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PHYSICS

TARGET IIT JEE

ELECTROSTATICS - 1

THEORY AND EXERCISE BOOKLET

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JEE SYLLABUS :

Coulomb's law; Electric field and potential; Electrical Potential energy of a system of point charges and of electrical dipoles in a uniform electrostatic field, Electric field lines;

1. INTRODUCTION :

(a) Introduction : Electromagnetism is a science of the combinatin of electrical and magnetic phenomenon. Electromagnetism can be divided into 2 parts :

(1) Electrostatics : It deals with the study of charges at rest.

(2) Electrodynamics : It deals with the study of charges in motion (discusses magnetic phenomenon).

In this chapter we will be dealing with charges at rest i.e. electrostatics.

(b) Structure of Atom :

An atom consists of two parts (i) nucleus (ii) extra nuclear part. Nucleus consists of neutrons and protons and extra nuclear part has electrons revolving around nucleus.

In a neutral atom.

number of electrons = number of protons.

charge of electrons = charge of protons = 1.602×10^{-19} coulomb.

Normally positive charges are positron, proton and positive ions. In nature practically free existing positive charge are positive ions and negative charges are electrons.

(c) Electric Charge

Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of naturally charged particles are electron, proton, α-particle etc.

(d) Types of Charge

(i) Positive charge : It is the deficiency of electrons compared to protons.

(ii) Negative charge : It is the excess of electrons compared to protons.

(e) Units of Charge

Charge is a derived physical quantity. Charge is measured in coulomb in S.I. unit. In practice we use mC (10^{-3}C), μC (10^{-6} C), nC (10^{-9}C) etc.

C.G.S unit of charge = electrostatic unit = esu.

1 coulomb = 3×10^9 esu of charge

Dimensional formula of charge = $[M^oL^oT^1I^1]$

(f) Properties of Charge

(I) Charge is a scalar quantity : It adds algebrically and represents excess, or deficiency of electrons.

(II) Charge is transferable : Charging a body implies transfer of charge (electrons) from one body to another. Positively charged body means loss of electrons, i.e. deficiency of electrons. Negatively charged body means excess of electrons. This also shows that mass of a negatively charged body > mass of a positively charged identical body.

(III) Charge is conserved : In an isolated system, total charge (sum of positive and negative) remains constant whatever change takes place in that system.

(IV) Charge is quantized : Charge on any body always exists in integral multiples of a fundamental unit of electric charge. This unit is equal to the magnitude of charge on electron ($1e = 1.6 \times 10^{-19}$ coulomb). So charge on anybody $Q = \pm ne$, where n is an integer and e is the charge of the electron. Millikan's oil drop experiment proved the quantization of charge or atomicity of charge.

☛

- Recently, the existence of particles of charge $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$ has been postulated. These particles are called quarks but still this is not considered as the quantum of charge because these are unstable (They have very short span of life.)

(v) Like point charges repel each other while unlike point charges attract each other.

(vi) Charge is always associated with mass, i.e., charge can not exist without mass though mass can exist without charge. The particle such as photon or neutrino which have no (rest) mass can never have a charge.

(vii) Charge is relativistically invariant : This means that charge is independent of frame of reference, i.e., charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with increase in speed.

(viii) A charge at rest produces only electric field around itself; a charge having uniform motion produces electric as well as magnetic field around itself while a charge having accelerated motion emits electromagnetic radiation.

(g) Conductors and Insultators :

Any object can be broadly classified in either of the following two categories :

(i) Conductors

(ii) Insulators

(i) Conductors : These are the materials that allow flow of charge through them. This category generally comprises of metals but may sometimes contain non-metals too. (ex. Carbon in form of graphite.)

(ii) Insulators : These are the materials which do not allow movement of charge through them.

(h) Charging of Bodies :

An object can be charged by addition or removed of electrons from it. In general an object can either be a conductor or insulator. Thus we are going to discuss the charging of a conductor and charging of an insultor in brife.

(i) Charging of Conductors :

Conductors can be charged by

(a) Rubbing or frictional electricity

(b) Conduction & Induction (will be studied in later sections)

(c) Thermionic emission (will be study the topic "heat")

(d) Photo electric emission (will be studied under the topic modern physics)

(ii) Charging of Insulators :

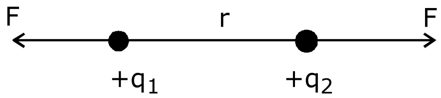
Since charge cannot flow through insulators, neither conduction nor induction can be used to charge, insultators, so in order to charge an insulator friction is used. Whenever an insulator is rubbed against a body exchange of electrons takes place between the two. This results in apperance of equal and opposite charges on the insulator and the other body. Thus the insulator is charged. For example rubbing of plastic with fur, silk with glass causes charging of these things.

☛ To charge the bodies through friction one of them has to be an insultator.

2. COULOMB'S LAW :

Coulomb, through his experiments found out that the two charges 'q_1' and 'q_2' kept at distance 'r' in a medium as shown in figure-1 exert a force 'F' on each other. The value of force F is given by

$$F = \frac{Kq_1q_2}{r^2}$$



This law gives the net force experienced by q_1 and q_2 taking in account the medium surrounding them.

Where

F gives the magnitude of electrostatic force.

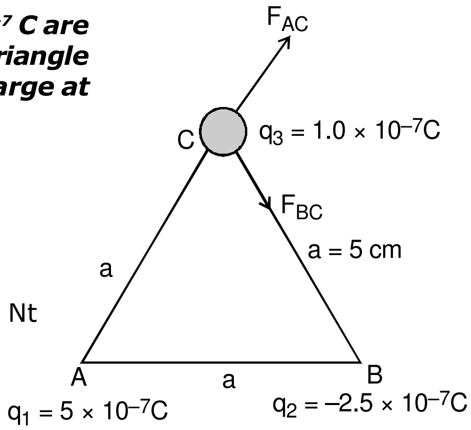
q_1 and q_2 are the magnitudes of the two interacting charges.

K is electrostatic constant which depends upon the medium surrounding the two charges.

This force F acts along the line joining the two charges and is repulsive if q_1 and q_2 are of same sign and is attractive if they are of opposite sign.

Let us take some examples on application of coulomb's Law.

Ex.1 ***Charge 5.0×10^{-7} C, -2.5×10^{-7}C and 1.0×10^{-7} C are fixed at the corners A, B and C of an equilateral triangle of side 5.0 cm. Find the electric force on the charge at C due to the rest two.***



$$F_{AC} = \frac{9 \times 10^9 \times 5 \times 10^{-7} \times 1 \times 10^{-7}}{(.05)^2} = 0.18 \text{ Nt}$$

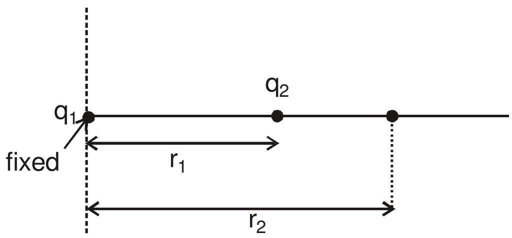
$$F_{BC} = \frac{9 \times 10^9 \times -2.5 \times 10^{-7} \times 1 \times 10^{-7}}{(.05)^2} = -.09 \text{ Nt}$$

Net force on C is $\vec{F}_{Net} = \vec{F}_{AC} + \vec{F}_{BC}$

$$\left|\vec{F}_{Net}\right| = \sqrt{(F_{AC})^2 + (F_{BC})^2 + 2(F_{AC})(F_{BC})\cos\theta} \qquad [\theta = 120°]$$

$$= 0.15588 \text{ Nt}$$

Ex.2 ***If charge q_1 is fixed and q_2 is free to move then find out the velocity of q_2 when it reaches distance r_2 after it is release from a distance of r_1 from q_1 as shown in figure (Assume friction is absent).***

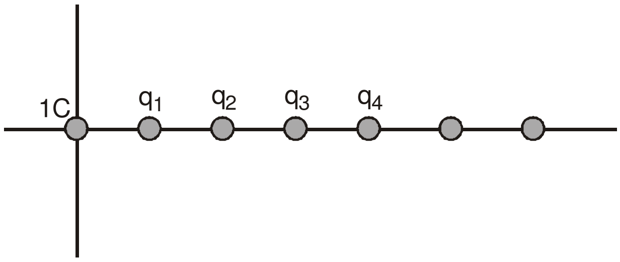


Find v of q_2 when it reaches distance r_2 after it is released from rest.

Sol. $a = \dfrac{kq_1q_2}{mx^2}$

$$\int_0^v vdv = \frac{kq_1q_2}{m}\int_{r_1}^{r_2}\frac{dx}{x^2} \Rightarrow \frac{v^2}{2} = \frac{kq_1q_2}{m}\left[\frac{1}{r_1}-\frac{1}{r_2}\right] \Rightarrow v = \frac{2kq_1q_2}{m}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Ex.3 Ten charged particles are kept fixed on the X axis at point x = 10 mm, 20 mm, 30 mm, ... 100 mm. The first particle has a charge 10^{-8} C, the second 8×10^{-8} C, the third 27×10^{-8}C and so on. The tenth particle has a charge 1000×10^{-8}C. Find the magnitude of electric force acting on a 1 C charge placed at the origin.

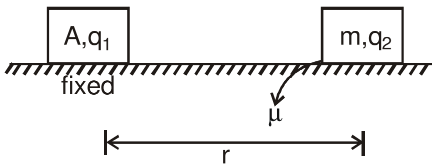


Sol. Force of 1C charge = $\dfrac{Kq_1 \times 1}{(10\times10^{-3})^2} + \dfrac{Kq_2 \times 1}{(20\times10^{-3})^2} + \dfrac{Kq_3 \times 1}{(30\times10^{-3})^2} + ...$

$$= \frac{K\times10^{-8}}{10^{-4}}\left[\frac{1^3}{1^2}+\frac{2^3}{2^2}+\frac{3^3}{3^2}+......\frac{10^3}{10^2}\right] = 9 \times 10^9 \times 10^{-4} \times 55 = 4.95 \times 10^7 \text{ Nt}$$

[This example explains that the concept of superposition holds in the case of electric forces. Net electric force at the origin is equal to sum of the individual electric forces on the 1 C charge]

Ex.4 A block 'A' of charge q_1 is fixed and second block of mass m and charge q_2 is allowed to free on the floor findout the range of q_2 for which the particle is at rest.

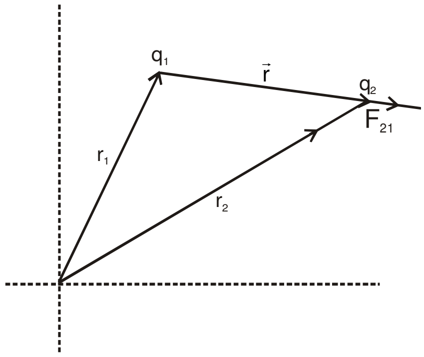


Sol Maximum friction = μ mg

$$\mu mg = \frac{kq_1q_2}{r^2} \Rightarrow q_2 = \frac{\mu mgr^2}{kq_1}$$

$$-\frac{\mu mgr^2}{kq_1} < q < \frac{\mu mgr^2}{kq_1}$$

2.1 Vector forms of Coloumb's law



(F_{21} : force on q_2 due to q_1)

$$\vec{F}_{21} = \frac{k\,q_1q_2}{r^2}\hat{r} = \frac{k\,q_1q_2}{r^3}\vec{r}$$

$$\vec{F}_{21} = \frac{k\,q_1q_2}{\left[\vec{r}_2 - \vec{r}_1\right]^3}\left(\vec{r}_2 - \vec{r}_1\right)$$

- Head of $\vec{r}$ points at that position where force has to be calculated.
- $\vec{r}_2$ & $\vec{r}_1$ depend on origin but $\vec{r}$ does not.
- q_1 and q_2 should be put along with sign.

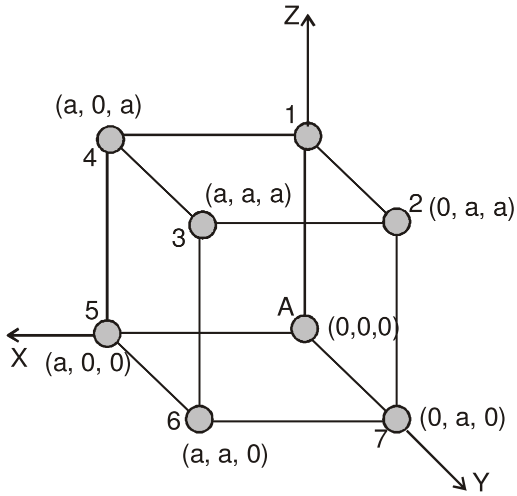
Ex.5 ***Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.***

Sol. The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube.

For this we use vector form of coulomb's law

$$\vec{F} = \frac{Kq_1q_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}(\vec{r}_1 - \vec{r}_2)$$

From the figure the different forces acting on A are given as



$$\vec{F}_{A1} = \frac{Kq^2(-a\hat{k})}{a^3}$$

$$\vec{F}_{A2} = \frac{Kq^2(-a\hat{j} - a\hat{k})}{(\sqrt{2}a)^3}$$

$$\vec{F}_{A3} = \frac{Kq^2(-a\hat{i} - a\hat{j} - a\hat{k})}{(\sqrt{3}a)^3}$$

$$\vec{F}_{A4} = \frac{Kq^2(-a\hat{i} - a\hat{k})}{(\sqrt{2}a)^3} \quad , \quad \vec{F}_{A5} = \frac{Kq^2(-a\hat{i})}{a^3}$$

$$\vec{F}_{A6} = \frac{Kq^2(-a\hat{i} - a\hat{j})}{(\sqrt{2}a)^3}$$

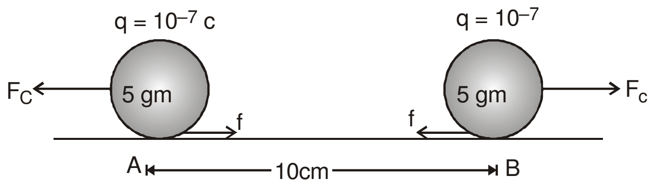
$$\vec{F}_{A7} = \frac{Kq^2(-a\hat{j})}{a^3}$$

The net force experienced by A can be given as

$$\vec{F}_{net} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_3} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_6} + \vec{F}_{A_7}$$

$$= \frac{-Kq^3}{a}\left[\left(\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right)(\hat{i} + \hat{j} + \hat{k})\right]$$

Ex.6 ***Two particles, each having a mass of 5 gm and charge 1.0 × 10⁻⁷C, stay in limiting equilirbium on a horizontal table with a separation of 10 cm between them. The coefficient of friction between each particle and the table is the same. Find the value of this coefficient.***



Sol. Consider particle A. Forces acting on A are coulombic force and frictional force under limiting condition friction will be limiting and will be equal to coulombic force.

$$F_C = \frac{Kq^2}{r^2} = \frac{9\times10^9\times(10^{-7})^2}{(10\times10^{-2})^2} = 9\times10^{-3}\text{N}$$

$$f = \mu N = \mu mg = \mu(5 \times 10^{-3} \times 10) = \mu\,(5 \times 10^{-2}\ \text{N})$$

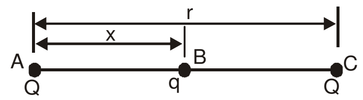
For equilibrium, we have $F_c = f$

$$9 \times 10^{-3} = \mu\,(5 \times 10^{-2}) \Rightarrow \quad \mu = \frac{9\times10^{-3}}{5\times10^{-2}} = 0.18$$

Ex.7 ***Two identical charge, Q each, are kept at a distance r from each other. A third charge q is placed on the line joining the above two charges such that all the three charges are in equilibrium. What is the magnitude, sign and position of the charge q ?***

Sol. Suppose the three charges be placed in the manner, as shown in fig.

The charge q will be in equilibrium if the forces exerted on it by the charges at A and C are equal and opposite.



$$k.\frac{Qq}{x^2} = k.\frac{Qq}{(r-x)^2} \text{ or } x^2 = (r - x)^2$$

or $\quad x = r - x \quad$ or $\quad x = \dfrac{r}{2}$

Since the charge at A is repelled by the similar charge at C, so it will be in equilibrium if it is attracted by the charge q at B, i.e., the sign of charge q should be opposite to that of charge Q.

