JEE Main

Joint Entrance Examination

Question Booklet Code **P**

Practice Set 2

Duration: 3 Hours *Max. Marks: 360*

Read the Following Instructions Carefully

1. *Immediately fill the particulars on this page of the test booklet with blue / black ball point pen. Use of pencil is strictly prohibited.*
2. *The test is of 3 hours duration.*
3. *The test booklet consists of 90 questions. The maximum marks are 360.*
4. *There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.*
5. *Candidates will be awarded marks as stated in above instructions for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. There is no negative marking for unattampted questions.*
6. *There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per above instructions.*
7. *No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.*
8. *Rough work is to be done on the space provided for this purpose in the test booklet only. This space is given at the bottom of pages.*

Name of the Candidate (in Capital Letters)_____

Roll Number (in Figures) | | | | | | | | | in Words _____

PART A Physics

1. **A metal rod of cross-sectional area A and length L is fixed between two rigid walls. If the rod is heated by temperature ΔT, then what shall be the reaction generated by walls given that Young's modulus of rod is Y and coefficient of thermal expansion is α?**

(a) $\frac{\alpha AY(\Delta T)}{2}$ (b) $\alpha AY(\Delta T)$ (c) $\frac{2\alpha(\Delta T)}{AY}$ (d) $\frac{\alpha(\Delta T)}{2AY}$

2. **At what temperature does a lamp (100 W, 200 V) work, if it is given that resistance of cold tungsten filament is 40 Ω. Take $\alpha = 3 \times 10^{-3}/^{\circ}C$ and 25°C as normal room temperature.**

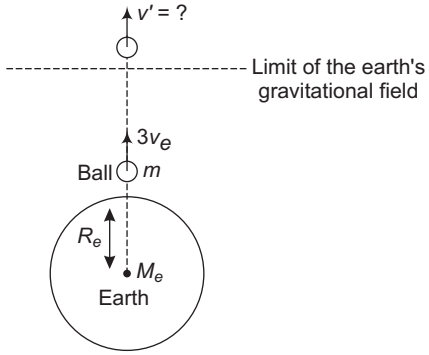
(a) 3000°C (b) 325°C (c) 3025°C (d) 300°C

3. **Energy levels A, B and C of a certain atom corresponding to increase values of energy i.e., $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are the wavelengths of radiations corresponding to transition C to B, B to A and C to A respectively. Then, which of the following statements/relations is correct?**

(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

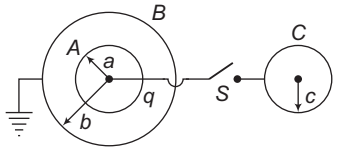
(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

4. **A ball is thrown upwards from the earth's surface with a speed $3v_e$, when the ball crosses the earth's gravitational field, then [v_e = escape velocity]**



(a) $v' = 2v_e$ (b) $v' = \sqrt{7}\,v_e$ (c) $\sqrt{2}\,v_e = v'$ (d) $v' = v_e$

5. **A spherical capacitor of inner radius a = 1 cm and outer radius b = 2 cm is earthed as shown. It can be connected to an isolated metallic sphere of radius c = 1 cm through a switch S and a very long conducting wire. If initial charge on inner sphere is q = 30 μC, then charge on the sphere of radius c, when switch S is closed, will be**



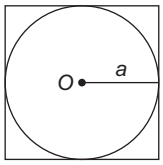
(a) 10 μC (b) 20 μC (c) 25 μC (d) 30 μC

6. **Let n_R and n_B are the number of photons emitted by a red bulb and a blue bulb of equal power in a given time. Then,**

(a) $n_R < n_B$ (b) $n_R = n_B$

(c) $n_R > n_B$ (d) Data is insufficient

7. A spherical ball is placed in a cube. The radius of the sphere is a and the side of cube is 2a. The centre of mass of the system



(a) must lie at O
(b) must lie left of O
(c) must lie right of O
(d) Could lie anywhere inside the cube

8. A particle of charge – q and mass m moves in a circle of radius r around an infinitely long line charge of linear density $+\lambda$. Then, the time period will be

(a) $T = 2\pi r\sqrt{\dfrac{m}{2k\lambda q}}$ (b) $T^2 = \dfrac{4\pi^2 m}{2k\lambda q}\cdot r^3$ (c) $T = \dfrac{1}{2\pi r}\sqrt{\dfrac{2k\lambda q}{m}}$ (d) $T = \dfrac{1}{2\pi r}\sqrt{\dfrac{m}{2k\lambda q}}$

where, $k = \dfrac{1}{4\pi\varepsilon_0}$

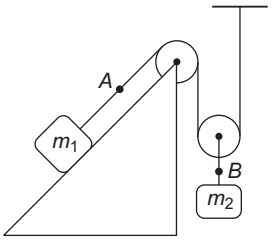
9. In Young's double slit experiment, the intensity on the screen at a point whose distance from the central maxima is 2.25 times of fringe width is found to be
(Given that I_0 is the intensity of single beam)

(a) $\dfrac{I_0}{2}$ (b) I_0 (c) $2I_0$ (d) $4I_0$

10. Two cones A and B are made of two different materials, the density of A being greater than that of B. The height of B is greater than that of A but their base areas and masses are the same. The correct statement about the moment of inertia of the two cones about their axes, is

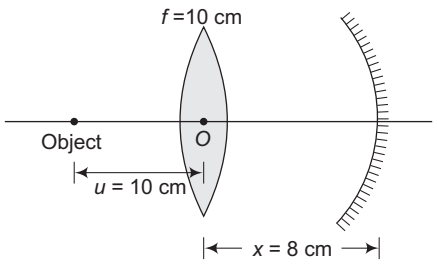
(a) A will have larger moment of inertia than B
(b) B will have larger moment of inertia than A
(c) in such a situation, it is dependent upon the height of the cone, the mass of the cone and radius of the base
(d) the moment of inertia of the two will be the same as it is not dependent upon height of the cone but depends only upon the mass and the base area

11. If we produce standing waves in wire A and B and the ratio of lengths of wire A and B is 4 : 1, then ratio of frequencies of fundamental waves produced in two wires is



(a) $\sqrt{2}:1$ (b) 2 : 1
(c) 1 : 1 (d) $1:2\sqrt{2}$

12. An object is placed at 10 cm from the lens. If the final image is formed at the optical centre of lens, then focal length of mirror will be



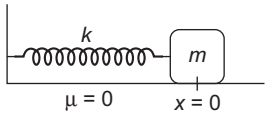
(a) + 10 cm
(b) – 8 cm
(c) + 8 cm
(d) – 10 cm

13. **If a non-conservative as well as a conservative forces is acting in a system**

(a) $W_C + W_{NC} = \Delta U$ (b) $W_C + W_{NC} = \Delta \text{KE}$

(c) W_{NC} = loss of mechanical energy (d) Both (b) and (c) are correct

14. **The block is stretched two times with two different amplitudes A_1 and $A_2 (A_1 > A_2)$. Then, time periods T_1 and T_2 can be related as**



(a) $T_1 > T_2$ (b) $T_1 < T_2$

(c) $T_1 = T_2$ (d) Information is insufficient

15. **Electric field vector in the vacuum associated with an electromagnetic wave is given by**

$$\mathbf{E = (60\ N/C)\sin(44 \times 10^4 x + 132 \times 10^{12} t)}$$

Then, its wavelength and frequency in a medium of refractive index $\mu = 1.4$ would be

(a) 10^{-5} m and 2.1×10^{13} s^{-1} (b) 1.4×10^{-5} m and 1.5×10^{12} s^{-1}

(c) 10^{-5} m and 1.5×10^{12} s^{-1} (d) 10^{-5} m and 1.32×10^{12} s^{-1}

16. **If the road is banked and circular, choose the correct option.**

(a) There is no need of friction for centripetal force

(b) If $N \sin\theta > F_C$, $(f_s)_{max}$ must act in the upward direction

(c) If $N \sin\theta < F_C$, $(f_s)_{max}$ must act in the downward direction

(d) None of the above

17. **Second law of thermodynamics**

(a) says that law of conservation of energy is not valid

(b) says that $\Delta Q = \Delta U$

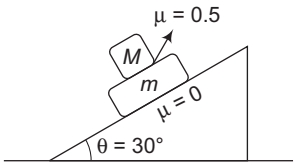
(c) says that $\Delta Q = \Delta U + \Delta W$ is not always valid

(d) is consistent with the first law

18. **Current through an AC series circuit is 4 A if operated at resonant frequency ω_0. At $\omega_0/2$, current reduces to 2 A. Then, at $2\omega_0$, current in the circuit shall be**

(a) $\sqrt{2}$ A (b) 1 A (c) 2 A (d) 3 A

19. **M = 2 kg, m = 1 kg. The force of friction between M and m is**



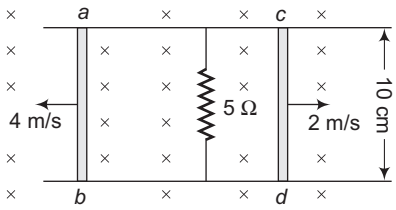
(a) 5 N (b) $\frac{5\sqrt{3}}{2}$ N (c) 10 N (d) zero

20. **The average translational energy and the rms speed of molecules in a sample of oxygen at 300 K are 6.21×10^{-21} J and 484 m/s, respectively. The corresponding value at 600 K are nearly (assuming ideal gas behaviour).**

(a) 12.42×10^{-21} J, 968 m/s (b) 6.21×10^{-21} J, 968 m/s

(c) 8.78×10^{-21} J, 684 m/s (d) 12.42×10^{-21} J, 684 m/s

21. Consider the given case



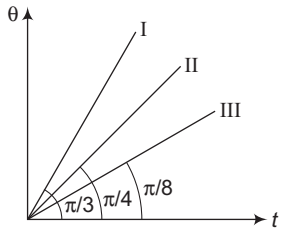
Magnetic field of 1 Tesla is applied to this arrangement. If connecting wires and rods have negligible resistance. Then, current through 5 Ω resistor is

(a) 120 mA (b) 40 mA
(c) 80 mA (d) 60 mA

22. At $t = 0$, three particles A, B and C are located at the origin of the coordinate system. Then, they start moving simultaneously, A moves with a constant velocity of $3\hat{i}$ (m/s) and B moves under a constant acceleration of $2\hat{k}$ (m/s^2) with an initial velocity of $8\hat{j}$ (m/s). Particle C moves with constant velocity v_0 in such a way that B and C collide at $t = 4$ s. Then,

(a) v_0 is $8\hat{j} + 4\hat{k}$
(b) position vector of location where two particles collide is $16\hat{i} + 32\hat{k}$
(c) Both (a) and (b) are correct
(d) it is not possible that *B* and *C* collide with each other for any value of v_0

23. Three bodies A, B and C of masses m, m and $\sqrt{3}$ m respectively are supplied heat at a constant rate. The change in temperature θ versus time t graph for A, B and C are shown by I, II and III respectively. If their specific heat capacities are S_A, S_B and S_C respectively then which of the following relation is correct? (Initial temperature of each body is 0°C)

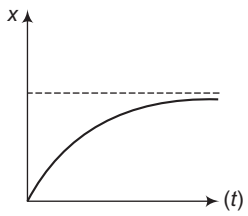


(a) $S_A > S_B > S_C$ (b) $S_B = S_C < S_A$
(c) $S_A = S_B = S_C$ (d) $S_A = S_B > S_C$

24. The materials suitable for making electromagnets should have

(a) high retentivity and high coercivity (b) low retentivity and low coercivity
(c) high retentivity and low coercivity (d) low retentivity and high coercivity

25. The displacement x of a particle as a function of time t is shown in the figure. The figure indicates

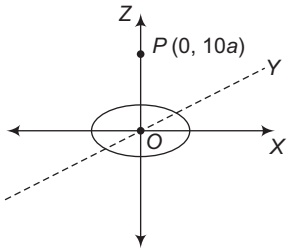


(a) the particle starts with a certain velocity, but the motion is retarded, finally the particle stops
(b) the velocity of the particle is constant throughout
(c) the acceleration of the particle is constant throughout
(d) the particle starts with certain velocity and then its motion is accelerated

26. **Two point sources, which are in phase and separated by distance $D = 1.5\lambda$, emit identical sound waves of wavelength λ. If a circle with a radius much greater than D, centered on the mid-point between the sources. What is the number of points around the circle at which the interference is fully constructed?**

(a) 2 (b) 4 (c) 6 (d) 8

27. **A ring of radius a is carrying a current I is placed at the origin. Then, the ratio of magnetic field produced at the centre and at 10a distance away from centre will be**

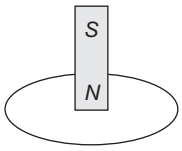


(a) 10 : 1 (b) 100 : 1 (c) 1000 : 1 (d) 2000 : 1

28. $\mathbf{F_{ext} = \frac{P}{t}}$

(a) above expression is dimensionally correct
(b) above expression is dimensionally incorrect
(c) above expression is a correct expression
(d) None of these

29. **Statement I A magnet is dropped along the axis of a circular conducting loop as shown in figure. Then, the acceleration of magnet is always more than g.**



Statement II According to Lenz's law, the direction of magnetic field induction effect is such as to oppose the cause of the effect.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are false

30. **Statement I In elastic collision, the KE of the system always remains constant.**

Statement II The momentum of the system of colliding bodies remains conserved during collision.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are false

PART B Chemistry

31. **$Na_2B_4O_7 \cdot 10H_2O$ is correctly represented as**

(a) $Na_2[B_4O_5(OH)_4] \cdot 8H_2O$ (b) $2NaBO_2 \cdot Na_2B_2O_3 \cdot 10H_2O$

(c) $Na_2[B_4(H_2O)_4O_7] \cdot 6H_2O$ (d) None of these

32. **An element undergoes a reaction as follows $X + 2e^- \longrightarrow X^{2-}$, energy released $= 30.86$ eV/atom. If the energy released is used to dissociate 4 g of H_2 of molecule, equally into H^+ and H*, where H* is excited state of H atoms where the electron travels in orbit whose circumference equal to four times its de-Broglie's wave length. Determine the least moles of X that would be required.**
Given, ionisation energy of H = 13.6 eV/atom
Binding energy of H_2 = 4.52 eV/molecule.

(a) 2 (b) 4 (c) 8 (d) 16

33. **Which of the following will undergo benzoin condensation?**

(a) Furfural (b) 4-nitro benzaldehyde

(c) p.NN- dimethylamino benzaldehyde (d) All of these

34. **In the electrolysis of aquous NaCl solution, side reactions taking place are**

I. $2OH^- + Cl_2 \longrightarrow 2OCl^- + H_2$

II. $2Na + 2H_2O \longrightarrow 2NaOH + H_2$

III. $4OH^- \longrightarrow O_2 + 2H_2 + 4e^-$

Select the correct alternative.

