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PHYSICS

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HEAT- I

THEORY AND EXERCISE BOOKLET

CONTENTS

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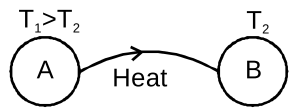
Syllabus :

Thermal expansion of solids, liquids and gases; Calorimetry, latent heat; Heat conduction in one dimension; Elementary concepts of convection and radiation; Newton's law of cooling;

CALORIMETRY

1. HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of receiving body. If should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of body" are meaningless.



When we say that a body is heated it means that its molcules begin to move with greater kinetic energy.

S.I. unit of heat energy is joule (J). Another common unit of heat energy is calorie (cal).

1.1 Mechanical Equivalent of Heat

In early days heat was not recongnised as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experimets were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write,

W = JH

Where J is called mechanical equivalent of heat. J is expressed in joule/calories. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

1 calorie : The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 °C at one atmospheric pressure is 1 calorie.

1 calorie = 4.186 Joule

1.2 Specific Heat

Specific heat of substances is equal to heat gain or released by that substance to raise or fall its temperature by 1°C for a unit mass of substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to :

(a) the mass of the body $\Delta Q \propto m$

(b) rise or fall of temperature of the body $\Delta Q \propto \Delta T$

$\Delta Q \propto m\,\Delta T$ or $\boldsymbol{\Delta Q \propto m\, s\, \Delta T}$

or $dQ \propto m\, s\, dT$ or $Q = m \int s\, dT$

where s is a constant and is known as the specific heat of the body $s = \dfrac{Q}{m\Delta T}$. S.I. unit of s is joule/kg-kelvin and C.G.S unit is cal/gm °C

Specific heat of water : s = 4200 J/kg°C = 1000 cal/kg°C = 1 Kcal/kg°C = 1 cal/gm°C

Specific heat of steam = half of specific heat of water = specific heat of ice

Ex .1 Heat required to increases the temperature of 1 kg water by 20°C

Sol. heat required = $\Delta Q = ms\Delta\theta$

$= 1 \times 20 = 20$ Kcal.

$\therefore$ S = 1 cal/gm°C = 1 Kcal/kg°C

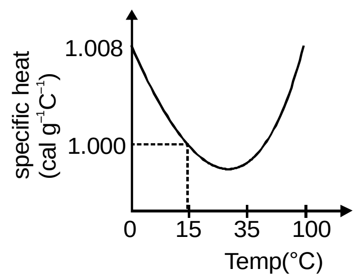
Important Points :

(a) We know, $s = \frac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$), the $s = Q/0 = \infty$. Thus the specific heat of a substance when it melts or boils at constant temprature is infinite.

(b) If the temperature of the substance changes without the transfer of heat ($Q = 0$) then $s = \frac{Q}{m\Delta T} = 0$. Thus when liquid in the thermos flask is shaken, its temperature increases without the tranfer of heat and hence and the specific heat of liquid in the thermos flask is zero.

(c) To raise the temperature of saturated water vapour, heat (Q) is withdrawn. Hence, specific heat of saturated water vapour is negative. (This is for your information only and not in the course)

(d) The slight variation of specific heat of water with temperature is shown in the graph at 1 atmosphere pressure. Its variation is less than 1% over the interval form 0 to 100°C.



1.3 Heat capacity or Thermal capacity :

Heat capacity of a body is defined as the amount of heat required to rasie the temperature of that body by 1°C. If 'm' is the mass and 's' the specific heat of the body, then

Heat capaicty = m s.

Units of heat capacity in : CGS system is, **cal °C^{-1} ;** SI unit is, **JK^{-1}**

1.4 Relation between Specific heat and Water equivalent :

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$ms\,\Delta T = m_w\,S_w\,\Delta T \Rightarrow m_w = \frac{ms}{s_w}$

In calorie $s_w = 1$

$\therefore \quad m_w = ms$

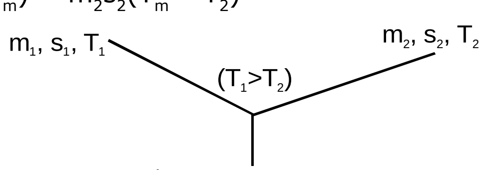
m_w is also represented by W

so W = ms

2. LAW OF MIXTURE :

When two substances at different temperatures are mixed together, the exchange of heat continues to take place till their temperatues become equal. This tempeature is then called final temperature of mixtue. Here, **Heat taken by one substance = Heat given by another substance**

$\Rightarrow m_1s_1(T_1 - T_m) = m_2s_2(T_m - T_2)$



Ex.2 ***An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it loses 25 % energy to surroundings. Then find the temperature rise of the block (Take sp. heat of iron 470 J/kg°C)***

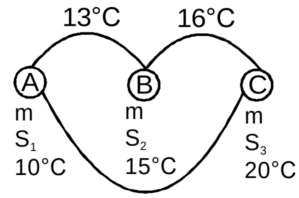
Sol. $mS\Delta\theta = \frac{1}{4}mgh \Rightarrow \Delta\theta = \frac{10 \times 10}{4 \times 470}$

Ex.3 ***The temperature of equal masses of three different liquids A, B, and C are 10°C 15°C and 20°C respectively. The temperatue when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?***

Sol. when A and B are mixed

$mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13)$

$3S_1 = 2S_2$ ...(1)



when B and C are mixed

$S_2 \times 1 = S_3 \times 4$ ...(2)

when C and A are mixed

$S_1(\theta - 10) = S_3 \times (20 - \theta)$ ...(3)

by using equation (1), (2) and (3)

we get $\theta = \frac{140}{11}{}^\circ C$

Ex.4 ***If three different liquid of different masses specific heats and temperature are mixed with each other and then what is the temperature mixtrue at thermal equilibrium.***

$m_1, s_1, T_1 \rightarrow$ specification for liquid

$m_2, s_2, T_2 \rightarrow$ specification for liquid

$m_3, s_3, T_3 \rightarrow$ specification for liquid

Sol. Total heat lost or gain by all substance is equal to zero

$\Delta Q = 0$

$m_1s_1(T - T_1) + m_2s_2(T - T_2) + m_3s_3(T - T_3) = 0$

so $T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$

3. PHASE CHANGE :

Heat required for the change of phase or state,

Q = mL, L = latent heat.

(a) **Latent heat (L) :** The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

(b) **Latent heat of Fusion (L_f) :** The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion.

