



### ELECTROSTATICS - 2

THEORY AND EXERCISE BOOKLET

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#### JEE SYLLABUS :

Flux of electric field; Gauss's law and its application in simple cases, such as, to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.



#### 1. ELECTRIC FLUX :

Any group of electric lines of forces passing through a given surface, we call electric flux and it is denoted by  $\phi.$ 

#### • Area as a Vector :

Till now we have considered area of a surface as a scalar quantity but for further analysis we treat area of a surface as a vector quantity whose direction is along the normal to the surface. The area vector  $\vec{S}$  of a surface which has surface area S can be written as

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Where  $\,\hat{n}\,$  is the unit vector in the direction along normal to the surface.

If a surface is three dimensional we consider a small elemental area dS on this surface and direction of this elemental area vector is along the local normal of the surface at the point where elemental area is chosen as shown. Thus

 $\vec{dS} = dS\hat{a}$ 

Here  $\hat{a}$  is the unit vector in the direction along the normal at elemental area dS.

#### Electric Field Strength in Terms of Electric Flux :

Earlier we've defined that the density of electric lines gives the magnitude of electric field strength. Mathematically the numerical value of electric field strength at a point in the region of electric field can be give as the electric flux passing through a unit normal area at that point.

Flux = 
$$\phi = \int \vec{E} \cdot \vec{dA}$$
, If  $\vec{E}$  is constant,  $\phi = \vec{E} \cdot \vec{A}$ 

In a uniform electric field shown in figure. If  $\phi$  be the flux passing through an area S which is normal to the electric field lines, the value of electric field strength at this surface can be given as

$$E = \frac{\phi}{S}$$

or flux through the surface can be given as

 $\phi = \mathsf{ES}$ 

If in an electric field, surface is not normal as shown in figure. Here the are ABCD is inclined at an angle  $\theta$  from the normal to electric field. Here we resolve the area ABCD in two perpendicular components as shown in figure. One is S cos  $\theta$ , which is area ABC'D' normal to electric field direction and other is S sin  $\theta$ , which is area CDC'D'

along the direction of electric field.











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Here the total flux passing through the given area ABCD is same which is passing through its normal component S cos  $\theta$ , thus here the flux  $\phi$  through the area can be given as

 $\phi = ES \cos \theta$  [S cos  $\theta$  = area of ABC'D']

If we consider the direction of area vector normal to the area surface, as shown in figure,  $\theta$  would be the angle between  $\vec{s}$  and

 $\vec{E}$ . Thus flux through the surface ABCD can be given as

 $\phi = \vec{E} \cdot \vec{S}$ 

#### (a) Electric Flux in Non-uniform Electric Field :

In non-uniform electric field, we can calculate electric flux through a given surface by integrating the above expression for elemental surface area of the given surface.

For this consider the situation shown in figure. If we wish to calculate electric flux through the surface M shown in figure.

For this we consider an elemental area dS on the surface M as shown. At this position if electric field is E then the electric flux through this elemental area dS can be given as

 $d\phi = EdS \cos \theta$ 

Total flux through the surface M can be given as

$$\phi = \int d\phi = \int_{M} EdS \cos \theta$$

#### (b) Electric Flux Through a circular Disc :

TION

Figure shows a point charge q placed at a distance  $\ell$  from a disc of radius R. Here we wish to find the electric flux through the disc surface due to the point charge q. We know a point charge q originates electric flux in radially outward direction. The flux of q which is originated in cone shown in figure passes through the disc surface.



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To calculate this flux, we consider an elemental ring on disc surface of radius x and width dx as shown. Area of this ring (strip) is

 $dS = 2\pi x dx$ 

The electric field due to q at this elemental ring is given as

$$\mathsf{E} = \frac{\mathsf{K}\mathsf{q}}{(\mathsf{x}^2 + \ell^2)}$$

If  $d\phi$  is the flux passing through this elemental ring, we have

 $d\phi = EdS\cos\theta$ 

$$= \frac{Kq}{(x^{2} + \ell^{2})} \times 2\pi x \, dx \times \frac{\ell}{\sqrt{x^{2} + \ell^{2}}} = \frac{2\pi kq \ell \, x dx}{(\ell^{2} + x^{2})^{3/2}}$$

Total flux through the disc surface can be given by integrating this expression over the whole area of disc thus total flux can be given as

$$\begin{split} \phi &= \int d\phi = \int_{0}^{R} \frac{q\ell}{2 \,\epsilon_{0}} \, \frac{x \, dx}{(\ell^{2} + x^{2})^{3/2}} \\ &= \frac{q\ell}{2 \,\epsilon_{0}} \, \int_{0}^{R} \frac{x \, dx}{(\ell^{2} + x^{2})^{3/2}} \, = \frac{q\ell}{2 \,\epsilon_{0}} \, \left[ -\frac{1}{\sqrt{\ell^{2} + x^{2}}} \right]_{0}^{R} \, = \frac{q\ell}{2 \,\epsilon_{0}} \, \left[ \frac{1}{\ell} - \frac{1}{\sqrt{\ell^{2} + R^{2}}} \right]_{0}^{R} \end{split}$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's Law, shortly we'll discuss it.

#### (c) Electric Flux Through the Lateral Surface of a Cylinder due to a Point Charge :

Figure. shows a cylindrical surface of length L and radius R. On its axis at its centre a point charge q is placed. Here we wish to find the flux coming out from the lateral surface of this cylinder due to the point charge q.

For this we consider an elemental strip of width dx on the surface of cylinder as shown. The area of this strip is

 $dS = 2\pi R \cdot dx$ 

The electric field due to the point charge on the strip can be given as

$$\mathsf{E} = \frac{\mathsf{K}\mathsf{q}}{(\mathsf{x}^2 + \mathsf{R}^2)}$$

If  $d\phi$  is the electric flux through the strip, we can write

$$d\phi = E dS \cos \theta$$

$$=\frac{Kq}{(x^2+R^2)}\times 2\pi Rdx\times \frac{R}{\sqrt{x^2+R^2}}$$

$$= 2\pi \ \text{KqR}^2 \times \frac{dx}{(x^2 + R^2)^{3/2}}$$



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Total flux through the lateral surface of cylinder can be given by integrating the above result for the complete lateral surface, which can be given as

$$\phi = \int d\phi = \frac{qR^2}{2\epsilon_0} \int_{-L/2}^{+L/2} \frac{dx}{(x^2 + R^2)^{3/2}}$$
$$\phi = \frac{q}{\epsilon_0} \cdot \frac{\ell}{\sqrt{\ell^2 + 4R^2}}$$

or

The solution of above integration is left for students as exercise. This situation can also be easily handled by using the concepts of Gauss's Law, we'll discuss in next section.



The figure. shows a point charge placed at the centre of a spherical surface of radius R from which electric lines are originated and coming out of the surface of sphere. For clarity and convenience only lower half of sphere is drawn in the picture. As the charge q is inside the sphere, whatever flux it originates will come out from the spherical surface. To find the total flux, we consider an elemental area dS on surface. The electric field on the points on surface of sphere can be given as

$$\mathsf{E} = \frac{\mathsf{K}\mathsf{q}}{\mathsf{R}^2}$$

The electric flux coming out from the surface dS is

$$d\phi = \vec{E} \cdot dS = EdS$$

Thus

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Total flux coming out from the spherical surface is

 $d\phi = \frac{Kq}{R^2}dS$ 

$$\phi = \int d\phi = \int \frac{Kq}{R^2} dS$$

At every point of spherical surface, magnitude of electric field remains same hence we have

[As  $\theta = 0$  shown in figure]

 $[As \int dS = 4\pi R^2]$ 

$$\phi = \frac{Kq}{R^2} \int dS$$
$$\phi = \frac{Kq}{R^2} \times 4\pi R^2$$
$$\phi = \frac{q}{R^2}$$

or

$$\phi = \frac{q}{\in_0}$$

Thus total flux, the charge q originates is  $\frac{q}{\epsilon_0}$ . Similarly a

charge –q absorbs  $\frac{q}{\epsilon_0}$  electric lines (flux) into it.

Figure shows a charge q enclosed in a closed surface S of random shape. Here we can say that the total electric flux emerging out from the surface S is the complete flux which charge q is originating, hence flux emerging from surface is

$$\phi_{\mathsf{S}} = \frac{\mathsf{q}}{\mathsf{e}_0}$$

The above result is independent of the shape of surface it only depends on the amount of charge enclosed by the surface.

(e) Flux Calculation in the Region of Varying Electric field : In a region electric field depends on x dierection as

In the cube of edge a shown in figure from front face electric flux goes in which can be given as

$$\begin{split} \varphi_{in} &= E_0 \; (2a)^2 \; . \; a^2 = 4E_0 a^4 \\ \text{From the other surface flux coming out can be given as} \\ \varphi_{out} &= E_0 (3a)^2 \; . \; a^2 \; = 9 \; E_0 a^4 \end{split}$$

Here  $\phi_{out} > \phi_{in}$  for the cubical surface hence net flux  $= \phi_{out} - \phi_{in} = 5E_0a^4$ 





#### 2. CONCEPT OF SOLID ANGLE :

Solid angle is the three dimensional angle enclosed by the lateral surface of a cone at its vertex as shown in figure shown. Solid angle can also be defined as the three dimensional angle subtended by a spherical section at its centre of curvature. As in the figure shown point a is the centre of curvature of a "photoecone" of a cone at the three dimensional angle and the figure shown point a state three dimensional angle of a cone at its centre of curvature of a "photoecone" of the three dimensional angle at point A.

#### (a) Relation in Half Angle of cone and Solid Angle at Vertex :

Consider a spherical section M of radius R, which subtend a half angle  $\phi$  (radian) at the centre of curvature. To find the area of this section, we consider an elemental strop on this section of radius R sin  $\theta$  and angular width d $\theta$  as shown in figure. The surface area of this strip can be given as

 $dS = 2\pi R \sin \theta \times R d\theta$ 

The total area of spherical section can be given by integrating the area of this elemental strip within limits from O to  $\phi.$ 

Total area of spherical section is

$$S = \int dS = \int_{0}^{\theta} 2\pi R^2 \sin \theta d\theta$$

$$= 2\pi R^2 \left[ -\cos \theta \right]_{\alpha}^{\phi}$$

 $= 2\pi R^2 (1 - \cos \phi)$ 

If solid angle subtended by this section at its centre O is  $\Omega$  then its area can be given as S =  $\Omega R^2$ 

$$S = \Omega R$$

From equation (1) we have

$$\Omega R^2 = 2\pi R^2 \left(1 - \cos\phi\right)$$

$$\Omega = 2\pi \left(1 - \cos\phi\right)$$

Equation (2) gives the relation in half angle of a cone  $\phi$  and the solid angle enclosed by the lateral surface of cone at its vertex.