$\therefore$ Force of repulsion between charges at A and C

= Force of attraction between charges at A and B

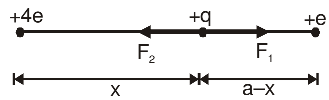
$$k\frac{Q.q}{(r/2)^2} = k\frac{Q.Q}{r^2} \quad \text{or} \quad q = \frac{Q}{4}$$

Ex.8 ***Two point charges +4e and +e are fixed a distance 'a' apart. Where should a third point charge q be placed on the line joining the two charges so that it may be in equilibrium ? In which case the equilibrium will be stable and in which unstable.***

Sol. Suppose the three charges are placed as shown in fig.

Let the charge q be positive.

For the equilibrium of charge +q, we must have



Force of repulsion F_1 between + 4e and +q = Force of repulsion F_2 between + e and +q

or $\quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{4e\times q}{x^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e\times q}{(a-x)^2}$

or $\quad 4(a - x)^2 = x^2$

or $\quad 2(a - x) = \pm x$

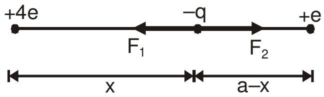
$\therefore \quad x = \dfrac{2a}{3}$ or $\quad 2a.$

As the charge q is placed between +4e and +e, so only x = 2a/3 is possible. Hence for equilibrium, the charge q must be placed at a distance 2a/3 from the charge +4e.

We have considered the charge q to be positive.

If we displace it slightly towards charge e, from the equilibrium position, then F_1 will decrease and F_2 will increase and a net force $(F_2 - F_1)$ will act on q towards left i.e., towards the equilibrium position. Hence the equilibrium of position q is stable.

Now if we take charge q to be negative, the force F_1 and F_2 will be attractive, as shown in fig.



The charge –q will still be in equilibrium at x = 2a/3. However, if we displace charge – q slightly towards right, then F_1 will decrease and F_2 will increase. A net force (F_2 – F_1) will act on –q towards right i.e., away from the equilibrium position. So the equilibrium of the negative q will be unstable.

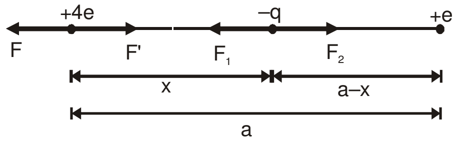
Ex.9 ***Two 'free' point charges +4e and +e are placed a distance 'a' apart. Where should a third point charge –q be placed between them such that the entire system may be in equilirbium ? What should be the magnitude and sign of q ? What type of a equilibrium will it be ?***

Sol. Suppose the charges are placed as shown in fig.

As the charge +e exerts repulsive force F on charge +4e, so for the equilibrium of charge + 4e, the charge –q must exert attraction F′ on +4e. This requires the charge q to be negative.

For equilibrium of charge +4e,

$$F = F'$$



$$\frac{1}{4\pi\varepsilon_0}\frac{4e \times e}{a^2} = \frac{1}{4\pi\varepsilon_0}.\frac{4e \times q}{x^2}$$

or $\quad q = \dfrac{ex^2}{a^2}$

For equilibrium of charge –q,

F_1 between +4e and –q

F_2 between + e and – q

$$\frac{1}{4\pi\varepsilon_0}\frac{4e \times e}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{e \times q}{(a-x)^2}$$

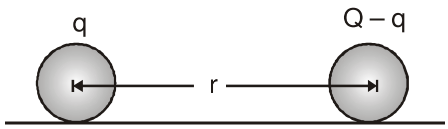
or $\quad x^2 = 4(a - x)^2 \qquad \therefore x = 2a/3$

Hence $q = \dfrac{ex^2}{a^2} = \dfrac{e}{a^2}.\dfrac{4a^2}{9} = \dfrac{4e}{9}$

Ex.10 ***A charge Q is to be divided in to two smalll objects. What should be the value of the charge on the objects so that the force between the objects will be maximum.***

Sol. Let one body have charge q and other hence Q – q

Here force between the charges $F = \dfrac{Kq(Q-q)}{r^2}$



For F to be maximum $\quad \dfrac{dF}{dq} = 0$

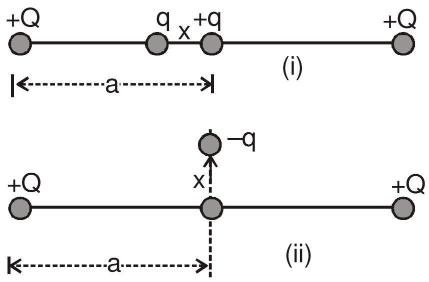
$$\frac{d}{dq}\left(\frac{KqQ}{r^2} - \frac{Kq^2}{r^2}\right) = 0$$

$$\frac{KQ}{r^2} - \frac{2Kq}{r^2} = 0 \Rightarrow \quad q = \frac{Q}{2}$$

Thus we have to divide charges equally on the objects.

Ex.11 ***Two identical positive point charges Q each are fixed apart at a distance 2a. A point charge q lies mid way between the fixed charges. Show that***
(i) For small displacement (relative to a) along line joining the fixed charges, the charge q executes SHM if it is +ve and
(ii) For small lateral displacement, it executes SHM if it is –ve. Compare the frequencies of oscillation in the two cases.

Sol. The two situations are shown in figure



(i) Let x be the displacement of the charge +q from the mean position. Now net force acting on the charge q toward its equilirbium position is

$$F = \frac{KQq}{(a-x)^2} - \frac{KQq}{(a+x)^2}$$

$$= \frac{4KQqax}{(a^2-x^2)^2} \approx \frac{4KQqax}{a^4} \quad [\text{As } x << a]$$

$$\approx \frac{4KQqx}{a^3}$$

Restoring acceleration, $a = \frac{F}{m} = -\frac{4KQqx}{ma^3}$ [– ve sign shows restoring tendency]

$a = -\omega^2 x$ [where m is the mass of the charge]

As acceleration is directly proportional to displacement, hence the motion is SHM. Its time period T_1 is given by

$$T_1 = \frac{2\pi}{\omega}$$

$$T_1 = 2\pi\sqrt{\left(\frac{ma^3}{4QqK}\right)} = 2\pi\sqrt{\frac{\pi \in_0 ma^3}{qQ}} \qquad ...(1)$$

(ii) Restoring force on –q toward Q is given by

$$F = \frac{2KQq}{(a^2+x^2)} \cdot \frac{x}{\sqrt{(a^2+x^2)}} = \frac{2KQq}{(a^2+x^2)^{3/2}} \approx \frac{2KQqx}{a^3} \quad [\text{As } x << a]$$

Restoring acceleration $a = \frac{F}{m} = -\frac{2KQq}{ma^3}x$

$a = -\omega^2 x$

Hence the motion is SHM. Its time period T_2 is igven by

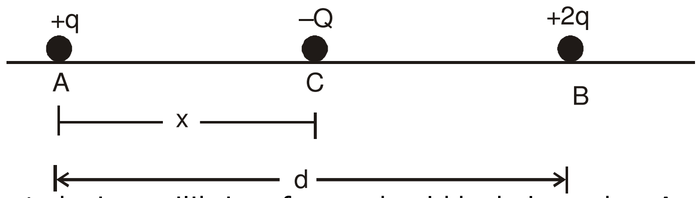
$$T_2 = \frac{2\pi}{\omega}$$

$$T_2 = 2\pi\sqrt{\left(\frac{ma^3}{2QqK}\right)} = 2\pi\sqrt{\frac{2\pi \in_0 ma^3}{qQ}} \qquad ...(2)$$

Now, $\frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{2}$

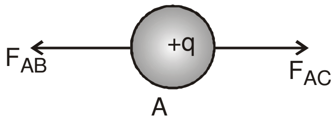
Ex.12 ***Two particles A and B having charges q and 2 q respectively are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such a way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped ?***

Sol.



For the charges to be in equilibrium forces should be balanced on A as well as on B.
Balancing forces on A

$$F_{AB} = \frac{Kq(2q)}{d^2}$$

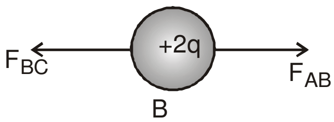


$$F_{AC} = \frac{KqQ}{x^2} \quad \text{or} \quad \frac{2q}{d^2} = \frac{Q}{x^2}$$

or $\quad Q = \dfrac{2qx^2}{d^2}$ ...(1)

Balancing force on B

$$F_{BC} = \frac{2Kq(Q)}{(d-x)^2}$$



or $\quad \dfrac{2Kq(Q)}{(d-x)^2} = \dfrac{Kq(2q)}{d^2}$

or $\quad \dfrac{(Q)}{(d-x)^2} = \dfrac{q}{d^2}$

Solving equation (1) and (2) we get

$$\frac{2qx^2}{d^2} = \frac{q}{d^2}(d-x)^2$$

or $\quad 2x^2 = (d-x)^2$

or $\quad 2x^2 = d^2 + x^2 - 2xd$

or $\quad x^2 + 2xd - d^2 = 0$

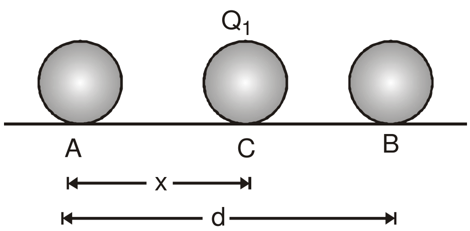
or $\quad x = (\sqrt{2} - 1)\, d$ or $-\, d\,(1 + \sqrt{2})$

The negative value implies that the particle C will lie toward left of A at a distance ($\sqrt{2}$ – 1) d from A (as x was measured from A)

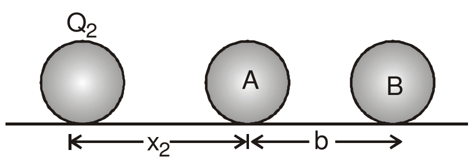
For the position $x = x_1 = (\sqrt{2} - 1)\, d$. $\qquad Q = Q_1 = -\, q(6 - \sqrt{2})$

and for $x = x_2 = -\, d\,(\sqrt{2} + 1)$ $\qquad Q = Q_2 = -\, q\,(6 + 4\sqrt{2})$

Thus be two possibilities are shown in figure

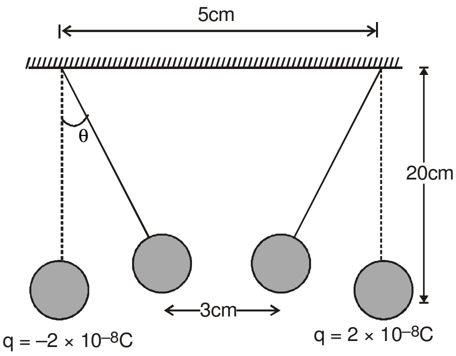


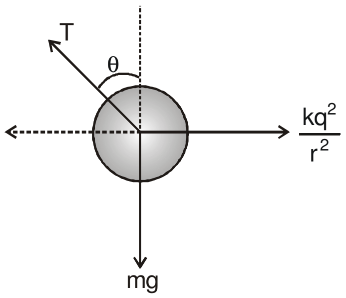
or



Ex.13 Two identical pitch balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each. The separation between the suspension points being 5 cm. In equilibrium the separation between the balls is 3 cm. Find the mass of each ball and the tension in the string. The charge on each ball has magnitude 2×10^{-8} C.

Sol. As the balls are rubbed against each other they will acquire equal and opposite charges. The FBD of left ball is shown in figure which shows all the forces acting on ball in equilibrium position.





Here for equilibrium of each bob. we have

$$T \sin \theta = \frac{kq^2}{r^2} \qquad ...(1)$$

$$T \cos \theta = mg \qquad ...(2)$$

or $\tan \theta = \frac{kq^2}{r^2\, mg}$ or $\frac{1}{\sqrt{(20)^2 - 1^2}} = \frac{K(2\times10^{-8})^2}{(3\times10^{-2})^2 m\times10}$

or $m = 7.96$ gm

From equation (2)

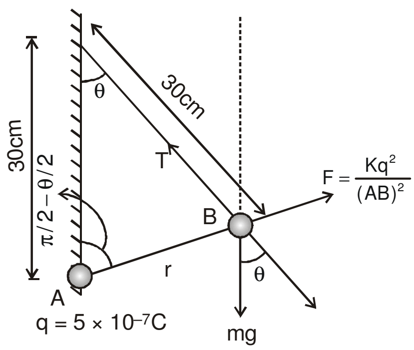
$$T = \frac{mg}{\cos\theta} = \frac{7.96\times10^{-3}\times10\times20}{\sqrt{(20)^2-1}} = 7.72 \times 10^{-2}\ N$$

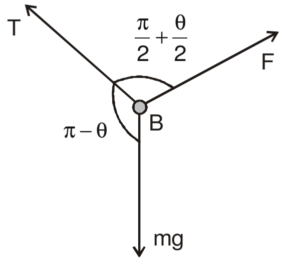
Ex.14 A particle A having a charge $q = 5 \times 10^{-7}$ C is fixed on a vertical wall. A second particle B of mass 100 g and having equal charge is suspended by a silk thread of length 30 cm from the wall. The point of suspension is 30 cm above the particle A. Find the angle of thread with vertical when it stays in equilibrium.

Sol. The situation shown in figure

Here the forces acting on bob B can be shown as

FBD of B is





Using Lami's theorem, we get

$$\frac{mg}{\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)} = \frac{F}{\sin(\pi-\theta)}$$

or $$\frac{mg}{\cos\frac{\theta}{2}} = \frac{Kq^2}{2\times0.30\times\sin\frac{\theta}{2}\times\sin\theta}$$

or $$\frac{mg}{\cos\frac{\theta}{2}} = \frac{Kq^2}{0.60\sin\frac{\theta}{2}\times2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

or $$\sin^2\frac{\theta}{2} = \frac{Kq^2}{2mg(0.60)}$$

On solving we get $\theta = 17°$

2.2 Coloumb's law in a medium :

(i) Relative Permittivity

When two charges are placed in vacuum or when the same set of charges are placed in a medium, the net force experienced by the charges will be different. The effect of presence of medium is accounted in the proportionality constant K. This electrostatic constant K is defined as

$$K = \frac{1}{4\pi \epsilon} \qquad \text{where } \epsilon = \epsilon_0 \, \epsilon_r$$

where ϵ = absolute permittivity of medium

ϵ_0 = permittivity of free space. having a constant value = 8.85×10^{-12} coul^2/N-m^2

$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ = relative permittivity of medium with respect to free space, also termed as dielectric constant.

For free space $\epsilon_r = 1$ and $K = \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \frac{N-m^2}{coul^2}$

(ii) Force dependency on Medium

We can say that when two charges are placed in vacuum (or air) the force experienced by the charges can be given as

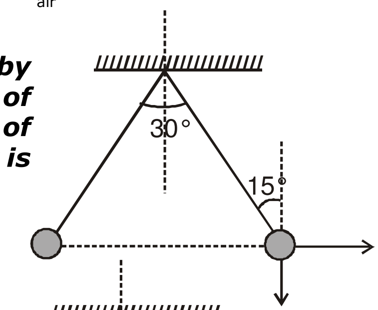
$$F_{air} = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2}$$

When these charges are submerged in a medium, having dielectric constant ϵ_r, the force becomes

$$F_{med} = \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2}$$

or $\quad F_{med} = \frac{F_{air}}{\epsilon_r} \quad$ as $\quad \epsilon_r > 1 \quad \Rightarrow \quad F_{med} < F_{air}$

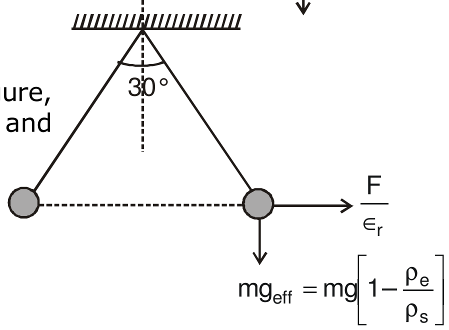
Ex.15 ***Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 gm/cc the angle remains same. What is the dielectric constant of liquid. Density of sphere = 1.6 gm/cc***



Sol. When set up shown in figure is in air, we have

$$\tan 15° = \frac{F}{mg}$$

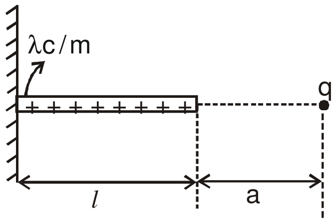
When set up is immersed in the medium as shown in figure, the electric force experienced by the ball will reduce and will be equal to $\frac{F}{\epsilon_r}$ and the effective gravitational force will become $mg\left(1 - \frac{\rho_\ell}{\rho_s}\right)$



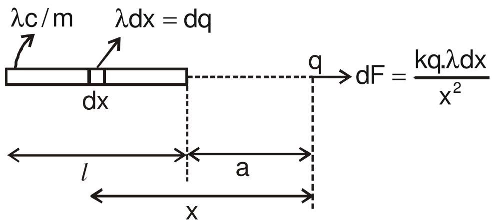
Thus we have $\quad \tan 15° = \frac{F}{mg\epsilon_r\left(1 - \frac{\rho_\ell}{\rho_s}\right)}$

$$\epsilon_r = \frac{1}{1 - \frac{\rho_\ell}{\rho_s}} = 2$$

Ex.16 ***Find the total force on charge q due to a charge rod having linear charge density λ C/m placed as shown in figure.***



Sol.



force $F = \int dF = \int_a^{a+\ell} \frac{kq'\lambda.dx}{x^2} = kq\lambda\left[-\frac{1}{x}\right]_a^{a+\ell} = kq\lambda\left[\frac{1}{a} - \frac{1}{a+\ell}\right]$

3. ELECTRIC FIELD :

The figure shown a charge q is lying in free space.

