad \text{--- (1)}$$

$$y = u \sin \theta \cdot t - \frac{1}{2} g t^2 \quad \text{--- (2)}$$

$$(1) \Rightarrow t = \frac{x}{u \cos \theta} \quad \text{Substituting in (2),}$$

$$(2) \Rightarrow y = u \sin \theta \cdot \frac{x}{u \cos \theta} - \frac{1}{2} g \cdot \frac{x^2}{u^2 \cos^2 \theta}$$

$$\Rightarrow y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

$$\text{or } y = x \tan \theta \left[1 - \frac{x}{R} \right]$$

$$\text{where, } R = \frac{2u^2 \sin \theta \cos \theta}{g}$$

This is the equation relating y and x and being a quadratic in x , it is the equation of a parabola. It gives the various points in space through which the projectile passes.

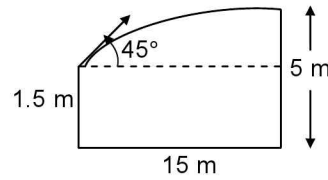
Suppose for example, there is a tall wall at a particular horizontal distance x from the point of projection, substitute that value of x , determine y from the equation and see whether y is greater than, equal to or less than the height of the wall given. If y is more, it means the projectile clears the wall. If y is less, it hits the wall. If y is equal, it just grazes the top of the wall.

For such and similar applications, we can use the equation of the trajectory of the projectile.

CONCEPT STRAND

Concept Strand 4

A boy 1.5 m tall throws a stone at an angle 45° to the horizontal against a wall 5 m in height and 15 m away. What is the minimum velocity with which he has to throw the stone so that the stone just clears the wall?



Solution

Using the equation for a projectile, viz.,

$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2} \sec^2 \theta \quad \left(\because \frac{1}{\cos^2 \theta} = \sec^2 \theta \right)$$

We get $5 - 1.5$

$$= 15 \times 1 - \frac{1}{2} \times 10 \times \frac{15^2}{u^2} \times 2$$

$$\Rightarrow u \approx 14 \text{ m s}^{-1}$$

FOR ADDITIONAL READING

Envelope of a projectile

Suppose a particle is projected at a particular speed, i.e., u is fixed. What are the points in space where the projectile will never reach? In other words, you can throw several stones at a constant u , varying the projection angle θ , but still you will not be able to reach some points in space.

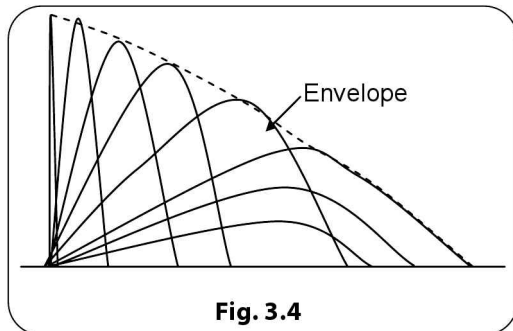


Fig. 3.4

First you throw the projectile at 90° , i.e., vertically up. The projectile will reach maximum height $= \frac{u^2}{2g}$. Next reduce θ slightly. Now the height will be less than the maximum, but the range increases. As you go on gradually reducing θ , $y_{\max}$ will decrease. At $\theta = 45^\circ$, x will be maximum

Further reduction of θ will not only reduce the range but also $y_{\max}$. If you join all the top points ($y_{\max}$ values) till $\theta = 45^\circ$ and then on follow the trajectory of $\theta = 45^\circ$, you

get the dotted line curve (See Fig. 3.4) which is called the envelope of the projectile.

Beyond this envelope, for a given value of u , you will never be able to hit any point with your projectile (i.e., the trajectory equation $y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$ will never be satisfied) whatever be the value of θ .

Below this envelope curve, any point can be hit by choosing a θ , which can be determined from the trajectory equation.

How can we determine the equation of this envelope?

$$\text{Take } y = x \tan \theta - \frac{1}{2} g \frac{x^2 \sec^2 \theta}{u^2} \quad \text{--- (1)}$$

For a given u and x , let us determine the maximum possible value of y .

$$\text{i.e., } \frac{dy}{d\theta} = 0.$$

$$\frac{dy}{d\theta} = x \sec^2 \theta - \frac{1}{2} g \frac{x^2}{u^2} \cdot 2 \sec \theta \cdot \sec \theta \tan \theta = 0$$

$$\Rightarrow \tan \theta = \frac{u^2}{gx}$$

Substitute in (1), put $\sec^2 \theta = 1 + \tan^2 \theta$

$$\Rightarrow y = \frac{u^2}{2g} - \frac{gx^2}{2u^2}$$

$$y = \left[\frac{R_{\max}}{2} - \frac{x^2}{2R_{\max}} \right]$$

$$\text{or } y = H_{\max} - \frac{x^2}{4H_{\max}}$$

$$\text{or } y = H_{\max} - \frac{x^2}{2R_{\max}}$$

where, $H_{\max}$ = maximum height, $R_{\max}$ = maximum range.

This is the equation of the envelope (the dotted curve shown in the diagram).

This equation gives $y_{\max}$ for any given x and also conversely it gives $x_{\max}$ (i.e., $R_{\max}$) for any given y .

∴ If we put $y = -h$, we get

$$-h = \frac{u^2}{2g} - \frac{gR_{\max}^2}{2u^2}$$

$$\Rightarrow R_{\max} = \frac{u}{g} \sqrt{u^2 + 2gh}$$

Projectiles launched horizontally (Horizontal projectiles)

Consider a particle thrown horizontally from the top of a tower of height H (Fig. 3.5), with an initial velocity u (angle of projection with horizontal $\theta = 0$). The time taken by it to reach the ground is called its time of flight (T). The velocity with which it hits the ground is called the terminal velocity or velocity of impact ($v_{\max}$). The horizontal distance of the

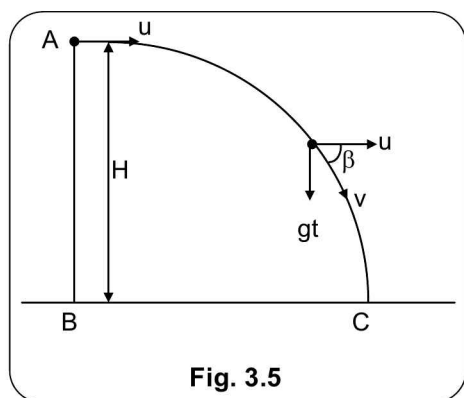


Fig. 3.5

point where it hits the ground from the foot of the tower is called the horizontal range. Clearly, the maximum height of fall (H) is the height of the tower from where it was launched horizontally. The horizontal component of initial velocity $u_x = u$ and vertical component of initial velocity $u_y = 0$ respectively.

(i) Velocity of the projectile t second after projection (v)

Let its velocity be v , after a time t second, before it reaches the ground.

Its horizontal component = $v_x = u$ and vertical component

$$v_y = 0 + gt = gt$$

$$\therefore v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{u^2 + g^2 t^2}$$

angle

$$\beta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{gt}{u} \right)$$

(ii) Time of flight (T)

The time of flight depends on the vertical distance (H) to be traversed during the flight.

The time taken to reach the ground,

$$T = \sqrt{\frac{2H}{g}}$$

is the time of flight of this projectile.

(iii) Horizontal Range (R)

Here horizontal range R = horizontal velocity $\times$ Time of flight

Hence

$$R = u \times \sqrt{\frac{2H}{g}}$$

CONCEPT STRAND

Concept Strand 5

A plane flying horizontally with a speed u at an altitude H has to drop a bomb targeting a military installation on the ground. Find the horizontal distance from the target at which the bomb has to be dropped.

Solution

$$\frac{1}{2}gt^2 = H \Rightarrow t = \sqrt{\frac{2H}{g}}$$

$$\therefore d = ut = \sqrt{\frac{2u^2H}{g}}$$

Projectiles launched on inclined planes

(i) Range of a projectile on an inclined plane (R)

Consider a particle projected upwards at an angle θ with the horizontal from the foot of an inclined plane of angle of inclination α with the horizontal (Fig. 3.6). Let the particle hit the point A on the inclined plane at a distance R along the plane. Now, consider the equation for the trajectory of the projectile.

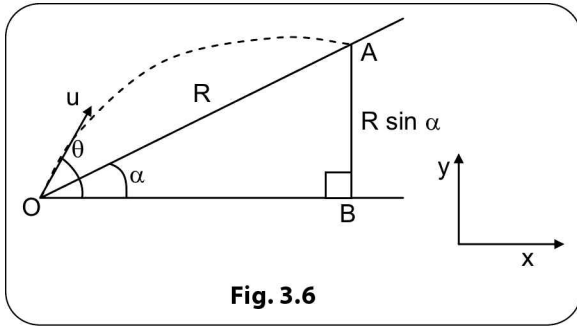


Fig. 3.6

$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

Co-ordinates of A are $(R \cos \alpha, R \sin \alpha)$, where R is the range required.

Substitute $y = R \sin \alpha$; $x = R \cos \alpha$

$$\Rightarrow R \sin \alpha = R \cos \alpha \tan \theta - \frac{1}{2} g \cdot \frac{R^2 \cos^2 \alpha}{u^2 \cos^2 \theta}$$

$$\begin{aligned} \Rightarrow R &= \frac{2u^2 \cos^2 \theta}{g \cos^2 \alpha} [\cos \alpha \cdot \tan \theta - \sin \alpha] \\ &= \frac{2u^2 \cos^2 \theta}{g \cos^2 \alpha} \left[\cos \alpha \frac{\sin \theta}{\cos \theta} - \sin \alpha \right] \\ &= \frac{2u^2 \cos^2 \theta}{g \cos^2 \alpha} \frac{[\sin \theta \cos \alpha - \cos \theta \cdot \sin \alpha]}{\cos \theta} \end{aligned}$$

$$\Rightarrow R = \frac{2u^2 \cos \theta}{g \cos^2 \alpha} \cdot \sin(\theta - \alpha)$$

$$\Rightarrow R = \frac{u^2}{g \cos^2 \alpha} [\sin(2\theta - \alpha) - \sin \alpha]$$

This formula for R can also be derived from fundamentals, as shown below: Choose the x-axis along the incline and the y-axis perpendicular to the incline (Fig. 3.7).

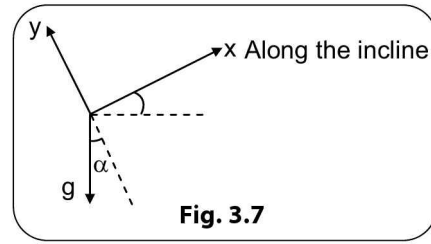


Fig. 3.7

Resolve $\vec{u}$ into $u_x = u \cos(\theta - \alpha)$

$$u_y = u \sin(\theta - \alpha)$$

Resolve $\vec{g}$ into $a_x = -g \sin \alpha$

$$a_y = -g \cos \alpha$$

(ii) Time of flight of a projectile on an inclined plane (T)

When the projectile reaches A,

$$y = 0, x = R$$

$\therefore$ Time of flight T can be obtained from

$$0 = u_y \cdot T + \frac{1}{2} a_y T^2$$

$$\Rightarrow 0 = u \sin(\theta - \alpha) T + \frac{1}{2} (-g \cos \alpha) T^2$$

$$\Rightarrow T = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$$

Verification of expression for range on inclined plane (R) using T

$$\begin{aligned} \therefore R &= u_x T + \frac{1}{2} a_x T^2 \\ &= \left[u \cos(\theta - \alpha) + \frac{1}{2} (-g \sin \alpha) \frac{2u \sin(\theta - \alpha)}{g \cos \alpha} \right] \\ &\quad \times \frac{2u \sin(\theta - \alpha)}{g \cos \alpha} \\ &= \frac{2u^2 \sin(\theta - \alpha)}{g \cos \alpha} \left[\cos(\theta - \alpha) - \frac{\sin \alpha \sin(\theta - \alpha)}{\cos \alpha} \right] \\ &= \frac{2u^2 \sin(\theta - \alpha)}{g \cos^2 \alpha} [\cos \alpha \cos(\theta - \alpha) - \sin \alpha \sin(\theta - \alpha)] \\ &= \frac{2u^2 \sin(\theta - \alpha)}{g \cos^2 \alpha} \cos(\alpha + \theta - \alpha) \\ &= 2u^2 \cos \theta \cdot \frac{\sin(\theta - \alpha)}{g \cos^2 \alpha} \\ &= \frac{u^2}{g \cos^2 \alpha} [\sin(2\theta - \alpha) - \sin \alpha] \end{aligned}$$

(iii) Maximum range on an inclined plane

For given u and α , R is maximum when

$$\sin(2\theta - \alpha) = 1$$

$$\Rightarrow 2\theta - \alpha = 90^\circ$$

$$\Rightarrow \theta = 45^\circ + \frac{\alpha}{2}$$

and $R_{\max}$ is

$$R_{\max} = \frac{u^2}{g \cos^2 \alpha} [1 - \sin \alpha]$$

$$\text{Using } \cos^2 \alpha = 1 - \sin^2 \alpha = (1 + \sin \alpha)(1 - \sin \alpha)$$

$$\Rightarrow R_{\max} = \frac{u^2}{g(1 + \sin \alpha)}$$

For a given u , it can be proved that there are two angles of projection, giving the same range, which are equally inclined to the direction of projection for maximum range.

$$R = \frac{u^2}{g \cos^2 \alpha} [\sin(2\theta - \alpha) - \sin \alpha]$$

For given u , α and R , $\sin(2\theta - \alpha)$ is constant.

$\therefore$ Two values of θ namely θ_1 and θ_2 will give the same value of R .

Two values of θ_1 and θ_2 are related as $2\theta_1 - \alpha = 180^\circ - (2\theta_2 - \alpha)$

$$\Rightarrow \theta_1 - \left(45^\circ + \frac{\alpha}{2}\right) = \left(45^\circ + \frac{\alpha}{2}\right) - \theta_2$$

i.e., θ_1 and θ_2 are equally inclined to $\left(45^\circ + \frac{\alpha}{2}\right)$ which is the angle of projection for maximum R on the inclined plane.

Note:

For a projectile launched downwards on an inclined plane, change α into $-\alpha$ in the above results.

CONCEPT STRAND

Concept Strand 6

From the top of an inclined plane of inclination 30° with the horizontal, a body is thrown horizontally with a speed of 3 ms^{-1} . If the body lands exactly at the foot of the incline, find the angle with the horizontal at which the body lands.

Solution

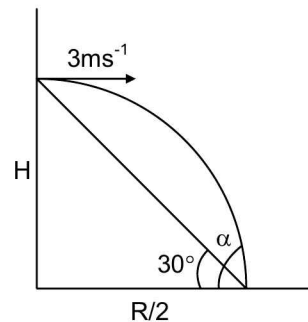
For a body landing at the foot of the incline,

$$\frac{H}{R/2} = \tan 30^\circ \quad (1) \text{ (from the given figure)}$$

$$R \tan \alpha = 4H \quad (2) \text{ (from the equations of a projectile)}$$

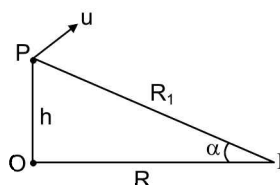
$$\text{Combining } \frac{\tan \alpha}{\tan 30^\circ} = 2$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{2}{\sqrt{3}} \right)$$



(iv) Projection from height h

Let a particle be projected from a height h with speed u . We want to find the maximum horizontal range OI (R). In this case the value PI (R_1) is also maximum, since h is a constant.



Now

$$R_1 = \frac{u^2}{g} \times \frac{1}{1 - \sin \alpha} \text{ and } \sin \alpha = \frac{h}{R_1}$$

$$\text{But } R_1 = \sqrt{R^2 + h^2}$$

From maximum range on the inclined plane,

$$R_1 = \frac{u^2}{g} \times \frac{1}{1 - \sin \alpha}$$

$$\frac{u^2}{g} = R_1 - R_1 \sin \alpha = \sqrt{R^2 + h^2} - h$$

$$\Rightarrow R = \frac{u \sqrt{u^2 + 2gh}}{g}$$

3.10 Motion in Two Dimensions

CONCEPT STRAND

Concept Strand 7

A gun of muzzle velocity $100\sqrt{2} \text{ m s}^{-1}$ is mounted on the edge of a cliff, 210 m high. Can it cover a strait 2.5 km wide?

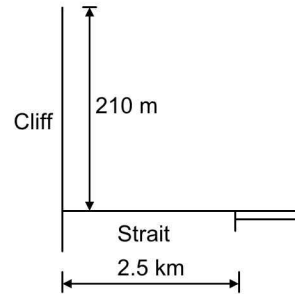
Solution

$$R = \frac{u \sqrt{u^2 + 2gh}}{g}$$

$$= \frac{100\sqrt{2} \sqrt{100^2 \times 2 + 2 \times 10 \times 210}}{10}$$

$$= \frac{100\sqrt{2} \times 10\sqrt{2} \times 11}{10} = 2200 < 2500 \text{ m}$$

∴ It cannot cover the strait.



RELATIVE MOTION IN TWO DIMENSIONS

Consider two railway trains running on parallel tracks in the same direction at different uniform speeds. For a person on the faster train, the slower train will appear to move backwards at a uniform speed, whereas a stationary observer will find it moving forward. Thus the speed of a body measured by observers depend on the frame of reference.

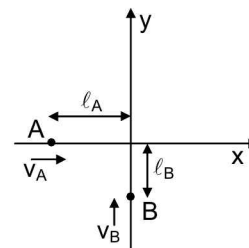
Consider two particles A and B travelling on a plane surface.

- (1) The relative positions of B with respect to A is given by $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$, where $\vec{r}_A$ = position of A and $\vec{r}_B$ = position of B respectively, with respect to the origin.
- (2) The relative positions of A with respect to B is given by $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$, where $\vec{r}_A$ = position of A and $\vec{r}_B$ = position of B respectively, with respect to the origin.
- (3) The relative velocity of B with respect to A is given by $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$, where $\vec{v}_A$ = velocity of A and $\vec{v}_B$ = velocity of B respectively, with respect to the origin.
- (4) The relative velocity of A with respect to B is given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$, where $\vec{v}_A$ = velocity of A and $\vec{v}_B$ = velocity of B respectively, with respect to the origin.
- (5) The relative acceleration of B with respect to A is given by $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$, where $\vec{a}_A$ = acceleration of A and $\vec{a}_B$ = acceleration of B respectively, with respect to the origin.
- (6) The relative acceleration of A with respect to B is given by $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$, where $\vec{a}_A$ = acceleration of A and $\vec{a}_B$ = acceleration of B respectively, with respect to the origin.

CONCEPT STRANDS

Concept Strand 8

Two particles A and B travel along the coordinate axes with speeds v_A and v_B as shown and they are at distances ℓ_A and ℓ_B away from the origin. Find their distance of closest approach.



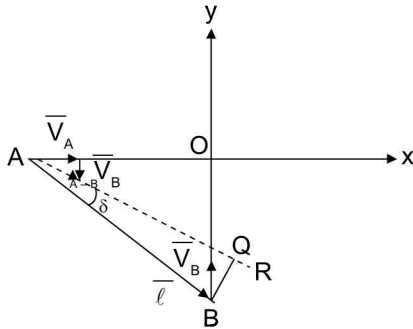
Solution

$$\vec{v}_A = v_A \hat{i} \quad \vec{v}_B = v_B \hat{j}$$

The relative velocity $\vec{v}_{1-2} = \vec{v}_A - \vec{v}_B$ and relative distance

$$\vec{\ell} = \vec{\ell}_A - \vec{\ell}_B = -\ell_A \hat{i} + \ell_B \hat{j}$$

The component of relative distance along the relative velocity will change, and that perpendicular to relative velocity will be unchanged. Thus the perpendicular component is the closest approach distance.



Let A move with velocity $\vec{v}_A$ and B move with velocity $\vec{v}_B$. From figure it is clear that relative velocity of A with respect to B, i.e., $\vec{v}_{AB}$ is as shown along path AR. $AB = |\vec{\ell}|$ is the initial separation. Now for B, A moves with a relative velocity $\vec{v}_{AB}$ along AR. From B draw BQ normal to AR. BQ is the distance of closest approach.

If we consider unit vector $\hat{v}_{AB}$ along AQ, then

$|\vec{\ell} \times \hat{v}_{AB}| = \ell \times 1 \sin \delta = BQ = \text{distance of closest approach}$

$$\vec{\ell} = \ell_A \hat{i} - \ell_B \hat{j}; \quad \vec{v}_{AB} = v_A \hat{i} - v_B \hat{j}$$

$$\Rightarrow \therefore \hat{v}_{AB} = \frac{v_A \hat{i} - v_B \hat{j}}{\sqrt{v_A^2 + v_B^2}}$$

$$\begin{aligned} \therefore \vec{\ell} \times \hat{v}_{AB} &= (\ell_A \hat{i} - \ell_B \hat{j}) \times \frac{(v_A \hat{i} - v_B \hat{j})}{\sqrt{v_A^2 + v_B^2}} \\ &= \frac{-\hat{k} \ell_A v_B + \hat{k} \ell_B v_A}{\sqrt{v_A^2 + v_B^2}} \end{aligned}$$

$$\therefore BQ = |\vec{\ell} \times \hat{v}_{AB}| = \left| \frac{\ell_A v_B - \ell_B v_A}{\sqrt{v_A^2 + v_B^2}} \right|$$

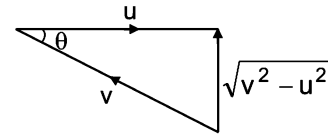
Concept Strand 9

Consider a boat of speed v crossing a river of width ℓ flowing with a speed of u ($v > u$). Find

- the time required to cross via the shortest path.
- time required to cross in the shortest time.

Solution

- The shortest path is perpendicular to the banks (the width of the river)

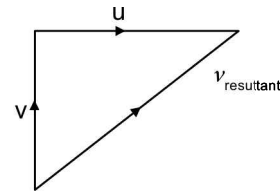


To cancel the drift due to flow of river, we have to align the boat velocity as shown, so that $v \cos Q = u$. Hence the component of velocity of boat normal to the flow (i.e., perpendicular to the bank) is $v \sin Q = \sqrt{v^2 - u^2}$.

$\Rightarrow$ Time to cross

$$= \frac{\ell}{\sqrt{v^2 - u^2}}$$

- For shortest time boat velocity is normal to the bank. Shortest time is $t = \frac{\ell}{v}$. In this case, the boat will drift.



Concept Strand 10

A person walking at 3 kmph finds rain falling vertically. When he walks in the opposite direction with the same speed, the rain appears to fall at 45° to the vertical. What is the speed of rain?

Solution

Let the velocity of rain be $\vec{v}_r = v_x \hat{i} + v_y \hat{j}$

Case (i)

Velocity of man $\vec{v}_m = 3\hat{i}$

3.12 Motion in Two Dimensions

Relative velocity of rain $\vec{v}_{rm} = (v_x - 3)\hat{i} + v_y\hat{j}$

Since rain appears to fall vertically x component is zero.

$$\therefore v_x = 3 \text{ kmph}$$

Case (ii)

Velocity of man $\vec{v}'_m = -3\hat{i}$

Relative velocity of rain $\vec{v}'_{rm} = (v_x + 3)\hat{i} + v_y\hat{j} = 6\hat{i} + v_y\hat{j}$

Since rain appears to fall at 45° to the vertical,

$$\frac{|v_y|}{6} = \tan 45^\circ \Rightarrow |v_y| = 6 \text{ kmph}$$

$\therefore$ speed of rain with respect to earth is

$$v_x = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 6^2} = 3\sqrt{5} \text{ kmph}$$

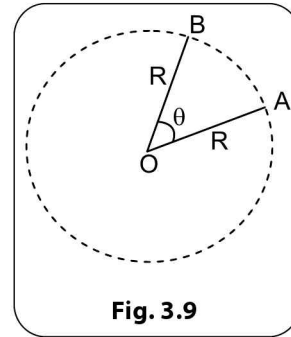
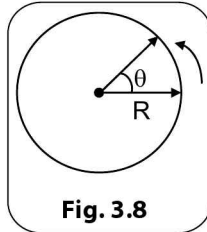
CIRCULAR MOTION

A particle is said to be in circular motion when it moves along the circumference of a circle.

in Fig. 3.9. When the speed is constant, the particle is in uniform circular motion.

Angular displacement

When a particle moves along the circumference of a circle, the radius vector drawn from the centre to the particle's position, rotates through a certain angle. The angle through which the radius vector turns in an interval of time t is called the angular displacement θ . The SI unit of angular displacement is radian (rad).



Angular velocity

Angular displacement per unit time is called angular velocity, $\omega = \left(\frac{d\theta}{dt}\right)$, where $d\theta$ is the infinitesimal angle through which the radius vector rotates in the infinitesimal interval of time dt . It is a vector quantity and its direction is along the axis of rotation and is given by the right hand screw rule. In Fig. 3.8, the motion is shown to be anti-clockwise. Here the direction of ω is along the axis of rotation and out of the plane of the paper. SI unit of angular velocity is radian per second (rad s^{-1}).

Circular motion could be of two types, uniform circular motion and non-uniform circular motion.

Uniform circular motion

Let a particle travel along the circumference of a circle with centre O and radius R with a uniform speed v , as shown

in Fig. 3.9. When the speed is constant, the particle is in uniform circular motion. Let it cover the arc from A to B in t second. θ is the angular displacement. Then arc length $AB = R\theta$. Then the linear speed is $v = \frac{R\theta}{t}$.

In this relation, while v is 'linear speed', the ratio $\frac{\theta}{t}$ is called 'angular speed' and is given a symbol ω (omega).

$$\frac{\theta}{t} = \omega$$

ω in this case is constant angular speed. If ω has the same magnitude at all points along the circle.

$$\frac{d\theta}{dt} = \omega = \frac{v}{R}$$

The instantaneous angular speed is same at all points. Angular speed ω is actually called angular velocity, $\vec{\omega}$, because it is a vector whose direction is perpendicular to the plane of motion (the plane of circle, the plane of paper).

When the circular motion is taking place in the XY plane it is along the +Z direction if the angular displacement $d\theta$ is counter-clockwise and along -Z if $\Delta\theta$ is clockwise. Hence the vector relation defining $\vec{v}$ is $\vec{v} = \vec{\omega} \times \vec{R}$.

$$\text{Time period to traverse a circle} = T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$$

Frequency is the number of cycles per second i.e., number of times the circle is traversed in one second). The SI unit of frequency is hertz (Hz), which is the reciprocal of time in second. 1 Hz = 1 cycle per second.

$$= \frac{1}{T} = \frac{v}{2\pi R} = \frac{\omega}{2\pi}$$

Frequency is represented by ν .

If you have noticed, we have been calling v as uniform speed and not uniform velocity. This is because, even though its magnitude remains a constant, the direction varies. The tangent to the circle at any point on the circular path gives the direction of velocity of the particle at that point.

If you take two points A and A' separated by a small angle $\Delta\theta$ and covered during a small time interval Δt as shown in Fig. 3.10,

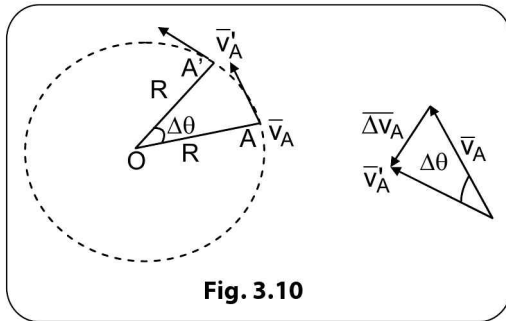


Fig. 3.10

The change in velocity $\Delta\vec{v}_A$ is $\vec{v}_{A'} - \vec{v}_A$ as shown.

$$|\vec{v}_A| = |\vec{v}_{A'}| = v$$

If we take $\Delta\theta$ as small as possible, $|\Delta\vec{v}_A| = v\Delta\theta$ and $\Delta\vec{v}_A$ is perpendicular to $\vec{v}_A$ i.e., radially along AO. The acceleration vector $\vec{a} = \text{Rate of change of velocity} = \frac{\Delta\vec{v}_A}{\Delta t}$.

∴ Direction of $\vec{a}$ is direction of $\Delta\vec{v}_A$, i.e., radially inward and

$$|\vec{a}| = \left| \frac{\Delta\vec{v}_A}{\Delta t} \right| = \frac{v\Delta\theta}{\Delta t} = v\omega = \frac{v \cdot v}{R} = \frac{v^2}{R}$$

Because it is radially inward, it is called centripetal acceleration and its magnitude is $\frac{v^2}{R}$ or $v\omega$ or $\omega^2 R$.

Therefore, a particle in uniform circular motion is under centripetal acceleration. The acceleration vector has a constant magnitude but varying direction because at any instant it is along the radius vector at that point (and towards centre).

Centripetal acceleration is generally given by a symbol a_r (a_{radial}) or a_n (a_{normal}).

Non-uniform circular motion (Circular motion with variable speed)

In non-uniform circular motion, when the particle moves along the circumference of a circle, both the magnitude of its linear velocity (speed) and the direction of its velocity change continuously. Thus, in this type of motion, there are two types of changes in velocity, which will result in two types of accelerations.

- Acceleration due to the directional change in velocity called the centripetal acceleration $|\vec{a}_r| = \frac{v^2}{r}$ and it is defined as the vector $\vec{a}_r = \vec{\omega} \times \vec{v}$.
- Acceleration due to the change in magnitude of velocity is called the tangential acceleration $\vec{a}_t = \left(\frac{dv}{dt} \right)$

Now consider a particle in circular motion (centre O, radius R) with tangential acceleration a_t ($a_{\text{tangential}}$) of constant magnitude.

Fig. 3.11 gives the velocity vectors at A and A' separated by an angular displacement $\Delta\theta$ in Δt .

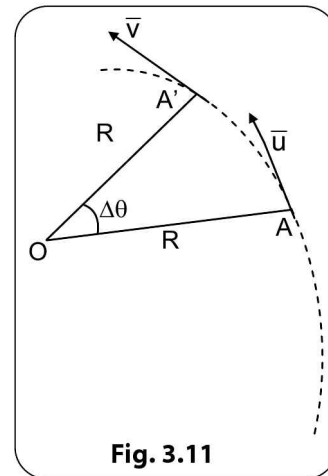


Fig. 3.11

$$\vec{v} = \vec{u} + \vec{a}_t \Delta t$$

3.14 Motion in Two Dimensions

$$\Rightarrow \frac{\bar{v}}{R} = \frac{\bar{u}}{R} + \frac{\bar{a}_t}{R} \Delta t$$

$$\Rightarrow \bar{\omega} = \bar{\omega}_0 + \bar{\alpha} \Delta t \text{ where } \bar{\omega}_0 \text{ is the initial angular velocity and } \bar{\omega} \text{ is the final angular velocity. } \bar{\alpha} = \frac{\bar{a}_t}{R} \text{ is called}$$

the **angular acceleration** of the particle and it is a vector quantity directed along the axis of rotation. Its SI unit is radian per second squared (rad s^{-2}). It must be noted that when the magnitude of the tangential acceleration of the particle is constant, the angular acceleration of the particle is constant in magnitude and direction. Such a particle is said to execute circular motion with uniform angular acceleration (sometimes called as uniformly accelerated circular motion).

Vector diagram

In Fig. 3.12

$$\Delta \bar{v} = \bar{v} - \bar{u}$$

$\Delta \bar{v}$ can be considered as sum of two components $\Delta \bar{v}_r + \Delta \bar{v}_t$.

$$\Delta \bar{v} = \Delta \bar{v}_r + \Delta \bar{v}_t$$

$$\frac{\Delta \bar{v}}{\Delta t} = \frac{\Delta \bar{v}_r}{\Delta t} + \frac{\Delta \bar{v}_t}{\Delta t}$$

$$\bar{a} = \bar{a}_r + \bar{a}_t$$

= centripetal acceleration vector + tangential acceleration vector.

This resultant acceleration $\bar{a}$ is along the plane of the circle but not directed radially inwards but at an angle ϕ with the radius, where $\tan \phi = \frac{a_t}{a_r}$.

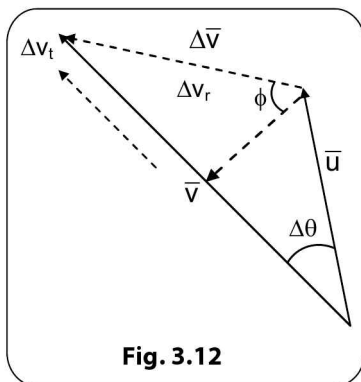


Fig. 3.12

For a particle in circular motion with tangential acceleration of constant magnitude (i.e., constant angular acceleration), the kinematic equations of motion are as below:

$$\bar{\omega} = \bar{\omega}_0 + \bar{\alpha} t \text{ (similar to) } \bar{v} = \bar{u} + \bar{a} t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \text{ (similar to } S = ut + \frac{1}{2} at^2)$$

$$\theta = \left(\frac{\omega_0 + \omega}{2} \right) \cdot t \text{ (similar to } S = \left(\frac{u + v}{2} \right) \cdot t)$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta \text{ (similar to } v^2 - u^2 = 2aS)$$

$$\text{total acceleration vector } \bar{a} = \bar{a}_r + \bar{a}_t$$

Fig. 3.13 shows path of a particle P in a uniformly accelerated circular motion, where a_t and a_r are the tangential acceleration and centripetal acceleration respectively at a particular instant of time. The net acceleration will have the magnitude.

$$a_{\text{net}} = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r} \right)^2 + \left(\frac{dv}{dt} \right)^2}$$

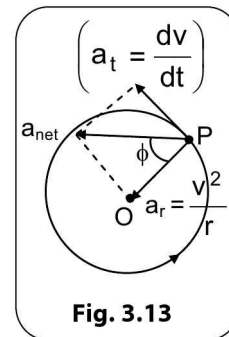


Fig. 3.13

The direction of net acceleration will be as shown in Fig. 3.13.

(Alternate) Derivation of Velocity and Acceleration of a Particle in Circular Motion

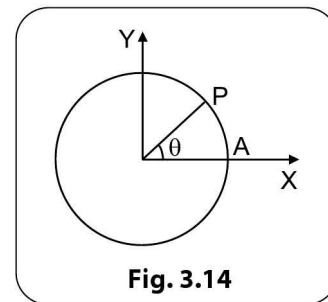


Fig. 3.14

Consider a particle in circular motion, with centre O and radius R. At time $t = 0$, the particle is at A and at time t , at P, moving counter clockwise.

At the instant t , the position vector of P can be written as $\vec{r} = R \cos \theta \hat{i} + R \sin \theta \hat{j}$

$$= R \cos \omega t \hat{i} + R \sin \omega t \hat{j} \quad \text{--- (1)}$$

($\because \theta = \omega t$ where $\omega = \frac{d\theta}{dt}$)

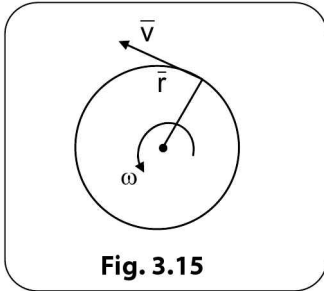


Fig. 3.15

(i) Velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = -R\omega \sin \omega t \hat{i} + R\omega \cos \omega t \hat{j}$$

$$= \vec{\omega} \times \vec{r} \quad \text{--- (2)}$$

$$(\because \vec{\omega} = \omega \hat{k}; \vec{r} = R \cos \omega t \hat{i} + R \sin \omega t \hat{j})$$

As can be seen, $\vec{v}$, $\vec{\omega}$ and $\vec{r}$ are mutually perpendicular. $\vec{v}$ is tangential at P . Its magnitude is ωR and its direction is given by the right hand rule.

$\vec{\omega}$ is outwards from the plane of the paper.

(ii) Acceleration

From (2),

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega R \left(-\cos \theta \frac{d\theta}{dt} \hat{i} - \sin \theta \frac{d\theta}{dt} \hat{j} \right)$$

$$+ \frac{d\omega}{dt} R (-\sin \theta \hat{i} + \cos \theta \hat{j})$$

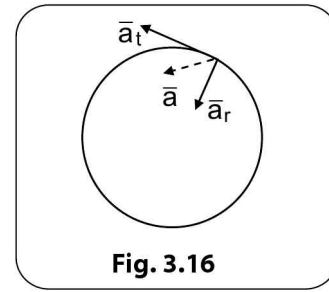


Fig. 3.16

But $\frac{d\theta}{dt} = \omega$; $\frac{d\omega}{dt} = \alpha$, the angular acceleration.

$$\therefore \vec{a} = -\omega^2 R (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$+ \alpha R (-\sin \theta \hat{i} + \cos \theta \hat{j})$$

$$= -\omega^2 \vec{r} + \vec{\alpha} \times \vec{r} \quad (\because \vec{\alpha} = \alpha \hat{k})$$

$$= \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r} \quad \text{--- (3)}$$

The first term is the centripetal acceleration. Its magnitude is $\omega^2 R$ and acts radially inwards in a direction opposite to $\vec{r}$; hence the negative sign in the expression $-\omega^2 \vec{r}$. The second term is the tangential acceleration. Its direction is tangential at that point P , being perpendicular to $\vec{r}$.

$$\therefore \vec{a} = \vec{a}_r + \vec{a}_t \quad \text{--- (4)}$$

(iii) Uniform circular motion

A special case is when the particle does not have tangential acceleration; i.e., the particle is in uniform circular motion; $a_t = 0$.

$$(\because \frac{d\omega}{dt} = \alpha = 0)$$

$$\vec{a} = \vec{a}_r = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

In this case, the particle is subject only to centripetal acceleration.

CONCEPT STRAND

Concept Strand 11

A car running on a straight road at 60 kmph was brought to stop uniformly in a distance of 10 m. What was the uniform angular deceleration of the tyres of radii 30 cm?

Solution

v = Speed of car = 60 kmph

$$\therefore \text{Angular speed of tyre, } \omega = \frac{v}{r}$$

$$= \frac{60 \times 10^3}{3600} \times \frac{1}{0.3} = \frac{10^3}{18} \text{ rad s}^{-1}$$

$$\text{The angle covered during braking, } \theta = \frac{S}{r} = \frac{10}{0.3} \text{ rad}$$

$\therefore$ Using $\omega^2 - \omega_0^2 = -2\alpha\theta$, we get

$$\omega_0^2 = 2\alpha \frac{10}{0.3} \Rightarrow \left(\frac{10^3}{18} \right)^2 = -2\alpha \frac{10}{0.3}$$

$$\therefore -\alpha \approx 46.3 \text{ rad s}^{-2}$$

SUMMARY

$$R = \frac{u^2 \sin 2\theta}{g}; R_{\max} = \frac{u^2}{g} \text{ for } \theta = 45^\circ$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$T = \frac{2u \sin \theta}{g}$$

$$v = \sqrt{u^2 + g^2 t^2 - 2u \sin \theta g t}$$

$$\beta = \tan^{-1} \left(\frac{u \sin \theta - g t}{u \cos \theta} \right)$$

$$y = x \tan \theta - \left(\frac{g x^2}{2u^2 \cos^2 \theta} \right)$$

$$y = x \tan \theta \left[1 - \frac{x}{R} \right]$$

$$y = H_{\max} - \frac{x^2}{4H_{\max}} = \frac{R_{\max}}{2} - \frac{x^2}{2R_{\max}}$$

$$= H_{\max} - \frac{x^2}{2R_{\max}} = \frac{u^2}{2g} - \frac{g x^2}{2u^2}$$

$$R = u \sqrt{\frac{2h}{g}}$$

$$v = \sqrt{u^2 + g^2 t^2}$$

$$\beta = \tan^{-1} \left(\frac{g t}{u} \right)$$

$$R = \frac{2uv}{g}$$

$$\tan \theta = \frac{4H}{R}$$

$$L = p H$$

$R \rightarrow$ Range $q =$ angle of projection with horizontal $u =$ initial speed of projectile.

$H \rightarrow$ Maximum Height

$T \rightarrow$ Time of flight (T)

$v \rightarrow$ Velocity of a projectile after 't' seconds of projection from ground

$\beta \rightarrow$ Angle made by the velocity after 't' seconds of the projection with the horizontal.

Equation for the trajectory of a projectile launched from at angle 'q' with horizontal.

Equation for the envelope of a projectile

$R \rightarrow$ Range of a horizontal projectile projected, with initial horizontal velocity u from a height h above ground.

Velocity after 't' second in the case of a body thrown horizontally from a height H above ground, with initial horizontal velocity, u .

Angle made by the velocity with the horizontal in the case of a body thrown horizontally from a height H above ground.

Where, R is the range of a projectile which is projected vertically upwards with velocity v from another body moving horizontally with velocity u .

$R \rightarrow$ Range

$H \rightarrow$ Maximum height

$\theta =$ angle of projection with horizontal

$L \rightarrow$ Angular momentum of a projectile at maximum height about the point of projection

$p \rightarrow$ Linear momentum at maximum height

$H \rightarrow$ Maximum height

$$R = \frac{2u^2 \sin(\theta - \alpha_0) \cos \theta}{g \cos^2 \alpha}$$

$$T = \frac{2u \sin(\theta - \alpha_0)}{g \cos \alpha}$$

$$f = \frac{1}{T}, T = \frac{2\pi}{\omega}, \omega = \frac{2\pi}{T} = 2\pi f$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$v = r\omega$$

$$a = \frac{v^2}{r}$$

$$a = v\omega$$

$$a = r\omega^2$$

$$\vec{a}_T = \vec{\alpha} \times \vec{r}$$

$$\vec{a}_r = \vec{\omega} \times \vec{v}$$

$$\begin{aligned} \vec{a} &= \vec{a}_r + \vec{a}_T \\ &= (\vec{\alpha} \times \vec{r}) + (\vec{\omega} \times \vec{v}) \end{aligned}$$

$$\omega_f = \omega_i \pm \alpha t$$

$$\theta = \omega_i t \pm \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_i^2 \pm 2 \alpha \theta$$

$$\theta_n = \omega_i \pm \frac{\alpha}{2} [2n - 1]$$

$$\theta = \left(\frac{\omega_i + \omega_f}{2} \right) t$$

$$N = \frac{\theta}{2\pi}$$

$R \rightarrow$ Range on an inclined plane, α - angle of projection with the horizontal

$\alpha^0 \rightarrow$ inclination of the plane with the horizontal.

$T \rightarrow$ Time of flight of a projectile launched on an inclined plane.

f = frequency of circular motion.

T = time period of revolution.

ω = angular speed of circular motion.

$\vec{v} \rightarrow$ linear velocity.

$\vec{r} \rightarrow$ radius of circular path.

$\vec{\omega} \rightarrow$ angular velocity.

$a \rightarrow$ centripetal acceleration

$v \rightarrow$ linear velocity

$r \rightarrow$ radius of circular path

ω = angular speed of circular motion.

$\vec{a}_T$ = tangential acceleration

$\vec{a}_r$ = centripetal or radial acceleration
= acceleration

$\vec{\omega}$ = angular velocity

$\vec{\alpha}$ = angular acceleration

$\vec{r}$ = (radius) position vector

$\vec{v}$ = tangential velocity

ω_i = initial angular velocity

ω_f = final angular velocity

t = time interval

a = uniform angular acceleration

θ = angular displacement

θ_n = angular displacement in the n th second

N = total number of revolution

CONCEPT CONNECTORS

Connector 1: A shell fired at an angle of 15° to the horizontal hits the ground 3 km away. By adjusting its angle of projection, can it hit a target 8.6 km away?

Solution: Let v_0 be the initial velocity of the shell. When fired the range is 3 km when the angle of projection $\theta = 15^\circ$

$$R_1 = \frac{v_0^2 \sin 2\theta}{g} = 3 \text{ km}$$

$$\Rightarrow \frac{R_{\max}}{R_1} = \frac{1}{\sin 2\theta}$$

$$\Rightarrow R_{\max} = \frac{R_1}{\sin 2\theta} = 2R_1 = 2 \times 3 = 6 \text{ km}$$

The maximum range is 6 km; therefore it is not possible to hit a target 8.6 km away by adjusting the angle of projection.

Connector 2: A baseball player can throw a ball at a maximum horizontal distance of 100 m. What is the maximum vertical distance the baseball player can throw the ball?

Solution: Maximum horizontal distance = 100 m

$$R_{\max} = \frac{v_0^2}{g}$$

$$\text{Maximum height is given by the relation, } H_{\max} = \frac{v_0^2}{2g} = \frac{R_{\max}}{2}$$

$$H_{\max} = 50 \text{ m}$$

The maximum vertical distance to which the baseball player can throw the ball is 50 m.

Connector 3: A pipe lying horizontally on ground, pumps water upwards towards a wall, 8 m away from the end of the pipe, with a velocity of 20 m s^{-1} . If the water stream makes an angle of 37° with the horizontal, find the height at which the water will hit the wall. ($\tan 37^\circ = \frac{3}{4}$, $g = 10 \text{ m s}^{-2}$)

Solution: Here $u = 20 \text{ m s}^{-1}$ and $\theta = 37^\circ$

$$u_x = u \cos \theta = 20 \cos 37^\circ = 16 \text{ m s}^{-1}$$

$$u_y = u \sin \theta = 20 \sin 37^\circ = 12 \text{ m s}^{-1}$$

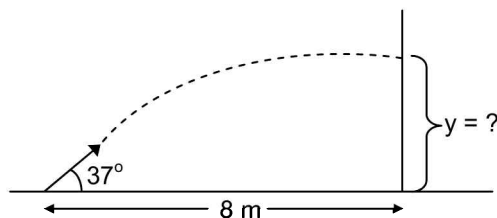
$$\text{Horizontal displacement } x = u_x \times t$$

$$\text{i.e., } 8 = 16 \times t$$

$$\therefore t = \frac{8}{16} = 0.5 \text{ s}$$

$$\therefore \text{Height } y = u_y t - \frac{1}{2} g t^2$$

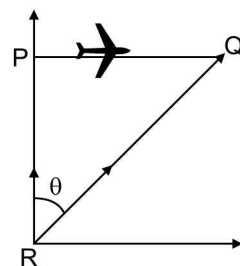
$$= 12 \times \frac{1}{2} - \frac{1}{2} \times 10 \left(\frac{1}{2} \right)^2 = 6 - 1.25 = 4.75 \text{ m}$$



Connector 4: A fighter jet plane flying horizontally at an altitude of 2 km at a speed of 200 m s^{-1} passes directly above an anti-aircraft gun. What is the minimum muzzle speed and at what angle from the vertical should the gun be fired for the shell to hit the plane? ($g = 10 \text{ m s}^{-2}$)

Solution: Speed of the plane = 200 m s^{-1}

The plane is at 2000 m above the anti-aircraft gun at $t = 0$ and let the shell hit the plane at t seconds. The horizontal distance covered by the plane in t s is $PQ = v \times t = 200 t$.



Meanwhile the shell reaches Q and the distance PQ covered by the shell = $u \sin \theta \times t$ where θ is the angle from the vertical at which the shell is fired.

$$PQ = 200 t = u \sin \theta \times t$$

$$\Rightarrow u_x = 200 \text{ m s}^{-1}$$

$$\frac{u_y^2}{2g} = 2000 \Rightarrow u_y = 200 \text{ m s}^{-1}$$

($\because$ For minimum muzzle speed, the shell hits the plane at the highest point in its trajectory)

$$\Rightarrow u_y = 200 \text{ m s}^{-1}$$

$$\frac{u_x}{u_y} = \tan \theta = 1 \Rightarrow \theta = 45^\circ$$

To hit the plane, the shell should be fired at an angle of 45° to the vertical with muzzle speed of $200\sqrt{2} \text{ m s}^{-1}$.

Connector 5: A golf ball is projected with a velocity of $40\sqrt{2} \text{ m s}^{-1}$ at an angle of 45° . Find the position and velocity of the golf ball after 2 seconds. ($g = 10 \text{ m s}^{-2}$).

Solution: $v_0 = 40\sqrt{2} \text{ m s}^{-1}$; $\theta = 45^\circ$; $t = 2 \text{ s}$

Now, horizontal component of velocity = $v_x = v_0 \cos 45^\circ$

$$= 40\sqrt{2} \times \frac{1}{\sqrt{2}} = 40 \text{ m s}^{-1}$$

Vertical component of velocity = $v_y = v_0 \sin 45^\circ$

$$= 40\sqrt{2} \times \frac{1}{\sqrt{2}} = 40 \text{ m s}^{-1}$$

The vertical component of velocity after 2 s

$$\begin{aligned} v_y &= (v_0 \sin 45^\circ) - gt \\ &= 40\sqrt{2} \times \frac{1}{\sqrt{2}} - (10)(2) = 40 - 20 = 20 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} \therefore \text{The resultant velocity after 2 s} &= \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 20^2} \\ &= 20\sqrt{5} \text{ m s}^{-1} \end{aligned}$$

Direction of resultant velocity with the X-axis

$$\tan \alpha = \frac{v_y}{v_x} = \frac{20}{40} = \frac{1}{2}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{1}{2} \right)$$

$$\Rightarrow \alpha = 26.57^\circ$$

Horizontal displacement $x = v_0 \cos 45^\circ \times t$

$$= 40\sqrt{2} \cdot \frac{1}{\sqrt{2}} \cdot 2 = 80 \text{ m}$$

Vertical displacement $y = (v_0 \sin 45^\circ) t - \frac{1}{2} gt^2$

$$\begin{aligned} &= 40\sqrt{2} \cdot \frac{1}{\sqrt{2}} \cdot 2 - \frac{1}{2} \times 10 \times 2^2 \\ &= 80 - 20 = 60 \text{ m} \end{aligned}$$

The position after 2 s is (80 m, 60 m) with respect to the point of projection as origin.

Connector 6: A bird is sitting on the top of a tree 40 m tall. A stone thrown from a distance of 80 m from the foot of the tree hits the bird horizontally. What was the magnitude and direction of the velocity with which the stone was thrown? ($g = 10 \text{ m s}^{-2}$).

3.20 Motion in Two Dimensions

Solution: Let AB be the tree of height 40 m and the stone be thrown from P with velocity v_0 at an angle θ .

$$AB = 40 \text{ m}$$

$$\text{Range (R)} = PQ = 80 \times 2 = 160 \text{ m}$$

$$H = \frac{u^2 \sin^2 \theta}{2g} = 40 \quad \text{--- (1)}$$

$$R = \frac{u^2 \sin 2\theta}{g} = 160 \quad \text{--- (2)}$$

Dividing (1) by (2)

$$\frac{40}{160} = \frac{u^2 \sin^2 \theta / 2g}{u^2 \sin 2\theta / g} = \frac{\sin^2 \theta}{2 \sin 2\theta} = \frac{\sin^2 \theta}{2(2 \sin \theta \cos \theta)}$$

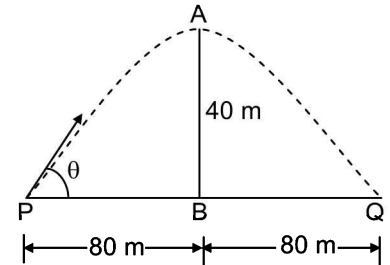
$$\frac{1}{4} = \frac{1}{4} \frac{\sin \theta}{\cos \theta}$$

$$\therefore \tan \theta = 1$$

$$\Rightarrow \theta = \tan^{-1}(1) = 45^\circ$$

$$\text{since } \theta = 45^\circ, R = \frac{u^2}{g} = 160$$

$$u = 40 \text{ m s}^{-1}$$



Connector 7: A body is thrown with a velocity of 15 m s^{-1} making an angle 37° with the horizontal in the downward direction from the top of a tower of height 80 m. Find

- The time taken to reach the ground.
 - Velocity with which it hits the ground.
 - The angle made by the velocity with the ground at the time of impact.
- ($g = 10 \text{ ms}^{-2}$)

Solution: Initial horizontal component of velocity $v_x = u \cos 37^\circ = 15 \times \frac{4}{5} = 12 \text{ m s}^{-1}$

Initial vertical component $v_y = u \sin 37^\circ = 15 \times \frac{3}{5} = 9 \text{ m s}^{-1}$

$$a_x = 0 \text{ and } a_y = g$$

We have $v_y(t)^2 = v_y(0)^2 + 2gy$ where $y = 80 \text{ m}$

$$v_y(t)^2 = 9^2 + 2 \times 10 \times 80$$

$$v_y(t)^2 = 81 + 1600 = 1681$$

$$\therefore v_y(t) = \sqrt{1681} = 41 \text{ m s}^{-1}.$$

$\therefore$ Time taken to attain this velocity $v_y(t) = v_y(0) + gt$

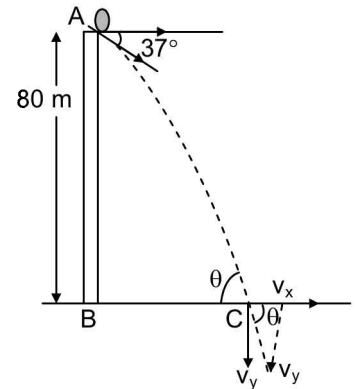
$$\therefore t = \frac{v_y(t) - v_y(0)}{g}$$

$$t = \frac{41 - 9}{10} = 3.2 \text{ s}$$

Here the angle made by the velocity of impact with the horizontal will be $\tan \theta = \frac{v_y}{v_x}$

$$\Rightarrow \tan \theta = \frac{41}{12} = 3.4$$

$$\Rightarrow \therefore \theta = \tan^{-1}(3.4) = 73.61^\circ = 73^\circ 36' 38''$$



Note:

Here in this problem horizontal separation of the point of impact from the point of projection can be obtained as $BC = u \cos \theta \times t = 12 \times 3.2 = 38.4 \text{ m}$.

Connector 8: A bullet is fired upwards from the top of a tower 100 m tall with a velocity 80 m s^{-1} at an angle 30° with horizontal. Find the horizontal distance between the vertical line through the gun and the point where the shot strikes the ground ($g = 10 \text{ m s}^{-2}$).

Solution: Let G be the position of the gun and u be the Initial velocity of the bullet. Then the horizontal and vertical components of velocity are $u \cos \theta$ and $u \sin \theta$ and t is the time taken to reach the ground.

The vertical displacement, $S = -h = -100 \text{ m}$

$$a = g = 10 \text{ m s}^{-2}; u_y = u \sin \theta = 80 \sin 30^\circ = 40 \text{ m s}^{-1}$$

Consider the equation, $S = ut + \frac{1}{2}gt^2$

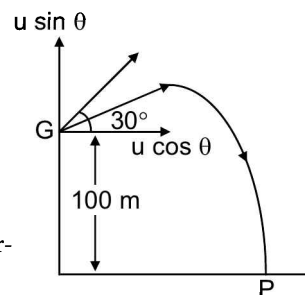
$$-100 = 40t - \frac{1}{2} \times 10 \times t^2 \Rightarrow 5t^2 - 40t - 100 = 0$$

$$\Rightarrow t^2 - 8t - 20 = 0 \Rightarrow (t + 2)(t - 10) = 0$$

Solving the quadratic equation, we get $t = 10 \text{ s}$, admissible solution Now during the time, the shot moves horizontally with a uniform velocity

$\therefore$ Displacement = Velocity $\times$ time

$$\text{Horizontal displacement} = 80 \cos 30^\circ \times 10 = 80 \times \frac{\sqrt{3}}{2} \times 10 = 400\sqrt{3} \text{ m}$$



Connector 9: When a plane is at an altitude of about 4800 m at a point P bomb is released. If the plane was moving horizontally at a speed of 200 m s^{-1} , find the distance from P to the target. ($g = 9.8 \text{ m s}^{-2}$) (Neglect air drag)

Solution: The aeroplane and bomb have the same horizontal velocity at the point of dropping. Horizontal velocity of the bomb at the time of release = 200 m s^{-1} .

Time taken by the bomb to reach the ground

Initial velocity = $u_y = 0$

$$g = 9.8 \text{ m s}^{-2}$$

$$S = 4800 \text{ m}$$

$$S_y = ut + \frac{1}{2}gt^2$$

$$\Rightarrow 4800 = 0 + \frac{1}{2} \times 9.8t^2$$

$$\Rightarrow 4800 = 4.9t^2 \Rightarrow t^2 = 980$$

$$\therefore t = \sqrt{980} = 31.3 \text{ s}$$

$$\Rightarrow S_x = \text{Horizontal distance covered in } 31.3 \text{ s} = 200 \times 31.3 = 6260 \text{ m}$$

$$\text{Distance} = \sqrt{S_y^2 + S_x^2} = \sqrt{(4800)^2 + (6260)^2} = 7888 \text{ m}$$

Connector 10: An aircraft flying horizontally at 100 m s^{-1} at a height of 500 m releases a bomb from it. Find the time taken by the bomb to reach its target and the velocity with which it hits the target on the ground. ($g = 10 \text{ m s}^{-2}$)

Solution: Time taken by the bomb to reach the ground is the same as time for free fall.

Now, Initial velocity = $u_y = 0$

$$g = 10 \text{ m s}^{-2}$$

$$s_y = 500 \text{ m}$$

$$s_y = ut + \frac{1}{2}gt^2$$

$$\Rightarrow 500 = 0 + \frac{1}{2}(10)t^2$$

$$\Rightarrow t^2 = 100 \Rightarrow t = 10 \text{ s}$$

$\therefore$ Time taken by bomb to reach ground = 10 s

3.22 Motion in Two Dimensions

Horizontal component of velocity, $v_x = 100 \text{ m s}^{-1}$ which remains uniform throughout the flight

Vertical component of velocity, $v_y = u + at$

$$v_y = 0 + 10t = (10)(10) = 100 \text{ m s}^{-1}$$

$$\begin{aligned}\therefore \text{ Resultant velocity} = v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{(100)^2 + (100)^2} = 100\sqrt{2} \text{ m s}^{-1}\end{aligned}$$

If θ is the angle of v with the horizontal

$$\tan\theta = \frac{v_y}{v_x} = \frac{100}{100} = 1$$

$$\Rightarrow \theta = \tan^{-1}(1) = 45^\circ$$

Connector 11: A body is thrown horizontally with a speed u from the top of an inclined plane of inclination θ with the horizontal. Find the separation between the points of projection and impact on the inclined plane.

Solution: Consider the motion in x direction

Initial velocity = u

Acceleration = 0

$$\therefore x = ut$$

For motion in y direction

$$u = 0$$

$$a = g$$

$$\therefore y = \frac{1}{2}gt^2$$

From (1) and (2)

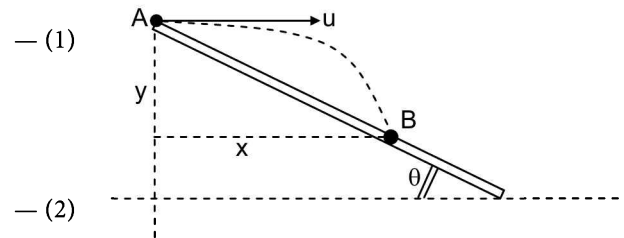
$$y = \frac{1}{2}g \frac{x^2}{u^2}$$

Also from the figure $y = x \tan\theta$

$$\therefore \frac{gx^2}{2u^2} = x \tan\theta \Rightarrow x = \frac{2u^2 \tan\theta}{g}$$

$$\text{Also } y = x \tan\theta = \frac{2u^2 \tan^2\theta}{g}$$

$$\begin{aligned}\therefore \text{ Distance AB} &= \sqrt{x^2 + y^2} \\ &= \frac{2u^2}{g} \tan\theta \sqrt{1 + \tan^2\theta} = \frac{2u^2}{g} \tan\theta \cdot \sec\theta\end{aligned}$$



Connector 12: A target on a hill has an elevation of 30° and is 90 m away (horizontally) from a gun.

- Find the acceleration of the shell normal to the plane when the shell is fixed to hit the target.
- Find the minimum required muzzle velocity ($g = 10 \text{ m s}^{-2}$)
- The time required to hit the target after fixing

Solution: (i) Normal acceleration = $g \cos \alpha$ (downward)

$$\text{Velocity (normal)} = u \sin(\beta - \alpha)$$

$$\Rightarrow \text{Time of flight} = \frac{2u \sin(\beta - \alpha)}{g \cos \alpha}$$

- (ii) Let A be the position of the gun and B be the target

Date: AC = 90 m; Inclination of AB = 30°

$$\frac{AC}{AB} = \cos 30^\circ \Rightarrow AB = \frac{AC}{\cos 30^\circ} = \frac{90}{\sqrt{3}/2} = 104 \text{ m}$$

Let velocity u give maximum range on AB

$$\therefore \frac{u^2}{g(1 + \sin \alpha)} = AB \Rightarrow u^2 = 10(1 + \sin 30^\circ) \times 104 = 1560$$

$$u \cong 39.5 \text{ m s}^{-1}$$

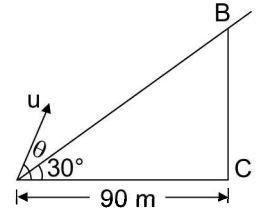
Time to reach target (T):

$$\text{Angle of projection } \theta = 45^\circ + \frac{\alpha}{2} = 45^\circ + 15^\circ = 60^\circ$$

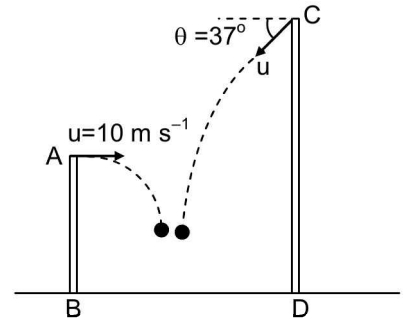
$$\therefore T = \frac{AC}{u \cos \theta} = \frac{90}{39.5 \times \cos 60^\circ} = 4.56 \text{ s}$$

Aliter:

$$T = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha} = \frac{2 \times 39.5 \sin(60^\circ - 30^\circ)}{10 \times \cos 30^\circ} = 4.56 \text{ s}$$



Connector 13: AB and CD are two towers in the same vertical plane having heights 20 m and 30 m respectively. From the top of AB a body is thrown with velocity $u = 10 \text{ m s}^{-1}$ horizontally towards CD and at the same instant another body is thrown downwards from the top of CD towards AB with the same velocity, but making an angle 37° with the horizontal as shown. If these bodies collide in air, find the separation between the towers. $\left(\tan 37^\circ = \frac{3}{4} \right)$



Solution:

Let the bodies collide after t second. In this time, if x_1 and x_2 are the displacements in the horizontal directions for the bodies. Separation between the towers.

$$d = x_1 + x_2 \text{ where}$$

$$x_1 = u \times t \text{ and } x_2 = u \cos 37^\circ \times t$$

$$\text{i.e., } d = x_1 + x_2 = t(u + u \cos 37^\circ)$$

— (1)

During this time ' t ', in the Y direction, Relative acceleration = 0

$$\Rightarrow \text{Uniform motion, Relative velocity} = u \sin 37^\circ$$

$$(y_2 - y_1) = 10 \text{ m} = u \sin 37^\circ \times t \Rightarrow t = \frac{10}{6} \text{ s}$$

$$(x_1 + x_2) = t \times u (1 + \cos 37^\circ) = \frac{10}{6} \times 10 \times 1.8 = 30 \text{ m}$$

Aliter:

$\vec{r}_{\text{rel}}$ and $\vec{v}_{\text{rel}}$ to be in the same direction

$$\frac{10}{d} = \frac{u \sin 37^\circ}{u + u \cos 37^\circ}$$

$$d = \left(\frac{1.08}{0.6} \right) \times 10 = 30 \text{ m}$$

3.24 Motion in Two Dimensions

Connector 14: A river flows due east at 6 m s^{-1} . A boat sails with velocity of 8 m s^{-1} due south relative to the river i.e., its resultant velocity in the river is due south.

(i) What is the speed of the boat relative to a stationary observer on the ground?

A car goes down the road along the bank due west at 11 m s^{-1} .

(ii) What is the velocity of the car relative to the boat?

Solution:

$$\vec{v}_{\text{river}} = \vec{v}_r$$

$$\vec{v}_{\text{boat}} = \vec{v}_b$$

$$\vec{v}_{\text{car}} = \vec{v}_c$$

$$\vec{v}_b = \vec{v}_{br} + \vec{v}_r$$

$$(i) \Rightarrow v_b = \sqrt{6^2 + 8^2} = 10 \text{ m s}^{-1}$$

$$\theta = \tan^{-1} \frac{6}{8} = 37^\circ$$

$$\phi = 90^\circ - 37^\circ = 53^\circ$$

$$(ii) \quad v_{a/b} = \sqrt{11^2 + 10^2 + 2 \times 11 \times 10 \times \cos \phi}$$

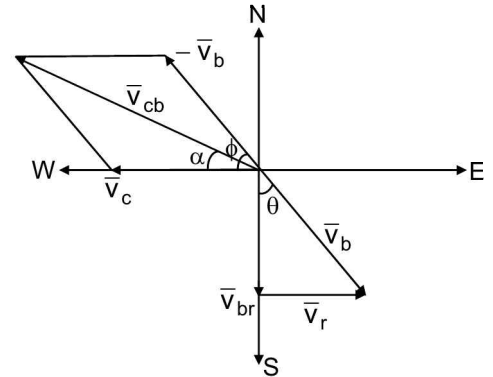
$$= \sqrt{221 + 220 \times \frac{3}{5}} = \sqrt{353} \text{ m s}^{-1}$$

$$\tan \alpha = \frac{10 \sin \phi}{11 + 10 \cos \phi} = \frac{10 \cos 37^\circ}{11 + 10 \sin 37^\circ}$$

$$= \frac{10 \cdot \frac{4}{5}}{11 + 10 \cdot \frac{3}{5}} = \frac{8}{17}$$

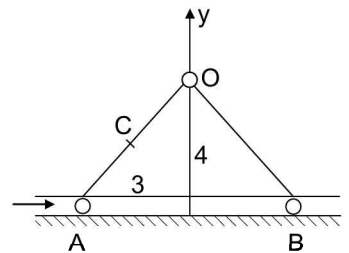
$$\alpha = \tan^{-1} \frac{8}{17}$$

(α being angle from West to North).



Connector 15: Consider the mechanism shown in the figure. O is a sliding hinge which can move vertically up or down along y the rod kept on y – a x is and A and B can slide along slot in the x direction. If $\vec{v}_O = 4\hat{j}$, find, at the given instant.

- $\vec{v}_A$, the velocity of A
- $\vec{v}_{AO}$, the velocity of A relative to O.
- $\vec{v}_{CO}$, the velocity of C relative to O.
- $\vec{v}_C$, the velocity of C, mid point of O A.



Solution:

(i) Equation for OA $\Rightarrow x^2 + y^2 = 5^2$

$$x \dot{x} + y \dot{y} = 0$$

$$\dot{x} = v_A = \left| \frac{y}{x} \dot{y} \right| = \left| \frac{y}{x} v_O \right| = \text{Obviously, when O moves up A moves to right } (+\hat{i}) \text{ and B moves to left } (-\hat{i})$$

$$\vec{v}_A = \frac{4}{3} \times 4\hat{i} = \frac{16}{3}\hat{i}$$

$$(ii) \quad \vec{v}_{AO} = \vec{v}_A - \vec{v}_O = \frac{16}{3}\hat{i} - 4\hat{j}$$

$$(iii) \quad \vec{v}_{AO} = \frac{16}{3}\hat{i} - 4\hat{j}$$

$$\overline{OA} = -3\hat{i} - 4\hat{j}$$

$$\overline{v_{AO}} \cdot \overline{OA} = -\frac{16}{3}\hat{i}^2 + 16 = 0$$

$$\overline{v_{AO}} \text{ perpendicular to } \overline{OA}$$

$$\Rightarrow \overline{v_{CO}} \text{ parallel to } \overline{v_{AO}}$$

$$\Rightarrow \overline{v_{CO}} = \frac{\overline{v_{AO}}}{2} = \frac{1}{2} \left(\frac{16}{3}\hat{i} - 4\hat{j} \right) = \frac{8}{3}\hat{i} - 2\hat{j}$$

$$(iv) \overline{v_C} = \overline{v_{CO}} + \overline{v_O} = \frac{8}{3}\hat{i} - 2\hat{j} + 4\hat{j} = \frac{8}{3}\hat{i} + 2\hat{j}$$

Connector 16: The motion of a particle in XY plane is given by $\vec{r} = (3t)\hat{i} + (3t^2 - 6t)\hat{j}$, where distances are in metre and time t in second. If the particle starts motion at t = 0, when the particle crosses X axis again, find the

- time t.
- position coordinates of the particle.
- velocity of the particle.
- acceleration of the particle. (tangential, normal and total) at time t.

Solution:

$$(i) x = 3t \quad y = 3t^2 - 6t$$

$$\text{Particle crosses x axis if } y = 0 \text{ i.e., } 3t^2 - 6t = 0$$

$$\Rightarrow t = 0 \text{ (initial position) or } t = 2 \text{ s}$$

$$(ii) \text{ At } t = 2, x = 3 \times 2 = 6 \text{ m, } y = 0$$

$$\therefore (6 \text{ m, } 0)$$

$$(iii) v_x = \frac{dx}{dt} = 3 \text{ ms}^{-1}; v_y = \frac{dy}{dt} = 6t - 6 = 6 \text{ m s}^{-1} \text{ at } t = 2 \text{ s}$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = 3\sqrt{5} \text{ m s}^{-1}$$

$$(iv) a_x = \frac{d^2x}{dt^2} = \frac{dv_x}{dt} = 0; a_y = \frac{d^2y}{dt^2} = \frac{dv_y}{dt} = 6$$

$$\therefore a = \sqrt{0^2 + 6^2} = 6 \text{ m s}^{-2}$$

$$\vec{a} = 6\hat{j} \text{ m s}^{-2}$$

$$\vec{v} = (3\hat{i} + 6\hat{j}) \text{ m s}^{-1}; \hat{v} = \left(\frac{\hat{i} + 2\hat{j}}{\sqrt{5}} \right)$$

$$\vec{a}_t = (a \cos \theta) \hat{v} = (\vec{a} \cdot \hat{v}) \hat{v} = \left[(6\hat{j}) \cdot \left(\frac{\hat{i} + 2\hat{j}}{\sqrt{5}} \right) \right] \left(\frac{\hat{i} + 2\hat{j}}{\sqrt{5}} \right)$$

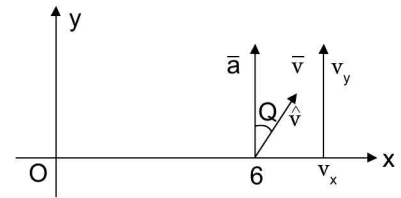
$$(\because \hat{a}_t = \hat{v} \text{ and } \theta = \text{angle between } \vec{a} \text{ and } \vec{a}_t = \text{angle between } \vec{a} \text{ and } \hat{v})$$

$$= \frac{2}{\sqrt{5}} \cdot 6 \left(\frac{\hat{i} + 2\hat{j}}{\sqrt{5}} \right) = \frac{12(\hat{i} + 2\hat{j})}{5}$$

$$\vec{a}_n = \vec{a} - \vec{a}_t = 6\hat{j} - \frac{12(\hat{i} + 2\hat{j})}{5} = \frac{-12\hat{i}}{5} + \frac{6\hat{j}}{5}$$

$$|\vec{a}_t| = \frac{12}{\sqrt{5}} \text{ m s}^{-2}$$

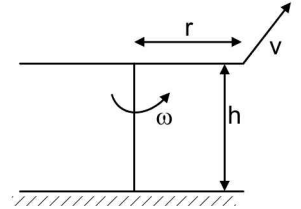
$$|\vec{a}_n| = \frac{6}{\sqrt{5}} \text{ m s}^{-2}$$



3.26 Motion in Two Dimensions

Connector 17: A sprinkler system starts from rest and accelerates with uniform angular acceleration α about a vertical axis as shown in figure.

- Assuming water has a very small speed with respect to the sprinkler arm, find its initial speed relative to the ground in terms of θ , the angle through which the arm has turned.
- Find the horizontal range of the water, in terms of θ , h , α , r and g .
- Find the horizontal distance of the point of impact of water on the ground from the axis of rotation, in terms of θ , h , α , r and g .



Solution:

- Angular velocity $= \omega = \alpha t$

$$\text{Angular displacement} = \theta = \frac{1}{2} \alpha t^2$$

$$\Rightarrow \omega = \sqrt{2\alpha\theta}$$

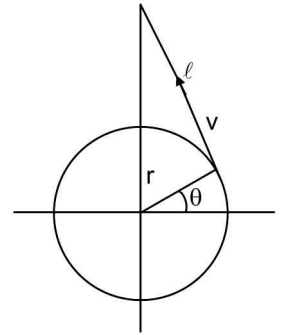
$$\Rightarrow \text{speed} = r\omega = r\sqrt{2\alpha\theta}$$

$$\text{(ii) Time of flight} = \sqrt{\frac{2h}{g}}$$

$$\text{speed} = r\sqrt{2\alpha\theta}$$

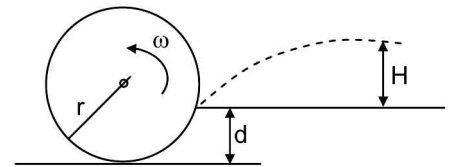
$$\therefore \text{Range} = \ell = \sqrt{\frac{2h}{g}} r \sqrt{2\alpha\theta} = 2r \sqrt{\frac{h\alpha\theta}{g}}$$

$$\text{(iii) The position is given by } \sqrt{r^2 + \ell^2} = r \sqrt{\frac{1 + 4h\alpha\theta}{g}}$$



Connector 18: Consider the wheel of a grinding machine shown below.

- Find the speed at which the particles leave the wheel.
- Find the angle with the horizontal, at which the particles leave the wheel
- Find the maximum height (H shown in figure) reached by the particle.



Solution:

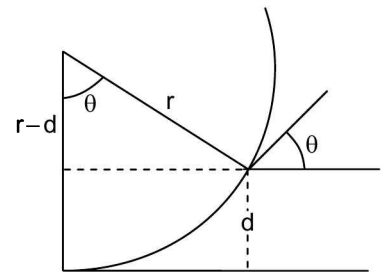
- Speed $= r\omega$

$$\text{(ii) } \cos \theta = \frac{r-d}{r}$$

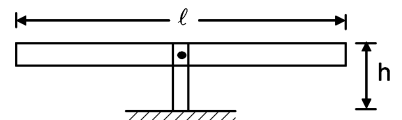
$$\Rightarrow \theta = \cos^{-1} \left(1 - \frac{d}{r} \right)$$

$$\text{(iii) } H = \frac{u^2 \sin^2 \theta}{2g} \text{ and } \sin \theta = \frac{\sqrt{r^2 - (r-d)^2}}{r}$$

$$\Rightarrow H = r^2 \omega^2 \frac{[r^2 - (r-d)^2]}{2g r^2} = \frac{\omega^2 d (2r-d)}{2g}$$



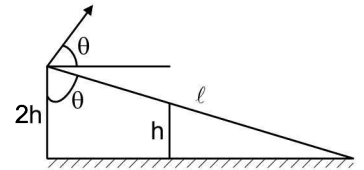
Connector 19: Consider a seesaw of length ℓ and height h as shown. A small boy sits at one end of the seesaw. Another boy pushes down the other end of the seesaw. At the instant when the other end hits the ground, the angular velocity of the seesaw is ' ω ' and the boy is airborne.



- (i) At what angle to the horizontal does he take off?
- (ii) What is his initial velocity?
- (iii) What is the maximum height reached by the boy?
(Neglect the mass of the boy)

Solution:

$$\begin{aligned}
 \text{(i)} \quad \cos \theta &= \frac{2h}{\ell} \\
 \Rightarrow \theta &= \cos^{-1} \left(\frac{2h}{\ell} \right) \\
 \text{(ii)} \quad v &= \frac{\ell}{2} \omega \\
 \text{(iii)} \quad H &= \frac{v^2 \sin^2 \theta}{2g} = \left(\frac{\ell}{2} \omega \right)^2 \cdot \frac{1}{2g} \left(\frac{\sqrt{\ell^2 - (2h)^2}}{\ell} \right)^2 \\
 &= \frac{\ell^2 \omega^2}{4} \cdot \frac{1}{2g} \cdot \frac{\ell^2 - 4h^2}{\ell^2} = \frac{\omega^2 (\ell^2 - 4h^2)}{8g}
 \end{aligned}$$



Connector 20: A particle is on top of a smooth hemispherical surface of radius R . It is given a horizontal velocity v .

- (i) What is the minimum value of v such that the particle never touches the hemisphere again?
If the above condition is satisfied,
- (ii) Find the time required for the particle to reach the ground.
- (iii) Find the horizontal range of the particle and the distance from the base.

Solution:

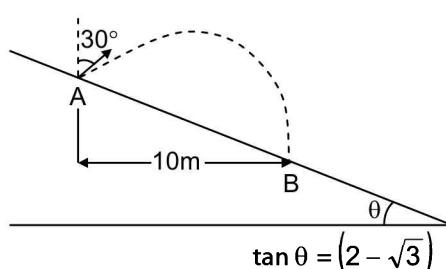
- (i) Let the radius of curvature of the path be r .
The normal acceleration is given by $\frac{v^2}{r}$.
But normal acceleration = g
For no touching, $\frac{v^2}{r} > g \Rightarrow v > \sqrt{gR}$
- (ii) Time of flight = $\sqrt{\frac{2R}{g}}$
- (iii) Range = speed $\times$ time = $\sqrt{gR} \cdot \sqrt{\frac{2R}{g}} = \sqrt{2}R$
Distance from base = $\sqrt{2}R - R = (\sqrt{2} - 1)R$

TOPIC GRIP



Subjective Questions

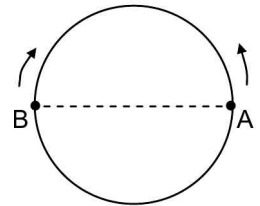
- A ball is thrown upwards from the ground with a speed u for maximum range.
 - Find the maximum height reached by it.
 - Find the angle of elevation at maximum height.
- A projectile launched from the ground with an initial velocity $20\sqrt{5} \text{ ms}^{-1}$, just clears two walls, each 50 m high and separated by 100 m. What is the time of passage between the walls? ($g = 10 \text{ m s}^{-2}$)
- If a particle is projected from origin with the X and Y axes along the horizontal and vertical directions respectively and a target is located at (a, b) .
 - If the velocity of projection is $\sqrt{2ga}$, show that it is impossible to hit the target if $b > \frac{3}{4}a$.
 - if the velocity of projection is $\sqrt{2gb}$, show that it is impossible to hit the target.
 - If the velocity of projection is $\sqrt{ngb}$ where, n is a positive constant, show that it is impossible to hit the target if $a > \left(\sqrt{n(n-2)}\right)b$.
- Two shells were fired one after another with the same initial velocity from a point on the ground, but at different angles with horizontal. Both hit the ground target, 1920 m away. There was a wall of height $11\frac{5}{2} \text{ m}$ at some distance from the point of firing. The first shell just grazed the top of the wall; the second shell comfortably cleared the wall by a margin of $\frac{7}{9}$ times the height of the wall. How far away is the wall from the point of firing? ($g = 10 \text{ m s}^{-2}$)
- A projectile is launched from ground at an angle ' θ ' with the horizontal. If the projectile's velocity, when it was at its maximum height, was $\frac{3}{\sqrt{11}}$ times its velocity when it was at one-third of the maximum height, what was the angle of projection with horizontal?
- A ball is dropped vertically on to an inclined plane at A (inclination $\theta = \tan^{-1}(2 - \sqrt{3})$). The direction of rebound forms an angle 30° with the vertical. The ball strikes the inclined plane again at B, 10 m horizontally away from A. Calculate,



$\tan \theta = (2 - \sqrt{3})$

- velocity of rebound at A.
 - time taken to travel from A to B. ($g = 10 \text{ m s}^{-2}$)
- Two shells were fired upwards from a cannon mounted on a elevated platform, one after the other (with a time delay), with the same initial speed of 200 m s^{-1} , the first at 45° and the second at 30° to horizontal. Both hit the same target simultaneously. What was the time delay in firing the shells? ($g = 10 \text{ m s}^{-2}$).

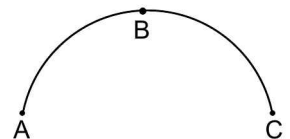
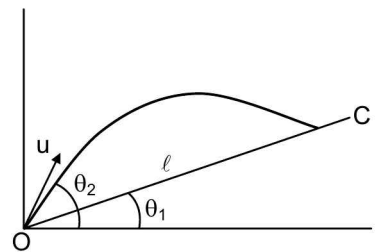
8. Ships A and B are in the high seas. Ship A is sailing from South towards North at 5 m s^{-1} . Ship B is sailing at $5\sqrt{10} \text{ m s}^{-1}$ in a direction making an angle θ towards West of South such that $\tan \theta = \sqrt{1.5}$. At $t = 0$, ship B was at 2000 m directly North of A. Determine the time and distance of their closest approach. Treat high seas as a flat, plane surface.
9. A particle starts from rest and moves along a circle of radius 20 cm with a constant tangential acceleration of 5 cm s^{-2} . How much time is needed after motion begins for the normal acceleration a_n of the particle to be
- equal to its tangential acceleration?
 - double its tangential acceleration?
10. Particle A is moving with constant angular velocity of $3\pi \text{ rad s}^{-1}$ along the horizontal circle in counter clockwise direction. Particle B starts from rest at $t = 0$ and moves along the same circle but clockwise and with angular acceleration of $\pi/4 \text{ rad s}^{-2}$. At $t = 0$, they are at the ends of a diameter as shown. Find the time at which they meet. (Take $\sqrt{38} = 6\frac{1}{6}$)



Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

11. The range of two projectiles launched from ground with the same initial velocity u and angles of projection θ_1 and θ_2 are equal. The ratio of maximum heights reached by them is $\frac{h_1}{h_2} = \frac{1}{3}$. The range R is equal to
- $\frac{u^2}{2g}$
 - $\frac{\sqrt{3} u^2}{2g}$
 - $\frac{u^2}{4g}$
 - $\frac{2u^2}{g}$
12. A body is projected horizontally from a height h above ground with an initial velocity of 10 m s^{-1} . If the body hits the ground with a velocity of 20 m s^{-1} then the value of h is ($g = 10 \text{ m s}^{-2}$)
- 10 m
 - 15 m
 - 30 m
 - 40 m
13. A body is projected upwards on an inclined plane OC at an angle of projection θ_2 to the horizontal. The angle of inclination of the inclined plane is θ_1 ($\theta_1 < \theta_2$). If T is the time of flight and u is the initial velocity, then ' ℓ ' is
- $u \cos(\theta_2 + \theta_1)T$
 - $u \frac{\cos \theta_1}{\cos \theta_2} T$
 - $\frac{u \cos \theta_2 T}{\cos \theta_1}$
 - $\frac{2u \cos \theta_2 T}{\cos \theta_1}$
14. A particle has an initial velocity of $(6\hat{i} + 5\hat{j}) \text{ m s}^{-1}$. The particle accelerates at $(0.5\hat{i} + 0.35\hat{j}) \text{ m s}^{-2}$. Its speed 20 s later is
- 10 m s^{-1}
 - 16 m s^{-1}
 - 20 m s^{-1}
 - 11 m s^{-1}
15. A particle starts from rest on a horizontal semi-circular path at point A and goes to point C with uniform rate of change of speed. The ratio of centripetal and tangential acceleration at the half-way point B is
- $\sqrt{\pi^2 + 1}$
 - π^2
 - π
 - $\frac{1}{\pi}$





Assertion–Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- (c) Statement-1 is True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True

16. Statement 1

If at any time during the flight of a projectile, it gets reflected by a vertical wall as per laws of reflection of a light ray, its time of flight does not change.

and

Statement 2

Such a reflection does not change horizontal speed.

17. Statement 1

A projectile projected from top of a building will always take more time to reach ground if projected obliquely upward than if obliquely downward, whatever be the respective velocities of projection.

and

Statement 2

The situation is equivalent to dropping from different heights.

18. Statement 1

Time of flight of a projectile with initial velocity u at angle θ with a plane is

T , if the plane is horizontal

T_1 , if the plane is inclined, sloping upwards

T_2 , if the plane is inclined, sloping downwards

Then $T_1 T_2 = T^2$

and

Statement 2

Time of flight of a projectile over an inclined plane of angle α with initial velocity u at angle θ to horizontal is $\frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$.

19. Statement 1

If two projectiles, projected simultaneously from different points in space, collide at a certain instant of time, they would have done so even if the space was gravity free.

and

Statement 2

The path of a particle relative to another particle is independent of the magnitude of the constant acceleration with which both move.

20. Statement 1

If two identical swimmers on directly opposite points on the banks of a flowing river, start to swim simultaneously across in shortest possible times with same speed, and if the river washes away their caps at the starting points, then each swimmer can pick up the other's cap immediately at end of journey.

and

Statement 2

Crossing in shortest time means drifting at speed of river.

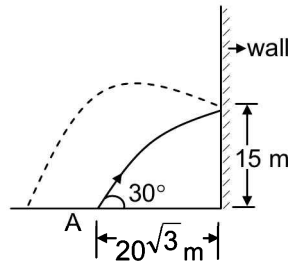


Linked Comprehension Type Questions

Directions: This section contains 2 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage 1

A small steel ball is projected at point A making an angle 30° with the horizontal, towards a vertical smooth steel wall. The ball strikes the steel wall at height 15 m above the ground and is reflected ($g = 10 \text{ m s}^{-2}$).

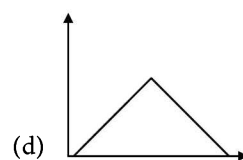
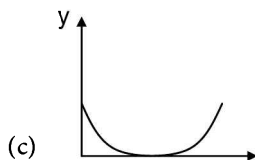
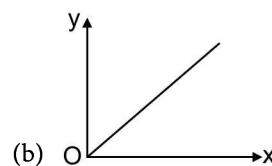
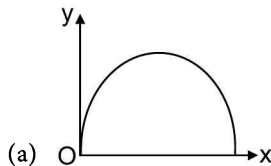


21. The speed of projection of the ball is (m s^{-1})
 (a) 10 (b) 20 (c) 30 (d) 40
22. The time taken by the ball to reach the ground after hitting the wall is (s)
 (a) 1 (b) 2 (c) 4 (d) 3
23. The velocity component of the ball along the wall at the time it hits the wall is (m s^{-1})
 (a) 10 (b) 20 (c) 30 (d) 40

Passage II

A particle moves in the xy plane according to the law $x = at$, $y = at(1 - \alpha t)$, where a and α are positive constants and t is time

24. The variation of y as a function of x is given by



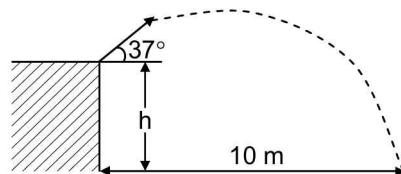
25. The value of x for which y takes maximum value is
 (a) $\frac{a}{2\alpha}$ (b) $\frac{a}{\alpha}$ (c) $\frac{2a}{\alpha}$ (d) $\frac{4a}{\alpha}$
26. The maximum value of y is
 (a) $\frac{a}{3\alpha}$ (b) $\frac{a}{4\alpha}$ (c) $\frac{a}{2\alpha}$ (d) $\frac{a}{\alpha}$



Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

27. A particle projected from ground at an angle θ with the horizontal, reaches a height h in 3 s and reaches the same height again after another 4 s. Then
- time of flight = 10 s
 - $h = 105$ m
 - maximum height = 120 m
 - if the angle of projection is 37° , range = 2000 m
28. A particle is projected from the edge of a platform slightly raised above ground level, with an initial velocity of 10 m s^{-1} and at an angle of projection $\theta = 37^\circ$ with the horizontal. The projectile falls at a distance of 10 m from the foot of the platform.
- Height of the platform = $5/16$ m
 - Height of the platform = 0.52 m
 - Time of flight = 1.25 s
 - Max height reached from ground is 2.11 m
29. A particle starts from rest and moves along a circular path with constant angular acceleration. During the time interval from $t = 4$ s to $t = 4.8$ s, its average angular velocity was 44 rad s^{-1} .
- Its angular velocity at 4 s is 40 rad s^{-1} .
 - Its angular velocity at 4 s is 42 rad s^{-1} .
 - Its angular acceleration is 10 rad s^{-2} .
 - During the first 1 s it makes $\frac{2.5}{\pi}$ revolution.



