Class 11

2017-18

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8. SIMPLE HARMONIC MOTION AND ELASTICITY

SIMPLE HARMONIC MOTION

1. INTRODUCTION

There are so many examples of oscillatory or vibrational motion in our world. E.g. the vibrations of strings in a guitar or a sitar, the vibrations in the speakers of a music system, the to and fro motion of a pendulum, vibration in a suspension bridge as a vehicle passes on it, the oscillations in a tall building during an earthquake etc. Simple harmonic motion (SHM) is a type of oscillatory or vibrational motion. Every kind of oscillation or vibration of a particle or a system is not necessarily simple harmonic. The particle executing SHM like any other oscillatory motion has a variable acceleration, but this variation is different in different kinds of oscillations. The study of SHM is very useful and forms an important tool in understanding the characteristics of sound and light waves and alternating currents. Any oscillatory motion which is not simple harmonic can be expressed as a superposition of several simple harmonic motions of different frequencies.

2. PERIODIC AND OSCILLATORY MOTION

Periodic Motion: A motion which repeats itself after equal intervals of time is called periodic motion.

Oscillatory Motion: A body is said to possess oscillatory or vibratory motion if it moves back and forth repeatedly about a mean position. For an oscillatory motion, a restoring force is required.

Examples of Periodic and Oscillatory motion are revolution of earth around sun and motion of bob of a simple pendulum respectively.

PLANCESS CONCEPTS

All Oscillatory motions are periodic but all Periodic motions need not be oscillatory.

A body experiencing force $F = -k(x-a)^n$ is in Oscillatory motion only if n is odd and its mean position is x=a. As, if n is odd only then we would have restoring force.

Vaibhav Krishan (JEE 2009, AIR 22)

2.1 Periodic Functions

A function is said to be periodic if it repeats itself after time period T i.e. the same function is obtained when the variable t is changed to t + T. Consider the following periodic functions:

$$f(t) = \sin\frac{2\pi}{T}t \quad \text{and} \quad g(t) = \cos\frac{2\pi}{T}t$$

Here T is the time period of the periodic motion. We shall see that if the variable t is changed to t + T, the same function results.

$$f(t+T) = \sin\left[\frac{2\pi}{T}(t+T)\right] = \sin\left[\frac{2\pi t}{T} + 2\pi\right] = \sin\left(\frac{2\pi t}{T}\right) \qquad \therefore\ f(t+T) = f(t)$$

Similarly, $g(t+T) = g(t)$

It can be easily verified that: $f(t+nT) = f(t)$ and $g(t+nT) = g(t)$

where n = 1,2,3,........

PLANCESS CONCEPTS

These functions could be used to represent periodic motion i.e. Periodic functions represent periodic motion

T is the period of the above function.

To find periodicity of summation of two or more periodic functions the periodicity would be the L.C.M of the periodicities of the each function

Vaibhav Gupta (JEE 2009, AIR 54)

Illustration 1: Find the period of the function, $y = \sin\omega t + \sin 2\omega t + \sin 3\omega t$ **(JEE MAIN)**

Sol: The function with least angular frequency will have highest time period.

The given function can be written as, $y = y_1 + y_2 + y_3$

Here $y_1 = \sin\omega t$, $T_1 = \frac{2\pi}{\omega}$ $y_2 = \sin 2\omega t$, $T_2 = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$, and $y_3 = \sin 3\omega t$

$T_3 = \frac{2\pi}{3\omega}$ $\therefore T_1 = 2T_2$ and $T_1 = 3T_3$

So, the time period of the given function is T_1 or $\frac{2\pi}{\omega}$.

Because in time $T = \frac{2\pi}{\omega}$, first function completes one oscillation, the second function two oscillations and the third, three.

3. SIMPLE HARMONIC MOTION

Simple Harmonic Motion is a periodic motion in which a body moves to and fro about its mean position such that its restoring force or its acceleration is directly proportional to the displacement from its mean position and is directed towards its mean position. It can be expressed mathematically as, $F = m\frac{d^2x}{dt^2} = -kx$. Where m is the mass on which a restoring force F acts to impart an acceleration $\frac{d^2x}{dt^2}$ along x-axis such that the restoring force F or acceleration is directly proportional to the displacement x along x-axis and k is a constant. The negative sign shows that the restoring force or acceleration is directed towards the mean position.

The differential equation of a simple harmonic motion is given by, $\frac{d^2x}{dt^2}+\left(\frac{K}{m}\right)x=0$ or $\frac{d^2x}{dt^2}+\omega^2x=0$

Where $\omega=\sqrt{\frac{K}{m}}=\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$

The time period T, to complete one complete cycle by a body undergoing simple harmonic motion is given by

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{\text{acceleration}}{\text{displacement}}}=2\pi\sqrt{\frac{m}{K}}$$

3.1 Types of SHM

Two Types of Simple Harmonic Motion

(a) Linear SHM (b) Angular SHM

Important among all oscillatory motion is the simple harmonic motion. A particle executing linear simple harmonic motion oscillates in straight line periodically in such a way that the acceleration is proportional to its displacement from a fixed point (called equilibrium), and is always directed towards that point.

If a body describes rotational motion in such a way that the direction of its angular velocity changes periodically and the torque acting on is always directed opposite to the angular displacement and magnitude of the torque is directly proportional to the angular displacement, then its motion is called angular SHM.

4. REPRESENTATION OF SIMPLE HARMONIC MOTION

If a point mass m is moving with uniform speed along a circular path of radius a, it's projection on the diameter of the circle along y-axis represents its simple harmonic motion (see Fig. 8.1). $y=a\sin\omega t$

Where ω is the uniform angular velocity of the body of mass m along a circular path of radius a than ωt is angle covered by the radius in time t from the initial position A at $t=0$ to the position B. As ∠AOB = ∠OBC = ωt, the foot of perpendicular from B to the diameter YOY′ gives the projection at the point C such that $y=OC$ is the projection of this body on the diameter and represents the displacement of the body executing SHM along y-axis. If the body does not start its motion from the point A but at a point A′ so that ∠AOA′ is the phase angle ϕ,

then $y=a\sin(\omega t\pm\phi)$ ... (i)

Where ϕ is the phase angle which may be positive or negative. The phase angle represents the fraction of the angle by which the motion of the body is out of step between the initial position of the body and the mean position of simple harmonic motion. The phase difference is the fraction of angle 2π or time period T of SHM by which the body is out of step initially from the mean position of the body.

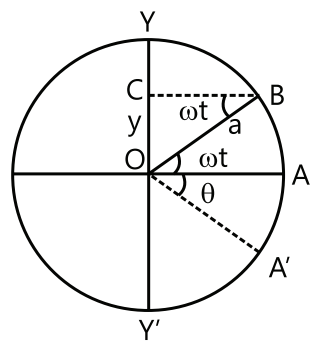


Figure 8.1: Particle moving in a circle with angular speed ω in X-Y plane

Differentiating equation (i), $\frac{dy}{dt}=v=a\omega\cos(\omega t\pm\phi)$

As $\sin(\omega t\pm\phi)=\frac{y}{a},\cos(\omega t\pm\phi)=\sqrt{1-\sin^2(\omega t\pm\phi)}=\sqrt{1-\frac{y^2}{a^2}}=\frac{\sqrt{a^2-y^2}}{a}$

$\therefore v=\omega\sqrt{a^2-y^2}$ ... (ii)

Differentiating (ii), the acceleration = $\frac{d^2y}{dt^2}=-a\omega^2\sin(\omega t\pm\phi)$

$\therefore \frac{d^2y}{dt^2}=-\omega^2y$

It represents the equation of simple harmonic motion where $\omega = \sqrt{\left(\frac{d^2y}{dt^2}\Big/y\right)} = \sqrt{\frac{\text{acceleration}}{\text{displacement}}}$

Time period, $T = 2\pi/\omega = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

4.1 Alternative Method for Finding Velocity and Acceleration in SHM

Let v be the velocity of the reference particle at P. Resolve velocity V into two rectangular components $V\cos\theta$ parallel to YOY′ and $V\sin\theta$ perpendicular to YOY′ (see Fig. 8.2). The velocity v of the projection N is clearly $V\cos\theta$.

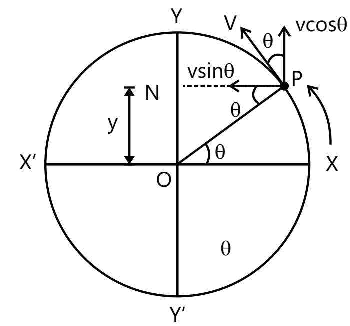


Figure 8.2: Relation between v and ω.

$\therefore\ v = V\cos\theta = A\omega\cos\omega t$ or $v = A\omega\sqrt{1-\sin^2\omega t}$

or $v = A\omega\sqrt{1-\frac{y^2}{A^2}}$ or $v = \cancel{A}\omega\sqrt{\frac{A^2-y^2}{\cancel{A^2}}}$ or $v = \omega\sqrt{A^2-y^2}$

The centripetal acceleration $\frac{V^2}{A}$ of the particle at P can be resolved into two rectangular components $-\frac{V^2}{A}\cos\theta$ Perpendicular to YOY′ and $\frac{V^2}{A}\sin\theta$ anti-parallel to YOY′ Acceleration of $N = -\frac{V^2}{A}\sin\theta$

or $\text{Acceleration} = -\frac{V^2}{A^2}(A\sin\theta) = -\omega^2(A\sin\omega t)$

or $\text{Acceleration} = -\omega^2 y$

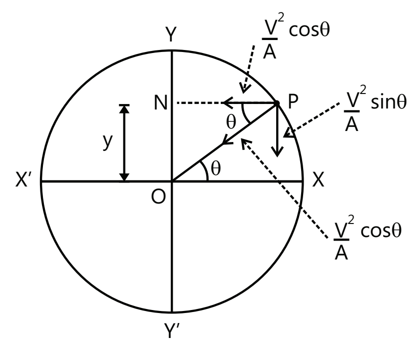


Figure 8.3: Direction of centripetal acceleration of particle

4.2 Time Period or Periodic Time of SHM

It is the smallest interval of time at which the details of motion repeat. It is generally represented by T.

$$'x'\text{ at }(t+T) = A\cos\left(2\pi\frac{t+T}{T}+\phi_0\right) = A\cos\left(\frac{2\pi t}{T}+2\pi+\phi_0\right) \quad \text{... (i)}$$

It is clear from here that the details of motion repeat after time T. Time period may also be defined as the time taken by the oscillating particle to complete one oscillation. It is equal to the time taken by the reference particle to complete one revolution. In one revolution, the angle traversed by the reference particle is 2π radian and T is the time taken. If ω be the uniform angular velocity of the reference particle, then $\omega = \frac{2\pi}{T}$ or $T = \frac{2\pi}{\omega}$

4.3 Frequency

It is the number of oscillations (or vibrations) completed per unit time. It is denoted by f. In time T second, one vibration is completed.

In 1 second, $\frac{1}{T}$ vibrations are completed or $f = \frac{1}{T}$ or $fT = 1$

Also, $\omega = \frac{2\pi}{T} = 2\pi\times\frac{1}{T} = 2\pi f$ So, equation (i) may also be written as under

$$x = A\cos(2\pi ft + \phi_0) \quad \text{... (ii)}$$

The unit of f is s^{-1} or hertz or 'cycles per second' (cps). $\therefore\ \phi s^{-1} = \phi Hz = \phi cps$.

4.4 Angular Frequency

It is frequency f multiplied by a numerical quantity 2π. It is denoted by ω so that $\omega = 2\pi f = \frac{2\pi}{T}$. Equation (vi) may be written as $x = A\cos(\omega t + \phi)$

4.5 Phase

Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

The argument of the cosine in equation $x = A\cos(\omega t + \phi_0)$ gives the phase of oscillation at time t.

It is denoted by ϕ. $\therefore\ \phi = 2\pi\frac{t}{T} + \phi_0$ or $\phi = \omega t + \phi_0$

It is clear that phase ϕ is a function of time t. The phase of a vibrating particle can be expressed in terms of fraction of the time period that has elapsed since the vibrating particle left its initial position in the positive direction. Again, $\phi - \phi_0 = \omega t = \frac{2\pi t}{T}$. So, the phase change in time t is $\frac{2\pi t}{T}$. The phase change in T second will be 2π which actually means a 'no change in phase'. Thus, time period may also be defined as the time interval during which the phase of the vibrating particle changes by 2π.

PLANCESS CONCEPTS

The phase difference between acceleration and displacement is 180°. In SHM phase difference between velocity and acceleration is $\pi/2$ and velocity and displacement is $\pi/2$.

Fig. 8.4 (a) displacement, (b) velocity and (c) acceleration vs. time in SHM.

$v = \pm\omega\sqrt{A^2 - y^2}$. Graphical variation of v with y is an ellipse.

Max velocity at $y = 0$ i.e. at mean position and

$V_{max} = A\omega$; $a = -\omega^2 y$

Graph between acceleration and displacement of a particle executing SHM is straight line.

Max acceleration at $y = A$ i.e. at extreme position and $a_{max} = A\omega^2$

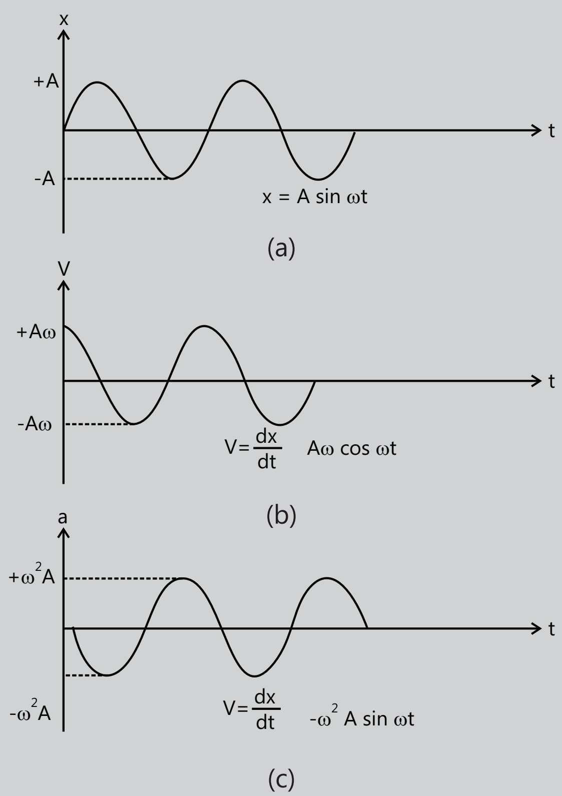


Figure 8.4

Nivvedan (JEE 2009, AIR 113)

Illustration 2: A particle executes simple harmonic motion about the point $x = 0$. At time $t = 0$ it has displacement $x = 2$ cm and zero velocity. If the frequency of motion is $0.25\, s^{-1}$, find (a) the period, (b) angular frequency, (c) the amplitude, (d) maximum speed, (e) the displacement at $t = 3s$ and (f) the velocity at $t = 3s$. **(JEE MAIN)**

Sol: The standard equation for displacement in SHM is $x = A\sin(\omega t + \phi)$. When velocity is zero, the particle is at maximum displacement.

(a) Period $T = \frac{1}{f} = \frac{1}{0.25 s^{-1}} = 4s$

(b) Angular frequency $\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2} rad/s = 1.57 rad/s$

(c) Amplitude is the maximum displacement from mean position. Hence, $A = 2 - 0 = 2cm$

(d) Maximum speed $v_{max} = A\omega = 2.\frac{\pi}{2} = \pi cm/s = 3.14\ cm/s$

(e) The displacement is given by $x = A\sin(\omega t + \phi)$

Initially at t=0; $x = 2cm$, then $2 = 2\sin\phi$ or $\sin\phi = 1 = \sin 90^\circ$ or $\phi = 90^\circ$

Now, at $t = 3s$ $x = 2\sin\left(\frac{\pi}{2} \times 3 + \frac{\pi}{2}\right) = 0$

(f) Velocity at $x = 0$ is v_{max} i.e.,3.14 cm/s.

Illustration 3: Two particles move parallel to x-axis about the origin with the same amplitude and frequency. At a certain instant, they are found at distance $\frac{A}{3}$ from the origin on opposite sides but their velocities are found to be in the same direction. What is the phase difference between the two? **(JEE ADVANCED)**

Sol: The standard equation for displacement in SHM is $x = A\sin(\omega t + \phi)$. Displacement on opposite sides of the mean position has opposite signs. Equation for velocity is $v = A\omega\cos(\omega t + \phi)$. Velocities in same direction have same sign.

Let equations of two SHM be $x_1 = A\sin\omega t$ ... (i)

$x_2 = A\sin(\omega t + \phi)$ ... (ii)

Give that $\frac{A}{3} = A\sin\omega t$ and $-\frac{A}{3} = A\sin(\omega t + \phi)$ Which gives $\sin\omega t = \frac{1}{3}$ ... (iii)

$\sin(\omega t + \phi) = -\frac{1}{3}$ ... (iv)

From Eq.(iv), $\sin\omega t\cos\phi + \cos\omega t\sin\phi = -\frac{1}{3}$; $\frac{1}{3}\cos\phi + \sqrt{1 - \frac{1}{9}}\sin\phi = -\frac{1}{3}$

Solving this equation, we get or $\cos\phi = -1, \frac{7}{9}$; $\phi = \pi$ or $\cos^{-1}\left(\frac{7}{9}\right)$

Differentiating Eqs. (i) and (ii), we obtain; $v_1 = A\omega\cos\omega t$ and $v_2 = A\omega\cos(\omega t + \phi)$

If we put $\phi = \pi$, we find v_1 and v_2 are of opposite signs. Hence, $\phi = \pi$ is not acceptable.

$\phi = \cos^{-1}\left(\frac{7}{9}\right)$

Illustration 4: With the assumption of no slipping, determine the mass m of the block which must be placed on the top of a 6 kg cart in order that the system period is 0.75s. What is the minimum coefficient of static fraction μ_s for which the block will not slip relative to the cart if the cart is displaced 50mm from the equilibrium position and released? Take $\left(g = 9.8\,m/s^2\right)$. **(JEE ADVANCED)**

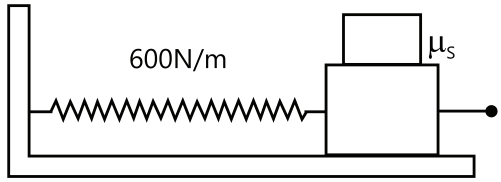


Figure 8.5

Sol: $\omega = \sqrt{\dfrac{k}{M}}$ where M is the total mass attached to the spring. The maximum restoring force on the blocks will be at the extreme position. The limiting friction on mass m should be greater than or equal to the maximum restoring force required for mass m.

(a) $T = 2\pi\sqrt{\dfrac{m+6}{600}} \quad \left(\because T = 2\pi\sqrt{\dfrac{m}{k}}\right) \quad \therefore \quad 0.75 = 2\pi\sqrt{\dfrac{m+6}{600}}\ ;\ m = \dfrac{(0.75)^2 \times 600}{(2\pi)^2} - 6 = 2.55\text{kg}$

(b) Maximum acceleration of SHM is $a_{max} = \omega^2 A \quad (A = \text{amplitude})$

i.e., maximum force on mass 'm' is $m\omega^2 A$ which is being provided by the force of friction between the mass and the cart. Therefore, $\mu_s mg \ge m\omega^2 A$ or $\mu_s \ge \dfrac{\omega^2 A}{g}$ or $\mu_s \ge \left(\dfrac{2\pi}{T}\right)^2 \dfrac{A}{g}$ or $\mu_s \ge \left(\dfrac{2\pi}{0.75}\right)^2 \left(\dfrac{0.05}{9.8}\right)$ $(A = 50\text{mm})$

or $\mu_s \ge 0.358$. Thus, the minimum value of μ_s should be 0.358.

5. ENERGY IN SHM

The displacement and the velocity of a particle executing a simple harmonic motion are given by

$x = A\sin(\omega t + \delta)$ and $v = A\omega\cos(\omega t + \delta)$. The potential energy at time t is, therefore,

$U = \dfrac{1}{2}kx^2$ and $k = m\omega^2$ Therefore $U = \dfrac{1}{2}m\omega^2 x^2 = \dfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \delta)$, and the kinetic energy at time t is $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}mA^2\omega^2\cos^2(\omega t + \delta)$

The total mechanical energy time t is $E = U + K$

$= \dfrac{1}{2}m\omega^2 A^2\left[\sin^2(\omega t + \delta) + \left(\cos^2(\omega t + \delta)\right)\right] = \dfrac{1}{2}m\omega^2 A^2$ Average value of P.E. and K.E

By equation (i) P.E. at distance x is given by

$U = \dfrac{1}{2}m\omega^2 x^2 = \dfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$ $\left\{\text{since at time t}, x = A\sin(\omega t + \phi)\right\}$

The average value of P.E. of complete vibration is given by

$$U_{average} = \frac{1}{T}\int_0^T U dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) = \frac{m\omega^2 A^2}{4T}\int_0^T 2\sin^2(\omega t + \phi)dt = \frac{1}{4}m\omega^2 A^2$$

Because the average value of sine square or cosine square function for the complete cycle is 0.

Now, KE at x is given by $K.E. = \dfrac{1}{2}m\left(\dfrac{dx}{dt}\right)^2 = \dfrac{1}{2}m\left[\dfrac{d}{dt}\left\{A\sin(\omega t + \phi)\right\}\right]^2 = \dfrac{1}{2}m\omega^2 A^2\cos^2(\omega t + \phi)$

The average value of K.E. for complete cycle $K.E._{average} = \frac{1}{T}\int_0^T \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi)dt$

$$= \frac{m\omega^2 A^2}{4T}\int_0^T \{1 + \cos 2(\omega t + \phi)\} dt = \frac{m\omega^2 A^2}{4T}.\ T = \frac{1}{4} m\omega^2 A^2$$

PLANCESS CONCEPTS

Thus average values of K.E. and P.E. of harmonic oscillator are equal to half of the total energy.

The total mechanical energy is constant but the kinetic energy and potential energy of the particle are oscillating

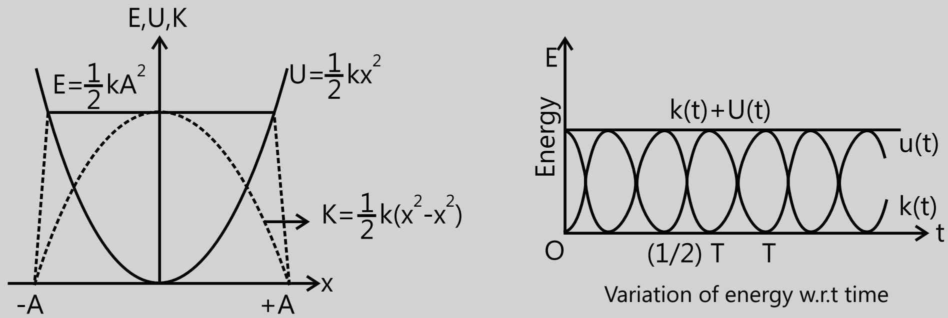


Figure 8.6

Graph for Energy of SHM: Figure 8.6 shows the variation of total energy (E), Potential energy (U) and kinetic energy (K) with Displacement (x).

Chinmay S Purandare (JEE 2012, AIR 698)

At a glance

S.No.	Name of the equation	Expression of the Equation	Remarks
1.	Displacement-time	$x = A\cos(\omega t + \phi)$	X varies between +A and –A
2.	Velocity-time $\left(V = \frac{dx}{dt}\right)$	$v = -A\omega \sin(\omega t + \phi)$	v varies between $+A\omega$ and $-A\omega$
3.	Acceleration-time $\left(a = \frac{dv}{dt}\right)$	$a = -A\omega^2 \cos(\omega t + \phi)$	a varies between $+A\omega^2$ and $-A\omega^2$
4.	Kinetic energy-time $\left(K = \frac{1}{2}mv^2\right)$	$K = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi)$	K varies between 0 and $\frac{1}{2}mA^2\omega^2$
5.	Potential energy-time $\left(U = \frac{1}{2}m\omega^2 x^2\right)$	$K = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$	U varies between $\frac{1}{2}mA^2\omega^2$ and 0
6.	Total energy-time $(E = K + U)$	$E = \frac{1}{2}m\omega^2 A^2$	E is constant

S.No.	Name of the equation	Expression of the Equation	Remarks
7.	Velocity-displacement	$v = \omega\sqrt{A^2 - X^2}$	$v = 0$ at $x = \pm A$ and at $x = 0$ $v = \pm A\omega$
8.	Acceleration-displacement	$a = -\omega^2 x$	$a = 0$ at $x = 0$ $a = \pm\omega^2 A$ at $x = \mp A$
9.	Kinetic energy-displacement	$K = \frac{1}{2}m\omega^2\left(A^2 - X^2\right)$	$K = 0$ at $x = \mp A$ $K = \frac{1}{2}m\omega^2 A^2$ at $x = 0$
10.	Potential energy-displacement	$U = \frac{1}{2}m\omega^2 x^2$	$U = 0$ at $x = 0$ $U = \frac{1}{2}m\omega^2 A^2$ at $x = \pm A$
11.	Total energy-displacement	$E = \frac{1}{2}m\omega^2 A^2$	E is constant

PLANCESS CONCEPTS

At mean position $\rightarrow$ K is the maximum and U is the minimum (it may be zero also, but it is not necessarily zero). At extreme positions $\rightarrow$ K is zero and U is the maximum.

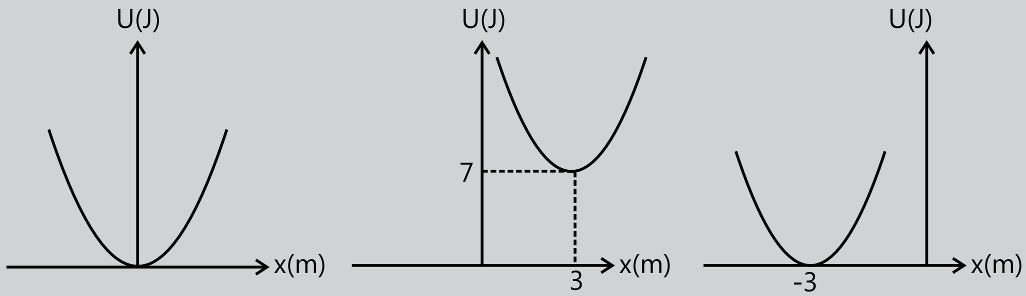


Figure 8.7

B Rajiv Reddy (JEE 2012, AIR 11)

Illustration 5: The potential energy of a particle oscillating on x-axis is given as $U = 20 + \left(x - 2\right)^2$

Here, U is in joules and x in meters. Total mechanical energy of the particle is 36J. **(JEE MAIN)**

(a) State whether the motion of the particle is simple harmonic or not.

(b) Find the mean position.

(c) Find the maximum kinetic energy of the particle.

Sol: At the mean position the kinetic energy is the maximum and potential energy is the minimum. The sum of kinetic energy and potential energy is constant throughout the SHM, equal to the total mechanical energy.

(a) $F = -\frac{dU}{dx} = -2\left(x - 2\right)$

By assuming $x-2=X$, we have $F=-2x$, Since, $F\propto -X$

The motion of the particle is simple harmonic

(b) The mean position of the particle is $X=0$ or $x-2=0$, which gives $x=2\,m$

(c) Maximum kinetic energy of the particle is, $K_{max}=E-U_{min}=36-20=16J$

Note U_{min} is 20J at mean position or at $x=2m$.

Illustration 6: A block with mass M attached to a horizontal spring with force constant k is moving with simple harmonic motion having amplitude A_1. At the instant when the block passes through its equilibrium position a lump of putty with mass m is dropped vertically on the block from a very small height and sticks to it.

(JEE ADVANCED)

(a) Find the new amplitude and period.

(b) Repeat part (a) for the case in which the putty is dropped on the block when it is at one end of its path.

Sol: Sticking of putty constitutes an inelastic collision. Kinetic energy at equilibrium position converts into potential energy at extreme position, $\frac{1}{2}mv^2=\frac{1}{2}kA^2$.

(a) Before the lump of putty is dropped the total mechanical energy of the block and spring is $E_1=\frac{1}{2}kA_1^2$. Since, the block is at the equilibrium position, $U=0$, and the energy is purely kinetic. Let v_1 be the speed of the block at the equilibrium position, we have $v_1=\sqrt{\frac{k}{M}}A_1$

During the process momentum of the system in horizontal direction is conserved. Let v_2 be the speed of the combined mass, then $(M+m)v_2=Mv_1$; $v_2=\frac{M}{M+m}v_1$

Now, let A_2 be the amplitude afterwards. Then, $E_2=\frac{1}{2}kA_2^2=\frac{1}{2}(M+m)v_2^2$

Substituting the proper values, we have $A_2=A_1\sqrt{\frac{M}{M+m}}$

Note: $E_2<E_1$, as some energy is lost into heating up the block and putty. Further, $T_2=2\pi\sqrt{\frac{M+m}{k}}$

(b) When the putty drops on the block, the block is instantaneously at rest. All the mechanical energy is stored in the spring as potential energy. Again the momentum in horizontal direction is conserved during the process, but now it is zero just before and after putty is dropped. So, in this case, adding the extra mass of the putty has no effect on the mechanical energy, i.e.,

$E_2=E_1=\frac{1}{2}kA_1^2$ and the amplitude is still A_1. Thus, $A_2=A_1$ and $T_2=2\pi\sqrt{\frac{M+m}{k}}$

6. ANGULAR SIMPLE HARMONIC MOTION

When a particle executes SHM on a curve path, then it is said to be angular SHM. E.g. - Simple pendulum. In this case, to find out the time period, we find out restoring torque and hence angular acceleration.

i.e. $\tau=-k\theta$ Where k is a constant $\Rightarrow$ $I\alpha=-k\theta$ where I is moment of inertia ... (i)

$\Rightarrow \alpha=\frac{-k}{I}\theta$ ... (ii)

Also, the equation of SHM for angular SHM, is $\alpha=-\omega^2\theta$. Comparing (i) and (ii), we get ω, hence the time period.

Problem solving strategy

Step 1: Find the stable equilibrium position which usually is also known as the mean position. Net force or torque on the particle in this position is zero. Potential energy is the minimum.

Step 2: Displace the particle from its mean position by a small displacement x (in case of a linear SHM) or θ (in case of an angular SHM).

Step 3: Find net force or torque in this displaced position.

Step 4: Show that this force or torque has a tendency to bring the particle back to its mean position and magnitude of force or torque is proportional to displacement, i.e.,

$F \propto -x$ or $F = -kx$ ...(i); $\tau \propto -\theta$ or $\tau = -k\theta$ ...(ii)

This force or torque is also known as restoring force or restoring torque.

Step 5: Find linear acceleration by dividing Eq.(i) by mass m or angular acceleration by dividing Eq.(ii) by moment of inertia I.

Hence, $a = -\frac{k}{m}.x = -\omega^2 x$ or $\alpha = -\frac{k}{I}\theta = -\omega^2\theta$

Step 6: Finally, $\omega = \sqrt{\left|\frac{a}{x}\right|}$ or $\sqrt{\left|\frac{\alpha}{\theta}\right|}$ or $\frac{2\pi}{T} = \sqrt{\left|\frac{a}{x}\right|}$ or $\sqrt{\left|\frac{\alpha}{\theta}\right|}$

$\therefore\ T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$ or $2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|}$

Energy Method: Repeat step 1 and step 2 as in method 1. Find the total mechanical energy (E) in the displaced position. Since, mechanical energy in SHM remains constant. $\frac{dE}{dt} = 0$ By differentiating the energy equation with respect to time and substituting $\frac{dx}{dt} = v$, $\frac{d\theta}{dt} = \omega$, $\frac{dv}{dt} = a$, and $\frac{d\omega}{dt} = \alpha$ we come to step 5. The remaining procedure is same.

Note: (i) E usually consists of following terms:

(a) Gravitational PE (b) Elastic PE (c) Electrostatic PE (d) Rotational KE and (e) Translational KE

(ii) For gravitational PE, choose the reference point (h=0) at mean position.

Illustration 7: Calculate the angular frequency of the system shown in Fig 8.8. Friction is absent everywhere and the threads, spring and pulleys are massless. Given, that $m_A = m_B = m$. **(JEE ADVANCED)**

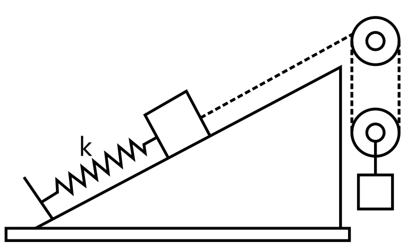


Figure 8. 8

Sol: This problem can be solved either by restoring force method or by the energy method. The gain in kinetic energy is at the cost of decrease in gravitational and/or elastic potential energy.

Let x_0 be the extension in the spring in equilibrium. Then equilibrium of A and B give $T = kx_0 + mg\sin\theta$ ... (i)

and $2T = mg$ ... (ii)

Here, T is the tension in the string. Now, suppose A is further displaced by a distance x from its mean position and v be its speed at this moment. Then B lowers by $\frac{x}{2}$ and speed of B at this instant will be $\frac{v}{2}$. Total energy of the system in this position will be,

$$E=\frac{1}{2}k\left(x+x_0\right)^2+\frac{1}{2}m_A v^2+\frac{1}{2}m_B\left(\frac{V}{2}\right)^2+m_A g h_A-m_B g h_B$$

or $E=\frac{1}{2}k\left(x+x_0\right)^2+\frac{1}{2}mv^2+\frac{1}{8}mv^2+mgx\sin\theta-mg\frac{x}{2}$

or $E=\frac{1}{2}k\left(x+x_0\right)^2+\frac{5}{8}mv^2+mgx\sin\theta-mg\frac{x}{2}$

Since, E is constant, $\frac{dE}{dt}=0$ or $0=k\left(x+x_0\right)\frac{dx}{dt}+\frac{5}{4}mv\left(\frac{dv}{dt}\right)+mg(\sin\theta)\left(\frac{dx}{dt}\right)-\frac{mg}{2}\left(\frac{dx}{dt}\right)$

Substituting, $\frac{dx}{dt}=v$; $\frac{dv}{dt}=a$ and $kx_0+mg\sin\theta=\frac{mg}{2}$ [From Eqs. (i) and (ii)]

We get, $\frac{5}{4}ma=-kx$ Since, $a\propto -x$

Motion is simple harmonic, time period of which is, $T=2\pi\sqrt{\left|\frac{x}{a}\right|}=2\pi\sqrt{\frac{5m}{4k}}$ $\therefore$ $\omega=\frac{2\pi}{T}=\sqrt{\frac{4k}{5m}}$

7. SIMPLE PENDULUM

It is an example of angular simple harmonic motion. Let's calculate its time period. Let us suppose that a bob of mass m is executing SHM (see Fig. 8.9). The length of the pendulum is ℓ, which is the distant between the point of oscillation and the center of mass of the bob. Torque acting on the bob about the point O.

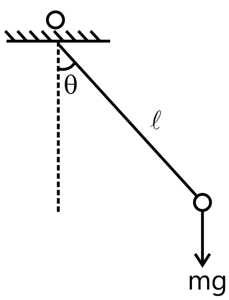


Figure 8.9: Oscillations of simple Pendulum

$\Gamma=mg\,\ell\sin\theta$ (And for small θ, $\sin\theta\simeq\theta$)

$\Rightarrow\Gamma=mg\ell\theta\Rightarrow I\alpha=-mg\ell\theta\Rightarrow\alpha=-\frac{mg\ell}{I}\theta$

where α is angular acceleration $=-\frac{mg\ell}{m\ell^2}\theta$; $\alpha=-\frac{g}{\ell}\theta$ ... (i)

The equation of SHM is $\alpha=-\omega^2\theta$ ... (ii)

Comparing (i) and (ii), we get $\omega^2=\frac{g}{\ell}$; $\omega=\sqrt{\frac{g}{\ell}}$ $\Rightarrow$ $\frac{2\pi}{T}=\sqrt{\frac{g}{\ell}}$; $T=2\pi\sqrt{\frac{\ell}{g}}$

PLANCESS CONCEPTS

The period is independent of the mass of the suspended particle.

Nitin Chandrol (JEE 2012, AIR 134)

Various scenarios: Time period

Pendulum in a lift descending with acceleration "a", $T=2\pi\sqrt{\frac{\ell}{(g-a)}}$

Pendulum in a lift ascending with acceleration "a", $T=2\pi\sqrt{\frac{\ell}{(g+a)}}$

Pendulum suspended in a train accelerated with ''a'' uniformly in horizontal direction $T=2\pi\sqrt{\frac{\ell}{\left(a^2+g^2\right)^{\frac{1}{2}}}}$

Pendulum suspended in car taking turn with velocity v in a circular path of radius r, $T = 2\pi\sqrt{\dfrac{\ell}{\left(\left(\dfrac{v^2}{r}\right)^2 + g^2\right)^{\frac{1}{2}}}}$

Note: If the pendulum is suspended in vacuum, then the time period of the pendulum decreases.

Illustration 8: A simple pendulum consists of a small sphere of mass m suspended by a thread of length ℓ. The sphere carries a positive charge q. The pendulum is placed in a uniform electric field of strength E directed vertically upwards. With what period will pendulum oscillate if the electrostatic force acting on the sphere is less than the gravitational force? **(JEE MAIN)**

Sol: The electrostatic force is acting opposite to the weight of the block. So the effective value of acceleration due to gravity will be less than the actual value of g.

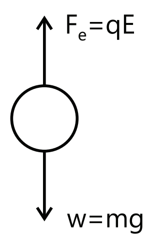


Figure 8.10

The two forces acting on the bob are shown in Fig 8.10.

g_{eff} in this case will be $\dfrac{w - F_e}{m}$ or $g_{eff} = \dfrac{mg - qE}{m} = g - \dfrac{qE}{m}$

$\therefore\ T = 2\pi\sqrt{\dfrac{\ell}{g_{eff}}} = 2\pi\sqrt{\dfrac{\ell}{g - \dfrac{qE}{m}}}$

PLANCESS CONCEPTS

In case of a pendulum clock, time is lost if T increase and gained if T decreases. Time lost or gained in time t is given by.

$\Delta t = \dfrac{\Delta T}{T'} t$ e.g., if $T = 2s, T' = 3s$, then $\Delta T = 1s$

$\therefore$ Time lost by the clock in 1 hr. $\Delta t = \dfrac{1}{3} \times 3600 = 1200s$

Second pendulum is a with its time period precisely 2 seconds

Vaibhav Gupta (JEE 2009, AIR 54)

Illustration 9: A simple pendulum of length l is suspended from the ceiling of a cart which is sliding without friction on an inclined plane of inclination θ. What will be the time period of the pendulum? **(JEE MAIN)**

Sol: The cart accelerates down the plane with acceleration a = g sinθ.

$g_{eff} = |\vec{g} - \vec{a}| = \sqrt{g^2 + 2g^2 \sin\theta \cos(90^o + \theta) + g^2 \sin^2\theta} = g\cos\theta$

Here, point of suspension has acceleration. $\vec{a} = g\sin\theta$ (down the Plane). Further, $\vec{g}$ can be resolved into two components g sin θ (along the plane) and g cos θ (perpendicular to plane)

$\therefore \vec{g}_{eff} = \vec{g} - \vec{a} = g\cos\theta$

(perpendicular to plane)

$\therefore T = 2\pi\sqrt{\dfrac{\ell}{|\vec{g}_{eff}|}} = 2\pi\sqrt{\dfrac{\ell}{g\cos\theta}}$

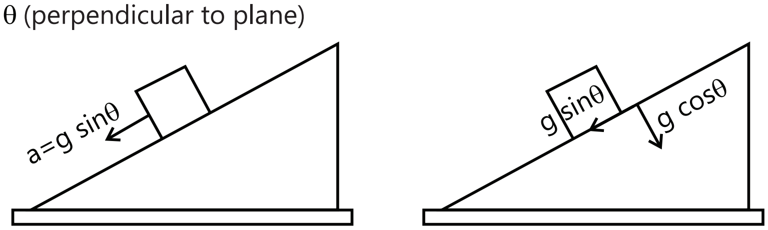


Figure 8.11

Note: If $\theta = 0^\circ, T = 2\pi\sqrt{\frac{\ell}{g}}$ which is quiet obvious.

8. PHYSICAL PENDULUM

Any rigid body mounted so that it can swing in a vertical plane about some axis passing through it is called a physical pendulum (see Fig. 8.12).

A body of irregular shape is pivoted about a horizontal frictionless axis through P and displaced from the equilibrium position by an angle θ. (The equilibrium position is that in which the center of mass C of the body lies vertically below P).

