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# PHYSICS

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# ELASTICITY & THERMAL EXPANSION

*THEORY AND EXERCISE BOOKLET*

## CONTENTS




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## 1. DEFINATION

Elasticity is that property of the material of a body by virtue of which the body opposes any change in its shape or size when deforming forces are applied to it, and recovers its original state as soon as the deforming forces are removed.

**On the basis of defination bodies may be classified in two types :**

**(a) Perfectly Elastic (P.E.) :** If body regains its original shape ans size completely after removal of force.

**Nearest approach P.E. :** quartz-fibre

**(b) Perfectly Plastic (P.P.) :** If body does not have tendency to recover its original shape and size.

**Nearest Approach P.P. :** Peetty

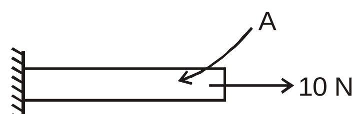
**Limit of Elasticity :** The maximum deforming force upto which a body retains its property of elasticity is called the limit of elasticity of the material of the body.

## 2. STRESS

When a deforming force is applied to a body, it reacts to the applied force by developing a reaction (or restoring force which, from Newton's third law, is equal in magnitude and opposite in direction to the applied force. *Thereaction force per unit area of the body which is called into play due to the action of the applied force is called stress.* Stress is measured in units of force per unit area, i.e. $Nm^{-2}$. Thus.

$$\text{Stress} = \frac{F}{A}$$

where F is the applied force and A is the area over which it acts.




**Unit of stress : $N/m^2$**

**Dimension of stress : $M^1L^{-1}T^{-2}$**

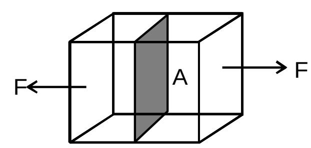
### 2.1 Types of stress :

**Three Types of Stress :**

**(A) Tensile Stress :** Pulling force per unit area.

It is applied parallel to the length

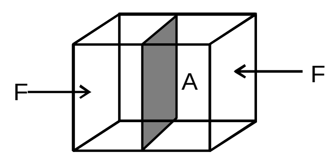
It causes increase in length or volume




**(B) Compressive Stress :** Pushing force per unit area.

It is applied parallel to the length

It causes decrease in length or volume




**(C) Tangential Stress :** Tangential force per unit area. It causes shearing of bodies.

**Note :**

**1.** If the stress is normal to surface called normal stress.

**2.** Stress is always normal to surface in case of change in length of a wire or volume of body.

**3.** When external force compresses the body $\Rightarrow$ Nature of atomic force will be repulsive.

**4.** When external forces expanses the body $\Rightarrow$ Nature of atomic force will be attractive.

**Difference between Pressure v/s Stress :**

| S. No. | Pressure | Stress |
|---|---|---|
| 1 | Pressure is always normal to the area. | Stress can be normal or tangential |
| 2 | Always compressive in nature | May be compressive or tensile in nature. |
| 3 | Scalar | Tensor |

***Ex.1*** ***A 4.0 m long copper wire of cross sectional area 1.2 $cm^2$ is stretched by a force of $4.8 \times 10^3$ N stress will be -***

***(A) $4.0 \times 10^7$ N/$mm^2$ (B) $4.0 \times 10^7$ KN/$m^2$ (C) $4.0 \times 10^7$ N/$m^2$ (D) None***

**Sol.** **[C]**

$$\text{Stress} = \frac{F}{A} = \frac{4.8 \times 10^3 N}{12 \times 10^{-4} m^2} = 4.0 \times 10^7 \ N/m^2$$

## 3. STRAIN

When a deforming force is applied to a body, it may suffer a change is size or shape. Strain is defined as the ratio of the change in size or shape to the original size or shape of the body. Strain is a number; it has no units or dimensions.

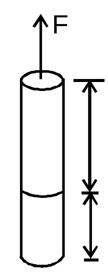
The ratio of the change in length to the original length is called *longitudinal strain.* The ratio of the change in volume to the original volume is called *volume strain.* The strain resulting from a change in shape is called shearing strain.

$$\text{Strain} = \frac{\Delta L}{L_0} = \frac{\text{final length} - \text{original length}}{\text{original length}} = \alpha \Delta T,$$

**Note : Original and final length should be at same temperature.**

### 3.1 Types of strain :

**Three Types of Strain :**

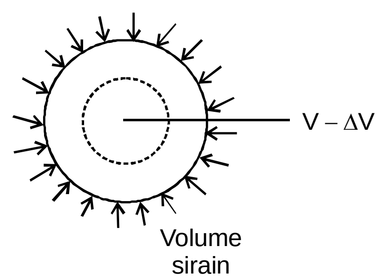



**(A)** **Linear Strain :** Change in length per unit length is called linear strain

$$\textbf{Linear Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$= \frac{\Delta L}{L}$$