Matrix-Match Type Question

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

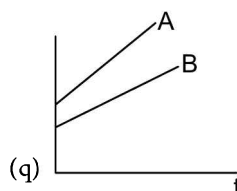
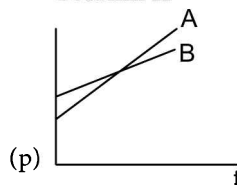
30. A and B are two particles in motion along a straight line such that at time $t = 0$, A is behind B. v_A and v_B are their velocities and a_A and a_B are their accelerations. The origin of the frame of reference is neither on A nor on B. The graphs can have their y axis representing position, velocity or acceleration.

Column I

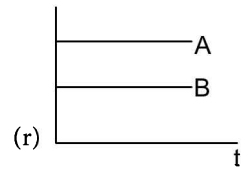
- (a) Uniform motion $v_A > v_B$

- (b) Uniformly accelerated motion; at $t = 0$, $v_A > v_B$; $a_A > a_B$

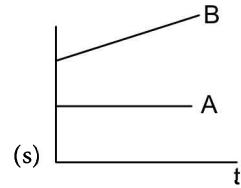
Column II



(c) Uniformly accelerated motion; at $t = 0$, $v_A < v_B$; $a_A > a_B$



(d) Uniformly accelerated motion for B only; at $t = 0$ $v_A < v_B$;



IIT ASSIGNMENT EXERCISE

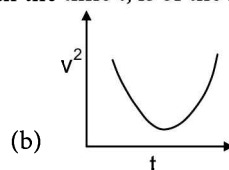
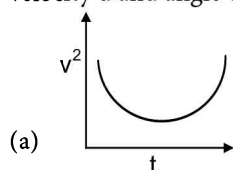


Straight Objective Type Questions

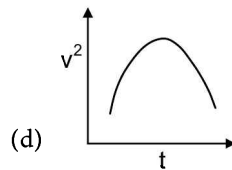
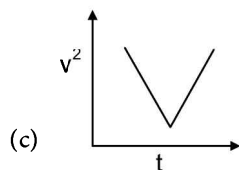
Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

31. At the maximum height of a projectile launched upwards from ground at an angle with horizontal,
 - (a) velocity is zero but acceleration is not zero
 - (b) velocity and acceleration both are zero
 - (c) vertical component of velocity is zero, but acceleration is not zero
 - (d) vertical component of velocity as well as acceleration are zero
32. In the case of a projectile launched from level ground, the time taken to reach the maximum height will be
 - (a) $\frac{1}{4}$ time of flight
 - (b) $\frac{1}{2}$ time of flight
 - (c) $\frac{3}{4}$ of time of flight
 - (d) equal to time of flight
33. When a body is thrown with a velocity u making an angle θ with the horizontal, change in velocity between the points of projection and impact is
 - (a) only in horizontal direction.
 - (b) only in vertical direction.
 - (c) both in horizontal and vertical direction.
 - (d) neither in horizontal nor in vertical direction.
34. For angles of projection $(45 - \alpha)^\circ$ and $(45 + \alpha)^\circ$ with the horizontal, of a projectile launched upwards from level ground, if the range of the projectiles are R_1 and R_2 respectively for the same velocity of projection, then
 - (a) $R_1 = R_2$
 - (b) $R_1 = \frac{3R_2}{2}$
 - (c) $R_2 = \frac{R_1}{2}$
 - (d) $R_2 = 2R_1$
35. A bullet leaves a rifle with a speed of 160 m s^{-1} . The maximum horizontal distance the bullet reaches is ($g = 10 \text{ ms}^{-2}$)
 - (a) 2560 m
 - (b) 5620 m
 - (c) 2150 m
 - (d) 5260 m
36. In the above question, if the bullet is fired for maximum horizontal range, the maximum height the bullet can reach is ($g = 10 \text{ ms}^{-2}$)
 - (a) 320 m
 - (b) 640 m
 - (c) 821 m
 - (d) 934 m
37. At time $t = \frac{T}{4}$, $\frac{H}{H_{\max}}$ is (where T is the total time of flight, $H_{\max}$ is the maximum height, and H is the height at time t of a projectile launched from ground)
 - (a) $\frac{1}{2}$
 - (b) $\frac{3}{4}$
 - (c) $\frac{3}{8}$
 - (d) $\frac{5}{8}$
38. The angle of projection with horizontal of a projectile launched upwards from level ground is 30° . The ratio of horizontal range to maximum height is
 - (a) $4\sqrt{3}$
 - (b) $3\sqrt{4}$
 - (c) $2\sqrt{3}$
 - (d) $3\sqrt{2}$
39. If the maximum range of a gun on a horizontal plane is 16 km, the muzzle velocity of the shell is ($g = 10 \text{ m s}^{-2}$)
 - (a) 200 m s^{-1}
 - (b) 400 m s^{-1}
 - (c) 800 m s^{-1}
 - (d) 1600 m s^{-1}
40. A large number of bullets are fired in all directions from ground, with the same speed v . The maximum area on the ground the bullets can spread is
 - (a) $\frac{\pi v^2}{g}$
 - (b) $\frac{\pi v^4}{g^2}$
 - (c) $\frac{\pi v^2}{g^2}$
 - (d) $\frac{\pi v^2}{g^4}$

41. A fielder on the ground throws a ball at an angle of 15° to the horizontal with velocity v_A at the wicket to dismiss the batsman. Had he thrown the ball at 45° , with a speed v_B to hit the wicket, then $\frac{v_B}{v_A}$ is
- (a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{2}}$
42. For a projectile launched upwards from level ground, if the speed of a projectile at the maximum height is half the speed of projection u , its horizontal range is
- (a) $\frac{u^2}{\sqrt{3}g}$ (b) $\frac{2u^2}{\sqrt{3}g}$ (c) $\frac{\sqrt{3}u^2}{2g}$ (d) $\frac{\sqrt{3}u^2}{g}$
43. The range of a projectile launched from ground is R on the surface of the earth. For the same initial velocity and angle of projection, on a planet where g is one fourth of that on earth, the range will be
- (a) $\frac{R}{4}$ (b) $4R$ (c) $\frac{R}{16}$ (d) $16R$
44. A cricket ball is hit at 45° to the horizontal with a velocity u . Its speed at the highest point is
- (a) u (b) $u/2$ (c) $u \cos 45^\circ$ (d) 0
45. A particle is projected vertically upwards with a velocity given by $v = \sqrt{gR}$, where g is the acceleration due to gravity. Then the maximum height ascended by the particle is (assume g remains same throughout)
- (a) $R/2$ (b) R (c) $R/4$ (d) R^2
46. It is possible to project a particle from level ground with a given velocity in two possible ways so as to make it pass through a point on the ground at a distance r from the point of projection. The product of time taken to reach this point in the two possible ways is then proportional to
- (a) r (b) r^2 (c) $\frac{1}{r}$ (d) $\frac{1}{r^2}$
47. An object is thrown upwards from ground at an angle of 45° with the horizontal. The horizontal range of the particle is equal to
- (a) maximum height (b) half the maximum height
(c) twice the maximum height (d) four times the maximum height
48. The range of a projectile when launched upwards from ground at an angle of 15° with the horizontal is 1.5 km. The range of the projectile launched upwards from the same point at an angle of 45° to the horizontal with the same speed is
- (a) 1.5 km (b) 3.0 km (c) 6.0 km (d) 4.5 km
49. The maximum height and horizontal range are equal for a projectile launched upward from ground, at an angle with horizontal equal to
- (a) 60° (b) 90° (c) $\tan^{-1}8$ (d) $\tan^{-1}4$
50. For a projectile launched from ground at an angle θ with the vertical, the maximum height reached is
- (a) directly proportional to $\sin^2\theta$ (b) directly proportional to $\cos^2\theta$
(c) directly proportional to $1/\sin\theta$ (d) directly proportional to $1/\sin^2\theta$
51. The variation of the square of the speed at any instant of a projectile (launched upwards from ground with an initial velocity u and angle of projection θ with the horizontal) with the time t , is of the form



3.36 Motion in Two Dimensions

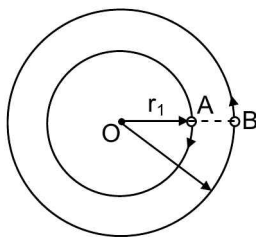


52. An aeroplane flying horizontally with a speed of 360 kmph releases a bomb at a height of 490 m above the ground. The horizontal distance travelled by the bomb before hitting the ground is ($g = 9.8 \text{ m s}^{-2}$)
 (a) 490 m (b) 780 m (c) 1000 m (d) 1250 m
53. For very small angles of projection α with horizontal, the ratio of the range to the maximum height of a particle projected upwards from level ground with initial speed u is
 (a) $\frac{\alpha}{4}$ (b) $\frac{4}{\alpha}$ (c) $\frac{4u}{g\alpha}$ (d) $\frac{4\alpha}{gu}$
54. A projectile is projected upwards from level ground at an angle of 60° with horizontal with an initial speed of 150 m s^{-1} . Then the time after which its inclination with the horizontal becomes 45° is ($g = 10 \text{ m s}^{-2}$, $\sqrt{3} = 1.7$)
 (a) 2.75 s (b) 5.5 s (c) 11 s (d) 15 s
55. A body is projected upwards from level ground such that its velocity makes an angle of 45° with the horizontal at half the maximum height. The initial angle of projection with horizontal is $\tan^{-1}x$, where x is
 (a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{2}$ (d) 2
56. A body is projected upwards from level ground with an initial velocity of 20 m s^{-1} at an angle of 30° to the horizontal. Then the magnitude of its velocity after 1 second in m s^{-1} is ($g = 10 \text{ m s}^{-2}$)
 (a) $10\sqrt{3}$ (b) 10 (c) $10(\sqrt{3} - 1)$ (d) 20
57. A stone is thrown upwards from ground at an angle of 45° with the horizontal. It rises to a maximum height of 10 m. Its horizontal range is
 (a) 10 m (b) 20 m (c) 40 m (d) 60 m
58. A gun can impart a maximum speed of 100 m s^{-1} to a bullet. A boy holding this gun is separated from a stationary target by a horizontal distance of 50 m. In order to hit the target in minimum time, the barrel of the gun is directed towards a point P, vertically above the target. The vertical separation between P and the target is (Neglect length of the barrel of gun and take $g = 10 \text{ m s}^{-2}$)
 (a) 1.8 m (b) 1.6 m (c) 1.4 m (d) 1.25 m
59. A cannon on a level plane is aimed at an angle θ above the horizontal and a shell is fired with a muzzle velocity v_0 towards a vertical cliff at a horizontal distance D away. Then the height above the bottom of the cliff, at which the shell strikes the sidewalls of the cliff is
 (a) $D \sin\theta - \frac{gD^2}{2v_0^2 \sin^2\theta}$ (b) $D \cos\theta - \frac{gD^2}{2v_0^2 \cos^2\theta}$ (c) $D \tan\theta - \frac{gD^2}{2v_0^2 \cos^2\theta}$ (d) $D \tan\theta - \frac{gD^2}{2v_0^2 \sin^2\theta}$
60. "The instantaneous acceleration vector of a particle projected from the ground is always in the direction of motion". This statement is
 (a) true (b) false
 (c) true or false depending on the angle of projection (d) Acceleration vector for a projectile is zero
61. Two seconds after projection upwards from level ground, a projectile is moving at 30° to the horizontal; after one more second, its velocity is horizontal. The velocity of projection was in m s^{-1} ($g = 10 \text{ m s}^{-2}$)
 (a) $10\sqrt{3}$ (b) 20 (c) 10 (d) $20\sqrt{3}$

62. A body is thrown horizontally with a velocity of 10 m s^{-1} from the top of a tower. It takes 5 second to reach the ground. The height of the tower is ($g = 10 \text{ m s}^{-2}$)
 (a) 125 m (b) 50 m (c) 250 m (d) 275 m
63. If a particle is projected with an initial velocity u , making an angle θ with the horizontal, the speed t second after projection is given by
 (a) $u - gt$ (b) $\sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$
 (c) $\sqrt{2ugt \sin \theta}$ (d) Zero
64. From the top of a tower of height H , two identical bodies, A and B, are thrown horizontally with velocities 100 m s^{-1} and 50 m s^{-1} in the same direction at the same instant. Then
 (a) A will reach ground first.
 (b) B will reach the ground first.
 (c) Both will reach the ground simultaneously.
 (d) All of the above cases may be true depending on the height of the tower.
65. In the above question the separation between the two bodies on the ground will be
 (a) $100 \sqrt{\frac{2H}{g}}$ (b) $50 \sqrt{\frac{2H}{g}}$ (c) $\frac{\sqrt{2H}}{g} \times 100$ (d) $\frac{100}{\sqrt{2H}} \times g$
66. When a body is thrown horizontally from the top of a tower of height h with a velocity u , its horizontal component of velocity after t second will be
 (a) zero (b) $u \sin \theta$ (c) u (d) $\frac{u}{t}$
67. In the above case, the vertical component of velocity after t second will be
 (a) u (b) zero (c) $\frac{1}{2} gt^2$ (d) gt
68. In the case described in Q.No. 66, the angle made by the velocity of the body with the horizontal 't' second after the throw will be
 (a) $\tan^{-1} \left(\frac{u}{gt} \right)$ (b) $\tan^{-1} \left(\frac{gt}{u} \right)$ (c) $\tan^{-1}(gt)$ (d) $\tan^{-1} \left(\frac{u^2}{g^2 t^2} \right)$
69. A bullet fired horizontally, with a velocity v from a gun, located on a hill of height h , reaches the ground in a time t proportional to
 (a) $\frac{1}{\sqrt{v}}$ (b) $\sqrt{h}$ (c) $\sqrt{v}$ (d) $\frac{1}{\sqrt{h}}$
70. A point on an inclined plane can be reached by projecting a particle from another point on it at either 40° or at 60° with the horizontal, with the same initial speeds. The angle of inclination of the plane is
 (a) 50° (b) 25° (c) 10° (d) 15°
71. A boy throws stones with a speed of 12 m s^{-1} . If he releases them at a height 1.25 m, the maximum distance from his feet where they land is
 (a) 14.4 m (b) 7.2 m (c) 15.6 m (d) 13.2 m
72. A body is dropped from the window of a train. The time taken by the dropped body to reach the ground will be
 (a) large if the train is moving.
 (b) large if the train is at rest.
 (c) less if the train is moving.
 (d) independent of the state of the train (whether moving or at rest).

3.38 Motion in Two Dimensions

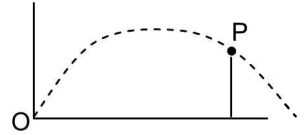
73. The X and Y co-ordinates of a particle moving in XY plane at time 't' are given by $x = ct^2$ and $y = bt^2$. The speed of the particle at any instant is given by
 (a) $2t(c + b)$ (b) $2t\sqrt{c^2 + b^2}$ (c) $t\sqrt{c^2 + b^2}$ (d) $2t\sqrt{c^2 + b^2}$
74. The path of a projectile observed from a second projectile will be a
 (a) parabola (b) ellipse (c) straight line (d) circle
75. A man in a train moving with constant speed drops a ball on to the platform. The path of the ball seen by an observer standing on the platform will be a/an
 (a) straight line (b) circle (c) parabola (d) ellipse
76. The speed of a body, constrained to move in a smooth horizontal circular groove of radius $r = 1$ m is increased at the rate of 1 m s^{-2} starting from rest. After 5 s, the acceleration experienced by the body is (in m s^{-2}) approximately.
 (a) 1 (b) $25\sqrt{2}$ (c) 25 (d) 50
77. The velocity of a body under uniform circular motion changes by $|\Delta v| = v$, where v is the speed of the body and during this time its angular position changes by θ . The value of θ is
 (a) 30° (b) 45° (c) 60° (d) 75°
78. The angular velocity of a body under uniform circular motion is doubled, without changing its centripetal acceleration. If v was the original speed of the body, its new speed is
 (a) $4v$ (b) $2v$ (c) $\sqrt{2} v$ (d) $\frac{v}{2}$
79. Two bodies A and B are moving in concentric circular paths in uniform circular motion with common centre O, in clockwise and anticlockwise directions respectively. The ratio of their radii is $r_B/r_A = 1.5$. They cross each other at points where their radii coincide (see figure), with a relative velocity $2v$ where, v is the speed of A. Then the ratio of angular velocities ω_A/ω_B is



- (a) 2 (b) 4 (c) 1.5 (d) 2.25
80. A body starts from rest and moves in a circular path with angular acceleration 4 rad s^{-2} . The angle between the velocity and acceleration will be 45° in time
 (a) 2 s (b) $\frac{1}{2} \text{ s}$ (c) 1 s (d) $\frac{1}{4} \text{ s}$
81. A projectile launched upwards from level ground with an initial velocity u has a range, time of flight and maximum height attained equal to R_1 , T_1 and H_1 respectively for angle of projection 30° with horizontal and R_2 , T_2 and H_2 respectively for angle of projection 60° with horizontal. Then
 (a) Both ranges and times of flight are same. (b) Range is same but $\frac{T_1}{T_2} = \frac{1}{3}$
 (c) $\frac{H_1}{H_2} = \frac{1}{\sqrt{3}}$ (d) $\frac{H_1}{H_2} = \left(\frac{T_1}{T_2}\right)^2$

82. Under conditions of maximum range two projectiles launched upwards from level ground have their times of flight in the ratio 1:4. Then their ranges are in the ratio
- (a) 1:2 (b) 1:3 (c) 1:4 (d) 1:16

83. A projectile is projected upwards from level ground, with an initial velocity u and angle of projection 60° with horizontal. It is found that at a particular position P on its path the radial distance from the point of projection O is twice the vertical height of P. The distance of OP is



- (a) $\frac{u^2}{\sqrt{3}g}$ (b) $\frac{u^2}{2g}$
(c) $\frac{9u^2}{16g}$ (d) $\frac{2u^2}{3g}$

84. For a projectile launched upwards from level ground, with initial velocity u and angle of projection 60° with the horizontal, the height above ground will be greater than the horizontal distance covered, up to time

- (a) $\frac{u}{\sqrt{2}g}$ (b) $\frac{u}{\sqrt{3}g}$ (c) $\frac{u}{g}(\sqrt{3}-1)$ (d) $\frac{u}{2g}(\sqrt{2}-1)$

85. A particle projected from the top of a tower at an initial velocity 40 m s^{-1} and angle of projection 30° falls at a distance of $200\sqrt{3} \text{ m}$ from the foot of the tower. The height of the tower is: ($g = 10 \text{ m s}^{-2}$)

- (a) 160 m (b) 200 m (c) 300 m (d) 400 m

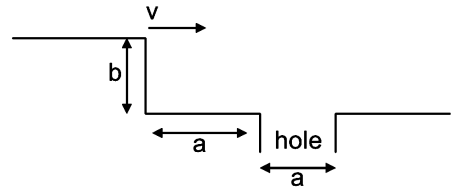
86. A body is projected from the top of a tower with a horizontal velocity of 10 m s^{-1} . Taking the bottom of tower as origin, horizontal direction as +X and vertical direction as +Y respectively, the projectile has co-ordinates $x = 10 \text{ m}$, $y = 95 \text{ m}$ at a particular instant. The height of the tower is ($g = 10 \text{ m s}^{-2}$)

- (a) 950 m (b) 9.5 m (c) 100 m (d) 200 m

87. Two bodies are dropped from two different heights h_1 and h_2 . One of them is dropped directly down, while the other is projected with a horizontal velocity of 10 m s^{-1} . It is found that both reach the ground with same speed ($g = 10 \text{ m s}^{-2}$). Then

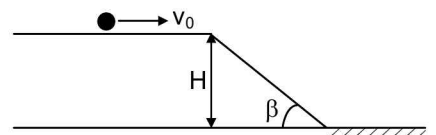
- (a) $h_1 = h_2$ (b) $\frac{h_1}{h_2} = \sqrt{10}$
(c) $h_1 - h_2 = 5 \text{ m}$ (d) given condition is impossible to achieve.

88. A particle is projected horizontally from a height b . The range of values of horizontal speed v so that it falls in the hole of length a is



- (a) $\left(a\sqrt{\frac{g}{b}}, 2a\sqrt{\frac{g}{b}}\right)$ (b) $\left(a\sqrt{\frac{g}{2b}}, a\sqrt{\frac{2g}{b}}\right)$
(c) $\left(a\sqrt{\frac{g}{b}}, 3a\sqrt{\frac{g}{b}}\right)$ (d) $\left(b\sqrt{\frac{g}{a}}, a\sqrt{\frac{g}{b}}\right)$

89. A block is in the form of a flat and inclined plane. A body travels with a speed v_0 as shown. The minimum value of angle β so that the particle does not hit the incline is



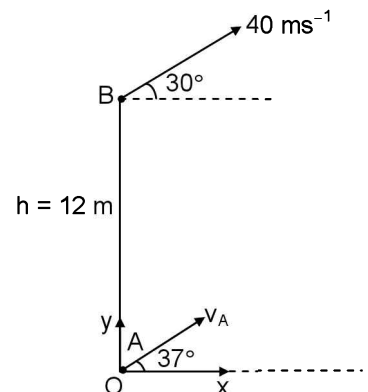
- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\tan^{-1} \frac{v_0}{\sqrt{2gH}}$ (d) $\tan^{-1} \left(\frac{1}{v_0} \sqrt{\frac{gH}{2}} \right)$

3.40 Motion in Two Dimensions

90. For a projectile launched upwards from level ground, if R/T (where R is range and T is total time of flight) equals 20 m s^{-1} and initial velocity $u = 40 \text{ m s}^{-1}$, angle of projection with horizontal is
 (a) 45° (b) 60° (c) 20° (d) 15°
91. A particle is projected upwards from level ground at an angle with horizontal such that its range is twice the maximum height reached by it. The angle made by its velocity vector with the horizontal at a horizontal distance $\frac{R}{4}$ from the point of projection (R is the horizontal range) is
 (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{12}$
92. A projectile launched upwards from ground has a range of 180 m and its time of flight is 10 s . The velocity (in m s^{-1}) after 5 s from start is (angle of projection with horizontal is θ)
 (a) $18 \cos \theta$ (b) 18 (c) 736 (d) $\frac{18}{\cos \theta}$
93. A particle is projected upwards from ground with a speed u at an angle α with horizontal. At the same time, another particle is dropped from a height h . If they collide at a horizontal distance s from the point of projection, h is
 (a) $s - \frac{u^2}{g} \tan \alpha$ (b) $s + \frac{u^2 \sin 2\alpha}{g}$ (c) $s \tan \alpha$ (d) $\sqrt{s^2 + \left(\frac{u^2}{g}\right)^2}$
94. A dog on the horizontal ground at certain distance near the foot of a tall building of height h is threatened by a boy at the top of the building and runs along the horizontal X direction with a uniform velocity of magnitude v_d . At the same instant the boy throws a stone with horizontal velocity of magnitude v_b parallel to X axis and manages to hit the dog. The magnitude of the relative velocity of the stone with respect to the dog, when it hits the dog is
 (a) $v_b - v_d$ (b) $\sqrt{|v_b|^2 - |v_d|^2}$ (c) $v_b + v_d + \sqrt{2gh}$ (d) $\sqrt{(v_b - v_d)^2 + 2gh}$
95. Two bodies A and B are projected upwards from ground with the same velocity but at different angles of projection θ_1 and θ_2 ($\theta_1 < \theta_2$) with horizontal and they have same range. If their vertical separation during the time of flight of A is h , then
 (a) h rises gradually and reaches maximum when it is at $\frac{1}{2}$ the range.
 (b) h increases linearly with time.
 (c) h is varying randomly with time.
 (d) h is constant.
96. A boy, travelling in a car moving with uniform velocity, projects a ball outside. If the ball returns to the boy after some time, he projected the ball
 (a) vertically upward with respect to the car.
 (b) at 45° with horizontal with a certain velocity.
 (c) horizontally with a velocity equal to the velocity of the car.
 (d) with horizontal velocity opposite to that of the car.

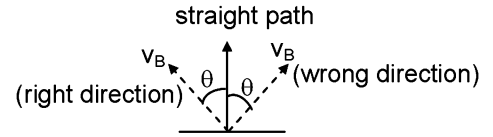
97. Two projectiles A and B are projected as shown, (A from origin on ground and B from height $h = 12 \text{ m}$ above A), at the same instant and they meet in mid-air. The relative velocity of B with respect to A ($\vec{v}_{B/A}$) when they meet is in m s^{-1})

- (a) $8\hat{j}$
 (b) $2\hat{i} + 3\hat{j}$
 (c) $5\hat{i} + 3\hat{j}$
 (d) $-6\hat{j}$



98. A river $5\sqrt{3}$ km wide flows with a velocity of 5 kmph. A boat which can move with a velocity of 10 kmph in still water crosses the river, without any drift. For this the boat has to tilt its velocity by θ° with respect to the straight direction. If the boat's velocity is tilted by same angle θ in the wrong direction, the drift will be

(a) 8.1 km (b) 9.1 km (c) 10 km (d) 12 km

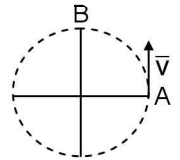


99. Rain drops falling with actual velocity of 5 m s^{-1} on a windy day appear to fall vertically down on a man moving Eastward with a speed of 3 m s^{-1} . If the man reverses his direction with the same speed, the raindrops will fall on him with a speed of (in m s^{-1})

(a) $\sqrt{35}$ (b) $\sqrt{52}$ (c) $\sqrt{47}$ (d) $\sqrt{83}$

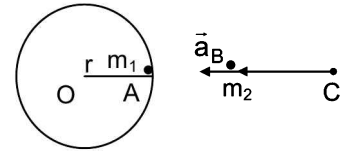
100. A body is moving with a uniform speed v in a circular path of radius r as shown in figure. T is the period of rotation. The average velocity in segment AB is

(a) $\frac{2v}{T}$ (b) $\frac{4v}{T}$
(c) $\frac{4v}{\pi}$ (d) $\frac{2\sqrt{2}v}{\pi}$



101. A body m_1 rotating in a circle of radius r with uniform angular velocity, has period T . Another body m_2 moves from C towards O (see figure) with uniform acceleration a_B . The relative acceleration of m_2 with respect to m_1 at the instant shown is

(a) $a_B - \left(\frac{2\pi}{T}\right)^2 r$ (b) $a_B r$
(c) $a_B + \left(\frac{2\pi}{T}\right)^2 r$ (d) $\sqrt{a_B^2 + \left(\left(\frac{2\pi}{T}\right)^2 r\right)^2}$



102. A stone is tied to a string and whirled in a horizontal circle of radius 1 m with a constant angular velocity ω . The change in velocity of the stone in time t is

(a) ωt (b) $2\cos\frac{\omega t}{2}$ (c) $2\omega\sin\frac{\omega t}{2}$ (d) $\frac{\omega}{t}$

103. A wheel at rest is turned with a uniform angular acceleration, such that a particle on the rim of the wheel experiences a tangential acceleration of $20\pi \text{ m s}^{-2}$. The radius of the wheel is 2 m. The instantaneous period of rotation at the end of 4 s is

(a) 0.01 s (b) 0.02 s (c) 0.025 s (d) 0.05 s

104. In the above case, the angular position of the particle with respect to its initial position after 1 s is

(a) 45° (b) 90° (c) 180° (d) 65°

105. Two particles A and B in motion along same circle in same sense. At $t = 0$, they are at the ends of a diameter, and have equal speeds of $\pi \text{ rad s}^{-1}$ but travel with constant but different tangential accelerations; B's acceleration is twice that of A. They collide at $t = 1 \text{ s}$. If they had travelled in opposite sense, they would have collided at $t =$

(a) $\frac{1}{6} \text{ s}$ (b) $\frac{1}{4} \text{ s}$
(c) $\frac{1}{3} \text{ s}$ (d) $\frac{1}{2} \text{ s}$

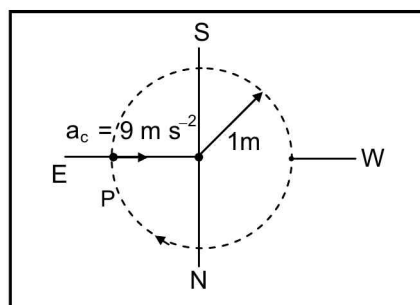


Fig. 1

Fig. 2

3.42 Motion in Two Dimensions

106. A particle in uniformly accelerated circular motion of radius 1 m has a constant tangential acceleration. At $t = 0$, the speed is u , and its acceleration vector makes an angle $\frac{\pi}{4}$ with the velocity vector. At $t = 1$ s, its acceleration vector makes an angle $\tan^{-1} 4$ with the velocity vector. u is (in m s^{-1})
 (a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2
107. A particle revolving uniformly at 170 rad s^{-1} is brought to rest by a uniform angular retardation of $\alpha = 5 \text{ rad s}^{-2}$. The time required to stop is
 (a) 10 s (b) 34 s (c) 17 s (d) 85 s
108. In the above problem, the number of revolutions the particle makes before coming to rest is
 (a) $\frac{400}{\pi}$ (b) $\frac{1300}{3\pi}$
 (c) $\frac{1445}{\pi}$ (d) $\frac{700}{\pi}$
109. A body is kept on a smooth table and tied to a string 1 m long and rotated clockwise on the horizontal plane of the table. Just when it reaches position P it experiences a centripetal acceleration of 9 m s^{-2} . At that instant the string is cut. The body will fly now
 (a) eastward with an acceleration of 9 m s^{-2} .
 (b) westward with an acceleration of 9 m s^{-2} .
 (c) eastward with velocity of 3 m s^{-1} .
 (d) southward with a velocity of 3 m s^{-1} .
110. A ball is kept on a horizontal table with a smooth surface and is tied to a string of adjustable length, which is attached to a vertical axis. When the ball is rotated with a constant angular velocity ω and radius of rotation R , the magnitude of the acceleration of the ball is a . If the magnitude of angular velocity is doubled, the radius of rotation to keep the same magnitude of the acceleration is
 (a) $\frac{R}{2}$ (b) $\frac{R}{4}$ (c) $2R$ (d) $\sqrt{2}R$



Assertion–Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
 (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
 (c) Statement-1 is True, Statement-2 is False
 (d) Statement-1 is False, Statement-2 is True

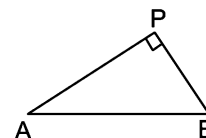
111. Statement 1

If APB is an inclined plane and if a projectile, launched at A lands at B via P, the angle of projection at A with the horizontal cannot be less than 45° .

and

Statement 2

For a ground to ground projectile, the angle of projection with horizontal and angle of landing with horizontal are equal.



112. Statement 1

For a particle in motion on a plane with constant acceleration, if displacement from origin is zero at time $t = t_1$ and at $t = t_2$, such that $t_1, t_2 \neq 0$, then for $0 < t < t_1$

$\hat{r} = -\hat{v}$ where, $\vec{r}$ and $\vec{v}$ are instantaneous position and velocity vectors respectively.

and

Statement 2

For a particle in motion along a plane, $\vec{r}$ and $\frac{d\vec{r}}{dt}$ will be along the same straight line.

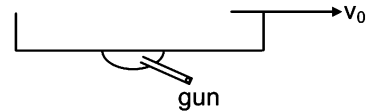
113. Statement 1

A gun mounted on the underside of an aircraft flying horizontally with constant speed has equal range in all directions.

and

Statement 2

Time of flight of the bomb depends only on the height from which it is released.


Linked Comprehension Type Questions

Directions: This section contains a paragraph. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage I

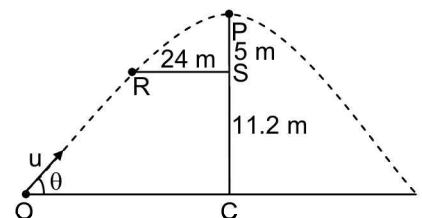
An elevator is going up with an upward acceleration of 0.2 m s^{-2} . At the instant when its velocity is 2 m s^{-1} a stone is projected upwards from its floor with a speed of 2 m s^{-1} at an angle of 30° with horizontal, relative to the elevator ($g = 9.8 \text{ m s}^{-2}$).

114. The time taken by the stone to return to the floor is (s)
 (a) 0.2 (b) 0.1 (c) 0.4 (d) 0.12
115. The distance travelled by the floor of the lift from the time the stone was projected to the time when the stone hits the floor, is
 (a) 0.404 m (b) 0.400 m (c) 0.412 m (d) 0.416 m
116. The path of the projectile as observed by an observer outside the lift is
 (a) parabola (b) straight line (c) circle (d) hyperbola


Multiple Correct Objective Type Questions

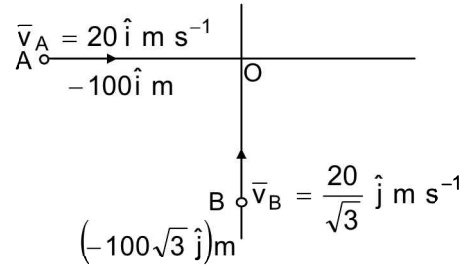
Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

117. A projectile launched from O on ground, just clears the obstacle CSPR, in the shape shown, reaching its maximum height at P. Then
 (a) $u = 30 \text{ m s}^{-1}$
 (b) time of flight = 3.6 s
 (c) time of flight cannot be ascertained with the given data
 (d) $\theta = 30^\circ$



3.44 Motion in Two Dimensions

118. Each of two boats A and B, can move at 20 kmph in still water. The river they want to cross is $2\sqrt{3}$ km wide and flows at 10 kmph. A crosses by shortest distance and B crosses by shortest time.
- They reach the other side with a time difference of 10 minute.
 - They reach the other side with a separation of $\sqrt{3}$ km.
 - If boat B decides to reduce its drift speed (i.e., resultant speed in the direction of flow of the river) to 5 kmph, then it will reach at a position of $\left(\frac{2}{\sqrt{5}}\right)$ km away from A
 - If B moves as stated in option (c), its resultant speed is 20 kmph
119. Position of two vehicles A and B with reference to origin O and their velocities are as shown. Then, if they move with the same velocities shown
- they will collide in 8 s.
 - distance of closest approach is 100 m.
 - their relative velocity is $\frac{40}{\sqrt{3}} \text{ m s}^{-1}$ (modulus value).
 - they reach the closest position in 7.5 s.



Matrix-Match Type Question

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

120. Let ℓ be the applicable length

Column I	Column II
(a) Time for a boat to cross a river of width ℓ by the shortest distance ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water)	(p) $\frac{\ell}{ \vec{v} + \vec{u} }$
(b) Time for two railway trains of velocities $\vec{v}$ and $\vec{u}$ running in opposite directions to cross each other. (ℓ initial separation of with ℓ)	(q) $\frac{\ell}{\sqrt{v^2 - u^2}}$
(c) Time for a boat to cross a river in the shortest time ($\vec{v}$ -velocity of boat with respect to water $\vec{u}$ -velocity of water).	(r) $\frac{\ell}{ \vec{v} + \vec{u} }$
(d) Time for a boat to travel a distance ℓ downstream ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water).	(s) $\frac{\ell}{ \vec{v} }$

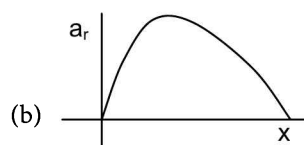
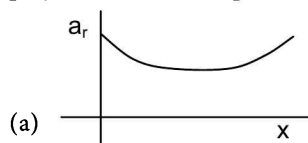
ADDITIONAL PRACTICE EXERCISE



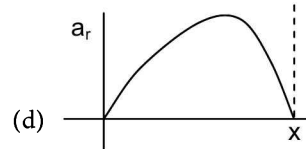
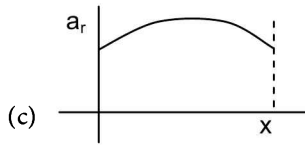
Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

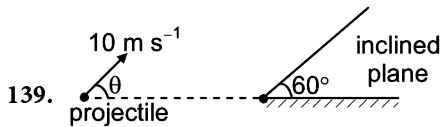
121. The maximum height reached by a projectile launched upwards from level ground is half the horizontal range. Then, its angle of projection with horizontal is
 (a) $\tan^{-1}1$ (b) $\tan^{-1}2$ (c) $\tan^{-1}\frac{1}{2}$ (d) $\tan^{-1}4$
122. In a cricket ground a ball thrown by one player reaches another in 2 s. The maximum height attained by the ball above the point of projection will be [$g = 10 \text{ m s}^{-2}$]
 (a) 2.5 m (b) 7.5 m (c) 10 m (d) 5 m
123. During the projectile motion the quantity that remains unchanged is
 (a) horizontal velocity. (b) vertical velocity. (c) distance. (d) angular velocity.
124. Two bullets are fired from ground at angles θ and $(90 - \theta)$ to the horizontal with the same initial speeds. The ratio of their times of flight is
 (a) 1 : 1 (b) $\tan\theta : 1$ (c) $1 : \tan\theta$ (d) $\cos\theta : 1$
125. Two particles are projected upwards from level ground with the same initial velocity v_0 at two different angles of projection with horizontal, such that their horizontal ranges are the same. The ratio of the heights of their highest points will be (let $\theta_1 =$ angle of projection of the first particle with horizontal)
 (a) $\frac{1}{\cos\theta_1}$ (b) $\sin\theta_1$ (c) $\tan^2\theta_1$ (d) $\cos\theta_1$
126. The time of flight of a projectile launched upwards from ground is maximum when the angle of projection is
 (a) 45° (b) 60° (c) 90° (d) 30°
127. For a projectile launched upwards from ground, the condition of horizontal distance travelled being equal to the vertical height at some point in its trajectory,
 (a) can be achieved for any angle of projection with the horizontal with appropriate u .
 (b) can be achieved only if angle of projection with horizontal is less than 60° .
 (c) can be achieved only if angle of projection with horizontal is greater than 45° .
 (d) cannot be achieved at all.
128. A particle is projected upwards from level ground with an initial speed u at an angle of $\frac{\pi}{3}$ radian with horizontal. The time required for the velocity vector to turn by $\frac{\pi}{12}$ radian is
 (a) $\frac{u}{g}$ (b) $\frac{u}{4g}$ (c) $\frac{u}{2g}$ (d) $\left(\frac{\sqrt{3}-1}{2}\right)\frac{u}{g}$
129. The plot of normal acceleration (component of acceleration perpendicular to velocity) vs horizontal distance for a projectile launched upwards from level ground at an angle with horizontal, is given by



3.46 Motion in Two Dimensions



130. If a ground projectile changes its direction such that at $t = \frac{T}{3}$ ($T \rightarrow$ time of flight), the direction of its velocity is at 60° with vertical upward direction, then what is the other angle of projection to obtain the same range as in the first case, assuming same initial speed in both cases.
 (a) 30° (b) 60° (c) $\cot^{-1} 3$ (d) $\tan^{-1} 3$
131. When the angle of projection of a projectile is increased from θ_1 to θ_2 , both range and maximum height attained increase for same initial velocity of projection. Then the condition to be satisfied is
 (a) $\theta_1 < 45^\circ, \theta_2 < 45^\circ$ (b) $\theta_1 < 45^\circ, \theta_2 < 90^\circ$ (c) $\theta_1 + \theta_2 < 90^\circ$ (d) $\theta_1 > 45^\circ, \theta_2 > 45^\circ$
132. A body thrown horizontally from top of a building reaches ground with a velocity of 22.5 m s^{-1} at an angle 22.5° with the ground. If the same throw was made vertically downwards, the body's velocity on reaching the ground would have been (in m s^{-1})
 (a) $22.5 \times 2^{\frac{1}{4}}$ (b) $22.5 \times \sqrt{2}$ (c) 45 (d) 22.5
133. A projectile is projected upwards from level ground for maximum range. Unfortunately, in that region horizontal wind blows, exerting an additional constant horizontal acceleration of 10 m s^{-2} . The path of the projectile is ($g = 10 \text{ m s}^{-2}$)
 (a) a straight line (b) a parabola (c) either (a) or (b) (d) neither (a) nor (b)
134. In above the question, the range obtained is
 (a) zero (b) twice the aimed range
 (c) either (d) None of the above
135. A stone projected from ground so as to reach a maximum height of H , hits a bird at height $\frac{H}{2}$ on its way up, due to which its velocity is halved without change of direction. The maximum height reached by it is
 (a) H (b) $\frac{7H}{8}$ (c) $\frac{3H}{4}$ (d) $\frac{5H}{8}$
136. Two projectiles, projected upwards from level ground with the same initial velocity, have the same range but maximum heights reached by them are in the ratio 3 : 4. The difference in their angles of projection is given by $\theta =$
 (a) $\tan^{-1} \frac{1}{\sqrt{3}}$ (b) $\tan^{-1} \frac{1}{2\sqrt{3}}$ (c) $\tan^{-1} \frac{1}{3\sqrt{3}}$ (d) $\tan^{-1} \frac{1}{4\sqrt{3}}$
137. Two projectiles A and B projected upwards from level ground at 30° and 60° with horizontal and same initial speeds respectively achieve equal range, attain maximum heights H_A and H_B and times of flight T_A and T_B respectively. Then,
 (a) $\frac{H_A}{T_A} < \frac{H_B}{T_B}$ (b) $\frac{H_A}{T_A} = \frac{H_B}{T_B}$ (c) $\frac{H_A}{T_A} > \frac{H_B}{T_B}$ (d) cannot be concluded
138. At a height 1 m from ground, the velocity of a projectile launched upwards from level ground is $2\hat{i} + 3\hat{j} \text{ m s}^{-1}$ where the X and Y-axes are along the horizontal and vertical directions respectively. Angle of projection with horizontal is ($g = 10 \text{ m s}^{-2}$)
 (a) $\tan^{-1} \frac{2}{\sqrt{27}}$ (b) $\tan^{-1} \sqrt{7}$ (c) $\tan^{-1} \frac{\sqrt{21}}{2}$ (d) $\tan^{-1} \frac{\sqrt{29}}{2}$

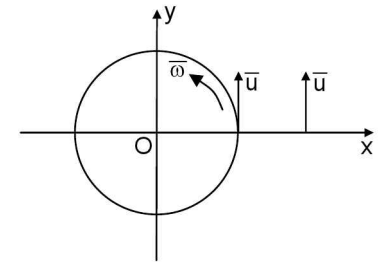
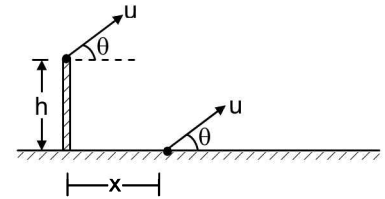


The projectile hits the inclined plane perpendicularly after $\frac{2}{\sqrt{3}}$ s. θ is

- (a) 30° (b) 45° (c) 60° (d) 75°
140. Two projectiles projected upwards from level ground, at same initial speeds have same range but different times of flight. The ratio of the product of times of flight to their range is (g = acceleration due to gravity)
- (a) g (b) $\frac{g}{2}$ (c) $\frac{1}{g}$ (d) $\frac{2}{g}$
141. A stone projected from ground at 37° with horizontal hits the top of the pole when its velocity is least. The angle of elevation of the top of a pole from the point of projection of the stone is
- (a) $\tan^{-1} \frac{1}{4}$ (b) $\tan^{-1} \frac{1}{3}$ (c) $\tan^{-1} \frac{3}{8}$ (d) $\tan^{-1} \frac{1}{2}$
142. Two stones A and B are projected horizontally from top of a building at 20 m s^{-1} and 25 m s^{-1} respectively.
- (a) A will reach ground first. (b) B will reach ground first.
(c) Both will reach ground simultaneously. (d) Cannot be concluded without knowing height.
143. If a projectile is projected upwards from ground with an initial velocity u and at angle θ to vertical, its speed after t second is
- (a) $u - gt$ (b) $\sqrt{u^2 + g^2 t^2 - 2u \sin \theta \cdot gt}$
(c) $\sqrt{u^2 + g^2 t^2 - 2u \cos \theta \cdot gt}$ (d) $\sqrt{2u \sin \theta \cdot gt}$
144. A projectile, projected at the foot of an inclined plane so as to obtain maximum range on horizontal, obtained half of that value as range on the inclined plane. The angle of inclination α of the inclined plane with horizontal is given by
- (a) $\cos^2 \alpha - 2 \cos \alpha - 2 \sin \alpha = 0$ (b) $\cos^2 \alpha - \cos \alpha - 2 \sin \alpha = 0$
(c) $\cos^2 \alpha - 2 \cos \alpha + 2 \sin \alpha = 0$ (d) $\cos^2 \alpha - 2 \cos \alpha + \sin \alpha = 0$
145. A projectile is projected from the foot of an inclined plane of inclination α with the horizontal, with a velocity u at an angle θ over the inclined plane. When it is at maximum height over the incline plane, its velocity is
- (a) less than $u \cos (\theta + \alpha)$ (b) equal to $u \cos (\theta + \alpha)$
(c) more than $u \cos (\theta + \alpha)$ (d) any of above is possible
146. In the above question, at the location described, its velocity is
- (a) less than $u \cos \theta$ (b) equal to $u \cos \theta$
(c) more than $u \cos \theta$ (d) any of above is possible
147. Rain drops falling vertically down appear to fall at 45° on a man when he moves horizontally at 5 m s^{-1} and at 30° with vertical when he reduces his speed. His new speed is (in m s^{-1})
- (a) $\frac{5}{\sqrt{3}}$ (b) $\frac{5}{\sqrt{2}}$ (c) $\sqrt{\frac{25}{14}}$ (d) $\frac{3}{5}$
148. Ship B is moving at 16 kmph due East and ship A is moving with a velocity $\vec{v}_A$ due South. At a particular instant B is at 100 km away from A in South – West direction from A. The relative velocity of A with respect to B is $v_{r(A/B)} = 32 \text{ kmph}$. The distance of closest approach between them (in km) is
- (a) $100 \sin 32^\circ$ (b) $100 \sin 18^\circ$ (c) $100 \cos 81^\circ$ (d) $100 \sin 15^\circ$

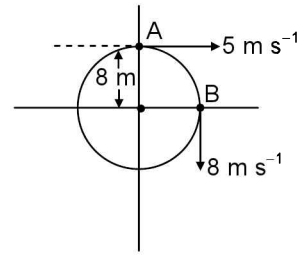
3.48 Motion in Two Dimensions

149. In the above case, if the velocity of B is increased to v_B' the two ships will meet each other. Then v_B' is (in kmph)
- (a) 38 (b) $16\sqrt{2}$ (c) $16\sqrt{5}$ (d) $16\sqrt{3}$
150. A particle in uniform circular motion on XY plane, crosses X axis at (9 m, 0) with velocity $5\hat{j}$ m s⁻¹ and crosses Y axis with velocity $-3\hat{i} - 4\hat{j}$ m s⁻¹. The magnitude of its acceleration in m s⁻² is
- (a) 5 (b) 10 (c) 15 (d) 25
151. Two projectiles are projected, not simultaneously, as shown in the same vertical plane. They collide one second after projecting the second one. The horizontal distance between them when the second one is projected is
- (a) $\frac{hu \sin \theta}{g}$ (b) 0
(c) $\frac{hu \tan \theta}{g}$ (d) $\frac{hu \cot \theta}{g}$
152. Treating horizontal ground as XY plane, Z axis vertically upwards, $g = 10$ m s⁻², the position vector of the point of landing on ground by a projectile projected from position vector $3\hat{i} + 2\hat{j} + 2\hat{k}$ (metre) with velocity $\hat{i} + 2\hat{j} + 3\hat{k}$ (m s⁻¹)
- (a) $7\hat{i} + 10\hat{j}$ (m) (b) $6\hat{i} + 8\hat{j}$ (m) (c) $5\hat{i} + 6\hat{j}$ (m) (d) $4\hat{i} + 4\hat{j}$ (m)
153. A man, as a daily routine, crosses a river in time t . On a particular day, when he was unaware that the river is flowing at double its usual speed, he takes time t' . Assume he takes the same direction with the flow of river as he routinely does
- (a) $t' < t$ (b) $t' > t$
(c) $t' = t$ (d) any of the above is possible
154. Relative velocity between a particle in uniform motion and particle in circular motion which are projected at the same instant with the same speed as shown, is given by: $11 \sin 11t$ ms⁻¹, (t in s). The radius of the circle is (in m)
- (a) 0.25 (b) 0.5
(c) 0.75 (d) 1
155. A man crosses a flowing river in shortest time with average speed v and immediately crosses back by shortest path with average speed v' which is
- (a) $< v$ (b) $= v$
(c) $> v$ (d) cannot be concluded
156. A particle, during its circular motion with constant tangential acceleration on XY plane, successively crosses. X axis with velocity $\hat{j}$ m s⁻¹
Y axis with velocity $2\hat{i}$ m s⁻¹
and X axis with velocity $\bar{v}$ m s⁻¹ and acceleration $\bar{a}$ m s⁻² respectively.
The angle between $\bar{v}$ and $\bar{a}$ is
- (a) $\tan^{-1}\pi$ (b) $\tan^{-1}3\pi$ (c) $\tan^{-1}\left(\frac{7\pi}{3}\right)$ (d) $\tan^{-1}4\pi$



157. A particle is performing uniformly accelerated circular motion along a circle of radius 8 m. It moves from point A to point B in time 0.5 s. Its acceleration at B is (in m s^{-2})

(a) 10
(b) 8
(c) 6
(d) 2



158. Velocity of a particle with time t is given by

$$\vec{v} = t\hat{i} + t^2\hat{j} \text{ m s}^{-1}$$

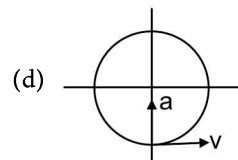
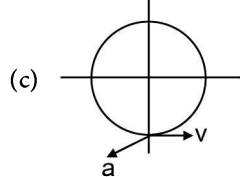
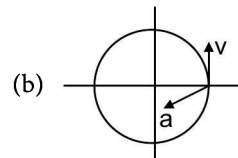
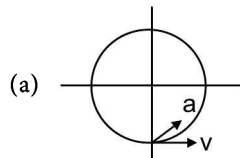
Its radial acceleration at $t = 1$ s is (in m s^{-2})

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

159. A particle starts from rest and executes a uniformly accelerated circular motion. If θ is the angle such that the angle between the velocity and acceleration is the same as the angular displacement, the condition for θ is

(a) $\theta = \tan \theta$ (b) $\theta = \sin \theta$ (c) $2\theta = \tan \theta$ (d) $2\theta = \cot \theta$

160. A particle is undergoing uniformly accelerated circular motion with angular retardation $\pi \text{ rad s}^{-2}$. If the angular velocity of the particle at $t = 0$ is $2\pi \text{ rad s}^{-1}$, the velocity and acceleration vectors of the body at $t = 1$ s are best represented by



Assertion-Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

161. Statement 1

The path of a projectile launched from earth is initially for a very short time linear and then a parabola and

Statement 2

$$y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta} \text{ contains a term linear in } x$$

3.50 Motion in Two Dimensions

162. Statement 1

The angle of elevation of a particle from the point of projection always falls.
and

Statement 2

The acceleration due to gravity always has a component perpendicular to the line of sight.

163. Statement 1

The shape of the trajectory of a particle projected on the Earth and the Moon, with the same velocity and at the same angle will be the same.
and

Statement 2

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

164. Statement 1

On a vertical pole which is at a horizontal distance of 12 m from a point P on the ground, A and B are two points at heights of 5 m and 9 m respectively. The minimum speeds required to reach these points from P are in the ratio $\sqrt{\frac{13}{15}}$.

and

Statement 2

For a projectile, the time required to reach the maximum height H is $\sqrt{\frac{2H}{g}}$.

165. Statement 1

If two particles are projected on an inclined plane at angle α with the inclined plane, one up and the other down the inclined plane, the maximum height reached can be different.
and

Statement 2

The trajectory of the two particles is symmetric about a normal to the inclined plane.

166. Statement 1

To achieve maximum distance from the inclined plane, a particle should be projected perpendicular to the inclined plane.
and

Statement 2

The acceleration due to gravity is not perpendicular to the inclined plane.

167. Statement 1

A boat with any velocity v in still water can cross a river by the shortest distance, if it directs its course upstream.
and

Statement 2

To cross the river by the shortest distance, the relative velocity of boat with respect to water has to be along the shortest distance across the river.

168. Statement 1

The motion of a shell fired from the ground that hits an aeroplane in level flight can be a straight line as observed from the aeroplane.

and

Statement 2

The horizontal component of the shell's motion is uniform.

169. Statement 1

The centripetal acceleration of a particle in non-uniform circular motion is constant, in order to maintain it in a circular trajectory.

and

Statement 2

The tangential acceleration of a particle in non-uniform circular motion changes continuously.

170. Statement 1

The angle between velocity and acceleration for a body in uniformly accelerated circular motion always tends to 90° .

and

Statement 2

The radial acceleration of a body in uniformly accelerated circular motion always increases.

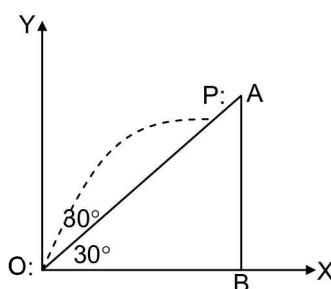


Linked Comprehension Type Questions

Directions: This section contains 3 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage I

OAB is an inclined plane with a large base. OB is along X-axis and AB is parallel to Y-axis. The inclined plane makes angle 30° with the X-axis. The particle is projected at a speed of 10 m s^{-1} , (Take $g = 10 \text{ m s}^{-2}$) at an angle of 30° with the surface of the inclined plane to hit the point P as shown in figure.



171. The time taken by the particle to hit the target is (in second).

(a) $\frac{2}{\sqrt{3}}$

(b) $\frac{1}{\sqrt{3}}$

(c) 2

(d) $\frac{\sqrt{3}}{2}$

172. The magnitude of velocity with which the particle strikes the point P is

(a) 10 m s^{-1}

(b) $5\sqrt{3} \text{ m s}^{-1}$

(c) $10\sqrt{3} \text{ m s}^{-1}$

(d) $\frac{10}{\sqrt{3}} \text{ m s}^{-1}$

173. The angle made by the particle with the inclined plane at the instant it strikes the plane is

(a) 15°

(b) 45°

(c) 30°

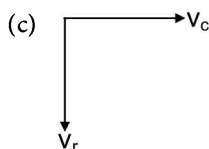
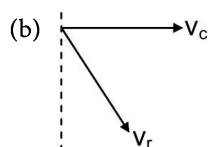
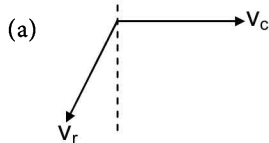
(d) 60°

3.52 Motion in Two Dimensions

Passage II

To a cyclist riding with uniform velocity along a horizontal road, the rain appears to fall at 37° to vertical. When he doubles his speed, rain appears to fall at 53° to vertical.

174. If the velocity of the cyclist is to the right, the direction of rain is represented correctly as given in: ($v_c \rightarrow$ velocity of cyclist, $v_r \rightarrow$ velocity of rain)



(d) can be (a) or (b)

175. The angle that the direction of rain makes to vertical is

(a) $\tan^{-1}3$ (b) $\tan^{-1}\frac{1}{6}$ (c) can be (a) or (b) (d) neither (a) nor (b)

176. The ratio of speed of rain to that of the cyclist (before doubling) is

(a) $\frac{\sqrt{13}}{5}$ (b) $\frac{2\sqrt{41}}{7}$ (c) $\frac{2\sqrt{37}}{7}$ (d) $\frac{2\sqrt{13}}{5}$

Passage III

A car moves on a circular road with a uniform speed of 11 m s^{-1} . A stone projected upwards from the car hits the car after 3 s and when the car has completed a quarter circle. Treat the car as a particle.

Use $\pi = \frac{22}{7}$, $g = 10 \text{ m s}^{-2}$

177. Acceleration of the car is nearly (in m s^{-2})

(a) 3 (b) 4 (c) 5 (d) 6

178. The velocity of projection relative to the car is (in m s^{-1}) nearly

(a) 16 (b) 17 (c) 18 (d) 19

179. The stone hits the car at an angle to the vertical,

(a) between 0° and 30° (b) between 30° and 45° (c) between 45° and 60° (d) between 60° and 90°

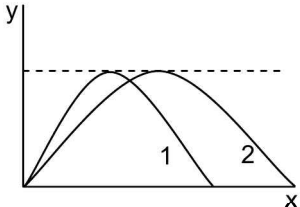


Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

180. A ball is projected upwards from ground at a certain angle α with the horizontal. Which of the following statements is/are correct? At the highest point

(a) velocity of the projectile is $u \cos \alpha$.
(b) acceleration of the projectile is 'g'.

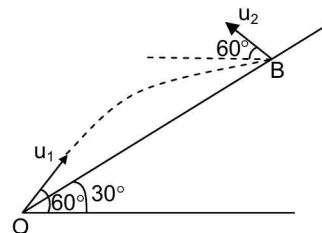
- (c) velocity of the projectile is not in the horizontal direction.
 (d) acceleration of the projectile is vertically upwards.
181. Choose the correct statement/statements from the following. The vertical displacement of the projectile launched upwards from level ground depends upon.
 (a) angle of projection. (b) the initial velocity of the projection.
 (c) the acceleration due to gravity. (d) the mass of the projectile.
182. A body is projected horizontally from a certain height at time $t = 0$ with a velocity v_0 .
 (a) The horizontal distance covered in time t is $v_0 t$.
 (b) The vertical distance covered in time is $\frac{1}{2}gt^2$.
 (c) The trajectory of the path projected horizontally from any height above the ground is parabolic.
 (d) The angle α which the resultant velocity vector makes with the vertical direction is given by $\tan \alpha = \frac{gt}{v_0}$.
183. Trajectories of two projectiles are shown in figure. Let T_1 and T_2 be the time periods and u_1 and u_2 , their initial speeds of projection. Which is/are wrong among the options?
- 
- (a) $T_2 > T_1$
 (b) $T_1 = T_2$
 (c) $u_1 > u_2$
 (d) $u_1 < u_2$
184. A body is projected from the top of a tower 30 m tall, with an initial velocity u and the maximum range achieved on the ground from the foot of the tower is 40 m. If the body is projected from ground with the same velocity u , on an inclined plane 37° inclined up, then
 (a) The maximum range on the inclined plane is 12.5 m.
 (b) The angle of projection from the ground plane for maximum range is 60° .
 (c) $u = 10\sqrt{2} \text{ m s}^{-1}$.
 (d) $u = 10 \text{ m s}^{-1}$.
185. A wheel rotating with an initial angular velocity ω about a fixed vertical axis is brought under constant angular deceleration of α at $t = 0$ and it comes to halt in 8 s. In the first 2 s it makes 112 revolutions. Then
 (a) $\alpha = 16 \pi \text{ rad s}^{-2}$
 (b) Number of revolutions per second initially is 64.
 (c) Total number of revolutions it makes before halt is 250.
 (d) Total number of revolutions it makes before halt is 280.
186. A body projected horizontally from the top of an inclined plane with an initial velocity of $u = 20 \text{ m s}^{-1}$, falls on the inclined plane after 3 s. Then
 (a) The angle of inclination of the inclined plane with horizontal = 37°
 (b) The angle of inclination of the inclined plane with horizontal = 30°
 (c) Final velocity when it reaches on the incline = 50 m s^{-1}
 (d) The range on the inclined plane = 75 m
187. A particle P is projected vertically upwards from level ground with an initial velocity of upwards 100 m s^{-1} . One second later another particle Q is projected vertically upwards from the same point with an initial velocity of 90 m s^{-1} . Before either of the particle strikes the ground ($g = 10 \text{ m s}^{-2}$), which of the following is/are the wrong statements?
 (a) Both particles are at rest with respect to each other.
 (b) After 3 s from launch of P, the distance between the particles is 95 m.
 (c) When the particle P is at the highest position, particle Q is moving.
 (d) When the particle P is at highest point particle Q will be moving downwards.



Matrix-Match Type Questions

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

188. A projectile is launched from O at the foot of an inclined plane inclined at 30° to the horizontal, with an initial velocity u_1 , at angle of projection 60° to the horizontal to reach a point B on the top of the incline. From the position B it is launched with velocity u_2 at 60° to the horizontal to reach back O at the foot. If t_1 , and t_2 are the times of flight in 1st and 2nd cases respectively, then match the following:



Column I

(a) $\frac{u_1}{u_2}$

(b) $\frac{t_1}{t_2}$

(c) Modulus value of acceleration normal to the plane (in ms^{-2})(d) Modulus value of acceleration parallel to the plane (in ms^{-2})

Column II

(p) $5\sqrt{3}$

(q) $\frac{1}{\sqrt{2}}$

(r) $\sqrt{2}$

(s) same for both

189. At $t = 0$, a projectile is launched vertically upward and another is launched at an angle θ to the horizontal from same point on ground. Their time of flight is 4 s, same for both. Match the following:

Column I

(a) Maximum height achieved (in metre)

(b) Time duration in which they move with acceleration (in second)

(c) Time duration in which they move with deceleration (in second)

(d) Minimum velocity achieved

Column II

(p) 2

(q) 20

(r) same for both

(s) not same for both

190. The following are the parameters of circular motion of a body: $v \rightarrow$ speed of the body; $\vec{R} \rightarrow$ radius vector; $\vec{a} \rightarrow$ total acceleration; $\vec{a}_R \rightarrow$ radial acceleration; $\vec{a}_T \rightarrow$ tangential acceleration; $\vec{\omega} \rightarrow$ angular velocity. Match the following:

Column I

(a) $\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{R})$

(b) $\vec{a} = \vec{\alpha} \times \vec{R}$

(c) $|\vec{a}| = \sqrt{2} |\vec{a}_R|$

(d) $\vec{a} = \vec{\omega} \times \vec{v}$

Column II

(p) $|\vec{a}_T| = 0$

(q) $|\vec{a}_R| = \frac{v^2}{R}$

(r) $|\vec{a}_T| = |\vec{a}_R|$

(s) $\frac{dv}{dt} = a$

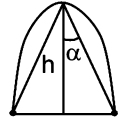


Subjective Questions

191. A stone is projected upwards from ground level with an initial velocity of 20 m s^{-1} . After 0.4 s, it just clears (grazes the top) of a wall 4 m high. There is another wall 5 m tall, at a horizontal distance of 25.6 m from the first wall. Will the stone clear that wall? ($g = 10 \text{ m s}^{-2}$)
192. A ball is to be thrown upwards from the ground over a wall of height 40 m, at a horizontal distance of 30 m from the point of throw.

- (i) Find the minimum value of initial velocity required.
 - (ii) Find the angle of projection with the horizontal for this initial velocity.
- Take $g = 10 \text{ m s}^{-2}$

193. A projectile is fired upwards from the base of a cone shaped hill of half apex angle α and height h in order to just clear it



- (i) Express the range in terms of maximum height h and α .
- (ii) Find an expression for the angle of projection with horizontal in terms of α .
- (iii) Find an expression for u in terms of h and α .

194. A body is projected horizontally with a speed of 90 kmph from the top of an inclined plane of angle 45° .

- (i) Write the equation for the path of the projectile.
- (ii) Write the equation for the inclined plane.
- (iii) Find the co-ordinates of the point P where the particle lands on the inclined plane and the distance of that point P from the point of projection. ($g = 10 \text{ m s}^{-2}$)

195. A cart is moving along a level road with uniform velocity V . A dust particle that gets thrown from the highest point of the rear wheel lands at the highest point of the front wheel. If the radii of rear and front wheels are R_1 and R_2 ($R_1 > R_2$) respectively and the distance between their centres is ℓ , find the velocity of the cart. [Consider that the speed with which the dust particle leaves horizontally from top of rear wheel is $2V$]

196. A train is moving on straight horizontal track above ground level with a velocity of 5 m s^{-1} . At time $t = 0$, an object is thrown horizontally out of a window (which is 5 m above ground level) at an angle 60° to direction of motion of the train. It lands at a point on the ground and the above reformed window of the train overtakes that point, (that is, crosses the normal from that point to direction of motion) at $t = 1.5 \text{ s}$. What is the velocity with which the object was thrown relative to the ground? ($g = 10 \text{ m s}^{-2}$).

197. An aeroplane flies horizontally with speed v at height h . A gun vertically below the plane fires at it.

- (i) What is the horizontal speed of the shell to hit the plane?
- (ii) What is the minimum vertical speed to hit the plane?
- (iii) What is the angle of projection and muzzle velocity?

198. A particle is projected upwards from origin on level ground, with speed u at an angle α with horizontal to hit a target on the ground. From the target, another particle is projected at the same angle and same initial velocity after some time. These two particles collide in air.

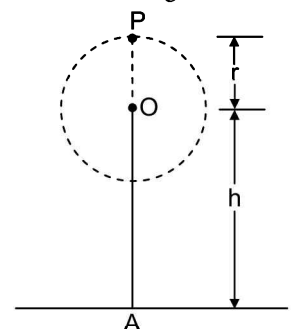
- (i) Write an expression for the trajectory of the first particle.
- (ii) If they collide at half the maximum height of the first particle, write an expression to find the horizontal distance from the origin to the point of collision.
- (iii) Find the ratio of the horizontal distances travelled by the 1st and 2nd particles till collision.

199. A car enters a circular road (radius 100 m) with a speed of 54 kmph and increases the speed at a uniform rate of 5 m s^{-2} . Calculate:

- (i) the angular velocity while entering the circular road.
- (ii) the angular acceleration.
- (iii) angular velocity at a point on the road, at which the car has travelled 100 m on the road from entering the curve. Find the centripetal and total acceleration at that point.

200. A stone is tied to one end of a light, inelastic string and rotated in a vertical plane as shown. The centre of the circle 'O' is at a height 'h' vertically above a point 'A' on the ground. When the stone is at the maximum height 'P', its angular velocity was ' ω '. At that instant, the string snaps.

- (i) What is the magnitude and direction of velocity of the stone when the string snaps?
- (ii) If the stone hits the ground at point B, what is the distance AB?



SOLUTIONS

ANSWER KEYS

Topic Grip

1. (i) $\frac{u^2}{4g}$
(ii) $26^\circ 33' 54''$
2. $2\sqrt{5} \text{ s}$
3. $-$
4. 80 m or 1840 m
5. 30°
6. (i) 10 m s^{-1}
(ii) 2 s
7. 6.6 s
8. 80 s, 1264.9 m
9. (i) 2 s
(ii) $2\sqrt{2} \text{ s}$
10. $\frac{1}{3} \text{ s}$
11. (b) 12. (b) 13. (c)
14. (c) 15. (c) 16. (b)
17. (a) 18. (d) 19. (a)
20. (a) 21. (d) 22. (d)
23. (a) 24. (a) 25. (a)
26. (b)
27. (a), (b)
28. (a), (c), (d)
29. (a), (c), (d)
30. (a) $\rightarrow$ (p), (r)
(b) $\rightarrow$ (q), (r)
(c) $\rightarrow$ (p), (r)
(d) $\rightarrow$ (s)

IIT Assignment Exercise

31. (c) 32. (b) 33. (b)
34. (a) 35. (a) 36. (b)
37. (b) 38. (a) 39. (b)
40. (b) 41. (d) 42. (c)
43. (b) 44. (c) 45. (a)
46. (a) 47. (d) 48. (b)
49. (d) 50. (b) 51. (b)
52. (c) 53. (b) 54. (b)
55. (c) 56. (a) 57. (c)

58. (d) 59. (c) 60. (b)
61. (d) 62. (a) 63. (b)
64. (c) 65. (b) 66. (c)
67. (d) 68. (b) 69. (b)
70. (c) 71. (c) 72. (d)
73. (d) 74. (c) 75. (c)
76. (c) 77. (c) 78. (d)
79. (c) 80. (b) 81. (d)
82. (d) 83. (d) 84. (c)
85. (c) 86. (c) 87. (c)
88. (b) 89. (d) 90. (b)
91. (c) 92. (b) 93. (c)
94. (d) 95. (b) 96. (a)
97. (d) 98. (c) 99. (b)
100. (d) 101. (a) 102. (c)
103. (d) 104. (c) 105. (c)
106. (b) 107. (b) 108. (c)
109. (d) 110. (b) 111. (b)
112. (d) 113. (d) 114. (a)
115. (a) 116. (a)
117. (a), (b)
118. (b), (c), (d)
119. (b), (c), (d)
120. (a) $\rightarrow$ (p), (q)
(b) $\rightarrow$ (r)
(c) $\rightarrow$ (s)
(d) $\rightarrow$ (p), (r)
163. (a) 164. (d) 165. (b)
166. (b) 167. (d) 168. (a)
169. (d) 170. (c) 171. (a)
172. (d) 173. (d) 174. (a)
175. (b) 176. (c) 177. (d)
178. (b) 179. (b)
180. (a), (b)
181. (a), (b), (c)
182. (a), (b), (c)
183. (b), (c)
184. (a), (c)
185. (a), (b)
186. (a), (d)
187. (c), (d)
188. (a) $\rightarrow$ (r)
(b) $\rightarrow$ (q)
(c) $\rightarrow$ (p), (s)
(d) $\rightarrow$ (s)
189. (a) $\rightarrow$ (q), (r)
(b) $\rightarrow$ (p), (r)
(c) $\rightarrow$ (p), (r)
(d) $\rightarrow$ (s)
190. (a) $\rightarrow$ (p), (q)
(b) $\rightarrow$ (q), (s)
(c) $\rightarrow$ (q), (r)
(d) $\rightarrow$ (p), (q)
191. No.
192. (i) 30 m s^{-1}
(ii) $71^\circ 30'$
193. (i) $R = 2h \tan \alpha$
(ii) $\theta = \tan^{-1}(2 \cot \alpha)$
(iii) $u = \sqrt{2gh \left(1 + \frac{\tan^2 \alpha}{4} \right)}$

Additional Practice Exercise

121. (b) 122. (d) 123. (a)
124. (b) 125. (c) 126. (c)
127. (c) 128. (d) 129. (c)
130. (a) 131. (c) 132. (d)
133. (c) 134. (c) 135. (d)
136. (d) 137. (a) 138. (d)
139. (c) 140. (d) 141. (c)
142. (c) 143. (c) 144. (c)
145. (c) 146. (a) 147. (a)
148. (d) 149. (d) 150. (a)
151. (b) 152. (d) 153. (c)
154. (a) 155. (a) 156. (c)
157. (a) 158. (b) 159. (c)
160. (b) 161. (d) 162. (a)

194. (i) $y = \frac{-x^2}{125} \text{ m}$
(ii) $y = -x$
(iii) P(125 m, -125 m), OP = 176.8 m

$$195. V = \sqrt{\frac{\ell^2 - (R_1 - R_2)^2}{4(R_1 - R_2)}} g$$

196. At 8.66 m s^{-1} horizontally, making an angle of 30° with the direction of motion of the train.

197. (i) $v_H = v$

(ii) $v_V = \sqrt{2gh}$

(iii) muzzle velocity

$$= \sqrt{2gh + v^2}$$

$$\tan^{-1} \frac{\sqrt{2gh}}{v}$$

198. (i) $y = x \tan \alpha - \frac{1}{2} \frac{gx^2}{u^2 \cos^2 \alpha}$

(ii) $x = \frac{gx^2}{24^2 \cos^2 \alpha} - x$
 $\tan \alpha + \frac{u^2 \sin^2 \alpha}{hg} = 0$

(iii) $3 + 2\sqrt{2}$

199. (i) 0.15 rad s^{-1}

(ii) 0.05 rad s^{-2}

(iii) $\omega = 0.35 \text{ rad s}^{-1}$,
 $a_r = 12.25 \text{ m s}^{-2}$

$$a = 13.20 \text{ m s}^{-2}$$

$$\phi = 22^\circ \text{ to radial direction}$$

200. (i) $V = r\omega$, in the horizontal direction

(ii) $AB = r\omega \sqrt{\frac{2(h+r)}{g}}$.

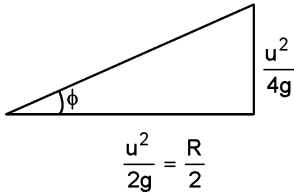
HINTS AND EXPLANATIONS

Topic Grip

1.

(i) Angle of projection = 45°

$$\Rightarrow h = \frac{u^2 \cdot \sin^2 45^\circ}{2g} = \frac{u^2}{4g}$$

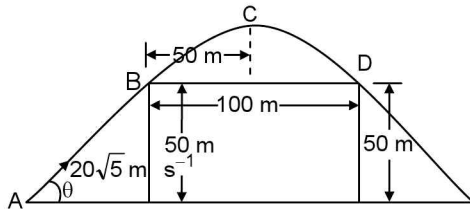


$$(ii) R = \frac{u^2}{g} \Rightarrow \frac{R}{2} = \frac{u^2}{2g}$$

$$\tan \phi = \frac{\frac{u^2/4g}{u^2/2g}}{1} = \frac{1}{2}$$

$$\Rightarrow \phi = \tan^{-1} \frac{1}{2} = 26.56^\circ = 26^\circ 33' 54''$$

2.



Let horizontal component of velocity be called v_{Ax} and vertical component be v_{Ay} .

Let time between B and C be T .

$$v_{By}^2 = v_{Ay}^2 - 2g \times 50 \text{ (using } v^2 = u^2 - 2gh\text{)} \\ = v_{Ay}^2 - 1000$$

$$v_{Bx} = v_{Ax} \Rightarrow v_{Bx}^2 = v_{Ax}^2; v_B^2 = v_{By}^2 + v_{Bx}^2 \text{ and}$$

$$v_A^2 = v_{Ay}^2 + v_{Ax}^2$$

$$\therefore v_B^2 = v_{Ay}^2 - 1000 + v_{Ax}^2$$

$$\Rightarrow v_B^2 = v_A^2 - 1000 = 2000 - 1000 \\ = 1000 = v_{Bx}^2 + v_{By}^2$$

$$T = \frac{2v_{By}}{g}$$

$$\Rightarrow \text{Distance moved } BD = v_{Bx} \cdot T = \frac{2v_{Bx} v_{By}}{g} = 100 \text{ (data)}$$

$$2v_{Bx} v_{By} = 1000$$

$$= v_{Bx}^2 + v_{By}^2 \Rightarrow (v_{Bx} - v_{By})^2 = 0$$

$$\Rightarrow v_{Bx} = v_{By}$$

$$\Rightarrow v_{By} = \sqrt{500} = 10\sqrt{5} \text{ m s}^{-1}$$

$$\Rightarrow T = \frac{2v_{By}}{g} = \frac{2 \times 10\sqrt{5}}{10} = 2\sqrt{5} \text{ s}$$

3.

$$(i) \text{ Using } y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta},$$

$$b = a \tan \theta - \frac{1}{2} g \frac{a^2}{2ga} \sec^2 \theta$$

$$= a \tan \theta - \frac{a}{4} (1 + \tan^2 \theta)$$

$$\text{(using } \sec^2 \theta = 1 + \tan^2 \theta)$$

$$\Rightarrow \tan^2 \theta - 4 \tan \theta + \left(1 + \frac{4b}{a}\right) = 0$$

For real $\tan \theta$,

Discriminant should be ≥ 0

$$\text{i.e., } 16 - 4 - 16 \frac{b}{a} \geq 0 \Rightarrow \frac{b}{a} \leq \frac{3}{4} \Rightarrow b \leq \frac{3}{4}a$$

Hence if $b > \frac{3}{4}a$, the target cannot be hit.

(ii) $\sqrt{2gb}$ is the velocity of vertical projection to reach a height b . Hence, unless $a = 0$, the target cannot be hit.

$$(iii) \text{ Using } y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta},$$

$$b = a \tan \theta - \frac{1}{2} g \frac{a^2 \sec^2 \theta}{ngb};$$

$$\text{Multiply by } \frac{2nb}{a^2}$$

$$\Rightarrow 2n \frac{b^2}{a^2} = 2n \frac{b}{a} \tan \theta - (1 + \tan^2 \theta)$$

$$\tan^2 \theta - \frac{2nb}{a} \tan \theta + 1 + 2n \frac{b^2}{a^2} = 0$$

$$\text{For real } \tan \theta \quad 4n^2 \frac{b^2}{a^2} - 4 - 8n \frac{b^2}{a^2} \geq 0$$

$$\Rightarrow \frac{b^2}{a^2}(n^2 - 2n) \geq 1$$

$$\Rightarrow b \geq \frac{a}{\sqrt{n(n-2)}}$$

$$\Rightarrow a \leq \sqrt{n(n-2)} b$$

$\therefore$ If $a > (\sqrt{n(n-2)}) b$, the target cannot be hit.

4. Using $y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$

$$\frac{115}{2} = x \tan \theta_1 - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta_1} \quad \text{--- (1)}$$

$$\frac{115}{2} \left(1 + \frac{7}{9} \right) = x \tan(90^\circ - \theta_1) - \frac{1}{2} g \frac{x^2}{u^2 \cos^2(90^\circ - \theta_1)}$$

($\because$ both have same R; $\theta_2 = 90^\circ - \theta_1$)

$$\frac{115}{2} \times \frac{16}{9} = x \cot \theta_1 - \frac{1}{2} g \frac{x^2}{u^2 \sin^2 \theta_1} \quad \text{--- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{9}{16} = \frac{x \left(\frac{\sin \theta}{\cos \theta} - \frac{1}{2} g \frac{x}{u^2 \cos^2 \theta} \right)}{x \left(\frac{\cos \theta}{\sin \theta} - \frac{1}{2} g \frac{x}{u^2 \sin^2 \theta} \right)}$$

$$= \frac{\left(\frac{u^2 \sin \theta \cos \theta - \frac{1}{2} g x}{u^2 \cos^2 \theta} \right)}{\left(\frac{u^2 \sin \theta \cos \theta - \frac{1}{2} g x}{u^2 \sin^2 \theta} \right)} = \tan^2 \theta$$

$$\therefore \tan \theta = \frac{3}{4} \Rightarrow \theta = 37^\circ, \sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}$$

$$\text{Range} = 1920 = \frac{u^2 \sin 2\theta}{g} = u^2 \times 2 \times \frac{3}{5} \times \frac{4}{5} \times \frac{1}{10}$$

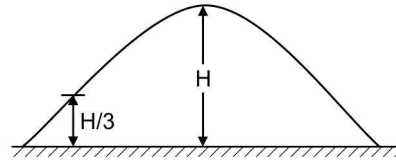
$$\Rightarrow u = 100\sqrt{2} \text{ m s}^{-1}$$

$$\frac{115}{2} = x \cdot \frac{3}{4} - \frac{1}{2} \cdot \frac{10 \times x^2 \times 25}{20000 \times 16}$$

$$\Rightarrow x^2 - 1920x + 147200 = 0$$

$$x = 80 \text{ m or } 1840 \text{ m}$$

5. If H is the maximum height reached, vertical component of velocity of projection = $\sqrt{2gH}$



If θ is angle of projection, horizontal component of

$$\text{velocity} = \frac{\sqrt{2gH}}{\sin \theta} \cos \theta = \sqrt{2gH} \cot \theta$$

$$\therefore \text{Velocity at } H = \sqrt{2gH} \cot \theta$$

$$\text{Velocity at } \frac{H}{3}$$

Vertical component : (use $v^2 - u^2 = 2g \frac{H}{3}$; $N^2 = 2gH$)

$$= \sqrt{(2gH) - 2 \times g \times \frac{H}{3}} = \sqrt{\frac{4}{3} gH} = \sqrt{\frac{2}{3}} \sqrt{2gH}$$

$$\text{Horizontal component} = \sqrt{2gH} \cot \theta$$

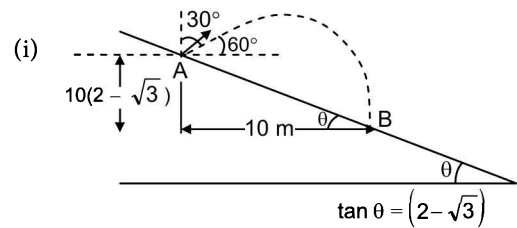
$$\text{It is given that, } \frac{\sqrt{2gH} \cot \theta}{\sqrt{\frac{2}{3} \cdot 2gH + 2gH \cot^2 \theta}} = \frac{3}{\sqrt{11}}$$

$$\Rightarrow \frac{\cot \theta}{\sqrt{\frac{2}{3} + \cot^2 \theta}} = \frac{3}{\sqrt{11}}$$

$$\Rightarrow \cot^2 \theta = \frac{9}{11} \left(\frac{2}{3} + \cot^2 \theta \right)$$

$$\Rightarrow \cot \theta = \sqrt{3} \Rightarrow \theta = 30^\circ$$

6.



Let A be origin

$$\text{Using } y = x \tan \phi - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \phi} \quad [\phi = 60^\circ]$$

$$-10(2 - \sqrt{3}) = 10 \cdot \tan 60^\circ - \frac{1}{2} g \frac{10^2}{u^2 \cos^2 60^\circ}$$

$$= 10\sqrt{3} - \frac{500}{u^2} \times 4$$

$$\Rightarrow \frac{2000}{u^2} = 10\sqrt{3} + 20 - 10\sqrt{3}$$

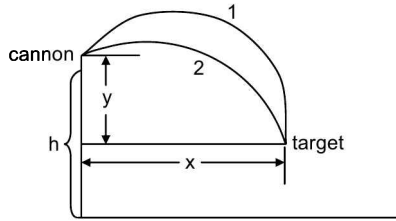
$$\Rightarrow u = 10 \text{ m s}^{-1}$$

3.60 Motion in Two Dimensions

(ii) Using $u \cos 60^\circ t = 10 \text{ m}$

$$\therefore t = 2 \text{ s}$$

7.



Both projectiles, launched upwards with same initial speed, at angles of 45° and 30° with the horizontal, can hit the same target, only if the point of projection is above the ground. In this case, it must be noted that angle of projection ' θ ' for maximum horizontal range will not be 45° but less than 45° . Let the first shell take t second to hit the target and the second shell $(t - \Delta t)$ second. Then, obviously horizontal component $u \cos 45^\circ < 4 \cos 30^\circ$, hence time to hit for angle of projection 45° is more.

$$200 \times \cos 45^\circ \times t = 200 \times \cos 30^\circ \times (t - \Delta t)$$

$$\Rightarrow \frac{t - \Delta t}{t} = \frac{\sqrt{2}}{\sqrt{3}} \quad \text{---(1)}$$

$$\text{and } \Delta t = \frac{t(\sqrt{3} - \sqrt{2})}{\sqrt{3}} \quad \text{---(2)}$$

$$200 \times \sin 45^\circ \times t - (1/2)gt^2 = 200 \times \sin 30^\circ \times (t - \Delta t) - (1/2)g(t - \Delta t)^2$$

Substitute for $t - \Delta t$ from equation --- (1)

$$\Rightarrow \frac{200}{\sqrt{2}}t - \frac{1}{2}gt^2 = \frac{200\sqrt{2}}{2\sqrt{3}}t - \frac{1}{2}g\frac{2}{3}t^2$$

$$\Rightarrow \frac{200}{\sqrt{2}}t \left(1 - \frac{1}{\sqrt{3}}\right) = \frac{gt^2}{6}$$

$$\Rightarrow t = \frac{6 \times 200}{g\sqrt{2}} \left(1 - \frac{1}{\sqrt{3}}\right) = \frac{120}{\sqrt{2}} \left(1 - \frac{1}{\sqrt{3}}\right) = 35.9 \text{ s}$$

From eqn (2)

$$\therefore \Delta t = \left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}}\right)t = 6.6 \text{ s}$$

8.

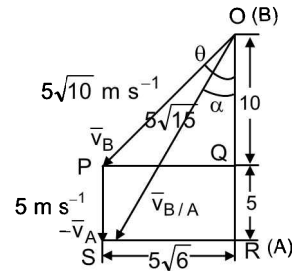
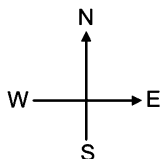


Fig. 1

Let ship A be at R and B be at O

Take due East $+\hat{i}$ due North $+\hat{j}$

$$\vec{v}_A = 5\hat{j}; \vec{v}_B = -5\sqrt{10}\sin\theta\hat{i} - 5\sqrt{10}\cos\theta\hat{j}$$

$$\text{Given: } \tan\theta = \frac{\sqrt{6}}{2} \Rightarrow$$

$$\Rightarrow \sin\theta = \frac{\sqrt{6}}{\sqrt{10}} \text{ and } \cos\theta = \frac{2}{\sqrt{10}}$$

$$\vec{v}_B = -5\sqrt{6}\hat{i} - 10\hat{j}$$

$$\vec{v}_{B/A} = -5\sqrt{6}\hat{i} - 15\hat{j}$$

$$\Rightarrow \left|\vec{v}_{B/A}\right| = \sqrt{375} \Rightarrow OS = \sqrt{375} = 5\sqrt{15} \text{ m s}^{-1}$$

Direction of $\vec{v}_{B/A}$

$$\vec{OP} = \vec{v}_B; \vec{PS} = -\vec{v}_A; \vec{OS} = \vec{v}_B - \vec{v}_A = \vec{v}_{B/A}$$

From Fig. 1.

$$\sin\alpha = \frac{5\sqrt{6}}{5\sqrt{15}} = \sqrt{\frac{6}{15}}$$

$$\cos\alpha = \frac{15}{5\sqrt{15}} = \frac{3}{\sqrt{15}}$$

Ref. Fig. 3. Distance of closest approach:

$$AD = AB \sin\alpha = 2000 \cdot \frac{\sqrt{6}}{\sqrt{15}} = 1264.9 \text{ m}$$

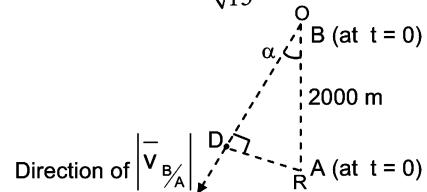


Fig. 3

$$BD = AB \cos\alpha = 2000 \cdot \frac{3}{\sqrt{15}} = 1549.2 \text{ m}$$

Time of closest approach

$$= \frac{BD}{v_{B/A}} = \frac{2000 \cdot \frac{3}{\sqrt{15}}}{5\sqrt{15}} = 80 \text{ s}$$

9. Let radius be R

$$\omega = \omega_0 + \alpha t$$

$$\text{since } \omega_0 = 0, \omega = \alpha t; v = R\omega = a_t t \text{ \& } a_t = R\alpha$$

$$\Rightarrow R\omega = R\alpha t; \text{ and } v^2 = a_t^2 t^2$$

$$\Rightarrow R a_n = a_t^2 t^2 \text{ (since } a_n = \frac{v^2}{R} \text{)}$$

$$\therefore t = \frac{\sqrt{R a_n}}{a_t}$$

$$(i) a_n = a_t \therefore t = \sqrt{\frac{R}{a_t}} = \sqrt{\frac{20 \times 10^{-2}}{5 \times 10^{-2}}} = \sqrt{4} = 2 \text{ s}$$

$$(ii) a_n = 2a_t \therefore t = \sqrt{\frac{2R}{a_t}} = 2\sqrt{2} \text{ s}$$

10. Assume origin at position of A at $t = 0$ and counter clockwise direction as positive

$$\text{For any } t, \theta_A = +3\pi t; \theta_B = \pi - \frac{1}{2} \times \frac{\pi}{4} \times t^2$$

(measured from A)

They meet when $\theta_A = \theta_B$

$$\Rightarrow 3\pi t = \pi - \frac{\pi}{8} t^2 \Rightarrow t^2 + 24t - 8 = 0, t > 0$$

$$\Rightarrow t = \frac{1}{3} \text{ s} \quad \left(\text{using } \sqrt{38} = 6\frac{1}{6} \right)$$

11. Since the range is equal for the same initial velocity.
 $\theta_2 = (90 - \theta_1)$ where θ_1 and θ_2 are the angles of projection.

$$\frac{h_1}{h_2} = \frac{u^2 \sin^2 \theta_1}{2g} \times \frac{2g}{u^2 \sin^2 (90 - \theta_1)} = \frac{\sin^2 \theta_1}{\cos^2 \theta_1} = \tan^2 \theta_1$$

$$\text{But } \frac{h_1}{h_2} = \frac{1}{3}$$

$$\therefore \tan \theta_1 = \frac{1}{\sqrt{3}}; \theta_1 = 30^\circ$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 60^\circ}{g} = \frac{\sqrt{3} u^2}{2g}$$

12. When the body hits the ground, the horizontal velocity $v_H = 10$.

$$\text{Final vertical velocity } v_v = \sqrt{2gh} = \sqrt{2 \times 10 \times h}$$

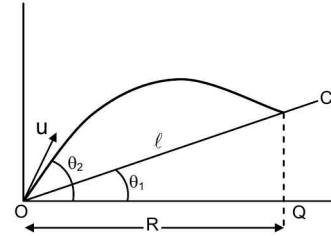
$$\therefore \text{Final velocity} = \sqrt{v_v^2 + v_H^2} = \sqrt{20h + 100} = 20 \text{ m s}^{-1} \text{ (data)}$$

$$\therefore 20h + 100 = 400 \Rightarrow h = 15 \text{ m}$$

13. Let R be the horizontal distance.

$$R = u \cos \theta_2 T$$

$$\frac{R}{\ell} = \cos \theta_1$$



$$\therefore \ell = \frac{R}{\cos \theta_1} = \frac{u \cos \theta_2 T}{\cos \theta_1}$$

$$14. \bar{u} = 6\hat{i} + 5\hat{j} \text{ m s}^{-1} \therefore u_x = 6 \text{ m s}^{-1}, u_y = 5 \text{ m s}^{-1}$$

$$\bar{a} = 0.5\hat{i} + 0.35\hat{j} \text{ or } a_x = 0.5 \text{ m s}^{-2},$$

$$a_y = 0.35 \text{ m s}^{-2}$$

Speed after $t = 20$ second is:

$$v_x = u_x + a_x t = 6 + 20 \times 0.5 = 16 \text{ m s}^{-1}$$

$$v_y = u_y + a_y t = 5 + 0.35 \times 20 = 12 \text{ m s}^{-1}$$

$$\therefore v = \sqrt{16^2 + 12^2} = 20 \text{ m s}^{-1}$$

$$15. v^2 = 2a_t \times \frac{\pi R}{2} \Rightarrow \frac{v^2}{R} = a_t = \pi \times a_t$$

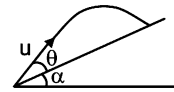
$$\Rightarrow \frac{a_t}{a_t} = \pi$$

16. Such a reflection does not change vertical velocity, which is why the time of flight is unchanged.

17. If time of flight for oblique upward be T_u , and the other T_d , then maximum T_d is when vertical downward velocity is zero. (i.e., dropping from h)

For the oblique upward projectile, maximum height reached h' will be $> h$

- 18.