The distance from the pivot to the center of mass is d. The moment of inertia of the body about an axis through the pivot is I and the mass of the body is M. The restoring torque about the point P,

$\tau = Mgd\theta$ (if θ be very small, $\sin\theta = \theta$)

$\tau = Mgd\theta$; $I\alpha = -Mgd\theta$; $\alpha = -\frac{Mgd}{I}\theta$......(i)

Comparing with the equation of SHM

$\omega^2 = \frac{Mgd}{I}$; $\omega = \sqrt{\frac{Mgd}{I}}$; $2\pi / T = \sqrt{\frac{Mgd}{I}}$; $T = 2\pi\sqrt{\frac{I}{Mgd}}$

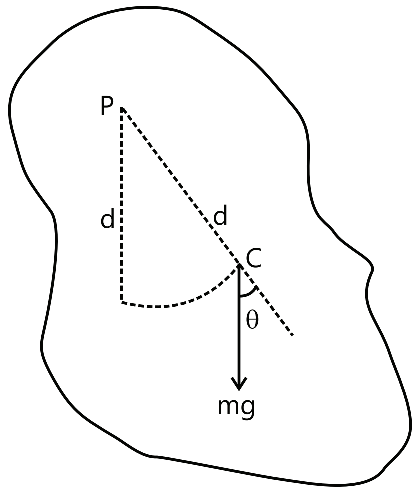


Figure 8.12: Rotation of Physical Pendulum

PLANCESS CONCEPTS

It may be necessary to use parallel axis theorem to find Moment of Inertia about the pivoted axis $I = I_G + ml^2$

Yashwanth Sandupatla (JEE 2012, AIR 821)

9. TORSIONAL PENDULUM

In torsional pendulum, an extended body is suspended by a light thread or a wire (see Fig. 8.13). The body is rotated through an angle about the wire as the axis of rotation. The wire remains vertical during this motion but a twist is produced in the wire. The lower end of the wire is rotated through an angle with the body but the upper end remains fixed with the support. Thus, a twist θ is produced. The twisted wire exerts a restoring torque on the body to bring it back to its original position in which the twist θ in the wire is zero. This torque has a magnitude proportional to the angle of twist which is equal to the angle rotated by the body. The proportionality constant is called the torsional constant of the wire. Thus, if the torsional constant of the wire is κ and the body is rotated through an angle θ, the torque produced is $\Gamma = -\kappa\theta$. If I be the moment of inertia of the body about the vertical axis, the angular acceleration is $\alpha = \frac{\Gamma}{I} = -\frac{\kappa}{I}\theta = -\omega^2\theta$ where $\omega = \sqrt{\frac{\kappa}{I}}$

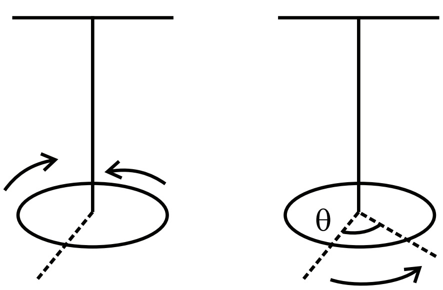


Figure 8.13: Torsional pendulum

Thus, the motion of the body is simple harmonic and the time period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$

Illustration 10: A ring of radius r is suspended from a point on its circumference. Determine its angular frequency of small oscillations. **(JEE ADVANCED)**

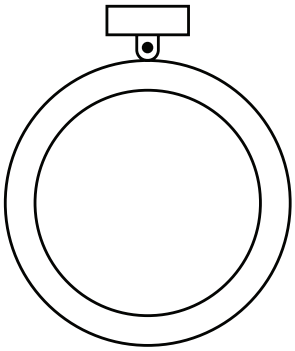


Figure 8.14

Sol: This is an example of a physical pendulum. Find moment of inertia about point of suspension and the distance of the point of suspension from the center of mass.

It is physical pendulum, the time period of which is, $T = 2\pi\sqrt{\frac{I}{mgl}}$

Here, I = moments of inertia of the ring about point of suspension

$= mr^2 + mr^2 = 2mr^2$

and l = distance of point of suspension from centre of mass = r

$$\therefore \quad T = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}; \quad \therefore \text{Angular frequency} \quad \omega = \frac{2\pi}{t} \quad \text{or} \quad \omega = \sqrt{\frac{g}{2r}}$$

Illustration 11: Find the period of small oscillations of a uniform rod with length l, pivoted at one end. **(JEE MAIN)**

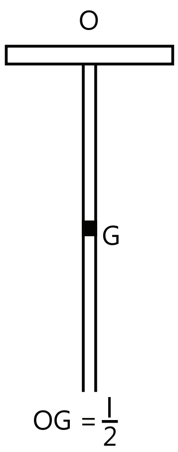


Figure 8.15

Sol: This is an example of a physical pendulum. Find moment of inertia about point of suspension and the distance of the point of suspension from the center of gravity.

$$T = 2\pi\sqrt{\frac{I_\circ}{mg(OG)}} \qquad \text{Here, } I_\circ = \frac{1}{3}ml^2 \quad \text{and } OG = \frac{l}{2}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\left(\frac{1}{3}ml^2\right)}{(m)(g)\left(\frac{l}{2}\right)}} \qquad \text{or } T = 2\pi\sqrt{\frac{2l}{3g}}$$

Illustration 12: A uniform disc of radius 5.0 cm and mass 200 g is fixed at its center to a metal wire, the other end of which is fixed with a clamp. The hanging disc is rotated about the wire through angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire. **(JEE MAIN)**

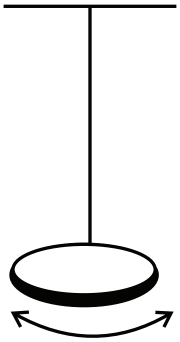


Figure 8.16

Sol: This is an example of a torsional pendulum. Find moment of inertia about the axis passing through the wire.

The Situation is shown in Fig 8.16. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200\text{kg})\left(5.0\times10^{-2}\text{m}\right)^2}{2} = 2.5\times10^{-4}\text{kg.m}^2$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{K}}; \qquad K = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2\left(2.5\times10^{-4}\text{kg}-\text{m}^2\right)}{(0.20\text{s})^2} = 0.25\frac{\text{kg}-\text{m}^2}{\text{s}^2}$$

10. SPRING - MASS SYSTEM

As shown in the Fig. 8.17 a mass m is attached to a massless spring. It is displaced from its mean position to a distance x. The restoring force is given by

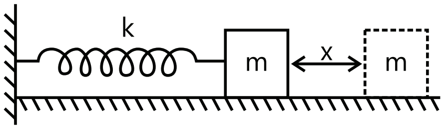


Figure 8.17: Block of mass m attached to spring

$F = -kx$ where k is the force constant.

$\Rightarrow ma = -kx; \quad a = -x\dfrac{k}{m}$ ...(i)

$\Rightarrow a \propto -x$, $\therefore$ Motion is SHM

$\Rightarrow \omega^2 = \dfrac{k}{m}$ or $\omega = \sqrt{\dfrac{k}{m}}$; $T = 2\pi\sqrt{\dfrac{m}{k}}$

10.1 Series and Parallel Combination of Springs

10.1.1. Serial Combination of Springs

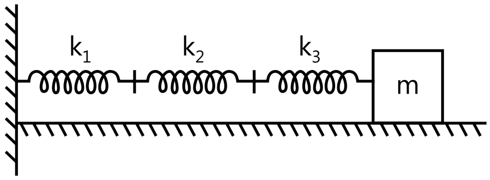


Figure 8.18: Series combination of springs

If springs are connected in series, having force constants k_1, k_2, k_3 then the equivalent force

constant is $\dfrac{1}{k_{eff}} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{k_3}$

10.1.2 Parallel Combination of Springs

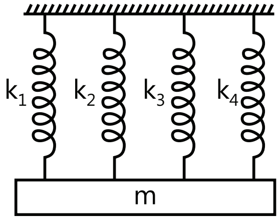


Figure 8.19: Parallel combination of springs

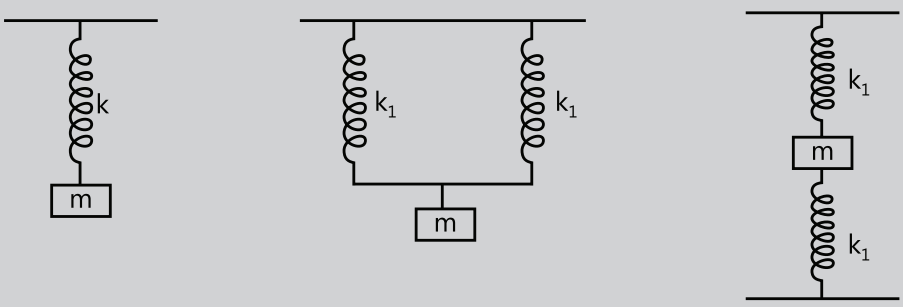
If springs are connected in parallel, then the effective force constant is given by

$k_{eff} = k_1 + k_2 + k_3 + \ldots\ldots\ldots\ldots$

The force constant of a spring is inversely proportional to its length. If a spring of spring constant k is cut into two equal parts, the spring constant of each part becomes 2k. In general, if a spring of spring constant k is divided into n equal parts, the spring constant of each part is nk.

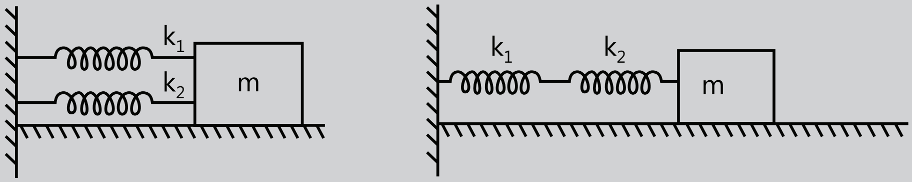
PLANCESS CONCEPTS

Time Period for various scenarios at glance

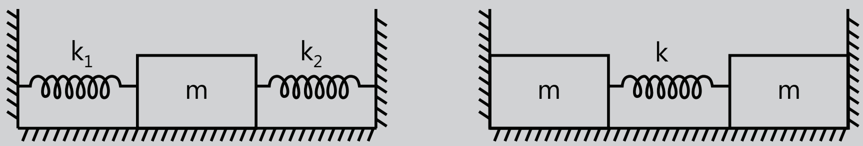


$T = 2\pi\sqrt{\dfrac{m}{k}}$ $\quad T = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$ $\quad T = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$

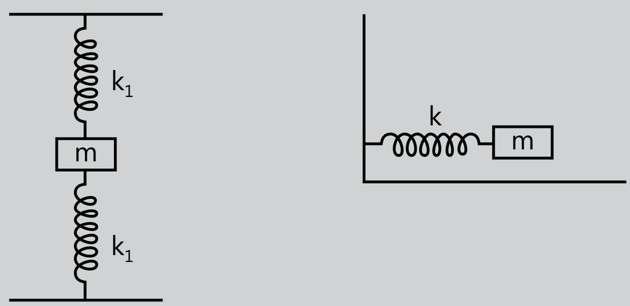
PLANCESS CONCEPTS



$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}} \qquad T = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$$



$$T = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}} \qquad T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$



$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Figure 8.20

GV Abhinav (JEE 2012, AIR 329)

Illustration 13: For the arrangement shown in Fig 8.21, the spring is initially compressed by 3 cm. When the spring is released the block collides with the wall and rebounds to compress the spring again. **(JEE ADVANCED)**

(a) If the coefficient of restitution is $\frac{1}{\sqrt{2}}$, find the maximum compression in the spring after collision.

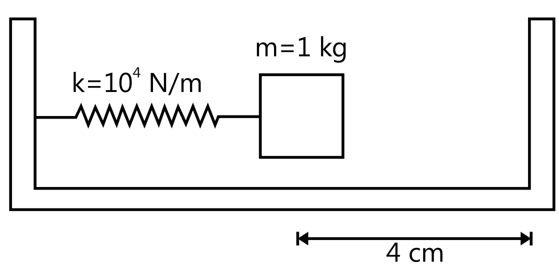


Figure 8.21

Sol: Conserve energy to find the velocity of the block. Use equation of restitution for collision of block with the wall.

(a) Velocity of the block just before

collision, $\frac{1}{2}mv_0^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2$ or $v_0 = \sqrt{\frac{k}{m}\left(x_0^2 - x^2\right)}$

Here, $x_0 = 0.03m$, $x = 0.01m$, $k = 10^4 N/m$, $m = 1kg$ $\therefore$ $v_0 = 2\sqrt{2} m/s$

After collision, $v = ev_0 = \frac{1}{\sqrt{2}} 2\sqrt{2} = 2m/s$

Maximum compression in the spring

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \quad \text{or} \quad x_m = \sqrt{x^2 + \frac{m}{\kappa}v^2} = \sqrt{(0.01)^2 + \frac{1(2)^2}{10^4}}m \quad = 2.23cm$$

Illustration 14: Figure 8.22 shows a system consisting of a massless pulley, a spring of force constant k and a block of mass m. If the block is slightly displaced vertically down from its equilibrium position and released, find the period of its vertical oscillation in case (a), (b) and (c). **(JEE ADVANCED)**

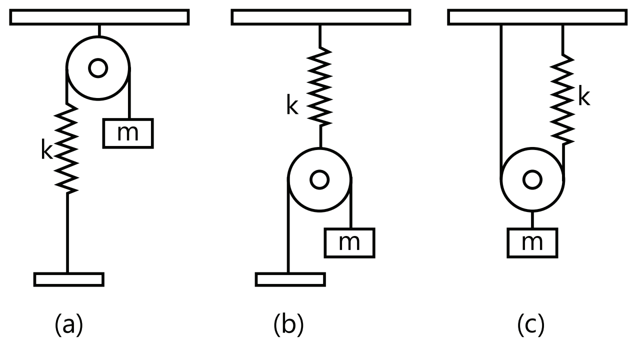


Figure 8.22

Sol: The restoring force on the block will depend on the elongation of the spring. For a small displacement of block find the elongation in the spring.

(a) In equilibrium, $kx_0 = mg$ ...(i)

When further depressed by an amount x, net restoring force (upwards) is,

$$F = -\{k(x + x_0) - mg\} = -kx \quad (\text{as } kx_0 = mg)$$

$$a = -\frac{k}{m}x \quad \therefore \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

(b) In this case if the mass m moves down a distance x from its equilibrium position, then pulley will move down by $\frac{x}{2}$. So, the extra force in spring will be $k\frac{x}{2}$. Now, as the pulley is massless, this force $\frac{kx}{2}$ is equal to extra 2 T or T = $\frac{kx}{4}$. This is also the restoring force of the mass. Hence,

$$F = -\frac{kx}{4}; \quad a = -\frac{k}{4m}x \quad \text{or} \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} \quad \text{or} \quad T = 2\pi\sqrt{\frac{4m}{k}}$$

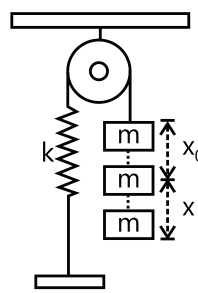


Figure 8.23

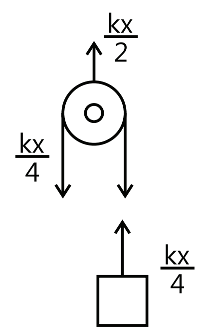


Figure 8.24

(c) In this situation if the mass m moves down a distance x from its equilibrium position, the pulley will also move by x and so the spring will stretch by 2x. Therefore, the spring force will be 2kx. The restoring force on the block will be 4kx.

Hence, $F = -4kx$ or $a = -\frac{4k}{m},x$

$$\therefore \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{4k}}$$

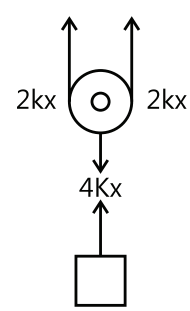


Figure 8.25

Illustration 15: A Spring mass system is hanging from the ceiling of an elevator in equilibrium. The elevator suddenly starts accelerating upwards with acceleration 'a' Find: (a) The frequency and (b) The amplitude of the resulting SHM. **(JEE MAIN)**

Sol: The time period of spring mass system does not depend on g or acceleration of elevator.

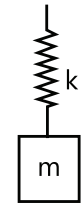


Figure 8.26

(a) Frequency = $2\pi\sqrt{\frac{m}{k}}$ (Frequency is independent of g in spring)

(b) Extension in spring in equilibrium in initial $= \frac{mg}{k}$

Extension in spring in equilibrium in accelerating lift $= \frac{m(g+a)}{k}$

$\therefore$ Amplitude $= \frac{m(g+a)}{k} - \frac{mg}{k} = \frac{ma}{k}$

11. BODY DROPPED IN A TUNNEL ALONG EARTH DIAMETER

Assume earth to be a sphere of radius R and center O. Let a tunnel be dug along the diameter of the earth as shown in Fig. 8.27. If a body of mass m is dropped at one end of the tunnel, the body executes SHM about the center of the earth. Let, at any instant body in the tunnel is at a distance y from the center O of the earth. Only the inner sphere of radius y will exert gravitational force F on the body as the body is inside the earth. The force F serves as the restoring force that tends to bring the body to the equilibrium position O.

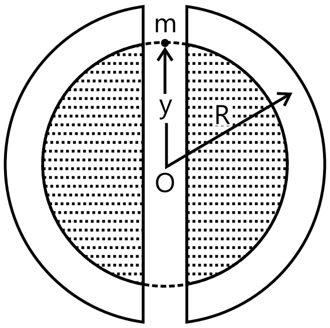


Figure 8.27: Body moving along diameter of earth

$\therefore$ Restoring force, $F = -G\frac{\left(4/3\,\pi y^3 \rho\right)m}{y^2}$

Where ρ is the density of the earth. The negative sign is assigned because the force is of attraction.

Acceleration of the body, $a = \frac{F}{m} = -\left(\frac{4}{3}\pi G\rho\right)y$ ... (i)

Now the quantity $(4/3)\pi G\rho$ is constant so that: $a \propto -y$

Thus the acceleration of the body is directly proportional to the displacement y and its direction is opposite to the displacement. Therefore, the motion of the body is simple harmonic.

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$ or $T = \sqrt{\frac{3\pi}{G\rho}}$ ... (ii)

12. DAMPED AND UNDAMPED OSCILLATIONS

Damped oscillations is shown in the Fig 8.28 (a) given below. In such a case, during each oscillation, some energy is lost. The amplitude of the oscillation will be reduced to zero as no compensating arrangement for the less is provided. The only parameters that will remain unchanged are the frequency or time period. They will change only according to the circuit parameters.

As shown in Fig 8.28 (b), undamped oscillations have constant have amplitude oscillations.

Damping Force, $F_d = -bV = -b\frac{dx}{dt}$ where b is a constant giving the strength of damping. We can write Newton's law, now including damping force along with the restoring force. For a spring-mass system, we have,

$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$ or $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$; $x = ae^{-bt/2m}\cos(\omega t + \phi)$ ... (i) E.q. (i) describes sinusoidal motion whose amplitude (a) decreases exponentially with time. How fast the amplitude drops depends on the damping constants b and m. The frequency of this damped motion is given by: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

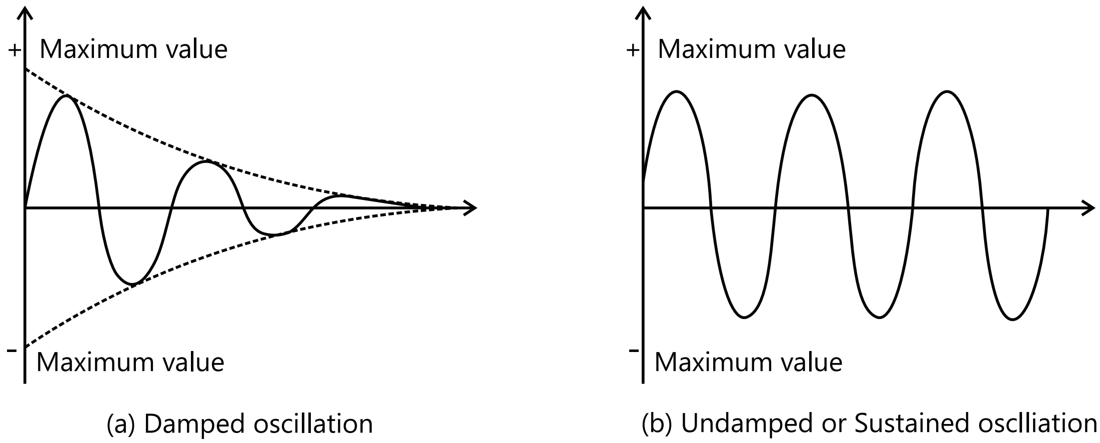


Figure 8.28: Damped and undamped oscillation

If the frictional forces are absent, b=0 so that: $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (undamped oscillations)

13. FREE, FORCED AND RESONANT OSCILLATIONS

(a) **Free oscillations** are executed by an oscillating body that vibrates with its own frequency.

For example, when a simple pendulum is displaced from its mean position and then left free, it executes free oscillations. The natural frequency of the simple pendulum depends upon its length and is given by; $f=\frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

(b) **Forced oscillations** - When a body is maintained in a state of oscillations by an external periodic force of frequency other than the natural frequency of the body, it executes forced oscillations.

The frequency of forced oscillations is equal to the frequency of the periodic force. The external applied force on the body is called the driver and the body set into oscillations is called driven oscillator.

Examples. (a) When the stem of a vibrating tuning fork is held in hand, only a feeble sound is heard. However, if the stem is pressed against a table top, the sound becomes louder. It is because the tuning fork forces the table to vibrate with fork's frequency. Since the table has a large vibrating area than the tuning fork, these forced oscillations produce a more intense sound.

Fig 8.29 shown the graph of forced oscillations as a function of ω.

At $\omega=\omega_a$, the value of A_0 is $\left(\frac{f_0}{b\omega}\right)$

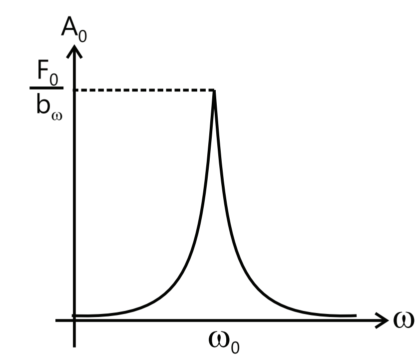


Figure 8.29: Forced oscillation

PLANCESS CONCEPTS

Notice that amplitude of motion A_0 is directly proportional to the amplitude of driving force.

GV Abhinav (JEE 2012, AIR 329)

Mathematical analysis: Most of the oscillations that occur in systems (e.g. machinery) are forced oscillations; oscillations that are produced and sustained by an external force. The simplest driving force is one that oscillates as a sine or a cosine. Suppose such an external force F_{ext} is applied to an oscillator that moves along x axis such as a block connected to a spring. We can represent the external forces as: $F_{ext} = F_0 \cos\omega t$ Where F_0 is the maximum magnitude of the force and $\omega(= 2\pi f)$ is the angular frequency of the force. Then the equation of motion (with damping) is $ma = -kx - bV + F_0 \cos\omega t$. This equation can be written as

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \cos\omega t \quad \text{or} \quad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos\omega t \qquad \text{... (i)}$$

The solution of eq. (i) is $x = A_0 \cos(\omega t + \phi)$ Where $A_0 = \dfrac{F_0/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\dfrac{b\omega}{m}\right)^2}}$ ... (ii)

and $\omega_0 = \sqrt{k/m}$ is the frequency of undamped (b=0) oscillator i.e., natural frequency.

(iii) Resonant oscillations: When a body is maintained in a state of oscillations by a periodic force having the same frequency as the natural frequency of the body, the oscillations are called resonant oscillations. The phenomenon of producing resonant oscillations is called resonance.

(b) The amplitude of motion (A_0) depends on the difference between the applied frequency (ω) and natural frequency(ω_0). The amplitude is the maximum when the frequency of the driving force equals the natural frequency i.e., when $\omega = \omega_0$. It is because the denominator in eq. (ii) is the minimum when $\omega = \omega_0$. This condition is called resonance. When the frequency of the driving force equals ω_0, the oscillator is said to be in resonance with the driving force.

$$A_0 = \frac{F_0/m}{\sqrt{\left(\omega^2 - \omega_0^2\right) + \left(\dfrac{b\omega}{m}\right)^2}} \qquad \text{At resonance, } \omega = \omega_0 \text{ and } A_0 = \frac{F_0/m}{\sqrt{(b\omega/m)^2}} = \frac{F_0}{b\omega}$$

PROBLEM-SOLVING TACTICS

To verify SHM see whether force is directly proportional to y or see if $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$ in cases when the equation is directly given compare with general equation to find the time period and other required answers

FORMULAE SHEET

1. Simple Harmonic Motion (SHM):

$F = -kx^n$

n is even - Motion of particle is not oscillatory

n is odd - Motion of particle is oscillatory.

If n = 1, F = -kx or $F \propto -x$. The motion is simple harmonic.

x = 0 is called the mean position or the equilibrium position.

Condition for SHM $\dfrac{d^2x}{dt^2} \propto -x$

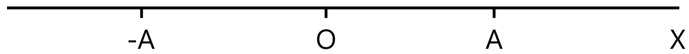


Figure 8.30

Acceleration, $a = \frac{F}{m} = -\frac{k}{m}x = -\omega^2 x$

Displacement $x = A\cos\underbrace{(\omega t + \phi)}_{\text{phase angle}}$ (A is Amplitude)

Time period of SHM $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Frequency ν of SHM $\nu = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Velocity of particle $v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$

Acceleration of particle $a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$

2. Energy in SHM:

Kinetic energy of particle $= \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$

Potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$

Total energy E= P.E + K.E = $\frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2$

E is constant throughout the SHM.

3. Simple pendulum: Time period $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$

Here, ℓ is length of simple pendulum and $\vec{g}_{eff} = \vec{g} - \vec{a}$ where $\vec{g}$ is acceleration due to gravity and $\vec{a}$ is acceleration of the box or cabin etc. containing the simple pendulum.

4. Spring-block system: Time period $T = 2\pi\sqrt{\frac{m}{k}}$

5. Physical pendulum: Time period $T = 2\pi\sqrt{\frac{I}{mg\ell}}$

Here I is the moment of inertia about axis of rotation and ℓ is the distance of center of gravity from the point of suspension.

6. Torsional Pendulum:

$$T = 2\pi\sqrt{\frac{I}{k}}$$

I is the moment of Inertia about axis passing through wire, k is torsional constant of wire.

7. Springs in series and parallel

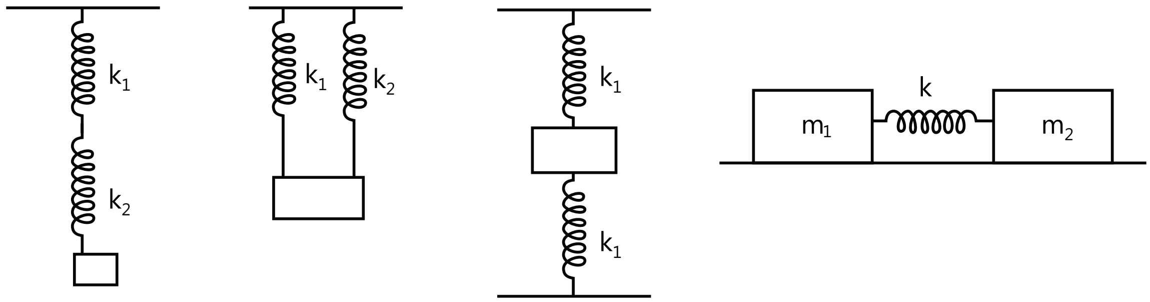


Figure 8.31

Series combination $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}$

Parallel combination $k=k_1+k_2$

8. For two blocks of masses m_1 and m_2 connected by a spring of constant k:

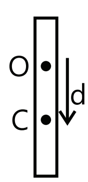
Time period $T=2\pi\sqrt{\frac{\mu}{k}}$

where $\mu=\frac{m_1m_2}{m_1+m_2}$ is reduced mass of the two-block system.

Solved Examples

JEE Main/Boards

Example 1: What is the period of pendulum formed by pivoting a meter stick so that it is free to rotate about a horizontal axis passing through 75 cm mark?



Sol: This is an example of a physical pendulum. Find moment of inertia about point of suspension and the distance of the point of suspension from the center of gravity.

Let m be the mass and ℓ be the length of the stick. $\ell=100\text{cm}$ The distance of the point of suspension from center of gravity is $d=25\,\text{cm}$

Moment of inertia about a horizontal axis through O is

$$I=I_c+md^2=\frac{m\ell^2}{12}+md^2$$

$$T=2\pi\sqrt{\frac{I}{mgd}};\quad T=2\pi\sqrt{\frac{\frac{m\ell^2}{12}+md^2}{mgd}}$$

$$T=2\pi\sqrt{\frac{\ell^2+12d^2}{12gd}}=2\pi\sqrt{\frac{\ell^2+12(0.25)^2}{12\text{x}9.8\text{x}0.25}}=153\text{ s.}$$

Example 2: A particle executes SHM.

(a) What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude?

(b) At what value of displacement are the kinetic and potential energies equal?

Sol: The sum of kinetic energy and potential energy is the total mechanical energy which is constant throughout the SHM.

We know that $E_{total}=\frac{1}{2}m\omega^2A^2$

$KE=\frac{1}{2}m\omega^2\left(A^2-X^2\right)$ and $U=\frac{1}{2}m\omega^2x^2$

(a) When $x=\frac{A}{2}$, $KE=\frac{1}{2}m\omega^2\frac{3A^2}{4}\Rightarrow\frac{KE}{E_{total}}=\frac{3}{4}$

At $x=\frac{A}{2},\ U=\frac{1}{2}m\omega^2\frac{A^2}{4}\ \Rightarrow \frac{PE}{E_{total}}=\frac{1}{4}$

(b) Since, $K=U$, $\frac{1}{2}m\omega^2\left(A^2-x^2\right)=\frac{1}{2}m\omega^2x^2$;

$2x^2=A^2$ or $x=\frac{A}{\sqrt{2}}=0.707\,A$

Example 3: Show that the period of oscillation of simple pendulum at depth h below earth's surface is inversely proportional to $\sqrt{R-h}$, where R is the radius of earth. Find out the time period of a second pendulum at a depth $R/2$ from the earth's surface?

Sol: As we go at a depth below the earth surface, the acceleration due to gravity decreases. The value of g inside the surface of earth is directly proportional to the radial distance from the center of the earth.

At earth's surface the value of time period is given by

$$T=2\pi\sqrt{\frac{L}{g}}\ \text{or}\ T\propto\frac{1}{\sqrt{g}}$$

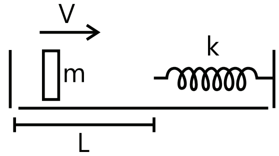
At a depth h below the surface, $g'=g\left(1-\frac{h}{R}\right)$

$$\therefore\ \frac{T'}{T}=\sqrt{\frac{g}{g'}}=\sqrt{\frac{1}{\left(1-\frac{h}{R}\right)}}=\sqrt{\frac{R}{R-h}}\ \therefore\ T'=T\sqrt{\frac{R}{R-h}}$$

or $T'\propto\frac{1}{\sqrt{R-h}}$ Hence Proved.

Further, $T_{R/2}=2\sqrt{\frac{R}{R-R/2}}=2\sqrt{2}\,s$

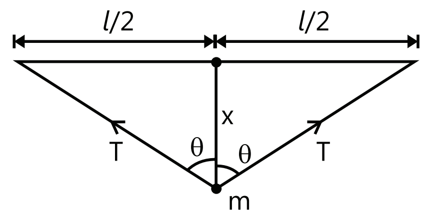
Example 4: Describe the motion of the mass m shown in figure. The walls and the block are elastic.



Sol: As the collision of the block with the wall is elastic, there will not be any loss in the kinetic energy and block will execute periodic motion of constant time period.

The block reaches the spring with a speed 'v'. It now compresses the spring. The block is decelerated due to the spring force, comes to rest when $\frac{1}{2}mv^2=\frac{1}{2}kx^2$ and return back. It is accelerated due to the spring force till the spring acquires its natural length. The contact of the block with the spring is now broken. At this instant it has regained its speed v (towards left) as the spring is not stretched and no potential energy is stored. This process takes half the period of oscillation, i.e. $\pi\sqrt{m/k}$. The block strikes the left wall after a time L/v and as the collision is elastic, it rebounds with the same speed v. After a time L/v, it again reaches the spring and the process is repeated. The block thus undergoes periodic motion with time period $\pi\sqrt{m/k}+\frac{2L}{v}$.

Example 5: A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.



Sol: The amplitude in case of combination of two or more SHMs in same direction and same frequency is obtained by vector addition of the amplitudes of individual SHMs. The angle of each of the individual amplitude with the x-axis is equal to the phase constant of the respective SHM.

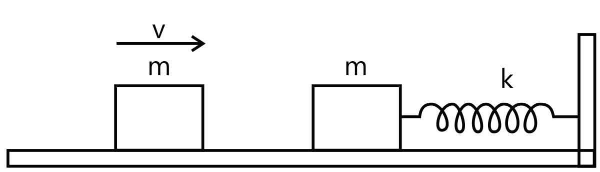
Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. If the phase difference between the two motion is δ,

$$A=\sqrt{A^2+A^2+2A.A.\cos\delta}$$

or $A=A\sqrt{2\left(1+\cos\delta\right)}=A\cos\delta/2$

or $\cos\frac{\delta}{2}=\frac{1}{2}$ or $\delta=2\pi/3$

Example 6: The figure shown below a block collides in-elastically with the right block and sticks to it. Find the amplitude of the resulting simple harmonic motion.



Sol: Conserve momentum before and after collision. The kinetic energy of blocks after collision is converted into elastic potential energy of the spring at the instant of maximum compression. Maximum compression is equal to amplitude of resulting SHM.

Assuming the collision to last for a small interval only, we can apply the principle of conservation of momentum. The common velocity after the collision is $\frac{v}{2}$. The kinetic energy $=\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2=\frac{1}{4}mv^2$. This is also the total energy of vibration as the spring is un-stretched at this moment. If the amplitude is A, the total energy can also be written as $\frac{1}{2}kA^2$.

Thus, $\frac{1}{2}kA^2 = \frac{1}{4}mv^2$, giving $A = \sqrt{\frac{m}{2k}}v$.

Example 7: Find the time period of small oscillations in a horizontal plane performed by a ball of mass 40 g fixed at the middle of a horizontally stretched string 1.0 m in length. The tension of the string is assumed to be constant and equal to 10 N.

Sol: Use the restoring force method to find the angular frequency.

Consider a ball of mass m placed at the middle of a string of length l and tension T. The components of tension T towards mean position is $T\cos\theta$.

The force acting on the ball $= 2T\cos\theta$

$$\therefore ma=-\frac{2Tx}{\sqrt{\left((l^2/4)+x^2\right)}}$$

$$\because T=F \text{ and } \cos\theta=\frac{x}{\sqrt{\left((l^2/4)+x^2\right)}}$$

As x is small, x_2 can be neglected in the denominator.

$$\therefore a=-\frac{2Tx}{m(l/2)}=-\left(\frac{4T}{ml}\right)x=-\omega^2x$$

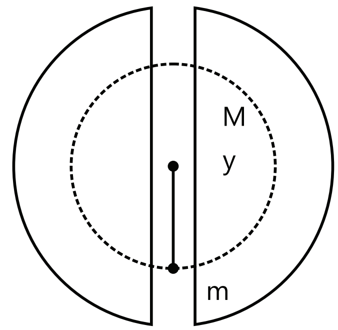
The acceleration is directly proportional to negative displacement x and is directed towards the mean position. Hence the motion is SHM

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{(4T/ml)}}=\pi\sqrt{\left(\frac{ml}{T}\right)}$$

Substituting the given values, we get

$$T=3.14\,x\sqrt{\left(\frac{(4x10^{-2})(1.0)}{10}\right)}=0.2\,s$$

Example 8: If a tunnel is dug through the earth from one side to the other side along a diameter. Show that the motion of a particle dropped into the tunnel is simple harmonic motion. Find the time period. Neglect all the frictional forces and assume that the earth has a uniform density.



$G=6.67\,x10^{-11}\,Nm^2kg^{-2}$; Density of earth $=5.51\,x10^3\,kgm^{-3}$

Sol: Use the restoring force method to find the angular frequency.

Consider a tunnel dug along the diameter of the earth. A particle of mass m is placed at a distance y from the center of the earth. There will be a gravitational attraction of the earth experienced by this particle due to the mass of matter contained in a sphere of radius y. Force acting on particle at distance y from center

$$F=\frac{GM}{R^3}.y$$

$$\Rightarrow ma=-\frac{GMm}{R^3}.y$$

$$\Rightarrow a=-\frac{GM}{R^3}.y=-\frac{G\times d\times\frac{4}{3}\pi R^3}{R^3}y=-\frac{4\pi G}{3}.d.y$$

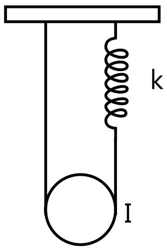
As the force is directly proportional to the displacement and is directed towards the mean position, the motion is simple harmonic.

$$\Rightarrow \omega^2=\frac{4}{3}\pi dG. \text{ and } T=2\pi\sqrt{\left(\frac{3}{4\pi dG}\right)}$$

$$=\sqrt{\left(\frac{3\pi}{dG}\right)}=\sqrt{\left(\frac{3x3.14}{5.51x10^3x6.67x10^{-11}}\right)}$$

$$=5062\,s=84.4\,min$$

Example 9: The pulley shown in figure below has a moment of inertia I about its axis and mass m. find the time period of vertical oscillation of its center of mass. The spring has spring constant k and the string does not slip over the pulley.



Sol: For a small displacement of the pulley find the extension in the spring. Use the energy method to find the angular frequency.

Let us first find the equilibrium position. For rotational equilibrium of the pulley, the tensions in the two strings should be equal. Only then the torque on the pulley will be zero. Let this tension be T. The extension of the spring will be $y = T/k$, as the tension in the spring will be the same as the tension in the string. For translational equilibrium of the pulley,

$$2T = mg \quad \text{or,} \quad 2ky = mg \quad \text{or,} \quad y = \frac{mg}{2k}.$$

The spring is extended by a distance $\frac{mg}{2k}$ when the pulley is in equilibrium.

Now suppose, the center of the pulley goes down further by a distance x. The total increase in the length of the string plus the spring is 2x (x on the left of the pulley and x on the right). As the string has a constant length, the extension of the spring is 2x. The energy of the system is

$$U = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 - mgx + \frac{1}{2}k\left(\frac{mg}{2k} + 2x\right)^2$$

$$= \frac{1}{2}\left(\frac{I}{r^2} + m\right)v^2 + \frac{m^2g^2}{8k} + 2kx^2.$$

As the system is conservative, $\frac{dU}{dt} = 0$,

giving $0 = \left(\frac{I}{r^2} + m\right)v\frac{dv}{dt} + 4kxv$

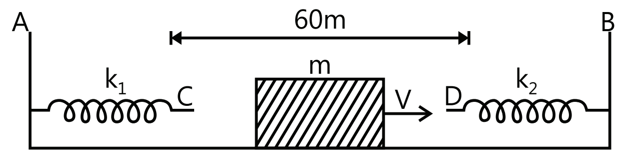
Or, $\frac{dv}{dt} = -\frac{4kx}{\left(\frac{I}{r^2} + m\right)}$

or $a = -\omega^2 x$, where $\omega^2 = \frac{4k}{\left(\frac{I}{r^2} + m\right)}$

Thus, the center of mass of the pulley executes a simple harmonic motion with time period

$$T = 2\pi\sqrt{\left(\frac{I}{r^2} + m\right)/(4k)}.$$

Example 10: Two light springs of force constant k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in figure.



The distance CD between the free ends of the springs 60 cm. If the block moves along AB with a velocity 120 cm/sec in between the springs, calculate the period of oscillation of the block

$(k_1 = 1.8\,N/m,\ k_2 = 3.2\,N/m, m = 200\,gm)$

If initially block is mid-way of CD.

Sol: As there are no dissipative forces the motion of the block is oscillatory with constant time period. Add the time of motion of different segments to get the time period.