Now a charge q′ is brought near it.

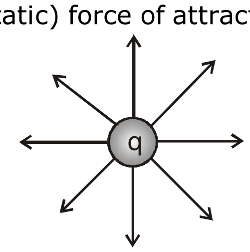
By columb's law we know that the charge q experiences a force and it exerts an equal force on q′.

How does q become aware of the presence of q′ ???

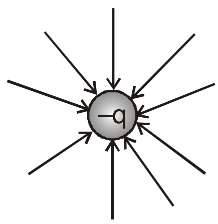
(We don't expect q to have sensory organs just as we have)

The answer is electric field !!!

Electric field is the space surrounding an electric charge q in which another charge q′ experiences a (electrostatic) force of attraction, or repulsion.



Electric field for a positive charge



Electric field for a negative charge

The direction of electric field is radially outwards for a positive charge and is radially inwards for a negative charge as shown in the figure above. There are some points always to be kept in mind. These are

(1) Electric field can be defined as a space surrounding a charge in which another static charge experiences a force on it.

(2) In a region electric field is said to exist if an electric force is exerted on a static charge placed at that point.

(3) It is important to note that with every charge particle, there is an electric field associated which extends up to infinity.

(4) No charged particle experiences force due to its own electric field.

$$E_p = \frac{F}{q_0} \text{ N/c}$$

☛ A very small positive charge which does not produce its significant electric field is called a test charge. Thus electric field strength at point can be defined generally as "Electric field strength at only point in space to be the electrostatic force per unit charge on a test charge."

If a charge q_0 placed at a point in electric field, experiences a net force $\vec{F}$ on it, then electric field strength at that point can be

or $\vec{E} = \frac{\vec{F}}{q_0}$ ....(1) [$q_0 \rightarrow$ test charge]

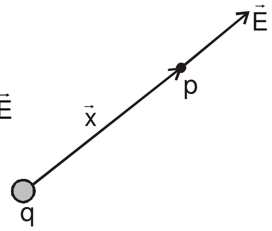
(a) Electric Field Strength due to Point Charge :

As discussed earlier, if we find electric field due to a point charge at a distance x from it. Its magnitude can be given as

$$E = \frac{Kq}{x^2} \qquad ...(2)$$

(b) Vector Form of Electric field due to a Point Charge :

As shown in figure, the direction of electric field strength at point P is along the direction of $\vec{x}$. Thus the value of $\vec{E}$ can be written as



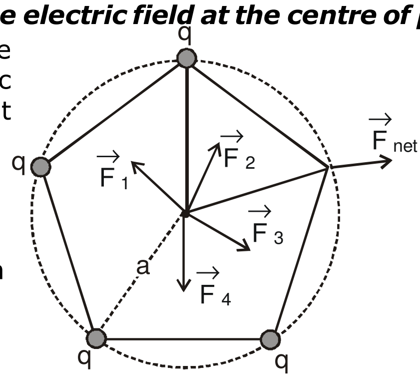
$$\vec{E} = \frac{Kq}{x^2}.\hat{x}$$

or $$\vec{E} = \frac{Kq}{x^3}.\vec{x} \qquad ...(3)$$

☛ It should be noted that the expression in equation (2) and (3) are only valid for point charges. We can not find electric field strength due to charged extended bodies by concentrating their whole charge at geometric centre and using the result of a point charge.

Ex.17 Four particles each having a charge q are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is 'a'. Find the electric field at the centre of pentagon.

Sol. We can calculate the electric field at centre by the superposition method i.e., by adding vectorially the electric field due to all the 4 charges at centre which will come out to be :



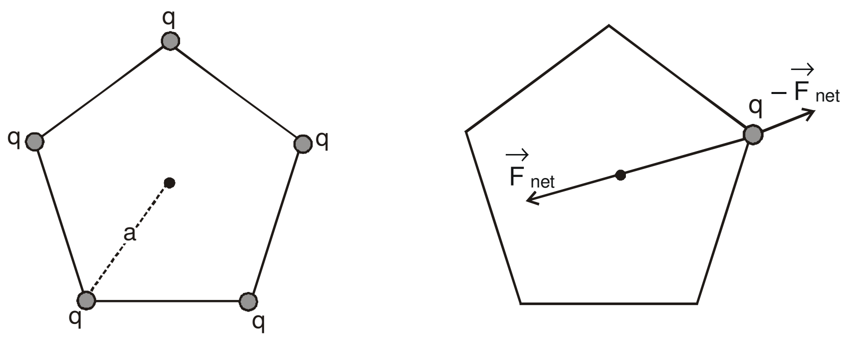
$$\vec{F}_{centre} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \frac{Kq}{a^2}$$

In the direction of the vector with no charge as shown in figure shown.

Alternate :

Consider pentagon with charges on all vector.

Now, E.F. at centre must be zero due to symmetry



(a) (b)

Thus E.F. due to 4 charge + E. F. due to 1 charge = 0

or E.F. due to 4 charges = – E.F. due to 1 charge

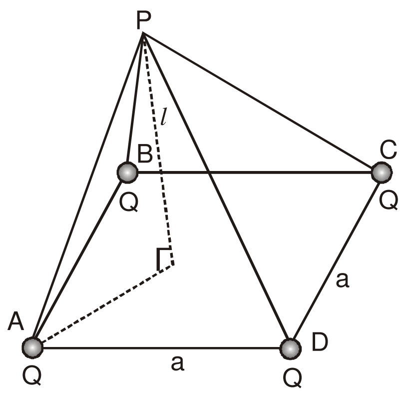
Where – sign denotes that both the forces are in opposite direction.

Thus E.F. dut to 4 charges = – E.F. due to 1 charge = $\frac{Kq}{a^2}$

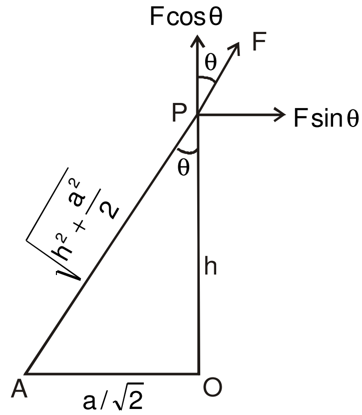
[Another good example of superposition theorem]

Ex.18 Four equal positive charges each of value Q are arranged at the four corners of a square of side a. A unit positive charge mass m is placed at P, at a height h above the centre of the square. What should be the value of Q in order that this unit charge is in equilibrium.

Sol. The situation is shown in figure (a)



(a)



(b)

Force experienced by unit positive charge placed at P due to a charge Q at A is given by

$$F = \frac{K(Q \times 1)}{\left(h^2 + \frac{a^2}{2}\right)}$$

Similarly, equal forces act on unit positive charge at P due to charge at B, C and D. When these forces are resolved in horizontal and vertical directions, the horizontal component (F sin θ) cancel each other and the net vertical force is 4F cos θ.

Thus net upward force $= \dfrac{4KQ}{\left(h^2 + \frac{a^2}{2}\right)} . \cos\theta$

For the equilibrium of unit positive charge at P,

Upward force = Weight of unit charge

$$\frac{4KQ}{\left(h^2 + \frac{a^2}{2}\right)} . \cos\theta = mg$$

From figure (b)

$$\cos\theta = \left\{\frac{h}{\sqrt{(h^2 + a^2/2)}}\right\}$$

or $$\frac{4KQh}{\left(h^2 + \frac{a^2}{2}\right)^{3/2}} = mg$$

or $$Q = \frac{mg}{4kh}\left(h^2 + \frac{a^2}{2}\right)^{3/2}$$

Ex.19 ***A particle of mass 9 × 10^{-31} kg and a negative charge of 1.6 × 10^{-19} coulomb projected horizontally with a veloicty of 10^{5} m/s into a region between two infinite horizontal parallel plates of metal. The distance between the plates is 0.3 cm and the particle enter 0.1 cm below the top plate. The top and bottom plates are connected respectively to the positive and negative terminals of a 30 volt battery. Find the component of the velocity of the particle just before it hits one on the plates.***

Sol. We known that between two parallel plates electric field can be given as

$$E = \frac{V}{d}$$

Here V = 30 volt and d = 0.3 cm = 3×10^{-3} m

Thus we have $E = \frac{30}{3\times10^{-3}} = 10^4$ N/C

Force on the particle of negative charge moving between the plates

$$F = e \times E = 1.6 \times 10^{-19} \times 10^4 = 1.6 \times 10^{-15} \text{ newton}$$

The direction of force will be towards the positive plate i.e., upward.

Now acceleration of the particle is

$$a = \frac{eE}{m}$$

or $a = (1.6 \times 10^{-15}) / (9 \times 10^{-31})$

or $a = 1.77 \times 10^{15}$ m/sec^2

As the electric intensity E is acting in the vertical direction the horizontal velocity v of the particle remains same. if y is the displacement of the particle, in upward direction, we have

$$y = \frac{1}{2}at^2$$

Here, y = 0.1 cm = 10^{-3} m, a = 1.77×10^{15} m/sec^2

Thus $10^{-3} = \frac{1}{2} \times (1.77 \times 10^{-15})(t^2)$

Solving we get t = 1.063×10^{-10} second

component of velocity in the direction of field is given by

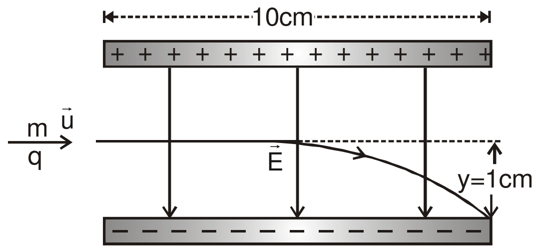
$v_y = at$

$= (1.77 \times 10^{15})(1.063 \times 10^{-10})$

$= 1.881 \times 10^4$ m/s.

Ex.20 ***A particle having a charge of 1.6 × 10^{-19} C enters midway between the plates of a parallel plate condenser. The initial velocity of particle is parallel to the plates. A potential difference of 300 volts is applied to the capacitor plates. If the length of the capacitor plates is 10 cm and they are separated by 2cm. Calculate the greatest intial velocity for which the particle will not be able to come out of the plates. The mass of the particle is 12 × 10^{-24} kg.***

Sol. The situation is shown in figure.



Here we know the electric field can be given as

$$E = \frac{V}{d} = \frac{300}{2/100} = 15000 \text{ v/m}$$

As the particle does not come out, its maximum deflection in vertical direction can be

y = 1 cm = 10^{-2} m

we known that $y = \frac{1}{2}at^2 = \frac{1}{2}.\frac{qE}{m}\left(\frac{l}{u}\right)^2$ [As $a = \frac{qE}{m}$ and $t = \frac{\ell}{u}$]

or $u^2 = \frac{1}{2}.\frac{qE}{my}.x^2$

$$= \frac{1}{2}\frac{(1.6\times10^{-19})(15000)}{(12\times10^{-24})(10^{-2})}\left(\frac{1}{10}\right)^2 = 10^8$$

$u = 10^4$ m/s

Ex.21 ***A uniform electric field E is created between two parallel charged plates as shown in figure shown. An electron enter the field symmetrically between the plates with a speed u. The length of each plate is ℓ, find the angle of deviation of the path of the electron as it comes out of the field.***

Sol. The situation is shown in figure.

Here we know in X-direction speed of electron remains uniform

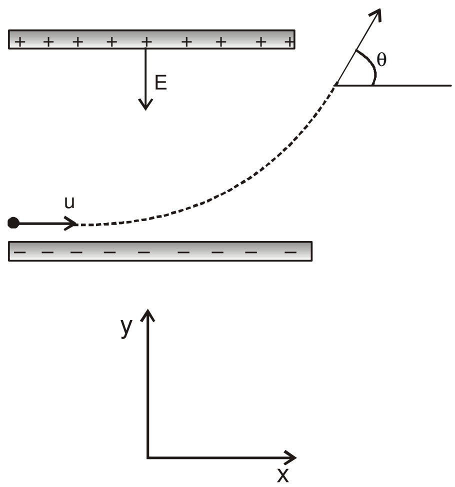
In X direction

$u_x = u$

In Y direction $v_{y\ initial} = 0$

Acceleration in y-direction of electron is

$$a = \frac{eE}{m}$$



$$v_{y_{fifnal}} = u_{y_{intial}} + at$$

$$v_y = \left(\frac{eE}{m}\right)\left(\frac{\ell}{u}\right)$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{eE\ell}{mu}\Big/u$$

$$\theta = \tan^{-1}\left(\frac{eE\ell}{mu^2}\right)$$

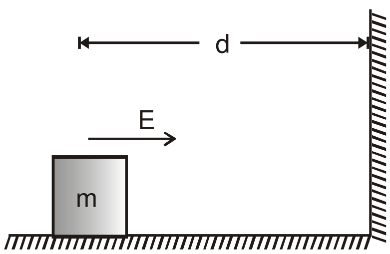
Ex.22 ***A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure. The distance of the block from the wall is d. A horizontal electric field 'E' towards right is switched on. Assuming elastic collision (if any) find the time period of resulting oscillatory motion. Is it a simple harmonic motion.***

Sol. Here acceleration of block is $a = \frac{qE}{m}$

Time taken by block to reach wall

$$d = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$t = \sqrt{\frac{2dm}{qE}}$$



Velocity at the time of impact is

$$v = \sqrt{2ad} \quad \text{or} \quad v = \sqrt{\frac{2qEd}{m}}$$

When the block will rebound time taken by block in coming to rest.

$$0 = \sqrt{\frac{2qEd}{m}} - \left(\frac{qE}{m}\right)t$$

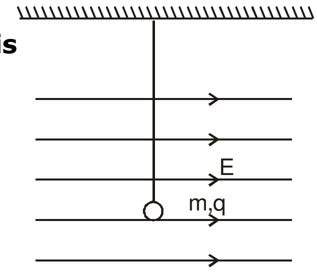
$$t = \frac{\sqrt{\frac{2qEd}{m}}}{\frac{qE}{m}} = \sqrt{\frac{2md}{qE}}$$

Thus time period of oscillation of block is

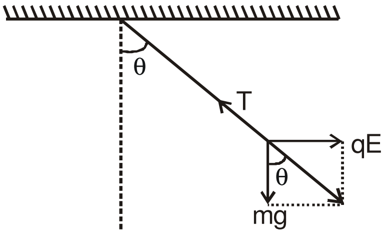
$$T = 2t = 2\sqrt{\frac{2md}{qE}}$$

Since the restoring force is independent of x, the displacement from mean position, this is not a simple harmonic motion

Ex.23 Find out the time period of oscillation when the bob is slightly shift through an angle θ from it mean position.

