JEE Main

Joint Entrance Examination

Question Booklet Code **P**

Practice Set 3

Duration: 3 Hours *Max. Marks: 360*

Read the Following Instructions Carefully

1. *Immediately fill the particulars on this page of the test booklet with blue / black ball point pen. Use of pencil is strictly prohibited.*
2. *The test is of 3 hours duration.*
3. *The test booklet consists of 90 questions. The maximum marks are 360.*
4. *There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.*
5. *Candidates will be awarded marks as stated in above instructions for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. There is no negative marking for unattampted questions.*
6. *There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per above instructions.*
7. *No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.*
8. *Rough work is to be done on the space provided for this purpose in the test booklet only. This space is given at the bottom of pages.*

Name of the Candidate (in Capital Letters)_____

Roll Number (in Figures) | | | | | | | | | in Words _____

PART A Physics

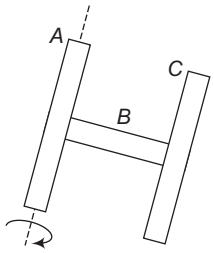
1. **If ω be the angular speed of electron and v be its speed in the nth orbit of hydrogen atom. Then,**

(a) $\omega \propto \frac{1}{n^2}$ and $v \propto \frac{1}{n}$ (b) $\omega \propto n^{1/2}$ and $v \propto \frac{1}{n^2}$ (c) $\omega \propto \frac{1}{n^3}$ and $v \propto \frac{1}{n}$ (d) $\omega \propto n^{-1/2}$ and $v \propto 1/n$

2. **A spherical conductor of radius 3 m is charged to a potential of 90 V. It is now placed inside another hollow spherical conductor of radius 6 m. Calculate the potential of bigger sphere if the smaller sphere is made to touch the bigger sphere.**

(a) 40 V (b) 45 V (c) 50 V (d) 90 V

3. **A rigid body is made of three identical thin rods, each of length L fastened together in the form of letter H. The body is free to rotate about a horizontal axis that runs along the length of one of the legs of the H shaped body. The body is allowed to fall from rest to a position in which the plane of H is horizontal. What is the angular speed of the body when the plane of H is vertical?**



(a) $\sqrt{\frac{g}{L}}$ (b) $\frac{1}{2}\sqrt{\frac{g}{L}}$

(c) $\frac{3}{2}\sqrt{\frac{g}{L}}$ (d) $2\sqrt{\frac{g}{L}}$

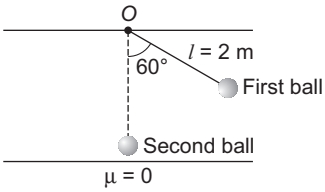
4. **The maximum wavelength of radiation that can produce photoelectric effect in a certain metal is 200 nm. The maximum KE acquired by electron due to radiation of wavelength 100 nm will be**

(a) 6.2 eV (b) 12.4 eV (c) 100 eV (d) 200 eV

5. **The source of sound emit a sound wave of frequency f_0. An observer is moving towards the source with speed v. The apparent frequency listen by the observer is f. The wavelength of the sound listen by observer is λ. If the original wavelength of sound was λ_0, then**

(a) $\lambda < \lambda_0, f > f_0$ (b) $\lambda = \lambda_0, f = f_0$ (c) $\lambda = \lambda_0, f > f_0$ (d) $\lambda > \lambda_0, f > f_0$

6. **A ball is tied with a string and left from some height as shown in the figure. A second ball is placed on the floor just below the pivoted point O. The first ball hit the second ball and the collision is inelastic with the coefficient of restitution $3/\sqrt{5}$. The height raised by the first ball after the collision is**



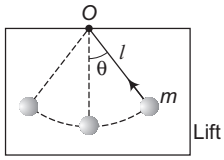
(a) 1.2 m (b) 1.52 m

(c) 1.38 m (d) 1.6 m

7. **Sodium emits photons of wavelength of 5890Å when electrons jump from excited state n = 4 to n = 3. Suppose two sources of sodium are used in double slit experiment. Then, the intensity observed at a distance of 2945 Å from the centre of screen is (Given, I_0 is the intensity of single beam)**

(a) zero (b) I_0 (c) $2I_0$ (d) $4I_0$

8. **A bob connected with a light extensible rod is executing SHM in a stationary lift as shown in figure. When the bob was at the extreme position the lift comes in free fall, then**



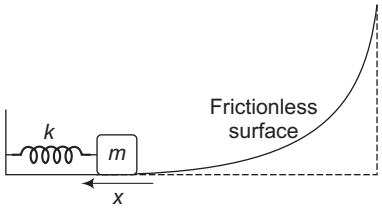
(a) the bob will still execute SHM

(b) the tension in the rod $T = mg$ at the next moment

(c) the bob will execute SHM making an angle θ with the vertical

(d) the bob will remain at rest

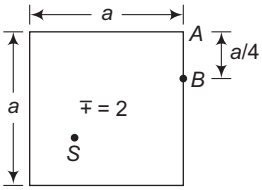
9. **A block is placed over a horizontal surface and it is in touch with a spring. The spring is compressed by x and then released and the block moves along the curved surface and achieved a maximum height**

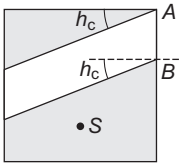


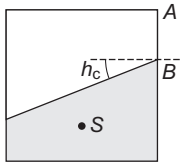
(a) $W_{gravity} = W_{spring}$ (b) $W_{spring} = -W_{gravity}$ (c) H_{max} achieved $= \frac{kx^2}{mg}$ (d) None of these

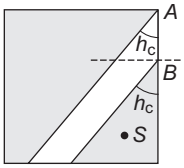
10. **Figure shows a transparent block of front sides a and a. The third dimension of the block is negligible. A point source S which can emit light in all directions can move inside the block. It is desired that no light of S should pass through AB.**

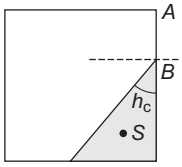
The region in which S should be present to satisfy this condition shown by shaded region. Choose the correct option.



(a) 

(b) 

(c) 

(d) 

11. **Choose the correct statement.**

(a) A system could have some heat energy.

(b) A system could have some work.

(c) A ball is moving with speed v, here $\frac{1}{2}mv^2$ is its internal energy.

(d) None of the above

12. **For an oscillating simple pendulum. Choose the correct option.**

(a) Tension T is always greater than mg (b) $0 < T < mg$

(c) $T_{min} = mg$ (d) $T_{max} > mg$

13. **When 50V AC and DC are applied to a solenoid, then the value of current flowing through is was found to be 1 A and 0.5 A. If the frequency of AC source is 100 Hz, then the inductance of solenoid is**

(a) 1.4 H (b) 0.14 H (c) 2.4 H (d) 0.24 H

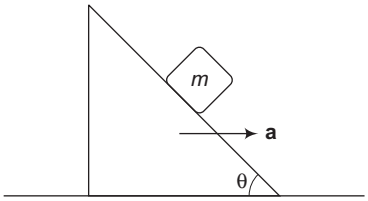
14. **One mol of an ideal gas with molar heat capacity 4R undergoes a process in which work done by gas for small change in temperature is given by the relation dW = 2RdT. Then, the ratio $\frac{C_p}{C_v}$ is**

(a) 7/5 (b) 5/3 (c) 3/2 (d) 2

15. **By what acceleration, the wedge should move so that the block moves upwards along the surface of wedge with an acceleration of g sin θ**

(a) $2g \cos\theta$ (b) $g \sin\theta$

(c) $2g \tan\theta$ (d) $g \cos\theta$



16. **The flux linked with coil at any instant t is given by $\phi = 5t^3 - 20t + 100$, then the induced emf at t = 2 s is**

(a) 60 (b) 40 (c) – 40 (d) – 60

17. **A rod of length l (laterally thermally insulated) of uniform cross-sectional area A consists of a material whose thermal conductivity varies with temperature as $K = \frac{k_0}{a + bT}$, where, k_0, a and b are constants. T_1 and T_2 ($< T_1$) are the temperature of two ends of rod. Then, rate of flow of heat across the rod is**

(a) $\frac{Ak_0}{bl}\left(\frac{a + bT_1}{a + bT_2}\right)$ (b) $\frac{Ak_0}{bl}\left(\frac{a + bT_2}{a + bT_1}\right)$

(c) $\frac{Ak_0}{bl}\ln\left[\frac{a + bT_1}{a + bT_2}\right]$ (d) $\frac{Ak_0}{al}\ln\left[\frac{a + bT_2}{a + bT_1}\right]$

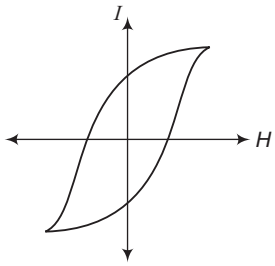
18. **The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by**

(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$ (c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

19. **Consider the figure given below**

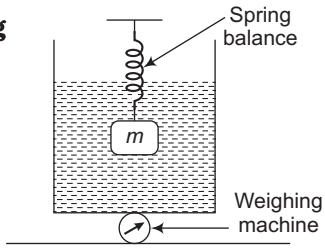
It is the hysteresis loop of a magnetic substance. Then, the integral $\oint H.dI$ represents

(a) energy gained by the substance during complete cycle
(b) energy lost as heat during complete cycle
(c) energy lost per unit volume during complete cycle
(d) its value is equal to zero as magnetic forces are conservative

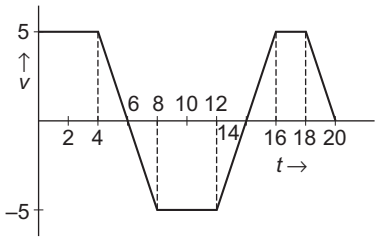


20. **If the weighing machine reading is 20 N and spring balance reading is 20 N, then**

(a) buoyant force = 40 N
(b) weight of block = 20 N
(c) buoyant force = 30 N
(d) weight of block = 40 N



21. **Velocity v (m/s) versus time graph of a cyclist moving along the x-axis is shown from time t = 0 to t = 20 s. The average velocity and the average speed on the cyclist during this time interval are (in m/s)**



(a) $\frac{3}{4}, \frac{15}{4}$ (b) $\frac{3}{4}, \frac{3}{4}$

(c) $0, \frac{15}{4}$ (d) $\frac{15}{4}, \frac{15}{4}$

22. **A charge q is moving with a velocity $v_1 = 1\,\hat{i}$ m/s at a point in magnetic field and experiences a force $F = q[-\hat{j} + 1\hat{k}]$ N. If the charge is moving with velocity $v_2 = 2\hat{j}$ m/s at the same point, it experiences a force $F_2 = q(1\hat{i} - 1\hat{k})$ N. The magnetic induction B at that point is**

(a) $(\hat{i} + \hat{j} + \hat{k})$ Wb/m^2 (b) $(\hat{i} - \hat{j} + \hat{k})$ Wb/m^2

(c) $(-\hat{i} + \hat{j} - \hat{k})$ Wb/m^2 (d) $(\hat{i} + \hat{j} - \hat{k})$ Wb/m^2

23. **Bulk modulus of metal is 10^{10} N/m^2 and its density is 11 g/cm^3. The density of this metal under a pressure of 20000 N/cm^2 will be (in g/cm^3)**

(a) $\frac{550}{49}$ (b) $\frac{610}{51}$

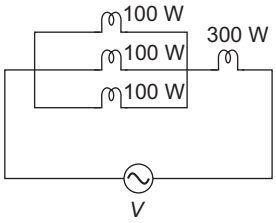
(c) $\frac{555}{49}$ (d) $\frac{450}{41}$

24. **The dimensional formula of surface tension is**

(a) $[ML^2T^{-2}]$ (b) $[MLT^{-2}]$

(c) $[MT^{-2}]$ (d) $[ML^2]$

25. **Four bulbs of different power readings are arranged in a circuit as shown below**



The resultant power of combination if rated voltage is applied across the combination is

(a) 50 W (b) 600 W

(c) 150 W (d) 33.3 W

26. **When a satellite shifts from the orbit of radius r to the orbit of radius 2r, then**

(a) PE of satellite will increase (b) KE of satellite will decrease

(c) total mechanical energy of satellite will increase (d) options (a), (b) and (c) all are correct

27. **A radioactive nucleus can decay by two different processes. The half-life for the first process is t_1 and that for the second process is t_2. Then, the effective half-life t of the nucleus can be given by**

(a) $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$ (b) $\frac{1}{t} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

(c) $\frac{1}{t} = \frac{1}{t_1} - \frac{1}{t_2}$ (d) $\frac{1}{t} = \frac{1}{t_1^2} - \frac{1}{t_2^2}$

28. **Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is**

(a) $\frac{1}{4}C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}C(V_1^2 + V_2^2)$

(c) $\frac{1}{4}C(V_1 - V_2)^2$ (d) $\frac{1}{4}C(V_1 + V_2)^2$

29. **Statement I The translation motion of a body can be described with the help of centre of mass.**

Statement II The CM of a regular uniform density body lies on the geometric centre of the body.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are false

30. **Statement I A nucleus in a large sample of identical radioactive nuclei did survive 4 half-life. Then, the probability of this nuclei to survive next half-life will be $\frac{1}{16}$.**

Statement II After 4 half-life, the quantity of sample will reduce to $\frac{1}{16}$ of initial quantity.

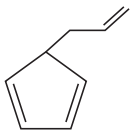
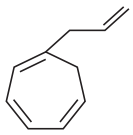
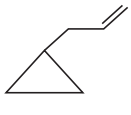
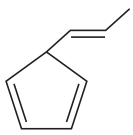
(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are false

PART B Chemistry

31. **The total number of neutrons present in 108 mL $H_2O(l)$ are**

(a) $32\,N_A$ (b) $48\,N_A$ (c) $16\,N_A$ (d) $8\,N_A$

32. **What will be the product when most acidic species among following will react with 3-chloro prop 1-ene?**

(a) 
(b) 
(c) 
(d) 

33. **Nitrogen exist as N_2 but phosphorous exist as P_4 this is due to**

(a) tripple bond exist between phosphorous atom (b) $p_\pi - p_\pi$ bond is weak in P_4
(c) $p_\pi - p_\pi$ bond is strong in P_4 (d) multiple bonds form easily

34. **For a 3 s orbital,**

$$\psi\,(3s) = \frac{1}{9\sqrt{3}}\left(\frac{1}{a_0}\right)^{3/2}(6 - 5\sigma + \sigma^2)\,e^{-\sigma/2}$$

where, $\sigma = \frac{2r.Z}{3a_0}$ What will be maximum radial distance of node from nucleus?