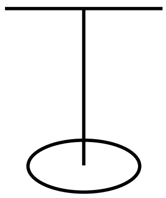
If initially block is mid-way of CD their the time period T is equal to sum of time to travel 30 cm to right, time in contact with spring k_2, time to travel 60 cm to left, time in contact with spring k_1 and time to travel 30 cm to right.

$$\therefore\ T = \frac{30}{120} + \frac{1}{2}\left[2\pi\sqrt{\left(\frac{m}{k_2}\right)}\right] + \frac{60}{120} + \frac{1}{2}\left[2\pi\sqrt{\left(\frac{m}{k_1}\right)}\right] + \frac{30}{120}$$

$$= 0.25 + \pi\sqrt{\left(\frac{0.2}{3.2}\right)} + 0.5 + \pi\sqrt{\left(\frac{0.2}{1.8}\right)} + 0.25$$

$$= 0.25 + \pi/4 + 0.5 + \pi/3 + 0.25 = 2.83\text{ s.}$$

Example 11: The moment of inertia of the disc used in torsional pendulum about the suspension wire is $0.2\ kg-m^2$. It oscillates with a period of 2s. Another disc is played over the first one and the time period of the system becomes 2.5 s. Fine the moment of inertia of the second disc about the wire.



Sol: As another disc is placed on the first disc moment of inertia about the axis passing through the wire increases and thus time period increases.

Let the torsional constant of the wire be k. The moment of inertia of the first disc about the wire is 0.2 $kg-m^2$. hence, the time period is

$$2s = 2\pi\sqrt{\frac{I}{K}} = 2\pi\sqrt{\frac{0.2kg-m^2}{k}} \qquad ...(i)$$

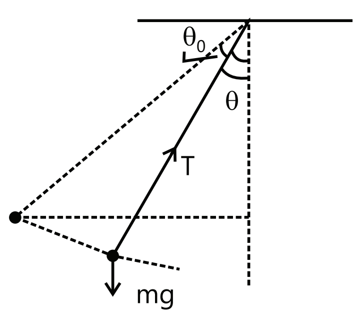
When the second disc having moment of inertia I_1 about. The wire is added, the time period is

$$2.5s = 2\pi\sqrt{\frac{0.2kg-m^2+I_1}{0.2kg-m^2}} \qquad ...(ii)$$

From (i) and (ii), $\frac{6.25}{4} = \frac{0.2kgm-m^2+I_1}{0.2kg-m^2}$.

This gives $I_1 = 0.11kg-m^2$.

Example 12: A simple pendulum having a bob of mass m undergoes small oscillations with amplitude θ_0. Find the tension in the string as a function of the angle made by the string with the vertical. When is this tension maximum, and when is it minimum?



Sol: The forces acting on the bob are tension due to string and weight mg. The bob moves in a circular path. The acceleration of the bob has both radial and tangential component.

Suppose the speed of the bob at angle θ is υ. Using conservation of energy between the extreme position and the position with angle θ,

$$\frac{1}{2}mv^2 = mgl(\cos\theta - \cos\theta_0) \qquad ...(i)$$

As the bob moves in a circular path, the force towards the center should be equal to mv^2 / l. Thus,

$T - mg\cos\theta = mv^2/l$.

Using (i),

$T - mg\cos\theta = 2mg(\cos\theta - \cos\theta_0)$

or $\quad T = 3mg\cos\theta - 2mg\cos\theta_0$.

Now $\cos\theta$ is maximum at $\theta = 0$ and decreases as $|\theta|$ increases $(\text{for} |\theta| < 90^o)$.

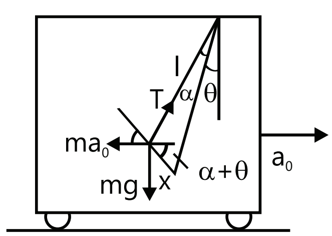
Thus, the tension is maximum when $\theta = 0$, i.e., at the mean position and is minimum when $\theta = \pm\theta_0$, i.e., at extreme positions.

JEE Advanced/Boards

Example 1: A simple pendulum is suspended from the ceiling of a car accelerating uniformly on a horizontal road. If the acceleration is a_0 and the length of the pendulum is l, find the time period of small oscillations about the mean position.

Sol: The car accelerates with acceleration a. In the reference frame of car the effective value of acceleration due to gravity is

$$g_{eff} = |\vec{g} - \vec{a}| = \sqrt{g^2 + a^2}$$



We shall work in the car frame. As it is accelerated with respect to the road, we shall have to apply a pseudo force $m\,a_0$ on the bob of mass m.

For mean position, the acceleration of the bob with respect to the car should be zero. If θ be the angle made by the string with the vertical, then tension, weight and the pseudo force will add to zero in this position.

Suppose at some instant during oscillation, the string is further deflected by an angle α so that the displacement of the bob is x. Taking the components perpendicular to the string, component of T = 0, component of $mg = mg\sin(\alpha+\theta)$ and component of $ma_0 = -ma_0\cos(\alpha+\theta)$. Thus, the resultant component $F = m\left[g\sin(\alpha+\theta) - a_0\cos(\alpha+\theta)\right]$.

Expanding the sine and cosine and putting $\cos\alpha \approx l$, $\sin\alpha \approx x/l$, we get,

$$F = m\left[g\sin\theta - a_0\cos\theta + (g\cos\theta + a_0\sin\theta)\frac{x}{l}\right]$$

At $x = 0$, the force F on the bob should be zero, as this is the mean position. Thus by (i),

$$0 = m\left[g\sin\theta - a_0\cos\theta\right] \qquad \text{...(ii)}$$

Giving $\tan\theta = \frac{a_0}{g}$

$$\text{Thus, } \sin\theta = \frac{a_0}{\sqrt{a_0^2 + g^2}} \qquad \text{...(iii)}$$

$$\cos\theta = \frac{g}{\sqrt{a_0^2 + g^2}} \qquad \text{...(iv)}$$

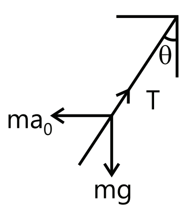
Putting (ii), (iii) and (iv) in

(i), $F = m\sqrt{g^2 + a_0^2}\,\frac{x}{l}$ or, $F = m\omega^2 x$, where $\omega^2 = \frac{\sqrt{g^2 + a_0^2}}{l}$.

This is an equation of simple harmonic motion with

time period $t = \frac{2\pi}{\omega} = 2x\frac{\sqrt{l}}{\left(g^2 + a_0^2\right)^{1/4}}$.

As easy working rule may be found out as follows. In the mean position, the tension, the weight and the pseudo force balance. From figure, the tension is



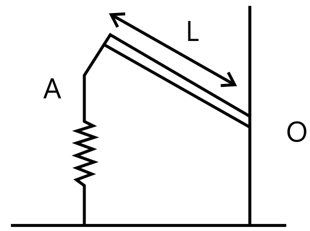
$$T = \sqrt{(ma_0)^2 + (mg)^2}$$

or, $\frac{T}{m} = \sqrt{a_0^2 + g^2}$.

This plays the role of effective 'g'. Thus the time period

is $t = 2\pi\sqrt{\frac{I}{T/m}} = 2\pi\frac{\sqrt{I}}{\left[g^2 + a_0^2\right]^{1/4}}$.

Example 2: A long uniform rod of length L and mass M is free to rotate in a vertical plane about a horizontal axis through its one end 'O'. A spring of force constant k is connected vertically between one end of the rod and ground. When the rod is in equilibrium it is parallel to the ground.



(a) What is the period of small oscillation that result when the rod is rotated slightly and released?

(b) What will be the maximum speed of the displaced end of the rod, if the amplitude of motion is θ_0 ?

Sol: The rod executes angular SHM. Use restoring torque method to find angular frequency of SHM.

(a) Restoring torque about 'O' due to elastic force of the spring

$$\tau = -FL = -kyL \qquad (F = ky)$$

$$\tau = -kL^2\theta \qquad (\text{as } y = L\theta)$$

$$\tau = I\alpha = \frac{1}{3}ML^2\frac{d^2\theta}{dt^2}$$

$$\frac{1}{3}ML^2\frac{d^2\theta}{dt^2} = -kL^2\theta;\quad \frac{d^2\theta}{dt^2} = -\frac{3k}{M}\theta$$

$$\omega = \sqrt{\frac{3k}{M}} \Rightarrow T = 2\pi\sqrt{\frac{M}{3k}}$$

(b) In angular SHM, maximum angular velocity

$$\left(\frac{d\theta}{dt}\right)_{max} = \theta_0,\quad \omega = \theta_0\sqrt{\frac{3k}{M}},\quad v = r\left(\frac{d\theta}{dt}\right)$$

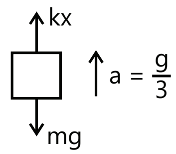
$$\text{So, } v_{max} = L\left(\frac{d\theta}{dt}\right)_{max} = L\theta_0\sqrt{\frac{3k}{M}}$$

Example 3: A block with mass of 2 kg hangs without vibrating at the end of a spring of spring constant 500 N/m, which is attached to the ceiling of an elevator. The elevator is moving upwards with acceleration $\frac{g}{3}$. At time t=0, the acceleration suddenly ceases.

(a) What is the angular frequency of oscillation of the block after the acceleration ceases?

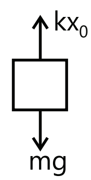
(b) By what amount is the spring stretched during the time when the elevator is accelerating?

(c) What is the amplitude of oscillation and initial phase angle observed by a rider in the elevator? Take the upward direction to be positive. Take $g = 10.0\,m/s^2$.



Sol: The angular frequency of the spring block system in vertical oscillations does not depend on the acceleration due to gravity or the acceleration of the elevator. The equilibrium position depends on the acceleration due to gravity and the elevator. When the acceleration of the elevator ceases the block moves to the new equilibrium position.

(a) Angular frequency



$$\omega = \sqrt{\frac{k}{m}} \text{ or } \omega = \sqrt{\frac{500}{2}}$$

or $\omega = 15.81$ rad / s

(b) Equation of motion of the block (while elevator is accelerating) is,

$$kx - mg = ma = m\frac{g}{3}$$

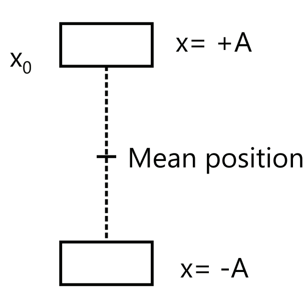
$$\therefore\ x = \frac{4mg}{3k} = \frac{(4)(2)(10)}{(3)(500)} = 0.053m$$

or $x = 5.3\,cm$

(c) (i) In equilibrium when the elevator has zero acceleration, the equation of motion is

$$kx_0 = mg \text{ or } \quad x_0 = \frac{mg}{k} = \frac{(2)(10)}{500} = 0.04m$$

= 4cm



$\therefore$ Amplitude $\quad A = x - x_0$

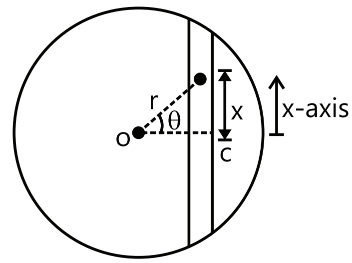
$= 5.3 - 4.0$

= 1.3 cm

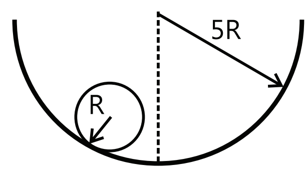
(ii) At time $t = 0$, block is at $x = -A$. Therefore, substituting $x = -A$ and $t = 0$ in equation,

$x = A\sin(\omega t + \phi)$ We get initial phase $\phi = \frac{3\pi}{2}$

Example 4: A solid sphere (radius = R) rolls without slipping in a cylindrical through (radius = 5R). Find the time period of small oscillations.



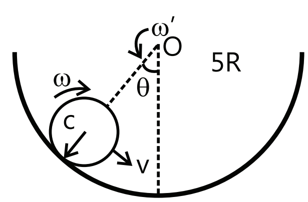
Sol: The sphere executes pure rolling in the cylinder. The mean position is at the lowest point in the cylinder. Find the acceleration for small displacement from the mean position and compare with standard equation of SHM to find angular frequency.



For pure rolling to take place, v = Rω

ω' = Angular velocity of COM of sphere C about O

$$= \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$$



$$\therefore\ \frac{d\omega'}{dt} = \frac{1}{4}\frac{d\omega}{dt} \text{ or } \alpha' = \frac{\alpha}{4}$$

$\alpha = \frac{a}{R}$ for pure rolling;

Where, $a = \frac{g\sin\theta}{I + mR^2} = \frac{5g\sin\theta}{7}$

As, $I = \frac{2}{5}mR^2 \quad \therefore \quad \alpha' = \frac{5g\sin\theta}{28R}$

For small θ, $\sin\theta = \theta$, being restoring in nature,

$$\alpha' = -\frac{5g}{28R}\theta \qquad \therefore T = 2\pi\sqrt{\left|\frac{\theta}{\alpha'}\right|} = 2\pi\sqrt{\frac{28R}{5g}}$$

Example 5: Consider the earth as a uniform sphere of mass M and radius R. Imagine a straight smooth tunnel made through the earth which connects any two points on its surface. Show that the motion of a particle of mass in along this tunnel under the action of gravitation would be simple harmonic. Hence, determine the time that a particle would take to go from one end to the other through the tunnel.

Sol: Use the restoring force method to find the angular frequency.

Suppose at some instant the particle is at radial distance r from center of earth O. Since, the particle is constrained to move along the tunnel, we define its position as distance x from C. Hence, equation of motion of the particle is, $ma_x = F_x$

The gravitational force on mass m at distance r is,

$$F = \frac{GMmr}{R^3} \text{ (Towards O)}$$

$$\text{Therefore, } F_x = -F\sin\theta = -\frac{GMmr}{R^3}\left(\frac{x}{r}\right)$$

Since $F_x \propto -x$, motion is simple harmonic in nature. Further,

$$ma_x = -\frac{GMm}{R^3}.x \quad \text{or} \quad a_x = -\frac{GM}{R^3}.x$$

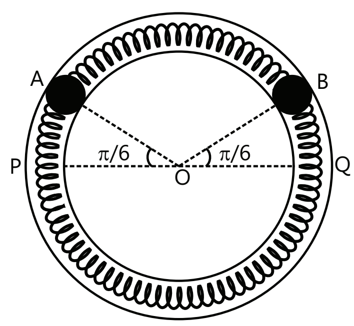
$\therefore$ Time period of oscillation is,

$$T = 2\pi\sqrt{\frac{x}{a_x}} = 2\pi\sqrt{\frac{R^3}{GM}}$$

The time taken by particle to go from one end to the other is $\frac{T}{2}$

$$\therefore \quad t = \frac{T}{2} = \pi\sqrt{\frac{R^3}{GM}}$$

Example 6: Two identical balls A and B, each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centers of the balls can move in a circle of radius 0.06 m. Each spring has a natural length 0.06 π m and spring constant 0.1 N/m. Initially both the balls are displaced by angle $\pi/6$ radian with respect to the diameter PQ of the circle and released from rest.



(a) Calculate the frequency of oscillation of ball B.

(b) Find the speed of the ball A when A and B are at the two ends of diameter PQ

(c) What is the total energy of the system

Sol: Here the two balls connected by the springs are free to oscillate along the length of the springs, so the time period will depend on the reduced mass of the two-ball system.

(a) Restoring force on A or B $= k\Delta x + k\Delta x = 2k\Delta x$.

Where Δx is compression in the spring at one end? Effective force constant = 2k

$$\text{Frequency } \nu = \frac{1}{2\pi}\sqrt{\frac{2k}{\mu}}$$

Where μ is reduced mass of system.

$$\text{reduced mass. } \mu = \frac{mm}{m+m} = \frac{m}{2}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{2k}{m/2}} = \frac{1}{3.14}\sqrt{\frac{0.1}{0.1}} = \frac{1}{3.14}s$$

(b) P and Q are equilibrium position. Balls A and B at P and Q have only kinetic energy and it is equal the potential energy at extreme positions.

Potential energy at extreme position

$$= \frac{1}{2}k(2\Delta x)^2 + \frac{1}{2}k(2\Delta x)^2 = 4k(\Delta x)^2$$

$$\text{Where } \Delta x = R x \frac{\pi}{6}$$

$$\Rightarrow \text{P.E.} = \frac{\pi^2 kR^2}{36} = \frac{(3.14)^2 x 0.1 x (0.06)^2}{36} \approx 3.94 x 10^{-4} J$$

When the balls A and B are at points P and Q respectively.

$$KE_{(A)} + KE_{(B)} = PE. ; \ 2KE_{(A)} = P.E.$$

$$2x\frac{1}{2}mv^2 = 3.94 x 10^{-4}$$

$$\Rightarrow v = \left(\frac{3.94}{0.1}\right)^{\frac{1}{2}} x 10^{-2} = 6.28 x 10^{-2} = 0.0628 ms^{-1}$$

(c) Total potential and kinetic energy of the system is equal to total potential energy at the extreme position=$3.94x10^{-4}$J.

JEE Main/Boards

Exercise 1

Q.1 A simple harmonic motion is represented by y(t)=10 sin (20t+0.5). Write down its amplitude, angular frequency, time period and initial phase, if displacement is measured in meters and time in seconds.

Q.2 A particle executing SHM along a straight line has a velocity of 4 ms^{-1}, when at a distance of 3 m from its mean position and 3 ms^{-1}, when at a distance of 4 m from it. Find the time it takes to travel 2.5 m from the positive extremity of its oscillation.

Q.3 A simple harmonic oscillation is represented by the equation.

Y=0.4sin (440t+0.61)

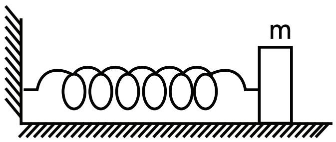
Here y and t are in m and s respectively. What are the values of (i) amplitude (ii) angular frequency (iii) frequency of oscillation (iv) time period of oscillation and (v) initial phase?

Q.4 A particle executing SHM of amplitude 25 cm and time period 3 s. What is the minimum time required for the particle to move between two points 12.5 cm on either side of the mean position?

Q.5 A particle executes SHM of amplitude a. At what distance from the mean position is its K.E. equal to its P.E?

Q.6 An 8 kg body performs SHM of amplitude a. At what distance from the mean position is its K.E. equal to its P.E?

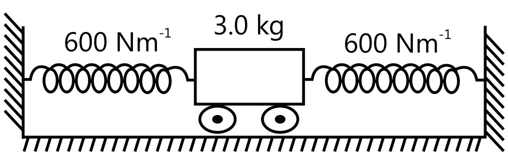
Q.7 A spring of force constant 1200 Nm^{-1} is mounted on a horizontal table as shown in figure. A mass of 3.0 kg is attached to the free end of the spring. Pulled sideways to a distance of 2cm and released, what is



(a) The speed of the mass when the spring is compressed by 1.0 cm?

(b) Potential energy of the oscillating mass.

Q.8 A trolley of mass 3.0 kg is connected to two identical springs each of force constant 600 Nm^{-1} as shown in figure. If the trolley is displaced from its equilibrium position by 5.0 cm and released, what is the total energy stored?



Q.9 A pendulum clock normally shows correct time. On an extremely cold day, its length decreases by 0.2%. Compute the error in time per day.

Q.10 Two particles execute SHM of same amplitude and frequency on parallel lines. They pass one another when moving in opposite directions and at that time their displacement is one third their amplitude. What is the phase difference between them?

Q.11 What is the frequency of a second pendulum in an elevator moving up with an accelerating of g/2?

Q.12 Explain periodic motion and oscillatory motion with illustration.

Q.13 What is a simple pendulum? Find an expression for the time period and frequency of a simple pendulum.

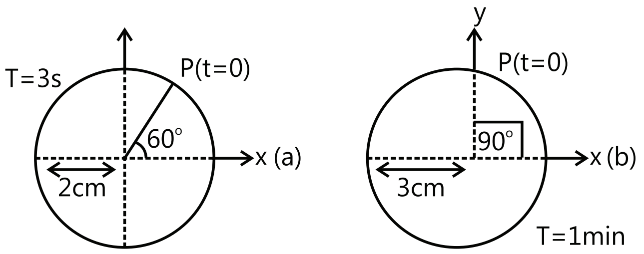
Q.14 Explain the oscillations of a loaded spring and find the relations for the time period and frequency in case of (i) horizontal spring (ii) vertical spring

Q.15 What is a spring factor? Find its value in case of two springs connected in (i) series and (ii) parallel.

Q.16 Explain phase and phase difference, angular frequency, displacement in periodic motion with illustrations.

Q.17 Explain displacement, velocity, acceleration and time period in SHMs. Find the relation between them.

Q.18 From the figure (a) and (b). Obtain the equation of simple harmonic motion of the y-projection of the radius vector of the revolving particle P in each case.



Q.19 Two particles execute SHM of the same amplitude and frequency along does parallel lines. They pass each other moving in opposite directions, each time their displacement in half their amplitude. What is their phase difference?

Q.20 A body oscillates with SHM according to the equation, X=6 cos $(3\pi t+\pi/3)$ metres. What is (a) amplitude and (b) the velocity at t=2s.

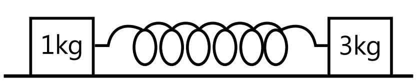
Q.21 A bob of simple pendulum executes SHM of period 20 s. Its velocity is 5 ms^{-1}, two seconds after it has passed through its mean position. Determine the amplitude of SHM.

Q.22 A particle is moving in a straight line with SHM Its velocity has values 3 ms^{-1} and 2 ms^{-1} when its distance from the mean positions are 1 m and 2 m respectively. Find the period of its motion and length of its path.

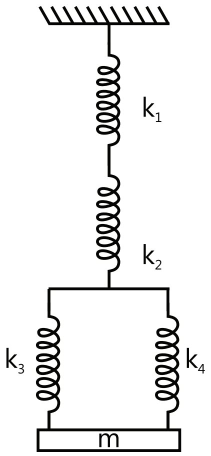
Q.23 A particle executes SHM with an amplitude 4 cm. Locate the position of point where its speed is half its speed is half its maximum speed. At what displacement is potential energy equal to kinetic energy?

Q.24 A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant 50 N m^{-1}. The block is pulled to a distance x=10 cm from its equilibrium position at x=0 on a frictionless surface at t=0. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.

Q.25 Two point masses of 3.0 kg and 1.0 kg are attached to opposite ends of a horizontal spring whose spring constant in 300 Nm^{-1} as shown in figure. Find the natural frequency of vibration of the system.



Q.26 A system of springs with their spring constants are as shown in figure . What is the frequency of oscillations of the mass m?



Exercise 2

Single Correct Choice Type

Q.1 A simple harmonic motion having an amplitude A and time period T is represented by the equation: $y = 5\sin\pi(t+4)m$

Then the values of A (in m) and T (in s) are:

(A) A = 5; T = 2 (B) A = 10; T = 1

(C) A = 5; T = 1 (D) A = 10; T = 2

Q.2 The maximum acceleration of a particle in SHM is made two times keeping the maximum speed to be constant. It is possible when

(A) Amplitude of oscillation is doubled while frequency remains constant

(B) Amplitude is doubled while frequency is halved

(C) Frequency is doubled while amplitude is halved

(D) Frequency is doubled while amplitude remains constant

Q.3 A stone is swinging in a horizontal circle 0.8 m in diameter at 30 rev/min. A distant horizontal light beam causes a shadow of the stone to be formed on a nearly vertical wall. The amplitude and period of the simple harmonic motion for the shadow of the stone are

(A) 0.4 m, 4 s (B) 0.2 m, 2 s

(C) 0.4 m, 2 s (D) 0.8 m, 2 s

Q.4 A small mass executes linear SHM about O with amplitude a and period T. Its displacement from O at time T/8 after passing through O is:

(A) a/8 (B) $a/2\sqrt{2}$ (C) a/2 (D) $a/\sqrt{2}$

Q.5 The displacement of a body executing SHM is given by $x = A\sin(2\pi t + \pi/3)$. The first time from t=0 when the velocity is maximum is

(A) 0.33 s (B) 0.16 s (C) 0.25 s (D) 0.5 s

Q.6 A particle executes SHM of period 1.2 s. and amplitude 8 cm. Find the time it takes to travel 3cm from the positive extremely of its oscillation.

(A) 0.28 s (B) 0.32 s (C) 0.17 s (D) 0.42 s

Q.7 A particle moves along the x-axis according to $: x = A[I + \sin\omega t]$. What distance does it travel between? $t = 0 \text{ and } t = 2.5\pi/\omega$?

(A) 4A (B) 6A (C) 5A (D) None

Q.8 Find the ratio of time periods of two identical springs if they are first joined in series & then in parallel & a mass m is suspended from them:

(A) 4 (B) 2 (C) 1 (D) 3

Q.9 The amplitude of the vibrating particle due to superposition of two SHMs,

$y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin\omega t$ is:

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

Q.10 Two simple harmonic motions y_1=A sin ωt are superimposed on a particle of mass m. The total mechanical energy of the particle is:

(A) $\frac{1}{2}m\omega_2 A_2$ (B) $m\omega_2 A_2$

(C) $\frac{1}{4}m\omega_2 A_2$ (D) Zero

Q.11 A block of mass 'm' is attached to a spring in natural length of spring constant 'k'. The other end A of the spring is moved with a constant velocity v away from the block. Find the maximum extension in the spring.

(A) $\frac{1}{4}\sqrt{\frac{mv^2}{k}}$ (B) $\sqrt{\frac{mv^2}{k}}$

(C) $\frac{1}{2}\sqrt{\frac{mv^2}{k}}$ (D) $2\sqrt{\frac{mv^2}{k}}$

Q.12 In the above question, the find amplitude of oscillation of the block in the reference frame of point A of the spring.

(A) $\frac{1}{4}\sqrt{\frac{mv^2}{k}}$ (B) $\frac{1}{2}\sqrt{\frac{mv^2}{k}}$

(C) $\sqrt{\frac{mv^2}{k}}$ (D) $2\sqrt{\frac{mv^2}{k}}$

Q.13 For a particle acceleration is defined as

$\vec{a} = \frac{-5x\vec{i}}{|x|}$ for $x \neq 0$ and $\vec{a} = 0$ for $x = 0$.

If the particle is initially at rest (a, 0) what is period of motion of the particle.

(A) $4\sqrt{2a/5}$ sec. (B) $8\sqrt{2a/5}$ sec.

(C) $2\sqrt{2a/5}$ sec. (D) Cannot be determined

Q.14 A mass m, which is attached to a spring with spring constant k, oscillates on a horizontal table, with amplitude A. At an instant when the spring is stretched by $\sqrt{3A/2}$, a second mass m is dropped vertically onto the original mass and immediately sticks to it. What is the amplitude of the resulting motion?

(A) $\frac{\sqrt{3}}{2}A$ (B) $\sqrt{\frac{7}{8}}A$

(C) $\sqrt{\frac{13}{16}}A$ (D) $\sqrt{\frac{2}{3}}A$

Q.15 A particle is executing SHM of amplitude A, about the mean position x=0. Which of the following cannot be a possible phase difference between the positions of the particle at x=+ A/2 and $x = -A/\sqrt{2}$

(A) 75° (B) 165° (C) 135° (D) 195°

Previous Years' Questions

Q.1 A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is ***(1987)***

(A) $f/2$ (B) f (C) $2f$ (D) $4f$

Q. 2 Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the one amplitude of vibration of M to that of N is ***(1988)***

(A) k_1/k_2 (B) $\sqrt{k_2/k_1}$

(C) k_2/k_1 (D) $\sqrt{k_1/k_2}$

Q.3 A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn. Block A executes small oscillations. The time period of which is given by ***(1992)***

(A) $2\pi\sqrt{M\eta L}$ (B) $2\pi\sqrt{\frac{M\eta}{L}}$

(C) $2\pi\sqrt{\frac{ML}{\eta}}$ (D) $2\pi\sqrt{\frac{M}{\eta L}}$

Q.4 One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to ***(1993)***

(A) $2\pi(m/k)^{1/2}$ (B) $2\pi\sqrt{\frac{m(YA+kL)}{YAk}}$

(C) $2\pi[(mYA/kL)^{1/2}$ (D) $2\pi[(mL/YA)^{1/2}$

Q.5 A particle of mass m is executing oscillation about the origin on the x-axis. Its potential energy is $U(x)=k|x|^3$, Where k is a positive constant. If the amplitude of oscillation is a then its time period T is ***(1998)***

(A) Proportional to $1/\sqrt{a}$

(B) Independent of a

(C) Proportional to $\sqrt{a}$

(D) Proportional to $a^{3/2}$

Q.6 A spring of force constant k is cut into two pieces such that one piece is double the length of the other the long piece will have a force constant of ***(1999)***

(A) 2/3 k (B) 3/2 k (C) 3k (D) 6k

Q.7 A particle free to move along the x – axis has porential energy by $U(x)=k[1-\exp(-x^2)]$ for $-\infty \le x \le +\infty$ Where k is a positive constant of appropriate dimensions. Then ***(1999)***

(A) At points away from the origin, the particle is in unstable equilibrium

(B) For any finite non-zero value of x, there is a force directed away from the origin

(C) If its total mechanical energy is k/2, it has its minimum kinetic energy at the origin

(D) For small displacements from x=0, the motion is simple harmonic

Q.8 The period of oscillation of simple pendulum of length L suspended from the roof of the vehicle which moves without friction, down an inclined plane of inclination α, is given by ***(2000)***

(A) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (B) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

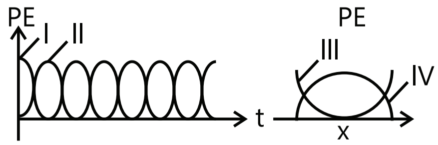
(C) $2\pi\sqrt{\frac{L}{g}}$ (D) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

Q.9 A particle executes simple harmonic motion between x= -A and x= + A. The time taken for it to go from O to A/2 is T_1 and to go from A/2 to A is T_2, then ***(2001)***

(A) $T_1 < T_2$ (B) $T_1 > T_2$

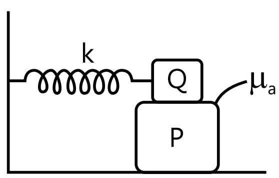
(C) $T_1 = T_2$ (D) $T_1 = 2T_2$

Q.10 For a particle executing SHM the displacement x is given by $x=A\ \cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement. ***(2003)***



(A) I, III (B) II, IV (C) II, III (D) I, IV

Q.11 A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by a distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks? ***(2004)***



(A) kA (B) kA

(C) μ_s mg (D) Zero

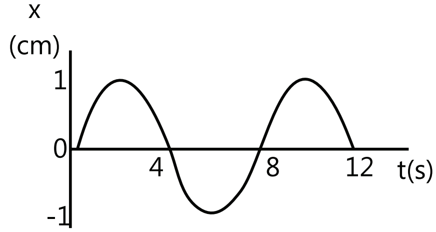
Q.12 A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2, (k = 1 m/s^2)$ where y is the vertical displacement.

The time period now beomes T_2.

The ratio of $\frac{T_1^2}{T_2^2}$ is (Take $g = 10 m/s^2$) ***(2005)***

(A) 6/5 (B) 5/6 (C) 1 (D) 4/5

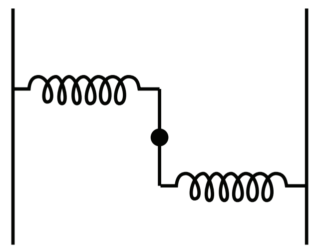
Q.13 The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t=4/3 s is ***(2009)***



(A) $\frac{\sqrt{3}}{32}\pi^2\ cms^{-2}$ (B) $\frac{-\pi^2}{32}cms^{-2}$

(C) $\frac{\pi^2}{32}cms^{-2}$ (D) $-\frac{\sqrt{3}}{32}\pi^2\ cms^{-2}$

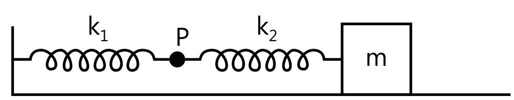
Q.14 A uniform rod of length L and mass M is pivoted at the center. Its two ends are attached to two springs of equal spring constants k. The spring are fixed to rigid supports as shown in the Fig, and rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is ***(2009)***



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

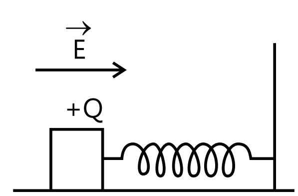
Q.15 The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is ***(2009)***



(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$

(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

Q.16 A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be ***(2011)***



(A) Of the same frequency and with shifted mean position

(B) Of the same frequency and with the same mean position

(C) Of changed frequency and with shifted mean position

(D) Of changed frequency and with the same mean position

Q.17 A point mass is subjected to two simultaneous sinusoidal $\sqrt{2A, \frac{3\pi}{4}}$ displacements in x-direction

$x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$.

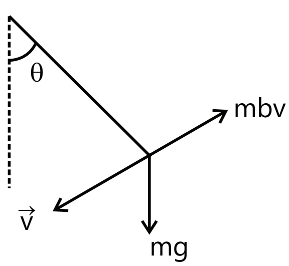
Adding a third sinusoidal displacement

$x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are ***(2011)***

(A) $A, \frac{6\pi}{3}$ (B) $A, \frac{4\pi}{3}$

(C) $\sqrt{3A, \frac{5\pi}{6}}$ (D) $A, \frac{\pi}{3}$

Q.18 If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = Os to $t = \tau s$, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with ' b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds: ***(2012)***



(A) $\frac{0.693}{b}$ (B) b

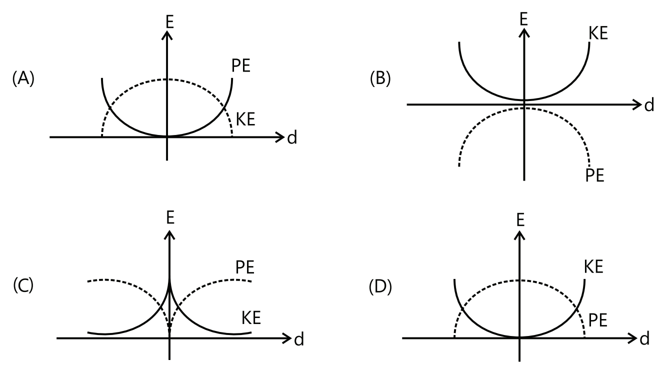
(C) $\frac{1}{b}$ (D) $\frac{2}{b}$

Q.19 The amplitude of a damped oscillator decreases to0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals. ***(2013)***

(A) 0.81 (B) 0.729 (C) 0.6 (D) 0.7

Q.20 For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?

(Graphs are schematic and not drawn to scale) ***(2015)***



Q.21 A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is: ***(2016)***

(A) 3 A (B) $A\sqrt{3}$

(C) $\frac{7A}{3}$ (D) $\frac{A}{3}\sqrt{41}$

JEE Advanced/Boards

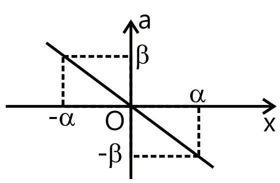
Exercise 1

Q.1 A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1:3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

Q.2 A body undergoing SHM about the origin has its equation is given by $x = 0.2\cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ sec.

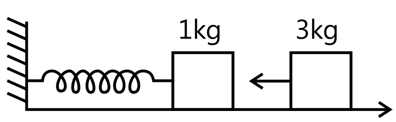
Q.3 Two particles A and B execute SHM along the same line with the same amplitude a, same frequency and same equilibrium position O. If the phase difference between them is $\phi = 2\sin^{-1}(0.9)$, then find the maximum distance between the two.

Q.4 The acceleration-displacement $(a - x)$ graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



Q.5 A point particle of mass 0.1kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its K.E. is 8×10^{-3} J. Obtain the equation of motion of this particle if the initial phase of oscillation is $45°$.

Q.6 One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to x-axis. A block of mass m=1 kg is attached to free end of the spring and it is performing SHM. Equation of position of block in coordinate system shown is $x = 10 + 3\sin 10t$, is in second and x in cm. Another block of mass M=3kg, moving towards the origin with velocity 30cm/s collides with the block performing SHM at t=0 and gets stuck to it, calculate:

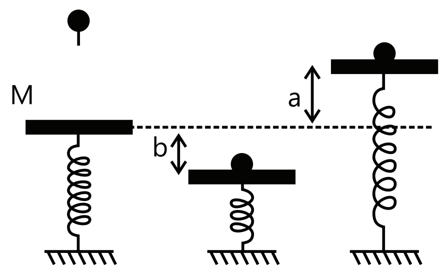


(i) New amplitude of oscillations.

(ii) New equation for position of the combined body.

(iii) Loss of energy during collision. Neglect friction.

Q.7 A mass M is in static equilibrium on a massless vertical spring as shown in the figure. A ball of mass m dropped from certain height sticks to the mass M after colliding with it. The oscillations they perform reach to height 'a' above the original level of scales & depth 'b' below it.

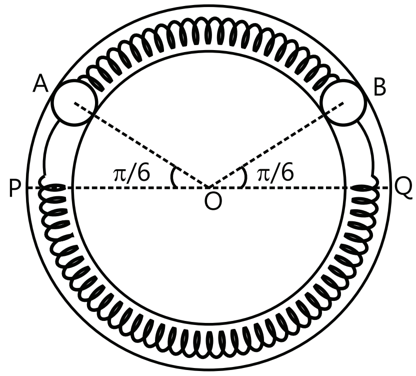


(a) Find the constant of force of the spring;

(b) Find the oscillation frequency.

(c) What is the height above the initial level from which the mass m was dropped?

Q.8 Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in figure. The pipe is fixed in a horizontal plane. The centers of the ball can move in a circle of radius 0.06m. Each spring has a natural length 0.06π m and force constant 0.1N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest

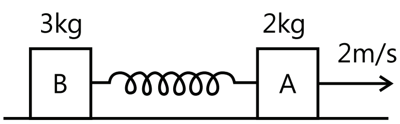


(a) Calculate the frequency of oscillation of the ball B.

(b) What is the total energy of the system?

(c) Find the speed of the ball A when A and B are at the two ends of the diameter PQ.

Q.9 Two blocks A(2kg) and B(3kg) rest up on a smooth horizontal surface are connected by a spring of stiffness 120 N/m. Initially the spring is unreformed. A is imparted a velocity of 2m/s along the line of the spring away from B. Find the displacement of A, t seconds later.



Q.10 A force $F = 10x + 2$ acts on a particle of mass 0.1 kg, where 'k' is in m and F in newton. If it is released from rest at $x = 0.2m$, find :

(a) Amplitude; (b) time period; (c) equation of motion.

Q.11 Potential Energy (U) of a body of unit mass moving in one-dimension conservative force field is given by, $U = (x^2 - 4x + 3)$. All units are in S.I.

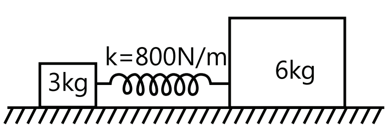
(i) Find the equilibrium position of the body.

(ii) Show that oscillations of the body about this equilibrium position are simple harmonic motion & find its time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

Q.12 A body is executing SHM under the action of force whose maximum magnitude is 50N. Find the magnitude of force acting on the particle at the time when its energy is half kinetic and half potential.

Q.13 The system shown in the figure can move on a smooth surface. The spring is initially compressed by 6cm and then released. Find



(a) Time period

(b) Amplitude of 3kg block

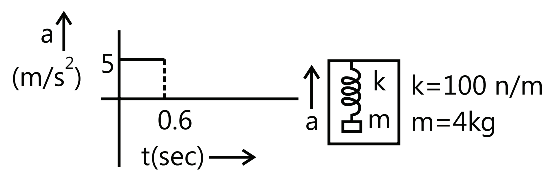
(c) Maximum momentum of 6kg block

Q.14 The resulting amplitude A' and the phase of the vibrations δ

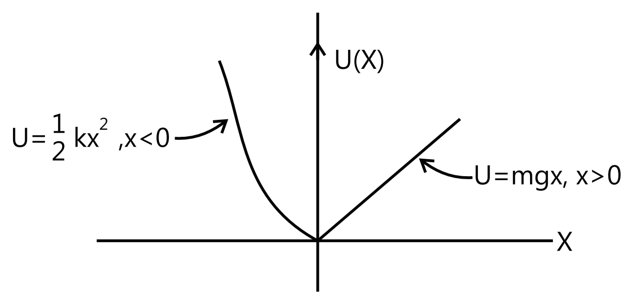
$$S = A\cos(\omega t) + \frac{A}{2}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{A}{4}\cos(\omega t + \pi)$$
$$+ \frac{A}{8}\cos\left(\omega t + \frac{3\pi}{2}\right) = A'\cos(\omega t + \delta)$$

are ___________ and __________ respectively.