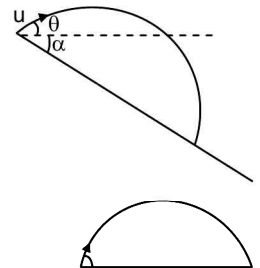


$$T_1 = \frac{2u \sin \theta}{g \cos \alpha}$$

$$T_2 = \frac{2u \sin \theta}{g \cos(-\alpha)} = T_1$$

$$T = \frac{2u \sin \theta}{g T^2}$$

$$\therefore T_1 T_2 = \frac{4u^2 \sin^2 \theta}{g^2 T^4}$$



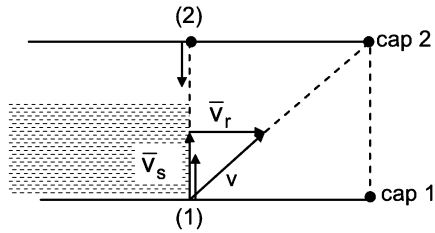
$$19. \bar{s}_{1/2} = \bar{s}_0 + (\bar{u}_1 - \bar{u}_2)t + \left(\frac{1}{2}gt^2 - \frac{1}{2}gt^2 \right)$$

If $s_{1/2} = 0$ at some t , it would be so at any value of g .

3.62 Motion in Two Dimensions

20. Vector diagram for the shortest time is as shown.

For any time t the drift of the swimmers and the caps along the stream direction is same.



($\vec{v}_s \rightarrow$ Swimmer velocities.)

$$21. y = x \tan \theta - \frac{g x^2}{2u^2 \cos^2 \theta}$$

$$15 = 20\sqrt{3} \times \frac{1}{\sqrt{3}} - \frac{10 \times (20\sqrt{3})^2}{2 \times u^2 \times \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow u = 40 \text{ m s}^{-1}$$

22. At collision no change in vertical velocity

$$\text{Time of flight} = \frac{2u \sin \theta}{g} = \frac{2 \times 40 \times \frac{1}{2}}{10} = 4 \text{ s.}$$

$$\text{Time to reach wall} = \frac{20\sqrt{3}}{40 \times \frac{\sqrt{3}}{2}} = 1 \text{ s}$$

$$\therefore \text{Time to reach ground} = 4 - 1 = 3 \text{ s}$$

23. The vertical component of velocity after 1 s of projection is

$$v_y = u \sin \theta - gt = 40 \times \frac{1}{2} - 10 \times 1 = 10 \text{ m s}^{-1}$$

24. $x = at$ $y = at [1 - \alpha t]$

$$y = x - \frac{\alpha}{a} x^2$$

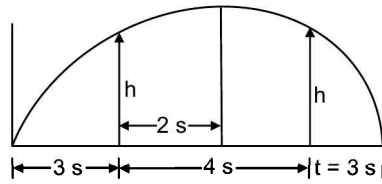
projectile motion

$$25. \left(\frac{dy}{dx}\right) = 0 \Rightarrow 1 - 2\frac{\alpha x}{a} = 0$$

$$x = \frac{a}{2\alpha}$$

$$26. y = \left(\frac{a}{2\alpha}\right) - \frac{a}{4\alpha} = \frac{a}{4\alpha}$$

27.



From the figure, time of flight

$$T = 10 \text{ s}$$

$$\text{Maximum height } H = \frac{1}{2}g\left(\frac{T}{2}\right)^2$$

$$= \frac{1}{2} \times 10 \times 25 = 125 \text{ m}$$

Consider free fall from topmost point till height h .

$$h = 125 - \frac{1}{2}gt^2 \Rightarrow t = 2 \text{ s (see figure)}$$

$$\therefore h = 125 - \frac{1}{2} \times 10 \times 4 = 105 \text{ m}$$

$$4H = R \tan \theta \Rightarrow R = 4 \times 125 \cot \theta = 500 \cot \theta$$

$$\text{If } \theta = 37^\circ, \text{ then } R = 500 \times \frac{4}{3} = \frac{2000}{3} \text{ m}$$

Ans: (a), (b)

$$28. y = x \tan \theta - \frac{1}{2}g \frac{x^2}{u^2 \cos^2 \theta}$$

$$= 10 \times \frac{3}{4} - \frac{1}{2} \times 10 \times \frac{(10)^2}{(10)^2 \left(\frac{4}{5}\right)^2} = \frac{15}{2} - \frac{125}{16} = -\frac{5}{16} \text{ m}$$

(a) is correct

$$\text{Time of flight} \Rightarrow T = \frac{10}{u \cos \theta} = \frac{10}{10 \times 0.8} = 1.25 \text{ s}$$

(c) is correct

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{100 \times (0.6)^2}{20} = 1.8 \text{ m}$$

$$H' \text{ from ground} = 1.8 + \frac{5}{16} \approx 2.11 \text{ m}$$

(d) is correct

$$29. \omega(4.8) = \alpha \times 4.8$$

$$\omega(4) = \alpha \times 4$$

$$\omega_{av} = \frac{\omega(4.8) + \omega(4)}{2}$$

$$= \frac{4.8\alpha + 4\alpha}{2} = 44 \text{ rad s}^{-1}$$

$$\frac{(4.8+4)}{2}\alpha = 44$$

$$\Rightarrow \alpha = 10 \text{ rad s}^{-2}$$

(c) is correct

$$\omega \text{ at } 4s = \alpha t = 10 \times 4 = 40 \text{ rad s}^{-1}$$

(a) is correct

$$\text{Revolution in } 1s \Rightarrow \theta' = \frac{1}{2} \times 10 \times 1 = 5 \text{ rad}$$

$$n = \frac{5}{2\pi} = \left(\frac{2.5}{\pi} \right)$$

(d) is correct

30. (a) $s = vt$ (p) can be $s - t$ graph with $a_A = a_B = 0$
 $v_A > v_B$ (r) can be $v - t$ graph.
 $\therefore (a) \rightarrow p, r$
 (b) $v = u + at$ $\therefore$ (q) can be $v - t$ graph
 $a_A > a_B$ (r) can be $a - t$ graph
 (c) $v = u + at$ (p) can be $v - t$ graph
 $a_A > a_B$ (r) can be $a - t$ graph
 (d) $v = u + at$ (s) can be $v - t$ graph and no other matches.

IIT Assignment Exercise

31. For a projectile launched upwards from ground at an angle with the horizontal, the horizontal component of velocity remains unchanged throughout the flight, but vertical component is continuously reduced by the force of gravity. At maximum height vertical component of velocity becomes zero but acceleration is not zero.
32. For projectile launched from level ground, time of flight is the time taken by the projectile to return to the same level from where it was thrown.
 $\therefore$ Time of flight = Twice the time taken by the projectile to reach the maximum height.
 ($\because$ time of ascent = time of descent)
 $\therefore$ Time of flight $T = 2t \Rightarrow t = \frac{T}{2}$
33. The horizontal velocity remains constant. The vertical velocity changes from $u \sin \theta$ to $-u \sin \theta$.

34. Range $R = \frac{u^2 \sin 2\theta}{g}$
 $\theta_1 = 45^\circ - \alpha$ and $\theta_2 = 45^\circ + \alpha$
 $R_1 = \frac{u^2 \sin 2(45^\circ + \alpha)}{g} = \frac{u^2 \cos 2\alpha}{g}$
 $R_2 = \frac{u^2 \sin 2(45^\circ - \alpha)}{g}$
 $= \frac{u^2 \cos(-2\alpha)}{g} = \frac{u^2 \cos 2\alpha}{g} = R_1.$

35. Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$
 For range to be maximum $\sin 2\theta = 1$
 $\therefore R_{\max} = \frac{u^2}{g} = \frac{160^2}{10} = 2560 \text{ m}$

36. $R = R_{\max}$ when $\theta = 45^\circ$

$$H_{\max} = \frac{u^2 (\sin 45^\circ)^2}{2g} = \frac{u^2}{4g} = 640 \text{ m}$$

37. $H = u_y t - \frac{1}{2} g t^2$, $u_y = u \sin \theta$

$$T = \frac{2u \sin \theta}{g}$$

$$\frac{T}{4} = t = \frac{u \sin \theta}{2g}$$

$$\Rightarrow H = \frac{u^2 \sin^2 \theta}{2g} \left(1 - \frac{1}{4} \right) = \frac{3}{4} H_{\max}$$

$$\Rightarrow \frac{H}{H_{\max}} = \frac{3}{4}$$

38. Horizontal range = $\frac{u^2 \sin 2\theta}{g}$

$$\text{Maximum height} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Ratio } \frac{R}{H} = \frac{4}{\tan \theta} = \frac{4}{\tan 30^\circ} = 4\sqrt{3}$$

39. $R_{\max} = \frac{v^2}{g} \Rightarrow v = \sqrt{g R_{\max}}$
 $= \sqrt{10 \times 16 \times 1000} = 400 \text{ m s}^{-1}$

40. The maximum distance a bullet can fall from the point of projection is $R_{\max}$, which is the same in all directions. $\Rightarrow$ Circle of radius $R_{\max}$

$$\text{Area} = \pi (R_{\max})^2$$

$$= \pi \left(\frac{v^2}{g} \right)^2 = \frac{\pi v^4}{g^2}$$

41. $R = \frac{v_A^2 \sin 2\theta_A}{g} = \frac{v_B^2 \sin 2\theta_B}{g}$

$$\frac{v_B^2}{v_A^2} = \frac{\sin 2\theta_A}{\sin 2\theta_B} = \frac{\sin 30^\circ}{\sin 90^\circ} = \frac{1}{2}$$

$$\Rightarrow \frac{v_B}{v_A} = \frac{1}{\sqrt{2}}$$

42. At the maximum height, vertical component is zero

$$u \cos \theta = \frac{u}{2} \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

3.64 Motion in Two Dimensions

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 120}{g}$$

$$= \frac{\sqrt{3}u^2}{2g}$$

43. $R_E = \frac{u^2 \sin 2\alpha}{g} = R$ on earth,
on the planet $R_p = \frac{u^2 \sin 2\alpha}{\frac{g}{4}} = 4R_E = 4R$

44. If v is the velocity of projection, at the highest point, vertical component of velocity is zero.
 $\Rightarrow v = u \cos 45^\circ$

45. For a particle projected vertically upwards with velocity $v = \sqrt{gR}$,

Velocity = 0 at maximum height.

$$\therefore (v')^2 - v^2 = -2gh.$$

$$v' = 0 \Rightarrow v^2 = 2gh$$

$$\Rightarrow h = \frac{v^2}{2g} = \frac{gR}{2g} = \frac{R}{2}$$

Aliter:

$$H_{\max} = \frac{v^2}{2g} = \frac{gR}{2g} = \frac{R}{2}$$

46. Particles projected at angles θ and $(90-\theta)$ with the horizontal have the same range r where, $r = \frac{u^2 \sin 2\theta}{g}$
(since $\sin 2\theta = \sin(180 - 2\theta)$)

$$t_1 = \frac{2u \sin \theta}{g}, t_2 = \frac{2u \sin(90 - \theta)}{g} = \frac{2u \cos \theta}{g}$$

$$t_1 t_2 = \frac{4u^2 \sin \theta \cos \theta}{g^2} = \frac{2r}{g}$$

$$\therefore t_1 t_2 \propto r$$

47. Horizontal range $R = \frac{u^2}{g}$, $\sin 2 \times 45 = \frac{u^2}{g}$

$$H_{\max} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R}{4} \Rightarrow R = 4H.$$

48. Range = $\frac{u^2 \sin 2\theta}{g} \Rightarrow 1.5 = \frac{u^2 \sin 30}{g}$ — (1)

Let $x = \frac{u^2 \sin 90}{g}$ — (2)

$$\frac{(1)}{(2)} \Rightarrow \frac{1.5}{x} = \frac{1}{2}$$

$$\Rightarrow x = 3 \text{ km}$$

49. Maximum height $H = \frac{u^2 \sin^2 \theta}{2g}$;

$$\text{Horizontal range } R = \frac{u^2 \sin 2\theta}{g}$$

$$\text{When } H = R, \frac{1}{2} \sin^2 \theta = \sin 2\theta$$

$$\Rightarrow \frac{\sin \theta}{2} = 2 \cos \theta \Rightarrow \tan \theta = 4.$$

$$\theta = \tan^{-1} 4$$

50. Maximum height reached by a projectile making an angle θ with the vertical

$$= \frac{u^2 \sin^2(90 - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

$$\therefore H \propto \cos^2 \theta$$

51. At any instant 't', speed 'v' is given by $v_x = u \cos \theta$, $= u \sin \theta - gt$; $v_y \Rightarrow v^2 = v_x^2 + v_y^2$
 $= u^2 (\sin^2 \theta + \cos^2 \theta) + g^2 t^2 - 2u \sin \theta \cdot gt$
 $\therefore v^2 = u^2 + g^2 t^2 - 2u g t \sin \theta \Rightarrow v^2$ versus t graph is parabolic

52. Height = 490 m

The bomb reaches the ground in

$$\sqrt{\left(\frac{2H}{g}\right)} \text{ s } (\because H = \frac{1}{2} g t^2)$$

$$\text{i.e., in } \sqrt{\frac{2 \times 490}{9.8}} = 10 \text{ s}$$

$$\text{The horizontal velocity} = \frac{360 \times 10^3}{3600} = 100 \text{ m s}^{-1}.$$

$$\text{Range} = \text{horizontal distance covered in 10 second} = 100 \times 10 \text{ m} = 1000 \text{ m}.$$

53. For small α , $R = \frac{u^2 2\alpha}{g}$ and $H = \frac{u^2 \alpha^2}{2g}$

$$\therefore \frac{R}{H} = \frac{4}{\alpha}$$

Aliter:

$$R \tan \alpha = 4H; \tan \alpha = \alpha \text{ for small angle}$$

$$\frac{R}{H} = \frac{4}{\alpha}$$

54. $\therefore v_x = v \cos \theta.$

$$v_y = v \sin \theta - gt$$

$$\beta = 45^\circ \Rightarrow v_y = v_x$$

$$\tan \beta = \frac{v_y}{v_x} = \frac{v \sin \theta - gt}{v \cos \theta} = 1$$

$$t = \frac{v(\sin \theta - \cos \theta)}{g} = \frac{150 \times (\sqrt{3} - 1)}{2 \times 10} \Rightarrow t = 5.5 \text{ s.}$$

55. Let the maximum height of the projectile be H .
The vertical component of velocity at height $H_{1/2}$ =
 $\tan 45^\circ u \cos \theta = u \cos \theta$

$$u^2 \sin^2 \theta - 2g \frac{H}{2} = u^2 \cos^2 \theta$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\Rightarrow \therefore u^2 \sin^2 \theta - gu^2 \frac{\sin^2 \theta}{2g} = u^2 \cos^2 \theta$$

$$\Rightarrow \tan^2 \theta = 2 \Rightarrow \theta = \tan^{-1} \sqrt{2}$$

56. Horizontal component of velocity remains unchanged.

$$v \cos \theta = 20 \cos 30^\circ = 20 \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ m s}^{-1}$$

Vertical velocity after 1 second,

$$v' = v \sin \theta - g = 20 \times \frac{1}{2} - g = 0.$$

$$\therefore \text{Resultant velocity is } 10\sqrt{3} \text{ m s}^{-1}$$

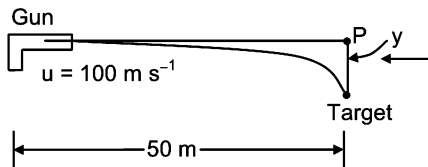
57. $\theta = 45^\circ$, maximum height

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g}$$

$$\text{Horizontal range } R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 90^\circ}{g} = \frac{u^2}{g} = 4H$$

$$= 4 \times 10 = 40 \text{ m}$$

- 58.



$u \cos \theta \approx u = 100 \text{ m s}^{-1}$ Time required to cover 50 m horizontally $t \approx \frac{50}{100} = \frac{1}{2} \text{ s}$, as speed is very high, compared to the vertical velocity component due to gravity

$$y \approx \frac{1}{2} \times 10 \times t^2 = \frac{1}{2} \times 10 \times \left(\frac{1}{2}\right)^2 = 1.25 \text{ m}$$

59. $y = tv_0 \sin \theta - \frac{1}{2} gt^2$ — (1)

$$D = t \times v_0 \cos \theta,$$

$$t = \frac{D}{v_0 \cos \theta} \quad \text{--- (2)}$$

Substituting (2) in (1),

$$y = \frac{Dv_0 \sin \theta}{v_0 \cos \theta} - \frac{1}{2} \frac{gD^2}{v_0^2 \cos^2 \theta}$$

$$= D \tan \theta - \frac{1}{2} \frac{gD^2}{v_0^2 \cos^2 \theta}$$

Aliter:

$u = v_0$; $x = D$; using trajectory equation,

$$y = D \tan \theta - \frac{1}{2} \frac{gD^2}{v_0^2 \cos^2 \theta}$$

60. In projectile motion, in general, the velocity makes an angle with the acceleration (as opposed to motion in 1D). For a particle projected vertically upward, the velocity and acceleration are always along the same line, but not always along the same direction. Only for a particle fired vertically downwards, its velocity and accelerations are in the same direction.

61. $\tan 30^\circ = \frac{\text{vertical component}}{\text{horizontal component}}$

$$= \frac{u \sin \alpha - 2g}{u \cos \alpha} \quad \text{--- (1)}$$

At maximum height after 3 seconds,

$$0 = u \sin \alpha - 3g \quad \text{--- (2)}$$

Substituting (2) in (1),

$$\tan 30^\circ = \frac{3g - 2g}{u \cos \alpha} = \frac{g}{u \cos \alpha}$$

$$\therefore u \cos \alpha = \sqrt{3}g \quad \text{--- (3)}$$

Combining (2) and (3),

$$u^2 = 9g^2 + 3g^2 = 12g^2$$

$$u = 2\sqrt{3}g = 20\sqrt{3} \text{ m s}^{-1}$$

62. Since the initial velocity is horizontal, the vertical position is free fall with zero component

$$\therefore H = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times 25 = 125 \text{ m.}$$

63. t seconds after projection vertical component of velocity is $(u \sin \theta - gt)$. The horizontal component continues to be $u \cos \theta$.

$\therefore$ Speed after t seconds is:

$$\sqrt{(u \sin \theta - gt)^2 + (u \cos \theta)^2}$$

$$= \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$$

3.66 Motion in Two Dimensions

64. The vertical component of velocity at the instant of release is zero. The vertical motion is free fall. Hence they will reach simultaneously

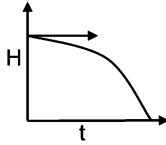
65. Let the height of the tower be H

$$H = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2H}{g}}$$

Distance between the two points of impact is $-(V_1 - V_2)t$

Distance between foot of the tower and the point of impact is :

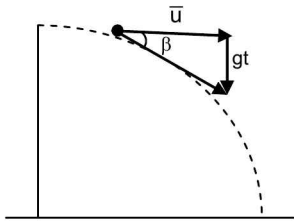
$$x = vt = (100 - 50) \sqrt{\frac{2H}{g}} = 50 \sqrt{\frac{2H}{g}}$$



66. Horizontal component of velocity remains unchanged (for projectile motion)

67. $v = 0 + gt = gt$.

68. After time t , vertical component is gt . Horizontal component u is unchanged.



$\therefore$ Tangent of the angle made by the velocity is

$$\frac{\text{vertical component}}{\text{horizontal component}} = \frac{gt}{u}$$

$$\text{i.e., } \beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

69. For vertical motion, $h = \frac{1}{2}gt^2 \therefore t \propto \sqrt{h}$

70. The angles are equally inclined (say by δ) to $45^\circ + \frac{\alpha}{2}$

$$45 + \frac{\alpha}{2} + \delta = 60 \quad \text{---(1)}$$

$$45 + \frac{\alpha}{2} - \delta = 40 \quad \text{---(2)}$$

Add (1) + (2)

$$\alpha + 90^\circ = 60^\circ + 40^\circ \Rightarrow \alpha = 10^\circ$$

71. The maximum horizontal range for a particle projected from height h is

$$R = \frac{u\sqrt{u^2 + 2gh}}{g} = \frac{12 \times \sqrt{12^2 + 2 \times 10 \times 1.25}}{10} = \frac{12 \times 13}{10} = 15.6 \text{ m}$$

72. A body dropped from a moving train has a horizontal velocity same as that of the train and no vertical component of velocity. Hence the trajectory is that of the downward motion of a projectile

A body dropped from a stationary train falls under gravity. In both the cases above, the time that the body takes to reach the ground is independent of the state of the train.

73. $x = ct^2$; $y = bt^2$

$$v_x = \frac{dx}{dt} = 2ct; v_y = \frac{dy}{dt} = 2bt$$

$$v = \sqrt{v_x^2 + v_y^2} = 2t\sqrt{b^2 + c^2}$$

74. Relative acceleration = 0

$$\vec{r}_1 - \vec{r}_2 = (\vec{u}_1 - \vec{u}_2)t + \vec{r}_0$$

$x - x_0 = u_x t$, $y - y_0 = u_y t$, where u_x and u_y are the components of the initial relative velocity

$\therefore t = \frac{x - x_0}{u_x} = \frac{y - y_0}{u_y}$ is the equation of a straight line.

75. Stone dropped from a moving train will have a horizontal velocity equal to that of the train and zero vertical component of velocity. These two conditions are those for parabolic motion after the release. An observer on the platform will see a body projected horizontally from a height

Equations of motion are:

$$x = ut \quad \text{--- (1)}$$

$$y = \frac{1}{2}gt^2 \quad \text{--- (2)}$$

Substituting (1) in (2),

$$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 \Rightarrow x^2 = \frac{2u^2}{g}y$$

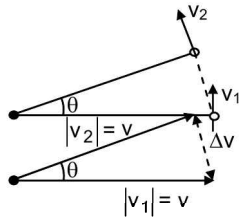
$x^2 = ky \Rightarrow$ equation of a parabola.

76. After 5 s $\rightarrow v = at = 1 \times 5$
 $= 5 \text{ m s}^{-1}$

$$\text{Centripetal acceleration} = \frac{v^2}{R} = 25 \text{ m s}^{-2}$$

$$\text{Total acceleration} = \sqrt{(25)^2 + (1)^2} \approx 25 \text{ m s}^{-2}$$

77.

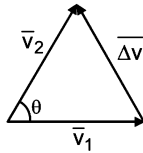


$$\Delta v = \left[v^2 + v^2 - 2vv \cos \theta \right]^{\frac{1}{2}}$$

$$= \left[2v^2 (1 - \cos \theta) \right]^{\frac{1}{2}} = \left(2v^2 2 \sin^2 \frac{\theta}{2} \right)^{\frac{1}{2}}$$

$$= 2v \sin \frac{\theta}{2} = v(\text{given data})$$

$$\sin \frac{\theta}{2} = \frac{1}{2} \Rightarrow \frac{\theta}{2} = 30^\circ \Rightarrow \theta = 60^\circ$$

Aliter:

 $\Rightarrow$ vector diagram $\Rightarrow$

Data given:

$$|\vec{v}_1| = |\vec{v}_2| = |\Delta \vec{v}|$$

 equilateral triangle $\Rightarrow \theta = 60^\circ$

$$78. \omega^2 r = 4\omega^2 r' \Rightarrow r' = \frac{r}{4}; v'$$

$$= \omega' r' = 2\omega \times \frac{r}{4}$$

$$v' = \frac{\omega r}{2} = \frac{v}{2}$$

Aliter:

$$a_r = \omega v = \omega' v' \Rightarrow v' = \frac{\omega v}{\omega'} = \frac{\omega v}{2\omega} = \frac{v}{2}$$

79. Because the bodies are at the same radial line at the positions of crossing each other, their velocities are along the normal to the same line but directed in opposite direction $\rightarrow v_A = \omega_A r$; $v_B = \omega_B \cdot 1.5r$

$$\text{Relative velocity} = v_B + v_A = \omega_B \cdot 1.5r + \omega_A r$$

$$= 2v_A (\text{given data})$$

$$\omega_B \cdot 1.5r + \omega_A r = 2\omega_A r \Rightarrow \frac{\omega_B}{\omega_A} = 1.5$$

80. The radial (centripetal) acceleration

$$a_r = \omega^2 r$$

$$\omega = \alpha t \text{ (since the body starts from rest)}$$

The tangential acceleration

$$a_t = \alpha r$$

Let angle be θ . The velocity of a body in circular motion is in the tangential direction

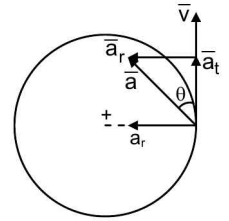
$$\tan \theta = \left(\frac{a_r}{a_t} \right);$$

$$\theta = \frac{\pi}{4} \Rightarrow a_r = a_t$$

$$\omega^2 r = r\alpha \Rightarrow \omega^2 = \alpha$$

$$\Rightarrow \alpha^2 t^2 = \alpha$$

$$\Rightarrow t = \sqrt{\frac{1}{\alpha}} = \frac{1}{2} \text{ s}$$



$$81. T_1 = \frac{2u \sin 30^\circ}{g} = \frac{2u}{g} \cdot \frac{1}{2} = \frac{u}{g}; u \text{ same,}$$

$$\theta_2 = 90^\circ - \theta_1 (\because 60^\circ = 90^\circ - 30^\circ) \Rightarrow R_1 = R_2$$

$$T_2 = \frac{2u \sin 60^\circ}{g} = \frac{2u}{g} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}u}{g}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{1}{\sqrt{3}} \Rightarrow T_1 \neq T_2$$

$$H_1 = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{u^2}{2g} \cdot \frac{1}{4}$$

$$H_2 = \frac{u^2 \sin^2 60^\circ}{2g} = \frac{u^2}{2g} \cdot \frac{3}{4}$$

$$\frac{H_1}{H_2} = \frac{1}{3} = \left(\frac{T_1}{T_2} \right)^2$$

82. The maximum range condition $\theta = 45^\circ$; is the same for both

$$R = \frac{u^2}{g}; t = \frac{2u \sin 45^\circ}{g}; \frac{t_1}{t_2} = \frac{u_1}{u_2} = \frac{1}{4}$$

$$\frac{R_1}{R_2} = \left(\frac{u_1}{u_2} \right)^2 = \left(\frac{1}{4} \right)^2$$

$$\therefore R_1 : R_2 = 1 : 16$$

Aliter:

$$R \tan \theta = 4 H_{\max} = \frac{1}{2} g T^2$$

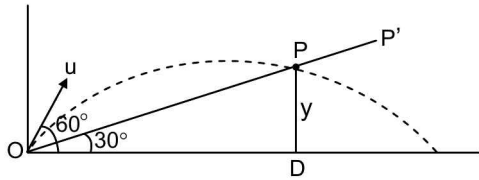
$$\tan \theta = \tan 45^\circ = 1$$

$$\therefore R = \frac{1}{2} g T^2$$

$$\therefore \frac{R_1}{R_2} = \frac{T_1^2}{T_2^2} = \frac{1}{16}$$

3.68 Motion in Two Dimensions

83.



Let $OP = r$; Given that $OP = 2y \Rightarrow \angle POD = 30^\circ$

$$\therefore x = r \cos 30^\circ = r \frac{\sqrt{3}}{2}$$

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\theta = 60^\circ$$

$$y = \frac{r}{2} = \sqrt{3} \cdot \frac{\sqrt{3}}{2} r - \frac{g}{2u^2} \frac{3r^2}{4} \frac{1}{4}$$

$$\Rightarrow \frac{3gr^2}{2u^2} = \left(\frac{3}{2} - \frac{1}{2} \right) r$$

$$\Rightarrow \frac{g}{u^2} \frac{3}{2} r = \left(\frac{3}{2} - \frac{1}{2} \right)$$

$$\Rightarrow r = \frac{2u^2}{3g}$$

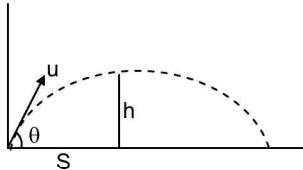
Aliter:

Range on the inclined plane = OP

$$= \frac{2u^2 \cos \theta \sin (\theta - \alpha)}{g \cos^2 \alpha}$$

$$= \frac{2u^2 \cos 60^\circ \sin 30^\circ}{g \cos^2 30^\circ} = \frac{2u^2}{3g}$$

84.



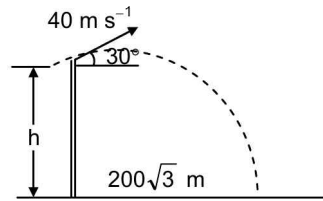
$$\text{Horizontal distance} = u \cos \theta = \frac{u}{2} t$$

$$\text{Vertical distance} = u \sin \theta - \frac{1}{2} g t^2 = u \frac{\sqrt{3}}{2} t - \frac{g}{2} t^2$$

$$\text{For the given condition } \frac{\sqrt{3}u}{2} t - \frac{g}{2} t^2 > \frac{u}{2} t$$

$$u \left[\frac{\sqrt{3}}{2} - \frac{1}{2} \right] > \frac{g}{2} t \quad t < \frac{u}{g} (\sqrt{3} - 1)$$

85.



$$v \cos \theta t = 200\sqrt{3} \text{ m}$$

$$40 \times \frac{\sqrt{3}}{2} \times t = 200\sqrt{3}$$

$$t = 10 \text{ s}; -h = v \sin \theta t - \frac{1}{2} g t^2$$

$$-h = 40 \times \frac{1}{2} \times 10 - \frac{1}{2} \times 10 \times 100$$

$$-h = 200 - 500 = -300$$

$$h = 300 \text{ m}$$

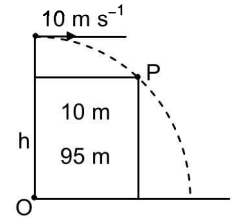
86. For the projectile $x = ut = 10t$

$$\Rightarrow t = 1$$

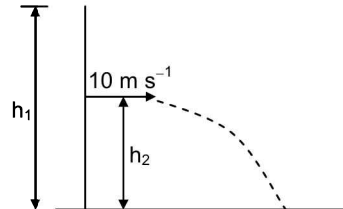
$$y = h - \frac{1}{2} g t^2$$

$$\therefore 95 = h - \frac{1}{2} 10 \times 1^2$$

$$h = 100 \text{ m}$$



87.



Since the modulus $|v|$ is the same

In the first case

$$|v| = \sqrt{2gh_1}$$

In the other case:

$$v_2^2 = (10)^2 + 2gh_2 = v^2 \text{ (given)} = 2gh_1$$

$$2g(h_1 - h_2) = 10^2$$

$$\Rightarrow h_1 - h_2 = \frac{100}{20} = 5 \text{ m}$$

$$88. t = \sqrt{\frac{2b}{g}}$$

$$a < vt < 2a$$

$$\Rightarrow a < v \sqrt{\frac{2b}{g}} < 2a$$

$$v \in \left(a \sqrt{\frac{g}{2b}}, a \sqrt{\frac{2g}{b}} \right)$$

89. If it does not hit the incline, it travels a vertical distance of H , so that the time of flight is

$$t = \sqrt{\frac{2H}{g}}$$

The initial velocity is horizontal, so the horizontal

$$\text{range is } = v_0 t = v_0 \sqrt{\frac{2H}{g}}$$

The minimum value of range so that it does not hit the incline is $\frac{H}{\tan \beta}$

$$\Rightarrow \tan \beta = \frac{H}{v_0 \sqrt{\frac{2H}{g}}} = \frac{1}{v_0} \sqrt{\frac{gH}{2}}$$

$$\beta = \tan^{-1} \left(\frac{1}{v_0} \sqrt{\frac{gH}{2}} \right)$$

90. $\frac{R}{T} = u \cos \theta = 20$

$$u = 40, \cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \frac{1}{2} = 60^\circ$$

91. $\frac{R}{H} = \frac{4}{\tan \alpha} = 2 \Rightarrow \tan \alpha = 2$
 $v_y = u \sin \alpha - gt$

$$t = \frac{R}{4u \cos \alpha} = \frac{u^2}{g} \cdot \frac{2 \sin \alpha \cos \alpha}{4u \cos \alpha} = \frac{u \sin \alpha}{g \cdot 2}$$

$$\Rightarrow \therefore v_y = u \sin \alpha - gt = \frac{u \sin \alpha}{2}$$

$$\Rightarrow \tan \beta = \frac{v_y}{v_x} = \frac{\frac{u \sin \alpha}{2}}{u \cos \alpha} = \frac{\tan \alpha}{2}$$

$$\text{Angle } \beta = \tan^{-1} \left(\frac{\tan \alpha}{2} \right) = \frac{\pi}{4} [\because \tan \alpha = 2]$$

92. $u \cos \theta T = R$ where T is the time of flight and R is the range.

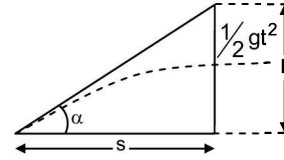
Velocity at peak position

$$u \cos \theta = \frac{R}{T} = \frac{180}{10} = 18 \text{ ms}^{-1}$$

93. For collision, angle of elevation of $h = \alpha$

$$\Rightarrow \frac{h}{s} = \tan \alpha$$

$$h = s \tan \alpha$$



Aliter:

Let the time for collision be t [$\because$ both fall by $\frac{1}{2}gt^2$]

$$u \cos \alpha t = s$$

Relative velocity is $u \sin \alpha$

$$\Rightarrow \frac{h}{u \sin \alpha} = t = \frac{s}{u \cos \alpha} \Rightarrow h = s \tan \alpha$$

94. Taking the dog's direction as positive X axis and upward direction as Y -axis:

Velocity of the stone when it reaches ground = $\hat{i}v_b - \hat{j}\sqrt{2gh}$. Velocity of the dog = $\hat{i}v_d$ (v_b and v_d must be in the same direction along horizontal, otherwise the stone will not hit the dog)

$\therefore$ Relative velocity of the stone with respect to dog $v_b - v_d$, where v_b is the final velocity of the stone.

$$(\hat{i}v_b - \hat{j}\sqrt{2gh}) - \hat{i}v_d = \hat{i}(v_b - v_d) - \hat{j}\sqrt{2gh}$$

Hence the magnitude will be $[(v_b - v_d)^2 + 2gh]^{1/2}$

95. Since they have positive same range for initial velocity u , $\theta_2 = (90 - \theta_1)$ at any time t , vertical distances are

$$h_1 = u \sin \theta_1 t - \frac{1}{2}gt^2; h_2 = u \sin \theta_2 t - \frac{1}{2}gt^2$$

$$= u \cos \theta_1 t - \frac{1}{2}gt^2$$

$$h = (h_2 - h_1)$$

$$= u \cos \theta_1 t - \frac{1}{2}gt^2 - (u \sin \theta_1 t - \frac{1}{2}gt^2)$$

$$= ut (\cos \theta_1 - \sin \theta_1)$$

$[u(\cos \theta_1 - \sin \theta_1)]$ is a constant, hence h increases linearly with time.

96. For the ball to return to the car, it should not be projected with any additional horizontal velocity. Hence the ball should be projected vertically upwards with respect to the boy.

3.70 Motion in Two Dimensions

97. Since they meet in air their horizontal velocities should be same.

$$v_A \cos 37^\circ = 40 \cos 30^\circ$$

$$\Rightarrow v_A = 40 \times \frac{\sqrt{3}}{2} \times \frac{1}{0.8} = 25\sqrt{3} \text{ m s}^{-1}$$

vertical component of their velocities are:

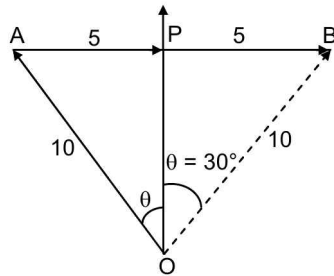
$$v_{A-v} = 25\sqrt{3} \times \sin 37^\circ = 15\sqrt{3} \text{ m s}^{-1}$$

$$v_{B-v} = 40 \sin 30^\circ = 20 \text{ m s}^{-1}$$

Since they move under same acceleration and since their horizontal velocities are same, their relative velocity remain same and is equal to

$$\vec{v}_{B/A} = \vec{v}_{B-v} - \vec{v}_{A-v} = (20 - 15\sqrt{3})\hat{j} \approx -6\hat{j} \text{ m s}^{-1}$$

98.



Let $\overline{OA}$ represent boat velocity $\vec{v}_b = 10 \text{ kmph}$, $\overline{AP}$ represent river velocity $\vec{v}_r = 5 \text{ kmph}$. For zero drift resultant $\overline{OP}$ normal to $\overline{AP}$. Hence θ is $\Rightarrow \sin \theta = \frac{AP}{OA} = \frac{5}{10} \Rightarrow \theta = 30^\circ$

$$\text{Time to cross} = \frac{d}{|\overline{OP}|} = \frac{5\sqrt{3}}{10 \cos 30^\circ} = 1 \text{ hour}$$

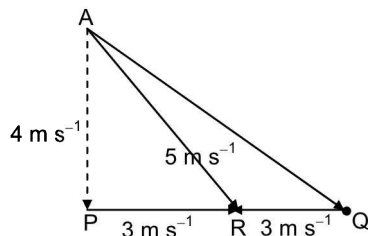
When tilted wrongly the boat velocity will be along OB

Hence drift velocity = $10 \sin 30^\circ + 5 = 10 \text{ kmph}$

Time to cross remains same because $10 \cos 30^\circ$ is in same direction.

$$\therefore \text{Total drift} = 10 \times 1 = 10 \text{ km}$$

99.



Let $\overline{AR}$ represent rain velocity and $\overline{PR}$ represent man's velocity. Then the resultant velocity

$\overline{AP} = \overline{AR} - \overline{PR} = \overline{AR} + \overline{RP}$. Given that raindrops fall vertically $\Rightarrow$

$$\therefore \angle APR = 90^\circ$$

$$\therefore AP = \sqrt{25 - 9} = 4 \text{ m s}^{-1}$$

When he reverses his direction, his velocity becomes $\overline{QR}$. Hence relative velocity is $\overline{AR} - \overline{QR} = \overline{AR} + \overline{RQ} = \overline{AQ}$

$$\text{From } \triangle APQ \Rightarrow AQ^2 = AP^2 + PQ^2 = 4^2 + 6^2 = 52$$

$$\therefore AQ = \sqrt{52} \text{ ms}^{-1}$$

Key (b)

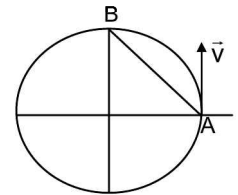
100. Displacement is along AB.

Total displacement AB

$$= \sqrt{r^2 + r^2} = \sqrt{2} r$$

$$\text{Time } t = \frac{2\pi/4}{\omega} = \frac{2\pi r}{4 \times v} = \frac{\pi r}{2v}$$

$$\text{Average velocity} = \frac{\sqrt{2}r \cdot 2v}{\frac{\pi r}{2v}} = \frac{2\sqrt{2}v}{\pi}$$



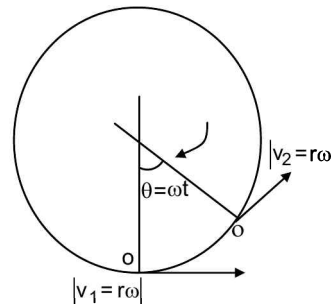
101. At A the acceleration of m_1 is

$$\omega^2 r = \left(\frac{2\pi}{T} \right)^2 r \text{ directed along } \overline{AO}.$$

The acceleration of m_2 is also directed in the same direction. Therefore relative acceleration of m_2 with respect to m_1

$$\vec{a}_R = \vec{a}_B - \frac{4\pi^2}{T^2} r = \vec{a}_B - \left(\frac{2\pi}{T} \right)^2 r$$

102.



$$\overline{AC} = \overline{AB} + \overline{BC} = \vec{v}_2 - \vec{v}_1$$

$\therefore AC$ represents change in velocity.

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cos \theta$$

$$\text{Let } |\mathbf{v}_1| = |\mathbf{v}_2| = v$$

$$AC^2 = v^2 + v^2 - 2v^2(1 - 2\sin^2 \theta/2) \\ = 2v^2(2\sin^2 \theta/2) = 4v^2 \sin^2 \theta/2$$

$$v = r\omega = 1 \times \omega$$

$$AC = 2v \sin \omega t/2 = 2\omega \sin \omega t/2$$

$$103. r\alpha = 20\pi \Rightarrow \alpha = \frac{20\pi}{2} = 10\pi \text{ rad s}^{-2}$$

$$\omega = 0 + \alpha t = 10\pi \text{ t}; \text{ At } t = 4 \text{ s};$$

$$\omega = 40\pi; \frac{2\pi}{T} = 40\pi,$$

$$T = \frac{2}{40} = 0.05 \text{ s}$$

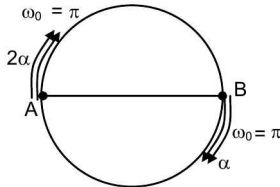
$$104. \theta = \frac{1}{2}\alpha t^2, \alpha = 10\pi \text{ (see above solution)}$$

$$\therefore \text{ at } t = 1 \text{ s}, \theta = \frac{1}{2} \times 10\pi \times 1 = 5\pi \text{ rad}$$

This corresponds to 2 full revolutions and a half revolution.

$$\therefore \theta = 180^\circ \text{ with respect to initial position.}$$

105.



Relative $\left(\frac{B}{A}\right)$ equations: α_r = relative acceleration;

Consider the diagram shown. B is π radian ahead of A in the clockwise direction. When they meet, angular separation is 0

Also relative $\omega_r = 0$ initially:

$$\theta = \theta_0 + \omega_r t + \frac{1}{2}\alpha_r t^2$$

$$\Rightarrow \pi = (\pi - \pi)1 + \frac{1}{2}(2\alpha - \alpha)1^2$$

$$\Rightarrow \alpha = 2\pi \text{ rad s}^{-2}$$

If they travel in opposite sense;

$$\Rightarrow \omega_r = \pi + \pi$$

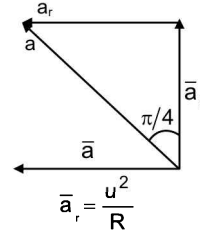
$$\alpha_r = (\alpha + 2\alpha)$$

$$\pi = (2\pi)t + \frac{1}{2}(2\alpha + \alpha)t^2$$

$$\pi = 2\pi t + \frac{1}{2}6\pi t^2 \Rightarrow t = \frac{1}{3} \text{ s}$$

106. At $t = 0$;

$$\therefore a_r = \frac{u^2}{R} = \frac{u^2}{1} = u^2; \frac{a_r}{a_t} = \tan\left(\frac{\pi}{4}\right) = 1 \rightarrow a_r = a_t = u^2$$



$$\text{At } t = 1 \text{ s}; v = u + a_t \times t = u + u^2 \times 1$$

$$\therefore v = u + u^2$$

$$\text{Given at } t = 1 \text{ s}, \frac{a_r}{a_t} = 4 \text{ (but } a_t = a_r = u^2)$$

$$a_r = \frac{v^2}{R} = \frac{(u + u^2)^2}{1} = (u + u^2)^2$$

$$\Rightarrow \frac{(u + u^2)^2}{u^2} = 4$$

$$(u + 1)^2 = 4$$

$$\Rightarrow u = 1 \text{ m s}^{-1}$$

$$107. \omega = \omega_0 + \alpha t$$

$$t = \frac{\omega_0}{-\alpha} = \frac{170}{-5} = 34 \text{ s}$$

$$108. \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \frac{-\omega_0^2}{2\alpha} = \frac{170^2}{2 \times 5} = 2890 \text{ rad}$$

$$= \frac{2890}{2\pi} \text{ revolution} = \frac{1445}{\pi}$$

Aliter:

$$\theta = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{(0 + 170)}{2}\right)34 = 2890 \text{ rad}$$

$$\therefore N = \frac{\theta}{2\pi} = \frac{2890}{2\pi} = \frac{1445}{\pi} \text{ revolution}$$

$$109. \text{ The tangential velocity is } \frac{v^2}{r} = 9; v^2 = 9$$

$v = 3 \text{ m s}^{-1}$. When the string is cut the body will move with just the tangential velocity. Since it is moving clockwise, it will move towards South.

$$110. a_1 = \omega_1^2 R_1$$

$$a_2 = \omega_2^2 R_2 = 4\omega_1^2 R_2$$

$$a_1 = a_2 \Rightarrow R_2 = \frac{R}{4}$$

3.72 Motion in Two Dimensions

111. Angle of projection $> \angle A$. Similarly, angle of landing $> \angle B$.

$$\therefore \text{Their sum} > \angle A + \angle B = 90^\circ$$

$$\Rightarrow 2\theta > 90^\circ \Rightarrow \theta > 45^\circ$$

[(via P or not is immaterial)]

112. $\vec{r}$ and $\frac{d\vec{r}}{dt}$ will have same or opposite directions only if 1D motion.

clearly the particle is in 1D motion (otherwise it cannot cross same point twice)

113. The gun has higher range in the forward

114. $a_{\text{rel}} = 9.8 + 0.2 = 10 \text{ m s}^{-2}$

$$T = \frac{2u_{\text{rel}} \sin \theta}{a_{\text{rel}}} = \frac{2 \times 2 \times \sin 30}{10} = \frac{2}{10} = 0.2 \text{ s}$$

115. $t = 0.2$, $u = 2 \text{ m s}^{-1}$ (lift) $a = 0.2 \text{ m s}^{-2}$

$$S = ut + \frac{1}{2}at^2 = 2 \times 0.2 + \frac{1}{2} \times 0.2 \times 0.04$$

$$= 0.404 \text{ m}$$

116. The initial velocity of the particle with respect to the ground is $\vec{u}_{\text{ground}} = \vec{u}_{\text{rel}} + \vec{u}_{\text{lift}}$

$$\Rightarrow \vec{u}_{\text{ground}} = \frac{2\sqrt{3}}{2}\hat{i} + \left(2 \times \frac{1}{2} + 2\right)\hat{j} = \sqrt{3}\hat{i} + 3\hat{j}$$

Since initial velocity is at an angle to the vertical, the path is a parabola.

117. Since P is the highest point of the projectile,

$$PS = \frac{1}{2}gt_1^2 = 5 \Rightarrow t_1 = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ s}$$

$t_1 \rightarrow$ time to reach from R to P,

$$\therefore t_1 \times u \cos \theta = RS$$

$$1 \times u \cos \theta = 24$$

— (1)

$$\frac{u^2 \sin^2 \theta}{2g} = H = 11.2 + 5 = 16.2 \text{ m}$$

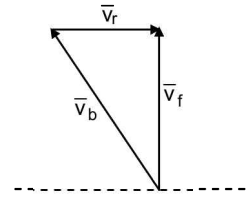
$$u \sin \theta = \sqrt{16.2 \times 20} = 18$$

$$\Rightarrow \tan \theta = \frac{18}{24} = \frac{3}{4} \Rightarrow \theta = 37^\circ$$

$$u \cos \theta = 24 \Rightarrow u = \frac{24}{0.8} = 30 \text{ m s}^{-1}$$

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 18}{10} = 3.6 \text{ s}$$

118. A moving by shortest distance $\rightarrow$



$\vec{v}_r \rightarrow$ river velocity

$\vec{v}_b \rightarrow$ boat velocity

$\vec{v}_f \rightarrow$ final resultant velocity

$\vec{v}_f \rightarrow$ normal to the river flow

$$v_f = \sqrt{v_b^2 - v_r^2} = \sqrt{400 - 100} = 10\sqrt{3} \text{ kmph}$$

$$\text{time to cross} = \frac{2\sqrt{3}}{10\sqrt{3}} = \frac{1}{5} \text{ h} = 12 \text{ minute}$$

B moving by shortest time $\Rightarrow$

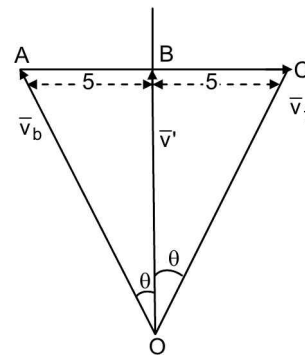
$$t_B = \frac{2\sqrt{3}}{20} = \frac{\sqrt{3}}{10} \text{ h} = 0.1732 \text{ h} = 10.4 \text{ minute}$$

(a) $\rightarrow$ wrong

Drift of B = time taken to cross $\times$ flow speed of river

$$\Rightarrow \frac{\sqrt{3}}{10} \times 10 = \sqrt{3} \text{ km} \quad (\text{b) is correct}$$

When B reduces drift velocity to 5 kmph the velocity vector diagram will be as shown.



$$\Rightarrow |\vec{v}_f| = |\vec{v}_b|$$

(d) is correct

$$|\vec{v}'| = OB \Rightarrow \frac{BC}{OC} = \frac{5}{20} = \sin \theta$$

$$\therefore \sin \theta = \frac{1}{4} \Rightarrow \cos \theta = \frac{\sqrt{15}}{4} \Rightarrow v' = 20 \frac{\sqrt{15}}{4}$$

$$= 5\sqrt{15} \text{ kmph}$$

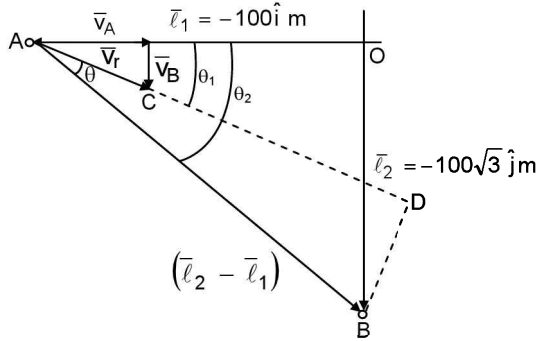
Time to cross

$$= \frac{2\sqrt{3}}{5\sqrt{15}} = \frac{2}{5\sqrt{5}} \Rightarrow \text{Drift} = \frac{2}{5\sqrt{5}} \times 5 = \frac{2}{\sqrt{5}} \text{ km}$$

(c) is correct

$\therefore$ (b), (c), (d) are correct

119.



Initial distance between them = AB

$$\overline{AB} = \overline{l}_2 - \overline{l}_1$$

Let $\overline{AC}$ represent relative velocity of A with reference to B $\Rightarrow \overline{AC} = \overline{v}_A - \overline{v}_B = \overline{v}_r$

Closest approach BD = AB sin θ

$$= (\overline{l}_2 - \overline{l}_1) \times \hat{v}_r$$

$$\overline{l}_2 - \overline{l}_1 = (-\hat{j}\sqrt{3} \cdot 100 + \hat{i}100)$$

$$\overline{v}_r = \overline{v}_A - \overline{v}_B = \left(20\hat{i} - \frac{20}{\sqrt{3}}\hat{j}\right)$$

$$\Rightarrow |\overline{v}_r| = \sqrt{(20)^2 + \left(\frac{20}{3}\right)^2} = \left(\frac{40}{\sqrt{3}}\right) \text{ m s}^{-1}$$

$$\hat{v}_r = \frac{20\hat{i} - \frac{20}{\sqrt{3}}\hat{j}}{\left(\frac{40}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$$

$$(\overline{l}_2 - \overline{l}_1) \times \hat{v}_r = (100\hat{i} - \sqrt{3} \cdot 100\hat{j}) \times \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$$

$$= 50\hat{k} + 150\hat{k} = 100\hat{k}$$

$$\therefore BD = |(\overline{l}_2 - \overline{l}_1) \times \hat{v}_r| = 100 \text{ m}$$

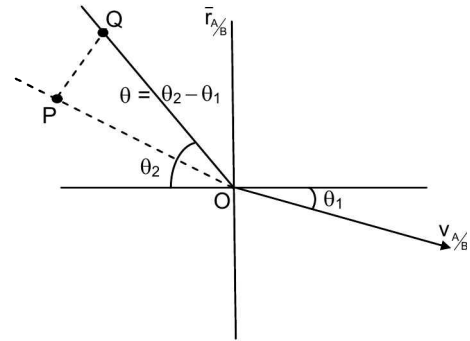
Time to reach closest position

$$= \frac{AD}{|\overline{v}_r|} = \frac{(\overline{l}_2 - \overline{l}_1) \cdot \hat{v}_r}{|\overline{v}_r|}$$

$$= \frac{(100\hat{i} - \sqrt{3}100\hat{j}) \cdot \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)}{\left(\frac{40}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3}}{40} \left(100 \times \frac{\sqrt{3}}{2} + 100 \frac{\sqrt{3}}{2}\right) = \frac{300}{40} = 7.5 \text{ s}$$

Aliter:



$$\overline{v}_{A/B} = \overline{v}_A - \overline{v}_B = 20\hat{i} - \frac{20}{\sqrt{3}}\hat{j}$$

$$\tan \theta_1 = \frac{1}{\sqrt{3}} \Rightarrow \theta_1 = 30^\circ$$

$$\overline{r}_{A/B} = \overline{r}_A - \overline{r}_B$$

$$= -100\hat{i} + 100\sqrt{3}\hat{j} = 20a_0 \left(-\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

$$\tan \theta_2 = \sqrt{3} \Rightarrow \theta_2 = 60^\circ$$

$$|\overline{v}_{A/B}| = \frac{20}{\cos 30^\circ} = \frac{40}{\sqrt{3}}$$

$$\theta = \theta_2 - \theta_1 = 30^\circ$$

$$\frac{PQ}{OQ} = \frac{1}{2} \Rightarrow \text{Distance of closest approach} = PQ = 100 \text{ m}$$

Time to reach closest position

$$= \frac{OP}{|\overline{v}_{A/B}|} = \frac{OQ \cdot \cos 30^\circ}{\frac{20}{\cos 30^\circ}} = \frac{40}{\sqrt{3}}$$

$$= \frac{200}{20} \cdot \cos^2 30^\circ = 7.5 \text{ s}$$

3.74 Motion in Two Dimensions

$$120. (a) \quad t = \frac{\ell}{|\vec{v} + \vec{u}|} = \frac{\ell}{\sqrt{v^2 - u^2}}$$

$\therefore (a) \rightarrow p, q$

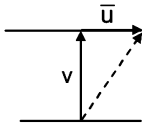
(b)



$$t = \frac{\ell_1 + \ell_2}{|\vec{v} - (-\vec{u})|} = \frac{\ell}{|\vec{v}| + |\vec{u}|}$$

$\therefore (b) \rightarrow r$

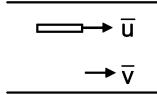
(c)



$$t = \frac{\ell}{|\vec{v}|}$$

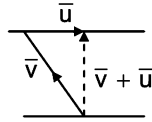
$\therefore (c) \rightarrow s$

(d)



$$t = \frac{\ell}{|\vec{v} + \vec{u}|} = \frac{\ell}{|\vec{v}| + |\vec{u}|}$$

$\therefore (d) \rightarrow p, r$



123. Horizontal motion takes place with constant velocity, as acceleration is the acceleration due to gravity, which is vertical. Horizontal velocity remains constant.

$$124. \quad \frac{T_1}{T_2} = \frac{\frac{2u \sin \theta}{g}}{\frac{2u \sin(90 - \theta)}{g}} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

125. Angles are θ_1 and $90 - \theta_1$.

$$\frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2(90 - \theta_1)} = \tan^2 \theta_1$$

$$126. \quad \text{Time of flight } T = \frac{2v \sin \theta}{g}$$

T is maximum when $\sin \theta = 1 \Rightarrow \theta = 90^\circ$

127. h is the vertical distance.

$$h = u \sin \theta t - \frac{1}{2} g t^2$$

horizontal distance S,

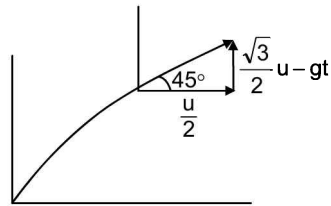
$$S = u \cos \theta t.$$

To achieve the given condition, $u \cos \theta t = u \sin \theta t - \frac{1}{2} g t^2$

$\Rightarrow u(\sin \theta - \cos \theta) = \frac{g}{2} t$. RHS is positive. For the LHS to be positive $(\sin \theta - \cos \theta)$ would be positive. $\therefore \theta > 45^\circ$.

Above 45° only $\sin \theta > \cos \theta$ upto 90°

128.



Angle at the instant, angle made by velocity

$$\frac{\pi}{3} - \frac{\pi}{12} = \frac{\pi}{4}$$

$$\Rightarrow \frac{u \left(\frac{\sqrt{3}}{2} \right) - gt}{\frac{u}{2}} = \tan \frac{\pi}{4} = 1$$

$$gt = \frac{u}{2} (\sqrt{3} - 1)$$

$$\Rightarrow t = \frac{u}{g} \left(\frac{\sqrt{3} - 1}{2} \right)$$

Additional Practice Exercise

$$121. \quad H = \frac{1}{2} R \Rightarrow \frac{u^2 \sin^2 \theta}{2g} = \frac{1}{2} \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \sin \theta = 2 \cos \theta$$

$$\Rightarrow \tan \theta = 2, \theta = \tan^{-1} 2$$

Aliter:

$$R \tan \theta = 4 H_{\max}$$

$$\therefore \tan \theta = \frac{4 H_{\max}}{R} = \frac{4}{R} \times \frac{R}{2} = 2$$

$$\theta = \tan^{-1} 2$$

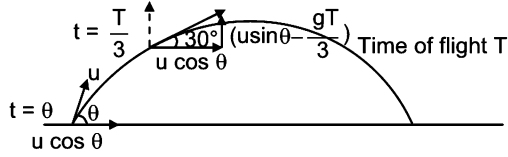
$$122. \quad \text{Time of flight } T = \frac{2u \sin \theta}{g} = 2s \Rightarrow u \sin \theta = 10$$

Maximum height

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{10^2}{2 \times 10} = 5 \text{ m}$$

129. $a_r = g \cos \theta$, is minimum (> 0) $x = 0$ and $x = R$ and maximum at $x = \frac{R}{2}$ and symmetric

130.



$$\text{given: } \frac{u \sin \theta - \frac{gT}{3}}{u \cos \theta} = \tan 30^\circ \quad \text{--- (1)}$$

$$T = \frac{2u \sin \theta}{g}$$

$$\therefore u \sin \theta = \frac{gT}{2} \quad \text{--- (2)}$$

substitute for gT in (1)

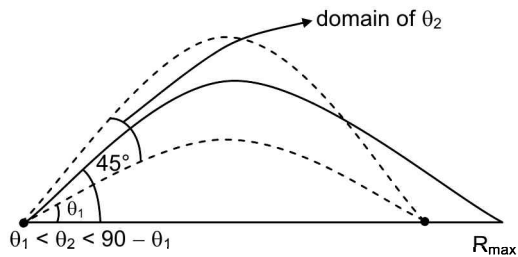
$$\Rightarrow \frac{u \sin \theta - \frac{2u \sin \theta}{3}}{u \cos \theta} = \tan 30^\circ$$

$$\Rightarrow \tan \theta = 3 \tan 30^\circ$$

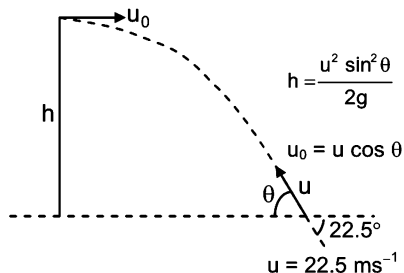
$$= \sqrt{3} \Rightarrow \theta = 60^\circ$$

$\therefore$ the other angle for same range is 30°

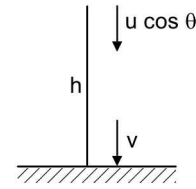
131.



132. Assume reverse process: projectile reaching maximum height: then:



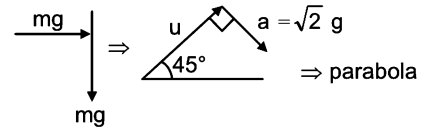
When projected down with same initial velocity u_0 .



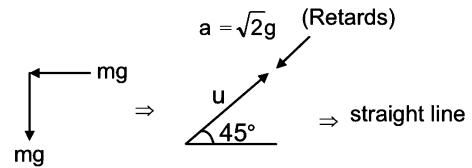
$$v^2 = u^2 \cos^2 \theta + 2gh$$

$$= u^2 \cos^2 \theta + u^2 \sin^2 \theta = u^2$$

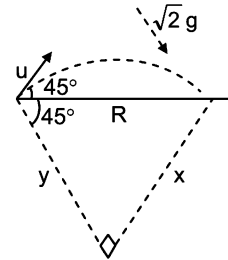
133. Direction of wind not known. Two possibilities



or



134. Among the two possibilities,
Case 2: comes back to initial point $\Rightarrow$ Range = 0
Case 1:



$$y = \frac{1}{2} \sqrt{2} g t^2 = x = u t$$

$$\Rightarrow t = \frac{\sqrt{2} u}{g} \Rightarrow x = y = \frac{\sqrt{2} u^2}{g}$$

$$\Rightarrow R = \sqrt{2} x = \frac{2u^2}{g} = 2 \cdot R_{\max}$$

(aimed range = $R_{\max}$)

Aliter:

No change in time of flight because of wind acting horizontally.

3.76 Motion in Two Dimensions

$$T = 2 \left(\frac{u}{\sqrt{2}} \right) \cdot \frac{1}{g} = \frac{\sqrt{2}u}{g}$$

$R = u_x T \mp \frac{1}{2} g T^2$ depending on whether g' the acceleration due to wind is acting along horizontal component of u on apposing it. $|g'| = g = 10 \text{ m s}^{-2}$, or zero.

$$= \frac{u}{\sqrt{2}} \cdot \frac{\sqrt{2}u}{g} + \frac{1}{2} \cdot g \left(\frac{\sqrt{2}u}{g} \right)^2 = \frac{2u^2}{g}$$

135. Consider only vertical component of velocities

At start: u_0

At $\frac{H}{2}$, before bird hit u_1 :

Hence $u_1^2 = 2g \frac{H}{2}$ (since originally it will travel another $\frac{H}{2}$, if there was no bird hit) $\Rightarrow u_1^2 = gH$

After bird hit: $u_1' = \frac{u_1}{2}$ [Since velocity is halved both horizontal and vertical components are halved].

$$\therefore \frac{u_1'^2}{4} = 2gh \Rightarrow \frac{gH}{4} = 2gh \Rightarrow h = \frac{H}{8}$$

$$H_{\text{total}} = H + h = \frac{H}{2} + \frac{H}{8} = \frac{5H}{8}$$

136. $R = 4H \cot \theta$

$$R = 4H' \cot (90 - \theta) = 4H' \tan \theta$$

$\Rightarrow \tan^2 \theta = \frac{H}{H'} = \frac{3}{4}$ or $\frac{4}{3} \Rightarrow$ since the angles are complementary: $\tan \theta_1 = \cot \theta_2$

$$\Rightarrow \tan \theta_1 = \sqrt{\frac{4}{3}}; \tan \theta_2 = \sqrt{\frac{3}{4}}$$

$$\begin{aligned} \tan(\theta_1 - \theta_2) &= \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2} = \frac{\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}}}{1 + 1} \\ &= \frac{4 - 3}{2\sqrt{12}} = \frac{1}{4\sqrt{3}} \end{aligned}$$

$$137. \frac{H}{T} = \frac{u^2 \sin^2 \theta / 2g}{2u \sin \theta / g} \propto u \sin \theta$$

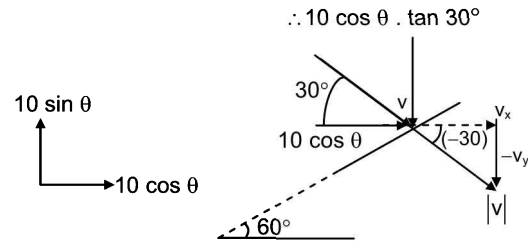
$$\frac{H_A}{T_A} < \frac{H_B}{T_B} \left[\because \sin 30^\circ > \sin 60^\circ \right]$$

$$138. u_y = \sqrt{3^2 + 2g \cdot 1} = \sqrt{29} \text{ m s}^{-1}$$

(using $v^2 = u^2 + 2gh$); $u_x = 2 \text{ m s}^{-1}$

$$\theta = \tan^{-1} \frac{\sqrt{29}}{2}$$

139.



When the projectile hits the inclined plane normally, the velocity vector will be as shown, hence $\bar{v}_y$ is negative

$$\therefore \frac{v_y}{v_x} = \tan(-30^\circ) = -\frac{1}{\sqrt{3}}$$

$$\begin{aligned} v_y &= 10 \sin \theta - gt \\ &= 10 \sin \theta - 10 \frac{2}{\sqrt{3}}; v_x = 10 \cos \theta \end{aligned}$$

$$\frac{10 \sin \theta - 10 \times \frac{2}{\sqrt{3}}}{10 \cos \theta} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \sin \theta - \frac{2}{\sqrt{3}} = -\frac{\cos \theta}{\sqrt{3}}$$

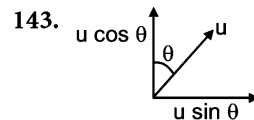
$$\frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta = 1 \Rightarrow \sin(\theta + 30^\circ) = 1$$

by inspection $\theta = 60^\circ$

$$\begin{aligned} 140. t_1 t_2 &= \frac{2u \sin \theta}{g} \cdot \frac{2u \cos \theta}{g} \\ &= \frac{2}{g} \frac{u^2 \sin 2\theta}{g} = \frac{2}{g} R \end{aligned}$$

$$141. R = 4H \cot \theta \Rightarrow \frac{H}{R/2} = \frac{\tan \theta}{2} = \frac{3}{8}$$

142. Time of flight depends on height only, which is same for both particles.



$$v(t) = \sqrt{(u \cos \theta - gt)^2 + (u \sin \theta)^2}$$

144. $R_{\max}$ on horizontal = $\frac{u^2}{g}$

R on inclined plane

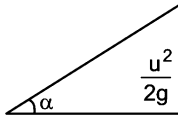
$$= \frac{u^2}{g \cos^2 \alpha} [\sin(2\theta - \alpha) - \sin \alpha]$$

$$= \frac{R_{\max}}{2} = \frac{u^2}{2g}$$

$$\therefore \frac{1}{2} = \frac{1}{\cos^2 \alpha} [\sin(2 \times 45 - \alpha) - \sin \alpha]$$

$$\Rightarrow \cos^2 \alpha = 2(\cos \alpha - \sin \alpha)$$

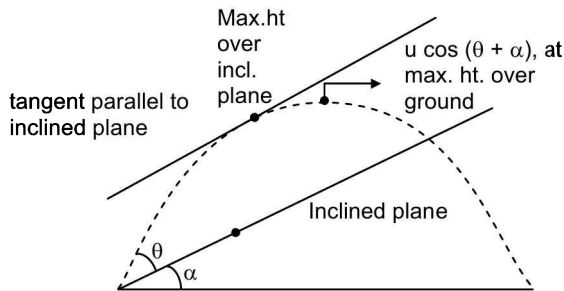
Aliter:



$$\frac{u^2}{2g} \cdot \sin \alpha = \frac{u^2}{2g} \cos \alpha \cdot 1 - \frac{g \left(\frac{u^2}{2g} \cos \alpha \right)^2}{2 \cdot u^2 \left(\frac{1}{2} \right)}$$

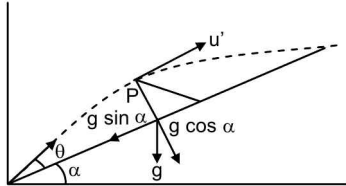
$$\Rightarrow \cos^2 \alpha = 2(\cos \alpha - \sin \alpha)$$

145.



Clearly option (c)

146.



At that point, horizontal component of velocity is $u \cos(\theta + \alpha)$

At P, when it is at maximum height above inclined plane, its velocity normal to the inclined plane is zero.

$$u \sin \theta - g \cos \alpha \cdot t = 0 \Rightarrow t = \frac{u \sin \theta}{g \cos \alpha}$$

At P velocity parallel to the plane, (which is the only velocity) is:

$$u \cos \theta - g \sin \alpha \cdot t = u \cos \theta - g \sin \alpha \cdot u \frac{\sin \theta}{g \cos \alpha}$$

$$= u(\cos \theta - \sin \theta \tan \alpha) < u \cos \theta$$

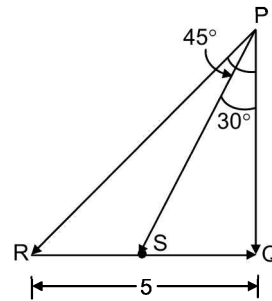
$$[\text{Actual } v_p = u \frac{(\cos \theta \cos \alpha - \sin \theta \sin \alpha)}{\cos \alpha}]$$

$$= \frac{u \cos(\theta + \alpha)}{\cos \alpha}]$$

Aliter:

Since $(g \sin \alpha)$ retardation happens up to P on the velocity components $u \cos \theta$ it is obviously less than $u \cos \theta$.

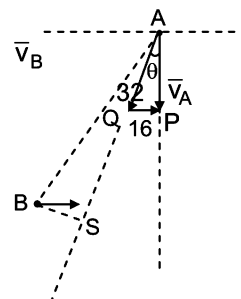
147.



Let $\overline{PQ}$ represent velocity of rain drops falling vertically down and $\overline{RQ}$ represent the man's initial velocity. Resultant relative velocity of rain $\overline{PR} = \overline{PQ} - \overline{RQ}$. Given $\angle RPQ = 45^\circ$ and $\angle PQR = 90^\circ$. Hence $\triangle PRQ$ is a right angled isosceles triangle $\Rightarrow PQ = RQ = 5 \text{ ms}^{-1}$. Let $\overline{SQ}$ represent the reduced velocity so that resultant $PS = PQ - SQ$. Given $\angle QPS = 30^\circ$. $\Rightarrow$

$$\frac{SQ}{PQ} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow SQ = \frac{PQ}{\sqrt{3}} = \frac{5}{\sqrt{3}} \text{ m s}^{-1}$$

148.



3.78 Motion in Two Dimensions

Let $\overline{AP}$ represent $\vec{v}_A$ (due South) and $\overline{QP}$ represent $\vec{v}_B$ (due East, 16 kmph)

$\overline{AQ}$ is the relative velocity $v_{r(A/B)}$ $|\overline{AQ}| = 32$ kmph

$$\overline{AQ} = \overline{AP} - \overline{QP} = \overline{AP} + \overline{PQ}$$

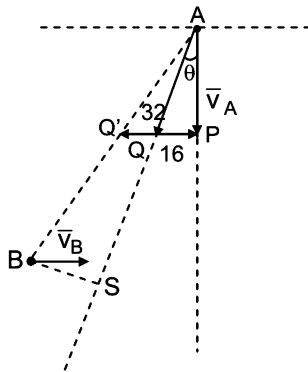
$$\text{From } \triangle AQP \Rightarrow \sin \theta = \frac{QP}{AQ} = \frac{16}{32} = \frac{1}{2}$$

$$\theta = 30^\circ \Rightarrow \angle PAB = 45^\circ \text{ (given data)}$$

$\therefore \angle BAQ = (45 - 30) = 15^\circ$. Draw BS normal to the relative velocity $\overline{AQ}$ extended. BS is the distance of closest approach.

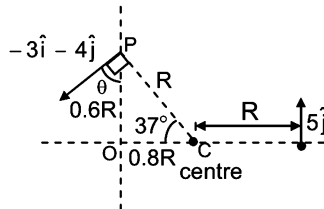
$$BS = AB \sin 15^\circ = 100 \sin 15^\circ$$

149.



From the previous problem : $v_A = 32 \cos 30^\circ = 32 \cdot \frac{\sqrt{3}}{2} = 16\sqrt{3}$ kmph. Let $\overline{Q'P}$ represent the new velocity $\vec{v}_B$. Resultant velocity $\overline{AQ'}$ should be along AB for the two ships to meet each other. $\triangle APQ'$ is right angled isosceles triangle. $\therefore PQ' = AP = 16\sqrt{3}$. $\overline{Q'P}$ is the required velocity

150.



key (a)

At any instant radius vector is perpendicular to velocity vector. At P velocity is as shown

$$\Rightarrow \tan \theta = \frac{3}{4} \Rightarrow \theta = 37^\circ$$

$$\Rightarrow OC = R \cos 37 = 0.8 R$$

$$1.8 R = 9 \text{ m} \Rightarrow R = 5 \text{ m}$$

$$\therefore \frac{v^2}{R} = \frac{25}{5} = 5 \text{ ms}^{-2}$$

151. Horizontal velocity of both projectiles are the same. So if they have to meet, P_1 should have been vertically above P_2 when P_2 is projected.

$$\Rightarrow \text{distance} = 0$$

$$152. \vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a} = -10\hat{k}$$

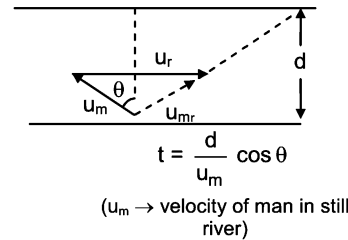
$$\begin{aligned} \vec{r} &= \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2 \\ &= 3\hat{i} + 2\hat{j} + 2\hat{k} + (\hat{i} + 2\hat{j} + 3\hat{k})t - 5t^2\hat{k} \end{aligned}$$

For ground, $\hat{k}$ component = 0

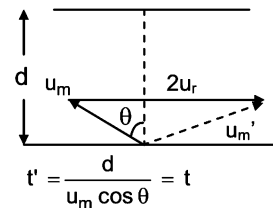
$$\Rightarrow 2 + 3t - 5t^2 = 0 \Rightarrow t = 1 \text{ s}$$

$$\Rightarrow \vec{r} = 4\hat{i} + 4\hat{j} \text{ (m)}$$

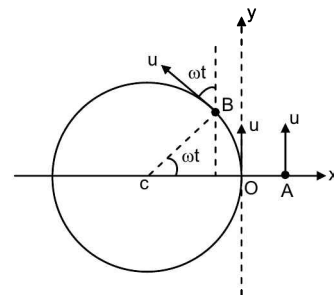
153.



That day:



154.



Note that $v_{rel} = 0$ at $t = 0$

$\Rightarrow$ speed same and at $t = 0$ they are parallel.

Conveniently assume

$$\vec{u}_A = u\hat{j}$$

At t ,

$$\vec{u}_B = u \sin \omega t (-\hat{i}) + u \cos \omega t (\hat{j})$$

$$v_{rel} = |\vec{u}_A - \vec{u}_B| = u \sqrt{\sin^2 \omega t + (1 - \cos \omega t)^2}$$

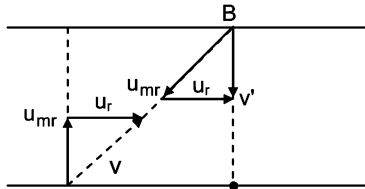
$$= u \sqrt{2 - 2 \cos \omega t}$$

$$= 2u \sin \frac{\omega t}{2} = 11 \sin 11t \text{ (given)}$$

$$\Rightarrow u = 5.5 \text{ m s}^{-1}, \omega = 22 \text{ rad s}^{-1}$$

$$\Rightarrow R = \frac{u}{\omega} = \frac{5.5}{22} = 0.25 \text{ m}$$

155.



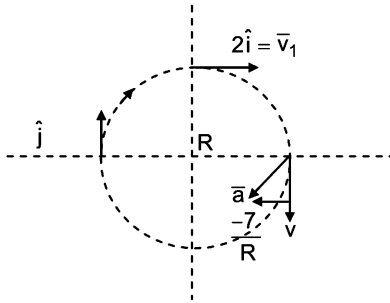
(in A
shortest time)

Key (a)

$$v^2 = u_{mr}^2 + u_r^2$$

$$v'^2 = u_{mr}^2 - u_r^2 < v^2$$

156.



$$\text{Using } v_1^2 - u^2 = 2as \Rightarrow 2^2 - 1^2 = 2 \cdot a_t \cdot \frac{\pi}{2} R$$

$$\Rightarrow a_t = \frac{3}{\pi R}$$

$$v^2 - v_1^2 = 2a_t s \rightarrow v^2 = v_1^2 + 2a_t s$$

$$= 2^2 + 2 \times \frac{3}{\pi R} \cdot \frac{\pi}{2} R = 7$$

$$v = \sqrt{7} \text{ m s}^{-1}$$

$$a_r = \frac{v^2}{R} = \frac{7}{R}$$

$$\vec{a} = \frac{7}{R} (-\hat{i}) + \frac{3}{\pi R} (-\hat{j})$$

$$\text{Angle required} = \tan^{-1} \left(\frac{7/R}{3/\pi R} \right) = \tan^{-1} \left(\frac{7\pi}{3} \right)$$

$$157. a_t = \frac{8-5}{0.5} = 6 \text{ m s}^{-2}$$

$$a_r = \frac{8^2}{8} = 8 \text{ m s}^{-2}$$

$$a = \sqrt{a_r^2 + a_t^2} = 10 \text{ m s}^{-2}$$

key (a)

$$158. \vec{v} = t\hat{i} + t^2\hat{j} = \hat{i} + \hat{j} \Big|_{\text{at } t=1}$$

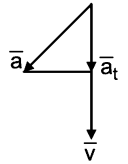
$$= \sqrt{2} \text{ m s}^{-1}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \hat{i} + 2t\hat{j} = \hat{i} + 2\hat{j} \Big|_{\text{at } t=1}$$

$$= \sqrt{5} \text{ m s}^{-2} \text{ at } t=1 \text{ s}$$

$$\vec{a}_t \Big|_{\text{at } t=1} = \frac{\vec{a} \cdot \vec{v}}{v} = \frac{(\hat{i} + 2\hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ m s}^{-1}$$

$$a_r \Big|_{\text{at } t=1} = \sqrt{a^2 - a_t^2} = \sqrt{5 - \frac{9}{2}} = \frac{1}{\sqrt{2}} \text{ m s}^{-2}$$



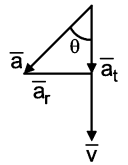
$$159. a_r = \omega^2 r$$

$$a_t = r\alpha$$

$$\tan \theta = \frac{a_r}{a_t} = \frac{\omega^2}{\alpha}$$

$$\omega^2 = 2\alpha \theta$$

$$\frac{\omega^2}{\alpha} = 2\theta = \tan \theta$$



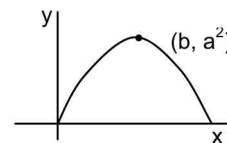
160. $\vec{a}_r$ must be towards the centre of the circle.

$\rightarrow$ (c) wrong

$\vec{a}_T$ must be opposite to $\vec{v} \rightarrow$ (a), (d) wrong.

$\therefore$ correct answer (b)

161.



$$y = a^2 - (x - b)^2$$

represents a parabola when $a = b$,

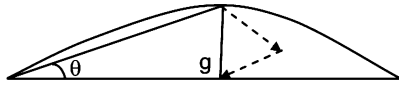
$$y = 2ax - x^2$$

3.80 Motion in Two Dimensions

statement 1 is wrong,

statement 2 is correct but not an explanation for 1.

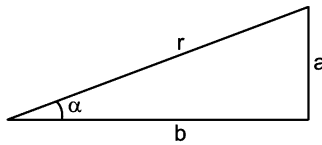
162.



$$163. (gy) = (gx) \tan \theta - \frac{1}{2} \frac{(gx)^2}{u^2 \cos^2 \theta}$$

Both x and y are scaled by the same factor, g
 $\Rightarrow$ shape is the same

164.



$$\frac{u^2}{g(1 + \sin \alpha)} = r$$

$$u^2 = g(r + r \sin \alpha) = g(r + a)$$

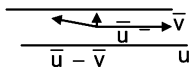
$$\left(\frac{u_1}{u_2}\right)^2 = \left(\frac{13 + 5}{15 + 9}\right) = \frac{18}{24} \Rightarrow \frac{u_1}{u_2} = \frac{\sqrt{3}}{2}$$

165. Statement (1) is correct because for small α , the upper point of projection could be higher. Statement (2) is also correct but not a reason for statement (1).

Key (b)

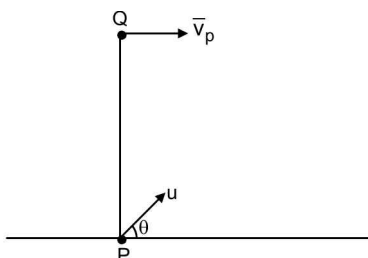
166. The distance from the plane is determined by the initial velocity perpendicular to the plane.

167. Statement 2 is correct. The time to cross by the shortest distance is $t = \frac{d}{\sqrt{v^2 - u^2}}$ where u is the water velocity.



Statement 1 is wrong because for crossing across (the shortest distance) $v > u$

168.



If the canon at P on ground fires at the aeroplane exactly above it (ie. at Q) and manages to hit it, obviously their relative horizontal velocity is zero. $u \cos \theta = v_p$. $\therefore$ As observed from the plane, the velocity of the canon is $\hat{i} (u \cos \theta - v_p) + \hat{j} (u \sin \theta - \frac{1}{2} gt^2) = \hat{j} (u \sin \theta - \frac{1}{2} gt^2)$. Hence its motion is in the vertical direction only with varying vertical velocity.

$$169. |\bar{a}_r| = \frac{v^2}{r} \text{ and } \bar{a}_t = \frac{dv}{dt}$$

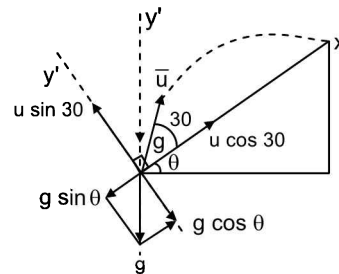
both depend on v. Therefore, statement 1 is wrong. Statement 2 is correct.

$$170. \omega = \omega_0 + \alpha t$$

$$\text{and } a_r = \omega^2 r,$$

ω may initially increase or decrease, but eventually magnitude becomes large.

171.



With respect to inclined plane as x' axis.

$$T = \frac{2u \sin 30^\circ}{g \cos 30^\circ} = \frac{2 \times 10 \times 1/2}{10 \times \sqrt{3}/2} = \frac{2}{\sqrt{3}} \text{ s}$$

$$172. v_x^1 = u \cos 30^\circ - g \sin 30^\circ t$$

$$= \frac{10\sqrt{3}}{2} - 10 \times \frac{1}{2} \frac{2}{\sqrt{3}} = 5\sqrt{3} - \frac{10}{\sqrt{3}}$$

$$= \frac{5}{\sqrt{3}} \text{ m s}^{-1} \text{ (along the inclined plane)}$$

$$v_y^1 = u \sin 30^\circ - g \cos 30^\circ t$$

$$= 10 \times \frac{1}{2} - 10 \times \frac{\sqrt{3}}{2} \frac{2}{\sqrt{3}}$$

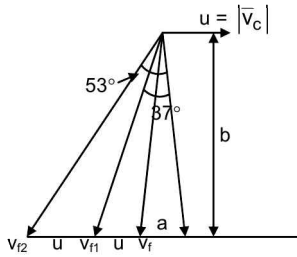
$$= -5 \text{ m s}^{-1} \text{ (normal to the inclined plane in to the plane)}$$

$$v = \sqrt{v_x^2 + v_y^2} = 5\sqrt{\frac{1}{3} + 1} = \frac{10}{\sqrt{3}} \text{ m s}^{-1}$$

$$173. \tan \alpha = \frac{v_y^1}{v_x^1} = \sqrt{3}$$

$\alpha = 60^\circ$ with the inclined plane.

174.



Let the condition be as in option a

 Let a be the horizontal component of rain velocity $\bar{v}_r$

 b $\rightarrow$ vertical component of rain velocity $\bar{v}_r$

 u $\rightarrow$ velocity of cyclist (modular value)

 $\bar{v}_f \rightarrow$ final relative velocity (i.e. $\bar{v}_{f1}$ and $\bar{v}_{f2}$)

Taking modulus values, for trigonometric equations:

Case 1:

$$\frac{a+u}{b} = \tan 37^\circ = \frac{3}{4} \Rightarrow 4a + 4u = 3b \quad (1)$$

Case 2:

$$\frac{a+2u}{b} = \tan 53^\circ = \frac{4}{3} \Rightarrow 3a + 6u = 4b \quad (2)$$

 Solving (1) & (2) $\Rightarrow 4 \times (1) - 3 \times (2) \Rightarrow$

$$16a + 16u - 9a - 18u = 12b - 12b = 0$$

$$7a = 2u \Rightarrow a = \frac{2}{7}u; b = \frac{12u}{7}$$

Note:

If you assume condition, as in option (b) and solve, you will get -ve value for a; implying the vector has to be in the opposite direction of the chosen direction ie, as in case (a)

175. key (b)

$$\tan^{-1} \frac{a}{b} = \tan^{-1} \frac{2}{12} = \frac{1}{6}$$

176.

$$v_r = \sqrt{a^2 + b^2} = \frac{2}{7}u\sqrt{1^2 + 6^2}$$

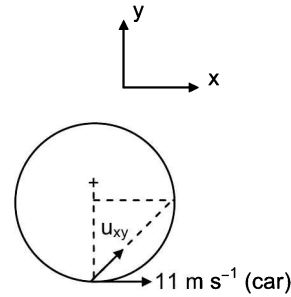
$$\Rightarrow \therefore \text{ratio } \frac{v_r}{u} = \frac{2\sqrt{37}}{7}$$

$$177. \frac{\pi}{2} \text{ in } 3 \text{ s} \Rightarrow \omega = \frac{\pi}{6} = \frac{v}{R} = \frac{11}{R}$$

$$\Rightarrow R = \frac{66}{\pi} = \frac{66}{22/7} = 21 \text{ m}$$

$$\Rightarrow \text{acceleration} = \frac{v^2}{R} = \frac{121}{21} = 6 \text{ ms}^{-2}$$

178. xy plane



$$\text{Stone: } u_2 = g \frac{t}{2} = 10 \cdot \frac{3}{2} = 15 \text{ m s}^{-1}$$

$$u_{xy} = \frac{\sqrt{2}R}{t} = 7\sqrt{2}$$

$$\Rightarrow \bar{u}_{xy} = 7\hat{i} + 7\hat{j} \text{ m s}^{-1}$$

Relative to car

$$\Rightarrow \bar{u}_{vel} = 7\hat{i} + 7\hat{j} + 15\hat{k} - 11\hat{i}$$

$$= -4\hat{i} + 7\hat{j} + 15\hat{k}$$

$$u_{rel} = \sqrt{4^2 + 7^2 + 15^2} = \sqrt{290} \approx \sqrt{289} = 17 \text{ m s}^{-1}$$

 179. Velocity of stone at hitting = $7\hat{i} + 7\hat{j} - 15\hat{k}$

$$\theta \text{ to vertical} = \tan^{-1} \frac{7\sqrt{2}}{15} \approx \tan^{-1} \frac{1}{\sqrt{2}} > \tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$$

180. key (a), (b)

181. key (a), (b), (c)

182. key (a), (b), (c)

183. Maximum height and time of flight depends on vertical components of initial velocity

$$H_1 = H_2$$

$$\Rightarrow u_{y1} = u_{y2}$$

$$T_1 = T_2$$

$$\text{Range } R = \frac{u^2 \sin 2\theta}{g} = \frac{2(u \sin \theta)(u \cos \theta)}{g}$$

$$= \frac{2u_x u_y}{g}$$

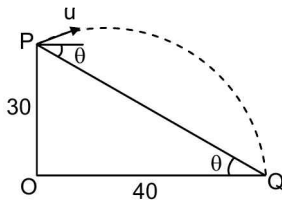
$$R_2 > R_1 \quad u_{x2} > u_{x1}$$

$$u_2 > u_1$$

Key (b), (c)

3.82 Motion in Two Dimensions

184.