#### (b) Electric Flux Calculation due to a Point Charge Using solid Angle :

Figure shows a point charge q placed at a distance  $\ell$  from the centre of a circular disc of radius R. Now we wish to find the electric flux passing through the disc surface due to the charge q.



We know from a point charge q, total flux originated is  $\frac{q}{\epsilon_0}$  in all directions or we can say that from a

point charge q,  $\frac{q}{\epsilon_0}$  flux is originated is  $4\pi$  solid angle.

Here the solid angle enclosed by cone subtended by disc at the point charge can be given as

$$\Omega = 2\pi(1-\cos\phi) = 2\pi \left(1-\frac{\ell}{\sqrt{\ell^2+R^2}}\right)$$







Now we can easily calculate the flux of q which as passing through the disc surface as

$$\phi_{disc} = \frac{q/\epsilon_0}{4\pi} \times \Omega = \frac{q/\epsilon_0}{4\pi} \times 2\pi \left(1 - \frac{\ell}{\sqrt{\ell^2 + R^2}}\right)$$

or

## *Ex.1* Find the electric flux coming out from one face of a cube of edge a, centre of which a point charge q is placed.

**Sol.** Here the total solid angle subtended by cube surface at the point charge q is  $4\pi$ . As q is at centre of cube, we can say the each face of cube subtend equal solid angle at the centre, thus solid angle subtended by each force at point charge is

$$\Omega_{\text{face}} = \frac{4\pi}{6} \text{ steradian}$$

 $\phi_{disc} = \frac{q}{2\epsilon_0} \left( 1 - \frac{\ell}{\sqrt{\ell^2 + R^2}} \right)$ 

Thus electric flux through each face is

$$\phi_{\text{face}} = \frac{q / \epsilon_0}{4\pi} \times \Omega_{\text{face}}$$

$$= \frac{q/\epsilon_0}{4\pi} \times \frac{4\pi}{6} = \frac{q}{6\epsilon_0}$$



Ex.2 A point light source of 100 W is placed at a distance x from the centre of a hole of radius R in a sheet as shown in figure. Find the power passing through the hole in sheet.



#### Sol.

From figure, the solid angle of cone shown in figure can be given as

$$\Omega = 2\pi (1 - \cos \theta) = 2\pi \left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

Power in hole = power given in solid angle  $\Omega$ 

$$P = \frac{100}{4\pi} \times \Omega$$
$$= \frac{100}{4\pi} \times 2\pi \left( 1 - \frac{x}{\sqrt{R^2 + x^2}} \right)$$
$$= 50 \left( 1 - \frac{x}{\sqrt{R^2 + x^2}} \right) \text{watt}$$



#### 3. GAUSS'S LAW :

This law is the mathematical analysis of the relation between the electric flux from a closed surface and its enclosed charge.

This law states that the total flux emerging out from a closed surface is equal to the product of sum of

enclosed charge by the surface and the constant  $\frac{1}{\epsilon_n}$ 

Mathematically Gauss's law is written as

$$\oint_{M} \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{\sum q_{encl}}{\epsilon_0}$$

Here the sign  $\oint$  represents the integration over a closed surface M which encloses a total charge  $\sum \alpha$ 

 $\Sigma q_{encl}$ 

Let us consider a surface M shown in figure which encloses three charges  $q_1 - q_2$  and  $q_5$ . For the surface M if we find surface integral of electric field  $\oint_{M} \stackrel{\rightarrow}{E} \cdot \stackrel{\rightarrow}{dS}$ , it gives the total electric flux coming out from the surface, which can be given as

$$\oint_{M} \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{q_1 + q_5 - q_3}{\epsilon_0} \qquad [Gauss's Law]$$



Here electric field  $\overrightarrow{E}$  is the net electric field at the points on the surface of M. Remember that the electric field we use to find the flux must be the net electric field of the system due to all the charges but the total flux coming out from the surface is the flux originated by the charges enclosed in the closed surface.

Using Gauss law we can find electric field strength due to some symmetrical distribution of charges.

For appllication of Gauss's Law, we choose a closed surface over which we apply Gauss law, called Gaussian surface.

Gauss Law can be used to calculate electric field strength, for this we first choose a proper Gaussian surface on which the electric field strength is to be calculated.

Some times a random Gaussion surface is chosen then the integral  $\oint \vec{E} \cdot \vec{dS}$  involves complex calcula-

tions. To make these calculations easier, we choose a Gaussian surface keeping following points in mind.

(i) The Gaussion surface should be chosen in such a way that at every point of surface the magnitude of electric field is either uniform or zero.

(ii) The surface should be chosen in such a way that at every point of surface electric field strength is either parallel or perpendicular to the surface.



Following example will illustrate the applications of Gauss's Law in calculation of electric field in the surrounding of some charge configurations.

Gauss Law is a very helpful tool in finding the electric field strength due to various distribution of charges. We start with a very simple example. Now we try to find the electric field strength due to a point charge q at a distance x, using Gauss's law.



To find electric field strength at P, we first consider a Gaussian surface so that point P will be on its surface. But the question is what should be the shape of Gaussion surface. Look at the following figure shown.



If we apply Gauss's Law to the above two cases, it will require laborious calculations to find  $\oint \vec{E} \cdot \vec{dS}$ .

The Gaussion surface should be chosen in such a way to minimize the calculations. Now consider a spherical surface shown in figure. at every point of this surface electric field due to the charge q is

$$\mathsf{E} = \frac{\mathsf{k}\mathsf{q}}{\mathsf{x}^2}$$

Here if we use Gauss Law for the spherical surface, we have

$$\oint \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{q}{\epsilon_0} \implies E \int dS = \frac{q}{\epsilon_0}$$
$$E \cdot 4\pi x^2 = \frac{q}{\epsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} = \frac{Kq}{x^2}$$

Here we can see that at every point of sphere electric field vector is parallel to  $\overrightarrow{dS}$  and also the magnitude of  $\overrightarrow{E}$  is uniform at every point, thus the integral  $\oint \overrightarrow{E} \cdot \overrightarrow{dS}$  can be easily evaluted.

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Basically flux is the count of number of lines of electric field crosing an area.

- For open surface we choose one direction as a area vector & stick to it for the whole problem



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*Ex.3* In figure (a) a charge q is pla ed just outside the centre of a closed hemisphere. In figure (b) the same charge q is placed just inside the centre of the closed hemisphere and in figure (c) the charge is plced at the centre of hemisphere open from the base. Find the electric flux passing through the hemisphere in all the three cases.



- *Ex.4* A charge q is placed at point D of the cube. Find the electric flux passing through the face EFGH and face AEHD.



**Sol.**  $\phi = q/6\epsilon_0$ 

#### (i) Electric field due to a point charge

The electric field due to a point charge is every where radial. We wish to find the electric field at a distance r from the charge q. We select Gaussain surface, a sphere at distance r from the charge. At every point of this sphere the electric field has the same magnitude E and it is perpendicular to the surface itself. Hence we can apply the simplified form of Gauss law,





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#### (ii) Electric Field Strength due to a Long Charged Wire :

If we wish to find electric field strength due to a long charged wire having a linear charge density  $\lambda$  coul/m at a point P situated at a distance x from the wire.

For this application of Gauss's Law we consider a cylindrical Gaussian surface of length  $\ell$  and radius x as shown in figure. If we apply Gauss's Law on this surface, we have

$$\oint \vec{E} \cdot \vec{dS} = \frac{q_{encl}}{\epsilon_0} \qquad \dots (1)$$

Here the closed Gaussian surface is made of three parts, I, II and III, two flat circular faces and one cylindrical lateral surface. Here we split the closed surfae integration in three parts as

$$\oint \vec{E} \cdot \vec{dS} = \int_{T} \vec{E} \cdot \vec{dS} + \int_{T} \vec{E} \cdot \vec{dS} + \int_{T} \vec{E} \cdot \vec{dS}$$

Here we know for part I and III, electric field strength vector is perpendicular to the area vector as shown in figure hence no flux will come out of these parts. Thus we have

$$\int_{I} \overrightarrow{E} . \overrightarrow{dS} = \int_{III} \overrightarrow{E} . \overrightarrow{dS} = 0$$

Now from equation (1) we have

$$\mathsf{E}\!\!\int_{\mathsf{II}}\mathsf{d}\mathsf{S}=\frac{\lambda\ell}{\epsilon_0}$$

[As enclosed charged is  $q_{encl} = \lambda l$ ] For lateral surface as  $\vec{F}$  is parallel to  $d\vec{S}$  is parallel

$$\mathsf{E}\!\!\!\int_{\mathrm{II}} \mathsf{d}s = \frac{\lambda l}{\varepsilon_0}$$

or

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$$\mathsf{E.}\,2\pi\mathsf{x}\,\ell=\frac{\lambda\ell}{\varepsilon_0}$$

or



#### (iii) Electric Field Strength due to a Long Uniformly Charged Cylinder :







#### Case - I : Conducting Cylinder

Figure shows a long cylinder of radius R which is uniformly charged on its surface with surface charge density  $\sigma$  coul/m<sup>2</sup>

We know at interior points of a metal body electric field strength is zero. For finding electric field strength at outer points at a distance x from the axis of the cylinder, we consider a cylindrical Gaussian surface of radius x and length  $\ell$  as shown in figure. Now we apply Gauss's Law on this surface we have

$$\oint \overrightarrow{E} . d\overrightarrow{S} = \frac{q_{encl}}{\epsilon_0}$$

Here enclosed charge in the cylindrical Gaussian surface can be given

$$q_{encl} = \sigma \cdot 2\pi R\ell$$

Here also similar to previous case the electric flux through the circular faces is zero, hence according to Gauss law, we have

$$\int_{II} \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{\sigma \cdot 2\pi R\ell}{\varepsilon_0} \text{ or } \qquad E \int_{II} dS = \frac{\sigma \cdot 2\pi R\ell}{\varepsilon_0} \qquad \text{ or } \qquad E = \frac{\sigma R}{\varepsilon_0 x}$$

#### Case II : Uniformly Charged Non-conducting Cylinder

Figure shows a long cylinder of radius R, charged uniformly with volume charge density p coul/m<sup>3</sup>. To find electric field strength at a distance x from the cylinder axis we again consider a cylindrical Gaussian surface shown in figure.

