McGRAW HILL EDUCATION WINNING EDGE (WE) SERIES

2ND EDITION

UNDERSTANDING OPTICS

For JEE Main and Advanced

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UNDERSTANDING OPTICS

For JEE Main and Advanced

M K Sinha

Ex. Sr. Faculty, Bansal Classes, Kota

McGraw Hill Education (India) Private Limited

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Preface to the Second Edition

Optics as a subject has a significant weightage in the JEE Physics syllabus and this book aims to meet the requirements of the aspirant on this topic.

The second revised edition of *Optics for JEE Main and Advanced* has been enhanced with new chapters on Optical Instruments and Wave Optics.

Besides, there are plenty of solved numericals and practice problems which provide aspirants enough exposure and practice and make them prepared to face any type of question asked in the examination.

This edition also has the solved questions on Optics from the 2014 JEE Main and Advanced Papers.

I hope the aspirants would find the revised edition useful for their preparation.

Wish you all the best!

M K Sinha

Preface to the First Edition

It is with great pleasure that I place before you the first edition of *Understanding Optics*. Optics is a subject which carries nearly 15% weightage in IIT-JEE. To provide aspirants with a thorough understanding of Optics, I have prepared this comprehensive book on this subject. I am hopeful that my efforts would make the topic easy and interesting for the aspirants. The title is a reflection of the aim and purpose of my book.

My twelve years of experience in teaching and interaction with students in Kota (Rajasthan) have enabled me to polish all concepts and conceptualise the important areas of the subject where students sometimes feel confused.

All minute and important concepts have been highlighted in this book. The theoretical part is followed by solved numerical examples which give the aspirant a thorough grounding of the topic.

Keeping in mind the latest trend of the IIT-JEE question paper, a large number of objective questions **(including Assertion-Reason, Matrix-Match type, Multiple Answer Type, etc.)** have been included at the end of chapters. All these questions have been solved with detailed analysis so that no ambiguity remains in the mind of students. I have included tips and tricks of solving problems to help students save valuable time in the examination hall.

Finally, I would like to share some useful tips for students preparing for the IIT-JEE.

Some Useful Tips While Preparing for IIT-JEE

- Be an active learner and be conceptually strong in the topics.
- Finish reading all the material—text, notes, etc., at least three days prior to the exam and prepare short notes that will help you during revision.
- Three days prior to the exam, set aside time each day for self-testing, practice problems, and review of notes.

Useful Tools for Last Minute Revision

- End of chapter summaries
- Solved numericals
- Practice problems
- Short notes prepared by you

Analyse your weaknesses and create a list of topics or problems that need your attention. Focus on these topics a little more than the others.

Have a daily healthy routine of eating nutritious food and getting sufficient sleep before the examination. Do not strain yourself much on the eve of the examination.

Wish you all success in your endeavours.

M K SINHA

Acknowledgements

I am deeply grateful to my wife **Aparna Sinha** as without her love, support, encouragement and patience, this book could never have been written, and my little kids **Sona**, **Kanha** and **Suchi** who were patient enough to tolerate me while I spent hours and hours writing this book.

My regards are due to my parents, Mr. Sidhi Prasad Sinha and Mrs. Sarvila Sinha, and my in-laws Mr. Haribansh Prasad and Mrs. Vidya Sinha. I am grateful for the encouragement provided by Mr. Rahul Ranjan (working in Qual Com., U.S.A.), Archana Sinha (working in American Bank, U.S.A.), Dr. Rajesh Kumar, Ragini Sinha and Mini.

I thank my student, Aviral Jain (IIT-Madras), my brother Ramesh Kumar Sinha and Akash Pandey for checking the calculations and solutions. I am indeed grateful to Sanjeev Kumar for taking so keen an interest in bringing the text to this nice form.

Lastly, I would like to thank Mr. V K Bansal (Founder, Bansal Classes, Kota) with whom I have worked and gained a lot of knowledge of the ways in which to teach.

Last but not least, I would like to thank my mentor Mr. Rajan Khare (Ex-HoD Chemistry at Bansal Classes, Kota) for guiding me all the way.

M K SINHA

Contents

CHAPTER

1 Reflection

Ray optics is the simplest theory of light. Here light is described in terms of rays, and we ignore the wave and photon character of light. We also do not bother about the nature of light, but are only interested to understand how it behaves on a large scale. The basic concept here is that a light ray travels along a well-defined path. The optics of light rays involves only geometric considerations and the laws are formulated using the language of geometry. Therefore, ray optics is also called **geometrical optics.** The ray description is adequate when the wavelengths involved are smaller compared with the dimensions of the objects. Ray optics is just an approximation, but it is of great importance technically. It is concerned with the location and direction of light rays. Hence it is useful in studying image formation by mirrors, lenses and the working of many optical instruments and devices. However, many aspects of the behavior of light cannot be explained on the basis of ray theory; for example we cannot explain the phenomenon of interference, diffraction, and polarization using ray concept. G. Kirchhoff, A. Sommerfeld and others showed that the ray model of propagation light could be derived from the basic electromagnetic theory. Ray optics is based mainly on the following three simple laws:

1.1 THE THREE LAWS

1. **Law of rectilinear propagation of light:**
   It is a common sight that an object kept in the path of light coming from a point source produces a sharp shadow. Similarly, on misty mornings sunlight passing through the dense foliage of trees appears as light shafts. There are many such instances where light appears to exhibit, rectilinear medium light propagation that states that in an optically homogeneous medium light propagates in a straight line A medium is said to be optically homogeneous if its refractive index is the same everywhere. This law is approximate and when light passes through very small opening, deviations from a straight line are observed.
2. **Law of independence of light rays:**
   Light rays do not disturb each other when they intersect. If several rays are passing through a medium simultaneously in different directions, then the path of any ray is the same as it would be if all other were absent. The intersection of rays does not hinder the rays from propagating independently of each other. Rays of light always preserve their individuality. The rays pass through each other simultaneously without mutual interaction. For example when we view an object, light rays passing in other directions do not obstruct the light that comes to us from the object.

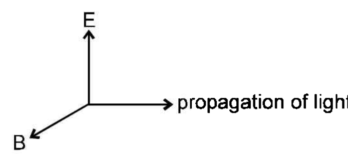
3. Law of reversibility of path :

If the path of a light ray is reversed, it will exactly retrace its path, irrespective of the number of reflections and refractions.

1.2 PROPERTIES OF LIGHT

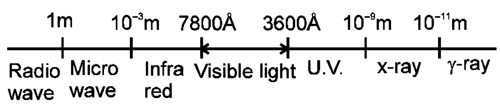
(i) Speed of light in vacuum, denoted by c, is equal to 3×10^8 m/s approximately.

(ii) Light is an electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.

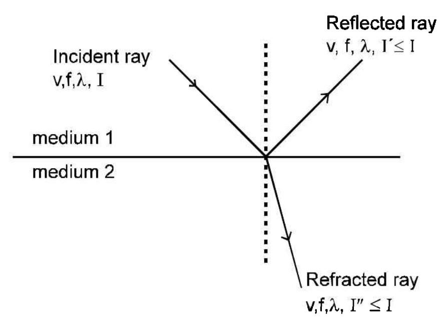


(iii) Light carries energy and momentum.

(iv) The formula $v = f\lambda$ is applicable to light.



Electromagnetic spectrum



(v) When light gets reflected in the same medium, it suffers no change in frequency, speed and wavelength.

(vi) Frequency of light remains unchanged when it gets reflected or refracted.

1.3 LIGHT RAY

The concept of energy travelling out from a source along with rays can be used to explain the rectilinear propagation of light. We may define a light ray as a narrow stream of radiation that travels in a line, never diverging or converging. A light ray is represented by a line drawn in the direction in which the light energy is travelling. We can think of a broad beam of light as a bundle of parallel rays. Rays concept was used to describe light long before its wave nature was firmly established. The ray model is highly useful in studying reflection, refraction and formation of images by mirrors and lenses.

Note: Light rays as such do not exist, nor can they be isolated experimentally; they exist only in theory. If we attempt to isolate a ray by means of a series of small holes, all that we can isolate are pencils of light. If the holes are made very small (of a few wavelengths size), light ceases to act as a ray and behaves more like a wave.

1.4 TYPE OF RAYS

(A) Paraxial Rays : Rays which are close to the principal axis and *make* small angles with it i.e. they are nearly parallel to the principle axis, are called paraxial rays. Our treatment of spherical mirror will be restricted to such rays which means that the formula for connecting u, v and f for spherical mirrors whose sizes are very small compared to their radii of curvature.

$\sin\theta \approx \theta$; $\tan\theta \approx \theta$; $\cos\theta \approx 1$ Where θ is the angle made by the rays with normal

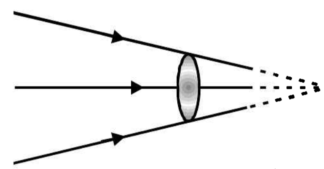
(B) Marginal rays: Rays which makes large angles with it.

Note: (i) All formulae of geometrical optics are only applicable for paraxial rays & not for marginal rays.

(ii) The laws of geometrical optics are applicable for both types of rays, i.e. (paraxial and margined)

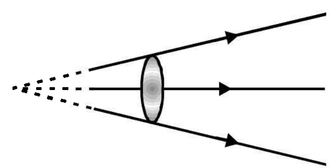
BEAM : A bundle or bunch of rays is called a beam. It is of following three types:

(A) CONVERGENT BEAM:- In this case diameter of beam decreases in the direction of ray.



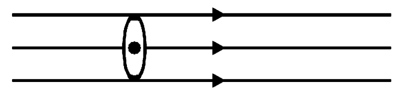
Convergent beam of light

(B) DIVERGENT BEAM:- It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.



Divergent beam of light

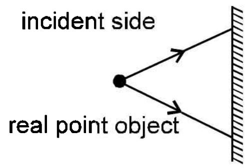
(C) PARALLEL BEAM:- It is a beam in which all the rays constituting the beam move parallel to each other and diameter of the beam remains same. In this case object and image distance both become infinity.

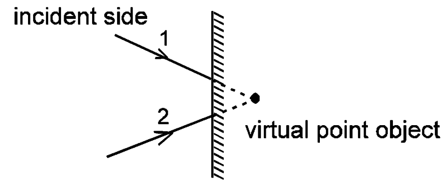


Parallel beam of light

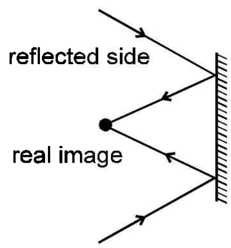
1.5 OBJECT AND IMAGE

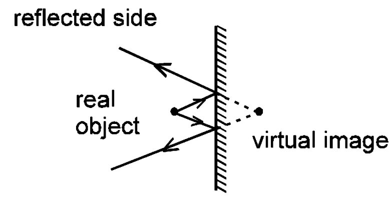
Object is defined as point of intersection of **incident** rays. **Image** is defined as point of intersection of **reflected** rays (in case of reflection) or **refracted** rays (in case of refraction). Let us call the side in which incident rays are present as incident side and the side in which reflected (refracted) rays are present, as reflected (refracted) side. An object is called **real** if it lies on incident side otherwise it is called **virtual** .





An image is called **real** if it lies on reflected or refracted side otherwise it is called **virtual.**





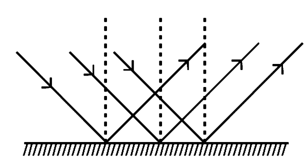
The image formed by an optical component may be real or virtual. The eye sees a virtual image as well as it sees a real image. . A real image contains light energy; the image can be received and seen on a screen. and also, the real image can be photographed by simply placing a photographic film (or plate) at the position of the image. Real images are essential for photography.

On the other hand, a virtual image cannot be received on a screen. The light rays never actually pass through the virtual image; hence, the image cannot be received on a screen. For the same reason , it cannot be recorded on a photographic plate placed at its position. Virtual image can be photographed only with the help of a converging optical system, which uses the virtual image as a virtual object and forms its real image.

1.6 REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called *Reflection of Light.*

(a) Regular Reflection:

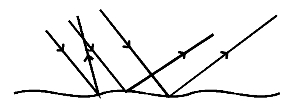


Regular Reflection

When the reflection takes place from a perfect plane surface it is called *Regular Reflection*. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.

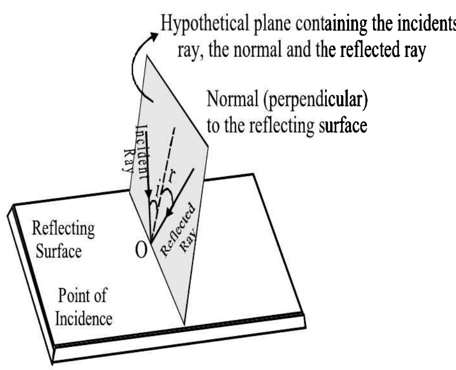
(b) Diffused Reflection

When the surface is rough, we do not get a regular behavior of light. Although at each point light ray gets reflected irrespective of the overall nature of the surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as diffused reflection.


Diffused Reflection

For example reflection from a wall, from a news paper etc. This is why you can not see your face in news paper and in the wall.

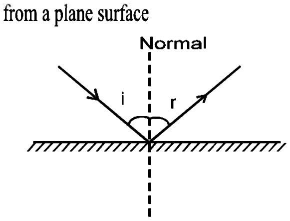
1.7 LAW OF REFLECTION



Reflection from a plane surface

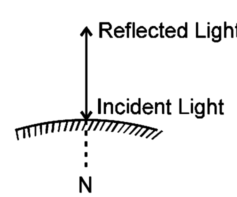
(a) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the *plane of incidence* (or *plane of reflection*). This condition can be expressed mathematically as

$\vec{R}\cdot(\vec{I}\times\vec{N}) = \vec{N}\cdot(\vec{I}\times\vec{R}) = \vec{I}\cdot(\vec{N}\times\vec{R}) = 0$ where $\vec{I}$, $\vec{N}$ and $\vec{R}$ are vectors of any magnitude along the incident ray, the normal and the reflected ray respectively.

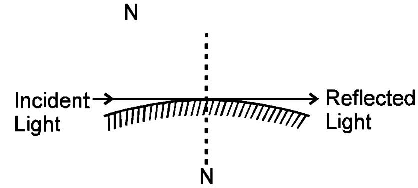


(b) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e. $\angle \mathbf{i} = \angle \mathbf{r}$

Special Cases :

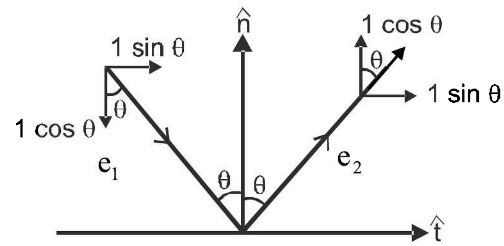


Normal Incidence : In case light is incident normally,
$i = r = 0$
$\delta = 180°$



Grazing Incidence : In case light strikes the reflecting surface tangentially,
$i = r = 90 \qquad \delta = 0°$ or $360°$

Write the law of reflection of light beam from a mirror in vector form, using the direction unit vectors $\hat{e}_1$ and $\hat{e}_2$ of the incident and reflected beams and the unit vector $\hat{n}$ of the outside normal to the mirror surface.



$\hat{e}_1$ = unit vector along the incident ray.

$\hat{e}_2$ = unit vector along the reflected ray

$\hat{t}$ = unit vector along tangential direction

$\hat{n}$ = unit vector along outside normal

Since, $|\hat{e}_1| = |\hat{e}_2| = 1$; we can write

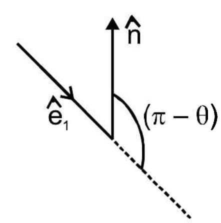
$\hat{e}_1$ as the vector form i.e. $\hat{e}_1 = 1 \sin\theta\ \hat{t} - 1\cos\theta\ \hat{n}$ .... (i)

and $\hat{e}_2 = 1\sin\theta\ \hat{t} + 1\cos\theta\ \hat{n}$ .... (ii)

From equation (i) & (ii) we get,

$\hat{e}_2 - \hat{e}_1 = (2\cos\theta)\,\hat{n}$

$\Rightarrow \quad \hat{e}_2 = \hat{e}_1 + 2(\cos\theta)\,\hat{n}$ ....(iii)



By concept of dot product

$\hat{e}_1.\hat{n} = (1)(1)\cos(\pi - \theta) \quad \Rightarrow \quad \hat{e}_1.\hat{n} = -\cos\theta$

By putting the value of cos θ in the equation (iii) we get,

$[\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1.\hat{n})\hat{n}]$ **The vector of law of refraction**

Ex. 1 Two plane mirrors are combined to each other as such one is in (y - z) plane and other is in (x - z) plane. A ray of light along vector $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ is incident on the first mirror. Find the unit vector in the direction of emergence ray after successive reflections through the mirrors.

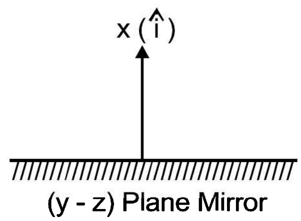
Sol. From observations of previous problem. In reflected ray only direction of normal unit vector to the plane mirror changed other unit vector remained unchanged. Hence, when a light ray is incident on the (y - z) plane mirror after reflection only sign of $\hat{i}$, normal to the (y - z) plane mirror changes while others remain unchanged then, $\hat{e}_2 = -\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

and finally after reflection through the (x - z) plane mirror,

$$\hat{e}_3 = -\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \qquad \textbf{Ans.}$$

Ex. 2 **A ray of light is incident on the (y - z) plane mirror along a unit vector $\hat{e}_1 = \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$. Find the unit vector along the reflected ray.**

Sol. The normal unit vector to the (y - z) plane mirror is i.e. $\hat{n} = \hat{i}$



Then, $\hat{e}_2$ = unit vector along the reflected ray.

The relation between $\hat{e}_2$ and $\hat{e}_1$: $\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1.\hat{n})\hat{n}$

$$\hat{e}_2 = \left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right) - 2\left(\frac{1}{\sqrt{3}}\right)\hat{i}$$

$$\Rightarrow \qquad \hat{e}_2 = -\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$$

Observation : only the sign of normal unit vector changes, others remain unchanged.

Ex. 3 **A light beam reflected from three mutually perpendicular plane mirrors in succession reverses its direction.**

Sol. We can assume three mutually perpendicular plane mirror

To the	(x - y) plane	Normal unit vector is $\hat{k}$
	(y - z) plane	Normal unit vector is $\hat{i}$
	(z - x) plane	Normal unit vector is $\hat{j}$

$\hat{e}_1 = a\hat{i} + b\hat{j} + c\hat{k}$ is a unit vector incident on the first mirror after reflection through (x - y) plane mirror the unit vector of reflected ray. $\hat{e}_2 = a\hat{i} + b\hat{j} - c\hat{k}$

and this ray incidents on the mirror (y - z) plane and reflected ray $\hat{e}_3 = -a\hat{i} + b\hat{j} - c\hat{k}$ and finally this ray incidents on the mirror (z - x) plane after reflection $\hat{e}_4 = -a\hat{i} - b\hat{j} - c\hat{k}$

i.e. $\hat{e}_4 - \hat{e}_1$ This means, final ray reverses its direction.

Ex. 4 **Prove that when incident ray suffers two successive reflection from two mutually perpendicular mirrors final ray is antiparallel to the original ray.**

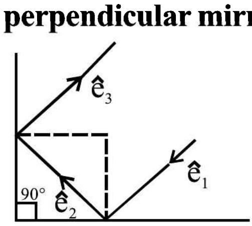
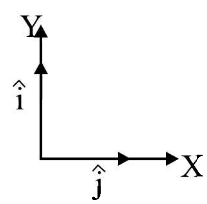


Fig. (a)



From vector form of laws of reflection

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1.\hat{j})\hat{j}$$

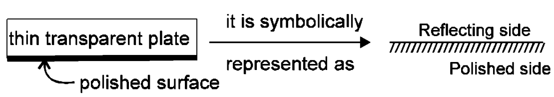
Similarly, $\hat{e}_3 = \hat{e}_2 - 2(\hat{e}_2.\hat{i})\hat{i}$ $\Rightarrow$ we get, $\hat{e}_3 = \hat{e}_1 - 2(\hat{e}_1.\hat{j})\hat{j} - 2\left[\left\{\hat{e}_2 - 2(\hat{e}_2.\hat{i}).\hat{i}\right\}\right]\hat{i}$

$$= \hat{e}_1 - 2(\hat{e}_1.\hat{j}) - 2(\hat{e}_1.\hat{i})\hat{i} - 4\left[(\hat{e}_1.\hat{j})\hat{j}.\hat{i}\right]\hat{i} = \hat{e}_1 - 2\left[(\hat{e}_1.\hat{i})\hat{i} + (\hat{e}_1.\hat{j})\hat{j}\right], \ (\because \hat{i}.\hat{j} = 0)$$

or, $\hat{e}_3 = \hat{e}_1 - 2\hat{e}_1$ $\quad$ $\hat{e}_3 = -\hat{e}_1$ **Hence, $\hat{e}_3$ is antiparallel to $\hat{e}_1$.**

PLANE MIRROR

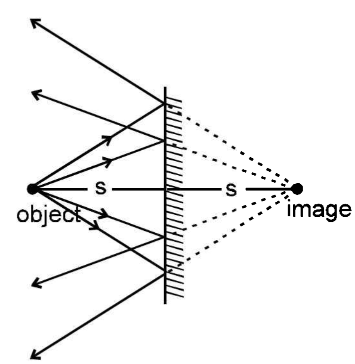
Plane mirror is formed by polishing one surface of a plane thin glass plate. It is also said to be silvered on one side.



PLANE MIRROR

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

1.8 POINT OBJECT



Characteristics of image due to Reflection by a Plane Mirror :

(i) Distance of object from mirror = Distance of image from the mirror.

All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called an image.

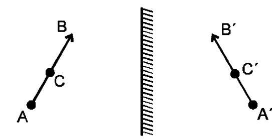
(ii) The line joining a point object and its image is normal to the reflecting surface.

(iii) The size of the image is the same as that of the object.

(iv) For a real object the image is virtual and for a virtual object the image is real

Extended object :

An extended object like AB shown in figure is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A′ will be image of A, C′ will be image of C, B′ will be image of B etc. All point images together form an extended image. Thus an extended image is formed from an extended object.

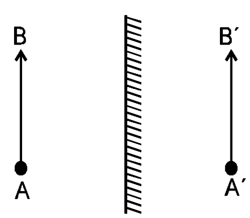


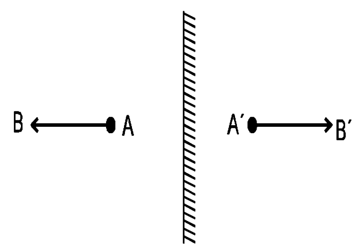
Properties of image of an extended object, formed by a plane mirror:

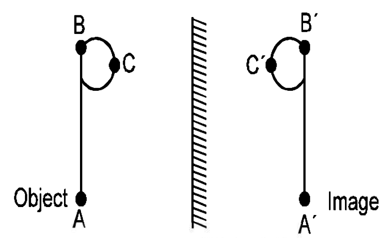
(1) Size of extended object = size of extended image.

(2) The image is upright, if the extended object is placed parallel to the mirror.

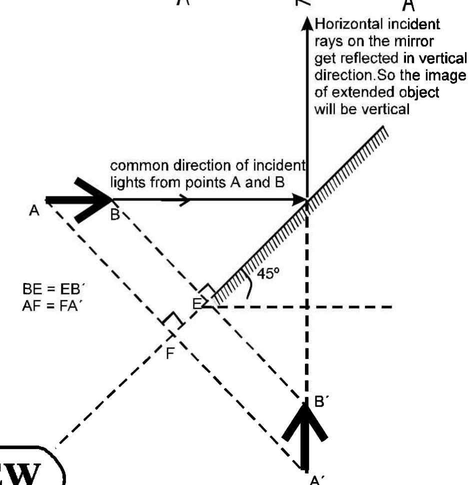
(3) The image is inverted if the extended object lies perpendicular to the plane mirror.







(4) If an extended horizontal object is placed in front of a mirror inclined at 45° with the horizontal the image formed will be vertical. See figure.

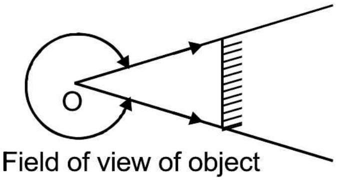


1.9 CONCEPT OF FIELD OF VIEW

Field of view (FOV) :

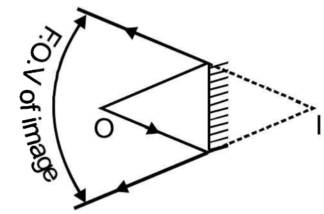
- **It is the region between two extreme reflected rays in the case of reflection through a mirror or two extreme refracted rays in the case of refraction through a lens.**
- **This is the region where diverging rays from objected image are present.**
- **If our eyes are present in the field of view then only we can see the object and image.**
- **Virtual object has no field of view hence it cannot be seen.**

(F. O. V.) For object :

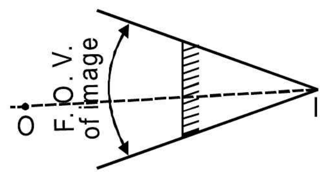


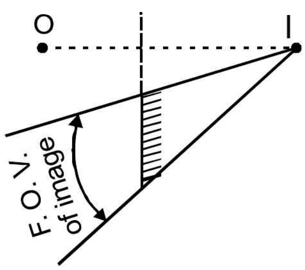
Field of view of object

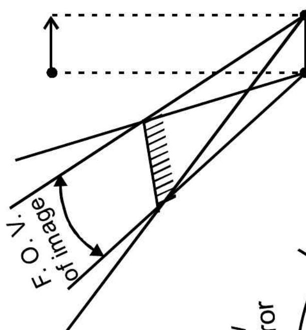
(F. O. V.) For image in the case of reflection :

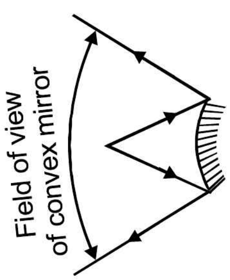


Shortcut method : Draw position of image in the plane mirror and then join through the mirror



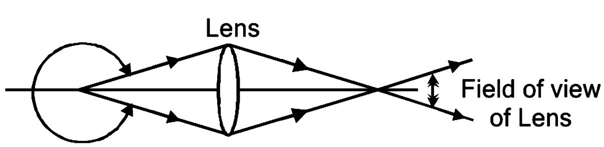


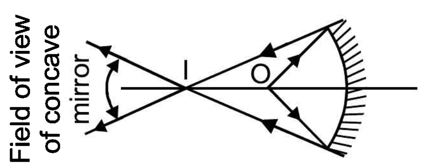


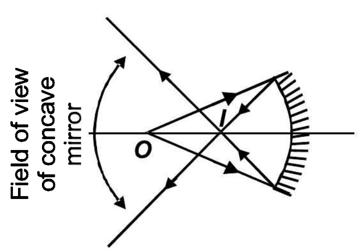


[since plane mirror is always perpendicular and bisects the line joining the object and image.]

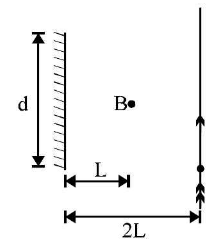
• Field of view for an extended object :







Ex. 5 A point source of light B is placed at a distance L in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown. The greatest distance over which he can see the image of the light source in the mirror is



(A) d/2 **(B) d** **(C) 2d** **(D) 3d**

Sol. Locate the position of image and by the help of shortcut method of locating field of view join the mirror through lines, and that region is the field of view then, by the similar triangle concept the triangles IPQ and IMN are similar,

$$\frac{PQ}{MN} = \frac{3L}{L} \quad \Rightarrow \quad PQ = 3d$$

The ray diagram is shown in figure (a)

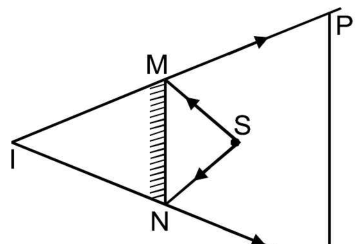
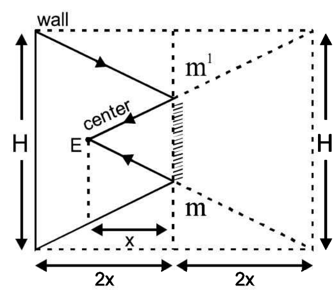
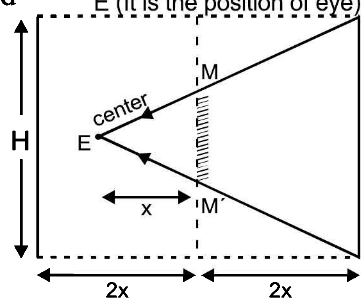


Figure (a)

Ex. 6 Find the minimum size of mirror required to see the full image of a wall behind a man standing at the centre of room, where H is the height of wall.



E (It is the position of eye)



Sol. For solving question draw shortcut diagram

- First locate the position of image of wall.
- Joint the eye (E) position of through the image of wall.
- The shaded portion is the minimum size of mirror required to see the full image of a wall behind the man
- Then apply the concept of similar triangles

From Shortcut diagram :
(From similar triangles concept)

$$\frac{H}{MM'} = \frac{3x}{x} \quad \Rightarrow \quad MM' = \frac{H}{3} \quad \textbf{Ans.}$$

Ex. 7 A child is standing in front of a straight plane mirror. His father is standing behind him as shown in figure (1). The height of father is double the child. What is the minimum length of mirror required so that (a)The child can completely see his father image in the mirror (b) If father wishes to see his child image completely then find minimum size of mirror required.
[Given that the height of father is 2H]

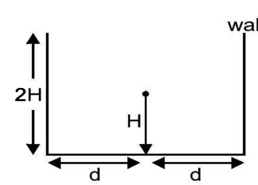
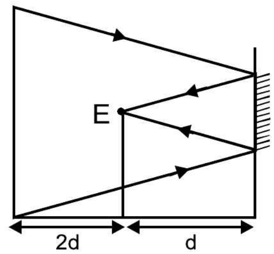


fig.1

Sol. (a) Ray diagram for the question of part (a)

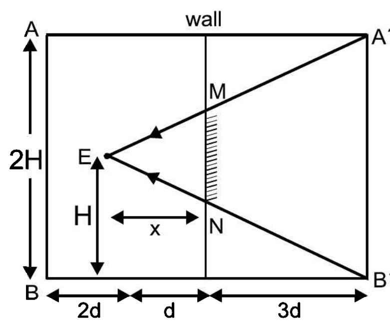


Shortcut Solution :

For solving question (a) draw the shortcut diagram.

- Locate the position of image of father
- Join the actual eye (E) of child position through the image of father.
- The shaded portion is the minimum size of mirror required to see the full image of father.

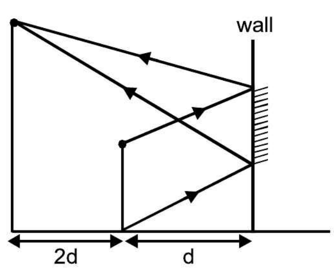
The triangles A′B′E and MEN are similar.



then, $\dfrac{A'B'}{MN} = \dfrac{4d}{d}$

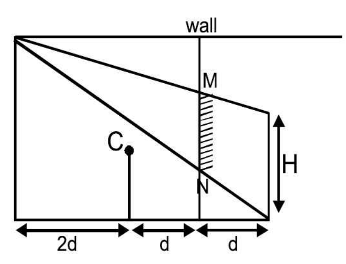
$$MN = \frac{A'B'}{4} = \frac{2H}{4} = \frac{H}{2} \qquad \textbf{Ans.}$$

(b) Ray diagram of the question of part (b)



Shortcut solution : For solving the question part (b) draw the shortcut diagram.

- Locate the position of image of child
- Join the actual eye position of father through the image of child.
- The shaded region is the required minimum length of mirror to see full image of child by father.



From the similar triangle concept :

$$\frac{H}{MN} = \frac{4d}{3d} \qquad \Rightarrow \qquad MN = \frac{3H}{4} \qquad \textbf{Ans.}$$

Ex. 8 A person's eye level is 1.5 m. He stands in front of a 0.3 m long plane mirror which is 0.8 m above the ground. The length of the image he sees of himself is

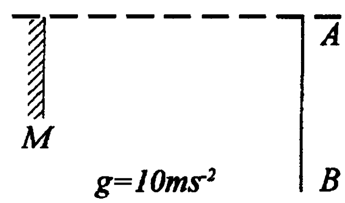
(A) 1.5 m (B) 1.0 m (C) 0.8 m (D) 0.6 m

Sol. **(D)** In the figure AB is the length of image he sees in the mirror. From the similar triangles EAB and EMN i.e

$$\frac{AB}{MN} = \frac{2d}{d}$$

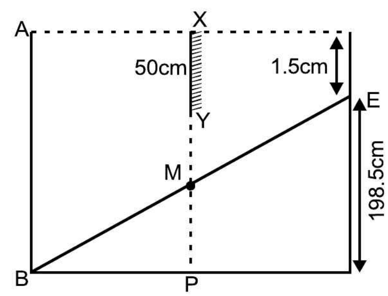
$\Rightarrow$ AB = (2) MN = (2) (30) = 60 cm = 0.6 m

Ex. 9 AB is a man of height 2m and M is a mirror of length 0.5 m and mass 0.1 kg. Initially M and A are at the same level and M starts falling freely always remaining vertical. If the level of the eyes of the man is 1.5 cm below, the time after which the man sees the reflection of his feet is ________ sec.



Sol. When the lowest part of the plane mirror i.e. Y reaches at M as shown in the figure then the man sees the reflection of his feet.

From the similar triangle concepts



$$MP = \frac{198.5}{2} = 99.25 \text{ cm}$$

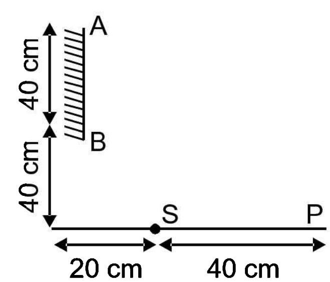
Then, YM = 200 – 50 – 99.25 = 50.75 cm

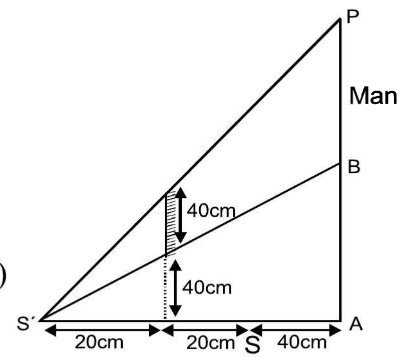
Apply, $s = at + \frac{1}{2} at^2$

$\Rightarrow s = 0 + \frac{1}{2} \times 10 \times t^2$

$= 50.75 \times 10^{-2} = \frac{1}{2} \times 10 \times t^2 \quad t = \sqrt{\frac{50.75 \times 2 \times 10^{-2}}{10}}$ t = 0.318 sec.

Ex. 10 In the figure shown AB is a plane mirror of length 40 cm placed at a height 40 cm from ground. There is a light source S at a point on the ground. Find the minimum and maximum height of a man (eye height) required to see the image of the source if he is standing at a point P on ground shown in figure.



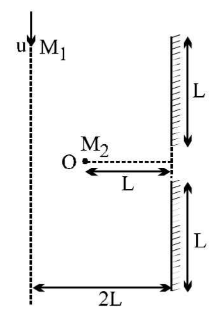


Sol. From the similar triangle concept :

$$\frac{AB}{40} = \frac{80}{20} \Rightarrow AB = 160 \text{ cm (Minimum height)}$$

and $\dfrac{AP}{80} = \dfrac{80}{20} \Rightarrow AP = 320$ cm (maximum height)

Ex. 11 Two plane mirrors of length L are separated by distance L and a man M_2 is standing at distance L from the connecting line of mirrors as shown in figure. A man M_1 is walking along a straight line at distance 2 L parallel to mirrors at speed u, then man M_2 at O will be able to see image of M_1 for total time:



(A) $\dfrac{4L}{u}$ **(B)** $\dfrac{3L}{u}$ **(C)** $\dfrac{6L}{u}$ **(D)** $\dfrac{9L}{u}$

Sol. **(C)** When the man M_1 is in the region RP and QS, then man M_1 sees the image of M_2.

From the similar triangle

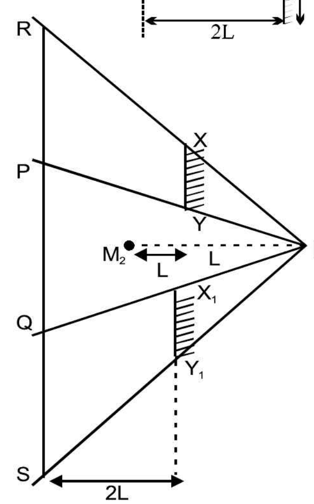
PIQ and XIX_1 $\Rightarrow \dfrac{PQ}{XX_1} = \dfrac{3L}{L} \Rightarrow PQ = 3L$

From the similar triangle,

RIS and XIY_1 $\Rightarrow \dfrac{RS}{XY_1} = \dfrac{3L}{L} \Rightarrow RS = 9L$

Then RP + QS = RS – PQ = 9L – 3L = 6L

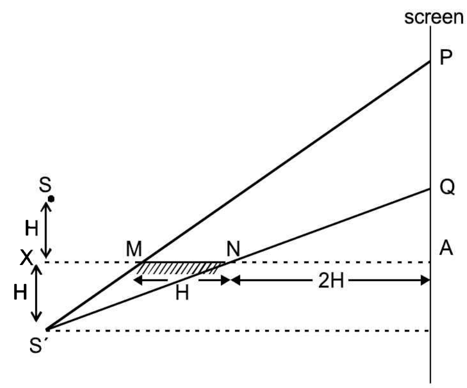
$\therefore$ Time $= \dfrac{6L}{u}$



Ex. 12 A point source has been placed as shown in the figure. What is the length on the screen that will receive reflected light from the mirror ?

(A) 2H **(B) 3H** **(C) H** **(D) None of these**

Sol. Here, PQ is the length of reflected light from the mirror. From the concept of similar triangles, PMA and MXS′ $\Rightarrow \dfrac{PA}{XS'} = \dfrac{3H}{H}$



Then, PA = 3 (XS′) = 3H

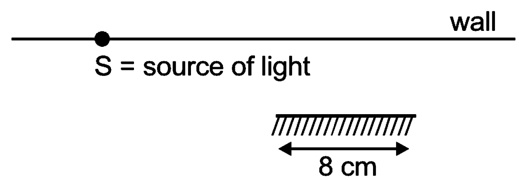
And from the similar triangles concept

QNA and XNS′ $\dfrac{QA}{XS'} = \dfrac{2H}{2H}$

$\Rightarrow$ QA = H $\Rightarrow$ QA = H

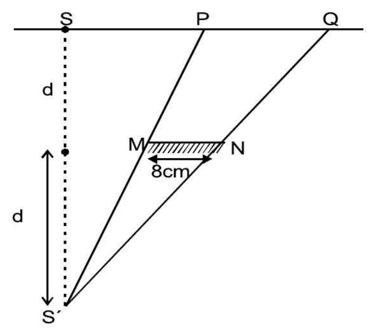
PQ = PA – QA = 3H – H = 2H

Ex. 13 A plane mirror of length 8 cm is present near a wall in situation as shown in figure. Then the size of spot formed on the wall is



(A) 8 cm **(B) 4 cm** **(C) 16 cm** **(D) None of these**

Sol. **(C)** Here, PA is the size of spot formed on the wall. From the similar triangle

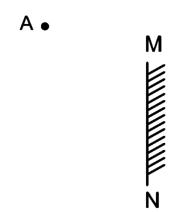


PS′Q and MS′N

$$\frac{PQ}{MN} = \frac{2d}{d}$$

PQ = (2) MN = 2 × 8 = 16 cm

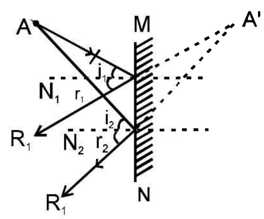
Ex. 14 Figure shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing a ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called field of view).

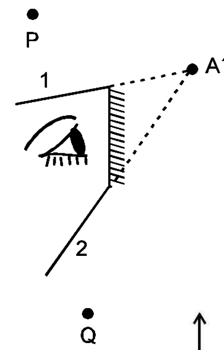


Sol. See figure, consider any two rays emanating from the object. N_1 and N_2 are normals; $i_1 = r_1$ and $i_2 = r_2$

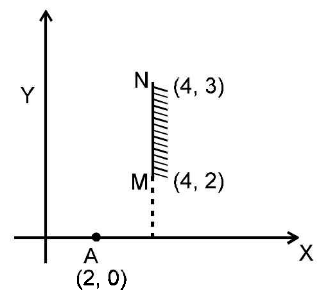
The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror such that their meeting point is A′. To obtain the region in which reflected rays are present, join A′ with the ends of mirror and extend.

The following figure shows this region as shaded. In the figure there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q cannot see the image because they do not receive any reflected ray.]

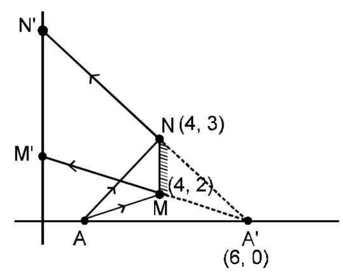




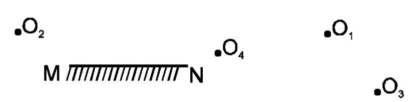
Ex. 15 Find the region on Y axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown.



Sol. The image of point A, in the mirror is at A' (6, 0).
Join A' M and extend to cut Y axis at M' (Ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join A'N and extend to cut Y axis at N' (Ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A') .
From Geometry.
M' ≡ (0, 6)
N' ≡ (0, 9). **M'N' is the region on Y axis in which reflected rays are present.**



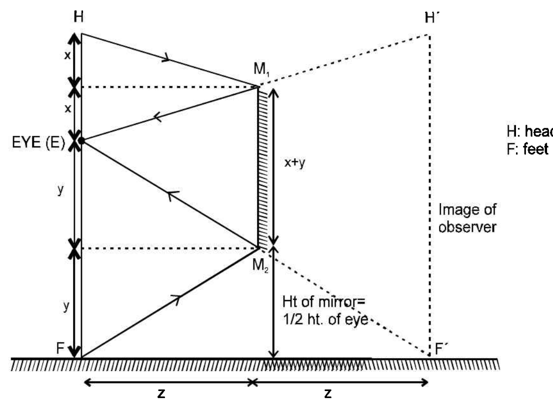
Ex. 16 See the following figure. Which of the object(s) shown in figure will not form its image in the mirror.



Ans. O_3

Ex. 17 Show that the minimum size of a plane mirror, required to see the full image of an observer is half the size of that of the observer.

Sol. See the following figure. It is self explanatory if you consider lengths 'x' and 'y' as shown in figure.



Aliter :

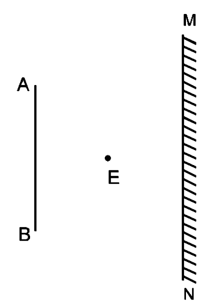
$\Delta E\ M_1, M_2$ and $\Delta E\ H'F'$ are similar,

$$\therefore \quad \frac{M_1M_2}{H'F'} = \frac{z}{2z}$$

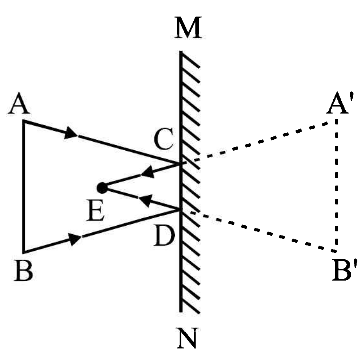
or $M_1 M_2 = H'\ F' / 2 = HF / 2$

Note : That the height of the mirror is half the height of eye as shown in figure.

Ex. 18 See the following figure. Which of the object(s) shown in figure will not form its image. The figure shows an object AB and a plane mirror MN placed parallel to the object. Indicate the mirror length required to see the image of the object if observer's eye is at E.



Sol.



Part CD is used to view the image.

1.10 LOCATING ALL THE IMAGES FORMED BY TWO PLANE MIRRORS

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha+\beta$ as shown in figure

I_1 = Image of object P formed by the mirror M_1.

I_2 = Image of object P formed by the mirror M_2.

$I_{21} = I_2$ will act as an object for mirror M_1.

$I_{12} = I_1$ will act as an object for mirror M_1. and so on

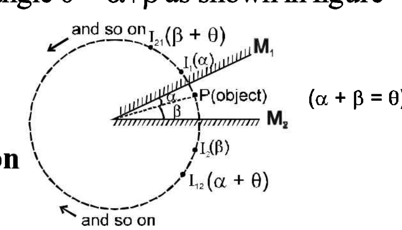
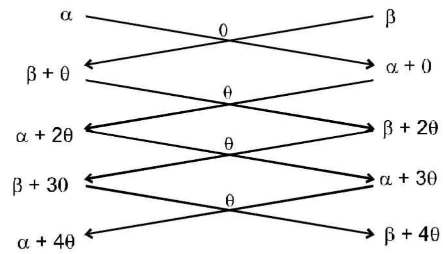


Image formed by Mirror M_1 (angles are measured from the mirror M_1.)	Image formed by Mirror M_2 (angles are measured from the mirror M_2.)



Stop if next angle will be more than 180° or equal | Stop if next angle will be more than 180° or equal

To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Therefore in this case the last images coincide. Therefore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 -1 (as the last images coincide)

Number of images formed by two inclined mirrors

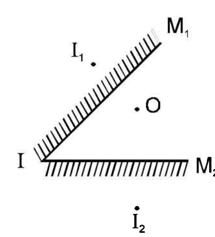
(i) If $\frac{360^\circ}{\theta}$ = even number ; number of image = $\frac{360^\circ}{\theta} - 1$

(ii) If $\frac{360^\circ}{\theta}$ = odd number ; number of image = $\frac{360^\circ}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\frac{360^\circ}{\theta}$ = odd number ; number of image = $\frac{360^\circ}{\theta}$, if the object is not placed on the angle bisector.

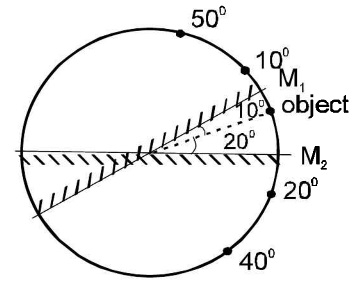
(iv) If $\frac{360^\circ}{\theta} \neq$ integer, then count the number of images as explained above.

Note : Figure shows two inclined plane mirrors M_1 and M_2 and an object O. Their images formed in mirrors M_1 and M_2 individually are I_1 and I_2 respectively. Show that I_1 and I_2 and O lie on the circumference of a circle with centre at O. [This result can be extended to show that all the images will also lie on the same circle. Note that this result is independent of the angle of inclination of mirrors.]



Ex. 19 Two mirrors are inclined by an angle 30^0. An object is placed making 10^0 with the mirror M_1. Find the positions of the first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.

Sol. Figure is self explanatory. **Number of images**



(i) Using direct formula : $\dfrac{360^\circ}{30^\circ} = 12$ (even number)

$\therefore$ number of images = 12 – 1 = 11

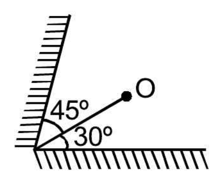
(ii) By counting . See the following table

Image formed by Mirror M_1 (angles are measured from the mirror M_1.)		Image formed by Mirror M_2 (angles are measured from the mirror M_2.)
10°	+30°	20°
50°	+30°	40°
70°	+30°	80°
110°	+30°	100°
130°	+30°	140°
170°		160°
Stop because next angle will be more than 180° or eqal		Stop because next angle will be more than 180° or eqal

To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide.Here last angles made by the mirrors + the angle between the mirrors = 160° + 170° + 30° = 360°. Therefore in this case the last images coincide.

Therefore the number of images = number of images formed by mirror M_1
+ number of images formed by mirror M_2 -1
(as the last images coincide)= 6 + 6 –1 = 11.

Ex. 20 Two plane mirrors are inclined at an angle of 75° to each other. Find the total number of images formed when an object is placed as shown in figure.



Sol. $I_{21} = I_2$ will act as an object for mirror M_1

I_1 = Image of object P formed by the mirror M_1.

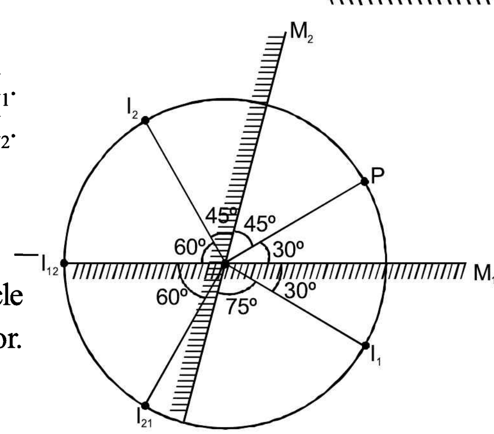
I_2 = Image of object P formed by the mirror M_2.

$I_{21} = I_2$ will act as an object for mirror M_1.

$I_{12} = I_1$ will act as an object for mirror M_1.

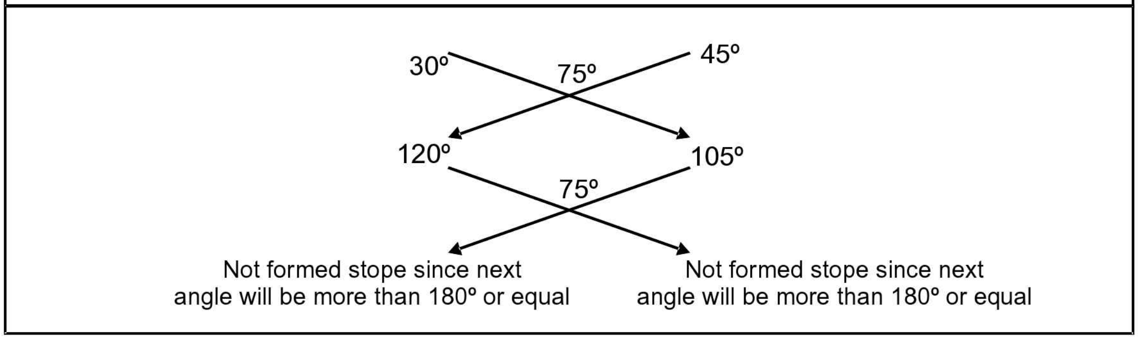
and so on

• Note : all images and object lies in the same circle whose centre is the intersection point of two mirror.



Shortcut solution :

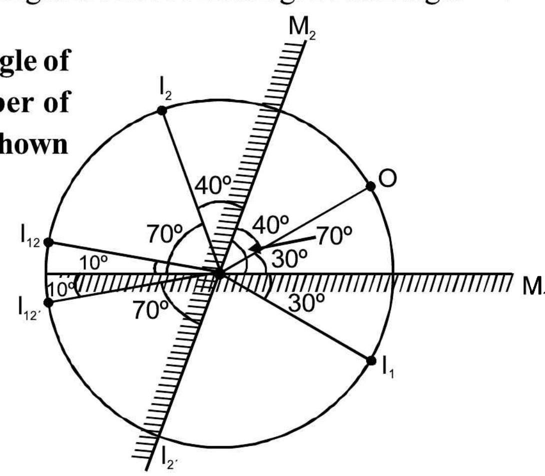
Image formed by Mirror M_1 (angles are measured from the mirror M_1.)	Image formed by Mirror M_2 (angles are measured from the mirror M_2.)



Check the image coincides or not :

120 + 105 + 35 = 300. Which is not 360. Then, images are not coinciding. Total images = 4

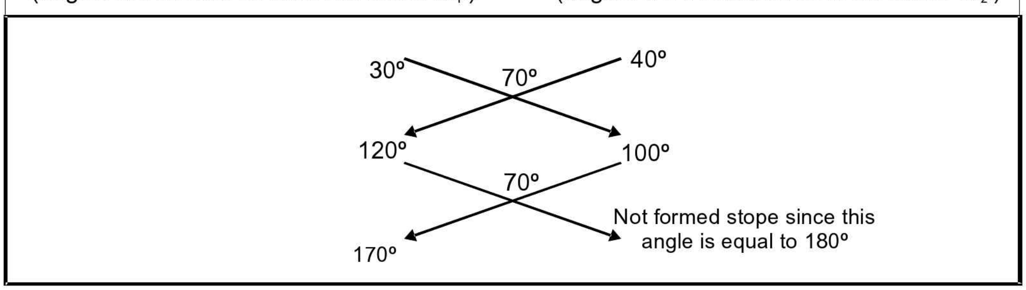
Ex. 21 Two plane mirrors are inclined at an angle of 70° to each other. Find the total number of images formed when object is placed as shown in figure.



Total images = 5

Shortcut solution :

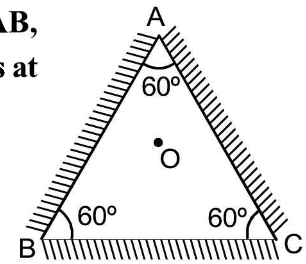
Image formed by Mirror M_1 (angles are measured from the mirror M_1.)	Image formed by Mirror M_2 (angles are measured from the mirror M_2.)



Therefore in this case last images not coinciding.

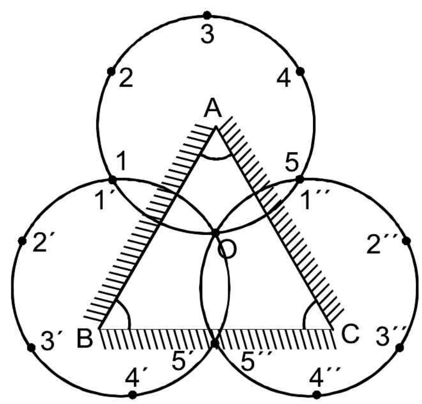
Since (170 + 100 + 70) = 340, which is not 360. Total images = 5.

Ex. 22 Find the number of images formed by three mirrors AB, BC and AC in situation as shown in figure. The object is at the centre of triangle.



Solution :

Combination of mirrors	**Images**
AB and BC	1′, 2′, 3′, 4′, 5′
AC and BC	1′′, 2′′, 3′′, 4′′, 5′′
AB and AC	1, 2, 3, 4, 5



For the mirror AB and BC $\frac{360}{60} = 6$

Hence, number of images formed by the two plane mirrors

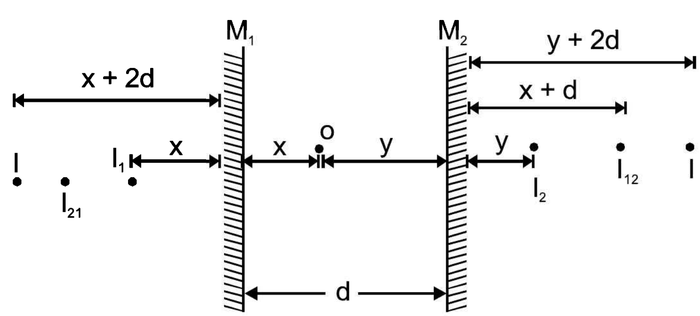
AB and BC = 5. These images will lie on a circle.

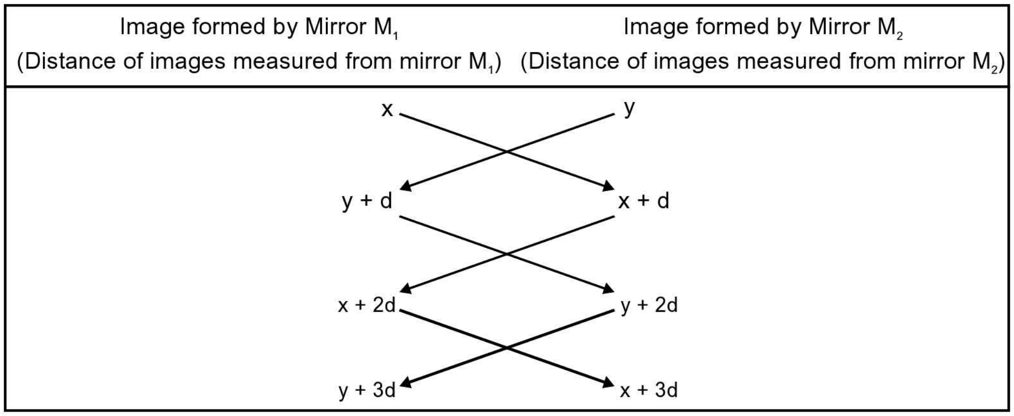
Three images

(i) 5 and 1′′, (ii) 5′ and 5′′, (iii) 1 and 1′′

will coincide hence the number of images formed by the three mirror AB, BC and AC = (5 × 3 – 3) = 12

1.11 IMAGES FORMED BY THE TWO PLANE MIRRORS





and so on.

I_1 = Image formed

$I_{21} = I_2$ will act as an object for mirror M_1

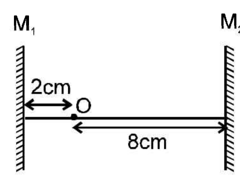
I_1 = Image of object P formed by the mirror M_1.

I_2 = Image of object P formed by the mirror M_2.

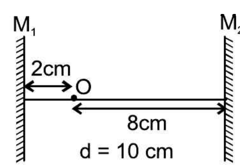
$I_{21} = I_2$ will act as an object for mirror M_1.

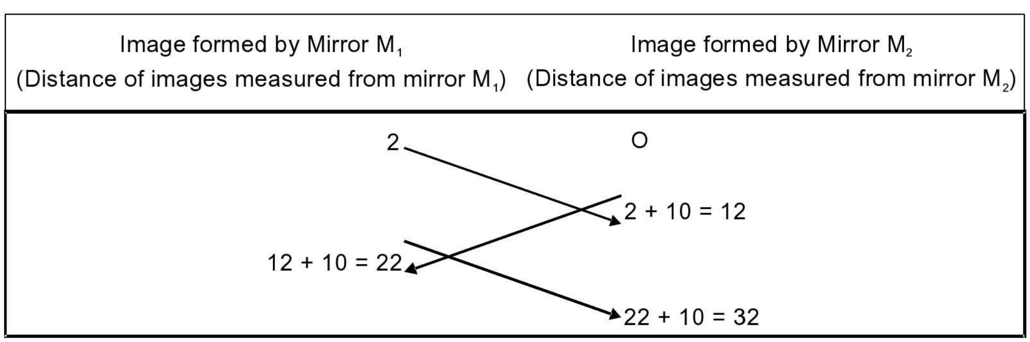
$I_{12} = I_1$ will act as an object for mirror M_1. and so on

Ex. 23 Figure shows a point O i.e. the object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two
mirrors considering reflection on mirror M_1 first.



Sol. Since given that reflection on mirror M_1 first than





1.12 CONCEPT OF VELOCITY OF IMAGE

Concept of velocity of image for a moving object in the plane mirror. There are three components of velocity of image for a moving object.

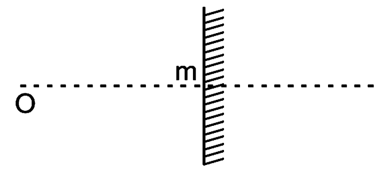
(i) Perpendicular to the plane mirror

(ii) Other two are parallel to the plane mirror

Concept of perpendicular component of velocity of image in the plane mirror:

$(S_{IM})_\perp = -(S_{OM})_\perp$

Differentiating both sides w.r.t. time, we get



$(V_{IM})_\perp = -(V_{OM})_\perp$

$(V_{IG})_\perp - (V_{MG})_\perp = [(V_{OG})_\perp - (V_{MG})_\perp]$

$$(V_{MG})_\perp = \frac{(V_{OG})_\perp + (V_{IG})_\perp}{2}$$

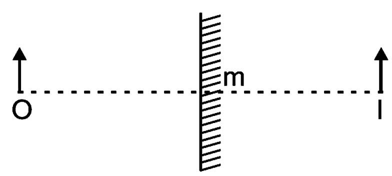
If mirror is an $(x - y)$ plane and the perpendicular component is along 'z' direction.

Concept of parallel components of velocity of image in the plane mirror :

$(S_{IM})_\parallel = -(S_{OM})_\parallel$

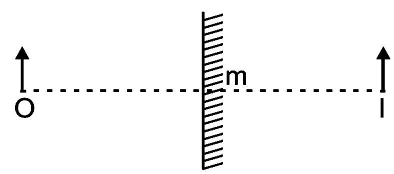
Differentiating both sides w.r.t. time, we get

$(V_{IM})_\parallel = (V_{OM})_\parallel \Rightarrow (V_{IG})_\parallel - (V_{MG})_\parallel = (V_{OG})_\parallel - (V_{MG})_\parallel$

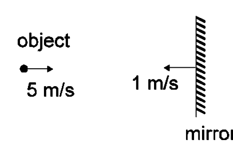


$(V_{IG})_\parallel = (V_{OG})_\parallel$

That implies, parallel component of velocity of image w.r.t. ground or mirror will remain same as velocity of object w.r.t. ground or mirror. If mirror has (x – y) plane then x and y component of velocity of image are the parallel components.



Ex. 24 **An object moves with 5 m/s towards right while the mirror moves with 1m/s towards the left as shown. Find the velocity of image.**



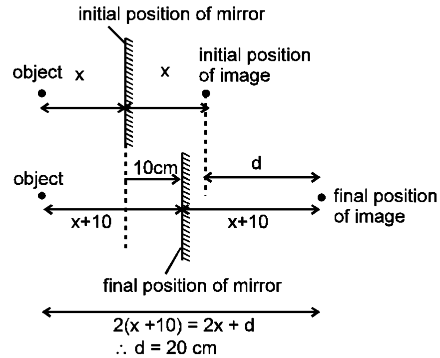
Sol. Take $\rightarrow$ as + direction. $v_i - v_m = v_m - v_0$

$v_i - (-1) = (-1) - 5$

$\therefore \quad v_i = -7$ m/s. $\Rightarrow$ 7 m/s and direction towards left.

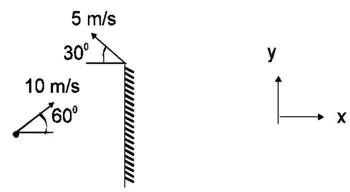
Ex. 25 **There is a point object and a plane mirror. If the mirror is moved 10 cm away from the object, find the distance by which the image will move.**

Sol. We know that $x_{im} = -x_{om}$ or $x_i - x_m = x_m - x_o$ or $\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$. In this Q. $\Delta x_o = 0$; $\Delta x_m = 10$ cm. Therefore $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.



Ex. 26 An object is kept fixed in front of a plane mirror which is moved by 10 m/s away from the object, find the velocity of the image.

Ans. 20 m/s in the direction of motion of mirror.



Ex. 27 In the situation shown in figure, find the velocity of image.

Sol. Along x direction, applying $v_i - v_m = -(v_0 - v_m)$

$$v_i - (-5\cos 30^\circ) = -(10\cos 60^\circ - (-5\cos 30^\circ)), \quad \therefore \quad v_i = -5\left(1+\frac{\sqrt{3}}{2}\right) \text{ m/s}$$

Along y direction $\quad v_0 = v_i$, $\therefore$ $v_i = 10 \sin 60^\circ = 5\sqrt{3}$ m/s

$\therefore$ Velocity of the image $= -5\left(1+\frac{\sqrt{3}}{2}\right)\hat{i} + 5\sqrt{3}\,\hat{i}$ m/s.

1.13 CONCEPT FOR THE ACCELERATION OF IMAGE

Again, differentiating the velocity equation w.r.t. time we get,the acceleration of image.

Ex. 28 A plane mirror is moving with velocity $-2\hat{i} - 3\hat{j} + 4\hat{k}$. A point object in front of the mirror move with a velocity $-3\hat{i} + 4\hat{j} - 4\hat{k}$. There $\hat{j}$ is along the normal to the plane mirror and facing towards the object. Find velocity of image.

Sol. Given that $\overrightarrow{V_{OG}} = -2\hat{i} - 3\hat{j} + 4\hat{k}$, $\overrightarrow{V_{MG}} = -3\hat{i} + 4\hat{j} - 5\hat{k}$

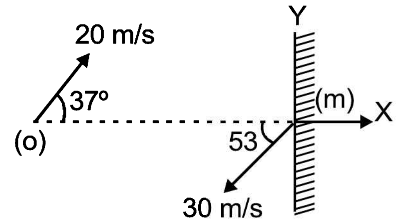
For the perpendicular (y) component :

$$(V_{MG})_y = \frac{(V_{OG})_y + (V_{IG})_y}{2} \Rightarrow 5 = \frac{4 + (V_{IG})_y}{2} \quad (V_{IG})_y = -14$$

And, other x and z component of velocity of image will remain the same as the velocity of the object w.r.t. ground, it does not depend on the velocity of mirror.

Means, $(V_{IG})_x = (V_{OG})_x = -2$; $(V_{IG})_Z = (V_{OG})_Z = 4$. Then, $\overrightarrow{V_{IG}} = -2\hat{i} - 14\hat{j} + 4\hat{k}$

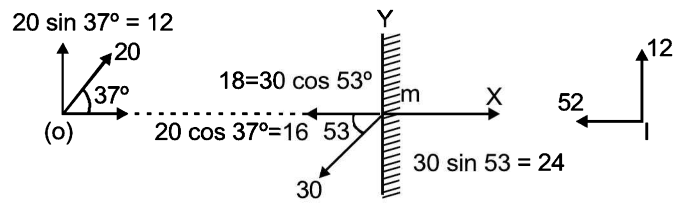
Ex. 29 Find the velocity of the image of a moving object in situation as shown in figure :



(Given velocity w.r.t. ground)

Sol. For the perpendicular component :

$$(V_{MG})_{\perp} = \frac{(V_{OG})_{\perp} + (V_{IG})_{\perp}}{2}$$



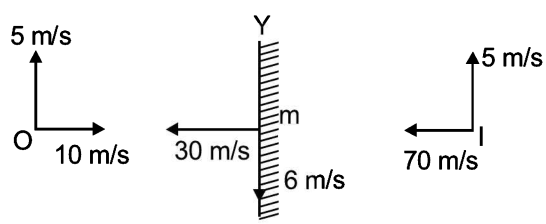
$$-18 = \frac{16 + (V_{IG})_{\perp}}{2}$$

$$(V_{IG})_{\perp} = -52$$

And, the parallel component (y) of velocity of image is same as that of object w.r.t. ground.

$$\overrightarrow{V_{IG}} = -52\hat{i} + 12\hat{j}$$

Ex. 30 Find the velocity of the image of a moving object in situation shown in figure.
(Given velocity w.r.t. ground)



Sol. For the perpendicular component

$$(V_{MG})_{\perp} = \frac{(V_{OM})_{\perp} + (V_{IG})_{\perp}}{2} \Rightarrow -30 = \frac{10 + (V_{IG})_{\perp}}{2} \Rightarrow (V_{IG})_{\perp} = -70$$

And, the parallel component of velocity of image is same as that of object w.r.t. ground. It does not depend on the velocity of mirror.

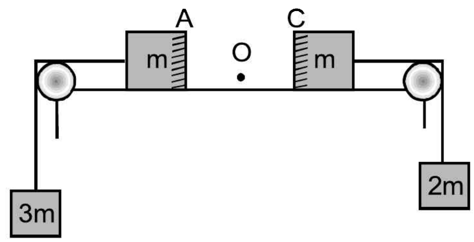
Ex. 31 Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object 0 is kept at rest on the table. The sides AB & CD of the two blocks are made reflecting. The acceleration of the two images formed on those two reflecting surfaces w.r.t. each other is :

(A) 5g/6 **(B) 5g/3** **(C)g/3** **(D) 17g/6**

Sol. For equation for the mirror A : $T = ma_A$ and $3mg - T = 3ma_A$ $\therefore$ $a_A = \dfrac{3g}{4}$

Similarly, the force equation for the mirror C :

$T = ma_C \Rightarrow 2mg - T = 2ma_C \Rightarrow \therefore a_C = \dfrac{2g}{3}$



For the perpendicular component :

$$(a_{MG})_\perp = \frac{(a_{OG})_\perp + (a_{MG})_\perp}{2}$$

$(a_{OG})_\perp$ = Perpendicular component for acceleration of object w.r.t. to the plane mirror

And parallel component,

$(a_{IG})_\parallel = (a_{OG})_\parallel$ and $(a_{IM})_\parallel = (a_{OM})_\parallel$

For the perpendicular component :

$(a_{MG})_\perp = \dfrac{(a_{OG})_\perp + (a_{IG})_\perp}{2}$ for the acceleration of image formed by the mirror A :

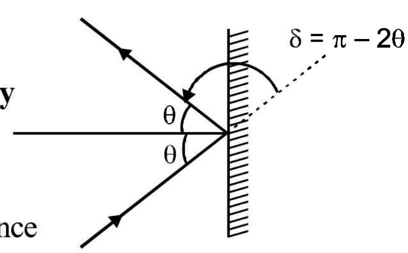
$$\frac{3g}{4} = \frac{0 + (a_{IG})_\perp}{2} \quad \Rightarrow \quad (a_{IG})_\perp = \frac{3g}{2} \text{ [left side]}$$

Similarly, for the acceleration of image formed by the mirror C. $(a_{IG})_\perp = \dfrac{4g}{3}$ [right side]

Then relative acceleration w.r.t. each other $= \dfrac{3g}{2} + \dfrac{4g}{3} = \left(\dfrac{9+8}{6}\right)g = \dfrac{17g}{6}$

(added since their direction are opposite to each other)

1.14 ANGLE OF DEVIATION



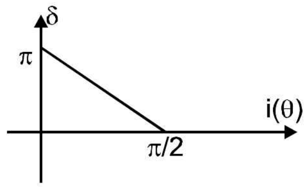
Concept of angle of deviation suffered by incident ray due to reflection :

(i) By single reflection :

Graph between deviation versus angle of incidence

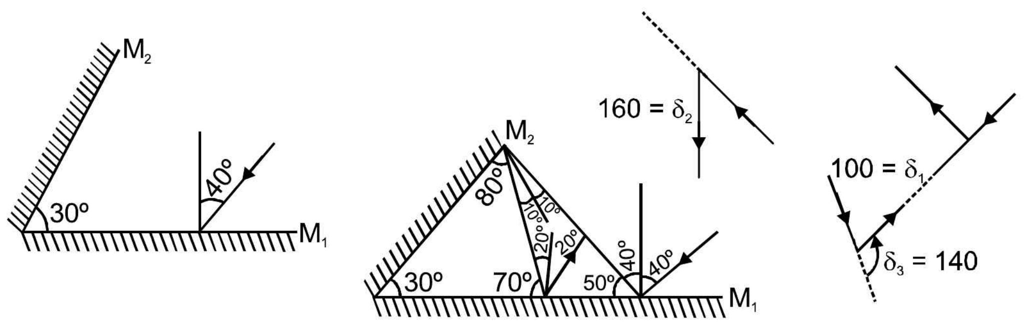
$\delta = \pi - 2\theta$, $y = \pi - 2x$ [substitute y in the place of δ and x in the place of θ]

This is the equation of straight line



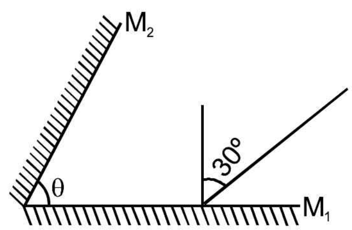
(ii) Multiple reflection : In this case net deviation suffered by an incident ray is the sum of deviation due to individual reflections, Sense of rotation is taken into account.

Ex. 32 Two plane mirrors are inclined to each other at an angle 30° to each other. A ray of light is incidenting at an angle of 40° to the mirror (m_1). Find the total angle of deviation of the ray after the third successive reflection due to mirrors.



$\delta = 100 \circlearrowleft + 160 \circlearrowright + 140 \circlearrowleft \Rightarrow 100 \circlearrowleft + 300 \circlearrowleft \Rightarrow 200 \circlearrowleft \Rightarrow$ or $160 \circlearrowright$

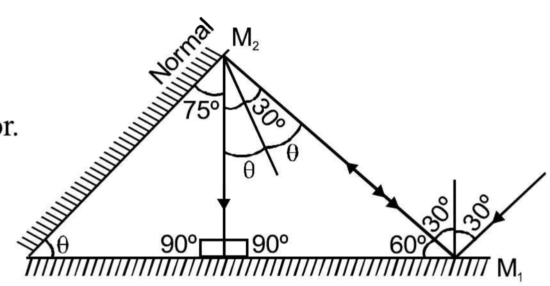
Ex. 33 Two plane mirrors are inclined to each other as shown. A ray after the three successive reflection falls on the mirror (M_1) and finally retraces its path. Calculate the angle between the two plane mirror.



Sol. $2\alpha = 30 \Rightarrow \alpha = 15$, then from triangle $75 + \theta + 90 = 180$

$\Rightarrow \theta = 15$ **Ans.**

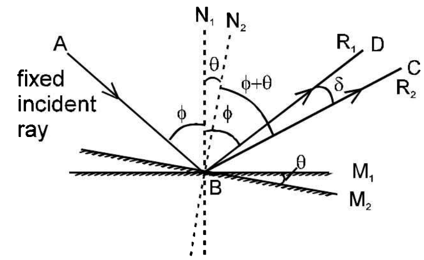
Concept : The light ray retraces its path only when it falls normally to the mirror.



1.15 ROTATION OF PLANE MIRROR

Case (1) : When direction of incident ray is kept fixed.

See figure M_1, N_1 and R_1 indicating the initial position of mirror, initial normal and initial direction of reflected light ray respectively. M_2, N_2 and R_2 indicate the final position of mirror, final normal and final direction of reflected light ray respectively.

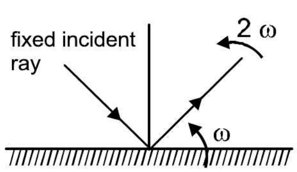


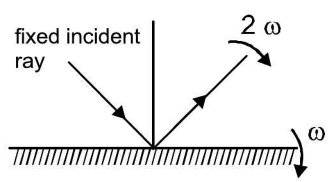
From figure it is clear that $\angle ABC = 2\phi + \delta = 2(\phi + \theta)$ or $\delta = 2\theta$.

That means when incident ray is fixed, and mirror rotates through the angle θ :

(i) Then reflected ray rotates through the angle 2θ in the same sense as the mirror rotates.

(ii) The angular velocity and angular acceleration of new reflected ray becomes twice as that of mirror.





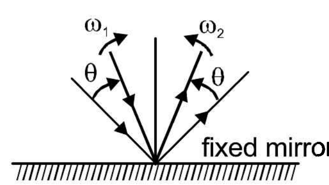
$\omega_1 = 2\omega_2 \Rightarrow \omega_2 =$ Angular velocity of mirror $\Rightarrow \omega_1 =$ Angular velocity of reflected ray

Case (2) : When mirror is fixed and incident ray rotates :

$\omega_1 =$ Angular velocity of incident ray

$\omega_2 =$ Angular velocity of reflected ray

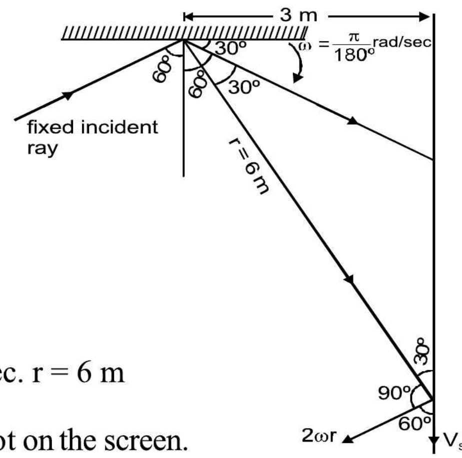
$\omega_1 = -\,\omega_2$



[negative sign shows that the direction of angular velocity is opposite to each other]

Ex. 34: Figure shows a torch producing a straight light beam falling on a plane mirror at an angle of 60°. The reflected beam makes a spot P on the screen along Y-axis. If at t = 0, mirror starts rotating clockwise about the hinge A with an angular velocity Ω = 1° per second. Find the speed of the spot on screen after time t = 15 s.

Sol.



Concept : When incident ray is fixed the angular velocity of reflected ray becomes twice in the same sense as that of reflected ray. After t = 15 sec, the mirror rotates 15° then reflected ray rotates 30° in the same sense.

$\cos 60° = \frac{3}{r}$, $\omega = 1$ degree per sec $= \frac{\pi}{180}$ rad/sec. r = 6 m

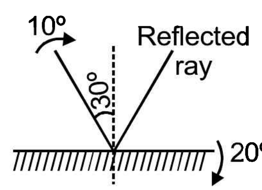
Then, there are two components of velocity of spot on the screen.

(i) perpendicular component

$$V_s \cos 60° = 2\omega r \quad \Rightarrow V_s \times \frac{1}{2} = 2 \times \frac{\pi}{180} \times 6 \quad \Rightarrow \quad V_s = \left(\frac{2\pi}{15}\right) \text{ m/s}$$

(ii) radial component which increases the length of r i.e. $V_s \sin 60° = \frac{dr}{dt}$

Ex. 35 Figure shows a plane mirror onto which a light ray is incidenting. If the incidenting light ray is turned by 10° and the mirror by 20°, as shown, find the angle turned by the reflected ray.



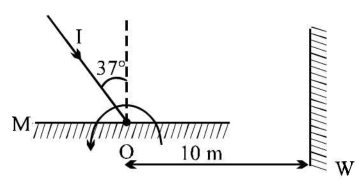
Sol. (i) When the incident ray is fixed and mirror rotates through angle 20° clockwise then reflected ray rotates clockwise through 40° angle.

(ii) When mirror (M) in fixed and incident ray rotates through angle 10° clockwise then reflected ray rotates through angle 10° anticlockwise.

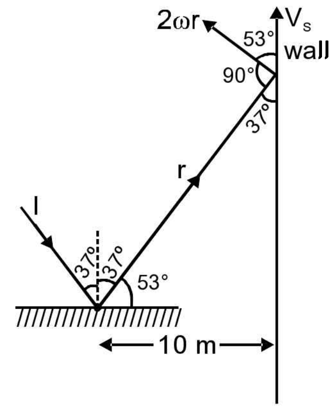
Total angle turned by the reflected ray

$= 40° + 10° \Rightarrow \quad 30° \quad \Rightarrow \quad 30°$(clockwise)

Ex. 36 A light ray I is incidenting on a plane mirror M. The mirror is rotated in the direction as shown in the figure by an arrow at the frequency $\frac{9}{\pi}$ rev/sec. The light reflected by the mirror is received on the wall W that is at a distance 10 m from the axis of rotation. When the angle of incidence becomes 37°, find the speed of the spot (a point) on the wall



Sol. **Concept :** When the incident ray is fixed and if we rotate the mirror the angular velocity ω and reflected ray get rotated by angular velocity 2ω.



$$\omega = \frac{9}{\pi} \text{ rev/sec} = \frac{9}{\pi} \times 2\pi = 18 \text{ rad/sec.}$$

$$\Rightarrow \qquad \cos 53 = \frac{10}{r} \Rightarrow \frac{3}{5} = \frac{10}{r} \Rightarrow r = \frac{50}{3}$$

Let V_S be the velocity of spot on the wall. Then there are two components of V_S

(i) $V_S \cos 53 = 2\,\omega r$ (that is the transverse component)

$$V_S \times \frac{3}{5} = 2 \times 18 \times \frac{50}{3} \qquad \Rightarrow \qquad V_S = 1000 \text{ m/s}$$

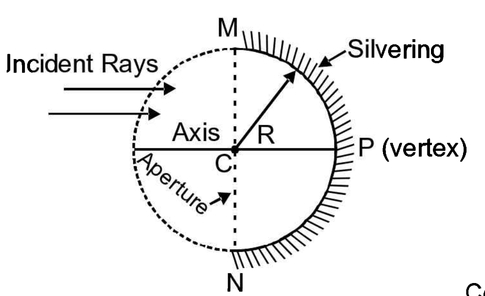
(ii) $V_S \sin 53° = \dfrac{dr}{dt}$ [radial component which increases the length of r]

SPHERICAL MIRROR

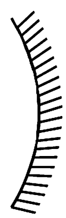
Spherical Mirror is formed by polishing one surface of a part of sphere. Depending upon which part is shining the spherical mirror is classified as

(a) Concave mirror, if the side towards center of curvature is shining and

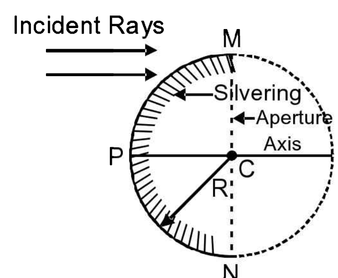
(b) Convex mirror, if the side away from the center of curvature is shining.



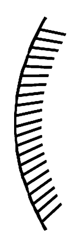
Concave Mirror



Concave Mirror



Convex Mirror



Convex Mirror

1.16 CONCEPT OF UMBRA & PENUMBRA

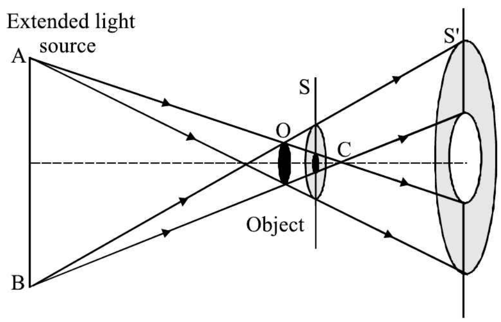
The shadow consists of two parts:

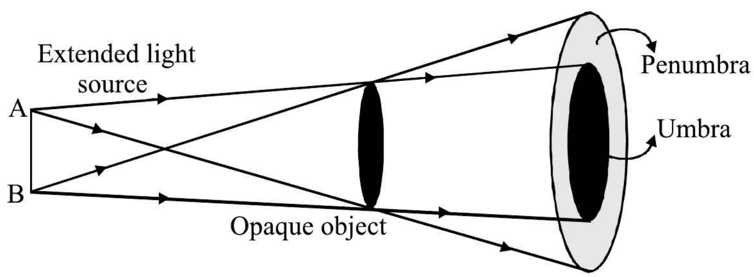
(1) One called Umbra **(2) Other called Penumbra**

UMBRA : It is the region in which there is not any light coming from the source.

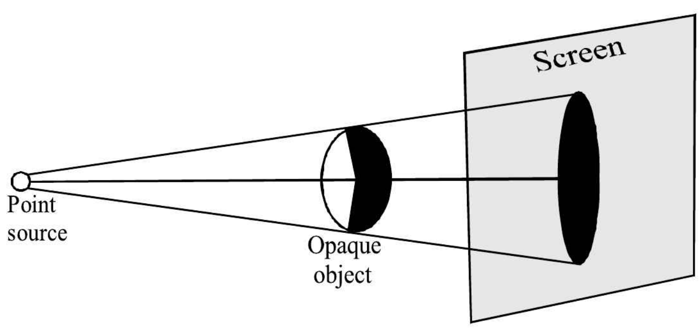
PENUMBRA : Surrounding umbra is called penumbra, where there is partial illumination.

Shadow formation for the extended Source:



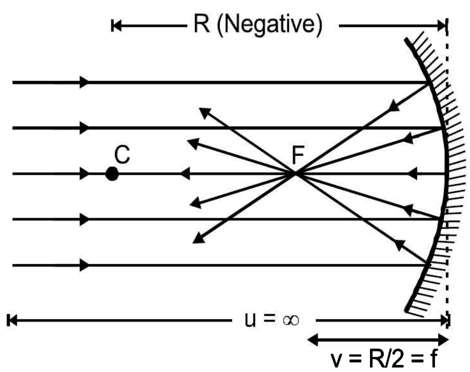


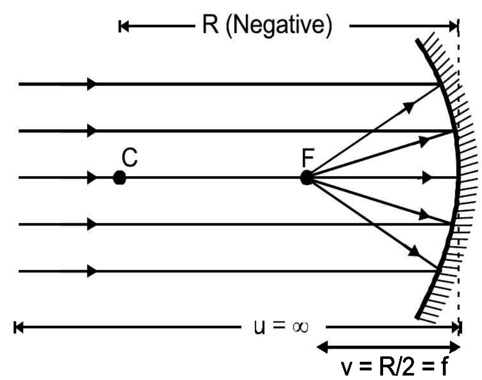
Shadow formation : Shadow formation is explained by the law of rectilinear propagation of light which states that in a homogeneous medium light travels along straight paths. Thus, an opaque object placed between a point source of light and screen will cast a shadow with a *sharply defined* boundary.



1.17 BASIC TERM

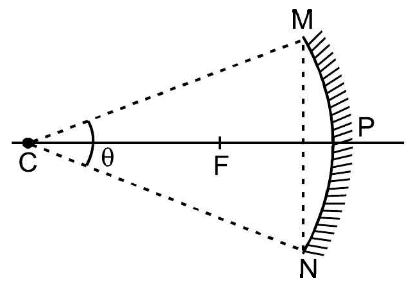
The basic terms associated with spherical mirrors are as follows:
(i) The **centre of curvature, C** is the centre of the sphere of which the mirror is a small segment.
(ii) The **vertex (or pole), P** is the midpoint of the mirror.
(iii) The **radius of curvature, R** is the radius of the sphere of which the mirror is a small section.
(iv) The **principal axis** is the line passing through the centre of curvature C and the vertex P. It extends indefinitely in both directions.
(v) With a concave mirror, all rays parallel to the principal axis pass through a single point F after reflection. F is called the **principal focus.** The parallel rays converge to F after reflection (See fig.). However, in case of a convex mirror, the reflected rays appear to emerge from F behind the mirror. Thus, the reflected rays appear to diverge from the principal focus.





(vi) **Focal plane** is a plane passing through the point F and perpendicular to the principal axis.
(vii) The **focal length, f** of the mirror is the distance, PF, between the vertex P and the principal focus F. It will be shown later that $f = \frac{R}{2}$.
(viii) The reciprocal of the focal length is called the **power** of the mirror. Thus, $P = -\frac{1}{f}$.
(f in meter). Then unit of the power is diopter.
(ix) The diameter MN of the circular outline of the mirror is called the **aperture** of the mirror. The unobstructed surface area of the mirror, having circular rim and available for reflection, represents the aperture (See fig.).

It is measured in terms of the angle θ, subtended by the extremities of the mirror at the centre of curvature, as shown in fig. The aperture determines the amount of light energy that is received by the mirror. In other words, it determines the light gathering capability of the mirror.



1.18 PARAXIAL RAYS AND PARAXIAL APPROXIMATION

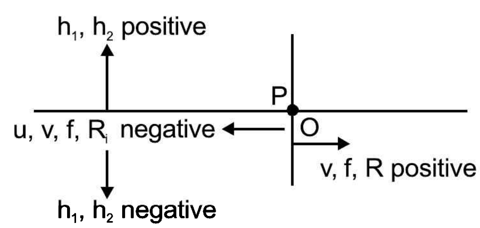
A ray is said to be paraxial if the angle θ between the ray and the principal axis (or symmetry axis) is small. Only rays inclined at less than about 10° to the principal axis are considered as paraxial rays and they lie close to the axis throughout the distance from the object to the image. As the angles are small, and when expressed in radians, we can set the cosines equal to unity and the sines and tangents equal to the angles. This is known as the small angle approximation. Accordingly, we can set

$\cos\theta \cong 1$, $\sin\theta \cong \theta$, $\tan\theta \cong \theta$.

This is also known as **paraxial approximation.** As we are approximating $\sin\theta$ to θ, paraxial approximation is also called as the **first-order theory.** A small area in the immediate surrounding of the optical axis is known as the **paraxial region.** Gauss, in 1841, was the first to give a systematic analysis of the formation of images under paraxial approximation. The results are known as first-order, paraxial or **Gaussian optics.** If we restrict the rays to the paraxial rays, a good image is formed with monochromatic light by an optical system. Therefore, we restrict ourselves to paraxial rays in our study of mirrors and lenses.

1.19 SIGN CONVENTION

In order to specify the position of the object and the images, we need a reference point and sign convention. There are many sign conventions in use in different books, which cause a lot of confusion in the minds of the students.



We adopt the following sign convention known as Cartesian sign convention in our study. This is a relatively a simpler convention, widely used and easy to remember. We use this convention uniformly in the present and next chapters.

(a) The diagrams are drawn with the incident light travelling from left to right.
(b) The distances are measured by taking the vertex as the origin.
(c) The distances measured in the direction of the incident light are considered positive while those measured in the direction opposite to the incident light are taken as negative. (All quantities measured to the right of P are positive and all those to its left are negative.)
(d) Heights measured upward and perpendicular to the principal axis are taken as positive while those measured downward are considered negative.
(e) angles measured clockwise, beginning at the optic axis, are negative. Angles measured counterclockwise are positive.

In order to derive the formula for the spherical mirrors (and lenses), we first draw a diagram and treat all the distances and angles as positive, and from the geometry of the diagram deduce an equation relating to the quantities. When the final equation is obtained, we apply the above sign convention and change the signs in front of quantities that are negative in the figure. This procedure gives us an algebraic equation, which can be applied to any possible case.

1.20 EXTENDED OBJECT

Now suppose we have an object of finite size, represented by the arrow AB in figure (1) perpendicular to the optic axis. A real and inverted image A′B′ is formed between C and F. The triangles ABC and A′B′C are similar and hence

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \quad \text{.............(a)}$$

Also, the triangles ABP and A′ B′ (P) are similar. Hence

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \quad \text{...............(b)}$$

From equations (a) and (b), we get

$$\frac{CB}{CB'} = \frac{PB}{PB'} \qquad \text{or} \qquad \frac{PB - PC}{PC - PB'} = \frac{PB}{PB'}$$

$$\therefore \quad \frac{u - R}{R - v} = \frac{u}{v} \quad \therefore \quad uR + vR = 2uv$$

Dividing throughout by uvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

Using the sign convention, u = –ve, R = – ve and v = –ve into the above equation, we get

$$\frac{1}{-u} + \frac{1}{-v} = \frac{2}{-R} \qquad \frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

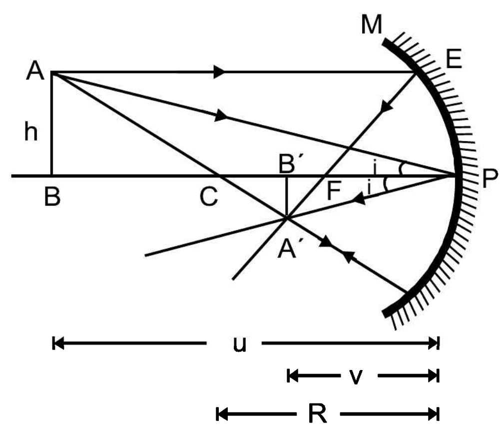


Figure (1)

Using the relation (R = 2f) in the above equation, we obtain $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

This relation is the Mirror formula.

Note : This mirror formula is only applicable for paraxial rays and not for the marginal rays.

Figure (2) shows the formation (image) with an extended object. The object AB is in front of the mirror and a virtual and erect image A' B' is formed behind the mirror. The triangles ABC and A' B' C are similar. Hence we have

$\frac{AB}{A'B'}=\frac{CB}{CB'}$ .... (a) Also, the triangles ABP and A' B' P' similar. Hence we have

$\frac{AB}{A'B'}=\frac{PB}{PB'}$ .... (b) From (a) and (b), we get

$\frac{CB}{CB'}=\frac{PB}{PB'}$ or $\frac{PB+PC}{PC-PB'}=\frac{PB}{PB'}$ $\therefore \frac{u+R}{R-v}=\frac{u}{v}$ or $vR-uR=-2uv$

Dividing the above equation throughout by u v R, we obtain $\frac{1}{u}-\frac{1}{v}=-\frac{2}{R}$

Using the sign convention, u = –ve,
v = +ve and R = +ve, we get

$$\frac{1}{-u}-\frac{1}{v}=-\frac{2}{R}$$

or $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

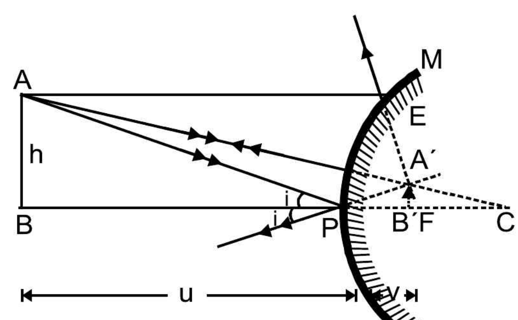
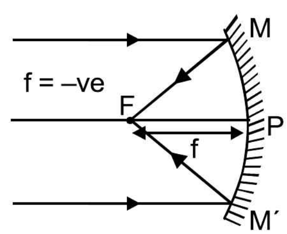


Figure (2)

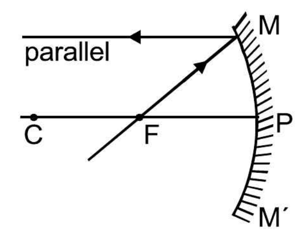
1.21 RULES FOR IMAGE FORMATION (FOR PARAXIAL RAYS ONLY)

(These rules are based on the laws of reflection, $\angle i = \angle r$)

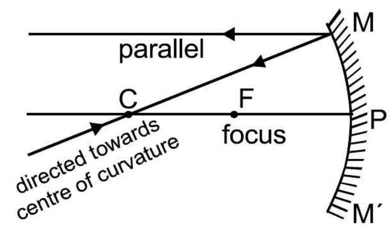


1. A ray parallel to the principal axis after reflection from the mirror passes or appears to pass through its focus (by definition of focus).

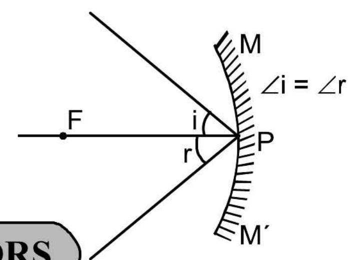
2. A ray passing through or directed towards the focus, after reflection from the mirror, became parallel to the principal axis.



3. A ray passing through or directed towards the centre of curvature, after reflection from the mirror, retraces its path (as for it $\angle i = 0$ and so $\angle r = 0$).



4. Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis.



1.22 RELATIONS FOR SPHERICAL MIRRORS

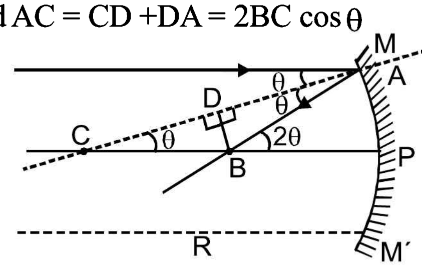
Relation between f and R for the spherical mirror

1. **For Marginal rays,** In Δ ABC, AB = BC and AC = CD +DA = 2BC cos θ

$\Rightarrow$ R = 2BC cos θ

$\Rightarrow$ BC = $\dfrac{R}{2\cos\theta}$ and

BP = PC- BC = R – $\dfrac{R}{2\cos\theta}$



Note : B is not the focus; it is just a point where a marginal ray after reflection meets.

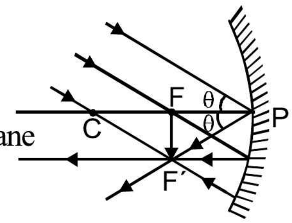
2. **For paraxial rays (parallel to principal axis)**

(θ small so $\sin\theta \approx \theta$, $\cos\theta \approx 1$, $\tan\theta \approx \theta$). Hence BC = $\dfrac{R}{2}$ and BP = $\dfrac{R}{2}$. Thus, point

B is the midpoint of PC (i.e. radius of curvature) and is defined as FOCUS so BP = f= $\dfrac{R}{2}$

(**Definition :** Paraxial rays that are parallel to the principal axis after reflection from the mirror, meet the principal axis at focus)

3. **BP is the focal length (f)** $f = \frac{R}{2}$



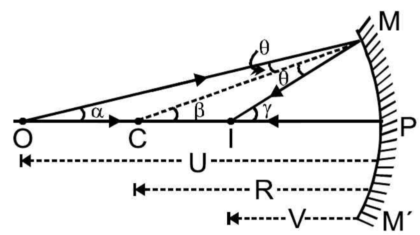
4. **Paraxial rays (not parallel to principal axis)**

Such rays after reflection meet at a point on the focal plane (F), such that

$$\frac{FF'}{FP} = \tan\theta \approx \theta \Rightarrow \frac{FF'}{f} = \theta \quad \Rightarrow \quad FF' = f\theta$$

An object is placed at a distance u from the pole of a mirror and its image is formed at a distance v (from the pole)

If angle is very small : $\alpha = \frac{MP}{u}, \beta = \frac{MP}{R}, \gamma = \frac{MP}{v}$



from Δ CMO, $\beta = \alpha + \theta \quad \Rightarrow \theta = \beta - \alpha$

from Δ CMI, $\gamma = \beta + \theta \quad \Rightarrow \theta = \gamma - \beta$

so we can write $\beta - \alpha = \gamma - \beta \quad \Rightarrow 2\beta = \gamma + \alpha$

$$\therefore \quad \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \quad \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

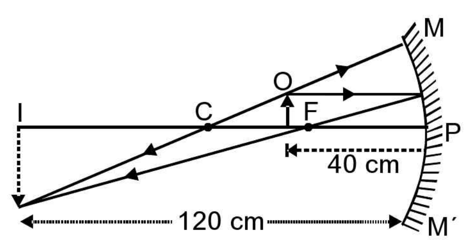
Sign convention for object / image for spherical mirrors

Real object	u – ve	Real image	v – ve
Virtual object	u + ve	Virtual Image	v + ve

Ex. 37 The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

Sol. As the object is in front of the mirror it is real and for real object the magnified image formed by concave mirror can be inverted (i.e., real) or erect (i.e, virtual), so there are two possibilities.

(a) If the image is inverted (i.e., real)



$$m = \frac{f}{f - u}$$

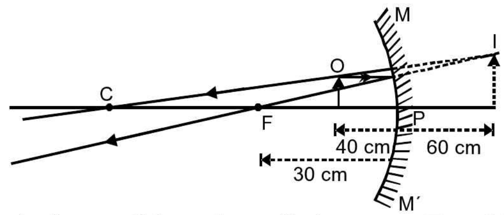
$$-3 = \frac{-30}{-30 - u} \qquad \Rightarrow u = -40 \text{ cm}$$

Object must be at a distance of 40 cm in front of the mirror (in between C and F).

(b) If the image is erect (i.e. virtual)

$$m = \frac{f}{f-u} \Rightarrow 3 = \frac{-30}{-30-u}$$

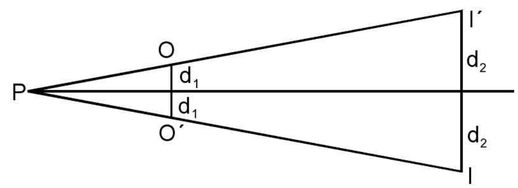
$\Rightarrow u = -20$ cm



Object must be at a distance of 20 cm in front of the mirror (in between F and P).

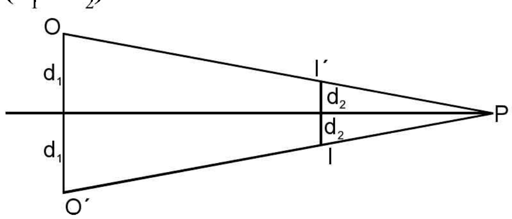
1.23 HOW TO LOCATE

(i) Position of mirror
(ii) Position of centre of curvature
(iii) Position of focus if the position of object and image is given w.r.t. principal axis. For position of mirror :
$(d_2 > d_1)$

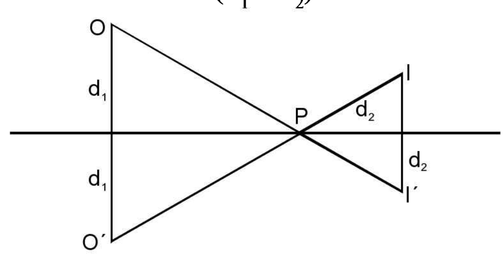


Step-I : Draw a perpendicular line from object and image to the principle axis and cut equal distance d_1 and d_2 as shown in figure.
Step-II : Join the line O′I or I′O and extend it to the principle axis, it cuts at the point P. This point P is the position of mirror.

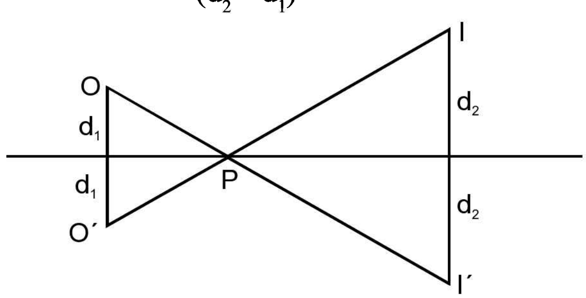
$(d_1 > d_2)$



$(d_1 > d_2)$

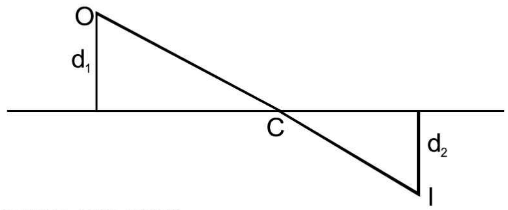


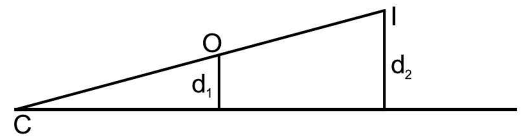
$(d_2 > d_1)$



FOR CENTRE OF CURVATURE:

Step : Join object and image through a line, the point C where the line OI cuts the principle axis, is the centre of curvature of curved mirror.





FOR FOCUS :

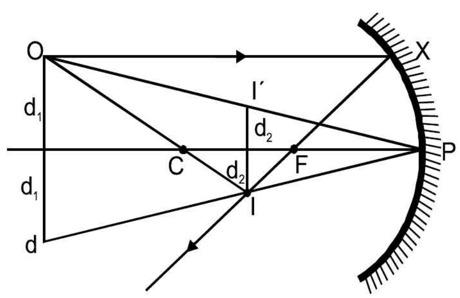
Step-I : Draw a parallel line from the object w.r.t. principle axis to the spherical mirror where it cuts at point X on the mirror.

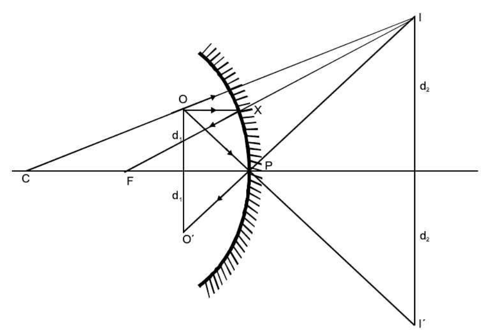
Step-II : Extend the line XI, let it cut the principle axis at point F. The point F is the focal point of the spherical mirror.

Ex. 38 The positions of the real object O and image I are given with respect to the principle axis then locate : (i) position of mirror (nature of mirror)

(ii) position of centre of curvature for, real object case (1) $d_1 > d_2$, case (2), $d_2 > d_1$

Sol.



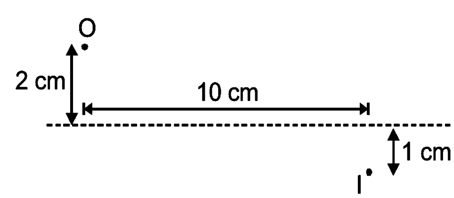


Step-I : Draw a line perpendicular to the principle axis.

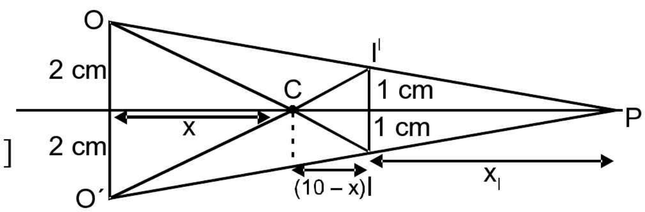
Step-II : Extend the line OI′ and O′I. Let it cut at point P to the principle axis. Then point P is the pole it mirror.

Step-III : Joint O and I through a line, extend it to the principle axis, i + cut at point c. Then point C is the centre of curvature of the mirror.

Ex. 39 The principal axis of a spherical mirror is shown by dotted line. O is the point object whose real image is I. Find the distance of the pole and centre of curvature of the mirror from the object measured along the principal axis by drawing ray diagram.



Sol. Here C, is the position of the curved mirror. The triangles OCO′ and CII′ are similar Then,



$$\frac{OO'}{II'} = \frac{x}{10-x}$$

$$\frac{4}{2} = \frac{x}{10-x} \qquad \left[\text{then} \quad x = \frac{20}{3}\text{cm}\right]$$

Then, distance of centre of curvature from object $= \frac{20}{3}$ cm

For pole the triangles OO′P and II′P are similar

$$\text{Then,} \qquad \frac{OO'}{II'} = \frac{10+x_I}{x_I} \Rightarrow \frac{4}{2} = \frac{10+x_I}{x_I} \Rightarrow x_I = 10 \text{ cm}$$

Then, the distance of pole from object = 30 cm

1.24 MAGNIFICATION

Concept of Magnification :

Magnification : $\frac{\text{size of image}}{\text{size of object}}$

• For transverse magnification size means height and for longitudinal magnification size means length of object.

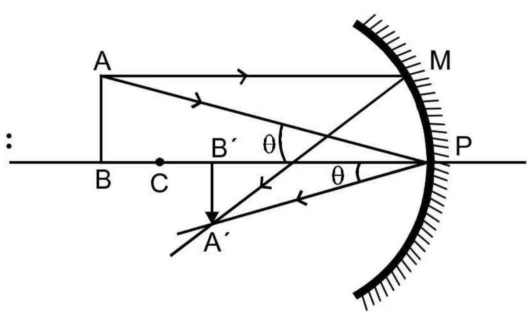
• FOR 1 DIMENSION OBJECT

There are two type of magnification

(a) Transverse or Lateral magnification

(b) Longitudinal magnification

(a) Transverse or Lateral magnification :



$$m = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_o}$$

Δ ABP and A′B′P are similar. So,

$$\frac{-h_i}{h_o} = \frac{-v}{-u} \Rightarrow \frac{-h_i}{h_o} = +\frac{v}{u} \Rightarrow m = \frac{+h_i}{h_o} = -\frac{v}{u}$$

• Erect image means that both object and image are formed on the same side of principle axis.

• Inverted image means that both object and image are formed on the opposite side of principle axis.

Magnification	Nature of Image
$\lvert m \rvert > 1$	enlarged
$\lvert m \rvert < 1$	diminished
$m > 0$	erect
$m < 0$	inverted

(b) Longitudinal magnification :

• If one dimensional object is placed along the principle axis then linear magnification is called longitudinal magnification.

Longitudinal magnification : $m_\ell = \dfrac{\text{length of image}}{\text{length of object}} = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right)$

for small objects only : $m_\ell = \dfrac{-dv}{du}$

By differentiation of mirror equation on both side with respect to u we get,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow -\frac{1}{v^2}\frac{dv}{du}-\frac{1}{u^2}=0 \Rightarrow \frac{dv}{du}=-\frac{v^2}{u^2} \Rightarrow m_\ell = -\frac{dv}{du}=\frac{v^2}{u^2}=m^2$$

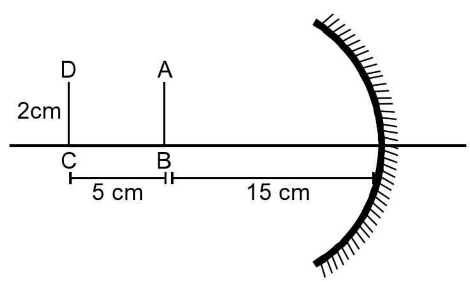
Length of image = (m^2) (length of object)

(for only small object)

• For small one dimension object :

m_ℓ = (Transverse magnification)

Ex. 40 A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure. Find the total length of the image.



Sol. For AB $m_1 = \dfrac{f}{f-u} = \dfrac{(-10)}{(-10)-(-15)} = -2$

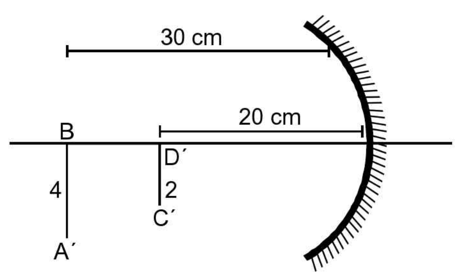
$v_1 = -m_1u = (-)(-2)(-15) = -30$

$h_1 = m_1h_o = -4$, For CD

$m_2 = \dfrac{f}{f-u} = \dfrac{(-10)}{(-10)-(-20)} = -1$

$v_2 = -m_2u = (-)(-1)(-20) = -20$

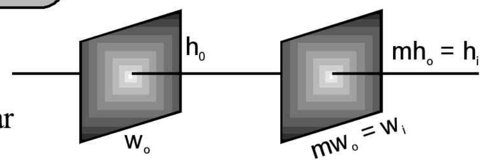
$h_2 = m_2h_o = -2 \Rightarrow$ Total length of image = 2 + 10 + 4 = 16 cm.



1.25 SUPERFICIAL MAGNIFICATION

(a) Magnification in the area of object

(b) For two dimension object



Case (1) : When an object is placed perpendicular to the principle axis.

In this case both dimensions height of i. e. the object and width of the object are perpendicular to the principle axis. then,

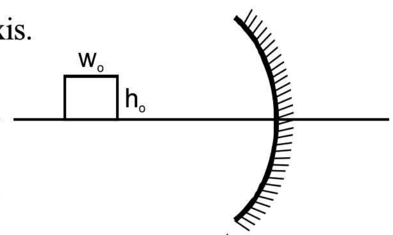
$h_i = mh_o$ [h_0 = height of object] ; $w_i = mw_o$ [w_0 = width of object]

Area of image : $A_{image} = h_i \times w_i = mh_o \times mw_o \Rightarrow A_{image} = m^2 A_{object}$

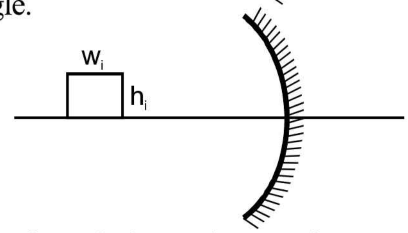
Superficial magnificat0ion = m^2

Case (2) : When object is placed along the principle axis.

• for the small object :



All dimensions will be magnified equally because all dimensions are almost at the same distance from mirror. Hence, the image of a rectangular kept in front of a concave mirror (as shown in the figure) will be rectangle.



$h_i = mh_o$ (transverse magnification)

$w_i = m^2h_o$ (Longitudinal magnification for small objects only)

then, area of image = $(w_i)(h_i) = (m^2 w_o)(mh_o)$

area of image = (m^3) (area of object)

But, when a large object is kept in front of a concave mirror, its image is trapezium.

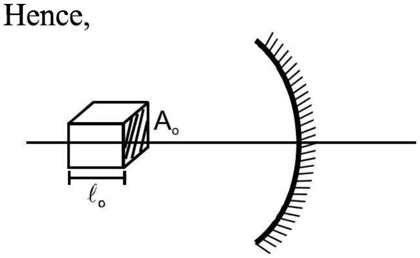
1.26 VOLUME MAGNIFICATION

For small cubical objects only, all dimensions will be magnified equally because all dimensions are almost at the same distance from the mirror, hence final image is also a cube.

Area of object is perpendicular to the principle axis. Hence,

Area of image $(A_i) = m^2A_o$

Length of image (for small object)



$\Rightarrow \ell_i = m^2\ell_o$

Then, Volume of image = $(A_i)(\ell_i)$

$\Rightarrow (m^2A_o)(m^2\ell_o) \Rightarrow m^4(A_o \ell_o)$

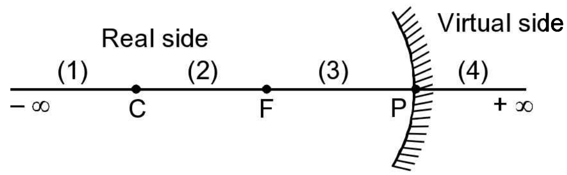
volume of image = m^4v_o

Ex. 41 A cube of side length 1 mm is placed on the axis of a concave mirror at a distance of 45 cm from the pole as shown in the figure. One edge of the cube is parallel to the axis. The focal length of the mirror is 30 cm. Find the volume of the image.

Ans. 16 mm^3

1.27 CONCEPT OF POSITION OF IMAGE FOR GIVEN OBJECT IN CONCAVE MIRROR

- Let us define the region from
  $-\infty$ to C as region .... (1)
  C to F as region .... (2)
  F to P as region .... (3)
  P to $+\infty$ as region .... (4)



- Any object in region (1) will have its image in region (2) and vice versa.
- Similarly, any object in region (3) will have its image in region (4) and vice versa.

	Position of Object	Position of image	Magnification
a	Region (1) (Real side) Real object	Region (2) (Real side) Real image, inverted $m < 0$	Region of image (2) is smaller as compared to region of object (1) (diminished image)
b	Region (2) (Real side) Real object	Region (1) (Real side) Real image, inverted $m < 0$	Region of image (1) is larger as compared to region of object (2) (enlarged image)
c	Region (3) (Real side) Real object	Region (4) (Virtual side) virtual image, erect image $m > 0$	Region of image (4) is larger as compared to region of object (3) (enlarged image)
d	Region (4) (Virtual side) Virtual object	Region (3) (Real side) Real image, erect image $m > 0$	Region of images (3) is smaller as compared to region of object (4) (diminished image)

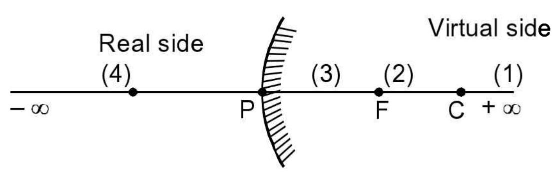
$\Rightarrow$ For region (1), (2) and (3) u is always (–)ve and for region (4) it is (+)ve. Similarly v can be concluded from diagram of concave mirror.

- Formula of magnification is $m = -\dfrac{v}{u}$

(a) For same signs of u and v, negative magnification ($m < 0$) implies inverted image.
(b) For opposite signs of u and v, positive magnification ($m > 0$) implies erect image.
$\rightarrow$ In concave mirror for virtual object, nature of image always real.

1.28 CONCEPT OF POSITION OF IMAGE FOR GIVEN OBJECT IN CONVEX MIRROR

- Let us define the region from
  C to $+\infty$ as region .... (1)
  F to C as region .... (2)
  P to F as region .... (3)
  $-\infty$ to P as region .... (4)



- Any object in the region (1) will have its image in region (2) and vice versa.
- Any object in the region (3) will have its image in region (4) and vice versa.

	Position of Object	Position of image	Magnification
a	Region (1) (Virtual side) Virtual object	Region (2) (Virtual side) Virtual image, inverted $m < 0$	Region of image (2) is smaller as compared to region of object (1) (diminished image)
b	Region (2) (Virtual side) Virtual object	Region (1) (Virtual side) Virtual image, inverted $m < 0$	Region of image (1) is larger as compared to region of object (2) (enlarged image)
c	Region (3) (Virtual side) Virtual object	Region (4) (Real side) Real image, erect image $m > 0$	Region of image (4) is larger as compared to region of object (3) (enlarged image)
d	Region (4) (Real side) Real object	Region (3) (Virtual side) Virtual image, erect image $m > 0$	Region of images (3) is smaller as compared to region of object (4) (diminished image)

$\Rightarrow$ In convex mirror real object always formed virtual image.

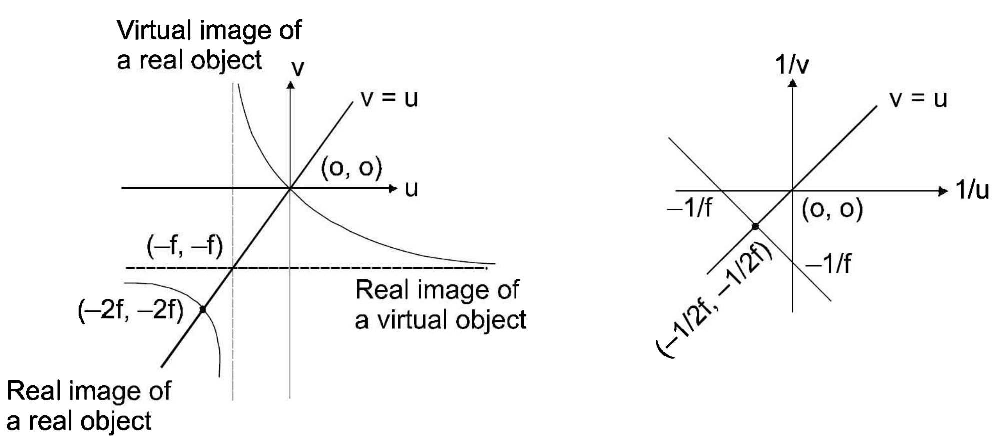
1.29 CONCEPT OF (u – v) GRAPH

(a) For concave mirror : Apply mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{y}+\frac{1}{x}=-c \text{ [Since } f<0,\ c=\frac{1}{|f|}\text{]} \quad \Rightarrow \quad \frac{1}{y}=-c\frac{-1}{x}$$

$$\Rightarrow \frac{1}{y}=-\frac{cx-1}{x} \quad \Rightarrow y=\frac{-1}{c}\left[\frac{cx}{cx+1}\right] \quad \Rightarrow y=-\frac{1}{c}\left[\frac{cx+1-1}{cx+1}\right] \Rightarrow -cy=1-\frac{1}{cx+1}$$

$$\Rightarrow \frac{1}{cx+1}=1+cy \qquad \Rightarrow \qquad (cx+1)(cy+1)=1$$



and this is the equation of rectangular hyperbola whose origin shifts.

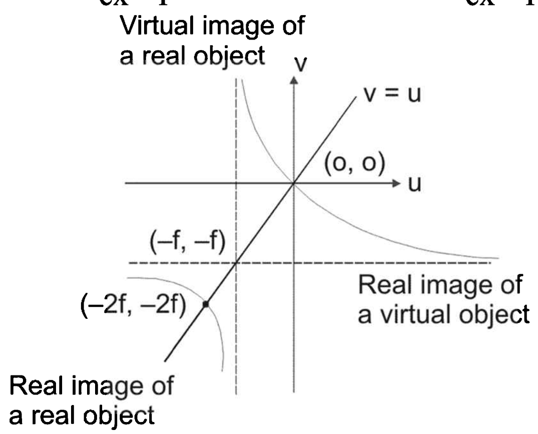
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad y + x = -c$$

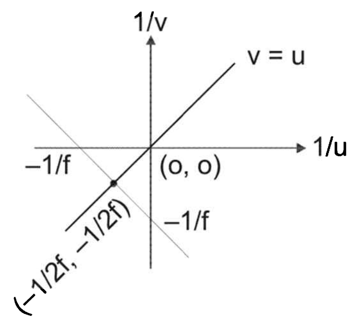
(b) For convex mirror : **Apply mirror formula**

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{y}+\frac{1}{x} = +c \text{ [Since } f > 0,\ c = \frac{1}{|f|}\text{]}$$

$$\frac{1}{y} = +c - \frac{1}{x} \quad \Rightarrow \quad \frac{1}{y} = \frac{cx-1}{x} \quad \Rightarrow \quad y = \frac{1}{c}\left[\frac{cx}{cx-1}\right]$$

$$cy = 1 - \frac{(cx-1+1)}{cx-1} \quad \Rightarrow \quad cy = 1 + \frac{1}{cx-1} \quad \Rightarrow \quad (cx-1)(cy-1) = 1$$





This is the equation of rectangular hyperbola whose origin shifts.

$$\frac{1}{v}+\frac{1}{u}=+c \quad \Rightarrow \quad y + x = +c$$

1.30 VELOCITY OF IMAGE OF MOVING OBJECT (IN SPHERICAL MIRROR)

→ There are two components of velocity of an image

(a) Velocity component along the principle axis that means perpendicular to the mirror

(b) Velocity component perpendicular to the principle axis that means parallel to the mirror

(a) Velocity component along the axis:

Differentiating mirror formula on the both side w.r.t. time we get,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$-\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0 \text{ (Since focal length of the mirror remains constant)}$$

$$\Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt} \qquad \Rightarrow \qquad (v_{Im})_{ll} = -\frac{v^2}{u^2}(v_{om})_{ll}$$

$(v_{Im})_{ll}$ = Velocity of image w.r.t. mirror along the principle axis.

$(v_{om})_{ll}$ = Velocity of an object w.r.t. mirror along the principle axis.

- Direction of velocity of an object and image w.r.t. mirror is opposite to each other.
- Any object in the region (1) will have its image in region (2) and vice versa
- Similarly to region (3) and (4).
- When an object moves from $-\infty$ to c, the image moves from F to C as shown in figure (a).

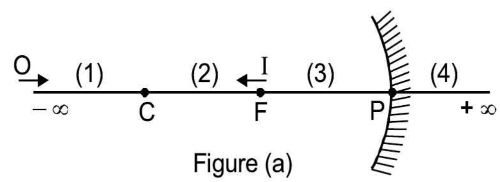


Figure (a)

- When an object moves from F to P, the image moves from $+\infty$ to P as shown in figure (b).

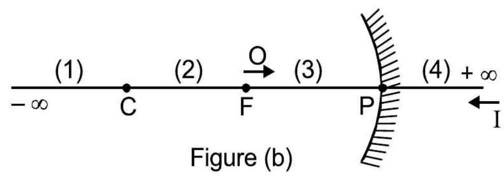


Figure (b)

- When an object moves from $-\infty$ to P, image moves from F to P as shown in figure (c).

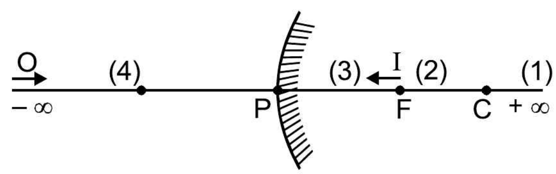


Figure (c)

- When an object moves from P to F, the image moves from P to $+\infty$ as shown in figure (d)

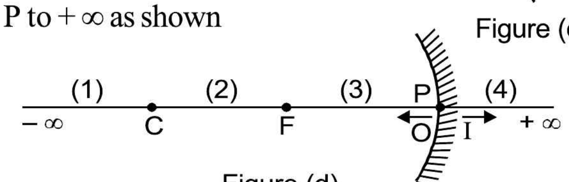


Figure (d)

- When an object moves from C To F, the image moves from C to $-\infty$ as shown in figure (e).

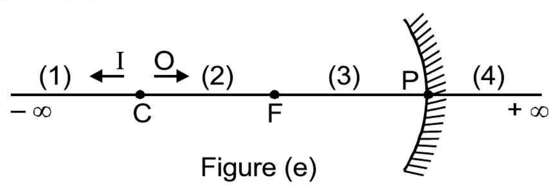


Figure (e)

(b) Velocity component perpendicular to axis:

$$m = \frac{h_i}{h_o} \qquad \Rightarrow \qquad hi = mho$$

Differentiating this equation on the both side w.r.t. time we get,

$$\frac{dhi}{dt} = \frac{mdho}{dt} + ho\frac{dm}{dt}$$

$$\Rightarrow (V_{IM})_{\perp} = m\,(V_{om})_{\perp} + ho\frac{dm}{dt}$$

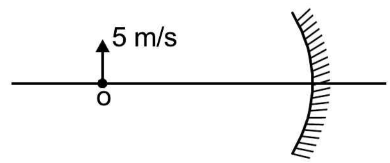
$(V_{IM})_{\perp}$ = Velocity of image w.r.t mirror perpendicular to the principle axis.
$(V_{om})_{\perp}$ = Velocity of image w.r.t mirror perpendicular to the principle axis.

$\frac{dm}{dt}$ = Rate of change of magnification unit (per sec)

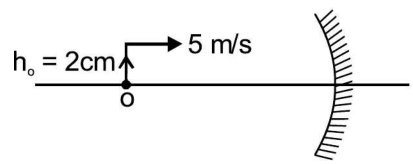
$m = -\frac{v}{u}$ By differentiating this equation on both the sides w.r.t. time we get,

$$\frac{dm}{dt} = -\left[\frac{u(V_{IM})_{\parallel} - v(V_{om})_{\parallel}}{u^2}\right]$$

Type -1 : In this type $h_o = 0$ but $\frac{dho}{dt} = 5$ m/s

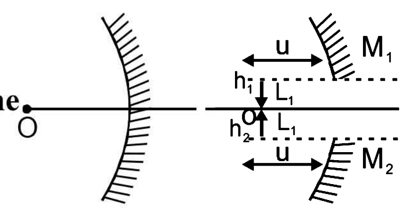


Type -2 : In this type $h_o = 2$ cm but $\frac{dho}{dt} = 0$

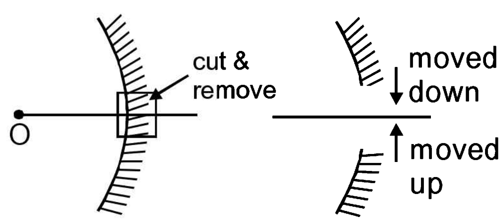


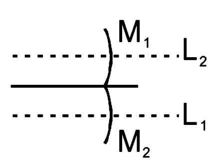
1.31 CUTTING OF SPHERICAL MIRROR

(i) Bisplit cutting
(ii) Billetsplit cutting
(i) Bisplit cutting involves cutting and moving the mirror Perpendicular to principal axis. L_1 behaves as P.A. for M_1 and similarly for M_2.



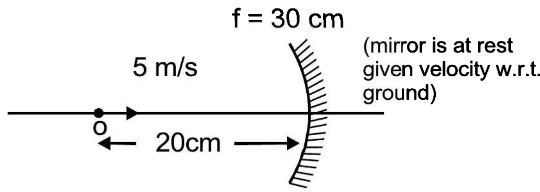
(ii) Billet split cutting involves cutting and removing a part of mirror from between L_1 behaves as P.A. for M_1 and similarly for M_2.





Note : Both of these types can be for convex mirror also.

Ex.42 Find the velocity of image w.r.t. ground.



Sol. For the velocity component along the principle axis

$$(V_{IM})_{\|} = -\frac{v^2}{u^2}(V_{om})_{\|}$$

Apply mirror equation :

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{-20}=\frac{1}{-30} \Rightarrow v = +60 \text{ cm} \Rightarrow m = -\frac{v}{u} = \frac{-60}{-20} = 3$$

Shortcut solution : $m = \dfrac{f}{f-u} = \dfrac{-30}{(-30)-(-20)} = +3$

$$v = -mu = -(3)(-20) = +60 \text{ cm}$$

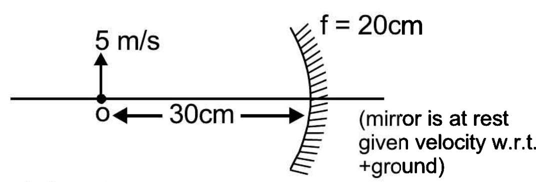
Since mirror is at rest (given that) :

$$(V_{IM})_{\|} = -\left(\frac{60}{-20}\right)^2(5) = -45 \text{ m/s} \Rightarrow \overrightarrow{V_{IG}} = \overrightarrow{V_{IM}} + \overrightarrow{V_{MG}} \Rightarrow = -45 + 0 = -45 \text{ m/s}$$

that means image approaches the mirror with velocity 45 m/s.

Ex.43 Find velocity of image w.r.t. ground

Sol. Apply mirror equation :



$$m = \frac{f}{f-u} = \frac{(-20)}{(-20)-(-30)} = -2$$

$v = -mu = (-)(-2)(-30) = -60$ cm

For the velocity component along the principle axis :

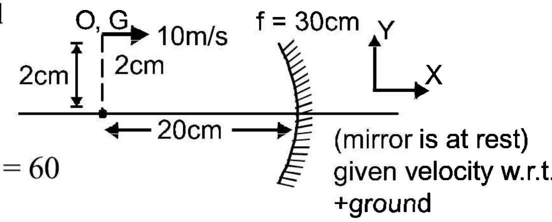
$$(V_{IM})_{\|} = -\frac{v^2}{u^2}(V_{om})_{\|} \Rightarrow (V_{IM})_{\|} = 0 \quad [\text{Since } (V_{om})_{\|} = 0]$$

For the velocity component perpendicular to the principle axis :

$$(V_{IM})_{\perp} = m(V_{om})_{\perp} + h_o\frac{dm}{dt} \Rightarrow (V_{IM})_{\perp} = (-2)(5) = -10 \text{ m/s} \quad [\text{Since, } h_0 = 0]$$

$$\overrightarrow{V_{IG}} = \overrightarrow{V_{IM}} + \overrightarrow{V_{MG}} \Rightarrow -10 + 0 = -10 \text{ m/s} \Rightarrow 10 \text{ m/s (moving downwards)}$$

Ex.44 Find the velocity of image w.r.t. ground



Sol. By applying mirror formula

$$m = \frac{f}{f-u} = \frac{-30}{(-30)-(-20)} = 3 \Rightarrow v = 60$$

$$(V_{IM})_{\|} = -\frac{v^2}{u^2}(V_{om})_{\|} \Rightarrow (V_{IM})_{\|} = \left(\frac{60}{-20}\right)^2 (10) = -90 \text{ m/s}$$

$$(V_{IM})_{\perp} = m\,(V_{om})_{\perp} + h_0\frac{dm}{dt} \Rightarrow [(V_{om})_{\|} = 0] \qquad \left[\text{but } \frac{dm}{dt} \neq 0\right]$$

$$\frac{dm}{dt} = -\left[\frac{u(V_{IM})_{\|} - v(V_{OM})_{\|}}{u^2}\right] \Rightarrow \frac{dm}{dt} = \left[\frac{(-20)\times10^{-2}\times(-90)-(60)\times10^{-2}(10)}{(-20\times10^{-2})^2}\right]$$

$$\Rightarrow \frac{dm}{dt} = -3\times10^2 \text{ per sec.}$$

Then, $(V_{IM})_{\perp} = m(V_{om})_{\perp} + h_0\dfrac{dm}{dt} \Rightarrow (V_{IM})_{\perp} = 0 + 2\times10^{-2}\times(-3\times10^2) = -6$ m/s

$$\overrightarrow{V_{IM}} = (V_{IM})_{\|}\hat{i} + (V_{IM})_{\perp}\hat{j} \qquad \overrightarrow{V_{IM}} = -90\hat{i} - 6\hat{j} \Rightarrow \overrightarrow{V_{IG}} = \overrightarrow{V_{IM}} + \overrightarrow{V_{MG}}$$

$$\therefore \quad \overrightarrow{V_{IG}} = (-90\hat{i} - 6\hat{j}) \text{ m/s}$$

Q.1 An object 1 cm high is placed at 10 cm in front of a concave mirror of focal length 15 cm. Find the position, height and nature of the image.

Sol. location = 45 cm

h_i = 3 cm

Ex.45 The image of a real object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the position of object and magnification.

Sol. Now $u = \dfrac{vf}{v-f} = \dfrac{(4)(12)}{4-12} = -6$ cm

The negative sign shows that the object is real and it is placed in front of the mirror.

The magnification, $m = -\dfrac{v}{u} = \dfrac{-(+4)}{-6} = \dfrac{2}{3}$

Ex.46 When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1. The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2. If $\dfrac{m_1}{m_2} = 4$, then calculate the focal length of the mirror.

