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PHYSICS
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CURRENT ELECTRICITY

THEORY AND EXERCISE BOOKLET

CONTENTS

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Syllabus

Ohm's law; Series and parallel arrangements of resistances and cells; kirchhoff's laws and simple applications.

1. INTRODUCTION

- **Conductor**

In some materials, the outer electrons of each atoms or molecules are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.

- **Insulator :**

Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

- **Semiconductor :**

In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller then that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

2. ELECTRIC CURRENT AND CURRENT DENSITY

When there is a transfer of charge from one side of an area to the other, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion, if they are negative, the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt, we define the average electric current through the area during this time as

$$i = \frac{\Delta Q}{\Delta t}$$

The current at time t is

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Thus, electric current through an area is the rate of transfer of charge from one side of the area to the other. The SI unit of current is ampere. If one coulomb of charge crosses an area in one second, the current is one ampere. It is one of the seven base units accepted in SI.

Ex.1 If $q = 2t^2 + 3$, find current at t = 2 sec ?

Sol. $i = \frac{dq}{dt}$

$i = 4t$

$\therefore$ i at 2 sec = $4 \times 2 = 8$ A

We shall now define a vector quantity known as electric current density at a point. To define the current density at a point P, we draw a small area ΔS through P perpendicular to the flow of charges(shown in figure) If Δi be the current through the area ΔS, the average current density is

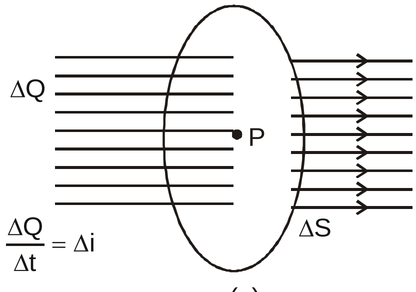
$$\vec{j} = \frac{\Delta i}{\Delta \vec{S}}$$

The current density at the point P is

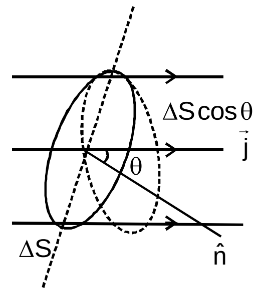
$$j = \lim_{\Delta S \to 0} \frac{\Delta i}{\Delta S} = \frac{di}{dS}$$

The direction of the current density is the same as the direction of the current. Thus, it is along the motion of the moving charges, if the charges are positive and opposite to the motion of the charges, if the charges are negative. If a current i is uniformly distributed over an area S and is perpendicular to it,

$$j = \frac{i}{S}$$



(a)



Now let us consider an area ΔS which is not necessarily perpendicular to the current (figure shown) If the normal to the area makes an angle θ with the direction of the current, the current density is,

$$j = \frac{\Delta i}{\Delta S \cos\theta}$$

or, $\Delta i = j\ \Delta S \cos\theta$

where Δi is the current through ΔS, If $\Delta\vec{S}$ be the area vector corresponding to the area ΔS, we have

$$\Delta i = \vec{j}.\Delta\vec{S}$$

For a finite area,

$$i = \int \vec{j}.d\vec{S}$$

Note carefully that an electric current has direction as well as magnitude but it is not a vector quantity. It does not add like vectors. Therefore current is neither a vector quantity nor a scalar quantity but a tensor quantity. The current density is a vector quantity.

Ex.2 An electron beam has an aperature 1.0 mm². A total of 6.0 × 10¹⁰ electrons go through any perpendicular cross-section per second. Find (a) the current and (b) the current density in the beam.

Sol. The total charge crossing a perpendicular cross-section in one second is

$q = ne$

$= 6.0 \times 10^{16} \times 1.6 \times 10^{-19}$ C

$= 9.6 \times 10^{-3}$ C

The current is

$$i = \frac{q}{t} = \frac{9.6 \times 10^{-3}\,C}{1s} = 9.6 \times 10^{-3}\ A$$

As the charge is negative, the current is opposite to be direction of motion of the beam.

(b) The current density is

$$j = \frac{i}{S} = \frac{9.6 \times 10^{-3}\,A}{(1.0mm)^2} = \frac{9.6 \times 10^{-3}\,A}{1.0 \times 10^{-6}\,m^2} = 9.6 \times 10^{3}\ A/m^2$$

3. DRIFT SPEED

A conductor contains a large number of loosely bound electrons which we call free electrons or conduction electrons. The remaining material is a collection of relatively heavy positive ions which we call lattice. These ions keep on vibrating about their mean positions. The average amplitude depends on the temperature. Occasionally, a free electron collides or interacts in some other fashion with the lattice. The speed and direction of the electron changes randomly at each such event. As a result, the electron moves in a zig-zag path. As there is a large number of free electrons moving in random directions, the number of electrons crossing an area ΔS from one side very nearly equals the number crossing from the other side in any given time interval. The electric current through the area is, therefore, zero.

When there is an electric field inside the conductor, a force acts on each electron in the direction opposite to the field. The electrons get biased in their random motion in favour of the force. As a result, the electrons drift slowly in this direction. At each collision, the electron starts afresh in a random direction with a random speed but gains an additional velocity v' due to the electric field. This velocity v' increases with time and suddenly becomes zero as the electron makes a collision with the lattice and starts afresh with a random velocity. As. the time ,t between successive collisions is small, the electron "slowly and steadily drifts opposite to the applied field (shown figure). If the electron drifts a distance ℓ in a long time t, we define drift speed as

$$v_d = \frac{l}{t}$$

If τ be the average time between successive collisions, the distance drifted during this period is

$$l = \frac{1}{2}\alpha(\tau)^2 = \frac{1}{2}\left(\frac{eE}{m}\right)(\tau)^2$$

The drift speed is $\quad v_d = \frac{l}{\tau} = \frac{1}{2}\left(\frac{eE}{m}\right)\tau$

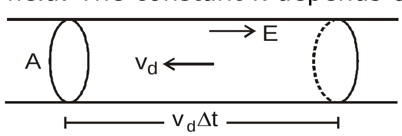
It is proportional to the electric field E and to the average collision-time τ.

The random motion of free electrons does not contribute to the drift of these electrons. Also, the average collision-time is constant for a given material at a given temperature. We, therefore, make the following assumption for our present purpose of discussing electric current.

When no electric field exists in a conductor, the free electrons stay at rest ($V_d = 0$) and when a field E exists, they move with a constant velocity

$$v_d = \frac{e\tau}{2m}E = kE \qquad \text{...(1)}$$

opposite to the field. The constant k depends on the material of the conductor and its temperature.



Let us now find the relation between the current density and the drift speed. Consider a cylindrical conductor of cross-sectional area A in which an electric field E exists. Consider a length $v_d\,\Delta t$ of the conductor (figure shown). The volume of this portion is $Av_d\Delta t$. If there are n free electrons per unit volume of the wire, the number of free electrons in this portion is $nAv_d\Delta t$. All these electrons cross the area A in time Δt. Thus, the charge crossing this area in time Δt is

$$\Delta Q = nAv_d\,\Delta t\, e$$

or, $$i = \frac{\Delta Q}{\Delta t} = nAv_d e$$

and $$j = \frac{i}{A} = nev_d \qquad \text{...(2)}$$

Ex.3 Calculate the drift speed of the electrons when 1 A of current exists in a copper wire of cross-section 2 mm². The number of free electrons in 1 cm³ of copper is 8.5 × 10²².

Sol. We have

$$j = nev_d$$

or, $$v_d = \frac{j}{ne} = \frac{i}{Ane} = \frac{1A}{(2\times10^{-6}m^2)(8.5\times10^{22}\times10^6\,m^{-3})(1.6\times10^{-19}\,C)} = 0.036\,mm/s$$

We see that the drift speed is indeed small.

4. OHM'S LAW

Using equations (1) and (2)

$$j = nev_d = \frac{ne^2\tau}{2m}E$$

or, $j = \sigma E$ ...(3)

where $\sigma = \dfrac{ne^2\tau}{2m}$

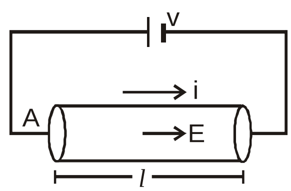
where σ depends only on material of the conductor and its temperature. This constant is called the electrical conductivity of the material. Equation (3) is known as Ohm's law.

The resistivity of a material is defined as

$$\rho = \frac{1}{\sigma} = \frac{2m}{ne^2\tau} \quad ...(4)$$

Ohm's law tells us that the conductivity (or resistivity) of a material is independent of the electric field existing in the material. This is valid for conductors over a wide range of field.

Suppose we have a conductor of length l and uniform cross-sectional area A (figure shown) Let us apply a potential difference V between the ends of the conductor. The electric field inside the conductor is $E = \dfrac{V}{l}$. If the current in the conductor is i, the current density is $j = \dfrac{i}{A}$. Ohm's law j = σE then becomes



(a)

(b)

$$\frac{i}{A} = \sigma\frac{V}{l}$$

or, $V = \dfrac{1}{\sigma}\dfrac{l}{A}i = \rho\dfrac{l}{A}i$ ...(5)

or, V = R i ...(6)

R is called the resistance of the given conductor. The quantity $1/R$ is called conductance.

Equation (5) is another form of Ohm's law which is widely used in circuit analysis. The unit of resistance is called ohm and is denoted by symbol Ω. An object of conducting material, having a resistance of desired value, is called a resistor.

From equation (5) and (6)

$$R = \frac{\rho\ell}{A} \quad ...(7)$$

From equation (7), the unit of resistivity ρ is ohm-metre, also written as Ω-m. The unit of conductivity (σ) is (ohm-m) written as mho/m.

from eq. (4) & (7)

$$R = \frac{2m\ell}{ne^2\tau A} = \frac{\rho\ell}{A}$$

where ρ = resistivity $\left(\text{where } \rho = \dfrac{2m}{ne^2\tau}\right)$

ℓ = length along the direction of current

A = Area of the cross section perpendicular to direction of current

n = no. of free charges per unit volume.

τ = relaxation time

m = mass of electron

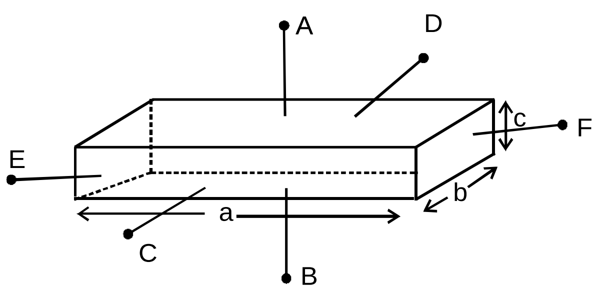
Ex.4 Calculate the resistance of an aluminium wire of length 50 cm and cross-sectional area 2.0 mm². The resistivity of aluminium is $\rho = 2.6 \times 10^{-8}\ \Omega$-m

Sol. The resistance is $R = \rho\dfrac{\ell}{A}$

$$= \frac{(2.6\times10^{-8}\,\Omega-\text{m})\times(0.50\,\text{m})}{2\times10^{-6}\,\text{m}^2} = 0.0065\,\Omega$$

We arrived at Ohm's law by making several assumptions about the existence and behaviour of the free electrons. These assumption are not valid for semiconductors, insulators, solutions etc. Ohm's law cannot be applied in such cases.

Ex.5 The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.

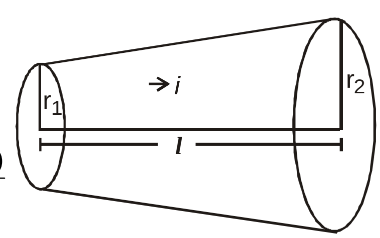


Sol. $R_{AB} = \dfrac{\rho c}{ab},\ R_{CD} = \dfrac{\rho b}{ac}, R_{EF} = \dfrac{\rho a}{bc}$

Ex.6 A portion of length L is cut out of a conical solid wire. The two ends of this portion have circular cross-sections of radii r_1 and r_2 ($r_2 > r_1$). It is connected lengthwise to a circuit and a current i is flowing in it. The resistivity of the material of the wire is ρ. Calculate the resistance of the considered portion and the voltage developed across it.

Sol. If follows from the figure, that

$$\tan\theta = \frac{r_2 - r_1}{L}$$



$$\therefore\ r = r_1 + x\tan\theta = r_1 + x\left(\frac{r_2 - r_1}{L}\right) = \frac{r_1L + x(r_2 - r_1)}{L}$$

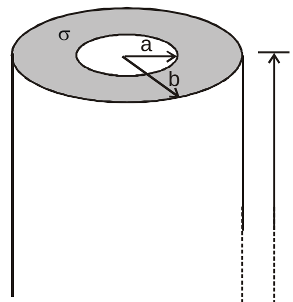
$$\therefore \quad A = \pi r^2 = \frac{\pi}{L^2}[r_1L + (r_2 - r_1)x]^2$$

$$dR = \frac{\rho dx}{\pi r^2} = \frac{\rho dx L^2}{\pi[r_1L + (r_2 - r_1)x]^2} \quad \Rightarrow R = \int dR = \frac{\rho L^2}{\pi}\int_0^L \frac{dx}{[r_1L + (r_2 - r_1)x]^2}$$

$$= \frac{\rho L^2}{\pi}\left[\{r_1L + (r_2 - r_1)x\}^{-1}\right]_0^L\left(-\frac{1}{(r_2 - r_1)}\right)$$

$$= \frac{-\rho L}{\pi(r_2 - r_1)}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{\rho L}{\pi(r_1 r_2)} \qquad \therefore \quad V = IR = \frac{I\rho L}{\pi r_1 r_2}$$

Ex.7 The space between two coaxial cylinders, whose radii are a and b (where a < b as in (figure shown) is filled with a conducting medium. The specific conductivity of the medium is σ.



(a) Compute the resistance along the length of cylinder.

(b) Compute the resistance between the cylinders in the radial direction. Assume that the cylinders are very long as compared to their radii, i.e., L >> b, where L is the length of the cylinders.

