BRAIN MAP

WAVES

Every piece of music you hear, from Hindustani classical to film songs, depends on performers producing waves and your detection of those waves.

Wave Motion

A means of transferring momentum and energy from one point to another without any actual transportation of matter

Progressive Wave Parameters

- Displacement, $y = A\sin(\omega t + kx)$

$$y = A\sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right) = A\sin\frac{2\nu}{\lambda}(vt+x)$$

- Phase, $\phi = 2\pi\left(\frac{\not\in t}{T}+\frac{x}{\lambda}\right)+\phi_0$

where ϕ_0 is the initial phase.

- Phase change with time,

$$\Delta\phi = \frac{2\nu}{T}\Delta t.$$

- Phase change with position,

$$\Delta\phi = \frac{2\nu}{\lambda}\Delta x.$$

- Instantaneous particle velocity,

$$u = \frac{dy}{dt} = \frac{2\nu A}{T}\cos 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$

- Velocity amplitude,

$$u_0 = \frac{2\nu A}{T} = \omega A$$

- Instantaneous particle acceleration,

$$a = \frac{du}{dt} = -\frac{4\nu^2}{T^2}A\sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$
$$= -\omega^2 y$$

- Acceleration amplitude,

$$a_0 = \frac{4\nu^2}{T^2}A = \omega^2 A$$

Wave Travelling Along a String

- Speed, $v ; \sqrt{\frac{T}{m}}$,

where, T = tension in the string,
m = mass per unit length.

- Average rate at which kinetic energy or potential energy transported $= \frac{1}{4}\frac{\upsilon^2 A^2 T}{v}$
- Average power transmitted along the string by a sine wave

$$P_{av} = \frac{1}{2}\frac{\upsilon^2 A^2 T}{v} = 2\pi^2 mvA^2\upsilon^2$$

Principle of Superposition of Waves

- According to the principle of superposition of waves, when any number of waves interact at a point in a medium, the net displacement of the point at a given time is the algebraic sum of the displacements due to each wave at that instant of time.

Stationary Waves

- The stationary wave formed by the superposition of incident wave and reflected wave is given by

$$y = 2A\sin\frac{2\nu x}{\lambda}\cos\frac{2\pi t}{T}$$

Nodes are formed at the positions

$$x = 0,\ \frac{\varphi}{2},\ \lambda,\ \frac{3\lambda}{2}....$$

and anti nodes are formed at

$$x = \frac{\varphi}{4},\ \frac{3\lambda}{4},\ \frac{5\lambda}{4}...$$

Transverse and Longitudinal Waves

- A transverse wave is one in which the disturbance occurs perpendicular to the direction of travel of the wave.
- A longitudinal wave is one in which the disturbance occurs parallel to the line of travel of the wave.

Velocity of Longitudinal Waves

- Velocity of longitudinal waves in a solid of bulk modulus κ, modulus of rigidity η and density ρ is

$$v = \sqrt{\frac{\iota + \frac{C}{3}\eta}{\rho}}$$

- Velocity of longitudinal waves in a long solid rod of Young's modulus Y and density ρ is given by

$$§ ; \sqrt{\frac{\breve{G}}{\rho}}$$

- Velocity of longitudinal waves in a fluid of bulk modulus κ and density ρ is

$$§ = \sqrt{\frac{\iota}{\rho}}$$

- Newton's formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\iota_{x\sim}}{\rho}} = \sqrt{\frac{P}{\rho}}.$$

here, P = pressure of the gas

- Laplace formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{ps\,xp}}{\rho}} = \sqrt{\frac{\varepsilon P}{\rho}},$$

- Intensity of sound waves

$$I = \frac{1}{2}\frac{\upsilon^2 A^2\kappa}{v} = \frac{2\pi^2\kappa}{v}A^2 v^2 = \frac{P_0^A v}{2\kappa} = \frac{P_0^2}{2\rho v}$$

Factors Affecting Velocity of Sound through Gases

- Effect of density, $v \cdot \frac{1}{\sqrt{\rho}}$

i.e., $\frac{v_2}{v_1} = \sqrt{\frac{\pi_1}{\rho_2}}$

- Effect of temperature, $v \cdot \sqrt{T}$

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273)\ t}{273}}$$

- No change in velocity of sound with change in pressure provided temperature is kept constant.

Doppler's Effect

- If v, v_o, v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency,

$$\upsilon' = \frac{v \Leftrightarrow v_m \pm v_o}{v \pm v_m \rightsquigarrow v_s}\times\upsilon$$

- If the medium is at rest, $(v_m = 0)$ then

$$\upsilon' = \frac{v \Leftrightarrow v_o}{v \rightsquigarrow v_s}\times\upsilon$$

- Upper sign on v_s (or v_o) is used when source (observer) moves towards the observer (source) while lower sign is used when it moves away.

Organ Pipes

- Open organ pipe
Fundamental mode,

$$\sigma_1 = \frac{v}{2L} = \upsilon\ \text{(First harmonic)}$$

Second mode, $\upsilon_2 = 2\upsilon$
(Second harmonic or first overtone)

n^{th} mode, $\upsilon_n = \frac{nv}{2L}$
(n^{th} harmonic or $(n-1)^{th}$ overtone)

- Closed organ pipe
Fundamental mode,

$$\sigma_1 = \frac{v}{4L} = \upsilon\quad \text{(First harmonic)}$$

Second mode, $\upsilon_2 = 3\upsilon$
(Third harmonic or first overtone)
Third mode, $\upsilon_3 = 5\upsilon$
(Fifth harmonic or second overtone)
n^{th} mode, $\upsilon_n = (2n-1)\upsilon$
$[(2n-1)^{th}$ harmonic or $(n-1)^{th}$ overtone$]$

- Laplace correction $e = 0.6r$ (in closed pipe) and $2e = 1.2r$ (in open pipe)

$$\upsilon = n\left[\frac{v}{2(l+1.2r)}\right]\ \text{(in open pipe)}$$

$$\upsilon = n\left[\frac{v}{4(l+0.6r)}\right]\ \text{(in closed pipe)}$$

Modes of Vibration of Strings

- String fixed at both ends
Frequency of vibration

$$\sigma = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

where L = length of string
n = mode of vibration
Fundamental frequency

$$\sigma_0 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{m}},$$

Second harmonic or 1st overtone, $\upsilon_2 = 2\upsilon_0$
Third harmonic or 2nd overtone, $\upsilon_3 = 3\upsilon_0$
and so on.

- String fixed at one end
Frequency of vibration

$$\upsilon = \left(n+\frac{1}{2}\right)\frac{v}{2L} = \frac{\left(\not\in n+\frac{1}{2}\right)}{2L}\sqrt{\frac{T}{m}}$$

Fundamental frequency,

$$\sigma_0 = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{T}{m}}$$

- Law of length
υL = constant
or $\upsilon_1 L_1 = \upsilon_2 L_2$

Beats

- Beat frequency = Number of beats/sec
= Difference in frequencies of two sources.
$\upsilon_{beat} = (\upsilon_1 - \upsilon_2)$ or $(\upsilon_2 - \upsilon_1)$
$\therefore\quad \upsilon_2 = \upsilon_1 \pm \upsilon_{beat}$
- The ± sign is decided by loading/filing any of the prongs of either tuning fork.
- On loading a fork, its frequency decreases and on filing, its frequency increases.