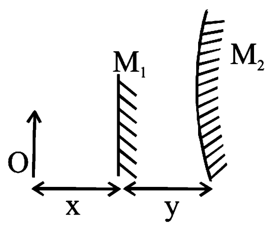
Sol. Since $\frac{m_1}{m_2} = 4$, therefore $\frac{f+40}{f+25} = 4$

thus $f + 40 = 4f + 100$

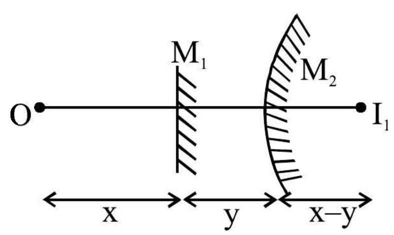
or $f = -20$cm

The negative sign shows that the mirror is concave.

Ex.47 An object O is placed in front of a small plane mirror M_1 and a large convex mirror M_2 of focal length *f*. The distance between O and M_1 is x, and the distance between M_1 and M_2 is y. The images of O formed by M_1 and M_2 coincide. Find the focal length of the mirror.



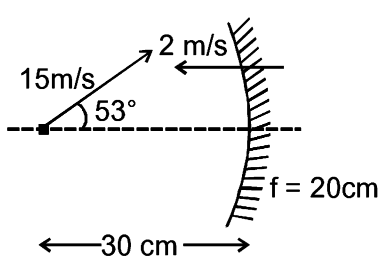
Sol.



$$\frac{1}{x-y} + \frac{1}{-(x+y)} = \frac{1}{f} \Rightarrow f = \frac{x^2 - y^2}{2y}$$

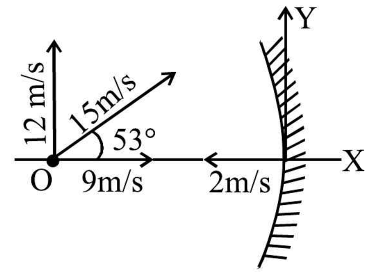
Now, as this is convex mirror, f is (+)ve, hence magnitude of $f = \frac{x^2 - y^2}{2y}$

Ex.48 Find the velocity of image in a situation as shown in the figure.



Sol. $\vec{V}_O$ = Velocity of object = $(9\hat{i}+2\hat{j})m/s$

$\vec{V}_m$ = Velocity of mirror = $-2\hat{i}m/s$



$$m = \frac{f}{f-u} = \frac{-20}{-20-(-30)} = -2$$

For velocity component parallel to optical axis

$$\left(\vec{V}_{I/m}\right)_{\parallel} = -m^2\left(\vec{V}_{O/m}\right)_{\parallel}$$

$$\left(\vec{V}_{I/m}\right)_{\parallel} = (-2)^2 11\hat{i} = -44\hat{i}\, m/s$$

For velocity component perpendicular to optical axis

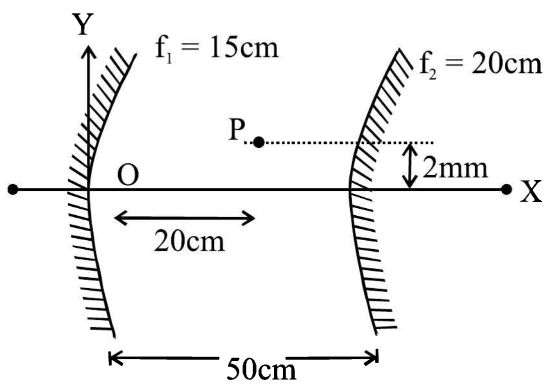
$$\left(\vec{V}_{I/m}\right)_{\perp} = \left(\vec{V}_{O/m}\right)_{\perp} \qquad = (-2)12\hat{j} = -24\hat{j}\, m/s$$

$\therefore\ \vec{V}_{I/m}$ = Velocity of image w.r.t. mirror

$= \left(\vec{V}_{I/m}\right)_{\parallel} + \left(\vec{V}_{I/m}\right)_{\perp} \qquad = (-44i - 24\hat{j})m/s$ Also, $\vec{V}_{I/m} = \vec{V}_I - \vec{V}_m$

or $\quad \vec{V}_I = (-44\hat{i} - 24\hat{j}) - 2\hat{i} \qquad = (-46\hat{i} - 24\hat{j})m/s$

Q.2 Find the co-ordinates of the image of point object P formed after two successive reflections in a situation as shown in the figure considering first reflection at a concave mirror and then at a convex mirror.



Ex.49 A point object located at a distance of 20 cm from the pole of a concave mirror of focal length 30 cm with height 2 cm is moving with a velocity ($\overrightarrow{V_{OG}} = 4\hat{i} - 5\hat{j}$) m/s and velocity of the mirror is ($\overrightarrow{V_{mg}} = -6\hat{i} + 10\hat{j}$) m/s as shown. Find the velocity of image w.r.t. ground.

Sol. From the velocity diagram of object w.r.t. mirror is

$(V_{om})_{\|} = 10$ m/s, $\Rightarrow$ $(V_{om})_{\perp} = -15$ m/s $\Rightarrow$ $m = \dfrac{f}{f-u} = 3 \Rightarrow v = 60$ cm

$(V_{IM})_{\|} = -\dfrac{v^2}{u^2}(V_{om})_{\|} \Rightarrow (V_{IM})_{\|} = -\left(\dfrac{60}{-20}\right)^2(10) = -90$ m/s

$$\frac{dm}{dt} = -\left[\frac{u(V_{IM})_{\|} - V(V_{OM})_{\|}}{u^2}\right] = \left[\frac{(-20)x10^2x(-90)-(60)x10^{-2}x(10)}{(-20x10^{-2})^2}\right] \text{ per sec.}$$

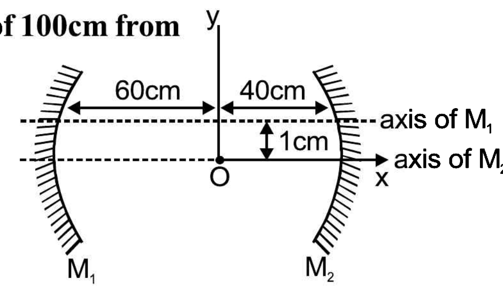
$\dfrac{dm}{dt} = -3 \times 10^2$ per sec. $\Rightarrow$ $(V_{IM})_{\perp} = h_0\dfrac{dm}{dt} + m(V_{om})_{\perp}$

$(V_{IM})_{\perp} = 2 \times 10^{-2} \times (-3 \times 10^2) + (3)(-15)$ $\Rightarrow$ $-6 - 45 = -5$ m/s

$\overrightarrow{V_{IM}} = (V_{IM})_{\|}\hat{i} + (V_{IM})\hat{j}$ $\Rightarrow$ $\overrightarrow{V_{IM}} = -90\hat{i} - 51\hat{j}$

$\overrightarrow{V_{IG}} = (-90\hat{i} - 51\hat{j}) + (-6\hat{i} + 10\hat{j}) \Rightarrow$ $\therefore$ $\overrightarrow{V_{IG}} = (-96\hat{i} - 41\hat{j})$ m/s

Ex.50 Two concave mirrors, each having radius of curvature 40 cm are placed such that their principal axes are parallel to each other and at a distance of 1 cm to each other. Both the mirrors are at a distance of 100cm from each other. Considering first reflection at M_1 and then at M_2, find the coordinates of the image thus formed. Take the location of the object as the origin.



Sol. For the mirror M_1:

$m = \dfrac{f}{f-u} = \dfrac{-20}{(-20)-(-60)} = \dfrac{-20}{40} = -\dfrac{1}{2}$ $\Rightarrow$ $v = -mu = -\left(-\dfrac{1}{2}\right)(-60) = -30$ cm

h_1 = (height of image from the principle axis of mirror M_1)

$= mh_0 = -\dfrac{1}{2}x1 = -0.5$ cm $\qquad I_1 \equiv (-30, -0.5)$ cm

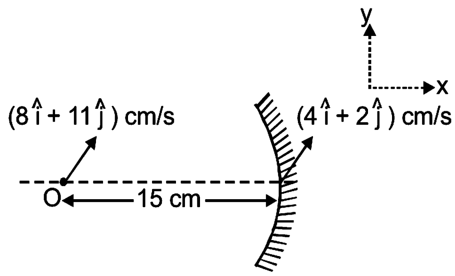
For the mirror M_2:

$m = \dfrac{f}{f-u} = \dfrac{-20}{(-20)-(-70)} = \dfrac{-20}{50} = -\dfrac{2}{5}$ $\Rightarrow$ $v = -mu = (-)\left(-\dfrac{2}{5}\right)x(-70) = -28$cm

$h_2 = mh_0 = \left(-\dfrac{2}{5}\right)(1.5) = -\dfrac{2}{5}x\dfrac{3}{2} = -\dfrac{3}{5}$ cm

Position of final image is 28 cm from mirror M_2 and $\dfrac{3}{5}$ cm of below the principle axis of mirror M_2. $I_2 \equiv (12, -\dfrac{3}{5})$ cm.

Ex.51 **A point object located at a distance of 15 cm from the pole of a concave mirror of focal length 10 cm on its principal axis is moving with a velocity $(8\hat{i}+11\hat{j})$cm/s and velocity of the mirror is $(4\hat{i}+2\hat{j})$cm/s as shown. If $\vec{V}$ is the velocity of the image. Then find the value of $|\vec{V}|$ (in cm/s)**



Sol. Apply mirror formula (For magnification)

$$m=\frac{f}{f-u}=\frac{-10}{(-10)-(-15)}=\frac{-10}{5}=-2$$

$v=-mu=(-)(-2)(-15)=-30$ cm

For the parallel component of velocity of image to the principle axis:

$(V_{IM})_{\parallel} = -m_2(V_{om})_{\parallel} \Rightarrow -(-2)^2(4)=-16$ cm/s

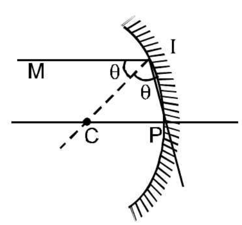
For the perpendicular component of velocity of image to the principle axis:

$$(V_{Im})_{\perp}=m\,(V_{om})_{\perp}+h_o\frac{dm}{dt} \quad\Rightarrow\quad (-2)(9)=-18\text{ cm/s} \quad\Rightarrow\quad (\text{since } h_0=0)$$

$$\vec{V}_{IM}=(V_{om})_{\parallel}\hat{i}+(V_{om})_{\parallel}\hat{j} \quad\Rightarrow\quad \vec{V}_{IM}=16\hat{i}-18\hat{j} \quad\Rightarrow\quad \vec{V}_{IG}=\vec{V}_{Im}+\vec{V}_{mG}$$

$$=(-16\hat{i}-18\hat{j})+4\hat{i}-2\hat{j} \quad\Rightarrow\quad -12\hat{i}-16\hat{j} \quad\Rightarrow\quad \left|\vec{V}_{IG}\right|=\sqrt{12^2+16^2}=20\text{ cm/sec.}$$

Ex.52 **Find the angle of incidence of a ray as it passes through the pole, given that MI || CP.**

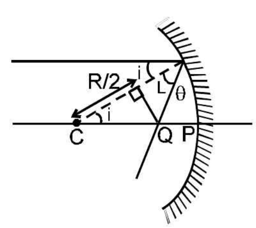


Sol. $\angle MIC=\angle CIP=\theta \Rightarrow$ MI || CP $\angle MI\theta=\angle ICP=\theta$

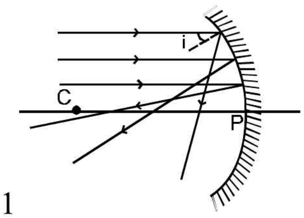
CI = CP $\Rightarrow$ $\angle CIP=\angle CPI=\theta$

$\therefore$ In ΔCIP all angle are equal $\Rightarrow$ $3\theta=180^\circ \Rightarrow \theta=60^\circ$

Ex.53 **Find the distance CQ if incident light ray parallel to the principal axis is incidenting at an angle i. Also find the distance CQ if $i\to 0$.**



Sol. $\cos i = \dfrac{R}{2CQ}$, $CQ = \dfrac{R}{2\cos i}$

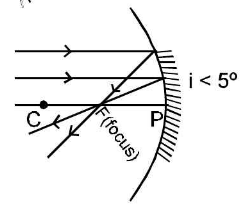


As i increases cos i decreases.

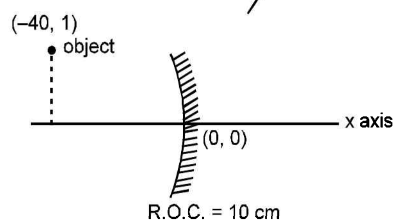
Hence CQ increases. If i is a small angle $\cos i \approx 1$

$\therefore$ $CQ = R/2$

So, paraxial rays meet at a distance equal to R / 2 from center of curvature, which is called focus.



Ex.54 Figure shows a spherical concave mirror with its pole at (0, 0) and principal axis along x axis. There is a point object at (–40 cm, 1cm), find the position of image.



Sol. According to sign convention,

$u = -40$ cm $\Rightarrow$ $h_1 = +1$ cm $\Rightarrow$ $f = -5$ cm. $\Rightarrow$ $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \dfrac{1}{v} + \dfrac{1}{-40} = \dfrac{1}{-5}$; $v = \dfrac{-40}{7}$ cm. ; $\dfrac{h_2}{h_1} = \dfrac{-v}{u} \Rightarrow$ $h_2 = -\dfrac{-v}{u} \times h_1 = \dfrac{-\left(-\dfrac{40}{7}\right) \times 1}{-40}$

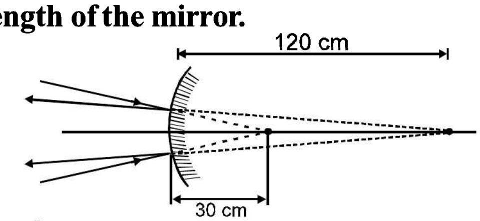
$= -\dfrac{1}{7}$ cm. $\Rightarrow$ $\therefore$ The position of image is $\left(\dfrac{-40}{7}\text{cm}, -\dfrac{1}{7}\text{cm}\right)$

Ex.55 Converging rays are incidenting on a convex spherical mirror such that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam such that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

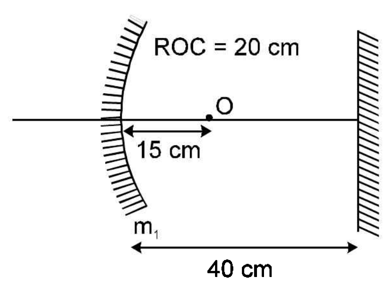
Sol. In this case

$u = +30$ $v = +120$

$\therefore \dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{120} + \dfrac{1}{30} \Rightarrow f = 24$ cm



Ex.56 Find the position of the final image after three successive reflections taking the first reflection on m_1.



Sol. **I reflection :**

Focus of mirror $= -10$ cm $\Rightarrow$ $u = -15$ cm

Applying mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow v=-30$ cm.

For II reflection on plane mirror :

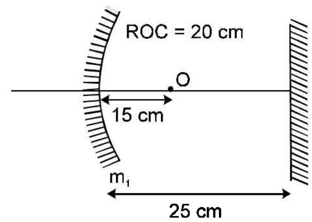
$u=-10$ cm $\therefore$ $v=10$ cm

For III reflection on curved mirror again :

$u=-50$ cm $f=-10$ cm

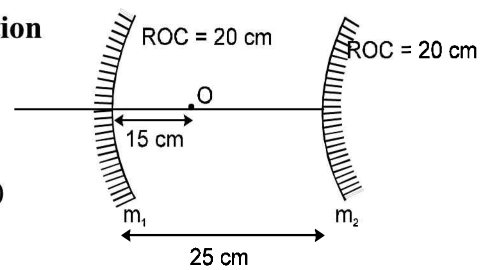
Applying mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow v=-12.5$ cm.

Q.3 Find the position of the final image after three successive reflections taking the first reflection on m_1.



Ans. At the centre of curvature of curved mirror.

Q.4 Find the position of the final image after three successive reflections taking the first reflection on m_1.



Ans. Final image will be formed at distance 30 cm in front of m1.

Ex.57 An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from pole. Find the lateral magnification produced.

Sol. $u=-15$ cm $f=-10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we get, $v=-30$ cm

$\therefore$ $m=-\frac{v}{u}=-2.$

Aliter : $m=\frac{f}{f-u}=\frac{-10}{-10-(-15)}=-2$

Ex.58 A person looks into a spherical mirror. The size of the image of his face is twice the actual size of his face. If the face is at a distance 20 cm then find the nature of radius of curvature of the mirror.

Sol. Person will see his face only when the image is virtual. Virtual image of real object is erect.

Hence $m=2$

$$\therefore \qquad \frac{-v}{u} = 2 \qquad \Rightarrow \qquad v = 40 \text{ cm}$$

Applying $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; $f = -40$ cm or $R = 80$ cm.

Aliter : $m = \frac{f}{f-u} \Rightarrow 2 = \frac{f}{f-(-20)} \Rightarrow f = -40$ cm or $R = 80$ cm

Ex.59 An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance 5 cm then the image becomes triple its size. Find the nature and ROC of the mirror.

Sol. Since the image is formed on screen it is real. Real object and real image implies concave mirror.

Applying $m = \frac{f}{f-u}$ or $-2 = \frac{f}{f-(u)}$ ....(1)

After shifting $-3 = \frac{f}{f-(u+5)}$ ....(2)

[Why u + 5 ?, why not u – 5 : In a concave mirror, are size of real image will increase, only when the real object is brought closer to the mirror. In doing so, its x coordinate will increase]

From (1) & (2) we get, $f = -30$ cm or $R = 60$ cm

Ex.60 A coin is placed 10 cm in front of a concave mirror . The mirror produces a real image that has diameter 4 times that of the coin. What is the image distance?

Ans. 40 cm.

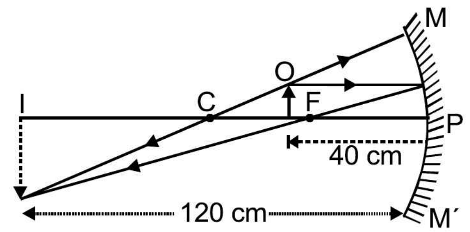
Ex.61 A small statue has a height of 1 cm and is placed in front of a spherical mirror. The image of the statue is inverted and is 0.5cm tall and located 10 cm in front of the mirror. Find the focal length and nature of the mirror.

Ans. $\frac{20}{3}$ cm, concave

Ex.62 The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, such that the image is three times the size of the object.

Sol. As the object is in front of the mirror, it is real and for a real object the magnified image formed by a concave mirror can be inverted (i.e., real) or erect (i.e, virtual), so there are two possibilities.

(a) If the image is inverted (i.e., real)



$$m = \frac{f}{f-u}$$

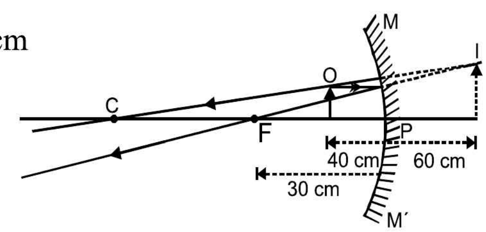
$$-3 = \frac{-30}{-30-u} \qquad \Rightarrow u = -40 \text{ cm}$$

Object must be at a distance of 40 cm in front of the mirror (in between C and F).

(b) If the image is erect (i.e. virtual)

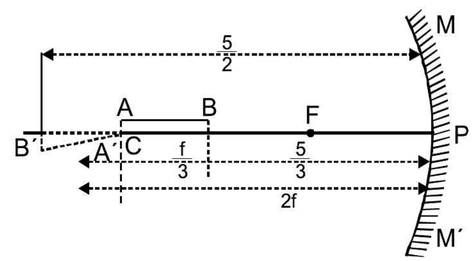
$$m = \frac{f}{f-u} \Rightarrow 3 = \frac{-30}{-30-u} \quad \Rightarrow u = -20 \text{ cm}$$

object must be at a distance of 20 cm in front of the mirror (in between F and P).



Ex.63 A thin rod of length $\frac{f}{3}$ is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. What is magnification?

Sol. Image is real and enlarged, the object must be between C and F. One end A' of the image coincides with the end A of itself, so



$v_A = u_{A'}$, $\frac{1}{v_A} + \frac{1}{v_A} = \frac{1}{-f}$ i.e, $v_A = u_{A'} = -2f$

so it clear that the end A is at C.

$\because$ the length of rod is $\frac{f}{3}$

$\therefore$ distance of the other end B from P is $u_B = 2f - \frac{f}{3} = \frac{5}{3}f$

if the distance of image of end B from P is v_B then $\frac{1}{v_B} + \frac{1}{-\frac{5}{3}f} = \frac{1}{-f} \Rightarrow v_B = -\frac{5}{2}f$

$\therefore$ the length of the image $|v_B| - |v_A| = \frac{5}{2}f - 2f = \frac{1}{2}f$ and magnification

$$m = \frac{|v_B| - |v_A|}{|u_B| - |u_A|} = \frac{\frac{1}{2}f}{-\frac{1}{3}f} = -\frac{3}{2}$$

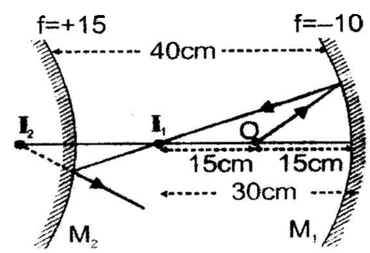
Ex.64 A concave mirror of focal length 10 cm and concave mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirror on their common axis and 15 cm from the concave mirror. Find the position of image produced by the first reflection at concave mirror and then at convex mirror.

Sol. For M_1 mirror O act as a object, let its image is I_1 then,

$u = -15\text{cm}, f = -10$ cm $\quad \Rightarrow \frac{1}{v} + \frac{1}{-15} = \frac{1}{-10} \quad \Rightarrow v = -30$ cm

Image I_1 will act as a object for mirror M_2 its distance from mirror M_2.

$u_1 = -(40 - 30)$ cm $= -10$ cm



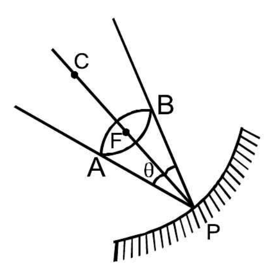
so $\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f} \quad \Rightarrow \frac{1}{v_1} + \frac{1}{10} = \frac{1}{15} \Rightarrow v_1 = +6\text{cm}$

so final image I_2 is formed at a distance 6 cm behind the convex mirror and is virtual.

Ex.65 The sun subtends an angle θ radians at the pole of a concave mirror of focal length f. What is the diameter of the image of the sun formed by the mirror?

Sol. Since the sun is at large distance very distant, u is very large and so $\frac{1}{u} \approx 0$

$\therefore \quad \Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \frac{1}{v} = -\frac{1}{f} \quad \Rightarrow v = -f$



The image of sun will be formed at the focus and will be real, inverted and diminished

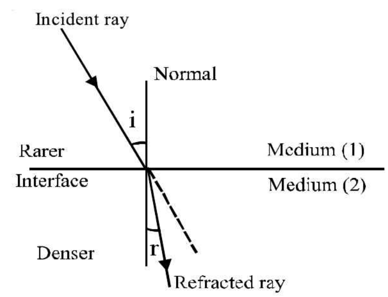
A'B' = height of image and

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{A'B'}{FP} \quad \Rightarrow \theta = \frac{d}{f} \Rightarrow d = f\theta$$

CHAPTER

2 Refraction

The phenomenon by virtue of which light suffers change of speed at the interface separating two media when light travels from one medium to another medium is called refraction. If the light is incidenting obliquely as shown in fig. (1), the change of speed of light results in a change in its path. There is no change of path in case of normal incidence.



2.1 REFRACTIVE INDEX (R.I.)

→ **More refractive index implies less speed of light in that medium, which therefore is called denser medium.**

→ **Less refractive index implies more speed of light in that medium, which therefore is called rarer medium.**

There are following two types of refractive indices:

(a) Absolute (R.I) :- This is the refractive index of a medium with respect to air or vacuum.

$$\mu = \frac{\text{Speed of light in air or vacuum}}{\text{Speed of light in medium}} \quad \Rightarrow \quad \mu = \frac{c_a}{c_m}$$

(b) Relative (R.I.) :- This is the refractive index of a medium with respect to mediums another medium other than air or vacuum.

$_2\mu_1$ = R.I of medium (1) w.r.t medium (2)

$$= \frac{\text{Speed of light in medium (2)}}{\text{Speed of light in medium (1)}} = \frac{c_2}{c_1} \text{ or } {}_2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{\frac{c_a}{c_1}}{\frac{c_a}{c_2}} = \text{ Also, } {}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{c_1}{c_2}$$

IMPORTANT POINTS:

(1) Refractive index is different for different wavelengths. $\mu = f(\lambda) = A + \frac{B}{\lambda^2}$

Where A and B are constants depending upon medium.

(2) Absolute R.I. of any medium is more than one because speed of light is maximum in air or vacuum.

(3) Relative refractive index can be less than one. example ${}_g\mu_w = \frac{\mu_w}{\mu_g} = \frac{8}{9} < 1$

(4) A medium which is optically rarer is acoustically denser and vice-versa.

(5) Refraction is never 100% at any interface. There is some partial refraction.

(6) ${}_2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{c_2}{c_1} = \frac{\lambda_2}{\lambda_1}$

[$\because$ f = frequency of light which is independent from medium]

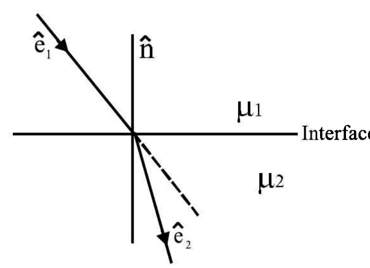
(7) Frequency of light does not change during refraction.

2.2 LAWS OF REFRACTION

There are following two types of laws of refraction:

(1) First law :- The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie in one plane and that plane is perpendicular to the interface separating the two media.

Vector form of Snell's law



$$\hat{e}_1 \cdot (\hat{e}_2 \times \hat{n}) = 0$$

Where, $\hat{e}_1$ = unit vector along the incidence ray

$\hat{e}_2$ = unit vector along the refracted ray

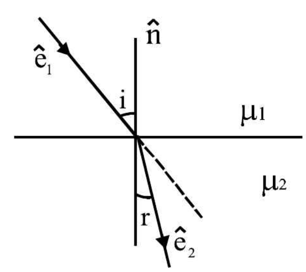
$\hat{n}$ = unit vector along the normal.

(2) Second law :- The sine of the angle of incidence bears a constant ratio with the sine of angle of refraction.

$$\frac{\sin i}{\sin r} = \text{constant} = \frac{\mu_2}{\mu_1} \qquad \mu_1 \sin i = \mu_2 \sin r$$

$\mu \sin\theta$ = constant. This law is known as **Snell's law.**

2.2(A) VECTOR FORM OF SNELL'S LAW



$\mu_1 \sin = \mu_2 \sin r$ (Snell's law) (1)

From the concept of cross product,

$\hat{e}_1 \times \hat{n} = (1)(1)\sin(\pi - i) \Rightarrow \hat{e}_1 \times \hat{n} = \sin i$

Similarly, $\hat{e}_2 \times \hat{n} = (1)(1)\sin(\pi - r) \Rightarrow \hat{e}_2 \times \hat{n} = \sin r$

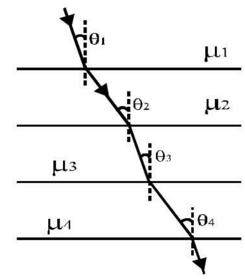
After putting the value of sin i and sin r in the above equation (1) we get

$\mu_1(\hat{e}_1 \times \hat{n}) = \mu_2(\hat{e}_2 \times \hat{n})$ This is the vector form of Snell's law.

Where, $\hat{e}_1$ = unit vector along the incidence ray

$\hat{e}_2$ = unit vector along the refracted ray, $\hat{n}$ = unit vector along the normal

2.2(B) APPLICATION OF SNELL'S LAW



(1) Snell's law is valid for any interface, plane or curved.

(2) When light propagates through a series of parallel layers of different media as shown in the figure, then according to Snell's law, we may write $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = \mu_4 \sin\theta_4$

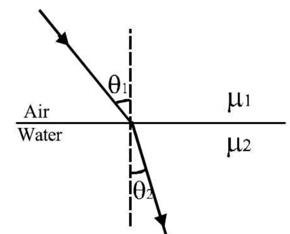
In general, $\mu \sin\theta$ = constant

(3) When light passes from a rarer to denser medium it bends to toward the normal as shown in figure.

Using Snell's law: $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_2}{\mu_1} > 1$$

Thus, if $\mu_2 > \mu_1$ then $\theta_2 < \theta_1$



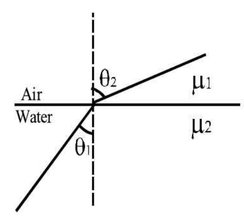
In a denser medium, light bends towards the normal

When a light ray passes from denser to rarer medium it bends away from the normal as shown in the figure.

From the Snell's law, we know that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_2}{\mu_1} < 1$$

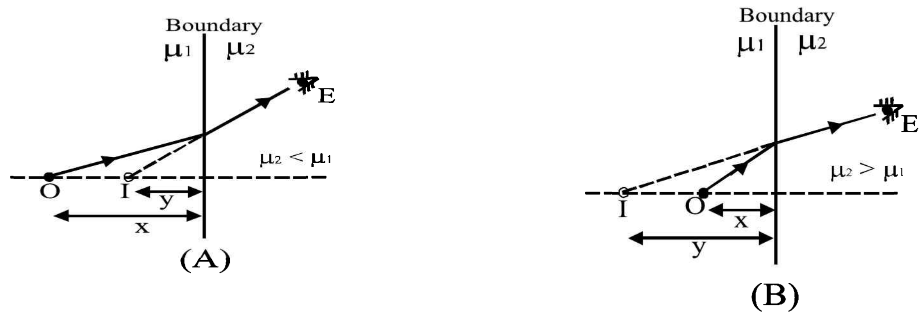
Thus, if $\mu_2 < \mu_1$ then $\theta_2 > \theta_1$



In a rarer medium, light bends away from the normal

(4) **Principle of Reversibility of Light Rays :** A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus, the incident and the refracted ray are mutually reversible.

(5) Relation between an object and image distance (for normal incidence) :An object in medium 1 (refractive index μ_1) is viewed from the medium 2 (refractive index μ_2). It is important to note that the object and image both are formed on the same side of the boundary. distance x is related as $\Rightarrow$ $y = \left(\frac{\mu_2}{\mu_1}\right)x$



→ If $\mu_2 > \mu_1$, that is, when the object is observed from a denser medium, it appears to be farther from the interface, i.e. $y > x$.

→ If $\mu_2 < \mu_1$, that is, when the object is observed from a rarer medium, it appears to be closer to the interface, i.e. $y < x$.

Note : That the above formula is only applicable for normal view or paraxial ray assumption.

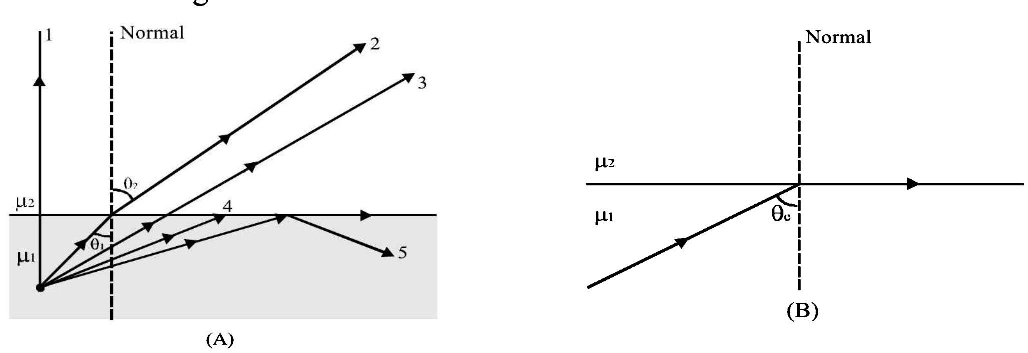
2.3 TOTAL INTERNAL REFLECTION

Light is partially reflected and partially transmitted *at an interface* between two materials with different index of refraction. Under certain circumstances, however, all the light might be reflected back from the interface, with none of it being transmitted, even though the second material is transparent. The fig. shows several rays radiating from a point source P kept in a material having a refractive index μ_2. The ray strikes the surface of the second material having refractive index $\mu_1(\mu_2 > \mu_1)$.

From Snell's law of refraction

$$\sin\theta_2 = \frac{\mu_2}{\mu_1}\sin\theta_1$$

Because μ_2 / μ_1 is greater than unity, $\sin\theta_2 > \sin\theta_1$. Therefore, the ray is bent away from the normal. Thus, there must be some value of θ_1 less than 90º for which Snell's law gives S $\sin\theta_2 = 1$ and $\theta_2 = 90º$. Ray 3 as shown in the diagram emerges just grazing the surface at an angle of refraction of 90º.



The angle of incidence, from which the refracted ray emerges as a tangent to the surface, is called the **critical angle**, denoted by θ_{crit}. If the angle of incidence is greater than the critical angle, the sine of the angle of refraction, as computed by Snell's law, would have to be greater than unity, which is impossible. Beyond the critical angle, the ray cannot pass to the upper material; it is trapped in the lower material and gets reflected completely at the boundary surface. This is called **total internal reflection.**

We can find the critical angle for two given materials by setting $\theta_2 = 90^o$ in Snell's law. We then have

$$\sin\theta_{crit} = \frac{\mu_I}{\mu_2} = \frac{\mu_r}{\mu_d}$$

Light propagating within the glass ($\mu \cong 1.52$) will get totally reflected if it strikes the glass-air surface at an angle of about 41° or more.

When a beam of light enters from one end of a transparent rod, the light can be totally reflected internally if the index of refraction of the rod is greater than that of the surrounding material. The light is trapped within the rod even if the rod is curved. Such a rod is called a light pipe.

2.4 DEVIATION

(i) The figure shows a light ray travelling from a denser to a rarer medium at an angle α greater than the critical angle θ_c.

The deviation δ of the light ray is given by

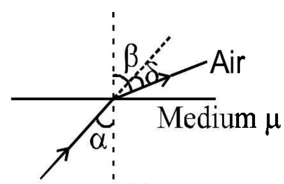
$\delta = \beta - \alpha$

Since $\mu \sin\alpha = \sin\beta$, therefore

$\delta = \sin^{-1}(\mu \sin\alpha) - \alpha$

This is a non-linear increasing equation.

The maximum value of δ occurs when $\alpha = \theta_c$, and is equal to



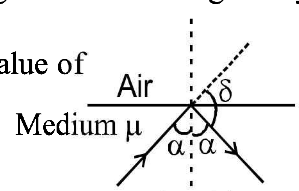
($\alpha < \theta_c$)
Deviation of a transmitted ray

$$\delta_{max} = \frac{\pi}{2} - \theta_c$$

(ii) If the light is incidenting at an angle $\alpha > \theta_c$, then the angle of deviation is given by

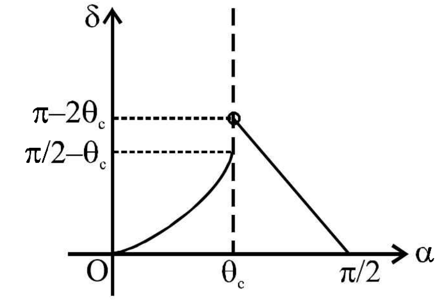
$$\delta = \pi - 2\alpha$$

This is a linearly decreasing function. The maximum value of δ occurs when $\alpha = \theta_c$ and is equal to $\delta_{max} = \pi - 2\theta_c$



($\alpha > \theta_c$)
Deviation of reflected ray

The variation of δ with the angle of incidence α is plotted in the figure.



Variation of the angle of derivation δ with the angle of incidence

2.5 REFRACTION AT SPHERICAL SURFACES

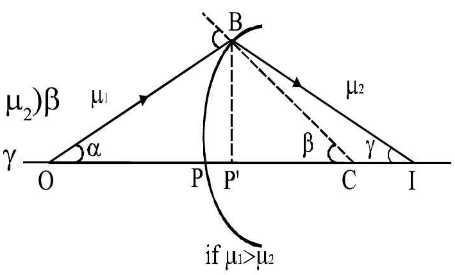
Consider the point object O placed in a medium with refractive index equal to μ_1.

As $\mu_1 \sin i = \mu_2 \sin r$, and for small aperture i, r $\to \theta$

$\Rightarrow \mu_1 i = \mu_2 r \quad \Rightarrow i = \alpha + \beta,\ \beta = \gamma + p$

$\Rightarrow \mu = (\alpha + \beta) = \mu_2(\beta - \gamma) \Rightarrow \mu_1 \alpha + \mu_2 \gamma = (\mu_1 - \mu_2)\beta$

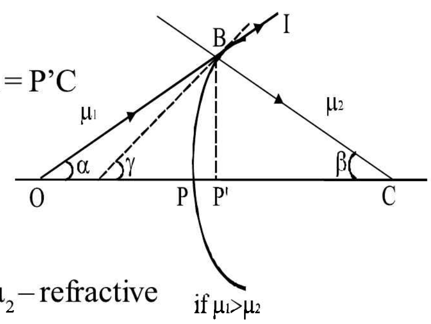
As aperture is small, $\alpha \approx \tan\alpha$, $\beta \approx \tan\beta$, $\gamma \approx \tan\gamma$



$\Rightarrow \mu_1 \tan\alpha + \mu_2 \tan\gamma = (\mu_2 - \mu_1) \tan\beta$

$$\Rightarrow \frac{\mu_1}{P'O} + \frac{\mu_2}{P'I} = \frac{\mu_2 - \mu_1}{P'C}$$

Applying sign convention, i.e., u = P'O, v = P' I, R = P'C



$\therefore$ For a spherical surface, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

The symbols should be carefully remembered as : μ_2 – refractive index of the medium into which light rays are entering ; μ_1 – refractive index of the medium from which light rays are coming. Care should also be taken while applying the sign convention to R.

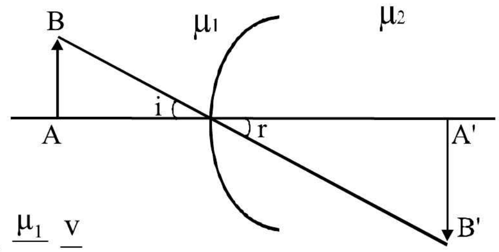
2.6 LATERAL MAGNIFICATION FOR REFRACTING SPHERICAL SURFACE

Lateral magnification, $m = \frac{\text{Image height}}{\text{Object height}} = \frac{-(A_1B_1)}{AB}$, Now, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

For small angles of incidence and thus refraction, sin i = tan i and sin r = tan r

$\Rightarrow \frac{\tan i}{\tan r} = \frac{\mu_2}{\mu_1}$; in triangles ABP and A_1 B_1 P we have

$$\frac{AB \mid PA}{A_1B_1 \mid PA_1} = \frac{\mu_2}{\mu_1}$$



$\Rightarrow \frac{A_1B_1}{AB} = -\frac{\mu_1}{\mu_2}\frac{PA_1}{PA}$ $\Rightarrow \frac{\mu_1}{\mu_2}\frac{v}{(-u)} = \frac{\mu_1}{\mu_2}\frac{v}{u}$

2.7 LONGITUDINAL MAGNIFICATION

In general, objects are not linear but are distributed or extended. Therefore, in addition to lateral magnification, longitudinal magnification also occurs. The ratio of a short length of the **image dv**, measured along the axis, to the conjugate length **du** of the **object,** is known as **longitudinal** or axial **magnification.** Thus,

Longitudinal magnification $L = \frac{dv}{du}$, But $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Differentiating the above expression, we get $-\frac{\mu_2}{v^2} \cdot \frac{dv}{du} + \frac{\mu_1}{u^2} = 0$

or $\frac{dv}{du} = \frac{\mu_1}{u^2} \cdot \frac{v^2}{\mu_2} = \frac{\mu_1}{\mu_2} \cdot \frac{v^2}{u^2}$, $\therefore L = \frac{\mu_1}{\mu_2} \cdot \frac{v^2}{u^2}$ We can rewrite the above expression as

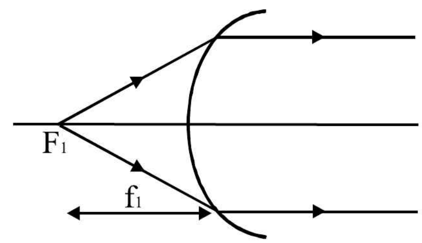
$$L = \left[\frac{\mu_1 v}{\mu_2 u}\right]^2 . \frac{\mu_2}{\mu_1} \qquad \therefore \qquad L = [m]^2 . \frac{\mu_2}{\mu_1}$$

The longitudinal magnification is always positive. If the object extends from left to right, the image likewise extends from left to right.

2.8 PRINCIPAL FOCUS

We can use equ. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ to determine the first and second principal foci. F_1 is the position of the object for which the image lies at infinity, while F_2 is the position of the image for which the object lies at infinity.

(See the Fig. given below)



2.8(A) FIRST PRINCIPAL FOCUS

If the object is placed at F_1 such that the refracted rays become parallel to the principal axis, $v = \infty$. The object distance u is denoted by f_1 and is called the **first principal focal length** of the refracting surface.

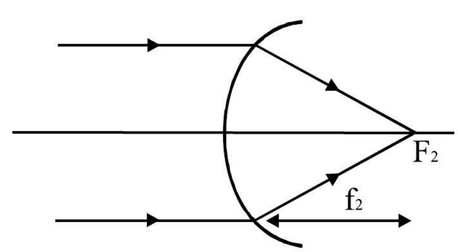
$$\frac{\mu_2}{\infty} - \frac{\mu_1}{f_1} = \frac{\mu_2 - \mu_1}{R} \qquad \therefore \qquad f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$$

The position of the axial point object whose image is formed at infinity is known as the **first principal focus of the refracting surface.**

2.8(B) SECOND PRINCIPAL FOCUS

If the object is at infinity, then $u = \infty$ and $1/u = 0$. Let f_2 be the value of v corresponding to $u = \infty$. Then,

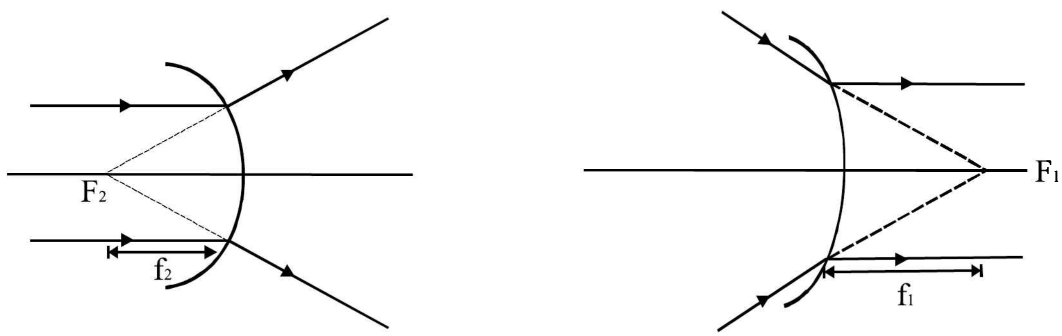
$$\frac{\mu_2}{f_2} = \frac{\mu_2 - \mu_1}{R} \quad \text{or} \quad f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$$



This value of $v = f_2$ is termed as the **second principal focal length** and the position of the image corresponding to the object at infinity is called the second principal focus of the refracting surface.

It is easy to see that the first focal length f_1 of the spherical refracting surface is not equal to the second focal length f_2.

In case of a convex surface, R is positive, f_1 is negative and f_2 is positive. On the other hand, for a concave surface R is negative and hence f_1 is positive and f_2 is negative. When f_1 is positive the system is called **convergent,** and when f_1 is negative it is called **divergent.**



2.9 VARIOUS RELATIONS INVOLVING f_1 AND f_2

(i) The sum of the two focal lengths equals the radius of curvature.

$$f_1 + f_2 = \left[\frac{-\mu_1}{\mu_2 - \mu_1}\right]R + \left[\frac{\mu_2}{\mu_2 - \mu_1}\right]R = R$$

(ii) We can write $\dfrac{\mu_1}{f_1} = \dfrac{\mu_2 - \mu_1}{R} - \dfrac{\mu_2}{f_2}$

Use the sign convention in the above formula.

This is known as **Gauss' formula for a spherical surface.**

(iii) We may write

$$\frac{f_1}{\mu_1} = \frac{-R}{\mu_2 - \mu_1}, \quad \frac{f_2}{\mu_2} = \frac{R}{\mu_2 - \mu_1}, \quad \text{Adding,} \quad \frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} = 0 \text{ or } \frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}$$

As μ_1 and μ_2 are always positive and unequal, f_1 and f_2 are always of opposite sign and are unequal. The ratio of focal lengths f_1 / f_2 is equal to the ratio μ_1 / μ_2 of the corresponding refractive indices.

(iv) We may write $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ as $\frac{\mu_2 R}{v(\mu_2 - \mu_1)} - \frac{\mu_1 R}{u(\mu_2 - \mu_1)} = 1 \quad \therefore \quad \frac{f_2}{v} + \frac{f_1}{u} = 1$

2.10 POWER

The **refracting power** of a spherical surface is defined as the inverse of the reduced focal length. Any distance divided by the refractive index of the space in which it is measured is called a **reduced distance.** Therefore, the ratios f_1/μ_1 and f_2/μ_2 are the reduced focal lengths. Since,

$\frac{f_1}{\mu_1} = -\frac{f_2}{\mu_2}$, the refracting power is given by, $P = \frac{\mu_1}{f_1} = -\frac{\mu_2}{f_2} \qquad \therefore \quad P = \frac{\mu_2 - \mu_1}{R}$

Power is measured in **diopters.** Use the sign convention in the above formula.

2.11 REFRACTION THROUGH A PARALLEL GLASS SLAB

In this case we can apply the formula of spherical surface

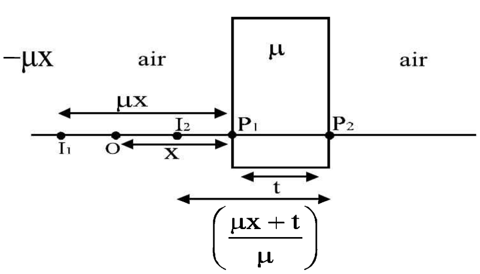
$$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

for each refracting surfaces separately taking, ($R = \infty$)

2.11(A) CONCEPT OF SHIFT

For pole P_1, applying the formula of spherical surface i.e $\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{\mu}{V_1} - \frac{1}{-x} = 0 \; (\because R = \infty) \qquad \Rightarrow V_1 = -\mu x$



Here, I_1 behave like object for pole P_2:

Then, $\frac{1}{V_2} - \frac{\mu}{-(\mu x + t)} = 0 \quad \Rightarrow \quad V_2 = -\frac{(\mu x + t)}{\mu}$

$$\text{Shift} = OI_2 = OP_2 - I_2P_2 = (x + t) - \left(\frac{\mu x + t}{\mu}\right) = t\left(1 - \frac{1}{\mu}\right)$$

2.11(B) HOW TO KNOW THE DIRECTION OF SHIFT?

If shift becomes positive then the direction of shift is along the incident ray and if shift becomes negative then it is opposite to the incident ray w.r.t. object.

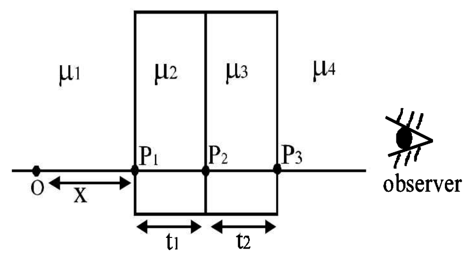
2.11(C) REFRACTION ACROSS MULTIPLE SLABS

Let us solve the problem taking one step at a time, by summation of shift = $\sum t\left(1-\frac{1}{\mu}\right)$

Where, $\mu = \frac{\mu_{incident}}{\mu_{observer}}$ and taking summation at (n) times where (n) is the number of poles or number of interfaces. In the first summation, t = distance of object from pole P_1. In the second summation, t = distance of pole P_1 from pole P_2 and, in the third summation , t = distance of pole P_2 from pole P_3 and so on.

Ex.1 Refraction across multiple slabs
Calculate the shift for the given object as shown in fig.

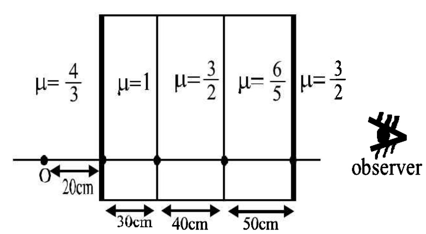
Sol. $\text{Shift} = \sum t\left(1-\frac{1}{\mu}\right)$



$$\text{Shift} = x\left(1-\frac{1}{\frac{\mu_1}{\mu_4}}\right)+t_1\left(1-\frac{1}{\frac{\mu_2}{\mu_4}}\right)+t_2\left(1-\frac{1}{\frac{\mu_3}{\mu_4}}\right)$$

Shift is always measured from the object to image. If it is positive then it is along the incident ray from the object at a distance equal to shift and if it becomes negative then opposite to the incident ray from object.

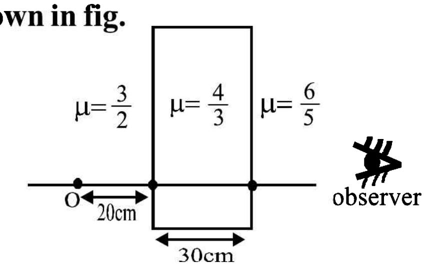
Ex.2 Calculate the shift for the given object that is shown in fig.



Sol. $\text{Shift} = \sum t\left(1-\frac{1}{\mu}\right)$, where $\mu = \frac{\mu_{incident}}{\mu_{observer}}$

$$\text{Shift} = 20\left(1-\frac{1\times\frac{3}{2}}{\frac{4}{3}}\right)+30\left(1-\frac{1\times\frac{3}{2}}{1}\right)+40\left(1-\frac{1\times\frac{3}{2}}{\frac{3}{2}}\right)+50\left(1-\frac{1\times\frac{3}{2}}{\frac{6}{5}}\right)$$

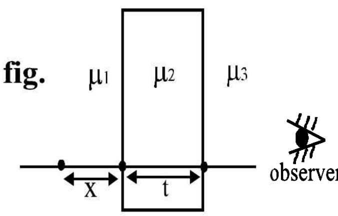
Ex.3 Calculate the shift for the given object that is shown in fig.



Sol. $\text{Shift} = \sum t\left(1-\frac{1}{\mu}\right)$

$$\text{Shift} = 20\left(1-\frac{1\times\frac{6}{5}}{\frac{3}{2}}\right)+30\left(1-\frac{1\times\frac{6}{5}}{\frac{4}{3}}\right)$$

Ex.4 Calculate the shift for the given object that is shown in fig.



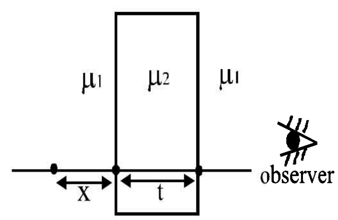
Sol. $\text{Shift} = x\left(1-\frac{1\times\mu_3}{\mu_1}\right)+t\left(1-\frac{1\times\mu_3}{\mu_2}\right)$

Remarks :

(1). When the media of both sides are not same then shift is not independent to the object distance.

(2). When the media of both sides are same then shift is independent to the object distance.

Ex.5 Calculate the shift for the given object when the media on both the sides is same as shown in fig.



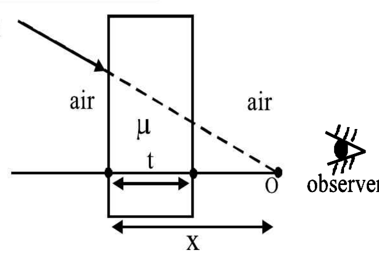
Sol. $\text{Shift} = \sum t\left(1-\frac{1}{\mu}\right)$

$$\text{Shift} = x\left(1-\frac{1\times\mu_1}{\mu_1}\right)+t\left(1-\frac{1\times\mu_1}{\mu_2}\right)$$

$\text{Shift} = t\left(1-\frac{\mu_1}{\mu_2}\right)$ (which is independent to the object distance, x)

2.11(D) SHIFT IN THE CASE OF VIRTUAL OBJECT

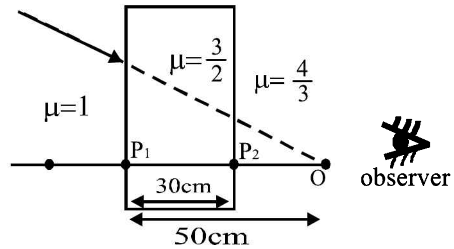
Ex.6 Calculate the shift for the given virtual object shown in fig. when the medium on both sides is same.



Sol. $\text{Shift} = -x\left(1-\frac{1\times 1}{1}\right)+t\left(1-\frac{1\times 1}{\mu}\right)$

$\text{Shift} = t\left(1-\frac{1}{\mu}\right)$ [Independent to the object distance, x]

Ex.7 Calculate the shift for the given virtual object when medium on both sides are different as shown in fig.



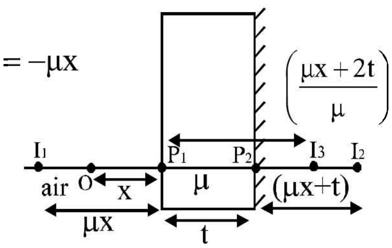
Sol. $\text{Shift} = \sum t\left(1-\frac{1}{\mu}\right)$

$$\text{Shift} = (-50)\left(1-\frac{1\times\frac{4}{3}}{1}\right)+30\left(1-\frac{1\times\frac{4}{3}}{\frac{3}{2}}\right)$$

2.11(E) CONCEPT OF SLAB WHEN ONE SURFACE IS SILVERED

Applying the formula of spherical surface for pole P_1:

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \Rightarrow \frac{\mu}{V_1}-\frac{1}{-x}=0(\because R=\infty) \quad \Rightarrow V_1=-\mu x$$



Here, I_1 behaves like object for pole P_2 (Plane mirror): for this I_2 is the image from reflection through the plane mirror

Again for pole P_1, I_2 behaves as an object, then apply

$$\frac{\mu_2}{V}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \quad (\text{for pole } P_1) \quad \Rightarrow \frac{1}{V_3}-\frac{\mu}{-(\mu x+2t)}=0(\because R=\infty)$$

$$\Rightarrow V_3=-\frac{(\mu x+2t)}{\mu} \quad \text{(This is the final image formed by the combination)}$$

2.12 LATERAL DISPLACEMENT OF EMERGENT BEAM THROUGH A GLASS SLAB

Fig. (1) represents the refraction of a ray AO incident on the slab at an angle of incidence i through the glass slab EFGH. At face EF, the incident ray AO is refracted along OP, the angle of refraction being r. If e is the angle of emergence, then from Snell's law at faces EF and HG,

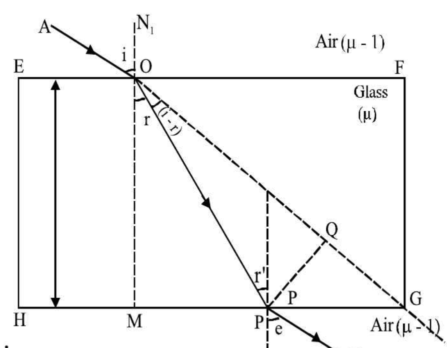


fig. (1)

$$\mu_a \sin i = \mu \sin e \quad \text{and} \quad \mu \sin r' = \mu_a \sin e$$

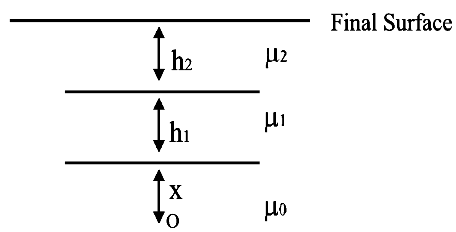
$r' = r$ and $\mu_a = 1$, we have, $\sin i = \sin e$ or $e = i$.

That is the emergent ray is parallel to the incident ray.

(ii) Let PQ be the perpendicular dropped from P on incident ray produced. Then, the lateral displacement caused by the plate,

$$P = PQ = OP\sin(i-r) = \frac{OM}{\cos r}\sin(i-r) \quad = \frac{t\sin(i-r)}{\cos r}$$

2.13 CONCEPT OF APPARENT DEPTH



$$\text{Apparent depth} = \sum \frac{t}{\mu}\left(\text{where, } \mu = \frac{\mu\,\text{incident}}{\mu\,\text{observer}}\right)$$

$$\text{Apparent depth} = \frac{x}{\left(\dfrac{\mu_0}{\mu_3}\right)} + \frac{h_1}{\left(\dfrac{\mu_1}{\mu_3}\right)} + \frac{h_2}{\left(\dfrac{\mu_2}{\mu_3}\right)}$$

This is the final position of final image from the final surface.

2.14 CONCEPT OF REFRACTION IN A MEDIUM OF VARIABLE REFRACTIVE INDEX

In the previous case, we have assumed that the refractive index of slab is constant. This is not true in atmosphere. The atmosphere becomes thinner as we move up. When we move up from the earth, refractive index decreases. We assumed the situation where refractive index changes only in one direction, medium can be considered as a collection of large number of the thin layers. Let refractive index be a function of y i.e., $\mu = f(y)$ then the medium can be considered as to be made up of large number of thin slabs placed parallel to x-axis and optical normal at any interface is parallel to y-axis. Similarly if $\mu = f(x)$ then slabs are parallel to y-axis and optical normal at any interface is parallel to x-axis.

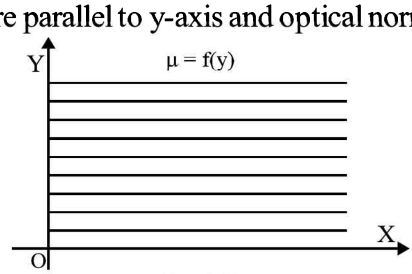


fig. (A)

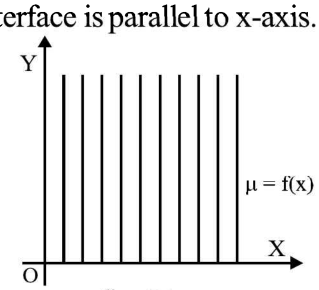
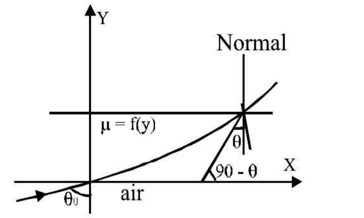


fig.(B)

2.14(A) LET US CONSIDER A SITUATION

$\mu = f(y)$. Now the medium can be divided into thin slices parallel to x axis and optical normal parallel to y axis.

Let θ_0 be the angle of incident in the variable medium at point (0, 0). And θ is the angle made by tangent with normal parallel to y axis at any point (x, y) on the trajectory.



Trajectory of light ray when $\mu = f(y)$

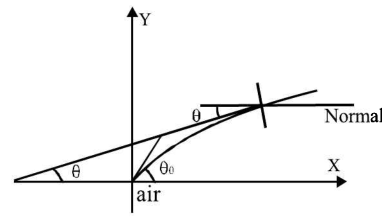
From the snell's law: $1.\ \mathrm{Sin}\,\theta_0 = \mu \sin\theta$ ......(1)

Geometrically, relate the slope of this tangent to the angle θ i.e $\frac{dy}{dx} = \tan(90-\theta)$...(2)

Substitute for θ from equation (1) and determine $\frac{dy}{dx}$ as a function of y. Integrate and obtain an expression of y as a function of x.

2.14(B) LET US CONSIDER A SITUATION

$\mu = f(x)$. Now the medium can be divided into thin slices parallel to y axis and optical normal parallel to axis.



Trajectory of light ray when $\mu = f(x)$

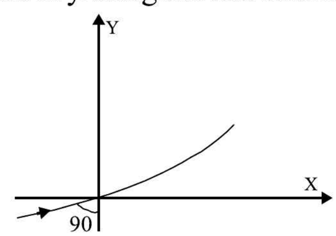
Trajectory of light ray when $\mu = f(x)$

From the snell's law: $1.\mathrm{Sin}\theta_0 = \mu \sin\theta$ (1) [Since $\mu = 1$ at $x = 0$]

Geometrically, relate the slope of this tangent to the angle of incidence θ i.e $\frac{dy}{dx} = \tan\theta$ (2)

Substitute θ from equation (1) and determine $\frac{dy}{dx}$ as a function of y. Integrate and obtain an expression of y as a function of x.

Ex.8 Find the variation of Refractive index assuming it to be a function of y such that a ray entering origin at grazing incident follows a parabolic path y = x^2 as shown in fig:



Sol. Draw a tangent at any point (x, y) which makes an angle θ with optical normal parallel to y axis.

From the Snell's law: $1.\ \mathrm{Sin}\ 90 = \mu \sin\theta \Rightarrow \sin\theta = \frac{1}{\mu}$ (1)

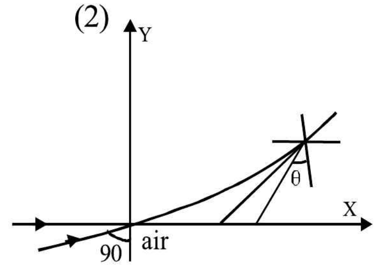
Geometrically, $\frac{dy}{dx} = \tan(90-\theta) \Rightarrow \frac{dy}{dx} = \cot\theta$ (2)

Given that, $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

[By Differentiating the above equation]

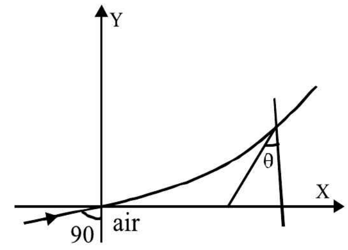
$\cot\theta = 2x$ [From equation (2)]

Hence, $\mu = \frac{1}{\sin\theta} = \mathrm{cosec}\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+4x^2}$



Ex.9 A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu = \sqrt{(1+y)}$. If the thickness of the slab is d, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.

Sol. Draw a tangent at any point (x, y) on the trajectory which makes an angle θ with optical normal parallel to y axis [$\because \mu = f(y)$]



From the Snell's law:

1. $\text{Sin } 90 = \mu \sin\theta \Rightarrow \sin\theta = \frac{1}{\mu} = \frac{1}{\sqrt{1+y}}$ ....(1)

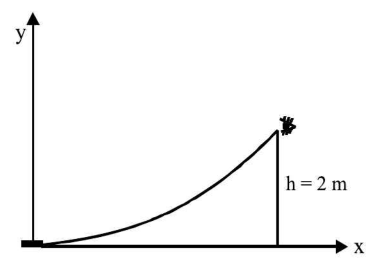
Geometrically, $\frac{dy}{dx} = \tan(90-\theta) \Rightarrow \frac{dy}{dx} = \cot\theta$ ....(2)

From equation (1) and (2) we get, $\frac{dy}{dx} = y^{\frac{1}{2}} \Rightarrow \int_0^y \frac{dy}{y^{\frac{1}{2}}} = \int_0^x dx$

$\Rightarrow 2\sqrt{y} = x \Rightarrow y = \frac{x^2}{4}$

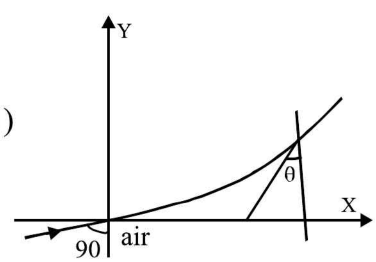
The ray will exit at point (x, y) where, $x = \sqrt{4y} = 2\sqrt{d}$ [$\because y = d$] and $y = d$

Ex.10 Due to a vertical temperature gradient in the atmosphere, the index of refraction varies. Suppose index of refraction varies as $n = n_0\sqrt{1+ay}$, where n_0 is the index of refraction at the surface and a 2.0×10^{-6} m^{-1}. A person of height h = 2.0 m stands on a level surface. Beyond what distance will he not see the runway?



Sol. Draw a tangent on the trajectory at any point (x,y) which makes an angle θ with optical normal.

From the snell's law:



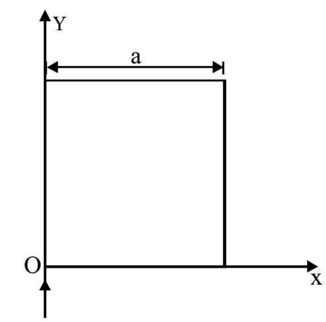
$n_0 \sin 90 = n_0\sqrt{1+ay}\sin\theta \quad \Rightarrow \sin\theta = \frac{1}{\sqrt{1+ay}}$ ......(1)

Geometrically, $\frac{dy}{dx} = \tan(90-\theta)$ ....(2)

[Slope of tangent] From equations (1) and (2) we get

$$\frac{dy}{dx} = \sqrt{ay} \Rightarrow \int \frac{dy}{\sqrt{ay}} = \int dx \quad \Rightarrow \quad x = 2\sqrt{\frac{y}{a}} \Rightarrow x_{max} = (2000)m$$ **Ans.**

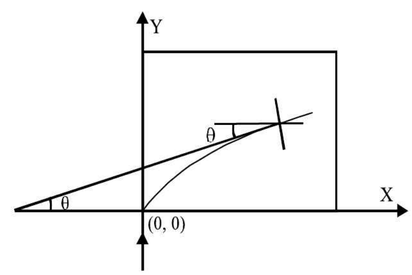
Ex.11 The refraction index of an anisotropic medium varies as $\mu = \mu_0\sqrt{(x+1)}$, where $0 \le x \le a$. A ray of light is incident at the origin just along y-axis (shown in figure). Find the equation of ray in the medium.



Sol. Draw a tangent at any point (x, y) which makes an angle θ with optical normal parallel to x axis.
From the Snell's law:

$$\mu_0 \sin 90 = \left(\mu_0\sqrt{x+1}\right)\sin\theta \qquad [\because \text{At } x = 0, \mu = \mu_0]$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{x+1}} \qquad \text{............(1)}$$



Geometrically, $\frac{dy}{dx} = \tan\theta$ ....(2)

[Slope of tangent]

From equations (1) and (2) we get, $\frac{dy}{dx} = \frac{1}{\sqrt{x}} \Rightarrow \int dy = \int \frac{dx}{\sqrt{x}} \Rightarrow y = 2\sqrt{x}$ **Ans.**

2.15 CONCEPT OF VELOCITY OF OBJECT AND IMAGE IN THE REFRACTION THROUGH THE PLANE SURFACE AND SPHERICAL SURFACE

Apply the formula of spherical surface $\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

On differentiating the above equation with respect to time, we get,

$$\mu_2\left(-\frac{1}{V^2}\right)\frac{dv}{dt} - (\mu_1)\left(-\frac{1}{u^2}\right)\frac{du}{dt} = 0 \quad \Rightarrow \frac{dv}{dt} = \left(\frac{V^2}{u^2}\right)\left(\frac{\mu_1}{\mu_2}\right)\frac{du}{dt} \quad \Rightarrow$$

$$V_{IP} = \left(\frac{V^2}{u^2}\right)\left(\frac{\mu_1}{\mu_2}\right) V_{O,P} \qquad \text{...........(1)}$$

Where, V_{IP} = Velocity of image w.r.t pole of refracting surface
V_{OP} = Velocity of object w.r.t pole of refracting surface

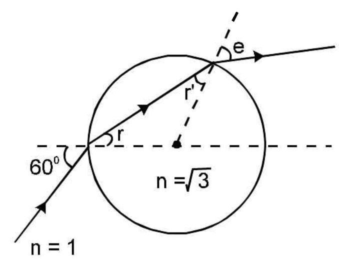
Special case: For the plane surface

$$\frac{\mu_2}{V}-\frac{\mu_1}{u}=0\ (\because R=\infty) \qquad \Rightarrow \frac{V}{u}=\frac{\mu_2}{\mu_1} \qquad ....(2)$$

From equation (1) and (2) we get,

$$V_{I,P}=\left(\frac{\mu_2}{\mu_1}\right)^2\left(\frac{\mu_1}{\mu_2}\right)(V_{O,P}) \Rightarrow V_{I,P}=\left(\frac{\mu_2}{\mu_1}\right)(V_{O,P})$$ Final result is only applicable for the plane surface only.

Ex.12 A light ray is incident on a glass sphere at an angle of incidence 60^0 as shown. Find the angles r, r',e and the total deviation after two refractions.



Sol. Applying Snell's law $1\sin 60^0 = \sqrt{3}\sin r$

$\Rightarrow$ $r = 30^0$ From symmetry $r' = r = 30^0$.

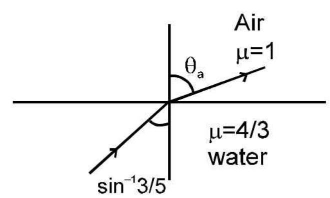
Again applying snell's law at the second surface $1\sin e = \sqrt{3}\sin r$

$\Rightarrow$ $e = 60^0$

Deviation at first surface $= i - r = 60^0 - 30^0 = 30^0$

Deviation at second surface $= e - r' = 60^0 - 30^0 = 30^0$

Therefore total deviation $= 60^0$.



Ex.13 Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in figure.

Sol. Snell's Law

$$\mu_W \sin\theta_W = \mu_a \sin\theta_a \quad \Rightarrow \quad \frac{4}{3}\times\frac{3}{5}=1\sin\theta_a \quad \Rightarrow \quad \sin\theta_a = \frac{4}{5} \quad \Rightarrow \quad \theta_a = \sin^{-1}\frac{4}{5}$$

Ex.14 Find the speed of light in medium 'a' if speed of light in medium 'b' is $\frac{c}{3}$ where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45^o and 60^o respectively with the normal.

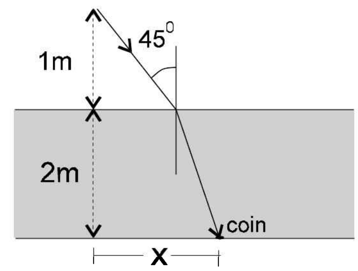
Snell's Law $\qquad \mu_a \sin\theta_a = \mu_b \sin\theta_b \qquad \frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b.$

$$\frac{c}{v_a}\sin 45^o = \frac{c}{c/3}\sin 60^o. \qquad v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$$

Q.1 **A light ray deviates by 30⁰ (which is one third of the angle of incidence) when it gets refracted from vacuum to a medium. Find the refractive index of the medium.**

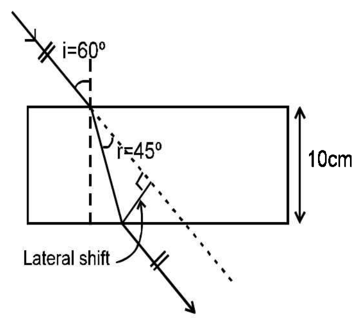
Ans. $\mu = \dfrac{2}{\sqrt{3}}$

Q.2 **A coin lies on the bottom of a lake 2m deep at a horizontal distance x from the spotlight (a source of thin parallel beam of light) situated 1 m above the surface of the liquid of refractive index $\mu = \sqrt{2}$ and height 2m. Find x.**



Ans. $\left(1+\dfrac{2}{\sqrt{3}}\right)$

Ex.15 **Find the lateral shift of light ray while is passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.**



Sol. $d = \dfrac{t\sin(i-r)}{\cos r} = \dfrac{10\sin(60°-45°)}{\cos 45°}$

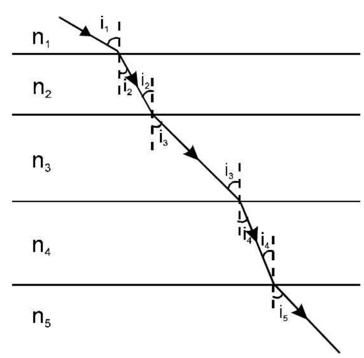
$= \dfrac{10\sin 15°}{\cos 45°} = 10\sqrt{5}\ \sin 15°.$

Q.3 **A ray of light falls at an angle of 30° onto a plane-parallel glass plate and leaves it parallel to the initial ray. The refractive index of the glass is 1.5. What is the thickness d of the plate if the distance between the rays is 3.82 cm?**

[Given : $\sin^{-1}\left(\dfrac{1}{3}\right)$ = 19.5° ; cos 19.5° = 0.94 ; sin 10.5° = 0.18]

Ans. d = 0.2 m

Q.4 **A light passes through many parallel slabs one-by-one as shown in figure. Prove that $n_1\sin i_1 = n_2\sin i_2 = n_3\sin i_3 = n_4\sin i_4$. Also prove that if $n_1 = n_4$ then the light rays in medium n_1 and in medium n_4 are parallel.**



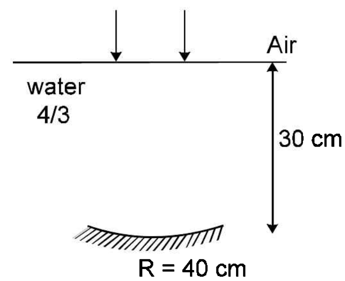
Ex.16 An object lies 100 cm inside water. It is viewed from air nearly normally. Find the apparent depth of the object.

Sol: $$d' = \frac{d}{n_{relative}} = \frac{100}{\frac{4/3}{1}} = 75 \text{ cm}$$

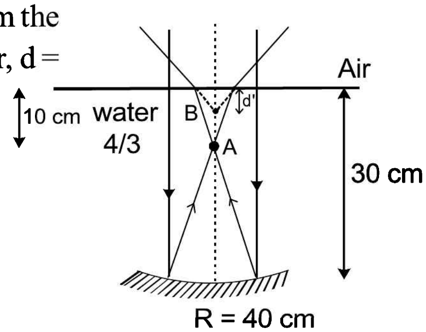
Q.6 An object lies 90 cm in air above water surface. It is viewed from water nearly normally. Find the apparent height of the object.

Ans. 120 cm.

Ex.17 **A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of the concave mirror. Find the position of the final image.**

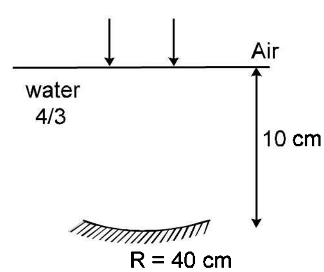


Sol. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 cm from the mirror. Now for the interface between water and air, d = 10 cm.



$$\therefore \quad d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5 \text{ cm}$$

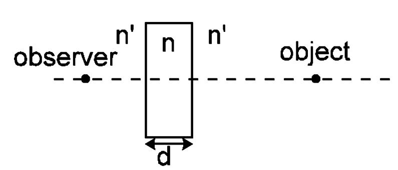
Q.7 A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



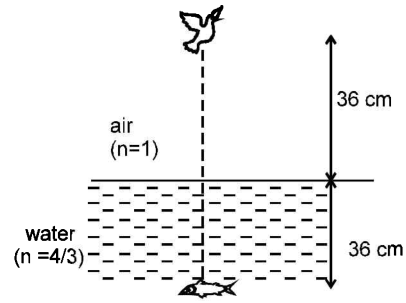
Ans. 7.5 cm above the water surface

Q.8 Prove that the shift in position of object due to parallel slab is given

by shift $= d\left(1 - \frac{1}{n_{rel}}\right)$ **where** $n_{rel} = \frac{n}{n'}$.



Ex.18 See the figure



(i) Find apparent height of the bird

(ii) Find apparent depth of fish

(iii) At what distance will the bird appear to the fish?

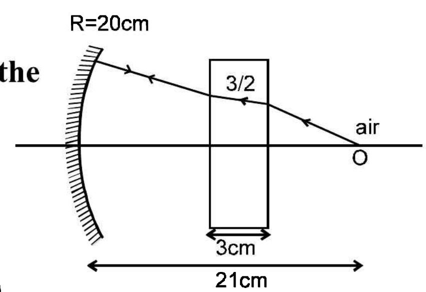
(iv) At what distance will the fish appear to the bird?

Sol. (i) $d'_B = \dfrac{36}{\dfrac{1}{\left(\dfrac{4}{3}\right)}} = \dfrac{36}{3/4} = 48$ cm (ii) $d'_F = \dfrac{36}{4/3} = 27$ cm

(iii) For fish : $d_B = 36 + 48 = 84$ cm
$d_B = 36 + 48 = 84$ cm

(iv) For bird : $d_F = 27 + 36 = 63$ cm.
$d_F = 27 + 36 = 63$ cm.

Ex.19 See the figure.
Find the distance of the final image formed by the mirror

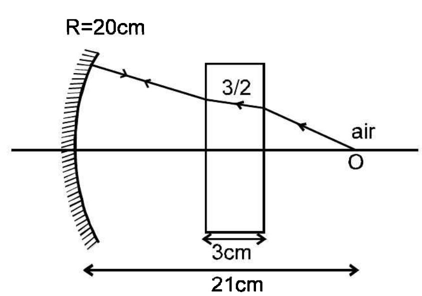


Sol. $\text{Shift} = 3\left(1 - \dfrac{1}{3/2}\right) \quad = 3\left(1 - \dfrac{1}{3/2}\right)$

For mirror, object is at a distance $= 21 - 3\left(1 - \dfrac{1}{3/2}\right) = 20$ cm

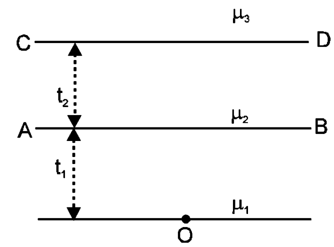
$\therefore$ Object is at the centre of curvature of mirror. Hence the light rays will be retrace and image will formed on the object itself.

Ex.20 See figure. Find the apparent depth of object seen below surface AB.



Sol. $D_{app} = \sum \dfrac{d}{\mu} = \dfrac{20}{\left(\dfrac{2}{1.8}\right)} + \dfrac{15}{\left(\dfrac{1.5}{1.8}\right)} = 18 + 18 = 36$ cm.

Q.9 **Find the apparent depth of object O below surface AB, seen by an observer in medium of refractive index μ_2**



Ans. $\dfrac{t_1}{\mu_1/\mu_2}$

Q.10 **In the above question, what is the depth of object corresponding to incident rays striking on surface CD in medium μ_2.**

Ans. $t_2 + \dfrac{t_1}{\mu_1/\mu_2}$.

Q.11 **In the above question, if observer is in medium μ_3, what is the apparent depth of object seen below surface CD.**

Ans. $\dfrac{t_2 + \dfrac{t_1}{\mu_1/\mu_2}}{\mu_2/\mu_3} = \dfrac{t_2}{\mu_2/\mu_3} + \dfrac{t_1}{\mu_1/\mu_3} = \sum \dfrac{t}{\mu_{rel}}$ where $\mu_{rel} = \dfrac{\mu \text{ of corresponding medium}}{\mu \text{ of observer medium}}$

Ex.21 **Find the max. angle that can be made in glass medium ($\mu = 1.5$) if a light ray is refracted from glass to vacuum.**

Sol. $1.5 \sin C = 1 \sin 90^\circ$, where C = critical angle.

$\sin C = 2/3 \qquad C = \sin^{-1} 2/3$

Ex.22 **Find the angle of refraction in a medium (m = 2) if light is incident in vacuum, making angle equal to twice the critical angle.**

Sol. Since the incident light is in rarer medium. Total Internal Reflection can not take place.

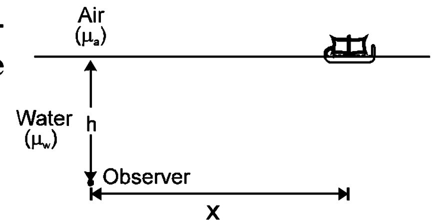
$C = \sin^{-1} \dfrac{1}{\mu} = 30^\circ \qquad \therefore \qquad i = 2C = 60^\circ$

Applying Snell's Law. $1 \sin 60^\circ = 2 \sin r \Rightarrow \sin r = \dfrac{\sqrt{3}}{4} \Rightarrow r = \sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$.

Q.12 **Find the radius of C.O.I., if a luminous object is placed at a distance h from the interface in a denser medium.**

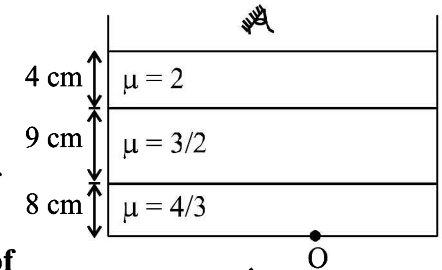
Ans. $r = h \tan C = \dfrac{\mu_r h}{\sqrt{\mu_d^2 - \mu_r^2}}$.

Q.13 A ship is sailing in a river. An observer is situated at a depth h in water (μ_w). If x >> h, find the angle made from vertical, to the line of sight of ship.



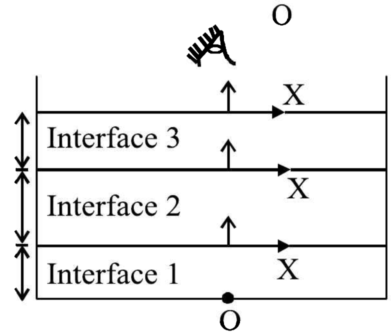
Ans. $\sin^{-1}\dfrac{\mu_a}{\mu_w}$

Ex.23 A tank contains three layers of immiscible liquids (figure). The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is an oil with refractive index 3/2 and thickness 9cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.



Sol. Case- 1: Method of interfaces:

A ray of light from the object undergoes refraction at three interfaces. (1) Water-oil, (2) Oil-glycerine (3) Glycerine-air. The coordinate system for each of the interfaces is shown in figure.



Water-Oil Interface: $d_1 = -8$ cm, $\mu_1 = 4/3$, $\mu_2 = 1.5$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$ we get $d_2 = -9$cm

Oil-Glycerine Interface: $d_1 = -(9 + 9) = -18$cm, $\mu_1 = 1.5$, $\mu_2 = 2$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$ we get $d_2 = -24$ cm

Glycerine-Air Interface: $d_1 = -(4 + 24) = -28$ cm, $\mu_1 = 2$, $\mu_2 = 1$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$ we get $d_2 = -14$ cm

Thus, the final image is 14cm below the glycerine - air interface.

Case-2 : Method of elements:

The system now comprises three slabs—one of water, one of oil and one of glycerine. As discussed in this Section and given by equation, the net shift of the system is the sum of the individual shifts in each of the slabs assuming they were in air.

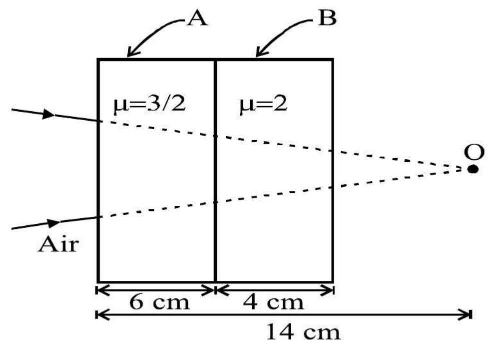
Therefore,

Net shift $$s = t_1\left[1-\frac{1}{\mu_1}\right] + t_2\left[1-\frac{1}{\mu_2}\right] + t_3\left[1-\frac{1}{\mu_3}\right]$$

$$s = 8\left[1-\frac{3}{4}\right] + 9\left[1-\frac{2}{3}\right] + 4\left[1-\frac{1}{2}\right] = 7 \text{ cm.}$$

The direction of the shift is in the direction of the incident rays which is upwards. Therefore, the final position of the object is = (4 + 9 + 8) – 7 = 14cm below the glycerine-air interface.

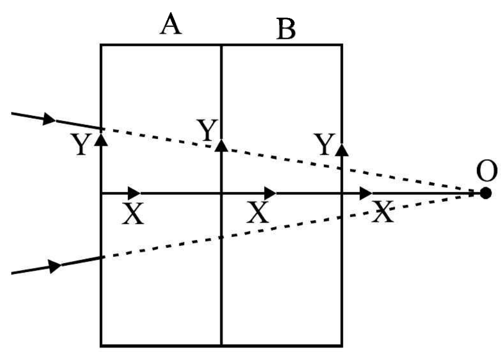
Ex.24 A convergent beam is incidenting on two slabs placed in contact with each others (as shown in figure.) Where will the rays finally converge?



Sol.

Case-I Method of Interfaces :

A ray of light from the object undergoes refraction at three interfaces. (1) Air-Medium A, (2) Medium A-Medium B (3) Medium B-Air. The coordinate system for each interface is shown in the figure.



Air-Medium A Interface:

$d_1 = +14$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$ we get $d_2 = +21$cm

Medium A – Medium B Interface : $d_1 = (21 - 6) = 15$ cm, $\mu_1 = 1.5$, $\mu_2 = 2$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$ we get $d_2 = +20$ cm

Medium B - Air Interface : $d_1 = (20 - 4) = +16$ cm, $\mu_1 = 2$, $\mu_2 = 1$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$ we get $d_2 = +8$ cm

Thus, the final image is 8 cm in front of the medium B - air interface.

Case-2 : Method of Elements:

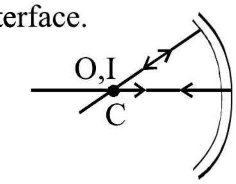
The system now comprises of two slabs. The net shift is

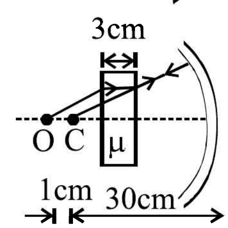
$$s = t_1\left[1-\frac{1}{\mu_1}\right] + t_2\left[1-\frac{1}{\mu_2}\right] = 6\left[1-\frac{2}{3}\right] + 4\left[1-\frac{1}{2}\right] = 4 \text{ cm}$$

The direction of the shift is in the direction of the incident rays which is to the right. Therefore, the rays will finally converge to a point.

= (14 – 6 – 4) + 4 = 8 cm to the right of the Medium B-air interface.

Ex..25 The image of an object kept at a distance 30cm in front of a concave mirror is found to coincide with itself. If a glass slab (μ = 1.5) of thickness 3cm is introduced between the mirror and the object, then





(i) identify, the direction in which the mirror should be displaced so that the final image may again coincide with the object itself.

(ii) Find the magnitude of displacement.

Sol. (i) Since the apparent shift occurs in the direction of incident light, therefore, the mirror should be displaced away from the objects

(ii) The magnitude of displacement is equal to the apparent shift, i.e.,

$$s = t\left(1-\frac{1}{\mu}\right) = 3\left(1-\frac{1}{3/2}\right) = 1 \text{ cm}$$

Ex.26 A bird in air is diving vertically over a tank with speed 6 cm/s. The base of the tank is silvered. The fish in the tank is rising upward along the same line with speed 4 cm/s. [Take: μ_{water} = 4/3]. Find:

(A) The speed of the image of the fish as seen directly by the bird.

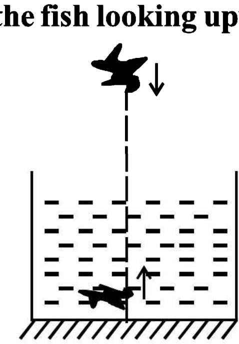
(B) The speed of the image of the bird relative to the fish looking upwards.

Sol. (A) Velocity of fish in air $= 4 \times \frac{3}{4} = 3\uparrow$

Velocity of fish w.r.t bird $= 3 + 6 = 9\uparrow$

(B) Velocity of bird in water $= 6 \times \frac{4}{3} = 8\downarrow$

w.r.t fish $= 8 + 4 = 12\downarrow$

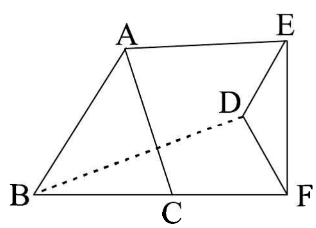


CHAPTER 3 Prism

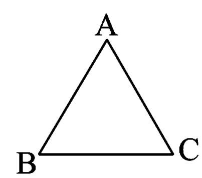
3.1 PRISM

Prism is made by a transparent material bounded by any number of surfaces (plane or curved) such that the surface on which a ray is incidenting and the surface through which the ray emerges are plane and non-parallel.

3-D view of Triangular Prism:



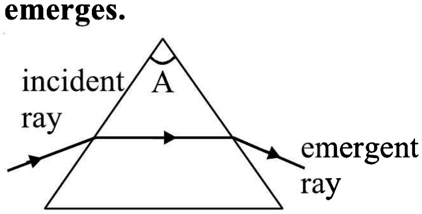
Here, ABC is the principle section of a prism which is shown as

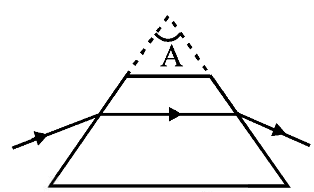


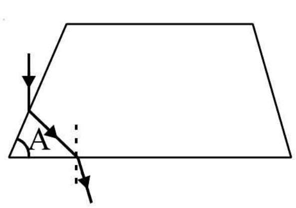
3.2 DEFINITION OF REFRACTING ANGLE OF PRISM

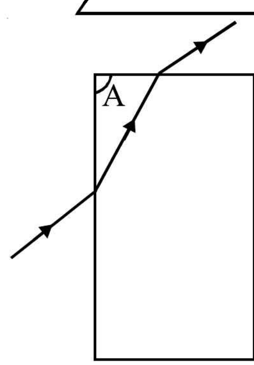
It is the angle made by two refracting surfaces through which refraction takes place.

(or) The angle between the faces on which light is incidenting and from which it emerges.







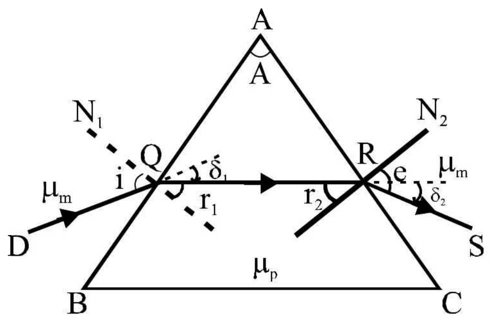


In all above figure A is the angle of Prism :

IN A PRISM, WE CAN APPLY

(i) Snell's law at the two separate refracting surfaces.

(ii) Basic geometry for a selected triangle.



PQ = incident ray, QR = Refracted ray, RS = emergent ray

A = Prism angle (Refracting angle of Prism), i = incident angle on face AB

r_1 = Refracted angle on face AB, r_2 = incident angle on face AC

e = emergent angle on face AC, **δ_1 = Angle of deviation on face AB = $(i - r_1)$**

δ_2 = Angle of deviation on face AC = $(e - r_2)$

δ = Total angle of deviation = $\delta_1 + \delta_2$ **(since δ_1 & δ_2 both are in the same sense)**

$\delta = (i + e) - (r_1 + r_2)$

In the Δ PQR (sum of all three angles = 180°)

$A + 90 - r_1 + 90 - r_2 = 180°$, $A = r_1 + r_2$..............(1)

$\delta = \delta_1 + \delta_2$, $\delta = i - r_1 + e - r_2 = i + e - (r_1 + r_2)$, $\delta = i + e - A$ (since, $A = r_1 + r_2$)

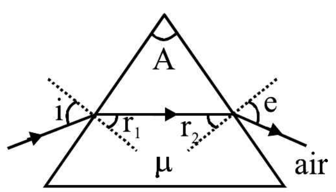
$\delta + A = i + e \Rightarrow \delta = i + e - A$..............(2)

μ_m = Refractive index of medium, μ_p = Refractive index of medium

$\mu_m \sin i = \mu_p \sin r_1$ (Snell's law at surface AB)

$\mu_p \sin r_2 = \mu_m \sin e$ (Snell's law at surface AC)

3.3 CONCEPT OF MINIMUM DEVIATION



$A = r_1 + r_2$ .... (i)

$\delta = i + e - A$ .... (ii)

Applying **Snell's law** at 1st interface, we get

$1 \sin i = \mu \sin r_1 \quad \Rightarrow \quad i = \sin^{-1}(\mu \sin r_1)$ .... (iii)

Applying Snell's law at second interface, we get

$\mu \sin r_2 = 1 \sin e \quad \Rightarrow \quad e = \sin^{-1}(\mu \sin r_2)$

$\Rightarrow \; e = \sin^{-1}[\mu \sin(A - r_1)]$ .... (iv) (since, $A = r_1 + r_2$)

Then, $\delta = i + e - A$

From equation (iii) & (iv) $\Rightarrow \; \delta = [\sin^{-1}(\mu \sin r_1) + \sin^{-1}[\sin(A - r_1)] - A$

Differentiating with respect to r_1, we get

$$\frac{d\delta}{dr_1} = \frac{\mu \cos r_1}{\sqrt{[1-\mu^2 \sin^2 r_1]}} - \frac{\mu \cos(A - r_1)}{\sqrt{[1-\mu^2 \sin^2(A - r_1)]}} = \frac{\mu\sqrt{[1-\sin^2 r_1]}}{\sqrt{[1-\mu^2 \sin^2 r_1]}} - \frac{\mu\sqrt{[1-\sin^2(A-r_1)]}}{\sqrt{[1-\mu^2 \sin^2(A - r_1)]}}$$

(Since, $0 < r_1 < \frac{\pi}{2}$ and $< r_1 < \frac{\pi}{2}$, so that $\cos r_1$ and $\cos r_2$ are essentially positive.

For minimum deviation $\quad \frac{d\delta}{dr_1} = \mathbf{0}$, $\quad \frac{\mu\sqrt{[1-\sin^2 r_1]}}{\sqrt{[1-\mu^2 \sin^2 r_1]}} = \frac{\mu\sqrt{[1-\sin^2(A-r_1)]}}{\sqrt{[1-\mu^2 \sin^2(A - r_1)]}}$

[From above condition]

By squaring the above equation, we get , $(\mu^2 - 1)\sin^2 r_2 = (\mu^2 - 1)\sin r_1$

$$\sin^2 r_2 = \sin^2 r_1 \quad \Rightarrow \quad r_1 = r_2 \quad \therefore \quad r_1 = A - r_1 \quad \text{or} \quad r_1 = r_2 = \frac{A}{2}$$

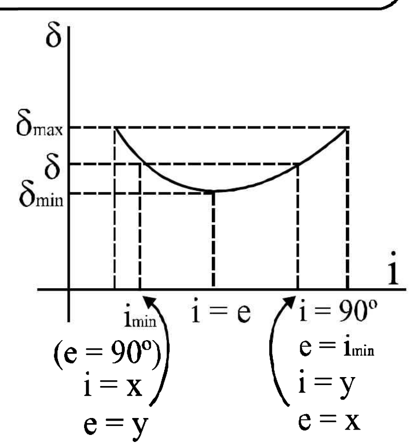
Hence, $i = e \quad (\because r_1 = r_2) \quad \therefore \quad \delta_{min} = 2i - A \quad$ or $\quad i = \frac{\delta_{min} + A}{2}$

Using Snell's law, we get the refractive index of prism material with respect to the surrounding medium which is given by, $\quad \mu = \dfrac{\sin\left(\dfrac{\delta_{min} + A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$ **(where $\mu = \frac{\mu_p}{\mu_m}$)**

This equation can be used to measure refractive index of the prism accurately as angles A and δ_m can be measured experimentally with good precision.

3.4 GRAPH BETWEEN δ VERSUS I

(shown in diagram)



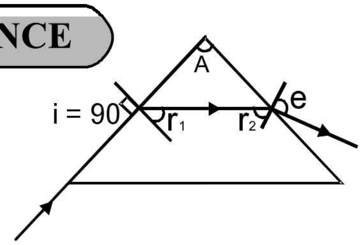
3.5 CHARACTERISTIC OF GRAPH

(1) For one δ (except δ min) there are two values of angle of incidence. If i and e are interchanged then we get the same value of δ because of the reversibility principle of light.

(2) There is one and only one angle of incidence for which the angle of deviation is minimum when, $\delta = \delta$ min (the angle of minimum deviation) then $i = e$ and $r_1 = r_2 = r$ (say). The ray passes symmetrically w.r.t. the refracting surfaces.

When a prism is in minimum deviation position, the refracted ray inside the prism is parallel to the base and passes symmetrically through the prism provided base angles are equal (in the equilateral or isosceles prism).

3.6 CONDITION OF GRAZING INCIDENCE



when, $i = 90$, $r_1 = \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$

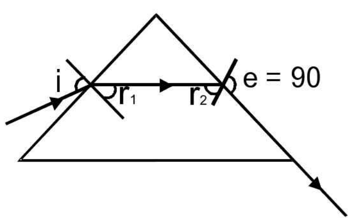
Now, $A = r_1 + r_2 \Rightarrow r_2 = A - r_1 = A - \theta_c$

Applying snell's law at second interface

$\mu \sin r_2 = 1 \sin e \Rightarrow \mu \sin (A - \theta_c) = 1 \sin e$

Then, $d = (i + e - A) = (90 + e) - A$

3.7 CONDITION OF GRAZING EMERGENCE

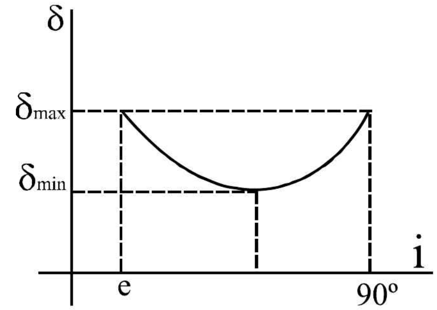


When , $e = 90$, $r_2 = \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$. Now, $A = r_1 + r_2 \Rightarrow r_1 = A - r_2 = A - \theta_c$

Applying Snell's law at first interface $1 \sin i = \mu \sin r_1 \Rightarrow 1 \sin i = \mu \sin(A - \theta_c)$

3.8 CONDITION OF MAXIMUM DEVIATION

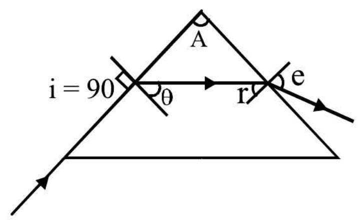
It is clear from the graph that as we move away from the angle of incidence giving minimum deviation (δ min), the deviation increases. Hence, when $i = \frac{\pi}{2}$, δ is maximum $\delta = i + e - A$



$\delta_{max} = \left(\frac{\pi}{2} + e - A\right)$ [when $i = \frac{\pi}{2}$]. From **the principle of reversibility,**

when, $e = \frac{\pi}{2}$, δ is also maximum

$\delta_{max} = (i + \frac{\pi}{2} - A)$ [when $e = \frac{\pi}{2}$]

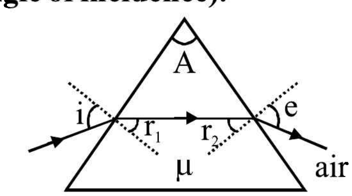


3.9 CONDITION OF NO EMERGENCE

A ray of light incidenting on a prism of angle A and refractive index μ will not emerge out of the prism (whatever may be the angle of incidence).

If $A > 2\theta_C$, (where θ_c is the critical angle)

Proof :- By the geometry (In Δ PQR)



$A + 90 - r_1 + 90 - r_2 = 180 \quad \Rightarrow \quad A = r_1 + r_2$

for the maximum value of $i = 90$, $(r_1)_{max} = \theta_c$

$(r_2)_{min} = A - (r_1)_{max} \quad \Rightarrow \quad (r_2)_{min} = A - \theta_c$

Then, for no emergence $(r_2)_{min}$ should be greater than the critical angle.

$(r_2)_{min} > \theta_c \Rightarrow \quad A - \theta_c > \theta_c \quad \Rightarrow \quad A > 2\theta_c$

then $\theta_c > \sin^{-1}\left(\frac{\mu_r}{\mu_d}\right)$; $\mu_r = 1$, $\mu_d = \mu \quad \Rightarrow \quad \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\frac{A}{2} > \theta_c$ (condition of no emergence);$\sin\frac{A}{2} > \sin\theta_c$ (By taking sin e on both the sides)

$\sin\frac{A}{2} > \frac{1}{\mu}$ $(\because \sin\theta_c = \frac{1}{\mu})$; $\mu > \frac{1}{\sin\left(\frac{A}{2}\right)}$

3.10 CONCEPT OF SMALL ANGLE PRISM

When prism angle and angle of incidence are very small then the angle of refraction r_1 will also be very small. As A is small, r_2 must also be very small.
From snell's law, we have

$1 \sin i = \mu \sin r_1 \quad \Rightarrow \quad$ **$i = \mu r_1$ (for small angle prism)**

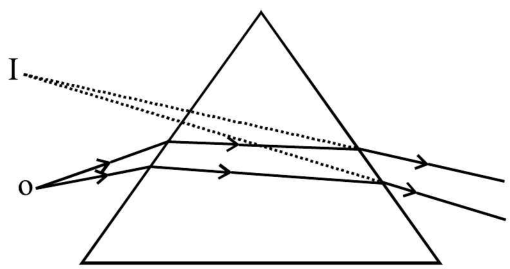
Also, $\mu \sin r_2 = \sin e$; **$\mu r_2 = e$ (for small angle prism)**

Hence, $\delta = i + e - A$, $\delta = \mu r_1 + \mu r_2 - A = \mu (r_1 + r_2) - A$; $\delta = (\mu - 1) A$ ($\because r_1 + r_2 = A$)

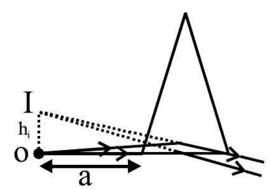
Hence, $\delta = (\mu - 1) A$. This relation is valid for any position of prism.

3.11 CONCEPT OF IMAGE FORMATION BY A PRISM

A prism forms a virtual image as shown in fig. :



Virtual image by a prism



For the small angle prism :

$\tan \delta = \dfrac{h_i}{a}$ **[Form the right angle triangle]**

$h_i = a\,\delta$ ($\because$ for small angle prism $\delta <<$ and $\tan \delta = \delta$) ; h_i (height of the image formed by the prism)

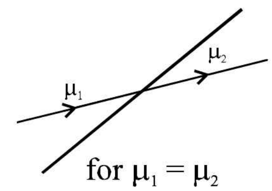
3.12 CAUCHY'S RELATION

An approximate empirical relation as proposed by Cauchy is given by, $\mu = A + \dfrac{\beta}{\lambda^2}$
where A and B are known as Cauchy's constant .The value of A and B depends upon the material of the prism.
we know that,

$\lambda_{red} > \lambda_{violet} \qquad \mu_{red} < \mu_{violet} \qquad \delta_{red} < \delta_{violet}$

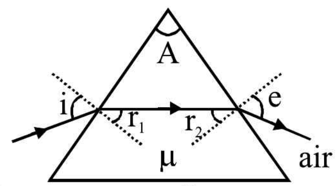
3.13 CONDITION OF NO. DEVIATION THROUGH THE INTERFACE

1. **If the refractive index is same as that of the other side of interface then for any angle of incidence there is no deviation through the interface.**



for $\mu_1 = \mu_2$

2. **If light falls normally (means i = 0) on the interface, for any μ_1 and μ_2 there is no any deviation through the interface.**



Note **Prism formula :**

$A = r_1 + r_2$ and $\delta = i + e - A$ are not always applicable.

In this fig : From Δ PQR

$90 + r_1 + 90 - r_2 + A = 180$

$A = r_2 - r_1$ ....(1)

For angle of deviation,

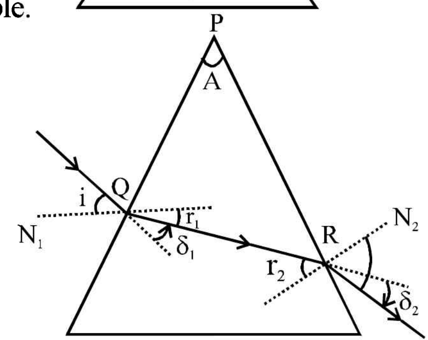
$\boldsymbol{\delta = \delta_1 - \delta_2}$ **($\because$ δ_1 and δ_2 both are in opposite sense)**

$\delta = (i - r_1) - (e - r_2)$

$\delta = i - e + r_2 - r_1$

$\boldsymbol{\delta = (i - e + A)}$ **(In the anticlockwise direction)**

In the above fig. general prism formula is not applicable.



3.14 DISPERSION

Dispersion By A Prism

When a beam of white light, passes through a prism it is split up into its constituent colours. Different colours suffer different amounts of deviation. The spread of colours is called **dispersion of light**. The image thus formed on a screen is called a **spectrum**. The medium, which produces dispersion, is called a **dispersive medium**.

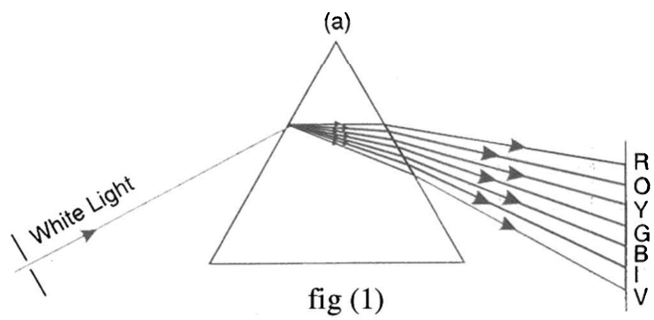


fig (1)

The spectrum consists of visible and invisible regions. In the visible region, the order of the colours is from violet to red. The principal colours are given by the word. **VIBGYOR**

(Violet, Indigo, Blue, Green, Yellow, Orange and Red). The deviation produced for the violet rays of light is maximum and for red rays of light it is minimum. Fig (1) represents the dispersion of a white ray of light by a prism in the visible region. The region of the spectrum of wavelengths shorter than **violet** is called ultraviolet and the region of wavelength longer than red is called **infrared**. In the present chapter, the discussion relates only to the visible region of the spectrum.
The refractive index of the material of a prism (or a lens) is different for different wavelengths (or colours). The deviation and hence, the refractive index is more for blue rays of light than the corresponding values for red rays of light. The deviation and the refractive index of the yellow constituent are taken as the mean values.

REFRACTION THROUGH A PRISM :
The refractive index of the material of prism is given by

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Where A is the angle of the prism and δ_m is the angle of minimum deviation. When the angle of the prism is very small, the prism is said to be a **thin prism**. For a thin prism, the values of the angles A and δ are so small that the sines of the angles may be taken equal to the angles. Therefore, the above equation becomes.

$$\mu = \frac{A+\delta_m}{A} \quad \text{or} \quad \delta_m = (\mu - 1)A$$

3.15 ANGULAR DISPERSION

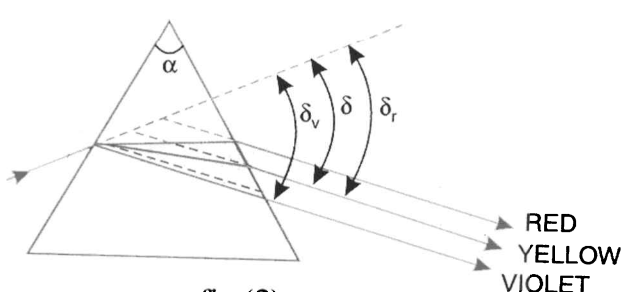


fig (2)

The refraction and dispersion do not bear a simple relationship with one another. It is known that some glasses have a high index of refraction and little dispersion while others have low index of refraction and high dispersion. Therefore, to characterize a material, it is necessary to use more than one refractive index. In practice three colours are chosen; one in the middle and two at the two extremes of the visible spectrum. Thus, the yellow colour is taken as the mean colour and violet and red colours are taken at the two extremes. The refractive indices corresponding to these colours are indicated by μ, μ_v and μ_R. The deviation δ corresponding to yellow colour is taken as the mean deviation.

Fig. (2) shows the angles of deviation δ_v ,δ and δ_R produced in the violet, mean yellow and red rays of light.

The deviation δ_v, δ and δ_R can be written as

$\delta = (\mu - 1)A$, $\delta_V = (\mu_V - 1)A$, $\delta_R = (\mu_R - 1)A$

The total angle through which the spectrum is spread is known as the angular dispersion. It is measured by the difference in deviation between the two extreme colours of the spectrum. Therefore, the angular dispersion is given by

$$\theta = \delta_V - \delta_R = (\mu_V - \mu_R)A$$

Thus, the **angular dispersion** depends on the nature of the material and the angle of the prism.

3.16 DISPERSIVE POWER

Dispersive power indicates the ability of the material of the prism to disperse the light rays. It is defined as the ratio of the angular dispersion to the deviation of the mean ray.

$$\text{Dispersive power, } \omega = \frac{\delta_V - \delta_R}{\delta} \qquad \text{............. (1)}$$

Using the set of above equations, we can rewrite equation (1) as $\omega = \dfrac{\mu_V - \mu_R}{\mu - 1}$

This is the expression for **dispersive power** used mainly in geometric optics. It is seen that the dispersive power is independent of the angle of the prism and the angle of incidence.

3.17 ANGULAR AND CHROMATIC DISPERSIONS

Now we note the definition of dispersive power as used in the wave theory of light. The variation of refractive index μ of a given medium with wavelength is called dispersion. If μ varies rapidly with wavelength, the medium is said to be highly dispersive. The refractive index μ of the material of a prism is given by

$$\mu = \frac{\sin\dfrac{A+\theta}{2}}{\sin A/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where A is the angle of the prism and θ is the angle of minimum deviation.

$$\sin\frac{A+\theta}{2} = \mu\sin\frac{A}{2} \qquad \therefore \qquad \cos\frac{A+\theta}{2} = \sqrt{1-\mu^2\sin^2\frac{A}{2}}$$

If dθ is the difference in the angle of deviation between two spectral lines of wavelengths λand λ+ dλ, then $\frac{d\theta}{d\lambda}$ is called the angular dispersion between the wavelength λ and λ+ dλ

Differentiating equation (1) with respect to μ on the left-hand side and θ on the right hand side, we obtain,

$$d\mu = \frac{\frac{1}{2}\cos\dfrac{A+\theta}{2}}{\sin A/2}.d\theta \qquad \Rightarrow \qquad \frac{d\mu}{d\lambda} = \frac{\cos\dfrac{A+\theta}{2}}{2\sin A/2}.\frac{d\theta}{d\lambda}$$

$$\frac{d\theta}{d\lambda} = \frac{2\sin\dfrac{A}{2}}{\cos\dfrac{A+\theta}{2}}.\frac{d\mu}{d\lambda} \qquad \text{or} \qquad \frac{d\theta}{d\lambda} = \frac{2\sin\dfrac{A}{2}}{\sqrt{1-\mu^2\sin^2\dfrac{A}{2}}}.\frac{d\mu}{d\lambda} \quad \ldots\ldots\ldots (2)$$

$\frac{d\mu}{d\lambda}$ is called chromatic dispersion of the material of the prism. It depends on the angle of the prism . In general $\frac{d\mu}{d\lambda}$ is not constant and decreases as λ increases. Using a spectrometer and the given prism, a graph is drawn between μ and λ(μ along the y-axis and λ along the x-axis). The tangent to the curve at any point measures the chromatic dispersion dμ/dλ for the particular wavelength. Substituting this value of dμ/dλin equation (2), dθ /dλ can be calculated.

3.18 ACHROMATIC COMBINATION OF PRISM-DEVIATION WITHOUT DISPERSION

An achromatic prism is a combination of two appropriate prisms constructed such that it doesn't show any colours. It is noted that the dispersive powers of different materials are different. Flint glasses have higher dispersive power than crown glasses. Hence, it is possible to combine two prisms of different materials and specified angles such that a ray of white light may pass through the combination without dispersion, though it may suffer deviation. Such a combination is called an achromatic combination.

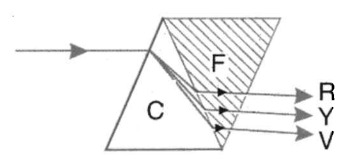


fig (a)

Fig.(a) shows an arrangement of two prisms, which do not cause dispersion though it may produce deviation. Let the first prism be made of crown and the second of flint. Let δ_V and δ_R be the deviations and μ_V and μ_R the refractive indices for the violet and red rays in case of crown glass prism. Let the angle of the prism be A. The angular dispersion produced by the crown glass prism is

$$\delta_V - \delta_R = (\mu_V - \mu_R)\,A$$

Let the refracting angles of the second prism of lint glass be A'. It is arranged in such a way that it produces a deviation in a direction opposite to that of the first prism. Let δ_V' and δ_R' be the deviations and μ_V' and μ_R' the refractive indices for the violet and red rays. The angular dispersion produced by the flint glass prism is

$$\delta_V' - \delta_R' = (\mu_V' - \mu_R')A'$$

For achromatism the net dispersion produced by the two prism together must be zero.

$$\therefore \qquad (\mu_V - \mu_R)A + (\mu_V' - \mu_R')A' = 0 \qquad \text{.......... (1)}$$

This is the condition for achromatism.

$$\therefore \qquad A = -\left(\frac{\mu_V' - \mu_R'}{\mu_V - \mu_R}\right)A'$$

The above expression gives the relation between the angles of the two prisms in order that the combination may function as an achromatic combination. The negative sign shows that the two prisms should be placed with their refracting angles in opposite directions. Equation (1) may be written as

$$\frac{(\mu_V - \mu_R)}{\mu - 1} \cdot (\mu - 1)\,A + \left(\frac{\mu_V' - \mu_R'}{\mu - 1}\right) \cdot (\mu' - 1)\,A' = 0$$

Using the equation of dispersive power we can write the above equation as

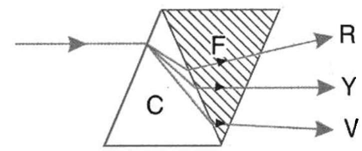
$$\omega\delta = -\,\omega'\delta' \qquad \text{............ (2)}$$

where ω and ω' are the dispersive powers of the two prisms and δ and δ' are their mean deviations. Equation (2) indicates that the dispersive powers of the prism should be in the inverse ratio of their mean deviations. It is found that when a crown glass prism of refracting angle 60° is combined with a flint glass prism of an angle of 29°17', the combination becomes achromatic though the combination is achromatic, yet it produces deviation.

3.19 DISPERSION WITHOUT DEVIATION

Achromatic prisms are not of much practical used. A more useful combination of prism is the one that produces dispersion but no deviation. It is called a direct vision prism.

It consists of two prisms of different dispersive powers and different angles, combined in such a way that the combination does not produce any deviation of the mean rays. If the angles of the crown and flint glass prism are adjusted such that the deviation produced for the mean rays by the first prism is equal and opposite to that produced by the second prism, then the final beam will be parallel to the incident beam. However , such a combination of two prism produces dispersion of the incident beam without deviation.



Let us consider two thin prisms, one made of crown glass and the other of flint glass. Let A and A' be the refracting angles of the prism and μ and μ' the refractive indices. Let δ and δ' be the deviations produced for the mean ray by the two prisms respectively. The deviations are given by

$$\delta = (\mu - 1)A \quad , \qquad \delta' = (\mu' - 1)A'$$

The net deviation produced by the prism combination is to be zero.

That is, $\qquad \delta + \delta' = 0 \qquad \Rightarrow \qquad$ Or $(\mu - 1)A + (\mu' - 1)A' = 0$

$$\therefore \qquad A = -\left(\frac{\mu'-1}{\mu-1}\right)A' \qquad \text{............ (1)}$$

The negative sign shows that the refracting angles of the two prisms are in opposite direction. We calculate the net angular dispersion produced by the combination of prism as follows.

The total deviation for the violet rays rays is given by

$$\delta_V = (\mu_V - 1)A + (\mu_V' - 1)A'$$

and the total deviation for the red rays is given by

$$\delta_R = (\mu_R - 1)A + (\mu_R' - 1)A'$$

Hence, the total dispersion is equal to

$$\delta_V - \delta_R = (\mu_V - \mu_R)A + (\mu_V' - \mu_R')A'$$

Using equation (1) into the above equation, we obtain

$$\delta_V - \delta_R = (\mu_V - \mu_R)A + (\mu_V' - \mu_R')\left(-\frac{\mu-1}{\mu'-1}\right)A$$

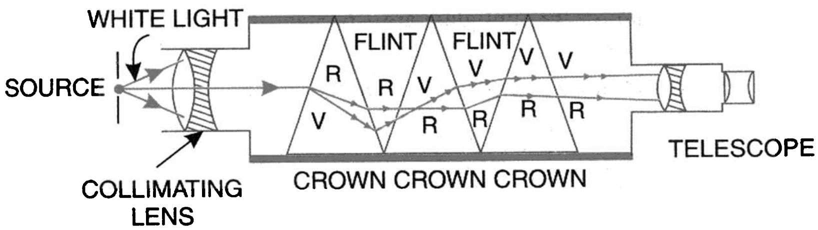
or $$\delta_V - \delta_R = \left[\frac{(\mu_V - \mu_R)}{(\mu-1)} - \frac{(u_V' - \mu_R')}{\mu'-1}\right](\mu - 1)A$$

In follows from equation of dispersive power that

$$\delta_V - \delta_R = (\omega - \omega')(\mu - 1)A$$

Thus, the resultant dispersion is equal to the difference between the two dispersions.

3.20 DIRECT VISION SPECTROSCOPE



A direct spectroscope consists of three crown and two flint glass prisms of suitable refracting angles. The prisms are fixed in a metal tube. A collimating lens is fixed at one end of the tube and a telescope at the other. The angles of the prisms are such that the total deviation produced for the mean rays is zero. The refracting angles of crown and flint glass prisms are in opposite directions.

Thus, the deviation produced by the crown glass prisms in one direction is equal and opposite to that produced by the flint glass prisms in the opposite direction. The result will be dispersion and the dispersed beam will almost be parallel to the incident beam. The prisms are cemented together with canada balsam (μ= 1.54) to minimize the reflection losses at the interfaces. A spectroscope of this type is very handy and is used to qualitatively study the spectra of different sources of light. Use of more than two prisms increases the resolving power of the instrument i.e. the spectral lines will appear well separated from one another.

Ex.1 Calculate the dispersive power for crown and flint glass from the following data.

	C	D	F
Crown	1.5145	1.5170	15230
Flint	1.6444	1.6520	1.6637

Sol.
$$\omega_1 = \frac{\mu_F - \mu_C}{\mu_D - 1} = \frac{1.5230 - 1.5145}{1.5170 - 1} = 0.01644,$$

$$\omega_1 = \frac{\mu_F - \mu_C}{\mu_D - 1} = \frac{1.6637 - 1.6444}{1.6520 - 1} = 0.02961$$

Ex.2 For a crown and flint glass for C and F lines $\mu_C = 1.515$ and $\mu_F = 1.523$ and $\mu_C = 1.644$, $\mu_F = 1.664$ respectively. Calculate the angle of flint glass prism which may be combined with crown glass prism having refracting angle 20µ such that the combination is achromatic for C and F rays.

Sol. The condition for the combination to be achromatic is that

$$(\mu_F - \mu_C)A = (\mu_F' - \mu_C')A'$$

$$(1.523 - 1.515) \times 20^\circ = (1.664 - 1.644)\,\alpha' \quad \therefore \quad \alpha' = \frac{0.008}{0.02} \times 20^\circ = 8^\circ$$

Ex.3 A crown glass prism of refracting angle 8° is combined with a flint glass prism to obtain deviation without dispersion. If the refractive indices for red and violet rays for the crown glass are 1.514 and 1.524 and for the flint glass are 1.645 and 1.665 respectively, find the angle of flint glass prism and net deviation.

Sol. The condition for deviation without dispersion is

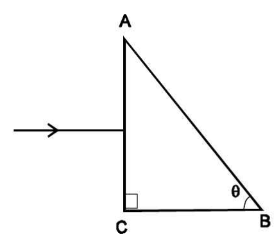
$$(\mu_V - \mu_R)A = (\mu_V' - \mu_R')A' \quad \therefore \quad A' = \frac{(1.524 - 1.514) \times 8^\circ}{(1.665 - 1.645)} = \frac{0.08^\circ}{0.02} = 4^\circ$$

Mean refractive index for crown glass $\mu = \dfrac{1.514 + 1.524}{2} = 1.519$

Mean refractive index for flint glass $\mu = \dfrac{1.645 + 1.665}{2} = 1.655$

$\therefore$ The net deviation $(\delta - \delta') = (\mu - 1)A - (\mu' - 1)A' = 0.519 \times 8^\circ - 0.655 \times 4^\circ = \mathbf{1.53^\circ}$

Ex.4 What should be the value of angle θ such that light entering normally through the surface AC of a prism (n=3/2) does not cross the second refracting surface AB.

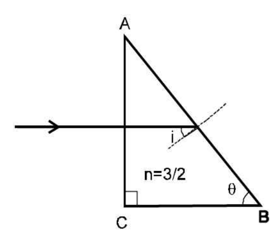


Sol. Light ray will pass through the surface AC without bending since, it is incidenting normally. Suppose it strikes the surface AB at an angle of incidence i. $i = 90 - \theta$

For the required condition: $90^\circ - \theta > C$

or $\sin(90^\circ - \theta) > \sin C$

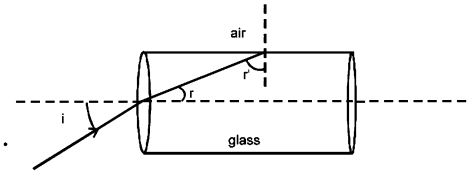
or $\cos\theta > \sin C = \dfrac{1}{3/2} = \dfrac{2}{3}$ or $\theta < \cos^{-1}\dfrac{2}{3}$.



Ex.5 What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod.

Sol. It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle ....(A)
Angle r' is minimum when r is maximum i.e. C(why ?).Therefore the minimum value of r' is 90-C.
From condition (A) :
$90° - C > C$ or $C < 45°$

$\sin C < \sin 45°$; $\frac{1}{n} < \frac{1}{\sqrt{2}}$ or $n > \sqrt{2}$.

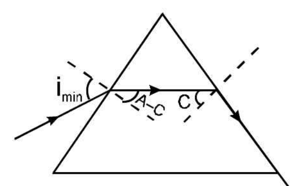


Ex.6 Show that if $A > A_{max}$ (= 2 C), then Total internal reflection occurs at second refracting surface PR for any value of 'i'.

Sol. For T.I.R. at second surface $r' > C$
$\Rightarrow$ $(A - r) > C$ or $A > (C + r)$
The above relation will be fulfilled if
or $A > C + r_{max}$ or $A > C + C$ or $A > 2C$

(j) On the basis of above example and similar reasoning, it can be shown that (you should try the following cases (ii) and (iii) yourself.)
(i) If $A > 2C$, all rays are reflected back from the second surface.
(ii) If $A \le C$, no rays are reflected back from the second surface i.e. all rays are refracted from second surface.
(iii) If $2C \ge A > C$, some rays are reflected back from the second surface and some rays are refracted from second surface, depending on the angle of incidence..

(k) δ is maximum for two values of i $\Rightarrow$ i_{min} (correspondingto $e = 90°$) and $i = 90°$ (corresponding to e_{min}).
For i_{min} : $n_s \sin i_{min} = n_p \sin(A - C)$
If $i < i_{min}$ then T.I.R. takes place at second refracting surface PR.



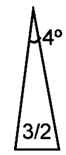
Ex.7 Refracting angle of a prism A = 60° and its refractive index is, n = 3/2, what is the angle of incidence i to get minimum deviation. Also find the minimum deviation. Assume the surrounding medium to be air (n = 1).

Sol. For minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30°$.

applying snell's law at I surface $1 \times \sin i = \frac{3}{2} \sin 30° \Rightarrow i = \sin^{-1}\left(\frac{3}{4}\right)$

$d_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}$

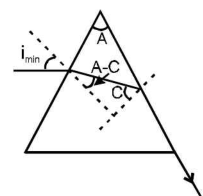
Ex.8 See the figure
Find the deviation caused by a prism having refracting angle 4° and refractive index $\frac{3}{2}$.



Sol. $d = (\frac{3}{2} - 1) \times 4^0 \quad = 2^0$

Ex.9 For a prism, A = 60°, n = $\sqrt{\frac{7}{3}}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also find δ_{max}.

Sol. In minimum incidence case the angles will be as shown in figure

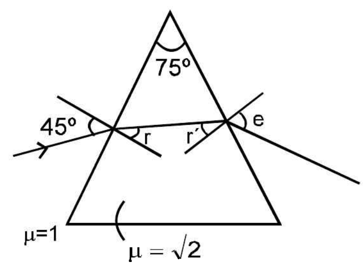


Applying snell's law : $\quad 1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin(A - C)$

$$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C) \quad = \sqrt{\frac{7}{3}} \left(\sin 60\sqrt{1-\frac{3}{7}} - \cos 60\sqrt{\frac{3}{7}}\right) = \frac{1}{2}$$

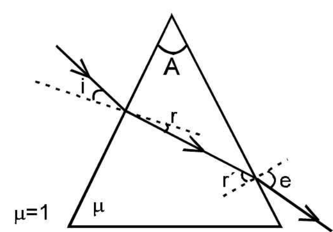
$\therefore \quad i_{min} = 30° \quad \therefore \quad \delta_{max} = i_{min} + 90^0 - A \quad = \quad 30^0 + 90^0 - 60^0 = 60^0$.

Q.1 Find r, r′, e, δ for the case shown in figure.



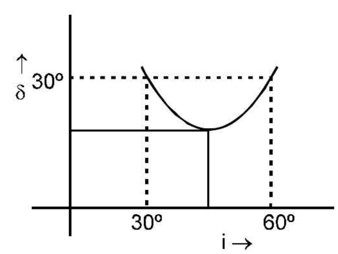
Sol. r = 30° ; r′ = 45° ; e = 90° ; δ = 60°

Q.2 For the case shown in figure prove the relations r′ – r = A and δ = | (i – e) + A | (do not try to remember these relations because the prism is normally not used in this way).



Sol. By the Geometry.

Q.3 From the graph of angle of deviation d versus angle of incidence i, find the prism angle



Sol. 60°

Ex.10 The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively. Find the angular dispersion produced by a thin prism of flint glass with refracting angle 5°.

Sol. Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion $= \delta_v - \delta_r = (\mu_v - \mu_r)A$

$= \quad (1.632 - 1.613) \times 5^0 \quad = \quad 0.095^0$.

Deviation of beam (also called mean deviation)

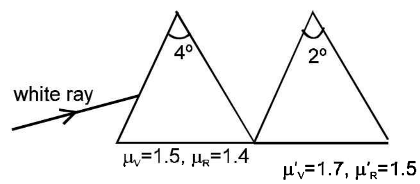
$$\delta = \delta_y = (n_y - 1)A$$

n_v, n_r and n_y are R. I. of material for violet, red and yellow colours respectively.

Ex.11 Refractive indices of glass for red and violet colours are 1.50 and 1.60 respectively. Find (A) the ref. index for yellow colour, approximately
(B) Dispersive power of the medium.

Sol. (A) $\mu_r \simeq \dfrac{\mu_v + \mu_R}{2} = \dfrac{1.50 + 1.60}{2} = 1.55$ (B) $\omega = \dfrac{\mu_v - \mu_R}{\mu_r - 1} = \dfrac{1.60 - 1.50}{1.55 - 1} = 0.18.$

Ex.12 If two prisms are combined, as shown in figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Sol. Both prisms will turn the light rays towards their bases and hence in same direction. Therefore, turnings caused by both prisms are additive.

Total angular dispersion

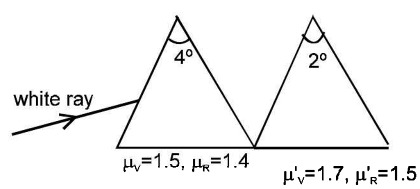
$= \theta + \theta' = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' \quad = (1.5 - 1.4)\,4° + (1.7 - 1.5)2° = 0.8°$

Total derivation $= \delta + \delta'$

$$= \left(\frac{\mu_V + \mu_R}{2} - 1\right)A + \left(\frac{\mu'_V + \mu'_R}{2} - 1\right)A' = \left(\frac{1.5 + 1.4}{2} - 1\right)0.4° + \left(\frac{1.7 + 1.5}{2} - 1\right)0.2°$$

$= (1.45 - 1)\,0.4° + (1.6 - 1)\,0.2° = 0.45 \times 0.4° + 0.6 \times 0.2° = 1.80 + 1.2 =$ **3.0° Ans.**

Q.4 If two prisms are combined, as shown in figure, find the net angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Sol. $\theta = 0°, \ \delta = 0.6°$

Ex.13 Two thin prisms are combined to form an achromatic combination. For I prism A = 4°, μ_R = 1.35, μ_Y = 1.40, μ_v = 1.42. for II prism μ'_R= 1.7, μ'_Y = 1.8 and μ'_R = 1.9 find the prism angle of II prism and the net mean derivation.

Sol. Condition for achromatic combination.

$$\theta = \theta', \qquad (\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$$

$$\therefore \quad A' = \frac{(1.42-1.35)4^\circ}{1.9-1.7} = 1.4^\circ \quad , \quad \delta_{Net} = \delta \sim \delta' = (\mu_Y - 1)A \sim (\mu'_Y - 1)A'$$

$= (1.40 - 1)\,4^\circ \sim (1.8 - 1)\,1.4^\circ \quad = 0.48^\circ. \;, \quad q = q'$

$$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A' \quad \therefore \quad A' = \frac{(1.42-1.35)4^\circ}{1.9-1.7} = 1.4^\circ$$

$\delta_{Net} = \delta \sim \delta' = (\mu_Y - 1)A \sim (\mu'_Y - 1)A' = (1.40 - 1)\,4^\circ \sim (1.8 - 1)\,1.4^\circ = 0.48^\circ.$

Ex.14 A crown glass prism of angle 5^0 is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.514, 1.517 and 1.523 respectively for crown glass and 1.613,1.620 and 1.632 for flint glass.

Sol. The deviation produced by the crown prism is $\delta = (\mu - 1)A$
and by the flint prism is : $\delta' = (\mu' - 1)A'$.
The prisms are placed with their angles inverted with respect to each other. The deviations are also in the opposite directions. Thus, the net deviation is :

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'. \qquad \text{.................(1)}$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'. \qquad \text{or,} \quad A' = \frac{(\mu-1)}{(\mu'-1)}A = \frac{1.517-1}{1.620-1} \times 5^0$$

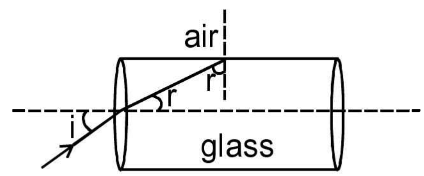
(b) The angular dispersion produced by the crown prism is : $\delta_v - \delta_r = (\mu_v - \mu_r)A$
and that by the flint prism is, $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$
The net angular dispersion is, $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A$
$= (1.523 - 1.514) \times 5^0 - (1.632 - 1.613) \times 4.2^0 \quad = -0.0348^0$.
The angular dispersion has magnitude 0.0348^0.

Q.5 The dispersive powers of crown and flint glasses are 0.03 and 0.05 respectively. The refractive indices for yellow light for these glasses are 1.517 and 1.621 respectively. It is desired to form an achromatic combination of prisms of crown and flint glasses which can produce a deviation of 1^0 in the yellow ray. Find the refracting angles of the two prisms needed.

Sol. $Ac = 4.8^0$, $Af = 2.4^0$

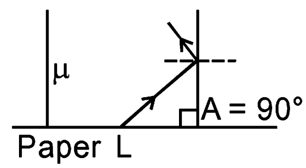
Ex.15 What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod.

Sol. It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle ... (A)
Angle r' is minimum when r is maximum i.e. C (why?). Therefore, the minimum value of r' is 90 – C.
From condition (A) :



$90° - C > C$ or $C < 45°$, $\sin C < \sin 45°$; $\frac{1}{n} < \frac{1}{\sqrt{2}}$ or $n > \sqrt{2}$

Ex.16 A rectangular block of refractive index μ is placed on a printed page lying on a horizontal surface as shown in figure. Find the minimum value of μ so that the letter L on the page is not visible from any of the vertical sides.