 $[As q_{encl} = \rho.\pi R^2 \ell]$ 

If we apply Gauss Law on this surface, we have

$$\oint \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{q_{encl}}{\epsilon_0}$$

 $\int_{II} \overrightarrow{E} . \overrightarrow{dS} = \frac{\rho . \pi R^2 \ell}{\varepsilon_0}$ 

or

$$E \int dS = \frac{\rho \pi R^2 \ell}{\varepsilon_0}$$

or

or

 $\mathsf{E.}2\pi \mathsf{x}\ell = \frac{\rho\pi\mathsf{R}^2\ell}{\epsilon_0}$ 

To find electric field inside the cylinder at a distance x from the axis, we consider a small cylindrical Gaussian surface of radius x and length l. If we apply Gauss Law for this surface, we have

 $E.2\pi x\ell = \frac{\rho\pi x^2\ell}{\varepsilon_0} \text{ or } E = \frac{\rho R^2}{2\varepsilon_0 x}$ 

$$\oint \overrightarrow{\mathsf{E}} \cdot \overrightarrow{\mathsf{dS}} = \frac{\mathsf{q}_{\mathsf{encl}}}{\in_0}$$

 $E\int_{II} dS = \frac{\rho \pi x^2 \ell}{\epsilon_0}$ 

 $E = \frac{\rho x}{2 \in \Omega}$ 

or

or

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or

 $\sigma$  coul / m<sup>2</sup>

#### (iv) Electric field Strength due to a Non-conducting Uniformly Charged Sheet :

To find the electric field strength at a point P infront of the charged sheet we consider a cylindrical Gaussian surface as shown in figure of face area S. If we apply Gauss law for this surface, we have

$$\oint \vec{E} \cdot \vec{dS} = \frac{q_{encl}}{\epsilon_0}$$
$$\int_{T} \vec{E} \cdot \vec{dS} + \int_{T} \vec{E} \cdot \vec{dS} + \int_{TT} \vec{E} \cdot \vec{dS} = \frac{\sigma S}{\epsilon_0} \quad [As here q_{encl} = \sigma S]$$

In this case  $\int \vec{E} \cdot \vec{dS} = 0$  as the lateral surface of cylinder is

parallel to the direction of electric field strength, no flux is coming out from the lateral surface, hence we have

$$\int_{I} EdS + \int_{II} EdS = \frac{\sigma S}{2 \epsilon_0}$$



or

$$E = \frac{\sigma}{2 \in \Omega}$$

## area S II $2ES = \frac{\sigma S}{\epsilon_{o}}$ [As electric field is uniform on both sides]

#### (v) Electric field Stength due to a Charged Conducting Sheet :

Figure show a large charged conducting sheet, charged on both the surfaces with surface charge density  $\sigma$  coul/m<sup>2</sup>. As we know in the metal sheet there is no charge within the volume of the sheet and also the electric field inside the metal sheet is zero. To find electric field strength at a point P in front of the sheet we consider a cylindrical Gaussian surface having one face at point P where electric field is required and other face is within the volume of sheet. If we apply Gauss's Law on this surface, we have

$$\oint \vec{E} \, \vec{dS} = \frac{q_{encl}}{\epsilon_{o}}$$

 $Or \quad \int_{\tau} \overrightarrow{E} \cdot \overrightarrow{dS} + \int_{\tau I} \overrightarrow{E} \cdot \overrightarrow{dS} + \int_{III} \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{\sigma S}{\epsilon_0}$ [As here  $q_{encl} = \sigma S$ ] Here on surface I of the Gaussian surface E = 0 hence  $\int \overrightarrow{E} \cdot \overrightarrow{dS} = 0$  and  $\int \overrightarrow{E} \cdot \overrightarrow{dS} = 0$  as no electric flux is coming out

from the lateral surface of cylinder ( $\vec{E}$  is perpendicular to area vector of curved surface). Hence we have total flux coming out is

$$\int_{II} \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{\sigma S}{\epsilon_0}$$

or  $ES = \frac{\sigma S}{\epsilon_0}$  or  $E = \frac{\sigma}{\epsilon_0}$ 









#### 4. CONDUCTORS :

A conductors contains free electrons, which can move freely in the material, but cannot leave it.

On applying an external electric field on a conductor charges of a conductor adjust themselves in such a fashion that the net electric field inside the conductor is zero under electrostatics conditions.



Net  $\vec{E} = 0 \Rightarrow$  Potential is constant

#### ... Conductor behaves as an equipotential surface

Being an equipotential surface, electric field lines will terminate or originate perpendicularly Let us now consider the interior of a charged conducting object. Since it is a conductor, the electric field in the interior is everywhere zero. Let we analyse a Gaussian surface inside the conductor as close as possible to the surface of the conductor. Since the electric intensity  $\vec{E}$  is zero everywhere inside the conductor, it must be zero for every point of the Gaussian surface. Hence the flux through the surface.

 $\oint \vec{E}.d\vec{S}$  will be zero. Therefore, according to Gauss's law ( $\oint \vec{E}.d\vec{S} = \frac{q}{\epsilon_0}$ ), the net charge inside the

Gaussian surface and hence inside the conductor must be zero. Since there can be no charge in the interior of the conductor charge given to the conductor will reside on the surface of the conductor.

#### All the charge given to the conductor reside on the surface of the conductor



Till now we have only discussed the case of uniform shaped bodies on which the charge distribute itself uniformly.

But what about the charge distribution on irregular shaped bodies ? Does in this case also uniform charge distribution take place ? ...... **NO** 



Let us consider a random shaped body and find Electric field due to a small portion of this body. However the  $\sigma$  is not uniform everywhere but for a small area dA, we can assume that  $\sigma$  is constant. Considering a cylindrical gaussian surface, we will calculate flux pasing through the cross section dA.

$$\phi_{net} = \oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}$$

$$\begin{split} \varphi_{net} &= \, \varphi_{curved \, surface} + \varphi_{outer \, flat \, surface} + \, \varphi_{\, inner \, flat \, surface} \\ \varphi_{curved \, surface} &= \, 0 \end{split}$$

because no flux is passing through lateral surface (electric field lines are perpendicular to area vector.)

$$\vec{E}.d\vec{s} = 0$$

 $\phi_{\text{inner flat surface}} = 0$ 

because  $\vec{E}$  inside conductor = 0

$$\therefore \frac{q_{in}}{\epsilon_0} = \phi_{outer \, flat \, \, surface}$$

$$\frac{\sigma dA}{\varepsilon_0} = \vec{E}.d\vec{A} \implies \vec{E} = \frac{\sigma}{\varepsilon_0}$$

#### 5. ELECTRIC PRESSURE :

#### (a) Electric pressure on a Charged Metal Surface :

We know when some charge is given to a metal body it will spread on the outer surface of the body due to mutual repulsion in the charge. When on surface every charge experiences an outward repulsive force due to remaining charges, every part of body experiences an outward pressure. This pressure which acts on every part of charged metal body surface due to remaining charges on the body is called electric pressure.

To calculate this we consider a small segment AB on body surface of area dS as shown. If  $\sigma$  be the surface charge density on AB, charge on it is



#### $dq = \sigma dS$

Now we consider two points M and N just outside and inside of section AB as shown in figure. At the two point if  $E_1$  be the electric field due to section. AB then direction of the electric fields at M and N can be given as shown in figure. If we remove section AB from the body then due to removing body ACB, if  $E_2$  be the electric field strength at point M and N, the direction of  $E_2$  can be given as shown in figure.









(b) Due to complete body we know net electric fields at just outside and inside points can be given as

$$E_{M} = E_{1} + E_{2} = \frac{\sigma}{\varepsilon_{0}} \qquad \dots(1)$$
  
and 
$$E_{N} = E_{1} - E_{2} = 0 \qquad \dots(2)$$
  
Solving equations (1) and (2) we get  
$$E_{1} = E_{2}$$
  
and 
$$E_{1} = E_{2} = \frac{\sigma}{2\varepsilon_{0}}$$

a

Thus electric field at the location of section AB due to remaining body ACB is  $\frac{\sigma}{2\epsilon_0}$ , using which we can find the outward force on section AB, due to the rest of the body ACB as

 $dF = dq E_2 = \sigma dS \times \frac{\sigma}{2\epsilon_0}$ Force on AB is Thus pressure experienced by the section AB can be given as

$$P_e = \frac{df}{ds} = \frac{\sigma^2}{2\epsilon_0}$$

As net electric field outside the surface is

$$E_{net} = \frac{\sigma}{\epsilon_0}$$

Thus we have

$$\mathsf{P}_{\mathsf{e}} = \frac{(\mathsf{E}_{\mathsf{net}})^2}{2 \,\epsilon_0}$$





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#### (b) $\vec{E}$ due to conducting hollow sphere, conducting solid sphere & non conducting hollow sphere.

For the above mentioned bodies, any excess charge given to body gets distributed uniformly over its outer surface. Since the charge lines must point radially outward & also the field strength will have the same value at all points on any imaginary spherical surface concentric with the charged conducting sphere or the shell this is the symmetry which leads us to choose the gaussian surface to be a sphere.

Any arbitrary element of area  $d\vec{s}$  is parallel to the local  $\vec{E}$  so  $\vec{E}.d\vec{s} = Eds$  at all points on the surface.

### (c) Electric field Strength due to a Conducting (solid and hollow) Sphere, Non conducting hollow sphere

#### Case I : x > R

To find electric field at an outer point at a distance x from the centre of sphere, we cosider a spherical Gaussian surface of radius x. If electric field strength at every point of this surface is E, using Gauss's law we have



Here we have

or

Simlarly for surface points we can consider a spherical Gaussian surface of radius R which gives electric field strength on the sphere surface as

$$\mathsf{E}_{\mathsf{S}} = \frac{1}{4\pi \in_0} \frac{\mathsf{Q}}{\mathsf{x}^2}$$

To find electric field strength at an interior point of the sphere, we consider an inner spherical Gaussian surface of radius x(x < R).