Sol. (a) $R = \frac{\rho l}{A} = \frac{l}{\sigma A} = \frac{l}{\sigma(\pi b^2 - \pi a^2)} = \frac{l}{\pi\sigma(b^2 - a^2)}$

(b) From Ohm's law, we have

$$\vec{J} = \sigma\vec{E}$$

Assuming radial current density. $\vec{J}$ becomes

$$\vec{J} = \frac{I}{2\pi r L}\hat{r} \text{ for } a < r < b$$

and, therefore, $\vec{E} = \frac{I}{2\pi\sigma r L}\hat{r}$

Here we have used the assumption that L >> b so that $\vec{E}$ and $\vec{J}$ are in cylindrically symmetric form.

The potential drop across the medium is thus :

$$V_{ab} = -\int_b^a \vec{E}(r).\vec{dr} = -\frac{I}{2\pi\sigma L}\int_b^a \frac{dr}{r} = \frac{I}{2\pi\sigma L}\ln\left(\frac{b}{a}\right)$$

The resistance

$$R_{ab} = \frac{V_{ab}}{I} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma L}$$

Method 2 : We split the medium into differential cylindrical shell elements of width dr, in series. The current flow is cylindrically symmetric (L >> b). The area through which the current flows across a shell of radius r is A(r) = $2\pi rL$. The length the current flows, passing through a shell of radius r is dr. Therefore, the resistance of the shell of radius r is :

$$dR = \frac{1}{\sigma}\frac{dr}{2\pi rL}$$

Since the shells are connected in a series, we have

$$R_{ab} = \int_a^b dR = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma L}$$

Effect of Temperature on Resistance

(a) Resistance of Pure Metals

(i) We know that $R = \left[\frac{2m}{ne^2\tau}\right]\frac{l}{A}$

For a given conductor, l, A and n are constant, hence $R \propto (1/\tau)$

If λ represents the mean free path (Average distance covered between two successive collisions) of the electron and v_{rms}, the root-mean-square speed, then

$$\tau = \frac{\lambda}{v_{rms}}, \qquad \text{Hence } R \propto \frac{V_{rms}}{\lambda}$$

Now,

(a) λ decreases with rise in temperature because the amplitude of vibrations of the +ve ions of the metal increases and they create more hindrance in the movement of electrons and,

(b) (i) v_{rms} increases because $v_{rms} \propto \sqrt{T}$. Therefore, **Resistance of the metallic wire increases with rise in temperature.** As $\rho \propto R$ and $\sigma \propto (1/\rho)$, hence resistivity increases and conductivity decreases with rise in temperature of the metallic of the metallic wires.

(ii) If R_0 and R_t represent the resistances of metallic wire at 0°C and t°C respectively then R_t is given by the following formula :

$$R_t = R_0(1 + \alpha t)$$

where α is called as the **Temperature coefficient of resistance** of the material of the wire.

α depends on material and temperature but generally it is taken as a constant for a particular material for small change.

$$R_t - R_0 = R_0\,\alpha\,t$$

for very small change in temperature $dR = R_0\,\alpha\,dt$

(c) Resistance of semiconductors

(i) There are certain substances whose conductivity lies in between that of insulators and conductors, higher than that of insulators but lower than that of conductors. These are called as semiconductors, e.g., silicon, germanium, carbon etc.

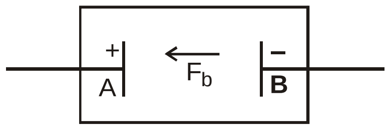
(ii) The resistivity of semiconductors decreases with increase in temperature i.e., **α for semiconductors is –ve and high.**

(iii) Though at ordinary temperature the value of n (no. of free electrons per unit volume) for these materials is very small as compared to metals, but increases very rapidly with rise in temperature (this happens due to breaking of covalent bonds). Though τ decreases but factor of n dominates. Therefore, the resistance

$$R = \frac{ml}{ne^2\tau A}$$ **goes on decreasing with increase in temperature.**

5. BATTERY AND EMF

A battery is a device which maintains a potential difference between its two terminals A and B. Figure shows a schematic diagram of a battery. Some internal mechanism exerts forces on the charges of the battery material. This force drives the positive charges of the battery material towards A and the negative charges of the battery material towards B. We show the force on a positive charge q as $\vec{F}_b$. As positive charge accumulates on A and negative charge on B, a potential difference develops and grows between A and B. An electric field $\vec{E}$ is developed in the battery material from A to B and exerts a force $\vec{F}_e = q\vec{E}$ on a charge q. The direction of this force is opposite to that of $\vec{F}_b$ In steady state, the charge accumulation on A and B is such that $F_b = F_e$. No further accumulation takes place.



If a charge q is taken from the terminal B to the terminal A , the work done by the battery force F_b is $W = F_b d$ where d is the distance between A and B. The work done by the battery force per unit charge is

$$E = \frac{W}{q} = \frac{F_b d}{q}$$

This quantity is called the emf of the battery. The full form of emf is electromotive force. The name is misleading in the sense that emf is not a force, it is work done/charge. We shall continue to denote this quantity by the short name emf. If nothing is connected externally between A and B,

$$F_b = F_e = qE$$

or, $$F_b d = qEd = qV$$

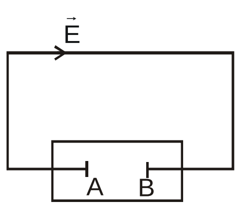
where V = Ed is the potential difference between the terminals. Thus,

$$E = \frac{F_b d}{q} = V$$

Thus, the emf of a battery equals the potential difference between its terminals when the terminals are not connected externally.

Potential difference and emf are two different quantities whose magnitudes may be equal in certain conditions. The emf is the work done per unit charge by the battery force F_b which is non-electrostatic in nature. The potential difference originates from the electrostatic field created by the charges accumulated on the terminals of the battery.

A battery is often prepared by putting two rods or plates of different metals in a chemical solution. Such a battery, using chemical reactions to generate emf, is often called a cell.



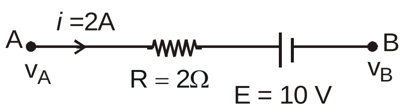
Now suppose the terminals of a battery are connected by a conducting wire as shown in above figure. As the terminal A is at a higher potential than B, there is an electric field in the wire in the direction shown in the figure. The free electrons in the wire move in the opposite direction .and enter the battery at the terminal A. Some electrons are withdrawn from the terminal B which enter the wire through the right end. Thus, the potential difference between A and B tends to decrease. If this potential difference decreases, the electrostatic force F_e inside the battery also decreases. The force

F_b due to the battery mechanism remains the same. Thus, there is a net force on the positive charges of the battery material from B to A. The positive charges rush towards A and neutralise the effect of the electrons coming at A from the wire. Similarly, the negative charges rush towards B. Thus, the potential difference between A and B is maintained.

For calculation of current, motion of a positive charge in one direction is equivalent to the motion of a negative charge in opposite direction. Using this fact, We can describe the above situation by a simpler model. The positive terminal of the battery supplies positive charges to the wire. These charges are pushed through the wire by the electric field and they reach the negative terminal of the battery. The battery mechanism drives these charges back to the positive terminal against the electric fIeld existing in the battery and the process continues. This maintains a steady current in the circuit

Current can also be driven into a battery in the reverse direction. In such a case, positive charge enters the battery at the positive terminal, moves inside the battery to the· negative terminal and leaves the battery from the negative terminal. Such a process is called charging of the battery. The more common process in which. the positive charge comes out of the battery from the positive terminal is called discharging of the battery.

Ex.8

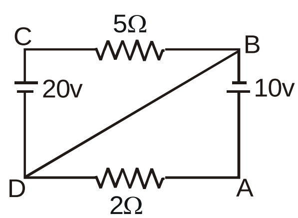


Find $v_A - v_B$

Sol. $v_A - iR - E = v_B$

$v_A - v_B = iR + E = 4 + 10 = 14$ volt

Ex.9 Shown in the figure. Find out the current in the wire BD

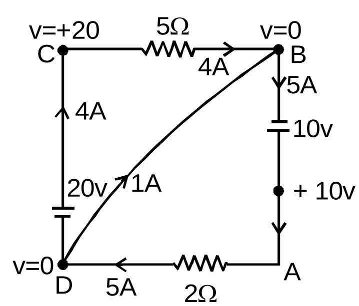


Sol. Let at point D potential = 0 and write the potential of

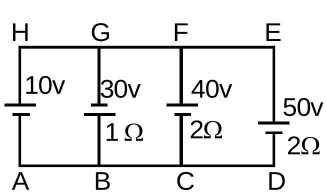
other points then current in wire AD = $\frac{10}{2}$ = 5A from A to D

current in wire CB = $\frac{20}{5}$ = 4A from C to B

$\therefore$ current in wire BD = 1 A from D to B



Ex.10 Find the current in each wire



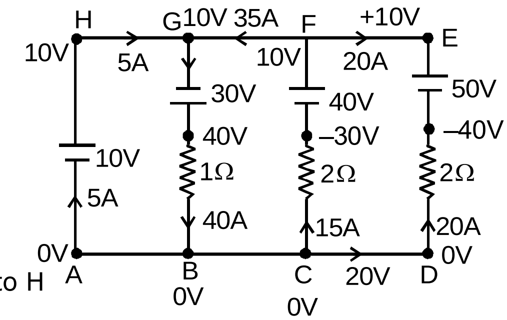
Sol. Let potential at point A is 0 volt then potential of other points is shown in figure.

Current in BG = $\frac{40-0}{1}$ = 40 A from G to B

Current in FC = $\frac{0-(-30)}{2}$ = 15A from C to K

current in DE = $\frac{0-(-40)}{2} = 20A$ from D to E

current in wire AH = 40 – 35 = 5 A from A to H



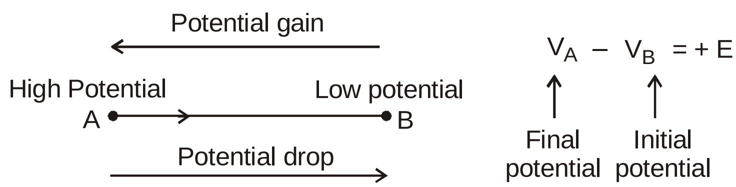
6. KIRCHHOFF'S LAWS FOR CIRCUIT ANALYSIS

Before moving on to the statement of Kirchhoff's law, we state some conventions to be followed in circuit analysis :

(1) Direction of conventional current is from high potential to low potential terminal.

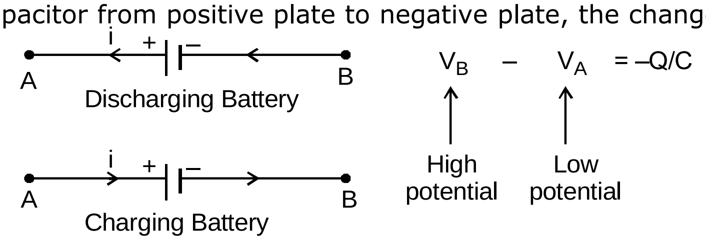
(2) Current flows from high potential node A to low potential node B. if we traverse from point A to B, there is drop of potential; similarly from B to A, there is gain of potential.

If we traverse from point A to B, there is drop of potential; similarly from B to A, there is gain of potential. If a source of emf is traversed from negative to positive terminal, the change in potential is +E.

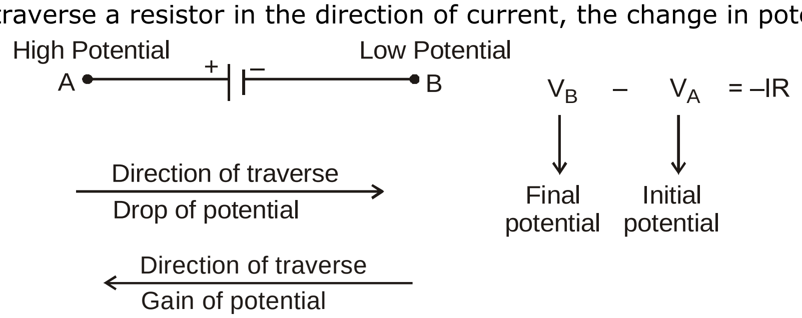


While discharging, current is drawn from the battery, the current comes out from positive terminal and enters negative terminal, while charging of battery current is forced from positive terminal of the battery to negative terminal. Irrespective of direction of current through a battery the sign convention mentioned above holds.

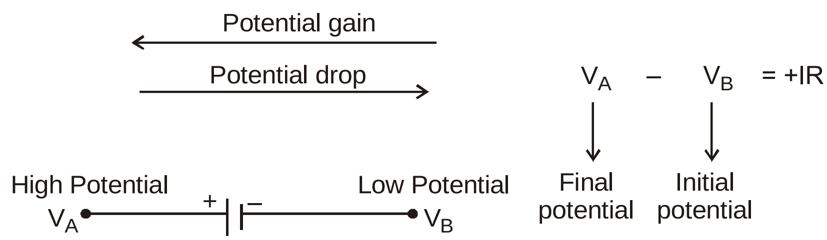
The positive plate of a capacitor is at high potential and negative plate at low potential. If we traverse a capacitor from positive plate to negative plate, the change in potential is –Q/C



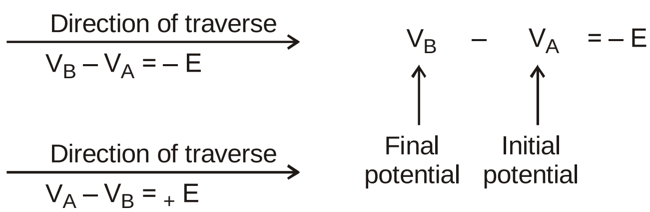
If we traverse a resistor in the direction of current, the change in potential is –IR.



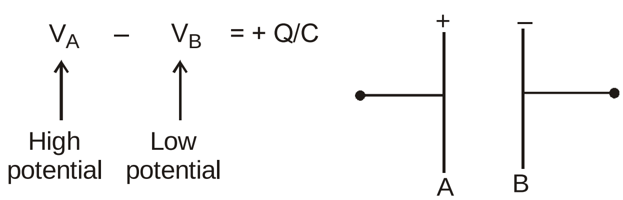
If we traverse a resistor in the direction opposite to the direction of current, the change in potential is +IR.



Positive terminal of source of emf is at high potential and negative terminal at low potential. If we traverse a source of emf from the positive terminal to negative terminal, the change in potential is –E.