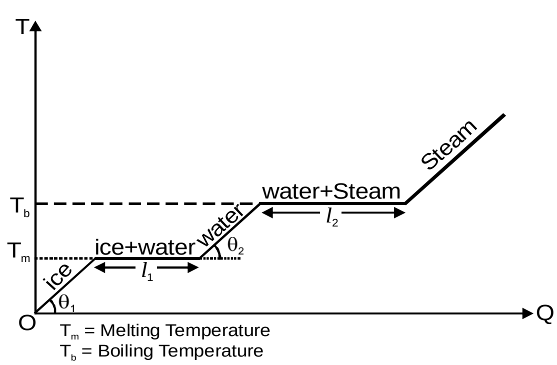
(c) **Latent heat of vaporisation (L_v) :** The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization.

If in question latent heat of water are not mentioned and to solve the problem it require to assume that we should consider following values.

Latent heat of ice : L = 80 cal/gm = 80 Kcal/kg = 4200 × 80 J/kg

Latent heat of steam : L = 540 cal/gm = 540 Kcal/kg = 4200 × 540 J/kg

The given figure, represents the change of state by different lines



$\theta_2 > \theta_1$

$l_2 > l_1$

Note : If we increases the temperature of liquid (phase) K.E ↑ & temp. ↑ but at a later time K .E. stop increasing and the phase of the liquid starts changing.

Ex.5 Find amount of heat released if 100 g ice at – 10°C is converted into 120°C, 100 g steam.

–10°C, 100 gm ice ⟶ 120° 100 gm steam

$Q = ms\Delta T$

$= \frac{1}{2} \times 100 \times 10$

= 500 cal.

↓

0°, 100 gm ice

$Q = mL_f$

= 100 × 80

= 8000 cal.

↓

0°, 100 gm water

$Q = ms\Delta T$

= 1×100 × 100

= 10000 cal.

↓

100°, 100 gm water

$Q = mL_v$

= 100 × 540

= 54000 cal

↓

100°, 100 gm Steam

↑

$Q = ms\Delta T$

$= \frac{1}{2} \times 100 \times 20$

= 1000 cal.

Q_{net} = 73.5 Kcal.

Ex.6 500 gm of water at 80°C is mixed with 100 gm steam at 120°C. Find out the final mixture.

Sol. 120°C steam ⟶ 100°C steam

Req. heat = 100 × $\frac{1}{2}$ ×20 = 1 kcal

80°C water ⟶ 100°C water

Req. heat = 500 × 1 × 20 = 10 kcal

100gm steam ⟶ 100 gm water at 100°C

Req. heat = 100 × 540 = 54 kcal

Total heat = 55 kcal.

Remaining heat = 55 – 10 = 45 kcal

Now we have 600 gm water at 100°C ⇒ 4500 = m × 540 ⇒ $m = \frac{250}{3}$ gm

So at last we have $\frac{250}{3}$ gm steam and $\left(600 - \frac{250}{3}\right)$gm of water

HEAT TRANSFER

4. INTRODUCTION

Heat is energy in transit which flows due to temperature difference; from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three routes.

(i) Conduction **(ii)** Convection **(iii)** Radiation

(a) CONDUCTION

(i) Requires Medium

(ii) Energy is transmitted from one particle to another particle without displaced of particle

(iii) No transfer of particle

(b) CONVECTION

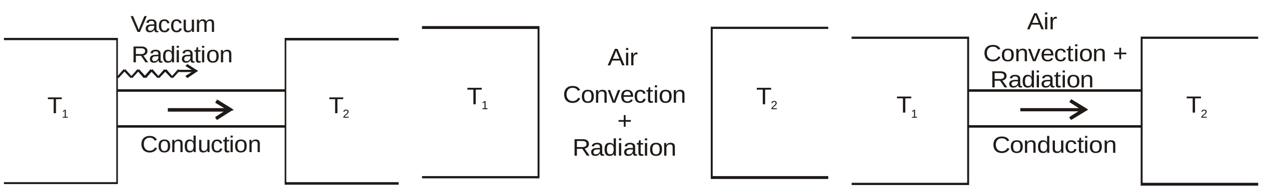
(i) Requires Medium

(ii) Enegy is transfer through movement of the particle of medium.

(c) RADIATION

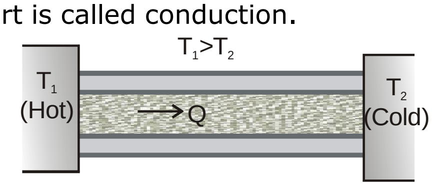
(i) Does not requires any medium

(ii) Enegy is transfer through Electromagnetic waves.



5. CONDUCTION

Figure shows a rod whose ends are in thermal contact with a hot reseroir at temperature T_i and a cold reservoir at temperature T_2. The sides of the rod are covered with insulating medium, so the transport of heat is along the rod, not through the sides. The molecules at the hot reservoir have greater vibrational energy. This energy is transferred by collisions to the atoms at the end face of the rod. These atoms in turn transfer energy to their neighbours further along the rod. Such transfer of heat through a substance in which heat is transported without direct mass transport is called conduction.



Most metals use another, more effective mechanism to conduct heat. The free electrons, which move throughout the metal can rapidly carry energy from the hotter to cooler regions, so metals are generally good donductors of heat. The presence of 'free' electrons also causes most metals to be good electrical conductors. A metal rod at 5°C feels colder than a piece of wood at 5°C because heat can flow more easily from your hand into the metal.

Heat transfer occurs only between regions that are at different temperatures, and the rate of heat flow is $\frac{dQ}{dt}$. This rate is also called the heat current, denoted by H. Experiments show that the heat current is proportional to the cross-section area A of the rod and to the temperature gradient $\frac{dT}{dx}$, which is the rate of change of temperature with distance along the bar. In general

$$H = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

The negative sign is used to make $\frac{dQ}{dt}$ a positive quantity since $\frac{dT}{dt}$ is negative. The constant k, called the thermal conductivity is a measure of the ability of a material to conduct heat. A substance with a large thermal conductivity k is a good heat conductor. The value of k depends on the temperature, increasing slightly with increasing temperature, but k can be taken to be practically constant throughout a substance if the temperature difference between its ends is not too great.

Let us apply Eq. (i) to a rod of length L and constant cross sectional area A in which a steady state has been reached. In a steady state the temperature at each point is constant in time. Hence.

$$-\frac{dT}{dt} = T_1 - T_2$$

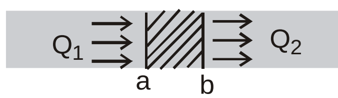
Therefore, the heat ΔQ transferred in time Δt is

$$\Delta Q = kA\left(\frac{T_1 - T_2}{L}\right)\Delta t$$

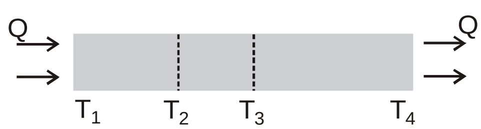
Here, ΔT = temperature difference (TD) and $R = \frac{l}{kA}$ =thermal resistance of the rod.