**(B)** **Volume Strain :** Change in volume per unit volume is called volume strain.

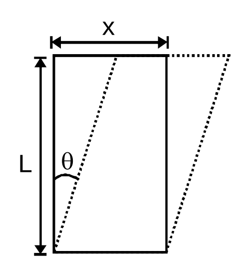



**Volume Strain**

$$= \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

**(C)** **Shear Strain :** Angle through which a line originally normal to fixed surface is turned.

$$\phi = \frac{x}{L}$$



Shear strain

**Note :** Strain is unitless.

***Ex.2*** ***A copper rod 2m long is stretched by 1mm. Strain will be -***

***(A) $10^{-4}$, volumetric*** ***(B) $5 \times 10^{-4}$, volumetric***

***(C) $5 \times 10^{-4}$, longitudinal*** ***(D) $5 \times 10^{-3}$, volumetric***

**Sol.** **[C]** Strain $= \dfrac{\Delta \ell}{\ell} = \dfrac{1 \times 10^{-3}}{2} = 5 \times 10^{-4}$, longitudinal

## 4. THERMAL STRESS

If the ends of a rod are rigidly fixed and its temperature is changed, then compressive stresses are set up in the rod. These developed stress are called thermal stress.

Thermal Stress = Y $\alpha$ $\Delta t$

Y $\rightarrow$ modulus of elasticity, $\alpha \rightarrow$ Coefficient of linear expansion

$\Delta t \rightarrow$ change in temperature

## 5. WORK DONE IN STRETCHING A WIRE

In stretching a wire work is done against internal restoring forces. This work is stored in body as elastic potential energy or strain energy.

If L = length of wire &

A = Cross-sectional Area.

$$Y = \frac{F/A}{x/L} \Rightarrow F = \frac{YA}{L}\ x$$

work done to increase dx length

$$dW = Fdx = \frac{YA}{L}\ xdx$$

Total work done = W = $\int_0^{\Delta L} \frac{YA}{L}\ xdx = \frac{1}{2}\ \frac{YA}{L}\ (\Delta L)^2$

Work done per unit volume = $\frac{W}{V} = \frac{1}{2}\ Y \left(\frac{\Delta L}{L}\right)^2$ $\qquad [\because\ V = AL]$

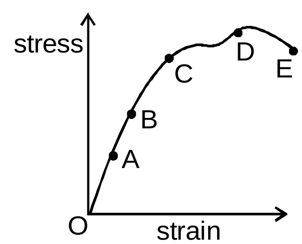
$$\frac{W}{V} = \frac{1}{2}\ Y\ (\text{strain})^2$$

$$\frac{W}{V} = \frac{1}{2} \text{x stress x strain} \qquad [\because\ Y = \frac{\text{Stress}}{\text{Strain}}]$$

$$\frac{W}{V} = \frac{1}{2}\ \frac{(\text{stress})^2}{Y} \Rightarrow \frac{W}{AL} = \frac{1}{2}\ \frac{F}{A} \times \frac{\Delta L}{L}$$

$$W = \frac{1}{2}\ F \times \Delta L = \frac{1}{2} \text{ load x elongation}$$

## 6. STRESS-STAIN CURVE




If we increase the load gradually on a vertical suspended metal wire,

**In Region OA :**

Strain is small (< 2%)

Stress $\propto$ Strain $\Rightarrow$ Hook's law is valid.

Slope of line OA gives Young's modulus Y of the material.

**In Region AB :** Stress is not proportional to strain, but wire will still regain its original length after removing of stretching force.

**In region BC :** Wire yields $\Rightarrow$ strain increases rapidly with small change in stress. This behavior is shown up to point C known as yield point.

**In region CD :** Point D correspondes to maximum stress, which is called point of breaking or tensile strength.

**In region DE :** The wire literally flows. The maximum stress corresponding to D after which wire begin to flow.

In this region strain increase even if wire is unloaded and rupture at E.