(a), (c)

$$\tan \theta = \frac{30}{40} \Rightarrow \theta = 37^\circ$$

This is equal to a projectile for maximum range on an inclined plane, inclined 37° down

$$\text{Maximum range } R = PQ = \sqrt{30^2 + 40^2} = 50 \text{ m}$$

$$\Rightarrow \frac{u^2}{g(1 - \sin 37^\circ)} = 50$$

$$\frac{u^2}{g} = 50 \times 0.4 = 20 \Rightarrow u = \sqrt{200} = 10\sqrt{2} \text{ m s}^{-1}$$

For an inclined plane inclined upward 37°

$$R_{\max} = \frac{u^2}{g(1 + \sin \beta)} = \frac{20}{1.6} = 12.5 \text{ m}$$

Angle of projection for maximum range

$$= 45^\circ + \frac{\theta}{2} = 45^\circ + \frac{37^\circ}{2} = 63.5^\circ$$

185. If we consider equal intervals 2 s, there are 4 such intervals in 8 s. Hence the rotations will be in the ratio 7n, 5n, 3n & n, where n is the rotations made in the

$$\text{last 2 s} \Rightarrow \text{For the last 2 s} \Rightarrow \frac{1}{2} \alpha t^2 = \frac{1}{2} \times \alpha \times 4 = n \times 2\pi$$

$$2\alpha = \frac{112}{7} \times 2\pi [\because 7n = 112] \Rightarrow \alpha = 16\pi \text{ rad s}^{-2}$$

$$\omega = \alpha t = 16\pi \times 8 = 128\pi \Rightarrow n = \frac{\omega}{2\pi} = 64 \text{ revolution s}^{-1}$$

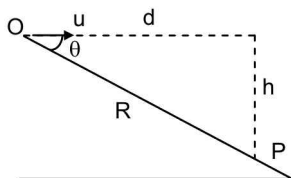
$$\text{Total angle turned } \theta = \frac{1}{2} \alpha t^2 = \frac{1}{2} \times 16\pi \times 64$$

$$= 512\pi \text{ radian}$$

$$n = \frac{512\pi}{2\pi} = 256 \text{ revolution}$$

key (a), (b)

186.



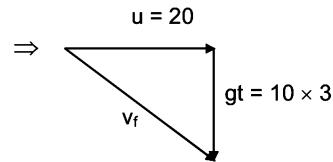
$$\tan \theta = \frac{h}{d} \Rightarrow h = \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times 9 = 45 \text{ m}$$

$$d = u \times t = 20 \times 3 = 60 \text{ m}$$

$$\tan \theta = \frac{45}{60} = \frac{3}{4} \Rightarrow \theta = 37^\circ$$

$$\text{Range } R = OP = \sqrt{45^2 + 60^2} = 75 \text{ m}$$

Final velocity when it reaches the inclined plane



$$\Rightarrow v_f = \sqrt{(20)^2 + (30)^2} = 36 \text{ m s}^{-1}$$

key (a), (d)

187. To find out the difference in distance after 3 s

Note:

At the instant Q is projected, velocity of P, $v_p = u - gt = 100 - 10 \times 1 = 90 \text{ ms}^{-1}$. Hence their relative velocity is zero. The distance of separation between P and Q remains constant through out their flight and is equal to the distance travelled by P in its first second of motion)

$$u_1 = 100 \text{ m s}^{-1}$$

At 3s after launch of P:

$$s_1 = 100 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = 300 - 45 = 255 \text{ m}$$

In the second case

$$u_2 = 90 \text{ m s}^{-1}$$

$$s_2 = 90 \times 2 - \frac{1}{2} \times 10 \times 4 = 180 - 20 = 160 \text{ m}$$

The difference in distance = $255 - 160 = 95 \text{ m}$

(c) and (d) are the wrong statements. ($\because$ both are at their highest positions at the same time)

188. For up the plane: Range

$$= \frac{2u_1^2}{g} \cos 60^\circ \frac{\sin(60^\circ - 30^\circ)}{\cos^2 30^\circ} = OB \quad \text{--- (1)}$$

For down the plane: Range

$$= \frac{2u_2^2}{g} \cos 60^\circ \frac{\sin(60^\circ + 30^\circ)}{\cos^2 30^\circ} = OB \quad \text{--- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{u_1^2}{u_2^2} \sin 30^\circ = 1 \Rightarrow \frac{u_1}{u_2} = \sqrt{2}$$

$$t_1 = \frac{2u_1 \sin(60^\circ - 30^\circ)}{g \cos 30^\circ}; t_2 = \frac{2u_2 \sin(60^\circ + 30^\circ)}{g \cos 30^\circ}$$

$$\frac{t_1}{t_2} = \frac{u_1}{u_2} \sin 30^\circ = \sqrt{2} \cdot \frac{1}{2} = \frac{1}{\sqrt{2}}$$

Acceleration normal to the plane = $g \cos \alpha$

$$= 10 \cos 30^\circ = 10 \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ m s}^{-2}$$

Modulus value of acceleration parallel to the plane = $g \sin \alpha$

$$= 10 \sin 30^\circ = 10 \times \frac{1}{2} = 5 \text{ m s}^{-2}$$

189. Maximum height $H = \frac{1}{2}g\left(\frac{T}{2}\right)^2 = 5 \times (2)^2 = 20 \text{ m}$, same for both.

They move in deceleration during half the flight time, and in acceleration in the next half. Minimum velocity is zero for vertical projectile and non-zero for the other (i.e., $u \cos \alpha$)

190. (a) $\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{R}) \Rightarrow \omega^2 R = \frac{v^2 R}{R^2}$
 $= \frac{v^2}{R}$ and $\vec{a}_T = 0$

$\therefore (a) \rightarrow p, q$

(b) $\vec{a} = \vec{\alpha} \times \vec{R} \Rightarrow$ since total acceleration is only $a = \alpha R$, $a_R = 0$, which implies $v = 0$ $\therefore v = 0$ and $a_R = \frac{v^2}{R} = 0$ and $a = \frac{dv}{dt}$

$\therefore (b) \rightarrow q, s$

(c) $|\vec{a}| = \sqrt{2}|\vec{a}_R| \Rightarrow \vec{a}_T$ and $\vec{a}_R$ are equal and perpendicular

$\therefore (c) \rightarrow q, r$

(d) $\vec{a} = (\vec{\omega} \times \vec{v}) = \vec{a}_R \Rightarrow \vec{a}_R = \frac{v^2}{R}$. Same as (a)
 $\therefore (d) \rightarrow p, q$

191. $4 = 20 \sin \theta (0.4) - (1/2) \times 10 \times (0.4)^2$

$$(\therefore y = u_y t - \frac{1}{2}gt^2)$$

$$\Rightarrow \sin \theta = \frac{3}{5}; \cos \theta = \frac{4}{5}$$

$\therefore$ Horizontal distance of first wall from point of projection = $20 \times \cos \theta (0.4)$

$$= 20 \times (4/5) \times (0.4) = 6.4 \text{ m}$$

$\therefore$ Horizontal distance of second wall from point of projection = $6.4 + 25.6 = 32 \text{ m}$

$$\text{Using } y = x \tan \theta - \frac{1}{2}g \frac{x^2}{u^2 \cos^2 \theta}$$

$$y = 32 \times \frac{3}{4} - \frac{1}{2} \times 10 \times \frac{32^2}{20^2 \times \frac{16}{25}} = 4 \text{ m}$$

Since the wall is 5 m high, the stone will not clear the wall.

192. $R = \frac{u^2}{g} \times \frac{1}{1 + \sin \alpha}$

$$50 = \frac{u^2}{10} \times \frac{1}{1 + 0.8}$$

(i) $u^2 = 18 \times 50 = 900$

$$u = 30 \text{ m s}^{-1}$$

(ii) Angle = $45^\circ + \frac{\alpha}{2} = 45^\circ + \frac{53^\circ}{2}$
 $= 71.5^\circ = 71^\circ 30'$

193. (i) $R = 2h \tan \alpha$

(ii) $h = \frac{u^2 \sin^2 \theta}{2g}$

$$R = \frac{u^2 \sin 2\theta}{g} = 2h \tan \alpha$$

$$\Rightarrow \frac{u^2 2 \sin \theta \cos \theta}{g} = \frac{2u^2 \sin^2 \theta}{2g} \tan \alpha$$

$$\Rightarrow \tan \alpha = 2 \cot \theta$$

$$\Rightarrow \tan \theta = 2 \cot \alpha$$

$$\therefore \theta = \tan^{-1}(2 \cot \alpha)$$

Aliter:

$$R \tan \theta = 4h \rightarrow \tan \theta = \frac{4h}{R}$$

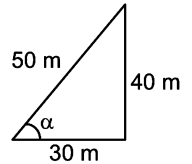
$$= \frac{4h}{2h \tan \alpha} = 2 \cot \alpha$$

(iii) $h = \frac{u^2 \sin^2 \theta}{2g}$

$$u^2 = \frac{2gh}{\sin^2 \theta} = \frac{2gh}{1 - \cos^2 \theta}$$

$$\left(\because \cos^2 \theta = \frac{1}{\sec^2 \theta} = \frac{1}{\tan^2 \theta + 1} = \frac{1}{4 \cot^2 \alpha + 1} \right)$$

$$= \frac{2gh}{1 - \frac{1}{1 + 4 \cot^2 \alpha}} = \frac{(1 + 4 \cot^2 \alpha) 2gh}{4 \cot^2 \alpha}$$

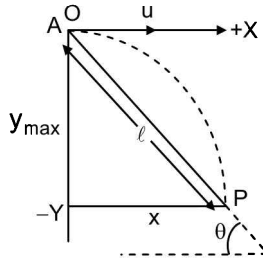


3.84 Motion in Two Dimensions

$$u^2 = 2gh \left(1 + \frac{\tan^2 \alpha}{4} \right)$$

$$\Rightarrow u = \sqrt{2gh \left(1 + \frac{\tan^2 \alpha}{4} \right)}$$

194.



(i) $y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$ (Taking the point of projection as origin, +X = horizontal direction of motion and -Y = vertical downward direction)

$\alpha = 0 \Rightarrow y = \frac{-gx^2}{2u^2} = \frac{-10x^2}{2 \times 25} = \frac{-x^2}{125} \text{ m}$

(ii) $y = -x$

(iii) $-x = \frac{-gx^2}{2u^2}$

$$\Rightarrow x = \frac{2u^2}{g} = \frac{2 \times 25^2}{10} = 125 \text{ m}$$

$$y = -x = \frac{-2u^2}{g}$$

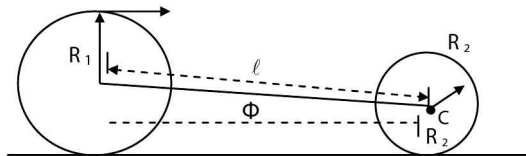
$\therefore P = (125 \text{ m}, -125 \text{ m})$

Distance = $\sqrt{x^2 + y^2}$

$$= \frac{2u^2}{g} \sqrt{2}$$

$$= \frac{2\sqrt{2}}{10} \left(90 \times \frac{5}{18} \right)^2 = 176.8 \text{ m}$$

195.



$$\sin \phi = \frac{R_1 - R_2}{l};$$

$$\cos \phi = \sqrt{1 - \sin^2 \phi} = \frac{\sqrt{l^2 - (R_1 - R_2)^2}}{l}$$

$$H = 2(R_1 - R_2)$$

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 2(R_1 - R_2)}{g}} = 2\sqrt{\frac{R_1 - R_2}{g}}$$

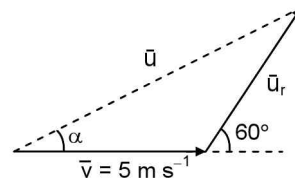
speed relative to cart = $2v - v = v$

Relative horizontal distance = $l \cos \phi$

$$t = \frac{l \cos \phi}{v} \Rightarrow v = \frac{l \cos \phi}{t} = \frac{\sqrt{l^2 - (R_1 - R_2)^2}}{2\sqrt{(R_1 - R_2)}} \times \sqrt{g}$$

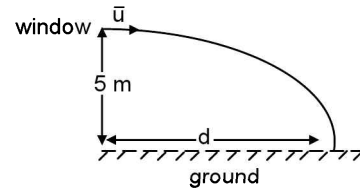
$$\therefore v = \sqrt{\frac{[l^2 - (R_1 - R_2)^2]g}{4(R_1 - R_2)}}$$

196.



Let $\bar{v}$ be velocity of train and $\bar{u}_r$ be velocity with which passenger throws the object.

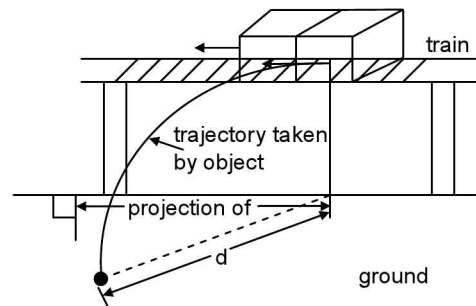
Then $\bar{u}$ the initial velocity of object with respect to ground is given by $\bar{u} = \bar{u}_r + \bar{v}$

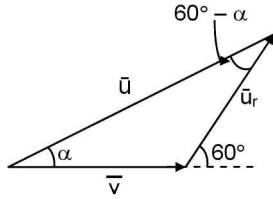


Now consider the plane of trajectory of the object:

$$\therefore \text{time to land on ground} = \sqrt{\frac{2 \times 5}{g}} = 1 \text{ s}$$

$$\therefore \text{distance } d = u \cdot t = u \text{ (i.e., } d = u)$$





The train takes 1.5 s to travel the projection of $d = d \cos \alpha$

$$\therefore d \cos \alpha = 1.5 \times v$$

$$\therefore \cos \alpha = \frac{1.5v}{d} = \frac{1.5v}{u} \Rightarrow u = \frac{1.5}{\cos \alpha} v \quad (\because d = u)$$

$$\frac{u}{\sin 120^\circ} = \frac{v}{\sin(60^\circ - \alpha)} = \frac{u_r}{\sin \alpha} \quad (\text{From property of triangles})$$

$$\Rightarrow \frac{1.5v}{\cos \alpha \sin 120^\circ} = \frac{v}{\sin(60^\circ - \alpha)}$$

$$\Rightarrow \frac{\sin(60^\circ - \alpha)}{\cos \alpha} = \frac{\sqrt{3}}{2 \cdot (1.5)} = \frac{1}{\sqrt{3}}$$

[using $\sin(A - B) = \sin A \cos B - \cos A \sin B$]

$$\Rightarrow \frac{\sqrt{3}}{2} - \frac{\tan \alpha}{2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \alpha = \sqrt{3} - \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^\circ$$

$$\therefore u = \frac{1.5v}{\cos \alpha} = \frac{1.5 \times 5}{\cos 30^\circ} = 8.66 \text{ m s}^{-1}$$

197. (i) Since horizontal acceleration of the shell is zero, the speed has to be equal to the speed of the plane
 $\Rightarrow v_H = v$

- (ii) The minimum vertical speed should be such that it hits at the highest point.

$$v_v^2 = 2gh; \quad v_v = \sqrt{2gh}$$

$$(iii) \tan \theta = \frac{v_v}{v_H} = \frac{\sqrt{2gh}}{v}$$

$$\Rightarrow \theta = \tan^{-1} \frac{\sqrt{2gh}}{v} \quad \text{with horizontal}$$

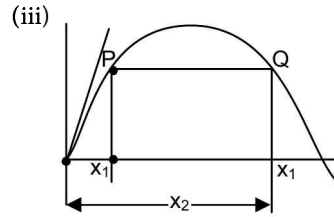
$$u = \sqrt{v_v^2 + v_H^2} = \sqrt{2gh + v^2}$$

$$198. (i) y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

$$(ii) h = \frac{u^2 \sin^2 \alpha}{2g}$$

$$y = \frac{h}{2} \Rightarrow \frac{u^2 \sin^2 \alpha}{4g} = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

$$\Rightarrow \frac{gx^2}{2u^2 \cos^2 \alpha} - x \tan \alpha + \frac{u^2 \sin^2 \alpha}{4g} = 0$$



From the above equation in (ii) we get:

$$\frac{gx^2}{2u^2 \cos^2 \alpha} - x \tan \alpha + \frac{u^2 \sin^2 \alpha}{4g} = 0$$

Solving the quadratic equation in x , we get:

$$x = \frac{\tan \alpha \pm \sqrt{\tan^2 \alpha - \frac{4g}{2u^2 \cos^2 \alpha} \frac{u^2 \sin^2 \alpha}{4g}}}{\left(2g/2u^2 \cos^2 \alpha\right)}$$

$$x = \frac{\tan \alpha \pm \sqrt{\tan^2 \alpha - \frac{\tan^2 \alpha}{2}}}{\left(g/u^2 \cos^2 \alpha\right)}$$

$$x = \frac{\tan \alpha \left[1 \pm \sqrt{\frac{1}{2}}\right]}{g/u^2 \cos^2 \alpha}$$

The two values of x say x_1 and x_2 will correspond to, lower value of x (i.e., x_1) travelled by second projectile and higher value of x (i.e., x_2) travelled by first projectile (see Fig.)

$$\therefore \text{Ratio} = \frac{x_2}{x_1} = \frac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = 3 + 2\sqrt{2}$$

199.

$$(i) v_0 = 54 \times \frac{5}{18} = 15 \text{ ms}^{-1}$$

$$\omega_0 = \frac{v_0}{R} = \frac{15}{100} = 0.15 \text{ rad s}^{-1}$$

$$(ii) \alpha = \frac{a_T}{R} \left(\text{acceleration}_{\text{tangential}} \right) \\ = \frac{5}{100} = 0.05 \text{ rad s}^{-2}$$

(iii) **Method 1**

$$s = 100 \text{ m}$$

$$v^2 = 15^2 + 2 \times 5 \times 100 = 1225$$

3.86 Motion in Two Dimensions

$$\Rightarrow v = 35 \text{ m s}^{-1}$$

$$\omega = \frac{v}{R} = 0.35 \text{ rad s}^{-1}$$

Method 2

$$\theta = \frac{s}{R} = \frac{100}{100} = 1 \text{ rad}$$

$$\text{Using } \omega^2 = \omega_0^2 + 2 \times \alpha \times \theta$$

$$\omega^2 = (0.15)^2 + 2 \times (0.05) \times 1 = 0.1225$$

$$\Rightarrow \omega = 0.35 \text{ rad s}^{-1}$$

$$\text{Centripetal acceleration (a}_r\text{)} = \frac{v^2}{R} \text{ or } \omega^2 R$$

$$= (0.35)^2 \times 100 = 12.25 \text{ m s}^{-2}$$

$$\text{Tangential acceleration (a}_t\text{)} = 5 \text{ m s}^{-2} \text{ (given)}$$

$$\therefore \text{ Total acceleration} = \sqrt{(12.25)^2 + 5^2} \\ = 13.2 \text{ m s}^{-2}$$

$$\text{Direction: } \tan^{-1} \frac{5}{12.25} = 22^\circ \text{ to the radial direction at that point.}$$

200. (i) $v = v_n = r\omega$ at the instant when the string snaps. It is in the horizontal direction

$$(ii) H = h + r \text{ and } T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2(h+r)}{g}}$$

$$\text{Distance AB} = v_n T = r\omega \sqrt{\frac{2(h+r)}{g}}$$

CHAPTER

4

LAWS OF MOTION

■■ CHAPTER OUTLINE

Preview

STUDY MATERIAL

Introductory Concepts

- Concept Strands (1-2)

Newton's Laws of Motion

- Concept Strands (3-6)

Basic Forces in Nature

Common Forces in Mechanics

- Concept Strand (7)

Application of Laws of Motion to Solve Problems

- Concept Strands (8-15)

Problems Involving Constraints

- Concept Strands (16-17)

Reference Frame: Inertial and Accelerated Frames

- Concept Strands (18-19)

Friction

- Concept Strands (20-23)

Circular Motion and Centripetal Force

CONCEPT CONNECTORS

- 20 Connectors

TOPIC GRIP

- Subjective Questions (10)
- Straight Objective Type Questions (5)
- Assertion–Reason Type Questions (5)
- Linked Comprehension Type Questions (6)
- Multiple Correct Objective Type Questions (3)
- Matrix-Match Type Question (1)

IIT ASSIGNMENT EXERCISE

- Straight Objective Type Questions (80)
- Assertion–Reason Type Questions (3)
- Linked Comprehension Type Questions (3)
- Multiple Correct Objective Type Questions (3)
- Matrix-Match Type Question (1)

ADDITIONAL PRACTICE EXERCISE

- Subjective Questions (10)
- Straight Objective Type Questions (40)
- Assertion–Reason Type Questions (10)
- Linked Comprehension Type Questions (9)
- Multiple Correct Objective Type Questions (8)
- Matrix-Match Type Questions (3)

INTRODUCTORY CONCEPTS

Mass (m)

Mass is a measure of the quantity of the matter present in a body. It is a scalar quantity.

Its SI unit is kilogram (kg) and its dimensional formula is M.

Inertia

Inertia of a body can be defined as “*the property of a body by virtue of which it opposes any change in its state of rest or of uniform motion*”.

If the same amount of push or pull (i.e., a constant force) is applied on a toy car and a real car, the toy car will easily change its state of rest or uniform motion. However, it may not be able to change the state of rest or uniform motion of a real car. It is because toy car has very small mass compared to the real car. Thus inertia of a body is proportional to the mass of the body. This mass of the body is called as ‘the inertial mass of the body’.

There are three types of inertia, namely inertia of rest, inertia of motion and inertia of direction.

(a) Inertia of rest

“*The inability of a body to change its state of rest by itself is called the inertia of rest*”.

For example:

- (i) A book lying on a table remains on the table till some one picks it up.
- (ii) When a doormat is beaten with a stick, the dust is separated from the mat because the force is applied on the mat, which moves the mat. However, the dust due to its inertia of rest remains in its original position. Thus when the mat moves away, the dust gets separated.
- (iii) Leaves and fruits fall from a tree when it is shaken.
- (iv) A person standing in a bus tends to fall backward when the bus starts suddenly because the lower part of the person, which is in contact with the bus moves along with the bus, while the upper part of the body continues in its state of rest due to inertia.

(b) Inertia of motion

The inability of a body to change by itself its state of motion is called inertia of motion.

For example:

- (i) An athlete running in a race continues to run even after reaching the finishing point.
- (ii) When a man gets out of a running bus, he falls forward. When he touches the ground, his feet come to rest but the top portion of his body continues to move due to inertia of motion, which makes him fall forward.
- (iii) A person riding a horse will fall forward if the horse stops suddenly.
- (iv) When a fan is switched off, it continues to rotate for some more time before coming to rest.

(c) Inertia of direction

The inability of a body to change its direction of motion by itself is called inertia of direction.

For example:

- (i) If a bus moving along a straight road takes a right turn, the passengers inside the bus are thrown towards left. This is because due to inertia of direction, the passengers continue to travel along the same straight line, even though the bus has turned to the right.
- (ii) When a boy rides a bicycle on a muddy road, the mud flies off tangentially to the rim of the cycle due to inertia of direction.

Linear momentum ($\vec{p}$)

The quantity of motion contained in a body is called the linear momentum of the body. “*Linear momentum of a body is defined as the product of its mass and velocity*”. Since mass is a scalar quantity and velocity a vector, the product of mass and velocity (i.e., linear momentum) is a vector quantity. It is denoted by $\vec{p}$.

$$\vec{p} = m\vec{v}$$

where, m = mass of body and $\vec{v}$ = velocity of the body. The direction of linear momentum ($\vec{p}$) is the same as the direction of velocity of the body ($\vec{v}$). The SI unit of linear momentum is kilogram metre per second (kg m s^{-1}). Its dimensional formula is MLT^{-1} . Whenever the velocity of a body changes (either in magnitude or in direction or in both), then the linear momentum of the body also changes.

A mass of 2 kg travelling at 10 m s^{-1} and a mass of 200 kg travelling at 0.1 m s^{-1} possess momenta of equal magnitude; $p_1 = p_2$, ($= 2 \times 10 = 200 \times 0.1 = 20 \text{ kg m s}^{-1}$). If their velocities are in the same direction, then $\vec{p}_1 = \vec{p}_2$. (Linear

momentum is sometimes called momentum. Here in this chapter, when we say 'momentum' or 'momenta' (plural of momentum), it means 'linear momentum'.

CONCEPT STRANDS

Concept Strand 1

Determine the momentum of a body of mass 5 kg, moving with a velocity of $(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ m s}^{-1}$.

Solution

$$\vec{p} = m\vec{v} = 5(2\hat{i} + 3\hat{j} + 6\hat{k}) = 10\hat{i} + 15\hat{j} + 30\hat{k}$$

$$p = \sqrt{10^2 + 15^2 + 30^2} = \sqrt{1225} = 35 \text{ kg m s}^{-1}$$

Being a vector, momentum can be resolved along the co-ordinate axes. It will have components along each co-ordinate axis, as illustrated by the above example.

$$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

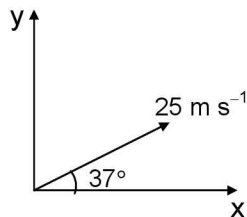
$$p_x = 10 \text{ kg m s}^{-1}$$

$$p_y = 15 \text{ kg m s}^{-1}$$

$$p_z = 30 \text{ kg m s}^{-1}$$

Concept Strand 2

A body of mass 10 kg is projected with a velocity of 25 m s^{-1} at an angle 37° to the horizontal. Find $\vec{p}$ in $(\hat{i}, \hat{j})$ direction taking origin at point of projection ($g = 10 \text{ m s}^{-2}$, $\hat{i}$ is along the horizontal direction and $\hat{j}$ is along the vertically upward direction respectively)



- At the instant of projection.
- 1 s after projection.
- Just before it reaches the ground.

Solution

$$(i) \vec{u} = u \cos 37^\circ \hat{i} + u \sin 37^\circ \hat{j}$$

$$= 25(0.8)\hat{i} + 25(0.6)\hat{j} = (20\hat{i} + 15\hat{j}) \text{ m s}^{-1}$$

$$\therefore \vec{p}_i = m\vec{u} = (200\hat{i} + 150\hat{j}) \text{ kg m s}^{-1} (\because m = 10 \text{ kg})$$

$$p_i = \sqrt{200^2 + 150^2} = 250 \text{ kg m s}^{-1}$$

$$(ii) \vec{v} = u \cos 37^\circ \hat{i} + (u \sin 37^\circ - gt)\hat{j}$$

$$= 25(0.8)\hat{i} + [25(0.6) - 10.1]\hat{j} = (20\hat{i} + 5\hat{j}) \text{ m s}^{-1}$$

$$\therefore \vec{p}_{ii} = m\vec{v} = (200\hat{i} + 50\hat{j}) \text{ kg m s}^{-1} (\because m = 10 \text{ kg})$$

$$p_{ii} = \sqrt{200^2 + 50^2} = 50\sqrt{17} \text{ kg m s}^{-1}$$

$$(iii) \text{ Velocity at the time of landing is:}$$

$$\vec{v} = u \cos 37^\circ \hat{i} - u \sin 37^\circ \hat{j}$$

$$\vec{p}_{iii} = m\vec{v} = (200\hat{i} - 150\hat{j}) \text{ kg m s}^{-1}$$

$$p_{iii} = \sqrt{200^2 + 150^2} = 250 \text{ kg m s}^{-1}$$

Note that the momenta at $t = 0$ and $t = T$, where, T is the time of flight, are not equal because their directions are different, even though their magnitudes are same. Momentum of a body measured by different observers can be different. For example, with respect to an observer traveling at 10 m s^{-1} in the direction of positive X-axis, the momentum of the projectile of concept strand 2 as a function of time t will be

$$\vec{p}_{\text{rel}} = m\vec{v}_{\text{rel}} = m[20\hat{i} + (15 - gt)\hat{j} - 10\hat{i}]$$

$$= 100\hat{i} + (150 - 100t)\hat{j} (\because m = 10 \text{ kg and } g = 10 \text{ m s}^{-2})$$

$$= 100\hat{i} + 150\hat{j} \text{ kg m s}^{-1} \text{ at } t = 0$$

$$100\hat{i} + 50\hat{j} \text{ at } t = 1 \text{ s and } 100\hat{i} - 150\hat{j} \text{ at } t = T$$

NEWTON'S LAWS OF MOTION

The dynamics of motion or the question 'What causes motion of bodies?' is explained using three laws formulated by Sir Issac Newton. These laws are known as Newton's laws of motion. Earlier, on the basis of experiments conducted on bodies moving on inclined planes, Galileo had established a law of inertia, which stated that no force is required to keep a body moving with uniform velocity. Newton's analysis of motion extended the ideas of Galileo. Newton's first law gives a qualitative definition of force and conditions for a frame of reference to be inertial; the second law gives the quantitative and dimensional definition of force and the third law explains the nature of the force. Newton's laws of motion provide the basis by which we can determine the nature of the motion of a body subject to a single force or a set of forces.

When a force acts on a particle or a system of particles, it can cause translatory motion, rotational motion about a specific point or a combination of both. In this chapter, we will restrict our study to translatory effects of forces by assuming the body to be a particle or a point mass. The size and shape of the body will be ignored.

First law of motion

"Every body continues in a state of rest or of uniform motion along a straight line unless it is acted upon by an external force". The state of rest or of uniform motion of a particle is called its 'inertial state'. So, the first law is a statement of the concept of inertia and introduces the concept of force as an external entity causing a change of state.

Force

According to Newton's first law of motion, to overcome the inertia of a body an external force has to be applied on it. Thus a qualitative definition of force can be made as follows. *"Force is that external agency, which when acting on a body changes or tends to change the state of rest or uniform motion of the body along a straight line".*

When the law says external force, it means the 'net force' or the resultant force or the vector sum of all the forces acting on the body.

Translational equilibrium

When the resultant force on a body is equal to zero, the body is said to be in translational 'equilibrium'. *When a body is said to be in translational equilibrium, it means ei-*

ther the body is at rest (static equilibrium) or in uniform motion (dynamic equilibrium) and the net force on the body is zero. i.e., Net Force = 0 is the condition for translational equilibrium of a body.

Second law of motion

When the same force is made to act on two bodies of different masses for the same time interval, the observed effects are different. Lighter body moves faster than the heavier body. But the change in momentum of both the bodies is the same, which leads to Newton's second law of motion.

Newton's second law of motion states that *"the rate of change of momentum of a body is directly proportional to the external force applied on the body and the change in momentum takes place in the direction of the force"*.

This law gives a method for measuring the external unbalanced force acting on a body and hence this law gives a quantitative definition of force. Consider a body of mass 'm', moving with a velocity $\vec{v}$, subjected to an external unbalanced force $\vec{F}$. The linear momentum of the body is $\vec{p} = m\vec{v}$.

According to Newton's second law of motion, $\vec{F} \propto \frac{d}{dt}(\vec{p})$

$$\text{i.e., } \vec{F} \propto \frac{d}{dt}(m\vec{v})$$

Constant mass situation

$\vec{F} \propto m \frac{d\vec{v}}{dt}$ (When the mass 'm' remains constant for the body)

i.e., $\vec{F} = km \frac{d\vec{v}}{dt}$, where 'k' is the constant of proportionality

i.e., $\vec{F} = k.m\vec{a}$, where $\vec{a} = \frac{d\vec{v}}{dt}$ is the acceleration produced in the motion of the body.

$\vec{a} \propto \frac{\vec{F}}{m} \Rightarrow \vec{a} \propto \frac{1}{m}$ for a given force $\vec{F}$. Now unit force is

defined as that force which when acting on a body of unit mass, produces unit acceleration.

i.e., $F = 1$ unit when mass = 1 unit and acceleration = 1 unit
 $\therefore 1 = k \cdot 1 \times 1 \rightarrow k = 1$

Thus 'k' becomes equal to 1 with the definition of unit force.

$$\therefore \bar{F} = m\bar{a}$$

$$\text{i.e., } \bar{F} = \frac{m(\bar{v} - \bar{u})}{t}$$

Thus the unbalanced force acting on a body is measured by the product of mass of the body and the acceleration produced on it by the force acting on the body.

Net force $\bar{F}$ on the body causes acceleration $\bar{a}$ of the body given by $\bar{a} = \frac{\bar{F}}{m}$. The mass of a particle obtained from

the relation $m = \frac{|\bar{F}|}{|\bar{a}|}$, is called as **the inertial mass** of the particle. In terms of components of the vectors,

if $\bar{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and net force $\bar{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$, then,

$$F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = m(a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$$

$$\therefore F_x = ma_x; \quad F_y = ma_y; \quad F_z = ma_z$$

If any force component is zero then component of acceleration along that direction is also zero. In SI, the unit of force is 'newton' (N). One newton (1 N) is defined as that force which when acting on a body of mass 1 kg, produces an acceleration of magnitude 1 m s⁻² on it.

$$1 \text{ N} = 1 \text{ kg} \times 1 \text{ m s}^{-2}$$

$$\therefore 1 \text{ N} = 1 \text{ kg m s}^{-2}$$

The dimensional formula for force is MLT⁻². 'newton' (N) is called the absolute unit of force in SI. There is another unit of force that is also frequently used. It is called the gravitational unit of force. Earth attracts all bodies and exerts a force on them, which produces the same magnitude of acceleration on all of them that is directed towards the centre of Earth. Near the surface of Earth, the acceleration produced on different masses is of the same magnitude, $g = 9.8 \text{ m s}^{-2}$. The force, which produces this acceleration on bodies, is called the gravitational force.

$\therefore$ Gravitational force on a body of mass 'm' is given by $F_g = \text{mass} \times \text{acceleration produced} = mg$ newton. Unit of gravitational force is called '1 kilogram weight' (kg wt) or 1 kilogram force (kgf). It is the force with which the Earth attracts a body of mass 1 kg that is near the surface of the Earth.

$$1 \text{ kgf} = 1 \text{ kg} \times g' \text{ m s}^{-2} = g \text{ kg m s}^{-2} = g' \text{ N}$$

$$\therefore 1 \text{ kgf} = 9.8 \text{ N}$$

Variable mass situation

$\bar{F} = \frac{d\bar{p}}{dt}$ according to Newton's second law of motion. But $\bar{p} = m\bar{v}$. We get $\bar{F} = m\bar{a}$ only if the mass of the body in motion remains the same always. However, there are situations where the mass of the body in motion varies with respect to time.

For example:

- Motion of rockets.
- Materials like sand, cement, gravel etc., poured on to a conveyor belt, which is moving at a constant speed.
- A falling rain drop collecting the mist present in the atmosphere.

$$\text{In such cases, } \bar{F} = \frac{d\bar{p}}{dt} = \frac{d}{dt}(m\bar{v})$$

$$\bar{F} = \frac{dm}{dt}(\bar{v}) + m \frac{d\bar{v}}{dt}$$

If $\bar{v}$ remains constant with respect to time, $\frac{d\bar{v}}{dt} = 0$

In such cases, $\bar{F} = \left(\frac{dm}{dt}\right)\bar{v}$ and the magnitude of the force

$$F = v \left(\frac{dm}{dt}\right)$$

Note:

Newton's second law of motion is strictly applicable to a particle or point mass. In the case of a system of particles or rigid bodies, $\bar{F}$ is the total external force acting on the system and all internal forces of the system are not to be included in $\bar{F}$. Further, $\bar{a}$ in such cases, is the acceleration of the centre of mass of the system (explained in later sections in this course).

Impulse ($\bar{J}$)

Impulse is the 'aggregate effect' of an external force (or net force) on a body and it is measured as the product of the average force acting on the body and the time interval for which it acts on the body. Its effect is to produce a change in the linear momentum of the body. Being the change in momentum, it is a vector, given by

$$\bar{J} = \Delta\bar{p} = \bar{F}\Delta t$$

4.6 Laws of Motion

where $\bar{F}$ is the average force acting on the particle for a small duration of time Δt , producing a change in linear momentum $\Delta \bar{p}$. The direction of impulse on the particle is the direction of change in its linear momentum $\Delta \bar{p}$. Its SI unit is newton second (N s) which is the same as that of $\Delta \bar{p}$ (kg m s^{-1}). Its dimensional formula is MLT^{-1} .

Impulsive force

When a very large force acts on a body for a very short duration of time (eg: the blow of a hammer, the collision between two billiard balls etc), it may not be possible to measure either the force or the time interval for which the force acts on the body. However, it will be possible to measure the change in momentum of the body in such cases.

An impulsive force is very large force acting for a very short time on a body so that the change in position of body during the time the force acts on it may be neglected. Consider a body of mass 'm', subjected to a constant force $\bar{F}$ for a duration of time 't'. If $\bar{a}$ is the acceleration produced on the body, then from Newton's second law of motion, we have $\bar{F} = m\bar{a}$. From the definition of impulse, we have impulse during the time interval 't' given by $\bar{J} = \bar{F}t = m\bar{a}t$
 $\therefore \bar{J} = m\bar{a}t$ — (1)

If $\bar{u}$ was the initial velocity of the body (at the time of application of the force $\bar{F}$) and $\bar{v}$ is its velocity after time interval 't', then

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

$$\therefore \bar{a} = \frac{\bar{v} - \bar{u}}{t} \quad \text{--- (2)}$$

Substituting the value of $\bar{a}$ from (2) in (1), we get

$$\begin{aligned} \bar{J} &= mt \frac{(\bar{v} - \bar{u})}{t} = m(\bar{v} - \bar{u}) = m\bar{v} - m\bar{u} \\ &= \bar{p}_f - \bar{p}_i \quad (m\bar{v} = \text{final momentum of body} = \bar{p}_f \text{ and } m\bar{u} = \text{initial momentum of body} = \bar{p}_i) \\ \therefore \text{Impulse} &= \text{Final momentum} - \text{initial momentum} \\ &= \text{change in momentum during the time interval.} \end{aligned}$$

Hence the total change in the momentum of a body during a time interval is equal to the impulse of the force acting on it during the same interval of time. This is known as **impulse-momentum theorem**.

For example:

- (i) In athletic events, a thick foam mattress is kept over the pits of high jump and pole-vault events. When the athlete falls over the foam mattress, he hits it with certain velocity, compresses the foam mattress and comes to rest. Thus there is a change in momentum of the athlete due to the force exerted by the mattress on him. Since this force is exerted over certain time interval 't' in which the mattress is getting compressed, the magnitude of the force exerted on the athlete is reduced considerably. This prevents injuries to the athlete. $\bar{F} = \frac{m(\bar{v} - \bar{u})}{t}$. Longer the duration of 't', smaller will be the force on the athlete.
- (ii) When a cricketer takes a catch, he moves his hands in the direction of motion of the ball to increase the time interval in which the catch is taken. This reduces the impact of the ball in his hands and prevents injuries.
- (iii) Fragile items like glass, crockery etc. are wrapped in straw or paper strips and kept in boxes for shipment to avoid breakage.
- (iv) Automobiles are fitted with springs and shock absorbers to reduce jerk while moving on uneven roads.
- (v) A person falling on a cemented floor receives more injuries than the person falling on a sandy floor.

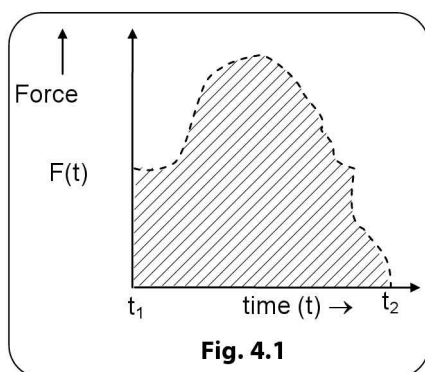
Consider a body of mass 2 kg, travelling with a velocity of 10 m s^{-1} . To bring it to rest, a stopping 'force' has to be applied till it comes to rest, i.e., for the body's momentum to change by $\Delta p = -20 \text{ kg m s}^{-1}$. If the force is small, the force has to be applied for a longer duration than when the force is large. If the stopping force is large, then the time during which the force has to act will be smaller so as to have the same change in momentum of the body.

The above example concerns the stopping of a moving body. It is equally applicable to reducing/increasing the body's momentum; in general, changing a body's momentum. To summarize,

$$\bar{J} = \Delta \bar{p} = \bar{F} \times \Delta t$$

where, $\bar{F}$ is the force applied.

It is quite possible that the external agent may not be applying a constant force throughout the Δt period. It may be a variable force as shown in Fig. 4.1.



In this case, the impulse is given by

$$\bar{J} = \int_{t_1}^{t_2} \bar{F}(t) dt$$

where, $\bar{F}$ is the instantaneous force and t_1 and t_2 are the instants at which the action of the force starts and stops (i.e., $t_2 - t_1$ is the interval of time in which the force acts). The area enclosed by the force vs time graph (the shaded area) represents the impulse i.e., the change in momentum of the particle during the time interval from t_1 to t_2 .

CONCEPT STRANDS

Concept Strand 3

A man pushes a trolley of mass 150 kg from rest along a smooth straight road. He applies a constant force of 25 N for the first 20 s, takes rest for the next 30 s, and applies an opposite force of 20 N for the next 10 s. Find the velocity of the trolley at the end of this 60 s period.

Solution

$$\Delta p = \sum \bar{F} \Delta t = (25 \times 20 + 0 \times 30 + (-20) \times 10) \text{ N s} \\ = 300 \text{ N s}$$

Initial momentum = 0

$\therefore$ Final momentum = 300 Ns

$$\therefore \text{Final velocity} = \frac{300 \text{ N s}}{150 \text{ kg}} = 2 \text{ ms}^{-1}$$

Thus, force is what is applied by an external agency on a body and which 'tries to cause' change in momentum of

the body. If two equal and opposite forces are applied on the trolley, each force 'tries to cause' change in momentum of the body but since the changes are oppositely directed, net change in momentum produced is zero.

Concept Strand 4

In a billiards game, if the cue has been in contact with the ball of mass 0.2 kg for 0.05 s and the ball starts from rest to attain a uniform velocity of 25 m s⁻¹, what is the average force exerted by the cue on the ball?

Solution

$$\text{Average force} = \frac{\int F dt}{\int dt} = \frac{\Delta p}{\Delta t} \\ = \frac{5 \text{ kg ms}^{-1}}{0.05 \text{ s}} = 100 \text{ N}$$

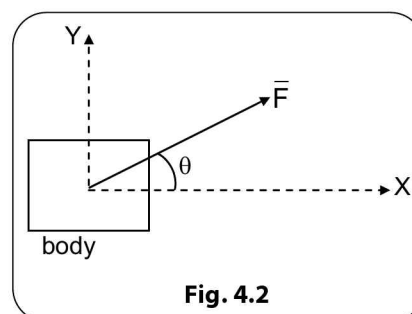
Resolution of force

Force, being a vector, can be resolved along the chosen co-ordinate axes. It will have components along each co-ordinate axis.

In the example shown in Fig. 4.2, a force $\bar{F}$ acts on a body at an angle θ to the x-axis. This can be resolved in the X and Y directions and written as $\bar{F} = F_x \hat{i} + F_y \hat{j}$; where $F_x = F \cos \theta$ and $F_y = F \sin \theta$.

$$\therefore \bar{F} = F \cos \theta \hat{i} + F \sin \theta \hat{j}.$$

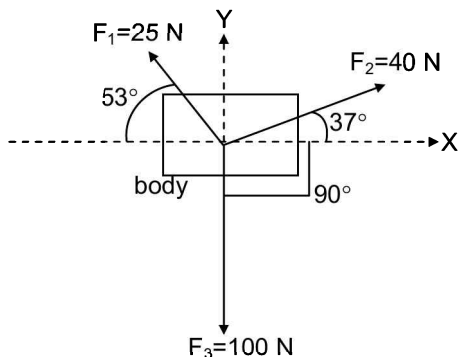
If more than one force act on a body, they can be replaced by a single resultant force.



CONCEPT STRANDS

Concept Strand 5

Consider the body acted upon by three forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ as shown in the figure.



Solution

If $\vec{F}$ is the resultant, i.e., $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$\vec{F} = F_x \hat{i} + F_y \hat{j}, \text{ where}$$

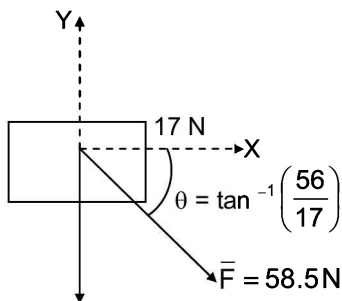
$$F_x = F_{1x} + F_{2x} + F_{3x} = -25 \cos 53^\circ + 40 \cos 37^\circ + 100 \cos 90^\circ = 17 \text{ N, and}$$

$$F_y = F_{1y} + F_{2y} + F_{3y} = 25 \sin 53^\circ + 40 \sin 37^\circ - 100 = -56 \text{ N}$$

where, F_{1x}, F_{2x}, F_{3x} and F_{1y}, F_{2y}, F_{3y} are the X and Y components respectively.

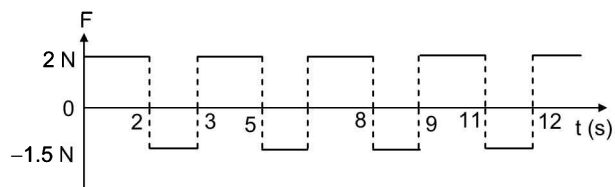
$$\therefore \text{Resultant } |\vec{F}| = \sqrt{F_x^2 + F_y^2} = 58.5 \text{ N}$$

making angle θ with positive X-axis such that $\theta = \tan^{-1} \left(\frac{-56}{17} \right)$. So, a single force as shown in the figure can replace the set of three forces.



Concept Strand 6

A periodic force in the form of pulses, as shown in the diagram, is applied on a body of mass 5 kg lying at rest on a smooth fixed table at $t = 0$. What is the velocity of the body at the end of 10 second?



Solution

Method 1 (by finding acceleration)

$$a_1 = \text{Magnitude of acceleration during positive cycle} = \frac{2 \text{ N}}{5 \text{ kg}} = 0.4 \text{ ms}^{-2}, t_1 = 2 \text{ s}$$

$$a_2 = \text{Magnitude of acceleration during negative cycle} = \frac{-1.5 \text{ N}}{5 \text{ kg}} = -0.3 \text{ ms}^{-2}, t_2 = 1 \text{ s}$$

Velocity gain after each cycle of 3 second = $a_1 t_1 - a_2 t_2 = 0.4 \times 2 - 0.3 \times 1 = 0.5 \text{ m s}^{-1}$ ($\because a_1$ and a_2 are in opposite directions)

$$\begin{aligned} \text{Velocity at the end of 10 second} &= \text{velocity gain in 9 s} \\ &+ \text{velocity gain in the 10th second} \\ &= (0.5 \times 3) + 0.4 = 1.9 \text{ m s}^{-1} \end{aligned}$$

Method 2

(Impulse method)

$$\begin{aligned} \Delta p = J &= \sum_i F_i (\Delta t)_i = (2 \times 2) \times 3 - (1.5 \times 1) \times 3 + 2 \times 1 \\ &= 9.5 \text{ N s} \end{aligned}$$

$\therefore$ Velocity at the end of 10 second

$$= \frac{9.5 \text{ N s}}{5 \text{ kg}} = 1.9 \text{ ms}^{-1}$$

Third law of motion

Newton's third law of motion states that *"To every action, there is an equal and opposite reaction"*. As per this law, whenever there is an interaction between two objects, there is a force acting on each other.

When object 1 interacts with another object 2, if the force exerted by object 1 on object 2 (let us call it as action) is $\vec{F}_{12}$, then the object 2 also exerts an equal and opposite force on object 1. Then we can call this force $\vec{F}_{21}$ as reaction and as per Newton's third law,

$$\text{Action} = - \text{Reaction or } \vec{F}_{12} = -\vec{F}_{21} \text{ i.e., } \vec{F}_{12} + \vec{F}_{21} = 0$$

It must be noted that the *action and reaction are acting on different bodies and hence they do not cancel each other*. As per Newton's third law, forces in nature always occur in pairs. Also, action and reaction occur simultaneously. There is no time delay between action and reaction.

For example:

- (i) A bird flies in the air by pushing the air downwards (called action) with its wings. The air, in turn, pushes

the wings upwards (reaction) which makes the bird go up.

- (ii) When a tennis ball falls on to the floor, it exerts a force on the floor downwards (action) and the floor exerts a force on the tennis ball upward (reaction). This reaction of floor makes the tennis ball bounce back.
- (iii) A man on a boat at rest near the riverbank exerts a force on the boat, directed away from the shore (action). The boat, in turn, exerts a force on the man directed towards the shore (reaction). The reaction of the boat on the man makes the boat drift away from the shore. Hence it is necessary to tie the boat to the shore to prevent it from drifting away.
- (iv) A table resting on the floor exerts a downward force on the floor which is equal to its weight ' $\vec{W}$ '. The floor, in turn, exerts an equal upward force on the table which is the reaction of the floor ' $\vec{W}$ ' such that ' $\vec{W}' = -\vec{W}$ '
- (v) Recoil of gun, propulsion of rocket etc., are governed by Newton's third law of motion.

BASIC FORCES IN NATURE

The basic forces in nature are

- (a) Gravitational force.
- (b) Electromagnetic force.
- (c) Strong nuclear force.
- (d) Weak nuclear force.

Gravitational force

Gravitational force exists between any pair of bodies in the Universe and is due to the mass of the bodies. It is a non-contact force, which can act through vacuum or any medium. This force is always attractive in nature and the magnitude of the force between any pair of bodies can be calculated using Newton's law of universal gravitation. The gravitational force between a pair of bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. This force is independent of the medium through which the force acts. It is the weakest force in nature but it is a long range force. The formation of stars and galaxies,

the motion of planets around the Sun, motion of artificial and natural satellite are all governed by the gravitational force, which is universally attractive.

Electromagnetic force

Electric and magnetic forces are collectively called electromagnetic force. These are also non-contact forces.

(i) Electrostatic force

It is the force between two charged particles at rest. Between like charges the force is repulsive and between unlike charges, the force is attractive.

(ii) Magnetic force

It is the force between the magnetic poles of different magnets. In a later chapter you will learn that current carrying wires exert magnetic force on each other. Magnetic force may be attractive or repulsive.

Strong nuclear force

The force, which keeps the protons and neutrons inside the nucleus of any atom, is called the strong nuclear force. It is the strongest force in nature and is about 10^{38} times stronger than the gravitational force. However it is a short-range force and is confined to distances, which are approximately the size of the atomic nuclei (of the order of $1\text{ fm} = 10^{-15}\text{ m}$). It is the force between the nucleons inside a nucleus and can be either attractive or repulsive. This force is charge independent.

Weak nuclear forces

The force between elementary particles emitted during radioactive decay of a radioactive substance is known as weak nuclear force. In negative β decay, an electron and an uncharged particle called anti-neutrino (chargeless particle) are emitted. The force between the electron and anti-neutrino is called weak nuclear force. Similarly in positive β -decay, a positron (a charged particle having the mass of an electron and charge of a proton) and an uncharged particle called neutrino are emitted. The force between positron and neutrino is also called weak nuclear force.

COMMON FORCES IN MECHANICS

Gravitational force (weight)

The force exerted by the Earth on a body is called the weight of the body. If m is the mass of the body and g is the acceleration due to gravity at that place, then weight of the body is given by $W = mg$. This force acts vertically downwards and is a non-contact force on the body.

Contact forces

Contact forces are forces arising when the two interacting objects are physically in contact with each other. These are usually of electromagnetic origin.

For example:

When a block of wood is placed on a table, the weight of the block acts on the table and the table in turn exerts a normal force on the wooden block.

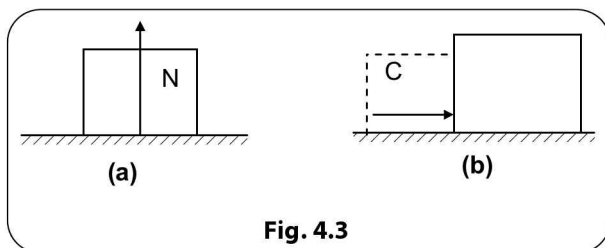


Fig. 4.3

Contact forces can usually be resolved as Normal reaction (which acts normal to the contact surfaces and Friction (which acts tangential to the contact surfaces). Friction is dealt in detail separately in this chapter.

The normal reaction on a body by another surface always acts towards that body, trying to compress that body. i.e., any other body or surface in contact with a given body can only push it and not pull it. When the contact surfaces are horizontal, the normal reaction is vertical and towards the body [see N in figure 4.3(a)].

When the contact surfaces are vertical, the normal reaction is horizontal and towards the body [see C in Fig. 4.3(b)].

When the contact surfaces are horizontal, the normal reaction may be equal to the weight of the body, less than its weight, more than its weight or even zero, depending upon how the bodies or surfaces in contact are moving relative to each other.

Tension

Force can be applied on a body through strings or ropes. When a load is suspended from a string whose other end is fixed to a support, restoring forces are built up in string, which is called as tension denoted by T . For a light inextensible string $T = W$, the weight of the load suspended from it. A string or rope can only pull a body and it cannot push the body. See T in Fig. 4.4.

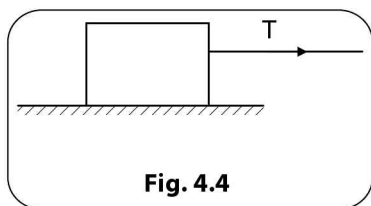


Fig. 4.4

For a light (negligible mass), inextensible string connecting two bodies of masses m_1 and m_2 respectively, the tension in the string will be the same throughout its length. See T in figure 4.4. If the string is not light or if the string is elastic, then the tension in the string may not be the same along its length. If a light, inelastic string passes over a light, smooth pulley, the tension in the string on either side of the pulley will be the same. If the pulley is not light (i.e., mass of pulley is not negligible) and the surface of pulley over which the string passes is not smooth, the tension in the string on either side of the pulley will not be the same.

Spring force

When a spring is neither compressed nor elongated/extended, its length is called its natural length. Consider a spring of natural length ' ℓ ' fixed at one end and hanging vertically. Now, if a body of mass ' m ' is connected to the free-end of the spring and slowly released, the mass will move down slowly, thereby stretching the spring along with it. When the mass stops moving, let the length of the spring be ' $\ell + x$ '. Then the extension of the spring is given by $(\ell + x) - \ell = x$. In this position, the mass ' m ' is in equilibrium

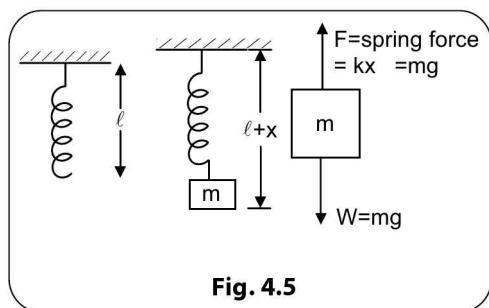


Fig. 4.5

(i.e., the net external force on it is zero). One of the forces on this mass is its weight ' mg ' acting downwards. It means there must be a balancing force of ' mg ' acting upwards on this mass. This force is exerted by the spring on the mass and it is called as spring force.

When the displacement ' x ' is downwards, the spring force is acting upwards. Hence the spring force and displacement are in opposite direction. i.e., $\vec{F} \propto \vec{x}$, where ' F ' is the spring force. This is called Hooke's law, which states that the restoring force of a spring is directly proportional to the extension of the spring. i.e., $\vec{F} = -k\vec{x}$, where ' k ' is the constant of proportionality, called spring constant.

$$k = \left| \frac{F}{x} \right|. \text{ If } x = 1 \text{ unit, } k = F. \text{ Hence the spring constant}$$

of a spring is numerically equal to the force required to produce unit extension in the spring. SI unit of k is newton per meter (N m^{-1}) and its dimensional formula is ML^0T^{-2} . Spring constant of a spring is also called force constant of the spring.

[In actual practice, when a spring is compressed or elongated by a large amount, the spring will permanently stay in the deformed state. That is, when you remove the external force, the spring will not come back to its natural length; it will no more be elastic. The limit upto which the spring will have its elastic property, is called elastic limit and the restoring force equation $F = kx$ is called Hooke's law which is applicable only within the elastic limit].

A spring balance works on this principle of restoring force $F = kx$. When you suspend a mass m from the spring balance (with spring constant k), the system will be in equilibrium when the spring's extension is, say x , then $mg = kx$. The spring balance is graduated directly in kg. The pointer attached to the spring moves over the graduated scale when the spring gets extended and reads the mass in kg (which in fact is proportional to its weight). The weighing scales that we find in railway stations, bus stand, and hospitals are nothing but spring balances – a spring supporting a platform on which when you stand, the spring gets compressed, the graduated circular scale reading the equivalent mass.

CONCEPT STRAND

Concept Strand 7

A boy of mass 50 kg steps on a weighing scale causing a displacement of 1 mm of the spring. What is the spring constant? Take $g = 10 \text{ m s}^{-2}$

Solution

$$m = 50 \text{ kg; } x = 1 \text{ mm}$$

$$= 1 \times 10^{-3} \text{ m, } g = 10 \text{ m s}^{-2}$$

Taking the platform of the scale as reference,

$$k = \frac{mg}{x} = \frac{500}{10^{-3}} = 5 \times 10^5 \text{ N m}^{-1}$$

APPLICATION OF LAWS OF MOTION TO SOLVE PROBLEMS

Solving problems in this context means determining the motion of body/bodies by using Newton's Laws. This involves the following steps:

(i) Identify the system

The system could be a single particle, a single block, a combination of blocks, ropes, pulleys etc. While deciding a system, ensure that every component of the system must move with the same acceleration.

(ii) Identify and list all the forces acting on the system

It must be ensured that the *forces exerted by the system on the surroundings are not included* in this list.

(iii) Draw the Free Body Diagram

This is a diagram, which shows all the forces acting on the system and explained further below:

(iv) Assume or predict the direction of acceleration of the system

Even if your assumed directions are wrong, in the final analysis, the correct direction will be obtained! Use constraint theory, if required, to relate the accelerations of various bodies in the system. See concept strand 16 and 17 for further explanation of the constraint theory.

(v) Select a suitable co-ordinate system

(vi) Resolve the forces along the axes of the co-ordinate system and determine the resultant force in each direction (X, Y and Z).

(vii) Calculate the acceleration/net force on the system

Using the relation $\vec{F} = m\vec{a}$, the component of acceleration in each direction and hence the total acceleration of the system can be calculated.

Free body diagram (FBD)

In a free body diagram, all the forces acting on a body are shown with their magnitudes and direction. This means to

- (i) Draw the body (of our subject of interest) as a point mass or a small block etc.
- (ii) Show the immediate surroundings (i.e., other objects/bodies in contact with our subject body) preferably in dotted line.
- (iii) List out the various forces experienced by the body, disregarding the forces exerted by the body because, they will appear as forces experienced by some other body in the system,
- (iv) Draw all the forces acting on the body as arrows with directions clearly marked, ignoring forces exerted by

this body on others, because the motion of a body is solely determined by forces acting on it,

- (v) Show, by means of arrows, alongside the body, the parameters of motion of the body (velocity/acceleration).

This is called a free body diagram. The purpose of the free body diagram is to ensure that we consider all the forces acting on the body properly without missing out any. If there is more than one body in the problem, each body requires a FBD. In general, any body having mass needs one FBD.

As an example, the FBD of a block of mass m_1 on a block of mass m_2 is shown in Fig. 4.6

Applying Newton's second law:

$$\text{Acceleration} = \frac{\text{Resultant force}}{\text{Mass}} \quad \text{i.e.,} \left(\bar{a} = \frac{\bar{F}}{m} \right).$$

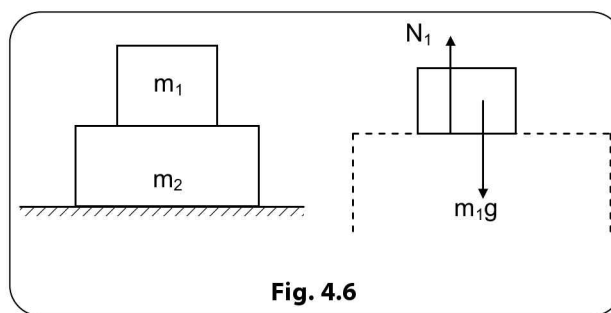


Fig. 4.6

In general, there will be two such equations – one for the X-direction, and the other for the Y-direction. This means, while drawing diagrams, the forces shall be resolved into their components along the chosen axes and so also the parameters of motion. In any situation, we must have as many equations as there are unknowns. If the equations are insufficient, we look for geometric relations to provide the remaining equations (constraint method).

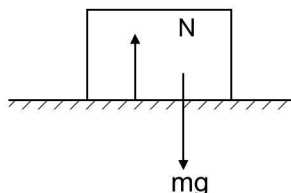
CONCEPT STRANDS

Concept Strand 8

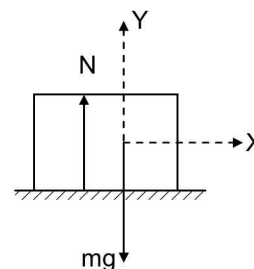
Consider a block of mass m kept on a smooth horizontal surface. What is the normal reaction exerted by the horizontal surface on the block?

Solution

Step 1: Draw FBD.



Step 2: Choose the co-ordinate axes and resolve forces along the chosen axes. X and Y axes are chosen as shown. Forces are already along the chosen axes:



Step 3: Mark motion parameters, namely accelerations along the chosen axes. Since no forces along X-axis, there will be no acceleration along the X-axis. There will be no acceleration along negative Y-axis, i.e., into the horizontal supporting surface, i.e., vertically downward. It is obvious that there can be no motion vertically upwards. This is embodied in the rule:

Along a particular direction, if the only force acting on a body is a normal reaction (or a contact force) exerted by a fixed surface or another stationary body, there can be no motion of the body in that direction.

4.14 Laws of Motion

Applying this rule, there can be no acceleration in the upward (vertical) direction either.

Step 4: Write force equation:

$$\sum F_y = -mg + N = ma_y = 0$$

$$\Rightarrow N = mg$$

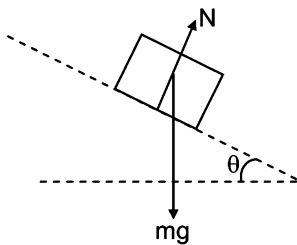
Concept Strand 9

Consider a block of mass m kept on a smooth inclined plane at an angle θ to the horizontal. Determine

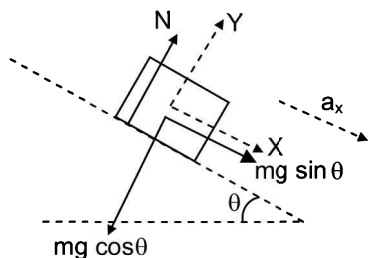
- The acceleration of the block (magnitude and direction).
- If the height of the inclined plane is h and if the body is released from rest from the top, determine its velocity when it reaches the bottom and the time to do so.
- What is the significance of the two limiting cases of the inclination, $\theta = 90^\circ$ and $\theta = 0^\circ$?

Solution

Step 1: Draw FBD



Step 2: Choose axes, resolve forces and mark acceleration.



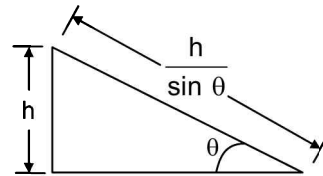
In inclined plane problems, it is convenient to choose X-axis along the plane and Y-axis normal to the plane.

$$(a) \sum F_y = N - mg \cos \theta = ma_y = 0$$

$$\sum F_x = mg \sin \theta = ma_x$$

$$\Rightarrow a_x = g \sin \theta$$

- (b) Since $a_x = g \sin \theta$ is constant, we can use $v^2 = u^2 + 2as$ and $v = u + at$ or $s = ut + \frac{1}{2}at^2$ to get v and t at the bottom.



$$v^2 = 0 + 2g \sin \theta \frac{h}{\sin \theta}$$

$$\Rightarrow v = \sqrt{2gh}$$

(same result as we get for free fall through height h).

$$\sqrt{2gh} = 0 + g \sin \theta \cdot t$$

$$\Rightarrow t = \frac{1}{\sin \theta} \cdot \sqrt{\frac{2h}{g}} \quad [\text{For free fall } t = \sqrt{\frac{2h}{g}}; \text{ hence the time taken is more than that for a free fall by a factor of } \frac{1}{\sin \theta} \text{ which is } > 1]$$

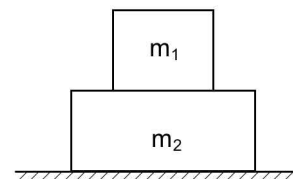
$$(c) \theta = 90^\circ \Rightarrow a = g \sin 90^\circ = g; v = \sqrt{2gh};$$

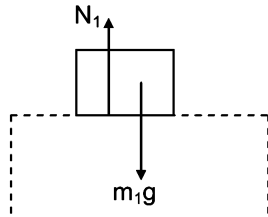
$$t = \sqrt{\frac{2h}{g}} \quad (\text{vertical free fall}).$$

$$\theta = 0^\circ \Rightarrow a = 0; v = 0 = u; t = \infty \quad (\text{at rest for infinitely long time}).$$

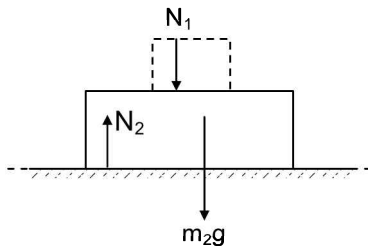
Concept Strand 10

Consider a block of mass m_1 kept on top of another block of mass m_2 , which is kept at rest on a smooth horizontal surface. Find the normal reaction exerted by m_2 on m_1 and by the floor on m_2 .



SolutionFBD of m_1 :

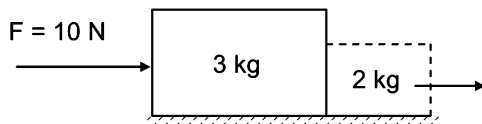
$$\sum F_y = N_1 - m_1g = ma_y = m_1 \cdot 0 = 0 \Rightarrow N_1 = m_1g$$

FBD of m_2 :Note that N_1 is the third-law reaction force by m_1 on m_2 .

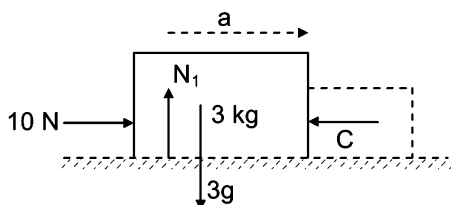
$$\begin{aligned} \sum F_y &= N_2 - N_1 - m_2g = m_2 \cdot 0 = 0 \\ \Rightarrow N_2 &= N_1 + m_2g = (m_1 + m_2)g \end{aligned}$$

Concept Strand 11

A force of 10 N is applied on a system of bodies as shown. Determine the acceleration of the bodies.

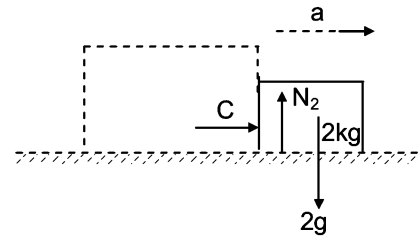
**Solution**

- (a) Since F acts in the direction so as to maintain contact between the bodies, the acceleration of both bodies will be same.

FBD of 3 kg block: $F = 10$ N

FBD of 2 kg block:

Force eqns:



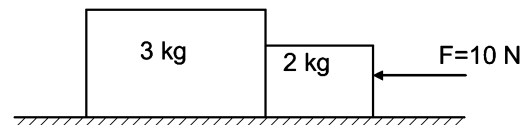
$$\text{For 3 kg body: } Y: N_1 - 3g = 0 \quad \text{--- (1)}$$

$$X: 10 - C = 3a \quad \text{--- (2)}$$

$$\text{For 2 kg body: } Y: N_2 - 2g = 0 \quad \text{--- (3)}$$

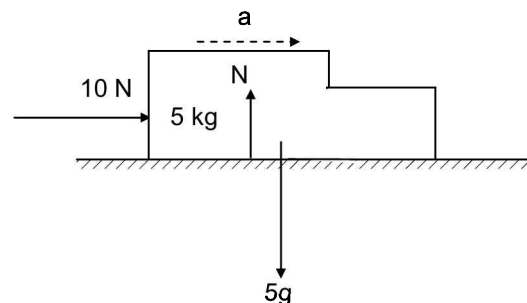
$$X: C = 2a \quad \text{--- (4)}$$

There are 4 equations and 4 unknowns (N_1 , N_2 , C , a); solving, $N_1 = 30$ N, $N_2 = 20$ N, $C = 4$ N, $a = 2$ m s⁻². Since the problem does not ask for values of N_1 and N_2 the Y-axis equations (1) and (3) need not be written. In fact the FBD's need not show forces along Y-axis ($3g$, $2g$, N_1 , N_2). Now, if the applied force $F = 10$ N acts on 2 kg body instead of 3 kg body, the acceleration a of the bodies will be same as before but is opposite direction.



a will be 2 m s⁻², but C will now be 6 N and not 4 N.

- (b) The acceleration can easily be determined by assuming a single block of $3 + 2 = 5$ kg instead of two separate masses of 3 kg, 2 kg, especially since the problem does not ask for value of the contact force between them.



$$X: 10 = 5a \Rightarrow a = 2 \text{ m s}^{-2}.$$

In other words, we are treating it as a single system; the contact force C is a pair of equal and opposite internal

4.16 Laws of Motion

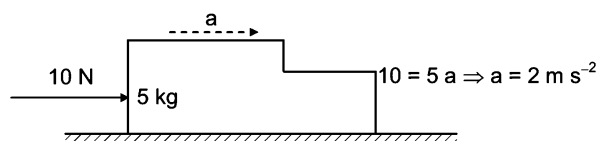
force within the system and is not an external force on the system. It is interaction force between two parts of the system, analogous to intermolecular forces inside a body.

Such a method of treating it as a single system makes problem solving much easier, especially if C need not be determined.

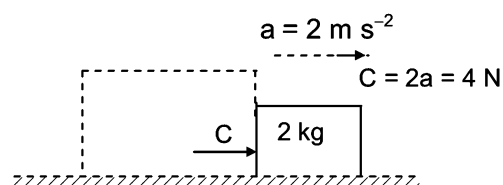
Note that sum of equations (2) and (4) written for the separate blocks gives the equation for the system, cancelling the equal and opposite internal force C .

(c) Even if C is to be determined, this method is still quicker, as the demonstrated below:

(i) Draw FBD of combined mass

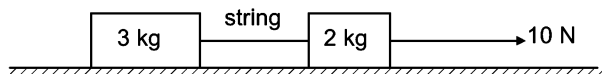


(ii) Draw FBD of only 2 kg mass.

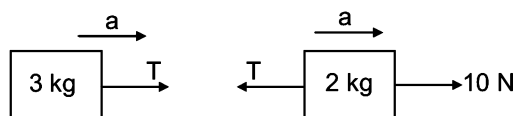


Concept Strand 12

A 3 kg block and 2 kg block are connected by a string and the 2 kg block is pulled by a force of 10 N. Calculate the tension in the string.



Solution



F.B.D

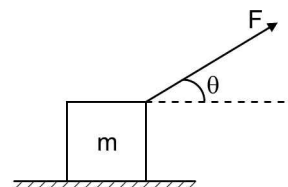
$$3a = T \quad \text{--- (1)}$$

$$2a = 10 - T \quad \text{--- (2)}$$

$$\text{Solving (1) and (2) simultaneously, } a = 2 \text{ m s}^{-2}, \\ T = 6 \text{ N}$$

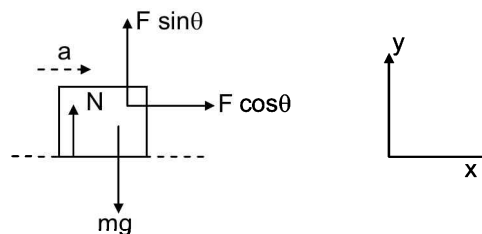
Concept Strand 13

Force F is applied on a mass m at an angle θ to horizontal. Determine acceleration of the mass and the normal reaction exerted by the surface on the mass.



Solution

FBD: Resolve F along X, Y:



$$\text{Y : } N + F \sin \theta - mg = 0 \\ \Rightarrow N = mg - F \sin \theta \quad \text{--- (1)}$$

$$\text{X : } F \cos \theta = ma \\ \Rightarrow a = \frac{F \cos \theta}{m} \quad \text{--- (2)}$$

We note two points:

(i) The normal reaction N is less than mg (unlike the previous example where the pulling force F was horizontally applied). This is logical because F when applied at an angle has a vertical component, which tends to lift the mass away from the surface and thus reduces the normal reaction.

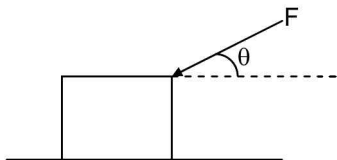
(ii) If we increase the value of F , the acceleration along X direction also increases, ($\because a = \frac{F \cos \theta}{m}$),

but N decreases ($\because N = mg - F \sin \theta$) and when N becomes zero, the block loses contact with the surface and lifts off the surface.

What is the value of F at which this happens and what is the horizontal acceleration when it happens?

$$N = 0 \Rightarrow mg = F \sin \theta$$

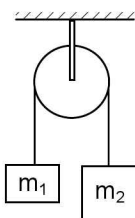
$$\Rightarrow F = \frac{mg}{\sin\theta} \text{ and } a = \frac{F \cos\theta}{m} = \frac{mg}{\sin\theta} \times \frac{\cos\theta}{m} = g \cot\theta$$



Suppose the applied F is in the opposite direction: Then N will always be more than mg .

Concept Strand 14

Two blocks of masses m_1 and m_2 ($m_2 > m_1$) are connected by an inextensible, massless string which goes around a light, smooth fixed pulley as shown. Determine the accelerations of the blocks.

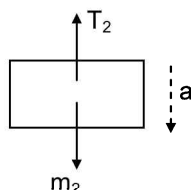


Solution

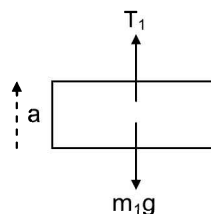
Before we attempt to solve the problem, let us understand the role of pulleys. A pulley is used to change the direction of applied force. A body can be lifted up by using a downward force by use of a pulley and a rope, similar to drawing water from a well. In the above problem, it can be easily seen that the accelerations of the two bodies are equal in magnitude with m_2 downward ($\because m_2 > m_1$) and m_1 , upward. This is because the string is inextensible. $\therefore$ During a time interval t , if displacement of m_2 is $-y$, then that of m_1 is $+y$

so that at t , $v_{m_2} = \frac{-dy}{dt} = -v_{m_1}$ and $a_{m_2} = \frac{-d^2y}{dt^2} = -a_{m_1}$

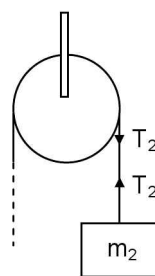
FBD of m_2 :



FBD of m_1 :

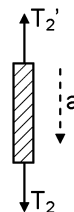


T_2 is the tension in the right part of the string (the part between the pulley and the mass m_2).



If the string is light, i.e., massless, then tension will be same at every point of that string. The proof is below:

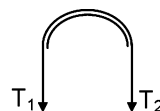
FBD of the RHS of the string between the pulley and m_2 :



Force equation: $T_2 - T_2' = \text{mass} \times a$.

But if mass of the string is zero (i.e., light string, massless) $\Rightarrow T_2 = T_2'$.

$\therefore$ FBD of the part of the string around the pulley:



If the surface of the pulley (i.e., the groove of the pulley) is smooth i.e., frictionless, then T_1 will equal T_2 (for massless string), and the pulley will not rotate. Suppose there is friction between string and pulley. Then, the pulley rotates when the string moves over it. If it does and given that there is no relative motion between string and

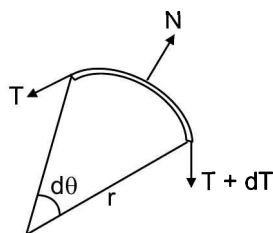
4.18 Laws of Motion

pulley (i.e., the rope does not slip over the pulley), then the rope has a linear displacement S , and the pulley's angular displacement is $\frac{S}{R}$ where R is the radius of the pulley. If pulley is given as light/massless, then $T_1 = T_2$. Therefore if pulley has mass and the rope moves with acceleration, $T_1 \neq T_2$ (and T_1 will be $< T_2$, given direction of 'a' as assumed).

Case (i) Smooth Pulley

$$(T + dT) \sin \frac{d\theta}{2} + T \sin \frac{d\theta}{2} = N$$

$$N = T d\theta$$

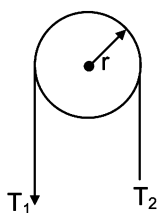


$$(T + dT) \cos \frac{d\theta}{2} - T \cos \frac{d\theta}{2} = 0$$

$$dT = 0 \quad T = \text{constant}$$

Case (ii) Massless Pulley

$(T_1 - T_2)r = I\alpha$, where I = moment of inertia of pulley about axis of rotation (explained in a later chapter)



$$I = 0 \text{ (massless)} \Rightarrow T_1 = T_2$$

Force equations:

$$T_2 = T_1 = T \quad \text{--- (1)}$$

$$m_2 g - T = m_2 a \quad \text{--- (2)}$$

$$T - m_1 g = m_1 a$$

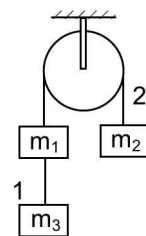
$$\Rightarrow a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

and

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

Concept Strand 15

Consider masses m_1 , m_2 and m_3 connected by inextensible strings as shown in the diagram. Strings are massless and inextensible and pulley is frictionless and light. Determine accelerations of the bodies. Will the tensions in the two strings be same or different?



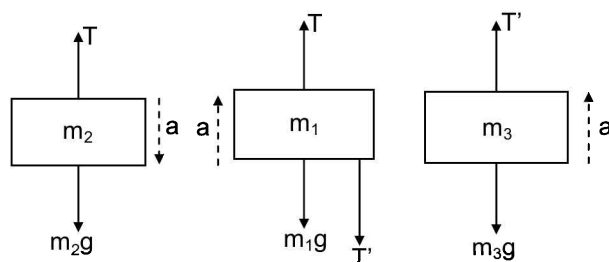
Solution

Tensions in strings 1 and 2 will be different.

Tension will be same only in any part of the same string.

It will be different if it is not the same string. This is proved below:

FBDs:



We arbitrarily assume downward acceleration for m_2 . After determining 'a' with numerical data, if it turns out to be negative, it will only mean that the actual direction is reverse of assumed direction, but with the same magnitude as obtained. There is no need to recalculate the magnitude with the reversed direction except in cases involving friction.

$$\text{Equations: } m_2 g - T = m_2 a \quad \text{--- (1)}$$

$$T - m_1 g - T' = m_1 a \quad \text{--- (2)}$$

$$T' - m_3 g = m_3 a \quad \text{--- (3)}$$

Solving,

$$a = \frac{m_2 - (m_1 + m_3)}{m_2 + (m_1 + m_3)} \cdot g$$

$$T = \frac{2m_2 \cdot (m_1 + m_3)}{m_2 + (m_1 + m_3)} \cdot g$$

$$T' = \frac{2m_2 m_3}{m_2 + (m_1 + m_3)} \cdot g$$

The expression for a and T are similar to the two body problem treating combined mass $m_1 + m_3$, with T' as internal force between them.

PROBLEMS INVOLVING CONSTRAINTS

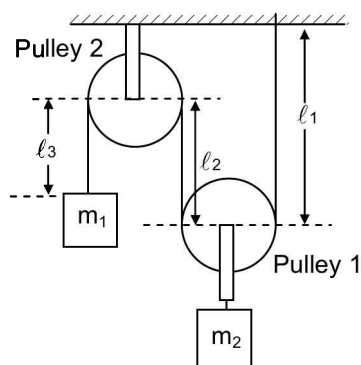
CONCEPT STRANDS

Concept Strand 16

Two blocks of masses m_1 and m_2 are connected by a light inextensible string passing over two light and frictionless pulleys, one fixed and the other movable as shown in the diagram. Determine the acceleration of the blocks.

Solution

Here the constraint is that the string is inextensible, i.e., the length of the string is constant, whatever be the motion of the bodies. This is represented by the equation



$\ell_1 + \pi R_1 + \ell_2 + \pi R_2 + \ell_3 = \text{constant}$ — (i) where R_1 and R_2 are the radii of the pulley 1 and pulley 2 respectively.

i.e., $\ell_1 + \ell_2 + \ell_3 = \text{constant}$

($\because \pi R_1 + \pi R_2$ are constants)

By differentiating (i), we get

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} = 0 \quad \text{— (ii)}$$

We have

$$\frac{d\ell_1}{dt} = \text{rate of increase in } \ell_1 = \text{velocity of } m_2 = v_{m_2}$$

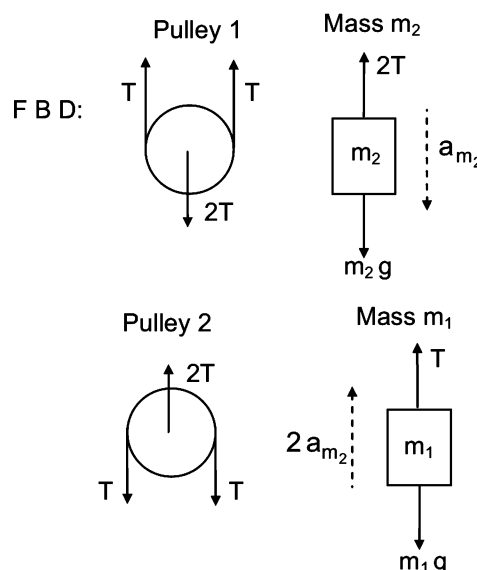
$$\frac{d\ell_2}{dt} = \text{rate of increase in } \ell_2 = \text{velocity of } m_2 = v_{m_2}$$

$$\frac{d\ell_3}{dt} = \text{rate of increase in } \ell_3 = \text{velocity of } m_1 = v_{m_1}$$

$$v_{m_2} + v_{m_2} + v_{m_1} = 0 \Rightarrow v_{m_1} = -2v_{m_2}$$

Differentiating once more, $a_{m_1} = -2a_{m_2}$

We note that the constraint in the problem helps find a relationship between the dynamical variables. We may now draw the FBD as follows:



The system of equations are

$$m_2 g - 2T = m_2 a_{m_2} \quad \text{— (iii)}$$

$$T + 2m_1 a_{m_2} = m_1 g \quad \text{— (iv)}$$

$$\text{Solving, we get } a_{m_2} = \frac{(2m_1 - m_2)}{(4m_1 - m_2)} g;$$

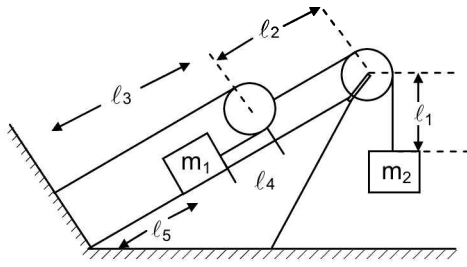
$$T = \frac{m_1 m_2}{(4m_1 - m_2)} g$$

Note that in problems involving masses, strings and pulleys, *constraint relations among the string lengths should be so chosen that they directly refer to accelerations of the bodies*. It is, therefore, necessary to adopt the rule that when choosing a portion of length of the string, one end point must be fixed and the other end point moving. This point is illustrated in the following example.

Concept Strand 17

Strings passing over pulleys as shown in the diagram connect two blocks of masses m_1 and m_2 to each other. Describe the motion of the blocks.

4.20 Laws of Motion



Solution

The constraints are

$$\ell_1 + \ell_2 = \text{constant} \quad \text{--- (i)}$$

$$\ell_2 + \ell_3 = \text{constant} \quad \text{--- (ii)}$$

$$\ell_3 + \ell_4 = \text{constant} \quad \text{--- (iii)}$$

ℓ_1 refers to acceleration a_2 of mass m_2 and ℓ_4 to a_1 of m_1 . However, ℓ_4 does not conform to the rule that one end point be fixed; both ends of ℓ_4 are moving points. Instead

of ℓ_4 , therefore, we may choose ℓ_5 , which satisfies the criterion. We write equation (iii) in the form

$$\ell_3 + (\ell_3 - \ell_5) = 2\ell_3 - \ell_5 = \text{constant} \quad \text{--- (iv)}$$

Eliminating ℓ_2 and ℓ_3 from equation (i), (ii) and (iv)

We get

$$2\ell_1 - \ell_5 = \text{constant} \Rightarrow 2a_2 - a_1 = 0$$

$$\text{i.e., } a_1 = 2a_2$$

But what happens if we still use the constraint equation

$$\ell_3 + \ell_4 = \text{Constant instead of the equation } \ell_3 - \ell_5 =$$

Constant. ℓ_4 will refer to $-a_1$ provided the pulley to which it is connected is at rest. In other words, ℓ_4 will refer $-a_{12}$ (negative of acceleration of m_1 relative to m_2) i.e., $-(a_1 - a_2)$;

Using the set equations (i) (ii) and $\ell_3 + \ell_4 = \text{Constant}$ (iii),

eliminating ℓ_2 and ℓ_3 , $\ell_1 + \ell_4 = \text{Constant}$.

$$\Rightarrow a_2 - a_1 + a_2 = 0 \Rightarrow a_1 = 2a_2 \text{ (same result)}$$

REFERENCE FRAME: INERTIAL AND ACCELERATED FRAMES

We have so far seen many situations where forces act on bodies; applied Newton's laws of motion and have determined the accelerations of these bodies. In other words, we know everything we need to know, because once we know the acceleration of a body, the initial conditions such as velocity, and the displacement at $t = 0$, we can determine its velocity and displacement at any instant.

Inertial frames

An inertial frame of reference, by definition, is one in which Newton's first law of motion is valid. Hence strictly speaking, all inertial frames are either at rest or moving with uniform velocity. That is, the absolute acceleration of an inertial frame is zero. The accelerations, which we have determined by the laws of motion, are relative to Earth, or objects fixed to Earth such as the ceiling to which the pulley is attached over which a string with bodies attached to its ends go round; such as the floor on which a 5 kg and 3 kg bodies one over the other move; such as an observer stationary on the railway platform beside which a train moves with acceleration, and so on. Practically acceleration of a body relative to Earth or points fixed to Earth is treated as 'absolute acceleration' of the body because we assume

Earth as fixed frame of reference in space (ignoring the rotation of Earth and its translatory motion in space which produce only negligibly small accelerations).

Practically, frames of reference such as points fixed/attached to Earth, i.e., frames which are at rest relative to Earth are called 'inertial frames'. Practical 'Inertial frames' also include those frames, which are not at rest relative to Earth, but move with uniform velocity relative to Earth.

Let us take an example.

A train is passing through a station with an acceleration of 2 m s^{-2} . At $t = 0$ the engine of the train is at origin and in line with

- (A) an observer standing on the platform, and
- (B) a car moving with a constant velocity of 5 m s^{-1} in the same direction as the train along a road parallel to the track; also let the velocity of the train at $t = 0$ be 10 m s^{-1}

$\therefore$ At $t = 3 \text{ s}$, the displacement of train from origin is

$$\left(\text{using } ut + \frac{1}{2}at^2\right) 10 \times 3 + \frac{1}{2} \times 2 \times 9 = 39 \text{ m}$$

Relative to the car, at $t = 0$, velocity of train is $10 - 5 = 5 \text{ m s}^{-1}$

At $t = 3 \text{ s}$, displacement of train relative to car is $39 - (5 \times 3) = 24 \text{ m}$

∴ Relative to car, acceleration of train can be calculated (using relative values in $S = ut + \frac{1}{2}at^2$)

$$24 = 5 \times 3 + \frac{1}{2}a' \times 9 \Rightarrow a' = 2 \text{ ms}^{-2}$$

which is the same value of acceleration that is relative to the stationary observer.

∴ To summarize, inertial frames are

- Frames, which are at rest relative to other inertial frames.
- Frames, which have uniform velocity relative to other inertial frames.