Here if we apply Gauss Law for this surface, we have



Thus

For points outside the sphere, the field is same as that of a point charge at the centre of sphere.

#### Case II x < R



#### Non-conducting Uniformly Charged Sphere

For outer and surface points the electric field strength can be calculated by using Gauss Law similar to the previous case of conducting sphere.

For interior points of sphere, we consider a spherical Gaussian surface of radius x as shown. If we apply Gauss Law for this surface, we have

$$\oint \overrightarrow{\mathsf{E}} \cdot \overrightarrow{\mathsf{dS}} = \frac{\mathsf{q}_{\mathsf{encl}}}{\epsilon_0}$$

Here enclosed charge can be given as

$$q_{encl} = \rho \times \frac{4}{3} \pi x^3$$

E.  $4\pi x^2 = \frac{\rho \times \frac{4}{3}\pi x^3}{\epsilon_0}$ 

 $\mathsf{E} = \frac{\mathsf{p}\mathsf{x}}{\mathsf{3}\mathsf{e}_0}$ 

Thus

or

0



To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius r(r > R). This surface encloses the entire charged sphere. So, from Gauss's law, we have

$$E(4\pi r^{2}) = \frac{Q}{\varepsilon_{0}}$$

$$r, \qquad E = \frac{1}{4\pi\varepsilon_{0}} \frac{Q}{r^{2}} \qquad \dots(13F)$$

The field at points outside the sphere is the same as that of a point charge at the centre.

#### Variation of E with the distance from the centre (r)









#### Electric Potential Inside a Metal Body :

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As we've already discussed whenever charge is given to a metal body, it is distributed on its outer surface in such a way that net electric field at every interior point of body is zero. Thus if inside a metal body, a charge is displaced, no work is done in the process as electric field at every point is zero. Hence we can say that the whole metal body is equipotential

At the interior points of sphere as at every point electric field is zero, we can state that this is an

On the basis of above explanation we can state that a region in which at every point electric field is zero, can be regarded as equipotential region.

#### • Electric Potential due to a Charged Sphere : Case I : Conducting Sphere

As we know for outer points of a charged sphere we can assume whole charge is concentrated at its centre thus electric potential at a distance x from the centre of sphere outside can be given as

$$V = \frac{KQ}{x}$$

At the points on surface of sphere, the potential can be given as

$$V_s = \frac{KQ}{R}$$



#### Variation of Potential with Distance from centre of Sphere :



**Note** : Above results are also valid for a uniformly charged hollow sphere.

#### Case II : Non-conducting Uniformly Charged Sphere

For outer and surface points here also we can say that the potnetial remains same as that of a conducting sphere as

$$V_{out} = \frac{KQ}{x} \text{ (for } x > R)$$
$$V_{s} = \frac{KQ}{R} \text{ (for } x = R)$$

For an interior point unlike to a conducting sphere, potential will not remain uniform as electric field exists inside region. We known inside a uniformly charged sphere electric field is in radially outward direction thus as we move away from

centre, in the direction of electric field potential decreases. As shown in figure if there is a point P at a distance x from the centre of sphere, the potential difference between points P and S can be given as







$$V_{p} - V_{s} = \int_{x}^{R} \frac{KQx}{R^{3}} dx$$

or

$$V_{p} - \frac{KQ}{R} = \frac{KQ}{2R^{3}}(R^{2} - x^{2})$$

or

$$V_{p} = \frac{KQ}{2R^{3}}(R^{2} - x^{2}) + \frac{KQ}{R}$$

$$V_p = \frac{KQ}{2R^3}(3R^2 - x^2)$$

Here at x = 0, we have potential at centre of sphere is

$$V_{c} = \frac{3KQ}{2R} = \frac{3}{2}V_{s}$$

Thus at centre, potential is maximum and is equal to 3/2 times that on the surface.

#### Variation of Potential in a Uniformly Charged Sphere with Distance :



#### 6. FIELD ENERGY OF ELECTROSTATIC FIELD



Consider a situation shown in figure. A small body of mass m and charge q placed in an electric field E. When the body is released it starts moving in the direction of electric field due to the electric field qE acting on it. The body will gain some kinetic energy due to its motion. Who is giving energy to this particle ? Answer is simple-electric field. This shows that electric field must posses some energy in the region where field exist due to which it can do work on any charged body placed in it. This energy we call field energy of electric field. Wherever electric field exist, field energy also exsit in space. Let us calculate the amount of energy stored in the space where electric field exist.

#### (a) Field Energy Density of Electric field :

As discussed in previous section in every region where electric field is present, energy must exist. This field energy we can calculate by an example given here.

Consider a charged conducting body shown in figure. Its surface M is having a charge distributed on it. We know the electric field just outside the surface M at a point can be given as

$$\mathsf{E}=\,\frac{\sigma}{\epsilon_0}$$





We also know that on the surface of metal body experience an outward electric pressure which is given as

$$\mathsf{P}_{\mathsf{e}} = \frac{\sigma^2}{2 \in \mathfrak{o}} = \frac{1}{2} \in \mathfrak{o} \mathsf{E}^2$$

Now if we consider that the metal surface M is flexible and allowed to expand due to electric pressure upto a small limit to M'. Here if we check electric field associated with the body, we known inside the body there is no electric field. Initially electric field only exist from surface M to infinity. Hence the field energy also exist from the surface M to infinity. When the surface expand to M' then in the final stage the electric field as well as field energy exist from surface M' to infinity. This implies that during expansion of surface field energy in the shaded volume (say dV) vanishes as before expansion there was electric field in this region and after expansion electric field becomes zero in the region as there is no electric field inside the body.

We also know that the expansion is done by electric force in the body (electric pressure) hence the work done by electric field during expansion is equal to the loss in field energy in the shaded volume dv.

If  $P_e$  is the electric pressure on the body surface then in the small expansion in body volume dV, work done can be given as

$$dW = P_dV$$

 $\frac{dU}{dV} = P_e$ 

And if dU the field energy stored in this volume dV then we can use

$$dU = dW = P_e dV$$

or

$$u = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2 joul/m^3$$

Here  $u = \frac{dU}{dV}$  is the field energy stored per unit volume in the space where electric field E exist and is called field energy density of electric field.

If in a region electric field is uniform, the total field energy stored in a given volume V of space can be given as

$$U = \frac{1}{2} \in_0 E^2 \times V$$

If electric field in a region is non-uniform, the total field energy stored in a given volume of space can be calulated by integrating the field energy in an elemental volume dV of space as

$$dU = \frac{1}{2} \in_0 E^2 \times dV$$

And total field energy in a given volume can be given as

$$U=\int dU=\int \frac{1}{2}\in_{_{0}}E^{2}dV$$

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#### (b) Self energy of a Hollow, conducting, solid conducting & hollow non conducting sphere.

We've dicussed whenever a system of charges is assembled, some work is done and this work is stored in the form of electrical potential energy of the system. Now we consider an example of charging a conducting sphere of radius R.

In the process of charging we bring charge to the sphere from infinity in steps of elemental charges dq. The charge on sphere opposes the elemental charge being brough to it. Let us assume that at an instant sphere has charge q, due to which it has a potential given as

$$V = \frac{Kq}{R}$$

If now a charge dq is brought to its surface from infinity work done in this process can be given as

$$dW = dqV = \frac{Kq}{R}dq$$

Total work done in charging the sphere can be given as

$$W = \int dW = \int_{0}^{Q} \frac{Kq}{R} dq$$
$$W = \frac{KQ^{2}}{2R} \qquad \dots (1)$$

Equation (1) gives the total work done in charging the sphere of radius R.

We've discussed that in space wherever electric field exist, there must be some field energy stored which has energy density, given as

$$u = \frac{1}{2} \in_0 E^2 J/m^3$$

Here we can see that when the sphere was uncharged, there was no electric field in its surroundings. But when the sphere is fully charged, electtric field exist in its surrounding from its surface to infinity. Let us calculate the field energy associated with this charged conducting sphere.

We know electric field due to a sphere at outer points varies

with distance from centre as

$$E = \frac{KQ}{x^2}$$

To find the total field energy due to this sphere, we consider an elemental spherical shell of radius x and width dx as shown in figure. The volume enclosed in this shell is

$$dV = 4\pi x^2. dx$$

Thus the field energy stored in the volume of this elemental shell is

$$dU = \frac{1}{2} \in_0 E^2.dV$$

$$= \frac{1}{2} \in_0 \left[ \frac{KQ}{x^2} \right]^2 \times 4\pi x^2 dx = \frac{KQ^2}{2x^2} dx$$









Thus total field energy associated with the sphere can be calculated by integrating this expression from surface of sphere to infinity as electric field inside the sphere is zero.

Total field energy in the surrounding of sphere is

$$U = \int dU = \int_{R}^{\infty} \frac{KQ^2}{2x^2} dx$$
$$= \frac{KQ^2}{2} \left[ -\frac{1}{x} \right]_{R}^{\infty} = \frac{KQ^2}{2R} \qquad \dots (2)$$

Here we can see that this result is same as equation (1). We can conclude by this total whatever work is done in charging a body is stored in its surrounding in the form of its field energy and can be regarded as self energy of that body. Once a body is charged in a given configuration, its self energy is fixed, if the body is now displaced or moved in any manner keeping its shape and charge distribution constant, its self energy does not charge. as discussed above we can say that

"Self energy of a charged body is the total field energy, associated with the electric field due to this body in its surrounding.

#### (c) Self Energy of a Uniformly Charged Non-conducting Sphere :

We know in outside region of a non-conducting uniformly charged sphere, every point is same as that of a conducting sphere of same radius. Thus field energy in the surrounding of this sphere from surface to infinity can be given as

$$\mathsf{U}_{\mathsf{R}\to\infty} = \frac{\mathsf{K}\mathsf{Q}^2}{2\mathsf{R}}$$

Unlike to the case of conducting sphere, in nonconducting sphere at interior point  $E \neq 0$ . Thus field energy also exist in the interior region. This can be calculated by considering an elemental shell inside the sphere as shown.