(a) $\frac{5Za_0}{2}$ (b) $\frac{6Za_0}{7}$ (c) $\frac{9Za_0}{2}$ (d) $\frac{7Za_0}{3}$

35. A saturated organic compound contains 80% C and 20% H. What will be the product when this organic compound undergo reaction with Cl_2 in presence of sunlight?
(Hint consider one equivalent of Cl_2 only)

(a) CH_3CH_2Cl (b) C_2H_6 (c) CH_4 (d) CH_3Cl

36. An inorganic compound (X) made of two most occurring elements in the earth's crust and used in building construction when made to react with carbon, forms a poisonous gas (Y) which is most stable diatomic molecule. Compound (X) and gas (Y) will be

(a) SiO_2, CO_2 (b) Si, CO_2 (c) SiO_2, CO (d) Si, CO

37. What will be the magnitude of the charge on the oxygen atom in water molecule if dipole moment of water is 1.85 D and bond length of O—H is 0.94 Å ?

(a) 3.23×10^{-10}esu (b) 0.323×10^{-10}esu (c) 2.32×10^{-10}esu (d) 3.32×10^{-7}esu

38. Which of the following pairs of complexes are isomeric with each other but their aqueous solutions exhibit different molar conductivities?

(a) $[Co(NH_3)_5NO_2]Cl_2$ and $[Co(NO_3)_5ONO]Cl_2$ (b) $[Co(NH_3)_5Br]SO_4$ and $[Co(NH_3)_5SO_4]Br$
(c) $[PtCl_2(NH_3)_4]Br_2$ and $[Pt(NH_3)_4Br_2]Cl_2$ (d) $[Co(NH_3)_4Cl_2]NO_2$ and $[Co(NH_3)_4ClNO_2]Cl$

39. Equivalent weight of H_3PO_2, when it disproportionate into PH_3 and H_3PO_3 is

(a) $\frac{5M}{4}$ (b) $\frac{7M}{4}$ (c) $\frac{3M}{4}$ (d) $\frac{9M}{4}$

40. A mixture of 4 chloro 3,5-dimethyl phenol and α terpineol is commonly used in our daily life the mixture is known as

(a) bithional (b) dettol (c) lysol (d) None of these

41. Select the codes accordingly.

I.	HCO_2^-	A.	Obtained through Solvay process
II.	K_2CO_3	B.	Green colour due to $[Cr(H_2O)_6]^{3+}$ ion
III.	$S_2O_3^{2-}$ + $FeCl_3$ solution	C.	Reduces $[Cu(C_4H_4O_6)_2]^{2-}$ to red ppt.
IV.	SO_3^{2-} + $K_2Cr_2O_7/H^+$	D.	Green colouration
V.	Na_2CO_3	E.	Melts at 850°C

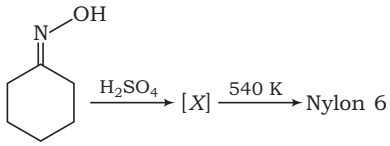
	A	B	C	D	E
(a)	V	III	I	II	IV
(c)	V	IV	I	III	II

	A	B	C	D	E
(b)	IV	I	II	III	V
(d)	II	I	V	IV	III

42. 500 mL of (0.2 M) $CuSO_4$ (aq) solution was electrolyzed using inert electrodes by passing current till the pH of the resulting solution was 3. The solution after electrolysis was neutralized and then treated with excess KI and formed I_2 is titrated with 0.04 M $Na_2S_2O_3$. Calculate the required volume (in mL) of $Na_2S_2O_3$ [Assume concentrate ion $[H^+]$ produced will double on passing current]

(a) 255.5 litre (b) 237.5 litre (c) 305.5 litre (d) 407.5 litre

43. What will be the [X] for following reaction?



(a) cyclohexanone (b) hexamethylene diamine
(c) hexamethylene di-isocyanide (d) caprolactum

44. 100 mL of 0.5 M acetic acid is shaken with 2 gm activated carbon. The final concentration of the solution after adsorption is 0.4 M. What is the amount of acetic acid adsorbed per gram of carbon?

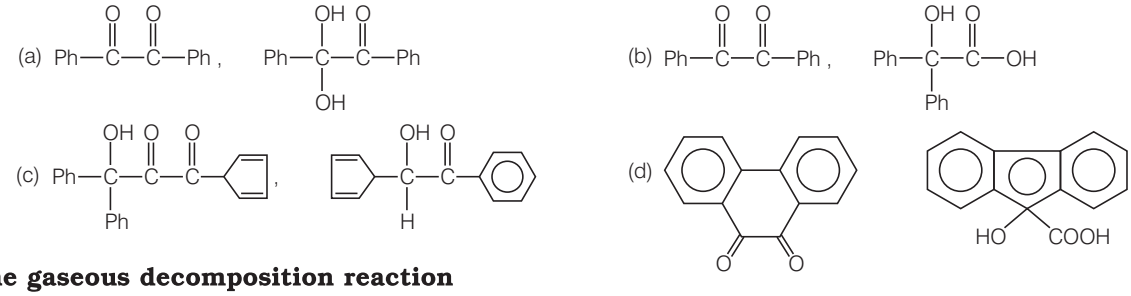
(a) 0.3 (b) 0.4 (c) 0.8 (d) 0.5

45. Which of the following set of compounds represents isostructural characteristics?

(a) SF_4, CH_4, NH_3 (b) NF_3, BCl_3, NH_3 (c) $BF_3, NF_3, AlCl_3$ (d) BF_3, BCl_3, BBr_3

46. An organic diketone (A) having molecular formula $C_{14}H_{10}O_2$ when undergo reaction with hydroxyl ion (OH^-) converted into benzilic (B) acid after acidification.

The molecular structure of (A) and (B) are



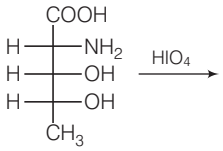
47. The gaseous decomposition reaction

$A_{(g)} \rightarrow 2B_{(g)} + C_{(g)}$ is observed to first order over the excess of liquid water at 28°C. It is found that after 20 min the total pressure of the system is 200 torr and after a long time it is 380 torr. The rate constant of the reaction in (min^{-1}) is

[Given vapour pressure of H_2O = 20 torr, ln 2 = 0.3010 ln 3 = 0.4771]

(a) 0.005 min^{-1} (b) 0.004 min^{-1} (c) 0.003 min^{-1} (d) 0.006 min^{-1}

48. Choose the correct option regarding product (s) of following reaction.



(a) CHO—COOH + CH_3CHO + HCOOH (b) 4COOH + CH_3CHO
(c) 3 COOH + CH_3CHO (d) 2COOH + CHO—COOH + CH_3CHO

49. When aqueous NaOH is added to an aqueous solution of chromium (III) ions, a green blue precipitate is first formed which re-dissolves to give a green solution. The green colour is due to

(a) $[Cr(H_2O)_6]^{3+}$ (b) $[Cr(OH)_4]^-$ (c) CrO_4^{2-} (d) $[Cr(OH)_3(H_2O)_3]$

50. At a certain temperature and 4 atm pressure equilibrium constant (k_p) is 25 for the reaction

$$SO_2\,(g) + NO_2(g) \rightleftharpoons SO_3(g) + NO\,(g)$$

Initially if we take 2 moles of each of the four gases and 2 moles of inert gas, what would be the equilibrium partial pressure of SO_2?

(a) 2 atm (b) 0.17 atm (c) 0.37 atm (d) 0.27 atm

51. Which of the following reaction does not occur in Bessemer's converter?

(a) $FeO + SiO_2 \longrightarrow FeSiO_3$ (b) $2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$

(c) $2Cu_2S + 5O_2 \longrightarrow 2CuSO_4 + 2CuO$ (d) $2CuFeS_2 + O_2 \longrightarrow Cu_2S + 2FeS + SO_2$

52. Which of the following will produce an insecticide on reaction with the product of reaction (I)?

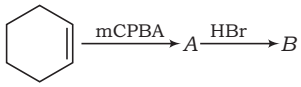
$$CaOCl_2 + \text{acetone} \longrightarrow \text{product}$$

(a) phenol (b) nitric acid (c) amine (d) acetone

53. A 200 mL of solution of I_2 is divided into two unequal parts. I part reacts with hypo solution in acidic medium, 30 mL of 0.2 M hypo was consumed, II part was added with 50 mL of 0.6 M NaOH solution. What was initial concentration of I_2?

(a) 0.525 M (b) 0.625 M (c) 0.225 M (d) 0.50 M

54. Identify the A and B for the following reaction



A and B are

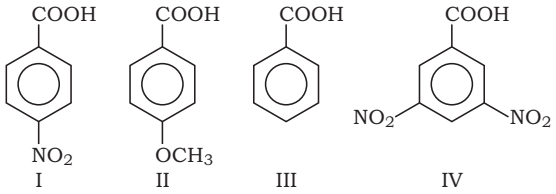
(a) 1, 2-epoxycyclohexane, *trans*-2-bromocyclohexanol (b) 1, 2 epoxycylohexane, *cis*-2-bromocyclohexanol

(c) *trans*-2-bromocyclohexanol, 1,2-epoxyethane (d) *cis*-2-bromocyclohexanol, 1,2-epoxyethane

55. Out of following set of molecules which will show paramagnetic character?

(a) B_2, O_2, N_2 (b) B_2, O_2, NO (c) B_2, F_2, O_2 (d) B_2, O_2, Li_2

56. Arrange the following in their increasing order of acidity



(a) II < III < I < IV (b) IV < III < I < II (c) III < II < I < IV (d) III < II < IV< I

57. The vapour pressure of two pure liquids A and B, that form an ideal solution are 100 and 400 torr respectively at temperature T. This liquid solution of A and B is composed of 1 mole of A and 1 mole of B. What will be the pressure when 1 mole of mixture has been vapourized?

(a) 500 torr (b) 600 torr (c) 700 torr (d) 200 torr

58. Arrange the following in correct order of Lewis acidity BF_3, BCl_3, BBr_3.

(a) $BF_3 > BBr_3 > BCl_3$ (b) $BF_3 > BCl_3 > BBr_3$ (c) $BF_3 < BCl_3 < BBr_3$ (d) $BBr_3 < BF_3 < BCl_3$

59. Statement I The 3-chloro 3,4-dimethyl hex-1-ene undergo reaction with base to give butadiene.

Statement II This is due to Sytzeff rule which states that more substituted product will be the more stable product.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I

(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I

(c) Statement I is false but Statement II is true

(d) Both Statement I and Statement II are false

60. **Statement I The fluorides of alkaline earth metals are almost insoluble in water.**
Statement II The lattice energies of the fluorides of alkaline earth metals are very high.

(a) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I
(b) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are false

PART C Mathematics

61. **Locus of a point z in argand plane satisfying $|z^2 - (\bar{z})^2| = |z|^2$, $\text{Re}(z) \geq 0$, $\text{Im}(z) \geq 0$ is**

(a) point (b) pair of straight line (c) hyperbola (d) ellipse

62. **The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solution is**

(a) zero (b) infinite (c) unique (d) None of these

63. **The locus of the middle points of the normal chords of the parabola, $y^2 = 4ax$ is**

(a) $x + 2a = \frac{y^2}{2a} + \frac{4a^3}{y^2}$ (b) $x + 2a = \frac{y^2}{2a} - \frac{4a^3}{y^2}$ (c) $x - 2a = \frac{y^2}{2a} + \frac{4a^3}{y^2}$ (d) None of these

64. **If $y = y(x)$ and $\frac{2 + \sin x}{(y+1)}\frac{dy}{dx} = -\cos x$, $y(0) = 1$, then $y(\pi/2)$ is equal to**

(a) $\frac{1}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{4}$ (d) $\frac{2}{5}$

65. **Let a, b, c are positive real numbers different from 1. If $\log_a 100, 2\log_b 10, 2\log_c 5 + \log_c 4$ are in HP then a, b, c are in**

(a) AP (b) GP (c) HP (d) None of these

66. **A card is lost from a pack of 52 playing cards. From the remainder of the pack, one card is drawn and is found to be spade. The probability that the missing card is a spade is**

(a) $\frac{5}{17}$ (b) $\frac{4}{17}$ (c) $\frac{3}{17}$ (d) $\frac{2}{17}$

67. **If $\int \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2} dx = A \ln|\cos x + \sin x - 2| + Bx + C$. Then, ordered triplet A, B, λ is**

(a) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$ (b) $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$ (c) $\left(\frac{1}{2}, -1, -\frac{3}{2}\right)$ (d) $\left(\frac{3}{2}, -1, \frac{1}{2}\right)$

68. **If a circle of radius r is concentric with ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the common tangent is inclined to major axis at an angle**

(a) $\tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$ (b) $\tan^{-1}\sqrt{\frac{r^2 - a^2}{b^2 - r^2}}$ (c) $\tan^{-1}\sqrt{\frac{r^2 - b^2}{r^2 - a^2}}$ (d) $\tan^{-1}\sqrt{\frac{r^2 - a^2}{r^2 - b^2}}$

69. **In a certain town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statement in this regard.**

1. 10 % families own both a car and a phone **2. 35% families own either a car or a phone**

3. 40,000 families live in the town

Which statement (s) is/are correct?