## 7. HOOKES' LAW

Hookes' law states that, within the elastic limit, the stress developed in a bodyis proportional to the strain produced in it. Thus the ratio of stress to strain is a constant. This constant is called the modulus of elasticity. Thus

Modulus of elasticity = $\frac{\text{stress}}{\text{strain}}$

Since strain has no unit, the unit of the modulus of elasticity is the same as that of stress, namely, $Nm^{-2}$

## 8. YOUNG'S MODULUS

Suppose that a rod of length l and a uniform crossectional area a is subjected to a logitudinal pull. In other words, two equal and opposite forces are applied at its ends.

$$\text{Stress} = \frac{F}{A}$$

The stress in the present case is called linear stress, tensile stress, or extensional stress. If the direction of the force is reversed so that $\Delta L$ is negative, we speak of compressional strain and compressional stress. If the elastic limit is not exceeded, then from Hooke's law

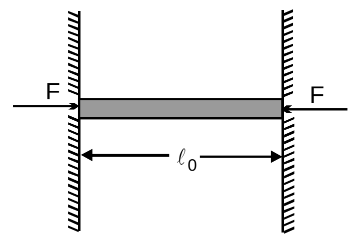
$$\text{Stress} \propto \text{strain}$$

or $$\text{Stress} = Y \times \text{strain}$$

or $$Y = \frac{\text{stress}}{\text{strain}} = \frac{F}{A}.\frac{L}{\Delta L} \qquad ...(1)$$

where Y, the constant of proportionality, is called the Young's modulus of the material of the rod and may be defined as the ratio of the linear stess to linear strain, provided the elastic limit is not exceeded. Since strain has no unit, the unit of Y is $Nm^{-2}$.

Consider a rod of length $\ell_0$ which is fixed between to rigid end separated at a distance $\ell_0$ now if the temperature of the rod is increased by $\Delta\theta$ then the strain produced in the rod will be :




$$\text{strain} = \frac{\text{length of the rod at new temperatrue} - \text{natural length of the rod at new temperature}}{\text{natural length of the rod at new temperature}}$$

$$= \frac{\ell_0 - \ell_0(1+\alpha\Delta\theta)}{\ell_0(1+\alpha\Delta\theta)} = \frac{-\ell_0\alpha\Delta\theta}{\ell_0(1+\alpha\Delta\theta)}$$

$\therefore$ $\alpha$ is very small so

strain = $-\alpha\,\Delta\,\theta$ (negative sign in the answer represents that the length of the rod is less than the natural length that means is compressed by the ends.)

We know that $\gamma = \frac{\text{stress}}{\text{strain}}$ then $F = \gamma\,\alpha\,\Delta T\,A$

**Note :**

**(A) For Loaded Wire :**

$$\Delta L = \frac{FL}{\pi r^2 Y} \qquad \left[\because Y = \frac{FL}{A\Delta L} \;\&\; A = \pi r^2\right]$$

for rigid body $\Delta L = 0$ so $Y = \infty$ i.e.
elasticity of rigid body is infinite.

**(B)** If same stretching force is applied to different wire of same material.

$$\Delta L \propto \frac{L}{r^2} \qquad \text{[As F and Y are const.]}$$

Greater the value $\Delta L$, greater will be elongation.

**(C) Elongation of wire by its own weight :**

In this case F = Mg acts at CG of the wire so length of wire which is stretched will be L/2

$$\Delta L = \frac{FL}{AY} = \frac{(Mg)\times L/2}{\pi r^2 Y} = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

[$\because$ $M = \rho AL$]

$$\Delta L = \frac{\rho g L^2}{2Y}$$

**Ex.3** ***A wire of length 1m and area of cross section 4 x $10^{-8}$ $m^2$ increases in length by 0.2 cm when a force of 16 N is applied. Value of Y for the material of the wire will be***
***(A) 2 × $10^6$ $N/m^2$ (B) 2 × $10^{11}$ $kg/m^2$ (C) 2 × $10^{11}$ $N/mm^2$ (D) 2 × $10^{11}$ $N/m^2$***

**Sol.** **[D]** By Hook's law

$$Y = \frac{F/A}{\ell/L} = \frac{FL}{A\ell}$$

$$Y = \frac{16 \times 1}{(4\times10^{-8})\,(0.2\times10^{-2})} = 2 \times 10^{11}\ N/m^2$$

**7.2 Bulk Modulus :**

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{\Delta P}{-\frac{\Delta V}{V}} \Rightarrow B = -\frac{V\Delta P}{\Delta V}$$

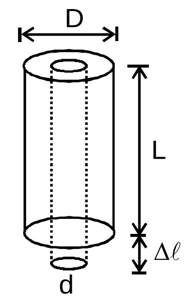
**7.3 Compressibility :**

$$k = \frac{1}{B} = -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)$$

**7.4 Modulus of Rigidity :**

$$\eta = \frac{\text{tangential stress}}{\text{tangential strain}} \Rightarrow \eta = \frac{F/A}{\phi}$$

Only solid can have shearing as these have definite shape.