For practical purposes of measuring motion relative to Earth, we can treat Earth as an inertial frame.

Non-inertial frames

Non-inertial frames are those frames, which are in accelerated motion relative to inertial frames. Newton's first law of motion is not valid in non-inertial frames.

Take the same example but with a small change – the car with an acceleration of 0.5 m s^{-2} . Displacement of the train (at $t = 3 \text{ s}$) relative to car will now be

$$39 - \left(5 \times 3 + \frac{1}{2} \times 0.5 \times 9 \right) = 21.75 \text{ m}$$

∴ Recalculate a'

$$21.75 = 5 \times 3 + \frac{1}{2} a' \times 9 \Rightarrow a' = 1.5 \text{ m s}^{-2} \text{ which is different from } a = 2 \text{ m s}^{-2}$$

This value of $a' = 1.5 \text{ m s}^{-2}$ is obvious to us due to earlier knowledge of relative acceleration viz. $\bar{a}_{A/B} = \bar{a}_A - \bar{a}_B = 2 - 0.5 = 1.5 \text{ m s}^{-2}$. But we took up the illustration to show that non-inertial frames are accelerated frames, and the values of acceleration measured from non-inertial frames are different from those measured with respect to inertial frames.

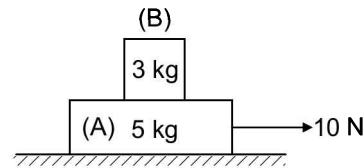
Non-inertial (accelerated) frames and pseudo forces

Forces produce accelerations. The natural forces acting on the bodies (train, the observer, the car) remain the same whatever be the frame of reference. Then why do the two frames of reference give two different acceleration values? The answer to this question is that Newton's second law of motion (i.e., identifying the forces and methods of

calculating acceleration) is not valid in non-inertial frames, if only the natural forces are considered. Another way of putting it is that we can apply the second law of motion to non-inertial frames only if we assume the existence of fictitious extra forces. To see this consider again the accelerated frame fixed on the moving train relative to the stationary frame fixed on the platform. If the acceleration of the train is ' a ', a box lying on the platform will appear to be moving with acceleration $-a$ for a person in the train. This can be accounted for by assuming a force $-ma$ acting on the box. The assumed force $-ma$ is called a pseudo force. (pseudo means artificial; it does not really exist but assumed to exist).

Let us take an example:

A body of mass 3 kg is placed on another body of mass 5 kg , which rests on ground. A force of 10 N is applied on the 5 kg body as shown. No friction anywhere.



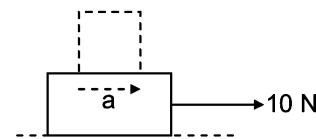
Let us calculate the accelerations of the bodies with respect to inertial frame (say ground)

FBD of (A): 5 kg
(we will ignore Y-direction)

$$10 = 5a$$

$$\Rightarrow a = 2 \text{ m s}^{-2}$$

— (1)

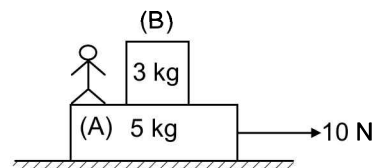


FBD of (B): 3 kg

No forces

$$\therefore a = 0$$

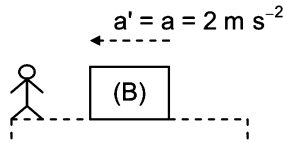
Now, let us calculate with respect to an observer stationary with respect to the block (A)



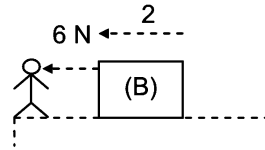
(5 kg). Assume that the observer is standing on (A)

4.22 Laws of Motion

FBD of (B)

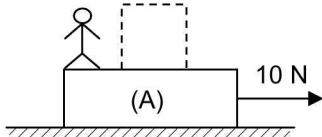


The observer registers that the block (B) is moving towards him with $a = 2 \text{ m s}^{-2}$, but there is no force acting on the block. This is not possible unless he assumes that a pseudo force equal to $3 \text{ kg} \times 2 \text{ m s}^{-2} = 6 \text{ N}$ acting as shown.

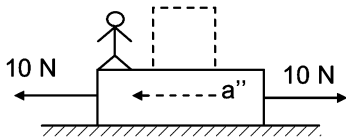


Pseudo force = mass of block $\times$ acceleration of the non-inertial frame

Similarly, FBD of block A:



The observer finds (A) at rest relative to him, but a force of 10 N acting.



$\therefore$ This is not possible unless a pseudo force
 $= 5 \text{ kg} \times 2 \text{ m s}^{-2} = 10 \text{ N}$ acts as shown.

$\therefore a'' = 0$.

To summarise:

A non-inertial observer has to assume pseudo force in order to apply the law $F = ma$ and the 'a' thus calculated is always the acceleration relative to the non-inertial frame. But, for an inertial observer or frame, there is no need for any pseudo force, and the acceleration he calculates is always the 'absolute' acceleration.

A usual phenomenon that we experience is that when we go up in a lift, we feel slightly heavier and when the lift goes down, we feel somewhat lighter. This happens when the lift moves up/moves down with acceleration. It does not happen when the lift moves with uniform velocity or when it is at rest. While most lifts are designed to move with uniform velocity, in actual operation, due to equip-

ment factors such as less than 100% efficiency, acceleration occurs, especially at the starting or stopping of the lift. Our weight is what we 'feel'. Our weight is the sum of body forces acting on us. When we are not accelerating, the body force is mg , and it is numerically equal to the normal reaction offered by the floor. When we are accelerating, every part of our body feels a pseudo force, so that the total pseudo force is ma . This may act along or against the gravity force, causing an increase or decrease in apparent weight. When we stand on a weigh scale in doctor's room, it reads mg (i.e., normal reaction = mg). When we stand on the scale within a lift which moves up with an acceleration a , the normal reaction will be $m(g + a)$ and therefore the scale will read $m(g + a)$. As we said earlier, the scale is graduated to directly read mass units and not force units. Therefore, in the above case, the scale will show

$$\frac{m}{g}(g + a) = m \left(1 + \frac{a}{g} \right) \text{ i.e., the scale will convert the force}$$

$m(g + a)$ into mass units by dividing by g . Now let us understand why it is so. FBD of a man on a lift is represented in Fig. 4.7.

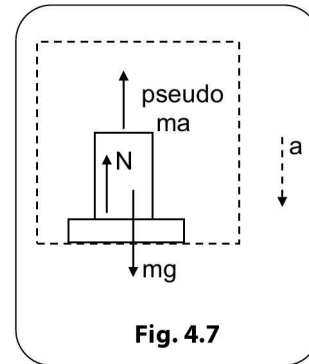


Fig. 4.7

If the lift is having acceleration a downwards, since the man is in an accelerated frame, and at rest relative to the accelerated frame, applying pseudo force = ma upwards and writing force equation for equilibrium condition,

$$\therefore ma + N = mg \quad \text{--- (1)}$$

$$\therefore N = m(g - a)$$

If the lift is having acceleration 'a' upwards, then pseudo force ma will be downwards

$$\Rightarrow mg + ma = N \Rightarrow N = m(g + a).$$

Therefore, it does not matter whether the lift is moving up or down, i.e., the direction of lift's velocity is not relevant. What is relevant is only the direction of the lift's acceleration. When the lift is going up and braking to stop at a floor, (i.e., decelerating upwards, i.e., accelerating downwards) we feel lighter. Similarly, when going down and about to stop,

we feel heavier. The above example illustrates the general method of solving problems involving accelerated frames. Whenever we need to analyze motion of bodies, which are in accelerating frames, we can follow any one of the two methods:

- (i) Usual method of inertial frame, which does not need the introduction of pseudo force
- (ii) Applying pseudo force method in non-inertial frames. *But we must remember that the acceleration of the subject body is not its absolute acceleration, but relative acceleration.*

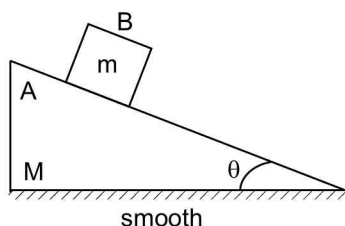
Any method can be followed for any problem; but a general guideline is:

- (i) Whenever a problem requires determination of the condition for a body to be in equilibrium with an accelerating frame, follow the pseudo force method,
 - (ii) Otherwise follow the inertial frame method.
- FBD remains same as in pseudo force method except that we mark its acceleration a downward. Therefore, the equation now becomes
- $$\text{Net force} = mg - N = ma \Rightarrow N = m(g - a)$$
- Recall that the equation under pseudo force method was:
- $$\text{Net force} = 0 \quad (\because \text{relative acceleration} = 0)$$
- $$\Rightarrow \text{Pseudo force } ma + N = mg \Rightarrow N = m(g - a)$$
- We see that both methods lead to the same equation.

CONCEPT STRANDS

Concept Strand 18

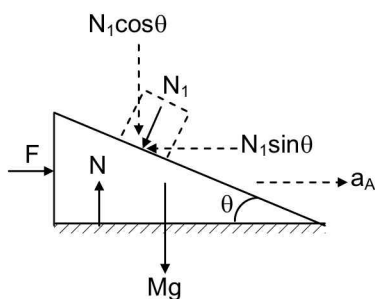
The block B of mass m is on the smooth inclined surface of wedge A (of angle θ) and mass M that is on a smooth floor. What should be the magnitude of a horizontal force F to be applied on A so that B will not slide on the inclined surface?



Solution

It is required that B is at rest relative to A. The (net) force acting on B is $mg \sin \theta$ down the incline.

$\therefore$ The pseudo force is to oppose this.

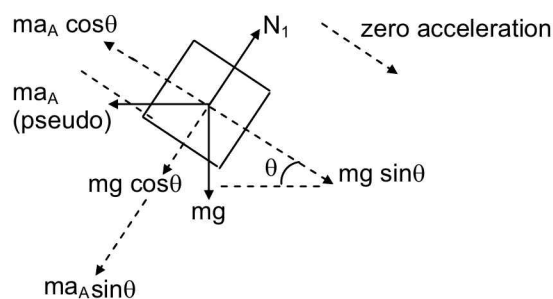


$\therefore$ F to be applied on A towards right.

FBD of A:

$$F - N_1 \sin \theta = Ma_A \quad \text{--- (1)}$$

FBD of B:



$$N_1 = mg \cos \theta + ma_A \sin \theta \quad \text{--- (2)}$$

$$ma_A \cos \theta = mg \sin \theta \quad \text{--- (3)}$$

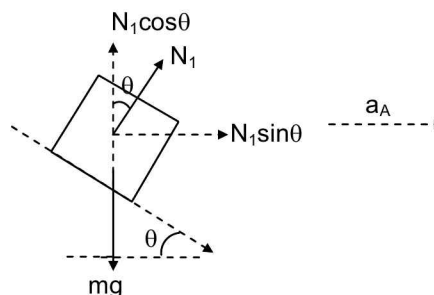
3 equations and 3 unknowns (N_1 , a_A , F). Solving,

$$F = (M + m) g \tan \theta.$$

Let us now solve the same problem without assuming pseudo force. (i.e., from inertial frame)

B to be at rest relative to A means B to have same acceleration vector as that of A.

FBD of A remains same; so is the equation



4.24 Laws of Motion

$$F - N_1 \sin \theta = Ma_A \quad \text{--- (1)}$$

FBD of B : (with respect to inertial frame $\Rightarrow$ no pseudo force).

$$N_1 \cos \theta = mg \quad \text{--- (4)}$$

$$N_1 \sin \theta = ma_A \quad \text{--- (5)}$$

$$\frac{(5)}{(4)} \Rightarrow a_A = g \frac{\sin \theta}{\cos \theta} = g \tan \theta$$

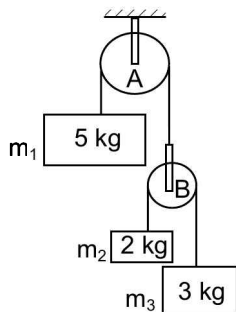
$$\begin{aligned} (1) \Rightarrow F &= N_1 \sin \theta + Ma_A = ma_A + Ma_A \\ &= mg \tan \theta + Mg \tan \theta \\ &= (M + m) g \tan \theta \end{aligned}$$

which is the same result as by the first method.

We notice that the set of equations (2) and (3) is same as the set (4) and (5).

$(5) \times \cos \theta - (4) \times \sin \theta$ gives (3). Therefore both methods are equivalent. *Any problem, which can be solved by accelerated frame approach, can also be solved by inertial frame approach.*

Concept Strand 19



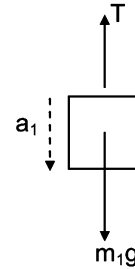
Masses of 2 kg and 3 kg are connected by a massless inextensible string, passed over a smooth and light pulley B, which in turn is connected to a mass of 5 kg by a massless, inextensible string passing over another smooth, light fixed pulley A. Determine the accelerations of the masses.

Solution

Since the total mass on either side is same ($5 \text{ kg} = 2 \text{ kg} + 3 \text{ kg}$), we may be tempted to conclude that the 5 kg mass will have no acceleration, the pulley B will therefore have no acceleration and 3 kg will have downward acceleration and 2 kg will have equal but upward acceleration. But this need not be true.

FBD of $m_1 = 5 \text{ kg}$

Let us assume the downward direction as positive and arbitrarily assume acceleration of m_1 downward.

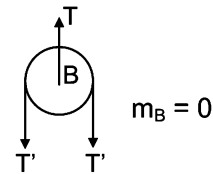


$$m_1 g - T = m_1 a_1 \quad \text{--- (1)}$$

$\therefore$ Pulley B has acceleration $= a_1$ upward.

Let accelerations of m_2 and m_3 relative to the pulley be $a_{2/B}$ upward and $a_{3/B}$ downward.

Free body diagram for pulley B.



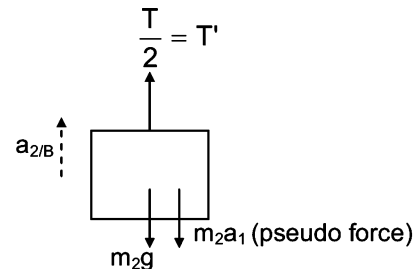
$$\therefore T = T' + T' = 2T'$$

$$\therefore T' = \frac{T}{2}$$

$\therefore$ We should apply FBD with pseudo forces for $m_2 = 2 \text{ kg}$:

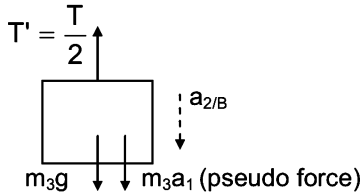
$$-\frac{T}{2} + m_2 g + m_2 a_1 = -m_2 a_{2/B}$$

$$\Rightarrow \frac{T}{2} - m_2 (g + a_1) = m_2 a_{2/B} \quad \text{--- (2)}$$



FBD of $m_3 = 3 \text{ kg}$:

Note that relative to pulley B, m_2 and m_3 will have accelerations of equal magnitude
(Constraint)



$$m_3g + m_3a_1 - \frac{T}{2} = m_3 a_{2/B}$$

$$\Rightarrow m_3(g + a_1) - \frac{T}{2} = m_3 a_{2/B} \quad \text{--- (3)}$$

$$(2) + (3) \Rightarrow a_{2/B} = \frac{m_3 - m_2}{m_2 + m_3}(g + a_1) \quad \text{--- (4)}$$

$$\text{and } \frac{T}{2} = \frac{2m_2m_3}{m_2 + m_3}(g + a_1) \quad \text{--- (5)}$$

(4) and (5) are familiar expressions similar to the case of two bodies hung from single pulley except that g is now replaced with $(g + a_1)$. (Recall the lift example. Here, pulley functions like the lift).

$$\Rightarrow T = \frac{4m_2m_3}{m_2 + m_3}(g + a_1)$$

$$\therefore (1) \Rightarrow m_1g - \frac{4m_2m_3}{m_2 + m_3}(g + a_1) = m_1a_1$$

$$\Rightarrow a_1 = \left(\frac{m_1 - \frac{4m_2m_3}{m_2 + m_3}}{m_1 + \frac{4m_2m_3}{m_2 + m_3}} \right) \cdot g$$

This looks similar to fixed pulley-two body expression except that the second mass is replaced by an equivalent mass, $\frac{4m_2m_3}{m_2 + m_3}$.

In fact, it is a reduced mass compare with to $m_2 + m_3$, because $4m_2m_3$ is always $< (m_2 + m_3)^2$.

$\therefore 4m_2m_3 = (m_2 + m_3)^2 - (m_2 - m_3)^2 = (m_2 + m_3)^2 - (\text{a positive quantity})$ and T can be obtained by substituting for a_1 . Now the absolute accelerations (with respect to Earth frame or say with respect to pulley A).

Substituting values in equations (5) and (4),

$$a_1 = \frac{5 - \frac{4 \times 2 \times 3}{2 + 3}}{5 + \frac{4 \times 2 \times 3}{2 + 3}} \times 10 = \frac{10}{49} \text{ m s}^{-2} \text{ (downward);}$$

$$a_B = \frac{-10}{49} \text{ m s}^{-2} = \frac{10}{49} \text{ m s}^{-2} \text{ (upward)}$$

$$a_{2/B} = \frac{1}{5} \times \left(10 + \frac{10}{49} \right) = \frac{100}{49} \text{ m s}^{-2} \text{ (upward)}$$

$$\therefore a_2 = a_{2/B} + a_B = \frac{-100}{49} + \frac{-10}{49} = \frac{-110}{49} \text{ m s}^{-2}$$

$$= \frac{110}{49} \text{ ms}^{-2} \text{ (upward)}$$

$$\text{and, } a_3 = a_{3/B} + a_B = \left(\frac{100}{49} + \frac{-10}{49} \right) = \frac{90}{49} \text{ m s}^{-2}$$

(downward)

Note:

Downward (+) and upward (-) has been considered in this solution.

FRICTION

The force, which opposes the relative motion or tendency for relative motion between two surfaces in contact, is called **friction**. Thus friction is a form of contact force. When the surfaces of two bodies are in contact and there is no relative motion between them or no tendency for rela-

tive motion between them, the contact force between the two bodies will be normal to the surface of contact. This force is called the normal force (N). This is the reaction of one surface over the other and it is normal to the contact surface.

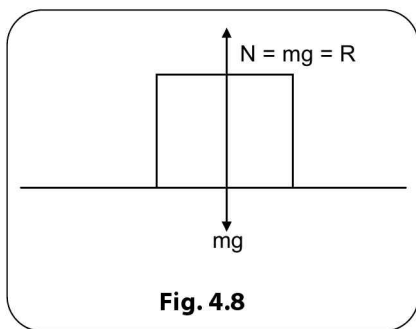


Fig. 4.8

Friction is an important force in mechanics and is important to our everyday life in many ways. Frictional force can operate between

- (i) Two solid surfaces in contact.
- (ii) A solid surface and a fluid surface.
- (iii) Two fluid surfaces in which case it is called viscous force.

Friction—a self-adjusting force

Consider a solid body X placed on the horizontal surface of a table as shown in Fig. 4.9.

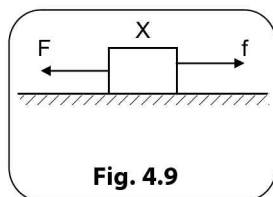


Fig. 4.9

Initially, the body is at rest as the normal reaction force exerted by the table balances its weight. It is found to be stationary if a small horizontal force F is applied on it. Here the frictional force f prevents the motion of X shown in the figure. As the applied force F is gradually increased, the frictional force f also increases such that $F = f$ in magnitude as long as the body X is in equilibrium. Hence we can say that friction is a self-adjusting force.

Origin of frictional force

The surface of any solid, no matter how polished, has many irregularities (crests and troughs or hills and valleys), which may be perhaps seen only under a powerful microscope. It is these irregularities that make contact with each other. Thus interlocking of a large number of irregularities takes place when two surfaces are in contact. This is shown in Fig. 4.10.

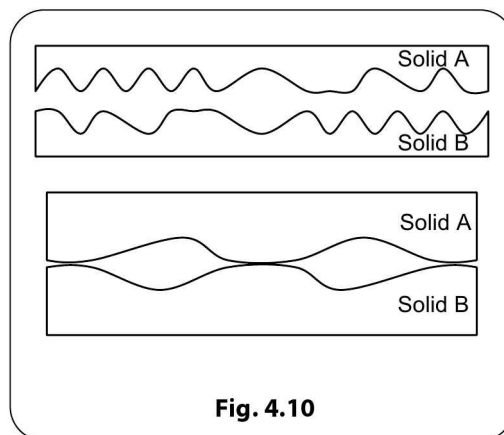


Fig. 4.10

Due to the small area of contact at the irregularities, the pressure (force per unit area of contact) at the irregularities is very high and this results in a 'cold-welded condition' between the two surfaces. This is called surface adhesion. Any external force, applied to the surfaces to produce relative motion to the contact surfaces has to rupture these interlockings. This opposing force is the force of friction. The frictional force always opposes relative motion of contact surfaces (in case of actual relative motion) or it opposes the direction of impending motion (direction in which the applied external force would cause motion, if there was no friction).

It must be noted that friction is not a fundamental force of nature. It is a secondary force, which originates from the electromagnetic forces between atoms. The friction between two objects converts a part of their kinetic energy into atomic vibrations (or heat).

There are some important characteristics of frictional force.

- (i) It is a contact force and is tangential (or parallel) to the contact surfaces.
- (ii) It *opposes relative motion of the contact surfaces* or tendency for relative motion of the contact surfaces. It should be noted that friction may not oppose the motion of a particle (because it opposes only relative motion of the contact surfaces) and friction can even cause the motion of a particle, if it is an unbalanced force (or net force) on that particle.
- (iii) It is a self-generating force, which automatically comes into play whenever there is a tendency for relative motion or actual relative motion between the contact surfaces.
- (iv) It is a self-adjusting force till actual relative motion takes place between the contact surfaces.

Types of frictional forces

The three types of frictional forces are:

- (i) Static frictional forces called static friction.
- (ii) Kinetic frictional forces called kinetic friction or dynamic friction.
- (iii) Rolling frictional forces called rolling friction. Let us examine these three types of friction.

Static friction

Static friction occurs when the two surfaces in contact are not moving relative to each other.

For example:

A book kept on a table, a heavy box kept on a railway platform, a car at rest on the road etc. all experience static friction (provided a force is acting on it). If F be the applied external force and f_s the force of static friction between the surfaces in contact then $f_s = F$ when the body is in equilibrium. When the applied force is gradually increased, the force of static friction also increases and attains a maximum value $f_s^{\max}$ (limiting friction).

The limiting friction is the maximum value of the static friction, which comes into play when a body is just going to slide over the surface of another body. The limiting friction between these contact surfaces decides the initial force needed to get an object moving relative to another object.

Sliding friction (or kinetic friction)

The frictional force, which opposes the actual relative sliding motion between two contact surfaces, is called sliding friction or kinetic friction. It is denoted as ' f_k '. It is also called dynamic friction. If the applied external force is greater than $f_s^{\max}$, then there is actual relative motion between the contact surfaces and kinetic friction ' f_k ' comes into play. It is seen that for the same normal reaction ' N ', $f_k < f_s^{\max}$. i.e., kinetic friction ' f_k ' is less than limiting friction $f_s^{\max}$ for a given pair of contact surfaces and same normal reaction.

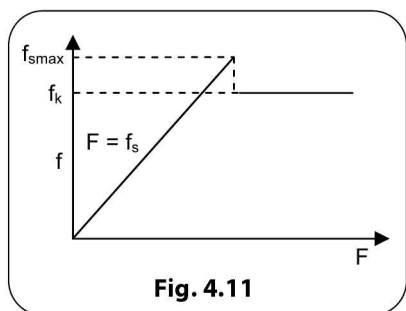


Fig. 4.11

A graph of frictional force f against the applied force F is as shown in Fig. 4.11.

Laws of limiting friction

Based on experimental observations, the following laws of limiting friction have been established.

- (i) The force of limiting friction (or any frictional force) between two surfaces in contact is always opposite to the direction of the applied tangential force (or net external tangential force).
- (ii) The force of limiting friction (or any frictional force) acts tangential to the contact surfaces.
- (iii) The magnitude of the limiting friction is directly proportional to the normal reaction between the contact surfaces.
- (iv) The magnitude of the limiting friction between two contact surfaces is independent of the shape or area of contact surfaces, provided the magnitude of the normal reaction remains the same.
- (v) The limiting friction depends upon the material and the nature of the surfaces in contact.

Coefficient of friction

According to law of friction, the limiting friction $f_s^{\max}$ is directly proportional to the normal reaction N between the contact surfaces. i.e., $f_s^{\max} \propto N$. i.e., $f_s^{\max} = \mu_s N$, where μ_s is the constant of proportionality, called co-efficient of static friction.

$$\therefore \mu_s = \frac{f_s^{\max}}{N} = \frac{\text{Limiting friction}}{\text{Normal reaction}}$$

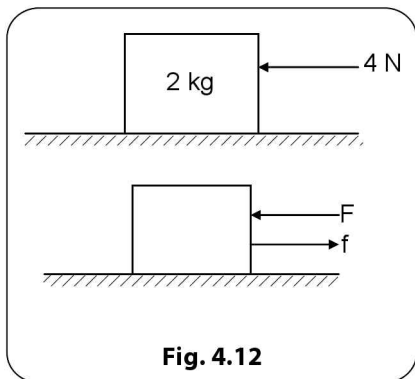
Similarly when the body is in motion $f_k \propto N$ or $f_k = \mu_k N$ where f_k is the kinetic friction and μ_k is the coefficient of kinetic friction.

It is found that

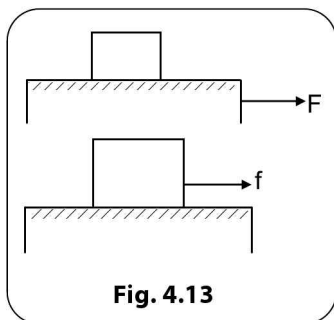
- Rough surfaces have high values of $\mu \sim 0.3$ to 0.6
- An ice block on metal surface has very low friction; therefore, μ is small.
- A rubber mat on the doorstep has a high value of μ .
- Tyre on concrete may have μ as large as 1.7 .
- Teflon on teflon has $\mu \sim 0.04$.
- $\mu = 0$ is practically impossible.

Let us illustrate with an example how friction acts. A block of mass 2 kg is at rest on a fixed table. For this pair of surfaces let the static co-efficient friction be $\mu_s = 0.3$ and the co-efficient kinetic friction be $\mu_k = 0.2$

4.28 Laws of Motion



The direction of friction force is always so as to oppose relative motion as shown in Fig. 4.12. Suppose the body is kept on the table and we try to move the table under the body as shown. The direction of f acting on the body is along same direction as F .



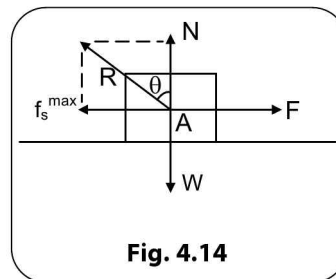
The direction of motion of block relative to table is towards left. Friction will oppose relative motion. Hence, f will act towards right.

Angle of friction

The angle made by the resultant of the force of limiting friction ($f_s^{\max}$) and the normal reaction (N) with the nor-

mal reaction (N) is called the angle of friction denoted by θ .

Consider a body A kept over the horizontal surface.



When a constant force F is applied on A such that the body is about to move, then the magnitude of limiting friction, $F = f_s^{\max}$. The resultant of $f_s^{\max}$ and N is R . The angle between

R and N gives θ . From the figure $\tan \theta = \frac{f_s^{\max}}{N}$.

But $\frac{f_s^{\max}}{N} = \mu_s$, the coefficient of static friction.

$$\therefore \tan \theta = \mu_s$$

Thus the tangent of angle of friction is equal to the coefficient of static friction. When a body is pulled along a rough surface with the pulling force applied at an angle with the horizontal, the upward component of the pulling force reduces the normal reaction on the body. This reduces the limiting friction between the contact surfaces. However, if the body is pushed along the rough surface with the pushing force applied at an angle with the horizontal, then the vertical component of this force increases the normal reaction on the body. This increases the frictional force on the body. This situation is explained in the numerical example below. Thus, *it is easy to pull a body than to push it along a rough surface.*

CONCEPT STRAND

Concept Strand 20

Two identical rollers, each of mass 100 kg, are on a rough horizontal ground. Two persons A and B are moving these rollers on the ground with same horizontal acceleration of 1 m s^{-2} , along the ground. The coefficient of friction between the roller and ground is $\mu_s = 0.25$ and $\mu_k = 0.20$. A is

pulling it at 30° with horizontal and B is pushing it at 30° with horizontal. ($g = 10 \text{ m s}^{-2}$)

- What is the net force acting on the rollers used by A and B?
- Out of A and B, who is exerting the minimum force and maximum force on the roller? Explain.

Solution

- (a) The net force acting on roller

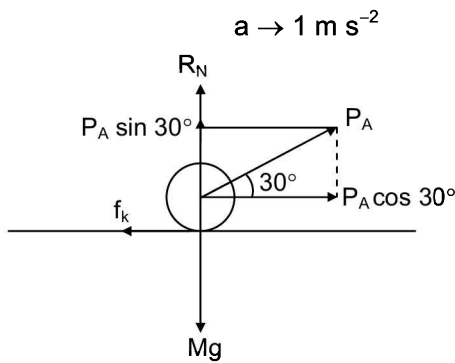
$$F_1 = \text{mass of roller} \times \text{acceleration} = Ma$$

$$= 100 \text{ kg} \times 1 \text{ m s}^{-2} = 100 \text{ N}$$

- (b) For roller used by A

Force applied by A is P_A ; its vertical component is $P_A \sin 30^\circ$ and horizontal component is $P_A \cos 30^\circ$.

$$\text{Normal reaction, } R_N = (Mg - P_A \sin 30^\circ)$$



$$\text{Frictional force } f_k = \mu_k R_N = \mu_k (Mg - P_A \sin 30^\circ)$$

$$\text{For horizontal motion, } P_A \cos 30^\circ - f_k = Ma$$

$$\therefore P_A \cos 30^\circ - \mu_k (Mg - P_A \sin 30^\circ) = Ma$$

$$\therefore P_A (\cos 30^\circ + \mu_k \sin 30^\circ) = \mu_k Mg + Ma$$

$$= 200 + 100 = 300 \text{ N}$$

$$\therefore P_A = \frac{300}{(\cos 30^\circ + 0.2 \times \sin 30^\circ)} = \frac{300}{(0.866 + 0.1)}$$

$$= 311.6 \text{ N}$$

For roller used by B, Force applied by

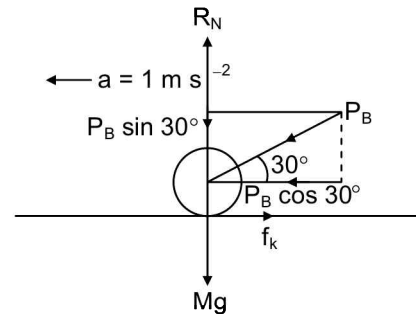
B = P_B (push)

Horizontal component = $P_B \cos 30^\circ$

Vertical component = $P_B \sin 30^\circ$ (downwards)

Normal reaction, $R_N = (Mg + P_B \sin 30^\circ)$

Frictional force $f_k = \mu_k R_N$



$$= \mu_k Mg + \mu_k P_B \sin 30^\circ$$

For horizontal motion, $P_B \cos 30^\circ - f_k = Ma$

$$\therefore P_B (\cos 30^\circ - \mu_k \sin 30^\circ) = Ma + \mu_k Mg$$

$$\therefore P_B = \frac{300}{\cos 30^\circ - 0.1 \times \sin 30^\circ} = \frac{300}{0.766} = 367.63 \text{ N}$$

Hence $P_A < P_B$

Hence the force exerted by 'A' (pulling) is less than the force exerted by B (pushing).

Angle of repose

The angle made by an inclined plane with the horizontal surface such that the body placed on the inclined plane is just at the verge of sliding, is called the *angle of repose*, denoted by α . $\tan \alpha = \tan \theta$ (where θ is the angle of friction for the contact surfaces).

Rolling friction

When a body rolls without sliding over the surface of another body (i.e., pure rolling), the frictional force which comes into play to oppose the rolling motion of the body is called **rolling friction** (f_r).

For example:

The wheels of a car rolling on ground, a cylinder rolling down an inclined plane etc. For the same magnitude of normal re-

action N, the rolling friction f_r is very much less than sliding friction f_k , i.e., $f_r \ll f_k$. This knowledge has been used in the design and construction of wheel and equipment moving on wheels. $\therefore$ Rolling friction $\ll$ sliding friction $<$ limiting

friction i.e., $f_r \ll f_k < f_s^{\max}$ Rolling friction $f_r = \mu_r \times \frac{R_N}{r}$,

where, r is the radius of the rolling body, R_N is the normal reaction and μ_r is a constant called the coefficient of rolling friction.

Unit of μ_r is metre.

Advantages and disadvantages of friction

Friction is considered as a 'necessary evil'. This means that friction has both advantages as well as disadvantages.

Advantages of friction

- (i) Friction enables us to walk on Earth. It prevents us from slipping and falling down.
- (ii) The frictional grip between the tyres and the road enables the automobiles to roll on the road. If the frictional grip is more, the rolling will be better. This is the reason for providing zig-zag grooves on the tyres of automobiles.
- (iii) The brakes of vehicles use the frictional force to stop them.
- (iv) Nails and screws, fixed to the wall and other objects, are held in place due to the frictional force.

Disadvantages of friction

- (i) Friction is responsible for wear and tear in machinery parts.
- (ii) Since extra energy is required to overcome friction, there is a reduction in efficiency of machines due to friction.
- (iii) The friction between various components converts a part of the energy into heat. It is harmful to the machine.

Methods of reducing friction

The following methods are generally used for reducing friction.

(a) Polishing

By polishing the contact surfaces within specified tolerances, the surface imperfections can be considerably reduced. This reduces the 'interlocking' of the surface imperfection and reduces friction.

(b) Lubrication

In this method a lubricant, such as oil is introduced between the contact surfaces. This lubricant fills up the im-

perfection in the contact surfaces and considerably reduces the coefficient of friction.

This reduces the frictional force considerably. For light machine, like sewing machines, watches etc., a thin oil (liquid) is used as lubricant while thick oil or paste (like grease) are used in heavy machinery and fast moving machines. Solid lubricants like graphite or talc are also used in some cases.

(c) Streamlining

Bodies that move at high speeds (like aeroplanes, rockets, cars, trains etc.) experience a large opposing force due to air friction (called as air drag or air resistance). Giving the special shape to the bodies of these objects can considerably reduce this resistance.

(d) Use of ball-bearings

Since rolling friction (f_r) is very much smaller than sliding friction (f_s) for the same normal reaction (N), by converting the sliding motion of two contact surfaces into rolling motion, we can considerably reduce the frictional force. This is achieved by using ball bearings or roller bearings. These devices use small steel balls, which roll (along with rotating parts of machinery) fixed parts or stationary parts of the machine.

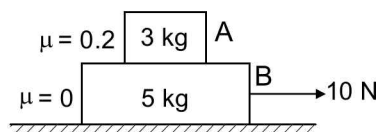
(e) By preventing ingress of dust and moisture on the contact surface

The ingress of foreign particle like sand, dust or even moisture can considerably increase friction of the contact surfaces. The presence of this film of oxide, water vapour and other absorbed materials increase the bond strength of the interlocking imperfections of the contact surfaces. Thus by preventing these impurities from entering the contact surface through suitable filters, anti-oxidising agents etc and the frictional force can be reduced significantly.

CONCEPT STRANDS**Concept Strand 21**

A 5 kg block rests on fixed, horizontal, smooth floor. A 3 kg block rests on 5 kg block; μ between them = 0.2 (take $\mu = \mu_s = \mu_k = 0.2$). Initially the blocks are at rest. Now a 10 N force is applied on 5 kg block towards right. Deter-

mine accelerations of the blocks and the friction force acting on the blocks.

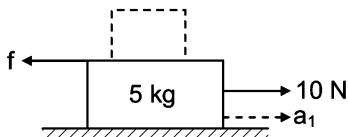


Solution

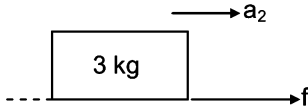
Let us first analyse this case from an inertial frame on ground. Since there is no friction on the floor, we know that the 5 kg block will definitely move. But will it drag the 3 kg block also along with it? Will the 3 kg block move with same acceleration as the 5 kg block so that it continues to sit on 5 kg at the same place on 5 kg block without any relative motion with respect to the 5 kg block? Or will the 3 kg block move (towards right) with an acceleration different (may be more, may be less) from that of 5 kg? Or will the 5 kg block slide underneath the 3 kg block with the 3 kg block having no motion at all. All these questions will be answered in our step-by-step approach.

Step 1: Draw free body diagrams.

FBD of 5 kg:



FBD of 3 kg:



Step 2: Apply Newton's second law

$$10 - f = 5a_1 \quad \text{---(1)}$$

$$f = 3a_2 \quad \text{---(2)}$$

$$f = f_{\max} = \mu N = 0.2 \times 30 = 6 \text{ N} \quad \text{---(3)}$$

($\therefore$ Using a_1, a_2 , i.e., different accelerations for the bodies means we assume relative motion between blocks $\Rightarrow f = f_{\max}$).

Therefore, solving, we get $a_1 = 0.8 \text{ m s}^{-2}$ and $a_2 = 2 \text{ m s}^{-2}$. But this is not valid.

When you apply a pulling force on a block B, sandwiched between table and a block A sitting on top, common sense tells that the top block cannot have more acceleration than that of the bottom one (the one to which we are applying the pulling force). But we should not rely on common sense. Let us prove this as shown below:

If $a_1 = 0.8 \text{ m s}^{-2}$ and $a_2 = 2 \text{ m s}^{-2}$

This means relative acceleration of 3 kg block with respect to 5 kg block is towards right $\Rightarrow f$ on 3 kg should be towards left – contrary to what has been used in the equations.

$\Rightarrow$ There cannot be relative motion between the blocks

$$\Rightarrow a_1 = a_2 = a$$

$$\Rightarrow f < f_{\max}$$

Now rewrite the equations as:

$$10 - f = 5a \quad \text{---(1)}$$

$$f = 3a \quad \text{---(2)}$$

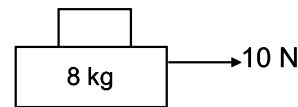
$$\text{Solving; } a = 1.25 \text{ m s}^{-2}$$

$$f = 3.75 \text{ N}$$

$\therefore$ Answer is both bodies move with 1.25 m s^{-2} acceleration and f between them = 3.75 N with directions as marked.

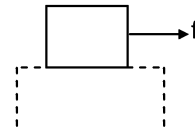
We could also solve the problem by starting with the assumption that the bodies move together.

$$\therefore a = \frac{10}{8} = 1.25 \text{ m s}^{-2}$$

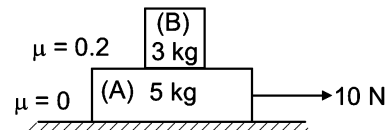


$$\therefore f = 3a = 3.75 \text{ N} < f_{\max} = 6 \text{ N}$$

Hence our starting assumption is correct.



Let us analyze the previous example from a non-inertial frame and compare the result with what we worked out already with respect to inertial frame.



We already found out that the two bodies move together with $a = 1.25 \text{ m s}^{-2}$ and friction force between them = $f = 3.75 \text{ N}$. Let us now understand how $a_{B/A}$ is zero (i.e., B at rest relative to A). Since A is a non-inertial frame, we apply pseudo force as shown:

Magnitude of pseudo force

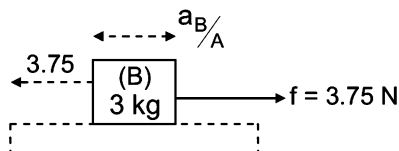
$$= \text{mass of that block (3 kg)} \times \text{acceleration of the non-inertial frame (1.25 m s}^{-2}\text{)}$$

$$= 3.75 \text{ N.}$$

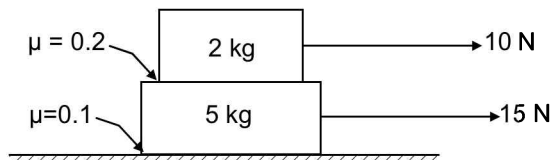
Direction of pseudo force is always opposite to the non-inertial frame's acceleration.

$$\therefore a_{B/A} = 0.$$

4.32 Laws of Motion



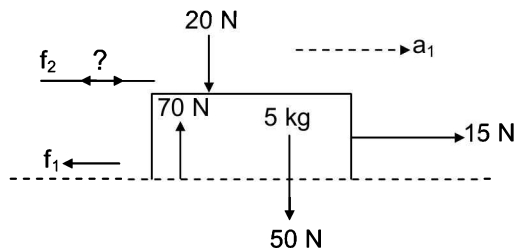
Concept Strand 22



A 2 kg body rests on a 5 kg body, which lies on a horizontal surface. The coefficient of friction between the 2 kg body and the 5 kg body is 0.2 and that between the 5 kg body and the horizontal surface is 0.1. Forces of 10 N and 15 N are applied in the same direction on the 2 kg body and the 5 kg body, respectively. Calculate the accelerations of the bodies.

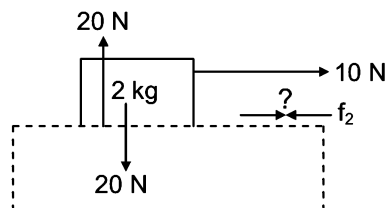
Solution

FBD of 5 kg body:



here is no difficulty in choosing direction of f_1 (between bottom surface and the 5 kg body). But what will be the direction of f_2 (between the two bodies)?

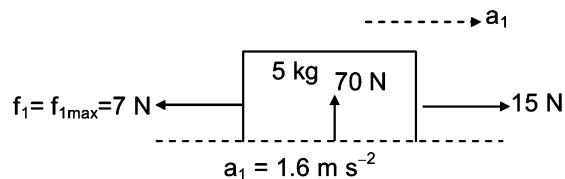
FBD of 2 kg body:



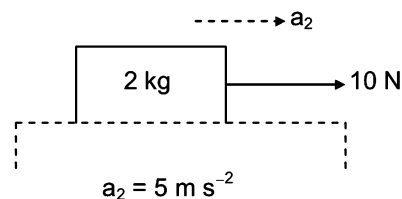
Step 1: Assume no friction between the two bodies.,

Step 2: Determine motion (in this case acceleration) of the bodies, and hence determine relative motion of the bodies.

FBD of 5 kg:



FBD of 2 kg:

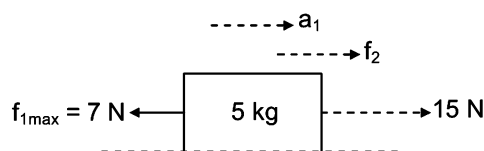


Step 3: Direction of friction will be opposing relative motion.

$\therefore$ Relative motion of 5 kg is towards left. $\therefore f_2$ on 5 kg mass is towards right.

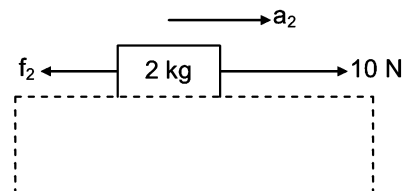
Step 4: Incorporate friction forces and recalculate accelerations, by following the procedure as per previous example.

FBD of 5 kg:



$$8 + f_2 = 5a_1 \quad \text{--- (1)}$$

FBD of 2 kg:



$$10 - f_2 = 2a_2 \quad \text{--- (2)}$$

Let there be relative motion:

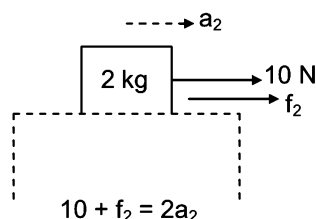
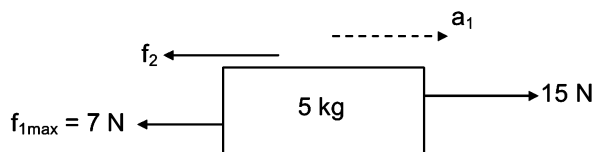
$$\Rightarrow f_2 = f_{2\max} = 4 \text{ N}; \text{ substituting in (1)}$$

$$\Rightarrow 12 = 5a_1 \Rightarrow a_1 = 2.4 \text{ m/s}^2$$

$$\text{substituting in (2)} \quad 6 = 2a_2 \Rightarrow a_2 = 3 \text{ m/s}^2$$

⇒ Relative motion direction is same as assumed. However, note that even if this procedure is not followed and if we assume the direction of friction arbitrarily, the correct solution can be arrived at eventually as shown below:

Let us assume direction of f_2 as shown.



Let there be relative motion

$$\Rightarrow f_2 = f_{2\max} = 4 \text{ N}$$

$$\Rightarrow a_1 = 0.8 \text{ m s}^{-2}$$

$$a_2 = 7 \text{ m s}^{-2}$$

⇒ Relative motion of 5 kg body is to the left

⇒ f_2 on 5 kg should be to the right

⇒ direction of f_2 is to be reversed and a_1, a_2 to be recalculated $\Rightarrow a_1 = 2.4 \text{ m s}^{-2}, a_2 = 3 \text{ m s}^{-2}$.

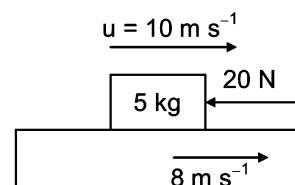
An important point to be noted is that the direction of friction force is so as to oppose relative motion i.e., relative velocity and not relative acceleration. In all the examples we encountered so far, the bodies were initially assumed to be at rest and hence direction of relative acceleration is same as that of relative velocity.

Concept Strand 23

A 5 kg body is moving on a rough surface ($\mu = 0.2$) along a straight line; at $t = 0$, its velocity is 10 m s^{-1} . From $t = 0$, a constant force $F = 20 \text{ N}$ is applied on the body in the direction opposite to its velocity. Also from $t = 0$, the plane surface is pulled along the same direction as the body's velocity with a constant velocity of 8 m s^{-1} . Determine the body's velocity at $t = 4 \text{ s}$.

Solution

Take motion towards the right as positive. Let, v_r be velocity of body relative to surface



$$u_r = (10 - 8) = 2 \text{ m s}^{-1}$$

FBD of body (from $t = 0$ to $t =$ instant at which body comes to relative rest).

$$f_{\max} = \mu mg = 0.2 \times 5 \times 10 = 10 \text{ N}$$

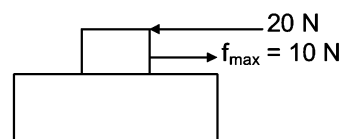


$$\therefore a = \frac{-30}{5} = -6 \text{ m s}^{-2}; v_r = u_r + at$$

$$\Rightarrow v_r = 0 \text{ at } t = \frac{2}{6} = \frac{1}{3} \text{ s}$$

∴ At $t = \frac{1}{3} \text{ s}$, the body is momentarily at rest with respect to the surface.

$$\text{FBD after } t = \frac{1}{3} \text{ s. } F_{\text{net}} = -20 + 10 = -10 \text{ N}$$



$$\therefore a = \frac{-10}{5} = -2 \text{ m s}^{-2}$$

$$\therefore v_r \text{ at } t = 4 \text{ s} = 0 - 2 \times \left(4 - \frac{1}{3}\right) = -\frac{22}{3} \text{ m s}^{-1}.$$

Velocity of the body after 4s is

$$\therefore v = v_r + \text{velocity of the surface} = -\frac{22}{3} + 8 = \frac{2}{3} \text{ m s}^{-1} \text{ (to the right)}$$

CIRCULAR MOTION AND CENTRIPETAL FORCE

In circular motion of radius R a particle has a radial acceleration known as centripetal acceleration, $a_r = \frac{v^2}{R}$. If v is uniform speed, a_r is constant in magnitude; if v is variable a_r is instantaneous radial acceleration and v is the instantaneous velocity. R of course is constant for circular motion. Since a resultant force causes acceleration, centripetal acceleration (we may use 'radial' or 'centripetal' interchangeably for circular motion) is caused by

centripetal force = mass $\times$ centripetal acceleration

$$= m \cdot \frac{v^2}{R} = m\omega^2 R = mv\omega$$

The direction of centripetal force is radial and towards the centre.

Note:

Without a centripetal force, no circular motion is possible. The centripetal force is provided by real forces in nature (friction, tension in string, gravitational force, magnetic force etc). Centripetal force can be a contact force or a non-contact force.

Consider a particle of mass m attached to one end of a string, performing uniform circular motion on a horizontal plane with the other end of the string as the centre of the circle. Length of the string is R and the horizontal plane is a frictionless surface of a table. So, in the vertical direction mg is balanced by table's normal reaction N . On the horizontal plane, the FBD is as shown in Fig. 4.15.

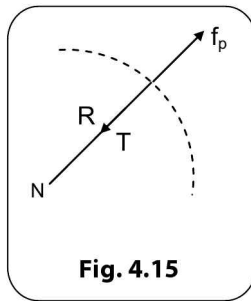


Fig. 4.15

So the force equation relative to an inertial frame is

$$T = \frac{mv^2}{R}$$

From this equation, it is clear that the tension in the string will adjust itself to the required value depending upon velocity of the particle. If v is low, tension will be low;

if v is high, T will be high. If v is zero, T will be zero; i.e., no tension, i.e., the string will slacken. If the particle is in circular motion with varying speed v , then instantaneous acceleration

$$a_t = a_{\text{tangential}} = \frac{dv}{dt}.$$

Relative to the rotating frame of the body (non-inertial frame), the radial inward tension is cancelled by the radial outward centrifugal force (explained later in this section), giving net zero acceleration.

Applications of centripetal force

(a) Cyclist on a curve

While riding on straight roads, cyclists keep their cycles normal to the road so that the weight ' mg ' of the cyclist and cycle is balanced by the normal reaction ' N ' of the ground. No other force is theoretically needed to keep the cycle moving with constant velocity.

However, while riding on curved roads, a centripetal force is needed to keep the cyclist in the circular path. This centripetal force can be provided in two ways.

(i) By frictional force between the tyre and the road

Let ' m ' be the mass of cycle and rider, v = speed of the cycle on the curved road, r = radius of curvature of the road, g = acceleration due to gravity and μ = co-efficient of friction between the tyre and road.

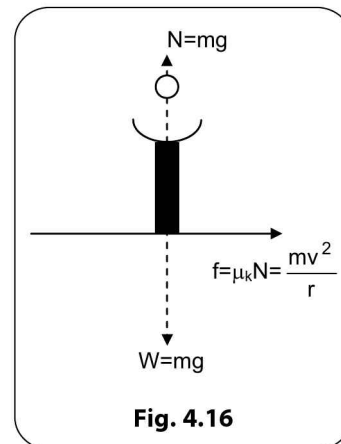


Fig. 4.16

If the cycle is kept normal to the road, weight $w = mg$ acts on the road and the normal reaction $N = W = mg$ acts on the cycle. The limiting friction force between road and cycle is equal to $f = \mu N = \mu mg$. If the frictional force acts radially on the cycle (i.e., it is directed towards the centre of the curved path), then it can provide the centripetal force

$$F_c, \text{ if } f = F_c. \text{ But } F_c = \frac{mv^2}{r}$$

$$\therefore f = \mu mg = F_c = \frac{mv^2}{r}$$

$$\therefore \mu g = \frac{v^2}{r} \quad (\text{Note: It is immaterial what the mass of the cycle is. It could be any object moving with a velocity } v)$$

$$\therefore v^2 = \mu rg \text{ or}$$

$v = \sqrt{\mu rg}$. This is the maximum speed with which the cyclist can travel on the curved road.

Note:

Problems of blocks at rest, on rough rotating horizontal platforms, can also be solved by the same method.

(ii) By inclining the cycle at an angle ' θ ' with the vertical

The required centripetal force can be produced by the cyclist by inclining his body (and the cycle) at an angle ' θ ' with the vertical.

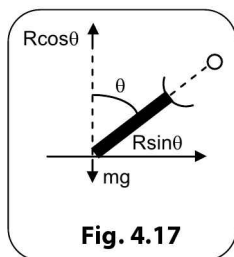


Fig. 4.17

Now the cycle is exerting a force ' F ' on the ground at an angle ' θ ' with the vertical. The reaction ' R ' of the ground is equal and opposite. This reaction has a vertical component, called normal reaction N , which is equal to $R \cos \theta$.

This component balances the weight ' mg ' of the cycle.

$$\therefore N = R \cos \theta = mg \quad \text{--- (1)}$$

The horizontal component of the reaction $F_H = R \sin \theta$ provides the centripetal force for circular motion.

$$\therefore F_H = F_c = \frac{mv^2}{r}$$

$$\therefore R \sin \theta = \frac{mv^2}{r} \quad \text{--- (2)}$$

$$\text{Equation (2)} \div \text{(1)} \rightarrow \frac{R \sin \theta}{R \cos \theta} = \frac{mv^2}{r} \times \frac{1}{mg}$$

$$\therefore \tan \theta = \frac{v^2}{rg} \quad (\text{Again mass is immaterial})$$

$$\therefore \theta = \tan^{-1} \left(\frac{v^2}{rg} \right) \text{ or } v = \sqrt{rg \tan \theta}$$

For a given road of fixed value of ' r ', larger value of $\tan \theta$ will enable the cyclist to travel with larger speed. Usually in motorcycle race, the speed ' v ' of the vehicles are very high. Hence the θ (angle of inclination with vertical) for the rider is also very high as seen in sports events.

(b) Banking of roads and rail-tracks

While it is possible for a cyclist to incline his body at an angle ' θ ' with the vertical on curved roads, it may not be possible with heavy vehicles like trucks, cars, trains etc. Hence to enable heavy vehicles to move with suitable speeds on curved tracks, the track (or road) itself is given some slope so that vehicles entering them will be automatically inclined at certain angle ' θ ' with the vertical. The outer edge of the road is kept slightly raised than the inner edge so that there is a slope ' θ ' for the road. This process of making the roads and railway tracks inclined at curves is called "banking of curved tracks".

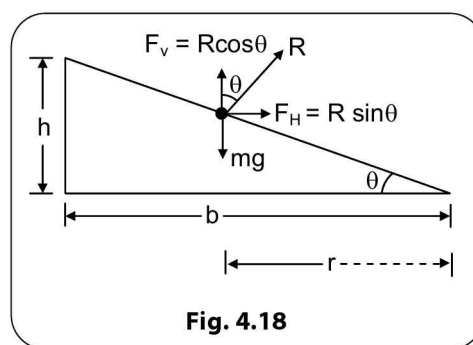


Fig. 4.18

$$\tan \theta = \frac{h}{b}, \text{ where } h = \text{height of outer edge of road}$$

above the inner edge and

$b = \text{width of the road}$

R is the reaction of road on vehicle. Horizontal component of R is, $F_H = R \sin \theta$. This component provides the centripetal force $\frac{mv^2}{r}$, where $r = \text{radius of circular track}$.

4.36 Laws of Motion

$$\therefore F_H = R \sin\theta = \frac{mv^2}{r} \quad \text{--- (1)}$$

The vertical component $F_v = R \cos\theta$ balances the weight 'mg'

$$\therefore F_v = R \cos\theta = mg \quad \text{--- (2)}$$

$$(1) \div (2) \rightarrow \frac{R \sin\theta}{R \cos\theta} = \frac{mv^2}{r} \div mg$$

$$\therefore \tan\theta = \frac{v^2}{rg}$$

$$\text{But } \tan\theta = \frac{h}{b}, \text{ so } \frac{h}{b} = \frac{v^2}{rg}$$

$$\therefore h = \frac{bv^2}{rg} \text{ is the height of far end of road above the}$$

near end.

$$\text{Also } v = \sqrt{\frac{rgh}{b}} \text{ is the limiting speed of the vehicle on}$$

the banked road. This is independent of the mass of the vehicle. In the above analysis, we have not considered the friction between the vehicle and the road. If friction between the wheels of the vehicle and the contact surface on the banked road is not negligible, then the friction also must be considered, with proper signs, in the force analysis. If the tendency for the vehicle is to skid down the banked road, friction will act along the road in the upward direction; if the tendency for the vehicle is to skid up the banked road, then the friction will act along the road in the downward direction.

Centrifugal force

A particle in rotational motion is in accelerated motion due to the centripetal acceleration. Hence any frame of reference attached to a body in circular motion (or rotational motion) is an accelerating frame of reference and hence it is a non-inertial frame of reference. Bodies inside non-inertial frame of reference experience a fictitious force or pseudo force. This fictitious force or pseudo force, experienced by bodies in circular motion or rotational motion, is called the centrifugal force. It is fictitious in the sense that it has no physical origin i.e., it is not caused by one of the basic interaction in nature. This "action" does not have the "reaction" as required by Newton's third law.

Explanation of centrifugal force

Consider a man B of mass 'm', standing at a distance 'r' from the centre, on a horizontal rough table which rotates about a vertical axis through its centre, with a constant angular velocity ' ω '. According to an observer A on the ground (or the inertial frame), the man B is moving along a circular path with a velocity ' v ' such that $v = r\omega$.

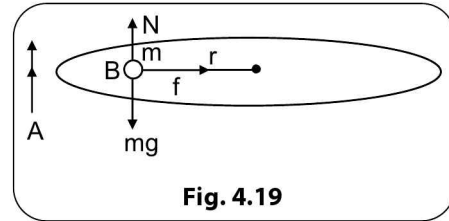


Fig. 4.19

The friction 'f' between B and the table provides the centripetal force on B. i.e., $f = \frac{mv^2}{r}$. According to B, standing on the rotating table, he is at relative rest on the table, so that the resultant force acting on B is zero.

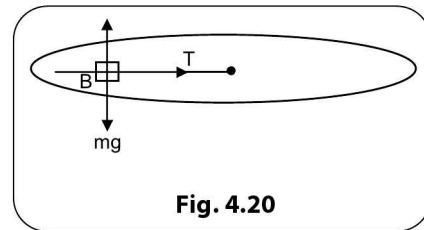


Fig. 4.20

Since the resultant force measured by observer B is zero, he will feel that an outward force equal to friction 'f' acts on B, which is called the **centrifugal force**. $\therefore$ Centrifugal force + friction = 0 (for observer B). Centrifugal force is equal and opposite to centripetal force only in the rotating frame of reference. They are not action and reaction pair because they act on the same body in the rotating frame of reference (non-inertial frame).

Applications of centrifugal force

- (i) Centrifugal pumps and centrifugal drying machines make use of centrifugal force in their working.
- (ii) Centrifuges make use of centrifugal force for separation of heavy suspended particles from liquids.

SUMMARY

$$p = mv$$

$$F = \frac{dp}{dt}$$

$$F = ma$$

$$F = v \frac{dm}{dt}$$

$$J = F\Delta t$$

$$R_1 = m(g+a)$$

$$R_2 = m(g-a)$$

$$F_s = \mu_s N$$

$$F_k = \mu_k N$$

$$\tan \theta = \mu_s$$

$$v = \frac{-m}{M} V$$

$$T = -u \frac{dm}{dt}$$

$$a_1 = \frac{1}{M} u \frac{dm}{dt}$$

$$a_2 = \frac{1}{M} u \frac{dm}{dt} - g$$

$p \rightarrow$ linear momentum, $m \rightarrow$ mass, $v \rightarrow$ velocity

$F \rightarrow$ applied force

$p \rightarrow$ momentum

$F \rightarrow$ applied force

$m \rightarrow$ mass (constant)

$a \rightarrow$ acceleration

$v \rightarrow$ velocity

$\frac{dm}{dt} \rightarrow$ rate of change of mass

$J \rightarrow$ impulse

$F \rightarrow$ force

$\Delta t \rightarrow$ time interval

$R_1 \rightarrow$ apparent weight when a body is accelerating upwards.

$a \rightarrow$ upward acceleration of the body.

$R_2 \rightarrow$ apparent weight when a body is accelerating downwards.

$a \rightarrow$ downward acceleration of the body.

$F_s \rightarrow$ Limiting friction.

$\mu_s \rightarrow$ coefficient of static friction.

$N \rightarrow$ normal reaction

$F_k \rightarrow$ Kinetic friction.

$\mu_k \rightarrow$ coefficient of kinetic friction.

$\theta \rightarrow$ angle of friction (angle of repose)

$\mu_s \rightarrow$ coefficient of static friction

$v \rightarrow$ recoil velocity of gun

$V \rightarrow$ velocity of bullet

$m \rightarrow$ mass of bullet

$M \rightarrow$ mass of gun

$T \rightarrow$ thrust on rocket

$u \rightarrow$ Relative speed of exhaust gas w.r.t rocket

$\frac{dm}{dt} \rightarrow$ rate of decrease of mass (or rate of fuel consumption)

$a_1 \rightarrow$ acceleration of rocket in the absence of gravitational force

$a_2 \rightarrow$ acceleration of rocket in the presence of gravitational force

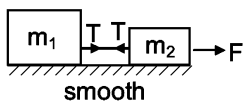
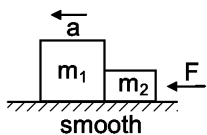
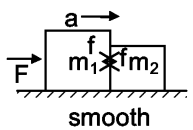
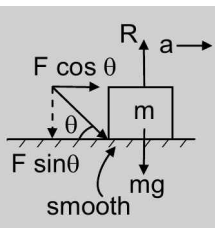
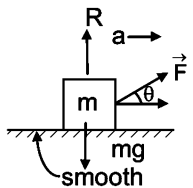
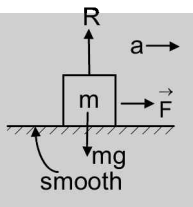
$M \rightarrow$ mass of the rocket plus mass of fuel remaining

4.38 Laws of Motion

$$a = \frac{v^2}{r}$$

$$F = \frac{mv^2}{r}$$

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$



$a \rightarrow$ centripetal acceleration

$v \rightarrow$ linear velocity

$r \rightarrow$ radius of circular path

$F \rightarrow$ centripetal force

$m \rightarrow$ mass of particle

$\theta \rightarrow$ Banking angle or inclination of cyclist to the vertical

$v \rightarrow$ linear velocity

$r \rightarrow$ radius of curvature

$g \rightarrow$ acceleration due to gravity

$$R = mg$$

$R \rightarrow$ Normal reaction

$$a = \frac{F}{m}$$

$F =$ horizontal pull

$F =$ inclined pull

$$R = mg - F \sin \theta$$

$$a = \frac{F \cos \theta}{m}$$

$$R < mg$$

$F =$ inclined push

$$R = mg + F \sin \theta$$

$$a = \frac{F \cos \theta}{m}$$

$$R > mg$$

$$f = \frac{m_2 F}{m_1 + m_2}$$

$f \rightarrow$ force of contact

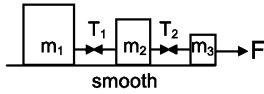
$$a = \frac{F}{(m_1 + m_2)}$$

$$f = \frac{m_1 F}{m_1 + m_2}, f \rightarrow \text{contact force}$$

$$a = \frac{F}{(m_1 + m_2)}$$

$$a = \frac{F}{m_1 + m_2}$$

$$T = \frac{m_1 F}{m_1 + m_2}$$

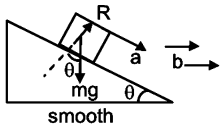


$$a = \frac{F}{m_1 + m_2 + m_3}$$

$$T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$$

$$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$$

F = horizontal pull

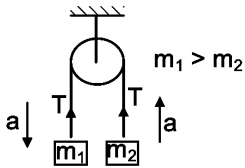


When the plane is moving horizontally with an acceleration b

$$a = g \sin \theta - b \cos \theta$$

$$R = m (g \cos \theta + b \sin \theta)$$

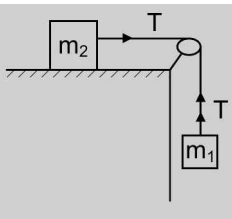
R = normal reaction



Light and smooth pulley, light inextensible thread

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$



For smooth surface

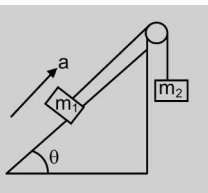
$$a = \frac{m_1 g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2}$$

If the surface is rough

$$a = \frac{(m_1 - \mu m_2)g}{m_1 + m_2}$$

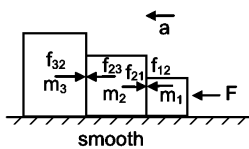
$$T = \frac{m_1 m_2 g}{m_1 + m_2} (1 + \mu)$$



All contact surfaces are smooth

$$a = \frac{(m_2 - m_1 \sin \theta)g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2} (1 + \sin \theta)$$



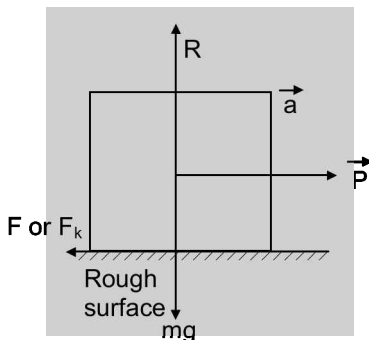
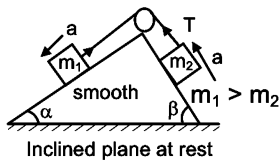
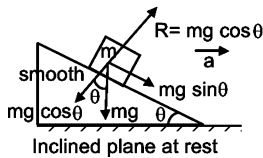
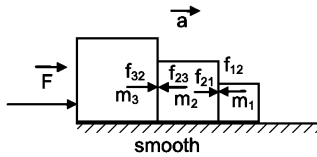
$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

= acceleration of system

$$f_3 = m_3 a = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$

$$(F - f_1) = m_1 a$$

4.40 Laws of Motion



$$f_{12} = (m_2 + m_3)a = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$$

f_{12} and f_{23} are the contact forces at the relevant surface

$$\frac{f_{12}}{f_{23}} = \frac{m_2 + m_3}{m_3}$$

$$a = \frac{F}{(m_1 + m_2 + m_3)}$$

$$f_{12} = m_1 a = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

$$f_{23} = (m_2 + m_1)a = \frac{(m_2 + m_1)F}{(m_1 + m_2 + m_3)}$$

$$\frac{f_{12}}{f_{23}} = \frac{m_1}{(m_1 + m_2)}$$

Block moving on smooth inclined plane at rest.

Acceleration along the plane

$$a = \frac{mg \sin \theta}{m} = g \sin \theta$$

Reaction of plane, $R = mg \cos \theta$

Block moving on smooth inclined plane at rest.

Acceleration of blocks,

$$a = \frac{(m_1 \sin \alpha - m_2 \sin \beta)g}{(m_1 + m_2)}$$

$$T = \frac{m_1 m_2 g (\sin \alpha + \sin \beta)}{(m_1 + m_2)}$$

where, T = tension

Block of mass ' m ', is moving with an acceleration ' a ', on a rough horizontal surface

P = applied force

F = limiting friction

R = Normal reaction

μ_s = coefficient of static friction

μ_k = coefficient of kinetic friction

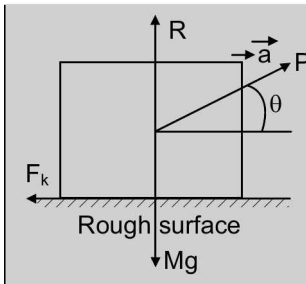
F_k = kinetic friction or sliding friction

$F = \mu_s R = \mu_s mg$

$F_k = \mu_k R = \mu_k mg$

$$a = \frac{(P - F_k)}{m} = \frac{P}{m} - \mu_k g$$

If block is moving with uniform velocity, $a = 0$. Then $P = F_k = \mu_k mg$



The applied force 'P' on block is applied at angle θ with the horizontal. P is a pull

$$R + P\sin\theta = mg$$

$$\text{or } R = mg - P\sin\theta$$

$$F_k = \mu_k R = \mu_k (mg - P\sin\theta)$$

$$ma = P\cos\theta - F_k$$

$$= P\cos\theta - \mu (mg - P\sin\theta)$$

$$\theta = \tan^{-1} \mu$$

$$P_{\min} = \frac{mg}{\sqrt{1 + \mu^2}};$$

P is a push at an angle ' θ ' with horizontal.

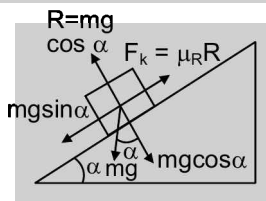
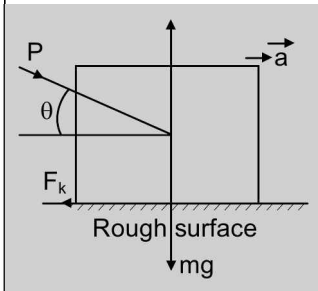
$$R = mg + P\sin\theta$$

$$F_k = \mu R = \mu [mg + P\sin\theta]$$

$$Ma = P\cos\theta - F_k$$

$$= P\cos\theta - \mu (mg + P\sin\theta)$$

It is more easy to pull than to push a block on rough surface.



Block sliding down a rough inclined plane (α = angle of incline with horizontal)

$$R = mg \cos\alpha$$

$$F_k = \mu_k R = \mu_k mg \cos\alpha$$

$$ma = mg \sin\alpha - F_k$$

$$= mg \sin\alpha - \mu_k mg \cos\alpha$$

$$a = g[\sin\alpha - \mu_k \cos\alpha]$$

For limiting friction, $\tan\alpha = \mu$ and block slides down the plane with uniform velocity.

Block being pulled up by a parallel force P, along a rough inclined plane.

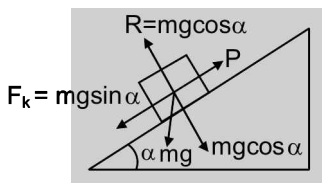
$$R = mg \cos\alpha$$

$$F_k = \mu_k R = \mu_k mg \cos\alpha$$

$$ma = P - [mg \sin\alpha + F_k]$$

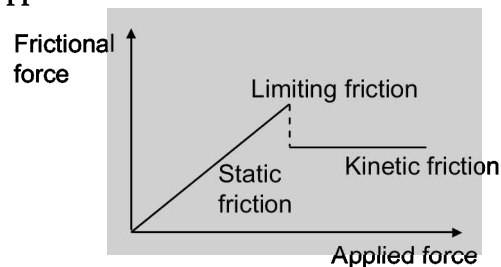
$$= P - (mg \sin\alpha + \mu_k mg \cos\alpha)$$

$$= P - mg[\sin\alpha + \mu_k \cos\alpha]$$



Important Graphs

Variation of frictional force with applied force



CONCEPT CONNECTORS

Connector 1: The force $\vec{F}$ exerted on a particle of mass 5 kg is given by $\vec{F} = (5\hat{i} + 15\hat{j})$ N. If the particle starts from rest, from the origin, find its position at a time $t = 3$ s.

Solution: $\vec{F} = m\vec{a}$

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{S} = \frac{1}{2}\vec{a}t^2 = \frac{1}{2}\left(\frac{\vec{F}}{m}\right)t^2$$

Position after 3 second is given by $\vec{r} = \frac{1}{2} \frac{(5\hat{i} + 15\hat{j})}{5} \times 3^2 = (4.5\hat{i} + 13.5\hat{j})$ m

Connector 2: A force produces an acceleration a_1 on a body of mass m_1 and a_2 on a body of mass m_2 . Find the acceleration produced by the same force on the combined mass $(m_1 + m_2)$.

Solution: Let F be the force acting

$$F = m_1 a_1 \Rightarrow m_1 = \frac{F}{a_1} \quad \text{--- (1)}$$

$$F = m_2 a_2 \Rightarrow m_2 = \frac{F}{a_2} \quad \text{--- (2)}$$

Then $F = (m_1 + m_2) a$ --- (3) where a is the acceleration of combined mass

$$F = \left(\frac{F}{a_1} + \frac{F}{a_2} \right) a \Rightarrow a = \frac{a_1 a_2}{a_1 + a_2}$$

Connector 3: A body of mass 10 kg is acted upon by a force 10 N for 2 second, then 20 N for 3 second and 30 N for 4 second in the same direction. Find the final velocity of the body.

Solution: Velocity = $\frac{\text{Total Impulse}}{\text{mass}}$

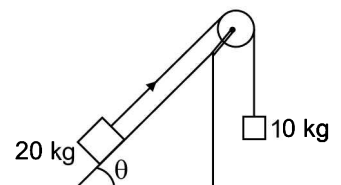
$$= \frac{F_1 t_1 + F_2 t_2 + F_3 t_3}{m} = \frac{10 \times 2 + 20 \times 3 + 30 \times 4}{10} = \frac{200}{10} = 20 \text{ m s}^{-1}$$

Connector 4: A ball of mass 0.2 kg moving with a velocity 10 m s^{-1} hits a bat normally and returns with a velocity of 20 m s^{-1} . If the duration of impact is 0.01 s, find the average force exerted by the bat on the ball.

Solution: Average force = $\frac{\text{change in momentum}}{\text{time}} = \frac{(mu_2 - mu_1)}{t} \left\{ \begin{array}{l} u_2 = 20 \text{ m s}^{-1} \\ u_1 = -10 \text{ m s}^{-1} \end{array} \right\}$

$$= \frac{0.2(20 - (-10))}{0.01} = \frac{6}{0.01} = 600 \text{ N}$$

Connector 5: Two bodies of masses 10 kg and 20 kg are connected by a light inextensible string over a smooth light pulley at the top of an incline as shown. If the system is at rest, find the angle of incline and force exerted by the incline on the 20 kg mass.



Solution: Let $m_1 = 20$ kg and $m_2 = 10$ kg. Let T be the tension in the string, and θ the angle of the incline.

Here we can consider the mass m_2 as a system.

$$\text{i.e., } T = m_2 g \quad \text{--- (1)}$$

Now considering the mass m_1 as a system, we can write

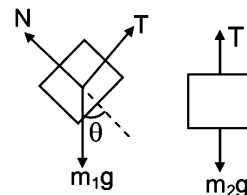
$$T = m_1 g \sin \theta \quad \text{--- (2)}$$

$$N = m_1 g \cos \theta \quad \text{--- (3)}$$

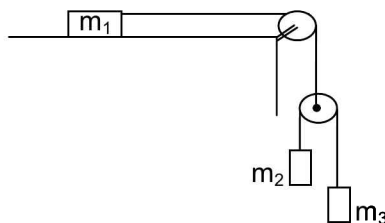
$$\text{From (1) \& (2) } m_2 g = m_1 g \sin \theta$$

$$\text{or } \sin \theta = \frac{m_2}{m_1} \text{ or } \theta = \sin^{-1} \left(\frac{m_2}{m_1} \right) = \sin^{-1} \left(\frac{10}{20} \right) = \frac{\pi}{6} \text{ rad}$$

$$\text{and from (3), } N = m_1 g \cos \theta = 20 \times 10 \times \frac{\sqrt{3}}{2} = 100\sqrt{3} \text{ N}$$



Connector 6: Three bodies of masses m_1 , m_2 and m_3 ($m_3 > m_2 > m_1$) are connected as shown in the figure. Neglecting the force of friction, find the acceleration of the body of mass m_1 . Pulleys are massless and frictionless, strings are light and inextensible.



Solution: Let a_1 be the acceleration of the body of mass m_1 towards right. The acceleration of the pulley (downwards) will also be the same. Since $m_3 > m_2$, m_2 will move upward and m_3 downward. Let a_2 be the acceleration of m_2 and m_3 with respect to the pulley. (Acceleration of m_2 upwards and that of m_3 downwards)
The pulley is considered massless. The forces on the pulley are T upwards and $2T'$ downwards.

$$\therefore 2T' - T = 0 \Rightarrow T' = \frac{T}{2} \quad \text{--- (1)}$$

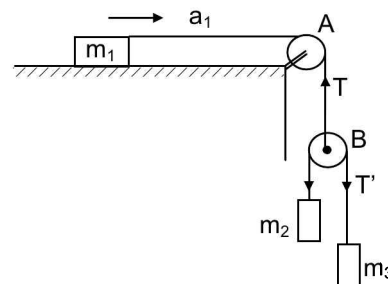
Consider the motion of m_1 . Here $T = m_1 a_1$ --- (2)

Consider the motion of m_2 . Acceleration of m_2 is $(a_1 - a_2)$ downward.

$$\therefore m_2 g - T' = m_2 (a_1 - a_2) \text{ or } m_2 g - \frac{T}{2} = m_2 (a_1 - a_2) \quad \text{--- (3)}$$

For the motion of m_3 acceleration is $(a_1 + a_2)$ downward.

$$\therefore m_3 g - \frac{T}{2} = m_3 (a_1 + a_2) \quad \text{--- (4)}$$



$$\text{Using equation (2), (3) and (4) we have, } a_1 = \frac{g}{1 + \frac{m_1}{4} \left(\frac{1}{m_2} + \frac{1}{m_3} \right)}$$

Connector 7: In the given set up the pulley is massless, smooth and fixed and the string is light and inextensible. Find the tension in the string connecting m_2 and m_3 . Given $m_1 < (m_2 + m_3)$

4.44 Laws of Motion

Solution: Let T be the tension in the string between m_1 and m_2 and T_1 the tension in the string between the bodies m_2 and m_3 . Also, let a be the acceleration. Then

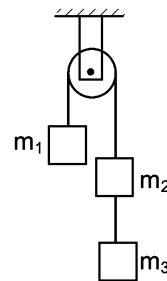
$$T - m_1 g = m_1 a \quad (1)$$

$$(m_2 + m_3)g - T = (m_2 + m_3) a \quad (2)$$

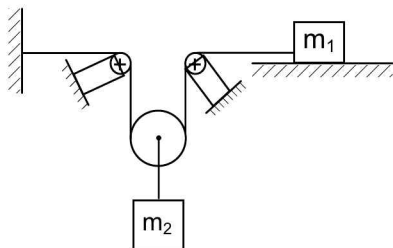
$$a = \left(\frac{m_3 + m_2 - m_1}{m_1 + m_2 + m_3} \right) g$$

$$m_3 g - T_1 = m_3 a \Rightarrow T_1 = m_3 (g - a) = \frac{m_3 g ((m_1 + m_2 + m_3 + m_1) - (m_2 + m_3))}{m_1 + m_2 + m_3}$$

$$T_1 = \frac{2m_1 m_3}{m_1 + m_2 + m_3} g$$



Connector 8: The pulleys are light, frictionless and the strings are light and inextensible. Find the accelerations of m_1 and m_2 . No friction anywhere.



Solution: F.B.D

Let acceleration of m_1 be a_1 and acceleration of m_2 be a_2 .

$$a_1 = 2a_2 \quad (\text{By constraint method})$$

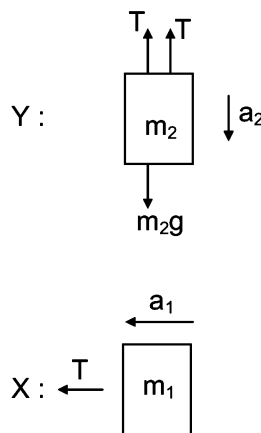
$$\text{For } m_2: m_2 g - 2T = m_2 a_2$$

$$\text{For } m_1: T = m_1 a_1$$

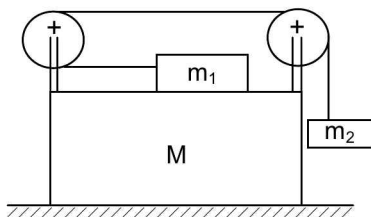
$$m_2 g = 2m_1 a_1 + m_2 a_2 = 2m_1 \times 2a_2 + m_2 a_2 = (4m_1 + m_2)a_2 \quad (\because a_1 = 2a_2)$$

$$a_2 = \left(\frac{m_2 g}{4m_1 + m_2} \right)$$

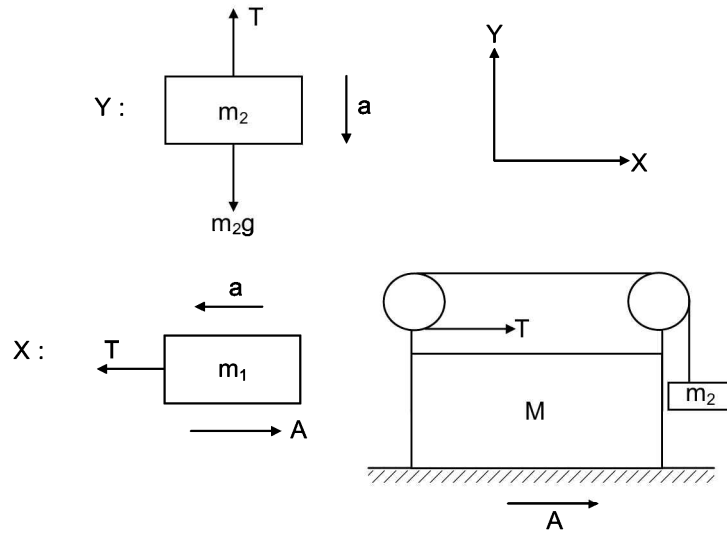
$$a_1 = \left(\frac{2m_2}{4m_1 + m_2} g \right)$$



Connector 9: Find the vertical acceleration of m_2 . Pulleys are massless and strings are light and inextensible. M moves to the right with acceleration A . There is no friction anywhere.



Solution: F.B.D



$m_2g - T = m_2a$ (where a is the downward acceleration of m_2)

$$T = m_1(a - A)$$

$$(m_2 + M)A = T$$

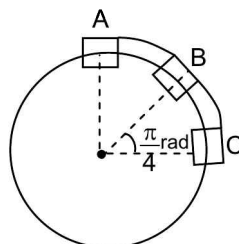
$$\Rightarrow \frac{T}{m_1} = a - A \text{ and } \frac{T}{m_2 + M} = A$$

$$\Rightarrow T \left(\frac{1}{m_1} + \frac{1}{m_2 + M} \right) = a$$

$$m_2g = m_2a + T = \left(m_2 + \frac{1}{\frac{1}{m_1} + \frac{1}{m_2 + M}} \right) a \Rightarrow$$

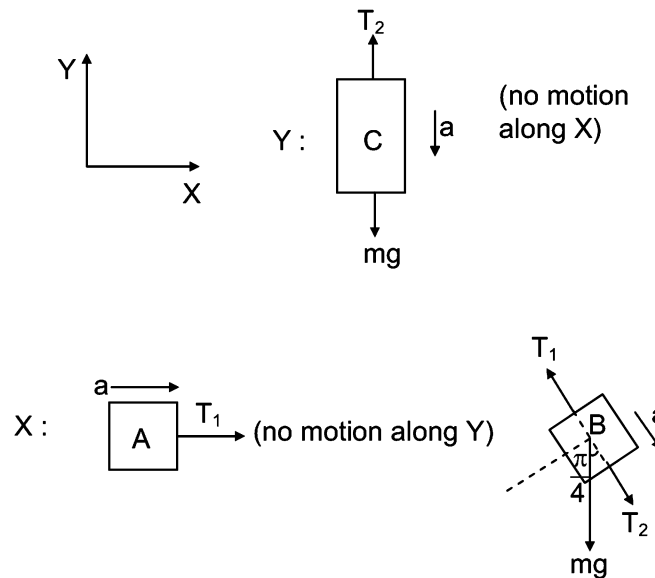
$$a = \frac{g}{1 + \frac{1}{\frac{m_2}{m_1} + \frac{m_2}{m_2 + M}}} = \frac{g}{\left[\frac{m_1(m_2 + M)}{m_2(m_1 + m_2 + M)} + 1 \right]}$$

Connector 10: Three equal masses are placed on a smooth vertical circular ring as shown and connected by strings. Find the initial acceleration of the masses and the tension in the strings, if the strings lie along the circumference.



4.46 Laws of Motion

Solution: Resolving forces along the tangent as strings are inextensible, a is the same.



For mass A,

$$T_1 = m a$$

For mass B,

$$T_2 - T_1 + mg \frac{1}{\sqrt{2}} = ma$$

For mass C,

$$mg - T_2 = ma$$

adding

$$3 ma = mg \left(1 + \frac{1}{\sqrt{2}} \right) \Rightarrow a = \frac{g \left(1 + \frac{1}{\sqrt{2}} \right)}{3}$$

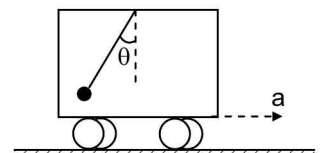
$$T_1 = ma = \frac{mg}{3} \left(1 + \frac{1}{\sqrt{2}} \right);$$

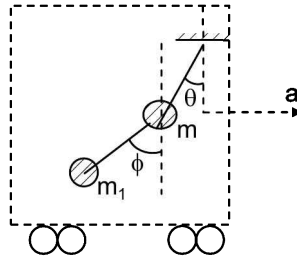
$$T_2 = mg - ma$$

$$= mg \left(1 - \frac{1}{3} \left(1 + \frac{1}{\sqrt{2}} \right) \right) = \frac{mg}{3} \left(2 - \frac{1}{\sqrt{2}} \right)$$

Connector 11: (i)

A body of mass m is suspended by a light inextensible thread from the ceiling of a truck, which moves with a horizontal acceleration ' a '. Find the angle θ , which the thread makes with the vertical at equilibrium.





- (ii) If the pendulum consists of two masses connected by another string (as shown) what is θ and what is ϕ ?

Solution:

(using accelerated frame)

(i) $T \cos \theta = mg$ — (1)

$T \sin \theta = ma$ — (2)

$\frac{(2)}{(1)} \Rightarrow \tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1} \frac{a}{g}$

- (ii) From FBD of mass 'm', we get

$T \cos \theta = mg + T' \cos \phi$ — (3)

$T \sin \theta = ma + T' \sin \phi$ — (4)

From FBD of mass ' m_1 ', we get

$T' \cos \phi = m_1 g$ — (5)

$T' \sin \phi = m_1 a$ — (6)

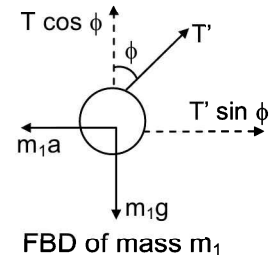
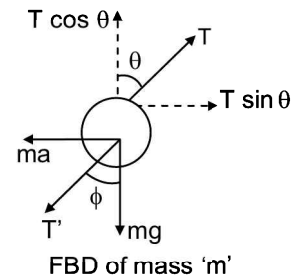
$\frac{(6)}{(5)} \Rightarrow \tan \phi = \frac{a}{g} \Rightarrow \phi = \tan^{-1} \frac{a}{g}$

Substituting (5) in (3) and (6) in (4)

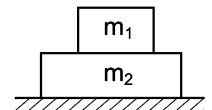
$(3) \Rightarrow T \cos \theta = (m_1 + m)g$

$(4) \Rightarrow T \sin \theta = (m + m_1)a$

Dividing, $\tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1} \frac{a}{g} = \phi$



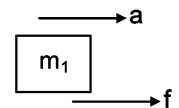
Connector 12: A body of mass m_2 is at rest on which another body of mass m_1 is kept as shown in the figure. Find the maximum acceleration with which the lower body can move without the upper body slipping. The coefficient of friction between the two bodies is μ . The floor is smooth.



Solution: For no slipping, the upper body should move with the same acceleration as the lower body. Let this value be a .

i.e., $f = m_1 a$. But $f_{\max} = \mu m_1 g$.

$m_1 a_{\max} = \mu m_1 g \Rightarrow a_{\max} = \mu g$



Connector 13: A trolley carries a body of weight 25 kg, which is kept at a distance of 2.5 m from the rear end of the trolley. The trolley starts from rest and accelerates at a constant rate of 2 m s^{-2} . If the coefficient of friction between the trolley and the body is 0.1, find the distance of the trolley from the starting point where the body will just slip out of the trolley. (Take $g = 10 \text{ m s}^{-2}$)

4.48 Laws of Motion

Solution: The force which moves the body is the frictional force.