Here field energy in the volume of this elemental shell can be given as

$$dU = \frac{1}{2} \in_0 \left[ \frac{KQx}{R^3} \right]^2 \times 4\pi x^2 dx \qquad [As E_{in} = \frac{KQx}{R^3}]$$
$$= \frac{KQ^2}{2R^6} x^4 dx$$

Total field energy inside the sphere can be given as

$$U = \int dU = \frac{KQ^2}{2R^6} \int_0^R x^4 dx \implies U = \frac{KQ^2}{2R^6} \left[ \frac{x^5}{5} \right]_0^R$$
$$U_{0 \to R} = \frac{KQ^2}{10R}$$

Thus total self energy of this sphere can be given as

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$$U_{self} = \frac{KQ^2}{10R} + \frac{KQ^2}{2R} \implies U_{self} = \frac{3}{5}\frac{KQ^2}{R}$$



#### 7. CHARGE INDUCTION IN METAL CAVITIES :

We've discussed that there can never be any electric field inside a conductor due to static charges. Hence no electric line of force can enter into a conducting body. Consider a point charge +q inside a spherical cavity at centre within a metal body shown in figure.

The total electric flux originated by +q is  $\frac{q}{\epsilon_0}$ . Due to this charge at the inner surface of cavity a

charge – q is induced on which this complete flux will terminate and no electric line of force exists into the metal body. A point, A inside the metal volume we know net electric field is zero. Thus the electric field at A due to the point charge +q is nullified by the electric field due to the negative induced charges on the inner surface of cavity and the positive charge induced on outer surface is automatically distributed on the surface in such a way that it does not produce any electric field with in the metal body.



From the above analysis we can conclude some points about the charge induction when a charge is placed inside the cavity of a metal body. These are.

(1) Whenever a charge is placed inside a metal cavity, an equal and opposite charge is induced on the inner surface of cavity.

(2) A similar charge is induced on the outer surface of body with surface charge density inversely proportional to radius of curvature of body

(3) When the charge inside is displaced, the induced charge distribution on inner surface of body changes in such a way that its centre of charge can be assumed to be at the point charge so as to nullify the electric field in outer region.

(4) Due to movement in the point charge inside the body. The charge distribution on outer surface of body does not change as shown in figure.



(5) If another charge is brought to the body from outside, it will only affect the outer distribution of charges not on the charge distribution inside the cavity as shown in figure







Now consider the situation shown in figure. Inside a  $R_2$ , a point charge q is placed at a distance x from the centre as shown. The electric potential at centre due to this system can be given as

$$V_{c} = \frac{kq}{r} - \frac{Kq}{R_{1}} + \frac{Kq}{R_{2}}$$

If we find electric field and potential at a distance r from the centre outside the shell, it will be only due to the charge on outer surface as induced charge on inner surface of cavity always nullifies the effect of point charge inside it. Thus it can be given as

$$E_{_{out}} = \frac{kq}{r^2}$$
 and  $V_{_{P}} = \frac{kq}{r}$ 

Cavity in a conducting Material

consider the system shown in figure. As we know that when a charge is given to a conductor it resides on its outer surface.

Let us find  $\vec{E}$  at a point distanced r

#### Case I

where  $r < R_1$ 

 $\vec{E} = 0$ [because net charge within this region = 0] **Case II** 

When  $R_1 < r < R_2$ 

$$\vec{E} = 0 \ [q_{net} = 0, \ \phi = \frac{q_{in}}{\epsilon_0} = 0 \Rightarrow E = 0 \ ]$$

#### Case III

When  $r > R_2$ 

 $\vec{E} = \frac{kQ}{r^2}$  [It is similar to case of hollow charged sphere]

Now we consider a case when charge is placed inside a conductor. For such case charge distribution will be as follows

For simplicity in the calculation, we could bifurgate the above system as  $\label{eq:constraint}$ 

- A point charge
- A hollow sphere  $S_1$  with charge -Q
- A hollow sphere  $S_2$  with charge +Q electric field at A

Due to Q =  $\frac{KQ}{r^2}$ 

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Due to  $S_1 = 0$  [ $\because$  Point lies inside the hollow sphere] Due to  $S_2 = 0$  [ $\because$  Point lies inside the hollow sphere]









Electric field at B

Due to  $S_1 = \frac{KQ}{r^2} \rightarrow$ Due to  $S_2 = 0$   $\vec{E}_{net}$  at B = 0Electric field at C Due to  $Q = \frac{kq}{r^2} \rightarrow$ Due to  $S_1 = \frac{kq}{r^2} \leftarrow$ Due to  $S_2 = \frac{kq}{r^2} \rightarrow$  $\vec{E}_{net}$  at  $C = \frac{kq}{r^2} \rightarrow$ 

 $\Rightarrow$ 

At point C, net  $\vec{E}$  due to  $S_1 \& q$  is zero.  $\vec{E}$  at C is only due to outside charge  $(S_2)$ . If we place an external charge at point C, then effect of  $S_1 \& Q$  on external charge is zero or we can say that effect of external charge on  $S_1 \& Q$  is zero.

Or we can say charge placed inside the conductor & the charge induced on the inner surface of the conductor does not get affected by any external electric field this is known as electrostatic shielding that is why, equipment sensitive towards electric field are placed inside a conductor. External electric field only affects the charge distributed on the surface of conductor.

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We again go back to the case when a charge was placed in the conductor

Potential at A  
Due to 
$$Q = \frac{kQ}{r}$$
  
Due to  $S_1 = \frac{-kQ}{R_1}$   
due to  $S_2 = \frac{kQ}{R_2}$   
 $V_{net} = kQ\left[\frac{1}{r} - \frac{1}{R_1} + \frac{1}{R_2}\right]$   
Potential at B  
Due to  $Q = \frac{kq}{r}$   
Due to  $S_1 = \frac{-kQ}{r}$   
Due to  $S_2 = \frac{kQ}{R_2}$   $\Rightarrow V_{net} = \frac{kQ}{R_2}$ 



Potential at C Due to Q =  $\frac{kQ}{r}$ Due to S<sub>1</sub> =  $-\frac{-kQ}{r}$ Due to S<sub>2</sub> =  $\frac{kQ}{r}$ V<sub>net</sub> =  $\frac{kQ}{r}$ 

While writing potential at various points is case of cavity in a conducting material first distribute charge on various surfaces & then the potential due to induced charges is also considered.

#### 8. CAVITY IN A NON CONDUCTING SPHERE

(a) Electric field due to a Non-uniformly Radially Charged Solid Non-conducting Sphere : If a sphere of radius R is charged with a non-uniform charge density which varies with the distance x from centre x as

$$\rho = \frac{\rho_0}{x} \operatorname{coul} / \operatorname{m}^3$$

Here if we wish to find electric field strength at a point situated at a distance r from centre of sphere outside it, at point P shown in figure. This can be given as

$$E_p = \frac{KQ}{r^2}$$
 (where Q is the total charge of sphere)

For outer points we can assume whole charge of sphere to be at its centre. Now Q can be calculated by integrating the charge of an elemental shell of radius x and width dx as shown in figure. The charge dq in this shell can be given as

$$dq = \rho. 4\pi x^2 dx = \frac{\rho_0}{x} \times 4\pi x^2 dx = 4\pi \rho_0 x dx$$

Total charge of sphere can be given as

$$Q = \int dq = \int_{0}^{R} 4\pi \rho_{0} x dx = 4\pi \rho_{0} \left[ \frac{x^{2}}{2} \right]_{0}^{R} = 2\pi \rho_{0} R^{2}$$

Thus electric field strength at outer points can be given as

$$\mathsf{E}_{\mathsf{p}} = \frac{\mathsf{K}(2\pi\rho_0\mathsf{R}^2)}{\mathsf{r}^2} = \frac{\rho_0\mathsf{R}^2}{2\,\varepsilon_0\,\mathsf{r}^2}$$

To find electric field strength at an interior point at a distance r from the centre of sphere, we first find the charge enclosed within the inner sphere of radius r of which point P is on the surface. Thus enclosed charge can be given as

$$q_{encl} = \int_{0}^{r} \frac{\rho_{0}}{R} .4\pi x^{2} dx = 2\pi \rho_{0} r^{2}$$

Here electric field strength at point P can be given as

$$E_{p} = \frac{Kq_{encl}}{r^{2}} = \frac{K(2\pi\rho_{0}r^{2})}{r^{2}}$$
$$E_{p} = \frac{\rho_{0}}{2\epsilon_{p}}$$

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Here we can see that the above expression is independent of distance from centre.

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ELECTROSTATICS - 2





#### (i) Force acting on the point charge q due to the sphere.

#### (ii) Force acting on the sphere due to the point charge.

(i) Electric field at the position of point charge Sol.

$$\vec{\mathsf{E}} = \frac{\mathsf{K}\mathsf{Q}}{\mathsf{r}^2}\hat{\mathsf{r}}$$

so, 
$$\vec{F} = \frac{KqQ}{r^2}\hat{r}$$
  $|\vec{F}| = \frac{KqQ}{r^2}$ 

(ii) Since we know that every action has equal opposite reaction so

$$\vec{F}_{sphere} = -\frac{KqQ}{r^2}\hat{r}$$
$$|\vec{F}_{sphere}| = \frac{KqQ}{r^2}$$

Ex.6 Figure shows a uniformly charged thin sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere. Find out the following : (i) Force on charge q (ii) Electric field intensity at A. (iii) Electric field intensity at B. Sol. (i) Electric field at the centre of the uniformly charged hollow sphere = 0



So force on charge q = 0

(ii) Electric field at A

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$$\vec{E}_{A} = \vec{E}_{Sphere} + \vec{E}_{q}$$
$$= 0 + \frac{Kq}{r^{2}} ; r = CA$$

E due to sphere = 0, because point lies inside the charged hollow sphere.

(iii) Electric field  $\vec{E}_{B}$  at point B =  $\vec{E}_{Sohere} + \vec{E}_{a}$ 

$$= \frac{KQ}{r^2}\hat{r} + \frac{Kq}{r^2}\hat{r}$$
$$= \frac{K(Q+q)}{r^2}\hat{r} \quad ; \ r = CB$$

- Here we can also assume that the total charge of sphere is concentrated at the centre, for calculation of electric field at B.
- Two concentric uniformly charged spherical shells of Ex.7 radius  $R_1$  and  $R_2$  ( $R_2 > R_1$ ) have total charges  $Q_1$  and  $Q_2$ respectively Derive an expression of electric field as a function of r for following positions.