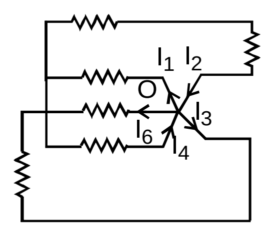
If a capacitor is traversed from negative plate to positive plate, the change in potential is +Q/C.



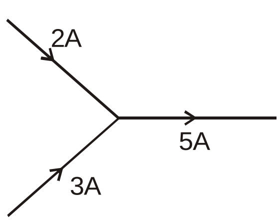
(a) The Kirchhoff's Current Law

The Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering the junction must equal to sum of the currents leaving the junction. From the standard point of physics, KCL is a statement of charge conservation.

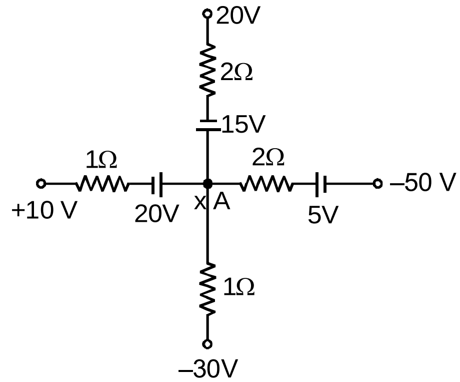
The KCL applied to junction O yields.



$$\underbrace{I_1 + I_5 + I_3}_{\text{Outgoing current}} = \underbrace{I_2 + I_4}_{\text{Incoming current}}$$



Ex.11 **Find the potential at point A**



Sol. Let potential at A = x, applying kirchhoff current law at junction A

$$\frac{x-20-10}{1}+\frac{x-15-20}{2}+\frac{x-5+50}{2}+\frac{x+30}{1}=0$$

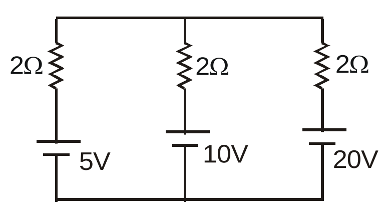
$$\Rightarrow \quad \frac{2x-60+x-35+x+45+2x+60}{2}=0$$

$$\Rightarrow \quad 6x + 10 = 0$$

$$\Rightarrow \quad x=-\frac{5}{3}$$

Potential at A = $\frac{-5}{3}$ V

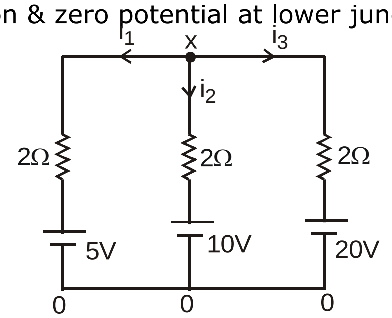
Ex.12



Find the current in every branch ?

Sol. Let we assume x potential at the top junction & zero potential at lower junction

As from KCL,net current on a junction is O



$$i_1 + i_2 + i_3 = 0$$

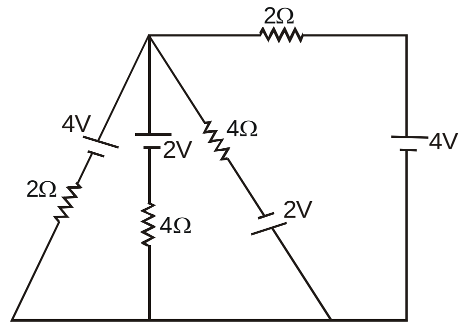
$$\frac{x-5}{2}+\frac{x-10}{2}+\frac{x-20}{2}=0$$

$$3x = 35 \Rightarrow x=\frac{35}{3}$$

$$\therefore \quad i_1=\frac{\frac{35}{3}-5}{2}=\frac{10}{3}\text{ A}$$

Similarly, $i_2 = \frac{5}{6}$ A ; $i_3 = -\frac{25}{6}$ A.

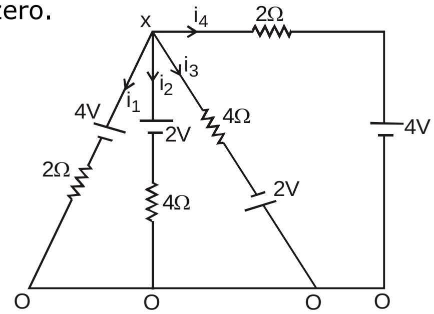
Ex.13



Find the current in every branch ?

Sol. Assume x potential at the upper junction & zero potential at the lower junction.

By KCl, we know that net current on a junction is zero.



$\therefore \quad i_1 + i_2 + i_3 + i_4 = 0$

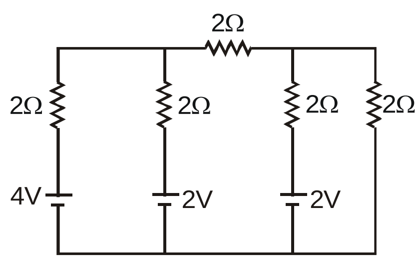
$$\frac{x-4}{2} + \frac{x-2}{4} + \frac{x+2}{4} + \frac{x-4}{2} = 0$$

$2x - 8 + x - 2 + x + 2 + 2x - 8 = 0$

$6x - 16 = 0 \quad \Rightarrow \quad x = \frac{8}{3}V$

$\therefore \quad i_1 = -\frac{2}{3}$ A, $\quad i_2 = +\frac{1}{6}$ A, $\quad i_3 = \frac{7}{6}$ A, $\quad i_4 = -\frac{2}{3}$ A

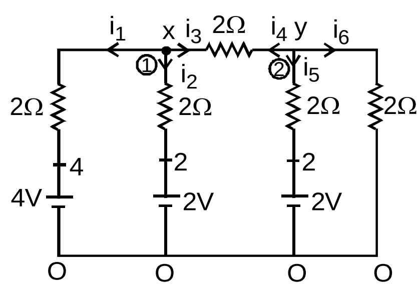
Ex.14



Find the current in every branch ?

Sol. The above question could be solved by assuming potential x & y at the top junctions & zero potential at lower junctions

At the junction 1 applying KCL,



$i_1 + i_2 + i_3 = 0$

$$\frac{x-4}{2} + \frac{x-2}{2} + \frac{x-y}{2} = 0$$

$\Rightarrow \quad 3x - y = 6 \qquad \ldots(1)$

At the junction 2 applying KCL,

$i_4 + i_5 + i_6 = 0$

$$\frac{y-x}{2} + \frac{y-2}{2} + \frac{y}{2} = 0$$

$\Rightarrow 3y - x = 2 \qquad \ldots(2)$

Solving (1) & (2)

$9x - 3y = 18 \Rightarrow 3y - x = 2$

$\Rightarrow \quad 8x = 20$

$x = \frac{5}{2}, \;\; y = \frac{3}{2}$

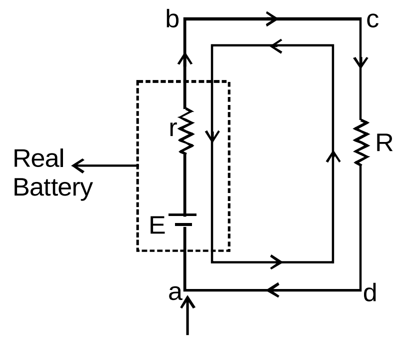
Just put the values of x & y & then the evaluate the current in every branch

(b) The Kirchhoff's Voltage Law

The Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential difference around any closed loop of an electric circuit is zero. **The KVL is a statement of conservation of energy**. The KVL reflects that electric force is conservative, the work done by a conservative force on a charge taken around a closed path is zero.

- We can move clockwise or anticlockwise, it will make no difference because the overall sum of the potential difference is zero.
- We can start from any point on the loop, we just have to finish at the same point.
- An ideal battery is modelled by an independent voltage source of emf E and an internal resistance r as shown in figure A real battery always absorbs power when there is a current through it, thereby offering resistance to flow of current.

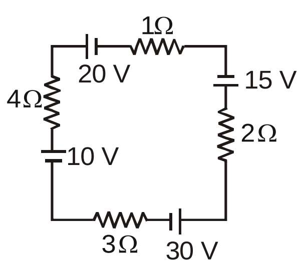
Applying KVL around the single loop in anticlockwise direction, starting from point A, we have



+ IR	+	Ir	–	E	=	0
In the opposite direction to current		In the opposite direction to current		From positive to negative terminal		

Hence, $I = \frac{E}{R + r}$

Ex.15 Find current in the circuit



Sol. ∴ all the elements are connected in series

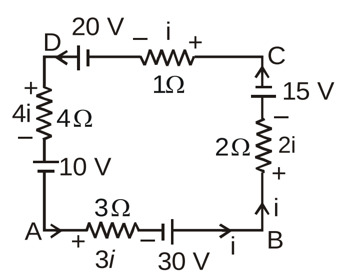
∴ current in all of them will be same

let current = i

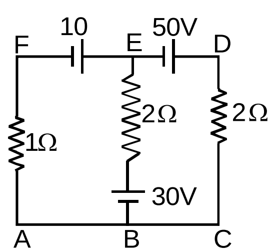
Applying kirchhoff voltage law in ABCDA loop

10 + 4i – 20 + i + 15 + 2i – 30 + 3i = 0

10 i = 25 ⇒ i = 2.5 A



Ex.16 Find the current in each wire applying only kirchhoff voltage law



Sol. Applying kirchhoff voltage law in loop ABEFA

$i_1 + 30 + 2(i_1 + i_2) - 10 = 0$

$3i_1 + 2i_2 + 20 = 0$ ...(i)

Applying kirchhoff voltage law in BCDEB

$+ 30 + 2(i_1 + i_2) + 50 + 2i_2 = 0$

$4i_2 + 2i_1 + 80 = 0$

$2i_2 + i_1 + 40 = 0$ ...(ii)

Solving (i) and (ii)

$3[-40 - 2i_2] + 2i_2 + 20 = 0$

$-120 - 4i_2 + 20 = 0$

$i_2 = -25$ A

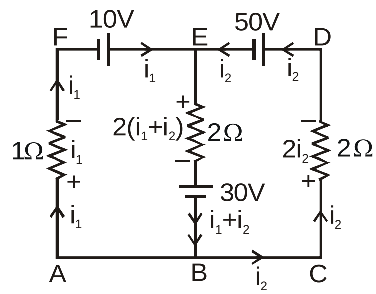
and $i_1 = 10$ A

$\therefore$ $i_1 + i_2 = -15$ A

current in wire AF = 10 A from A to E

current in wire EB = 15 A from B to E

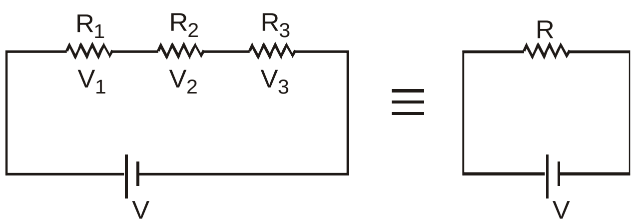
current in wire DE = 25 A from D to C



7. COMBINATION OF RESISTANCE

A number of resistance can be connected in a circuit and any complicated combination can be, in general, reduced essentially to two different types, namely series and parallel combinations.

(a) Resistance in Series



(i) In this combination the resistance are joined end to end. The second end of each resistance is joined to first end of the next resistance and so on. A cell is connected between the first end of first resistance and second end of last resistance. Figure shows three resistances R_1, R_2 and R_3 connected in this way. Let V_1, V_2 and V_3 are the potential differences across these resistances.

(ii) In this combination current flowing through each resistance will be same and will be equal to current supplied by the battery.

(iii) As resistances are different and current flowing through them is same, hence potential differences across them will be different. Applied potential difference will be distributed among three resistances directly in their ratio.

As i is constant, hence $V \propto R$

i.e., $V_1 = iR_1$, $V_2 = iR_2$, $v_3 = iR_3$

(iv) If the potential difference between the points A and D is V, then

$V = V_1 + V_2 + V_3 = i(R_1 + R_2 + R_3)$

(v) If the combination of resistances between two points is replaced by a single resistance R such that there is no change in the current of the circuit in the potential difference between those two points, then the single resistance R will be equivlaent to combination and V = i R i.e.,

$iR = i(R_1 + R_2 + R_3)$ or $\mathbf{R = R_1 + R_2 + R_3}$

(v) Thus in series combination of resistances, important conclusion are

(a) Equivalent Resistance > highest individual resistance

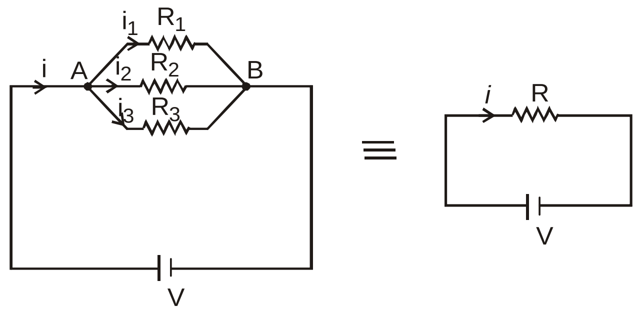
(b) Current supplied by source = Current in each resistance

$$\text{or } \frac{V}{R_1+R_2+R_3} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3}$$

(c) The total potential difference V between points A and B is shared among the three resistances directly in their ratio

$$V_1 : V_2 : V_3 = R_1 : R_2 : R_3$$

(b) Resistance in Parallel



(i) When two or more resistance are combined in such a way that their first ends are connected to one terminal of the battery while other ends are connected to other terminal, then they are said to be connected in parallel. Figure shows three resistances R_1, R_2 and R_3 joined in parallel between two points A and B. Suppose the current flowing from the battery is i. This current gets divided into three parts at the junction A. Let the currents in three resistance R_1 , R_2 and R_3, are i_1, i_2, i_3 respectively.

(ii) Suppose potential difference between points A and B is V. Because each resistance is connected between same two points A and B, hence potential difference across each resistance will be same and will be equal to applied potential difference V.

(iii) Since potential difference across each resistance is same, hence current approaching the junction A is divided among three resistances reciprocally in their ratio.