Sol. The letter L will not be visible from the vertical sides if the light ray does not emerge through it. We can apply the condition of no emergence for a prism of angle A = 90°.

Thus, $\mu > \dfrac{1}{\sin\frac{A}{2}}$ or $\mu > \dfrac{1}{\sin\frac{90°}{2}}$ or $m > \sqrt{2}$

Ex. 17 What is the apex angle of the conical region through which a person in water views the outside object ?

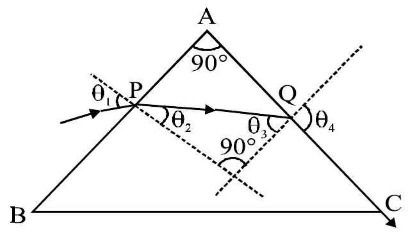
Sol. For water-air interface, the critical angle is given as, $i_c = \sin^{-1}(3/4) = 48.6°$.
∴ The required apex angle is 2 × 48.6°.

Ex.18 A prism with angle A = 60° produces a minimum deviation of 30°. Find the refractive index of the material.

Sol. We know that , $$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Here A = 60°, δ_{min} = 30° ∴ $$\mu = \frac{\sin\left(\frac{90+30}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 60°}{\sin 30°} = \sqrt{2}$$

Ex.19 Figure shows a triangular prism of refracting angle 90°. A ray of light incident at face AB at an angle θ refracts at point Q with an angle of refraction 90°. (a) What is the refractive index of the prism in terms of θ ? (b) What is the maximum value that the refractive index can have? What happens to the light at Q if the incident angle at Q is (c) increased slightly and (d) decreased slightly ?



Sol. Thin prisms

In thin prisms the distance between the refracting surfaces is negligible and the angle of prism (A) is very small.

Since $A = r_1 = r_2$, therefore, if A is small then both r_1 and r_2 are also small, and the same is true for i_1 and i_2.

According to Snell's law $\quad \sin i_1 = \mu \sin r_1 \quad$ or $\quad i_1 = \mu r_1$

$\sin i_2 = \mu \sin r_2 \quad$ or $\quad i_2 = \mu r_2$

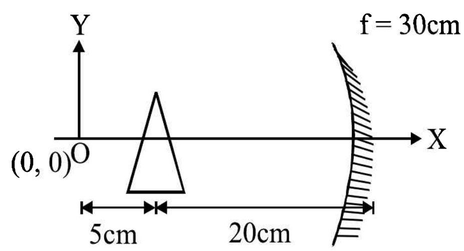
Therefore, deviation $\delta = (i_1 - r_1) + (i_2 - r_2)$, $\delta = (r_1 + r_2) + (i_2 - r_2)$, $\delta = A(\mu - 1)$

Note : The deviation for a small angled prism is independent of the angle of incidence.

Ex.20 A thin prism of angle A = 6° produces a deviation δ = 3°. Find the refractive index of the material of prism.

Sol. We know that $\delta = A(\mu - 1)$ or $\mu = 1 + \frac{\delta}{A}$, Here $A = 6°$, $\delta = 3°$, $\Rightarrow \mu = 1 + \frac{3}{6} = 1.5$

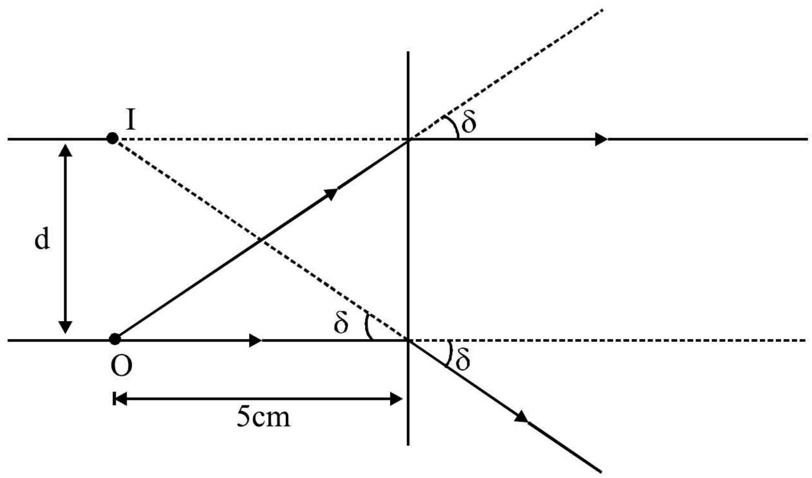
Ex.21 Find the co-ordinates of image of the point object 'O' formed after reflection from concave mirror as shown in figure assuming prism to be thin and small in size of prism angle 2°. Refractive index of the prism material is 3/2.



Sol. Consider image formation through prism. All incident rays will be deviated by

$$\delta = (\mu - 1)A = \left(\frac{3}{2} - 1\right)2° = 1° = \frac{\pi}{180}\text{rad}$$

As prism is thin, object and image will be in the same plane as shown in figure.



It is clear $\frac{d}{5} = \tan\delta \approx \delta$ ($\therefore$ δ is very small) or $d = \frac{\pi}{36}$ cm

Now this image will act as an object for concave mirror.

$u = -25$ cm, $f = -30$ cm , $\therefore\ v = \frac{uf}{u-f} = 150$ cm , Also, $m = \frac{-v}{u} = = +6$

$\therefore$ Distance of image from principal axis $= \frac{\pi}{36} \times 6 = \frac{\pi}{6}$ cm

Hence, co-ordinates of image formed after reflection from concave mirror are $\left(175\text{cm}, \frac{\pi}{6}\text{cm}\right)$

CHAPTER 4 Lens

A lens is a piece of transparent material with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

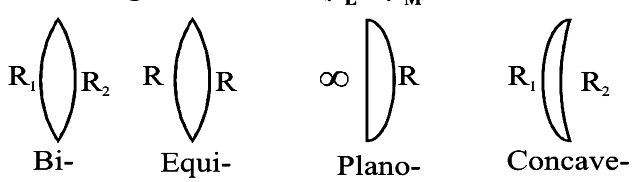
If the curved surface (or surfaces) of a lens is spherical, the lens is called spherical and if its thickness is small the lens is called thin.

We have to study about thin lenses. A lens is said to be thin if the gap between the two surfaces is very small.

4.1 TYPES OF LENS

(1) Converging lens (convex lens):

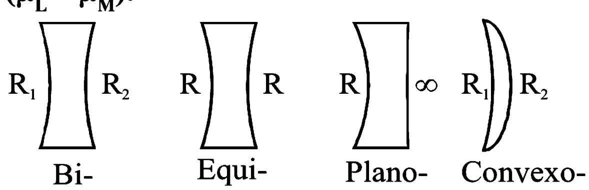
A thin spherical lens with refractive index greater than that of the surroundings behaves as a convergent or convex lens, i.e. converges parallel ray **if its central portion is thicker than marginal one with ($\mu_L > \mu_M$).**



Convex lens if ($\mu_L > \mu_M$)

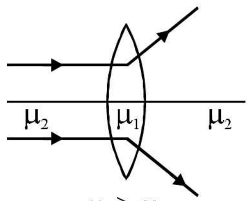
(2) Diverging lens (concave lens):

A thin spherical lens with refractive index greater than that of surroundings behaves as divergent or concave lens **if the central portion of a lens is thinner than marginal with ($\mu_L > \mu_M$).**

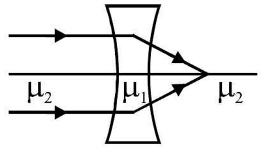


Concave Lens if ($\mu_L > \mu_m$)

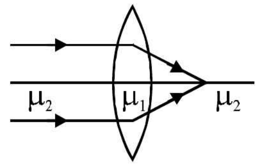
The behavior of a lens also depends on the surrounding medium. If R. I of surrounding medium is more than the R.I of lens material, then a converging lens will behave like a diverging lens or vice-versa.



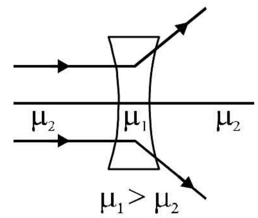
$\mu_2 > \mu_1$



$\mu_2 > \mu_1$

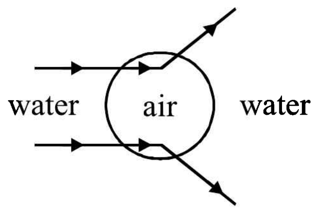


$\mu_1 > \mu_2$

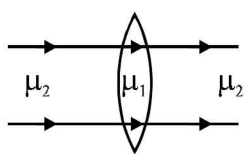


$\mu_1 > \mu_2$

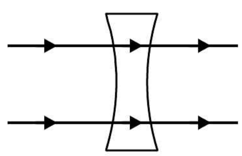
- **An air bubble in water behaves like a diverging lens as shown in figure.**



- **If R.I. of lens material is same as that of the surrounding medium then the lens will be neither converging nor diverging.**



$\mu_1 = \mu_2$

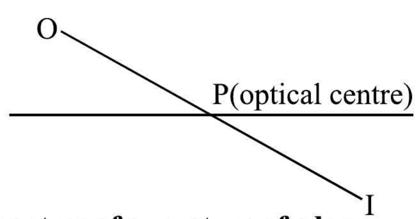


$\mu_1 = \mu_2$

4.2 BASIC DEFINITIONS

(1) Optical centre : For every lens, there is a point on the principal axis for which the rays passing through it are not deviated by the lens. Any ray passing through it emerges in a direction to incident ray. Such a point is called optical centre. When the lens is thin and the radii of curvature of the two refracting surfaces are equal, then the geometrical center of lens becomes **the optical centre of lens.**

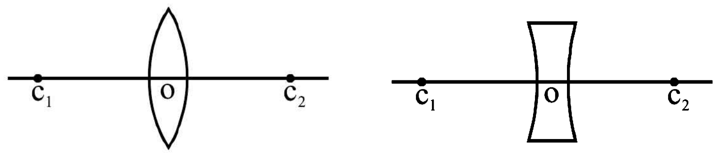
When only position of object and image are given with principal axis then, join object and image through a line. The point p where the line OI cuts the principle axis, is **the optical centre.**



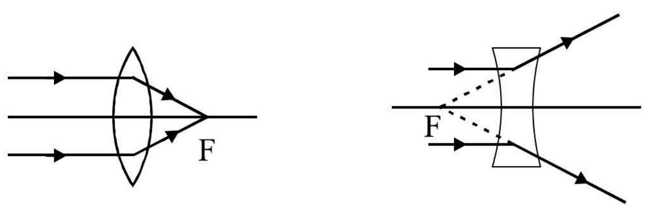
(2) Centre of curvature (c_1 and c_2) : There are two **centres of curvature of a lens**, one each belonging to both surfaces. Centre of curvature of a surface of a lens is defined as the centre of that sphere of which that surface forms a part.

(3) Radius of curvature (R_1 and R_2) : There are two radii of curvature of a lens, each belonging to both the surfaces. **Radius of curvature of a surface of a lens** is defined as the radius of that sphere of which the surface forms a part.

(4) Principal axis : A line joining the two centres of curvature and passing through the optical centre is called **principal axis.**

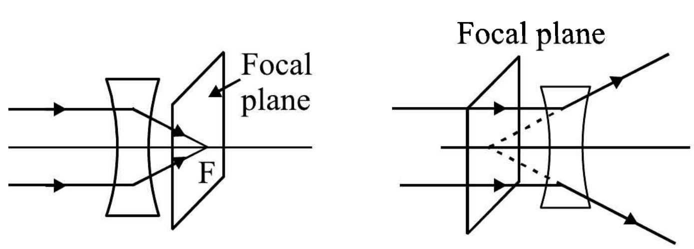


(5) Principal focus (F) : Principal focus of a lens is a point at which, a beam of light coming parallel to principal axis actually meets or appears to meet after refraction through the lens.



(6) Focal length (f) : The distance between the focal point (F) and the optical centre of the lens is called the **focal length** of the lens.

(7) Focal plane : The plane perpendicular to the principal axis of lens and passing through its focal point is known as the **focal plane**.

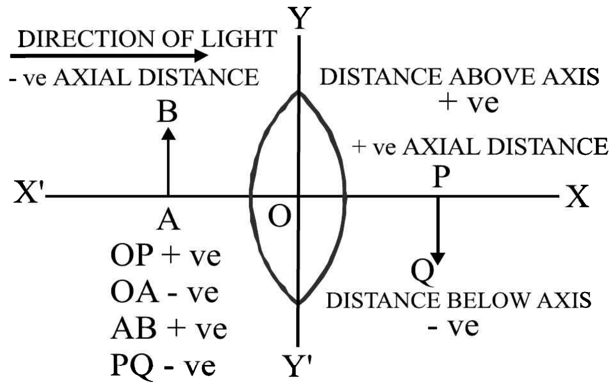


(8) Aperture : In reference to a lens, **aperture** means the **effective diameter of its light transmitting area.** So, the brightness, i.e., intensity of image formed by a lens which depends on the light passing through the lens will **depend on the square of aperture, $I \propto$ $(Aperture)^2$**

4.3 SIGN CONVENTION

The following convention of the signs is adopted for obtaining the relation between these quantities.

(a) The diagrams are drawn showing the incident light travelling from left to right.



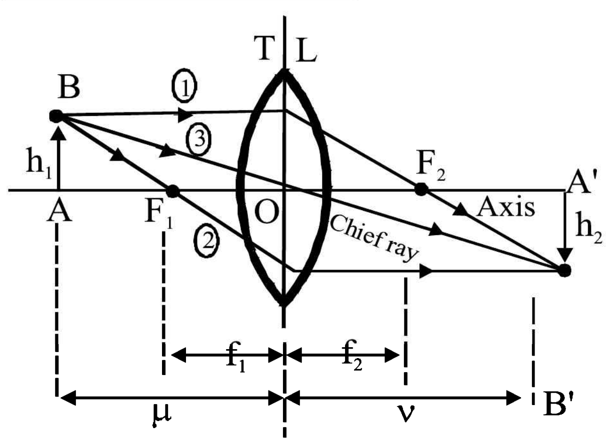
(b) The distances are measured by taking the optical centre of the lens as the origin.

(c) The distances measured in the direction of the incident light are considered positive while those measured in the direction opposite to the incident light are taken as negative. (All quantities measured to the right of P are positive and all those to its left are negative.)

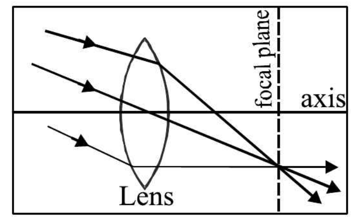
(d) Heights measured upward and perpendicular to the principal axis are taken as positive while those measured downward are considered negative.

(e) The angle made by a ray with the principal axis is taken to be positive if the ray has to be rotated anti-clockwise to become coincident with the axis, otherwise it is negative.

4.4 IMAGE TRACING



We may use graphical ray tracing to determine the position of the image formed by a lens. To find the image, we take the help of characteristic rays shown in figure.



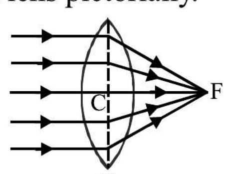
(i) One is the ray parallel to the principal axis, which after refraction, passes through focal point F_2.
(ii) Second ray is the ray that passes through the first focal point F_1 of the lens ; after refraction, it travels parallel to the principal axis.
(iii) The third ray, usually called chief ray goes through the optical centre of the lens and emerges without deviation.

Thus, we have three characteristic rays whose paths are known. Using any two of the three characteristic rays, we can readily determine the image of any object-point or of any extended object.

4.5 LOCATION OF THE IMAGE

A convex lens produces a real or virtual image depending on the location of the object. A concave lens always produces virtual images of real object. Figure illustrates the behaviour of a convex lens pictorially.

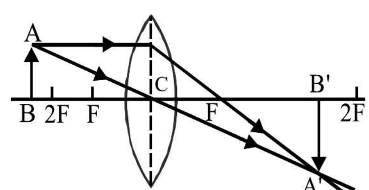
Object at infinity



(i)

Image at F point, real and inverted

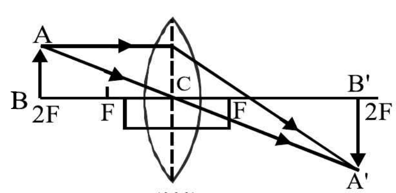
Object beyond 2F



(ii)

Image between F and 2F Real, inverted and diminished

Object at 2F



(iii)

Image at 2F Real, inverted and of same size

Object between F and 2F

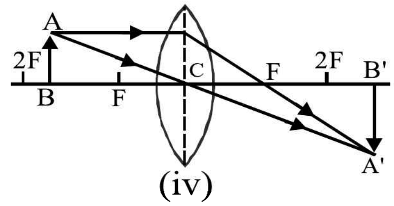


Image beyond 2F Real, inverted and ımagnified

Object at F

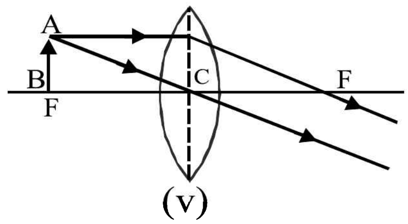
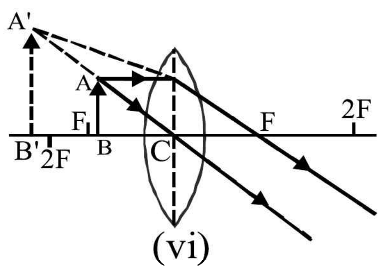


Image at infinity Real, inverted and very much magnified

Object between F and C



Beyond 2F and behind the object, Virtual, erect and ımagnified

As the object approaches the lens, it is seen that the real image moves away from it.

(i) When the object is very far away, the image is just to the right of the focal plane. The image is real, inverted, and smaller in size than the object ($m < 1$).

(ii) As the object approaches the lens, the image moves away to the right of the focal plane getting larger and larger. The image is real and inverted. This is the configuration for cameras and eyeballs.

(iii) When the object is at 2F, the image is real, inverted and of the same size as the object ($m = 1$). This is the configuration of a photocopier.

(iv) When the object is in between 2F and F, the image is enlarged ($m > 1$), real and inverted. This configuration corresponds to the film projector.

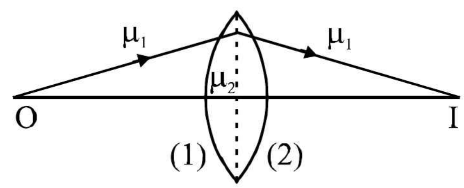
(v) When the object is precisely at F, there is no image as the emerging rays are parallel and in effect the image is at infinity.

(vi) With the object is between F and lens the image reappears. It is virtual, erect and enlarged (m > 1). This is the configuration of the magnifying glass.

It may be noted that as an image-forming device a convex lens is similar to a concave mirror. On the other hand, a concave lens resembles a convex mirror.

4.6 LENS MAKER'S FORMULA

Consider the thin lens shown here with the two refracting surfaces having radii of curvature equal to R_1 and R_2 respectively.



The refractive indices of the surrounding medium and of the material of the lens are μ_1 and μ_2 respectively. Now using the result that we obtained for refraction at single spherical surface, we get

For first surface, $\dfrac{\mu_2}{v_1} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$ .... (1)

For second surface, $\dfrac{\mu_1}{v} - \dfrac{\mu_2}{v_1} = \dfrac{\mu_1 - \mu_2}{R_2}$ .... (2)

Adding (1) and (2), $\dfrac{\mu_1}{v} - \dfrac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right] \Rightarrow \dfrac{1}{v} - \dfrac{1}{u} = \left(\dfrac{\mu_2}{\mu_1} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

When , $u = \infty$, $v = f$, (By definition) $\therefore \dfrac{1}{f} = \left(\dfrac{\mu_2}{\mu_1} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$ Also, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Note : A lens is converging if its focal length is positive and diverging if focal length is negative (in Cartesian sign convention). From this we can conclude that a convex lens need not necessarily be a converging and a concave lens diverging

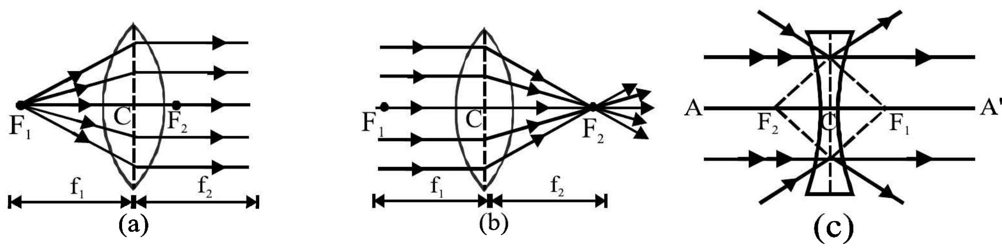
Limitations of the lens maker's formula :

1. The lens should be thin so that the separation between the two refracting surface should be small.
2. The medium on either side of the lens should be same. If any of the limitation is violated then we have to use the refraction at the curved surface formula for both the surfaces.

4.7 POSITION OF THE PRINCIPAL FOCI

Each individual surface of the lens has its own focal points and planes and the lens as a whole has its own pair of focal points and focal planes. The focal points and focal plane of the lens are known as principal focal points and principal focal plane. We are interested in knowing the locations of these principal focal points and principal focal planes. They are obtained as follows.

(i) If a point object is placed on the principal axis such that the rays refracted by the lens are parallel to the axis, then the position of the point object is called the first principle focus F_1 of the lens. The distance at the first principal focus from the optical centre C of the lens is called first principal focal length, f_1. We can find f_1 as follows.



Using $u = f_1$, and $v = \infty$ into equation, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ we get

$$\frac{1}{\infty} - \left(\frac{1}{f_1}\right) = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{or} \qquad \frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{..................} (1)$$

The plane perpendicular to the axis and passing through the first focal point is known as the **first principal focal plane**.

(ii) If the object is situated at infinity, the position of the image on the axis is known as the **second principal focus F_2**. The distance of the second principal focus from the optical centre C is called the second principal focal length, f_2, using $u = \infty$ and $v = f_2$ in eq.

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ we get}$$

$$\frac{1}{f_2} - \frac{1}{\infty} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]; \qquad \frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{..................} (2)$$

The plane perpendicular to the axis and passing through the second focal point is known as the **second principal focal plane**. It follows from eq. (1) and (2) that

$$f_1 = f_2 \qquad \text{........} (3)$$

Thus, every thin lens in air has two focal points (F_1 and F_2), one on each side of the lens and equidistant from the centre.

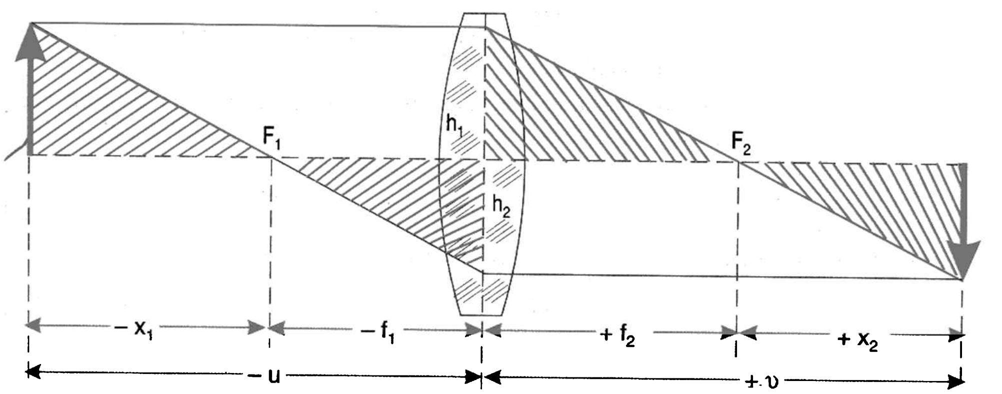
It will be seen that the second focal length (f_2) of a converging lens is positive and the first (f_1) negative, while for a diverging lens the reverse is true. The two focal lengths of thin lens in air are numerically equal. Frequently, one refers simply to **'focal length'** of a lens ; it will be assumed that this always refers to the **second focal length** so that the focal length of a converging lens is positive, while for a diverging lens it is negative.

Note : For lenses having a large diameter or aperture, the image of a point is not a point. Incident rays that are parallel to the principal axis intersect at different points, depending on their distance from that axis. The refracted rays intersect over a conical surface called the refraction caustic.

4.8 NEWTON'S LENS EQUATION

Let us obtain a lens equation for the practical applications. It relates the focal length to the object and image distances. This is also known as lens user's formula or **Newton's lens equation**.

Let h_1 be the height of the object and h_2 be the image height. From the similar triangles to the left of the lens we find that $\frac{h_1}{x_1} = \frac{h_2}{f_1}$



To the right of the lens we have $\frac{h_1}{f_2} = \frac{h_2}{x_2}$

Rearranging both equations and combining them by eliminating h_1/h_2 gives $\frac{x_1}{f_1} = \frac{f_2}{x_2}$

or $\qquad x_1x_2 = f_1f_2$

This is known as **Newton's lens equation**. The terms x_1 and x_2 are extra focal object and image distance respectively. They are measured from the foci rather than from the lens. Consequently, **Newton's equation** can be used with both 'thin' and 'thick' lenses.

When the medium is the same on both sides of the lens, the above equation reduces to

$$x_1 x_2 = f^2$$

Note : In the Gaussian formula the object and image distances are measured from the lens centre, while in the Newton's formula they are measured from focal points.

4.9 MAGNIFICATION

Lenses are used to form a magnified image of an object. Magnification is defined as

$$\text{Magnification, } m = \frac{\text{size of the image}}{\text{size of the object}}$$

The image of a three-dimensional object will occupy a three-dimensional region of space. The lens can affect the transverse, longitudinal and angular dimensions of the image. Since the image and object relationship is expressible in terms of axial, lateral and angular distances, we distinguish three types of magnification, namely lateral magnification, longitudinal magnification and angular magnification.

4.9(A) LATERAL MAGNIFICATION

Lateral or **transverse magnification** of a lens is defined as the ratio of the length of the image to the length of the object size, both the lengths being measured perpendicular to the principal axis. Thus, $m = \frac{h_2}{h_1}$

According to sign convention, the distances above the principal axis of the lens are taken positive and those below the axis are negative. Hence, the lateral magnification is positive for an erect image and negative for an inverted image. If the refractive indices on either side of the lens are the same, then $\frac{h_1}{u} = \frac{h_2}{v}$ $\quad \therefore \quad m = \frac{v}{u}$

The lateral magnification corresponding to Newton's formula may be written as

$$m = \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2}$$

4.9(B) LONGITUDINAL MAGNIFICATION

The longitudinal magnification is defined as the ratio of an infinitesimal axis length in the region of the image to the corresponding length in the region of the object. Thus, if dv and du are the extensions of the object and the image along the principal axis respectively, then the longitudinal magnification of the lens is given by, $m_L = \frac{dv}{du}$

Differentiating lens eq., i.e. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get,

$$-\frac{1}{v^2}\frac{dv}{du}+\frac{1}{u^2}=0 \Rightarrow \frac{dv}{du}=\frac{v^2}{u^2}=m^2 \quad ; \quad \text{Then } m_L=\frac{dv}{du}=m^2$$

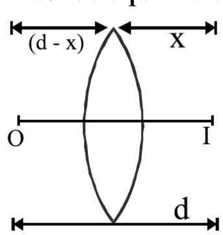
4.9(C) ANGULAR MAGNIFICATION

Angular magnification is defined as the ratio of slopes of emergent ray and conjugate incident ray with the principal axis. If the incident ray and the conjugate emergent ray make angles θ_1 and θ_2 with the principal axis respectively, then the angular magnification of the lens is given by

$$\gamma=\frac{\tan\theta_2}{\tan\theta_1}$$

4.10 SMALLEST SEPARATION OF OBJECT AND REAL IMAGE

We find that it is not always possible for a convex lens to form a real image of the object on a screen, although the object and the screen may both be at a greater distance from the lens than its focal length. A convex lens forms a real or a virtual image depending on the distance of the object from the lens. We now show that the distance between an object and a screen must be equal to or greater than '4f' if a real image is to be formed with a convex lens.



Suppose I is the real image of a point object O in a converging lens. If the image distance is x, and the distance OI = d, the object distance [d – x]. Thus, v = x and u = (d – x). Substituting these in the lens equation, we get

$$\frac{1}{x}+\frac{1}{d-x}=\frac{1}{f} \qquad \therefore \qquad \frac{d}{x(d-x)}=\frac{1}{f} \qquad \therefore \qquad x^2-xd+fd=0$$

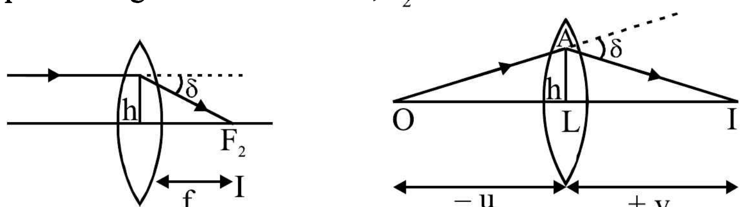
If the image is to be real, the roots of this quadratic equation for x must be real.

$\therefore\ d^2-4fd>0 \qquad \text{or} \qquad d>4f.$

Hence, the minimum distance between an object and its real image for a convex lens must be 4f where f is the focal length of the lens.

4.11 DEVIATION BY A THIN LENS

A lens may be considered to be made up of a large number of prisms placed one above the other. As the action of the lens is to **deviate** the incident rays of light, it is necessary to find the deviation produced by a particular section of the lens. Let a ray of monochromatic light parallel to the principal axis be incident on a thin lens at a height h above the axis and let f be the focal length of the lens. As the ray is parallel to the principal axis, after refraction it will pass through the second focus, F_2.



(a) (b)

The deviation suffered by the ray is given by, $\tan\delta = \dfrac{h}{f}$

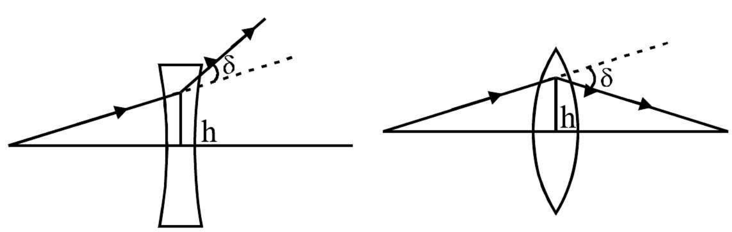
In the paraxial region δ being small, $\tan\delta \approx \delta \quad \therefore \quad \delta = \dfrac{h}{f}$

Next, consider a luminous point object O and its corresponding image I. The deviation suffered by the ray OA incident at A is given by

$$\delta = \angle AOL + \angle AIL \Rightarrow \delta = \frac{h}{-u} + \frac{h}{+v} = h\left[\frac{1}{v} - \frac{1}{u}\right] = +h\left[\frac{1}{f}\right] \quad \therefore \quad \delta = \frac{h}{f}$$

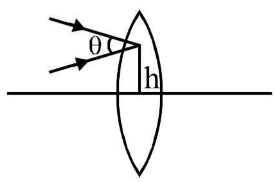
This show that the deviation produced by a lens is independent of the position of the object. Equation is valid for all rays incident at the same point A of a lens, whatever may be the direction of incidence. All the rays suffer the same deviation in refraction through the lens.

- **For the positive focal length (convex lens) the sense of deviation will be clockwise.**
- **For the negative focal length (concave lens) the sense of deviation will be anticlockwise.**



$$d = \frac{h}{f}$$

Example: Find the angle between two emergent rays for convex lens shown in figure.



Ans. θ

4.12 POWER

The **power** of a lens is the measure of its ability of produce convergence of a parallel beam of light. A convex lens of larger focal length produces a smaller converging effect and a convex lens of smaller focal length produces a larger converging effect. Due to this reason, the power of a convex lens is taken as positive and a convex lens of smaller focal length has higher power. On the other hand, a concave lens produces divergence. Therefore, its power is taken as negative.

The unit in which the power of a lens is measured is called a **diopter** (D). A convex lens of focal length one meter has a power = + 1 **diopter** and a convex lens of focal length 2 m has a power = + 0.5 **diopter**.

Mathematically, $$\text{power} = \frac{1}{\text{Focal length in metres}}$$

The concept power is useful because it allows us to work out the effective focal length of a combination of lenses very easily. The power of a pair of lenses, of focal lengths f_1 and f_2 placed in contact is simply the sum of their individual powers.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \therefore \qquad P = P_1 + P_2$$

Where P_1 and P_2 are the powers of the two lenses and P is the equivalent power.

- When surrounding medium of lens is air then, $P = \frac{1}{f_{air}}$
- When surrounding medium of lens is water then, $P = \frac{\mu_w}{f_w}$

  Where, μ_w = Refractive index of water f_w = focal length of lens in water

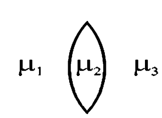


Fig (1)

- When medium of lens are different on both sides as shown in figure

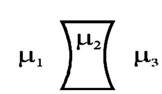


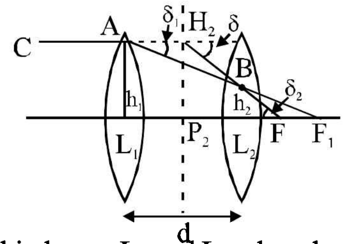
Fig. (2)

Power for fig.(1) $P = \left(\frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{-R_2}\right)$

Then, Power for Fig.(2) $P = \left(\frac{\mu_2 - \mu_1}{-R_1} + \frac{\mu_3 - \mu_2}{R_2}\right)$

4.13 EQUIVALENT FOCAL LENGTH OF TWO THIN LENSES

When two thin lenses are arranged coaxially, the image formed by the first lens system becomes the object for a second lens system and the two systems act as a single optical system forming the final image from the original object.



Let us consider a simple optical system that consists of two thin lenses L_1 and L_2, placed on a common axis and separated by a distance d, as shown in figure. The lenses are separated by a distance and have focal lengths f_1 and f_2. We are interested to know how the combination works. We find that two lenses, separated by a finite distance, can be replaced by a single thin lens called on **equivalent lens**. The equivalent lens, when placed at a suitable fixed point, will produce an image of the same size as that produced by the combination of the two lenses. The focal length of equivalent lens is called **equivalent focal length**. We now derive an expression for the equivalent focal length, f, of the combination of two lenses.

Let a ray CA of monochromatic light parallel to the principal axis be incident on the first lens L_1 at a height h_1 above the axis. The ray CA is deviated through an angle δ_1 by the lens L_1. The incident ray CA, after refraction, is directed toward F_1, which is the **second principal focus** of lens L_1. The deviation produced by the first lens is given by $\delta_1 = \frac{h_2}{f_1}$

The emergent ray, AB, from the first lens is incident on the lens L_2 at a height h_2. On refracting, the ray is deviated through an angle δ_2 by the lens L_2 and meets the principal axis at F. Since the incident ray CA is parallel to the principal axis and after refraction through the optical system meets the axis at F. F must be the **second principal focus** of the combined lens system. The deviation produced by the second lens is given by $\delta_2 = \frac{h_2}{f_2}$

If the incident ray CA is extended forward and the final emergent ray BF backward, they meet at a point H_2. It is clear that a single thin lens placed at P_2 will produce the same deviation as that produced by the two lenses put together. The lens of focal length P_2F placed at P_2 is termed as the **equivalent lens**, which can replace the two lenses $L_1$and L_2.

The deviation produced by the equivalent lens is $\delta = \frac{h_1}{f}$

4.13(A) FOCAL LENGTH OF THE EQUIVALENT LENS

Deviation produced by the first lens L_1 is $\delta_1 = h_1/f_1$

Deviation produced by the second lens L_2 is $\delta_2 = h_2/f_2$

But $\quad \delta = \delta_1 + \delta_2 \qquad \therefore \qquad \dfrac{h_1}{f} = \dfrac{h_1}{f_1} + \dfrac{h_2}{f_2}$ .... (1)

The Δ^{les} AL_1F_1 and BL_2F_1 are similar. $\quad \therefore \quad \dfrac{AL_1}{L_1F_1} = \dfrac{AL_2}{L_2F_1}$

or $\quad \dfrac{h_1}{f_1} = \dfrac{h_2}{f_1 - d}$ .... (2) $\qquad$ or $\qquad h_2 = \dfrac{h_1(f_1 - d)}{f_1}$ .... (3)

using equation (1) into equ. (3), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2} \quad \Rightarrow \quad \frac{1}{f} = \frac{1}{f_1} + \frac{f_1 - d}{f_1 f_2} \quad \Rightarrow \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Therefore, the equivalent focal length is given by, $f = \dfrac{f_1 f_2}{f_1 + f_2 - d}$ ...... (4)

or $\quad f = \dfrac{-f_1 f_2}{\Delta}$

where $\Delta = d - (f_1 + f_2)$ and is known as the **optical interval** between the two lenses. It is numerically equal to the distance between the second principal focus of the first lens and the first principal focus of the second lens.

From figure, it may be seen that the rays converge at F and appear to have come from a plane EP_2. Plane EP_2 is therefore, the position of a single lens, the effect of which is the same that of the two lenses, L_1 and L_2. The distance L_2F is the **equivalent focal length**. The equivalent focal length is independent of the direction from which the light enters the system. The behavior of the system cannot be reproduced in all respects by the equivalent thin lens placed in a fixed position. It will have to be placed in the position H_2P_2 when light enters from left and in position H_1P_1 when light enters from the right.

Note :

1. Equation (4) shows that if the distance d between the two convex lenses exceeds the sum of their focal lengths $(f_1 + f_2)$, the system becomes divergent, because of negative focal length.

2. If the medium between the two convex lenses is other than air, then the equation (4) from equivalent focal length would become

$$f = \frac{f_1 f_2}{f_1 + f_2 - d/\mu}$$ where μ is the refractive index of the medium.

This can be written as : $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{\mu f_1 f_2}$

4.13(B) DISTANCE OF EQUIVALENT LENS FROM L_2

Let us say the plane EP_2 is located at a distance of L_2P_2 from the second lens L_2. Now consider the similar Δ^{les} EP_2F and BL_2F. $\dfrac{P_2F}{L_2F} = \dfrac{EP_2}{BL_2}$

From the figure, it is seen that $P_2F = f$, $L_2F = f - L_2P_2$ and $EP_2 = AL_1 = h_1$

$$\frac{h_1}{h_2} = \frac{f}{f - L_2P_2} \qquad \text{............... (5)}$$

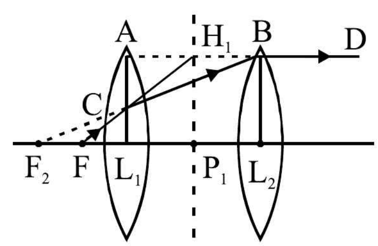
Comparing equations (4) and (2), we obtain

$$\frac{f}{f - L_2P_2} = \frac{f_1}{f_1 - d} \quad \Rightarrow \quad f f_1 - fd = f f_1 + (L_2P_2)\, f_1 \quad \text{or} \quad L_2P_2 = \frac{fd}{f_1} \qquad \text{............. (6)}$$

4.13(C) DISTANCE OF EQUIVALENT LENS FROM L_1

The distance of equivalent lens from L_1 is given by

$$L_1P_2 = (d - L_2P_2) = d - \frac{fd}{f_1} = d\left(1 - \frac{f}{f_1}\right) \qquad \text{........ (7)}$$



If we consider a parallel beam of light to be incident from right and falls first on lens L_2, the equivalent lens moves to the left, as shown in figure.

The equivalent lens is now at a distance of $L_1P_1 = \dfrac{fd}{f_2}$ .... (8)

from the lens L_1. These two positions of the equivalent lens are called the **principal planes**.

4.13(D) POWER

When two thin lenses of focal lengths f_1 and f_2 are placed coaxial and separated by a distance d, the equivalent focal length is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \qquad \therefore \qquad P = P_1 + P_2 - d \, . \, P_1 P_2$$

If the medium between the two lenses is other than air, then the above equation for equivalent power would become

$$P = P_1 + P_2 - \frac{dP_1P_2}{\mu}$$ where μ is the refractive index of the medium.

4.14 CONCEPT OF (μ-V) GRAPH FOR THE LENS

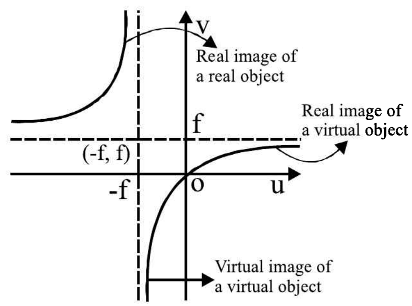
(a) For the convex lens

Apply lens formula i.e $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{y}-\frac{1}{x}=c \qquad \left(\text{Since } f>0,\ c=\frac{1}{|f|}\right)$$

$$\Rightarrow y=\frac{x}{cx+1} \Rightarrow (cy-1)(cx+1)=-1$$

Then graph will be plotted as :

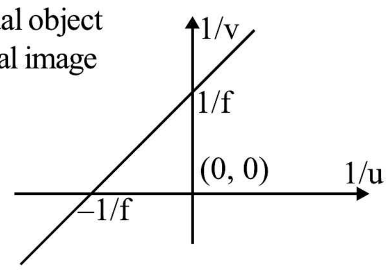


For Convex lens

While interpreting these graphs remember following points :
(i) u is negative for real object and u is positive for virtual object
(ii) v is positive for real image and u is negative for virtual image

$$y - x = +c \ \left(\because f>0,\ c=\frac{1}{|f|}\right) \quad ; \quad \frac{x}{-c}+\frac{y}{+c}=1$$

Then graph will be plotted as :

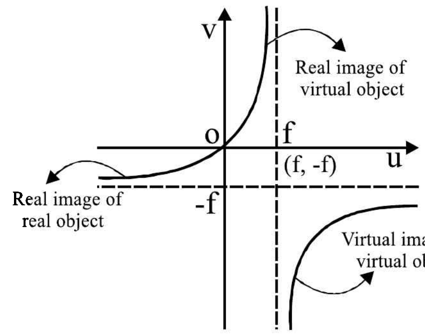


(b) For concave lens : Apply lens formula i.e. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \qquad \frac{1}{y} - \frac{1}{x} = -c \qquad (\because f < 0,\ c = \frac{1}{|f|})$$

$$\Rightarrow \qquad y = \frac{x}{1 - cx} \ \Rightarrow \ (cx - 1)(y + 1) = -1$$

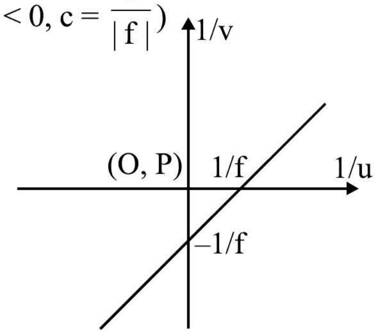
Then graph will be plotted as :



For Concave lens

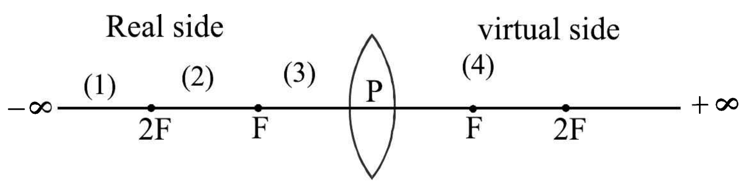
$$y - x = -c \quad (\because f < 0,\ c = \frac{1}{|f|})$$

$$\frac{x}{c} + \frac{y}{-c} = 1$$



4.15 CONCEPT OF POSITION OF IMAGE FOR GIVEN OBJECT

(A) For Convex Lens



- **Let us define the region for object in the convex lens**

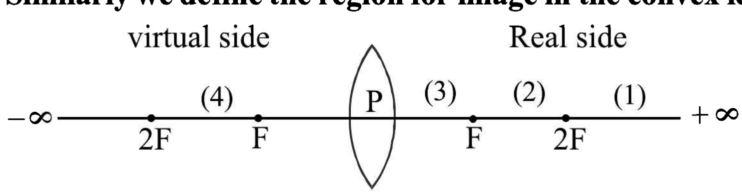
from $\quad -\infty$ to 2F as region .... (1)

2F to F as region .... (2)

F to P as region .... (3)

P to $+\infty$ as region .... (4)

- **Similarly we define the region for image in the convex lens**



from $\quad -\infty$ to P as region .... (4)

P to F as region .... (3)

F to 2F as region ....(2)

2F to $+\infty$ as region .... (1)

- Any object in region (1) will have its image in region (2) and vice-versa.
- Similarly, any object in region (3) will have its image in region (4) and vice-versa.

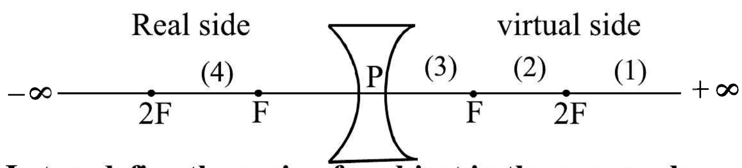
	Position of Object	Position of image	Magnification
a	Region (1) (Real side) Real object	Region (2) (Real side) Real image, inverted $m < 0$	Region of image (2) is smaller as compared to region of object (1) (diminished image)
b	Region (2) (Real side) Real object	Region (1) (Real side) Real image, inverted $m < 0$	Region of image (1) is larger as compared to region of object (2) (enlarged image)
c	Region (3) (Real side) Real object	Region (4) (Virtual side) virtual image, erect image $m > 0$	Region of image (4) is larger as compared to region of object (3) (enlarged image)
d	Region (4) (Virtual side) Virtual object	Region (3) (Real side) Real image, erect image $m > 0$	Region of images (3) is smaller as compared to region of object (4) (diminished image)

$\rightarrow$ For region (1), (2) and (3) u is always (–)ve and for region (4) it is (+)ve.
$\rightarrow$ For region (1), (2) and (3) v is always (+)ve and for region (4) it is (–)ve.

- Formula of magnification is $m = \dfrac{v}{u}$

(a) For same sign of u and v, positive magnification ($m > 0$) implies eract image.
(b) For opposite sign of u and v, negative magnification ($m < 0$) implies inverted image.
$\rightarrow$ In convex lens for virtual object, nature of image always real formed.

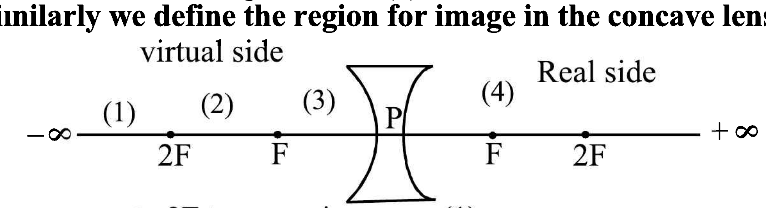
(B) Concept of position of image for given object in concave lens :
For Concave Lens



- **Let us define the region for object in the concave lens**

from 2F to $+\infty$ as region .... (1)
F to 2F as region .... (2)
P to F as region .... (3)
$-\infty$ to P as region .... (4)

Similarly we define the region for image in the concave lens



from $-\infty$ to 2F to as region .... (1)
2F to F as region .... (2)
F to P as region .... (3)
P to $+\infty$ as region .... (4)

- Any object in the region (1) will have its image in region (2) and vice versa.
- Any object in the region (3) will have its image in region (4) and vice versa.

	Position of Object	Position of image	Magnification
a	Region (1) (Virtual side) Virtual object	Region (2) (Virtual side) Virtual image, inverted $m < 0$	Region of image (2) is smaller as compared to region of object (1) (diminished image)
b	Region (2) (Virtual side) Virtual object	Region (1) (Virtual side) Virtual image, inverted $m < 0$	Region of image (1) is larger as compared to region of object (2) (enlarged image)
c	Region (3) (Virtual side) Virtual object	Region (4) (Real side) Real image, erect image $m > 0$	Region of image (4) is larger as compared to region of object (3) (enlarged image)
d	Region (4) (Real side) Real object	Region (3) (Virtual side) Virtual image, erect image $m > 0$	Region of images (3) is smaller as compared to region of object (4) (diminished image)

→ **In concave lens real object always formed virtual image.**

4.16 VELOCITY OF IMAGE OF MOVING OBJECT (IN LENS)

→ There are two components of velocity of image

(a) Velocity component along the principal axis that means perpendicular to the lens.

(b) Velocity component perpendicular to the principal axis that means parallel to the lens.

(a) Velocity component along the Principal (Optical) axis :

Differentiating mirror formula on the both side w.r.t. time we get, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

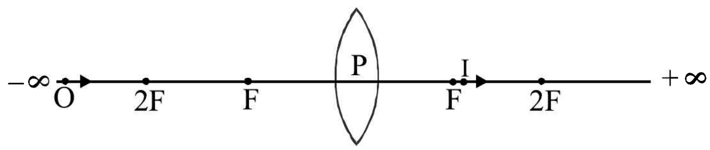
$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0 \quad \text{(Since focal length of the lens remains constant)}$$

$$\Rightarrow \frac{dv}{dt} = +\frac{v^2}{u^2}\frac{du}{dt} \qquad \Rightarrow \qquad (v_{IL})_{\parallel} = +\frac{v^2}{u^2}(v_{OL})_{\parallel}$$

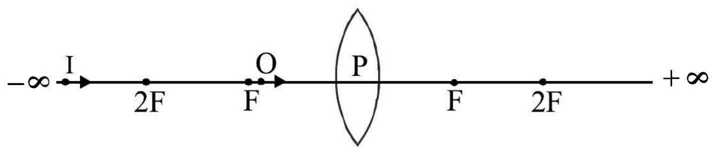
$(v_{IL})_{\parallel}$ = Velocity of image w.r.t. lens along the principal axis.

$(v_{OL})_{\perp}$ = Velocity of object w.r.t. lens along the principal axis.

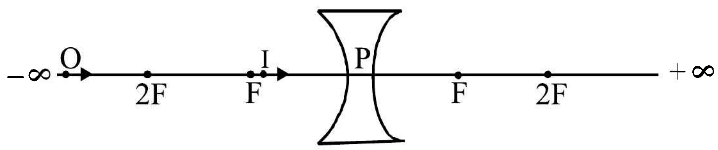
- Direction of velocity of object and image w.r.t. lens is same to each other.
- Any object in the region (1) will have its image in region (2) and vice versa
- Similarly for region (3) and (4).
- When object is moving from $-\infty$ to 2F, the image is coming from F to 2F as shown in the figure.



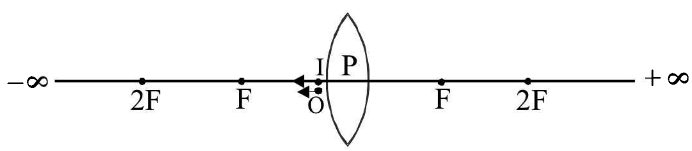
• When the object shifts from F to P. The image formed, moves from (– ∞) to P, as shown in figure.



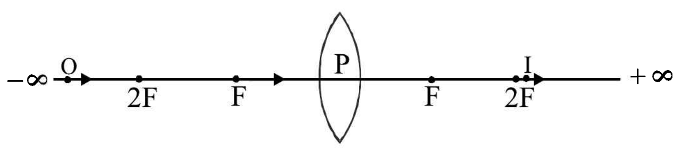
• When the object shifts from (– ∞) to P, the image moves from F to P, as shown in figure.



• When the object shifts from P to F, the image moves from P to (– ∞), as shown in figure.



• When the object shifts from 2F to F, the image moves from 2F to (+ ∞), as shown in figure.



(b) Velocity component perpendicular to axis : $m = \dfrac{h_i}{h_o} \Rightarrow h_i = mh_o$

Differentiating this equation on the both side w.r.t. time, we gets

$$\frac{dh_i}{dt} = \frac{mdh_o}{dt} + h_o\frac{dm}{dt} \quad ; \quad (V_{IL})_\perp = m\,(V_{OL})_\perp + h_o\frac{dm}{dt}$$

$(V_{IL})_\perp$ = Velocity of image w.r.t lens perpendicular to the principal axis.

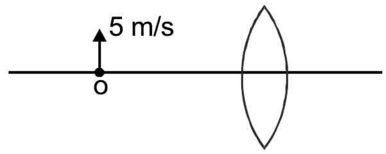
$(V_{OL})_\perp$ = Velocity of object w.r.t lens perpendicular to the principal axis.

$\dfrac{dm}{dt}$ = Rate of change of magnification unit (per sec) will be

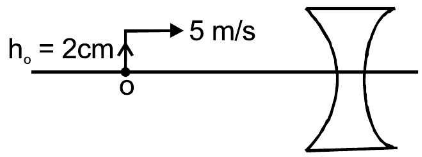
$$m = +\frac{v}{u}$$

By differentiating this equation on the both sides w.r.t. time, we gets

$$\frac{dm}{dt} = \left[\frac{u(V_{IL})_{\parallel} - v(V_{OL})_{\parallel}}{u^2}\right]$$

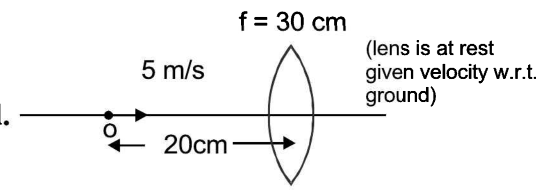
Type -1 : 

In this type, $h_o = 0$ but $\frac{dh_o}{dt} = 5$ m/s

Type -2 : 

In this type, $h_o = 2$ cm but $\frac{dh_o}{dt} = 0$

Ex.1 Find the velocity of image w.r.t. ground.



Sol. For the velocity component along the principal axis $(V_{IL})_{\parallel} = +\frac{v^2}{u^2}(V_{OL})_{\parallel}$

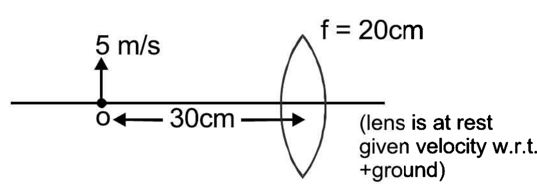
Apply lens equation : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-20} = \frac{1}{+30} \Rightarrow v = -60$ cm

$m = +\frac{v}{u} = \frac{-60}{-20} = 3$; $(V_{IM})_{\parallel} = +\left(\frac{-60}{-20}\right)^2(5) = +45$ m/s ; $\vec{V_{IG}} = \vec{V_{IL}} + \vec{V_{LG}}$

$= +45 + 0 = +45$ m/s (Since lens is at rest given that)

Hence, image approaches the lens with a velocity 45 m/s.

Ex.2 Find the velocity of image w.r.t. ground



Sol. Apply lens equation : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} - \frac{1}{-30} = \frac{1}{+20} \Rightarrow v = +60$ cm

For the velocity component along the principal axis :

$$(V_{IL})_{\parallel} = +\frac{v^2}{u^2}(V_{OM})_{\parallel} \quad ; \qquad (V_{IL})_{\parallel} = 0 \qquad [\text{Since } (V_{OL})_{\parallel} = 0]$$

For the velocity component perpendicular to the principal axis :

$$(V_{IL})_{\perp} = m(V_{OL})_{\perp} + h_o \frac{dm}{dt}$$

$$(V_{IL})_{\perp} = (-2)\,(5) = -10 \text{ m/s} \quad [\text{Since, } h_0 = 0] \;\; \left[m = \frac{v}{u} = \frac{+60}{-30} = -2\right]$$

$$\overrightarrow{V_{IG}} = \overrightarrow{V_{IL}} + \overrightarrow{V_{LG}} = -10 + 0 = -10 \text{ m/s} = 10 \text{ m/s (moving downwards)}$$

4.17 HOW TO LOCATE

[When position of object and image are given w.r.t. principal axis of the lens.]

(i) Position of lens (ii) Position of Focus (iii) Nature of lens

(i) For Position of lens : Join object and image through a line, the point P where line OI cuts the principal axis (optical axis), is the pole of lens. (Position of lens)

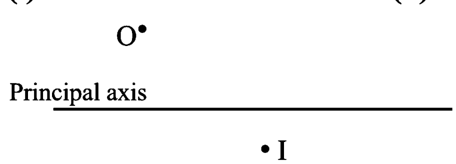
(ii) For Position of Focus : Step 1 : Draw a parallel line from object w.r.t principal axis to the lens it cuts at point 'x' to the lens.

Step 2 : Extend the line XI, let it cut the principal axis at point F. F is the focal point of lens.

(iii) Nature of lens : If the ray is converging, the lens is convex and if the ray is diverging, the lens is concave.

Ex.3 The position of real object O and image I w.r.t. the principal axis is shown. Then locate the

(i) Position of lens (ii) Position of Focus (iii) Nature of lens



Sol. (1) Join object and image with a line, that intersects at pole of lens.

(2) Draw a line parallel to the position of lens that cuts at point x to the lens.

(3) Extend the line XI, and let it cut principal axis at point F. F is the focal point of lens.

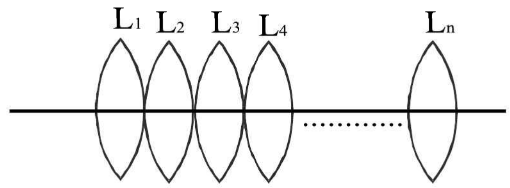
(4) Since this ray is converging, hence the lens is convex.

4.18 EQUIVALENT FOCAL LENGTH

Equivalent focal length of combination of thin lenses when placed adjacent to each other in homogenous medium. Consider a system of 'n' thin lenses $L_1, L_2, L_3, ...L_n$ having focal length $f_1, f_2, f_3,....... f_n$ respectively, placed adjacent to each other.

The equivalent focal length is given by

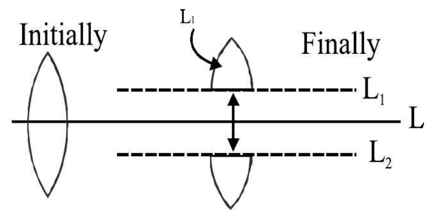
$$\frac{1}{F_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots\ldots\ldots \frac{1}{f_n}$$



4.19 CUTTING OF LENS

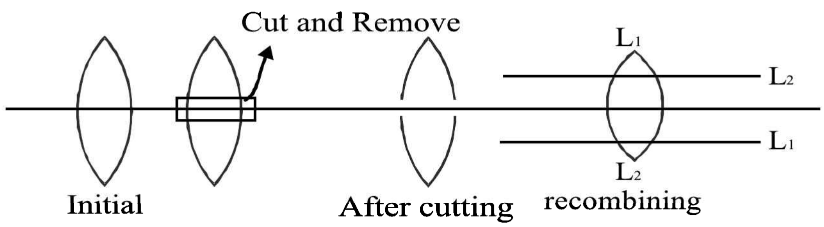
Types of cutting
(i) Bisplit Cutting
(ii) Billetsplit Cutting

(i) **Bisplit Cutting :-** This type of cutting involves the seperation of lens in two halves through a plane parallel to the principal axis and dividing it into two parts. After cutting, the lenses are moved apart perpendicular to principal axis.



Initially, line L is the principal axis but now for part L_1, L_1 behaves as the principal axis similarly for L_2.

(ii) **Billetsplit Cutting :-** This type of cutting involves cutting and removing a part of lens from middle and then recombining the two internal surface.

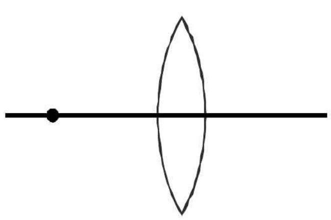


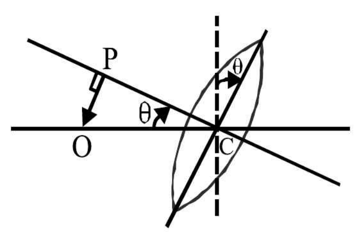
Here L_1 is P.A. of L_1 and similarly for L_2

Note. **All calculations here also must be done considering the new P.A.**

4.20 ROTATION OF LENSES

When a thin lens is rotated in a uniform medium by an angle θ, the P.A. also rotates by an angle θ in the same direction.





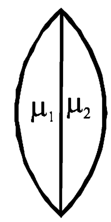
Now, CP is the new object distance ; OP is the object height

Note. All calculations to be performed now according to the new system of principal axis.

4.21 LENSES OF UNIFORM OPTICAL DENSITY

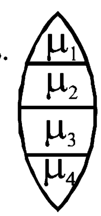
(multiple focal lengths)
Variation of optical density can be

(i) Along the principal axis
(ii) Perpendicular to the principal axis

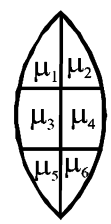


(i) **Along principal axis :-** Only the focal length affects the image formation and only one image is formed.
Only one focal length exist hence number of image is one

(ii) **Perpendicular to principal axis :-** Due to different R.I. there exist multiple principal foci and hence multiple images are formed.
Here four images are formed due to four different foci that exists.



Here three images are formed due to different refractive indices.

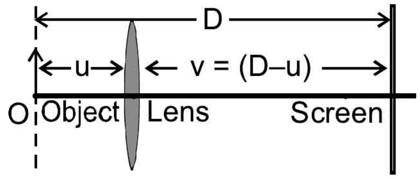


4.22 DISPLACEMENT OF LENS WHEN OBJECT AND SCREEN ARE FIXED

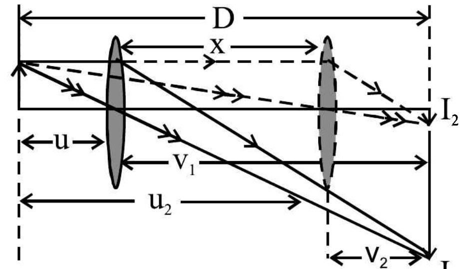
This method is called **'Displacement method'** and is used in laboratory to determine the focal length of convergent lens. In this case, if the object is at a distance u from the lens, the distance of image from the lens v = (D–u). So from lens-formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we have $\frac{1}{(D-u)} - \frac{1}{-u} = \frac{1}{f}$ i.e. $u^2 - Du + Df = 0$

So that $u = \frac{1}{2}[D \pm \sqrt{D(D-4f)}]$ .... (1)



(A)



(B)

Now, there are three possibilities :

(a) If D <4f ; u will be imaginary, so physically no position of lens is possible.

(b) If D - 4f: In this situation u = D/2 = 2f. So only one position is possible and in this situation, v = D – u = 4f– 2f =2f) (= u)

(c) If D > 4f ; in this situation both the roots of Eq. (1) will be real, i.e.,

$$u_1 = \frac{1}{2}\left[D - \sqrt{D(D-4f)}\right] \quad \text{and} \quad u_2 = \frac{1}{2}\left[D + \sqrt{D(D-4f)}\right] \qquad \text{..........(2)}$$

So if D > 4f: there are two positions of lens at distances u_1 and u_2 from the object for which real image is formed on the screen.

(i) If the distances between two positions of the lens is x

$$x = u_2 - u_1 = \sqrt{D(D-4f)} \;\; [\text{From Eq. 2}] \;\; \text{i.e.} \;\; x^2 = D^2 - 4Df \text{ so } f = \frac{(D^2 - x^2)}{4D} \qquad \text{......(3)}$$

(ii) the image distances corresponding to two positions of lens will be

$$v_1 = D - u_1 = D - \frac{1}{2}\left[D - \sqrt{D(D-4f)}\right] = \frac{1}{2}\left[D + \sqrt{D(D-4f)}\right] = u_2$$

$$\text{and } v_2 = D - u_2 = D - \frac{1}{2}\left[D + \sqrt{D(D-4f)}\right] = \frac{1}{2}\left[D - \sqrt{D(D-4f)}\right] = u_1$$

i.e. for two positions of the lens object and image distances are interchangeable.

(iii) As $x = u_2 - u_1$ and $D = v_1 + u_1 = u_2 + u_1$ [as $v_1 = u_2$] i.e. $u_1 (= v_2) = (D - x)/2$ and $u_2 (= v_1) = (D + x)/2$

So the magnification for the two positions of the lens will be

$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \left[\frac{D+x}{D-x}\right] \quad \text{and} \quad m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \left[\frac{D-x}{D+x}\right] \qquad \text{........... (4)}$$

And hence :

(a) $m_1 \times m_2 = (I_1 I_2 / O^2) = 1$, i.e. $O = \sqrt{I_1 I_2}$ ....(5)

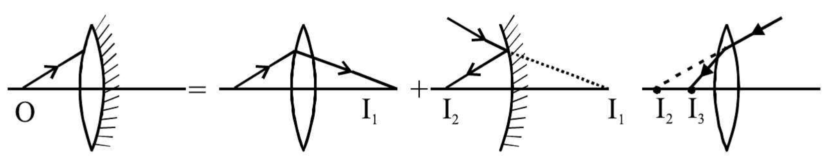
(b) $\dfrac{m_1}{m_2} = \dfrac{I_1}{I_2} = \left[\dfrac{D+x}{D-x}\right] \times \left[\dfrac{D+x}{D-x}\right] = \left[\dfrac{D+x}{D-x}\right]^2$ ....(6)

(c) $m_1 - m_2 = \left[\dfrac{D+x}{D-x}\right] - \left[\dfrac{D-x}{D+x}\right] = \dfrac{4Dx}{D^2 - x^2}$ which in the light of Eq. (3) yields

$$m_1 - m_2 = \frac{x}{f} \text{ i.e. } f = \frac{x}{m_1 - m_2} \quad ...(7)$$

4.23 LENS WITH ONE SILVERED SURFACE

If the back surface of a lens is silvered and an object is placed in front of it then :
(i) First, light will pass through the lens and it will form the image I_1



(ii) The image I_1 will act as an object for silvered surface which acts as curved mirror and forms an image I_2 of object I_1.
(iii) The light after reflection from silvered surface will again pass through the lens and lens will form the final image I_3 of object I_2. This is shown in the above figure.
In this situation :
Power of the silvered lens will be $P_e = P_L + P_M + P_L = 2P_L + P_M$

with, $P_L = \dfrac{1}{f_L}$ where, $\dfrac{1}{f_L} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

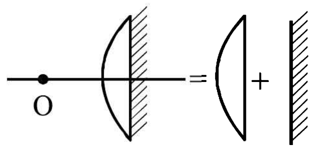
$P_M = -\dfrac{1}{f_M}$ where $f_M = \dfrac{R_2}{2}$ Use the sign convention in the formula of focal length.

So the system will behave as a curved mirror of focal length F_e given by, $F_e = -\dfrac{1}{P_e}$

To determine the final position of image use the mirror formula for equivalent focal length F_e i.e. $\dfrac{1}{v_f} + \dfrac{1}{u_i} = \dfrac{1}{F_e}$

Consider the following examples :

Ex.4 When the plane surface is silvered and the object is in front of the curved surface.



Sol. P_e = Equivalent power of silvered lens will be

$P_e = 2P_L + P_M$

$$P_L = \frac{1}{f_L} = \left(\frac{\mu-1}{1}\right)\left(\frac{1}{R}-\frac{1}{\infty}\right) = \left(\frac{\mu-1}{R}\right) \qquad \text{(By Lens Maker's formula)}$$

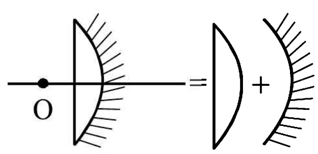
$$P_M = -\frac{1}{f_m} = -\frac{1}{\infty} = 0 \text{ [since } f_M = \frac{R}{2} = \frac{\infty}{2} = \infty \text{ (Plane mirror)]}$$

$$P_e = \frac{2(\mu-1)}{R} \qquad \text{So,} \qquad F_e = -\frac{1}{P_e} = -\frac{R}{2(\mu-1)}$$

Now, to find the final position of image use. $\frac{1}{v_f}+\frac{1}{u_i}=\frac{1}{F_e}$

Since, combination (silvered lens) behave as curved mirror, use the mirror formula to find v, the final position of image i.e. v

Ex.5 When the curved surface is silvered and the object is in front of the plane surface.



Sol. Then, P_e = Equivalent power of silvered lens will be $P_e = 2P_L + P_M$

$$P_L = \frac{1}{f_L} \qquad \text{where, } \frac{1}{f_L} = \left(\frac{\mu-1}{1}\right)\left(\frac{1}{\infty}-\frac{1}{-R}\right) \qquad \text{[by lens maker's formula]}$$

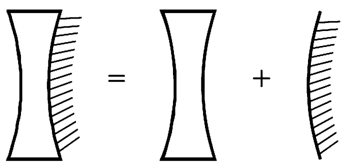
Then, $P_L = \left(\frac{\mu-1}{R}\right)$; $P_M = -\frac{1}{f_M}$ where, $f_M = -\frac{R}{2}$ (concave mirror). So, $P_M = +\frac{2}{R}$

And hence, equivalent power of silvered lens, $P_e = 2P_L + P_M = 2\left(\frac{\mu-1}{R}\right)+\frac{2}{R}=\frac{2\mu}{R}$

$F_e = -\dfrac{1}{P_e}$ Then $F_e = -\dfrac{R}{2\mu}$ (since, $P_e = \dfrac{2\mu}{R}$)

Now, to find the final position of the image, use $\dfrac{1}{v_f} + \dfrac{1}{u_i} = \dfrac{1}{F_e}$

Since, combination (silvered lens) behave as curved mirror, use the mirror formula to find the final position of the image i.e. v

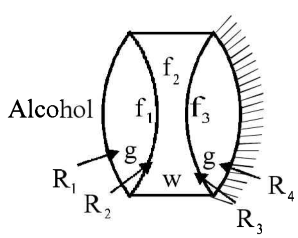
Ex.6 

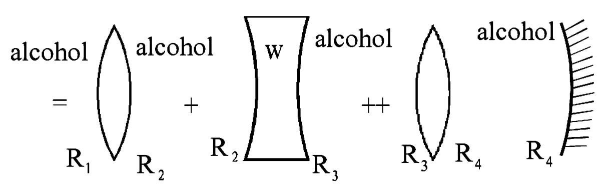
Sol. $P_e = 2P_L + P_M$; $P_L = \dfrac{1}{F_L}$ where $\dfrac{1}{F_L} = \left(\dfrac{\mu-1}{1}\right)\left(\dfrac{1}{-R} - \dfrac{1}{+R}\right)$ (By lens maker's formula)

$\dfrac{1}{F_L} = -\dfrac{2(\mu-1)}{R}$, Then $P_L = \dfrac{1}{F_L} = -\dfrac{2(\mu-1)}{R}$; $P_M = -\dfrac{1}{f_M}$ where $F_M = +\dfrac{R}{2}$ (convex mirror)

$\Rightarrow \quad P_M = -\dfrac{2}{R}$, Then, $P_e = 2P_L + P_M$, $P_e = 2\left[\dfrac{-2(\mu-1)}{R}\right] - \dfrac{2}{R}$

$P_e = \dfrac{-4(\mu-1)}{R} - \dfrac{2}{R}$; $P_e = \dfrac{-4\mu+4-2}{R} = \left(\dfrac{-4\mu+2}{R}\right)$; $P_e = \left(\dfrac{2-4\mu}{R}\right)$

Ex.7 



Sol. When the system in dipped in alcohol : $P_e = 2\,(Pg_1 + Pw_2 + Pg_3) + P_m$

$Pg_1 = \dfrac{\mu_{A\ell}}{f_2}$ where $\dfrac{1}{f_I} = \left(\dfrac{\mu_g - \mu_{A\ell}}{\mu_{A\ell}}\right)\left(\dfrac{1}{R_I} + \dfrac{1}{R_2}\right)$, Then, $P_{g_1} = (\mu_g - \mu_{A\ell})\left(\dfrac{1}{R_I} + \dfrac{1}{R_2}\right)$

$P_{\omega_2} = \dfrac{\mu_{A\ell}}{f_2}$ where $\dfrac{1}{f_2} = \left(\dfrac{\mu_g - \mu_{A\ell}}{\mu_{A\ell}}\right)\left(-\dfrac{1}{R_I} - \dfrac{1}{R_3}\right)$, Then, $P_{\omega_2} = (\mu_\omega - \mu_{A\ell})\left(-\dfrac{1}{R_I} - \dfrac{1}{R_3}\right)$

$$P_{g_3} = \frac{\mu_{A\ell}}{f_3} \text{ where } \frac{1}{f_3} = \left(\frac{\mu_g - \mu_{A\ell}}{\mu_{A\ell}}\right)\left(\frac{1}{R_3} + \frac{1}{R_4}\right), \text{Then} \quad P_{g_3} = (\mu_g - \mu_{A\ell})\left(\frac{1}{R_3} + \frac{1}{R_4}\right)$$

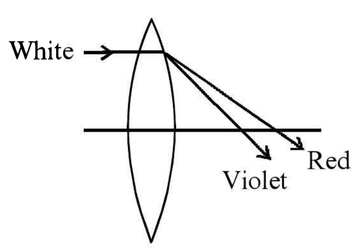
$$P_m = -\frac{1}{f_m} \text{ where } f_m = -\frac{R_4}{2} \quad \text{Then,} \quad P_m = +\frac{2}{R_4} \quad \text{(Since, the mirror is concave)}$$

$$\text{Then} \quad F_e = \frac{1}{P_e} \quad ; \quad F_e = \text{Equivalent focal length of the combined system.}$$

Use $\frac{1}{v_f} + \frac{1}{u_i} = \frac{1}{F_e}$ to determine the final position of the image.

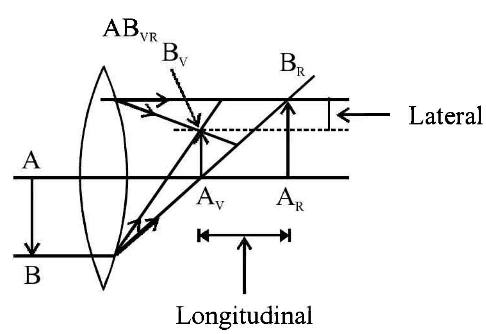
4.24 CHROMATIC ABERRATION

The transparent material of an optical system has different refractive indices for different colours (wavelengths) of the spectrum. Therefore, different colours are focussed at different points, producing series of images which results in blurred image. This aberration is called *chromatic aberration.* When polychromatic **(VIBGYOR)** light passes through a prism, violet is deviated more than red. This prismatic action is observed in a simple lens and it is more towards its edge. The result is that violet light is focussed nearest to the lens as shown in Fig.. The variation of focal length of a lens with wavelength produces axial or longitudinal chromatic aberration as well as lateral chromatic aberration (Fig). The horizontal distance between the extreme image points is called longitudinal chromatic aberration and the difference in heights of extreme colours (in this case violet and red) is called lateral chromatic aberration. For convergent lens, longitudinal chromatic aberration is positive and for divergent it is negative.



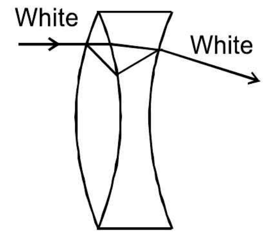
(a)

(Chromatic aberration)



(b)

(Lateral and longitudinal chromatic aberrations)



Achromatic doublet

4.24(A) Reduction or removal of chromatic aberration. A system free of chromatic aberration is called as achromatic. Generally, a doublet (combination of suitable convergent and divergent lens) is used to remove chromatic aberration (Fig.)

4.24(B) Axial or longitudinal chromatic aberration. For a thin lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where f is the focal length, μ is the refractive index of the material of lens, r_1 and r_2 are the radii of curvatures of lens. When the wavelength is changed, μ becomes $(\mu + \delta\mu)$. Therefore,

$$\delta\left(\frac{1}{f}\right) = \delta\mu\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{\delta\mu}{\mu - 1}\frac{1}{f} = \frac{\omega}{f}$$

where $\omega = \delta\mu/(\mu - 1)$ is the dispersive power of the material of the lens. Thus

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{\omega}{f}$$

where f_r, f_v and f are focal lengths for red, violet and yellow (or mean) colours. From this we can write

$$\frac{f_r - f_v}{f_v f_r} = \frac{\omega}{f} \qquad \text{with } f_v f_r \approx f^2 \text{ and } f_r - f_v = \omega f$$

Thus, axial chromatic aberration of a thin lens for parallel rays is the product of geometrical mean of extreme focal lengths and the dispersive power of the material of the lens.

4.24(C) Two lenses in contact. When two thin lenses with focal lengths f_1 and f_2 are in contact, the focal length of the combination is f, where,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{and} \quad \delta\left(\frac{1}{f}\right) = \delta\left(\frac{1}{f_1}\right) + \delta\left(\frac{1}{f_2}\right)$$

To remove the chromatic aberration, focal lengths of the extreme colours should be same, i.e $\delta(1/f)$ must be zero. Thus, the condition for achromatism is

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \text{ or } \frac{f_2}{f_1} = -\frac{\omega_2}{\omega_1}$$

4.24(D) Two lenses at a finite distance. Two thin lenses made up of same glass separated by a fixed distance can produce achromatic system. For convenience, we can write

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ as $P = (\mu - 1)\,k_1$, where P is the power of the lens.

When two lenses are separated by a distance d

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ or $P = P_1 + P_2 - dP_1P_2$ which can be written as

$P = (\mu_1 - 1)\,k_1 + (\mu_2 - 1)\,k_2 - d\,(\mu_1 - 1)\,(\mu_2 - 1)k_1k_2$

As stated earlier, the lenses are made up of same glass, therefore

$P = (\mu - 1)\,(k_1 + k_2) - d\,(\mu - 1)^2\,k_1k_2$

We want to make the focal length independent of the variation of μ with wavelength, i.e. $dP/d\mu$ must be zero. Thus

$$\frac{dP}{d\mu} = k_1 + k_2 - 2d\,(\mu - 1)\,k_1k_2 = 0$$

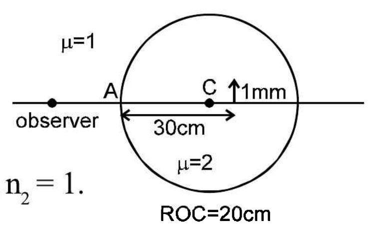
Therefore $(\mu - 1)\,k_1 + (\mu - 1)k_2 - 2d\,(\mu - 1)^2\,k_1k_2 = 0$

Then $\mu = \mu_1 = \mu_2 \Rightarrow (\mu_1 - 1)\,k_1 + (\mu_2 - 1)\,k_2 - 2d\,(\mu_1 - 1)k_1\,(\mu_2 - 1)\,k_2 = 0$

Hence, $P_1 + P_2 - 2dP_1P_2 = 0$. Thus, $d = \frac{P_1 + P_2}{2P_1P_2} = \frac{f_1 + f_2}{2}$

Two lenses made up of same glass separated by half of the sum of their focal lengths can produce a chromatic system.

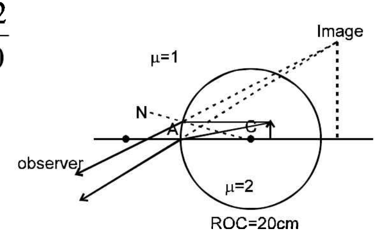
Ex.8 Find the position, size and nature of image, for the situation shown in figure. Draw ray diagram.



Sol. For refraction near point A, u = – 30 ; R = – 20; $n_1 = 2$; $n_2 = 1$.

Applying the refraction formula

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{v} - \frac{2}{-30} = \frac{1-2}{-20}$$



v = – 60 cm

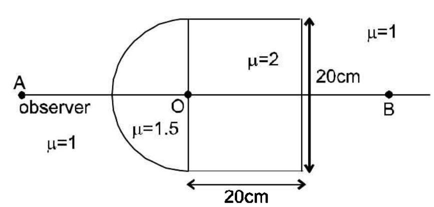
$$m = \frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} = \frac{2(-60)}{1(-30)} = 4$$

$\therefore h_2 = 4$ mm.

Q.1 Consider the situation shown in figure

(1) Find the position of image as seen by observer A.

(2) Find the position of image as seen by observer B.



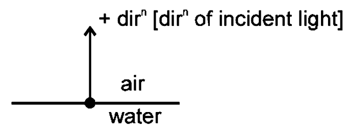
Sol. (1) At O (2) 10 cm right of O.

Ex.9 **Using formula of spherical surface or otherwise, find the apparent depth of an object placed 10 cm below the water surface, if seen normally from air.**

Sol. Put R = ∞ in the formula of the Refraction at Spherical Surfaces we get,

$$v = \frac{un_2}{n_1} \qquad u = -10 \text{ cm}$$

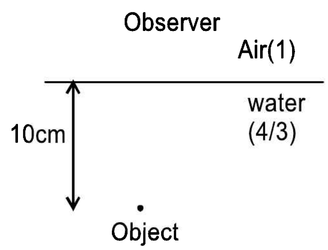
$$n_1 = \frac{4}{3} \qquad n_2 = 1$$



$$v = -\frac{10\times 1}{4/3} = -7.5 \text{ cm}$$

–ve sign implies that the image is formed in water.

Aliter :



$$d_{app} = \frac{d_{real}}{\mu_{rel}} = \frac{10}{4/3} = \frac{30}{4} = 7.5 \text{ cm.}$$

Ex.10 **Find the behavior of a concave lens placed in a rarer medium.**

Sol. Factor (A) is **negative**, since the lens is **concave**.
Factor (B) is **positive**, since the **lens is placed in a rarer medium**.
Therefore the focal length of the lens, which depends on the product of these factors , is negative and hence the lens will behave as a diverging lens.

Q.2 **Find the focal length of a double-convex lens with R_1 = 15 cm and R_2 = –25 cm. The refractive index of the lens material n = 1.5.**

Sol. 0.188 m

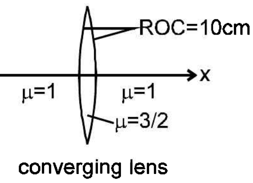
Q.3 **Find the focal length of a plano-convex lens with R_1 = 15 cm and R_2 = ∞. The refractive index of the lens material n = 1.5.**

Sol. 0.30 m

Q.4 **Find the focal length of a concavo-convex lens (positive meniscus) with R_1 = 15 cm and R_2 = 25 cm. The refractive index of the lens material n = 1.5.**

Sol. 0.75 m

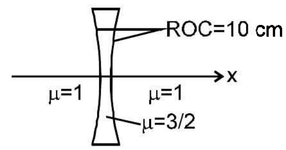
Ex.11 Find the focal length of the lens shown in the figure.



Sol. $\because \quad \frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \frac{1}{f} = (3/2 - 1)\left(\frac{1}{10} - \frac{1}{(-10)}\right)$

$$\frac{1}{f} = \frac{1}{2} \times \frac{2}{10} \qquad f = +10 \text{ cm.}$$

Ex.12 Find the focal length of the lens shown in figure



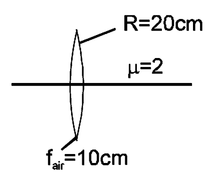
Sol. $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{10}\right) \Rightarrow f = -10 \text{ cm}$

Ex.13 Point object is placed on the principal axis of a thin lens with parallel curved boundaries i.e., having same radii of curvature. Discuss the position of the image formed.

Sol: $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0 \qquad [\because R_1 = R_2]$

$\frac{1}{v} - \frac{1}{u} = 0$ or $v = u$ i.e. rays pass without appreciable bending.

Ex.14 Focal length of a thin lens in air, is 10 cm. Now medium on one side of the lens is replaced by a medium of refractive index μ=2. The radius of curvature of surface of lens, in contact with the medium, is 20 cm. Find the new focal length.



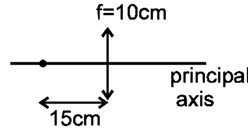
Sol. Let the radius of first surface be R_1 and refractive index of lens be μ and parallel rays be incident on the lens. Applying refraction formula at first surface

$$\frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \quad ...(1) \;; \text{ At II surface } \quad \frac{2}{V} - \frac{\mu}{V_1} = \frac{2 - \mu}{-20} \quad ...(2)$$

Adding (1) and (2) $\Rightarrow \frac{\mu}{V_1} - \frac{1}{\infty} + \frac{2}{V} - \frac{\mu}{V_1} = \frac{\mu - 1}{R_1} + \frac{2 - \mu}{-20}$

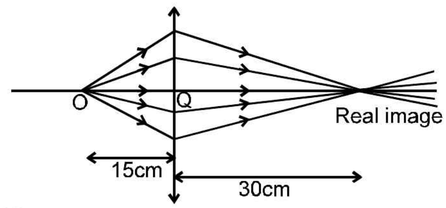
$$= (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{\mu - 1}{20} - \frac{2 - \mu}{20} = \frac{1}{f} \text{ (in air)} + \frac{1}{20} - \frac{2}{20} \Rightarrow v = 40 \text{ cm} \Rightarrow f = 40 \text{ cm}$$

Ex.15 Figure shows a point object and a converging lens. Find the final image formed.

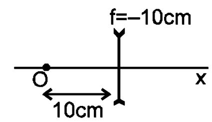


Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30} \quad \Rightarrow \quad v = +30$ cm

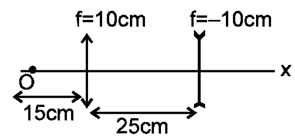


Q.5 Figure shows a point object and a diverging lens. Find the final image formed.



Sol. – 5 cm.

Ex.16 See the figure.
Find the position of final image formed.

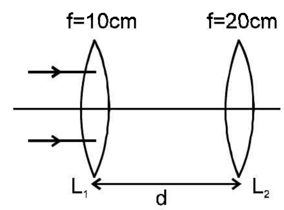


Sol. For converging lens

$u = -15$ cm, $f = 10$ cm, $v = \frac{fu}{f+u} = 30$ cm

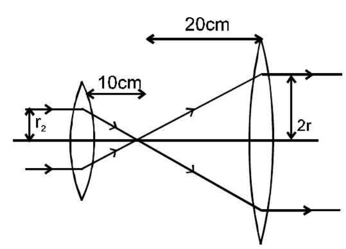
For diverging lens $u = 5$ cm, $f = -10$ cm $\therefore v = \frac{fu}{f+u} = 10$ cm

Ex.17 Figure shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel.

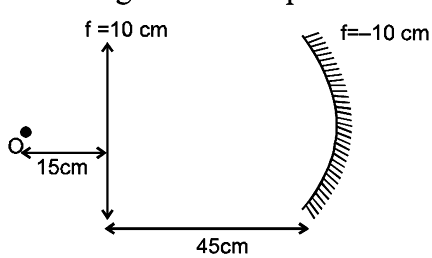


Sol. Final rays should be parallel. For this the II focus of L_1 must coincide with the I focus of L_2.

$d = 10 + 20$
$= 30$ cm
Here the diameter of ray beam becomes wider.



Ex.18 See the figure. Find the position of final image formed.



Sol. For lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \quad \Rightarrow v = +30 \text{ cm}$$

Hence, it is object for mirror

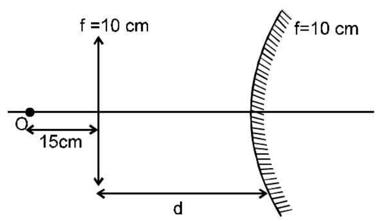
$$u = -15 \text{ cm}, \ \frac{1}{v} + \frac{1}{-15} = \frac{1}{-10} \quad \Rightarrow \quad v = -30 \text{ cm}$$

Now, it passes through lens second time

$$u = -15 \text{ cm}, \ v = ? \ ; \ f = 10 \text{ cm} \quad \Rightarrow \quad \frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \quad \Rightarrow v = +30$$

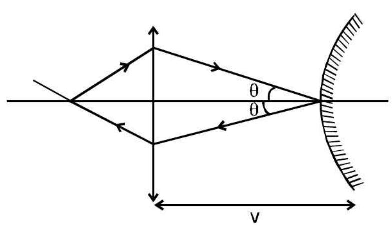
Hence, final image will form at a distance 30 cm from the lens towards left.

Ex.19 What should be the value of d so that image is formed on the object itself.

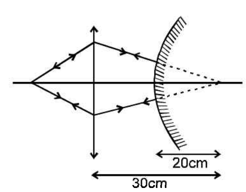


Sol. For lens : $\frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \quad \Rightarrow \quad v = +30 \text{ cm}$

Case I : If d = 30, the object for mirror will be at pole and its image will be formed there itself.



Case II : If the rays strike the mirror normally, they will retrace and the image will be formed on the object itself



$\therefore$ d = 30 – 20 = 10 cm

Ex.20 An extended real object of size 2 cm is placed perpendicular to the principal axis of a converging lens of focal length 20 cm. The distance between the object and the lens is 30 cm.

(i) Find the lateral magnification produced by the lens.

(ii) Find the height of the image.

(iii) Find the change in lateral magnification, if the object is brought closer to the lens by 1 mm along the principal axis.

Sol. (i) Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ and $m = \frac{v}{u}$ we get $m = \frac{f}{f+u}$ ....(A)

$\therefore\ m = \frac{+20}{+20+(-30)} = \frac{+20}{-10} = -2$ (–ve sign implies that the image is inverted.)

(ii) $\frac{h_2}{h_1} = m \quad \therefore \quad h_2 = mh_1 = (-2)\,(2) = -4$ cm

(iii) Differentiating (A) we get, $dm = \frac{-f}{(f+u)^2}\,du = \frac{-(20)}{(-10)^2}\,(0.1) = \frac{-2}{100} = -.02$

Note that the method of differential is valid only when the changes are small.

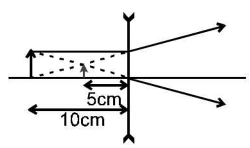
Aliter u (after displacing the object) $= -(30 + 0.1) = -29.9$ cm

Applying the formula, $m = \frac{f}{f+u} \Rightarrow m = \frac{20}{20+(-29.9)} = -2.02$

$\therefore$ Change in 'm' = – 0.02.

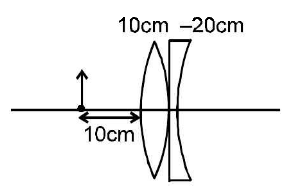
Since in this method differential is not used, this method can be used for any changes, small or large.

Q.6 An extended real object is placed perpendicular to the principal axis of a concave lens of focal length –10 cm, such that the image found is half the size of object.
(a) Find the object distance from the lens
(b) Find the image distance from the lens and draw the ray diagram
(c) Find the lateral magnification if object in moved by 1 mm along the principal axis towards the lens.

Sol. (a) 10 cm, (b) –5 cm 

(c) 0.5025

Ex.21 Find the lateral magnification produced by the combination of lenses shown in the figure.

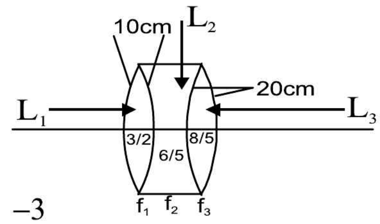


Sol. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow f = +20$

$\therefore \quad \frac{1}{v} - \frac{1}{-10} = \frac{1}{20} \quad \Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{10}$

$\Rightarrow \quad \frac{-1}{20} = -20$ cm $\therefore \quad m = \frac{-20}{-10} = 2$

Ex.22 Find the focal length of equivalent system.

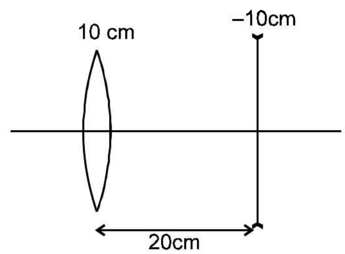


Sol. $$\frac{1}{f_1} = \left(\frac{3}{2}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right) = \frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$$

$$\Rightarrow \frac{1}{f_2} = \left(\frac{6}{5}-1\right)\left(\frac{-1}{10}-\frac{1}{20}\right) = \frac{1}{5} \times \left(\frac{-30}{10\times 20}\right) = \frac{-3}{100}$$

$$\Rightarrow \frac{1}{f_3} = \left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{3}{50}$$

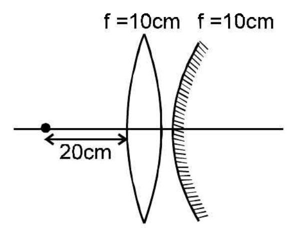
$$\Rightarrow \frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3} = \frac{1}{10}+\frac{-3}{100}+\frac{3}{50} \Rightarrow f = \frac{100}{13}$$



Q.7 Find the equivalent focal length of the system for paraxial rays parallel to axis.

Sol. $f_e = -5$ cm

Ex.23 Find the position of the final image formed. (The gap shown in figure is of negligible width)

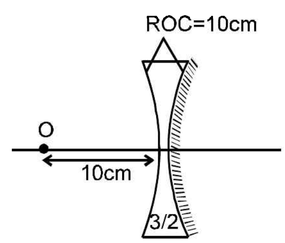


Sol. $$\frac{1}{f_{eq}} = \frac{1}{10} - \frac{2}{10} = \frac{-1}{10}$$

$f_{eq} = -10$ cm

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-10} \quad \Rightarrow \quad v = -20 \text{ cm}$$

Hence image will be formed on the object itself



Q.8 Find the equivalent focal length of the combination shown in the figure and position of the image.

Sol. $f_e = 2.5$ cm, $v = 2$ cm.

Ex.24 A spherical surface of radius R separates two media of refractive indices μ_1 and μ_2 as shown in figure. Where should an object be placed in the medium 1 so that a real image is formed in medium 2 at the same distance?

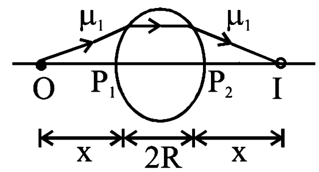
Sol. Let x be the distance of the object from the pole in medium 1.

Here we have, $u = -x$, $v = +x$ and $R = +R$

Using the equation we get, $\frac{\mu_2}{+x} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{-R}$ or $x = \left(\frac{\mu_2 + \mu_1}{\mu_2 - \mu_1}\right)R$

Note that the real image is formed only when $\mu_2 > \mu_1$.

Ex.25 A sphere of radius R made of material of refractive index μ_2. Where should an object be placed so that a real image is formed equidistant from the sphere?

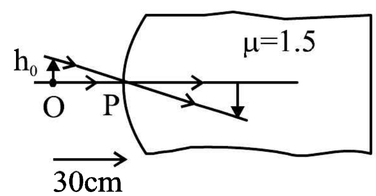


Sol. Let the object be placed at a distance x from the pole P_1 of the sphere. If a real image is to be formed equidistant from the sphere, then the ray must pass symmetrically through the sphere, as shown in the figure. Applying the equation at the first surface, we get

$$\frac{\mu_2}{+\infty} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{-R} \quad \text{or} \quad x = \left(\frac{\mu_1}{\mu_2 - \mu_1}\right)R$$

Note that the real image is formed only when the refractive index of the sphere is more than that of the surrounding. i.e. $\mu_2 > \mu_1$.

Ex.26 A small object of height 0.5 cm is placed in front of a convex surface of glass (μ = 1.5) of radius of curvature 10 cm. Find the height of the image formed in glass.



Sol. According to Cartesian Sign convention

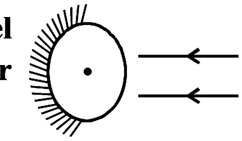
u = - 30 cm, R = + 10 cm ; $\mu_1 = 1$; $\mu_2 = 1.5$

Applying equation, we get $\frac{1.5-1}{v} - \frac{1}{-30} = \frac{1.5-1}{+10}$ or v = 90 cm (real image)

Let h_1 be the height of the image, then $\frac{h_i}{h_0} = \frac{\mu_1 v}{\mu_2 u} = \frac{(1)(90)}{(1.5)(-30)} = -2$

$h_i = -2h_0 = -2(0.5) = -1$ cm, The negative sign shows that the image is inverted.

Ex. 27 A glass sphere of index 1.5 and radius 40 cm has half its hemispherical surface silvered. The point where a parallel beam of light, coming along a diameter, will focus (or appear to) after coming out of sphere, will be:

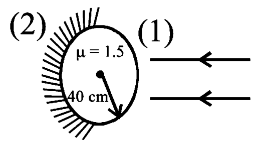


(A) 10 cm to the left of centre **(B) 30 cm to the left of centre**

(C) 50 cm to the left of centre **(D *) 60 cm to the left of centre**

Sol. R = 40cm

For refracting surface



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}, \quad n = 120 \text{ cm}$$

for mirror (2) $\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{40} = \frac{1}{(-20)} \Rightarrow v = -40/3$

for refracting surface ... (1)

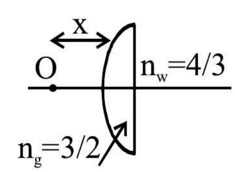
$$u = 80 - \frac{40}{3} = \frac{200}{3} \Rightarrow \frac{1}{v} - \frac{1.5}{-(200/3)} = \frac{1-1.5}{-40} \Rightarrow \frac{1}{v} + \frac{4.5}{200} = \frac{0.5}{40}$$

$$\frac{1}{v} = \frac{1}{80} - \frac{4.5}{200} \Rightarrow \quad v = -100 \text{ cm}$$

So the distance of image from the centre will be 100 - 40 = 60 cm (left of the centre)

Ex.28 An object 'O' is kept in air in front of a thin plano convex lens of radius of curvature 10 cm. It's refractive index is 3/2 and the medium towards right of plane surface is water of refractive index 4/3. What should be the distance 'x' of the object so that the rays become parallel finally.

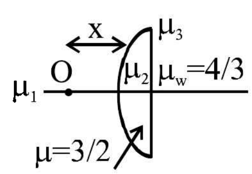
(A) 5 cm (B) 10 cm (C*) 20 cm (D) none of these



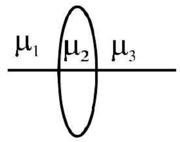
Sol. $$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

$$\frac{\mu_3}{\infty} - \frac{1}{x} = \frac{1.5-1}{10} + \frac{4/3 - 3/2}{\infty}$$

$$\frac{-1}{x} = \frac{0.5}{10} \qquad x = -20 \text{ cm}$$



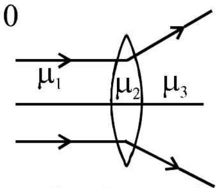
Ex.29 The diagram shows an equiconvex lens. What should be the condition on the refractive indices so that the lens becomes diverging?



(A) $2\mu_2>\mu_1-\mu_3$ **(B*) $2\mu_2<\mu_1+\mu_3$** **(C) $2\mu_2>2\mu_1-\mu_3$** **(D) $2\mu_2>\mu_1+\mu_3$**

Sol. $\dfrac{\mu_3}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R_1}+\dfrac{\mu_3-\mu_2}{R_2} \Rightarrow \dfrac{\mu_3}{v}-\dfrac{\mu_1}{\infty}=\dfrac{\mu_2-\mu_1}{R}+\dfrac{\mu_3-\mu_2}{(-R)}$

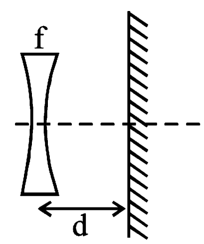
$\dfrac{\mu_3}{v}=\dfrac{\mu_2-\mu_1}{R}-\dfrac{(\mu_3-\mu_2)}{R}<0 \Rightarrow$ For diverging nature $v<0$



$\dfrac{\mu_2-\mu_1}{R}<\dfrac{\mu_3-\mu_2}{R} \Rightarrow 2\mu_2<\mu_1+\mu_3$

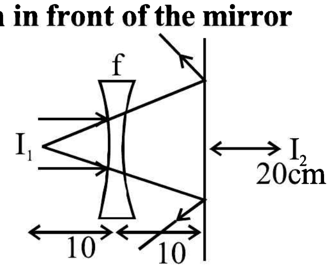
Ex.30 A diverging lens of focal length 10 cm is placed 10 em in front of a plane mirror as shown in the figure. Light from a very far away source falls on the lens. The final image is at a distance

(A) 20 cm behind the mirror (B) 7.5 em in front of the mirror
(C) 7.5 cm behind the mirror (D) 2.5 cm in front of the mirror



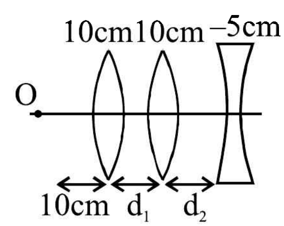
Sol. $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f} \Rightarrow \dfrac{1}{v}-\dfrac{1}{-30}=+\dfrac{1}{-10}$

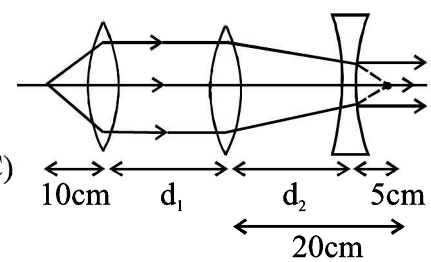
$\Rightarrow \dfrac{1}{v}=\dfrac{-1}{10}-\dfrac{1}{30}=-\dfrac{4}{30} \Rightarrow v=-7.5$



Ex.31 The values of d_1 & d_2 for final rays to be parallel to the principal axis are: (focal lengths of the lenses are written on the lenses)

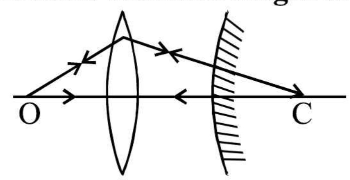
(A) $d_1 = 10$cm, $d_2 = 15$ cm **(B) $d_1 = 20$ cm, $d_2 = 15$ cm**
(C) $d_1 = 30$ cm, $d_2 = 15$ cm **(D) None of these**





Sol. (ABC) $d_2 = 20 - 5 = 15$ cm, d_1 can take any value

Ex.32 An object is placed at a distance of15cm from a convex lens of focal length 10 cm on the other side of the lens, a convex mirror is placed at its focus such that the image formed by combination coincides with the object itself. The focal length of concave mirror

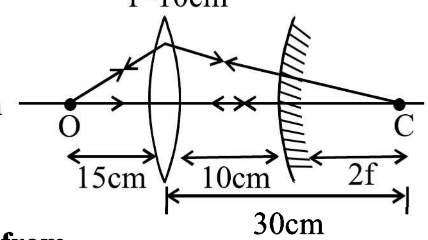


(A)20cm (B)10cm (C) 15cm (D)30cm

Sol. (B) For retracing of ray ; ray must fall normally on mirror i.e. towards the centre of curvature.

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow u=-15 \Rightarrow \frac{1}{v}+\frac{1}{15}=\frac{1}{10} \Rightarrow f=10$ cm

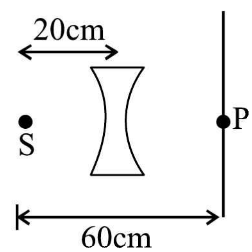
$v=30$ cm , for mirror $\Rightarrow \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow u=2f$



$\frac{1}{v}+\frac{1}{2f}=\frac{1}{f}$ $v=2f$ hence form the ray diagram

$2f=30-10$ $f=10$ cm

Ex.33 A point source of light is placed 60cm away from screen. Intensity detected at point P is I. Now a diverging lens of focal length 20 cm is placed 20 cm away from S between S and P. The lens transmits 75% of light incident on it. Find the new value of intensity at P.

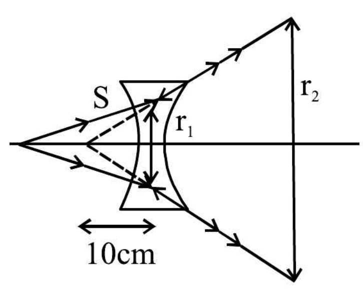


Sol. $u=-20, f=-20$, gives, $v=-10$

Let P = Power of source, $I=\frac{P}{4\pi(60)^2}$ ...(1)

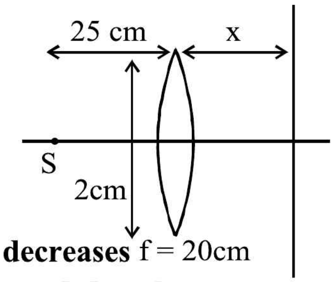
Energy received by lens $E_2=\frac{P}{4\pi(20)^2}A_1$

$\therefore I_2=\frac{0.75E_2}{A_2}$ ... (2)



From similar triangles $\frac{A_2}{A_1}=25$ $\therefore I_2=0.27\,I$

Ex.34 A point source of light S is placed on the axis of a lens of focal length 20 cm as shown. A screen is placed normal to the axis of lens at a distance x from it. Treat all rays as paraxial.



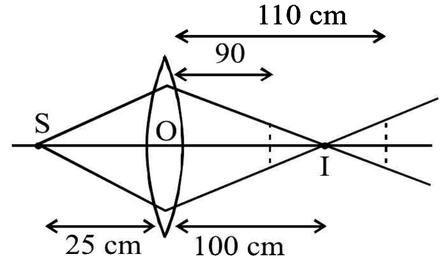
(A) As x is increased from zero, intensity continuously decreases

(B) As x is increased from zero, intensity first increases and then decreases

(C) Intensity at centre of screen for x = 90 cm and x = 110 cm is same

(D) Radius of bright circle obtained on screen is equal to 1 cm for x = 200 cm

Sol. (BCD)



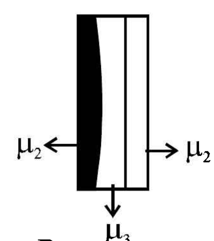
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{(-25)}=\frac{1}{20}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{20}-\frac{1}{25}=\frac{1}{100} \Rightarrow V = 100 \text{ cm}$$

From O to I intensity increases and then decreases.

At x = 90 cm & 110 cm the intensity is same. Radius at x = 200 cm is equal to the radius of lens.

Ex.35 The adjacent figure shows a thin plano-convex lens of refractive index μ_1 and a thin plano-concave lens of refractive index μ_2, both having same radius of curvature R of their curved surfaces. The thin lens of refractive index μ_3 has radius of curvature R of both its surfaces. This lens is so placed in between the plano-convex and plano-concave lenses that the plane surfaces are parallel to each other. The focal length of the combination is



(A) $\dfrac{R}{(\mu_1+\mu_2-\mu_3-1)}$ **(B)** $\dfrac{R}{(\mu_1+\mu_2+\mu_3)}$ **(C)** $\dfrac{R}{(\mu_1-\mu_2)}$ **(D)** $\dfrac{R}{(\mu_1-\mu_2-\mu_3-3)}$

Sol. $\dfrac{1}{f_1}=(\mu_1-1)\left(\dfrac{1}{\infty}+\dfrac{1}{R}\right)=\dfrac{\mu_1-1}{R} \quad \Rightarrow \quad \dfrac{1}{f_1}=(\mu_3-1)\left(-\dfrac{1}{R}+\dfrac{1}{R}\right)=0$

$$\frac{1}{f_3}=(\mu_2-1)\left(-\frac{1}{R}-\frac{1}{\infty}\right)=-\frac{(\mu_2-1)}{R}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}=\frac{(\mu_1-1)}{R}-\frac{(\mu_2-1)}{R}=\frac{1}{R}(\mu_1-\mu_2) \Rightarrow \quad f=\frac{R}{(\mu_1-\mu_2)} \text{ Ans.}$$

Ex.36 A glass slab of thickness 3cm and refractive index 1.5 is placed in front of a concave mirror of focal length 20cm. Where should a point object be placed if it is to image on to itself ?

Sol. The glass slab and the concave mirror are shown in figure. Let the distance of the object from the mirror be x. We know that the slab simply shifts the object. The shift being equal to

$$s = t\left[1-\frac{1}{\mu}\right] = 1 \text{ cm}$$

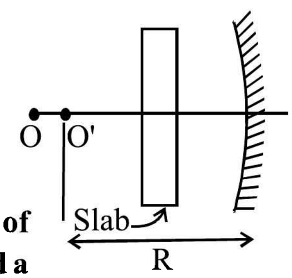
The direction of shift is towards the concave mirror.

$\therefore$ the apparent distance of the object from the mirror is $x - 1$.

If the rays are to retrace their paths, the object should appear to be at the center of curvature of the mirror.

$$\therefore\ x - 1 = 2f = 40 \text{ cm}$$

or $x = 41$ cm from the mirror.



Ex.37 A convex lens of focal length 20 cm is placed 10 cm in front of a convex mirror of radius of curvature 15 cm. Where should a point object be placed in front of the lens so that it images on to itself ?

The convex lens and the convex mirror are shown in the figure.

The combination behaves like a concave mirror

Let the distance of the object from the lens be x.

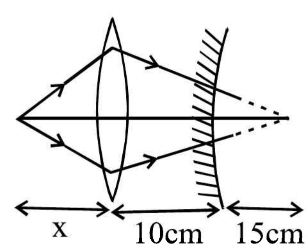
For the ray to retrace its path it should be incident normally on the convex mirror, or in other words the rays should pass through the center of curvature of the mirror.

From the diagram we see that for the lens

$$u = -x,\ f = +20 \text{ cm}, \qquad v = +10 + 15 = +25 \text{ cm}$$

From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{25} - \frac{1}{-x} = \frac{1}{20} \qquad \text{or} \qquad x = 100 \text{ cm}$$



Ex.38 A convex lens of focal length 10cm is placed 30 cm in front of a second convex lens also of the same focal length. A plane mirror is placed behind the two lenses. Where should a point object be placed in front of the first lens so that it images on to itself?

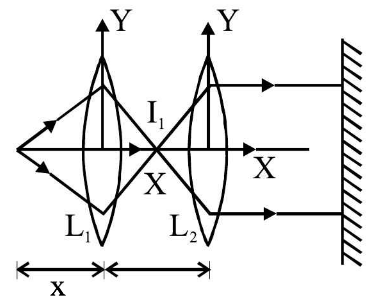
Sol. The convex lenses and the plane mirror are shown in figure.

The combination behave like a mirror.

Let the distance of the object from the first lens be x.

For the ray to retrace its path it should be incident normally on the plane mirror.

From the diagram we see that for lens L_2, from the lens equation we get



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ or } u = -10 \text{ cm}$$

From the diagram we see that for lens L_1,

$v = 30 - 10 = 20$ cm, $f = +10$ cm, $u = -x$

From the lens equation we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{20} - \frac{1}{-x} = \frac{1}{10} \quad \text{or} \quad x = 20 \text{ cm}$$

CHAPTER

5 Optical Instruments

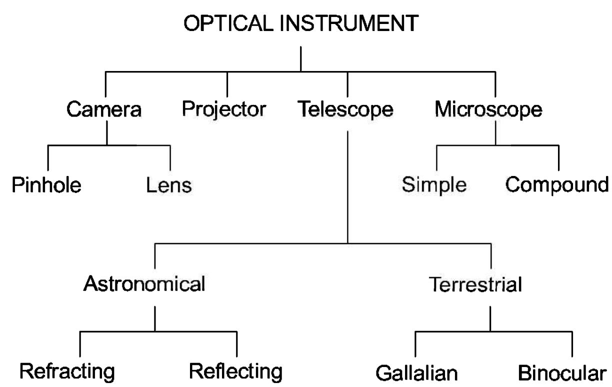
5.1 INTRODUCTION

Definition

Optical instruments are used primarily to assist the eye in viewing an object.

Types of Instruments

Depending upon the use, optical instruments can be categorised in the following way :



5.2 FILM OR SLIDE PROJECTOR

It projects real, inverted and magnified image of an object, when the object is placed between F & 2F and screen between 2F and ∞.

Magnification

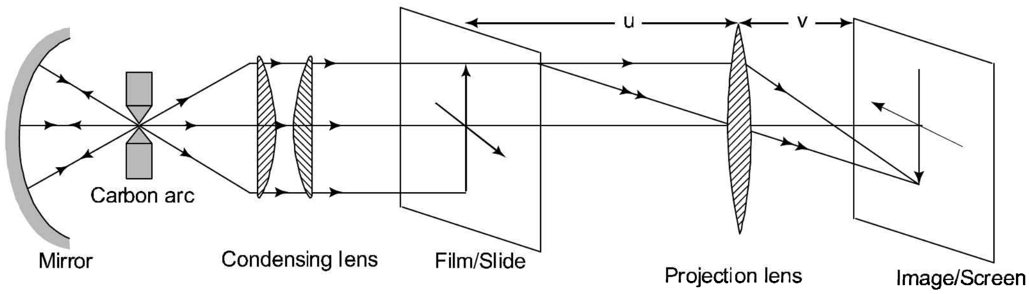
If the sides of slide are a and b and magnification of projection lens is m, each side will become m times so that the area of image formed.

$$A_i = ma \times mb = m^2\,ab = m^2 A_0$$

Intensity

As area of image becomes m^2 times that of object, the intensity of image will become $1/m^2$ times that of object, as

$$IA_0 = I_i A_i = I_i\, m^2 A_0 \quad \Rightarrow \quad I_i = (I/m^2)$$



Examples based on Projector

Ex.1 A slide projector lens has a focal length 10 cm. It throws an image of a 2cm × 2 cm slide on a screen 5 m from the lens. Find (a) the size of the picture on the screen and (b) the ratio of illumination of the slide and the picture on the screen.

Sol. (a) As here f = 10 cm and v = 5m = 500 cm

So from lens formula $\frac{1}{v}=\frac{1}{u}=\frac{1}{f}$, we have

$$\frac{1}{500}-\frac{1}{u}=\frac{1}{10} \text{ i.e., } \quad u=-\left[\frac{500}{49}\right]\text{cm}$$

So that $m=\frac{v}{u}=\frac{(500)}{(-500/49)}=-49$

Here negative sign means the image is inverted with respect to object. Now as here object is (2 cm × 2 cm) so the size of picture on the screen

$A_i = (2 \times 49 \text{ cm}) \times (2 \times 49\text{cm})$

$= (98 \times 98) \text{ cm}^2$

(b) As light energy passing per sec through slide is equal to that in the picture on the screen.

$IA = I_i A_i$

So $\quad \frac{I}{I_i}=\frac{A_i}{A}=\frac{(ma)\times(mb)}{(a\times b)}=m^2$

i.e. $\quad \frac{I}{I_i}=(-49)^2=(49\times 49)$

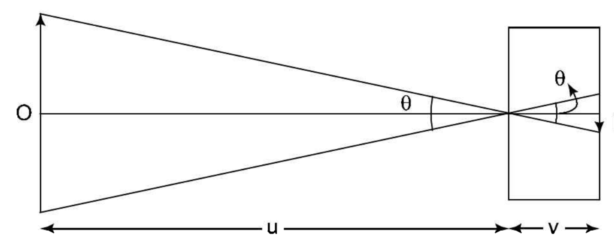
i.e. intensity in picture on the screen will be [1/(49 × 49)] times lesser than that on the slide. This is why in case of projector for observing image on a screen the source of light must be very powerful and the room dark.

5.3 CAMERA

Camera is of two types viz. pin hole camera and lens camera.

Pinhole Camera

It is based on recti linear propagation of light and forms real, inverted image on the screen.

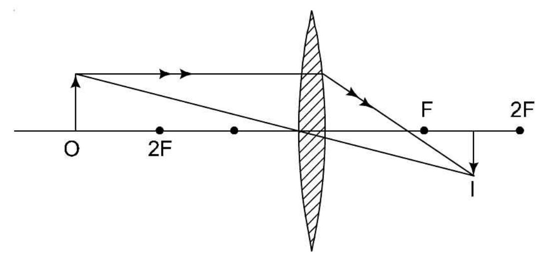


$$\theta = \frac{h_0}{u} = \frac{h_i}{v}$$

$$\Rightarrow \quad \frac{h_i}{h_0} = \frac{u}{v}$$

Note:

The image formed on the screen is neither a shadow (as it is not dark) nor a true image as the rays do not intersect each other and can not be seen as an aerial image in absence of screen.



In it a converging lens whose aperture and distance from the film can be adjusted is used. Usually object is real and between ∞ and 2F; so the image is real, inverted, diminished and between F and 2F.

(A) Exposure:

For a particular film, a definite amount of light energy must i.e. incident on the film for proper exposure. i.e.,

$I \times S \times t$ = constant where, I = intensity of light, S = area of aperture of lens and t is the exposure time.

i.e. $ID^2 t$ = constant where D = aperture

(B) f-number:

It is the ratio of focal length to the aperture of lens. i.e. f - no. = f/D

Examples based on Camera

Ex.2 A pinhole camera with 75 mm high film is to be used to take a picture of a 10 m high tree. The film is 150 mm from the pinhole. How far should the camera be from the tree to include the full height of the tree?

Sol. In case of pinhole camera

$$\theta = \frac{1}{v} = \frac{O}{u} \text{ i.e., } \quad u = v \times \frac{O}{I}$$

Here I = 75 mm, O = 10 m and v = 150 mm,

So $$u = (10\text{ m}) \times \frac{(150\text{ mm})}{(75\text{ mm})} = 20\text{ m}$$

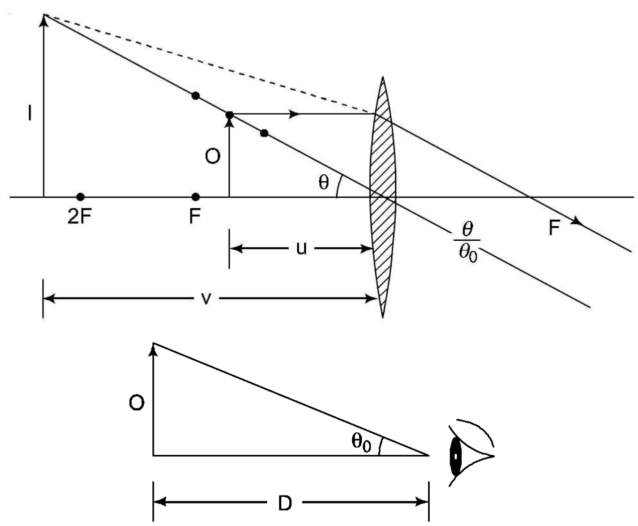
5.4 MICROSCOPE

It is an optical instrument used to increase the visual angle of near objects which are too small to be seen by naked eye.

Microscopes are of two types viz. simple microscope and compound microscope.

Simple Microscope

It is also known as magnifying glass or **magnifier** and consists of a convergent lens with object between its focus and optical centre and eye close to it. The image formed by it is erect, virtual, enlarged and on same side of lens between object and infinity.



Here

Magnifying power

$$= \frac{\text{Visual angle with instrument}}{\text{Maximum visual angle for unaided eye}} = \frac{\theta}{\theta_0}$$

Now, $\theta = \frac{h_i}{v} = \frac{h_0}{u}$ with, $\theta_0 = h_0/D$

$$M.P. = \frac{\theta}{\theta_0} = \frac{h_0}{u} \times \frac{D}{h_0} = \frac{D}{u}$$

Now, two possibilities are there:

(A) Image is at infinity (far point)

If $v = \infty,\ u = f$ (from lens formula)

So, $M.P. = \frac{D}{u} = \frac{D}{f}$

Note:

Here parallel beam of light enters the eye i.e., eye is least strained.

(B) Image is at D (Near point)

In this situation $v = -D$, so that, $\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$

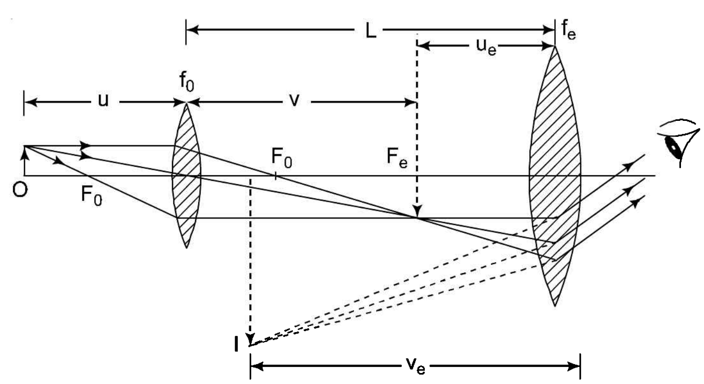
or, $\frac{D}{u} = 1 + \frac{D}{f}$

So, $M.P. = 1 + \frac{D}{f}$

Note:

Here final image is closest to eye i.e. , eye is under maximum strain.

Compound Microscope



It consists of two convergent lenses of short focal lengths and apertures arranged co-axially. Lens f_0 is the **objective** or **field lens** and f_e is the **eye-piece** or **ocular**. Objective has smaller aperture and focal length than eye-piece. The separation between objective and eye-piece can be varied.

$$\text{Magnifying Power} = \frac{\theta}{\theta_0} = \frac{h_i}{u_e} \times \frac{D}{h_0} = \frac{h_i}{h_0} \times \frac{D}{u_e}$$

But for objective, $m = \frac{v}{u}$ i.e., $\frac{h_i}{h_0} = -\frac{v}{u}$

so, $\text{M.P.} = -\frac{v}{u}\left[\frac{D}{u_e}\right]$ where, $u_e + u = L$

Now two possibilities are there :

(A) Final image is at infinity (far point)

$u_e = f_e \quad \Rightarrow \quad \text{M.P.} = -\frac{v}{u}\left[\frac{D}{f_e}\right]$

where $L = u + f_e$

Note:

A microscope is usually considered to operate in this mode unless stated otherwise.

(B) Final image is at D (near point)

For eye - piece . $v_e = D$,

$$\Rightarrow \quad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{D}\left(1 + \frac{D}{f_e}\right)$$

$$\therefore \quad \text{M.P.} = -\frac{v}{u}\left(1 + \frac{D}{f_e}\right) \text{ with } L = v + \frac{f_e D}{f_e + D}$$

Note:

In case, $u \cong f_0$ and, $L = v + u_e \cong v$ so that

$$|\text{M.P.}| \cong \frac{L}{f_0} \times \frac{D}{f_e}$$

IMPORTANT POINTS

1. As **magnifying power** is negative, the image seen in a microscope is always truly inverted, i.e., left is turned right with upside down simultaneously.
2. **Resolving Power:** The minimum distance between two lines at which they are just distinct is called limit of resolution and reciprocal of limit of resolution is called resolving power.

$$\therefore \quad \text{R.P.} = \frac{1}{\Delta x} \propto \frac{1}{\lambda} = 2\mu \sin\theta/\lambda$$

Examples based on Microscope

Ex.3 A man with normal near point (25 cm) reads a book with small print using a magnifying glass, a thin convex lens of focal length 5 cm. (a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass ? What is the maximum and minimum MP possible using the above simple microscope ?

Sol. **(a)** As for normal eye far and near points are ∞ and 25 cm respectively so for magnifier $v_{max} = -\infty$ and $v_{min} = -25$ cm. However, for a lens as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{i.e.} \quad u = \frac{f}{(f/v) - 1}$$

So u will be minimum when v = min = – 25 cm

$$\text{i.e. } (u)_{min.} = \frac{5}{-(5/25) - 1} = -\frac{25}{6} = -4.17 \text{ cm}$$

And u will be maximum when v = max = ∞

$$\text{i.e. } (u)_{max.} = \frac{5}{(5/\infty) - 1} = -5 \text{ cm}$$

So the closest and farthest distances of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) As in case of simple magnifier MP = (D/u). So MP will be minimum when

u = max = 5 cm.

$$\text{i.e. } (MP)_{min} = \frac{-25}{-5} = 5 \left[= \frac{D}{f} \right]$$

And MP will be maximum when u = min = (25/6) cm i.e. $(MP)_{max} = \dfrac{-25}{-(25/6)} = 6 \left[= 1 + \dfrac{D}{f} \right]$

Ex.4 **If the focal length of a magnifier is 5 cm calculate (a) the power of the lens (b) the magnifying power of the lens for relaxed and strained eye.**

Sol. **(a)** As power of a lens is reciprocal of focal length in m,

$$P = \frac{1}{\left(5 \times 10^{-2}\ \mathrm{m}\right)} = \frac{1}{0.05}\ \text{diopter} = 20\ \mathrm{D}$$

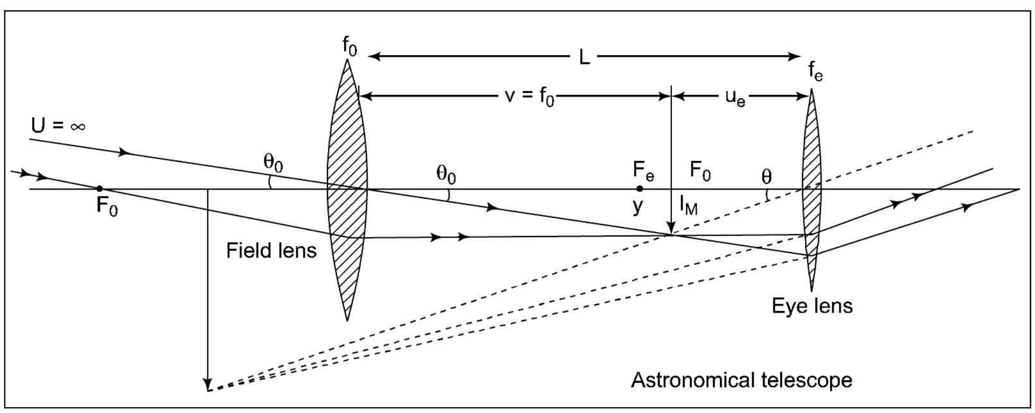
(b) For relaxed eye, MP is minimum and will be

$$MP = \frac{D}{f} = \frac{25}{5} = 5$$

While for strained eye, MP is maximum and will be

$$MP = 1 + \frac{D}{f} = 1 + 5 = 6$$

5.5 TELESCOPE



Astronomical telescope

It is an optical instrument used to increase the visual angle of distant large objects. Telescopes mainly are of two types viz. astronomical and terrestrial.

Astronomical Telescope

It consists of two converging lenses placed coaxially with objective having large aperture and a large focal length while the eye- piece having smaller aperture and focal length. The separation between eye- piece and objective can be varied.

Magnifying power

$$= \frac{\text{visual angle with instrument}}{\text{visual angle for unaided eye}} = \frac{\theta}{\theta_0},$$

$\theta_0 = h/f_0$ & $\theta = h/(-u_e)$

$$\Rightarrow \quad \text{M.P.} = -\left[\frac{f_0}{u_e}\right] \quad \text{with } L = f_0 + u_e$$

Now two possibilities are there.

(A) Final image is at infinity (far point)

Here, $v = \infty \quad \Rightarrow \quad u_e = f_e$

So, M.P. $= -(f_0/f_e)$ with, $L = f_0 + f_e$

Note:

Usually, a telescope operates in this mode unless stated otherwise.

(B) Final image is at D (near point)

$$\text{Here, } v = D \quad \Rightarrow \quad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

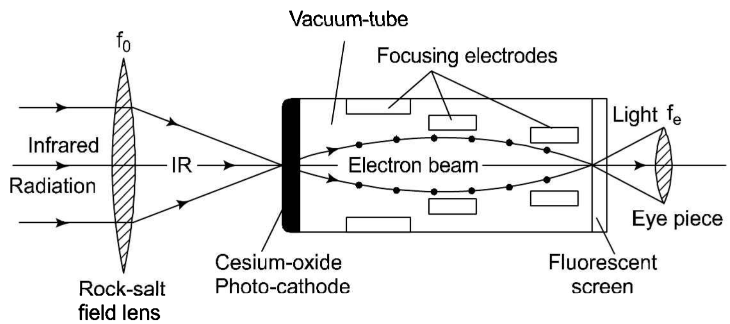
$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

$$\text{So, M.P.} = -\frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]$$

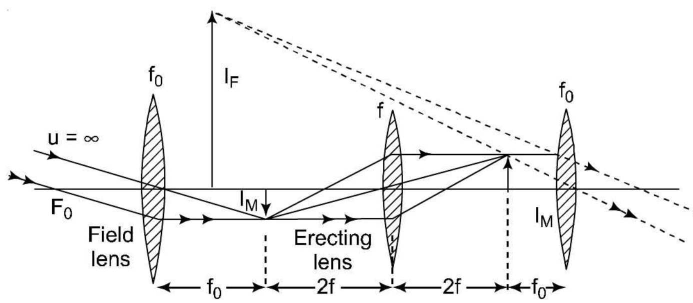
$$\text{with } L = f_0 + \frac{f_e D}{f_e + D}$$

Note:

1. The above discussion is that of the **refracting telescope**.
2. **Reflecting Telescope:**
   If the field lens of refracting telescope is replaced by a converging mirror, then the telescope becomes a reflecting one, where M.P. $= f_0/f_e$
3. **Infrared Telescope:**
   It is used to see distant objects in dark.



Terrestrial Telescope

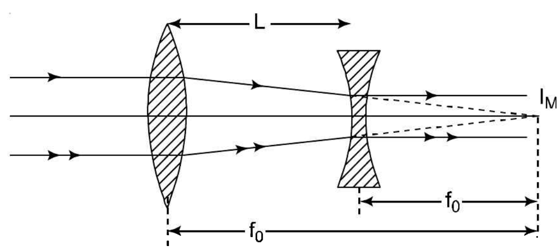


If a lens of short focal length f is placed at 2f from the intermediate image at a distance 2f on the other side of it and this image will act as an object for eye- lens which will produce erect image with respect to the object; this lens is called **erecting lens** and as for it

m = – 1, the MP and length of telescope for relaxed eye will be

$$M.P. = -\frac{f_0}{f_e}(-1) = \frac{f_0}{f_e}, \; L = f_0 + f_e + 4f$$

(A) Galilean Telescope



Here the convergent eye- piece of astronomical telescope is replaced by a divergent lens. Here M.P. = f_0 / f_e with , L = $f_0 - f_e$

Note:

In this telescope as the intermediate image is outside the tube, the telescope cannot be used for measurements. This was not the case for all previous telescopes.

(B) Binocular

When two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called a binocular. Here, the length of each tube is reduced by using a set of totally reflecting prisms which provide intense, erect image free from lateral inversion. Binocular gives proper 3-D image.

Examples based on Telescope

Ex.5 An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eye-piece is 36 cm and the final image is formed at infinity. Determine the focal length of objective and eye-piece.

Sol. In case of astronomical telescope if object and final image both are at infinity.

MP = $-(f_0/f_e)$ and L = $f_0 + f_e$

So here $-(f_0/f_e) = -5$ and $f_0 + f_e = 36$

Solving these for f_0 and f_e, we get

$f_0 = 30$ cm and $f_e = 6$ cm

Ex.6 A telescope has an objective of focal length 50 cm and an eye- piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale 2 m away from the objective. Calculate (a) magnification produced and (b) separation between objective and eye- piece.

Sol. As objective has focal length 50 cm and object is 2 m from it, it will form the image of object at a distance v such that

$$\frac{1}{v} - \frac{1}{-200} = \frac{1}{50} \quad \text{i.e., } v = \frac{200}{3} \text{ cm.}$$

$$\text{with } m_0 = \frac{v}{u} = \frac{(200/3)}{-200} = -\frac{1}{3}$$

and as focal length of eye- piece is 5 cm and it forms as image 25 cm in front of it, the distance of object (image formed by objective) from it will be

$$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

$$\text{i.e.,} \quad u_e = -\frac{25}{6} \text{ cm}$$

$$\text{with} \quad m_e = \frac{v_e}{u_e} = -\frac{25}{-(25/6)} = 6$$

and hence

(a) Magnification $m = m_0 \times m_e = (-1/3) \times 6 = -2$, i.e., final image is inverted, virtual, double of object and is at a distance of 25 cm in front of eye lens.

(b) As distance of intermediate image (which is between the two lenses), from objective is (200/3) cm while from the eye- lens is (25/6) cm., so separation between the objective and eye-piece,

$$L = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.83 \text{ cm.}$$

Solved Examples

Ex.1 A 35 mm film is to be projected on a 20 m wide screen situated at a distance of 40 m from the film- projector. Calculate the distance of the film from the projection lens and focal length of projection lens.

Sol. As in case of projector,

$$m = \frac{I}{O} = \frac{v}{u}$$

$$\text{So } -\frac{(200 \times 100 \text{ cm})}{(3.5 \text{ cm})} = \frac{40 \times 100}{u}$$

i.e., $u = -7$ cm

i.e., film is at a distance of 7 cm in front of projection lens.