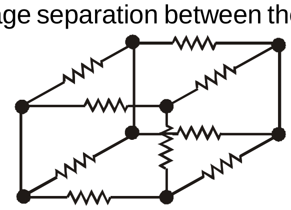

## 8. POISSION'S RATIO

$$\sigma = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{d/D}{\Delta L/L} \Rightarrow \sigma = \frac{dL}{\Delta LD}$$

Interatomic force constant = Young Modulus x Interatomic distance.

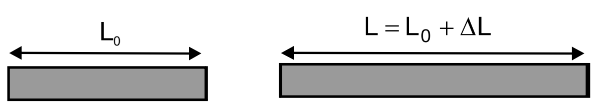
## 9. THERMAL EXPANSION

Most substances expand when they are heated. Thermal expansion is a consequence of the change in average separation between the constituent atoms of an object. Atoms of an object can be imagined to be connected to one another by stiff springs as shown in figure. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately $10^{-11}$ m. The average specing between the atom is about $10^{-10}$ m. As the temperature of solid increases, the atoms oscillate with greater amplitudes, as a result the average separation between them increases, consequently the object expands.



### 9.1 LINEAR EXPANSION

When the rod is heated, its increase in length $\Delta L$ is proportional to its original length $L_0$ and change in temperture $\Delta T$ where $\Delta T$ is in °C or K.




Before heating After heating

$dL = \alpha L_0 dT \Rightarrow \Delta L = \alpha L_0 \Delta T$ If a $\Delta T$ << 1

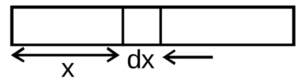
$\alpha = \frac{\Delta L}{L_0 \Delta T}$ where $\alpha$ is called the coefficient of linear expansion whose unit is $°C^{-1}$ or $K^{-1}$.

$L = L_0(1 + \alpha\Delta T)$ . Where L is the length after heating the rod.

**Variation of $\alpha$ with temperatue and distance**

**(a)** If $\alpha$ varies with distance, $\alpha = ax + b$

Then total expansion = $\int (ax + b)\,\Delta T dx$



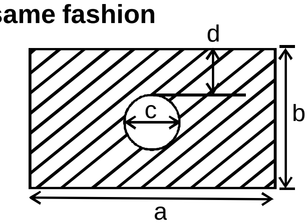

**(b)** If $\alpha$ varies with temperature, $\alpha = f(T)$

Then $\Delta L = \int \alpha L_0 dT$

**Note :**

- **Actually thermal expansion is always 3-D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one dimension and it is known as linear expansion.**
- **Avery linear dimenstions of the object changes in the same fashion**

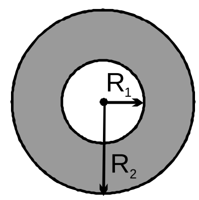
**Ex.4** ***A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.***




**Sol.** Distance between any two point on an object increases with increase in temperature. So, all dimensions a,b,c and d will increase.

**Ex.5** ***In the given figure, when temperature is increased then which of the following increases***

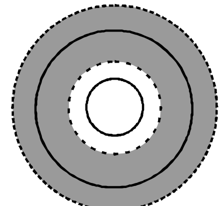
***(A) $R_1$*** ***(B) $R_2$*** ***(C) $R_2 - R_1$***




**Ans.** All of the above

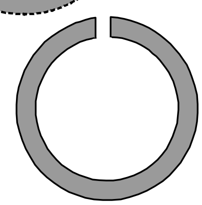
- - - - - - represents expanded Boundary

----------- represents original Boundary



As the intermolecular distance between atoms increases on heating the inner and outer perimeter increases. Also if the atomic arrangement in radial direction is observed then we can say that it also increases hence all A, B, C are true.

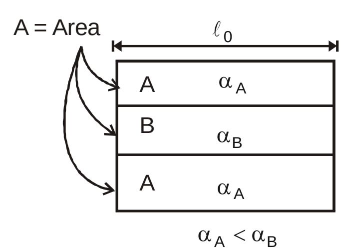
**Ex.6** ***A small ring having small gap is shown in figure on heating what will happen to size of gap.***



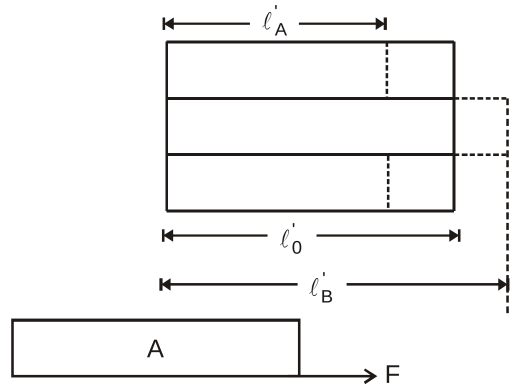
**Sol.** Gap will also increase. The reason is same as in above example.