(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3

70. **The value of $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3)$ is**

(a) 17 (b) -15 (c) 15 (d) 16

71. **The value of a and b respectively so that the function**

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x & ; \quad 0 \le x < \dfrac{\pi}{4} \\ 2x\cot x + b & ; \quad \dfrac{\pi}{4} \le x \le \dfrac{\pi}{2} \\ a\cos 2x - b\sin x & ; \quad \dfrac{\pi}{2} < x \le \pi \end{cases}$$

is continuous for $x \in [0, \pi]$

(a) $\frac{\pi}{6}, \frac{\pi}{12}$ (b) $\frac{\pi}{6}, \frac{-\pi}{12}$ (c) $\frac{-\pi}{4}, \frac{\pi}{12}$ (d) $\frac{\pi}{4}, \frac{\pi}{6}$

72. **The line $\frac{x}{k} = \frac{y}{2} = \frac{z}{-12}$ makes an isosceles triangle with the planes $2x + y + 3z - 1 = 0$ and $x + 2y - 3z - 1 = 0$, then value of k is**

(a) 3 (b) -2 (c) 5 (d) 0

73. **If A, B and C are three points position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ respectively, then perpendicular distance of A from the line joining B and C is**

(a) $\dfrac{|\mathbf{a}\times\mathbf{b}\times\mathbf{c}|}{2(\mathbf{b}\times\mathbf{c})}$ (b) $\dfrac{|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}{2|(\mathbf{b}-\mathbf{c})|}$ (c) $\dfrac{|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}{|(\mathbf{b}-\mathbf{c})|}$ (d) None of these

74. **In how many ways can 6 coins be chosen from 20 one rupees coins, 10 fifty paise coins, 7 twenty paise coins?**

(a) 26 (b) 27 (c) 28 (d) 20

75. **If the line $x = y = z$ intersect the line $\sin Ax + \sin By + \sin Cz = 2d^2$, $\sin 2Ax + \sin 2By + \sin 2Cz = d^2$ then $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ is equal to (where, $A + B + C = \pi$)**

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$ (c) $\frac{1}{13}$ (d) $\frac{1}{15}$

76. **The equation $x^3 - 6x^2 + 15x + 3 = 0$ has**

(a) only one positive root (b) two positive and one negative roots

(c) no positive root (d) None of these

77. **If, $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then**

(a) $\mathrm{Re}(z) = 0$ (b) $\mathrm{Im}(z) = 0$ (c) $\mathrm{Re}(z) > 0, \mathrm{Im}(z) > 0$ (d) $\mathrm{Re}(z) > 0, \mathrm{Im}(z) < 0$

78. **In an isosceles triangle ABC, the coordinate of the point B and C on the base BC are respectively (2, 1) and (1, 2). If the equation of the line AB is $y = \frac{x}{2}$, then equation of the line AC is**

(a) $2y = x + 3$ (b) $y = 2x$ (c) $y = \frac{1}{2}(x - 1)$ (d) $y = x - 1$

79. **The solution of the curves $\frac{x\,dx + y\,dx}{x\,dy - y\,dx} = \sqrt{\frac{1 - x^2 - y^2}{x^2 + y^2}}$ are**

(a) circles passing through the origin (b) parabola

(c) circles of radius $\frac{1}{2}$ through the origin (d) not circle

80. **If α, β are the roots $\lambda(x^2 + x) + x + 5 = 0$ and λ_1, λ_2 are two values of λ for which α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4$ then the value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}$ is equal to**

(a) 254 (b) 482 (c) 784 (d) 782

81. **The component of $\hat{i}$ in the direction of the vector $\hat{i} + \hat{j} + 2\hat{k}$ is**

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{2}{\sqrt{6}}$ (c) $\frac{3}{\sqrt{6}}$ (d) $\frac{5}{\sqrt{6}}$

82. **The value of integral $\int_{-\frac{\pi}{4}}^{\frac{-3\pi}{4}} \frac{\sin x + \cos x}{e^{x-\frac{\pi}{4}} + 1} dx$ is equal to**

(a) 0 (b) 1 (c) 3 (d) None of these

83. **In ΔABC, AD is an altitude from A on side BC. If $b > c$, $\angle C = 23°$ and $AD = \frac{abc}{b^2 - c^2}$. Then, $\angle B$ is**

(a) 113° (b) 110° (c) 117° (d) 112°

84. **Let, $f : R \to R$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$, then f is**

(a) one-one and onto (b) one-one and into (c) many one and onto (d) many one and into

85. **Suppose X has a binomial distribution $B\left(6, \frac{1}{2}\right)$ then which of the outcome is most likely?**

(a) $X = 0$ and $X = 6$ (b) $X = 3$ (c) $X = 0$ (d) $X = 6$

86. **The value of $\lim_{x \to \infty} \left(\sqrt{3x^2 + \sqrt{3x^2 + \sqrt{3x^2}}} - \sqrt{3x^2}\right)$ is**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{-3}{2}$ (d) $\sqrt{3}$

87. **If the normal at an end of a latus rectum of an ellipse passes through the other end of the minor axis, then**

(a) $e^4 + e^2 = 2$ (b) $e^4 + e^3 = 1$ (c) $e^4 + e^2 = 1$ (d) $e^4 + e^2 = 4$

88. **The set of all 2×2 matrices which commutes with the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ with respect to matrix multiplication**

(a) $\left\{\begin{bmatrix} a & b \\ c & a-b \end{bmatrix} : a, b, c \in R\right\}$ (b) $\left\{\begin{bmatrix} a & b \\ b & c \end{bmatrix} : a, b, c \in R\right\}$

(c) $\left\{\begin{bmatrix} a-b & a \\ a & c \end{bmatrix} : a, b, c \in R\right\}$ (d) $\left\{\begin{bmatrix} a & b \\ b & a-b \end{bmatrix} : a, b \in R\right\}$

89. **The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On review, it was found that an observation 8 was incorrect.**

Statement I The correct standard deviation if the wrong item is omitted is 1.997.

Statement II The correct standard deviation if 8 is replaced by 12 is 1.990.

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of the Statement I

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

90. **Let, A be a 2×2 matrix with non-zero entries and let $A^2 = I$ where, I is 2×2 identity matrix.**

Define, T_r (A) = Sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement I T_r (A) = 0

Statement II $|A| = 1$

(a) Statement I is true, Statement II is also true and Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is also true and Statement II is not the correct explanation of the Statement I

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

Analytical Explanations

Physics

1. (*c*) **Idea** In Bohr model,
angular momentum, $L_n = \frac{nh}{2\pi}$
radius of n^{th} orbit, $r_n \propto n^2$

Bohr model tells us that angular momentum in n^{th} orbit is n times h (or $h/2\pi$)

So, $$L_n = \frac{nh}{2\pi}$$

$$\Rightarrow \quad I\omega = \frac{nh}{2\pi}$$

$$\Rightarrow \quad (mr^2)\,\omega = \frac{nh}{2\pi}$$

$[\because I = \text{moment of inertia} = m \times r^2]$

As, $r \propto n^2$ [r = radius of n^{th} orbit]

So, $$\omega \propto \frac{n}{n^4} \Rightarrow \omega \propto \frac{1}{n^3}$$

Also, $$\omega = v/r$$

$\therefore$ $v = \omega r,\ r \propto n^2$

and $$\omega \propto \frac{1}{n^3}$$

$\therefore$ $$v \propto \frac{1}{n}$$

TEST Edge Questions related to angular momentum, velocity, radius, energy etc., are frequently asked.

In Bohr's theory,

Angular momentum $L_n = \frac{nh}{2\pi}$

Speed of electron $v_n = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{Z}{n}$

Radius $r_n = (0.53\ \text{Å})\frac{n^2}{Z}$

Energy, $E_n = (-13.6)\frac{Z^2}{n^2}$

2. (*b*) Potential $= \frac{kq}{r}$

$$\Rightarrow \quad \frac{kq}{3} = 90 \Rightarrow kq = 270$$

Now if smaller sphere is touched to bigger sphere, whole charge will transfer to bigger sphere.

Hence, $$V_{\text{Bigger}} = \frac{kq}{R} = \frac{kq}{6} = \frac{270}{6}$$

$$\Rightarrow \quad V_{\text{Bigger}} = 45\ \text{V}$$

3. (*c*) **Idea** Moment of inertia of discrete masses at a distance L is given by ML^2 and moment of inertia of a rod about one end is $ML^2/3$ and total mechanical energy is conserved.

Moment of inertia of the system about the given axis $I = I_A + I_B + I_C$

Now, as rod is thin $I_A = \Sigma m \times (0)^2 = 0$

Rod B is rotating about one end

$\therefore$ $$I_B = \frac{ML^2}{3}$$

and for rod call points are always at distance L from the axis of rotation, so

$$I_C = \Sigma mL^2 = ML^2$$

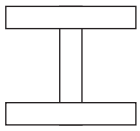
$$I = 0 + \frac{ML^2}{3} + ML^2 = \frac{4}{3}ML^2$$

So, if ω is the desired angular speed, gain in kinetic energy due to rotation of H from horizontal to vertical position.

So, $$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{4}{3}ML^2\right]\omega^2$$

$$= \frac{2}{3}ML^2\omega^2$$

and loss in potential energy of the system in doing so $= 0 + Mg\frac{L}{2} + MgL = \frac{3}{2}MgL$



So, by conservation of mechanical energy

$$\frac{2}{3}ML^2\omega^2 = \frac{3}{2}MgL$$

$$\Rightarrow \quad \omega = \frac{3}{2}\sqrt{\frac{g}{L}}$$

TEST Edge Questions involving concept such as angular momentum conservation, ($I\omega = \text{const.}$) energy conservation etc., and parallel axis theorem ($I = I_0 + Md^2$) and perpendicular axis theorem ($I_z = I_x + I_y$) are frequently asked, so important results related to moment of inertia from axis of rotation of important object [*i.e.*, moment of inertia from one end $= \frac{ML^2}{3}$ etc.] must be memorized.

4. (*a*) **Idea** In photoelectric effect maximum kinetic energy of e^- is given by

$$(KE)_{max} = h\nu - W$$

Work function of the metal is $W = \frac{hc}{\lambda_0}$.

It is given that $\lambda_0 = 200$ nm because 200 nm corresponds the wavelength which is just able to emit electrons from the metal.

Now KE = incident energy of radiation – work function

$$\therefore \quad KE = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \text{ in Joules}$$

$$KE \text{ (in eV)} = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \text{eV}$$

$$= 12375 \text{ eV/A°} \times \left[\frac{1}{100} - \frac{1}{200}\right]$$

$$= 6.18 \text{ eV} \approx 6.2 \text{ eV}$$

TEST Edge Question involving $(KE)_{max} = h\nu - W$ is almost asked every year also dependency of photocurrent on intensity etc., are also asked.

5. (*c*) When the source remains stationary and emit some waves, the wavelength of the waves do not change and as the observer is moving towards the source apparent frequency will increase.

$$\Rightarrow \quad \lambda = \lambda_0 \quad \text{and} \quad f > f_0$$

6. (*c*) **Idea** The change in potential energy will be converted to kinetic energy.

From conservation of mechanical energy

$$mg(l - l\cos\theta) = \frac{1}{2}mv^2$$

$$v = \sqrt{2gl(1 - \cos\theta)}$$

$$= \sqrt{2 \times 10 \times 2\left(1 - \frac{1}{2}\right)} = 2\sqrt{5} \text{ m/s}$$

$\Rightarrow$ After the collision,

$$e = \frac{|v_1 - v_2|}{|v - 0|}$$

$$\frac{v_3}{\sqrt{5}} = v_1 - v_2$$

$$v_1 - v_2 = 6 \qquad \ldots(i)$$

From momentum conservation

$$m \times v = m(v_1 + v_2)$$

$$v_1 + v_2 = 2\sqrt{5} \qquad \ldots(ii)$$

$\Rightarrow$ Solving two expressions, $v_1 = 5.25$ m/s

$\Rightarrow$ From mechanical energy conservation

$$h_1 = \frac{v_1^2}{2g} = 1.38 \text{ m}$$

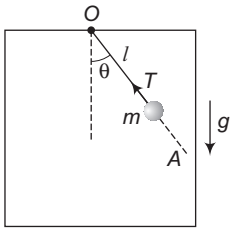
TEST Edge Question involving work-energy theorem may also be asked, to solve these type of problem, student must know that change in kinetic energy of a particle is equal to the work done on it by the net force acting on the particle.

7. (*c*) Note that two sources of sodium, although produce EM waves of same wavelength but still they do not act as coherent sources because there is no definite relation between their initial phases of EM waves produced.

Hence, no interference pattern will be observed and thus there will be no minima and maxima. Intensity everywhere will be just $2I_0$ [$= Z_0 + Z_0$].

8. (*c*) **Idea** For small extension restoring force ($F = -kx$) act on body.

When lift comes in free fall the ball will execute SHM along the line *OA* in vertical plane due to tension in rod.



TEST Edge Question related to simple hormonic motion is asked frequently such as given equation or given motion is SHM or not, to solve such problem, student must know that in SHM acceleration ∝-(displacement).

9. (*b*) $W_{spring} = \frac{1}{2}kx^2$

This work done will convert into KE

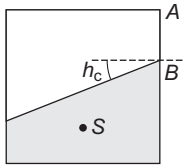
$$\Rightarrow \quad \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

Gravity will do negative work and this KE will convert into PE

$$\Rightarrow \quad W_{gravity} = -W_{spring}$$

10. (*b*) **Idea** When light rays go from denser to rarer medium, total internal reflection will occur if angle of incidence is greater than critical angle.

If *S* is anywhere in the shaded region, the light rays from *S* will strike *AB* making an angle more than critical angle and hence reflected back in the same region.



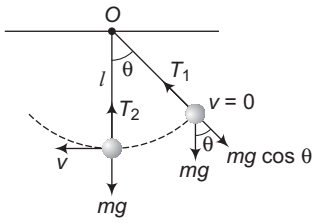
TEST Edge Other question including concept of total internal reflection such as mirage, early visibility of sun etc., are also asked. Student must remember total internal reflection occurs when ray goes from denser to rarer medium and angle must be greater than critical angle if angle will be less than critical angle then refraction will occur.

11. (*d*) Heat energy is the energy in transition, a system could have some internal energy or we could give system some heat energy and system could do some work to change its internal energy.

$\frac{1}{2}mv^2$ is the mechanical energy of the ball not its internal energy.

12. (*d*) At extreme position

$$T_1 = mg\cos\theta = T_{\min}$$



At the lowest position,

$$T_2 = mg + \frac{mv^2}{l} = T_{\max}$$

So, $T_{\max} > mg$

13. (*b*) Value of resistance solenoid is

$$R = \frac{50}{1} = 50\,\Omega$$

[∵ only inductive reactance is zero for DC.]

$$\text{Impedance} = \frac{50}{0.5} = 100\,\Omega$$

Now, $Z^2 = X_L^2 + R^2$

$\Rightarrow$ $X_L^2 = 100^2 - 50^2 = 7500$

$\Rightarrow$ $X_C = \sqrt{75}\times 10 \approx 8.7\times 10 = 87$

Now, $2\pi\nu \times L = 87$

$\therefore$ $L = \frac{87}{2\times 3.14\times 100} = 0.14\,\text{H}$

14. (*c*) **Idea** In an ideal gas, for small heat change $dQ = du + dw$ and $\frac{C_p}{C_V} = \gamma$

For small change, $dQ = dU + dW$

$$nCdT = nC_V dT + 2nRdT$$

$\therefore$ $C = C_V + 2R$, $4R = C_V + 2R$

Given, molar heat capacity = $4R$

$\therefore$ $C_V = 2R$

Also, $C_V = \frac{R}{\gamma - 1}$

$\therefore$ $\frac{R}{\gamma - 1} = 2R$

$\Rightarrow$ $2\gamma - 2 = 1$

$\Rightarrow$ $\gamma = \frac{3}{2} = C_p/C_V$

TEST Edge Question involving concept of work done are also asked *e.g.*, work done during adiabatic change is given by

$$W = \frac{nR}{(\gamma-1)}(T_1 - T_2) = \frac{p_1V_1 - p_2V_2}{\gamma - 1} \text{ etc.}$$

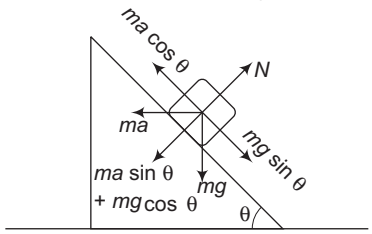
15. (*c*) $N = ma\sin\theta - mg\cos\theta$

$(ma\cos\theta - mg\sin\theta) = m \times a_{\text{upwards}}$

$ma\cos\theta - mg\sin\theta = mg\sin\theta$

$ma\cos\theta = 2mg\sin\theta$

$a = 2g\tan\theta$



16. (*c*) **Idea** Emf induced in a coil $e = -\frac{d(\phi)}{dt}$, ϕ is flux linked with the coil.

