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PHYSICS

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ERRORS

THEORY AND EXERCISE BOOKLET

CONTENTS

394,50 - Rajeev Gandhi Nagar Kota, Ph. No. : 93141-87482, 0744-2209671
IVRS No : 0744-2439051, 52, 53, www. motioniitjee.com , info@motioniitjee.com

ERRORS

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity.

(1) random and (2) systematic errors

1. Random errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of random error can be some what reduced by taking the average of measured values. Random errors have no fixed sign or size.

2. Systematic error occur due to error in the procedure, or miscalibration of the instrument etc. Such errors have same size and sign for all measurements. Such errors can be determined.

A measurement with relatively small random error is said to have high precision. A measurement with small random error and small systematic error is said to have high accuracy.

The experimental error [uncertainty] can be expressed in several standard ways.
Error limits $Q \pm \Delta Q$ is the measured quantity and ΔQ is the magnitude of its limit of error. This expresses the experimenter's judgement that the 'true' value of Q lies between $Q - \Delta Q$ and $Q + \Delta Q$. This entire interval within which the measurement lies is called the range of error. Random errors are expressed in this form.

Absolute Error

Error may be expressed as absolute measures, giving the size of the error in a quantity in the same units as the quantity itself.
Least Count Error :- If the instrument has known least count, the absolute error is taken to be half of the least count unless otherwise stated.

Relative (or Fractional) Error

Error may be expressed as relative measures, giving the ratio of the quantity's error to the quantity itself. In general,

$$\text{relative error} = \frac{\text{absolute error in a measurement}}{\text{size of the measurement}}$$

We should know the error in the measurement because these errors propagate through the calculations to produce errors in results.

A. **Systematic errors :** They have a known sign. The systematic error is removed before begining calculations bench error and zero error are examples of systematic error.

B. **Random error:** They have unknown sign. Thus they are represented in the form $A \pm a$.

Here we are only concerned with limits of error. We must assume a "worst-case" combination. In the case of substraction, $A - B$, the worst-case deviation of the answer occurs when the errors are either $+a$ and $-b$ or $-a$ and $+b$. In either case, the maximum error will be $(a + b)$.
For example in the experiment on finding the focal length of a convex lens, the object distance(u) is found by subtracting the positions of the object needle and the lens. If the optical bench has a least count of 1 mrn, the error in each position will be 0.5 mm. So, the error in the value of u will be 1 mm.

1. ***Addition and subtraction rule :*** The absolute random errors add.
Thus if $R = A + B$, $\quad r = a + b$
and if $R = A - B$, $\quad r = a + b$

2. ***Product and quotient rule :*** The relative random errors add.

Thus if $R = AB$, $\quad \frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

and if $R = \frac{A}{B}$, then also $\quad \frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

3. **Power rule :** When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q. This also holds for negative powers.

If $R = Q^p$, $\frac{r}{R} = P \times \frac{q}{Q}$

4. The quotient rule is not applicable if the numerator and denominator are dependent on each other.

e.g. if $R = \frac{XY}{X+Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows $\frac{1}{R} = \frac{1}{X} + \frac{1}{Y}$. Differentiating both the sides, we get

$-\frac{dR}{R^2} = -\frac{dX}{X^2} - \frac{dY}{Y^2}$.   Thus $\frac{r}{R^2} = \frac{x}{X^2} + \frac{y}{Y^2}$

Examples

1. ***A student finds the constant acceleration of a slowly moving object with a stopwatch. The equation used is S = (l/2) AT². The time is measured with a stopwatch, the distance, S with a meter stick. What is the acceleration and its estimated error?***

S = 2 ± 0.005 meter.

T = 4.2 ± 0.2 second.

Sol. We use capital letters for quantities, lower case for errors. Solve the equation for the result, a.

$A = 2S/T^2$. Its random-error equation is $\frac{a}{A} = 2\frac{t}{T} + \frac{s}{S}$

Thus $A = 0.23 \pm 0.02$ m/s².

SIGNIFICANT DIGITS

Significant figures are digits that are statistically significant. There are two kinds of values in science:

1. Measured Values
2. Computed Values

The way that we identify the proper number of significant figures in science are different for these two types.