Q.15 A spring block (force constant k=1000N/m and mass m=4kg) system is suspended from the ceiling of an elevator such that block is initially at rest. The elevator begins to move upwards at t=0. Acceleration time graph of the elevator is shown in the figure. Draw the displacement x (from its initial position taking upwards as positive) vs time graph of the block with respect to the elevator starting from t=0 to t=1 sec. Take $\pi^2 = 10$.

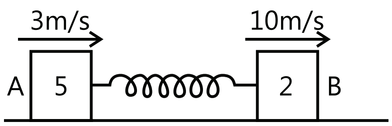


Q.16 A particle of mass m moves in the potential energy U shown below. Find the period of the motion when the particle has total energy E.



Q.17 The motion of a particle is described by x=30 $\sin(\pi t + \pi/6)$, where x is in cm and t in sec. Potential energy of the particle is twice of kinetic energy for the first time after t=0 when the particle is at position __________ after _______ time.

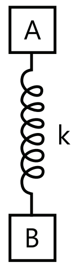
Q.18 Two blocks A (5kg) and B (2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



(a) Find the maximum extension of the spring.

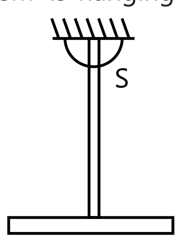
(b) When does the first maximum compression occurs after start.

Q.19 Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



Q.20 (a) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2\ J/rad$. The moment of inertia of rigid body is $10 kg\text{-}m^2$ about the axis of rotation.

(b) A simple pendulum of length $l = 0.5m$ is hanging from ceiling of a car. The car is kept on a horizontal plane The car starts accelerating on the horizontal road with acceleration of $5 m/s^2$. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.



Q.21 An object of mass 0.2kg executes SHM along the x-axis with frequency of $(25/\pi)Hz$. At the point $x = 0.04m$ the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is ___________.

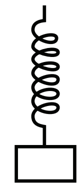
Q.22 A body of mass 1kg is suspended from a weightless spring having force constant 600N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

Q.23 A body A of mass $m_1 = 1kg$ and a body B of mass m_2 = 4kg are attached to the ends of a spring. The body a performs vertical simple harmonic oscillations of amplitude a=1.6 cm and angular frequency $\omega = 25$ rad/s. Neglecting the mass of the spring determine the maximum and minimum values of force the system exerts on the surface on which it rests. [Take $g = 10 m/s^2$]

Q.24 A spring mass system is hanging from the ceiling of an elevator in equilibrium Elongation of spring is l. The elevator suddenly starts accelerating downwards with accelerating $g/3$ find

(a) The frequency and

(b) The amplitude of the resulting SHM.



Exercise 2

Single Correct Choice Type

Q.1 A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the numerical value of magnitude of acceleration is equal to the numerical value of magnitude of velocity. The frequency of SHM (in $second^{-1}$) is:

(A) $2\pi\sqrt{3}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2\pi}$ (D) $\frac{1}{2\pi\sqrt{3}}$

Q.2 A particle executed SHM with time period T and amplitude A. The maximum possible average velocity in time $\frac{T}{4}$ is

(A) $\frac{2A}{T}$ (B) $\frac{4A}{T}$ (C) $\frac{8A}{T}$ (D) $\frac{4\sqrt{2}A}{T}$

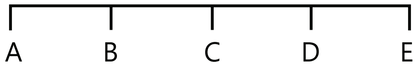
Q.3 A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance a/2 from the extreme position is

(A) a/T (B) 2a/T (C) 3a/T (D) a/2T

Q.4 Two particles are in SHM on same straight line with amplitude A and 2A and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles

(A) 45° (B) 90° (C) 135° (D) 180°

Q.5 A body performs simple harmonic oscillations along the straight line ABCDE with C as the midpoint of AE. Its kinetic energies at B and D are each one fourth of its maximum value. If AE=2R, the distance between B and D is



(A) $\frac{\sqrt{3}R}{2}$ (B) $\frac{R}{\sqrt{2}}$

(C) $\sqrt{3}R$ (D) $\sqrt{2}R$

Q.6 In an elevator, a spring clock of time period T_s (mass attached to a spring) and a pendulum clock of time period T_p are kept. If the elevator accelerates upwards

(A) T_s well as T_p increases

(B) T_s remain same, T_p increases

(C) T_s remains same, T_p decreases

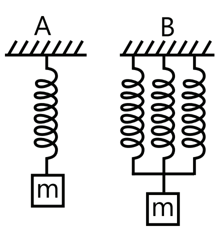
(D) T_s as well as T_p decreases

Q.7 Two bodies P & Q of equal mass are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the maximum velocities of them are equal during their motion, the ratio of amplitude of P to Q is:

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$

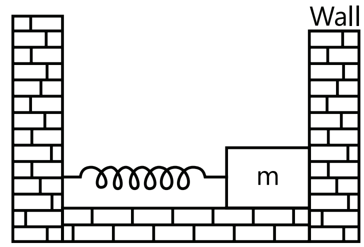
(C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

Q.8 The spring in figure. A and B are identical but length in A is three times each of that in B. the ratio of period T_A/T_B is



(A)√3 (B) 1/3 (C) 3 (D) 1/√3

Q.9 In the figure the block of mass m, attached to the spring of stiffness k is in contact with the completely elastic wall, and the compression in the spring is 'e'. The spring is compressed further by 'e' by displacing the block towards left and is then released. If the collision between the block and the wall is completely elastic then the time period of oscillations of the block will be:



(A) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m}{k}}$

(C) $\frac{\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$

Q.10 A 2 kg block moving with 10 m/s strikes a spring of constant $\pi 2$ N/m attached to 2 Kg block at rest kept on a smooth floor. The time for which rear moving block remain in contact with spring will be

(A) $\sqrt{2}$ sec (B) $\frac{1}{\sqrt{2}}$ sec

(C) 1sec (D) $\frac{1}{2}$ sec

Q.11 In the above question, the velocity of the rear 2 kg block after it separates from the spring will be:

(A) 0 m/s (B) 5 m/s

(C) 10 m/s (D) 7.5 m/s

Q.12 A rod whose ends are A & B of length 25 cm is hanged in vertical plane. When hanged from point A and point B the time periods calculated are 3 sec & 4 sec respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in ratio 9:4 at points A and B. Find the distance of the center of mass from point A.

(A) 9 cm (B) 5 cm (C) 25 cm (D) 20 cm

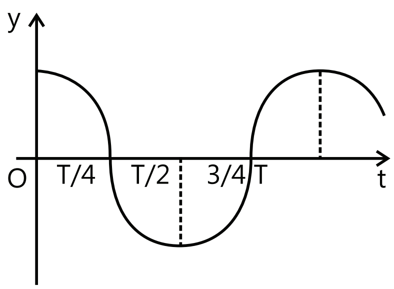
Q. 13 A circular disc has a tiny hole in it, at a distance z from its center. Its mass is M and radius R (R > z). A horizontal shaft is passed through the hole and held fixed so that the disc can freely swing in the vertical plane. For small disturbance, the disc performs SHM whose time period is the minimum for z =

(A) R/2 (B) R/3

(C) $R/\sqrt{2}$ (D) $R/\sqrt{3}$

Multiple Correct Choice Type

Q.14 The displacement-time graph of a particle executing SHM is shown which of the following statement is/are true?



(A) The velocity is maximum at t=T/2

(B) The acceleration is maximum at t=T

(C) The force is zero at t= 3T/4

(D) The potential energy equals the oscillation energy at t=T/2.

Q.15 The amplitude of a particle executing SHM about O is 10 cm. Then:

(A) When the K.E. is 0.64 of its max. K.E. its displacement is 6cm from O.

(B) When the displacement is 5cm from O its K.E.is 0.75 of its max. P.E.

(C) Its total energy at any point is equal to its maximum K.E.

(D) Its velocity is half the maximum velocity when its displacement is half the maximum displacement.

Q.16 A particle of mass m performs SHM along a straight line with frequency f and amplitude A.

(A) The average kinetic energy of the particle is zero.

(B) The average potential energy is $m\pi 2f2A^2$.

(C) The frequency of oscillation of kinetic energy is 2f.

(D) Velocity function leads acceleration by $\pi/2$

Q.17 A system is oscillating with undamped simple harmonic motion. Then the

(A) Average total energy per cycle of the motion is its maximum kinetic energy.

(B) Average total energy per cycle of the motion is $\frac{1}{\sqrt{2}}$ times its maximum kinetic energy.

(C) Root means square velocity $\frac{1}{\sqrt{2}}$ times its maximum velocity.

(D) Mean velocity is $\frac{1}{2}$ of maximum velocity.

Q.18 A spring has natural length 40 cm and spring constant 500 N/m. A block of mass 1 kg is attached at one end of the spring and other end of the spring is attached to ceiling. The block released from the position, where the spring has length 45cm.

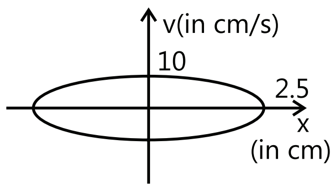
(A) The block will performs SHM of amplitude 5 cm.

(B) The block will have maximum velocity $30\sqrt{5}\,cm/sec$.

(C) The block will have maximum acceleration $15\,m/s^2$

(D) The minimum potential energy of the spring will be zero.

Q.19 The figure shows a graph between velocity and displacement (from mean position) of a particle performing SHM:



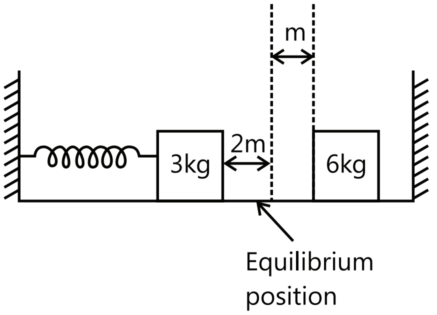
(A) The time period of the particle is 1.57s

(B) The maximum acceleration will be $40cm/s^2$

(C) The velocity of particle is $2\sqrt{21}cm/s$ when it is at a distance 1 cm from the mean position.

(D) None of these

Q.20 Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg block is attached to A Spring with a force constant



$k = 900Nm^{-1}$ Which is compressed 2 m from beyond the equilibrium position. The 6 kg mass is at rest at 1m from mean position 3kg mass strikes the 6kg mass and the two stick together.

(A) Velocity of the combined masses immediately after the collision is $10ms^{-1}$

(B) Velocity of the combined masses immediately after the collision is $5ms^{-1}$

(C) Amplitude of the resulting oscillations is $\sqrt{2}$m

(D) Amplitude of the resulting oscillation is $\sqrt{\frac{5}{2}}$m.

Q.21 A particle is executing SHM with amplitude A. time period T, maximum acceleration a_0 and maximum velocity v_0. Its starts from mean position at t-0 and at time t, it has the displacement A/2, acceleration a and velocity v then

(A) t=T/12 (B) $a = a_0 / 2$

(C) $v = v_0 / 2$ (D) t=T/8

Q.22 For a particle executing SHM, x=displacement from equilibrium position, v= velocity at any instant and a = acceleration at any instant, then

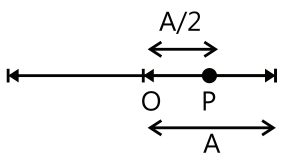
(A) v-x graph is a circle

(B) v-x graph is an ellipse

(C) a-x graph is a straight line

(D) a-v graph is an ellipse

Q.23 A particle starts from a point P at a distance of A/2 from the mean position O & travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A then the equation of motion of particle is:



(A) $x = A\sin\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$ (B) $x = A\sin\left(\frac{2\pi}{T}t+\frac{5\pi}{6}\right)$

(C) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$ (D) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{3}\right)$

Q.24 Two particles execute SHM with amplitude A and 2A and angular frequency ω and 2ω respectively. At t=0 they starts with some initial phase difference. At, t= difference is: $\frac{2\pi}{3\omega}$. They are in same phase. Their initial phase

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) π

Q.25 A mass of 0.2 kg is attached to the lower end of a massless spring of force-constant 200 N/m, the upper end of which is fixed to a rigid support. Which of the following statements is/are true?

(A) In equilibrium, the spring will be stretched by 1cm.

(B) If the mass is raised till the spring is in not stretched state and then released, it will go down by 2 cm before moving upwards.

(C) The frequency of oscillation will be nearly 5 Hz.

(D) If the system is taken to moon, the frequency of oscillation will be the same as on the earth.

Q.26 The potential energy of particle of mass 0.1kg, moving along x-axis, is given by U=5x(x-4)J where x is in meters. It can be concluded that

(A) The particle is acted upon by a constant force.

(B) The speed of the particle is maximum at x=2m

(C) The particle executes simple harmonic motion

(D) The period of oscillation of the particle is π /5 s

Q.27 The displacement of a particle varies according to the relation $x=3\sin 100t + \cos^2 50t$. Which of the following is/are correct about this motion.

(A) The motion of the particle is not SHM

(B) The amplitude of the SHM of the particle is 5 units

(C) The amplitude of the resultant SHM is $\sqrt{73}$ units.

(D) The maximum displacement of the particle from the origin is 9 units.

Q.28 The equation of motion for an oscillating particle is given by x=3sin (4πt) + 4cos (4πt), where x is in mm and t is in second

(A) The motion is simple harmonic

(B) The period of oscillation is 0.5 s

(C) The amplitude of oscillation is 5 mm

(D) The particle starts its motion from the equilibrium

Q.29 A linear harmonic oscillator of force constant $2 \times 10^6 Nm^{-1}$ and amplitude 0.01 m has a total mechanical energy of 160 J. Its

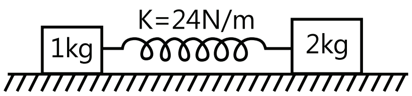
(A) Maximum potential energy is 100 J

(B) Maximum kinetic energy is 100 J

(C) Maximum potential energy is 160

(D) Minimum potential energy is zero.

Q.30 The two blocks shown here rest on a frictionless surface. If they are pulled apart by a small distance and released at t=0, the time when



1 kg block comes to rest can be

(A) $\frac{2\pi}{3}$sec (B) π *sec.*

(C) $\frac{\pi}{2}$sec (D) $\frac{\pi}{9}$sec

Assertion Reasoning Type

Q.31 Statement-I: A particle is moving along x-axis. The resultant force F acting on it at position x is given by F=-ax-b. Where a and b are both positive constants. The motion of this particle is not SHM.

Statement-II: In SHM restoring force must be proportional to the displacement from mean position.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.32 Statement-I: For a particle performing SHM, its speed decreases as it goes away from the mean position.

Statement-II: In SHM, the acceleration is always opposite to the velocity of the particle.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and Statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.33 Statement-I: Motion of a ball bouncing elastically in vertical direction on a smooth horizontal floor is a periodic motion but not an SHM.

Statement-II: Motion is SHM when restoring force is proportional to displacement from mean position.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true

Q.34 Statement-I: A particle, simultaneously subjected to two simple harmonic motions of same frequency and same amplitude, will perform SHM only if two SHM's are in the same direction

Statement-II: A particle, simultaneously subjected to two simple harmonic motions of same frequency and same amplitude, perpendicular to each other the particle can be in uniform circular motion.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.35 Statement-I: In case of oscillatory motion the average speed for any time interval is always greater than or equal to its average velocity.

Statement-II: Distance travelled by a particle cannot be less than its displacement.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

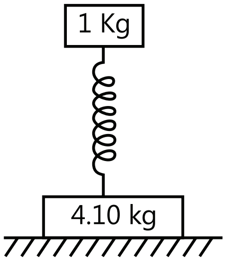
(B) Statement-I is true. statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Comprehension Type

Paragraph 1: When force acting on the particle is of nature $F = -kx$, motion of particle is SHM, Velocity at extreme is zero while at mean position it is maximum. In case of acceleration situation is just reverse. Maximum displacement of particle from mean position on both sides is same and is known as amplitude. Refer to figure One kg block performs vertical harmonic oscillations with amplitude 1.6 cm and frequency 25 rad s^{-1}.



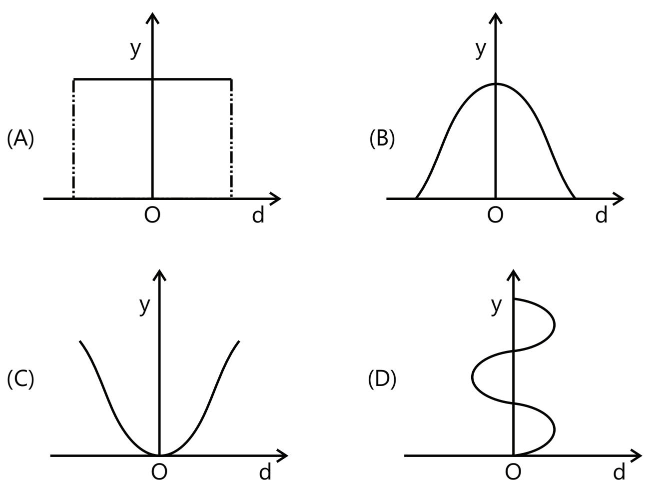
Q.36 The maximum value of the force that the system exerts on the surface is

(A) 20 N (B) 30 N (C) 40 N D) 60 N

Q.37 The minimum force is

(A) 20 N (B) 30 N (C) 40 N (D) 60 N

Paragraph 2: The graphs in figure show that a quantity y varies with displacement d in a system undergoing simple harmonic motion.



Which graphs best represents the relationship obtained when Y is

Q. 38 The total energy of the system

(A) I (B) II (C) III (D) IV

Q.39 The time

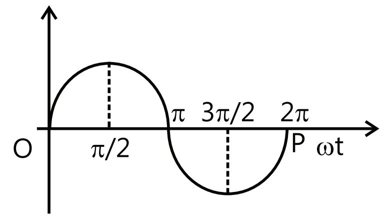
(A) I (B) II (C) III (D) IV

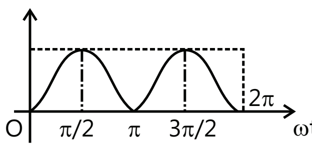
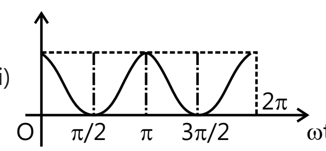
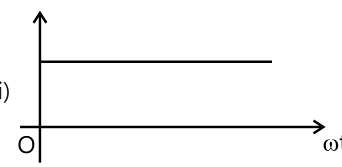
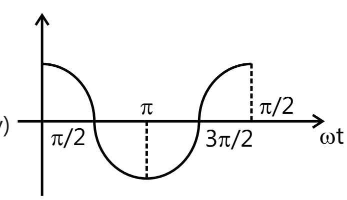
Q.40 The unbalanced force acting on the system

(A) I (B) II (C) III (D) None

Match the Columns

Q.41 The graph plotted between phase angle (ϕ) and displacement of a particle from equilibrium position (y) is a sinusoidal curve as shown below. Then the best matching is



Column A	Column B
(a) K.E. versus phase angle curve	(i) 
(b) P.E. versus phase angle curve	(ii) 
(c) T.E. versus phase angle curve	(iii) 
(d) Velocity versus phase angle curve	(iv) 

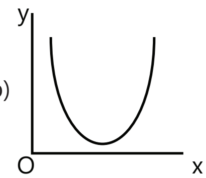
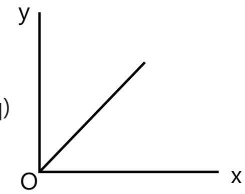
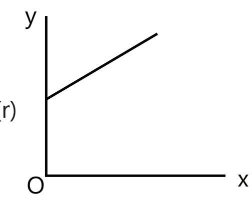
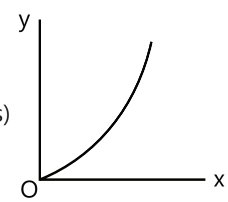
(A) (a)-(i), (b)-(ii), (c)-(iii) & (d)-(iv)

(B) (a)-(ii), (b)-(i), (c)-(iii) & (d)-(iv)

(C) (a)-(ii), (b)-(i), (c)-(iv) & (d) – (iii)

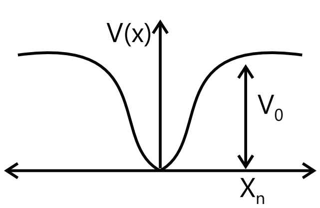
(D) (a)-(ii), (b)-(iii), (c)-(iv) & (d)-(i)

Q.42 Column I is a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 x 4 matrix given in the ORS.

Column I	Column II
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)	(p) 
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	(q) 
(C) Range of projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle.	(r) 
(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)	(s) 

Previous Years' Questions

Paragraph 1: When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $v(x) = \alpha x^2 (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \ge X_0$ (see figure below) ***(2010)***



Q.1 If the total energy of the particle is E, it will perform periodic motion only if

(A) E< 0 (B) E>0

(C) $V_0 > E > 0$ (D) $E > V_0$

Q.2 For periodic motion of small amplitude A, the time period t of this particle is proportional to

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Q.3 The acceleration of this particle for $|x| > X_0$ is

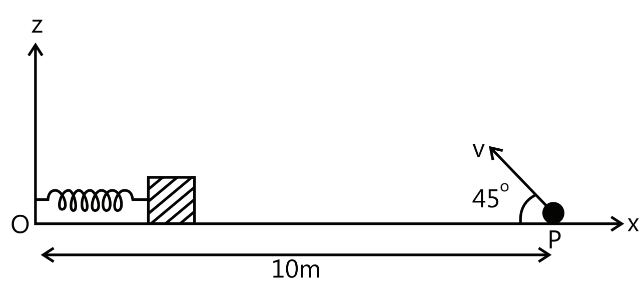
(A) Proportional to V_0

(B) Proportional to $\frac{V_0}{mX_0}$

(C) Proportional to $\sqrt{\frac{V_0}{mX_0}}$

(D) Zero

Q.4 A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v form point P at an angle of $45°$ as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s²) ***(2012)***



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s

(C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Q.5 A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0. It collides elastically with a rigid wall. After this collision, ***(2013)***

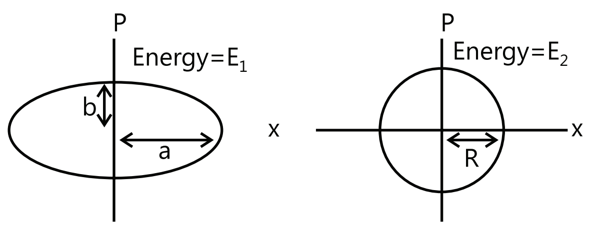
(A) The speed of the particle when it returns to its equilibrium position is u_0

(B) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(C) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) The time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

Q.6 Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are) ***(2015)***



(A) $E_1\,\omega_1 = E_2\,\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\,\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

Q.7 A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (< M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M? ***(2016)***

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases

Q.8 Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situations in column I with the characteristics in column II. ***(2007)***

Column I	Column II
(A) The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x_2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration α. The motion of the object is observed from the elevator during the period it maintain this acceleration.	(r) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

Q.9 A linear harmonic oscillator or force constant 2×10^6 N/m and amplitude 0.01m has a total mechanical energy of 160 J. Its ***(1989)***

(A) Maximum potential energy is 100 J

(B) Maximum kinetic energy is 100J

(C) Maximum potential energy is 160J

(D) Maximum potential energy is zero

Q.10 Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by $45°$, then ***(1999)***

(A) The resultant amplitude is $\left(1+\sqrt{2}\right)a$

(B) The phase of the resultant motion relative to the first is 90°

(C) The energy associated with the resulting motion is $\left(3+2\sqrt{2}\right)$ times the energy associated with any single motion

(D) The resulting motion is not simple harmonic

Q.11 Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represent SHM ***(2006)***

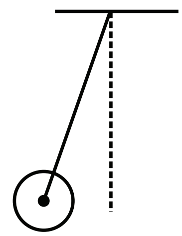
(A) For any value of A, B and C (except C=0)

(B) If A=-B, C=2B, amplitude=$\left|B\sqrt{2}\right|$

(C) If A=B; C=0

(D) If A=B; C=2B, amplitude=$\left|B\right|$

Q.12 A metal rod of length L and mass m is pivoted at one end. A thin disk of mass M and radius R $(<L)$ is attached at its center to the free end of the rod. Consider two ways the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true? ***(2011)***



(A) Restoring torque in case A=Restoring torque in case B

(B) Restoring torque in case A<Restoring torque in case B

(C) Angular frequency for case A>Angular frequency for case B

(D) Angular frequency for case A<, angular frequency for case B

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 7 Q.8 Q.20
Q.24 Q.25

Exercise 2

Q.3 Q.9 Q.15

Previous Years' Questions

Q.9 Q.10 Q.11
Q.14 Q.15 Q.16

JEE Advanced/Boards

Exercise 1

Q.4 Q.6 Q.8
Q.18 Q.23 Q.24

Exercise 2

Q.1 Q.2 Q.4
Q.5 Q.9 Q.14
Q.15 Q.20 Q.25
Q.29 Q.30 Q.42

Answer Key

JEE Main/Boards

Exercise 1

Q.1 0.5 rad **Q.2** 1.048 s **Q.3** 0.61 rad

Q.4 0.5s **Q.5** 0.71 a **Q.6** 7.56J

Q.7 $0.35\,ms^{-1}$, 0.06 J **Q.8** 1.5J **Q.9** 86.4s

Q.10 $141°.4'$ **Q.11** $0.612\,s^{-1}$

Q.18 (a) $y = 2\sin\left(\frac{2\pi t}{3} + \frac{\pi}{3}\right)$ (b) $y = 3\cos\left(\frac{\pi}{30}t\right)$ **Q.19** $2\pi / 3$rad

Q.20 (a) 6m (b) $-48.99ms^{-1}$ **Q.21** 19.67m **Q.22** 4.86s; 5.06m

Q.23 $2\sqrt{3}$cm; $2\sqrt{2}$cm **Q.24** 0.1875 J; 0.0625 J, 0.25J **Q.25** 3.2 Hz

Q.26 $\frac{1}{(2\pi)}\left[\frac{k_1k_2(k_3+k_4)}{\{(k_1+k_2)\times(k_3+k_4)+k_1k_2\}m}\right]^{1/2}$

Exercise 2

Single Correct Choice Type

Q.1 A **Q.2** C **Q.3** C **Q.4** D **Q.5** A **Q.6** C

Q.7 C **Q.8** B **Q.9** C **Q.10** B **Q.11** B **Q.12** C

Q.13 A **Q.14** B **Q.15** C

Previous Years' Questions

Q.1 C **Q.2** B **Q.3** D **Q.4** B **Q.5** A **Q.6** B

Q.7 D **Q.8** A **Q.9** A **Q.10** A **Q.11** A **Q.12** A

Q.13 D **Q.14** C **Q.15** D **Q.16** A **Q.17** B **Q.18** D

Q.19 B **Q.20** C **Q.21** C

JEE Advanced/Boards

Exercise 1

Q.1 $1/\sqrt{3}$ **Q.2** 2 m/s **Q.3** 1.8a

Q.4 $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$ **Q.5** $x = 0.1\sin(4t + \pi / 4)$

Q.6 3cm, $x = 10 - 3\sin 5t; \Delta E = 0.135J$

Q.7 (a) $k = \frac{2mg}{b-a}$: (c) $\frac{ab}{b-a}$, (b) $\frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(M+m)}}$

Q.8 $f = \frac{1}{\pi}; E = 4\pi^2 \times 10^{-5} J; v = 2\pi \times 10^{-2} m/s$

Q.9 $0.8t + 0.12 \sin 10t$

Q.10 (a) 0.4 m, (b) $\frac{\pi}{5}$sec., (c) x = 0.2 – 0.4 cosωt

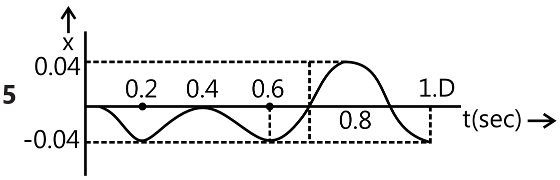
Q.11 (i) $x_0 = 2m$; (ii) $T = \sqrt{2}\pi$ sec.; (iii) $2\sqrt{3}$ m

Q.12 $25\sqrt{2}N$

Q.13 (a) $\frac{\pi}{10}$sec, (b) 6cm (c) 2.40kgm / s.

Q.14 $\frac{3\sqrt{5}A}{8}\tan^{-1}\left(\frac{1}{2}\right)$

Q.15



Q.16 $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

Q.17 $10\sqrt{6}$cm, $\frac{1}{\pi}\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) - \frac{1}{6}$sec

Q.18 (a) 25cm, (b) $3\pi/56$ seconds

Q.19 $2\pi\sqrt{\frac{17L}{18g}}$

Q.20 (a) 2sec, (b) $T = \frac{2}{5^{1/4}}$sec

Q.21 0.06m

Q.22 10πHz, $\frac{5\sqrt{37}}{6}$cm

Q.23 60N, 40N

Q.24 (a) $\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, (b) $\frac{L}{3}$

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** D **Q.3** C **Q.4** C **Q.5** C **Q.6** C
Q.7 B **Q.8** C **Q.9** A **Q.10** C **Q.11** A **Q.12** D
Q.13 C

Multiple Correct Choice Type

Q.14 B, C, D **Q.15** A, B, C **Q.16** B, C **Q.17** A, C **Q.18** B, C, D **Q.19** A, B, C
Q.20 A, C **Q.21** A, B **Q.22** B, C, D **Q.23** B, D **Q.24** B, C **Q.25** A, B, C, D
Q.26 B, C, D **Q.27** B, D **Q.28** A, B, C **Q.29** B, C **Q.30** A, B, C

Assertion Reasoning Type

Q.31 D **Q.32** C **Q.33** A **Q.34** D **Q.35** A

Comprehension Type

Paragraph 1: **Q.36** D **Q.37** C

Paragraph 2: **Q.38** A **Q.39** D **Q.40** D

Match the Columns

Q.41 B

Q.42 A → p, s; B → q, r, s; C → s; D → q

Previous Years' Questions

Q.1 C	**Q.2** B	**Q.3** D	**Q.4** A
Q.5 A, D	**Q.6** B, D	**Q.7** A, B, D	**Q.8** A → p; B → q, r; C → p; D → r, q
Q.9 A	**Q.10** A, C	**Q.11** A, B, D	**Q.12** A, D

Solutions

JEE Main/Boards

Exercise 1

Sol 1: y(t) = 10 sin (20t + 0.5)

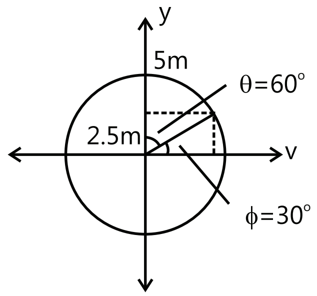
A = 10 m

ω = 20 rad./sec

ϕ = 0.5 radians

$f = \frac{w}{2\pi} = \frac{20}{2\pi} = \frac{10}{\pi}$ hz

$T = \frac{1}{f} = \frac{\pi}{10}$ sec



Sol 2: $V = \omega\sqrt{A^2 - y^2}$

$4 = \omega\sqrt{A^2 - 9}$

$3 = \omega\sqrt{A^2 - 16}$

$T = \frac{2\pi}{\omega}$ = 2π secω = 1 sec

$t = \frac{\theta}{360^\circ} \times T = \frac{60}{360^\circ} \times 2\pi$ sec

$t = \frac{\pi}{3}$ sec

Sol 3: y = 0.4 sin (440 t + 0.61)

(i) Amplitude = 0.4 m

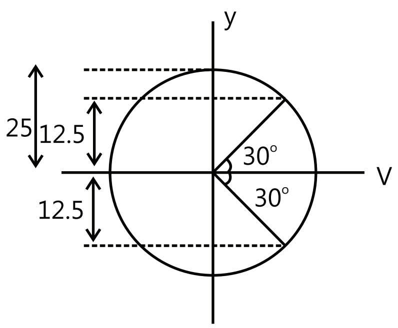
(ii) ω = 440 rad.sec

(iii) $f = \frac{\omega}{2\pi}, \frac{220}{\pi}$ hz

(iv) $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{\pi}{220}$ sec

(v) Initial phase = 0.61 radians

Sol 4: A = 25 cm, T = 3s



$\Rightarrow t = \frac{60^\circ}{360^\circ} \times = \frac{1}{2}$ sec.

Sol 5: Amplitude = 0

Total energy = $\frac{1}{2}$ Ka^2

Potential energy = $\frac{1}{2}$ Kx^2

$\frac{1}{2}Kx^2 = \frac{1}{2} \times \frac{1}{2}Ka^2 \Rightarrow x = \frac{a}{\sqrt{2}}$

Sol 6: m = 8 kg

a = 30 cm

$k \times 0.3 = 60 \Rightarrow k = \frac{60}{0.3} = 200$ n/m

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T = 2\pi\sqrt{\frac{8}{200}} = \frac{2\pi}{5} = 0.4\pi$

(a) T = 0.4 π sec.

(b) $a = \frac{-k}{m}x$

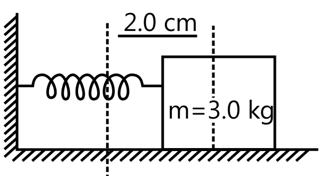
$a = \frac{-200}{84} \times 0.12 = 3 m/sec^2$

P.E. = $\frac{1}{2}kx^2 = \frac{1}{2} \times 200 \times (0.12)^2 = 1.44$ J

K.E. = $\frac{1}{2}k(A^2 - x^2)$

= 100 x (0.09 – 0.0144) = 7.56 J

Sol 7: k = 1200 Nm^{-1}



(a) $w = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20$ rad/s.

$v = \omega\sqrt{A^2 - x^2} = 20\frac{\sqrt{4-1}}{100} = \frac{\sqrt{3}}{5}$

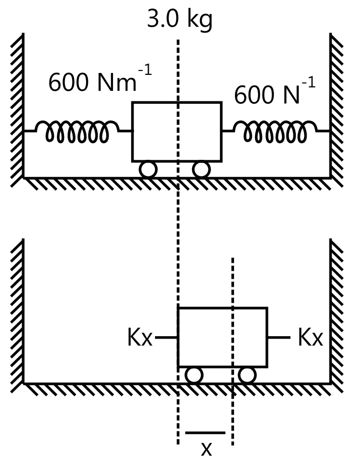
v = 0.35 m/s

(b) P.E. = $\frac{1}{2}kx^2 = \frac{1}{2} \times 1200 \times \left(\frac{1}{100}\right)^2$

$= 600 \times \frac{1}{100 \times 100}$

P.E. = 0.06 J

Sol 8:



$k_{eq} = 2k = 1200\ Nm^{-1}$

Total energy stored = $\frac{1}{2}kA^2$

$= \frac{1}{2} \times 1200 \times \left(\frac{1}{2}\right)^2 = 1.5$ Joules.

Sol 9: $T = 2\pi\sqrt{\frac{\ell}{g}}$; $T \propto \ell^{1/2}$

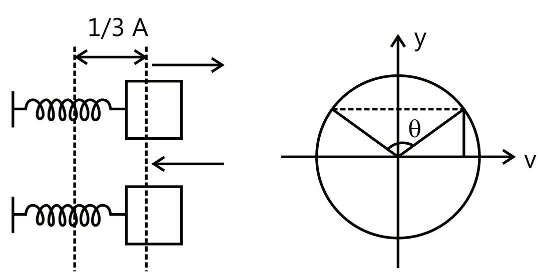
$\ell \rightarrow 0.998$ l

$\ell \rightarrow (0.998)^{1/2}$ T

T → 0.999 T

Error in a day = 0.001 × (60 × 60 × 24) = 86.4 sec

Sol 10:



Phase Difference = θ = 2 $\cos^{-1}$ (1/3) = 141.05°

Sol 11: Elevator moving up

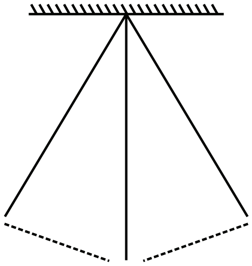
Frequency of seconds pendulum = f_0= 0.5 Hz

$g_{eff} = g + \frac{g}{2} = \frac{3}{2}g$

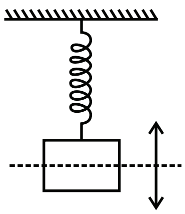
$f = \frac{1}{2\pi}\sqrt{\frac{g_{eff}}{\ell}} = \sqrt{\frac{3}{2}}f_0 = \sqrt{\frac{3}{2}} \times 0.5$ Hz; f = 0.61 Hz

Sol 12: Periodic motion: A motion which repeats itself after equal intervals of time is called periodic motion

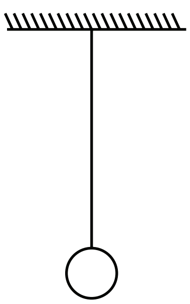
eq; motion of a pendulum



Oscillatory motion: A body is said to possess oscillatory or vibratory motion if it moves back and forth repeatedly about a mean position. For an oscillatory motion, a restoring force is required.



Sol 13: Simple Pendulum: A simple pendulum is a weight suspended from a pivot so that it can swing freely.



Time period = $2\pi\sqrt{\frac{\ell}{g}}$

$\ell \rightarrow$ length of pendulum

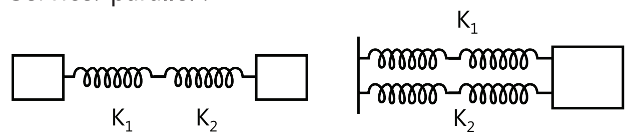
g $\rightarrow$ acceleration due to gravity

Frequency = f = $\frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

Sol 14: Refer spring mass system and ex.3

Sol 15: Spring factory: It is a mesure of the stiffness of a spring

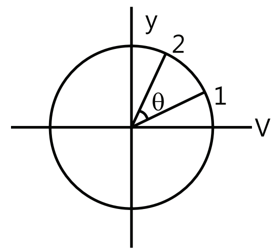
Service:-parallel :



$k = \frac{k_1k_2}{k_1+k_2}$ $k = k_1 + k_2$

Sol 16: Phase: Phase of a vibrating particle at any instant is the state of the vibrating particle regarding it's displacement and direction of vibration at that particular instant. It is denoted by ϕ.

Phase difference is the difference in phases of two vibrating particles at a given time.



Particle 1 lags in phase by θ.

i.e. $\phi_2 - \phi_1 = \theta$

Angular frequency:- It is frequency f multiplied by a numerical quantity ω. It is denoted by ω.

$\omega = 2\pi f = \frac{2\pi}{T}$

f $\rightarrow$ frequency

T $\rightarrow$ Time period

Displacement in periodic motion: It is the displacement from the mean/equilibrium position.

Sol 17: $a = -\frac{d^2x}{dt^2} = -\omega^2x$

$a = -\omega^2 x \omega = \frac{2\pi}{T}$

$a = -\frac{4\pi^2x}{T^2}$ x $\rightarrow$ displacement

T $\rightarrow$ time period

$v = \omega\sqrt{A^2 - x^2}$ v $\rightarrow$ velocity

A $\rightarrow$ Amplitude

x $\rightarrow$ Displacement

Sol 18: Figure (a) Initial phase = $\phi = 60° = \frac{\pi}{3}$

$y = 2\sin(\omega t + \phi)$

$y = 2\sin\left(\frac{2\pi}{3}t + \frac{\pi}{3}\right)$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$

Figure (b) initial phase = $\phi = \frac{\pi}{2}$

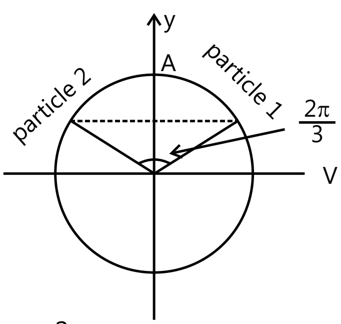
A = 3 cm

$\omega = \frac{2\pi}{60} = \frac{\pi}{30}$

$y = A\sin(\omega t + \phi)$

$y = 3\sin\left(\frac{\pi t}{3} + \frac{\pi}{2}\right)$ cm

Sol 19:



$\phi = \frac{2\pi}{3}$

Sol 20: $x = 6\cos(3\pi t + \pi/3)$ metres

(a) A = 6m

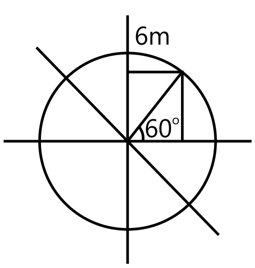
(b) $V = \omega\sqrt{A^2 - x^2}$

w = 3π

$T = \frac{2\pi}{\omega}$ = 2/3 sec.

At t = 2s particle will complete 3 orcillations

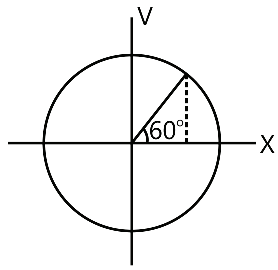
So the position will be same as at t = 0 s.