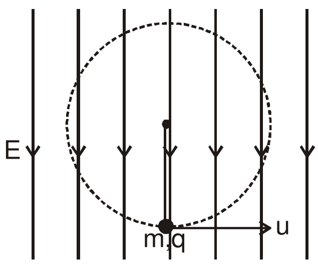


Sol.



$$\mathbf{g_{eff}} = \frac{\mathbf{T_{M.D.}}}{\mathbf{m}} = \sqrt{(mg)^2 + (qE)^2} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2} \Rightarrow \qquad T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

Ex.24



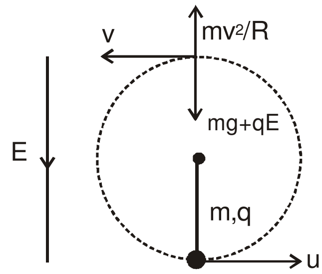
Find u_{min} so that particle will complete vertical circle

Sol. Applying force balance

$$T + mg + qE = \frac{mv^2}{R}$$

but for u_{min} $T = 0$

$$\therefore\ mg + qE = \frac{mv^2}{R}$$



using energy conservation

$$w_{mg} + w_T + w_{E.f} = \Delta k$$

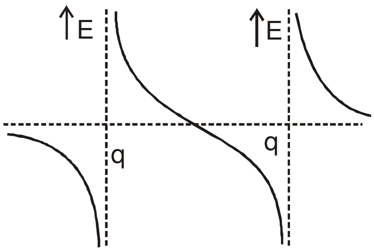
$$-\ mg.2R + 0 - qE.\ 2R = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$-\ 4\ R\ (mg + qE) = R\ (mg + qE) - mu^2$$

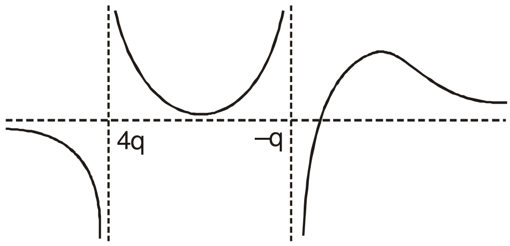
$$u = \sqrt{5\left(g + \frac{qE}{m}\right)R}$$

3.1 Graph of electric field due to binary charge configuration

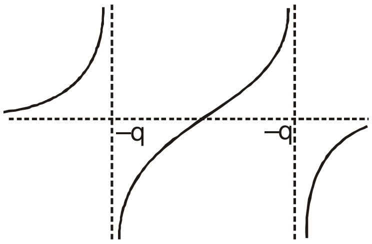
1.



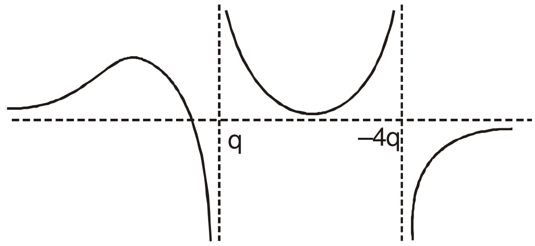
2.



3.

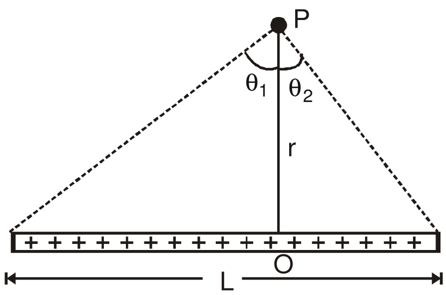


4.



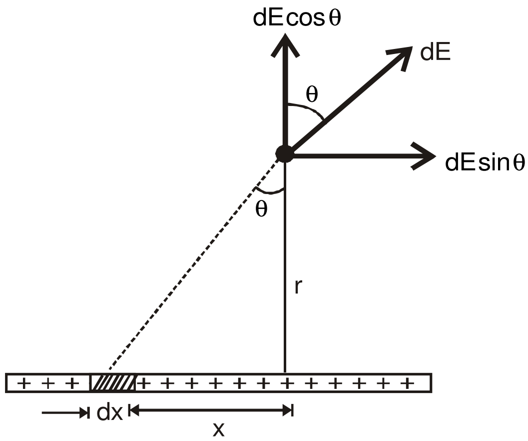
3.2 Electric field Strength at a General Point due to a Uniformly Charged Rod :

As shown in figure, if P is any general point in the surrounding of rod, to find electric field strength at P, again we consider an element on rod of length dx at a distance x from point O as shown in figure.



Now if dE be the electric field at P due to the element, then it can be given as

$$dE = \frac{Kdq}{(x^2 + r^2)}$$



Here $dq = \frac{Q}{L}dx$

Now we resolve electric field in components.
Electric field strength in x-direction due to dq at P is

$dE_x = dE \sin\theta$

or $$dE_x = \frac{Kdq}{(x^2 + r^2)}\sin\theta$$

$$= \frac{KQ\sin\theta}{L(x^2 + r^2)}dx$$

Here we have $x = r\tan\theta$
and $dx = r\sec^2\theta d\theta$

Thus we have $$dE_x = \frac{KQ}{L}\frac{r\sec^2\theta d\theta}{r^2\sec^2\theta}\sin\theta$$

Strength $$= \frac{KQ}{Lr}\sin\theta d\theta$$

Net electric field strength due to dq at point P in x-direction is

$$E_x = \int dE_x = \frac{KQ}{Lr}\int_{-\theta_2}^{+\theta_1}\sin\theta d\theta$$

or $$E_x = \frac{KQ}{Lr}\left[-\cos\theta\right]_{-\theta_2}^{+\theta_1}$$

or $$E_x = \frac{KQ}{Lr}\left[\cos\theta_2 - \cos\theta_1\right]$$

Similarly, electric field strength at point P due to dq in y-direction is

$dE_y = dE\cos\theta$

or $$dE_y = \frac{KQdx}{L(r^2 + x^2)} \times \cos\theta$$

Again we have $x = r\tan\theta$
and $dx = r\sec^2\theta\, d\theta$

Thus we have $$dE_y = \frac{KQ}{L}\cos\theta \times \frac{r\sec^2\theta}{r^2\sec^2\theta}d\theta = \frac{KQ}{Lr}\cos\theta d\theta$$

Net electric field strength at P due to dq in y-direction is

$$E_y = \int dE_y = \frac{KQ}{Lr}\int_{-\theta_2}^{+\theta_1}\cos\theta d\theta$$

or $$E_y = \frac{KQ}{Lr}\left[+\sin\theta\right]_{-\theta_2}^{+\theta_1}$$

or $$E_y = \frac{KQ}{Lr}\left[\sin\theta_1 + \sin\theta_2\right]$$

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles θ_1 and θ_2 at the two corners of rod can be given as

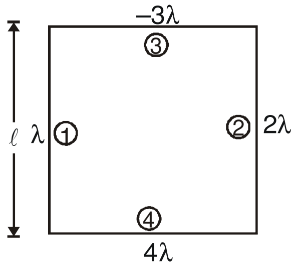
in ||-direction $E_x = \frac{KQ}{Lr}(\cos\theta_2 - \cos\theta_1) = \frac{k\lambda}{r}(\cos\theta_2 - \cos\theta_1)$

in $\perp$ -direction $E_y = \frac{KQ}{Lr}(\sin\theta_1 + \sin\theta_2) = \frac{k\lambda}{r}(\sin\theta_1 + \sin\theta_2)$

- r is the perpendicular distance of the point from the wire
- θ_1 and θ_2 should be taken in opposite sense

Ex.25 ***In the given arrangement of a charged square frame find field at centre. The linear charged density is as shown in figure***

Sol. E.F. due to 1 = $\frac{2K\lambda}{\ell}(\sin 45° + \sin 45°)\hat{i} = \frac{2\sqrt{2}K\lambda}{\ell}\hat{i}$



E.F. due to 2 = $-\frac{4\sqrt{2}K\lambda}{\ell}\hat{i}$

E.F. due to 3 = $\frac{6\sqrt{2}K\lambda}{\ell}\hat{j}$

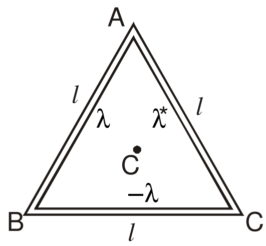
E.F. due to 4 = $\frac{8\sqrt{2}K\lambda}{\ell}\hat{j}$

$$\vec{E}_{net} = \vec{E}_{due\ to\ 1} + \vec{E}_{due\ to\ 2} + \vec{E}_{due\ to\ 3} + \vec{E}_{due\ to\ 4}$$

$$= \left(\frac{2\sqrt{2}K\lambda}{\ell} - \frac{4\sqrt{2}K\lambda}{\ell}\right)\hat{i} + \left(\frac{6\sqrt{2}K\lambda}{\ell} + \frac{8\sqrt{2}K\lambda}{\ell}\right)\hat{j}$$

$$= \frac{-2\sqrt{2}K\lambda}{\ell}\hat{i} + \frac{14\sqrt{2}K\lambda}{\ell}\hat{j}$$

Ex.26 ***Given an equilateral triangle with side ℓ. Find E at the centroid. The linear charge density is as shown in figure.***



Sol. The electric field strength due to the three rods AB, BC and CA are as shown in figure

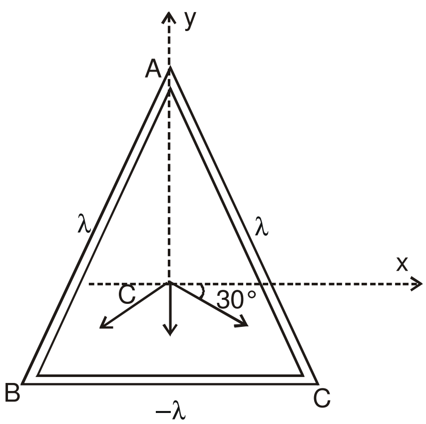
$$\vec{E}_{AC} = \frac{-2K\lambda}{\ell/\sqrt{3}}(2\sin 30°)(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$\vec{E}_{AB} = \frac{2K\lambda}{\ell/\sqrt{3}}(2\sin 30°)(\cos\theta\hat{i} - \sin\theta\hat{j})$$

$$\vec{E}_{BC} = \frac{2K\lambda}{\ell/\sqrt{3}}(2\sin 30°)\hat{j}$$

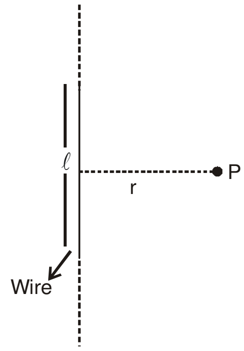
$$\vec{E}_{net} = \vec{E}_{AC} + \vec{E}_{AB} + \vec{E}_{BC}$$

$$\vec{E}_{net} = \frac{-\lambda}{2\pi\epsilon_0\ \ell}\hat{j}$$



3.3 Electric field due to infinite wire (l >> r)

Here we have to find the electric field at point p due to the given infinite wire. Using the formula learnt in above section which



$$E_{||} = \frac{k\lambda}{r}\left(\cos\theta_2 - \cos\theta_1\right)$$

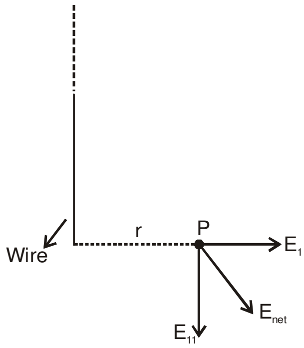
$$E_{\perp} = \frac{k\lambda}{r}\left(\sin\theta_2 + \sin\theta_1\right)$$

For above case, $\theta_1 = \theta_2 = \frac{\pi}{2}$

$$\therefore \quad E_{net} \text{ at } P = \frac{k\lambda}{r}(1+1) = \frac{2k\lambda}{r}$$

3.4 Electric field due to semi infinite wire

For this case



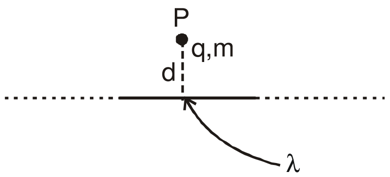
$$\theta_1 = \frac{\pi}{2}, \quad \theta_2 = 0^o$$

$$\therefore \quad E_r = \frac{k\lambda}{r};\ E_{||} = \frac{k\lambda}{r}$$

$$E_{net} \text{ at } P = \frac{\sqrt{2}\,k\lambda}{r} s$$

Ex.27 Consider the system shown below

If the charge is slightly displaced perpendicular to the wire from its equilibrium position then find out the time period of SHM.



Sol : At equilibrium position weight of the particle is balanced by the electric force

$$\Rightarrow \quad mg = qE$$

$$mg = q\frac{2k\lambda}{d} \qquad ....(1)$$

Now if the particle is slightly displaced by a distance x_λ (where x << d) net force on the body,

$$F_{net} = \frac{2k\lambda q}{d+x} - mg$$

from (1)

$$F_{net} = \frac{2k\lambda q}{d+x} - \frac{2k\lambda q}{d} = \frac{-2k\lambda q x}{d(d+x)}$$

As $x << d$ $\quad F_{net} \approx \frac{-2k\lambda q x}{d^2} \quad \Rightarrow \quad a = -\frac{-2k\lambda q x}{md^2}$

for SHM

$a = -\omega^2 x$

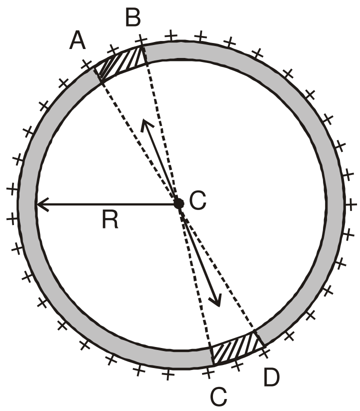
$$\therefore \quad \omega^2 = \frac{2k\lambda q}{md^2} \Rightarrow \omega = \sqrt{\frac{2k\lambda q}{md^2}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{md^2}{2k\lambda q}}$$

3.5 Electric field due to Uniformly Charged Ring :

Case - I : At its Centre

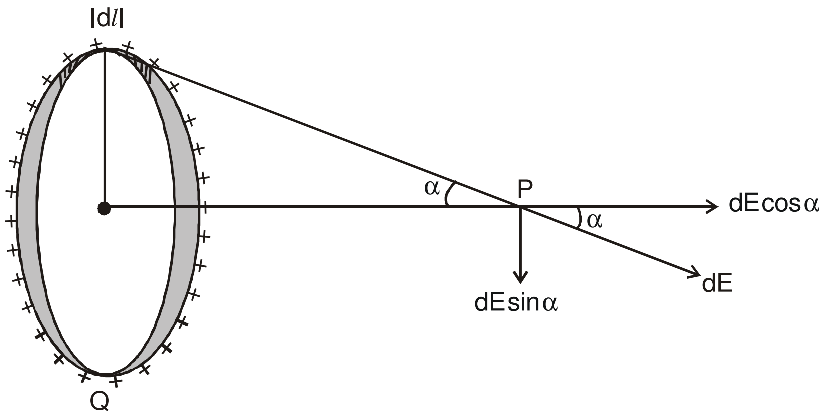
Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. As shown in figure. The electric field strength at centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is zero



Case II : At a Point on the Axis of Ring

For this look at the figure. There we'll find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this we consider a small section of length dl on ring as shown. The charge on this elemental section is

$$dq = \frac{Q}{2\pi R}\, d\ell \qquad [Q = \text{total charge of ring}]$$



Due to the element dq, electric field strength dE at point P can be given as

$$dE = \frac{Kdq}{(R^2 + x^2)}$$

The component of this field strength dE sin α which is normal to the axis of ring will be cancelled out due to the ring section opposite to dl. The component of electric field strength along the the axis of ring dE cosα due to all the sections will be added up. Hence total electric field strength at point P due to the ring is

$$E_p = \int dE \cos\alpha$$

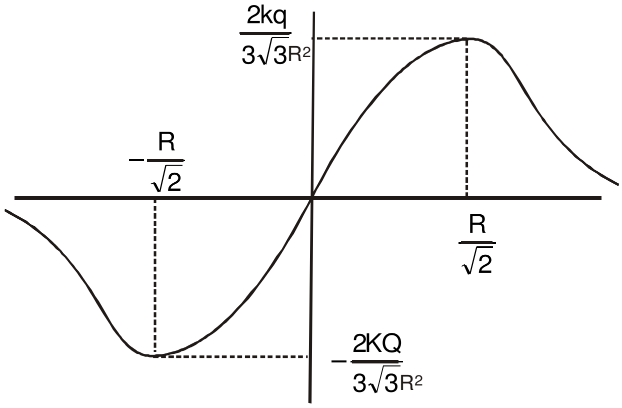
$$= \int_0^{2\pi R} \frac{Kdq}{(R^2 + x^2)} \times \frac{x}{\sqrt{R^2 + x^2}}$$

or $$E_p = \int_0^{2\pi R} \frac{KQx}{2\pi R(R^2 + x^2)^{3/2}} dl$$

$$= \frac{KQx}{2\pi R(R^2 + x^2)^{3/2}} \int_0^{2\pi R} d\ell$$

$$= \frac{KQx}{2\pi R(R^2 + x^2)^{3/2}} [2\pi R]$$

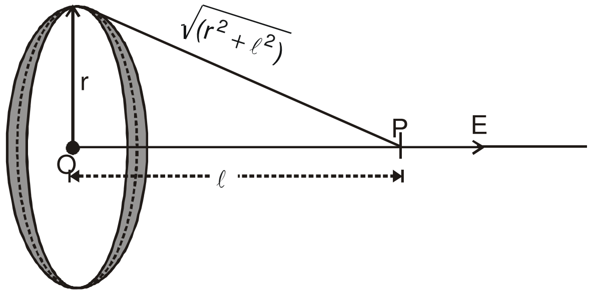
$$E_P = \frac{KQx}{(R^2 + x^2)^{3/2}}$$



Ex.28 ***A thin wire ring of radius r carries a charge q. Find the magnitude of the electric field strength on the axis of the ring as function of distance l from centre. Investigate the obtained function at l >> r. Find the maximum strength magnitude and the corresponding distance ℓ.***

Sol. See figure (Modify for maximum E)

We know due to ring electric field strength at a distance ℓ from its centre on its axis can be given as



$$E = \frac{Kq\ell}{(\ell^2 + r^2)^{3/2}} \quad ....(1)$$

For $\ell >> r$, we have $E = \frac{1}{4\pi \epsilon_0} \times \frac{q}{\ell^2}$

Thus the ring behaves like a point charge.