MEASURED VALUES

Identifying a measured value with the correct number of significant digits requires that the instrument's calibration be taken into consideration. The last significant digit in a measured value will be the first estimated position. For example, a metric ruler is calibrated with numbered calibrations equal to 1 cm. In addition, there will be ten unnumbered calibration marks between each numbered position. (each equal to 0.1 cm). Then one could with a little practice estimate between each of those marking. (each equal to 0.05 cm). That first estimated position would be the last significant digit reported in the measured value. Let's say that we were measuring the length of a tube, and it extended past the fourteenth numbered calibration half way between the third and fourth unnumbered mark. The metric ruler was a meter stick with 100 numbered calibrations. The reported measured length would be 14.35 cm. Here the total number of significant digits will be 4.

COMPUTED VALUE

The other type of value is a computed value. The proper number of significant figures that a computed value should have is decided by a set of conventional rules. However before we get to those rules for computed values we have to consider how to determine how many significant digits are indicated in the numbers being used in the math computation.

A. Rules for determining the number of significant digits in number with indicated decimals.

1. All non-zero digits (1-9) are to be counted as significant.
2. Zeros that have any non-zero digits anywhere to the LEFT of them are considered significant zeros.
3. All other zeros not covered in rule (ii) above are NOT be considered significant digits.

For example: 0.0040000

The 4 is obviously to be counted significant (Rule-1), but what about the zeros ? The first three zeros would not be considered significant since they have no non-zero digits anywhere to their left (Rule-3). The last four zeros would all be considered significant since each of them has the non-zero digit 4 to their left (Rule-2). Therefore the number has a total of five significant digits.

Here is another example: 120.00420

The digit 1, 2, 4 and 2 are all considered significant (Rule-1). All zeros are considered significant since they have non-zero digits somewhere to their left (Rule-2). So there are a total of eight significant digits. If in the question, we are given a number like 100, we will treat that the number has only one significant digit by convention.

B. Determining the number of significant digits if number is not having an indicated decimal.

The decimal indicated in a number tells us to what position of estimation the number has been indicated.

But what about 1,000,000 ?

Notice that there is no decimal indicated in the number. In other words, there is an ambiguity concerning the estimated position. This ambiguity can only be clarified by placing the number in exponential notation.

For example: If! write the number above in this manner.

1.00×10^6

I have indicated that the number has been recorded with three significant digits. On the other hand, if write the same number as : 1.0000×10^6

I have identified the number to have 5 significant digits. Once the number has been expressed in exponential notation form then the digits that appear before the power of ten will all be considered significant.

So for example : 2.0040×10^4 will have five significant digits. Thus means that unit conversion will not change the number of significant digits. Thus 0.000010 km = 1.0 cm = 0.010 m = 1.0×10^{-2} m = 1.0×10^{-5} km

Rule for expressing proper number of significant digits in an answer from multiplication or division

For multiplication AND division there is the following rule for expressing a computed product or quotient with the proper number of significant digits.

The product or quotient will be reported as having as many significant digits as the number involved in the operation with the least number of significant digits.

For example : $0.000170 \times 100.40 = 0.017068$

The product could be expressed with no more that three significant digits since 0.000170 has only three significant digits, and 100.40 has five. So according to the rule the product answer could only be expressed with three significant digits. Thus the answer should be 0.0 171 (after rounding off)

Another example : $2.000 \times 10^4 / 6.0 \times 10^{-3} = 0.33 \times 10^7$

The answer could be expressed with no more that two significant digits since the least digited number involved in the operation has two significant digits.

Sometimes this would required expressing the answer in exponential notation.

For example : $3.0 \times 800.0 = 2.4 \times 10^3$

The number 3.0 has two significant digits and then number 800.0 has four. The rule states that the answer can have no more than two digits expressed. However the answer as we can all see would be 2400. How do we express the answer 2400 while obeying the rules ? The only way is to express the answer in exponential notation so 2400 could be expressed as : 2.4×10^3.

Rule for expressing the correct number of significant digits in an addition or substraction :

The rule for expressing a sum or difference is considerably different than the one for multiplication of division. The sum or difference can be no more precise than the least precise number involved in the mathematical operation. Precision has to do with the number of positions to the RIGHT of the decimal. The more position to the right of the decimal, the more precise the number. So a sum or difference can have ho more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

For example : 160.45 + 6.732 = 167.18 (after rounding off)

The answer could be expressed only to two positions to the right of the decimal, since 160.45 is the least precise.