$x = 6\sin(3xt + 5\pi/6)$

$x = 6\cos 60° = 6\sin 30°$

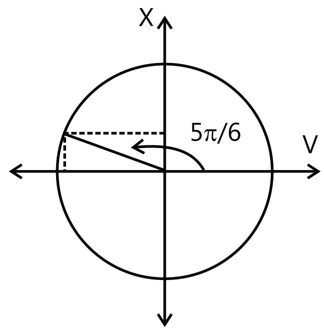
x = 3cm



$v = \omega\sqrt{A^2 - x^2} = 3\pi\sqrt{36 - 9}$

$v = 6\sqrt{3}\,\pi$

$v = -48.97\ ms^{-1}$



Sol 21: T = 20 s V = 5 ms^{-1}

$\Rightarrow \theta = \frac{2}{20} \times 360° \Rightarrow \theta = 36°$

$5 = A\omega\cos 36°$

$\omega = \frac{2\pi}{20} = \frac{\pi}{\omega}$

$5 = A\frac{\pi}{10}\cos 36°$

$A = \frac{50}{\pi\cos 36°} = 19.68$ m

Amplifide of SHM = 19.68 m

Sol 22: x = 1m v = 3 ms^{-1}

x = 2mv = 2 ms^{-1}

$v = 3 = \omega\sqrt{A^2 - 1}$

$3 = \omega\sqrt{A^2 - 1}$

$2 = \omega\sqrt{A^2 - 4}$

$\Rightarrow \frac{9}{4} = \frac{A^2-1}{A^2-4} \Rightarrow 9A^2 - 36 = 4A^2 - 4$

$\Rightarrow 5A^2 = 32 \Rightarrow A = \sqrt{\frac{32}{5}} = \sqrt{6.4}$

$\Rightarrow A = 2.53$ m

$3 = \omega\sqrt{6.4-1}$

$w = \frac{3}{\sqrt{5.4}} = 1.29$ rad/s

Period of motion: $T = \frac{2\pi}{\omega} = 4.86$ s

Length of path = 2A = 5.06 m

Sol 23: A = 4 cm

$v_{max} = A\omega$

$v = \frac{v_{max}}{2} = \frac{A\omega}{2} = \omega\sqrt{A^2-\omega^2}$

$A^2 - x^2 = \frac{A^2}{4}$

$x = \frac{\sqrt{3}}{2}A$

$x = \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3} = 3.464$ cm

P.E. = K.E.

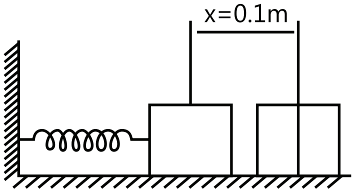
$\Rightarrow P.E. = \frac{1}{2} \times \frac{1}{2}kA^2$

$\frac{1}{2}kx^2 = \frac{1}{4}kA^2$

$x = 2\sqrt{2} = 2.828$ m

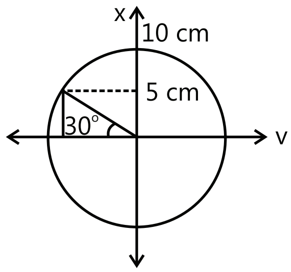
Sol 24: $k = 50\ Nm^{-1}$

m = 1kg



$\omega^2 = \frac{k}{m} = \frac{50}{1} = 50$

$\omega = 5\sqrt{2}$ rad/s



$v = -A\omega\cos 30°$

$v = \frac{10}{100} \times 5\sqrt{2} \times \frac{\sqrt{3}}{2}$; $v = \sqrt{\frac{3}{2}}$

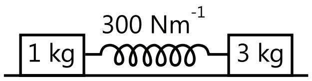
$v = 0.61\ ms^{-1}$

$K.E. = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times 1 \times \frac{3}{8} = 0.1875$ J

$P.E. = \frac{1}{2} \times 50 \times (0.05)^2 = 0.0625$ J

Total energy $= \frac{1}{2} \times 50 \times (0.1)^2 = 0.25$ J

Sol 25:



For two mass system.

We take effective mass instead of mass to calculate frequency.

$\mu = \frac{m_1m_2}{m_1+m_2} = \frac{1\times 3}{1+3} = \frac{3}{4}$ kg

$\omega^2 = \frac{k}{\mu} = \frac{300}{3/4} = 400$

$\omega = 20$ rad/sec.

$f = \frac{10}{\pi}$ Hz $\cong$ 3.2 Hz

Sol 26: $k_{34} = k_3 + k_4$

$\frac{1}{k_{1234}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_{34}}$

$= \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{(k_3+k_4)}$

$= \frac{k_2(k_3+k_4)+k_1(k_3+k_4)+k_1k_2}{k_1k_2(k_3+k_4)}$

$\frac{1}{k_{1234}} = \frac{(k_1+k_2)(k_3+k_4)+k_1k_2}{k_1k_2(k_3+k_4)}$

$$\omega = \left(\frac{k_{1234}}{m}\right)^{1/2}$$

$$f = \frac{1}{2\pi}\left(\frac{k_{1234}}{m}\right)^{1/2}$$

$$f = \frac{1}{2\pi}\left(\frac{k_1k_2(k_3+k_4)}{(k_1+k_2)(k_3+k_4)+(k_1k_2)m}\right)^{1/2}$$

Exercise 2

Single Correct Choice Type

Sol 1: (A) $y = 5\sin(\pi t + 4\pi)$

$A = 5\ T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$ sec

$A = 5$; $T = 2$ sec

Sol 2: (C) $a_{max.} = A\omega^2$

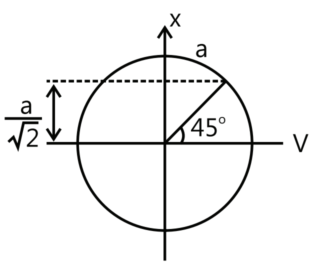
$v_{max.} = A\omega$

Double ω; half the amplitude

Sol 3: (C) $A = \frac{0.8}{2} = 0.4$ m

$f = \frac{30}{60} = \frac{1}{2}$ hz $T = 2$ sec

Sol 4: (D)



Sol 5: (A) $2\pi t + \frac{\pi}{3} = \pi$

$2\pi t = \frac{2\pi'}{3}$

$t = 1/3$ sec

Sol 6: (C) $T = 1.2$ sec

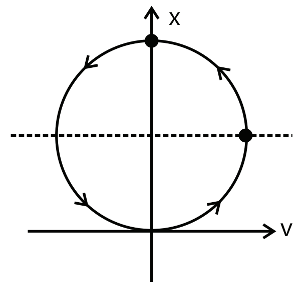
$A = 8$ cm

$\theta = \cos^{-1}\frac{5}{8}$; $\theta = 51.31°$

$t = \frac{\theta}{360} \times 1.2$; $t = 0.17$ sec

Sol 7: (C) $x = A + a\sin\omega t$

$t = \frac{5}{4}T$



Distance in one rev. = 4A

Total distance covered = 4A + A = 5A

Sol 8: (B) $k_1 = \frac{k\times k}{k+k} = \frac{k}{2}$

$k_2 = k + k = 2k$

$t \propto k^{-1/2}\ \frac{T_1}{T_2} = \sqrt{\frac{k_2}{k_1}} = 2$

Sol 9: (C) $y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ $y_2 = \sin\omega t$

$y_1 + y_2\ 2\sin\left(\omega t + \frac{\pi}{6}\right)\cos\frac{\pi}{6}$

$= \sqrt{3}\sin\left(\omega t + \frac{\pi}{6}\right)$

Sol 10: (B) $y = A\sin\omega t + A\cos\omega t$

$= 2A\left(\sin\omega t + \sin\left(\frac{\pi}{2} + \omega t\right)\right)$

$= 2A\sin\left(\omega t + \frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) = \sqrt{2}A\sin\left(\omega t + \frac{\pi}{4}\right)$

$T.E. = \frac{1}{2} \times m\omega^2 \times \left(\sqrt{2}A\right)^2$

$T.E. = m\omega^2A^2$

Sol 11: (B) $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 \Rightarrow A = \sqrt{\frac{mv^2}{k}}$

Sol 12: (C) Amplitude dose not depend on frame of reference.

Sol 13: (A) $s = ut + \frac{1}{2}at^2 \Rightarrow -a = 0 + \frac{1}{2} \times (-5) \times t^2$

$t = \sqrt{\frac{2a}{5}}$

$T = 4t = 4\sqrt{\frac{2a}{5}}$

Sol 14: (B) $\frac{1}{2}kA^2\left(1-\frac{3}{4}\right) = \frac{1}{2}mv^2$

$\Rightarrow v = \left(\frac{k}{m}\right)^{1/2}\frac{A}{2}$

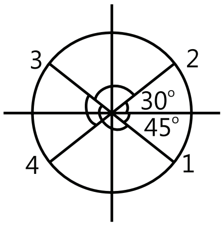
$\Rightarrow v^1 = \left(\frac{k}{m}\right)^{1/2}\frac{A}{4}, \omega_2 = \frac{\omega_1}{\sqrt{2}}$

$T.E. = \frac{3}{8}kA^2 + \frac{2m}{2}\left(\frac{k}{m}\right)\frac{A^2}{16} = \frac{7}{16}kA^2$

$\Rightarrow \frac{7}{16}kA^2 = \frac{1}{2}kA'^2$

$A' = \sqrt{\frac{7}{8}}A$

Sol 15: (C)



$\theta_{24} = 195°$

$\theta_{12} = 75°$

$q_{31} = 165°$

Previous Years' Questions

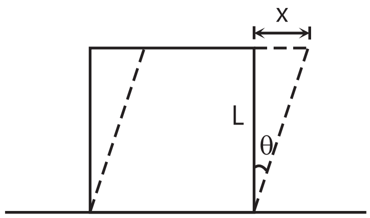
Sol 1: (C) In SHM frequency with which kinetic energy oscillation is two times the frequency of oscillation of displacement.

Sol 2: (B) $(v_M)_{max} = (v_N)_{max}$

$\therefore \omega_M A_M = \omega_N A_N$

or $\frac{A_M}{A_N} = \frac{\omega_N}{\omega_M} = \sqrt{\frac{k_2}{k_1}} \left(\because \omega = \sqrt{\frac{k}{m}}\right)$

Sol 3: (D)



Modulus of rigidity, $\eta = F/A\theta$

Here, $A = L^2$ and $\theta = \frac{x}{L}$

Therefore, restoring force force is

$F = -\eta A\theta = -\eta Lx$

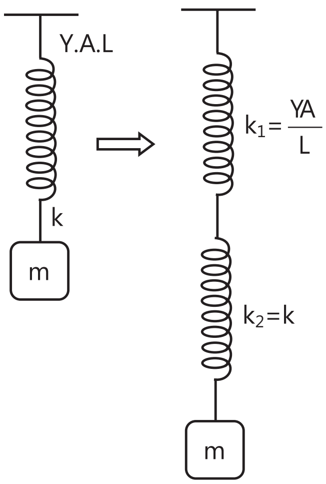
Or acceleration, $a = \frac{F}{M} = -\frac{\eta L}{M}x$

Since, $a \propto -x$, oscillations are simple harmonic in nature, time period of which is given by

$T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|} = 2\pi\sqrt{\left|\frac{x}{a}\right|}$

$= 2\pi\sqrt{\frac{M}{\eta L}}$

Sol 4: (B) $K_{e\theta} = \frac{k_1k_2}{k_1+k_2} = \frac{\frac{YA}{L}}{\frac{YA}{L}+k} = \frac{YAk}{YA+Lk}$



$\therefore T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(YA+Lk)}{YAk}}$

Note Equivalent fore constant for a wire is given by k $= \frac{AY}{L}$. Because in case of a wire. F = $\left(\frac{AY}{L}\right)\Delta L$ and in case of spring F = k.Δx. Comparing these two, we find k of wire = $\frac{AY}{L}$

Sol 5: (A) U(x) = k|x|^3

$\therefore [k] = \frac{[U]}{[x^3]} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$

Now, time period may depend on

$T \propto (\text{mass})^x(\text{amplitude})^y(k)^z$

$[M^0L^0T]=[M]^x[L]^y[ML^{-1}T^{-2}]^z=[M^{x+z}L^{y-2}T^{-2z}]$

Equating the powers, we get

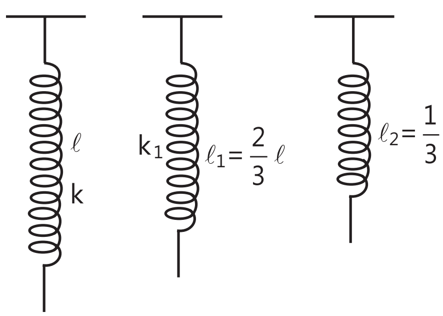
$-2z = 1$ or $z = -1/2$

$y - z = 0$ or $y = z = -½$

Hence, $T \propto (\text{amplitude})^{-1/2} \propto (a)^{-1/2}$

or $T \propto \frac{1}{\sqrt{a}}$

Sol 6: (B)



$\lambda_1 = 2\lambda_2$

$\therefore \lambda_1 = \frac{2}{3}l$

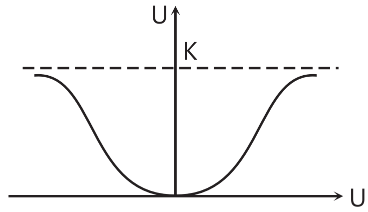
Force constant $k \propto \frac{1}{\text{length of spring}}$

$\therefore k_1 = \frac{3}{2}k$

Sol 7: (D) $U(x) = k(1 - e^{-x^2})$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore, U versus x graph will be as shown. At origin.

Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is zero because.



$F = \frac{-dU}{dx} = -$ (slope of U-x graph) = 0.

Therefore, origin is the stable equilibrium position. Hence, particle will oscillate simple harmonically about x = 0 for small displacement. Therefore, correct option is (d).

(a), (b) and (c) options are wrong due to following reasons.

(a) At equilibrium position $F = \frac{-dU}{dx} = 0$ i.e., slope of U-x graph should be zero and from the graph we can see that slope is zero at x = 0 and x = $\pm\infty$

Now among these equilibriums stable equilibrium position is that where U is minimum (Here x=0). Unstable equilibrium position is that where U is maximum (Here none).

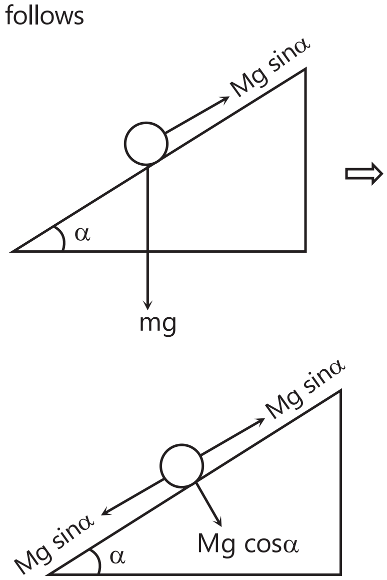
Neutral equilibrium position is that where U is constant (Here x = $\pm\infty$)

Therefore, option (a) is wrong.

(b) For any finite non-zero value of x, force is directed towards the origin because origin is in stable equilibrium position. Therefore, option (b) is incorrect.

(c) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, option (c) is also wrong.

Sol 8: (A) Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as follows



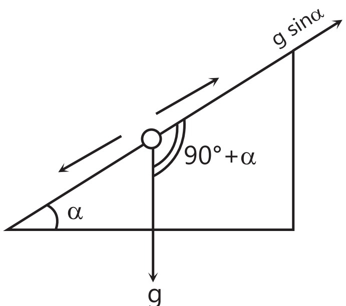
$\therefore$ Net force on the bob is F_{net} = mg cosα

or Net acceleration of the bob is g_{eff} = g cosα

$$T = 2\pi\sqrt{\frac{L}{g_{eff}}}$$

$$\text{or } T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

Note: Whenever point of suspension is accelerating



$$\text{Take } T = 2\pi\sqrt{\frac{L}{g_{eff}}}$$

Where $\vec{g}_{eff} = \vec{g} - \vec{a}$

$\vec{a}$ = acceleration of point of suspension.

In this question $\vec{a}$ = g sin α (down the plane)

$\therefore |\vec{g} - \vec{a}| = g_{eff}$

$$= \sqrt{g^2 + (g\sin\alpha)^2 + 2(g)(g\sin\alpha)\cos(90° + \alpha)}$$

= g cosα

Sol 9: (A) In SHM, velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme positions. Therefore, the time taken for the particle to go from O to A/2 will be less than the time taken to go it from A/2 to A, or $T_1 < T_2$

Note From the equation of SHM we can show that

$t_1 = T_{0-A/2}$ = T/12

and $t_2 = T_{A/2-A}$ = T/6

So, that $t_1 = t_2 = T_{0-A}$ = T/4

Sol 10: (A) Potential energy is minimum (in this case zero) at mean position (x = 0) and maximum at extreme positions (x = ± A).

At time t = 0, x = A. Hence, PE should be maximum. Further in graph III, PE is minimum at x = 0 Hence, this is also correct.

Sol 11: (A) Angular frequency of the system,

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Maximum acceleration of the system will be, ω^2 A or $\frac{kA}{2m}$

This acceleration to the lower block is provided by friction.

Hence, $f_{max} = ma_{max}$

$$= m\omega^2 A = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$$

Sol 12: (A) y = kt^2

$\frac{d^2y}{dt^2}$ = 2k or a_y = 2m/s^2(as k = 1 m/s^2)

$$T_1 = 2\pi\sqrt{\frac{\ell}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{\ell}{g+a_y}}$$

$$\therefore \frac{T_1^2}{T_2^2} = \frac{g+a_y}{g} = \frac{10+2}{10} = \frac{6}{5}$$

Sol 13: (D) T = 8s, $\omega = \frac{2\pi}{T} = \left(\frac{\pi}{4}\right)$ rads^{-1}

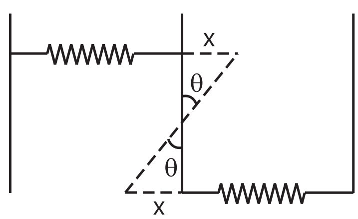
x = A sin ωt

$$\therefore a = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$$

Substituting t = $\frac{4}{3}$ s, we get

$$a = -\left(\frac{\sqrt{3}}{32}\pi^2\right)\text{cms}^{-2}$$

Sol 14: (C)



$$x = \frac{L}{2}\theta$$

Restoring torque = – (2kx). $\frac{L}{2}$

$$\alpha = -\frac{kL(L/2\theta)}{I} = -\left[\frac{kL^2/2}{ML^2/12}\right]\theta = -\left(\frac{6k}{M}\right)\theta$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

Sol 15: (D) $x_1 + x_2 = A$ and $k_1x_1 = k_2x_2$

or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$

Solving these equations, we get

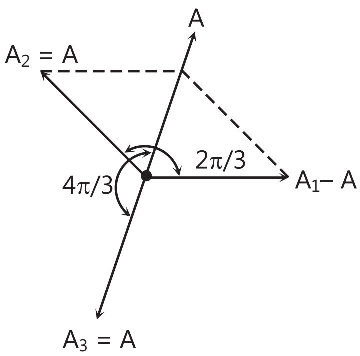
$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right)A$$

Sol 16: (A) Frequency or time period of SHM depends on variable forces. It does not depend on constant external force. Constant external force can only change the mean position. For example, in the given question mean position is at natural length of spring in the absence of electric field. Whereas in the presence of electric field mean position will be obtained after a compression of x_0. Where x_0 is given by

$Kx_0 = QE$

or $x_0 = \frac{QE}{K}$

Sol 17: (B)



Resultant amplitude of x_1 and x_2 is A at angle $\left(\frac{\pi}{3}\right)$ from A_1. To make resultant of x_1, x_2 and x_3 to be zero. A_3 should be equal to A at angle $\phi = \frac{4\pi}{3}$ as shown in figure.

Alternate solution: It we substitute, $x_1 + x_2 + x_3 = 0$

or $A \sin \omega t + A \sin\left(\omega t + \frac{2\pi}{3}\right) + B \sin(\omega t + \phi)$

Then by applying simple mathematics we can prove that

$B = A$ and $\phi = \frac{4\pi}{3}$.

Sol 18: (D) As retardation = bv

$\therefore$ Retarding force = mbv

$\therefore$ Net restoring torque when angular displacement is θ is given by

$= -mg\,\ell \sin\theta + mbv\,\ell$

$\therefore I\alpha = -mg\,\ell \sin\theta + mbv\,\ell$

Where, $I = m\ell^2$

$$\therefore \frac{d^2\theta}{dt^2} = \alpha = -\frac{g}{\ell}\sin\theta + \frac{bv}{\ell}$$

for small damping, the solution of the above differential equation will be

$$\therefore \theta = \theta_0 e^{-\frac{bt}{2}} \sin(wt + \phi)$$

$\therefore$ Angular amplitude will be $= \theta.e^{\frac{-bt}{2}}$

According to question, in τ time (average life–time),

Angular amplitude drops to $\frac{1}{e}$ value of its original value (θ)

$$\therefore \frac{\theta_0}{e} = \theta_0 e^{-\frac{6\tau}{2}} \Rightarrow \frac{6\tau}{2} = 1$$

$$\therefore \tau = \frac{2}{b}$$

Sol 19: (B) $A = A_0 e^{-kt}$

$\Rightarrow 0.9 A_0 = A_0 e^{-5k}$

and $\alpha A_0 = A_0 e^{-15k}$

Solving $\Rightarrow \alpha = 0.729$

At mean position, K.E. is maximum where as P.E. is minimum.

Sol 20: (C) $3\omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \omega\sqrt{A_1^2 - \left(\frac{2A}{3}\right)^2}$

$$\therefore A_1 = \frac{7A}{3}$$

Sol 21: (C) $v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

$v = \sqrt{5}\frac{A\omega}{3}$

$v_{new} = 3v = \sqrt{5}A\omega$

So the new amplitude is given by

$V_{new} = \omega\sqrt{A_{new}^2 - x^2} \Rightarrow \sqrt{5}\ A\omega = \omega\sqrt{A_{new}^2 - \left(\frac{2A}{3}\right)^2}$

$A_{new} = \frac{7A}{3}$

JEE Advanced/Boards

Exercise 1

Sol 1: $T \propto \frac{1}{k^{1/2}}$; $T = 2\pi\sqrt{\frac{m}{k}}$

$k_1 = 4k$; $k_2 = \frac{4k}{3}$

By $k_1\lambda_1 = k_2\lambda_2 = kl$

$T_1 = \frac{T}{2}$; $T_2 = \frac{T\sqrt{3}}{2}$

$\frac{T_1}{T_2} = \frac{1}{\sqrt{3}}$

Sol 2: $x = 0.2 \cos 5\pi t$

velocity $= \frac{dx}{dt} = -\pi \sin 5\pi t$

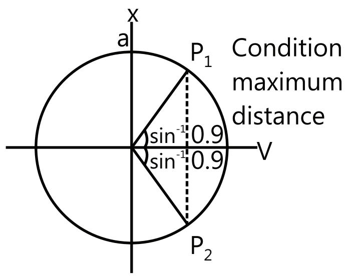
speed $= \pi\ |\sin 5\pi t|$

$v_{avg} = \frac{\pi\int_0^{0.7}|\sin 5\pi t|\,dt}{0.7}$

$= \frac{\pi}{0.7} \times 7 \times \int_0^{0.1} \sin 5\pi t\ dt = \frac{10\pi}{5\pi}\left[-\cos 5\pi t\right]_0^{0.1}$

$v_{avg} = 2$ m/s

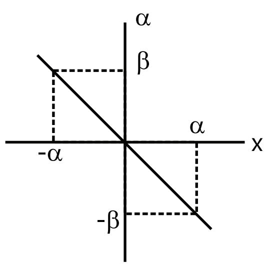
Sol 3: $\phi = 2 \sin^{-1}(0.9)$



P_1P_2 || y-axis

Max. Distance = 1.8 a

Sol 4:



$a = -\omega^2 x$

$-\omega^2 = \frac{\beta}{\alpha}$ = slope of a-x graph

$\omega = \sqrt{\frac{\beta}{\alpha}}$

Frequency $= \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

Sol 5: m = 0.1 kg

A = 0.1 m

$\frac{1}{2} \times mv_{max}^2 = 8 \times 10^{-3}$ J

$0.1\times v_{max}^2 = 16 \times 10^{-3} \Rightarrow v_{max} = 0.4$ m/s

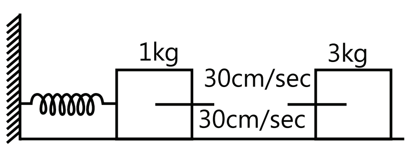
$A\omega = 0.4$

$0.1\times \omega = 0.4 \Rightarrow \omega = 4$

$x = A \sin(\omega t + \phi)$

$x = 0.1 \sin(4t + \pi/4)$

Sol 6: (i)



x = 10 + 3 sin 10 t

At t = 0 s block 1 is at equilibrium position.

$v_1 = A\omega = 3 \times 10 = 30$ cm/s

$v_2 = 30$ cm/s

Conservation of momentum

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

$-1 \times 30 + 3 \times 30 = 4 \times v$

v = 15 cm/s

Final velocity is in opposite direction of initial velocity of block 1. This causes a phase change of π.

$\omega \propto m^{-1/2}$

$\omega' = 5$ rad/s

$A'\omega' = 15$; $A' = 3$ cm

New amplitude = 3 cm

(ii) New equation

$x = 10 + 3 \sin (5t + \pi)$

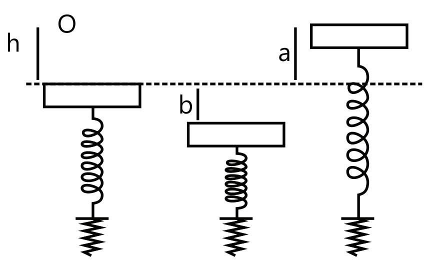
(iii) Loss of energy

$$= \left(\frac{1}{2}\times 1\times 30^2 + \frac{1}{2}\times 3\times 30^2 - \frac{1}{2}\times 4\times 15^2\right) \times 10^{-4} \text{ J}$$

$$= \frac{1}{2}(900+2700-900)\times 10^{-4} \text{ J} = 1350 \times 10^{-4} \text{ J}$$

$\Delta E_{loss} = 0.1350$ J

Sol 7:



(a) $mgh + \frac{1}{2} k \left(\frac{Mg}{k}\right)^2$

$$= \frac{1}{2} k \left(\frac{Mg}{k} + b\right)^2 - (M + m) gb$$

$$= \frac{1}{2} k \left(\frac{Mg}{k} - a\right)^2 + (M + m) ga$$

Equalising energies in 3 states

$$\frac{1}{2} k \left(\frac{Mg}{k} + b\right)^2 - (M + m) gb$$

$$= \frac{1}{2} k \left(\frac{Mg}{k} - a\right)^2 + (M + m) ga$$

$$k\left(\frac{2mg}{k}(b+a) + b^2 - a^2\right) = 2 (M + m)g (a + b)$$

$$2Mg (b + a) + k (b^2 - a^2) = 2 (M + m) g (a + b)\ k = \frac{2mg}{b-a}$$

Constant of force of spring = $\frac{2mg}{b-a}$

(b) $\omega = \sqrt{\frac{k}{(M+m)}} = \sqrt{\frac{2mg}{(M+m)(b-a)}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{2mg}{(M+m)(b-a)}}$$

(c) $mgh = \frac{1}{2} k \left[\left(\frac{Mg}{k} - a\right)^2 - \left(\frac{Mg}{k}\right)^2\right] + (M + m) ga$

$$mgh = -\frac{1}{2} k \left[a \times \left(\frac{2Mg}{k} - a\right)\right] + (M + m) ga$$

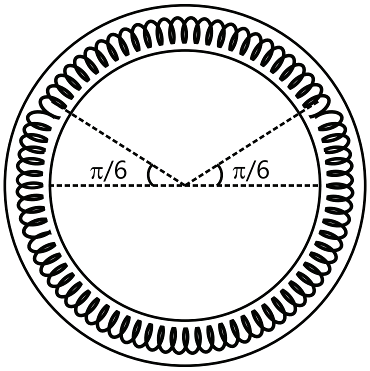
$$mgh = \frac{-ka}{2}\left(\frac{2mg}{k} - a\right) + (M + m) ga$$

$$mgh = -Mga + \frac{ka^2}{2} + (M + m) ga$$

$$mgh = mga + \frac{2mga^2}{(b-a)2}$$

$$h = a + \frac{a^2}{(b-a)} = \frac{ab}{b-a}$$

Sol 8:



(a) Frequency

Displace by $d\theta$

$\Delta x = 2Rd\theta$

$d\alpha = -2\,\dfrac{k}{m} \times \dfrac{2Rd\theta}{R}$

$d\alpha = -\omega^2\,d\theta$

$\omega^2 = \dfrac{4k}{m}\quad \omega = 2\sqrt{\dfrac{k}{m}} = 2$

$f = \dfrac{2}{2\pi} = \dfrac{1}{\pi}$

(b) Total energy $= 2 \times \dfrac{1}{2}\,k\left(R\dfrac{\pi}{3}\right)^2$

$= 2 \times \dfrac{1}{2} \times 0.1 \times \left(\dfrac{0.06\times\pi}{3}\right)^2$

$= 3.94 \times 10^{-4}\ J/4\pi^2 \times 10^{-5}\ J$

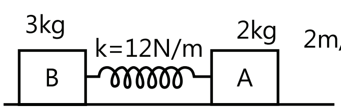
(c) $\quad 2 \times \dfrac{1}{2}\,mv^2 = 4\pi^2 \times 10^{-5}$

$v^2 = \dfrac{4\pi^2 \times 10^{-5}}{0.1}$

$v^2 = 4\pi^2 \times 10^{-4}$

$v = 2\pi \times 10^{-2} = 0.02\,\pi$ m/sec

Sol 9:



$V_{com} = \dfrac{2\times2+3\times0}{5} = 0.8$ m/s

$x_A = v_{com}\,t + A\sin\omega t$

At maximum expansion

$\dfrac{1}{2}\times5\times(0.8)^2 + \dfrac{1}{2}kx^2 = \dfrac{1}{2}\times2\times2^2$

$kx^2 = 8 - 3.2 = 4.8$

$x = 0.2$

$A = \dfrac{3}{5}\,x = \dfrac{3}{5} \times 0.2 = 0.12$

$\mu = \dfrac{3\times2}{3+2} = 1.2$

$\omega = \sqrt{\dfrac{k}{\mu}} = \sqrt{\dfrac{120}{1.2}} = 10$

$x_A = 0.8\,t + 0.12\sin 10\,t$

Sol 10: (a) m = 0.1 kg

$F = 10\,x + 2$

Only variable force causes SHM

(a) $F(x) = 10\,x + 2$

$a(x) = 100\,x + 20$

$v(x) = 50\,x^2 + 20\,x + c$

$v(0.2) = 0$

$50 \times 0.04 + 20 \times 0.2 + c = 0$

$c = -6$

$v = 50\,x^2 + 20\,x - 6$ ⟨ $x=0.2$, $x=-0.6$

$A = \dfrac{0.2-(-0.6)}{2} = 0.4$ m

Amplitude = 0.4 m

(b) $\omega = \sqrt{\dfrac{10}{0.1}} = 10$ rad/sec

$T = \dfrac{2\pi}{\omega} = \dfrac{\pi}{5}$ sec.

(c) $x = 0.2 - A\cos\omega t$

$x = 0.2 - 0.4\cos\dfrac{5t}{\pi}$

Sol 11: $u = (x^2 - 4x + 3)$

(i) $F = -\dfrac{dU}{dx}$

$F = -2x + 4$

At equilibrium F = 0

$-2x + 4 = 0 \Rightarrow x = 2$ m

(ii) $dF = -2dx$ similar to $dF = -\omega^2 dx$ as in SHM

$2 = \dfrac{\omega^2}{m} = \omega^2$

$\omega = \sqrt{2}$

$T = \dfrac{2\pi}{\omega} = \sqrt{2}\,\pi$ sec

(iii) $A\omega = 2\sqrt{6}$

$A = \dfrac{2\sqrt{6}}{\sqrt{2}} \Rightarrow A = 2\sqrt{3}$ m

Sol 12: $F_{max} = m\omega^2 A$

P.E. $= \dfrac{1}{2}$ K.E.

$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2} \times \frac{1}{2}kA^2$

$\Rightarrow x = \frac{A}{\sqrt{2}}$

$F = m\omega^2 \frac{A}{\sqrt{2}} = \frac{F_{max}}{\sqrt{2}}$

$F = 25\sqrt{2}$ N

Sol 13: (a) $T = 2\pi\sqrt{\frac{\mu}{k}}$

$\mu = \frac{3\times6}{3+6} = \frac{18}{9} = 2$ kg

$T = 2\pi\sqrt{\frac{1}{400}} = \frac{\pi}{10}$ sec

(b) A = 6 cm

(c) $v_{cm} = 0$; $v_B = \frac{-1}{2}v_A$

$\frac{1}{2} \times k \times A^2 = \frac{1}{2} \times 3 \times (-2v_B)^2 + \frac{1}{2} \times 6 \times v_B^2$

$800 \times (0.06)^2 = 12 v_B^2 + 6v_B^2$

$v_B^2 = \frac{8\times0.36}{18}$

$v_B = \frac{2\times0.6}{3}$

$V_{Bmax.} = 0.4$ m/s

$P_{Bmax} = 0.4 \times 6$

$P_{Bmax} = 2.4$ kg ms^{-1}

Sol 14: $s = \left(A - \frac{A}{4}\right)\cos\omega t - \left(\frac{A}{2} - \frac{A}{8}\right)\sin\omega t$

$s = \frac{3A}{4}\cos\omega t - \frac{3A}{8}\sin\omega t$

$s = \frac{3A}{8}(2\cos\omega t - \sin\omega t)$

$s = \frac{3\sqrt{5}}{8}A\left(\frac{2}{\sqrt{5}}\cos\omega t - \frac{1}{\sqrt{5}}\sin\omega t\right)$

$s = \frac{3\sqrt{5}}{8}A\cos\left(\omega t + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$

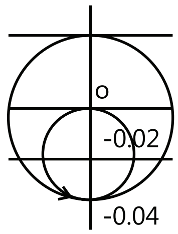
$A' = \frac{3\sqrt{5}}{8}A$; $\delta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Sol 15: $T = 2\pi\sqrt{\frac{m}{k}} = 0.4$ sec

$\omega = 5\pi$

For 0 < t < 0.6 sec

$x = -\frac{mg}{2k} + \frac{mg}{2k}\sin(5\pi t + \frac{\pi}{2})$



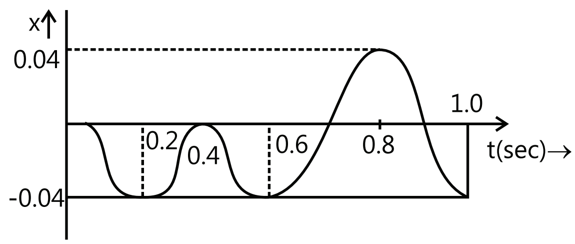
$\frac{mg}{2k} = \frac{4\times10}{2\times1000} = 0.02$ m

for 0 < t < 0.6 sec

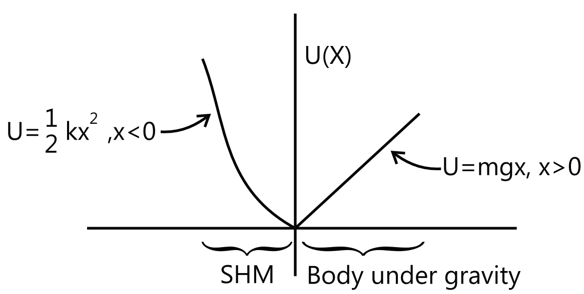
$x = -0.02 + 0.02\sin(5\pi t + \pi/2)$

for 0.6 < t 1 sec

$x = -0.04 + 0.04\sin(5\pi t)$



Sol 16:



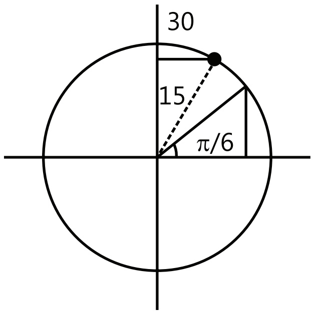
$T = \frac{1}{2} \times 2\pi\sqrt{\frac{m}{k}} + \frac{2v}{g}$

$\because E = \frac{1}{2}mv^2$; $T = \pi\sqrt{\frac{m}{k}} + \frac{2}{g}\sqrt{\frac{2E}{m}}$

$\because v = \sqrt{\frac{2E}{m}}$; $T = \pi\sqrt{\frac{m}{k}} + \frac{2\sqrt{2}}{g}\sqrt{\frac{E}{m}}$

Sol 17: $x = 30\sin\left(\pi t + \frac{\pi}{6}\right)$

$T = \frac{2\pi}{\pi} = 2$



P.E. = 2 K.E.

P.E. = $\frac{2}{3}$ T.E.

$x = \sqrt{\frac{2}{3}}\ A$

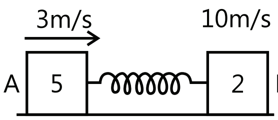
Position: $x = \sqrt{\frac{2}{3}} \times 30$

$x = 10\sqrt{6}$ cm

$$t = \frac{\left(\sin^{-1}\sqrt{\frac{2}{3}} - \frac{\pi}{6}\right)}{2\pi} \times 2 \text{ sec}$$

$$t = \frac{1}{\pi}\left(\sin^{-1}\sqrt{\frac{2}{3}} - \frac{\pi}{6}\right) \text{ sec}$$

Sol 18: (a)



$v_{cm} = \frac{5\times3+10\times2}{7} = 5ms^{-1}$

$\frac{1}{2}\ 5 \times 3^3 + \frac{1}{2} \times 2 \times 10^2$

$= \frac{1}{2}\ 7 \times 5^2 + \frac{1}{2}\ kx^2$

$45 + 200 = 175 + kx^2$

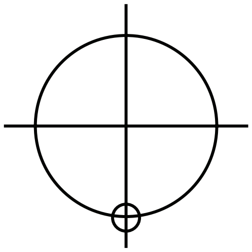
$kx^2 = 70$

$x^2 = \frac{70}{1120}$ $x = \frac{1}{4}$ m

Maximum extension = 0.25 m

(b) $t = \frac{3}{4}\ T$

$T = 2\pi\sqrt{\frac{\mu}{k}}$



$\mu = \frac{5\times2}{7} = \frac{10}{7}$

$T = 2\pi\sqrt{\frac{10}{7\times1120}} = \frac{2\pi}{28} = \frac{\pi}{14}$

Time for first maximum compression

$= \frac{3}{4} \times \frac{\pi}{14} = \frac{3\pi}{56}$ sec

Sol 19: $T = 2\pi\sqrt{\frac{I}{mgx}}$

$I = \frac{m\ell^2}{3} + \left(\frac{m\ell^2}{12} + m\ell^2\right)$

$x = \frac{mx\frac{1}{2}+mx\ell}{2m} = \frac{3\ell}{4} = \frac{4m\ell^2}{12} + \frac{13}{12}m\ell^2$

$I = \frac{17}{12}\ m\ell^2$

$T = 2\pi\sqrt{\frac{\frac{17}{12}m\ell^2}{2mg\frac{3\ell}{4}}}$;$T = 2\pi\sqrt{\frac{17\ell}{18g}}$

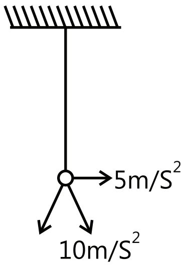
Sol 20: (a) $I\alpha = -k\theta$

$\alpha = -\frac{k}{I}\ \theta$

$\omega^2 = \frac{k}{2}$ $\omega = \sqrt{\frac{k}{I}} = \pi$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi}$;T = 2 sec

(b)



$g_{eff.} = \sqrt{10^2 + 5^2} = \sqrt{125}$

$g_{eff.} = 5\sqrt{5}$

$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.5}{5\sqrt{5}\times 10}}$

$T = \frac{2}{5^{1/4}}$ sec

Sol 21: m = 0.2 kg $f = \frac{25}{\pi}$ Hz

P.E. = $\frac{4}{9}$ T.E.