(a) All I, II, III (b) I, II (c) II, III (d) I, III

35. **Calculate the emf in volts of the cell**

$$Pt\,|H_2(g)\,(0.1\text{ atm})\,|\,BOH\,(aq)\,(1\text{ M})\,||HA\,(aq)\,(0.1\text{ M})\,|H_2(g)\,(1\text{ atm})\,|Pt$$

Given, $K_a(HA) = 10^{-7}, K_b = (BOH) = 10^{-5}$

(a) 0.395 V (b) 0.495 V (c) 0.240 V (d) 0.595 V

36. **Which of the following will not produce the benzyl amine?**

(a) Benzamide $\xrightarrow{LiAlH_4}$ (b) Phenyl nitrile $\xrightarrow{LiAlH_4}$

(c) 2-phenyl ethyl isonitrile $\xrightarrow{LiAlH_4}$ (d) 2-phenyl ethoxamide $\xrightarrow{Br_2 + KOH}$

37. **$[Co(NH_3)_4(NO_2)_2]$ Cl exhibits**

(a) linkage, geometrical and ionisation isomerism

(b) linkage, ionisation and optical isomerism

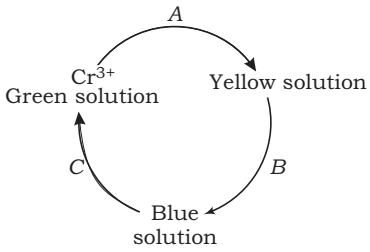
(c) linkage, geometrical and optical isomerism

(d) ionisation, geometrical and optical isomerism

38. **If 0.1 M H_2SO_4 (aq) solution shows freezing point $-0.39°C$, then what is K_{a_2} for H_2SO_4 (Given, molality = molarity and $K_{f\,(H_2O)} = 1.86$ kg mol^{-1})**

(a) 0.012 (b) 0.015 (c) 0.018 (d) 2.0

39. Identify A, B and C in the following sequence.



(a) Alkaline H_2O_2, acidified H_2O_2, on standing
(b) Alkaline O_3, acidified O_3, Zn/HCl
(c) Acidified H_2O_2, alkaline H_2O_2, heat
(d) Alkaline O_3, heat, NH_4OH

40. 23.2 g of an organic compound having molecular formula C_nH_{2n+2} is burnt in excess of O_2(g) initially taken in a 44.82 L steel vessel. Before reaction the gaseous mixture w at 273 K with pressure of 2 atm. After complete combustion and loss of considerable amount of heat the mixture of product and excess of O_2 had a temperature of 546 K and 4.6 atm pressure. The formula of compound is

(a) C_5H_{12} (b) C_6H_{14} (c) C_3H_8 (d) C_4H_{10}

41. Which of the following statement(s) is/are true about product, obtained by treatment of CCl_3CHO with chlorobenzene in presence of H_2SO_4?

I. It has one chiral centre.
II. It is used as an insecticide
III. It is not easily metabolised by animals.
IV. Its name is p,p-dichlorodiphenyltrichloroethane.
Choose the correct option.

(a) I and II (b) II and III (c) II, III and IV (d) only IV

42. Salt A + S $\longrightarrow$ B $\xrightarrow{BaCl_2}$ white ppt.

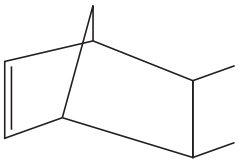
A is paramagnetic in nature and contains about 55% of potassium (K). Thus, A is

(a) KO_2 (b) K_2O_2 (c) K_2O (d) K_2CO_3

43. Under the influence of an electric field, the particles in a solution migrate towards cathode. The coagulation of the same solution is studied using NaCl, Na_2SO_4 and Na_3PO_4 solution. Their coagulating values will be in maximum for

(a) Na_3PO_4 (b) NaCl
(c) Na_2SO_4 (d) None of these

44. IUPAC name and degree of unsaturation of compound X is



(a) 2, 3- dimethyl bicyclo [2,2,1] hept-5 ene,2
(b) 1, 2- dimethyl bicyclo [2,2,1] hept-4 ene,3
(c) 5, 6- dimethyl bicyclo [2,2,1] hept-2 ene,3
(d) 4, 5- dimethyl bicyclo [2,2,1] hept-1 ene,2

45. For a complex reaction

$$P \xrightarrow{k} \text{Products}$$

E_{a_1} = 200 kJ/mol, E_{a_2} = 90 kJ/mol, E_{a_3} = 80 kJ/mol

Overall rate constant k is related to individual rate constant by the equation $k = \left(\frac{k_1 k_2}{k_3}\right)^{2/3}$.

Activation energy (kJ/mol) for the overall reaction is

(a) 140 kJ/mol (b) 240 kJ/mol (c) 370 kJ/mol (d) 30 kJ/mol

46. Which of the following statement is correct?

(a) NO_2 on reaction with NaOH gives a mixture of two salts.

(b) N_2O is laughing gas and is angular in shape.

(c) NO_2 is a sweet smelling and is angular in shape.

(d) NO is a colourless gas and acidic in nature.

47. Which of the following statements is correct regarding the following reaction?

$$CH_3—C\equiv C—H \xrightarrow{Sia_2BH} A \xrightarrow{H_2O_2/OH^-} B$$

(a) Product will show tautomerism and metamerism

(b) Product will show geometrical isomerism only

(c) Product will show metamerism only

(d) None of the above

48. In a planar tetra-atomic molecule PQ_3, P is at the centroid of the equivalent triangle formed by the atoms, Q. If the P—Q bond distance is 2 Å, what is the distance between the centres of any two Q atoms?

(a) $2\sqrt{3}$ Å (b) $\sqrt{3}$ Å (c) $4\sqrt{3}$ Å (d) $\frac{1}{\sqrt{3}}$ Å

49. Phenol on reaction with a mixture of conc. HNO_3 and conc. H_2SO_4 produces a compound. What is the degree of unsaturation present in the compound and nature of compound?

(a) 5, acidic (b) 7, basic

(c) 6, neutral (d) 7, acidic

50. Copper sulphate is prepared by blowing a current of air through Cu scrap and dilute H_2SO_4, Dilute HNO_3 is also added

(a) which combines with H_2SO_4 to give a very strong oxidising mixture and oxidises Cu to Cu^{2+}

(b) to oxidise Cu to Cu^{2+} which then form $CuSO_4$ with dilute H_2SO_4

(c) to increase ionisation of H_2SO_4 to give SO_4^{2-} ions

(d) to oxidise Fe^{2+} to Fe^{3+} sulphate, which remains in solution after crystallisation of $CuSO_4$

51. The chlorate ion can disproportinate basic solution according to the reaction

$$2ClO_3^- \rightleftharpoons ClO_2^- + ClO_4^-$$

What is the equilibrium concentration of perchlorate ions from a solution initially at 0.1M in chlorate ions at 298 K? Given, $E^\circ_{ClO_4^-/ClO_3^-}$ = 0.39 V and $E^\circ_{ClO_3^-/ClO_2^-}$ = 0.36 V at 298 K.

(a) 3.8×10^{-2} (b) 3.6×10^{-3}

(c) 1.9×10^{-2} (d) 4.2×10^{-5}

52. **Which of the following compounds are slightly soluble or insoluble in NH_4OH solution?**

I. $Ni(OH)_2$ II. Ag_2CrO_4 III. $Fe(OH)_3$ IV. $Al(OH)_3$ V. Ag_2CO_3

(a) V only (b) I and III

(c) I, III, IV, V (d) III, IV

53. **An organic compound having molecular formula C_4H_5Cl, which does not contain any of ring or tripple bond. What will be the product when this organic compound undergo polymerisation?**

(a) Duprene (b) Natural rubber (c) Buna-*S* (d) Buna-*N*

54. **One atom of an element y weighs 6.64×10^{-23} g. Then, find the numbers of moles of atom in 20 kg**

(a) 300 (b) 100 (c) 200 (d) 500

55. **What will be the empirical formula of organic compound. If in Carius method 0.099 g organic compound gave 0.287 g AgCl and it contain 24% carbon and 4.3% of hydrogen?**

(a) CH_2Cl_2 (b) CH_2Cl (c) $CHCl_3$ (d) $C_2H_4Cl_2$

56. **$AlCl_3$ is an electron deficient compound but AlF_3 is not this is because**

(a) $AlCl_3$ exists as a dimer but AlF_3 does not

(b) Atomic size of F is smaller than Cl which makes AlF_3 more covalent

(c) $AlCl_3$ is a covalent compound while AlF_3 is an ionic compound

(d) Al in $AlCl_3$ is in sp^3 hybridised state but Al in AlF_3 is in sp^2 state

57. **What will be the product obtained when acetophenone will treated with ammonium polysulphide in aqueous medium?**

(a) 2-phenyl acetamide (b) 2-phenyl acetic acid (c) Benzamide (d) Benzophenone

58. **When an electron makes a transition from $(n+1)$ state to nth state, the frequency of emitted radiations is related to n according to $(n >> 1)$.**

(a) $\frac{4RCZ^2}{n^2}$ (b) $\frac{2RCZ^2}{n^3}$ (c) $\frac{2RCZ^3}{n^3}$ (d) $\frac{4RCZ^3}{n^3}$

59. **Statement I Conc. H_2SO_4 cannot be used to prepare HBr or HI from KBr or KI.**

Statement II Conc. H_2SO_4 is a strong oxidising agent while HBr or HI are reducing agent.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I

(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I

(c) Statement I is false but Statement II is true

(d) Both Statement I and Statement II are false

60. **Statement I 2-acetoxy benzoic acid can cause ulcer in stomach when taken in empty stomach.**

Statement II This compound is aspirin which prevents platelets coagulation as it has blood clotting agent.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I

(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I

(c) Statement I is true but Statement II is false

(d) Both Statement I and Statement II are false

PART C Mathematics

61. **The variance of first 30 natural numbers, is**

(a) 74.92 (b) 37.98 (c) 38.98 (d) None of these

62. **Set of all real values of x satisfying the inequation $\frac{\log_2(x^2 - 5x + 4)}{\log_2(x^2 + 1)} > 1$ is**

(a) $\left(-\infty, \frac{3}{5}\right) - \{0\}$ (b) $(-\infty, 1) - \{0\}$ (c) $\left(\frac{3}{5}, \infty\right)$ (d) $(4, \infty)$

63. **The value of a for which the function $(a + 2)x^3 - 3ax^2 + 9ax - 1$ decrease monotonically throughout for all real x, are**

(a) $a < -2$ (b) $a > -2$ (c) $-3 < a < 0$ (d) $-\infty < a < -3$

64. **The degree of the differential equation satisfying $\sqrt{1 - x^2} + \sqrt{1 - y^2} = b(x - y)$ is**

(a) 1 (b) 3 (c) 4 (d) 0

65. **$z = a + ib, a, b \in R, b \neq 0$ and $|z| = 1$, then $z = \frac{c + i}{c - i}$, where c is equal to**

(a) $\frac{a}{b}$ (b) $\frac{a-1}{b}$ (c) $\frac{a+1}{b}$ (d) $\frac{a+1}{b+1}$

66. **If $k = \lim_{x \to \infty} \left(\frac{\sum_{k=1}^{1000} (x + k)^m}{x^m + 10^{1000}} \right)$, then k is (m > 101)**

(a) 10 (b) 10^2 (c) 10^3 (d) 10^4

67. **Let $A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$, where ϕ is a null set, then**

(a) $\phi \subseteq A, \phi \in A, \{\phi\} \in A, \{\phi\} \subseteq A$ is true (b) $\phi \in A$ but $\phi \not\subset A$

(c) $\{\phi\} \in A$ but $\phi \not\subset A$ (d) A is a null set

68. **Consider $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then the value of $\lim_{n \to \infty} \frac{A^n}{n}$ (where $\theta \in R$) is equal to**

(a) 10 (b) zero matrix (c) symmetric matrix (d) 4

69. **The coordinates of the point on the parabola $y^2 = 8x$, which is at minimum distance from circle $x^2 + (y - 6)^2 = 1$ are**

(a) (2, 14) (b) (12, 18) (c) (2, 4) (d) (4, 2)

70. **The equation of the line passing through the points $(1, -2, 3)$ and parallel to the plane $x - y + 2z - 5 = 3x + y + z - 6 = 0$ is**

(a) $\frac{x-1}{-3} = \frac{y+2}{5} = \frac{z-3}{4}$ (b) $\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z-3}{2}$ (c) $\frac{x+2}{-3} = \frac{y-3}{5} = \frac{z+3}{4}$ (d) None of these

71. **$\int_{-\frac{\pi}{2}}^{0} \sqrt{\cos x - \cos^3 x}\, dx$**

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

72. **Let r be a unit vector satisfying $\mathbf{r} \times \mathbf{a} = \mathbf{b}$, where $|\mathbf{a}| = \sqrt{3}$ and $|\mathbf{b}| = \sqrt{2}$, then**

(a) $\mathbf{r} = \frac{2}{3}(\mathbf{a} + \mathbf{a} \times \mathbf{b})$ (b) $\mathbf{r} = \frac{1}{3}(\pm\mathbf{a} + \mathbf{a} \times \mathbf{b})$ (c) $\mathbf{r} = \frac{1}{4}(\pm\mathbf{a} + \mathbf{a} \times \mathbf{b})$ (d) $\mathbf{r} = \frac{2}{3}(\pm\mathbf{a} + \mathbf{a} \times \mathbf{b})$

73. **If a, b, c are in AP, b, c, d in GP and c, d, e in HP then a, c, e are in**

(a) AP (b) GP (c) HP (d) None of these

74. **The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersect the curve $xy = c^2, z = 0$ if c is equal to**

(a) $\pm\sqrt{5}$ (b) ± 4 (c) $\pm 1/2$ (d) $\pm 1/\sqrt{3}$

75. **If R and R′ are symmetric relations (not disjoint) on a set A, then the relation $R \cap R'$ is**

(a) reflexive (b) symmetric (c) transitive (d) None of these

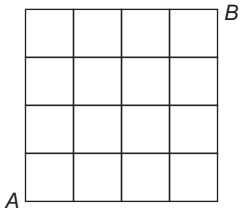
76. **The minimum value of $(\sin x)^{\sin x}$ when $0 < x < \pi$ is**

(a) $e^{1/e}$ (b) e^{-e} (c) $e^{-1/e}$ (d) None of these

77. **A line L passes through the points (1, 1) and (2, 0) and another line L′ passes through $\left(\frac{1}{2}, 0\right)$ and perpendicular to L. Then, area of the triangle formed by the line $L \cdot L'$ and y-axis is**