As V is constant, hence $i \propto (1/R)$ i.e.,

$$i_1 = \frac{V}{R}, \qquad i_2 = \frac{V}{R_2} \text{ and } i_3 = \frac{V}{R_3}$$

(iv) Because i the main current which is divided into three parts i_1, i_2 and i_3 at the junction A.

$$\text{hence, } i = i_1 + i_2 + i_3 = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

If the equivalent resistance between the points A and B is R, then $i = \frac{V}{R}$

$$\text{Thus, } \frac{V}{R} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

(v) Thus in parallel combination of resistance important conclusion are :

(a) Equivalent resistance < lowest individual resistance

(b) Applied potential difference = Potential difference across each resistance.

$$\text{or } iR = i_1R_1 = i_2R_2 = i_3R_3$$

(c) Current approaching the junction A = Current leaving the junction B and current is shared among the three resistances in the inverse ratio of resistances

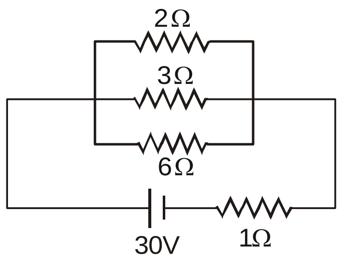
$$i_1 : i_2 : i_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

☛ **(i) If two or more resistance are joined in parallel then $i_1R = i_1R_2 = i_3R_3$...........**
i.e., iR = constant i.e., a low resistance joined in parallel always draws a higher current.
(ii) When two resistance R_1 and R_2 are joined in parallel, then

$$\frac{i_1R_1}{i_2R_2} = 1 \quad \text{or} \quad \frac{i_1^2R_1^2}{i_2^2R_2^2} = 1 \quad \text{or} \quad \frac{i_1^2R_1t}{i_2^2R_2t} = \frac{R_2}{R_1} \quad \text{or} \quad \frac{H_1}{H_2} = \frac{R_2}{R_1}$$

i.e., heat produced will be maximum in the lowest resistance.

Ex.17 Find current which is passing through battery.



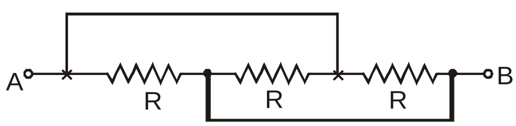
Sol. Here potential difference across each resistor is not 30 V

∵ battery has internal resistance here the concept of combination of resistors is useful.

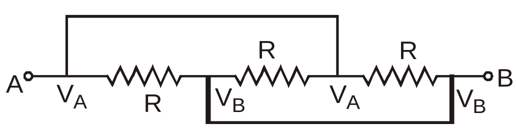
$R_{eq} = 1 + 1 = 2\Omega$

$i = \frac{30}{2} = 15A$

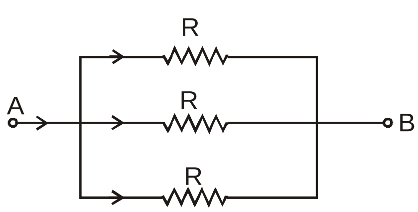
Ex.18 Find equivalent Resistance



Sol.

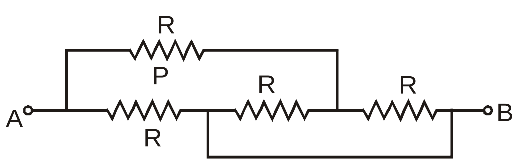


Here all the Resistance are connected between the terminals A and B. So, Modified circuit is

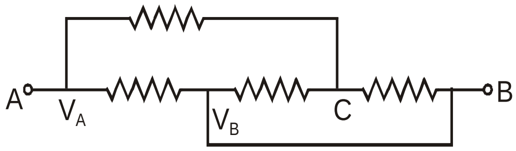


So $\quad R_{eq} = \frac{R}{3}$

Ex.19 Find the current in Resistance P if voltage supply between A and B is V volts

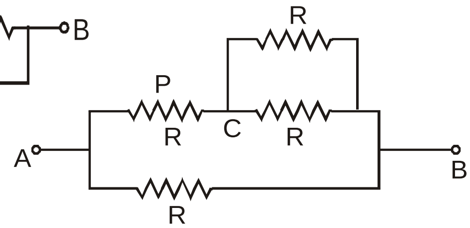


Sol. $R_{eq} = \frac{3R}{5}$

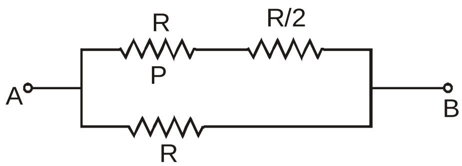


$I = \frac{5V}{3R}$

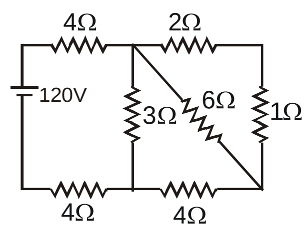
Modified circuit



Current in $P = \frac{R \times \frac{5V}{3R}}{1.5R + R} = \frac{2V}{3R}$



Ex.20 Find the current in 2 Ω resistance.



Sol. 2Ω, 1Ω in series = 3Ω

3Ω, 6Ω in parallel = $\frac{18}{9} = 2\Omega$

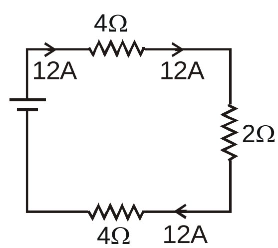
2Ω, 4Ω in series = 6Ω

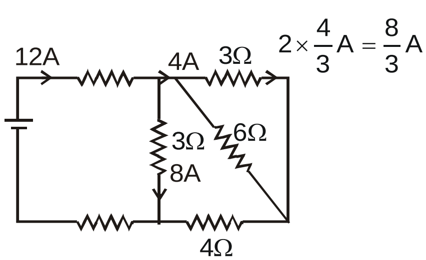
6Ω, 3Ω is parallel = 2Ω

$R_{eq} = 4 + 4 + 2 = 10\ \Omega$

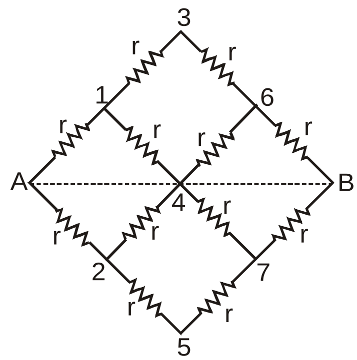
$i = \frac{120}{10} = 12A$

So current in 2Ω Resistance = $\frac{8}{3}A$





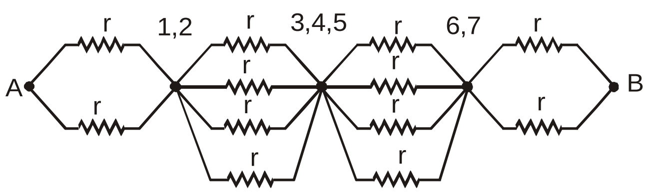
- **SPECIAL PROBLEMS**



We wish to determine equivalent resistance between A and B. In figure shown points (1,2) (3, 4, 5) and (6, 7) are at same potential Equivalent circuit can be redrawn as in figure shown.

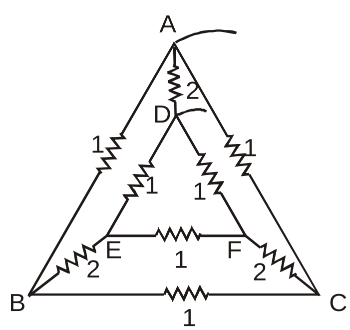
The equivalent resistance of this series combination is

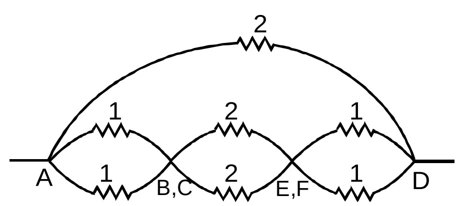
$$R_{eq.} = \frac{r}{2}+\frac{r}{4}+\frac{r}{4}+\frac{r}{2}=\frac{3r}{2}$$



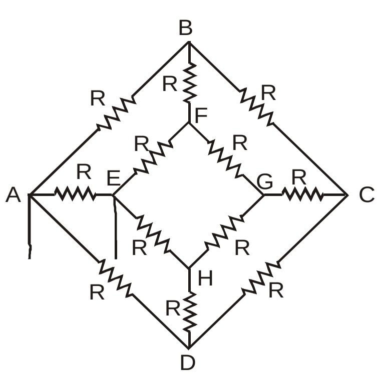
➢ In the figure shown, the resistances specified are in ohms. We wish to determine the equivalent resistance between point A and D. Point B and C, E and F are the the same potential so the circuit can be redrawn as in figure shown.

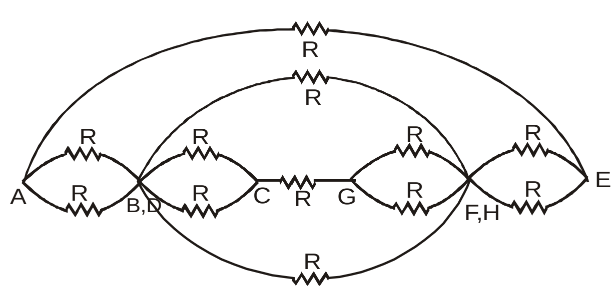
Thus the equivalent resistance is 1 Ω.





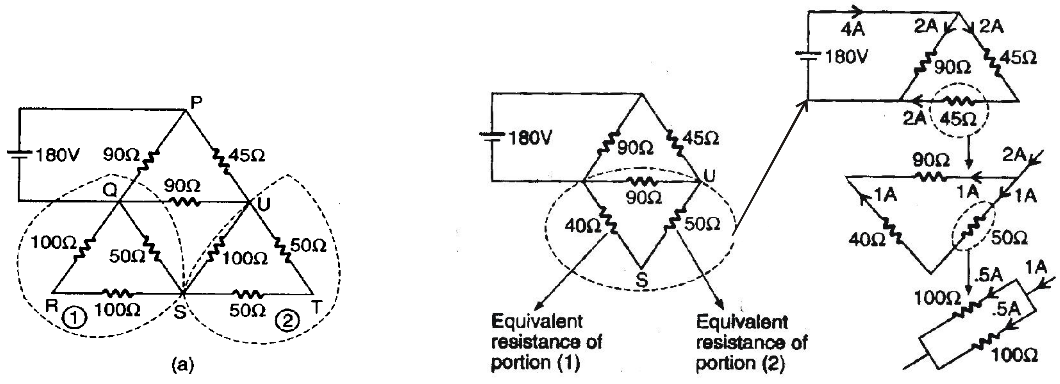
➢ In the network shown in figure shown all the resistances are equal, we wish to determine equivalent resistance between A and E. Point B and D have same potential, similarly F and H have same potential. The equivalent circuit is shown in figure shown. The equivalent resistance of network is 7R/2.





Ex.21 In the circuit shown in figure. (a) find the current flowing through the 100 Ω resistor connecting points U and S.

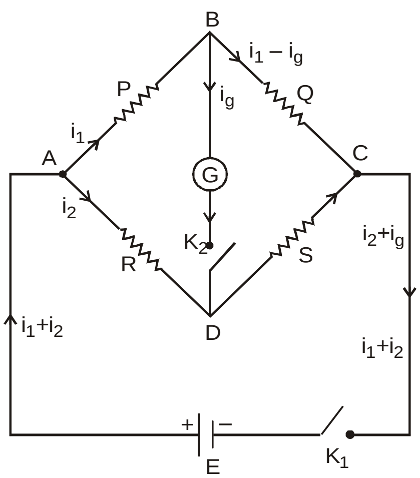
Sol. Figure (b) shows simplified circuit. The battery is directly attached to resistor 90Ω hence current in it is 2 A, see figure (c), The total resistance of second branch is also 90 Ω, hence current divides equally. Now current through 45 Ω resistor is 2 A and it is a combination of two equal 90 Ω resistors. Once again current divides equally. 90 Ω resistor is a series combination of 40 Ω and 50 Ω, hence current through them is equal, i.e.,



(a) (b) (c)

1 A. As 50 Ω resistor is a parallel combination of two equal 100 Ω resistors, they must have the same current i.e., 0.5 A

8. WHEATSTONE'S BRIDGE



(i) Wheatstone designed a network of four resistances with the help of which the resistance of a given conductor can be measured. Such a network of resistances is known as Wheastone's bridge.

(ii) In this bridge, four resistance P, Q, R and S are so connected so as to form a quadrilateral ABCD. A sensitive galvanometer and key K_2 are connected between diagonally opposite corners B and D, and a cell and key K_1 are connected between two other corners A and C (figure shown)

(iii) When key K_1 is pressed, a current i flows from the cell. On reaching the junction A, the current i gets divided into two parts i_1 and i_2. Current i_1 flows in the arm AB while i_2 in arm AD. Current i_1, on reaching the junction B gets further divided into two parts ($i_1 - i_g$) and i_g, along branches BC and BD respectively. At junction D, currents i_2 and i_g are added to give a current ($i_2 + i_g$), along branch DC. ($i_2 - i_g$) and ($i_2 + i_g$) add up at junction C to give a current ($i_1 + i_2$) or i along branch CE. In this way, currents are distributed in the different branches of bridge. In this position, we get a deflection in the galvanometer.

(iv) Now the resistance P,Q,R and S are so adjusted that on pressing the key K_2, deflection in the galvanometer becomes zero or current i_g in the branch BD becomes zero. In this situation, the bridge is said to be balanced.

(v) In this balanced position of bridge, same current i_1 flows in arms AB and BC and similarly same current i_2 in arms AD and DC. In other words, resistances P and Q and similarly R and S, will now be joined in series.

(vi) **Condition of balance :** Applying Kirchhoff's 2nd law to mesh ABDA, $i_1P + i_gG - i_2R = 0$...(1)

Similarly, for the closed mesh BCDB, we get, $(i_1 - i_g)\,Q - (i_2 + i_g)S - i_gG = 0$ ...(2) When bridge is balanced, $i_g = 0$. Hence eq. (1) & (2) reduce to

$i_1P - i_2R = 0$ or $i_1P = i_2R$ ....(3)

$i_1Q - i_2S = 0$ or $i_1Q = i_2S$ ...(4)

Dividing (3) by (4), we have, $\frac{P}{Q} = \frac{R}{S}$ ....(5)

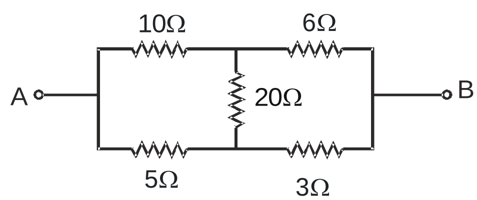
This is called as condition of balanced for Wheatstone's Bridge.