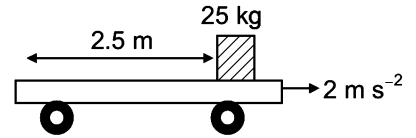
$$f_{\max} = \mu mg \Rightarrow a_{\max} = \mu g = 0.1 \times 10 \\ = 1 \text{ m s}^{-2} < 2 \text{ m s}^{-2}$$

$\Rightarrow$ slipping

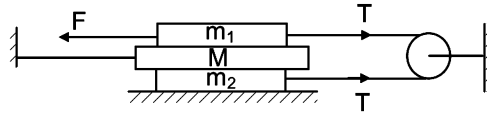
$$\text{Relative acceleration} = 2 - 1 = 1 \text{ m s}^{-2}$$

$$2.5 = 0 \times t + \frac{1}{2} \times 1 \times t^2 \Rightarrow t^2 = 5 \Rightarrow t = \sqrt{5} \text{ s}$$

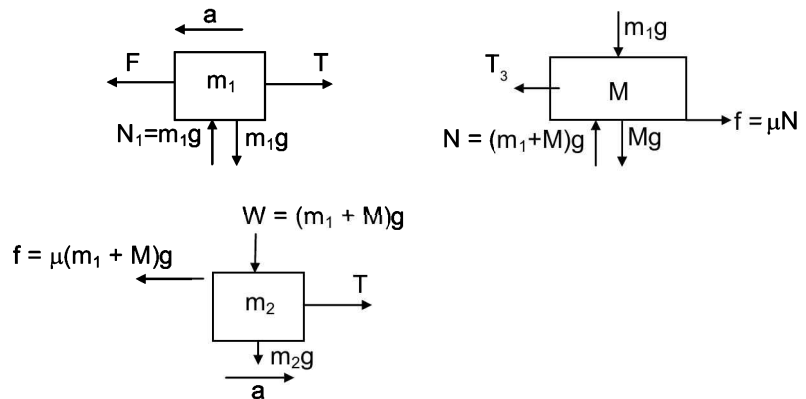
$$\text{Distance travelled by the trolley in this time} = s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times 5 = 5 \text{ m}$$



Connector 14: Find the acceleration of m_1 if the coefficient of friction between M and m_2 is μ and zero everywhere else. The pulley is light and smooth, strings are light and inextensible.



Solution: F.B.D



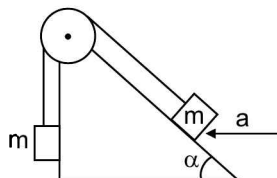
$$T = \mu (m_1 + M)g + m_2 a$$

$$F - T = m_1 a \Rightarrow F = T + m_1 a$$

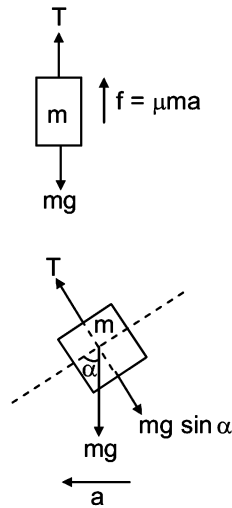
$$\Rightarrow F = \mu(m_1 + M)g + (m_1 + m_2) a$$

$$a = \frac{F - \mu(m_1 + M)g}{m_1 + m_2}$$

Connector 15: Two equal masses m and m are connected by a light inextensible string passing over a smooth massless pulley at the top of an incline of angle α . The acceleration is $\bar{a}$ as shown. Only the vertical face has friction coefficient μ . What is the minimum acceleration required so that there is no relative motion?



Solution: FBD



$$mg - T = \mu ma$$

$$T = mg \sin \alpha + ma \cos \alpha$$

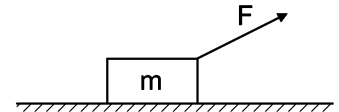
$$mg = mg \sin \alpha + \mu ma + ma \cos \alpha$$

$$a(\mu + \cos \alpha) = g(1 - \sin \alpha)$$

$$a = g \left(\frac{1 - \sin \alpha}{\mu + \cos \alpha} \right)$$

Connector16: A body of mass m is kept on a horizontal rough surface of static friction μ .

- What is the minimum force F required to pull the mass?
- What is the angle at which F is to be applied?



Solution:

$$\text{Normal reaction } N = mg - F \sin \theta$$

When F is just enough to pull the mass,

$$F \cos \theta = f = \mu N = \mu(mg - F \sin \theta)$$

$$\Rightarrow F = \frac{\mu mg}{\cos \theta + \mu \sin \theta}$$

For F to be minimum, the denominator on the R.H.S is maximum

$$\therefore \frac{d}{d\theta} (\cos \theta + \mu \sin \theta) = 0$$

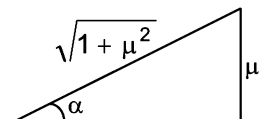
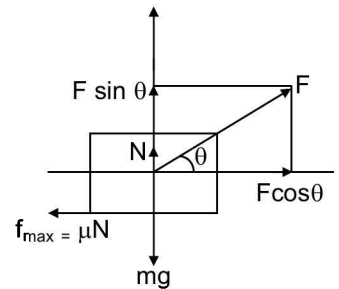
$$\Rightarrow -\sin \theta + \mu \cos \theta = 0 \Rightarrow \mu = \tan \theta \quad (1)$$

In terms of the angle of friction α ,

$$\mu = \tan \alpha \quad (2)$$

combining (1) and (2) we get

$$\sin \theta = \frac{\mu}{\sqrt{1 + \mu^2}}; \cos \theta = \frac{1}{\sqrt{1 + \mu^2}}$$



4.50 Laws of Motion

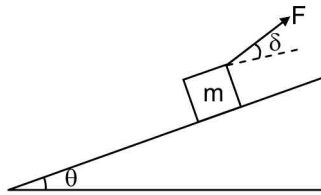
$$\therefore F = \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} = \frac{\mu mg}{\sqrt{1+\mu^2}} = mg \sin \alpha$$

Answer

- (i) $F = \frac{\mu mg}{\sqrt{1+\mu^2}} = mg \sin \alpha$ (α = Angle of friction)
 (ii) $\theta = \alpha = \tan^{-1} \mu$

Connector17: A body of mass m is kept on a rough inclined plane of angle θ and coefficient of friction μ .

- (i) What is the minimum force F required to pull the mass up the inclined plane?
 (ii) What is the angle δ with respect to the inclined plane at which the force has to be applied?



Solution:

Normal reaction $N = mg \cos \theta - F \sin \delta$

Max. frictional force $f = \mu N$

In the limiting case, when F just pulls up the mass m ,

$$F \cos \delta = mg \sin \theta + f = mg \sin \theta + \mu mg \cos \theta - \mu F \sin \delta$$

$$\therefore F(\cos \delta + \mu \sin \delta) = mg \sin \theta + \mu mg \cos \theta$$

$$\therefore F = \frac{mg \sin \theta + \mu mg \cos \theta}{\cos \delta + \mu \sin \delta}$$

F is minimum when denominator is maximum

$$\therefore \frac{d}{d\delta}(\cos \delta + \mu \sin \delta) = 0 = -\sin \delta + \mu \cos \delta$$

$$\therefore \mu = \tan \delta = \tan \alpha \text{ where } \alpha \text{ is the angle of friction.}$$

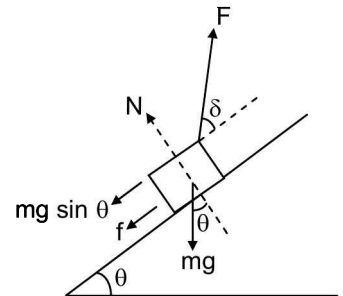
$$\therefore \cos \delta = \frac{1}{\sqrt{1+\mu^2}}; \sin \delta = \frac{\mu}{\sqrt{1+\mu^2}}$$

$$\text{and } F = \frac{mg \sin \theta + \mu mg \cos \theta}{\sqrt{1+\mu^2}}$$

$$\therefore F = (mg \sin \theta + \mu mg \cos \theta) \cos \alpha$$

Answer

- (i) $F = mg (\sin \theta + \mu \cos \theta) \cos \alpha$ (α = angle of friction)
 (ii) $\delta = \alpha$, where $\alpha = \tan^{-1} \mu$



Connector 18: Find the force required to be applied parallel to the inclined plane, to push a 10 kg body up an incline of 30° with a constant acceleration of 5 m s^{-2} . The coefficient of friction is 0.2 (Take $g = 10 \text{ m s}^{-2}$)

Solution:

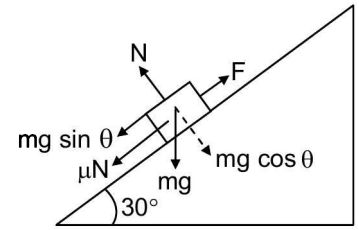
$$N = mg \cos \theta$$

$$F - (mg \sin \theta + \mu N) = ma$$

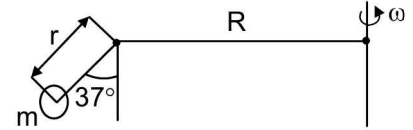
$$F = ma + mg \sin \theta + \mu N$$

$$= 10 \left(5 + 10 \times \frac{1}{2} + 0.2 \times 10 \times \frac{\sqrt{3}}{2} \right)$$

$$= 10 (5 + 5 + \sqrt{3}) = 117.3 \text{ N}$$



Connector 19: A child sits on a horse hung by a light rod of length r in a carousel of radius R . If the angle made by the horse with the vertical is 37° , when the carousel is rotating with a uniform angular speed, find the angular speed of the carousel.

**Solution:**

Consider the frame rotating with angular velocity ω

$$r' = R + r \sin 37^\circ$$

If 'T' is the tension in the rod,

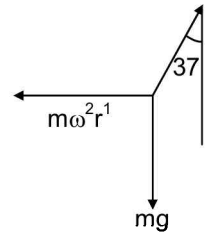
$$T \cos 37^\circ = mg \text{ and}$$

$$T \sin 37^\circ = m\omega^2 r' \Rightarrow \tan 37^\circ = \frac{m\omega^2 r'}{mg}$$

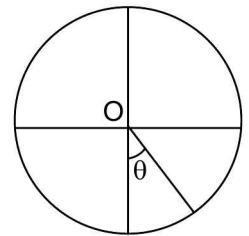
$$\frac{3}{4} = \omega^2 \frac{\left(R + r \frac{3}{5} \right)}{g}$$

$$\omega^2 = \frac{3}{4} g \frac{5}{(5R + 3r)}$$

$$\omega = \sqrt{\frac{15}{4} \frac{g}{(5R + 3r)}}$$



Connector 20: A particle of mass 'm' travels in a vertical circular path of radius r under the action of a tangential force. If the speed of the particle at a position when the radius vector makes an angle ' θ ' with vertical as shown in figure, is $v = v_0 \sin \theta$, find the maximum normal reaction (given $v_0^2 = g r$).

**Solution:**

$$\frac{mv^2}{r} = N - mg \cos \theta$$

$$N = \frac{mv^2}{r} + mg \cos \theta = \frac{mv_0^2}{r} \sin^2 \theta + mg \cos \theta$$

$$\frac{v_0^2}{r} = g$$

$$\Rightarrow N = mg (\sin^2 \theta + \cos \theta) = mg (1 - \cos^2 \theta + \cos \theta)$$

$$= mg \left[\frac{5}{4} - \left(\cos \theta - \frac{1}{2} \right)^2 \right]$$

$$\Rightarrow N_{\max} = \frac{5}{4} mg$$

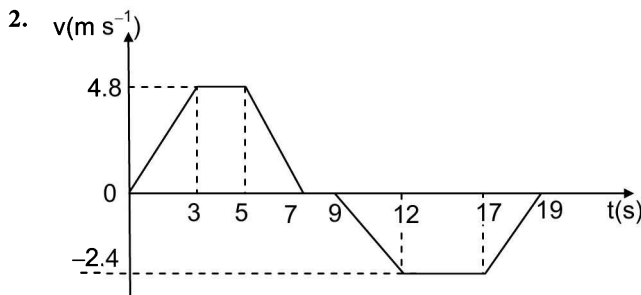
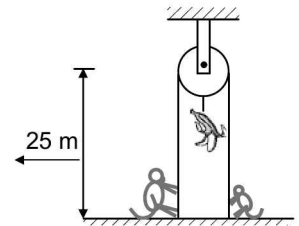
TOPIC GRIP



Subjective Type Questions

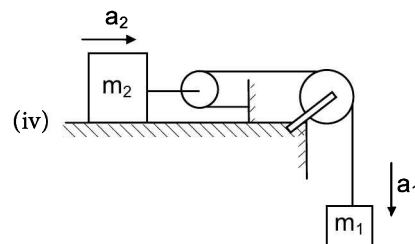
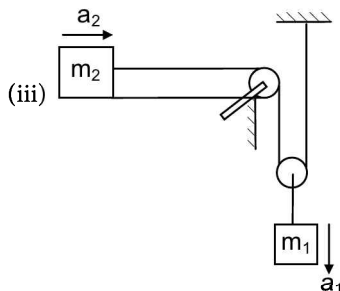
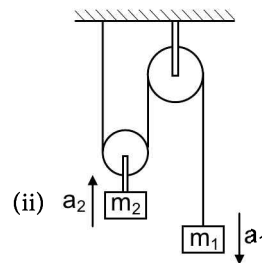
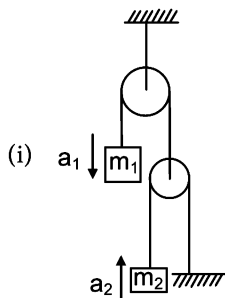
1. Two monkeys of masses 16 kg and 25 kg respectively are standing on the horizontal floor as shown. They stand holding a rope passed over a smooth, light pulley from which a banana is hung. Both monkeys get lifted up together, when the larger monkey climbs up with an acceleration of 1.2 m s^{-2} .

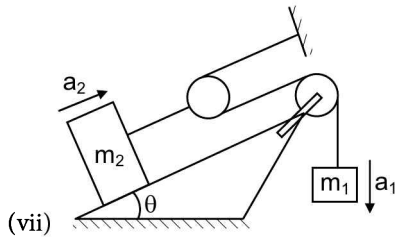
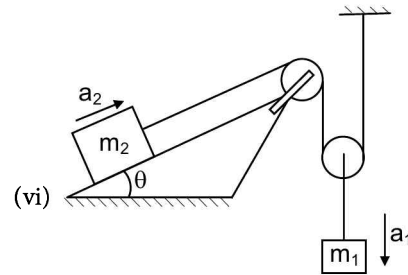
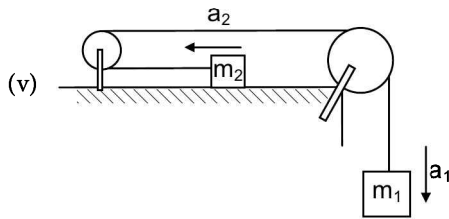
- Which monkey will reach the banana first?
 - What is the height of the other monkey from the floor when this happens?
- ($g = 10 \text{ m s}^{-2}$)



A lift with passengers is in motion. The mass of lift when empty is 500 kg and the mass of passengers is 500 kg. The graph of the velocity of the lift vs time is given in the figure (upward positive)

- What are the maximum and minimum values of tension in the rope connecting the lift to the motor?
 - If the lift is designed for accelerations between $+3 \text{ m s}^{-2}$ and -3 m s^{-2} and if the rope is designed to withstand 19500 N, what is the maximum allowable mass of passengers in the lift? ($g = 10 \text{ m s}^{-2}$)
3. In each of the following the system is released from rest. Derive the relation between acceleration of the masses involved.





4. In the arrangement shown, masses $m_1 = 40 \text{ kg}$ and $m_2 = 30 \text{ kg}$ are connected on an inclined plane $\left(\theta = \tan^{-1} \left(\frac{3}{4} \right) \right)$. Friction coefficients between m_1 and the plane is

$\mu_1 = 0.2$ and that between m_2 and the plane is $\mu_2 = 0.4$; $g = 10 \text{ m s}^{-2}$. If the system is released from rest, find

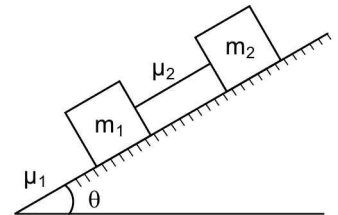
- the acceleration of the masses
- the tension in the light, inextensible connecting rope.

If the coefficients of friction are reversed, i.e., if $\mu_1 = 0.4$ and $\mu_2 = 0.2$, what are the

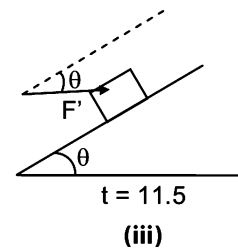
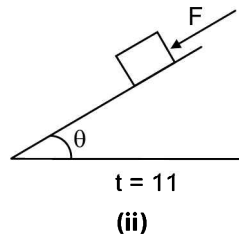
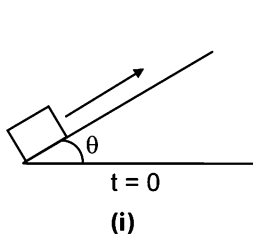
- accelerations of the masses?
- tension in the rope?.

If instead of rope, the masses are interconnected by a light rod, (and $\mu_1 = 0.4$ and $\mu_2 = 0.2$) what are the

- accelerations of the masses?
- force in the rod?



5. A body of mass 1000 kg is moved up an inclined plane of angle $\theta = \cos^{-1} 0.8$. Its displacement s from the foot of the inclined plane is given by $s = 0.6t + 0.2t^2$. At $t = 11 \text{ s}$, the upward force is removed and a constant force F is applied as shown in figure (ii). In addition, friction is present ($\mu = 0.20$). If the body comes to stop at $t = 11.5 \text{ s}$,

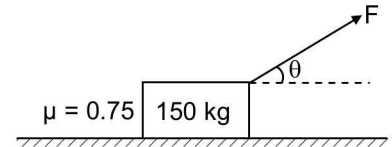


- What is the value of F ?
 - After coming to stop, the force F is withdrawn. What is the minimum value of F' to be applied (see Fig (iii)) to prevent the body from sliding down?
 - What is the maximum value of F' for which the body remains at rest?
- ($g = 10 \text{ m s}^{-2}$)

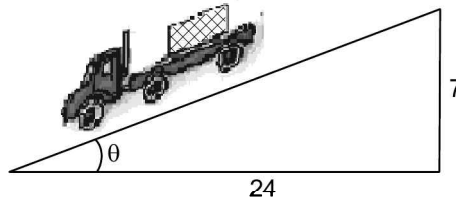
4.54 Laws of Motion

6. A body of mass 150 kg is at rest on a floor with coefficient of friction $\mu = 0.75$. A force F is applied as shown at an angle θ to horizontal.

- (i) What is the minimum value of F required to move the box? At what angle should it act?
- (ii) At that value of θ , what is the maximum value of F so that the body does not leave the floor? ($g = 10 \text{ m s}^{-2}$, $\tan^{-1} 0.75 = 37^\circ$)

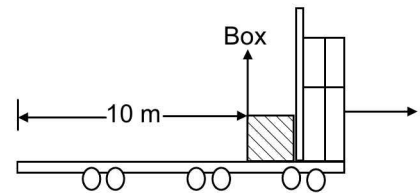


7. A truck travels down a gradient as shown with a speed of 48 kmph. If it decelerates to 30 kmph in 2.5 s, find the maximum value of the friction coefficient μ between a box kept on the floor of the truck and the floor of the truck such that it still slides forward during this motion. ($g = 10 \text{ m s}^{-2}$)



8. A truck is moving with uniform speed of 5 m s^{-1} along a straight horizontal road. At $t = 0$, it accelerates at the rate of 2 m s^{-2} . A box of mass 50 kg is kept in the truck at initial position close to the cabin. The length of the truck is 10 m. ($g = 10 \text{ m s}^{-2}$). Friction coefficient between box and truck floor is μ . At what value of t does the box fall off the truck if

- (i) there is no friction ($\mu = 0$)
- (ii) $\mu = 0.15$
- (iii) $\mu = 0.25$.



9.

- (i) A uniform, thick rope of length ℓ kept on a smooth horizontal floor has two forces F_1 and F_2 acting at its ends as shown in Fig. 1. Find the tension in the rope at a point x distance away from the end where, F_2 acts. Given $F_1 > F_2$

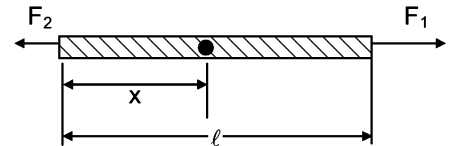


Fig. 1

- (ii) A uniform, thick rope of mass m and length ℓ is hanging freely from a support as shown in Fig. 2. Find the tension in the rope at a point x from top.

- (iii) Solve problem (ii) if a point mass of m_1 is attached to the bottom end of the rope.

- (iv) A uniform, thick rope of mass m and length ℓ is whirled around in a horizontal circle with one end as centre of rotation, with constant angular velocity ω . What is the tension in the rope at a point x from the centre of rotation?

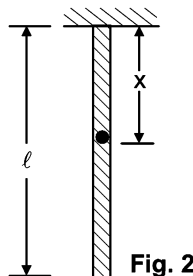
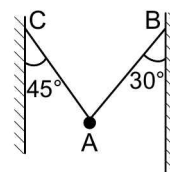


Fig. 2

- (v) Solve problem (iv) if a point mass m_1 is attached to the outer end.

10. A ball of mass 2 kg is held as shown by two light wires BA and CA. The ball is at rest.

- Find the tension in the wires BA and CA.
- Find the tension in AB immediately after CA is cut. ($g = 10 \text{ m s}^{-2}$).
- Find the acceleration of the ball, immediately after CA is cut.



Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

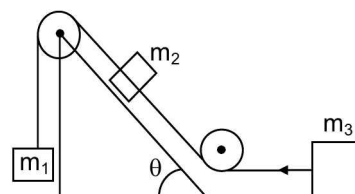
- If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be
 - going up with increasing speed
 - going up with uniform speed
 - going down with uniform speed
 - (b) or (c)
- Two bodies of masses $m_1 = 30 \text{ g}$ and $m_2 = 40 \text{ g}$ are attached to the ends of an inextensible string of negligible mass and suspended from a light, frictionless, fixed pulley. The magnitude of the acceleration of the bodies is
 - 1.40 m s^{-2}
 - 1.40 cm s^{-2}
 - 2.80 m s^{-2}
 - 2.80 cm s^{-2}
- A particle of small mass m is connected to a very heavy body by a light, inextensible string passing over a light smooth, fixed pulley. Both bodies are free to move vertically. The total force exerted on the pulley by the string when the system is released from rest is
 - mg
 - $2mg$
 - $3mg$
 - $4mg$

- Three masses are connected by light, inextensible strings which pass over smooth light, fixed pulleys. The masses m_1 and m_3 are 4 kg and 6 kg respectively and the coefficient of friction for all surfaces in contact is 0.3. The minimum mass of m_2

and the tension in the horizontal part of the string $\left(\theta = \sin^{-1} \left(\frac{4}{5} \right) \right)$ for no motion

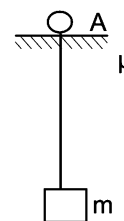
($g = 10 \text{ m s}^{-2}$) are respectively

- 2.2 kg, 18 N
- 4.2 kg, 20 N
- 3 kg, 30 N
- 6.1 kg, 25 N



- A mass is hung from a massless pivot, A which rests on a support of co-efficient of friction μ . The mass is now given a horizontal velocity such that A just slips. The tension in the string when this happens is

- μmg
- $\frac{\mu}{\sqrt{1 + \mu}} mg$
- mg
- $\frac{mg}{\sqrt{1 + \mu^2}}$



Assertion–Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- Statement-1 is True, Statement-2 is False
- Statement-1 is False, Statement-2 is True

4.56 Laws of Motion

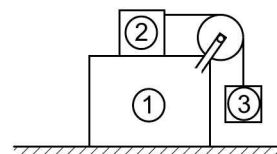
16. Statement 1

No friction anywhere. When the system is released from rest, body (3) and (1) will have equal horizontal leftward acceleration.

and

Statement 2

When normal reaction exists between two surfaces, it means they are in contact.



17. Statement 1

A box is successfully moved up an inclined plane by pulling it with a string, parallel to the incline but if the experiment is done with the angle of the inclined plane lowered a little, the string breaks. Then either the angle of inclination of plane with horizontal is very high or surface must be very rough.

and

Statement 2

$\frac{d}{d\theta} (\sin\theta + a \cos\theta) < 0$ if θ is large or a is large.

18. Statement 1

In a tug of war game, the rope exerts a larger horizontal force on the losing team than on the winning team.

and

Statement 2

A unbalanced external force is necessary to cause motion of a body at rest.

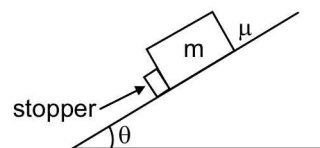
19. Statement 1

If $\tan\theta > \mu$, then the frictional force acting on the block is $\mu mg \cos\theta$.

and

Statement 2

Friction and normal reaction are self-adjusting forces.



20. Statement 1

A vehicle moves under control on a curved road. Banking of the road increases the resultant force that the road exerts on the vehicle, compared to an unbanked road.

and

Statement 2

Due to banking, rainwater if any drains off increasing the friction force on vehicle.

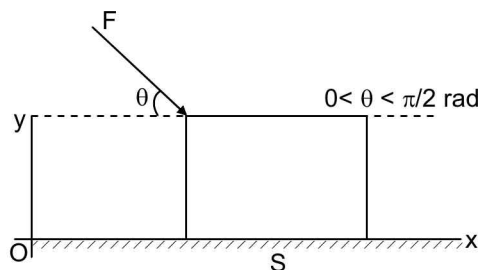


Linked Comprehension Type Questions

Directions: This section contains 2 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage I

The force of friction between two contact surfaces is an important asset. In many applications, it is desired that the surfaces do not slide with respect to each other, no matter how the other forces are changed. This condition is called 'Friction Locking'. When the bodies are friction locked, the force of friction is always static. This condition is very much desired in many



applications to avoid dissipation of energy and damage due to accidents. To illustrate the concept, the figure shows a block of mass m kept on a horizontal surface of coefficient of static friction μ . It is acted upon by a force F . It is desired to find the range of values of θ such that there is friction locking no matter how large F is.

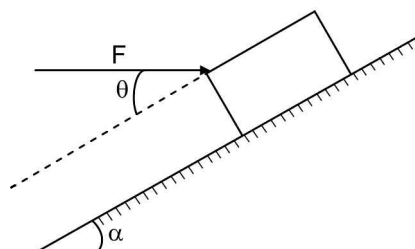
21. The force of friction acting on the surface S is

- (a) $\mu mg \hat{i}$ (b) $-F \cos \theta \hat{i}$ (c) $F \cos \theta \hat{i}$ (d) $\mu F \cos \theta \hat{i}$

22. The condition for friction locking is

- (a) $\tan \theta \geq \mu$ (b) $\tan \theta \geq \frac{1}{\mu}$ (c) $\tan \theta \leq \frac{1}{\mu}$ (d) $\tan \theta \leq \mu$

23. If the block is kept on an inclined surface as shown, the condition that the block does not slide up the plane irrespective of the value of F is

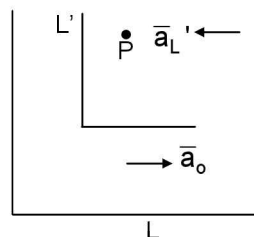


- (a) $\tan (\theta + \alpha) \geq \mu$ (b) $\tan (\theta + \alpha) \geq \frac{1}{\mu}$ (c) $\tan (\theta + \alpha) \leq \mu$ (d) $\tan \theta \geq \frac{1}{\mu}$

Passage II

Newton's laws of motion are applicable to most of the practical situations that we encounter, but they will fail in certain situations. One example is the application of the first and 2nd laws in an accelerated frame, technically called non-inertial frame.

Consider a frame L that is at rest, called an inertial frame. Now another frame L' moves with respect to L with an acceleration $\vec{a}_0$. Hence L' is a non-inertial frame. A particle P in L' is found to have an acceleration $\vec{a}_L'$ when observed in frame L' . Obviously when observed from frame L , P will have an acceleration $\vec{a}_L = \vec{a}_L' + \vec{a}_0$. This violates Newton's 2nd law because same particle acted upon by the same forces shows two different accelerations in two different frames of references. However by certain manipulations we can solve this anomaly.



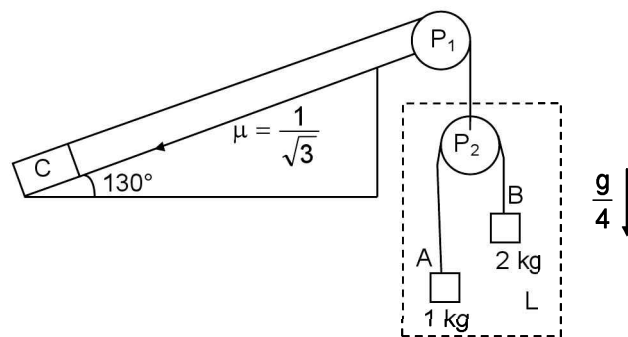
4.58 Laws of Motion

$$\text{Since } \vec{a}_{L'} = \vec{a}_L - \vec{a}_0 \Rightarrow m\vec{a}_{L'} = m\vec{a}_L - m\vec{a}_0$$

$$m\vec{a}_L = \text{total force acting on the body as per Newton's law} = \vec{F}$$

$$\therefore \vec{a}_{L'} = \frac{\vec{F} - m\vec{a}_0}{m}. \text{ Hence we can conclude that if we add with all the forces acting on P, an extra term, Newton's laws}$$

work in an accelerated frame also. The term $-m\vec{a}_0$ is called "Pseudo force". This principle can be used in many complicated situations like study of accelerated galaxies. They can also be used in solving simple day-to-day problems. An example is that, a man in a lift accelerated upward will weigh more because the pseudo force 'ma' act downwards along with mg. Now consider the situation shown, where the frame L (though imaginary) consisting of a weightless pulley P_2 and two masses A and B connected by an inextensible weightless string, is moving down with an acceleration $\frac{g}{4}$



Take upward direction as positive in each case.

24. What is the acceleration of A in the reference frame L?

- (a) $\frac{g}{3}$ (b) $\frac{g}{4}$ (c) $\frac{g}{12}$ (d) 0

25. For an outside observer, the acceleration of A is

- (a) $-\frac{g}{2}$ (b) $+\frac{g}{2}$ (c) 0 (d) $+\frac{g}{12}$

26. For an outside observer, the acceleration of B is

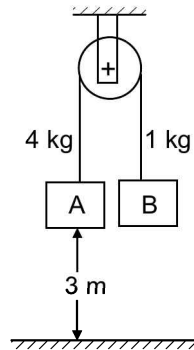
- (a) $-\frac{g}{2}$ (b) $+\frac{g}{2}$ (c) 0 (d) $\frac{7g}{12}$



Multiple Correct Objective Type Questions

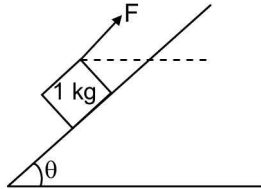
Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

27. The system is released from the position shown at $t = 0$. The pulley is massless and frictionless the string is massless and inextensible. A hits the ground and gets stuck to the ground. ($g = 10 \text{ m s}^{-2}$)



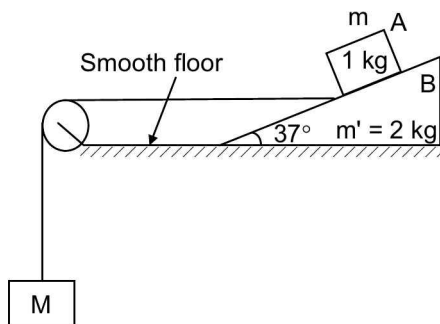
- (a) The impulse A exerts on ground $J = 24 \text{ N s}$
- (b) B reaches a maximum height of 7.8 m
- (c) B reaches maximum height from the ground 7.8 s after release
- (d) B reaches maximum height from the ground 2.1 s after release

28.



A body weighing 1 kg is being pulled up on an inclined plane of angle $\theta = 37^\circ$, by employing minimum force F . The coefficient of friction on the plane is $\mu = 0.75$. ($g = 10 \text{ m s}^{-2}$)

- (a) whatever may be the value of μ and θ , the minimum force F , has to be applied vertically upward
 - (b) whatever may be the value of μ and θ , the minimum force F , has to be parallel to the plane.
 - (c) for the given value of μ and θ the minimum force F has to be applied parallel to the inclined plane.
 - (d) For the given value of μ and θ , minimum value of $F = 9.6 \text{ N}$
29. Wedge B kept on a smooth floor is being pulled by the weight M , connected to B by a light inextensible string as shown. Body A of mass $m = 1 \text{ kg}$ is kept on the smooth surface of the wedge. Wedge angle $\theta = 37^\circ$ with horizontal



- (a) If $M = 10 \text{ kg}$, acceleration of A with respect to B, is up the plane
- (b) If $M = 9 \text{ kg}$, acceleration of A wrt wedge B is zero
- (c) If $M < 9 \text{ kg}$, acceleration of A wrt B is down the plane
- (d) If wedge angle $\theta \geq 45^\circ$, A can only move down the plane whatever be the value of M (take $g = 10 \text{ m s}^{-2}$)



Matrix-Match Type Question

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

30. Match the following

Column I

- (a) centrifugal force
- (b) tension force
- (c) centripetal force
- (d) spring force

Column II

- (p) pseudo force
- (q) elastic force
- (r) radial force
- (s) reaction force

IIT ASSIGNMENT EXERCISE



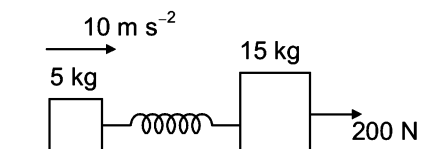
Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

31. A constant force F applied on a body of mass 5 kg, changes its velocity from 2 m s^{-1} to 7 m s^{-1} in 10 second. If the direction of motion is unchanged, applied force F is
 (a) 2.5 N (b) 5 N (c) 7.5 N (d) 10 N
32. A body, initially at rest, is acted upon by a constant force. Let p be the momentum of the body after time t_1 . The momentum at time $3t_1$ is
 (a) $3p$ (b) $2p$ (c) $\sqrt{2} p$ (d) $\sqrt{3} p$
33. A body of mass 2 kg is at rest. The impulse required to impart a velocity of 8 m s^{-1} to it in 5 s is
 (a) 16 N s (b) 40 N s (c) 80 N s (d) 20 N s
34. A force of 100 N acts on a mass of 5 kg for 10 s. If the mass starts from rest, its velocity at the end of 10 s is
 (a) 2 m s^{-1} (b) 20 m s^{-1} (c) 200 m s^{-1} (d) 50 m s^{-1}
35. A constant force F acts on a mass of 6 kg and changes its velocity from 12 m s^{-1} to 20 m s^{-1} in 1 minute. The magnitude of F is
 (a) 0.8 N (b) 2.6 N (c) 3.8 N (d) 5.9 N
36. A given force acts on two objects of different masses, one heavy (M) and the other light (m), for the same duration of time. Which of the following statements is correct? [p_M and p_m are the change in momenta of heavy and light body respectively]
 (a) $p_M > p_m$ (b) $p_M < p_m$ (c) $p_M = p_m$ (d) $p_M \geq p_m$
37. A car starting from rest attains a speed of 72 kmph when it covers a distance 100 m on a straight road. If the mass of the car is 500 kg, the average force exerted on the car is
 (a) 500 N (b) 1000 N (c) 250 N (d) 627 N
38. A force of 1 N acts on a body of mass 1 kg, which is able to move freely. Then the body acquires
 (a) a speed of 1 m s^{-1} (b) an acceleration of 1 m s^{-2}
 (c) an acceleration of 9.8 m s^{-2} (d) an acceleration of 1.02 m s^{-2}
39. The force that should be applied on a 5 kg body so that it has a downward acceleration of 4 m s^{-2} is
 (a) 69 N upward (b) 69 N downward (c) 29 N upward (d) 29 N downward.
40. Inertia of a body is that property by virtue of which the body is unable to change by itself, its
 (a) state of rest. (b) state of uniform motion.
 (c) direction of motion. (d) all the above
41. When a body is at rest, which of the following statements is true?
 (a) Every force acting on the body is zero. (b) The force acting on it is not in contact with the body.
 (c) The resultant force acting in the body will be zero. (d) All of the above.
42. A motor car of mass 1000 kg, cruising at a speed of 72 kmph, has to be brought to a stop in a straight distance of 40 m. The average force required to stop the car is
 (a) 7200 N (b) 5000 N (c) $\frac{10000}{72} \text{ N}$ (d) $\frac{720}{1000} \text{ N}$

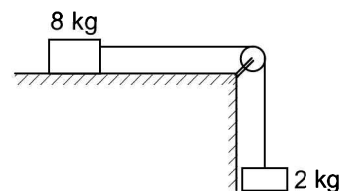
4.62 Laws of Motion

43. When a horse pulls a cart, the force which helps the horse to move forward is the force exerted by
 (a) The cart on the horse. (b) The ground on the horse.
 (c) The ground on the cart. (d) The horse on the ground.
44. A certain force applied to a mass m_1 gives it an acceleration of 15 m s^{-2} . The same force produces an acceleration of 10 m s^{-2} in another mass m_2 . If the two masses stick together, the acceleration produced by the same force is
 (a) 5 m s^{-2} (b) 6 m s^{-2} (c) 25 m s^{-2} (d) 12.5 m s^{-2}
45. A body of mass 2 kg is sliding with a constant velocity of 4 m s^{-1} on a frictionless horizontal table. The force required to keep the body moving with the same velocity is
 (a) 8 N (b) 6 N (c) zero (d) $\frac{1}{2} \text{ N}$
46. A force of 6 N acts on a body of mass 1 kg , initially at rest, for some time during which the body attains a velocity of 30 m s^{-1} . The time (in second) for which the force acts on the body is
 (a) 2 (b) 5 (c) 6 (d) 30
47. A car travelling at a speed of 30 kmph is brought to a halt in 8 m by a constant force on applying the brakes. If the same car is travelling at 60 kmph , it can be brought to a halt with the same braking force in
 (a) 8 m (b) 16 m (c) 24 m (d) 32 m
48. A man getting down from a running bus falls forward because
 (a) due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road.
 (b) due to inertia of rest, road is left behind and man reaches forward.
 (c) he leans forward as a matter of habit.
 (d) of the combined effect of all the three factors stated (a), (b), (c).
49. A bullet of mass 10 g moving with a velocity 40 m s^{-1} strikes a fixed uniform wooden plank of thickness 5 cm and emerges with a velocity 38 m s^{-1} . Average resistance offered by the plank is
 (a) 5.2 N (b) 10.2 N (c) 15.6 N (d) 0.2 N
50. A body of mass 3 kg is acted upon by two perpendicular forces of 3 N and 4 N respectively. The magnitude and direction of acceleration of the body are ($\tan^{-1}(4/3) = 53^\circ$)
 (a) $\frac{3}{5} \text{ m s}^{-2}$, at 53° to the 3 N force. (b) $\frac{5}{3} \text{ m s}^{-2}$, at 53° to the 3 N force.
 (c) $\frac{3}{5} \text{ m s}^{-2}$, at 53° to the 4 N force. (d) $\frac{5}{3} \text{ m s}^{-2}$, at 53° to the 4 N force.
51. A 10 kg block is subjected to two forces $(10\hat{i} + 20\hat{j}) \text{ N}$ and $(6\hat{i} - 8\hat{j}) \text{ N}$. The magnitude of the resultant acceleration is
 (a) 2 m s^{-2} (b) 3 m s^{-2} (c) 4 m s^{-2} (d) 5 m s^{-2}
52. A block of mass m is placed on a smooth inclined plane of inclination α with the horizontal and the whole system is accelerated horizontally. If the body does not slip, the force exerted by the inclined plane on the body is
 (a) mg (b) $mg \sin \alpha$ (c) $mg \cos \alpha$ (d) $mg \sec \alpha$
53. Two masses of 5 kg and 15 kg are connected by a massless spring as shown. At the instant shown in the figure, the 5 kg mass has an acceleration of magnitude 10 m s^{-2} . The magnitude of the acceleration of the 15 kg mass is
 (a) 5 m s^{-2} (b) 10 m s^{-2}
 (c) 15 m s^{-2} (d) 20 m s^{-2}

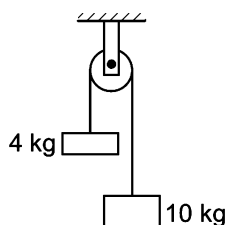


54. A body of mass 0.4 kg falls through air with an acceleration of 5 m s^{-2} . The frictional force of air on the body is ($g = 10 \text{ m s}^{-2}$)
- (a) 0.02 N (b) 0.1 N (c) 1 N (d) 2 N

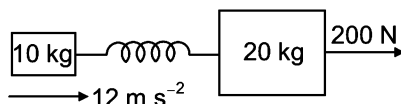
55. An 8 kg block is placed on a smooth horizontal table is connected by a light string passing over a frictionless pulley to a 2 kg block. The acceleration of the 8 kg block is ($g = 10 \text{ m s}^{-2}$)
- (a) 2 m s^{-2} (b) 4 m s^{-2}
(c) 6 m s^{-2} (d) 8 m s^{-2}



56. In the above question the tension in the string is
- (a) 2 N (b) 4 N (c) 8 N (d) 16 N
57. Two masses of 4 kg and 10 kg respectively are connected at the ends of a light inextensible string that goes over a smooth fixed pulley. The acceleration of the bigger mass is ($g = 10 \text{ m s}^{-2}$)



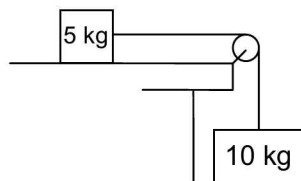
- (a) 1.5 m s^{-2} (b) 2.7 m s^{-2} (c) 3.4 m s^{-2} (d) 4.3 m s^{-2}
58. In the above question the tension in the string will be
- (a) 36 N (b) 48 N (c) 57 N (d) 63 N
59. Two blocks of masses of 2 kg and 1 kg are placed on a smooth horizontal surface in contact with each other. A horizontal force of 3 N is applied on 2 kg block so that the system moves with a constant acceleration. Then the force between the blocks is
- (a) 3 N (b) 2 N
(c) 1 N (d) 0.5 N
60. A monkey of mass 20 kg is holding a vertical rope, which breaks under the weight of a mass of 25 kg. The maximum acceleration with which the monkey can climb up the rope is ($g = 10 \text{ m s}^{-2}$)
- (a) 2 m s^{-2} (b) 2.5 m s^{-2} (c) 5 m s^{-2} (d) 7.5 m s^{-2}
61. Two masses of 10 kg and 20 kg, are connected by a massless spring. A force of 200 N acts on the 20 kg mass. At the instant shown, the 10 kg mass has an acceleration of magnitude of 12 m s^{-2} . The magnitude of the acceleration of the 20 kg mass is



- (a) 12 m s^{-2} (b) 4 m s^{-2} (c) 10 m s^{-2} (d) zero
62. A block of mass M is pulled along a smooth horizontal surface with a rope of mass m . If the force applied at the free end of the rope is F , the acceleration of the block will be
- (a) $\frac{F}{(M + m)}$ (b) $F \times \frac{m}{M}$ (c) $\frac{F}{M}$ (d) $\frac{F}{(M - m)}$

4.64 Laws of Motion

63. A heavy, uniform rope of length L lies on a smooth horizontal table. If the rope is pulled at an end by a horizontal force of F , the tension at a point distant x from the point of application of force is
 (a) $\frac{FL}{x}$ (b) $\frac{F(L-x)}{L}$ (c) $\frac{FL}{L-x}$ (d) $\frac{Fx}{L-x}$
64. Two weights W_1 and W_2 are suspended from the ends of a light string passing over a smooth fixed pulley. If the pulley is pulled up with an acceleration g , the tension in the string will be
 (a) $\frac{4W_1W_2}{W_1+W_2}$ (b) $\frac{2W_1W_2}{W_1+W_2}$ (c) $\frac{W_1W_2}{W_1+W_2}$ (d) $\frac{W_1W_2}{2(W_1+W_2)}$
65. An elevator has a mass of 5 ton. When the tension in the supporting cable is $6 \times 10^4 \text{ N}$ the acceleration of the elevator is ($g = 10 \text{ m s}^{-2}$)
 (a) 2 m s^{-2} (b) 4 m s^{-2} (c) 6 m s^{-2} (d) 8 m s^{-2}
66. The force along the plane required to push a body of weight 100 N up a smooth 30° incline with an acceleration of 3 m s^{-2} is ($g = 10 \text{ m s}^{-2}$)
 (a) 20 N (b) 40 N (c) 60 N (d) 80 N
67. In the above problem, if the surface is not smooth, but has a coefficient of kinetic friction equal to 0.05 , the force to be applied to produce acceleration of 3 m s^{-2} is
 (a) 46.3 N (b) 57.8 N (c) 72.9 N (d) 84.3 N
68. A body rolling on the surface of Earth comes to rest after some time because
 (a) it has mass (b) it experiences friction (c) it has inertia of rest (d) All of the above
69. A block of mass 2 kg rests on a horizontal surface. A gradually increasing force is applied horizontally on the body by a light inextensible string tied to the body. If the body just begins to slide at 5 N , the coefficient of static friction is ($g = 10 \text{ m s}^{-2}$)
 (a) 0.025 (b) 0.25 (c) 0.01 (d) 0.4
70. A cart moves with acceleration ' a ' along the horizontal so that a mass m pressing against it on a vertical wall does not fall. The coefficient of friction between the cart and the block is
 (a) $\frac{g}{a}$ (b) $< \frac{g}{a}$ (c) $< \frac{a}{g}$ (d) $\geq \frac{g}{a}$
71. A body of mass 5 kg is placed on a horizontal table having $\mu = 0.4$. The mass of 10 kg is arranged to move vertically. The string connecting the two masses is inextensible and the pulley over which the string is passed is smooth. The acceleration of the system is ($g = 10 \text{ m s}^{-2}$)

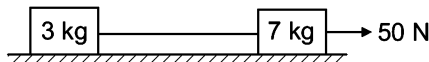


- (a) 3.33 m s^{-2} (b) 5.33 m s^{-2} (c) 0.533 m s^{-2} (d) 10 m s^{-2}
72. A block of mass 0.15 kg rests on an inclined plane of coefficient of friction 0.3 . If the angle of the plane with the horizontal is 30° , the minimum force parallel to the plane necessary to prevent it from slipping is ($g = 10 \text{ m s}^{-2}$)
 (a) 0.5 N (b) 0.36 N (c) 1.5 N (d) 0.25 N
73. Frictional force between two surfaces has maximum value for
 (a) Limiting friction (b) Sliding friction (c) Rolling friction (d) Static friction

74. A man in a lift weighs more when
 (a) the lift begins to go up. (b) the lift is going up steadily.
 (c) the lift is slowing down. (d) He will always weigh the same.
75. "The acceleration of a body is measured to be non zero when the resultant force acting on the body is zero"
 (a) Statement is true.
 (b) The watch used for the measurement was slow.
 (c) Scale used for the measurement may be longer than the standard.
 (d) Measurement was done in a non inertial frame.
76. A man of mass 60 kg stands on a weighing scale in a lift, which is moving upwards with a uniform speed of 10 m s^{-1} . The reading of the scale will be
 (a) 50 kg wt (b) 60 kg wt (c) 70 kg wt (d) 80 kg wt
77. In the above problem if the lift moves down with uniform acceleration 3 m s^{-2} , the reading of the scale will be
 (a) 60 kg wt (b) 55.7 kg wt (c) 50.2 kg wt (d) 41.6 kg wt
78. A boy sitting on the top most berth in the compartment of a train, which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother sitting vertically below his hands at a distance of about 2 m. The apple will fall
 (a) precisely on the hand of his brother.
 (b) slightly away from the hand of his brother in the direction of motion of the train.
 (c) slightly away from the hand of his brother in the direction opposite to the direction of motion of the train.
 (d) slightly away from the hand of his brother in the direction perpendicular to the motion of the train.
79. A small coin is placed at a distance of 0.2 m from the centre of a turntable. If the coefficient of friction between the coin and table is 0.5, the maximum angular speed the turntable may have without the coin sliding off is ($g = 10 \text{ m s}^{-2}$)
 (a) 5 rad s^{-1} (b) 8 rad s^{-1} (c) 3.5 rad s^{-1} (d) 15 rad s^{-1}
80. A train takes a circular bend, banked at an angle θ with the horizontal ($\tan \theta = 0.1$), with a safe speed of 72 kmph. The radius of the bend is ($g = 10 \text{ m s}^{-2}$)
 (a) 400 m (b) 141 m (c) 200 m (d) 282 m
81. The equation $v^2 - u^2 = 2aS$ ($a \neq 0$) is applicable for a body of constant mass, when
 (a) no force is acting on it.
 (b) any force is acting on it.
 (c) only a force, constant in magnitude and direction is acting on it.
 (d) particle is in dynamic equilibrium.
82. Newton's second law can give a measure of
 (a) force (b) acceleration
 (c) rate of change of momentum (d) All of the above
83. A body of mass 2 kg at O has an initial velocity of 3 m s^{-1} along OE and it is subjected to a force of 4 N in a direction perpendicular to OE. The distance of body from O after 4 second will be
 (a) 12 m (b) 20 m (c) 8 m (d) 6 m
84. A car weighing 10^4 N is stopped in 5 second, covering 25 m with uniformly decelerated motion. The braking force is ($g = 10 \text{ m s}^{-2}$)
 (a) 2000 N (b) 1940 N (c) 1400 N (d) 1500 N
85. A bullet moving at 900 kmph penetrates 0.1 m into a tree before it stops. If the force exerted by the tree is uniform and if the mass of the bullet is 10 g, then the force exerted by the bullet on the tree is
 (a) 625 N (b) 3125 N (c) 6250 N (d) 920 N

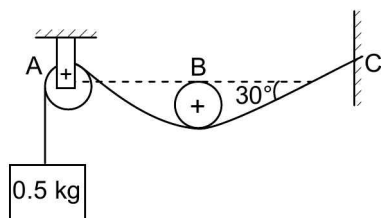
4.66 Laws of Motion

86. Two blocks of masses 3 kg and 7 kg, connected by a light inextensible string, are on a smooth horizontal floor. A horizontal force of 50 N is applied on the 7 kg mass. If the surface is smooth, the tension in the string is



- (a) 50 N (b) 15 N (c) 25 N (d) 35 N
87. A 20 kg weight hangs at the end of a long light inextensible rope. If the support can move it along a vertical line, its acceleration when the tension in the rope is 180 N is ($g = 10 \text{ m s}^{-2}$)
- (a) 2.3 m s^{-2} , upward (b) 1.3 m s^{-2} , upward (c) 1 m s^{-2} , downward (d) 1 m s^{-2} , upward

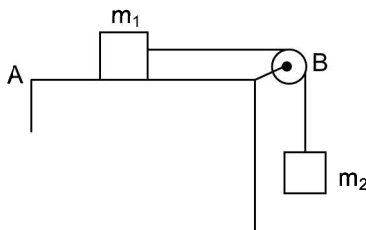
88.



A 0.5 kg mass is connected to a point C over a smooth, fixed, light pulley A such that the string AC is horizontal. Now a smooth, light, pulley B is used to deflect it at the midpoint by 30° with horizontal. The force on B is ($g = 10 \text{ m s}^{-2}$)

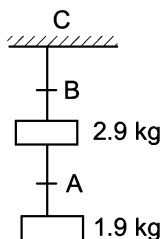
- (a) 2 N (b) 5 N (c) 1.5 N (d) 2.5 N

89. A mass ($m_1 = 0.5 \text{ kg}$) is placed on a smooth table with a light, inextensible string attached to it. The string goes over a frictionless, light, fixed pulley and is connected to another mass m_2 (0.2 kg). At $t = 0$, the mass m_1 is moving with a speed of 0.5 m s^{-1} towards B. Its speed at $t = 1 \text{ s}$ is



- (a) -2.3 m s^{-1} (b) -2.4 m s^{-1} (c) 2.8 m s^{-1} (d) 3.30 m s^{-1}

90. Two blocks of masses 2.9 kg and 1.9 kg are suspended from a support C by 2 inextensible fixed wires. The upper wire has negligible mass and the lower wire has a mass of 0.2 kg. The support has an upward acceleration of 0.2 m s^{-2} . The tensions at the mid points of the wires A and B are



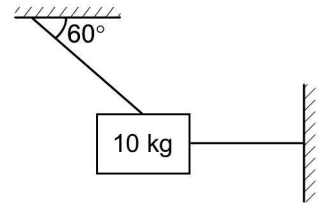
- (a) 20 N, 50 N (b) 50 N, 20 N (c) 30 N, 60 N (d) 45 N, 55 N

91. Two masses of 1 kg and 2 kg are connected by a light, inextensible string which passes over a light, frictionless, fixed pulley. The masses are initially at rest. The velocity of the mass at the instant one of the masses has moved up a distance of 7.5 meter is (m s^{-1})

(a) 12 (b) 7 (c) 10 (d) 5

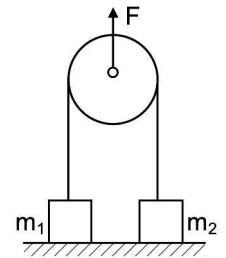
92. A 10 kg mass is hung from two light, inextensible strings as shown in figure. The tension in the horizontal string is: ($g = 10 \text{ m s}^{-2}$)

(a) 115.5 N
(b) 58 N
(c) 100 N
(d) 0 N



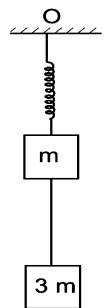
93. Two masses, (just touching the ground) are connected by a light, inextensible string, passing over a light, frictionless pulley. The minimum force F needed to move the pulley ($m_2 > m_1$) is

(a) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$ (b) $\left(\frac{m_2}{m_1 + m_2}\right)g$
(c) $2m_1g$ (d) $\left(\frac{m_1 + m_2}{m_1 - m_2}\right)g$

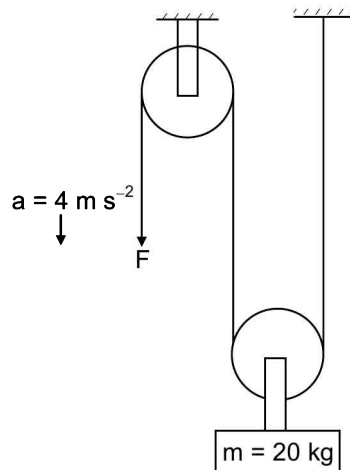


94. A light spring is fixed to a point 'O' in the ceiling and a block of mass 'm' is suspended from its other end. A larger block of mass 3 m is suspended from m by a light, inextensible thread as shown in figure. At an instant when the string connecting 'm' and 3 m snaps suddenly, the acceleration of blocks 3 m and m are respectively (g = acceleration due to gravity)

(a) ' g ' downward and ' $3g$ ' downward
(b) ' g ' downward and ' $\frac{g}{3}$ ' upward
(c) ' g ' downward and ' $3g$ ' upward
(d) ' g ' downward and ' $4g$ ' upward



95. In the system shown, all pulleys are light and smooth, all strings are light and inextensible. A block of mass $m = 20 \text{ kg}$ is attached to the movable pulley. A constant force F , applied at the free end of the string, makes the point of application of force move down with a constant acceleration of 4 m s^{-2} . The applied force F is ($g = 10 \text{ m s}^{-2}$)



(a) 40 N (b) 80 N (c) 120 N (d) 160 N

4.68 Laws of Motion

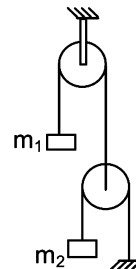
96. A block of mass m slides down on rough inclined plane of angle θ with the horizontal, with an acceleration a along the plane. The value of the reaction is

- (a) $m(a + g \sin \theta)$ (b) $m \cdot \frac{(a + g \sin \theta)}{g \cos \theta} \cdot a$
(c) $mg \cos \theta$ (d) $m\sqrt{g^2 + a^2 - 2ag \sin \theta}$

97. Pulleys are light and frictionless, strings are light and inextensible. The acceleration of m_2 , if

$m_1 = \frac{m_2}{2}$ is.

- (a) $\frac{2}{5}g$ (b) $\frac{2}{9}g$
(c) $\frac{2g}{3}$ (d) $\frac{g}{6}$



98. A body slides down a fixed rough inclined plane that is at an angle of 45° with the horizontal. The distance x travelled by the body in time t is given by $x = \sqrt{3}t^2$. The coefficient of friction between the body and the inclined surface is nearly

- (a) 0.3 (b) 0.4 (c) 0.5 (d) 0.6

99. If a minimum force of 500 N is required to move a box of mass 100 kg resting on a horizontal floor, the coefficient of friction between the box and the floor is ($g = 10 \text{ m s}^{-2}$)

- (a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{2}{3}$

100. A chain of length 1 m lying lengthwise on a fixed table is partially hanging vertically outside the edge of the table. If the coefficient of friction between the chain and the table is 0.5, the maximum length of the chain that can hang from the table without slipping is

- (a) 0.5 m (b) 0.8 m (c) 0.33 m (d) 1.5 m

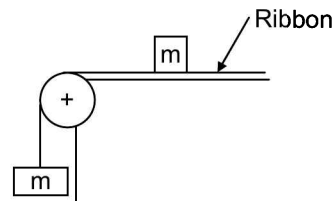
101. A block of mass 3 kg slides down an inclined plane which makes an angle of 30° with the horizontal. The coefficient of kinetic friction between the block and surface is $\frac{1}{2\sqrt{3}}$. The forces that must be applied to the block for it

- (i) to move down the plane without any acceleration
(ii) to move up the plane without any acceleration are respectively ($g = 10 \text{ m s}^{-2}$)

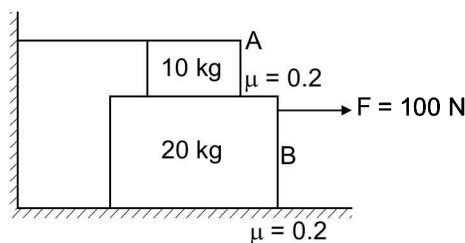
- (a) 7.5 N, 22.5 N (b) 40 N, 25 N (c) 20 N, 50 N (d) 15 N, 21 N

102. A mass ' m ' is connected to a thin ribbon which is passed over a pulley. Another mass m is placed over the ribbon as shown. The floor is smooth. The small friction coefficient between m and the ribbon is μ . The acceleration of the lower mass m is

- (a) g (b) $\frac{g}{2}$
(c) μg (d) $(1 - \mu)g$

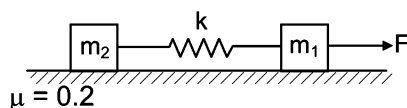


103. Block A of mass 10 kg rests on block B of mass 20 kg, which rests on a horizontal tabletop. A horizontal force of 100 N pulls on block B, while block A is attached to a wall by a light, inextensible cord. The coefficient of friction between all surfaces is 0.2. The acceleration of block B and tension in the cord are: ($g = 10 \text{ m s}^{-2}$)



- (a) 3 m s^{-2} , 20 N. (b) 1 m s^{-2} , 20 N. (c) 1 m s^{-2} , 60 N. (d) 3 m s^{-2} , 60 N.

104. Two masses $m_1 = m_2 = 5 \text{ kg}$ connected by a light spring of constant $k = 100 \text{ N m}^{-1}$ are placed on a horizontal floor with coefficient of friction $\mu = 0.2$. If the force F is increased gradually to 15 N, the extension of the spring is



- (a) 0 (b) 15 cm (c) 10 cm (d) 5 cm

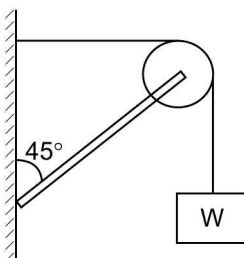
105. An elevator ascending with a uniform acceleration of 2 m s^{-2} , has a fixed smooth inclined plane of angle 45° inside. The base of the inclined plane is 0.5 m wide. If a body is released at the top of the inclined plane, its velocity in the lift (in m s^{-1}) when it reaches the bottom is ($g = 10 \text{ m s}^{-2}$)

- (a) 2 (b) $(\sqrt{3} + 1)^2$ (c) $2\sqrt{3}$ (d) $\left(\frac{\sqrt{3}}{2} + 1\right)^2$

106. A boy weighing 45 kg stands on a spring scale in a lift that is going down with a constant speed of 3 m s^{-1} . If the lift is brought to halt by a constant deceleration, in a distance of 10 m, the scale reading during this period ($g = 10 \text{ m s}^{-2}$) is

- (a) 47 kg (b) 55 kg (c) 50 kg (d) 44 kg

107. A small, smooth, fixed pulley of weight 'w' is connected by a bracket to the wall as shown. A block of weight 'W' is connected to a light, inextensible string as shown. If $\frac{w}{W} = \frac{1}{2}$, the magnitude of the force exerted by the bracket on the pulley is

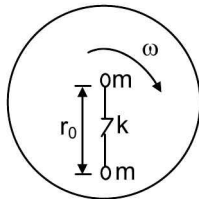


- (a) $\sqrt{2} W$ (b) $2 W$ (c) $\frac{\sqrt{13}}{2} W$ (d) $\frac{3}{2} W$

108. Two identical masses are placed on a horizontal disc, one of them at the centre and the other at r_0 from it and connected by a spring of constant k . The wheel starts from rest and gains speed as it rotates about a vertical axis

4.70 Laws of Motion

through its centre. If friction coefficient for contact surfaces is μ , the angular velocity ω when the central mass moves, is



- (a) $\sqrt{\frac{k}{\mu m}}$ (b) $\sqrt{\frac{2\mu g}{\frac{\mu mg}{k} + r_0}}$ (c) $\sqrt{\frac{2\mu g}{r_0}}$ (d) $\sqrt{\frac{2k}{m}}$

109. A large vertical drum of radius R spins very fast about a vertical axis through the centre of its circular base, so as to pin every one inside it against the vertical wall of the drum without falling (co-efficient of friction between contact surfaces is μ). The minimum angular velocity needed is

- (a) $\mu\sqrt{\frac{g}{R}}$ (b) $\sqrt{\frac{\mu g}{\sqrt{1 + \mu^2} R}}$ (c) $\sqrt{\frac{g}{\mu R}}$ (d) $\sqrt{\frac{(1 + \mu^2)g}{R}}$

110. For a body moving in a horizontal circular path on level ground, the condition for no skidding is

- (a) $\left(\frac{mv^2}{r}\right) \leq \mu mg$ (b) $\left(\frac{mv^2}{r}\right) \geq \mu mg$
(c) $\left(\frac{v}{r}\right) = \mu g$ (d) $\left(\frac{mv^2}{r}\right) = \mu mg$



Assertion–Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

111. Statement 1

Even a child can move an Olympic weightlifter resting on floor if there is no friction between the weight lifter and the floor, but the same weightlifter cannot hold a sting perfectly horizontal if the child's nappy is suspended from it, with the other end of the string fixed to a wall.

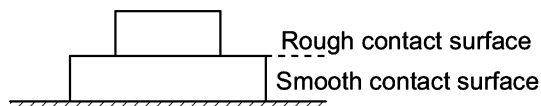
and

Statement 2

For every action there is an equal and opposite reaction.

112. Statement 1

If you find the above two bodies moving as a single unit, there must be external force(s) acting on either or both bodies horizontally, or having a horizontal component.



and

Statement 2

When an opposing force is present, motion is possible only when an external force overcomes it.

113. Statement 1

If a string breaks when used to move a box at rest on floor, it is most likely to break if the box is suspended from it and used to lift vertically.

and

Statement 2

Coefficient of static friction in most cases is less than 1.



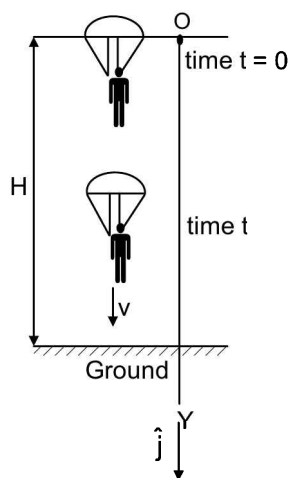
Linked Comprehension Type Questions

Directions: This section contains a paragraph. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Read the following passage and answer the questions.

Passage I

Parachute jumping is a common military and sports activity. The figure shows a man with the parachute opened at a height H starting from rest at time $t = 0$ from the origin O . As the man speeds up towards the ground, the atmospheric drag force F acts on the parachute, which is opposite to the instantaneous velocity v and proportional to the square of v . It is assumed that the weight remains constant.



- 114.** At any instant, the force acting on the man with parachute is
(m = mass of the system)

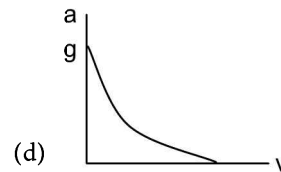
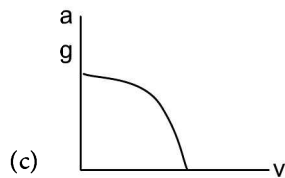
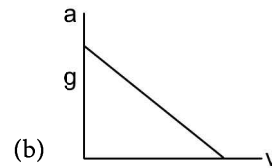
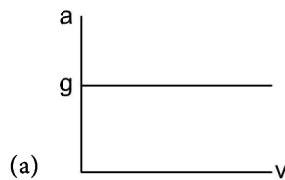
(a) $(mg + F) \hat{j}$ (b) $mg \hat{j}$ (c) $(mg - F) \hat{j}$ (d) $-F \hat{j}$

- 115.** At any instant t , the equation governing the motion is (k = positive constant)

(a) $kv^2 + mg = m \frac{dv}{dt}$ (b) $mg - kv^2 = m \frac{dv}{dt}$ (c) $-mg - kv^2 = m \frac{dv}{dt}$ (d) $kv^2 - mg = m \frac{dv}{dt}$

4.72 Laws of Motion

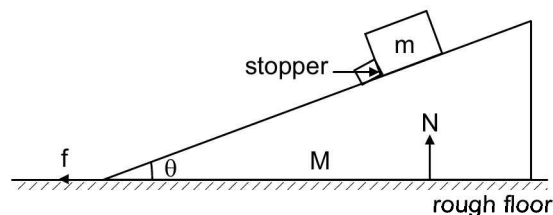
116. The variation of the acceleration with velocity v is given by.



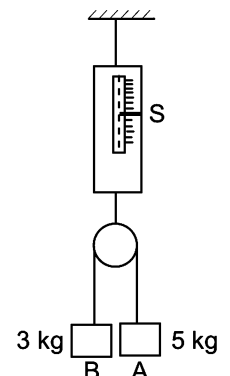
Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

117. A wedge of mass M is kept on a rough horizontal floor and is always stationary. A body of mass m is kept on the smooth surface of wedge M and is held stationary by a stopper.



- (a) when the stopper is removed, the normal reaction the floor exerts on the wedge is $N = Mg + mg \cos^2 \theta$
 (b) when the stopper is removed, the normal reaction the floor exerts on the wedge is $N = Mg + mg \cos \theta \sin \theta$
 (c) Whether stopper is removed or not, $N = (M + m)g$
 (d) when the stopper is removed, the frictional force between the wedge and the floor, $f = mg \cos \theta \sin \theta$
118. A mass of 1 kg is gently dropped on a horizontal conveyor belt moving with a velocity of 4 m s^{-1} and an acceleration of 1 m s^{-2} at that instant. The coefficient of friction between the conveyor belt and the mass is 0.5 ($g = 10 \text{ m s}^{-2}$).
 (a) The mass slips on the belt a distance of 2 m, before it is at relative rest with the belt.
 (b) The mass slips on the belt a distance of $10/3$ m before it is at relative rest with the belt.
 (c) Time in which the relative motion stops is 1 s.
 (d) Time interval in which the frictional force on the 1 kg mass vanishes is 1 s.
119. In the arrangement shown S is a spring balance hanging from the ceiling of a lift. Pulley is light and frictionless and thread is light and inextensible. Then ($g = 10 \text{ m s}^{-2}$)
 (a) when lift is moving up or down with uniform velocity, the reading of the balance is 8 kg.
 (b) when the lift moves up with uniform deceleration of 4 m s^{-2} the reading is 10.5 kg.
 (c) when the lift moves up with uniform acceleration of 4 m s^{-2} the reading is 12 kg.
 (d) when the lift moves down with uniform acceleration of 4 m s^{-2} the reading is 4.5 kg.



**Matrix-Match Type Question**

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

120. (m: mass of body, g: acceleration due to gravity)

Column I

- (a) Body remaining stationary relative to an inclined plane.
- (b) Body on an elevator floor that is coming down with acceleration g .
- (c) Body at rest on an accelerating horizontal conveyor belt.
- (d) Body in a space capsule near Earth accelerating downward at the rate of g .

Column II

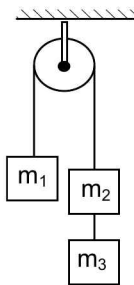
- (p) Apparent weight of the body is zero.
- (q) Force of friction helps to maintain the state.
- (r) A pseudo force of mg can be evoked.
- (s) Acceleration of the system helps to maintain the state.

ADDITIONAL PRACTICE EXERCISE

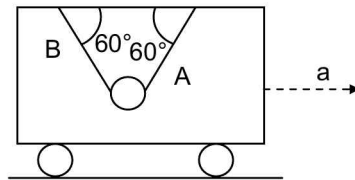
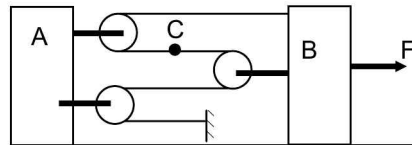


Subjective Questions

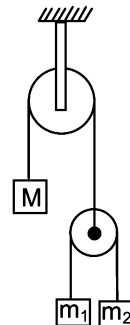
121. A 25 kg box initially at rest, is gently dropped on to the body of a van moving at a speed of 54 kmph. The coefficient of friction between the contact surfaces is 0.5.
- What are the force and acceleration on the box?
 - Calculate how far the van will move before the box stops slipping. ($g = 10 \text{ m s}^{-2}$)
122. In the arrangement shown, the pulley is light and frictionless and the strings are light and inextensible. $m_1 = 5 \text{ kg}$; $m_2 = 2 \text{ kg}$; $m_3 = 1 \text{ kg}$. The system is released from rest. Calculate the

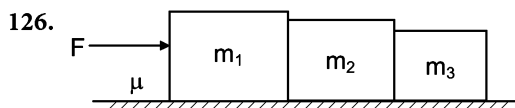


- Acceleration of the masses.
 - Tensions in the strings.
 - If the tension in the string connecting m_2 and m_3 cannot exceed 11.25 N, what is the maximum allowable mass of m_1 ? ($g = 10 \text{ m s}^{-2}$)
123. In the system shown, pulleys have negligible mass and friction, strings are light and inextensible, floor is smooth. Find the
- accelerations of blocks A and B.
 - acceleration of point C on the string ($F = 73.5 \text{ N}$, $m_B = 20 \text{ kg}$, $m_A = 8 \text{ kg}$).
124. A steel ball is suspended from the roof of an accelerating truck by the two cords A and B. Determine the acceleration a of the truck which will cause
- the tension in A to be twice that in B.
 - the tension in B to be zero.
 - the tension in B to be more than that in A.



125. In the arrangement shown, the system is released from rest. Pulleys are massless, frictionless; strings are light and inextensible.
- Calculate accelerations of M , m_1 and m_2 . ($m_1 > m_2$)
 - Calculate tensions in the strings.
 - Analyse accelerations when
 - $m_1 = m_2$
 - $M = \frac{4m_1m_2}{(m_1 + m_2)}$
 - both (a) and (b) hold good.

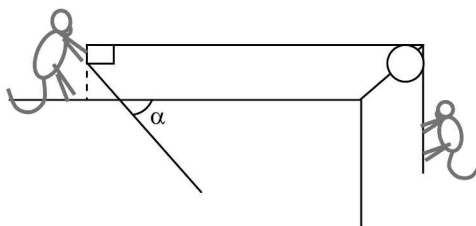




Three masses $m_1 = 10 \text{ kg}$, $m_2 = 6 \text{ kg}$, $m_3 = 4 \text{ kg}$ are in contact with each other and rest on a surface with coefficient of friction $\mu = 0.1$ between surface and the blocks. A force F is applied in the direction shown. Calculate acceleration of the blocks, interaction forces between the blocks and frictional forces on each block for various values of F given below ($g = 10 \text{ m s}^{-2}$). There is no friction on the contact faces between the blocks.

- (i) $F = 25 \text{ N}$
- (ii) $F = 8 \text{ N}$
- (iii) $F = 12 \text{ N}$
- (iv) $F = 20 \text{ N}$

127. A monkey of mass 10 kg climbs down a rope with an acceleration of 6 m s^{-2} . Another monkey of mass 8 kg supports this monkey. If the coefficient of friction on the horizontal floor is 0.75 , find the angle α made by the 8 kg monkey's leg with horizontal, assuming resultant reaction is along the leg.



128. Two blocks of masses m_1 and m_2 respectively are connected by a light, inextensible thread and placed on a rough inclined plane, inclined at an angle ' α ' with the horizontal. The co-efficient of friction between the blocks and inclined plane are μ_1 and μ_2 respectively. If the string is taut, find
- (i) the common acceleration ' a ' of the blocks along the plane.
 - (ii) the effective coefficient of friction of the system.
129. A truck going at 72 kmph around a horizontal curve of radius 200 m , inwardly banked at $\theta = \sin^{-1} 0.1$, carries a box of mass 100 kg . The coefficient of static friction between the box and the truck surface is 0.4 . Determine the frictional force acting on the box. ($g = 10 \text{ m s}^{-2}$)
130. A small sphere of mass 500 g is attached to a light, inextensible string of length 2 m , the other end being fixed to a ceiling. The sphere is made to describe a horizontal circle of radius 1 m .
- (i) Calculate the time period of this pendulum.
 - (ii) What is the magnitude of the tension in the string in this position?
 - (iii) At what rotational speed will the string rotate in a horizontal circle of radius $\sqrt{2} \text{ m}$?
 - (iv) At what rotational speed can the string become horizontal?



Straight Objective Type Questions

Directions: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

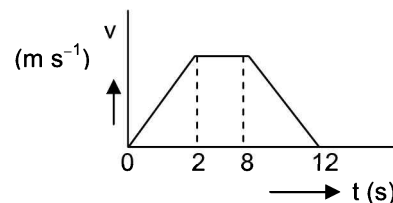
131. A bullet penetrates a distance d of a plank. If the initial momentum of the bullet is doubled, the distance penetrated will be
- (a) $2d$
 - (b) $4d$
 - (c) $8d$
 - (d) $16d$

4.76 Laws of Motion

132. A lift of mass 160 kg moves from ground floor to 10th floor as per the v-t graph shown.

When passing through 2nd floor and 8th floor, the tensions in the rope differ by 120 N. Assuming equidistant floors, height of each floor is (in m)

- (a) 2.7 (b) 3.2
(c) 3.6 (d) 4



133. A body subjected to three constant forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ is moving with uniform velocity, How many of the following statements must necessarily be true?

- (i) $|\vec{F}_1| + |\vec{F}_2| > |\vec{F}_3|$
(ii) $|\vec{F}_1| - |\vec{F}_2| < |\vec{F}_3|$
(iii) $(\vec{F}_1 \times \vec{F}_2) \cdot \vec{F}_3 = 0$

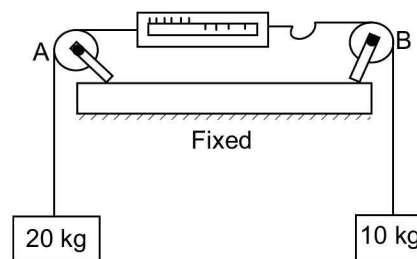
- (a) zero (b) one (c) two (d) three

134. The displacement of a 1 kg mass moving along the X axis is given by the equation $x = t^3 - 2t^2 - 9$ where x is in metre and t in second. At t = 0, force on the mass is [Take +x as +ve direction]

- (a) -4 N (b) -6 N (c) 3 N (d) 8 N

135. Take $g = 10 \text{ m s}^{-2}$. Pulleys A and B are identical, smooth, light and at the same horizontal levels. The reading of the spring balance (of negligible mass), when the system is released from rest is (strings are light and inextensible)

- (a) 10 kg wt (b) 15 kg wt
(c) $\frac{40}{3}$ kg wt (d) 30 kg wt



136. Two monkeys move up with constant accelerations along either part of a light inextensible rope passing over a horizontal branch of a tree. The heavier monkey is moving up; its acceleration is $\frac{g}{4}$ and their relative acceleration is $\frac{g}{2}$.

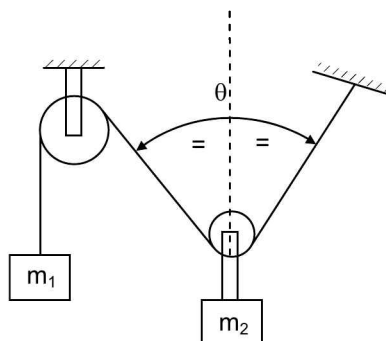
The ratio of the masses of the monkeys (larger: smaller) is

- (a) $\frac{7}{5}$ (b) $\frac{8}{3}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

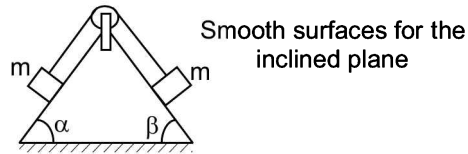
137. Pulleys are light and smooth, string is light and inelastic and both blocks are at rest.

The angle θ is

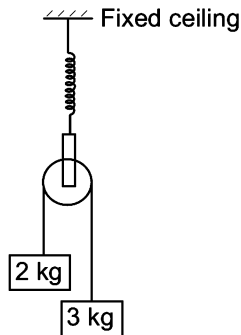
- (a) $\cos^{-1} \frac{m_2}{2m_1}$ (b) $\sin^{-1} \frac{m_2}{m_1}$ (c) $2 \cos^{-1} \frac{m_2}{2m_1}$ (d) $2 \sin^{-1} \frac{m_2}{m_1}$



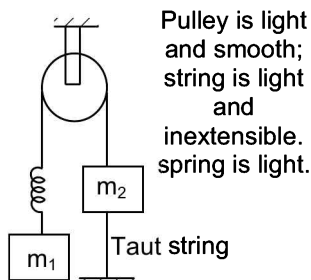
138. Pulley is light, smooth and fixed. String is light and inextensible. If the string can withstand a maximum tension of $\frac{mg}{2}$, then $\sin\alpha + \sin\beta$ cannot exceed



- (a) $\frac{1}{2}$ (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$
139. The pulley is light and smooth and fixed to a light spring as shown. Two blocks of masses 3 kg and 2 kg are connected by a light, inextensible string, passing over the pulley. When the system is released from rest, the spring stretches by 2 cm. The force constant of the spring in N m^{-1} is (Take $g = 10 \text{ m s}^{-2}$)



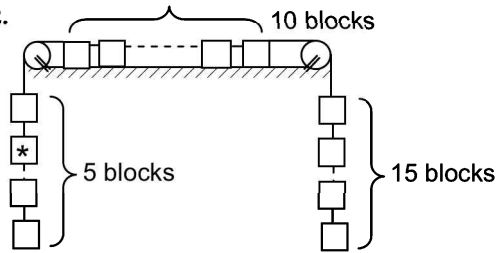
- (a) 2400 (b) 4800 (c) 1200 (d) 3600
140. In the above problem, When the blocks are set in motion and the spring has reached maximum extension, the
- apparent weight of 3 kg block is more than apparent weight of 2 kg block.
 - apparent weight of 2 kg block is more than the apparent weight of 3 kg block.
 - apparent weights of 3 kg block and 2 kg block are equal to their weights when they were at rest.
 - apparent weights of 3 kg block and 2 kg block are same.
141. Immediately on cutting the taut string, the acceleration of one of the blocks is found to be 2 m s^{-2} . If $m_1 = 6 \text{ kg}$, m_2 is equal to ($g = 10 \text{ m s}^{-2}$)



- (a) 4 kg (b) 5 kg (c) 2 kg (d) 8 kg

4.78 Laws of Motion

142.



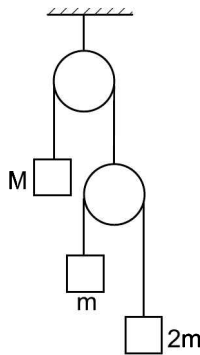
Pulley are light,
strings are light and
inextensible

All blocks are identical except that one of the blocks on the table is a broken block (of less mass). No friction. $g = 10 \text{ m s}^{-2}$. How many among the following are acceptable values of acceleration of the block marked * ?

2.5 m s^{-2} , 3.5 m s^{-2} , 4.5 m s^{-2}

- (a) zero (b) one (c) two (d) three

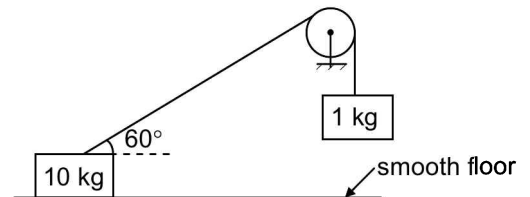
143. In the arrangement shown M is stationary if M is equal to



- (a) 3 m (b) $\frac{11}{4} \text{ m}$ (c) $\frac{11}{3} \text{ m}$ (d) $\frac{8}{3} \text{ m}$

144. The masses are released in the set up as shown and the floor is smooth. The string is light and inextensible; pulley is light and smooth. The acceleration of the 10 kg mass is (in m s^{-2}): Take $g = 10 \text{ m s}^{-2}$

- (a) $\frac{11}{23}$ (b) $\frac{14}{17}$
(c) $\frac{18}{21}$ (d) $\frac{20}{41}$



145. A monkey of weight mg sits in a rocket moving up with an acceleration of $5g$. The apparent weight of the monkey is $x mg$ where, x is

- (a) 5 (b) 4 (c) 3 (d) 6

146. A block of mass 0.5 kg is placed on a smooth wedge of inclination 60° with the horizontal. The wedge is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block is: ($g = 10 \text{ m s}^{-2}$)

- (a) 5 N (b) 10 N (c) 20 N (d) 8 N

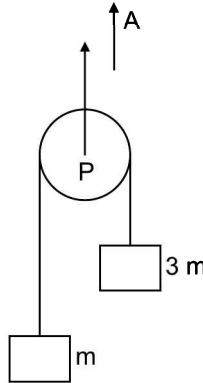
147. A small sphere is suspended by a string from the ceiling of a car, which is in uniform motion. If the car decelerates, the tension in the string

- (a) Increases (b) decreases (c) remains same (d) becomes zero

148. A block placed on an inclined plane within a lift does not slide down, if the lift is stationary. Will it slide down if the lift is given an acceleration upwards?

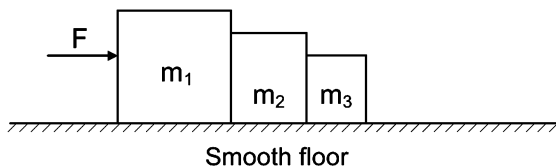
- (a) It will (b) It will not
(c) It may or may not (d) It will actually slide up.

149. In the figure shown, the pulley is moving up with an acceleration A and the mass $3m$ is stationary. The acceleration of m is (modulus value)



- (a) $\frac{g}{4}$ (b) $\frac{g}{3}$ (c) g (d) $2g$

150.

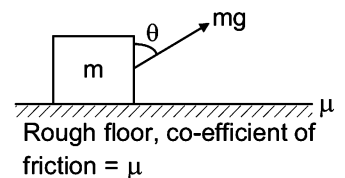


All contact surfaces are smooth. The ratio of the magnitude of the contact force between m_2 and m_3 to the magnitude of the contact force between m_1 and m_2 is (given $m_1 : m_2 : m_3 = 7 : 5 : 3$)

- (a) $\frac{5}{7}$ (b) $\frac{3}{5}$ (c) $\frac{8}{15}$ (d) $\frac{3}{8}$

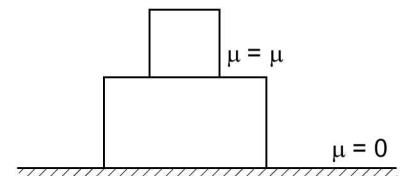
151. In which of the following cases can the body move?

- (a) $\tan \frac{\theta}{2} \geq \mu$ (b) $\cot \frac{\theta}{2} \geq \mu$
(c) $\tan \theta \geq \mu$ (d) $\cot \theta \geq \mu$



152. Total mass of both the blocks is M . Initially the system is at rest. A horizontal force F is applied on one of the blocks. The maximum value of F so that the blocks do not get separated is

- (a) Mg
(b) μMg
(c) cannot be determined unless the individual masses are known.
(d) cannot be determined unless the individual masses and the body on which the force is applied are known



4.80 Laws of Motion

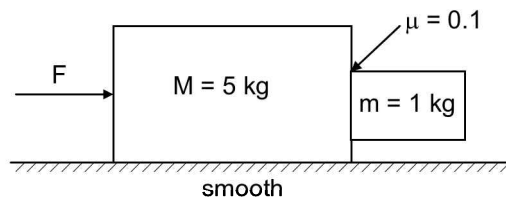
153. Two identical inclined planes have same height but different angles of inclination with horizontal, 53° and 37° respectively. One incline is smooth. A body's time of descent on the smooth plane is same as that of another body on the rough plane. The coefficient of friction between contact surfaces of rough plane is

(a) $\frac{1}{3}$ (b) $\frac{7}{12}$ (c) $\frac{2}{5}$ (d) $\frac{3}{4}$

154. The same amount of force needs to be applied to prevent a block from sliding down a fixed inclined plane of angle of inclination θ with the horizontal, whether it is applied parallel to the plane or horizontally. The coefficient of friction between the plane and the block is

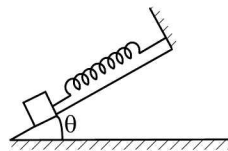
(a) $\tan \frac{\theta}{2}$ (b) $\sin \frac{\theta}{2}$ (c) $\cos \frac{\theta}{2}$ (d) $\tan \theta$

155. The co-efficient of friction between M and m (of masses 5 kg and 1 kg respectively) is 0.1. What is the magnitude of the minimum horizontal force F to be applied on M, so that m does not slip on M? Take $g = 10 \text{ m s}^{-2}$.



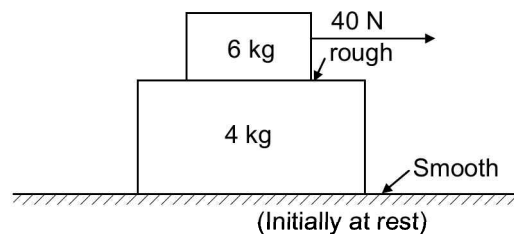
(a) 500 N (b) 10 N (c) 60 N (d) 600 N

156. When $\theta = 20^\circ$, the spring is at natural length. When $\theta = 90^\circ$, spring extends by 1 m at equilibrium, when $\theta = 30^\circ$ spring extends by x m at equilibrium. How many among 0.5, 0.3, 0.18 could be acceptable values of x? ($\tan 20^\circ = 0.36$)



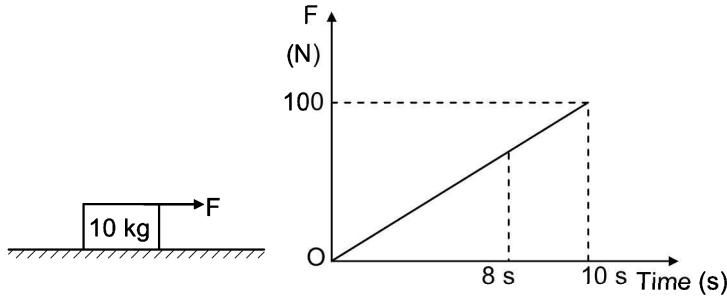
(a) none (b) one (c) two (d) three

157. The accelerations are equal. Then how many among the following are acceptable values of the coefficient of friction? ($g = 10 \text{ m s}^{-2}$)
0.24, 0.25, 0.26



(a) zero (b) one (c) two (d) three

158.