• ***Important Points in conduction***

1. Consider a section ab of a rod as shown in figure. Suppose Q_1 heat enters into the section at 'a' and Q_2 leaves at 'b', then $Q_2 < Q_1$. Part of the energy $Q_1 - Q_2$ is utilized in raising the tempeature of section ab and the remaining is lost to atmosphere thorugh ab. If heat is continuously supplied from the left end of the rod, a stage comes when temperature of the section becomes constant. In that case, $Q_1 = Q_2$ if rod is insulated from the surroundings (or loss thorugh ab is zero). This is called the **steady state** condition. Thus, in steady state temperature of different sections of the rod becomes constant (but not same).



Hence, in the figure :



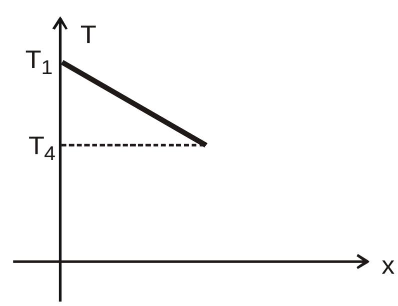
Insulated rod in steady state

T_1 = constant, T_2 = constant etc.

and $T_1 > T_2 > T_3 > T_4$

Now, a natural question arises, why the temperature of whole rod not becomes equal when heat is being continuously supplied ? The answer is : there must be a temperature difference in the rod for the heat flow, same as we require a potential difference across a resistance for the current flow thorugh it.

In steady state, the temperature varies linearly withd istance along the rod if it is insluated.



2. Comparing equation number (iii), i.e., heat current

$$H = \frac{dQ}{dt} = \frac{\Delta T}{R} \qquad \left(\text{where } R = \frac{l}{kA}\right)$$

with the equation, of current flow through a resistance,

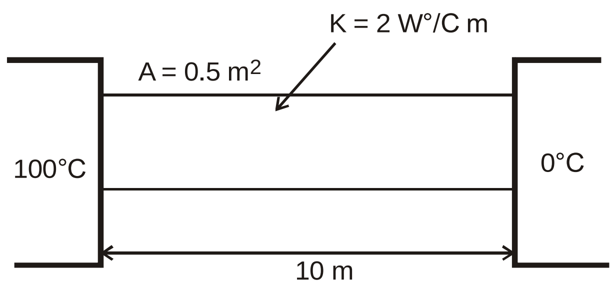
$$i = \frac{dq}{dt} = \frac{\Delta V}{R} \qquad \left(\text{where } R = \frac{l}{\sigma A}\right)$$

We find the following similarities in heat flow through a rod and current flow through a resistance.

Heat flow through a conducting rod	Current flow thorugh a resistance
Heat current H = $\frac{dQ}{dt}$ = rate of heat flow	Electric current i = $\frac{dq}{dt}$ = rate of charge flow
$H = \frac{\Delta T}{R} = \frac{TD}{R}$	$i = \frac{\Delta V}{R} = \frac{PD}{R}$
$R = \frac{l}{kA}$	$R = \frac{l}{\sigma A}$
k = thermal conductivity	σ = electrical conductivity

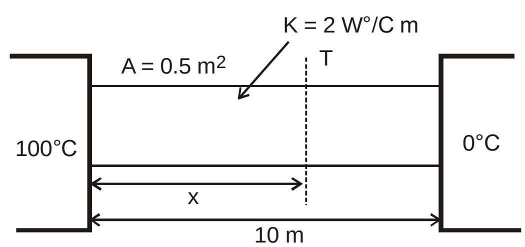
From the above table it is evident that flow of heat through rods in series and parallel is analogous to the flow of current through resistances in series and parallel. This analogy is of great importance in solving complicated problems of heat conduction.

Ex.7



Find out the heat current and temperature at any distance x.

Sol. $R = \frac{10}{2 \times 0.5} = 10 = \frac{\ell}{KA}$



$i = \frac{100}{10} = 10 = \frac{kA\Delta T}{\ell}$

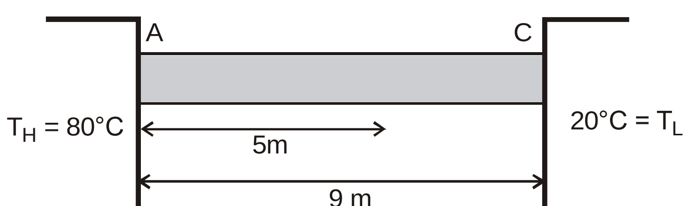
and temperature at any distance x.

$q = \frac{kA(100 - T)}{x} = \frac{KA\Delta T}{\ell} \quad \Rightarrow \quad \frac{(100 - T)}{x} = \frac{(100 - 0)}{\ell}$

$100\,\ell - T\,\ell = 100\,x$

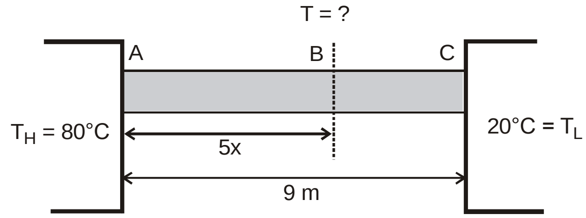
$T = \frac{100(\ell - x)}{\ell}$

Ex.8



Find out the temperature at distance 5 m.

Sol. Heat current is same. so,

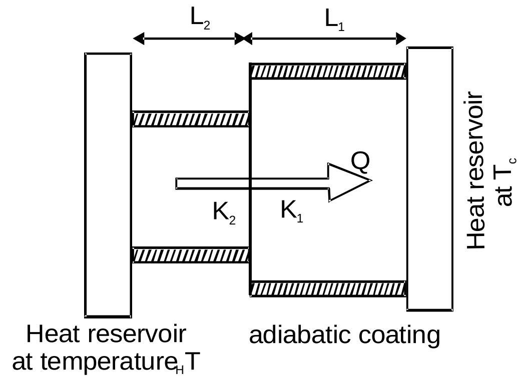


$$\frac{T_H - T_L}{\ell} = \frac{T_H - T}{x} \Rightarrow \frac{80-20}{9} = \frac{80-T}{5}$$

$$T = \frac{140}{3}°C$$

5.1 SLABS IN PARALLEL AND SERIES

(a) *Slabs in series (in steady state)*



Consider a composite slab consisting of two materials having different thickness L_1 and L_2 different cross-sectional areas A_1 and A_2 and different thermal conductivities K_1 and K_2. The temperature at the outer surface of the states are maintained at T_H and T_C, and all lateral surfaces are covered by an adiabatic coating.