(a) $\frac{15}{8}$ (b) $\frac{25}{4}$ (c) $\frac{25}{8}$ (d) $\frac{25}{16}$

78. **In figure given, person can move towards right or in upward direction only how many ways are there to reach the point B from A**



(a) 70 (b) 90 (c) 8 (d) 16

79. **For any two random events A and B with $0 < P(A), P(B) < 1$, which of the following statement(s) is/are always true?**

(a) $P(A/B) > P(A) \Rightarrow P(B/A) > P(B)$

(b) $P\left(\frac{B}{A}\right) + P(\bar{B}/\bar{A}) = 1$

(c) $P(B/A) \neq P(B/A')$ only when A, B are independent

(d) $P(A/B) = P(A'/B)$ only when A, B are mutually exclusive

80. **If $\mathbf{A} = \{\theta : 2\cos^2\theta + \sin\theta \leq 2\}$ and $\mathbf{B} = \left\{\theta : \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}\right\}$. Then, $\mathbf{A} \cap \mathbf{B}$ is equal to**

(a) $\{\theta : \pi/2 \leq \theta \leq 5\pi/6\}$ (b) $\{\theta : \pi \leq \theta \leq 3\pi/2\}$

(c) $\{\theta : \pi/2 \leq \theta \leq 5\pi/6\} \cup \{\theta : \pi \leq \theta \leq 3\pi/2\}$ (d) None of these

81. **If in a triangle ABC, $\mathbf{AB} = \frac{\mathbf{u}}{|\mathbf{u}|} - \frac{\mathbf{v}}{|\mathbf{v}|}$ and $\mathbf{AC} = \frac{2\mathbf{u}}{|\mathbf{u}|}$, where $|\mathbf{u}| \neq |\mathbf{v}|$, then**

(a) $1 + \cos 2A + \cos 2B + \cos 2C = 0$ (b) $1 + \cos 2A + \cos 2B = 0$

(c) $1 + \cos 2A + \cos 2B + \cos 2C = 3$ (d) $1 + \cos 2A + \cos 2B = 4$

82. **The tangent at a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets one of its directrix in F. If PF subtends an angle θ at the corresponding focus, then θ equals**

(a) $3\pi/4$ (b) $\pi/2$ (c) $3\pi/2$ (d) $\pi/6$

83. **If both the roots of $4x^2 - 20\lambda x + 25\lambda^2 + 15\lambda - 66 = 0$ are less than 2, then λ lie on the interval**

(a) $\left(\frac{4}{3}, 2\right)$ (b) $(2, \infty)$ (c) $\left(-1, -\frac{4}{5}\right)$ (d) $(-\infty, -1)$

84. **In ΔABC, the tangent of half the difference of two angles is one-third the tangent of half the sum of the angles. Then, the ratio of the side opposite to the angles is**

(a) 2 : 1 (b) 1 : 3 (c) 1 : 5 (d) 1 : 2

85. **If $n \in N$ and C_r, C_{r-1}, C_{r-2} have their usual meaning, then the value of the expression $^{n-2}C_r + 2^{n-2}C_{r-1} + {}^{n-2}C_{r-2}$ equals**

(a) $^{n-2}C_{r-2}$ (b) $^{n}C_{r-1}$ (c) $^{n-1}C_{r-1}$ (d) $^{n}C_{r}$

86. **The equation of the circle which cuts each of the three circles $x^2 + y^2 = 4$, $(x-1)^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ orthogonally is**

(a) $x^2 + y^2 + x + 2y + 4 = 0$ (b) $x^2 + y^2 + x - 2y + 4 = 0$
(c) $x^2 + y^2 - x - 2y + 4 = 0$ (d) $x^2 + y^2 - x + 2y + 4 = 0$

87. **If A and B are square matrices of same order and A is non-singular then for a positive integer n, $(A^{-1}BA)^n$ equals**

(a) $A^nB^nA^{-n}$ (b) $A^{-1}B^nA$ (c) $A^{-n}B^nA^n$ (d) None of these

88. **If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, then $\frac{xy'}{y}$ is**

(a) $\frac{1}{a-x} + \frac{1}{b-x} + \frac{1}{c-x}$ (b) $\frac{x}{a-x} + \frac{x}{b-x} + \frac{x}{c-x}$ (c) $\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}$ (d) $\frac{a}{x-a} + \frac{b}{x-b} + \frac{c}{x-c}$

89. **Statement I If p is false statement and q is true statement, then**
Statement II $\sim p \wedge q$ is equivalent to $\sim(p \vee \sim q)$.

(a) Statement I is true and Statement II is true. Statement II is the correct explanation for Statement I
(b) Statement I is true and Statement II is true. Statement II is not the correct explanation for Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

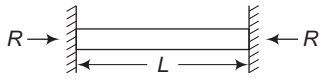
90. **Statement I $[(m+1)^7 - m^7 - 1]$ is divisible by 7 for each $m \in N$.**
Statement II $[m^7 - m]$ is divisible by 7 for each $m \in N$.

(a) Statement I is true and Statement II is true. Statement II is the correct explanation for Statement I
(b) Statement I is true and Statement II is true. Statement II is not the correct explanation for Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

Analytical Explanations

Physics

1. (*b*) **Idea** This question is based on the concept of thermal expansion and on Hooke's law. Combining these two concepts, students can easily solve this question.



Due to heating, there shall be expansion in rod given by, $\Delta L_1 = \alpha (\Delta T) L$

But this expansion is resisted by rigid rod. If R is the reaction produced then, stress in rod is $\frac{R}{A}$.

So, $\text{strain} = \frac{\text{stress}}{Y} \Rightarrow \frac{\Delta L_2}{L} = \frac{R/A}{Y}$

[ΔL_2, compression by stress due to wall]

$$\Rightarrow \quad \Delta L_2 = \frac{R}{AY} \cdot L$$

Since walls are rigid so expansion due to heating should be equal to compression by stress due to walls

$$\Rightarrow \quad \Delta L_1 = \Delta L_2 \Rightarrow L\alpha \cdot (\Delta T) = \frac{RL}{AY}$$

$$\Rightarrow \quad R = \alpha AY (\Delta T)$$

TEST Edge This question is based on linear thermal expansion and on Hooke's law and could be asked separately.

2. (*c*) **Idea** This question is based on the variation of resistance with temperature.

Power $= \frac{V^2}{R}$

Resistance of hot tungsten $= \frac{V^2}{P} = \frac{200 \times 200}{100}$

$$R_{\text{hot}} = 400\ \Omega$$

$$R_{\text{cold}} = 40\ \Omega$$

Now, $R_{\text{hot}} = R_{\text{cold}} (1 + \alpha \cdot \Delta T)$

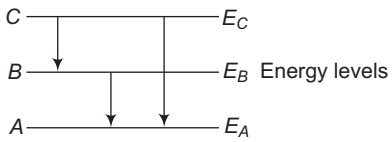
$\therefore \quad \frac{400}{40} = 1 + \alpha \times \Delta T$

$$\Delta T = \frac{9}{3 \times 10^{-3}} = 3000^\circ\text{C}$$

$\therefore \quad T_{\text{hot}} = 3000^\circ + 25^\circ = 3025^\circ\text{C}$

TEST Edge Similar questions in which some resistance value is given at a particular temperature and the value of resistance at other temperature could be asked.

3. (*b*) **Idea** This question is based on the energy levels of an atom and the transition of the electrons between them.



From the figure, $(E_A - E_B) + (E_B - E_C) = (E_C - E_A)$

$$\Rightarrow \quad \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = \frac{hc}{\lambda_3}$$

$$\Rightarrow \quad \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

TEST Edge Similar questions based on the energy levels of hydrogen atom are normally asked in JEE Main.

4. (*b*) **Idea** This question is based on conservation of mechanical energy. Here, one should observe that the summation of gravitational PE and KE will remain constant.

We can apply mechanical energy conservation

$$(\text{TME})_i = (\text{TME})_f$$

$$-\frac{GM_e m}{R_e} + \frac{1}{2} m \left(3 \times \sqrt{\frac{GM_e}{R_e}} \right)^{1/2} = (\text{PE})_\infty + \frac{1}{2} m v'^2$$

$$-\frac{GM_e m}{R_e} + \frac{9}{2} \frac{GM_e m}{R_e} = \frac{1}{2} m v'^2$$

$$\frac{7}{2} \frac{GM_e}{R_e} = \frac{1}{2} v'^2$$

$$v' = \sqrt{\frac{7GM_e}{R_e}} = \sqrt{7}\, v_e$$

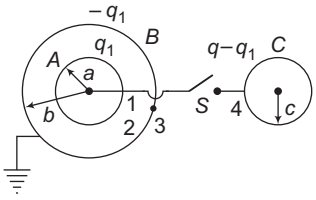
TEST Edge Similar questions where an object is thrown upwards from the earth's surface or where two objects are attracted towards each other could be asked. These questions could be solved easily by just applying the conservation of mechanical energy.

5. (*a*) **Idea** This question is judging the knowledge of Electrostatics of conductor and **Electric potential.** Here after connecting two spheres, their potential will be same.

When switch S is closed, the charge on sphere A will be redistributed on A and C in the ratio of their capacitances,

$$\frac{q_1}{q-q_1}=\frac{4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)}{4\pi\,\varepsilon_0\,c}$$

$$q_1=\frac{q\,ab}{ab+bc-ca}$$



Taking $q = 30\,\mu C$

$a = 1$ cm, $b = 2$ cm, $c = 1$ cm

We get, $q_1 = 20\,\mu C$

$\therefore \quad q - q_1 = 30 - 20 = 10\,\mu C$

TEST Edge Similar questions where two concentric shells are connected with conducting wires and all the charge on inner shell would flow on the outer shell could be asked. Here, one should remember that positive charge will flow towards lower potential and *vice-versa*.

6. (*c*) **Idea** This question is based on the power emitted by the light sources.

Energy of a photon is $E=\frac{hc}{\lambda}=h\nu$

If N be the number of photons emitted/s, then

$$\text{Power} = N\,h\,\nu$$

Let n_R and n_B be the number of photons emitted by red bulb and blue bulb per second. As power of both bulbs is same, so

$$n_R\,h\,\nu_R = n_B\,h\,\nu_B$$

$$\Rightarrow \quad \frac{n_R}{n_B}=\frac{\nu_B}{\nu_R} \text{ as } \nu_B > \nu_R$$

So, $n_R > n_B$

In time t, we have $n_R \times t > n_B \times t$

or $n_R > n_B$

TEST Edge Similar questions based on intensity (energy emitted per second by per unit area) of radiation could be asked.

7. (*d*) **Idea** This question is based on the concept of centre of mass. Here, one has to remember that the centre of mass of a system with uniform mass density lie at their geometrical centre but if the mass density is non-uniform then centre of mass will not lie at geometrical centre.

Here there is no relation given between the densities of spherical ball and cube, so the centre of mass of the two could lie anywhere inside the cube.

TEST Edge Different questions where one has to find out the centre of mass of a system could also be asked.

8. (*a*) **Idea** Here, the electric force on the charge due to infinitely long wire acts as a centripetal force.

Electric field around an infinite line charge is calculated using Gauss'law and it is

$$E=\frac{\lambda}{2\pi\,\varepsilon_0\,r}$$

Now, particle is moving in a circle. So, centripetal force is provided by electric force.

$$\Rightarrow \quad \frac{mv^2}{r}=\frac{\lambda}{2\pi\,\varepsilon_0\,r}\cdot q$$

$$\Rightarrow \quad v=\left(\frac{2k\,\lambda\,q}{m}\right)^{1/2} \quad \left[\text{using } \frac{1}{4\pi\,\varepsilon_0}=k\right]$$

Now, time period $T=\frac{2\pi r}{v}$

$$=2\pi r\times\sqrt{\frac{m}{2k\,\lambda\,q}}$$

TEST Edge The questions based on the electric field of a charged sheet or charged conducting shell could be asked.

9. (*b*) **Idea** Here, students must remember that the resultant intensity in YDSE depends on the phase difference between the two waves.

Fringe width is $\beta=\frac{\lambda D}{d}$

Given distance from central maxima $= 2.25\times\frac{\lambda D}{d}$

Also, we know that path difference Δx between two interfering waves at a distance y from the centre is given by

$$\Delta x=\frac{yd}{D}, \qquad \left[\text{given } y = 2.25\,\frac{\lambda D}{d}\right]$$

Hence, $\Delta x=\frac{2.25\times D\lambda}{d}\times\frac{d}{D}$

$$=2.25\,\lambda$$

Hence, phase difference is $\phi=\frac{2\pi}{\lambda}\times\Delta x$

$$=\frac{2\pi}{\lambda}\times 2.25\,\lambda$$

$$=4.5\,\pi$$

Now, $I = I_{max}\cos^2\phi/2$

$$=I_{max}\times\cos^2\left(\frac{4.5\,\pi}{2}\right)$$

$$=I_{max}\cos^2\frac{\pi}{4}$$

$\Rightarrow \quad I = \frac{I_{max}}{4}$

Now, $I_{max} = 4I_0$

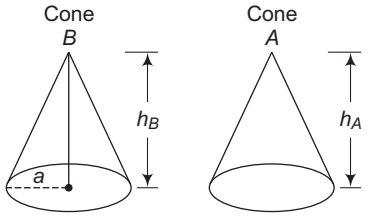
So, $I = \frac{4I_0}{4} = I_0$

TEST Edge In YDSE, different types of questions could be asked. The questions based on dark and bright fringes could be asked.

10. (*b*) **Idea** In this question one should understand that the moment of inertia of an rotating object depends on the mass distribution about the axis of rotation.

$h_A < h_B$ (heights)

$d_A > d_B$ (densities)



B has larger shape and smaller density so, the same volume of *B* as volume of *A* will have smaller value of its mass due to increase density.

So, the particles of *B* will be distributed far from the axis of rotation in comparison of *A*.

So, $I_B > I_A$.

TEST Edge Questions where moment of inertia of a rotating object could be asked. These questions could be solved by integrating method or by the use of parallel and perpendicular axes theorems.

11. (*d*) **Idea** Here, two concepts are used in this problem. One is of constraint motion and the other is of wave speed $\left(v = \sqrt{\frac{T}{\mu}}\right)$.