Induced Emf $= -\frac{d\phi}{dt}$

$$\text{Emf} = -[15t^2 - 20]$$

Emf at $t = 2$ s $= -[15\times 4 - 20] = -40$ V

TEST Edge Question related to induced emf can also be asked other ways, so student must know that emf can be induced by changing area of coil, magnetic flux of the coil and angle between area vector and magnetic field.

17. (*c*) **Idea** Rate of flow of heat is given by

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

As we know that, $\frac{dQ}{dt} = -KA\frac{dT}{dx}$

$$\frac{dQ}{dt} = -\frac{k_0 A}{a + bT}\frac{dT}{dx}$$

On integrating both sides within the proper limits.

$$\frac{dQ}{dt}\int_0^l dx = -k_0 A\int_{T_1}^{T_2}\frac{dT}{a + bT}$$

This gives $\frac{dQ}{dt} = \frac{Ak_0}{bl}\ln\left[\frac{a + bT_1}{a + bT_2}\right]$

TEST Edge Question related to equivalent thermal conductivity of two or more rods in series and parallel at various temperature can be asked.

In series equivalent conductivity is given by $K_{eq} = \frac{K_1K_2(L_1 + L_2)}{(L_1K_1 + L_2K_2)}$

In parallel equivalent conductivity is given by

$$K_{eq} = \left(\frac{K_1A_1 + K_2A_2}{A_1 + A_2}\right)$$

18. (*b*) Given, $x = \alpha t^3$, $y = \beta t^3$

Then, $v_x = \frac{dx}{dt} = 3\alpha t^2$

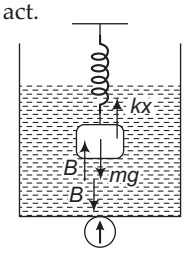
and $v_y = \frac{dy}{dt} = 3\beta t^2$

Resultant velocity

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2t^4 + 9\beta^2t^4}$$
$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

19. (*b*) $\oint H \cdot dl$ represents the energy lost as heat during the complete cycle of magnetization.

20. (*d*) **Idea** Here concept of force balance and Newton's third law is used also whenever a block is dipped in water force of buoyancy will act.



Here, $mg = B + kx$

where, B = reading of weighing machine = 20 N

$$kx = 20 \text{ N}$$

$\Rightarrow$ $$mg = 20 + 20$$
$$= 40 \text{ N}$$

TEST Edge Question based on Archimede's principal can also be asked which is when a solid body is wholly or partly immersed in a fluid, it experiences an upward thrust or buoyant force equal to the weight of the fluid displaced by it.

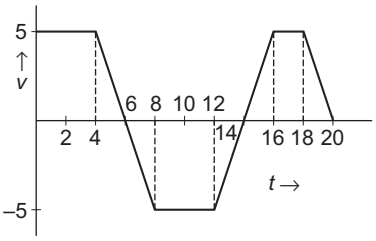
21. (*a*) **Idea** Area of velocity-time graph with sign gives displacement and without sign gives distance.

$$< v > = \frac{s}{\Delta t} = \frac{\text{Area of } v\text{-}t \text{ graph}}{\Delta t}$$

$$= \frac{5 \times 4 + \frac{1}{2} \times 5 \times 2 - \frac{1}{2} \times 5 \times 2 - 5 \times 4 - \frac{1}{2} \times 5 \times 2 + \frac{1}{2} \times 5 \times 2 + 5 \times 2 + \frac{1}{2} \times 5 \times 2}{20}$$

$$= \frac{15}{20} = \frac{3}{4} = \text{m/s}$$

(Here note that proper signs are taken)



$$< v > = \frac{\text{distance travelled}}{\Delta t}$$

$$= \frac{\text{Area of } s\text{-}t \text{ graph}}{\Delta t}$$

$$= \frac{5 \times 4 + \frac{1}{2} \times 5 \times 2 + \frac{1}{2} \times 5 \times 2 + 5 \times 4 + \frac{1}{2} \times 5 \times 2 + \frac{1}{2} \times 5 \times 2 + 5 \times 2 + \frac{1}{2} \times 5 \times 2}{20}$$

$$= \frac{70}{20} = \frac{15}{4} \text{ m/s}$$

TEST Edge Questions from kinematic including various graph like *v*-*t* graph, *a*-*t* graph etc., are frequently asked, student must know important concept related to it such as slope of displacement *versus* time gives instantaneous velocity and slope of velocity-time gives instantaneous acceleration etc.

22. (*a*) **Idea** Force on a charged particle in a uniform magnetic field is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{F})$$

Let, magnetic field is $\mathbf{B} = B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}}$

Applying $\mathbf{F_m} = q(\mathbf{v} \times \mathbf{B})$, we have

$$q[-\hat{\mathbf{j}} + \hat{\mathbf{k}}] = q[\hat{\mathbf{i}} \times (B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}})]$$

$\Rightarrow$ $$-\hat{\mathbf{j}} + \hat{\mathbf{k}} = B_2\hat{\mathbf{k}} - B_3\hat{\mathbf{j}}$$

By comparing, we have $B_2 = 1$ and $B_3 = 1$

Further $q[\hat{\mathbf{i}} - \hat{\mathbf{k}}] = q[\hat{\mathbf{j}} \times (B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}})]$

$$= q[-B_1\hat{\mathbf{k}} + B_3\hat{\mathbf{i}}]$$

Again by comparing, we have $B_1 = 1$ and $B_3 = 1$

$\therefore$ $$\mathbf{B} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \text{ Wb/m}^2$$

TEST Edge Question in which both electric field and magnetic field is applied on a charged particle in such case force on charged particle is given by

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

23. (*a*) Bulk modulus, $K = \dfrac{-\Delta p}{\Delta V/V}$

$$\Rightarrow \quad \Delta V = -\frac{\Delta pV}{K} = -\frac{2\times10^8\times V}{10^{10}} = \frac{-V}{50}$$

New volume of metal is $= V + \Delta V = \dfrac{49V}{50}$

As mass of metal will remain constant.

So, $\rho V = \rho' V'$

[where, ρ, V are its initial density and volume while ρ' and V' are density and volume after application of pressure]

$$\rho' = \frac{\rho V}{V'} = \frac{11\times V\times 50}{49\times V} = \frac{550}{49}\ \text{g/cm}^3$$

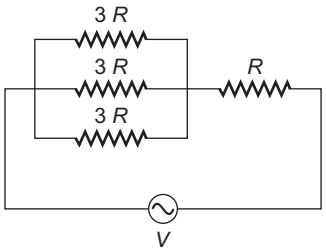
24. (*c*) Surface tension $= \dfrac{\text{Force}}{\text{Length}} = \dfrac{\text{MLT}^{-2}}{\text{L}} = [\text{MT}^{-2}]$

25. (*c*) **Idea** In an electrical circuit, $P = \dfrac{V^2}{R}$ also in series connection $P_1 : P_2 : P_3 = R_1 : R_2 : R_3$ and in parallel connection

$$P_1;\ P_2 :\ P_3 = \frac{1}{R_1} :\ \frac{1}{R_2} :\ \frac{1}{R_3}$$

Let resistance of 300 W bulb is R.

Hence $\dfrac{V^2}{R} = 300$ W then, as $R = \dfrac{V^2}{P}$



∴ Resistance of 100 W bulb should be $3R$.

So, we have

∴ Equivalent resistance is $R_{eq} = 2R$

Hence, $(\text{Power})_{eq} = \dfrac{V^2}{(2R)} = \dfrac{1}{2}\times\dfrac{V^2}{R} = \dfrac{300}{2} = 150$ W

TEST Edge Question related to brightness of bulb etc., can also be asked, so important concept such as in series connection, a bulb of less wattage will give more light than bulb of greater wattage.

26. (*d*) **Idea** Potential energy of satellite revolving around earth is negative and inversely proportional to radius of orbit also kinetic energy and total energy is positive and inversely proportional to radius of orbit.

$$\text{PE} = -\frac{GmM_e}{r},$$

$$\text{KE} = \frac{GmM_e}{2r_e}$$

$$\text{TE} = \text{KE} + \text{PE}$$

$$\text{TE} = \frac{-GmM_e}{2r}$$

So, as r increase

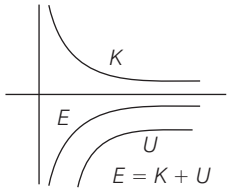
$\text{PE} \rightarrow$ increase

$\text{KE} \rightarrow$ decrease

$\text{TE} \rightarrow$ increase

TEST Edge Student must know important formulae for energy of satellite like $\text{TE} = \dfrac{-GmMe}{2\pi}$, $\text{TE} = \text{KE} + \text{PE}$ etc.

Graph related to energy of satellite can also be asked which is as follow.



27. (*a*) The decay constant for the first process is $\lambda_2 = \dfrac{\ln 2}{t_1}$ and for the second process it is $\lambda_2 = \dfrac{\ln 2}{t_1}$.

The probability that an active nucleus decay by the first process in a time interval dt is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ, the probability is also equal to $\lambda\, dt$. Thus,

$$\lambda dt = \lambda_1\, dt + \lambda_2 dt$$

or $\quad \lambda = \lambda_2 + \lambda_2 \ \Rightarrow\ \dfrac{1}{t} = \dfrac{1}{t_1} + \dfrac{1}{t_2}$

28. (*c*) **Idea** When two isolated capacitors having different charges are combined then

$$\begin{pmatrix}\text{charge lost by}\\ \text{one capacitor}\end{pmatrix} = \begin{pmatrix}\text{charge gained by}\\ \text{other capacitor}\end{pmatrix}$$

and charge will arrange in such a way that they reach common potential also energy is lost during process.

Initial energy of the system

$$U_i = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2$$

When the capacitors are joined, common potential,

$$V = \frac{CV_1 + CV_2}{2C} = \frac{V_1 + V_2}{2}$$

Final energy of the system,

$$U_f = \frac{1}{2}(2C)V^2$$

$$= \frac{1}{2} 2C \left(\frac{V_1 + V_2}{2}\right)^2$$
$$= \frac{1}{4} C (V_1 + V_2)^2$$

Decreasing in energy $= U_i - U_f = \frac{1}{4} C (V_1 - V_2)^2$

TEST Edge Question related to change in potential charge and energy can also be asked in which battery may also be connected with the circuit. Important relations like $q = CV$, $E = \frac{1}{2} CV^2$ etc., must be memorised to solve such relations.

29. (*b*) Both Statement I and Statement II are correct but Statement II is not explanation of the Statement I.

30. (*c*) **Idea** Here concept of radioactive reactions and basic concept of probability is involved. Student must know that nuclear disintegration is not affected by physical condition and nearby nucleus

Radioactivity is an independent activity thus does not depend upon the quantity of substance remaining. A nuclei can disintegrate at any moment irrespective of what is happening to its neighbouring atoms/nuclei. So the probability for a particular nuclei to disintegrate in half life time will always be 1/ 2 also quantity of substance after *n* half life is given by

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \begin{bmatrix} N, \text{ final quantity} \\ N_0, \text{ initial quantity} \\ n, \text{ number of half-life} \end{bmatrix}$$

So, after 4 half-life $N = \frac{N_0}{16}$

TEST Edge Radioactive disintegration is not affected by changing physical condition like temperature, pressure etc., student must know about relation between half-life and concentration, questions involving these concepts are frequently asked.

Chemistry

31. (*b*) **Idea** First of all calculate the number of moles of H_2O present then calculate number of neutrons present in O and H followed by number of neutrons present on H_2O. Finally, multiply these two results and get the final answer.

Number of moles of $H_2O(l) = \frac{108}{18} = 6$

(density = 1.0 g/mL of H_2O)

H has no neutron.

Number of neutrons in $H_2O = 6 \times 8 \times 6 \times 10^{23}$

$= 48 N_A$

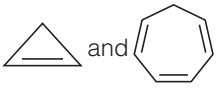
Number of neutrons in O = 16 – 8 = 8

TEST Edge By solving this question, you will be able to calculate numbers of moles present on any element, ion or molecule present in any solution.

32. (*a*) **Idea** This problem includes conceptual mixing of acidic character, aromaticity and nucleophilic substitution reaction.

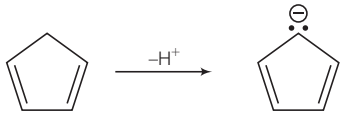
Students are advised to identify the most stable intermediate obtained among all (after the removal of H^+) keeping in mind the concept of conjugation and aromaticity. Then, complete the reaction further using concept of nucleophilic substitution reaction.

Acidic character The species which easily donate its hydrogen and produces stable conjugate base is acid. The species which produces more stabler conjugate base is more stronger acid.

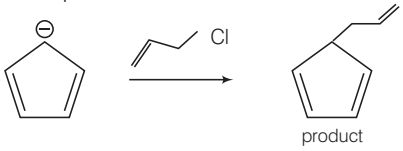

and do not looses H^+ hence are not


acidic. looses the H^+ easily and produces

more stabler aromatic cyclopentadienyl anion.



Now, cyclopentadienyl anion on reaction with 3-chloro prop1-ene produces the product *via* nucleophilic substitution reaction.

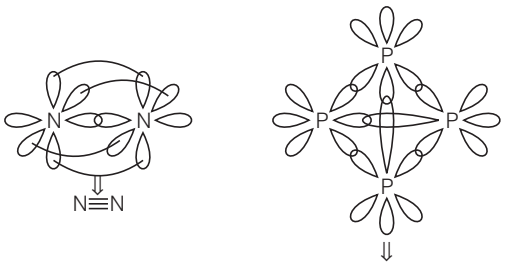


TEST Edge Generally, in JEE Main the problems related to conjugation, aromaticity and nucleophilic substitution reaction are asked frequently. hence, students are advised to understand the concept of aromaticity, conjugation and various chemical reactions of aromatic and aliphatic intermediates such as cation, anion, radicals etc.

33. (*b*) **Idea** This problem includes conceptual mixing of existance of phosphorous and reason of their existance. Students are advised to go through the concept of $p_\pi - p_\pi$ bonding and characteristics of element to show $p_\pi - p_\pi$ bonding to a greater extent than 3rd period element.

Existance of N as N_2 is due to strong $p_\pi - p_\pi$ bonding between smaller sized *p*-orbitals of N.

Existance of phosphorous as P is due to existance of weak $p_\pi - p_\pi$ bonding due to large size of *p*-orbital of phosphorous atom.



Discrete unit of P_4 due to large size of *p*-orbital of phosphorous, it show.