**Note : Original and final length should be at same temperature.**

**Ex.7** ***Find the equillibrium length for the system after increasing temperature by $\Delta T$.***




**Sol.** here $\ell'_A$ and $\ell'_B$ are the natural length of the rod A and B after increase in temperature by $\Delta T$, and $\ell'_0$ is actual length after temperature increase by $\Delta T$.




So strain in A = $\dfrac{\ell'_0 - \ell'_A}{\ell'_A}$

and in B = $\dfrac{\ell'_B - \ell'_0}{\ell'_B}$

Now force balance

Now $\quad \dfrac{F}{A} = \gamma_A \dfrac{\ell'_0 - \ell'_A}{\ell'_A}$ ...(1)

and $\quad \dfrac{2F}{A} = \gamma_B \dfrac{\ell'_B - \ell'_0}{\ell'_B}$ ....(2)

(1) ÷ (2)

$$\frac{1}{2} = \frac{\gamma_A}{\gamma_B}\frac{[\ell'_0 - \ell_0(1+\alpha_A\Delta T)]\,\ell_0(1+\alpha_B\Delta T)}{\ell_0(1+\alpha_A\Delta T)[\ell_0(1+\alpha_B\Delta T) - \ell'_0\,]}$$

$$\ell'_0 = \frac{\ell_0(\gamma_B + 2\gamma_A)[1+(\alpha_B+\alpha_A)\Delta T)}{2\ell_A(1+\alpha_B\Delta T) + \gamma_B(1+\alpha_A\Delta T)}$$

## 9.2 Measurement of length by metallic scale :

**case (i)**

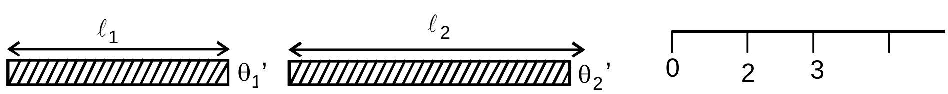
When object is expanded only

$\ell_2 = \ell_1\{1+\alpha_0(\theta_2 - \theta_1)\}$

$\ell_1$ = actual length of object at $\theta_1$°C = measure length of object at $\theta_1$°C.

$\ell_2$ = actual length of object at $\theta_2$°C = measured length of object at $\theta_2$°C

$\alpha_0$ = linear expansion coefficient of object




**case (ii)**

When only measureal instrument is expaneded actual length of object will not change but measured value (MV) decreases.

MV = $\ell_1\{1-\alpha_s(\theta_2 - \theta_1)\}$

$\alpha_s$ = linear expansion coefficient of measuring instrument.

**case (iii)**

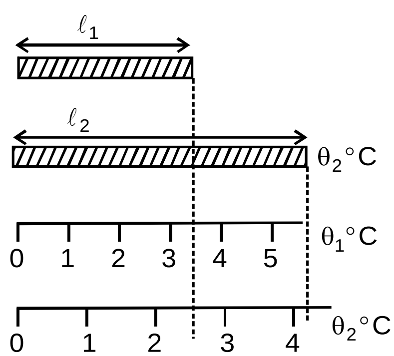
If both expanded simultaneously

MV = $l_1\{1 + (\alpha_0 - \alpha_s)(\theta_2 - \theta_1)\}$

(i) if $\alpha_0 > \alpha_s$, then measured value is more than actual value at $\theta_1$°C

(ii) If $\alpha_0 < \alpha_s$, then measured value is less than actual value at $\theta_1$°C

at $\theta_1$ °C MV = 3.4, $\quad \theta_2$ °C MV = 4.1




## 9.3 Effect of temperature on the time period of a pendulum :

The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{or} \qquad T \propto \sqrt{l}$$

As the temperature is increased length of the pendulum and hence, time period gets increased or a pendulum clock becomes slow and it loses the time,

$$\frac{T'}{T} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{l+\Delta l}{l}}$$

Here, we put $\Delta l = l\,\alpha\,\Delta\,\theta$ in place of $l\,\alpha\,\Delta T$ so as to avoid the confusion with change in time period. Thus,

$$\frac{T'}{T} = \sqrt{\frac{l + l\alpha\Delta\theta}{l}} = (1+\alpha\Delta\theta)^{1/2} \qquad \text{or} \qquad T' \approx T\left(1 + \frac{1}{2}\alpha\Delta\theta\right)$$

or $\Delta T = T' - T = \frac{1}{2} T\alpha\Delta\theta$

Time lost in time t (by a pendulum clock whose actual time period is T and the changed time period at some higher temperature T′) is

$$\Delta t = \left(\frac{\Delta T}{T'}\right)t$$

Similarly, if the temperature is decreased the length and hence, the time period gets decreased. A pendulum clock in this case runs fast and it gains the time.