(vii) It is clear from above equation that if ratio of the resistance P and Q, and the resistance R are known, then unknwon resistance S can be determined. This is the reason that arms P and Q are called as ratio arms, arm AD as known arm and arm CD as unknown arm.

(viii) When the bridge is balanced then on inter-changing the positions of the galvanometer and the cell there is no effect on the balance of the bridge. Hence the arms BD and AC are called as conjugate arms of the bridge.

(ix) The sensitivity of the bridge depends upon the value of the resistances. **The sensitivity of bridge is maximum when all the four resistances are of the same order.**

Ex.22 Find equivalent resistance of the circuit between the terminals A and B.

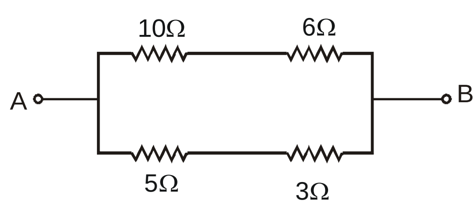


Sol. Since the given circuit is wheat stone bridge and it is in balance condition.

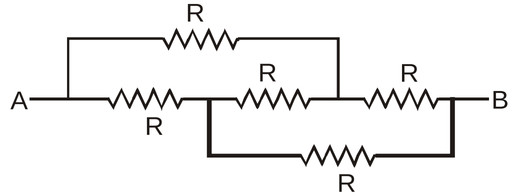
$\because$ $10 \times 3 = 30 = 6 \times 5$

hence this is equivalent to

$$R_{eq} = \frac{16 \times 8}{16 + 8} = \frac{16}{3}\Omega$$

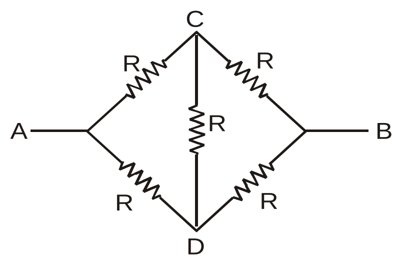


Ex.23 Find the equivalent resistance between A and B



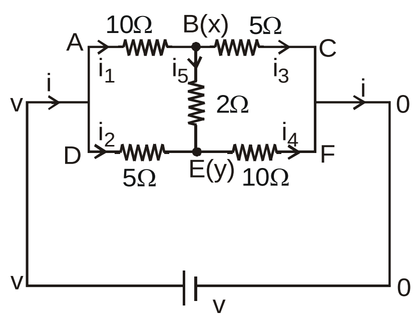
Sol. This arrangement can be modified as shown in figure since it is balanced wheat stone bridge.

$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$



8.1 Unbalanced Wheatstone Bridge

Ex.24



Find equivalent resistance ?

Sol. Let potential at point B is x and E is Y

$$R_{eq} = \frac{v}{i}$$

Applying KCl at point B

$$\frac{x-v}{10} + \frac{x-y}{2} + \frac{x-0}{5} = 0$$

$8x - 5y = v$ ...(1)

Applying KCL at point E

$$\frac{y-v}{5} + \frac{y-x}{2} + \frac{y-0}{10} = 0$$

$\Rightarrow$ $8y - 5x = 2v$ ...(2)

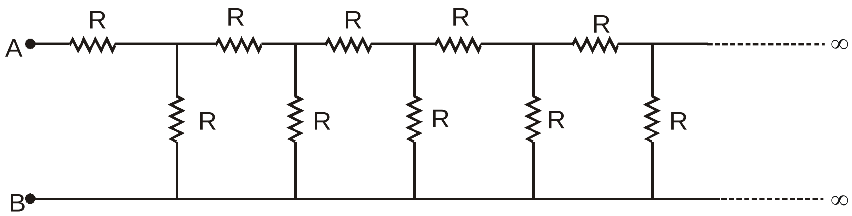
solving x & y $x = \frac{6v}{13}$, $y = \frac{7v}{13}$

current from branches BC & EF adds up to give total current (i) flowing in the circuit.

$$i = i_3 + i_4 = \frac{x-0}{5} + \frac{y-0}{10} = \frac{19v}{130}$$

$\because$ $i = \frac{V}{R_{eq}}$ $\therefore$ $R_{eq.} = \frac{130}{19}$

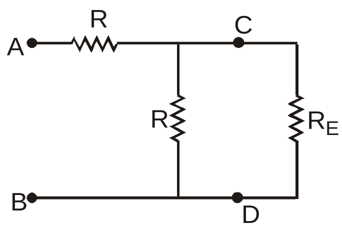
Ladder Problem :



Find the effective resistance between A & B ?

Sol. Let the effective resistance between A & B be R_E since the network is infinite long, removal of one cell from the chain will not change the network. The effective resistance between points C & D would also be R_E.

The equivalent network will be as shown below



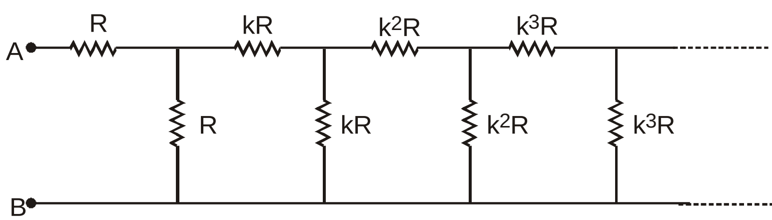
The original infinite chain is equivalent to R in series with R & R_E in parallel.

$$R_E = R + \frac{RR_E}{R+R_E}$$

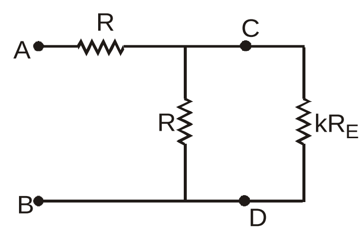
$R_ER + R_E^2 = R^2 + 2RR_E \Rightarrow R_E^2 - RR_E - R^2 = 0$

$$R_E = \frac{R(1+\sqrt{5})}{2}$$

Ex.25



Find the equivalent resistance between A & B ?



Sol. As moving rom one section to next one, resistance is increasing by k times. Since the network is infinitely long, removal of one section from the chain will bring a little change in the network. The effective resistance between points C & D would be kR_E (where R_E is the effective resistance)

$\therefore$ Effective R between A & B.

$$R_E = R + \frac{R(kR_E)}{kR_E + R}$$

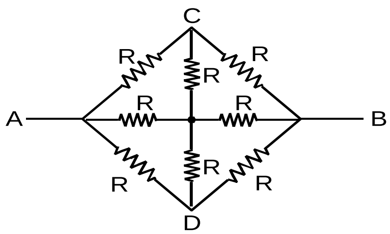
On solving we get

$$R_E = \frac{2kR - R + \sqrt{(R-2kR)^2 + 4kR^2}}{2k}$$

15. Symmetrical Circuits :

Some circuits can be modified to have simpler solution by using symmetry if they are solved by traditional method of KVL and KCL then it would take much time.

Ex.26 Find the equivalent Resistance between A and B

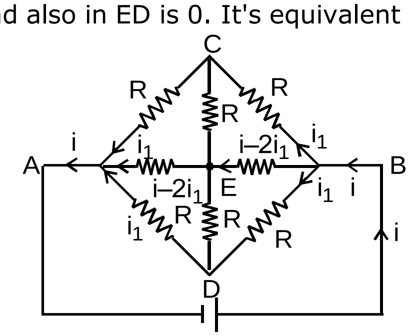


Sol. I Method : MIRROR SYMMETRY

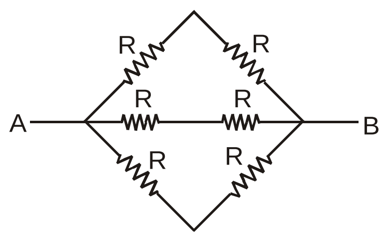
The branches AC and AD are symmetrical

$\therefore$ current through them will be same.

The circuit is also similar from left side and right side like mirror images with a mirror placed alone CD therefore current distribution while entering through B and an exiting from A will be same. Using all these facts the currents are as shown in the figure. It is clear that current in resistor between C and E is 0 and also in ED is 0. It's equivalent is shown in figure (b)



(fig.a)



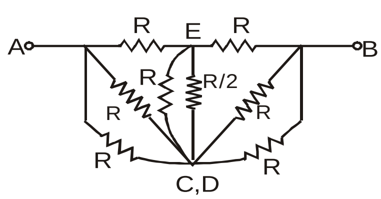
(fig.b)

$$R_{eq} = \frac{2R}{3}$$

II Method : FOLDING SYMMETRY

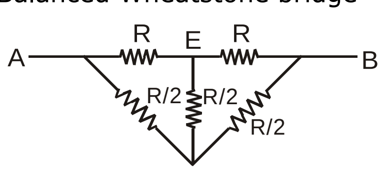
$\therefore$ The potential difference in R between (B, C) and between (B, D) is same $V_C = V_D$

Hence the point C and D are same hence circuit can be simplified as



This called folding.

Now , it is Balanced Wheatstone bridge

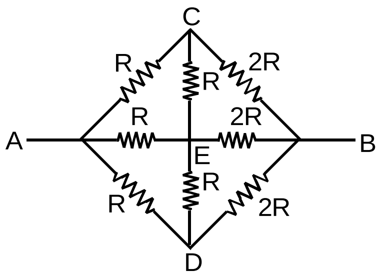


(C,D)

$$R_{eq} = \frac{2R \times R}{2R + R} = \frac{2R}{3}$$

☛ In II Method it is not necessary to know the currents in CA and DA

Ex.27 Find the equivalent Resistance between A and B



Sol. In this case the circuit has symmetry in the two branches AC and AD at the input

$\therefore$ current in them are same but from input and from exit the circuit is not similar

($\because$ on left R and on right 2R)

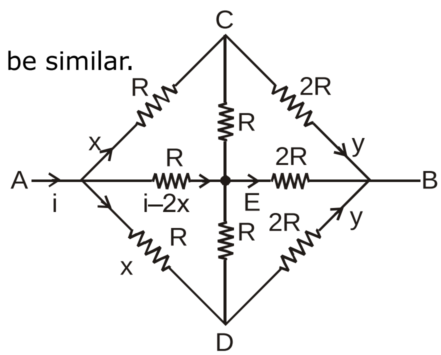
$\therefore$ on both sides the distribution of current will not be similar.

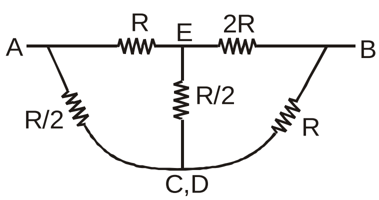
Here $V_c = V_d$

hence C and D are same point

So, the circuit can be simplified as

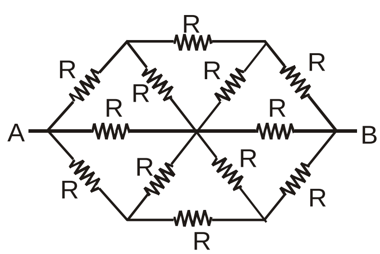
Now it is balanced wheat stone bridge.



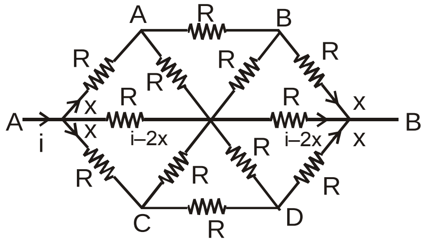


$$R_{eq} = \frac{3R \times \frac{3R}{2}}{3R + \frac{3R}{2}} = \frac{\frac{9}{2}R}{\frac{9}{2}} = R$$

Ex.28 Find the equivalent Resistance between A and B

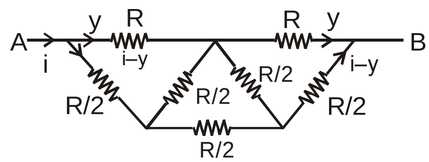


Sol.

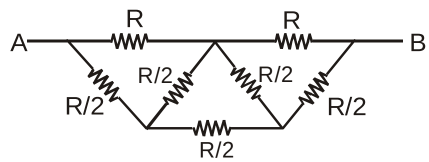


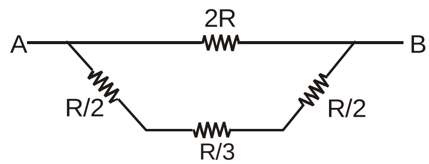
Here $V_A = V_C$ and $V_B = V_D$

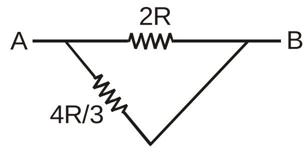
Here the circuit can be simplified as



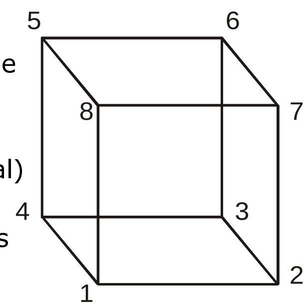
$$R_{eq}\ \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}} = \frac{4R}{5}$$ **Ans.**





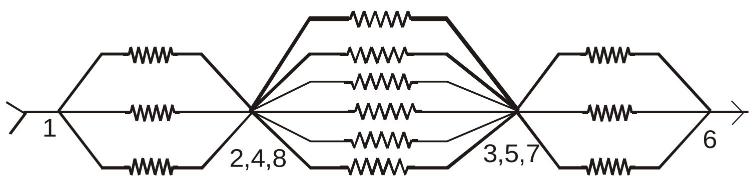


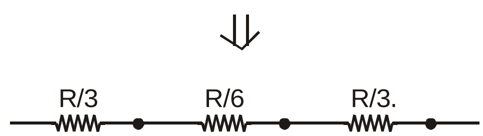
Ex.29 Twelve equal resistors each R Ω are connected to form the edges of a cube. Find the equivalent resistances of the network.