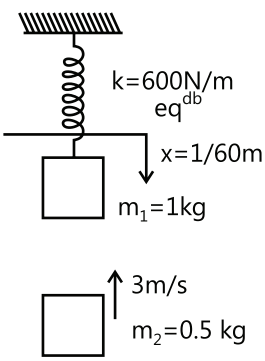
$\frac{1}{2}kx^2 = \frac{4}{9}\times\frac{1}{2}kA^2; x = \frac{2}{3}A$

$A = \frac{3}{2}x = \frac{3}{2}\times 0.04$

A = 0.06 m

Sol 22: 0.5 × 3 = 1.5 × v

v = 1 m/s



$\omega = \sqrt{\frac{600}{1.5}} = \sqrt{400} = 20$ rad./sec

$f = \frac{20}{2\pi} = \frac{10}{\pi}$ Hz

$\frac{1}{2}kx^2 + \frac{1.5\times 1^2}{2}$

$= \frac{1}{2}kh^2 - 1.5\times 10\times\left(h - \frac{1}{60}\right)$

$\frac{1}{2}\times 600\times\frac{1}{60^2} + \frac{1.5}{2}$

$= \frac{1}{2}\times 600\times h^2 - 15\times\left(h - \frac{1}{60}\right)$

$60h^2 - 3h - 7/60 = 0$

$h = \frac{1.5}{60} + \frac{\sqrt{37}}{120}$ m

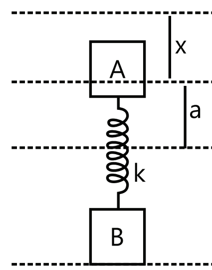
$A = h - \frac{1.5}{60} = \frac{\sqrt{37}}{120}$ m

$A = \frac{\sqrt{37}}{120}\times 100 \text{ cm} = \frac{5\sqrt{37}}{60}$ cm

Sol 23: $m_1 = 1$kg; $m_2 = 4$kg

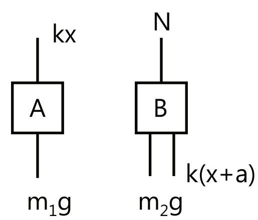
a = 1.6 cm

$kx = m_1 g$



$k = \omega^2 m_1 = 25^2 \times 1 = 625$ N/m

$N_{max} = m_2 g + k(x + a)$



$= (m_1 + m_2)g + ka = 50 + \frac{625\times 1.6}{100}$

$N_{max} = 60$ N

$N_{min} = (M_1 + M_2)\,g - ka$

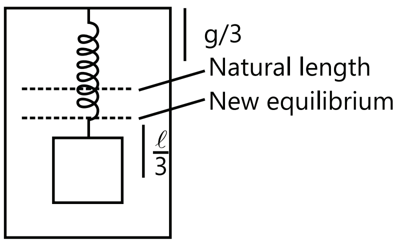
$N_{min} = 40$N

Sol 24: $k\ell = mg$; $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{9}{\ell}}$

$f = \frac{1}{2\pi}\left(\frac{g}{\ell}\right)^{1/2}$

$A = \ell/3$; $m \times \frac{2g}{3} = k \times x$

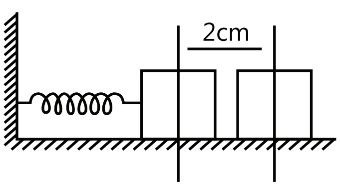
$x = \frac{2}{3}\frac{mg}{k} = \frac{2}{3}l$



Exercise 2

Single Correct Choice Type

Sol 1: (C)

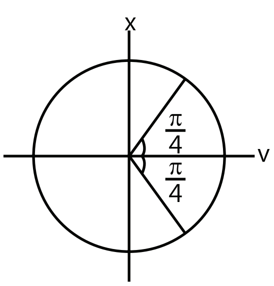


$\omega\sqrt{4-1} = \omega^2 \times 1$

$\omega = \sqrt{3}$

$F = \frac{\sqrt{3}}{2\pi}$ Hz

Sol 2: (D)

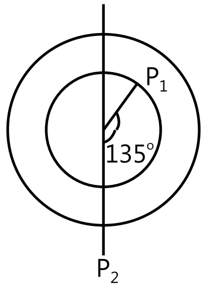


$v_{avg} = \frac{\text{displacement}}{\text{time}} = \frac{\sqrt{2}\times A}{T/4}$

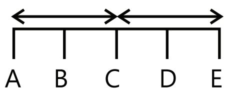
$v_{avg} = \frac{4\sqrt{2}A}{T}$

Sol 3: (C) $v_{mean} = \frac{a/2}{T/6} = \frac{3a}{T}$

Sol 4: (C)



Sol 5: (C)



$V_B^2 = \frac{1}{4}V_A^2$

$\frac{1}{4}R^2\omega^2 = \omega^2(R^2 - x^2) \Rightarrow R^2 = (R^2 - x^2)\,4$

$x = \frac{\sqrt{3}}{2}R$

$d_{BD} = 2x = \sqrt{3}R$

Sol 6: (C) $T_S = 2\pi\sqrt{\frac{m}{k}}$ T_s doesn't depend on g.

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$; $T_p \propto g^{-1/2}$

$\therefore T_p$ decreases

Sol 7: (B) $v_{max} = A\omega = \frac{A\sqrt{k}}{\sqrt{m}}$

$\frac{A_1\sqrt{k_1}}{\sqrt{m}} = \frac{A_2\sqrt{k_2}}{\sqrt{m}}$

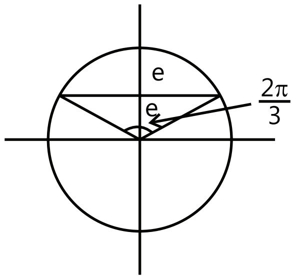
$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

Sol 8: (C) $k_A = k/3$; $k_B = 3k$

$T_A \propto k^{-1/2}$; $\frac{T_A}{T_B} = 3$

Sol 9: (A) $T = 2\pi\sqrt{\frac{m}{k}} \times \frac{2\pi/3}{2\pi}$

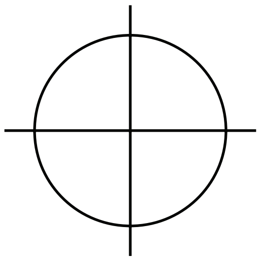
$T = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$



Sol 10: (C) $t = \frac{T}{4}$

$t = \frac{\pi}{2}\sqrt{\frac{\mu}{k}}$

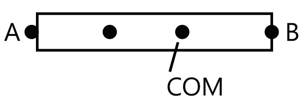
$t = \frac{\pi}{2}\sqrt{\frac{1}{\pi^2}}$; $t = \frac{1}{2}$ sec



Sol 11: (A) Both block have speed same as

v_{cm} = 5m/s

Sol 12: (D)



$T = 2\pi\sqrt{\frac{I}{mg\ell}}$

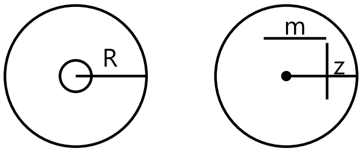
$\frac{T_A}{T_B} = \frac{3}{4} = \sqrt{\frac{9}{4}\times\frac{\ell_B}{\ell_A}}$

$\frac{3}{4} = \frac{3}{2}\sqrt{\frac{\ell_B}{\ell_A}}$

$\frac{\ell_B}{\ell_A} = \frac{1}{4}$

$\ell_A = \frac{4}{5} \times 25 = 20$ cm

Sol 13: (C)



$T = 2\pi\sqrt{\frac{I}{mgz}}$ $\Rightarrow I = \frac{mR^2 + 2mz^2}{2}$

$T = \sqrt{\frac{2\pi}{g}}\sqrt{m\frac{R^2}{2} + 2mz}$

$\frac{mR^2}{2}$ = 2mz for minimum T

$z = \frac{R}{\sqrt{2}}$

Multiple Correct Choice Type

Sol 14: (B, C, D) v = 0 at t = T/2

a is maximum at extremes

F = 0 at $t = \frac{3T}{4}$

K.E. = 0 at t = T/2

Sol 15: (A, B, C) K.E. = 0.64 $KE_{max.}$

$v = 0.8\ v_{max.}$

$\therefore$ x = 0.6 A = 6 cm

$x = \frac{A}{2}$ P.E. $= \frac{PE_{max}}{4}$ KE $= \frac{3}{4}$ PE_{max}

KE_{max} = TE at mean position

$x = \frac{A}{2}$ $v = \frac{\sqrt{3}v_{max}}{2}$

Sol 16: (B, C) (A) KE_{avg} is never zero in SHM

(B) $PE_{avg} = \frac{1}{2}$ TE = $m\pi^2 t^2 A^2$

(C) Frequency of occurrence of mean position =2f

(D) Acceleration leads

Sol 17: (A, C) $v_{rms} = \sqrt{\frac{\int_0^T v^2 dt}{T}} = \frac{v}{\sqrt{2}}$

$$v_{mean} = \frac{\int_0^T v\,dt}{T} = \frac{\sqrt{8}}{\pi} V$$

Sol 18: (B, C, D) A = 3cm

$\frac{1}{2} mv_m^2 = \frac{1}{2} \times 500 \times 9$

$v_m = 3 \times 10\sqrt{5}$ cm/s ; $\omega = \sqrt{500} = 10\sqrt{5}$

$a_{max.} = \omega v_m$

$= 10\sqrt{5} \times 30\sqrt{5}$ cm/s^2

$= 15$ m/s^2

$PE_{min} = 0$ at mean position

Sol 19: (A, B, C) A = 2.5

$\omega = \frac{v_{max}}{A} = 4;\ T = \frac{2\pi}{4} = \frac{\pi}{2} = 1.57$ s

$v_{max} = 16 \times 2.5 = 40$ cm/s^2

$v = \omega\sqrt{A^2 - x^2} = 4\sqrt{2.5^2 - 1^2}$

$= 4\sqrt{5.25}$ cm/s $= 2\sqrt{21}$ cm/s

Sol 20: (A, C) Energy conservation:

$\frac{1}{2} \times 900 \times 2^2 = \frac{1}{2} \times 900 \times 1^2 + \frac{1}{2} \times 3\, v_1^2$

$2700 = 3 \times v_1^2$

$v_1^2 = 900$

$v_1 = 30$ m/s

Conservation of momentum:-

$3 \times 30 + 6 \times 0 = 9 \times v$

$v = 10$ ms^{-1}

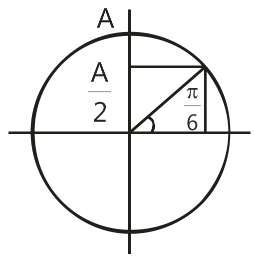
Energy conservation:-

$\frac{1}{2} \times 900 \times 1^2 + \frac{1}{2} \times 9 \times 10^2 = \frac{1}{2} \times 900 \times A^2$

$A^2 = 2;\quad A = \sqrt{2}$ m

Sol 21: (A, B) $t = \frac{\pi/6}{2\pi} \times T \Rightarrow t = \frac{T}{12}$

$v = \frac{\sqrt{3}}{2} V_0 \Rightarrow a \propto x \Rightarrow a = a_0/2$

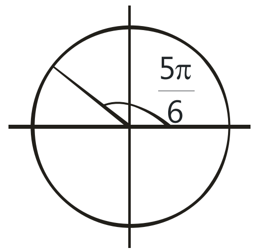


Sol 22: (B, C, D) $v^2 = \omega^2 (A^2 - x^2)$

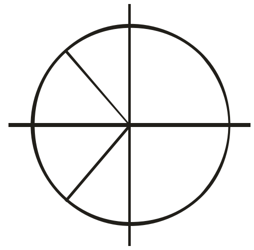
$a = -\omega^2 x$

$v^2 = \omega^2 \left(A^2 - \frac{a^2}{\omega^4}\right)$

Sol 23: (B, D) $x = A \sin\left(\frac{2\pi t}{T} + \frac{5\pi}{6}\right) = A \cos\left(\frac{2\pi t}{T} + \frac{\pi}{3}\right)$



Sol 24: (B, C)



Initial phase difference = 0, $\frac{2\pi}{3}, \frac{4\pi}{3}$

Sol 25: (A, B, C, D) $x = \frac{mg}{k} = \frac{0.2 \times 10}{200} = 1$ cm

Amplitude = 1 cm

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0.2}} = 10\sqrt{10}$

$f = \frac{10\sqrt{10}}{2\pi} = \frac{5\sqrt{10}}{\pi} \cong 5$Hz

Amplitude changes, frequency remains the same.

Sol 26: (B, C, D) m = 0.1 kg

$U = 5x(x - 4)$

$F = -\frac{dU}{dx} = 20 - 10x$

P.E. minimum at x = 2 m

Force is linear function of x with negative slope.

$\omega^2 = \frac{10}{m}$

$\omega = \sqrt{\frac{10}{0.1}} = 10$ rad/s

$T = \frac{2\pi}{10} = \frac{\pi}{5}$ sec

Sol 27: (B, D) $x = 3 \sin 100\,t + 8 \cos^2 50\,t$

$= 3 \sin 100\,t + 4 \cos 100\,t + 4$

$x = 5 \sin (100\,t + \sin^{-1} 4/5) + 4$

Sol 28: (A, B, C) $x = 5 \sin (4\pi t + \sin^{-1} 4/5)$ mm

$T = \frac{2\pi}{4\pi} = 0.5$ s

A = 5 mm

$\phi = \sin^{-1} (4/5)$

Sol 29: (B, C) $k = 2 \times 10^6$ Nm^{-1}

A = 0.01 m

T.E. = 160 J

$PE_{max} = 160$ J

when KE = 0 J

i.e. at equilibrium

$KE_{max} = \frac{1}{2} \times 2 \times 10^6 \times 10^{-4} = 100$ J

$PE_{min} = 60$ J

Sol 30: (A, B, C) $t = n\frac{T}{2}$

T $= 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{3\times 24}} = \frac{\pi}{3}$

Assertion Reasoning Type

Sol 31: **(D)** The motion is SHM with $\omega = \sqrt{\frac{a}{m}}$

If the force is linear w.r.t. x and slope is negative. The motion is always SHM.

Sol 32: (C) When particle moves from extreme to mean position velocity and acceleration have same direction.

Sol 33: (A) Statement-II is the correct explanation.

Sol 34: (D) Phase remains same and SHMs are perpendicular**.**

Sol 35: (A) Statement-II is the correct explanation.

Comprehension Type

Paragraph 1:

Sol 36: (D) $\omega = 25$ rad/s

$k = m\omega^2 = 1 \times 625 = 625\ Nm^{-1}$

$F_{max} = 1\times 9.8 + 625\times\frac{16}{100\times 10} + 4.1\times 9.8$

$= 59.98\ N \cong 60\ N$

$F_{min} = 5.1 \times 9.8 - 10 \cong 40\ N$

Sol 37: (C) Minimum force on the surface = (50 – 10) N = 40 N

Sol 38: (A) TE of system is constant

Sol 39: (D) $d = A \sin (\omega t + \phi)$

Sol 40: (D) $F = -kx + c$

$k > 0$

Match the Columns

Sol 41: (B) (a) y = A sin (t)

v = A cos (t)

$KE = c \times \cos^2 (t)$

(a) → (ii)

(b) → (i) PE + KE = const.

$PE = c \times \sin^2 t$

(c) → (iii) TE constant always

(d) → (iv) v = A cos t

Sol 42: (A) $PE \propto x^2$ (A) → p, s

(B) $s = ut + \frac{1}{2}at^2$

q, r when a = 0 ; S when $a \neq 0$

(C) Range $= \frac{v^2 \sin 2\theta}{g}$

(D) $T^2 = \frac{4\pi^2 \ell}{g}$

Previous Year's Questions

Sol 1: (C) If $E > V_B$, particle will escape. But simultaneously for oscillations, $E > 0$

Hence, the correct answer is $V_0 > E > 0$

Or the correct option is (c)

Sol 2: (B) $[\alpha] = \left[\frac{PE}{x^4}\right] = \left[\frac{ML^2T^{-2}}{L^4}\right] = [ML^{-2}T^{-2}]$

$\therefore \left[\frac{m}{\alpha}\right] = [L^2T^2]$; $\quad \therefore \left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = [T]$

As dimensions of amplitude A is [L]

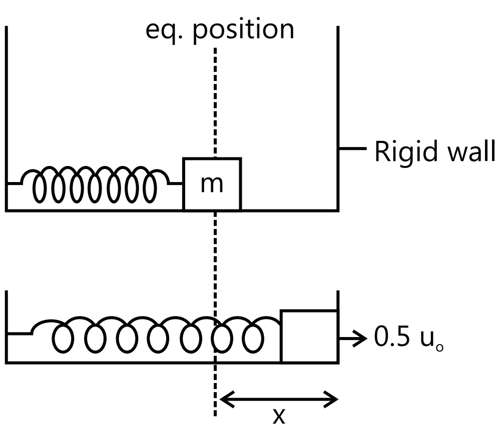
Sol 3: (D) For $|x| > x_0$, potential energy is constant. Hence, kinetic energy, speed or velocity will also remain constant.

$\therefore$ Acceleration will be zero

Sol 4: (A) $\frac{2v\sin 45°}{g} = 1$

$\therefore v = \sqrt{50}\ m/s$

Sol 5: (A, D)



$$\frac{1}{2}mu_0^2 = \frac{1}{2}kx^2 + \frac{1}{2}\times m0.25\,u_0^2 \qquad \text{....(i)}$$

After elastic collision

Block speed is $0.5\,u_0$

So when it will come back to equilibrium point its speed will be u_0 as (A)

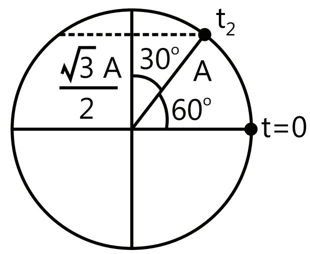
Amplitude $\frac{1}{2}mu_0^2 = \frac{1}{2}kA^2$

$$A = \frac{u_0}{\sqrt{k}}$$

Value of x from eq. (i)

$$\frac{3}{4}\times\frac{1}{2}mu_0^2 = \frac{1}{2}kx^2$$

$$x = \frac{\sqrt{3}u_0}{2}\sqrt{\frac{m}{k}}$$



Time to reach eq. position first time $\Rightarrow \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

Second time it will reach at time $\Rightarrow$

$$\frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{T}{2} \Rightarrow \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{2\pi\sqrt{m}}{\sqrt{k}\times 2} \Rightarrow \frac{5\pi}{3}\sqrt{\frac{m}{k}} \text{ as (D)}$$

For max. compression time is t_2

$$t_2 = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{T}{4}$$

$$= \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{2\pi}{\sqrt{k}}\frac{\sqrt{m}}{k} = \frac{7\pi}{6}\sqrt{\frac{m}{k}}$$

Sol 6: (B, D)

$$E_1 = \frac{1}{2}m\omega_1^2 a^2 = \frac{b^2}{2m} \qquad \frac{a}{b} = \frac{1}{m\omega_1} = n^2 \qquad \text{... (i)}$$

$$E_2 = \frac{1}{2}m\omega_2^2 R^2 = \frac{R^2}{2m} \qquad m\omega_2 = 1 \qquad \text{... (ii)}$$

From (i) and (ii) $\frac{\omega_2}{\omega_1} = n^2$

$$\frac{E_1}{E_2} = \left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{a}{R}\right)^2 = \frac{1}{n^2}\cdot\frac{\omega_1}{\omega_2}\cdot n^2 \Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Sol 7: (A, B, D)

(A) $\omega_i = \left(\frac{K}{M}\right)$ and $\omega_f = \left[\frac{K}{(M+m)}\right]$

Case I: $v_f = \frac{Mv_i}{M+m}\ \frac{1}{2}Mv^2 = \frac{1}{2}KA_i^2$

$\Rightarrow A_i^2 = \frac{M}{K}v_1^2$ and $\frac{1}{2}(M+m)v_f^2 = \frac{1}{2}KA_i^2$

$\Rightarrow A_f^2 = \frac{Mv^2}{K}\cdot\frac{M}{M+m} \Rightarrow \frac{A_f}{A_i} = \sqrt{\frac{M}{M+m}}$

(B) $T_f = 2\pi\sqrt{\frac{M}{M+m}}$ for both

(C) $TE_{\text{case I}} = \frac{1}{2}(M+m)v_f^2 = \frac{1}{2}Mv^2\left(\frac{M}{M\ \ m}\right)$

$TE_{\text{case II}} = \frac{1}{2}KA_f^2 = \frac{1}{2}KA_i^2$

(D) VEP = $A_f\omega_f$: Decreases in both cases.

Sol 8: A → p; B → q, r; C → p; D → r, q

Sol 9: (A) The total mechanical energy = 160 J

The maximum PE will be 160 J at the instant when KE = 0

Sol 10: (A, C) By principle of superposition $y = y_1 + y_2 + y_3$

$= a\sin(\omega t + 45°) + a\sin\omega t + a\sin(\omega t - 45°)$

$= a\sin(\omega t + 45°) + a\sin(\omega t - 45°) + a\sin\omega t$

$= 2a\sin\omega t\cos 45° + a\sin\omega t$

$= \sqrt{2}a\sin\omega t + a\sin\omega t = \left(1+\sqrt{2}\right)a\sin\omega t$

∴ Amplitude of resultant motion = $\left(1+\sqrt{2}\right)$ a ...(i)

(b) The option is incorrect as the phase of the resultant motion relative to the first is 45°.

(c) Energy is SHM is proportional to (amplitude)^2

$\therefore \frac{E_R}{E_S} = \frac{\left(1+\sqrt{2}\right)^2 a^2}{a^2} \quad \therefore \frac{E_R}{E_S} = \frac{\left(1+2+2\sqrt{2}\right)}{1}$

or $E_R = \left(3+2\sqrt{2}\right)E_S$

(d) Resultant motion is $y = \left(1+\sqrt{2}\right)a\sin\omega t$

Ii is SHM.

Sol 11: (A, B, D)

$x = \frac{A}{2}(1-\cos 2\omega t) + \frac{B}{2}(1+\cos 2\omega t) + \frac{C}{2}\sin 2\omega t$

For A = 0, B = 0

$x = \frac{C}{2}\sin 2\omega t$

A = -B and C = 2B

X = B cos 2ωt + B sin 2ωt

Amplitude = $\left|B\sqrt{2}\right|$

For A = B; C = 0

X = A,

Hence this is not correct option.

For A = B, C = 2B

X = B + B sin 2ωt

It is also represent SHM.

Sol 12: (A, D) Restoring torque is same in both cases

$\alpha = \frac{T}{I} = -\omega^2\theta$

In case A the moment of inertia is more as compared to B, so $\omega_B > \omega_A$

ELASTICITY

1. INTRODUCTION

We have learnt that the shape and size of a rigid body does not change but this is an ideal concept. Actually a rigid solid does experience some kind of deformation under the action of external forces and if the magnitude of forces cross a certain limit, the deformation is so severe that the material of the solid loses its rigidity. We say that the material has broken-down or failure has happened. In this chapter we learn about the properties of solid bodies by virtue of which they resist the deformation in their shape and size. These properties constitute the strength of a material and the knowledge of these is very essential in constructing small and large structures like houses, tall buildings, bridges, railway tracks etc.

2. MOLECULAR STRUCTURE OF A MATERIAL

Matter is made up of atoms and molecules. An atom is made up of a nucleus and electrons. Nucleus contains protons and neutrons (collectively known as "nucleons"). Nuclear forces are responsible for the structure of nucleus. Likewise, forces between different atoms and molecules are responsible for the structure of a material.

2.1 Interatomic and Intermolecular Forces

The forces that are responsible for holding the atoms/molecules in place in a solid or liquid are called interatomic and intermolecular forces. The interaction between any isolated pair of atoms and molecules may be represented by a curve that shows how the potential energy varies with the separation between them as shown in the Fig. 8.32

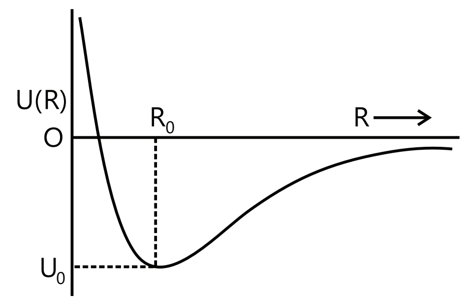


Figure 8.32: Potential energy versus separation

We see that as the distance R decreases, the attractive force first increases and then decreases to zero at a separation R_0 where the potential energy is the minimum. For smaller distance, force is repulsive.

The above picture of interatomic or intermolecular force is an over simplification on the actual situation. However, it provides a reasonable visualisation.

The force between the atoms can be found from the potential energy using the relation,

$$F(R) = -\frac{dU}{dR}$$

The resulting force curve is shown in Fig. 8.33.

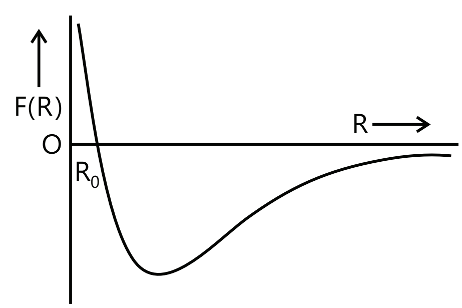


Figure 8.33: Graph of force versus separation

Force is along the line joining the atoms or molecules, and is shown negative for attraction & positive for repulsion.

2.2 Classification of Matter

Matter can be classified into three states:- solids, liquids and gases.

Solids: A solid is that state of matter whose atoms and molecules are strongly bound so as to preserve their original shape and volume. Solids are of two types-crystalline & amorphous.

(a) **Crystalline solid:** A crystalline solid is one which has regular & periodic arrangement of atoms or molecules in three dimensions. Examples of crystalline solids are diamond, rock salt, mica, sugar etc.

(b) **Amorphous solids or glassy solids:** The word 'amorphous' literally means 'without any form'. There is no 'order' in arrangement of atoms in such a solid. Example - glass.

In solids, the intermolecular forces are so strong that there is no change in shape and size easily.

Liquids: The intermolecular forces are comparably less than that in solids, so the shape can easily be changed.

But volume of a given mass of a liquid is not easy to change. It needs quite an effort to change the density of liquids.

Liquids are not able to produce reaction forces to applied forces in arbitrary directions.

Gases: This is the third state of matter which cannot support compressive, tensile, or sharing forces. Densities of gases change very rapidly with increase in temperature.

Liquids and gases are together classified as fluids: The word ''fluid" comes from a Latin word meaning ''to flow".

On an average, the atoms or molecules in a gas are far apart, typically about ten atomic diameters at room temperature and pressure. They collide much less frequently than those in a liquid. Gases in general are compressible.

3. INTRODUCTION TO ELASTICITY

When external forces are applied to a body which is fixed to a rigid support, there is a change in its length, volume or shape. When the external forces are removed, the body tends to regain its original shape and size. Such a property of a body by virtue of which a body tends to regain its original shape or size, when the external forces are removed, is called elasticity.

If a body completely regains its shape and size, it is called perfectly elastic. If it does not regain its shape and size completely, it is called inelastic material. Those materials which hardly regain their shape are called plastic material.

An **elastic** body is one that returns to its original shape after a deformation. Eg- golf ball, rubber band, soccer ball.

An **inelastic** body is one that does not return to its original shape after a deformation. Eg – dough or bread, clay, inelastic ball.

PLANCESS CONCEPTS

Microscopic reason of elasticity

Each molecule in a solid body is acted upon by forces due to neighboring molecules. When all molecules are in a state of stable equilibrium, the solid takes a particular shape. When the body is deformed, molecules are displaced from their stable equilibrium positions. The intermolecular distances change and restoring forces start acting which drives the molecules to come back to its original shape.

Vaibhav Krishnan (JEE 2009, AIR 22)

One can compare this situation to a spring-mass system. Consider a particle connected to several particles through spring. If this particle is displaced a little, the spring exerts a resultant force which tries to bring the particle towards its natural position. In fact, the particle will oscillate about this position. In due course, the oscillations will be damped out and the particle will regain its original position.

3.1 Stress and Strain

Stress: Elastic bodies regain their original shape due to internal restoring forces. This internal restoring force, acting per unit area of a deformed body is called a stress.

i.e. Stress = $\dfrac{\text{Restoring force}}{\text{Area}}$

SI unit of stress is N/m^2 and Dimensional formula of stress is $[ML^{-1}T^{-2}]$

An object can be deformed in different ways.

PLANCESS CONCEPTS

Misconception: People often get confused between pressure and stress.

Difference between pressure v/s stress:

S. No.	Pressure	Stress
1	Pressure is always normal to the area	Stress can be normal or tangential
2	Always compressive in nature	May be compressive or tensile in nature

Nivvedan (JEE 2009, AIR 113)

3.1.1 Types of Stress

There are 2 types of stresses – NORMAL stress and SHEAR stress

Normal stress – When the force applied is perpendicular to the area of application of force, it is called normal stress. Normal stress usually leads to a change in length (**longitudinal stress**) or a change in volume.

Normal stress can be of two types – tensile stress and compressive stress.

(a) **Tensile Stress**: Pulling force per unit area. It is applied parallel to the length.

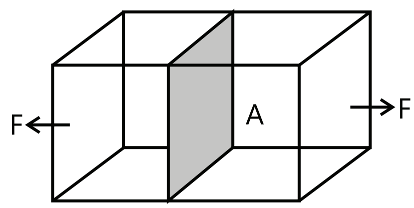


Figure 8.34: Tensile stress

It causes increase in length or volume.

(b) **Compressive Stress**: Pushing force per unit area. It is applied parallel to the length.

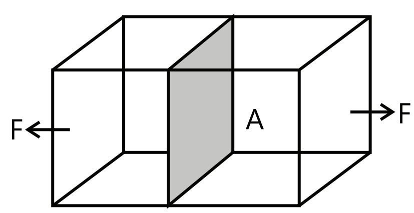


Figure 8.35: Compressive stress

It causes decreases in length or volume.

If the force is applied tangentially to one face of a rectangular body keeping the opposite face fixed, the stress is called tangential or shearing stress.

Stress is measured in units of $1 N/m^2$. $1 N/m^2 = 10$ dynes/cm^2.

Strain: The fractional or relative change in shape, size or dimensions of body is called the strain.

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

There are three types of strains:

(i) **Longitudinal strain**: It is the ratio of the change in length, $\Delta\ell$, to the original length, ℓ i.e. $\frac{\Delta\ell}{\ell}$.

(ii) **Volume strain**: It is the ratio of change in volume, ΔV, to the original volume V i.e. $\frac{\Delta V}{V}$

(iii) **Shearing strain**: The angular deformation, θ, in radians of a face of a rectangular body is called shearing strain.

If a tangential force F is used to displace upper face of rectangular body through a small angle θ such that the upper face is displaced through distance Δx where ℓ is height of the body, then shearing strain = $\theta \approx \tan\theta = \frac{\Delta x}{\ell}$

Strain is a ratio of two similar quantities and does not have any units.

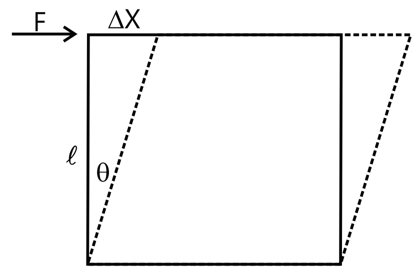


Figure 8.36: Shearing strain

Illustration 1: A 4.0 m long copper wire of cross sectional area 1.2 cm^2 is stretched by a force of 4.8×10^3 N. Stress will be- **(JEE MAIN)**

(A) 4.0×10^7 N/mm^2 (B) 4.0×10^7 kN/m^2 (C) 4.0×10^7 N/m^2 (D) None

Sol: (C) Stress is restoring force per unit area of cross-section.

$$\text{Stress} = \frac{F}{A} = \frac{4.8\times10^3 N}{1.2\times10^{-4} m^2} = 4.0 \times 10^7 \text{ N/m}^2$$

Illustration 2: A copper rod 2m long is stretched by 1mm. Strain will be **(JEE MAIN)**

(A) 10^{-4}, volumetric (B) 5×10^{-4}, volumetric (C) 5×10^{-4}, longitudinal (D) 5×10^{-3}, volumetric

Sol: Longitudinal strain is equal to change in length per unit length.

(C) $\text{Strain} = \frac{\Delta\ell}{\ell} = \frac{1\times10^{-3}}{2} = 5 \times 10^{-4}$, Longitudinal

Illustration 3: A lead of 4.0 kg is suspended from a ceiling through a steel wire of radius 2.0 mm. Find the tensile stress developed in the wire when equilibrium is achieved. Take g = 3.1π ms^{-2}. **(JEE MAIN)**

Sol: Stress is restoring force per unit area of cross-section.

Tension in the wire is $F = 4.0 \times 3.1\,\pi$ N.

The area of cross section is $A = \pi r^2 = \pi \times (2.0 \times 10^{-3}\text{ m})^2 = 4.0\,\pi \times 10^{-6}\text{ m}^2$.

Thus, the tensile stress developed $\frac{F}{A} = \frac{4.0\times3.1\pi}{4.0\pi\times10^{-6}}$ N/m^2 = 3.1×10^6 N/m^2.

Illustration 4: Find the stress on a bone (1 cm in radius and 50 cm long) that supports a mass of 100kg. Find the strain on the bone if it is compressed 0.15 mm by this weight. Find the proportionality constant C for this bone. **(JEE MAIN)**

Sol: Stress is restoring force per unit area of cross-section. Strain is equal to change in length per unit length. Strain $\propto$ Stress

Stress = F/A = (100kg) (9.8 m/s^2) / π × (0.01 m)^2 = 3.1 × 10^6 N/m^2

Strain = $\Delta L/L_0$ = (0.15 × 10^{-3}m) / (0.5m) = 3.0 × 10^{-4}

Since strain = C × stress, C = strain / stress = 0.96 × 10^{-10} m^2/N.

4. HOOKE'S LAW AND MODULI OF ELASTICITY

Hooke's Law: It states that for small deformations, stress is directly proportional to strain within elastic limits and the ratio is a constant called modulus of elasticity.

$$\frac{\text{Stress}}{\text{Strain}} = \text{modulus of Elasticity} = E$$

4.1 Young's Modulus

Young's modulus is a measure of the resistance of a solid to a change in its length when a force is applied perpendicular to its surface. Consider a rod with an unstressed length L_0 and cross-sectional area A, as shown in the Fig. 8.37. When it is subjected to equal and opposite forces F_n along its axis and perpendicular to the end faces, its length changes by ΔL. These forces tend to stretch the rod. The tensile stress on the rod is defined as $\sigma = \frac{F_n}{A}$ Forces acting in the opposite direction, as shown in Fig. 8.37, would produce a compressive stress. The resulting strain is defined as the dimensionless ratio, $\varepsilon = \frac{\Delta L}{L_0}$ Young's modulus Y for the material of the rod is defined as the ratio of tensile stress to tensile strain.

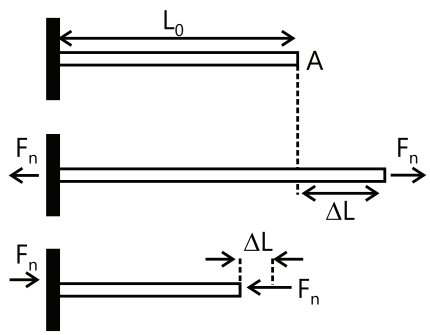


Figure 8.37: Variation in length of rod

So Young's Modulus = $\frac{\text{Tensile stress}}{\text{Tensile strain}}$; $Y = \frac{\sigma}{\varepsilon} = \frac{F_n / A}{\Delta L / L_0} = \frac{F_n L_0}{A\Delta L}$

A force applied normal to the end face of a rod cause a change in length.

PLANCESS CONCEPTS

(a) For loaded wire: $\Delta L = \frac{FL}{\pi r^2 \gamma}$

For rigid body $\Delta L = 0$ so $Y = \infty$ i.e. Elasticity of rigid body is infinite.

(b) If same stretching force is applied to different wires of same material, $\Delta L \propto \frac{L}{r^2}$ [As F and Y are const.]

Greater the value ΔL, greater will be elongation.

Following conclusions can be drawn from γ=stress/strain:

(i) $E \propto$ stress (for same strain), i.e. if we want the equal amount of strain in two different materials, the one which needs more stress is having more E.

PLANCESS CONCEPTS

(ii) $E \propto \frac{1}{\text{strain}}$ (for same stress), i.e., if the same amount of stress is applied on two different materials, the one having less strain is having more Elasticity. Rather we can say that, the one which offers more resistance to the external forces is having greater value of E. So, we can see that modulus of elasticity of steel is more than that of rubber or $E_{steel} > E_{rubber}$

(iii) E = stress for unit strain $\left(\frac{\Delta x}{x} = 1 \text{ or } \Delta x = x\right)$, i.e. suppose the length of a wire is 2m, then the Young's modulus of elasticity (Y) is the stress applied on the wire to stretch the wire by the same amount of 2m.

Chinmay S Purandare (JEE 2012, AIR 698)

Illustration 5: Two wires of equal cross section but one made of steel and the other of copper, are joined end to end. When the combination is kept under tension, the elongations in the two wires are found to be equal. Find the ratio of the lengths of the two wires. Young's modulus of steel = 2.0×10^{11} Nm^{-2}. **(JEE ADVANCED)**

Sol: The wires joined together have same stress and same elongation. Ratio of stress and young's modulus is strain. As young's modulus for steel and copper is different, strains of the wires will be different.

As the cross sections of the wires are equal and same tension exists in both, the stresses developed are equal. Let the original lengths of the steel wire and the copper wire be L_S and L_C respectively and the elongation in each wire be ℓ.

$$\frac{\ell}{L_S} = \frac{\text{stress}}{2.0 \times 10^{11} Nm^{-2}} \qquad \text{... (i)}$$

And $$\frac{\ell}{L_C} = \frac{\text{stress}}{1.1 \times 10^{11} Nm^{-2}} \qquad \text{... (ii)}$$

Dividing (ii) by (i), $L_S/L_C = 2{\cdot}0 / 1{\cdot}1 = 20:11$.

Illustration 6: A solid cylindrical steel column is 4.0 m long and 9.0 cm in diameter. What will be decrease in length when carrying a load of 80000 kg? Y = 1.9×10^{11} Nm^{-2}. **(JEE MAIN)**

Sol: The stress will be equal to load per unit cross section. Strain is the ratio of stress and young's modulus.

Let us first calculate the cross-sectional area of column = $\pi r^2 = \pi(0.045m)^2 = 6.36 \times 10^{-3}\ m^2$

Then, from $Y = \frac{F/A}{\Delta L/L}$ we have $\Delta L = \frac{FL}{AY} = \frac{[(8\times10^4)(9.8N)](4.0m)}{(6.36\times10^{-3}m^2)(1.9\times10^{11}Nm^{-2})} = 2.6 \times 10^{-3}$ m.

Illustration 7: A load of 4.0 kg is suspended from a ceiling through a steel wire of length 20 m and radius 2.0 mm. It is found that the length of the wire increases by 0.031 mm as equilibrium is achieved. Find Young's modulus of steel. Take g = 3.1 π m/s^2. **(JEE MAIN)**

Sol: The stress will be equal to load per unit cross section. Strain is the change in length per unit length. Young's modulus is the ratio of stress and strain.

The longitudinal stress = $\frac{(4.0kg)(3.1\pi ms^{-2})}{\pi(2.0\times10^{-3}m)^2} = 3.1 \times 10^6\ N/m^2$

The longitudinal strain = $\frac{0.031\times10^{-3}\text{m}}{2.0\text{m}}$ = 0.0155 × 10^{-3}

Thus Y = $\frac{3.1\times10^{6}\text{Nm}^{-2}}{0.0155\times10^{-3}}$ = 2.0 × 10^{11} N/m^2.

Illustration 8: A bar of mass m and length ℓ is hanging from point A as shown in Fig. 8.38. Find the increase in its length due to its own weight. The Young's modulus of elasticity of the wire is Y and area of cross-section of the wire is A. **(JEE ADVANCED)**

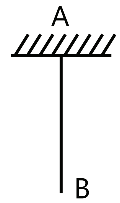


Figure 8.38

Sol: Find the elongation for an elementary length dx of the wire due to tension in the wire at the location of the element.