$f = \frac{nv}{2L}$

For $n = 1$

$f_1 = \frac{v}{2L}$

$\Rightarrow \quad \frac{(f_1)_A}{(f_2)_B} = \frac{\sqrt{T_1/\mu}/2L_1}{\sqrt{T_2/\mu}/2L_2} = \sqrt{\frac{T_1}{T_2}} \times \frac{L_2}{L_1}$

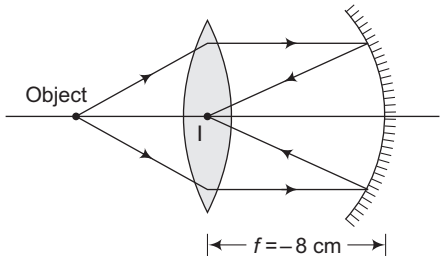
$= \sqrt{\frac{T_1}{2T_1}} \times \frac{1}{4} \qquad \left[\because T_1 = \frac{T_2}{2}\right]$

$= \sqrt{\frac{1}{2}} \times \frac{1}{2} = \frac{1}{2\sqrt{2}}$

TEST Edge Questions based on standing waves on a string or in a organ pipe are usually asked in JEE Main.

12. (*b*) **Idea** This question is based on the combination of mirror and lens. By drawing ray diagram one can easily solve this question.

From the ray diagram, it is easy to understand the situations.



TEST Edge Different questions where the combination of two lenses or two mirrors could be asked from this topic.

13. (*d*) **Idea** To solve this question one must know the difference between conservative and non-conservative forces. One must also know work-energy theorem.

If conservative and non conservative forces are acting in the system

$\Rightarrow \quad W_{net} = W_C + W_{NC} = \Delta KE$

$\Rightarrow \quad W_{non\text{-}conservative} =$ loss in mechanical energy

TEST Edge When only conservative forces are acting in the system, the total mechanical energy will remain conserved. If conservative as well as non-conservative forces are acting in the system then there will be the loss of mechanical energy due to non-conservative forces.

14. (*c*) $T = 2\pi\sqrt{\frac{m}{k}}$

Time period do not depend on the amplitude.

So, $T_1 = T_2$

15. (*a*) **Idea** This question is based on electromagnetic wave. Here, students must know about how to find frequency, speed and wavelength of a electromagnetic wave.

Standard equation of electromagnetic wave is

$E = E_0 \sin(k_0 x + \omega_0 t)$

So, $k_0 = 44 \times 10^4 \text{ m}^{-1}$

and $\omega_0 = 132 \times 10^{12} \text{ s}^{-1}$

Now, $k_0 = \frac{2\pi}{\lambda_0} \Rightarrow 2 \times \frac{22}{7} \times \frac{1}{\lambda_0} = 44 \times 10^4$

$\Rightarrow \quad \lambda_0 = \frac{1}{7} \times 10^{-4} \text{ m}$

and $\omega_0 = 132 \times 10^{12} \text{ s}^{-1}$

$\Rightarrow \quad 2\pi\nu_0 = 132 \times 10^{12}$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times \nu_0 = 132 \times 10^{12}$$

$$\Rightarrow \quad \nu_0 = 2.1 \times 10^{13}\ s^{-1}$$

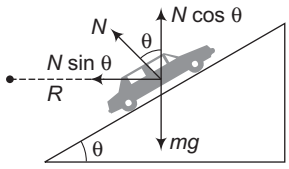
Now in a medium of $\mu = 2$ only the λ_0 will change to $\lambda' = \frac{\lambda_0}{\mu}$ but no change to ν_0 shall occur.

So, $\lambda' = \frac{\lambda_0}{1.4} = \frac{10^{-4}}{1.4 \times 7} = \frac{10^{-4}}{9.8} \simeq 10^{-5}$ m

and $\nu_0 = 2.1 \times 10^{13}\ s^{-1}$

TEST Edge Questions on displacement current and about the nature of EM wave and about the permittivity and permeability of free space (or a medium) could be asked.

16. (*d*) **Idea** For this question student must know about the dynamics for the motion of a car on a circular banked road.



If $N \sin\theta = \frac{mv^2}{R}$, then there is no need of friction.

If $N \sin\theta > \frac{mv^2}{R}$, then f_s will act in upward direction along the inclined surface.

$\Rightarrow$ But $(f_s)_{max}$ will act for the minimum speed.

In the same way if $N \sin\theta < \frac{mv^2}{R}$, then f_s will act in downward direction along the inclined plane.

But $(f_s)_{max}$ will act for the maximum speed.

TEST Edge Different questions could be asked on the same concept. One can replace the banked road from a rotating hemispherical bowl and with the same concept of normal reaction and friction, students can solve this problem.

17. (*c*) Second law of thermodynamics says that as first law of thermodynamics is based on conservation of energy $[\Delta Q = \Delta U + \Delta W]$ but first law of thermodynamics is not always valid in nature.

For example, if we leave a ball from same height it will bounce again and again on the floor and finally comes to rest. It losses all its mechanical energy in the form of heat and sound.

But its reverse is not possible means the energy lost will not come back itself and the ball will not start bouncing again. But this process is possible according to first law of thermodynamics but not valid in nature (by IInd law of thermodynamics).

18. (*c*) **Idea** This question is based on the resonant frequency in an AC circuit.

At resonance $X_C = X_L$

At $\omega_0/2$, $X_C' = 2X_C$

and $X_L' = \frac{X_L}{2}$

$[\because X_C = \frac{1}{\omega C}$ *i.e.*, $X_C \propto \frac{1}{\omega}$ and $X_L = \omega L$ *i.e.*, $X_L \propto \omega]$

Then, $Z' = \sqrt{X_C'^2 + X_L'^2}$

$$= \sqrt{4X_C^2 + \frac{X_L^2}{4}} = \frac{\sqrt{17}}{2} \cdot X_C \quad [\because X_C = X_L]$$

i.e., $Z' = \frac{\sqrt{17}}{2} X_C$

Now, when frequency is $2\omega_0$ then $X_C'' = \frac{X_C}{2}$ and $X_L'' = 2X_L$

So, $Z'' = \sqrt{X_C''^2 + X_L''^2} = \frac{\sqrt{17}}{2} X_C = Z'$

Since impedance remains the same at $\omega = \frac{\omega_0}{2}$ and at $\omega = 2\omega_0$.

So, current shall be 2 A.

TEST Edge A question on quality factor could be asked from the same topic.

19. (*d*) **Idea** Here, one must understand that the force arises only where is an impending or relative motion occur between two surfaces. Here there is no relative motion between the two blocks so force of friction between two blocks will be zero.

Here the relative acceleration between two blocks is zero, so there will be no friction between the blocks.

TEST Edge Different types of questions could be asked on frictional force *i.e.,* friction between the wedge block, friction between two blocks.

20. (*d*) As $KE = \frac{3}{2}kT$ and $v_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore \quad KE_2 = 2\,KE_1 = 2 \times 6.21 \times 10^{-21}$

$$= 12.42 \times 10^{-21}\ J$$

$$\frac{v_{rms,\,2}}{v_{rms,\,1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{2}$$

$\therefore \quad v_{rms,\,2} = \sqrt{2} \times v_{rms,\,1} = 684$ m/s

21. (*b*) Potential difference induced in both the rods would have different polarity.

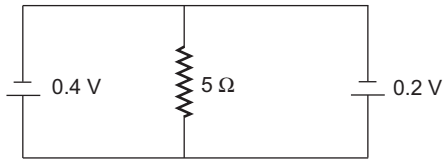
PD (potential difference) in

$$ab = Blv = 1 \times 0.1\ m \times 4\ m/s = 0.4\ V$$

PD (potential difference) in

$$cd = Blv = 1 \times 0.1 \text{ m} \times 2 \text{ m/s} = 0.2 \text{ V}$$

So, the arrangement would act like



$$\therefore \quad \text{Current } I = \frac{0.4 - 0.2}{5} = \frac{0.2}{5} = 0.04 \text{ A} = 40 \text{ mA}$$

22. (*a*) **Idea** This question is based on motion. Here, one must know the resolution of vectors and about unit vectors.

$$\mathbf{v}_A = 3\,\hat{\mathbf{i}}$$

$\mathbf{v}_B = 8\hat{\mathbf{i}} + 2t\,\hat{\mathbf{k}}$, where $\mathbf{v}_B$ is velocity of B at any time t.

Location of B at any time t is given by,

$$\mathbf{r}_B = (8t)\,\hat{\mathbf{j}} + \frac{1}{2}(2t^2)\,\hat{\mathbf{k}}$$

Location of C at any time t is given by,

$$\mathbf{r}_C = \mathbf{v}_0 \times t$$

So, from given condition, $\mathbf{r}_B\,(t = 4) = r_C\,(t = 4)$

$$\Rightarrow \qquad (8 \times 4)\,\hat{\mathbf{j}} + 4^2\hat{\mathbf{k}} = \mathbf{v}_0 \times 4$$

or $$\mathbf{v}_0 = (8\,\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\,\text{m/s}$$

TEST Edge From this lesson questions on relative velocity in 2D could be asked. The river man problems could also be asked.

23. (*d*) If R is rate of heating, $\Delta Q = ms(\theta)$

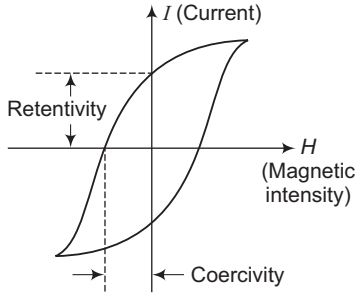
$$R\,t = ms\theta \Rightarrow \theta = \left(\frac{R}{ms}\right)t$$

Slope of θ-t curve $= \dfrac{R}{ms} = \tan\phi$

From it we conclude, $S_A = S_B > S_C$

Because $\dfrac{R}{ms}$ for both A and B are same.

24. (*c*) Electromagnets are made up of soft iron. The soft iron has high retentivity and low coercivity. In electromagnets hysteresis loss should be low and retentivity must be high.



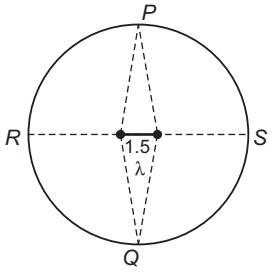
Note Retentivity of a substance is a measure of magnetisation remaining in the substance when magnetising field is removed.

Coercivity of a substance is the measure of the reverse magnetising field required to destroy the residual magnetism of substance.

25. (*a*) The slope of x-t graph at $t = 0$ is positive *i.e.*, initial velocity of the particle is +ve and x-t graph is concave down, so it means acceleration is negative but whenever it is constant or not it cannot be predicted from given information. As v and acceleration are in opposite directions, then motion is the retarded one and finally the particle stops.

26. (*b*) **Idea** This question is based on the concept of constructive and destructive interference of waves and its relation with path difference.

At point P and Q path difference is equal to zero. So, point P and Q are parts y constructive interference, Now in points R and S path difference is equal to 1.5 λ, according to question path difference of 1.5 λ also correspond, to point of constructive difference.



So, P, Q, R and S are 4 points around circle and are in constructive interference.

TEST Edge Questions based on phase difference, path difference, constructive interference and destructive interference are frequently asked.

27. (*c*) Field at centre $= \dfrac{\mu_0 i}{2a}$

Field on the axis at a distance >> radius of ring is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\mathbf{m}}{d^3}$$

Here, $\mathbf{m}$ is magnetic moment $= i \times$ area of ring

$$\Rightarrow \qquad m = \pi a^2 \cdot i$$

So, $$B_{\text{at}P} = \frac{\mu_0}{4\pi} \times \frac{2 \times \pi a^2 \times i}{(10a)^3}$$

$$= \frac{\mu_0 i}{2 \times 1000\, a}$$

$$\therefore \quad \text{Ratio is } \frac{B_0}{B_P} = \frac{\mu_0 i}{2a} \times \frac{2 \times 1000\, a}{\mu_0 i}$$

$$= 1000 : 1$$

28. (*a*) **Idea** From dimensional analysis, we can check the dimensional accuracy of the equation but it could not tell that the expression is correct or not.

Given expression is dimensionally correct but it is not correct otherwise.

The correct expression is $\mathbf{F}_{ext} = \frac{d\mathbf{P}}{dt}$

TEST Edge Questions on different applications of dimentional analysis could be asked in JEE Main.

29. (*c*) **Idea** It is based on Lenz's law and motion under gravity.

If we apply Lenz's law, initially current will flow in counter clockwise direction, as flux is increasing. So upper face of ring will act as north pole. So ring will repel the magnet. When magnet is releasing, then the current will flow in clockwise direction as flux now is decreasing.

So lower face will act as North pole which would attract the South pole of magnet.

So, acceleration of magnet shall always be less than g.

TEST Edge Another questions where the direction of induced current are asked can be solved by Lenz's law.

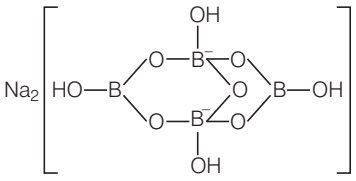
30. (*c*) Statement I is false but Statement II is true.

The KE of the system converts in PE during collision. So, even in elastic collision the KE of the system is not conserved during collision. But momentum of the system is conserved even during collision.

Chemistry

31. (*a*) **Idea** This problem is based on molecular formula representation and structure of borax, students are advised to go through the structure of borax and their molecular formula representation to solve the question.

Borax molecule is actually made of two tetrahedra and two triangular units joined as follows



So, correct representation of borax is

$$Na_2[B_4O_5(OH)_4]\cdot 8H_2O$$

TEST Edge These types of questions are asked in exam to judge the knowledge of student on structure of a various inorganic molecules, students are advised to go through the understanding of structure of various inorganic compounds. Question related to structure of compounds like P_4O_{10}, P_4O_6, etc., may also be asked.

32. (*a*) **Idea** This problem includes conceptual mixing of de-Broglie equation and energy consideration during formation of molecule. So, students are advised to calculate the energy required and energy released during process and then by using de-Broglie equation and concept of energy consideration during formation of molecule. Calculate the required parameter.

$X + 2e^- \longrightarrow X^{2-}$ energy released = 30.86 eV

Total energy released = number of moles of molecule × energy released by one moles of molecule

$= y \times 30.86$ eV

Number of moles of $H_2 = \frac{4g}{2g} = 2$

$$2H_2 \longrightarrow 2H^+ + 2H^*$$

According to de-Broglie, $2\pi r = n\lambda \Rightarrow 2\pi r = 4\lambda$

Total energy required

= total energy required to dissociate two moles of H_2 + total energy required in ionisation of two H^- to two H^+ + total energy required in ionisation of two H to 4th excited energy level.

$= 2 \times 4.52 \times N_A + 2 \times 13.6\, N_A + 2 \times 13.6 \times \left(1 - \frac{1}{16}\right) \times N_A$

$= N_A\,(9.04 + 27.2 + 27.2 \times 0.93)$

$= N_A\,(61.53)$ eV

We know that during formation of H^+ and H^* in above reaction.