(a)When current enters at 1 & leaves at 6 (body diagonal)

Sol. Here 2, 4, 8 are equipotential points (if we move from 1 → 2, 4, 8 it comes along the edge & 6 → 2, 4, 8 it comes along face diagonal). Similarly 3, 5, 7 are equipotential points.

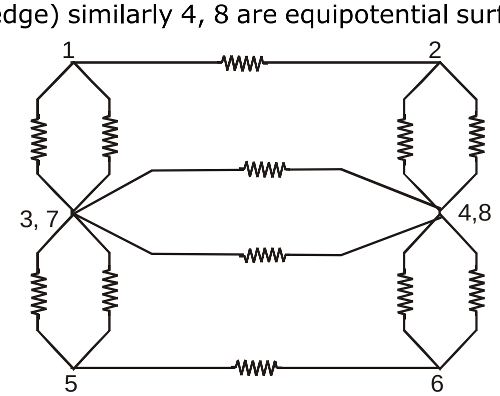




$$R_{eq} = \frac{5R}{6}$$

(b) When current enters at 1 and leaves at 2

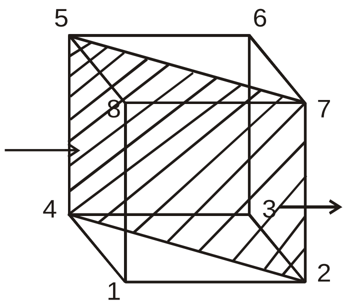
Sol. Here 3, 7 are equipotential surface (if we move from 1 → 3, 7 we have along face and 2, →3, 7 we move along edge) similarly 4, 8 are equipotential surface.

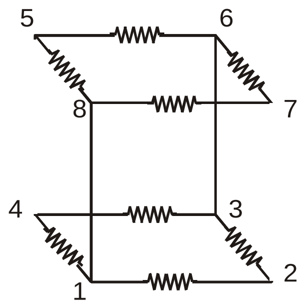


$$R_{eq} = \frac{7R}{12}$$

(c) When current enters at 1 and leaves at 3

Sol. If we cut the cube along the plane passing through 2, 4, 5, 7 then by mirror symmetry, the final configuration will be





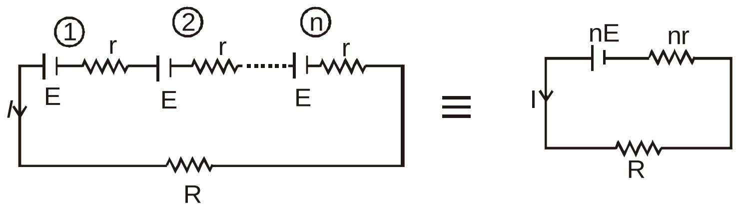
$$R_{eq} = \frac{3R}{4}$$

9. COMBINATIONS OF CELLS

A cell is used to maintain current in an electric circuit. We cannot obtain a strong current from a single cell. Hence need arises to combine two or more cells to obtain a strong current. Cells can be combined in three possible ways :

(A) In series, **(B)** In parallel, and **(C)** In mixed grouping.

(A) Cells in Series



In this combination, cells are so connected that –ve terminal of each cell is connected with the +ve terminal of next and so on. Suppose n cells are connected in this way. Let e.m.f and internal resistance of each cell are E and r respectively.

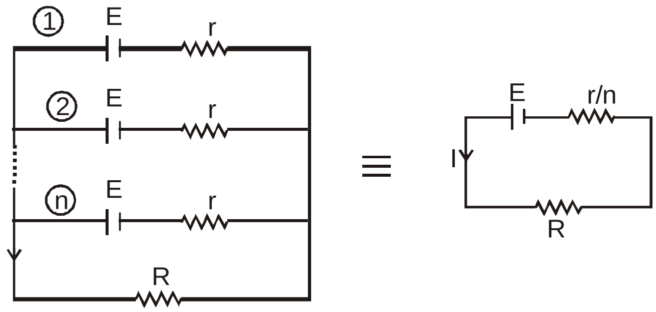
Net e.m.f of the cells = nE. Total internal resistance = nr. Hence total resistance of the circuit = nr + R.

If total current in the circuit is I, then $I = \frac{\text{net e.m.f}}{\text{Total Resistance}} = \frac{nE}{nr + R}$ ...(1)

Case (i) : If nr < < R, then $I \cong nE/R$ i.e., if total internal resistance of the cells is far less than external resistance, then current obtained from the cells is approximately equal to n times the current obtained from a single cell. Hence cells, whose total internal resistance is less than external resistance, just be joined in series to obtain strong current.

Case (ii) : If nr >> R, then $I \cong \frac{nE}{nr} = \frac{E}{r}$ i.e., if total internal resistance of the cells is much greater than the external resistance, then current obtained from the combination of n cells is nearly the same as obtained from a single cell. Hence there is no use of joining such cells in series.

(B) Cells in Parallel



(I) When E.M.F's and internal resistance of all the cells are equal : In this combination, positive terminals of all the cells are connected at one point and negative terminals at other point. Figure shown such cells connected in parallel across some external resistance R. Let e.m.f and internal resistance of each cell are E and r respectively.

Because all the cells are connected in parallel between two points, hence e.m.f of battery = E.

Total internal resistance of the combination of n cells = r/n

Because external resistance R is connected in series with internal resistance, hence total resistance of the circuit = (r/n) + R

If current in external resistance is I, then

$$I = \frac{\text{net E.M.F}}{\text{Total resistance}} = \frac{E}{(r/n)+R} = \frac{nE}{r+nR}$$

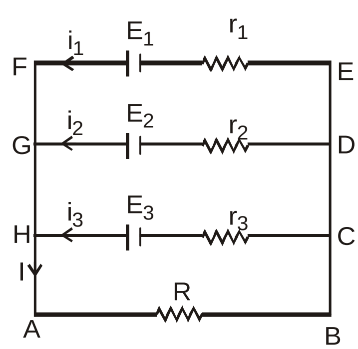
Case (I) : If r << R, the $I \cong \frac{nE}{nR} = \frac{E}{R}$ i.e., if internal resistance of the cells is much less than external resistance, then total current obtained from combination is nearly equal to current given by one cells only. Hence there is no use of joining cells of low internal resistance in parallel.

Case (II) : If r >> R, then $I \cong \frac{nE}{r}$ i.e., if the internal resistance of the cells is much higher than the external resistance, then total current is nearly equal to n times the current given by one cell. Hence cells of high internal resistance must be joined in parallel to get a strong current.

(II) When emf's and internal resistance of all the cells connected in parallel are different.

In this case, total current in external resistance is obtained with the help of Kirchhoff's laws. Figure shows three cells of e.m.f E_1, E_2 and E_3 and internal resistances r_1, r_2 and r_3 connected in parallel across some external resistance R. Suppose currents given by three cells are i_1, i_2 and i_3. Hence according to Kirchhoff's first law, total current I in external resistance R, is given by

$$I = i_1 + i_2 + i_3 \qquad \text{...(1)}$$



Applying Kirchhoff's 2nd law to closed mesh ABEF we get

$$IR + i_1 r_1 = E_1 \quad \text{or} \quad i_1 = \left[\frac{(E_1 - IR)}{r_1}\right] \text{...(2)}$$

Similarly, for closed meshes ABDG and ABCH, we get

$$i_2 = \frac{E_2 - IR}{r_2} \qquad \text{....(3)}$$

and

$$i_3 = \frac{E_2 - IR}{r_3} \qquad \text{....(4)}$$

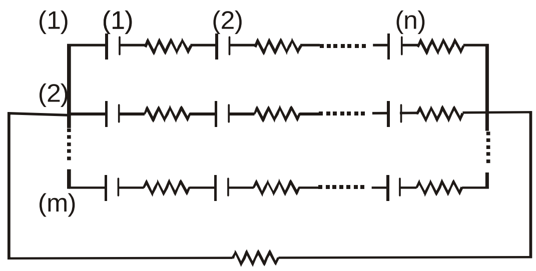
Substituting eq. (2), (3) and (4) in eq. (1), we have

$$I = \frac{E_1 - IR}{r_1} + \frac{E_2 - IR}{r_2} + \frac{E_3 - IR}{r_3} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} - IR\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

or $$I\left[1 + R\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)\right] = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}$$ or $$I = \frac{(E_1/r_1) + (E_2/r_2) + (E_2/r_3)}{1 + R(1/r_1 + 1/r_2 + 1/r_3)}$$

If n cells are joined in parallel, then $I = \dfrac{\sum_i \frac{E_i}{r_i}}{1 + R\sum_i \frac{1}{r_i}}$ and $E_{eq.} = \dfrac{\sum \frac{E_i}{r_i}}{\sum \frac{1}{r_i}}$, $r_{eq.} = \dfrac{1}{\sum \frac{1}{r_i}}$

(C) Cells in Mixed Grouping



In this combination, a certain number of cells are joined in series in various rows, and all such rows are then connected in parallel with each other.

Suppose n cells, each of e.m.f E and internal resistance r, are connected in series in every row and m such rows are connected in parallel across some external resistance R, as shown in figure.

Total number of cells in the combination = mn. As e.m.f. of each row = nE and all the rows are connected in parallel, hence net e.m.f of battery = nE.

Internal resistance of each row = nr. As m such rows are connected in parallel, hence total internal resistance of battery = $\left(\frac{nr}{m}\right)$

Hence total resistance of the circuit = $\left[\left(\frac{nr}{m}\right) + R\right]$

If the current in external resistance is I, then

$$I = \frac{\text{net e.m.f}}{\text{Total resistance}} = \frac{nE}{(nr/m) + R} = \frac{mnE}{nr + mR}$$

$$= \frac{mnE}{\left(\sqrt{nr} - \sqrt{mr}\right)^2 + 2\sqrt{nmrR}}$$

It is clear from above equation that I will be maximum when

$[(\sqrt{nr} - \sqrt{mR})^2 + 2\sqrt{nmrR}]$ is minimum.

This will be possible when the quantity $[\sqrt{nr} - \sqrt{mR}]^2$ is minimum. Because this quantity is in square, it can not be negative, hence its minimum value will be equal to zero, i.e.,

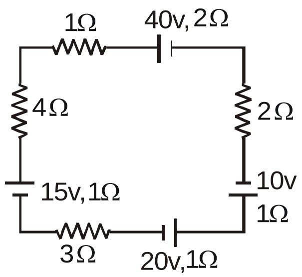
$$mR = nr \qquad \text{or} \qquad R = \frac{nr}{m}$$

i.e., In mixed grouping of cells, current in external resistance will be maximum when total internal resistance of battery is equal to external resistance.

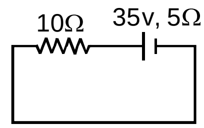
☛ **Because power consumed in the external resistance or load = I^2R, hence when current in load is maximum, consumed power in it is also maximum, Hence consumed power in the load will also be maximum when R = $\frac{nr}{m}$.**

$$I_{max} = \frac{mnE}{2mR} \quad \text{or} \quad \frac{mnE}{2nr} = \frac{nE}{2R} \quad \text{or} \quad \frac{mE}{2r}$$

Ex.30 Find the current in the loop.



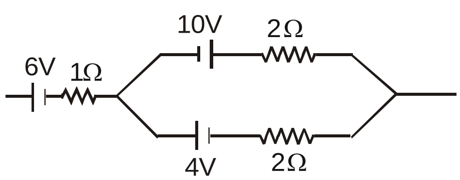
Sol. The given circuit can be simplified as



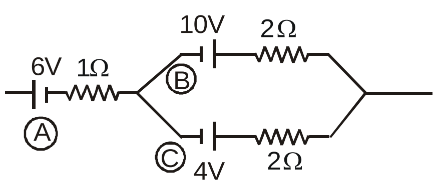
$$i = \frac{35}{10+5} = \frac{35}{15}$$

$$= \frac{7}{3}A \quad \Rightarrow \quad I = \frac{7}{3}A$$

Ex.31 Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



Sol.

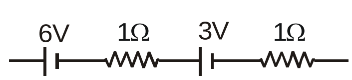


Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent

$$\epsilon_{BC} = \frac{\frac{10}{2} + \frac{-4}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{5-2}{1} = 3V$$

$r_{BC} = 1\ \Omega$

Now, $\epsilon_{ABC} = 6 - 3 = 3V$



$r_{ABC} = 2\Omega$ **Ans.**

10. ELECTRICAL POWER

The energy liberated per second in a device is called its power, the electrical power P delivered by an electrical device is given by

$$P = \frac{dq}{dt} V = VI$$

Power consumed by a resistor.

$$P = VI \quad = I^2R \quad = \frac{V^2}{R} \text{watt}$$

The power P is in watts when I is in amperes, R is in ohms and V is in volts.

The practical unit of power is 1 kW = 1000 W.

The formula for power $P = I^2R = VI = \frac{V^2}{R}$ is true only when all the electrical power is dissipated as heat and not converted into mechanical work, etc. simultaneously.

If the current enters the higher potential point of the device then electric power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source.)

(A). JOULE'S LAW OF ELECTRICAL HEATING

When an electric current flows through a conductor electrical energy is used in overcoming the resistance of the wire. If the potential difference across a conductor of resistance R is V volt and if a current of I ampere flows the energy expanded in time t seconds is given by

$$W = VIt \text{ joule}$$
$$= I^2Rt \text{ joule} = \frac{V^2}{R} t$$

The electrical energy so expanded is converted into heat energy and this conversion is called the heating effect of electric current.

The heat generated in joules when a current of I amperes flows through a resistance of R ohm for t seconds is given by

$$H = I^2Rt \text{ joule} = \frac{I^2Rt}{4.2} \text{ cal.}$$

This relation is known as Joule's law of electrical heating.

Ex.32 If bulb rating is 100 watt and 220 V then determine

(a) Resistance of filament

(b) Current through filament

(c) If bulb operate at 110 volt power supply then find power consume by bulb.