Consider a small section dx of the bar at a distance x from B. The weight of the bar for a length x is,

$$W = \left(\frac{mg}{\ell}\right)x$$

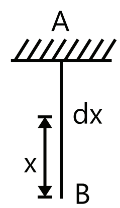


Figure 8.39

Elongation in section dx will be $d\ell = \left(\frac{W}{AY}\right)dx = \left(\frac{mg}{\ell AY}\right)x\,dx$

Total elongation in the bar can be obtained by integrating this expression for x = 0 to x = ℓ.

$$\therefore \quad \Delta\ell = \int_{x=0}^{x=\ell} d\ell = \left(\frac{mg}{\ell AY}\right)\int_0^{\ell} x\,dx \quad \text{or} \quad \Delta\ell = \frac{mg\ell}{2AY}$$

Illustration 9: One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. Find the longitudinal strain in both the wires. Area of cross section of each wire is 0.005 cm^2 and Young modulus of the metal is 2.0 × 10^{11} N m^{-2}. Take g = 10 ms^{-2}. **(JEE ADVANCED)**

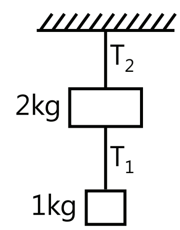


Figure 8.40

Sol: Find the tension in each wire. Stress is tension per unit area of cross section. Strain is the ratio of stress and Young's modulus.

The situation is described in Fig. 8.40. As the 1kg mass is in equilibrium, the tension in the lower wire equals the weight of the load.

Thus T_1 = 10N; Stress = 10N/0.005 cm^2 = 2 × 10^7 N/m^2

Longitudinal strain = $\frac{\text{stress}}{Y} = \frac{2\times10^{7}\ \text{N/m}^2}{2\times10^{11}\ \text{N/m}^2} = 10^{-4}$

Considering the equilibrium of the upper block, we can write, T_2 = 20N + T_1 or T_2 = 30N

Stress = 30 N/0.005 cm^2 = 6 × 10^7 N/m^2

Longitudinal strain = $\frac{6\times10^{7}\ \text{N/m}^2}{2\times10^{11}\ \text{N/m}^2}$ = 3 × 10^{-4}.

Illustration 10: Each of the three blocks P, Q and R shown in Figure has a mass of 3 kg. Each of the wires A and B has cross-sectional area 0.005 cm^2 and Young modulus 2 × 10^{11} N/m^2. Neglect friction. Find the longitudinal strain developed in each of the wires. Take g = 10 m/s^2. **(JEE ADVANCED)**

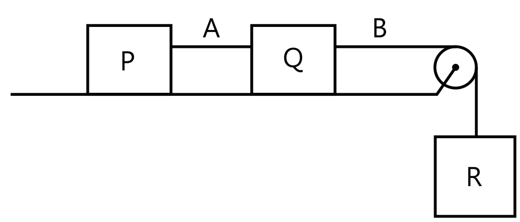


Figure 8.41

Sol: Find the tension in each wire. Stress is tension per unit area of cross section. Strain is the ratio of stress and Young's modulus.

The block R will descend vertically and the blocks P and Q will move on the frictionless horizontal table. Let the common magnitude of the acceleration be a. Let the tensions in the wires A and B be T_A and T_B respectively.

Writing the equations of motion of the blocks P, Q and R, we get,

$$T_A = (3\text{kg})\,a \quad \text{.... (i)}$$

$$T_B - T_A = (3\text{kg})\,a \quad \text{.... (ii)}$$

$$\text{And } (3\text{kg})g - T_B = (3\text{kg})a \quad \text{.... (iii)}$$

By (i) and (ii), $T_B = 2T_A$; By (i) and (iii), $T_A + T_B = (3\text{kg})\,g = 30\text{ N}$

or $3T_A = 30\text{N}$ or, $T_A = 10\text{N}$ and $T_B = 20\text{ N}$.

$$\text{Longitudinal strain} = \frac{\text{Longitudinal stress}}{\text{Young modulus}}$$

Strain in wire A $= \dfrac{10\text{N}/0.005\text{ cm}^2}{2\times10^{11}\text{ N}/\text{m}^2} = 10^{-4}$; And strain in wire B $= \dfrac{20\text{N}/0.005\text{ cm}^2}{2\times10^{11}\text{ N}/\text{m}^2} = 2\times10^{-4}$.

PLANCESS CONCEPTS

In practical life, we often hear something like elastic band is usually referred to a rubber band because it is easily stretchable and a steel rod is not.

However, here elasticity has some different meaning. Being more elastic means, the material will resist more to any external force which tries to change its configuration.

That is why $E_{steel} > E_{rubber}$.

Nitin Chandrol (JEE 2012, AIR 134)

4.2 Shear Modulus

The shear modulus of a solid measures its resistance to a shearing force, which is a force applied tangentially to a surface, as shown in the Fig. 8.42. (Since the bottom of the solid is assumed to be at rest, there is an equal and opposite force on the lower surface). The top surface is displaced by x relative to the bottom surface.

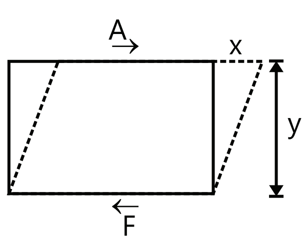


Figure 8.42: Shearing stress

The shear stress is defined as, Shear stress $= \dfrac{\text{Tangential force}}{\text{Area}} = \tau = \dfrac{F_t}{A}$ where A is the area of the surface.

The shear strain is defined as Shear strain $= \dfrac{x}{y}$

where y is the separation between the top and the bottom surfaces.

The shear modulus G is defined as

$$\text{Shear modulus} = \frac{\text{Shear Stress}}{\text{Shear Strain}}; \qquad G = \frac{F_t/A}{x/y} = \frac{Fy}{Ax}$$

Illustration 11: A box shaped piece of gelatin dessert has a top area of 15 cm^2 and a height of 3cm. When a shearing force of 0.50 N is applied to the upper surface, the upper surface displaces 4 mm relative to the bottom surface. What are the shearing stress, the shearing strain and the shear modulus for the gelatin? **(JEE MAIN)**

Sol: Shearing stress is tangential force per unit area of surface. Shearing Strain is the ratio of displacement of the surface to the distance of the surface from the fixed surface. Shear modulus is the ratio of shearing stress to shearing strain.

$$\text{Shear stress} = \frac{\text{tangential force}}{\text{area of face}} = \frac{0.50\,\text{N}}{15\times10^{-4}\,\text{m}^2} = 333\ \text{N/m}^2$$

$$\text{Shear stress} = \frac{\text{Displacement}}{\text{height}} = \frac{0.4\,\text{cm}}{3\,\text{cm}} = 0.133$$

$$\text{Shear modulus G} = \frac{\text{stress}}{\text{strain}} = \frac{333}{0.133}$$

$$= 2.5\times10^3\ \text{N/m}^2 \qquad (1\ \text{Pa} = 1\ \text{N/m}^2)$$

4.3 Bulk Modulus

The bulk modulus of a solid or a fluid indicates its resistance to a change in volume. Consider a cube of some material, solid or fluid, as shown in the Fig. 8.43. We assume that all faces experience the same force F_n normal to each face. (One way to accomplish this is to immerse the body in a fluid-as long as the change in pressure over the vertical height of the cube is negligible). The pressure on the cube is defined as the normal force per unit area $p = \frac{F_n}{A}$

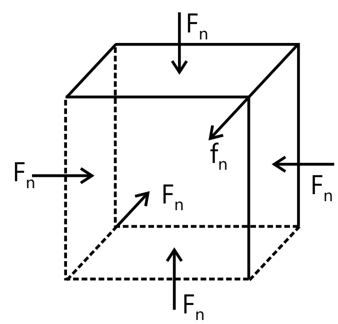


Figure 8.43: Determination of bulk modulus of an object

The SI unit of pressure is N/m^2 and is given the name Pascal (Pa).

The change in pressure ΔP is called the volume stress and the fractional change in volume $\Delta V / V$ called the volume strain. The bulk modulus B of the material is defined as

$$\text{Bulk modulus} = \frac{\text{Volume stress}}{\text{Volume strain}} \quad \text{or} \quad B = \frac{-\Delta P}{\Delta V / V}$$

The negative sign is included to make B a positive number since an increase in pressure $(\Delta p > 0)$ leads to a decrease in volume $(\Delta V < 0)$.

The inverse of B is called the compressibility factor $k = \frac{1}{B}$

Elastic properties of matter

Sate	**Shear Modulus**	**Bulk Modulus**
Solid	Large	Large
Liquid	Zero	Large
Gas	Zero	Small

PLANCESS CONCEPTS

Bulk Modulus has very important applications in case of fluids. Actually, it has various applications in adiabatic expansion of gases. Also, while calculating speed of sound through air, one would find that it would come out to be directly proportional to square root of bulk modulus of air. (In general, speed of sound depends of elastic properties of matter. A more general statement is that mechanical waves' speed depends on elastic properties of matter)

B Rajiv Reddy (JEE 2012, AIR 11)

Illustration 12: Find the decrease in the volume of a sample of water from the following data. Initial volume = 1000 cm^3, initial pressure = 10^5 Nm^{-2}, final pressure = 10^6 Nm^{-2}, compressibility of water = 50×10^{-11} m^2N^{-1}. **(JEE MAIN)**

Sol: Using the formula for bulk modulus deduce the value for decrease in volume.

The change in pressure = $\Delta P = 10^6\ Nm^{-2} - 10^5\ Nm^{-2} = 9 \times 10^5\ Nm^{-2}$.

Compressibility = $\dfrac{1}{\text{Bulk modulus}} = -\dfrac{\Delta V / V}{\Delta P}$ or,

$$50 \times 10^{-11}\ m^2N^{-1} = -\frac{\Delta V}{(10^{-3}\ m^3)\times(9\times10^5\ Nm^{-2})}$$

or, $\Delta V = -50 \times 10^{-11} \times 10^{-3} \times 9 \times 10^5\ m^3 = -4.5 \times 10^{-7}\ m^3 = -0.45\ cm^3$.

Thus the decrease in volume is 0.45 cm^3.

PLANCESS CONCEPTS

A solid will have all the three moduli of elasticity Y, B and η. But in case of a liquid or a gas, only B can be defined because a liquid or a gas cannot be framed into a wire or no shear force can be applied on them. For a liquid or a gas, $B = \left(\dfrac{-dP}{dV / V}\right)$

So, instead of P, we are more interested in change in pressure dP.

In case of a gas, $B = XP$

Anand K (JEE 2011, AIR 47)

5. THE STRESS-STRAIN CURVE

The stress-strain graph of a ductile metal is shown in Fig. 8.44 Initially, the strain graph is linear and it obeys the Hooke's Law up to the point P called the proportional limit. After the proportional limit, the $\sigma - \varepsilon$ graph is non-linear but it still remains elastic up to the yield point Y where the slope of the curve is zero. At the yield point, the material starts deforming under constant stress and it behaves like a viscous liquid. The yield point is the beginning of the plastic zone. After the yield point, the material starts gaining strength due to excessive deformation and this phenomenon is called strain hardening. The point U shows the

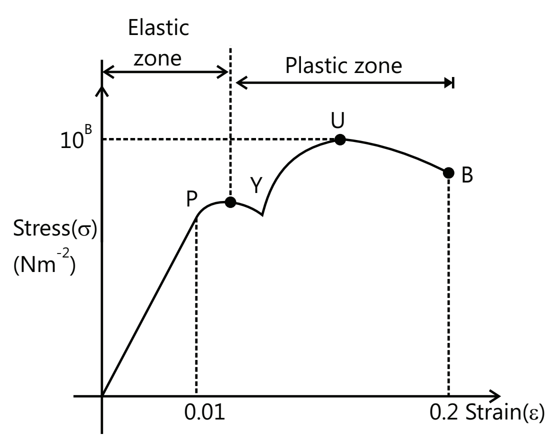


Figure 8.44

ultimate strength of the material. It is the maximum stress that the material can sustain without failure. After the point U the curve goes down towards the breaking point B because the calculation of the stress is based on the original cross-sectional area whereas the cross-sectional area of the sample actually decreases.

PLANCESS CONCEPTS

It is generally thought that strain results from stress, or many say that Hooke's law states wrong statement that stress is directly proportional to strain.

However, we must not worry because Hooke's law is correct. Going deeper to a microscopic level will help us understand better. It appears that external force cause strain in the body on which it is applied. However, stress is defined as restoring force (at equilibrium) per unit area. There can be no restoring force if there is no strain. Hence, strain is the cause and not stress. The only glitch here is that restoring force is equal to the force applied because (again not to forget) body is in equilibrium. So, it creates confusion but we must not take it for granted and understand the minute concepts.

Yashwanth Sandupatla (JEE 2012, AIR 821)

6. RELATION BETWEEN LONGITUDINAL STRESS AND STRAIN

For small deformations, longitudinal stress is directly proportional to the longitudinal strain. What if the deformation is large? The stress-strain relation gets more complicated in that case and depends on the material under study. Let's take a metal wire and a rubber piece as example and study the same.

Metal Wire: The Fig. 8.45 shows the relation between stress and strain as the deformation gradually decreases in a stretched wire.

Up to a strain < 0.01, Hooke's law is valid and Young's modulus is defined. Point a represents proportional limit up to which stress is proportional to strain.

Point b is called the yield point or elastic limit up to which stress is not proportional to strain (a to b) but elasticity still holds true.

The wire shows plastic behavior after point b where there is a permanent deformation in the wire and it does not return back to its original dimensions.

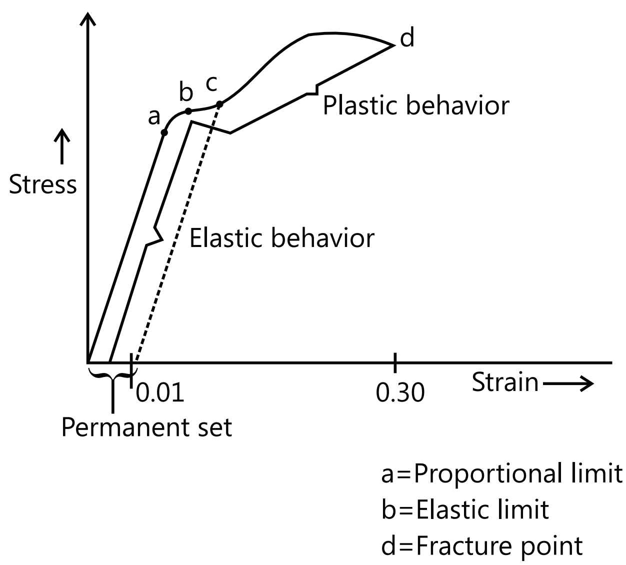


Figure 8.45: Graph of Stress versus Strain

The wire breaks at d which is the fracture point if stretched beyond point c. The corresponding stress is called breaking stress.

PLANCESS CONCEPTS

If large deformation takes place between the elastic limit and the fractured point, the material is called ductile. If it breaks soon after the elastic limit is crossed, it is called brittle.

Yashwanth Sandupatla (JEE 2012, AIR 821)

Rubber: Vulcanized rubber shows a very different stress-strain behavior. It remains elastic even if it is stretched to 8 times its original length. There are 2 important phenomena to note from the above Fig 8.46. Firstly, stress is nowhere proportional to strain during deformation. Secondly, when external forces are removed, body comes back to original dimensions but it follows a different retracing path.

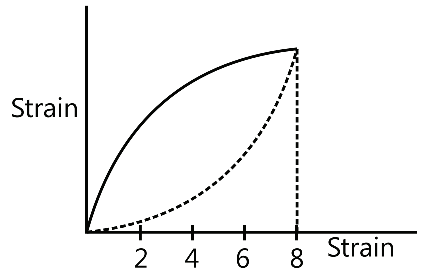


Figure 8.46: Stress versus Strain curve for rubber

The work done by the material in returning to its original shape is less than the work done by the deforming force when it was deformed. A particular amount of energy is, thus, absorbed by the material in the cycle which appears as heat. This phenomenon is called elastic hysteresis.

Elastic hysteresis has an important application in shock absorbers.

PLANCESS CONCEPTS

The material which has smaller value of Y is more ductile, i.e., it offers less resistance in framing it into a wire. Similarly, the material having the smaller value of B is more malleable. Thus, for making wire, we choose a material having less value of Y.

GV Abhinav (JEE 2012, AIR 329)

7. POISSON'S RATIO

When a longitudinal force is applied on a wire, its length increases but its radius decreases. Thus two strains are produced by a single force.

(a) Longitudinal strain = $\frac{\Delta l}{l}$ and (b) Lateral strain = $\frac{\Delta R}{R}$

The ratio of these two strains is called the Poisson's ratio.

Thus, the Poisson's ratio $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\Delta R / R}{\Delta l / l}$

Negative sign in σ indicates that radius of the wire decreases as the length increases.

PLANCESS CONCEPTS

Relation between Y, B, η and σ : Following are some relations between the four

(a) $B = \frac{Y}{3(1-3\sigma)}$ (b) $\eta = \frac{Y}{2(1+\sigma)}$ (c) $\sigma = \frac{3B-2\eta}{2\eta+6B}$ (d) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$

Anurag Saraf (JEE 2011, AIR 226)

8. ELASTIC POTENTIAL ENERGY OF A STRAINED BODY

When a body is in its natural shape, potential energy due to molecular forces is minimum and assumed to be zero. When deformed, internal forces come into existence and work is done against these forces. Thus potential energy of the body increases. This is called elastic potential energy.

8.1 Work Done in Stretching a Wire

If a force F is applied along the length of a wire of length l, area of cross-section A and Young's modulus Y, such that the wire is extended through a small length x, then $Y=\frac{Fl}{Ax}$ or $F=\frac{YAx}{l}$

The work done, W, in extending the wire through length Dl is given by

$$W=\int_0^{\Delta l} Fdx = \frac{YA}{l}\int_0^{\Delta l} xdx = \frac{YA(\Delta l)^2}{2l} = \frac{1}{2}\left(\frac{Y\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right)(Al) = \frac{1}{2}\times \text{stress} \times \text{strain} \times \text{volume}$$

Also $W=\frac{1}{2}\left(\frac{YA\Delta l}{l}\right)\Delta l=\frac{1}{2}\times \text{force} \times \text{extension}$

This work is stored in the wire as elastic potential energy.

Work done per unit volume = $\frac{1}{2}\left[\frac{Y\Delta l}{l}\right]\times\frac{\Delta l}{l}=\frac{1}{2}\times Y \times (\text{strain})^2=\frac{1}{2}\text{x stress x strain}$.

Illustration 13: Spring is stretched by 3 cm when a load of 5.4 × 10^6 dyne is suspended from it. Work done will be

(A) 8.1 × 10^6 J (B) 8 × 10^6 J (C) 8.0 × 10^6 ergs (D) 8.1 × 10^6 ergs **(JEE MAIN)**

Sol: Work done in stretching the spring is equal to the elastic potential energy stored in the spring.

(D) $W=\frac{1}{2}\times$ load × elongation W = 8.1 × 10^6 ergs =0.81 J

Illustration 14: A steel wire of length 2.0 m is stretched through 2.0 mm. The cross-sectional area of the wire is 4.0 mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young modulus of steel = 2.0 × 10^{11} N/m^2. **(JEE MAIN)**

Sol: We know the formula to find the elastic potential energy stored per unit volume of the wire. Calculate the volume of the wire and find the energy stored in the entire wire.

The strain in the wire $\frac{\Delta l}{l}=\frac{2.0\text{mm}}{2.0\text{m}}=10^{-3}$.

The stress in the wire = Y × strain = 2.0 × 10^{11} N m^{-2} × 10^{-3} = 2.0 × 10^8 N/m^2.

The volume of the wire = (4 × 10^{-6} m^2) × (2.0 m) = 8.0 × 10^{-6} m^3.

The elastic potential energy stored $=\frac{1}{2}\times$ stress × strain × volume

$=\frac{1}{2}\times 2.0\times10^8\ \text{Nm}^{-2}\times10^{-3}\times8.0\times10^{-6}\ \text{m}^3 = 0.8$ J

PLANCESS CONCEPTS

This energy can also be thought of as elastic potential energy of a spring. You just need to calculate spring constant.

A simple way would be considering Δl=x and rearranging terms of Hooke's law in the form of F=-kx. Remember F here is restoring force. Now energy is simply $\frac{1}{2}kx^2$

Vijay Senapathi (JEE 2011, AIR 71)

9. THERMAL STRESS AND STRAIN

A body expands or contracts whenever there is an increase or decrease in temperature. No stress is induced when the body is allowed to expand and contract freely. But when deformation is obstructed, stresses are induced. Such stresses are called thermal/ temperature stresses. The corresponding strains are called thermal/temperature strains.

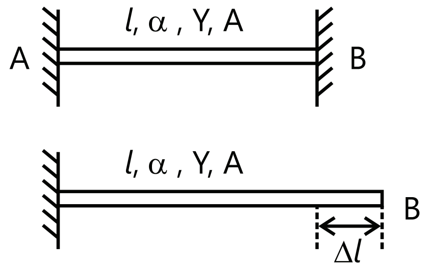


Figure 8.47: Thermal expansion of a rod

Consider a rod AB fixed at two supports as shown in Fig. 8.47.

Let l = Length of rod

A = Area of cross-section of the rod

Y = Young's modulus of elasticity of the rod

And α = Thermal coefficient of linear expansion of the rod

Let the temperature of the rod is increased by an amount t. The length of the rod would had increased by an amount Δl, if it were not fixed at two supports. Hence $\Delta l = l\alpha t$

But since the rod is fixed at the supports, a compressive strain will be produced in the rod. Because at the increased temperature, the natural length of the rod is $l + \Delta l$, while being fixed at two supports its actual length is l.

Hence, thermal strain $\varepsilon = \frac{\Delta l}{l} = \frac{l\alpha t}{l} = \alpha t$ or $\varepsilon = \alpha t$

Therefore, thermal stress $\sigma = Y\varepsilon$ (stress = Y × strain)

or $\sigma = Y\alpha t$ or force on the supports, $F = \sigma A = YA\alpha t$

This force F is in the direction shown:

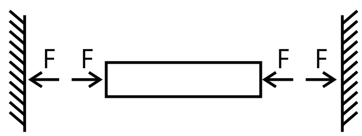


Figure 8.48: Thermal stress on a rod

Illustration 16: A wire of cross sectional area 3 mm^2 is just stretched between two fixed points at a temperature of 20°C. Determine the tension when the temperature falls to 20°C. Coefficient of linear expansion $\alpha = 10^{-5}$ / °C and Y = 2 × 10^{11} N/m^2. **(JEE MAIN)**

(A) 120 kN (B) 20 N (C) 120 N (D) 12 N

Sol: Thermal stress is equal to product of young's modulus and thermal strain. Tension is product of area of cross-section and stress.

(C) $F = YA\alpha\Delta t = 2 \times 10^{11} \times 3 \times 10^{-6} \times 10^{-5} \times 20$; F = 120 N

10. DETERMINATION OF YOUNG'S MODULUS IN LABORATORY

The given Fig. 8.49 shows an experimental set up of a simple method to determine Young's modulus in laboratory. A 2-3 metres long wire is suspended from a fixed support. It carries a graduated scale and below it a heavy fixed load. This load keeps the wire straight. Wire A is the reference wire whereas wire B serves as the experimental wire. A Vernier scale is placed at the end of the experimental wire.

Now the stress due to the weight Mg at the end is

Stress = $\frac{Mg}{\pi r^2}$ and strain $= \frac{l}{L}$; Thus, $Y = \frac{MgL}{\pi r^2 l}$

All the quantities on the right-hand side are known and hence Young's modulus Y may be calculated.

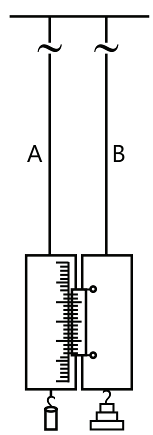


Figure 8.49: Searle's method for determination of young's modulus

PROBLEM-SOLVING TACTICS

- Be careful while using the Hooke's law of elasticity. Always remember that this law is not valid for an elastic material when it is stretched beyond its elastic limit. Stress is proportional to strain only when the material is stretched up to a certain limit.
- Always keep the stress-strain graph in mind while solving elasticity problems.
- The extent of ductility of a material can be calculated using the strain formulae. Greater the elongation, greater the ductility of the material. This concept can be used in questions where one is asked to arrange the elastic material in the order of increasing brittleness or ductility.
- Conservation of energy principle can be used to solved many problems where elastic potential energy gets converted to other forms of energy in the given problem system.
- Elongation and compression can be thought as analogous to a spring (refer to Plancess concept to how to do it) in appropriate limits.
- Direct questions may be asked on relation between Poisson's ratio and modulus of elasticity, so it would be nice if you learn them.

FORMULAE SHEET

Elasticity:

Stress: Stress (σ) = $\dfrac{\text{Restoring force}}{\text{Area}}$

SI units = N/m^2

Normal/ longitudinal stress $s_n = \dfrac{F_n}{A}$

F_n is the normal force

A is the cross-sectional area

Tangential / shearing stress $s_t = \dfrac{F_t}{A}$

F_t is the tangential force

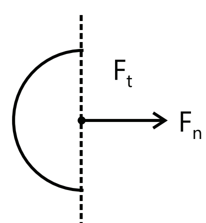


Figure 8.50

Volume stress $s_V = \dfrac{F}{A}$

Note: This is the stress developed when body is immersed in a liquid.

Strain: Longitudinal strain $\varepsilon = \dfrac{\Delta l}{l}$

Volumetric strain $\varepsilon = \dfrac{\Delta V}{V}$

$\Delta\ell$ and ΔV are change in length and volume respectively.

Shearing strain $\varepsilon = \dfrac{\Delta X}{X}$

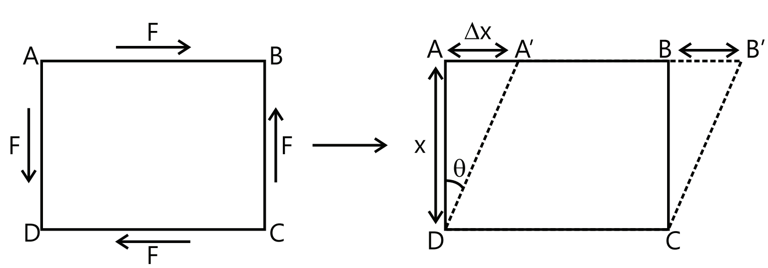


Figure 8.51

Hooke's Law: stress $\propto$ strain

stress = (E) (strain) (E is modulus of elasticity)

E is constant for a particular type of strain for a particular material. SI unit of E is N/m^2.

Young's modulus of elasticity (Y) $Y = \dfrac{\text{longitudinal stress}}{\text{Longitudinal strain}} = \dfrac{(F_n / A)}{(\Delta l / l)}$

Bulk modulus of elasticity (B) $B = \dfrac{-\text{Volume stress}}{\text{Volume strain}} = \dfrac{-F/A}{\Delta V / V} = \dfrac{-P}{\Delta V / V}$

For a liquid or gas $B = -\dfrac{dp}{(dV / V)}$

Compressibility = $\left(\dfrac{1}{\beta}\right)$

Modulus of rigidity (η) $\eta = \dfrac{\text{shearing stress}}{\text{shearing strain}} = \dfrac{F_t / A}{(\Delta x / x)} = \dfrac{F_t / A}{\theta}$ (See Fig. 8.52)

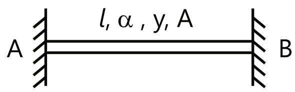


Figure 8.52

Elastic potential energy stored per unit volume in a stretched wire

$u = \dfrac{1}{2}$ (stress x strain)

Thermal stress and strain $\in = \dfrac{\Delta l}{l} = \alpha \Delta T$

$$Y = \frac{\sigma}{\in} = \frac{F}{A \in} = \frac{F}{A\alpha\Delta T}$$

α is thermal coefficient of linear expansion of rod. ΔT is change in temperature of the rod.

Variation of density with pressure: As pressure on a body increases, its density also increases. When pressure increases by dp, the new density ρ' in terms of the previous density ρ is $\rho' = \dfrac{\rho}{1 - \dfrac{dp}{B}}$ where B is the Bulk modulus.

Poisson's ratio: As the length of a wire of circular cross-section increases, its radius decreases.

Poisson's ratio is defined as $\sigma = \dfrac{\text{lateral strain}}{\text{longitudinal strain}} = -\dfrac{\Delta R / R}{\Delta l / l}$

Relation between Y, B, η and σ

$$B = \frac{Y}{3(1-2\sigma)}; \quad \sigma = \frac{3B - 2\eta}{2\eta + 6B}; \quad \eta = \frac{Y}{2(1+\sigma)}; \quad \frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$$

Solved Examples

JEE Main/Boards

Example 1: A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg-wt. Find the increase in its length, if the Young's modulus of steel is 2.4×10^{12} dyne/cm^2.

Sol: From the formula for Young's modulus deduce the change in length.

Here, l = 4 m = 400 cm, 2r = 5 mm

or r = 2.5 mm = 5mm

F = 5 kg-wt = 5000 g-wt = 5000 × 980 dyne

Δl = ?, Y = 2.4×10^{12} dyne/cm^2

$$\text{As } Y = \frac{F}{\pi r^2} \times \frac{l}{\Delta l}$$

$$\Delta l = \frac{(5000 \times 980) \times 400}{(22/7) \times (0.25)^2 \times 2.4 \times 10^{12}} = 0.0041 \text{ cm}$$

$\Delta l = 4.1 \times 10^{-5}$ m

Example 2: One end of a wire 2 m long and 0.2 cm^2 in cross section is fixed in a ceiling and a load of 4.8 kg is attached to the free end. Find the extension of the wire. Young's modulus of steel = 2.0×10^{11} N/m^2.

Take g = 10 m/s^2.

Sol: From the formula for Young's modulus deduce the extension in wire.

$$\text{We have } Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L}$$

With symbols having their usual meanings. The extension is $l = \frac{TL}{AY}$

As the load is in equilibrium after the extension, the tension in the wire is equal to the weight of the load = 4.8 kg × 10 ms^{-2} = 48 N

$$\text{Thus, } l = \frac{(48\text{N})(2\text{m})}{(0.2 \times 10^{-4}\text{m}^2) \times (2.0 \times 10^{11}\text{Nm}^{-2})}$$

$= 2.4 \times 10^{-5}$ m.

Example 3: A steel wire 4.0 m in length is stretched through 2.0 mm. The cross-sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} N/m^2. Find

(a) The energy density of wire,

(b) The elastic potential energy stored in the wire.

Sol: Find the stress and strain and use the formula for energy density. Product of energy density and volume is energy stored in entire wire.

Here, l = 4.0 m, $\Delta l = 2 \times 10^{-3}$ m,

A = 2.0×10^{-6} m^2, Y = 2.0×10^{11} N/m^2

(a) The energy density of stretched wire

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$$

$$= \frac{1}{2} \times 2.0 \times 10^{11} \times \left(\frac{(2 \times 10^{-3}}{4}\right)^2$$

$= 0.25 \times 10^5 = 2.5 \times 10^4$ J/m^3.

(b) Elastic potential energy = energy density × volume= $2.5 \times 10^4 \times (2.0 \times 10^{-6}) \times 4.0$ J

$= 20 \times 10^{-2} = 0.20$ J

Example 4: The bulk modulus of water is

2.3×10^9 N/m^2.

(a) Find its compressibility.

(b) How much pressure in atmosphere is needed to compress a sample of water by 0.1%?

Sol: Compressibility is inverse of bulk modulus. From the formula for bulk modulus deduce the change in pressure required to produce the given change in volume.

Here, B = 2.3×10^9 N/m^2

$$= \frac{2.3 \times 10^9}{1.01 \times 10^5} \text{atm} = 2.27 \times 10^4 \text{ atm}$$

$$\text{(a) Compressibility} = \frac{1}{B}$$

$$= \frac{1}{2.27 \times 10^4} = 4.4 \times 10^{-5} \text{ atm}^{-1}$$

(b) Here, $\frac{\Delta V}{V} = -0.1\% = -0.001$

Required increase in pressure,

$$\Delta P = B \times \left(-\frac{\Delta V}{V}\right)$$

= 2.27 × 10^4 × 0.001= 22.7 atm

Example 5: One end of a nylon rope of length 4.5 m and diameter 6 mm is fixed to a tree-limb. A monkey weighing 100 N jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in the diameter. Young's modulus of nylon = 4.8 × 10^{11} Nm^{-2} and Poisson ratio of nylon = 0.2.

Sol: From the formula for Young's modulus deduce the change in length of the rope. From the formula for Poisson ratio deduce the change in diameter.

As the monkey stays in equilibrium, the tension in the rope equals the weight of the monkey. Hence,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L} \text{ or } l = \frac{TL}{AY}$$

or, elongation

$$l = \frac{(100\text{ N})\times(4.5\text{ m})}{(\pi\times9\times10^{-6}\text{ m}^2)\times(4.8\times10^{11}\text{ Nm}^{-2})}$$

= 3.32 × 10^{-5} m

$$\text{Again, Poisson ratio } = \frac{\Delta d/d}{l/L} = \frac{(\Delta d)L}{ld}$$

$$\text{or, } 0.2 = \frac{\Delta d\times4.5\text{ m}}{(3.32\times10^{-5}\text{ m})\times(6\times10^{-3}\text{ m})}$$

$$\text{or, } \Delta d = \frac{0.2\times6\times3.32\times10^{-8}\text{ m}}{4.5} = 8.8 \times 10^{-9}\text{ m}$$

Example 6: A solid lead sphere of volume 0.5 m^3 is taken in the ocean to a depth where the water pressure is 2 × 10^7 N/m^2. If the bulk modulus of lead is 7.7 × 10^9 N/m^2. Find the fractional change in the radius of the sphere.

Sol: From the formula for bulk modulus deduce the change in volume for the given increase in pressure.

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta r}{r} = \frac{1}{3}\frac{\Delta V}{V}$$

$$\text{Bulk modulus } K = -\frac{\Delta P}{(\Delta V/V)}$$

$$\text{or } \frac{\Delta V}{V} = -\frac{\Delta P}{K}$$

$$\text{or } \frac{\Delta r}{r} = -\frac{1}{3}\frac{\Delta P}{K} = -\frac{1}{3}\times\frac{2\times10^7}{7.7\times10^9}$$

= – 0.87 × 10^{-3}.

The negative sign indicates that the radius decreases.

Example 7: Find the greatest length of steel wire that can hang vertically without breaking. Breaking stress of steel =8.0 × 10^8 N/m^2.

Density of steel = 8.0 × 10^3 kg/m^3.

Take g = 10 m/s^2.

Sol: Breaking stress gives the maximum weight per unit area of cross-section that the wire can withstand.

Let l be the length of the wire that can hang vertically without breaking. Then the stretching force on it is equal to its own weight. If therefore, A is the area of cross-section and ρ is the density, then

$$\text{Maximum stress } (s_m) = \frac{\text{weight}}{A}$$

$$\left(\text{stress} = \frac{\text{force}}{\text{area}}\right) \text{ or } \sigma_m = \frac{(Al\rho)g}{A}$$

$$\therefore \quad l = \frac{\sigma_m}{\rho g} \text{ Substituting the values}$$

$$l = \frac{8.0\times10^8}{(8.0\times10^3)(10)} = 10^4\text{ m}$$

Example 8: A copper wire of negligible mass, length 1 m and cross-sectional area 10^{-6} m^2 is kept on a smooth horizontal table with one end fixed. A ball of mass 1 kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad/s. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus of copper. If on increasing the angular velocity to 100 rad/s, the wire breaks down, obtain the breaking stress.

Sol: The stress developed in the wire will be due to the centrifugal force. Ratio of stress and strain is the Young's modulus. The breaking stress will be due to the centrifugal force at increased angular velocity.

The stretching force developed in the wire due to rotation of the ball is

F = mrw^2 = 1 × 1 × $(20)^2$ = 400 N

$$\text{Stress in the wire} = \frac{F}{A} = \frac{400}{10^{-6}}\text{ N/m}^2 \text{ Strain in the wire}$$

$$= \frac{10^{-3}}{1} = 10^{-3}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{400}{10^{-6}\times10^{-3}} = 4\times10^{11}\text{ N/m}^2$$

Breaking stress = $\frac{1\times1\times(100)^2}{10^{-6}}$ = 10^{10} N/m^2.

Example 9: (a) A wire 4 m long and 0.3 mm in diameter is stretched by a force of 100 N. If extension in the wire is 0.3 mm, calculate the potential energy stored in the wire.

(b) Find the work done in stretching a wire of cross-section 1 mm^2 and length 2 m through 0.1 mm, Young's modulus for the material of wire is 2.0×10^{11} N/m^2.

Sol: Work done in stretching the wire is equal to the elastic potential energy stored in the wire. (a) Energy stored

$$U=\frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$

or $$U=\frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta l}{l}\right)(Al) = \frac{1}{2}F\cdot\Delta l$$

$$= \frac{1}{2}(100)\left(0.3\times10^{-3}\right) = 0.015\text{ J}$$

(b) Work done = potential energy stored

$$= \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2\left(\text{as } k=\frac{YA}{l}\right)$$

Substituting the values, we have

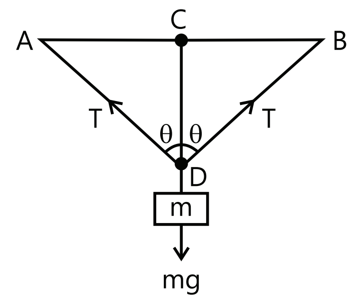
$$W=\frac{1}{2}\frac{(2.0\times10^{11})(10^{-6})}{(2)}(0.1\times10^{-3})^3$$

$= 5.0 \times 10^{-4}$ J

Example 10: A steel wire of diameter 0.8 mm and length 1 m is clamped firmly at two points A and B which are 1 m apart and in the same horizontal plane. A body is hung from the middle point of the wire such that the middle point sags 1 cm lower from the original position. Calculate the mass of the body. Given Young's modulus of the material of wire = 2×10^{12} dynes/cm^2.

Sol: Tension in the wire is the product of stress and area of cross-section. Stress is the product of Young's modulus and strain. The vertical components of tensions in the two parts of the wire will balance the weight of the body hung from the wire.

Let the body be hung from the middle point C so that it sags through 1cm to the point D as shown in the figure.



$\therefore$ AD^2 = AC^2 + CD^2 = $(50)^2 + (1)^2$

or AD = 50.01 cm

Increase in length = 0.01 cm

Strain = $\frac{0.01}{50} = 2\times10^{-4}$

Stress = $2 \times 10^{12} \times 2 \times 10^{-4}$

$\therefore$ Stress = 4×10^8 dynes/cm^2

Tension T = Stress × Area of cross-section

= $4 \times 10^8 \times \pi \times (0.08)^2$

Since the mass m is in equilibrium

mg = 2T cos θ or m = $\frac{2T\cos\theta}{g}$

$$= \frac{2\times4\times10^8\times\pi(0.08)^2\times(1/50.01)}{980} = 82\text{ gm.}$$

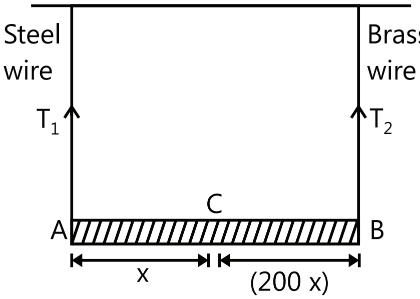
JEE Advanced/Boards

Example 1: A light of rod of length 200 cm is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section 0.1 sq cm and the other is of brass of cross-section 0.2 sq. cm. Find the position along the rod at which a weight may be hung to produce (a) equal stresses in both wires and (b) equal strains in both wires.

(Y_{brass} = 10×10^{11} dynes/cm^2.

Y_{steel} = 20×10^{11} dynes/cm^2).

Sol: Net torque of the tensions in the wires about the point of suspension of the weight on the rod must be zero.



Let AB be the rod and let C be the point at which the weight is hung.

(a) Stress in steel wire = $\frac{T_1}{0.1}$

Stress in brass wire = $\frac{T_2}{0.2}$

As the two stresses are equal,

$$\frac{T_1}{0.1} = \frac{T_2}{0.2} \text{ or } \frac{T_1}{T_2} = 0.5 \qquad \text{... (i)}$$

Taking moments about C,

$$T_1 = T_2 (200 - x) \text{ or } \frac{T_1}{T_2} = \frac{200 - x}{x} \qquad \text{... (ii)}$$

Equations (i) and (ii) give

$$\frac{200 - x}{x} = 0.5$$

Or x = 133.3 cm = 1.33 m

(b) Strain = $\frac{\text{Stress}}{Y} = \frac{T}{AY}$

As the strain in both wires are equal,

$$\frac{T_1}{A_1Y_1} = \frac{T_2}{A_2Y_2} \text{ or } \frac{T_1}{T_2} = \frac{A_1Y_1}{A_2Y_2} = \frac{0.1\times20\times10^{11}}{0.2\times10\times10^{11}}$$

$\therefore T_1 = T_2$

Now, $T_1x = T_2 (200 - x) \Rightarrow x = 200 - x$

or x = 100 cm =1 m.