$$\frac{T'}{T} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{l - l\alpha\Delta\theta}{l}} \approx 1 - \frac{1}{2}\alpha\Delta\theta \quad \text{or} \quad T' = T\left(1 - \frac{1}{2}\alpha\Delta\theta\right)$$

$$\Delta T = T - T' = \frac{1}{2}T\alpha\Delta\theta$$

and time gained in time t is the same, i.e.,

$$\Delta t = \left(\frac{\Delta T}{T'}\right)t$$

**Ex.8** ***A second's pendulum clock has a steel wire. The clock is calibrated at 20°C. How much time does the clock lose or gain in one week when the temperature is increased to 30°C? $\alpha_{steel} = 1.2 \times 10^{-5}\ (°C)^{-1}$.***

**Sol.** The time period of second's pendulum is 2 second. As the temperature increases length and hence, time period increases. Clock becomes slow and it loss the time. The change in time period is

$$\Delta T = \frac{1}{2}T\alpha\Delta\theta = \left(\frac{1}{2}\right)(2)(1.2 \times 10^{-5})(30° - 20°) = 1.2 \times 10^{-4}\ \text{s}$$

∴ New time period is ,

$$T' = T + \Delta T = (2 + 1.2 \times 10^{-4}) = 2.0012\ \text{s}$$

∴ Time lost in one week

$$\Delta t = \left(\frac{\Delta T}{T'}\right)t = \frac{(1.2 \times 10^{-4})}{(2.00012)}(7 \times 24 \times 3600) = 36.28\ \text{s}$$

## 9.4 SUPERFICIAL OR AREAL EXPANSION

When a solid is heated and its area increases, then the thermal expansion is called superficial or areal expansion. Consider a solid plate of side $l_0$ and linear expansion coefficient $\alpha_s$. Then $A_i = ab$

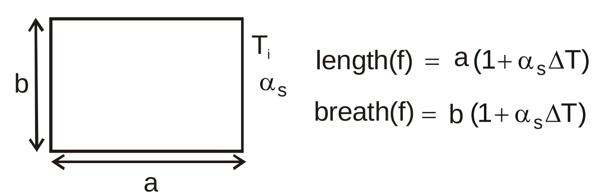
final Area = $l \times b = ab(1 + \alpha_s \Delta T)^2$

$= ab(1 + 2\alpha_s \Delta T) = ab(1 + \beta \Delta T)$

$A_f = A_i (1 + \beta \Delta T)$

$\beta = 2\alpha$

$\beta$ = coefficent of Area expansion.




***Isotropic Material***

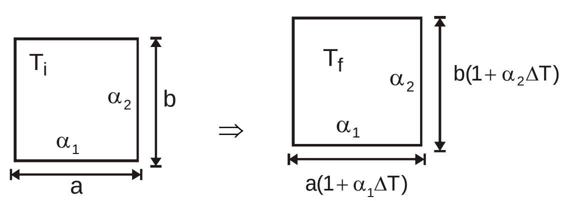
Material having coefficient of linear Expansion is same in all the direction.

***An isotropic Material***

Material having coefficient of linear Expansion is different for different direction.

**Note :** • Most of the time we take material as the isotropic material

For an isotropic material




$A_i = ab$

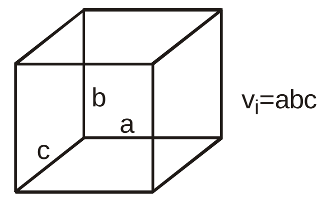
$A_f = ab(1 + \alpha_1 \Delta T)(1 + \alpha_2 \Delta T) = ab(1 + (\alpha_1 + \alpha_2)\Delta T + \alpha_1\alpha_2 \Delta T^2$

$= ab(1 + (\alpha_1 + \alpha_2)\Delta T) = A_i(1 + (\alpha_1 + \alpha_2)\Delta T)$

## 9.5 VOLUME OR CUBICAL EXPANSION

When a solid is heated and its volume increases, then the expansion is called volume expansion or cubical expansion.

**Note :** Now after increase in temp by $\Delta T$



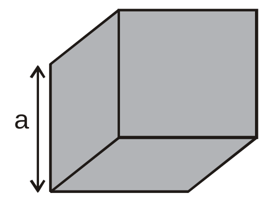

$V_f = a' \, b' c'$

$= a[1 + \alpha\Delta T]^3 bc$

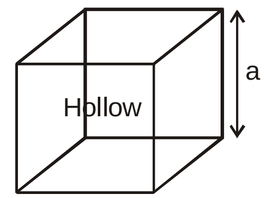
$= abc\,[1 + 3\,\alpha\,\Delta T] \qquad \therefore\ \alpha\Delta T << 1$

$v_f = v_i\,[1 + 3\alpha\,\Delta T]$

So $3\alpha = \gamma$ = coefficient of volume expansion.