And from lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, here we have

$$\frac{1}{4000} - \frac{1}{-7} = \frac{1}{f}$$

or $f \cong 7$ cm $= 70$ mm

[as (1/4000) << (1/7)]

i.e., focal length of projection lens is 70 mm.

Ex.2 A compound microscope has an objective of focal length 2 cm and an eye-piece of focal length 5 cm. If an object is placed at a distance of 2.4 cm in front of the field lens, find the magnifying power of the instrument and length of the tube if (a) final image is at infinity (b) final image is at least distance of distinct vision (= 25 cm).

Sol. As object is at a distance of 2.4 cm. in front of field lens of focal length 2 cm. field lens will form its image at distance v such that

$$\frac{1}{v}-\frac{1}{-2.4}=\frac{1}{2} \text{ i.e., } v = 12 \text{ cm.}$$

so that $m=\frac{v}{u}=\frac{12}{-2.4}=-5$

(a) If final image is at infinite (far point)

For eye- piece, $\frac{1}{\infty}-\frac{1}{u_e}=\frac{1}{5}$

i.e., $u_e = -5$ cm.
and $m_\theta = (D/F_e) = (25/5) = 5$
So, $MP = m \times m_\theta = -5 \times 5 = -25$
and $L = v + u_e = 12 + 5 = 17$ cm.

In this situation eye is said to be relaxed and for a given microscope MP is minimum while length of tube maximum.

(b) If final image is at D (Near point)

For eye- piece, $\frac{1}{-25}-\frac{1}{u_e}=\frac{1}{5}$

i.e., $u_e=\frac{-25}{6}=-4.17$ cm.

and, $m_\theta = [1 + (D/f_e)] = [1 + (25/5)] = 6$
So, $MP = m \times m_\theta = -5 \times 6 = -30$
and, $L = v + u_e = 12 + 4.17 = 16.17$ cm.

In this situation eye is said to be strained and for a given microscope MP is maximum while length of tube is minimum.

Note: In case (b) $m_e=\frac{D}{u_e}=\frac{-25}{(-25/6)}=6=m_\theta$

So here $MP = m \times m_\theta = m \times m_e =$ linear magnification

Ex.3 In a compound microscope the objective and the eye- piece have focal lengths of 0.95 cm and 5 cm respectively, and are kept at a distance of 20 cm. The last image is formed at a distance of 25 cm from the eye- piece. Calculate the position of object and the total magnification.

Sol. As final image is at 25 cm in front of eye piece

$$\frac{1}{-25}-\frac{1}{u_e}=\frac{1}{5} \text{ i.e., } u_e=-\frac{25}{6}$$

And so, $m_e=\frac{v_e}{u_e}=\frac{-25}{(-25/6)}=6$ ...(1)

Now for objective,

$v = L - u_e = 20 - (25/6) = (95/6)$

So if object is at a distance u from the objective,

$$\frac{6}{95}-\frac{1}{u}=\frac{1}{0.95}$$

i.e., $u=-\frac{95}{94}$ cm

i.e. object is at a distance (95/94) cm in front of field lens.

Also, $m=\frac{v}{u}=\frac{(95/6)}{(-95/94)}=-\left[\frac{94}{6}\right]$ ...(2)

So total magnification,

$M = m \times m_e = -\left[\frac{94}{6}\right] \times (6) = -94$ i.e., final image is inverted virtual and 94 times that of object.

Ex.4 A Galilean telescope consists of an objective of focal length 12 cm and eye-piece of focal length 4 cm. What should be the separation of the two lenses when the virtual image of a distant object is formed at a distance of 24 cm from the eye-piece? What is the magnifying power of telescope under this condition?

Sol. As object is distant, i.e., $u = -\infty$, so

$$\frac{1}{v}-\frac{1}{-\infty}=\frac{1}{f_0} \text{ i.e. } \quad v=f_0=12 \text{ cm}$$

i.e. objective will form the image I_M at its focus which is at a distance of 12 cm from O. Now as eye- piece of focal length – 4 cm forms image I at a distance of 24 cm from it,

$$\frac{1}{-24}-\frac{1}{u_e}=\frac{1}{-4} \quad \Rightarrow \quad u_e=\frac{24}{5}=4.8 \text{ cm}$$

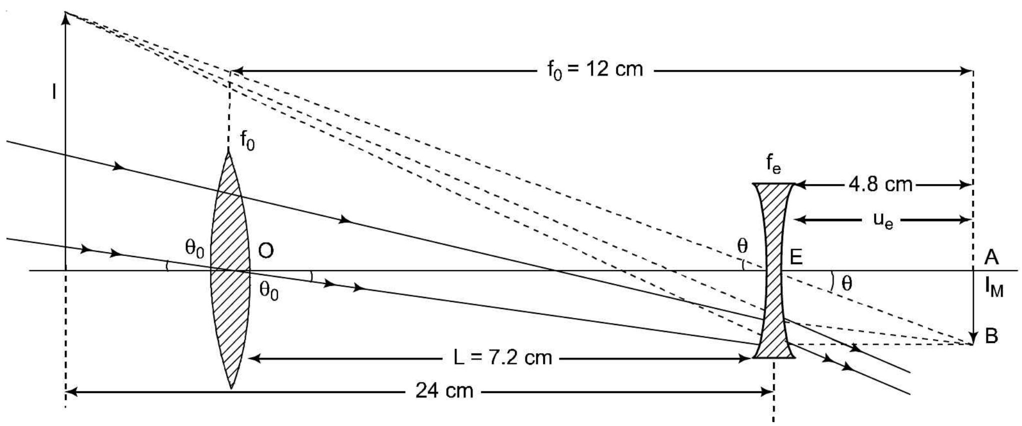
i.e, the distance of I_M from eye lens EA is 4.8 cm. So the length of tube

L = OA – EA = 12 – 4.8 = 7.2 cm.

Now by definition:

$$MP = \frac{\theta}{\theta_0} \cong \frac{\tan\theta}{\tan\theta_0} = \frac{(AB/EA)}{(AB/OA)} = \frac{OA}{EA}$$

So, $MP = \frac{f_0}{u_e} = \frac{12}{4.8} = \frac{10}{4} = 2.5$ i.e., the image is erect, virtual, and is at a distance of 24 – 7.2 = 16.8 cm in front of objective.



LEVEL 1

Questions based on Projector

Q.1 **A film projector magnifies a 100 sq. cm. film strip on a screen. If the linear magnification is 4, the area of the magnified film on the screen is**

(A) 1600 sq. cm **(B) 400 sq. cm** **(C) 800 sq. cm** **(D) 200 sq. cm.**

Questions based on Camera

Q.2 **It is desired to photograph the image of an object placed at a distance of 3 m from a plane mirror the camera which is at a distance of 4.5 m from the mirror should be focussed for a distance of –**

(A) 3 m **(B) 4.5 m** **(C) 6 m** **(D) 7.5 m**

Q.3 **A photographer changes the aperture of his camera so that the new diameter of the aperture is twice the initial one, the ratio of new exposures time to the initial one is –**

(A) 1 : 2 **(B) 2 : 1** **(C) 1 : 4** **(D) 4 : 1**

Q.4 **The exposure time of a camera lens at the f / 2.8 setting is (1/200) s. the correct time of exposure of f / 5.6 is –**

(A) 0.04 s **(B) 0.20 s** **(C) 0.40 s** **(D) 0.02 s**

Q.5 **An object 5 cm long and a pencil 10 cm long are placed in front of a pin hole camera such that their images have the same length. The ratio of the distance of the object from the pin hole to that of the pencil is–**

(A) 3: 2 **(B) 1: 2** **(C) 5: 2** **(D) 1: 9**

Q.6 **The 'f' number of a camera lens is 4.5. Which of the following statement is correct?**

(A) The focal length of lens is 4.5

(B) It is the reciprocal of the focal length expressed in m

(C) The ratio of focal length to aperture is 4:5

(D) The aperture of the lens is 4.5 cm

Questions based on Microscope

Q.7 **The focal length of the objective of a microscope is–**

(A) Greater than the focal length of eye piece

(B) Lesser than the focal length of the eye piece

(C) Equal to the focal length of the eye piece

(D) Any of (A) (B) and (C)

Q.8 **When length of a microscope tube increases its magnifying power–**

(A) Decreases **(B) Increases**

(C) Does not change **(D) May increase or decrease**

Q.9 **In a simple microscope, if the final image is located at infinity then its magnifying power–**

(A) 25/F **(B) 25/D** **(C) F/25** **(D) (1+ 25/F)**

Q.10 **In a simple two lens refracting microscope, the intermediate image in normal use is–**

(A) Virtual, erect and magnified **(B) Real, erect and magnified**

(C) Real, inverted and magnified **(D) Virtual, inverted and magnified**

Q.11 **The magnifying power of compound microscope in terms of the magnification m_0 due to objective, and the magnification power m_E by the eye-piece is given by**

(A) $\frac{m_0}{m_E}$ **(B)** $m_0 \times m_E$ **(C)** $m_0 + m_E$ **(D)** $m_0 - m_E$

Q.12 **The final image produced by a compound microscope is**

(A) Real and erect **(B) Virtual and erect**

(C) Virtual and inverted **(D) Real and inverted**

Q.13 In a simple microscope, if the final image is located at 25 cm from the eye placed close to the lens, then the magnifying power is

(A) 25/F (B) 1 + 25/F (C) F/25 (D) F/25 + 1

Questions based on Telescope

Q.14 In astronomical telescope, the final image is formed at –

(A) The least distant of distinct vision (B) The focus of objective ions

(C) The focus of the eye lens (D) Infinity

Q.15 If f_0 and f_e are the focal lengths of the objective and eye-piece respectively for a telescope, the magnifying power of the telescope will be–

(A) f_0 / f_e (B) $f_0 \times f_e$ (C) f_0 / f_e (D) $f_0 + f_e$

Q.16 In which of the following the length of the tube is equal to the sum of the focal lengths of the field and eye lenses–

(A) Astronomical telescope (B) Galilean telescope

(C) Terrestrial telescope (D) Microscope

Q.17 Two convex lenses of focal length 0.3 m and 0.05 m are used to make a telescope. The distance kept between them is equal to–

(A) 0.35 m (B) 0.25 m (C) 0.175 m (D) 0.15 m

Q.18 An astronomical telescope has a magnifying power 10. The focal length of the eye piece is 20 cm. The focal length of the objective is–

(A) 2 cm (B) 200 cm (C) (1/2) cm (D) (1/200) cm

Q.19 The magnifying power of an astronomical telescope can be increased, if we–

(A) Increase the focal length of the objective

(B) Increase of the focal length of the eye piece

(C) Decreases the focal length of the objective

(D) Decrease the focal length of the objective and at the same time increase the focal length of the eye piece.

Q.20 When the length of an astronomical telescope tube increases its magnifying power–

(A) Decreases (B) Increases

(C) Does not change (D) May increase or decrease

Q.21 The resolving power of a telescope depends upon–

(A) Length of telescope (B) Focal length of objective

(C) Diameter of objective (D) Focal length of the eye piece

Q.22 If an astronomical telescope has objective and eye-pieces of focal lengths 200 cm and 4 cm respectively, then the magnifying power of the telescope for the normal vision is

(A) 42 (B) 50 (C) 58 (D) 204

Q.23 In question no. 22, the length of the telescope for normal vision is

(A) 204 cm (B) 200 cm (C) 196 cm (D) 203.45 cm

Q.24 In question no.22, the magnifying power of the telescope for distinct vision is

(A) 42 (B) 50 (C) 58 (D) 204

Q.25 In question no.22, the length of the telescope for distinct vision is

(A) 204 cm (B) 200 cm (C) 196 cm (D) 203.45 cm

Q.26 If F_0 and F_e are the focal lengths of the objective and eye-piece respectively for a Galilean telescope, its magnifying power is about

(A) $F_0 + F_e$ (B) $F_0 \times F_e$ (C) F_0/F_e (D) $\frac{1}{2}F_0 + F_e$

Q.27 The numerical value of the length of Galilean telescope for normal vision is (assuming f_0 and f_e as positive length)

(A) $f_0 \times f_e$ (B) f_0/f_e (C) $f_0 + f_e$ (D) $f_0 - f_e$

LEVEL 2

Q.1 The objective of a small telescope has focal length 120 cm and diameter 5 cm. The focal length of the eye piece is 2 cm. The magnifying power of the telescope for distant object is –

(A) 12 (B) 24 (C) 60 (D) 300

Q.2 With a simple microscope if the lens is held at a distance d from the eye and the image is formed at the least distance of distinct vision from the eye, then the magnifying power is

(A) $\frac{D}{f}$ (B) $1+\frac{D}{f}$ (C) $1+\frac{D-d}{f}$ (D) $1+\frac{D+d}{f}$

Q.3 To remove the chromatic aberration, the combination of lenses should be such that

(A) $F_R + F_V = 0$ (B) $F_R > F_V$ (C) $F_R < F_V$ (D) $F_R - F_V = 0$

Q.4 The distance between the objective lens and the eye lens of an astronomical telescope when adjusted for parallel light is 100 cm. The measured value of the magnification is 19. The focal lengths of the lenses are–

(A) 85 and 15 cm (B) 82 and 18 cm

(C) 95 and 5 cm (D) 50 and 50 cm

Q.5 If we double the aperture of a photographic camera, the new exposure should be–

(A) 1/4 (B) 1/2

(C) 2 (D) Same as before

Q.6 **When the object is self- luminous, the resolving power of a microscope is given by the expression–**

(A) $\frac{2\mu\sin\theta}{\lambda}$ (B) $\frac{\mu\sin\theta}{\lambda}$ (C) $\frac{2\mu\cos\theta}{\lambda}$ (D) $\frac{2\mu}{\lambda}$

Q.7 **A Galilean telescope measures 9 cm from the objective to the eye-piece. The focal length of the objective is 15 cm. Its magnifying power is–**

(A) 2.5 **(B) 2/5** **(C) 5/3** **(D) 0.4**

Q.8 **In order to increase the magnifying power of a telescope–**

(A) The focal powers of the objective and the eye-piece should be large

(B) Objective should have small focal length and the eye-piece large

(C) Both should have large focal length

(D) The objective should have large focal length and the eye-piece should have small

Q.9 **An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eye-piece is 36 cm and the final image is formed at infinity. The focal length f_0 of the objective and f_e of the eye-piece are –**

(A) $f_0 = 45$ **cm and** $f_e = -9$ **cm** **(B)** $f_0 = 50$ **cm and** $f_e = 10$ **cm**

(C) $f_0 = 7.2$ **cm and** $f_e = 5$ **cm** **(D)** $f_0 = 30$ **cm and** $f_e = 6$ **cm**

LEVEL 3

Each of the questions given below consist of Statement - I and Statement - II. Use the following key to choose the appropriate answer.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement - I.

(B) If both Statement - I and Statement - II are true but Statement - II is not the correct explanation of Statement - I.

(C) If Statement - I is true but Statement - II is false.

(D) If Statement - I is false but Statement - II is true.

Q.1 **Statement I: A simple microscope of smaller focal length is preferred.**

Statement II: For a simple microscope

$$m = \frac{1}{f} + D$$

Q.2 **Statement I: The focal length of objective of telescope is larger than that of eye lens.**

Statement II: Magnifying power of a telescope increases when the aperture's diameter is reduced.

Q.3 Statement I: The image formed by the simple microscope cannot be projected on the screen.

Statement II: The image formed by simple microscope is always virtual.

Q.4 Statement I: In a simple microscope, image formed is virtual, erect and magnified.

Statement II: Object to be seen is held between optical centre and focus of a convex lens.

Q.5 Statement I: Magnifying power of a compound microscope is negative.

Statement II: Final image is inverted with respect to the object.

Q.6 Statement I: In normal adjustment, length of a telescope is $L = f_0 + f_c$.

Statement II: The length can always be adjusted.

Q.7 Statement I: To increase the depth of focus, photographer decreases the aperture and increases the time of exposure.

Statement II: More light enters the camera when aperture is small.

Passage

The focal lengths of the objective and the eye-piece of a compound microscope are 1 cm and 5 cm respectively. An object is placed at a distance of 1.1 cm from the objective.

Q.8 If the final image is formed at the least distance of distinct vision, the magnifying power is –

(A) 60 **(B) 50**
(C) 40 **(D) none of these**

Q.9 In previous question, what is the distance between the two lenses?

(A) 16 cm **(B) 15.17 cm** **(C) 11 cm** **(D) 6 cm**

Q.10 If the final image is formed at inifinity, the magnifying power and the distance between the lenses are respectively given by –

(A) 60, 16 cm **(B) 50, 16 cm** **(C) 50, 15.7 cm** **(D) 60, 16.7 cm**

True and False type questions :

Q.11 For the two given statements:

(1) The length of an astronomical telescope for normal vision is $f_0 + f_e$.

(2) An astronomical telescope consisting of an objective of focal length 60 cm and a single eye lens of focal length 5 cm, is focussed on a distant object such that parallel

rays emerge from the eye-lens. If the object subtends an angle of 2° at the objective, the angular width of the image is 24°.

(A) 1 is true, 2 is false (B) 2 is true, 1 is false

(C) Both are true (D) Both are false

LEVEL 4

(Questions asked in previous years' AIEEE & IIT-JEE)

Section - A

Q.1 An astronomical telescope has a large aperture to – [AIEEE-2002]

(A) Reduce spherical aberration

(B) Have high resolution

(C) Increase span of observation

(D) Have low dispersion

Q.2 Wavelength of light used in an optical instument are $\lambda_1 = 4000A$ and $\lambda_2 = 5000A$, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is– [AIEEE-2002]

(A) 16 : 25 (B) 9 : 1 (C) 4 : 5 (D) 5 : 4

Q.3 The image formed by an objective of a compound microscope is– [AIEEE-2003]

(A) Real and diminished (B) Real and enlarged

(C) Virtual and enlarged (D) Virtual and diminished

Q.4 Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm] [AIEEE-2005]

(A) 5 m (B) 1 m (C) 6 m (D) 3 m

Q.5 An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by– [AIEEE-2008]

(A) A standard laboratory scale

(B) A meter scale provided on the microscope

(C) A screw gauge provided on the microscope

(D) A vernier scale provided on the microscope

Section - B

Q.1 A planet is observed by an astronomical reflecting telescope having an objective of focal length 16 m and an eye-piece of focal length 2 cm [IIT - 92]

(A) The distance between the objective and the eye-piece is 16.02 m
(B) The angular magnification of the planet is 800
(C) The image of planet is inverted
(D) All of these

Q.2 **The resolving power of an electron microscope is higher than that of an optical microscipe because the wavelength of electrons is .... than the wavelength of visible light.** **[IIT - 92]**

(A) smaller **(B) larger**
(C) of any size **(D) none of these**

Q.3 **In a compound microscope, the intermediate image is:** **[IIT 2000]**

(A) Virtual erect and magnified **(B) Real erect and magnified**
(C) Real, inverted and magnified **(D) Virtual, erect and reduced**

Q.4 **Focal length of objective & eyepiece of a telescope are f_O & f_e respectively.** **[IIT 2006]**

Column - I	**Column - II**
(A) Intensity of the image	**(P) Radius of curvature (R)**
(B) Angular magnification	**(Q) Dispersion of lens**
(C) Length of telescope	**(R) Focal length f_O & f_e**
(D) Sharpness of image	**(S) Spherical aberration**

Answer Key

Level 1

1. A	**2.** D	**3.** C	**4.** D	**5.** B	**6.** C
7. B	**8.** A	**9.** A	**10.** C	**11.** B	**12.** C
13. B	**14.** D	**15.** C	**16.** A	**17.** A	**18.** B
19. A	**20.** A	**21.** C	**22.** B	**23.** A	**24.** C
25. D	**26.** C	**27.** D			

Level 2

1. C	**2.** C	**3.** D	**4.** C	**5.** A	**6.** A
7. A	**8.** D	**9.** D			

Level 3

1. C	**2.** C	**3.** A	**4.** A	**5.** A	**6.** B
7. C	**8.** A	**9.** B	**10.** B	**11.** C	

Level 4

Section A

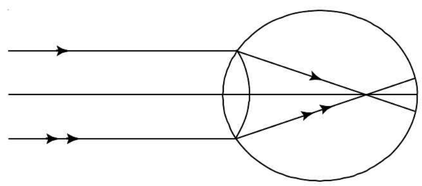
1. B	**2.** D	**3.** C	**4.** A	**5.** D

Section B

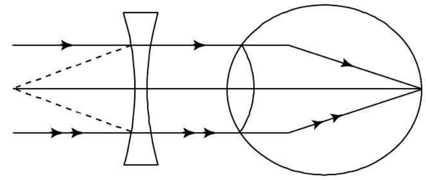
1. D **2.** A **3.** C **4.** A → P; B → R; C → R; D → P, Q, S

Defects of Eyes

MYOPIA [or Short-sightedness or Near-sightedness]



Defective-eye
(a)

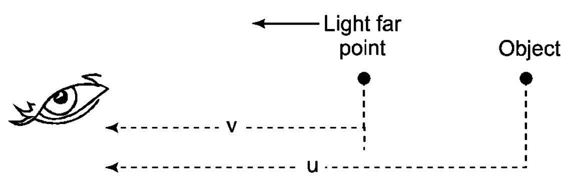


Corrected -eye
(b)

(i) Distant object are not clearly visible, but near object are clearly visible because image is formed before the ratina.

(ii) To remove the defect concave lens is used.

- The maximum distance which a person can see without help of spectacles is known as far point.



- If the reference of object is not given then it is taken as infinity.
- In this case image of the object is formed at the far point of person.

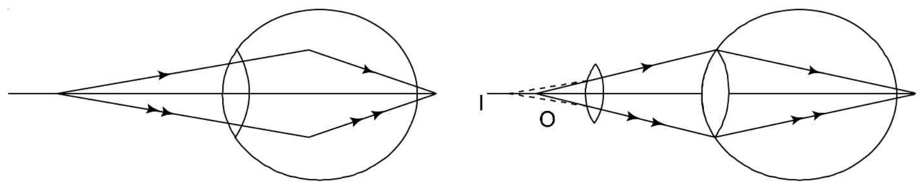
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=P \Rightarrow \frac{1}{\text{distance of far point (in m)}}$$

$$-\frac{1}{\text{distance of object (in m)}}=\frac{1}{f}=P$$

$$\Rightarrow \frac{100}{\text{distance far point (in cm)}}-\frac{100}{\text{distance of object (in cm)}}=P$$

distance of far point = –ve ,
distance of object = –ve, P = –ve

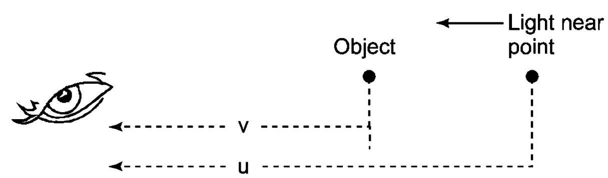
Long-Sightedness or Hypermetropia



(i) Near object are not clearly visible but far object are clearly visible.
(ii) The image of near object is formed behind the retina.
(iii) To remove this defect convex lens is used.

Near Point:-

The minimum distance which a person can see without help of spectacles.



- In this case image of the object is formed at the near point.
- If reference of object is not given it is taken as 25 cm.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=P \Rightarrow \frac{1}{\text{distance of near point (in m)}}$$

$$-\frac{1}{\text{distance of object (in m)}}=\frac{1}{f}=P$$

$$\Rightarrow \frac{100}{\text{distance of near point (in cm)}}-\frac{100}{\text{distance of object (in cm)}}=P$$

distance of near point = –ve ,
distance of object = –ve, P = + ve

Presbyopia

In this case both near and far object are not clearly visible.
To remove this defect two separate spectacles one for myopia and other for hypermetropia are used or bifocal lenses are used.

Astigmatism

This defect arises due to imperfection of complete spherical shape of eye lens. In this defect eye can not see object in two orthogonal directions clearly. It can be removed by using cylinderical lens in perticular direction.

Ex.4 A person can not see clearly an object kept at a distance beyond of 100 cm. Find the nature and the power of lens to be used for seeing clearly the object at infinity.

Sol. For lens u = – ∞ and v = –100 cm

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} - \frac{1}{f} \quad \Rightarrow \quad f = v = -100 \text{ cm}$$

$$\therefore \quad \text{Power of lens } P = \frac{1}{f} = \frac{1}{1} = -1\text{D}$$

Ex.5 A far sighted person has a near point of 60 cm. What power lens should be used for eye glasses such that the person can read this book at a distance of 25 cm.

Sol. It is given that his far sight is good beyond 60 cm. His near sight is bad. If he desires to read book at u = –25 cm, then for good vision image distance should be at a distance of 60 cm or more. For minimum power lens, v = – 60 cm, u = –25 cm.

Therefore the focal length of the lens used should be such that

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = -\frac{1}{60} + \frac{1}{25} \quad \Rightarrow \quad f = \frac{300}{7} \text{ cm}$$

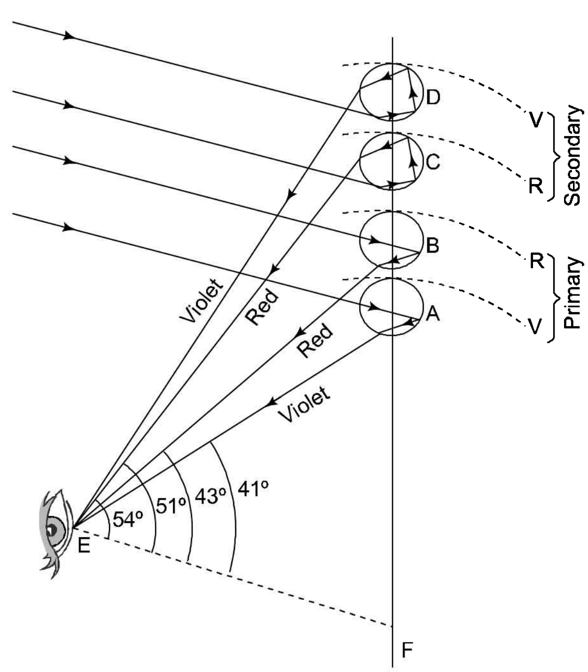
$$\therefore \quad \text{Power } P = \frac{1}{f\text{ (in metre)}} = \frac{1}{(3/7)} = +2.33 \text{ D}$$

Rainbow

(i) After the rain, the sky is almost clear and there is bright Sunshine, with clouds in the east.

(ii) The Sun is at the back of the observer.

Figure shows four rain drops A, B, C and D. The light suffers only one total internal reflection in drops A and B. While the least deviated violet ray is received from drop A, the least deviated red ray is received from drop B. The red and violet rays are received at angles of 43° and 41° respectively with the line EF parallel to the rays of the Sun. If EA is rotated about EF to describe a cone with vertex at E, then A would describe a vertical circle whose centre would be at F. The observer would receive the least deviated violet rays from all the drops above the horizon and on the surface of the cone just described. So, a violet arc will be observed.



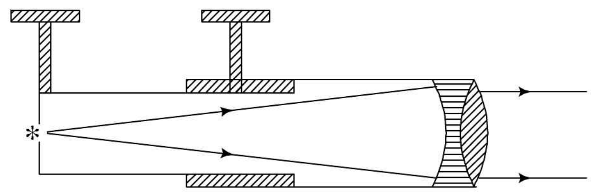
(1) Following are the **main features** of a primary rainbow:
   (i) The red colour is at the outer edge.
   (ii) The inner edge is violet. The violet rays make an angle of 41° with the Sun rays.
   (iii) The angle between the red rays having minimum deviation and the Sun rays is nearly 43°.
   (iv) All other colours lie between these two boundaries, i.e., within 2°.

(2) Following are the main features of a secondary rainbow:
   (i) The outer edge is violet.
   (ii) The violet rays make an angle of 54° with the Sun rays
   (iii) The inner edge is red. The red rays make an angle of 51° with the Sun rays.
   (iv) All other colours lie between these two edges.

Spectrometer

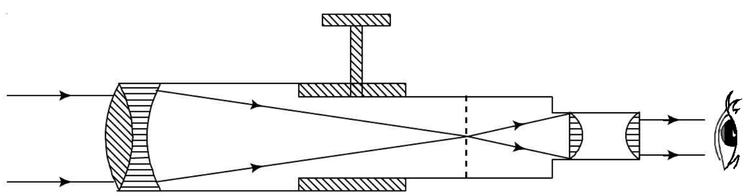
It essentially consists of three parts–Collimator, Prism table and Telescope. All the three parts are mounted in such a way that they can revolve around a common vertical axis.

(i) Collimator (C): It is a metallic tube having an adjustable slit at one end and an achromatic convergent lens at the other end. The width of the slit can be adjusted with a fine screw. The distance between the slit and the lens can be adjusted with the help of rack and pinion arrangement. In the adjusted position, the slit is at the principal focus of the lens. The source is placed a little beyond the slit. A parallel beam of light comes out of the collimator.

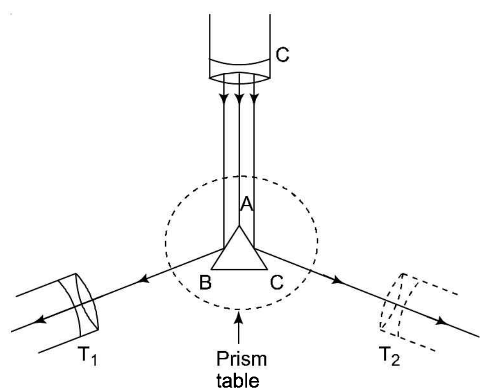


(ii) Prism table (P): It is small metallic table mounted on a vertical support with the help of which the table can be raised or lowered to a convenient height.

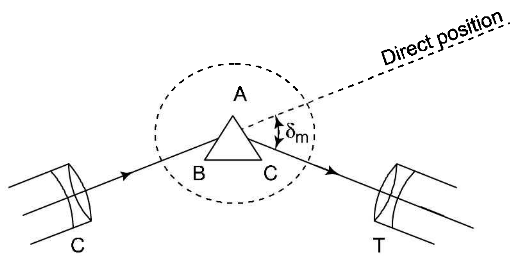
(iii) Teloscope (T): It consists of a metallic tube having an achromatic objective lens at one end and an eyepiece at the other end. With the help of rack and pinion arrangement, the eyepiece (along with cross wires) can be moved backward of forward. The telescope is supported in the same horizontal plane as that of the collimator. For the adjustments, the telescope is provided with a tangent and a clamping screw.



Determination of angle of prism. The prism ABC is placed on the prism table so that its refracting edge A coincides with the centre of the prism table. Now, the rays coming from the collimator will fall simultaneously on the faces AB and AC of the prism and get reflected from these faces. Keeping the prism table fixed, the telescope is turned to position T_1 so that the image of the slit coincides with the cross-wires of the telescope. This position of the telescope is noted on the circular scale. Let the telescope be now turned to the position T_2 on the other side such that the image of the slit is again formed at the cross-wires of the telescope. This position is again noted on the circular scale. Half the difference of the two readings give the angle of prism.



Determination of the angle of minimum deviation. Place the prism symmetrically on the prism table so that light from the collimator falls on one of the surfaces of the prism and the refracted light emerges from the other surface. The telescope is turned to receive the refracted light so that the spectrum is observed through the eyepiece. Now, turn the prism table so that the image of the slit as seen through the telescope moves in the direction of the incident light. The telescope is also turned so that the reflected image of the slit remains clearly visible. If prism table and telescope continue to be rotated slowly in this way, then a stage will be reached where the refracted image of the slit will become stationary. [Any further rotation of the prism in the same direction will start the 'turning back' of the refracted image.] At this stage, the prism is said to be in the minimum deviation position. The reading is taken on the circular scale. The prism is now removed and the telescope is turned towards collimator till the cross-wires coincide with the image of the slit. Reading is again taken on the circular scale. The difference between two readings gives the angle δ_m of minimum 7 deviation.



CHAPTER

6 Wave Optics

6.1 INTRODUCTION

In 1680 Christian Huygens suggested that light propagates in the form of waves. Huygens did not know anything about the nature of the light wave ; whether it is a transverse wave or a longitudinal wave ; he had no knowledge about the speed of light or its wavelength. The wave theory of light received the first experimental evidence in 1801 from the interference experiments conducted by Thomas Young. Using the principle of superposition and Huygen's wave concept, Young explained the interference effects observed in various instances such as double slit experiment, the striking colours observed on oil slicks exposed to sunlight and Newton's rings.

6.2 WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

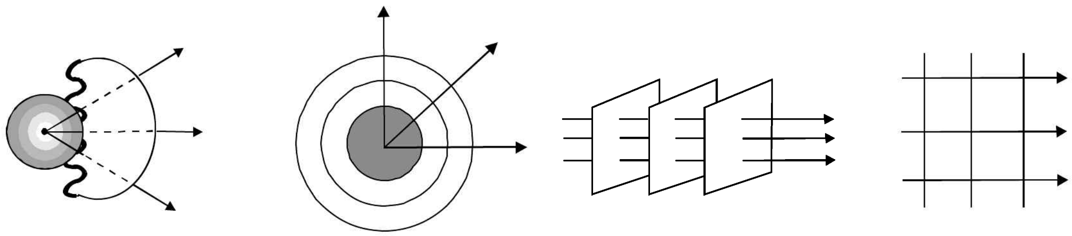
Figure (2.1a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e.. radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (2.1b)

A linear source such as a slit Illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wave fronts may be regarded as planar.



(a) (b)

Figure 2.1 : Wavefronts and the corresponding rays in. two cases: (a) diverging spherical wave. (b) plane wave. The figure on the left shows a wave (e.g.. light) in three dimensions. The figure on the right shows a wave in two dimensions (a water surface).

6.3 HUYGEN'S PRINCIPLE

1. Every point on a wavefront vibrates in same phase and with the same frequency.
2. Every point on a wavefront acts like a secondary source and sends out a spherical wave, called a secondary wave.
3. Wavefronts move in space with the velocity of wave in that medium.

6.4 COHERENCE

Two sources which vibrate with a fixed phase difference between them are said to be coherent. The phase differences between light coming from such sources does not depend on time.

6.4(A) COHERENCE SOURCES

Two sources are said to be coherent, if they produce waves of same frequency with a constant phase difference. Unlike sound waves, two independent sources of light cannot be coherent. Since sound is a bulk property of matter, therefore, two independent sources of sound can be identical in all respects and can produce coherent waves. On the contrary, light is not a bulk property of matter, it is a property of each individual atom. As the individual atoms emit light randomly and independently, therefore, two independent sources of light cannot be coherent.

6.4(B) HOW TO PRODUCE COHERENT SOURCES OF LIGHT WAVES?

Two coherent sources are produced from a single source of light by adopting any of following two methods.

• **Division of Wavefront**

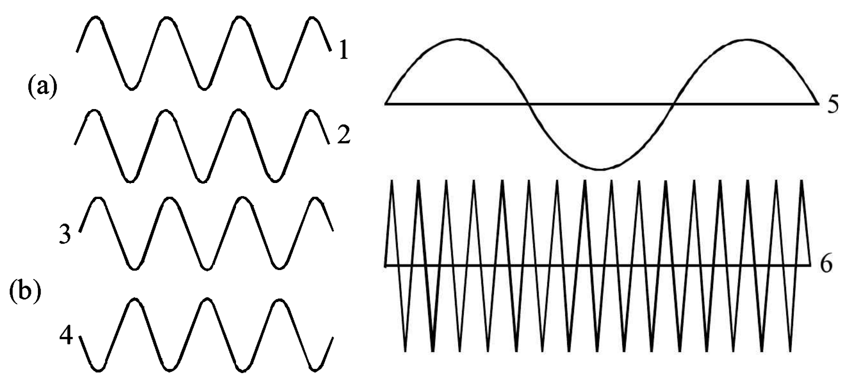
It is done in Young's double slit experiment, Fresnel's biprism and LIoyd's mirror, etc.

• Division of Amplitude

Usually by partial reflection and transmission at a boundary, as it occurs in thin film interference, Newton's rings, etc.

6.5 PHASE DIFFERENCE AND COHERENCE

Suppose two waves are passing through a point in space. If the frequencies of the two waves are different, the phase difference between the vibrations changes with time. The waves will drift out of phase because the crests of the higher frequency wave will arrive ahead of the crests of the lower frequency wave (see Fig.). Also, if one (or both) of the waves undergoes changes in frequency irregularly, the phase difference changes irregularly. Under these conditions the two waves are said to be incoherent. The light emitted by most of the light sources is incoherent as the frequency of light changes abruptly and irregularly, though we can think of an average frequency associated with the light wave. On the other hand, if we consider two waves of same frequency, they may differ in the amplitudes but they maintain a predictable phase relationship (see Fig.). The difference in their phases may have any value from zero radians to a maximum of 2π radians; but the phase difference remains constant. Thus, two or more waves of the same frequency can maintain the same phase or constant phase difference over a distance and time. Such waves are said to be coherent waves.



(a) & (b) Coherent waves (c) In coherent waves

(a) Waves and 2 are in-phase, (b) Waves 3 and 4 are in opposite phase & (c) Waves 5 and 6 are incoherent

When coherent waves rise or fall together, reaching the crest (or trough) at the same time, they are said to be in phase, as shown in Fig. The phase difference will be 0 or 2π radians, which remains constant as the waves propagate in space. The path difference between the waves will then be zero or an integral multiple of a wavelength λ. Such waves move through space with a crest-to-crest correspondence. When one wave reaches its crest while the other falls to its trough, then the phase difference between the waves is π and the waves are said to be in opposite phase and the phase difference remains 180º (see Fig.). They are inverted with respect to each other everywhere. Then the path difference between the waves will be $\lambda/2$ or an odd integral multiple of $\lambda/2$.

The waves may have also any constant phase difference other than zero or π radians. If two (or more) waves maintain a constant phase difference over a long distance and time, then they are said to be coherent. Two waves of different frequency (see Fig.) can never maintain a constant phase difference, because their phase difference goes on fluctuating and changes arbitrarily. They are said to be incoherent, as the phase difference between the waves fluctuates with time.

Thus, coherence means the coordinated motion of several waves in a medium maintaining a fixed and predictable phase relationship over a length of time.

The waves shown in Fig. (a) and (b) are coherent waves. Sources, which produce coherent waves, are called coherent sources.

In the study of interference, we focus our attention mainly on the two specific types of disposition of the waves, namely in phase and opposite phase dispositions, at the point of observation.

6.6 OPTICAL PATH AND PHASE CHANGE

Optical path length indicates the number of light waves that fit into that path. Thus; $\Delta = N\lambda$, where Δ is the optical path length and N is an integer or a mixed fraction. Optical path is related to the geometric path and the relation may be found as follows. The distance traversed by light in a medium of refractive index μ in time t is given by

$$L = \upsilon t$$

where υ is the velocity of light in the medium. The distance travelled by light in a vacuum in the same time t, is $\Delta = c\,t = c\,.\dfrac{L}{\upsilon} = \mu L$

The distance L is called the geometric path length (GPL). Δ is the equivalent distance in a vacuum and is called optical path length (OPL). Thus,

$$\text{O.P.L.} = \mu \times \text{G.P.L.} \qquad \text{or} \qquad \Delta = \mu L$$

The above result means that more number of waveforms are accommodated along the optical path than in the corresponding geometrical path. Therefore, in the study of interference we always must calculate the optical paths travelled by light rays.

(a) Effect of Optical path : Optical path determines the phase of a light wave arriving at a point. We know that if a wave covers a distance of one wavelength in air i.e. 1λ, its phase changes by 2π radians. Therefore, we compute that if a wave travels a distance L in air, its phase change is given by

$$\delta = \frac{2\pi L}{\lambda} \qquad \text{...... (i)}$$

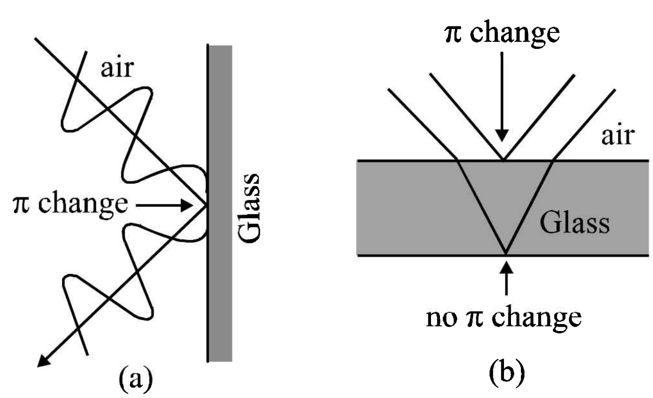
When the wave travels the distance L in a medium, then $\delta = \dfrac{2\pi\Delta}{\lambda} = \dfrac{2\pi\mu L}{\lambda}$ ....(ii)

where Δ is the optical path or optical path difference.
Comparing equ. (i) and (ii), we find that a light path of geometric length L in a medium of refractive index μ produces the same phase change as a light path of length μL in a vacuum.

(b) Effect of reflection : The process of reflection also affects the phase of a light wave. When light is incident on a surface, part of the light gets reflected while a major portion may be transmitted or absorbed. The quantity characterizing the reflectivity of a surface is called the reflection coefficient, ρ. ρ depends on the nature of the surface, the angle of incidence of light and many other factors. Augustin Jean Fresnel, about 150 years ago, derived a set of expressions which allow us to calculate the amount of light reflected and transmitted at an interface. For normal incidence it was shown that

$$\rho = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \qquad \text{.......... (3)}$$

It may be seen that ρ is positive when $\mu_2 < \mu_1$. It implies that the oscillations in the incident and reflected waves occur in the same phase. On the other hand, when $\mu_2 > \mu_1$, ρ is negative signifying that the oscillations in the incident and reflected waves are in opposite phase. Hence, we draw the following conclusions :



phase change due to reflection

(i) A light wave travelling from a rarer medium (μ_1) to a denser medium (μ_2) undergoes a phase change of π radians when it gets reflected at the boundary of denser medium, as shown in Fig.. The wave loses a half-wave on reflection at the boundary of rarer to-denser medium.

(ii) A light wave travelling from a denser medium (μ_2) to a rarer medium (μ_1) does not undergo a change in phase on reflection at the boundary of denser-to-rarer medium (Fig.). Therefore, the change in path is zero.

6.7 SUPERPOSITION OF WAVE

Frequently it is necessary to find the resultant disturbance at a point when a number of disturbances arrive simultaneously. According to the principle of superposition -

When two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

It means that the resultant is simply the sum of the disturbances. The principle of superposition applies to electromagnetic waves also and is the most important principle in wave optics. In case of electromagnetic waves, the term displacement refers to the amplitude of the electric field vector. Interference is an important consequence of superposition of coherent waves.

6.8 INTERFERENCE

The term interference refers to any situation in which two or more waves overlap in space. When this occurs, the displacement at any point at any instant of time is governed by the principle of superposition, which states:

When two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

Interference effects are most easily seen when we combine sinusoidal waves with a single frequency f and wavelength λ. In optics, sinusoidal waves are characteristic of monochromatic light (light of a single colour.) As we analyze characteristics of interference and diffraction, we will assume that we are working with monochromatic waves (unless we explicitly state otherwise.)

The conditions for sustained interference are given as follows:

(a) The initial phase difference between the interfering waves must remain constant.

(b) The frequencies and the wavelengths of the two waves should be equal. If not, the phase difference will not remain constant and so the interference will not sustain.

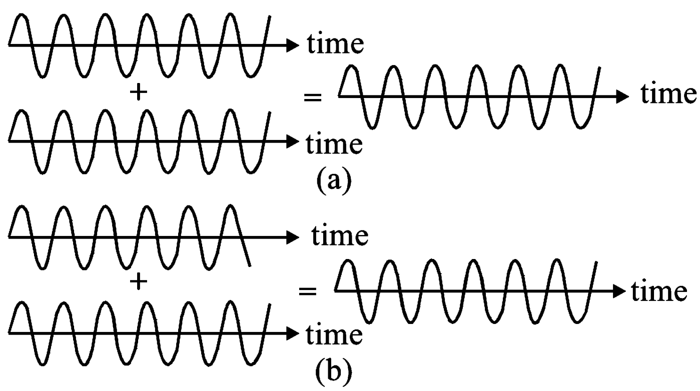
(c) The light will be monochromatic. This eliminates overlapping of patterns as each wave length corresponds to one interference pattern.

With sound waves, as we have studied, the interference pattern can be observed without much difficulty because the two interfering waves maintain a constant phase relationship; this is also the case with the microwaves. Since process of emission occurs at the atomic level, one cannot observe any interference between waves from two independent sources. Interference can be observed by deriving two interfering sources from the same source. The methods to achieve this can be classified under two broad categories as

(i) Division of wave front

(ii) Division of amplitude.

INTERFERENCE : If two or more light waves of the same frequency overlap at a point, the resultant effect depends on the phase of the waves as well as their amplitudes. The resultant wave at any point at any instant of time is governed by the principle of superposition. The combined effect at each point of the region of superposition is obtained by adding algebraically the amplitudes of the individual waves. Let us assume here that the component waves are of the same amplitude.



At certain points, the two waves may be in phase.The amplitude of the resultant wave will then be equal to the sum of the amplitudes of the two waves, as shown in Fig. 14.5 (a).Thus, the amplitude of the resultant wave, $A_R = A + A = 2A$.

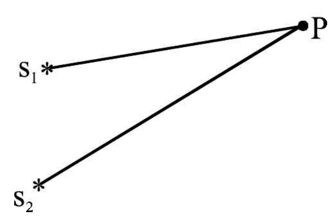
Hence, the intensity of the resultant wave is, $I_R \propto A_R^2 = 2^2 A^2 = 2^2 I$.

It is obvious that the resultant intensity is greater than the sum of the intensities due to individual waves. $I_R > I + I = 2I$

Therefore, the interference produced at these points is known as constructive interference.

A stationary bright band of light is observed at the points of constructive interference.

At certain other point, the two waves may be in opposite phase. The amplitude of the resultant wave will then be equal to the sum of the amplitudes of the two waves, as shown in Fig. 14.4 (b). Thus, the amplitude of the resultant wave, $A_R = A - A = 0$.
Hence, the intensity of the resultant wave is, $I_R \propto 0^2 = 0$.
It is obvious that the resultant intensity is less than the sum of the intensities due to individual waves. $I_R < 2I$
Therefore, the interference produced at these points is known as destructive interference. A stationary dark band of light is observed at points of destructive interference. Thus, we see that a redistribution of energy took place in the region.
Thus, when two or more coherent waves of light are superposed, the resultant effect is brightness in certain regions and darkness at other regions.The regions of brightness and darkness alternate and may take the form of straight bands, or circular rings or any other complex shape. The alternate bright and dark bands are called interference fringes.The phenomenon of redistribution of light energy due to the superposition of light waves from two or more coherent sources is known as interference.
Whether the condition (equation) occurs at a point is solely determined by the difference in the optical paths traversed by the waves that are superposing at that point.
Let us consider two sources of light $S_1$and S_2, as shown in Fig. Let us assume that the sources are identical and produce harmonic waves of same wavelength and that the waves are in the same phase at $S_1$and S_2. Light from these sources travel along different paths, S_1P and S_2P, and meet at a point P. We now wish to know whether we get brightness or darkness at P due to the superposition of waves.
Referring to Fig. we find that the waves move along the geometric paths $S_1P = r_1$ and $S_2P = r_2$, which are different in length. Also, the media through which the two waves travelled, may be different. As a result, the optical path lengths are different. If μ_1 is the refractive index of the medium in which the ray S_1P travelled, the corresponding optical path length is $\mu_1 r_1$. Similarly, if μ_2 is the refractive index of the medium in which the ray S_2P travelled, the corresponding optical path length is $\mu_2 r_2$.
These paths accommodate different number of waveforms along their lengths. The optical path difference between the waves at the point P is $(\mu_2 r_2 - \mu_1 r_1)$. It may come to few full waves or a mixed fraction of waves. It means that though the waves started with the same phase, they may arrive at P with different phases because they travelled along different optical path lengths.



If the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to zero or an integral multiple of wavelength λ , then the waves arrive in phase at P and superpose with crest - to - crest correspondence.

That is, if $\Delta = \pm n\lambda$

where n is an integer and takes values, m = 0, 1, 2, 3, 4, 5, ...,then the waves are in phase (see Fig.) and their overlapping at P produces constructive interference or brightness. On the other hand, if the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to an odd integral multiple of half-wavelength, $\lambda / 2$, then the waves arrive out of phase at P and superpose with crest-to-trough correspondence. That is, if $\Delta = \pm (2n - 1)\lambda / 2$

When n is an integer and takes values, n = 1, 2, 3, 4, 5,........., then the waves are inverted with respect to each other (see Fig.) and their overlapping at P produces destructive interference or darkness.

The regions of brightness and darkness are also known as regions of maxima and minima.

6.9 THEORY OF INTERFERENCE

(a) Analytical Method : Let us assume that the two sinusoidal waves arriving at point P vary with time as $Y_1 = A_1 \sin \omega t$ and $Y_2 = A_2 \sin(\omega t + \delta)$

where δ is the phase difference between them. According to Young's principle of superposition, the resultant electric field at the point P due to the simultaneous action of the two waves is given by

$Y_R = Y_1 + Y_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \delta)$

$= A_1 \sin \omega t + A_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta) = (A_1 + A_2 \cos \delta) \sin \omega t + A_2 \sin \delta \cos \omega t$ ....(1)

Equ (1). shows that the superposition of two sinusoidal waves having the same frequency but with a phase difference, produces a sinusoidal wave with the same frequency but with a different amplitude E.

Let $A_1 + A_2 \cos \delta = A \cos \phi$ ......(2)

and $A_2 \sin \delta = A \sin \phi$ ......(3)

where A is the amplitude of the resultant wave and ϕ is the new initial phase angle. In order to solve for A and ϕ, we square the equ. (2) & (3) and add them.

$(A_1 + A_2 \cos \delta)^2 + A_2^2 \sin^2 \delta = A^2 (\cos^2 \phi + \sin^2 \phi)$ or

$A^2 = A_1^2 + A_2^2 \cos^2 \delta + 2A_1A_2 \cos \delta + A_2^2 \sin^2 \delta$ or $A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \delta$

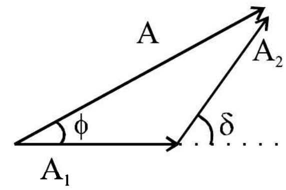
Thus, it is seen that the square of the amplitude of the resultant wave is not a simple sum of the squares of the amplitudes of the superposing waves, there is an additional term which is known as the interference term.

ALTERNATE SOLUTION BY THE (Phasor diagram method) :

$Y_1 = A_1 \sin \omega t$ and $Y_2 = A_2 \sin(\omega t + \delta)$; $Y = Y_1 + Y_2 = A \sin(\omega t + \phi)$

Where , A = Amplitude of resultant wave, ϕ = New initial phase angle.

Phasor diagram :



By the right angle triangle :

$A^2 = (A_1 + A_2 \cos\delta)^2 + (A_2 \sin\delta)^2$

$\Rightarrow A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\delta$

6.10 INTENSITY DISTRIBUTION

The intensity of a light wave is given by the square of its amplitude.

Using this relation into above equation, we get , $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\delta$

We see that the resultant intensity at P on the screen is not just the sum of the intensities due to the separate waves. The term $2\sqrt{I_1I_2}\cos\delta$ is known as the **interference term.** Whenever the phase difference between the waves is zero, i.e. $\delta = 0$, we have maximum amount of light. Thus, $\quad I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2}$ ....(4)

When $I_1 = I_2 = I_0 \quad ; \quad I_{max} = 4I_0$

It means that the resultant intensity I will be more than the sum of the intensities due to the two sources. When the phase difference is δ = 180° , cos 180° = –1 and we have a minimum amount of light. $I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2}$

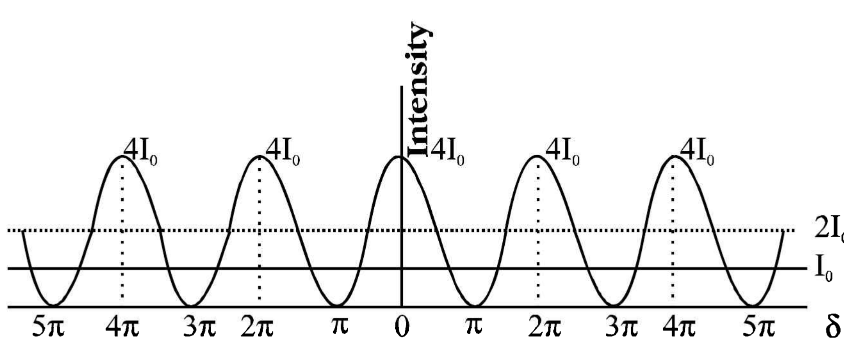
Which , when $I_1 = I_2$, becomes, $I_{min} = 0$

It means that the resultant intensity I will be less than the sum of the intensities due to the two sources. At points that lie between the maxima and minima, when $I_1 = I_2 = I_0$, we get $I = I_0 + I_0 + 2I_0 \cos\delta = 2I_0(1 + \cos\delta)$

Then using the identity $1 + \cos\delta = 2\cos^2\left(\frac{1}{2}\delta\right)$, we get $I = 4I_0\cos^2\left(\frac{1}{2}\delta\right)$ ....(5)

Equ.(5) shows that the intensity varies along the screen in accordance with the law of cosine square. Figure shows the variation of intensity as a function of phase angle δ.

It is seen from the plot that the intensity varies from zero at the fringe minima to $4I_0$ at the fringe maxima.



NOTE : In interference, energy is neither created nor destroyed but it is conserved.

In case of interference, intensity at a point is given by

$$I = I_1 + I_2 + 2\left(\sqrt{I_1 I_2}\cos\delta\right)\cos\delta$$

Now, as at a given point phase difference can have any value between 0 and 2π, the average value of intensity will be

$$I_{av}\left[\frac{\int_0^{2\pi} I\, d\delta}{\int_0^{2\pi} d\delta}\right] = \frac{1}{2\pi}\int_0^{2\pi} (I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta)\, d\delta$$

$$I_{av} = I_1 + I_2 \quad ; \quad \left[\text{as } \int_0^{2\pi} (\cos\delta)\, d\delta = 0\right]$$

As the average value of intensity is equal to the sum of individual intensities, in interference energy is neither created nor destroyed but is redistributed. Hence in interference, energy is conserved.

6.11(A) SUPERPOSITION OF INCOHERENT WAVES

Incoherent waves are the waves that do not maintain a constant phase difference.Then the phase of the waves fluctuate irregularly with time and independent of each other. In case of light waves, the phase fluctuates randomly at a rate of about 10^8 per second. Light detectors such as human eye, photographic film etc cannot respond to such rapid changes. The detected intensity is always the average intensity, averaged over a time interval which is very much larger than the time of fluctuation. Thus,

$$I_{ave} + I_1 + I_2 + 2\sqrt{I_1 I_2}\,(\cos\delta)$$

The average value of the cosine over a large time interval will be zero and hence the interference term becomes zero. Therefore, the average intensity of the resultant wave is

$I_{ave} = I_1 + I_1$; If $I_1 = I_2$, then $I_{ave} = 2I$

It implies that the superposition of incoherent waves does not produce interference but gives a uniform illumination.The average intensity at any point is simply equal to the sum of the intensities of the component waves.

6.11(B) SUPERPOSITION OF MANY COHERENT WAVES

The equation (3) may be written as, $I_{max} = 2^2 I_0$

which gives the resultant intensity when two coherent waves superpose. The resultant maximum intensity due to N coherent waves will be therefore, $I_{max} = N^2 I_0$

and the minimum intensity $I_{min} = 0$, where N represents the number of coherent waves superposing at a point.

6.12 CONDITIONS FOR OBSERVING SUSTAINED INTERFERENCE WITH GOOD CONTRAST

1. The initial phase difference between the interfering waves must remain constant otherwise the interference will not be sustained.
2. The frequencies and wavelengths of the two waves should be equal. If not, the phase difference will not remain constant and so the interference will not be sustained.
3. The light must be monochromatic. This eliminates overlapping of pattern as each wavelength corresponds to one interference pattern.
4. The amplitudes of the interfering waves must be equal. This improves constant with $I_{max} = 4I_0$ and $I_{min} = 0$.
5. The sources must be close to each other, otherwise due to small fringe width fringes may be so close to each other that the eye cannot resolve them, resulting in uniform illumination.
6. The sources must be narrow. A broad source will be equal to a large number of narrow sources and each set of two sources will give its own pattern and overlapping of patterns will result in uniform illumination.

6.13 TECHNIQUES OF OBTAINING INTERFERENCE

The phase relation between the waves emitted by two independent light sources rapidly changes with time and therefore they can never be coherent, though the sources are identical in all respects. However, if two sources are derived from a single source by some device, then any phase change occurring in one source is simultaneously accompanied by the same phase change in the other source. Therefore , the phase difference between the waves emerging from the two sources remains constant and the sources are coherent. The techniques used for creating coherent sources of light can be divided into the following two broad classes.

(a) Wavefront splitting :

One of the methods consists in dividing a light wavefront, emerging from a narrow slit, by passing it through two slits closely spaced side by side. The two parts of the same wavefront travel through different paths and reunite on a screen to produce fringe pattern. This is known as interference due to division of wavefront. This method is useful only with narrow sources. Young's double slit, Fresnel's double mirror, Fresnel's biprism, Lloyd's mirror, etc employ this technique.

(b) Amplitude splitting :

Alternately, the amplitude (intensity) of a light wave is divided into two parts, namely reflected and transmitted components, by partial reflection at a surface. The two parts travel through different paths and reunite to produce interference fringes.This is known as interference due to division of amplitude. Optical elements such as beam splitters, mirrors are used for achieving amplitude division. Interference in thin films (wedge, Newton's rings etc.), Michelson's interferometer etc interferometers utilize this method. This method requires extended source.

6.14 YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E.)

In 1802 Thomas Young devised a method to produce a stationary interference pattern. This was based upon division of a single wavefront into two; these two wavefronts acted as if they emanated from two sources having a fixed phase relationship. Hence, when they were allowed to interfere, stationary interference pattern was observed.

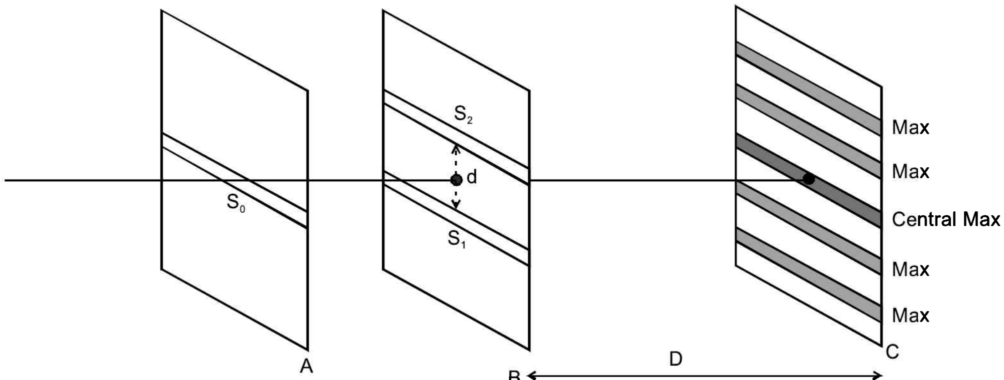


Figure : (A) *Young's Arrangement to produce stationary interference pattern by division of wave front S_0 into S_1 and S_2*

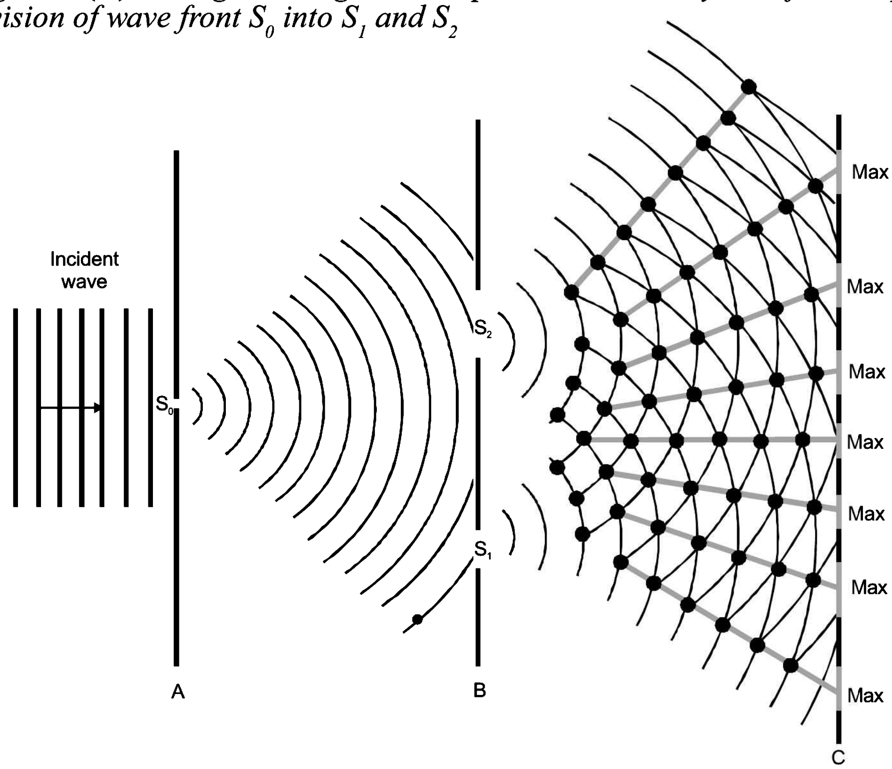


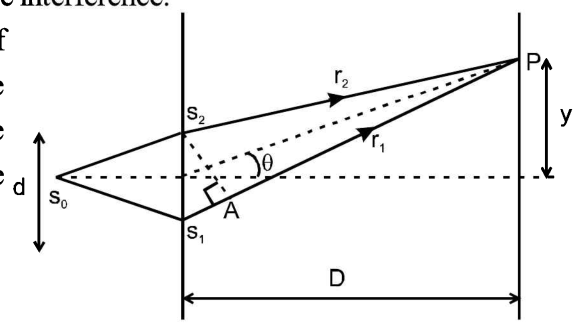
Figure : (B) In Young's interference experiment, light diffracted from pinhole S_0 encounters pinholes S_1 and S_2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producing an interference pattern on screen C.

(A) Analysis of Interference Pattern :

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However, the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former.

We have already discussed the phase-difference arising due to path difference. If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference.

If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.



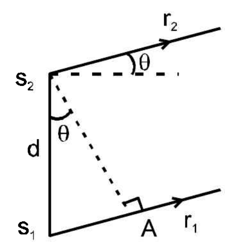
Path difference $\Delta p = S_1P - S_2P = \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2}$

Approximation I :

For D >> d, we can approximate rays $\vec{r}_1$ and $\vec{r}_2$ as being approximately parallel, at angle θ to the principal axis.

Now, $S_1P - S_2P = S_1A = = S_1S_2 \sin\theta$

$\Rightarrow$ path difference = $d \sin\theta$



Approximation II :

Further if $\lambda << d$ (Not comparable)

Then, θ in small we can write ($\sin\theta \approx \tan\theta$)

$\sin\theta = \tan\theta = \frac{y}{D}$ and hence, path difference = $d\sin\theta = \frac{dy}{D}$

Approximation III :

Further if $\lambda = d$ or $\lambda = 2d$ or $\lambda = 3d$ (Means comparable)

Then θ is not small. We can not write as ($\sin\theta \approx \tan\theta$)

In this case, if $\sin\theta = \frac{2}{3}$; then $\tan\theta = \frac{2}{\sqrt{5}}$ (By the right angle triangle)

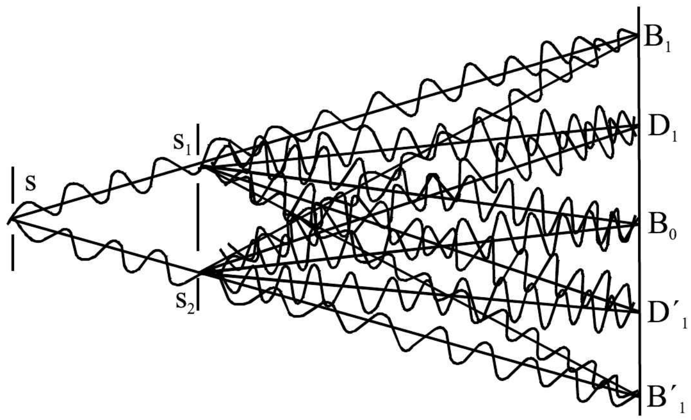
BRIGHT FRINGES :

Bright fringes occur wherever the waves from S_1 & S_2 interfere constructively. The first time this occurs is at O, the axial point. There, the waves from S_1& S_2travel the same optical path length to O and arrive in phase. The next bright fringe occurs when the wave from S_2travels one complete wavelength further the wave from S_1. In general constructive interference occurs if S_1P and S_2P differ by a whole number of wavelengths.

The condition for finding a bright fringe at P is that $S_2P - S_1P = \pm n\lambda : n = 0, 1, 2, 3, ...$

Using the equation, it means that, $\frac{yd}{D} = \pm n\lambda$

where m is called the order of the fringe.



The bright fringe B_0(at O), corresponding to $n = 0$, is called the zero-order fringe. It means the path difference between the two waves reaching at O is zero. Fringe at B_1 is the first order bright fringe from the axis corresponding to n = 1; the path difference between the two waves reaching B_1 is one λ. The second order bright fringe (n = 2) will be located where the path difference is 2λ and so on.

DARK FRINGES :

The first dark fringe occurs when ($S_2P - S_1P$) is equal to $\lambda / 2$. The waves are now in opposite phase at P.The second dark fringe occurs when ($S_2P - S_1P$) equals $3\lambda/2$. The mth dark fringe occurs when, $(S_2P - S_1P) = \pm (2n-1) \frac{n}{2}$

The condition for finding a dark fringe is $\frac{yd}{D} = \pm (2n-1)\frac{\lambda}{2}$

The first - order dark fringe D_1 (Fig.) from the axis corresponds to n = 1, where the path difference between the two waves is $\lambda/2$. The second order dark fringe (n=2) will be

produced where the path difference is $3\lambda/2$ and so on. For maxima (constructive interference)
$\Delta P = \pm n\lambda$; n = 0, 1, 2, 3, 4, ...
Here, n = 0 corresponds to the control, n = 1 corresponds to the control
n =1 correspond to the 1st maxima, n = 2 correspond to the 2nd maxima and soon.

If D>>d and l<<d. Then, $\Delta p = \frac{dy}{D}$; For the maxima : $\Delta P = \pm n\lambda$

$\Rightarrow \frac{dy}{D} = \pm n\lambda \quad \Rightarrow y = \pm \frac{n\lambda D}{d}$; n = 0,1,2,3,4......... Here,
n = 1 corresponds to first maxima, n = 2 corresponds to second minimum and soon.

If D>>d and λ<< d, than $\Delta p = \frac{yd}{D}$; For the maxima, $\Delta P = \pm (2n-1)\frac{\lambda}{2}$

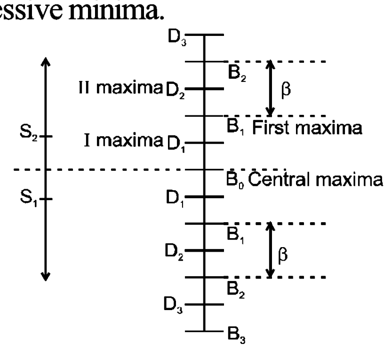
$\Rightarrow \frac{yd}{D} = \pm (2n-1)\frac{\lambda}{2}$

6.15 FRINGE WIDTH

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

(B) Fringe width :
It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

$\beta = y_n - y_{n-1}$, fringe width $\beta = \frac{\lambda D}{d}$



fringe pattern in YDSE

- Notice that it is directly proportional to the wavelength and inversely proportional to the distance between the two slits.

From the above equation, following points can be noted.
(i) Fringe width is independent of n, i.e., all the interference fringes have equal width in experiments where there is a division of wavefront of the incoming waves.

(ii) Fringe width is directly proportional to the wavelength of light used, i.e., $\beta \propto \lambda$. So, fringes for red light are wider than those for blue light.

(iii) Fringe width is inversely proportional to the separation between the slits, i.e., $\beta \propto (1/d)$. Thus, with increase in separation between the sources, fringe width decreases.

(iv) With increase in distance between screen and plane of slits, fringe width β increases linearly with D. However, with increase in D, intensity of light sources and hence of interfering waves is adversely affected.

(v) If the interference experiment is performed in a medium of refractive index μ (say water) instead of air, the wavelength of light will change from λ to (λ/μ) and

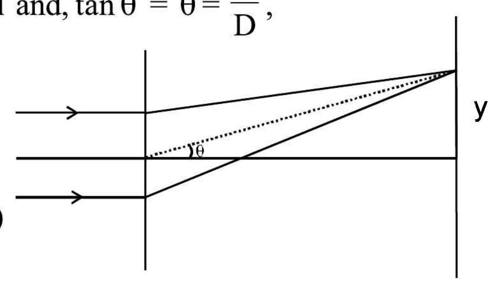
$$\beta' = \frac{D}{d}\left[\frac{\lambda}{\mu}\right] = \frac{\beta}{\mu}$$

(vi) That is, fringe width reduces and becomes $(1/\mu)$ times of its value in air.
If monochromatic light is replaced by white light, due to overlapping of patterns (each corresponding to a single wavelength with fringe width $\beta \propto$), central band, i.e., principal maxima will be white with red edges.
On either side of it, we shall get a few colored bands and then uniform illumination.

6.16 ANGULAR FRINGE WIDTH

It is the angular separation between two maxima of successive order on one side of the central maxima. This is equal to angular separation between two minima of successive order on one side of central maxima.

$\tan\theta = \frac{y}{D}$, since, θ in small then, $\tan\theta$ in small and, $\tan\theta = \theta = \frac{y}{D}$,



Angular fringe width $= \theta_n - \theta_{n-1} = \frac{y_n}{D} - \frac{y_n - 1}{D}$

$= \frac{1}{D}(y_n - y_{n-1}) = \frac{\beta}{D}$ (sine, $\beta = y_n - y_{n-1} = \frac{\lambda D}{d}$)

Angular fringe width $= \frac{\lambda}{d}$. Notice that it is independent to D (distance between slit and screen)

6.17 SHAPE OF INTERFERENCE FRINGES IN YDSE

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

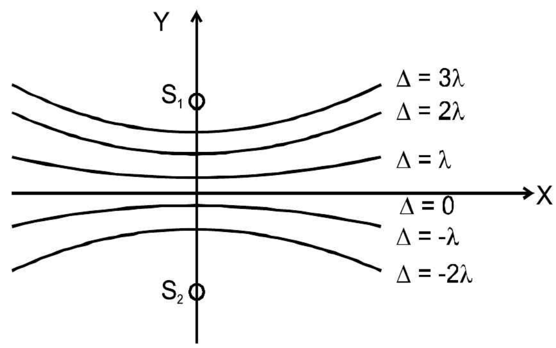
Fringes are locus of points which move in such a way that its path difference from the two slits remains constant. $S_2P - S_1P = \Delta = \text{constant}$ ....(5.1)

If $\Delta = \pm \dfrac{\lambda}{2}$, the fringe represents 1st minima.

If $\Delta = \pm \dfrac{3\lambda}{2}$ it represents 2nd minima

If $\Delta = 0$ it represents central maxima,

If $\Delta = \pm \lambda$, it represents 1st maxima etc.



Equation represents a hyperbola with its two foci at S_1 and S_2

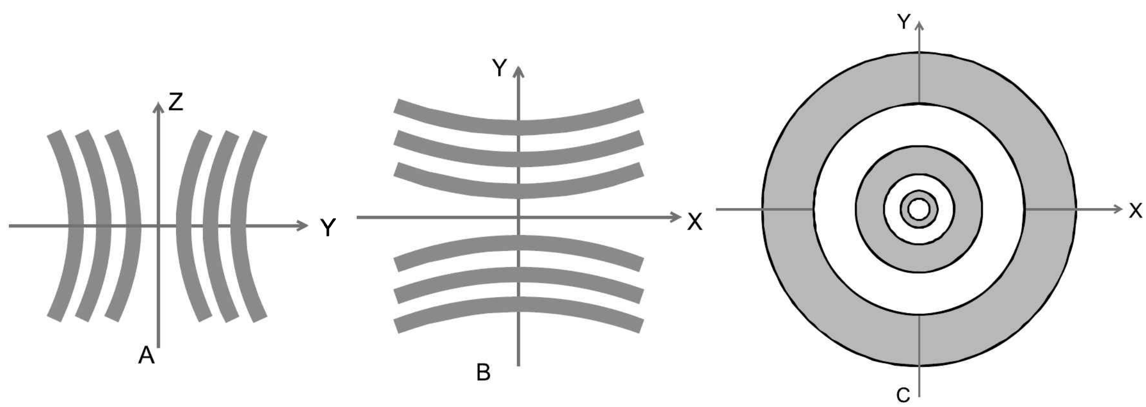
The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

(a) If the screen is $\perp$ ar to the X axis, i.e. in the YZ plane, as is generally the case, fringes are hyperbolic with a straight central section.

(b) If the screen is in the XY plane, again fringes are hyperbolic.

(c) If screen is $\perp$ ar to Y axis (along S_1S_2), ie in the XZ plane, fringes are concentric circles with center on the axis S_1S_2; the central fringe is bright if

$S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n - 1)\dfrac{\lambda}{2}$.



6.18 YDSE WITH WHITE LIGHT

The central maxima will be white because all wavelengths will constructively interference here. However slightly below (or above) the position of central maxima fringes will be coloured. For example if P is a point on the screen such that

$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190$ nm,

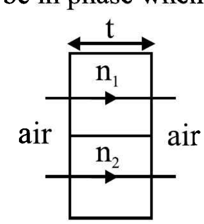
completely destructive interference will occur for violet light. Hence, we will have a line devoid of violet colour that will appear reddish. And if $S_2P - S_1P = \frac{\lambda_{red}}{2} = 350$ nm, completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe. For these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination. for example if $S_2P - S_1P = 3000$ nm,

then constructive interference will occur for wavelengths $\lambda = \frac{3000}{n}$ nm. In the visible region these wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish–yellow), 430 nm (violet). Clearly such a light will appear white to the unaided eye.

Thus, with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white. In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

6.19 OPTICAL PATH LENGTH IN TERMS OF WAVE LENGTHS

In the fig. two light rays of identical wavelength and initially in phase in air travel through two different media of refractive indices n_1 and n_2, and same thickness t. The wavelengths of the waves will be different in the two media ; so the two waves will no longer be in phase when they emerge.



Number of wavelengths in medium 1, $N_1 = \frac{t}{\lambda_1} = \frac{t}{\lambda/n_1} = \frac{n_1 t}{\lambda}$

Number of wavelength in medium 2, $N_2 = \frac{t}{\lambda_2} = \frac{t}{\lambda/n_2} = \frac{n_2 t}{\lambda}$

To find a new phase difference, we subtract the number of wavelengths of the waves in the two media ; assuming $n_2 > n_1$, we have $N_2 - N_1 = (n_2 - n_1)\frac{t}{\lambda}$

Phase difference corresponding to difference of one wavelength is 2π ; hence phase difference corresponding to $N_2 - N_1$ is, $\Delta\phi = \frac{2\pi}{\lambda}(n_2 - n_1)\,t$

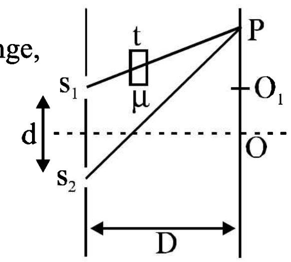
In other words, we can say that the equivalent optical paths of waves in media 1 and 2 are $n_1 t$ and $n_2 t$ respectively. Thus the path difference, $\Delta x = n_2t - n_1t$.

6.20 DISPLACEMENT OF FRINGE ON INTRODUCTION OF A GLASS SLAB IN THE PATH OF THE LIGHT COMING FROM THE SLITS

On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by $t(\mu - 1)$. Now the path difference between waves coming from S_1 and S_2 at any point P is

$\Delta p = S_2P - (S_1P + t(\mu - 1)) = (S_2P - S_1P) - t(\mu - 1)$

$\Rightarrow \Delta p = d \sin\theta - t(\mu - 1)$ if $d << D$



and $\Delta p = \dfrac{yd}{D} - t(\mu - 1)$ if $y << D$ as well. For central bright fringe,

$$\Delta p = 0 \Rightarrow \frac{yd}{D} = t(\mu - 1)$$

$$\Rightarrow y = OO_1 = (\mu - 1)\, t\, \frac{D}{d} = (\mu - 1)\, t \left(\frac{\beta}{\lambda}\right)$$

The whole fringe pattern gets shifted by the same distance, $\Delta = (\mu - 1)\, t\, \dfrac{D}{d} = (\mu - 1)\, t\, \dfrac{\beta}{\lambda}$

• Note that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

6.21 BEHAVIOUR OF THE WAVEFRONTS

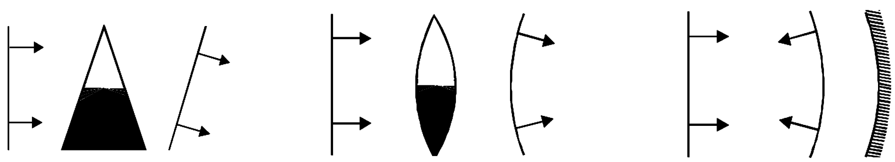
Once we have the laws of reflection and refraction, the behaviour of prisms. lenses, and mirrors can be understood. These topics are discussed in detail in the previous Chapter. Here we just describe the behaviour of the wavefronts in these three cases (Fig.)

(i) Consider a plane wave passing through a thin prism. Clearly, the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most. Since light travels more slowly in glass. This explains the tilt in the emerging wavefront.

(ii) Similarly, the central part of an incident plane wave traverses the thickest portion of a convex lens and is delayed the most. The emerging wavefront has a depression at the centre. It is spherical and converges to a focus,

(iii) A concave mirror produces a similar effect. The centre of the wavefront has to travel a greater distance before and after getting reflected, when compared to the edge. This again produces a converging spherical wavefront.

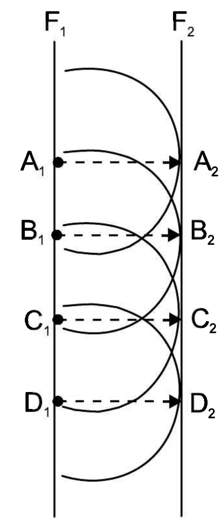
(iv) Concave lenses and convex mirrors can be understood from time delay arguments in a similar manner. One interesting property which is obvious from the pictures of wavefronts is that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray (Fig.). For example, when a convex lens focuses light to form a real image, it may seem that rays going through the centre are shorter. But because of the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.



(a) (b) (c)

6.22 HUYGEN'S CONSTRUCTION

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. We can guess that he must have seen water waves many times in the canals of his native place Holland. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wavefront as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets. Huygens' principle is illustrated in (Fig.) in the simple case of a plane wave.



(i) At time t = 0, we have a wavefront F_1, F_1 separates those parts of the medium which are undisturbed from those where the wave has already reached.

(ii) Each point on F_1 acts like a new source and sends out a spherical wave. After a time 't' each of these will have radius vt. These spheres are the secondary wavelets.

(iii) After a time t, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Notice that F_2 is a surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.

(iv) The secondary wavelet from the point A_1 on F_1 touches F_2 at A_2. Draw the line connecting any point A_1 on F_1 to the corresponding point A_2 on F_2. According to Huygens, A_1A_2 is a ray. It is perpendicular to the wavefronts F_1 and F_2 and has length vt. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts is the same along any ray. In our example, the speed 'v' of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.

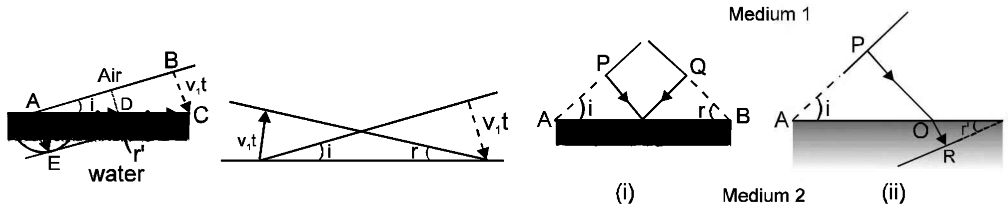
(v) This geometrical construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on. This is known as Huygens' construction.

Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbours into oscillation, and therefore acts as a secondary source. But what if there is no medium, as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction works in this case as well.

10.1 REFLECTION AND REFRACTION :

We can use a modified form of Huygens' construction to understand reflection and refraction of light. Figure (10.2a) shows an incident wavefront which makes an angle 'i' with the surface separating two media, for example, air and water. The phase speeds in the two media are v_1 and v_2. We can see that when the point A on the incident wavefront strikes the surface, the point B still has to travel a distance BC = AC sin *i*, and this takes a time t = BC/v_1 = AC (sin i) / v_1. After a time t, a secondary wavefront of radius v_2t with A as centre would have travelled into medium 2. The secondary wavefront with C as centre would have just started, i.e.. would have zero radius. We also show a secondary wavelet originating from a point D in between A and C. Its radius is less than v_2t. The wavefront in medium 2 is thus a line passing through C and tangent to the circle centered on A. We can see that the angle r' made by this refracted wavefront with the surface is given by AE = v_2t = AC sin r'. Hence, t = AC (sin r'} / v_2. Equating the two expressions for 't' gives us the law of refraction in the form sin i/sin r' = v_1/v_2. A similar picture is drawn in Fig. (10.2 b) for the reflected wave which travels back into medium 1. In this case, we denote the angle made by the reflected wavefront with the surface by r, and we find that i = r. Notice that for both reflection and refraction, we use secondary wavelets starting at different times. Compare this with the earlier application (Fig.) where we start them at the same time.

The preceding argument gives a good physical picture of how the refracted and reflected waves are built up from secondary wavelets. We can also understand the laws of reflection and refraction using the concept that the time taken by light to travel along different rays from one wavefront to another must be the same. Fig. shows the incident and reflected wavefronts when a parallel beam of light falls on a plane surface. One ray POQ is shown normal to both the reflected and incident wavefronts. The angle of incidence i and the angle of reflection r are defined as the angles made by the incident and reflected rays with the normal. As shown in Fig. (c), these are also the angles between the wavefront and the surface.



(a) (b) (c)

(Fig.) (a) Huygens' construction for the (a) refracted wave. (b) Reflected wave. (c) Calculation of propagation time between wavefronts in (i) reflection and (ii) refraction.

We now calculate the total time to go from one wavefront to another along the rays. From Fig. (c), we have, Total time for light to reach from P

$$= \frac{PO}{v_1} + \frac{OQ}{v_1} = \frac{AO \sin i}{v_1} + \frac{OB \sin r}{v_1}$$

$$= \frac{OA \sin i + (AB - OA) \sin r}{v_1} \qquad = \frac{AB \sin r + OA(\sin i - \sin r)}{v_1}$$

Different rays normal to the incident wavefront strike the surface at different points O and hence have different values of OA. Since the time should be the same for all the rays, the right side of equation must actually be independent of OA. The condition, for this to happen is that the coefficient of OA in Eq. (should be zero, i.e., sin i = sin r. We, thus. have the law of reflection, i = r. Figure also shows refraction at a plane surface separating medium 1 (speed of light v_1) from medium 2 (speed of light v_2). The incident and refracted wavefronts are shown, making angles i and r' with the boundary. Angle r' is called the angle of refraction. Rays perpendicular to these are also drawn. As before, let us calculate the time taken to travel between the two wavefronts along any ray.

$$\text{Time taken from P to R} = \frac{PO}{v_1} + \frac{OR}{v_2}$$

$$= \frac{OA \sin i}{v_1} + \frac{(AC - OA) \sin r'}{v_2} = \frac{AC \sin r'}{v_2} + OA\left(\frac{\sin i}{v_1} - \frac{\sin r'}{v_2}\right)$$

This time should again be independent of the ray we consider. The coefficient of OA in

Equation is, therefore, zero,. That is, $\frac{\sin i}{\sin r'} = \frac{v_1}{v_2} = n_{21}$

where n_{21} is the refractive index of medium 2 with respect to medium 1. This is the Snell's law of, refraction that we have already dealt with from Eq. n_{21} is the ratio of speed of light in the first medium (v_1) to that in the second medium (v_2). Equation is, known as the Snell's law of refraction. If the first medium is vacuum, we have $\frac{\sin i}{\sin r'} = \frac{c}{v_2} = n_2$

where n_2 is the refractive index of medium 2 with respect to vacuum, also called the absolute refractive index of the medium. A similar equation defines absolute refractive index n_1 of the first medium. From Eq. we then get $n_{21} = \frac{v_1}{v_2} = \left(\frac{c}{n_1}\right) / \left(\frac{c}{n_2}\right) = \frac{n_2}{n_1}$

The absolute refractive index of air is about 1.0003, quite close to 1. Hence, for all practical purposes, absolute refractive index of a medium may be taken with respect to air. For water, $n_1 = 1.33$, which means $v_1 = \frac{c}{1.33}$, i.e. about 0.75 times the speed of light in vacuum. The measurement of the speed of light in water by Foucault (1850) confirmed this prediction of the wave theory.

Ex. 1 A beam of light consisting of wavelengths 6000Å and 4500Å is used in a YDSE with D = 1m and d = 1 mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

Sol. $\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6 \text{ mm} \qquad \Rightarrow \beta_2 = \frac{\lambda_2 D}{d} = 0.45 \text{ mm}$

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

then, $y = n_1 P_1 = n_2 P_2 =$ LCM of β_1 and β_2 $\Rightarrow$ y = LCM of 0.6 cm and 0.45 mm

y = 1.8 mm **Ans.**

At this point, 3rd maxima for 6000 Å & 4th maxima for 4500 Å coincide

Ex. 2 White light is used in a YDSE with D = 1m and d = 0.9 mm. Light reaching the screen at position y = 1 mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

Sol. $\Delta p = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3} \text{ m} = 900 \text{ nm}$

for minima $\Delta p = (2n - 1)\lambda/2 \Rightarrow \lambda = \frac{2\Delta P}{(2n-1)} = \frac{1800}{(2n-1)} = \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7}$ ....

of these 600 nm and 360 nm lie in the visible range. Hence, these will be missing lines in the visible spectrum.

6.23 ORDER OF INTERFERENCE FRINGES

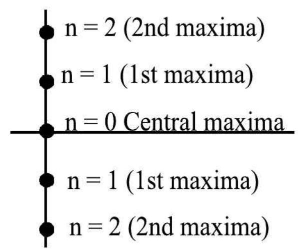
Ex. 3 In a YDSE, D = 1m, d = 1mm and $\lambda = \frac{1}{3}$mm. (means, d = 3λ)

(i) Find the distance between the first and central maxima on the screen.
(ii) Find the number of maxima and minima obtained on the screen.

Sol. Approximation (I) applicable When, $\lambda = \frac{1}{3}$mm and d = 1 mm **(means, d = 3λ)** (which is comparable) then, θ is not small, $\Rightarrow \sin\theta \neq \tan\theta$, for the maxima : $\Delta P = \pm n\lambda$

In a YDSE Path difference i.e. $\Delta P = d\sin\theta \Rightarrow \text{Sin}\theta = \pm\frac{n\lambda}{d}$, $\text{Sin}\theta = \pm\frac{n}{3}$ (n = 0, 1, 2, 3)

For the 1st maxima : n = 1 $\text{Sin}\theta = \pm\frac{1}{2}$ (one is above the central maxima and other is below the central maxima).



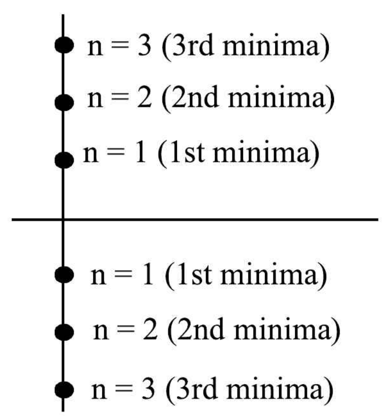
Then, $\tan\theta = \frac{1}{\sqrt{3}} = \frac{y}{D} \Rightarrow y = \frac{D}{\sqrt{3}} = \frac{1}{\sqrt{3}}$ m

(ii) For the no. of maxima,

$\text{Sin}\theta = \pm\frac{n}{3}$ Only possible values of n = 0, 1, 2, 3

When, n = 0, Sin θ = 0 $\Rightarrow$ θ = 0, n = 3, Sin $\theta = \pm 1 \Rightarrow \theta = \pm 90$

n = 3; is not possible value as interference takes place on the screen.
n = 0; Corresponds to Central maxima
n = 1; corresponds to 1st maxima, one above and other is below the central maxima.
n = 2; corresponds to 2nd maxima, one above and other is below the central maxima.
n = 3; is not possible. Since at θ = 90, fringe are not formed on the screen.
Hence, Total no. of maxima = 5



For calculation of no. of minima $\Delta P = \pm(2n-1)\frac{\lambda}{2}$

and $\Delta P = d\sin\theta$ In YDSE $\Rightarrow$ $\text{Sin}\theta = \pm\frac{(2n-1)\lambda}{2d}$

$\Rightarrow \text{Sin}\theta = \pm\frac{(2n-1)}{6}$

Where, n = 1, 2, 3 are only possible values
Hence, Total no. of minima = 6

Ex. 4 In a YDSE, D = 1m, d = 1 mm and $\lambda = \frac{1}{2}$ mm.

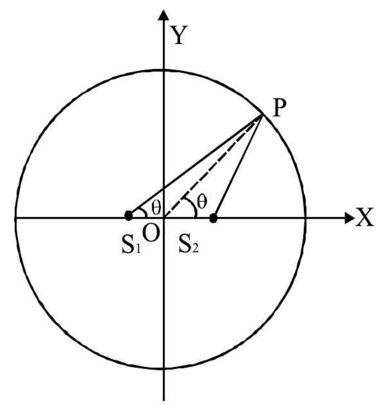
(i) Find the distance between the first and central maxima on the screen.

(ii) Find the number of the maxima and the number of maxima obtained on the screen.

Ans. (i) $y = \frac{1}{\sqrt{3}}$ m (ii) Total no. of maxima = 3, Total no. of maxima on the screen = 4

Ex. 5 Two sources are placed on x-axis at a separation d = '4λ'. An observer starts moving on a circular track of radius R (R >> d). How many bright points and dark points will be observed? Also find the angular position of maxima and minima.

Sol. Path difference at point P i.e. $\Delta x = d\cos\theta$



For the maxima : $\Delta x = \pm\ n\lambda$

Then, $d\cos\theta = \pm\ n\lambda \Rightarrow \quad \cos\theta = \pm\frac{n\lambda}{d} = \pm\frac{n}{4}$

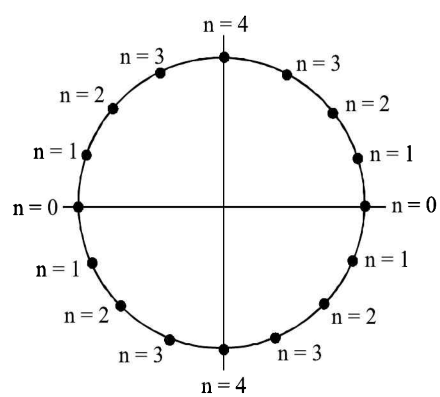
Only possible values of n = 0, 1, 2, 3, 4

For n = 0 , θ = 90, For n = 4 , θ = 0

Hence, Total no. of maxima = 16 &

Total no. of minima = 16

For, $0 < \theta < \pi$, Total maxima = 7



For, $0 \le \theta \le \pi$, Total maxima = 9

For, $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$, Total maxima = 9

For the angular position,

$$\theta = \cos^{-1}\left(\frac{\Delta x}{d}\right)$$

Think if d = 3.5 λ in the above question.

Ex. 6 Two sources are placed on x-axis at a separation d = 3λ. An observer moving on a circular screen of large radius (R>>d). How many bright points and dark points will be observed?

Ans. Total bright points = 12 Total dark points = 12

Think if (d = 2.5 λ) in the above question.

Ex. 7 **Two point sources are placed on a straight line separated by a distance $d = 4\lambda$. Both the sources are placed at a distance L from a wall which is perpendicular to the straight line. Both the sources are sending waves of equal intensity. How many bright points and dark points will be observed on the screen?**

Sol. Path difference at point i.e. P $\Delta x = S_1P - S_2P = d\cos\theta$

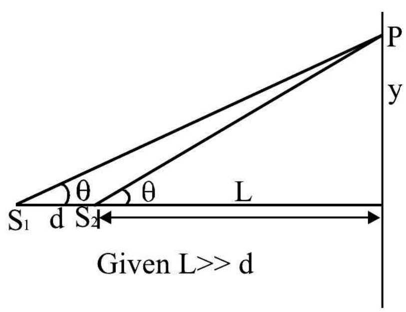
For bright points : $\Delta x = \pm n\lambda \Rightarrow d\cos\theta = \pm n\lambda \Rightarrow \cos\theta = \pm\dfrac{n\lambda}{d} = \pm\dfrac{n}{4}$

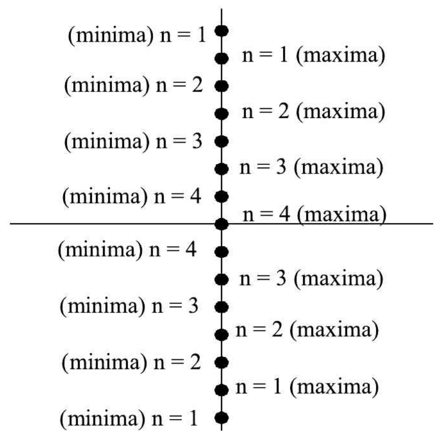
only possible value of n = 0, 1, 2, 3, 4

For n = 0 , $\theta = 90$ This is not formed on the screen.

For n = 4 , $\theta = 0$

Hence, no of fringes formed on the screen as shown in the fig :





Total bright points = 7 ; Total dark points = 8

THINK IF d = 2.5 λ IN THE ABOVE QUESTION.

Ex. 8 **Two point sources are placed on a straight line separated by a distance $d = 3\lambda$. Both the sources are placed at a distance L from a wall which is perpendicular to the straight line. Both the sources are sending waves of equal intensity. How many bright points and dark points will be observed on the screen?**

Ans. Bright points = 5 ; Dark points = 6

SPECIAL CASE IN THE YDSE EXPERIMENTS :

(1) When source is at infinite distance and oblique incidence above the axis:

Path difference between the rays reaching point P i.e.

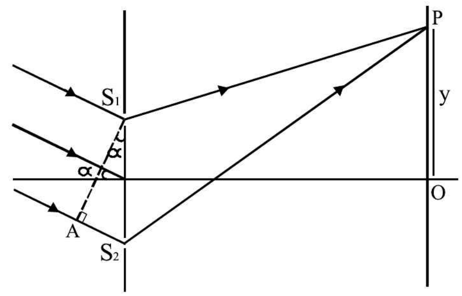
$\Delta x = (AS_2 + S_2P) - S_1P$

$\Delta x = AS_2 + (S_2P - S_1P)$

$$\Delta x = d\sin\alpha + \frac{yd}{D} \qquad \left[\therefore S_2P - S_1P = \frac{yd}{D}\right]$$

And for, maxima $\Delta x = \pm n\lambda$

for, minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$



(2) When source is at infinite distance and oblique incidence below the axis :
Path difference between the rays reaching point P,

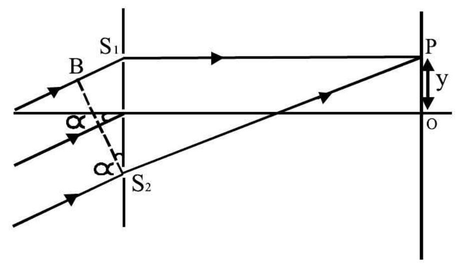
$\Delta x = (S_2P) - (BS_1 + S_1P)$

$\Delta x = (S_2P - S_1P) - BS_1$

$$\Delta x = \frac{yd}{D} - d\sin\alpha \qquad \left[\because S_2P - S_1P = \frac{yd}{D}\right]$$

And for, maxima $\Delta x = \pm n\lambda$

for, minima $\Delta x = (2n-1)\frac{\lambda}{2}$



(3) When we introduce a thin transparent plate in front of upper slit in YDSE
Path difference between the rays reaching point P in air i.e

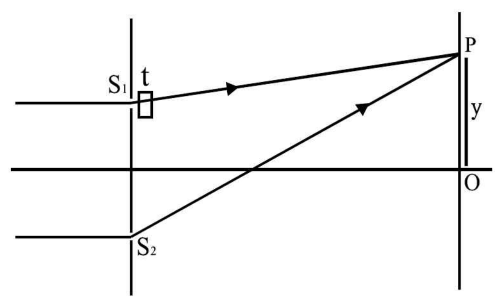
$\Delta x = (S_2P) - (S_1P - t + \mu t)$

$\Delta x = (S_2P - S_1P) - t(\mu - 1)$

$$\Delta x = \frac{yd}{D} - t(\mu-1) \qquad \left[\because S_2P - S_1P = \frac{yd}{D}\right]$$

And for maxima, $\Delta x = \pm n\lambda$

for maxima, $\Delta x = (2n-1)\frac{\lambda}{2}$



(4) When we introduce a thin transparent plate in front of lower slit in YDSE
Path difference between the rays reaching point P in air i.e

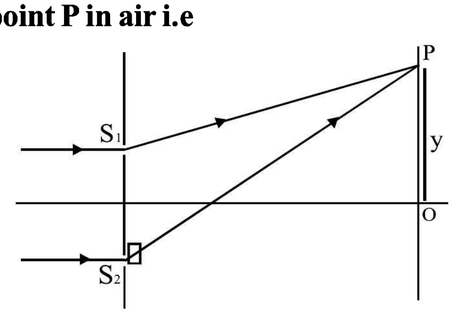
$\Delta x = (S_2P - t + \mu t) - S_1P$

$\Delta x = (S_2P - S_1P) - t + \mu t$

$$\Delta x = \frac{dy}{D} + t(\mu-1) \qquad \left[\because S_2P - S_1P = \frac{yd}{D}\right]$$

And for, maxima $\Delta x = \pm n\lambda$

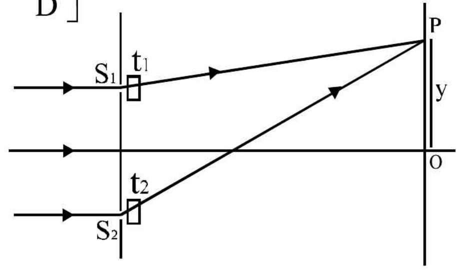
for, minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$



(5) When we introduce a thin transparent plate in front of the lower slits in YDSE Path difference between the rays reaching point P, in air i.e.

$\Delta x = (S_2P - t_2 + \mu_2 t_2) - (S_1P - t_1 + \mu_1 t_1)$ $\Delta x = (S_2P - S_1P) + (t_1 - t_2) + (\mu_2 t_2 - \mu_1 t_1)$

$$\Delta x = \frac{yd}{D} + (t_1 - t_2) + (\mu_2 t_2 - \mu_1 t_1)\left[\because S_2P - S_1P = \frac{yd}{D}\right]$$



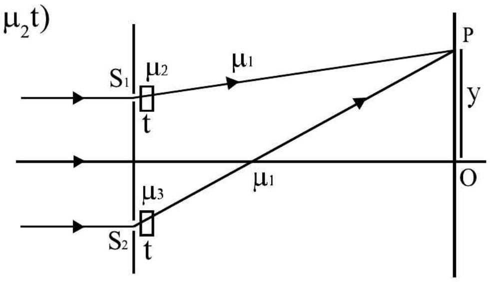
And for, maxima $\Delta x = \pm n\lambda$

for, minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$

(6) Path difference between the rays reaching point P in air i.e.

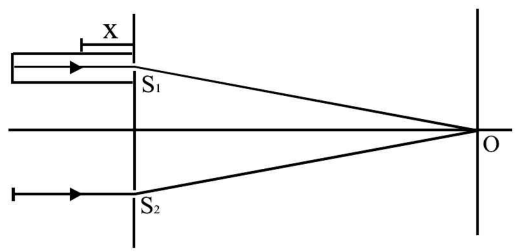
$\Delta x = [(S_2P - t)\,\mu_1 + \mu_3 t)] - [(S_1P - t)\,\mu_1 + \mu_2 t)$

where, $(S_2P - S_1P) = \frac{yd}{D}$



(7) Light incident on the slit S_1 passes through a medium of variable refractive index $\mu = 1 + ax$ (where 'x' is the distance from the plane of slits as shown, up to a distance 'L' before falling on S_1). If at 'O' a maxima is formed. Find the minimum value of 'a'. For variable refractive index, optical path = $\int \mu dx$

Path difference at point O, in air



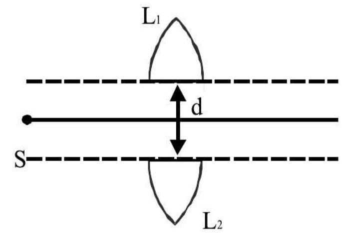
$$\Delta x = [L + S_2O] - \left[\int_0^L (L + ax)dx + S_1O\right]$$

$$\Delta x = L - \int_0^L (L + ax)dx\,[\because S_2O - S_1O = 0]$$

at point 'O' $\Delta x = L + \left(L + \frac{aL^2}{2}\right) = -\frac{aL^2}{2}$

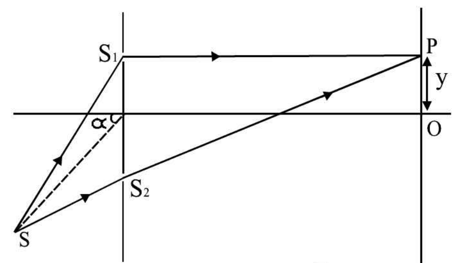
Then, for maxima 'a', $\Delta x = -\frac{\lambda}{2} \Rightarrow \frac{aL^2}{2} = \frac{\lambda}{2} \Rightarrow a = \frac{\lambda}{L^2}$

(8) A thin lens of circular shape and focal length 'f' is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. Two halves are placed symmetrically about the central line with the gap 'd'. In this case, there are two image of S by lens L_1 and L_2 which behave like two slits of YDSE.



(9) When the source is at finite distance Path difference at point O in air i.e.

$$\Delta x = [(SS_2 + S_1P) - (SS_1 + S_1P)]$$
$$\Delta x = (SS_2 - SS_1) + (S_2P - S_1P)$$



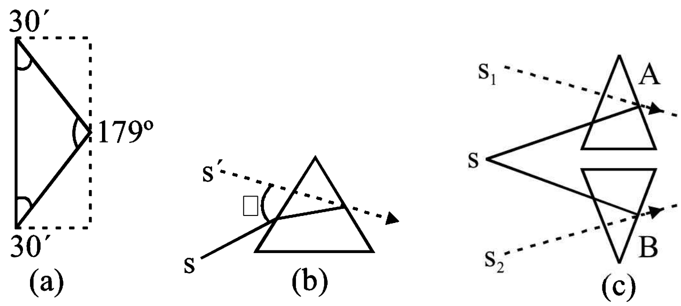
$$\Delta x = -\,d\sin\alpha + \frac{yd}{D} \quad \left[\because S_2P - S_1P = \frac{yd}{D} \text{ and } SS_1 - SS_2 = d\sin\alpha\right]$$

Here, we assume that interference pattern formed at point P is above the point 'O'.

6.24 FRESNEL'S BIPRISM

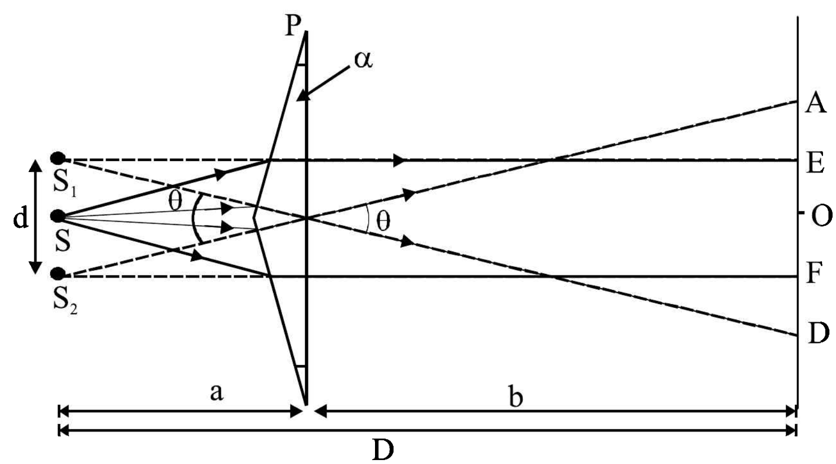
Fresnel used a biprism to show interference phenomenon. The biprism consists of two prisms of very small refracting angles joined base to base. In practice, a thin glass plate is taken and one of its faces is ground and polished till a prism (Fig.) is formed with an obtuse angle of about 179° and two side angles of the order of 30'.

When a light ray is incident on an ordinary prism, the ray is bent through an angle called the angle of deviation. As a result, the ray emerging out of the prism appears to have emanated from a source S' located at a small distance above the real source, as shown in Fig. We say that the prism produced a virtual image of the source. A biprism, in the same way, creates two virtual sources S_1 and S_2, as seen in Fig. These two virtual sources are images of the same source S produced by refraction and are hence coherent.



6.24(A) EXPERIMENTAL ARRANGEMENT

The biprism is mounted suitably on an optical bench. An optical bench consists of two horizontal long rods, which are kept strictly parallel to each other and at the same level. The rods carry upright on which the optical components are positioned. A monochromatic light source such as sodium vapor lamp illuminates a vertical slit S. Therefore, the slit S acts as a narrow linear monochromatic light source. The biprism is placed in such a way that its refracting edge is parallel to the length of the slit S. A single cylindrical wavefront impinges on both prisms. The top portion of wavefront is refracted downward and appears to have emanated from the virtual image S_1. The lower segment, falling on the lower part of the biprism, is refracted upwards and appears to have emanated from the virtual source S_2. The virtual source S_1 and S_2 are coherent (see fig.a), and hence the light waves are in the position to interference in the region beyond the biprism. If a screen is held there, interference fringes are seen. In order to observe fringes, a micrometer eyepiece is used.



(a)

THEORY :

The theory of the interference and fringe formation in case of Fresnel biprism is the same as described in the double-slit. As the point O is equidistant from S_1 and S_2, the central bright fringe of maximum intensity occurs there. On both sides of O, alternate bright and dark fringes, are produced. The width of the dark or bright fringe is given by equation.

$\beta = \frac{\lambda D}{d}$ where $D(= a + b)$ is the distance of the sources from the eye-piece.

6.24(B) DETERMINATION OF WAVELENGTH OF LIGHT

The wavelength of the light can be determined using the equ. For using the relation, the values of β , D and d are to be measured. These measurements are done as follows.

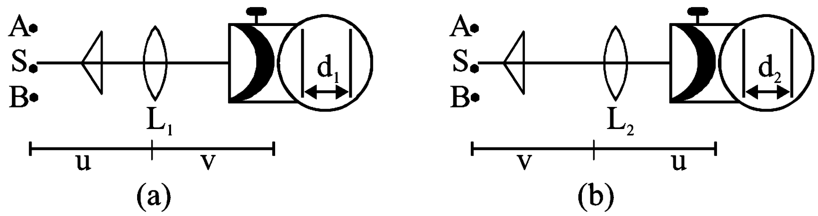
ADJUSTMENTS :

A narrow adjustable slit S, the biprism, and a micrometer eyepiece are mounted on the uprights and are adjusted to be at the same height and in a straight line.The slit is made vertical and parallel to the refracting edge of the biprism by rotating it in its own plane. It is illuminated with the light from the monochromatic source. The biprism is moved along the optical bench till, on looking through it along the axis of the optical bench, two equally bright vertical slit images are seen. Then the eyepiece is moved till the fringes appear in the focal plane of the eyepiece.

(i) Determination of fringe width β : When the fringes are observed in the field view of the eyepiece, the vertical cross-wire is made to coincide with the centre of one of the bright fringes. The position of the eyepiece is read on the scale, say x_0. The micrometer screw of the eyepiece is moved slowly and the number of the bright fringes N that pass across the cross - wire is counted . The position of the cross-wire is again read, say x_N . The fringe width is then given by $\beta = \frac{x_N - x_0}{N}$.

(ii) Determination of 'd' :

(a) A convex lens of short focal length is placed between the slit and the eyepiece without disturbing their positions. The lens is moved back and forth near the biprism till a sharp pair of images of the slit is obtained in the field view of the eyepiece. The distance between the images is measured. Let it be denoted by d_1.



(a) (b)

Measurement of the distance between the two virtual source

If u is the distance of the slit and υ that of the eyepiece from the lens (Fig.a) , then the magnification is $\frac{v}{u} = \frac{d_1}{d}$ ....(1)

The lens is then moved to a position nearer to the eyepiece, where again a pair of images of the slit is seen. The distance between the two sharp images is again measured . Let it be d_2.

Again magnification is given by $\frac{u}{v} = \frac{d_2}{d}$ ....(2)

Note that the magnification in one position is the reciprocal of the magnification in the other position.

Multiplying the equations (1) and (2), we obtain $\frac{d_1 d_2}{d^2} = 1 \quad \Rightarrow \quad d = \sqrt{d_1 d_2}$

Using the values of β, d and D in the equation ($\beta = \frac{\lambda D}{d}$), the wavelength λ can be computed.

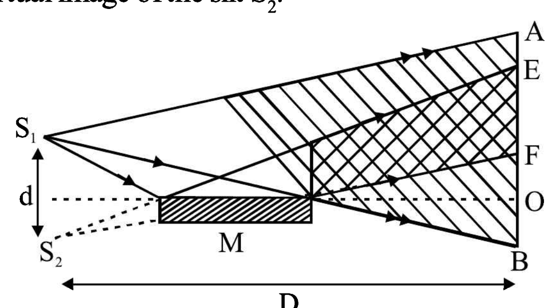
(b) Alternatively , the value of d can be determined as follows. The deviation δ produced in the path of a ray by a thin prism is given by, $\delta = (\mu - 1)\,\alpha$

where α is the refracting angle of the prism. From the Fig. of biprism, it is seen that $\delta = \theta/2$. Since d is very small, we can also write $d = a\theta$.

$$\therefore \frac{\theta}{2} = \frac{d}{2a} = (\mu - 1)\,\alpha \qquad \therefore d = 2a\,(\mu - 1)\,\alpha$$

6.25 LLOYD'S SINGLE MIRROR

In 1834, Lloyd devised an interesting method of producing interference, using a single mirror and using almost grazing incidence. The Lloyd's mirror consists of a plane mirror about 30 cm in length and 6 to 8 cm in breadth (see Fig.). It is polished on the front surface and blackened at the back to avoid multiple reflections. A cylindrical wavefront coming from a narrow slit S_1 falls on the mirror which reflects a portion of the incident wavefront, giving rise to a virtual image of the slit S_2.



Another portion of the wavefront proceeds directly from the slit S_1 to the screen .The slit S_1 and S_2 act as two coherent sources. Interference between direct and reflected waves occurs within the region of overlapping of the two beams and fringes are produced on the screen placed at a distance D from S_1 in the shaded portion EF.

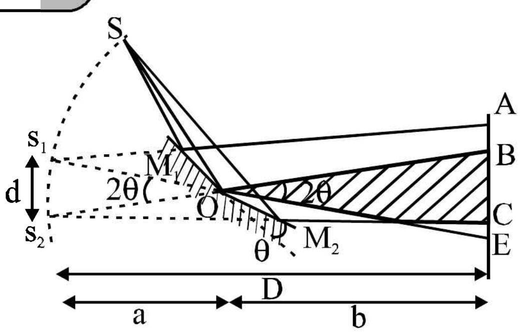
The point O is equidistant from S_1 and S_2. Therefore, central (zero-order) fringe is expected to lie at O (the perpendicular bisector of S_1S_1) and it is also expected to be bright . However it is not usually seen since the point O lies outside the region of interference (only the directly light and not the reflected light reaches O).

By moving the screen nearer to the mirror such that it comes into contact with the mirror, the point O can be just brought into the region of interference.

With white light the central fringe at O is expected to be white but in practice it is dark. The occurrence of dark fringe can be understood taking into the consideration of the phase change of π that light suffers when reflected from the mirror. The phase change leads to a path difference of $\lambda/2$ and hence destructive interference occurs there.

6.26 FRESNEL'S DOUBLE MIRROR

Fresnel's double mirror is an arrangement for obtaining two coherent sources by using the phenomenon of reflection. If consists of two plane mirrors inclined to each other at a very small angle , as shown in the Fig. The mirrors are silvered on their front surfaces and are arranged at nearly 180º such that their surfaces are nearly coplanar.



Fresnel's Mirrors

A narrow slit S is placed parallel to the line of intersection of the mirror surfaces and is illuminated with monochromatic light . One portion of the cylindrical wavefront coming from slit S is reflected from the first mirror and another portion of the wavefront is reflected from the second mirror. After reflection, the light appears to diverge from S_1 and S_2, which are the virtual images of S. As the images S_1 and S_2 of the slit are derived from the same source S, they behave as two coherent sources, placed at a distance d apart. The waves diverging from S_1 and S_2 overlap and interference fringes are produced in the overlapping region BC on the screen. The fringes are of equal width.

6.26(A) FRINGE WIDTH

It is seen from the geometry of the figure (Fig.) that $OS_1 = OS_2 = OS$. That is, S_1, S_2 and S lie on circle with O as a centre. Let 'a' be the distance of the sources and 'b' be the distance of the screen from O. Then the fringe width is given by

$$\beta = \frac{\lambda D}{d} = \frac{(a+b)\lambda}{d} \qquad \text{..........(1)}$$

OE and OB are the reflected rays from OM_1 and OM_2 respectively, corresponding to the incident ray So. Therefore, the angle between OE and OB is twice the angle between the mirrors. Hence, $\angle S_1OS_2 = \angle BOE = 2\theta$. Now, Arc $S_1S_2 = a \times 2\theta$

$\therefore\ d = a \times 2\,\theta$

Using the above result into equ. (1), we get $\beta = \frac{(a+b)}{2a\theta}\lambda$

Comparison between the fringes produced by biprism and double mirror :

The fringes in both cases are similar in appearance. However, the double mirror fringes are narrower than the biprism fringes.

6.27 PLANE PARALLEL FILM

A transparent thin film of uniform thickness bounded by two parallel surfaces is known as a plane parallel thin film.

When light is incident on a parallel thin film, a small portion of it gets reflected from the top surface and a major portion is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it is transmitted from the lower surface of the film. Thin films transmit incident light strongly and reflect only weakly. After two reflections, the intensities of reflected rays drop to a negligible strength. Therefore, we consider the first two reflected rays only (see Fig. 1). These two rays are derived from the same incident ray but appear to come from two sources located below the film. The sources are virtual coherent sources. The reflected waves 1 and 2 travel along parallel paths and interfere at infinity. This is a case of two - beam interference.

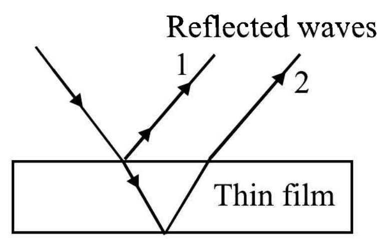


fig. (1)

The condition for maxima and minima can be deduced once we have calculated the optical path difference between the two rays at the point of their meeting.

6.27(A) INTERFERENCE DUE TO REFLECTED LIGHT

Let us consider a transparent film of uniform thickness 't' bounded by two parallel surfaces as shown in (Fig. 2). Let the refractive index of the material be μ. The film is surrounded by air on both the sides. Let us consider plane waves from a monochromatic source falling on the thin film at an angle of incidence 'i'. Part of a ray such as AB is reflected along BC, and part of it is transmitted

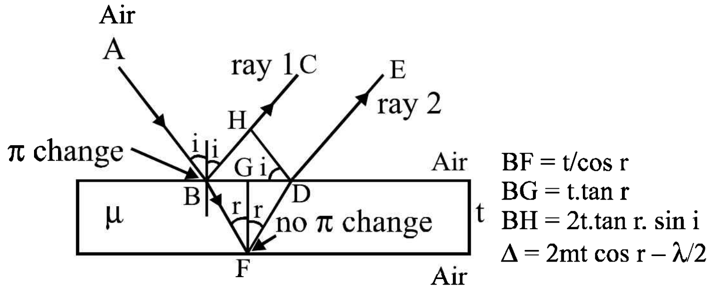


fig. (2)

into the film along BF. The transmitted ray BF makes an angle 'r' with the normal to the surface at the point G The ray BF is in turn partly reflected back into the film along FD while a major part refracts into the surrounding medium along FK. Part of the reflected ray FD is transmitted at the upper surface and travels along DE. Since the film boundaries are parallel, the reflected rays BC and DE will be parallel to each other. The waves travelling along the paths BC and BFDE are derived from a single incident wave AB . Therefore they are coherent and can produce interference if they are made to overlap by condensing lens or the eye.

(i) Geometrical Path Difference : Let DH be normal to BC. From points H and D onwards, the rays HC and DE travel equal path. The ray BH travels in air while the ray BD travels in the film of refractive index μ along the path BF and FD. The geometric path difference between the two rays is

BF + FD – BH.

(ii) Optical Path Difference :

Optical path difference $\Delta_a = \mu L$, $\quad \therefore \Delta_a = \mu\,(BF + FD) - 1\,(BH)$

$\therefore$ In the Δ BFD , $\angle BFG = \angle GFD = \angle r$, $\quad BF = FD$

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}, \quad \therefore BF + FD = \frac{2t}{\cos r}$$

Also, $BG = GD$, $\quad \therefore BD = 2BG$, $\quad BG = FG \tan r = t \tan r$, $\quad \therefore \quad BD = 2\,t \tan r$

In the Δ^{le} BHD $\quad \angle HBD = (90 - i) \quad$ and $\quad \angle BHD = 90^\circ$

$\therefore \quad \angle BDH = i$, $\quad \therefore \quad BH = BD \sin i = 2\,t \tan r \sin i$

From Snell's law, $\quad \sin i = \mu \sin r$, $\quad \therefore BH = 2t \tan r\,(\mu \sin r) = \dfrac{2\mu t \sin^2 r}{\cos r}$

Using the above equations, we get $\Delta_a = \mu\left[\dfrac{2t}{\cos r}\right] - \left[\dfrac{2\mu t \sin^2 r}{\cos r}\right]$

$$= \frac{2\mu t}{\cos r}[1-\sin^2 r] \qquad = \frac{2\mu t}{\cos r}\cos^2 r \;\Rightarrow\; \Delta_a = 2\mu\, t \cos r$$

(iii) Correction on account of phase change at reflection :

When a ray is reflected at the boundary of a rarer to denser medium, a path change of $\lambda/2$ occurs for the ray BC (see Fig. 2). There is no path difference due to transmission at D. Including the change in path difference due to reflection, the true path difference

$$\Delta_t = 2\mu\, t \cos r - \frac{\lambda}{2}$$

CONDITIONS FOR MAXIMA (BRIGHTNESS) AND MINIMA (DARKNESS)

Maxima occur when the optical path difference $\Delta = m\lambda$. If the difference in the optical path between the two rays is equal to an integral number of full waves, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves interfere constructively.

Thus, when $\quad 2\mu\, t \cos r - \frac{\lambda}{2} = m\lambda$

the reflected rays undergo constructive interference to produce brightness or maxima at the point of their meeting. $\quad 2\mu\, t \cos r = m\lambda + \lambda/2$

or $\quad 2\mu\, t \cos r = (2m+1)\lambda/2$ **Condition for Brightness**

Minima occur when the optical path difference is $\Delta = (2m+1)\lambda/2$. If the difference in the optical path between the two rays is equal to an odd integral number of half-waves, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and the waves interfere destructively. Thus, then

$2\mu\, t \cos r - \lambda/2 = (2m+1)\lambda/2$

the reflected rays undergo destructive interference to produce darkness. Above Equ. may be rewritten as $\quad 2\mu\, t \cos r = (m+1)\lambda$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves. Therefore $(m+1)\lambda$ can as well be replaced by $m\lambda$ for simplicity in expression. Thus, $\; 2\mu\, t \cos r = m\lambda$ **Condition for Darkness**

SOME IMPORTANT POINTS

(a) It is seen that the conditions of interference depend on four parameters, namely μ, t, λ and r. In the case of constant thickness (parallel) film, (μt) is constant. When a parallel beam of light is incident on such a film, r also remains constant. Then the interference conditions solely depend on the wavelength λ.

(b) When monochromatic light falls on a parallel beam, the whole film will appear uniformly dark or bright. If the condition of constructive interference is satisfied, the film will show intense colour corresponding to the incident light.

(c) If a parallel beam of white light falls on a parallel film, those wavelengths for which the path difference is mλ, will be absent from the reflected light. The other colours will be reflected. Therefore, the film will appear uniformly coloured with one colour being absent.

6.27(B) NARROW LIGHT SOURCE VERSUS EXTENDED LIGHT SOURCE

In case of Fresnel's biprism and Lloyd's mirror, interference fringes were produced by two coherent source. The initial source is narrow. The fringes obtained on a screen are viewed with an eyepiece. In case of a thin film a narrow source limits the area of the film that can be viewed.

Consider a thin film illuminated by a narrow source of light S (Fig. 3).The ray 1 produces

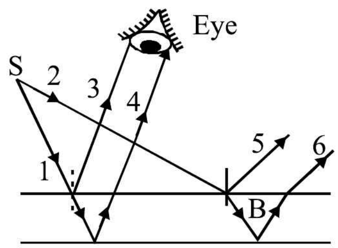


fig. (3)

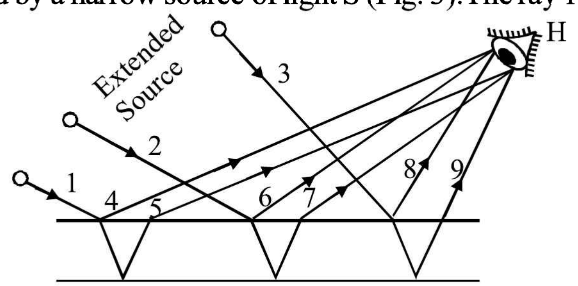


fig. (4)

interference fringes because rays 3 and 4 reach the eye. The ray 2 is incident on the surface of the film at a different angle and is reflected along 5 and 6. The rays 5 and 6 do not reach the eye. Similar is the case for other rays incident at different angles on the film surface. The reflected rays do not reach the eye. Thus, only the portion A of the film is visible to the eye. If an extended (or broad) light source is used to illuminate the film, as in (Fig. 4), a larger area of the film surface is observed .The ray 1 after reflection from the upper and the lower surface of the film emerges as rays 4 and 5 , which reach the eye. Ray 2 from some other point of the source after reflection from the upper and lower surfaces of the film emerge as rays 6 and 7 which also reach the eye. Also, ray 3 from some other point of the source after reflection from the upper and lower surfaces of the film emerge as rays 8 and 9 which also reach the eye. Therefore, the rays incident at different angles on the film are accommodated by the eye and the field of view is large. Therefore, a broad source of light is required to observe to observe interference in thin films.

6.27(C) INTERFERENCE DUE TO TRANSMITTED LIGHT

Consider a thin transparent film of thickness t and refractive index μ . A ray SA after refraction goes along AB. At B it is partly reflected along BC and partly refracted along BR. The ray BC, after reflection at C, finally emerges along DQ. Here at B and C reflection takes place at the rarer medium.

Therefore, no phase change occurs. Draw BM normal to CD and DN normal to BR. The optical path different between DQ and BR is given by

(Fig. 5)

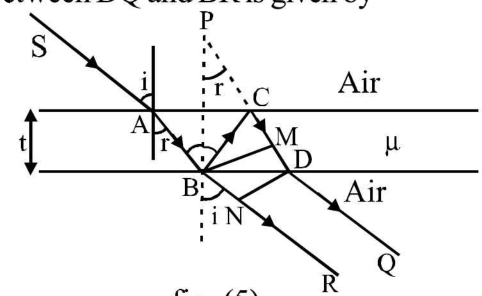


fig. (5)

$\Delta = \mu\ (BC + CD) - BN$

Also, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{BN}{MD}$ or $BN = \mu\ .\ MD$, In (Fig. 5), $\angle\ BPC = r$ and $CP = BC = CD$

$\therefore\ BC + CD = PD$, $\therefore\ \Delta = \mu\ (PD) - \mu\ (MD) = \mu\ (PD - MD) = \mu\ .\ PM$

In Δ BPM, $\cos r = \dfrac{PM}{BP}$ or $PM = BP.\ \cos r$

But $BP = 2t$, $\therefore\ PM = 2\ t \cos r$, $\therefore\ \Delta = \mu\ .\ PM = 2\mu\ t \cos r$

Bright Fringes :

When the optical path difference $\Delta = m\,\lambda$, bright fringe occurs.

$\therefore$ $2\ \mu\ t \cos r = m\lambda$, where $m = 0,1,2,3,$.......

Dard Fringes :

When the optical path difference $\Delta = (2m + 1)\ \lambda / 2$, dark fringe occurs.

$$\therefore \quad 2\mu\ t \cos r = \frac{(2m+1)}{2}\lambda$$

where $m = 0,1,2,3,$.........

In case of transmitted light, the fringes are less distinct because the difference in amplitudes of BR and DQ is very large. However, when the angle of incidence is nearly 45° the fringes are more distinct.

6.28 VARIABLE THICKNESS (WEDGE - SHAPED) FILM

Let us now study the interference of light in a film of varying thickness. A thin film having zero thickness at one end and progressively increasing to particular thickness at the other end is called a wedge. A thin wedge of air film can be formed by two glass slides resting on each other at one edge and separated by a thin spacer at the opposite edge.

The arrangement for observing the interference pattern in a wedge shaped film is shown in Fig. 6(a) .The wedge angle is usually very small and of the order of a fraction of a degree. When a parallel beam of monochromatic light illuminates the wedge from above, the rays reflected from its two bounding surface will not be parallel. They appear to diverge from a point near the film. The path difference between the rays reflected from the upper and lower surfaces of the air film varies along its length due to variation in film thickness . Therefore, alternate bright and dark fringes are observed on its top surface (see fig. 6 b) .The fringes are localized at the top surface of the film.

When the light is incident on the wedge from above, it gets partly reflected from the glass - to - air boundary at the top of the air film. Part of the light is transmitted through the air film and gets reflected partly at the air - to - glass boundary, as shown in (Fig. 7). The two rays BC and DE, thus reflected from the top and bottom of the air film , are coherent as they are derived from the same ray AB through division of amplitude. The rays are close enough if the thickness of the film is of the order of a wavelength of light. For small film thickness the rays interfere producing darkness or brightness depending on the phase difference. The thickness of the glass plates is large compared

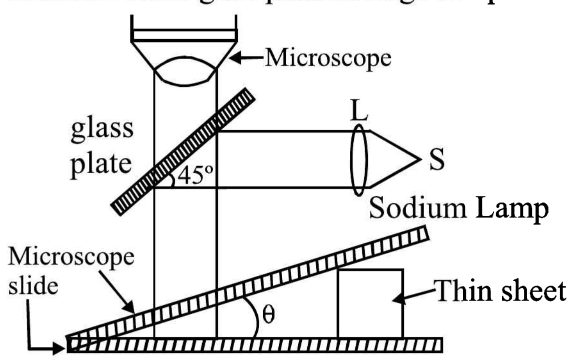


fig. 6(a)

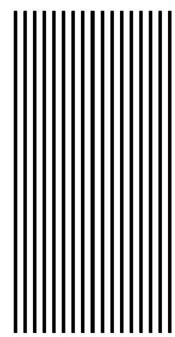


fig. 6(b)

with the wavelength of the incident light. Hence, the observed interference effects are entirely due to the wedge - shaped air film.

The optical difference between the two rays BC and DE is given by

$\Delta = 2\mu\, t \cos r - \lambda / 2$

where $\lambda /2$ takes account the gain of half - wave due to the abrupt jump of π radians in the phase of the wave reflected from the bottom boundary of air - to - glass.

Maxima occur when the optical path difference $\Delta = m\lambda$ .If the difference in the optical path between the two rays is equal to an integral number of full waves, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves interfere constructively. This needs that

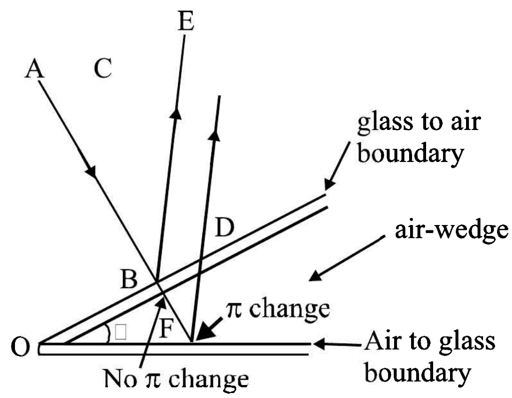


fig. (7)

$\Delta = 2\mu\, t \cos r - \lambda / 2$

Minima occur when the optical path difference is $\Delta = (2m + 1)\ \lambda / 2$. If the difference in the optical path between the two rays is equal to an odd integral number of half - waves, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and the waves interfere destructively. If needs that

$2\ \mu\ t \cos r = m\lambda.$

Referring to (Fig. 8), let us say a dark fringe occurs at A where the relation

$2\ \mu\ t \cos r = m\lambda$

is satisfied. If normal incidence is assumed, $\cos r = 1$ and if the thickness of air film at A is denoted by t_1. then at A , $\quad 2\mu\ t_1 = m\lambda$ ....(1)

The next dark fringe will occur, say, at C where the thickness $CL = t_2$. Then at C

$2\ \mu\ t_2 = (m + 1)\ \lambda$ ....(2)

Subtracting eq. (1) from eq. (2) , we get

$2\mu\ (t_2 - t_1) = \lambda \ \Rightarrow \quad$ But $(t_2 - t_1) = BC \ \Rightarrow \ \therefore \quad 2\mu\ (BC) = \lambda$

or $\quad BC = \dfrac{\lambda}{2\mu}$, $\quad$ From the Δ^{le} ABC, $\angle$ CAB = θ and BC = AB $\tan\theta$

$\therefore \quad (AB) \tan\ \theta = \dfrac{\lambda}{2\mu}$ ....(3)

AB is the distance between successive dark fringes and it also equals the separation of the successive bright fringes. It is, therefore, called the **fringe width, β.** That is $AB = \beta$. We may write eq. (3) as $\quad \beta = \dfrac{\lambda}{2\mu \tan\theta}$

For small values of θ , $\tan\theta \approx \theta$. $\Rightarrow \quad \therefore\ \beta = \dfrac{\lambda}{2\mu\theta}$ ....(4)

As the quantities on the right side of the above equation are all constant, β is constant for a given wedge angle. According to equ. (4), an increase in the angle θ makes the fringes move closer. At an angle $\theta \approx 1°$, the interference pattern vanishes. On the other hand, if θ is gradually decreased, the fringe separation increases, and ultimately the fringes disappear as the faces of the film become parallel.

The interference pattern has the following salient features.

(i) Fringe at the apex is dark. **(ii) Fringes are straight and parallel.**

(iii) Fringes are equidistant. **(iv) Fringes are localized.**

(v) Fringes are of equal thickness. **(iv) Fringe at the apex is dark :**

At the apex, the two glass slides are in contact with each other. Therefore , the thickness of the air film at the contact edge is negligible ($t \cong 0$) . The optical path difference there becomes.