Another example : 45.621 + 4.3 – 6.41 = 43.5 (after rounding off)

The answer could be expressed only to one position to the right of the decimal, since the number 4.3 is the least precise number (i.e. having only one position to the right of its decimal). Notice we aren’t really determining the total number of significant digits in the answer with this rule.

Rules for rounding off digits :

There are a set of conventional rules for rounding off.

1. Determine according to the rule what the last reported digit should be.

2. Consider the digit to the right of the last reported digit.

3. If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.

4. If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.

5. If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

For example if we wish to round off the following number to 3 significant digits : 18.3682

The last reported digits would be the 3. The digit to its right is a 6 which is greater than 5. According to the Rule-4 above, the digit 3 is increased by one and the answer is : 18.4

Another example : Round off 4.565 to three significant digits.

The last reported digit would be the 6. The digit to the right is a 5 followed by nothing. Therefore according to Rule-5 above since the 6 is even it remains so and the answer would be 4.56.

EXPERIMENT

(i) MEASUREMENT OF LENGTH

The simplest method measuring the length of a straight line by means of a meter scale. But there exists some limitation in the accuracy of the result :

(i) the dividing line ....finite thickness

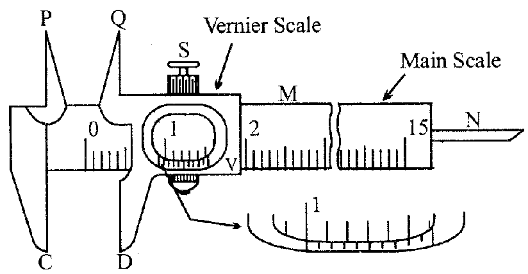
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like

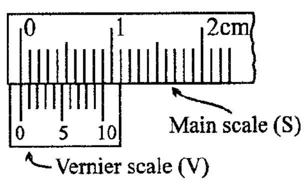
(a) Vernier callipers

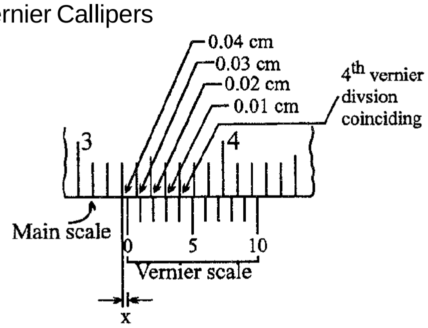
(b) micrometer scales (screw gauge) are used

VERNIER CALLIPERS :



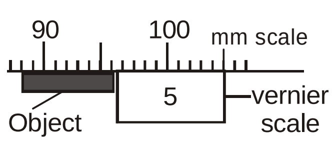
Vernier Callipers

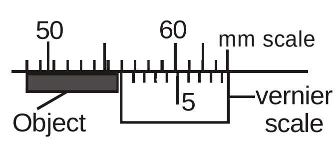




Principle of Vernier

Reading a vernier with 4th division coinciding





It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being shorter than the divisions of the main scale.

Least count of Vernier Callipers

The least count of Vernier Constant (v.c) is the minimum value of correct estimation of length without eye estimation. If N division of vernier coincides with (N-1) division of main scale, then

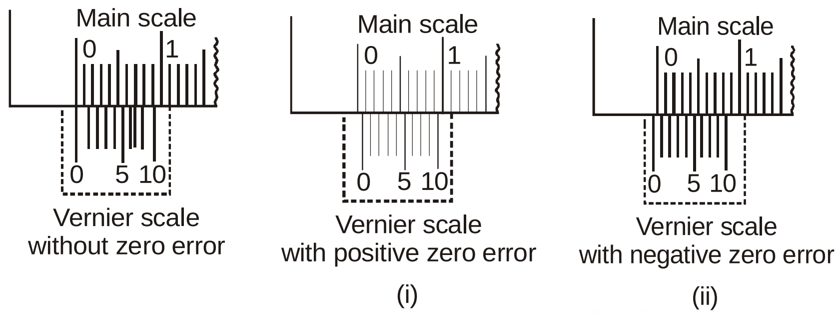
$N(VS) = (N - 1)\ ms \Rightarrow 1VS = \frac{N-1}{N}\ ms$

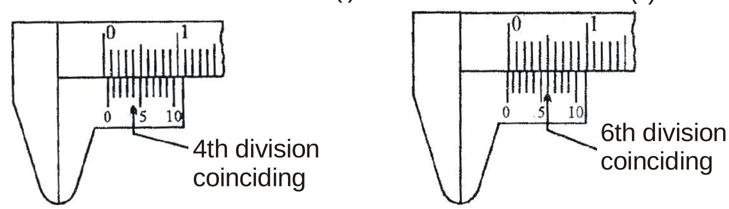
Vernier constant = 1 ms – 1 vs = $\left(1 - \frac{N-1}{N}\right)$ ms = $\frac{1ms}{N}$, which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

Length as measured by Vernier Callipers

The formula for measuing the length is L= main scale reading + least count of vernier scale × Vernier scale division coinciding with a main scale division Main scale reading is given by the zeroth division of the vernier scale as shown in the figure.