Solid cube



Cubical Container of same material

1. When temperature changes the volume of the container and volume of the cube change in the same fashion because a changes in the same fashion.
2. In volume expansion of container we use $\gamma$ of the container material.

**For Isotroptic**

$$v_f = v_i\,(1 + 3\alpha\,\Delta T)$$

**For Isotroptic**

$$v_f = v_i[1 + (\alpha_1 + \alpha_2 + \alpha_3)\Delta T]$$

**Note :-**

(i) $\alpha : \beta : \gamma = 1 : 2 : 3$

(ii) And they are dependent of temperature.

## 9.6 Effect of temperature on density :

If the initial density of the body is $\rho_i$ having mass m and volume v then

$$\rho_i = \frac{m}{v}$$

If the temperature increases then volume should be changes and the final volume is given by $v_f = v\,(1 + \gamma\,\Delta T)$. So the final density

$$\rho_f = \frac{m}{v_f} \Rightarrow \rho_f = \frac{m}{v(1 + \gamma\Delta T)}$$

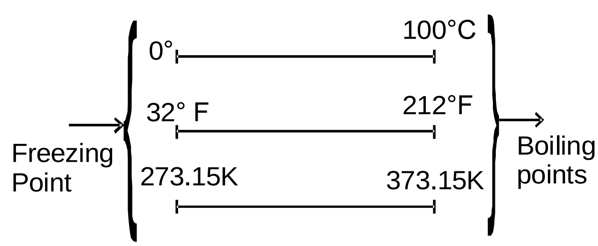
$\rho_f = \rho_i(1 + \gamma\Delta T)^{-1}$

from binomial theorem

$\rho_f = \rho_i\,(1 - \gamma\Delta T)$

## 9.7 Temperature scale

Relation between different scales.




K = Kelvin

C = Centigrade

F = Fahrenheit

100°C difference = 180° F difference

1°C difference = $\frac{9}{5}^{\circ}$ F difference

$$\frac{9C}{5} = F - 32$$

$$K = C + 273.15$$

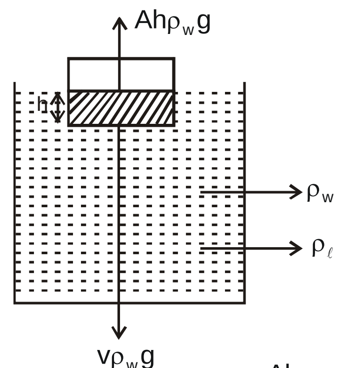
Relation between temperature. on two difference scales.
L.F. value = Lower fixed value
U.F. value = Upper fixed value

$$\frac{\text{Temperature on } S_1 \text{ scale} - \text{L.F. value of } S_1}{\text{U.F value of } S_1 - \text{L.F. value of } S_1} = \frac{\text{Temp. on } S_2 \text{ Scale} - \text{L.F. value of } S_2}{\text{U.F.value of } S_2 - \text{L.F. value of } S_2}$$

***Ex.9*** ***A faulty thermometer reads 5° at freezing point and 95° at boiling point then findout original reading in °C when it reads 50°.***

**Sol.** $\frac{50-5}{95-5} = \frac{x-0}{100} \Rightarrow \frac{45}{90} = \frac{x}{100} \Rightarrow x = 50$

**Effect of temperature of Buoyancy Force**




Initially at temperature T

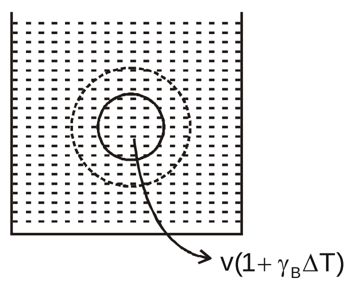
$$F_B = v\,\rho_l\, g$$

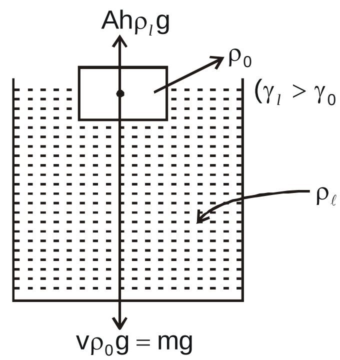
on increase temperature at $\Delta T$ then

$$F_B = \frac{V(1+\gamma_B\Delta T)\rho_\ell g}{(1+\gamma_\ell\Delta T)} = v\rho_\ell g\frac{(1+\gamma_B\Delta T)}{(1+\gamma_\ell\Delta T)}$$