$\Delta = 2\mu t - \lambda / 2 = 0 - \lambda /2 = -\lambda / 2$

In implies that a path difference of $\lambda / 2$ or a phase difference of π occurs between the reflected waves at the edge. The two waves interference destructively. Therefore, the fringe at the apex is always dark

(ii) Straight and parallel fringes :

Each fringe in the pattern is produced by the interference or rays reflected from sections of the wedge having the same thickness .The locus of points having the same thickness lie along lines parallel to the contact edge . Therefore, the fringes are straight . Since the fringes are equidistant, they will be parallel

(iii) Equidistant fringes : The fringe width β is given by $\beta \approx \lambda / 2\theta$

where λ is the wavelength of the incident light and θ is the angle of the wedge. As the quantities λ and θ are constants, β is constant for a given wedge angle. Therefore, the fringes are equidistant.

(iv) Localized fringes :

The fringes form very close to the top surface of the wedge and can be seen with a microscope.

(v) Fringes of equal thickness :

In thin films of thickness of the order of a few λ , the rays from various parts of the film have almost the same inclination and hence the path difference between the overlapping waves changes mainly due to change of thickness .The fringes produced in such cases are mainly due to the variation in thickness of the film. Each fringe will be the locus of points of the same thickness. Such fringes are called **fringes of equal thickness.**

DETERMINATION OF THE WEDGE ANGLE

The wedge angle θ can be experimentally determined with the help of a travelling microscope. Using the microscope, the positions of dark fringes at two distant points Q and R are noted (Fig. 8). Let the distance OQ be x_1 and OR be x_2. Let the thickness of the wedge be t_1 at Q and t_2 at R.

The dark fringe at Q is given by $\Rightarrow \quad 2\mu\, t_1 = m\,\lambda$

But as θ is very small, we can write

$t_1 = x_1 \tan\theta \cong x_1\theta$, $\quad \therefore\ 2\mu\, x_1\theta = m\lambda$

Similarly we can write for the dark fringe at R as

$\therefore\ 2\mu x_2 \theta = (m + N)\lambda$

where N is the number of dark fringes lying between the position Q and R .

Subtracting above equ., we get

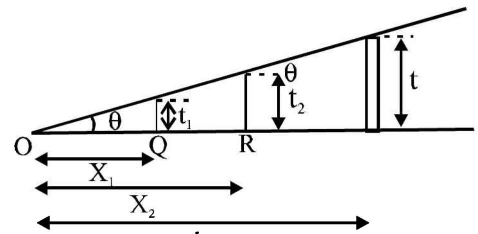


fig. (8)

$$2\mu (x_2 - x_1)\theta = N\lambda \quad \Rightarrow \quad \therefore\ \theta = \frac{N\lambda}{2\mu(x_2 - x_1)}$$

In case of air $\mu = 1$ and the above relation reduces to $\theta = \dfrac{N\lambda}{2(x_2 - x_1)}$

DETERMINATION OF THE THICKNESS OF THE SPACER

The thickness of the spacer used to form the wedge shaped air film between the glass slides can be determined from the above measurements. If 't' is the thickness of the spacer (foil or wire) used, we can write from (Fig. 8) that $t = L \tan\theta \cong L\theta$

where L is the length of the air wedge. Using the above equ., we obtain, $\therefore t = \dfrac{lN\lambda}{2(x_2 - x_1)}$

6.29 NEWTON'S RINGS

Newton's rings are an example of fringes of equal thickness. Newton's rings are formed when a plano - convex lens P of a large radius of curvature placed on a sheet of plane glass AB is illuminated from the top with monochromatic light (Fig. 9). The combination forms a thin circular air film of variable thickness in all directions around the point of contact of the lens and the glass plate. The locus of all points corresponding to specific of air film falls on a circle whose centre is at O. Consequently, interference fringes are observed in the form of a series of concentric rings with their centre at O. Newton originally observed these concentric circular fringes and hence they are called **Newton's rings.**

The experimental arrangement for observing Newton's rings is shown in Fig. 9.

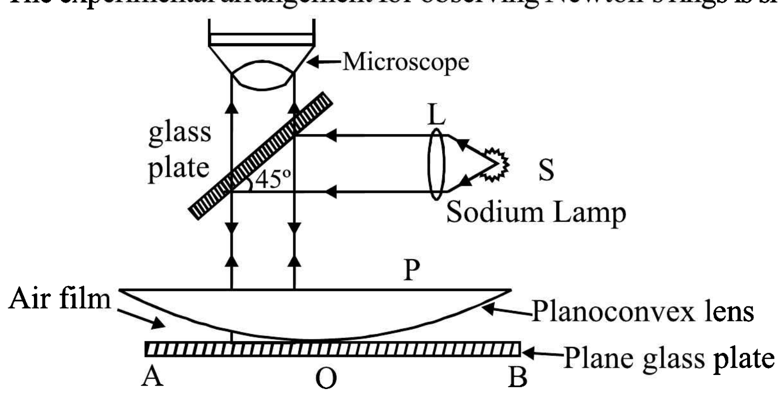


fig. (9)

CONDITION FOR BRIGHT AND DARK RINGS

The optical path difference between the rays is given by $\Delta = 2\,\mu\, t \cos r - \lambda/2$. Since $\mu = 1$ for air and $\cos r = 1$ for normal incidence of light,

$\Delta = 2t - \lambda / 2$

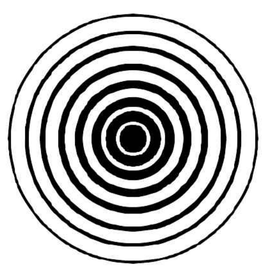
Intensity maxima occur when the optical path difference $\Delta = m\,\lambda$. If the difference in the optical path between the two rays is equal to an integral number of full waves, then the rays meet each other in phase. The crests of one wave falls on the crests of the other and the waves interfere constructively. Thus, if $2t - \lambda / 2 = m\lambda$,

$2\,t = (2m + 1)\,\lambda / 2$ bright fringe is obtained.

Intensity minima occur when the optical path difference is $\Delta = (2m + 1)\,\lambda / 2$. If the difference in the optical path between the two rays is equal to an odd integral number of half-waves, then the rays meet each other in opposite phases. The crests of one wave fall on the troughs of the other and the waves interfere destructively.

6.29(A) CIRCULAR FRINGES

In Newton's ring arrangement, a thin air film is enclosed between a plano -convex lens and a glass plate. The thickness of the air film at the point of contact is zero and gradually increases as we move outwards. The locus of points where the air film has the same thickness then fall on a circle whose centre is the point of contact. Thus, the thickness of air film is constant at points on any circle having the point of lens - glass plate contact as the centre. The fringes are therefore circular.



6.29(B) RADII OF DARK FRINGES

Let R be the radius of curvature of the lens (Fig.10).Let a dark fringe be located at Q. Let the thickness of the air film at Q be PQ = t. Let the radius of the circular fringe at Q be $OQ = r_m$. By the Pythagorus theorem,

$PM^2 = PN^2 + MN^2$, $\quad\therefore\; R^2 = r_m^{\,2} + (R-t)^2$

or $\quad r_m^{\,2} = 2Rt - t^2$

As $R >> t$, $2Rt >> t^2$, $\therefore$ $r_m^2 \cong m\lambda R$

$r_m = \sqrt{m\lambda R}$

The radii of dark fringes can be found by inserting values 1,2,3,............for m. Thus,

$r_1 = \sqrt{1\lambda R}$ or $r_1 \propto \sqrt{1}$

$r_2 = \sqrt{2\lambda R}$ or $r_2 \propto \sqrt{2}$

$r_3 = \sqrt{3\lambda R}$ or $r_3 \propto \sqrt{3}$ and so on

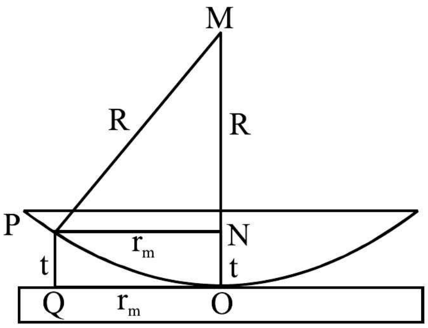


fig. (10)

It means that the radii of the dark rings are proportional to under root of the natural numbers.

The above relation also implies that $r_m \propto \sqrt{\lambda}$

Thus, the radius of the mth dark ring is proportional to under root of wavelength.

Ring Diameter :

Diameter of mth dark ring $D_m = 2r_m \Rightarrow D_m = 2\sqrt{2Rt} \Rightarrow D_m = 2\sqrt{m\lambda R}$

6.29(C) NEWTON'S RINGS IN TRANSMITTED LIGHT

Newton's rings in transmitted light may be observed with the arrangement made as shown in the Fig. The condition for maxima or bright rings is $2\mu t \cos r = m\lambda$

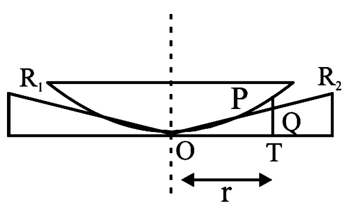
and for dark rings is $2\mu t \cos r = (2m + 1)\lambda / 2$

As $\mu = 1$ and $r = 0$ for normal observation, the above expressions may be simplified to

For bright fringes $2t = \mu\lambda$ and for dark rings $2t = (2m + 1)\lambda / 2$

As $t = \dfrac{r^2}{2R}$, the radius for the bright ring is given by $r_m^2 = m\lambda R$

and the radius for dark rings is given by $r_m^2 = (2m + 1)\lambda R / 2$



Ex. 9 In YDSE experiment the distance between slits is d = 0.25 cm and the distance of screen is D = 120 cm from slits. If the wavelength of light used is λ = 6000 Å and I_0 is the intensity of central maximum, at what distance from the centre of the intensity will be $\dfrac{I_0}{2}$?

(A) 4.8×10^{-5} m **(B)** 7.2×10^{-5} m **(C)** 3.6×10^{-5} m **(D)** 5.4×10^{-5} m

Ans. B

Ex. 10 In a YDSE experiment, I_0 is given to be the intensity of the central bright fringe and β is the fringe width. Then, at a distance y from the Central Bright Fringe, the intensity will be

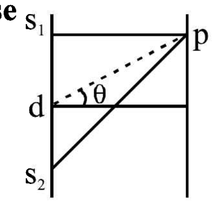
(A) $I_0 \cos(\pi y/\beta)$ **(B)** $I_0 \cos^2(\pi y/\beta)$ **(C)** $I_0 \cos(2\pi y/\beta)$ **(D)** $I_0 \cos^2(\pi y - 2\beta)$

Ans. B

Ex. 11 In a Young's Double Slit Experiment, the separation between the slits is d, distance between the slit and screen is (D >> d). In the interference pattern, there is a maxima exactly in front for each slit.Then the possible wavelength (s) used in the experiment are

(A) d^2/D, $d^2/2D$, $d^2/3D$ **(B)** d^2/D, $d^2/3D$, $d^2/5D$

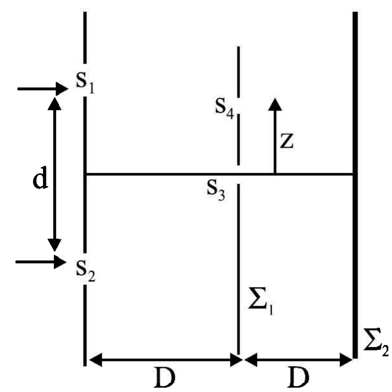
(C) $d^2/2D$, $d^2/4D$, $d^2/6D$ **(D) none of these**



Sol. $s_2p - s_1p = \frac{d \times y}{D} = \frac{d \times (d/2)}{D} = \frac{d^2}{2D} \Rightarrow \frac{d^2}{2D} = n\lambda$

$\lambda = \frac{d^2}{2nD}$, n = 1, 2, ... $\Rightarrow \lambda = \frac{d^2}{2D}, \frac{d^2}{4D}, \frac{d^2}{6D}$

Ex. 12 Consider the situation shown in the figure. The two slits S_1 and S_2 placed symmetrically around the central line are illuminated by a monochromatic light of wavelength λ. The separation between the slits is d. The light transmitted by the slits falls on a screen Σ_1 placed at a distance D from the slits. The slit S_3 is at the central line and the slit S_4 is at a distance z from S_3. Another screen Σ_2 is placed a further distance D away from Σ_1. Find the ratio of the maximum to minimum intensity observed on Σ_2 if z is equal to



(a) $\frac{D\lambda}{2d}$ (b) $\frac{\lambda D}{4d}$

Sol. **(a)** Let I is the intensity due to slits S_1 and S_2 on the screen. Further, intensity at any point on the screen Σ_1 is given by

$I_p = 4I\cos^2\left(\frac{\varphi}{2}\right)$ At slit S_3, $\phi = 0$ $\therefore Is_3 = 4I$

At slit S_4, $\Delta x = \frac{dz}{D} = \frac{\lambda}{2}$; $\therefore \phi = \pi \Rightarrow Is_3 = 0$

Now on screen Σ_2 $\quad I_{max} = \left(\sqrt{Is_3} + \sqrt{Is_4}\right)^2 = 4I$; $I_{min} = \left(\sqrt{Is_3} - \sqrt{Is_4}\right)^2 = 4I \quad \therefore \dfrac{I_{max}}{I_{min}} = 1$

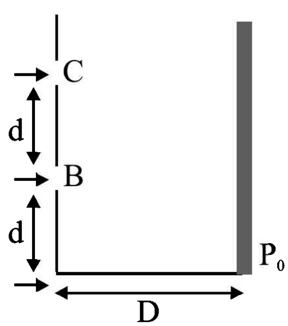
Ans

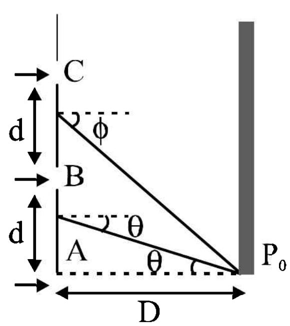
(b) $z = \dfrac{D\lambda}{4d}$ $\qquad Is_3 = 4I$ $\qquad$ At slit S_4, $\Delta x = \dfrac{dz}{D} = \dfrac{\lambda}{4}$ $\qquad \therefore \ \phi = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{4} = \dfrac{\pi}{2}$

$\therefore \ Is_4 = 41 \cos^2\left(\dfrac{\pi}{4}\right) = 2I$; $\quad \therefore \ I_{max} = \left(\sqrt{Is_3} + \sqrt{Is_4}\right)^2 = \left(\sqrt{41} + \sqrt{21}\right)^2 = I\left(2 + \sqrt{2}\right)^2$

Similarly, $I_{min} = \left(\sqrt{Is_3} + \sqrt{Is_4}\right)^2 = \left(\sqrt{41} + \sqrt{21}\right)^2 = I\left(2 - \sqrt{2}\right)^2 \quad \therefore \ \dfrac{I_{max}}{I_{min}} = \left(\dfrac{2 + \sqrt{2}}{2 - \sqrt{2}}\right)^2$ **Ans.**

Ex. 13 Figure shows three equidistant slits being illuminated by a monochromatic parallel beam of light. Let $BP_0 - AP_0 = \lambda/3$ and $D >> \lambda$. (a) Show that in this case $d = \sqrt{2\lambda D/3}$. (b) Show that the intensity at P_0 is three times the intensity due to any of the three slits individually.





Sol. $\quad BP_0 - AP_0 = \dfrac{\lambda}{3} \quad$ or $\quad d \sin\theta = \dfrac{\lambda}{3} \quad$ or $\quad d \tan\theta = \dfrac{\lambda}{3}$

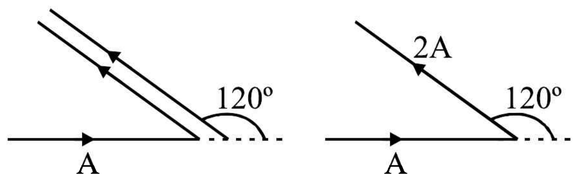
(For small angle $\tan\theta \approx \sin\theta \approx \theta$), $\qquad \therefore \qquad d = \sqrt{\dfrac{2\lambda D}{3}}$

(b) $\qquad \Delta x_{A/B}$ = path difference between waves coming from A and B $= \dfrac{\lambda}{3}$

$\therefore \qquad \phi_{A/B}$ = phase difference, $\qquad = \dfrac{2\pi}{\lambda} \Delta x_{AB} = \dfrac{2\pi}{3}$

Similarly, $\Delta x_{BC} = d \sin\phi \qquad , \qquad = d\left(\dfrac{3d/2}{D}\right) = \dfrac{3d^2}{2D} = 1$, $\qquad \therefore \qquad \phi_{BC} = 2\pi$

Now, phase diagram of the waves arriving at P_0 is as shown below :

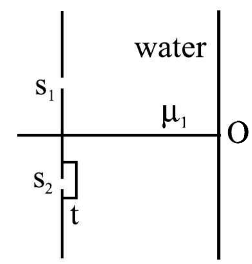


$\therefore$ Amplitude of resultant wave is given by

$A' = \sqrt{A^2 + (2A)^2 + 2(A)(2A)\cos 120^\circ} = \sqrt{3}\,A$ As intensity (I) α A^2

$\therefore$ Intensity at P_0 will be three times the intensity due to any of the three slits individually.

Ex. 14 A Young's Double Slit Experiment is conducted in water (μ_1) as shown in the figure, and a glass plate of thickness t and refractive index μ_2 is placed in the path of S_2. Wavelength of light in water is λ. Find the magnitude of the phase difference between waves coming from S_1 and S_2 at 'O'.

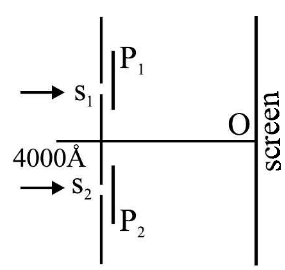


(A) $\left|\left(\frac{\mu_2}{\mu_1}-1\right)t\right|\frac{2\pi}{\lambda}$ (B) $\left|\left(\frac{\mu_1}{\mu_2}-1\right)t\right|\frac{2\pi}{\lambda}$

(C) $|(\mu_2-\mu_1)\,t|\,\frac{2\pi}{\lambda}$ (D) $|(\mu_2-1)\,t|\,\frac{2\pi}{\lambda}$

Ans A

Ex. 15 P_1, P_2 are transparent plates having equal thickness 20μm and refractive indices $\mu_1 = 1.6$, $\mu_2 = 1.5$. P_1 transmits 75 % whereas P_2 transmits 50 % of energy incident. Without P_1 and P_2 intensity at O, $I_0 = 4I$. Find intensity at O after placing P_1 and P_2. S_1, S_2 are identical slits.



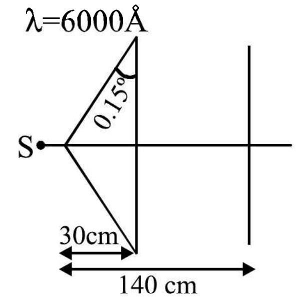
Sol. $S_2P - S_1P = (\mu_1 - 1)\,t - (\mu_2 - 1)\,t = 0.1\,t = 2 \times 10^{-6}$ m,

$$\Delta\phi = \frac{\Delta x \times 2\pi}{\lambda} = \frac{2\times10^{-6}\times2\pi}{4000\times10^{-10}} = 10\pi$$

$I_1 = 0.75I$, $I_2 = 0.5I$, $I = 0.75I + 0.5I + 2\left(\sqrt{0.75\times0.5}\right) I\cos\Delta\phi = \frac{51}{4} + I\sqrt{\frac{3}{2}}$

Example No. 16 to 20 (5 questions)

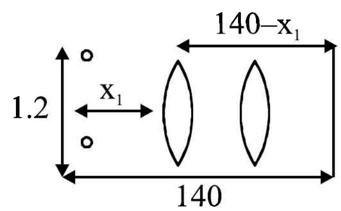
Refracting angles of biprism are both 0.15°. Distance of biprism from a source is 30 cm. Distance between the source and the screen is 140 cm. The Fresnel's biprism arrangement shown in the figure uses a source of light of wavelength 6000 Å. When a converging lens is moved between the biprism and the screen, two images of S are focused on the screen for two positions of the lens. the separations between the two images for the two positions are found to be 0.9 mm and 1.6 mm.



Ex. 16 The focal length of the lens is

(A) $\frac{120}{7}$ **cm** **(B)** $\frac{120}{7}$ **cm** **(C) Data not sufficient** **(D) None**

Sol. **(B)**



$h_0 = \sqrt{h_1 h_2} = \sqrt{1.6 \times 0.9} = 1.2$ mm, $h_1 = 1.6$, $h_0 = 1.2$, $m_1 = \frac{h_1}{h_2} = \frac{1.6}{1.2} = \frac{140 - x_1}{x_1} \Rightarrow x_1$ $= 80$

$\Rightarrow \frac{1}{60} - \frac{1}{-80} = \frac{1}{f} \quad \Rightarrow \quad f = \frac{240}{7}$ cm & $\quad x_2 = 60$]

Ex. 17 Gap between two positions of the lens is

(A) 80 cm **(B) 60 cm** **(C) 40 cm** **(D) 20**

Sol. (D) $(x_2 - x_1) = 20$cm]

Ex. 18 Refractive index of the prism material is approximately

(a) 1.46 **(B) 1.56** **(C) 1.66** **(D) 1.76**

Sol. **(D)** $h_0 = \sqrt{h_1 h_2} = \sqrt{1.6 \times 0.9} = 1.2 \text{ mm} = 2a\delta \quad \Rightarrow \quad 1.2 = 2aA(\mu - 1)$

$A = \frac{0.05}{180} \times \pi$, $a = 30$ cm $\quad \Rightarrow \quad \mu \approx 1.76$

Ex. 19 The fringe width of the pattern on the screen (after the lens has been removed) is

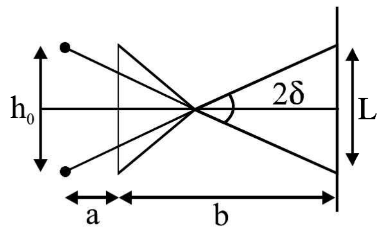
(A) 0.7 mm **(B) 0.35 mm** **(C) 1.4 mm** **(D) None**

Sol. (A) $d = 1.2$ mm, $D = 140$ cm

Ex. 20 The total number of fringes obtained on the screen are

(A) $\frac{4400}{7}$ **(B)** $\frac{400}{7}$ **(C)** $\frac{44}{7}$ **(D) None**

Sol. **(C)**



$$L = 2\delta b \;;\; n = \frac{2\delta b}{\beta} = \frac{h_0 b}{a\beta} \Rightarrow \frac{44}{7}$$

Ex. 21 ***IN BILLET'S LENS ARRANGEMENT*** **a convex lens of focal length 50 cm is cut along the diameter into two identical halves A and B and in the process a layer C of the lens thickness 1 mm is lost. Then the two halves A and B are put together to form a composite lens. Now, infront of this composite lens a source of light emitting wavelength λ = 6000 Å is placed at a distance of 25 cm as shown in the figure. Behind the lens there is a screen at a distance 50 cm from it. Find the fringe width of the interference pattern obtained on the screen.**

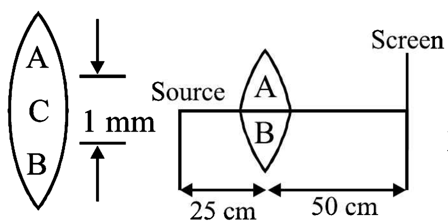
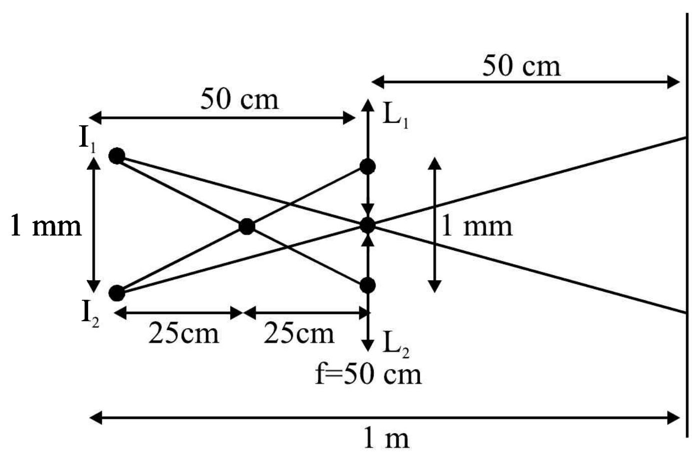


Fig.

Sol.



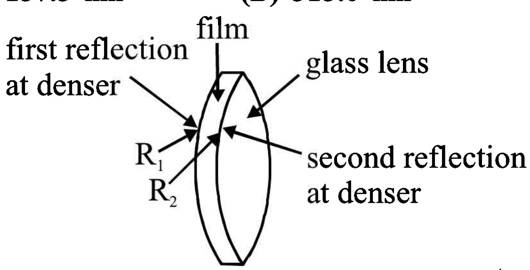
from the question, u = – 25 $\Rightarrow \frac{1}{v} - \frac{1}{-25} = \frac{1}{50} \Rightarrow \frac{1}{v} = -\frac{1}{50} \Rightarrow v = -50$

$\beta = \frac{6\times10^{-7}\times1}{10^{-3}} = 6 \times 10^{-4} = 0.6$ mm

Ex. 22 Many people's glasses appear to be blue-green in colour when viewed under reflected light. A thin film of index of refraction n = 1.35 is applied to the outside surface of the glass so that the film/glass interface does not reflect any red light incident near normal of wavelength λ = 630 nm. What thickness must the film layer be in order to achieve this ? Take the index of refractions of air and glass to be 1.0 and 1.6 respectively.

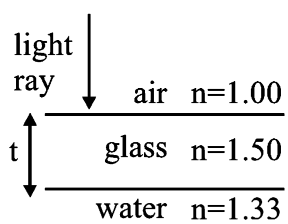
(A) 157.5 nm (B) 315.0 nm (C) 233.3 nm (D) 116.7 nm

Sol. **(D)**



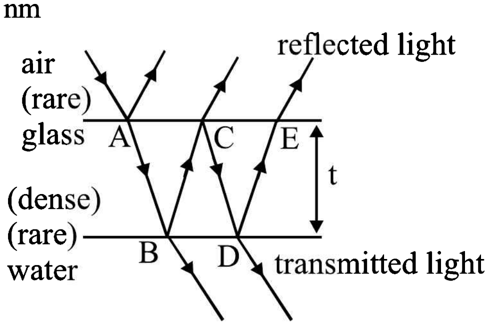
$\Delta X_{net} = 2\mu t = \lambda / 2$; $t = \lambda / 4\,\mu = \frac{(630\times10^{-9})}{4\times1.35} = 116.7$ nm

Ex. 23 A light ray is incident normal to a thin layer of glass. In the given the figure, what is the minimum thickness of the glass so that the reflected light gives distructive interference (λ_{air} = 600 nm) ?



(A) 50 nm (B) 100 nm
(C) 150 nm (D) 200 nm (E) 500 nm

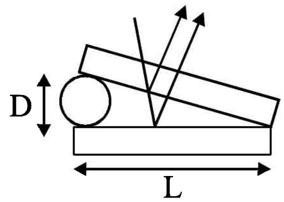
Sol. For reflected light to have orangish color, rays from A, C, E must be out of phase for λ = 600 nm



or $\delta = (2n-1)\,\frac{\lambda}{2}$ or $2\mu_g t - \frac{\lambda}{2} = (2n-1)\,\frac{\lambda}{2}$

or $\quad t = \dfrac{n\lambda}{2\mu_g}$ $\quad$ or $\quad t_{min} = \dfrac{\lambda}{2\mu_g} = 200 \text{ nm}$

Ex. 24 A wedge-shaped film of air is produced by placing a fine wire of diameter D between the ends of two flat glass plates of length L = 20 cm, as shown in the fig. When the air film is illuminated with light of wavelength λ = 550 nm, there are 12 dark fringes per centimeter. Find D



Sol. As indicated in the fig. only one of the reflected ray suffers a phase inversion.At the thin end of the wedge, where the thickness is less than $\lambda / 4$, the two rays interfere destructively.This region is dark in the reflected light. The condition for destructive interference in the reflected light is $\quad 2t = m\lambda \quad m = 0, 1, 2, ...$
the change in thickness between adjacent dark fringes is $\Delta t = \lambda/2$. The horizontal spacing between fringes $d = 1 / 12 \text{ cm} = 8.3 \times 10^{-4}$ m. From figure we see that $D / L = \Delta t / d$, (from similar triangle)

$$\Delta = \frac{\lambda L}{2d} = \frac{(5.5\times10^{-7}\text{m})(0.2\text{m})}{16.6\times10^{-4}\text{m}} \quad \Rightarrow \quad \text{Thus} \quad D = 6.6 \times 10^{-5}\text{ m}$$

Ex. 25 In an experiment on Newton's rings the light has a wavelength of 600 nm. The lens has a refractive index of 1.5 and a radius of curvature of 2.5 m. Find the radius of the 5th bright fringe.

Sol. If R is the radius of curvature of the lens, then from Fig. of **Newton's ring in theory** we see that $r^2 = R^2 - (R - t)^2$, where r is the radius of a fringe, and t is the thickness of the film. Since t is very small. we may drop terms in t^2 to obtain. $\Rightarrow \quad r^2 \approx 2Rt$ **......(i)**
In order to find r, we must first find t. The condition for a bright fringe is

$$2t = \left(m + \frac{1}{2}\right)\lambda_F \quad \textbf{.......(ii)}$$

We note that n = 1 for the air film (the index for the glass is irrelevant) and that m = 4 for the fifth bright fringe. Thus, from (ii) $\Rightarrow \quad t = \dfrac{(4.5)(6\times10^{-7})}{2} = 1.35 \times 10^{-6}$ m

Substituting this into (i), we find $r = \sqrt{2Rt} = 2.6 \times 10^{-3}$ m.

EXERCISE – I

Q.1 Two coherent waves are described by the expressions.

$$E_1 = E_{0\sin}\left(\frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6}\right) \quad ; \qquad E_2 = E_{0\sin}\left(\frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}\right)$$

Determine the relationship between x_1 and x_2 that produces constructive interference when the two waves are superposed?

Sol. In interference, $E_r = E_1 + E_2$ (by superposition principle)

$$\phi_1 = \frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6} \quad ; \qquad \phi_2 = \frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}$$

Phase difference at t = 0, $\Delta\phi = \left(\frac{2\pi x_1}{\lambda} + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} + \frac{\pi}{8}\right)$

For constructive interference, $\Delta\phi = \pm 2n\pi$ (where n = 0, 1, 2, 3 ....)

$$\Rightarrow \pm 2n\pi = \frac{2\pi}{\lambda}(x_1 - x_2) + \frac{\pi}{24} \Rightarrow \pm\left(n - \frac{1}{48}\right)\lambda = (x_1 - x_2)$$

[Ans. $\left(n - \frac{1}{48}\right)\lambda = x_1 - x_2$**]**

Q.2 In a Young's Double Slit Experiment for interference of light, the slits are 0.2 cm apart and are illuminated by yellow light ($\lambda = 600$ nm). What would be the fringe width on a screen placed 1 m from the plane of slits if the whole system is immersed in water of index 4/3 ?

Sol. Fringe width depends on medium D and d. $\Rightarrow$ Fringe width $= \frac{\lambda_m D}{d}$ [λ_m = wavelength in the medium)

By the definition of refractive index : $\mu_m = \frac{\lambda_a}{\lambda_m} \Rightarrow \lambda_m = \frac{\lambda_a}{\mu_m} = \frac{600}{\frac{4}{3}} = \left(\frac{1800}{4}\right)$mm

Given that, D = 1m ; d = 0.2 cm

Then, Fringe width $= \frac{\lambda_m D}{d} = \left(\frac{1800\times10^{-9}}{4\times0.2\times10^{-2}}\right) = \left(\frac{9}{4}\times10^{-4}\right)$m $= 0.225$ mm

[Ans. 0.225 mm]

Q.3 **In Young's double slit experiment the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slit. It is found that the 9th bright fringe is at a distance of 7.5 mm from the second dark fringe from the center of the fringe pattern on same side. Find the wavelength of the light used.**

Sol. In Young's Double Slit Experiment, (YDSE), for bright pattern: $Y_B = \pm \frac{n\lambda D}{d} (n = 0, 1, 2, ...)$

For dark pattern, $y_d = \pm \frac{(2n-1)\lambda D}{2d}$ (n = 1, 2, 3, ...)

For the second dark pattern above the point 'O', $y_{2nd} = \frac{3\lambda D}{2d}$

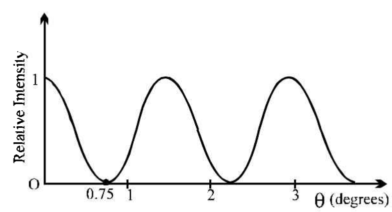
For the ninth bright pattern above the point 'O', $Y_{9^{th}} = \frac{9\lambda D}{d}$

Given that, $Y_9 - Y_2 = 7.5$ mm

$$\Rightarrow \frac{9\lambda D}{d} - \frac{3\lambda D}{2d} = 7.5 \times 10^{-3} \quad \Rightarrow \quad \lambda = 5000 Å$$

[Ans. 5000 Å]

Q.4 **Light of wavelength 520 nm passing through a double slit, produces interference pattern of relative intensity versus deflection angle θ as shown in the figure. Find the separation d between the slits.**



Sol. Formula of intensity, $I = 4I_0 \cos^2 \frac{\Delta\phi}{2}$, Where $I_1 = I_2 = I_0$ then, $\frac{I}{4I_0} = \cos^2 \frac{\Delta\phi}{2}$

So, $I_r = \cos^2 \frac{\Delta\phi}{2}$, here $I_r = \frac{I}{4I_0}$

In the given question, when $\theta = 0.75$, $I_r = 0$. By concept of YDSE, we know that, $\Delta x = d\sin\theta = d\theta$

then, $\Delta x = \frac{\lambda}{2} (\text{For } I_r = 0) \Rightarrow \frac{\lambda}{2} = d\theta \Rightarrow d = \frac{\lambda}{2\theta}$ (where θ is in radian)

Putting the values, $\lambda = 520 \times 10^{-9}$m, $\theta = \frac{0.75 \times \pi}{180}$ rad. In the above equation, we get $d = 1.99 \times 10^{-5}$ m

[Ans. 1.99x 10^{-2} mm]

Q.5 In a YDSE apparatus, d = 1mm, λ = 600nm and D = 1m. The slits produce same intensity on the screen. Find the minimum distance between two points on the screen having 75% intensity of the maximum intensity.

Sol. For minimum distance, one point should be above the central maxima & the other point should be below the central maxima $I = 4I_0\cos^2\frac{\Delta\phi}{2}$ (Where $I_1 = I_2 = I_0$)

Given that, $I = 75\%$ of maximum intensity $= \frac{75}{100} \times 4I_0 = 3I_0$

We need the path difference at the point where intensity $= 3I_0$. For this we need $\Delta\phi$ at that point, $\Delta\phi = \frac{\pi}{3}$ (From the above equation)

So, $\frac{2\pi}{\lambda}(\Delta x) = \frac{\pi}{3} \quad \Rightarrow \Delta x = \frac{\lambda}{6} \quad \& \quad \Delta x = \frac{\lambda}{6} = \frac{dy}{D} \Rightarrow y = \frac{yD}{6d}$

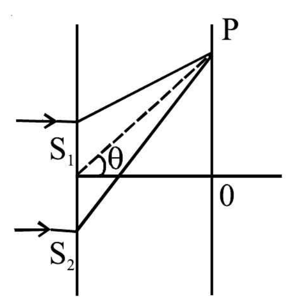
Since one point is above & one below the central maxima, so, minimum distance, $2y = 0.2$ mm

[Ans. 0.2 mm]

Q.6 The distance between two slits in a YDSE apparatus is 3mm. The distance of the screen from the slits is 1m. Microwaves of wavelength 1 mm are incident on the plane of the slits normally. Find the distance of the first maxima on the screen from the central maxima.

Sol. In YDSE, Path difference at a point P, $\Delta x = S_2P - S_1P$

$= d\sin\theta$ For the maxima, $\Delta x = \pm n\lambda \Rightarrow \sin\theta = \pm\left(\frac{n\lambda}{d}\right)$.

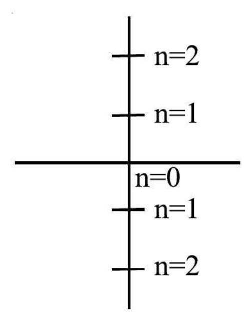


Since λ and d are both comparable, $\sin\theta \neq \theta$

($\because$ θ is not small) $\Rightarrow \sin\theta = \pm\frac{n}{3}$

For I[st] maxima, $n = 1 \quad \Rightarrow \sin\theta = \frac{1}{3}$

[For above the central maxima]



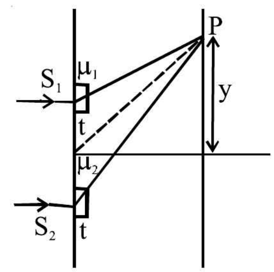
$\Rightarrow \tan\theta = \frac{1}{\sqrt{8}} = \frac{y}{D} \quad \therefore \; y = \frac{D}{\sqrt{8}} = 35.35$ cm

& n = 0, 1, 2, only for satisfying the equation $\sin\theta = \pm\frac{n}{3}$

($n = 3$ not possible as $\theta = \frac{\pi}{2}$) $\Rightarrow \quad \therefore$ 5 maxima present.

[Ans. 35.35 cm app., 5]

Q.7 **One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of refractive index 1.7. The point on the screen, where central bright fringe was formed before the introduction of the glass sheets, is now occupied by the 5^{th} bright fringe. Assuming that both the glass plates have the same thickness and wavelength of light used is 4800 Å, find their thickness.**



Sol. Path difference at a point P, $\Delta x = (S_2P - t + \mu_2 t) - (S_1P - t + \mu_1 t)$

$= (S_2P - S_1P) + (\mu_2 - \mu_1)t$

$= \dfrac{dy}{D} + (\mu_2 - \mu_1)t$; ($\because S_2P - S_1P = d\sin\theta \simeq d\tan\theta = \dfrac{dy}{D}$)

For 5th bright fringe, $\Delta x = 5\lambda$; $\therefore 5\lambda = \dfrac{dy}{D} + (\mu_2 - \mu_1)t$

Given that 5th bright pattern is formed at the point where central bright fringe was formed before introducing the glass sheet.

$$\therefore 5\lambda = (\mu_2 - \mu_1)t \ (\because y = 0) \Rightarrow t = \frac{5\lambda}{\mu_2 - \mu_1} = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8 \times 10^{-6}\text{ m}$$

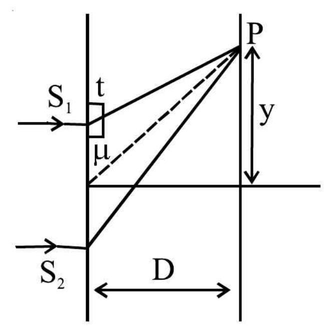
[Ans. 8 μm]

Q.8 **A monochromatic light of λ = 5000 Å is incident on two slits separated by a distance of 5 × 10^4m. The interference pattern is seen on a screen placed at a distance of 1 m from the slits . A thin glass plate of thickness 1.5 × 10^{-6} m & refractive index μ = 1.5 is placed between one of the slits & the screen. Find the intensity at the centre of the screen, if the intensity there is I_0 in the absence of the plate . Also find the lateral shift of the central maximum.**

Sol. Path difference at a point P, $\Delta x = (S_2P) - (S_1P - t + \mu t) = (S_2P - S_1P) + (t - \mu t)$

$= \dfrac{dy}{D} - t(\mu - 1)$

For finding intensity at the center of the screen, Put y = 0.



$\therefore |\Delta x| = t(\mu - 1)$; $\therefore \Delta\phi = \dfrac{2\pi}{\lambda}|\Delta x| = \dfrac{2\pi}{\lambda} \times t(\mu - 1)$

$I = I_0\cos^2\dfrac{D\phi}{2}$ Putting the values of $\Delta\phi$

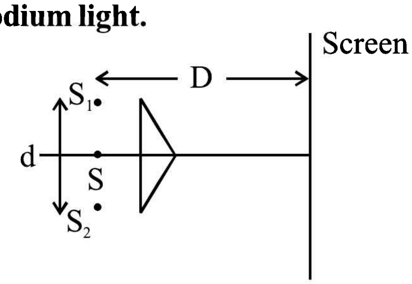
in the above equation we get, I = 0

For lateral shift, Put n = 0 (for the central maxima)

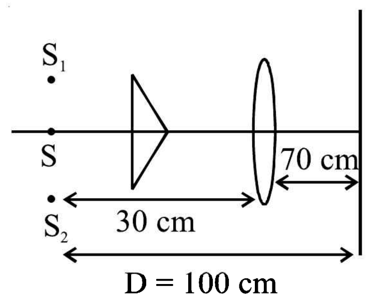
$\therefore \Delta x = \pm n\lambda = 0 \Rightarrow \dfrac{dy}{D} = t(\mu - 1) \Rightarrow y = \dfrac{Dt(\mu - 1)}{d}$ Putting respective values, y = 1.5 mm

[Ans. 0, 1.5 mm]

Q.9 In a Biprism experiment with sodium light, bands of width 0.0195 cm are observed at 100 cm from slit. On introducing a convex lens 30 cm away from the slit between biprism and screen, two images of the slit are seen 0.7 cm apart at 100 cm distance from the slit. Calculate the wavelength of sodium light.



Sol. In the Biprism experiment –



Here, S is the source of light and S_1 and S_2 are the images of S through the biprism.
d = Distance between S_1 and S_2 ; D = Distance between slits and the screen.

Then β (φρινγε width) = $\frac{\lambda D}{d}$

Introduce a convex lens 30 cm away from shift between biprism. From the given diagram, for convex lens, u = – 30 m ; v = 70 cm

Then, $m = \frac{v}{u} = \frac{d'}{d} = \frac{h_i}{h_0} \Rightarrow \frac{70}{-30} = \frac{0.7}{d}$, Given that, d' = 0.7 cm ⇒ d = –0.3 cm

–ve sign shows that S_1 & S_2 positions have been interchanged. Also β = fringe width = 0.0195 cm

$$\therefore \quad \beta = \frac{\lambda D}{d} \quad \Rightarrow \quad \lambda = \frac{d\beta}{D}$$

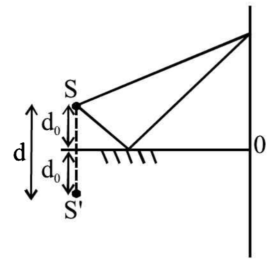
Putting values in the above equation we get, $\lambda = 5.85 \times 10^{-7}$ m = 5850 Å

[Ans. λ = 5850 Å]

Q.10 A long narrow horizontal slit lies 1mm above a plane mirror. The interference pattern produced by the slit and its image is viewed on a screen distant 1m from the slit. The wavelength of light is 600nm. Find the distance of first maximum above the mirror.

Sol. S' is the image of S. Now, S' & S will function as two slits as in YDSE.

$\therefore\ \Delta x = (S'P - SP) + \frac{\lambda}{2}$ (Where $\frac{\lambda}{2}$ is the extra path changed due to reflection from mirror)



Given that,
$d_0 = 1$ mm
$\therefore\ d = 2$ mm

For Ist maxima, $\Delta x = \lambda$; $\therefore\ \lambda = \frac{d\lambda}{D} + \frac{\lambda}{2}$

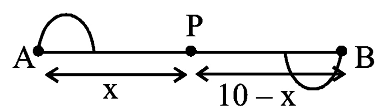
$\Rightarrow \frac{\lambda}{2} = \frac{dy}{D}$; $\therefore\ y = \frac{\lambda D}{2d} = \frac{600 \times 10^{-19} \times 1}{2 \times 2 \times 10^{-3}} = 0.15$ mm

For bright pattern, $\Delta x = \pm n\lambda$ putting n = 0, we get y < 0 which means central bright is shifted below 0. **[Ans. 0.15 mm]**

Q.11 One radio transmitter A operating at 60.0 MHz is 10.0 m from another similar transmitter B that is 180^0 out of phase with transmitter A. How far must an observer move from transmitter A towards transmitter B along the line connecting A and B to reach the nearest point where the two beams are in phase ?

Sol. Path difference at a point P, $\Delta x = |AP - BP| = |x - (10 - x)| = |2x - 10|$ So, $\Delta\phi = \frac{2\pi}{\lambda}$.

$\Delta x + \pi$ (Since sources are out of phase)



For waves to be in the same phase, $\Delta\phi = 2n\pi$ where, n = 0, ± 1, ± 2

$\Rightarrow \frac{2\pi}{\lambda}\Delta x + \pi = 2n\pi \quad \Rightarrow \frac{2\Delta x}{\lambda} = (2x - 1) \quad \Rightarrow \frac{2|x - 10|}{\lambda} = (2n - 1)$

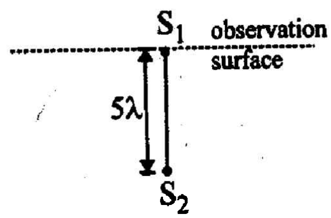
We know that $C = f\lambda \quad \Rightarrow 3 \times 10^8 = 60 \times 10^5 \times \lambda \quad \Rightarrow \lambda = 5$ m

$\therefore\ \frac{2}{5}|2x - 10| = (2n - 1) \quad \Rightarrow \frac{2}{5}(2x - 10) = (2n - 1) \quad \Rightarrow \quad x = \frac{5(2n - 1)}{4} + 5$

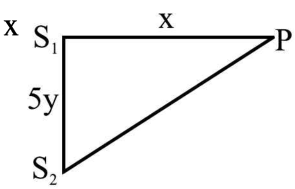
Putting n = 0, $x = \frac{15}{4}$m $\Rightarrow$ Putting n = 1, $x = 5 + \frac{5}{4} = \frac{25}{4}$m

Putting n = – 1, $x = \frac{5}{4}$ m, Putting n = – 2, x < 0 which is not possible. So minimum value of $x = \frac{5}{4}$m = 1.25 m **[Ans. 1.25 m]**

Q.12 Two microwave coherent point sources emitting waves of wavelength λ are placed at 5λ distance apart. The interference is being observed on a flat non-reflecting surface along a line passing through one source, in a direction perpendicular to the line joining the second source (refer figure) Considering λ as 4 mm, calculate the positions of maxima and draw shape of interference pattern. Take initial phase difference between the two sources to be zero.



Sol. Path difference at a point i.e. P, $\Delta x = S_2P - S_1P = \sqrt{(5\lambda)^2 + x^2} - x$



$\therefore\ \Delta x = \pm n\lambda$

For right of S_1, we take +ve values of n.

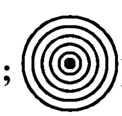
$\therefore\ 25\lambda^2 + x^2 = (x + n\lambda)^2$

Putting n = 1, $25\lambda^2 + x^2 = (x + \lambda)^2 \Rightarrow x = 12\lambda = 12 \times 4 = 48$ mm

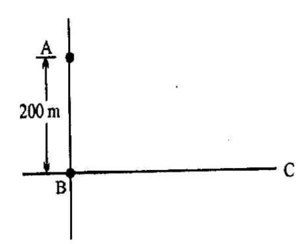
Putting n = 2, $25\lambda^2 + x^2 = (x + 2\lambda)^2 \Rightarrow x = \dfrac{21\lambda^2}{4\lambda} = \dfrac{21\lambda}{4} = 21$ mm

Putting n = 3, $25\lambda^2 + x^2 = (x + 3\lambda)^2 \Rightarrow 16\lambda^2 - 6x\lambda = 0 \Rightarrow x = \dfrac{16\lambda}{6} = \dfrac{16 \times 4}{6} = \dfrac{32}{3}$ mm

Putting different values of n, we get x = 48 mm, 21 mm, $\dfrac{32}{3}$ mm, $\dfrac{9}{2}$ mm, 0 mm and shape of fringe is circular in nature since line joining S_1 & S_2 is $\perp$ to the screen

[Ans. 48, 21, $\dfrac{32}{3}$, $\dfrac{9}{2}$ 0,m.m ;  **]**

Q.13 Two radio antennas radiating wave in phase are located at points A and B, 200m apart (Figure). The radio waves have a frequency of 5.80 MHz. A radio receiver is moved out from point B along a line perpendicular to the line connecting A and B (line BC shown in figure). At what distance from B will there be destructive interference ?

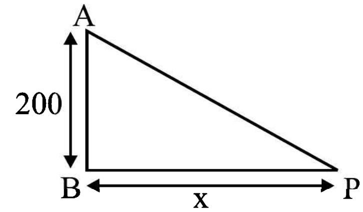


Sol. Let x be the distance of point P from point B where interference is destructive.

Path difference = AP – BP, Path destructive = $\sqrt{2 \times 10^4 + x^2} - x$

For destructive interference, $\sqrt{2\times10^4+x^2}-x=(2n-1)\dfrac{\lambda}{2}$

For the wavelength of radio antennas, $v=f\lambda$



$$\Rightarrow \lambda=\frac{300\times10^6}{5.8\times10^6}\ (51.72)\ \text{m}$$

For, $n=1$ $x=760$, For, $n=2$ $x=21.9$ m

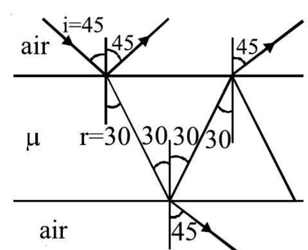
For, $n=3$ $x=89.4$ m ,For, $n=4$ $x=19.6$ m

[Ans. 760 m, 21.8 m, 89.4 m, 19.6 m]

Q.14 Our discussion of the techniques for determining constructive and destructive interference by reflection from a thin film in air has been confined to rays striking the film at nearly normal incidence. Assume that a ray is incident at an angle of 45^0 (relative to the normal) on a film with an index of refraction of $\sqrt{2}$. Calculate the minimum thickness for constructive interference if the light is sodium light with a wavelength of 600 nm.

Sol. By the Snell's law : $1\sin i=\mu\sin r\Rightarrow 1\sin 45=\sqrt{2}\ \sin r,\ \therefore\ r=30$

Path difference in air between two reflected ray, $\Delta x=2\mu t\cos r+\left(0-\dfrac{\lambda}{2}\right)$, For constructive



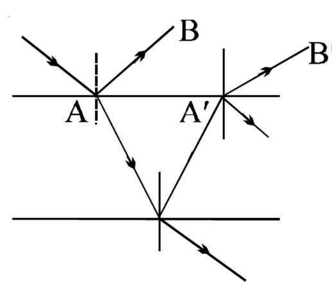
interference, $2\mu t\cos r-\dfrac{\lambda}{2}=n\lambda\Rightarrow 2\mu t\cos r=\dfrac{(2n+1)\lambda}{2}$

$$\Rightarrow t=\frac{(2n+1)\lambda}{4\mu\cos r}$$

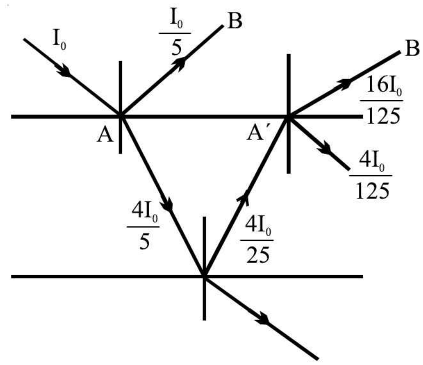
$$(t)_{min}=\frac{\lambda}{4\mu\cos r}\ (n=0 \text{ for minimum thickness})=\frac{\lambda}{4\sqrt{2}\times\frac{\sqrt{3}}{2}}=\frac{\lambda}{2\sqrt{6}}=(122.4)\ \text{nm}$$

[Ans. 122.4 nm]

Q.15 A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in the figure. It undergoes partial reflection and refraction. At each reflection 20% of incident energy is reflected. The rays AB and A′ B′ undergo interference. Find the ratio I_{max}/I_{min}.



Sol. According to the question, Intensity of ray AB, $I_1 = \frac{I_0}{5}$, and Intensity of ray A′B′,



$I_2 = \frac{16I_0}{125}$, $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \frac{81}{125}I_0$,

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \frac{I_0}{125}$, $\frac{I_{max}}{I_{min}} = 81$

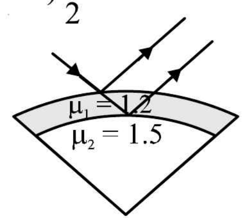
[Ans. 81 : 1]

Q.16 A lens (μ =1.5) is coated with a thin film of refractive index 1.2 in order to reduce the reflection from its surface at λ = 4800 Å. Find the minimum thickness of the film which will minimize the intensity of the reflected light.

Sol. Path difference between two reflected ray in air,

$$\Delta X = 2\mu_1 t + \left(\frac{\lambda}{2} - \frac{\lambda}{2}\right) = 2\mu_1 t$$

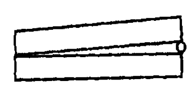
For the minimum intensity, $\Delta X = (2n-1)\frac{\lambda}{2} \Rightarrow 2\mu_1 t = (2n-1)\frac{\lambda}{2}$



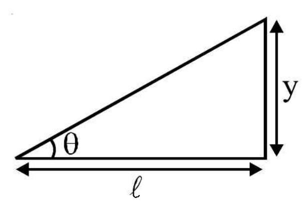
$$t = \frac{(2n-1)\lambda}{4\mu_1} = \frac{\lambda}{4\mu_1} \quad \text{(For minimum thickness, n = 1)}$$

$$t = \frac{\lambda}{4\mu_1} = \frac{4800\times10^{-10}}{4\times1.2} = 10^{-7}\text{ m}$$ **[Ans. 10^{-7} m]**

Q.17 A broad source of light of wavelength 680nm illuminates normally two glass plates 120mm long that meet at one end and are separated by a wire 0.048 mm in diameter at the other end. Find the number of bright fringes formed over the distance of 120 mm.



Sol. In the case of variable thickness (wedge shaped) film,



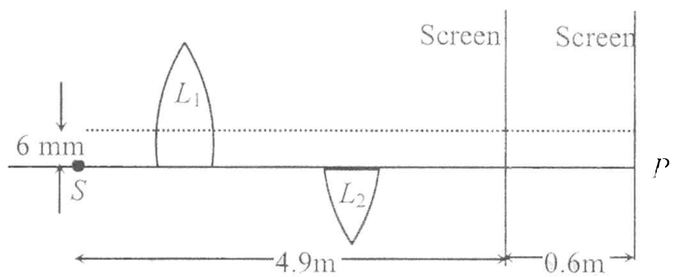
Fringe width, $\beta = \frac{\lambda}{2\mu\theta} \Rightarrow \tan\theta = \theta = \frac{y}{\ell}$

Let N be the no. of fringes formed over length (ℓ)

Then, $\ell = N\beta$, $\ell = N\left(\frac{\lambda}{2\mu\theta}\right) \Rightarrow N = \frac{2\mu\ell\theta}{\lambda}$

$\Rightarrow N = \frac{2\mu y}{\lambda}$ $(\because \theta = \frac{y}{\ell}) \Rightarrow N = \frac{2\times1\times0.048\times10^{-3}}{680\times10^{-9}} = (141)$ $(\because \mu = 1)$ **[Ans. 141]**

Q.18 A thin convex lens of focal length f = 0.6 m is cut into two unequal parts L_1 and L_2. One part is shifted along the cutting plane axis as shown in the figure. A monochromatic line source S, perpendicular to the plane of paper, emitting light of wavelength λ = 600 nm, is placed on the cutting plane axis. A screen with slits where the images of S is formed by these two pieces of the lens separately is placed perpendicular to the optical axis from the source at 4.9 m. There is an another screen placed at distance 0.6 m normal to optical axis where fringes are observed due to interference of the light passing through the holes. Find the position of central maximum from P. [Dotted line represents the principal axis of lens L_1]



Sol. Let x = distance between lenses, D is distance between the source and first screen. As images of the source due to both pieces of lens are obversed on the screen, from displacement method,

$$x = \sqrt{D(D-4f)} = \sqrt{4.9(4.9-2.4)} = 3.5 \text{ m}$$

Let m_1 and m_2 are the magnification by the lenses L_1 and L_2 $m_1 = -\dfrac{D+x}{D-x} = -6$, $m_2 = -\dfrac{1}{6}$

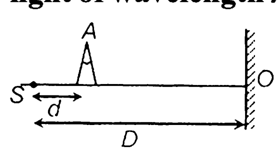
Let S_1 is the image formed by L_1 of S and h_1 is the height of S_1 form $P' \Rightarrow h_1 = m_1 h = 36$ mm

Let S_2 is the image formed by L_2 of S and h_2 is the height of S_2 form $P' \Rightarrow h_2 = m_2 h = 1.0$ mm

Distance between S_1 and S_2 (d) = $h_1 - h_2$ = 36.0 – 1.0 = 35.0 mm

Position of central maximum O from P is $6+1+\dfrac{35}{2} = 24.5$ mm

Q.19 A prism having angle A° (which is very small) is placed in front of a point source S at a small distance d. A screen is placed at a large distance D as shown in the figure. Find the fringe width of interference pattern given that source is emitting light of wavelength λ and refractive index of prism is μ.



Sol. Fringe width = $\dfrac{D\lambda}{(\mu-1)Ad}$

EXERCISE – II

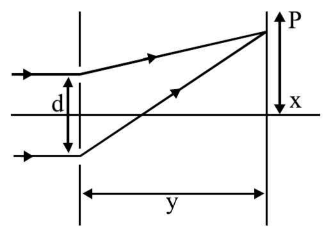
Q.1 If slits of the double slit were moved symmetrically apart with relative velocity v, calculate the rate at which fringes pass a point at a distance x from the center of the fringe system formed on a screen y distance away from the double slits if wavelength of light is λ. Assume y >> d & d >> λ.

Sol. No. of fringes passing through point 'P' = No. of maxima passing through point P

Path difference at a point i.e. P, $\Delta x = \frac{xd}{y}$, For the maxima, $n\lambda = \frac{xd}{y}$

Differentiating w.r.t. time on both sides we get,

$$\lambda \frac{dn}{dt} = \left(\frac{x}{y}\right)\left(\frac{d(d)}{dt}\right)$$



$$\Rightarrow \frac{dn}{dt} = \frac{xv}{y\lambda}\left[\because \frac{d(d)}{dt} = v \text{ given}\right]$$

[Ans. $\frac{x}{\lambda y}v$**]**

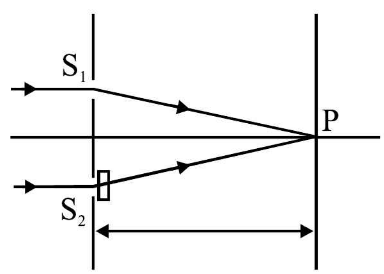
Q.2 A thin glass plate of thickness t and refractive index μ is inserted between screen & one of the slits in a Young's experiment. If the intensity at the center of the screen is I, what was the intensity at the same point prior to the introduction of the sheet.

Sol. Path difference at a point a P, $\Delta x = (S_2P - t + \mu t) - (S_1P) \Rightarrow \Delta x = (S_2P - S_1P) + t(\mu - 1)$

$\Delta x = t(\mu - 1)$ [since, $(S_2P - S_1P) = \frac{yd}{D}$ and at point P, y = 0]

Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \quad \Rightarrow \quad \Delta\phi = \frac{2\pi}{\lambda}(t)(\mu - 1)$$



Intensity, $I = 4I_0 \cos^2\frac{\Delta\varphi}{2}$ (where, $I_1 = I_2 = I_0$)

$$\Rightarrow I = 4I_0 \cos^2\left[\frac{\pi t(\mu - 1)}{\lambda}\right] \text{.............. (1)}$$

When slit is not present, Path difference at a point, P $\Delta x = 0$; $\Delta\phi = 0$
Putting the value of I_0 (from equation (1) we get).

Then, Intensity $= 4I_0 = \dfrac{I}{\cos^2\left[\dfrac{\pi t(\mu-1)}{\lambda}\right]}$ **[Ans.** $\mathbf{I_0 = I\sec^2\left[\dfrac{\pi(\mu-1)t}{\lambda}\right]}$**]**

Q.3 In Young's experiment, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength. [JEE '97(I)]

Sol. (i) Path difference due to the glass slab, $\Rightarrow \Delta x = (\mu - 1)t = (1.5 - 1)t = 0.5\,t$
Due to this slab, 5 red fringes have been shifted upwards. Therefore
$\Delta x = 5\lambda_{red}$ or $\quad 0.5\,t = (5)\,(7 \times 10^{7}\,\text{m})$
$\therefore$ t = thickness of glass slab $= 7 \times 10^{-6}$ m
(ii) Let μ' be the refractive index for green light then, $\Delta x' = (\mu' - 1)\,t$
Now the shifting is of 6 fringes of red light. Therefore,
$\Delta x' = 6\lambda_{red} \qquad \therefore \qquad t(\mu - 1) = 6\lambda_{red}$

$$\therefore \quad (\mu' - 1) = \frac{(6)\times(7\times10^{-7})}{7\times10^{-6}} = 0.6 \qquad \Rightarrow \qquad \mu' = 1.6$$

(iii) In part (i), shifting of 5 bright fringes was equal to 10^{-3} m, which implies that

$$5\omega_{red} = 10^{-3}\text{ m} \qquad [\text{Here } \omega = \text{Fringe width}] \Rightarrow \quad \omega_{red} = \frac{10^{-3}}{5}\text{ m} = 0.2 \times 10^{-3}\text{ m}$$

$$\text{Now since } \omega = \frac{\lambda D}{d} \text{ or } \omega \propto \lambda \qquad \Rightarrow \therefore \qquad \frac{\omega_{green}}{\omega_{red}} = \frac{\lambda_{green}}{\lambda_{red}}$$

$$\therefore \quad \omega_{green} = \omega_{red}\,\frac{\lambda_{green}}{\lambda_{red}} = (0.2 \times 10^{-3})\left(\frac{5\times10^{-7}}{7\times10^{-7}}\right) \quad \Rightarrow \omega_{green} = 0.143 \times 10^{-3}\text{ m}$$

$$\therefore \quad \Delta\omega = \omega_{green} - \omega_{red} = (0.143 - 0.2) \times 10^{-3}\text{ m} \ \Rightarrow \ \Delta\omega = -\,5.71 \times 10^{-5}\text{ m}$$

[Ans. 7 μm, 1.6, 400 / 7μm (decrease)]

Q.4 In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate having the same thickness as the first one but having refractive index 1.7 . Interference pattern is observed using light of wavelength 5400 Å . It is found that the point P on the screen where the central maximum (n = 0) fell before the glass plates were inserted now has 3/4 the original intensity . It is further observed that what used to be the 5th maximum earlier, lies below the point P while the 6th minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected). [JEE '97 (II)]

Sol. $\mu_1 = 1.4$ and $\mu_2 = 1.7$ and let t be the thickness of each glass plate.

Path difference at O, due to insertion of glass plates will be

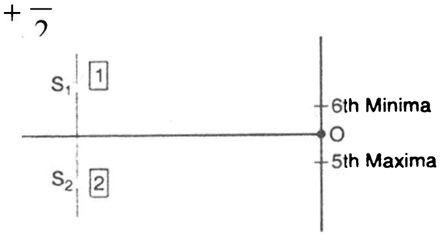
$\Delta x = (\mu_2 - \mu_1)\, t = (1.7 - 1.4)\, t = 0.3\, t$ .... (1)

Now, since 5th maxima (earlier) lies below O and 6th minima lies above O.

This path difference should lie between 5λ and $5\lambda + \frac{\lambda}{2}$

So, let $\Delta x = 5\lambda + \Delta$ .... (2)

where $\Delta < \frac{\lambda}{2}$



Due to the path difference Δx, the phase difference at O will be

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(5\lambda + \Delta) = (10\pi + \frac{2\pi}{\lambda}.\Delta) \quad \textbf{.......... (3)}$$

Intensity at O is given $\frac{3}{4} I_{max}$ and since

$$I(\phi) = I_{max}\cos^2\left(\frac{\varphi}{2}\right) \Rightarrow \therefore \frac{3}{4} I_{max} = I_{max}\cos^2\left(\frac{\varphi}{2}\right), \text{ or } \frac{3}{4} = \cos^2\left(\frac{\varphi}{2}\right) \quad \textbf{........... (4)}$$

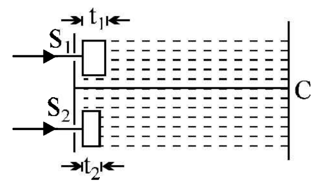
From eq. (3) and (4), we find that

$$\Delta = \frac{\lambda}{6} \Rightarrow \text{ i.e., } \Delta x = 5\lambda + \frac{\lambda}{6} = \frac{31}{6}\lambda = 0.3\, t \ (\Delta x = 0.3\, t)$$

$$\therefore \ t = \frac{31\lambda}{6(0.3)} = \frac{(31)(5400 \times 10^{-10})}{1.8} \text{ m}, \quad \text{or} \quad t = 9.3 \times 10^{-6} \text{ m} = 9.3\ \mu\text{m}$$

[Ans. 9.3 μm]

Q.5 **A screen is at a distance D = 80 cm from a diaphragm having two narrow slits S_1 and S_2 which are d = 2 mm apart. Slit S_1 is covered by a transparent sheet of thickness t_1 = 2.5 μm and S_2 by another sheet of thickness t_2 = 1.25 μm as shown in the figure. Both sheets are made of same material having refractive index μ = 1.40. Water is filled in the space between diaphragm and screen. A monochromatic light beam of wavelength λ = 5000 Å is incident normally on the diaphragm. Assuming intensity of beam to be uniform and slits of equal width, calculate ratio of intensity at C to maximum intensity of interference pattern obtained on the screen, where C is foot of perpendicular bisector of S_1S_2. (Refractive index of water, μ_w = 4/3)**



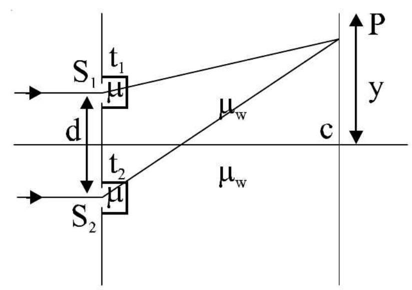
Sol. Path difference at a point P in air,

$\Delta x = (S_2P - t_2 + \mu t_2) - (S_1P - t_1 + \mu t_1) \quad \Rightarrow \Delta x = (S_2P - S_1P) - (t_2 - t_1)\,\mu_1 + (t_2 - t_1)\mu_2$

We know that, $(S_2P - S_1P) = \dfrac{yd}{D}$ At point C, y = 0 Hence, $S_2P - S_1P = 0$ at point C,

Then, $\quad \Delta x = (t_2 - t_1)\,\mu_2 - (t_2 - t_1)\mu_1$

$\Delta x = -\left(1.25 \times \dfrac{1}{15}\right)\mu m \quad ; \quad \Delta\phi = \dfrac{2\pi}{\lambda_{air}}\Delta x = \left(-\dfrac{\pi}{3}\right)$



Intensity at a point C,

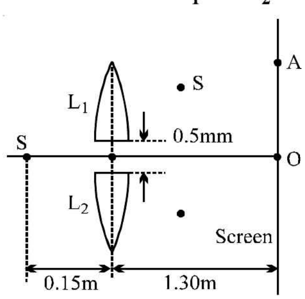
$I_c = 4I_0 \cos^2 \dfrac{\Delta\varphi}{2} \Rightarrow I_c = 3I_0$ (since, $\Delta\phi = -\dfrac{\pi}{3}$)

Ratio = $\dfrac{I_c}{I_{max}} = \dfrac{3I_0}{4I_0} = \dfrac{3}{4}$ **[Ans. 3/4]**

Q.6 **In the given figure S is a monochromatic point source emitting light of wavelength = 500 nm . A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter . The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm . The distance along the axis from S to L_1 and L_2 is 0.15 m, while that from L_1 & L_2 to O is 1.30 m. The screen at O is normal to SO.**

(i) If the third intensity maximum occurs at a point A on the screen, find the distance OA.

(ii) If the gap between L_1 & L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease or remain the same ?



Sol. (i) For the lens, u = – 0.15 m ; f = + 0.10 m therefore, using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have

$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{(-0.15)} + \frac{1}{(0.10)}$ or v = 0.3 m Linear magnification, $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images S_1 and S_2 of S will be formed at 0.3 m from the lens as shown in the figure. Image S_1 due to part 1 will be formed at 0.5 mm above its optical axis (m = – 2). Similarly, image S_2 due to part 2 is formed 0.5 mm below the optical axis of this part as shown.

Hence, d = distance between S_1 and S_2 = 1.5 mm. D = 1.30 – 0.30 = 1.0 m = 10^3 mm; λ = 500 nm = 5×10^{-4} mm. Therefore, fringe width,

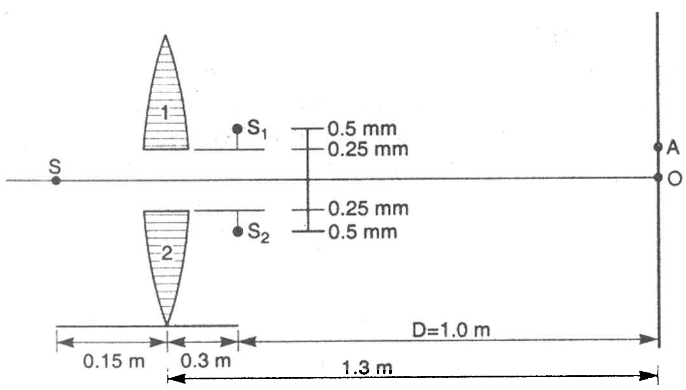
$\omega = \frac{\lambda D}{d} = \frac{(5 \times 10^{-4})(10^3)}{(1.5)}$ mm $= \frac{1}{3}$ mm. Now, as the point A is at the third maxima

OA = 3ω = 3 (1/3) mm or OA = 1mm

Concept :

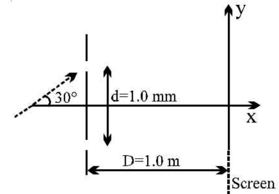
(i) The language of the question is slightly confusing. The third intensity maximum may be understood as second order maximum (zero order, first order and the second order). In that case OA = 2ω = 2 (1/3) mm = 0.67 mm.

(ii) If the gap between L_1 and L_2 is reduced, d will decrease. Hence, the fringe width ω will increase or the distance OA will increase.



[Ans. (i) 1 mm (ii) increase]

Q.7 A coherent parallel beam of microwaves of wavelength λ = 0.5 mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on the screen placed parallel to the plane of the slits at a distance of 1.0 m from it, as shown in the figure.

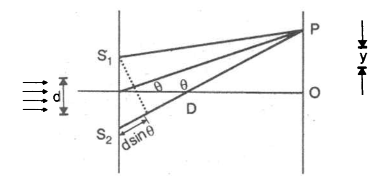


(a) **If the incident beam falls normally on the double slit apparatus, find the y−coordinates of all the interference minima on the screen**

(b) **If the incident beam makes an angle of 30° with the x−axis (as in the dotted arrow shown in the figure), find the y−coordinates of the first minima on either side of the central maximum.** **[JEE '98]**

Sol. Given $\lambda = 0.5$ mm, $d = 1.0$ mm, $D = 1$ m

(a) When the incident beam falls normally : Path difference between the two rays S_2P and S_1P is, $\Delta x = S_2P - S_1P \approx d\sin\theta$, For minimum intensity,



$$d\sin\theta = (2n-1)\frac{\lambda}{2},\ n = 1, 2, 3, ...$$

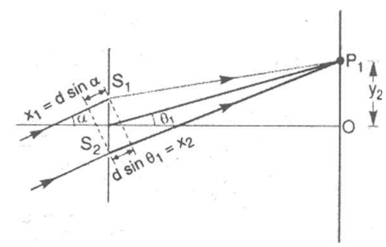
or $\quad \sin\theta = \dfrac{(2n-1)\lambda}{2d} = \dfrac{(2n-1)0.5}{2\times1.0} = \dfrac{2n-1}{4}$

As $\sin\theta \le 1$ therefore $\dfrac{(2n-1)}{4} \le 1$ or $n \le 2.5$, $\Rightarrow$ So, n can be either 1 or 2

When $n = 1$, $\sin\theta_1 = \dfrac{1}{4}$ or $\tan\theta_1 = \dfrac{1}{\sqrt{15}}$

when $n = 2$, $\sin\theta_2 = \dfrac{3}{4}$

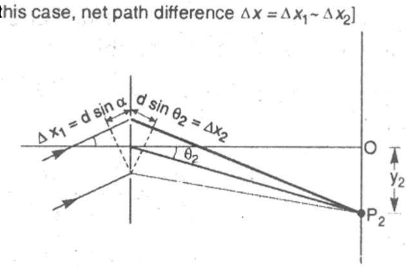
or $\quad \tan\theta_2 = \dfrac{3}{\sqrt{7}}$



[In this case, net path difference $\Delta x = \Delta x_1 \sim \Delta x_2$]

$\therefore\ \ y = D\tan\theta = \tan\theta\ (D = 1\text{ m})$

So, the position of minima will be :



[In this case, $\Delta x = \Delta x_1 + \Delta x_2$]

$y_1 = \tan\theta_1 = \dfrac{1}{\sqrt{15}}$ m $= 0.26$ m

$y_2 = \tan\theta_2 = \dfrac{3}{\sqrt{7}}$ m $= 1.13$ m

And as minima can be on either side of centre O. Therefore there will be four minimas at positions ± 0.26 m and ± 1.13 m on the screen.

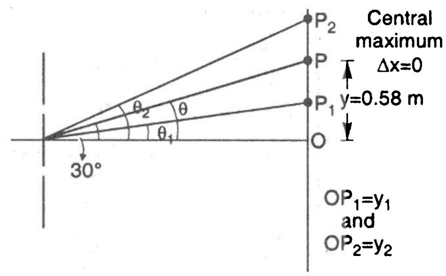
(b) When $\alpha = 30°$, path difference between the rays before reaching S_1 and S_2 is $\Delta x_1 = d\sin\alpha = (1.0)\sin 30° = 0.5\text{ mm} = \lambda$. So, there is already a path difference of λ between the rays.

Position of central maximum : Central maximum is defined as a point where net path difference is zero. So

$$\Delta x_1 = \Delta x_2 \quad \text{or} \quad d\sin\alpha = d\sin\theta \quad \text{or} \quad \theta = \alpha = 30^\circ \text{ or} \quad \tan\theta = \frac{1}{\sqrt{3}} = \frac{y_0}{D}$$

$$\therefore \quad y_0 = \frac{1}{\sqrt{3}} \text{ m} \quad [D = 1 \text{ m}]$$

$$y_0 = 0.58 \text{ m}$$



At point P, $\Delta x_1 = \Delta x_2$

Above point P $\Delta x_2 > \Delta x_1$ and

Below point P $\Delta x_1 > \Delta x_2$

Now, let P_1 and P_2 be the minimas on either side of central maxima. Then, for P_2

$$\Delta x_2 = \Delta x_1 + \frac{\lambda}{2} \quad \text{or} \quad \Delta x_2 = \Delta x_1 + \frac{\lambda}{2} = \lambda + \frac{\lambda}{2} = \frac{3\lambda}{2} \quad \text{or} \quad d\sin\theta_2 = \frac{3\lambda}{2}$$

$$\text{or} \quad \sin\theta_2 = \frac{3\lambda}{2d} = \frac{(3)(0.5)}{(2)(1.0)} = \frac{3}{4} \quad \therefore \quad \tan\theta_2 = \frac{3}{\sqrt{7}} = \frac{y_2}{D} \quad \text{or} \quad y_2 = \frac{3}{\sqrt{7}} \text{ m} = 1.13 \text{ m}$$

Similarly for P_1, $\Delta x_1 - \Delta x_2 = \frac{\lambda}{2}$ or $\Delta x_2 = \Delta x_1 - \frac{\lambda}{2} = \lambda - \frac{\lambda}{2} = \frac{\lambda}{2}$

$$\text{or} \quad d\sin\theta_1 = \frac{\lambda}{2} \quad \text{or} \quad \sin\theta_1 = \frac{\lambda}{2d} = \frac{(0.5)}{(2)(1.0)} = \frac{1}{4}$$

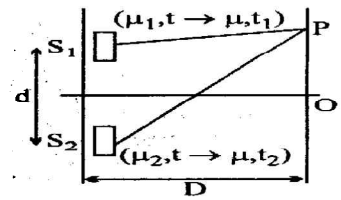
$$\therefore \quad \tan\theta_1 = \frac{1}{\sqrt{15}} = \frac{y_1}{D} \quad \text{or} \quad y_1 = \frac{1}{\sqrt{15}} \text{ m} = 0.26 \text{ m}$$

Therefore, y-coordinates of the first minima on either side of the central maximum are y_1 = 0.26 m and y_2 = 1.13 m.

Note : In this problem the relation $\sin\theta \approx \tan\theta \approx \theta$ is not valid as θ is large.

[Ans. (a) $\pm\frac{1}{\sqrt{15}}$ **m,** $\pm\frac{3}{\sqrt{7}}$ **m (b)** $+\frac{1}{\sqrt{15}}$ **m,** $+\frac{3}{\sqrt{7}}$ **m]**

Q.8 In a YDSE with visible monochromatic light two thin transparent sheets are used in front of the slits S1 and S_2. μ_1=1.6 and μ_2 =1.4. If both sheets have thickness t, the central maximum is observed at a distance of 5 mm from center O. Now the sheets are replaced by two sheets of same material of refractive index $\frac{\mu_1+\mu_2}{2}$ but having thickness t_1 & t_2 such that $t = \frac{t_1+t_2}{2}$.



Now central maximum is observed at a distance of 8 mm from center O on the same side as before. Find the thickness t1 (in μm) [Given :d =1 mm. D = 1 m].

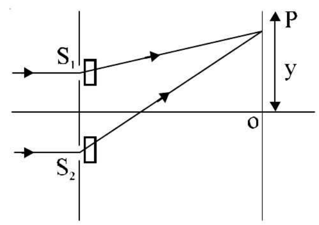
Sol. Path difference at a point P,

$\Delta x = (S_2P - t + \mu t) - (S_1P - t + \mu t)$

$\Rightarrow \Delta x = (S_2P - S_1P) + (\mu_2 - \mu_1)t$

For the central maxima, $\Delta x = 0 \Rightarrow \dfrac{yd}{D} + (\mu_2 - \mu_1)t = 0$

$y = \dfrac{(\mu_1 - \mu_2)tD}{d}$ .... (1)



In the other case,

$\Delta x = (S_2P - t_2 + \mu t_2) - (S_1P - t_1 + \mu t_1)$

$\Delta x = (S_2P - S_1P) + (t_1 - t_2) - \mu (t_2 + t_1)$

For the central maxima, $\Delta x = 0$

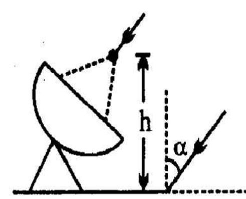
$\Rightarrow \dfrac{yd}{D} - (t_2 - t_1) - \mu(t_1 - t_2) \Rightarrow y = [(t_2 - t_1) + \mu(t_1 - t_2)]\dfrac{D}{d}$ .... (2)

From the equation (1), we get $t_1 + t_2 = 50\ \mu m$, From the equation (2), we get $t_1 - t_2 = 16\ \mu m$

Then, $t_1 = 33\ \mu m$

[Ans. 33]

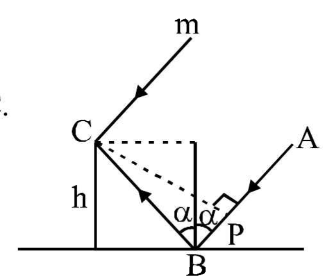
Q.9 **Radio waves coming vertically at $\angle\alpha$ are received by a radar after reflection from a nearby water surface & directly. What should be the height of antenna from water surface so that it records a maximum intensity. (wavelength = λ).**



Sol. For the maximum intensity path difference, $\Delta x = n\lambda$

At the point C, two rays interfere, one is mc and other is ABC.

Then, path difference, $\Delta x = (ABC + \dfrac{\lambda}{2}) - (mC)$



(since, in the case of reflection at point B, there is an extra path change equal to $\dfrac{\lambda}{2}$)

$\Rightarrow \Delta x = (ABC - mC) + \dfrac{\lambda}{2} \Rightarrow \Delta x = (PB + BC) + \dfrac{\lambda}{2}$ ($\because$ AP = mc)

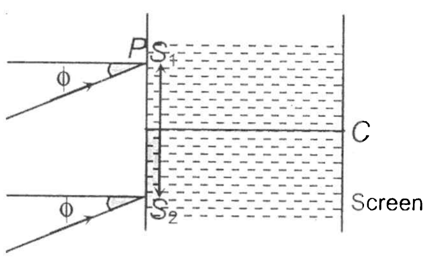
$$\Rightarrow \qquad \Delta x = \left[\frac{h}{\cos\alpha} + \frac{h}{\cos\alpha}(\cos 2\alpha)\right] + \frac{\lambda}{2} \qquad \Rightarrow \qquad \Delta x = \frac{h}{\cos\alpha}(1+\cos 2\alpha) + \frac{\lambda}{2}$$

$$\Rightarrow \qquad \Delta x = \frac{2h\cos^2\alpha}{\cos\alpha} + \frac{\lambda}{2} \qquad \Rightarrow \qquad \Delta x = 2h\cos\alpha + \frac{\lambda}{2}$$

For the maximum intensity, $\Delta x = n\lambda \Rightarrow 2\,h\cos\alpha + \frac{\lambda}{2} = n\lambda \Rightarrow h = \frac{\lambda}{4\cos\alpha}$ (for, n = 1)

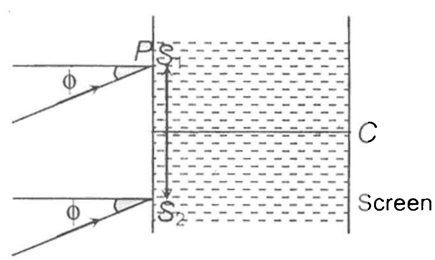
[Ans. $\frac{\lambda}{4\cos\alpha}$ **]**

Q.10 In a young's double slit is experiment a parallel beam containing wavelengths λ = 4000 Å and λ_2 = 5600 Å is incident at an angle $\phi = 30^\circ$ on a diaphragm having narrow slits at a separation d = 2 mm. The screen is placed at a distance D = 40 cm from the slits. A mica slab of thickness t = 5 mm is placed in front of one of the slits and the whole apparatus is submerged in water. If the central bright fringe is observed at C, which is equidistant from both the slits. Calculate
(a) the refractive index of the slab.
(b) the distance of the first black line from C.



Sol. (a) Total phase difference at C, ? $\phi = kd\sin\phi - Kt(\mu' - 1)$ for central maxima at

$$C,\ \Delta\phi = 0 \left[\text{Here } \mu' = \frac{\mu_s}{\mu_w} \ \&\ k = \frac{2\pi}{\lambda}\right] \quad t = \frac{d\sin\varphi}{(\mu'-1)}$$



$$\Rightarrow \frac{2\times10^{-3}\times\sin 30^\circ}{(\mu'-1)} = 5\times10^{-3} \qquad \mu' = 1.2$$

$\Rightarrow \mu = 1.2 \times (4/3) = 1.6$

Hence refractive index of mica slab = 1.6

(b) A black line is formed at the position where dark fringe is formed for both the wave-length.

The distance of the first black line from center bright line $y = \dfrac{(2n-1)\lambda D}{2d}$ ....(i)

For black line ; $\dfrac{(2n_1 - 1)\lambda_1' D}{2d} = \dfrac{(2n_2 - 1)\lambda_1' D}{2d}$

$\dfrac{(2n_2 - 1)\lambda_1' D}{2d} = \dfrac{(2n_2 - 1)\lambda_2' D}{2d}$ $\quad \dfrac{(2n_1 - 1)}{(2n_2 - 1)} = \dfrac{\lambda_2'}{\lambda_1'}$; where $\lambda_1' = \dfrac{\lambda_1}{\mu_\omega}$ and $\lambda_2' = \dfrac{\lambda_1}{\mu_\omega}$

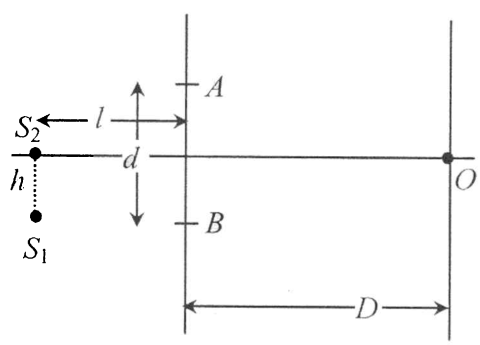
$\dfrac{(2n_1 - 1)}{(2n_2 - 1)} = \dfrac{7}{5}$ $\qquad$ For minimum value, $n_1 = 4$ and $n_2 = 3$.

Hence distance of the first black line

$$y = \frac{(2 \times 4 - 1)4000 \times 10^{-10} \times 40 \times 10^{-2} \times 3}{2 \times 2 \times 10^{-3} \times 4} = 2.1 \times 10^{-4} \text{m} = 210\ \mu\text{m}$$

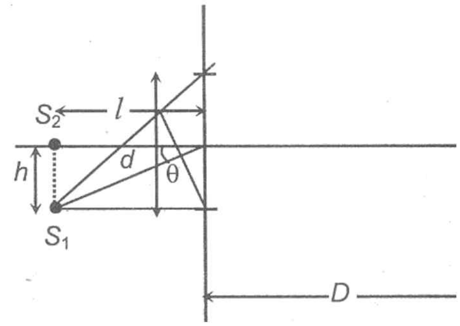
Q.11 In a Young Double Slit Experiment, a source S_1 of white light is kept at a distance h from the central line such that at the central point O, green light is missing. When another monochromatic green light source S_2 is kept on central line it forms central maximum at O.

(a) Find the minimum distance, h_{min} of source from central line.
(b) If $h = 2\ h_{min}$, at what minimum distance from point O maximum intensity of green light will appear. Assume intensity of the monochromatic source and that of green light in white light source to be same. ($D = 1$ m, $d = 1$ mm, $l = 0.5$ m, $\lambda_{green} = 500$ nm)



Sol. Here

(a) $\Delta x = d \sin\theta = d \tan\theta = d \times \dfrac{h}{l}$ For green light to be missing



$$\Delta x = \frac{n\lambda_g}{2} \Rightarrow h = \frac{n\lambda_g l}{2d}$$

For minimum h, n should be equal to 1

$$\text{or } h_{min} = \frac{\lambda_g l}{2d} = \frac{5\times10^{-7}\times0.5}{2\times10^{-3}} = 1.25\times10^{-4}\,\text{m}$$

$$\text{Here fringe width } \beta = \frac{\lambda_g D}{d} = \frac{5\times10^{-7}\times1}{10^{-3}} = 5\times10^{-4}\text{ m.}$$

(b) If intensity due to S_2 at any point on the screen is, $I_2 = 4\,I_0 \cos^2\dfrac{\varphi}{2}$, then intensity due to S_1 at the same point $I_1 = 4I_0\cos^2\left[\dfrac{\varphi+\varphi_1}{2}\right]$ where, $\phi_1 = \left(\dfrac{hd}{l}\right)\times\dfrac{2\pi}{\lambda} = 2\times\dfrac{\lambda l}{2d}\times\dfrac{d}{l}\times\dfrac{2\pi}{\lambda} = 2\pi$

i.e. $I_1 = 4I_0\cos^2\left[\dfrac{\varphi+2\pi}{2}\right] = 4\,I_0\cos^2\dfrac{\varphi}{2} = I_2$ $\therefore$ total intensity $I = I_1 + I_2 = 8\,I_0 = \cos^2\dfrac{\varphi}{2}$

$\therefore$ minimum distance of maximum from O $= \dfrac{\beta}{2} = 2.5\times10^{-4}\,\text{m}$

EXERCISE – III

Q.1 **As a wave propagates,**

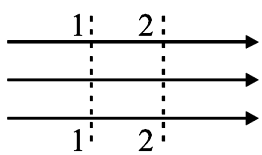
(A) the wave intensity remains constant for a plane wave

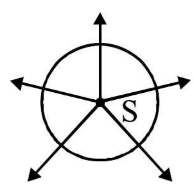
(B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

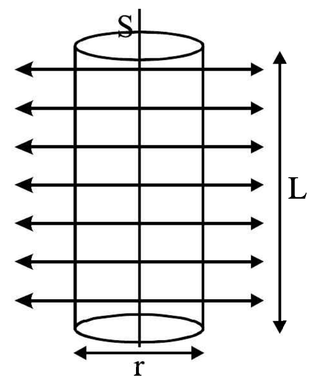
(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) total intensity of the spherical wave over the spherical surface centered at the source remains same at all times. **[JEE '99]**

Sol.(ACD) For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points. But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by







$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2}$$

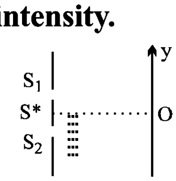
Note : For a line source $I \propto \frac{1}{r}$ because $I = \frac{P}{\pi r L}$

Q.2 **The Young's Double Slit Experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown.** **[JEE'99]**

(a) **Find the location of the central maximum (bright fringe with zero path difference) on the y-axis.**

(b) **Find the light intensity at point O relative to the maximum fringe intensity.**

(c) **Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O. [All wavelengths in this problem are for the given medium of refractive index 4/3. Ignore dispersion]**

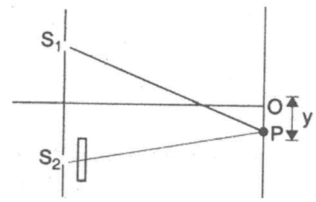


Sol. Given $\lambda = 600$ nm $= 6 \times 10^{-7}$ m, $d = 0.45$ mm $= 0.45 \times 10^{-3}$ m, $D = 1.5$ m

Thickness of glass sheet, $t = 10.4\ \mu m = 10.4 \times 10^{-6}$ m.

Refractive index of medium, $\mu_m = 4/3$ and refractive index of glass sheet, $\mu_g = 1.5$

(a) Let central maximum is obtained at a distance y below point O.



Then $\Delta x_1 = S_1P - S_2P = \frac{yd}{D}$

Path difference due to glass sheet

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right) t$$

Net path difference will be zero when

$$\Delta x_1 = \Delta x_2 \text{ or } \frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t \quad \therefore \quad y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

Substituting the values, we have

$$y = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4 \times 10^{-6}(1.5)}{0.45 \times 10^{-3}} \quad ; \quad y = 4.33 \times 10^{-3} \text{ m} \quad \text{or we can say } y = 4.33 \text{ mm.}$$

(b) At O, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$ $\therefore$ Net path difference, $\Delta x = \Delta x_2$

Corresponding phase difference, $\Delta\phi$ or simple $\phi = \frac{2\pi}{\lambda}.\Delta x$

Substituting the values, we have $\phi = \frac{2\pi}{6 \times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6})$; $\phi = \left(\frac{13}{3}\right)\pi$

$$\text{Now,} \quad I(\phi) = I_{max}\cos^2\left(\frac{\varphi}{2}\right) \quad \therefore \quad I = I_{max}\cos^2\left(\frac{13\pi}{6}\right) \quad ; \quad I = \frac{3}{4}I_{max}$$

(c) At O, path difference is $\Delta x = \Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

For maximum intensity at O, $\Delta x = n\lambda$ (Here n = 1, 2, 3, ...)

$$\therefore \quad \lambda = \frac{\Delta x}{1}, \frac{\Delta x}{2}, \frac{\Delta x}{3} \text{ ...... and so on}$$

$$\Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6}\,m) = \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-3}\,nm)\,;\ \Delta x = 1300\ nm$$

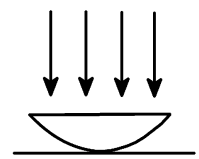
$\therefore$ Maximum intensity will be corresponding to

$$\lambda = 1300\ nm, \frac{1300}{2}nm, \frac{1300}{3}nm, \frac{1300}{4}nm\ ..$$

The wavelengths in the range 400 to 700 nm are 650 nm and 433.33 nm.

Ans. **(a) y= – 13/3 mm, (b) intensity at O = 0.75I_{max} ; (c) 650 nm, 433.33 nm**

Q.3 **A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate as shown. The observed interference fringes from this combination shall be [JEE '99]**



(A) straight
(B) circular
(C) equally spaced
(D) having fringe spacing which increases as we go outwards.

[Ans. **A]**

Q.4 **In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern [JEE '(Scr.) 2000]**
(A) the intensities of both the maxima and the minima increase.
(B) the intensity of the maxima increases and the minima has zero intensity.
(C) the intensity of the maxima decreases and that of the minima increases.
(D) the intensity of the maxima decreases and the minima has zero intensity.

Sol.(A) In inteference we know that $I_{max} = (\sqrt{I_1} + \sqrt{I_2}\,)^2$ and $I_{min} = (\sqrt{I_1} - \sqrt{I_2}\,)^2$

Under normal conditions (when the widths of both the slits are equal)

$I_1 \approx I_2 = I$ (say) $\qquad \therefore \qquad I_{max} = 4I$ and $I_{min} = 0$

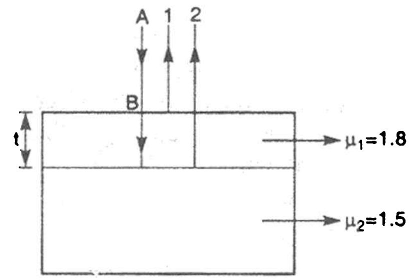
When the width of one of the slits is increased, intensity due to that slit would increase, while that of the other will remain the same. So, let :

$I_1 = I$ and $I_2 = \eta I$ $\ (\eta > 1)$ Then, $I_{max} = I\,(1 + \sqrt{\eta}\,)^2 > 4I$ and $I_{min} = I\,(\sqrt{\eta} - 1)^2 > 0$

$\therefore$ Intensity of both maxima and minima is increased.

Q.5 **A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer . It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If λ = 648 nm, obtain the least value of t for which the rays interfere constructively. [JEE '2000]**

Sol. Incident ray AB is partly reflected as ray 1 from the upper surface and partly reflected as ray 2 from the lower surface of the layer of thickness t and refractive index $\mu_1 = 1.8$ as shown in the figure. Path difference between the two rays would be



$$\Delta x = 2\mu_1 t$$
$$= 2\,(1.8)\,t = 3.6\,t$$

Ray 1 is reflected from a denser medium, therefore, it undergoes a phase change of π, whereas the ray 2 gets reflected from a rarer medium, therefore, there is no change in phase of ray 2. Hence, phase difference between rays 1 and 2 would be $\Delta\phi = \pi$ Therefore, condition of constructive interference would be

$$\Delta x = \left(n - \frac{1}{2}\right)\lambda \qquad \text{where } n = 1, 2, 3 \text{ .... or} \qquad 3.6\,t = \left(n - \frac{1}{2}\right)\lambda$$

Least values of t is corresponding to n = 1 or

$$t_{min} = \frac{\lambda}{2\times 3.6} \qquad \text{or} \qquad t_{min} = \frac{648}{7.2}\text{nm} \qquad \text{or} \qquad t_{min} = 90 \text{ nm}$$

Note : (i) For a wave (whether it is sound or electromagnetic), a medium is denser or rarer is decided from the speed of wave in that medium. In a denser medium, speed of wave is less. For example, water is rarer for sound, while denser for light compared to air because speed of sound in water is more than in air, while speed of light is less.

(ii) In transmission / refraction, no phase change takes place. In reflection, there is a change of phase of π when it is reflected by a denser medium and phase change is zero if it is reflected by a rarer medium.

(iii) If two waves in phase interfere having a path difference of Δx ; then condition of maximum intensity would be $\Delta x = n\lambda \quad n = 0, 1, 2, ...$ But if two waves, which are already out of phase (a phase difference of π) intefere with path difference Δx, then condition of maximum intensity would be, $\Delta x = \left(n - \frac{1}{2}\right)\lambda \qquad n = 1, 2, ...$

[Ans. $\mathbf{t = \frac{\lambda}{7.2}, \frac{3\lambda}{7.2}, ...t_{minimum} = \frac{\lambda}{7.2} = 90nm}$ **]**

Q.6 **Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is**

[JEE (Scr.) 2001]

(A) 2I **(B) 4I** **(C) 5I** **(D) 7I**

Sol.(B) $I(\phi) = I_1 + I_2 + 2\sqrt{I_1I_2}\ \cos\phi$ .... (1) Here, $I_1 = I$ and $I_2 = 4I$. At point A, $\phi = \frac{\pi}{2}$

$\therefore\ I_A = I + 4I = 5I$ At point B, $\phi = \pi$ $\therefore\ I_B = I + 4I - 4I = I$ $\therefore\ I_A - I_B = 4I$

Note : Eq. (1) for resultant intensity can be applied only when the sources are coherent. In the question, it is given that the rays interfere. Interference takes place only when the sources are coherent. That is why we applied equation number (1). When the sources are incoherent, the resultant intensity is given by $I = I_1 + I_2$.

Q.7 In a Young Double Slit Experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by [JEE (Scr.) 2001]

(A) 12 (B) 18 (C) 24 (D) 30

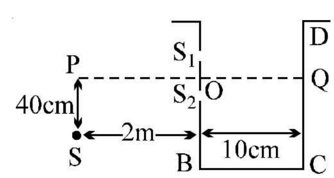
Sol.(B) Fringe width, $\omega = \frac{\lambda D}{d} \propto \lambda$

When the wavelength is decreased from 600 nm to 400 nm, fringe width will also decrease by a factor of $\frac{4}{6}$ or $\frac{2}{3}$ or the number of fringes in the same segment will increase by a factor of 3/2. Therefore, number of fringes observed in the same segment $= 12 \times \frac{3}{2} = 18$

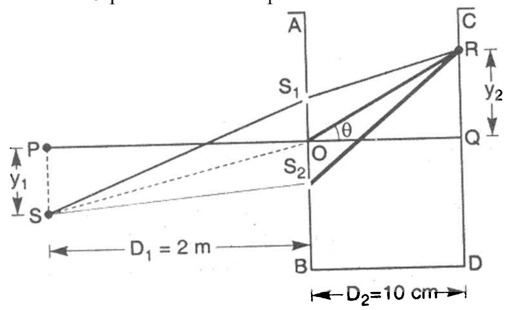
Concept :

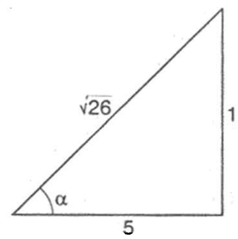
Since $\omega \propto \lambda$, and if YDSE apparatus is immersed in a liquid of refractive index μ, the wavelength λ and thus the fringe width will decrease μ times.

Q.8 A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure below. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled up to OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid. [JEE'2001]



Sol. Given $y_1 = 40$ cm, $D_1 = 2\text{m} = 200$ cm, $D_2 = 10$ cm, $\tan\alpha = \dfrac{y_1}{D_1} = \dfrac{40}{200} = \dfrac{1}{5}$





$\therefore\ \alpha = \tan^{-1}(1/5)\ \ \sin\alpha = \dfrac{1}{\sqrt{26}} \approx \dfrac{1}{5} = \tan\alpha$ Path difference between SS_1 and SS_2 is

$\Delta X_1 = SS_1 - SS_2$ or $\Delta X_1 = d\sin\alpha = (0.8\text{ mm})\left(\dfrac{1}{5}\right)$

or $\Delta X_1 = 0.16$ mm .... (1)

Now, let at point R on the screen, central bright fringe is observed (i.e., net path difference = 0).

Path difference between S_2R and S_1R would be

$\Delta X_2 = S_2R - S_1R$ or $\Delta X_2 = d\sin\alpha$ .... (2)

Central bright fringe will be observed when net path difference is zero.

or $\Delta X_2 - \Delta X_1 = 0$ or $\Delta X_2 = \Delta X_1$

or $d\sin\theta = 0.16$ or $(0.8)\sin\theta = 0.16$

or $\sin\theta = \dfrac{0.16}{0.8} = \dfrac{1}{5}$ $\tan\theta = \dfrac{1}{\sqrt{24}} \approx \sin\theta = \dfrac{1}{5}$

Hence, $\tan\theta = \dfrac{y_2}{D_2} = \dfrac{1}{5}$ $\therefore$ $y_2 = \dfrac{D_2}{5} = \dfrac{10}{5} = 2$ cm

Therefore, central bright fringe is observed at 2 cm above point Q on side CD.

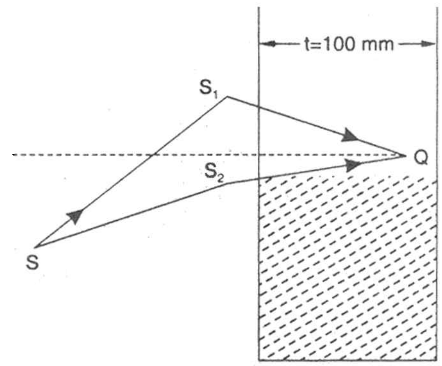
Alternate solution

ΔX at R will be zero if $\Delta X_1 = \Delta X_2$ or $d\sin\alpha = d\sin\theta$

or $\alpha = \theta$ or $\tan\alpha = \tan\theta$; $\dfrac{y_1}{D_1} = \dfrac{y_2}{D_2}$

or $y_2 = \dfrac{D_2}{D_1}.y_1 = \left(\dfrac{10}{200}\right)(40)$ cm or $y_2 = 2$cm

(ii) The central bright fringe will be observed at point Q. If the path difference created by the liquid slab of thickness t = 10 cm or 100 mm is equal to ΔX_1, so that the net path difference at Q becomes zero.



So, $(\mu - 1)\,t = \Delta X_1$

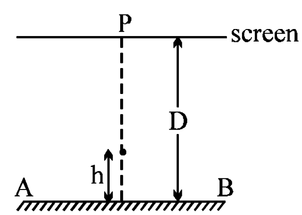
or $(\mu - 1)(100) = 0.16$

or $\mu - 1 = 0.0016$

or $\mu = 1.0016$

[Ans. (i) y =2 cm (ii) m = 1.0016]

Q.9 A point source S emitting light of wavelength 600 nm is placed at a very small height h above the flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it. [JEE'2002]



(a) What is the shape of the interference fringes on the screen?

(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

(c) If the intensities at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

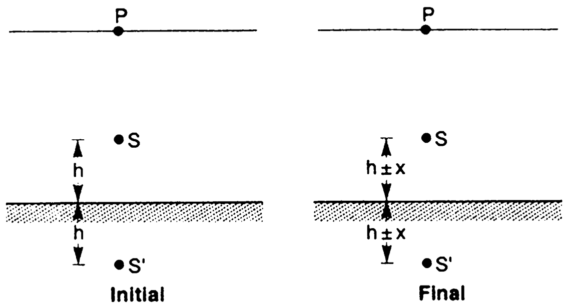
Sol. (a) Shape of the interference fringes will be circular.

(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is

$$I_2 = 36\% \text{ of } I_0 = 0.36\, I_0 \quad \therefore \quad \frac{I_1}{I_2} = \frac{I_0}{0.36 I_0} = \frac{1}{0.36} \text{ or } \sqrt{\frac{I_1}{I_2}} = \frac{1}{0.6}$$

$$\therefore \quad \frac{I_{min}}{I_{max}} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$

(c) Initially path difference at P between two waves reaching from S and S′ is



Therefore, for maximum intensity at P : $2h = \left(n - \frac{1}{2}\right)\lambda$ .... (1)

Now, let the source S is displaced by x (away or towards mirror) then new path difference will be 2h + 2x or 2h – 2x. So, for maximum intensity at P

$2h + 2x = \left[n + 1 - \frac{1}{2}\right]\lambda$ .... (2) or $2h - 2x = \left[n - 1 - \frac{1}{2}\right]\lambda$ .... (3)

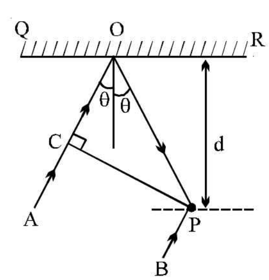
Solving eq. (1) and (2) or eq. (1) and (3), we get $x = \frac{\lambda}{2} = \frac{600}{2} = 300$ nm

Note : Here, we have taken the condition of maximum intensity at P as :

Path difference $\Delta x = \left(n - \frac{1}{2}\right)$? Because the reflected beam suffers a phase difference of π.

[Ans. (a) circular, (b) 1/16, (c) 3000 Å]

Q.10 In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray OP. [JEE (Scr.) 2003]



(A) $\cos\theta = \frac{3\lambda}{2d}$ **(B)** $\cos\theta = \frac{\lambda}{4d}$

(C) $\sec\theta - \cos\theta = \frac{\lambda}{d}$ **(D)** $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

Sol.(B) PR = d

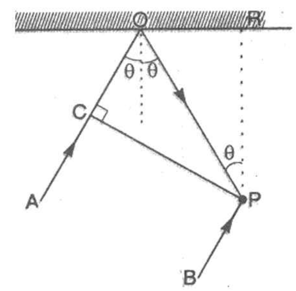
$\therefore$ PO = d sec θ and CO = PO cot 2θ = d sec θ cos 2θ

path difference between the two rays is, $\Delta x = CO + PO = (d\sec\theta + d\sec\theta\cos 2\theta)$

path difference between the two rays is, $\Delta\phi = \pi$ (one is reflected, while another is direct)

Therefore, condition for constructive interference should be

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2} \; ......$$

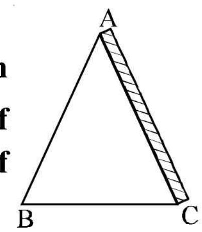


or $$d\sec\theta\,(1 + \cos 2\theta) = \frac{\lambda}{2}$$

or $$\left(\frac{d}{\cos\theta}\right)(2\cos^2\theta) = \frac{\lambda}{2}$$

or $$\cos\theta \; \frac{\lambda}{4d}$$

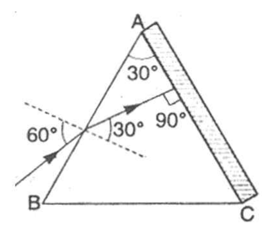
Q.11 A prism ($\mu_P = \sqrt{3}$) has an angle of prism A = 30°. A thin film (μ_f = 2.2) is coated on face AC as shown in the figure. Light of wavelength 550 nm is incident on the face AB at 60° angle of incidence. Find [JEE' 2003]



(i) the angle of its emergence from the face AC and

(ii) the minimum thickness (in nm) of the film for which the emerging light is of maximum possible intensity.

Sol. (a) $\sin i_1 = \mu \sin r_1$ or $\sin 60° = \sqrt{3}\sin r_1$



$\therefore$ $\sin r_1 = \frac{1}{2}$ or $r_1 = 30°$

Now, $r_1 + r_2 = A$ $\therefore$ $r_2 = A - r_1 = 30° - 30° = 0°$

Therefore, ray of light falls normally on the face AC and angle of emergence $i_2 = 0°$.

(b) Multiple reflection occurs between surfaces of the film. Intensity will be maximum if interference takes place in the transmitted wave. For maximum thickness

$\Delta x = 2\mu t = \lambda$ (t = thickness) $\therefore$ $t = \frac{\lambda}{2\mu} = \frac{6600}{2 \times 2.2} = 150$ Å

[Ans. 0, 125 nm]

Q.12 **In a YDSE bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between the two successive regions of complete darkness is [JEE' 2004 (Scr)]**

(A) 4 mm **(B) 5.6 mm** **(B) 14 mm** **(D) 28 mm**

[Ans. D]

Q.13 **In a Young's Double Slit Experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? Take $D/d = 10^3$. Symbols have their usual meanings. [JEE 2004]**

Sol. Let n_1 bright fringe corresponding to wavelength $\lambda_1 = 500$ nm coincides with n_2 bright fringe corresponding to wavelength $\lambda_2 = 700$ nm.

$$\therefore \qquad n_1 \frac{\lambda_1 D}{d} = n_2 \frac{\lambda_2 D}{d} \qquad \text{or} \qquad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

This implies that 7th maxima of λ_1 coincides with 5th maxima of λ_2. Similarly 14th maxima of λ_1 will coincide with 10th maxima of λ_2 and so on.

$$\therefore \qquad \text{Minimum distance} = \frac{n_1 \lambda_1 D}{d} = 7 \times 5 \times 10^{-7} \times 10^{-3} = 3.5 \times 10^{-3}\ \text{m} = 3.5\ \text{mm}$$

[Ans. 3.5 mm]

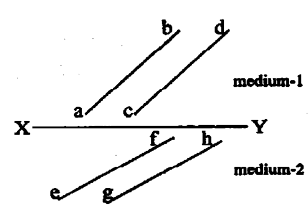
Q.14 **In Young's Double Slit Experiment maximum intensity is I, then the angular position where the intensity becomes $\frac{I}{4}$ is : [JEE' 2005 (Scr)]**

(A) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ **(B)** $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ **(C)** $\sin^{-1}\left(\frac{\lambda}{2d}\right)$ **(D)** $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

[Ans. B]

Paragraph for Question Nos. 15 to 17 (3 questions)

The figure shows a surface XY separating two transparent media, medium-1 and medium-2, The lines ab and cd represent wavefronts of a light wave traveling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



Q.15 Light travels as a

(A) parallel beam in each medium

(B) convergent beam in each medium

(C) divergent beam in each medium

(D) divergent beam in one medium and convergent beam in the other medium

Sol.(A) Wavefronts are parallel in both media. Therefore, light which is perpendicular to wavefront travels as a parallel beam in each medium. Hence, the correct option is (a).

Q.16 The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$.

(A) ϕ_c cannot be equal to ϕ_d **(B) ϕ_d can be equal to ϕ_e**

(C) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$ **(D) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$**

Sol.(C) All points on a wavefront are at the same phase.

$\therefore$ $\phi_d = \phi_c$ and $\phi_f = \phi_e$ $\therefore$ $\phi_d - \phi_f$ and $\phi_c - \phi_e$ Hence, the correct option is (c).

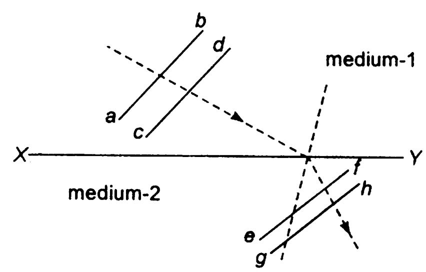
Q.17 Speed of light is

(A) the same in medium-1 and medium-2

(B) larger in medium-1 than in medium-2

(C) larger in medium-2 than in medium-1

(D) different at b and d



Sol.(B) In medium-2 wavefront bends away from the normal after refraction. Therefore, ray of light which is perpendicular to wavefront bends towards the normal in medium-2 during refraction. So, medium-2 is denser or its speed in medium-1 is more. Hence, option (b) is correct.

Q.18 In a Young's Double Slit Experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice (s).

(A) If d = λ, the screen will contain only one maximum

(B) If λ<d<2λ, at least one more maximum (besides the central maximum) will be observed on the screen.

(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase

(D) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright will increase.

Sol.(AB) (a) If $d = \lambda$ then for maxima, $d \sin\theta = n\lambda$

$$\Rightarrow \sin\theta = \pm\frac{n\lambda}{d} = \pm n \text{ Only possible values of } n = 0, 1$$

For $n = 0, \theta = 0$ (central maxima) For $n = 1, \theta = 90$ (which is not formed on the screen) Hence, only central maxima is formed.

(b) Again, $\sin\theta = +\frac{n\lambda}{d}$. If $\lambda < d < 2\lambda$

Then, only possible values of $n = 0, 1$; For $n = 0, \theta = 0$ (central maxima)
For $n = 1, \theta < 90$ (which is possible) Hence, three maxima formed on the screen.

(c) When I_1 is reduced so that it becomes equal to that of the slit 2,

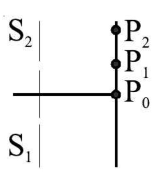
$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$, decreases ; $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$, decreases

(d) When I_2 is increased so that it become equal to that of intensity of slit 1,

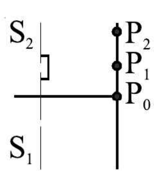
$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$, increases ; $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$, decreases

Q.19 Column I shows four situations of standard Young's double slit arrangement with the screen placed far away from the slit S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, S_1P_1-$S_2P_1 = \lambda/4$ and S_1P_2-$S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on the slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by δ (P) and the intensity by I(P). Match each situation given in Column-I with the statement(s) in ColumnII valid for that situation.

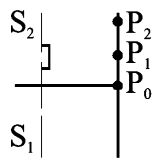
Column-I **Column-II**

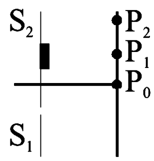
(A) 

(p) $\delta(P_0) = 0$

(B) $(\mu - 1)t = \lambda/4$ 

(q) $\delta(P_1) = 0$

(C) $(\mu-1)t=\lambda/2$  (r) $I(P_1)=0$

(D) $(\mu-1)t=3\lambda/4$  (s) $I(P_0)>I(P_1)$

(t) $I(P_2)>I(P_1)$

Sol. $I(P) = 4I_0\cos^2\left(\frac{\varphi}{2}\right)$ where I_0 is the intensity of each interfering beam and ϕ is the phase difference given by $\phi = \frac{2\pi}{\lambda}\delta(P)$

(a) At P_0, $\delta(P_0) = 0$, therefore $\phi(P_0) = 0$ and $I(P_0) = 4I_0$. At P_1, $\delta(P_1) = \frac{\lambda}{4} \Rightarrow \phi = \frac{\pi}{2}$.

Therefore, $I(P_1) = 4I_0\cos^2\left(\frac{\pi}{4}\right) = 2I_0$, Hence, $I(P_0) > I(P_1)$.

At P_2, $\delta(P_2) = \frac{\lambda}{3} \Rightarrow \phi = \frac{2\pi}{3}$. Therefore, $I(P_2) = 4I_0\cos^2\left(\frac{\pi}{3}\right) = I_0$

Hence $I(P_2) < I(P_1)$. So, the correct choices are (p) and (s).

(b) At P_0, $\delta(P_0) = \frac{\lambda}{4} \Rightarrow \phi = \frac{\pi}{2}$. Therefore,

$I(P_0) = 2I_0$, At P_1, $\delta(P_1) = 0$. Therefore, $I(P_1) = 4I_0$

At P_2, $\delta(P_2) = \frac{\lambda}{3} - \frac{\lambda}{4} = \frac{\lambda}{12} \Rightarrow \phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{12} = \frac{\pi}{6}$, Therefore $I(P_2) = 4I_0\cos^2\left(\frac{\pi}{12}\right) \simeq 3.7I_0$

So $I(P_1) > I(P_2)$. So the only correct choice is (q).

(c) At P_0, $\delta(P_0) = \frac{\lambda}{4}$. Therefore $I(P_0) = 0$, At P_1, $\delta(P_1) = \frac{\lambda}{2} - \frac{\lambda}{4} = \frac{\lambda}{4}$. Therefore $I(P_1) = 2I_0$

At P_2, $\delta(P_2) = \frac{\lambda}{2} - \frac{\lambda}{3} = \frac{\lambda}{6} \Rightarrow \phi = \frac{\pi}{3}$. Therefore $I(P_2) = 4I_0\cos^2\left(\frac{\pi}{6}\right) = 3I_0$

So, the only correct choice is (t).

(d) At P_0, $\delta(P_0) = \frac{3\lambda}{4} \Rightarrow I(P_0) = 2I_0$ At P_1, $\delta(P_1) = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} \Rightarrow I(P_1) = 0$

At P_2, $\delta(P_2) = \frac{3\lambda}{4} - \frac{\lambda}{3} = \frac{5\lambda}{12} \Rightarrow \phi = \frac{2\pi}{\lambda} \times \frac{5\lambda}{12} = \frac{5\pi}{6}$

Therefore, $I(P_2) = 4I_0 \cos^2\left(\frac{5\pi}{12}\right) = 0.27\, I_0 \neq 0$. So, the correct choices are (r), (s) and (t).

[Ans. (A) p,s; (B) q; (C)t; (D) r, s, t]

OBJECTIVE QUESTION BANK

ONLY ONE CHOICE CORRECT

Take approx. 2 minutes for answering each question.

Q.1 Two coherent monochromatic light beams of intensities I and 4I are superposed. The maximum and minimum possible intensities in the resulting beam are :

(A) 5I and I (B) 5I and 3I (C) 9I and I (D) 9I and 3I

Sol.(C) $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$. For the maximum intensity, $\cos\Delta\phi = +1$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ For the minimum intensity, $\cos\Delta\phi = -1$

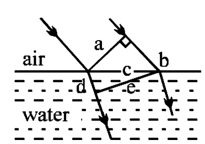
$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ In the question : $I_1 = I$, $I_2 = 4I$ $\therefore I_{max} = (\sqrt{I} + \sqrt{4I})^2 = (3\sqrt{I})^2 = 9I$

and $I_{min} = (\sqrt{I} - \sqrt{4I})^2 = (\sqrt{I})^2 = I$. Then, $I_{max} = 9I$ and $I_{min} = I$

Q.2 Figure shows plane waves refracted form air to water using Huygen's principle. a, b, c, d, e are lengths on the diagram. The refractive index of water w.r.t. air is the ratio:

(A) a/e (B) b/e

(C) b/d (D) d/b



Sol.(C) By the Huygen's principle :

Time for the path length (b) in air = Time for the path length (d) in water

$\Rightarrow \frac{b}{c_a} = \frac{d}{c_w}$ (Where, C_a = speed of light in air, C_w = speed of light in water)

$\Rightarrow \frac{C_a}{C_w} = \frac{b}{d}$ From the definition of Refractive index, $\mu_w = \frac{C_a}{C_w} = \frac{b}{d}$

Q.3 When light is refracted into a denser medium,

(A) its wavelength and frequeny both increases

(B) its wavelength increases but freqnency remains unchanged

(C) its wavelength decreases but freqnency remains unchanged

(D) its wavelength and freqnency both decrease.

Sol.(C) Frequency of light only depends on the source, it is independent of the medium.

$V = f\lambda$ Hence, $v \propto \lambda$

In the denser medium, speed decreases and wavelength also decreases.

Q.4 In YDSE how many maxima can be obtained on the screen if wavelength of light used is 200nm and d = 700 nm:

(A) 12 **(B) 7** **(C) 18** **(D) none of these**

Sol.(B) Here, λ and d both are comparable, Hence Path difference i.e. $\Delta x = d \sin\theta$

For the maxima, $\Delta x = \pm n\lambda$

$$\Rightarrow d\sin\theta = \pm n\lambda \quad \Rightarrow \sin\theta = \pm\frac{2n}{7}$$

[where only possible values of n = 0,1,2,3]

Hence, total maxima = 7

Third maxma	n = 3
Second maxma	n = 2
First maxma	n = 1
Central maxima	n = 0
First maxima	n = 1
Second maxima	n = 2
Third maxima	n = 3

Q.5 In a YDSE, the central bright fringe can be identified :

(A) as it has greater intensity than the other bright fringes.

(B) as it is wider than the other bright fringes.

(C) as it is narrower than the other bright fringes.

(D) by using white light instead of single wavelength light.

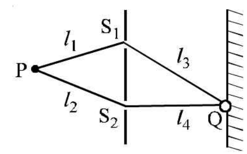
Sol.(D) Because, 1st maxima and maxima other than central maxima are different points for different wavelenght. But, central maxima is only one point for different wavelength. We know that white light has all wavelengths, hence the central bright fringe can be identified by using white light instead of single wavelength light.

Q.6 In Young's Double Slit Experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light, is :

(A) 1 **(B) 2** **(C) 3** **(D) 4**

Sol.(B) For the bright band, $y = \frac{n\lambda D}{d}$. According to the question, $(y_n)_R = (y_{n+1})_B$

$\Rightarrow \frac{n\lambda_R D}{d} = \frac{(n+1)\lambda_B D}{d}$ $\Rightarrow 6n = 4n + 4$ $\Rightarrow n = 2$



Q.7 Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point source P.
If, as shown in the diagram above, the light is then allowed to fall on a screen, and if n is a positive integer, the condition for destructive interference at Q is that

(A) $(l_1 - l_2) = (2n + 1)\lambda/2$ **(B) $(l_3 - l_4) = (2n + 1)\lambda/2$**
(C) $(l_1 + l_2) - (l_2 + l_4) = n\lambda$ **(D) $(l_1 + l_3) - (l_2 + l_4) = (2n + 1)\lambda/2$**

Sol.(D) For destructive interference, $\Delta x = (2n+1)\frac{\lambda}{2}$

At Q, $\Delta x = (\ell_1 + \ell_3) - (\ell_2 + \ell_4)$

$\Rightarrow (\ell_1 + \ell_3) - (\ell_2 + \ell_4) = (2n + 1)\frac{\lambda}{2}$

Q.8 In Young's Double Slit Experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same setup the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is

(A) 1 : 1 **(B) 2 : 1** **(C) 4 : 1** **(D) none of these**

Sol.(B) For the coherent source, $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$, $I = 4I_0\cos^2\frac{\Delta\varphi}{2}$ ($\because I_1 = I_2 = I_0$ for equal amplitude) Then $I_{av} = \frac{\int_0^{2\pi} I d\varphi}{\int_0^{2\pi} d\varphi} = 4I_0$ For the incoherent source,

$I = I_1 + I_2$ [since, interference term becomes zero]. Then, $I_{av} = 2I_0$ ($\because I_1 = I_2 = I_0$)

Q.9 In a Young's Double Slit Experiment, a small detector measures an intensity of illumination of I units at the center of the fringe pattern. If one of the two (identical) slits is now covered, the measured intensity will be

(A) 2I **(B) I** **(C) I/4** **(D) I/2**

Sol.(C) In the YDSE experiment

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$$

Let $I_1 = I_2 = I_0$ (for identical slits)

Then, $I = 4I_0$ ($\because$ At the center of fringe pattern, $\Delta\phi = 0$)

When, one of two identical slits is now covered

$$I_{new} = I_0 (\because\ I_2 = 0 \text{ and } \Delta\phi = O) \Rightarrow I_{now} = \frac{I}{4} \text{ (From the above equation)}$$

Q.10 In a young double slit experiment, D equals the distance of screen and d is the separation between the slit. The distance of the nearest point to the central maximum where the intensity is same as that due to a single slit, is equal to

(A) $\frac{D\lambda}{d}$ **(B)** $\frac{D\lambda}{2d}$ **(C)** $\frac{D\lambda}{3d}$ **(D)** $\frac{2D\lambda}{d}$

Sol.(C) In such type of question, we should obtain phase difference ($\Delta\phi$) at the given point.

$$I = 4I_0\cos^2\frac{\Delta\varphi}{2} \quad \Rightarrow \quad I_0 = 4I_0\cos^2\frac{\Delta\varphi}{2} (\because I = I_0 \text{ Given in the question})$$

$$\Rightarrow \Delta\phi = \frac{2\pi}{3} \quad \text{We know that, } \Delta\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \text{In the Y.D.S.E. experiment}, \Delta x = \frac{yd}{D}$$

$$\Rightarrow \frac{yd}{D} = \frac{\lambda}{3} \text{ [From the above relation]} \Rightarrow y = \frac{\lambda D}{3d}$$

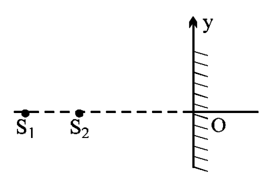
Q.11 Two point monochromatic and coherent sources of light of wavelength l are placed on the dotted line in front of an large screen. The source emit waves in phase with each other. The distance between S_1 and S_2 is 'd' while their distance from the screen is much larger. Then,

(1) → If d = 7λ/2, O will be a minima

(2) → If d = 4.3λ, there will be a total of 8 minima on y axis.

(3) → If d = 7λ, O will be a maxima.

(4) → If d = λ, there will be only one maxima on the screen.



Which is the set of correct statement :

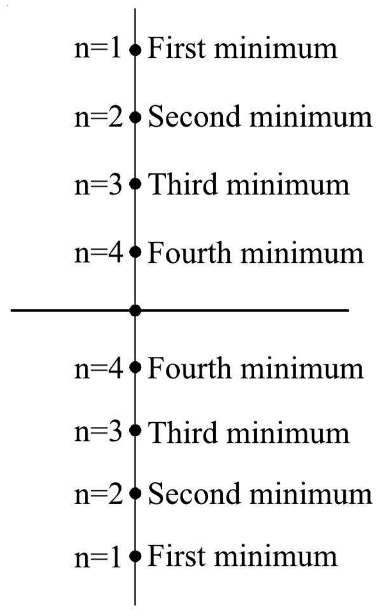
(A) 1, 2 & 3 **(B) 2, 3 & 4** **(C) 1, 2, 3 & 4** **(D) 1, 3 & 4**

Sol.(C) Path difference at point P, $\Delta x = S_1P - S_2P$, $\Delta x = d\cos\theta$

(i) Path difference at point 'O' i.e. $\Delta x = d$ ($\because \theta = 0$ at point 'O')

For the maxima, path difference

$$\Delta x = \frac{(2n-1)\lambda}{2} \Rightarrow d = \frac{(2n-1)\lambda}{4} \quad (\text{since } \Delta x = d)$$



$\Rightarrow n = 4$ (Fourth maxima will be at point 'O')

$$\Rightarrow \frac{7\lambda}{2} = \frac{(2n-1)\lambda}{2} \quad (\because d = \frac{7\lambda}{2})$$

(ii) $\Delta x = d\cos\theta$ (at any point, P)

$$\Rightarrow d\cos\theta = \frac{(2n-1)\lambda}{2} \text{(For the maxima)}$$

$$\Rightarrow \cos\theta = \frac{2n-1}{8.6}. \text{ Hence, } n = 1,2,3,4,$$

only possible values for the above equations.

(iii) $d\cos\theta = n\lambda$ (For the maxima) $\cos\theta = \dfrac{n\lambda}{d}$

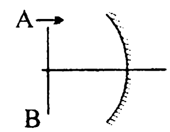
$\Rightarrow \cos\theta\ \dfrac{n}{7}$ At point, O [$\cos\theta = 1$]

which is possible from the above equation $\cos\theta = \dfrac{n}{7}$ (For, $n = 7$)

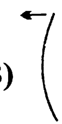
(iv) $d\cos\theta = n\lambda$ (for the maxima) $\cos\theta = n$ ($\because d = \lambda$)

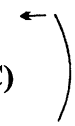
[$n = 0,1$] only possible values of above equation $n = 0$, is not possible, (since $\theta = 90$ at $n = 0$) which does not form any fringe pattern on the screen. $n = 1$, is only possible value (since $\theta = 0$ at $n = 1$ which form at point, O)

Q.12 **A plane wavefront AB is incident on a concave mirror as shown. Then, the wavefront just after reflection is**



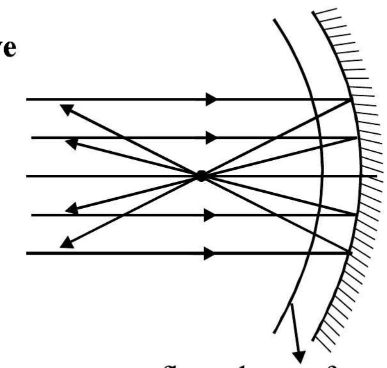
(A) 

(B) 

(C) 

(D) None of the above

Sol.(C) Make reflected rays & perpendicular to reflected rays will be reflected wavefront.



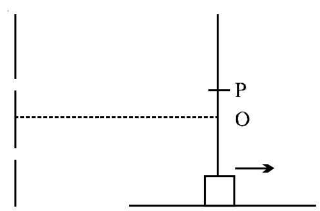
Q.13 **In a Young's Double Slit Experiment, first maxima is observed at a fixed point P on the screen. Now the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point O (center of the screen)**

(A) remains constant

(B) keeps on decreasing

(C) first decreases and then increases

(D) First decreases and then becomes constant

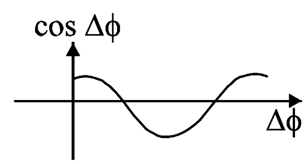


Sol.(C) $I = 2I_0(1 + \cos\Delta\phi)$

Where, $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x = \dfrac{2\pi}{\lambda}\left(\dfrac{yd}{D}\right)(\because \Delta x = \dfrac{yd}{D}) \Rightarrow \Delta\phi \propto \dfrac{1}{D}$

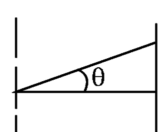
At the 1st maxima, $\cos\Delta\phi = +1$

from the graph of $(\cos\Delta\phi)$



Intensity at point, P first decreases and then increases.

Q.14 **Two slits are separated by 0.3 mm. A beam of 500 nm light strikes the slits producing an interference pattern. The number of maxima observed in the angular range $-30° < \theta < 30°$.**



(A) 300 **(B) 150**

(C) 599 **(D) 149**

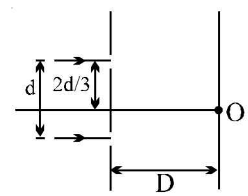
Sol.(C) In the YDSE, path difference, $\Delta x = d \sin\theta$

For the maxima, $d \sin\theta = n\lambda$ $\Rightarrow$ $\sin\theta = \frac{n\lambda}{d} = n \times \frac{5}{3} \times 10^{-3}$

$\Rightarrow$ $\therefore \sin\theta \in (-30, 30)$ $\Rightarrow$ $\sin\theta \; n \times \frac{5}{3} \times 10^{-3} \in (-\frac{1}{2}, \frac{1}{2})$

$\therefore$ $n_{max} = \frac{1}{2} \times \frac{3}{5} \times 10^{3} = 300$ Possible value of $n = (-299, 299) = 599$

Q.15 In the given figure, if a parallel beam of white light is incident on the plane of the slits then the distance of the white spot on the screen from O is [Assume $d << D, \lambda << d$]



(A) 0 **(B) d/2**
(C) d/3 **(D) d/6**

Sol.(D) If a parallel beam of white light is incident on the plane, then there is only one point where all light falls on the screen, which in the central maxima. Hence, position of the central maxima is at bisector of the two slits, which is at $\frac{d}{6}$ distance above the point 'O'.

Q.16 In the above question if the light incident is monochromatic and point O is a maxima, then the wavelength of the light incident cannot be

(A) $d^2/3D$ **(B)** $d^2/6D$ **(C)** $d^2/12D$ **(D)** $d^2/18D$

Sol.(A) In the YDSE experiment, $\Delta x = \frac{yd}{D}$, For the maxima, $\Delta x = n\lambda \Rightarrow \frac{yd}{D} = n\lambda$

$\Rightarrow y = \frac{n\lambda D}{d}$. In the question, $y = \frac{d}{6}$. Then, $\frac{d}{6} = \frac{n\lambda D}{d} \Rightarrow \lambda = \frac{d^2}{6nD}$ where, $n = 1,2,3,4......$

Q.17 In Young's Double Slit arrangement, water is filled in the space between the screen and the slits. Then :

(A) fringe pattern shifts upwards but fringe width remains unchanged.
(B) fringe width decreases and central bright fringe shifts upwards.
(C) fringe width increases and central bright fringe does not shift.
(D) fringe width decreases and central bright fringe does not shift.

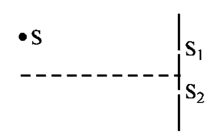
Sol.(D) Fringe width, $\beta = \frac{\lambda D}{d}$ (where, λ = wavelength of light in the medium). In the water medium, $\lambda_w = \frac{\lambda a}{\mu_w}$ ($\downarrow$) hence, β ($\downarrow$). Central maxima is a point where path difference = 0, which does not change when water is filled in space between slit and screen. Hence it remains unchanged.