Let temperature at the junction be T, since steady state has been achieved thermal current through each slab will be equal. Then thermal current through the first slab.

$$i = \frac{Q}{t} = \frac{T_H - T}{R_1} \quad \text{or } T_H - T = iR_1 \qquad ..(5.1)$$

and that through the second slab,

$$i = \frac{Q}{t} = \frac{T - T_C}{R_2} \quad \text{or} \quad T - T_C = iR_2 \qquad ..(5.2)$$

adding eqn. 5.1 and eqn 5.2

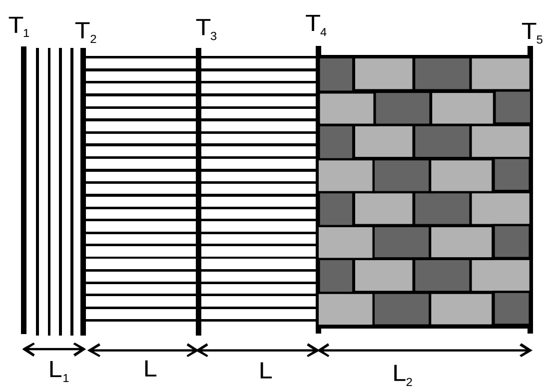
$$T_H - T_L = (R_1 + R_2)\, i \quad \text{or} \quad i =$$

Thus these two slabs are equivalent to a single slab of thermal resistance $R_1 + R_2$

If more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$$R = R_1 + R_2 + R_3 + ... \qquad (5.3)$$

Ex.9 ***The figure shows the cross-section of the outer wall of a house built in a hill-resort to keep the house insulated from the freezing temperature of outside. The wall consists of teak wood of thickness L_1 and brick of thickness ($L_2 = 5L_1$), sandwitching two layers of an unknown material with identical thermal conductivities and thickness. The thermal conductivity of teak wood is K_1 and that of brick is ($K_2 = 5K$). Heat condcution through the wall has reached a steady state with the temperature of three surfaces being known. ($T_1 = 25°C$, $T_2 = 20°C$ and $T_5 = -20°C$.) Find the interface temperature T_4 and T_3.***



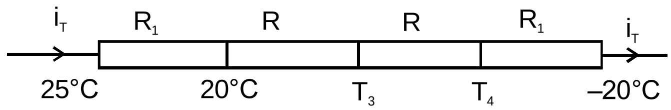
Sol. Let interface area be A. the thermal resistance of wood.

$R_1 = \frac{L_1}{K_1A}$

and that of brick wall

$R_2 = \frac{L_2}{K_2A} = \frac{5L_1}{5K_1A} = R_1$

Let thermal resistance of the each sand witch layer = R. Then the above wall can be visualised as a circuit



thermal current through each wall is same.

Hence $\frac{25-20}{R_1} = \frac{20-T_3}{R} = \frac{T_3-T_4}{R} = \frac{T_4+20}{R_1}$

$\Rightarrow 25 - 20 = T_4 + 20 \Rightarrow T_4 = -15°C$ **Ans.**

also, $20 - T_3 = T_3 - T_4 \Rightarrow T_3 = \frac{20+T_4}{2} = 2.5°C$ **Ans.**

Ex.10 ***In example 3, $K_1 = 0.125$ W/m–°C, $K_2 = 5K_1 = 0.625$ W/m–°C and thermal conductivity of the unknown material is K = 0.25 W/m°C. L_1 = 4cm, $L_2 = 5L_1$ = 20 cm and L = 10 cm. If the house consists of a single room of total wall area of 100 m², then find the power of the electric heater being used in the room.***

Sol. $R_1 = R_2 = \frac{(4\times10^{-2}m)}{(0.125w/m–°C)(100m^2)} = 32 \times 10^{-4}$ °C/w

$R = \frac{(10\times10^{-2}m)}{(0.25W/m–°C)(100m^2)} = 40 \times 10^{-4}$°C/w

the equivalent thermal resistance of the entire wall = $R_1 + R_2 + 2R = 144 \times 10^{-4}$ °C/W

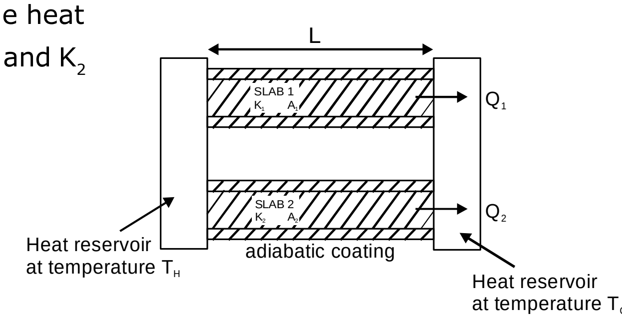
$\therefore$ Net heat current, i.e. amount of heat flowing out of the house per second $= \frac{T_H - T_C}{R}$

$= \frac{25°C-(-20°C)}{144\times10^{-4}°C/w} = \frac{45\times10^4}{144}$ watt

Hence the heater must supply 3.12 kW to compensate for the outflow of heat. **Ans.**

(b) Slabs in parallel :

Consider two slabs held between the same heat reservoirs, their thermal conductivities K_1 and K_2 and cross-sectional areas A_1 and A_2



then $R_1 = \frac{L}{K_1A_1}$, $R_2 = \frac{L}{K_2A_2}$

thermal current through slab 1

$i_1 = \frac{T_H - T_C}{R_1}$

and that through slab 2

$i_2 = \frac{T_H - T_C}{R_2}$

Net heat current from the hot to cold reservoir

$$i = i_1 + i_2 = (T_H - T_C)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Comparing with $i = \dfrac{T_H - T_C}{R_{eq}}$, we get,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

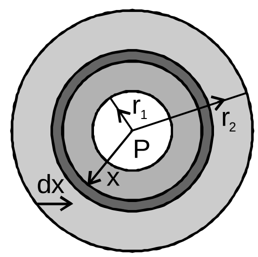
If more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots \qquad \ldots\ldots(5.4)$$

Ex.11 ***Two thin concentric shells made from copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P.***