Zero error :





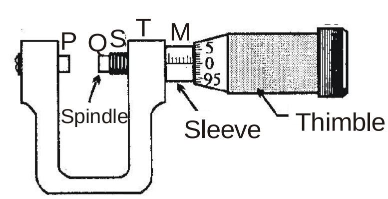
Positive zero error(+0.04cm) and its correction

Negative zero error = (–0.04cm) and its correction

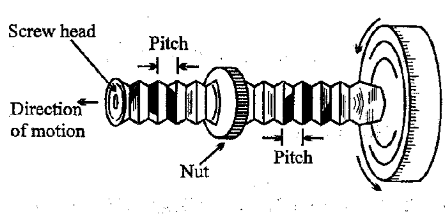
If the zero marking of main scale and vernier callipers do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument.
If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.
The zero error is always subtracted from the reading to get the corrected value.
If the zero error is positive, its value is calculated as we take any normal reading. If the zero error negative (the zero of vernier scale lies to the left of the zero of main scale),
negative zero error = – [Total no. of vsd – vsd coinciding] × L.C.
Do not try to read the main scale at the point where the lines match best. This has no meaning. Read from the vernier scale instead. Sometimes it is difficult to tell whether the best match of liners is for vernier marks 9, 0 or 1. Make your best estimate, but realize that the final result including the vernier must round off to the result you best estimate, but realize that the final result including the vernier must round off to the result you would choose if there was no vernier. If the mark is close to 3.20 on the main scale and the vernier is 9, the length is 3.19 cm. If the mark is close to 3.2 on the main scale and vernier 1, the length is 3.21 cm.

SCREW GAUGE (OR MICROMETER SCREW)

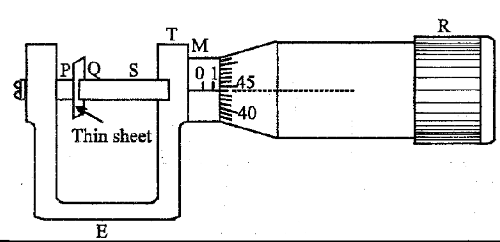
In general vernier callipers can measure accurately upto 0.01 cm and for greater accuracy micrometer screw devices e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by running it axially.

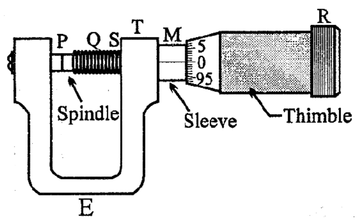


The instrument is provided with two scales :

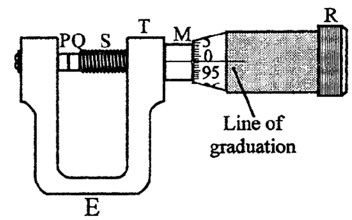


Principle of a micrometer

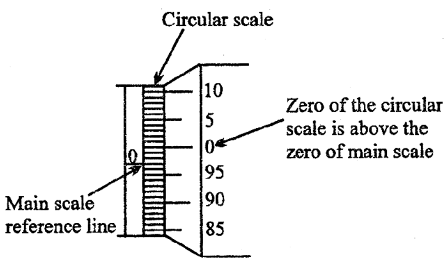


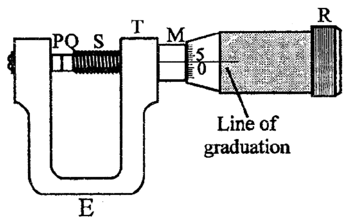


Screw gauge with no zero error

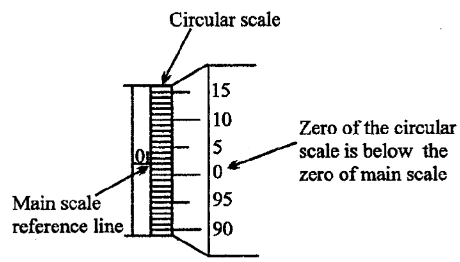


Negative zero error
(3 division error) i.e.,-0.003 cm





Positive zero error
(2 division error) i.e.,+0.002 cm



(i) The main scale or pitch scale M graduated along the axis of screw.
(ii) The cap-scale or head scale H round the edge of the screw head.
Constants of the Screw Gauge

(a) **Pitch :** The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for 10 rotation of cap = 5 mm, then pitch = 0.5 mm.