Q.18 Light of wavelength λ in air enters a medium of refractive index μ. Two points in this medium, lying along the path of this light, are at a distance x apart. The phase difference between these points is :

(A) $\frac{2\pi\mu x}{\lambda}$ **(B)** $\frac{2\pi x}{\mu\lambda}$ **(C)** $\frac{2\pi(\mu-1)x}{\lambda}$ **(D)** $\frac{2\pi x}{(\mu-1)\lambda}$

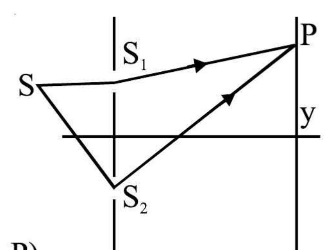
Sol.(A) Phase difference, $\Delta\phi = \frac{2\pi}{\lambda_m} \times$ (Path difference in the medium) where, λ_m = wavelength of light in the medium $\Delta\phi = \left(\frac{2\pi}{\lambda_m}\right)x = \frac{2\pi\mu x}{\lambda}$ $\left(\because \frac{\lambda_a}{\lambda_m} = \mu\right)$

Q.19 In YDSE, the source placed symmetrically with respect to the slit is now moved parallel to the plane of the slits so that it is closer to the upper slit, as shown. Then,



(A) the fringe width will increase and fringe pattern will shift down.
(B) the fringe width will remain same but fringe pattern will shift up.
(C) the fringe width will decrease and fringe pattern will shift down.
(D) the fringe width will remain same but fringe pattern will shift down.

Sol.(D) Fringe width does not depend on the initial path change or initial phase change. When the source is now moved parallel to the plane of slits, only initial path changes. Hence, fringe width remains unchanged. But fringe pattern depends on the total path difference.



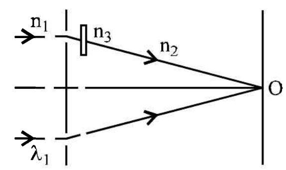
Path difference at point P,

$\Delta x = (SS_2 + S_2P) - (SS_1 + SP)$, $\Delta x = (SS_2 + SS_1) - (S_2 + S_1P)$

$\Delta x = (SS_2 + SS_1) + \frac{yd}{D}(S_2 + S_1P)$, at the central maxima, $\Delta x = 0$

Hence, $y = < 0$ [From the above relation]. That means fringe pattern will shift down.

Q.20 In YDSE (as shown in the Fig.), a parallel beam of light is incident on the slit from a medium of refractive index n_1. The wavelength of light in this medium is λ_1. A transparent slab of thickness 't' and refractive index n_3 is put infront of one slit. The medium between the screen and the plane of the slits is n_2. The phase difference between the light waves reaching point 'O' (symmetrical, relative to the slits) is :



(A) $\dfrac{2\pi}{n_1\lambda_1}(n_3 - n_2)\,t$ **(B)** $\dfrac{2\pi}{\lambda_1}(n_3 - n_2)\,t$

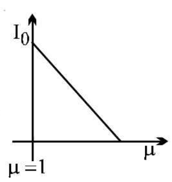
(C) $\dfrac{2\pi n_1}{n_2\lambda_1}\left(\dfrac{n_3}{n_2} - 1\right)t$ **(D)** $\dfrac{2\pi n_1}{\lambda_1}(n_3 - n_1)\,t$

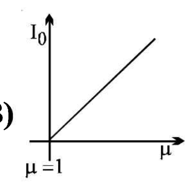
Sol.(A) Path difference in air at point O, $\Delta x = [\,(S_1O - t)\,n_2 + tn_3 - (S_2O)\,n_2]t$

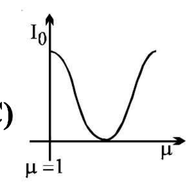
$\Delta x = [S_1O - S_2O)\,n_2 + (n_3 - n_2)\,t]$, $\Delta x = (n_3 - n_2) + (\because S_1O - S_2O = 0$ at point, 'O')

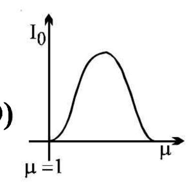
Phase difference, $\Delta\phi = \dfrac{2\pi}{\lambda_a} \times$ Path difference in air, $\Delta\phi = \dfrac{2\pi}{n_1\lambda}(n_3 - n_2)$, $\left(\because n_1 = \dfrac{\lambda_a}{\lambda}\right)$

Q.21 In a YDSE experiment, if a slab whose refractive index can be varied, is placed in front of one of the slits then the variation of resultant intensity at mid-point of screen with 'μ' will be best represented by ($\mu \geq 1$).[Assume slits of equal width and there is no absorption by slab]

(A)


(B)


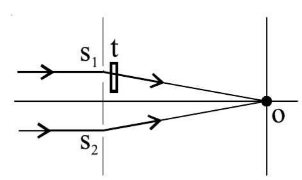
(C)


(D)


Sol.(C) $(\Delta x)_{air} = [S_1O - t + \mu t - S_2O]$, $(\Delta x)_{air} = [(\mu - 1)\,t + (S_1O - S_2O)]$

$(\Delta x)_{air} = (\mu - 1)\,t$ ($\because$ $S_1O = S_2O$ at point 'O'), $\Delta\phi = \dfrac{2\pi}{\lambda_a}(\Delta x)_{air}$

$\Rightarrow \Delta\phi = \dfrac{2\pi}{\lambda_a}(\mu - 1)\,t$. Intensity, $I = 4I_0\cos^2\dfrac{\Delta\varphi}{2}$

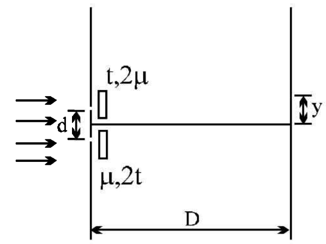


Hence, graph must be function of $\cos^2\theta$

Q.22 In the YDSE shown the two slits are covered with thin sheets having thickness t & 2t and refractive index 2μ and μ. Find the position (y) of central maxima.

(A) zero **(B)** $\frac{tD}{d}$

(C) $-\frac{tD}{d}$ **(D)** None



Sol.(B) Path difference at point P i.e. $\Delta x = [(S_2P - 2t + 2\mu t) - (S_1p - t + 2\mu t)]$

$$\Delta x = (S_2P - S_1P) - t \;\; ; \;\; \Delta x = \frac{yd}{D} - t \; (\because \; S_2P - S_1P = \frac{yd}{D})$$

At the central maxima, $\Delta x = 0$; $y = \frac{tD}{d}$

Q.23 In a YDSE with two identical slits, when the upper slit is covered with a thin, perfectly transparent sheet of mica, the intensity at the center of the screen reduces to 75% of the initial value. Second minima is observed to be above this point and third maxima below it. Which of the following can not be a possible value of phase difference caused by the mica sheet?

(A) $\frac{\pi}{3}$ **(B)** $\frac{13\pi}{3}$ **(C)** $\frac{17\pi}{3}$ **(D)** $\frac{11\pi}{3}$

Sol.(A) $I = 4I_0\cos^2\frac{\Delta\varphi}{2} \Rightarrow 4I_0 \times \frac{75}{100} = 4I_0\cos^2\frac{\Delta\varphi}{2} \; (\because I = 4\,I_0 \times \frac{75}{100}) \Rightarrow \cos^2\frac{\Delta\varphi}{2} = \frac{3}{4}$

$$\Rightarrow \cos\frac{\Delta\varphi}{2} = \pm\frac{\sqrt{3}}{2} \Rightarrow \frac{\Delta\varphi}{2} = \left(\frac{\pi}{6}\right), \left(\pi - \frac{\pi}{6}\right), \left(\pi + \frac{\pi}{6}\right), \left(2\pi - \frac{\pi}{6}\right), \left(2\pi + \frac{\pi}{6}\right), \left(3\pi - \frac{\pi}{6}\right)$$

$$\frac{\Delta\varphi}{2} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \Rightarrow \Delta\phi = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \ldots\ldots$$

According to the question, $3\pi < \Delta\phi < 6\pi$ (Since second maxima is observed to be above this point and third maxima below it). Then possible values of $\Delta\phi = \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$.

Q.24 Two monochromatic and coherent point sources of light are placed at a certain distance from each other in the horizontal plane. The locus of all those points in the horizontal plane which have construct interference will be

(A) a hyperbola **(B) family of hyperbolas**

(C) family of straight lines **(D) family of parabolas**

Sol.(B) Path difference at any point, P in the plane $\Delta x = (S_2P - S_1P)$

For constructive interference, $\Delta x = n\lambda \quad \Rightarrow \quad S_2P - S_1P = n\lambda$

This is the condition of family of hyperbola for different values of n.

Q.25 A circular planar wire loop is dipped in a soap solution and after taking it out, held with its plane vertical in air. Assuming thickness of film at the top very small, as sunlight falls on the soap film, & observer receives reflected light

(A) the top portion appears dark while the first colour to be observed as one moves down is red.

(B) the top portion appears violet while the first colour to be observed as one moves down is indigo.

(C) the top portion appears dark while the first colour to be observed as one move down is violet.

(D) the top portion appears dark while the first colour to be observed as one move down depends on the refractive index of the soap solution.

Sol.(C) Central spot is dark as seen by reflection and radius of n^{th} dark ring is proportional to under root of wavelength. Since $(\lambda_v < \lambda_R)$ then first colour to be observed as one move down is violet.

Q.26 A thin film of thickness *t* and index of refraction 1.33 coats a glass with index of refraction 1.50. What is the least thickness *t* that will strongly reflect light with wavelength 600 nm incident normally ?

(A) 225 nm **(B) 300 nm** **(C) 400 nm** **(D) 450 nm**

Sol.(A) When reflected rays 1 and 2 interfere at point, P interference pattern is formed.

$\Delta x = 2 \times 1.33 \times t + \left(\frac{\lambda}{2} - \frac{\lambda}{2}\right)$ $\because$ In the (1) reflected ray extra path change, $\frac{\lambda}{2}$ and In the (2) reflected ray same extra path change, $\frac{\lambda}{2}$ which cancelled each other.

For the strong reflection (Maxima) : $\Delta x = n\lambda \Rightarrow 2 \times 1.33 \times t = n\lambda$

$$\Rightarrow t = \frac{n\lambda}{2\times 1.33} = \left(\frac{n\times 600}{2\times\frac{4}{3}}\right) \text{nm, [for n = 1]} \Rightarrow t_{least} = (225)\ \text{nm}$$

Q.27 It is necessary to coat a glass lens with a non-reflecting layer. If the wavelength of the light in the coating is λ, the best choice is a layer of material having an index of refraction between those of glass and air and a thickness of

(A) $\frac{\lambda}{4}$ **(B)** $\frac{\lambda}{2}$ **(C)** $\frac{3\lambda}{8}$ **(D)** λ

Sol.(A) For the interference of two transmitted rays : $\Delta x = 2\mu t \Rightarrow 2\mu t = n\lambda$ (For maxima)

$$\Rightarrow t = \frac{n\lambda}{2\mu} \quad ; \text{ Least value of (t)} = \frac{\lambda}{2\mu} \quad \text{[For n = 1], For air, (t)} = \frac{\lambda}{2}$$

For glass, $(t) = \frac{2\lambda}{6} = \frac{\lambda}{3}$ Hence, minimum thickness of coat, $\frac{\lambda}{2} < t < \frac{\lambda}{3}$

Q.28 A parallel coherent beam of light falls on fresnel biprism of refractive index μ and angle α. The fringe width on a screen at a distance D from biprism will be (wavelength = λ)

(A) $\frac{\lambda}{2(\mu-1)\alpha}$ **(B)** $\frac{\lambda D}{2(\mu-1)\alpha}$ **(C)** $\frac{D}{2(\mu-1)\alpha}$ **(D)** none

Sol. (A) In the biprism experimental : Fringe width, $\beta = \frac{\lambda(D+a)}{2a\alpha(\mu-1)}$

where, a = distance between source and biprism
D = distance between biprism and screen

α = Angle of prism. For the parallel coherent beam, $\underset{a\to\infty}{\text{Lt}}$

Then, $\beta = \frac{\lambda}{2\alpha(\mu-1)}$ [Taking $\underset{a\to\infty}{\text{Lt}}$]

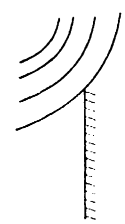
Q.29 **In a double slit experiment, the separation between the slits is d = 0.25 cm and the distance of the screen D = 100 cm from the slits. If the wavelength of light used is λ = 6000 Å and I_0 is the intensity of the central bright fringe, the intensity at a distance $x = 4 \times 10^{-5}$ m from the central maximum is**

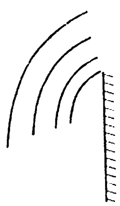
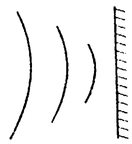
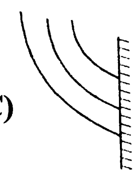
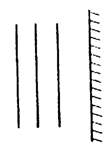
(A) I_0 **(B) $I_0/2$** **(C) $3I_0/4$** **(D) $I_0/3$**

Sol.(C) Path difference $= \dfrac{xd}{D} = 10^{-6}$ m. Phase difference $= \left(\dfrac{2\pi}{\lambda}\right) \times$ (path difference)

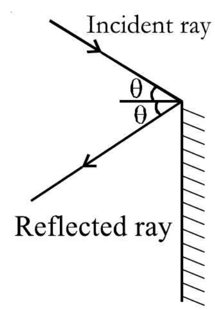
$$\Delta\phi = \frac{10\pi}{3} \quad \Rightarrow \quad I_0 \cos^2\frac{\Delta\varphi}{2} = \left(\frac{3I_0}{4}\right)$$

Q.30 **Spherical wave fronts shown in figure, strike a plane mirror. Reflected wave fronts will be as shown in**



(A)  **(B)**  **(C)**  **(D)** 

Sol.(C) Incident and reflected ray both are perpendicular to the wavefront. Make incident ray and reflected ray and perpendicular. The reflected rays will be reflected wavefront.



ASSERTION & REASON

Q.1 ***Statement-1*** **: In YDSE, as shown in the figure, central bright fringe is formed at O. If a liquid is filled between plane of slits and the screen, the central bright fringe is shifted in upward direction.**

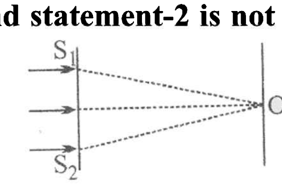
Statement-2 **: If path difference at O increases y-coordinate of central bright fringe will change.**

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(A) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true



Sol.(D) Central bright fringe will be formed at a point where path difference = 0. When liquid is filled between the plane of slits and the screen, path difference at 'O' remains uncharged.

• If path difference at O increases then O will not be the central maxima. Then some other point on the screen will become central maxima.

Q.2 ***Statement-1*** **: In glass, red light travels faster than blue light**
Statement-2 **: Red light has a wavelength longer than blue light**
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(A) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false
(D) Statement-1 is false, statement-2 is true

Sol.(A) Since frequency of light is independent of the medium, it only dependends on the source.

$$V = f\lambda \quad \Rightarrow \quad V \propto \lambda \quad \because \quad \lambda_R > \lambda_B \quad , \quad \text{Hence} \quad V_R > V_B$$

Where, λ_R = wavelength of red light, λ_B = wavelength of blue light

V_R = Velocity of red light, V_B = Velocity of blue light

Q.3 ***Statement-1*** **: In standard YDSE set up with visible light, the position on screen where phase difference is zero appears bright.**
Statement-2 **: In YDSE set up magnitude of electromagnetic field at central bright fringe is not varying with time.**
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1
(A) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false
(D) Statement-1 is false, statement-2 is true

Sol.(C) $\rightarrow$ Where phase difference is zero appears central bright for all colours.
$\rightarrow$ Magnitude of Electromagnetic field is always varriable.

ONE OR MORE THAN ONE OPTION MAY BE CORRECT

Take approx. 3 minutes for answering each question.

Q.1 To observe a stationary interference pattern formed by two light waves, it is not necessary that they must have :

(A) the same frequency **(B) same amplitude**

(C) a constant phase difference **(D) the same intensity**

Sol.(BD) Amplitude and intensity need not be the same for coherent sources and coherent sources is the necessary condition for interference pattern formed.

Q.2 In a YDSE apparatus, if we use white light then :

(A) the fringe next to the central fringe will be red

(B) the central fringe will be white

(C) the fringe next to the central fringe will be violet

(D) there will be no completely dark fringe

Sol.(BCD) → For all colours, Path difference is equal to zero at one point. Hence, $y_v < y_R$

$$[\because\ y = y = \frac{\lambda D}{d} \Rightarrow y \propto \lambda \text{ and } (\lambda_V < \lambda_R)\ ]$$

→ There will be no completely dark fringe, since all other fringes except central fringe, bright fringes overlaps each other for different colours.

Q.3 If the source of light used in a Young's Double Slit Experiment is changed from red to blue, then

(A) the fringes will become brighter

(B) consecutive fringes will come closer

(C) the number of maxima formed on the screen increases

(D) the central bright fringe will become a dark fringe

Sol.(BC) When wavelength of light is changed from red to blue, it means wavelength of light is decreased then, $\beta = \dfrac{\lambda D}{d}$ also decreases.

→ Then consecutive fringes will come close

→ When fringe width (β) is decreased, the number of maxima on the screen increases.

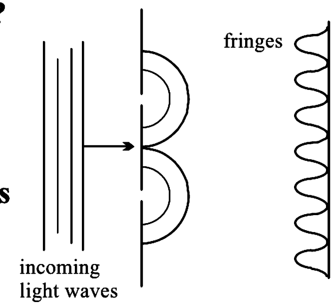
Q.4 In a Young's Double Slit Experiment, green light is incident on the two slits. The interference pattern is observed on a screen. Which of the following changes would cause the observed fringes to be more closely spaced?

(A) Reducing the separation between the slits

(B) Using blue light instead of green light

(C) Used red light instead of green light

(D) Moving the light source further away from the slits



Sol.(B) For fringes to be more closely spaced fringe width (β) must be decreased. For this wavelength must be decrease.

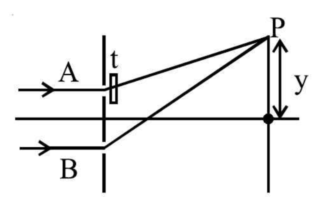
Q.5 In a Young's Double Slit Experiment, let A and B be the two slits. A thin film of thickness t and refractive index μ is placed in front of A. Let $\beta =$ fringe width. The central maximum will shift:

(A) towards A **(B) towards B** **(C) by $t(\mu - 1)\frac{\beta}{\lambda}$** **(D) by $\mu t \frac{\beta}{\lambda}$**

Sol.(AC) Path difference at a point P i.e. $\Delta x = (BP) - (AP - t + \mu t)$

$\Rightarrow \Delta x = (BP - AP) - t(\mu - 1)$

$\Rightarrow \Delta x = \frac{dy}{D} - t(\mu - 1)$ For the central maxima,



$\Delta x = 0 \quad \Rightarrow y = \frac{tD(\mu - 1)}{d}$

Since, y becomes positive hence, central maxima shift towards A.

Shift of central maxima i.e. $y = \frac{tD(\mu-1)}{d} = \frac{t(\mu-1)\beta}{\lambda}$; $\left(\because \beta = \frac{\lambda\theta}{d}\right)$

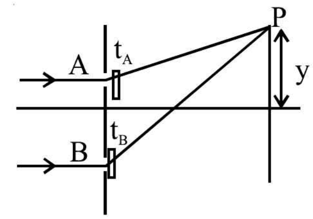
Q.6 In the previous question, films of thicknesses t_A and t_B and refractive indices μ_A and μ_B, are placed in front of A and B respectively. If $\mu_A t_A = \mu_B t_B$, the central maximum will :

(A) not shift **(B) shift towards A**

(C) shift towards B **(D) option (B), if $t_B > t_A$; option (C) if $t_B < t_A$**

Sol.(D) Path difference at point P, $\Delta x = (BP - t_B + \mu_B t_B) - (AP - t_A + \mu_A t_A)$

$\Rightarrow \Delta x = (BP - AP) + (t_A - t_B) + (\mu_B t_B - \mu_A t_A)$



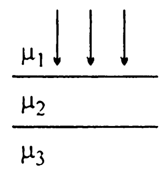
$\Rightarrow \Delta x = \frac{yd}{D} + (t_A - t_B)$ [$\because \mu_B t_B = \mu_A t_A$ and $BP - AP = \frac{yd}{D}$]

For the position of central maxima, $\Delta x = 0$

$\Rightarrow \quad 0 = \frac{yd}{D} + (t_A - t_B) \quad \Rightarrow y = \frac{(t_B - t_A)D}{d}$

If $t_B > t_A$, $y > 0$ [central maxima will shift towards A]
If $t_A > t_B$, $y < 0$ [central maxima will shift towards B]

Q.7 Consider a case of thin film interference as shown in the figure. Thickness of film is equal to wavelength of light in μ_2.



(A) Reflected light will be maxima if $\mu_1 < \mu_2 < \mu_3$
(B) Reflected light will be maxima if $\mu_1 > \mu_2 > \mu_3$
(C) Transmitted light will be maxima if $\mu_1 < \mu_2 < \mu_3$
(D) Transmitted light will be maxima if $\mu_1 > \mu_2 > \mu_3$

Sol.(AD) (1) Path difference in air of reflected light if $\mu_1 < \mu_1 < \mu_3$, $\Delta x = 2\,\mu_2 t + \left(\frac{\lambda_a}{2} - \frac{\lambda_a}{2}\right) = n\lambda_a$

$\Rightarrow 2\,\mu_2 \left(\frac{\lambda_a}{\mu_2}\right) + 0 = n\lambda_a \left(\because t = \frac{\lambda_a}{\mu_2}\right) \qquad \Rightarrow n = 2$, which is possible

(2) Path difference in air in case of reflected light if $\mu_1 < \mu_2 > \mu_3$. $\Delta x = 2\mu_2 t + (0 - \frac{\lambda_a}{2})$

$= n\lambda_a$ (since, $t = \frac{\lambda_a}{\mu_2}$) $\Rightarrow n = \frac{3}{2}$, which is not possible, since (n) shoud be integer.

(3) Path difference in air in case of transmitted light if $\mu_1 > \mu_2 > \mu_3$

$\Delta x = 2\,\mu_2 t + \left(\frac{\lambda_a}{2} - 0\right) = n\lambda_a \;\Rightarrow n = \frac{5}{2}$, which is not possible (since, $t = \frac{\lambda_a}{\mu_2}$)

(4) Path difference in case of transmitted light, if $= \mu_1 > \mu_2 < \mu_3$,

$\Delta x = 2\,\mu_2 t + \left(\frac{\lambda_a}{2} + \frac{\lambda_a}{2}\right) = n\,\lambda_a \qquad \Rightarrow \qquad n = 3$ (which is impossible)

Q.8 If one of the slits of a standard YDSE apparatus is covered by a thin parallel sided glass slab so that it transmits only one half of the light intensity of the other, then :

(A) the fringe pattern will get shifted towards the covered slit

(B) the fringe pattern will get shifted away from the covered slit

(C) the bright fringes will be less bright and the dark ones will be more bright

(D) the fringe width will remain unchanged

Sol.(ACD) If one of the slits is covered by a thin glass slab, Path difference i.e. $\Delta x = \frac{yd}{D} - (\mu - 1)t$

[when it is placed in front of upper slit] $\Rightarrow$ $y > 0$ [$\because \Delta x = 0$, For central maxima]

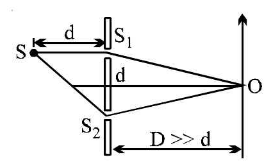
Hence, fringe pattern will shift towards upper slit.

For intensity :

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$, $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

According to the question, $(I_1 = I_0)$ then, $I_{max} < 4I_0$ and $I_{min} > 0$

Q.9 To make the central fringe at the center 'O' a mica sheet of refractive index 1.5 is introduced. Choose the correct statement(s).



(A) The thickness of sheet is $2(\sqrt{2} - 1)d$ **in front of S_1.**

(B) The thickness of sheet is $(\sqrt{2} - 1)d$ **in front of S_2.**

(C) The thickness of sheet is $2\sqrt{2}d$ **in front of S_1.**

(D) The thickness of sheet is $(2\sqrt{2} - 1)d$ **in front of S_1.**

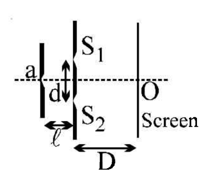
Sol.(A) For the central maxima at the center 'O', path difference i.e. $\Delta x = 0$ should be at this point. From the given fig, there is some extra path length $\left(\sqrt{2} - 1\right)d$ at point S_2, then by introducing the mica sheet at S_1 we would increase the path length $(\mu t - t)$ which cancells the path length at S_2.

$(\mu t - t) = (\sqrt{2} - 1)d$ [from the above statements]

$\Rightarrow \quad t(1.5 - 1) = (\sqrt{2} - 1)d \quad \Rightarrow \quad t = 2(\sqrt{2} - 1)d$

Question No. 10 to 12 (3 questions)

The figure shows a schematic diagram showing the arrangement of Young's Double Slit Experiment



Q.10 Choose the correct statement(s) related to the wavelength of light used
(A) Larger the wavelength of light, larger the fringe width
(B) The position of central maxima depends on the wavelength of light used
(C) If white light is used in YDSE, then the violet colour forms its first maxima closest to the central maxima
(D) The central maxima of all the wavelengths coincide

Sol.(ACD) $\rightarrow$ Fringe width $(\beta) = \dfrac{\lambda D}{d} \Rightarrow \beta \propto \lambda$

$\rightarrow$ At the position of central maxima, $\Delta x = 0$ which is independent of the wavelength of light used.
$\rightarrow \lambda_v < \lambda_R$, Then violet colour forms its first maxima closer to the central maxima.
$\rightarrow$ Central maxima for all colours coincides, since $\Delta x = 0$ for all colours formed at the same position.

Q.11 If the distance D is varied, then choose the correct statement(s)
(A) The angular fringe width does not change
(B) The fringe width changes in direct proportion
(C) The change in fringe width is same for all wavelengths
(D) The position of central maxima remains unchanged

Sol.(ABD) $\rightarrow$ Angular fringe width $= \dfrac{\lambda}{d}$, which is independent of D.

$\rightarrow \beta \propto D \left(\because \beta = \dfrac{\lambda D}{d} \right)$

$\rightarrow$ Fringe width depends on λ
$\rightarrow$ At the position of central maxima, $\Delta x = 0$ which is independent of D.

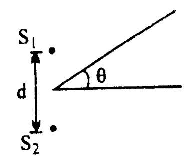
Q.12 If the distance d is varied, then identify the correct statement
(A) The angular width does not change
(B) The fringe width changes in inverse proportion
(C) The positions of all maxima change
(D) The positions of all minima change

Sol.(BD) $\rightarrow$ Angular fringe width $= \dfrac{\lambda}{d}$, which depends on d.

$\rightarrow \beta \propto \dfrac{1}{d} \left(\because \beta = \dfrac{\lambda \theta}{d} \right)$

$\rightarrow$ Position of central maxima remains unchanged.
$\rightarrow$ Position of minima changes.

Q.13 In an interference arrangement similar to Young's Double Slit Experiment, the slits S_1 & S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance d = 150.0 m. The intensity I (θ) is measured as a function of θ at a large distance from S_1 & S_2, where θ is defined as shown. If I_0 is the maximum intensity, then I (θ) for $0 \le \theta \le 90°$ is given by :



(A) $I(\theta) = \frac{I_0}{2}$ for θ = 30° (B) $I(\theta) = \frac{I_0}{4}$ for θ = 90°

(C) $I(\theta) = I_0$ for θ = 0° (D) I(θ) is constant for all values of θ.

Sol.(AC) $I(\theta) = I_{max}\cos^2\left(\frac{\Delta\varphi}{2}\right) \Rightarrow I(\theta) = I_0\cos^2\left(\frac{\Delta\varphi}{2}\right)$, where, $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)$.

Then, $\Delta\phi = \frac{2\pi}{\lambda}(d\sin\theta) \because c = f\lambda \Rightarrow \lambda = \frac{3\times10^8}{10^6} = 300\text{ m} \Rightarrow \Delta\phi = \left(\frac{2\pi}{300}\right)\left(150\times\frac{1}{2}\right)$

(at θ = 30) $\Rightarrow \Delta\phi = \frac{\pi}{2}$, at θ = 30°, $I(\theta) = I_o\cos^2\frac{\pi}{4} = \frac{I_o}{2}\left(\because \Delta\varphi = \frac{\pi}{2}\right)$

at θ = 0°, $I(\theta) = I_0 \left(\because \Delta\varphi = 0\right)$ at θ = 90°, $I(\theta) = 0, \left(\because \Delta\varphi = \pi\right)$

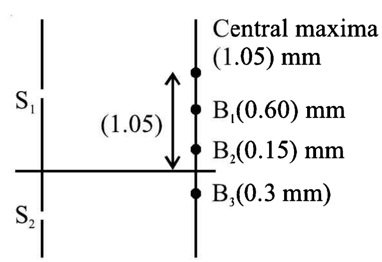
Q.14 In a standard YDSE apparatus a thin film (μ = 1.5, t = 2.1 μm) is placed in front of the upper slit. How far above or below, the center point of the screen are two nearest maxima located ?

[Take D = 1 m, d = 1 mm, λ = 4500 Å. (Symbols have usual meaning]

(A) 1.5 mm (B) 0.6 mm (C) 0.15 mm (D) 0.3 mm

Sol.(CD) When a thin film (μ = 1.5, t = 2.1 μ m) is placed in front of the upper slit,

Path difference i.e. $\Delta x = (S_2P) - (S_1P - t + \mu t)$



$\Rightarrow 0 = \frac{dy}{D} - t(\mu - 1)$ [For central maxima, Δx = 0]

y = (1.05) mm [Position of central maxima]

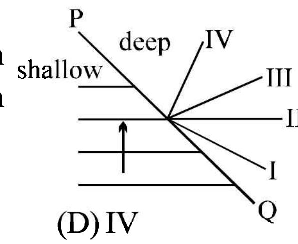
Fringe width, $\beta = \frac{\lambda D}{d} = 0.45$ mm

∴ Position of second bright and third bright is nearest maxima from the center of the screen.

ONLY ONE CHOICE CORRECT

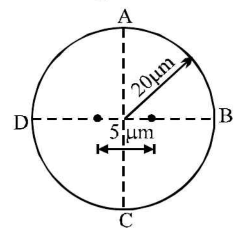
Take approx. 2 minutes for answering each question.

Q.1 Figure, shows wave fronts in still water, moving in the direction of the arrow towards the interface PQ between a shallow region and a deep(denser) region. Which of the lines shown may represent one of the wave fronts in the deep region?



(A) I (B) II (C) III (D) IV

Q.2 Two point sources separated by d = 5 μm emit light of wavelength λ = 2 μm in phase. A circular wire of radius 20 μm is placed around the source as shown in figure.



(A) Point A and B are dark and points C and D are bright
(B) Points A and B are bright and point C and D are dark
(C) Points A and C are dark and points B and D are bright
(D) Points A and C are bright and points B and D are dark

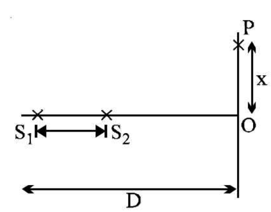
Q.3 Plane microwaves from a transmitter are directed normally towards a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima, the detector travels a distance 0.13 m. If the velocity of light is 3×10^8 m/s, find the frequency of the transmitter.

(A) 1.5×10^{10} Hz (B) 10^{10} Hz (C) 3×10^{10} Hz (D) 6×10^{10} Hz

Q.4 Two monochromatic (wavelength = a/5) and coherent sources of electromagnetic waves are placed on the x-axis at the points (2a, 0) and (–a, 0). A detector moves in a circle of radius R(>>2a) whose center is at the origin. The number of maximas detected during one circular revolution by the detector are:

(A) 60 (B) 15 (C) 64 (D) None

Q.5 Two coherent narrow slits emitting light of wavelength λ in the same phase are placed parallel to each other at a small separation of 3λ. The light is collected on a screen S which is placed at a distance D (>> λ) from the slits. The smallest distance x such that the P is a maxima is:



(A) $\sqrt{3D}$ (B) $\sqrt{8}\,D$

(C) $\sqrt{5}\,D$ (D) $\sqrt{5}\dfrac{D}{2}$

Q.6 Two coherent sources of light are placed at points $(-\frac{5a}{2}, 0)$ and $(+\frac{5a}{2}, 0)$. Wavelength of the light is $\lambda = \frac{4a}{3}$. How many maximas will be obtained on a CD planar circle of large radius with center at origin?
(A) 12 (B) 15 (C) 16 (D) 14

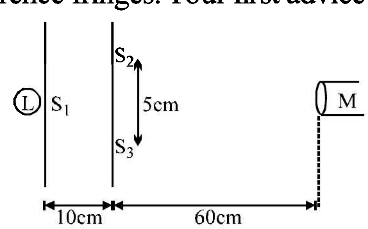
Q.7 In YDSE how many maxima can be obtained on the screen if wavelength of light used is 200nm and d = 700 nm:
(A) 12 (B) 7 (C) 18 (D) none of these

Q.8 In Young's Double Slit Experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light, is :
(A) 1 (B) 2 (C) 3 (D) 4

Q.9 If the Young's Double Slit Experiment is performed with white light, then which of the following is not true?
(A) the central maximum will be white
(B) there will be no completely dark fringe
(C) the fringe next to the central fringe will be red
(D) the fringe next to the central fringe will be violet

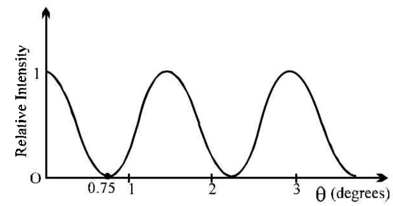
Q.10 Imagine a Young's Double Slit interference experiment performed with waves associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease maximum if,
(A) the accelerating voltage in the electron gun is decreased
(B) the accelerating voltage is increased and the distance of the screen from the slits is decreased
(C) the distance of the screen from the slits is increased
(D) the distance between the slits is decreased

Q.11 A student is asked to measure the wavelength of monochromatic light. He sets up the apparatus sketched below. S_1, S_2, S_3 are narrow parallel slits, L is a sodium lamp and M is a microscope eyepiece. The student fails to observe interference fringes. Your first advice to him will be:
(A) increase the width of S_1
(B) decrease the distance between S_2 and S_3
(C) replace L with a white light source
(D) replace M with a telescope
(E) make S_2 and S_3 wider.



Q.12 Light of wavelength 520 nm passing through a double slit, produces interference pattern of relative intensity versus deflection angle θ as shown in the figure. The separation d between the slits is:

(A) 2×10^{-2} mm (B) 5×10^{-2} mm
(C) 4.5×10^{-2} mm (D) 1.1×10^{-2} mm



Q.13 In Young's Double Slit Experiment, the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slit. It is found that the 9th bright fringe is at a distance of 7.5 mm from the second dark fringe from the center of the fringe pattern. The wavelength of the light used is:

(A) $\frac{2500}{7}$ Å (B) 2500 Å (C) 5000 Å (D) $\frac{5000}{7}$ Å

Q.14 In a YDSE apparatus, two identical slits are separated by 1 mm and distance between slits and screen is 1 m. The wavelength of light used is 6000 Å. The minimum distance between two points on the screen having 75% intensity of the maximum intensity is :

(A) 0.45 mm (B) 0.40 mm (C) 0.30 mm (D) 0.20 mm

Q.15 In a Young Double Slit Experiment, D equals the distance of screen and d is the separation between the slits. The distance of the nearest point to the central maximum where the intensity is same as that due to a single slit, is equal to:

(A) $\frac{D\lambda}{d}$ (B) $\frac{D\lambda}{2d}$ (C) $\frac{D\lambda}{3d}$ (D) $\frac{2D\lambda}{d}$

Q.16 A beam of light consisting of two wavelength 6300 Å and λ Å is used to obtain interference fringes in a Young's Double Slit Experiment. If 4^{th} bright fringe of 6300 Å coincides with 5^{th} dark fringe of λ Å, the value of λ (in Å) is:

(A) 5200 (B) 4800 (C) 6200 (D) 5600

Q.17 A beam of light consisting of two wavelengths 6500Å and 5200Å is used to obtain interference fringes in Young's Double Slit Experiment. The distance between slits is 2 mm and the distance of screen from slits is 120 cm. What is the least distance from central maximum where the bright fringe due to both wavelength coincides?

(A) 0.156 cm (B) 0.312 cm (C) 0.078 cm (D) 0.468 cm

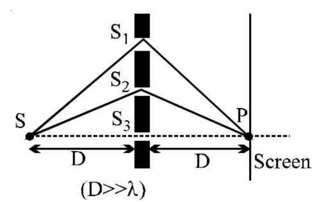
Q.18 In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If separation between the slits is 10^{-3}m, the wavelength of light used is:
(A) 6000 Å (B) 5000 Å (C) 3000 Å (D) 4500 Å

Q.19 The ratio of the intensity at the center of a bright fringe to the intensity at a point one–quarter of the fringe width from the center is:
(A) 2 (B) 1/2 (C) 4 (D) 16

Q.20 In a YDSE, let S_1 and S_2 be the two slits, and C be the center of the screen. If θ is the angle S_1CS_2 and λ is the wavelength, the fringe width will be :

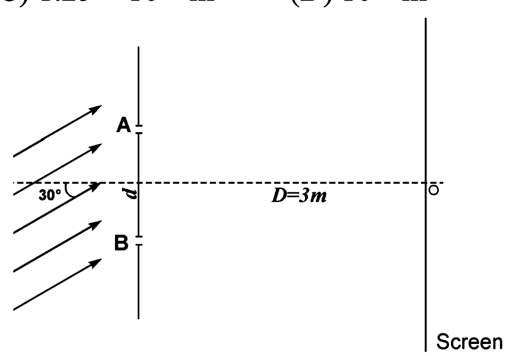
(A) $\frac{\lambda}{\theta}$ (B) $\lambda\theta$ (C) $\frac{2\lambda}{\theta}$ (D) $\frac{\lambda}{2\theta}$

Q.21 A monochromatic light source of wavelength λ is placed at S. Three slits S_1, S_2 and S_3 are equidistant from the source S and the point P on the screen. $S_1P - S_2P = \lambda/6$ and $S_1P - S_3P = 2\lambda/3$. If I be the intensity at P when only one slit is open, the intensity at P when all the three slits are open is:
(A) 3 I
(B) 5 I
(C) 8 I
(D) zero



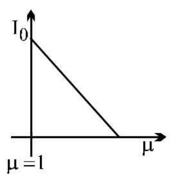
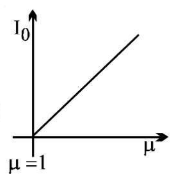
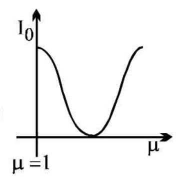
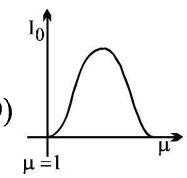
Q.22 In a Young's Double Slit Experiment, the value of λ = 500 nm. The value of d = 1 mm, D = 1 m. Then, the minimum distance from central maximum for which the intensity is half the maximum intensity will be:
(A) 2.5×10^{-4} m (B) 2×10^{-4} m (C) 1.25×10^{-4} m (D) 10^{-4} m

Q.23 A parallel beam of light 500nm is incident at an angle 30° with the normal to the slit plane in a Young's Double Slit Experiment. The intensity due to each slit is Io. Point O is equidistant from S_1 and S_2. The distance between slits is 1mm.

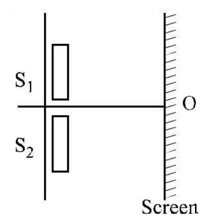


(A) the intensity at O is 4Io
(B) the intensity at O is zero.
(C) the intensity at a point on the screen 4mm from O is 4Io
(D) the intensity at a point on the screen 4mm from O is zero.

Q.24 In a YDSE experiment, if a slab whose refractive index can be varied is placed in front of one of the slits, then the variation of resultant intensity at the mid-point of a screen with 'μ' will be best represented by ($\mu \geq 1$).[Assume slits of equal width and there is no absorption by slab]

(A) 
(B) 
(C) 
(D) 

Q.25 Young's Double Slit Experiment is carried with two thin sheets of thickness 10.4 μm each and refractive index $\mu_1 = 1.52$ and $\mu_2 = 1.40$ covering the slits S_1 and S_2, respectively. If white light of range 400 nm to 780 nm is used then, which wavelength will form maxima exactly at point O, the center of the screen ?



(A) 416 nm only (B) 624 nm only
(C) 416 nm and 624 nm only (D) none of these

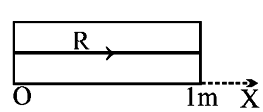
Q.26 A light of wavelength 6300Å shines on two narrow slits separated by a distance 1.0 mm and illuminates a screen at a distance 1.5 m away. When one slit is covered by a thin glass of refractive index 1.8 and other slit by a thin glass plate of refractive index μ, the central maxima shifts by 6°. Both plates have same thickness of 0.5 mm. The value of refractive index μ of the plate is:
(A) 1.6 (B) 1.7 (C) 1.5 (D) 1.4

Q.27 Minimum thickness of a mica sheet having $\mu = \frac{3}{2}$ which should be placed in front of one of the slits in YDSE is required to reduce the intensity at the center of the screen to half of maximum intensity is:
(A) $\lambda/4$ (B) $\lambda/8$ (C) $\lambda/2$ (D) $\lambda/3$

Q.28 The figure shows a transparent slab of length 1m placed in air whose refractive index in x direction varies as $\mu = 1 + x^2 (0 \leq x \leq 1)$. The optical path length of ray R will be:

(A) 1 m (B) $\frac{2}{3}$ m

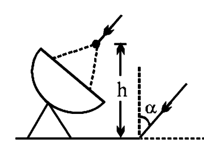
(C) $\frac{4}{3}$ m (D) $\sqrt{2}$ m



Q.29 A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate with the curved surface downwards. Monochromatic light is incident normally from the top. The observed interference fringes from this combination do not follow one of the following statements:
(A) the fringes are straight and parallel to the length of the piece
(B) the line of contact of the cylindrical glass piece and the glass plate appears dark
(C) the fringe spacing increases as we go outwards
(D) the fringes are formed due to the interference of light rays reflected from the curved surface of the cylindrical piece and the top surface of the glass plate

Q.30 Radio waves coming at $\angle\, \alpha$ to vertical are received by a radar after reflection from a nearby water surface & directly. What should be height of antenna from water surface so that it records a maximum intensity? (wavelength = λ).

(A) $\dfrac{\lambda}{2\cos\alpha}$ (B) $\dfrac{\lambda}{2\sin\alpha}$
(C) $\dfrac{\lambda}{4\sin\alpha}$ (D) $\dfrac{\lambda}{4\cos\alpha}$



Q.31 In a Biprism experiment, the distance of source from biprism is 1 m and the distance of screen from biprism is 4 meters. The angle of refraction of biprism is 2×10^{-3} radians. μ of biprism is 1.5 and the wavelength of light used is 6000Å. How many fringes will be seen on the screen?
(A) 4 (B) 5 (C) 3 (D) 6

Q.32 In a Biprism experiment using sodium light $\lambda = 6000$ Å, an interference pattern is obtained in which 20 fringes occupy 2 cm. On replacing sodium light by another source of wavelength λ_2 without making any other change 30 fringes occupy 2.7 cm on the screen. What is the value of λ_2?
(A) 4500 Å (B) 5400 Å (C) 5600 Å (D) 4200 Å

ONE OR MORE THAN ONE CHOICES MAY BE CORRECT

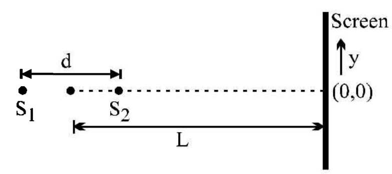
Take approx. 3 minutes for answering each question.

Q.33 A light of wavelength 600nm in air enters a medium of refractive index 1.5. Inside the medium :
(A) its frequency is 5×10^{14} Hz (B) its frequency is 7.5×10^{14} Hz
(C) its wavelength is 400nm (D) its wavelength is 900nm

Q.34 Four monochromatic and coherent sources of light, emitting waves in phase of wavelength λ, are placed at the points $\rightarrow x = 0$, d, 2d and 3d on the x-axis. Then,
(A) points having $|x| >> d$ appear dark if $d = \lambda/4$
(B) points having $|x| >> d$ appear dark if $d = \lambda/8$
(C) points having $|x| >> d$ appear maximum bright if $d = \lambda/4$
(D) points having $|x| >> d$ appear maximum bright if $d = \lambda/8$

Q.35 In the above question, the intensity of the waves reaching a point P far away on the +x axis from each of the four sources is almost the same, and equal to I_0. Then,
(A) If $d = \lambda/4$, the intensity at P is $4I_0$ (B) If $d = \lambda/6$, the intensity at P is $3I_0$
(C) If $d = \lambda/2$, the intensity at P is $3I_0$ (D) none of these is true

Q.36 The figure shows two point sources which emit light of wavelength λ in phase with each other and are at a distance $d = 5.5\ \lambda$ apart along a line which is perpendicular to a large screen at a distance L from the center of the source. Assume that d is much less than L. Which of the following statements is (are) correct?
(A) Only five bright fringes appear on the screen
(B) Only six bright fringes appear on the screen
(C) Point y = 0 corresponds to the bright fringe
(D) Point y = 0 corresponds to the dark fringe



0;p

Q.37 White light is used to illuminate two slits in a YDSE. The separation between the slits is d and the screen is at a distance D (D >> d) from the slits. At a point on the screen directly in front of one of the slits, which of the following wavelengths are missing.

(A) $\frac{d^2}{D}$ (B) $\frac{2d^2}{D}$ (C) $\frac{d^2}{3D}$ (D) $\frac{2d^2}{3D}$

Q.38 In a Double Slit Experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then in the interference pattern :
(A) the intensities of both the maxima and minima increases
(B) the intensity of the maxima increases and the minima has zero intensity
(C) the intensity of the maxima decreases and that of minima increases
(D) the intensity of the maxima decreases and the minima has zero intensity

Q.39 In a YDSE, if the slits are of unequal width :
(A) fringes will not be formed
(B) the positions of minimum intensity will not be completely dark
(C) bright fringe will not be formed at the center of the screen
(D) distance between two consecutive bright fringes will not be equal to the distance between two consecutive dark fringes

Answer Key

ONLY ONE CHOICE CORRECT

Q.1	A	Q.2	D	Q.3	A	Q.4	A	Q.5	D	Q.6	D	Q.7	B
Q.8	B	Q.9	C	Q.10	B	Q.11	B	Q.12	A	Q.13	C	Q.14	D
Q.15	C	Q.16	D	Q.17	A	Q.18	A	Q.19	A	Q.20	A	Q.21	A
Q.22	C	Q.23	A	Q.24	C	Q.25	C	Q.26	A	Q.27	C	Q.28	C
Q.29	C	Q.30	D	Q.31	B	Q.32	B						

ONE OR MORE THAN ONE CHOICES MAY BE CORRECT

Q.33	A,C	Q.34	A	Q.35	B	Q.36	A,D
Q.37	A,C	Q.38	A	Q.39	B		

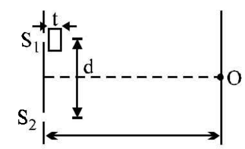
EXERCISE – I

Q.1 In a Young's Double Slit Experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of the light is changed to 400 nm, find the number of fringes observed in the same segment.

Q.2 In an ideal double slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position which the central maximum occurred previously remains unchanged. find the minimum thickness of the glass plate.

Q.3 Three identical monochromatic point sources of light emit light of wavelength λ coherently and in phase with each other. They are placed on the x-axis at the points $x = -d$, 0 and d. Find the minimum value of d/λ for which there is destructive interference with almost zero resultant intensity at points on the x-axis having $x >> d$.

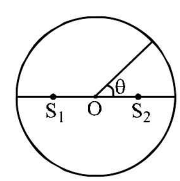
Q.4 The figure shows a Young's Double Slit Experiment with a mica sheet of thickness t and refractive index μ, introduced in front of S_1. If the mica sheet is removed from the earlier position and placed in front of S_2 find the number of fringes crossing O.



Q.5 The central fringe of the interference pattern produced by the light of wavelength 6000 Å is found to shift to the position of 4^{th} bright fringe after a glass sheet of refractive index 1.5 is introduced. Find the thickness of the glass sheet.

Q.6 A broad source of light of wavelength 680nm illuminates normally two glass plates 120mm long that meet at one end and are separated by a wire 0.048 mm in diameter at the other end. Find the number of bright fringes formed over the 120mm distance.

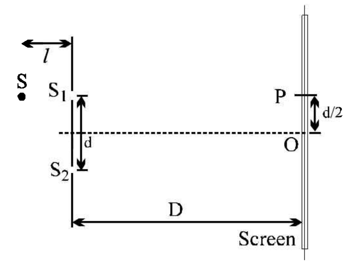
Q.7 Two coherent sources S_1 and S_2 separated by distance 2λ emit light of wavelength λ in phase as shown in the figure. A circular wire of radius 100λ is placed in such a way that S_1S_2 lies in its plane and the midpoint of S_1S_2 is at the center of wire. Find the angular positions θ on the wire for which intensity reduces to half of its maximum value.



Q.8 In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5}. If the distance between the slits is 10^{-3} m, calculate the wavelength of the light used.

EXERCISE – II

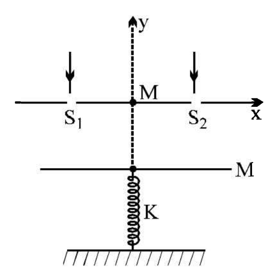
Q.1 A source S is kept directly behind the slit S_1 in a double slit apparatus. What will be the phase difference at P, if a liquid of refraction index μ is filled: (wavelength of light in air is λ due to the source, assume $l >> d$, $D >> d$).



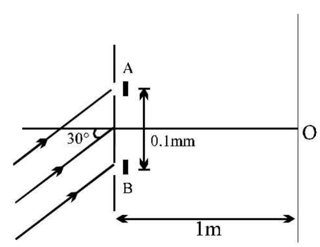
(i) between the screen and the slits.

(ii) between the slits & the source S. In this case, find the minimum distance between the points on the screen where the intensity is half the maximum intensity on the screen.

Q.2 Two slits S_1 & S_2 on the x – axis & symmetric with respect to y-axis are illuminated by a parallel monochromatic light beam of wavelength λ. The distance between the slits is d (>> λ). Point M is the mid point of the line S_1S_2 & this point is considered as the origin. The slits are in horizontal plane. The interference pattern is observed on a horizontal plate (acting as the screen) of mass M, which is attached to one end of a vertical spring of spring constant K. The other end of the spring is fixed to the ground. At t = 0, the plate is at a distance D(>>d) below the plane of slits & the spring is of natural length. The plate is left from rest from its initial position. Find the x & y co-ordinates of the nth maxima on the plate as a function of time. Assume that spring is light & plate always remains horizontal.



Q.3 In a YDSE, a parallel beam of light of wavelength 6000 Å is incident on slits at angle of incidence 30°. A & B are two thin transparent films, each of refractive index 1.5. Thickness of A is 20.4 µm. Light coming through A & B have intensities I & 4I respectively on the screen. Intensity at point O which is symmetric relative to the slits is 3 I. The central maxima is above O.



(a) What is the maximum thickness of B to do so? Assuming thickness of B to be that found in part (a) answer the following parts:

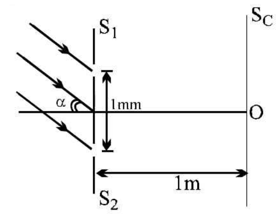
(b) Find fringe width, maximum intensity & minimum intensity on screen.

(c) Distance of nearest minima from O.

(d) Intensity at 5 cm on either side of O.

Q.4 In a YDSE experiment, the distance between the slits & the screen is 100 cm . For a certain distance between the slits, an interference pattern is observed on the screen with the fringe width 0.25 mm. When the distance between the slits is increased by Δd = 1.2 mm, the fringe width decreased to n = 2/3 of the original value. In the final position, a thin glass plate of refractive index 1.5 is kept in front of one of the slits & the shift of central maximum is observed to be 20 fringe width. Find the thickness of the plate & wavelength of the incident light.

Q.5 A plane wave of mono chromatic light of wavelength 6000Å is incident on the plane of two slits s_1 and s_2 at angle of incidence $\alpha = (1.8/\pi)^0$. The widths of S_1 and S_2 are w and 2w respectively. A thin transparent film of thickness 4μm and R.I. 3/2 is placed in front of S_1. It absorbs 50% of light energy and transmits the remaining. The interference is observed on the screen. Point O is equidistant from S_1 and S_2. If the maximum intensity on the screen is I, then find:



(i) intensity at O (ii) Minimum intensity
(iii) fringe width (iv) Distance of nearest maxima from O
(v) Distance of central maxima from O (vi) intensity at 4mm from O upwards.

Q.6 A central portion with a width of d = 0.5 mm is cut out of a convergent lens having a focal length of 20 cm. Both halves are tightly fitted against each other and a point source of monochromatic light (A = 2500Å) is placed in front of the lens at a distance of 10 cm. Find the maximum possible number of interference bands that can be observed on the screen.

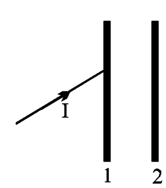
Q.7 In a Young's Double Slit Experiment, a parallel light beam of wavelength λ_1 = 4000Å and λ_2 = 5600Å is incident on a diaphragm having two narrow slits. Separation between the slits is d = 2 mm. If distance between diaphragm and screen is D = 40 cm, calculate :
(i) distance of first black line from the central bright fringe
(ii) distance between two consecutive black lines

Q.8 A plastic film with index of refraction 1.80 is put on the surface of a car window to increase the reflectivity and thereby to keep the interior of the car cooler. The window glass has index of refraction 1.60.

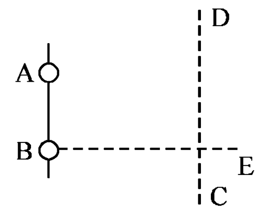
(a) What minimum thickness is required, if light of wavelength 600 nm in air reflected from the two sides of the film is to interfere constructively?

(b) It is found to be difficult to manufacture and install coatings as thin as calculated in part (a). What is the next greatest thickness for which there will also be constructive interference?

Q.9 A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in the figure. Another identical glass plate is kept close to the first one & parallel to it . Each glass plate reflects 25 % of the light incident on it & transmits the remaining. Find the ratio of the minimum & the maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate.



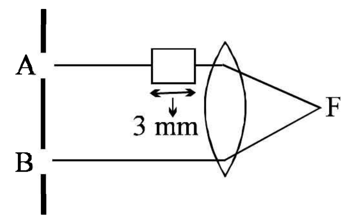
Q.10 Two coherent monochromatic sources A and B emit light of wavelength λ. The distance between A and B is $d = 4\lambda$.



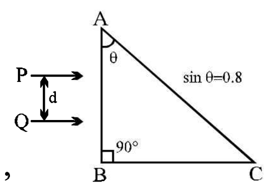
(i) If a light detector is moved along a line CD parallel to AB, what is the maximum number of minima observed ?

(ii) If the detector is moved along a line BE perpendicular to AB and passing through B, what is the number of maxima observed ?

Q.11 Two identical monochromatic light sources A and B of intensity 10^{-15} W/m^2 produce wavelength of light $4000\sqrt{3}$ Å. A glass of thickness 3mm is placed in the path of the ray as shown in the fig. The glass has a variable refractive index $n = 1 + \sqrt{x}$ where x (in mm) is distance of plate from left to right. Calculate resultant intensity at focal point F of the lens.



Q.12 Two parallel beams of light P & Q (separation d) each containing radiations of wavelengths 4000 Å & 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in the figure. The refractive index of the prism as a function of wavelength is given by the relation, $\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$, where λ is in Å & b is a positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wavelength & is not satisfied for the other. Find the value of b. A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper & the lower beams immediately after transmission from the face AC, are 4I & I respectively, find the resultant intensity at the focus.



ANSWER KEY

EXERCISE – I

Q.1 18 **Q.2** 2λ **Q.3** 1/3 **Q.4** $\frac{2(\mu-1)t}{\lambda}$

Q.5 4.8 μm **Q.6** 141

Q.7 $\pm\cos^{-1}\left(\frac{2n+1}{8}\right)$, n = 0, 1, 2, 3 & $\pi \pm \cos^{-1}\left(\frac{2n+1}{8}\right)$ n = 0, 1, 2, 3

Q.8 6000 Å

EXERCISE – II

Q.1 (i) $\Delta\phi = \left(\frac{1}{l}+\frac{\mu}{D}\right)\frac{\pi d^2}{\lambda}$ (ii) $\Delta\phi = \left(\frac{\mu}{l}+\frac{1}{D}\right)\frac{\pi d^2}{\lambda}$; $D_{min} = \frac{\beta}{2} = \frac{\lambda D}{2d}$;

Q.2 X - coordinate = $\frac{n\lambda D'}{d}$; Y–coordinate = – D', Where D' = D + Mg/K (1 – cosωt)

Q.3 (a) t_B = 120 μm (b) β = 6mm; I_{max} = 9I, I_{min} = I (c) β/6 = 1mm (d) I (at 5cm above 0) = 9I,
I (at 5 cm below 0) = 3I **Q.4** λ = 600 nm, t = 24 μm

Q.5 (i) I/3, (ii) I/9, (iii) 0.6 mm, (iv) 0.2mm downwards , (v) 8mm down, (vi) I

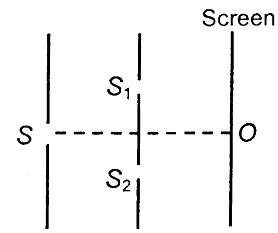
Q.6 5 **Q.7** (i) 280 μm, (ii) 560 μm

Q.8 (a) 8.33×10^{-8} m, (b) 2.5×10^{-7} m **Q.9** 1 : 49

Q.10 (i) 8, (ii) 4 **Q.11** 4×10^{-15} W/m **Q.12** 8.0×10^{5} Å^2 , 9 I

MATCHING COLUMN TYPE

1. Figure shows a set up to perform a Young's Double Slit Experiment. A monochromatic source of light is placed at S. S_1 and S_2 act as coherent screen. Match Column-I with column-II in regard to interference in Young's Double Slit Experiment :



Column-I	**Column-II**
I. A thin transparent plate is placed in front of S_1. Assuming negligible absorption by the plate	(A) Interference fringes disappear
II. S_1 is closed	(B) There is uniform illumination on a large part of the screen
III. A thin transparent plate is placed in front	(C) The zero order fringe will not form at O
IV. S is removed and two real sources emitting light of same wavelength are placed at S_1 and S_2.	(D) Central maxima is formed below O

2. Given

Column-I	**Column-II**
I. Diamond	(A) Total Internal Reflection
II. Air bubble is water	(B) Diverging lens
III. An equiconvex lens made of glass kept in a liquid of ($\mu = 1.70$, $\mu_g = 1.5$)	(C) Converging lens
IV. An equiconvex lens made of glass kept in in water ($\mu_g = 1.5$, $\mu_{water} = 1.33$)	(D) Refraction through curved surface

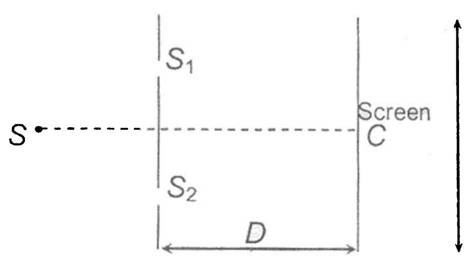
3. Match the following

Column-I	**Column-II**
I. Young's Double Slit Experiment uses	(A) Incoherent sources
II. Sources of variable phase difference	(B) Coherent source
III. A point on a wave front behaves as a light source	(C) Superposition
IV. Net displacement is the vector sum of individual displacement	(D) Huygens principle

4. Consider a linear extended object that could be real or virtual with its length at right angles to the optic axis of a lens. With regard to image formation by lenses, match the Column-I with Column-II :

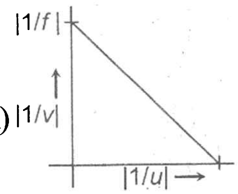
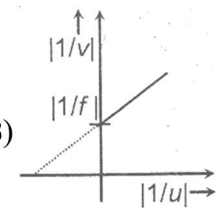
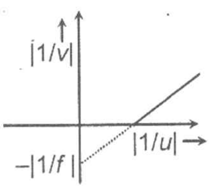
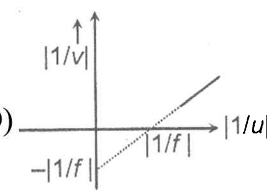
Column-I	**Column-II**
I. Image of the same size as the object	(A) Concave lens in case of real object
II. Virtual image of a size greater than the object	(B) Convex lens in case of real object
III. Real image of a size smaller than the object	(C) Concave lens in case of virtual object
IV. Real and erect image	(D) Convex lens in case of virtual object

5. In a YDSE setup, light of wavelength of light 4000Å is used. Distance of screen from the slits is 2m and distance between the slits is 1 mm. There are four slabs, slab 1 (thickness 2mm, refractive index 2), slab 2 (thickness 1mm, refractive index 3), slab 3 (thickness 4mm, refractive index 3/2) and slab 4 (thickness 3mm, refractive index 5/3).



Column-I	**Column-II**
I. If slab 1 is placed in front of slit S_1	(A) Central maxima is at C.
II. If slab 2 is placed in front of slit S_2 along with condition (A)	(B) Central maxima is above C.
III. If slab 3 is placed behind slit S_1 along condition (B)	(C) Fringe width = 0.8 mm
IV. If slab 4 is placed behind slit S_2 along with condition (C)	(D) No. of fringes crossing the central maxima as a result of slab placing is 5000.

6. Match the graphs of $\left|\frac{1}{v}\right| vs \left|\frac{1}{u}\right|$ where v is the image distance and u is the object distance for an object which could be real or virtual, under the conditions given below:

Column-I	**Column-II**
I. In concave mirror when object is real or virtual and image is real	(A) 
II. In concave mirror when object is real and image is virtual	(B) 
III. In concave mirror when object is real or virtual and image is virtual	(C) 
IV. In concave mirror when object is virtual and image is real	(D) 

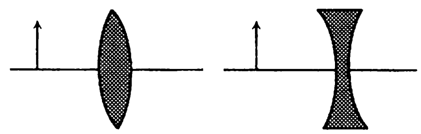
7. Match the following

Column-I	**Column-II**
I. Concave mirror, real object	(A) Real image
II. Convex mirror, real object	(B) Virtual image
III. Concave lens, real object	(C) Magnified image
IV. Convex lens, real object	(D) Diminished image

8. A real object is kept in front of a lens. The object is a linear extended object with its length perpendicular to the optic axis of lens. With reference to different cases of image formation by lenses, match column-I with column-II.

Column-I	**Column-II**
I. The image has a magnification – 2.5	(A) Image is virtual
II. Magnification of the image is + 0.5	(B) Image is real
III. Length of image is the same as that of object	(C) Power of lens is positive
IV. Length of image is four times the length of the object	(D) Power of lens is negative

9. Figure A shows a lens X and figure B shows another lens Y. In each case, a real object is kept in front of the lens. The object is a linear extended object with its length perpendicular to the optic axis of the lens. With regard to image formation by lenses, match column-I with column-II.



(A) (B)

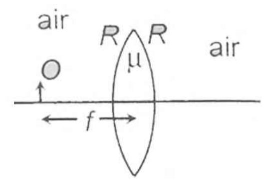
Column-I	**Column-II**
I. In figure A, assume that refractive index of lens relative to the surrounding is greater than 1	(A) The lens shows divergent behaviour
II. In figure A, assume that refractive index of lens relative to the surrounding is less than 1	(B) The lens shows convergent behaviour
III. In figure B, assume that refractive index of lens relative to the surrounding is less than 1	(C) The image formed cannot be real
IV. In figure B, assume that refractive index of the lens relative to the surrounding is less than 1	(D) The image formed cannot be virtual

10. Match the following :

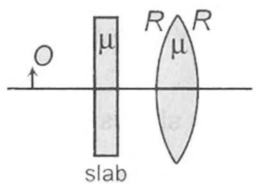
Column-I	**Column-II**
I. Object placed between optic center and 1st principle focus in a diverging lens	(A) Image is inverted
II. Object placed between optic center and 1st principle focus of a converging lens	(B) Image is Erect
III. Object placed between optic center and 2nd principle focus of a diverging lens	(C) Image is of greater size than the object
IV. Object placed between optic center and 2nd principle focus of a converging lens	(D) Image is of smaller size than the object

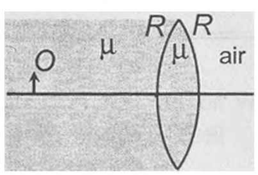
11. An object O (real) is placed at focus of an equi-biconvex lens as shown in the figure. The refractive index of lens is $\mu = 1.5$ and the radius of curvature of either surface of lens is R. The lens is surrounded by air.

In each statement of Column-I some changes are made to situation given above and information regarding final image formed as a result is given in Column-II. The distance between the lens and object is unchanged in all statements of Column-I. Match the statements in Column-I with resulting image in Column-II.

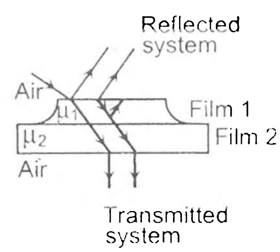


Column-I	**Column-II**
I. If the refractive index of the lens is doubled (that is, made 2μ) then	(A) Final image is real
II. If the radius of curvature is doubled (that is, made 2R) then	(B) Final image is virtual
III. If a glass slab of refractive index $\mu = 1.5$ is introduced between the object and lens as shown, then	(C) Final image becomes smaller in size in comparison to size of image before the change was made
IV. If the left side of lens is filled with a medium of refractive index $\mu = 1.5$ as shown, then	(D) Final image is of the same size as that of the object





12. For the situation shown in the figure match the entries of Column-I with Column-II.

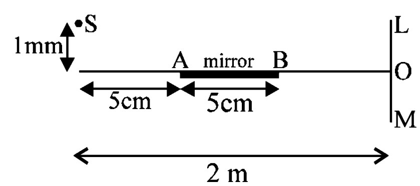


Column-I	**Column-II**
I. $\mu_1 = \mu_2$	(A) Film 1 appears shiny from the reflected system
II. $\mu_1 > \mu_2$	(B) Film 1 appears dark from the reflected system
III. $\mu_1 < \mu_2$	(C) Film 1 appears shiny from the transmitted system
IV. $\mu_1 \neq \mu_2$	(D) Film 1 appears dark from the transmitted system

13. Light rays are incident on devices which may cause either reflection or refraction or both. The nature of the incident light and the devices are described in Column I. Some possible results of this on the rays are given in Column II.

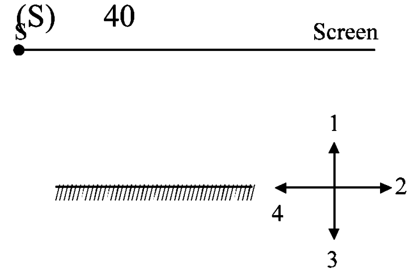
Column-I	**Column-II**
I. A ray of white light is incident on one face of an equilateral glass prism	(A) Divergent beam
II. A ray of white light is incident at an angle on a thick glass sheet	(B) Total internal reflection
III. A ray of white light passes from an optically denser medium to an optically rarer medium	(C) Lateral shift
IV. A parallel beam of monochromatic light passes symmetrically through a glass lens	(D) Dispersion

14. The arrangement of Lloyd's mirror experiment is shown in the figure. S is a point source of frequency 6×10^{14} Hz. A and B are the two ends of a mirror placed horizontally, and LOM represents the screen.



Column I	***Column II***
(a) The minimum height, above point O, at which fringes will be formed is (in mm)	(P) 39
(b) The maximum height, above point O, at which fringes will be formed is (in mm)	(Q) 19
(c) Total no. of fringes are	(R) 500
(d) Fringe width (in μm) is	(S) 40

15. A plane mirror is arranged parallel to the screen and a point source S is kept on the screen as shown in the figure. The mirror is moved towards the screen. Corresponding to the direction of motion of the mirror match the column. (Consider the moment, the motion starts)

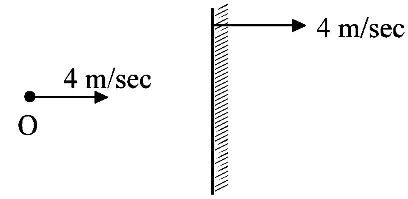


Column - I	**Column- II**
(A) Direction of motion of mirror along 1	(p) length of spot increases
(B) Direction of motion of mirror along 2	(q) length of spot decreases
(C) Direction of motion of mirror along 3	(r) length of spot does not change
(D) Direction of motion of mirror along 4	(s) brightness of spot increases

16. **Column 1** | **Column 2**

	Column 1		Column 2
(i)	Conversing system	(A)	convex lens
(ii)	Diversing system	(B)	concave lens
(iii)	Virtual Image is formed by	(C)	concave mirror
(iv)	Magnification < 1 is possible with	(D)	convex mirror

17. A point object is moving with velocity 4 m/sec towards the plane mirror which is moving with 4 m/sec away from the object as shown in the figure.

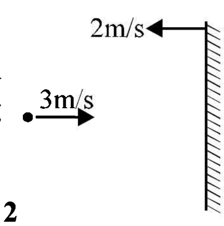


	Column I		Column - II
(A)	speed of image w.r.t. ground	(p)	8 m/sec towards right
(B)	speed of image w.r.t. mirror	(q)	0
(C)	speed of image w.r.t. object	(r)	4 m/sec towards left
(D)	speed of mirror w.r.t. object	(s)	4 m/sec towards right

18. In Young's Double Slit Experiment if the distance between slits is d, distance between slit and the screen is D, wavelength of light used is λ. Then match Column 1 and Column 2.

	Column 1		Column 2
(i)	condition for bright fringe	(A)	$\frac{D\lambda}{2d}$
(ii)	condition for dark fringe	(B)	$\frac{D}{d}(\mu-1)t$
(iii)	displacement of fringe when glass plate of thickness t is placed	(C)	Path difference = $n\lambda$
(iv)	Distance between central maxima and first dark fringe when glass plate of thickness t is used.	(D)	Path difference = $(2n-1)\lambda/2$

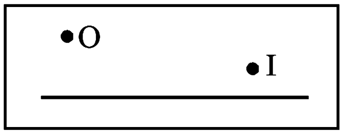
19. A point object is placed in front of a plane mirror as shown and moving with velocity 3 m/s towards the mirror. Mirror is moving with speed 2 m/s towards the object, then :

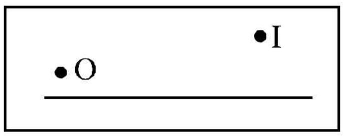


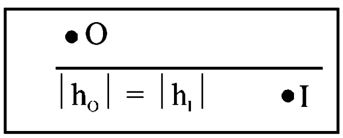
	Column 1		Column 2
(i)	speed of image w.r.t. ground	(A)	10 m/s
(ii)	speed of image w.r.t. mirror	(B)	5 m/s
(iii)	speed of image w.r.t. object	(C)	14 m/s
(iv)	speed of mirror w.r.t. object	(D)	7 m/s

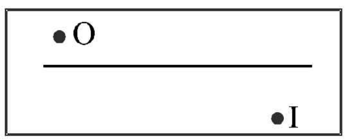
20. **Column I** shows four diagrams of real object point O, image point I and principal axis(optical axis). Select the proper optical system from **Column II,** which can produce the required image. (Image may be real or virtual)

Column I

(A) 

(B) 

(C) 

(D) 

Column II

(p) Diverging lens

(q) Converging lens

(r) Concave mirror

(s) Convex mirror

21. Regarding power of an optical instrument, match the following table.

	Column (I)		**Column (II)**
(A)	Convex mirror	(p)	Positive
(B)	Concave lens	(q)	Negative
(C)	Concave mirror	(r)	Infinite
(D)	Plane glass plate	(s)	Zero

22. There is an equi biconvex lens, made of refractive index 3/2 and radius of curvature 60 cm.

	Column 1		**Column 2**
(A)	Its focal length is	(p)	Infinite
(B)	When placed in a liquid of $\mu = 5/2$, its focal length is	(q)	less than 60 cm
(C)	When placed in a liquid of $\mu = 5/4$, its focal length is	(r)	more than 60 cm
(D)	When placed in a liquid of $\mu = 3/2$, its focal length is	(s)	60 cm

23. For a real object, match the following. m is lateral magnification, f is the focal length, u is the object distance

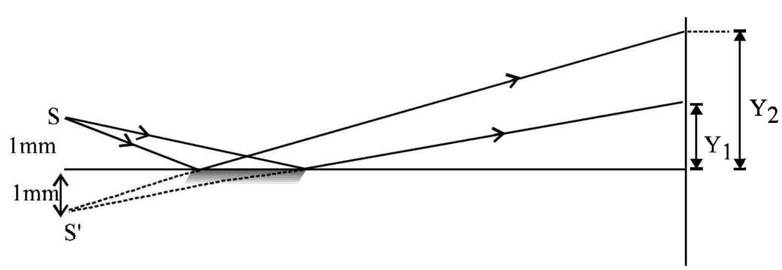
Column 1		**Column 2**	
(A)	Convex mirror	(p)	$|m| > 1$
(B)	Concave mirror	(q)	$|m| < 1$
(C)	Plane mirror	(r)	$|m| = 1$
(D)	Transparent slab	(s)	$m = \dfrac{f}{f-u}$

24. Angle between two mirrors 'θ' and location of object is given in Column I and some possible number of images are given in Column II. Match the two columns.

Column I		**Column - II**	
(A)	$\theta = 30°$ and object is on bisector	(p)	9
(B)	$\theta = 36°$ and object is on bisector	(q)	11
(C)	$\theta = 22.5°$ and object is on bisector	(r)	7
(D)	$\theta = 50°$ and object is on bisector	(s)	15

ANSWER KEY

1. (I) - (C); (II) - (A), (B); (III) - (C), (D); (IV) - (A), (B)
2. (I) - (A); (II) - (A), (B), (D); (III) - (B), (D); (IV) - (C), (D)
3. (I) - (B), (C); (II) - (A); (III) - (D); (IV) - (C)
4. (I) - (B), (C); (II) - (B), (C); (III) - (B), (D); (IV) - (C), (D)
5. (I) - (B), (C), (D); (II) - (A), (C); (III) - (B), (C), (D); (IV) - (A), (C)
6. (I) - (A), (B); (II) - (D); (III) - (A), (B); (IV) - (C)
7. (I) - (A), (B), (C), (D); (II) - (B), (D); (III) - (B), (D); (IV) - (A), (B), (C), (D)
8. (I) - (B), (C); (II) - (A), (D); (III) - (B), (C); (IV) - (A), (B), (C)
9. (I) - (B); (II) - (A), (C); (III) - (A), (C); (IV) - (B)
10. (I) - (B), (D); (II) - (B), (C); (III) - (B), (D); (IV) - (B), (C)
11. (I) - (A), (C); (II) - (B), (C); (III) - (B), (C); (IV) - (B), (C)
12. (I) - (B), (C); (II) - (B), (D); (III) - (A), (D); (IV) - (A), (B), (C), (D)
13. (I) - (A), (B), (D); (II) - (C); (III) - (A), (B), (D); (IV) - (A), (B)
14. (a)-(Q), (b)-(P), (c)-(S), (d)-(R)



Using geometry $Y_1 = 190 \times \dfrac{0.1}{10} = 1.9$ cm

$$Y_2 = 195 \times \frac{0.1}{5} = 3.9 \text{ cm}$$

The region in which interference occurs is $Y_2 - Y_1 = 2\text{cm}$

Fringe width $= \omega = \dfrac{\lambda D}{2d} = 0.5$ mm

no. of fringes $= \dfrac{Y_2 - Y_1}{\omega} = \dfrac{20}{0.5} = 40.$

15. (A)- r,s (B) - r (C) - r (D)- r, s

16. (i) - ABC ; (ii) - ABD ; (iii) - ABCD ; (iv) - ABCD

17. (A)-(s), (B)-(q), (C)-(q), (D)-(q)

18. (i) - C ; (ii) - D ; (iii) - B ; (iv) - A

19. (i) - D ; (ii) - B ; (iii) - A ; (iv) - B

20. $A \to p, s$ $\quad B \to q, r$ $\quad C \to q$ $\quad D \to q, r$

21. $A \to q$ $\quad B \to q$ $\quad C \to p$ $\quad D \to s$

22. (A)-(s), (B)-(q), (C)-(r), (D)-(p, r)

23. (A)-(q,s), (B)-(p,q, r,s), (C)-(r), (D)-(r)

24. (A)-(q), (B)-(p), (C)-(s), (D)-(r)

ASSERTION AND REASON

Each question contains STATEMENT -1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B) , (C) and (D) out of which **ONLY ONE** is correct.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1.

(B) Statement-1 is True, Statement -2 is True; Statement -2 **is not** a correct explanation for Statement -1.

(C) Statement -1 is True, Statement -2 is False.

(D) Statement -1 is False, Statement -2 is True.

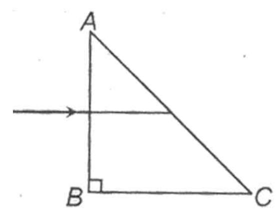
1. **STATEMENT-1 :** Although the surfaces of goggle lens are curved, it does not have any power.

STATEMENT-2 : In case of goggles, both the curved surfaces have equal radii of curvature and have center of curvature on the same side.

(A) A (B) B (C) C (D) D

2. **STATEMENT-1 :** White light is incident on face AB of an isosceles right angled prism as shown. Colours, for which refractive index of material of prism is more than 1.414, will be able to emerge from the face AC.

STATEMENT-2 : Total internal reflection cannot happen for light travelling from a rarer medium to a denser medium.



(A) A

(B) B

(C) C

(D) D

3. **STATEMENT-1 :** Monochromatic light is incident on a prism at an angle of incidence 60°. Refracting angle of prism is 60° and its refractive index for given light is 1.732. In this situation, deviation suffered by light will be minimum.

STATEMENT-2 : Deviation continuously decreases as the angle of incidence is continuously increased.

(A) A (B) B (C) C (D) D

4. **STATEMENT-1 :** A lens L (shown in the figure), kept in surrounding medium X, has a power + 10 D. If the same lens is kept in a surrounding medium Y, its power is found to be + 12.5 D. Also the same lens is placed in a surrounding medium Z, its power is now measured to be – 3.5 D ; then $\mu_Z > \mu_X > \mu_Y$.

STATEMENT-2 : In different surroundings, power of a given lens has different values but the same sign.

(A) A (B) B

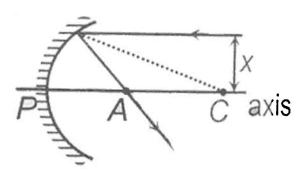
(C) C (D) D

5. **STATEMENT-1 :** When a thin transparent sheet is placed in front of one of the slits in Young's Double Slit Experiment, the fringe of zero order shifts to some other position.
STATEMENT-2 : On placing a thin transparent sheet in front of a slit, the position of zero path difference on the screen changes.
(A) A (B) B (C) C (D) D

6. **STATEMENT-1 :** Image formed by a concave lens is not always virtual.
STATEMENT-2 : Image formed by a lens is real if the image is formed in the direction of ray of light with respect to the lens.
(A) A (B) B (C) C (D) D

7. **STATEMENT-1 :** Light from an object falls an a concave mirror forming a real image of the object. If both the object and mirror are immersed in water, there is no change in the position of the image.
STATEMENT-2 : The formation of image by reflection does not depend on surrounding medium, so there is no change in the position of the image.
(A) A (B) B (C) C (D) D

8. **STATEMENT-1 :** As the distance x of a parallel ray from axis increases, angle of incidence decreasess.
STATEMENT-2 : As x increases, the distance of point of intersection of the reflected ray with axis from pole decreases.



(A) A
(B) B
(C) C
(D) D

9. **STATEMENT-1 :** A concave mirror of focal length f in air is used in a medium of refractive index 2. The focal length of the mirror in the medium becomes double.
STATEMENT-2 : The radius of curvature of a mirror is double of the focal length.
(A) A (B) B (C) C (D) D

10. **STATEMENT-1 :** A plano-convex lens is silvered on plane surface. It can act as a diverging mirror.
STATEMENT-2 : Focal length of concave mirror is independent of medium.
(A) A (B) B (C) C (D) D

11. **STATEMENT-1 :** If the phase difference between the waves emerging from two slits in Young's Double Slit Experiment is π-radian, the central point will have maximum intensity.
STATEMENT-2 : Phase difference is equal to $\frac{2\pi}{\lambda}$ times the path difference.
(A) A (B) B (C) C (D) D

13. **STATEMENT-1 :** In a Young's Double Slit Experiment, the two slits are at a distance d apart. Interference pattern is observed on a screen at distance D from the slits. At a point on the screen which is directly opposite to one of the slits, a dark fringe is observed. Then the wavelength of wave is proportional to square of distance between the two slits.
STATEMENT-2 : The light rays coming from two slits do not interfere at the screen.
(A) A (B) B (C) C (D) D

14. **STATEMENT-1 :** Light can show interference.
STATEMENT-2 : Light can show diffraction.

15. **STATEMENT-1 :** For observing traffic at our back, we prefer to use a convex mirror.
STATEMENT-2 : A convex mirror has a more larger field of view than a plane mirror or concave mirror.

16. **STATEMENT-1 :** Spherical aberration is a defect of a spherical mirror, in which not all rays focus at a single point.
STATEMENT-2 : The laws of reflection are not valid for all rays.

17. **STATEMENT-1 :** while passing through a lens or prism, the phase difference between two waves does not change.
STATEMENT-2 : The optical path lengths of all rays are same.

18. **STATEMENT-1 :** A convex lens may be diverging.
STATEMENT-2 : The nature of a lens depends upon the refractive index of the material of lens and surrounding medium besides geometry.

19. **STATEMENT-1 :** The power of a thin lens does not depend upon the surrounding medium.
STATEMENT-2 : Power of a thin lens $= \frac{\mu}{f}$

20. **STATEMENT-1 :** If white light is used in YDSE, coloured fringes are obtained.
STATEMENT-2 : The fringe width is proportional to the wavelength (color) of the light.

21. **STATEMENT-1 :** When a sound wave in air is reflected from a wall, it does not suffer a phase change.
STATEMENT-2 : For sound waves, air is denser as compared to wall.

22. **STATEMENT-1 :** The two slits in a YDSE are illuminated by two different sodium lamps emitting light of same wavelength. No interference pattern will be observed.
STATEMENT-2 : Two independent light sources (except LASER) cannot be coherent.

23. **STATEMENT-1 :** A virtual image can be photographed.
STATEMENT-2 : Only a real image can be formed on a screen.

24. **STATEMENT-1 :** The focal length of a lens does not depend on the medium in which it is submerged.

STATEMENT-2 : $\frac{1}{f} = \frac{\mu_2 - \mu_1}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

25. **STATEMENT-1 :** The minimum slit separation d for interference to produce at least one maximum other than central maximum in YDSE is 3λ.
STATEMENT-2 : For a maximum, path difference = $n\lambda$. The maximum value of path difference = d, slit separation.

26. **STATEMENT-1 :** In calculating the disturbance produced by a pair of superimposed incoherent wave trains, you can add their intensities.

STATEMENT-2 : $I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$. The average value of $\cos\delta = 0$ for incoherent waves.

27. **STATEMENT-1 :** A single lens cannot be free from chromatic aberration.
STATEMENT-2 : When light passes through single lens, dispersion must occur.

28. **STATEMENT-1 :** Superposition takes place only between those waves emitted by coherent sources.
STATEMENT-2 : All coherent sources emit energy in proper order.

29. **STATEMENT-1 :** When a glass prism is immersed in water, the deviation caused by the prism decreases.
STATEMENT-2 : Refractive index of glass prism relative to water is less than air.

30. **STATEMENT-1 :** An air bubble in water shines.
STATEMENT-2 : When light is incident from water to air, total internal reflection takes place at outer surface of the bubble.

31. **STATEMENT-1 :** Thin films such as soap bubble or thin layer of oil spread on water show beautiful colors when illuminated by white light.
STATEMENT-2 : It is due to interference of Sun's light reflected from upper and lower surfaces of the film.

32. **STATEMENT-1 :** In a YDSE, central fringe is always a bright fringe.
STATEMENT-2 : If path difference at central fringe is zero, then it will be a bright fringe.

33. **STATEMENT-1 :** When white light passes through a prism, deviation of violet light is more than green light.
STATEMENT-2 : In a prism, average deviation is measured as deviation of yellow light.

34. **STATEMENT-1 :** A real object is kept on principle axis of mirror. Size of image measured is equal to size of the object. The mirror must be a plane mirror.
STATEMENT-2 : For a plane mirror magnification is unity.

35. **STATEMENT-1 :** Different colours travel with different speed in vacuum
STATEMENT-2 : Wavelength of light depends on refractive index of the medium.

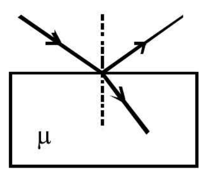
36. **STATEMENT-1 :** We can not produce a real image by plane or convex mirrors under any circumstances is.
STATEMENT-2 : The focal length of a convex mirror is always taken as positive.

37. **STATEMENT-1 :** A fish inside a pond will see a person standing outside taller than he actually.
STATEMENT-2 : Light bends away from the normal as it enters water from air.

38. **STATEMENT-1 :** Hollow prism forms no spectra as a solid equilateral prism of glass.
STATEMENT-2 : Neglecting the thickness of hollow glass surface. The media is same. So dispersion is not to take place.

39. **STATEMENT-1 :** In a Young's Double Slit Experiment, if whole set up is immersed in a liquid, then fringe width is decreased.
STATEMENT-2 : Wavelength of light entering in a liquid increases.

40. **STATEMENT-1 :** A light ray is incident on a glass slab. Some portion of it is reflected and some is refracted. Refracted and reflected rays are always perpendicular to each other.
STATEMENT-2 : Angle of incidence is equal to angle of reflection.



41. **STATEMENT-1 :** The critical angle in case of total internal reflection depends on the pair of medium chosen.
STATEMENT-2 : The critical angle in case of total internal reflection is independent of pair of medium chosen.

42. **STATEMENT-1 :** If we increase the separation, between slits, then angular fringe width decreases.
STATEMENT-2 : Angular fringe width increases by increasing slit separation.

43. **STATEMENT-1 :** A concave mirror has f = 40 cm in air. It has f = 30 cm in water.
STATEMENT-2 : Focal length of mirror is independent of medium.

44. **STATEMENT-1 :** Air bubble in glass medium behaves as concave lens
STATEMENT-2 : Lens formula is $\frac{1}{f} = \left(\frac{\mu_{lens}}{\mu_{med}} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

45. **STATEMENT-1 :** Interference phenomena is based upon the conservation of energy principle.
STATEMENT-2 : All the bright fringes are of the same intensity in a YDSE.

46. **STATEMENT-1 :** Angular width of central maximum in a YDSE is independent of D i.e., distance between the source and the screen.
STATEMENT-2 : Fringe width of central maximum is double of the first maxima on the screen.

47. **STATEMENT-1 :** In a Young's Double Slit Experiment, interference pattern disappears when one of the slits is closed.
STATEMENT-2 : Interference occurs due to superimposition of light wave from two coherent sources.

48. **STATEMENT-1 :** Power of a lens depends on the nature of material of lens, medium in which it is placed and radii of curvature of its surface.
STATEMENT-2 : It follows the relation $p=\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

49. **STATEMENT-1 :** Law of reflection is applicable for all types of mirrors.
STATEMENT-2 : Rays which are parallel to principle, axis are known as paraxial rays.

50. **STATEMENT-1 :** In a Young's Double Slit Experiment, if intensity of each source is I_0, then minimum and maximum intensity is zero and $4I_0$ respectively.
STATEMENT-2 : In a Young's Double Slit Experiment, energy conservation is not followed.

51. **STATEMENT-1 :** Image formed by a convex mirror is always smaller in size.
STATEMENT-2 : It is always virtual.

52. **STATEMENT-1 :** Fringe width depends upon refractive index of the medium.
STATEMENT- 2 : Refractive index changes the optical path of ray of light forming fringe pattern.

53. **STATEMENT-1 :** The images formed by total internal reflections are much brighter than those formed by mirrors or lenses.
STATEMENT-2 : There is no loss of intensity in total internal reflection.

54. **STATEMENT-1 :** In a Young's experiment, the fringe width for dark fringes is same as that of the white fringes.
STATEMENT-2 : In a Young's Double Slit Experiment, the fringes are performed with a source of white light, then only black and bright fringes are observed.

55. **STATEMENT-1 :** The focal length of lens does not change when red light is replaced by blue light.
STATEMENT-2 : The focal length of lens depends on the colour of light used.

56. **STATEMENT-1 :** A convex lens of focal length f(μ = 1.5) behaves as a diverging lens when immersed in carbon di-sulphide of higher refractive index (μ = 1.65).
STATEMENT-2 : The focal length of lens does not depend on the colour of light used.

57. **STATEMENT-1 :** When a light wave travels from a rarer to a denser medium, it loses speed. The reduction in speed implies a reduction in energy carried by the light wave.
STATEMENT-2 : The energy of a wave is proportional to wave frequency.

58. **STATEMENT-1 :** In interference all the fringes are of same width.
STATEMENT-2 : In interference , fringe width is independent of position of fringe.

59. **STATEMENT-1 :** A red object appears dark in the yellow light.
STATEMENT-2 : Red colour is scattered less.

60. **STATEMENT-1 :** Stars twinkle while the planets do not.
STATEMENT-2 : The stars are much bigger in size than the planets.

61. **STATEMENT-1 :** The air bubble shines in water.
STATEMENT-2 : Air bubble in water shines due to refraction of light.

62. **STATEMENT-1 :** In a movie, ordinarily 24 frames are projected per second from one end to the other of the complete film.
STATEMENT-2 : The image formed on retina of eye is sustained upto 1/10 second after the removal of stimulus.

63. **STATEMENT-1 :** Blue colour of sky appears due to scattering of blue colour.
STATEMENT-2 : Blue colour has shortest wavelength in visible spectrum.

64. **STATEMENT-1 :** The setting sun appears to be red.
STATEMENT-2 : Scattering of light is directly proportional to the wavelength.

65. **STATEMENT-1 :** Different colours travel with different speed in vacuum.
STATEMENT-2 : Wavelength of light depends on refractive index of the medium.

66. **STATEMENT-1 :** Diamond glitters brillianlty.
STATEMENT-2 : Diamond does not absorb light.

67. **STATEMENT-1 :** A cloud in the sky generally appear to be whitish.
STATEMENT-2 : Diffraction due to cloud is efficient and is equal measure at all wavelength.

68. **STATEMENT-1 :** By roughening the surface of a glass sheet, its transparency can be reduced.
STATEMENT-2 : Glass sheet with rough surface absorbs more light.

69. **STATEMENT-1 :** The frequencies of incident, reflected and refracted beam of monochromatic light incident from one medium to another are same.
STATEMENT-2 : The incident, reflected and refracted rays are coplanar.

70. **STATEMENT-1 :** The refractive index of a prism depends only on the kind of glass of which it is made of and the colour of light.
STATEMENT-2 : The refractive index of a prism depends upon the refracting angle of the prism and the angle of minimum deviation.

71. **STATEMENT-1 :** Corpuscular theory fails in explaining the velocities of light in air and water.
STATEMENT-2 : According to Corpuscular theory, light should travel faster in denser medium than in rarer medium.

72. **STATEMENT-1 :** Thin films such as a soap bubble or a thin layer of oil on water show beautiful colours when illuminated by while light.
STATEMENT-2 : It happens due to interference of light reflected from the upper surface of the thin film.

73. **STATEMENT-1 :** Average energy in the interference pattern is the same as it would be if, there were no interference .
STATEMENT-2 : Interference is the only rare phenomenon in which law of conservation of energy does not hold good.

74. **STATEMENT-1 :** Geometrical optics can be regarded as the limiting case of wave optics.
STATEMENT-2 : When size of obstacle or opening is very large compared to the wavelength of light, then wave nature can be ignored and light can be assumed to be travelling in a straight line.

75. **STATEMENT-1 :** A single ray can't be isolated from a source however small it may be.
STATEMENT-2 : The concept of single ray is hypothetical.

76. **STATEMENT-1 :** Virtual images can be photographed.
STATEMENT-2 : Rays from virtual images are diverging.

77. **STATEMENT-1 :** For sustained interference sources must be coherent.
STATEMENT-2 : Two independent and identical sources can't act as coherent sources.

78. **STATEMENT-1 :** Virtual object can't be seen by human eye.
STATEMENT-2 : Virtual object is formed by converging rays.

79. **STATEMENT-1 :** A convex mirror is used as a rear view mirror.
STATEMENT-2 : Convex mirror always forms virtual, erect and diminished image.

80. **STATEMENT-1 :** The behavior of any lens depends on the surrounding medium.
STATEMENT-2 : A lens can be looked upon as a collection of small prism with varying prism angle.

81. **STATEMENT-1 :** Human eye can see virtual object.
STATEMENT-2 : Virtual object is formed by apparent intersection of incident rays.

82. **STATEMENT-1 :** Real image is formed by real intersection of reflection or refracted rays.
STATEMENT-2 : Real image can't be obtained on the screen.

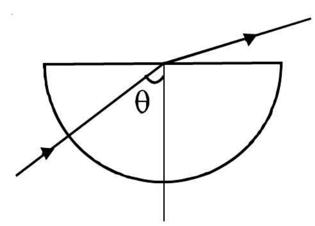
83. **STATEMENT-1 :** If a portion of lens of mirror is blocked or removed, then intensity of image reduces.
STATEMENT-2 : As every portion of lens or mirror forms image, hence blocking or removing a portion will result in intensity reduction.

84. **STATEMENT-1 :** Light is used to kill light in anti-reflecting films.
STATEMENT-2 : Interferences is destructive when path difference is odd multiple of half of the wavelength.

85. **STATEMENT-1 :** A rectangular glass slab produces no deviation and no dispersion.
STATEMENT-2 : Dispersive power of glass is zero.

86. **STATEMENT-1 :** Although the surfaces of goggles, lenses are curved, it does not have any power.
STATEMENT-2 : In case of goggle both the curvature and the center of curvature are on the same side.

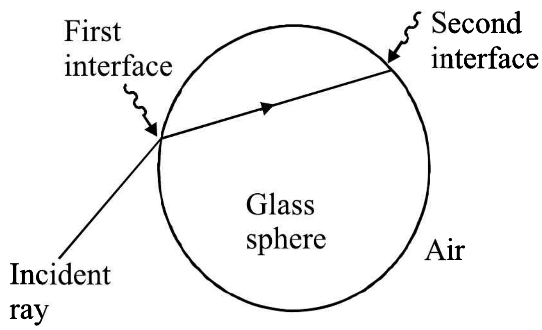
87. **STATEMENT-1 :** A beam of white light enters the curved surface of a semicircular piece of glass along the normal. The incoming beam is moved clockwise (so that the angle θ increases), such that the beam always enters along the normal to the curved side. Just before the refracted beam disappears, it becomes predominantly red.



STATEMENT-2 : The index of refraction for light at the red end of the visible spectrum is more than at the violet end.

88. **STATEMENT-1 :** There exist two angles of incidence for the same magnitude of deviation (except minimum deviation) by a prism kept in air.
STATEMENT-2 : In a prism kept in air, a ray is incident on the first surface and emerges out of the second surface. Now, if another ray is incident on the second surface (of prism) along the previous emergent ray. This principle is called principle of reversibility of light.

89. **STATEMENT-1 :** A ray is incident from outside on a glass sphere surrounded by air as shown. This ray may suffer total internal reflection at the second interface.



STATEMENT-2 : For a ray going from a denser to rarer medium, the ray may suffer total internal reflection.

90. **STATEMENT-1 :** Keeping a point object fixed, if a plane mirror is moved, the image will also move.
STATEMENT-2 : In case of a plane mirror, distance of object and its image is equal from any point on the mirror.

91. **STATEMENT-1 :** Lights of different colours travel with different speeds in vacuum.
STATEMENT-2 : Speed of light depends on the medium.

92. **STATEMENT-1 :** Dry sand appears bright.
STATEMENT-2 : Dry sand is smooth.

93. **STATEMENT-1 :** A beam of light rays has been reflected from a rough surface.
STATEMENT-2 : Amplitude of incident and reflected rays would be different.

ANSWER KEY

1.	A	2.	D	3.	C	4.	C	5.	A	6.	B
7.	A	8.	D	9.	D	10.	B	11.	D	13.	C
14.	B	15.	A	16.	C	17.	C	18.	A	19.	D
20.	D	21.	A	22.	A	23.	B	24.	D	25.	D
26.	A	27.	A	28.	D	29.	A	30.	A	31.	A
32.	D	33.	B	34.	D	35.	D	36.	D	37.	C
38.	A	39.	C	40.	D	41.	C	42.	C	43.	D
44.	A	45.	C	46.	C	47.	A	48.	A	49.	C
50.	C	51.	B	52.	B	53.	A	54.	C	55.	D
56.	B	57.	D	58.	A	59.	B	60.	B	61.	C
62.	A	63.	A	64.	C	65.	D	66.	B	67.	C
68.	C	69.	B	70.	C	71.	A	72.	C	73.	C
74.	A	75.	B	76.	A	77.	B	78.	A	79.	A
80.	B	81.	D	82.	C	83.	A	84.	A	85.	C
86.	A	87.	C	88.	A	89.	D	90.	D	91.	D
92.	A	93.	B								

HINT & SOLUTIONS

14. Conceptual.

15. The nature of the image formed due to a convex mirror does not change with changing distance of the object. It is always virtual, erect and smaller.

21. Since reflection is at a boundary, the other side of which is rarer there is no phase change.

23. The rays of light are diverging out from a virtual image. These can be easily converged onto the film of a concave lens by convergent action of its lens.

24. As can be seen from the expression of f, it depends upon the refractive index of the medium in which the lens is submerged.

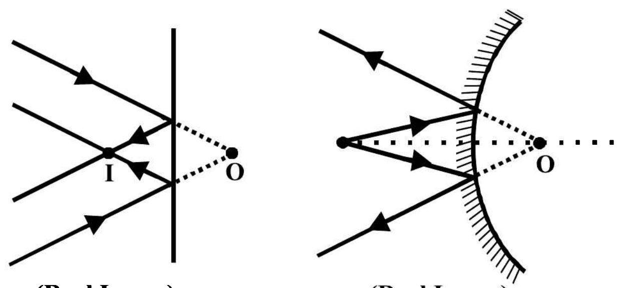
25. $\Delta x = d = n\lambda$

    For $n = 1$, $d = \lambda$ and we will have three maximum.

26. Average value of cos $\delta = t = \frac{\int_{t}^{t=T} \cos(\phi_1 - \phi_2)dt}{T} = 0$. Here ϕ_1 and ϕ_2 are constantly randomly, fluctuating phases of the two wave trains and integral is taken over a long time (relative to periods of the individual waves).

35. The velocity of light of different colours (all wavelength) is same in vacuum and $\mu \propto \frac{1}{\lambda}$

36. We can produce a real image by a plane or a convex mirror.



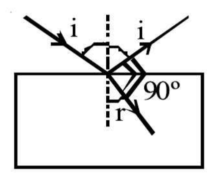
(Real Image) (Real Image)

Focal length of convex mirror is taken positive.

37. Since light bends towards normal on entering water from air.

39. $\beta' = \frac{\lambda_n D}{d}$; $\lambda_m = \frac{\lambda_{air}}{\mu}$ $\Rightarrow$ $\beta' = \frac{\lambda_{air} D}{d\,\mu}$; $\beta' = \frac{\beta}{\mu}$

40. $1 \times \sin i = \sin r = \mu \sin(90 - i)$
$\sin i = \mu \cos i$; $\tan i = \mu$
So refracted and reflected rays are perpendicular if $\tan i = \mu$.



41. $\theta_C = \sin^{-1}(1/\theta)$. and μ is taken relative, hence it depends on pair of medium chosen.

42. Angular fringe width is inversely proportional to separation of both slits.

44. For air bubble, glass medium is denser and $\frac{\mu_i}{\mu_{glass}} = \frac{1}{\left(3/2\right)}$ is < 1 value.

46. Angular width $\theta = \frac{\text{fringe width}}{D} \Rightarrow \theta = \frac{\lambda}{d}$

47. When one of the slits is closed, there appears general illumination from a single source. Interference does not take place.

48. Both statement I and statement II are true and correct explanation.

49. Rays which are near and parallel to the principal axis are known as paraxial rays.

50. In Young's Double Slit Experiment, energy remains conserved.

52. Wavelength in a medium of refractive index μ. $\lambda' = \frac{\lambda}{\mu}$ where λ is wavelength in air.

Fringe width $\theta = \frac{\lambda D}{d}$.

53. 100% of incident light is reflected back into the same medium, and there is no loss of intensity, while in reflection from mirrors and refraction from lenses, there is always some loss of intensity. Therefore, images formed by total internal reflection are much brighter than those formed by mirrors or lenses.

54. In Young's experiments, fringe width for dark and white fringes are same. While in Young's Double Slit Experiment, when a white light is used as a source, the central fringe is bright around which few colored fringes are observed on either side.

55. The focal length of a Lens is given by the formula $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

As $\mu_b > \mu_R \therefore f_b < f_r$. Therefore, focal length of lens decreases when red light is replaced by blue light.

56. Since $\mu = \frac{\mu_g}{\mu_{cs}} = \frac{1.5}{1.65} \angle 1$. From $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow f < 0$

Therefore the lens behaves as a diverging lens. Hence (B) is the correct option.

57. When a light wave travels from a rarer to a denser medium it loses speed, but energy carried by the wave does not depend on its speed.

58. As given in the expression, fringe width $\beta = \frac{\lambda D}{d}$, fringe width β is independent of 'n' (position).

COMPREHENSION TYPE

Direction : This section contains paragraphs. Based upon these paragraphs, multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE choice is correct.

PARAGRAPH (FOR QUESTIONS 1 to 4):

A Young's double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and interference pattern is observed on the screen at a distance 1.33 m from plane of slits. The wavelength in air is 6300 Å.

1. Calculate the fringe width.

(A) 0.63 mm (B) 1.26 mm (C) 1.67 mm (D) 2.2 mm

2. Find the distance of seventh bright fringe from third bright fringe lying on the same side of central bright fringe.

(A) 2.52 mm (B) 4.41 mm (C) 1.89 mm (D) 1.26 mm

3. One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to interchange the position of minima and maxima

(A) 2.575 μm (B) 1.575 μm (C) 3.275 μm (D) 4.185 μm

4. One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the fringe width.

(A) 0.63 mm (B) 1.26 mm (C) 1.67 mm (D) 2.2 mm

PARAGRAPH (FOR QUESTIONS 5 to 8):

In physics laboratory, two friends Peter and Paul are trying to calculate the focal lengths of two thin lenses. One is convex lens and the other is plano-convex lens. But only one optical bench is available. Peter started experiment with convex lens. He measures the distance between a screen and a light source lined up on the optical bench to be 120 cm. When Peter shifts the convex lens along the axis of optical bench, sharp images of the source is obtained at two lens positions. He also measures the ratio of these two magnifications to be 1 : 9. Meanwhile, Paul splits the plano-convex lens into two equal halves along its optical axis. One of the halves is shifted optical axis. Paul measures, the separation between object and image planes to be 1.8 m. The magnification of the image formed by one of the half lens is 2.

5. The focal length of the convex lens measured by Peter is

(A) 22.5 cm (B) 30 cm (C) 45 cm (D) None of these

6. Which image, as seen by Peter is brighter ?

(A) smaller (B) bigger

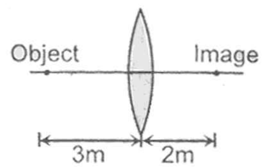
(C) both are equally bright (D) can not be judged

7. The focal length of the lens used by Paul is
(A) 50 cm (B) 40 cm (C) 60 cm (D) 70 cm

8. The separation between the two halves as measured by Paul is
(A) 50 cm (B) 40 cm (C) 60 cm (D) 70 cm

PARAGRAPH (FOR QUESTIONS 9 to 12):

An object is placed at a distance 3 meter from an equi-convex lens. Its image is formed at 2 m from the lens as shown in the figure.



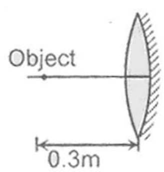


Fig.(a)

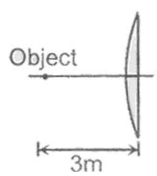


Fig.(b)

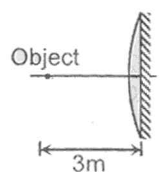


Fig.(c)

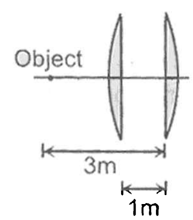


Fig.(d)

9. Find the position of image if one face the lens is silvered [figure (a).]
(A) 12 m from the lens towards right (B) 12 m from the lens towards left
(C) 6 m from the lens towards right (D) at infinity

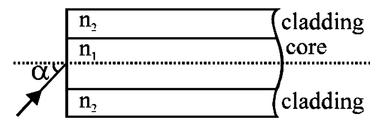
10. Find the position of image if lens is cut into two symmetrical plano convex lens and one of the plano convex lens is removed [figure b]
(A) 12 m from the lens towards right (B) 12 m from the lens towards left
(C) 6 m from the lens towards right (D) 6 m from the lens towards left

11. Find the position of image if lens is cut into two symmetrical plano-convex lens and one of the plano-convex lens is removed and the plane surface is silvered [figure (c)]
(A) 12 m from the lens towards right (B) 12 m from the lens towards left
(C) 2 m from the lens towards right (D) at infinity

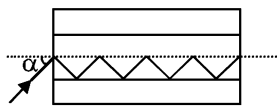
12. Find the position of image (approx.) if plano convex surfaces are displaced by 100 cm. [figure (d)]
(A) 2.0 m towards right of second lens (B) 2.94 m towards right of first lens
(C) 3.31 m towards right of second lens (D) 4.31 m towards right of first lens

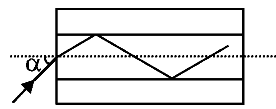
PARAGRAPH (FOR QUESTIONS 13 and 14):

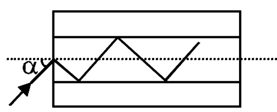
Figure shows the cross section of an optical fiber consisting of a solid cylindrical glass core of refractive index n_1 surrounded by a glass cladding of refractive index $n_2 < n_1$. Light enters the fiber at an angle α to the end face and propagates down the core of the fiber using total internal reflection at the boundary between the core and cladding.

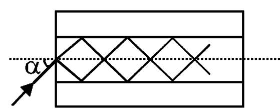


13. Which of the following shows the path of the light through the fiber ?

(A) 

(B) 

(C) 

(D) 

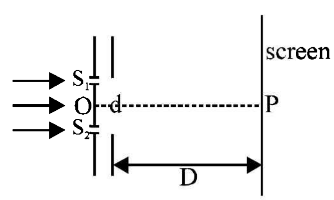
14. The maximum value of the angle α for which light propagates down the core is given by :

(A) $\sin^2\alpha = n_1^2 + n_2^2$ (B) $\sin^2\alpha = n_1^2 - n_2^2$

(C) $\cos^2\alpha = n_1^2 - n_2^2$ (D) $\cos^2\alpha = n_2^2 - n_1^2$

PARAGRAPH (FOR QUESTIONS 15 to 17):

A student is performing Young's double slit experiment. There are two slits S_1 and S_2. Separation between them is d. There is large screen at a distance D(D >>d) from the slits. The set-up is shown in the following figure . A parallel beam of light is incident upon it. A monochromatic light of wavelength λ is used. The initial phase difference between the two slits which behave as two coherent sources of light is zero. The intensity of light waves on the screen coming out of S_1 and S_2 are same and is I_0. In this situation, the principal maximum is formed at the point P. At the point on screen where principal maximum is formed, phase difference between two interfering waves will be zero.



15. Initially the distance of third minima from principal maxima will be :

(A) $\frac{3}{2}\frac{\lambda D}{d}$ (B) $\frac{3\lambda D}{d}$ (C) $\frac{5}{4}\frac{\lambda D}{d}$ (D) $\frac{5}{2}\frac{\lambda D}{d}$

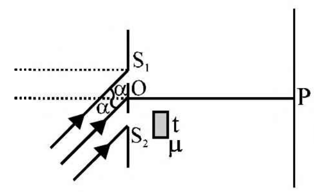
16. A glass slab of thickness t and refractive index μ is introduced before S_2, now P does not remain the point of principal maximum. Suppose principal maximum forms at a point P' on screen. Then PP' is equal to :

(A) $\frac{tD(\mu-1)}{d}$ (B) $\frac{tD(\mu-1)}{2d}$ (C) $\frac{D(\mu-1)}{t}$ (D) $\frac{D(\mu-1)}{d}$

17. Use statement given in previous question. Now parallel beam is incident at an angle α w.r.t. line OP, such that principal maximum again comes at point P. The figure is shown. The value of α is :

(A) $\sin^{-1}\frac{t(\mu-1)}{d}$ (B) $\cos^{-1}\frac{t(\mu-1)}{d}$

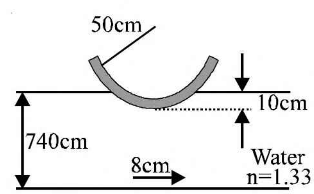
(C) $\sin^{-1}\frac{t(\mu-1)D}{d}$ (D) $\sin^{-1}\frac{tD}{d}$



PARAGRAPH (FOR QUESTIONS 18 and 19):

A thin hemispherical bowl of clear plastic floats on water in a tank. The radius of the bowl is 50 cm and the depth of the bowl in water is 10 cm. The depth of the water (R.I. = 1.33) in the tank is 740 cm.

An object 8 cm long is on the bottom of the tank directly below the bowl. The object is viewed from directly above the bowl. Ignore the refractive index of the plastic .



18. In figure, the position of the image below the water level, in cm, is closest to :

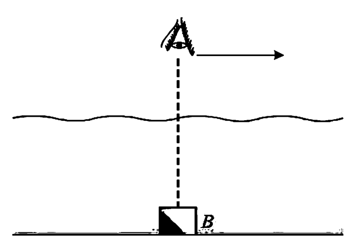
(A) 130 (B) 210 (C) 120 (D) 160

19. In figure, the size of the image, in cm, is closest to :

(A) 1.7 (B) 3.4 (C) 5.1 (D) 2.6

PARAGRAPH (FOR QUESTIONS 20 TO 24):

" Rakesh has a red coloured box and a blue coloured box. One by one he puts the box in a large tub filled with water at position B shown in figure. Initially he looks at the box from the normal position as shown in figure. Now he starts moving his eye away from this normal in horizontal direction keeps looking at the box. "



20. When Rakesh's eyes are along normal he studied the apparent depth of the two boxes blue and red and found :
(A) apparent depth of blue box is more
(B) apparent depth of red box is more
(C) apparent depth of both boxes equal
(D) we cannot compare the apparent depths of the two boxes as these are of different colours

21. During observation Rakesh will find that :
(A) blue box will appear as slightly green and red box will appear colourless
(B) blue box will appear reddish and red box will appear bluish
(C) blue box will appear yellowish and red box will appear greenish
(D) blue box will appear blue and red box will appear red

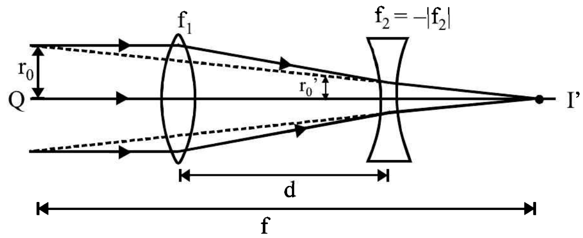
22. As Rakesh's eye is shifted to right, he 'll find :
(A) apparent depth of the box will remain same and there will be a horizontal shift in position of box image.
(B) apparent depth of the box will remain same and there will be no horizontal shift in position of box image.
(C) apparent depth of box will also change and there is a horizontal shift in the position of box image
(D) apparent depth of object will only change and there is no horizontal shift in the position of box image

23. When Rakesh's eye is in normal position and if the temperature of water in the tub is increased
(A) there will be no effect on apparent depth of boxes
(B) apparent depth of both boxes will decrease
(C) apparent depth of both boxes will increase
(D) apparent depth of blue box will increase and that of red box will decrease

24. When Rakesh's eye is moving away in horizontal direction and keeps looking at the box :
(A) after sometime the box will disappear
(B) during motion of eye apparent depth continuously decreases
(C) during motion of eye apparent depth first increases then decreases
(D) none of these

PARAGRAPH (FOR QUESTIONS 25 to 27):

The following figure shows a simple version of a zoom. The converging lens has a focal length f_1 and the diverging lens has focal length $f_2 = -|f_2|$. The two lens are separated by a variable distance d that is always less than f_1, also the magnitude of the focal length of the diverging lens satisfies the inequality $|f_2| > (f_1 - d)$.



If the rays that emerge from the diverging lens and reach the final image point are extended backward to the left of the diverging lens, they will eventually expand to the original radius r_0 at the same point Q. To determine the effective focal length of the combination lens consider a bundle of parallel rays of radius r_0 entering the emerging lens.

25. At the point where ray enters the diverging lens, the radius of the ray bundle decreases to

(A) $r_0' = \left(\dfrac{f_1 - d}{f_1}\right) r_0$ (B) $r_0 = \left(\dfrac{f_1 - d}{f_1}\right) r_0'$ (C) $r_0' = \left(\dfrac{f_1 - f_2}{f_1}\right) r_0$ (D) $r_0' = r_0$

26. To the right of the diverging lens the final image I' is formed at a distance S_2' given by

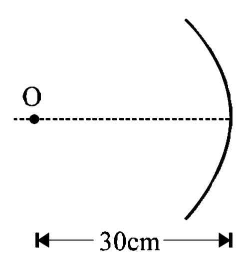
(A) $\dfrac{(f_1 - f_2)d}{f_1 - f_2 + d}$ (B) $\dfrac{(f_1 - d)\ f_2}{|f_2| + d - f_1}$ (C) $\dfrac{f_1 - f_2 + d}{f_1 - f_2}$ (D) $\dfrac{(d - f_1)f_2}{f_1 - f_2 - d}$

27. The effective focal length f is given by

(A) $\dfrac{f_1 |f_2|}{|f_2| - f_1 + d}$ (B) $\dfrac{f_1 f_2}{f_1 - f_2 - d}$ (C) $\dfrac{f_1}{f_2 - f_1 + d}$ (D) $\dfrac{f_2}{f_1 + f_2 + d}$

PARAGRAPH (FOR QUESTIONS 28 to 30):

An object is present on the principal axis of a concave mirror at a distance 30 cm from it. Focal length of mirror is 20 cm.



28. Image formed by mirror is

(A) At a distance 60 cm in front of mirror

(B) At a distance 60 cm behind the mirror

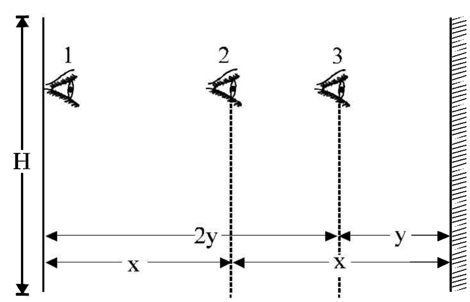
(C) At a distance 12 cm in front of mirror
(D) At a distance 12 cm behind the mirror.

29. If object starts moving with 2 cms^{-1} along principal axis towards the mirror then,
(A) image starts moving with 8 cms^{-1} away from the mirror
(B) image starts moving with 8 cms^{-1} towards the mirror
(C) image starts moving with 4 cms^{-1} towards the mirror
(D) image starts moving with 4 cms^{-1} away from the mirror

30. If object starts moving with 2 cms^{-1} perpendicular to principal axis above the principal axis then,
(A) image moves with velocity 4 cms^{-1} below the principal axis
(B) image moves with velocity 4 cms^{-1} above the principal axis
(C) image moves with velocity 8 cms^{-1} below the principal axis
(D) image moves with velocity 8 cms^{-1} above the principal axis

PARAGRAPH (FOR QUESTIONS 31 to 33):

A man is standing in a room and a plane mirror is fixed on the wall in front of him. The height of the wall is H. Man has to see the complete image of wall behind him at a time.



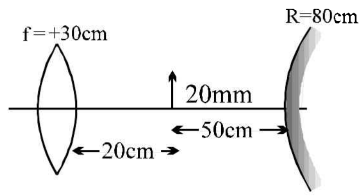
31. When the man is standing near wall i.e. at position 1, the minimum length of the mirror required is
(A) H (B) H/2 (C) H/3 (D) H/4

32. When the man is standing at the middle of the room i.e. at position 2, the minimum length of the mirror required is
(A) H (B) H/2 (C) H/3 (D) H/4

33. When the man is at position 3, the minimum length of the mirror required is
(A) H (B) H/2 (C) H/3 (D) H/4

PARAGRAPH (FOR QUESTIONS 34 and 35):

An optical system comprises in turn, from left to right : an observer, a lens of focal length + 30 cm, an object 20 mm high, and a convex mirror of radius 80 cm. The object is between the lens and the mirror, 20 cm from the lens and 50 cm from the mirror. The observer views the image formed first by reflection and then by refraction



34. In figure, the character of the intermediate image formed by the mirror, with respect to the erect object, is

(A) virtual and diminished (B) real and erect
(C) indeterminate (D) virtual and inverted

35. In figure, the position of the final image, measured from the mirror, in cm, is closest to

(A) 90 (B) 138 (C) 114 (D) 126

ANSWER KEY

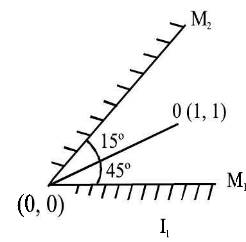
1.	A	2.	A	3.	B	4.	A	5.	A	6.	A
7.	B	8.	C	9.	D	10.	A	11.	C	12.	A
13.	B	14.	B	15.	D	16.	A	17.	A	18.	A
19.	A	20.	B	21.	D	22.	C	23.	C	24.	B
25.	A	26.	B	27.	A	28.	A	29.	A	30.	A
31.	B	32.	C	33.	D	34.	A	35.	C		

PRACTICE QUESTIONS

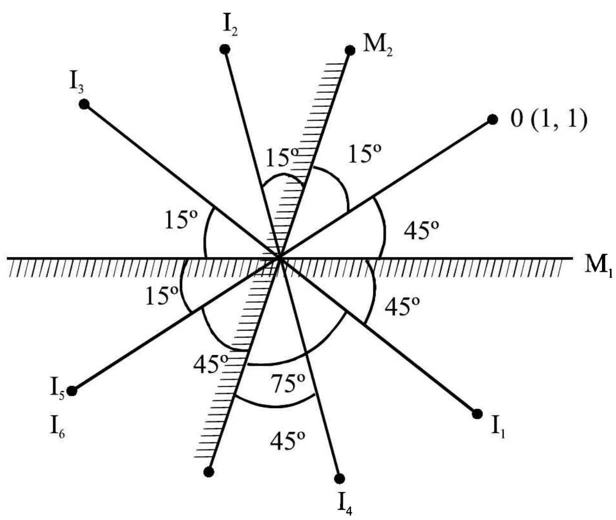
Subjective Type (Fully Solved)—Level I

1. Two flat mirrors have reflecting surfaces facing each other, an edge of one mirror in contact with the edge of the other, so that the angle formed between the mirrors is 60^0. Find all the angular positions of the image with respect to x-axis. Take the case when the point object is between the mirrors at (1, 1), Point of intersection is (0 ,0) and 1st mirror is along x axis.

Sol. All images lie on a circle because distance of each image is equal from origin.



Method 1:



(I_5, I_6 coincide)
No. of images = 5.

Method : 2

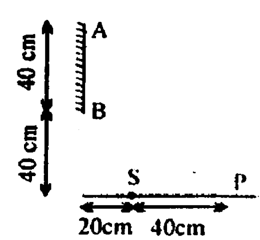
Find the initial angular positions of the images when the object suffers reflection for the first time from mirrors M_1 & M_2. Now go on adding θ (θ is the angle between the two mirrors) till the angular position obtained exceeds 180°.

Image formed by Mirror M_1 (angles measured from the mirror M_1.)		Image formed by Mirror M_2 (angles measured from the mirror M_2.)
45°	+60°	15°
75°	+60°	105°
165°	+60°	135°
195°		225°
Stop because next angle will be more than 180° or equal to 180°		Stop because next angle will be more than 180° or equal to 180°

To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors.
If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide.
Here last angles made by the mirrors + the angle between the mirrors = 165° + 135° + 60° = 360°.
Therefore in this case the last images coincide.
Therefore the number of images
= number of images formed by mirror M_1
+ number of images formed by mirror M_2 -1
(as the last images coincide)= 3 + 3 –1 = 5.

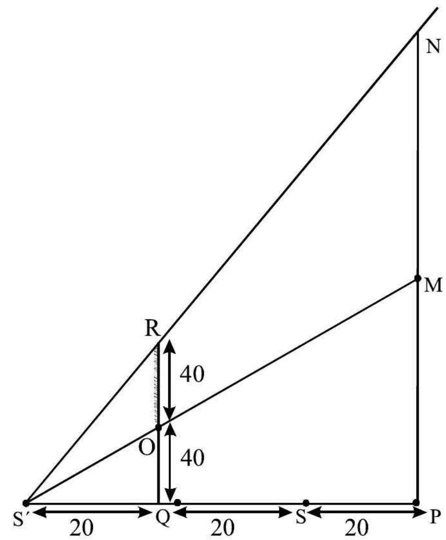
Ans. **(i) 75°, (ii) 165°, (iii) 195°, (iv) 285°, (v) 315°**

2. **In the figure, AB is a plane mirror of length 40 cm placed at a height 40 cm from the ground. There is a light source S at a point on the ground. Find the minimum and the maximum height of a man (eye height) required to see the image of the source if he is standing at a point P on ground as shown in figure.**



Sol. Let S′ be the image of S on the plane mirror AB, PM the minimum height of the man & PN the maximum height of the man in order to see the image S′.

Drawing field of view of image of S let M be the point of lower limit, N be the upper limit by similarity of Δ SOQ and ΔSPM



$$\frac{S'Q}{OQ} = \frac{S'P}{PM} \Rightarrow \frac{20}{40} = \frac{80}{PM} \quad \Rightarrow PM = 160 \text{ cm}$$

Also Δ S'QR $\approx$ Δ S'PN

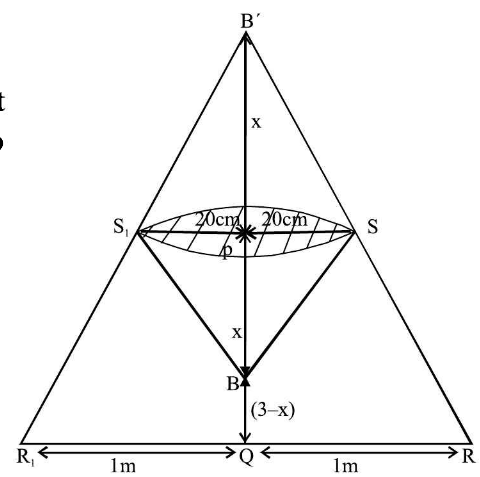
$$\frac{S'Q}{QR} = \frac{S'P}{PN} \Rightarrow \frac{20}{80} = \frac{80}{PN} \Rightarrow PN = 320 \text{ cm}$$

Ans. **160 cm ; 320 cm**

3. **A plane mirror of circular shape with radius r = 20 cm is fixed to the ceiling. A bulb is to be placed on the axis of the mirror. A circular area of radius R = 1 m on the floor is to be illuminated after reflection of light from the mirror. The height of the room is 3 m. What should be the maximum distance from the centre of the mirror and the bulb so that the required area is illuminated ?**

Sol. Let B' represent the image of bulb B in mirror

For complete illumination of region on ground it must be in the field of view of image of the bulb



Since Δ B′PS ~ Δ B′QR

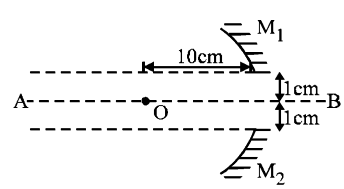
$$\frac{x}{20} = \frac{x+300}{100} \quad \text{(Similar triangles)}$$

$\Rightarrow$ $100x = 20x + 6000 \Rightarrow 80x = 6000$

$$x = \frac{6000}{80} \text{ cm} \Rightarrow \frac{6}{8} \text{ m} = 0.75 \text{ m}$$

Ans. **75 cm**

4. **A concave mirror of focal length 20 cm is cut into two parts from the middle and these two parts are moved perpendicularly by a distance 1 cm from the previous principal axis AB. Find the distance between the images formed by the two parts ?**



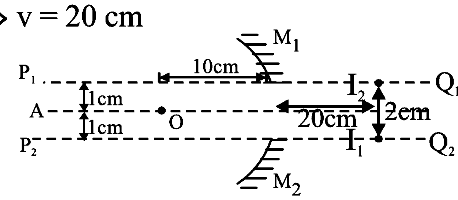
Sol. Let P_1Q_1 = principal axis for the mirror M_1 & P_2Q_2 = principle axis for the mirror M_2

(I_1 = Image formed by M_1, I_2 = Image formed by M_2)

Height of object and image are major with respect to principal axis of respective mirror.

Applying convention we have, f = – 20 cm (concave), u = – 10 cm

and by mirror formula $\Rightarrow \dfrac{1}{-10}+\dfrac{1}{v}=\dfrac{1}{-20} \quad \Rightarrow v = 20$ cm



$m=\dfrac{-v}{u}=\dfrac{-(20)}{-10}=2=\dfrac{h_i}{h_o} \Rightarrow h_i = 2$ cm

(+ve magnification means image & object are on the same side w.r.t. principal axis.)

h_{I_1} = 2 cm(↓) (2 cm below the principal axis P_1Q_1)

h_{I_2} = 2 cm(↑) (2 cm above the principal axis P_2Q_2) $\Rightarrow I_1I_2$ = 2 cm **[Ans. 2 cm]**

5. **A balloon is rising up along the axis of a concave mirror of radius of curvature 20 m. A ball is dropped from the balloon at a height 15 m from the mirror when the balloon has a velocity 20 m/s. Find the speed of the image of the ball formed by concave mirror after 4 seconds? [Take : g=10 m/s²]**

Sol. This is a kinematics approach problem. At first find the final velocity of the ball after 4 seconds and also the final position of the ball at this time. Then apply the formula for velocity calculation.

After t = 4 seconds ; use the equation of kinematics :

v= 20 – 10 x 4 (v = u + at) = – 20 m/s (20m/s ↓)

(S = 20 x 4 -1/2 x 10 x 16= 0 (S = ut +½ at²)

$(V_{I/m})_\perp = -(m)^2(V_{0/m})_\perp$ & $(V_{I/G})_\perp = -m^2(V_{0/G})_\perp$ (Since, $v_m = 0$)

Magnification, $m=\dfrac{f}{f-u}=\dfrac{-10}{-10-(-15)}=-2$

$V_{I/G} = -(-2)^2(-20) = 80$ m/s **[Ans. 80 m/s]**

6. **A thin rod of length d/3 is placed along the principal axis of a concave mirror of focal length = d such that its image, which is real and elongated, just touches the rod. Find the length of the image?**

Sol. Image of A is formed at the same position (since is at C.) For image of B we will use mirror formula. According to question, only fig (ii) is possible since image is enlarged & real.

Applying mirror formula for image of B.

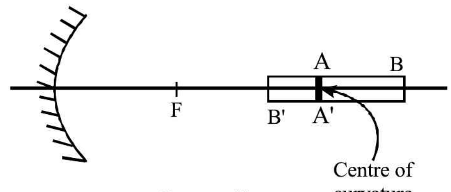


figure (i)

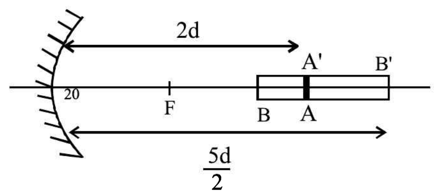


Figure (ii)

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}+\frac{1}{-\frac{5d}{3}}=\frac{1}{-d} \qquad \Rightarrow v=-\frac{5d}{2}$$

(–ve sign shows that image is formed at the real side as shown in figure)

Hence length of image $A'B'=\frac{5d}{2}-2d=\frac{d}{2}$ **Ans.** $\frac{\mathbf{d}}{\mathbf{2}}$

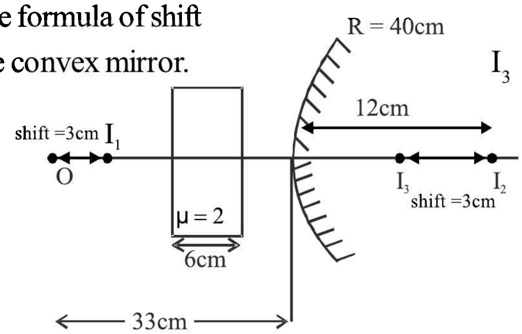
7. A point object is placed 33 cm from a convex mirror of curvature radius = 40 cm. A glass plate of thickness 6 cm and index 2.0 is placed between the object and the mirror, closer to the mirror. Find the distance of final image from the object.

Sol. I_1 = image formed by the glass slab through the formula of shift

I_2 = image of I_1 formed by the reflection by the convex mirror.

= image of I_2 formed by the refraction through glass slab. Shift by glass slab in air :

$$\text{shift}=t\left(1-\frac{1}{\mu}\right)=6\left(1-\frac{1}{2}\right)=3\text{cm}.$$



Apply mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ (For calculations of I_2)

$\frac{1}{v}+\frac{1}{-30}=\frac{1}{20}$ (u = – 30 cm ; f = 20 cm, convex mirror) $\Rightarrow$ v = + 12 cm

+ 12 indicates the position of I_2 as shown in figure. Again, shift = +3 (using shift = $t\left(1-\frac{1}{\mu}\right)$.

+ ve sign shows the shift of I_2 towards left along the incident ray to obtain I_3.

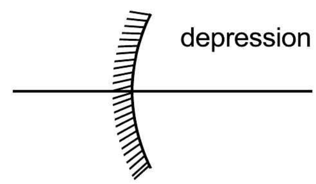
So, OI_3 = 33 + 9 = 42 cm. **[Ans. 42 cm]**

8. A large temple has a depression in one wall. On the floor plane it appears as a indentation having spherical shape of radius 2.50 m . A worshiper stands on the centre line of the depression, 2.00m out from its deepest point, and whispers a prayer. Where is the sound concentrated after reflection from the back wall of the depression ?

Sol. Since sound follows laws of reflection we can consider the system to behave as optical system and maxima will be heard at the point where image is formed.