Sol. Heat flowing per second through each cross-section of the sphere = P = i

Thermal resistance of the spherical shell of radius x and thickness dx,

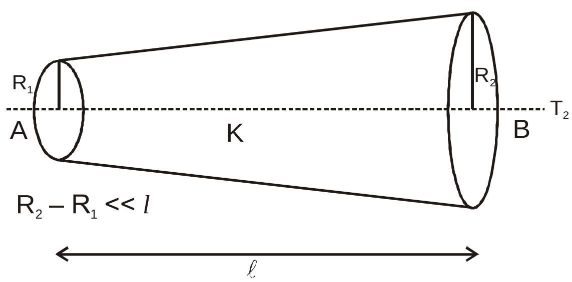


$$dR = \frac{dx}{K.4\pi x^2} \Rightarrow R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2.K} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

thermal current

$$i = P = \frac{T_H - T_C}{R} = \frac{4\pi K(T_H - T_C)r_1 r_2}{(r_2 - r_1)} \quad \textbf{Ans.}$$

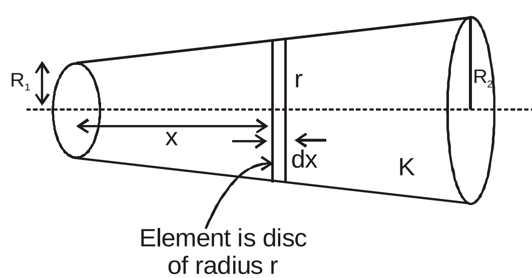
Ex.12



Find out the equivalent thermal resistance between point A and B.

Sol. $dR = \dfrac{dx}{k\pi r^2}$

$$\frac{r_1}{y} = \frac{r_2}{y+\ell} = \frac{r}{y+x} \quad \Rightarrow r_1 y + r_1 \ell = r_2 y$$



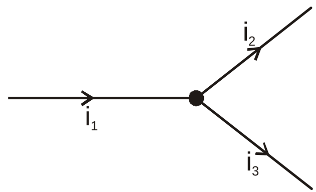
$$y = \frac{-r_1\ell}{(r_1 - r_2)} = \frac{r_1\ell}{(r_2 - r_1)} \Rightarrow r = \frac{r_2(y+x)}{y+\ell} = \frac{r_2\left(\dfrac{r_1\ell}{r_2 - r_1} + x\right)}{\left(\dfrac{r_1\ell}{r_2 - r_1} + \ell\right)}$$

$$dR = \frac{dx}{k\pi r^2} \Rightarrow \int_0^{\ell} \frac{dx}{k.\pi r^2} = \int_0^{\ell} \frac{dx}{k.\pi\left(r_1 + \dfrac{(r_2 - r_1)x}{\ell}\right)^2} \qquad \ldots(i)$$

5.2 Junction Law

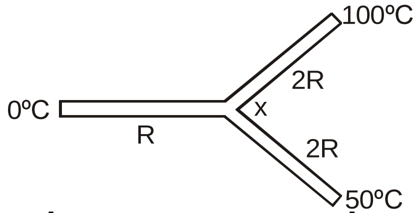
Heat current is a Tensor quantity because, it doesn't follow the vector laws but the direction of heat current matter.

According to the Junction law the sum of all the heat current directed towards a point is equal to the sum of all the heat currents directed away from the points.



$i_1 = i_2 + i_3$

Ex.13

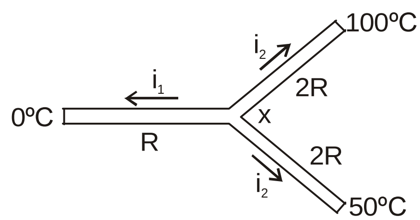


Find out the temperature at point x.

Sol. $i_1 + i_2 + i_3 = 0$

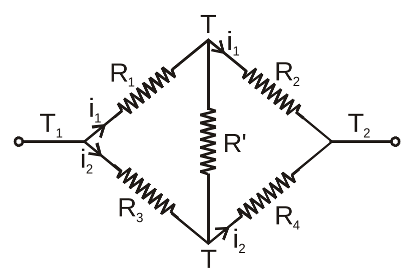
$i_1 = \dfrac{(x-0)}{R}$, $i_2 = \dfrac{(x-100)}{2R}$, $i_3 = \dfrac{(x-50)}{2R}$

$4x = 150 \Rightarrow x = 37.5^\circ C$



WHEAT STONE BRIDGE

Ex.14 ***Find out the relation between R_1, R_2, R_3 and R_4, so there is no that current in R'.***



Sol.

$\Delta T = T_1 - T_2$

$T_1 - T = i_1 R_1$ ...(i)

$T_1 - T = i_2 R_3$ ...(ii)

Eq. (i)/(ii)

$\Rightarrow \dfrac{i_1 R_1}{i_2 R_3} = 1$ ...(iii)

$T - T_2 = i_1 R_2$ ...(iv)

$T - T_2 = i_2 R_4$ ...(v)

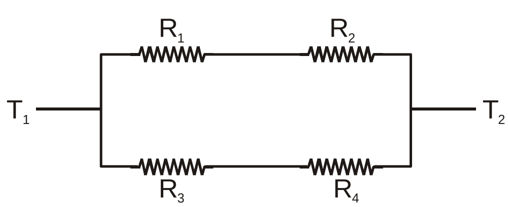
Eq. (4) / eq. (5)

$i_1 R_2 = i_2 R_4$ ...(vi)

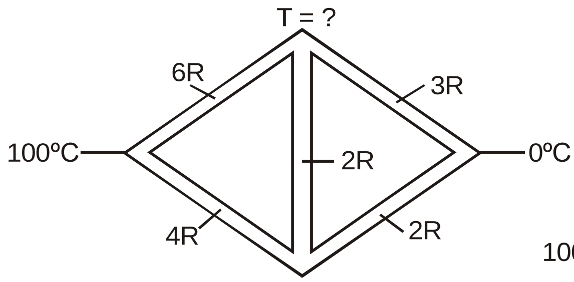
Eq. (iii)/(vi)

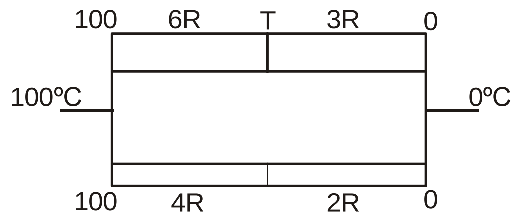
$\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4} \Rightarrow R_1 R_4 = R_2 R_3$