TEST Edge In JEE Main, the questions related to the concept of back bonding are asked frequently so students are advised to go through the study of condition of happening back bonding.

Lower the difference between size of atomic orbitals undergoing back bonding greater will be extent of overlapping between those orbitals.

For *e.g.,* 2nd period elements such as oxygen, nitrogen, contain double and tripple bond respectively while S and P form single bond with itself.

34. (*c*) Radial nodes occurs where probability of finding electron is zero

$$\psi^2 = 0 \text{ or } \psi = 0$$
$$\sigma^2 - 5\sigma + 6 = 0$$
$$\sigma^2 - 3\sigma - 2\sigma + 6 = 0$$
$$\sigma(\sigma - 3) - 2(\sigma - 3) = 0$$
$$\sigma = 2 \text{ or } 3$$

For maximum distance $\sigma = 3$

$$3 = \frac{2}{3}\frac{rZ}{a_0}$$
$$r = \frac{9a_0}{2}Z = \frac{9Za_0}{2}$$

35. (*a*) **Idea** This problem includes conceptual mixing of determination of molecular formula and their chemical reaction. Students are advised to calculate the simplest ratio of number of atoms present in molecule/compound then to identify the possible molecular formula of compound keeping in mind the types of products given in option and simplest ratio of atoms.

Symbol	% age	At mass	Relative number of atom	Simplest ratio
C	80	12	$\frac{80}{12} \approx 7.0$	$\frac{7}{1} = 1$
H	20	1	$\frac{20}{1} = 20$	$\frac{20}{7} \approx 3$

Empirical formula = CH_3; Molecular formula = C_2H_6

Reaction of ethane When ethane is treated with Cl_2 in sunlight it produces CH_3CH_2Cl

$$CH_3CH_3 \xrightarrow{Cl_2/h\nu} CH_3CH_2Cl$$

TEST Edge In JEE Main, the questions related to conceptual mixing of molecular formula determination and chemical formula of compound are asked very frequently, so students are advised to go through calculation of molecular formula determination and various chemical properties of organic compounds.

36. (*c*) **Idea** Students are recommended to see the product given in option and think that what is the possibility of starting material according to information provided in the question and chemical properties of starting material.

As the most occurring element in the earth crust is silicon as SiO_2 and the chemical reaction of SiO_2 with carbon produces CO which is a poisonous gas as shown below

$$SiO_2 + 2C \longrightarrow Si + 2\underset{\substack{\uparrow \\ \text{Poisonous gas and stable} \\ \text{diatomic molecule}}}{CO}$$

TEST Edge Generally, in JEE Main these types of questions are based on the concept of occurrence of element and their chemical properties are asked therefore students are advised to go through study of occurrence of (such as Al, Sn, Cu etc.) and their chemical properties.

37. (*a*) **Idea** This problem involves conceptual mixing of structure, bond angle, dipole moment, charge on water molecule. To solve this type of problem student should determine the structure and bond angle of molecule. Now use the simple triangle law to calculate the bond length then calculate the charge on molecule using formula

$$\mu = e \times l$$

where, μ = dipole moment

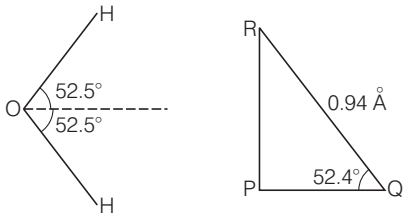
e = charge on molecule, l = bond length.

Structure of water molecule Structure of water molecule can be determined as

$$H = \frac{V + M - C + A}{2} = \frac{6 + 2}{2} = 4$$

Hybridisation = sp^3

Due to lone pair-lone pair repulsion the bond angle decreases to 105°.



From the value of bond angle and vector moment, e can be calculated as

$$PQ = QR \cos 52.5°$$
$$= 0.94 \times 0.605 = 0.572 \text{ Å}$$
$$= 0.572 \times 10^{-8} \text{ cm}$$

We know that $\mu = e \times l = e \times AB$

$$e = \frac{\mu}{AB} = \frac{1.85 \times 10^{-18} \text{ esu cm}}{0.572 \times 10^{-8} \text{ cm}}$$

$$e = 3.23 \times 10^{-10} \text{ esu}$$

TEST Edge Questions related to dipole moment, charge and bond length are asked very frequently in JEE Main. The problems on properties and application of dipole moment will also be asked therefore students are recommended to go through study of these concepts.

38. (*b*) **Idea** This problem includes conceptual mixing of type of isomerism shown by coordination compound and their molar conductivity.

- Identify the types of isomerism in coordination compound.
- Determine the number of ions produced by coordination compound in the aqueous solution.
- Now use the concept of electrochemistry to solve the problem.

Greater the charge on ions produced by coordination compound greater will be its molar conductivities. Molecule having different charge on ions have different molar conductivity.

$$[Co(NH_3)_5Br]\,SO_4 \longrightarrow [Co(NH_3)_5Br]^{++}_{+}\; SO_4^{2--}$$

$$[Co(NH_3)_5\; SO_4]Br \longrightarrow [Co(NH_3)_5SO_4]^{+} + Br^{-}$$

TEST Edge In JEE Main, problem related to isomerism in coordination compound and conductivity in coordination compound both are asked independently as well as combinely sometimes questions having only molecular composition is given and molar conductivity is also asked so student should deep study these topic by relating these concepts.

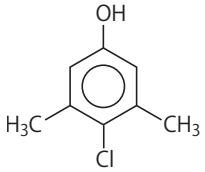
39. (*c*) n-factor $= \frac{4 \times 2}{4 + 2} = \frac{4}{3}$

$$H_3PO_2 \longrightarrow \underset{\substack{\text{Oxidation number}\\ \text{of P} = -3}}{PH_3} + \underset{\substack{\text{Oxidation number}\\ \text{of P} = +2}}{H_3PO_3}$$

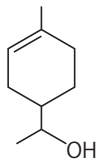
So, equivalent weight $= \frac{\text{mol weight}}{n\text{-factor}} = \frac{M}{\frac{4}{3}} = \frac{3M}{4}$

40. (*b*) This problem involves properties of dettol as antiseptic disinfectant and its chemical constitution.

Antiseptic and Disinfectant Dettol is a most commonly used antiseptic which is a mixture of 4, chloro, 3, 5, dimethyl phenol *i.e.*, chloroxylenol and terpineol. It is chloroxylenol and terpineol. It is chloroxylenol which is responsible for its antiseptic and disinfectant properties.



Chloroxylenol



α-terpineol

41. (*c*) **Idea** This problem is based on conceptual mixing of preparation physical and chemical properties of various inorganic compound. Try to find the exact relation between compound and information regarding compounds keep a clear idea in your mind regarding concept of preparation and properties of inorganic compound.

Exact relation can be determined by using the information given in both column one by one.

(i) Fehling solution :

$$CuSO_4 + NaK \longrightarrow C_4H_4O_6 + NaOH$$
(Rochell's salt)

(ii) K_2CO_3 (m. pt) = 850° C

(iii) $FeCl_3 + 2S_2O_3^{--} \longrightarrow \underset{\text{violet solution}}{[Fe(S_2O_3)_2]^{-}} + 3Cl^{-}$

$$[Fe(S_2O_3)_2]^{-} + Fe^{3+} \longrightarrow \underset{\text{(green)}}{2Fe^{2+}} + S_4O_6^{2-}$$

(iv) $\underset{\text{(orange)}}{Cr_2O_7^{2-}} + 8H^{+} + 3SO_3^{--} \longrightarrow \underset{\text{(green)}}{2Cr^{3+}} + 3SO_4^{--} + 4H_2O$

(v) Na_2CO_3 : Solvay process.

TEST Edge This type of question is asked in JEE Main to know the clear concept of students regarding preparation and properties of compounds, so the students are advised to go through deep and clear students study of preparation and properties of compound.

42. (*b*) Initial moles of $Cu^{2+} = 500 \times 0.02 = 10$

mole equivalents or milli moles of H^+ produced

$$= 500 \times 10^{-3} = 2 \times 10^{-1} = 0.5 \times 2 = 1.0$$

milli moles of Cu^{2+} converted into $Cu = 1/2 = 0.5$

milli moles of Cu^{2+} remaining in solution

$$= 10 - 0.5 = 9.5$$

$$2\,Cu^{2+} + 4I^- \longrightarrow Cu_2I_2 + I_2$$

and $I_2 + 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$

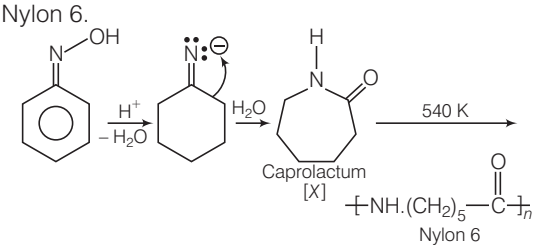
milli moles of Cu^{2+} remaining

= millimoles of $Na_2S_2O_3$

$$9.5 = 0.4 \times V \text{ or } V = 237.5\,L$$

43. (*d*) This problem involves conceptual mixing of Beckman arrangement and polymerisation.

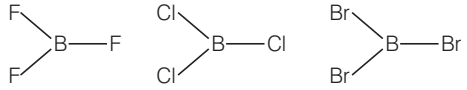
Beckman rearrangement The acid catalysed conversion of N. hydroxyl oxime to N substituted amine is known as Beckman rearrangement. The chemical sequence of the reaction is as follows in which [*X*] is caprolactum which polymerises to Nylon 6.



44. (*a*) Mass of acetic acid adsorbed by 2 g charcoal $= 100 \times 10^{-3} \times (0.5 - 0.4) \times 60 = 0.6$ (molecular wieght of $CH_3COOH = 60$) $\frac{x}{m} = \frac{0.6}{2} = 0.3$

45. (*d*) This problem involves conceptual mixing of structure of compound. As, trihalides of boron have same structure due to same value of H. (hybrid orbitals) $H = \frac{V + M - C + A}{2} = \frac{3+3}{2} = 3$

Hybridisation = sp^2

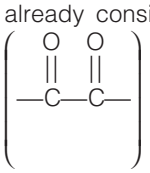


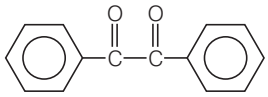
46. (*b*) **Idea** This problem include conceptual mixing of molecular structure determination and benzilic acid rearrangement. *This problem can be solved by using following sequential step.*

- Calculate the degree of unsaturation and then determine the appropriate molecular structure.
- Complete the reaction using the concept of benzilic acid rearrangement in which diketone undergo benzilic acid rearrangement in presence of base to produce corresponding benzilic acid.

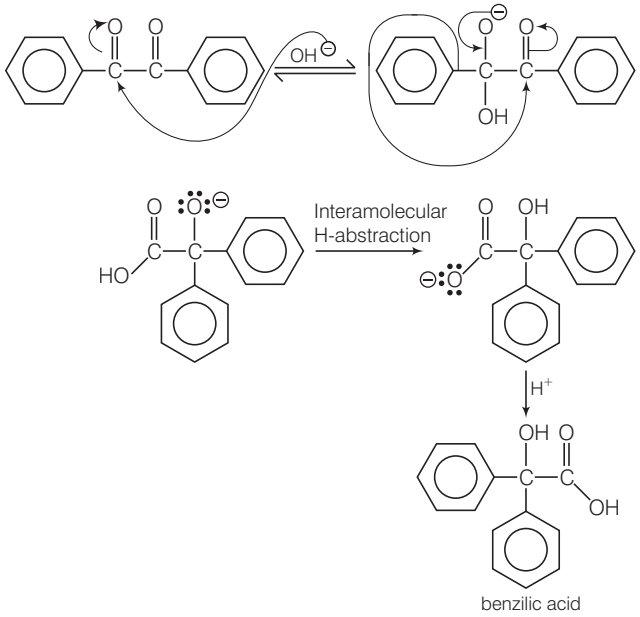
Molecular structure determination Molecular structure of compound having molecular formula $C_{14}H_{10}O_2$ is determined by calculating degree of unsaturation

$$u = (C + 1) - \frac{H}{2} + \frac{N}{2} = (14 + 1) - \frac{10}{2} = 15 - 5 = 10$$

degree of unsaturation is 10 in which 2 units are already considered to be used as diketo group  . Rest 8 unit of unsaturation may be satisfied by two phenyl ring each having $u = 4$. Hence correct structure may be



Benzilic acid rearrangement Conversion of benzil to benzilic acid in presence of base is known as benzilic acid rearrangement in general benzilic acid is α hydroxy carboxylic acid the reaction is believed to occur as



TEST Edge JEE Main examination include this type of question to judge the knowledge of student in rearrangement reaction of ketone which are asked generally. Therefore, students are advised to go through study of rearrangement reaction of carbonyl compound such as Beckman rearrangement, Pinacol-Pinacolone rearrangement etc.

47. (*d*) **Idea** This problem is based on rate law expression and rate constant of 1st order gaseous reaction. *This problem can be solved using following steps*

- Write the chemical reaction
- Calculate the total pressure, initial pressure and final pressure.
- Now, calculate the value of rate constant using equation of 1st order rate constant *i.e.,*

$$k = \frac{2.303}{t}\log\left(\frac{p_0}{p_0 - x}\right)$$

where, p_0 = initial pressure

$p_0 - x$ = final pressure.

Let initial pressure = p_0

$$A(g) \longrightarrow 2B(g) + C(g)$$

After 10 mins $p_0 - x \quad 2x \quad x$

At long time , $t \to \infty \quad 0 \quad 2p_0 \quad p_0$

As given, $p_0 - x + 2x + x$ = vapour pressure of water = 200

$$p_0 + 2x + 20 = 200$$

$$p_0 + 2x = 180$$

and $2p_0 + p_0 + 20 = 380$

$$3p_0 = 360$$

$$p_0 = 120 \text{ torr}$$

$$120 + 2x = 180$$

$$2x = 60$$

$$x = 30 \text{ torr}$$

$$k = \frac{1}{t}\ln\left(\frac{p_0}{p_0 - x}\right) = \frac{1}{20}\ln\left(\frac{120}{90}\right)$$

$$k = \frac{1}{20}(\ln 12 - \ln 9)$$

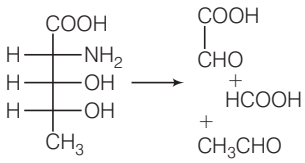
$$= \frac{1}{20}(\ln 4 + \ln 3 - 2\ln 3)$$

$$= \frac{1}{20}(2\ln 2 - \ln 3) = 0.006 \text{ min}^{-1}$$

TEST Edge In JEE Main, these types of problems are included to judge the quantitative and theoretical knowledge of student about concern topic, so students are advised to practice more and more in determination of value of rate constant. The question relating half-life time and quarter life time may also be asked.

48. (*a*) This problem includes conceptual mixing of malaprade oxidation of aminohydroxyl compound.