(i) 
$$r < R_1$$
 (ii)  $R_1 \le r < R_2$  (iii)  $r \ge R_2$ 



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(ii)

Sol. (i) for  $r < R_1$ ,

therefore point lies inside both the spheres

 $E_{net} = E_{inner} + E_{outer} = 0$ for  $R_1 \leq r < R_2$ ,

therefore point lies outside inner sphere but inside outer sphere :

$$\mathsf{E}_{\mathsf{net}} = \mathsf{E}_{\mathsf{inner}} + \mathsf{E}_{\mathsf{outer}} = \frac{\mathsf{K}\mathsf{Q}_1}{\mathsf{r}^2}\hat{\mathsf{r}} + 0 \qquad = \quad \frac{\mathsf{K}\mathsf{Q}_1}{\mathsf{r}^2}\hat{\mathsf{r}}$$

(iii) for  $r \ge R_2$ 

point lies outside inner as well as outer sphere therefore.

$$E_{Net} = E_{inner} + E_{outer}$$
$$= \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} = \frac{K(Q_1 + Q_2)}{r^2}\hat{r}$$

- Ex.8 A solid non conducting sphere of radius R and uniform volume charge density ρ has its centre at origin. Find out electric field intensity in vector form at following positions :
  - (i)  $(\frac{R}{2}, 0, 0)$  (ii)  $(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0)$  (iii) (R, R, 0)

(i) at  $(\frac{R}{2}, 0, 0)$ : Distance of point from centre =  $\sqrt{\left(\frac{R}{2}\right)^2 + 0^2 + 0^2} = \frac{R}{2} < R$ , so point lies inside the Sol. sphere so

$$\vec{\mathsf{E}} = \frac{\rho \vec{\mathsf{r}}}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0} \left[\frac{\mathsf{R}}{2}\,\hat{\mathsf{i}}\right]$$

(ii) At 
$$\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$$
; distance of point from centre =  $\sqrt{\left(\frac{R}{\sqrt{2}}\right)^2 + \left(\frac{R}{\sqrt{2}}\right)^2 + 0^2} = R = R$ 

so point lies at the surface of sphere, therefore

$$\vec{\mathsf{E}} = \frac{\mathsf{K}\mathsf{Q}}{\mathsf{R}^3}\,\vec{\mathsf{r}} = \frac{\mathsf{K}\frac{4}{3}\,\pi\mathsf{R}^3\rho}{\mathsf{R}^3} \left[\frac{\mathsf{R}}{\sqrt{2}}\,\hat{\mathsf{i}} + \frac{\mathsf{R}}{\sqrt{2}}\,\hat{\mathsf{j}}\right] = \frac{\rho}{3\varepsilon_0} \left[\frac{\mathsf{R}}{\sqrt{2}}\,\hat{\mathsf{i}} + \frac{\mathsf{R}}{\sqrt{2}}\,\hat{\mathsf{j}}\right]$$

(iii) The point is outside the sphere

so, 
$$\vec{E} = \frac{KQ}{r^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3 \rho}{(\sqrt{2}R)^3}[R\hat{i} + R\hat{j}] = \frac{\rho}{6\sqrt{2}\epsilon_0}[R\hat{i} + R\hat{j}]$$

#### (b) Electric Field Inside a Cavity of Non-conducting Charged Body :



Consider the sphere shown in figure charged uniformly with charge density  $\rho$  coul/m<sup>3</sup>. Inside the sphere a spherical cavity is created with centre at C.

Now we wish to find electric field strength inside the cavity. For this we consider a point P in the cavity at a position vector  $\vec{x}$  from the centre of sphere and at a position vector  $\vec{y}$  from the centre of cavity as shown.

If  $\vec{E}_1$  be the electric field strength at P due to the complete charge of the sphere (inside cavity also) then we known electric field strength inside a uniformly charged sphere is given as

$$\vec{\mathsf{E}}_1 = \frac{\rho \vec{x}}{3 \in_0}$$

Similarly if we assume charge is only there in the region of cavity, this will also be a uniformly charged small sphere. If  $\vec{E}_2$  be the electric field only due to the cavity charge, it can be given as

$$\mathsf{E}_{_{2}}=\frac{\rho\vec{y}}{3\epsilon_{_{0}}}$$

Now the electric field due to the charged sphere in the cavity at point P can be given as

$$\vec{E}_{net} = \vec{E}_1 - \vec{E}_2$$
 [As now charge of cavity is removed]

$$= \frac{\rho \bar{a}}{3 \epsilon_0} \qquad [A]$$

As 
$$\vec{x} - \vec{y} = \vec{a}$$
]

This shows that the net electric field inside the cavity is uniform and in the direction of  $\vec{a}$  i.e. along the line joining the centre of spheres and cavity.

Similarly we can find the electric field strength inside a cyclindrical cavity of a long uniformly charged cylinder. If cavity axis is displaced from axis of cylinder by a displacement vector  $\vec{a}$ , by the analysis we've done for a sphere, we can say that the electric field strength inside the cavity is also uniform and can be given as

$$\vec{E} = \frac{\rho \vec{a}}{2\epsilon_0}$$







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#### (c) Some other important results for a closed conductor :

(i) If a charge q is kept in the cavity then -q will be induced on the inner surface and +q will be induced on the outer surface of the conductor (it can be proved using gauss theorem)

(ii) If a charge q is kept inside the cavity of a conductor and conductor is given a charge Q then -q charge will be induced on inner surface and total charge on the outer surface will be q + Q. (it can be proved using gauss theorem)



(iii) Resultant field, due to q (which is inside the cavity) and induced charge on  $S_1$ , at any point outside  $S_1$  (like B, C) is zero. Resultant field due to q + Q on  $S_2$  and any other charge outside  $S_2$ , at any point inside of surface  $S_2$  (like A, B) is zero



(iv) Resultant field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also this result is true. No charge will be induced on the inner most surface of the conductor.

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(v) Charge distribution for different types of cavities in conductors 0



#### ELECTROSTATICS - 2



Using the result that  $\vec{E}_{res}$  in the conducting material should be zero and using result (iii) We can show that

## In all cases charge on inner surface $S_1 = -q$ and on outer surface $S_2 = q$ . The distribution of charge on $S_1'$ will not change even if some charges are kept outside the conductor (i.e. outside the surface $S_2$ ). But the charge distribution on $S_2'$ may change if some charges(s) is/are kept outside the conductor.

Electric field due at 'A' due to -q of  $S_1$  and +q of  $S_2$  is zero individually because they are uniformly distributed

At point B :

$$V_{\rm B} = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2}, \ E_{\rm B} = 0$$

At point C :

$$V_{c} = \frac{Kq}{OC}, \vec{E}_{c} = \frac{Kq}{OC^{3}} \overrightarrow{OC}$$
(ii) Force on point charge Q :

Here force on 'Q' will be only due to 'q' of S<sub>2</sub> see result (iii)  $\vec{F}_{q} = \frac{KqQ}{r^{2}}\hat{r}$  (r = distance of 'Q' from centre 'O') Force on point charge q :  $\vec{F}_{q} = 0$  (using result (iii) & charge on S<sub>1</sub> uniform)



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Ex.9 An uncharged conductor of inner radius R, and outer radius R, contains a point charge q placed at point P (not at the centre) as shown in figure? Find out the following :

- (i) V<sub>c</sub> (iv) E<sub>A</sub>
- (ii)  $V_A$  (iii)  $V_B$ (v)  $E_B$  (vi) force on charge Q if it is placed at B
- (i)  $V_{c} = \frac{Kq}{CP} + \frac{K(-q)}{R_{1}} + \frac{Kq}{R_{2}}$ Sol.

-q on S<sub>1</sub> in nonuniformly distributed still it produces potential  $\frac{K(-q)}{R_1}$  at 'C' because 'C' is at distance 'R<sub>1</sub>' from each points of  $'S_1'$ .

(ii)  $V_A = \frac{Kq}{R_a}$ (iii)  $V_{B} = \frac{Kq}{CB}$ 

(iv)  $E_A = O$  (point is inside metallic conductor)

(v)  $E_{B} = \frac{Kq}{CB^{2}}CB^{2}$ (vi)  $F_{Q} = \frac{KQq}{CB^2}CB^2$ 

#### Combination of conducting spherical shells.

Let us consider a system of concentric conducting shelk with charge  $q_1$  on inner shell &  $q_2$  on outer shell.

- Potential at A =  $\frac{kq_1}{R_1} + \frac{kq_2}{R_2}$ Potential at B =  $\frac{kq_1}{r} + \frac{kq_2}{R_2}$ Potential at C =  $\frac{kq_1}{r} + \frac{kq_2}{r}$ Potential of  $S_1 = \frac{kq_1}{R_1} + \frac{kq_2}{R_2}$ Potential of  $S_2 = \frac{kq_1}{R_1} + \frac{kq_2}{R_2}$
- In case of combination of concentric conducting spherical shell, we do not consider the potential due to induced charge distribution as we used to consider in the case of cavties in conducting materials.

#### Ex.10 Consier the following system & find $V_c - V_A$



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Sol. 
$$V_c = \frac{R(4q)}{4R} + \frac{k(2q)}{4R} + \frac{k(q)}{4R} = \frac{7kq}{4R}$$
  
 $V_A = \frac{kq}{R} + \frac{k(2q)}{2R} + \frac{k(4q)}{4R} = \frac{3kq}{R}$   
 $V_c - V_A = \frac{7kq}{4r} - \frac{3kq}{R} = \frac{7kq - 12kq}{4R} = -\frac{5kq}{4R}$ 

#### 9. **CONNECTION OF TWO CONDUCTING MATERIAL:**

Two conducting hollow spherical shells of radii  $R_1$  and  $R_2$  having charges  $Q_1$  and  $Q_2$  respectively and placed separately by large distance, are joined by a conducting wire



Let final charges on spheres are  $q_1$  and  $q_2$  respectively. Potential on both spherical shell become equal after joining, therefore

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2}$$
$$\frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad \dots (i)$$

and by charge conservation,  $q_1 + q_2 = Q_1 + Q_2$ ....(ii) from (i) and (ii)

$$q_1 = \frac{(Q_1 + Q_2)R_1}{R_1 + R_2}$$
,  $q_2 = \frac{(Q_1 + Q_2)R_2}{R_1 + R_2}$ 

ratio of charges  $\frac{q_1}{q_2} = \frac{R_1}{R_2} \Rightarrow \frac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \frac{R_1}{R_2}$ 

ratio of surface charge densities  $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$ 

Ratio of final charges  $\frac{q_1}{q_2} = \frac{R_1}{R_2}$ 

$$\sigma_1 R_2$$

Ratio of final surface charge densities.  $\frac{1}{\sigma_2} = \frac{1}{R_1}$ 

- If two concentric hollow sphere are connected by a wire then all the charge from inner sphere will reside to the outer sphere.
- Ex.11 The two conducting spherical shells are joined by a conducting wire and cut after some time when charge stops flowing. Find out the charge on each sphere after that.