A force F , increasing from 0 as shown in graph is acting on a 10 kg mass kept on a rough horizontal floor, in a direction parallel to the floor. At $t = 8$ s, the body starts moving. At $t = 10$ s its velocity is (in m s^{-1})

- (a) 0.8 (b) 0.95 (c) 1.1 (d) 2

159. The minimum force required to pull a mass of 10 kg kept on a horizontal rough surface (coefficient of friction : μ) is 60 N. On a rough inclined plane having same μ value and variable inclination angle, the angle of repose is ($g = 10 \text{ m s}^{-2}$)

- (a) 30° (b) 45° (c) $\sin^{-1} \frac{1}{3}$ (d) 37°

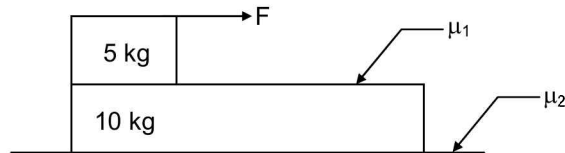
160. The minimum force required to pull up a 10 kg mass kept on a rough inclined plane ($\mu = 0.75$) inclined at an angle of inclination 45° with the horizontal is approximately ($g = 10 \text{ m s}^{-2}$)

- (a) 80 N (b) 85 N (c) 90 N (d) 99 N

161. In the above case, the frictional force developed is just less than

- (a) 60 N (b) 55 N (c) 20 N (d) 8.5 N

162. In the figure shown, the minimum ratio of $\frac{\mu_1}{\mu_2}$ so that the masses moves together is

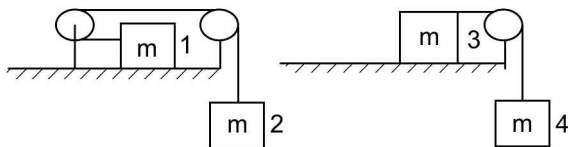


- (a) 5 (b) 4 (c) 3 (d) 2

163. A body slides up an inclined plane of angle θ with the horizontal with an initial velocity v . If the coefficient of friction between the contact surfaces μ , ($\mu < \tan \theta$) is small, the time taken by the body to return to the starting point is

- (a) $2 \frac{v}{g \sin \theta}$ (b) $2 \frac{v}{g \sin \theta} \frac{1}{(\mu \cos \theta + \sin \theta)}$
 (c) $\frac{v}{g \cos \theta} \left(\frac{1}{(\mu + \tan \theta)} + \frac{1}{\sqrt{-\mu^2 + \tan^2 \theta}} \right)$ (d) $\frac{v}{g \cos \theta}$

164.



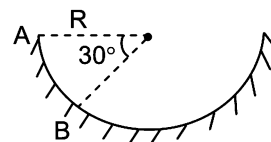
Identical blocks 1,2,3 and 4 have accelerations: a_1 , a_2 , a_3 and a_4 respectively.

If all surfaces are smooth,

- (a) $a_1 = a_2 \neq a_3 = a_4$ ($a_i \neq 0$) (b) $a_1 = a_2 = a_3 = a_4$ (c) $a_1 \neq a_2 \neq a_3 = a_4$ (d) $a_1 = 0 = a_2 \neq a_3 = a_4$

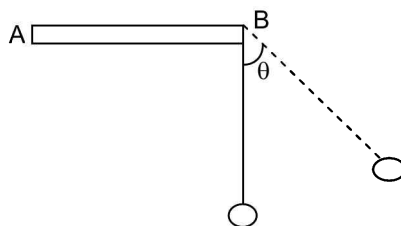
4.82 Laws of Motion

165. A particle is released at point A of the semi circular vertical smooth track. When it reaches B, $v = \sqrt{gR}$, the fraction of the normal reaction which provides centripetal force is



- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) 1

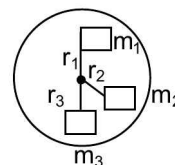
166. A horizontal rod AB of length 1 m, with a 1 m long light, inextensible string with a bob attached to it and suspended from end B is rotated on the horizontal plane about point A at a constant angular velocity of $\sqrt{10} \text{ rad s}^{-1}$, so that the string makes angle θ with vertical. ($g = 10 \text{ m s}^{-2}$) θ lies between



- (a) 0° and 30° (b) 30° and 45° (c) 45° and 60° (d) 60° and 90°
167. A uniform thin rod of length ℓ and mass m , is rotated on horizontal plane about a hinge at one end, with a constant angular velocity. If the force exerted by the rod on the hinge at one end is F , the angular velocity of the rod is

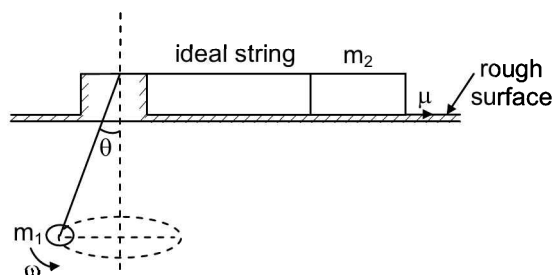
- (a) $\left(\frac{F}{m\ell}\right)^{\frac{1}{2}}$ (b) $\left(\frac{2F}{m\ell}\right)^{\frac{1}{2}}$ (c) $\left(\frac{2F}{3m\ell}\right)^{\frac{1}{2}}$ (d) $\left(\frac{3F}{2m\ell}\right)^{\frac{1}{2}}$

168. Three masses m_1, m_2, m_3 with base areas a_1, a_2, a_3 are kept on the rough surface of a horizontal platform, which can rotate about a central vertical axes as shown. r_1, r_2 and r_3 are the radial distances from the centre to the masses, respectively. The platform starts rotating about the central axis and the angular frequency ω is increased gradually. The mass that slips first is ($\mu = \text{same for all masses}$):



- (a) The one with largest $\frac{ma}{r}$ value
(b) The one with largest value of $m \times r$
(c) The largest mass
(d) The one at largest radial distance r

169.



In the arrangement shown, m_1 is rotating like a conical pendulum and the string makes an angle θ with the vertical. The string is massless, inextensible and passes through a smooth orifice and is attached to m_2 and m_1 as shown. m_2 rests

on a rough surface, with coefficient friction μ . $\frac{m_1}{m_2} = \frac{1}{2}$. The angular velocity ω of the conical pendulum is gradually increased and in the limiting case, when m_2 is about to slip, $\theta = 45^\circ$. Then μ is

- (a) $\frac{1}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{1.1}}$

170. An aeroplane is flying in a horizontal circular path of radius 2 km at a place where $g = 10 \text{ m s}^{-2}$. If the speed of the plane remains constant at 720 kmph, the angle with the horizontal at which the wings of the plane are inclined is equal to

- (a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$ (c) $\tan^{-1}\left(\frac{1}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{3}\right)$



Assertion–Reason Type Questions

Directions: Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
 (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
 (c) Statement-1 is True, Statement-2 is False
 (d) Statement-1 is False, Statement-2 is True

171. Statement 1

Impulse is the product of the average force and the time for which the force acts.

and

Statement 2

Impulse is equal to the change in momentum of a body during that time.

172. Statement 1

Equilibrium means either the body is at rest (static equilibrium) or in uniform motion (dynamic equilibrium).

and

Statement 2

When the resultant force is zero or there is no unbalanced force the body is said to be in equilibrium.

173. Statement 1

When a person is pedaling a bicycle on level ground at constant speed and the bicycle is moving East, then the direction of friction on the front wheel is towards West and on the rear wheel is towards East.

and

Statement 2

If pedalling is stopped, both wheels experience a force of friction in the backward direction.

174. Statement 1

A frame of reference is known as an inertial frame if Newton's first law of motion holds good in it.

and

Statement 2

A frame of reference is a non-inertial frame, if pseudo forces are to be considered in applying Newton's second law of motion in it.

175. Statement 1

When two highly polished metallic surfaces are in contact, the coefficient of friction is often more than one.

4.84 Laws of Motion

and

Statement 2

This is because the original friction lies in inter atomic attraction and when two polished surfaces are so close, their atoms form a cold weld.

176. Statement 1

A tablecloth can be pulled horizontally off the table without dislodging the table or the dishes on top of it, if there is no friction between the contact surfaces.

and

Statement 2

Friction opposes motion of contact surfaces.

177. Statement 1

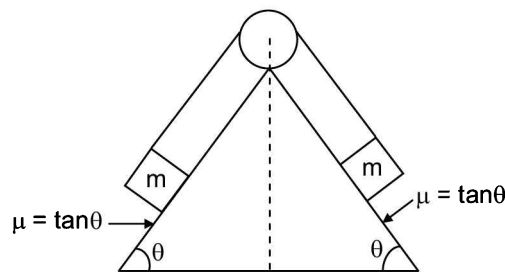
If the acceleration $\vec{a}$ of a particle (as seen from an inertial frame) is zero, then no force is acting on that particle.

and

Statement 2

$\frac{d\vec{p}}{dt} = m\vec{a}$ and when $\vec{a} = 0$, $\frac{d\vec{p}}{dt} = 0$. ($\vec{p}$ is the linear momentum and m is the mass of the particle)

178. Statement 1



In the set up shown, two equal masses are kept on two rough inclined planes and are connected by an ideal string which passes over an ideal pulley. The angle of incline θ is equal to angle of repose. If the masses are put in motion to left or right the masses will not continue to move with uniform velocity.

and

Statement 2

At the angle of repose: $mg\sin\theta = \mu mg\cos\theta$

179. Statement 1

When the passengers sitting inside a stationary bus push the seats in front of them simultaneously in the same direction, the bus does not move.

and

Statement 2

The net force exerted by the passengers on the bus is zero.

180. Statement 1

In a rotating frame of reference, centripetal force on a particle is equal and opposite to the centrifugal force on it.

and

Statement 2

Newton's third law of motion is applicable only in inertial frames and a rotating frame of reference is a non-inertial frame.

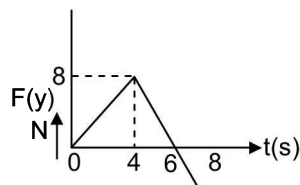
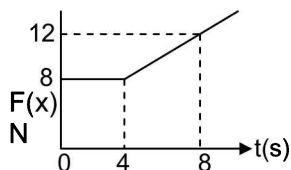


Linked Comprehension Type Questions

Directions: This section contains 3 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Passage I

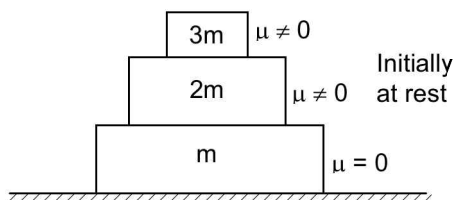
A particle is at rest at $t = 0$ at origin on XY plane. Forces F_x and F_y along respective axes are applied on the particle as per graphs shown:



181. The velocity of the particle makes 45° with x axis at $t =$ (approximately)
 (a) 7.6 s (b) 14.5 s (c) 29.3 s (d) never
182. The velocity of the particle is parallel to X axis at $t =$
 (a) 9.5 s (b) 8 s (c) 6 s (d) never
183. The velocity of the particle is parallel to Y axis at $t =$
 (a) 9.5 s (b) 8 s (c) 6 s (d) never

Passage II

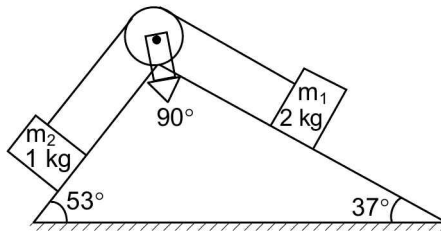
When a horizontal force is applied on the bottom block, the accelerations of the block (in m s^{-2}) are 1, 2 and 3 respectively ($g = 10 \text{ m s}^{-2}$)



184. μ between m and $2m$ is
 (a) 0.14 (b) 0.23 (c) 0.32 (d) 0.41
185. The applied force in N is
 (a) 7 m (b) 10 m (c) 14 m (d) 16 m
186. If the same force had been applied horizontally on the top block, the smallest of the three accelerations would be (in m s^{-2})
 (a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $\frac{4}{7}$ (d) 1

Passage III

Consider the masses m_1 and m_2 (of 2 kg and 1 kg respectively) kept on fixed, rough, inclined planes, connected by a light, inextensible string passing over a smooth, light pulley. Coefficient of friction of the contact surfaces is 0.5 ($g = 10 \text{ m s}^{-2}$)



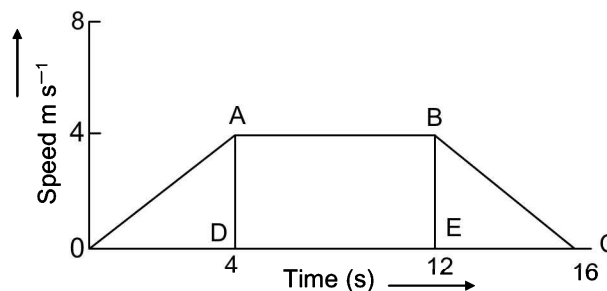
187. When released
- The bodies will move with acceleration, with 2 kg block going down the plane.
 - The bodies will move with acceleration, with 1 kg block going down the plane.
 - They move with uniform velocity, either down the plane or up the plane.
 - They will not move
188. If m_1 is given an initial velocity of $\sqrt{7} \text{ m s}^{-1}$ along the downward direction on the plane, the distance moved by it in 2 s is:
- 1 m
 - 1.5 m
 - 2 m
 - $2\sqrt{7} \text{ m}$
189. If m_2 is given an initial velocity of $\sqrt{15} \text{ m s}^{-1}$ along the downward direction on the plane, the distance moved by it in 2 s is:
- 1 m
 - 1.5 m
 - 2 m
 - $2\sqrt{15} \text{ m}$



Multiple Correct Objective Type Questions

Directions: Each question in this section has four suggested answers out of which ONE OR MORE answers will be correct.

190. A lift is going up. The variation of the speed of the lift with time is shown in figure. The total mass of the lift and passengers is 1000 kg. ($g = 10 \text{ m s}^{-2}$)

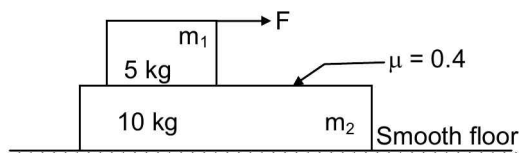


- the tension in the rope of the lift at 2 s will be 11,000 N
 - the tension in the rope at $t = 8 \text{ s}$ will be 10,000 N
 - the tension in the rope at $t = 14 \text{ s}$ will be 9,000 N
 - the height up to which the lift takes passengers is 96 m
191. The coefficient of friction between the wheels of the car and the ground is 0.2. The car starts from rest and moves along a perfectly horizontal road. ($g = 10 \text{ m s}^{-2}$) The car
- can acquire a maximum acceleration of 2 m s^{-2} without slipping.
 - can attain a speed of 20 m s^{-1} while moving with maximum acceleration, in a minimum distance of 100 m.

- (c) can go up to a speed of 10 m s^{-1} in 4 s.
 (d) after acquiring a speed of 50 m s^{-1} , when the engine is switched off, it can come to rest in a time of 25 s.

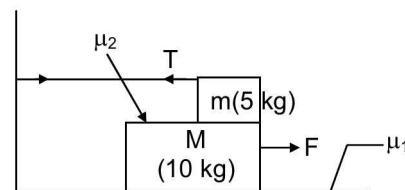
192. A body is projected up, along a rough inclined plane of angle θ , from its bottom, with an initial velocity v . The body comes to halt somewhere on the inclined plane at a height h . As μ is varied, the maximum height is H . Then
- The body will reach H for maximum value of μ
 - The body will reach H for μ just greater than $\tan \theta$
 - H is independent of θ
 - $H = \frac{v^2}{4g} \sin \theta$

193.

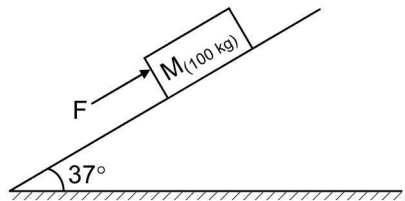


In the figure, the coefficient of friction between m_1 and m_2 is $\mu = 0.4$ and the floor is smooth. Force F is applied as shown. $g = 10 \text{ m s}^{-2}$. Then

- m_1 will start slipping on m_2 when $F > 20 \text{ N}$
 - m_1 will start slipping on m_2 when $F > 30 \text{ N}$
 - The maximum acceleration possible for m_2 is 2 m s^{-2}
 - If $F = 40 \text{ N}$, m_1 will move 1 m over m_2 , each second.
194. In the arrangement shown, the static and dynamic coefficients of friction between M and the floor is $\mu_{1s} = 0.3$, $\mu_{1k} = 0.2$ respectively and similarly between m and M is $\mu_{2s} = 0.4$ and $\mu_{2k} = 0.3$ respectively. ($g = 10 \text{ m s}^{-2}$)
- M will start moving when F is just above 60 N
 - When M moves its acceleration is $\geq 2 \text{ m s}^{-2}$
 - The maximum tension on the string when M is stationary is 15 N
 - The maximum tension on the string when M moves is 15 N .

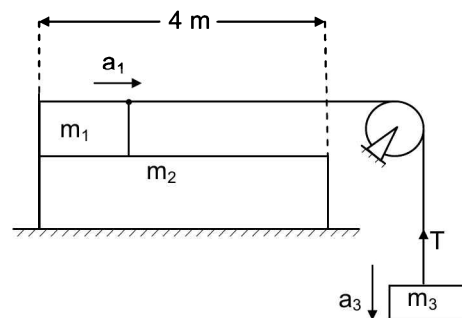


195. A force F is applied as shown on a block of mass $M = 100 \text{ kg}$, kept on a fixed, rough incline of angle $\theta = 37^\circ$ with horizontal. The force is gradually increased and when $F = 840 \text{ N}$, the block starts moving up and covers a distance of 7.5 m with an average velocity of 1.5 m s^{-1} . ($g = 10 \text{ m s}^{-2}$)



- The coefficient of static friction between block and incline is $\mu_s = 0.3$.
- The coefficient of dynamic friction between block and incline is $\mu_k = 0.28$
- The coefficient of dynamic friction between block and incline is $\mu_k = 0.225$
- The minimum acceleration with which M will move is 0.6 m s^{-2} .

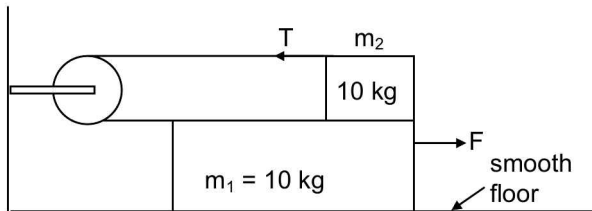
196. In the system shown, $m_1 = m_2 = m_3 = 10 \text{ kg}$. Frictional coefficient between m_1 and m_2 is $\mu_1 = 0.4$, and between m_2 and floor is $\mu_2 = 0.1$. At $t = 0$, the system is released. The string is light and inextensible. Pulley is light and smooth ($g = 10 \text{ m s}^{-2}$)



- Acceleration a_1 of m_1 is 3 m s^{-2}
- Acceleration a_2 of m_2 is 2 m s^{-2}
- The tension of the string is $T = 60 \text{ N}$
- m_1 will fall off m_2 in time $t = 3 \text{ s}$

4.88 Laws of Motion

197.



In the arrangement shown, floor is smooth and coefficients of static and dynamic friction between m_1 and m_2 are 0.6 and 0.4 respectively. A force F applied as shown is increased gradually. String is light and inextensible, pulley is light and smooth. $g = 10 \text{ m s}^{-2}$.

- (a) Minimum value of F at which they move is 120 N
- (b) Minimum value of F at which they move is 100 N
- (c) The maximum tension on the string when the masses are stationary is 60 N
- (d) m_1 and m_2 can move with minimum relative acceleration of 4 m s^{-2} only



Matrix-Match Type Questions

Directions: Match the elements of Column I to elements of Column II. There can be single or multiple matches.

198. $\vec{F}$ is the total resultant force acting on a body given in Table I. Match the columns.

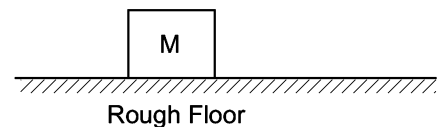
Column I

- (a) $F = 0$
- (b) $\vec{F} = \text{non-zero constant}$
- (c) $|\vec{F}| = \text{non-zero constant}$ (r)
- (d) $\vec{F} \propto x$

Column II

- (p) projectile motion
- (q) uniform motion
- uniform circular motion
- (s) Accelerated motion along x - axis

199. A block of mass ' M ' is on a rough horizontal floor. $\vec{p}$ indicates the linear momentum of the block in the horizontal direction. Match the columns.



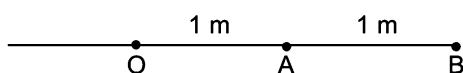
Column I

- (a) $\frac{d\vec{p}}{dt} = \text{constant}$
- (b) $\frac{d\vec{p}}{dt} = \text{constant} > 0$
- (c) $\frac{d\vec{p}}{dt} = 0$
- (d) $\vec{p} = 0$

Column II

- (p) zero friction
- (q) static friction
- (r) kinetic friction
- (s) Limiting friction

200.



Two particles A and B of same mass are connected by light inextensible strings OA and AB, each of length 1 m. With O as centre, A and B rotate in a horizontal circle with constant angular velocity ' ω '.

Tension in OA = T_A (N), tension in AB = T_B (N), speed of A = V_A (m s^{-1}), speed of B = V_B (m s^{-1}). Match the entries in column I with the correct entries in column II

Column I

- (a) Magnitude of centripetal acceleration of A:
Magnitude of centripetal acceleration of B
- (b) $\frac{T_B}{T_A}$
- (c) $\frac{V_A}{V_B}$
- (d) In the rotating frame, centrifugal force on B/centrifugal force on A

Column II

- (p) Greater than 1
- (q) Less than 1
- (r) $\frac{1}{2}$
- (s) $\frac{2}{3}$

SOLUTIONS

ANSWERS KEYS

1. (i) Monkey of mass 16 kg reaches the top first
(ii) 4 m
2. (i) $T_{\max} = 11,600 \text{ N}$;
 $T_{\min} = 7,600 \text{ N}$
(ii) 1000 kg
3. (i) $a_2 = 2a_1$
(ii) $a_2 = \frac{a_1}{2}$
(iii) $a_2 = 2a_1$
(iv) $a_2 = \frac{a_1}{2}$
(v) $a_2 = a_1$
(vi) $a_2 = 2a_1$
(vii) $a_2 = \frac{a_1}{2}$
4. (i) $\frac{26}{7} \text{ m s}^{-2}$, downwards along the plane
(ii) $27\frac{3}{7} \text{ N}$ (tension)
(iii) $a_{m_1} = 2.8 \text{ m s}^{-2}$, downward along the plane
 $a_{m_2} = 4.4 \text{ m s}^{-2}$ downward along the plane
(iv) $T = 0$
(v) 3.5 m s^{-2} , downwards along the plane
(vi) $F = 27\frac{3}{7} \text{ N}$ (compression)
5. (i) 2400 N
(ii) 4783 N
(iii) 11,176 N
6. (i) 900 N, at 37° with the surface of inclined plane, directed upwards
(ii) 2500 N
7. $\mu = 0.5$
8. (i) $t = \sqrt{10} \text{ s}$
(ii) $t = 2\sqrt{10} \text{ s}$
(iii) $t = \infty$
9. (i) $F_2 + \frac{(F_1 - F_2)x}{\ell}$
(ii) $T = \frac{mg}{\ell}(\ell - x)$
(iii) $T = \frac{mg}{\ell}(\ell - x) + m_1g$
(iv) $T = \frac{m\omega^2(\ell^2 - x^2)}{2\ell}$
(v) $T = \frac{m\omega^2(\ell^2 - x^2)}{2\ell} + m_1\omega^2\ell$
10. (i) $T_{CA} = \frac{20\sqrt{2}}{(1 + \sqrt{3})} \text{ N}$
 $T_{BA} = \frac{40}{(1 + \sqrt{3})} \text{ N}$
(ii) $10\sqrt{3} \text{ N}$
(iii) 5 m s^{-2}
11. (d) 12. (a) 13. (d)
14. (a) 15. (d) 16. (d)
17. (a) 18. (d) 19. (b)
20. (d) 21. (b) 22. (b)
23. (d) 24. (b) 25. (c)
26. (a) 27. (a), (b) 28. (d)
29. (a), (b), (c), (d)
30. (a) (p), (r)
(b) (q), (s)
(c) (r)
(d) (q), (s)
31. (a) 32. (a) 33. (a)
34. (c) 35. (a) 36. (c)
37. (b) 38. (b) 39. (c)
40. (d) 41. (c) 42. (b)
43. (b) 44. (b) 45. (c)
46. (b) 47. (d) 48. (a)
49. (c) 50. (b) 51. (a)
52. (d) 53. (b) 54. (d)
55. (a) 56. (d) 57. (d)
58. (c) 59. (c) 60. (b)
61. (b) 62. (a) 63. (b)
64. (a) 65. (a) 66. (d)
67. (d) 68. (b) 69. (b)
70. (d) 71. (b) 72. (b)
73. (a) 74. (a) 75. (d)
76. (b) 77. (d) 78. (b)
79. (a) 80. (a) 81. (c)
82. (d) 83. (b) 84. (a)
85. (b) 86. (b) 87. (c)
88. (b) 89. (d) 90. (a)
91. (b) 92. (b) 93. (c)
94. (c) 95. (c) 96. (d)
97. (c) 98. (c) 99. (b)
100. (c) 101. (a) 102. (d)
103. (b) 104. (d) 105. (c)
106. (a) 107. (c) 108. (b)
109. (c) 110. (a) 111. (b)
112. (d) 113. (a) 114. (c)
115. (b)
116. (c)
117. (a), (d)
118. (a), (c)
119. (b), (d)
120. (a) - (q), (s)
(b) - (p), (r), (s)
(c) - (q), (s)
(d) - (p), (r), (s)
121. (i) 125 N , 5 m s^{-2}
(ii) 45 m
122. (i) 2.5 m s^{-2} (downwards for 5 kg, upwards for 2 kg and 1 kg)
(ii) 37.5 N (between 5 kg and 2 kg)
12.5 N (between 2 kg and 1 kg)
(iii) 3.86 kg
123. (i) $a_A = 2.25 \text{ m s}^{-2}$, $a_B = 3 \text{ m s}^{-2}$
(ii) $a_C = 1.5 \text{ m s}^{-2}$
124. (i) $\frac{g}{3\sqrt{3}}$ unit
(ii) $\frac{g}{\sqrt{3}}$ unit
(iii) $a < 0$ (or retardation)

125. (i)

$$a_M = \frac{4m_1m_2 - M(m_1 + m_2)g}{4m_1m_2 + M(m_1 + m_2)}$$

downwards;

$$a_{m_1} = \frac{4m_1m_2 + M(m_1 - 3m_2)g}{4m_1m_2 + M(m_1 + m_2)},$$

downwards;

$$a_{m_2} = \frac{4m_1m_2 + M(m_2 - 3m_1)g}{4m_1m_2 + M(m_1 + m_2)}$$

downwards

$$(ii) T_1 = \frac{4Mm_1m_2g}{4m_1m_2 + M(m_1 + m_2)} \text{ (in}$$

the string connecting m_1 and m_2)
 $T_2 = 2T_1$ (in the string connecting
 M and pulley)

$$(iii) (a) a_1 = a_2 = A$$

$$(b) A = 0$$

126. (i) $a = 0.25 \text{ m s}^{-2}$, $f_1 = 10 \text{ N}$,

$$f_2 = 6 \text{ N}, f_3 = 4 \text{ N},$$

$$R_{12} = 12.5 \text{ N}, R_{21} = 12.5 \text{ N},$$

$$R_{23} = 5 \text{ N}, R_{32} = 5 \text{ N}$$

$$(ii) a = 0, f_1 = 8 \text{ N}; f_2, f_2, R_{12}, R_{23} = 0$$

$$(iii) a = 0, f_1 = 10 \text{ N}, f_2 = 2 \text{ N}, R_{12} = R_{21} = 2 \text{ N},$$

$$f_3 = 0, R_{23} = R_{32} = 0$$

$$(iv) a = 0, f_1 = 10 \text{ N}, f_2 = 6 \text{ N},$$

$$f_3 = 6 \text{ N}, R_{12} = R_{21} = 10 \text{ N},$$

$$R_{23} = R_{32} = 4 \text{ N}$$

$$127. \alpha = \tan^{-1}(2)$$

128. (i)

$$a = g \left[\sin \alpha - \frac{(\mu m_1 + \mu_2 m_2)}{(m_1 + m_2)} \cos \alpha \right]$$

$$(ii) \mu = \frac{\mu_1 m_1 + \mu_2 m_2}{(m_1 + m_2)}$$

129. 100 N

130 (a) 2.64 s

(b) 5.66 N

(c) 25.11 rpm

(d) Infinity (∞)

131. (b) 132. (a) 133. (d)

134. (a) 135. (c) 136. (a)

137. (c) 138. (b) 139. (a)

140. (d) 141. (b) 142. (a)

143. (d) 144. (d) 145. (d)

146. (b) 147. (a) 148. (b)

149. (d) 150. (d) 151. (b)

152. (d) 153. (b) 154. (a)

155. (d) 156. (b) 157. (a)

158. (d) 159. (d) 160. (d)

161. (d) 162. (c) 163. (c)

164. (b) 165. (b) 166. (d)

167. (b) 168. (d) 169. (c)

170. (b) 171. (b) 172. (a)

173. (b) 174. (b) 175. (a)

176. (c) 177. (d) 178. (b)

179. (c) 180. (c) 181. (c)

182. (a) 183. (d) 184. (a)

185. (b) 186. (d) 187. (d)

188. (b)

189. (b)

190. (a), (b), (c)

191. (a), (b), (d)

192. (b), (c)

193. (b), (c), (d)

194. (b), (d)

195. (a), (c), (d)

196. (a), (b)

197. (a), (c), (d)

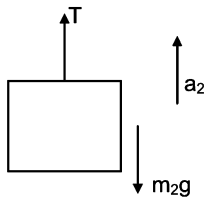
198. (a) $\rightarrow$ (q)(b) $\rightarrow$ (p), (s)(c) $\rightarrow$ (p), (r), (s)(d) $\rightarrow$ (s)199. (a) $\rightarrow$ (p), (q), (r), (s)(b) $\rightarrow$ (r)(c) $\rightarrow$ (p), (q), (r), (s)(d) $\rightarrow$ (p), (q), (s)200. (a) $\rightarrow$ (q), (r)(b) $\rightarrow$ (q), (s)(c) $\rightarrow$ (q), (r)(d) $\rightarrow$ (p)

HINTS AND EXPLANATIONS

Topic Grip

1.

(i)



Let the heavier monkey (mass m_2) move up with an acceleration a_2

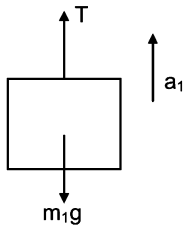
$$T - m_2g = m_2a_2$$

$$T = m_2(a_2 + g)$$

$$T - m_1g = m_1a_1$$

$$m_2(g + a_2) - m_1g = m_1a_1$$

$$a_1 = \left(\frac{m_2}{m_1}\right) \cdot (g + a_2) - g$$



$$\frac{25}{16} (10 + 1.2) - 10$$

$$a_1 = 7.5 \text{ m s}^{-2}$$

⇒ The smaller monkey will reach the top first.

$$(ii) \quad S = \frac{1}{2} a t^2$$

$$25 = \frac{1}{2} a_1 t^2$$

$$h = \frac{1}{2} a_2 t^2$$

$$\frac{h}{25} = \left(\frac{a_2}{a_1}\right) = \left(\frac{1.2}{7.5}\right)$$

$$h = 4 \text{ m}$$

2.

$$(i) \quad T - 1000g = 1000a$$

$$\Rightarrow T = 1000(g + a)$$

where a could be positive or negative.

The extreme values of a from the graph are

$$\frac{4.8}{3} = +1.6 \text{ m s}^{-2} \quad (t = 0 \text{ to } 3 \text{ s})$$

$$\text{and } \frac{-4.8}{2} = -2.4 \text{ m s}^{-2}$$

$$(t = 5 \text{ s to } t = 7 \text{ s})$$

$$\therefore T_{\max} = 1000(10 + 1.6) = 11,600 \text{ N}$$

$$T_{\min} = 1000(10 - 2.4) = 7,600 \text{ N}$$

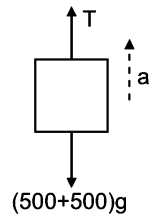
(ii) If p is passenger mass,

$$(500 + p)(g + a_{\max}) \leq 19500$$

$$\Rightarrow (500 + p)(10 + 3) \leq 19500$$

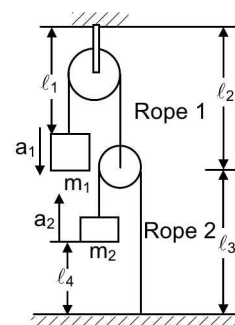
$$\Rightarrow 500 + p \leq \frac{19500}{13} = 1500$$

$$\Rightarrow p \leq 1000 \text{ kg}$$



3.

(i)



$$\text{Rope 1 constraint: } \ell_1 + \ell_2 = \text{constant} \quad \text{--- (1)}$$

$$\text{Rope 2 constraint: } 2\ell_3 - \ell_4 = \text{constant} \quad \text{--- (2)}$$

$$\text{Fixed constraint: } \ell_2 + \ell_3 = \text{constant} \quad \text{--- (3)}$$

Multiply (1) by 2 and add to (2)

$$\Rightarrow 2\ell_1 + 2\ell_2 + 2\ell_3 - \ell_4 = \text{constant}$$

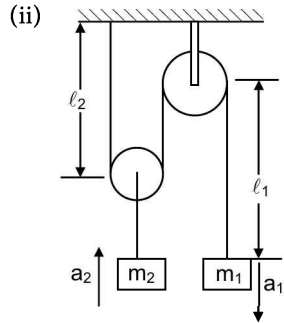
Using (3),

$$\Rightarrow 2\ell_1 - \ell_4 = \text{constant}$$

$$\Rightarrow \frac{2d^2\ell_1}{dt^2} = \frac{d^2\ell_4}{dt^2}$$

$$\frac{d^2\ell_1}{dt^2} = a_1; \frac{d^2\ell_4}{dt^2} = a_2$$

$$\Rightarrow a_2 = 2a_1$$



Rope constraint:

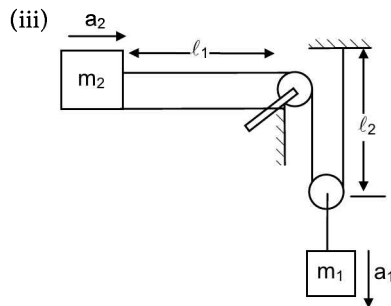
$$2\ell_2 + \ell_1 = \text{constant}$$

$$\frac{2d^2\ell_2}{dt^2} + \frac{d^2\ell_1}{dt^2} = 0$$

$$\frac{d^2\ell_1}{dt^2} = a_1; \frac{-d^2\ell_2}{dt^2} = a_2$$

$$\therefore -2a_2 + a_1 = 0$$

$$\Rightarrow a_2 = \frac{a_1}{2}$$



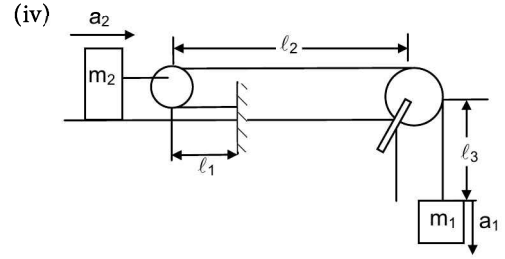
Rope constraint:

$$\ell_1 + 2\ell_2 = \text{constant}$$

$$\Rightarrow \frac{d^2\ell_1}{dt^2} + 2\frac{d^2\ell_2}{dt^2} = 0$$

$$\frac{-d^2\ell_1}{dt^2} = a_2, \frac{d^2\ell_2}{dt^2} = a_1$$

$$\therefore -a_2 + 2a_1 = 0 \Rightarrow a_2 = 2a_1$$

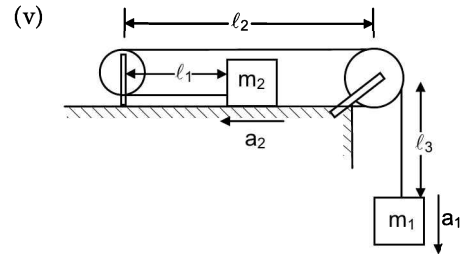


Rope constraint: $\ell_1 + \ell_2 + \ell_3 = \text{constant}$

$$\Rightarrow \frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_2}{dt^2} + \frac{d^2\ell_3}{dt^2} = 0$$

$$\frac{-d^2\ell_1}{dt^2} = a_2, \frac{-d^2\ell_2}{dt^2} = a_2, \frac{d^2\ell_3}{dt^2} = a_1$$

$$\therefore a_1 - 2a_2 = 0 \Rightarrow a_2 = \frac{a_1}{2}$$



Rope constraint:

$$\ell_1 + \ell_2 + \ell_3 = \text{constant}$$

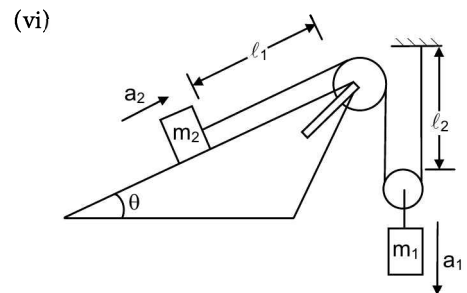
Fixed constraint:

$$\ell_2 = \text{constant}$$

$$\therefore \ell_1 + \ell_3 = \text{constant} \Rightarrow \frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_3}{dt^2} = 0$$

$$\frac{-d^2\ell_1}{dt^2} = a_2, \frac{d^2\ell_3}{dt^2} = a_1$$

$$\therefore -a_2 + a_1 = 0 \Rightarrow a_2 = a_1$$



Rope constraint:

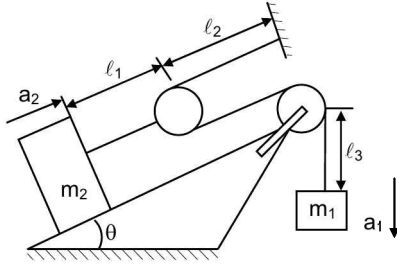
$$\ell_1 + 2\ell_2 = \text{constant} \Rightarrow \frac{d^2\ell_1}{dt^2} + 2\frac{d^2\ell_2}{dt^2} = 0$$

4.94 Laws of Motion

$$\frac{-d^2\ell_1}{dt^2} = a_2; \quad \frac{d^2\ell_2}{dt^2} = a_1$$

$$\therefore -a_2 + 2a_1 = 0 \Rightarrow a_2 = 2a_1$$

(vii)



Rope constraint:

Rope1: $\ell_1 = \text{constant}$

Rope2: $2\ell_2 + \ell_3 = \text{constant}$

$$\frac{d^2}{dt^2}(\ell_1 + \ell_2) = -a_2 \Rightarrow \frac{d^2\ell_2}{dt^2} = -a_2$$

$$\frac{d^2\ell_3}{dt^2} = a_1$$

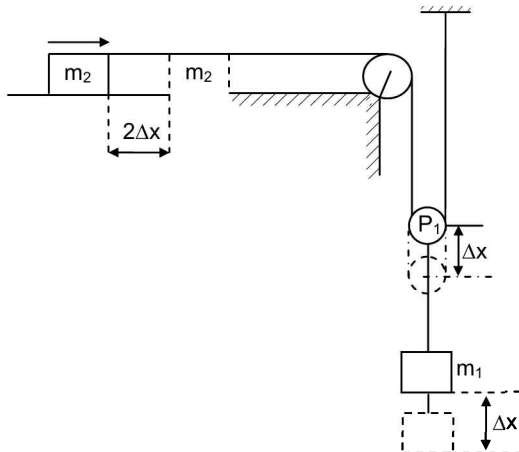
$$\frac{2d^2\ell_2}{dt^2} + \frac{d^2\ell_3}{dt^2} = 0$$

$$2 \times (-a_2) + a_1 = 0 \Rightarrow a_2 = \frac{a_1}{2}$$

Aliter:

Whenever the movable pulley attached to mass m_1 is displaced by Δx , the string is released or used up from both sides of the pulley (i.e., total length is $2\Delta x$) which is used for the movement of the other mass say m_2 . Hence m_2 moves by $2\Delta x$, while m_1 moves by Δx .

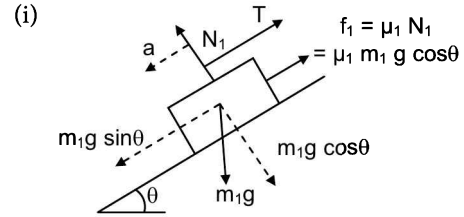
Hence : a_1 of $m_1 = \frac{1}{2}(a_2 \text{ of } m_2)$. Typical example, consider case (iii).



When P_1 moves down by Δx , m_1 moves down by Δx . But the string is used upon both sides of P_1 by Δx , i.e., total $2\Delta x$. Hence m_2 is pulled by $2\Delta x$.

$$\therefore a_2 = 2a_1$$

4.



Let the acceleration of both masses be 'a' downwards, along the plane. FBD of m_1 :

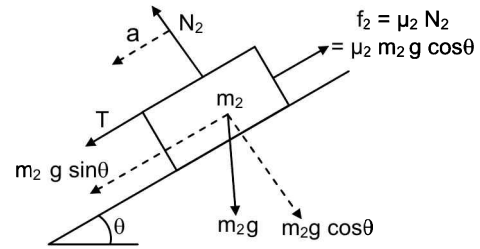
$$m_1 g (\sin \theta - \mu_1 \cos \theta) - T = m_1 a$$

$$\Rightarrow 40 \times 10 \times (0.6 - (0.2)(0.8)) - T = 40a$$

$$\Rightarrow 176 - T = 40a$$

— (1)

FBD of m_2 :



$$T + m_2 g (\sin \theta - \mu_2 \cos \theta) = m_2 a$$

$$\Rightarrow T + 30 \times 10 \times (0.6 - (0.4)(0.8)) = 30a$$

$$\Rightarrow T + 84 = 30a$$

— (2)

$$(1) + (2) \Rightarrow a = \frac{260}{70} = \frac{26}{7} \text{ m s}^{-2}$$

$$\text{Using } a = \frac{26}{7} \text{ in (2)}$$

$$(ii) \quad T = 30 \times \frac{26}{7} - 84 = 27\frac{3}{7} \text{ N}$$

$$(iii) \quad \mu_1 = 0.4, \mu_2 = 0.1$$

Equation (1) becomes

$$40 \times 10 \times (0.6 - (0.4)0.8) - T = 40a$$

$$\Rightarrow 112 - T = 40a$$

— (3)

Equation (2) becomes

$$T + 30 \times 10 (0.6 - (0.2)0.8) = 30a$$

$$\Rightarrow T + 132 = 30a$$

— (4)

Adding (3) and (4)

$$a = \frac{244}{70} \text{ but } T \text{ becomes negative } (-27) \text{ which is}$$

not possible because rope can not take compression. Hence bodies move independently $\Rightarrow T = 0$

Putting $T = 0$ in (3) and (4)

$$a_{m_1} = \frac{112}{40} = 2.8 \text{ m s}^{-2}; a_{m_2} = \frac{132}{30} = 4.4 \text{ m s}^{-2}$$

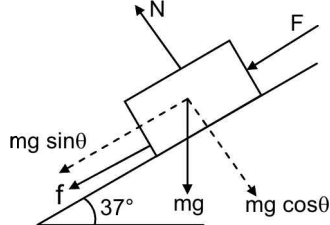
(iv) $T = 0$

(v) Instead of rope, if rod is used, it can take compressive stress and therefore common acceleration of the masses = $a = \frac{244}{70} = 3.5 \text{ m s}^{-2}$

(vi) $T = -27\frac{3}{7} \text{ N}$ (compression)

5.

(i)



$$s = 0.6t + 0.2t^2$$

$$\left. \frac{ds}{dt} \right|_{t=0} = u = 0.6 \text{ m s}^{-1}$$

$$\frac{d^2s}{dt^2} = a = 0.4 \text{ m s}^{-2}, v = u + at$$

$$\Rightarrow v = 0.6 + (0.4)11 = 5 \text{ m s}^{-1} \text{ at } t = 11 \text{ s}$$

$$N = mg \cos \theta = 1000 \times 10 \times 0.8 = 8000 \text{ N}$$

$$f = \mu N = (0.2) 8000 = 1600 \text{ N}$$

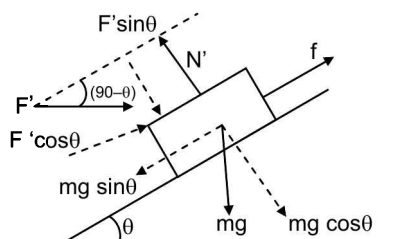
$$\text{Retardation } a = \frac{5 \text{ m s}^{-1}}{0.5 \text{ s}} = 10 \text{ m s}^{-2}$$

$$\therefore F + f + mg \sin \theta = ma$$

$$\Rightarrow F + 1600 + 1000 \times 10(0.6) = 1000 \times 10$$

$$\Rightarrow F = 10000 - 7600 = 2400 \text{ N}$$

(ii)



$$N' = mg \cos \theta + F' \sin \theta$$

$$= 1000 \times 10 \times (0.8) + F' (0.6)$$

$$\therefore f_{\max} = \mu N' = 0.2(8000 + 0.6F')$$

$$= 1600 + 0.12F'$$

$$\text{For equilibrium, } F' \cos \theta + f = mg \sin \theta$$

$$\Rightarrow f = mg \sin \theta - F' \cos \theta$$

$$= 6000 - 0.8F'$$

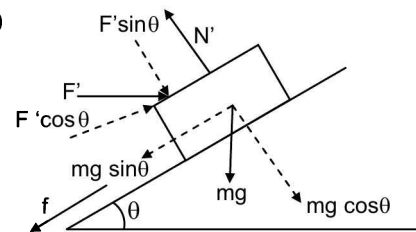
$$f \leq f_{\max} \Rightarrow 6000 - 0.8F' \leq 1600 + 0.12F'$$

$$\Rightarrow 0.92F' \geq 4400$$

$$\Rightarrow F' \geq \frac{4400}{0.92}$$

$$\Rightarrow F'_{\min} = 4783 \text{ N}$$

(iii)



As F' increases, f decreases reaches zero. Further increase in F' will tend to move body up the incline, causing f to change direction.

$$f_{\max} = 1600 + 0.12F'$$

$$\text{For equilibrium, } F' \cos \theta = mg \sin \theta + f$$

$$f = F' \cos \theta - mg \sin \theta = 0.8F' - 6000$$

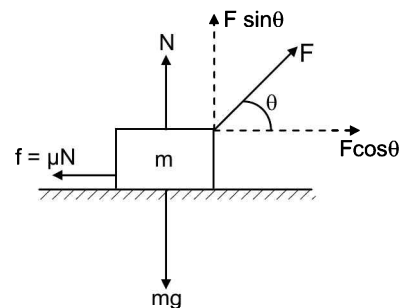
$$f \leq f_{\max} \Rightarrow 0.8F' - 6000 \leq 1600 + 0.12F'$$

$$\Rightarrow 0.68F' \leq 7600 \Rightarrow F' \leq \frac{7600}{0.68}$$

$$\Rightarrow F'_{\max} = 11176 \text{ N}$$

6.

(i)



$$N = mg - F \sin \theta; f = \mu N$$

$$\Rightarrow f = \mu mg - \mu F \sin \theta$$

4.96 Laws of Motion

For body to move,

$$F \cos \theta = f$$

$$= \mu mg - \mu F \sin \theta$$

$$\Rightarrow F (\cos \theta + \mu \sin \theta) = \mu mg$$

$$\Rightarrow F = \frac{\mu mg}{(\cos \theta + \mu \sin \theta)}$$

Let $\mu = \tan \alpha$, [$\alpha \rightarrow$ angle of friction]

$$F = \frac{\sin \alpha / \cos \alpha \cdot mg}{\cos \theta + \frac{\sin \alpha}{\cos \alpha} \sin \theta}$$

$$= \frac{\sin \alpha \cdot mg}{\cos \alpha \cos \theta + \sin \alpha \sin \theta}$$

$$F = \frac{mg \sin \alpha}{\cos(\theta - \alpha)} \Rightarrow \text{minimum when denominator is maximum}$$

is maximum

$$\Rightarrow \cos(\theta - \alpha) = 1$$

$$\Rightarrow \theta = \alpha$$

$$F_{\min} = mg \sin \alpha = mg \frac{\mu}{\sqrt{1 + \mu^2}}$$

$$\Rightarrow F_{\min} = \frac{\mu mg}{\sqrt{1 + \mu^2}} = \frac{\frac{3}{4} \times 150 \times 10}{\sqrt{1 + \left(\frac{3}{4}\right)^2}}$$

$$= 900 \text{ N}; \theta = \alpha = \tan^{-1} \mu = \tan^{-1} 0.75 = 37^\circ$$

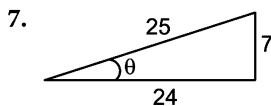
(ii) For body to move on floor $N \geq 0$

$$\Rightarrow mg - F \sin \theta \geq 0 \Rightarrow F \leq \frac{mg}{\sin \theta}$$

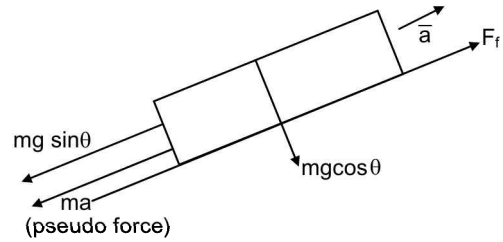
$$\text{At } \theta = \tan^{-1} \mu, F \leq \frac{mg \sqrt{1 + \mu^2}}{\mu}$$

$$F_{\max} = \frac{mg}{\mu} \sqrt{1 + \mu^2} = \frac{150 \times 10}{\frac{3}{4}} \sqrt{1 + \left(\frac{3}{4}\right)^2}$$

$$= 2500 \text{ N}$$



Free body diagram of the box



$$F_f = \mu mg \cos \theta$$

$$\Delta v = (48 - 30) \times \frac{5}{18} = 5 \text{ m s}^{-1}$$

$$\Rightarrow a = \frac{\Delta v}{\Delta t} = \frac{5}{2.5} = 2 \text{ m s}^{-2}$$

For the box not to slide down

$$F_f > mg \sin \theta + ma$$

In the limiting case, substituting the values

$$\mu mg \cos \theta = mg \sin \theta + ma \Rightarrow$$

$$\mu \times 10 \times \frac{24}{25} = 10 \times \frac{7}{25} + 2 \Rightarrow \mu = 0.5$$

8. **Case (i)** since no friction, there is no force in horizontal direction acting on the box.

$\therefore$ Initial relative velocity of box = 0

a = Relative acceleration = -2 m s^{-2}

(i.e., direction to left)

Relative distance to cover = -10 m

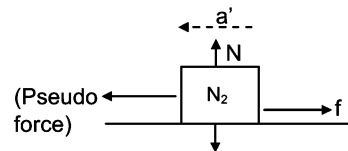
$$\Rightarrow s = \frac{1}{2} at^2, \text{ where } t = \text{time taken}$$

$$\therefore t^2 = 2 \cdot \frac{(-10)}{-2} = 10$$

$$\Rightarrow t = \sqrt{10} \text{ s}$$

case (ii)

FBD of box:



$$a = 2 \text{ m s}^{-2}$$

$$\text{friction } f = \mu N = \mu \cdot 50g = 0.15 \times 500 = 75 \text{ N}$$

$\therefore$ acceleration a' on the reference frame of the truck, (this is relative to truck, so we apply pseudo force) is given by

$$50a - f = 50a' \Rightarrow 50 \times 2 - 75 = 50a'$$

$$\Rightarrow a' = \frac{25}{50} = 0.5 \text{ m s}^{-2}; t^2 = \frac{2s}{a'}$$

$$\Rightarrow t^2 = \frac{2 \times 10}{0.5}$$

$$\Rightarrow t = \sqrt{40} = 2\sqrt{10} \text{ s}$$

case (iii) $f_{\max} = \mu N = 0.25 \times 500 = 125 \text{ N}$ is more than pseudo force = 100 N

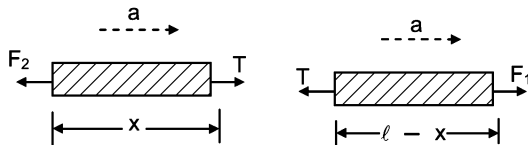
$\therefore$ f will be $< f_{\max}$ and will equal 100 N. There will be no relative motion of the box.

$\Rightarrow$ It will not fall off.

$\therefore$ $t = \text{infinity } (\infty)$

9.

(i) The rope can be thought of as two parts and the FBD's are below.



Let μ be mass per unit length of rope

$$T - F_2 = \mu x \cdot a$$

$$F_1 - T = \mu(\ell - x) \cdot a$$

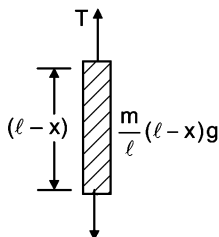
$$\text{Adding, } a = \frac{F_1 - F_2}{\mu \ell}$$

Substituting,

$$T = F_2 + \mu x \cdot \frac{(F_1 - F_2)}{\mu \ell} = F_2 + (F_1 - F_2) \cdot \frac{x}{\ell}$$

(ii) The rope can be thought of as two parts and the FBD for the lower part is as below:

$$T = mg \frac{(\ell - x)}{\ell}$$

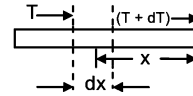


(iii) If a point mass m_1 is attached at end, the tension will increase by $m_1 g$

$$\therefore T = mg \frac{(\ell - x)}{\ell} + m_1 g$$

(iv) Direction away from the origin is positive

If the mass per unit length is μ , for a small length dx at a distance x , mass = μdx and centripetal acceleration = $-\omega^2 x$



$$-(T + dT) + (T) = -\mu \omega^2 x dx$$

$$-dT = -\mu \omega^2 x dx$$

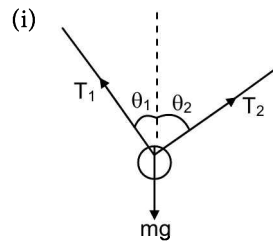
$$\mu = \frac{m}{\ell} \Rightarrow \int dT = \int_x^\ell \frac{m}{\ell} \omega^2 x dx$$

$$T = \frac{m \omega^2 (\ell^2 - x^2)}{2\ell}$$

(v) If a point mass m_1 is attached to end, tension will increase by $m_1 \omega^2 \ell$

$$\therefore T = \frac{m \omega^2 (\ell^2 - x^2)}{2\ell} + m_1 \omega^2 \ell$$

10.



$$T_1 \cos \theta_1 + T_2 \cos \theta_2 = mg$$

$$T_1 \sin \theta_1 = T_2 \sin \theta_2$$

$$\theta_1 = 45^\circ, \theta_2 = 30^\circ$$

$$\therefore T_1 \sin 45^\circ = T_2 \sin 30^\circ$$

$$\Rightarrow \frac{T_1}{\sqrt{2}} = T_2 \cdot \frac{1}{2} \Rightarrow T_2 = \sqrt{2} T_1$$

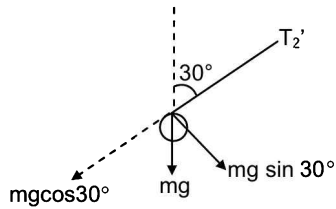
$$T_1 \cos 45^\circ + T_2 \cos 30^\circ = 2 \times 10$$

$$\Rightarrow \frac{T_1}{\sqrt{2}} + \sqrt{2} T_1 \cdot \frac{\sqrt{3}}{2} = 20$$

$$T_1 = \frac{20\sqrt{2}}{(1 + \sqrt{3})} \text{ N and } T_2 = \frac{40}{(1 + \sqrt{3})} \text{ N}$$

4.98 Laws of Motion

(ii)



Immediately on cutting CA, the ball starts to swing in a circle with B as centre.

$$T_2' - mg \cos 30^\circ = m\omega^2 R$$

But $\omega = 0$, just after cutting CA.

$$\therefore T_2' = mg \cos 30^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ N}$$

(iii) $mg \sin 30^\circ = ma_t$

where, a_t is the tangential acceleration

$$\therefore a_t = g \sin 30^\circ = 10 \times \frac{1}{2} = 5 \text{ m s}^{-2}$$

11. When the elevator has constant velocity, either going up or going down, $T = mg$

12. For mass m_1 ,

$$T - m_1 g = m_1 a \quad (1)$$

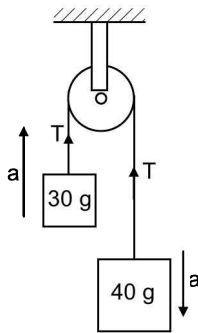
For mass m_2 ,

$$m_2 g - T = m_2 a \quad (2)$$

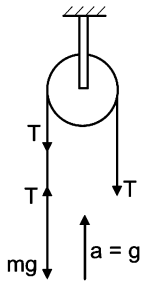
$$(1) + (2) \Rightarrow (m_2 - m_1) g = (m_1 + m_2) a$$

$$\Rightarrow a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

$$= \frac{40 - 30}{40 + 30} \times 9.8 = 1.4 \text{ m s}^{-2}$$



13.



The downward acceleration of the heavy body = g
= upward acceleration of light body

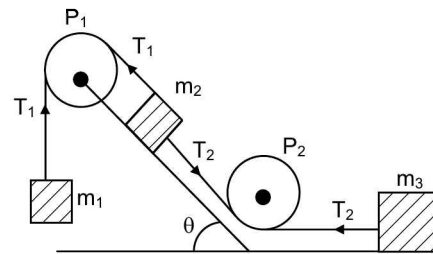
If T = tension of the string

$$T - mg = ma = mg$$

$$T = 2mg$$

The downward force on the pulley is $2T = 4mg$

14.



$$T_1 = m_1 g,$$

$$T_1 = T_2 + m_2 g \sin \theta + \mu m_2 g \cos \theta$$

Also $T_2 = \mu m_3 g$

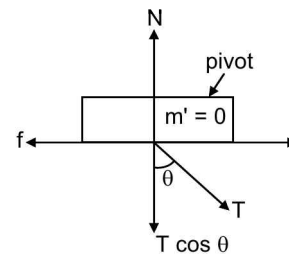
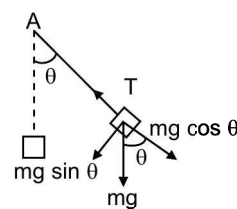
$$= 0.3 \times 6 \times 10 = 18 \text{ N}$$

$$m_1 g = \mu m_3 g + m_2 g \sin \theta + \mu m_2 g \cos \theta$$

$$m_2 = \frac{m_1 - \mu m_3}{\sin \theta + \mu \cos \theta}$$

$$= \frac{4.0 - 6 \times 0.3}{\sin 53^\circ + 0.3 \cos 53^\circ} = \frac{4 - 1.8}{0.8 + 0.3 \times 0.6} = \frac{2.2}{0.98} \approx 2.2 \text{ kg}$$

15.



When it just slips, $f = T \sin \theta$

$$\Rightarrow \mu T \cos \theta = T \sin \theta$$

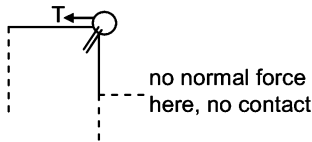
$$\Rightarrow \mu = \tan \theta \text{ [if } \mu > \tan \theta, \text{ no slippage] } \theta \text{ is the maximum angle reached for a given } \mu$$

$$\begin{aligned} \text{Tension } T &= mg \cos \theta \\ &= \frac{mg}{\sqrt{1 + \mu^2}} \end{aligned}$$

$$\left(\because \cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{1 + \tan^2 \theta}} = \frac{1}{\sqrt{1 + \mu^2}} \right)$$

16. Body (1) will have leftward acceleration

[Its FBD:



Body (3) will have no horizontal force.

[imagine: If you assume normal force, then no other horizontal force on (3) $\Rightarrow$ it should have horizontal rightward acceleration $\Rightarrow$ it should lose contact $\Rightarrow$ no normal force]

17. $T = mg \sin \theta + \mu mg \cos \theta$

$$\frac{dT}{d\theta} = mg [\cos \theta - \mu \sin \theta]$$

Given as θ decreases,

T increases

$$\Rightarrow \frac{dT}{d\theta} < 0 \Rightarrow \cos \theta < \mu \sin \theta$$

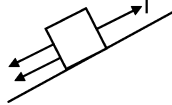
$$\Rightarrow \mu > \cot \theta,$$

possible if μ is large or if θ is large.

[$\because$ if θ high, $\tan \theta$ high, $\cot \theta$ small, μ can be small but $> \cot \theta$.

or

μ has to be > 1 , but θ small, $\tan \theta$ small, $\cot \theta$ large but can be less than μ]



18. In tug of war game, the rope exerts the same force on each team, as the tension in the string is the same
19. Both friction and normal reaction from the stopper support the block against gravity; and both are self adjusting. So that

$$F_f + F_N = mg \sin \theta$$

20. Banking causes normal reaction to have a centripetal component, which is otherwise provided by frictional force. Same force as long as vehicle is under control, i.e., there is no skidding

21. $\vec{f} + F \cos \theta \hat{i} = 0 \rightarrow \vec{f} = -F \cos \theta \hat{i}$

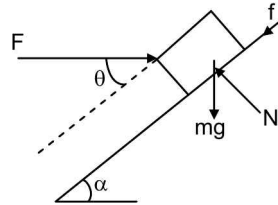
22. The condition for static friction (friction locking) is $F \cos \theta \leq \mu (mg + F \sin \theta)$

$$F (\cos \theta - \mu \sin \theta) \leq \mu mg$$

If $\mu \sin \theta \geq \cos \theta$ this inequality is satisfied irrespective of the value of F , since μmg is positive.

$$\therefore \tan \theta \geq \frac{1}{\mu}$$

- 23.



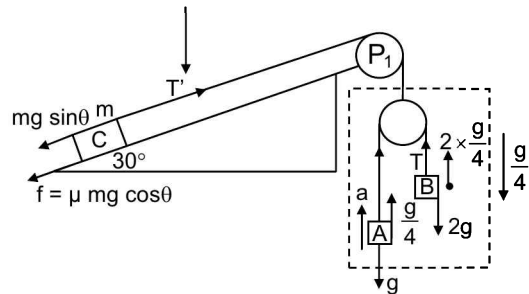
$$N = mg \cos \alpha + F \sin \theta$$

$$f = F \cos \theta - mg \sin \alpha$$

$$f \leq \mu N$$

we get the same condition $\tan \theta \geq \frac{1}{\mu}$

- 24.



Pseudo force $m g/4$ acts on each mass as shown

$$2 \left(g - \frac{g}{4} \right) - T = 2a \quad \text{--- (1)}$$

$$T - \left(g - \frac{g}{4} \right) = a \quad \text{--- (2)}$$

(b) solving we get $a = \frac{g}{4}$

25. For an outside observer acceleration of A

$$= a - a_L = \frac{g}{4} - \frac{g}{4} = 0$$

26. Similarly for B $= a_B = \frac{g}{4} + \frac{g}{4} = \frac{g}{2}$ (down) i.e., $-\frac{g}{2}$

27. $a = \frac{(4-1)}{(4+1)} g = 6 \text{ m s}^{-2}$

4.100 Laws of Motion

When A hits ground $v^2 = 2as = 2 \times 6 \times 3$

$$\Rightarrow v = 6 \text{ m s}^{-1}$$

$$\therefore \text{impulse } J = 6 \times 4 = 24 \text{ N s}$$

At that instant B is at 6 m from ground. The distance B travels further. $\therefore v^2 = 2gs$

$$\Rightarrow 36 = 2 \times 10 \times s$$

$$\Rightarrow s = \frac{36}{20} = 1.8 \text{ m}$$

$$\therefore \text{Total height} = 6 + 1.8 = 7.8 \text{ m}$$

After A stops, time to reach max height

$$v = gt \Rightarrow t_2 = \frac{6}{10} = 0.6 \text{ s}$$

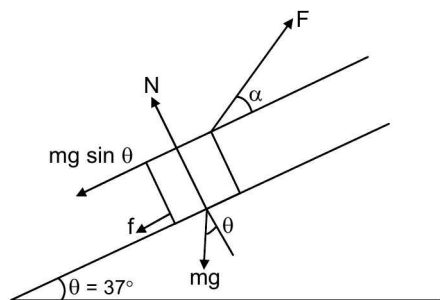
When A strikes ground $\Rightarrow v = at$

$$\Rightarrow t = \frac{v}{a} = \frac{6}{6} = 1 \text{ s}$$

$$\therefore \text{Total time} = 1 + 0.6 = 1.6 \text{ s}$$

ans. (a), (b)

28.



$$f = \mu N = \mu mg \cos \theta$$

for the body to just move up, α should be such that:

$$\mu = \tan \alpha$$

[This theory is already dealt in study material, repeated here for reference]

$$F \cos \alpha = mg \sin \theta + \mu N$$

$$N = mg \cos \theta - F \sin \alpha$$

$$\therefore F \cos \alpha = mg \sin \theta + \mu(mg \cos \theta - F \sin \alpha)$$

$$F(\cos \alpha + \mu \sin \alpha) = mg \sin \theta + \mu mg \cos \theta$$

$$F_{\min} = \frac{mg \sin \theta + \mu mg \cos \theta}{\cos \alpha + \mu \sin \alpha}$$

$$\Rightarrow F_{\min} \rightarrow \text{denominator max: } \frac{d}{dx}(\cos \alpha + \mu \sin \alpha)$$

$$= 0 - \sin \alpha + \mu \cos \alpha = 0 \Rightarrow \mu = \tan \alpha$$

$$\Rightarrow \alpha = \tan^{-1} \mu \Rightarrow \tan^{-1} 0.75 = 37^\circ$$

Hence F should be applied at 37° to the plane =

$$\sin \alpha = 0.6$$

$$\cos \alpha = 0.8$$

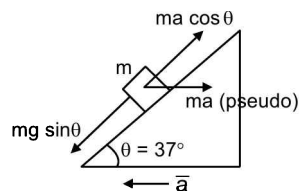
$$\sin \theta = 0.6$$

$$\cos \theta = 0.8$$

$$F_{\min} = \frac{(10 \times 0.6) + (0.75 \times 10 \times 0.8)}{0.8 + (0.75 \times 0.6)}$$

$$= \frac{12}{0.8 + 0.45} = \frac{12}{1.25} = 9.6 \text{ N}$$

29.



The condition for the mass m to be stationary on the wedge when the wedge moves with an acceleration a is given by:

$$mg \sin \theta = ma \cos \theta \text{ ('ma' is the pseudo force on m)}$$

$$\therefore 10 \sin 37^\circ = a \cos 37^\circ$$

$$\Rightarrow a = 10 \tan 37^\circ = 7.5 \text{ m s}^{-2}$$

when the mass is stationary on the wedge, they move together. Hence the acceleration of the total mass is given by:

$$\therefore Mg = (\text{Total mass}) a$$

$$10 M = (M + (2 + 1)) 7.5 \Rightarrow 20 M = 15 M + 45 \Rightarrow$$

$M = 9 \text{ kg}$. This is the critical value of M for the mass m to be stationary on the wedge.

If $M > 9 \text{ kg}$; m moves up the wedge

If $M < 9 \text{ kg}$; m moves down the edge

If $\theta = 45^\circ$, for m to be stationary

$$\Rightarrow \frac{a}{g} = \tan 45^\circ = 1; a = g$$

Not possible to achieve for finite value of M.

30. (a) When a body moves in a circle, centrifugal force acts along the radius vector and away from the centre of the circle. It has to be assumed only when motion is considered in the rotating frame of reference of the body.

Hence (a) $\rightarrow$ (p), (r)

- (b) Tension force appears as a reaction to stretching of a string and it arises due to the elasticity of the string. Hence (b) $\rightarrow$ (q), (s)

- (c) As in (a) but it is not a pseudo force. Hence (c) $\rightarrow$ (r)
 (d) As in (b). Hence (d) $\rightarrow$ (q), (s)

IIT Assignment Exercise

31. $v = u + at \Rightarrow a = \frac{5}{10} \text{ m s}^{-2}$, $F = ma = 2.5 \text{ N}$

32. $p = mv$, $a = \frac{F}{m}$.
 $v = at$, $v' = (3t)a \Rightarrow p'$
 $= mv' = m \times 3at = 3p$

Aliter:

$$p = Ft_1 \rightarrow t_1 = \frac{p}{F}$$

$$p' = Ft_2 = F \cdot 3t_1 = F \cdot 3 \frac{p}{F} = 3p$$

33. $J = m \cdot v = 2 \times 8 = 16 \text{ N s}$.

34. $F = ma \Rightarrow a = \frac{100}{5} = 20 \text{ m s}^{-2}$

$$v = u + at \Rightarrow v = 20 \times 10 = 200 \text{ m s}^{-1}.$$

Aliter:

$$v = \frac{p}{m} = \frac{Ft}{m} = \frac{100 \times 10}{5} = 200 \text{ m s}^{-1}$$

35. $F = ma = m \frac{v - u}{t} = \frac{6 \times 8}{60} = 0.8 \text{ N}$

Aliter:

$$F = \frac{\Delta p}{t} = \frac{m(v - u)}{t}$$

36. Same force acts for same time

$$F = \frac{dp}{dt};$$

$$\Delta p = \int F dt$$

$$p_M = p_m$$

37. $v = \frac{72 \times 10^3}{3600} = 20 \text{ m s}^{-1}$,

$$u = 0; v^2 - u^2 = 2aS$$

$$\Rightarrow \left(\frac{72 \times 10^3}{3600} \right)^2 = 2a \times 100$$

$$\Rightarrow a = \frac{20^2}{200} = 2 \text{ m s}^{-2}$$

$$F = ma = 500 \times 2 = 1000 \text{ N}$$

38. $m = 1 \text{ kg}$, $F = 1 \text{ N}$.

$$a = \frac{F}{m} = 1 \text{ m s}^{-2}$$

39. $mg_{\text{down}} - F_{\text{up}} = ma_{\text{down}} \Rightarrow F_{\text{up}} = m(g - a)$
 $= 5(9.8 - 4) = 29 \text{ N upwards}$.

40. Inertia is the property by virtue of which a body is unable to change by itself, its state of rest or uniform linear motion.

41. When a body is stationary the resultant force acting on the body will be zero.

42. $u = \frac{72 \times 10^3}{3600} = 20 \text{ m s}^{-1}$

$$v^2 - u^2 = 2aS \Rightarrow 0 - u^2 = 2aS$$

$$\therefore -a = \frac{u^2}{2S} = \frac{400}{80} = 5 \text{ m s}^{-2}$$

$$F = 1000 \times 5 = 5000 \text{ N}.$$

43. When a horse pulls a cart, the horizontal component of the reaction of the ground on the horse makes the cart and the horse move forward.

44. $15m_1 = 10m_2 = F$

We may write $150m_1 = 10F$ and $150m_2 = 15F$ so that

$$150(m_1 + m_2) = 25F$$

$$\Rightarrow \frac{F}{(m_1 + m_2)} = \frac{150}{25} = 6 \text{ m s}^{-2}.$$

Aliter:

$$m_1 = \frac{F}{a_1}; m_2 = \frac{F}{a_2}$$

$$m_1 + m_2 = F \left[\frac{1}{a_1} + \frac{1}{a_2} \right]$$

$$a = \frac{F}{(m_1 + m_2)} = \frac{1}{\left[\frac{1}{a_1} + \frac{1}{a_2} \right]} = \frac{a_1 a_2}{a_1 + a_2}$$

$$= \frac{15 \times 10}{(15 + 10)} = 6 \text{ m s}^{-2}$$

45. $m = 2 \text{ kg}$

$$v = 4 \text{ m s}^{-1} = \text{constant}$$

$$a = \frac{dv}{dt} = 0 \Rightarrow F = 0$$

4.102 Laws of Motion

46. $F = ma \Rightarrow a = 6/1 = 6 \text{ m s}^{-2}$.

$$v = u + at \Rightarrow t = \frac{v - u}{a} = \frac{30 - 0}{6} = 5 \text{ s}$$

Aliter:

$$t = \frac{\Delta p}{F} = \frac{mv}{p} = \frac{1 \times 30}{6} = 5 \text{ s}$$

47. $v^2 - u^2 = 2aS \Rightarrow$

$$0 - u^2 = 2aS, \left(\frac{u_1}{u_2} \right)^2 = \left(\frac{S_1}{S_2} \right) \Rightarrow S_2 = \left(\frac{60}{30} \right)^2 \times 8 = 32 \text{ m}$$

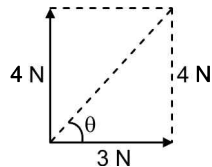
48. When a man gets down a running bus, he falls forward. Due to inertia of motion, upper part of his body continues to be in motion in forward direction while feet comes to rest as soon as they touch the road.

49. $v^2 - u^2 = 2aS$

$$a = \frac{40^2 - 38^2}{2 \times 0.05} \text{ (retardation)}$$

$$F = ma = \frac{10}{1000} \times \frac{40^2 - 38^2}{2 \times 0.05} = 15.6 \text{ N}$$

50.



$$\text{Resultant force} = \sqrt{4^2 + 3^2} = 5 \text{ N}$$

$$\text{Acceleration} = \frac{F}{m} = \frac{5}{3} \text{ m s}^{-2}$$

$$\tan \theta = \frac{4}{3} \Rightarrow 53^\circ \text{ with } 3 \text{ N force}$$

51. $\vec{F}_1 = (10\hat{i} + 20\hat{j}) \text{ N}$ and $\vec{F}_2 = (6\hat{i} - 8\hat{j}) \text{ N}$

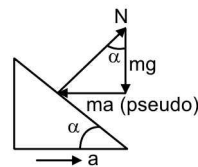
$$\text{Resultant } \vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\therefore \vec{F} = (16\hat{i} + 12\hat{j})$$

$$\text{Magnitude of resultant} = \sqrt{16^2 + 12^2} = 20 \text{ N}$$

$$\therefore a = \frac{F}{m} = \frac{20}{10} = 2 \text{ m s}^{-2}$$

52. For no slipping



$$N \cos \alpha = mg$$

$$\Rightarrow N = mg \sec \alpha$$

53. $F = m_1 a_1 + m_2 a_2$

$$200 = 5 \times 10 + 15 \times a_2 \Rightarrow a_2 = 10 \text{ m s}^{-2}$$

54. $mg - f = ma$

$$\text{Frictional force} = 0.4 \times (10 - 5) = 2 \text{ N}$$

55. For a body of mass $m_1 = 8 \text{ kg}$, $T = m_1 a$ — (1)

For a body of mass $m_2 = 2 \text{ kg}$, $m_2 g - T = m_2 a$ — (2)

Adding (1) and (2), $m_2 g = (m_1 + m_2) a$

$$a = \frac{m_2}{m_1 + m_2} g = \frac{2}{2 + 8} \times 10 = 2 \text{ m s}^{-2}$$

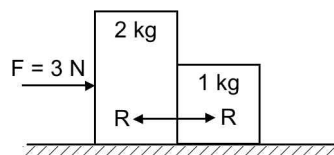
56. In the above solution, equation (1) $\Rightarrow$

$$T = m_1 a = 8 \times 2 = 16 \text{ N}$$

57. $a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g = \frac{6}{14} \times 10 = 4.3 \text{ m s}^{-2}$

58. $T = \frac{2m_1 m_2 g}{(m_1 + m_2)} = \frac{2 \times 10 \times 4}{14} \times 10 = 57 \text{ N}$

59.



Using Newton's 2nd law of motion,

$$F = (m_1 + m_2) a$$

$$3 = (2 + 1) a \Rightarrow a = 1 \text{ m s}^{-2}$$

Let R be the force on common face.

Considering the motion of smaller block only

$$R = 1 \times a = 1 \text{ N}.$$

60. $T - mg = ma$

$$25 \times 10 - 20 \times 10 = 20a$$

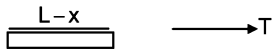
$$\Rightarrow a = \frac{50}{20} = 2.5 \text{ m s}^{-2}$$

61. $F = m_1 a_1 + m_2 a_2$

$$200 = 20a_1 + 10 \times 12$$

$$\Rightarrow a_1 = \frac{200 - 120}{20} = 4 \text{ m s}^{-2}$$

62. $F = (M+m)a$

63. 

$$a = \frac{F}{M}$$

$$T = \left(\frac{L-x}{L} \right) \times M \times \frac{F}{M} = \frac{F(L-x)}{L}$$

64. Tension in the string, when pulley is stationary,

$$T = \frac{2m_1m_2}{m_1 + m_2}g = \frac{2(m_1g)(m_2g)}{m_1g + m_2g}$$

$$= \frac{2W_1W_2}{W_1 + W_2}$$

When pulley is accelerated upwards with acceleration g , a pseudo force mg acts downwards and each weight is doubled as

$$W' = m(g+g) = 2mg$$

$$\therefore \text{Required tension} = \frac{2 \times 2W_1 \times 2W_2}{2W_1 + 2W_2}$$

$$= \frac{4W_1W_2}{W_1 + W_2}$$

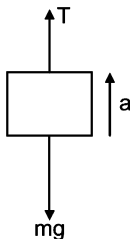
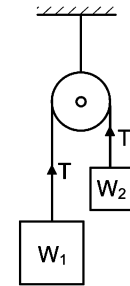
65. Here Tension $T > mg$.

So the elevator will move upwards

$$T - mg = ma$$

$$\therefore a = \frac{60000 - 50000}{5000}$$

$$= 2 \text{ m s}^{-2}$$



66. $mg = 100 \text{ N}$

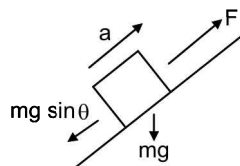
From the figure,

$$F - mg\sin\theta = ma$$

$$F = mg\sin\theta + ma$$

$$= 100 \times \frac{1}{2} + \frac{100}{10} \times 3$$

$$= 80 \text{ N}$$



67. For a body sliding up a rough inclined plane

$$F - mg\sin\theta - \mu mg\cos\theta = ma$$

$$\therefore F = mg\sin\theta + \mu mg\cos\theta + ma$$

$$= 100 \times \frac{1}{2} + 0.05 \times 100 \times \frac{\sqrt{3}}{2} + \frac{100}{10} \times 3$$

$$= 84.3 \text{ N}$$

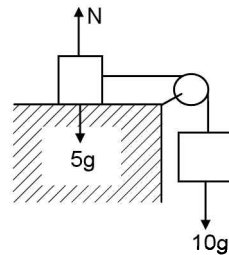
68. A rolling body on the surface of the Earth comes to rest due to friction.

69. When the body begins to slide $\mu mg = F$

$$\text{Here } \mu \times 2 \times g = 5 \Rightarrow \mu = 0.25$$

70. $F \leq \mu_s R \Rightarrow mg \leq \mu_s ma \Rightarrow \mu_s \geq \frac{g}{a}$

71.



$$10g - T = 10a$$

$$T - \mu 5g = 5a$$

$$10g - \mu 5g = 15a$$

$$\Rightarrow 10 \times 10 - 0.4 \times 5 \times 10 = 15a$$

$$\Rightarrow a = 5.33 \text{ m s}^{-2}$$

72. $F_{\min} = m(g\sin\alpha - \mu g\cos\alpha)$

$$= 0.15 \times 10 \times \left(\frac{1}{2} - 0.3 \times \frac{\sqrt{3}}{2} \right)$$

$$= 0.75 (1 - 0.3 \times \sqrt{3}) = 0.36 \text{ N}$$

73. Maximum value of frictional force is called limiting friction.

74. A man in an elevator weighs more when the lift begins to go up.

$$R - mg = ma$$

$$\therefore \text{Apparent weight} = m(g+a)$$

75. Since Newton's first law is violated, measurement is done in a non-inertial frame.

76. $a = 0$, $m = 60 \text{ kg}$

For uniform velocity the reading will show actual weight 60 kg wt.

4.104 Laws of Motion

77. When the lift moves downwards with uniform acceleration apparent weight of the person becomes less than the actual weight of the person.

$$\text{i.e., } mg' = m(g - a) = 60(9.8 - 3) = 408 \text{ N}$$

$$\frac{408}{9.8} = 41.6 \text{ kg wt.}$$

78. Since the train is retarding, the apple will fall slightly away from the hand of his brother in the direction of motion of the train. This is because at the instant of dropping of the apple, it had a particular speed, which it maintains during the fall while the boy's hand below at the next instance has a smaller velocity due to continued action of the brake (deceleration).

$\therefore$ The apple will fall slightly in front of the boy's hand.

79. Force providing centripetal acceleration of coin is the frictional force.

$$\therefore \frac{mv^2}{r} = \mu mg \Rightarrow v^2 = \mu rg = 0.5 \times 0.2 \times 10 = 1$$

$$\Rightarrow \omega = \frac{v}{r} = \frac{1}{0.2} = 5 \text{ rad s}^{-1}.$$

80. $v = 72 \text{ kmph} = 20 \text{ m s}^{-1}$

$$N \sin \theta = \frac{mv^2}{R}$$

$$N \cos \theta = mg$$

$$\tan \theta = \frac{v^2}{Rg}$$

$$R = \frac{v^2}{g \tan \theta} = \frac{20 \times 20}{10 \times 0.1} = 400 \text{ m}$$

81. The equation $v^2 - u^2 = 2aS$ is applicable only when 'a' is constant i.e.,

$$ma = \text{constant or } \bar{F} \text{ is constant.}$$

82. Newton's second law $\bar{F} = m\bar{a}$ gives a measure of the

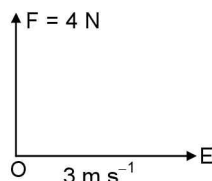
force $\bar{F}$; $\bar{F} = \frac{d\bar{p}}{dt}$, $\bar{a} = \frac{\bar{F}}{m}$, when 'm' is constant and

$$\bar{F} = \frac{d\bar{p}}{dt}.$$

83. $OE \rightarrow x$

perpendicular $OE \rightarrow y$

$$a_y = \frac{F}{m} = \frac{4}{2} = 2 \text{ m s}^{-2}$$



$$S_y = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 4^2 = 16 \text{ m}$$

$$S_x = 3 \times t = 3 \times 4 = 12 \text{ m}$$

$$\Rightarrow S = \sqrt{S_x^2 + S_y^2} = 20 \text{ m}$$

$$84. m = \frac{10^4}{10} = 1000 \text{ kg; } S = \frac{1}{2}at^2 = 25 \text{ m} \Rightarrow a = \frac{2S}{t^2}$$

$$F = ma = \frac{2Sm}{t^2} = \frac{2 \times 25 \times 1000}{5^2} = 2000 \text{ N}$$

$$85. u = \frac{900 \times 5}{18} = 250 \text{ m s}^{-1}, S = 0.1 \text{ m, } v = 0$$

$$\therefore a = \frac{(250)^2}{2 \times 0.1} = 3125 \times 10^2 \text{ m s}^{-2}$$

The force on the bullet is $F = ma$

$$= \frac{10}{1000} \times 3125 \times 10^2 = 3125 \text{ N}$$

$$86. \begin{array}{|c|} \hline M_1 \\ \hline \end{array} \longrightarrow \begin{array}{|c|} \hline M_2 \\ \hline \end{array} \longrightarrow F$$

$$\begin{array}{|c|} \hline M_1 \\ \hline \end{array} \longrightarrow T$$

$$\text{Acceleration } a = \frac{F}{M_1 + M_2} = \frac{50}{10} = 5 \text{ m s}^{-2}$$

For horizontal motion of mass M_1

$$\text{Tension } T = M_1 a = 3 \times 5 = 15 \text{ N}$$

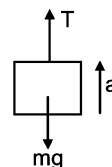
87. $a = \text{upward acceleration}$

$$T - mg = ma$$

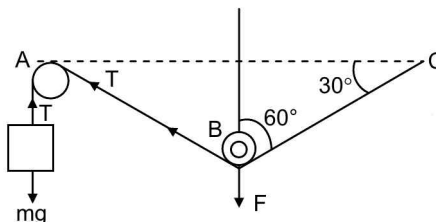
$$\Rightarrow 180 - 200 = 20a$$

$$a = -1 \text{ m s}^{-2}$$

$$\Rightarrow 1 \text{ m s}^{-2} \text{ downwards}$$



- 88.



$$F = 2T \cos 60^\circ;$$

$$T = mg = 5 \text{ N;}$$

$$F = 2 \times 5 \times 0.5 = 5 \text{ N}$$

89. $0.5a = T, 0.2g - T = 0.2a$

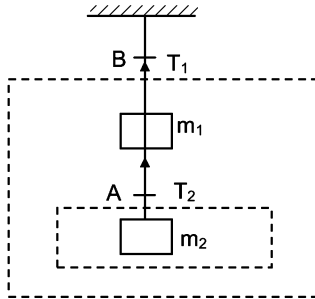
$$\therefore 0.2g = (0.5 + 0.2)a$$

$$\Rightarrow a = \frac{2}{7}g \text{ m s}^{-2}$$

$$u = 0.5 \text{ m s}^{-1}, v = u + at$$

$$\text{After 1 s: } v = 0.5 + \frac{2}{7}g \times 1 = 3.36 \text{ m s}^{-1}$$

90.



$$(m_1 + m_2 + m)a = T_1 - (m_1 + m_2 + m)g.$$

$$\Rightarrow T_1 = (m_1 + m_2 + m_3)(a + g) \\ = (2.9 + 1.9 + 0.2) \times (0.2 + 9.8) \text{ N} = 50 \text{ N}.$$

Let the mass of the rope be m and x the fraction of the second wire below A

$$(m_2 + xm)a = T_2 - (m_2 + xm)g$$

$$\Rightarrow T_2 = (m_2 + xm)(a + g).$$

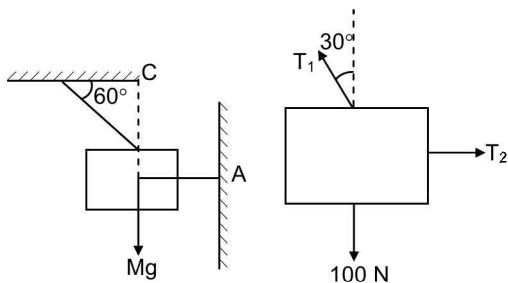
$$\text{For } x = \frac{1}{2} \Rightarrow T_2 = (1.9 + 1/2 \times 0.2) \times 10 \text{ N} = 20 \text{ N}.$$

$$91. a = \frac{m_2 - m_1}{m_1 + m_2}g$$

$$a = \frac{1}{3}g$$

$$v^2 = 2as \Rightarrow v = \sqrt{\frac{2gs}{3}} = \sqrt{\frac{2 \times 9.8 \times 7.5}{3}} = 7 \text{ m s}^{-1}$$

92.



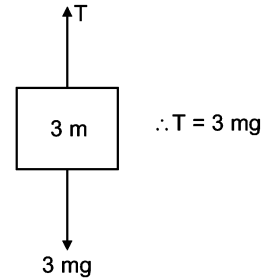
$$T_1 \cos 30^\circ = mg$$

$$T_1 \sin 30^\circ = T_2$$

$$\frac{T_2}{mg} = \tan 30^\circ$$

$$T_2 = \frac{10 \times 10}{\sqrt{3}} = 58 \text{ N}$$

$$93. T = m_1g, F_1 = T + m_1g = 2m_1g$$

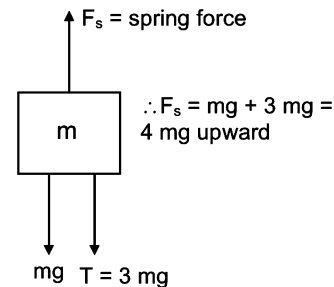
 94. FBD of $3m$


When string snaps, $T = 0$

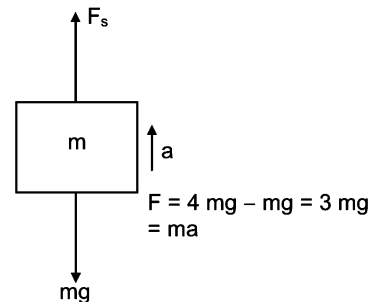
$\Rightarrow F = \text{unbalanced force} = 3mg$

$$\therefore \text{Acceleration} = \frac{F}{\text{mass}} = \frac{3mg}{3m} = g, \text{ downward}$$

FBD of m



When string snaps, $T = 0$, so FBD is



$$\therefore a = \frac{F}{m} = \frac{3mg}{m} = 3g, \text{ upwards}$$

4.106 Laws of Motion

95. When free end of string moves down by 'x' the movable pulley moves up by $\frac{x}{2}$.

Hence if 'a' is the acceleration of free end of string, acceleration of movable pulley is $\frac{a}{2}$ in the opposite direction. Tension in string is 'F' through out.