Total energy required = Total energy released

$\therefore \quad 61.53 \times N_A$ eV $= -30.86y$ eV

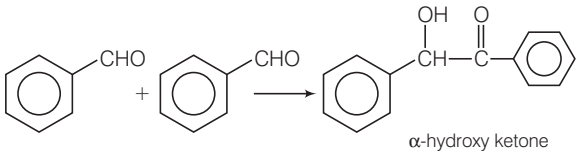
$$y = \frac{61.53}{30.86} \simeq 2$$

Hence, number of moles required = 2

TEST Edge In JEE Main, this question is asked to Judge the depth of knowledge of student. So, students are advised to study the subject in such a way that question including depth of theory and concept would be solved easily. Students are advised to study the Hisenberg uncertainty principle and photoelectric effect.

33. (*a*) **Idea** This problem includes conceptual mixing of benzoin condensation and effect of inductive effect on substrate presence of benzoin condensation. To solve such types of questions, students are advised to understand the role of various electrons withdrawing group and electron releasing group on substrate.

Benzoin condensation This is a phenomena of self condensation of aromatic aldehyde in presence of KCN. This process produces α-hydroxy ketone as a final product.



α-hydroxy ketone

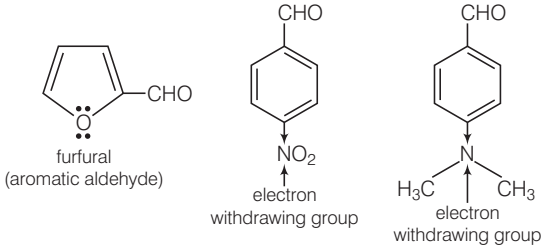
Inductive effect There are two kinds of inductive effect + *I* effect shown by electron donating group and – *I* effect shown by electron withdrawing group *e.g.*,

+ *I* effect ⟶ OCH_3

– *I* effect ⟶ NO_2, CHO, CN. etc.

Effect of strong electron donating group or strong electron withdrawing group present on substrate of benzoin condensation play an important role in occurrence of reaction.

Aromatic aldehyde having either strong electron withdrawing group or electron with drawing group do not give this reaction.



Hence, only furfural will undergo benzoin condensation.

TEST Edge Generally, in JEE Main this type of questions are asked very frequently so students are advised to study the depth of various chemical rearrangement reactions such as pinacol_ pinacolone rearrangement, Beckmann rearrangement which may also be asked.

34. (*d*) **Idea** This problem is based on concept of electrolysis of NaCl (*aq*) and redox reaction. This problem completes in various sequential steps, students are advised to solve the question step by step.

Aqueous NaCl is NaCl + H_2O

$$NaCl \longrightarrow Na^+ + Cl^-$$

Strong electrolyte

$$H_2O \rightleftharpoons H^+ + OH^-$$

Weak electrolyte

So, OH^- combine with Cl_2 and OH^- will be oxidised.

TEST Edge This type of question is generally asked in JEE Main, so students are advised to go through concept of electrolysis and redox reaction. The question related to redox reaction may also be asked with conceptual mixing of acid-base concept.

35. (*a*) **Idea** This problem includes concept of equilibrium constant and nernst equation. Students are advised to go through step by step determination of concentration and then cell emf using nernst equation.

Nernst equation for given cell reaction may be written as

$$E^\circ_{cell} = \frac{0.059}{2}\log\frac{[H^+]^2_{RHS}\,[PH_2]_{LHS}}{[H^+]^2_{LHS}\,[PH_2]_{RHS}}$$

$$\alpha << 1$$

$$[OH^-] = \sqrt{K_b \times C}$$

$$[OH^-] = \sqrt{10^{-5} \times 10^{-1}} = 10^{-3}$$

$$[H^+]_{LHS} = 10^{-11}$$

$$[H^+][OH^-] = 10^{-14},\ [H^+] = 10^{-11}$$

$$[H^+]_{RHS} = \sqrt{K_a \times C} = \sqrt{10^{-7} \times 10^{-1}} = 10^{-4}$$

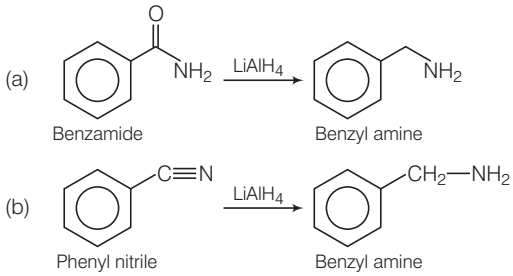
$$E^\circ_{cell} = \frac{0.059}{2}\log\frac{(10^{-4})^2 \times 0.1}{(10^{-11})^2} \times 1 = 0.395\ V$$

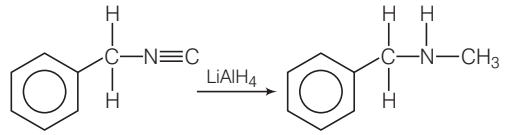
TEST Edge In general, the questions based on nernst equation are asked frequently, so students are advised to study the nernst equation for various types of cell and emf calculation. The question related to end and spontaniety of reaction may also be asked.

$$\Delta h = -nFE^\circ_{cell}$$

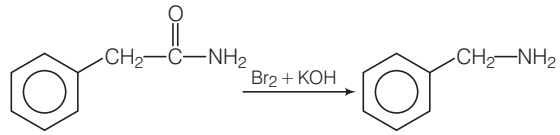
∴ Positive value of E°_{cell} cell show spontaneous reaction.

36. (*c*) **Idea** This problem includes conceptual mixing of nomenclature and reduction of acid amide and various important methods of preparation of amine. To solve this type of problem students are advised to complete the all given reactions and analyse the product obtained then to choose the correct option.



(c) 

2-phenyl ethyl isonitrile N-methyl benzyl amine

(d) 

2-phenyl ethoxamide Benzyl amine

Last process is a good example of Hoffmann bromide reaction in which carboxyl amide are converted into amine having one carbon atom less than carboxyl amide.

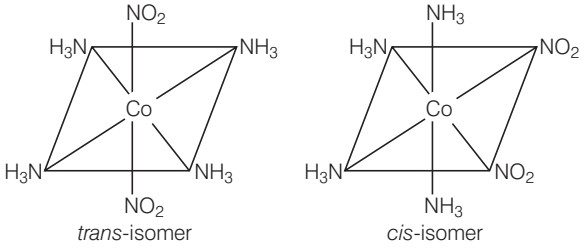
TEST Edge Generally, in JEE Main these types of questions are asked so students are recommended to understand the preparation as well as properties of amine question based on property of amine may also be asked.

37. (*a*) **Idea** This problem is based on isomerism in coordination compound, students are advised to keep in mind the concept of various types of isomerisms in coordination compounds.

(i) NO_2 is an ambidentate group. It can show linkage isomerism.

(ii) The complex can show ionisation isomerism with the complex $[Co(NH_3)_4NO_2Cl]NO_2$

(iii) The complex show geometrical isomerism.



trans-isomer *cis*-isomer

TEST Edge This concept is a good choice for JEE Main from where questions are asked in general so students are advised to study this concept very clearly and carefully, question involving reaction of various isomers may also be asked.

38. (*a*) **Idea** This problem is based on concept of Vant Hoff factor and dissociation constant. Students are advised to calculate Vant Hoff factor first followed by calculation of dissociation constant step by step.

$$\Delta T_f = K_f \times m \times i = 0.39 = i \times 1.86 \times 0.1$$

$$i \approx 2.1$$

$$H_2SO_4 \longrightarrow H^+ + HSO_4^-$$

$$0 \qquad c \quad c(1-\alpha)$$

$$HSO_4^- \rightleftharpoons H^+ + SO_4^{--}$$

$$c(1-\alpha) \qquad c\alpha \qquad c\alpha$$

$$i = \frac{c(1-\alpha) + c\alpha + c\alpha + c}{c} = 2.1$$

$$\alpha = 0.1$$

Now, $HSO_4^- \rightleftharpoons H^+ + SO_4^{--}$

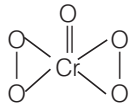
$$c(1-\alpha) \qquad (\alpha + c) \qquad c\alpha$$

$$K_{a_2} = \frac{c(1+\alpha) \times c\alpha}{c(1-\alpha)}$$

$$= \frac{1.1 \times 0.1 \times 0.1}{.9} \approx 0.012$$

TEST Edge Question involving concept of Vant Hoff factor and dissociation constant is asked in JEE Main generally. The question including calculation of pK_a may also be asked so students are advised to go through concept of pK_a also $pK_a = -\log K_a$

39. (*a*) (*A*) $2Ir^{3+} + 10OH^- + 3H_2O_2 \longrightarrow \underset{\text{Green}}{} \quad \underset{\text{Yellow}}{2CrO_4^{2-}} + H_2O$

(B) $CrO_4^{2-} + 2H_2O_2 + 2H^+ \longrightarrow$  $+ 3H_2O$

In aqueous solution, CrO_5 is unstable and it further decomposes.

(C) $$2CrO_5 \longrightarrow \underset{\text{(Amphoteric)}}{Cr_2O_3} + \frac{7}{2}O_2$$

$$Cr_2O_3 + 3H_2SO_4 \longrightarrow Cr_2(SO_4)_3 + 3H_2O$$

40. (*d*) **Idea** This problem can be solved by using concept of ideal gas equation and chemical equation involved in combustion reaction. Students are advised to follow steps.

- Write chemical equation involved in combustion of hydrocarbon.
- Assume initial pressure = p
- Calculate increased pressure
- Then, calculate number of mole using ideal gas equation.

Equation of combustion

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \longrightarrow n\,CO_2 + (n+1)H_2O$$

Initial pressure of C_nH_{2n+2} is p (assumed)

Increase in pressure

$$= p\left[(2n+1) - 1 - \left(\frac{3n+1}{2}\right)\right] = \left(\frac{n-1}{2}\right)p$$

$\because$ Mass of organic compound $= 14 \times n + 2$

546 K and 4.6 atm or 273 K 2.3 atm

Increase $\Rightarrow$ 2.3 – 2 = 0.3 atm

$$p = \frac{nRT}{V} = \frac{23.2}{M} \times \left(\frac{0.0821 \times 273}{44.82}\right)$$

$$= \frac{23.2}{M} \times \frac{0.5}{44.82}$$

$$= \frac{11.6}{(14n+2)} \times \frac{(n-1)}{2}$$

also $$= \frac{(n-1)}{2} \times \frac{11.6}{14n+2} = 0.3$$

On solving, $n = 4$

So, compound will be $C_4H_{4\times2+2} \Rightarrow C_4H_{10}$

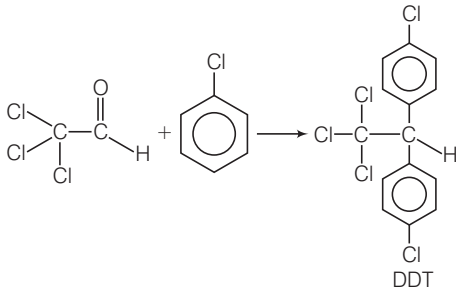
TEST Edge JEE Main includes this type of question in exam to judge the knowledge as well as numerical aptitude of students. So, students are advised to solve the question as much as possible.

Students are also advised to solve the problem having conceptual mixing of ideal gas equation and pressure and mole fraction determination.

41. (*c*) **Idea** This problem includes a conceptual mixing of preparation of DDT and its characteristics. To solve this problem use the concept of preparation and characteristic of DDT. First complete the given reaction and then read the characteristics given in question then choose the correct option.

Preparation of DDT

DDT is *p*, *p*-dichlorodiphenyl trichloroethane prepared by reaction of chlorobenzene with CCl_3—CHO in presence of H_2SO_4 through electrophilic substitution reaction.



Characteristics of DDT

- It is used as an insecticide to kill mosquito so that peoples are prevented by malaria.
- It is highly toxic towards fish.
- It is toxic for animals because it is not easily metabolised by animal.
- It has no any chiral centre.

TEST Edge The question from the application of halogen containing organic compounds are asked generally in JEE Main. So, students are advised to go through clear and carefully study of application of halogen containing organic compounds such as Freon, chloroform etc.

42. (*a*) $KO_2 \longrightarrow K^+ + O_2^-$

According to molecular orbital theory O_2^- will be paramagnetic and percentage of potassium

$$= \frac{39}{39+32} \times 100$$

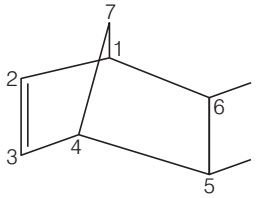
$$= \frac{39}{71} \times 100$$

$$\approx 55\%$$

43. (*b*) According to Hardy Schulz's rule, an ion will be migrated towards cathode so greater in the valency of effective ion smaller will be its coagulating value, PO_4^{---}, SO_4^{--}, Cl^-

So, NaCl will be answer.

44. (*c*) **Idea** This problem contains conceptual mixing of nomenclature of cyclic hydrocarbon and degree of unsaturation. This problem can be solved by identifying the parent chain, functional group, position of functional group, substituent and their position one by one and then write the name of compound according to IUPAC names then calculate degree of unsaturation.



Total carbon atom forming the bicyclic ring = 7 (hept.)

Functional group ⇒ double bond (ene)

Position of double bond ⇒ 2, 2-ene

Substituents ⇒ 2-methyl group ⇒ dimethyl

Position of substituents = 5, 6 ⟶ 5, 6-dimethyl

Number of cyclic chain = 2 ⟶ Bicyclo

3-bridges are of 2 carbons, 2 carbons and one carbon hence.

IUPAC name = 5, 6-dimethyl bicyclo [2, 2, 1] hept 2-ene

Molecular formula of compound is C_9H_{14}.

Degree of unsaturation can be calculated as

$$u = (C+1) - \frac{H}{2} + \frac{N}{2}$$

where, u = degree of unsaturation

C = number of carbons

H = number of hydrogens

N = number of nitrogen.

Hence, for a compound having molecular formula C_9H_{14} the degree of unsaturation may be calculated as

$$u = (9+1) - \frac{14}{2}$$

$$= 10 - 7 = 3$$

TEST Edge JEE Main includes this type of question very frequently so students are advised to solve such problems as well as the problem including the nomenclature of spiro and biphenyl compound also.