Now,



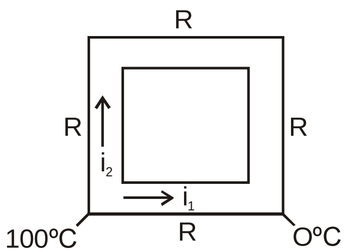
Ex.15





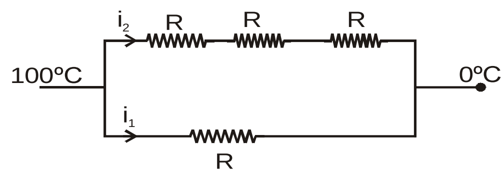
Sol. $\frac{100-T}{6R} = \frac{T-0}{3R} \Rightarrow T = 100/3\ °C$

Ex.16



Find $i_2/i_1 = ?$

Sol. $i_1 : i_2 = \frac{1}{R} : \frac{1}{3R} = 3 : 1$



$i_1 = \frac{3}{4} \times 100 = 75 \Rightarrow i_2 = \frac{1}{4} \times 100 = 25$

Ex.17 ***A container of negligible heat capacity contains 1 kg of water. It is connected by a steel rod of length 10 m and area of cross-section 10cm² to a large steam chamber which is maintaned at 100°C. If initial temperature of water is 0°C, find the time after which it becomes 50°C. (Neglect heat capacity of steel rod and assume no loss of heat to surroundings) (use table 3.1, take specific heat of water = 4180 J/kg°C)***

Sol. Let temperature of water at time t be T, then thermal current at time t,

$$i = \left(\frac{100-T}{R}\right)$$

This increases the temperature of water from T to T + dT

$$\Rightarrow i = \frac{dH}{dt} = ms\frac{dT}{dt} \Rightarrow \frac{100-T}{R} = ms\frac{dT}{dt}$$

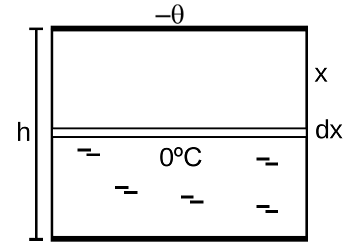
$$\Rightarrow \int_0^{50}\frac{dT}{100-T} = \int_0^{t}\frac{dT}{Rms} \Rightarrow -\ell n\left(\frac{1}{2}\right) = \frac{t}{Rms}$$

or $t = Rms\,\ell n\,2\text{sec} = \frac{L}{KA}\,ms\,\ell n\,2\text{sec}$

$$= \frac{(10m)(1kg)(4180\,J/kg\text{–}°C)}{46(w/m°C)\times(10\times10^{-4}m^2)} = \frac{418}{46}(0.69)\times10^5 = 6.27\times10^5 \text{ sec}$$

= 174.16 hours **Ans.**

Ex.18 ***On a cold winter day, the atmospheric temperature is -θ (on Celsius scale) which is below 0ºC. A cylindrical drum of height h made of a bad conductor is completely filled with water at 0ºC and is kept outside without any lid. Calculate the time taken for the whole mass of water to freeze. Thermal conductivity of ice is K and its latent heat of fusion is L. Neglect expansion of water on freezing.***



Sol. Suppose, the ice starts forming at time t = 0 and a thickness x is formed at time t. The amount of heat flown from the water to the surrounding in the time interval t to t + dt is

$$\Delta Q = \frac{KA\theta}{x}dt$$

The mass of the ice formed due to the loss of this amount of heat is

$$dm = \frac{\Delta Q}{L} = \frac{KA\theta}{xL}dt$$

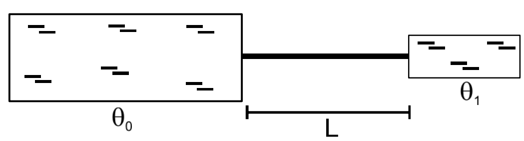
The thickness dx of ice formed in time dt is

$$dx = \frac{dm}{A\rho} = \frac{K\theta}{\rho xL}dt \qquad \text{or,} \qquad dt = \frac{\rho L}{K\theta}x\,dx$$

Thus, the time T taken for the whole mass of water to freeze is given by

$$\int_0^T dt = \frac{\rho L}{K\theta}\int_0^h x\,dx \qquad \text{or,} \qquad T = \frac{\rho L h^2}{2K\theta}.$$

Ex.19 ***Figure shows a large tank of water at a constant temperature θ_0 and a small vessel containing a mass m of water at an initial temperature $\theta_1 (< \theta_0)$. A metal rod of length L, area of cross-section A and thermal conductivity K connects the two vessels. Find the time taken for the temperature of the water in the smaller vessel to become $\theta_2 (\theta_1 < \theta_2 < \theta_0)$. Specific heat capacity of water is s and all other heat capacities are negligible.***



Sol. Suppose, the temperature of the water in the smaller vessel is θ at time t, In the next time interval dt, a heat ΔQ is transferred to it where

$$\Delta Q = \frac{KA}{L}(\theta_0 - \theta)dt \qquad ...(i)$$

This heat increases the temperature of the water of mass m to $\theta + d\theta$ where

$$\Delta Q = ms\,d\theta. \qquad ...(ii)$$

From (i) and (ii),

$$\frac{KA}{L}(\theta_0 - \theta)dt = ms\,d\theta$$

or, $$dt = \frac{Lms}{KA}\frac{d\theta}{\theta_0 - \theta} \qquad \text{or,} \qquad \int_0^T dt = \frac{Lms}{KA}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta_0 - \theta}$$

where T is the time required for the temperature of the water to become θ_2.

Thus, $$T = \frac{Lms}{KA}\ln\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}$$

RADIATION

6. RADIATION

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Propoerties of Radiation :

(a) All objects emit radiations simply because their temperature is above alsolute zero, and all objects absorb some of the radiation that falls on them from other objects.

(b) Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.

(c) More radiations are emitted at higher temperature of a body and lesser at lower temperature.

(d) The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing. Radiations from a body at NTP has predominantly infrared waves.

(e) Thermal radiations travels with the speed of light and move in a straight line.

(f) Radiations are electromagnetic waves and can also travel through vacuum.

(g) Similar to light, thermal radiations can be reflected, refracted, diffracted and polarized.

(h) Radiation from a point source obeys inverse square law (intensity $\alpha \frac{1}{r^2}$)

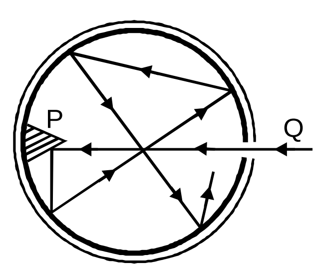
6.1. PREVOST THEORY OF EXCHANGE

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more then what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to temperature of its surroundings it radiates at the same rate as it absorbs.