Sol. Bulb rating in 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consume is 100 W

Here V = 220

$P = 100$

$$\frac{V^2}{R} = 100$$

So $R = 484\ \Omega$

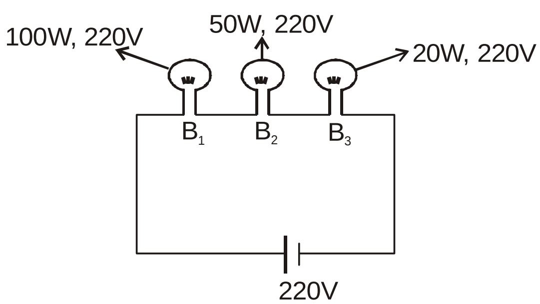
Since Resistance depends only on material hence it is constant for bulb

$$I = \frac{V}{R} = \frac{220}{22 \times 22} = \frac{5}{11}\text{Amp.}$$

power consumed at 110 V

$\therefore$ power consumed = $\frac{110 \times 110}{484} = 25\,W$

Ex.33 In the following figure, grade the bulb in order of their brightness :



Sol. $P_{rated} = \frac{V_{rated}^2}{R}$

$$R = \frac{V_{rated}^2}{P_{rated}}$$

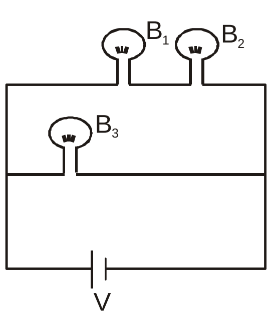
$\therefore \quad R_3 > R_2 > R_1$

Power = i^2R

As current passing through every bulb is same

$\therefore$ Brightness order is $B_3 > B_2 > B_1$

Ex.34



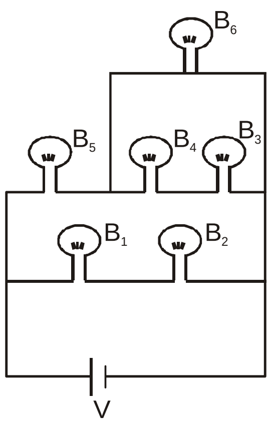
The above configuration shows three identical bulbs, Grade them in order of their brightness.

Sol. B_1 & B_2 withdraw less current as compared to B_3 because in series they give 2R resistance where as R is the resistance dut to B_3.

Power = i^2R

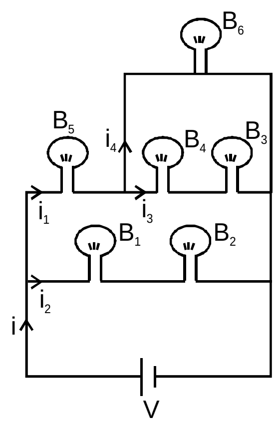
$\therefore$ Brightness order : $B_3 > B_2 = B_1$.

Ex.35



Grade the bulbs in order of their brightness (All bubls are identical)

Sol. As $i \propto \frac{1}{R}$



$i_1 : i_2 = \frac{3}{5R} : \frac{1}{2R} = 6 : 5$

$\therefore\ i_1 = \frac{6i}{11}, \quad i_2 = \frac{5i}{11}$

$i_3 : i_4 = \frac{1}{2R} : \frac{1}{R} = 1 : 2$

As $i_3 + i_4 = i_1$

$\therefore\ i_3 = \frac{2i}{11};\ i_4 = \frac{4i}{11}$

power = i^2R

$\therefore$ Order of Brightness : $B_5 > B_1 = B_2 > B_6 > B_4 = B_3$

(B) MAXIMUM POWER TRANSFER THEOREM

Let E be emf and r internal resistance of the battery. It is supplying current to an external resistance R

$$\text{current in circuit } I = \frac{E}{(R+r)}$$

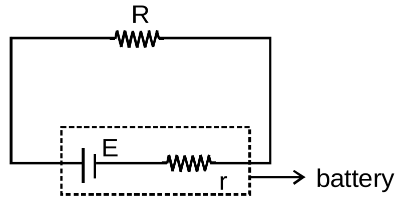
The power absorbed by load resistor R is

$$P = I^2R = \left(\frac{E}{R+r}\right)^2 R$$

For maximum power transfer we take the derivative of P w.r.t R, set it equal to zero and solve the equation for R.

$$\frac{dP}{dR} = 0$$

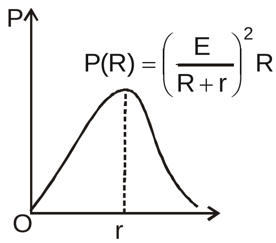
$$\frac{dP}{dR} = E^2 \frac{(R+r)^2 - R[2(R+r)]}{(R+r)^2} = 0$$

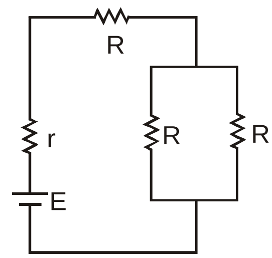


Solving for R, we have

$$(R + r)^2 - R\,(2)\,(R + r) = 0$$
$$(R + r) - 2R = 0$$
$$R = r$$

For a given real battery the load resistance maximizes the power if it is equal to the internal resistance of the battery.





The maximum power transfer theorem in general, holds for any real voltage source. The resitance R may be a single resistor or R may be the equivalent resistance of a collection of resistors.

11. INTRUMENTS

(A) AMMETER

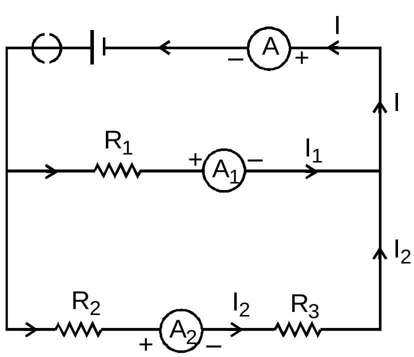
It is a device used to measure current and its always connected in series with the 'element' through which current is to be meaured, e.g., in figure (A) ammeter A_1 will measure the current (I_1) through resistance R_1, A_2 measures current (I_2) through R_2 and R_3 while A, measures current I(I_1 + I_2).

Regarding an ammeter it is worth noting that :

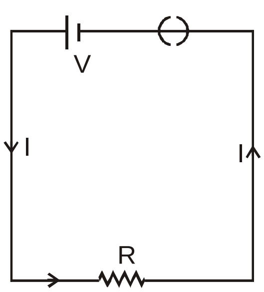
(1) The reading of an ammeter is always lesser than actual current in the circuit, e.g., true current in the resistance R in the circuit shown in figure (B) is $I = \frac{V}{R}$

However, when an ammeter of resistance r is used to measure current as shown in figure (C), the reading will be

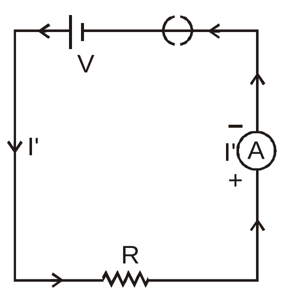
$$I' = \frac{V}{(R+r)} < I\left(=\frac{V}{R}\right)$$



(A)



(B)



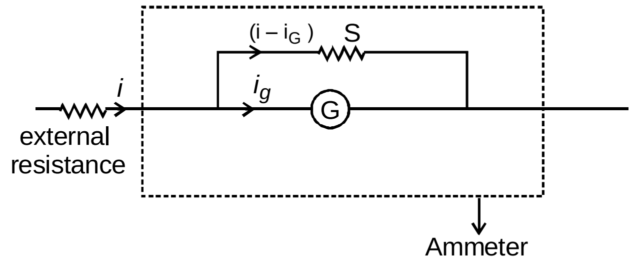
(C)

(2) Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be **ideal** if its resistance (r) is zero. However, as practically r ≠ 0, ideal ammeter cannot be realised in practice.

(3) To convert a galvanometer into an ammeter of a certain range say I, a small resistance S (called shunt) is connected in parallel with the galvanometer so that the current passing through the galvanometer of resistance G becomes equal to its full scale deflection value I_g. This is possible only if

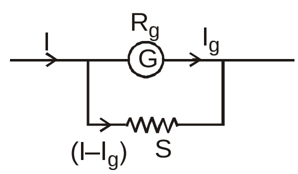
$$I_g G = (I - I_g)S$$

i.e., $$S = \frac{I_g}{(I - I_g)} G$$



Ex.36 What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm ?

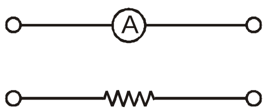
Sol. As in figure $R_g I_g = (I - I_g)S$



$\Rightarrow \quad 99 \times \frac{I}{10} = \left(I - \frac{I}{10}\right) \times S$

$\Rightarrow \quad S = 11\ \Omega$

For calculation it is simply a resistance

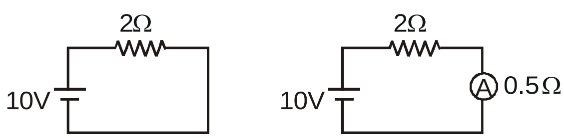


Resistance of ammeter

$$R_A = \frac{R_G.S}{R_G + S}$$

for $\quad S << R_G \qquad \Rightarrow \quad R_A = S$

Ex.37 Find the current in the circuit also determine percentage error in measuring in current through an ammeter (a) and (b).



(a) (b)

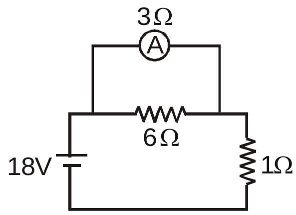
Sol. In A $\quad I = \frac{10}{2} = 5A$

In B $\quad I = \frac{10}{2.5} = 4A$

Percentage error is = $\frac{i - i'}{i} \times 100$ = 20% **Ans.**

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

Ex.38 Find the reading of ammeter. Is this the current through 6 Ω ?



Sol. $R_{eq} = \frac{3 \times 6}{3+6} + 1 = 3\Omega$

Current through battery

$$I = \frac{18}{3} = 6A$$

So, current through ammeter

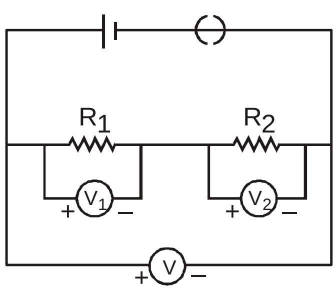
$$= 6 \times \frac{6}{9} = 4A$$

No, it is not the current through the 6Ω resistor.

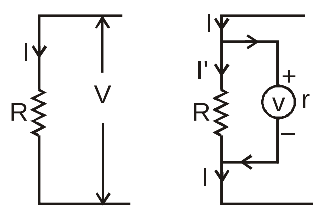
☛ **Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.**

(B) VOLTMETER

It is a device used to measure potential difference and is



(A)



(B)

always put in parallel with the 'circuit element' across which potential difference is to be measured e.g., in Figure (A) voltmeter V_1 will measure potential difference across resistance R_1, V_2 across resistance R_2 and V across $(R_1 + R_2)$ with $V = V_1 + V_2$

Regarding a voltmeter it is worth noting that :

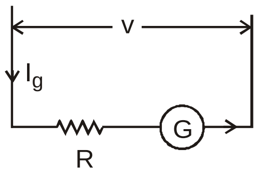
(1)The reading of a voltmeter is always lesser than true value, e.g., if a current I is passing through a resistance R [Fig. (B)], the true value V = IR. However, when a voltmeter having resistance r is connected across R, the current through R will become

$$I' = \frac{r}{(R+r)} I \quad \text{and so} \quad V' = I'R = \frac{V}{[1+(R/r)]}$$

and as voltmeter is connected across R its reading V' is lesser than V.

(2) Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be **ideal** if its resistance r is infinite, i.e., it draws no current from the circuit element for its operation. Ideal voltmeter has been realised in practice in the form of potentiometer.

(3) To convert a galvanometer into a voltmeter of certain range say V, a high resistance R is connected in series with the galvanometer so that current passing through the galvanometer of resistance G becomes equal to its full scale deflection value I_g. This is possible only if



$V = I_g (G + R)$ i.e., $R = \frac{V}{I_g} - G$

Ex.39 A voltmeter has a resistance of G ohm and range of V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

Sol. Full scale current $i_g = \frac{V}{G}$

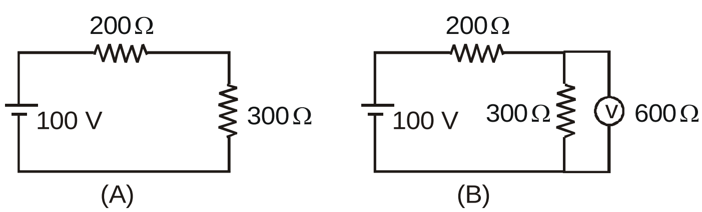
to change its range

$V_1 = (G + R_s)\, i_g$

$\Rightarrow$ $nV = (G + R_s)\, \frac{V}{G}$

$R_s = G(n - 1)$ **Ans.**

Ex.40 Find potential difference across the resistance 300 Ω in A and B.



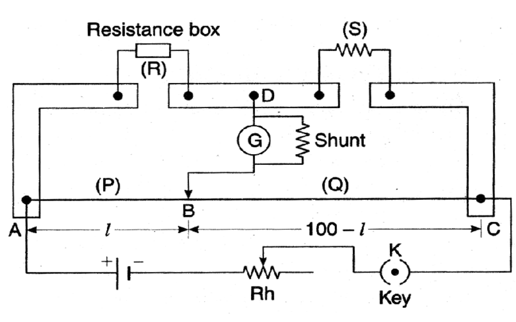
Sol. In (A) : Potential difference = $\frac{100}{200 + 300} = 300$ = 60 volt

In (B) Potential difference = $\frac{100}{200 + \frac{300 \times 600}{300 + 600}} \times \left[\frac{300 \times 600}{300 + 600}\right] = 50$ volt

We see that by connected voltmeter the voltage which was to be measured has charged. Such voltmeters are not good. If its resistance had been very large than 300 Ω then it would have not affected the voltage by much amount.

(C) METRE-BRIDGE

Metre-bridge is a sensitive device based on the principle of wheatstone-bridge, for the determination of the resistance of a conductor (wire). Its sensitivity is much more than that of the post-office box.