Example 2: A rod AD, consisting of three segments AB, BC and CD joined together, is hanging vertically from a fixed support at A. The lengths of the segments are respectively 0.1 m, 0.2 m and 0.15 m. The cross-section of the rod is uniformly equal to 10^{-4} m^2. A weight of 10 kg is hung from D. Calculate the displacements of the points B, C and D using the data on Young's moduli given below (neglect the weight of the rod).

$Y_{AB} = 2.5 \times 10^2$ N/m,

$Y_{BC} = 4.0 \times 10^2$ N/m and

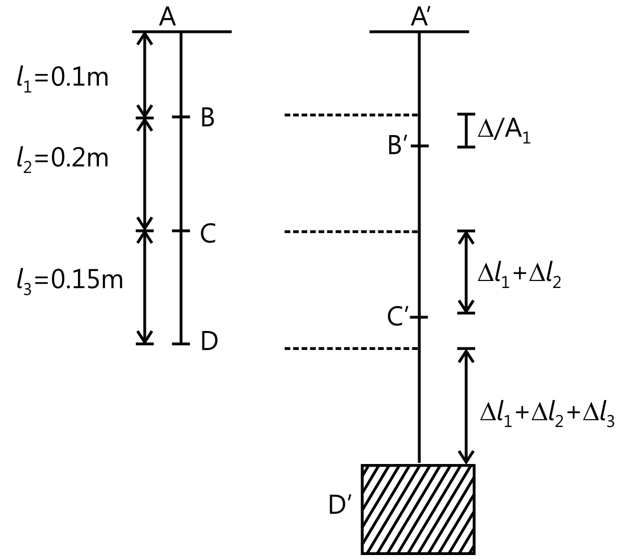
$Y_{CD} = 1.0 \times 10^2$ N/m

Sol: From the formula for Young's modulus deduce the elongation in each segment of the wire.

We know that

$$\Delta l = \frac{mgl}{AY} = \frac{10\times9.8\times0.1}{10^{-4}\times2.5\times10^{10}} = 3.92 \times 10^{-6} \text{ m}$$

This is the displacement of B.



For segment BC:

$$\Delta l_2 = \frac{10\times9.8\times0.2}{10^{-4}\times4.0\times10^{10}} = 4.9 \times 10^{-6} \text{ m}$$

Displacement of C: = $\Delta l_1 + \Delta l_2$

= 4.9×10^{-6} m

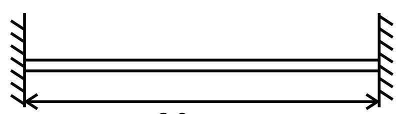
For segment CD:

$$\Delta l_3 = \frac{10\times9.8\times0.15}{10^{-4}\times1.0\times10^{10}} = 14.7 \times 10^{-6} \text{ m}$$

Displacement of D = $\Delta l_1 + \Delta l_2 + \Delta l_3$

= 23.52×10^{-6} m.

Example 3: A steel rod of length 6.0 m and diameter 20 mm is fixed between two rigid supports. Determine the stress in the rod, when the temperature increases by 80° C if



(a) The ends do not yield

(b) The ends yield by 1 mm.

Take Y = 2.0×10^6 kg/cm^2

And $\alpha = 12 \times 10^{-6}$ per °C.

Sol: Rise in temperature causes thermal strain and thermal stress. Use the formula for coefficient of thermal expansion to obtain thermal strain. Thermal stress is the product of Young's modulus and thermal strain.

Given, length of the rod l = 6 m

Diameter of the rod d = 20 mm = 2 cm

Increase in temperature t = 80°C

Young's modulus Y = 2.0×10^6 kg/cm^2

And thermal coefficient of linear expansion

$\alpha = 12 \times 10^{-6}$ per °C

(a) When the ends do not yield

Let, s_1 = stress in the rod

Using the relation $\sigma = \alpha t Y$

$\therefore s_1 = (12 \times 10^{-6})(80)(2 \times 10^6)$

= 1920 kg/cm^2 = 19.2×10^6 N

(b) When the ends yield by 1 mm

Increase in length due to increase in temperature Dl = $l$at

Of this 1mm or 0.1 cm is allowed to expand. Therefore, net compression in the rod

$\Delta l_{net} = (lt - 0.1)$

or compressive strain in the rod,

$$\varepsilon = \frac{\Delta l_{net}}{l} = \left(\alpha t - \frac{0.1}{l}\right)$$

$$\therefore \text{Stress } s_2 = Y\varepsilon = Y\left(\alpha t - \frac{0.1}{l}\right)$$

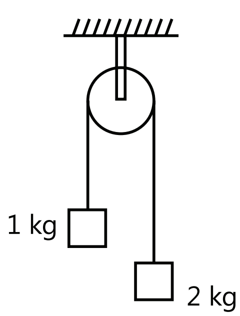
Substituting the values,

$$s_2 = 2 \times 10^6 \left(12 \times 10^{-6} \times 80 - \frac{0.1}{600}\right)$$

= 1587 kg/cm^2 = 15.8×10^6 N

Example 4: Two blocks of masses 1 kg and 2 kg are connected by a metal wire going over a smooth pulley as shown in figure The breaking stress of the metal is 2×10^9 Nm^{-2}. What should be the minimum radius of the wire used if it is not to break?

Take g = 10 ms^{-2}.



Sol: Find the tension in the metal wire due to the masses connected to it. The stress due to tension should not exceed the breaking stress.

The stress in the wire

$$= \frac{\text{Tension}}{\text{Area of crosssection}}$$

To avoid breaking, this stress should not exceed the breaking stress.

Let the tension in the wire be T. The equations of motion of the two blocks are,

T – 10 N = (1kg) a and 20 N – T = (2kg) a

Eliminating a from these equations,

T = (40/3) N

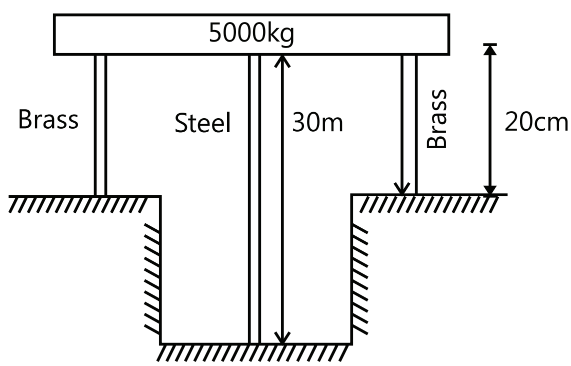
$$\text{The stress} = \frac{(40/3)\text{ N}}{\pi r^2}$$

If the minimum radius needed to avoid breaking is r,

$$2 \times 10^9 \frac{\text{N}}{\text{m}^2} = \frac{(40/3)\text{ N}}{\pi r^2}$$

Solving this, $r = 4.6 \times 10^{-5}$ m.

Example 5: A steel rod of cross-sectional area 16 cm^2 and two brass rods each of cross-sectional area 10 cm^2 together support a load of 5000 kg as shown in figure Find the stress in the rods. Take Y for steel = 2.0×10^6 kg/cm^2 and for brass = 1.0×10^6 kg/cm



Sol: Compression in the length of steel and brass rods is equal. From the formula for Young's modulus deduce the compression in length of each rod and equate them to get the relation between respective stresses.

Given area of steel rod A_S = 16 cm^2

Area of two brass rods

$A_B = 2 \times 10 = 20$ cm^2

Load, F = 5000 kg

Y for steel $Y_S = 2.0 \times 10^6$ kg/cm^2 Y for brass $Y_B = 1.0 \times 10^6$ kg/cm^2

Length of steel rod l_S = 30 cm

Length of steel rod l_B = 20 cm

Let s_S = stress in steel

and σ_B = stress in brass

Decrease in length of steel rod = decrease in length of brass rod

or $\frac{\sigma_S}{Y_S} \times l_S = \frac{\sigma_B}{Y_B} \times l_B$

or $s_S = \frac{Y_S}{Y_B} \times \frac{l_B}{l_S} \times \sigma_B$

$= \frac{2.0 \times 10^6}{1.0 \times 10^6} \times \frac{20}{30} \times \sigma_B$

$\therefore \quad s_S = \frac{4}{3}\, \sigma_B$ .... (i)

Now, using the relation,

$F = s_s A_s + \sigma_B A_B$ or

$5000 = s_S \times 16 + \sigma_B \times 20$ .... (ii)

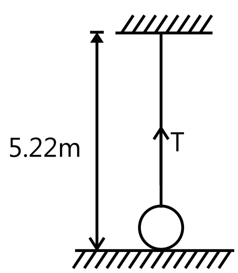
Solving eq. (i) and (ii), we get

$\sigma_B = 120.9$ kg/cm^2 and $s_S = 161.2$ kg/cm^2

Example 6: A sphere of radius 0.1 m and mass 8π kg is attached to the lower end of a steel wire length 5.0 m and diameter 10^{-3} m. The wire is suspended from 5.22 m high ceiling of a room. When sphere is made to swing as a simple pendulum, it just grazes the floor at its lowest point. Calculate velocity of the sphere at the lowest position. Young's modulus of steel is 1.994 $\times 10^{11}$ N/m^2.

Sol: The elongation in the wire is known, thus the corresponding stress can be calculated. The stress in turn gives the tension in the wire. At the lowest point the net acceleration of the sphere is centripetal, i.e. directed vertically upwards. Apply Newton's second law at the lowest point to find the speed of the sphere.

Let Dl be the extension of wire when the sphere is at mean position. Then, we have



$l + \Delta l + 2r = 5.22$

or $\Delta l = 5.22 - l - 2r$

$5.22 - 5 - 2 \times 0.1 = 0.02$ m

Let T be the tension in the wire at mean position during oscillations, $Y = \frac{T/A}{\Delta l / l}$

$\therefore \quad T = \frac{YA\Delta l}{l} = \frac{Y\pi r^2 \Delta l}{l}$

Substituting the values, we have

$$T = \frac{(1.994 \times 10^{11}) \times \pi \times (0.5 \times 10^{-3})^2 \times 0.02}{5}$$

= 626.43 N

The equation of motion at mean position is,

$$T - mg = \frac{mv^2}{R}$$

Hence, $R = 5.22 - r = 5.22 - 0.1 = 5.12$ m

and $m = 8\pi$ kg = 25.13 kg

Substituting the proper values in Eq. (i), we have

$$(626.43) - (25.13 \times 9.8) = \frac{(25.13)v^2}{5.12}$$

Solving this equation, we get V = 8.8 m/s

Example 7: A thin ring of radius R is made of a material of density ρ and Young's modulus Y. If the ring is rotated about its center in its own plane with angular velocity ω, find the small increase in its radius.

Sol: As the ring rotates each element of the ring of infinitesimal length experiences a centrifugal force, due to which the ring slightly expands, thus increasing its radius. The longitudinal strain in the ring produces a tensile stress or tension in the ring.

Consider an element PQ of length dl. Let T be the tension and A the area of cross-section of the wire.

Mass of element dm = volume × density = A (dl)ρ

The component of T, towards the center provides the necessary centripetal force

$\therefore \quad 2T\sin\left(\frac{\theta}{2}\right) = (dm)R\omega^2$ ... (i)

For small angles $\sin\frac{\theta}{2} \approx \frac{\theta}{2} = \frac{(dl/R)}{2}$

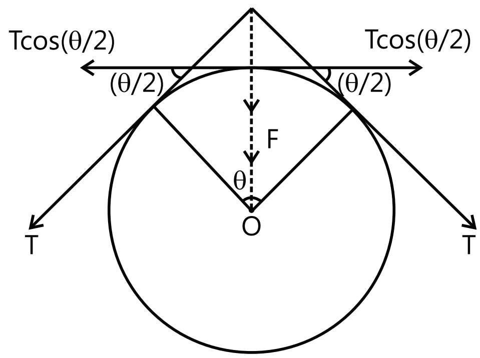
Substituting in eq. (i), we have

$T.\frac{dl}{R} = A(dl)\rho R\omega^2$ or $T = Arw^2R^2$

Let DR be the increase in radius,

Longitudinal strain

$$\frac{\Delta l}{l} = \frac{\Delta(2\pi R)}{2\pi R} = \frac{\Delta R}{R}$$



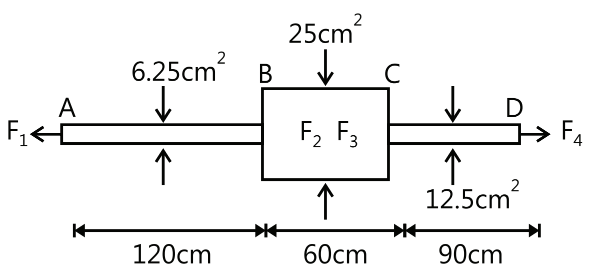
Now, $Y = \frac{T/A}{\Delta R/R}$

$$\therefore\ \Delta R = \frac{TR}{AY} = \frac{(A\rho\omega^2R^2)R}{AY} \text{ or } DR = \frac{\rho\omega^2R^3}{Y}$$

Example 8: A member ABCD is subjected to point loads F_1, F_2, F_3 and F_4 as shown in figure Calculate the force F_2 for equilibrium if F_1 = 4500 kg,

F_3 = 45000 kg and F_4 =13000 kg.

Determine the total elongation of the member, assuming modulus of elasticity to be 2.1 × 10^6 kg/cm^2.



Sol: Find the tension in each segment of the member ABCD. From the formula of Young's modulus, find the elongation in each segment.

Given

Area of part AB, A_1 = 6.25 cm^2

Area of part AB, A_2 = 25 cm^2

Area of part CD, A_3 = 12. 5 cm^2

Length of part AB, l_1 = 120 cm

Length of part BC, l_2 = 60 cm

Length of part CD, l_3 = 90 cm

Young's modulus of elasticity

Y = 2.1 × 10^6 kg/cm^2

Magnitude of the force F_2 for equilibrium

The magnitude of force F_2 may be found by equating the forces acting towards right to those acting towards left,

$F_2 + F_4 = F_1 + F_3$

F_2 + 13000 = 4500 + 45000

$\therefore$ F_2 = 36500 kg

Total Elongation of the member

For the sake of simplicity, the force of 36500 kg (acting at B) may be split up into two forces of 4500 kg and 32000 kg. The force of 45000 kg acting at C may be split into two forces of 32000 kg and 13000 kg. Now, it will be seen that the part AB of the member is subjected to a tension of 4500 kg, part BC is subjected to a compression of 32000 kg and part CD is subjected to a tension of 13,000 kg. Using the relation.

$$\Delta l = \frac{1}{Y}\left(\frac{F_1l_1}{A_1} - \frac{F_2l_2}{A_2} + \frac{F_3l_3}{A_3}\right)$$

With usual notation

$$\Delta l = \frac{1}{2.1\times10^6} x$$

$$\left(\frac{4500\times120}{6.25} - \frac{32000\times60}{25} + \frac{13000\times90}{12.5}\right)\text{cm}$$

= 0.049 cm or Dl = 0.49 mm

JEE Main/Boards

Exercise 1

Q.1 A wire is replaced by another wire of same length and material but of twice diameter.

(i) What will be the effect on the increase in its length under a given load?

(ii) What will be the effect on the maximum load which it can bear?

Q.2 Two wires are made of same metal. The length of the first wire is half that of the second wire and its diameter is double that of the second wire. If equal loads are applied on both wires, find the ratio of increase in their lengths.

Q.3 The breaking force for a wire is F. What will be the breaking forces for

(i) Two parallel wires of this size and

(ii) For a single wire of double thickness?

Q.4 What force is required to stretch a steel wire 1 sq. cm in cross section to double its length? $Y_{steel} = 2 \times 10^{11}$ Nm^{-2}.

Q.5 A structural steel rod has a radius of 10 mm and length of 1.0 m. A 100 kN force stretches it along its length. Calculate (a) stress, (b) elongation, and (c)% strain in the rod. Young's modulus of structural steel is 2.0×10^{11} Nm^{-2}.

Q.6 Find the maximum length of a steel wire that can hang without breaking.

Breaking stress = 7.9×10^{12} dyne/cm^2.

Density of steel = 7.9 g/cc.

Q.7 A spherical ball contracts in volume by 0.01%, when subjected to a normal uniform pressure of 100 atmosphere. Calculate the bulk modulus of the material.

Q.8 A sphere contracts in volume by 0.02% when taken to the bottom of sea 1 km deep. Find bulk modulus of the material of sphere. Density of sea water is 1000 kg/m^3.

Q.9 A metal cube of side 10 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity if the top of the cube is displaced by 0.05 cm with respect to its bottom.

Q.10 Calculate the increase in energy of a brass bar of length 0.2 m and cross sectional area 1cm^2 when combined with a load of 5kg weight along its length. Young's modulus of brass = 1.0×10^{11} Nm^{-2} and g = 9.8 ms^{-2}.

Q.11 A wire 30m long and of 2 mm^2 cross-section is stretched due to a 5kg-wt by 0.49 cm. Find

(i) The longitudinal strain

(ii) The longitudinal stress and

(iii) Young's modulus of the material of the wire.

Exercise 2

Single Correct Choice Type

Q.1 A wire of length 1m is stretched by a force of 10N. The area of cross-section of the wire is 2×10^{-6} m^2 & γ is 2×10^{11} N/m^2. Increase in length of the wire will be-

(A) 2.5×10^{-5} cm (B) 2.5×10^{-5} mm

(C) 2.5×10^{-5} m (D) None

Q.2 A uniform steel wire of density 7800 kg/m^3 is 2.5 m long and weighs 15.6×10^{-3} kg. It extends by 1.25 mm when loaded by 8kg. Calculate the value of young's modulus for steel.

(A) 1.96×10^{11} N/m^2 (B) 19.6×10^{11} N/m^2

(C) 196×10^{11} N/m^2 (D) None

Q.3 The work done in increasing the length of a one meter long wire of cross-sectional area 1mm^2 through 1mm will be (Y = 2×10^{11} N/m^2)

(A) 250 J (B) 10 J (C) 5 J (D) 0.1 J

Q.4 The lengths and radii of two wires of same material are respectively L, 2L, and 2R, R. Equal weights are applied on them. If the elongations produced in them are l_1 and l_2 respectively, then their ratio will be

(A) 2 : 1 (B) 4: 1 (C) 8 : 1 (D) 1 : 8

Q.5 What is the density of lead under a pressure of 2.0×10^8 N/m^2, if the bulk modulus of lead is 8.0×10^9 N/m^2 and initially the density of lead is 11.4g/cm^3 ?

(A) 11.69g/cm^3 (B) 11.92g/cm^3

(C) 11.55g/cm^3 (D) 11.862g/cm^3

Q.6 A rubber rod of density $1.3\times10^3 kg/m^3$ and Young's modulus $6\times10^6 N/m^2$ hangs from the ceiling of a room. Calculate the deviation in the value of its length from the original value 10m.

(A) 10.9 cm (B) 5.8 cm (C) 9.3 cm (D) 10.6 cm

Q.7 A metal rod is trapped horizontally between two vertical walls. The coefficient of linear expansion of the rod is equal to $1.2\times10^{-5}/^\circ C$ and its Young's modulus $2\times10^{11} N/m^2$. If the temperature of the rod is increased by 5°C, calculate the stress developed in it.

(A) 2.2×10^7 N/m^2 (B) 3.1×10^7 N/m^2

(C) 1.2×10^7 N/m^2 (D) 1.2×10^4 N/m^2

Previous Years' Questions

Q.1 The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied? ***(1981)***

(A) Length=50 cm, diameter=0.5 mm

(B) Length=100 cm, diameter = 1 mm

(C) Length=200cm, diameter= 2 mm

(D) Length=300 cm, diameter=3 mm

Q.2 A given quantity of an ideal gas is at pressure p and absolute temperature T. The isothermal bulk modulus of the gas ***(1998)***

(A) $\frac{2}{3}p$ (B) p (C) $\frac{3}{2}p$ (D) 2p

Q.3 The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is ***(2005)***

(A) $204.8\times10^5 Pa$ (B) $102.4\times10^5 Pa$

(C) $51.2\times10^5 Pa$ (D) $1.55\times10^5 Pa$

Q.4 A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to: (g = gravitational acceleration ***(2015)***

(A) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$ (B) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

(C) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (D) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$

JEE Advanced/Boards

Exercise 1

Q.1 A rubber cord has a cross-sectional area 1mm^2 and total unstretched length 10.0 cm. It is stretched to 12.0 cm and then released to project a missile of mass 5.0g. Taking Young's modulus Y for rubber as $5.0\times10^8 N/m^2$. Calculate the velocity of projection.

Q.2 Calculate the pressure required to stop the increase in volume of a copper block when it is heated from 50°C to 70°C. Coefficient of linear expansion of copper = 8.0 × 10–6/°C and the bulk modulus of elasticity = 10^{11} N/m^2.

Q.3 Calculate the increase in energy of a brass bar of length 0.2m and cross-sectional area 1.0 cm^2, when compressed with a load of 5kg-weight along its length. Young's modulus of brass $=1.0\times10^{11} N/m^2$ and g = 9.8 m/s^2.

Exercise 2

Q.1 A steel wire of uniform cross-section of 2mm^2 is heated upto 50° and clamped rigidly at two ends. If the temperature of wire falls to 30° then change in tension in the wire will be, if coefficient of linear expansion of steel is 1.1×10^{-5} /°C and young's modulus of elasticity of steel is 2×10^{11} N/m^2.

(A) 44 N (B) 88 N (C) 132 N (D) 22 N

Q.2 A metallic wire is suspended by suspending weight to it. If S is longitudinal strain and Y its young's modulus of elasticity. Potential energy per unit volume will be

(A) $\frac{1}{2}Y^2S^2$ (B) $\frac{1}{2}Y^2S$ (C) $\frac{1}{2}YS^2$ (D) $2YS^2$

Q.3 The compressibility of water is $5\times10^{-10} m^2/N$. Find the decrease in volume of 100 m*l* of water when subjected to a pressure of 15 mPa.

(A) 0.75 m*l* (B) 0.75 mm

(C) 0.75 mm (D) 7.5 mm

Q.4 The upper end of a wire 1 meter long and 2mm radius is clamped. The lower end is twisted through an angle of 45º. The angle of shear is

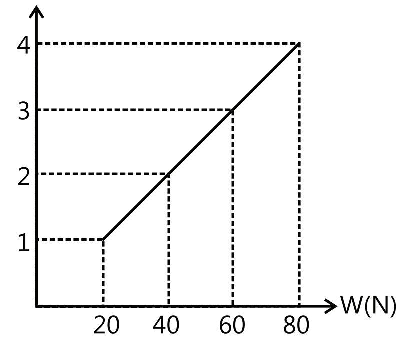
(A) 0.09° (B) 0.9° (C) 9° (D) 90°

Previous Years' Questions

Q.1 Two rods of different materials having coefficient of thermal expansion a_1, a_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $a_1 : a_2$ = 2:3, the thermal stresses developed in the two roads are equal provided $Y_1 : Y_2$ is equal to ***(1989)***

(A) 2 : 3 (B) 1 : 1 (C) 3 : 1 (D) 4 : 9

Q.2 The adjacent graph shows extension (D*l*) of a wire of length 1m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6}m^2, calculate from the graph the Young's modulus of the material of the wire. ***(2003)***

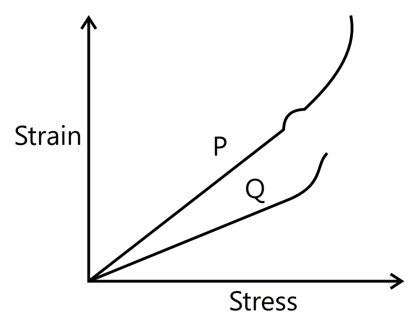


(A) 2×10^{11} N/m^2 (B) 2×10^{-11} N/m^2

(C) 2×10^{12} N/m^2 (D) 2×10^{13} N/m^2

Q.3 In Searle's experiment, which is used to find Young's modulus of elasticity, the diameter of experimental wire is D = 0.05 cm (measured by a scale of least count 0.001 cm) and length is L = 110 cm (measured by a scale of least count 0.1 cm). A weight of 50N causes an extension of *l* = 0.125 cm (measured by a micrometer of least count 0.001 cm). Find maximum possible error in the values of Young's modulus. Screw gauge and meter scale are free from error. ***(2004)***

Q.13 In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is(are**)** ***(2015)***



(A) P has more tensile strength than Q

(B) P is more ductile than Q

(C) P is more brittle than Q

(D) The Young's modulus of P is more than that of Q

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 1 Q.3 Q.7

Q.10

Exercise 2

Q.4 Q.6 Q.7

Previous Years' Questions

Q.9 Q.11

JEE Advanced/Boards

Exercise 2

Q.1 Q.4

Previous Years' Questions

Q.1 Q.2 Q.3

Answer Key

JEE Main/ Boards

Exercise 1

Q.2 1: 8

Q.3 (i) 2F (ii) 4F

Q.4 2×10^7 N

Q.5 (a) 3.18×10^8 N m^{-2} (b) 1.59 mm (c) 0.16%

Q.6 1.02×10^9 cm

Q.7 1.013×10^{11}

Q.8 4.9×10^{10}

Q. 9 2×10^6 Nm^{-2}

Q.10 2.4×10^{-5} J

Q.11 1.5×10^{11} N/m^2.

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** A **Q.3** D **Q.4** D **Q.5** A

Q.6 D **Q.7** C

Previous Years' Questions

Q.1 A **Q.2** B **Q.3** D **Q.4** D

JEE Advanced/Boards

Exercise 1

Q.1 20 m/s **Q.2** 1.728×10^8 N/m^2 **Q3** 2.4×10^{-5} J

Exercise 2

Q.1 B **Q.2** C **Q.3** A **Q.4** A

Previous Years' Questions

Q.1 C **Q.2** A **Q.3** 1.09×10^{10} N/m^2 **Q.4** A, B

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $\Delta L = \dfrac{fL}{Ay}$

If diameter is increased to twice then

(i) ΔL will decrease to ¼ value

(ii) $F = \dfrac{\Delta L}{L}\ A$

Maximum load capacity will decrease to ¼ of initial value.

Sol 2: $L_1 = \dfrac{L_2}{2}$

$d_1 = 2d_2$; $A_1 = 4A_2$

$F_1 = F_2$

$\Delta L = \dfrac{FL}{Ay}$

$$\frac{\Delta L_1}{\Delta L_2} = \frac{L_1A_2}{A_1L_2} = \frac{1}{2}\,\frac{1}{4} = \frac{1}{8}$$

Sol 3: Breaking force for two parallel wires of this size

(i) $F' = F_1 + F_2 = F + F = 2F$

(ii) If thickness is double that means area is 4 times.

$$F = \frac{\Delta L}{L}\ YA \Rightarrow F' = \frac{\Delta L}{L}\ Y\,4A = 4F$$

Sol 4: $F = y\ \dfrac{\Delta L}{L}\ A$

$\Delta L = 2L - L = L$

$F = 2 \times 10^{11}\ \dfrac{L}{L}\ 10^{-4} = 2 \times 10^7$ N

Sol 5: $r = 10 \times 10^{-3}$ m

$R = 10^{-2}$ m

$L = 1$ m

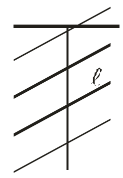
(a) Stress $= \dfrac{F}{A} = \dfrac{100\times10^3}{\pi(10^{-2})^2} = \dfrac{10^5}{\pi\times10^{-4}} = \dfrac{10^9}{\pi}$

$= 3.18 \times 10^8$ N/m^2

(b) Elongation $= \Delta L = \dfrac{\text{stress}\times\text{length}}{y}$

$= \dfrac{3.18\times10^8\times1}{2\times10^{11}} = 1.59 \times 10^{-3}$ m $= 1.59$ mm

(c) % Strain $= \dfrac{\Delta L}{L} \times 100 = \dfrac{1.59\times10^{-3}\times100}{1} = 0.159\%$



Sol 6: Stress$=7.9 \times 10^7$ N/cm^2 $= 7.9 \times 10^{11}$ N/m^2

Stress $= \dfrac{F}{A} = \dfrac{\rho\ell Ag}{A} = \rho\lambda g = 7.9 \times 10^{11}$

$\ell = \dfrac{7.9\times10^{11}}{7900\times10} = 10^7$ m $= 10^9$ cm

Sol 7: $\frac{\Delta V}{V} = -10^{-4}$

$P = 100 \times 10^5$ N/m^2

$B = \frac{-P}{\Delta V / V} = \frac{1.013 \times 10^7}{10-4} = 1.013 \times 10^{11}$ Nm^{-2}

Sol 8: Pressure at 1 km depth = P

$= P_0 + 1000 \times 98 \times 1000$

$= 10^5 + 98 \times 10^7 = 99 \times 10^5$ N/m^2

Bulk modulus $= \frac{-P}{\Delta V / V} = \frac{99 \times 10^5}{2 \times 10^{-4}}$

$= 4.9 \times 10^{10}$ Pa

Sol 9: Stream $= \frac{\Delta X}{L} = \frac{0.05}{10} = 5 \times 10^{-3}$

Modulus $= \frac{\text{stress}}{\text{strain}} = \frac{10^4}{5 \times 10^{-3}}$

$= \frac{10}{5} \times 10^6 = 2 \times 10^6$ N/m^2

Sol 10: Strain $= \frac{5 \times 10}{10^{-4} \times 10^{11}} = \frac{5 \times 10}{10^7} = 5 \times 10^{-6}$

Increase in energy = work done

$\frac{1}{2} \times \frac{50}{10^{-4}} \times 5 \times 10^{-6} \times 10^{-4} \times 0.2$

$= \frac{250 \times 10^{-11}}{10^{-4}} = 250 \times 10^{-7} = 2.5 \times 10^{-5}$ J

Sol 11: (i) The initial length of the wire

= L = 30m

The increase in length of the wire,

$l = 0.44 \times 10^{-2}$

Longitudinal stress

$= l / L = 1.633 \times 10^{-4}$.

(ii) The tension applied to the wire= Mg = 5 × 9.8 N

Area of cross section of the wire,

A = 2 mm^2 = 2×10^{-6} m^2

∴ Longitudinal stress

$= Mg / A = \frac{5 \times 9.8}{2 \times 10^{-6}}$

$= 2.45 \times 10^7$ N / m^2

(iii) Young's modulus $= \frac{\text{stress}}{\text{strain}}$

$\frac{2.45 \times 10^7}{1.633 \times 10^{-4}} = 1.5 \times 10^{11}$ N/m^2

Exercise 2

Sol 1: (C) Stress = F/A = $10/(2 \times 10^{-6})$

$= 5 \times 10^6$ N/m^2

Strain $= \frac{\text{Stress}}{Y} = \frac{5 \times 10^6}{2 \times 10^{11}}$

$= 2.5 \times 10^{-5}$

$l = L \times \text{strain} = 1 \times 2.5 \times 10^{-5}$

$l = 2.5 \times 10^{-5}$ m

Sol 2: (A) Volume = Mass / density

Area of cross-section = Volume/length

$= \frac{\text{mass}}{\text{density} \times \text{length}} = \frac{15.6 \times 10^{-3}}{7800 \times 2.5} = 8 \times 10^{-7}$ m^2

$Y = \frac{Fl}{A\Delta L} = \frac{8 \times 9.8 \times 2.5}{(8 \times 10^{-7}) \times 1.25 \times 10^{-3}}$

$Y = 1.96 \times 10^{11}$ N/m^2

Sol 3: (D) Work done on the wire

$W = \frac{1}{2} F \times l$

$= \frac{1}{2} \times$ stress × volume × strain

$W = \frac{1}{2} \times Y \times \text{strain}^2 \times \text{volume}$

$W = \frac{1}{2} \times Y \times \frac{\Delta L^2}{L^2} \times AL = \frac{YA\Delta L^2}{2L}$

$W = \frac{2 \times 10^{11} \times 10^{-6} \times 10^{-6}}{2 \times 1} = 0.1\,J$

Sol 4: (D) $\frac{l_1}{l_2} = \frac{L_1 r_2^2}{L_2 r_1^2}$

$L_1 = L$, $L_2 = 2L$, $r_1 = 2R.$, $r_2 = R$

$\therefore \frac{l_1}{l_2} = \frac{L}{2L} \frac{R^2}{4R^2} = \frac{1}{8}$

Sol 5: (A) The changed density, $\rho' = \frac{\rho}{1-\frac{dp}{B}}$

Substituting the value, we have

$$\rho' = \frac{11.4}{1-\frac{2.0\times10^8}{8.0\times10^9}}$$

$\rho' = 11.69\,g/cm^3 \approx 11.7 g/cm^3$

Sol 6: (D) Mass of the rod = $\frac{AL}{\rho}$ if A is its cross sectional area

Weight acts at the mid-point

$$\therefore\ Y = \frac{mg}{A}\times\frac{(L/2)}{\Delta L}$$

If L is the original length

$$\Rightarrow \Delta L = \frac{mgL}{2AY} = \frac{g\rho L^2}{2Y}$$

$$= \frac{9.8\times1.3}{120} = 10.6\,cm$$

Sol 7: (C) If L = initial length of the rod, increase in length caused by temperature increase

$= L\,\alpha\,\theta$

If this expansion is prevented by a compressive force, then

$$\text{Strain} = \frac{L\alpha\theta}{L}\alpha\theta = 6\times10^{-5}$$

$\therefore$ Stress developed in the rod

$= Y \times \text{strain} = 12 \times 10^6\ N/m^2$

$= 1.2 \times 10^7\ N/m^2$

Previous Years' Questions

Sol 1: (A) $\Delta l = \frac{Fl}{AY} = \left(\frac{Fl}{\left(\frac{\pi d^2}{4}\right)Y}\right)$ or $(\Delta l) \propto \frac{1}{d^2}$

Now, $\frac{1}{d^2}$ is maximum in option (A).

Sol 2: (B) In isothermal process

pV = constant

$\therefore$ pdV + Vdp = 0 or $\left(\frac{dp}{dV}\right) = -\left(\frac{p}{V}\right)$

$\therefore$ Bulk modulus,

$$B = -\left(\frac{dp}{dV/V}\right) = -\left(\frac{dp}{dV}\right)V$$

$$\therefore\ B = -\left[\left(-\frac{p}{V}\right)V\right] = p$$

$\therefore$ B = p

Note: Adiabatic bulk modulus is given by B = γp.

Sol 3: (D) From the definition of bulk modulus,

$$B = \frac{-dp}{(dV/V)}$$

Substituting the values, we have

$$B = \frac{(1.165-1.01)\times10^5}{(10/100)} = 1.55\times10^5\,Pa$$

Sol 4: (D)

Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{\ell+\Delta\ell}{g}}$$

$$\Delta\ell = \frac{Mg\ell}{AY} \Rightarrow T_M = 2\pi\sqrt{\frac{\ell+\frac{Mg\ell}{AY}}{g}}$$

$$\left(\frac{T_M}{T}\right)^2 = 1+\frac{Mg}{AY}$$

$$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

JEE Advanced/Boards

Exercise 1

Sol 1: Equivalent force constant of rubber cord.

$$k = \frac{YA}{l} = \frac{(5.0\times10^8)(1.0\times10^{-6})}{(0.1)} = 5.0\times10^3\,N/m$$

Now, from conservation of mechanical energy, elastic potential energy of cord

= Kinetic energy of missile

$\therefore \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}mv^2$

$\therefore v = \left(\sqrt{\frac{k}{m}}\right)\Delta l = \left(\sqrt{\frac{5.0\times10^3}{5.0\times10^{-3}}}\right)(12.0-10.0)\times10^{-2}$

= 20 m/s

Note: Following assumptions have been made in this problem:

(i) k has been assumed constant, even though it depends on the length (l).

(ii) The whole of the elastic potential energy is converting into kinetic energy of missile.

Sol 2: Let the initial volume of the block be V and v the increase in volume when it is heated t_1 to t_2. Then

$v = V\times\gamma\times(t_2 - t_1)$

Where γ is the coefficient of volume expansion. The volume strain is therefore,

$\frac{v}{V} = \gamma(t_2 - t_1)$

The bulk modulus is

$B = \frac{\text{change in pressure}}{\text{volume strain}}$

$B = \frac{P}{\gamma(t_2 - t_1)}$

$P = B\gamma(t_2 - t_1)$

Given B = 3.6 × 10^{11} N/m^2

$\gamma = 3\alpha$ = 3 × 8.0 × 10^{-6}

= 24 × 10^{-6} /°C

$(t_2 - t_1)$ = 70 – 50 = 20°C

∴ P (3.6 × 10^{11}) × (24 × 10^{-6}) × 20

= 1.728 × 10^8 N/m^2

Sol 3: Work done in compressing the bar is given by

$W = \frac{1}{2}Fl$

Where F is the force applied on the bar and l is the compression in the length of the bar. By Hooke's law, the Young's modulus of the material of the bar is given by

$Y = \frac{F/A}{l/L} = \frac{FL}{Al}$

Where A is the area of cross-section of the bar and L is the initial length

$\therefore l = \frac{FL}{AY}$

Hence from equation (i), we have

$W = \frac{F^2L}{2AY}$

Here F = 5kg, wt=5 × 9.8 N, L=0.2 m

A = 1.0 cm^2 = 1.0 × 10^{-6} m^2 and

Y = 1.0 × 10^{-5} N/m^2

$\therefore W = \frac{(5\times9.8)^2\times0.2}{2\times(1.0\times10^{-4})\times(1.0\times10^{11})}$

$= 2.4\times10^{-5}$ J

This is the increase in energy of the bar.

Exercise 2

Sol 1: (B)

F = Y α ΔtA; A = 2 × 10^{-6} m^2

Y = 2 × 11 N/m^2 ; α = 1.1 × 10^{-5}

T = 50 – 30 = 20°C

F = 2 × 10^{11}×1.1×10^{-5}×20 × 2 × 10^{-6} = 88 N

Sol 2: (C) Potential energy per unit volume = u

$= \frac{1}{2}\times\text{stress}\times\text{strain}$; But $Y = \frac{\text{stress}}{\text{strain}}$

∴ stress = Y × strain = Y × S

∴ Potential energy per unit volume = u

$= \frac{1}{2}\times(YS)S = \frac{1}{2}YS^2$

Sol 3: (A)

$\because$ Compressibility $= \frac{1}{K} = \frac{\Delta V}{V\times\Delta P}$

$\Delta V = (V\times\Delta P)\times\frac{1}{K}$

ΔV = (100 × 15 × 10^6) × 5 × 10^{-10}

ΔV = 0.75 ml

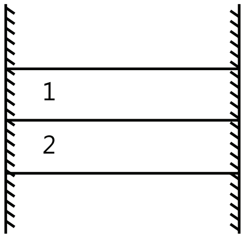
Sol 4: (A) $\theta = \frac{r\phi}{L} = \frac{(2/1000)45^\circ}{1}$ = 0.09°

Previous Years' Questions

Sol 1: (C) Thermal stress $\sigma = Y\alpha\Delta\theta$ Given, $\sigma_1 = \sigma_2$

$\therefore Y_1\alpha_1\Delta\theta = Y_2\alpha_2\Delta\theta$

or $\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$



Sol 2: (A) $\Delta l = \left(\frac{l}{YA}\right).W$

i.e., graph is a straight line passing through origin (as shown in question also), the slope of which is $\frac{l}{YA}$.

$\therefore$ Slope $= \left(\frac{l}{YA}\right)$

$\therefore Y = \left(\frac{l}{YA}\right)\left(\frac{1}{\text{slope}}\right)$

$= \left(\frac{1.0}{10^{-6}}\right)\frac{(80-20)}{(4-1)\times 10^{-4}}$

$= 2.0 \times 10^{11}$ N/m^2

Sol 3: Young's modulus of elasticity is given by

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{l/L} = \frac{FL}{lA} = \frac{FL}{l\left(\frac{\pi d^2}{4}\right)}$$

Substituting the values, we get

$$Y = \frac{50\times 1.1\times 4}{(1.25\times 10^{-3})\times\pi\times(5.0\times 10^{-4})^2}$$

$= 2.24\times 10^{11}$ N/m^2

Now, $\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$

$= \left(\frac{0.1}{110}\right) + \left(\frac{0.001}{0.125}\right) + 2\left(\frac{0.001}{0.05}\right) = 0.0489$

$\Delta Y = (0.0489)\,Y$

$= (0.0489) \times (2.24 \times 10^{11})$ N/m^2 $= 1.09 \times 10^{10}$ N/m^2

Sol 4: (A B)

$Y = \frac{\text{stress}}{\text{strain}}$

$\Rightarrow \frac{1}{Y} = \frac{\text{strain}}{\text{stress}} \Rightarrow \frac{1}{Y_P} > \frac{1}{Y_\theta} \Rightarrow Y_P < Y_Q$

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