Given that, f = –1.25 m, u = – 2 m

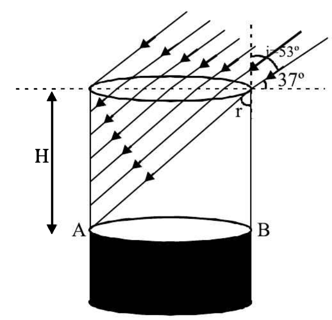
Apply the mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$



$\frac{1}{V}+\frac{1}{-2}=\frac{1}{-1.25}$ (mirror formula) $\Rightarrow v=\frac{-10}{3}$ m

The '– ve' sign shows that a real image in formed. **Ans.** $\left(\frac{\mathbf{10}}{\mathbf{3}}\mathbf{m}\right)$

9. An opaque cylindrical tank with an open top has a diameter of 3.00m and is completely filled with water. When the setting sun reaches an angle of 37° above the horizon, sunlight ceases to illuminate any part of the bottom of the tank. How deep is the tank ?

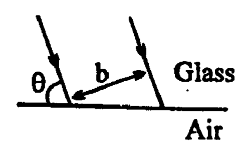


Sol. By Snell's law, 1 sin 53° = $\frac{4}{3}\sin r$,

$\frac{4}{5}=\frac{4}{3}\sin r$, $\sin r = \frac{3}{5} \Rightarrow r = 37°$

In ΔOAB, $\tan r=\frac{3}{H}=\tan 37° \Rightarrow H=4$ metres **Ans. 4 m**

10. A beam of parallel rays of width b propagates in glass at an angle θ to its plane face. The beam width after it goes over to air through this face is __________ if the refractive index of glass is μ.



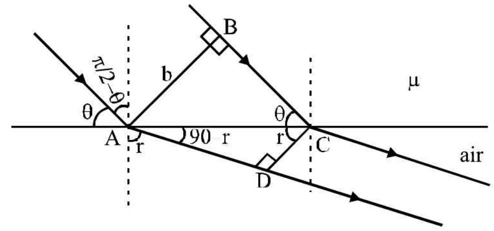
Sol. Applying Snell's law at the interface,

$\mu \sin(90-\theta) = 1 \sin r$

$\Rightarrow \mu \cos\theta = \sin r$ .... (i)

In ΔABC,

$\sin\theta = \frac{b}{AC} \Rightarrow AC = b \operatorname{cosec}\theta$

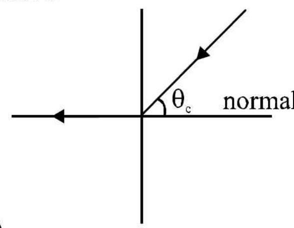


&, in Δ ADC $\cos r = \dfrac{CD}{AC}$ [where CD is the distance between parallel refracted rays.]

CD $= AC \cos r = b \operatorname{cosec}\theta \,.\, \cos r = b \operatorname{cosec}\theta \,.\, \sqrt{1-\mu^2\cos^2\theta}$ **[Ans.** $\mathbf{\dfrac{b\left(1-\mu^2\cos^2\theta\right)^{1/2}}{\sin\theta}}$**]**

11. **A room contains air in which the speed of sound is 340 m/s. The walls of the room are made of concrete, in which the speed of sound is 1700 m/s. (a) Find the critical angle for total internal reflection of sound at the concrete-air boundary. (b) In which medium should the sound be travel to undergo total internal reflection?**

Sol. Since sound wave follows snells' law for refraction, we have



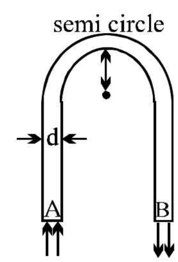
$$\frac{\mu_{wall}}{\mu_{air}} = \frac{C_{wall}}{C_{air}} \Rightarrow \frac{\mu_{wall}}{\mu_{air}} = \frac{1700}{340} = 5$$

$\Rightarrow \mu_{wall} = 5$ (Since, $\mu_{air} = 1$)

Apply Snell's law to find critical angle

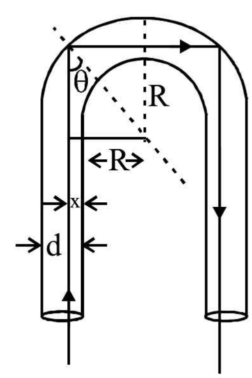
$5 \sin\theta_c = 1 \sin 90° \Rightarrow \theta_c = \sin^{-1}\dfrac{1}{5}$ **[Ans. (a)** $\sin^{-1}\left(\dfrac{1}{5}\right)$ **(b) air]**

12. **A rod made of glass (μ = 1.5) and of square cross-section is bent into the shape shown in the figure. A parallel beam of light falls perpendicularly on the plane flat surface A. Referring to the diagram, d is the width of a side & R is the radius of**
inner semicircle.



Find the maximum value of ratio $\dfrac{d}{R}$ so that all light entering the glass through surface A emerges from the glass through surface B. [REE '98]

Sol. This system is an optical fibre



$\sin\theta = \dfrac{R+x}{R+d}$ when $x \to 0$, then $\theta \to \theta_{min}$

If for θ_{min} ray suffers T.I.R. then all the other rays having $\theta > \theta_{min}$ will suffer T.I.R. Taking sin on both sides, $\sin\theta_{min} > \sin\theta_c \Rightarrow \dfrac{R}{R+d} \geq \dfrac{1}{\mu}$ on solving $\dfrac{d}{R} < \dfrac{1}{2}$ (taking $\mu = \dfrac{3}{2}$)

$\therefore \left(\dfrac{d}{R}\right)_{max} = \dfrac{1}{2}$ **[Ans.** $\mathbf{\left(\dfrac{d}{R}\right)_{max} = \dfrac{1}{2}}$**]**

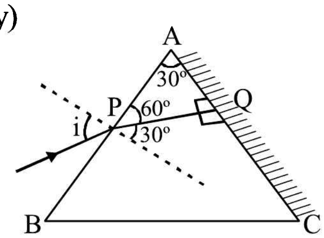
13. A prism of refractive index $\sqrt{2}$ has a refracting angle of 30°. One of the refracting surfaces of the prism is polished. For the beam of monochromatic light to retrace its path, find the angle of incidence on the refracting surface.

Sol. Refracting angle = angle of prism = 30° (given)

For any light ray to retrace its path, it must fall normally on the side AC. (Plane mirror)

By geometry, $\angle APQ = 60° \Rightarrow \therefore r = 30°$ (by geometry)

Using Snell's law :



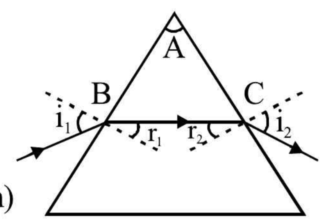
$$1 \times \sin i = \mu \sin 30 \Rightarrow 1 \times \sin i = \sqrt{2} \times \frac{1}{2}$$

$$\sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45°$$ **[Ans. 45°]**

14. An equilateral prism deviates a ray through 23° for two angles of incidence differing by 23°. Find μ of the prism ?

Sol. i_1 = angle of incidence for the first face

i_2 = angle of incidence for the other face



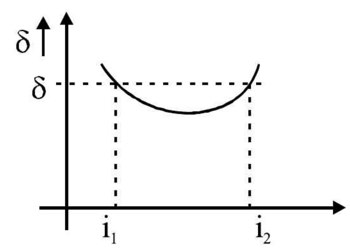
For same angle of deviation there are two angles of incidence, $i_2 > i_1$ (say)

$i_2 - i_1 = 23$ .... (i) (according to the question)

$\delta + A = i_1 + i_2$ .... (ii) (Prism formula)

$23 + 60 = i_1 + i_2$, On solving, $i_2 = 53°$, $i_1 = 30°$

$r_1 + r_2 = = 60°$



Now on applying Snell's law : At interface AB

$$\mu \sin r_1 = 1 \times \sin i_1 \Rightarrow \mu \sin r_1 = \frac{1}{2} \Rightarrow \sin r_1 = \frac{1}{2\mu}$$

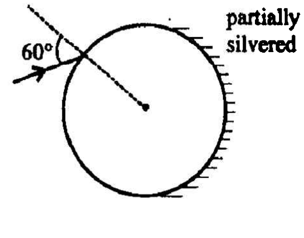
At interface AC $\Rightarrow \mu \sin r_2 = 1 \times \sin i_2 \Rightarrow \sin r_2 = \frac{4}{5} \Rightarrow r_2 = (60° - r_1)$

Applying sin on both sides :

$\sin(60° - r_1) = \sin r_2$; $\sin 60° \cos r_1 - \cos 60° \sin r_1 = \sin r_2$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{4\mu^2 - 1}}{2\mu} - \frac{1}{2} \cdot \frac{1}{2\mu} = \frac{4}{5\mu} \Rightarrow \text{On solving : } \mu = \frac{\sqrt{43}}{5}$$ **[Ans. $\frac{\sqrt{43}}{5}$]**

15. A ray is incident on a glass sphere as shown. The opposite surface of the sphere is partially silvered. If the net deviation of the ray transmitted at the partially silvered surface is 1/3rd of the net deviation suffered by the ray reflected at the partially silvered surface (after emerging out of the sphere), find the refractive index of the sphere.



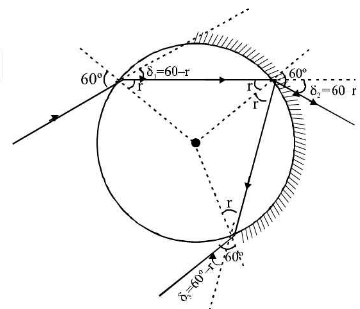
Sol. Since, the distance of all the points lying on the circle is constant from the centre. Hence, all the angles of refraction are same = r.

δ_1 = deviation suffered by the ray after first incidence.

δ_2 = deviation suffered by the transmitted ray through partially polished surface.

δ_3 = Deviation suffered by the ray emerging out of the sphere.

According to the question :



$$(60 - r) + (60 - r) = \frac{1}{3}\,[(60 - r) + (60 - r) + (180 - 2r)]$$

$$120 - 2r = \frac{1}{3}\,[300 - 4r] \Rightarrow 360 - 6r = 300 - 4r$$

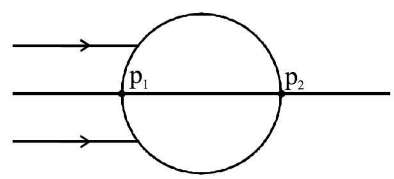
$$60 = 2r \Rightarrow r = 30°$$

Now on applying Snell's law :

$$1 \times \sin 60° = \mu \times \sin r \Rightarrow \frac{\sqrt{3}}{2} = \mu \times \frac{1}{2} \Rightarrow \mu = \sqrt{3} \qquad \textbf{[Ans. } \sqrt{3}\textbf{]}$$

16. A narrow parallel beam of light is incident to a transparent sphere of refractive index 'n'. If the beam finally gets focussed at a point situated at a distance = 2 × (radius of sphere) from the centre of the sphere, find n ?

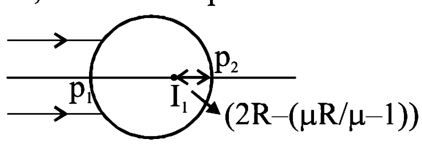
Sol. **For pole p_1** using formula of refraction for refraction for curved surface

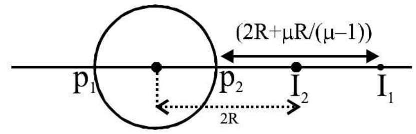


$$\frac{\mu_2}{v} - \frac{\mu_1}{\infty} = \frac{\mu_2 - \mu_1}{+R} \Rightarrow \frac{\mu_2}{v} - 0 = \frac{\mu_2 - 1}{R}$$

$$v = \left(\frac{\mu R}{\mu - 1}\right)$$

so, two case are possible





Now on using either of the case we find that the result is going to be the same

for pole, p_2 (apply the formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$)

$$\frac{1}{R} - \frac{\mu}{-\left(2R - \frac{\mu R}{\mu - 1}\right)} = \frac{1 - \mu}{-R} \Rightarrow \text{By solving the above equation we get, } \mu = \frac{4}{3} \quad \textbf{[Ans. } \frac{4}{3}\textbf{]}$$

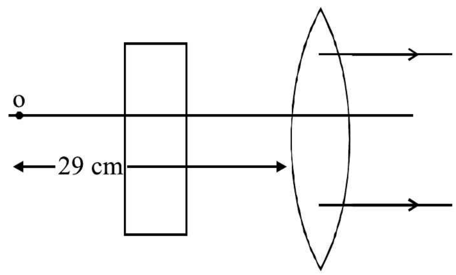
17. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness t and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. Find the thickness t ?

Sol. To solve this problem first find the object for lens. This object is the image formed by the glass slab.

Applying lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{\infty}-\frac{1}{20}=\frac{1}{f} \Rightarrow u=-20 \text{ cm}$$

Here –ve sign indicates that object is at the left of the lense.

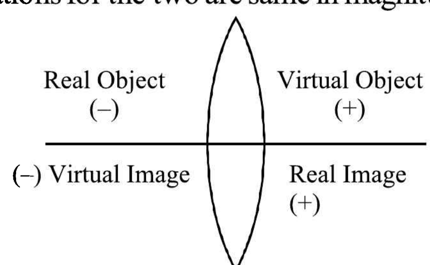


$\therefore$ shift of image by the glass slab = 25 – 20 = 5 cm

shift of image by the glass slab = $t\left(1-\frac{1}{\mu}\right) \Rightarrow 5 = t\left(1-\frac{1}{1.5}\right) \Rightarrow t = 15$ cm. **[Ans.15 cm]**

18. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, the image formed is virtual. If the size of the images formed are equal, then find the focal length of the lens ?

Sol. Now according to the question, magnifications for the two are same in magnitude.



$$m=\frac{h_i}{h_0}=\left|\frac{v_1}{-v}\right|=\frac{v_1}{16}$$

$$m=\frac{h_i}{h_0}=\left|\frac{-v_2}{-v_2}\right|=\frac{v_2}{6}$$

v_1 is the distance of the real image for a real object.

v_2 is the distance of the virtual image for a real object.

$$\frac{v_1}{16}=\frac{v_2}{6} \Rightarrow \quad 6v_1 = 16v_2 \text{ .... (i)}$$

$$\frac{1}{v_1}-\frac{1}{-16}=\frac{1}{f} \text{ .... (ii)}, \quad \frac{1}{-v_2}-\frac{1}{-6}=\frac{1}{f} \text{ .... (iii)} \quad \text{[From the lens formula]}$$

By solving above equations we get, f = 11 cm

+ve sign indicates that the lens is a convex (or converging) lens. **[Ans. 11 cm]**

19. A thin convex lens forms a real image of a certain object 'p' times its actual size. The size of real image becomes 'q' times that of object when the lens is moved nearer to the object by a distance 'a'. Find focal length of the lens ?

Sol. Displacement method (Experiment to find the focal length of lens)

Distance between the object and the screen is fixed = D

x_1 = Object distance for the 1st position of lens.

$D - x_1$ = Image distance for the 1st position of lens.

x_2 = Object distance for the 2nd position of lens.

$D - x_2$ = Image distance for the 2nd position of lens.

m_1 = Magnitude of magnification for the 1st position of lens.

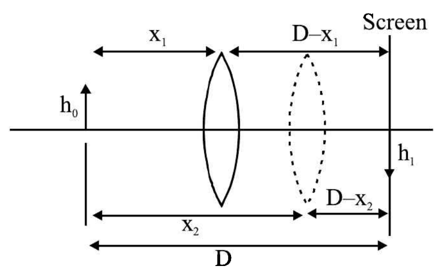
m_2 = Magnitude of magnification for the 2nd position of lens.

$m_1 = \frac{h_1}{h_0}, m_2 = \frac{h_2}{h_0}$ (h_1 and h_2 is the height of image for their respective position of lens)

Apply the lens formula :

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{D-x} - \frac{1}{-x} = \frac{1}{f}$ [General equation which satisfies x_1 and x_2]

$$\frac{x + D - x}{(D-x)x} = \frac{1}{f} \Rightarrow (D-x)x = Df$$



$$x^2 - Dx + Df = 0 \Rightarrow x = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$$

$$x_1 = \frac{D - \sqrt{D^2 - 4Df}}{2}, \; x_2 = \frac{D + \sqrt{D^2 - 4Df}}{2}$$

$$x_2 - x_1 = d \quad \Rightarrow \sqrt{D^2 - 4Df} = d \Rightarrow D^2 - 4Df = d^2 \Rightarrow f = \frac{D^2 - d^2}{4D}$$

$$x_1 + x_2 = D \Rightarrow m_1 = \frac{h_1}{h_0} = \frac{v_1}{u_1} = \frac{D - x_1}{x_1}, \; m_2 = \frac{h_2}{h_0} = \frac{v_2}{u_2} = \frac{D - x_2}{x_2}$$

Here, d = displacement of lens. Now,

$$m_1 m_2 = \left(\frac{h_1}{h_0}\right)\left(\frac{h_2}{h_0}\right) \quad \Rightarrow \left(\frac{D - x_1}{x_1}\right)\left(\frac{D - x_2}{x_2}\right) = \frac{h_1 h_2}{h_0^2}$$

Since, [$x_1 + x_2 = D$], $\Rightarrow x_1 = D - x_2, \; x_2 = D - x_1$ (applying above eq.)

Hence, $\frac{h_1 h_2}{h_0^2} = 1 \Rightarrow h_0 = \sqrt{h_1 h_2}$

$\Rightarrow m_1 m_2 = 1$

$|m_2 - m_1| = \frac{d}{f}$ [This is a very important result used to find the focal length of the lens]

$|m_2 - m_1| = \left|\frac{D - x_2}{x_2} - \frac{D - x_1}{x_1}\right|$ $\quad\Rightarrow \sqrt{h_1 h_2} = h_0$

$= \left|\frac{(D - x_2)x_1 - (D - x_1)x_2}{x_1 x_2}\right| = \left|\frac{D(x_1 - x_2)}{x_1 x_2}\right| = \frac{d}{f}$

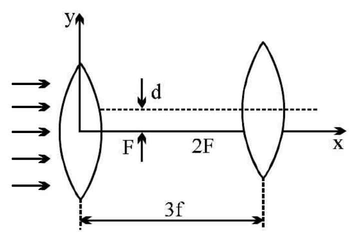
$h_1 = ph_0$ (According to the question), $h_2 = qh_0,$ $\quad h_0$ = height of the object

h_1, h_2 = height of images $\Rightarrow \sqrt{h_1 h_2} = h_0 \Rightarrow pq = 1$ [Using displacement method]

$|m_2 - m_1| = \frac{d}{f} \quad \Rightarrow \quad |p - q| = \frac{a}{f}$

$\Rightarrow f = \frac{a}{|p - q|}$ or $f = \frac{apq}{|p - q|}$ [$\because$ pq = 1] **[Ans. $\frac{apq}{(q-p)}$]**

20. **In the figure shown, the focal length f of the two thin convex lenses is the same. They are separated by a horizontal distance 3f and their optical axes are displaced by a vertical separation 'd' (d << f), as shown. Taking the origin of coordinates O at the centre of the first lens, find the x and y coordinates of the point where a parallel beam of rays coming from the left finally gets focussed ?**



Sol. The x-axes is the principal axis for first lens & PQ is the principal axis for second lens

Applying lens formula for first lens

$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$

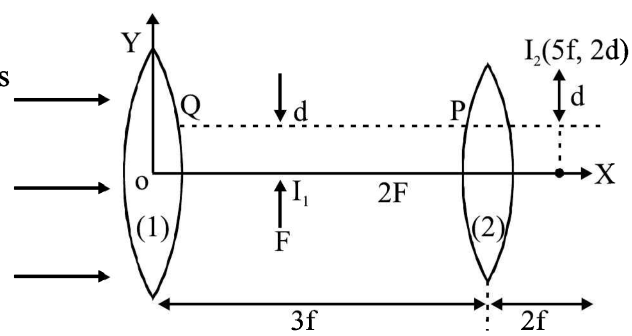
$v = f$

Applying lens formula for second lens

$\frac{1}{v} - \frac{1}{-2f} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{2f} \Rightarrow v = 2f$

$m = \frac{h_i}{h_0} = +\frac{v}{u} = \frac{+2f}{-2f} = -1$



–ve sign indicates that the image will be formed above the principal axis PQ for the second lens.

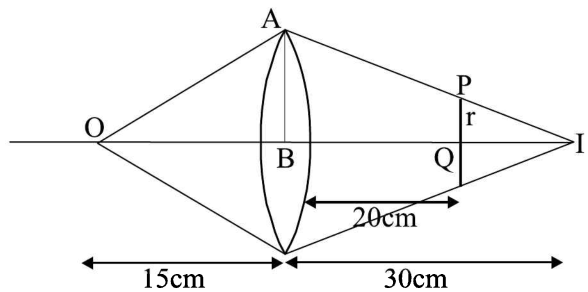
$\therefore$ Final coordinates of image are (5f, 2d) **[Ans. (5f, 2d)]**

21. A point source of light is kept at a distance of 15 cm from a converging lens, on its optical axis. The focal length of the lens is 10 cm and its diameter is 3 cm. A screen is placed on the other side of the lens, perpendicular to the axis of the lens, at a distance 20 cm from it. Find the area of the illuminated part of the screen ?

Sol. Applying lens formula :

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$ (I is the image of object O through the lens)

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} \Rightarrow v = +30 \text{ cm}$$



$$AB = \frac{\text{diameter}}{2} = 1.5 \text{ cm}$$

r = radius of the illuminated part

Using similar triangle concept

$\Delta PQI \sim \Delta ABI$

$$\frac{1.5}{r} = \frac{30}{10} \quad \Rightarrow \quad r = \frac{1}{2} \text{ cm},$$ So, Area of illuminated part = πr^2

$$= \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \text{ cm}^2$$ **[Ans. (π/4) cm^2]**

22. A double convex lens has focal length 25.0 cm in air. The radius of one of the surface is double the other. Find the radii of curvature if the refractive index of the material of the lens is 1.5.

Sol. Since the medium is same on both the side of lens and also the lens given is a thin lens, hence we can apply the lens makers formula

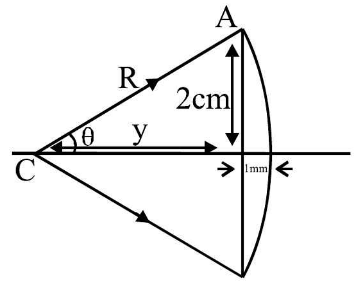
$$\frac{1}{f} = \left(\frac{\mu_L - \mu_m}{\mu_m}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{25} = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{R} - \frac{1}{-2R}\right) \Rightarrow \frac{1}{25} = (0.5)\left(\frac{3}{2R}\right) \Rightarrow 2R$$

$$= 3 \times 0.5 \times 25 \Rightarrow R = \frac{75}{4} \text{ cm}$$

And, radius of curvature for the other surface

$$= 2R = 2\times\left(\frac{75}{4}\right) = \frac{75}{2} \text{ cm}$$ **[Ans. 75/4 cm, 75/2 cm]**

23. A plano convex lens ($\mu = 1.5$) has a maximum thickness of 1 mm. If the diameter of its aperture is 4 cm. Find (i) Radius of the curvature of the curved surface; (ii) its focal length in air.



Sol. We cannot use lens maker formula or lens formula since the lens is thick.

$$\frac{2}{y} = \tan\theta \Rightarrow \qquad y = 2\cot\theta \;\; \text{[by geometry]}$$

$$R\sin\theta = 2$$

$$R - R\cos\theta = 0.1$$

$$R(1-\cos\theta) = 0.1 \quad \Rightarrow \quad 2\left(\frac{1-\cos\theta}{\sin\theta}\right) = 0.1$$

$$2\left(\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}\right) = 0.1 \Rightarrow 2\tan\frac{\theta}{2} = 0.1 \Rightarrow \tan\frac{\theta}{2} = \frac{1}{20}$$

$$\sin\theta = \frac{2\times\frac{1}{20}}{1+\frac{1}{20^2}} \quad [\because \sin\theta = \frac{2\tan\theta/2}{1+\tan^2\theta/2}] \quad \Rightarrow \quad \sin\theta = \frac{40}{401}$$

$$\Rightarrow \; R = \frac{2}{\sin\theta} = 20 \text{ cm}$$

Now on applying formula for curved surface

$$\frac{1.5}{v} - \frac{\mu_1}{\infty} = \frac{1.5-1}{\infty} \quad \Rightarrow \quad v = \infty$$

So, the ray emerges undeviated by plane surface

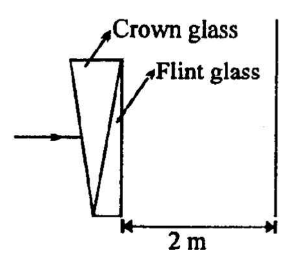
Now this ray acts as on object for the spherical surface.

$$\frac{1}{v} - \frac{1.5}{\infty} = \frac{1-1.5}{-20} \Rightarrow \frac{1}{v} = \frac{0.5}{20} = \frac{1}{40} \Rightarrow v = 40 \text{ cm}$$

This v is the focal length of lens since parallel rays striking on surface intersect at focus.

[Ans. (i) 0.2 m, (ii) 0.4 m]

24. The refractive indices of the crown glass for violet and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. A prism of angle 6^0 is made of crown glass. A beam of white light is incident at a small angle on this prism. The other thin flint glass prism is combined with the crown glass prism such that the net mean deviation is 1.5^0 anticlockwise.



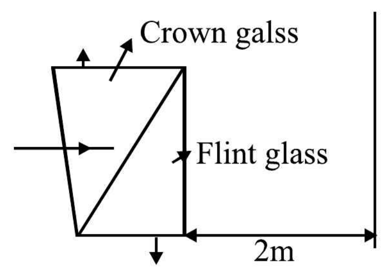
(i) Determine the angle of the flint glass prism.

(ii) A screen is placed normal to the emerging beam at a distance of 2m. from the prism combination. Find the distance between red and violet spot on the screen. Which is the topmost colour on screen?

Sol. **(i)** Since, the angle of prism and the angle of incidence are also small, the deviation is given by $\delta = (\mu - 1)\,A$, (the subscript c represents crown glass & f represents flint glass)

$$\mu_c = \frac{1.51+1.49}{2} = 1.5, \qquad \mu_f = \frac{1.77+1.73}{2} = 1.75$$

In the prism, since the final refracted ray always bends towards the base, means angle of deviation is always towards base.



$\delta_{net} = (\delta_C - \delta_F)\circlearrowleft$

For the crown glass $\delta_C = (\mu_C - 1)A \circlearrowleft$

For the flint glass $\delta_F = (\mu_F - 1)A \circlearrowright$

$\therefore\ \delta_{net} = (1.5 - 1)\,6 \circlearrowleft + (1.75 - 1) \circlearrowright$

$1.5 \circlearrowleft = 3 \circlearrowleft + 0.75\,A \circlearrowright$

$1.5 \circlearrowleft = 3 \circlearrowleft - 0.75\,A \circlearrowleft$

$0.75\,A = 1.5 \quad \Rightarrow \quad A = 2^\circ$

(ii) $\delta_V = A_C(\mu_{VC} - 1) - A_p(\mu_{VF} - 1)$, $\delta_r = 6\,(1.51 - 1) - 2\,(1.77 - 1)$

$= 6\,[0.51] - 2\,[0.77], = 3.06 - 1.54 = 1.52$

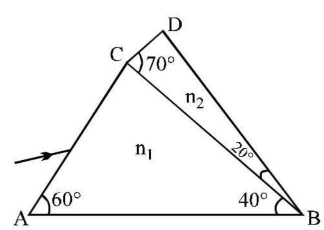
$\delta_R = 6\,(1.49 - 1) - 2\,(1.73 - 1) = 6\,(0.49) - 2\,(0.73) = 2.94 - 1.46 = 1.48$

Net shift $= r\,(\delta_v - \delta_r)$ [Since, $\tan \delta_r = \dfrac{\text{Shift of violet}}{r}$] (shift of violet) $= r \tan s\, \delta_r = r\,\delta_v$

$$\text{Net shift} = 2\,(1.52 - 1.48) \times \frac{\pi}{180} = \frac{0.08\pi}{180} = \frac{8}{18000}\pi = \frac{4}{9000}\pi\ (\text{m}) = \frac{4}{9}\pi\ \text{mm}$$

[Ans. (i) 2°, (ii) $\dfrac{4\pi}{9}$ mm]

25. **A prism of refractive index n_1 & another prism of refractive index n_2 are stuck together without a gap as shown in the figure. The angles of the prism are as shown . n_1 & n_2 depend on λ, the wavelength of light according to $n_1 = 1.20 + \dfrac{10.8\times10^4}{\lambda^2}$ & $n_2 = 1.45 + \dfrac{1.80\times10^4}{\lambda^2}$ where λ is in nm.**



(i) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface .

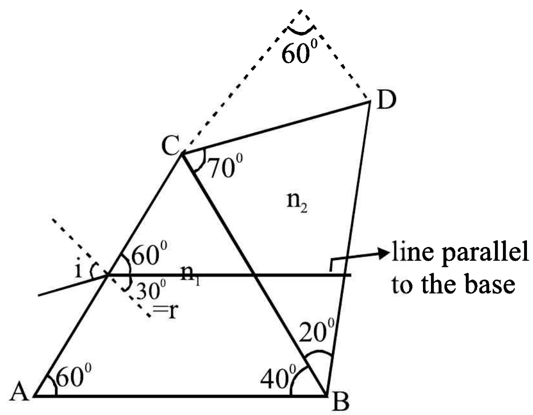
(ii) For the light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum. [JEE '98]

Sol. **(a)** Since the incident ray will not deviate at BC

So, $n_1 = n_2$

$$\Rightarrow 1.20 + \frac{10.8\times10^4}{\lambda_0^2} = 1.45 + \frac{10.8\times10^4}{\lambda_0^2}$$

$$\Rightarrow \frac{9\times10^4}{\lambda_0^2} = 0.25 \qquad \Rightarrow \lambda_0 = 600 \text{ nm.}$$



(b) For minimum deviation, the whole system behaves like an equilateral prism.

$r_1 + r_2 = A$

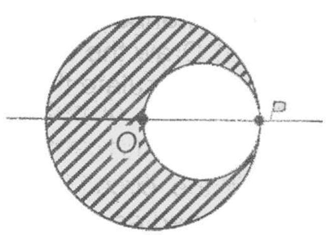
$r_1 = r_2$ (for minimum deviation)

$$r_1 = r_2 = 30^\circ, \qquad 1\times\sin i = n_1\times\sin 30 \Rightarrow \sin i = \left\{1.20 + \frac{10.8\times10^4}{(600)^2}\right\}\cdot\left(\frac{1}{2}\right) = \frac{3}{4} \;\therefore\; i = \sin^{-1}\left(\frac{3}{4}\right)$$

In the case of minimum deviation when the prism is equilateral, the ray inside the prism must be parallel to the base.

r = 30° (by geometry) **[Ans. (i) λ_0 = 600 nm, n = 1.5 (ii) i = $\sin^{-1}$ (0.75) = 48.59°]**

26. **A transparent sphere of radius R has a cavity of radius $\frac{R}{2}$ as shown in figure. Find the refractive index of the sphere if a parallel beam of light falling on left surface focuses at point P.**



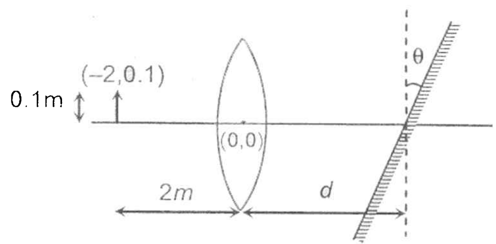
Sol. Let refractive index of glass be μ and after first refraction, image distance be v then

$$\frac{\mu}{v} - \frac{1}{\infty} = \frac{\mu-1}{R} \Rightarrow v = \frac{\mu R}{\mu-1}$$ Now second refraction will take place.

So distance of the first image from O is $u_1 = \frac{\mu R}{\mu-1} - R = \frac{R}{\mu-1}$ and the image is formed at R

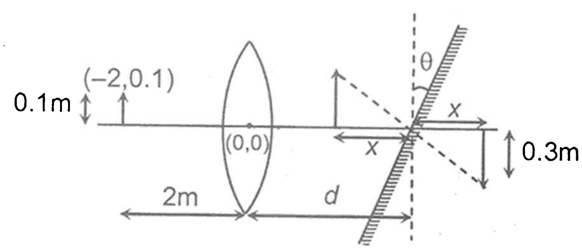
$$\therefore \frac{1}{R} - \frac{\mu(\mu-1)}{R} = \frac{2(1-\mu)}{R} \Rightarrow \mu^2 - 3\mu + 1 = 0. \qquad \text{So,} \quad \mu = \frac{3+\sqrt{5}}{2}$$

27. **A convex lens of focal length 1.5 m is placed in a system of coordinate axis such that its optical centre is at the origin and principal axis coinciding with the x-axis. An object and a plane mirror are arranged on the principal axis as shown in the figure. Find the value of d (in m) so that y-coordinate of image (after refraction and reflection) is 0.3 m. (Take tan θ = 0.3)**



Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-2} = \frac{1}{1.5}$; v = 6m

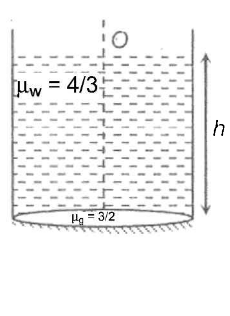


$m = \frac{v}{u} = -3$

$x = 6 - d$; $\tan\theta = \frac{0.3}{x}$

$\Rightarrow$ d = 5m.

28. **Bottom of a glass beaker is made of a thin equi-convex lens having bottom side silver polished as shown in the figure. Water is filled in the beaker upto a height of h = 4m. The image of point object, floating at middle point of beaker at the surface of water coincides with it. Find out the radius of curvature of the lens. ($_a\mu_g$ = 3/2, $_a\mu_w$ = 4/3)**



Sol. Power of equivalent mirror $P'_m = 2P_2 + P_M$

$$\frac{1}{f_2} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{8} \times \frac{2}{R} = \frac{1}{4R}, \; f_2 = 4R \text{ and } P_2 = \frac{1}{4R}$$

$$-\frac{1}{f'_m} = 2 \times \frac{1}{4R} + \frac{2}{R} = \frac{5}{2R} \quad \Rightarrow \quad f'_m = -\frac{2}{5}R, \qquad |h| = 2|f'_m|$$

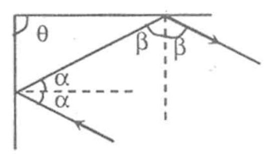
$$h = 2 \times \frac{2}{5}R = 4m \quad \Rightarrow \quad R = 5m$$

29. Find the angle between two flat mirrors positioned such that a beam of light incident on one of the mirrors at an arbitrary angle with a plane that is perpendicular to the mirror surface on reflection from both mirror surface becomes parallel to incident beam but in opposite direction.

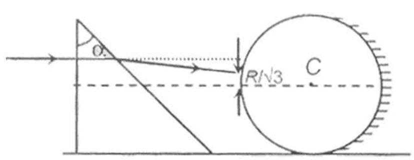
Sol. Total deviation of beam

$$2\alpha + 2\beta = \pi \qquad \Rightarrow \alpha + \beta = \frac{\pi}{2}$$

$$\theta = \pi - (\alpha - \beta) = \frac{\pi}{2}$$



30. A ray is incident normally on a right angle prism whose refractive index is $\sqrt{3}$ and prism angle $\alpha = 30°$. After crossing the prism, ray passes through a glass sphere. It strikes the glass sphere at $\frac{R}{\sqrt{3}}$ distance from principal axis, as shown in the figure. The sphere is half polished. Find the net angle of deviation of the incident ray.



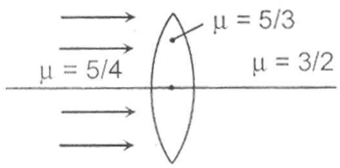
Sol. $\mu \sin 30° = \sin\beta \;\Rightarrow\; \sqrt{3} \cdot \frac{1}{2} = \sin\beta \Rightarrow \beta = 60°, \; \delta = 30°$

Point P where the ray strikes is $\frac{R}{\sqrt{3}} \;\Rightarrow\; \tan 30° = \frac{R}{x\sqrt{3}} \Rightarrow x = R$

$\Rightarrow$ Ray strikes normal to the spherical surface. It retraces the path.

$\therefore$ Angle of deviation = 180°.

31. A thin equi-convex lens having radius of curvature 10 cm is placed as shown in figure. Calculate focal length of the lens, if parallel rays are incident as shown.



Sol. Refraction at 1st surface, $u = \infty$, $\mu_1 = \frac{5}{4}$, $\mu_2 = \frac{5}{3}$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \Rightarrow \frac{5}{3v} = \frac{\frac{5}{3} - \frac{5}{4}}{10} \quad \Rightarrow v = 40 \text{ cm}$$

Refraction at second source $\mu_1 = -\frac{5}{3}$, $\mu_2 = \frac{3}{2}$ $u = +40$ $v = ?$, $R = -10$

$$\frac{3}{2V} - \frac{5}{3\times 40} = \frac{\frac{3}{2} - \frac{5}{3}}{-10}$$

After solving we get $V = \frac{180}{7}$ cm.

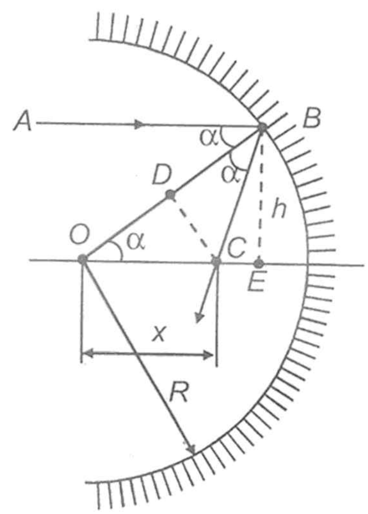
So focal length $= \frac{180}{7}$ cm.

32. An equi-convex lens of focal length 10 cm and refractive index ($\mu_g = 1.5$) is placed in a liquid whose refractive index varies with time as $\mu(t) = 1.0 + \frac{1}{10}t$. If the lens was placed in the liquid at t = 0, after what time will the lens act as concave lens of focal length 20 cm.?

Sol. $$\frac{f_e}{f_a} = \frac{({}_a\mu_g - 1)\,{}_a\mu_t}{{}_a\mu_g - {}_a\mu_t} \quad \Rightarrow \quad -\frac{20}{10} = \frac{(1.5-1)\,{}_a\mu_t}{1.5} \quad \Rightarrow \quad 3 = 1.5\,{}_a\mu_t \quad \Rightarrow t = 10 \text{ sec.}$$

33. Two rays are incident on a spherical concave mirror of radius R = 5 cm parallel to its optical axis at perpendicular distances 3 cm and 4 cm respectively. Determine the value Δx if distance between the points at which these rays intersect the optical axis after being reflected from the mirror is $\frac{\Delta x}{24}$ cm.

Sol. Let O be the centre of the spherical surface of the mirror, ABC the ray incident at a distance BE from the mirror axis, and OB = R. From the right triangle OBE, we find that $\sin\alpha = h / R$. The triangle OBC is isosceles since $\angle$ ABO = $\angle$ BOC and $\angle$ BOC = $\angle$ ABO as alternate - interior angles. Hence OD = BD = R/2. From the triangle ODC, we obtain



$$x = \frac{R}{2\cos\alpha} = \frac{R^2}{2\sqrt{R^2 - h^2}}$$

$$x_1 - x_2 = \frac{25}{24}\text{ cm} \therefore \Delta x = 25$$

Subjective Type (Fully Solved)—Level II

Q.1 An observer whose least distance of distinct vision is 'd', views his own face in a convex mirror of radius of curvature 'r'. Prove that the magnification produced can not exceed $\dfrac{r}{d+\sqrt{d^2+r^2}}$.

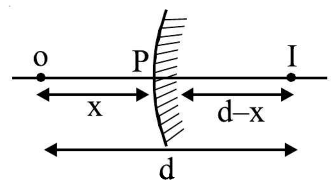
Sol. For clear vision, OI must be > d ; let OP be x (distance)

Apply mirror formula

$\dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f}$ where $v = +(d-x)$; $u = -x$; $f = +\dfrac{x}{2}$ (for $x > 0$)

$\dfrac{1}{d-x}-\dfrac{1}{x}=\dfrac{2}{r} \Rightarrow \dfrac{x-d+x}{(d-x)(x)}=\dfrac{2}{r} \Rightarrow -2x^2+2dx = 2rx - dr$

Forming a quadratic in x we have 2 values



$x = \dfrac{d-r\pm\sqrt{r^2+d^2}}{2}$,

rejecting $\left(\dfrac{d-r-\sqrt{r^2+d^2}}{2}\right)$ as it is –ve ; since only +ve value is accepted

$x = \dfrac{d-r+\sqrt{r^2+d^2}}{2}$, $= \dfrac{-(d-x)}{-x} = \dfrac{(d-r+\sqrt{r^2+d^2})}{2}$

= Rationalising the above equation, we get

$m = \dfrac{(d+r)-(\sqrt{r^2+d^2})^2}{(d-r+\sqrt{r^2+d^2})(d+r+\sqrt{r^2+d^2})} = \dfrac{2rd}{2d^2+2d(\sqrt{r^2+d^2})}$ **[m_{max}** $= \dfrac{r}{d+\sqrt{r^2+d^2}}$ **]**

Q.2 A thief is running away in a car with velocity of 20 m/s. A police jeep is following him, which is sighted by thief in his rear view mirror, a convex mirror of focal length 10 m. He observes that the image of the jeep is moving towards him with a velocity of 1 cm/s. If the magnification of the mirror for the jeep at that time is 1/10. Find

(a) the actual speed of the jeep

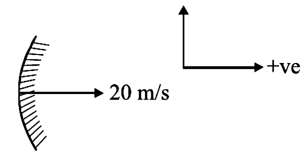
(b) the rate at which magnification is changing.

Assume that police jeep is on axis of the mirror.

Sol. $f = 10$ m

(a) $(V_{I/m})_{\perp} = -m^2 (V_{O/m})_{\perp}$ [From the velocity formula]

$$-1 \times 10^{-2} = \frac{-1}{10^2}(V_{O/m})_\perp$$



$(V_{O/m})_\perp = +1$ m/s $\quad = +1\hat{i}$

$\vec{V}_{O/G} = \vec{V}_{O/m} + \vec{V}_{m/G} = +1 + 20 = (+21 \text{ m/s})\ \hat{i}$

(b) $\quad m = \dfrac{f}{f-u} = \dfrac{1}{10}$ $\qquad$ [let u = – d where 'd' is a +ve quantity]

$$\frac{10}{10-(-d)} = \frac{1}{10} \Rightarrow \quad d = 90\ ;\ u = -90$$

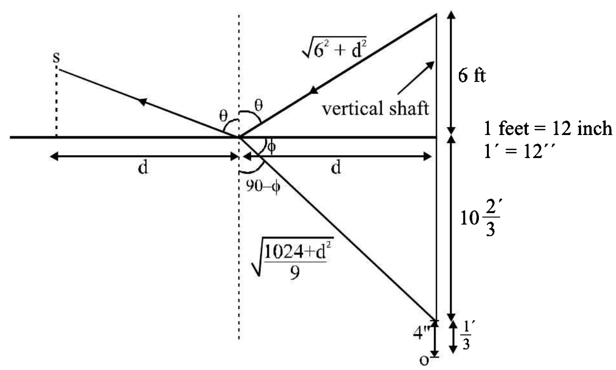
$$v = -mv = \frac{-1}{10} \times -90 = +9.$$

(c) $\quad \dfrac{dm}{dt} = \left[\dfrac{(-90)(-1\times10^{-2}) - 9(1)}{90^2}\right]$ per sec.

$= \left[\dfrac{-81}{10\times8100}\right] = +1 \times 10^{-3}$ per sec. **[Ans. (a) 21 m/s, (b) 1 x 10^{-3} / sec]**

Q.3 A surveyor on one bank of a canal observes the image of the 4 inch mark and 17 ft mark on a vertical staff, which is partially immersed in the water and held against the bank directly opposite to him. He sees that the refkected and refracted rays come from the same point which is the centre of the canal. If the 17ft mark and the surveyor's eye are both 6ft above the water level, estimate the width of the canal, assuming that the refractive index of the water is 4/3. Zero mark is at the bottom of the canal.

Sol. Applying Snell's law :



$$1 \times \sin\theta = \frac{4}{3}\sin(90-\varphi)$$

$$1\times\frac{d}{\sqrt{36+d^2}} = \frac{4}{3}\times\frac{d\times3}{\sqrt{1024+d^2}}$$

$\sqrt{1024+9d^2} = 4\sqrt{36+d^2}$

On squaring both sides :

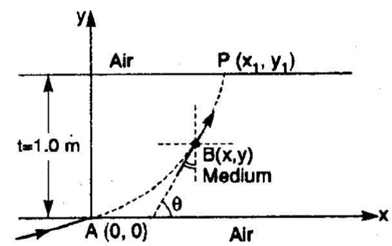
$1024 + 9d^2 = 16\ (36 + d^2)$, $1024 + 9d^2 = 576 + 16d^2$

$7\ d^2 = 448$, $d^2 = \dfrac{448}{7} = 64$

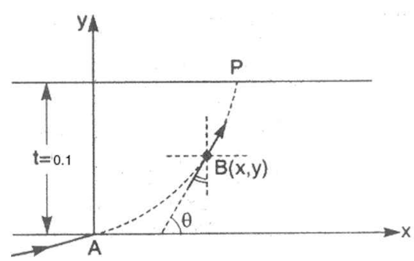
d = 8 feet, width = 2d = 16 feet. $\qquad$ **[Ans. 16 feet]**

Q.4 **A ray of light travelling in air is incident at grazing angle (Incident angle = 90°) on a long rectangular slab of a transparent medium of thickness t = 1.0 m. The point of incidence is the origin A(0, 0). The medium has a variable index of refraction n(y) given by $n(y) = [ky^{3/2} + 1]^{1/2}$ where $k = 1.0\ (\text{metre})^{-3/2}$. The refractive index of air is 1.0. [1995 ; 10M]**

(a) Obtain a relation between the slope of the trajectory of the ray at a point B(x, y) in the medium and the incident angle at the point.
(b) Obtain an equation for the trajectory y(x) of the ray in the medium.
(c) Determine the coordinates (x_1, y_1) of the point P, where the ray intersects the upper surface of the slab-air boundary
(d) Indicate the path of the ray subsequently.



Sol. (a) Refractive index is a function of y. It varies along y-axis i.e., the boundary separating two media is parallel to x-axis or normal at any point will be parallel to y-axis.
Secondly, refractive index increases as y is increased. Therefore, ray of light is travelling from rarer to denser medium i.e., it will bend towards the normal and shape of its trajectory will be as shown below.

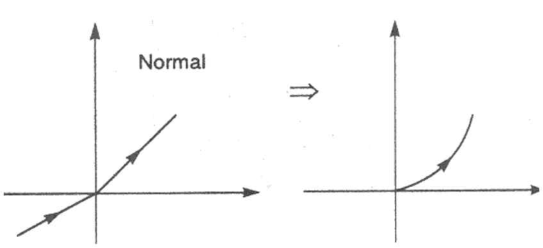


Now, refer to figure (1) :
Let i be the angle of incidence at any point B on its path
$\theta = 90° - i$, or $\tan\theta = \tan(90° - i) = \cot i$, or slope $= \cot i$

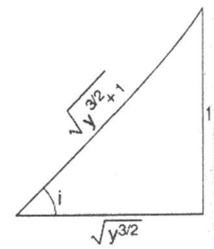
(b) but $\tan\theta = \dfrac{dy}{dx} \Rightarrow \therefore \dfrac{dy}{dx} = \cot i$ .. (1)

Applying Snell's law at A and B, $n_A \sin i_A = n_B \sin i_B$, $n_A = 1$ because $y = 0$, $\sin i_A = 1$ because $i_A = 90°$ (Grazing incidence)

$n_B = \sqrt{Ky^{3/2} + 1} = \sqrt{y^{3/2} + 1}$ because $K = 1.0\ (m)^{-3/2}$



$\therefore$ (1) (1) $= \sqrt{(y^{3/2}+1)}\,\sin i$ or $\sin i = \dfrac{1}{\sqrt{y^{3/2}+1}}$



$\therefore$ $\cot i = \sqrt{y^{3/2}}$ or $y^{3/4}$ .... (2)

Equating Eqs. (1) and (2), we get

$$\frac{dy}{dx} = y^{3/4} \quad \text{or} \quad y^{-3/4}\,dy = dx\text{, or } \int_0^y y^{-3/4}dy \int_0^y dx\text{, or } 4y^{1/4} = x \quad \textbf{... (3)}$$

The required equation of trajectory is $4y^{1/4} = x$.

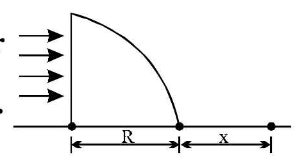
(c) At point P, where the ray emerges from the slab $y = 1.0$ m, $\therefore$ $x = 4.0$ m [From eq. 3]

Therefore, coordinates of point P are, P = (4.0 m, 1.0 m)

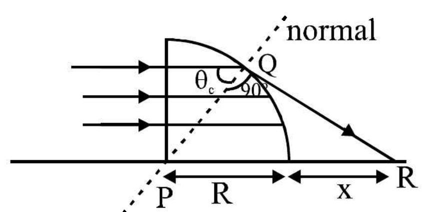
(d) As $n_A \sin i_A = n_p \sin i_p$ and as $n_A = n_p = 1$, therefore, $i_p = i_A = 90°$ i.e., the ray will emerge parallel to the boundary at P i.e., at grazing emergence.

[Ans. (i) $\tan\theta = \dfrac{dy}{dx} = \cot i$; (ii) 1; (iii) $y = k^2 (x/4)^4$; (iv) 4.0, 1; (v) It will become parallel to x-axis]

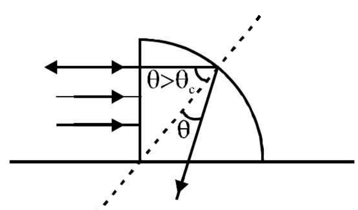
Q.5 A uniform, horizontal beam of light is incident upon a quarter cylinder of radius R = 5 cm, and has a refractive index $2/\sqrt{3}$. A patch on the table at a distance 'x' from the cylinder is illuminated. find the value of 'x'?



Sol. When the angle of incidence is θ_c, 'x' region will remain unilluminated. This is the limiting case.

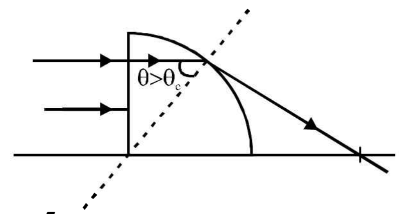


When the angle of incidence is greater than critical angle T. I. R. will take place. In this case also 'x' region is unilluminated.



When angle of incidence $\theta < \theta_c$, then the incident ray will refract and hence 'x' region will be illuminated. So, in limiting case :

In ΔPQR ; $\cos\theta_c = \dfrac{R}{R+x}$



$$\sin\theta_c = \frac{\mu_r}{\mu_d} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$$

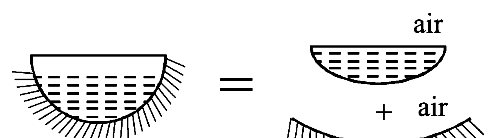
$\therefore\ \theta_c = 60°, R = 5$ cm (given) $\Rightarrow \cos 60 = \dfrac{5}{5+x} \Rightarrow 5 + x = 10 \Rightarrow x = 5$ cm. **[Ans. 5 cm]**

Q.6 A concave mirror has the form of a hemisphere with a radius of $R = 60$ cm. A thin layer of an unknown transparent liquid is poured into the mirror. The mirror-liquid system forms one real image and another real image is formed by mirror alone of the source in a certain position. One of them coincides with the source and the other is at a distance of $l = 30$ cm from the source. Find the possible value(s) of refractive index μ of the liquid.

Sol. For, concave mirror with unknown liquid

$P_e = 2P_L + p_m$

$$P_L = \frac{1}{f_L},\ \text{where } \frac{1}{f_L} = (\mu-1)\left(\frac{1}{\infty} - \frac{1}{-60}\right)$$



$$\therefore\ P_L = \left(\frac{\mu-1}{60}\right),\ \frac{1}{f_L} = \left(\frac{\mu-1}{60}\right)$$

$$P_m = -\frac{1}{f_m},\ \text{where } f_m = -\frac{60}{2}\ \therefore\ P_m = \frac{2}{60}\ \text{(For concave mirror)}$$

$$\Rightarrow P_e = 2P_L + P_m = 2\left(\frac{\mu-1}{60}\right) + \frac{2}{60} = \frac{2\mu}{60} = \frac{r}{30} \Rightarrow f_e = -\frac{1}{P_e} = -\frac{30}{\mu}$$

Case-I :

• Image formed by the mirror liquid system coincides with the source.

then, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f_e}$ (for, mirror liquid system) $\Rightarrow \dfrac{1}{-u} + \dfrac{1}{-u} = \dfrac{-\mu}{30} \Rightarrow u = \dfrac{60}{\mu}$

According to the question, $\dfrac{60}{\mu} + 30 =$ distance of the image formed by the mirror itself .... (1)

For calculation of image distance for mirror itself

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1\times\mu}{-60}=-\frac{1}{30} \Rightarrow v=\frac{60}{\mu-2}=-\left(\frac{60}{2-\mu}\right)$$

Image distance by the mirror itself $=\frac{60}{2-\mu}$, Then, from the equation (1)

$$\frac{60}{\mu}+30=\frac{60}{2-\mu} \Rightarrow \mu^2+2\mu-4=0 \Rightarrow \mu=-1\pm\sqrt{5}\ \ \mu=-1+\sqrt{5}$$

(since, μ can not be negative)

Case-II : Image formed by the mirror itself coincide with the source. For calculation of image distance for mirror itself

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-u}+\frac{1}{-u}=-\frac{1}{30} \Rightarrow u=60$$

For calculation of image distance for mirror liquid system

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_e} \text{ (mirror liquid system)} \Rightarrow \frac{1}{v}+\frac{1}{-60}=\frac{-\mu}{30} \Rightarrow v=\frac{60}{1-2\mu}=-\left(\frac{60}{2\mu-1}\right)$$

According to the question, $\frac{60}{2\mu-1}+30=60 \Rightarrow \mu=1.5$ **[Ans. 1.5 or $(\sqrt{5}-1)$]**

Q.7 A parallel beam of light falls normally on the first face of a prism of small angle. At the second face, it is partly transmitted and partly reflected. The reflected beam strikes at the first face again, and emerges from it in a direction making an angle $6^0 30'$ with the reversed direction of the incident beam. The refracted beam is found to have undergone a deviation of $1^0 15'$ from the original direction. Find the refractive index of the glass and the angle of the prism.

Sol. Using the given conditions, the problem has to be solved. Since, angle of the prism and the angle of incidence is small. $\delta = A(\mu-1)$ can be used. For transmission of ray :

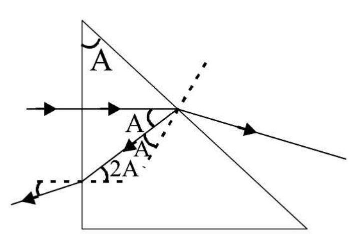
$\delta = A(\mu - 1) = \frac{5}{4}$ .... **(1)**

At the first surface where refraction takes place using Snell's law.

$\mu \sin 2A = 1 \times \sin r$

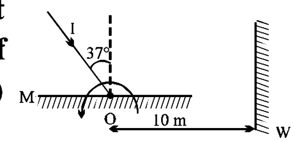
since, angles are small,

$r = 2A\mu. \Rightarrow \delta = 2A\mu = 6.5$ .... **(2)**



On solving (1) and (2) $\Rightarrow A = 2^o \Rightarrow \mu = \frac{13}{8}$ **[Ans. $\mu=\frac{13}{8}, A = 2^o$]**

Q.8 A light ray I is incident on a plane mirror M. The mirror is rotated in the direction as shown in the figure by an arrow at frequency $\frac{9}{\pi}$ rev/sec. The light reflected by the mirror is received on the wall W at a distance 10 m from the axis of rotation. When the angle of incidence becomes 37°, find the speed of the spot (a point) on the wall ?

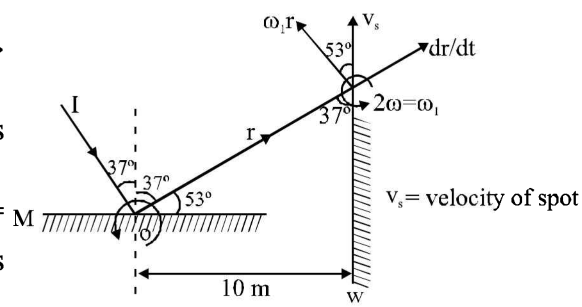


Sol. When the mirror is rotated by θ angle, keeping the incident ray fixed, reflected ray turns by 2θ in the same sense. If the mirror is rotated with frequency ω, reflected ray is rotated with frequency 2ω = (ω_1) in the same sense.

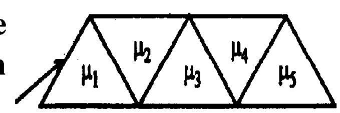
$\omega = \frac{9}{\pi}$ rev/s $= \frac{9}{\pi} \times \frac{2\pi}{1} = 18$ rad/s, $\omega_1 = 2\omega = 36$ rad/s, $\sin 37 = \frac{10}{r}$

(by geometry) $\Rightarrow r = \frac{10}{3/5} = 50/3$ m $\Rightarrow$

$\omega_1 = 36 \times \frac{50}{3} = 600$ m/s., then, $v_s \cos 53 = \omega_1 r \Rightarrow v_s \times \frac{5}{3} = 36 \times \frac{50}{3} \Rightarrow v_s =$ 1000 m/s The other component of v_s gives the dr/dt. **[Ans. 1000 m/s]**



Q.9 The diagram shows five isosceles right angled prisms. A light ray incident at 90° at the first face emerges at same angle with the normal from the last face. Find the relation between the refractive indices ?



Sol. Applying Snell's law at each interface :

$1 \times \sin 90 = \mu_1 \sin r_1$ .... (i) , $\mu_1 \sin (90 - r_1) = \mu_2 \sin r_2$ .... (ii)

$\mu_2 \sin (90 - r_2) = \mu_3 \sin r_3$ .... (iii), $\mu_3 \sin (90 - r_3) = \mu_4 \sin r_4$ .... (iv)

$\mu_4 \sin (90 - r_4) = \mu_5 \sin r_5$ .... (v), $\mu_5 \sin (90 - r_5) = 1 \times \sin 90$ .... (vi)

Reverse (ii), (iv), (vi) :

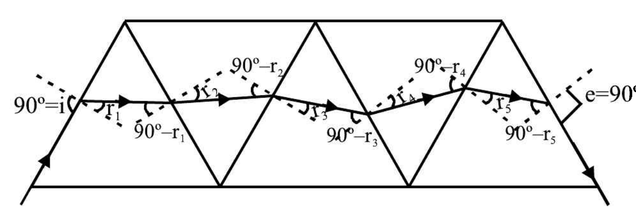
$1 = \mu_1 \sin r_1 \Rightarrow \mu_2 \sin r_2 = \mu_1 \cos r_1 \Rightarrow \mu_2 \cos r_2 = \mu_3 \sin r_3$

$\mu_4 \sin r_4 = \mu_3 \cos r_3$

$\mu_4 \cos r_4 = \mu_5 \sin r_5$

$\Rightarrow 1 = \mu_5 \cos r_5$

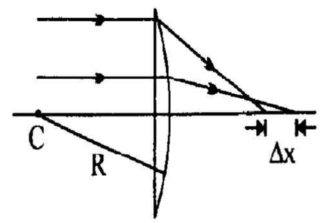
On squaring and adding :



$$\mu_1^2+\mu_3^2+\mu_5^2=2+\mu_2^2+\mu_4^2$$

[Ans. $\mu_1^2+\mu_3^2+\mu_5^2=2+\mu_2^2+\mu_4^2$]

Q.10 **Two rays travelling parallel to the principal axis strike a large plano-convex lens having a refractive index of 1.60. If the convex face is spherical, a ray near the edge does not pass through the focal point (spherical aberration occurs).**



If this face has a radius of curvature of magnitude 20.0 cm and the two rays are h_1= 0.500 cm and h_2 = 12.0 cm from the principal axis, find the difference in the positions where they cross the principal axis.

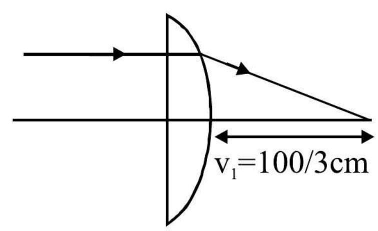
Sol. For the ray $h_1 = 0.5$ cm, Apply the refraction formula for the currved surface :

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}\Rightarrow\frac{1}{v_1}-\frac{1.6}{\infty}=\frac{1-1.6}{-20}\Rightarrow v_1=\left(\frac{100}{3}\right)\text{cm, from the curved surface}$$

For the ray $h_2 = 12$ cm (Near the edge) apply Snell's law at the curved surface., $1.6\sin i = 1\sin r$,

From the Δ ACB ; $\sin i=\frac{h}{R}=\frac{12}{20}=\frac{3}{5}$, then, $\sin r=\frac{24}{25}$, $\tan r=\frac{24}{7}$. In Δ ACB,

$$\tan i=\frac{h}{R-x}\Rightarrow\frac{3}{4}=\frac{12}{20-x}$$



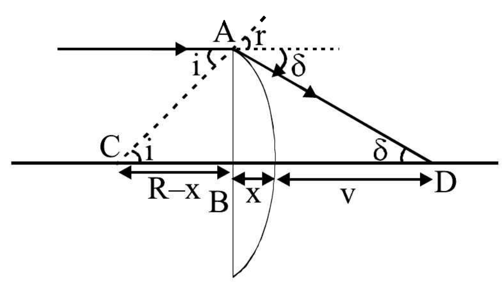
$\Rightarrow x = 4$ cm

In Δ ABD

$$\tan\delta=\frac{h}{v+x}\quad;\quad\tan\delta=\frac{12}{v+4}$$

By trigonometry : $\tan(\delta)=\tan(r-i)$

$$\tan\delta=\frac{\tan r-\tan i}{1+\tan r\tan i}\Rightarrow\tan\delta=\frac{\frac{24}{7}-\frac{3}{4}}{1-\frac{24}{7}\times\frac{3}{4}}$$



$$\tan\delta=\frac{3}{4}\Rightarrow\tan\delta=\frac{12}{v+4}$$

$$\Rightarrow\frac{3}{4}=\frac{12}{v+4},\ v=12\text{ cm}\ \Delta x=v_1-v=\frac{100}{3}-12=\left(\frac{64}{3}\right)\text{cm}\ \textbf{[Ans.}\ \frac{64}{3}\ \textbf{cm]}$$

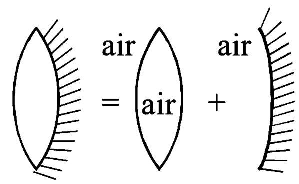
Q.11 Two thin similar watch glass pieces are joined together, front to front, with rear portion silvered and the combination of glass pieces is placed at a distance *a* = 60 cm from a screen. A small object is placed normal to the optical axis of the combination such that its two times magnified image is formed on the screen. If air between the glass pieces is replaced by water (μ = 4/3), calculate the distance through which the object must be displaced so that a sharp image is again formed on the screen.

Sol. When air is filled between the two similar glass pieces.

$P_e = 2P_L + P_M$

$$P_L = \frac{1}{f_L} \text{ where } \frac{1}{f_L} = \left(\frac{\mu_L - \mu_m}{\mu_m}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_L} = \left(\frac{1-1}{1}\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\Rightarrow \quad \frac{1}{f_L} = 0$$



Then, $P_L = 0$

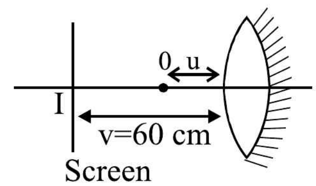
$$P_M = -\frac{1}{f_M} \quad \text{where } f_M = -\frac{R}{2} \text{ (since mirror is concave)}$$

$$P_M = +\frac{2}{R} \Rightarrow P_e = 2P_L + P_M = 2 \times 0 + \frac{2}{R} = \frac{2}{R}$$

Given that screen is placed at a distance 60 cm from the combination

$m = -\frac{v}{u} \Rightarrow$ Two time magnification means,

$m = -2$, then $v = 2u \Rightarrow 60 = 2u \Rightarrow u = 30$ cm



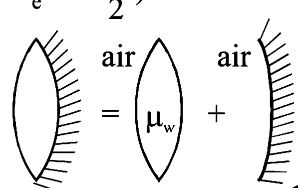
For equivalent combination, $P_e = -\frac{1}{f_e} \Rightarrow f_e = -\frac{1}{P_e} = -\frac{R}{2}$

Apply mirror formula to the equivalent combination of mirror.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_e} \Rightarrow \frac{1}{-60} + \frac{1}{-30} = \frac{1}{f_e} \Rightarrow -\frac{1}{60} - \frac{1}{30} = -\frac{2}{R} \text{ (since } f_e = -\frac{R}{2}\text{)}$$

$\Rightarrow R = 40$ cm

Again, if air between the glass pieces is replaced by water.



$P_e = 2P_L + P_M$

$$P_L = \frac{1}{f_L} \text{ where } \frac{1}{f_L} = \left(\frac{\mu_w - 1}{1}\right)\left(\frac{1}{40} - \frac{1}{-40}\right) \Rightarrow (\mu_w = 4/3) \Rightarrow P_L = \frac{1}{60} \Rightarrow P_M = -\frac{1}{f_M}$$

where $\frac{1}{f_M} = -\frac{R}{2}$ (for concave mirror), $P_M = \frac{2}{R} = \frac{2}{40} \Rightarrow P_e = 2P_L + P_M =$

$$2\times\frac{1}{60}+\frac{2}{40}=\frac{5}{60}=\frac{1}{12} \Rightarrow P_e = -\frac{1}{f_e} \Rightarrow f_e = -12 \text{ cm}$$

Again apply $\frac{1}{v}+\frac{1}{u}=\frac{1}{f_e} \Rightarrow \frac{1}{-60}+\frac{1}{u}=\frac{1}{-12} \Rightarrow u = -15$ cm

When air is filled between the gap, object distance = 30 when water is filled between the gas, object distance = 15 cm, Then, object is displaced by 15 cm towards the combination.

[Ans. 15 cm towards the combination]

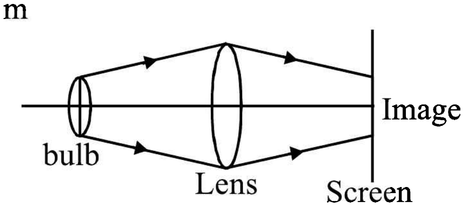
Q.12 A spherical lightbulb with a diameter of 3.0 cm radiates light equally in all directions, with a power of 4.5π W. (a) Find the light intensity at the surface of the bulb. (b) Find the light intensity 7.50 m from the centre of the bulb. (c) At 7.50-m, a convex lens is set up with its axis pointing toward the bulb. The lens has a circular face with a diameter of 15.0 cm and a focal length of 30.0 cm. Find the diameter of the image of the bulb formed on a screen kept at the location of the image. (d) Find the light intensity at the image.

Sol. (a) Light intensity, $I = \frac{P}{4\pi r^2} = \frac{4.5\pi}{4\pi(1.5\times10^{-2})^2} = (5000)$ w/m^2

(b) $I = \frac{P}{4\pi r_1^2} = \frac{4.5\pi}{4\pi(7.5)^2} = 0.02$ w/m^2, where, $r_1 = 7.5$ m from the centre of the bulb.

(c) Apply the lens formula for the image of bulb,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{-7.5}=\frac{1}{0.3} \Rightarrow v = \left(\frac{15}{48}\right)\text{m}$$



Magnification, $m = \frac{v}{u} = \frac{15/48}{-7.5} = -\frac{1}{24}$

$$\Rightarrow m = \frac{D_i}{D_0} \Rightarrow D_i = -\frac{1}{8} \text{ cm} = -0.125 \text{ cm}$$

$$\text{Intensity of image} = \frac{\text{Power converged by lens}}{\text{Area of image}},$$

Power converged by lens = Intensity at position of lens × Area of Lens

$$= \frac{p}{4\pi r_1^2}\times\frac{\pi d^2}{4} = \frac{pd^2}{16 r_1^2} = \frac{4.5\pi\times15\times10^{-4}}{16\times(7.5)^2}$$

where, d = diameter of lens = 15 cm

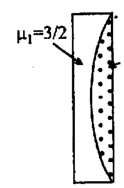
r_1 = distance of lens from the bulb = 7.5 m $\Rightarrow$ Intensity of image =

$$\frac{\text{power converged by lens}}{\text{Area of image}} = \frac{4.5\pi \times 15 \times 10^{-4}}{16 \times (7.5)^2 \times \pi \times \left(\frac{1}{16}\right)^2 \times 10^{-4}} = 288 \text{ w/m}^2$$

where, $\quad$ Area of image $= \frac{\pi D_i^2}{4} = \pi R_i^2 \Rightarrow R_i = \frac{D_i}{2} = \frac{1}{16}$ cm (radius of image)

Ans. **(a) 5000 W/m²** **(b) 0.02 W/m²** **(c) 0.125 W/m²** **(d) 288 W/m²**

Q.13 A thin plano-convex lens fits exactly into a plano concave lens with their plane surfaces parallel to each other as shown in the figure. The radius of curvature of the curved surface R = 30 cm. The lenses are made of different materials having refractive indices μ_1 = 3/2 and μ_2 = 5/4 as shown in the figure.



(i) if plane surface of the plano-convex lens is slivered, then calculate the equivalent focal length of this system and also calculate the nature of the equivalent mirror.

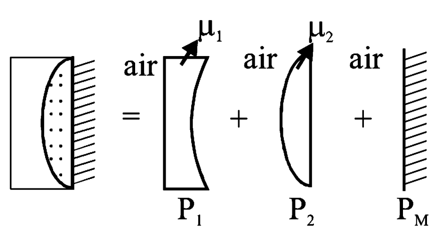
(ii) An object having transverse length 5 cm in placed on the axis of equivalent mirror (in part1), at a distance 15 cm from the equivalent mirror along the principal axis. Find the transverse magnification produced by equivalent mirror.

Sol. $P_{eq} = 2\,(P_1 + P_2) + P_M$

$P_1 = \frac{1}{f_1}$, where $\frac{1}{f_1} = \left(\frac{\mu_1 - 1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{30}\right)$ (By lens maker's formula)

Then, $P_1 = \left(-\frac{1}{60}\right) \Rightarrow P_2 = \frac{1}{f_2} \Rightarrow$ where $\frac{1}{f_2} = \left(\frac{\mu_2 - 1}{1}\right)\left(\frac{1}{30} - \frac{1}{\infty}\right)$

Then $P_2 = \left(\frac{1}{120}\right)$



$P_M = -\frac{1}{f_M} = 0$ (since $f_M = \frac{R}{2} = \frac{\infty}{2}$)

$P_{eq} = 2\,(P_1 + P_2) + P_M \Rightarrow P_{eq} = 2\left(\frac{-1}{60} + \frac{1}{120}\right) + 0 = -\frac{1}{60} \Rightarrow$ Now, $\quad P_e = -\frac{1}{f_e}$

$\Rightarrow f_e = -\frac{1}{P_e} = +60$ cm $\Rightarrow$ Nature of equivalent mirror in convex mirror

(ii) For the equivalent mirror, we can apply the mirror formula to find the final position of the image.

$\frac{1}{v_f}+\frac{1}{u_i}=\frac{1}{f_e} \Rightarrow \frac{1}{v_f}+\frac{1}{-15}=\frac{1}{60}$ (since $f_e = +60$), $v_f = +12$ cm

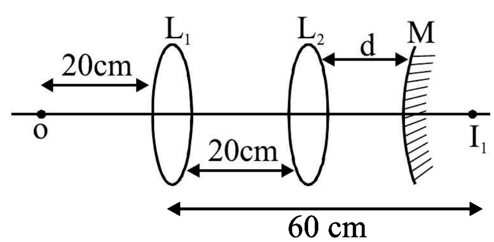
Then, Transvers magnification $= -\frac{v_f}{u_i}=\frac{-12}{-15}=\frac{4}{5}$ **[Ans. +60, + 4/5]**

Q.14 Two identical convex lenses L_1 and L_2 are placed at a distance 20 cm from each other on the common principal axis. The focal length of each lens is 15 cm and the lens L_2 is to the right of lens A. A point object is placed on the common axis of two lenses at a distance of 20 cm on the left of lens L_1, Find, where a convex mirror of radius of curvature 5 cm should be placed so that the final image coincides with the object ?

Sol. For L_1 :

Apply the lens formula for L_1 and take object distance (u) and image distance (v_1) from lens L_1.

$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f}$



$\Rightarrow \frac{1}{v_1}-\frac{1}{-20}=\frac{1}{15}$

$\Rightarrow v_1 = +60$ cm

Image form 60 cm right of lens L_1

For L_2 :

For lens L_2 object is I_1, Then, object distance for lens $L_2 = (60 - 20) = 40$ cm

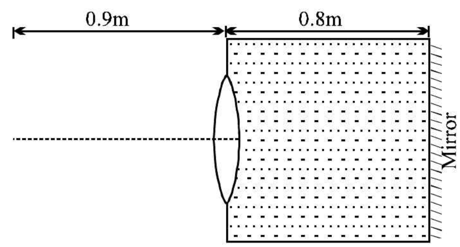
Apply the lens formula for L_2: $\frac{1}{v_2}-\frac{1}{+40}=\frac{1}{15} \Rightarrow v_2 = 10.9$ cm

For auto collination (Final image coincides with the object, v_2) (I_2) must be at centre of curvature of mirror or itself on pole of mirror.

Hence, d = 10.9 cm or (10.9 – 5) = 5.9 cm **[Ans. 5.9 cm, 10.9 cm]**

Q.15 A thin equiconvex lens of glass of refractive index **μ=3/2 & focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water (μ = 4/3). On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in the figure.**

The separation between the lens and the mirror is 0.8 m . A small object is placed outside the tank in front of the lens at a distance of 0.9 m from the lens along its axis . Find the position (relative to the lens) of the image of the object formed by the system. [JEE ' 97]



Sol. From lens Maker's formula

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \text{we have } \frac{1}{0.3} = \left(\frac{3}{2}-1\right)\left(\frac{1}{R}-\frac{1}{-R}\right)$$

(Here, $R_1 = R$ and $R_2 = -R$), $R = 0.3$, Now applying $\frac{\mu_2}{v}-\frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$ at the air glass surface, we get $\Rightarrow \frac{3/2}{v_1}-\frac{1}{-(0.9)} = \frac{3/2-1}{0.3}$

i.e., first image I_1 will be formed at 2.7 m from the lens. This will act as the virtual object for glass water surface. Therefore, applying $\frac{\mu_2}{v}-\frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$ at glass water surface, we have $\frac{4/3}{v_2}-\frac{3/2}{2.7} = \frac{4/3-3/2}{-0.3}$, $v_2 = 1.2$ m

i.e., second image I_2 is formed at 1.2 m from the lens or 0.4 m from the plane mirror. This will act as a virtual object for mirror. Therefore, third real image I_3 will be formed at a distance of 0.4 m in front of the mirror after reflection from it. Now this image will work as a real object for water-glass interface. Hence, applying

$$\frac{\mu_2}{v}-\frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$$

We get $\dfrac{3/2}{v_4} - \dfrac{4/3}{-(0.8-0.4)} = \dfrac{3/2-4/3}{0.3}$

$\therefore \quad v_4 = -0.54$ m

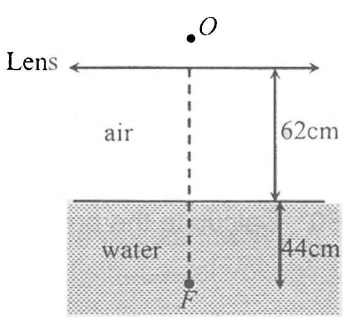
i.e., fourth image is formed to the right of the lens at a distance of 0.54 m from it. Now finally applying the same formula for glass-air surface,

$$\frac{1}{v_5} - \frac{3/2}{-0.54} = \frac{1-3/2}{-0.3}$$

$v_5 = -0.9$ m

i.e., position of final image is 0.9 m relative to the lens (towards right) or the image is formed 0.1 m behind the mirror. **[Ans. 90 cm from the lens towards right]**

Q.16 A stationary observer O looking at a fish F in water ($\mu_w = 4/3$) through a converging lens of focal length 90.0 cm. The lens is allowed to fall freely from a height 62.0 cm with its axis vertical. The fish and the observer are on the principal axis of the lens. The fish moves up with constant velocity 100 cm/s. Initially it was at a depth of 44.0 cm. Find the velocity (in cm/s) with which the fish appears to move with respect to lens, to the observer at t = 0.2 sec.(take g = 10 m/s²)



Sol. t = 0.2 sec, velocity of lens $V_1 = gt = 2$ m/s (downwards)

$\therefore$ for lens, the fish appears to approach with a speed of $2 + \left(1 \times \dfrac{3}{4}\right) = \dfrac{11}{4}$ m/s

At a distance of $\left[42 + \dfrac{24}{\left(\dfrac{4}{3}\right)}\right] = 60$ cm

$\therefore$ Image of fish w.r.t. lens,

$$V = \frac{-60 \times 90}{-60 + 90} = -180 \text{ cm.}$$

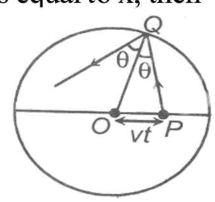
$\therefore$ Velocity of image w.r.t. lens

$$V_1 = \left(\frac{v^2}{u^2}\right)\frac{du}{dt} = \left(\frac{-180}{-60}\right)^2 \times \frac{11}{4} = \frac{99}{4} \text{ m/s} = 2475 \text{ cm/s}$$

Q.17 A point light source is moving with a constant velocity v inside a transparent thin spherical shell of radius R, which is filled with a transparent liquid. If at t = 0, light source is at the centre of the sphere, then at what time a thin dark ring will be visible to an observer outside the sphere. The refractive index of liquid with respect to that of the shell is $\sqrt{2}$.

Sol. Thin dark ring will be visible if ray from source suffers total internal reflection from the spherical shell. Let the source at any instant be at point P then at point Q ray will be totally reflected, if θ is equal to or greater than the critical angle. If QP is equal to x, then

$$z = \cos\theta = \frac{R^2 + x^2 - v^2t^2}{2Rx}$$

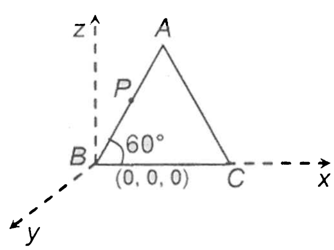


Q.18 An equilateral prism ABC is placed in air with its base side BC lying horizontally along x-axis as shown in the figure. A ray given by $\sqrt{3}z + x = 10$ is incident at a point P on the face AB of the prism.

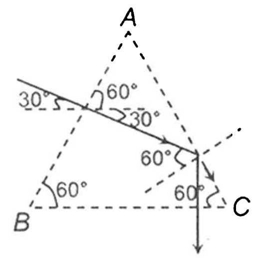
(a) Find the value of μ for which the ray grazes the face AC.

(b) Find the direction of the finally refracted ray if $\mu = \frac{3}{2}$.

(c) Find the equation of ray coming out of the prism if bottom BC is silvered ?



Sol. (a) $\sqrt{3}z + x = 10 \Rightarrow x = -\sqrt{3}z + 10 \Rightarrow \tan\theta = -\dfrac{1}{\sqrt{3}} \qquad \Rightarrow \theta = -30^\circ$



Thus the ray is incident normally on the face AB.

The angle of incidence on face AC is 60° . For

grazing $\mu \sin 60^\circ = 1 \sin 90^\circ \quad \Rightarrow \mu = \dfrac{2}{\sqrt{3}}$

(b) For $\mu = \dfrac{3}{2}$, the ray gets internally reflected. The ray is normal on face BC.

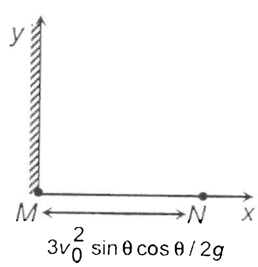
Hence finally refracted ray is parallel to z-axis.

(c) If BC is silvered, the ray will retrace its path. Hence equation of ray coming out of the prism is $\sqrt{3}z + x = 10$

Q.19 A plane mirror is placed at origin in the y-z plane as shown in the figure. A particle A of mass m is projected from origin with velocity v_0 at an angle θ with the horizontal in the x-y plane and another particle B with v_0 at $(180^\circ - \theta)$ with the horizontal in + x-direction from the point N in the same plane simultaneously.

(a) Find the velocity of the image of A w.r.t. the image of B when B is at $x = v_0^2 \sin\theta \cos\theta / 2g$.

(b) If the collision is perfectly inelastic, find the velocity of the image of A just after collision along x-axis.



Sol. (a) $\dfrac{3v_0^2 \sin\theta.\cos\theta}{2g} - \dfrac{v_0^2 \sin\theta.\cos\theta}{2g} = \dfrac{v_0^2 \sin\theta.\cos\theta}{g} = \dfrac{\text{Horizontal range}}{2}$

Thus particle B is at the highest point.

So will be the particle A

$\therefore\ \vec{V}_A = V\cos\theta\ \hat{i}$ and $\vec{V}_B = -V_0 \cos\theta\ \hat{i} \quad \therefore\ \vec{V}_A =$ velocity of image of A $= -v_0 \cos\theta\ \hat{i}$

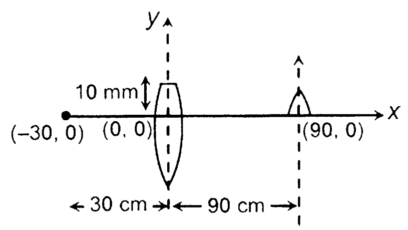
and V_B = velocity of image of B = + $v_0 \cos\theta\,\hat{i}$

$\therefore\ \vec{V}_{A'B'} = \vec{V}_{A'} - V_{B'} = -v_0\cos\theta\,\hat{i} - v_0\cos\theta\,\hat{i} = -2v_0\cos\theta\,\hat{i}$

(b) For a perfectly elastic collision, A and B will become a system. COM along x - axis.

$mv_x + (-mv_x) = 2mv'_x \Rightarrow$ Velocity of image of A along x just after collision = 0

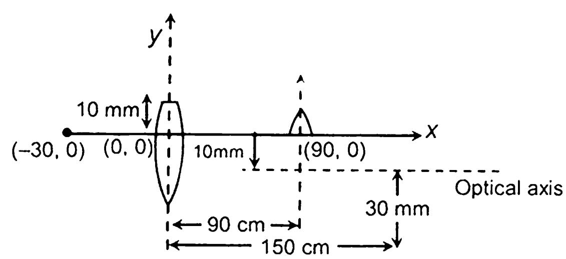
Q.20 A thin convex lens having focal length 20 cm is cut into two parts, 10 mm above the principle axis. The lower portion is placed with optical centre at the origin and upper portion at (90, 0) as shown in the figure. A point object is placed at (– 30, 0). Find the coordinates of the final image. Assuming paraxial ray approximation to remaim valid.



Sol. For first lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60} \Rightarrow v_1 = 60$ cm

For the second lens u = 30 cm, f = +20 cm

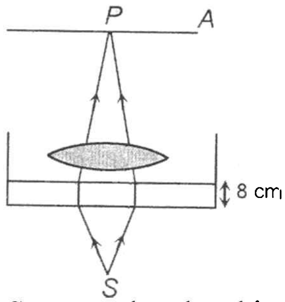
$\frac{1}{f} = \frac{1}{v_2} - \frac{1}{u} \Rightarrow \frac{1}{V} = \frac{1}{20} - \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{3-2}{60} = \frac{1}{60}$



$\therefore\ v_2 = 60$ cm $\Rightarrow m = -2$ So co-ordinates are (150 cm , – 30 mm)

Q.21 A thin convex lens of refractive index μ = 1.5 is placed between a point source of light S and a screen A, as shown in the figure. Light rays from the source S are brought to focus on the screen A, forming a point image P. The distance SP is equal to 50 cm. Water $\left(\mu = \frac{4}{3}\right)$ is now poured into a vessel interposed between the object and the lens, and it is observed that when the water level is 8 cm the screen

has to be moved up by a distance of 6 cm in order to get a sharp image. Find the focal length of the lens.

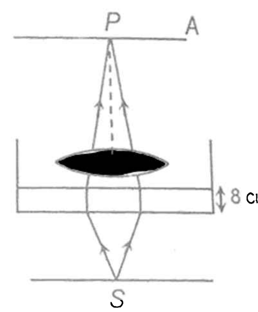


Sol. Suppose that the object distance (between the object and the lens) be u and the image distance v. Then,

$$|u| + |u| = \frac{1}{f} \text{......(i)}$$

When water is poured into the vessel,

$$\frac{1}{(|v|+6)} - \frac{1}{(|u|-2)} = \frac{1}{f} \text{.....(ii)}$$



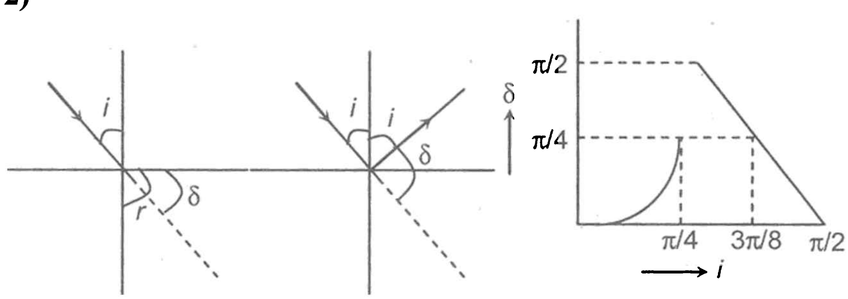
Solving the equation, we get

$|u| = 20\text{cm}$

$|v| = 30$ cm and $f = 12$ cm

Q.22 A ray of light is incident from a medium of refractive index $\sqrt{2}$ into vacuum. Find the range of angle of incidence for which the deviation suffered by the ray has a unique value (i.e. the same deviation does not occur for any other value of i) ($0 < i < \pi/2$)

Sol.



For $0 < i \leq i_c$ the angle of deviation is given by

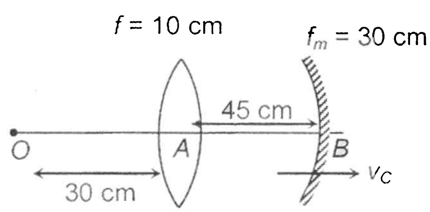
$\delta = r - i$ for $i = 0, r = 0 \Rightarrow \delta = 0$ For $i = i_c = \frac{\pi}{4}, r = \frac{\pi}{2}$

$\Rightarrow \delta = \frac{\pi}{4}$ Thus as i increases from 0 to $\frac{\pi}{4}$, δ increases continously from 0 to $\frac{\pi}{4}$.

For $i_c \left(= \frac{\pi}{4}\right) < i < \frac{\pi}{2}$, Total internal reflection takes place and angle of deviation is given by $\delta = \pi - 2i$. For this range of i, δ decreases continuously from $\frac{\pi}{2}$ to 0.

Hence unique values of δ are from $\frac{\pi}{2}$ to $\frac{\pi}{4}$ and these occur for the values of i (for reflection) in the range $\left(\frac{\pi}{4}, \frac{3\pi}{8}\right)$.

Q.23 An object O is placed 30 cm left of a convex lens of focal length 10 cm. A concave mirror starts moving towards right from point B and has a velocity $v_C = \frac{(45-x)^2}{300}$ at any point on the axis. Here x is the separation between the lens and the mirror.
(a) Find the velocity of the image (with respect to the ground) formed by the mirror.
(b) Also find the magnification of this image. Express your answer in terms of x.



Sol. $\frac{1}{u} + \frac{1}{v_1} = \frac{1}{f'} \Rightarrow v_1 = \frac{uf'}{u - f'} = \frac{30 \times 10}{20} = 15\text{cm}$

For mirror: let at any instant the mirror be x distance away from the lens.

The image formed by lens will act as object for the mirror.

$\therefore \frac{1}{V_2} - \frac{1}{x - v_1} = -\frac{1}{f'}$ $\therefore v_2 = \frac{(x-15)30}{(45-x)}$

$\therefore$ Speed of image is

$$\frac{dv_2}{dt} = \frac{(45-x)30 + (x-15)30}{(45-x)^2}\frac{dx}{dt} \quad \frac{dv_2}{dt} = \frac{900}{(45-x)^2} \times \frac{(45-x)^2}{300} = 3 \text{ m/s}$$

Velocity in ground frame $= 3 + \dfrac{(45-x)^2}{300}$

Magnificantion due to lens

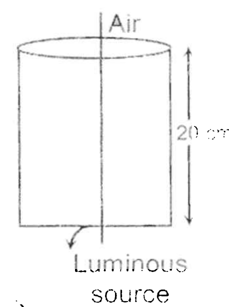
$$|m_1| = \left|\frac{v_1}{u}\right| + \frac{1}{2}$$

Magnification due to mirror $\quad |m_2| = \left|\dfrac{(x-15)(30)}{(45-x)(x-15)}\right| = \left|\dfrac{30}{45-x}\right|$

$\therefore$ Net magnification at that point

$$|m| = |m_1||m_2| = +\frac{1}{2} \times \left|\frac{(30)}{(45-x)}\right| = \frac{15}{45-x}$$

Q.24 A thin equiconvex glass lens ($\mu_g = 1.5$) is being placed on the top of a vessel of height h = 20 cm as shown in the figure. A luminous point source is being placed at the bottom of the vessel on the principal axis of the lens. When the air is on both the sides of the lens, the image of luminous source is formed at a distance of 20 cm from the lens out side the vessel. When the air inside the vessel is being replaced by a liquid of refractive index μ_1, the image of the same source is being formed at a distance 30 cm from the lens outside the vessel. Find the μ_1,



Sol. For the first case

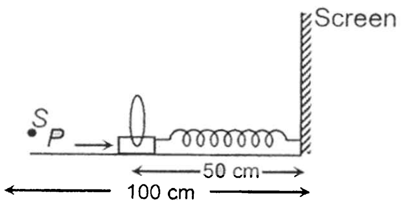
$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \frac{1}{20} - \frac{1}{-20} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow R = 10 \text{ cm}$$

For the second case, when the vessel is being filled with the liquid then

$$\frac{\mu_{air}}{v_1} - \frac{\mu_1}{u} = \frac{\mu_g - \mu_1}{R} + \frac{\mu_{air} - \mu_g}{-R} \qquad \frac{1}{30} - \frac{\mu_1}{-20} = \frac{1.5 - \mu_1}{10} + \frac{1 - 1.5}{-10}$$

$$\frac{\mu_1}{20} + \frac{\mu_1}{10} = \frac{3}{20} + \frac{1}{20} - \frac{1}{30} \qquad \frac{3\mu_1}{20} = \frac{1}{6} \qquad \mu_1 = \frac{10}{9} = 1.11$$

Q.25 A point object is located at a distance of 100 cm from a screen. A lens of focal length 23 cm mounted on a movable frictionless stand is kept between the source and the screen. The stand is attached to a spring of natural length 50 cm and spring constant 800 N/m as shown. Mass of the stand with lens is 2 kg. How much impulse P should be imparted to the stand so that a real image of the object is formed on the screen after a fixed time gap. Also find this time gap also. (Neglect the width of the stand)



Sol. Let the distance of the lens from the object be L when a real image is formed on the screen.

Then $\dfrac{1}{100-L} - \dfrac{1}{-L} = \dfrac{1}{23}$ On solving, we get $L = (50 \pm 10\sqrt{2})$ cm

Now, if the lens performs SHM and a real image is formed after a fixed time gap then this time gap must be one fourth of the time period.

$\therefore$ Phase difference between the two positions of real image must be $\dfrac{\pi}{2}$. As the two positions are symmetrically located about the origin, phase difference of any of these positions from origin must be $\dfrac{\pi}{4}$.

$\Rightarrow 10\sqrt{2}$ cm $= A \sin\dfrac{\pi}{4} \Rightarrow A = 20$ cm

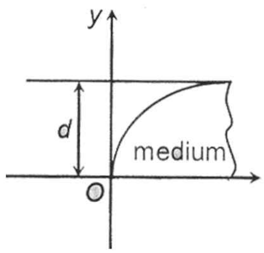
To achieve this velocity at the mean position

$$V_0 = A_\omega = A\sqrt{\frac{K}{m}}$$

$\therefore$ Required impulse $J = mv_0 = A\sqrt{Km} = 8$ kgm/s.

Q.26 A long rectangular slab of transparent medium of thickness d is placed on a table with length parallel to x-axis and width parallel to the y-axis. A ray of light is traveling along y-axis at origin. The refractive index μ of the medium varies as $\mu = \sqrt{1+e^{x/d}}$. The refractive index of the air is 1.

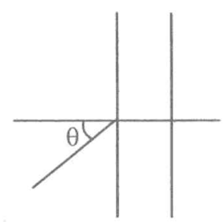
(a) Determine the x-coordinate of the point A, where the ray intersects the upper surface of the slab-air boundary.
(b) Write down the refractive index of the medium at A.



Sol. $\mu \sin\theta = 1 \times \sin 90^\circ \qquad \sin\theta = \dfrac{1}{\mu} \qquad \tan\theta = \dfrac{1}{\sqrt{\mu^2 - 1}} = \dfrac{1}{\sqrt{1 + e^{x/d} - 1}} = e^{-x/2d}$

$\dfrac{dy}{dx} = e^{-x/2d} \qquad y = 2d\,(1 - e^{-x/2d}) \qquad \text{When } y = d, \text{ then} \qquad \dfrac{1}{2} = 1 - e^{\frac{x}{2d}}$

$e^{\frac{x}{2d}} = 2 \qquad x = 2d\ \text{In}2$



Then, $\mu = \sqrt{1 + e^{\frac{2d\,\text{In}2}{d}}} = \sqrt{1+4} = \sqrt{5}$

Multiple Choice Questions

Fully Solved—With Only One Choice Correct

ONLY ONE OPTION IS CORRECT TYPE

Take approx. 2 minutes for answering each question.

1. **Two plane mirrors are inclined at 70°. A ray incident on one mirror at an angle θ after reflection falls on the second mirror and is reflected from there parallel to the first mirror, θ is :**

(A) 50° **(B) 45°** **(C) 30°** **(D) 55°**

2. **There are two plane mirrors with reflecting surfaces facing each other. Both the mirrors are moving with speed v away from each other. A point object is placed between the mirrors. The velocity of the image formed due to n-th reflection will be**

(A) nv **(B) 2nv** **(C) 2v** **(D) n/6**

3. **A man of height 'h' is walking away from a street lamp with a constant speed 'v'. The height of the street lamp is 3h. The rate at which the length of the man's shadow is increasing when he is at a distance 10h from the base of the street lamp is :**

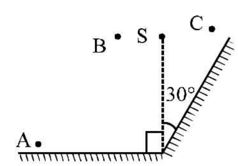
(A) v/2 **(B) v/3** **(C) 2v** **(D) v/6**

4. **A boy of height 1.5 m with his eye level at 1.4 m stands before a plane mirror of length 0.75 m fixed on the wall. The height of the lower edge of the mirror above the floor is 0.8 m. Then :**

(A) the boy will see his full image **(B) the boy cannot see his hair**

(C) the boy cannot see his feet **(D) the boy can see neither his hair nor his feet.**

5. **A point source of light S is placed in front of two large mirrors as shown. Which of the following observers will see only one image of S?**



(A) only A **(B) only C**

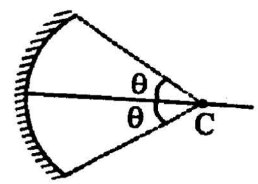
(C) Both A and C **(D) Both B and C**

6. **The reflection surface of a plane mirror is vertical. A particle is projected in a vertical plane which is also perpendicular to the mirror. The initial velocity of the particle is 10 m/s and the angle of projection is 60°. The point of projection is at a distance 5 m from the mirror. The particle moves towards the mirror. Just before the particle touches the mirror, the velocity of approach of the particle and its image is :**

(A) 10 m/s **(B) 5 m/s** **(C) $10\sqrt{3}$ m/s** **(D) $5\sqrt{3}$ m/s**

7. **The circular boundary of the concave mirror subtends a cone of half angle θ at its centre of curvature.**

The minimum value of θ for which ray incident on this mirror parallel to the principal axis suffers more than one reflection is



(A) 30^0 **(B) 45^0** **(C) 60^0** **(D) 75^0**

8. **The pasenger side-view mirror on an automobile often has the notation. "Objects seen in mirror are closer than they appear". Is the image really farther away than the object ?**

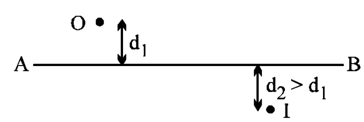
(A) Yes, the image is smaller and farther away than the object

(B) No, the image is smaller and closer than the object

(C) No, the image is larger and closer than the object

(D) Yes, the image is larger and farther away than the object

9. **In the figure shown, the image of a real object is formed at point I. AB is the principal axis of the mirror. The mirror must be :**



(A) concave & placed towards right I

(B) concave & placed towards left of I

(C) convex and placed towards right of I

(D) convex & placed towards left of I.

10. **A straight line joining the object point and the image point is always perpendicular to the mirror**

(A) if mirror is plane **(B) if mirror is concave**

(C) if mirror is convex **(D) irrespective of the type of mirror.**

11. **An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no gap between the images formed by the two mirrors. The radius of the convex mirror is :**

(A) 12.5 cm **(B) 25 cm** **(C) 50 cm** **(D) 100 cm**

12. **An infinitely long rod lies along the axis of a concave mirror of focal length f. The near end of the rod is at a distance u > f from the mirror. Its image will have a length**

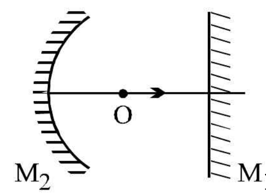
(A) $\frac{f^2}{u-f}$ (B) $\frac{uf}{u-f}$ (C) $\frac{f^2}{u+f}$ (D) $\frac{uf}{u+f}$

13. **A luminous point object is moving along the principal axis of a concave mirror of focal length 12 cm towards the mirror. When its distance from the mirror is 20 cm its velocity is 4 cm/s. The velocity of the image in cm/s at that instant is :**

(A) 6 towards the mirror **(B) 6 away from the mirror**

(C) 9 away from the mirror **(D) 9 towards the mirror**

14. **In the figure shown, if the object 'O' moves towards the plane mirror, then the image I (which is formed after successive reflections from M_1 & M_2 respectively) will move :**



(A) towards right **(B) towards left**

(C) with zero velocity **(D) cannot be determined**

15. **A point source of light is 60 cm from a screen and is kept at the focus of a concave mirror which reflects light on the screen. The focal length of the mirror is 20 cm. The ratio of average intensities of the illumination on the screen when the mirror is present and when the mirror is removed is :**

(A) 36 : 1 **(B) 37 : 1** **(C) 49 : 1** **(D) 10:1**

16. **Ray of sunlight enters a spherical water droplet (n = 4/3) at an angle of incidence 53° measured with respect to the normal to the surface. It is reflected from the back surface of the droplet and re-enters into air. The angle between the incoming and the outgoing ray is [Take sin 53° = 0.8]**

(A) 15° **(B) 34°** **(C) 138°** **(D) 30°**

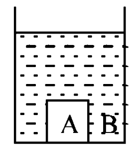
17. **A ray of light moving along the unit vector (– i – 2j) undergoes refraction at an interface of two media, which is the x-z plane. The refractive index for y > 0 is 2 while for y < 0, it is $\sqrt{5}/2$. The unit vector along which the refracted ray moves is :**

(A) $\frac{(-3\hat{i}-5\hat{j})}{\sqrt{34}}$ **(B)** $\frac{(-4\hat{i}-3\hat{j})}{5}$ **(C)** $\frac{(-3\hat{i}-4\hat{j})}{5}$ **(D) None of these**

18. **A bird is flying 3 m above the surface of water. If the bird is diving vertically down with speed = 6 m/s, its apparent velocity as seen by a stationary fish underwater is :**

(A) 8 m/s **(B) 6 m/s** **(C) 12 m/s** **(D) 4 m/s**

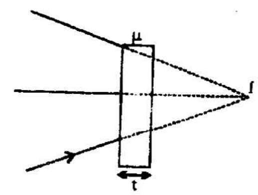
19. **A parallel sided block of glass of refractive index 1.5 which is 36 mm thick rests on the floor of a tank which is filled with water (refractive index = 4/3). The difference between apparent depth of floor at A & B when seen from vertically above is equal to**



(A) 2 mm **(B) 3 mm**

(C) 4 mm **(D) none of these**

20. **A beam of light is converging towards a point I. A plane parallel plate of glass of thickness t, refractive index μ is introduced in the path of the beam. The convergent point is shifted by (assume near normal incidence) :**



(A) $t\left(1-\frac{1}{\mu}\right)$ away (B) $t\left(1+\frac{1}{\mu}\right)$ away (C) $t\left(1-\frac{1}{\mu}\right)$ nearer (D) $t\left(1+\frac{1}{\mu}\right)$ nearer

21. **A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It forms a real image, also located at C (a condition called auto-collimation). If the mirror is now filled with water, the image will be:**

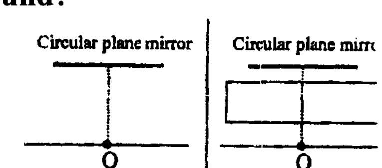
(A) real, and will remain at C

(B) real, and located at a point between C and ∞

(C) virtual, and located at a point between C and O.

(D) real, and located at a point between C and O.

22. **In the diagram shown below, a point source O is placed vertically below the centre of a circular plane mirror. The light rays starting from the source are reflected back from the mirror such that a circular region A on the ground receives light. Now, a glass slab is placed between the mirror and the source O. What will be the magnitude of the new area receiving light on the ground?**



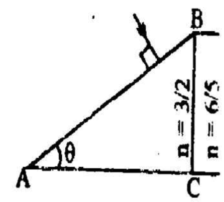
(A) A

(B) greater than A

(C) Less than A

(D) Cannot tell, as the information given is insufficien

23. **In the figure ABC is the cross section of a right angled prism and BCDE is the cross section of a glass slab. The value of θ such that light incident normally on the face AB does not cross the face BC is (given $\sin^{-1}(3/5) = 37^0$)**



(A) $\theta \le 37^0$ **(B)** $\theta < 37^0$ **(C)** $\theta \le 53^0$ **(D)** $\theta < 53^0$

24. **A small source of light is 4 m below the surface of a liquid of refractive index 5/3. In order to cut off all the light coming out of liquid surface, minimum diameter of the disc placed on the surface of liquid is :**

(A) 3m **(B) 4m** **(C) 6m** **(D)** ∞

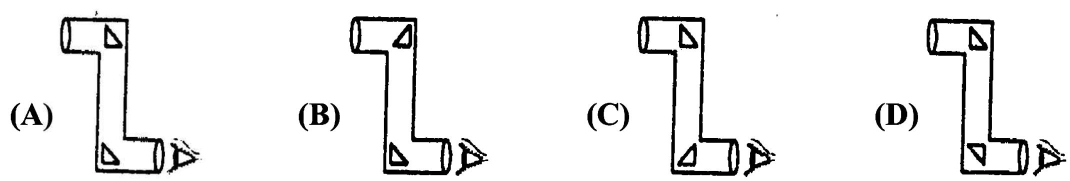
25. **The x-z plane separates two media A and B with refractive indices μ_1 and μ_2 respectively. A ray of light travels from A to B. Its directions in the two media are given by the unit vectors, $\vec{r}_A = a\hat{i} + b\hat{j}$ & $\vec{r}_B = \alpha\hat{i} + \beta\hat{j}$ respectively, where $\hat{i}$ & $\hat{j}$ are unit vectors in the x and y directions. Then**

(A) $\mu_1 a = \mu_2 \alpha$ **(B)** $\mu_1 \alpha = \mu_2 a$ **(C)** $\mu_1 b = \mu_2 \beta$ **(D)** $\mu_1 \beta = \mu_2 b$

26. **A vertical pencil of rays comes from bottom of a tank filled with a liquid. When it is accelerated with an acceleration of 7.5 m/s^2, the ray is seen to be totally reflected by liquid surface. What is the minimum possible refractive index of liquid?**

(A) slightly greater than 4/3 **(B) slightly greater than 5/3**
(C) slightly greater than 1.5 **(D) slightly greater than 1.75**

27. **Two glass blocks of triangular cross section (n =1.5) are used to make a periscope. Which of the following is the correct arrangement?**



28. **A ray of light from a denser medium strike a rarer medium. The angle of reflection is r and that of refraction is r'. The reflected and refracted rays make an angle of 90° with each other. The critical angle will be :**

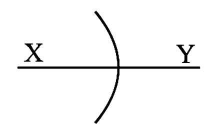
(A) $\sin^{-1}(\tan r)$ **(B)** $\tan^{-1}(\sin r)$ **(C)** $\sin^{-1}(\tan r')$ **(D)** $\tan^{-1}(\sin r')$

29. **Consider a common mirage formed by super-heated air just above a roadway. A truck driver is in a medium of $\mu = 1.0003$ looks forward. He perceives the illusion of a patch of**

water ahead on the road, where his line of sight makes an angle of 1.20^0 below the horizontal. Find the index of refraction of the air just above the road surface. (Hint : Treat this as a problem in total internal reflection.)

(A) 1.00006 (B) 1.0001 (C) 1.00008 (D) none of these

30. **A concave spherical surface of radius of curvature 10cm separates two media x & y of refractive index 4/3 & 3/2 respectively. If the object is placed along the principal axis in medium X then**

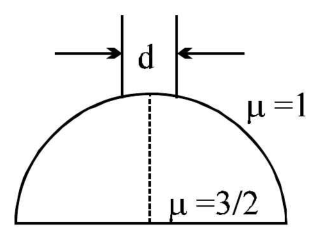


(A) image is always real
(B) image is real if the object distance is greater than 90cm
(C) image is always virtual
(D) image is virtual if the object distance is less than 90cm

31. **A fish is near the centre of a spherical water ($\mu = 4/3$) fish bowl filled with water. A child stands in air at a distance 2R (R is the radius of curvature of the sphere) from the centre of the bowl. At what distance from the centre would the child's nose appear to the fish situated at the centre:**

(A) 4R (B) 2R (C) 3R (D) 4R

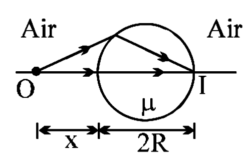
32. **A beam of diameter 'd' is incident on a glass hemisphere as shown. If the radius of curvature of the hemisphere is very large in comparison to d, then the diameter of the beam at the base of the hemisphere will be :**



(A) $\frac{3}{4}$ d (B) d (C) $\frac{d}{3}$ (D) $\frac{2}{3}$ d

Question No. 33 to 35(4 questions)

The figure, shows a transparent sphere of radius R and refractive index μ. An object O is placed at a distance x from the pole of the first surface, so that a real image is formed at the pole of the exactly opposite surface.



33. **If x = 2R, then the value of μ is**

(A) 1.5 (B) 2 (C) 3 (D) none of these

34. **If $x = \infty$, then the value of μ is**

(A) 1.5 **(B) 2** **(C) 3** **(D) none of these**

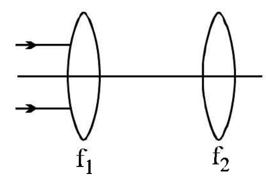
35. **If an object is placed at a distance R from the pole of first surface, then the real image is formed at a distance R from the pole of the second surface. The refractive index μ of the sphere is given by**

(A) 1.5 **(B) 2** **(C) $\sqrt{2}$** **(D) none of these**

36. **When the object is at distances u_1 and u_2, the images formed by the same lens are real and virtual respectively and of the same size. Then focal length of the lens is :**

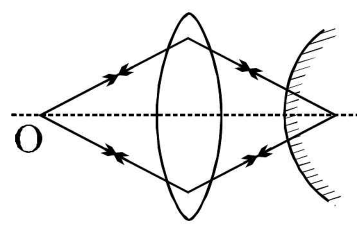
(A) $\frac{1}{2}\sqrt{u_1u_2}$ **(B)** $\frac{1}{2}(u_1+u_2)$ **(C)** $\sqrt{u_1u_2}$ **(D)** $2(u_1+u_2)$

37. **Parallel beam of light is incident on a system of two convex lenses of focal lengths f_1 = 20 cm and f_2 = 10 cm. What should be the distance between the two lenses so that rays after refraction from both the lenses pass undeviated :**



(A) 60 cm **(B) 30 cm** **(C) 90 cm** **(D) 40 cm**

38. **An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at its focus such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is**



(A) 20 cm **(B) 10 cm** **(C) 15 cm** **(D) 30 cm**

39. **A thin lens of focal length f and its aperture has a diameter d. It forms an image of intensity I. Now the central part of the aperture upto diameter (d/2) is blocked by an opaque paper. The focal length and image intensity would change to**

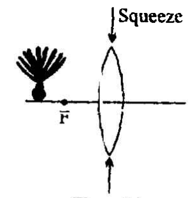
(A) f/2, I/2 **(B) f, I/4** **(C) 3f/4, I/2** **(D) f, 3I/4**

40. **An object is placed in front of a thin convex lens of focal length 30 cm and a plane mirror is placed 15 cm behind the lens. If the final image of the object coincides with the object, the distance of the object from the lens is**

(A) 60 cm **(B) 30 cm** **(C) 15 cm** **(D) 25 cm**

Question No. 41 to 43 (3 question)

A turnip is placed before a thin converging lens, outside the focal point of the lens. The lens is filled with a transparent gel so that it is flexible; by squeezing its ends toward its centre [as indicated in figure (a)], you can change the curvature of its front and rear sides.



Figı re(a)

41. When you squeezing the lens, the image.
(A) moves towards the lens (B) moves away from the lens
(C) shifts up (D) remains as it is

42. When you squeeze the lens, the lateral height of image.
(A) increases (B) decreases (C) remains same (D) data insufficient

43. Suppose a sharp image must be formed on a card which is at a certain distance behind the lens [figure(b)], while you move the turnip away from the lens, then you should
(A) decrease the squeeze of the lens
(B) increase the squeeze of the lens
(C) keep the card and lens as it is.
(D) move the card away from the lens

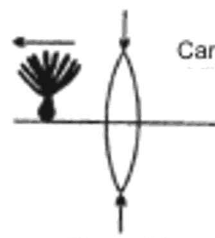
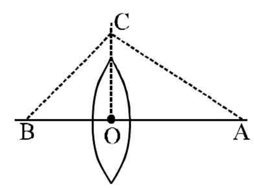


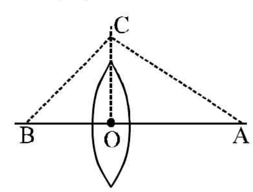
Figure (b)

44. If an object is placed at A (OA>f), where f is the focal length of the lens the image is found to be formed at B. A perpendicular is erected at O and C is chosen on it such that the angle ∠BCA is a right angle. The value of f will be
(A) AB/OC^2 (B) (AC)(BC)/OC (C) OC^2/AB (D) (OC)(AB)/AC+BC



45. The height of the image formed by a converging lens on a screen is 8cm. For the same position of the object and screen again, an image of size 12.5cm is formed on the screen by shifting the lens. The height of the object :
(A) 625/32cm (B) 64/12.5cm (C) 10cm (D) none

46. A converging lens of focal length 20 cm and diameter 5 cm is cut along the line AB. The part of the lens shown shaded in the diagram is now used to form an image of a point P placed 30 cm away from it on the line XY, which is perpendicular to the plane of the lens. The image of P will be formed.
(A) 0.5 cm above XY (B) 1 cm below XY (C) on XY (D) 1.5 cm below XY



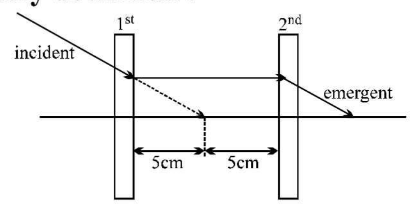
47. **A screen is placed at a distance of 90 cm from an bject. The image of this object is formed on the screen is formed by a convex lens at two different locations separated by 20 cm. The focal length of the lens is**

(A) 18 cm **(B) 21.4 cm** **(C) 60 cm** **(D) 85.6 cm**

48. **In the above problem, if the size of the images formed at the positions are 6 cm and 3 cm, then the highest object is**

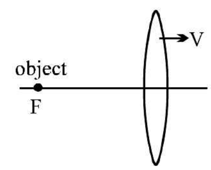
(A) 4.2 cm **(B) 4.5 cm** **(C) 5 cm** **(D) none of these**

49. **Look at the ray diagram shown, what will be the focal length of the 1st and the 2nd lens, if the incident light ray passes without any deviation ?**



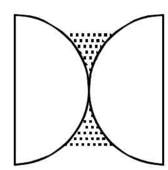
(A) –5cm and –10cm
(B) +5cm and +10cm
(C) –5cm and +5cm
(D) +5cm and +5cm

50. A point object is kept at the first focus of a convex lens. If the lens starts moving towards right with a constant velocity, the image will



(A) always move towards right
(B) always move towards left
(C) first move towards right & then towards left.
(D) first move towards left & then towards right.

51. Two planoconvex lenses each of focal length 10 cm & refractive index 3/2 are placed as shown. water (R.I. = 4/3 is filled between). The whole arrangement is in air. The optical power of the system is (in diapters):



(A) 6.67 (B) – 6.67 (C) 33.3 (D) 20

52. **The curvature radii of a concavo-convex glass lens are 20 cm and 60 cm. The convex surface of the lens is silvered. With the lens horizontal, the concave surface is filled with water. The focal length of the effective mirror is (μ of glass = 1.5, μ of water = 4/3)**

(A) 90/13 cm **(B) 80/13 cm** **(C) 20/3 cm** **(D) 45/8 cm**

53. **An object is placed in front of a symmetrical convex lens with refractive index 1.5 and radius of curvature 40 cm. The surface of the lens further away from the object is silvered. Under auto-collimation condition, the object distance is**

(A) 20 cm **(B) 10 cm** **(C) 40 cm** **(D) 5 cm**

54. **A planoconvex lens, when silvered at its plane surface is equivalent to a concave mirror of focal length 28cm. When its curved surface is silvered and the plane**

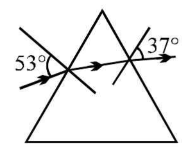
surface not silvered, it is equivalent to a concave mirror of focal length 10cm, then the refractive index of the material of the lens is :

(A) 9/14 (B) 14/9 (C) 17/9 (D) none

55. **A beam of monochromatic light is incident at $i = 50^0$ on one face of an equilateral prism, the angle of emergence is 40^0, then the angle of minimum deviation is:**

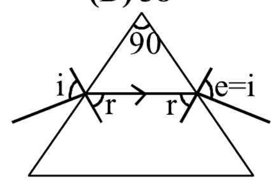
(A) 30^0 (B) $<30^0$ (C) $\leq 30^0$ (D) $\geq 30^0$

56. **A ray incident at an angle 53° on a prism emerges at an angle 37° as shown. If the angle of incidence is made 50°, which of the following is a possible value for the angle of emergence.**



(A) 35° (B) 42° (C) 40° (D) 38°

57. **A prism has a refractive index $\sqrt{\frac{3}{2}}$ and refracting angle 90°. Find the minimum deviation produced by the prism.**



(A) 40° (B) 45° (C) 30° (D) 49°

58. **A certtain prism is found to produce a minimum deviation of 38^0. It produces a deviation of 44^0 when the angle of incidence is either 42^0 or 62^0. What will be the angle of incidence when it undergoes minimum deviation?**

(A) 45° (B) 49° (C) 40° (D) 55°

59. **A thin prism of angle 5° is placed at a distance 10 cm from an object. What is the distance of the image from the object? (Given μ of prism = 1.5)**

(A) $\frac{\pi}{8}$ cm (B) $\frac{\pi}{12}$ cm (C) $\frac{5\pi}{36}$ cm (D) $\frac{\pi}{7}$ cm

60. **Light ray is incident on a prism of angle A = 60° and refractive index $\mu = \sqrt{2}$. The angle of incidence at which the emergent ray grazes the surface is given by**

(A) $\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ (B) $\sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$ (C) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

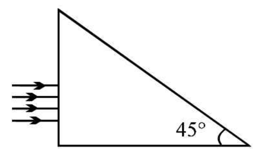
61. **When a monochromatic light ray is incident on a medium of refractive index μ with angle of incident θ_i, the angle of refraction is θ_r, If θ_i is hanged slightly by $\Delta\theta_i$, then the corresponding change in θ_r will be**

(A) $\Delta\theta_i$ (B) $\Delta\mu\theta_i$ (C) $\frac{1}{\mu}\frac{\cos\theta_i}{\cos\theta_r}\Delta\theta_i$ (D) $\mu\frac{\sin\theta_i}{\sin\theta_r}\Delta\theta_i$

62. **Two lenses in contact made of materials with dispersive powers in the ratio 2 : 1, behaves as an achromatic lens of focal length 10 cm. The individual focal lengths of the lense are:**

(A) 5 cm, –10 cm **(B) – 5 cm, 10 cm** **(C) 10 cm, – 20 cm** **(D) – 20 cm, 10 cm**

63. **A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will :**



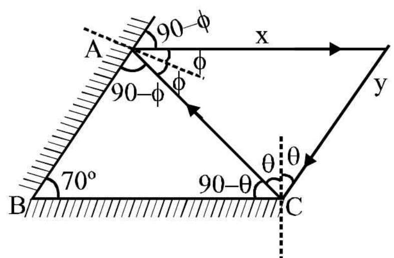
(A) separate part of the red color from the green and blue colors.
(B) separate part of the blue color from the red and green colours.
(C) separate all the three colors from the other two colors.
(D) not separate even partially any color from the other two colors.

Q.64 **It is desired to make an achromatic combination of two lenses (L_1 & L_2) made of materials having dispersive powers ω_1 and ω_2 ($<\omega_1$). If the combination of lenses is converging then**

(A) L_1 is converging **(B) L_2 is converging**
(C) Power of L_1 is greater than the power of L_2 **(D) None of these**

Solutions

1. (A) Reflected ray x is parallel to the base mirror BC.
In ΔABC, sum of the interior angles in 180.
$90 - \theta + 70 + 90 - \phi = 180^\circ$
$\Rightarrow \theta + \phi = 70^\circ$ .... (i)
Also ray x is || to the base mirror BC. So,
$90 - \phi = 70^\circ$
$\Rightarrow \phi = 20^\circ$ .... (ii)
From (i) & (ii), $\theta = 50^\circ$



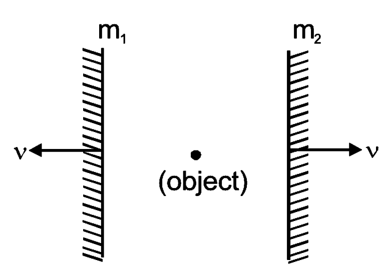
2. (B) For the plane mirror

$$\left(\overrightarrow{V_{I/m}}\right)_\perp = -\left(\overrightarrow{V_{O/m}}\right)_\perp$$

$$\Rightarrow \left(\vec{V}_{I/g} - \vec{V}_{m/g}\right)_\perp = -\left(\vec{V}_{obj/g} - \vec{V}_{m/g}\right)_\perp$$

(only perpendicular components)
On solving the above euqation,

$$\left(\vec{V}_{m/g}\right)_\perp = \frac{\left(\vec{V}_{obj/g}\right)_\perp + \left(\vec{V}_{I/g}\right)_\perp}{2}$$



Applying the above for mirror m_1 (first reflection),

$$-V = \frac{o + V_{I/g}}{2} \Rightarrow V_{I/g} = -2V = 2V(\leftarrow)$$

This image behaves like an object for reflection in mirror m_2.

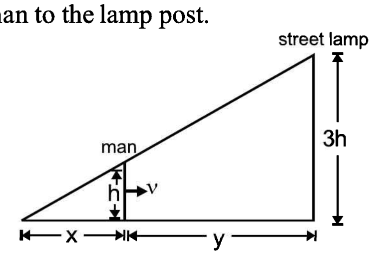
$$\text{So, } v = \frac{-2v + V_{I/g}}{2} \Rightarrow V_{I/g} = 4V\ (\rightarrow)$$

Now, this image is the object for third reflection in mirror m_1., $-v = \dfrac{1v + V_{I/g}}{2}$

$\Rightarrow V_{I/g} = 2 - 6V = 6V\ (\leftarrow)$, Similarly, for nth reflection, $V_{I/g} = 2nv$.

3.(A) 'x' is the length of the shadow when 'y' is the distance from the man to the lamp post.

By applying similar triangle concept,



$$\frac{h}{3h} = \frac{x}{x+y} \Rightarrow h\,(x + y) = 3k\,x$$

$$\Rightarrow x + y = 3x \Rightarrow h = \frac{y}{2}$$

On differentiating the above w.r.t. time, $\dfrac{dx}{dt} = \dfrac{1}{2} \cdot \dfrac{dy}{dt} = \dfrac{v}{2}$

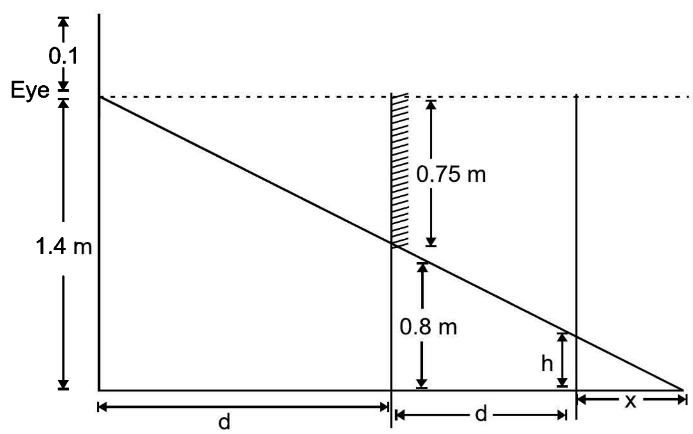
4.(C) $\dfrac{d+x}{0.8} = \dfrac{2d+x}{1.4}$ (By similar triangle) $\Rightarrow 1.4\,d + 1.4\,x = 1.6\,d + 0.8\,x \Rightarrow x = \dfrac{d}{3}$

$\therefore$ By similar triangles,

$$\frac{x}{h} = \frac{x+d}{0.8}$$

$$\frac{d}{3h} = \frac{4d}{2.4}$$

$$h = \frac{2.4}{3 \times 4} = 0.2\ \text{m}$$

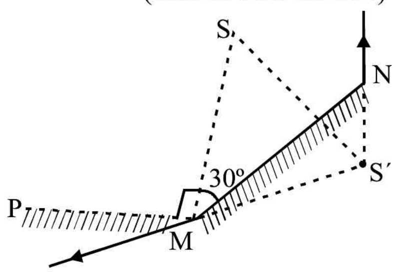


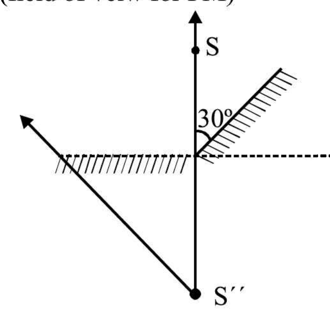
Hence the person cannot see his feet.

(fig.1)
(field of veiw for MN)

(fig.2)
(field of veiw for PM)

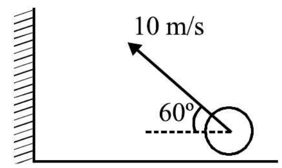
5. (B)

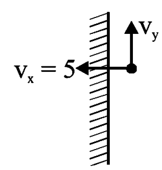




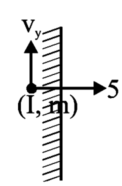
S′ and S′′ are the images of S w.r.t. Mirrors MN & PM respectively. After drawing the field of veiws of S for both the mirrors, we find that B and A lie on the field of veiw of both the mirrors whereas C lies on the field of veiw of only mirror MN. So, C will see only one image.

6.(A) Initially
$$\left[\begin{aligned} u_x &= 10\cos 60^\circ \\ &= \frac{10}{2} = 5 \\ u_y &= 10\cos 60^\circ \\ &= 5\sqrt{3} \end{aligned}\right.$$





Velocity of object w.r.t. mirror before striking the mirror,
(v_x remains unchanged however v_y changes w.r.t. time.)

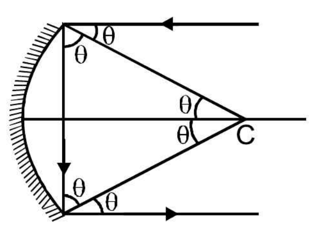


Velocity of image w.r.t. mirror before first collision :
So, velocity of approach of image and particle is 5 + 5 = 10 m/s

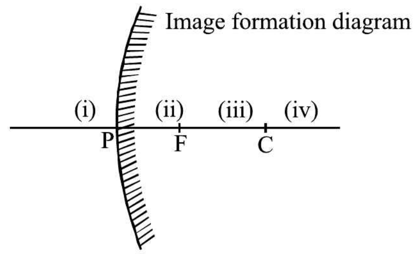
7.(B) $2\theta = 90^\circ$

$\Rightarrow \theta = 45^\circ$

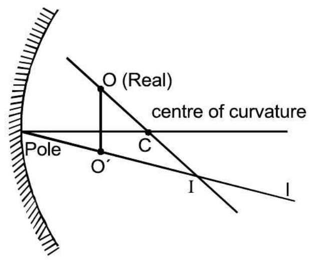
θ is min if incident ray and
reflected ray are mutually perpendicular



8.(B) Image of region (i) will always form at region (ii) & will be smaller in size. $\therefore$ It is smaller and closer than the object Automobiles have convex mirror and we know that a convex mirror forms virtual image which is diminished and situated between Pole and Focus



9.(B) The line joining object and image will cut principal axis at C & extension of object below the principal axis on joining with the image given the pole of the mirror (position of mirror).



10.(D) Straight line joining the object and the image is $\perp$ to the mirror because it passes through center of curvature in spherical mirror and in case of a plane mirror, it is a property that plane mirror bisects line joining the object and the image.

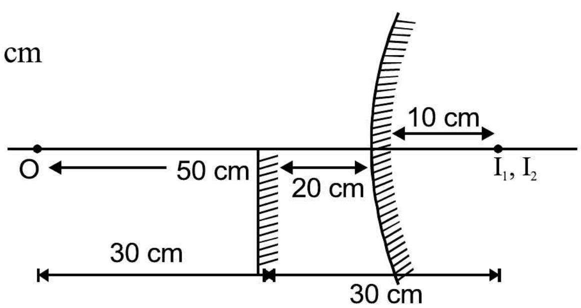
11.(B) I_1 is the image formed by the plane mirror at 30 cm behind the plane mirror & I_2 in the image formed by the convex mirror

From the above diagram

For the convex mirror u = – 50 cm

v = + 10 cm we have to find f

Applying mirror formula,



$$\frac{1}{-50}+\frac{1}{10}=\frac{1}{f}$$

$$\Rightarrow \frac{4}{50}=\frac{1}{f}, R = 2f = 25 \text{ cm}$$

12.(A) at infinity

Object diagram

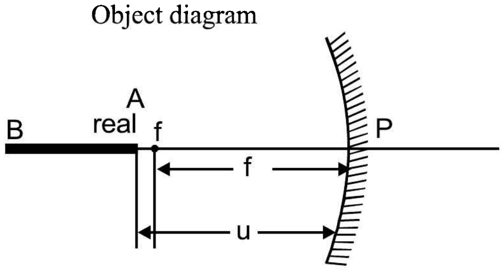


Image diagram

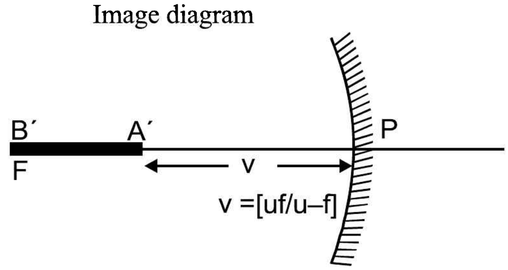
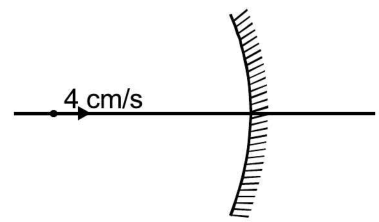


Image of B will form at focus since it is at infinity. Image of A will be at A which can be calculated by mirror formula.

$$\frac{1}{v}+\frac{1}{-u}=\frac{1}{-f} \Rightarrow v=\frac{fu}{f-u}=-\left[\frac{fu}{u-f}\right] \text{ Image length} = f - v = f-\left(\frac{fu}{f-u}\right)=\frac{f^2}{u-f}$$

13.(C) Given that u = – 20 ; f = – 12



$$\frac{1}{-20}+\frac{1}{v}=\frac{-1}{12} \text{ (mirror formula)} \Rightarrow \frac{1}{v}=\frac{1}{20}-\frac{1}{12}$$

$$\frac{1}{v}=\frac{3-5}{60} \Rightarrow v=-30$$

From velocity formula (perpendicular conponents for mirror)

$$(V_{I/m})_{\perp}=\frac{-v^2}{u^2}(V_{o/m})_{\perp} \Rightarrow (V_{I/m})_{\perp}=-\left(\frac{900}{400}\right)\times 4=-9=9 \text{ m/s (away from mirror)}$$

14.(A) I_1 is the image of O formed by a mirror M_1. I_2 is the image formed by a concave mirror M_2 for which I_1 is the object. Applying velocity formula for the concave mirror M_2, ($\perp$ components)

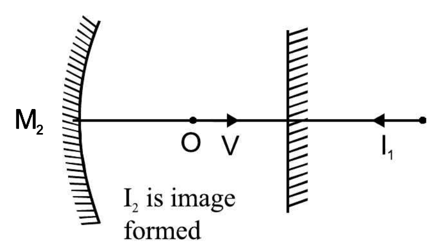
$$(V_{I/m_2})_{\perp}=-\frac{v^2}{u^2}(V_{O/m_2})$$

$$(V_{I/m_2})_{\perp}=-\frac{v^2}{u^2}(-v)$$

[since $(V_{o/m_2})=-v$ for mirror m_2]

So, $(V_{I/m_2})_{\perp}=\oplus$

[+ve quantity and is towards right]

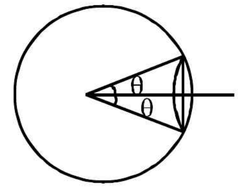


I_2 is image formed of I_1 from reflection on M_2

15.(D) Intensity $=\dfrac{\text{Energy}}{\text{Time}\times\text{Area}}=\dfrac{\text{Power}}{\text{Area}}$, area of solid sector of sphere $=\int_0^{\theta} 2\pi R\sin\theta . R d\theta$

$$= 2\pi R^2(1-\cos\theta), \text{ Power incident} = \frac{P}{4\pi R^2}\times(\text{Area of the solid sector})$$

$$=\frac{P}{4\pi R^2}\times 2\pi R^2(1-\cos\theta)=\frac{P}{2}(1-1+\frac{\theta^2}{2}+....)$$

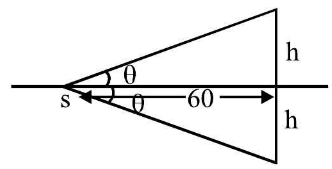


$(\because \cos\theta = 1 - \frac{\theta^2}{2} + \ldots) = \frac{P\theta^2}{4}$

$$\text{Intensity incident} = \frac{P\theta^2}{4\times(\text{area on which light is incident})} \quad \text{......... (i)}$$

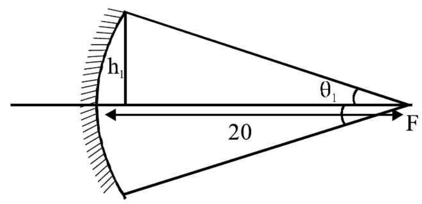
When the mirror is not present, light is reaching the screen upto high h. Maximum area on which light is incident $= \pi h^2$.