FBD of movable block

$$a' = \frac{a}{2} = 2 \text{ ms}^{-2} \uparrow$$

$$mg = 20 \times 10 = 200 \text{ N}$$

$$F_{\text{net}} = F + F - mg = 2F - mg = 2F - 200 = 2(F - 100)$$

$$\text{But } F_{\text{net}} = ma' = 20 \times 2 \Rightarrow 40 = 2(F - 100) \\ F = 120 \text{ N}$$

96. $F_f + ma = mg \sin \theta$

$$R = mg \cos \theta$$

$$\text{Total Reaction} = \sqrt{F_f^2 + R^2} = m\sqrt{g^2 + a^2 - 2ag \sin \theta}$$

97. When m_2 moves a distance x, centre of pulley supporting m_2 moves by $x/2$.

$\therefore$ acceleration of m_2 is double the acceleration of m_1 .

$$T_1 - m_1 g = \frac{m_1 a}{2}, m_2 g - T_2 = m_2 a$$

$$2T_2 - T_1 = 0 \text{ (as the pulley is massless)}$$

$$a = 2g \frac{2m_2 - m_1}{4m_2 + m_1} = \frac{2g(4m_1 - m_1)}{8m_1 + m_1}$$

$$(\because 2m_1 = m_2)$$

$$a = \frac{2g \times 3m_1}{9m_1} = \frac{2g}{3}$$

98. Since $x = \sqrt{3}t^2$

$$v = \frac{dx}{dt} = 2\sqrt{3}t; a = \frac{dv}{dt} = 2\sqrt{3}$$

$$\Rightarrow g(\sin 45^\circ - \mu \cos 45^\circ) = 2\sqrt{3}$$

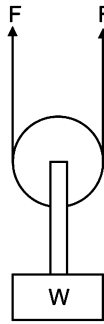
$$\Rightarrow \frac{g \times 1}{\sqrt{2}}(1 - \mu) = 2\sqrt{3} \Rightarrow \mu \approx 0.5.$$

99. (Refer Eg : 16 solution)

$$\therefore P_{\text{min}} = \sin \lambda mg \Rightarrow 500 = 1000 \cdot \sin \lambda$$

$$\Rightarrow \lambda = 30^\circ \text{ (angle of friction)}$$

$$\Rightarrow \mu = \tan \lambda = \frac{1}{\sqrt{3}}$$



100. Let λ be the mass per unit length of the chain of length L. The length of hanging portion is 'y'.

R is Normal reaction on chain on table

$$R = W_2 = \lambda(L - y)g.$$

Limiting frictional force (F_L) equals weight of hanging chains (λyg)

$$F_L = \mu R = \mu \lambda (L - y)g$$

$$= \lambda yg \Rightarrow \mu (L - y) = y$$

$$\Rightarrow y = \frac{\mu}{\mu + 1} L = \frac{0.5 \times 1}{1.50} = 0.33 \text{ m}$$

101. $mg \sin \theta = 3 \times 10 \times \sin 30 = 15 \text{ N}$

$$f = \mu mg \cos \theta = \frac{1}{2\sqrt{3}} \times 3 \times 10 \times \frac{\sqrt{3}}{2} = 7.5 \text{ N}$$

- (i) when moving down; net force (excluding F) = $(mg \sin \theta - f) = (15 - 7.5) \text{ N} = 7.5 \text{ N down.}$

$$\therefore F = 7.5 \text{ N up}$$

- (ii) To move up without acceleration:

$$F = (mg \sin \theta + f) = (15 + 7.5) \text{ N} = 22.5 \text{ N}$$

102. For both masses to move together, minimum friction is obtained as given below:

$$mg = 2ma \Rightarrow a = \frac{g}{2}$$

$$\therefore \text{frictional force } f = \mu mg = ma = mg/2$$

$$\therefore \mu = \frac{1}{2}. \text{ If } \mu < \frac{1}{2}; \text{ top mass slips and tension}$$

$$T = f = \mu mg$$

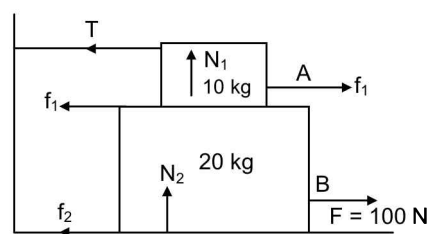
$$\therefore \text{for the lower hanging mass:}$$

$$mg - T = ma \Rightarrow$$

$$mg - \mu mg = ma$$

$$\Rightarrow a = g(1 - \mu)$$

- 103.



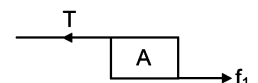
$$f_1 = \mu N_1 = 0.2 \times 10g = 100 \times 0.2 = 20 \text{ N}$$

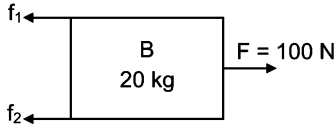
$$f_2 = \mu N_2 = 0.2 \times 30g = 300 \times 0.2 = 60 \text{ N}$$

For A

$$T = f_1 = 20 \text{ N} \text{ --- (1)}$$

For B





$$F - f_1 - f_2 = 20a$$

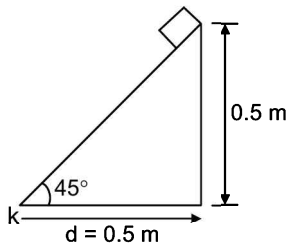
$$100 - 20 - 60 = 20a$$

$$a = \frac{20}{20} = 1 \text{ m s}^{-2}$$

104. $F < \mu (m_1 + m_2) g \Rightarrow$ The second mass will not move.

$$F > \mu m_1 g \Rightarrow \Delta = \frac{F - \mu m_1 g}{k} = 5 \text{ cm}$$

105.



Let the acceleration of the elevator be a_0 and the acceleration of the body along the plane be a .

$$mg \sin \theta + ma_0 \sin \theta = ma \Rightarrow a = (g + a_0) \sin \theta$$

S = The distance travelled by the particle along the plane = $d / \cos \theta$

$$v^2 = 2as = 2 \times (g + a_0) \sin \theta \times \frac{d}{\cos \theta}$$

$$= 2(g + a_0) d \tan \theta$$

$$= 2 \times (10 + 2) \times 0.5 \times 1 = 12$$

$$\Rightarrow v = \sqrt{12} = 2\sqrt{3} \text{ m s}^{-1}$$

106. $u = 3 \text{ m s}^{-1}, v = 0, s = 10 \text{ m}$

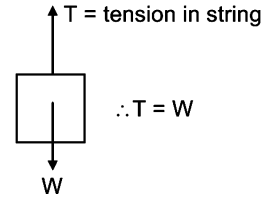
$$0 - 9 = 2 \times a \times 10$$

$$\Rightarrow a = \frac{-9}{20} = -0.45 \text{ m s}^{-2}$$

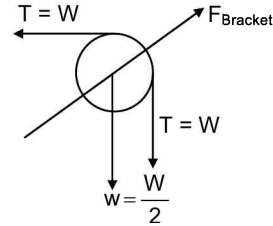
Effective value of $g = 10 + 0.45 = 10.45 \text{ m s}^{-2}$

$$\text{Apparent weight} = \frac{45 \times 10.45}{10} \cong 47 \text{ kg}$$

107. FBD of block



FBD of pulley



$$\therefore F_v = w + T = \frac{W}{2} + W = \frac{3W}{2}$$

$$F_H = T = W$$

$$\therefore F_R = \sqrt{F_v^2 + F_H^2} = \sqrt{\left(\frac{3W}{2}\right)^2 + W^2}$$

$$= \frac{\sqrt{13}}{2} W$$

F_{bracket} is equal to magnitude to F_R but opposite in direction

$$\therefore F_{\text{bracket}} = \frac{\sqrt{13}}{2} W$$

108. Central mass is 'first mass'

For the second mass not to halt:

$$m\omega^2 r_1 > \mu mg + k(r_1 - r_0) \quad \text{--- (1)}$$

For the first mass to move

$$k(r_1 - r_0) = \mu mg \quad \text{--- (2)}$$

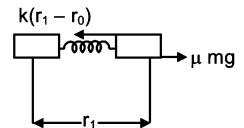
$$\Rightarrow \therefore m\omega^2 r_1 > 2 \mu mg$$

$$\text{from (2)} \Rightarrow r_1 = \frac{\mu mg}{k} + r_0;$$

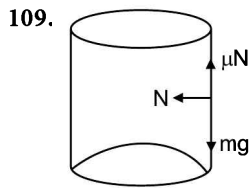
substituting value of r_1 in (1),

$$m\omega^2 \left(\frac{\mu mg}{k} + r_0 \right) > 2 \mu mg$$

$$\Rightarrow \omega^2 > \frac{2\mu g}{\frac{\mu mg}{k} + r_0} \Rightarrow \omega > \sqrt{\frac{2\mu g}{\frac{\mu mg}{k} + r_0}}$$



4.108 Laws of Motion



$$mg = \mu N$$

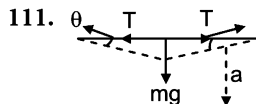
$$N = mR\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{g}{\mu R}}$$

110. The friction force should provide centripetal force

i.e., $F_f = \frac{mv^2}{r}$.

But $F_f \leq \mu mg \Rightarrow \frac{mv^2}{r} \leq \mu mg$



Due to Newton's II law

$a = \frac{F}{m}$, $a > 0$ however small resultant F may be and however large m may be

Resultant $\neq 0$,

however small m may be and however large T may be. In fact $T \rightarrow \infty$

For T to have a component vertically up to overcome mg ; it should have a small angle θ as shown.

112. Statement 2 is of course true. But what if they are moving at same constant velocity. (uniform motion) friction force is zero, even though rough.

113. Clearly there is friction ($\because$ otherwise box will move even for small tension)

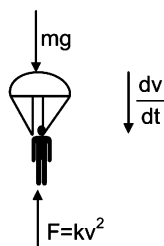
$\therefore T > \mu_s mg$, breaking strength $< \mu_s mg$)

If suspended $T' = mg > \text{breaking strength}$ ($\because$ usually $\mu_s < 1$)

114. At any instant F opposes mg

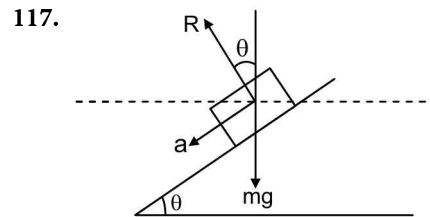
115. Newton's second law.

$$mg - kv^2 = m \frac{dv}{dt}$$



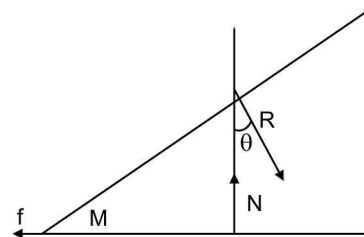
116. $a = \frac{dv}{dt} = g - \frac{k}{m}v^2$ as v increases, a decreases. Also

$$\frac{da}{dv} = -\frac{2kv}{m}, \text{ so that the initial slope is zero.}$$



Let R be the normal reaction on m , by the wedge
 $\Rightarrow R \sin \theta = ma \cos \theta$

$mg - R \cos \theta = ma \sin \theta \Rightarrow$ But $a = g \sin \theta$ since wedge is stationary



$$\therefore mg - R \cos \theta = mg \sin^2 \theta \Rightarrow mg(1 - \sin^2 \theta) = R \cos \theta$$

$$\therefore R = mg \cos \theta \Rightarrow \text{on the wedge}$$

$$N = Mg + R \cos \theta = Mg + mg \cos^2 \theta$$

(a) is correct

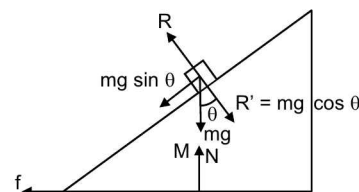
$$f = R \sin \theta$$

$$= mg \cos \theta \sin \theta$$

(d) is correct

$\therefore$ (a) and (d) are correct

Aliter:



When the stopper is removed; since the wedge is smooth on top and stationary, the acceleration of mass m is:

$ma = mg \sin \theta \Rightarrow$ Hence ' $mg \sin \theta$ ' is totally used up. (like free falling body has no gravity)

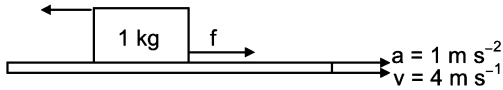
$$\therefore R = mg \cos \theta$$

$$\therefore N = Mg + R \cos \theta = Mg + mg \cos^2 \theta$$

for the wedge to be stationary:

$$f = R \sin \theta = mg \cos \theta \sin \theta$$

118. ma (pseudo)



The forces acting on the 1 kg mass are:

$$\text{frictional force } f = \mu mg = 0.5 \times 1 \times 10 = 5 \text{ N}$$

$$\text{pseudo force} = ma = 1 \times 1 = 1 \text{ N}$$

a_r = Its acceleration on the reference frame of the belt

$$\Rightarrow \frac{5-1}{1} = 4 \text{ m s}^{-2}$$

Its initial relative velocity $\Rightarrow u_r = 4 \text{ m s}^{-1}$ and final relative velocity $v_r = 0 \Rightarrow$ Distance moved

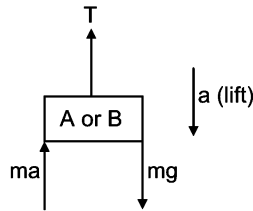
$$u_r^2 = 2a_r s \Rightarrow \text{distance } s = \frac{4^2}{2 \times 4} = 2 \text{ m}$$

Time over which it moves $\Rightarrow u_r = at$

$$\Rightarrow t = \frac{u_r}{a} = \frac{4}{4} = 1 \text{ s}$$

Even when there is no relative motion, frictional forces do not disappear, since $f = ma$

119.



When lift moves with uniform velocity or when it is at rest

$$T = \frac{2m_1 m_2}{m_1 + m_2} g = \frac{2 \times 5 \times 3 \times 10}{8} = 37.5 \text{ N} = 3.75 \text{ kg wt}$$

$$\therefore \text{Reading of the balance} = 2T = 7.5 \text{ kg}$$

When lift moves down with acceleration of $4 \text{ m s}^{-2} \rightarrow g' = 10 - 4 = 6 \text{ m s}^{-2}$

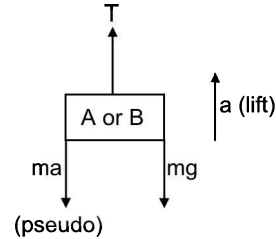
$$\begin{aligned} T &= \frac{2m_1 m_2 g'}{m_1 + m_2} = \frac{2 \times 5 \times 3 \times 6}{8} \\ &= \frac{180}{8} = 22.5 \text{ N} \\ &= 2.25 \text{ kg w} \end{aligned}$$

$$\therefore \text{Reading} = 2T = 4.5 \text{ kg}$$

$\therefore$ (d) is correct

When lift moves up with acceleration of 4 m s^{-2} , or when lift moves down with deceleration:

$$g'' = 10 + 4 = 14 \text{ m s}^{-2}$$



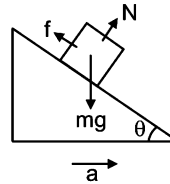
$$T = \frac{2 \times 5 \times 3}{8} \times 14 = 52.5 \text{ N} = 5.25 \text{ kg wt}$$

$$\text{Reading} = 2T = 10.5 \text{ kg wt}$$

$\therefore$ (b) is correct

Hence (b), (d) are correct

120.



- (a) either (i) $mg \sin \theta + ma \cos \theta = \mu N$
or (ii) $mg \sin \theta = \mu N$; $mg \cos \theta = N$ (if $a = 0$)

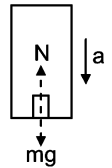
$\therefore$ (a) $\rightarrow$ (q), (s)

- (b) $ma = mg - N \Rightarrow N = m(g - a)$

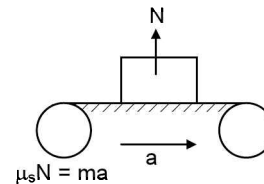
when $a = g$, $N = 0$

pseudo force $= ma = mg$

$\therefore$ (b) $\rightarrow$ (p), (r), (s)



(c)



$$\mu_s N = ma$$

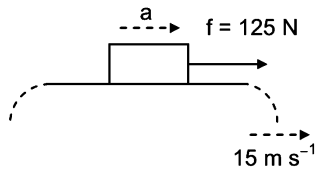
$\therefore$ (c) $\rightarrow$ (q), (s)

(d) as in (b)

$\therefore$ (d) $\rightarrow$ (p), (r), (s)

Additional Practice Exercise

121. (i)



Force acting on box = frictional force = μmg
= 125 N

$$\therefore \text{Acceleration, } a = \frac{f}{m} = \frac{125}{25} = 5 \text{ m s}^{-2}$$

(ii) When velocity of box reaches velocity of van, friction will cease because no relative movement.

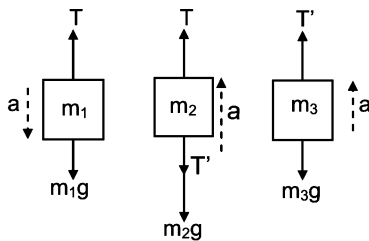
$$v = u + at, v = 15 \text{ m s}^{-1}, u = 0$$

$$\Rightarrow 15 = 5t \Rightarrow t = 3 \text{ s}$$

$\therefore$ Van would have moved

$$15 \text{ m s}^{-1} \times 3 \text{ s} = 45 \text{ m}$$

122.



FBD for m_1

$$(i) \quad m_1g - T = m_1a$$

$$\Rightarrow 50 - T = 5a \quad \text{--- (1)}$$

FBD for m_2 :

$$T - T' - m_2g = m_2a$$

$$\Rightarrow T - T' - 20 = 2a \quad \text{--- (2)}$$

FBD for m_3 :

$$T' - m_3g = m_3a \Rightarrow T' - 10 = a \quad \text{--- (3)}$$

$$(1) + (2) + (3) \Rightarrow 50 - 20 - 10 = 8a$$

$$\Rightarrow a = 2.5 \text{ m s}^{-2}$$

(ii) substituting in (3) $T' = 12.5 \text{ N}$

substituting in (1) $T = 37.5 \text{ N}$

(iii) $T' = 10 + a$ and $T' \leq 11.25 \text{ N}$

$$\Rightarrow 10 + a \leq 11.25 \Rightarrow a \leq 1.25 \text{ m s}^{-2}$$

Taking equation $m_1g - T = m_1a$

$$\Rightarrow m_1(g - a) - T = 0 \text{ and adding with (2) and}$$

(3) of (i) above,

$$\text{we get } m_1(g - a) - 20 - 10 = 3a$$

$$m_1(g - a) - 20 - 10 = 3a$$

Putting maximum value of a as derived above at

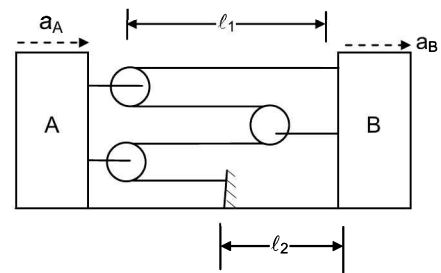
$$a = 1.25 \text{ m s}^{-2}$$

$$m_1(10 - 1.25) = 30 + 3.75$$

$$\Rightarrow m_1 = \frac{33.75}{8.75} = 3.86 \text{ kg}$$

This is the maximum allowed value for m_1

123. (i)



To find relation between a_A, a_B

string length constraint:

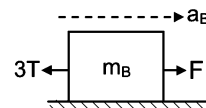
$$4\ell_1 - \ell_2 = \text{string length} = \text{constant}$$

$$\therefore 4 \frac{d^2 \ell_1}{dt^2} - \frac{d^2 \ell_2}{dt^2} = 0$$

$$\frac{d^2 \ell_1}{dt^2} = a_B - a_A; \frac{d^2 \ell_2}{dt^2} = a_B$$

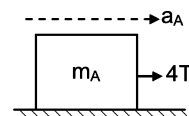
$$4(a_B - a_A) - a_B = 0 \Rightarrow a_A = \frac{3}{4}a_B \quad \text{--- (1)}$$

FBD of B:



$$F - 3T = m_B a_B \quad \text{--- (2)}$$

FBD of A:

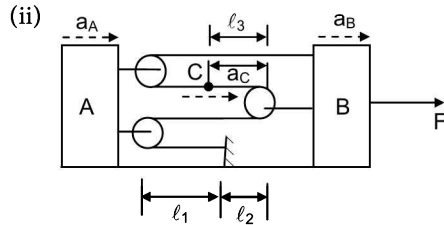


$$4T = m_A a_A = m_A \cdot \frac{3}{4}a_B \quad \text{--- (3)}$$

$$\text{Solving, } a_B = \frac{F}{m_B + \frac{9}{16}m_A} = \frac{73.5}{20 + \frac{9}{16} \cdot 8}$$

$$\Rightarrow a_B = 3 \text{ m s}^{-2}$$

$$\therefore a_A = \frac{3}{4}a_B = 2.25 \text{ m s}^{-2}$$



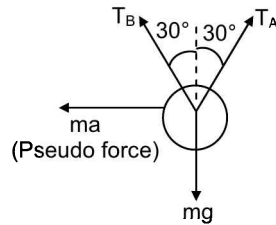
$$2\ell_1 + 2\ell_2 + \ell_3 = \text{constant}$$

$$\frac{d^2\ell_3}{dt^2} = -a_C = \frac{-2d^2\ell_1}{dt^2} - \frac{2d^2\ell_2}{dt^2}$$

$$-a_C = 2a_A - 2a_B = 4.5 - 6 = -1.5 \text{ m s}^{-2}$$

$$\Rightarrow a_C = 1.5 \text{ m s}^{-2}$$

124. (i)



$$T_A \cos 30^\circ + T_B \cos 30^\circ = mg$$

$$\Rightarrow \frac{\sqrt{3}}{2}T_A + \frac{\sqrt{3}}{2}T_B = mg$$

$$\Rightarrow T_A + T_B = \frac{2}{\sqrt{3}}mg \quad \text{--- (1)}$$

$$T_A \sin 30^\circ = T_B \sin 30^\circ + ma$$

$$\Rightarrow T_A - T_B = 2ma \quad \text{--- (2)}$$

$$T_A = 2T_B; \quad T_B = \frac{2mg}{3\sqrt{3}};$$

$$a = \frac{1}{2} \times \frac{2mg}{3\sqrt{3}m} = \frac{g}{3\sqrt{3}} \text{ ms}^{-2}$$

(ii) $T_B = 0$; From equations (1) & (2)

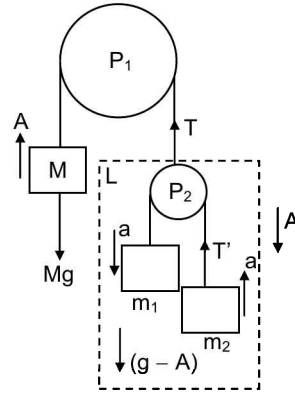
$$T_A = \frac{2mg}{\sqrt{3}} = 2ma$$

$$\Rightarrow a = \frac{g}{\sqrt{3}} \text{ ms}^{-2}$$

(iii) If $T_B > T_A$

$$2ma = T_A - T_B < 0 \Rightarrow a < 0 \text{ (Retardation if the truck is moving to right)}$$

125.



Consider the accelerated frame 'L' as marked where $g_{\text{eff}} = (g - A)$

where, A is the acceleration of the frame L

$$\therefore T' = \frac{2m_1 m_2}{(m_1 + m_2)}(g - A) \quad \text{--- (1)}$$

$$T = 2T' = \frac{4m_1 m_2}{(m_1 + m_2)}(g - A) \quad \text{--- (2)}$$

$$T - Mg = MA$$

$$\frac{4m_1 m_2}{m_1 + m_2}(g - A) - Mg = MA$$

$$A \left[M + \frac{4m_1 m_2}{(m_1 + m_2)} \right] = g \left(\frac{4m_1 m_2}{m_1 + m_2} - M \right) \quad \text{--- (3)}$$

$$A \left[\frac{M(m_1 + m_2) + 4m_1 m_2}{m_1 + m_2} \right] = g \left[\frac{4m_1 m_2 - M(m_1 + m_2)}{m_1 + m_2} \right]$$

$$A = g \frac{4m_1 m_2 - M(m_1 + m_2)}{4m_1 m_2 + M(m_1 + m_2)} \quad \text{--- (4)}$$

In reference frame of L;

$$\begin{aligned} a &= \left(\frac{m_1 - m_2}{m_1 + m_2} \right) (g - A) \\ &= \frac{(m_1 - m_2)}{m_1 + m_2} g \left[1 - \frac{4m_1 m_2 - M(m_1 + m_2)}{4m_1 m_2 + M(m_1 + m_2)} \right] \\ &= \frac{(m_1 - m_2)g \cdot 2M(m_1 + m_2)}{(m_1 + m_2) \{ 4m_1 m_2 + M(m_1 + m_2) \}} \quad \text{--- (5)} \end{aligned}$$

absolute acceleration of $m_1 \rightarrow a_1 = a + A$

4.112 Laws of Motion

$$a_1 = \frac{2Mg(m_1 - m_2)}{4m_1m_2 + M(m_1 + m_2)} + g \frac{4m_1m_2 - M(m_1 + m_2)}{4m_1m_2 + M(m_1 + m_2)}$$

$$= \frac{4m_1m_2 + M(m_1 - 3m_2)}{4m_1m_2 + M(m_1 + m_2)} g$$

Similarly, $a_2 = (A - a) = \frac{4m_1m_2 - M(m_1 - 3m_2)}{4m_1m_2 + M(m_1 + m_2)}$

(ii) Substituting (4) in (1) and (2),

$$T' = \frac{4Mm_1m_2}{[4m_1m_2 + M(m_1 + m_2)]} g$$

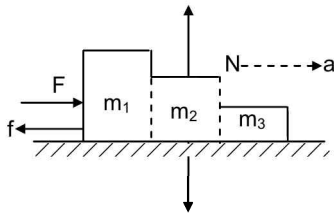
$$T = 2T' = \frac{8Mm_1m_2}{4m_1m_2 + M(m_1 + m_2)} g$$

(iii) If $m_1 = m_2 = m$ from equation (5) $a = 0$ They move together. Using formula:

$$A = \frac{2m - M}{2m + M} g$$

If $M = \frac{4m_1m_2}{m_1 + m_2}$ from equation (3) $A = 0$

126. (i)



FBD for system:

$$N = (m_1 + m_2 + m_3)g = (10 + 6 + 4)10 = 200 \text{ N}$$

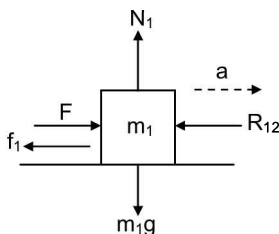
$$f = \mu N = (0.1)200 = 20 \text{ N}; F = 25 \text{ N}$$

$$F - f = (m_1 + m_2 + m_3)a$$

$$\Rightarrow 25 - 20 = 20a$$

$$\Rightarrow a = \frac{5}{20} = 0.25 \text{ m s}^{-2}$$

FBD for m_1



$$N_1 = m_1g = 100 \text{ N}$$

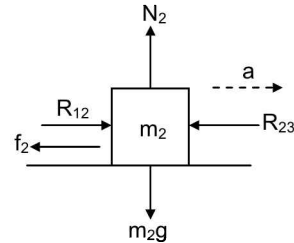
$$f_1 = \mu N_1 = (0.1)100 = 10 \text{ N}$$

$$F - f_1 - R_{12} = m_1a$$

$$\Rightarrow 25 - 10 - R_{12} = 10(0.25)$$

$$\Rightarrow R_{12} = 12.5 \text{ N}$$

FBD for m_2



$$N_2 = m_2g = 60 \text{ N}$$

$$f_2 = \mu N_2 = (0.1)60 = 6 \text{ N}$$

$$R_{12} - f_2 - R_{23} = m_2a$$

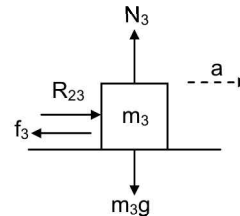
$$\Rightarrow 12.5 - 6 - R_{23} = 6(0.25)$$

$$\Rightarrow R_{23} = 5 \text{ N}$$

$$f = f_1 + f_2 + f_3$$

$$20 = 10 + 6 + f_3 \Rightarrow f_3 = 4 \text{ N}$$

FBD for m_3



$$N_3 = m_3g = 40 \text{ N}$$

$$f_3 = \mu N_3 = (0.1)40 = 4 \text{ N}$$

$$R_{23} - f_3 = m_3a$$

$$5 - 4 = 4(0.25) \text{ [verified]}$$

$$F = 25 \text{ N}$$

$$\Rightarrow a = 0.25 \text{ m s}^{-2}, f_1 = 10 \text{ N}$$

$$f_2 = 6 \text{ N}, f_3 = 4 \text{ N}, R_{12} = 12.5 \text{ N}$$

$$R_{23} = 5 \text{ N}$$

(ii) If $F = 8 \text{ N}$,

Since $f_{1\text{max}} = 10 \text{ N}$,

$$f_1 = 8 \text{ N and } a = 0$$

$$\therefore R_{12}, R_{23}, f_2, f_3 = 0$$

(iii) If $F = 12 \text{ N}$, $f_1 = 10 \text{ N}$, $R_{12} = 2 \text{ N}$, $a = 0$

$$f_2 = 2 \text{ N (less than } f_{2\text{max}})$$

$$R_{23} = 0, f_3 = 0$$

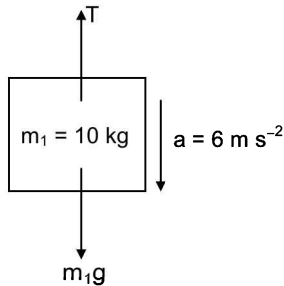
(iv) If $F = 20 \text{ N}$, $f_1 = 10 \text{ N}$, $R_{12} = 10 \text{ N}$,

$$f_2 = f_{2\text{max}} = 6 \text{ N}$$

$$R_{23} = 4 \text{ N}, f_3 = f_{3\text{max}} = 6 \text{ N}$$

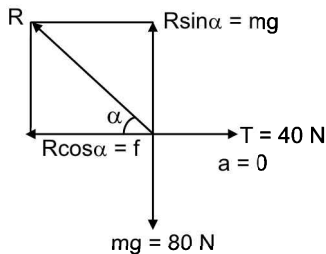
i.e. the applied force 20 N will just be able to overcome the total frictional force f_{max} of 20 N and $a = 0$, the system will be under equilibrium.

127. FBD of 10 kg monkey:



$$m_1g - T = m_1a \Rightarrow T = m_1(g - a) \\ = 10(10 - 6) = 40 \text{ N}$$

FBD of 8 kg monkey



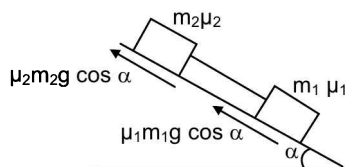
$$R\cos\alpha = \text{friction 'f'} = T (\because \text{system is at rest})$$

$$\therefore R\cos\alpha = 40 \text{ N}$$

$$R\sin\alpha = mg = 80 \text{ N}$$

$$\therefore \tan\alpha = \frac{80}{40} \Rightarrow \alpha = \tan^{-1}(2)$$

128.



$$\text{Total friction force} = \mu_1 m_1 g \cos\alpha + \mu_2 m_2 g \cos\alpha \\ = (\mu_1 m_1 + \mu_2 m_2) g \cos\alpha$$

Due to gravity $= (m_1 + m_2) g \sin\alpha$

Let the common acceleration along the plane be a
 $\Rightarrow (m_1 + m_2)a$

$$= (m_1 + m_2)g \sin\alpha - (\mu_1 m_1 + \mu_2 m_2)g \cos\alpha$$

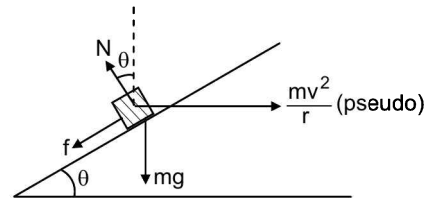
Note: Tension in the string is greater than zero, but it does not contribute to the acceleration of the system, as it is an internal force of the system

$$(i) \quad a = g \sin\alpha - \frac{(\mu_1 m_1 + \mu_2 m_2)}{(m_1 + m_2)} g \cos\alpha$$

(ii) Effective friction coefficient

$$= \mu = \frac{F_f}{(m_1 + m_2)g} = \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}$$

129.



$$v = 72 \times \frac{5}{18} = 20 \text{ m s}^{-1}$$

$$\sin\theta = 0.1,$$

$$\cos\theta \approx 1$$

$$N = mg \cos\theta + \frac{mv^2}{r} \sin\theta, \text{ Taking the rotating frame,}$$

where a centrifugal force $\frac{mv^2}{r}$ acts radially outwards.

$$= 100 \times 10 \times 1 + 100 \times \frac{20^2}{200} \times 0.1 = 1000 + 20 \\ = 1020 \text{ N}$$

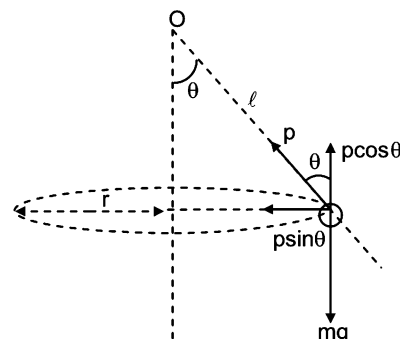
$$f_{\text{max}} = \mu N = 0.4 \times 1020 = 408 \text{ N}$$

$$f + mg \sin\theta = \frac{mv^2}{r} \cos\theta$$

$$f = 100 \times \frac{20^2}{200} \times 1 - 100 \times 10 \times 0.1 = 100 \text{ N} < f_{\text{max}}$$

$$\Rightarrow f = 100 \text{ N}$$

130.



4.114 Laws of Motion

Mass of body 'm' = 500 g = 0.5 kg

Length of string ' ℓ ' = 2 m

Radius of rotation ' r ' = 1 m

θ = angle which the string makes with the vertical
 p = tension in the string.

$$(a) \sin\theta = \frac{r}{\ell} = \frac{1}{2}$$

$$\therefore \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

The tension ' p ' can be resolved as ' $p \cos \theta$ ' vertically upwards (and this balances the weight ' mg ' of the body) and ' $p \sin \theta$ ' horizontally towards the centre of the circle (which provides the centripetal force $mr\omega^2$)

$$\therefore p \sin \theta = mr\omega^2 \quad \text{-----(1)}$$

$$p \cos \theta = mg \quad \text{-----(2)}$$

$$(i) (1) \div (2) \rightarrow \tan\theta = \frac{r\omega^2}{g}$$

$$\therefore \omega^2 = \frac{g \tan \theta}{r}$$

$$\omega = \sqrt{\frac{g \tan \theta}{r}}. \text{ If 'T' is the time period,}$$

$$\omega = \frac{2\pi}{T} \rightarrow \frac{2\pi}{T} = \sqrt{\frac{g \tan \theta}{r}}$$

$$\begin{aligned} \therefore T &= 2\pi \sqrt{\frac{r}{g \tan \theta}} = 2\pi \sqrt{\frac{1}{9.8 \times \tan 30^\circ}} \\ &= 2\pi \sqrt{\frac{1 \times \sqrt{3}}{9.8 \times 1}} \left(\because \tan 30^\circ = \frac{1}{\sqrt{3}} \right) \\ &= 2\pi \times \sqrt{\frac{1.732}{9.8}} = 2.64 \text{ s} \end{aligned}$$

Hence the time period of the conical pendulum is 2.64 s

(ii) From equation (2),

$$\begin{aligned} p &= \frac{mg}{\cos \theta} = \frac{0.5 \text{ kg} \times 9.8 \text{ ms}^{-2}}{\cos 30^\circ} \\ &= \frac{0.5 \times 9.8 \times 2}{\sqrt{3}} \left[\because \cos 30^\circ = \frac{\sqrt{3}}{2} \right] = 5.66 \text{ N} \end{aligned}$$

$\therefore$ Tension in the string = 5.66 N

(iii) $r_1 = \sqrt{2}$ m

$$\therefore \sin \theta_1 = \frac{r_1}{\ell} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\therefore \theta_1 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

$$\begin{aligned} \text{We know } \omega_1 &= \sqrt{\frac{g \tan \theta_1}{r_1}} \\ &= \sqrt{\frac{9.8 \times \tan 45^\circ}{\sqrt{2}}} = \sqrt{6.93} \\ &= 2.63 \text{ rad s}^{-1} \end{aligned}$$

$$\text{But } \omega_1 = 2\pi f_1 \rightarrow f_1 = \frac{\omega_1}{2\pi} = \frac{2.63}{2\pi}$$

$$\begin{aligned} \therefore f_1 &= 0.4186 \text{ revolution per second} \\ &= 0.4168 \times 60 \text{ rpm} = 25.11 \text{ rpm} \end{aligned}$$

So when the rotation speed of the sphere is 25.11 rpm, its radius will become $\sqrt{2}$ m.

(iv) When string becomes horizontal,

$$\theta = 90^\circ \text{ and } \tan \theta = \tan 90^\circ = \infty$$

$$\text{But } \tan \theta = \frac{r\omega^2}{g}$$

$$\therefore \frac{r\omega^2}{g} = \infty \text{ which is possible only if '}\omega\text{' is infinite.}$$

Hence at infinite speed of rotation, the string will become horizontal.

131. Let resistance force = F

$F \times t = \Delta p$; for $2\Delta p$, t is doubled.

$$\left(\frac{v+0}{2}\right)t = d \therefore \left(\frac{2v+0}{2}\right)2t = 4d.$$

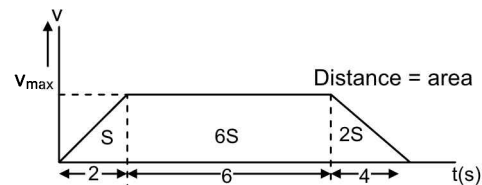
Aliter: Assuming constant retardation ' a ',

$$0 - u^2 = -2ad \rightarrow a = \frac{u^2}{2d}$$

$$0 - (2u)^2 = -2ad, \rightarrow d_1 = \frac{-4u^2}{-2a}$$

$$= \frac{2u^2 \times d}{(u^2)} = 4d$$

132.



The distance travelled are: $1 \times v_{\max}$, $6 \times v_{\max}$, and $2 \times v_{\max}$, respectively in 2, 6 and 4 seconds. Let $v_{\max} = S$ (numerically)

Let h be the height of each floor.

Then $10h = 9S$

$h = 0.9S$

So passing the second floor at height > 1.8 S, the lift is moving with uniform velocity $v_{\max}$

$$\therefore T_1 = mg$$

Passing through the eighth floor at height > 7.2 S, the lift is decelerating

$$\therefore T_2 = m(g - a)$$

$$\therefore T_1 - T_2 = ma = 120 \text{ N (given)}$$

$$\Rightarrow a = \frac{120}{160} = \frac{3}{4} \text{ m s}^{-2} (\because m = 160 \text{ kg})$$

From the deceleration curve,

$$\text{using the slope} = \frac{3}{4} = a$$

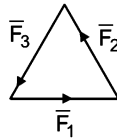
$$v_{\max} = 4 \times \frac{3}{4} = 3 \text{ m s}^{-1}$$

$$\therefore S = v_{\max} (\text{numerically}) = 3 \text{ m}$$

$$\therefore h = 0.9 S = 2.7 \text{ m}$$

133. Velocity is constant and non zero $\Rightarrow$ net force is zero

The three forces are coplanar and can be represented by the sides of triangle, for which all the three conditions hold good.



134. $x = t^3 - 2t^2 - 9$

$$\Rightarrow \frac{dx}{dt} = v = 3t^2 - 2 \times 2t = 3t^2 - 4t$$

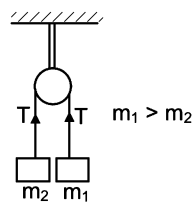
$$\frac{dv}{dt} = a = 3 \times 2t - 4 = 6t - 4$$

$$\Rightarrow F = ma = 1 \times (6t - 4) \text{ N}$$

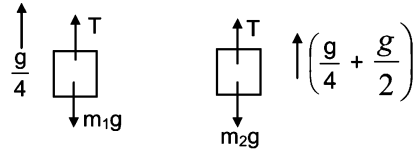
$$\text{at } t = 0, F = -4 \text{ N}$$

135. The reading of the spring balance shows the tension T in the string

$$= \frac{2m_1 m_2}{m_1 + m_2} g = \frac{2 \times 20 \times 10}{30} g = \frac{40}{3} \text{ kg wt.}$$

136. 

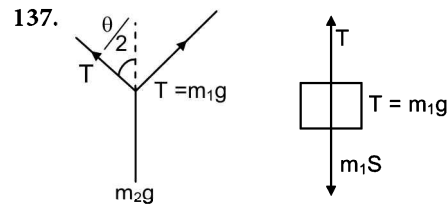
Clearly, $T > m_2 g$ and $T > m_1 g$, since both are climbing up. Also acceleration of m_2 is greater (lower mass)



$$\therefore T - m_1 g = m_1 \frac{g}{4} \text{ and } T - m_2 g = m_2 \frac{3g}{4}$$

$$\therefore m_1 g \left(1 + \frac{1}{4}\right) = m_2 g \left(1 + \frac{3}{4}\right)$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{7}{5}$$



Tension T everywhere.

$$\therefore T = m_1 g \text{ (From FBD of block } m_1 \text{)}$$

$$2T \cos \frac{\theta}{2} = m_2 g \text{ (From FBD of block } m_2 \text{)}$$

$$\Rightarrow \cos \frac{\theta}{2} = \frac{m_2}{2m_1}$$

$$\Rightarrow \theta = 2 \cos^{-1} \frac{m_2}{2m_1}$$

138. $T - mg \sin \alpha = ma$ — (1)

$$mg \sin \beta - T = ma$$
 — (2)

$$(1) - (2)$$

$$\Rightarrow 2T = mg(\sin \alpha + \sin \beta)$$

$$\Rightarrow T = \frac{1}{2} mg (\sin \alpha + \sin \beta): \text{ But } T \leq \frac{mg}{2} \text{ (data)}$$

$$T \leq \frac{mg}{2} \Rightarrow \sin \alpha + \sin \beta \leq 1$$

139. (a)

$$\text{Tension in string, } T = \frac{2m_1 m_2}{(m_1 + m_2)} g \text{ (standard result)}$$

$$= \frac{2 \times 3 \times 2}{(3 + 2)} \times 10 = 24 \text{ N}$$

$$F = \text{Pull on spring} = 2T = 2 \times 24 = 48 \text{ N}$$

$$\text{Extension, } \Delta \ell = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

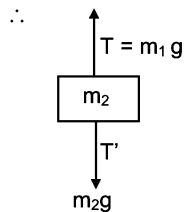
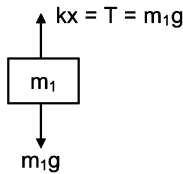
$$\therefore k = \frac{F}{\Delta \ell} = \frac{48}{2 \times 10^{-2}} = 2400 \text{ N m}^{-1}$$

4.116 Laws of Motion

140. (d)

The apparent weight of each block is equal to the tension in the string connected to that block. Since the tension in the string is same on both sides of the pulley, the apparent weight is the same for both blocks.

141. At rest



$$\therefore T' = (m_1 - m_2)g$$

$$\therefore m_1 > m_2$$

On cutting

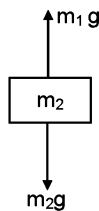
$$\therefore a = \left(\frac{m_1 - m_2}{m_2} \right) g$$

(upwards)

(m_1 's acceleration zero)

$$\therefore \text{given } \left(\frac{m_1 - m_2}{m_2} \right) g = 2$$

$$\Rightarrow \frac{m_1}{m_2} = 1.2$$



$$142. \text{ a is same for all } \Rightarrow 15mg - T_1 = 15ma \quad \text{--- (1)}$$

$$T_1 - T_2 = (10 - f)ma \quad \text{--- (2)}$$

$$T_2 - 5mg = 5ma \quad \text{--- (3)}$$

$$(1) + (2) + (3) \Rightarrow$$

$$15mg - 5mg = (30 - f)ma$$

$$\Rightarrow a = \frac{15mg - 5mg}{(30 - f)m}, \text{ where } f \text{ is the fraction broken}$$

$$\text{off } \Rightarrow m' = (1 - f)m$$

$$\Rightarrow a = \frac{10}{(30 - f)}g = \frac{100}{30 - f}$$

$$0 < f < 1$$

$$\Rightarrow a > \frac{100}{30} = 3.33 \text{ m s}^{-2} \text{ and}$$

$$\Rightarrow a < \frac{100}{29} = 3.4 \text{ m s}^{-2}$$

143. (d)

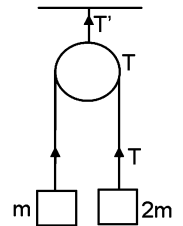
Consider a stationary pulley and masses m & $3m$.

$$T = \frac{2(m \times 2m)g}{(m + 2m)} = \frac{4}{3}mg$$

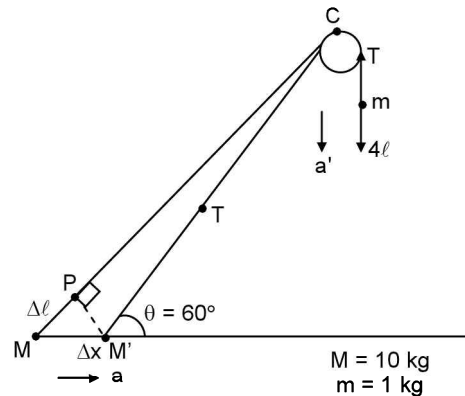
$$\Rightarrow T' = 2T = \frac{8}{3}mg$$

M will be stationary if

$$Mg = T' = \frac{8}{3}mg \Rightarrow M = \frac{8}{3}m$$



144.



When M moves a small distance Δx , the thread length released is $\Delta l = MP$ [$M'P$ is drawn normal to MC]

$$\frac{\Delta l}{\Delta x} = \cos 60^\circ$$

$$\Delta l = \Delta x \cos 60^\circ \Rightarrow \text{differentiate twice}$$

$$\therefore a' = a \cos 60^\circ = \frac{a}{2}$$

(The total acceleration of the 10 kg body along the ground a' is the component of the total acceleration along the string)

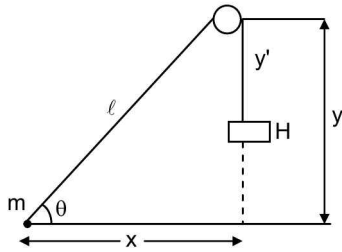
$$mg - T = ma' = m \frac{a}{2} \quad \text{--- (1)}$$

$$T \cos 60^\circ = Ma = 10a$$

$$T = 20a \text{ --- (2) } \Rightarrow \text{From equations (1) \& (2)}$$

$$1 \times 10 - 20a = 1 \times \frac{a}{2} \Rightarrow 10 = a(20.5)$$

$$= a = \frac{10}{20.5} = \frac{20}{41} \text{ m s}^{-2}$$

Alternate method:
 $\ell + y' = \text{constant (length of the string)}$

$$\ell = \sqrt{x^2 + y^2} \Rightarrow (x^2 + y^2)^{1/2} + y' = k$$

Differentiate:

$$\frac{1}{2}(x^2 + y^2)^{-1/2} \cdot 2x \frac{dx}{dt} + \frac{dy'}{dt} = 0$$

$$\Rightarrow \frac{dx}{dt} = v, \frac{dy'}{dt} = v'$$

$$v \cdot \frac{x}{\sqrt{x^2 + y^2}} = v'$$

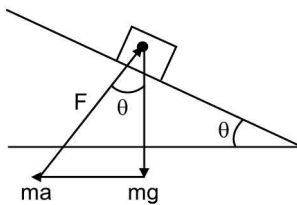
 $\Rightarrow$ differentiate and apply condition initial velocity is zero. $\Rightarrow a \cos \theta = a'$

$$\theta = 60^\circ \Rightarrow \frac{a}{2} = a' \text{ other steps, as above}$$

145. $R - mg = ma \Rightarrow R = m(g + 5g)$

 $mg' = \text{apparent weight} = 6 mg.$

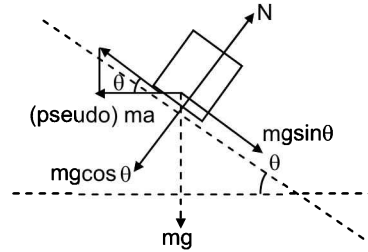
146.


 $F \rightarrow$ normal reaction

 $ma \rightarrow$ pseudo

For the accelerated frame on the wedge, the three forces acting on it form a triangle for the body to be stable.

$$\begin{aligned} \Rightarrow F &= \frac{mg}{\cos \theta} = \frac{0.5 \times 10}{\cos 60^\circ} \\ &= \frac{5}{1/2} = 10 \text{ N} \end{aligned}$$

Aliter:

For no slip, $ma \cos \theta = mg \sin \theta \Rightarrow a$

$$= g \frac{\sin \theta}{\cos \theta} = g \tan \theta$$

$$N = mg \cos \theta + ma \sin \theta$$

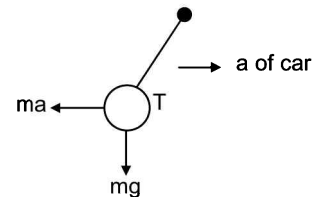
$$= m[g \cos \theta + a \sin \theta]$$

$$= m[g \cos \theta + g \tan \theta \sin \theta]$$

$$= mg \left[\cos \theta + \frac{\sin^2 \theta}{\cos \theta} \right] = mg \frac{[\cos^2 \theta + \sin^2 \theta]}{\cos \theta}$$

$$= \frac{mg}{\cos \theta} = \frac{0.5 \times 10}{\cos 60^\circ} = 10 \text{ N}$$

147. Accelerates or decelerates,
Apply pseudo force



$$\Rightarrow T = m\sqrt{g^2 + a^2} > mg$$

 $\Rightarrow$ Increases

148. $\mu \geq \tan \theta$. (for no sliding of the block)

Note that no 'g' term is involved,

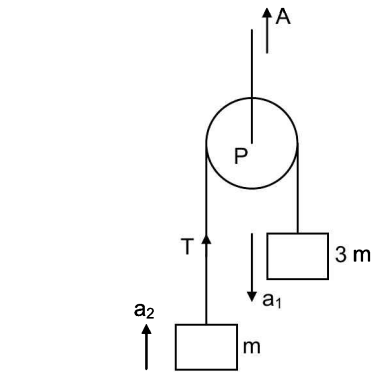
Hence no 'g + a' term involved either
(pseudo force ma does not change μ)

149. In the accelerated reference frame of the pulley, a pseudo forces ($3m \cdot A$) and ($m \cdot A$) act down on the masses $3m$ and m respectively.

$$\therefore g' = (g + A) \Rightarrow \therefore [\text{For } 3m \text{ mass}] a_1'$$

$$= \left(\frac{3m - m}{3m + m} \right) g' = \frac{2}{4} (g + A) = \frac{g + A}{2}$$

$$a_1 = a_1' - A$$



$$= \frac{g + A}{2} - A = \frac{g - A}{2} \Rightarrow \text{Given } a_1 = 0$$

$$\therefore \frac{g - A}{2} = 0 \Rightarrow g = A$$

$$\therefore a_2 = a_1' + A$$

$$= \frac{g + A}{2} + A = \frac{g + 3g}{2} = 2g$$

Aliter:

3 m is stationary (with respect to the accelerated frame of the pulley)

$$\rightarrow T = 3mg$$

$$T - mg = ma$$

$$a = 2g$$

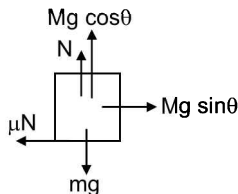
$$150. a = \frac{F}{(m_1 + m_2 + m_3)}$$

$$F_{32} = m_3 a = F_{23}$$

$$F_{21} = (m_2 + m_3)a = F_{12}$$

$$\therefore \frac{F_{32}}{F_{21}} = \frac{m_3 a}{(m_2 + m_3)a} = \frac{3}{(5+3)} = \frac{3}{8}$$

151.

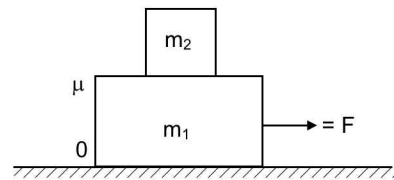


$$N = mg - mg \cos \theta$$

$$a \geq 0 \text{ if } mg \sin \theta \geq \mu mg (1 - \cos \theta)$$

$$\Rightarrow 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \geq \mu 2 \sin^2 \frac{\theta}{2} \Rightarrow \cot \frac{\theta}{2} \geq \mu$$

152. Case (1)

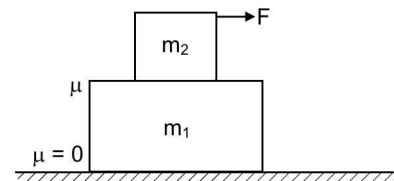


$$F_{\max} \text{ between } m_2 \text{ and } m_1 = \mu m_2 g$$

$$\therefore \text{maximum acceleration of } m_2 = \mu g$$

$$\therefore F_{\max} = \mu g (m_2 + m_1) = \mu Mg.$$

Case (ii)

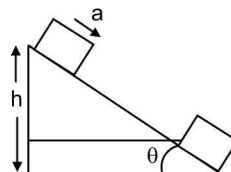


$$F_{\max} = \mu m_2 g$$

$$\therefore \text{maximum acceleration of } m_1 = \frac{\mu m_2 g}{m_1}$$

$$F_{\max} = (m_1 + m_2) \mu \frac{m_2 g}{m_1} = \mu Mg \cdot \frac{m_2}{m_1}$$

153.



$$l = \frac{h}{\sin \theta}; t = \sqrt{\frac{2l}{a}}$$

$$\Rightarrow t = \sqrt{\frac{2h}{\sin \theta a}}; t_1 = t_2,$$

$$h_1 = h_2$$

$$\text{For smooth : } a_1 = g \sin \theta_1$$

$$\text{For rough : } a_2 = g (\sin \theta_2 - \mu \cos \theta_2)$$

$$\therefore g \sin^2 \theta_1 = g \sin \theta_2 (\sin \theta_2 - \mu \cos \theta_2) (\because t_1 = t_2)$$

$$\text{Clearly } \theta_2 > \theta_1 \therefore \theta_2 = 53^\circ, \theta_1 = 37^\circ$$

$$\therefore (0.6)^2 = 0.8 (0.8 - 0.6\mu) \Rightarrow 0.48\mu = 0.64 - 0.36$$

$$= 0.28 \Rightarrow \mu = \frac{0.28}{0.64}$$

$$\therefore \mu = \frac{7}{12}$$

154. If parallel : $F = mg (\sin \theta - \mu \cos \theta)$ — (1)

If horizontal : $F \cos \theta$

$$= mg (\sin \theta - \mu \cos \theta) - \mu F \sin \theta$$

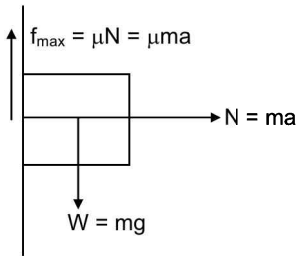
$$= F - \mu F \sin \theta \text{ (from (1))}$$

($\therefore$ normal reaction increases by $F \sin \theta$)

$$\Rightarrow F(1 - \mu \sin \theta) = F \cos \theta$$

$$\Rightarrow \mu = \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = \tan \frac{\theta}{2}$$

155. Let Acceleration of system be a



$$\therefore f_{\max} = \mu N = \mu ma = W = mg$$

$$\therefore a = \frac{g}{\mu} = \frac{10}{0.1} = 100 \text{ m s}^{-2}$$

$$F = (M + m)a = (5 + 1)100 = 600 \text{ N}$$

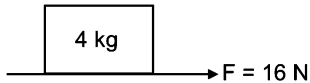
156. At $\theta = 90^\circ$, $k.l = mg \Rightarrow k = mg$

At $\theta = 20^\circ$, no extension because $\mu \geq 0.36$

$\therefore$ At $\theta = 30^\circ$, $kx = mg(\sin 30^\circ - \mu \cos 30^\circ)$ or 0

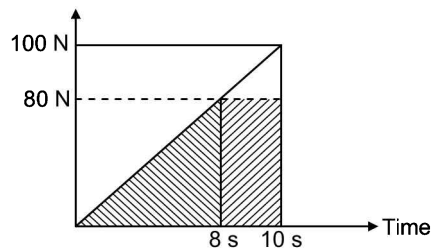
$$\Rightarrow x = \left(\frac{1}{2} - 0.36 \times \frac{\sqrt{3}}{2} \right) = 0.185 \text{ (max)}$$

157. $a = \frac{40}{10} = 4 \text{ m s}^{-2}$



$$\mu 6g \geq 16 \Rightarrow \mu \geq \frac{16}{60} = 0.266.$$

158.



Since it starts moving at 8 s, the force at that time is obviously 80 N. Hence the effective impulse is only the unshaded area as shown.

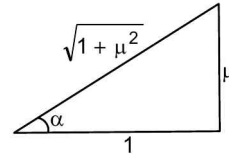
$$F \Delta t = \frac{1}{2} \times (100 - 80) \times (10 - 8)$$

$$= \frac{1}{2} \times 20 \times 2 = 20 \text{ Ns}$$

$$\therefore F \Delta t = mv$$

$$\therefore 10 \times v = 20 \Rightarrow v = 2 \text{ m s}^{-1}$$

159.



Let α be the angle of repose $\Rightarrow \mu = \tan \alpha \Rightarrow$

$$\sin \alpha = \frac{\mu}{\sqrt{1 + \mu^2}}$$

The minimum force required to move the load

$$F_{\min} = \frac{\mu mg}{\sqrt{1 + \mu^2}} = \sin \alpha (10 \times 10) = 60 \text{ N (data)}$$

$$\sin \alpha = 0.6, \alpha = 37^\circ$$

160. Let angle of friction be α

$$\tan \alpha = \mu = 0.75, \alpha = 37^\circ \Rightarrow$$

Minimum force required to pull up a mass m on a rough inclined plane of angle θ is

$$F_{\min} = (mg \sin \theta + \mu mg \cos \theta) \left(\frac{1}{\sqrt{1 + \mu^2}} \right)$$

$$= (mg \sin \theta + \mu mg \cos \theta) \cdot \cos \alpha$$

$$= \left(\frac{100}{\sqrt{2}} + \frac{0.75 \times 100}{\sqrt{2}} \right) \times 0.8$$

$$(\because 1 + \mu^2 = 1 + \tan^2 \alpha = \sec^2 \alpha)$$

$$\Rightarrow \frac{1}{\sqrt{1 + \mu^2}} = \frac{1}{\sec \alpha} = \cos \alpha = \cos 37^\circ = 0.8)$$

$$= \frac{1}{\sqrt{2}} (175 \times 0.8) = \frac{180}{\sqrt{2}} \approx 99 \text{ N}$$

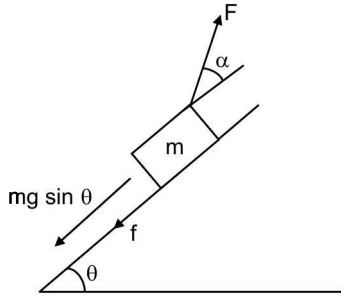
161. Frictional force f :

$$f < F \cos \alpha - mg \sin \theta$$

$$\alpha = 37^\circ, F = 99 \text{ N}$$

$$f < 99 \cos 37^\circ - 10 \times 10 \times \frac{1}{\sqrt{2}}$$

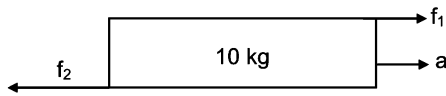
4.120 Laws of Motion



$$= 99 \times 0.8 - \frac{100 \times \sqrt{2}}{2}$$

$$f < 79.2 - 70.7 = 8.5 \text{ N}$$

162.



The free body diagram for 10 kg is

$$f_1 - f_2 = 10a$$

for RHS to be positive

(since the 10 kg has to move to the right)

$$f_1 > f_2 \Rightarrow \mu_1 5g > \mu_2 15g$$

$$\frac{\mu_1}{\mu_2} > 3$$

163. For upward journey;

$a_1 = g[\sin\theta + \mu\cos\theta]$, downwards along the plane

$$t_1 = \frac{v}{a_1} = \frac{v}{g(\sin\theta + \mu\cos\theta)} = \frac{v}{g\cos\theta(\tan\theta + \mu)}$$

Displacement along the plane upwards is

$$S = \frac{v^2}{2g(\sin\theta + \mu\cos\theta)}$$

For downward journey: $u = 0$

$a_2 = g(\sin\theta - \mu\cos\theta)$, downwards along the plane

To cover the distance S downwards, time taken

$$t_2 = \sqrt{\frac{2S}{a_2}}$$

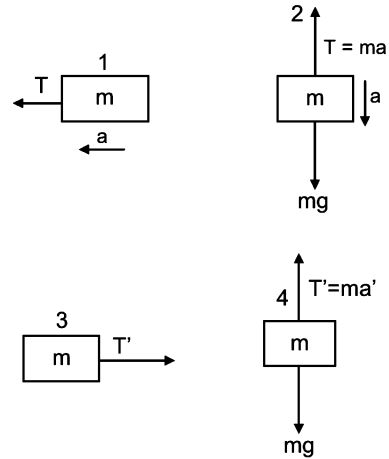
$$= \sqrt{\frac{2v^2}{2g(\sin\theta + \mu\cos\theta) \times g(\sin\theta - \mu\cos\theta)}}$$

$$= \frac{v}{g\sqrt{\sin^2\theta - \mu^2\cos^2\theta}}$$

$$= \frac{v}{g\cos\theta} \frac{1}{\sqrt{\tan^2\theta - \mu^2}}$$

$$\therefore t = t_1 + t_2 = \frac{v}{g\cos\theta} \left(\frac{1}{\tan\theta + \mu} + \frac{1}{\sqrt{\tan^2\theta - \mu^2}} \right)$$

$$164. a_1 = a_2 = a_3 = a_4 = \frac{g}{2}$$



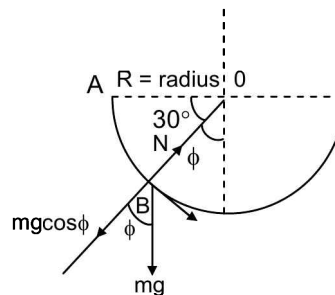
$$T = ma$$

$$\Rightarrow a = \frac{g}{2}$$

$$T' = ma', mg - ma' = ma'$$

$$\Rightarrow a = \frac{g}{2}$$

165.



$$\text{Centripetal force } F = \frac{mv^2}{R}$$

$$= \frac{m(\sqrt{gR})^2}{R} \quad (\because v = \sqrt{gR}, \text{ as per data})$$

$$= \frac{mgR}{R} = mg$$

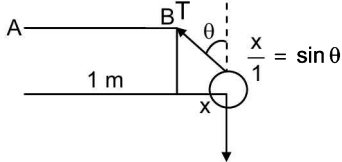
$$\text{But } F = N - mg\cos\phi$$

$$= N - mg \cos 60^\circ (\because \phi = 90^\circ - 30^\circ = 60^\circ)$$

$$\therefore N = mg \cos 60^\circ + F = \frac{mg}{2} + mg = \frac{3}{2}mg$$

$$\therefore \frac{F}{N} = \frac{mg}{\left(\frac{3}{2}mg\right)} = \frac{2}{3}$$

166.



$$T \cos \theta = mg \quad \text{--- (1)}$$

$$T \sin \theta = \text{centripetal force} = m r \omega^2;$$

$$r = 1\text{ m} + x = 1 + 1 \sin \theta$$

$$\therefore T \sin \theta = m \cdot \omega^2 (1 + \sin \theta) \quad \text{--- (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \tan \theta = \frac{\omega^2}{g} (1 + \sin \theta) = 1 + \sin \theta$$

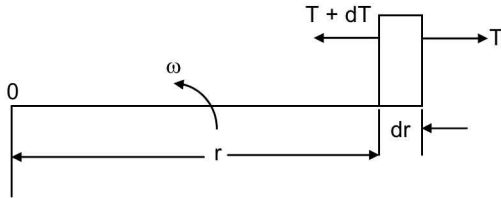
$$\Rightarrow \tan \theta - \sin \theta = 1 \Rightarrow \tan \theta > 1 \Rightarrow \theta > 45^\circ$$

($\tan \theta - \sin \theta$) increases with θ .

$$\text{Check with } \theta = 60^\circ, \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} < 1.$$

$$\therefore \theta \text{ should be } > 60^\circ$$

167.



$$\text{Let } \lambda \text{ be the mass per unit length. } \therefore \lambda = \frac{m}{\ell}$$

Centripetal force on an element of length 'dr' is

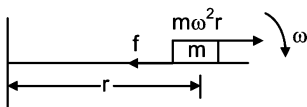
$$dT = (dm)\omega^2 r$$

$$= (\lambda dr) \omega^2 r$$

$$\therefore T = \int_0^\ell \lambda \omega^2 r dr = \frac{\lambda \omega^2 \ell^2}{2} = \frac{m \omega^2 \ell}{2} = F$$

$$\Rightarrow \omega = \left(\frac{2F}{m\ell} \right)^{1/2}$$

168.



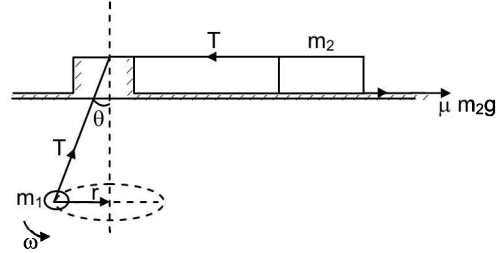
Centripetal force is provided by frictional force

$$\text{Centripetal force} = m \omega^2 r$$

$$\text{Frictional force } f_{\max} = \mu mg \Rightarrow \text{The body will slip where } \mu mg < m \omega^2 r$$

$$\mu g < \omega^2 r \Rightarrow \text{The one at largest } r \text{ will slip first}$$

169.



Let ω be the angular velocity:

$$T \sin \theta = m_1 \omega^2 r; T \cos \theta = m_1 g \Rightarrow \tan \theta = \frac{\omega^2 r}{g}$$

$$\text{When } \theta = 45^\circ \Rightarrow \omega^2 r = g \tan 45^\circ = g$$

$$T \sin 45^\circ = m_1 \omega^2 r \Rightarrow \frac{T}{\sqrt{2}} = m_1 g \Rightarrow T = \sqrt{2} m_1 g$$

$$\text{When } m_2 \text{ is about to slip } T = \mu m_2 g = f_{\max}$$

$$\therefore \sqrt{2} m_1 g = \mu m_2 g \Rightarrow \mu = \frac{m_1}{m_2} \sqrt{2} = \frac{1}{2} \sqrt{2} = \frac{1}{\sqrt{2}}$$

170. Let R be the reaction of air on plane

$$R \cos \theta = mg$$

$$R \sin \theta = \frac{mv^2}{r}$$

$$\therefore \tan \theta = \frac{v^2}{rg}$$

$$\text{Here } V = 720 \times \frac{5}{18} = 200 \text{ m s}^{-1}$$

$$r = 2000 \text{ m}$$

$$g = 10 \text{ m s}^{-2}$$

$$\therefore \tan \theta = \frac{200 \times 200}{2000 \times 10} = 2$$

$$\therefore \theta = \tan^{-1}(2)$$

171. (a)

172. (a)

173. (b)

174. (b)

175. (a)

4.122 Laws of Motion

176. Statement 2 is not correct. Friction opposes relative motion between contact surfaces and not motion of contact surfaces.

Ans. (c)

177. The net force is zero where, $\bar{a}$ is zero. Hence more than one force could act, if they cancel each other

178. Equation for uniform motion, say to right is:
 $mgsin\theta - \mu mgcos\theta - [mgsin\theta + \mu mgcos\theta] = 0$

$$\text{But } mgsin\theta = \mu mgcos\theta$$

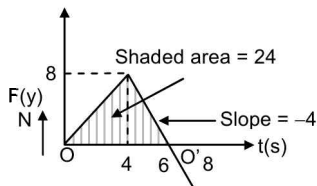
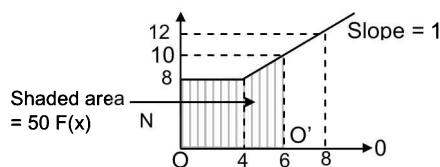
$$\therefore mgsin\theta = 3mgsin\theta$$

Impossible condition

179. The passengers and bus form one system and the force exerted by the passengers on the bus, is an internal force of the system. Since internal forces of a system cannot cause motion, the system remains at rest. Note that the bus exerts an equal and opposite force on the passengers. Hence the net force exerted by the passengers is not zero.

180. Newton's III law of motion is applicable for all real forces in nature only (i.e., all real forces in nature occur in equal and opposite pairs). Newton's III law can be applied to all real forces in inertial as well as non-inertial frames of references.

181.



$$\Delta p = \int F dt. \text{ Initially, velocity} = 0$$

$$\Rightarrow P_x = \int F_x dt \text{ and } P_y = \int F_y dt$$

Calculate areas and equate.

Shift origin to O' so that

$$T' = 0, \text{ at } t = 6 \text{ s}$$

Area up to 6 s is 50 and after that it is:

$$10t' + \frac{1}{2}t't' \quad [\because \text{slope} = +1]$$

$$\therefore P_x(t') = 50 + 10t' + \frac{1}{2}t'^2, \text{ where } t' = (t - 6)$$

For $p_{(y)}$ area upto 6 s is +24 and after that it is $\frac{1}{2}t'(-4t')$ (since slope is -4)

$$P_{(y)}(t') = 24 - 2t'^2$$

$$45^\circ \text{ to X axis} \Rightarrow p_x(t') = p_{(y)}(t') \text{ or } p_x(t') = -p_{(y)}(t')$$

$$\Rightarrow \text{case (i)} \quad 2\frac{1}{2}t'^2 + 10t' + 26 = 0$$

$$(\text{or}) \text{ case (ii)} \quad 1\frac{1}{2}t'^2 - 10t' - 74 = 0$$

$\Rightarrow$ no real t' for case (i)

$$\text{For case (ii)} \quad 1\frac{1}{2}t'^2 - 10t' - 74 = 0 \Rightarrow t' = 23.3 \text{ s}$$

$$\Rightarrow t = 29.3 \text{ s}$$

182. Parallel to X axis $\Rightarrow p_{(y)}(t') = 0 \Rightarrow 24 - 2t'^2 = 0$

$$\Rightarrow t'^2 = 12$$

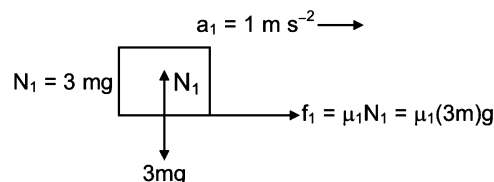
$$\Rightarrow t' = 2\sqrt{3} = 3.464 \Rightarrow t = t' + 6 = 9.464 \text{ s}$$

183. P_x is never zero.

184. Clearly accelerations are for 3 m : 1 m s^{-2} for 2 m : 2 m s^{-2}

for m : 3 m s^{-2}

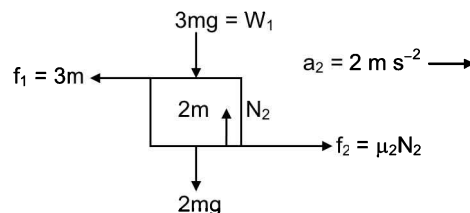
FBD of block mass 3m:



$$\text{But } f_1 = (3m)a_1 = 3m \times 1 = 3m \quad \text{--- (1)}$$

$$\Rightarrow \mu_1 = \frac{1}{g} = 0.1$$

FBD if block of mass 2m



$$N_2 = 3mg + 2mg = 5mg$$

$$(f_2 - f_1) = (2m)a_2 = 2m \times 2 = 4m$$

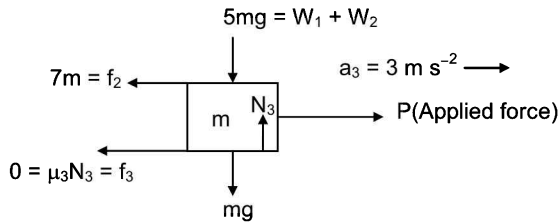
$$\therefore f_2 = 4m + f_1 = 4m + 3m = 7m$$

$$\therefore \mu_2 N_2 = 7m \Rightarrow \mu_2 \times 5mg = 7m$$

$$\therefore \mu_2 N_2 = 7m \Rightarrow \mu_2 \times 5mg = 7m$$

$$\therefore \mu_2 = \frac{7}{5g} = \frac{7}{50} = 0.14$$

185. FBD of block of mass m

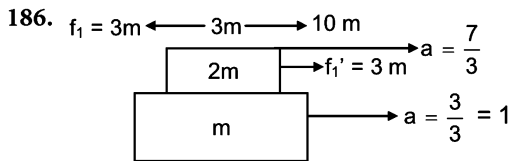


$$N_3 = 5mg + mg = 6mg; f_3 = \mu_3 N_3 = 0 \times 6mg = 0$$

$$P - (f_3 + f_2) = ma_3 = m \times 3 = 3m$$

$$\therefore P = 3m + f_2 + f_3 = 3m + 7m + 0 = 10m$$

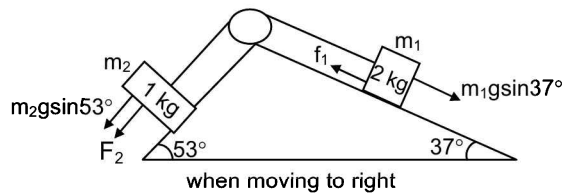
$$\therefore \text{Applied force } P = 10m$$



$\therefore$ 2m and m will move together at 1 m s^{-2} ,

since the frictional force f'_1 exerted on 2 m mass is, $f'_1 = 3 \text{ m}$, which is smaller than $f_{2\text{max}}$ ($= 7 \text{ m}$) between masses 2 m and m.

187.



If it has to move to right (consider the forces that aids motion to right as positive),

$$F = (m_1 g \sin 37^\circ - \mu m_1 g \cos 37^\circ) - (m_2 g \sin 53^\circ + \mu m_2 g \cos 53^\circ)$$

$$= 2(10 \times 0.6 - 0.5 \times 10 \times 0.8) - 1(10 \times 0.8 + 0.5 \times 10 \times 0.6)$$

$$= 2[6 - 4] - [8 + 3] = 4 - 11 = -7 \text{ N} \rightarrow \text{negative.}$$

Hence it cannot move to right

Similarly if it has to move to left, F

$$= (m_2 g \sin 53^\circ - \mu m_2 g \cos 53^\circ) - (m_1 g \sin 37^\circ + \mu m_1 g \cos 37^\circ)$$

$$F = 1(10 \times 0.8 - 0.5 \times 10 \times 0.6) - 2(10 \times 0.6 + 0.5 \times 10 \times 0.8)$$

$= (8 - 3) - 2(6 + 4) = 5 - 20 = -15 \text{ N}$ negative, hence it cannot move to left also. It is stationary

188. As seen above, the resultant force when it moves to

right is -7 N . Hence the retardation is $a = \frac{F}{m_1 + m_2}$

$$\text{i.e; } a = -\frac{7}{3} \text{ m s}^{-2} \Rightarrow v = at$$

$$\Rightarrow t = \frac{v}{a} = \frac{\sqrt{7}}{(7/3)} \gg 1.1 < 2 \text{ s} \Rightarrow \text{Hence maximum}$$

distance moved is:

$$v^2 = 2aS \Rightarrow 7 = 2 \times \frac{7}{3} \times S \Rightarrow S = 1.5 \text{ m}$$

189. As seen in 1 above; the resultant force when it moves to left is -15 N . Hence retardation a

$$= \frac{F}{(m_1 + m_2)} = -5 \text{ m s}^{-2} \Rightarrow \text{Time to halt} = t$$

$$= \frac{v}{a} = \frac{\sqrt{15}}{15/3} = \frac{3}{\sqrt{15}} \approx 0.774 < 2 \text{ s}$$

Hence the total distance moved:

$$v^2 = 2aS \Rightarrow 15 = 2 \times \frac{15}{3} \times S \Rightarrow S = 1.5 \text{ m}$$

190. Between $t = 0$ and $t = 4 \text{ s}$ the acceleration of the lift is

$$a = \frac{AD}{OD} = \frac{4 \text{ m s}^{-1}}{4} = 1 \text{ m s}^{-2}$$

Since the lift is accelerating upward the tension in the rope at $t = 2 \text{ s}$ (between $t = 0$ and $t = 4 \text{ s}$) is

$$T = m(g + a) = 1000(10 + 1) = 11,000 \text{ N.}$$

But $t = 4 \text{ s}$ and $t = 12 \text{ s}$ speed of the lift is constant. Hence $a = 0$,

$$T = mg = 1000 \times 10 = 10,000 \text{ N.}$$

Between $t = 12 \text{ s}$ and $t = 16 \text{ s}$ the lift is decelerating. Its deceleration is given by

$$a = \frac{BE}{EC} = \frac{4 \text{ m s}^{-1}}{4} = 1 \text{ m s}^{-2}$$

$$\therefore \text{Tension} = m(g - a) = 10,000(10 - 1) = 9,000 \text{ N.}$$

The height to which the lift rises = area of OABC

$$= \frac{1}{2} \cdot 4 \times 4 + 4 \times 8 + \frac{1}{2} \cdot 4 \times 4 = 48 \text{ m}$$

191. The maximum acceleration acquired without slipping is given by

$$ma = \mu mg \text{ or } a = \mu g = 0.2 \times 10 = 2 \text{ m s}^{-2}$$

4.124 Laws of Motion

With this maximum acceleration the minimum distance s covered to acquire a speed

$$= 20 \text{ m s}^{-1} \text{ is given by } v^2 = u^2 + 2as \text{ ie } (20)^2 - 0^2 = 2 \times 2 \times s$$

$$4s = 20 \times 20$$

$$s = 100 \text{ m}$$

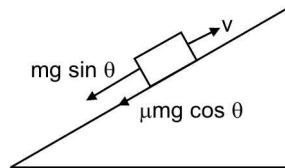
The time needed to come to rest from a speed of 50

$$\text{m s}^{-1} \text{ is } \frac{v - u}{a} = \frac{0 - 25}{-2} = 25 \text{ s}$$

($\because$ final velocity $v = 0$)

Hence a, b, d are correct.

192.



For the body not to slide back

$$\mu m g \cos \theta > m g \sin \theta$$

$\mu > \tan \theta \Rightarrow$ while climbing up from bottom, deceleration

$$a = (g \sin \theta + \mu g \cos \theta) \Rightarrow v^2 = 2as = 2(g \sin \theta + \mu g \cos \theta) s$$

Obviously, smaller the μ , larger the s .

Hence maximum s when $\mu = \tan \theta$

$$\Rightarrow s = \frac{v^2}{2 \times (2g \sin \theta)} = \frac{v^2}{4g \sin \theta}$$

$$\Rightarrow H = \frac{v^2}{4g \sin \theta} \sin \theta = \frac{v^2}{4g}$$

193. The maximum frictional force m_1 can exert on m_2 is $\mu m_1 g = 0.4 \times 5 \times 10 = 20 \text{ N}$. With 20 N applied the acceleration of m_2 is $\frac{20}{10} = 2 \text{ m s}^{-2}$. This is the maximum

acceleration possible for m_2 . At this acceleration, the total force required for both to move together is: $F' = (10 + 5) \times 2 = 30 \text{ N}$

(d) For $F = 40 \text{ N}$

$$\text{Acceleration of } m_1 \text{ is } a_1 = \frac{40 - 20}{5} = 4 \text{ m s}^{-2}$$

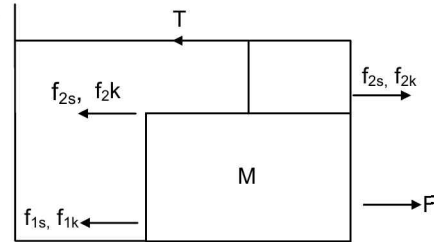
$$\text{Acceleration of } m_2 \text{ is } a_2 = \frac{20}{10} = 2 \text{ m s}^{-2}$$

$a_r = (4 - 2) = 2 \text{ m s}^{-2}$. Hence the distance m_1 moves m_2 each second is $\Rightarrow s$

$$= \frac{1}{2} a_r t^2 = \frac{1}{2} \times 2 \times (1)^2$$

ans. (b), (c), (d)

194.



Let f_{1s} and f_{1k} be the static and dynamic frictional forces between M and floor.

$$f_{1s} = \mu_{1s} (M + m)g = 0.3 \times 15 \times 10 = 45 \text{ N}$$

$$f_{1k} = \mu_{1k} (M + m)g = 0.2 \times 15 \times 10 = 30 \text{ N}$$

Let f_{2s} and f_{2k} be the static and dynamic frictional forces between M & m .

$$f_{2s} = \mu_{2s} (m)g = 0.4 \times 5 \times 10 = 20 \text{ N}$$

$$f_{2k} = 0.3 \times mg = 0.3 \times 5 \times 10 = 15 \text{ N}$$

$$M \text{ will move when } F > f_{1s} + f_{2s} \Rightarrow F_{\min} = 45 + 20$$

$$= 65 \text{ N}$$

When it starts moving, the friction changes to dynamic friction. Hence the frictional force on M is $= f_{1k} + f_{2k} = 30 + 15 = 45 \text{ N}$

Hence it has to move with minimum acceleration

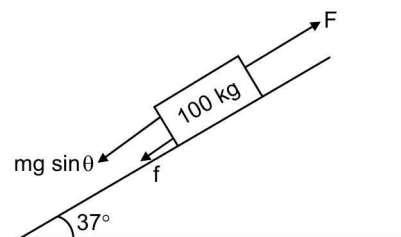
$$= \frac{65 - 45}{M}$$

$$a_{\min} = \frac{20}{10} = 2 \text{ m s}^{-2}. \text{ At that time tension on the string}$$

$T = f_{2k} = 15 \text{ N}$. But when M is stationary, maximum tension

$$T_{\max} = f_{2s} = 20 \text{ N}$$

195.



$$f_s = \mu_s Mg \cos \theta \text{ [static friction]}$$

The minimum force F required for the mass to move:

$$F_{\min} = Mg \sin \theta + (\mu_s Mg \cos \theta) = 100 \times 10 \times 0.6 + \mu_s \times 100 \times 10 \times 0.8$$

$$840 = 600 + 80 \mu_s \Rightarrow \mu_s = \frac{240}{800} = 0.3$$

When it starts moving, dynamic friction comes into play, so that

$$F_{\min} - Mg \sin \theta - f_k = Ma$$

$$\Rightarrow 840 - 600 - \mu_k 800 = 100 a \quad \text{--- (1)}$$

$$v_{\text{avg}} = 1.5 \text{ m s}^{-1}, S = 7.5 \text{ m} \Rightarrow t = \frac{7.5}{1.5} = 5 \text{ s}$$

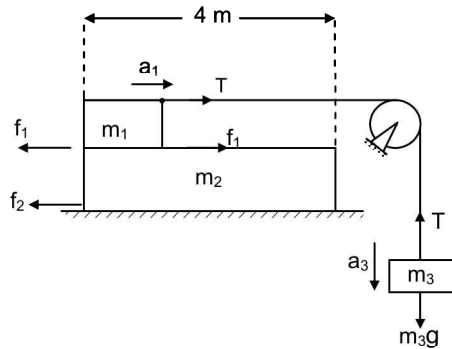
$$S = \frac{1}{2} at^2 \Rightarrow 7.5 = \frac{1}{2} \times a \times 25 \Rightarrow a = \frac{15}{25}$$

$$= 0.6 \text{ m s}^{-2}$$

substitute in equation (1) $\Rightarrow \mu_k$

$$= \frac{240 - 60}{800} = 0.225$$

196.



m_1 and m_3 will have same acceleration, say $a_1 \text{ m s}^{-2}$

$$m_1 = m_2 = m_3 = 10 \text{ kg (data)}$$

$$m_3 g - T = m_3 a_1 \Rightarrow 10 g - T = 10 a_1 \Rightarrow T$$

$$= 100 - 10 a_1 \quad \text{--- (1)}$$

$$T - f_1 = m_1 a_1 \Rightarrow f_1 = \mu_1 m_1 g = 0.4 \times 10 \times 10$$

$$= 40 \text{ N}$$

$$\text{Hence } 100 - 10 a_1 - 40 = 10 a_1 \Rightarrow a_1 = 3 \text{ m s}^{-2}$$

$$\text{From equation (1)} \rightarrow T = 70 \text{ N}$$

$$\frac{f_1 - f_2}{m_2} = a_2 \Rightarrow f_2 = \mu_2 (m_1 + m_2) g$$

$$= 0.1 \times 20 \times 10 = 20 \text{ N}$$

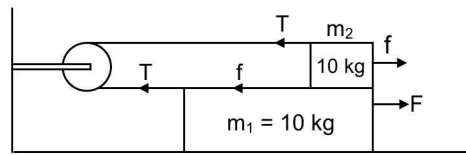
$$\therefore a_2 = \frac{40 - 20}{10} = 2 \text{ m s}^{-2}$$

$$a_{\text{rel}} = a_1 - a_2 = (3 - 2) = 1 \text{ m s}^{-2} \Rightarrow S$$

$$= \frac{1}{2} a_r \cdot t^2$$

$$t^2 = \frac{2 \times 4}{1} = 8 \Rightarrow t = 2\sqrt{2} \text{ s}$$

197.



The static friction developed between m_1 & m_2 is f_s
 $= \mu_s m_2 g = 0.6 \times 10 \times 10 = 60 \text{ N}$

The forces acting are as shown. At the limiting condition.

$$\text{For } m_2 : T = f_s, \text{ For } m_1 : T + f_s = F_{\max} = F'$$

$$F' = 2f_s = 2 \times 60 = 120 \text{ N} \Rightarrow \text{above } 120 \text{ N, they start moving and dynamic friction only acts.}$$

$$f_k = \mu_k m_2 g = 0.4 \times 10 \times 10 = 40 \text{ N}$$

Now they move in opposite direction with same acceleration, because of the string attached.

$$\text{For } m_2 : T - f_k = m_2 a = T - 40 = 10 a$$

$$\text{For } m_1 : F - T - f_k = m_1 a \Rightarrow 120 - T - 40 = 10 a$$

$$\text{Add : } 120 - 80 = 20 a \Rightarrow a = \frac{40}{20} = 2 \text{ m s}^{-2}$$

$a_r = 2a = 4 \text{ m s}^{-2}$. This is the minimum relative acceleration with which they can move. From equation for m_2 , tension $T = 60 \text{ N}$ [i.e., the maximum value in static condition]

198.

- (a) (q)
- (b) (p), (s)
- (c) (p), (r), (s)
- (d) (s)

$$199. \text{ (a) } \frac{d\bar{p}}{dt} = \text{constant} \Rightarrow \frac{d\bar{p}}{dt} = 0 \text{ or } > 0$$

$$\text{If } \frac{d\bar{p}}{dt} = 0, \bar{a} = 0 \Rightarrow \bar{V} = 0 \text{ or } \bar{V} = \text{constant} > 0$$

$$\frac{d\bar{p}}{dt} > 0 \Rightarrow \bar{a} = \text{constant} \Rightarrow \text{block is moving}$$

(p), (q) (r), (s) are possible

$$\text{(b) } \frac{d\bar{p}}{dt} = \text{constant} > 0 \Rightarrow \bar{a} = \text{constant} > 0$$

$\Rightarrow$ block is moving

$\therefore$ (r)

$$\text{(c) } \frac{d\bar{p}}{dt} = 0 \Rightarrow \text{same as in (a)}$$

$\Rightarrow$ (p), (q), (r), (s)

$$\text{(d) } \bar{p} = 0 \Rightarrow \text{block is at rest}$$

$\Rightarrow$ (p), (q), (s)

4.126 Laws of Motion

200. T_B = Centripetal force on B

$$= mR\omega^2 = (\because R = 2 \text{ metre})$$

$$= 2m\omega^2$$

$T_A = T_B$ + centripetal force on A

$$= 2m\omega^2 + m \times 1 \times \omega^2$$

$$= 3m\omega^2 \Rightarrow \frac{T_B}{T_A} = \frac{2}{3} (< 1)$$

$$v_A = r\omega = 1 \times \omega = \omega$$

$$v_B = R\omega = 2 \times \omega = 2\omega$$

$$\frac{v_A}{v_B} = \frac{1}{2} (< 1)$$

$$\frac{a_A}{a_B} = \frac{r\omega^2}{R\omega^2} = \frac{1}{2} (< 1)$$

$$\frac{\text{centrifugal force on B}}{\text{centrifugal force on A}} = \frac{mR\omega^2}{mr\omega^2} = \frac{2}{1}$$

(> 1)

(a) $\rightarrow$ q, r

(b) $\rightarrow$ q, s

(c) $\rightarrow$ q, r

(d) $\rightarrow$ p