(b) **Least count :** In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions. Thus in the Afore said Illustration.; if the total cap division is 100, then least count = 0.5 mm/100 =0.005 mm

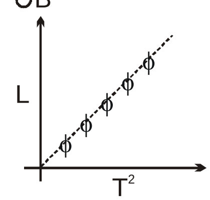
(c) **Measurement of length by screw gauge :**
L = n × pitch + f × least count, where n = main scale reading & f = caps scale reading
Zero Error : In a perfect instrument the zero of the heat scale coincides with the line of gradiation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line of reference line of the cap lies **below** the line of graduation and versa. The corresponding corrections will be just opposite.

(II) MEASUREMENT OF G USING A SIMPLE PENDULM

A small spherical bob is attached to a cotton thread and the combination is suspended from a point A. The length of the thread (L) is read off on a meter scale. A correction is added to L to include the finite size of the bob and the hook. The corrected value of L is used for futher calculation.

The bob is displaced slightly to one side and is alloswed to oscillate, and the total time taken for 50 complete oscillations is noted on a stop-watch. The time period (T) of a single oscillation is now calculated by division.

Observations are now taken by using different lenghts for the cotton thread (L) and pairs of values of L and T are taken. A plot of L v/s T^2, on a graph, is linear.



g is given by $g = 4\pi^2 \frac{L}{T^2}$

The major errors in this experiment are

(a) **Systematic :** Error due to finite amplitude of the pendulum (as the motion is not exactly SHM). This may be corrected for by using the correct numberical estimate for the time period. However the parctice is to ensure that the amplitude is small.

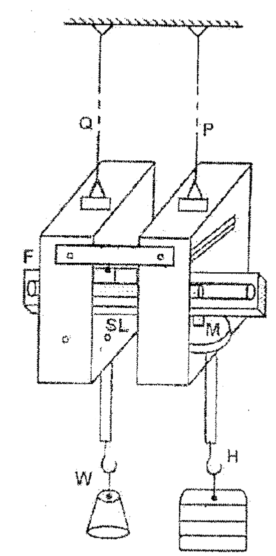
(b) **Statistical :** Errors arising from measurement of length and time.

$$\frac{\delta g}{g} = \frac{\delta L}{L} + 2\left(\frac{\delta T}{T}\right)$$

The contributions to δL, δT are both statistical and systematic. These are reduced by the process of averaging.
The systematic error in L can be reduced by plotting several values of L vs T^2 and fitting to a straight line.
The slope of this fit gives the correct value of L/T^2

(III) DTERMINATION OF YOUNG'S MODULUS BY SERALE'S METHOD

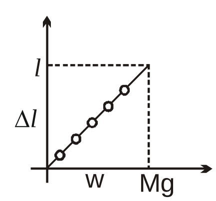
The experimental set up consists of two identical wires P and Q of uniform cross section suspended from a fixed rigid support. The free ends of these parallel wires are connected to a frame F as shown in the figure. The length of the wire Q remains fixed while the load L attached to the wire P through the frame F is varied in equal steps so as to produce extension along the length. The extension thus produced is measured with the help of spirit level SL and micrometer screw M attached to the F frame on the the side of experimental wire. On palacing the slotted weights on the hanger H upto a permissible value (half of the breaking force) the wire gets extended by small amount and the spirit level gets distrubed from horizontal setting. This increase in length is measured by turning the micrometer screw M upwards so as to restore the balance of the spirit level. If n be the number of turns of the micrometer screw and f be the difference in the cap reading, the increase in length M is obtained by



$$\Delta l = n \times \text{pitch} + f \times \text{least count}$$

In some situations, the change in length is obtained by vernier arrangment instead of the screw gauge. The load on the hanger is reduced in the same steps and spirit level is restored to horizontal position. The mean of these two observations gives the ture increase in length of the wire corresponding to the given value of load. This is to eliminate the effect of hysterisis.
From the data obtained, a graph showing extension (Δl) against the load (W) is plotted which is obtained as a straight line passing through the origin. The slope of the line gives

$$\tan\theta = \frac{l}{W} = \frac{l}{Mg}$$



Now, stress $= \frac{Mg}{\pi r^2}$ and strain $= \frac{l}{L}$

Y = Stress / strain $= \frac{MgL}{\pi r^2 l} = \frac{L}{\pi^2 r \tan\theta}$

With known values of initial length L, radius r of the experimental wire and tan θ, Young's modulus Y can be calculated.