Malaprade oxidation Amino alcohol compound having amino and hydroxyl compound are adjacent to each other undergo cleavage to give fragment product in a same way as in diol.



49. (*b*) **Idea** This problem includes conceptual mixing of chemical reactions of aqueous solution of transition metal ion (Co^{3+}) and colour of metal ion. Students are advised to determine the oxidation state of transition metal ion first followed by calculation of number of *d* electrons present in metal ion.

When aqueous solution of NaOH is added to aqueous solution of Cr (III) ion it produces $[Cr(H_2O)_3(OH)_3]^+$

$$[Cr(H_2O)_6]^{3+} + NaOH \longrightarrow \underset{\text{Light green ppt.}}{[Cr(OH)_3(H_2O)_3]} + H_2O$$

Which on further redissolves in *aq.* NaOH(in excess) to produce dark green solution due to formation of $[Cr(OH)_4]^-$

$$[Cr(OH)_3(H_2O)_3] + \underset{\text{Excess}}{OH^{\ominus}} \longrightarrow \underset{\text{Dark green solution}}{[Cr(OH)_4]^-}$$

TEST Edge The reactions of transition metal ions with common reagents such as NaOH, Na_2CO_3, HCl, H_2SO_4 etc., are asked in JEE Main very frequently, so students are advised to study the chemical reaction of various transition metal ions, such as $Fe^{2+}, Co^{3+}, Mn^{2+}, Cr^{6+}$ etc.

50. (*d*)

$$SO_2(g) + NO_2(g) \rightleftharpoons SO_3(g) + NO(g)$$

at $t = 0$	2	2	2	2
at equilibrium	$2 - x$	$2 - x$	$2 + x$	$2 + x$

Total number of moles of gases at equilibrium,

$$(2 - x) + (2 - x) + (2 + x) + (2 + x) + 2 \text{ (inert gas)}$$

$$= 8 + 2$$

$$= 10$$

$$k_p = \frac{p_{SO_3} \cdot p_{NO}}{p_{SO_2} \cdot p_{NO_2}}$$

$$25 = \frac{\left(\frac{2+x}{10}p\right)^2}{\left(\frac{2-x}{10}p\right)^2}$$

$$\Rightarrow \quad 5 = \frac{2+x}{2-x}, \; x = \frac{4}{3}$$

$$p_{SO_2} = \frac{2-x}{10} \times p_{Total} = \frac{2 - \frac{4}{3}}{10} \times 4$$

$$= \frac{8}{30} = 0.27 \text{ atm}$$

51. (*d*) **Idea** This problem is based on the various chemical reactions occuring in the Bessemer's converter. Students are advised to stick with concept of purification of metals. To solve this problem go through the reactions occuring in Bessemer process.

(a) $FeO + SiO_2 \xrightarrow[\text{flux}]{} \underset{\text{slag}}{FeSiO_3}$

This reaction shows slag formation reaction in which acidic flux SiO_2 is added to FeO which produces slag.

(b) $2\,Cu_2S + 3\,O_2 \longrightarrow 2\,Cu_2O + 2\,SO_2$

This reaction shows conversion of copper sulphide into copper oxide with evolution of SO_2 gas.

(c) $2\,Cu_2S + 5O_2 \longrightarrow 2\,CuSO_4 + 2\,CuO$

This reaction shows conversion of copper sulphide into copper sulphate which occurs in Bessemer's converter.

(d) $2\,CuFeS_2 + O_2 \longrightarrow Cu_2S + 2\,FeS + SO_2$

This process shows oxidation of impurity, which don't occur in Bessemer's converter.

TEST Edge In JEE Main, questions related to chemical reaction involved in purification of metal students are advised to go through study of various reactions involved in extraction and purification of metals.

52. (*b*) **Idea** This problem includes conceptual mixing of preparation of chloroform, chemical property of chloroform and use of compounds of $CHCl_3$. This type of trend related problem is sticked with preparation and properties of chloroform. This problem can be solved by completing the sequence of reaction. The student must have the knowledge regarding use of compound prepared.

Preparation of chloroform From bleaching powder chloroform are prepared by its reaction with acetone

$$CaOCl_2 + H_2O \longrightarrow Ca(OH)_2 + Cl_2$$

$$CH_3{-}\underset{\underset{O}{\|}}{C}{-}CH_3 + Ca(OH)_2 + Cl_2 \longrightarrow HCCl_3$$

Chemical properties of $CHCl_3$ $CHCl_3$ on reaction with nitric acid it produces chloropicrin which is used as an insecticide.

$$CHCl_3 + HNO_2 \longrightarrow \underset{\text{Chloropicrin}}{CCl_3 - NO_2}$$

TEST Edge This type of problem is asked in JEE Main examination to judge the knowledge of use of compound prepared by chemical reaction of chloroform property and uses of DDT, Freon, etc., may also be asked very frequently, so students are advised to study these topics.

53. (*c*) $I_2 + 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$ …(i)

milli moles of $Na_2S_2O_3$ consumed

$= 30 \times 0.2$

$= 60$ milli moles

milli moles of I_2 consumed $= \frac{60}{2} = 30$ milli moles

$3I_2 + 6NaOH \longrightarrow 5NaI + NaIO_3 + 3H_2O$ …(ii)

milli moles of I_2 reacted with NaOH,

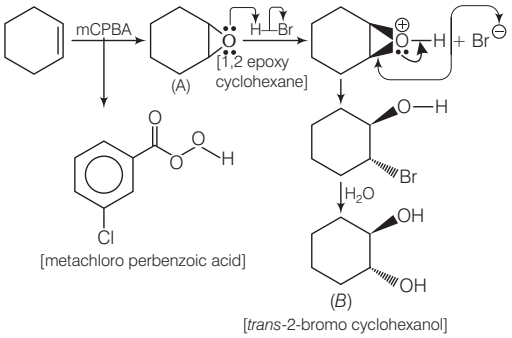
$$\frac{50 \times 0.6}{2} = \frac{30}{2} = 15 \text{ milli moles}$$

Total milli moles of I_2 consumed in reaction (i) and (ii) $= 15 + 30 = 45$ milli moles

molarity of $I_2 = \frac{45}{200} = 0.225$ M

54. (*a*) This problem includes conceptual mixing of epoxidation, ring opening and nomenclature.

Epoxidation When an organic compound containing double bond undergo reaction with per acid such as mCPBA it produces an epoxide. The first step of above reaction complete as follows.



55. (*b*) **Idea** This problem contain conceptual mixing of molecular orbital electronic configuration and magnetic properties of molecules or ions. The student is advised to stick with concept of molecular orbital electronic configuration, magnetic properties of diatomic molecule. *This problem can be solved by following sequential step*

- Write molecular orbital electronic configuration.
- Count then number of unpaired electrons. If molecule contain unpaired electron then it is paramagnetic.

Paramagnetic character of molecule or ions Molecules or ions which contains at least one unpaired electron in molecular orbital of molecule or ion shows paramagnetic character.

Molecular orbital electronic configuration (MOEC) of $B_2 = \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2p_x}^1 \equiv \pi_{2p_y}^1$

Unpaired electron = 2

Hence, B_2 is paramagnetic.

MOEC of $O_2 = \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p_z}^2$

$\pi_{2p_x}^2 \equiv \pi_{2p_y}^2\ \pi_{2p_y}^{*1} \equiv \pi_{2p_y}^{*1}$

Unpaired electron = 2

Hence, O_2 is paramagnetic.

MOEC of NO $= \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}$

$\sigma_{2p_z}^2\ \pi_{2p_x}^2 \equiv \pi_{2p_x}^{*1} \equiv \pi^*_{2p_y}$

$$\text{BO} = \frac{10-5}{2} = 2.5$$

Unpaired electron = 1

Hence, O_2 is also paramagnetic

MOEC of $N_2 = \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2\sigma}^{*2}$

$\pi_{2p_x}^2 \equiv \pi_{2p_y}^2 \sigma_{2p_z}^2$

$$\text{BO} = \frac{10-4}{2} = 3$$

Unpaired electron = 0

Molecule is diamagnetic

MOEC of $F_2 = \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{3s}^2\ \sigma_{2p_s}^{*2}\ \sigma_{2p_z}^2\ \pi_{2p_x}^2 \equiv \pi_{2py}^2$

$\pi_{2p_x}^2 \equiv \pi_{2p_y}^2$

Unpaired electron = 0

Molecule is diamagnetic.

MOEC of $Li_2 = \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^{*2}\ \pi_{2p_y}^2 \equiv \pi_{2p_y}^2$

Unpaired electron = 0

Molecule is diagmagnetic.

Hence, correct set of paramagnetic molecules are represented by (*b*).

TEST Edge These types of questions are commonly asked problem, students are recommended to go through study of magnetic property of diatomic molecule and molecular orbital electronic configuration of elements problems related to extent of paramagnetism can also be asked.

56. (*a*) **Idea** This problem includes conceptual mixing of acidic strength and inductive effect. To solve this problem identify the group attached to give system and then the position at which groups are attached then notice the effect of group in parent system (benzoic acid) here. Now, choose the correct choice.

Inductive effect The pull or push of electron density of any bond pair can be quantitatively expressed in term of inductive effect. *There are two types of inductive effect*

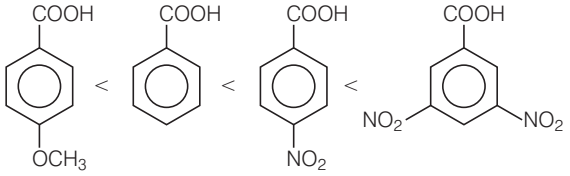
1. *+ I effect* It increases electron density towards most electronegative effect.

e.g., CH_3, OCH_3, Butyl. etc.

2. *– I effect* It decreases electron density towards most electropositive atom. *e.g.*, NO_2 , CHO etc.

OCH_3 is an electron pushing group hence increases electron density towards ring and causes decrease in acidic strength in comparison to benzoic acid.

NO_2 is an electron pulling group hence decreases electron density from ring and hence increases acidic strength in comparison to benzoic acid. Hence correct order is shown in choice (*a*).



acidity increases

TEST Edge In JEE Main, questions related to acidic strength order and basic strength order are asked very frequently, therefore students are recommended to understand the concept of inductive effect, resonance effect in acidic and basic strength of species.

57. (*d*) **Idea** This problem includes concept of Raoult's law. Students should understand the theory and numerical approach of Raoult's law to solve this type of problem using following steps.

- Write the data given in the question and then asked to answer in the question.
- Write the formula by which problem is solved.
- Calculate the required parameter using formula and must keep the accuracy of solution in the mind.

Let, n_B mole of *B* present in 1 mole of mixture that has been vapourised thus $Y_B = \frac{n_B}{1} X_B$ of *B* remains in liquid phase will be $X_B = \frac{1-n_B}{1}$

$$X_B = \frac{p - p_T^{\,0}}{p_B^0 - p_T^{\,0}} \quad \ldots(i)$$

$$\{p = p_T^0 + (p_B^0 - p_T^0)\, X_B\}$$

and $$Y_B = \frac{p_B}{p} = \frac{p_B^0\, X_B}{p} \quad \ldots(ii)$$

Putting X_B and Y_B in Eq. (i),

$$1 - n_B = \frac{p - p_T^0}{p_B^0 - p_T^0} \quad \ldots(iii)$$

$$n_B = \frac{(1-n_B)\, p_B^0}{p} \quad \ldots(iv)$$

or $$n_B = \frac{p_B^0}{p + p_B}$$

$$1-\frac{p_B^0}{p+p_B}=\frac{p-p_T^0}{p_B^0-p_T^0}$$

$$p=\sqrt{p_B^0\,p_T^0}=\sqrt{100\times400}$$

$$=200 \quad \text{torr}$$

TEST Edge Questions relating partial pressure and mole fraction are generally asked in JEE Main. Students are advised to understand the concept of colligative properties such as elevation in boiling point, depression in freezing point etc., and their applications.

58. (*c*) This problem contain conceptual mixing of Lewis acidity and back bond.

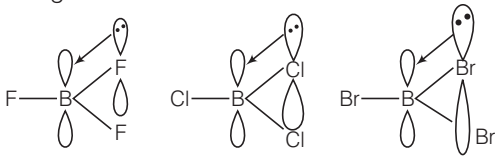
Transfer of electron from filled orbital of one atom to vacant orbital of another atom is termed as π back bonding. The back bonding are of three types.

(a) $p_\pi - p_\pi$ back bonding

(b) $p_\pi - d_\pi$ back bonding

(c) $d_\pi - d_\pi$ back bonding

In BF_3, BCl_3 and BBr_3 each will shown $p_\pi - p_\pi$ back bonding as follows



difference between size of overlapping p orbital increases

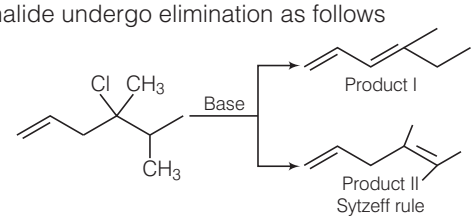
⟶

extent of π backbonding decreases

As extent of $p_\pi - p_\pi$ back bonding increases Lewis acidity decreases. Hence correct order of Lewis acidity is represented by option (c).

59. (*b*) This problem includes conceptual mixing of elimination reaction, Sytzeff rule and conjugation.

Elimination reaction In presence of base alkyl halide undergo elimination as follows



Product I is obtained according to conjugation which deal about stability of product to a greater extent than product II obtained according to inductive effect only.

Hence, correct Statement II is due to conjugation.

60. (*a*) Solubility is governed by lattice energy, hydration energy. Lower will be lattice energy more is the solubility but more will be lattice energy less will be solubility.

Mathematics

61. (*b*) **Idea** ∵ If $z = x + iy$, then $\bar{z} = x - iy$ and $i^2 = -1$

$$i = 0 + 1\cdot i$$

$$\Rightarrow \quad |i| = |0 + 1\cdot i| = \sqrt{0+1} = 1$$

$$\Rightarrow \quad |i| = 1$$

We have given that

$$|z^2 - (z)^2| = |z|^2$$

Let, $z = x + iy$

$$|(x+iy)^2 - (x-iy)^2| = x^2 + y^2$$

$$|x^2 - y^2 + 2ixy - x^2 + y^2 + 2ixy| = x^2 + y^2$$

$$|4xy| = x^2 + y^2 \qquad [\because |i| = 1]$$

$\Rightarrow$ $x^2 + y^2 = 4xy$, pair of straight line.

TEST Edge Generally, in JEE Main properties of complex number as modulus of complex number based question, students are advised to learn the properties of complex number.

62. (*c*)

$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

Above equation represent a system of three plane.

∵ Plane may have unique or infinite solution. So it can not have two solution.