(a) If $\gamma_B > \gamma_L$ & T $\uparrow$
then $F_B \downarrow$

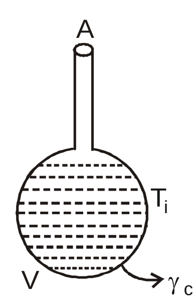
(a) If $\gamma_L > \gamma_B$ & T $\uparrow$
then $F_B \downarrow$, and T $\downarrow$
then $F_B \uparrow$




## 9.8 Barometer

Their is a capaillary tube which have coefficient of linear expansion $\alpha_c$ and a liquid of volume v of volume expansion coefficient v of volume expansion coefficient of $\gamma_\ell$ at temperature $T_i$. and given $3\alpha_c < \gamma_\ell$. The Area of cross-section of capillary tube is A.




Now temperature increases to $T_f$, So volume of liquid rises in the capillary. Let it rises to height H′. So volume rises in tube = $\Delta V$

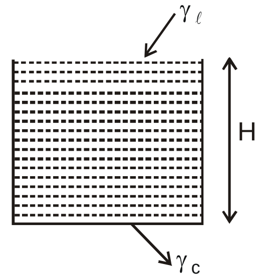
$\Delta V = V[1 + \gamma_\ell \Delta T] - V[1 + 3\alpha_c \Delta T] = V(\gamma_\ell - 3\alpha_c)\Delta T$

And Area of cross section of capillary = A′

$= A[1 + 2\alpha_C \Delta T]$

So height in capillary tube $H' = \frac{\Delta V}{A'} = \frac{V\Delta T(\gamma_\ell - 3\alpha_C)}{A(1+2\alpha_C\Delta T)}$

**Ex.10** ***What will happen to the water level if the vessel is heated ?***




**Sol.** (i) if $\gamma_\ell > \gamma_c$ then overflow occure and overflow

$= AH(1+\gamma_\ell \Delta t) - AH(1+\gamma_c \Delta T)$

(ii) if $\gamma_\ell < \gamma_c$

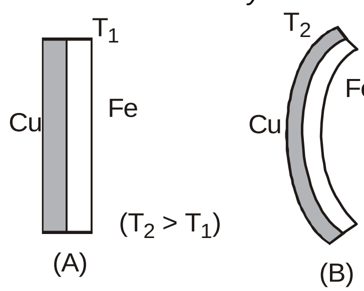
final volume $V_{fc} = AH(1+\gamma_C \Delta T)$

final volume $V_{f\ell} = AH(1+\gamma_\ell \Delta T)$

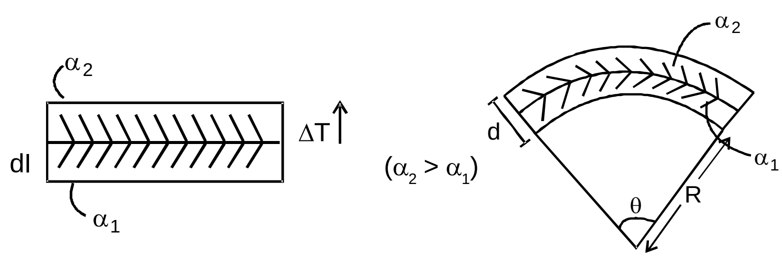
Now $A_F = A[1+2\alpha_C \Delta T]$

So $H'$ = final height = $\dfrac{H[1+y_\ell \Delta T]}{[1+2\alpha_c \Delta T]}$

**Note** If two strips of equal length but of different metals are placed on each other and riveted, the single strip so formed is called '**bimetallic strip'** [see given fig.]. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater $\alpha$ on outer side, i.e., convex side. This strip finds its application in auto-cut or thermostat in electric heating circuits. It has also been used as *thermometer by* calibrating its bending.




**Ex.11** ***When the two rods having expansion cofficient $\alpha_1$, $\alpha_2$ ($\alpha_2 > \alpha_1$) and width d are heated then the radius of the rod after expansion.***




$$R = \frac{d}{(\alpha_2 - \alpha_1)\Delta T}$$

**Proof :** $\ell_2 = \ell(1+\alpha_2 \Delta t) = (R+d)\theta \Rightarrow \ell_1 = \ell(1+\alpha_1 \Delta t) = R\theta$

$\dfrac{R+d}{R} = \dfrac{(1+\alpha_2 \Delta T)}{(1+\alpha_1 \Delta T)}$ from binomial theorem

$$R = \frac{d}{(\alpha_2 - \alpha_1)\Delta T}$$