(IV) SPECIFIC HEAT OF A LIQUID USING A CALORIMETER :

The principle is to take a known quantity of liquid in an insulated calorimeter and heat it by passing a known current (i) though a heating coil immersed within the liquid for a known length of time (t). The mass of the calorimeter (m_1) and, the combined mass of the calorimeter and the liquid (m_2) are measured. The potential drop across the heating coil is V and the maximum temperature of the liquid is measured to θ_2.
The specific heat of the liquid (S_l) is found by using the relation

$(m_2 - m_1) S_l (\theta_2 - \theta_0) + m_1 S_c (\theta_2 - \theta_0) = i. \ V. t$

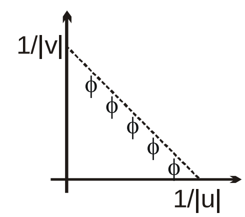
or $(m_2 - m_1) S_l + m_1 S_c = i. \ V. t / (\theta_2 - \theta_0)$ ....(i)

Here, θ_0 is the room temeperature, while S_c is the specific heat of the material of the calorimeter and the stirrer. If S_c is known, then S_l can be determined.
On the other hand, if S_c is unknown : one can either repeat the experiment with water or a different mass of the liquid and use the two equations to eliminate $m_1 S_c$.
The sources of error in this experiment are errors due to improper connection of the heating coil, radiation, apart from statistical errors in measurement.

Error analysis :
After correcting for systematic errors, equation (i) is used to estimate the remaining errors.

(V) FOCAL LENGTH OF CONCAVE MIRROR AND A CONVEX LENS USING THE U-V METHOD.

In this method one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder. The position of the lens is noted by reading the scale at the bottom of the holder. A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).
The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).
For the concave mirror, the position of the image is determined by placing a sharp object (a pin) on the optical bench such that the parallax between the object pin and the image is nil.



A plot of |u| versus |v| gives a rectangular hyperbola. A plot or $\frac{1}{|v|}$ vs $\frac{1}{|u|}$ gives a straight line.

The intercepts are equal to $\frac{1}{|f|}$, where f is the focal length.

Error : The systematic error in this experiment is mostly due to improper position of the object on the holder. This error may be eliminated by reversing the holder (rotating the holder by 180º about the vertical) and the taking the reading again. Averages are then taken.

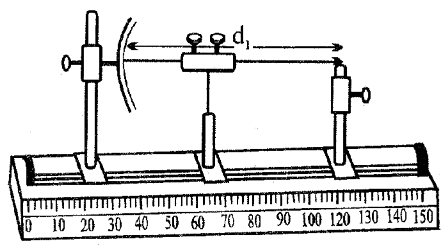
The equation for random errors gives : $\frac{\delta f}{f^2} = \frac{\delta u}{u^2} + \frac{\delta v}{v^2}$

The error δu, δv correspond to the error in the measurement of u and v. A ctually, we know the errors in the object position, lens position & image position. So, the errors in u & v too be estimated as described before.

Index Error or Bench Error and its correction : In an experiment using an optical bench we are requied to measure the object and image distances from the pole or vertex on the mirror. The distance between the tip of the needles and the pole of the mirror is the actual distance. But we practically measure distance between the indices with the help of the scale engraved on the bench. These distance are called the observed distances. The actual distances may not be equal to the observed distances and due to this reason an error creeps in the measurement of the distances. This error is called the index or the bench error. This error is estimated with the help of a needle of known length placed horizontally between the tip of the needle and the pole.

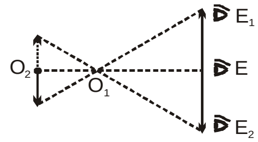
Index Error	=	Observed distance - actual distance and
Index Correction	=	Actual - observed distance

Note : Index correction whether position or negative, is always added algebraically to the observed distance to get the corrected distance.