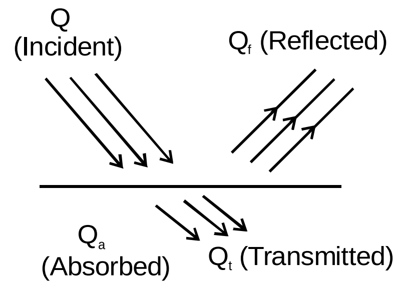
6.2 PERFECTLY BLACK BODY AND BLACK BODY RADIATION (FERY'S BLACK BODY)

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.

In actual practice, no natural object possesses strictly the properties of a perfectly black body. But the lamp-black and platinum black are good approximation of black body. They absorb about 99% of the incident radiation. The most simple and commonly used black body was designed by Fery. It consists of an enclosure with a small opening which is painted black from inside. The opeining acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little chance of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.



6.3 ABSORPTION, REFLECTION AND EMISSION OF RADIATIONS



$Q = Q_r + Q_t + Q_a$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

where r = reflecting power, a = absorptive power

and t = transmission power.

(i) r = 0, t = 0, a = 1, perfect black body

(ii) r = 1, t = 0, a = 0, perfect reflector

(iii) r = 0, t = 1, a = 0, perfect transmitter

(a) *Absorptive power :*

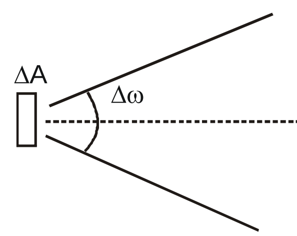
In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As all the raditions incident on a black body are absorbed, a = 1 for a black body.

(b) *Emissive power :*

Consider a small area ΔA of a body emitting thermal radiation. Consider a small solid angle $\Delta\omega$ about the normal to the radiating surface. Let the energy radiated by the area ΔA of the surface in the solid angle $\Delta\omega$ in time Δt be ΔU. We define emissive power of the body as



$$E = \frac{\Delta U}{(\Delta A)(\Delta\omega)(\Delta t)}$$

Thus, emissive power denotes the energy radiated per unit area per unit time per unit solid angle along the normal to the area.

(c) *Spectral Emissive power (E_λ) :*

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, E_λ. If E is the total emissive power and E_λ is spectral emissive power, they are related as follows,

$$E = \int_0^\infty E_\lambda d\lambda \quad \text{and} \quad \frac{dE}{d\lambda} = E_\lambda$$

(d) *Emissivity :*

$$e = \frac{\text{Emissive power of a body at temperature T}}{\text{Emissive power of a black body at same temperature T}} = \frac{E}{E_0}$$

7. STEFAN-BOLITZMANN'S LAW :

Consider a hot body at temperature T placed in an environment at a lower temperature T_0. The body emits more radiation than it absorbs and cools down while the surroundings absorb radiation from the body and warm up. The body is losing energy by emitting radiations and this rate.

$$\frac{d\theta}{dt} \propto T^4, \ \frac{d\theta}{dt} \propto A, \ \frac{d\theta}{dt} \propto e \Rightarrow \frac{d\theta}{dt} = \sigma e A T^4$$

$$P_1 = eA\sigma T^4$$

and is receiving energy by absorbing radiations and this absorption rate

$$\frac{d\theta}{dt} = P_2 = aA\sigma T_0^4$$

Here, 'a' is a pure number between 0 and 1 indicating the relative ability of the surface to absorbs radiation from its surroundings. Note that this 'a' is different from the absorptive power 'a'. In thermal equilibrium, both the body and the surrounding have the same temperture (say T_c) and,

$$P_1 = P_2$$

or $$eA\sigma T_c^4 = aA\sigma T_c^4$$

or $$e = a$$

Thus, when $T > T_0$, the net rate of heat transfer from the body to the surroundings is,

$$\text{Net heat loss} = \frac{dQ}{dt} = eA\sigma(T^4 - T_0^4)$$

or $$ms\left(\frac{dT}{dt}\right) = eA\sigma(T^4 - T_0^4) \Rightarrow \text{Rate of cooling}$$

$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}(T^4 - T_0^4) \quad \text{or} \quad \frac{dT}{dt} \propto (T^4 - T_0^4)$$

8. NEWTON'S LAW OF COOLING

According to this law, if the temperature T of the body is not very different from that of the surroundings T_0, then rate of cooling $-\frac{dT}{dt}$ is proportional to the temperature difference between them. To prove it let us assume that

$$T = T_0 + \Delta t$$

$$\frac{d\theta}{dt} = \sigma Ae\left[(T + \Delta T)^4 - T_0{}^4\right]$$

$$\frac{d\theta}{dt} = \sigma AeT_0^4\left[1 + \frac{4\Delta T}{T_0} - 1\right] = 4\sigma AT_0^3\Delta T$$

if the temperature difference is small.

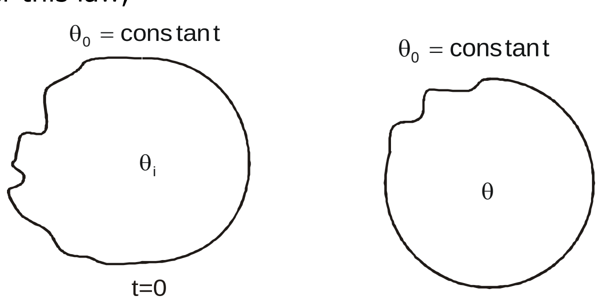
Thus, rate of colling

$$-\frac{dT}{dt} \propto \Delta T \quad \text{or} \quad -\frac{d\theta}{dt} \propto \Delta\theta$$

as $dT = d\theta$ or $\Delta T = D\theta$

8.1 Variation of temperature of a body according to Nerton's law

Suppose a body has a temperature θ_i at time t = 0. It is placed in an atmosphere whose temperature is θ_0. We are interested in finding the temperature of the body at time t, assuming Newton's law of cooling to hold good or by assuming that the temperature difference is small. As per this law,

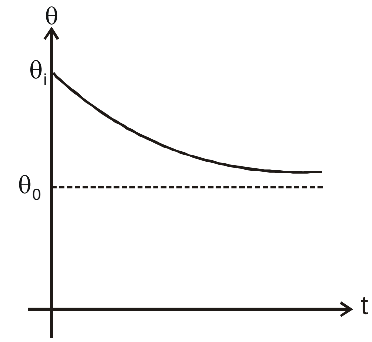


rate of cooling $\propto$ temperature difference or $\left(-\frac{d\theta}{dt}\right) = \left(\frac{eA\sigma}{mc}\right)(4\theta_0^3)(\theta - \theta_0)$ or $\left(-\frac{d\theta}{dt}\right) = \alpha(\theta - \theta_0)$