For E_{max} $\frac{dE}{d\ell} = O$. From equation we get

$$\frac{dE}{d\ell} = \frac{q}{4\pi \epsilon_0} \left[\frac{(r^2 + \ell^2)^{3/2}.1 - \frac{3\ell}{2}(r^2 + \ell^2)^{1/2} \times 2\ell}{(r^2 + \ell^2)^3} \right] = 0$$

or $$(r^2 + \ell^2)^{3/2} = \frac{3}{2}(r^2 + \ell^2)^{1/2} \times 2\ell^2$$

Solving we get, $\ell = \frac{r}{\sqrt{2}}$ ....(2)

Substituting the value of ℓ in equation (1) we get

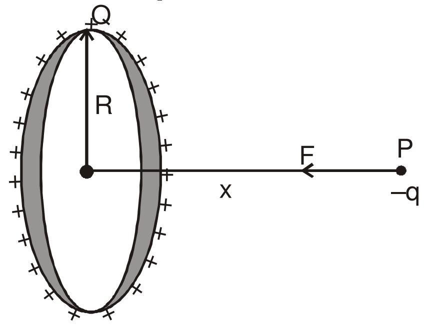
$$E = \frac{kq(r/\sqrt{2})}{(r^2 + r^2/2)^{3/2}} = \frac{2kq}{3\sqrt{3}\, r^2}$$

Ex.29 ***A thin fixed ring of radius 1 meter has a positive charge 1 × 10⁻⁵ coulomb uniformly distributed over it. a particle of mass 0.9 gm and having a negative charge of 1 × 10⁻⁶ coulomb is placed on the axis at distance of 1 cm from the centre of the ring. Shown that the motion of the negatively charged particle as approximately simple harmonic. Calculate the time period of oscillations.***

Sol. Let us first find the force on a – q charge placed at a distance x from centre of ring along its axis.

Figure shows the respective situation.

In this case force on particle P is



$$F_P = -qE = -q.\frac{KQx}{(x^2+R^2)^{3/2}}$$

For small x, x << R, we can neglect x, compared to R, we have

$$F = -\frac{KqQx}{R^3}$$

Acceleration of particle is $a = -\frac{KqQ}{mR^3}x$

[Here we have x = 1 cm and R = 1 m hence x << R can be used]

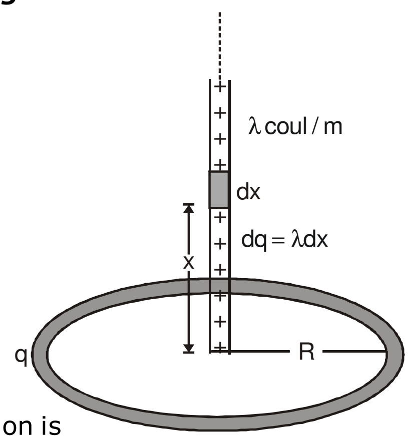
This shows that particle P excutes SHM, now comparing this acceleration with $a = -\omega^2 x$

We get $\omega = \sqrt{\frac{KqQ}{mR^3}}$

Thus time period of SHM is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mR^3}{KqQ}} = 2\pi\sqrt{\frac{0.9\times10^{-3}\times(1)^3}{9\times10^9\times10^{-5}\times10^{-6}}} = \frac{\pi}{5}\text{seconds}$

Ex.30 ***A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.***

Sol. Force df on the wire = dq $\vec{E}$



$$= \frac{Kqx}{(x^2+R^2)^{3/2}}.\lambda dx$$

$$F = Kq\lambda\int_0^\infty \frac{xdx}{(R^2+x^2)^{3/2}}$$

$$F = \frac{\lambda q}{4\pi\epsilon_0 R}$$

Alternate :

Due to wire electric field on the points of ring in y-direction is

$$E_y = \frac{K\lambda}{R}$$

Thus force on ring due to wire is

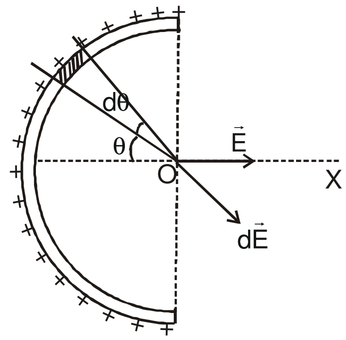
$$q\frac{K\lambda}{R} = \frac{Kq\lambda}{R} = \frac{\lambda q}{4\pi\epsilon_0 R}$$

and $E_x = 0$ [As cancelled out]

(Here x components of forces on small elements of rings are cancelled by the x component of diametrically opposite elements.)

Ex.31 ***A thin half-ring of radius R = 20 cm is uniformly charged with a total charge q = 0.70 nC. Find the magnitude of the electric field strength at the curvature centre of this half-ring.***

Sol. The situation is shown in figure



Here the semicircular wire subtend an angle π at the centre, we known that the electric field strength due to a circular arc subtending an angle φ at at it centre can be given as

$$E = \frac{2Kq\sin\phi/2}{\phi R^2} = \frac{2Kq}{\pi R^2} \qquad [\text{Here } \phi = \pi]$$

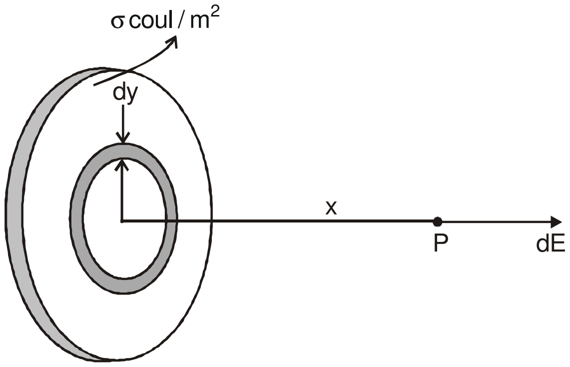
$$= \frac{q}{2\pi^2 \epsilon_0 R^2}$$

Substituting the value, we get

$$= \frac{7\times10^{-10}}{2\times(3.14)^2\times(8.85\times10^{-12})\times(0.2)^2} = 100 \text{ V/m}$$

3.6 Electric field Strength due to a Uniformly Surface Charged Disc :

If there is a disc of radius R, charged on its surface with surface charge density σ coul/m², we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.



To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in figure. Now the charge on this elemental ring dq can be given as

$$dq = \sigma\, 2\pi\, y\, dy \qquad [\text{Area of elemental ring } ds = 2\pi y\, dy]$$

Now we know that electric field strength due to a ring of radius R. Charge Q at a distance x from its centre on its axis can be given as

$$E = \frac{KQx}{(x^2+R^2)^{3/2}} \qquad [\text{As done earlier}]$$

Here due to the elemental ring electric field strength dE at point P can be given as

$$dE = \frac{Kdqx}{(x^2+y^2)^{3/2}} = \frac{K\sigma 2\pi y dy x}{(x^2+y^2)^{3/2}}$$

Net electric field at point P due to this disc is given by integrating above expression from O to R as

$$E = \int dE = \int_0^R \frac{K\sigma 2\pi x y dy}{(x^2+y^2)^{3/2}}$$

$$= K\sigma\pi x \int_0^R \frac{2y\,dy}{(x^2+y^2)^{3/2}} = 2K\sigma\pi x \left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^R$$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

Case : (i) If x >> R

$$E = \frac{\sigma}{2\varepsilon_0}[1 - \frac{x}{x\sqrt{\frac{R^2}{x^2}+1}}] = \frac{\sigma}{2\varepsilon_0}[1 - \left(1+\frac{R^2}{x^2}\right)^{-1/2}]$$

$$= \frac{\sigma}{2\varepsilon_0}[1 - 1 + \frac{1}{2}\frac{R^2}{x^2} + \text{higher order terms}]$$

$$= \frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

i.e. behaviour of the disc is like a point charge.

Case (ii) : If x << R

$$E = \frac{\sigma}{2\varepsilon_0}[1-0] = \frac{\sigma}{2\varepsilon_0}$$

i.e. behaviour of the disc is like ***infinite sheet.***

3.7 Electric Field Strength due to a Uniformly charged Hollow Hemispherical Cup :

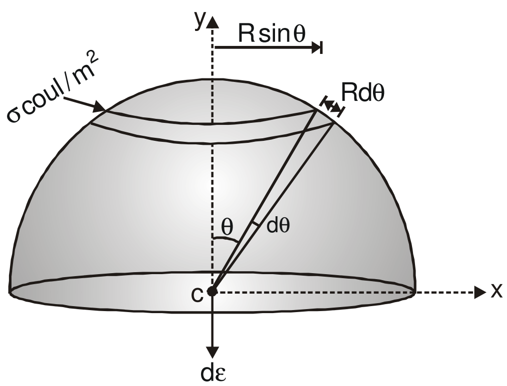
Figure shows a hollow hemisphere, uniformly charged with surface charge density σ coul/m². To find electric field strength at its centre C, we consider an elemental ring on its surface of angular width $d\theta$ at an angle θ from its axis as shown. The surface area of this ring will be

$$ds = 2\pi R \sin\theta \times R d\theta$$

Charge on this elemental ring is

$$dq = \sigma ds = \sigma . 2\pi R^2 \sin\theta \, d\theta$$

Now due to this ring electric field strength at centre C can be given as



$$dE = \frac{Kdq(R\cos\theta)}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

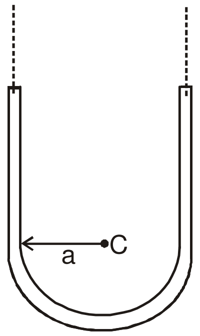
$$= \frac{K\sigma.2\pi R^2 \sin\theta d\theta.R\cos\theta}{R^3}$$

$$= \pi K\sigma \sin 2\theta \, d\theta$$

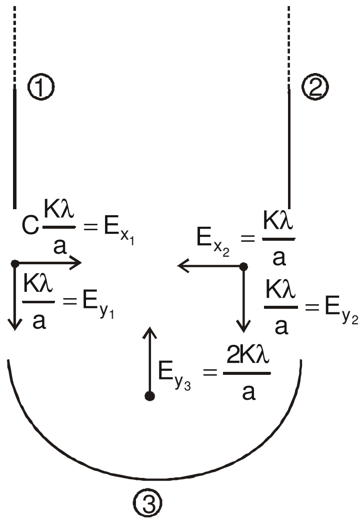
Net electric field at centre can be obtained by integrating this expression between limits 0 to $\frac{\pi}{2}$ as

$$E_0 = \int dE = \pi K\sigma \int_0^{\pi/2} \sin 2\theta d\theta = \frac{\sigma}{4\epsilon_0}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \frac{\sigma}{4\epsilon_0}\left[\frac{1}{2}+\frac{1}{2}\right] = \frac{\sigma}{4\epsilon_0}$$

Ex.32 ***In the given arrangement find the electric field at C in the figure (a). Here the U-shaped wire is uniformly charged with linear charge density λ.***



(a)



(b)

Sol. The electric field due to the three parts of U-shaped wire are shown in figure (b). Thus we have

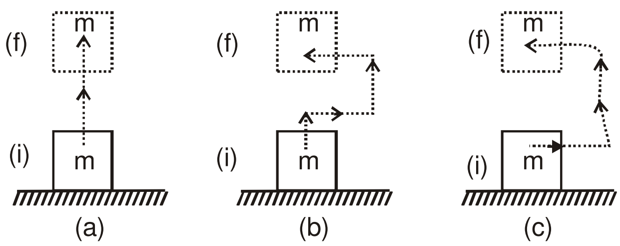
$$\vec{E}_{net} = (E_{x_1} + E_{x_2})\hat{i} + (E_{y_1} + E_{y_2} + E_{y_3})\hat{i}$$

$$\vec{E}_{net} = \left(\frac{K\lambda}{a} - \frac{K\lambda}{a}\right)\hat{i} + \left(\frac{2K\lambda}{a} - \frac{K\lambda}{a} - \frac{K\lambda}{a}\right)\hat{j} = 0$$

Thus E.F. due to given arrangement at C = 0

4. CONSERVATIVE FORCE

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.



Consider a body of mass m being raised to a height h vertically upwards as shown in above figure. The work done is mgh. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical path of the paths, we get the result mgh once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal path is zero. The work done along the vertical parts add up to mgh. Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the intial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative forces.

(i) Gravitational force, not only due to Earth due in its general form as given by the universal law of gravitation, is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

(iii) Electrostatic force between two electric charges is a conservative force.

(iv) Magnetic force between two magnetic poles is a conservative forces.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrosatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of Conservative forces

- **Work done by or against a conservative force depends only on the initial and final positions of the body.**
- Work done by or against a conservative force does not depend upon the nature of the path between initial and final positions of the body.

  If the work done by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.
- Work done by or against a conservative force in a round trip is zero.

  If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.

  The concept of potential energy exists only in the case of conservative forces.
- The work done by a conservative force is completely recoverable.

  Complete recoverability is an important aspect of the work of a conservative force.

- **Work done by conservative forces**

Ist format : (When constant force is given)

Ex.33 Calculate the work done to displace the particle from (1, 2) to (4, 5). if $\vec{F} = 4\hat{i} + 3\hat{j}$

Sol. dw = $\vec{F}.d\vec{r}$ ($d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$)

dw = $(4\hat{i} + 3\hat{j}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$ $\Rightarrow$ dw = 4dx + 3dy

$$\int_0^w dw = \int_1^4 4dx + \int_2^5 3dy \Rightarrow w = [4x]_1^4 + [3y]_2^5$$

w = (16 – 4) + (15 – 6) $\Rightarrow$ w = 12 + 9 = 21 Joule

II format : (When F is given as a function of x, y, z)

If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

then

dw = $(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$ $\Rightarrow$ dw = $F_x dx + F_y dy + F_z dz$

Ex.34 An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})$m ***to*** $\vec{r}_2 = (4\hat{i} + 6\hat{j})$m ***under a force*** $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$N***. Find the work done by this force.***

Sol. $W = \int_{r_i}^{\vec{r}_f} \vec{F}.d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2\hat{i} + 2y\hat{j}) \bullet (dx\hat{i} + dy\hat{j} + dz\hat{k}) = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2 dx + 2ydy) = [x^3 + y^2]_{(2,3)}^{(4,6)} = 83\,J$ **Ans.**

IIIrd format (perfect differential format)

Ex.35 If $\vec{F} = y\hat{i} + x\hat{j}$ then find out the work done in moving the particle from position (2, 3) to (5, 6)

Sol. dw = $\vec{F}.\vec{ds}$

dw = $(y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j})$

dw = ydx + xdy

Now ydx + xdy = d(xy) (perfect differential equation)

$\Rightarrow$ dw = d(xy)

for total work done we integrate both side

$$\int dw = \int d(xy)$$

Put xy = k

then at (2, 3) $k_i = 2 \times 3 = 6$

at (5, 6) $k_f = 5 \times 6 = 30$

then w = $\int_6^{30} dk = [k]_6^{30}$ $\Rightarrow$ w = (30 – 6) = 24 Joule

4.1 NON-CONSERVATIVE FORCES :

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by fricitional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force, magnetic force etc., are non conservative forces.