63. (*c*) **Idea** ∵ $y^2 = 4ax$; equation of normal at $(at^2, 2at)$ is $y + tx = 2at + at^3$

Here, equation of the normal chord at any point $(at^2, 2at)$ of the parabola is

$$y + tx = 2at + at^3 \qquad \ldots(i)$$

Equation of the chord with mid point (x_1, y_1) is $T = S_1$

$$yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$$

$$yy_1 - 2ax = y_1^2 - 2ax_1 \qquad \ldots(ii)$$

Since, Eq. (i) and (ii) are identical

$$\frac{1}{y_1}=\frac{t}{-2a}=\frac{2at+at^3}{y_1^2-2ax_1}$$

$$t=\frac{-2a}{y_1} \quad \text{and} \quad \frac{y_1^2-2ax_1}{-2a}=\frac{2at+at^3}{t}$$

$$=2a+a\left(\frac{-2a}{y_1}\right)^2$$

or $$\frac{-y_1^2}{2a}+x_1=2a+\frac{4a^3}{y_1^2} \Rightarrow x_1-2a=\frac{y_1^2}{2a}+\frac{4a^3}{y_1^2}$$

Hence, the locus of the middle point (x_1, y_1) is

$$x-2a=\frac{y^2}{2a}+\frac{4a^3}{y^2}$$

TEST Edge Locus of the given point in different way on normal of the parabola related questions are asked in JEE Main. To understand the basic concept of normal of parabola in different cases.

64. (*a*) Given that $\frac{2+\sin x}{(y+1)}\cdot\frac{dy}{dx}=-\cos x$

By variable separation, to separate variable x and y, we get

$$\frac{dy}{y+1}=\frac{-\cos x}{2+\sin x}\cdot dx$$

$$\int\frac{dy}{y+1}=\int\frac{-\cos x}{2+\sin x}dx$$

$$\log(y+1)=-\log(2+\sin x)+\log C$$

$$y+1=\frac{C}{2+\sin x}$$

$$y(0)=1\Rightarrow C=4$$

$$y=-1+\frac{4}{2+\sin x}$$

$$y\left(\frac{\pi}{2}\right)=-1+\frac{4}{2+\sin\frac{\pi}{2}}=-1+\frac{4}{3}=\frac{1}{3}$$

65. (*b*) **Idea** If a,b,c are in HP.

Then, $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in AP. Students are also use $\log_a^b=\frac{\log_e^b}{\log_e^a}$

We have given that,

$\log_a 100, 2\log_b 10, 2\log_c 5+\log_c 4$ are in HP

$$\therefore\ \frac{1}{\log_a 100},\frac{1}{\log_b 100},\frac{1}{\log_c 25+\log_c 4}\text{ are in AP}$$

$$\frac{1}{\log_a 100},\frac{1}{\log_b 100},\frac{1}{\log_c 100}\text{ are in AP}$$

$$\therefore\quad\frac{2}{\log_b 100}=\frac{1}{\log_a 100}+\frac{1}{\log_c 100}$$

$$\frac{2\log_e^b 100}{\log_e 100}=\frac{\log_e a}{\log_e 100}+\frac{\log_e c}{\log_e 100}$$

$$2\log_e b=\log_e a+\log_e c$$

$$2\log_e b=\log_e(ac)\ \Rightarrow\ b^2=ac$$

which implies a, b and c are in GP

TEST Edge In JEE Main, given terms are in AP, HP and GP related questions are asked. Students are advised to solve these types of questions to understand the relation between AP, GP and HP and also acquainted yourself with properties of log.

66. (*b*) Let, A and B are the events that card lost is spade and card drawn is spade.

$$P(A)=\frac{1}{4},P(\bar{A})=\frac{3}{4}$$

$$P(B/A)=\frac{12}{51},P(B/\bar{A})=\frac{13}{51}$$

$$P(A/B)=\frac{P(A).P(B/A)}{P(A)\cdot P(B/A)+P(\bar{A})P(B/\bar{A})}$$

$$=\frac{\frac{1}{4}\cdot\frac{12}{51}}{\frac{1}{4}\cdot\frac{12}{51}+\frac{3}{4}\cdot\frac{13}{51}}=\frac{12}{51}=\frac{4}{17}$$

67. (*b*) **Idea** Here $\therefore\int f(x)dx=g(x)$

Differentiating w.r.t. x, we get $f(x)=\frac{d}{dx}g(x)$

and $\frac{d}{dx}\log x=\frac{1}{x}$

We have given that,

$$\text{If }=\int\frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}dx$$

$$=A\ln|\cos x+\sin x-2|+Bx+C$$

$$\frac{d}{dx}(A\ln|\cos x+\sin x-2|+Bx+C)$$

$$=A\frac{(\cos x-\sin x)}{\cos x+\sin-2}+B$$

$$=\frac{A\cos x-A\sin x+B\cos x+B\sin x-2B}{\cos x+\sin x-2}$$

$$\therefore\frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}$$

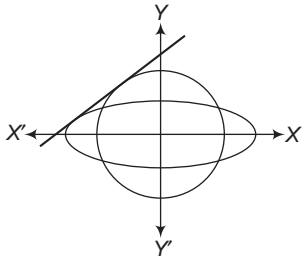
$$=\frac{(A+B)\cos x+(B-A)\sin x-2B}{\cos x+\sin x-2}$$

$$A+B=2,B-A=-1,\lambda=-2B$$

$$\Rightarrow\qquad A=\frac{3}{2},B=\frac{1}{2},\lambda=-1$$

TEST Edge Integration trigonometric function, polynomial function based question are asked. To understand the relation between integration and differentiation and also learn the formulae of integration.

68. (*a*) Let, equation of circle is $x^2+y^2=r^2$



Tangent to ellipse is $y=mx+\sqrt{a^2m^2+b^2}$

If it is a tangent to the circle, then it is perpendicular from (0,0) is equal to radius,

$$\therefore\qquad\frac{\sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}=r$$

$$\Rightarrow\qquad m=\frac{\sqrt{r^2-b^2}}{\sqrt{a^2-r^2}}\ \Rightarrow\ \theta=\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$$

69. (*c*) Let, P denotes the families who own a phone and C denotes the families who own a car. Then,

$$n(P) = 25\% \text{ and } n(C) = 15\%$$

$$n(P' \cap C') = 65\% \text{ and } n(P \cap C) = 2000$$

Now, since $n(P' \cap C') = 65\%$

$$\Rightarrow \quad n(P \cup C)' = 65\%$$

$$\Rightarrow \quad n(P \cup C) = 100 - 65 = 35\%$$

Now, $n(P \cup C) = n(P) + n(C) - n(P \cap C)$

$$\Rightarrow \quad 35 = 25 + 15 - n(P \cap C)$$

$$\Rightarrow \quad n(P \cap C) = 5\%$$

But, $n(P \cap C) = 2000$

$\therefore$ Total number of families $= \dfrac{2000 \times 100}{5} = 40000$

Since, $n(P \cup C) = 35\%$

and the total number of families = 40000

Hence, 2 and 3 are correct.

70. (*c*) **Idea** Here $\sec^2\theta = 1 + \tan^2\theta$, and $\text{cosec}^2\theta = 1 + \cot^2\theta$ and $\sin^{-1}(\sin x)^2 = x^2$

Consider, the given

$$= \sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3)$$
$$= \{\sec(\tan^{-1} 2)\}^2 + \{\text{cosec}(\cot^{-1} 3)\}^2$$
$$= \{\sec(\tan^{-1} 2)\}^2 + \{\text{cosec}(\cot^{-1} 3)\}^2$$
$$= \{\sec(\tan^{-1} 2)\}^2 + \{\text{cosec}(\cot^{-1} 3)\}^2$$
$$= \{\sec(\sec^{-1}\sqrt{5})]^2 + \{\text{cosec}(\text{cosec}^{-1}\sqrt{10})\}^2$$
$$= (\sqrt{5})^2 + (\sqrt{10})^2$$
$$= 15$$

TEST Edge Generally, in JEE Main trigonometric identities and inverse trigonometric functions related questions are asked. To learn the identities to solve the questions.

71. (*b*) **Idea** $\therefore f(x)$ is continuous in $[a, b]$. Then,

(i) $f(x)$ will be continuous in (a, b)

(ii) $\lim_{h \to 0}(a + h) = f(a)$

(iii) $\lim_{h \to 0}(b - h) = f(b)$

The given function is

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x & ; \quad 0 \le x < \dfrac{\pi}{4} \\ 2x \cot x + b & ; \quad \dfrac{\pi}{4} \le x \le \dfrac{\pi}{2} \\ a\cos 2x - b\sin x & ; \quad \dfrac{\pi}{2} < x \le \pi \end{cases}$$

At, $x = \dfrac{\pi}{4}$

$$\text{LHL} = \lim_{x \to \pi^-/4} f(x) = \lim_{x \to \pi^-/4} (x + a\sqrt{2}\sin x)$$
$$= \frac{\pi}{4} + a$$

$$\text{RHL} = \lim_{x \to \pi^+/4} f(x) = \lim_{x \to \pi^+/4} (2x \cot x + b)$$
$$= \frac{\pi}{2} + b$$

Also, $f\left(\dfrac{\pi}{4}\right) = 2\dfrac{\pi}{4}\cot\left(\dfrac{\pi}{4}\right) + b = \dfrac{\pi}{2} + b$

For continuity, these three must be equal.

$$\Rightarrow \quad \frac{\pi}{4} + a = \frac{\pi}{2} + b$$

$$\Rightarrow \quad a - b = \frac{\pi}{4} \qquad \ldots(i)$$

Now at, $x = \dfrac{\pi}{2}$

$$\text{LHL} = \lim_{x \to \pi^-/2} f(x) = \lim_{x \to \frac{\pi}{2}} (2x \cot x + b) = 0 + b$$

$$\text{RHL} = \lim_{x \to \pi^-/2} (a\cos 2x - b\sin x) = -a - b$$

Also, $f\left(\dfrac{\pi}{2}\right) = 0 + b$

So, for continuity, $f\left(\dfrac{\pi}{2}\right) = b = -a - b$

$$\Rightarrow \quad a + 2b = 0 \qquad \ldots(ii)$$

Solving (i) and (ii), we get

$$b = -\frac{\pi}{12}, \; a = \frac{\pi}{6}$$

TEST Edge The given function is continuous in open interval and at a point related questions are asked. To solve these types of questions students are advised to understand the concept of continuity and also acquainted yourself with formulae of limit.

72. (*b*) The given line will be parallel to one of the bisector planes of the given planes hence equation of

$$\text{Bisector planes} = \frac{2x + y + 3z - 1}{\sqrt{14}}$$
$$= \pm\frac{(x + 2y - 3z - 1)}{\sqrt{14}}$$

$$\Rightarrow \quad 3x + 3y - 2 = 0 \text{ and } x - y + 6z = 0$$

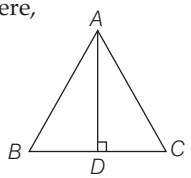
line $\dfrac{x}{k} = \dfrac{y}{2} = \dfrac{z}{-12}$ will be parallel to the plane

$$x - y + 6z = 0 \quad \Rightarrow \quad -2x + 2y - 12z = 0$$

Comparing direction ratio of line and plane

$$k = -2$$

73. (*c*) **Idea** Here,



The point A, B, C with position vectors **a, b** and **c** respectively.

$$\text{Area of triangle} = \frac{1}{2}|\mathbf{AD}|\cdot|\mathbf{BC}|$$

$$\frac{1}{2}|\mathbf{AC}\times\mathbf{BC}| = \frac{1}{2}|\mathbf{AD}|\cdot|\mathbf{BC}|$$

$$|\mathbf{AD}| = \frac{|\mathbf{AC}\times\mathbf{BC}|}{|\mathbf{BC}|}$$

We know that area of $\Delta ABC = \frac{1}{2} BC \cdot AD$

$$= \frac{1}{2}|\mathbf{AB}\times\mathbf{AC}|$$

$$AD = \frac{|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}{|(\mathbf{b}-\mathbf{c})|}$$

TEST Edge The distance between two points in a space, three points are collinear related questions are asked. To solve these types of questions students are advised to understand the concept of cross product of vectors.

74. (*c*) Mathematical distribution of above is

$$x + y + z = 6$$

Where, x, y and z represent one rupee, fifty paisa and twenty paisa coin respectively.

∴ Number of ways of choosing r things out of n things.

$$= {}^{n+r-1}C_{r-1} = {}^{6+3-1}C_{3-1} = {}^{8}C_2$$

$$= \frac{8!}{6!\cdot 2!} = \frac{8\times 7\times 6!}{6!\cdot 2!} = 28$$

Hence, there are 28 ways to choose six coins.

75. (*a*) Let, (k, k, k) be the point of intersection of two lines

$$\Rightarrow \quad k(\sin A + \sin B + \sin C) = 2d^2 \qquad \ldots(i)$$

and $k(\sin 2A + \sin 2B + \sin 2C) = d^2 \qquad \ldots(ii)$

From Eqs. (i) and (ii)

$$\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{A \sin A \sin B \sin C}{A \sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = \frac{1}{2}$$

(using trignometric identities)

$$\Rightarrow \frac{(2\sin A/2\cos A/2)(2\sin B/2\cos B/2)}{(2\sin C/2\cos C/2)\cos A/2\cos B/2\cos C/2} = \frac{1}{2}$$

$$\Rightarrow \quad \sin A/2 \sin B/2 \sin C/2 = \frac{1}{16}$$

76. (*c*) For the given question,

$$f(x) = x^3 - 6x^2 + 15x + 3$$

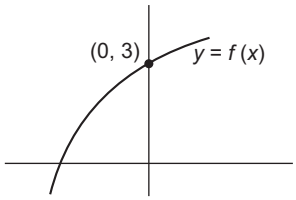
$$f'(x) = 3x^2 - 12x + 15$$

$$f'(x) = 3(x^2 - 2x + 5)$$

Hence, $a > 0$ *i.e.*, coefficient of x^2 is positive and $D < 0$

∴ $f'(x)$ is positive for all its point $f'(x) > 0$

$\Rightarrow f(x)$ is strictly increasing function.



Also, $f(0) = 3$

This implies $f(x)$ has no positive root.

77. (*b*) **Idea** Here,

$(x+a)^n + (x-a)^n = {}^nc_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4$ and $i^2 = -1, i^4 = 1$

It is given that

$$z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$$

$$= \frac{1}{2^5}[(\sqrt{3}+i)^5 + (\sqrt{3}-i)^5]$$

$$= \frac{1}{2^4}[{}^5C_0\ (\sqrt{3})^5 + {}^5C_2(\sqrt{3})^3 i^2 + {}^5C_4(\sqrt{3})i^4]$$

$$= \frac{1}{2^4}[\sqrt{3}\{(\sqrt{3})^4 + 10(\sqrt{3})^2(-1) + 5(-1)^2\}]$$

$$= \frac{1}{2^4}[\sqrt{3}(9 - 30 + 5)] = -\frac{16\sqrt{3}}{16}$$

$= -\sqrt{3}$, purely real number

∴ $\quad \text{Im}(z) = 0$

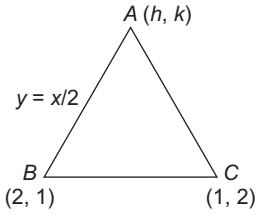
TEST Edge In JEE Main, properties of conjugate modulus and argument of complex number relative questions are asked from this concept. To solve these types of questions students are advised to understand the basic concept of conjugate modulus and argument of a complex number.