Sol. After cutting the wire, the potential of both the shells is equal Thus, potential of inner shell V<sub>in</sub> =  $\frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$ and potential of outer shell V<sub>out</sub> =  $\frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$ As  $V_{out} = V_{in}$ 

 $\frac{-KQ}{R} = \frac{K(x-2Q)}{2R} \quad \Rightarrow \quad -2Q = x - 2Q \quad \Rightarrow x = 0$  $\Rightarrow$ 

So charge on inner spherical shell = 0and outer spherical shell = -2Q.





#### ELECTROSTATICS - 2



Ex.13 Two conducting hollow spherical shells of radii R and 2R carry charges –Q and 3Q respectively. How much charge will flow into the earth if inner shell is grounded ?



Sol. When inner shell is grounded to the Earth then the potential of inner shell will become zero because potential of the Earth is taken to be zero.

$$\frac{Kx}{R} + \frac{K3Q}{2R} = 0$$

 $x = \frac{-3Q}{2}$ , the charge that has increased

$$=\frac{-3Q}{2}$$
 - (-Q)  $=\frac{-Q}{2}$  hence charge flows into the Earth  $=\frac{Q}{2}$ 



- Ex.14 An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time what is the amount of heat loss ?
- When two conducting spheres of equal radius are connected charge is equally distributed on them Sol. (Result VI). So we can say that heat loss of system

$$= \left(\frac{Q^2}{8\pi\epsilon_0 R} - 0\right) - \left(\frac{Q^2/4}{8\pi\epsilon_0 R} + \frac{Q^2/4}{8\pi\epsilon_0 R}\right) = \frac{Q^2}{16\pi\epsilon_0 R}$$

 $\Delta H = U_{c} - U_{c}$ 

#### 10. EARTHING OF CHARGED OR UNCHARGED METAL BODIES :

In electrical analysis, earth is assumed to be a very large conducting sphere of radius 6400 kms. If some charge Q is given to earth, its potential becomes

$$V_i = \frac{KQ}{R_e}$$

As R<sub>e</sub> is very large V<sub>e</sub> comes out to be a negligible value. Thus for very small bodies whose dimensions are negligible compared to earth we can assume that earth is always at zero potential

Keeping the above fact in mind if we connect a small body to earth, charge flow takes place between earth and the body till both will be at same potential, zero potential as potential of earth will always remain zero, no matter if charge flows into earth or from earth. This implies that if a body at some positive potential is connected to earth, earth will supply some negative charge to this body so that the final potential of body will become zero.

Consider a solid uncharged conducting sphere shown in figure. A point charge q is placed in front of the sphere centre at a distance x as shown. Here due to q, the potential at sphere is



Here we ignore induced charges due to q because potential due to induced charges on sphere is zero. If we close the switch S, earth supplies a charge  $q_e$  on to the sphere to make its final potential zero. Thus the final potential on sphere can be taken as



Here we ignore induced charges due to q because potential due to induced charges on sphere is zero. It we close the switch S, earth supplies a charge  $q_e$  on the the sphere to make its final potential zero. Thus the final potential on sphere can be taken as

$$V = \frac{Kq}{x} + \frac{Kq_e}{R} = 0$$
 or  $q_e = -\frac{qR}{x}$ 

Here it is obvious that earth has supplied a negative charge to devlop a negative potential on sphere to nullify the initial positive potential on it due to q.

Always remember whenever a metal body is connected to earth, we consider that earth suplies a charge to it (say  $q_e$ ) to make its final potential zero due to all the charges including the charge on body and the charges in its surrounding.



#### 11. CONDUCTOR AND IT'S PROPERTIES [FOR ELECTROSTATIC CONDITION] :

- (i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.
- (ii) In electrostatics conductors are always equipotential surfaces.
- (iii) Charge always resides on outer surface of conductor.
- (iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.
- (v) Electric field is always perpendicular to conducting surface.
- (vi) Electric lines of force never enter into conductors.
- (vii) Electric field intensity near the conducting surface is given by formula

$$\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n}$$

$$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0} \hat{n} ; \vec{E}_B = \frac{\sigma_B}{\varepsilon_0} \hat{n} \text{ and } \vec{E}_C = \frac{\sigma_C}{\varepsilon_0} \hat{n}$$

(viii) When a conductor is grounded its potential becomes zero.



- (ix) When an isolated conductor is grounded then its charge becomes zero.
- (x) When two conductors are connected there will be charge flow till their potential becomes equal.
- (xi) Electric pressure : Electric pressure at the surface of a conductor is given by formula  $P = \frac{\sigma}{2\varepsilon_0}$ where  $\sigma$  is the local surface charge density.
- Ex.14 There are 4 concentric shells A, B, C and D of radius of a, 2a, 3a, 4a respectively. Shells B and D are given charges +q and -q respectively. Shell C in now earthed. Find the potential difference  $V_{A} V_{C}$
- Sol Let shell C acquires charge 'q' which will be such that final potential of C is zero.

$$V_{c} = \frac{kq}{3a} + \frac{kq'}{3a} + \left(\frac{-kq}{4a}\right) = 0$$
$$\frac{kq}{3a} + \frac{kq'}{3a} = \frac{kq}{4a} \implies q' = 3q\left(\frac{1}{4} - \frac{1}{3}\right)$$
$$q' = -\frac{q}{4}$$
$$V_{c} = 0$$
$$V = V$$

As

 $V_A^{-} - V_C^{-} = V_A^{-}$ Now calculating V, we get

$$V_{A} = \frac{kq}{2a} - \frac{k(q/4)}{3a} - \frac{kq}{4a} \implies V_{A} = \frac{kq}{6a}$$
$$V_{A} - V_{C} = \frac{kq}{6a}$$



or

#### 12. COMBINATION OF CONDUCTING PLATES :

Let us consider two conducting plates placed parallel to each other. I plate is given a charge  $Q_1 \& II$  plate is given a charge  $Q_2$  which distributes itself as shown in figure above

where  $q_1 + q_2 = Q_1$ 

 $q_{3} + q_{4} = Q_{2}$ 

Now we take a rectangular gaussian surface ABCD.

Among the four faces, two faces AD & BC of this closed surfaces lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface AB & CD which are outside the conductor are parallel to the electric field i.e. their area vector is perpendicular to  $\vec{E}$  & hence the flux through these parts is also zero. The total flux of the electric field through the closed surface is therefore zero. From gauss's law, the total charge inside the closed surface should be zero. The charge on the inner surface of I should be equal & opposite to that on the inner surface of II.

So 
$$q_2 = -q_3$$

Now to find further relations between the charges distributed we find electric field at point P

Electric field at point P

due to 
$$q_1$$
 charge ayer =  $\frac{q_1}{2A\epsilon_0}$  (towards right)  $\begin{bmatrix} As \ \vec{E} \ due \ to \ a \ single \ layer \ of \ charge \\ is \ \frac{\sigma}{2e_0} = \frac{q_1}{2A\epsilon_0} \end{bmatrix}$ 

due to 
$$q_2$$
 charge layer =  $\frac{q_1}{2A\epsilon_0}$  [towards left]

due to 
$$q_3$$
 charge layer =  $\frac{q_3}{2A\epsilon_0}$  [towards left]

due to  $q_4$  charge layer =  $\frac{q_4}{2A\epsilon_0}$  [towards left]

$$\vec{E}_{net}$$
 at P =  $\frac{q_1}{2A\epsilon_0} - \frac{q_2}{2A\epsilon_0} - \frac{q_3}{2A\epsilon_0} - \frac{q_4}{2A\epsilon_0}$  [Towards right]

As the point P lies inside the conductor the field should be zero Hence

$$\frac{q_1}{2A\varepsilon_0} = \frac{q_2 + q_3 + q_4}{2A\varepsilon_0}$$
$$q_1 = q_2 + q_3 + q_4 \qquad (But q_2 = -q_3)$$
$$q_1 = q_4$$

Puting in equation above we get  $q_1 = q_4 = \frac{Q_1 + Q_2}{2}$ 







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## *Ex.15* Two large parallel conducting sheets (placed at finite distance) are given charges Q and 2Q respectively. Find out charges appearing on all the surfaces.

**Sol.** Let there is x amount of charge on left side of first plate, so on its right side charge will be Q - x, similarly for second plate there is y charge on left side and 2Q - y charge is on right side of second plate

 $E_{p} = 0$  (By property of conductor)

$$\Rightarrow \quad \frac{x}{2A\epsilon_0} - \left\{ \frac{Q-x}{2A\epsilon_0} + \frac{y}{2A\epsilon_0} + \frac{2Q-y}{2A\epsilon_0} \right\} = 0$$

we can also say that charge on left side of P = charge on right side of P

$$\mathbf{x} = \mathbf{Q} - \mathbf{x} + \mathbf{y} + 2\mathbf{Q} - \mathbf{y}$$

 $\Rightarrow x = \frac{3Q}{2} , \quad Q - x = \frac{-Q}{2}$ 

Similarly for point Q:

$$x + Q - x + y = 2Q - y$$
  
 $\Rightarrow x = 0/2, 20 = y = 30/2$ 

$$\Rightarrow$$
 y = Q/2, 2Q - y = 3Q/2

So final charge distribution of plates is :

## Ex.16 Figure shows three large metallic plates with charges –Q, 3Q and Q respectively. Determine the final charges on all the surfaces.