From equation (i)



$$\text{Intensity} = \frac{P\theta^2}{4\times\pi h^2} \text{ and } \tan\theta \simeq \theta \simeq \frac{h}{60} = \frac{ph^2}{4\pi h^2\times(60)^2}$$

$$= \frac{p}{4\pi\times(60)^2}$$



When the mirror in present,

$$\text{Similarly, Intensity} = \frac{P\theta_I^2}{4\pi h_I^2}$$

$$\text{and } \theta_1 \simeq \frac{h_I}{20} \Rightarrow \text{Intensity} = \frac{P}{4\pi(20)^2} \Rightarrow \therefore \text{ Ratio} = \frac{\text{Intensity when mirror present}}{\text{Intensity when mirror object}}$$

(when mirror is present, then intensity is reaching the screen directly from source and also

$$\text{from reflection on the mirror}) = \frac{\frac{P}{4\pi(20)^2} + \frac{P}{4\pi(60)^2}}{\frac{P}{4\pi(20)^2}} = \frac{10}{1}$$

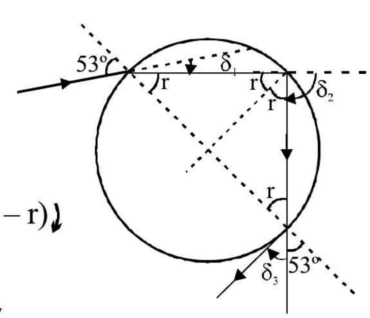
16.(C) Ray diagram according to the question.

δ_1 = angle of deviation of the incident ray after refraction.

δ_2 = angle of deviation of the ray after refraction from back surface.

δ_3 = angle of deviation of the energent ray.

Applying Snell's Law :



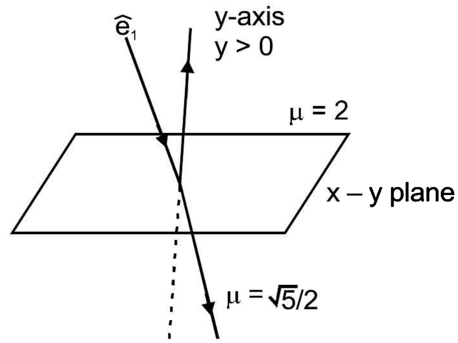
$1 \sin 53 = \frac{4}{3}\sin r \quad \sin r = \frac{3}{5} \quad r = 37^\circ$

$\delta_{net} = (\delta_1 + \delta_2 + \delta_3) = (53 - r) + (180 - 2r) + (53 - r)$

[taking clockwise direction]

$= (53 - 37) + (180 - (2\times 37)) + (53 - 37) = 138^\circ$

17.(B) Let $\hat{e}_1$ = unit vector along the incident ray.



$$= \frac{-\hat{i}-2\hat{j}}{\sqrt{5}} = \frac{-1}{\sqrt{5}}\hat{i} - \frac{-2}{\sqrt{5}}\hat{j}$$

Let $\hat{n}$ = unit vector along normal = $\hat{j}$

Let $\hat{e}_2$ = unit vector along the refracted ray.

let $\hat{e}_2 = a\hat{i} + b\hat{j} + c\hat{k}$

where, $a^2 + b^2 + c^2 = 1$

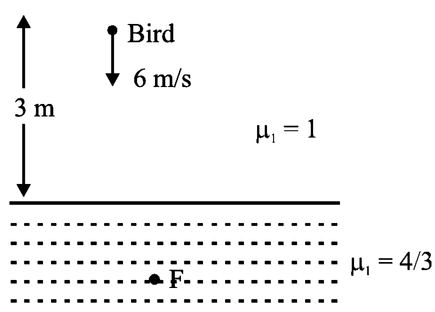
On applying Snell's law in the vector form : $\mu_1\ (\hat{e}_1 \times \hat{n}) = \mu_2\ (\hat{e}_2 \times \hat{n})$

$$2\left[\left(\frac{-1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right) \times (\hat{j})\right] = \frac{\sqrt{5}}{2}\left[(a\hat{i}+b\hat{j}+c\hat{k}) \times (\hat{j})\right].\ 2\left(\frac{-1}{\sqrt{5}}\hat{k} - 0\right) = \frac{\sqrt{5}}{2}\left[a\hat{k} - c\hat{i}\right]$$

On comparing coefficient of $\hat{i}$, $\hat{j}$, $\hat{k}$ on both sides : $\frac{-2}{\sqrt{5}} = \frac{a\sqrt{5}}{2} \Rightarrow a = \left(\frac{-4}{5}\right)$; $c = 0$

then, $a^2 + b^2 + c^2 = 1 \Rightarrow b = \pm\frac{3}{5}$, then value of $b = -\frac{3}{5}$ (since projection of $\hat{e}_2$ on the y axis in the –ve direction), $\therefore\ \hat{e}_2 = \left(\frac{-4}{5}\hat{i} - \frac{3}{5}\hat{j}\right)$

18.(A) Using the formula for the refraction only through curved surface :



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Differentiating on both sides w.r.t. time.

$$\mu_2\left(\frac{-1}{v^2}\right)\frac{dv}{dt} - \mu_1\left(\frac{-1}{u^2}\right)\frac{du}{dt} = 0$$

$$\frac{\mu_1}{u_2}\frac{du}{dt} = \frac{\mu_2}{v_2}\frac{dv}{dt}$$

$$\frac{dv}{dt} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v^2}{u^2}\right)\left(\frac{du}{dt}\right)$$

[$\frac{dv}{dt}$ = velocity of image w.r.t. surface ; $\frac{du}{dt}$ = velocity of object w.r.t. surface]

Using the formula for plane surface (whose $R = \infty$)

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0 \Rightarrow \frac{\mu_2}{v} = \frac{\mu_1}{u} \Rightarrow \frac{\mu_1}{\mu_2} = \frac{u}{v}$$

$$\Rightarrow \frac{du}{dt} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\mu_2}{\mu_1}\right)^2\left(\frac{du}{dt}\right) \Rightarrow \frac{dv}{dt} = \left(\frac{\mu_2}{\mu_1}\right)\left(\frac{dv}{dt}\right) \Rightarrow \frac{dv}{dt} = \left(\frac{\frac{4}{3}}{1}\right)(-6) = \frac{4}{3}\times -6^2 = -8\text{ m/s}$$

Here,–ve sign indicates that the fish is approaching towards the surface of water.

19.(B) A is the surface below the glass block

B is the surface below the water where the block is not present

Part-1 : for Image of floor A on the water, applying

$$\left(\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}\right) \Rightarrow \frac{4}{3v} - \frac{1.5}{(-36)} = 0 \text{ (since } R = \infty)$$

$\Rightarrow$ v = 32 mm below the top surface of glass block.

Again applying $\left(\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}\right)$ to find the final image of A:

$$\frac{1}{v_1} + \frac{4}{3(h-4)} = 0 \text{ (since, } R = \infty) \Rightarrow v_1 = \frac{3(4-h)}{4}$$

Part-2 : Image of floor B below the water surface (directly). Applying

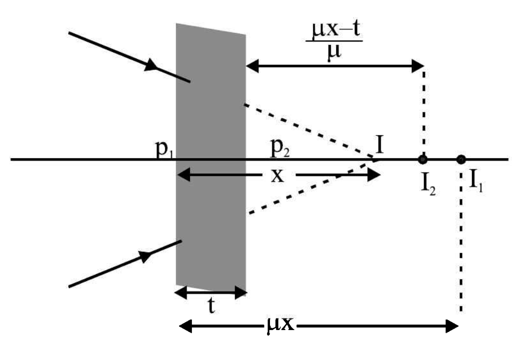
$$\left(\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}\right) \Rightarrow \frac{1}{v} + \frac{4}{3h} = 0 \Rightarrow v_2 = \frac{-3h}{4}$$

$$\therefore \text{ Difference in apparent depths} = \frac{3(4-h)}{4} + \frac{3h}{4} = 3\text{ mm}$$

20.(A) Here the object is a virtual object.

[I_1 = Image formed after refraction by p_1 ; I_2 = Image formed after refraction by p_2]

for p_1 : Applying $\left(\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}\right)$



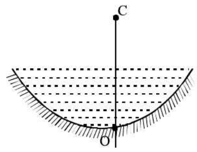
$$\frac{\mu}{v_1} - \frac{1}{+x} = 0 \quad \text{(since, } R = \infty)$$

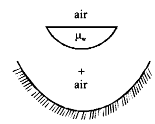
$$v_1 = +\mu x$$

for p_2 : Applying $\left(\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}\right)$

$$\frac{\mu}{v_2} - \frac{\mu}{+(\mu x - t)} = 0 \quad \text{(since, } R = \infty)$$

$v_2 = \left(\dfrac{\mu x - t}{\mu}\right)$, Shift of image $= P_2I_2 - P_2I = I_2I = \left(\dfrac{\mu x - t}{\mu}\right) - (x - t) = x - \dfrac{t}{\mu} + t - x \;.= t\left(1 - \dfrac{1}{\mu}\right)$ (along the incident ray)

21.(D) 

$\Rightarrow$ 

$p_e = 2P_L + P_M$

$$P_L = \frac{1}{f_L} \text{ where } \frac{1}{f_L} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-R}\right) \Rightarrow P_M = -\frac{1}{f_m} = +\left(\frac{2}{R}\right);\; P_L = \frac{1}{3R}$$

$$P_e = 2P_L + P_M = 2\left(\frac{1}{3R}\right) + \left(\frac{2}{R}\right) = \frac{8}{3R} \Rightarrow f_e = \frac{-1}{P_e} = \frac{-3R}{8} \Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f_e}$$

[Applying miror formula for the equivalent system, since equivalent system behave like a mirror]

$$\frac{1}{v} + \frac{1}{-R} = \frac{1}{-3R/8} \Rightarrow \frac{1}{v} = \frac{-8}{3R} + \frac{1}{R} = \frac{-16+6}{6R} \Rightarrow v = \frac{-6R}{10} = \frac{-3R}{5} = \frac{-3R}{5}$$

–ve sign indicates that the image is formed below C. Since the value is less than R, the final image is formed between C and O and the image is real.

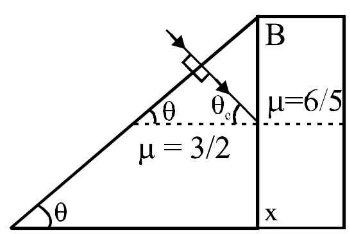
22.(A) The shift experienced by the rays cutering the glass slab will be cancelled by the shift experienced by the light rays that re-enter the glass slab after suffering reflection from the mirror. So the net shift is zero and the area illuminated will remain the same.

23.(A) $\sin\theta_c = \dfrac{\mu_r}{\mu_d}$ (Definition of critical angle)

[$\mu_r = \mu$ of rarer medium ; $\mu_d = \mu$ of denser medium]

θ_c is the critical angle for the BC boundary., $\sin\theta_c = \dfrac{6}{5} \times \dfrac{2}{3} = \dfrac{4}{5} \Rightarrow \theta_c = 53^\circ$

Let us calculate θ when light ray just graes the boundary xy.



$\theta = \dfrac{\pi}{2} - \theta_c = 90^\circ - 53^\circ = 37^\circ$,

Since the ray should not cross xy, so, $\theta \leq 37^\circ$.

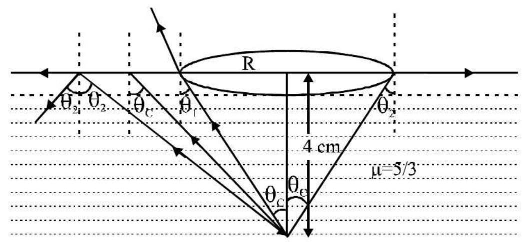
24.(C) In the fig. $\theta_1 < \theta_c < \theta_2$,

For angles greater than or equal to θ_c, light will not emerge out of surface.

For θ_1, light will come out of the surface., $\sin\theta_c = \frac{\mu_r}{\mu_d}$ (at the surface where refraction takes place) $\Rightarrow \sin\theta_c = \frac{1}{5/3} = 3/5 \Rightarrow \theta_c = 37°$, From the triangle,

$\tan\theta_c = \frac{R}{4} \Rightarrow R = \frac{3}{4} \times 4 = 3\text{m}.$

$\therefore$ diameter of the spherical part $= 2R = 6\text{m}.$

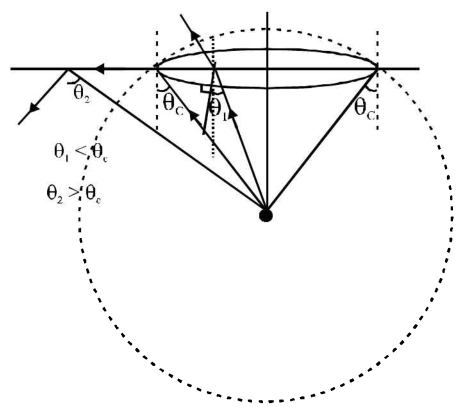


25.(D) Consider a sphere covering the maximum area that receives light from the source. Now, the power that emerges out from the surface of the liqid will be equal to the power emerging from the surface of the sphere.

$\therefore$ power emerging $= \frac{P}{4\pi R^2} \times$ [Area of the solid sector subtending a semi vertical angle θ_c]

where P is the total power of the source.

and $\sin\theta_c = \frac{\mu \text{ of rarer medium}}{\mu \text{ of denser medium}} = \frac{3}{4}$



$\therefore$ Power emerging $= \frac{P}{4\pi R^2}(2\pi R^2(1-\cos\theta_c))$

$= \frac{P(1-\cos\theta_c)}{2} = \frac{P}{2}\left(1 - \frac{\sqrt{7}}{4}\right)$

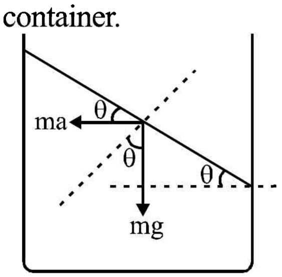
$\therefore$ percentage of power output $= 16.91\% \simeq 17\%$

26.(B) This is the position of the liquid surface w.r.t. frame of the tank.

Since any liquid surface cannot sustain any tangential force w.r.t. container.

$ma\cos\theta = mg\sin\theta$

$\tan\theta = \frac{a}{g} = \frac{7.5}{10} \Rightarrow \tan\theta = \frac{3}{4} = 37° \Rightarrow \theta_c < 37°$ (for T.I.R.)



Taking sine on both sides, we get from the above equation

$$\sin\theta_c < \sin 37^\circ \Rightarrow \frac{\mu_r}{\mu_d} < \sin 37^\circ \Rightarrow \frac{1}{\mu_L} < \frac{3}{5}\ \mu_L > \frac{5}{3}$$

27.(D) The two prisms must be set up in such a way that in both of them light ray suffers total internal reflection. In the prism used, light ray will suffer total internal reflection. Only if ray enters from a face that is not hypotense. Only (D) satisfies the above condition.

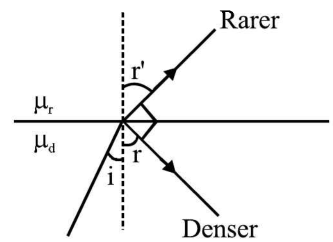
28.(A) From the law of reflection, i = r

From the question, $180^\circ - (r + r') = 90^\circ$

$\Rightarrow r + r' = 90^\circ$ ...... (i)

From Snell's law :

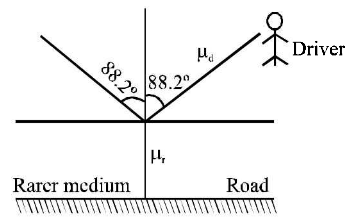
$\mu_d \cdot \sin r = \mu_r \cdot \sin r'$ (since, i = r)



$$\Rightarrow \frac{\mu_r}{\mu_d} = \frac{\sin_r}{\sin_{r'}} = \cot r' \Rightarrow \theta_c = \sin^{-1}\left(\frac{\mu_r}{\mu_d}\right) = \sin^{-1}(\cot r') = \sin^{-1}(\tan r)$$

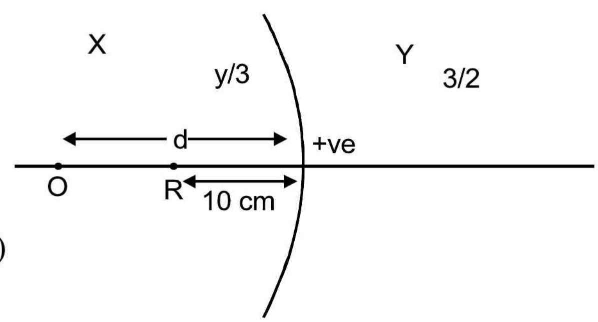
29.(C) The layer of air just above the road will get heated up and become rarer. So, 88.2° is the vertical angle if θ_c is the

vertical angle then, $\sin\theta_c = \dfrac{\mu \text{ of rarer medium}}{\mu \text{ of denser medium}} = \dfrac{\mu_r}{1.003}$



$$\Rightarrow \sin(88.2^\circ) = \frac{\mu_r}{1.003} \Rightarrow \mu_r = 1.0008$$

30.(C) Applying $\left(\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}\right)$



$$\frac{3}{2v} - \frac{4}{3u} = \frac{\frac{3}{2} - \frac{4}{3}}{-10}$$

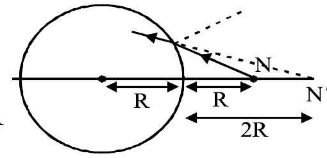
$$\frac{3}{2v} + \frac{4}{3d} = \frac{1}{-60} \qquad \text{(since } u = -d\text{)}$$

$$\frac{3}{2v} = \frac{-4}{3d} - \frac{1}{60} \Rightarrow \frac{3}{2v} = -\left(\frac{4}{3d} + \frac{1}{60}\right) \Rightarrow \frac{3}{2v} = -\left(\frac{80+d}{3060d}\right) \Rightarrow v = \frac{90d}{-(80+d)}$$

v is always –ve ⇒ virtual image always

31.(C) Here nose of the boy is the object & fish is the observer. So $\mu_2 = \mu_w$ and $\mu_1 = \mu_a = 1$
Applying the formula of refraction at curved surface.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{4}{3v} - \frac{1}{-R} = \frac{\frac{4}{3} - 1}{R} \Rightarrow v = -2R$$



That means image is formed on the same side as the nose of child
(N' is the image of nose) = 3R from the centre

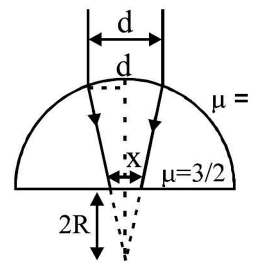
32.(D) Applying the formula for spherical curved surface :

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{3}{2v} - \frac{1}{\infty} = \frac{\frac{3}{2} - 1}{R}$$

$\Rightarrow v = 3R$

Let x be the diameter of the light falling at base.

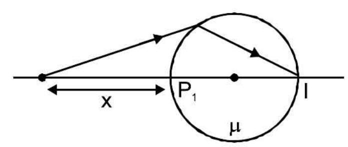
$$\frac{d}{x} = \frac{3R}{2R} \text{ (similar triangle concept)} \Rightarrow x = \frac{2}{3}d.$$



33.(C) Applying the formula of curved surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$\mu_2 = \mu$, $\mu_1 = 1$



$u = -x = -2R$ (given in the question), $v = +2R \Rightarrow \dfrac{\mu}{2R} - \dfrac{1}{-2R} = \dfrac{\mu - 1}{+R} \Rightarrow$ on solving, $\mu = 3$.

34.(B) $\mu_2 = \mu$, $\mu_1 = 1$, $x = \infty$, $R = +R$, (According to the question and diagram)

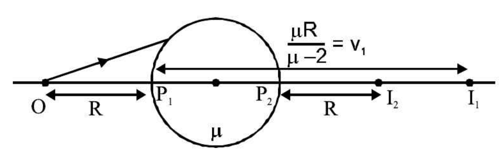
$$\Rightarrow \frac{\mu}{2R} - \frac{1}{-\infty} = \frac{\mu - 1}{R} \Rightarrow \mu = 2\mu - 2 \Rightarrow \mu = 2$$

35.(B) I_1 = Image from the pole P_1 , I_2 = Image from the pole P_2

For P_1, applying : $\left(\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}\right)$

$$\frac{\mu}{v_1} - \frac{1}{-R} = \frac{\mu - 1}{+R} \Rightarrow \frac{\mu}{v_1} + \frac{1}{R} = \frac{\mu - 1}{R} \Rightarrow \frac{\mu}{v_1} = \frac{\mu - 2}{R} \Rightarrow v_1 = \left(\frac{\mu R}{\mu - 2}\right)$$

For P_2, applying : $\left(\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}\right)$

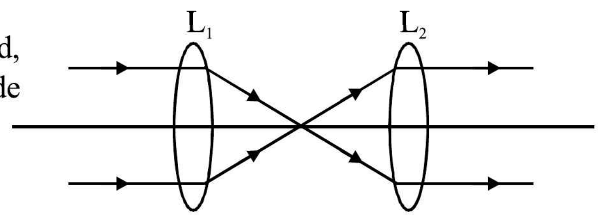


$$\Rightarrow \frac{1}{+R} - \frac{\mu}{\left(\frac{\mu R}{\mu - 2} - 2R\right)} = \frac{1-\mu}{-R}$$ On Solving ; $\mu = 2$.

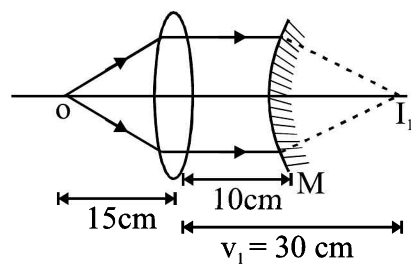
36.(B) Magnification, $m = \frac{f}{f+u}$ [use sign convention in this formula]

According to the question, $m_1 = -m_2 \Rightarrow \left(\frac{f}{f-u_1}\right) = -\left(\frac{f}{f-u_2}\right) \Rightarrow f = \frac{u_1 + u_2}{2}$

37.(B) For passing through system undeviated, second focus of lens (L_1) must coincide with primary focus of lens (L_2),
$\therefore\ d = f_1 + f_2 = 30$ cm



38.(B) For autocollination, final image coincides with the object. $V_1(I_1)$ for the mirror must be at centre of curvature of mirror or itself on the pole of mirror.
For the lens



$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v_1} - \frac{1}{-15} = \frac{1}{10} \Rightarrow v_1 = +30 \text{ cm}$$

Then, I_1 must be at the centre of curvature of the mirror and is not possible at the pole itself.

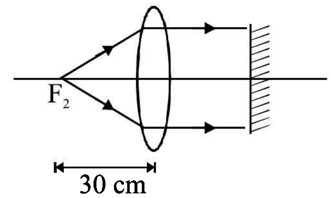
$\Rightarrow \quad R = 20$ cm, $f = 10$ cm

39.(D) Focal length of lens does not depends on the aperture of lens. It only depends on the **(i)** Refractive index of μ_c and μ_m **(ii)** Radius of curvature
For intensity, $I \propto (\text{aperture diameter})^2$

$$\Rightarrow I_{blocked} \propto \left(\frac{d}{2}\right)^2 \text{ and } I_0 \propto d^2$$

Hence, $I_{blocked} = \frac{I_0}{4}$. Final image intensity $= I_{original} - I_{blocked} = I_0 - \frac{I_0}{4} = \frac{3I_0}{4}$

40.(B) For autocallination by system, light should fall normally on plane mirror, since mirror is $\perp$r to principal axis, light must be parallel to principle axis. Hence the object must lie on secondary focus of the lens. From the Ray diagram:
u = 30 cm



41.(A) By squeezing the lens R (↓) and keeping object position (u) fixed
By the lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) \Rightarrow \text{when R (}\downarrow\text{), f (}\downarrow\text{), From then lens formula : } v = \frac{uf}{u+f} = \frac{u}{\frac{u}{f}+1}$$

when f(↓), v (↓), That means, image moves towards the lens.

42.(B) For the lateral height, $h_i = mh_0$, Magnification, $m = \frac{f}{f+u} = \frac{1}{1+\frac{u}{f}}$
when R ↓ , f ↓ , m(↓). Then, Lateral height = h_i (↓)

43.(A) Since, image position, is fixed ; by the lens formula :

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{u} + \frac{1}{f}, \text{ Differentiating the above formula on both sides :}$$

$$O = -\frac{1}{u^2}\frac{du}{df} - \frac{1}{f^2}, \frac{du}{df} = -\left(\frac{u}{f}\right)^2 \Rightarrow \frac{du}{df} < 0$$

That means when u ↑, f must be (↓) ; By the lens maker's formula
f(↓), R (↓)
Then, you should increase the squeeze of the lens.

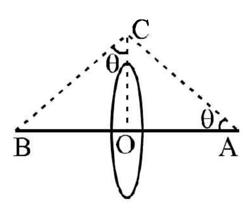
44.(D) Apply lens formula i.e. :

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{OB} - \frac{1}{-OA} = \frac{1}{f} \Rightarrow f = \frac{(OA)(OB)}{OA + OB} = \frac{(OA)(OB)}{AB}$$

Now, OC = BC cos θ and OC = AC sin θ

$(OC)^2 = (BC)\,(AC) \sin\theta \cos\theta = (OB)\,(OA)$

[since, OB = BC sin θ ; OA = AC cos θ] Than, $f = \frac{OC^2}{AB}$



45.(C) By the displacement method : $h_0 = \sqrt{h_1 h_2} = \sqrt{8 \times 12.5} = \sqrt{100} = 10$ cm

46.(D) For the shaded lens principal axis is 0.5 cm below the XY line.

Apply the lens formula :

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{-30}=\frac{1}{20}$$ (Since f remains unchanged),

$\therefore$ v = + 60 cm

$$\text{Now, } m=\frac{v}{u}=\frac{60}{-30}=-2$$

This (–)ve sign shows that the object and image are at opposite side with respect to principal axis

$h_i = mh_o = -1$ cm

Then, Image of P will be formed 1 cm below the principal axis and 1.5 cm below XY.

47.(B) By the displacement method :

$$f=\frac{D^2-d^2}{4D}\Rightarrow f=\frac{90^2-20^2}{4\times 90}=21.4 \text{ cm}$$

48.(A) $h_o = \sqrt{h_1h_2} = \sqrt{6\times 3} = \sqrt{18} = 4.2$ cm

49.(C) Angle of deviation through the lens $\delta=\frac{h}{f}$

if f > 0, δ (clockwise) and if f < 0, δ (anticlockwise)

Since, it is given that the emergent ray is parallel to the incident light ray.

$$S_{net}=O\Rightarrow\delta_1+\delta_2\Rightarrow\delta_1=-\delta_2=\frac{h}{f_1}=\frac{-h}{f_2}\Rightarrow \text{That means, } f_1=f_2$$

50.(D) For the velocity :

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$ [Differentiating this equation w.r.t. time]

$$-\frac{1}{v_2}\frac{dv}{dt}+-\frac{1}{u_2}\frac{du}{dt}=0\Rightarrow\frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}$$

$$(V_{IL})=\frac{v^2}{u^2}(V_{OL})$$ [For the perpendicular component to the lens]

$$V_{IG}-V_{LG}=\frac{v^2}{u^2}(V_{OG}-V_{LG}),\ V_{IG}-V=\frac{v^2}{u^2}(o-v)$$ [since $m=\frac{v}{u}$]

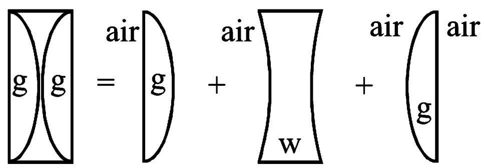
$V_{IG}-V = m^2(o-v)$, $V_{IG} = v(1-m^2)$

case (1), F < object < 2 F, m > 1, case (2), $-\infty$ < object < 2 F, m < 1

Hence, for the case (1), $V_{IG} < 0$, For the case (2), $V_{IG} > 0$

Then, Image will first move to towards left and then move towards right.

51.(A) Equivalent diagram for the system :



$$P_e = P_g + P_w + P_g\,,\ P_g = \frac{1}{f_g} \qquad \text{where, } \frac{1}{f_g} = \left(\frac{\mu_g - 1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\Rightarrow \frac{1}{10} = \frac{1}{2R} \quad \Rightarrow \text{Then, } P_g = \frac{1}{10} \Rightarrow R = 5\text{ cm,}$$

$$\Rightarrow P_w = \frac{1}{f_w} \text{ where, } \frac{1}{f_g} = \left(\frac{\mu_g - 1}{1}\right)\left(\frac{1}{-R} - \frac{1}{-R}\right)$$

$$\Rightarrow \frac{1}{f_w} = \left(\frac{1}{3}\right)\left(-\frac{2}{5}\right) = \frac{-2}{15} \text{ Now, } P_e = 2P_e + P_w \Rightarrow P_e = 2\left(\frac{1}{10}\right) + \left(-\frac{2}{15}\right) = \frac{6-4}{30} = \frac{1}{15(\text{cm})}$$

$$P_e = \frac{1}{15} \times 100 = 6.67 \text{ (diopters)}$$

52.(A) Equivalent diagram of the system :

$$P_e = 2\,(P_w + P_g) + P_m$$

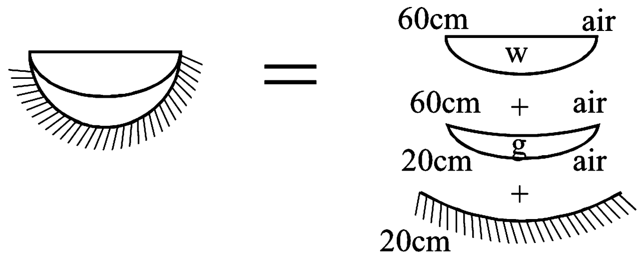
$$P_w = \frac{1}{f_w} \Rightarrow \text{where } \frac{1}{f_g} = \left(\frac{\mu_w - 1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-60}\right)$$

$$\Rightarrow \qquad P_w = \frac{1}{180}\ \frac{1}{f_w} = \frac{1}{180}$$

$$P_g = \frac{1}{f_g}\,, \text{ where } \frac{1}{f_g} = \left(\frac{\mu_g - 1}{1}\right)\left(\frac{1}{-60} - \frac{1}{-20}\right)$$

$$\Rightarrow \qquad P_g = \frac{1}{60}\,,\ \frac{1}{f_g} = \frac{1}{60}\,,\ P_m = \frac{-1}{f_m} = +\frac{2}{20} \text{ (since } f_m = -\frac{20}{2} \text{ concave mirror)}$$

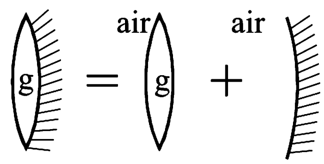
Then, $P_e = 2\,(P_w + P_g) + P_m$, $P_e = 2\left(\frac{1}{180}+\frac{1}{60}\right)+\frac{2}{20} \Rightarrow P_e = \frac{13}{90}$



Now, $P_e = -\frac{1}{f_e} \Rightarrow f_e = -\frac{90}{13}$ cm

53.(A) Equivalent diagram of the system :

$P_e = 2\,P_L + P_m$



$P_L = \frac{1}{f_L}$, where $\frac{1}{f_L} = \frac{1}{f_L} = \left(\frac{\mu_L - 1}{1}\right)\left(\frac{1}{40} - \frac{1}{-40}\right)$

$\Rightarrow P_L = \frac{1}{40}$, $\frac{1}{f_L} = \frac{1}{40}$

$P_m = -\frac{1}{f_m}$ where ($f_m = -\frac{40}{2}$ since mirror is concave)

$\Rightarrow P_m = \frac{2}{40}$, $P_e = 2P_L + P_m = 2\times\frac{1}{40}+\frac{2}{40} = \frac{1}{10} \Rightarrow$ Now, $f_e = -\frac{1}{P_e} = -10$ cm

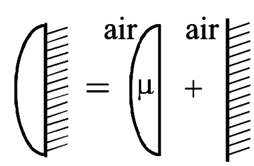
for the auto-collination condition ($v = -u$)

Then, apply the mirror formula

$\frac{1}{v}+\frac{1}{u} = \frac{1}{f_e} \Rightarrow \frac{1}{-u}+\frac{1}{-u} = -\frac{1}{10} \Rightarrow u = 20$ cm

54.(B) When plane surface is silvered

$P_e = 2P_L + P_m$

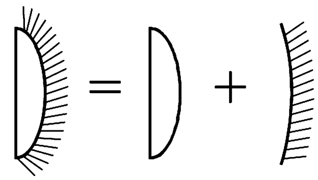


$P_L = \frac{1}{f_L}$, where $\frac{1}{f_L} = \left(\frac{\mu - 1}{1}\right)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$\Rightarrow P_L = \left(\frac{\mu - 1}{R}\right)$, $\frac{1}{f_L} = \left(\frac{\mu - 1}{R}\right)$

$P_m = -\frac{1}{f_m}$ = [since $f_m = \frac{\infty}{2}$] Then, $P_e = 2P_L + P_m$, $P_e = 2\left(\frac{\mu-1}{R}\right) + o = \frac{2(\mu-1)}{R}$

$f_e = -\frac{1}{P_e} = -\frac{R}{2(\mu-1)}$, Given that, $f_e = -28$ cm



Then, $28 = \frac{R}{2(\mu-1)}$ ...... **(1)**

When curved surface is silvered

$P_e = 2P_L + P_m$

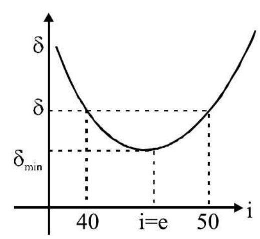
$P_L = \frac{1}{f_L}$ where $\frac{1}{f_L} = \left(\frac{\mu-1}{1}\right)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \left(\frac{\mu-1}{R}\right)$

$P_m = -\frac{1}{f_m}$ where ($f_m = -\frac{R}{2}$ for concave mirror)

$P_m = \frac{2}{R} \Rightarrow P_e = 2P_L + P_m \Rightarrow P_e = 2\left(\frac{\mu-1}{R}\right) + \frac{2}{R} = \left(\frac{2\mu}{R}\right) \Rightarrow f_e = -\frac{1}{P_e} = -\frac{R}{2\mu}$

Given that, $f_e = -10$ cm $\Rightarrow \frac{R}{2\mu} = 10$ .... (2), From equation (1) and (2) we get, $\mu = \frac{14}{9}$

55.(B) Graph between $(\delta - i)$ for prism

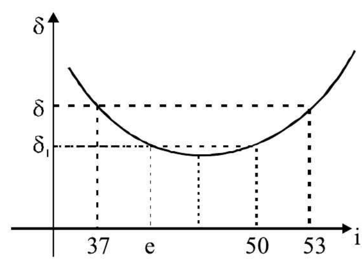


For the given condition

$\delta = (i + e - A) = 40 + 50 - 60$, $\delta = 30$

Then $\delta_{min} < \delta$, $\delta_{min} < 30$

56.(D) Graph between $(\delta - i)$ for the prism :



In the prism

$\delta = (i + e - A)$ where, δ is

the angle of deviation when $i = 53^o$; $e = 37^o$

$90 = (37 + 53 - A)$ .... (i)

$\delta_1 = (50 + e - A)$ .... (ii)

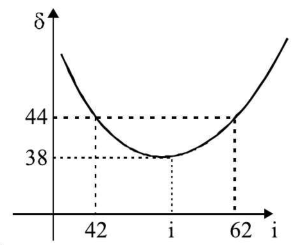
δ_1 = is the angle of deviation when $i = 50^o$;e= ?

From equation (i) and (ii), $\delta = \delta_1 + (40 - e)$. Since, $\delta > \delta_1$, $(40 - e)$ should be positive and $(40 - e) > 0$, $40 > e$, From the graph, e should be greater than 37^o. Hence, possible value of the angle of emergence $e = 38^o$

57.(C) For the minimum angle of deviation produced by the prism, $i = e$ and $r_1 = r_2 = r$, Prism formula : $r_1 + r_2 = A$ then, $r = 45°$ (since $A = 90°$), Apply Snell's law for the 1st interface,

$1.\sin i = \mu.\sin r \Rightarrow \sin i = \sqrt{\dfrac{3}{2}} \times \dfrac{1}{\sqrt{2}} \Rightarrow i = 60$, Then, $\delta_{min} = (i - r) + (e - r) = (60 - 45) + (60 - 45)$, $\delta_{min} = 30°$

58.(B) Graph between $(\delta - i)$ for the prism :

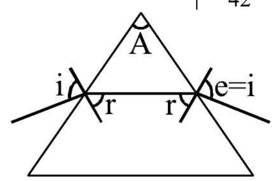


i = Angle of incidence when it is undergoes minimum deviation.

From the prism formula :

$$\delta = i + e - A$$
$$44 = 42 + 62 - A$$
$$\Rightarrow \quad A = 60°$$

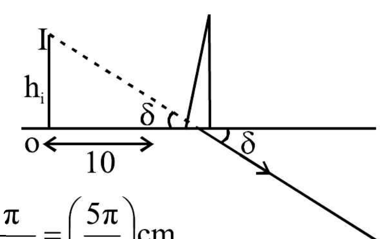
For the condition of minimum deviation



$$r_1 + r_2 = A \text{ (Prism formula)},$$
$$\Rightarrow \quad r = 30 \quad (\because r_1 = r_2 = r) \quad \delta_{min} = 2(i - r)$$
$$\Rightarrow \quad 38 = 2(i - 30) \quad \Rightarrow \quad i = 49$$

59.(C) For the thin prism : $\delta = A(\mu - 1)$ $\qquad \delta = 5(1.5 - 1) = \left(\dfrac{5}{2}\right)^{o} = \left(\dfrac{5}{2} \times \dfrac{\pi}{180}\right)$rad

Ray diagram for the thin prism :



h_i = height of image $\qquad \tan \delta = \dfrac{h_i}{10} \quad h_i = 10 \tan \delta$

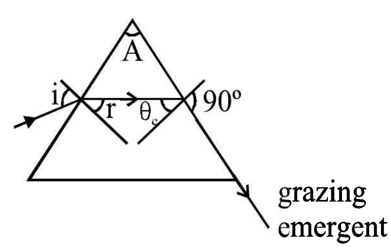
$h_i = 10\,\delta$ [since, δ is very small] $\Rightarrow \quad h_i = 10 \times \dfrac{5}{2} \times \dfrac{\pi}{180} = \left(\dfrac{5\pi}{36}\right)$cm

60.(A) Ray diagram for the grazing emergent :

From the prism formula : $\qquad r + \theta_c = A \quad \Rightarrow \quad r + \theta_c = 60$

Apply Snell's law at the second surface :

$\sqrt{2} \sin \theta_c = 1 \sin 90 \quad \Rightarrow \quad \sin \theta_c = \dfrac{1}{\sqrt{2}}$; $\theta_c = 45°$



Then, $r = A - \theta_c = 15°$

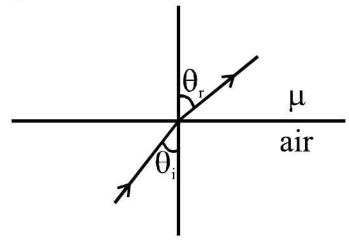
Again apply Snell's law at the 1st surface :

$$1 \sin i = \sqrt{2} \sin r \; ; \quad \sin i = \sqrt{2} \sin 15$$

$$i = \sin^{-1}\left(\frac{\sqrt{3} - 1}{2}\right) \qquad \text{since, } \sin 15° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

61.(C) Apply Snell's law at the interface : $1 \sin\theta_i = \mu \sin\theta_r$

differentiating this equation on the both side w.r.t θ_i.



$$1 \cos\theta_i = \mu(\cos\theta_r)\frac{d\theta_r}{d\theta_i} \Rightarrow d\theta_r = \frac{1}{\mu}\left(\frac{\cos\theta_i}{\cos\theta_r}\right)d\theta_i$$

Thus, for the small change : $\Delta\theta_r = \frac{1}{\mu}\left(\frac{\cos\theta_i}{\cos\theta_r}\right)\Delta\theta_i$

62.(A) For the achromatic combination of lens $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$ .... (1) where, ω_1 and ω_2 are the dispersive powers of the lenses. For the equivalent focal length, $\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f_e}$

$\Rightarrow \frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{10}$ ...... (2)[since, $f_e = 10$ cm] $\frac{\omega_1}{\omega_2}=\frac{2}{1}$ ..... (3) (Given in the question)

From equation (1), (2) and (3) we get, $f_2 = 5$ cm, $f_1 = -10$ cm

63.(A) By the Snell's law : $\sin\theta_c = \frac{1}{\mu}$, $\sin(\theta_c)_g = \frac{1}{\mu_g} = \frac{1}{1.44}$

Then, $(\theta_c)_g < 45°$ and $(\theta_c)_b < 45°$ and $(\theta_c)_r > 45°$, Hence, angle of incident = 45° for all the three clours. If, angle of incident is greater than critical angle then light will not emerge. Hence, blue and green light will not emerge. If, angle of incident is less than critical angle then light will emerge from the surface. Hence, red light will emerge.

64.(B) For the achromatic combination : $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0 \Rightarrow f_1 = \frac{-\omega_1}{\omega_2}f_2$

For the equivalent focal length of combination : $\frac{1}{f_e}=\frac{1}{f_1}+\frac{1}{f_2} \Rightarrow \frac{1}{f_e}=-\frac{\omega_2}{\omega_1 f_2}+\frac{1}{f_2}$

$\Rightarrow f_e = \left(\frac{\omega_1}{\omega_1-\omega_2}\right)f_2$ Given that, f_e is the converging lens means, $f_e > 0$

$\Rightarrow f_2 > 0$ (since $\omega_1 > \omega_2$) Hence, L_2 is converging.

From the above equation $f_1 < 0$, L_1 is diverging lens.

Assertion and Reason Type—Fully Solved

(A) Statement-1 is true, statement-2 is true; statement-2 is the correct explanation for statement-1.

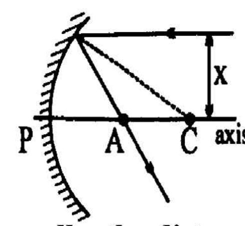
(B) Statement-1 is true, statement-2 is true, statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

1. Statement 1 : If a sourse of light is placed in front of a rough wall its image is not seen.
   Statement 2 : The wall does not reflect light.

2. Statement 1 : As the distance x of a parallel ray from axis increases, focal length decreses
   Statement 2 : As x increases, the distance from pole to the point of intersection of reflection of reflected ray with the principal axis decreases



3. Statement-1: when an object dipped in a liquid is viewed normally, the distance between the image and the object is independent of the height of the liquid above the object.
   Statement-2 : The normal shift is independent of the location of the slab between the object and the observer.

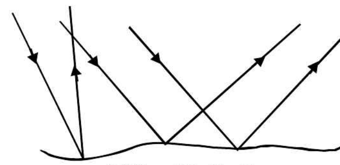
4. Statement-1: When two plane mirrors are kept perpendicular to each other as shown (O is the point object), 3 images will be formed.
   Statement-2 : In case of multiple reflections, image of one surface can act as an object for the next surface.

5. Statement-1 : Keeping a point object fixed, if a plane mirror is moved the image will definitely move.
   Statement-2 : In case of a plane mirror, distance of a point and its image from a given point on mirror is equal.

Solutions

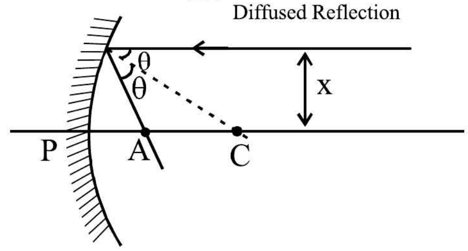
1.(C) When the surface is rough, we do not get a regular behavior of light . Although at each point light ray gets reflected irrespective of the overall nature of the surface. Difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite possible that these rays may move in different directions due to irregularity of the surface.This process enables us to see an object from any position.

Such a reflection is called diffused reflection. For example reflection from a wall, from a news paper etc. This is why we can not see our face in news paper and in the wall.



Diffused Reflection

2.(D) $AC = \dfrac{R}{2\cos\theta}$ (By geometry)



$PA = PC - AC = R - \dfrac{R}{2\cos\theta}$

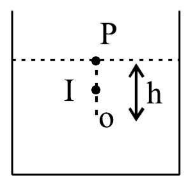
when $\theta\ (\uparrow)$ $\cos\ (\downarrow)$ and $PA\ (\downarrow)$

But, focal length of mirror remains constant.

3.(D) When an object dipped in a liquid is viewed normally.

h = height of liquid above the object.

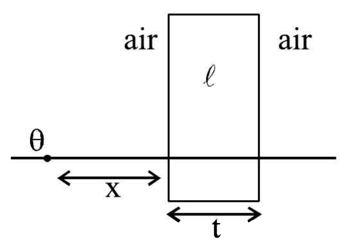
Apply the refraction formula for the curved surface :



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} - \frac{\mu_2}{-h} = 0 \quad (\text{since } R = \infty)$$

$$\Rightarrow v = \frac{-h}{\mu_2} \Rightarrow PI = \frac{h}{\mu_2} \qquad \text{Then, } OI = PO - PI = h - \frac{h}{\mu_2} = h\left(1 - \frac{1}{\mu_2}\right)$$

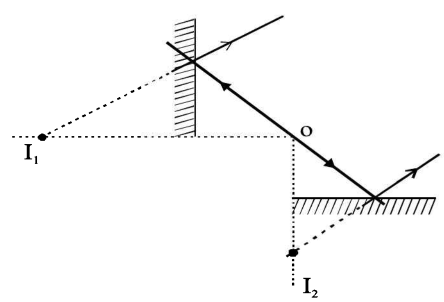
• which depends on h, hence statement-1 is false.



Calculation of shift : $\quad \text{Shift} = t\left(1 - \dfrac{1}{\mu}\right)$

• which is independent of x. Hence, statement-2 is true.

4.(D) Ray diagram



• Only two images are formed. Hence statement-1 is false.

• In the case of multiple reflections, image of one surfaces acts as an object for the next surface. Statement-2 is true.

5.(D) When mirror is moved parallel to the mirror as shown in figure (i) image will remains stationary But, when mirror is moved perpendicular to mirror as shown in figure (ii) image will definitely move. Hence, Statement-1 is false.

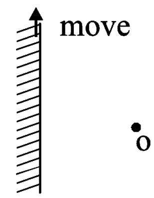


figure (i)

Multiple Choice Questions

Fully Solved—With One or More Choices Correct

Take approx. 3 minutes for answering each question.

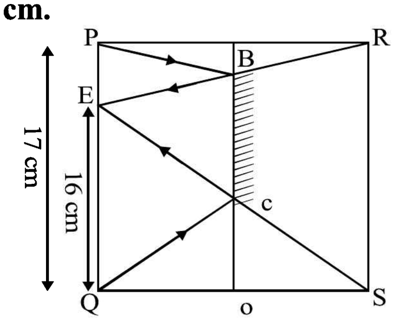
1. A man of height 170 cm wants to see his complete image in a plane mirror (while standing). His eyes are at a height of 160 cm from the ground.

(A) Minimum length of the mirror = 80 cm

(B) Minimum length of the mirror = 85 cm.

(C) Bottom of the mirror should be at a height 80 cm.

(D) Bottom of the mirror should be at a height 85 cm.



Sol.(BC) Ray diagram

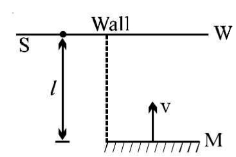
From the similar triangles RES and BEC :

$$\frac{RS}{BC} = \frac{2d}{d} \Rightarrow BC = 85 \text{ cm (minimum length of mirror)}$$

From the similar triangles SEQ and SCO :

$$\frac{EQ}{CO} = \frac{2d}{d} \Rightarrow CO = 80 \text{ cm (Bottom of the mirror should be at a height 80 cm or less)}$$

2. A flat mirror M is arranged parallel to a wall W at a distance *l* from it. The light produced by a point source S kept on the wall is reflected by the mirror and produces a light spot on the wall. The mirror moves with a velocity v towards the wall.



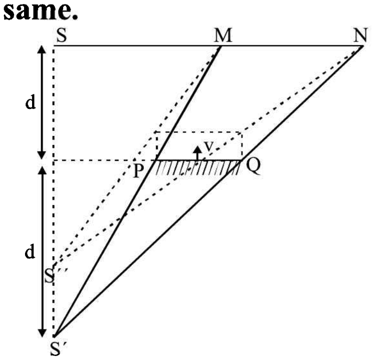
(A) The spot of light will move with the speed v on the wall.

(B) The spot of light will not move on the wall.

(C) As the mirror comes closer the spot of light will become larger and shift away from the wall with speed larger then v.

(D) The size of the light spot on the wall remains the same.

Sol.(BD) Ray diagram MN = Length of light spot on wall



From the similar triangles MNS′ and PQS′ : $\frac{MN}{PQ} = \frac{2d}{d}$

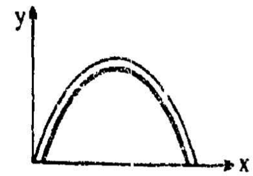
$\Rightarrow$ MN = 2PQ (which remains constant) where, PQ = length of mirror.

From the diagram, it is also clear that spot of light will not move on the wall.

3. A reflecting surface is represented by the equation $Y = \frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right), 0 \le x \le L$. A ray travelling horizontally becomes vertical after reflection.. The coordinates of the point (s) where this ray is incident are

(A) $\left(\frac{L}{4}, \frac{\sqrt{2}L}{\pi}\right)$ **(B)** $\left(\frac{L}{3}, \frac{\sqrt{3}L}{\pi}\right)$

(C) $\left(\frac{3L}{4}, \frac{\sqrt{2}L}{\pi}\right)$ **(D)** $\left(\frac{2L}{3}, \frac{\sqrt{3}L}{\pi}\right)$



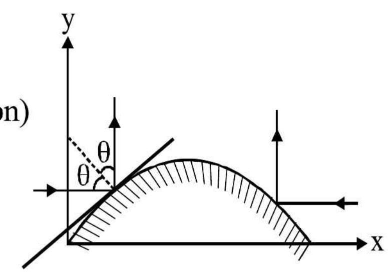
Sol.(BD) There are two possibilities $2\theta = 90$ (According to the question) $\theta = 45$

$\frac{dy}{dx}$ = Slope of the tangent = $\tan\theta$ $\frac{dy}{dx} = \tan 45 = 1$

Now, $\frac{dy}{dx} = \frac{2L}{\pi}\left(\frac{\pi}{L}\right)\cos\left(\frac{\pi x}{L}\right)$ (From the given equation)

$\Rightarrow \frac{dy}{dx} = 2\cos\frac{\pi x}{L} \Rightarrow \cos\frac{\pi x}{L} = \frac{1}{2}$ (since, $\frac{dy}{dx} = 1$)

$\Rightarrow \frac{\pi x}{L} = \frac{\pi}{3}, \frac{2\pi}{3} \Rightarrow x = \frac{L}{3}, \frac{2L}{3}$



4. A concave mirror cannot form

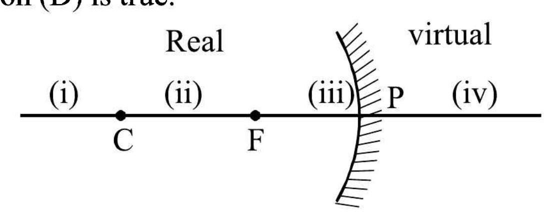
(A) virtual image of a virtual object **(B) virtual image of a real object**

(C) real image of a real object **(D) real image of a virtual object.**

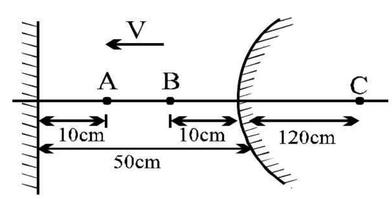
Sol.(A) • In the region (iv) : object is virtual. Image of the region (iv) is the region (iii) which is real. Hence, option (A) is false and option (D) is true.

• In the region (iii) : object is real. Image of the region (iii) is the region (iv) which is virtual. Hence, option (B) is true.

• In the region (i) : object is real. Image of the region (i) is the region (ii) which is real. Hence, option (C) is true.



5. In the figure shown consider the first reflection at the plane mirror and second at the convex mirror. AB is object.



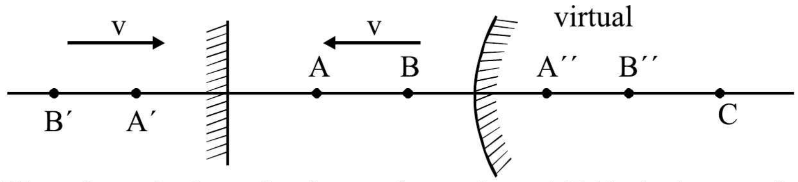
(A) the second image is real , inverted of 1/5 th magnification

(B) the second image is virtual and erect with magnification1/5

(C) the second image moves towards the convex mirror

(D) the second image moves away from the convex mirror.

Sol.(BC) Image Diagram



Here, from the 1st reflection at plane mirror A′B′ is the image of object AB.

For, the second reflection through the convex mirror, A′′B′′ is the image of object A′B′.

For the velocity of image for convex mirror :

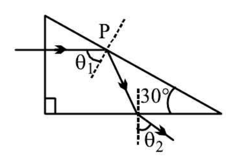
$(V_{Im}) = -m^2(V_{om})$ which shows its direction must be opposite. Hence, second image moves towards the convex mirror. **For image in the convex mirror :**

$\Rightarrow \frac{1}{v}+\frac{1}{-60}=\frac{1}{60}$ [for image of A′ and its image is A′′] $\Rightarrow$ v = 30 cm

$\frac{1}{v}+\frac{1}{-90}=\frac{1}{60}$ [for image of B′ and its image is B′′] $\Rightarrow$ $v=\frac{180}{5}$

Now, $A''B'' = \frac{180}{5} - 30$ = 6 cm; Magnification = $\frac{\text{size of image}}{\text{size of object}} = \frac{A''B''}{A'B'} = \frac{6}{30} = \frac{1}{5}$

6. **A ray of light is incident normally on one face of 30° – 60° – 90° prism of refractive index 5/3 immersed in water of refractive index 4/3 as shown in the figure.**

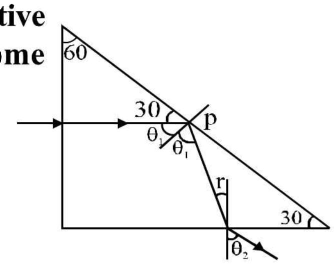


(A) The exit angle θ_2 of the ray is $\sin^{-1}(5/8)$

(B) The exit angle θ_2 of the ray is $\sin^{-1}(5/4\sqrt{3})$

(C) Total internal reflection at point P ceases if the refractive index of water is increased to $5/2\sqrt{3}$ by dissolving some substance.

(D) Total internal reflection at point P ceases if the refractive index of water is increased to 5/6 by dissolving some substance.



Sol.(AC) From the geometry of prism : $\theta_1 = 60 \quad r = 30$

Then apply Snell's law : $\frac{5}{3}\sin r = \frac{4}{3}\sin\theta_2 \quad \frac{5}{3}\times\frac{1}{2} = \frac{4}{3}\sin\theta_2$

$\Rightarrow \sin\theta_2 = \frac{5}{8} \quad \Rightarrow \theta_2 = \sin^{-1}\left(\frac{5}{8}\right)$ • Total internal reflection at the point P is only possible if $\mu_p > \mu_m$

7. Points A (0, 1) and B (12, 5) are object -image pair (one of the point acts as object and the other point as image). x-axis is the principal axis of the mirror. This object image pair is

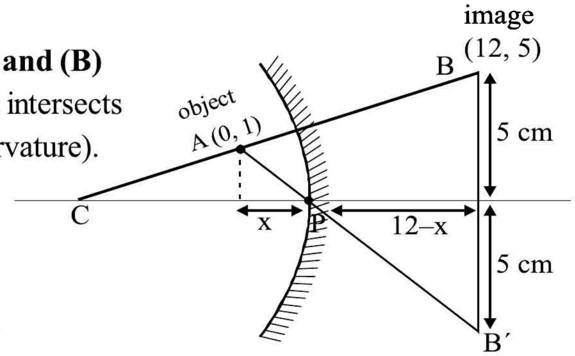
(A) due to a convex mirror of focal length 2.5 cm

(B) due to a concave mirror having its pole at (2,0)

(C) real virtual pair

(D) data is insufficient for options (A) and (B)

Sol.(BCD) Join, object and image with a line that intersects the principle axis at point C. (centre of curvature). From the similar triangles :



$$\frac{1}{x} = \frac{5}{12-x} \quad \Rightarrow \quad x = 2 \text{ cm}$$

If, A is the object and B is the image then, concave mirror having its pole at (2, 0) must be placed. If B is the object and A is the image then, convex mirror must be placed.

Apply mirror formula : $\Rightarrow \quad \frac{1}{2} + \frac{1}{-10} = \frac{1}{f} \quad \Rightarrow \quad f = 2.5$ cm

8. A ray of light in a liquid of refractive index 1.4, approaches the boundary surface between the liquid and air at an angle of incidence whose sine is 0.8. Which of the following statements is correct about the behaviour of the light

(A) It is impossible to predict the behavior of the light ray on the basis of the information supplied.

(B) The sine of the angle of refraction of the emergent ray will be less than 0.8.

(C) The ray will be internally reflected

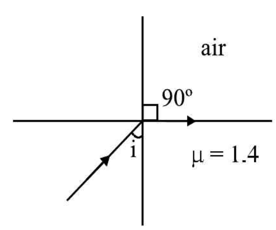
(D) The sine of the angle of refraction of the emergent ray will be greater than 0.8.

Sol.(C) At first we should check the critical angle for the interface : $\sin\theta_c = \frac{\mu_r}{\mu_d} = \frac{1}{1.4} = 0.71$

Given that, $\sin i = 0.8$ Hence, $i > \theta_c$

(Angle of incident is greater than critical angle)

Means, The ray will be internally reflected.



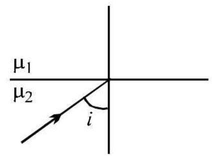
9. The figure shows a ray incident at an angle $i = \pi/3$. If the plot drawn shows the variation of $|r - i|$ versus $\frac{\mu_1}{\mu_2}$ = k, (r = angle of refraction)

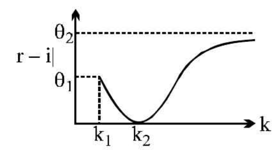
(A) the value of k_1 is $\dfrac{2}{\sqrt{3}}$

(B) the value of $\theta_1 = \pi/6$

(C) the value of $\theta_2 = \pi/3$

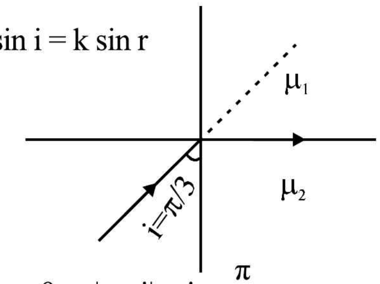
(D) the value of k_2 is 1





Sol.(BCD) Apply Snell's law : $\mu_2 \sin i = \mu_1 \sin r \Rightarrow \sin i = k \sin r$

From the given graph, angle of deviation decreases and becomes zero at $k = k_2$



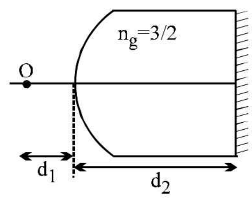
Hence, $\theta_1 = |r - i| = \dfrac{\pi}{6}$ (By geometry)

$\Rightarrow$ at $k = k_2$, $\theta = |r - i| = 0$ means, $k_2 = 1$

$\Rightarrow$ when $k = \infty$, $r = 0$, by the Snell's law. $\theta_2 = |r - i| = i = \dfrac{\pi}{3}$

$\Rightarrow$ k_1 = must be less than k_2 from the given graph.

10. In the figure shown a point object O is placed in air on the principal axis. The radius of curvature of the spherical surface is 60 cm. I_f is the final image formed after all the refractions and reflections.



(A) If d_1 = 120 cm, then 'I_f' is formed on 'O' for any value of d_2.

(B) If d_1 = 240 cm, then 'I_f' is formed on 'O' only if d_2= 360 cm.

(C) If d_1 = 240 cm, then 'I_f' is formed on 'O' for all values of d_2.

(D) If d_1 = 240 cm, then 'I_f' cannot be formed on 'O'.

Sol.(AB) Apply the refraction formula through the curved surface :

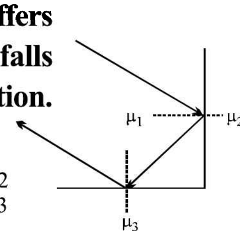
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \Rightarrow \quad \frac{3}{2v} - \frac{1}{-120} = \frac{\frac{3}{2} - 1}{+60} \quad \Rightarrow \quad v = \infty$$

That means after refraction light falls on the plane mirror normally for any value of d_2. This is the condition for object and image formed at the same point. When, d_1 = 240

$$\frac{3}{2v} - \frac{1}{-240} = \frac{\frac{3}{2} - 1}{+60} \quad \Rightarrow \quad v = 360 \text{ cm}$$

That means after refraction light falls on the pole of plane mirror if d_2 = 360 cm only. This condition is also for the object and the image formed at the same point.

11. In the diagram shown, a ray of light is incident on the interface between 1 and 2 at an angle slightly greater than critical angle. The light suffers total internal reflection at this interface. After that the light ray falls at the interface of 1 and 3, and again it suffers total internal reflection. Which of the following relations hold true?



(A) $\mu_1 < \mu_2 < \mu_3$ **(B)** $\mu_1^2 - \mu_2^2 > \mu_3^2$ **(C)** $\mu_1^2 - \mu_3^2 > \mu_2^2$ **(D)** $\mu_1^2 + \mu_2^2 > \mu_3^2$

Sol.(BCD) From the given condition : $\mu_1 > \mu_2, \mu_1 > \mu_3$ Since, μ_1, μ_2 and $\mu_3 > 1$

Hence, $\mu_1^2 > \mu_2^2$ $\mu_1^2 > \mu_3^2$ Then option (B), (C) and (D) hold true.

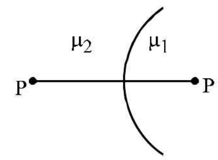
12. Two refracting media are separated by a spherical interface as shown in the figure. PP' is the principal axis, μ_1 and μ_2 are the refractive indices of medium of incidence and medium of refraction respectively. If

(A) $\mu_2 > \mu_1$, there cannot be a real image of real object.

(B) $\mu_2 > \mu_1$, there cannot be a real image of virtual object.

(C) $\mu_1 > \mu_2$, there cannot be a virtual image of virtual object.

(D) $\mu_1 > \mu_2$, there cannot be a real image of real object.



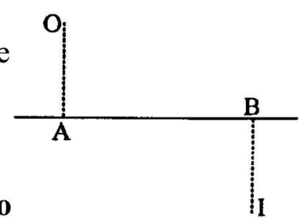
Sol.(AC) For real object : $\left(\frac{\mu_2}{v} - \frac{\mu_1}{-u} = \frac{\mu_2 - \mu_1}{R}\right) \Rightarrow \frac{\mu_1}{v} = \frac{-\mu_2}{u} + \frac{\mu_1 - \mu_2}{R}$

- If $\mu_2 > \mu_1$ then, $v < 0$ (must be) that means (it forms virtual image.)
- $\mu_1 > \mu_2$ then (v) can be +ve or –ve both.

For virtual object : $\frac{\mu_1 - \mu_2}{R} + \frac{\mu_2}{u} \Rightarrow \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R} + \frac{\mu_2}{u}$

- If $\mu_2 > \mu_1$ then (v) can be +ve or –ve both.
- If $\mu_1 > \mu_2$ then (v) > 0 (must be) that means real image is forme

13. A luminous point object is placed at O, whose image is formed at I as shown in the figure. AB is the optical axis. Which of the following statements are correct?



(A) If a lens is used to obtain an image, the lens must be co

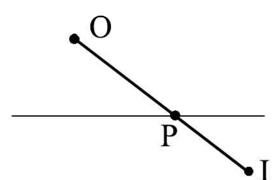
(B) If a mirror is used to obtain an image, the mirror must be a convex mirror having pole at the point of intersection of lines OI and AB

(C) Position of principal focus of mirror cannot be found

(D) I is a real image

Sol.(AD) For the lens : Line joining OI intersects at pole of the lens with AB (optical axis)

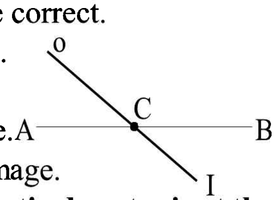
Then, lens must be converging, since ray must converge at point I. And the image is real. Hence, option (A) and (B) are correct.



For the mirror : Line OI intersects AB at centre of carvature, C.

Hence, option B, is not correct.

- For the convex mirror, real object always forms virtual image.
- For the concave mirror, object on left side of C must form real image.



14. A lens is placed in the XYZ coordinate system such that its optical centre is at the orgin and principal axis is along the x-axis. The focal length of the lens is 20 cm. A point object has been placed at the point (–40 cm,+1 cm, –1 cm.). Which of the following are correct about coordinates of the image?

(A) x = 40 cm **(B) y = +1 cm** **(C) z = + 1 cm** **(D) z = - 1 cm**

Sol.(AC) u = – 40 cm Since, principal axis of lens is along x-axis. For the image position (apply the lens formula) :⇒ $\frac{1}{v} - \frac{1}{-40} = \frac{1}{20}$ ⇒ v = 40 cm magnification, $m = \frac{v}{u} = \frac{40}{-40}$

Here, y and z coordinates of the object are the height of object.

$(h_i)_y = m\,(h_o)_y = (-1)(1) = -1$ $(h_i)_z = m\,(h_o)_z = (-1)(-1) = 1$

Co-ordinates of Image = (40, –1, 1)

15. Which of the following can form diminished, virtual and erect image of your face.

(A) Converging mirror (B) Diverging mirror (C) Converging lens (D) Diverging lens

Sol.(BD) • For converging mirror : The real object, image real or virtual both and image is real diminished but inverted. Hence, option (A) is not correct.

• For the Diverging mirror : The real object, image is virtual, erect and diminished. Hence, option (B) is correct.

• For the converging lens : Real object can form the virtual image but larger in size. Hence, option (C) is not correct.

• For the diverging lens : Real object always form virtual image, diminished and erect. Hence, option (D) is correct.

16. A convex lens forms an image of an object on a screen. The height of the image is 9 cm. The lens is now displaced until an image is again obtained on the screen. The height of this image is 4 cm. The distance between the object and the screen is 90 cm.

(A) The distance between the two positions of the lens is 30 cm.

(B) The distance of the object from the lens in its first position is 36 cm.

(C) The height of the object is 6 cm.

(D) The focal length of the lens is 21.6 cm.

Sol.(BCD) From the displacement method : $h_0 = \sqrt{h_1 h_2} = \sqrt{9 \times 4} = 6$ cm Hence, option (C) is correct $m_1 = \frac{h_1}{h_0} = \frac{9}{6} = \frac{3}{2}$ $m_2 = \frac{h_2}{h_0} = \frac{4}{6} = \frac{2}{3}$

From the displacement method $|m_1 - m_2| = \frac{d}{f}$ $\frac{3}{2} - \frac{2}{3} = \frac{d}{f}$ .... (1)

$f = \frac{D^2 - d^2}{4D} = \frac{90^2 - d^2}{4 \times 90}$ .... (ii) On solving equation (1) and (2) we get,

d = 18 cm, f = 21.6 cm Hence, option (D) is correct.

For the position of object : $x_2 - x_1 = d = 8$, $x_2 + x_1 = D = 90$

∴ $x_1 = 36$ cm, $x_2 = 54$ cm Here, x_1 and x_2 be the position of object for two positions of the lens. Hence, option (B) is correct.

17. A thin lens with focal length f is to be used as a magnifying glass. Which of the following statements regarding the situation is true?

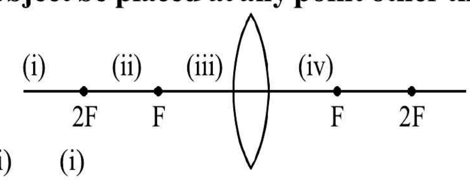
(A) A converging lens may be used, and the object be placed at a distance greater than 2f from the lens.

(B) A diverging lens may be used, and the object be placed between f and 2f from the lens.

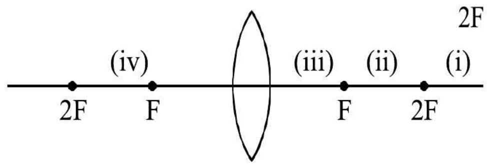
(C) A converging lens may be used, and the object be placed at a distance less than f from the lens.

(D) A diverging lens may be used, and the object be placed at any point other than the focal point.

Sol.(C) For the converging lens : For object :



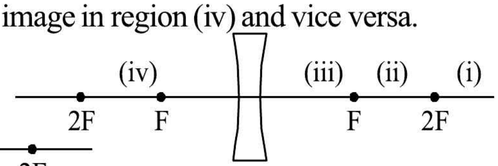
For image :



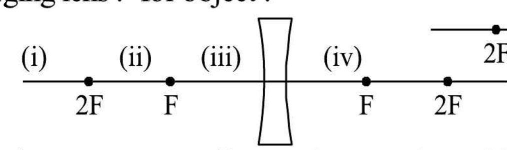
• Any object in region (i) will have its image in region (ii) and vice versa.

• Similarly any object in region (iii) will have its image in region (iv) and vice versa.

For the diverging lens : for object :



for image :



• Option (A) is wrong, according to the question, object lies in the region (i), its image is formed in the region (ii) which is smaller in size.

• Option (B) is wrong, according to the question, object lies in the region (ii), its image is formed in the region (iii) which is smaller in size.

• Option (C) is correct, according to the question, object lies in the region (iii), its image is formed in the region (iv) which is larger in size.

• Option (D) is wrong, since there is some region which form smaller size of image.

18. A man wishing to get a picture of a Zebra photographed a white donkey after fitting a glass with black streaks onto the objective of his camera.

(A) the image will look like a white donkey on the photograph.

(B) the image will look like a Zebra on the photograph.

(C) the image will be more intense compared to the case in which no such glass is used.

(D) the image will be less intense compared to the case in which no such glass is used.

Sol.(AD) In this case the image will be less intense compared to the case in which no such glass is used but image will look like a white donkey on the photograph not like a zebra.

19. For refraction through a small angled prism, the angle of deviation :

(A) increases with the increase in R.I. of the prism.

(B) will decrease with the increase in R.I. of the prism.

(C) is directly proportional to the angle of the prism.

(D) will be 2D for a ray of R.I.=2.4 if it is D for a ray of R.I.=1.2

Sol.(AC) For small angled prism : Deviation, $D = A(\mu - 1)$ where, $\mu = \frac{\mu_p}{\mu_m}$

Hence, D (↑) when μ (↑) (option A is correct) and $D \propto A^\circ$ (angle of prism) (option C is correct) $D_1 = A\,(2.4 - 1)$ $D_2 = A\,(1.2 - 1)$

$$\frac{D_1}{D_2} = \frac{2.4-1}{1.2-1} = \frac{1.4}{0.2} = 7 \quad \Rightarrow \quad D_1 = 7D_2 \text{ (option D is wrong)}$$

20. For the refraction of light through a prism

(A) For every angle of deviation there are two angles of incidences.

(B) The light travelling inside an equilateral prism is necessarily parallel to the base when prism is set for minimum deviation.

(C) There are two angles of incidences for maximum deviation.(for A < 20°C)

(D) Angle of minimum deviation will increase if refractive index of the prism is increased the keeping the outside medium unchanged if $\mu_P > \mu_S$.

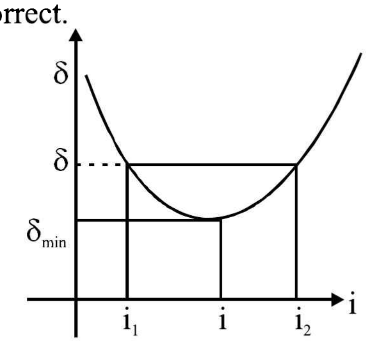
Sol.(BCD) Graph between $(\delta - i)$ for the prism :

• From the graph, δ_{min} is for only one angle of incident, hence option (A) is wrong.

• From the graph, δ_{min} is for two angles of incidences. Hence, option (C) is correct.

From the minimum deviation, $i_1 = i_2$, $r_1 = r_2$

This is only possible in the equilateral prism when light inside the prism is necessarily parallel to the base. Hence option (B) is correct.



• $$\frac{\mu_p}{\mu_m} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

δ_{min} (↑) when μ_p (↑) hence, option (D) is correct.

Multiple Choice Questions

With Only One Choice Correct

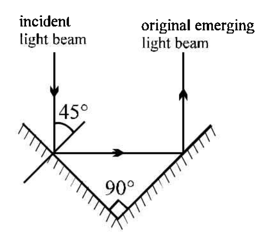
Take approx. 2 minutes for answering each question.

1. A person runs with a speed u towards a bicycle moving away from him with speed v. The person approaches his image in the plane mirror fixed at the rear of bicycle with a speed of

(A) u – v (B) u – 2v

(C) 2u – v (D) 2(u – v)

2. A beam of light strikes one mirror of a right angled mirror assembly at an angle of incidence 45° as shown in the figure. The right angled mirror assembly is rotated such that the angle of incidence becomes 60°. Which of the following statement is correct for the emerging light beam.



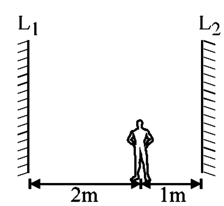
(A) It will move through an angle of 15° w.r.t. the original emerging beam.

(B) It will move through an angle of 30° w.r.t. the original emerging beam.

(C) It will move through an angle 45° w.r.t. the original beam.

(D) It will emerge parallel to the original emerging beam.

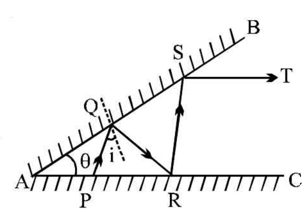
3. Two mirrors labelled L_1 for left mirror and L_2 for right mirror in the figure are parallel to each other and 3.0 m apart. A person standing 1.0 m away from the right mirror (L_2) looks into this mirror and sees a series of images. The second nearest image seen in the right mirror is situated at a distance :



(A) 2.0 m from the person (B) 4.0 m from the person

(C) 6.0 m from the person (D) 8.0 m from the person.

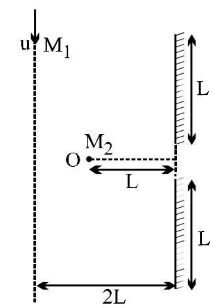
4. Two plane mirrors AB and AC are inclined at an angle $\theta = 20°$. A ray of light starting from point P is incident at point Q on the mirror AB, then at R on the mirror AC and again on S on AB. Finally the ray ST goes parallel to the mirror AC. The angle which the ray makes with the normal at point Q on the mirror AB is



(A) 20° (B) 30°

(C) 40° (D) 60°

5. Two plane mirrors of length L are separated by a distance L and a man M_2 is standing at a distance L from the connecting line of mirrors as shown in the figure. A man M_1 is walking in a straight line at a distance 2 L parallel to mirrors at speed u, then the man M_2 at O will be able to see the image of M_1 for total time:



(A) $\frac{4L}{u}$ (B) $\frac{3L}{u}$

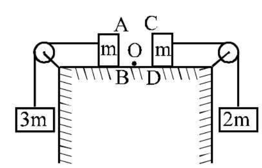
(C) $\frac{6L}{u}$ (D) $\frac{9L}{u}$

6. Two plane mirrors are placed parallel to each other at a distance L apart. A point object O is placed between them, at a distance L / 3 from one mirror. Both mirrors form multiple images. The distance between any two images cannot be

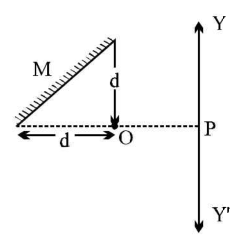
(A) 3L / 2 (B) 2L / 3 (C) 2L (D) None

7. Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB & CD of the two blocks are made reflecting. The acceleration of the two images formed in those two reflecting surfaces w.r.t. each other is:

(A) 5g / 6
(B) 5g / 3
(C) g / 3
(D) 17g / 6

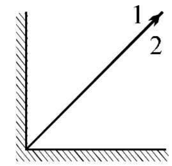


8. As shown in the figure a particle is placed at O in front of a plane mirror M. A man at P can move along path PY and PY'. Which of the following is true?



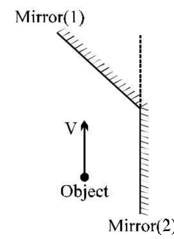
(A) For all point on PY man can see the image of O
(B) For all point on PY' man can see the image, but for no point on PY he can see the image of O
(C) For all point on PY' he can see the image but on PY he can see the image only upto distance d.
(D) He can see the image only upto a distance d on either side of P.

9. A two eyed man is looking at the junction of two large mutually perpendicular mirrors from a far off distance. Assume no reflection occurs from the edge. If both the eyes are open



(A) the eye 1 of man can see image of both eye 1 and eye 2.
(B) the eye 1 can see image of eye 1 only and eye 2 see image of eye 2 only.
(C) the eye 1 can see image of eye 2 only and eye 2 can see image of eye one only.
(D) all the above statements are false.

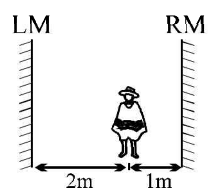
10. In the diagram shown, all the velocities are given with respect to earth. What is the relative velocity of the image in mirror (1) with respect to the image in the mirror (2)? The mirror (1) forms an angle β with the vertical.



(A) $2V\sin 2\beta$ (B) $2V\sin\beta$
(C) $2V / \sin 2\beta$ (D) none

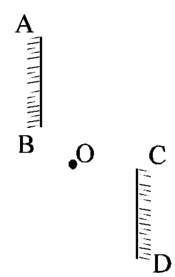
11. A point object is kept in front of a plane mirror. The plane mirror is doing SHM of amplitude 2 cm. The plane mirror moves along the x-axis and x-axis is normal to the mirror. The amplitude of the mirror is such that the object is always infront of the mirror. The amplitude of SHM of the image is
(A) zero (B) 2 cm (C) 4 cm (D) 1 cm

12. Two mirrors, labeled LM for left mirror and RM for right mirror in the adjacent figure, are parallel to each other and 3.0 m apart. A person standing 1.0 m from the right mirror (RM) looks into this mirror and sees a series of images. How far from the person is the second closest image seen in the right mirror (RM)?



(A) 10.0 m
(B) 4.0 m
(C) 6.0 m
(D) 8.0 m

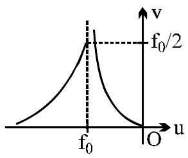
13. Two mirrors AB and CD are arranged along two parallel lines. The maximum number of images of object O that can be seen by any observer is

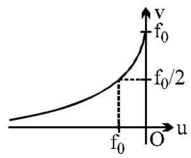


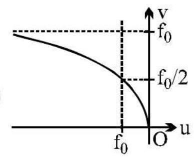
(A) One (B) Two
(C) Four (D) Infinite

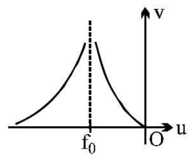
14. A concave mirror is used to form image of the sun on a white screen. If the lower half of the mirror were covered with an opaque card, the effect on the image on the screen would be
(A) negligible
(B) to make the image less brighter than before
(C) to make the upper half of the image disappear
(D) to make the lower half of the image disappear

15. A convex mirror of focal length 'f' is placed at the origin with its reflecting surface towards the negative x-axis. Choose the correct graphs between 'v' and 'u' for $u < 0$.

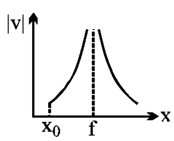
(A) 

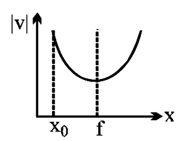
(B) 

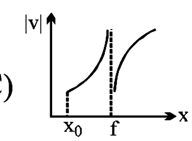
(C) 

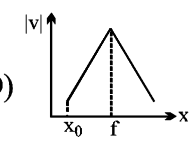
(D) 

16. A point source is situated at a distance $x < f$ from the pole of the concave mirror of focal length f. At time $t = 0$, the point source starts moving away from the mirror with constant velocity. Which of the graphs below represents best, the variation of image distance $|v|$ with the distance x between the pole of mirror and the source.

(A) 

(B) 

(C) 

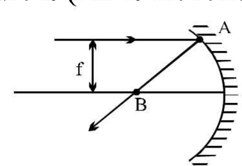
(D) 

17. A point object is between the Pole and Focus of a concave mirror, and moving away from the mirror with a constant speed. The velocity of the image is :
(A) away from the mirror and increasing in magnitude
(B) towards the mirror and increasing in magnitude
(C) away from the mirror and decreasing in magnitude
(D) towards the mirror and decreasing in magnitude

18. A ray of light is incident on a concave mirror. It is parallel to the principal axis and its height from principal axis is equal to the focal length of the mirror. The ratio of the distance of point B to the distance of the focus from the centre of curvature is (AB is the reflected ray)

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

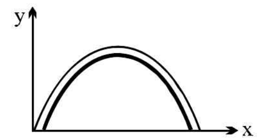


19. When an object is placed at a distance of 25 cm from a concave mirror, the magnification is m_1. The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2. If $m_1/m_2 = 4$, the focal length of the mirror is (Assume image is real, m_1, m_2 are numerical values)
(A) 10 cm (B) 30 cm (C) 15 cm (D) 20 cm

20. A reflecting surface is represented by the equation $Y = \frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right)$, $0 \le x \le L$. A ray travelling horizontally becomes vertical after reflection. The coordinates of the point (s) where this ray is incident are

(A) $\left(\frac{L}{4}, \frac{\sqrt{2}\,L}{\pi}\right)$ (B) $\left(\frac{L}{3}, \frac{\sqrt{3}\,L}{\pi}\right)$

(C) $\left(\frac{3L}{4}, \frac{\sqrt{2}\,L}{\pi}\right)$ (D) $\left(\frac{2L}{3}, \frac{\sqrt{3}\,L}{\pi}\right)$



21. The origin of x and y coordinates is the pole of a concave mirror of focal length 20 cm. The x-axis is the optical axis with $x > 0$ being the real side of mirror. A point object at the point (25 cm, 1 cm) is moving with a velocity 10 cm/s in positive x-direction. The velocity of the image in cm/s is approximately

(A) $-80\,i + 8\,j$ (B) $160\,i + 8\,j$ (C) $-160\,i + 8\,j$ (D) $160\,i - 4\,j$

22. All of the following statements are correct except (for real object):

(A) the magnification produced by a convex mirror is always less then or equal to one
(B) a virtual, erect, same sized image can be obtained using a plane mirror
(C) a virtual, erect, magnified image can be formed using a concave mirror
(D) a real, inverted, same sized image can be formed using a convex mirror.

23. The distance of an object from the pole of a concave mirror is equal to its radius of curvature. The image must be :

(A) real (B) inverted (C) same sized (D) erect

24. A concave mirror forms a real image three times larger than the object on a screen. Object and screen are moved until the image becomes twice the size of object. If the shift of object is 6 cm. The shift of the screen & focal length of mirror are

(A) 36 cm, 36cm (B) 36cm, 16cm (C) 72cm, 36cm (D) none of these

25. The distance of a real object from the focus of a convex mirror of radius of curvature 'a' is 'b'. Then the distance of the image from the focus is

(A) $\frac{b^2}{4a}$ (B) $\frac{a}{b^2}$ (C) $\frac{a^2}{4b}$ (D) none of these

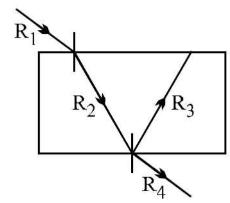
26. Choose the correct statement(s) related to the motion of an object and its image in the case of mirrors

(A) Object and its image always move along normal w.r.t. mirror in opposite directions
(B) Only in the case of convex mirror, it may happen that the object and its image move in the same direction
(C) Only in the case of concave mirror, it may happen that the object and its image move in the same direction
(D) Only in the case of plane mirrors, object and its image move in opposite directions

27. The x-z plane separates two media A and B with refractive indices μ_1 and μ_2 respectively. A ray of light travels from A to B. Its directions in the two media are given by the unit vectors, $\vec{r}_A = a\hat{i} + b\hat{j}$ & $\vec{r}_B = \alpha\hat{i} + \beta\hat{j}$ respectively where $\hat{i}$ & $\hat{j}$ are unit vectors in the x and y directions. Then

(A) $\mu_1 a = \mu_2 \alpha$ (B) $\mu_1 \alpha = \mu_2 a$ (C) $\mu_1 b = \mu_2 \beta$ (D) $\mu_1 \beta = \mu_2 b$

28. A ray R_1 is incident on the plane surface of the glass slab (kept in air) of refractive index $\sqrt{2}$ at angle of incident equal to the critical angle for this air glass system. The refracted ray R_2 undergoes partial reflection & refraction at the other surface. The angle between reflected ray R_3 and the refracted ray R_4 at that surface is :

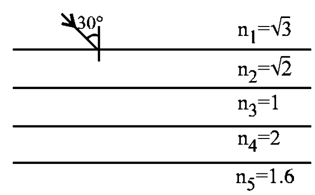


(A) 45° (B) 135°
(C) 105° (D) 75°

29. A tiny air bubble in a glass slab ($\mu = 1.5$) appears from one side to be 6 cm from the glass surface and 4 cm from other side. The thickness of the glass slab is

(A) 10 cm (B) 6.67 cm (C) 15 cm (D) one of these

30. In the figure shown the angle made by the light ray with the normal in the medium of refractive index 1 is :

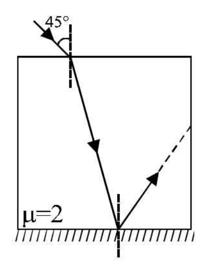


(A) 30° (B) 60°
(C) 90° (D) None of these

31. A plane glass slab is placed over various coloured letters. The letter which appears to be raised the least is

(A) red (B) yellow (C) violet (D) green

32. Bottom face of the glass cube is silvered as shown. A ray of light incident on top face of the cube is. Find the deviation of the ray when it comes out of the glass cube :



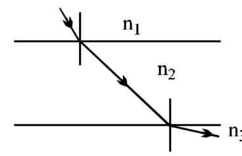
(A) 0 (B) 90°
(C) 180° (D) 270°

33. A ray of light is incident upon an air/water interface (it passes from air into water) at an angle of 45°. Which of the following quantities change as the light enters the water?

(I) wavelength (II) frequency
(III) speed of propagation (IV) direction of propagation

(A) I, III only (B) III, IV only (C) I, II, IV only (D) I, III, IV only

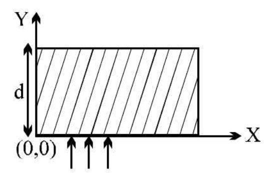
34. The figure shows the path of a ray of light as it passes through three different materials with refractive indices n_1, n_2 and n_3. The figure is drawn to scale. The refractive indices of the material satisfy the relation



(A) $n_3 < n_2 < n_1$ (B) $n_3 < n_1 < n_2$
(C) $n_2 < n_1 < n_3$ (D) $n_1 < n_3 < n_2$

35. The critical angle for glass to air refraction is least for which colour ?
(A) orange (B) blue (C) violet (D) red

36. A long rectangular slab of transparent medium is placed on a horizontal table with its length parallel to the x-axis and width parallel to the y-axis as shown in the figure. A ray of light travelling in air makes a normal incidence on the slab. The refractive index μ of the medium varies as $\dfrac{\mu_0}{1-(x/r)}$, where μ_0 and r (>d) are constants.



(A) The incident ray travels parabolically inside the slab.
(B) The incident ray travels in hyperbolic path inside the slab.
(C) The incident ray travels in circular path inside the slab.
(D) The incident ray travels in elliptical path inside the slab.

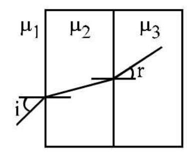
37. A ray of light travels from an optical denser medium to a rarer medium. The critical angle for the two media is C. The maximum possible deviation of the refracted light ray can be :
(A) $\pi - C$ (B) 2C (C) $\pi - 2C$ (D) $\dfrac{\pi}{2} - C$

38. A microscope is focused on a point object and then its objective is raised through a height of 2cm. If a glass slab of refractive index 1.5 is placed over this point object such that it is focused again, the thickness of the glass slab is :
(A) 6 cm (B) 3 cm (C) 2 cm (D) 1.5 cm

39. A paraxial beam of light is converging towards a point P on the screen. A plane parallel sheet of glass of thickness t and refractive index μ is introduced in the path of beam. The convergence point is shifted by :
(A) $t(1 - 1/\mu)$ away (B) $t(1 + 1/\mu)$ away
(C) $t(1 - 1/\mu)$ nearer (D) $t(1 + 1/\mu)$ nearer

40. A flat glass slab of thickness 6 cm and index 1.5 is placed in front of a plane mirror. An observer is standing behind the glass slab and looking at the mirror. The actual distance of the observer from the mirror is 50 cm. The distance of his image from himself, as seen by the observer is :
(A) 94 cm (B) 96 cm (C) 98 cm (D) 100 cm

41. In the figure shown $\frac{\sin i}{\sin r}$ is equal to :



(A) $\frac{\mu_2^2}{\mu_3 \mu_1}$ (B) $\frac{\mu_3}{\mu_1}$

(C) $\frac{\mu_3 \mu_1}{\mu_2^2}$ (D) none

42. An object is placed 20 cm in front of a 4 cm thick plane mirror. The image of the object is formed at a distance 45 cm from the object itself. The refractive index of the material of the unpolished side of the mirror is (considering near normal incidence)

(A) 1.5 (B) 1.6

(C) 1.4 (D) none of these

43. A ray of light is incident on a parallel slab of thickness t and refractive index n. If the angle of incidence θ is small, then the displacement in the incident and emergent ray will be :

(A) $\frac{t\theta(n-1)}{n}$ (B) $\frac{t\theta}{n}$ (C) $\frac{t\theta n}{n-1}$ (D) none

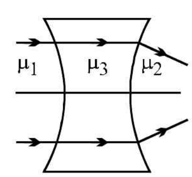
44. A ray of light is incident at an angle $75°$ into a medium having refractive index μ. The reflected and the refracted rays are found to suffer equal deviations in opposite direction μ, equals

(A) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (B) $\frac{\sqrt{3}+1}{2}$ (C) $\frac{2\sqrt{2}}{\sqrt{3}+1}$ (D) None of these

45. A small source of light is 4m below the surface of a liquid of refractive index 5/3. In order to cut off all the light coming out of the liquid surface, minimum diameter of the disc placed on the surface of liquid is :

(A) 3m (B) 4m (C) 6m (D) ∞

46. From the figure shown establish a relation between, μ_1, μ_2, μ_3.



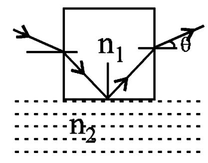
(A) $\mu_1 < \mu_2 < \mu_3$ (B) $\mu_3 < \mu_2$; $\mu_3 = \mu_1$

(C) $\mu_3 > \mu_2$; $\mu_3 = \mu_1$ (D) None of these

47. The critical angle of light going from medium A to medium B is θ. The speed of light in medium A is v. The speed of light in medium B is :

(A) $\frac{v}{\sin\theta}$ (B) $v \sin\theta$ (C) $v \cot\theta$ (D) $v \tan\theta$

48. A cubical block of glass of refractive index n_1 is in contact with the surface of water of refractive index n_2. A beam of light is incident on vertical face of the block (see figure). After refraction, a total internal reflection at the base and refraction at the opposite vertical face, the ray emerges out at an angle θ. The value of θ is given by :



(A) $\sin\theta < \sqrt{n_1^2 - n_2^2}$ (B) $\tan\theta < \sqrt{n_1^2 - n_2^2}$

(C) $\sin\theta < \dfrac{1}{\sqrt{n_1^2 - n_2^2}}$ (D) $\tan\theta < \dfrac{1}{\sqrt{n_1^2 - n_2^2}}$

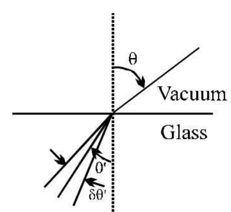
49. The flat bottom of cylinderical tank is silvered and water ($\mu = 4/3$) is filled in the tank upto a height h. A small bird is hovering at a height 3h from the bottom of the tank. When a small hole is opened near the bottom of the tank, the water level falls at the rate of 1 cm/s. The bird will perceive that its image's velocity is :

(A) 0.5 cm/s upward (B) 1 cm/s downwards
(C) 0.5 cm/s downwards (D) none of these

50. A ray of light is incident on one face of a transparent slab of thickness 15 cm. The angle of incidence is 60°. If the lateral displacement of the ray on emerging from the parallel plane is $5\sqrt{3}$ cm, the refractive index of the material of the slab is

(A) 1.414 (B) 1.532 (C) 1.732 (D) none

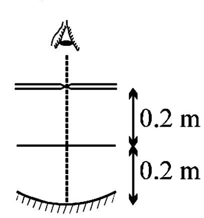
51. A beam of light has a small wavelength spread $\delta\lambda$ about a central wavelength λ. The beam travels in vacuum until it enters a glass plate at an angle θ relative to the normal to the plate, as shown in the figure. The index of refraction of the glass is given by $n(\lambda)$. The angular spread $\delta\theta'$ of the refracted beam is given by



(A) $\delta\theta' = \left|\dfrac{1}{n}\delta\lambda\right|$ (B) $\delta\theta' = \left|\dfrac{dn(\lambda)}{d\lambda}\delta\lambda\right|$

(C) $\delta\theta' = \left|\dfrac{\tan\theta'}{n}\dfrac{dn(\lambda)}{d\lambda}\delta\lambda\right|$ (D) $\delta\theta' = \left|\dfrac{\sin\theta}{\sin\theta'}\dfrac{\delta\lambda}{\lambda}\right|$

52. When a pin is moved along the principal axis of a small concave mirror, the image position coincides with the object at a point 0.5 m from the mirror, refer figure. If the mirror is placed at a depth of 0.2 m in a transparent liquid, the same phenomenon occurs when the pin is placed 0.4 m from the mirror. The refractive index of the liquid is



(A) 6/5 (B) 5/4 (C) 4/3 (D) 3/2

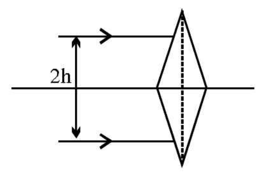
53. A light ray is incident on a transparent sphere of index = $\sqrt{2}$, at an angle of incidence = 45°. What is the deviation of a tiny fraction of the ray, which enters the sphere, undergoes two internal reflections, and then refracts out into air ?

(A) 270° (B) 240° (C) 120° (D) 180°

54. Two identical thin isosceles prisms of refracting angle 'A' and refractive index μ are placed with their bases touching each other. Two parallel rays of light are incident on this system as shown. The distance of the point where the rays converge from the prism is :

(A) $\dfrac{h}{\mu A}$ (B) $\dfrac{h}{A}$

(C) $\dfrac{h}{(\mu-1)A}$ (D) $\dfrac{\mu h}{(\mu-1)A}$

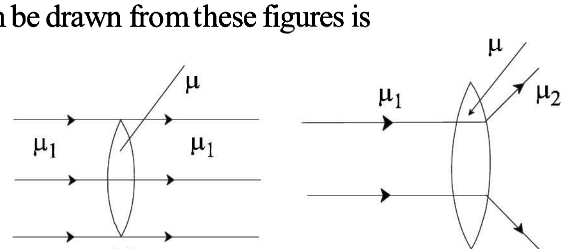


55. A ray of sunlight enters a spherical water droplet (n = 4/3) at an angle of incidence 53° measured with respect to the normal to the surface. It is reflected from the back surface of the droplet and re-enters into air. The angle between the incoming and the outgoing ray is [Take sin 53° = 0.8]

(A) 15° (B) 34° (C) 138° (D) 30°

56. The correct conclusion that can be drawn from these figures is

(A) $\mu_1 < \mu$ but $\mu < \mu_2$
(B) $\mu_1 > \mu$ but $\mu < \mu_2$
(C) $\mu_1 = \mu$ but $\mu < \mu_2$
(D) $\mu_1 = \mu$ but $\mu_2 < \mu$



(a) (b)

57. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at the point O, and PO = OQ. The distance PO is equal to :

(A) 5R (B) 3 R (C) 2 R (D) 1.5 R

58. A spherical surface of radius of curvature 10 cm separates two media X and Y of refractive indices 3/2 and 4/3 respectively. Centre of the spherical surface lies in denser medium. An object is placed in medium X. For the image to be real, the object distance must be

(A) greater than 90 cm (B) less than 90 cm.

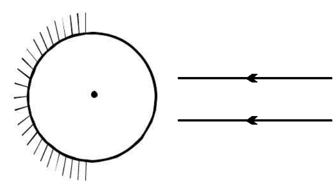
(C) greater than 80 cm (D) less than 80 cm.

59. A concave spherical refracting surface separates two media, glass and air ($\mu_{glass} = 1.5$). If the image is to be real, at what minimum distance, u should the object be placed in glass if R is the radius of curvature?

(A) $u > 3R$ (B) $u > 2R$ (C) $u < 2R$ (D) $u < R$

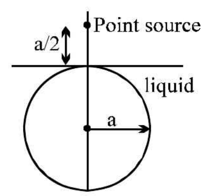
60. A glass sphere of index 1.5 and radius 40 cm has half its hemispherical surface silvered. The point where a parallel beam of light, coming along a diameter, will focus (or appear to) after coming out of sphere, will be:

(A) 10 cm to the left of centre
(B) 30 cm to the left of centre
(C) 50 cm to the left of centre
(D) 60 cm to the left of centre



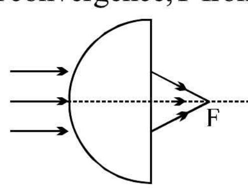
61. An opaque sphere of radius a is just immersed in a transparent liquid as shown in figure. A point source is placed on the vertical diameter of the sphere at a distance a/2 from the top of the sphere. One ray originating from the point source after refraction from the air liquid interface forms tangent to the sphere. The angle of refraction for that particular ray is 30°. The refractive index of the liquid is

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{3}{\sqrt{5}}$
(C) $\frac{4}{\sqrt{5}}$ (D) $\frac{4}{\sqrt{7}}$

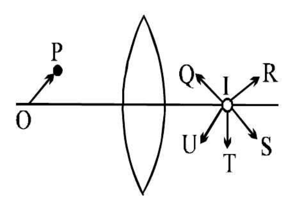


62. A paraxial beam is incident on a glass (n = 1.5) hemisphere of radius R = 6 cm in air as shown. The distance of point of convergence, F from the plane surface of hemisphere is

(A) 12 cm
(B) 5.4 cm
(C) 18 cm
(D) 8 cm

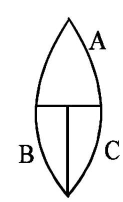


63. A point object O moves from the principal axis of a converging lens in the direction OP. I, the image of O, will move initially in the direction



(A) IQ (B) IR
(C) IS (D) IU

64. A thin symmetric double - convex lens of power P is cut into three parts A, B and C as shown. The power of



(A) A is P (B) A is 2P
(C) B is P (D) B is P/4

65. A lens behaves as a converging lens in air but a diverging lens in water, the refractive index(μ) of its material is
(A) $\mu > 4/3$ (B) $\mu > 3/2$ (C) $\mu < 4/3$ (D) $\mu < 3/2$

66. A parallel beam of white light falls on a convex lens. Images of blue, red and green light are formed on the other side of the lens at distances x, y and z respectively from the pole of the lens. Then :
(A) $x > y > z$ (B) $x > z > y$ (C) $y > z > x$ (D) None

67. A bi-concave glass lens having refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a
(A) convergent lens of focal length 3.5 R
(B) convergent lens of focal length 3.0 R
(C) divergent lens of focal length 3.5 R
(D) divergent lens of focal length 3.0 R

68. The power (in diopters) of an equiconvex lens with radii of curvature of 10 cm and refractive index of 1.6 is :
(A) -12 (B) $+12$ (C) $+1.2$ (D) -1.2

69. The focal length of a lens is greatest for which colour?
(A) violet (B) red (C) yellow (D) green

70. A converging lens forms an image of an object on a screen. The image is real and twice the size of the object. If the positions of the screen and the object are interchanged, leaving the lens in the original position, the new image size on the screen is
(A) twice the object size
(B) same as the object size
(C) half the object size
(D) can't say as it depends on the focal length of the lens.

71. A bi-concave symmetric lens made of glass has refractive index 1.5. It has both surfaces of same radius of curvature R. On immersion in a liquid of refractive index 1.25, it will behave as a
(A) Converging lens of focal length 2.5 R (B) Converging lens of focal length 2.0 R
(C) Diverging lens of focal length 4.5 R (D) None of these

72. A lateral object of height 0.5 cm is placed on the optical axis of bi-convex lens of focal length 80 cm, at an object distance = 60 cm. The image formed is :
(A) virtual, erect and 4 cm high (B) virtual, inverted and 2 cm high
(C) virtual, erect and 2 cm high (D) real, inverted and 2 cm high.

73. A concave mirror is placed on a horizontal surface and two thin uniform layers of different transparent liquids (which do not mix or interact) are formed on the reflecting surface. The refractive indices of the upper and lower liquids are μ_1 and μ_2 respectively. The bright point source at a height 'd' (d is very large in comparison to the thickness of the film) above the mirror coincides with its own final image. The radius of curvature of the reflecting surface therefore is

(A) $\frac{\mu_1 d}{\mu_2}$ (B) $\mu_1\mu_2 d$ (C) $\mu_1 d$ (D) $\mu_2 d$

74. An object is moving towards a converging lens on its axis. The image is also found to be moving towards the lens. Then, the object distance 'u' must satisfy

(A) $2f < u < 4f$ (B) $f < u < 2f$ (C) $u > 4f$ (D) $u < f$

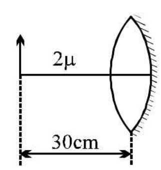
75. Two point sources P and Q are 24 cm apart. Where should a convex lens of focal length 9 cm be placed in between them so that the images of both sources are formed at the same place?

(A) 3 cm from P (B) 15 cm from Q (C) 9 cm from Q (D) 18 cm from P

76. If a concave lens is placed in the path of converging rays, real image will be produced if the distance of the pole from the point of convergence of incident rays lies between (f = magnitude of focal length of lens)

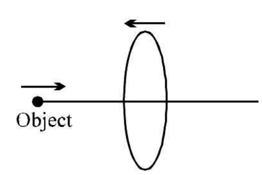
(A) 0 and f (B) f and 2f
(C) 2f and infinity (D) f and infinity

77. The diagram shows a silvered equiconvex lens. An object of length 1 cm has been placed in the front of the lens. What will be the properties of the final image? The refractive index of the lens is μ and the refractive index of the medium in which the lens has been placed is 2μ. Both the surfaces have the radius R.



(A) Half size, erect and virtual (B) same size, erect and real
(C) same size, erect and virtual (D) none

78. In the diagram shown, the lens is moving towards the object with a velocity V m/s and the object is also moving towards the lens with the same speed. What will be speed of the image with respect to the earth when the object is at a distance 2f from the lens? (f is the focal length.)



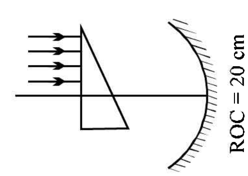
(A) 2V (B) 4V (C) 3V (D) V

79. You are given two lenses, a converging lens with focal length +10 cm and a diverging lens with focal length – 20 cm. Which of the following would produce a virtual image that is larger than the object?
(A) Placing the object 5cm from the converging lens.
(B) Placing the object 15cm from the converging lens.
(C) Placing the object 25cm from the converging lens.
(D) Placing the object 15cm from the diverging lens.

80. One of the refractive surfaces of a prism of angle 30° is silvered. A ray of light incident at an angle of 60° retraces it path. The refractive index of the material of prism is :
(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 3/2 (D) 2

81. On an equilateral prism, it is observed that a ray strikes grazingly at one face and if refractive index of the prism is 2, then the angle of deviation is
(A) 60° (B) 120° (C) 30° (D) 90°

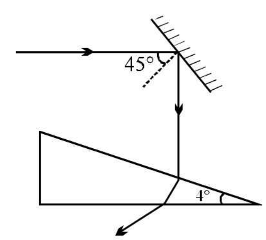
82. A parallel beam of light is incident on the upper part of a prism of angle 1.8° and R.I. 3/2. The light coming out of the prism falls on a concave mirror of radius of curvature 20 cm. The distance of the point (where the rays are focused after reflection from the mirror) from the principal axis is :



(A) 9 cm (B) 0.157 cm
(C) 0.314 cm (D) None of these

83. The refractive index of a prism is, $\cot\frac{A}{2}$ where A = angle of prism. The angle of minimum deviation is (in degrees)
(A) 2A (B) 90 – A (C) 180 – 2A (D) 0

84. A ray of light strikes a plane mirror at an angle of incidence 45° as shown in the figure. After reflection, the ray passes through a prism of refractive index 1.5, whose apex angle is 4°. The angle through which the mirror should be rotated if the total deviation of the ray is to be 90° is :



(A) 1° clockwise (B) 1° anticlockwise
(C) 2° clockwise (D) 2° anticlockwise

85. The refracting angle of prism is 60° and the index of refraction is 1/2 relative to the surrounding. The limiting angle of incidence of a ray that will be transmitted through the prism is :
(A) 30° (B) 45° (C) 15° (D) 50°

86. One face of a prism with a refracting angle of 30° is coated with silver. A ray incident on other face at an angle of 45° is refracted and reflected from the silvered coated face and retraces its path. The refractive index of the prism is :

(A) 2 (B) $\sqrt{3}$ (C) $\sqrt{3/2}$ (D) $\sqrt{2}$

87. An equilateral prism deviates a ray through 40° for two angles of incidences differing by 20°. The possible angles of incidences are :

(A) 40°, 60° (B) 50°, 30° (C) 45°, 55° (D) 30°, 60°

88. A beam of monochromatic light is incident at $i=50°$ on one face of an equilateral prism, the angle of emergence is 40°, then the angle of minimum deviation is :

(A) 30° (B) $<30°$ (C) $\leq 30°$ (D) $\geq 30°$

89. The dispersive powers of two lenses are 0.01 and 0.02. If focal length of one lens is + 10 cm, what should be the focal length of the second lens, so that they form an achromatic combination?

(A) Diverging lens with focal length 20 cm.
(B) Converging lens with focal length 20 cm
(C) Diverging lens with focal length 10 cm.
(D) Converging lens with focal length 10 cm

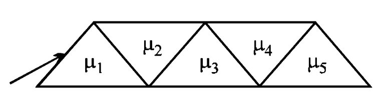
90. R.I. of a prism is $\sqrt{\dfrac{7}{3}}$ and the angle of prism is $60°$. The limiting angle of incidence of a ray that will be tansmitted through the prism is :

(A) 30° (B) 45° (C) 15° (D) 50°

91. For a prism of apex angle 45°, it is found that the angle of emergence is 45° for grazing incidence. Calculate the refractive index of the prism.

(A) $(2)^{1/2}$ (B) $(3)^{1/2}$ (C) 2 (D) $(5)^{1/2}$

92. The diagram shows five isosceles right angled prisms. A light ray incident at 90° at the first face emerges at same angle with the normal from the last face. Which of the following relations will hold for the refractive indices?



(A) $\mu_1^2+\mu_3^2+\mu_5^2=\mu_2^2+\mu_4^2$ (B) $\mu_1^2+\mu_3^2+\mu_5^2=1+\mu_2^2+\mu_4^2$

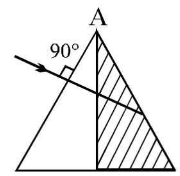
(C) $\mu_1^2+\mu_3^2+\mu_5^2=2+\mu_2^2+\mu_4^2$ (D) none

93. A certain prism is found to produce a minimum deviation of 38°. It produces a deviation of 44° when the angle of incidence is either 42° or 62°. What is the angle of incidence when it is undergoing minimum deviation?

(A) 45° (B) 49° (C) 40° (D) 55°

94. A ray of light is incident normally on the first refracting face of the prism of refracting angle A. The ray of light comes out at grazing emergence. If one half of the prism (shaded position) is knocked off, the same ray will

(A) emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/2\right)$



(B) not emerge out of the prism

(C) emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/4\right)$

(D) None of these

95. An achromatic convergent doublet of two lens in contact has a power of + 2 D. The power of the convex lens is + 5 D. What is the ratio of the dispersive powers of the convergent and divergent lenses?

(A) 2 : 5 (B) 3 : 5 (C) 5 : 2 (D) 5 : 3

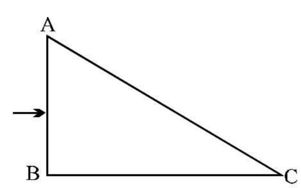
96. Two incident monochromatic waves whose wavelengths differ by a small amount $d\lambda$ are separated angularly at θ and $\theta + d\theta$. The dispersive power is given by

(A) $d\theta/d\lambda$ (B) $d\theta/\theta$ (C) $d\lambda/\lambda$ (D) $\lambda(d\lambda/d\theta)$

97. A ray of light is incident normally on a prism of refractive index 1.5, as shown. The prism is immersed in a liquid of refractive index 'μ'. The largest value of the angle ACB, so that the ray is totally reflected at the face AC, is 30°. Then the value of μ should be :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{5}{3}$

(C) $\frac{4}{3}$ (D) $\frac{3\sqrt{3}}{4}$



Multiple Choice Questions

With One or More Choices Correct

Take approx. 3 minutes for answering each question.

98 Two plane mirrors are at an angle such that a ray incident on a mirror undergoes a total deviation of 240° after two reflections.

(A) the angle between the mirror is 60°

(B) the number of images formed by this system will be 5, if an object is placed symmetrically between the mirrors.

(C) the no. of images will be 5 if an object is kept unsymmetrically between the mirrors.

(D) a ray will retrace its path after 2 successive reflections, if the angle of incidence on one mirror is 60°.

99 A concave mirror cannot form

(A) virtual image of virtual object

(B) virtual image of a real object

(C) real image of a real object

(D) real image of a virtual object.

Question No. 100 to 102 (3 questions)

A curved surface of radius R separates two media of refractive indices μ_1 and μ_2 as shown in figures A and B

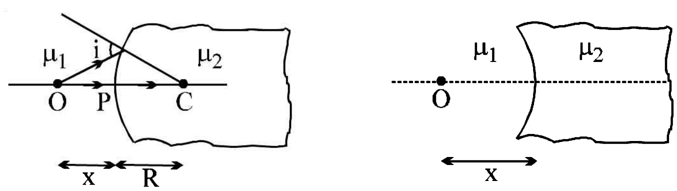


Fig. A Fig. B

100 Choose the correct statement(s) related to the real image formed by the object O placed at a distance x, as shown in figure A

(A) Real image is always formed irrespective of the position of the object if $\mu_2 > \mu_1$

(B) Real image is formed only when $x > R$

(C) Real image is formed due to the convex nature of the interface irrespective of μ_1 and μ_2

(D) None of these

101 Choose the correct statement(s) related to the virtual image formed by an object O placed at a distance x, as shown in figure A

(A) Virtual image is formed for any position of O if $\mu_2 < \mu_1$

(B) Virtual image can be formed if $x > R$ and $\mu_2 < \mu_1$

(C) Virtual image is formed if $x < R$ and $\mu_2 > \mu_1$

(D) None of these

102 Identify the correct statement(s) related to the formation of images of a real object O placed at x from the pole of the concave surface, as shown in figure B

(A) If $\mu_2 > \mu_1$, then virtual image is formed for any value of x

(B) If $\mu_2 < \mu_1$, then virtual image is formed if $x < \dfrac{\mu_1 R}{\mu_1 - \mu_2}$

(C) If $\mu_2 < \mu_1$, then real image is formed for any value of x

(D) none of these

103 A diminished image of an object is to be obtained on a large screen 1 m from it. This can be achieved by

(A) using a convex mirror of focal length less than 0.25 m

(B) using a concave mirror of focal length less than 0.25 m

(C) using a convex lens of focal length less than 0.25 m

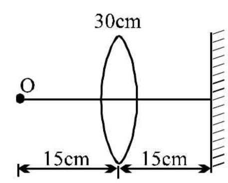
(D) using a concave lens of focal length less than 0.25 m

104 Which of the following quantities related to a lens depend on the wavelength of the incident light ?

(A) Refractive index (B) Focal length

(C) Power (D) Radii of curvature

105 An object O is kept infront of a converging lens of focal length 30cm behind which there is a plane mirror at 15cm from the lens.



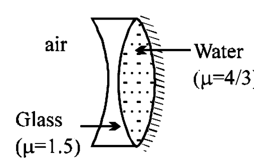
(A) the final image is formed at 60cm from the lens towards right of it

(B) the final image is at 60cm from lens towards left of it.

(C) the final image is real.

(D) the final image is virtual.

106 The radius of curvature of the left and right surface of the concave lens are 10cm and 15cm respectively. The radius of curvature of the mirror is 15cm.



(A) equivalent focal length of the combination is -18cm.

(B) equivalent focal length of the combination is +36cm.

(C) the system behaves like a concave mirror.

(D) the system behaves like a convex mirror.

Answer Key

ONLY ONE CHOICE CORRECT

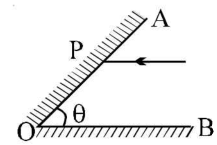
1. D	2. D	3. C	4. B	5. C	6. A
7. D	8. C	9. C	10. B	11. C	12. C
13. A	14. B	15. C	16. A	17. A	18. A
19. D	20. D	21. C	22. D	23. A	24. A
25. C	26. A	27. A	28. C	29. C	30. B
31. A	32. C	33. D	34. A	35. C	36. C
37. D	38. A	39. A	40. B	41. B	42. B
43. A	44. B	45. C	46. B	47. A	48. A
49. C	50. C	51. C	22. D	53. A	54. C
55. C	56. C	57. A	58. A	59. A	60. D
61. D	62. D	63. C	64. A	65. C	66. C
67. A	68. B	69. B	70. C	71. D	72. C
73. D	74. D	75. D	76. A	77. C	78. D
79. A	80. B	81. B	82. B	83. C	84. B
85. A	86. D	87. A	88. B	89. A	90. A
91. D	92. C	93. B	94. A	95. D	96. B
97. D					

ONE OR MORE THAN ONE CHOICES CORRECT

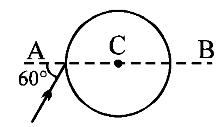
98. A,B,C,D	99. A	100. D	101. A,B
102. A.B	103. C	104. A,B,C	105. B,C
106. A,C			

Subjective Level # I

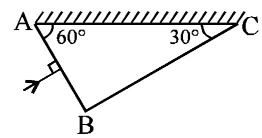
1. Two plane mirrors are inclined at an angle θ as shown in the figure. If a ray parallel to OB strikes the other mirror at P and finally emerges parallel to OA after two reflection then find θ.



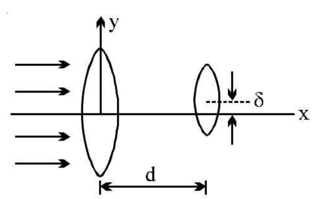
2. A ray of light falls on a transparent sphere with centre at C as shown in the figure. The ray emerges from the sphere parallel to the line AB. Find the refractive index of the sphere.



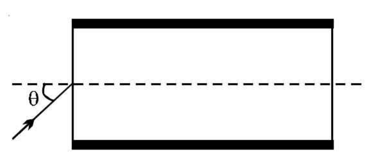
3. Face AC of a right angled prism ($\mu_\gamma = 1.5$) is coated with a thin film of liquid as shown in figure. Light is allowed to fall normally on the face AB of the prism. In order that the ray of light gets totally reflected, what can be the maximum refractive index of liquid?



4. A tiny air bubble inside a glass slab appears to be 6 cm deep when viewed from one side and 4 cm deep when viewed from the other side. Assuming $\mu_{glass} = 3/2$, find the thickness of slab.
5. A plano-convex lens, when silvered on the plane side, behaves like a concave mirror of focal length 30 cm. When it is silvered on the convex side, it behaves like a concave mirror of focal length 10 cm. Find the refractive index of the material of the lens.
6. Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d where ($d < f_1$, $d < f_2$) & their centres are displaced by a vertical separation δ as shown in the figure. Taking the origin of coordinates O, at the centre of the first lens, find the x & y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left.



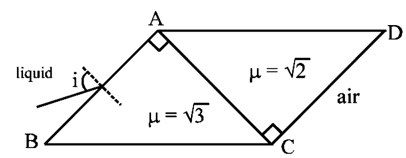
7. An object is kept on the principal axis of a convex mirror of focal length 10 cm at a distance 10 cm from the pole. The object starts moving at a velocity 20 mm/sec towards the mirror at an angle 30° with the principal axis. What will be the speed of its image and the direction with the principal axis at that instant?
8. A long solid cylindrical glass rod of refractive index 3/2 is immersed in a liquid of refractive index $\frac{3\sqrt{3}}{4}$. The ends of the rod are perpendicular to the central axis of the rod. Light enters one end of the rod at the central axis as shown in the figure. Find the maximum value of angle θ for which internal reflection occurs inside the rod?



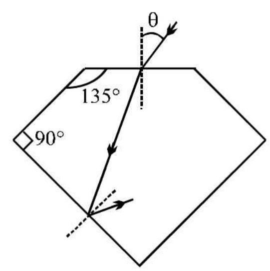
9. A ray of light moving along the unit vector $(-\mathbf{i}-2\,\mathbf{j})$ undergoes refraction at an interface of two media, which is the x-z plane. The refractive index for $y > 0$ is 2 while for $y < 0$ is $\sqrt{5}/2$. Find the unit vector along which the refracted ray moves?

10. A slab of glass of thickness 6 cm and index 1.5 is placed somewhere in between a concave mirror and a point object, perpendicular to the mirror's optical axis. The radius of curvature of the mirror is 40 cm. If the reflected final image coincides with the object, find the distance of the object from the mirror?

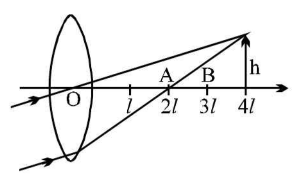
11. A ray of light from a liquid ($\mu = \sqrt{3}$) is incident on a system of two right-angled prisms of refractive indices $\sqrt{3}$ and $\sqrt{2}$ as shown in the figure. The ray of light suffers zero net deviation when it emerges into air from the surface CD. Find the angle of incidence?



12. A ray of light enters a diamond (n = 2) from air and is being internally reflected near the bottom as shown in the figure. Find the maximum value of angle θ possible?

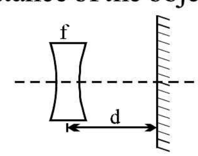


13. A thin converging lens L_1 forms a real image of an object located far away from the lens as shown in the figure. The image is located at a distance $4l$ and has height h. A diverging lens of focal length l is placed $2l$ from the lens L_1 at A. Another converging lens of focal length $2l$ is placed $3l$ from the lens L_1 at B. Find the height of the final image thus formed?

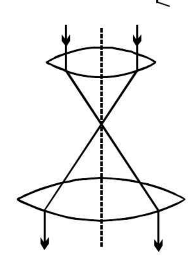


14. An object is placed at a certain distance from a screen. A convex lens of focal length 40 cm is placed between the screen and the object. A real image is formed on the screen for two positions of the lens, which differ by a distance $10\sqrt{17}$ cm. Find the distance of the object from the screen?

15. A diverging lens of focal length 10 cm is placed 10 cm in front of a plane mirror as shown in the figure. Light from a very far away source falls on the lens. Find the image of the source formed by plane mirror (before hitting lens again) at a distance from the mirror?

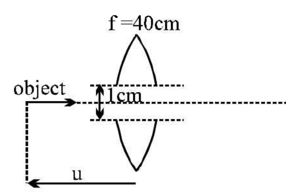


16. Consider a 'beam expander' which consists of two converging lenses of focal lengths 40 cm and 100 cm having a common optical axis. A laser beam of diameter 4 mm is incident on the 40 cm focal length lens. The diameter of the final beam will be (see figure)

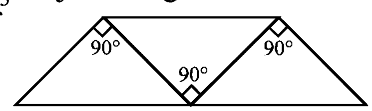


Subjective Level # II

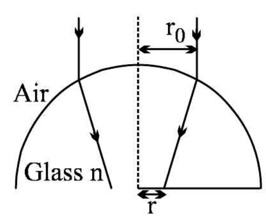
1. A thin converging lens is arranged between a small illuminated object & a screen so that an image of the object of linear magnification 3 is formed on a screen . The object and the screen are then 64 cm apart. A thin biconcave lens is then placed between the converging lens & the screen so that the lenses are coaxial & 6 cm apart . To restore a sharply focussed image on the image screen, the object was moved away from the converging lens through a distance of 14 cm . The biconcave lens has a surface of radii of curvature 14 cm & 21 cm. Calculate the focal length of the biconcave lens. Also, find the R. I. of the biconcave lens.
2. A ray of light refracted through a sphere, whose material has refractive index μ in such a way that it passes through the extremities of two radii which make an angle θ with each other. Prove that if α is the deviation of the ray caused by its passage through the sphere $\cos\frac{1}{2}(\theta-\alpha)=\mu\cos\frac{\theta}{2}$
3. In the figure shown, find the relative speed of approach/separation of the two final images formed after the light rays pass through the lens, at the moment when u = 30 cm. The speed of the object = 4 cm/s. The two lens halves are placed symmetrically w.r.t. the moving object.



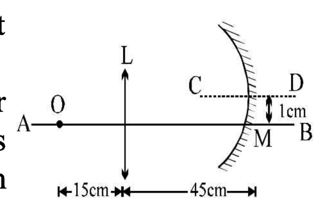
4. Three right angled prisms of refractive indices μ_1, μ_2 and μ_3 are joined together so that the faces of the middle prism in are in contact each with one of the outside prisms. If the ray passes undeviated, through the composite block show that $\mu_1^2+\mu_3^2-\mu_2^2=1$.



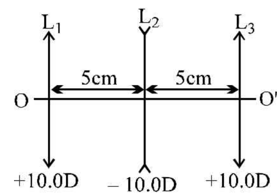
5. Two rays are incident on a spherical mirror of radius of R = 5 cm parallel to its optical axis at the distance $h_1 = 0.5$ cm and $h_2 = 3$ cm. Determine the distance Δx between the points at which these rays intersect the optical axis after being reflected at the mirror.
6. A beam of light is incident vertically on a glass hemisphere of radius R lying with its plane side on a table . The axis of the beam coincides with the vertical axis passing through the centre of the base of the hemisphere. The radius r_0 of the cross section of the beam is smaller than R.
Find the radius of the luminous spot formed on the table.



7. In the figure shown, L is a converging lens of focal length 10cm and M is a concave mirror of radius of curvature 20cm. A point object O is placed in front of the lens at a distance
15cm. AB and CD are optical axes of the lens and mirror respectively. Find the distance of the final image formed by this system from the optical centre of the lens.The distance between CD & AB is 1 cm.



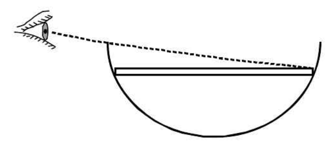
8. The figure illustrates an aligned system consisting of three thin lenses. The system is located in air. Determine:



(a) the position (relative to right most lens) of the point of convergence of a parallel ray incoming from the left after passing through the system ;

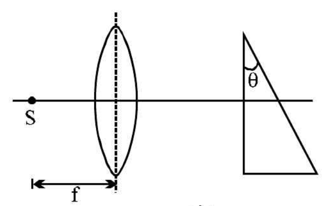
(b) The distance between the first lens and a point lying on the axis to the left of the system, at which this point and its image are located symmetrically with respect to the lens system?

9. A circular disc of diameter d lies horizontally inside a metallic hemispherical bowl of radius a. The disc is just visible to an eye looking over the edge. The bowl is now filled with a liquid of refractive index μ. Now, the whole of the disc is visible to the eye in the same position. Show that $d = 2a\dfrac{(\mu^2-1)}{(\mu^2+1)}$.

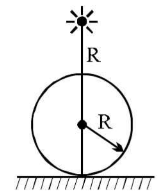


10. A luminous point P is inside a circle. A ray enters from P and after two reflections by the circle, return to P. If θ be the angle of incidence, a the distance of P from the centre of the circle and b the distance of the centre from the point where the ray in its course crosses the diameter through P, prove that $\tan\theta = \sqrt{\dfrac{a-b}{a+b}}$.

11. A glass wedge with a small angle of refraction θ is placed at a certain distance from a convergent lens with a focal length f, one surface of the wedge being perpendicular to the optical axis of the lens. A point sources S of light is on the other side of the lens at its focus. The rays reflected from the wedge (not from base) produce, after refraction in the lens, two images of the source displaced with respect to each other by d. Find the refractive index of the wedge glass.



12. An opaque sphere of radius R lies on a horizontal plane. On the perpendicular through the point of contact there is a point source of light at a distance R above the sphere.



(a) Show that the area of the shadow on the plane is $3\pi R^2$.

(b) A transparent liquid of refractive index $\sqrt{3}$ is filled above the plane such that the sphere is just covered with the liquid. Show that the area of shadow now becomes $2\pi R^2$.convergence of a parallel ray incoming from the left after passing through the system.

(c) The distance between the first lens and a point lying on the axis to the left of the system, at which that point and its image are located symmetrically with respect to the lens system?

ANSWER KEY

SUBJECTIVE LEVEL # I

1. 60°
2. $\sqrt{3}$
3. 1.3
4. 15 cm
5. 1.5
6. $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, \; y = \dfrac{\delta(f_1 - d)}{f_1 + f_2 - d}$
7. $\tan^{-1}\dfrac{2}{\sqrt{3}}$ with the principal axis, $\dfrac{\sqrt{7}}{4}$ cm/sec
8. $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$
9. $\dfrac{(-4\hat{i} - 3\hat{j})}{5}$
10. 42 cm
11. 45°
12. $\sin^{-1}\left(\dfrac{\sqrt{3}-1}{\sqrt{2}}\right)$
13. 2h
14. 1.70 m
15. 20 cm behind the mirror
16. 1 cm

SUBJECTIVE LEVEL # II

1. $f = -21$ cm, $\mu = 1.4$
3. 8/5 cm/s
5. 5/8 = 0.625 cm
6. $r = \dfrac{r_0}{\sqrt{\left\{1-(r_0/R)^2\right\}\left\{n^2-(r_0/R)^2\right\}} + (r_0/R)^2} = \dfrac{r_0}{n}$, if $r_0 << R$
7. $6\sqrt{26}$ cm
8. (a) 3.3 cm, (b) $l = (50/3)$ cm
11. $d/2f\theta$

Questions on Optics

JEE Advanced 2014—Paper I

SECTION - I: One or More Than One Choices Correct Type

This section contains 10 multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d) out of which only one & more than one choices is correct.

1. **A light source, which emits two wavelengths $\lambda_1 = 400\text{nm}$ and $\lambda_2 = 600\text{nm}$, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then**

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes for λ_1 is greater than λ_2

Sol. **(A,B,C)**

$\beta \propto \lambda$

$m \propto \dfrac{1}{\lambda}$

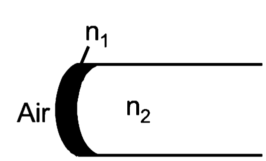
So $\beta_2 > \beta_1$ and $m_1 > m_2$

$$\frac{(2m-1)D\lambda_1}{2d} = \frac{nD\lambda_2}{d} \Rightarrow (2m-1) \times 2 = 6n$$

$\therefore$ $m = 5;\ n = 3$

$\therefore$ (A,B,C)

2. **A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then**



(A) $|f_1| = 3R$ **(B) $|f_1| = 2.8R$**

(C) $|f_2| = 2R$ **(D) $|f_2| = 1.4R$**

Sol. **(A,C)**

$$\frac{\mu_3}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2} \Rightarrow \frac{1.5}{f_1} - 0 = \frac{1.4-1}{R} + \frac{1.5-1.4}{R} = \frac{0.5}{R}$$

$f_1 = 3R$

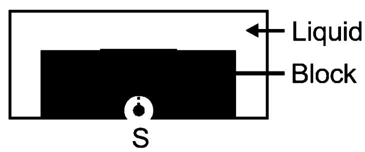
$$\frac{1}{f_2} - 0 = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R} = \frac{0.5}{R}$$

$f_2 = 2R$

JEE Advanced 2014—Paper II

Only One Choice Correct Type

1. **A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. If is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is**



(A) 1.21 **(B) 1.30** **(C) 1.36** **(D) 1.42**

Ans. (C)

Sol. $$\sin\theta = \frac{r}{\sqrt{r^2+h^2}} = \frac{\mu_{liq.}}{\mu_{Block}}$$

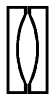
$$\mu_\ell = \frac{r}{\sqrt{r^2+h^2}} \times \mu_{Block}$$

Putting all values $\mu_\ell = 1.36$

Matrix-Match

2. **Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists.**

List I	List II
P. 	1. 2r
Q. 	2. r/2

R.  3. $-r$

S.  4. r

Code :

(A) P-1, Q-2, R-3, S-4 **(B) P-2, Q-4, R-3, S-1**

(C) P-4, Q-1, R-2, S-3 **(D) P-2, Q-1, R-3, S-4**

Ans. (B)

Sol. For equiconvex lens, f = radius of curvature [when surrounding medium is air and lens material is of refractive index = 1.5.)

JEE Main—2014

1. **A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is**

   (A) $f_2 > f$ and f_2 becomes negative

   (B) $f_2 > f$ and f_1 becomes negative

   (C) f_1 and f_3 both become negative

   (D) $f_1 = f_3 < f$

Sol. (A)

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)K$$

$$\frac{1}{f_1} = \left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)K$$

$$\frac{1}{f_2} = \left(\frac{\frac{3}{2}}{\frac{5}{3}} - 1\right)K$$

$$\frac{f_1}{f} = 4 \Rightarrow f_1 = 4f$$

$$\frac{f_2}{f} = -5 \Rightarrow f_2 = -5f$$

2. A green light is incident from the water to the air - water interface at the critical angle (θ). Select the correct statement

(A) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium

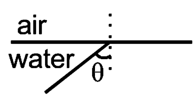
(B) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium

(C) The entire spectrum of visible light will come out of the water at various angles to the normal

(D) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal

Sol. (A)

$$\sin\theta = \frac{1}{\mu g}$$



$$\mu \propto \frac{1}{\lambda} \propto f$$

The frequencies more than green light will have lesser critical angle

3. Two beams, A and B, of plane polarized light with mutually perpenducular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beam appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals

(A) $\frac{3}{2}$ **(B)** 1

(C) $\frac{1}{3}$ **(D)** 3

Sol. (C)

$$I_A \cos^2 30 = I_B \cos^2 60$$

$$\frac{I_A}{I_B} = \cot^2 60 = \frac{1}{3}$$