Ex.36 Calculate the work done by the force $\vec{F} = y\hat{i}$ to move the particle from (0, 0) to (1, 1) in the following condition

(a) y = x (b) y = x^2

Sol. We know that

dw = $\vec{F}.\vec{ds}$ $\Rightarrow$ dw = $(y\hat{i}).(dx\,\hat{i})$

dw = ydx ...(1)

In equation (1) we can calculate work done only when we know the path taken by the particle.

either

y = x or y = x^2 so now

(a) when y = x

$$\int dw = \int_0^1 x dx \quad \Rightarrow \quad w = \frac{1}{2}\text{Joule}$$

(b) when y = x^2

$$\int dw = \int_0^1 x^2 dx \quad \Rightarrow \quad w = \frac{1}{3}\text{Joule}$$

Difference between conservative and Non-conservative forces

[redacted]

5. ELECTROSTATIC POTENTIAL ENERGY :

(a) Electrostatic Potential Energy :

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Before proceeding further, we should keep in mind the following points, which are useful in understanding potential energy in electric fields.

(i) Doing work implies supply of energy

(ii) Energy can neither be transferred nor be transformed into any other form without doing work

(iii) Kinetic energy implies utilization of energy where as potential energy implies storage of energy

(iv) Whenever work is done on a system of bodies, the supplied energy to the system is either used in form of KE of its particles or it will be stored in the system in some form, increases the potential energy of system.

(v) When all particles of a system are separated far apart by infinite distance there will be no interaction between them. This state we take as reference of zero potential energy.

Now potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles.

- **Electrostatic potential energy is defined in two ways.**

(i) Interaction energy of charged particles of a system.

(ii) Self energy of a charged object (will be discussed later)

(b) Electrostatic Interaction Energy :

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to a given configuration.

When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system hence final potential energy of system will be positive. If the force between the particles is attractive work will be done by the system and final potential energy of system will be negative.

Let us take some illustrations to understand this concept in detail.

(c) Interaction Energy of a System of Two Charged Particles :

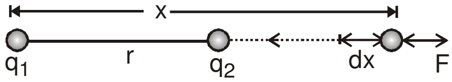


Figure shows two +ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing q_2 from infinity to the given separation from q_1. If can be calculated as

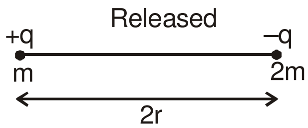
$$W = \int_{\infty}^{r} \vec{F} \cdot \vec{dx} = -\int_{\infty}^{r} \frac{Kq_1q_2}{x^2} dx \qquad \text{[–ve sign shows that x is decreasing]}$$

$$W = \frac{Kq_1q_2}{r} = U \qquad \text{[Interaction energy]}$$

If the two charges here are of opposite sign, the potential energy will be negative as

$$U = -\frac{Kq_1q_2}{r}$$

Ex.37 Find out speed of particles when separation between them is r.



Sol. Energy conservation :

$$0 - \frac{kq_1q_2}{2r} = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2 - \frac{kq_1q_2}{r}$$

Momentum conservation (as E.F is action-reaction pair)

$$mv_1 = 2mv_2 \Rightarrow \qquad v_2 = \frac{v_1}{2}$$

Ex.38 A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons in terms of mass of proton m and its charge e.

Sol. As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach both will move with same velocity. So if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two protons system.

$$mu = mv + mv \text{i.e.,} \qquad v = \frac{1}{2}u$$

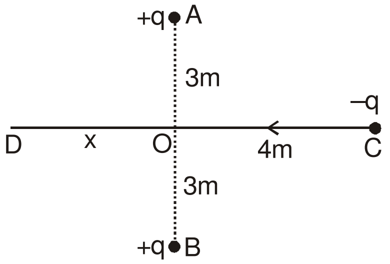
And by conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\Rightarrow \frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \qquad [\text{as } v = \frac{u}{2}]$$

$$\Rightarrow \frac{1}{4}mu^2 = \frac{e^2}{4\pi\varepsilon_0 r} \qquad \Rightarrow \qquad r = \frac{e^2}{\pi m\varepsilon_0 u^2}$$

Ex.39 ***Two fixed equal positive charges, each of magnitude 5×10^{-5} C are located at points A and B, separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.***



Sol. The kinetic energy is lost and converted to electrostatic potential energy of the system as the negative charge goes from C to D and comes to rest at D instantaneously.

Loss of K.E. = Gain in potential energy

$$4 = U_f - U_i$$

or, $$4 = \left[\frac{q.q}{4\pi\varepsilon_0(6)^2} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+x^2}}\right] - \left[\frac{q.q}{4\pi\varepsilon_0(6)^2} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+16}}\right]$$

or, $$4 = \frac{2q^2}{4\pi\varepsilon_0}\left[\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right]$$

or, $$4 = 2 \times (5 \times 10^{-5})^2 \times (9 \times 10^9)\left[\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right]$$

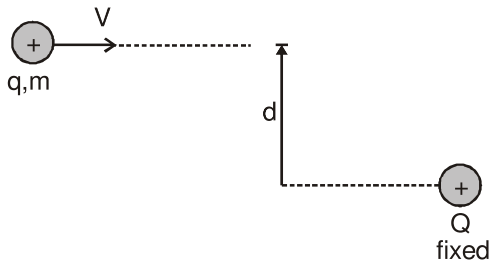
or, $$4 = 9 - \frac{45}{\sqrt{9+x^2}}$$

$\Rightarrow$ $$x = \sqrt{72} = 8.48\,\text{m}$$

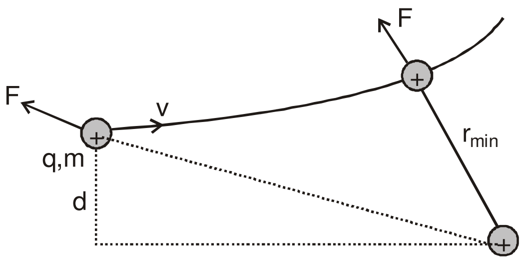
5.1 Motion of a Charge Particle and Angular Momentum Conservation :

We know that a system of particles when no external torque acts, the total angular momentum of system remains conserved. Consider following examples which explains the concept for moving charged particles.

Ex.40 ***Figure shows a charge +Q fixed at a position in space. From a large distance another charge particle of charge +q and mass m is thrown toward +Q with an impact parameter d as shown with speed v. find the distance of closest approach of the two particles.***



Sol.



Here we can see that as +q moves toward +Q, a repulsive force acts on –q radially outward +Q. Here as the line of action of force passes through the fix charge, no torque act on +q relative to the fix point charge +Q, thus here we can say that with respect to +Q, the angular momentum of +q must remain constant. Here we can say that +q will be closest to +Q when it is moving perpendicularly to the line joining the two charges as shown.

If the closest separation in the two charges is r_{min}, from conservation of angular momentum we can write

$$mvd = mv_0\, r_{min} \qquad ...(1)$$

Now from energy conservation, we have

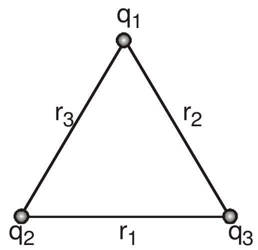
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \frac{KqQ}{r_{min}}$$

Here we use from equation (1) $v_0 = \dfrac{vd}{r_{min}}$

or $$\frac{1}{2}mv^2 = \frac{1}{2}mv^2\frac{d^2}{r_{min}^2} + \frac{KqQ}{r_{min}} \qquad ...(2)$$

Solving equation (2) we'll get the value of r_{min}.

5.2 Potential Energy for a System of charged Particles :



When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energy of all the pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure. The total interaction energy of this system can be given as

$$U = \frac{Kq_1q_2}{r_3} + \frac{Kq_1q_3}{r_2} + \frac{Kq_2q_3}{r_1}$$

- **Derivation for a system of point charges :**

(i) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebric sum of all the works.

Let W_1 = work done in bringing first charge

W_2 = work done in bringing second charge against force due to 1^{st} charge

W_3 = work done in bringing third charge against force due to 1^{st} and 2^{nd} charge.

PE = $W_1 + W_2 + W_3 + ...$ (This will contain $\dfrac{n(n-1)}{2} = {}^nC_2$ terms)

(ii) Method of calculation (to be used in problems)

U = sum of the interaction energies of the charges.

$= (U_{12} + U_{13} + ...+ U_{1n}) + (U_{23} + U_{24} + ...+U_{2n}) + (U_{34} + U_{35} + ...U_{3n})$ ....

(iii) Method of calculation useful for symmetrical point charge systems.

Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$= (U_{12} + U_{13} + \ldots\ldots\ldots + U_{1n})$

U_2 = PE of second charges due to all other charges.

$= (U_{21} + U_{23} + \ldots\ldots\ldots + U_{2n})$

U = PE of the system = $\dfrac{U_1 + U_2 + \ldots..}{2}$

6. ELECTRIC POTENTIAL :

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field, electric potential is defined as the interaction energy of a unit positive charge.

If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as

$$V = \frac{U}{q_0} \text{ joule/coulomb}$$

As potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces."

- **Properties :**

(i) Potential is a scalar quantity, its value may be positive, negative or zero.

(ii) S.I. Unit of potential is volt = $\dfrac{\text{joule}}{\text{coulomb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

(iii) Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e. infinity) to that point.

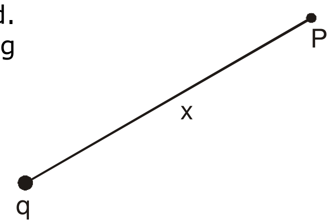
(iv) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinity. (taking $V_\infty = 0$)

(v) Potential decreases in the direction of electric field.

(a) Electric Potential due to a Point Charge in its Surrounding :

We know the region surrounding a charge is electric field. Thus we can also define electric potential in the surrounding of a point charge.

The potential at a point P at a distance x from the charge q can be given as



$$V_p = \frac{U}{q_0}$$

Where U is the potential energy of charge q_0, if placed at point P, which can be given as

$$U = \frac{Kqq_0}{x}$$

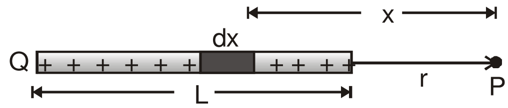
Thus potential at point P is

$$V_P = \frac{Kq}{x}$$

☛ The above result is valid only for electric potential in the surrounding of a point charge. If we wish to find electric potential in the surrounding of a charged extended body, we first find the potential due to an elemental charge dq on body by using the above result and then integrate the expression for the whole body.

(b) Electric Potential due to a Charge Rod :

Figure shows a charged rod of length L, uniformly charged with a charge Q. Due to this we will find electric potential at a point P at a distance r from one end of the rod shown in figure shown.



For this we consider an element of width dx at a distance x from the point P. Charge on this element is

$$dq = \frac{Q}{L}dx$$

The potential dV due to this element at point P can be given by using the result of a point charge as

$$dV = \frac{Kdq}{x} = \frac{KQ}{Lx}dx$$

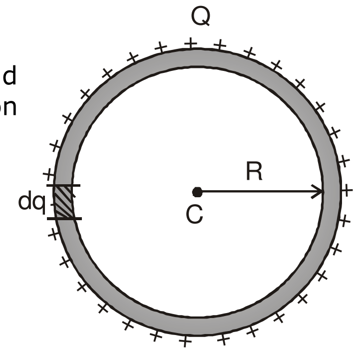
Net electric potential at point P can be given as

$$V = \int dV = \int_r^{r+L} \frac{KQ}{Lx}dx = \frac{KQ}{L}[\ell n\, x]_r^{r+L} = \frac{KQ}{L}\ell n\left(\frac{r+L}{r}\right)$$

(c) Electric Potential due to a Charged Ring :

Case I : At its centre

To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as



$$dV = \frac{Kdq}{R}$$

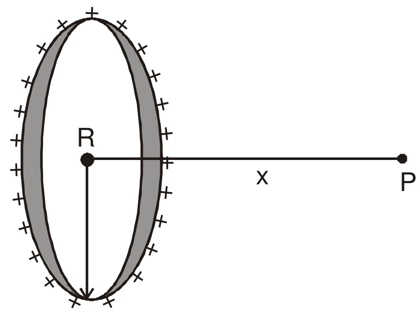
Total potential at C is $\quad V = \int dV$

$$= \int \frac{Kdq}{R} = \frac{KQ}{R}$$

As all dq's of the ring are situated at same distance R from the ring centre C, simply the potential due to all is added as being a scalar quantity, we can directly say that the electric potential at ring centre is $\frac{KQ}{R}$. Here we can also state that even if charge Q is non-uniformly distributed on ring, the electric potential at C will remain same.

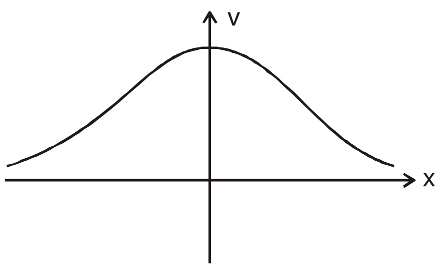
Case II : At a Point on Axis of Ring

If we wish to find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2+R^2}$ from the point P, thus the potential at P can be given as



$$V_P = \frac{KQ}{\sqrt{R^2+x^2}}$$

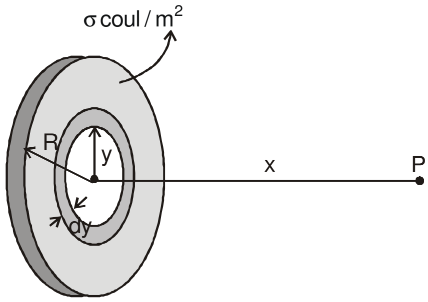
GRAPH



(d) Electric Potential due to a Uniformly Charged Disc :

Figure shows a uniformly charged disc of radius R with surface charge density σ coul/m². To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is

$$dq = \sigma.\ 2\pi y\ dy$$



Due to this ring, the electric potential at point P can be given as

$$dV = \frac{Kdq}{\sqrt{x^2+y^2}} = \frac{K.\sigma.2\pi y\,dy}{\sqrt{x^2+y^2}}$$

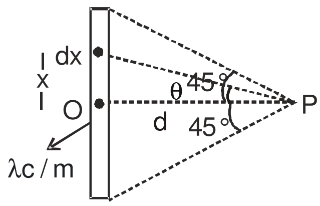
Net electric potential at point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\epsilon_0}.\frac{ydy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R$$

$$V_P = \frac{\sigma}{2\epsilon_0}[\sqrt{x^2+R^2} - x]$$

Ex.41 ***Consider the following rod & find the potential due to it at P***

$OP = d, \qquad x = d\tan\theta,\ dx = d\sec^2\theta\, d\theta$



$$dV = \frac{k\lambda dx}{d\sec\theta} \Rightarrow \int dV = \int_{-\pi/4}^{+\pi/4} \frac{kd\sec^2 d\theta\lambda}{d\sec\theta}$$

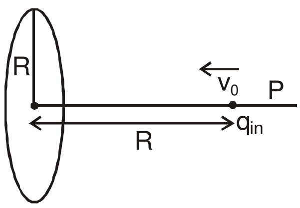
$$V = k\lambda \int_{-\pi/4}^{\pi/4} \sec\theta\, d\theta$$

$$V = k\lambda\ [l\,n(\sec\theta + \tan\theta)]_{-\pi/4}^{\pi/4}$$

$$V = k\lambda\ [ln\ (\sqrt{2}\ + 1)] - k\lambda\ [ln\ (\sqrt{2}\ - 1)]$$

$$V = k\lambda\, l\,n\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = k\lambda\ \ln(\sqrt{2}+1)^2$$

Ex.42



Find min velocity v_0 such that particle cross the ring.