78. (*b*) **Idea** Here equation of line passing through the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Given that the equation of line AB is

$$2y = x \qquad \ldots(i)$$



Let coordinate of A are (h, k)

$\because \quad AB = AC$

$\Rightarrow \quad AB^2 = AC^2$

$$(k-1)^2+(h-2)^2=(h-1)^2+(k-2)^2$$
$$h^2+k^2-2k-4h+1+4=h^2+k^2$$
$$-2h-4k+1+4$$
$$h=k$$

Also, A lies on equation (i)

$\therefore \quad 2k=k \Rightarrow k=0$

Simplifying above equation,

$$h=k=0$$

$\therefore$ Coordinate of A is (0,0)

Equation of line AC

$\Rightarrow \quad y-0=2(x-0)$

$$y=2x$$

TEST Edge Standard equation of straight line and its application related questions are asked. To solve these types of questions, students are advised to learn the formulae of equations of straight line and also acquainted yourself with its application.

79. (c) **Idea** This is a homogeneous differential equation to reduce to the separable variable type.

The parametric of the given equation let

$$x=r\cos\theta,\ y=r\sin\theta \qquad \ldots(i)$$

Differentiate to $d\theta$, we get

$\Rightarrow \quad dx=\cos\theta\, dr-r\sin\theta\, d\theta$

$\Rightarrow \quad dy=\sin\theta\, dr+r\cos\theta\, d\theta$

Then the given differential equation reduces

$$\frac{rdr}{r^2d\theta}=\frac{\sqrt{1-r^2}}{r}$$

By variable separation [To separate variable r and θ]

$$\frac{dr}{\sqrt{1-r^2}}=d\theta$$

Integrating both sides

$$\int\frac{1}{\sqrt{1-r^2}}dr=\int d\theta$$

$$\sin^{-1}r=\theta+\alpha \Rightarrow r=\sin(\theta+\alpha)$$
$$r=\cos\alpha\sin\theta+\sin\alpha\cos\theta$$

Multiplying by r,

$$r^2=r\sin\theta\cos\alpha+r\cos\theta\sin\alpha$$
$$x^2+y^2=y\cos\alpha+x\sin\alpha$$
$$x^2-x\sin\alpha+y^2-y\cos\alpha=0$$
$$x^2-x\sin\alpha+\frac{\sin^2\alpha}{4}+y^2-y\cos\alpha+\frac{\cos^2\alpha}{4}=\frac{1}{4}$$
$$\left(x-\frac{\sin\alpha}{2}\right)^2+\left(y-\frac{\cos\alpha}{2}\right)^2=\frac{1}{4}$$

So, It is clear that circles of radius $\frac{1}{2}$ passing through the origin.

TEST Edge Homogeneous equation, differential equation to reducible to the separable variable type based questions are asked. To solve these types of questions students are advised to understand the concept of these homogeneous equation.

80. (d) **Idea** If α and β are the roots of $ax^2+bx+c=0$ then $\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$

We have given equation as

$$\lambda(x^2+x)+x+5=0$$
$$\lambda x^2+(\lambda+1)x+5=0$$

Roots are α,β. Here, $\alpha+\beta=-\frac{\lambda+1}{\lambda}, \alpha\beta=\frac{5}{\lambda}$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=4\Rightarrow\alpha^2+\beta^2=4\alpha\beta$$

$\Rightarrow \quad (\alpha+\beta)^2=6\alpha\beta$

$\Rightarrow \quad \left(\frac{1+\lambda}{\lambda}\right)^2=6\times\frac{5}{\lambda}$

$\Rightarrow \quad \frac{1+\lambda^2+2\lambda}{\lambda^2}=\frac{30}{\lambda}$

$$1+\lambda^2+2\lambda=30\lambda$$

$\Rightarrow \quad \lambda^2-28\lambda+1=0$

$$\lambda_1+\lambda_2=28,\ \lambda_1\lambda_2=1$$
$$\frac{\lambda_1}{\lambda_2}+\frac{\lambda_2}{\lambda_1}=\frac{\lambda_1^2+\lambda_2^2}{\lambda_1\lambda_2}=\frac{(\lambda_1+\lambda_2)^2-2\lambda_1\lambda_2}{\lambda_1\lambda_2}$$
$$=\frac{(28)^2-2\times1}{1}=782$$

TEST Edge Generally, in JEE Main relation between roots of quadratic equation and some specific conditions, related questions are asked. To solve these types of questions students are advised to understand the basic concept of relation between the roots of quadratic equation.

81. (a) **Idea** Here, $\mathbf{a}=a_1\hat{\mathbf{i}}+a_2\hat{\mathbf{j}}+a_3\hat{\mathbf{k}}$ then a unit vector $\hat{\mathbf{a}}=\frac{\mathbf{a}}{|\hat{\mathbf{a}}|}$

The component of $\mathbf{b}$ along the $\mathbf{a}$ is $\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}$

First of all we find,

A unit vector along $\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}=\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{\sqrt{6}}$

Hence, the component of $\hat{\mathbf{i}}$ along the given vector

$$=\text{projection of }\hat{\mathbf{i}}\text{ on }\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{\sqrt{6}}=\frac{1}{\sqrt{6}}$$

TEST Edge The component of vector **a** along and perpendicular to vector **b** and projection of vector based questions are asked. To solve these types of questions students are advised to understand the concept of projection and component of vector.

82. (*a*) We have given that,

$$I = \int_{-\pi/4}^{3\pi/4} \frac{\sin x + \cos x}{e^{x-\frac{\pi}{4}} + 1}\, dx \qquad \ldots(i)$$

$$I = \int_{-\pi/4}^{3\pi/4} \frac{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}{(e^{\pi/4 - x} + 1)}$$

Using identity, $\int_a^b f(x)\, dx = \int_b^a f(a + b - x)\, dx$

$$= \int_{-\pi/4}^{3\pi/4} \frac{\cos x + \sin x}{e^{\pi/4 - x}(1 + e^{x - \pi/4})}\, dx$$

$$I = \int_{-\pi/4}^{3\pi/4} \frac{\cos x + \sin x}{(1 + e^{x - \pi/4})} e^{x - \pi/4}\, dx \qquad \ldots(ii)$$

Adding Eqs. (i) and (ii)

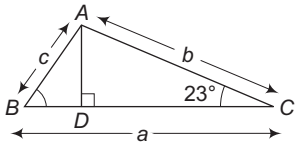
$$2I = \int_{-\pi/4}^{3\pi/4} = \frac{(\cos x + \sin x)\left(1 + e^{x - \frac{\pi}{4}}\right)}{(1 + e^{x - \pi/4})}\, dx$$

$$\int_{-\pi/4}^{3\pi/4} \frac{(\cos x + \sin x)\, dx}{[\sin x - \cos x]_{\pi/4}^{3\pi/4}}$$

$$I = 0$$

83. (*a*) **Idea** Here for any triangle ABC with sides a, b and c $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

In ΔABC, given that AD is an altitude from A on BC, $b > c$, $\angle C = 23°$ and $AD = \frac{abc}{b^2 - c^2}$



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{a}{2c} - \frac{b^2 - c^2}{2ac}$$

$$= \frac{a}{2c} - \frac{b^2 - c^2}{abc} \cdot \frac{b}{2} = \frac{a}{2c} - \frac{b}{2AD}$$

$$= \frac{\sin A}{2 \sin C} - \frac{1}{2 \sin C}$$

$$\left\{\because \text{In } \Delta ACD \; \frac{AD}{b} = \sin C \Rightarrow \frac{b}{AD} = \frac{1}{\sin C}\right\}$$

$$\Rightarrow \cos B = \frac{\sin A - 1}{2 \sin C}$$

$$\Rightarrow \quad 2 \cos B \sin C = \sin A - 1$$

$$\Rightarrow \quad \sin(B + C) - \sin(B - C) = \sin A - 1$$

$$\Rightarrow \quad \sin A - \sin(B - C) = \sin A - 1$$

$$\Rightarrow \quad \sin(B - C) = 1$$

$$\Rightarrow \quad B - C = \frac{\pi}{2}$$

$$\Rightarrow \quad B = C + \frac{\pi}{2} = 23 + 90 = 113°$$

TEST Edge Sine rule and cosine rule of any triangle properties of triangle related questions are asked. To solve these types of questions, students are advised to understand the concept of properties of triangle.

84. (*a*) **Idea** If $f(x)$ is one-one and onto then, it is one-one onto function.

Let, $f : R \to R$ be function defined

$$f(x) = x^3 + x^2 + 3x + \sin x,\ x \in R$$

$$f'(x) = 3x^2 + 2x + 3 + \cos x$$

$$f'(x) = g(x) + \cos x$$

$$g(x) > 0$$

$\because \quad D = 4 - 36 = -32 < 0$

Range of $g(x)$ is $= \left[\frac{-D}{4a}, \infty\right)$

$$\left[\frac{+32}{12}, \infty\right) = \left[\frac{8}{3}, \infty\right)$$

$\therefore \quad f'(x) > 0$

Hence, function is strictly increasing

$$\lim_{x \to \infty} f(x) = \infty$$

and $\lim_{x \to -\infty} gf(x) = -\infty$

$\therefore$ Function is one-one and onto as $f(x)$ is continuous function.

TEST Edge Types of functions such as one-one, onto many one etc., based questions are asked. To solve such type of question, students are advised to understand the basic concept of function and also acquainted yourself with differentiation of the different function.

85. (*b*) $\because$ Binomial variate is given as, $B\left(6, \frac{1}{2}\right)$

$\therefore \quad n = 6, p = \frac{1}{2}, q = \frac{1}{2}$

$$B(n, p) = n_{C_r} (p)^{n-r} (q^r)$$

$\because$ Binomial coefficient of middle term is greatest which is $r = 3$

$\therefore \quad P(X = 3)$ have highest probability.

86. (*a*) Consider, the given expression

$$\lim_{x\to\infty}\sqrt{3x^2+\sqrt{3x^2+\sqrt{3x^2}}}-\sqrt{3x^2}$$

$$=\lim_{x\to\infty}\frac{3x^2+\sqrt{3x^2+\sqrt{3x^2}}-3x^2}{\sqrt{3x^2+\sqrt{3x^2+\sqrt{3x^2}}}+\sqrt{3x^2}}$$

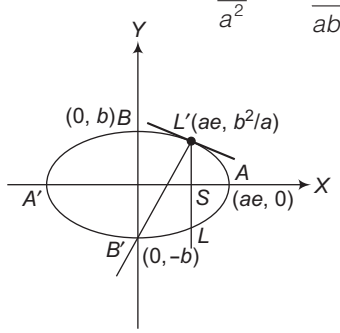
{by rationalisation}

$$=\lim_{x\to\infty}\frac{\sqrt{3+\sqrt{\frac{3}{x^2}}}}{\sqrt{3+\sqrt{\frac{3}{x^2}+\sqrt{\frac{3}{x^4}}}}+\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{3}+\sqrt{3}}=\frac{1}{2}$$

87. (*c*) **Idea** The equation of normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at (x_1,y_1) is $\frac{x-x_1}{x_1/a^2}=\frac{y-y_1}{y_1/b^2}$ and $b^2=a^2(1-e^2)$

Here, the equation of the normal at an end $L\left(ae,\frac{b^2}{a}\right)$ of a latus rectum of the ellipse,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } \frac{x-ae}{\frac{ae}{a^2}}=\frac{y-\frac{b^2}{a}}{\frac{b^2}{ab^2}}$$



$$\Rightarrow y-\frac{b^2}{a}=\frac{1}{e}(x-ae)\Rightarrow ay-b^2=\frac{ax}{e}-a^2$$

which will pass through $B'(0,-b)$

If $$-ab-b^2=0-a^2$$

$$ab=a^2-b^2$$

$$aa\sqrt{1-e^2}=a^2-a^2(1-e^2)=a^2e^2$$

$$\Rightarrow 1-e^2=e^4\Rightarrow e^4+e^2=1$$

TEST Edge Equation of the tangent at a point, equation of the chord with mid-point and equation of chord joining the two points related questions are asked. To solve these types of questions, students are advised to learn the formulae of above equation and understand the basic concept of the ellipse.

88. (*d*) Let, the required matrix be $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ such that

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a+b & a\\ c+d & c\end{bmatrix}=\begin{bmatrix}a+c & b+d\\ a & b\end{bmatrix}$$

$$\Rightarrow a=c+d,\ a=b+d$$

$$\Rightarrow d=a-b,\ b=c$$

Thus, set of all matrices that commute with $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ w.r.t. matrix multiplication

$$=\left\{\begin{bmatrix}a & b\\ b & a-b\end{bmatrix};a,b,\in R\right\}$$

89. (*b*) $n=20,\ \sigma=2$ and $\bar{X}=\frac{1}{n}\Sigma x_i$

$\therefore$ Variance $(\sigma^2)=4$

or $$\Sigma x_i=n\bar{X}$$

$$=20\times 10 \quad \{\because \bar{X}=10\}$$

$$=200$$

Incorrect $\Sigma x_i=200$

Also, $$\frac{1}{n}\Sigma x_i^2-(\text{mean})^2=\sigma^2 \quad \ldots(\text{i})$$

$$\frac{1}{20}\Sigma x_i^2-100=4$$

Incorrected $\Sigma x_i^2=2080$.

90. (*c*) **Idea** Here use the multiplication of two square matrices and corresponding elements are equal of a equal matrix.

Let, $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ where, $a,b,c,d\neq 0$

Now, $$A^2=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$\Rightarrow A^2=\begin{bmatrix}a^2+bc & ab+bd\\ ac+cd & bc+d^2\end{bmatrix}$$

also, it is given that $A^2=I$

$$a^2+bc=1,\ bc+d^2=1$$

and, $ab+bd=0=ac+cd$

$\Rightarrow b(a+d)=c(a+d)=0$

$\because b,c\neq 0$

$\Rightarrow a+d=0$

$\Rightarrow T_r(A)=0$

also $|A|=ad-bc=-a^2-bc=-1$

So, Statement I is true but Statement II is false.

TEST Edge Generally, in JEE Main addition, multiplication and its application related questions are asked. To solve these types of questions. students are advised to understand the operation on addition multiplication of matrix.