**Sol.** We assume that charge on surface 2 is x. Following conservation of charge, we see that surfaces 1 has charge (-Q - x). The electric field inside the metal plate is zero so fields at P is zero. Resultant field at P -

#### $E_{p} = 0$

- $\Rightarrow \qquad \frac{-Q-x}{2A\varepsilon_0} = \frac{x+3Q+Q}{2A\varepsilon_0}$  $\Rightarrow \qquad -Q-x = x+4Q$  $\Rightarrow \qquad x = \frac{-5Q}{2}$
- We see that charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be in general proved by gauss theorem also. Remember this it is important result. Thus the  $\frac{3Q}{2}$  final charge distribution on all

the surface is as shown in figure :







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# *Ex.17* An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces.

**Sol.** Let there is x charge on left side of plate and Q - x charge on right side of plate

 $E_{P} = 0$ 

$$\frac{x}{2A\epsilon_0} + E = \frac{Q - x}{2A\epsilon_0}$$

$$\Rightarrow \qquad \frac{x}{A\epsilon_0} = \frac{Q}{2A\epsilon_0} - E$$

$$\Rightarrow \qquad x = \frac{Q}{2} - EA\varepsilon_0 \text{ and } Q - x = \frac{Q}{2} + EA\varepsilon_0$$

A = X A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A = 0 A =

C

So charge on one side is  $\frac{Q}{2} - EA\epsilon_0$  and other side  $\frac{Q}{2} + EA\epsilon_0$ 

Solve this question for Q = 0 without using the above answer and match that answers with the answers that you will get by putting Q = 0 in the above answer.

#### (a) Earthing of a System of Parallel Plates :

Consider a large plate shown in figure charged with a charge Q. This is connected to earth with a switch S as shown. If switch S is closed, whole charge will flow to earth and the plate will become neutral as in the surrounding of a single earthed body no electric field exist.

Now consider the system of two plates A and B shown here. Plate A is given a charge Q and plate B is neutral the charge distribution on plates is as shown in figure. If the switch S is now closed the total charge on outer surface of the system of plates after earthing should become zero hence whole charge on plate A will transfer to its inner surface and hence on the inner surface of plate B an equal and opposite charge -Q is developed which is given by earth as shown in figure



If area of plates is A, the electric field between the system of plates can be given as

$$E_{f} = \frac{Q}{A \in_{0}}$$
  
Before earthing this electric field was  
$$E_{i} = \frac{Q}{2A \in_{0}} = \frac{E_{f}}{2}$$



Thus just after earthing the electric field between the plates is doubled and the potential difference between the two plates will also be doubled. as plate B is earthed, its potential is zero. The potential of plate. A can be given as

$$V_{A} = \frac{Q}{A \in _{0}} d$$

Now consider another example shown in figure. In a system of three parallel plates A, B and C the middle plate B is given a charge Q due to which charges are induced on plates A and C as shown. On the basis of discussion done in the previous section we can say that if switch  $S_1$  is closed whole charge of plate B will shift on its left surface and a charge -Q is flown through  $S_1$  toward plate A and final situation will be as shwon in figure (a)



If instead of switch  $S_1$ ,  $S_2$  is closed in the beginning the distribution of charges on the system will be obviously as shown according to the figure (b) and a charge -Q now flows through switch  $S_2$  from earth to plate C.

If we close both the switches simultaneously, the situation will be according to shown in figure. Now the charge on plate B is distributed on the two surface as shown and equal and oppsoite charges  $-q_1$  and  $-q_2$  are developed on the inner surfaces of plates A and C.

Here charges  ${\bf q}_{_1}$  and  ${\bf q}_{_2}$  can be calculated by equating the potential difference of plates A and B and C and B as



Thus if both the switches are closed simultaneously, charges  $-q_1$  and  $-q_2$  will flow through the switches  $S_1$  and  $S_2$  from each of plates A and C.

#### Ex.18 When a charge is given to a conducting plate, the charge distributes itself on two surface.

 $\frac{\sigma}{2\varepsilon_0}$  is the  $\vec{E}$  due to a single layer of charge but as in the case of conducting sheet there is generation

of two surfaces or two layers of charges.

 $\therefore$  electric field outside the conducting plate is  $\frac{\sigma}{\epsilon_0}$ 

### *Ex.*19 When a charge Q is given to a non conducting plate & conducting plate. Find the ratio of electric field produced by them ?

Sol. For non conducting plate For conducting plate

$$\vec{E} = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0}$$



#### 13. (A) TOTAL ELECTROSTATIC ENERGY OF A SYSTEM OF CHARGES :

Total electrostatic potential energy of system of charges can be given as

U =  $\Sigma$  self energy of all charged bodies +  $\Sigma$  Interaction energy of all pairs of charged bodies

Let us consider some cases to understand this concept. Figure shows two uniformly charged non  $\dots$  and  $R_2$  and charged with charges  $Q_1$  and  $Q_2$  respectively separated by a distance r. If we find the total electrostatic energy of this system, we can write as



#### (B) ELECTROSTATIC ENERGY OF A SYSTEM OF CONCENTRIC SHELLS :

Figure shown two concentric shells of radii a and b charged uniformly with charges  $q_1$  and  $q_2$ . Here the total energy of this

system can be given as

 $\rm U_{total}$  = self energy of inner shell + self energy of outer shell + interaction energy of the two shell

$$= \frac{Kq_1^2}{2a} + \frac{Kq_2^2}{2b} + \frac{Kq_1q_2}{b}$$

#### **Alternative Method :**

We know that the total electrostatic energy of a system in stored in the form of field energy of the system hence here we can calculate the total electrostatic energy of the system by integrating the field energy density in the space surrounding the shells where electric field exist.

Total field energy in the electric field associated with the system shown in figure can be given as

$$U = \int_{0}^{b} \frac{1}{2} \in_{0} \left(\frac{Kq_{1}}{x^{2}}\right)^{2} 4\pi x^{2} dx + \int_{0}^{\infty} \frac{1}{2} \in_{0} \left(\frac{K(q_{1} + q_{2})}{r^{2}}\right)^{2} 4\pi x^{2} dx$$
$$= \frac{1}{2} Kq_{1}^{2} \left[\frac{1}{a} - \frac{1}{b}\right] + \frac{1}{2} K(q_{1} + q_{2})^{2} \left[\frac{1}{b}\right]$$
$$= \frac{1}{2} \frac{Kq_{1}^{2}}{a} - \frac{Kq_{1}^{2}}{2b} + \frac{Kq_{1}^{2}}{b} + \frac{Kq_{2}^{2}}{2b} + \frac{Kq_{2}^{2}}{b} = \frac{Kq_{1}^{2}}{2b} + \frac{Kq_{2}^{2}}{2b} + \frac{Kq_{1}q_{2}}{b}$$

Ex.20 Figure shows a shell of radius R having charge  $q_1$ uniformly distributed over it. A point charge q is placed at the centre of shell. Find work required to increase radius of shell from R to  $R_1$  as shown in fiugre(b)

**Sol.** Work = 
$$U_1 - U_2$$

$$U_1 = SE_q + SE_{q_1} + IE = SE_q + \frac{Kq_1^2}{2R} + \frac{Kq_1q}{R}$$

$$Kq^2 \quad Kq q$$

$$U_{f} = SE_{q} + \frac{Kq_{1}}{2R_{1}} + \frac{Kq_{1}q}{R_{1}}$$





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work done 
$$= U_i - U_f = \frac{Kq_1^2}{2R} + \frac{Kq_1q}{R} - \frac{Kq_1^2}{2R_1} - \frac{Kq_1q}{R_1}$$

(Try this problem yourself using the energy density formula)

- Ex.21 A point charge  $q = 3 \mu C$  is located at the centre of the spherical layer of uniform isotropic dielectric with relative permittivity k = 3. The inside radius of the layer is equal to a = 250 mm and the outside radius is b = 500 mm. Find the electrostatic energy inside the dielectric layer.
- **Sol.** Consider a small elemental shell of thickness dx.

Volume =  $dV = 4\pi x^2 dx$ 

Electric field at 
$$x = \frac{Kq}{x^2k}$$

electric energy density =  $\frac{1}{2} \in_0 E^2 = \frac{1}{2} \in_0 \frac{K^2 q^2}{x^4 k^2}$ 

Thus energy content in the element shell is

$$= dE = \frac{1}{2} \epsilon_0 \frac{K^2 q^2}{x^4 k^2} 4\pi x^2 dx$$
$$E = \int_a^b \frac{4}{2} \epsilon_0 \frac{K^2 \pi q^2}{k^2 x^4} x^2 dx$$

$$= \frac{q^2}{2} \frac{K}{R^2} \int_{a}^{b} \frac{1}{x^2} dx = \frac{Kq^2}{2R^2} \left[ \frac{1}{b} - \frac{1}{a} \right] = \frac{Kq^2}{2R^2} \left[ \frac{1}{b} - \frac{1}{a} \right]$$



### Ex.22 Find the electrostatic energy stored in a cylindircal shell of length *l*, inner radius a and outer radius b, coaxial with a uniformly charged wire with linear charge density λC/m.

**Sol.** For this we consider an elemental shell of radius x and width dx. The volume of this shell dV can be given as

$$dV = 2\pi x \ell.dx$$

The electric field due to the wire at the shell is

$$\mathsf{E}=\frac{2K\lambda}{x}$$

The electrostatic field energy stored in the volume of this shell is

$$dU = \frac{1}{2} \epsilon_0 E^2 dV$$
$$dU = \frac{1}{2} \epsilon_0 \left(\frac{2K\lambda}{x}\right)^2 .2\pi x \lambda dx$$

or

The total electrostatic energy stored in the above mentioned volume can be obtained by integrating the above expression within limits from a to b as

$$U = \int dU = \int_{b}^{a} \frac{1}{2} \epsilon_{0} \left(\frac{2K\lambda}{x}\right)^{2} 2\pi x \ell dx$$
$$U = \frac{\lambda^{2}\ell}{4\pi\epsilon_{0}} \int_{a}^{b} \frac{1}{x} dx$$

or or

$$U = \frac{\lambda^2 \ell}{4\pi \epsilon_a} \int_{-\infty}^{b} \frac{1}{x} dx$$

 $U = \frac{\lambda^2 \ell}{4\pi \, \varepsilon_0} \, \ell \, n \! \left( \frac{b}{a} \right)$ 

or