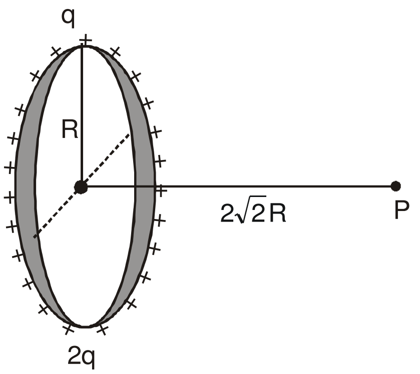
Sol. Potential at P = $\frac{kQ}{\sqrt{2}R}$

Applying energy conservation $\frac{1}{2}mv_0^2 + \frac{kQq}{\sqrt{2}R} = 0 + \frac{kqQ}{R}$

$$\Rightarrow \quad v_0 = \sqrt{\frac{2kQq}{mR}\left(1 - \frac{1}{\sqrt{2}}\right)}$$

Ex.43 ***A ring of radius R is having two charges q and 2q distributed on its two half parts. Find the electric potential at a point on its axis at a distance*** $2\sqrt{2}R$ ***from its centre.***

Sol.



Distance of P from periphery of ring is $\sqrt{R^2+(2\sqrt{2}R)^2} = 3R$

Electric potential = Potential due to upper half + Potential due to lower half

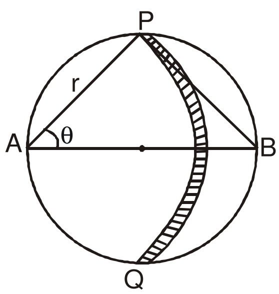
$$= \frac{Kq}{3R} + \frac{2Kq}{3R} \Rightarrow \frac{3Kq}{3R} = \frac{Kq}{R}$$

(e) Electric potential due to a closed disc at a point on the edge

Let us calculate the potential at the edge of a thin disc of radius 'R' carrying a uniformly distributed charge with surface density σ.

Let AB be a diameter and A be a point where the potential is to be calculated. From A as centre, we draw two arcs of radii r and r + dr as shown. The infinitesimal region between these two arcs is an element whose area is dA = $(2r\theta)$ dr, where 2θ is the angle subtended by this element PQ at the point A. Potential at A due to the element PQ is

$$dV = \frac{\sigma dA}{4\pi\varepsilon_0 r} = \frac{2\sigma r\theta dr}{4\pi\varepsilon_0 r} = \frac{2\sigma\theta dr}{4\pi\varepsilon_0}$$



From Δ APB, we have

$r = 2R\cos\theta$

or, $dr = -2R\sin\theta\, d\theta$

Hence

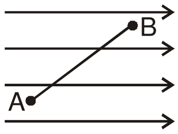
$$dV = \frac{-4\sigma\theta R\sin\theta d\theta}{4\pi\varepsilon_0}$$

$$V = -\int_{\pi/2}^{0} \frac{\sigma R\theta\sin\theta}{\pi\varepsilon_0} d\theta$$

$$= -\frac{\sigma R}{\pi\varepsilon_0} \left| -\theta\cos\theta + \sin\theta \right|_{\pi/2}^{0} = \frac{\sigma R}{\pi\varepsilon_0} \qquad ....(19C)$$

7. RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL :

(a) For uniform electric field :



(i) Potential difference between two points A and B

$V_B - V_A = -\vec{E}.\vec{AB}$

(b) Non uniform electric field

(i) $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

$\Rightarrow$ $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$= -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial y}V + \hat{k}\frac{\partial}{\partial z}V\right]$

$= -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V = -\nabla V = -\text{grad } V$

Where $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\frac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

(c) If electric potential and electric field depends only on one coordinate, say r :

(i) $\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$

where $\hat{r}$ is a unit vector along increasing r.

(ii) $\int dV = -\int \vec{E}.\vec{dr}$

$\Rightarrow$ $V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.\vec{dr}$

$\vec{dr}$ is along the increasing direction of r.

(iii) The potential of a point

$V = -\int_{\infty}^{r} \vec{E}.\vec{dr}$

- Area under E - x curve gives negative of change in potential.
- Negative of slope of V - x curve gives the electric field at that point.

Ex.44 ***V = x² + y, Find $\vec{E}$.***

Sol. $\frac{\partial V}{\partial x} = 2x$, $\frac{\partial V}{\partial y} = 1$ and $\frac{\partial V}{\partial z} = 0$

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) = -(2x\hat{i} + \hat{j})$$

Electric field is nonuniform.

Ex.45 ***For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$ find the potential at (x, y) if V at origin is 5 volts.***

Sol. $\int_5^V dV = -\int \vec{E}.\vec{dr} = -\int_0^x E_x dx - \int_0^y E_y dy$

$\Rightarrow \quad V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2} \quad \Rightarrow V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5$

Ex.46 ***The electric potential in a region is represented as V = 2x + 3y – z. Obtain expression for the electric field strength.***

Sol. We know

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Here, $\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}[2x + 3y - z] = 2$

$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}[2x + 3y - z] = 3$

$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}[2x + 3y - z] = -1$

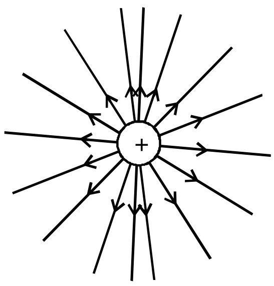
$\therefore \quad \vec{E} = -(2\hat{i} + 3\hat{j} - \hat{k})$

8. ELECTRIC LINES OF FORCE

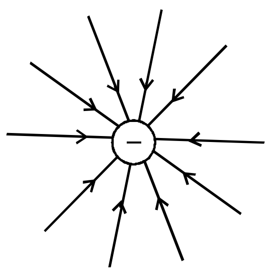
The idea of electric lines of force or the electric field lines introduced by Michael Faraday is a way to visualize electrostatic field geometrically.

The properties of electric lines of force are the following :

(i) The electric lines of force are continous curves in an electric field starting from a positively charged body and ending on a negatively charged body.



Electric lines of force due to positive charge

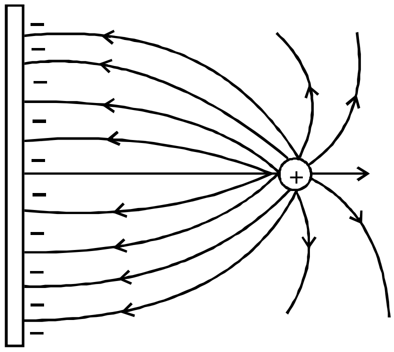


Electric lines of force due to negative charge

(ii) The tangent to the curve at any point gives the direction of the electric field intensity at that point.

(iii) Electric lines of force never intersect since if they cross at a point, electric field intensity at the point will have two directions, which is not possible.

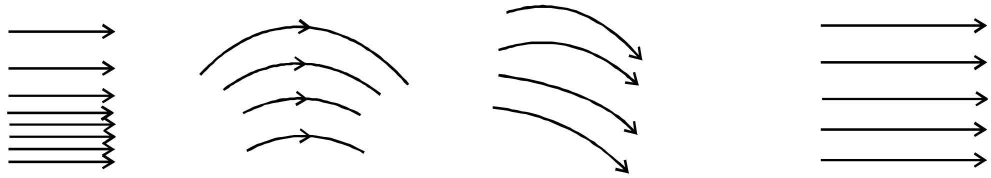
(iv) Electric lines of force do not pass but leave or end on a charged conductor normally. Suppose the lines of force are not perpendicular to the conductor surface. In this situation, the component of electric field parallel to the surface would cause the electrons to move and hence conductor will not remain equipotential which is an absurd as in electrostatics conductor is an equipotential surface.



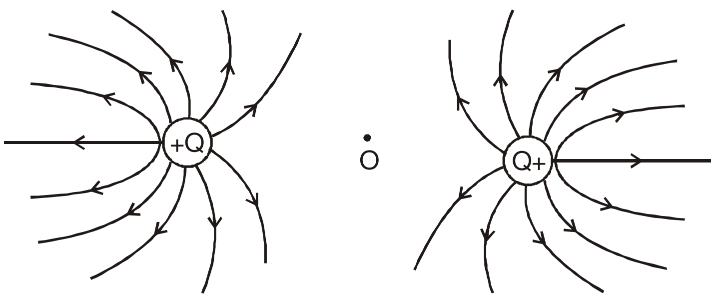
Fixed point charge near infinite metal plate

(v) The number of electric lines of force that originate from or terminate on a charge is proportional to the magnitude of the charge.

(vi) As number of lines of force per unit area normal to the area at point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field. Further, if the lines of force are equidistant straight lines, the field is uniform



Magnitude is not constant | Direction is not constant | Both magnitude and direction not constant | Both magnitude and direction constant



Electric lines of force due to two equal positive charges (field is zero at O). O is a null point

☛ A charge particle need not follow an ELOF.

☛ Electric lines of force produced by static charges do not form close loop.

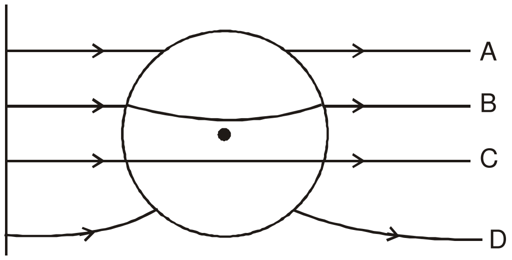
Ex.47 ***If number of electric lines of force from charge q are 10 then find out number of electric lines of force from 2q charge.***

Sol. No. of ELOF $\propto$ charge

$$\frac{q'}{q} = \frac{N'}{10} \Rightarrow N' = \frac{2q}{q} \times 10 = 20$$

So number of ELOF will be 20.

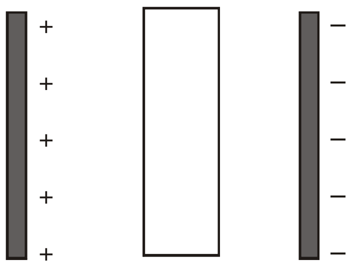
Ex.48 ***A solid metallic sphere is placed in a uniform electric field. Which of the lines A, B, C and D shows the correct representation of lines of force and why ?***



Sol. **(D)**

The line (A) is wrong as lines of force start or end normally on the surface of a conductor and here it is not so. Line (B) and (C) are wrong as lines of force does not exist inside a conductor and here it is not so. Also lines of force are not normal to the surface of the conductor. Line (D) represents the correct situation, as here line of force does not exist inside the conductor and start and end normally on its surface.

Ex.49 ***A metallic slab is introduced between the two charged parallel plates as shown below. Sketch the electric lines of force between the plates.***

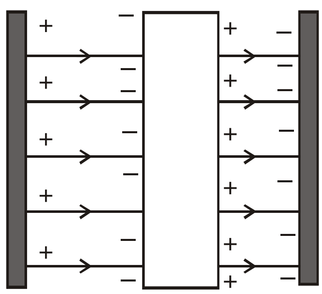


Sol. Keeping in mind that

(i) Electric lines of force start from positive charge and end on negative charge.

(ii) Electric lines of force start and end normally on the surface of a conductor.

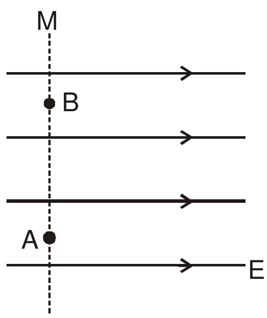
(iii) Electric lines of force do not exist inside a conductor, the lines of force are shown in the adjacent figure.



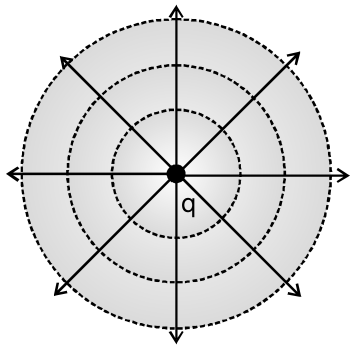
9. EQUIPOTENTIAL SURFACES :

As shown in figure if a charge is shifted from a point A to B on a surface. M which is perpendicular to the direction of electric field, the work done in shifting will obviously, be zero as electric force is normal to the direction of displacement.

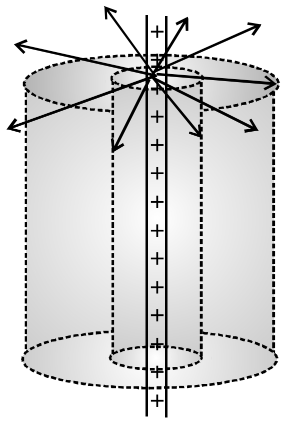
As no work is done in moving from A to B, we can say that A and B are at same potetials or we can say that all the points of surface M are at same potential or here we call surface M as equipotential surface.



Following figures show equipotential surfaces in the surrounding of point charge and a long charged wire



Point charge
Spherical equipotential surfaces

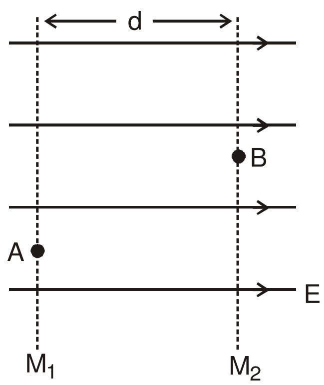


Line charge
Cylindrical equipotential surfaces

Every surface in electric field in which at every point direction of electric field is normal to the surface can be regarded as equipotential surface.

Figure shows two equipotential surfaces in a uniform electric field E. If we wish to find the potential difference between two points A and B shown in figure, we simply find the potential difference between the two equipotential surfaces on which the points lie, given as

$$V_A - V_B = Ed$$



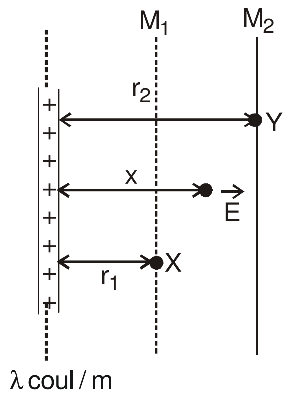


Figure shows a line charge with linear charge density λ coul/m. Here we wish to find potential difference between two points X and Y which lie on equipotential surfaces M_1 & M_2. To find the potential difference between these surfaces, we consider a point P at a distance x from wire as shown. The electric field at point P is

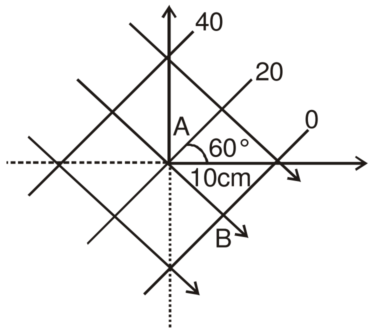
$$E = \frac{2K\lambda}{x}$$

Now the potential difference between surface M_1 and M_2 can be given as

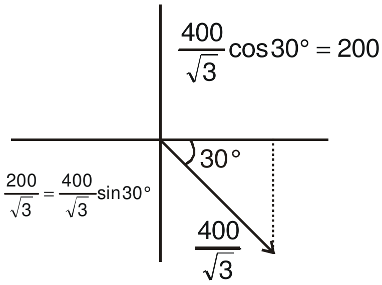
$$V_x - V_y = \int_{r_1}^{r_2} E dx = \int_{r_1}^{r_2} \frac{2K\lambda}{x} dx$$

$$V_X - V_Y = 2K\lambda \, \ell n\left(\frac{r_2}{r_1}\right)$$

Ex.50



Write down the Electric field in vector form ?

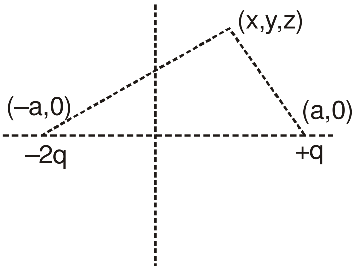


Sol. $v_A - v_B = E \times 0.1 \times \cos 30°$

$$20 = E \times 0.1 \times \frac{\sqrt{3}}{2} \Rightarrow E = \frac{400}{\sqrt{3}}$$

$$\text{E.F.} = 200\hat{i} - \frac{200}{\sqrt{3}}\hat{j}$$

Ex.51 Find out equipotential surface where potential is zero ?