Metre-bridge is shown in figure AC is one metre long wire of manganin or constantan which is fixed along a scale on a wooden base. The area of cross-section of the wire is same at all places. The ends A and C of the wire are joined to two L-shaped copper strips carrying binding-screws as shown. In between these strips, leaving a gap on either side, there is a third copper strip having three binding screws. The middle screw D is connected to a sliding jockey B through a shunted - galvanometer G. The knob of the jockey can be made to touch at any point on the wire.

To measure the unknown resistance, the connection as shown in figure are made.

A resistance R is taken out from the resistance box and the key K is closed. Now the jockey is slided along the wire and a point is determined such that, on pressing the jockey on the wire at that point there is no deflection in the galvanometer G. In this position the points B and D are at the same potential. The point B is called 'null-point'. The lengths of both the parts AB and BC of the wire are read on the scale. Suppose the resistance of the length AB of the wire is P and that of the length BC is Q. Then, by the principle of Wheastone-bridge. We have,

$$\frac{P}{Q} = \frac{R}{S}$$

Let the length AB be l cm. Then the length BC will be = $(100 - l)$ cm.

$\therefore$ resistance of AB, i.e. P = $\frac{\rho l}{A}$, and resistance of BC, Q = $\rho\,(100 - l)/A$

where ρ is the specific resistance of the material of the wire and 'A' is the area of cross-section of the wire. Thus

$$\frac{P}{Q} = \frac{l}{(100 - l)} \qquad \text{...(i)}$$

Substituting this value of $\frac{P}{Q}$ in eq. (i), we get

$$\frac{l}{(100 - l)} = \frac{R}{S} \quad \text{or} \quad S = \frac{R(100 - l)}{l}$$

R is the resistance taken in the resistance box and l is the length measured. Hence, the value of resistance S can be determined from the above formula.

A number of observations are taken for different resistances in the resistance box and for each observation the value of S is calculated.

Finally, the experiment is repeated by interchanging the unknown resistances S and the resistance box. The mean of the values of S is then obtained.

Ex.41 In a meter bridge experiment, the value of unknown resistance is 2Ω. To get the balancing point at 40 cm distance from the same end, then what will be the resistance in the resistance box ?

Sol. Apply condition for balance wheat stone bridge,

$$\frac{P}{Q} = \frac{\ell}{100 - \ell} = \frac{P}{2} = \frac{100 - 40}{40}$$

$$P = 3\,\Omega$$ **Ans.**

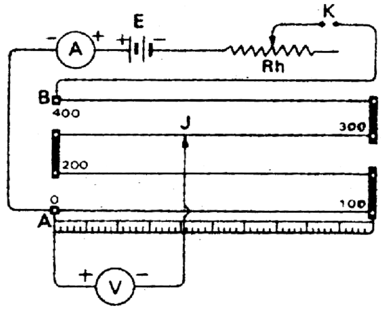
(D) POTENTIOMETER

A potentiometer is used to compare e.m.fs. of two cells or to measure internal resistance of a cell.

Principle : The potentiometer is based upon the principle that when a constant current is passed through a wire of uniform area of cross-section, the potential drop across any portion of the wire is directly proportional to the length of that portion.

Construction : A potentiometer consists of a number of segments of wire of uniform area of cross section stretched on a wooden board between two thick copper strips. Each segment of wire is 100 cm long. The wire is usually of constantan or manganin. A metre rod is fixed parallel to its length. A battery connected across the two end terminals sends current through the wire, which is kept constant by using a rheostat.

Theory : Let V be potential difference across certain portion of wire, whose resistance is R. If I is the current through the wire, then V = IR



We know that R = $\rho\frac{l}{A}$,

where l, A and ρ are length, area of cross-section and resistivity of the material of wire respectively.

$\therefore \quad V = I\rho\frac{l}{A}$

If a constant current is passed through the wire of uniform area of cross-section, then I and A are constants. Since, for a given wire, ρ is also constant, we have

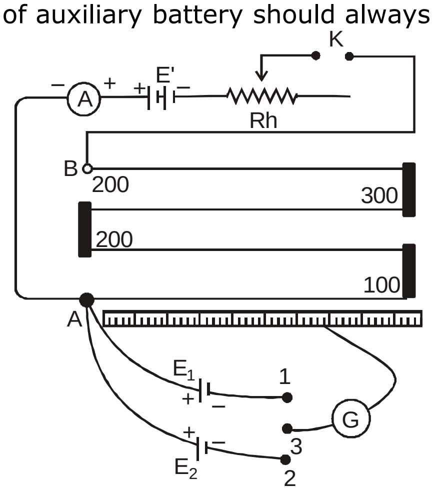
$V = \text{constant} \times l$

or $V \propto l$

Hence, if a constant current flows through a wire of uniform area of cross-section, then potential drop along the wire is directly proportional to the length of the wire.

Applications of a potentiometer. A potentiometer can be put to following uses :

1. **To compare e.m.fs. of two cells :** Two cells, whose e.m.fs. are E_1 and E_2, can be compared by making use of the ciruit as shown in figure. The positive poles of both the cells are connected to the terminal A of the potentiometer. The negative poles of the two cells are connected to terminals 1 and 2 of a two way key. while its common terminal is connected to a jockey j through a galvanometer G. An auxiliary or driver battery of e.m.f E′, an ammeter A, rheostat Rh and a one way key K are connected between the end terminals A and B of the potentiometer. Thus, the positive poles of the two cells as well as the positive pole of auxiliary battery are connected at the common point A. It may be pointed that the e.m.f of auxiliary battery should always be greater than the e.m.f of either of the two cells.



To compare the e.m.fs of the two cells, a constant current is passed through the potentiometer wire between points A and B. The current is kept constant by using the rheostat.

When the plug is put in the gap between the terminals 1 and 3 of the two way key, the cell of e.m.f. E_1 will come in the ciruit. Suppose the balancing length (between points A and J) is l_1. If x is the resistance per unit length of the potentiometer wire and I, the constant current flowing through it, then

$E_1 = (xl_1) I$

When the key is put in the gap between the terminals 2 and 3 and removed from the gap between 1 and 3, the cell of e.m.f E_2 wil be included in the circuit. Let the balancing length be l_2 in this case. Then,

$E_2 = (x\, l_2) I$

Dividing above equation

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

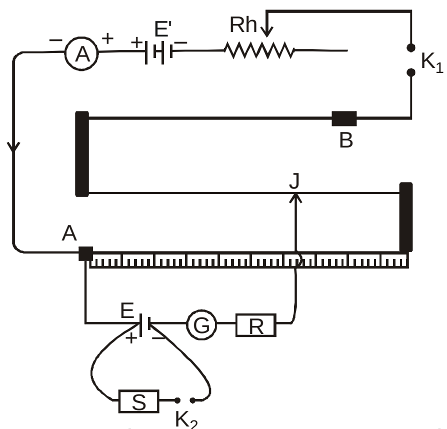
Note : It may be pointed out that the e.m.f of auxiliary battery should always be greater than the e.m.f. of the either of the two cells.

2. **To measure internal resistance of a cell.** The internal resistance of a cell may be found by using a potentiometer by setting up the circuit as shown in figure.

A constant current I is maintained through the potentiometer wire with the help of the rheostat.

Plug in the key K_2 is kept out and the jockey is moved over the potentiometer wire so as to balance the e.m.f. E of the cell, whose internal resistance is to be found. Let l_1 be the balancing length of the potentiometer wire between point A and jockey J. If x is resistance per unit length of the wire, then

$E = (x\, l_1) I$



With the help of resistance box S, introduce resistance say S and then put the plug in key K_2. Now find the balance point for the terminal potential difference V between the two poles of the cell. If l_2 is the balancing length, then

$V = (x\, l_2) I$

Dividing above equation , we have

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

The internal resistance* of the cell is given by

$$r = \left(\frac{E}{V} - 1\right) S$$

Using above equation , we have

$$r = \left(\frac{l_1}{l_2} - 1\right) S \quad \text{or} \quad r = \frac{l_1 - l_2}{l_2} \times S$$

Knowing the values of l_1, l_2 and S, the internal resistance r of the cell can be found.

Note : Apart from uses, a potentiometer can be used to compare unknown resistances and to calibrate a voltmeter or an ammeter.

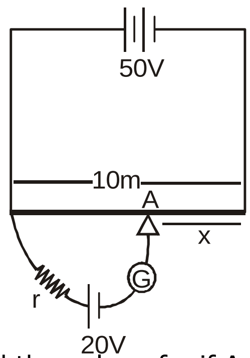
We use potentiometer for two tasks :

(i) to find emf of a cell

(ii) to find internal resistance of a celll

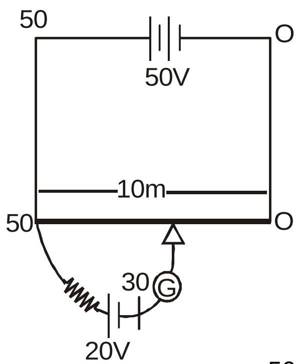
We will first analyse the first task → to find emf of a cell through some examples

Ex.42



Find the value of x if A is the null point ?

Sol.

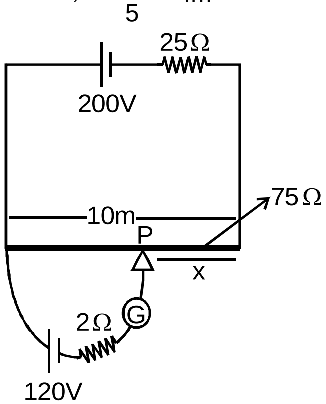


Potential gradient = $\frac{50}{10}$ = 5 V/m

$\therefore$ For 20V potential difference

$\Rightarrow \frac{20}{5} = 4m$ $\qquad \therefore \quad x = 10 - 4 = 6m$

Ex.43



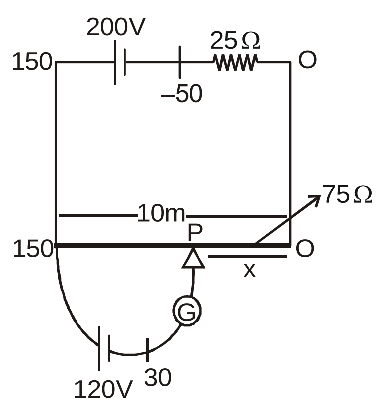
Find the value of x if P is a null point.

Sol. Current in the main circuit

200 – 25 i – 75 i = 0

i = 2 A

Potential gradient = $\frac{150V}{10m} = 15\,V/m$

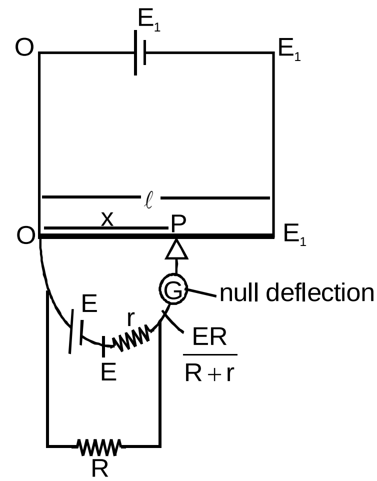


For 120 V $\Rightarrow \dfrac{120V/m}{15V} = 8m$

$\therefore$ x = 10 – 8 = 2m

Now we will analyse the other task to find internal resistance of the cell using potentiometer.

The main key point is that first analyse the main circuit then the auxillary circuit (supplementary circuit)



Potential gradient = $\dfrac{E_1}{\ell}$

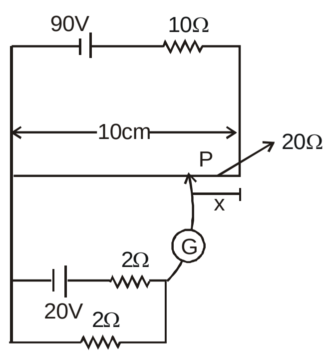
Now for the auxillary circuit

$$i = \frac{E}{R+r}$$

$E - ir = E - \dfrac{Er}{R+r} = \dfrac{ER}{R+r} \qquad \therefore \dfrac{ER}{R+r} = \dfrac{E_1}{\ell}x$

Let we take some examples to understand the topic in better way.

Ex.44

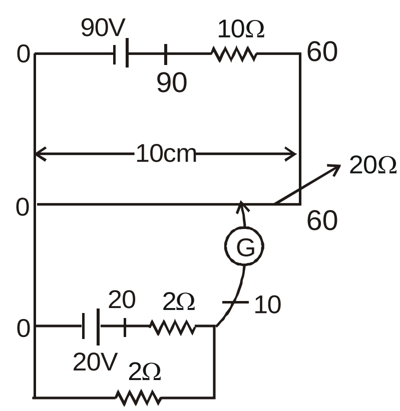


Find x if P is a null point?

Sol. First analysing the main circuit,

90 – 10i_1 – 20 i_1 = 0

i_1 = 3 A.



Potential gradient = $\dfrac{60V}{10m}$ = 6V/m.

Now analysing the auxillary circuit

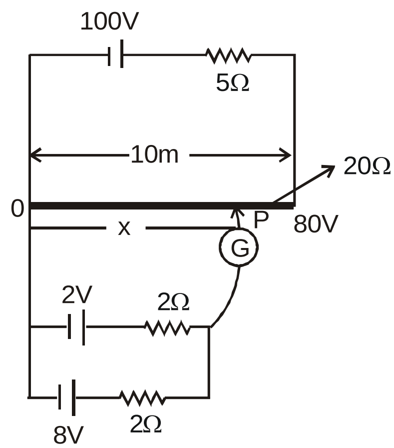
20 – 2i_2 – 2i_2 = 0

i_2 = 5 A.

For 10 $V_e \Rightarrow \dfrac{10}{6} = \dfrac{5}{3}$m

$\therefore$ x = $10 - \dfrac{5}{3} = \dfrac{25}{3}$ m

Ex.45



Find x if P is a null point ?

Sol. $100 - 5i_1 - 20\, i_1 = 0$

$i_1 = 4$ A

Potential gradient = $\frac{80}{10}$ = 8V/m

$8 - 2i_2 - 2i_2 - 2 = 0$

$i_2 = \frac{3}{2}$ A

for 5 volts $\Rightarrow \frac{5v}{8v/m} = \frac{5}{8}$m

$\therefore \quad x = \frac{5}{8}$m