Parallax

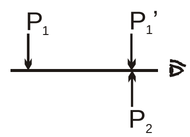
When two object O_1 and O_2 are placed in such a way that both of them lie in the same line of sight as shown in figure, then the object nearer to the eye covers the object farther from it. Their image on the retina are superimposed and therefore, it is impossible to decide which is the nearer object. To identify this fact, the observer displaces his eye to a position E_1 or E_2 until he is able to see two distinct objecs.



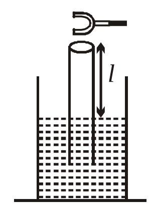
The more distant object O_2 apparently moves in the direction opposite to the displacement of the observer's eye with respect the nearer object O_1. This relative shift in the position of two object due to the shift in the position of the observer's eye is called parallax.

Parallax between the two objects disappear if they are at the same position.

The figure shows the tips of two pins P_1 and P_2 kept in the upright positions. The parallax between P_1 and P_2 is removed by shifting the position of observer's eye sideways. As the farther pin P_1 is displaced towards the pin P_2 the relative shift (parallax) between their position decreases as the position of eye is displaced sideways. The relative shift vanishes when the pin P_1 occupies the position P_1, that is, when the tips of the two are just coincident. At this position there is no parallax between the tips of the two pins.



(VI) SPEED OF SOUND USING RESONANCE COLUMN



A tuning fork of known frequency (f) is held at the mouth of a long tube, which is dipped into water as shown in the figure. The length (l_1) of the air column in the tube is adjusted until it reasonance with the tuning fork. The air temperature and humidity are noted. The length of the tube is adjusted again untile a second reasonace length (l_2) is found (provided the tube is long)

Then $l_2 - l_1 = \lambda / 2$, provided l_1, l_2 are resonance lengths for adjacent resonaces.

$\therefore \quad \lambda = 2(l_2 - l_1)$, is the wavelength of sound.

Since the frequency f, is known; the velocity of sound in air at the temperature (θ) and humidity (h) is given by

$C = f\lambda = 2(l_2 - l_1)f$

It is also possible to use a single measurement of the resonant length directly, but, then it has to be corrected for the "end effect".

$\lambda(\text{fundamental}) = 4(l_1 + 0.3d)$, where d = diameter

Errors : The major systematic error introduced are due to end effects in (end correction) and also due to excessive humidity.

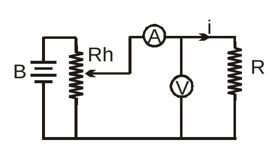
Random errors are given by

$$\frac{\delta C}{C} = \frac{\delta(l_2 - l_1)}{l_2 - l_1} = \frac{\delta l_2 + \delta l_1}{l_2 - l_1}$$

(vii) Verification of Ohm's law using voltmeter and ammeter

A voltmeter (V) and an ammeter (A) are connected in a circuit along with a resistance R as shown in the figure, along with a battery B and a rheostat, Rh

Simultaneous readings of the current i and the potential drop V are taken by chaning the resistance in the rheostat (Rh). A graph of V vs i is plotted and it is found to be linear (within errors). The magnitude of R is determined by either



(a) taking the ratio $\frac{V}{i}$ and then

(b) fitting to a straight line : V = iR, and determining the slope R.

Errors :

Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/ connections of the resistance. The effect of Joule heating may be minimsed by switching on the circuit for a short while only, while the effect of finite resistance of the voltameter can be overcome by using a high resistance insrument or a potentiometer. The lengths of connecting wires should be minimised as much as possible.

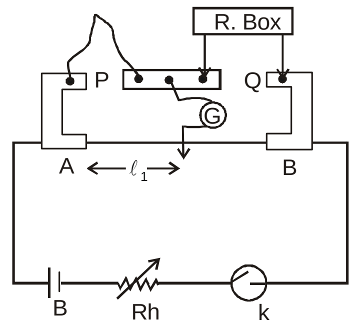
Error analysis :

The error in computing the ratio $R = \frac{V}{i}$ is given by $\left|\frac{\delta R}{R}\right| = \left|\frac{\delta V}{V}\right| + \left|\frac{\delta i}{i}\right|$

where δV and δi are of the order of the least count of the instruments used.