Sol. $$\frac{-2kq}{\sqrt{(x+a)^2+y^2+z^2}} + \frac{kq}{\sqrt{(x-a)^2+y^2+z^2}} = 0 \Rightarrow \frac{kq}{\sqrt{(x-a)^2+y^2+z^2}} = \frac{2kq}{\sqrt{(x+a)^2+y^2+z^2}}$$

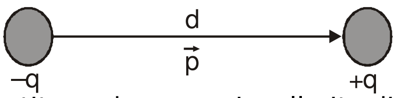
squaring both sides

$$\frac{1}{\sqrt{(x-a)^2+y^2+z^2}} = \frac{4}{\sqrt{(x+a)^2+y^2+z^2}}$$

10. ELECTRIC DIPOLE :

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges is given as.

$$\vec{p} = q\,\vec{d}$$

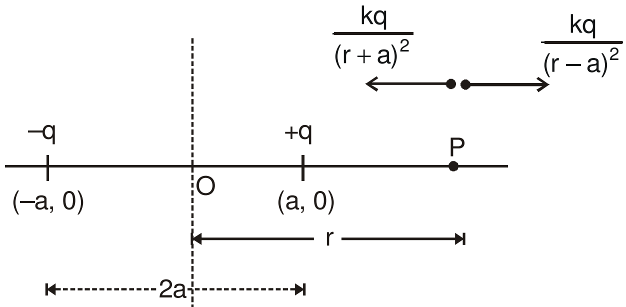


Dipole moment is a vector quantity and convensionally its direction is given from negative pole to positive pole.

(a) Electric field due to a Dipole

(1) At an axial point

Figure shows an electric dipole placed on x-axis at origin



Here we wish to find the electric field at point P having coordinates (r, o) (where r >> 2a). Due to positive charge of dipole electric field at P is in outward direction & due to negative charge it is in inward direction.

E_{net} at P = $\frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2} = \frac{4kqar}{(r^2-a^2)^2}$

As $\vec{P} = 2aq$

$\therefore$ E_{net} at P = $\frac{2kpr}{(r^2-a^2)^2}$

As r >> 2a

$\therefore$ we can neglect a w.r.t. r

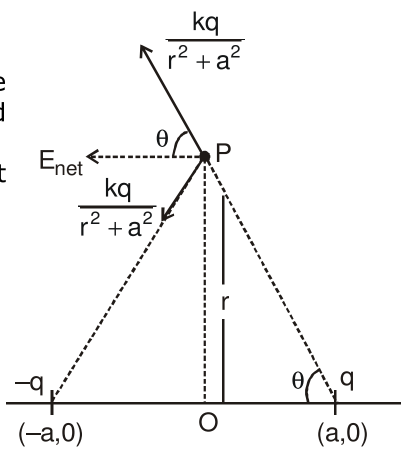
E_{net} at P = $\frac{2kp}{r^3}$

As we can observe that for axial point direction of field is in direction of dipole moment

$\therefore$ Vectorially, $\vec{E} = \frac{2k\vec{p}}{r^3}$

(2) At an equatorial point.

Again we consider the dipole placed along the x-axis & we wish to find, electric field at point P which is situated equatorially at a distance r (where r >> 2a) from origin. Vertical component of the electric field vectors cancel out each other.



$\therefore$ E_{net} at P = 2 E cos θ [where E = $\frac{kq}{r^2+a^2}$]

E_{net} at P = $\frac{2kq}{r^2+a^2} \cdot \frac{a}{\sqrt{r^2+a^2}}$ $\left[\because \cos\theta = \frac{a}{\sqrt{r^2+a^2}}\right]$

$E_{net} = \frac{2kqa}{(r^2+a^2)^{3/2}} = \frac{kp}{(r^2+a^2)^{3/2}}$ (As p = 2aq)

As we have already stated that r > > 2a

$\therefore$ E_{net} at P = $\frac{kp}{r^3}$

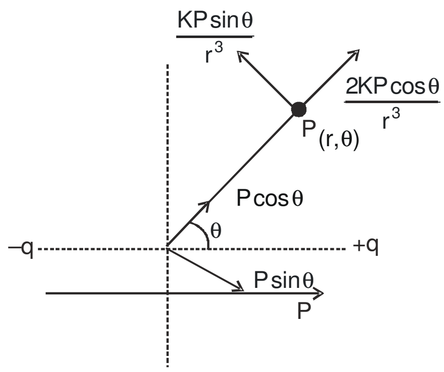
We can observe that the direction of dipole moment & electric field due to dipole at P are in opposite direction.

$\therefore$ Vectorially

$\vec{E} = \frac{-k\vec{P}}{r^3}$

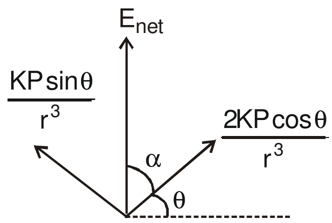
(b) Electric field at a general Point due to a dipole

Figure shows a electric dipole place on x-axis at origin & we wish to find out the electric field at point P with coordinate (r, θ)



E_{net} at

$$E_{net} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2} = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$$



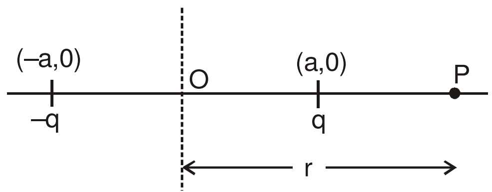
$$\tan\alpha = \frac{\frac{kp\sin\theta}{r^3}}{\frac{2kp\cos\theta}{r^3}}$$

$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$

(c) Electric potential due to a dipole.

1. At an axial point



(r >> 2a)

We wish to find out potential at P due to dipole (with p = 2aq)

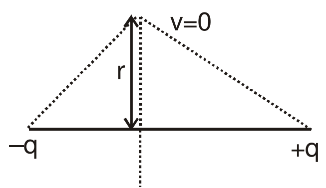
$$V_{net} = \frac{kq}{(r-a)} - \frac{kq}{(r+a)}$$

$$V_{net} = \frac{2akq}{(r^2 - a^2)}$$

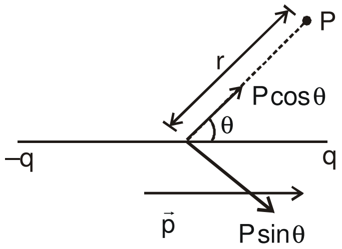
$$V_{net} = \frac{kp}{r^2} \text{ (As P = 2aq)}$$

2. At a point on perpendicular bisector

At an equatorial point, electric potential due to dipole is always zero because potential due to +ve charge is cancelled by –ve charge.



(3) Potential due to dipole at a general Point



Potential at P due to dipole = $\dfrac{kp\cos\theta}{r^2}$

• **BASIC TORQUE CONCEPT**

$\vec{\tau} = \vec{r} \times \vec{F}$

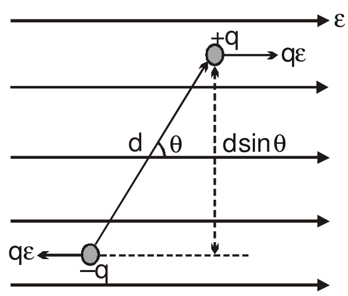
⇒ If the net transational force on the body is zero then the torque of the forces may or may not be zero but net torque of the forces about each point of universe is same

⇒ If we have to prove that a body is in equilibrium then first we will prove $\vec{F}_{net}$ is equal to zero & after that we will show τ_{net} about any point is equal to zero.

⇒ If the body is free to rotate then it will rotate about the axis passing through centre of mass & parallel to torque vector direction & of the body is hinged then it will rotate about hinged axis.

11. DIPOLE IN UNIFORM ELECTRIC FIELD :

Figure shows a dipole of dipole moment p placed at an angle θ to the direction of electric field. Here the charges of dipole experience forces qε in opposite direction as shown.



thus we can state that when a dipole is placed in a uniform electric field, net forces on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force × separation between lines of action of forces
= $q\varepsilon \times d\sin\theta$
= $p\varepsilon\sin\theta$

or vectorially we can write the torque on dipole is

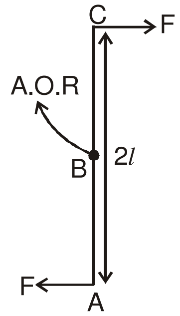
$\vec{\tau} = \vec{p} \times \vec{\varepsilon}$

Ex.52 ***Write down torque about A, B, C***

Sol. $\tau_A = 2Fl \otimes$

$\tau_C = 2Fl \otimes$

$\tau_B = Fl + Fl = 2Fl \otimes$

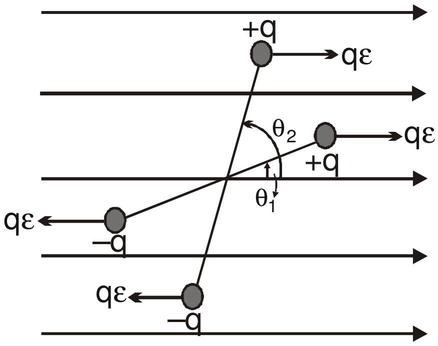


11.1 Potential Energy of a Dipole in Uniform Electric Field

When a dipole in an electric field at an angle θ, the torque on it due to electric field is

$$\tau = p\varepsilon \sin\theta$$

In the figure shown, the torque is in clockwise direction. If we rotate the dipole in anticlockwise direction from an angle θ_1 and θ_2 slowly, we have to apply an anticlockwise equal torque, then the work done in process will be given as



$$W = \int dW = \int \tau d\theta$$

$$= \int_{\theta_1}^{\theta_2} p\varepsilon \sin\theta d\theta$$

$$= p\varepsilon[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= p\varepsilon(\cos\theta_1 - \cos\theta_2)$$

$$W_{\text{external foce}} = -W_{\text{electricforce}}$$

$\therefore$ $W_{\text{electric field}} = -\,pE(\cos\theta_1 - \cos\theta_2)$

As we know that for conservative forces

$$W.D. = -\Delta U$$

$$\Delta U = pE(\cos\theta_1 - \cos\theta_2)$$

$$U_{\theta_2} - U_{\theta_1} = (-pE\cos\theta_2 - pE\cos\theta_1)$$

$\therefore$ We can generalise that

$$U_\theta = -\,pE\cos\theta$$

In vector notation we can write potential energy of dipole in electric field is

$$U = -\vec{p}.\vec{E}$$

[where potential energy at θ = 90° = 0]

11.2 Stable and Unstable equilibrium of a Dipole in Electric Field :

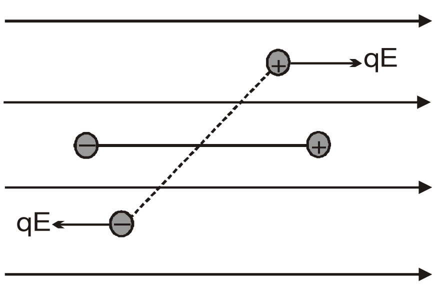
We've discussed that when a dipole in an electric field E, the potential energy of dipole can be given as

$$U = -p\varepsilon\cos\theta$$

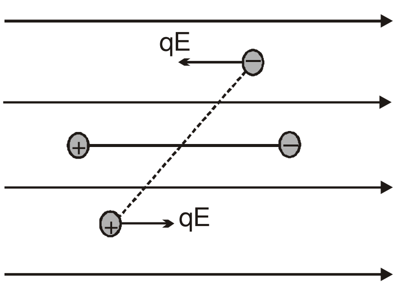
We also know that the net torque on a dipole in electric field can be given as

$$\tau = p\varepsilon\sin\theta$$

It shows that net torque on dipole in electric field is zero in two situations when $\theta = 0^\circ$ and $\theta = 180^\circ$ as shown in figure



(a)



(b)

We can see that when $\theta = 0$ as shown in figure(a) when torque on dipole is zero, the dipole is in equilibrium. We can verify that here equilibrium is stable. If we slightly tilt the dipole from its equilibrium position in anticlockwise direction as shown by dotted position. The dipole experiences a clockwise torque which tend the dipole to rotate back to its equilibrium position. This shows that at $\theta = 0$, dipole is in stable equilibrium. We can also find the potential energy of dipole at $\theta = 0$, it can by given as

$$U = -p\varepsilon \text{ (minimum)}$$

Here at $\theta = 0$, potential energy of dipole in electric field is minimum which favours the position of stable equilibrium.

Similarly when θ = 180º, net torque on dipole is zero and potential energy of dipole in this state is given as

$$U = p\varepsilon \text{ (maximum)}$$

Thus at θ = 180º, dipole is in unstable equilibrium. This can also be shown by figure(b). From equilibrium position if dipole is slightly displaced in anticlockwise direction, we can see that torque on dipole also acts in anticlockise direction away from equilirbium position. Thus here dipole is in unstable equilibrium.

11.3 Angular SHM or Dipole

When a dipole is suspended in a uniform electric field, it will align itself parallel to the field.

Now if it is given a small angular displacement θ about its equilibrium, the (restoring) couple will be

$$C = -pE\sin\theta$$

or, $$C = -pE\theta \text{ [as } \sin\theta \approx \theta\text{, for small } \theta]$$

or, $$I\frac{d^2\theta}{dt^2} = -pE\theta$$

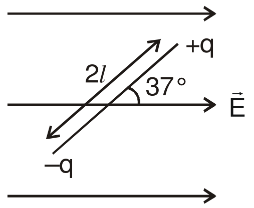
or, $$\frac{d^2\theta}{dt^2} = -\frac{pE}{I}\theta$$

or, $$\frac{d^2\theta}{dt^2} = -\omega^2\theta \qquad \text{where } \omega^2 = \frac{pE}{I}$$

This is standard equation of angular simple harmonic motion with time-period $T\left(=\frac{2\pi}{\omega}\right)$. So the dipole will execute angular SHM with time-period

$$T = 2\pi\sqrt{\frac{I}{pE}} \qquad ...(33)$$

Ex.53



Find out the angular frequency of the dipole when it crosses the mean position.

Sol. $0 - PE\cos 37° = \frac{1}{2}I\omega^2$

$$\frac{1}{2}I\omega^2 = \frac{PE}{5}$$

$$\frac{2ml^2}{2}.\omega^2 = \frac{2ql.E}{5} \Rightarrow \omega = \sqrt{\frac{2qE}{5ml}}$$

11.4 Force on an Electric Dipole in Non-uniform Electric Field :

If in a non-uniform electric field dipole is placed at a point where electric field is ε, the interaction energy of dipole at this point can be given as

$$U = -\vec{p}.\vec{\varepsilon}$$

Now the force on dipole due to electric field can be given as

$$F = -\Delta U$$

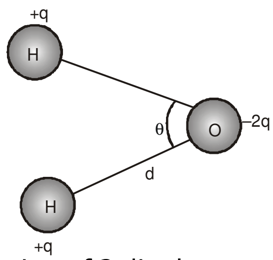
For unidirectional variation in electric field, we have

$$F = -\frac{d}{dx}(\vec{p}.\vec{\varepsilon})$$

If dipole is placed in the direction of electric field, we have

$$F = -p\frac{d\varepsilon}{dx}$$

Ex.54 ***A water molecule is placed at a distance ℓ from the line carrying linear charge density λ. Find the maximum force exerted on the water molecule. The shape of water molecule and the partial charges on H and O atoms as shown in figure.***



Sol. The figure can be resolved as combination of 2 dipoles.

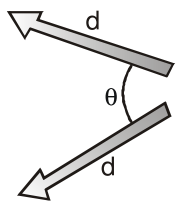
Dipole moments of each $p = qd$.

Here that dipole moment of system is

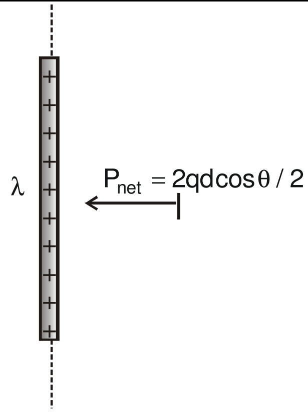
$$P_{net} = 2\,qd\cos\theta/2$$

Now

$$\vec{F} = \vec{P}_{net}.\frac{d\vec{\varepsilon}}{dx}$$



For maximum force, the angle between $\vec{P}_{net}$ and $\frac{d\vec{\varepsilon}}{dx}$ is 0º



$$F_{max} = 2qd\cos\frac{\theta}{2} \times \frac{d}{dx}\left(\frac{2k\lambda}{x}\right)$$

or $$F_{max} = 2qd\cos\frac{\theta}{2} \times 2k\lambda\left(-\frac{1}{x^2}\right)$$

$$= \frac{-4Kqd\lambda\cos\theta/2}{x^2}$$

$$|\vec{F}_{max}| = \frac{4Kqd\lambda\cos\theta/2}{x^2}$$

$$\left|\vec{F}_{max}\right| = \frac{4Kqd\lambda\cos\theta/2}{\ell^2}$$