45. (*a*) **Idea** Problem includes concept of Arrhenius equation for different rate constant of a common reaction. To solve this problem student is advised to write the Arrhenius equation for every rate constant. Then, put the value in $k = \left(\frac{k_1 k_2}{k_3}\right)^{2/3}$ to get the value of E_a

Putting $k = Ae^{-E_a/RT}$

$$k_1 = Ae^{-\frac{E_{a_1}}{RT}},\ k_2 = Ae^{-\frac{E_{a_2}}{RT}}$$

$$k_3 = Ae^{-\frac{E_{a_3}}{RT}}$$

$$k = \left(\frac{Ae^{-\frac{E_{a_1}}{RT}} \cdot Ae^{-\frac{E_{a_2}}{RT}}}{Ae^{-E_{a_3}/RT}}\right)^{2/3} = Ae^{-E_a/RT}$$

$$e^{\frac{1}{RT}\left[\frac{2}{3}(E_{a_3} - (E_{a_1} + E_{a_2}))\right]} = e^{-E_a/RT}$$

$$E_a = \frac{2}{3}[E_{a_1} + E_{a_2} - E_{a_3}]$$

$$E_a = \frac{2}{3}[200 + 90 - 80] = \frac{2}{3}[210]$$

$$= 140\ \text{kJ/mol}$$

TEST Edge Students are advised to study the concept of Arrhenius equation at different temperatures. Since, these types of questions are also asked.

46. (*a*) **Idea** This problem includes the concept of properties of oxides of nitrogen. This problem can be solved by using the properties of oxides of nitrogen.

$N_2O \Rightarrow$ laughing gas but linear in shape

$NO_2 \Rightarrow$ pungent order, angular

$NO \Rightarrow$ colourless and neutral

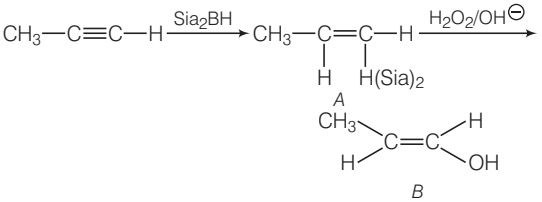
$$2NaOH + 2NO_2 \longrightarrow NaNO_2 + NaNO_3 + H_2O$$

TEST Edge This type of question is asked in JEE Main so students are advised to go through study of chemical properties of oxides of nitrogen as well as oxides of sulphur and halogen also which may also be asked.

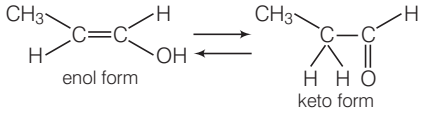
47. (*a*) **Idea** This problem includes a conceptual mixing of hydroboration oxidation and isomerism. Skill required to solve the problem includes command over electrophilic addition along in depth knowledge of isomerism.

Hydroboration oxidation

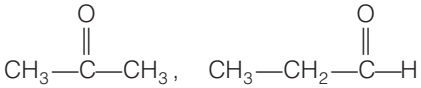
Alkene on reaction with borane produces alkyl borane which on oxidation produces alcohol. In the given reaction product *B* is obtained as follows

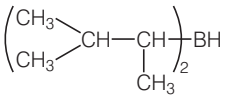


Tautomerism and metamerism Tautomerism is shown by a compound *B*. *B* interconverts to keto form as follows



Ketone also show metamerism due to difference in nature of alkyl group present on both sides of > C ═ O group possible metamers of *B* are

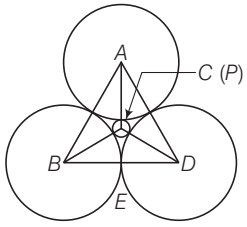


Structure of Sia_2BH is 

Name of Sia_2BH is disiamyl borane.

TEST Edge Students are advised to study information relating to oxymercuration demercuration and electrophilic addition of boron hydrides with cleavage of resultant product by peroxide, coupling reagent ($AgNO_3$ / NaOH) and protic agent. Hydroboration oxidation is the reaction of atmost importance as we can produce various hydrocarbons, alcohols and carboxyl compound from the reaction.

48. (*a*)



[*C* = centeroid]

$$\frac{BE}{BC} = \cos 30° = \frac{\sqrt{3}}{2},\ \frac{BE}{2} = \frac{\sqrt{3}}{2} \Rightarrow BE = \sqrt{3}$$

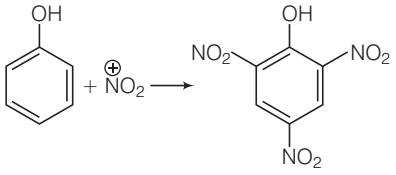
Distance between the centres of two *Q* atoms

$$= 2 \times BE = 2 \times \sqrt{3} = 2\sqrt{3}\ \text{Å}$$

49. (*d*) This problem includes conceptual mixing of electrophilic substitution reaction, degree of unsaturation and acidic/basic nature.

Electrophilic Substitution Reaction Phenol undergo electrophilic substitution reaction on reaction with a mixture of conc. HNO_3 and conc. H_2SO_4 (known as nitrating mixture). This reaction produces nitro compound and on successive nitration it produces trinitro phenol.

$$\text{conc. } HNO_3 + \text{conc. } H_2SO_4 \longrightarrow \overset{+}{N}O_2$$



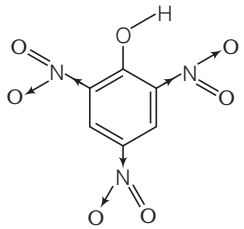
Picric acid

Molecular formula = $C_6H_3N_3O_7$

Degree of unsaturation $= (C+1) - \frac{H}{2} + \frac{N}{2}$

$$= (6+1) - \frac{3}{2} + \frac{3}{2} = 7 - 0 = 7$$

Nature of compound The compound is acidic in nature due to presence of three strong electron withdrawing groups (NO_2).



50. (*d*) $FeSO_4 \xrightarrow[HNO_3]{\text{Dilute}} \underset{\text{remains in solution}}{Fe_2(SO_4)_3} \downarrow +$

$CuSO_4$ crystallises out.

51. (*c*) **Idea** This problem includes conceptual mixing of Nernst equation and equilibrium constant determination, follow the following sequential step to solve the problem.

Write each steps involved in chemical transformation.

Calculate the value of emf using Nernst equation.

Now, calculate the equilibrium constant.

$$H_2O(l) + ClO_3^-(aq) = ClO_4^-(aq) + 2H^+(aq) + 2e^-$$

$$2H^+(aq) + ClO_3^-(aq) + 2e^- \longrightarrow 2ClO_2^-(aq) + H_2O(l)$$

$$2ClO_3^-(aq) \longrightarrow ClO_2^-(aq) + ClO_4^-(aq)$$

$$E^\circ_{cell} = -0.39 + 0.36 = -0.03$$

$$E^\circ_{cell} = \frac{RT}{2F} \ln K$$

$$-0.03 = \frac{0.06}{2} \log K \quad \text{or} \quad K = 0.1$$

$$\underset{0.1-2x}{2ClO_3^-} \rightleftharpoons \underset{x}{ClO_4^-} + \underset{x}{ClO_2^-}$$

$$\frac{x^2}{(0.1-2x)^2} = 0.1 = \frac{1}{10}$$

$$3.16\,x = 0.1 - 2x$$

$$5.16\,x = 0.1$$

$$x = \frac{0.1}{5.16} = 0.0193 = 1.9 \times 10^{-2}$$

TEST Edge Problems related to Nernst equation are generally asked in JEE Main, students are advised to study the application of Nernst equation for various concentration cell.

52. (*d*) ppt. of Ag_2CrO_4, Ag_2CO_3 are soluble in NH_4OH due to formation of $[Ag(NH_3)_2]^+$.

Green ppt. of $Ni(OH)_2$ is soluble in NH_4OH due to formation of $[Ni(NH_3)_6]^{2+}$

$$Ag_2CrO_4 + 4NH_4OH \longrightarrow 2[Ag(NH_3)_2]^+ + CrO_4^{2-} + 4H_2O$$

$$Ag_2CO_3 + 4NH_4OH \longrightarrow 2[Ag(NH_3)_2]^+ + CO_3^{2-} + 4H_2O$$

$$\underset{\text{Green ppt.}}{Ni(OH)_2} + 8NH_4OH \longrightarrow [Ni(NH_3)_6]^{2+} + 4OH^- + 6H_2O$$

$Fe(OH)_3$ is insoluble in NH_4OH.

$Al(OH)_3$ is insoluble in NH_4OH.

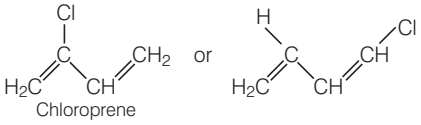
53. (*a*) **Idea** This problem contains conceptual mixing of molecular structure determination and polymerisation of identified monomer. To solve this problem students are advised to follow following steps.

- Calculate the value of degree of unsaturation.
- Draw the possible structures.
- Choose the correct structure among all possible structures.
- Then, complete chemical reaction according to given information.

Determination of molecular structure of C_4H_5Cl

Degree of unsaturation $= (4+1) - \frac{6}{2} = 5 - 3 = 2$

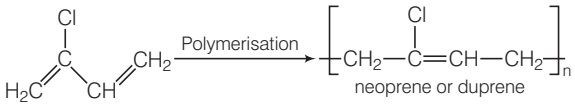
Since this molecule does not contain any ring or tripple bond hence it must be a diene.



Chloroprene

A *B*

A is chloroprene when it undergoes polymerisation it produces duprene also known as neoprene.



Chloroprene may undergo polymerisation *via* two modes either free radical polymerisation and Ziegler natta polymerisation.

TEST Edge Students are advised to go through the study of various polymerisation reaction and their characteristics properties such as nylon, bakelite etc.

54. (*d*) Atomic weight of element

$$y = 6.64 \times 10^{-23} \times N_A \approx 40$$

Number of moles of $y = \dfrac{20 \times 1000}{40} = 500$

55. (*b*) This problem includes conceptual mixing of determination of amount of percentage of element in organic compound and empirical formula.

Quantitative determination of amount of percentage of Cl

% of chlorine

$$= \frac{35.5}{143.5} \times \frac{\text{mass of AgCl}}{\text{mass of the compound}} \times 100$$

$$= \frac{35.5}{143.5} \times \frac{0.287}{0.099} \times 100 = 71.71\%$$

Determination of empirical formula

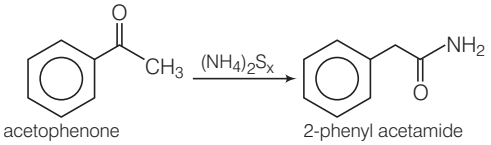
Symbol	Percentage	Atomic mass	Relative number of atoms
C	24	12	$\frac{24}{12} = 2$
H	4.3	1	$\frac{4.3}{1} = 4.3 \approx 4$
Cl	71.7	35.5	$\frac{71.7}{35.5} \approx 2$

Empirical formula = (CH_2Cl)

56. (*c*) $AlCl_3$ is covalent while AlF_3 (Al^{3+} $3F^-$) is an ionic compound.

57. (*a*) This problem includes conceptual mixing of Willgerodt reaction.

Willgerodt reaction When alkyl aryl ketone is treated with ammonium polysulphide it give amide as a major product.



58. (*b*) According to Rydberg's equation,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$n_1 = n,\ n_2 = n+1$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\frac{1}{\lambda} = \left(\frac{2n+1}{n^2(n+1)^2}\right)RZ^2$$

Since, $n >> 1$

We can write $2n + 1 \approx 2n$ and $(n+1)^2 \approx n^2$

$$\frac{1}{\lambda} = RZ^2\left(\frac{2n}{n^2 \cdot n^2}\right)$$

$$\frac{\nu}{C} = \frac{2RZ^2}{n^3} \text{ or } \nu = \frac{2CRZ^2}{n^3}$$

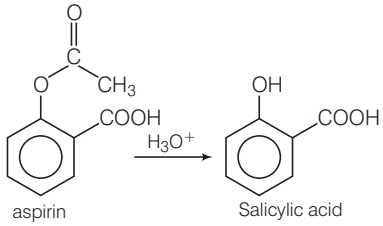
59. (*a*) Both Statement I and Statement II are correct and Statement II is correct explanation of Statement I.

H_2SO_4 oxidises KI to I_2 and KBr to Br_2.

60. (*c*) This problem includes concept of action of aspirin on empty stomach and its bad impact.

Action of aspirin on emtpy stomach

When aspirin *i.e.*, 2-acetoxy benzoic acid is taken by any one in empty stomach it causes hydrolysis of 2-acetoxy benzoic acid to form salicyclic acid.



Salicyclic acid has bad impact on our stomach wall. However, sodium and potassium salt of aspirin are less harmfull.

Mathematics

61. (*a*) We know that variance of *n* natural numbers is

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

$$\text{Variance}\,(\sigma^2) = \frac{1}{30}[1^2 + 2^2 + \ldots + 30^2] - \left[\frac{1}{30}(1 + 2 + \ldots + 30)\right]^2$$

$$= \frac{1}{30} \times \frac{30 \times 31 \times (2 \times 30 + 1)}{6} - \left[\frac{1}{30} \times \frac{30 \times 31}{2}\right]^2$$

$$= \frac{31 \times 61}{6} - \frac{31 \times 31}{4}$$

$$= \frac{31 \times 2 \times 61 - 31 \times 3 \times 31}{12}$$
$$= \frac{31}{12}[122 - 93] = \frac{31 \times 29}{12}$$
$$= \frac{899}{12} = 74.916$$
$$= 74.92$$

62. (*a*) **Idea** Students are advised to stick with definition of the function, to solve this question.
∴ Here $f(x) = \log_a x$ is defined when a > 0 and $x > 0$.