(VIII) SPECIFIC RESISTANCE OF THE MATERIAL OF A WIRE USING A METER BRIDGE :

A known length (*l*) of a wire is connected in one of the gaps (P) of a meter bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completer by using a battery (B), a Rheostat (Rh), a Key (K) and a galvanometer (G). The balance length (*l*) is found by closing key k and momentrily connecting the galvanometer until it gives zero deflection (null point).



Then, $\frac{P}{Q} = \frac{l}{100 - l}$ ...(i)

using the expression for the meter bridge at balance. Here, represents the resistance of the wire while Q represents the resistance in the resistanc box. The key K is open when the circuit is not in use.

The resistance of the wire, $P = \rho \frac{L}{\pi r^2} \Rightarrow \rho = \frac{\pi r^2}{L} P$ ....(ii)

Where r is the radius or wire and L is the length of the wire, r is meaused using a screw gauge while L is measured with a scale.

Errors : The major systematic errors in this experiment are due to the heating effect, end correction inturoduced due to shift of the zero fo the scale at A and B, and stray resistances in P and Q,are errors due to non-uniformity of the meter bridge wire.

Error analysis : End corrections can be estimated by including know resistance P_1 and Q_1 in the two ends and finding the null point :

$$\frac{P_1}{Q_1} = \frac{l_1 + \alpha}{100 - l_1 + \beta} \quad \text{...(ii),} \qquad \text{where } \alpha \text{ and } \beta \text{ are the end corrections.}$$

When the resistance Q_1 is placed in the left gap and P_1 in the right gap,

$$\frac{Q_1}{P_1} = \frac{l_2 + \alpha}{100 - l_2 + \beta} \quad \text{...(iii)}$$

which give two linear equation for finding α and β.

In order that α and β be measured accurately, P_1 and Q_1 should be as different from each other as possible. For the actual balance point,

$$\frac{P}{Q} = \frac{\ell + a}{100 - \ell + \beta} = \frac{l_1'}{l_2'}$$

Error due to non-uniformity of the meter bridge wire can be minimised by interchanging the resistances in the gaps P and Q.

$$\therefore \quad \frac{\delta P}{P} = \left|\frac{\delta l'_1}{l'_1}\right| + \left|\frac{\delta l'_2}{l'_2}\right|$$

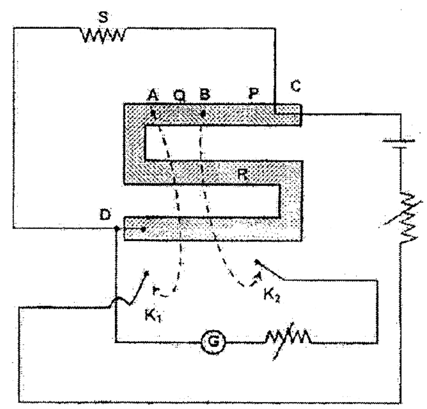
where $\delta l'_1$ and $\delta l'_2$ are of the order of the least count of the scale.

The error is, therefore, minimum if $l'_1 = l'_2$ i.e., when the balance point is in the middle of the bridge. The error is ρ is

$$\frac{\delta P}{P} = \frac{2\delta r}{r} + \frac{\delta L}{L} + \frac{\delta P}{P}$$

(IX) MEASUREMENT OF UNKNOWN RESISTANC USING A P.O. BOX

A P.O. Box can also be used to measure an unknown resistance. It is a Wheatstone Bridge with three arms P, Q and R; while the fourth arm(s) is the unknown resistance. P and Q are known as the ratio arms while R is known at the rheostat arm.
At balance, the unknown resistance



$$S = \left(\frac{P}{Q}\right)R \qquad ...(i)$$

The ratio arms are first adjusted so that they carry 100 Ω each. The resistance in the rheostat arm is now adjusted so that the galvanometer deflection is in one direction, if R = R_0(Ohm) and in the opposite direction when R = R_0 + 1 (Ohm).
This implies that the unknown resisttance, S lies between R_0 and R_0 + 1 (ohm). Now, the resistance in P and Q are made 100 Ω and 1000 Ω respecitlvey, and the process is repeated.
Equation (i) is used to compute S.
The ratio P/Q is progressively made 1 : 10, and then 1 : 100. The resistance S can be accurately measured.
Errors : The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating, and the insensitivity of the Wheatstone bridge,
These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating) and calculating the sensitivity.
In order that the sensitivity is maximum, the resistance in the arm P is close to the value of the resistance S.