Here $\alpha = \left(\frac{4eA\sigma\theta_0^3}{mc}\right)$ is a constant

$\therefore \quad \int_{\theta_i}^{\theta} \frac{d\theta}{\theta - \theta_0} = -\alpha \int_0^t dt$

$\therefore \quad \theta = \theta_0 + (\theta_i - \theta_0)e^{-\alpha t}$



From this expression we see that $\theta = \theta_i$ at $t = 0$ and $\theta = \theta_0$ at $t = \infty$, i.e., temperature of the body varies exponentially with time from θ_i to θ_0 ($< \theta_i$). The temperature versus time graph is a shown in figure.

Note : If the body cools by radiation from θ_1 to θ_2 in time t, then taking the approximation

$$\left(-\frac{d\theta}{dt}\right) = \frac{\theta_i - \theta_2}{t} \text{ and } \theta = \theta_{av} = \left(\frac{\theta_1 + \theta_2}{2}\right)$$

The equation $\left(-\frac{d\theta}{dt}\right) = \alpha(\theta - \theta_0)$ becomes

$$\frac{\theta_i - \theta_2}{t} = \alpha\left(\frac{\theta_i + \theta_2}{2} - \theta_0\right)$$

This form of the law helps in solving numerical problems related to **Newton's law of cooling.**

8.2 Limitations of Newton's Law of Cooling :

(a) The difference in temperature between the body and surroundings must be small

(b) The loss of heat from the body should be radiation only.

(c) The temperature of surroundings must remain constant during the cooling of the body.

Ex.20 ***A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30°C.***

Sol. $\Delta\theta_f = \Delta\theta_i e^{-kt}$

for the interval in which temperature falls from 40 to 35°C

$(35 - 20) = (40 - 20)\, e^{-k.10}$

$\Rightarrow e^{-10\,k} = \frac{3}{4} \quad \Rightarrow K = \frac{\ell n\frac{4}{3}}{10}$

for the next interval

$(30 - 20) = (35 - 20)\, e^{-kt} \quad \Rightarrow \quad e^{-10}\,k = \frac{2}{3}$

$\Rightarrow \; kt = \ell n\frac{3}{2} \quad \Rightarrow \quad \frac{\left(\ell n\frac{4}{3}\right)t}{10} = \ell n\frac{3}{2}$

$\Rightarrow t = 10\frac{\left(\ell n\frac{3}{2}\right)}{\left(\ell n\frac{4}{3}\right)}$ minute = 14.096 min **Ans.**

Aliter : (by approximate method)
for the interval in which temperature falls from 40 to 35°C

$<\theta> = \frac{40+35}{2}$ = 37.5°C

from equation (14.4) $\left\langle \frac{d\theta}{dt} \right\rangle = -k(<\theta> - \theta_0) \Rightarrow \frac{(35°C - 40°C)}{10(min)} = -K\,(37.5°C - 20°C)$

$\Rightarrow K = \frac{1}{35}(min^{-1})$

for the interval in which temperature falls from 35°C to 30°C

$<\theta> = \frac{35+30}{2}$ =32.5°C

from equation (14.4)

$\Rightarrow \frac{(30°C - 35°C)}{t} = -\,(32.5°C - 20°C)$

$\Rightarrow$ required time,

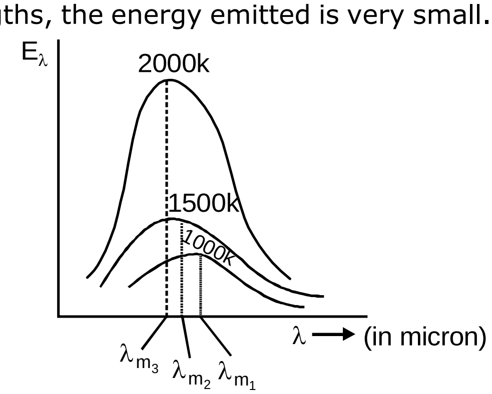
$t = \frac{5}{12.5} \times 35$ min = 14 min **Ans.**

9. NATURE OF THERMAL RADIATIONS : (WIEN'S DISPLACEMENT LAW)

From the energy distribution curve of black body radiation, the following conclusions can be drawn :

(a) The higher the temperature of a body, the higher is the area under the curve i.e. more amount of energy is emitted by the body at higher temperature.

(b) The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.



(c) For a given temperature, there is a paricular wavelength (λ_m) for which the energy emitted (E_λ) is maximum

(d) With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.

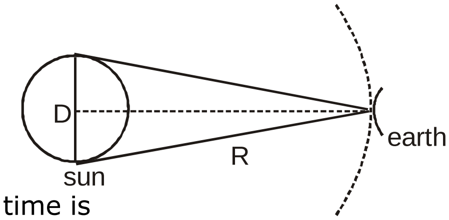
From the study of energy distribution of black body radiation discussed as above, it was established experimentally that the wavelength (λ_m) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body. i.e.

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

This is called Wien's displacement law.
Here b = 0.282 cm-K, is the Wien's constant.

Ex.21 ***The earth receives solar radiation at a rate of 8.2J/cm² - minute. Assuming that the sun radiates like a blackbody, calculate the surface temperature of the sun. The angle subtended by the sun on the earth is 0.53° and the Stefan constant $\sigma = 5.67 \times 10^{-8}$ W/ m² – K⁴.***

Sol. Let the diameter of the sun be D and its distance from the earth be R. From the questions.



$$\frac{D}{R} \approx 0.53 \times \frac{\pi}{180} = 9.25 \times 10^{-3} \qquad ...(i)$$

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\frac{D}{2}\right)^3 \sigma T^4 = \pi D^2 \sigma T^4$$

At distance R, this radiation falls on an area $4\pi R^2$ in unit time. the radiation received at the earth's surface per unit time per unit area is, therefore,

$$\frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2$$

Thus, $\frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2$ = 8.2 J/cm² - minute

or, $\frac{1}{4} \times \left(5.67 \times 10^{-8} \frac{W}{m^2 - K^4}\right) T^4 \times (9.25 \times 10^{-3})^2 = \frac{8.2}{10^{-4} \times 60} \frac{W}{m^2}$

or, T = 5794 K ≈ 5800 K.