Here, it is given that

$$\frac{\log_2(x^2 - 5x + 4)}{\log_2(x^2 + 1)} > 1$$
$$x^2 - 5x + 4 > 0 \Rightarrow (x - 4)(x - 1) > 0$$
$$\Rightarrow \quad x \in (-\infty, 1) \cup (4, \infty) \quad \ldots(i)$$
$$x^2 + 1 > 0 \text{ which is true } \forall\ x \in R \quad \ldots.(ii)$$
$$\log_2(x^2 - 5x + 4) > \log_2(x^2 + 1)$$
$$x^2 - 5x + 4 > x^2 + 1$$
$$-5x + 3 > 0$$
$$x < \frac{3}{5} \quad \ldots(iii)$$

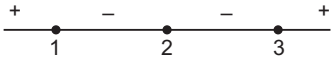
From Eqs. (i), (ii) and (iii), we get

$$x \in \left(-\infty, \frac{3}{5}\right) - \{0\}$$

TEST Edge Generally, in JEE Main, domain and range related question are asked from this concept. Students are advised to understand basic concept of the function given and also acquainted yourself with the concept of wavy curve method. *i.e.,*

Let $(x-1)(x-2)^2(x-3) \geq 0$

Then $x = 1, 2, 3$



Solutions $(-, \infty] \cup [3, \infty) \cup \{2\}$

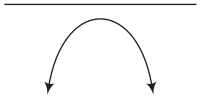
63. (*d*) For the function

$$f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$$
$$f'(x) = 3(a + 2)x^2 - 6ax + 9a$$

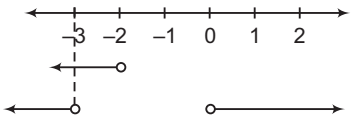
∵ Function is monotonically decreasing $\forall\ x \in R$

$$\therefore \quad f'(x) \leq 0 \quad \forall\ x \in R$$
$$3(a + 2)x^2 - 6ax + 9a \leq 0 \quad \forall\ x \in R$$
$$(a + 2)x^2 - 2ax + 3a \leq 0 \quad \forall\ x \in R$$

This is only possible when coefficient of x^2 is less than zero and discreminant is negative.



$$\Rightarrow a + 2 < 0 \text{ and } 4a^2 - 4(a + 2)(3a) < 0$$
$$\Rightarrow a < -2 \text{ and } 4a^2 - 12a^2 - 24a < 0$$
$$\Rightarrow a < -2 \text{ and } -8a^2 - 24a < 0$$
$$\Rightarrow a < -2 \text{ and } -8a(a + 3) < 0$$
$$\Rightarrow a < -2 \text{ and } a(a + 3) > 0$$



∴ Required solution is $(-\infty, -3)$.

64. (*a*) **Idea** To express given differential equation as polynomial in derivatives. Use trigonometric formulae.

Hence, we know that $\cos^2\theta + \sin^2\theta = 1$

Given that

$$\sqrt{1 - x^2} + \sqrt{1 - y^2} = b(x - y) \quad \ldots(i)$$

Put $x = \sin\alpha$, $y = \sin\beta$ in Eq. (i)

$$\Rightarrow \quad \alpha = \sin^{-1}x, \beta = \sin^{-1}y$$
$$\cos\alpha + \cos\beta = b(\sin\alpha - \sin\beta)$$

Use identities

$$2\cos\left(\frac{\alpha + \beta}{2}\right) \cdot \cos\left(\frac{\alpha - \beta}{2}\right)$$
$$= b \cdot 2\cos\frac{\alpha + \beta}{2} \cdot \sin\frac{\alpha - \beta}{2}$$
$$\alpha - \beta = 2\cot^{-1}b$$
$$\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}b$$

Differentiating w.r.t. x, we get

$$\frac{1}{\sqrt{1 - x^2}} - \frac{1}{\sqrt{1 - y^2}}\frac{dy}{dx} = 0$$

Degree of above differential equation is one.

TEST Edge Order and degree related to question are asked. To solve such types of questions students are advised to understand basic concept of differentiation and express the differential equation as a polynomial in derivatives.

65. (*c*) Let $z = \cos\theta + i\sin\theta$ | z | = 1

$$\because \quad z = \frac{c + i}{c - i}$$
$$\Rightarrow c = \frac{i(z + 1)}{z - 1} = \frac{i(\cos\theta + i\sin\theta + 1)}{(\cos\theta + i\sin\theta - 1)}$$
$$= \frac{i\left(2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)}{-2\sin^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{i\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{\left(-\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right)}\cot\frac{\theta}{2}$$

$$= \frac{i\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{i\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)} \cdot \frac{\cot\frac{\theta}{2}}{2} = \cot\frac{\theta}{2}$$

$$c = \frac{1+\cos\theta}{\sin\theta} = \frac{1+a}{b}$$

66. (*c*) It is given that

$$k = \lim_{x\to\infty}\left(\frac{\sum_{k=1}^{1000}(x+k)^m}{x^m + 10^{1000}}\right)$$

Divide numerator and denominator by x^m, we have

$$k = \lim_{x\to\infty}\left[\frac{\sum_{k=1}^{1000}\left(1+\frac{k}{x}\right)^m}{1+\frac{10^{1000}}{x^m}}\right]$$

$$= \frac{1+1+1+\ldots \text{upto 1000 times}}{1+0}$$

$$= 1000 = 10^3$$

So, correct option is (c).

67. (*a*) The given set is $A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

Since empty set is a subset of every set

$\therefore \quad \phi \subseteq A$

Also, ϕ is an element of the given set A

$\Rightarrow \quad \phi \in A.$

Now, since $\{\phi\}$ is an element of the set A

$\Rightarrow \quad \{\phi\} \in A$

Also $\{\phi\}$ is a subset of A being an element of a set A.

$\therefore \quad \{\phi\} \subseteq A$

So, correct option is (a).

68. (*b*) **Idea** $\because$ Let $A = [a_{ij}]$ be a $_{m\times n}$ matrix and k be any number, then $k_A = [k a_{ij}]_{m\times n}$ and use formula such as

$$\lim_{n\to\infty}\frac{\cos n\theta}{n} = \frac{\sin\theta}{n} = 0$$

Given that $A = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$$A^n = \begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$$

$$\frac{A^n}{n} = \begin{bmatrix}\frac{\cos n\theta}{n} & \frac{\sin n\theta}{n}\\ \frac{-\sin n\theta}{n} & \frac{\cos n\theta}{n}\end{bmatrix}$$

$$\lim_{n\to\infty}\frac{A^n}{n} = \begin{bmatrix}\lim_{n\to\infty}\frac{\cos n\theta}{n} & \lim_{n\to\infty}\frac{\sin\theta}{n}\\ \lim_{n\to\infty}\frac{-\sin n\theta}{n} & \lim_{n\to\infty}\frac{\cos n\theta}{n}\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = \text{zero matrix}$$

TEST Edge Properties of scalar multiplication and addition of matrices based questions are asked. These questions are solved with the help of properties of matrices.

69. (*c*) Here, the minimum distance occur along the common normal. Any normal to $y^2 = 8x$ at $(2t^2, 4t)$ is

$$tx + y = 4t + 2t^3$$

It passes through the centre (0, 6) of the circle

$$6 = 4t + 2t^3 \Rightarrow t = 1$$

So, required points (2, 4).

70. (*a*) If l, m, n be the direction cosines of the line, then as it lies in both the given planes it is perpendicular to their normal

i.e., $l - m + 2n = 0$ and $3l + m + n = 0$

or $$\frac{l}{-3} = \frac{m}{5} = \frac{n}{4}$$

So, equation of the line is $\frac{x-1}{-3} = \frac{y+2}{5} = \frac{z-3}{4}$

71. (*a*) **Idea** Understand the basic concept of modulus function.

Here. $\because$ we know that $-1 \le \sin x \le 1$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ but where $-\frac{\pi}{2} < x < 0$.

Then, $-1 \le \sin x \le 0$.

Hence, we know that $\cos^2\theta + \sin^2\theta = 1$

We have given that

$$\int_{-\frac{\pi}{2}}^{0}\sqrt{\cos x\,(1-\cos^2 x)}\,dx$$

$$\int_{-\pi/2}^{0}\sqrt{\cos x}\,|\sin x|\,dx$$

when $-\frac{\pi}{2} < x < 0$ then $-1 \le \sin x \le 0$

$\therefore \quad -\int_{-\pi/2}^{0}\sqrt{\cos x}\,\sin x\,dx$

Put $\cos x = t$, $-\sin x\,dx = dt$

$$\int_0^1 t^{1/2}\,dt$$

$$\left(\frac{t^{3/2}}{3/2}\right)_0^1 = +\frac{2}{3}$$

TEST Edge Let JEE Main properties of definite integrals based questions are asked to solve such types of questions students are advised that learn properties of definite integral and understand the concept of integration.

72. (*b*) Here $\mathbf{r} \times \mathbf{a} = \mathbf{b}$

$$\mathbf{a} \times (\mathbf{r} \times \mathbf{a}) = \mathbf{a} \times \mathbf{b}$$
$$3\mathbf{r} - (\mathbf{a} \cdot \mathbf{r})\mathbf{a} = \mathbf{a} \times \mathbf{b}$$
$$|\mathbf{r} \times \mathbf{a}| = |\mathbf{b}|$$
$$|\mathbf{r}| \times |\mathbf{a}| \sin\theta = |\mathbf{b}|$$
$$|\mathbf{r}|^2 \times |\mathbf{a}|^2 \sin^2\theta = |\mathbf{b}|^2$$
$$|\mathbf{r}|^2\, 3 \sin^2\theta = 2$$
$$|\mathbf{r}|^2 (1 - \cos^2\theta) = 2/3$$
$$|\mathbf{r}|^2 - \frac{2}{3} = |\mathbf{r}|^2 \cos^2\theta$$
$$\frac{1}{3} = \cos^2\theta \qquad [\because |\mathbf{r}| = 1]$$
$$3\mathbf{r} \mp \mathbf{a} = \mathbf{a} \times \mathbf{b}$$
$$\mathbf{r} = \frac{1}{3}(\mathbf{a} \times \mathbf{b} \pm \mathbf{a})$$

73. (*b*) **Idea** We know that if x, y and z are in AP, then $2y = x + z$ and for GP $y^2 = xz$, for HP $y = \frac{2x+z}{x+z}$.
Use these relations.

Since, a, b and c are in AP

$$\therefore \quad 2b = a + c$$

b, c and d are in GP

$$\therefore \quad c^2 = bd$$

and c, d and e are in HP

$$\therefore \quad d = \frac{2ce}{e + c}$$
$$\frac{c^2}{b} = \frac{2ce}{c + e}$$
$$\Rightarrow \quad c(c + e) = 2be$$
$$\Rightarrow \quad c(c + e) = (a + c)e$$
$$c^2 + ce = ae + ce$$
$$c^2 = ae$$

Hence, a, c, e are in GP.

TEST Edge Properties of series based questions are asked. To solve such types of questions students are stated that stick with definition and properties of AP, GP and HP and go through with relation between AP, GP and HP.

74. (*a*) **Idea** To find the points, the intersection of the given line and curve, put $z = 0$ in equation of line. These points lie on the curve $xy = e^2$.

For the points where the line intersects the curve, we have $z = 0$

$$\therefore \quad \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1}$$
$$\Rightarrow \quad \frac{x-2}{3} = 1 \text{ and } \frac{y+1}{2} = 1$$
$$\Rightarrow \quad x = 5,\ y = 1$$

Put $x = 5$, $y = 1$ in $xy = c^2$

$$c = \pm\sqrt{5}$$

TEST Edge The intersection of line and curve and a line lie on the plane related question are asked. Students are suggested to understand the concept of line and plane.

75. (*b*) **Idea** A relation R on set A is said to be symmetricif $(a,b) \in R \Rightarrow (b,a) \in R, \forall a,b \in A$

Since, it is given that $R \cap R'$ is not disjoint.

$\therefore$ There is atleast one ordered pair, say, (a, b) in $R \cap R'$. Then, $(a, b) \in R \cap R'$

$$\Rightarrow \quad (a, b) \in R \text{ and } (a, b) \in R'$$

Since, it is also given that R and R' are symmetric relations, we get

$$(b, a) \in R \quad \text{and} \quad (b, a) \in R'$$
$$\Rightarrow \quad (b, a) \in R \cap R'$$

$\Rightarrow R \cap R'$ is symmetric.

TEST Edge Identify relation, reflexive relation, symmetric relation related questions are asked. To solve these types of questions to understand the basic definition of these relations.

76. (*c*) **Idea** Here $\because y = x^x \Rightarrow \log y = x \log x = S$
Use above condition and then differential with respect to x.
$\because$ We know that for minimum $\frac{d^2S}{dx^2} > 0$

Let $f(x) = (\sin x)^{\sin x}$

$$S = \log f(x) = \sin x \log(\sin x)$$
$$\frac{dS}{dx} = \sin x \frac{\cos x}{\sin x} + \log(\sin x)\cos x$$
$$= \cos x + \cos x \log \sin x$$
$$= \cos x\,(1 + \log \sin x)$$
$$\frac{d^2S}{dx^2} = \cos x \frac{\cos x}{\sin x} + (1 + \log \sin x)(-\sin x)$$
$$\frac{d^2S}{dx^2} = \frac{\cos^2 x}{\sin x} - \sin x\,(1 + \log \sin x)$$
$$\frac{dS}{dx} = 0$$
$$\cos x\,(1 + \log \sin x) = 0$$
$$\Rightarrow \quad \cos x = 0 \quad \text{or} \quad 1 + \log \sin x = 0$$
$$x = \pi/2 \quad \text{or} \quad \log \sin x = -1$$
$$x = \frac{\pi}{2} \quad \text{or} \quad \sin x = e^{-1}$$
$$x = \frac{\pi}{2} \quad \text{or} \quad \sin x = \frac{1}{e}$$

$$\left(\frac{d^2S}{dx^2}\right)_{\sin x=\frac{1}{e}} = \frac{\left(1-\frac{1}{e^2}\right)}{\left(\frac{1}{e}\right)} - \frac{1}{e}(1+\log e^{-1})$$

$$= e\left(1-\frac{1}{e^2}\right) - \frac{1}{e}(1-1) = e\left(1-\frac{1}{e^2}\right) > 0$$

$\therefore$ $f(x)$ has minimum value at $\sin x = \frac{1}{e}$.

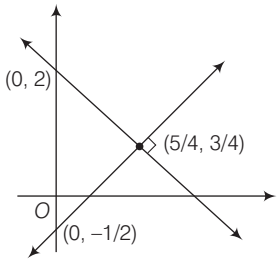
$\therefore$ Minimum value $= \left(\frac{1}{e}\right)^{1/e} = e^{-1/e}$

TEST Edge The function is maximum or minimum and maximum or minimum value related question are asked. Students are suggested that learn of maximum or minimum and also acquainted yourself the domain of some functions.

77. (*d*) **Idea** Equation of line passing through two given points (x, y) and (x_2, y_2) is $y - y_1 = \frac{x_2 - x_1}{y_2 - y_1}(x - x_1)$

Use this formula and find intersection point between two lines.

To solve this question, we have to come the above method.



Line L which passes through (1, 1) and (2, 0) is

$$(y-1) = \frac{-1}{1}(x-1)$$

$$y - 1 = -x + 1$$

$$x + y = 2 \quad \ldots(i)$$

Line L' is $(y-0) = 1\left(x - \frac{1}{2}\right)$

$$2y = 2x - 1$$

$$2x - 2y = 1 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), $x = \frac{5}{4}$, $y = \frac{3}{4}$

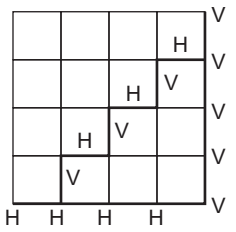
Required area is $= \frac{1}{2} \cdot \text{base} \times \text{height}$

$$= \frac{1}{2} \times \frac{5}{2} \times \frac{5}{4} = \frac{25}{16}$$

TEST Edge The area of triangle between two curves related questions are asked from this concept. To solve such types of questions, to understand basic definition of equation of line and also acquainted yourself with concept of graph transformation.

78. (*a*) We have shown two possible path in the figure.

If horizontal movement is written as H and vertical movement is written as V. Then, these paths can be written as



H H H H VVVV and HV HV HV HV

Hence, energy path is basically arrangement of 4 'H' and 4 'V'.

$\therefore$ Total number of ways $= \frac{8!}{4!\,4!}$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1 \times 4}$$

$$= 70$$

79. (*a*) **Idea** $\because P(A/B) =$ probability of occurrence of A given that B has already occur

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

We know that

$$P\left(\frac{A}{B}\right) > P(A)$$

$$\frac{P(A \cap B)}{P(B)} > P(A) \Rightarrow P(A \cap B) > P(A) \cdot P(B)$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} > \frac{P(A) \cdot P(B)}{P(A)}$$

$$P(B/A) > P(B)$$

TEST Edge Different types of questions based on conditional probability are asked in JEE Main. Students are learn the formulae and definition of conditional probability and intersection or union of sets to solve such types of questions.

80. (*c*) It is given that

$$2\cos^2\theta + \sin\theta \le 2$$

$$\Rightarrow \quad 2(1 - \sin^2\theta) + \sin\theta \le 2$$

$$\Rightarrow \quad 2\sin^2\theta - \sin\theta \ge 0$$

$$\Rightarrow \quad \sin\theta(2\sin\theta - 1) \ge 0$$

$$\Rightarrow \quad \sin\theta \ge 0 \text{ and } 2\sin\theta - 1 \ge 0$$

or $\sin\theta \le 0$ and $2\sin\theta - 1 \le 0$

***Case* I** When $\sin\theta \ge 0$ and $2\sin\theta - 1 \ge 0$

Now, $\sin\theta \ge 0$ and $2\sin\theta - 1 \ge 0$

$$\Rightarrow \quad \sin\theta \ge 0 \text{ and } \sin\theta \ge \frac{1}{2}$$

$$\Rightarrow \quad \frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$$

$$\therefore \quad A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\}$$

***Case* II** When $\sin\theta \le 0$ and $2\sin\theta - 1 \le 0$

Then, $\sin\theta \le 0$ and $\sin\theta \le \frac{1}{2}$

$$\Rightarrow \quad \sin\theta \le 0$$

$$\Rightarrow \quad \pi \le \theta \le 2\pi$$

$$\therefore \quad A \cap B = \{\theta : \pi \le \theta \le 3\pi/2\}$$

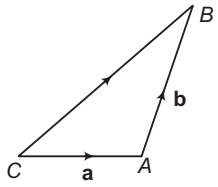
Thus, $A \cap B = \{\theta : \pi/2 \le \theta \le 5\pi/6\}$

$$\cup \{\theta : \pi \le \theta \le 3\pi/2\}$$

So, correct option is (c).

81. (*a*) **Idea** Use triangle law of addition of vectors

$\because$ In ΔABC



Then, $CB = CA + AB = \mathbf{a} + \mathbf{b}$

For a unit vector $(\hat{\mathbf{a}})$

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Here, $\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$

$$\mathbf{BC} = \frac{2\mathbf{u}}{|\mathbf{u}|} - \frac{\mathbf{u}}{|\mathbf{u}|} + \frac{\mathbf{v}}{|\mathbf{v}|}$$

$$= \frac{\mathbf{u}}{|\mathbf{u}|} + \frac{\mathbf{v}}{|\mathbf{v}|}$$

$$\mathbf{AB} \cdot \mathbf{BC} = \left(\frac{\mathbf{u}}{|\mathbf{u}|} - \frac{\mathbf{v}}{|\mathbf{v}|}\right)\left(\frac{\mathbf{u}}{|\mathbf{u}|} + \frac{\mathbf{v}}{|\mathbf{v}|}\right)$$

$$= (\hat{\mathbf{u}} - \hat{\mathbf{v}}) \cdot (\hat{\mathbf{u}} + \hat{\mathbf{v}})$$

$$= 1 - 1 = 0$$

$$\angle B = 90°$$

$\Rightarrow 1 + \cos 2A + \cos 2B + \cos 2C = 0$

TEST Edge Two vectors are perpendicular, parallel and points A, B and C are collinear. Related questions are asked to solve such types of questions to understand the product of vectors and using trigonometrics identities.

82. (*b*) Let directrix be $x = a/e$ and the focus be $S(ae, 0)$. Let $P(a\sec\theta, b\tan\theta)$ be any point on the curve.

Equation of tangent at P is

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

Let F be the intersection point of the tangent and the directrix, so that

$$F \equiv \left(\frac{a}{e}, \frac{b(\sec\theta - e)}{e\tan\theta}\right)$$

$$\Rightarrow \quad m_{SF} = \frac{b(\sec\theta - e)}{-a\tan\theta(e^2 - 1)},$$

$$m_{PS} = \frac{b\tan\theta}{a(\sec\theta - e)}$$

$$\Rightarrow \quad m_{SF} \cdot m_{PS} = -1$$

$$\theta = \frac{\pi}{2}$$

83. (*d*) **Idea** For a quadratic equation $ax^2 + bx + c = 0$ if roots are real then, $b^2 - 4ac \ge 0$

We have given that

$$4x^2 - 20\lambda x + 25\lambda^2 + 15\lambda - 66 = 0$$

$$(2x - 5\lambda)^2 = 66 - 15\lambda$$

The roots are

$$x = \frac{5\lambda \pm \sqrt{66 - 15\lambda}}{2}$$

Discriminant ≥ 0

$$\therefore \quad 66 - 15\lambda \ge 0 \Rightarrow \lambda \le \frac{22}{5} \qquad \ldots(i)$$

$$\frac{5\lambda + \sqrt{66 - 15\lambda}}{2} < 2$$

$$\Rightarrow \quad 5\lambda + \sqrt{66 - 15\lambda} < 4$$

$$\Rightarrow \quad \sqrt{66 - 15\lambda} < 4 - 5\lambda$$

$$\Rightarrow \quad 66 - 15\lambda < 16 - 40\lambda + 25\lambda^2$$

$$\Rightarrow \quad \lambda^2 - \lambda - 2 > 0$$

$$\Rightarrow \quad (\lambda + 1)(\lambda - 2) > 0$$

$$\Rightarrow \quad \lambda \in (-\infty, -1) \cup (2, \infty) \qquad \ldots(ii)$$

Also $4 - 5\lambda > 0 \Rightarrow \lambda < \frac{4}{5}$ $\ldots(iii)$

From above equations

$$-\infty < \lambda < -1$$

$$\therefore \quad \lambda \in (-\infty, -1)$$

TEST Edge Relation between the roots and nature of roots. Maximum and minimum values of $ax^2 + bx + c = 0$ related questions are asked. To solve these questions, students learn the formulae of above concept and acquainted yourself for wary curve method.

84. (*d*) **Idea** Here,

We use Napier's analogy and we know that

$$\tan\theta = \frac{1}{\cot\theta}$$

Here, given that in ΔABC

$$\tan\left(\frac{A - B}{2}\right) = \frac{1}{3}\tan\left(\frac{A + B}{2}\right) \qquad \ldots(i)$$

Using Napier's analogy,

$$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right) \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{3}\tan\left(\frac{A+B}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \quad \frac{1}{3}\cot\left(\frac{C}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$[\because\ A+B+C=\pi/2]$$

$$\Rightarrow \quad \frac{a-b}{a+b}=\frac{1}{3}$$

$$\Rightarrow \quad 3a-3b=a+b$$

$$2a=4b$$

$$\Rightarrow \quad \frac{a}{b}=\frac{2}{1} \Rightarrow \frac{b}{a}=\frac{1}{2}$$

Thus, the ratio of the sides opposite to the angle is $b:a=1:2$

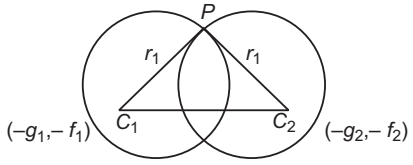
TEST Edge Relation between the sides and angle of triangle, related questions are asked from this concept. Students are advised to understand the proper using of trigonometric identities and stick with concept of properties of triangle.

85. (*d*) The given expression is

$$\begin{aligned}&{}^{n-2}C_r+2\,{}^{n-2}C_{r-1}+{}^{n-2}C_{r-2}\\&=({}^{n-2}C_r+{}^{n-2}C_{r-1})+({}^{n-2}C_{r-1}+{}^{n-2}C_{r-2})\\&={}^{n-2+1}C_r+{}^{n-2+1}C_{r-1}\\&={}^{n-1}C_r+{}^{n-1}C_{r-1}\\&={}^{(n-1+1)}C_r={}^{n}C_r\end{aligned}$$

So, correct option is (d).

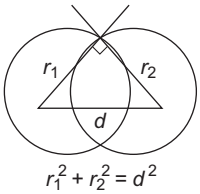
86. (*c*) **Idea** If two circles are intersect orthogonally as figure.



Then $C_1C_2^2=C_1P^2+C_2P^2$ Apply this condition to solve the question

Given, $x^2+y^2=4$

$(x-1)^2+y^2=4$ and $x^2+(y-2)^2=4$



$r_1^2+r_2^2=d^2$

Let the required circle be

$$(x-h)^2+(y-k)^2=r^2$$

Now, by condition of orthogonality,

$$h^2+k^2=r^2+4 \quad \ldots(i)$$

$$\Rightarrow \quad (h-1)^2+k^2=r^2+4 \quad \ldots(ii)$$

and $$h^2+(k-2)^2=r^2+4 \quad \ldots(iii)$$

$$\Rightarrow \quad h=\frac{1}{2} \text{ and } k=1$$

$$\therefore \quad r^2+4=\frac{1}{4}+1 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad r^2=-\frac{11}{4}$$

$\therefore$ Required equation of circle

$$\left(x-\frac{1}{2}\right)^2+(y-1)^2=-\frac{11}{4}$$

$$\Rightarrow \quad x^2+y^2-x-2y+4=0$$

So, correct option is (c).

TEST Edge Equation of circles in different cases and relation between the circle and conic section related questions are asked. To solve such types of questions students are advised to understand the concept of circle and conic section and how to circle and conic section are related with each other.

87. (*b*) Let A and B are square matrices of same order and A is non-singular.

For positive integer n, consider

$$(A^{-1}BA)^n=(A^{-1}BA)(A^{-1}BA)(A^{-1}BA)\underbrace{\ldots\ldots}_{n\text{ times}}(A^{-1}BA)$$

$$=A^{-1}B(AA^{-1})B(AA^{-1})B(AA^{-1})\ldots(AA^{-1})BA$$

$$=A^{-1}BI\,BI\,BI\ldots IBA \qquad \{\because AA^{-1}=BI\}$$

$$=A^{-1}\underbrace{B.B.B\ldots.}_{n\text{ times}}BA=A^{-1}B^nA$$

$$\Rightarrow \quad (A^{-1}BA)^n=A^{-1}B^nA$$

88. (*c*) **Idea** Here simply follow the rule of LCM and properties of logarithmic

$$\therefore \quad \log\frac{x^3}{pqr}=3\log x-\log p-\log q-\log v$$

Here, it is given that

$$y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{x}{x-c}+1$$

$$=\frac{ax^2+bx(x-a)}{(x-a)(x-b)(x-c)}+\frac{x}{x-c}$$

$$=\frac{ax^2+bx^2-abx+x(x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

$$=\frac{x^3}{(x-a)(x-b)(x-c)}$$

Thus, $$y=\frac{x^3}{(x-a)(x-b)(x-c)}$$

$\Rightarrow \ln y = 3\ln x - \ln(x-a) - \ln(x-b) - \ln(x-c)$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\cdot y' = \frac{3}{x} - \frac{1}{x-a} - \frac{1}{x-b} - \frac{1}{x-c}$$

$$= \left(\frac{1}{x} - \frac{1}{x-a}\right) + \left(\frac{1}{x} - \frac{1}{x-b}\right) + \left(\frac{1}{x} - \frac{1}{x-c}\right)$$

$$\Rightarrow \quad \frac{y'}{y} = \frac{a}{x(a-x)} + \frac{b}{x(b-x)} + \frac{c}{x(c-x)}$$

$$\Rightarrow \quad \frac{xy'}{y} = \frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}$$

So, correct option is (c).

TEST Edge Parametric differentiation logarithmic differentiation and higher order differentiation, related questions are asked. To solve such types of questions students are advised to understand the differentiation.

89. (*a*) Truth table is given below

p	q	$\sim p$	$\sim q$	$(p \vee \sim q)$	$\sim p \wedge q$	$\sim(p \vee \sim q)$
F	T	T	F	F	T	T

So, Statement I is true.

Since, $\sim[p \vee (\sim q)] = \sim p \wedge \sim(\sim q) = \sim p \wedge q$

So, Statement II is also true and Statement II is correct explanation of Statement I.

90. (*a*) **Idea** In step 1, put $n = 1$, the obtained result should be a multiple of 7. In step 2, put $n = k$, take equal to multiple of 7 with any non-zero constant. In step 3, put $n = k+1$ in the statement and solve till it becomes a multiple of 7.

Let $P(m) : m^7 - m$

$\therefore$ $P(1) : 1 - 1 = 0$ is divisible by 7.

and $P(2) : 128 - 2 = 126$ is divisible by 7.

and $P(K) : K^7 - K$ is divisible by 7.

$\Rightarrow$ $m^7 - m$ is divisible by 7. $\{\because\ m \in N \ \& \ K \in N\}$

$\therefore$ $P(K+1) : (K+1)^7 - (K+1)$

$\Rightarrow K^7 + {}^7C_1K^6 + \ldots + {}^7C_6(K+1) - K - 1$

$\Rightarrow$ $K^7 + {}^7C_1K^6 + \ldots + 7$

$\therefore P(K+1)$ is divisible by 7

Now, $(K+1)^7 - (K+1)$ is divisible by 7.

$\Rightarrow (m+1)^7 - (m+1)$ is divisible by 7.

$\Rightarrow (m+1)^7 - m^7 - 1 + (m^7 - m)$ is divisible by 7

$\Rightarrow (m+1)^7 - m^7 - 1$ is also divisible by 7.

TEST Edge In JEE Main, generally multiple and divisibility related questions are asked. To solve these types of questions follow the principle of mathematical induction.